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Essays on the Relationship between Macroeconomic Variables, Demographics and Inequality

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Date 31st January 2018

Alessandro Ferrari

Abstract

This PhD thesis is composed of three chapters, each one containing an essay on the relationship between macroeconomic variables, demographics and inequality.

The first chapter studies the effect of deep recessionary episodes on intergenerational inequality by making a quantitative assessment of the welfare effects of the Great Recession for households at different phases of the life cycle. The Great Recession was characterized by a fall in asset prices and in the employment rate: the former levy high welfare costs on older households who own financial wealth, the latter determined losses of human wealth for younger cohorts. Using an OLG model calibrated to the Great Recession I find that younger households had losses around 5.5% in consumption equivalent, seven times higher than those of older cohorts. The dynamics of individual level U.S. data on consumption (CE, PSID) and portfolio composition (SCF) between 2007 and 2013 support the main findings of the analysis.

The second chapter focuses on the effect of economic growth on intra-generational inequality through fertility and intergenerational transfers. The intra-cohort inequality among the young increased in the last twenty years, contributing to the overall increase in wealth inequality. This chapter quantitatively assesses the role of parents' expectations and their fertility decision in shaping wealth inequality. To this end the Barro-Becker model of fertility is solved in a setup with aggregate and idiosyncratic shocks and incomplete markets *à la* Bewley. A negative aggregate shock, by lowering expectations of children's income, determines a fall in fertility and an increase in *per capita* bequests which is greater in magnitude for the right tail of the wealth distribution. As a result wealth inequality in the next generation increases. The model is calibrated to the U.S. economy and shows that the slowdown in economic growth and the subsequent decline in fertility explain around 40 percent of the increase in wealth inequality observed in the data. Micro data (from SCF and PSID) provide empirical evidence that strongly supports the modeled mechanism.

Finally, the third chapter analyzes the effects of exogenous demographic changes to housing price, households debt and interest rate. I start from the correlation displayed in data between demographic variables and financial cycle: an increase in the working-age population is associated with an expansion of the financial cycle, that is, credit growth and increased housing prices. To account for this stylized fact, I use an OLG model with housing, life-cycle of income, and consumption. A transitory baby boom, which increases the working-age population, leads to higher housing prices and household borrowing.

Acknowledgements

Only failures use the word 'if'! You achieve greatness in life 'in spite of'.

— Father Nico, *Sweet Dreams*

I would like to express my special appreciation and thanks to my advisor Nicola Pavoni for the guidance and advice he has provided throughout my time as his student. His macroeconomics class in the first year was a milestone in my academic growth. As a mentor he always pushed me toward self-improvement and independence while being available to clarify any doubt and to give any suggestion that I needed. Any discussion that we had has been an occasion of professional enrichment. Being his Teaching Assistant for three years has been one of the most challenging yet rewarding experiences I had along the PhD.

For the the first chapter I am very thankful to Fabrizio Perri, Silvio Petriconi and Alessandro Secchi. I'm deeply indebted to Jonathan Heathcote and Andy Glover for disclosing codes of their original work. The paper has also benefited from useful comments by Antonella Trigari and participants at Bocconi's macro brownbag and Konstanz macro lunch seminar.

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The third chapter was written while I was working at the Bank of Italy. So, I wish to thank my colleagues Marco Casiraghi, Giuseppe Ferrero, Michele Loberto, Marcello Miccoli, Stefano Neri, Valerio Nispi Landi, Mario Pietrunti, Alessandro Secchi, Stefano Siviero and Andrea Tiseno. The work has benefited of insightful discussions from Tiziano Ropele, Ambrogio Cesa-Bianchi and Jouko Vilmunen. For their comments and suggestions I thank participants at the Bank of Italy Workshop *Secular Stagnation and Financial Cycles* and Bank of Finland-CEPR Conference *Demographics and the macroeconomy*.

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In addition, I am grateful to my colleagues of the Monetary Analysis unit at the Bank of Italy. Since I arrived one year ago they made me feel like part of the team, they encouraged and sustained my professional growth and, at the best of their possibilities, they allowed me to work on this thesis during the last months.

I owe a lot to my ‘step-parents’ Flavio and Giovanna who helped me at every stage of my personal and academic life. They have been for me a living example of marriage and parenthood. Giovanna always provided me with unconditional and maternal love. Flavio taught me how to be an alpinist, a man, a husband, a father.

I am indebted to my grandparents Giuliana and Luigi, my biggest fans since I was a child. I am especially grateful to my grandmother for having funded my bachelor and having given me the opportunity to attend a top university. She had made possible every subsequent academic and professional achievement.

Cecilia’s family deserve a special mention. Anna, Anna Claudia, Betta, Vittoria, Aldo, Giorgio, Lucia and Sara always supported us and provided help when needed. If I didn’t feel too guilty about spending holidays and summers working it was because I knew they were there, ready and willing to take care of Alberto. I am particularly thankful for making me feel part of the family in any occasion that we spent together.

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Contents

Part of the research contained in this volume was carried out while the author worked at the Bank of Italy. The views expressed herein are the author's only and do not necessarily reflect those of the Bank of Italy.

Introduction

This PhD thesis is composed of three research papers on the relationship between macroeconomic variables, demographics and inequality. Several aspects contribute to make this papers mutually itnerrelated. Chapters 1 and 2 are both focused on the effects of economic growth on inequality from complementary perspectives since Chapter 1 studies the effect of economic growth on intergenerational inequality while Chapter 2 analyzes the effects on intragenerational inequality. Chapters 2 and 3 deal on the relationship between demographics and macroeconomics variables from opposite perspective: Chpater 2 taking economic growth as exogenous studies its effects on demographics, Chapter 3 from the opposite perspective considers demographic shocks as exogenous and look at their implications for macroeconomic variables.

Chapter 1 ('Losers Amongst the Losers: the Welfare Effects of the Great Recession across Cohorts') studies what are the welfare effects of the contemporaneous fall in asset prices and in employment on households at different life-cycle phases. Individuals at the beginning of the cycle do not have financial wealth but only "human wealth", i.e. the present discounted value (PDV) of the stream of future labor incomes. As a consequence they are hit mainly by the fall in employment and the long-term effects associated to unemployment. On the other hand, older cohorts have cumulated assets throughout their life but their human wealth is almost zero and therefore their losses come mainly from the fall in asset prices.

Making an assessment between the two channels, Glover et al. (2014) found that during the Great Recession labor income losses of young generations were partially offset by "financial gains" identified in the work of Kiyotaki et al. (2011). Using data from *Survey of Consumer Finances*, they calibrate an OLG model and find that the large fall in asset prices allows young cohorts to buy assets underpriced by leveraging with cheap credit and, in turn, to enjoy a strong increase in their wealth in the future. On the contrary, older households suffer the most from the recession. According to their estimate households where the head is above 70 years old experience significant losses (around 10% in consumption terms) from the recession. By contrast, households that become active during a downturn suffer relatively less than other and, under some specific conditions, they may enjoy net welfare gains if compared to their counterparts that become active in normal times. A crucial assumption for their results is the temporaneity of the loss in labor income arising from the recession episode. Today's unemployment impacts human wealth only through labor income missed during the crisis, but it has no effect on the future stream of wages and employment.

Several empirical works (Ellwood, 1979; Jacobson et al., 1993; Gregg and Tominey, 2005; Bell and Blanchflower, 2011; Davis and von Wachter, 2011; Jarosch, 2014 among the others) have shown that earning losses from unemployment are persistent in time: a worker that suffered layoff and/or unemployment may have, ceteris paribus, lower labor income even after decades. There are two main reasons for this "permanent scar" on the stream of future labor income: less experience and on-the-job training, and loss in firm specific human capital caused by the displacement. The first channel determines higher losses during big economic downturns due to higher unemployment duration and it has been extremely relevant during the Great Recession, while the amplitude of the second effect is independent from the state of the economy.

This paper extends previous analyses keeping into account the permanent losses in earnings arising from unemployment and found that individuals in their twenties suffered welfare losses around 5.5% of consumption equivalent over the entire life-cycle, around seven times higher than households above seventy (0.82%) and more than three times higher than previously estimated in the literature (1.67%, according to Glover et al. (2014)).

Results are computed extending the framework developed by Glover et al. (2014) to incorporate a labor market friction and human capital that cumulates through on-the-job experience. The labor provided by each cohort is determined exogenously: labor market is characterized by an entry friction that affects the different cohorts in an heterogeneous manner and becomes less or more tight according to the state of the economy. Consistently with the observed data (Figure 1.4), the recession is associated with a lower level of employment for the young generations, reducing their accumulation of human capital and therefore their future earnings. Parameters that govern human capital accumulation are calibrated according to micro estimates from the empirical literature on long-term losses from unemployment for households of different age groups.

The analysis reveals that younger households are the most damaged in an economic downturn for two reasons: expected working life after the recession and the wealth effect on risk-taking behavior. The first one is a direct effect of the model: the higher the number of years left to work, the higher the loss in human wealth induced from unemployment. The calibration of the model amplifies this effect: for the same level of unemployment, the only age group that suffer higher percentage losses in human wealth than those in their twenties are households close to retirement with a shorter residual working life. The second effect is induced by the behavior of individuals: the loss in human wealth suffered by young housholds increases their degree of risk-aversion. As a result they buy a lower share of risky assets and they are less leveraged than a counterfactual cohort born in normal times. The channel identified by Kiyotaki et al. (2011), crucial for results of Glover et al. (2014), is therefore weak if permanent losses are included in the model.

Finally, the quantitative model has predictions on consumption and portfolio choice that can be tested in data. With respect to the former the model predicts a reduction of the share of aggregate consumption of individuals between 20 and 29 years old with respect to their counterparts in normal times. Due to higher human wealth losses the cohort born during a crisis is poorer than other cohorts. Consumption data disaggregated by age groups from *Consumption Expenditure Survey* (CE) and *Panel Study of Income Dynamics* (PSID) show a reduction in consumption share of households in their twenties between 2007 and 2010 of a magnitude in line with model predictions. With respect to portfolio choices the model predicts that during a downturn young households, compared with their counterparts in normal times, have a lower share of risky asset and a lower leverage. Data from *Survey of Consumer Finances* (SCF) exhibits the predicted pattern.

Chapter 2 ('Economic Growth and Wealth Inequality: the Role of Differential Fertility') focuses on the effect of economic growth on intra-generational inequality through fertility and intergenerational transfers. Wealth inequality has slowly but continuously increased in many developed countries in the last thirty years. In the US the Gini coefficient of wealth, computed using micro-data from the *Survey of Consumer Finances* has risen from 0.79 in 1992 to 0.85 in 2013. A vivid debate on its possible causes and driving forces of this trend has been spurred by the seminal work of Piketty and Zucman (2014) and by new evidence and data contained in the book Piketty (2014). Despite the great amount of literature that has studied wealth inequality, intra-cohort inequality has received far less scrutiny. In this chapter we provide evidence that inequality among younger households increased in the last twenty years, between 1995 and 2013 the Gini coefficient among households where the head is below 35 increased from 0.81 to more than 1, contributing to the overall increase in wealth inequality. In addition, Piketty (2014) shows that the relevance of bequests over life-time income has increased dramatically in the last decades. Using different data source he finds that, in France, from 1990 to 2010 the annual value of inheritance and intra-vivos transfers over household disposable income doubled, from around 10% to almost 20%.

Changes in intergenerational transfer are driven by changes in the amount of resources that parents choose to give to their heirs or by changes in the number of children. While changes in the overall amount of resources transfered to future generations has been greatly explored in the literature[1], its relationship with fertility has received much less attention.

This paper explores the relationship between economic growth, intergenerational transfers and fertility decision and their effects on intra-cohort wealth inequality. We found that fluctuations in total fertility rate are associated with fluctuations in total factor productivity and that the elasticity of fertility to economic growth is higher for wealthier households. We rationalize this evidence in a framework with altruistic parents *à la* Barro-Becker: parents like having children but they also care about children's prospects. Consistently with micro evidence, the number of children is increasing in the level of wealth of the housing unit, i.e. they are a normal 'good'. In this setup a 'persistent' decrease in TFP growth lowers parents' expectations on their children income. This is a negative wealth shock that induces a decrease in the number of children and an increase in the total amount of transfers (for inter-temporal optimization they move consumption from the present to the future). The magnitude of the wealth shock is correlated to the number of children that the household would have, and therefore is greater for wealthier households. To put it in another way, for the head of a 'dynasty' a decrease in expected income tomorrow determines an increase of the optimal quantity of intergenerational transfers (an increase of 'savings') and a decrease of heirs (he cares about their welfare and therefore he prefers to guarantee more consumption to fewer children).

We develop the analysis through several steps. Firstly, looking at aggregate data on fertility and TFP growth we find that changes in the trend of TFP growth are correlated to changes in the fertility rate (detrended by its historical declining path, determined by the level of GDP reached by the economy). In particular TFP growth *leads* the fertility rate: the correlation is statistically significant after 5 years and is robust to different definitions of TFP and fertility rate. We document such finding for both US and UK over the last century.

Then, the pattern observed in aggregate data are studied using a Barro-Becker model enriched with uninsurable idiosyncratic shocks and an aggregate shock on the level of income. The former is a necessary ingredient to generate heterogeneity in the level of wealth, the latter is used to study households reaction to changes in economic prospects. Under a standard calibration the model predicts that the elasticity of fertility and bequests to expectations about children income is larger in magnitude for wealthier households, and therefore a decrease in economic growth determines an increase in intra-cohort wealth inequality of the next generation. The model prediction is supported by micro-data from *Survey of Consumer Finances*

[1] See De Nardi (2015) and references therein.

(SCF) and *Panel Study of Income Dynamics* (PSID).

Finally, an extended version of the model with aggregate and idiosyncratic shocks is used to measure the change in wealth inequality determined by the fertility channel. We calibrate the model to match certain cross-sectional data moments of the US economy related to fertility and income/wealth inequality, such as the income elasticity of fertility and the average fertility level. The model does a fairly good job in capturing the salient features of the fertility-income-wealth distribution (also on data moments that were not targeted) and we use it to perform some counterfactual experiments. In the main quantitative result, we find that roughly 40 percent of the overall increase in wealth inequality can be accounted for by the decline in the fertility rate (stronger among wealthier households) induced by a revision in parents' expectations on future growth. We find a limited but not negligible role for *biased beliefs*. In particular we calibrate and solve a similar version of the quantitative model with the following modification: agents are not entirely rational and tend to overestimate the persistence of economic shocks. The idea is that a negative shock to TFP growth can have a longlasting effect on fertility if households perceive that slow economic growth will persist for many periods in the future.

Chapter 3 ('Looking Behind the Financial Cycle: the Neglected Role of Demographics') analyzes the effects of exogenous demographic changes to housing price, households debt and interest rate. I start from the correlation displayed in data between demographic variables and financial cycle: an increase in the working-age population is associated with an expansion of the financial cycle, that is, credit growth and increased housing prices. In this paper I explore the role of demographic trends as potential drivers of the financial cycle. Leaving aside the declining trend in the fertility rate as a potential explanation (e.g. Eggertsson and Mehrotra (2014), Carvalho et al. (2016)) for the fall in interest rates and asset prices, as suggested by the secular stagnation hypothesis, I focus on cyclical developments in fertility rates and show that a crucial element that triggers changes in the financial cycle is the relative size of the cohorts, determined by above or below trend population growth.

Firstly, I explore the demographic process in the last century in the US and I show that the detrended fertility rate features boom and bust episodes that, looking at the absolute number of births, have generated larger or smaller cohorts than those that would have been generated by an average population growth process. Then, I show that there is a positive correlation between the age composition of the population (the inverse dependency ratio[2]) and financial

[2]The inverse dependency ratio is the ratio between the number of people of working age and those out of the labour force, either because they are too young (below 15) or too old (above 64). It can be read as the number of workers that sustain an individual that is not in the labour force. The indicator takes into account the demographic

variables (such as the credit/GDP gap and house prices), with the former leading the latter. Finally, I build a three-period overlapping generations (OLG) model with demographic developments, exogenous fertility shocks and life-cycle patterns of consumption and income that, even if calibrated at micro-level, is able to match the correlation observed at aggregate level. In this model, ageing and mortality are deterministic factors: agents live for three periods and then death is a certainty. The size of the newborn cohort is determined by an exogenous shock; this is the only source of uncertainty in the model.

Newborn agents enter the model with no wealth; in the first period they earn an income, consume non-durable goods, and borrow to purchase housing. In the following period, when they are middle-aged, they earn a higher income, adjust their housing stock, pay back their debt, save for retirement and consume non-durable goods. In the last period, they become old, earn no income, get/inherit the housing of their parents (who have died) and use their accumulated wealth to finance consumption and housing. The steady state life cycle profiles of debt, consumption and housing are calibrated according to microdata evidence. Housing is in fixed supply.

The main conclusions are the following: a transitory positive demographic shock (i.e. a bigger cohort entering the economy) increases the share of workers in the economy, thus increasing consumption and per capita output. Since housing is assumed to be a complementary good and it is in fixed supply, increase in output leads to a higher demand that must be cleared through a price increase. As a result, the cohorts that were alive in the previous period and that have already bought houses become relatively richer. An agent from the newborn cohort, on the other hand, is relatively poorer and therefore borrows relatively less than an individual born in a steady state. The overall amount of credit increases when the demographic shock hits the economy because of the higher number of borrowers. Any agent from the baby boom cohort is poorer than an agent born in a normal size cohort and therefore owns a smaller amount of total wealth; on the contrary, the cohort of baby-boomers is richer on aggregate than a steady state size cohort and therefore owns a bigger share of wealth.

Since the shock is assumed to be temporary, when the baby-boomers become middle-aged, the new young cohort returns to its steady state size; the economy reaches a peak in terms of output, as the middle-aged agents are more productive than young workers. House prices also reach their maximum. Given that there is a partial no-arbitrage condition between housing and bonds[3], the negative perspective on house prices depresses the interest rate on bonds thus

structure but not the labour market status, and therefore is not affected by business cycle fluctuations.

[3] If the economy was populated by investors who can buy and resell houses without using them and eventually going short, a standard no-arbitrage would apply. Since housing is part of the utility function, the no-arbitrage condition includes the utility that provides the use of housing.

benefiting the newborn cohort with extremely accommodative credit conditions, due to the relatively larger size of the cohort that supplies credit with respect to the one that demands it.

With reference to the current debate that opposes the secular stagnation hypothesis[4] and to the financial cycle view[5], the paper offers two insights: on the one hand financial downturns may be triggered by changes in demographic developments (as in the secular stagnation hypothesis), on the other, the current phase of low interest rates can be part of a medium-frequency cycle (like the financial cycle) that will reverse (the model predicts the current phase of low interest rates as the temporary consequence of baby boomers ageing and its reversal in the near future as in the empirical work of Favero et al. (2016)). According to the model proposed here, the recent trends in interest rates and credit are neither the symptoms of a long-lasting "secular stagnation" nor the result of changes to regulation and monetary policy but the natural consequence of the demographic structure generated by a boom-bust demographic process.

[4]First developed by Hansen (1939) and nowadays championed by Summers (2014, 2016), advanced economies have entered a phase of low growth, high debt and low interest rates due to structural changes (mostly related to ageing populations and lower technological innovation growth) that are likely to persist in the future.

[5]The financial cycle view stresses the effect of debt overhang on sluggish growth with a particular focus on the role of loose monetary policy and financial regulation in driving the financial cycle (Lo and Rogoff (2015); Juselius et al. (2016)).

Chapter 1

Losers Amongst the Losers: the Welfare Effects of the Great Recession across Cohorts

1.1 Introduction

The Great Recession was characterized by a fall in house and financial assets prices (Figure 1.2) and a fall in employment (Figure 1.3). How this two contemporaneous collapses spread welfare costs among different cohorts? Who were the “losers amongst the losers”?

This paper studies what are the welfare effects of these two shocks on households at different life-cycle phases. Individuals at the beginning of the cycle do not have financial wealth but only “human wealth” (Figure 1.1), i.e. the present discounted value (PDV) of the stream of future labor incomes. As a consequence they are hit mainly by the fall in employment (Figure 1.4) and the long-term effects associated to unemployment. On the other hand, older cohorts have accumulated assets throughout their life but their human wealth is almost zero and therefore their losses come mainly from the fall in asset prices.

Making an assessment between the two channels, Glover et al. (2014) found that during the Great Recession labor income losses of young generations were partially offset by “financial gains” identified in the work of Kiyotaki et al. (2011). Using data from *Survey of Consumer Finances*, they calibrate an OLG model and find that the large fall in asset prices allows young cohorts to buy assets underpriced by leveraging with cheap credit and, in turn, to enjoy a strong increase in their wealth in the future. On the contrary, older households suffer the most from the recession. According to their estimate households where the head is above 70 years

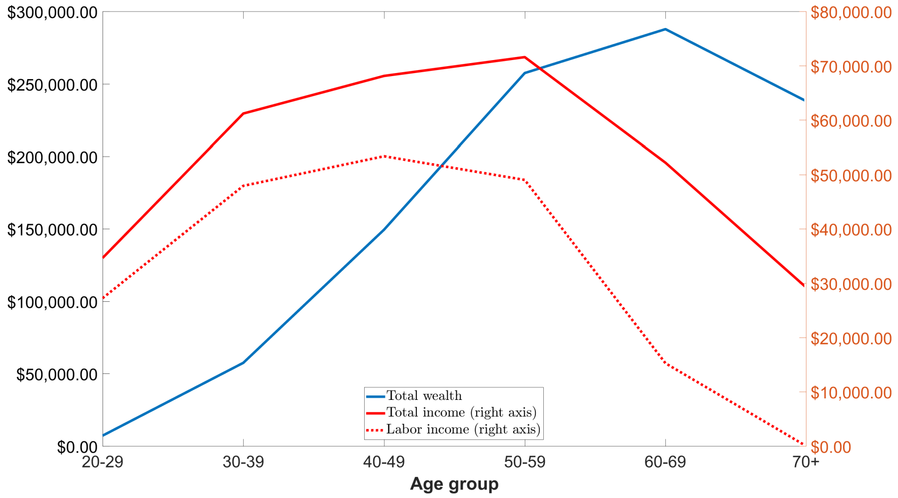

Figure 1.1: Life cycle profile of total wealth, labor income (right axis) and total income (right axis) computed with intra-cohort median values. Source: Survey of Consumer Finances, 2007

old experience significant losses (around 10% in consumption terms) from the recession. By contrast, households that become active during a downturn suffer relatively less than other and, under some specific conditions, they may enjoy net welfare gains if compared to their counterparts that become active in normal times. A crucial assumption for their results is the temporaneity of the loss in labor income arising from the recession episode. Today's unemployment impacts human wealth only through labor income missed during the crisis, but it has no effect on the future stream of wages and employment.

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This paper extends previous analyses keeping into account the permanent losses in earnings arising from unemployment and found that individuals in their twenties suffered welfare losses around 5.5% of consumption equivalent over the entire life-cycle, around seven times

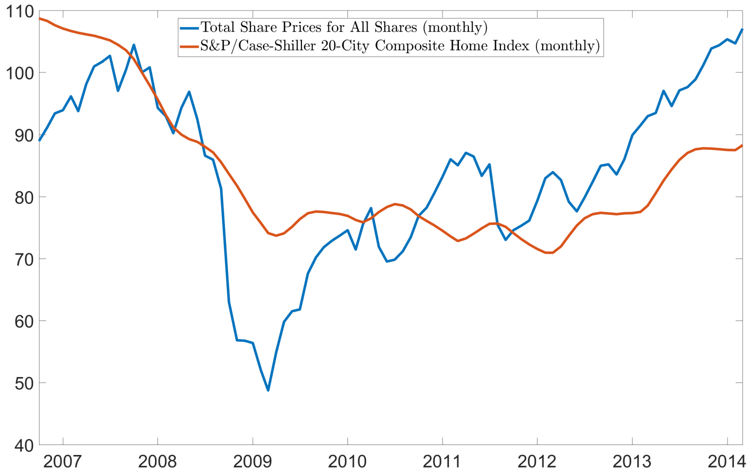


Figure 1.2: Stocks and housing prices during the Great Recession. Index scale, normalized to 100 on November 2007. Source: FRED, Federal Reserve Bank of St. Louis.

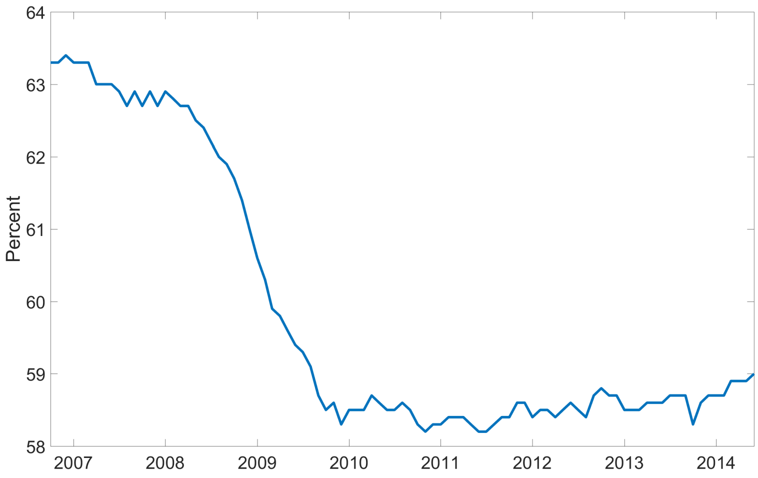


Figure 1.3: Civilian Employment-Population Ratio. Source: U.S. Bureau of Labor Statistics

higher than households above seventy (0.82%) and more than three times higher than previously estimated in the literature (1.67%, according to Glover et al. (2014)).

Results are computed extending the framework developed by Glover et al. (2014) to incorporate a labor market friction and human capital that cumulates through on-the-job experience. The labor provided by each cohort is determined exogenously: labor market is characterized by an entry friction that affects the different cohorts in an heterogeneous manner and becomes less or more tight according to the state of the economy. Consistently with the observed data (Figure 1.4), the recession is associated with a lower level of employment for the young generations, reducing their accumulation of human capital and therefore their future earnings. Parameters that govern human capital accumulation are calibrated according to micro estimates from the empirical literature on long-term losses from unemployment for households of different age groups.

The analysis reveals that younger households are the most damaged in an economic downturn for two reasons: expected working life after the recession and the wealth effect on risk-taking behavior. The first one is a direct effect of the model: the higher the number of years left to work, the higher the loss in human wealth induced from unemployment. The calibration of the model amplifies this effect: for the same level of unemployment, the only age group that suffer higher percentage losses in human wealth than those in their twenties are households close to retirement with a shorter residual working life. The second effect is induced by the behavior of individuals: the loss in human wealth suffered by young housholds increases their degree of risk-aversion. As a result they buy a lower share of risky assets and they are less leveraged than a counterfactual cohort born in normal times. The channel identified by Kiyotaki et al. (2011), crucial for results of Glover et al. (2014), is therefore weak if permanent losses are included in the model.

Finally, the quantitative model has predictions on consumption and portfolio choice that can be tested in data. With respect to the former the model predicts a reduction of the share of aggregate consumption of individuals between 20 and 29 years old with respect to their counterparts in normal times. Due to higher human wealth losses the cohort born during a crisis is poorer than other cohorts. Consumption data disaggregated by age groups from *Consumption Expenditure Survey* (CE) and *Panel Study of Income Dynamics* (PSID) show a reduction in consumption share of households in their twenties between 2007 and 2010 of a magnitude in line with model predictions. With respect to portfolio choices the model predicts that during a downturn young households, compared with their counterparts in normal times, have a lower share of risky asset and a lower leverage. Data from *Survey of Consumer Finances* (SCF) exhibits the predicted pattern.

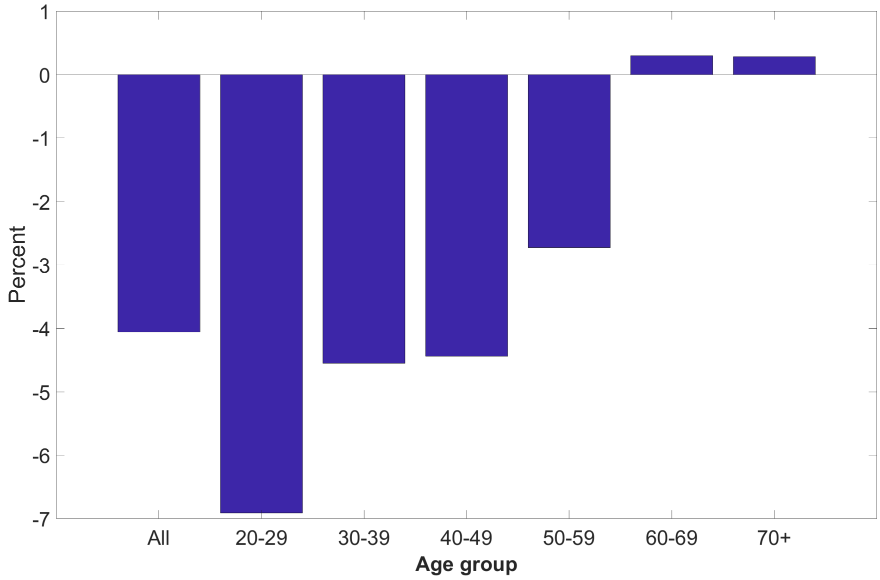


Figure 1.4: Employment-population ratio change by age group 2007:2-2010:2. Source: author's elaboration of U.S. Bureau of Labor Statistics and Census Bureau data

To the best of my knowledge, are few works study the welfare costs of an aggregate shock on different age groups to which this work can be related. The analysis conducted here is in the same spirit of Doepke and Schneider (2006a). They study the redistributive effect of a positive inflation shock in a quantitative OLG model. According to their work while the redistribution of wealth from borrowers to lenders is zero-sum, gainers and losers are characterized by different responses in terms of consumption which do not offset. They find that, on aggregate, positive inflation surprises increase the welfare of the economy: redistributive gains quantitatively offset the losses coming from monetary frictions estimated in other work (e.g. Lucas (2000)).

Hur (2014) also adopts the framework of Glover et al. (2014) but it extends their framework to study intra-cohort heterogeneity. This dimension is out of the scope of this paper that focuses only on age heterogeneity. Hur (2014) shows that a large fraction of young households, which are indebted and liquidity-constrained, couldn't take advantage of cheap assets. He shows that in 2013 portfolio data (the first available wave after the Great Recession) the amount of risky assets owned by the young has not increased. Estimates according to his model predict a much larger loss for the younger, up until 8% of lifetime consumption.

Menno and Oliviero (2014) study the welfare effects of worsened conditions of financial intermediation on borrowers and savers. Worsened conditions in the financial sector forced borrowers to deleverage and generated a pure redistribution from savers to borrowers. Therefore, the latter suffered higher welfare losses from the financial crisis with respect to the former. This paper is related to this work because, as previously discussed, up to a certain extent being borrower or saver depends on age but it broadens its analysis in order to take into account the effects on human wealth.

The rest of the paper is structured as follows: in the next subsections I firstly analyze data on income and wealth before and after the Great Recession for different age groups, then I discuss the temporary and permanent effects of a crisis across households at different ages; in section 2, I construct the model; in section 3, I explain the solution method and the calibration; in section 4, I present the welfare effects of the Great Recession; in section 5, I test the model's implications on micro data; finally, section 6 contains the conclusion.

1.1.1 Data on income and wealth across different cohorts during the Great Recession

The aim of this section is to roughly measure the change in income and wealth of households from different age group. To this end I use data from *Survey of Consumer Finances* that is a triennial cross-sectional survey of U.S. families. The survey data include information on families' balance sheets, pensions, income, and demographic characteristics.

The first rough attempt to measure the change in income and wealth can be done by looking at waves from 2007, immediately before the spread of the crisis, and the subsequent wave of 2010. Results are plotted in Figure 1.5 in absolute values and in Figure 1.6 in percentage terms.

The SCF is not a panel but a cross-sectional survey, nonetheless under random sampling assumption this should not be a problem when looking at mean across age groups. Nonetheless results do not contain only the effect of the shock on income and assets but also the behavioral reaction of agents of different cohorts to the new condition of the economy, both for portfolio choice and for labor supply decision.

From SCF data it is clear that, in absolute terms, the most damaged were the old who own a greater amount of wealth and an higher level of labor income. Nonetheless, if we look at percentage changes it is immediate to see how hard young cohorts have been hit by the crisis: the youngest cohort lost almost 50% of his wealth and roughly 17% of labor income.

A slight more sophisticated approach that can partially account for behavioral reaction on investments consists in using portfolio data from the wave of 2007 and then calculate wealth losses using the shocks observed in housing and financial markets. The technique is

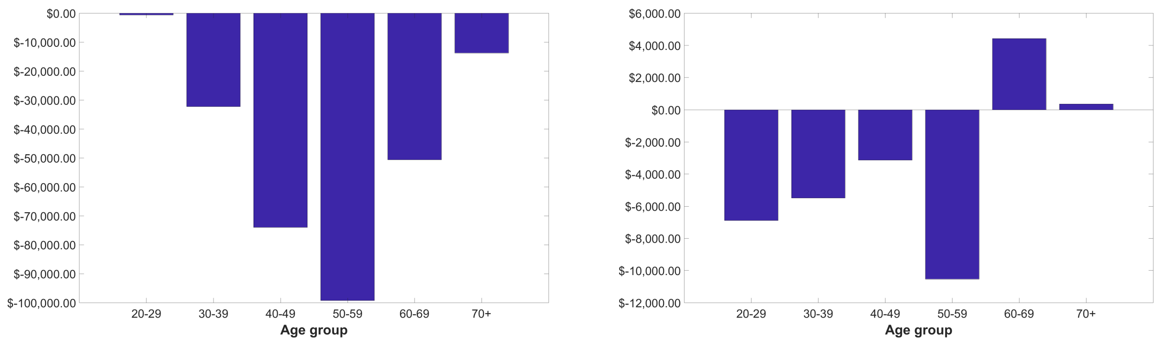


Figure 1.5: Change in labor income and financial wealth between 2007:2 and 2010:2. Source: Survey of Consumer Finances

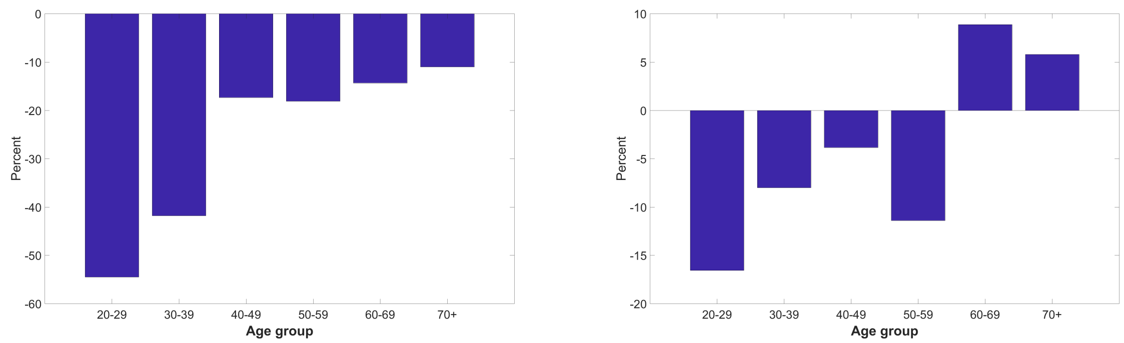


Figure 1.6: Percentage change in labor income and in financial wealth between 2007:2 and 2010:2. Source: Survey of Consumer Finances

similar to the one used by Doepke and Schneider (2006b) to compute the effects of unexpected inflationary episodes. The analysis has been already carried out by GHKR using a fall in the residential wealth value by 46.9% (minimum, 2009:1) and a fall in stock by 29.5% (minimum, 2009:1). Results are shown in Table 1.1.

With this computation the result of the welfare analysis is less unambiguous: losses of the old are higher (21.4% for the last cohort compared to 10% in SCF) and for the young are lower (34.3% compared to 50%).

1.1.2 Temporary and permanent effects of a crisis across households at different ages

The results of Glover et al. (2014) are based on the assumption that the shock arising from the Great Recession, while persistent (it lasts for 10 years in their model) and with sizeable

	Dollar Losses (1.000$)					Losses as % of	
Age of Head	Stocks	Residential Real Estate	Noncorporate Business	Nonresid. Property	Total Losses	Net Worth	Income
All	50.03	64.07	17.90	8.42	140.41	25.3	168.3
20-29	3.04	14.74	8.38	0.40	26.55	34.3	68.4
30-39	15.60	47.28	6.36	3.96	73.20	36.6	104.8
40-49	42.10	65.82	14.61	7.08	129.61	27.8	138.8
50-59	80.37	85.98	27.97	12.27	206.59	25.0	175.1
60-69	100.69	92.00	35.31	17.32	245.32	23.3	224.9
70+	58.72	71.04	16.36	9.67	155.79	21.4	270.6

Table 1.1: Estimated losses by age group from 2007:2 to 2010:2. Source: Glover, Heathcote, Kruger and Rios-Rull (2014)

redistribution effects between generations, does not have a permanent impact on the level of output and on the earning abilities of the agents.

This hypothesis conflicts with past experience and the preliminary analyses of the effects of the Great Recession. In this respect the study of Hall (2014) - which disaggregates U.S. output growth in its main components during the pre-crisis period (1990-2007) and measures the impact of the crisis on their trends (see Table 1) - observes that that GDP in 2013, i.e. 4 years after the end of the recession, is still significantly lower than the pre-crisis trend (13.3%). Furthermore it also concludes that, due to the Great Recession (as well as to other structural factors, including the impact of aging on the labor force participation rate), output will probably stay below trend permanently.

In assessing the impact of a crisis on households from different cohorts the impact on GDP, employment, wages and financial assets (as analyzed for example by Glover et al. (2014)) provide only a partial view. Indeed, there is another important channel to take into account namely the long-term negative effects of unemployment. The relevance of this channel is supported by many empirical works which find a negative and persistent impact of unemployment on human wealth.

In this respect, the positive correlation between unemployment, and especially youth unemployment, and future poor labor market performances is highly documented. While this evidence could be associated with the presence of "bad type" workers who, due to their low productivity, suffer the most the effects of a crisis and permanently earn lower wages, there is an abundant literature which points to a direct exogenous negative impact from unemployment or layoff to low future earnings.

The first attempt to quantitatively measure this effect has been done by Ellwood (1979),

Shortfall, percent									
Year	Output	Productivity	Capital contri-bution	Population	Labor force par-ticipation	Employment rate	Hours per week	Labor quality	Business fraction
2008	4.9	3.0	0.2	0.3	0.0	0.8	0.5	-0.3	0.3
2009	7.4	1.7	0.8	0.3	0.6	2.4	1.6	-0.4	0.4
2010	0.1	-1.6	1.0	0.3	0.6	0.3	-0.5	0.0	0.0
2011	0.5	0.3	0.8	0.4	0.5	-0.4	-0.2	0.1	-0.9
2012	-0.1	0.1	0.6	-0.1	0.4	-0.6	-0.4	0.0	-0.1
2013	0.5	0.1	0.5	0.3	0.3	-0.3	-0.2	0.3	-0.3
2007 through 2010	12.4	3.1	2.1	0.8	1.2	3.5	1.6	-0.6	0.7
2007 through 2013	13.3	3.5	3.9	1.3	2.4	2.2	0.8	-0.3	-0.5

Table 1.2: Components of the shortfall of output two and five years after the crisis. Source: Hall (2014)

despite a small and short dataset and old empirical methods he finds a negative effect of youth unemployment on future earnings. Using *National Longitudinal Survey of Youth* (1979) he estimates that the impact of youth unemployment on the probability of future employment exists but fades away in few years but the effects on future wages are persistent and significant.

Jacobson et al. (1993) study the effect of displacement on "high tenure" workers (six or more years), those that are more likely to have long-term losses in labor earnings. In order to do so they construct a longitudinal data set by merging several administrative records. They found that displaced workers' earning losses are around 25% per year, they are independent from age and sex and inversely correlated to employment growth in the local labor market.

Gregg and Tominey (2005) study the effect of youth unemployment in UK in a dataset that contains data on family background and skill characteristics that allow a control for unobservable heterogeneity. Furthermore, they use an IV approach based on the rate of youth unemployment in the area of residence when the individual is 16 years old and has a low mobility. They found that one year of youth unemployment (i.e. when aged between 16 and 23 as in the ILO definition) implies a decrease by 13-21% in earnings at 41. The fall is lower if unemployment is experienced only once.

Davis and von Wachter (2011) address the endogeneity issue focusing on mass layoff events. According to their definition a mass layoff is a reduction of employees in a firm of at least 30%. Moreover they focus on cases in which the reduction of activity is caused by "bad times" and, as a consequence, the probability of becoming unemployed is orthogonal to skills and other unobservable individual characteristics. They exploit a long panel dataset that

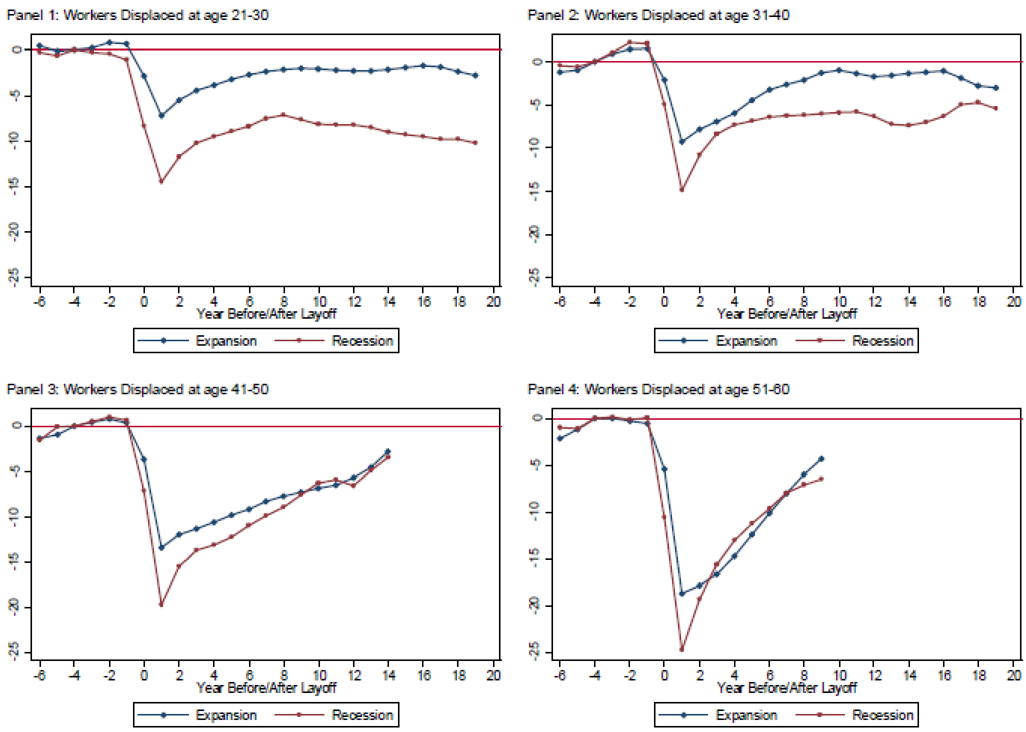


Figure 1.7: Effects on earnings of displacement during mass layoff. Expansions and recessions are determined using NBER definition. Source: Davis and von Wachter (2011).

allows to estimate the effects on earnings twenty years after. As it can be seen in Figure 1.7 older workers tend to have larger immediate losses than younger workers, nonetheless younger displaced workers have non-negligible negative effects that persist in all periods considered. What is more, if the layoff happened during a recession wage fall is considerably higher for those below 40 and especially for those in their twenties, suggesting that a correlation exists between labor market conditions and the magnitude of future earning loss. Finally, they try to use some variations of Diamond-Mortensen-Pissarides search and matching labor market model to explain numerical data but they couldn't replicate the persistency of the losses suffered by the young. Their analysis does not allow to disentangle the effects of time spent unemployed from those associated with a layoff.

Jarosch (2014) uses the same identifying strategy of Davis and von Wachter (2011) on German data and finds similar results: workers who suffered a layoff due to exogenous factors have a permanent decrease in earnings (effects are significant even 20 years after the unemployment spell). He estimates a fall in the present discounted value by 21.2%. He uses a search model with two dimensions (productivity and job security) to study the effects of

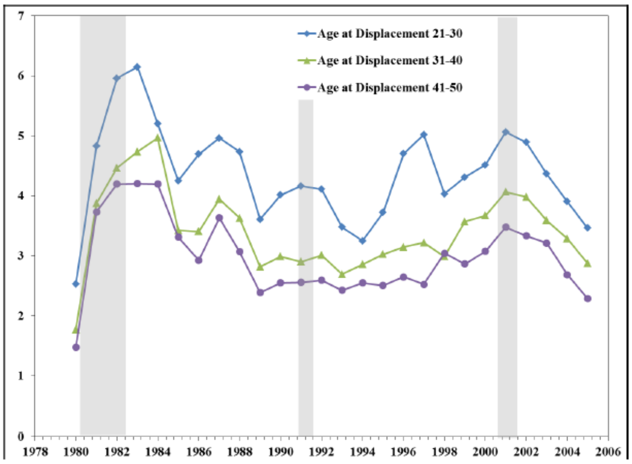


Figure 1.8: Displacement rate during mass layoff by age group. Source: Davis and von Wachter (2011)

unemployment benefits on labor market efficiency.

Overall the available evidence suggests that unemployment has a negative effect on future wages. There are no estimates on the effect of labor market tightness on human wealth loss, however the works of Jacobson et al. (1993), Davis and von Wachter (2011) and Jarosch (2014) suggest that losses are greater in magnitude for those being displaced during a recession. Therefore, it could be the case that bad labor market conditions determine a longer period of unemployment and therefore an higher deterioration (or less opportunity of accumulation) of human capital and in the end to higher losses in the future. Therefore, the deepness and the duration of the last recession suggests that a complete analysis on welfare effects can not disregard the long-term costs associated to unemployment.

1.2 The Model

The model is an OLG with T cohorts, in each period the agents have to choose between consumption and savings. Savings can be stored using a risky asset or a risk-free bond. Agents would like to inelastically supply one unit of time in each period but they face a state-dependent probability of staying unemployed due to labor market frictions. The cumulate human capital that augments effective labor supply through experience: the more they work the more they earns in future periods. Firms are perfectly competitive: they pay wages according to productivity. There is one unit of physical capital, fixed in all periods. The capital share goes to the owners of financial assets. Bonds are in fixed supply. The bond-holders receive a fixed return established in the previous period, the equity holders are residual claimants.

The model is an OLG *la* Samuelson, as in the seminal work of Auerbach and Kotlikoff (1987). This choice is made to capture the life cycle behavior in the savings-consumption choice that is not incorporated in the simpler Blanchard-Yaari setup (where all the individuals face the same probability of dying before the next period)[1].

There are T cohorts. Each cohort has a representative agent, i.e. the model entails inter-cohort but not intra-cohort heterogeneity. This modelling strategy is justified by the research question: asses the effect of the same aggregate shock at different moments of the life cycle, disregarding the other dimensions of heterogeneity among households. Agents live for T periods and then die with certainty.

Throughout the paper I use the following notation: x_t^i refers to the variable x of cohort with age i in period t[2].

1.2.1 The stochastic structure

The model entails only aggregate uncertainty. There is a random variable $\omega \in \Omega := \{\omega_L, \omega_H\}$ which follows a Markov process $\Gamma_{\omega'|\omega}$ and that represents the aggregate state of the economy (which can only be good or bad). All the other cohort-specific shocks of the model are functions of the aggregate state: this is a parsimonious representation that reduces the dimensionality of the problem. Being the effects of the same aggregate shock on different cohorts the main focus of this work, it is a plausible way of modelling.

1.2.2 Households

Households have standard time-separable preferences over stochastic consumption streams $(c_i)_{i=1}^T$ represented by:

$$U\left((c_i)_{i=1}^T\right) = \mathbb{E}\left[\sum_{i=1}^{T}\left(\prod_{j=1}^{i}\beta_j\right)\frac{c_i^{1-\sigma}-1}{1-\sigma}\right]$$

There is no uncertainty on lifetime, thus future periods consumption is simply discounted by the time-preference factors β. The utility function allows for different discount factors

[1]This modelling feature comes at a cost: for this family of models (usually) closed form solution cannot be computed.

[2]To make an example: x_t^2 refers to the cohort born in period $t-1$ which is now in her second period of life.

at different ages. This specification allows to calibrate $(\beta_i)_{i=1}^{T}$ to match life cycle profile of consumption.

Utility of future generations does not enter into the utility function and there is no uncertainty about death, thus there are neither voluntary nor involuntary bequests.

Labor market

Households have one unit of labor in each period that they supply inelastically (it does not have a cost in the utility function). Nonetheless there is a labor market friction and agents face an idiosyncratic probability of being employed. This probability is cohort and state specific, capturing the fact that younger cohorts suffer a disproportionately higher probability of becoming unemployed during recessions[3]. The probability for an agent of i-cohort of getting employed in period t is:

$$\varphi^i(\omega_t)$$

Given that agents of the same cohort are entirely homogenous, $\varphi^i(\omega_t)$ also represents the labor supply of the cohort representative agent, i.e. $l_t^i = \varphi^i(\omega_t)$.

The labor market friction is a one-to-one function of the aggregate state of the economy, i.e.

$$\Phi(\omega_t) := \begin{bmatrix} \varphi^1(\omega_t) \\ \varphi^2(\omega_t) \\ \vdots \\ \varphi^T(\omega_t) \end{bmatrix} : \Omega \longrightarrow [0,1]^T$$

Then, even if the shock is cohort specific, the model entails only aggregate uncertainty as discussed in the previous paragraph.

Human capital

Agents accumulate human capital through experience, human capital of i-cohort in period t is[4]:

$$h_t^i = h_{t-1}^{i-1}\left(1 + \chi^i l_{t-1}^i\right) \tag{1.1}$$

[3]The labor market representation is similar to Krusell et al. (1998), but here there is no idiosyncratic risk and the aggregate exogenous state is a sufficient statistic of the state of the economy.

[4]I choose the multiplicative cumulation because the χ^i has a more direct mapping with the long-term loss due to unemployment estimated by Davis and von Wachter (2011). Using a different functional form, as for example the additive representation adopted by Michelacci and Pijoan-Mas (2008), and a different calibration would not alter the results presented in this paper.

Where χ^i represents the return coming from experience and $h^1 = \overline{h}$ is a cohort-specific factor that captures the fact that experience has different returns at different stages of the life cycle. With this formulation χ^i, captures the loss in future earnings coming from unemployment.

The modelization of human capital is very simple since it is not chosen endogenously (individuals cannot augment human capital with education or training and they cannot increase labor supply to raise its accumulation), nonetheless it captures the effects on future earnings without increasing the number of endogenous variable of the model. With this specification h^i, is a function of the previous $i-1$ realizations of ω, then the exogenous state of the economy is not captured by ω but from the last T realizations of this variable.

The empirical literature shows that the damage of unemployment on future earnings comes from two sources: lower re-employment wages and higher probability of becoming unemployed. The modelization adopted in this paper does not take a stand on the origin of future earnings losses: the parameter χ capture the losses coming from both channels making any mix of the two observationally equivalent (and it is calibrated accordingly).

1.2.3 Representative firm

In the economy there is an infinite amount of identical firms with a constant returns to scale production technology. The production technology of the representative firm is:

$$Y_t = z\left(\omega_t\right) K_t^{\alpha} \mathfrak{L}_t^{1-\alpha}$$

where $z\left(\omega_t\right)$ is the aggregate TFP, which is a function of the aggregate state of the economy (ω), K_t is physical capital and $\mathfrak{L}_t$ is the aggregate labor supply, that is the sum of effective hours of different cohorts:

$$\mathfrak{L}_t := \sum_{i=1}^{T} \varepsilon^i h_t^i l_t^i = \sum_{i=1}^{T} \varepsilon^i h_t^i \varphi^i\left(\omega_t\right)$$

Where ε^i is the cohort specific productivity shock. K_t is fixed and is equal to 1 in all periods.

All factors of production are paid accordingly to their marginal productivity, thus the representative agent of i-cohort gets the following wage:

$$w_t^i = \frac{\partial Y_t}{\partial l_t^i} = (1-\alpha)\, \varepsilon^i z\left(\omega_t\right) K_t^{\alpha} \frac{h^i}{\mathfrak{L}_t^{\alpha}}$$

And thus, using also the fact that $K_t = 1$, the labor income of the i-cohort is:

$$l_t^i w_t^i = \varphi^i (\omega_t) (1 - \alpha) \varepsilon^i z (\omega_t) \frac{h^i}{\mathfrak{L}_t^\alpha}$$

1.2.4 Financial markets

The firm issues two financial assets to distribute the capital share: risk-free bonds and equity. Marginal productivity of capital is:

$$\begin{aligned} \frac{\partial Y_t}{\partial K_t} &= \alpha z_t K_t^{\alpha-1} \mathfrak{L}_t^{1-\alpha} \\ &= \alpha Y_t \end{aligned}$$

The return of the risk-free bond is endogenously established in the previous period, the return of the equity is state specific.

The amount of debt B issued by the firm is exogenous and equal in all periods. One unit of debt in period t costs q and pays back 1 in period $t+1$ whatever the state of the economy.

There is one unit of equity and shareholders are residual claimants of firm profits. Equity gives right to a dividend in each period:

$$d_t = \alpha Y_t - B (1 - q_{t-1})$$

The price of equity is p_t and it is endogenously determined in the model. The gross return of equity in period $t+1$ is given by:

$$\frac{p_{t+1} + d_{t+1}}{p_t}$$

Given the state of the economy in period t, only two states are possible in period $t+1$. Since the returns on bond and equity are independent, financial markets are complete.

1.2.5 History and "steady state"

The dynamics of human capital implies that the exogenous state of the model is given from the last T realizations of the random variable ω. Indeed, aggregate labor supply is a function of human capital of all alive cohorts (since they are all active in the labor market) and the human capital of cohort T is function of the previous $T-1$ realizations of ω while z and φ are functions of its realization at t. Then, to simplify notation (and the stochastic structure in the resolution algorithm), I introduce the variable $\eta_t = [\omega_{t-T+1}, \omega_{t-T+2}, \ldots, \omega_t]$. Notice that

$\eta \in \bigtimes_{j=1}^{T} \Omega_j$ and clearly $\left| \bigtimes_{j=1}^{T} \Omega_j \right| = |\Omega|^T$.

A Markov process on η , $\Gamma_{\eta'|\eta}$ can be defined starting from $\Gamma_{\omega'|\omega}$. Notice that $|\eta'|\eta| = |\Omega|$.

Since the aim of this paper is to study the effect of a big aggregate shock like the Great Recession on different cohorts, I do not analyze the Markovian equilibrium of the economy but I focus on a negative aggregate realization after a sequence, potentially infinite, of good realizations. This is the most reasonable way to frame the problem from an historical perspective since the Great Recession has been the greatest shock on output, unemployment and wealth from the Great Depression.

The model will be used to study when it is most harmful to live through a big negative shock: if during youth, when the probability of becoming unemployed is really high, or later in life when financial and housing wealth are severely hit from market collapses.

Notationally I will use $\eta^{SS} := [\omega_H, \omega_H, \dots, \omega_H, \dots]$.

1.2.6 Recursive problem and equilibrium

The recursive formulation of the problem of the i-cohort representative agent is:

$$v_i(\eta, A) = \max_{c_i, s_i, \varphi_i, a'_i} \left\{ u(c_i) + \beta_i \sum_{\eta' \in |\Omega|^T} \Gamma_{\eta'|\eta} v_{i+1}(\eta', A') \right\}$$

$$s.t.$$

$$c_i + s_i = w_i(\eta) + \mathcal{W}(\eta, A) a_i \tag{1.2}$$

$$a'_i = \frac{\left(\varphi_i \frac{[p(\eta', A') + d(\eta', A')]}{p(\eta, A)} + (1 - \varphi_i) \frac{1}{q(\eta, A)} \right) s_i}{\mathcal{W}(\eta', A')} \tag{1.3}$$

$$A' = \Phi(\eta, A, \eta') \tag{1.4}$$

Where $v_i(\eta, A)$ is the value function of i-cohort representative agent and is function of the two states of the economy: the exogenous shock η and the endogenous share of wealth owned by each cohort denoted by A. $A \in S^T$ is the vector that contains the share of wealth owned by cohort each one denoted by a^i, where $\sum_i a^i = 1$. $\mathcal{W}$ measures the total amount of wealth in the economy and is defined by:

$$\mathcal{W}(\eta, A) = p(\eta, A) + d(\eta) + B$$

c_i is consumption of the i-cohort and s_i are savings, $w_i(\eta)$ represents labor income. Finally φ_i denotes the fraction of wealth invested in risky assets by the cohort. The problem has three constraints: Equation (1.2) is the budget constraint, Equation (1.3) is the law of motion of cohort wealth (and is function of the portfolio allocation in the previous period), Equation (1.4) is the law of motion of the share of the endogenous state variable.

Now a recursive competitive equilibrium can be fully characterized.

Recursive competitive equilibrium A recursive competitive equilibrium is a set of value functions $\{v_i(\eta, A)\}_{i\in T}$ and a set of policy functions $\{a_i'(\eta, A)\}_{i\in T}$, $\{c_i(\eta, A)\}_{i\in T}$, $\{s_i(\eta, A)\}_{i\in T}$, $\{\varphi_i(\eta, A)\}_{i\in T}$, pricing functions $w_i(\eta)$, $d(\eta, A)$, $p(\eta, A)$, $q(\eta, A)$ and the aggregate law of motion $\Phi(\eta, A, \eta')$ such that $\forall i \in T$:

1. Given the pricing functions and the aggregate laws of motion, a set of value functions $\{v_i\}_{i\in T}$ solve the recursive problem of the households, and $\left\{c_i, s_i, a_i^{'}, \varphi_i\right\}_{i\in T}$ are the associated policy functions.

2. Wages and dividends satisfy

$$\forall i \in T: \; w_i(\eta) = \varepsilon_i l_i(\eta) \frac{\partial Y}{\partial l^i} = \varepsilon_i l_i(\eta) \left[Y(1-\alpha) \frac{h^i}{\mathcal{L}^\alpha} \right] \tag{1.5}$$

$$d(\eta, A) = \alpha Y(\eta) - [1 - q(\eta, A)] B \tag{1.6}$$

3. Markets clear:

$$\sum_{i\in T} c_i(\eta, A) = Y(\eta) \tag{1.7}$$

$$\sum_{i\in T} \varphi_i(\eta, A) s_i(\eta, A) = p(\eta, A) \tag{1.8}$$

$$\sum_{i\in T} [1 - \varphi_i(\eta, A)] s_i(\eta, A) = Bq(\eta, A) \tag{1.9}$$

4. The law of motion for the distribution of financial wealth is consistent with equilibrium decision rules:

$$\phi_1(\eta, A, \eta') = 0 \qquad \forall \eta' \tag{1.10}$$

$$\phi_i(\eta, A, \eta') = a_{i-1}^{'}(\eta, A, \eta') \qquad \forall \eta', i = 2, \ldots, T \tag{1.11}$$

The first condition requires that taking prices as given $\{c_i, s_i, a_i^{'}, \varphi_i\}_{i \in T}$ solve the household's problem. Equation (1.5) and Equation (1.6) have been widely discussed previously in 1.2.3. Equation (1.7) is the aggregate resource constraint: capital is fixed and therefore all the output is shared among cohorts for consumption. Equation (1.8) and Equation (1.9) are the clearing market conditions of financial markets. The former is the clearing condition for equity market and states that the sum of shares of savings invested in risky asset by living cohorts must be equal to the price of equity (multiplied by one, that is the normalized quantity of equity in fixed supply). Analogously, Equation (1.9) is the clearing condition of the bond markets. Finally, Equation (1.10) states that any newborn cohort has no wealth while Equation (1.11) characterize the law of motion of the share of wealth of all other cohorts.

1.3 Numerical computation

1.3.1 The algorithm

The state space of the problem includes $T-2$ continuous state variables and $|\omega|^T$ discrete states.

The algorithm for the estimation of the full model is taken from Brumm and Kubler (2013). Instead of solving the original problem the algorithm allows to find a policy function for the Negishi weights of the equivalent planner's problem. This is possible due to market completeness: there are two assets (with independent returns) and two possible states in the next period.

Complete markets imply a perfect aggregate risk sharing among the cohorts that were alive in the previous period and that could access the financial market. The algorithm identifies the policy function that, given the distribution of wealth among the other cohorts, assigns the right amount of resources to the new generation.

Brumm and Kubler (2013) show that the problem reduced to computing the policy function that satisfy the intertemporal budget constraint of newborn agents, thus reducing the burden of numerical computation.

The algorithm is explained in detail in appendix 1.7.1.

1.3.2 Calibration

One period in the model lasts 10 years. Agents enter the economy when they are 20 and live for $T = 6$ periods. Parameters defining households sector are risk aversion σ, time-preferences $(\beta^i)_{i=1}^T$, human capital accumulation factor $(\chi^i)_{i=1}^T$ and age specific wage - productivity gap and Social Security $(\varepsilon^i)_{i=1}^T$. The labor market friction is defined by $(\varphi^i)_{i=1}^T$, the share of capital income is determined from α and its split between equity and bond depends on B. TFP parameters are $(z(\omega))_{\omega\in\Omega}$ and transition probability matrix $\Gamma_{\omega'|\omega}$ for the state of the economy ω.

For the pre-crisis state the second quarter of 2007 is taken as a reference. This quarter is the last in which SCF data on wealth are available before the acceleration in house prices' fall and the beginning of financial crisis[5]. The through of the crisis is calibrated using the second quarter of 2010. This is not consistent with NBER recession definition but is consistent with Hall (2014) that argues this is a more natural end date when considering the dynamics of labor market, an important channel as discussed before and therefore a more appropriate target for this analysis.

The calibration strategy in a nutshell is the following: aggregate endowment processes $\{(z(\omega))_{\omega\in\Omega}, \Gamma_{\omega'|\omega}\}$ are set to math aggregate time series data; (α, B) are set to match empirically observed average portfolio share of risky assets and the aggregate wealth to income ratio in the 2007 SCF; σ is used to match the fall in stock and housing prices; $(\varphi^i)_{i=1}^T(\omega)$ match employment to population ratios by age before and during the crisis; $(\varepsilon^i)_{i=1}^T$ are taken from empirical estimates of Davis and von Wachter (2011); $\{(\beta^i)_{i=1}^T, (\varepsilon^i)_{i=1}^T\}$ are such that the model-implied life cycle profiles for labor earnings align with their empirical 2007 counterparts.

Parameters and calibration targets are listed in Table 1.3. Calibration details are presented throughout the remainder of the section.

Parameter(s)	Moment(s) to be matched
$\left((\varphi^i)_{i=1}^T(\omega)\right)_{\omega\in\Omega}$	Employment to population ratios Q2 2007 and Q2 2010
$(\chi^i)_{i=1}^T$	Loss in future incomes from unemployment
$(\varepsilon^i)_{i=1}^T$	Age specific wage-productivity gap and Social Security
$(z(\omega))_{\omega\in\Omega}$	Fall in output
σ	Fall in stock prices
$\{(\beta^i)_{i=1}^T$	Consumption profile
$\{\alpha, B, p, q\}$	Gross interest rate, wealth-to-labor income ratio, share of risky assets

Table 1.3: Overview of the moments to be matched with the calibration

[5]On August 9, 2007 BNP Paribas announced it was ceasing activity in three hedge funds that specialised in US mortgage debt, it is considered to be the beginning of financial crisis

Financial market

The calibration of financial market closely resembles Glover et al. (2014), thus keeping unchanged the channel that guarantees welfare gains to younger generations. The parameters to calibrate are α (the capital share of the production function), p (the steady state price of equity), q (the price of the bond), B (the amount of bonds in the economy). The empirical moments to match in the data are: the aggregate share of risky assets in households' net worth (which I denote with $\overline{\lambda}$), the aggregate wealth to labor income ratio ($\overline{W}$) and the gross interest rate ($\overline{R}$). Thus, the steady state relationships are:

$$\overline{\lambda} = \frac{p}{p+qB} \tag{1.12}$$
$$\overline{W} = \frac{p+qB}{1-\alpha} \tag{1.13}$$

In a deterministic world the two assets must satisfy the non-arbitrage condition. Therefore given the gross interest rate $\overline{R}$ the two following equations must hold:

$$\overline{R} = \frac{1}{q} \tag{1.14}$$
$$\overline{R} = \frac{p+\alpha Y+qB-B}{p} \tag{1.15}$$

Then, the calibration is done using four equations (Equation (1.12), Equation (1.13), Equation (1.14), Equation (1.15)) and four unknowns. The closed form solution of the four unknowns is:

$$p = \overline{\lambda}\frac{\overline{W}}{1+\left(\overline{R}-1\right)\overline{W}}$$
$$q = \frac{1}{\overline{R}}$$
$$B = \frac{\left(1-\overline{\lambda}\right)\overline{RW}}{1+\left(\overline{R}-1\right)\overline{W}}$$
$$\alpha = \frac{\left(\overline{R}-1\right)\overline{W}}{1+\left(\overline{R}-1\right)\overline{W}}$$

The empirical moments $\overline{\lambda}$ and $\overline{W}$ are taken from 2007 SCF. They are computed as the average across age-group averages because I want to replicate the distribution across cohorts that are assumed to be of equal size. And then:

$$\begin{aligned}\overline{\lambda} &= 0.918\\ \overline{W} &= 0.788\end{aligned}$$

The apparently low level of the wealth-to-labor income ratio reflects the fact that I am considering a 10-years period and therefore at the denominator is the cumulated labor income over the same time span.

The gross real return in the economy, $\overline{R}$ is computed as a weighted average return across asset classes. Piazzesi et al. (2007) estimate a real return on safe assets of 0.75% and on risky assets of 4.75% (which is an average between housing 2.52% and stocks of 6.94%). Given the 10-year period the implied $\overline{R}$ is:

$$\overline{R} = \overline{\lambda}\,(1.0475)^{10} + \left(1 - \overline{\lambda}\right)(1.0075)^{10} = 1.5485$$

The implied values are $\alpha = 0.3017$, $B = 0.0699$, $p = 0.6458$ and $q = 0.5050$. The value of the capital share is consistent with calibrations coming from estimates on micro data.

Labor market

The $\left(\varphi^i(\omega)\right)_{i=1}^T$ represent the probability that an agent of the i-cohort is employed given the state of the economy and therefore they are set to match the cohort-specific employment-to-population ratios in 2007 Q2 and in 2010 Q2.

The Bureau of Labor Statistics does not provide employment to population data for all the age brackets. For those that are missing I compute the data using the absolute number of employed (available for 5 years brackets) and population estimate by cohorts taken from Census Bureau. Data for the reference periods and the calibration of $\left(\varphi^i(\omega)\right)_{i=1}^T$ are shown in Table 1.4.

Age group	2007:Q2	2010:Q2	Change	$\varphi^i(\omega_H)$	$\varphi^i(\omega_L)$
20-29	73.89 %	66.97 %	-6.92 %	0.7389	0.6697
30-39	80.30 %	75.75 %	-4.55 %	0.8030	0.7575
40-49	81.07 %	76.62 %	-4.45 %	0.8107	0.7662
50-59	74.12 %	71.39 %	-2.73 %	0.7412	0.7139
60-69	42.12 %	42.42%	0.30 %	0.4212	0.4242
70+	9.93 %	10.22%	0.29 %	0.0993	0.1022

Table 1.4: Employment-to-population ratio anc calibration of $\left(\varphi^i(\omega)\right)_{i=1}^T$. Source: Bureau of Labor Statistics and Census Bureau

Human capital accumulation

The vector $(\chi^i)_{i=1}^T$ controls the human capital accumulation for workers of different age. What is more, the modelization makes χ^i equivalent to the loss in PDV of labor income over the life-cycle arising from today unemployment.

Claim 1. *Define W_e^i the PDV of future earnings of an individual of the i-cohort that has been employed in the period, W_u^i the one of an unemployed and L^i the loss in PDV for the i-cohort. Then $\forall i$, χ_i is inversely proportional to the loss in PDV induced by unemployment at age i, that is:*

$$\frac{W_u^i\left(\underline{\chi}\right)}{W_e^i\left(\underline{\chi}\right)} = 1 - L^i = \frac{1}{1+\chi^i}$$

Proof. The proof can be found in 1.7.2. □

To make some examples and to clarify the role of χ^i, effects on human capital profiles of one period spent in unemployment for an household between 20 and 29 and for an household between 40 and 49 are plotted in Figure 1.9 left and right panel respectively.

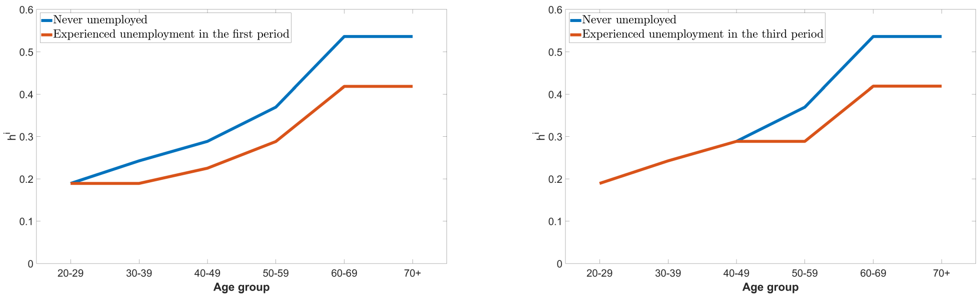


Figure 1.9: Comparison between human capital profile of a worker that is always employed and a worker that is unemployed in his twenties (left panel) or in his fourties (right panel)

Since human capital increases labor effectiveness and households of the same cohort have the same probability of being employed in the future, the wedge between the two profiles of human capital accumulation reflects in a wedge between the profiles of income.

Therefore, the calibration strategy of $(\chi^i)_{i=1}^T$ is the following: PDV of labor income losses implied by unemployment have to match those estimated by Davis and von Wachter (2011). Therefore, using equation (1.24) and filling L^i with estimate on the effect on labor income from layoff for the specific cohort it is possible to uniquely recouple χ^i. Estimate on the fall in PDV of labor income and calibration of $(\chi^i)_{i=1}^T$ can be found in Table 1.5.

Some remarks on the source of the micro-estimates and the assumption needed in this calibration are needed.

Firstly, the age groups used for micro-estimates do not perfectly overlap with those of the model, since the difference is of one year I use them without any adjustment. What is more there are no available estimates for the last age group of the model: I set the parameter to zero thus setting no long-term losses from unemployment experienced in the sixties. Given the small amount of labor income in the last period of life (the employment ratio is around 10% in both states, see Table 1.4) the assumption is not crucial for the results.

Secondly, there are several papers that try to estimate long term losses in labor income from unemployment. I choose those from the work of Davis and von Wachter (2011) since they are disaggregated by age and by state of the economy when the layoff happen. They are estimated using US data, while Jarosch (2014) finds losses of similar magnitude on German data.

Finally, notice that the loss in earnings under this calibration is a lower bound. Indeed, the representative agent of the cohort will suffer a loss proportional to the change in the employment ratio between 2007 Q2 and 2010 Q2 but some households were fired and in the meanwhile they found another job.

Calibration check can be found in Appendix 1.7.3.

Age of layoff (during recessions, with $u \geq 8\%$)	Fall in PDV of labor income (Davis and von Wachter (2011) estimate)	χ^i
21-30	22.0%	0.2821
31-40	16.0%	0.1905
41-50	21.9%	0.2804
51-60	31.1%	0.4514
61-70	N.A.	0

Table 1.5: Effects on future earnings of being fired during a mass layoff and calibration of χ^i

Cohort specific productivity

$(\varphi^i)_{i=1}^T$ and $(\chi^i)_{i=1}^T$, as calibrated before, are able to replicate with a good degree of approximation the gross labor income across households of different age that we observe in SCF data in 2007. Nonetheless, the $(\varepsilon^i)_{i=1}^T$ are needed to match labor income net of taxes and Social Security and Retirement transfers, particularly relevant for older cohorts.

The introduction of a government with a fiscal policy while increasing the burden of numerical computation, thus not improve the fit of households labor income[6].

Firstly, consider that the calibration of $(\varepsilon_i)_{i=1}^T$ in the model has an impact on the effective labor supply and therefore is deeply interconnected with the calibration of the initial level of human capital $\overline{h}$. Then, given that the targets of the calibration are the mean of cohort labor income net of transfers in 2007, therefore T moments, and there are $T+1$ parameters to be calibrated there is one degree of freedom. Indeed, for any initial level of human capital is possible to find a "neutral" value of ε^i, i.e. a $\overline{\varepsilon}$ equal for all cohorts and such that the sum of wages of different cohorts is not altered. More formally, $\forall \overline{h} \in \mathbb{R}_+$ $\exists \overline{\varepsilon}$ such that $\forall i$ $\varepsilon^i = \overline{\varepsilon}$ and $\left(\sum_{i=1}^{I} \varphi^i \left(\omega_H\right) \overline{\varepsilon} h^i \left(\eta^{SS}\right)\right) = \left(1-\alpha\right) Y \left(\eta^{SS}\right)$. I calibrate $\overline{h}$ in order to get $\overline{\varepsilon} = 1$, i.e. to get a neutral value of ε^i equal to 1 and making the interpretation of their calibration simpler.

Then $(\varepsilon^i)_{i=1}^T$ can be seen as a parsimonious representation of the Social Security system in the following way: if $\varepsilon^i > \overline{\varepsilon}$, i.e. if the i-cohort productivity is higher than the neutral one, it is as if the cohort is receiving a transfer from the government, At the opposite if $\varepsilon^i < \overline{\varepsilon}$ it is as if the cohort is paying a tax (non distortionary since labor supply is inelastic).

Three implications that must be discussed:

- The government pay actual pensioners using taxes of actual workers: this is consistent with the Pay-As-You-Go (PAYG) structure of the Social Security system;

- A lower h^i reduces the amount of transfers received from older households. h^i in this model records the working history of the cohort. The pension paid by Social Security system is, nowadays, directly related to earnings during the working life. Then, this feature of the modelization is consistent with the US system;

- A lower level of output reduces the value of actual transfers, nonetheless this modelization does not force period by period budget equilibrium of the government. It can be justified with budgetary pressure: the government has to reduce already defined benefits. Nonetheless, since the model does not have to satisfy a period-by-period budget constraint this modelization has a better match of 2010 data on income (see Appendix 1.7.3).

Then, I calibrate $(\varepsilon_i)_{i=1}^T$ to match the wage profile in 2007. Results are shown in Table 1.6. As a comparison I put also the vector of $(\tilde{\varepsilon_i})_{i=1}^T$, that would match 2007 data on gross labor income (net of government transfers).

[6] In Appendix 1.7.3 a complete fiscal policy with a government budget is introduced as a calibration check on ε^i withouth improving the fit of the model

As expected the ε^i are increasing with age and for the mid-cohorts are close to 1. What is more $\varepsilon^1 < 1$, i.e. the first cohort has a net income that is lower than its wage and ε^6 is much higher than 1, i.e. the oldest cohort rely heavily on social security transfers.

Cohort	ε^i	$\tilde{\varepsilon}^i$ (gross labor income)
1	0.6778	0.7803
2	0.9068	1.0372
3	0.9730	1.1000
4	1.0237	1.1306
5	1.0638	0.9112
6	2.1809	0.7048

Table 1.6: Calibration of $(\varepsilon^i)_{i=1}^T$

Aggregate shock

The level of $z(\omega_H)$ is normalized to 1 while $z(\omega_L)$ is calibrated to match the fall in output coming from the Great Recession. I do not match the actual fall in output (5.4% between the NBER recession dates) but the fall from the pre-crisis trend since the model is stabilized. I start from the work of (Hall, 2014) that estimates a trend between 1990-2007 for all the components of output growth and makes projections on 2010. It estimates a fall in GDP by 12.4% composed by:

- TFP: 3.1%
- Capital contribution: 2.1%
- Population: 0.8%
- Labor Force participation: 1.2% (of which 0.9% from ageing)
- Unemployment: 3.5%
- Labor Quality: -0.6%
- Hours per week: 1.6%
- Business fraction: 0.7%

I take away from the calibration the fall coming from decreasing population (0.8%) and from ageing labor (0.9%) (they relate to long-term processes on which the paper is not focused and that are not related to the Great Recession). Thus the implied fall is 10.7% of which 5.4% is related to labor and therefore is endogenously modeled. Then I impute to $z(\omega_L)$ the residual fall of 5.3%.

On the labor side, given ω_L, there is a fall in output automatically implied by the worsening condition of the labor market. In particular $\varphi(\omega_L)$ implies a fall in employment by 3.0%. This is roughly consistent with the estimate above, indeed the fall in employment rate is given by the sum of Labor Force participation and unemployment (4.7%) but 0.9% comes from ageing of the labor force. The residual discrepancy (0.8%) comes from the assumption of equal size of the cohorts[7]. There is no intensive margin in the model and therefore the change in hours is not endogenous. The (positive) contribution of labor quality is -0.6% also in the model (without considering the effect of ε^i, -1.0% considering them), an additional evidence of the good calibration of $(\chi^i)_{i=1}^T$.

1.4 Results

In this section I present the results of the welfare analysis and I compare them with those from the two main related works.

In the whole section I take as a starting point the steady state defined by η_{SS} and the implied consumption profile. To analyze the welfare effects of the Great Recession, starting from the steady state I perturb the economy with a negative shock for one period (using ω_L) and then I assume the economy is hit by a series of positive shocks (ω_H) which would eventally bring back the economy to the steady state.

Long term harm to the economy

The shock in this setup leaves a "scar" on the economy: the loss in human capital for households active when the recession cannot be compensated and therefore the economy will be below its potential until all the living cohorts will be replaced[8]. Output and its counterfactual are represented in Figure 1.10: after the shock output does not fully recover and a decade after the recession it is 1% below its counterfactual level, thus reducing consumption for all the living cohorts.

In this setup a loss of employment that hits 4% of the population determines a fall in potential output of 1% in the decade after the shock. What is more, the higher the loss in employment of the youngest generation the higher the loss in potential output for the years to come.

[7] The fall by 3.0% is computed as the average of the fall (or the increase) in all the cohorts, in order to match the Hall estimates one should take into account that it is a weighted average.

[8] Obviously, if in the meanwhile the economy is not hit by other negative shocks.

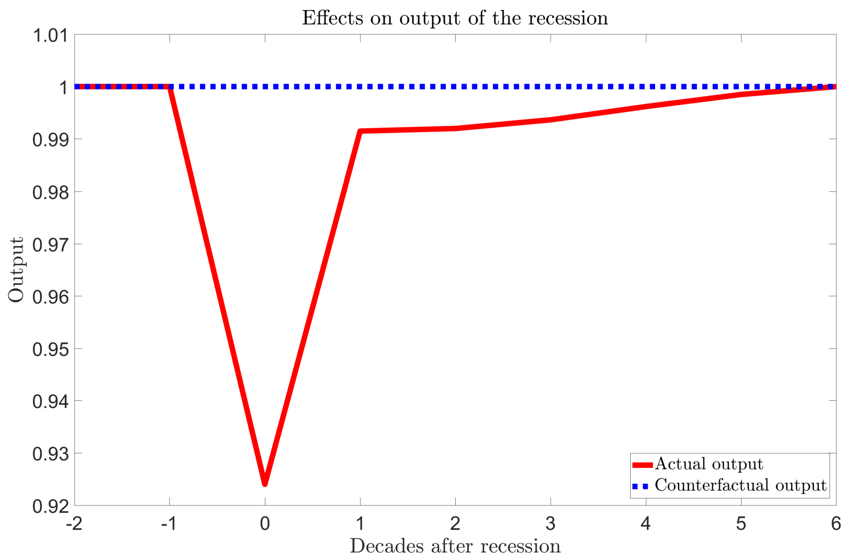


Figure 1.10: Recession long-term effects on output

Loss of human capital: effects on households

The human capital loss reduces effective labor supply also in periods after the recession and it has two effects on labor income of each age group going in opposite directions: on the one hand labor income is reduced due to reduced productivity, on the other hand all households suffered a loss in human capital and therefore the aggregate supply of (effective) labor is reduced thus increasing wages (capital is fixed).

The youngest age group of households is the most severely affected on their future incomes as a result of three main channels: the labor market is more tight for young workers during the recession thus increasing their unemployment, they are living a crucial phase for human capital accumulation, they have a longer horizon as a worker.

The first channel has already been discussed when looking at the cange in employment-to-population by different age group. The second channel is directly related to the calibration of $(\chi^i)_{i=1}^T$: the first cohort is one of those with the highest χ^i. That is, the age between 20 and 29 turns out to be a crucial age to learn by experience and a lay-off during this age leads to high labor market losses. As from Table 1.5 those in their fifties are the only that suffered more in percentage terms. However, they are close to retirement and therefore they have a shorter expected working life (third channel).

Financial gains and portfolio choice

Kiyotaki et al. (2011) argued that young households with almost zero wealth during the crisis have the opportunity of buying "underpriced" risky assets (housing and equity) by leveraging through cheap credit. Glover et al. (2014) have shown that this channel should have been particularly relevant during the Great Recession where the fall in asset prices have been much higher than the fall in output. This channel operates also here. Figure 1.11 reprsents the life-cycle evolution of the portfolio of households that became active during the recession relative to an household that never experienced a recession throughout his life (i.e. he spent his whole life with the economy in steady state). The effect of the recession is a small increase in equity holdings accompanied by a bigger decrease of leverage through safe bonds (around 4%).

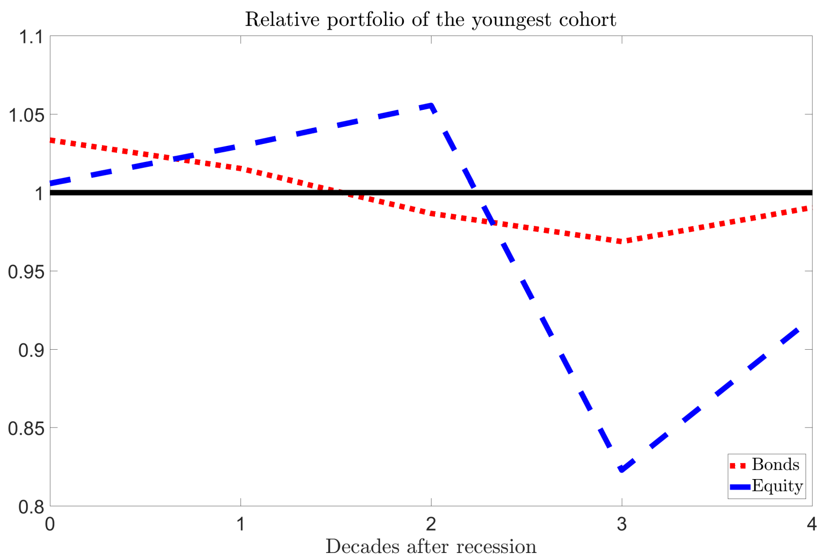


Figure 1.11: Relative portfolio composition during the life-cycle of households in their 20s during the Great Recession

Nonetheless, the introduction of long-term losses from unemployment weakens this channel. Young households, as previously discussed, are those that suffer the most negative wealth effect from loss in human capital. Given the CRRA utility function the level of wealth affects the risk-behavior: thus not only they reduce their savings but, relatively to other cohorts, they also become "more risk-averse" decreasing the fraction of equity holdings.

This model feature leads to replication under fully rational expectations of behavior that has been found in micro dataand has been explained as the result of extrapolative expectations[9].

Financial markets' evolution

The evolution of bond and equity prices is plotted in Figure 1.12. There are two main drivers of price fluctuations: aggregate output and wealth distribution among cohorts.

Firstly, as in the similar OLG model of Glover et al. (2014), the fall in asset prices is proportional to the intertemporal elasticity parameter of the CRRA utility function. More precisely, the fall in asset prices is σ times the fall in output.

Secondly, on the contrary in this model the fall in bond prices is less severe than the fall in stock prices. The negative effect on human wealth of all households and the reduction in prospected income from equity has a negative effect on all households thus increasing their risk aversion and determining, in equilibrium, an increase in the risk-premia.

The long-term effects on output maps into stocks prices indeed they provide a lower return. An important factor that determines their price is also the distribution of wealth among living cohorts, indeed younger cohorts (endogenously) *ceteris paribus* have an higher share of risky asset while they sell them before the last period of life in which they will have almost only asset income thus increasing their risk-aversion. The group of households that become economically active during the crisis is relatively poorer, for this reason they will have a smaller amount of risky asset than a cohort born in normal times. As a result their retirement (four decades after the crisis) and the sell-off of risky asset associated is accompanied by a peak in the stock market (the supply of asset is smaller than usual).

On the contrary, households that become active after the crisis have a welfare gain if compared to the counterfactual steady state. The loss in human capital experienced by households that are alive favors them through two different channels: higher wages and higher risk premia.

The first effect comes from the reduction in effective labor supply due to human capital loss in the previous period: it reduces the amount of labor supply increasing its price but the new generation is not affected by the crisis and therefore benefits of a positive income effect.

The second channel is similar to the one identified by Kiyotaki et al. (2011) and Glover et al. (2014) and discussed before. All the living cohorts during the crisis have seen a reduction in their human wealth (financial wealth recovers in the subsequent period) and therefore they

[9] Malmendier and Nagel (2011) show that individuals who have experienced low stock market returns throughout their lives so far report lower willingness to take financial risk, are less likely to participate in the stock market, invest a lower fraction of their liquid assets in stocks if they participate, and are more pessimistic about future stock returns.

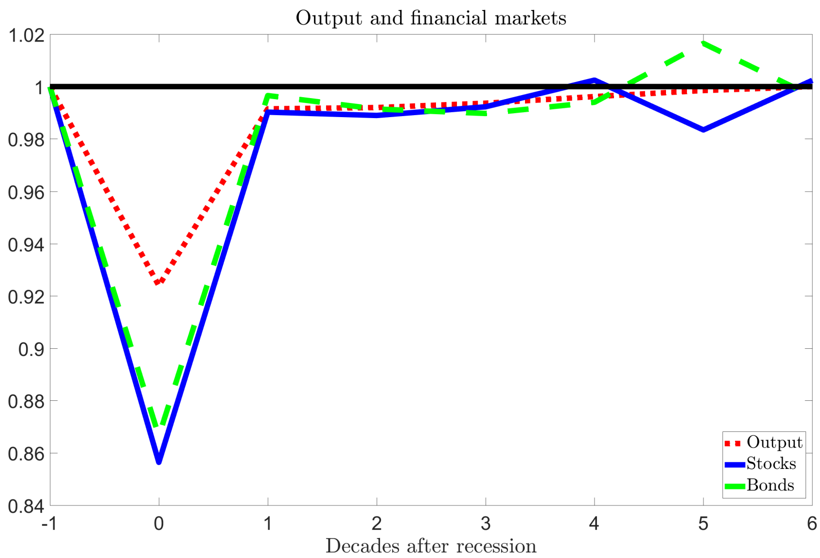


Figure 1.12: Evolution of output, stocks and bonds after the Great Recession

have increased their risk aversion. This lead to an higher risk premia in the economy that benefits the newborn cohorts that buy risky asset borrowing through a lower riskfree rate.

Therefore the cohort born one period after the crisis owns, throughout his life, an higher share of financial wealth. As a result, five periods after the crisis when they sell off risky asset in exchange of risk free assets before retirement stocks price decreases and the price of bonds increases.

1.4.1 Welfare analysis of the Great Recession

The worst age to experience the Great Recession

To compute what is the worst age to live in a recession I make an ex-ante computation of consumption equivalent between different consumption streams. I compute the percentage change in consumption (in all states and over all periods of life, also the past ones) to make the individuals indifferent between the current stream and the one of a counterfactual scenario without any great aggregate shock. The sequence of exogenous state realization[10] in different scenarios are shown in Table 1.7.

Results shown in Table 1.8 display univoquely that the sooner in life the recession is experienced the higher the ex-ante welfare costs. This is due to the fact that the sooner the

[10] H stands for ω_H and L stands for ω_L.

	Periods					
Age during recession	1	2	3	4	5	6
20-29	L	H	H	H	H	H
30-39	H	L	H	H	H	H
40-49	H	H	L	H	H	H
50-59	H	H	H	L	H	H
60-69	H	H	H	H	L	H
70+	H	H	H	H	H	L
Counterfactual	H	H	H	H	H	H

Table 1.7: Exogenous state sequences in different scenarios

Age during recession	Ex-ante welfare loss
20-29	-5.54%
30-39	-2.46%
40-49	-1.74%
50-59	-1.27%
60-69	-1.02%
70+	-0.82%

Table 1.8: Welfare losses in terms of consumption over the whole life cycle

	Periods					
Age during recession	1	2	3	4	5	6
20-29	L	H	H	H	H	H
Counterfactual 20-29	H	H	H	H	H	H
30-39	-	L	H	H	H	H
Counterfactual 30-39	-	H	H	H	H	H
40-49	-	-	L	H	H	H
Counterfactual 40-49	-	-	H	H	H	H
50-59	-	-	-	L	H	H
Counterfactual 50-59	-	-	-	H	H	H
60-69	-	-	-	-	L	H
Counterfactual 60-69	-	-	-	-	H	H
70+	-	-	-	-	-	L
Counterfactual 70+	-	-	-	-	-	H

Table 1.9: Exogenous state sequences in different scenarios

recession comes in the working life the longer the period in which it will have an effect on the individuals' income. The model highlights that the magnitude of losses when entering the labor market is two times the loss of 30-39 group.

Alternative welfare measures: losses over remaining periods of life

A different approach to measure welfare losses can be the one used by Glover et al. (2014). In this case welfare is measured as the percentage change in consumption (in all states and over all remaining periods of life) under a no recession scenario needed to make households indifferent between the current aggregate state being ω_H instead of ω_L.

The sequences of shocks compared in this case are shown in 1.9 and results of computation can be found in Table 1.10 together with results from the previous computation.

Welfare losses when computed on the remaining periods of life are hump-shaped and substantially higher for older cohorts. They are initially decreasing in age but then they are increasing in age and maximal for older cohorts.

The hump-shape is the result of two different effects at work. As I have shown in the previous computation ex-ante the loss suffered by younger cohorts is higher than those suffered by older cohorts. At the opposite, the computation on remaining periods of life shortens the number of period shortens the number of periods over which the negative shock can be diluted as it is clear from 1.9.

Age group	Welfare losses over remaining life	Ex-ante welfare loss
20-29	-5.54%	-5.54%
30-39	-3.49%	-2.46%
40-49	-3.99%	-1.74%
50-59	-5.24%	-1.27%
60-69	-7.15%	-1.02%
70+	-8.11%	-0.82%

Table 1.10: Consumption equivalent of welfare losses by cohort

There are pros and cons in the two computation methods.

The main strength of the first approach is that it has the same number of negative and positive shocks for each cohort and therefore makes consumption equivalent more meaningful. Nonetheless the magnitude of the negative shock on consumption depends on the calibration of betas and in particular the older cohorts have a lower cost in terms of consumption equivalent simply because it is discounted.

On the contrary, the computation of welfare losses only on remaining periods of life guarantees that the negative shock on consumption is not discounted but at the same time make welfare losses mechanically higher for older cohorts even if the amount of consumption in good and bad states is the same for all cohorts.

Making a general assessment and considering all the strength and the weaknesses of the different methods it can be stated that losses for newborn cohort are extremely high. When using ex-ante welfare losses they are more than twice in magnitude all the other cohorts. When computing losses only on remaining life even if they are the younger and therefore they can discount losses on an higher number of periods they have losses of the same magnitude of those above 60.

1.4.2 A comparison with the existing literature

Welfare analysis of the Great Recession have been done previously. The analysis closest to the one done here is the one of Glover et al. (2014).

In Table 1.11 I compare the two different computations of welfare losses across the two models[11].

[11] I take as a reference the version with endogenous portfolio choice and cohort specific earnings loss since the model has the same features of the one presented here. The computation of ex ante welfare losses has been done using replication codes while losses over the remaning periods of life can be found in their paper.

Age group	Ex ante welfare losses	Glover et al. (2014) ex ante	Welfare losses over remaining life	Glover et al. (2014) over remaining life
20-29	-5.54%	-1.72%	-5.54%	-1.67%
30-39	-2.46%	-1.79%	-3.49%	-2.11%
40-49	-1.74%	-1.63%	-3.99%	-2.50%
50-59	-1.27%	-1.72%	-5.24%	-3.66%
60-69	-1.02%	-1.51%	-7.15%	-5.45%
70+	-0.82%	-0.87%	-8.11%	-8.30%

Table 1.11: Ex ante welfare losses: comparison with Glover et al. (2014)

Age group	Model	CPS
20-29	-13.2%	-12.8%
30-39	-9.7%	-11.1%
40-49	-9.5%	-8.8%
50-59	-7.8%	-9.6%
60-69	-3.6%	-4.4%
70+	-1.5%	+0.3%

Table 1.12: Loss in earnings, comparison with CPS (used by Glover et al. (2014))

With respect to their calculation losses are on average higher for all cohorts and extremely higher for the newborn cohort, more than four times in magnitude independently from the method used.

Despite being endogenously determined by labor market friction labor earning losses in this model do not differ substantially from those imputed in their computation, a comparison can be founf in Table 1.12. Asset prices fall are similar since both models are calibrated to match the observed one.

Therefore the difference in results is entirely driven from the long-term losses coming from unemployment and it suggests that, despite the conservative calibration, it has relevant effect and in particular for young cohorts.

1.5 Testing model implications looking at the data

In this section the main model implications on consumption and portfolio choice are tested in data to eventually falsificate welfare losses computations. What is more, given that model previously used in the literature to make welfare analysis of the Great Recession episode have different implications micro data can be used also to make an assesment on which estimate is closer to reality.

1.5.1 Consumption data

The first check relates to consumption data. The model predicts that the difference between the share of consumption of households in their twenties before and after the crisis was -0.15%, i.e. that $\delta C^1_{2010-2007} = \frac{C^1_{2010}}{\sum_i C^i_{2010}} - \frac{C^1_{2007}}{\sum_i C^i_{2007}} = -0.15$.

There are two main sources for consumption data that can be used to recover an estimate of $\delta C^1_{2010-2007}$: the Consumer Expenditure Survey (CE) and the Panel Survey of Income Dynamics (PSID).

Consumer Expenditure Survey

The Consumer Expenditure Survey (CE) is a nationwide household survey conducted by the U.S. Bureau of Labor Statistics (BLS) on american households expenditures. It is the only federal government survey that provides information on the complete range of consumers' expenditures as well as their incomes and demographic characteristics. BLS publishes 12-month estimates of consumer expenditures twice a year with the estimates summarized by various income levels and household characteristics.

The age groups in which households are grouped do not match exactly with those used in the model: the youngest group of households in my calibration 20-29 partially overlaps with the groups 18-24 and 25-34. Unde the assumption that the relative dimension of cohorts is similar across years, that is reasonable given the small time span, the estimate of the change in consumption share for the cohort 18-24 and for the cohort 25-34 are diplayed in Table 1.13.

Age group	2007 vs 2008	2007 vs 2009	2007 vs 2010
18-24	-0.17%	-0.34%	-0.37%
18-34	-0.09%	-0.26%	-0.11%

Table 1.13: Share of total consumption by age group. Source: CE

Results show, independentely from the time span and the definition of the age group, that the change in consumption share is negative and the magnitude is extremely close to the one estimated from the quantitative model.

Panel Survey of Income Dynamics

Another source for data on consumption is the Panel Survey of Income Dynamics (PSID). The PSID is a household panel survey that began in 1968 to study the dynamics of income and poverty. To this end, the original sample contained two independent sub-samples: an over-sample of roughly 2000 poor families selected from the Survey of Economic Opportunities (SEO) and a nationally representative sample of roughly 3000 families designed by the Survey Research Center (SRC) at the University of Michigan. Survey waves are annual from 1968 to 1997, and biennial since then.

In the PSID data on consumption are not as detailed and complete as those of the CE. Nonetheless some specific extra-questions have been proven to be effective in matching consumption data in CE (Andreski et al. (2014)).

Using the sample from the SRC and the extra-questions mentioned before, I create 4 years age brackets[12] and I compute their mean (or median) of consumption in 2007. Then I compute the age group share of consumption over the sum of mean (or median) consumption of all age groups (thus assuming that all age groups are of equal size). Following the same households, exploiting the panel feature of the dataset, I compute the share of consumption in 2011 in the same way and I compare their share of consumption with those of the previous cohort of the same age in 2007. Results for the three cohorts closer to those the first cohort of the quantitative exercise are shown in Table 1.14. While being negative as predicted they are almost three times higher in magnitude pointing out that the losses estimated may be a lower bound.

Age group	% change in share (mean)	% change (median)
18-21	-0.43%	-0.54%
22-25	-0.71%	-0.90%
26-29	-0.46%	-0.62%

Table 1.14: Change in consumption share of households across the Great Recession. Source: Panel Survey of Income Dynamics

A comparison with predictions from Glover et al. (2014)

The model used in this paper and the one used in the previous estimate of Glover et al. (2014) have opposite prediction with respect to consumption share of the youngest cohort. As I said

[12] I cannot look at 10-year cohorts changes since last available data for consumption are those of 2011.

the model with long term effects of unemployments predicts a reduction in consumption share by -0.15%, on the contrary the model with only temporary fall in earnings lead to an increase in the newborn consumption share by 0.30%.

Under the complete market hypothesis that characterizes both models the reduction or the increase in the share of consumption is related with an increase or a reduction of the wealth owned by the youngest cohort. As I have shown in this section all available data points to a reduction in the consumption share of the youngest cohorts thus reinforcing the hypothesis that the financial channel previously identified is much weaker than the permanent loss of income.

1.5.2 Portfolio data

The second testable implication of the model is on portfolio choice of youngest housholds.

As previously discussed the cohort that become economically active during the downturn has a massive loss in its human wealth. Even considering the losses in financial wealth of other cohorts they becomes the poorest ones. As a result they becomes the relatively more risk-averse and they decrease their holdings of risky assets as well as their debt position, weakening the financial channel identified by Kiyotaki et al. (2011).

The quantitative model predicts that the cohort that enters the model during the recession will have a lower share of risky assets in their portfolio with respect to a cohort that enters in normal times by -4.2%. The empirical counterpart of the model estimates can be determined using the Survey of Consumer Finances (SCF).

The SCF is by far the best source of micro level fata on household-level assets and liabilities for the United States. It is conducted every three years by the Board of Governors of the Federal Reserve System and collects detailed information on income and assets. With respect to assets the survey is particularly detailed, it contains information on financial and non-financial assets, debts and capital gains. The survey has two parts: a standard random sample of US households, and a second sample that focuses on wealthy households, identified on the basis of tax returns.

In Table 1.15 mean and median of main portfolio datas for households where the head is between 20 and 29 and with a positive net worth are reported. Risky assets are computed as the sum of of housing and equity directly or indirectly held through mutual funds.

As predicted, households in their twenties reduced the share of risky asset in their portfolios by around -10% between 2007 and 2010 when looking at the median value. The median changes in the same direction but by a greater magnitude even if interpretation is complex because was greater than 2 in 2007 signaling an high leverage. The leverage, measured as the ratio between total debt and total assets, also points to a reduction in debt exposure of younger households, in line with model predictions.

Year	2007		2010		2013	
Variable	Mean	Median	Mean	Median	Mean	Median
Total value of debt held by household, 2013 dollars	59,986	6,961	48,339	5,251	39,051	3,700
Ratio of total debt to total assets	0.338	0.261	0.348	0.247	0.305	0.173
Total net worth of household, 2013 dollars	120,800	14,304	58,541	13,825	75,329	15,700
Total amount of risky assets of household, 2013 dollars	92,799	1,235	62,785	214.3	67,534	700
Share of risky assets in the portfolio	2.162	0.209	1.941	0.108	1.440	0.088

Table 1.15: 20-29 cohort portfolios across the Great Recession. Source: SCF

Therefore, also portofolio data display trends in line with model predictions and of greater magnitude signalling, as with consumption, that welfare losses estimated by the model should be considered a lower bound. Finally, data can be used as before to compare this estimate with the one of Glover et al. (2014). In the exercise that they perform the recession has no effect on asset allocation of the young generation and their welfare gains goes through the increase in the risk premium determined by the huge fall in equity prices. The data of Table 1.15 suggest that the financial gains that they suggest are probably small.

1.6 Conclusion

In this paper I analyzed the welfare losses of different age groups during the Great Recession taking into account both the financial disruption, that affected more the older generations, and the long-term losses of unemployment, that inflicted a greater damage to younger generations.

I found that the losses in human wealth suffered by households that become economically active during the downturn are greater than losses on financial wealth of older cohorts. As a result their risk aversion increased and the gains previously identified by Glover et al. (2014) are small and not significant to compensate them for the loss in human wealth.

In terms of consumption over the whole life-cycle, taking into account also consumption in the previous periods for cohorts that were alive before recession, households in their twenties suffered five times more than older cohorts.

Using consumption equivalent on the remaining periods of life increases the loss of older households. Nonetheless the loss of younger households is still sizeable especially considering the fact that they have many years over which recession can be diluted.

Model implication on consumption and portfolio choice can be compared with moments from data. Both CE and PSID consumption data provide evidence of a smaller consumption share going to younger households, supporting the model results. SCF data on portfolio confirm the deleveraging of the youngest cohort. Empirical moments on both consumption and portfolio choice goes in the same direction estimated from the model but are larger than predicted, pointing out that computed welfare losses should be considered as a lower bound.

There are two main policy implications.

Firstly, the quantitative analysis points out that welfare losses arising from employment fall dominate those arising from assets' markets collapse. Then, for a utilitarian social planner policy intervention should be focused on restoring employment level, and in particular youth employment, for two main reasons: it reduces the welfare loss of those cohorts that are most damaged and it reduces the welfare loss of all cohorts minimizing the loss on potential growth that affects welfare also fo those cohorts that are out of the labor market.

Secondly, those cohorts that were entering the labor market during the Great Recession suffered a huge welfare loss from the uninsurable shock of being born during a major downturn. This very specific market incompleteness call for a redistributive policy action of a social planner in favor of younger households. Since results of quantitative model suggests that households entering the economy after the crisis will have a net welfare gain from the same uninsurable shock, financing the above mentioned policy intervention through debt repaid by future taxes on next generation would be optimal from a utilitarian point of view.

1.7 Appendices

1.7.1 Appendix A: The algorithm

Given that there are complete markets, we can solve an equivalent problem in which instead of equity and bonds there are Arrow-Debreu securities. Denote with μ_i the Negishi weight associated to the i-cohort and with $q(\eta', \boldsymbol{\mu})$ the price of the Arrow-Debreu security that pays 1 when the exogenous state η' realizes and the Negishi weights in the previous period were $\boldsymbol{\mu}$. In the model there are no investments (K is in fixed supply) and therefore the whole output is consumed by the cohorts, i.e.

$$Y(\eta) = \sum_{i\in I} c_i(\eta, \boldsymbol{\mu}) = \sum_{i\in I} \mu_i(\eta, \boldsymbol{\mu}) Y(\eta) \tag{1.16}$$

If we normalize the weights to $\sum_i \mu_i = 1$ then the fraction of output assigned to consumption of i-cohort is $c_i(\eta, \boldsymbol{\mu}) = \mu_i(\eta, \boldsymbol{\mu}) Y(\eta)$. Then, given that there is perfect aggregate risk sharing among those cohorts that are alive when financial markets open we have:

$$\begin{aligned}
\forall i \in \{1, \ldots, T-1\}: \qquad (c_i)^{-\sigma} &= \beta_i \frac{1}{q(\eta, \boldsymbol{\mu}, \eta')} \left(c'_{i+1}\right)^{-\sigma} \\
\left(\mu'_{i+1} Y(\eta')\right)^{\sigma} &= \beta_i \frac{1}{q(\eta, \boldsymbol{\mu}, \eta')} \left(\mu_i Y(\eta)\right)^{\sigma} \\
\mu'_{i+1} Y(\eta') &= \left[\beta_i \frac{1}{q(\eta, \boldsymbol{\mu}, \eta')}\right]^{\frac{1}{\sigma}} \mu_i Y(\eta) \\
\mu'_{i+1} Y(\eta') &= \beta_i^{\frac{1}{\sigma}} \left[\frac{1}{q(\eta, \boldsymbol{\mu}, \eta')}\right]^{\frac{1}{\sigma}} \mu_i Y(\eta) \qquad (1.17)
\end{aligned}$$

Then, using the fact that $\sum_{i=1}^{T-1} \mu'_{i+1} = 1 - \mu'_1\left(\eta', \boldsymbol{\mu}\right)$ we have:

$$\begin{aligned}
\sum_{i=1}^{T-1} \mu'_{i+1} Y(\eta') &= \sum_{i=1}^{T-1} \left[\beta_i^{\frac{1}{\sigma}} \left[\frac{1}{q(\eta, \boldsymbol{\mu}, \eta')}\right]^{\frac{1}{\sigma}} \mu_i Y(\eta)\right] \\
Y(\eta') \left[1 - \mu'_1\left(\eta', \boldsymbol{\mu}\right)\right] &= \sum_{i=1}^{T-1} \left[\beta_i^{\frac{1}{\sigma}} \mu_i\right] \frac{Y(\eta)}{\left[q(\eta, \boldsymbol{\mu}, \eta')\right]^{\frac{1}{\sigma}}} \qquad (1.18)
\end{aligned}$$

Taking the ratio between equation 1.17 and equation 1.18 we get the following expression:

$$
\begin{aligned}
\frac{\mu'_{i+1} Y(\eta')}{Y(\eta')\left[1-\mu'_1(\eta',\boldsymbol{\mu})\right]} &= \frac{\beta_i^{\frac{1}{\sigma}}\left[\frac{1}{q(\eta,\boldsymbol{\mu},\eta')}\right]^{\frac{1}{\sigma}}\mu_i Y(\eta)}{\sum_{i=1}^{T-1}\left[\beta_i^{\frac{1}{\sigma}}\mu_i\right]\frac{Y(\eta)}{[q(\eta,\boldsymbol{\mu},\eta')]^{\frac{1}{\sigma}}}} \\
\frac{\mu'_{i+1}}{\left[1-\mu'_1(\eta',\boldsymbol{\mu})\right]} &= \frac{\beta_i^{\frac{1}{\sigma}}\left[\frac{1}{q(\eta,\boldsymbol{\mu},\eta')}\right]^{\frac{1}{\sigma}}\left[q(\eta,\boldsymbol{\mu},\eta')\right]^{\frac{1}{\sigma}}\mu_i}{\sum_{i=1}^{T-1}\left[\beta_i^{\frac{1}{\sigma}}\mu_i\right]} \\
\mu'_{i+1} &= \beta_i^{\frac{1}{\sigma}}\frac{\left[1-\mu'_1(\eta',\boldsymbol{\mu})\right]}{\sum_{i=1}^{T-1}\left[\beta_i^{\frac{1}{\sigma}}\mu_i\right]}\mu_i
\end{aligned}
$$

Since it holds for a generic i-cohort that is alive in the next period (i.e. $\forall i \in \{1,\ldots,T-1\}$) we have:

$$
\begin{aligned}
\forall i \in \{1,\ldots,T-1\}:\ \mu'_{i+1} &= \beta_i^{\frac{1}{\sigma}}\frac{\left[1-\mu'_1(\eta',\boldsymbol{\mu})\right]}{\sum_{i=1}^{T-1}\left[\beta_i^{\frac{1}{\sigma}}\mu_i\right]}\mu_i \qquad (1.19)\\
\mu'_i &= \Gamma_i\left(\eta',\boldsymbol{\mu}\right)
\end{aligned}
$$

Rearranging:

$$
\forall i \in \{1,\ldots,T-1\}:\ \frac{\mu'_{i+1}}{\mu_i} = \beta_i^{\frac{1}{\sigma}}\frac{\left[1-\mu'_1(\eta',\boldsymbol{\mu})\right]}{\sum_{i=1}^{T-1}\left[\beta_i^{\frac{1}{\sigma}}\mu_i\right]} \qquad (1.20)
$$

Under complete markets agents provide themselves insurance against idiosyncratic shocks and they share the same stochastic discount factor, that is:

$$
\forall i \in \{1,\ldots,T-1\}:\ \pi\left(\eta'|\eta\right)\beta_i c_{i+1}^{-\sigma} = q(\eta,\boldsymbol{\mu},\eta')\, c_i^{-\sigma}
$$

Then the price of an Arrow-Debreu security that pays 1 in the state $\eta^{'}$ given the current :

$$\begin{aligned}
q(\eta,\boldsymbol{\mu},\eta') &= \pi\left(\eta^{'}|\eta\right)\frac{\beta_i c_{i+1}^{-\sigma}}{c_i^{-\sigma}} \\
q(\eta,\boldsymbol{\mu},\eta') &= \pi\left(\eta^{'}|\eta\right)\frac{\beta_i\left[\mu_{i+1}^{'}Y\left(\eta^{'}\right)\right]^{-\sigma}}{\left[\mu_i Y(\eta)\right]^{-\sigma}} \\
q(\eta,\boldsymbol{\mu},\eta') &= \pi\left(\eta^{'}|\eta\right)\beta_i\left(\frac{\mu_{i+1}^{'}}{\mu_i}\right)^{-\sigma}\left(\frac{Y\left(\eta^{'}\right)}{Y(\eta)}\right)^{-\sigma}
\end{aligned}$$

Now, using equation (1.20):

$$\begin{aligned}
q(\eta,\boldsymbol{\mu},\eta') &= \pi\left(\eta^{'}|\eta\right)\beta_i\left[\beta_i^{\frac{1}{\sigma}}\frac{1-\mu_1^{'}\left(\eta^{'},\boldsymbol{\mu}\right)}{\sum_{i=1}^{T-1}\left(\beta_i^{\frac{1}{\sigma}}\mu_i\right)}\right]^{-\sigma}\left(\frac{Y\left(\eta^{'}\right)}{Y(\eta)}\right)^{-\sigma} \\
q(\eta,\boldsymbol{\mu},\eta') &= \pi\left(\eta^{'}|\eta\right)\left[\frac{1-\mu_1^{'}\left(\eta^{'},\boldsymbol{\mu}\right)}{\sum_{i=1}^{T-1}\left(\beta_i^{\frac{1}{\sigma}}\mu_i\right)}\right]^{-\sigma}\left(\frac{Y\left(\eta^{'}\right)}{Y(\eta)}\right)^{-\sigma} \qquad (1.21)
\end{aligned}$$

The whole numerical algorithm is based on equation (1.21). Following the Brumm and Kubler (2013) define the intra-period excess consumption of each cohort in the following way:

$$\begin{aligned}
\Psi_T(\eta,\boldsymbol{\mu}) &= c_T(\eta,\boldsymbol{\mu}) - w_I(\eta) \qquad (1.22) \\
\forall i\in\{1,\ldots,T-1\}:\ \Psi_i(\eta,\boldsymbol{\mu}) &= c_i(\eta,\boldsymbol{\mu}) - w_i(\eta) + \qquad (1.23) \\
&\quad \sum_{\eta^{'}} q(\eta,\boldsymbol{\mu},\eta')\,\Psi_{i+1}\left(\eta^{'},\boldsymbol{\mu}\right)
\end{aligned}$$

Proposition 1 Given the laws of motions for the distribution of consumption $\forall i\in\{2,\ldots,T\}$ defined by equation (1.19) and the consumption sharing rule in equation (1.16), the allocation defined by $\mu_1^{'}\left(\eta^{'},\boldsymbol{\mu}\right)$ is a competitive equilibrium if and only if $\forall\eta\in|\omega|^T, \forall\boldsymbol{\mu}\in S^T$: $\Psi_1(\eta,\boldsymbol{\mu})=0$.

Proof. The economy is isomorphic to an Arrow-Debreu economy, therefore an allocation is a competitive equilibrium if:

- Agents maximize their utility given the price of securities (equation (1.19) is derived using the FOCs of the households' problem);

- The aggregate resource constraint is satisfied ((1.16));
- The agent's budget constraints are satisfied at any age and financial wealth at the beginning of life is 0 (it is guaranteed by the set of equations (1.22)-(1.23)).

□

Notice that $\boldsymbol{\mu} \in S^T$ and therefore, WLOG, it can be projected in $\mathbb{R}_+^{T-1}$.

The aim of the numerical computation is to define the function $\mu_1^{'}\left(\eta^{'}, \boldsymbol{\mu}\right)$. Indeed, given $\mu_1^{'}\left(\eta^{'}, \boldsymbol{\mu}\right)$ and $\boldsymbol{\mu}$, the shares of consumption of the other cohorts (i.e. $\mu_i^{'}$ for $i = 2, \ldots T$) can be computed using equation (1.19). To characterize the function $\mu_1^{'}\left(\eta^{'}, \boldsymbol{\mu}\right)$ over the state space I use the following procedure:

1. Create the state space grid: generate a Smolyak grid of $\{\mu_i\}_{i=1}^{T-1}$ (a $T-1$ dimensional vector) for each exogenous state $\eta \in |\omega|^T$;
2. Create a guess of $\mu_1^{'}\left(\eta^{'}, \boldsymbol{\mu}\right)$ over the entire state space grid and an interpolating function $\hat{\mu}_1\left(\eta^{'}, \boldsymbol{\mu}\right)$ (Chebyshev polynomial) over the whole state space;
3. For any point of the grid, construct the feasible consumption histories, the price of the securities and the implied budget excesses for a cohort born in that state using the guess $\hat{\mu}_1\left(\eta^{'}, \boldsymbol{\mu}\right)$;
4. Use the fact that in a competitive equilibrium period-0 budget excess must be 0, i.e. $\Psi_1\left(\eta^{'}, \boldsymbol{\mu}\right) = 0$, to get an equation containing $\hat{\mu}_1\left(\eta^{'}, \boldsymbol{\mu}\right)$;
5. Repeat steps 3 and 4 for all points in the state-space grid and use a non-linear solver to get a new guess for $\hat{\mu}_1\left(\eta^{'}, \boldsymbol{\mu}\right)$
6. Repeat steps from 3 to 5 until the difference between guesses is small enough.

1.7.2 Appendix B: Mathematical appendix

Proof of Claim 1

Start with the defintion definition $\forall i$:

$$\frac{W_u^i\left(\underline{\chi}\right)}{W_e^i\left(\underline{\chi}\right)}=1-L^i$$

Where $\underline{\chi}$ is the vector $\left[\chi^1\ldots\chi^I\right]$. Then we have:

$$\frac{W_u^i\left(\underline{\chi}\right)}{W_e^i\left(\underline{\chi}\right)}=\frac{\sum\limits_{j=i+1}^{I}\frac{(1-\alpha)z\varepsilon^i\frac{h_u^j}{\mathfrak{L}^\alpha}}{(1+r)^{j-i}}}{\sum\limits_{j=i+1}^{I}\frac{(1-\alpha)z\varepsilon^i\frac{h_e^j}{\mathfrak{L}^\alpha}}{(1+r)^{j-i}}}$$

Considering an atomistic worker his employment status does not affect the aggregate labor supply and therefore $\mathfrak{L}^\alpha$ is the same in the two scenarios. With a similar argument the interest rate r at which the two flows are discounted is the same.

As a result, using Equation (1.1), the previous expression can be reduced to:

$$\begin{aligned}\frac{W_u^i\left(\underline{\chi}\right)}{W_e^i\left(\underline{\chi}\right)} &= \frac{\sum\limits_{j=i+1}^{I}h_u^j}{\sum\limits_{j=i+1}^{I}h_e^j}\\ &= \frac{\sum\limits_{j=i+1}^{I}h_u^i\prod\limits_{k=i}^{j}\left(1+\chi^k\right)}{\sum\limits_{j=i+1}^{I}h_e^i\prod\limits_{k=i}^{j}\left(1+\chi^k\right)}\end{aligned}$$

And collecting the constant terms h_u^i and h_e^i

$$\begin{aligned}\frac{W_u^i\left(\underline{\chi}\right)}{W_e^i\left(\underline{\chi}\right)} &= \frac{h_u^i}{h_e^i}\frac{\left[\sum\limits_{j=i+1}^{I}\prod\limits_{k=i}^{j}\left(1+\chi^k\right)\right]}{\left[\sum\limits_{j=i+1}^{I}\prod\limits_{k=i}^{j}\left(1+\chi^k\right)\right]}\\ &= \frac{h_u^i}{h_e^i}\end{aligned}$$

Finally, using equation 1.1 and considering the employment status of the two workers the expression becomes:

$$\frac{W_u^i\left(\underline{\chi}\right)}{W_e^i\left(\underline{\chi}\right)} = \frac{h^{i-1}}{h^{i-1}\left(1+\chi^i\right)} = \frac{1}{1+\chi^i}$$

Then the relationship between L^i and χ^i is $\forall i$:

$$1 - L^i = \frac{1}{1+\chi^i} \tag{1.24}$$

1.7.3 Appendix C: Calibration check

Human capital accumulation parameters

The calibration of the χ^i is crucial in this estimation and it is based on micro-estimates, therefore I test extensively how well it matches related moments in aggregate data. Ideally, since $(h^i)_{i=1}^T$ capture the return from experience and employment in the life cycle and $(\varphi^i)_{i=1}^T$ capture the employment rate of the age group one would expect this two parameters to provide a good representation of the income life cycle profile observed in data without using the cohort specific parameter ε^i. Then $\overline{h}$ (the initial levelof human capital) is used as a normalization term for the aggregate level of wages[13].

Then suppose a series of ω_H (that is consistent with the modelization of the "steady state") then, the implied profile of $(h^i)_{i=1}^T$, the $(w^i)_{i=1}^T$ implied by the model and the ones observed in data[14] are shown in Table 1.16.

Age group	h^i	w^i (model)	w^i (data, 2007)
20-29	0.1889	0.0974	0.0760
30-39	0.2283	0.1280	0.1328
40-49	0.2632	0.1490	0.1639
50-59	0.3230	0.1672	0.1890
60-69	0.4311	0.1268	0.1155
70+	0.4311	0.0299	0.0211
Total		0.6983	0.6983

Table 1.16: Wage profile implied by model calibration and actual wage profile

From Figure 1.13 we can see that the implied wage profile is similar to the one observed in the data. The model seems to overestimate the wage of younger workers and to underestimate the one of middle-aged workers. This is consistent with theory and empirical studies on wage-productivity gap during working life, according to which, younger workers receive a wage which is below their productivity in exchange for a wage higher than their productivity later in life[15].

Another important concern may be on the role played by the employment-to-population rate in driving the good fit. To fully understand the determinants of the match between model

[13] When considering the cohort specific productivity the role of $\overline{h}$ changes and it is discussed in the paragraph of $\left(\varepsilon^i\right)_{i=1}^T$ calibration.

[14] In this case I consider labor income the sum of wages and a fraction of $(1-\alpha)$ of business, farm and self-employment income.

[15] Seminal theoretical contribution from Lazear (1979).

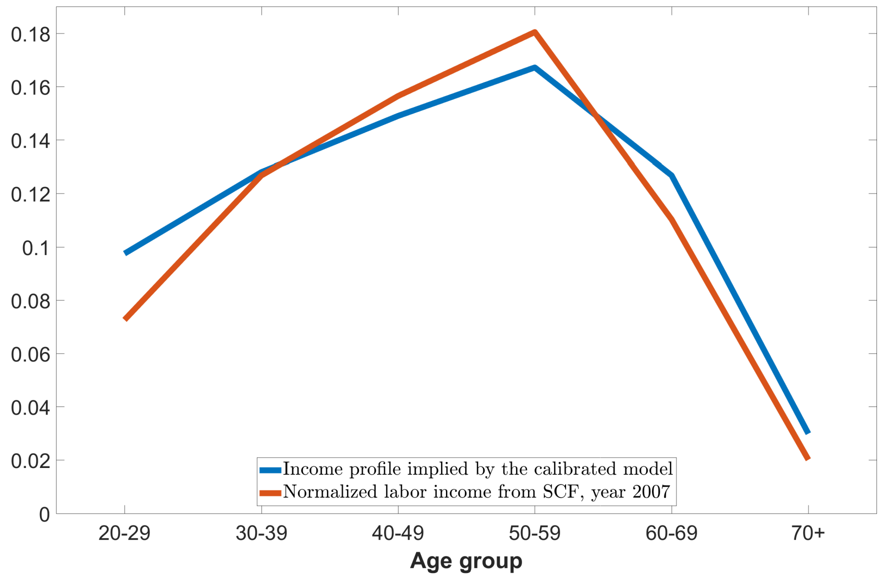


Figure 1.13: Wage profile in steady state

and data of labor income, I "normalize" (dividing them by their maximum value among the cohorts) $\varphi^i(\omega_H)$, w^i in the data, w^i implied from the model and I make a comparison between the two series. Results are shown in Figure 1.14. As it can be seen, relative employment is the main driver for the high fall in the last two cohorts but the good match of the hump-shaped wage profile comes from the calibration of χ^i and from the implied h^i profile.

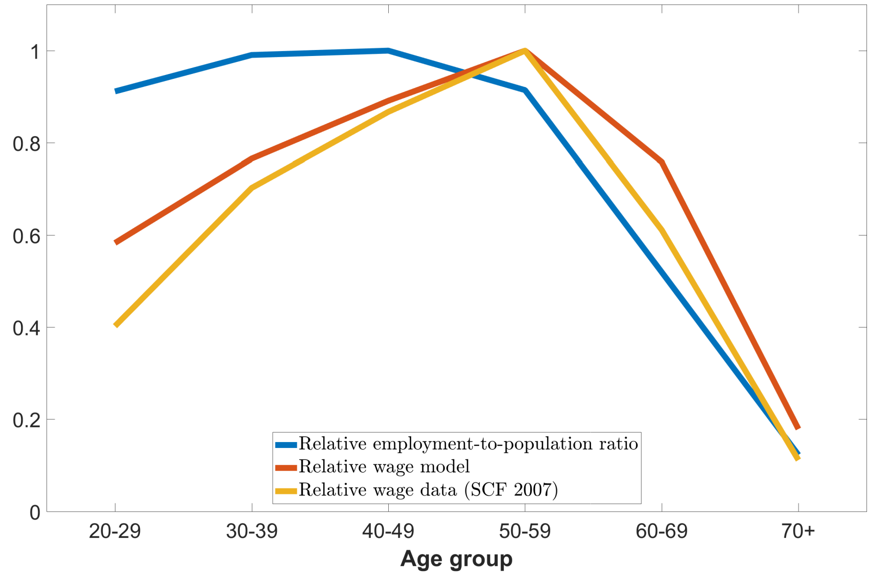


Figure 1.14: Relative wage implied from the model, relative wage in data and relative employment across age groups

Cohort specific productivity

I do two different calibration checks.

In the first one I impute the negative shock in the model (the calibration of the shock is discussed in the next subsection) and I compare the implied profile of labor income with the one observed in 2010 data (that is the year taken as a reference for the recession, the calibration of the exogenous state will be discussed in the next subsection). Results are shown in Table 1.17. The model does a good job in fitting the data.

Cohort	w^i (model)	w^i (data, normalized)	% difference
1	0.0548	0.0537	+1.96%
2	0.1002	0.0993	+0.83%
3	0.1253	0.1324	-5.61%
4	0.1508	0.1471	+2.48%
5	0.1243	0.1287	-3.61%
6	0.0613	0.0646	-5.38%

Table 1.17: Wage profiles after the shock

For an additional check I try to add a government to the model and I look at the implied

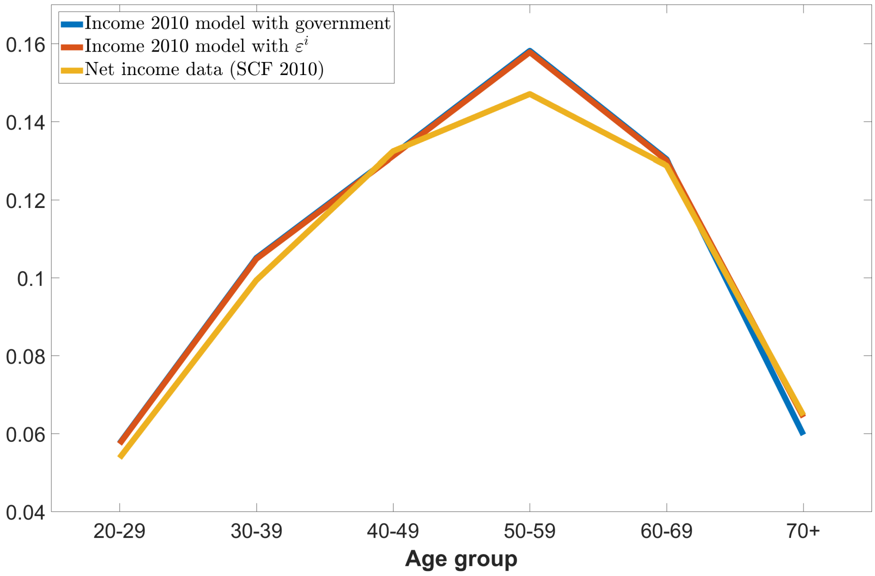


Figure 1.15: Wage profile after the shock as implied by the model with epsilon

profile of net labor income after the negative shock. The modelization is presented in appendix 1.7.3. Results are shown in Figure 1.15: the differences are negligible and if anything the social security is exacerbating the under-performance for the older cohorts. This is due to the fact that the simple fiscal policy rule does not allow for intertemporal redistribution through government debt, given that the last cohort takes all fiscal residuals any change in employment determines a one to one fall of transfers to the last cohort.

Adding the social security system to the model The representation through the ε^i of the social security system simplifies the model giving satisfactory results. Indeed alternative, simple methods for representing the transfers do not perform significantly better. The simplest alternative method would be a system of taxes (and subsidies) on gross wages. Unless we add public debt and a more complex fiscal rule (that increases the state dimensionality thus making computation more complex), the government balance has to be in equilibrium period by period. Therefore, when calibrating the fiscal policy rule I do not have 6 degrees of freedom but only 5[16]. Then, I model a simple fiscal rule with proportional tax rates (subsidies if τ^i is negative) on the first five cohorts, residuals are used to pay sixth cohort. The government

[16]I loose one degree of freedom adding the intra-temporal budget constraint of the government

budget constraint is:

$$\sum_{i=1}^{5} \tau^i l^i w^i = t_6$$

where t_6 is a lump sum transfer to the sixth cohort.

In order to calibrate $(\tau^i)_{i=1}^5$ and t_6 I minimize the distance between data and model in 2007 under the government budget constraint, results are shown in Table 1.18.

Cohort	τ^i
1	0.32
2	0.09
3	0.03
4	-0.02
5	-0.06

Table 1.18: Cohort specific tax rates calibration

Chapter 2

Economic Growth and Wealth Inequality: the Role of Differential Fertility

Co-authored with Alessandro Di Nola.

2.1 Introduction

Wealth inequality has slowly but continuously increased in many developed countries in the last thirty years. In the US the Gini coefficient of wealth, computed using micro-data from the *Survey of Consumer Finances* has risen from 0.79 in 1992 to 0.85 in 2013 (Figure 2.1).

A vivid debate on its possible causes and driving forces of this trend has been spurred by the seminal work of Piketty and Zucman (2014) and by new evidence and data contained in the book Piketty (2014). Despite the great amount of literature that has studied wealth inequality, intra-cohort inequality has received far less scrutiny. Figure 2.2 represents the intra-age group Gini coefficient for different years: the biggest increase in inequality is concentrated in the youngest age-groups, between 1995 and 2013 the Gini coefficient among households where the head is below 35 increased from 0.81 to more than 1[1]. Piketty (2014) shows that the relevance of bequests over life-time income has increased dramatically in the last decades (French data are displayed in Figure 2.3). Using different data source he finds that, in France, from 1990 to 2010 the annual value of inheritance and intra-vivos transfers over household disposable income doubled, from around 10% to almost 20%.

Changes in intergenerational transfer are driven by changes in the amount of resources that

[1]Other measures of inequality are displayed in appendix 2.7.1.

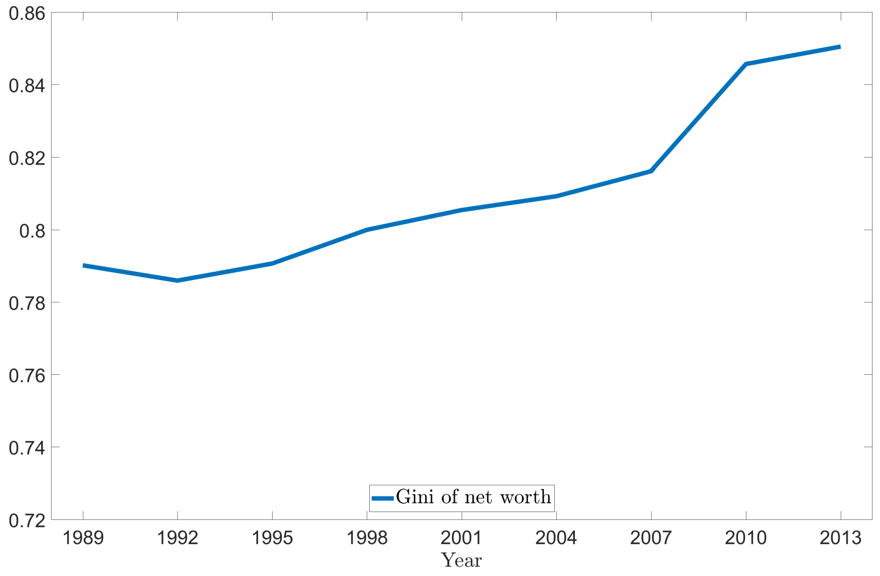


Figure 2.1: Gini coefficient of net wealth in the United States. Source: Survey of Consumer Finances.

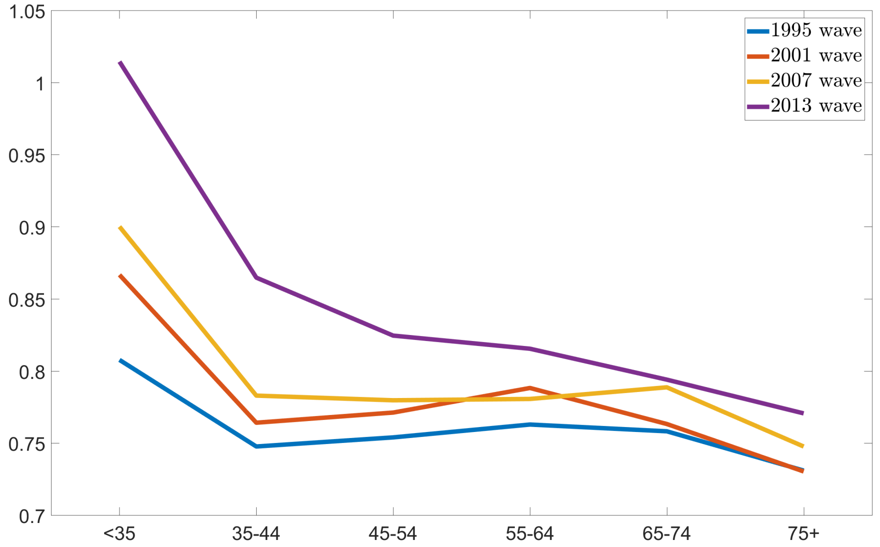


Figure 2.2: Intra-cohort Gini index across years in the United States. Source: Survey of Consumer Finances

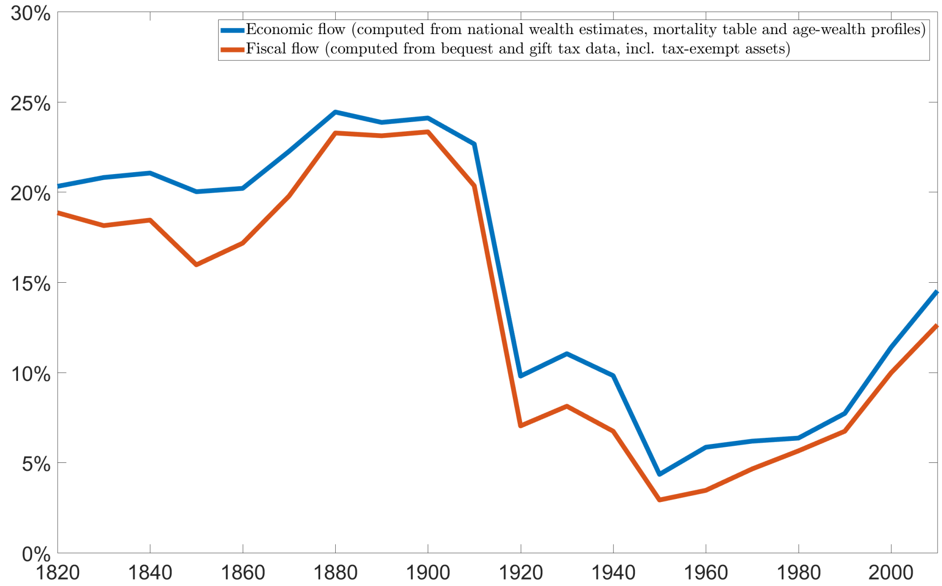


Figure 2.3: Annual inheritance flow as a fraction of household disposable income, France 1820-2010. Source: Piketty (2014)

parents choose to give to their heirs or by changes in the number of children. While changes in the overall amount of resources transfered to future generations has been greatly explored in the literature[2], its relationship with fertility has received much less attention.

This paper explores the relationship between economic growth, intergenerational transfers and fertility decision and their effects on intra-cohort wealth inequality. We found that fluctuations in total fertility rate are associated with fluctuations in total factor productivity and that the elasticity of fertility to economic growth is higher for wealthier households. We rationalize this evidence in a framework with altruistic parents *à la* Barro-Becker: parents like having children but they also care about children's prospects. Consistently with micro evidence, the number of children is increasing in the level of wealth of the housing unit, i.e. they are a normal 'good'. In this setup a 'persistent' decrease in TFP growth lowers parents' expectations on their children income. This is a negative wealth shock that induces a decrease in the number of children and an increase in the total amount of transfers (for inter-temporal optimization they move consumption from the present to the future). The magnitude of the wealth shock is correlated to the number of children that the household would have, and therefore is greater for wealthier households. To put it in another way, for the head of a 'dynasty'

[2]See De Nardi (2015) and references therein.

a decrease in expected income tomorrow determines an increase of the optimal quantity of intergenerational transfers (an increase of 'savings') and a decrease of heirs (he cares about their welfare and therefore he prefers to guarantee more consumption to fewer children).

We develop the analysis through several steps. Firstly, looking at aggregate data on fertility and TFP growth we find that changes in the trend of TFP growth are correlated to changes in the fertility rate (detrended by its historical declining path, determined by the level of GDP reached by the economy). In particular TFP growth *leads* the fertility rate: the correlation is statistically significant after 5 years and is robust to different definitions of TFP and fertility rate. We document such finding for both US and UK over the last century.

Then, the pattern observed in aggregate data are studied using a Barro-Becker model enriched with uninsurable idiosyncratic shocks and an aggregate shock on the level of income. The former is a necessary ingredient to generate heterogeneity in the level of wealth, the latter is used to study households reaction to changes in economic prospects. Under a standard calibration the model predicts that the elasticity of fertility and bequests to expectations about children income is larger in magnitude for wealthier households, and therefore a decrease in economic growth determines an increase in intra-cohort wealth inequality of the next generation. The model prediction is supported by micro-data from *Survey of Consumer Finances* (SCF) and *Panel Study of Income Dynamics* (PSID).

Finally, an extended version of the model with aggregate and idiosyncratic shocks is used to measure the change in wealth inequality determined by the fertility channel. We calibrate the model to match certain cross-sectional data moments of the US economy related to fertility and income/wealth inequality, such as the income elasticity of fertility and the average fertility level. The model does a fairly good job in capturing the salient features of the fertility-income-wealth distribution (also on data moments that were not targeted) and we use it to perform some counterfactual experiments. In the main quantitative result, we find that roughly 40 percent of the overall increase in wealth inequality can be accounted for by the decline in the fertility rate (stronger among wealthier households) induced by a revision in parents' expectations on future growth. We find a limited but not negligible role for *biased beliefs*. In particular we calibrate and solve a similar version of the quantitative model with the following modification: agents are not entirely rational and tend to overestimate the persistence of economic shocks. The idea is that a negative shock to TFP growth can have a longlasting effect on fertility if households perceive that slow economic growth will persist for many periods in the future.

The remainder of the paper is structured as follows: in the next subsection we review the relevant literature. In section 2.2 we present some aggregate evidence linking TFP to fertility.

In Section 2.3 we introduce a stripped-down version of the Barro-Becker model. We move on to Section 2.4 presenting a detailed analysis on microeconomic data. Section 2.5 contains the quantitative model and Section 2.6 concludes the paper.

2.1.1 Related literature

This paper is related to two main strands of literature: the first group of papers studies the determinants of wealth inequality and (possibly) its evolution over time. Prominent examples in this field are the papers of De Nardi (2004) who studies the role of bequests in amplifying wealth inequality and Kaymak and Poschke (2016) who analyze the relative importance of fiscal reforms and wage inequality behind the dramatic increase in wealth concentration in the US. Our paper shares the quantitative methodology with the papers cited above (and the importance given to intergenerational transfers as in (De Nardi, 2004)) but differs substantially as to incorporate the fertility channel, which is absent in De Nardi (2004) and Kaymak and Poschke (2016).
The second strand of literature our paper is connected to is fertility. The study of fertility decisions in economics started with the seminal work of Becker (1960) and was later embedded in a macroeconomic model by Barro and Becker (1989). This paper is related in particular to three main works in this area: De la Croix and Doepke (2003), Jones and Schoonbrodt (2016) and Cordoba et al. (2016). De la Croix and Doepke (2003) focus on the relationship between inequality and growth: they show that an increase in income inequality leads to more fertility differential between the rich and the poor, which in turn lowers aggregate education and, hence, growth. We take inspiration from them regarding the notion of fertility differential between the rich and the poor but the focus of our paper is orthogonal to them: they focus on income inequality, disregarding wealth inequality, while we do exactly the opposite. Jones and Schoonbrodt (2016) study the fertility response to economic growth in a Barro-Becker model with a representative agent and they have found that fertility is pro-cyclical, on the other hand Cordoba et al. (2016) study the inter-generational persistence of wealth inequality in a Barro-Becker model with incomplete markets *á la* Bewley but withouth aggregate shocks.

This paper builds a bridge between the two previous works by looking at intergenerational transfers in a Bewley world during fertility fluctuations determined by economic growth.

2.2 Aggregate evidence

This section studies at the empirical level the relationship between fertility and economic growth. Fertility and economic growth are correlated at different frequencies for different reasons, therefore we need to clarify what is the objective of this analysis.

Firstly, it has been documented and largely studied the long-run negative relationship between TFP growth (and more generally economic growth) and fertility. This relationship have been extensively studied in demography (e.g. Heer, 1966) as well as in economics (e.g. Jones et al., 2010, Boldrin et al., 2015), it is based on the 'Quantity-Quality' trade-off and it is beyond the scope of this paper.

At the opposite of the frequency spectrum we find the short run pro-ciclicality of fertility to business cycle fluctuations. It has been extensively studied (e.g. Ben-Porath, 1973 and Sobotka et al., 2011) and it has been explained as the result of fertility postponement during economic downturns. It has proven to be short-lived (as the business cycle) and it does not alter the life-long fertility decision of the couple.

In this paper we study the relationship between the fertility decision of a couple over its fertile years, around twenty, and fluctuations of TFP growth in the same time span. We are therefore interested on the medium to long frequency fluctuations of fertility and TFP. For this reason it cannot be addressed with standard TFP[3] and fertility series that spans 50 years. Therefore we have built an appropriate dataset that we describe in subsection 2.2.1. In subsection 2.2.2 we separate trend and cycle at different frequencies for the two series and finally in subsection 2.2.3 we show correlation of the two series.

2.2.1 Data sources

The only countries considered are the United States and the United Kingdom since they were the only ones for which we were able to reconstruct historical series of TFP. For the analysis we do not just consider the crude total fertility rate but we adjust it for child mortality, and we therefore assume that parents internalize the survival probabilities in their fertility choices[4]. Therefore we need three time series: total fertility rate (TFR), child mortality and TFP.

TFR and child mortality are the series constructed in Roser (2017a) and Roser (2017b) respectively: their historical sources are presented in detail in appendix 2.7.4. TFP series for the United Kingdom is taken from Bank of England project *A millennium of macroeconomic*

[3]For example the TFP series provided by (Fernald, 2012).

[4]Boldrin et al. (2015) shows that the fall in mortality played an important role in the overall fall in fertility rate.

data[5]. TFP for the United States is computed using the Chari et al. (2007) methodology on updated series[6].

2.2.2 Trend and cycle

Our research question is related to medium to long frequencies, for this reason we want to clean our series from short-term fluctuations of TFP and from the long-term decline of fertility. On the other hand fertility exhibits very small fluctuations in the short term and TFP growth does not exhibit a long-run trend. In order to preserve simplicity we do not perform a symmetric filtering but a simple HP filtering of opposite frequencies for the two series.

For the TFP we use an HP filter[7] with $\lambda = 20$ we separate yearly TFP growth trend from TFP growth cycle and we interpret the trend as the underlying measure of growth and innovation. With respect to total fertility rate we remove the decreasing linear trend[8], that is equivalent to HP filtering with $\lambda \to \infty$.

2.2.3 Aggregate correlation

The series and crosscorrelograms of all possible correlations of trends and cycle for United States and United Kingdom are in appendix 2.7.4.

Firstly, looking at the TFP series in the left panel of Figure 2.4 we can have an idea of what we are capturing with the filter we are applying. The peaks of TFP 'trend' coincides with the boom of the 1920s, the New Deal and the mobilization of the II World War in the 1940s, in 1960 reached another peak before steadily declining until 1980. Then another period of extaordinarily high growth persisted until the beginning of 2000s. The correlation between the two series can be spotted from the plot, and in particular the 'Baby Bust' of the 1920s and 1930s is clearly associated to the Great Depression and the subsequent stagnation as well as the 'Baby Boom' is associated to the TFP growth of the 1940s and 1950s. The repraisal of TFP growth in the 1980s is then associated with an incrase of the TFR. The correlogram on the

[5]The project is under development and it contains many data series starting from 1096. TFP, GDP and Central Bank balance sheet data starting in 1700.

[6]As in Jones et al. (2010).

[7]It is the optimal value for yearly data according to Mohr (2001) and an intermediate value between the 6.5 proposed by Ravn and Uhlig (2002) and the 100 proposed originally by Hodrick and Prescott (1997).

[8]According to economics of demographics it is the result of the increase in the level of income, since the aim of the model is to study fluctuations in growth we want to eliminate this trend. An alternative way of performing the same exercis, more complicated but probably with similar results, would be a cointegration between GDP and fertility assuming that, in line with the literature, any change in GDP has an effect of the same magnitude on fertility.

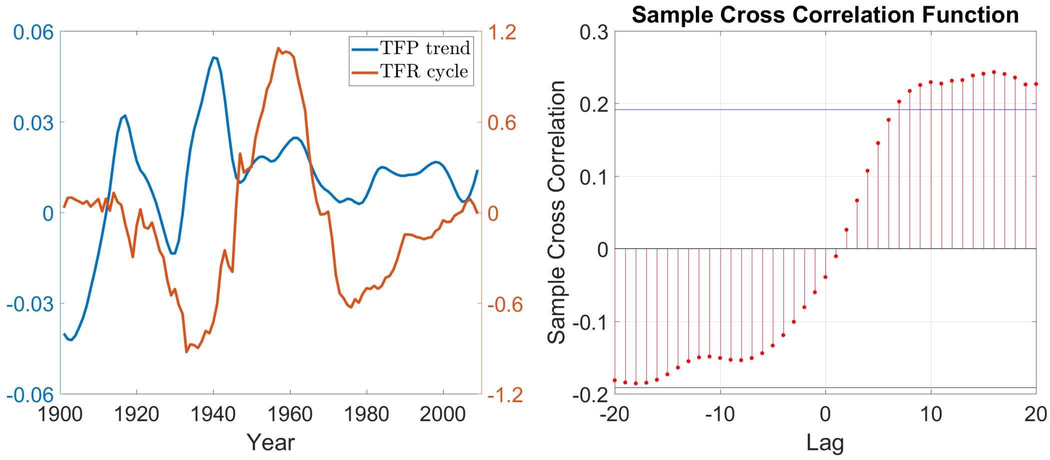


Figure 2.4: Fertility cycle and TFP trend in the United States and their crosscorrelogram

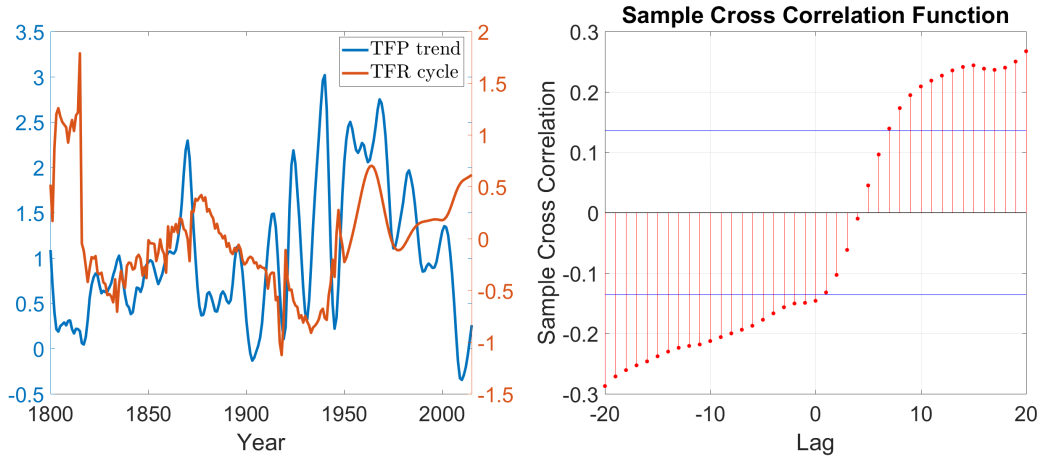


Figure 2.5: Fertility cycle and TFP trend in the United States and their crosscorrelogram

right hand side of Figure 2.4 shows that the correlation between the two series is statistically different from 0 after 5 years, with TFP trend leading TFR, and it is persistent from thereafter.

The same analysis has been conducted for the United Kingdom, exploiting a longer time series of TFP (up until the beginning of the 19^th century) provided by the Bank of England. Results are plotted in Figure 2.5. Reassuringly the correlation is of the same magnitude at the same lags in the two countries, even if the time span of the United Kingdom series is much longer.

2.3 Theoretical model

In this section we develop a simple two period model that allows us to study the main mechanism of the full quantitative model. We show that when the next period endowment is high enough, any further increase leads to a reduction in inequality even if the non-negativity constraints on bequests are not binding. On top of that, we show that, when there are households with the non-negativity constraints binding the level of growth required is even smaller.

2.3.1 Two periods model

The world lasts two period: in the first period there is only an individual with labor income ω and endowment τ. She has to choose the number of children that she wants to have (n) and the amount of per-capita bequests that she wants to leave to her offsprings (b). In the second period the offsprings consume all the endowments that they receive and the inherited bequests increased by their gross rate of return R. The cost of having children is an oppurtinity cost in terms of time where the time requested of having children is $\Phi(n) = \left(\frac{n}{\bar{n}}\right)^{\theta}$.

Given the modelization the cost of children is in term of time to be spent with them: therefore any change in wage has not only a wealth effect on the agent but also a substitution effect since it affects the marginal cost of having children. For this reason the modelization with τ allows to distinguish the wealth effect from the substition effect in comparative statics.

The problem is:

$$\max_{c,c',b,n} \quad \frac{(c)^{1-\sigma}}{1-\sigma} + \alpha\,(n)^{1-\eta}\,\frac{(c')^{1-\sigma}}{1-\sigma}$$
$$s.t.$$
$$c + bn \leq \omega\left[1 - \left(\frac{n}{\bar{n}}\right)^{\theta}\right] + \tau \qquad (\lambda)$$
$$c' \leq \omega' + bR \qquad (\lambda')$$
$$c, c', b, n \geq 0 \qquad (\phi_v)$$

Where the interpretation of non-negativity coefficients on consumption and children is straightforward while the non-negativity on bequests is a natural assumption given a number of legal (and moral) restrictions preventing parents from imposing debt obligations on their children (as in Cordoba et al. (2016)).

Proposition 1. If $\eta \geq \sigma > 1$ and $\theta > 1$ the problem has a solution and it is unique. What is more, the FOCs are not only necessary but also sufficient to characterize the equilibrium.

Proof. The proof can be found in appendix 2.7.5. □

Therefore, if $\eta \geq \sigma > 1$ and $\theta > 1$ the solution can be found by solving the system of equations together with the slackness conditions[9]

$$\begin{array}{rc} c: & c^{-\sigma} = \lambda \\ c^{'}: & \alpha\left(n\right)^{1-\eta}\left(c^{'}\right)^{-\sigma} = \lambda^{'} \\ b: & \lambda = R\lambda^{'} + \phi_{b} \\ n: & \alpha\left(1-\eta\right)\left(n\right)^{-\eta}\frac{\left(c^{'}\right)^{1-\sigma}}{1-\sigma} = \left[\Phi'\left(n\right)+b\right]\lambda \end{array}$$

A closed form solution cannot be computed but the four equations together with the constraints and the slackness conditions determine the optimal c^{*}, b^{*}, n^{*}, c'^{*}.

What are the effects of tomorrow endowment on inequality? It depends on the sign of $\frac{\partial^{2} n}{\partial\omega'\partial\tau}$ that characterizes *differential fertility*. Under the parametrization of Jones and Schoonbrodt (2010)[10] children are complementary to their *quality* and a better prospect for their consumption leads to an increase in the optimal choice of fertility, i.e. $\frac{\partial n}{\partial\omega'} > 0$. Distributional consequences can be explored looking at $\frac{\partial^{2} n}{\partial\omega'\partial\tau}$, i.e. how people with different level of wealth adjust their fertility to a change in tomorrow endowment. In particular, if $\frac{\partial^{2} n}{\partial\omega'\partial\tau} > 0$ then any increase in tomorrow endowment leads to a stronger increase in fertility for the wealthier households, leading to a more wide spreading of bequests and determing a decrease in the level of inequality in the next cohort.

The first step of the analysis is to determine the elasticity of fertility with respect to tomorrow endowment.

Lemma 2. If the non-negativity constraint on bequests is not binding, i.e. $b^{*} > 0$ and $\phi_{b}^{*} > 0$ then the elasticity of fertility with respect to tomorrow endowment is given by:

$$\frac{\frac{\partial n}{n}}{\frac{\partial\omega'}{\omega'}} = \frac{1}{(\theta-1)}\Gamma\left(b,n\right)$$

where $\Gamma\left(b,n\right) = \left[\frac{\omega^{'}}{\omega'+bR}\frac{b+\omega\Phi'(n)}{\omega\Phi'(n)}\right]$.

Proof. The proof can be found in appendix 2.7.5. □

[9]With respect to the non-negativity constraints we are including only the one on bequests since it is the only one that can be non-trivially binding

[10]It is discussed in appendix 2.7.2.

What is the economic interpretation of $\Gamma(b,n)$? It is composed by two terms and it captures the marginal benefits for parents from the increase in $\omega^{'}$ that allows to cut the per-capita bequest to next generation without decreasing her utility. It is composed by two terms:

- $\frac{\omega^{'}}{\omega'+bR}$: is the relative weight of tomorrow endowment on next generation consumption, the higher the amount of bequests that parents are leaving to them the less significant is the increase in consumption coming from an increase in the next period endowment, and therefore the amount of bequests that they can decrease leaving the next generation unaffected is smaller. As a result also the elasticity is of fertility to future endowment is low;

- $\frac{b+\omega\Phi'(n)}{\omega\Phi'(n)}$: is the relative weight of bequests in the marginal cost of having children. The higher is b, the higher is the benefit arising from the possible decrease in b and therefore it increases $\frac{\frac{\partial n}{n}}{\frac{\partial \omega^{'}}{\omega^{'}}}$.

The overall effect of being rich on $\Gamma(b,n)$ is therefore ambiguous. On the one hand the relative weight of next period endowment on future generation consumption is lower given that an high fraction of consumption is represented by bequests, but on the other hand bequests are representing a great fraction of the marginal cost of having children and therefore even a small reduction calls for an increase in fertility.

Now we can look at the distribution consequences of next period endowment.

Proposition 3. If the non-negativity constraints is not binding, i.e. $b^* > 0$, then $\exists\overline{\omega'}$ s.t. $\forall\omega' > \overline{\omega'}$, $\frac{\partial^2 n^*}{\partial\omega'\partial\tau} > 0$. That is, if tomorrow endowment is high enough the higher the level of initial wealth the higher the (positive) adjustment in fertility when the prospects for next period improves.

Proof. The proof can be found in appendix 2.7.5. □

The effect on non-negativity constraints

Even if $\Gamma_b(b,n) < 0$, the non-negativity constraint may generate an increase of wealth inequality. Indeed, $\Gamma(b=0,n) = 1$, Then if $\forall b > 0$, $\Gamma(b,n(n)) > 1$ the existence of the non-negativity constraint generates a decrease of wealth inequality when the next period endowment is higher.

In Figure 2.6 it is represented the comparative static of $\mathbb{E}[\omega']$ (when $\omega' \in \left\{\omega_h^{'},\omega_l^{'}\right\}$ and $Var\left[\omega^{'}\right]$ is kept fixed) on the differential fertility and on the differential level of bequests.

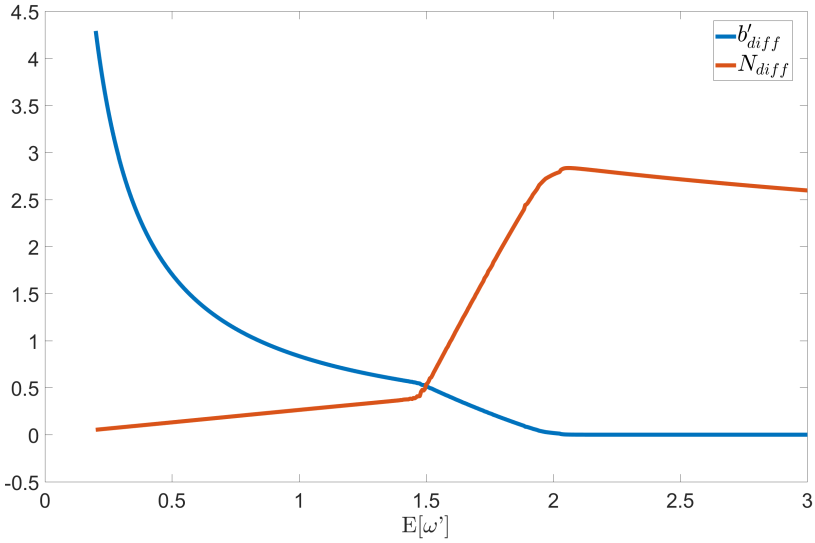


Figure 2.6: Comparative static on differential fertility of $\mathbb{E}[\omega']$ in absolute values and in percentage changes

More precisely, for each $\mathbb{E}\left[\omega^{'}\right]$ two individuals are considered, one with $\tau = 10$ and the other one with $\tau = 0$, this allow to study the pure wealth effect disentagling from the substitution effect generated from the opportunity cost of having children. In Figure are represented $n^{10} - n^{0}$ and $b^{10} - b^{0}$, that is the difference in the amount of children and in the amount of per-capita bequests left.

For low levels of $\mathbb{E}\left[\omega^{'}\right]$, an increase in the expected value of tomorrow endowment, *ceteris paribus*, decreases the amount of bequests left by both the rich and the poor agents and it increases the difference in the number of children. Notice that the fall in $b^{10} - b^{0}$ is decreasing with $\mathbb{E}\left[\omega^{'}\right]$. Around $\mathbb{E}[\omega'] = 1.5$ the non-negativity constraint of the poor household becomes binding. From that moment onward the poor does not benefit anymore from the increase in tomorrow endowment given that the optimal choice would require $b^{0} < 0$. For this reason children have a lower value and an higher cost that cannot be cut. This affect the Euler constraint of the agent that is prevented from borrowing and therefore $\lambda > R\mathbb{E}\left[\lambda_{s}^{'}\right]$. Then they reduce the number of children today and they increase their actual consumption. When $\mathbb{E}[\omega'] > 2$ the non-negativity constraints becomes binding also for the rich household and therefore she also starts reducing the number of children (the differencein the number of children starts to decrease).

2.3.2 Introducing uncertainty

In this case the next-period aggregate endowment is subject to risk, $\omega' \in \left\{\omega_h', \omega_l'\right\}$). She has to choose the number of children that she wants to have (N) and the total amount of bequests that she wants to leave to her offsprings (B). The dynastic problem[11] therefore is:

$$\max_{C, C_h', C_l', B, N'} \quad \frac{(C)^{1-\sigma}}{1-\sigma} + \alpha \left(N\right)^{\sigma-\eta} \mathbb{E}\left[\frac{(C')^{1-\sigma}}{1-\sigma}\right]$$
$$s.t.$$
$$C + B' \leq \omega \left[1 - \left(\frac{N}{\bar{N}}\right)^{\theta}\right] + \tau \qquad (\lambda)$$
$$C_s' \leq N\omega_s' + BR \qquad \left(\pi_s \lambda_s'\right)$$
$$C, B, N, C_h', C_s' \geq 0 \qquad (\phi_v)$$

Firstly, what are the conditions that guarantee that the problem has a solution and that the solution that we find is also unique? Under regular parametrization of the utility function[12] the objective function is strictly quasi-concave and therefore if the feasibility set is convex the solution exists and it is unique.

Proposition 4. If $eta \geq \sigma > 1$ and $\theta > 1$ the problem has a solution and it is unique.

Proof. The proof is almost identical to the proof of Proposition 1 (that can be found in 2.7.5). □

Then the FOCs are not only necessary but also sufficient, therefore the solution satisfies:

$$C: \quad C^{-\sigma} = \lambda$$
$$C_h': \quad \alpha \left(N\right)^{\sigma-\eta} \left(C_h'\right)^{-\sigma} = \lambda_h' \pi_h$$
$$C_l': \quad \alpha \left(N\right)^{\sigma-\eta} \left(C_l'\right)^{-\sigma} = \lambda_l' \pi_l$$
$$B: \quad \lambda = R\mathbb{E}\left[\lambda_s'\right] + \phi_b$$
$$N: \quad \left(\frac{\sigma-\eta}{1-\sigma}\right) \alpha \left(N\right)^{\sigma-\eta-1} \left(\pi_h C_h' + \pi_l C_l'\right) + \pi_h \lambda_h' \omega_h' + \pi_l \lambda_l' \omega_l' = \omega \frac{\theta}{\bar{N}^{\theta}} \left(N\right)^{\theta-1} \lambda$$

[11] The problem is written as the problem of the head of the dinasty that has to choose the total amount of consumption to allocate to the generation, it is a common way to write this problem (Alvarez (1999)).

[12] $\sigma = 2$, $\eta \geq \sigma$, see Jones and Schoonbroodt (2010) for a complete discussion on the effect of this calibration as opposed to Barro and Becker (1989) calibration (with $\sigma < 1$).

Where the last condition can be written more compactly as:

$$\left(\frac{\sigma-\eta}{1-\sigma}\right)\alpha\left(N\right)^{\sigma-\eta-1}\mathbb{E}\left[C_{s}^{'}\right]+\mathbb{E}\left[\lambda_{s}^{'}\omega_{s}^{'}\right]=\omega\frac{\theta}{\overline{N}^{\theta}}\left(N\right)^{\theta-1}\lambda$$

Study the effect of a reduction in the probability of high state tomorrow $\pi_h \downarrow$ leads to an increase in the expected marginal utility of tomorrow, i.e. $\mathbb{E}\left[\lambda_{s}^{'}\right]\uparrow$. From the Euler-equation (the FOC wrt B) it must follow $\lambda \uparrow$ and or $\phi_b \downarrow$. When the non-negativity constraint on bequests is not binding the only multiplier that must adjust is λ, and therefore we must have $\lambda \uparrow$, i.e. $C \downarrow$. From the budget constraint we have that the decrease in consumption can be compensated through an increase in the amount of bequests or an increase in the amount of children. Looking at the FOC wrt N we have that the marginal cost of children has increased due to the increase in λ and that the "marginal return" of children has decreased due to the fall of $\mathbb{E}\left[C_{s}^{'}\right]$ while the effect on $\mathbb{E}\left[\lambda_{s}^{'}\omega_{s}^{'}\right]$ is ambiguous. As a result

The comparative statics with respect to main parameters of the model is performed in appendix 2.7.3.

2.3.3 Comparative statics on differential fertility and differentials bequests

What are the effects of the different calibrations on the differential fertility? We perform the following analysis: we compute for different values of the parameter of interest the solution of the two period model, in one case we solve the model with $\tau = 0$ and in the other case we solve the model with $\tau = 10$, then we compare the optimal choice of fertility and per-capita bequests. Define for the generic variable x the solution of the problem when $\tau = t$ with x_t^*, for different values of the parameters of the model we plot $N_{diff} := N_{10}^* - N_0^*$, $b_{diff}^{'} := b_{10}^* - b_0^*$, $N_{diff}\% := \frac{N_{10}^* - N_0^*}{N_0^*}$ and $b_{diff}^{'} := \frac{b_{10}^* - b_0^*}{b_0^*}$. Results[13] are shown in Figure 2.7.

- $0.01 \leq \alpha \leq 0.60$: an increase in the discount factor of the parents uin absolute terms increases the difference in the number of children between rich and poors and increases the difference in per-capita bequests left to the children, nonetheless looking at the percentage differences we can see that the increase in patience leads to a reduction of per-capita bequests difference in percentage terms.

- $1.1 \leq \eta \leq 5$[14]: an increase in η reduces the differential fertility, the difference in

[13]Comparative static is performed using a non-linear solver and a filter to smooth the resulting lines.
[14]In this case I let $\sigma = 1.1$, this allow to study the effect of η

bequests is extremely small and disappear. Indeed an increase in η moves the preference of the parents from quality children to their quantity, this lead to a reduction in the quantity of bequests also from the rich.

- $1.5 \leq \sigma = \eta \leq 3.5$: σ captures the inverse of intertemporal elasticity of substitution and the degree of relative risk-aversion. An increase in σ determines an increase in the relative risk-aversion, given that children are the risky investment in this setup the differential fertility increases since the wealthier households will be relatively less risk-averse thus prefering children to bequests and determing a reduction in the percentage difference of bequests and an increase in the percentage difference in the number of children.

- $1 \leq R \leq 2$: the gross interest rate affects the investment opportunity in bonds instead of children. An increase in the interest rate increases the difference in bequests in absolute terms this is coming from the fact that when $R = 1.1$ the poor households has the non-negativity constraint on bequests binding. On the contrary the difference in the number of children is decreasing both in absolute and in relative terms.

- $1.1 \leq \theta \leq 2$: θ is a parameter that affects the "technology" of fertility and captures the degree of convexity of the time-cost function of children. An higher degree of concavity increases the marginal cost of having an additional child.

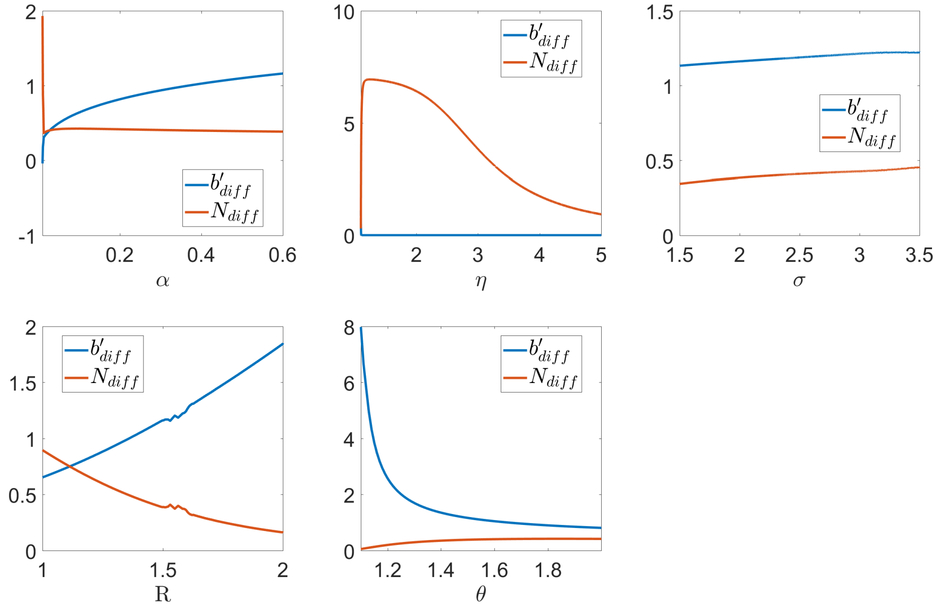


Figure 2.7: Effects on differential fertility and bequests of the main parameters of the model

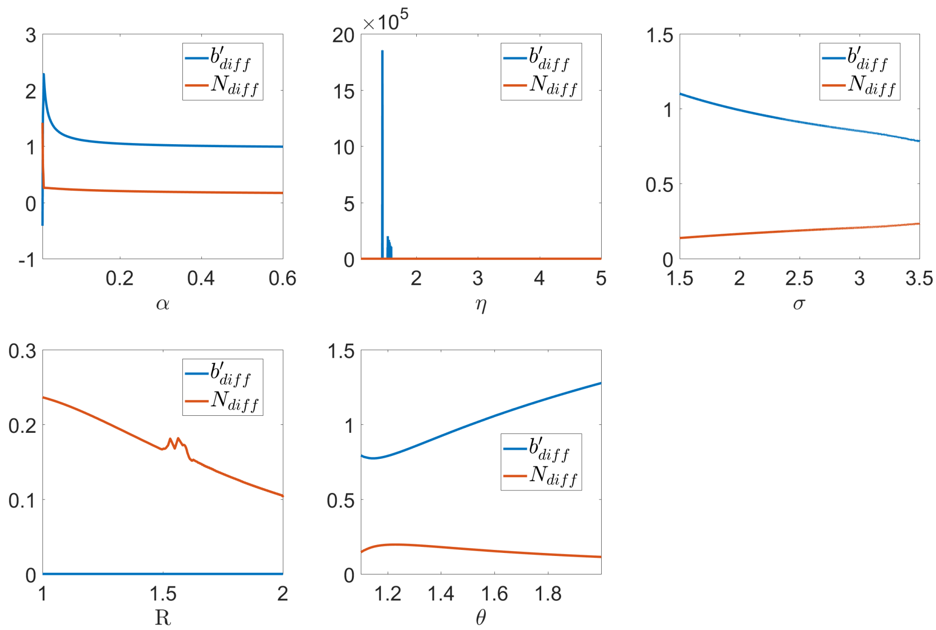


Figure 2.8: Effects on differential fertility and bequests of the main parameters of the model, % changes

2.4 Empirical evidence on microdata

The aim of this section is to get a measure of the effect of TFP growth on fertility at intra-cohort deciles (or terciles) of wealth. Therefore, data at the household level on wealth and family composition are needed. There are two main candidate datasets: Survey of Consumer Finances (SCF) and the Panel Survey of Income Dynamics (PSID). The PSID contains more detailed information on demographics variables, and in particular on the number of children, but the SCF contains more detailed information on wealth and it has the great advantage of over-sampling the wealthiest.

2.4.1 Different cohort specifications

Family units are considered in relative terms with the other units of the same cohort, this allow to control for any cohort-specific shock on income and wealth. We use two definitions of cohort: one is "cohort of the head of family" that simply assigns a family unit to the cohort of the head of family (father), the other is "family cohort" and it is the mean between the cohort of the father and the cohort of the mother of the unit. The first measure assumes that the most important drivers of family wealth are related to the father life cycle of income, the second measure assumes that the two have an equal weight in determing the wealth of the family.

Deciles (and terciles) computation

In each year we count how many unit belonging to the same cohort are in the dataset. If there are at least 20 units then we extend the population to those cohort that are born two years before and two year after (thus covering a 5 year window) and the unit of the cohort get imputed the decile of wealth and income to which they belong, this should guarantee a minimal sample over which it makes sense obtain a measure of relative wealth. The procedure is repeated for each cohort and for each year. Terciles computation is based upon identical procedure but the minimal number is 10.

2.4.2 Model specifications

Two main model specifications are possible: the first one can be used exploiting the panel structure of PSID and can take as dependent variable the change in the number of children in the family from the previous wave of the survey, the second one can focus on the total number of children.

With respect to the first model specification the dependent variable is $\Delta n^j_{t,z} = n^j_{t,z} - n^j_{t-1,z}$, i.e. the change in the number of children in household unit z, that belongs to decile (or tercile) of wealth j inside his cohort in period t. The regression must be run on those couples that may have a children and therefore those in which the women is between 20 and 45 years old. The main model specification is:

$$\Delta n^j_{t,z} = \alpha + \beta^j \Delta TFP^{trend}_{t-l} + \gamma X_{t,z} + \varepsilon_{t,z}$$

Where ΔTFP^{trend}_{t-l} is the change in TFP trend with lag l and $X_{t,z}$ is a bunch of demographic and economic controls. The parameters of interest in this specification are $(\beta^j)^{10}_{j=1}$, and in particular economic growth would decrease wealth inequality if $\beta^{10} > \beta^9 > \cdots > \beta^1$.

Alternatively, it is possible to look at the completed fertility of the couple and meausure how the mean of TFP trend during the fertility years affected their fertility decision. This regression is run only on those couple in which the female is above 45. In this model the dependent variable is n^j_z, i.e. the number of children that the family unit z in decile (or tercile) of wealth j has. The main specification is:

$$n^j_z = \alpha + \beta^j \left(K^{-1} \sum_{k=1}^{K} TFP^{trend}_{i+T-j} \right) + \gamma X^j_z + \varepsilon_z$$

In this specification the set of parameters of interest is $(\theta^j)^{10}_{j=1}$ and as before the relationship between growth and inequality is confirmed whenever $\theta^{10} > \theta^9 > \cdots > \theta^1$.

The former specification can be run only in the PSID where the panel structure of data allows to measure the change in family composition year after year. On the other hand the latter specification can be run also on SCF data that do not contains the age of the children but only their total number.

Linear specification

As specified in subsection 2.4.1 deciles are computed whenever a cohort has a minimal number of components and using a sort of rolling window computation. Nonetheless deciles are computed on a very small population that may affect results. There are two ways to address this problem: using terciles instead of deciles or making an assumption on the linearity of the coefficients. This second model specification assumes that $\frac{\beta^{10}}{\beta^9} = \frac{\beta^9}{\beta^8} = \cdots = \frac{\beta^2}{\beta^1}$ or alternatively that $\beta^j = \tilde{\beta}_f + j\tilde{\beta}$, that is it assumes that the elasticity of fertility with respect to growth is linearly increasing in the decile of wealth. It is a strong assumption but it can be used as a robustness check to addres the small sample problem.

2.4.3 Panel Survey of Income Dynamics

The PSID is a longitudinal survey of US individuals and their family units. It was started in 1968 to study the dynamics of income and poverty, for this reason it was originally designed with two different sub-samples: a specific sub-sample over-sampling poor families (2,000 family units) selected from the Survey of Economic Opportunities (SEO), a nationally representative sample (3,000 family units) designed by the Survey Research Center (SRC) at the University of Michigan. Individuals of family units in the original sample were followed also in the formation of new family units (the split-offs). The survey has annual waves until 1997 and then it became bi-annual.

The survey contains complete data on income of each family member, unfortunately it was not designed to study wealth. A specific set of questions on wealth was introduced in 1984 and it was repeated every five-years[15] until 1999 when it became part of the main set of questions and it was introduced in all waves. Details of wealth imputation are in appendix 2.7.6.

Sample restriction

All the regression for model 1 are run on those couples in which both partners are present and where the female has at most 45 years and the male has at most 50 years.

Results

Results from the main specification are shown in Table 2.1. The explanatory variable is the TFP trend with 5 years lag (i.e. the same explanatory variable suggested by the aggregate evidence analysis), in the first specification it is the only variable in the regression and it has the expected positive magnitude. In the second column the model contains also a set of dummies for the decile of wealth and their interaction with the TFP trend. As expected the higher deciles of income have an effect statistically different from the others. In the third column a linear trend (to control for decrease in overall fertility) and a set of dummies for the age of the head of family and his\her partners are added. In the fourth column also a

[15] The set of questions on wealth was conducted also in 1988.

set of dummies for labor income decile of the head of family are added. Finally in the last column also a set of dummies for deciles of income of the family are included in the controls. In all the specifications $0.0877 \leq \left(\hat{\beta}^{10} - \hat{\beta}^{1}\right) \leq 0.153$, and the effect is increasing in the decile of wealth even if the small sample size determines high variance and small statistical significance (limited to the highest deciles).

Robustness

In order to perform a robustness check and to address the small sample size that affects results in the main specification we perform some robustness checks on different specifications. In the first column of Table 2.2 only the interaction between the dummy of the 10^{th}decile and TFP is included and delivers a similar result. In the other two columns a linear specification of the model is estimated, in both cases the estimate of $\left(\hat{\beta}^{10} - \hat{\beta}^{1}\right)$ is around 0.10 in line with the previous estimates.

2.4.4 Survey of Consumer Finances

The Survey of Consumer Finances (SCF) is by far the best source of micro level fata on household-level assets and liabilities for the United States. It is conducted every three years by the Board of Governors of the Federal Reserve System and collects detailed information on income and assets. With respect to assets the survey is particularly detailed, it contains information on financial and non-financial assets, debts and capital gains. The survey has two parts: a standard random sample of US households, and a second sample that focuses on wealthy households, identified on the basis of tax returns. Only two waves of the sample have a panel-structure and are based on the previous samples: 1986 (same sample as 1983) and 2009 (special wave after 2007 to study the effects of the financial crisis). Both waves are excluded from the analysis.

The dependent variable and its measurement

The aim of the SCF is to represents the financial characteristcs of a subset of the household unit referred to as the “Primary Economic Unit” (PEU). In brief, the PEU consists of an economically dominant single individual or couple (married or living as partners) in a household and all other individuals in the household who are financially interdependent with that individual or couple. Clearly we are not interested to the children in the PEU but on the overall number of children that can inherit the wealth of the family, therefore the question on the

	Change in the number of children from the previous wave	Change in the number of children from th previous wave	Change in the number of children from the previous wave	Change in the number of children from the previous wave	Change in the number of children from the previous wave
2nd dec. of wealth X TFP tr.		0.0332	0.00766	0.00852	0.0236
		(0.0594)	(0.0581)	(0.0576)	(0.0584)
3rd dec. of wealth X TFP tr.		0.0485	0.0176	0.0108	0.0181
		(0.0453)	(0.0441)	(0.0442)	(0.0463)
4th dec. of wealth X TFP tr.		0.0776*	0.0312	0.0267	0.0170
		(0.0421)	(0.0399)	(0.0399)	(0.0428)
5th dec. of wealth X TFP tr.		0.0640	0.0317	0.0285	0.0320
		(0.0425)	(0.0394)	(0.0394)	(0.0408)
6th dec. of wealth X TFP tr.		0.0962**	0.0496	0.0469	0.0423
		(0.0395)	(0.0370)	(0.0369)	(0.0388)
7th dec. of wealth X TFP tr.		0.0988***	0.0677*	0.0602*	0.0538
		(0.0379)	(0.0361)	(0.0360)	(0.0385)
8th dec. of wealth X TFP tr.		0.0643	0.0211	0.0170	0.0179
		(0.0427)	(0.0409)	(0.0407)	(0.0429)
9th dec. of wealth X TFP tr.		0.101***	0.0693*	0.0686*	0.0665*
		(0.0384)	(0.0360)	(0.0360)	(0.0379)
10th dec. of wealth X TFP tr.		0.153***	0.0944**	0.0911**	0.0877**
		(0.0438)	(0.0419)	(0.0416)	(0.0424)
TFP tr.	0.0183***	-0.194***	-0.0755**	-0.0704**	-0.0653*
	(0.00687)	(0.0336)	(0.0318)	(0.0318)	(0.0343)
Observations	14454	3804	3804	3804	3438
R^2	0.001	0.052	0.131	0.136	0.142
Wealth decile	No	Dummies	Dummies	Dummies	Dummies
Age dummy	No	No	Head & Partner	Head & Partner	Head & Partner
Time control	No	No	Yes	Yes	Yes
Labor income decile	No	No	No	Head dummies	Head & family dummies

Table 2.1: Change in fertility from the previous wave, intra-cohort deciles are computed using the head of family unit as a reference, TFP trend is lagged by 5 years. Robust standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1. Source: PSID

	(1) Change in the number of children from the previous wave	(2) Change in the number of children from the previous wave	(3) Change in the number of children from the previous wave
10th decile of wealth × TFP trend 5 years ago	0.0622**		
	(0.0289)		
TFP trend 5 years ago	-0.0282***	-0.0839***	-0.0900***
	(0.00880)	(0.0219)	(0.0219)
Intra-cohort decile × TFP trend 5 years ago		0.00991***	0.0107***
		(0.00328)	(0.00331)
Constant	18.33***	18.38***	18.43***
	(1.571)	(1.571)	(1.570)
Observations	3804	3804	3804
R^2	0.080	0.081	0.078
Wealth decile	Dummies	Dummies	Linear
Time control	Yes	Yes	Yes
Head labor income decile	No	No	Linear
Family income decile	No	No	No

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2.2: Robustness check, different specifications of the model: only 10^{th} decile of wealth or linearity assumption

number of children in the PEU will underestimate the number of children in the family since only those that are financially dependent on the head of household or on the couple count. To get the number of children of interests we can exploit the bunch of other questions on demographics at the end of the survey. In particular question coded as X5910 is the answer to: "*Now I'd like to ask some questions about your family living elsewhere. Altogether, including children from previous marriages and adopted children, how many sons and daughters do you (or your {husband/wife/partner/spouse}) have who do not live with you?*".

Then, the variable "Children" that we will use to make the analysis will be the sum of the this two variables. The main shortfall of this approach is that we may be double-counting some children, indeed if the mother and the father of a children are divorced they will be two different PEU and the children in any case will be counted in both "families". On the other hand, this approach is necessary to extend the available data including in our sample families where the head of household is above 50. Indeed, in this case we are sure that any children even if economically independent will be counted. A different strategy would be to consider the number of children leaving in a family only for the people below a certain age. The main problem is that the life-cycle of people is dependent on their wealth: poorer people have children earlier and they leave the housing unit earlier than their wealthier counterparts. Therefore taking a snapshot of family unit of different wealth in the same moment may be misleading leading to under-estimation of the number of children of the rich if we look young ages or under-estimation of the children born in poorer family if we look older ages.

To address the issue of divorces and the potential increase in the number of children given by the fact that one may have descendants from multiple partners we exploit a specific question on the numer of years spent in previous marriages. We restrict the regression on those PEU in which none of the two partners have spent years in previous marriages.

The effect of TFP at different ages

Ideally one would like to regress the number of children on TFP trend at different ages, nonetheless given that we are using the trend there is an high correlation between contiguous years. In Table 2.3 the correlation between average TFP trend at 5 years age bracket in the subsample are shown. The correlation is above 0.70 for almost all contiguous time spans. Then, putting all spans in the same regression would result in multicollinearity. For this reason we proceed by using one time-span at a time and then we use the average between 20 and 35 years old.

Mean of TFP trend	15-20 years old	21-25	26-30	31-35	36-40
15-20 years old	1				
21-25	0.7626	1			
26-30	0.5571	0.8214	1		
31-35	0.5511	0.4934	0.7960	1	
36-40	-0.0500	-0.0972	0.1971	0.5314	1

Table 2.3: Correlation matrix between TFP trend at different ages of household in the sub-sample

Results

Results with deciles are shown in Table 2.4 for different age breaks. It can be seen that, as expected, the magnitude of coefficients is increasing in the deciles even if the difference between deciles is small, especially at the top. This is probably due to the small sample from which deciles are computed. Indeed, using terciles in Table 2.5 we get statistically significant results and strictly increasing in the tercile of wealth.

Finally, we use average TFP trend during fertility years (between 20 and 40 years old) as treatment variable. Results are shown in Table 2.6. The effect of economic growth is stronger for higher decile when using the set of dummies, results are robust when using the linearity assumption or an ordered probit specification.

	(1) Between 15 and 20 years old	(2) Between 21 and 25 years old	(3) Between 26 and 30 years old	(4) Between 31 and 35 years old	(5) Between 36 and 40 years old
Average TFP trend	-0.417***	-0.397***	-0.153**	-0.0709	-0.270***
	(0.0810)	(0.0617)	(0.0675)	(0.0704)	(0.0657)
2^{nd} decile × average TFP trend	0.143**	0.0102	0.0379	0.114	0.0681
	(0.0584)	(0.0525)	(0.0606)	(0.0711)	(0.0770)
3^{rd} decile × average TFP trend	0.233***	0.118**	0.119*	0.206***	0.182**
	(0.0651)	(0.0538)	(0.0613)	(0.0714)	(0.0761)
4^{th} decile × average TFP trend	0.294***	0.146***	0.180***	0.296***	0.271***
	(0.0694)	(0.0554)	(0.0628)	(0.0729)	(0.0760)
5^{th} decile × average TFP trend	0.262***	0.128**	0.175***	0.321***	0.312***
	(0.0718)	(0.0556)	(0.0626)	(0.0722)	(0.0756)
6^{th} decile × average TFP trend	0.280***	0.171***	0.229***	0.376***	0.421***
	(0.0739)	(0.0563)	(0.0627)	(0.0723)	(0.0759)
7^{th} decile × average TFP trend	0.288***	0.201***	0.254***	0.410***	0.393***
	(0.0741)	(0.0555)	(0.0617)	(0.0707)	(0.0734)
8^{th} decile × average TFP trend	0.347***	0.259***	0.309***	0.478***	0.494***
	(0.0744)	(0.0551)	(0.0609)	(0.0697)	(0.0725)
9^{th} decile × average TFP trend	0.354***	0.278***	0.326***	0.504***	0.551***
	(0.0750)	(0.0550)	(0.0605)	(0.0694)	(0.0730)
10^{th} decile × average TFP trend	0.366***	0.215***	0.246***	0.404***	0.425***
	(0.0761)	(0.0561)	(0.0609)	(0.0694)	(0.0734)
Constant	69.53***	76.47***	45.84***	22.07***	47.78***
	(4.757)	(4.636)	(5.023)	(4.031)	(2.247)
Observations	8387	9479	9479	9479	9479
R^2	0.053	0.050	0.049	0.053	0.052
Wealth decile	Dummies	Dummies	Dummies	Dummies	Dummies
Time control	Yes	Yes	Yes	Yes	Yes
Family income decile	Dummies	Dummies	Dummies	Dummies	Dummies

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2.4: Total number of children, intra-cohort deciles are computed using the head of family unit as a reference, average TFP trend at different ages, source: SCF

	(1) Between 15 and 20 years old	(2) Between 21 and 25 years old	(3) Between 26 and 30 years old	(4) Between 31 and 35 years old	(5) Between 36 and 40 years old
Average TFP trend	-0.222**	-0.433***	-0.135**	0.140**	-0.126**
	(0.103)	(0.0642)	(0.0643)	(0.0661)	(0.0619)
2^{nd} tercile $\times$ average TFP trend	0.187*	0.106*	0.110*	0.226***	0.278***
	(0.107)	(0.0625)	(0.0619)	(0.0678)	(0.0752)
3^{rd} tercile $\times$ average TFP trend	0.139	0.236***	0.225***	0.288***	0.356***
	(0.0990)	(0.0576)	(0.0560)	(0.0616)	(0.0682)
Constant	76.68***	95.65***	64.40***	26.57***	56.80***
	(5.481)	(5.664)	(5.947)	(4.771)	(2.546)
Observations	8544	9673	9673	9673	9673
R^2	0.057	0.057	0.055	0.059	0.057
Wealth tercile	Dummies	Dummies	Dummies	Dummies	Dummies
Time control	Yes	Yes	Yes	Yes	Yes
Family income tercile	Dummies	Dummies	Dummies	Dummies	Dummies

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2.5: Total number of children , intra-cohort terciles are computed using the head of family unit as a reference, average TFP trend at different ages, source: SCF

	(1) Total number of children	(2) Total number of children (Linear specification)	(3) Total number of children (Ordered probit)
Average TFP trend between 20 and 40 years old	0.0822	0.0620	0.0581
	(0.139)	(0.0930)	(0.0943)
2^{nd} decile $\times$ average TFP trend	-0.0765		-0.0376
	(0.157)		(0.106)
3^{rd} decile $\times$ average TFP trend	0.151		0.110
	(0.150)		(0.0999)
4^{th} decile $\times$ average TFP trend	0.221		0.163*
	(0.143)		(0.0959)
5^{th} decile $\times$ average TFP trend	0.412***		0.288***
	(0.140)		(0.0940)
6^{th} decile $\times$ average TFP trend	0.372***		0.263***
	(0.141)		(0.0948)
7^{th} decile $\times$ average TFP trend	0.486***		0.342***
	(0.135)		(0.0911)
8^{th} decile $\times$ average TFP trend	0.546***		0.388***
	(0.132)		(0.0885)
9^{th} decile $\times$ average TFP trend	0.685***		0.500***
	(0.132)		(0.0888)
10^{th} decile $\times$ average TFP trend	0.420***		0.315***
	(0.127)		(0.0862)
Intra-cohort decile $\times$ average TFP trend		0.0651***	
		(0.00834)	
Observations	9318	9318	9318
R^2	0.054	0.049	(0.0147)
Wealth decile	Dummies	Linear	Dummies
Time control	Yes	Yes	Yes
Family income decile	Dummies	Linear	Dummies

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2.6: Total number of children, intra-cohort deciles are computed using the head of family unit as a reference, average TFP trend between 20 and 40 years old, source: SCF

2.5 Quantitative Model

The economic mechanism connecting the fertility decline to wealth inequality that we explored from an empirical point of view in Sections 2.2 and 2.4 can be summarized as follows:

1. Agents experience lower economic growth during their fertility years. Hence they revise their expectations about future growth and decrease fertility.

2. The decrease in fertility leads to an increase in per-capita bequests which in turn translates into higher wealth inequality.

In this section we extend the economic model of endogenous fertility and wealth inequality introduced in Section 2.3 to rationalize the empirical evidence presented in Section 2.4. The first channel suggests that aggregate shocks experienced earlier in life matter significantly for fertility decisions. The quantitative model allows us to measure the impact of aggregate slowdown in TFP happened in the 1970s on the subsequent drop in fertility . In order to capture the second channel, we incorporate a **bequest motive** that connects each generation to the next one, along the lines of Barro and Becker (1989). The reminder of this section is organized as follows: Subsection 2.5.1 lays out the basic economic environment, subsection 2.5.2 presents the individual's maximization problem. We define the equilibrium in 2.5.3. After explaining the calibration procedure in subsection 2.5.4 we presents the quantitative results in subsection 2.5.5.

2.5.1 Economic Environment

We present now the dynastic extension of the simple two-period model of Section 2.3, building on Barro and Becker (1989) and Alvarez (1999). Economic agents lives for two periods: in the first period they are children and in the second period they are adults. All relevant decisions are taken in the second period: when adults, agents enjoy resources given by their parents bequests b and earn labor income $z\omega$. Labor income is given by the product of an aggregate shock z (as in Section 2.3) and an idiosyncratic shock ω. The latter shock captures idiosyncratic risks such as unemployment or health risk against which the agents cannot insure themselves. We explain below how we calibrate the stochastic process for the shocks; for the time being we define the joint distribution of the shocks as $F(z', \omega' \mid z, \omega)$. After observing the shocks agents decide on fertility n. We assume that each parent behaves altruistically and attaches a weight equal to $n^{1-\eta}$ to the children. The cost of raising children is given[16] by a

[16]In the previous version of the code we had $cost(n) = (n)^{1+\gamma}$.

factor that decreases labor income (we interpret it as an opportunity cost):

$$C(n) = \left(\frac{n}{\bar{n}}\right)^{\theta}$$

So that net labor income is given by:

$$z\omega\left[1 - C(n)\right] = z\omega\left[1 - \left(\frac{n}{\bar{n}}\right)^{\theta}\right]$$

The available resources $z\omega\left[1 - C(n)\right] + b(1+r)$ are then spent for consumption c and to leave bequests b' to each of the n children.

Aggregate and idiosyncratic shocks. The stochastic process for TFP is assumed to be given by:

$$\log z_t = \rho_z \log z_{t-1} + u_t,\ u_t \sim N(0, \sigma_z^2) \tag{2.1}$$

The autoregressive process in 2.1 is implemented in a simple way assuming a symmetric two-state Markov chain given by $\log z = [-z, z]$ and

$$\Pi_Z = \begin{bmatrix} \pi & 1-\pi \\ 1-\pi & \pi \end{bmatrix}$$

where z and π are function of (ρ_z, σ_z^2) estimated on the data[17]:

$$z = \sqrt{\frac{\sigma_z^2}{1-\rho_z}} \tag{2.4}$$

$$\pi = \frac{1+\rho_z}{2} \tag{2.5}$$

Regarding the idiosyncratic shock to labor income, ω, we assume that it follows the fol-

[17] To obtain formulas (2.4) and (2.5), simply use the method of moments and equate the variance and the serial correlation in the AR(1) and in the Markov chain:

$$\frac{\sigma^2}{1-\rho^2} = z^2 \tag{2.2}$$

$$\rho\frac{\sigma^2}{1-\rho^2} = (2\pi - 1)\, z^2 \tag{2.3}$$

lowing first-order autoregressive process:

$$\log \omega_t = \rho_\omega \log \omega_{t-1} + v_t,\ v_t \sim N(0, \sigma_\omega^2) \tag{2.6}$$

where ρ_ω is the coefficient on the intergenerational persistence of the shock and σ_ω is the dispersion.

2.5.2 Recursive Formulation

The problem faced by each individual can be written recursively as:

$$\begin{aligned} v(b, \omega, z) &= \max_{b', c, n} \frac{c^{1-\sigma}}{1-\sigma} + \beta n^{1-\eta} \mathbf{E} \{ v(b', \omega', z') | \omega, z \} \\ & \quad s.t. \\ c + nb' &\leq z\omega [1 - C(n)] + b(1+r) \\ b' &\geq 0 \end{aligned} \tag{2.7}$$

where z is the aggregate shock (as in Section 2.3) while ω is the idiosyincratic shock. The non-negativity constraint on desired bequests b' prevents parents from imposing debt obligation on their children and is consistent with the legal framework valid in most developed countries[18].

The model does not admit a closed form solution and will be solved numerically. We can however characterize analytically some properties of the model. To this end, let us derive the first order conditions with respect to the optimal choice of fertility and bequests:

$$n : u'(c) [b' + (1+\gamma) n^\gamma] = \beta (1-\eta) n^{-\eta} \mathbb{E} [v(b', z', \omega') \mid z, \omega] \tag{2.8}$$

$$b' : u'(c) \geq \beta n^{-\eta} \mathbb{E} [v_b(b', z', \omega') \mid z, \omega] \quad \text{with } = \text{ if } b^{'} > 0 \tag{2.9}$$

Negative relation between fertility and savings. Equation (2.9) makes clear that the marginal benefit of savings is decreasing in the number of children n: therefore larger households tend to leave fewer bequests. This property makes our model consistent with the quality-quantity tradeoff theroy originally proposed by Becker.

Fertility-income relation. According to the empirical evidence reported in Jones and Schoonbroodt (2010), there is a negative relationship between fertility and parents' earnings. Our model is consistent with such evidence provided that the parameter σ (the inverse of the

[18]Notice that such constraint breaks down the well-known equivalence result between OLG model with altruistic parents and the infinitely-lived agent model. See for example Blanchard and Fischer (1989), ch.3.

intertemporal elasticity of substitution) is low enough[19].

2.5.3 Equilibrium

In this section we provide a formal definition of the equilibrium and explain how to compute the distribution of heterogeneous households in the model.

Definition of recursive competitive equilibrium. *Given*[20] *interest rate r, a recursive competitive equilibrium is given by value and policy functions $v(b,\omega,z)$, $g_b(b,\omega,z)$, $g_n(b,\omega,z)$, $g_c(b,\omega,z)$ and an aggregate distribution $\lambda(b,\omega,z)$ such that:*

1. *The policy functions $b' = g_b(b,\omega,z)$ for bequests, $n = g_n(b,\omega,z)$ for fertility and $c = g_c(b,\omega,z)$ for consumption solve the individual problem defined in (2.7).*

2. *The aggregate distribution $\lambda(b,\omega,z)$ is induced by the exogenous stochastic processes for idiosyncratic and aggregate risk, summarized by $F(\omega',z' \mid \omega,z)$, as well as the optimal policy functions for bequests and fertility.*

We now give the explicit statement of (2) in the following. The Markov transition and the policy functions induce a transition equation for the distribution (so that the stationary distribution is simply the fixed point of such transition equation) given by:

$$\lambda_{t+1}(b',\omega',z') = \Pr(b',\omega',z') = \sum_b \sum_\omega \sum_z \Pr(b',\omega',z' \mid b,\omega,z)\,\lambda_t(b,\omega,z)$$

where:

$$\Pr(b',\omega',z' \mid b,\omega,z) = \begin{cases} F(\omega',z' \mid \omega,z)\,\frac{g_n(b,\omega,z)}{N} & \text{if } g_b(b,\omega,z) = b' \\ 0 & \text{otherwise} \end{cases} \tag{2.10}$$

and:

$$N = \sum_b \sum_\omega \sum_z g_n(b,\omega,z)\,\lambda_t(b,\omega,z)$$

So the element:

$$\frac{g_n(b,\omega)}{N}$$

[19]It is not possible to derive analytical results but this is confirmed in the numerical simulations.
[20]The reader should bear in mind that the model is laid out in partial equilibrium.

measures the fertility of the specific fraction of the population over the overall fertility of the population.

2.5.4 Calibration

We calibrate the model to match a number of cross-sectional targets related to fertility, income and wealth for the 1960-2010 U.S. economy. A detailed discussion of the targets and the identification strategy follows. See Table 2.8 for a summary.

	$\widehat{\rho}$	$\widehat{\sigma}$
TFP Fernald, from 1947 (trend, HP Filter $\lambda = 20$)	0.9893 (0.0245)	0.1548 (0.0186)
TFP Fernald adjusted for capacity utilitization, from 1947 (trend, HP Filter $\lambda = 20$)	0.9902 (0.0214)	0.1616 (0.0143)
TFP Chari et al. (2007) methodology, from 1900 (trend, HP Filter $\lambda = 20$)	0.9879 (0.0130)	0.0039 (0.0002)

Table 2.7: TFP process, different estimations

Stochastic process for aggregate productivity z. The concept of productivity in the model corresponds to aggregate TFP in the data. Following Fernald (2012), we estimate an AR(1) process on the yearly series of total factor productivity for US economy in the period 1960-2010. In particular we estimate the following equation:

$$\Delta TFP_t = \alpha + \rho \Delta TFP_{t-1} + \varepsilon_t$$

where $\varepsilon_t \sim \mathcal{N}(0, \sigma^2)$. We report the results in Table 2.7.

However, since the duration of a period in the model corresponds to 20 years in the data, we convert the yearly estimates as follows. Let $\tilde{\rho}$ and $\tilde{\sigma}^2$ be the yearly estimates; then the corresponding 20-years values $\hat{\rho}$ and $\hat{\sigma}^2$ are:

$$\hat{\rho} = \tilde{\rho}^{20}$$

$$\hat{\sigma}^2 = \left(\sum_{j=0}^{19} \hat{\rho}^{2j} \right) \tilde{\sigma}^2$$

The child discount factor is modelled as $\beta n^{1-\alpha}$, following Barro-Becker and Alvarez. We prefer this formulation to the exponential formulation of Cordoba et al. (2016) due to its simplicity. The pure discount factor β is chosen to match the earnings-income correlation whereas the elasticity parameter α targets the income elasticity of fertility. Of course in equilibrium all parameters affects all targets but for example β affects significantly the propensity to save therefore it has a strong impact on the correlation between income (given by earnings plus rb) and earnings.

The cost of raising children in the model takes the form $C(n) = \left(\frac{n}{\overline{n}}\right)^{\theta}$. The parameters θ and $\overline{n}$ are calibrated jointly with the others but in particular θ is meant to capture the cross-sectional dispersion in fertility choices whereas $\overline{n}$ targets mostly the average fertility level. We take data on fertility from Jones and Tertilt (2008). The paper reports an average fertility per household equal to 2.00. However in our model the relevant concept is fertility per person, therefore we set it equal to 1.00

Table 2.8: Parameters

Parameter	**Value**	**Target**	**Data**	**Model**
r interest rate	2	Annual interest rate of 4.5%	2.00	2.00
σ curvature utility	0.72	Gini bequests	0.82	0.80
β discount factor	0.25	Corr(earnings,income)	0.84	0.67
α child discount elasticity	0.57	Income elasticity of fertility	-0.20	-0.26
θ cost of raising children	1	Coeff. of variation fertility	0.60	1.75
$\overline{n}$ cost of raising children	10	Average fertility	1.00	1.13
ρ_ω persistence ability shock	0.50	Persistence wages	0.50	0.50
σ_ω dispersion ability shock	0.85	Gini earnings	0.64	0.63
ρ_z persistence TFP shock	0.82	Annual persistence (Fernald)	0.99	0.99
σ_z dispersion TFP shock	2.75	Annual volatility (Fernald)	0.16	0.16

Note: The child discounting takes the form $\beta n^{1-\alpha}$ and the cost of raising children is $(1 - \frac{n}{\bar{n}})^{\theta}$

The baseline calibration is summarized in Table (2.8). Overall our model does a good job of capturing the salient features of the fertility-income-wealth distribution. Given the features matched in the calibration, we also analyse how well the model does on other dimensions that were not explicitely targeted. Tables (2.9) and (2.10) report these overidentifying tests. We find this encouraging as it shows that the model provides an appropriate framework to study the macroeconomic implications of changes in agents' expectations about growth on fertility and, hence, wealth.

Table 2.9: Calibrated moments

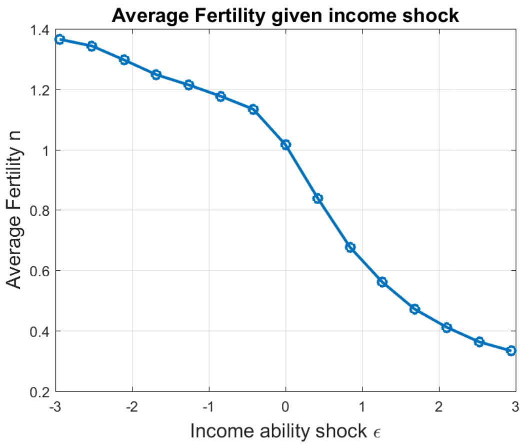
Moments	**Data**	**Model**
Gini of Wealth	**0.82**	**0.80**
Gini of Income	0.58	0.57
Gini of Earnings	0.60	0.59
Gini of Consumption	0.32	0.56

Note: Calibration targets are in boldface.

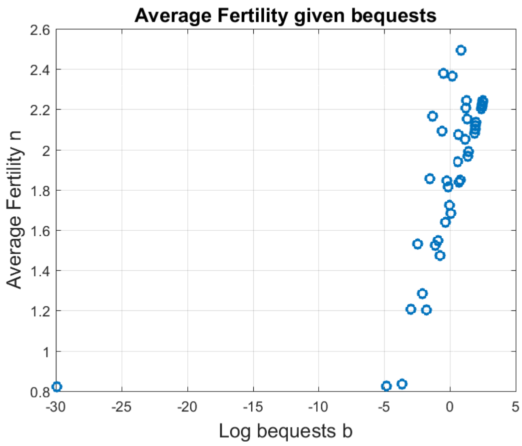
Table 2.10: Calibrated moments

Bequests	**Gini**	**Bottom 40%**	Top 20%	Top 10%	Top 1%
U.S. DATA	**0.82**	0.00	0.91	0.65	35.3
MODEL	**0.80**	0.00	0.83	0.64	0.19

Note: This Table compares the distribution of bequests between the U.S. data and the model. It indicates the Gini coefficient and the share of wealth owned by bottom and top percentiles of the U.S. population.



(a) Fertility vs income



(b) Fertility vs bequests

Figure 2.9: Optimal choice for fertility

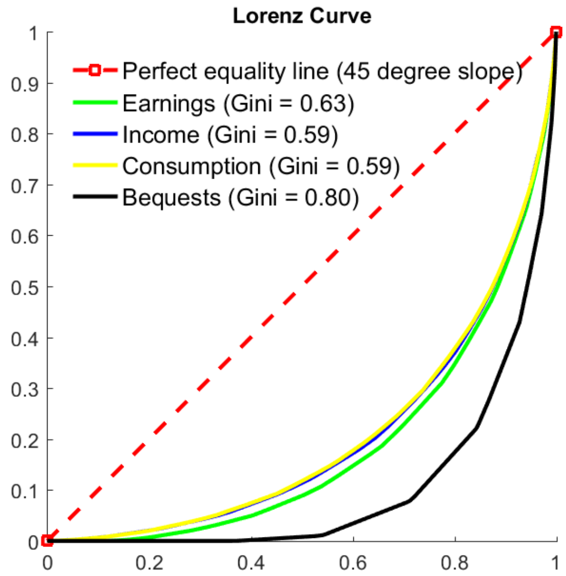


Figure 2.10: Inequality in the model

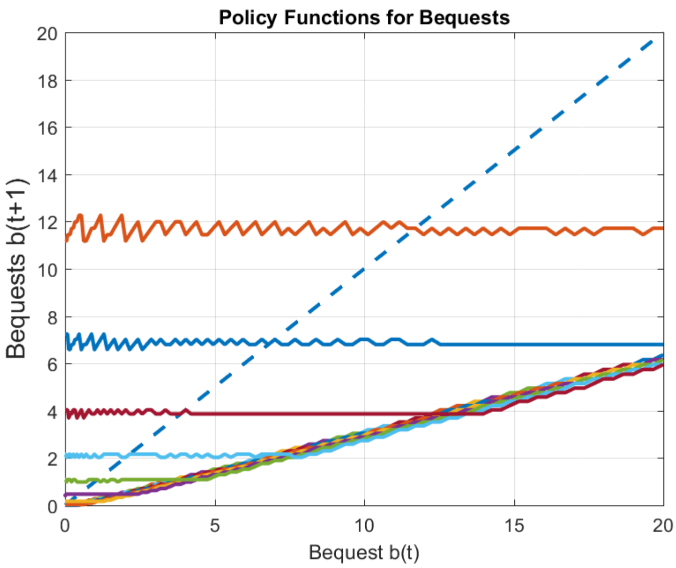


Figure 2.11: Bequests policy functions for different idiosyncratic shocks

2.5.5 Results

In order to highlight the importance of beliefs about future growth for shaping fertility and wealth accumulation decisions, we perform two main quantitative experiments. In the last section we further stress the role of beliefs by departing from fully rational expectations and introducing overpersistence bias (see Section 2.5.5).

Experiment 1

First we we simulate the model under two scenarios: *good* aggregate state vs *bad* aggregate state (this should capture the difference in the long-run distribution between periods of high and low economic growth).

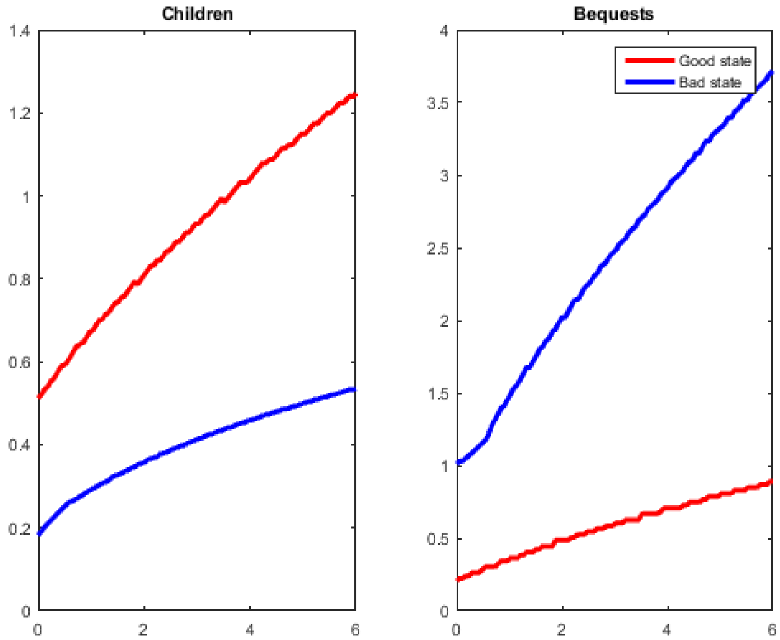


Figure 2.12: Policy functions for children and *per capita* bequests

We report the main findings in Figures (2.12)-(2.15). Figure (2.12) shows the effect of aggregate economic conditions (*good* vs *bad* times) on fertility choices and wealth accumulation through bequests. In particular, the left panel depicts the optimal choice for fertility (number of children) as a function of inherited wealth, given the two possible realizations of the aggregate shock[21]. The right panel of the same figure shows instead the optimal level of

[21] More precisely, we averaged out the idiosyncratic shocks.

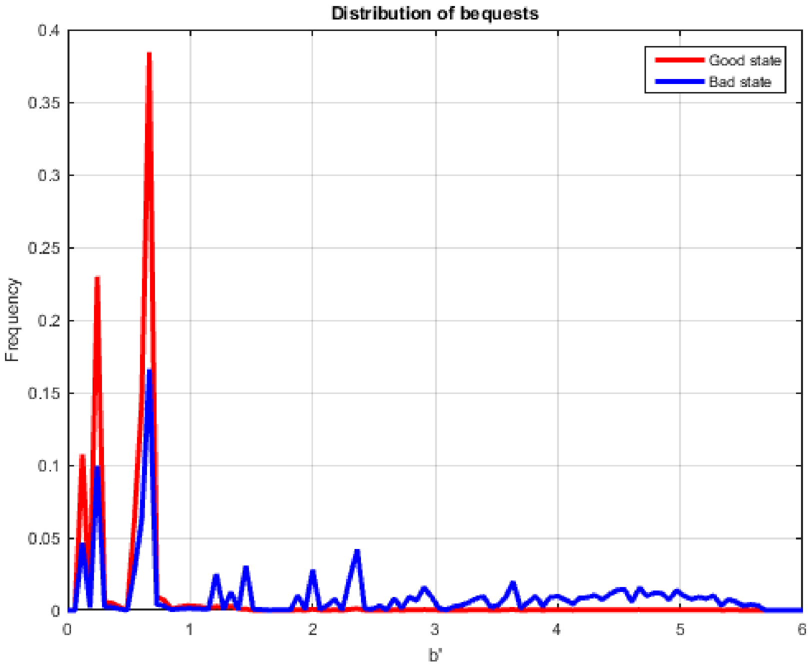


Figure 2.13: Bequests PDF good vs. bad state

bequests. The following pattern emerges:

First of all, bequests are higher in the bad state rather than in the good one for all levels of inherited wealth but the spread is higher the higher the level of initial wealth. Second, agents reduce fertility in the bad state. The figure illustrates nicely the main mechanism described in the introduction: when aggregate conditions are bad (and the agents overestimate the persistence of bad times) the negative perspective on future growth induces agents to leave a higher amount of bequests to a fewer number of children. Lower economic growth decreases the return of the ”children” asset inducing households to accumulate more bequests due to a precautionary motive.

Figure (2.13) and (2.14) provides a comparison of the distribution of bequests in the model conditional on the two realizations of aggregate TFP: consistently with the findings reported in Figure (2.12) in the stationary distribution[22] there is a higher concentration of mass among high levels of bequests when the state of the economy is bad. Under good economic conditions instead, agents never accumulate wealth higher than a certain threshold since the precautionary saving incentives are much less strong.

[22]Bear in mind that the stationary distribution is adjusted by the level of fertility of agents in each state, accordingly to equation (2.10).

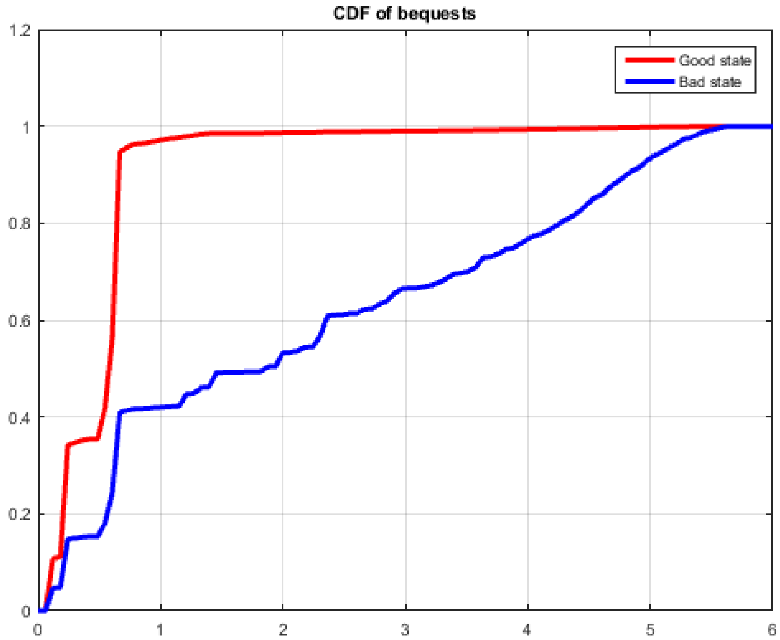


Figure 2.14: Bequests CDF good vs bad state

Finally Figure ((2.15) reports the Lorenz curve of bequests contrasting good versus bad times and shows that inequality is higher in the bad state than in the good state.

Inequality	z_H	z_L
Gini of income	0.63	0.63
Gini of wealth	0.60	0.85
Var of log income	0.21	0.21
Var of log wealth	0.44	1.26
P90/P50 of income	4.40	4.40
P90/P50 of wealth	2.66	18.14
P10/P50 of income	0.16	0.16
P10/P50 of wealth	0.01	0.00

Experiment 2

In the experiment described above we highlighted the role of aggregate shocks in shaping the incentives for fertility decisions and accumulation of wealth, showing that the mechanism we

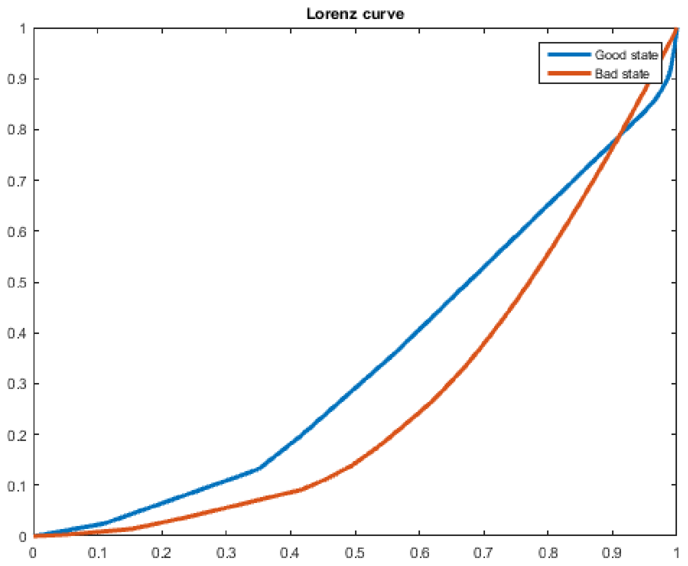


Figure 2.15: Lorenz curve bequests good vs bad state

propose is *qualitatively* relevant. In the present subsection we instead ask the model to answer a specific *quantitative* question: how much of the variation in fertility and wealth concentration that occurred in the US economy between 1960 and 2010 can be explained by the fall in aggregate TFP growth, and does this mechanism rely on whether agents overestimate changes in TFP. The third column of Table (2.11) answers the first part of question, showing the effect of changes in growth on fertility and wealth concentration within a standard mechanism of beliefs. The last column reports instead the results obtained by assuming the agents in the model hold biased beliefs along the lines described in section (2.5).

The TFP growth rate in the US economy averaged at 1.93% in the 1960-1970 decade, slowing down to 1.23% in the 2000-2010 decade. We capture such difference in TFP growth rates by calibrating the two-state Markov process for the aggregate shock accordingly. Furthermore the total fertility rate (adjusted by child mortality) fell by almost one-third, from 2.91 children per family down to 2.03, and the wealth concentration, measured as the share of wealth owned by the top 1%, sharply increased by around 10 percentage points (see the first column of Table 2.11 for additional details). Our model generates a fall in fertility rate after the negative shock to TFP growth rate that is roughly comparable to the decline found in the data: in particular the model generates a decrease in fertility of 24% whereas in the data the shrinkage is of 30%. The ability of the model to capture most of the variation in fertility hinges crucially on the income elasticity of fertility, which is one of the main calibration targets. Under the assumption of biased beliefs the change in fertility is somehow more pronounced, given that parents now overestimate the persistence of the negative shock. Turning our attention to wealth inequality, we observe first that our preferred measure of wealth inequality, namely the share of wealth owned by the richest 1% in the population[23], jumped up from 27% in the 1960s to 33% in the 2010s. Our model is then able to account for 40% of such increase in the baseline version (45% in the version with overpersistence bias). The main mechanism goes through an increase in per-capita bequests. Two effects play a role here: first all agents reduce their desired number of children and increase their bequests following a negative shock to economic growth. Second, the income elasticity of fertility is higher for wealthier families: therefore the increase in per-capita bequest is stronger among wealthy households, further exacerbating the increase in wealth inequality.

We conclude observing that the fertility channel explored in this experiment a significant part of the increase in wealth inequality. However more than half is left unexplained. Other factors such as changes in tax policies and the increase in labor earnings inequality are likely to play a role.

Experiment 3: Overpersistence bias

The first channel suggests that agents hold wrong beliefs about future economic growth, overestimating the persistence of aggregate income shocks. In order to capture this, our model features a **small departure from the rational expectation** hypothesis, that explains why transitory shocks can have long-lasting effects on fertility and demography

Overpersistence Bias. The idea that a negative shock experienced during the impressionable years affects disproportionately your future expectations can be rationalized with the following small deviation from rational expectations: agents overestimate the persistence of the aggregate income process. When they are hit by a negative shock they think such shock will last more that it actually does.

Formally, agents believe that the aggregate TFP shock is given by

$$\log z_t = \widehat{\rho_z} \log z_{t-1} + u_t, \;\; u_t \sim N\left(0, \sigma_z^2\right)$$

[23] We chose this statistic for two main reasons. First it has been popularized by Piketty (2014) and features prominently in the policy debate, and, second, data on other measures (such as the Gini coefficient of wealth distribution) are not available earlier than 1989.

whereas the true process is given by

$$\log z_t = \rho_z \log z_{t-1} + u_t, \quad u_t \sim N\left(0, \sigma_z^2\right)$$

with $\widehat{\rho_z} > \rho_z$. (They have correct beliefs about the variance σ_z^2). We implement these non-rational expectations in a simple way assuming that the TFP process is governed by a symmetric[24] two-state Markov chain given by $log(z) = [-z, z]$ and

$$\Pi_Z = \begin{bmatrix} \pi & 1-\pi \\ 1-\pi & \pi \end{bmatrix}$$

where z and π are given by $\left(\rho_z, \sigma_z^2\right)$ estimated on data[25]:

$$z = \sqrt{\frac{\sigma_z^2}{1-\rho_z}} \tag{2.11}$$

$$\pi = \frac{1+\rho_z}{2} \tag{2.12}$$

However agents have incorrect beliefs and think that the transition matrix is given by:

$$\widehat{\Pi}_Z = \begin{bmatrix} \widehat{\pi} & 1-\widehat{\pi} \\ 1-\widehat{\pi} & \widehat{\pi} \end{bmatrix}$$

where $\widehat{\pi} > \pi$. In the quantitative section we will show that the presence of biased beliefs improves the ability of the model to explain the data with respect to the baseline with rational expectations.

Regarding the numerical implementation we follow the procedure outlined in Rozsypal and Schlafmann (2017) with minimal modifications.

[24]A *symmetric* Markov chain implies however that the duration of the two states (high and low) is the same, which is not a good assmption for recessions and booms.

[25]To obtain formulas (2.11) and (2.12), simply use the method of moments and equate the variance and the serial correlation in the AR(1) and in the Markov chain:

$$\frac{\sigma^2}{1-\rho^2} = z^2$$

$$\rho\frac{\sigma^2}{1-\rho^2} = (2\pi - 1)\, z^2$$

Table 2.11: How a fall in growth expectations affects fertility and wealth concentration: Comparing the rational expectations model to the biased model

	U.S. data	**Rational Expectations**	**Biased Beliefs**
Delta TFP	-0.69%	-0.69%	-0.69%
Delta Fertility	-30.28%	-24.22%	-25.74%
Delta Wealth concentration	$+6.00\%$	$+2.40\%$	$+2.70\%$

Note: The source for US data on TFP is Fernald (2012). Here we use data on TFP corrected by capacity utilization. The data on fertility are adjusted by mortality rate and taken from Roser (2017a,b). The data on wealth concentration are taken from Rios-Rull and Kuhn (2016) and from the World Income Database (Alvaredo et al. 2017) . Wealth concentration is measured here as the top 1 percent share. See the appendix for alternative measures of wealth inequality.

2.6 Conclusions

The findings of our paper emphasize the role of decreasing fertility as a relevant channel for higher wealth inequality in the U.S. economy over the last 50 years.

Using the Barro-Becker model of fertility in a setup with aggregate and idiosyncratic shocks with incomplete markets we show that the slowdown in total factor productivity growth determines a fall of fertility by worsening parents' expectations about the future. This in turn leads to an increase in per-capita bequests for all households thus determining an increase in wealth inequality. We found empirical evidence of the phenomenon both in aggregate data on fertility and in micro-data on wealth from SCF and PSID. With the help of an extendive quantitative version of the baseline model, we are able to explain roughly 40 percent of the increase in welath inequality observed in the US data. The ability of the model to fit the increase in wealth inequality is somewhat enhanced when we relax the rational expectations hypothesis and we assume agents overestimate shocks persistence.

Our model provides a new perspective to evaluate a number of public policies that are used in many developed countries to incentivise fertility. Indeed our work points out an additional cost related to the decline in fertility that has taken place in many developed countries: lower fertility leads to a higher concentration of wealth through the accumulation of inheritances. However it should be considered as well that child-care subsidies, tax incentives and all other kind of fertility-related public policies typically stimulate fertility among poor families. To the extent that such policies increase the fertility differential between the rich and the poor, the effect on wealth inequality is ambiguous and could potentially go in the opposite direction.

A final caveat is in order. The model used in the quantitative analysis is set out in partial equilibrium for simplicity. We are aware that by doing so we neglect the impact of bequests accumulation on the interest rate. We leave the general equilibrium extension for future research.

2.7 Appendices

2.7.1 Appendix A: Additional evidence on the change in intra-cohort inequality

The increase in wealth inequality among households of all ages is robust to the exclusion of households with negative net worth from the sample. Figure 2.16, together with the standard Gini coefficient, contains the evolution of Gini coefficient computed excluding households with negative net worth. The Gini index by construction takes value between 0 and 1 when only null or positive values are used. If it is equal to 0 everyone has exactly the same amount of wealth, if it is equal to 1 then only one households owns the all wealth of the economy. When households with negative net worth are included the index can take also values bigger than one and thus it has a less obvious interpretation, for this reason sometimes they are excluded from the computation.

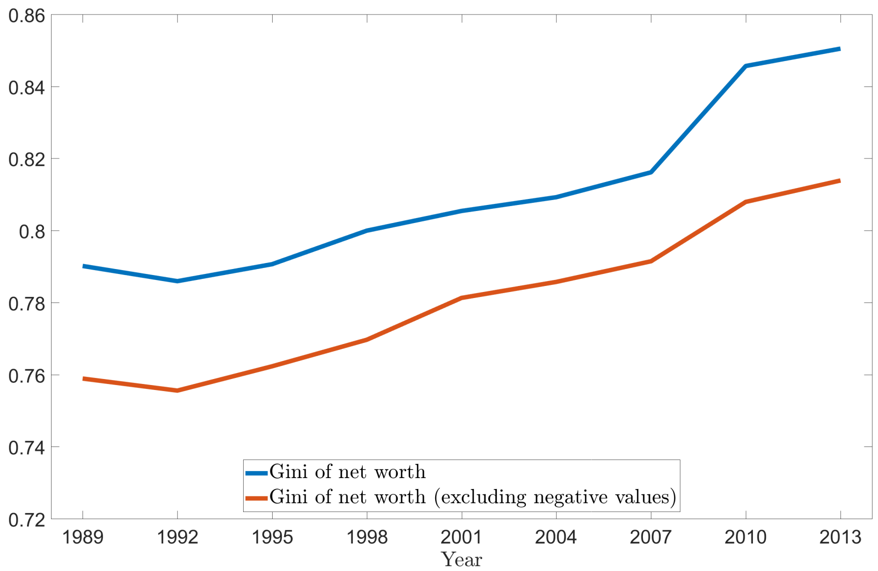


Figure 2.16: Gini coefficient of net wealth in the United States (including and excluding negative values), source: Survey of Consumer Finances.

The increase in wealth inequality in youngest cohorts is robust to different measures of inequality. In Figure 2.17 is plotted the 90-50 interpercentileand we can see that there has been a substantial increase in the last 20 years, despite the fall in 2013 after the Great Recession the index for under-35 is around 15 and it has almost doubled since 1995. The interpercentile at the bottom (i.e. the p50-p10 ratio) displayed in Figure 2.18 displays a similar pattern. We also consider the share of wealth owned by the richest 1 percent as a measure of wealth inequality. Figure 2.19 plots the evolution of the top 1 percent share over time for the US economy, whereas Figure 2.20 decomposes this statistic among cohorts with different ages.

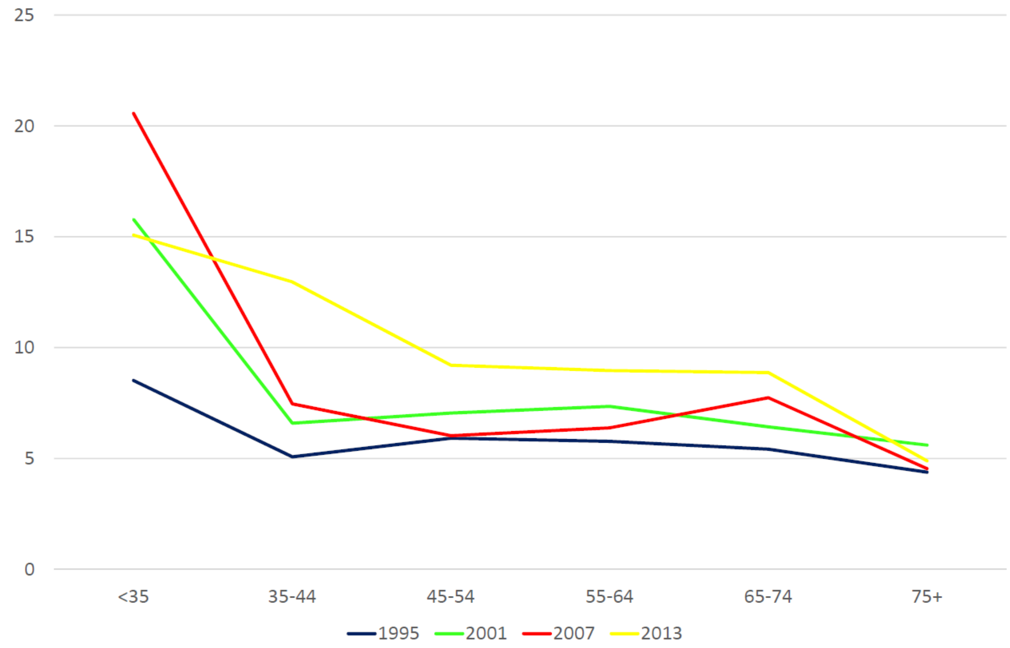


Figure 2.17: Intra-cohort interpercentile p90/p50 across years, source: Survey of Consumer Finances, source: Survey of Consumer Finances

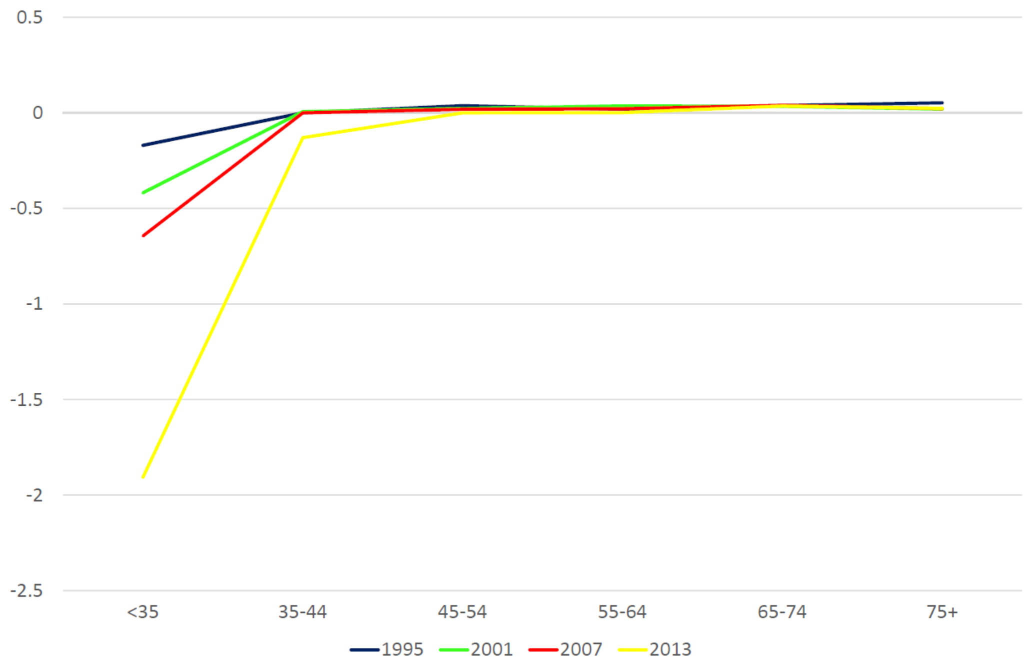


Figure 2.18: Intra-cohort interpercentile p10/p50 across years, source: Survey of Consumer Finances

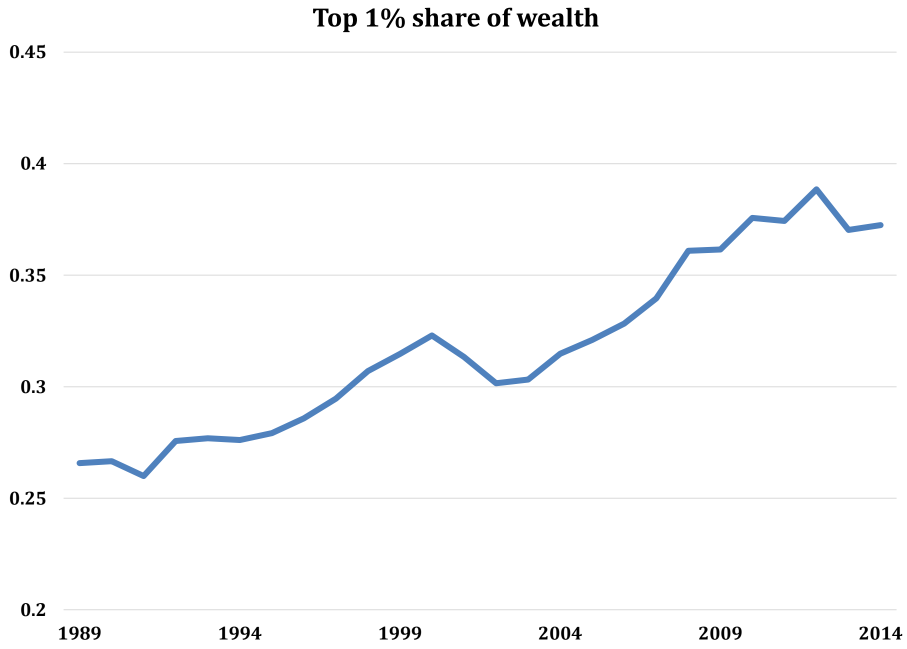


Figure 2.19: Share of wealth owned by the richest 1 percent, source: Survey of Consumer Finances

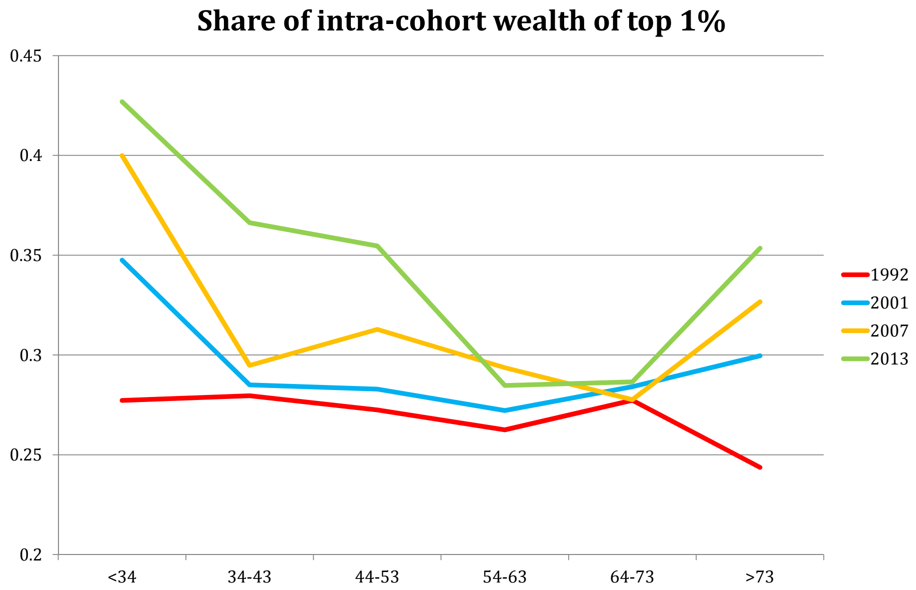


Figure 2.20: Intra-cohort share of wealth owned by the richest 1 percent, source: Survey of Consumer Finances

2.7.2 Appendix B: Barro-Becker model and its calibration

The household preferences for fertility are represented using the model proposed by Barro and Becker (1989), the model of endogenous fertility most used in the literature. In this section I will explain what are the main features of the model and make references to other papers that have analyzed is fit to aggregate data.

In the Barro-Becker world each cohort lives one period: decides consumption, savings (bequests) and fertility and then dies. Consider a simple case with an endowment economy. The value function associated to an agent of today generation is the following:

$$\begin{aligned} V(\omega, b) &= \max_{b', c, n' \geq 0} u(c) + n'a(n')\mathbb{E}\left[V(\omega', b')\right] \\ & s.t. \\ & c + n'b' + B(n') = \omega + b(1+r) \end{aligned}$$

Where $u(c)$ is a standard utility function and measures the utility from her own consumption, n' is the number of children that today cohort decides to have,$\mathbb{E}\left[V(\omega', b')\right]$ is the expected utility of each children (that enters the problem since agents are altruistic), $n'a(n')$ is the discount factor on which I will focus later on, $B(n')$ is a cost-function from having children, r is the real interest rate that the new generation gets from the endowment left from the past cohort and ω is the endowment of each agent of today cohort. The endogenous state variables are b and n'.

The discount factor $n'a(n')$ must have the following properties:

- An additional children, *ceteris paribus*, increases the utility of the parents;
- Having children increases utility at a marginally decreasing rate.

If we use CRRA functional form for utility from consumption good, i.e. $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ and an exponential form for the discount factor, i.e. $a(n') = \alpha(n')^{-\eta}$ as it has been deeply discussed in Jones and Schoonbroodt (2010) there are two alternatives calibration of σ and η that guarantees that the modelization satisfies the two assumptions above and the concavity of the value function:

- Assume that $u(c) \geq 0 \; \forall c \geq 0$, that $u(\cdot)$ is strictly increasing and strictly concave, and that $0 < 1 - \eta < 1$. Under this calibration the value function is increasing and concave in c and n if and only if $0 < 1 - \sigma \leq 1 - \eta < 1$

- Assume that $u(c) \leq 0 \; \forall c \geq 0$, that $u(\cdot)$ is strictly increasing and strictly concave, and that $1-\eta<0$. Under this calibration the value function is increasing and concave in c and n if and only if $0>1-\sigma \geq 1-\eta$

While the first calibration is the most used in the fertility literature the second calibration is in line with macro-literature calibration of the intertemporal elasticity of substitution (IES¡1) and implies the negative relationship between income and fertility that we have in data. Indeed the two calibrations have also different implications on the relationship between children and consumption goods: if $0<1-\sigma \leq 1-\eta<1$ the consumption goods and children are complements on the other hand if $0>1-\sigma \geq 1-\eta$ they are substitutes. This implies that if children have a cost in terms of time the higher the wage the lower the number of children that the household will have but it will guarantee to each of them an higher level of consumption (consistent with both individual and aggregate data on fertility[26]).

Inter-temporal trade off

The main difference of this model with respect to the standard model with a representative agent comes from the inter-temporal trade off. Usually the Euler-equation relates the marginal utility of consumption today with the marginal utility of consuption tomorrow, in this setup the inter-temporal optimality condition relates the marginal utility from consumption today with the marginal utility of the next generation and the marginal utility of having children. Therefore the number of children affects the discount factor making the interpretation of the Euler-equation less intuitive since children are, from a certain point of view, also "consumption" of the current generation (that indeed have children for having an higher utility).

For this reason the next subsections will try to shed a light on the mechanism of the model and on the effects that different parameters have on the optimal choice of children and bequests.

[26]At the individual level fixing a country and a year households with higher labor income have less children, at the aggregate data comparing country with different levels of income or a country across years we get the same result. The full discussion with data is in Jones and Schoonbrodt (2010)

2.7.3 Appendix C: Comparative statics of the two period model

A comparative statics exercise in order to fully understand the role of different parameters in driving the results.

- $0.01 \leq \alpha \leq 0.6$: an increase in α, *ceteris paribus*, increases the discount factor and therefore the weight that the agent gives to the next generation. Therefore, when α increases the agent reduces consumption today ($C \downarrow$) and increases both bequests and children ($N \uparrow$, $B' \uparrow$). Clearly also per-capita bequests increase.

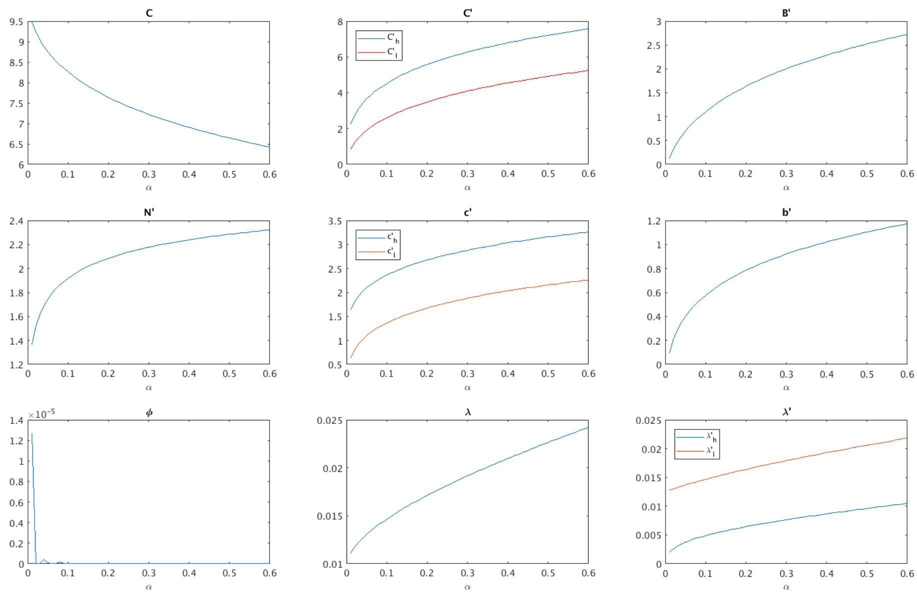


Figure 2.21: Comparative statics on α

- $1.5 \leq \sigma = \eta \leq 3.5$[27]: an increase in σ determines a reduction in intertemporal elasticity and a decrease in the risk aversion. In this setup children are the risky investment (their endowment depends on the state of the world tomorrow) while bequests are the safe asset, therefore an increase in σ determines a switch from children to bequests, leading to an increase in per-capita bequests.

[27] A calibration with $\eta < \sigma$ does not guarantee the existence of a solution, for this reason the comprative static on σ is conducted adjusting η in any step.

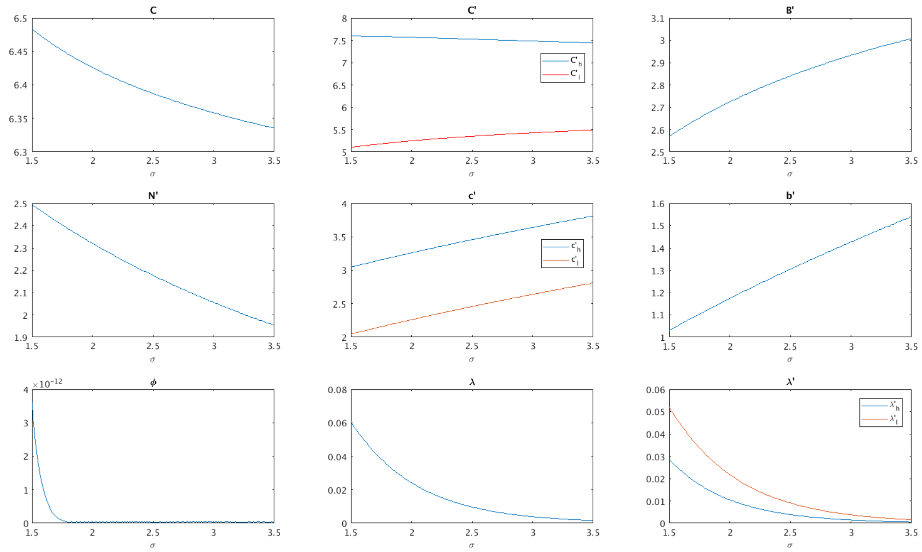


Figure 2.22: Comparative statics on σ

- $2 \leq \eta \leq 3.5$[28] determines the degree of concavity of the children component in the discount factor, when $\eta = \sigma$ the relative importance of the quality of children is maximal and it decreases when η increases. As a result an increase in η determines a reduction of the amount of bequests and an increase in the number of children and a decrease in consumption. Here there are two effects: on the one hand the quantity becomes more important than quality and therefore the agent increases the number of children as much as possible at the expenses of current consumption and bequests, at a certain point the increase in η determines also a decrease in the discount factor. For this reason the agent starts decreasing also children and slightly increase consumption. So the effect of η on consumption, children and bequests is not monotonic, notice that also the multiplier corresponding to the amount of bequests decreases.

- $1.1 \leq \theta \leq 2$: this affects the degree of convexity of the time cost of children. The higher the degree of convexity of the children cost function, the higher the number of children that the agents has.

[28] In this case I let $\sigma = 2$, this allow to study the separate effect of the parameter η

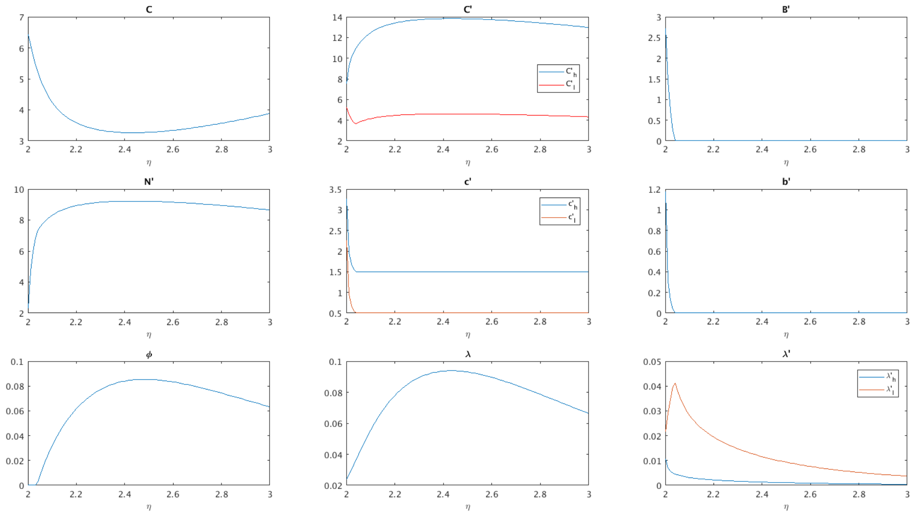


Figure 2.23: Comparative statics on η

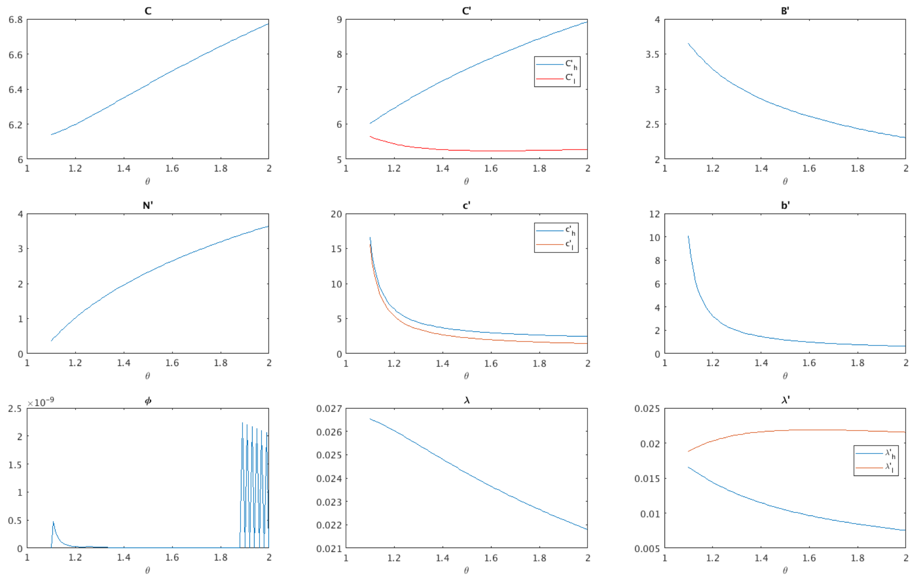


Figure 2.24: Comparative statics on θ

2.7.4 Appendix D: Aggregate evidence

TFR and child mortality: historical data sources

All the historical sources used to reconstruct the TFR of the United States are listed in Table 2.12. For some periods it has been reconstructed using the crude birth rate (CBR) and life tables provided by the Statistical office of the US.

Years	Source of TFR	Source of CBR	Frequency
1900	Computed from CBR	Statistical office of the US	1 year
1903-1908	Chesnais (1992)	(Statistical office of the US)	5 years
1909-1912	Computed from CBR	Statistical office of the US	1 year
1913	Chesnais (1992)	(Statistical office of the US)	1 year
1914-1916	Computed from CBR	Statistical office of the US	1 year
1917-1939	Heuser	(Statistical office of the US)	1 year
1940-1949	Statistical office of the US	(Statistical office of the US)	1 year
1950-2010	HFD (2013)	(UN Pop.)	1 year

Table 2.12: Data sources for fertility in the United States.

Data sources for child mortality in the United States, with which the TFR is adjusted, are listed in Table 2.13.

Years	Source	Frequency
1900-1923	Model based on Life Expectancy	1 year
1924-1932	-	-
1933-1949	HMD	1 year
1950-2010	CME (2014)	1 year

Table 2.13: Data sources for child mortality in the United States.

Sources for UK data on TFR are listed in Table 2.14 (together with the corresponding CBR source), those for child mortality are listed in Table 2.15.

Years	Source of TFR	Source of CBR	Frequency
1800-1871	Computed from CBR	Wrigley and Schofield (1989)	1 year
1873-1908	Chesnais (1992)	(Chesnais (1992))	5 years
1911-1949	Festy (1979)	-	1 year
1950-2010	UN Population Division (2013)	(UN Population Division (2008))	1 year

Table 2.14: Data sources for fertility in United Kingdom.

Years	Source	Frequency
1800-1831	Model based on Life Expectancy	1 year
1832-1840	-	-
1841-1921	HMD England	1 year
1922-1949	HMD	1 year
1950-2010	CME (2014)	1 year

Table 2.15: Data sources for child mortality in United Kingdom.

Trends and cycles in fertility and in TFP

Using the trend-cycle decomposition for TFP and TFR we get four different series: TFP cycle and TFP trend, TFR cycle and TFP trend. The analysis of the correlation among the two of interests have been already discussed in Section 2.2. For completeness we plot all the possible combinations of the series and their correlogram. Results for the United States care plotted in Figure 2.25 and those for the United Kingdom are plotted in Figure 2.26.

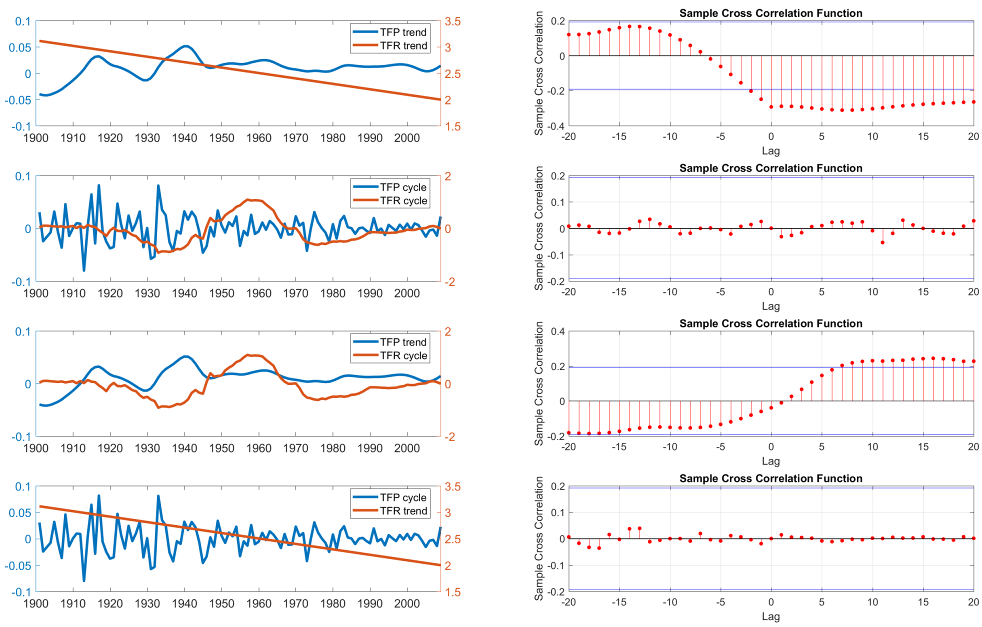


Figure 2.25: Fertility and TFP trend and cycle decomposition for the United States

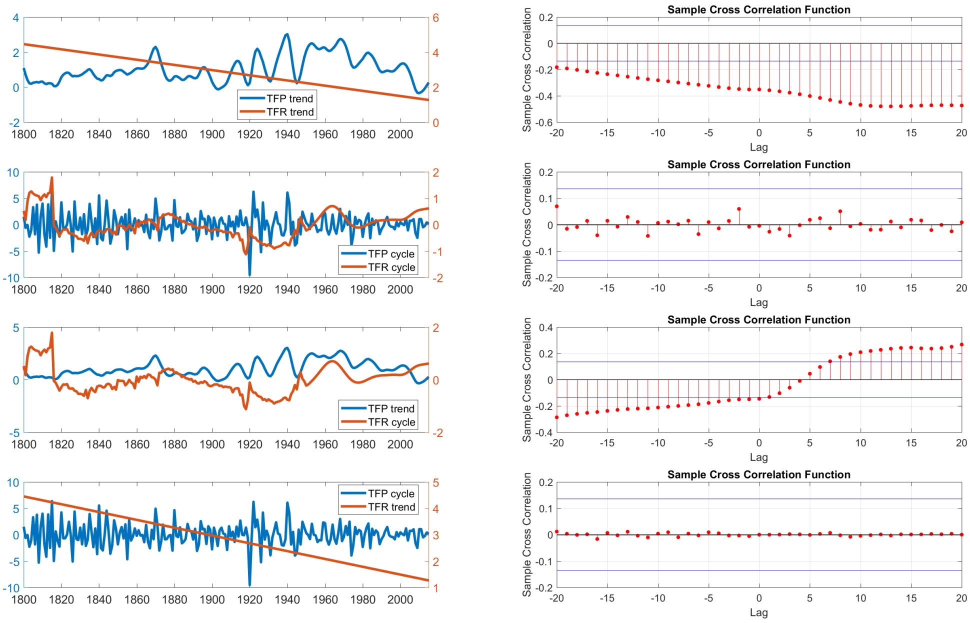


Figure 2.26: Fertility and TFP trend and cycle decomposition for the United Kingdom

Different time spans

As an additional check we perform the same experiment on different time spans. Firstly, in Figure 2.28 we plotted the TFP trend and TFR cycle for the United States after 1920 and 1950. Results are plotted in Figure 2.27 for the post-1920 period and in Figure 2.28 for the post-1950 period (that excludes the II World War period which may be problematic for several reasons). The correlation between the two series is similar to the one observed in subsection 2.2.3 and it only differs the lag at which they are maximally correlated: the recent period of the series is correlated at lower lags.

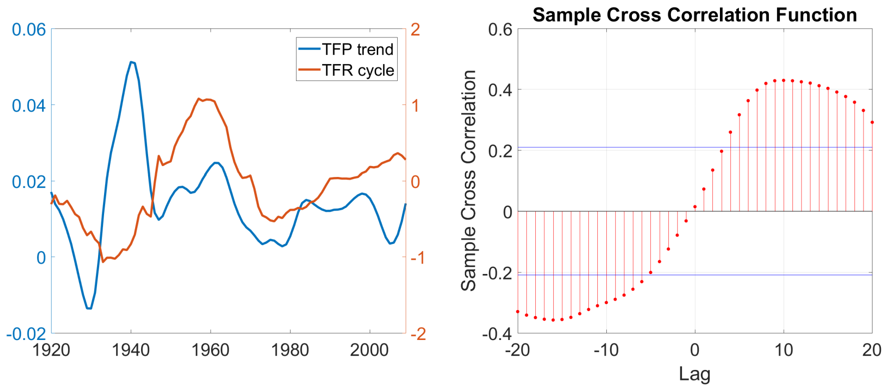


Figure 2.27: United States, post 1920 data

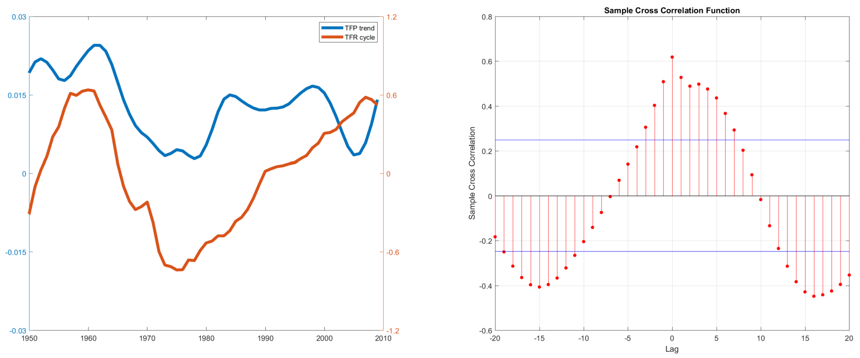


Figure 2.28: United States, post 1950 data

In Figure 2.29 we plot the series and the crosscorrelogram for UK starting from 1900, therefore discarding $19th$ century data on TFP that maybe more problematic. Results are similar to those previously found.

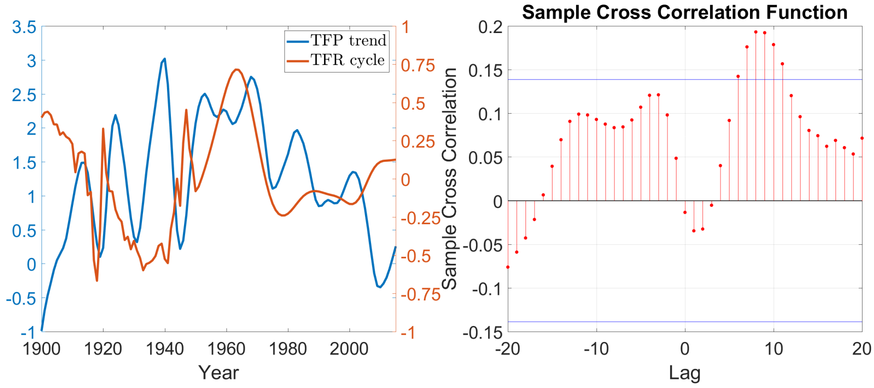


Figure 2.29: United Kingdom, post 1900 data

2.7.5 Appendix E: Mathematical appendix

Proof of Proposition 1

Proof. The proof can be split in two parts. Firstly we show that the problem is defined on a convex set, then that the objective function is strictly quasi-concave.

The feasibility set (i.e. the set of points that satisfies the constraints of the problem) is:

$$\mathcal{C} = \left\{ (c, c', b, n) \in \mathbb{R}^4_+ : \ c + bn \leq \omega \left[1 - \left(\frac{n}{\overline{n}} \right)^\theta \right] + \tau \ \wedge \ c' \leq \omega' + bR \right\}$$

The set is convex if and only if $\forall \breve{x}, \tilde{x} \in \mathcal{C} \ \Rightarrow \ \alpha\breve{x} + (1-\alpha)\,\tilde{x} \in \mathcal{C}$. Then, we have to check if a generic convex combination of two points belongs to the feasibility set. The non-negativity constraints are trivially satisfied. With respect to the other constraints, define $\hat{x} \equiv \alpha\breve{x} + (1-\alpha)\,\tilde{x}$ we have to check if the following inequalities are satisfied:

$$\begin{aligned} \hat{c} + \hat{b}\hat{n} &\leq \omega \left[1 - \left(\frac{\hat{n}}{\overline{n}} \right)^\theta \right] + \tau \\ \hat{c}' &\leq \omega' + \hat{b}R \end{aligned}$$

Applying the definition of $\hat{x}$:

$$\begin{aligned} \alpha\breve{c} + (1-\alpha)\,\tilde{c} + \left[\alpha\breve{b} + (1-\alpha)\,\tilde{b} \right] \left[\alpha\breve{n} + (1-\alpha)\,\tilde{n} \right] &\leq \omega \left[1 - \left(\frac{\alpha\breve{n} + (1-\alpha)\,\tilde{n}}{\overline{n}} \right)^\theta \right] + \tau \\ \alpha\breve{c}' + (1-\alpha)\,\tilde{c} &\leq \omega' + \left[\alpha\breve{b} + (1-\alpha)\,\tilde{b} \right] R \end{aligned}$$

With respect to the first equation, applying the definition of $\hat{x}$:

$$\alpha\breve{c} + (1-\alpha)\,\tilde{c} + \left[\alpha\breve{b} + (1-\alpha)\,\tilde{b} \right] \left[\alpha\breve{n} + (1-\alpha)\,\tilde{n} \right] \leq \omega \left[1 - \left(\frac{\alpha\breve{n} + (1-\alpha)\,\tilde{n}}{\overline{n}} \right)^\theta \right] + \tau$$

Given that $\breve{x}, \tilde{x} \in \mathcal{C}$ we have:

$$\begin{aligned} \alpha\breve{c} + \alpha\breve{b}\breve{n} &\leq \alpha\omega \left[1 - \left(\frac{\breve{n}}{\overline{n}} \right)^\theta \right] + \alpha\tau \\ (1-\alpha)\,\tilde{c} + (1-\alpha)\,\tilde{b}\tilde{n} &\leq (1-\alpha)\,\omega \left[1 - \left(\frac{\tilde{n}}{\overline{n}} \right)^\theta \right] + (1-\alpha)\,\tau \end{aligned}$$

Summing side by side we get:

$$
\begin{aligned}
\alpha\breve{c}+(1-\alpha)\,\tilde{c}+\alpha\breve{b}\breve{n}+(1-\alpha)\,\tilde{b}\tilde{n} &\leq \alpha\omega\left[1-\left(\frac{\breve{n}}{\overline{n}}\right)^{\theta}\right]+(1-\alpha)\,\omega\left[1-\left(\frac{\tilde{n}}{\overline{n}}\right)^{\theta}\right]+\tau \\
\alpha\breve{c}+(1-\alpha)\,\tilde{c}+\alpha\breve{b}\breve{n}+(1-\alpha)\,\tilde{b}\tilde{n} &\leq \omega\left\{\alpha\left[1-\left(\frac{\breve{n}}{\overline{n}}\right)^{\theta}\right]+(1-\alpha)\left[1-\left(\frac{\tilde{n}}{\overline{n}}\right)^{\theta}\right]\right\}+\tau \\
\alpha\breve{c}+(1-\alpha)\,\tilde{c}+\alpha\breve{b}\breve{n}+(1-\alpha)\,\tilde{b}\tilde{n} &\leq \omega\left\{1-\left[\frac{\alpha\,(\breve{n})^{\theta}+(1-\alpha)\,(\tilde{n})^{\theta}}{(\overline{n})^{\theta}}\right]\right\}+\tau \\
\alpha\breve{c}+(1-\alpha)\,\tilde{c}+\alpha\breve{b}\breve{n}+(1-\alpha)\,\tilde{b}\tilde{n} &\leq \omega\left\{1-\left[\frac{\alpha\,(\breve{n})^{\theta}+(1-\alpha)\,(\tilde{n})^{\theta}}{(\overline{n})^{\theta}}\right]\right\}+\tau \\
\hat{c}+\hat{n} &\leq \omega\left\{1-\left[\frac{\alpha\,(\breve{n})^{\theta}+(1-\alpha)\,(\tilde{n})^{\theta}}{(\overline{n})^{\theta}}\right]\right\}+\tau \\
\hat{c}+\hat{n} &\leq \omega-\frac{\omega}{(\overline{n})^{\theta}}\left[\alpha\,(\breve{n})^{\theta}+(1-\alpha)\,(\tilde{n})^{\theta}\right]+\tau
\end{aligned}
$$

The using the fact that $\theta > 1$:

$$
\alpha\,(\breve{n})^{\theta}+(1-\alpha)\,(\tilde{n})^{\theta} \geq [\alpha\breve{n}+(1-\alpha)\,\tilde{n}]^{\theta}\,\alpha\,(\breve{n})^{\theta}+(1-\alpha)\,(\tilde{n})^{\theta} \geq [\hat{n}]^{\theta}
$$

Then:

$$
\begin{aligned}
\omega-\frac{\omega}{(\overline{n})^{\theta}}\left[\alpha\,(\breve{n})^{\theta}+(1-\alpha)\,(\tilde{n})^{\theta}\right]+\tau &\leq \omega-\frac{\omega}{(\overline{n})^{\theta}}\,[\hat{n}]^{\theta}+\tau \\
\omega-\frac{\omega}{(\overline{n})^{\theta}}\left[\alpha\,(\breve{n})^{\theta}+(1-\alpha)\,(\tilde{n})^{\theta}\right]+\tau &\leq \omega\left[1-\left(\frac{\hat{n}^{\theta}}{\overline{n}^{\theta}}\right)^{\theta}\right]+\tau
\end{aligned}
$$

And then:

$$
\hat{c}+\hat{n} \leq \omega\left[1-\left(\frac{\hat{n}^{\theta}}{\overline{n}^{\theta}}\right)^{\theta}\right]+\tau
$$

With respect to the second equation, applying the definition of $\hat{x}$ and rearranging:

$$
\begin{aligned}
\alpha\breve{c}'+(1-\alpha)\,\tilde{c} &\leq \omega'+\left[\alpha\breve{b}+(1-\alpha)\,\tilde{b}\right]R \\
\alpha\breve{c}'+(1-\alpha)\,\tilde{c} &\leq \alpha\omega'+(1-\alpha)\,\omega'+\alpha\breve{b}R+(1-\alpha)\,\tilde{b}R
\end{aligned}
$$

That is satisfied since $\breve{x}, \tilde{x} \in \mathcal{C}$. Secondly, we have to prove quasi-concavity of the objective function. Fix a generic level t. Take two points from the upper contour set $\forall \breve{x}, \tilde{x} \in UCS\left(t\right)$, then we want to show that $f\left(\overline{x}\right) > t$, where $\overline{x} = \alpha \breve{x} + \left(1-\alpha\right)\tilde{x}$. We need to show:

$$\frac{\left(\beta \breve{c} + \left(1-\beta\right)\tilde{c}\right)^{1-\sigma}}{1-\sigma} + \alpha\left(\beta \breve{n} + \left(1-\beta\right)\tilde{n}\right)^{1-\eta}\left[\frac{\left(\beta \breve{c}' + \left(1-\beta\right)\tilde{c}'\right)^{1-\sigma}}{1-\sigma}\right] \quad > \quad t$$

Using the hypothesis $\sigma > 1$ we can easily show:

$$\frac{\left(\beta \breve{c} + \left(1-\beta\right)\tilde{c}\right)^{1-\sigma}}{1-\sigma} \quad > \quad \beta\frac{\left(\breve{c}+\right)^{1-\sigma}}{1-\sigma} + \left(1-\beta\right)\frac{\left(\tilde{c}+\right)^{1-\sigma}}{1-\sigma}$$

Using $\eta \geq \sigma > 1$ the end of the proof is trivial. □

Proof of Lemma 2

Proof. The utility function of the first generation is:

$$\overline{U}\left(c,b,n\right) = \frac{\left(c\right)^{1-\sigma}}{1-\sigma} + \alpha\left(n\right)^{1-\eta}\frac{\left(c^{'}\right)^{1-\sigma}}{1-\sigma}$$

The partial derivative with respect to today consumption is:

$$\frac{\partial \overline{U}}{\partial c} = c^{-\sigma}$$

The partial derivative with respect to children is:

$$\frac{\partial \overline{U}}{\partial n} = \alpha\left(1-\eta\right)\left(n\right)^{-\eta}\frac{\left(c^{'}\right)^{1-\sigma}}{1-\sigma}$$

And finally the partial derivative with respect to offsprings consumption is:

$$\frac{\partial \overline{U}}{\partial c'} = \alpha\left(n\right)^{1-\eta}\left(c^{'}\right)^{-\sigma}$$

On the other hand the the intertemporal budget constraint can be computed plugging the second onder period budget constraint of each children inside the first period budget constraint:

$$\begin{aligned} c+n\left[\frac{c_s^{'}-\omega^{'}}{R}\right] &\leq \omega\left[1-\left(\frac{n}{\overline{\overline{N}}}\right)^{\theta}\right]+\tau \\ c+\frac{n}{R}c_s^{'}-n\frac{\omega^{'}}{R} &\leq \omega\left[1-\left(\frac{n}{\overline{\overline{N}}}\right)^{\theta}\right]+\tau \\ c+\frac{n}{R}c_s^{'}-n\frac{\omega^{'}}{R}+\omega\left(\frac{n}{\overline{\overline{N}}}\right)^{\theta} &\leq \omega+\tau \end{aligned}$$

As a result, the relative price between $c^{'}$ and c is $\frac{n}{R}$. Then, the optimality condition between consumption today and consumption tomorrow can be derived as follow:

$$\frac{\frac{\partial \overline{U}}{\partial c^{'}}}{\frac{\partial \overline{U}}{\partial c}}=\frac{\alpha\left(n\right)^{1-\eta}\left(c^{'}\right)^{-\sigma}}{c^{-\sigma}}=\frac{n}{R}=\frac{p_{c^{'}}}{p_c}$$

Rearranging it we obtain:

$$\begin{aligned} \frac{\left(c^{'}\right)^{-\sigma}}{c^{-\sigma}} &= \frac{n}{\alpha R}\left(n\right)^{\eta-1} \\ c &= \left[\frac{1}{\alpha R}n^{\eta}\right]^{\frac{1}{\sigma}}c^{'} \end{aligned}$$

And using the second period budget constraint:

$$c=\left[\frac{1}{\alpha R}n^{\eta}\right]^{\frac{1}{\sigma}}\left(\omega^{'}+bR\right)$$

Applying the logarithm we have:

$$\begin{aligned} \log c &= \frac{1}{\sigma}\log\left[\frac{1}{\alpha R}n^{\eta}\right]+\log\left(\omega^{'}+bR\right) \\ \log c &= -\frac{1}{\sigma}\log\alpha-\frac{1}{\sigma}\log R+\frac{\eta}{\sigma}\log n+\log\left(\omega^{'}+bR\right) \end{aligned}$$

And therefore applying the partial derivative we get:

$$\frac{\partial c}{c}=\frac{\eta}{\sigma}\frac{\partial n}{n}+\frac{\omega^{'}}{\omega^{\prime}+bR}\frac{\partial\omega^{'}}{\omega^{\prime}}+\frac{bR}{\omega^{\prime}+bR}\frac{\partial b}{b}$$

On the other hand, looking at the ratio between marginal utilities and relative prices of c and n we have:

$$\frac{\frac{\partial\overline{U}}{\partial n}}{\frac{\partial\overline{U}}{\partial c}}=\frac{\alpha\left(1-\eta\right)\left(n\right)^{-\eta}\frac{\left(c^{'}\right)^{1-\sigma}}{1-\sigma}}{c^{-\sigma}}=b+\omega\theta\left(\frac{n}{\overline{\overline{N}}}\right)^{\theta}\frac{1}{n}=\frac{p_n}{p_c}$$

Rearranging it we get:

$$\begin{aligned}
\alpha\left(\frac{1-\eta}{1-\sigma}\right)\left(n\right)^{-\eta}\left(\omega^{'}+bR\right)\frac{\left(c^{'}\right)^{-\sigma}}{c^{-\sigma}} &= b+\omega\theta\left(\frac{n}{\overline{\overline{N}}}\right)^{\theta}\frac{1}{n}\\
\alpha\left(\frac{1-\eta}{1-\sigma}\right)\left(n\right)^{-\eta}\left(\omega^{'}+bR\right)\frac{1}{\alpha R}\left(n\right)^{\eta} &= b+\omega\theta\left(\frac{n}{\overline{\overline{N}}}\right)^{\theta}\frac{1}{n}\\
\left(\frac{1-\eta}{1-\sigma}\right)\left(\omega^{'}+bR\right)\frac{1}{R} &= b+\omega\theta\left(\frac{n}{\overline{\overline{N}}}\right)^{\theta}\frac{1}{n}\\
\left(\frac{1-\eta}{1-\sigma}\right)\left(\omega^{'}+bR\right)\frac{1}{R} &= b+\omega\theta\left(\overline{N}\right)^{-\theta}\left(n\right)^{\theta-1}
\end{aligned}$$

Applying the logarithm on both sides, using $\Phi'\left(n\right)=\theta\left(\overline{N}\right)^{-\theta}\left(n\right)^{\theta-1}$ to simplify the notation and taking the partial derivatives we have:

$$\begin{aligned}
\log\left[b+\omega\Phi'\left(n\right)\right] &= \log\left(\frac{1-\eta}{1-\sigma}\right)+\log\left(\omega^{'}+bR\right)-\log R\\
\frac{\omega\Phi'\left(n\right)}{b+\omega\Phi'\left(n\right)}\left(\theta-1\right)\frac{\partial n}{n} &= \frac{\partial\omega^{'}}{\omega'+bR}\\
\frac{\partial n}{n} &= \frac{1}{\left(\theta-1\right)}\frac{\partial\omega^{'}}{\omega^{'}}\left[\frac{b+\omega\Phi'\left(n\right)}{\omega\Phi'\left(n\right)}\frac{\omega^{'}}{\omega'+bR}\right]\\
\frac{\partial n}{n} &= \frac{1}{\left(\theta-1\right)}\frac{\partial\omega^{'}}{\omega^{'}}\Gamma\left(b,n\right)\\
\frac{\frac{\partial n}{n}}{\frac{\partial\omega'}{\omega^{'}}} &= \frac{1}{\left(\theta-1\right)}\Gamma\left(b,n\right)
\end{aligned}$$

□

Proof of Proposition 3

Proof. Given that $\frac{\partial b}{\partial \tau} > 0$ we can derive Γ by b and then look at the sign of the derivative. Then:

$$\begin{aligned} \frac{\partial}{\partial b}\Gamma(b,n) &= \frac{1}{\omega\Phi'(n)}\frac{\omega'}{\omega'+bR} - \frac{b+\omega\Phi'(n)}{\omega\Phi'(n)}\frac{R}{(\omega'+bR)^2} \\ &= \frac{\omega'\left(\omega'+bR\right) - R\left(b+\omega\Phi'(n)\right)}{\omega\Phi'(n)\left(\omega'+bR\right)^2} \end{aligned}$$

Given that $\omega\Phi'(n)\left(\omega'+bR\right)^2 > 0$ we have that $\frac{\partial}{\partial b}\Gamma(b,n) > 0$ if and only if:

$$\begin{aligned} \omega'\left(\omega'+bR\right) &> R\left(b+\omega\Phi'(n)\right) \\ \omega'^2 + \omega' bR - R\left(b+\omega\Phi'(n)\right) &> 0 \end{aligned}$$

That can be seen decomposed as:

$$\begin{aligned} &\left(\omega' - \frac{-bR+\sqrt{(bR)^2+4R\left(b+\omega\Phi'(n)\right)}}{2}\right)\times \\ &\left(\omega' - \frac{-bR-\sqrt{(bR)^2+4R\left(b+\omega\Phi'(n)\right)}}{2}\right) > 0 \end{aligned}$$

That has solution for $\omega' < \frac{-bR-\sqrt{(bR)^2+4R\left(b+\omega\Phi'(n)\right)}}{2}$ and $\omega' > \frac{-bR+\sqrt{(bR)^2+4R\left(b+\omega\Phi'(n)\right)}}{2}$. The first solution has no economic meaning, while the second one is:

$$\begin{aligned} \omega' &> \frac{-bR+\sqrt{(bR)^2+4R\left(b+\omega\Phi'(n)\right)}}{2} \\ \omega' &> \frac{-bR+bR\sqrt{\frac{1+4R\left(b+\omega\Phi'(n)\right)}{(bR)^2}}}{2} \\ \omega' &> \frac{bR\left[\sqrt{1+\frac{4R\left(b+\omega\Phi'(n)\right)}{(bR)^2}}-1\right]}{2} \end{aligned}$$

It exists a threshold of $\omega^{'}$ above which $\frac{\partial}{\partial b}\Gamma\left(b,n\right)>0$, and therefore:

$$\begin{aligned}\frac{\partial n}{n} &= \frac{1}{\left(\theta-1\right)}\frac{\partial\omega^{'}}{\omega^{'}}\Gamma_{b}\left(b,n\right)\partial b>0\\ \frac{\partial n}{n}\frac{b}{\partial b} &= \frac{1}{\left(\theta-1\right)}\frac{\partial\omega^{'}}{\omega^{'}}\Gamma_{b}\left(b,n\right)b>0\\ \frac{\partial^{2}n}{\partial\omega^{'}\partial b} &= \frac{n}{\left(\theta-1\right)\omega^{'}}\Gamma_{b}\left(b,n\right)>0\end{aligned}$$

And therefore: $\frac{\partial n}{\partial\omega^{'}\partial b}>0$. And given that $\frac{\partial b}{\partial\tau}\geq0$ by construction[29], i.e. the amount of per-capita bequests left is increasing in the level of wealth and $\frac{\partial b}{\partial\tau}>0$ when the non-negativity constraint is not binding we have $\frac{\partial^{2}n}{\partial\omega^{'}\partial\tau}>0$. □

[29] Look at appendix 2.7.2 for a discussion on the possible calibrations of Barro-Becker model and its implications.

2.7.6 Appendix F: Micro-evidence

Wealth imputation in the PSID

Given that before 1999 wealth data are available only every 5 years we decided to impute them in order to extend the number of waves in which the analysis can be conducted. We use the following rule:

$$\tilde{W}_t = \begin{cases} W_t & if\ W_t \neq \cdot \\ W_s & where\ s \in \arg\min_s |t-s|, W_s \neq \cdot, |t-s| \leq 5 \\ \cdot & elsewhere \end{cases}$$

That is:

1. Maximum 5 years of lag imputation (i.e. we take wealth data from not 5 years ahead as a maximum)

2. Choose the closer data among the one available

How data are imputed year-by-year can be seen in Table 2.16.

Wave	Wealth data available	Imputed from	Wave	Wealth data available	Imputed from	Wave	Wealth data available
1979	No	1984	1989	Yes		2001	Yes
1980	No	1984	1990	No	1989	2003	Yes
1981	No	1984	1991	No	1989	2005	Yes
1982	No	1984	1992	No	1994	2006	Yes
1983	No	1984	1993	No	1994	2007	Yes
1984	Yes		1994	Yes		2009	Yes
1985	No	1984	1995	No	1994	2011	Yes
1986	No	1984	1996	No	1994	2013	Yes
1987	No	1988	1997	No	1999		
1988	Yes		1999	Yes			

Table 2.16: Available data on wealth in PSID wave and imputations

Chapter 3

Looking Behind the Financial Cycle: the Neglected Role of Demographics

3.1 Introduction

The Great Recession has been the most severe episode of output, employment and asset prices fall since the Great Depression. The depth of this recessionary episode has moved the focus of the research from the routinary business cycle frequency to the medium to long frequencies of the economy. One of the major strands of literature was developed by the extended research (in space and in time) of Reinhart and Rogoff (Reinhart and Rogoff (2008, 2009a,b)). They showed the ciclcicality of financial crisis and how economies behave before and after this episodes: they are preceded by a period of asset (and housing) prices growth and leveraging and are followed by a deleverage process that decrease asset prices and keep interest rates low. They also compared the 2007 US experience to other episodes and they show similarities in debt and asset prices behavior with many preceding episodes.

But what are the drivers of this leverage and deleverage cycle? According to Reinhart and Rogoff this episodes are triggered by over-optimism that makes agents rapidly forget about past crises (the "*this time is different*" symptome) and leds to an excessive leverage of the economy that in the end becomes unsustainable. The argument has been push even forward and it has been claimed that financial deregulation and loose monetary policy in the end have favoured this leverage cycle and have worsened the crisis that followed.

In this paper I explore the role of demographic trends as potential drivers of the financial cycle. Leaving aside the declining trend in the fertility rate as a potential explanation (e.g. Eggertsson and Mehrotra (2014), Carvalho et al. (2016)) for the fall in interest rates and asset prices, as suggested by the secular stagnation hypothesis, I focus on cyclical developments in fertility rates and show that a crucial element that triggers changes in the financial cycle is the relative size of the cohorts, determined by above or below trend population growth.

Firstly, I explore the demographic process in the last century in the US and I show that the detrended fertility rate features boom and bust episodes that, looking at the absolute number of births, have generated larger or smaller cohorts than those that would have been generated by an average population growth process. Then, I show that there is a positive correlation between the age composition of the population (the inverse dependency ratio[1]) and financial variables (such as the credit/GDP gap and house prices), with the former leading the latter. Finally, I build a three-period overlapping generations (OLG) model with demographic developments, exogenous fertility shocks and life-cycle patterns of consumption and income that, even if calibrated at micro-level, is able to match the correlation observed at aggregate level. In this model, ageing and mortality are deterministic factors: agents live for three periods and then death is a certainty. The size of the newborn cohort is determined by an exogenous shock; this is the only source of uncertainty in the model.

Newborn agents enter the model with no wealth; in the first period they earn an income, consume non-durable goods, and borrow to purchase housing. In the following period, when they are middle-aged, they earn a higher income, adjust their housing stock, pay back their debt, save for retirement and consume non-durable goods. In the last period, they become old, earn no income, get/inherit the housing of their parents (who have died) and use their accumulated wealth to finance consumption and housing. The steady state life cycle profiles of debt, consumption and housing are calibrated according to microdata evidence. Housing is in fixed supply.

The main conclusions are the following: a transitory positive demographic shock (i.e. a bigger cohort entering the economy) increases the share of workers in the economy, thus increasing consumption and per capita output. Since housing is assumed to be a complementary good and it is in fixed supply, increase in output leads to a higher demand that must be cleared through a price increase. As a result, the cohorts that were alive in the previous period and that have already bought houses become relatively richer. An agent from the newborn cohort, on the other hand, is relatively poorer and therefore borrows relatively less than an individual born in a steady state. The overall amount of credit increases when the demographic shock hits the economy because of the higher number of borrowers. Any agent from the baby boom

[1]The inverse dependency ratio is the ratio between the number of people of working age and those out of the labour force, either because they are too young (below 15) or too old (above 64). It can be read as the number of workers that sustain an individual that is not in the labour force. The indicator takes into account the demographic structure but not the labour market status, and therefore is not affected by business cycle fluctuations.

cohort is poorer than an agent born in a normal size cohort and therefore owns a smaller amount of total wealth; on the contrary, the cohort of baby-boomers is richer on aggregate than a steady state size cohort and therefore owns a bigger share of wealth.

Since the shock is assumed to be temporary, when the baby-boomers become middle-aged, the new young cohort returns to its steady state size; the economy reaches a peak in terms of output, as the middle-aged agents are more productive than young workers. House prices also reach their maximum. Given that there is a partial no-arbitrage condition between housing and bonds[2], the negative perspective on house prices depresses the interest rate on bonds thus benefiting the newborn cohort with extremely accommodative credit conditions, due to the relatively larger size of the cohort that supplies credit with respect to the one that demands it.

With reference to the current debate that opposes the secular stagnation hypothesis[3] and to the financial cycle view[4], the paper offers two insights: on the one hand financial downturns may be triggered by changes in demographic developments (as in the secular stagnation hypothesis), on the other, the current phase of low interest rates can be part of a medium-frequency cycle (like the financial cycle) that will reverse (the model predicts the current phase of low interest rates as the temporary consequence of baby boomers ageing and its reversal in the near future as in the empirical work of Favero et al. (2016)). According to the model proposed here, the recent trends in interest rates and credit are neither the symptoms of a long-lasting "secular stagnation" nor the result of changes to regulation and monetary policy but the natural consequence of the demographic structure generated by a boom-bust demographic process.

The paper is structured as follows. Section 2 presents some stylized facts on financial and demographic cycles and life-cycle patterns of consumption and households' credit and housing. Section 3 presents the model and defines the equilibrium. Section 4 discusses the calibration and the solution of the model and section 5 presents the results. Section 6 offers some concluding remarks.

[2]If the economy was populated by investors who can buy and resell houses without using them and eventually going short, a standard no-arbitrage would apply. Since housing is part of the utility function, the no-arbitrage condition includes the utility that provides the use of housing.

[3]First developed by Hansen (1939) and nowadays championed by Summers (2014, 2016), advanced economies have entered a phase of low growth, high debt and low interest rates due to structural changes (mostly related to ageing populations and lower technological innovation growth) that are likely to persist in the future.

[4]The financial cycle view stresses the effect of debt overhang on sluggish growth with a particular focus on the role of loose monetary policy and financial regulation in driving the financial cycle (Lo and Rogoff (2015); Juselius et al. (2016)).

3.2 Selected stylized facts

Housing prices, private debt and demographics are correlated in many developed countries. Nishimura (2011) highlights the positive correlation existing between the inverse dependency ratio and house prices in many developed economies and argues that the most recent reversal in the dependency ratio has led to the beginning of financial crises in many of them. The work of Saita et al. (2013) and uses a panel on the USA and Japan to show that the direction of causality runs from the demographic structure (i.e. the inverse dependency ratio to housing prices. Piazzesi and Schneider (2016) highlight that fluctuations in mortgages explain a huge fraction of private debt fluctuations. Figure 3.1 plots the real house prices, the inverse dependency ratio and households' debt-to-GDP ratio in the USA; an analogous pattern for the UK is plotted in Figure 3.2. The three series are highly correlated in both countries. Indeed, the cross-correlograms (US, Figure 3.3; UK, Figure 3.4) show that correlations are high (statistically different from zero) at different lags.

As to the definition of financial cycle, several alternatives may be found in the literature. Drehmann et al. (2012) analyse many financial indicators with different techniques and find that a good proxy of the financial cycle[5] is given by the mean of the medium frequencies of house prices, real credit and the credit-to-GDP ratio. They propose a measure of the financial cycle which is the mean of the standardized band pass filter of these series with a lower limit of 32 quarters (8 years) and an upper limit of 120 quarters (30 years). An updated series for the United States and the United Kingdom, computed following their methodology, can be found in Figure 3.5 with the respective correlograms. For the United States, correlogram suggests that changes in the inverse dependency ratio anticipate changes in the financial cycle by 10 years while the lead is somewhat smaller (about 5 years) in the United Kingdom. These stylized facts only look at the correlation and do not focus on the direction of causality, the aim of the paper is to construct a model that can rationalize how demographic changes affect the financial cycle.

Why do changes in the population structure affect prices? There is age-specific heterogeneity in income and consumption at different stages of life. Figure 3.9 plots some stylized facts on consumption, income and wealth at different stages of life from the *Survey on Household Income and Wealth* (SHIW) in Italy, while Figure 3.10 plots the same data for US households (income and wealth are taken from the *Survey of Consumer Finances* while data on consumption are taken from the Consumption and Expenditures Survey). Young households are those with a higher marginal propensity to consume and with a higher amount of

[5]They look for a definition of financial cycle such that reversion of the cycle is associated with financial crises.

debt, income is at its maximum when agents become middle-aged and decreases thereafter, consumption of goods and housing follows a pattern similar to income, and older agents are the main owners of financial assets. The model will be calibrated in order to match these stylized facts.

Finally it is important to comment on housing prices. What is the main source of variations in housing prices? As highlighted by Piazzesi and Schneider (2016) the main source of variation in the price of housing in the US is the value of land and not the value of any structures built on it. Knoll et al. (2017) found quantitatively similar results by extending the analysis to 14 advanced economies over almost 150 years. Since the model aims to capture movements in housing prices, this empirical evidence justifies the fixed supply model (as if agents owned land).

3.2.1 Demographics

In the model proposed in this paper, demographic developments are assumed to be exogenous with respect to the financial cycle. While the role of the economic cycle on fertility has been extensively studied in the literature[6], and one might therefore object that demographic developments are not fully exogenous with respect to financial cycle developments. The assumption made here is required in order to study how shocks to the size of population cohorts may affect financial variables. The demographic process in the paper is modelled as a cycle, which is consistent with the detrended series of birth rates adjusted for mortality in the United States, represented in Figure 3.7. In the last century the (detrended and adjusted for child mortality[7]) birth rate fluctuated with a sequence of booms and busts of approximately 20 years; since the 1980s the fluctuations have decreased their magnitude and their duration as a consequence of a more stable socioeconomic environment. Indeed, the two biggest shocks in the series were generated by the Great Depression and WWII: between 1926 and 1945 economic conditions and the war led to a huge fall in the birth rate (almost 13 per cent below trend): on the other hand, the mobilization of women during WWII led to a tightened female labour market and then to younger women being crowded out of the labour market at the beginning of the 1950s, thereby generating the “baby boom”[8].

[6]Starting from the seminal work of Barro and Becker (1989), the literature has largely explored the relationship between business cycles and fertility from a theoretical and an empirical perspective, e.g. Jones and Schoonbrodt (2016); Jones and Schoonbroodt (2010); Jones et al. (2010) among others.

[7]Child mortality is computed as the number of children that die by the age of 5, that is the period of childhood where the probability of survival is minimal.

[8]Doepke et al. (2015) uses a quantitative model to show that this mechanism explains almost 80 per cent of the observed increase in fertility.

The picture does not change if instead of the birth rate we look at the total number of live births in the year (adjusted for child mortality[9]) that is represented in Figure 3.8.

3.2.2 Drivers of the inverse dependency ratio

The stylized facts shown above can rise concerns of a spurious correlation. Indeed, instead of being the demographic structure that drives financial variables one can think that there is a missing variable that can drive both series: economic growth.

An economic boom while driving housing prices and household debt attracts foreigners into the country (if the migration policy is open enough, as in US and UK) to benefit of higher wages and therefore it alters the demographic structure. When a recession comes the migration flux may stop or even reverse for working age population thus determing the inverse dependency ratio to fall together with housing prices and debt.

The inverse old-age dependency ratio is the ratio between the population in the working age (between 15 and 64 years old) and the older population (above 64 years old). Therefore, year by year changes in the ratio are determined by the fertility rate lagged by 15 periods (that determines the influx in the working age), the fertility rate lagged by 64 periods (that determines the outflows from the working age and the inflows in the old-age population), the old-age death rate (that determines the outflows from the old population) and net migration that are likely to affect the inflows in working age-population.

While data on overall death rate are rather easy to find, data on death rate above 64 years old are not available. For this reason I do not include the death rate in the regression. Since it should be exogenous the other variables it should not create any omitted variable bias and it is still possible to check wheter migration influx are a relevant driver of the dependency ratio.

Data on net migrations are from the World Bank. The dataset contains an observation any five years with the absolute number of net migrants in the five years before. In order to have a yearly standardized data that can be included in the regression I split equally among the previous 5 years the net-migration data and I divide it by total population in that year thus having a net migration rate in percentage terms.

[9]The series in child mortality for the United States begins in 1933; nonetheless, I have used data from Sweden (historically lower than United States) and France (historically higher) to reconstruct the first 25 years of data. Therefore the orange line uses Sweden's mortality rate up until 1933, the grey line uses French data until the same year and the yellow line uses a mean. The different hypotheses do not dramatically change the boom-bust picture.

Inv. Dependency Ratio	(1) United States	(2) United Kingdom
Lag of Inv. Dependency Ratio	0.940***	1.081***
	(0.0648)	(0.0602)
Net migration rate (% over population)	0.00562	-0.000334
	(0.00566)	(0.00270)
TFR lag 15 (adjusted by child mortality)	0.00922***	0.0950***
	(0.00289)	(0.0149)
TFR lag 64 (adjusted by child mortality)	-0.00398***	-0.0776***
	(0.00105)	(0.0176)
Constant	0.0480	-0.161
	(0.186)	(0.0974)
Observations	19	30
R^2	0.988	0.974

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 3.1: The role of migration and demographic trends in determing the inverse dependency ratio

I run the following specification:

$$InvDR_t = \alpha + \beta_1 InvDR_{t-1} + \beta_2 adjTFR_{t-15} + \beta_3 adjTFR_{t-64} + \beta_4 netMigration_t + \varepsilon_t$$

Where the total fertiliry rate is adjusted by child mortality (i.e. mortality under 5 years old, that is a non negligible phenomenon at the beginning of the sample).

Results of the regression in the two countries are shown in Table 3.1. The net migration rate is not significant in the two countries while the the total fertility rate lagged by 15 years is significant and positive and the same variable lagged by 64 years is significant and negative (as expected).

Therefore, regression results show that the main driver of the inverse dependency ratio is the (relatively) exogenous demographic structure and it is not significantly affected by migrations driven by economic conditions.

3.3 The Model

3.3.1 OLG structure

There are three cohorts: young, middle-aged and old. Agents stay in a cohort for one period (agents enter the model when they are 25 years old and one can imagine a period as lasting 20 years). The young and the middle-aged work while the old cohort does not earn any income from labour. Each generation lives for three periods and then dies with certainty. The size of the first cohort is determined by an exogenous process; the demographic variables are:

$$\begin{aligned} N_t^1 &= \overline{N} + \varepsilon_t \\ N_t^2 &= N_{t-1}^1 \\ N_t^3 &= N_{t-1}^2 \end{aligned}$$

Given that the demographic structure is the exogenous state variable of the economy, for notational convenience we can denote it with $\mathcal{N}_t = \{N_t^1, N_t^2, N_t^3\}$ the set that contains the dimensions of the three populations.

3.3.2 Households' preferences

In each period households get their utility from housing and consumption. Following the standard modelling, e.g. Piazzesi and Schneider (2016), I assume they have a CRRA utility function over a CES aggregator for housing and consumption. The intraperiod utility of a cohort i is given by $u\left(c_t^i, h_{t+1}^i\right)$:

$$u\left(c_t^i, h_{t+1}^i\right) = \frac{\left[x\left(h_{t+1}, c_t\right)\right]^{1-\sigma}}{1-\sigma}$$

where:

$$x\left(h_{t+1}, c_t\right) = \left[\left(1-\omega^h\right)\left(c_t\right)^{\frac{\eta-1}{\eta}} + \omega^h\left(h_{t+1}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

The parameter ω^h determines the relative share of income used to buy housing while η measures the elasticity of substitution between housing and consumption; σ measures the inverse of the IES.

3.3.3 First cohort

The first cohort is formed of newly born agents. They inelastically supply one unit of labour, earn an income w_t^1 and buy consumption goods c_t^1 and housing h_{t+1}^1. They can become indebted by going negative on a risk-free bond (that is sold at q_t^b and that is paid back in the subsequent period)[10]. Therefore the problem that the representative agent faces is:

$$V_t^1(\mathcal{N}_t) = \max_{c_t^1, h_{t+1}^1, d_{t+1}^1} u\left(c_t^1, h_{t+1}^1\right) + \beta_1 \mathbb{E}_t\left[V_{t+1}^2\left(d_{t+1}^1, h_{t+1}^1, \mathcal{N}_{t+1}\right)\right]$$

$$s.t.$$

$$c_t^1 + q_t^h h_{t+1}^1 + q_t^b d_{t+1}^1 \leq w_t^1 \qquad \left(\mu_t^1\right)$$

where $u\left(c_t^1, h_{t+1}^1\right)$ is the function described above.

3.3.4 Second cohort

When they are middle-aged, the inelastic labour supply of agents is still one unit of time and they earn an income w_t^2 which they use to buy consumer goods c_t^2 and eventually to adjust their amount of housing h_{t+1}^2. They have to pay back the debts incurred in the first period and can save money for their retirement by lending to the first cohort. Therefore the problem faced by the representative agent of the cohort is:

$$V_t^2\left(d_t^1, h_t^1, \mathcal{N}_t\right) = \max_{c_t^2, h_{t+1}^2, d_{t+1}^2} u\left(c_t^2, h_{t+1}^2\right) + \beta_2 \mathbb{E}_t\left[V_{t+1}^3\left(d_{t+1}^2, h_{t+1}^2, \mathcal{N}_{t+1}\right)\right]$$

$$s.t.$$

$$c_t^2 + q_t^h\left(h_{t+1}^2 - h_t^1\right) + q_t^b d_{t+1}^2 \leq w_t^2 + d_t^1 \qquad \left(\mu_t^2\right) \tag{3.1}$$

3.3.5 Third cohort

In the last period of their life agents do not work, inherit housing from their parents ($h_t^3 \frac{N_{t-1}^3}{N_t^3}$, where the second term takes into account that cohorts may differ in size, (which affects the amount of inheritance) and use their financial and real wealth to maximize their utility. The

[10]Notice that the bond market in principle allows the first cohort to save, and they become indebted only if it occurs endogenously $d_{t+1}^1 < 0$.

problem faced by the representative agent of the third cohort is:

$$\begin{aligned} V_t^3\left(d_t^2, h_t^2, \mathcal{N}_t\right) = & \max_{c_t^2, h_{t+1}^2} u\left(c_t^3, h_{t+1}^3\right) \\ & s.t. \\ & c_t^3 + q_t^h\left(h_{t+1}^3 - h_t^2\right) \leq d_t^2 + q_t^h h_t^3 \tfrac{N_{t-1}^3}{N_t^3} \quad \left(\mu_t^3\right) \end{aligned} \tag{3.2}$$

3.3.6 Representative firm

The main results of the paper can be obtained with a simpler age-dependent endowment economy. Nonetheless, in order to capture the effects of larger or smaller than usual cohorts the representative firm has a production function with a CES aggregator for labour input from young and middle-aged workers. In the literature there has been much discussion on the effect of cohort size in the labour market performances of an agent, and this model allows us to capture inter-cohort complementarities and intra-cohort substitutabilities of labour input. As a result the life-cycle profiles of income of baby boomers and baby busters will be affected: being born in a larger than usual cohort will reduce the per capita wage (intra-cohort substitutability), on the contrary the other cohorts at work will benefit from higher productivity (inter-cohort complementarity). Capital is assumed to be in fixed supply so as not to increase the state-space dimensionality; the role of this assumption will be discussed in section 3.6.1. The CES aggregator is calibrated by looking at empirical estimates on complementarity and substitutability between young and middle-aged workers[11].

The production sector includes one representative firm that produces consumption goods taking as its input young and middle-aged labour. The production does not require capital. The problem faced by the firm is:

$$\begin{aligned} \max_{N_t^1, N_t^2, K_t^1} \quad & Y_t - w_t^1 N_t^1 - w_t^2 N_t^2 \\ & s.t. \\ & Y_t = \left[\omega^y \left(N_t^1\right)^{\frac{\varepsilon-1}{\varepsilon}} + \omega^o \left(N_t^2\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \end{aligned}$$

Notice that the labour of the young and of the middle-aged have different levels of productivity and are complements in production; ε captures the level of complementarity, $\omega^y + \omega^o = 1$ captures the different levels of productivity through the life cycle while ε represents the elasticity of substitution between young and old workers in production. Therefore the FOCs

[11] For example Macunovich (1999), Murphy et al. (1984), Levine and Mitchel (1988) among others.

are:

$$
\begin{aligned}
w_t^1 &= \frac{Y_t}{N_t^1}\frac{\omega^y\left(N_t^1\right)^{\frac{\rho-1}{\rho}}}{\omega^y\left(N_t^1\right)^{\frac{\rho-1}{\rho}}+\omega^o\left(N_t^2\right)^{\frac{\rho-1}{\rho}}}\\
w_t^2 &= \frac{Y_t}{N_t^2}\frac{\omega^o\left(N_t^2\right)^{\frac{\rho-1}{\rho}}}{\omega^y\left(N_t^1\right)^{\frac{\rho-1}{\rho}}+\omega^o\left(N_t^2\right)^{\frac{\rho-1}{\rho}}}
\end{aligned}
$$

3.3.7 Housing market

There is a fixed housing supply (as already discussed it represents land); the exogenous amount of housing $\overline{H}$ is shared among all living individuals. The housing used by the third cohort is inherited by their children (the following cohort) at the end of the period.

In this economy housing plays a double role: agents get instant utility from owning it but it is also an investment that can be sold in the next period. For this reason the return of a housing good enters into the Euler equation of cohort 1 and 2 and there is a no-arbitrage condition between the price of housing and the price of the bond. Looking at the first cohort[12] The FOCs with respect to h_{t+1}^1 and d_{t+1}^1 are:

$$
\begin{aligned}
q_t^h\mu_t^1 &= \frac{\omega^h\left(h_{t+1}^1\right)^{-\frac{1}{\eta}}}{\left[\left(1-\omega^h\right)\left(c_t^1\right)^{\frac{\eta-1}{\eta}}+\omega^h\left(h_{t+1}^1\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta\sigma-1}{\eta-1}}}+\beta_1\mathbb{E}_t\left[\mu_{t+1}^2 q_{t+1}^h\right]\\
q_t^b\mu_t^1 &= \beta_1\mathbb{E}_t\left[\mu_{t+1}^2\right]
\end{aligned}
$$

Notice that if housing does not enter the utility function, the first term on the right hand side of the first equation equals zero and the model predicts a standard no-arbitrage condition:

$$
\frac{\mathbb{E}_t\left[\mu_{t+1}^2\right]}{\mu_t^1 q_t^b}=\frac{\mathbb{E}_t\left[\mu_{t+1}^2 q_{t+1}^h\right]}{\mu_t^1 q_t^h}
$$

In this case, since housing delivers utility today too, agents are willing to hold it even if the

[12] The same analysis can be conducted on the second cohort, and the main difference is in the discount factor (β_2 instead of β_1).

expected return is lower than the bond return

$$\frac{\omega^h \left(h_{t+1}^1\right)^{-\frac{1}{\eta}}}{\left[\left(1-\omega^h\right)\left(c_t^1\right)^{\frac{\eta-1}{\eta}}+\omega^h\left(h_{t+1}^1\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta\sigma-1}{\eta-1}}}>0 \implies \frac{\mathbb{E}_t\left[\mu_{t+1}^2\right]}{\mu_t^1 q_t^b}<\frac{\mathbb{E}_t\left[\mu_{t+1}^2 q_{t+1}^h\right]}{\mu_t^1 q_t^h}$$

Nonetheless the return on housing and bonds must be (partially) correlated.

3.3.8 Markets

There are three markets in this economy: the housing market, the bond market and the goods market. All agents participate in the housing and goods market while only the young and middle-aged trade on the financial market[13]. Market clearing conditions are:

$$\begin{aligned} N_t^1 h_{t+1}^1 + N_t^2 h_{t+1}^2 + N_t^3 h_{t+1}^3 &= \overline{H} \\ N_t^1 d_{t+1}^1 + N_t^2 d_{t+1}^2 &= 0 \\ N_t^1 c_t^1 + N_t^2 c_t^2 + N_t^3 c_t^3 &= Y_t \end{aligned}$$

3.3.9 Equilibrium of the economy

Given a sequence of shocks to the demographic process $\{\varepsilon_t\}_{t=0}^{\infty}$, an initial asset allocation $\{d_0^1, d_0^2\}$ and an initial split of housing between cohorts $\{h_0^1, h_0^2, h_0^3\}$, a competitive equilibrium for this economy is given by a sequence of allocations of housing and consumption $\left\{c_t^1, c_t^2, c_t^3, h_{t+1}^1, h_{t+1}^2, h_{t+1}^3\right\}_{t=0}^{\infty}$, a sequence of prices $\left\{q_t^h, q_t^b\right\}_{t=0}^{\infty}$ and a sequence of asset allocations $\left\{d_{t+1}^1, d_{t+1}^2\right\}_{t=0}^{\infty}$ such that $\forall t$:

1. The representative agent of the first cohort takes as given q_t^b and q_t^h and chooses c_t^1, h_{t+1}^1

[13] The old are free to participate in the financial market but they do not want to save since they do not display intergenerational altruism and no one is willing to lend to them since they will no longer be alive when the loan has to be repaid.

and d^1_{t+1} to solve her problem and therefore they satisfy:

$$\begin{aligned}
\mu^1_t &= \left(1-\omega^h\right)\left(c^1_t\right)^{-\frac{1}{\eta}}\left[\left(1-\omega^h\right)\left(c^1_t\right)^{\frac{\eta-1}{\eta}}+\omega^h\left(h^1_{t+1}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{1-\eta\sigma}{\eta-1}} \\
\mu^1_t q^h_t &= \omega^h\left(h^1_{t+1}\right)^{-\frac{1}{\eta}}\left[\left(1-\omega^h\right)\left(c^1_t\right)^{\frac{\eta-1}{\eta}}+\omega^h\left(h^1_{t+1}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{1-\eta\sigma}{\eta-1}} \\
&\quad +\beta_1\mathbb{E}_t\left[\mu^2_{t+1}q^h_{t+1}\right] \\
q^b_t\mu^1_t &= \beta_1\mathbb{E}_t\left[\mu^2_{t+1}\right] \\
w^1_t &= c^1_t+q^h_t h^1_{t+1}+q^b_t d^1_{t+1}
\end{aligned}$$

2. The representative agent of the second cohort takes as given the prices q^b_t and q^h_t and the amount of bonds and housing bought in the previous period (d^1_t and h^1_t) and chooses c^2_t, h^2_{t+1} and d^2_{t+1} to solve her problem. The associated FOCs are:

$$\begin{aligned}
\mu^2_t &= \left(1-\omega^h\right)\left(c^2_t\right)^{-\frac{1}{\eta}}\left[\left(1-\omega^h\right)\left(c^2_t\right)^{\frac{\eta-1}{\eta}}+\omega^h\left(h^2_{t+1}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{1-\eta\sigma}{\eta-1}} \\
\mu^2_t q^h_t &= \omega^h\left(h^2_{t+1}\right)^{-\frac{1}{\eta}}\left[\left(1-\omega^h\right)\left(c^2_t\right)^{\frac{\eta-1}{\eta}}+\omega^h\left(h^2_{t+1}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{1-\eta\sigma}{\eta-1}} \\
&\quad +\beta_2\mathbb{E}_t\left[\mu^3_{t+1}q^h_{t+1}\right] \\
\mu^2_t q^b_t &= \beta_2\mathbb{E}_t\left[\mu^3_{t+1}\right] \\
w^2_t+d^1_t &= c^2_t+q^h_t\left(h^2_{t+1}-h^1_t\right)+q^b_t d^2_{t+1}
\end{aligned}$$

3. The representative agent of the third cohort takes as given the prices q^b_t and q^h_t and the amount of bonds and housing bought in the previous period (d^2_t and h^2_t) together with the amount of housing left as a bequest by the parents h^3_t and chooses c^3_t and h^3_{t+1} to solve her problem. The associated FOCs are:

$$\begin{aligned}
\mu^3_t &= \left(1-\omega^h\right)\left(c^3_t\right)^{-\frac{1}{\eta}}\left[\left(1-\omega^h\right)\left(c^3_t\right)^{\frac{\eta-1}{\eta}}+\omega^h\left(h^3_{t+1}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{1-\eta\sigma}{\eta-1}} \\
q^h_t\mu^3_t &= \omega^h\left(h^3_{t+1}\right)^{-\frac{1}{\eta}}\left[\left(1-\omega^h\right)\left(c^3_t\right)^{\frac{\eta-1}{\eta}}+\omega^h\left(h^3_{t+1}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{1-\eta\sigma}{\eta-1}} \\
&\quad +\beta_3\left(q^h_t h^3_{t+1}\right)^{-\sigma}q^h_t \\
c^3_t+q^h_t h^3_{t+1} &= d^2_t+q^h_t h^2_t+q^h_t h^3_t\frac{N^3_{t-1}}{N^3_t}
\end{aligned}$$

4. The representative firm takes as given wages w^1_t and w^2_t and demands labour from the

two cohorts in order to maximize its profits:

$$
\begin{aligned}
w_t^1 &= \frac{Y_t}{N_t^1} \frac{\omega^y \left(N_t^1\right)^{\frac{\rho-1}{\rho}}}{\omega^y \left(N_t^1\right)^{\frac{\rho-1}{\rho}} + \omega^o \left(N_t^2\right)^{\frac{\rho-1}{\rho}}} \\
w_t^2 &= \frac{Y_t}{N_t^2} \frac{\omega^o \left(N_t^2\right)^{\frac{\rho-1}{\rho}}}{\omega^y \left(N_t^1\right)^{\frac{\rho-1}{\rho}} + \omega^o \left(N_t^2\right)^{\frac{\rho-1}{\rho}}}
\end{aligned}
$$

5. Housing market clears:

$$
N_t^1 h_{t+1}^1 + N_t^2 h_{t+1}^2 + N_t^3 h_{t+1}^3 = \overline{H}
$$

6. Goods market clears:

$$
N_t^1 c_t^1 + N_t^2 c_t^2 + N_t^3 c_t^3 = Y_t
$$

7. Financial market clears:

$$
N_t^1 d_{t+1}^1 + N_t^2 d_{t+1}^2 = 0
$$

3.4 Calibration and solution method

The model is calibrated to match the stylized facts described in section 3.2: agents borrow in the first period of their life; in the second period they repay their debt and they save for the next period thereby providing savings to the first generation; in the last period they consume all their wealth. Consumption of housing and goods is at its minimum when agents are young, it increases and reaches its peak in middle age and then decreases in the last period.

The value of the calibrated parameters is reported in Table 3.2, while the resulting steady state profile of consumption, housing and financial assets is shown in Figure 3.12[14]. In order to match the data, the middle-aged cohort earns a higher share of income ($\omega^o > \omega^y$) and agents discount at a higher rate from the first to the second period and at a lower rate from the second to the third period.

[14] It can be compared with the life cycle of consumption, housing and financial assets in the US and Italy in Figure 3.13.

Parameter	Calibration
η	0.5
ω^h	0.5
σ	2
β_1	1.3
β_2	0.5
ω^y	0.4
ω^o	0.6
ρ	0.7
$\overline{H}$	3.5
$\overline{N}$	1

Table 3.2: Parameters calibration

The model is solved using a third-order perturbation around the steady state.

I then consider a scenario in which an unexpected demographic shock of 10 per cent occurs. The population pyramids across periods are plotted in Figure 3.14: on the north-west the population pyramid is in a steady state, in which all cohorts are the same size, in period 1 the boomers generation is born, which is bigger than the other cohort and the population pyramid has a larger base (north-east); in period 2 a normal cohort is born and the pyramid has the bigger cohort at its centre (south-west); finally, in period 3 the boomers are at the top of the population pyramid that is now “reversed” (the largest cohort is at the top; south-east).

3.5 Results

Before discussing results of the simulation of the shock it is worthwile a brief discussion on the main channel of the model: consumption smoothing and intertemporal budget constraint.

3.5.1 The intertemporal budget constraint

Consider the problem of an agent born in period t. She solves an intertemporal maximization under the following intertemporal budget constraint[15]:

$$c_t^1 + \left(q_t^h - q_t^b q_{t+1}^h\right) h_{t+1}^1 + q_t^b c_{t+1}^2 + q_t^b \left(q_{t+1}^h - q_{t+1}^b q_{t+2}^h\right) h_{t+2}^2 + q_t^b q_{t+1}^b c_{t+2}^3$$
$$+ q_t^b q_{t+1}^b q_{t+2}^h h_{t+3}^3 \leq w_t^1 + q_t^b w_1^2 + q_t^b q_{t+1}^b q_{t+2}^h h_{t+2}^3 \frac{N_t^2}{N_t^1}$$

Firstly, notice that the housing price is similar to a “rental rate” in the first and second period while it is the pure housing price in the third period (since they do not re-sell it). Then, consider that the intertemporal wealth that the agent wants to consume smoothly according to their intertemporal preferences is:

$$w_t^1 + q_t^b w_1^2 + q_t^b q_{t+1}^b q_{t+2}^h h_{t+2}^3 \frac{N_t^2}{N_t^1}$$

Notice then that it is increasing in q_t^b and q_{t+1}^b, i.e. the lower the interest rate in the economy the higher the wealth for an agent that enters the economy. Notice that q_t^h does not enter into the period 0 income, therefore any shock to the housing price does not affect the wealth of the first cohort, while it affects the wealth of those cohorts that already own housing (check equation (3.1) and equation (3.2)).

The model works through wealth shocks created by the demographic shocks and their effects on housing prices and interest rates. Given the intertemporal preferences of different cohorts, it triggers changes in the financial cycle.

3.5.2 Demographic shock simulation

Figure 3.15 shows the reaction of the economy to a baby boom shock of 10 per cent in period 1. In any subplot the black line represents the steady state level.

When the baby-boomer cohort is born, output in the economy grows (Figure 3.15: third row, first column) as the labour force grows. Since consumption goods and housing are complementary goods but housing is in fixed supply, the extra demand for housing has to be cleared through a change in the relative price. Housing prices increase (more than proportionally due to the elasticity of the substitution of 0.5) (Figure 3.15: first row, first column), thus

[15] The mathematical derivation is in the appendix.

increasing the wealth of the cohorts that are already in the economy and bought housing in the previous period (2 and 3). The positive wealth effect and the substitution effect (due to the increase in q^h) work in the same direction for the second and the third cohort, and thus they increase non-durable consumption (c^2 and c^3, Figure 3.15: first row, third and quarter column respectively). On the contrary, the two effects work in opposite directions for housing: the substitution effect prevails and they (slightly) reduce its consumption (h^2 and h^3, Figure 3.15: second row, second and third column respectively). An agent from the first cohort, by contrast, is unfavourably affected by being born in a larger than usual cohort for two main reasons: she will have a lower wage (due to the CES aggregator of labour inside the production function) and will inherit a lesser amount of housing from her parents (there is one extra child in every ten on average and parents have less housing due to the increase in the population). The lower level of income throughout life reduces borrowing needs in the first period (wealth effect). Furthermore, the price of a bond is lower due to the non-arbitrage condition with housing that has expectations of a high return due to the (expected) increase in output (substitution effect). Notice that at the aggregate level the first cohort is getting a higher share of housing even if its per capita level of housing is smaller than a normal-size cohort.

In the second period the demographic shock reverses and a smaller (normal-sized)cohort is born after the boomers. The amount of consumer goods in the economy is at its maximum, since the boomers are now middle-aged and therefore the price of housing peaks (Figure 3.15: first row, first column). The newborns have a higher expected wealth with respect to a cohort born in "steady state" (i.e. after a high number of periods without demographic shocks) for two reasons: a higher wage when they enter the labour market and a higher amount of housing in the last period (when, in any case, it will not be worth as much as it is now). On top of the positive wealth effect, there is also a substitution effect: the relative abundance of credit supply and the no-arbitrage with housing that will have to decrease in the next period decreases the interest rate on bonds and promotes credit to the relatively impatient young households. Therefore, wealth and substitution effects lead to an increase of the per capita borrowing by the young cohort.

When the boom cohort retires, output falls and so do housing prices. In the fourth period the demographic structure reverts to its steady state level but other variables do not. Indeed, the state variables of this model are the demographic structure (exogenous) and the wealth distribution across cohorts (endogenous). When the baby boomers' cohort dies the demographic shock reverts but wealth shares are still affected by the previous shock and therefore it takes more time for the variables to return to their steady state level.

Figure 3.16 plots the housing prices (rescaled to fit the figure) with the inverse dependency ratio and the aggregate level of credit-to-output (i.e. taking into account cohort-dimension), i.e. the equivalent in the model of the series represented in Figure 3.1 for the United States and in Figure 3.2 for the United Kingdom. It can be seen that the model, despite its simplicity, is roughly able to replicate the dynamics observed in the data at the aggregate level.

3.6 Robustness

3.6.1 Alternative assumptions

In this subsection I briefly discuss two of the most restrictive assumptions of the model, housing inheritance and the fixed-capital production function, and their effect on the financial cycle.

With respect to housing inheritance, the crucial element for the model is the life cycle profile of borrowing and saving and the wealth shock arising from being born in a small or a big cohort. The fact that agents own a house also in the last period of their life is certainly realistic; therefore the problem to be addressed is where the housing they owned goes when they die. The most natural assumption is that their children inherit their house even if they do not have altruistic motives (introducing a bequest motive through an easy warm glow model[16] This would have a positive minor effect on the interest rate and on the price of housing due to the fact that the future also has a value now for the third generation), nonetheless one can make two alternative assumptions: either the houses are inherited by grandchildren (it is less natural but it means inheritance around the age of 45 which can be a reasonable age) or the government takes the whole amount of housing (one can imagine a 100 per cent tax on bequests) and share the proceedings among all the living cohorts. With respect to the first hypothesis, the results would not change, the first cohort would still find it optimal to become indebted in order to smooth consumption and the second cohort would probably increase their supply of savings and buy more houses in order to sustain their consumption in the third period where they would have no income at all. On the other hand, in the 100 per cent inheritance tax model everything depends on the sharing of the proceedings. If it does not alter the borrower\lender status of the cohort, i.e. if the first cohort is still a financial borrower (with a positive net wealth, due to the fact it owns housing) while the second cohort remains a financial lender, the results would not change. Nonetheless, the higher the share of proceedings that goes to the first cohort the higher the softening of the welfare effect since bigger cohorts benefit, at least partially, from increased housing prices and small cohorts are

[16] As in De Nardi (2004) and De Nardi and Yang (2014).

penalized.

With respect to the introduction of physical capital, one may argue that it would amplify the results of the model. Firstly, consider that physical capital is a risky asset since the return depends on the labour supply available in the next period, which is determined by an exogenous process on the population. For this reason the greatest share of it would be owned at the beginning of production by the middle-aged (who made investments when they were young)[17]. Therefore, with respect to the steady state without capital, the youngest generation would increase its short position on safe debt to finance investments (and housing), while clearly the oldest generation would not invest in physical capital since it will not be alive to benefit from the return. A demographic shock in this setup would favour the middle-aged who already own capital. On the one hand the investments will increase, leading to an increase in output in the subsequent period and therefore to an amplification of the house prices cycle (it is determined by the fact that housing is complementary to non-durable goods). The increase in investment would not fully compensate the increase in labour supply since the old cohort that benefits from the increase in housing prices would not want to invest in it. Therefore the risk-free interest rate would increase more than in the model without capital, increasing the negative welfare effect on baby boomers.

3.7 Conclusions

Starting from some stylized facts that show a correlation between demographic variables and financial variables, I have built an OLG model with housing and credit markets and an exogenous demographic process that is able to rationalize the stylized facts in data form.

In the model, a transitory baby boom triggers an expansion in the financial cycle: the entry of a large cohort has a positive impact on housing prices, favouring those cohorts that already own houses (middle-aged and old) and on the interest rate, given the relative scarcity of credit supply and the expected boom of house prices in the next period. The subsequent cohort, on the contrary, benefits from a larger credit supply and an expected fall in house prices that lowers the interest rate. As a result, newborns are relatively richer than a normal cohort even if their income in the first period is not very different; this increases credit demand in order to smooth consumption. For this reason, even when the cohort that becomes indebted is normal-sized, the amount of credit is still higher than that of a cohort in the steady state.

[17] As has been discussed in Glover et al. (2014), in an OLG model with portfolio choice and $\sigma > 1$ where the share of risky assets in a portfolio decreases with age. The result comes from the fact that a younger generation has a longer time horizon for investment and it counts on the fact that it can recoup losses at a later date. This is consistent with microdata on wealth in different countries.

With respect to the current debate between the secular stagnation and the financial cycle hypotheses, the paper provides evidence that the two can be partially reconciled: the medium-frequency cycle on interest rates and credit can be generated by the medium-frequency fluctuations in demographics that we observed in the last century. If this is the case, in the US the financial cycle will reverse as soon as the retirement phase of the baby-boomers ends and the smaller cohorts from the end of 1960 retire, and therefore sooner than expected by those who support the secular stagnation hypothesis[18]. Nonetheless, the level of interest rates observed during the 1980s should not be taken as a reference since it was determined by the entry of the baby boomers into the labor force. Thereafter the cycle should reduce its amplitude, given the relative stability of the demographic process since the mid-1980s[19].

In future research the analysis should focus on the link between demographics and the outbreak of housing bubbles. To this extent, the model should incorporate semi-rational agents or informational frictions that may trigger the ”rational exuberance” that has been used to justify the outbreak of the housing bubble (e.g. Kaplan et al. (2017)).

[18]The secular stagnation hypothesis was proposed in 1938 when the economic and demographic conditions were similar to today: fertility had fallen and the economy had collapsed after a financial crisis. Hansen in 1938 stated: *“it appears that prodigious growth of population in the nineteenth century was something unique in history. Gathering momentum with the progress of modern science and transportation, the absolute growth in western Europe mounted decade by decade until the Great World War [...] the advancing tide has come to a sudden halt and the accretions are dwindling toward zero”,* but ten years later the fertility rate started increasing and 60 years of unprecedented technological growth and development in human history followed (between 1945 and 2005).

[19]It is important to underline that there are fields of economics (as well as in demographics) that look at fertility as a boom and bust process with fluctuations, e.g. Jones and Schoonbrodt (2016). From this perspective one can see how the Great Moderation reflected the stabilization of the fertility rate.

3.8 Appendices

3.8.1 Appendix A: Derivation of the intertemporal budget constraint

Start from the budget constraint of the young cohort:

$$\begin{aligned} c_t^1 + q_t^h h_{t+1}^1 + q_t^b d_{t+1}^1 &\leq w_t^1 \\ c_t^1 + q_t^h h_{t+1}^1 &\leq w_t^1 - q_t^b d_{t+1}^1 \end{aligned}$$

Use the budget constraint of the second cohort to substitute for d_{t+1}^1:

$$c_{t+1}^2 + q_{t+1}^h \left(h_{t+2}^2 - h_{t+1}^1\right) + q_{t+1}^b d_{t+2}^2 - w_1^2 \leq d_{t+1}^1$$

And we get:

$$\begin{aligned} c_t^1 + q_t^h h_{t+1}^1 &\leq w_t^1 - q_t^b d_{t+1}^1 \\ c_t^1 + q_t^h h_{t+1}^1 &\leq \\ w_t^1 - q_t^b \left(c_{t+1}^2 + q_{t+1}^h \left(h_{t+2}^2 - h_{t+1}^1\right) + q_{t+1}^b d_{t+2}^2 - w_1^2\right) & \\ c_t^1 + q_t^h h_{t+1}^1 + q_t^b c_{t+1}^2 + q_t^b q_{t+1}^h \left(h_{t+2}^2 - h_{t+1}^1\right) &\leq w_t^1 + q_t^b w_1^2 \\ & \quad -q_t^b q_{t+1}^b d_{t+2}^2 \end{aligned}$$

Finally:

$$c_t^1 + \left(q_t^h - q_t^b q_{t+1}^h\right) h_{t+1}^1 + q_t^b \left(c_{t+1}^2 + q_{t+1}^h h_{t+2}^2\right) \leq w_t^1 + q_t^b \left(w_1^2 - q_{t+1}^b d_{t+2}^2\right) \tag{3.3}$$

Rearranging the old cohort budget constraint to get d_{t+2}^2 we have:

$$\begin{aligned} c_{t+2}^3 + q_{t+2}^h \left(h_{t+3}^3 - h_{t+2}^2\right) &\leq d_{t+2}^2 + q_{t+2}^h h_{t+2}^3 \frac{N_t^2}{N_t^1} \\ c_{t+2}^3 + q_{t+2}^h \left(h_{t+3}^3 - h_{t+2}^2\right) - q_{t+2}^h h_{t+2}^3 \frac{N_t^2}{N_t^1} &\leq d_{t+2}^2 \end{aligned}$$

And plugging it into (3.3):

$$\begin{aligned} c_t^1 + \left(q_t^h - q_t^b q_{t+1}^h\right) h_{t+1}^1 + q_t^b c_{t+1}^2 + q_t^b q_{t+1}^h h_{t+2}^2 \leq \\ w_t^1 + q_t^b w_1^2 - q_t^b q_{t+1}^b \left[c_{t+2}^3 + q_{t+2}^h \left(h_{t+3}^3 - h_{t+2}^2\right) - q_{t+2}^h h_{t+2}^3 \frac{N_t^2}{N_t^1}\right] \end{aligned}$$

And then:

$$c_t^1 + \left(q_t^h - q_t^b q_{t+1}^h\right) h_{t+1}^1 + q_t^b c_{t+1}^2 + q_t^b \left(q_{t+1}^h - q_{t+1}^b q_{t+2}^h\right) h_{t+2}^2 + q_t^b q_{t+1}^b c_{t+2}^3 + q_t^b q_{t+1}^b q_{t+2}^h h_{t+3}^3 \leq w_t^1 + q_t^b w_1^2 + q_t^b q_{t+1}^b q_{t+2}^h h_{t+2}^3 \frac{N_t^2}{N_t^1}$$

3.8.2 Appendix B: Figures

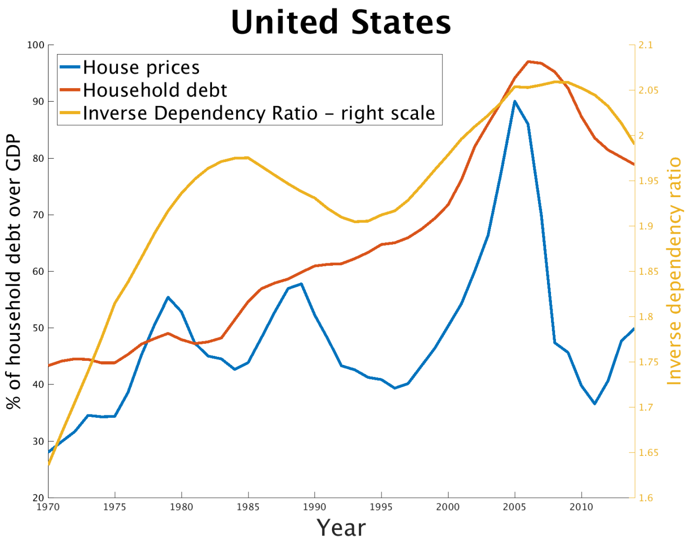


Figure 3.1: Historical series of house prices (BIS long series database), debt to GDP (IMF data) and inverse dependency ratio (World bank database) for United States

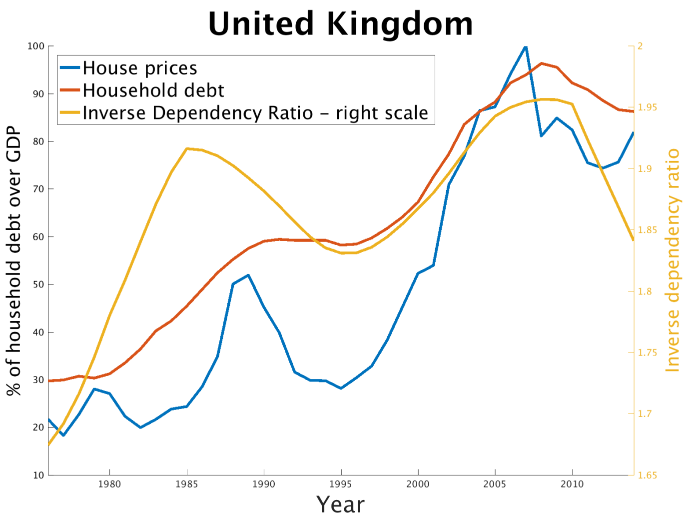


Figure 3.2: Historical series of house prices (BIS long series database), debt to GDP (IMF data) and inverse dependency ratio (World bank database) for United Kingdom

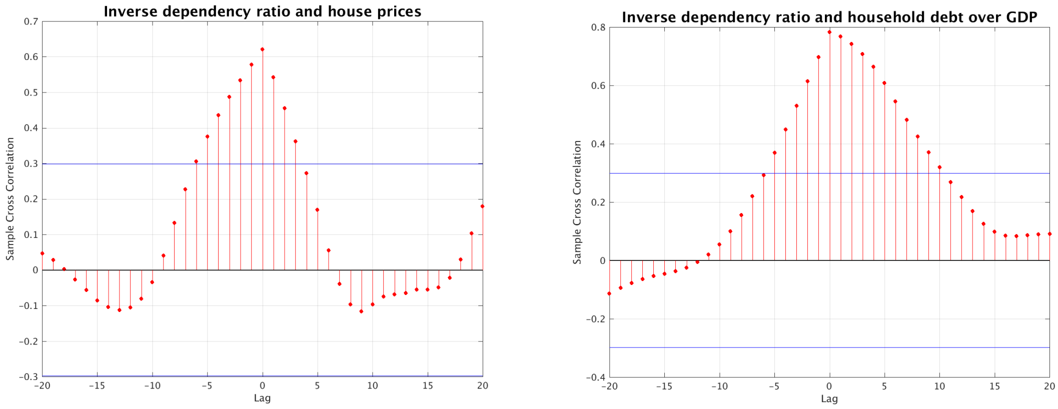


Figure 3.3: Cross correlograms of house prices and households debt to GDP with inverse dependency ratio in US (95% confidence bands are in blue)

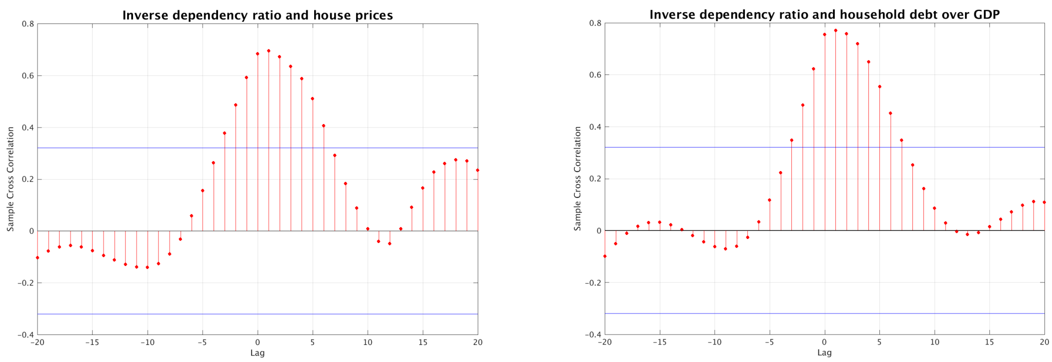


Figure 3.4: Cross correlograms of house prices and households debt to GDP with inverse dependency ratio in UK (95% confidence bands are in blue)

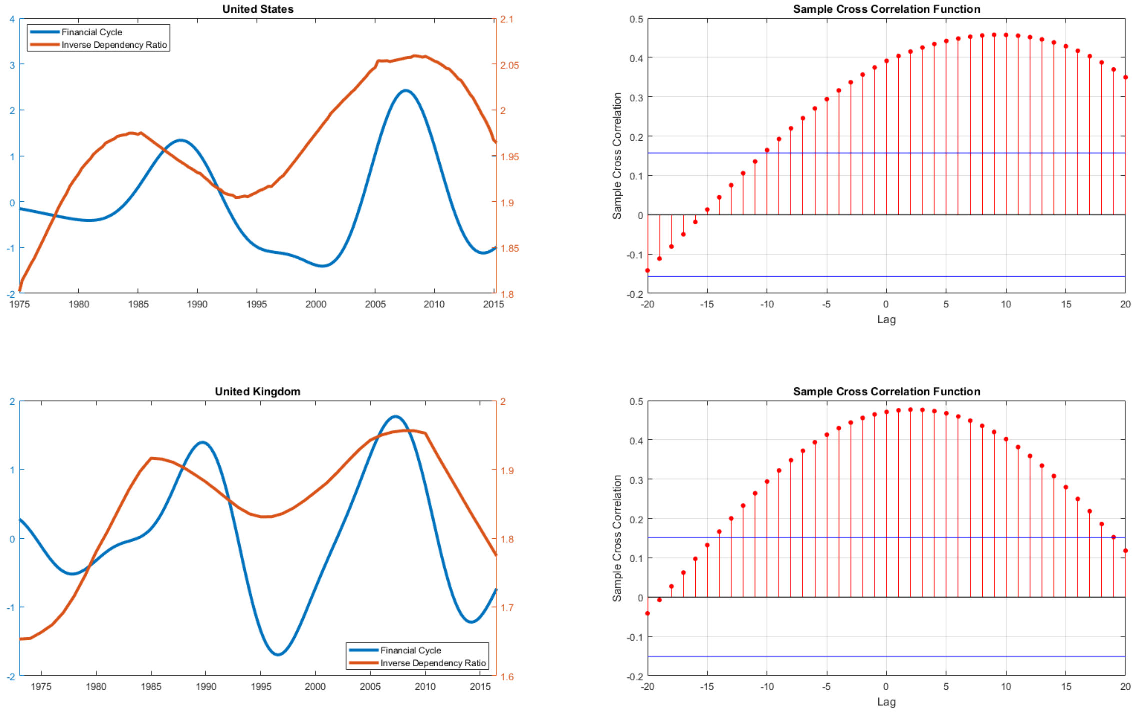


Figure 3.5: Financial (as measured by Drehmann, Borio and Tsatsaronis (2012)) cycle and inverse dependency ratio for the United States and United Kingdom and their respective correlograms.

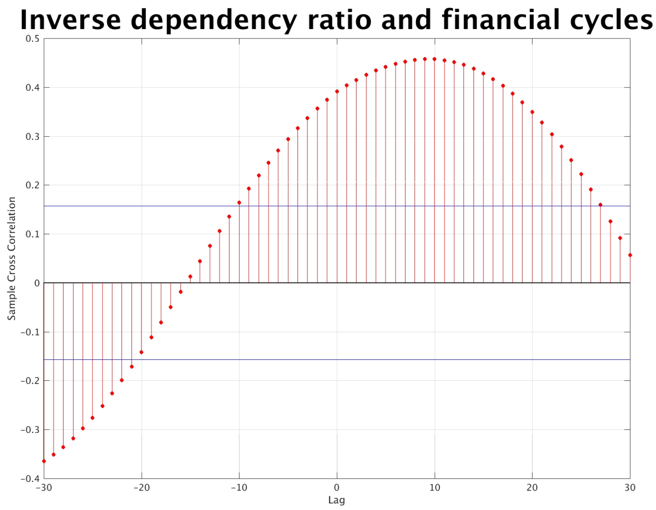


Figure 3.6: Correlogram between inverse dependency ratio and financial cycle

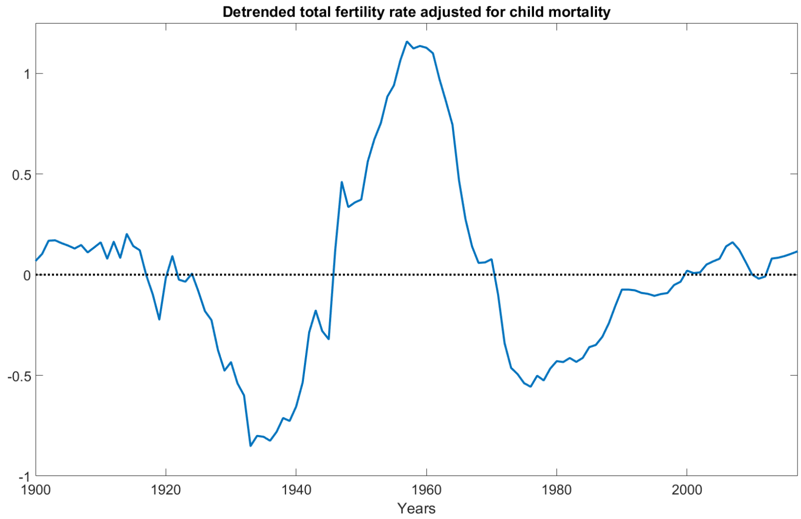


Figure 3.7: Detrended birth-rate, adjusted for children mortality in the United States

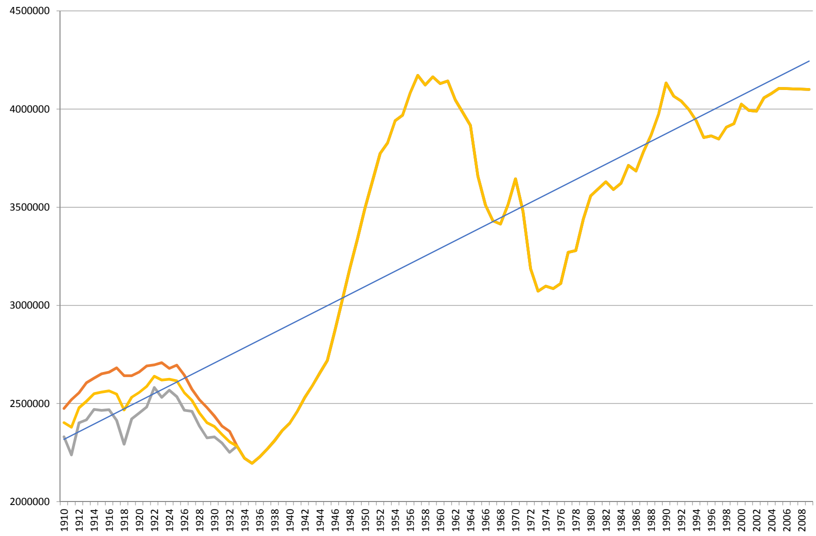


Figure 3.8: Live births in the US adjusted for children mortality under three alternatives hypothesis. The linear trend is traced under the central scenario.

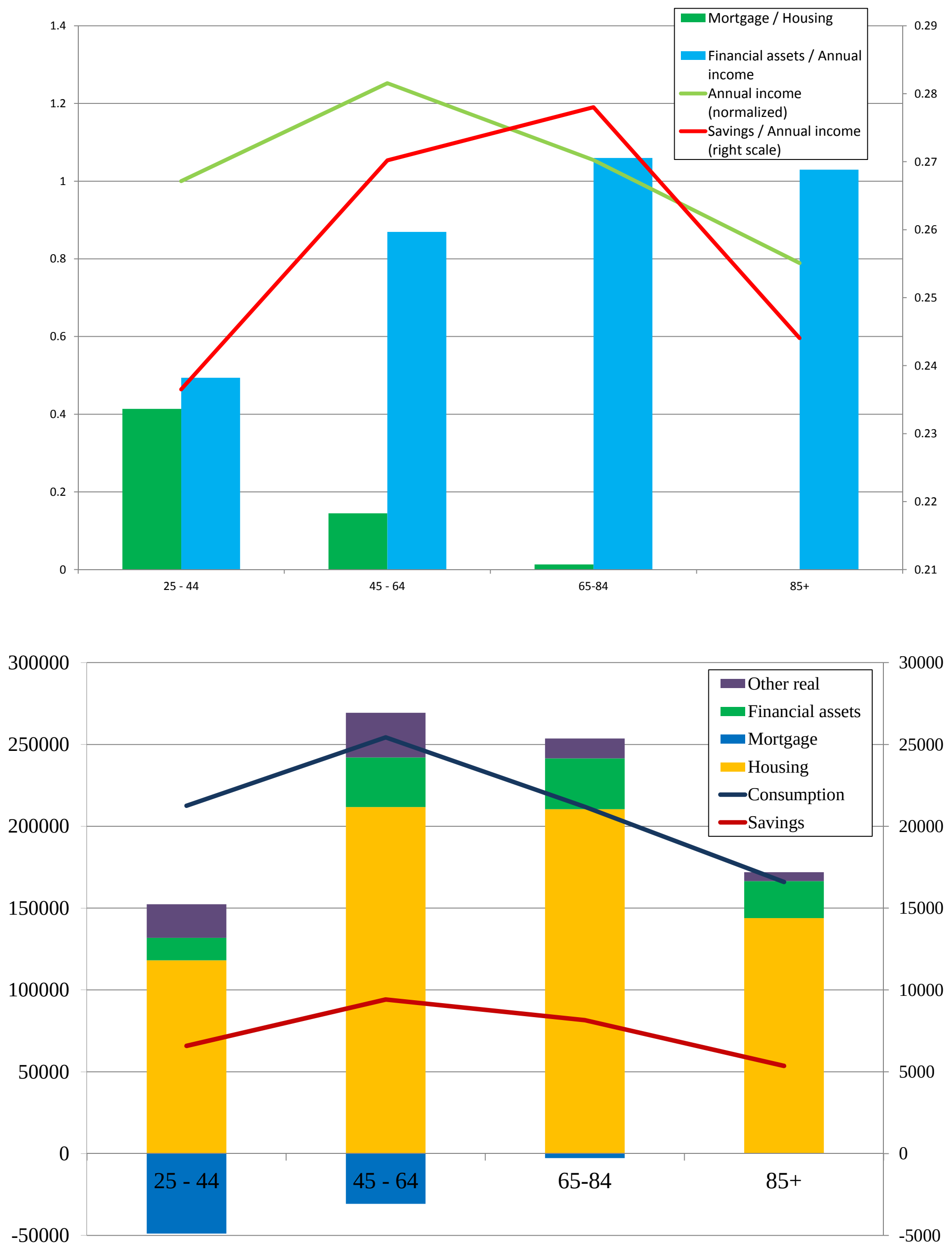


Figure 3.9: Life-cycle of income, savings and wealth in Italy 2014, source: SHIW

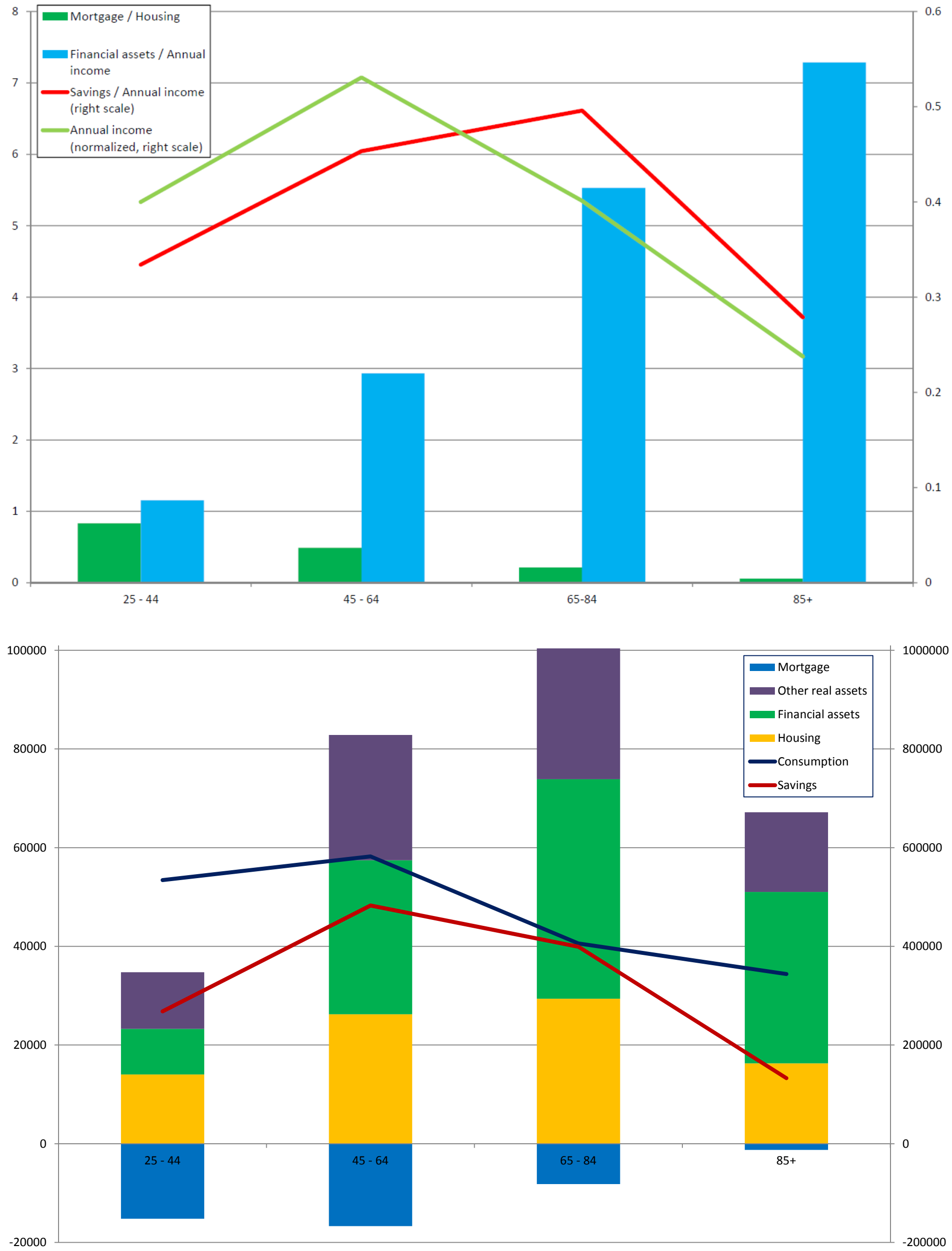


Figure 3.10: Life-cycle of income, savings and wealth in United States 2013, source: SCF and CE

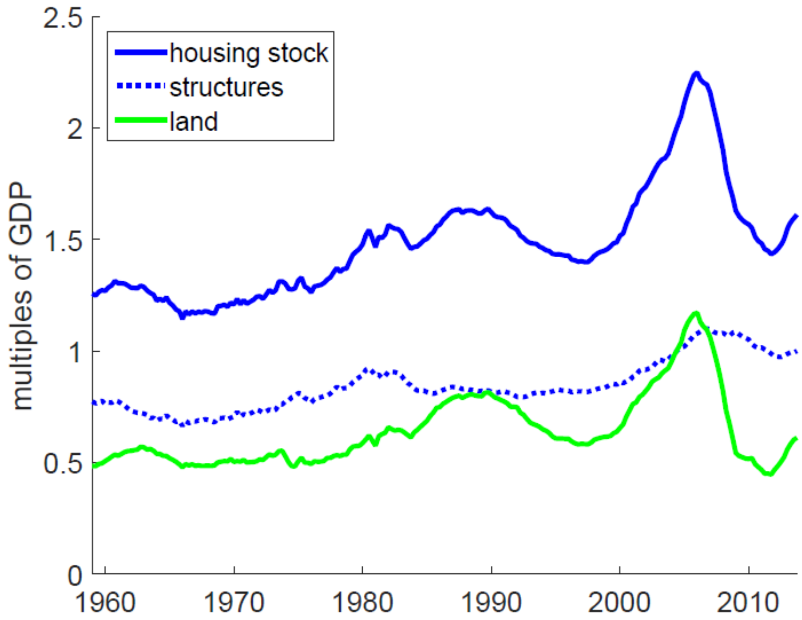


Figure 3.11: Historical decomposition of housing value in structures and land values (source: Piazzesi and Schneider, 2016)

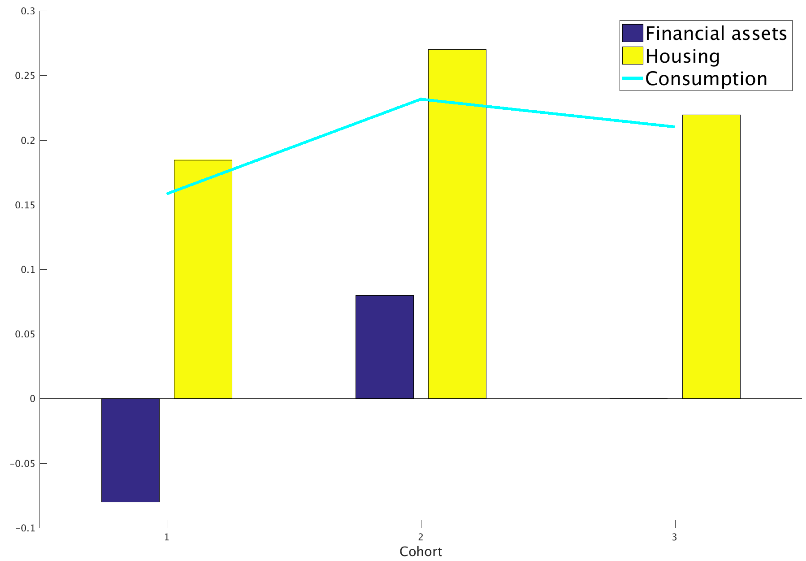


Figure 3.12: Steady state life-cycle of consumption, housing and financial assets in the model (end of period)

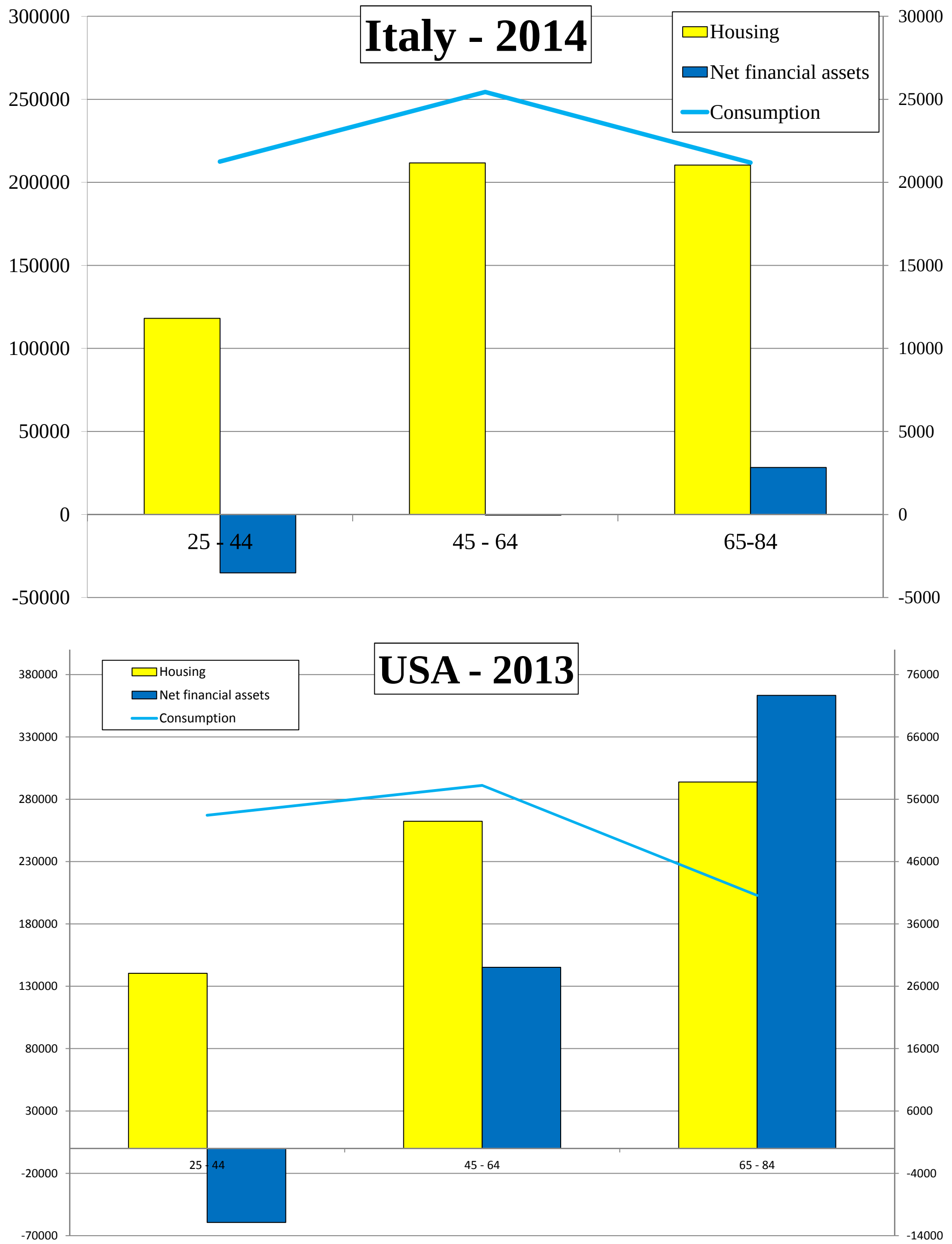


Figure 3.13: Life-cycle of consumption, housing and financial assets in data

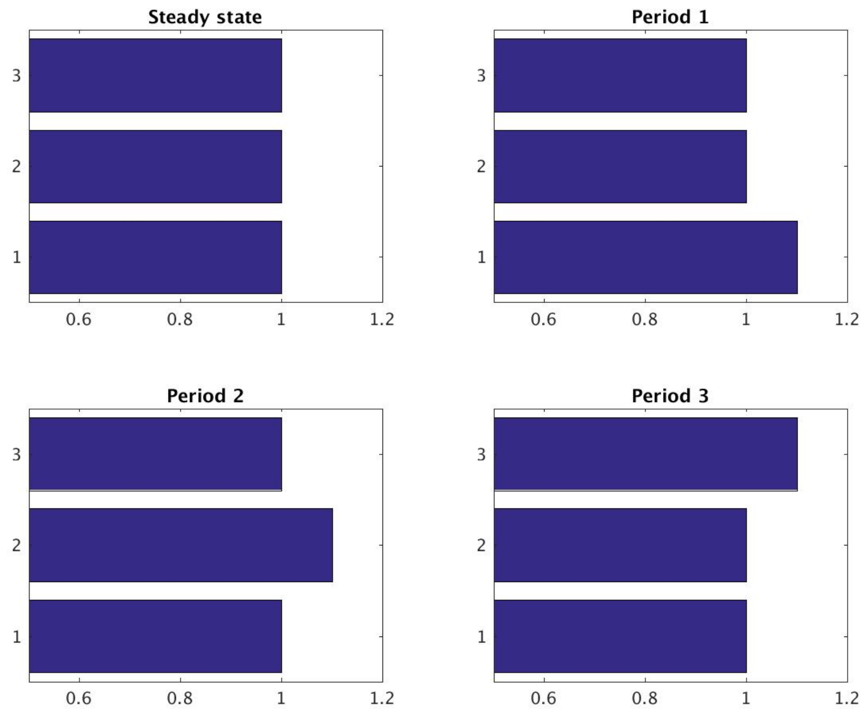


Figure 3.14: Population pyramid of the economy in steady state and in the following three periods.

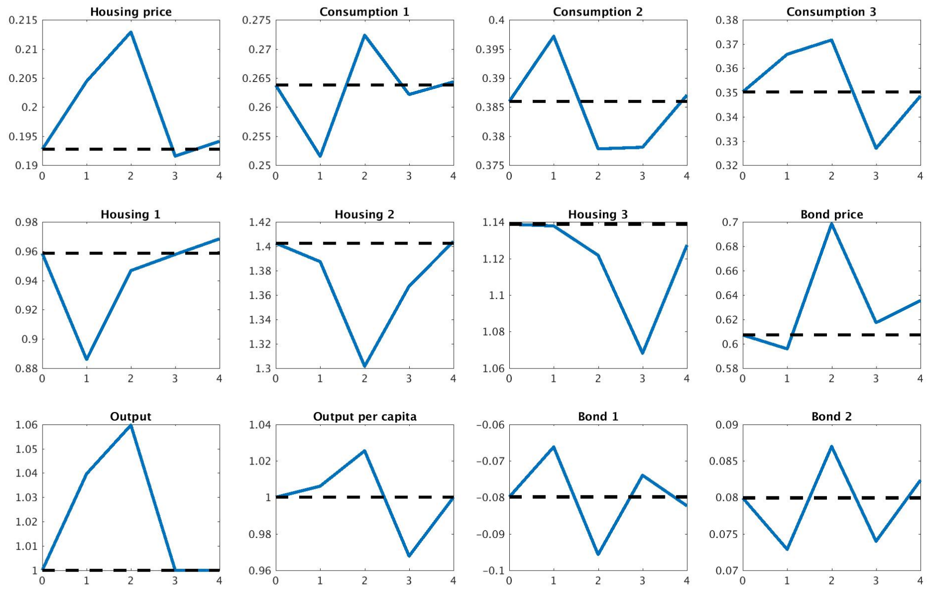


Figure 3.15: Patterns of the variables after a demographic shock in period 1, black lines are steady state values

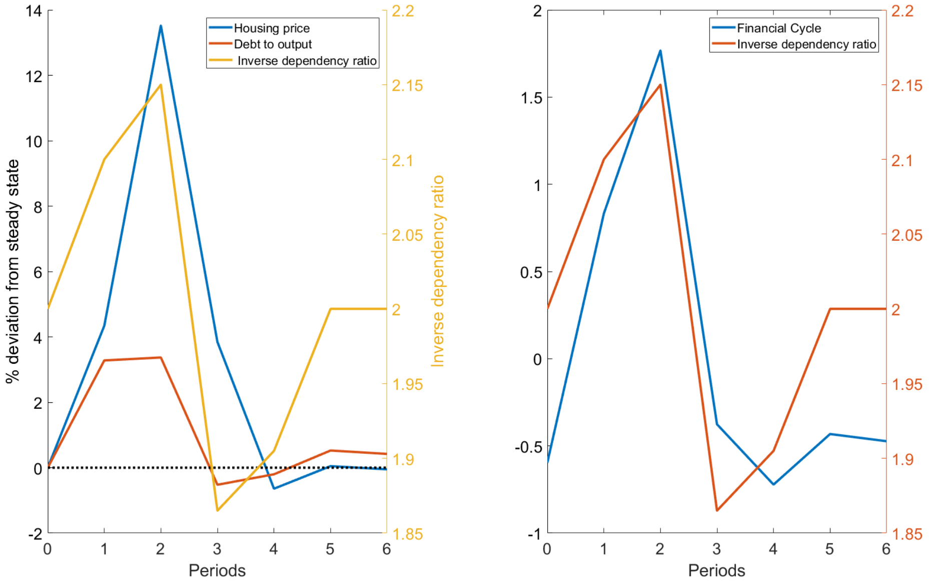


Figure 3.16: In the left panel housing price, debt-to-output and inverse dependency ratio (right hand scale) impulses to the imputed shock are plotted. In the right panel the financial cycle indicator is represented with the inverse dependency ratio (right hand scale). On the x axis time is measured in periods of the model.

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