Optimal Monetary Policy and Disclosure with an Informationally-Constrained Central Banker*

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Abstract

We study optimal monetary policy and central bank disclosure when the monetary authority has only incomplete information of the current economic state. Firms make their nominal pricing decisions under uncertainty. We find that implementing flexible-price allocations is both feasible and optimal despite an abundance of measurability constraints; we explore a series of different implementations. When monetary policy is sub-optimal, public information disclosure by the central bank is welfare-improving as long as either firm information or central bank information is sufficiently precise.

Keywords: monetary policy, nominal rigidities, monetary shocks, informational frictions, central bank disclosure, forward guidance.

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1 Introduction

We study optimal monetary policy and central bank disclosure with an informationally-constrained monetary authority. In particular, we consider the problem of a central banker who observes only an imperfect signal of the economic state each period and must make monetary policy decisions on the basis of their incomplete information set.

There are two broad motivations for introducing central bank incomplete information when considering optimal monetary policy. The first is simply realism. A large theoretical literature emphasizes the importance of firm informational frictions for business cycle fluctuations and policy; see e.g. Woodford (2003); Mankiw and Reis (2002); Mackowiak and Wiederholt (2009); Lorenzoni (2010); Angeletos and La'O (2020). It is typical in these models to assume that the policymaker sets the nominal interest rate under complete information of the aggregate state, putting the central banker at an enormous informational advantage relative to the firms.

Given the vast amount of resources that central bank research departments devote to estimating current economic conditions and forecasting future ones, one might question the realism of this assumption. While the central bank might pay more attention to the economy than, say, the average firm, it is unlikely that the central bank knows the underlying state of the economy with infinite precision.

A second motivation for considering an informationally-constrained central banker is to provide a coherent microfoundation for what others in the New Keynesian literature call monetary policy "shocks." Monetary policy shocks are modeled as unanticipated shocks to either the nominal interest rate or the money supply that are orthogonal to the underlying state of the economy. A large empirical literature attempts to estimate unanticipated monetary policy shocks; see, e.g. Romer and Romer (2004); Christiano, Eichenbaum, and Evans (2005); Gürkaynak, Sack, and Swanson (2005); Gertler and Karadi (2015); Nakamura and Steinsson (2018)

But what are these monetary policy "shocks" in real life? Certainly the FOMC does not choose an interest rate then, ex post, adds random noise. One rationalization for these shocks is that they are the byproduct of the central banker's incomplete information about the state. If the monetary authority receives a noisy, private signal about the aggregate state and fashions the nominal interest rate upon its signal, then any noise in this signal would translate into variation in the interest rate that is orthogonal to the underlying state. This variation would therefore appear to the econometrician as a "monetary policy shock."

However, under this interpretation, while monetary policy shocks are unanticipated from the point of view of the public, they are not at all random from the point of view of the central banker. If, for example, the central banker were to disclose its private information to the public, monetary policy shocks would all but disappear.

In order to tell whether or not it is socially desirable for the central bank to disclose its information, as well as understand how the central banker's incomplete information constrains optimal policy, one needs to explicitly model this friction.

Our framework. The economy we study is a relatively standard, microfounded, general equilibrium model with nominal rigidities. There is a representative household that consumes, saves, and supplies labor. Production takes place within a unit mass of differentiated, intermediate-good firms. Firms face a common, aggregate productivity shock—this is the only real shock in the model. It follows that the underlying flexible-price allocations are efficient.

Firms set nominal prices; we assume that the nominal rigidity takes the form of an informational friction along the lines of Woodford (2003); Mankiw and Reis (2002); Mackowiak and Wiederholt (2009); Angeletos and La'O (2020). Specifically, firms make their nominal pricing decisions under incomplete information of the aggregate state. The household, on the other hand, faces no frictions and makes its decisions under complete information.

Finally, there is a consolidated fiscal and monetary authority with full commitment. The tool of the fiscal authority is a constant revenue tax or subsidy; the tool of the monetary authority is the nominal interest rate. At the end of every period, the aggregate state is revealed to all agents—the firms, the household, and the policy maker—and markets clear.

We introduce two non-standard features into this model. The first is an informational constraint on the central banker. In any period, the central banker, like any firm, has only incomplete information about the current state. Specifically, we assume that the central banker receives a noisy private signal about current productivity and that the nominal interest rate can be contingent only on the central banker's incomplete information set at that point in time.

The second non-standard, but related, feature is that our model accommodates a form of "policy uncertainty." In particular, if the nominal interest rate is contingent on the central banker's private information, then from the point of view of the firms there exist monetary policy "shocks." When making their nominal pricing decisions, firms must therefore form beliefs not only over fundamentals but also over these contingencies of monetary policy.

We allow firms to have private information not only about aggregate productivity, but also about the information of the central banker. We model this as a conditional correlation between the private signals of the firms and the private signal of the central banker—conditional on fundamentals. One can interpret this conditional correlation as firm optimism or pessimism about monetary policy that is orthogonal to firm beliefs about fundamentals.

Optimal Monetary Policy. Our first set of results characterizes optimal monetary policy taking as given the informational constraints of the firms and of the central banker. We follow and adapt the classic Ramsey approach to our setting by specifying the policy instrument—here, the nominal interest rate—as a function of the exogenous information set of the central banker.

Our analysis follows in spirit that found in Correia, Nicolini, and Teles (2008); Correia, Farhi, Nicolini, and Teles (2013); Angeletos and La'O (2020). In particular, we first show that the first best efficient allocation is implementable under flexible prices with a subsidy that offsets the monopolistic mark-up. We then prove that the set of flexible-price allocations can be implemented under sticky prices.

While it may not be surprising that replicating flexible-price allocations under sticky prices is a desirable goal for policy, it might come as a surprise that it is feasible. We show that there exists paths for the nominal interest rate and prices that satisfy all measurability constraints imposed on the firms and the central banker yet, at the same time, implement allocations *as if* the firms and central banker made their respective decisions under complete information of the aggregate state.

We focus on two classes of implementations. The first class of implementations are ones in which the nominal interest rate does not condition on the central banker's private signal about current fundamentals. The second class of implementations are ones in which nominal interest rates do. Finally, we provide a characterization of the full set of flexible-price implementations.

We find that all flexible-price implementations share a common feature: price levels must respond to past economic fundamentals. It is precisely these future contingencies that are essential for circumventing present-day measurability constraints.

It follows that an optimal policy in our model is any monetary policy that implements flexible-price allocations coupled with a subsidy that offsets the constant mark-up—such a policy implements the first best. Furthermore, the necessity of price contingencies on past economic fundamentals implies that price stability is never optimal.

Welfare Effects of Central Bank Disclosure. The frictions we introduce allow us to study another question of interest: what is the social value of central bank information disclosure?

For this analysis we follow in the tradition of Morris and Shin (2002), Svensson (2006), Hellwig (2005), Angeletos, Iovino, and La'O (2016) and others that model central bank disclosure as public signals about macroeconomic conditions. Our model differs from all of these previous works in that the information disclosed by the central bank is itself the same incomplete information upon which the central bank sets monetary policy.

To answer this question, we restrict attention to a particular class of equilibria in which there exists a unique optimal policy that implements flexible-price allocations. We then consider the case in which monetary policy is sub-optimal; in particular, we let the interest rate rule deviate from its optimum.

We find that the welfare effect of central bank information disclosure in this context is generally ambiguous. We provide sufficient conditions under which central bank disclosure is welfare-improving—these are joint conditions on the preference structure of the household, the elasticity of substitution across goods, and the precisions of the firms' and central bank's private signals.

By publicly disclosing its signal about current economic conditions, the central bank reduces the uncertainty firms face about aggregate productivity. Disclosure of central bank information furthermore eliminates all monetary policy "shocks" as firms can perfectly anticipate the interest rate. These reductions in fundamental and interest rate uncertainty contributes to greater equilibrium welfare. However, there is an adverse welfare effect of disclosure stemming from greater coordination of prices on the public signal. If this adverse welfare effect is sufficiently strong, then public disclosure can be detrimental.

We find that as long as either firm information or central bank information is sufficiently precise, the benefits of central bank disclosure outweigh the costs. In this case, central bank disclosure is socially desirable.

Related Literature. Methodologically, we analyze optimal monetary policy following the primal approach. We thereby follow in the tradition of the primal approach conceived in public finance (Atkinson and Stiglitz, 1980), then imported into macro and adapted for studying the Ramsey problem (Lucas and Stokey, 1983; Chari, Christiano, and Kehoe, 1991, 1994; Chari and Kehoe, 1999). In particular, our analysis of optimal monetary policy closely mirrors those found in Correia, Nicolini, and Teles (2008); Correia, Farhi, Nicolini, and Teles (2013); Angeletos and La'O (2020); these papers similarly consider monetary policy in economies featuring nominal rigidity. Angeletos and La'O (2020) in particular study an economy in which firms face informational frictions. Our model differs from all of these previous works in that we explicitly model and study the implications of an informational constraint on the policymaker.[1]

In terms of the questions we address, this paper contributes to two literatures. One is the literature that considers how nominal rigidities originating in firm informational frictions affect the optimal conduct of monetary policy as in, e.g., Ball, Mankiw, and Reis (2005); Adam (2007); Lorenzoni (2010); Paciello and Wiederholt (2014); Angeletos and La'O (2020). The other is the literature that studies, in similar microfounded, general equilibrium models with nominal informational rigidities, the welfare effects of public information disclosures as in, e.g., Hellwig (2005); Walsh (2007); Lorenzoni (2010); Baeriswyl and Cornand (2010); Angeletos, Iovino, and La'O (2016).[2]

Relative to both of these literatures, our contribution is to revisit these questions in a setting

[1] Our framework furthermore differs from Angeletos and La'O (2020) in that we abstract from any real informational rigidities.

[2] In an earlier, extremely influential paper, Morris and Shin (2002) consider the welfare implications of public information disclosure in an abstract, linear-quadratic "beauty contest" game featuring strategic complementarities; see also the follow-up work by Svensson (2006); Morris, Shin, and Tong (2006); James and Lawler (2011). While this work has been paramount in catalyzing the literature on public information disclosure, its abstract framework lacks explicit microfoundations and therefore has limited power in answering applied, macroeconomic policy questions.

in which the central banker is informationally-constrained but at the same time controls the nominal interest rate. This implies that any information the central banker chooses to disclose to the public is the same information upon which monetary policy is based.

A closely related paper, in this respect, is Kohlhas (2020). Kohlhas (2020) likewise considers the problem of a policymaker with incomplete information but in a reduced-form, static, linear-quadratic beauty-contest game with dispersed information. Similar to our setting, the policymaker in Kohlhas (2020) can fashion its policy instrument to be contingent on its own information; it can also choose to disclose its information to the agents. Kohlhas (2020) furthermore considers a business cycle application based on the model of Hellwig (2005) in which firms set nominal prices, the household faces a cash-in-advance constraint, and the monetary authority sets the money supply.[3]

Relative to Kohlhas (2020), we study a fully microfounded, dynamic, general equilibrium model with nominal rigidities and assume that the policy instrument of the central banker is the nominal interest rate. We find that the conclusions of the business cycle application of Kohlhas (2020) do not apply in our context. The primary reason our results diverge is the difference in microfoundations, particularly the choice of tool of the central banker. In Kohlhas (2020), the money supply is constrained to be contingent on the central banker's information set. As a result, the complete-information first best cannot be implemented, and optimal policy achieves only a second best.

In our framework, we likewise assume that the tool of the central banker is constrained to be contingent on the central banker's information set. However, this tool is the nominal interest rate. By leveraging the *dynamic* nature of the Euler equation, we find that there exists paths for the nominal interest rate and prices that effectively circumvent the central banker's informational constraint and implement flexible-price allocations. By implication, the complete-information first best can be achieved. We furthermore show that for all possible implementations of flexible-price allocations, the money supply varies with the current economic state; it therefore does not satisfy the measurability restriction investigated in Kohlhas (2020).

Layout. This paper is organized as follows. In Section 2 we describe the model. In Section 3 we characterize the set of allocations that may be implemented in this economy as competitive equilibria under flexible prices; in Section 4 we characterize equilibria under sticky prices. In Section 5 we study optimal monetary policy. In Section 6 we analyze the welfare implications of central bank disclosure. Section 7 concludes.

[3] See Section 6.3 of Kohlhas (2020).

2 The Model

Time is discrete, indexed by $t = \{0, 1, \ldots\}$.

Production. There is a unit mass continuum of intermediate-good producers, indexed by $i \in I \equiv [0,1]$. Intermediate good firm $i \in I$ produces output y_{it} in period t according to the following CRS technology:

$$y_{it} = A_t \ell_{it}, \qquad \forall i \in I,$$

where ℓ_{it} is the labor input of firm i at time t, and $A_t > 0$ is an aggregate productivity shock. The profits of firm i at time t are given by

$$\pi_{it} = (1 - \tau) p_{it} y_{it} - W_t \ell_{it},$$

where p_{it} is the nominal price charged by firm i at time t, W_t is the nominal wage, and τ is a constant revenue tax.

There is a perfectly-competitive final good firm that aggregates intermediate goods according to the following CES production function:

$$Y_t = \left(\int_i y_{it}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}, \tag{1}$$

where $\theta > 1$ is the elasticity of substitution across intermediate goods. The output of the final good firm is consumed by the household.

The Household. There is a representative household with time-separable utility:

$$\sum_{t=0}^{\infty} \beta^t \left[U(C_t) - V(L_t) \right],$$

where the scalar $\beta \in (0,1)$ is the household's discount factor. At time t, the household draws utility from consumption C_t and disutility from labor L_t. We assume typical regularity conditions on the functions $U : \mathbb{R}_+ \to \mathbb{R}_+$ and $V : \mathbb{R}_+ \to \mathbb{R}_+$: they are twice continuously-differentiable with $U', V' > 0, U'' < 0, V'' > 0$, and satisfy the Inada conditions.

The household's budget constraint at time t is given, in nominal terms, by:

$$P_t C_t + B_t \leq W_t L_t + (1 + \iota_{t-1}) B_{t-1} + \int_i \pi_{it} di + T_t,$$

where P_t is the nominal price of the final consumption good at time t, W_t is the nominal wage, and B_t are risk free nominal bonds that pay $(1 + \iota_t) B_t$ one period later. The household furthermore receives dividends (profits) from owning all firms and collects lump-sum transfers T_t from the government.

The monetary and fiscal authority. The government consists of a consolidated monetary and fiscal authority with commitment. The government's budget constraint is given by:

$$\int_i \tau p_{it} y_{it} di + B_t = T_t + (1 + \iota_{t-1}) B_{t-1}.$$

We assume that the government can set the constant revenue tax τ and the gross nominal interest rate $1 + \iota_t$. We abstract from the zero lower bound on the nominal interest rate.

Market clearing. In any period t, the quantity consumed by the household must equal the total production of the final good, $C_t = Y_t$, and aggregate labor supply must equal aggregate labor demand, $L_t = \int_i \ell_{it} di$. Nominal bonds are in zero net supply: $B_t = 0$.

2.1 Shocks and Signals

The central banker and firms make their decisions under uncertainty. We model this uncertainty as follows.

The fundamental state. In each period t, Nature draws a random variable s_t from a finite set S. This random variable determines period t fundamentals, namely TFP. In particular, we write TFP at time t as a function, $A : S \to \mathbb{R}_+$, measurable in the current state: $A_t = A(s_t)$.

We call s_t the "fundamental state" and we assume s_t is Markov and evolves according to probability distribution $\mu(s_t|s_{t-1})$. We denote a history of states by $s^t = \{s_0, ..., s_t\} \in S^t$ and the unconditional probability of history s^t by $\mu(s^t)$.

Information of the Central Banker. In each period, the central banker observes a noisy private signal about the current fundamental state. We model this as follows.

In each period t, Nature draws a random variable ω_{pt} from a finite set Ω_p according to a probability distribution φ_p. We let $\varphi_p(\omega_{pt}|s_t)$ denote the probability of ω_{pt} conditional on s_t.[4] The variable ω_{pt} represents the "signal" the central banker observes in period t about the current fundamental state, s_t.

Information of the firms. Similarly, in each period, every firm receives private information about the current fundamental state *and* the information of the central banker. We model this as follows.

For every i, in each period t, Nature draws a random variable ω_{it} from a finite set Ω according to a probability distribution φ. We let $\varphi(\omega_{it}|s_t, \omega_{pt})$ denote the probability of ω_{it} conditional on (s_t, ω_{pt}). The variable ω_{it} represents the "signal" that firm i observes in period t; note that it can

[4] We use the subscript p to indicate the signal observed by the policymaker.

contain information about the fundamental state, s_t, as well as about the signal of the central banker, ω_{pt}.

Conditional on (s_t, ω_{pt}), we assume that the draws of ω_{it} are i.i.d. across firms and a law of large number applies so that $\varphi(\omega_{it}|s_t, \omega_{pt})$ is also the fraction of the population that receives the signal ω_{it}.[5]

The full aggregate state. The random variable s_t represents the fundamental state of the economy, namely aggregate TFP. However, we will soon impose measurability constraints such that the information of the central banker and the firms may affect their policy and nominal pricing decisions. As a result, these signals have the potential to affect equilibrium outcomes.

We therefore denote the *true*, full aggregate state by $\bar{s}_t \in \bar{S}$ and assume it is given by the set:

$$\bar{s}_t = \{s_t, \omega_{pt}, \varphi(\omega_{it}|s_t, \omega_{pt})\}.$$

That is, the full aggregate state in this economy includes not only current economic fundamentals, s_t, but also the private signal of the central banker, ω_{pt}, as well as the realized cross-sectional distribution of firm signals ω_{it} at time t. We denote a history of aggregate states by $\bar{s}^t = \{\bar{s}_0, \ldots, \bar{s}_t\} \in \bar{S}^t$.

We assume that the central banker and the firms learn the full aggregate state $\bar{s}_t$ at the end of the period. At that point, $\bar{s}_t$ becomes common knowledge.

2.2 Informational Constraints and Nominal Rigidities

We denote the monetary authority's information set at time t by ω_p^t and each firm i's information set at time t by ω_i^t. Following our previous discussion, these sets include the following objects:

$$\omega_p^t \equiv \{\omega_{pt}, \bar{s}^{t-1}\} \qquad \text{and} \qquad \omega_i^t \equiv \{\omega_{it}, \bar{s}^{t-1}\}.$$

That is, at time t, both the central banker and each firm have incomplete information about the current economic state but complete information about the history of past states.

We impose the following two measurability restrictions.

Assumption 1. *(i) The nominal price of intermediate good firm i at time t is constrained to be measurable in the firm's information set at time t:*

$$p_{it}(\omega_i^t), \qquad \forall \omega_i^t \in \Omega^t \equiv \Omega \times \bar{S}^{t-1}. \tag{2}$$

(ii) The nominal interest rate at time t is constrained to be measurable in the central banker's information set at time t:

$$\iota_t(\omega_p^t), \qquad \forall \omega_p^t \in \Omega_p^t \equiv \Omega_p \times \bar{S}^{t-1}. \tag{3}$$

[5] See Uhlig (1996) for an applicable law of large numbers with a continuum of draws.

Assumption 1 constitutes the two informational constraints in our model. Part (i) is a measurability constraint on the nominal price for each firm. This constraint encompasses the nominal rigidity in our model by imposing that each firm make its nominal pricing decision based on the firm's incomplete private information. Part (ii) is a measurability constraint on the nominal interest rate; it similarly imposes that the central banker must set the nominal interest rate based on its own incomplete private information about current economic conditions.

Timing and the household. The measurability constraints dictated by Assumption 1 boil down to an implicit "timing" assumption. Nature draws the full aggregate state, $\bar{s}_t \in \bar{S}$, at the beginning of period t. The central banker and firms observe their private signals and make their respective decisions within the period, under incomplete information about the aggregate state. Once the nominal interest rate and nominal prices are set, the aggregate state is revealed.

The household, on the other hand, makes its consumption, savings, and labor supply decisions at the end of the period. At this point, $\bar{s}_t$ is common knowledge and all real allocations adjust so that supply equals demand and markets clear. We thereby denote the household's information set at time t by the history $\bar{s}^t$.

We make the assumption that the central banker and the firms learn the aggregate state $\bar{s}_t$ at the end of the period for simplicity. However, this assumption is compatible with the notion that these agents observe endogenous outcomes—aggregate prices and market-clearing quantities—at the end of period, once their own choices have been set.

2.3 Examples and Interpretation

These measurability constraints may appear abstract. In what follows we provide examples of the nominal rigidity featured in our paper that may be more familiar to certain readers. We furthermore provide some interpretation of the new elements introduced in our framework.

The nominal rigidity. Consider part (i) of Assumption 1. This constraint, which requires p_{it} to be measurable in ω_i^t rather than s^t, introduces the same type of nominal rigidity as the one featured in Woodford (2003), Lorenzoni (2010), Angeletos and La'O (2020), and a large, growing literature that replaces Calvo-like sticky prices with an informational friction.[6]

Take, for example, the model of Correia, Nicolini, and Teles (2008) featuring a one-period version of sticky prices. In that economy, there are two types of firms: a fraction $\alpha \in (0, 1)$ of "sticky-price" firms set prices one period in advance; the remaining $1 - \alpha$ are "flexible-price"

[6] See, for example, Mankiw and Reis (2002); Ball, Mankiw, and Reis (2005); Hellwig (2005); Adam (2007); Nimark (2008); Mackowiak and Wiederholt (2009); Paciello and Wiederholt (2014); Angeletos, Iovino, and La'O (2016); Angeletos and Lian (2018); Angeletos and La'O (2020).

firms that choose prices contemporaneously. This setting can be directly nested in our framework by letting φ assign probability α to $\omega_i^t = \bar{s}^{t-1}$ and probability $1-\alpha$ to $\omega_i^t = \bar{s}^t$. In this case, a fraction α of sticky-price firms know the previous period's state with certainty but cannot react to the current economic state. On the other hand, a fraction $1-\alpha$ of firms perfectly observe the state at time t and set their prices accordingly.

Alternatively, consider models with "sticky information" as in Mankiw and Reis (2002) and Ball, Mankiw, and Reis (2005). In each period a fraction $\lambda \in (0,1)$ of randomly-selected firms observe perfectly the state of the economy while the remaining $1-\lambda$ firms continue to set their prices based on their past information. This setting can be nested in our framework by first dropping the assumption that the aggregate state is observed by all at the end of each period, and second by letting φ assign probability λ to $\omega_i^t = \bar{s}^t$ and probability $1-\lambda$ to $\omega_i^t = \omega_i^{t-1}$. In this case, λ is the probability with which a firm updates its information set in any given period, while $1-\lambda$ is the probability with which the firm is stuck with its previous information set.[7]

Finally, consider models with noisy Gaussian signals, as in Morris and Shin (2002); Woodford (2003); Hellwig (2005); Nimark (2008); Lorenzoni (2010); Angeletos et al. (2016). These models may be nested in our setting by specifying the underlying aggregate TFP shock as a Gaussian random variable and letting each firm observe a private noisy Gaussian signal about it. In Section 6, we consider an explicit example along these lines.[8]

The constraint on the central banker. We have argued that the nominal rigidity imposed in part (i) of Assumption 1 is essentially the same friction that appears throughout the broad literature incorporating informational frictions as a form of nominal rigidity. Relative to this literature, the key novelty of our framework is part (ii). This is the measurability constraint on the central banker: the central banker must set the nominal interest rate each period under incomplete information about current economic fundamentals.

As a concrete example, consider a simple, Gaussian setting. Suppose that log productivity, $\log A(s_t)$, is a Gaussian random variable. In each period t, the central banker observes a noisy private signal ω_{pt} about productivity given by:

$$\omega_{pt} = \log A(s_t) + \epsilon_{pt}, \tag{4}$$

[7]In order to strictly nest sticky information models, it is necessary that we drop our assumption that the aggregate state is observed by all firms at the end of period. However, underlying this is the implicit assumption that firms cannot observe or learn from their own market clearing quantities at the end of the period—an arguably unpalatable assumption.

[8]In order to strictly nest Gaussian settings, we must of course move from a discrete to a continuous state space and define states and signals as continuous random variables with associated probability density functions. At this level of generality, this adds unnecessary complication without delivering anything more in terms of results. We thus choose to work with discrete random variables for the majority of our analysis. It is only in Section 6 that we impose a continuous, Gaussian information structure and leverage this structure to obtain explicit, closed-form solutions for equilibrium prices and allocations.

where $\epsilon_{pt} \sim \mathcal{N}(0, \sigma_p^2)$ is the noise in the central banker's signal. Part (ii) of Assumption 1 dictates that the central banker sets the nominal interest rate at time t based only on its incomplete information set $\omega_p^t = (\omega_{pt}, \bar{s}^{t-1})$. An implication of this measurability constraint is that if the nominal interest rate is chosen to vary with the central banker's signal, ω_{pt}, then it varies with the noise in this signal, ϵ_{pt}.

Policy uncertainty. Finally, our model accommodates a form of "policy uncertainty." Firms face uncertainty not only over fundamentals, but also over the information of the central banker. If the central banker relies on its private signal ω_{pt} when setting nominal interest rates, the firms may try to form beliefs over these contingencies.

The information of each firm is embedded in its signal ω_{it}. Our framework allows these signals be correlated with the central banker's signal ω_{pt} even conditional on current fundamentals s_t. One can interpret this conditional correlation as public optimism or pessimism about monetary policy that is orthogonal to firm beliefs about fundamentals.

Take, for example, the Gaussian setting considered above in which the central banker receives a noisy private signal given by (4). In this example, the noise in the central banker's private signal, ϵ_{pt}, could be interpreted as an error or bias in the central bank's perception of the economy. Variation in ϵ_{pt} could be driven by new appointments to the Federal Reserve Board of Governors, a fresh rotation of Federal Reserve Bank presidents on the FOMC, or simply noise in the Fed research department's forecast of the economy.

Next, suppose each firm observes two Gaussian signals: the first is a private signal about TFP, the second is a private signal about the signal error of the central banker:

$$z_{it} = \epsilon_{pt} + \zeta_{it}^z,$$

where $\zeta_{it}^z \sim \mathcal{N}(0, \sigma_z^2)$ is idiosyncratic noise. Note that z_{it} contains no information about economic fundamentals s_t. Nevertheless, variation in z_{it} affects the firm's beliefs about the Fed's perception of the economy and can thereby influence the firm's nominal pricing decision. One could interpret this process as changes in the public's perception of the "hawkish" or "dovish" nature of FOMC members.

We return to this specific Gaussian example in Section 6. Regardless, these signals need not be taken so literally. They are simply modeling devices that allow us to introduce a form of policy uncertainty that is orthogonal to firm uncertainty about economic fundamentals.

2.4 Equilibrium Definitions

We close this section with formal definitions of equilibria. We denote a price system in this economy by the following set of producer prices, the price of the final consumption good, and the nominal wage:

$$\varrho = \left\{ \left\{ p_{it}(\omega_i^t) \right\}_{\omega_i^t \in \Omega^t}, P_t(\bar{s}^t), W_t(\bar{s}^t) \right\}_{\bar{s}^t \in \bar{S}^t}.$$

Given nominal prices, the Dixit-Stiglitz final good aggregator in (1), implies the typical downward-sloping CES demand function for intermediate goods given by:

$$y_{it}(\omega_i^t, \bar{s}^t) = \left(\frac{p_{it}(\omega_i^t)}{P_t(\bar{s}^t)} \right)^{-\theta} Y_t(\bar{s}^t), \tag{5}$$

with elasticity of substitution θ. Therefore, firm output (and labor) depends not only on its pre-set nominal price, but also on the aggregate state $\bar{s}^t$.

We denote an allocation in this economy by the following set:

$$\xi = \left\{ \left\{ \ell_{it}(\omega_i^t, \bar{s}^t), y_{it}(\omega_i^t, \bar{s}^t) \right\}_{\omega_i^t \in \Omega^t}, Y_t(\bar{s}^t), C_t(\bar{s}^t), L_t(\bar{s}^t) \right\}_{\bar{s}^t \in \bar{S}^t}$$

where individual firm output and labor input satisfy technology

$$y_{it}(\omega_i^t, \bar{s}^t) = A(s_t)\ell_{it}(\omega_i^t, \bar{s}^t), \qquad \forall \omega_i^t \in \Omega^t, \bar{s}^t \in \bar{S}^t \tag{6}$$

aggregate output and consumption is given by

$$C_t(\bar{s}^t) = Y_t(\bar{s}^t) = \left[\sum_{\omega_{it} \in \Omega} y_{it}(\omega_i^t, \bar{s}^t)^{\frac{\theta-1}{\theta}} \varphi(\omega_{it}|s_t, \omega_{pt}) \right]^{\frac{\theta}{\theta-1}}, \qquad \forall \bar{s}^t \in \bar{S}^t, \tag{7}$$

and aggregate labor is given by

$$L_t(\bar{s}^t) = \sum_{\omega_{it} \in \Omega} \ell_{it}(\omega_i^t, \bar{s}^t) \varphi(\omega_{it}|s_t, \omega_{pt}), \qquad \forall \bar{s}^t \in \bar{S}^t. \tag{8}$$

Finally, we denote a policy in this economy by the following set of nominal interest rates and taxes

$$\vartheta = \left\{ \tau, \iota_t(\omega_p^t), T_t(\bar{s}^t), B_t(\bar{s}^t) \right\}_{\bar{s}^t \in \bar{S}^t},$$

where $T_t(\bar{s}^t) = \tau \sum_{\omega_{it} \in \Omega} p_{it}(\omega_i^t) y_{it}(\omega_i^t, \bar{s}^t) \varphi(\omega_{it}|s_t, \omega_{pt})$ and bonds are in zero net supply: $B_t(\bar{s}^t) = 0$, for all states $\bar{s}^t \in \bar{S}^t$.

With these sets so defined, we define a competitive equilibrium in this economy as follows.

Definition 1. *A* ***sticky-price equilibrium*** *is a triplet $(\xi, \varrho, \vartheta)$ of allocations, prices, and policies such that: (i) prices and allocations jointly satisfy the CES demand function (5); (ii) given demand function (5) and policy, intermediate good nominal prices $p_{it}(\omega_i^t)$ maximize the firm's expected value of profits, conditional on its information set ω_i^t; (iii) given prices and policies, aggregate consumption, savings, and labor supply maximize the household's expected utility, conditional on its information set $\bar{s}^t$, subject to its budget set; (iv) the household's and government budget sets are both satisfied; (v) given prices, labor adjusts according to (6) in order to meet realized demand; and (vi) final goods and labor markets clear: (7) and (8).*

In addition to sticky price equilibria, we will also consider a hypothetical benchmark economy with no frictions. That is, we drop Assumption 1 and relax all measurability constraints on the firms and central banker so that they have complete information about current fundamentals s_t when making their respective decisions. Formally we call this the "flexible price" environment and define equilibria in this environment accordingly.

Definition 2. *A* ***flexible-price equilibrium*** *is a triplet $(\xi, \varrho, \vartheta)$ of allocations, prices, and policies such that in all periods t, intermediate good prices and the nominal interest rate are measurable in the fundamental state:*

$$\iota_t(s^t) \qquad \text{and} \qquad p_{it}(s^t), \quad \forall i,$$

and $(\xi, \varrho, \vartheta)$ satisfy the same conditions stated in Definition 1.

The flexible-price economy provides a useful benchmark for our subsequent analysis.

3 Flexible-Price Equilibrium

In this section we characterize the set of allocations that can be implemented as a competitive equilibrium under flexible prices. Recall that under flexible prices, the full aggregate state reduces to s_t. The household's intratemporal optimality condition is given by

$$\frac{V'(L_t(s^t))}{U'(C_t(s^t))} = \frac{W_t(s^t)}{P_t(s^t)}, \qquad \forall s^t \in S^t, \tag{9}$$

which sets the household's marginal rate of substitution between labor and consumption equal to the real wage. Under flexible prices, the firm faces no informational constraints and thereby sets its price equal to a constant markup over marginal cost:

$$p_{it}(s^t) = P_t(s^t) = \left(\frac{\theta-1}{\theta}\right)^{-1} \frac{1}{1-\tau} \frac{W_t(s^t)}{A(s_t)}, \qquad \forall s^t \in S^t. \tag{10}$$

Note that the firm's nominal marginal cost is equal to the nominal wage over aggregate productivity. Combining this condition with the household's optimality condition in (9), we obtain the following result.

Lemma 1. *An allocation is implementable as a flexible-price equilibrium if and only if there exists a strictly positive constant $\chi \in \mathbb{R}_+$ and two functions $\mathcal{C} : S \to \mathbb{R}_+$ and $\mathcal{L} : \mathcal{S} \to \mathbb{R}_+$ such that the allocation is given by:*

$$\ell_{it}(s^t) = L_t(s^t) = \mathcal{L}(s_t), \qquad \text{and} \qquad y_{it}(s^t) = Y_t(s^t) = C_t(s^t) = \mathcal{C}(s_t), \qquad \forall i \in I, s^t, \tag{11}$$

and $\chi, \mathcal{C}(\cdot), \mathcal{L}(\cdot)$ jointly satisfy:

$$U'(\mathcal{C}(s_t)) = \chi V'(\mathcal{L}(s_t)) \frac{1}{A(s_t)}, \qquad \text{and} \qquad \mathcal{C}(s_t) = A(s_t)\mathcal{L}(s_t) \qquad \forall s^t \in S^t, \tag{12}$$

Proof. See the appendix, Section A.2. □

Condition (11) states that in any flexible-price equilibrium, there is zero dispersion in output and labor inputs across intermediate good firms $i \in I$. All firms are ex-ante identical in their technology and, under flexible prices, they have common knowledge of productivity. As a result, all firms set the same nominal price (10). It follows that intermediate-good production is equalized across firms, as is intermediate good labor.

Furthermore, any flexible-price allocation ξ may be summarized by two functions $\{\mathcal{C}(s_t), \mathcal{L}(s_t)\}$, one for aggregate output and one for aggregate labor, that are jointly determined by the two conditions in (12). The first is the equilibrium intratemporal condition. In any state, the marginal rate of substitution between consumption and labor is equal to the marginal rate of transformation, modulo a constant wedge due to the constant revenue tax and the constant monopolistic markup:

$$\chi = \frac{1}{1-\tau}\left(\frac{\theta-1}{\theta}\right)^{-1} > 0. \tag{13}$$

The wedge χ characterizes the power of the fiscal authority to move around the equilibrium allocation (with its choice of τ). The second condition in (12) is simply the aggregate production function.

It is clear from condition (12) that it is only the current aggregate productivity shock $A(s_t)$ that moves around the aggregate allocation. As a result, the functions $\{\mathcal{C}(s_t), \mathcal{L}(s_t)\}$ are both history-independent and time-invariant. They are history-independent in the sense that they are functions of the current fundamental state, s_t, but not of the entire history of previous shocks, s^{t-1}. The functions $\{\mathcal{C}(\cdot), \mathcal{L}(\cdot)\}$, moreover, do not change over time and hence bear no time t subscript.

Finally, the nominal interest rate function $\iota(s^t)$ and the path of prices $P(s^t)$ must satisfy the following Euler equation:

$$\frac{U'(C_t(s^t))}{P_t(s^t)} = \beta(1+\iota(s^t))\mathbb{E}\left[\left.\frac{U'(C_{t+1}(s^{t+1}))}{P_{t+1}(s^{t+1})}\right| s^t\right].$$

Therefore, unlike the fiscal authority, in the flexible-price benchmark the monetary authority has no power to alter equilibrium allocations.

The first-best efficient allocation. We now characterize another benchmark: efficiency. We consider the problem of a planner who chooses the welfare-maximizing allocation among all feasible allocations. By feasibility, we mean allocations that satisfy all technology and resource constraints (6)-(8).

Lemma 2. *Let ξ^* denote the first best efficient allocation. The first best allocation ξ^* is the unique allocation that satisfies conditions (11) and (12) with $\chi = 1$.*

Proof. See the appendix, Section A.3. □

The following result is then immediate.

Theorem 1. *Let $\mathcal{X}^f$ denote the set of all flexible-price allocations; $\xi^* \in \mathcal{X}^f$.*

Proof. This result follows from Lemmas 1 and 2. □

As in the flexible-price equilibrium, the planner dictates zero dispersion in output and labor across intermediate good firms $i \in I$. Given the CES technology for final goods, any dispersion in intermediate good production is welfare-decreasing. Furthermore, it is optimal to equate the marginal rate of substitution between consumption and labor with the marginal rate of transformation. It follows that in our setting there are no missing tax instruments: the first-best efficient allocation can be implemented under flexible prices with a revenue subsidy that exactly offsets the markup.

4 Sticky-Price Equilibrium

We now reinstate Assumption 1—the measurability constraints on the firms and on the central banker—and consider the set of sticky-price equilibria in our environment.

Consider the firm's problem. The measurability constraint on the firm's pricing decision implies that the firm must choose its price in order to solve the following maximization problem:

$$\max_{p_i'} \mathbb{E}\left[\Lambda_t(\bar{s}^t)\left\{(1-\tau)p_i' y_{it}(\omega_i^t, \bar{s}^t) - W_t(\bar{s}^t)\frac{y_{it}(\omega_i^t, \bar{s}^t)}{A(s_t)}\right\}\middle|\, \omega_i^t\right],$$

subject to CES demand function (5), where we let $\Lambda_t(\bar{s}^t) \equiv U'(C_t(\bar{s}^t))/P_t(\bar{s}^t)$ denote the household's marginal value of nominal wealth in history $\bar{s}^t$. That is, the firm chooses a price that maximizes the value of the firm, given its information set at time t. The firm's optimal price is given by:

$$p_{it}(\omega_i^t) = \left(\frac{\theta-1}{\theta}\right)^{-1}\frac{1}{1-\tau}\mathbb{E}\left[q_{it}(\omega_i^t, \bar{s}^t)\left\{\frac{W_t(\bar{s}^t)}{A(s_t)}\right\}\middle|\, \omega_i^t\right],$$

where

$$q_{it}(\omega_i^t, \bar{s}^t) \equiv \frac{\Lambda_t(\bar{s}^t) y_{it}(\omega_i^t, \bar{s}^t)}{\mathbb{E}\left[\Lambda_t(\bar{s}^t) y_{it}(\omega_i^t, \bar{s}^t) | \omega_i^t\right]} \tag{14}$$

are the firm's risk weights. Therefore, the firm's optimal price is equal to its risk-weighted expectation of a constant markup over marginal cost, $W_t(\bar{s}^t)/A(s_t)$. The markup is again the result of the revenue tax and the monopolistic markup. Note that the risk weights defined in (14) satisfy $\mathbb{E}[q_{it}(\omega_i^t, \bar{s}^t)|\omega_i^t] = 1$ for all information sets $\omega_i^t \in \Omega^t$.

The following lemma provides a complete characterization of the set of sticky-price equilibria.

Lemma 3. *An allocation ξ, a policy ϑ, and price system ϱ are part of a sticky-price equilibrium if and only if the following four properties hold:*

(i) the following household optimality conditions are satisfied:

$$\frac{V'(L_t(\bar{s}^t))}{U'(C_t(\bar{s}^t))} = \frac{W_t(\bar{s}^t)}{P_t(\bar{s}^t)}, \qquad \forall \bar{s}^t \in \bar{S}^t, \tag{15}$$

$$\frac{U'(C_t(\bar{s}^t))}{P_t(\bar{s}^t)} = \beta(1 + \iota_t(\omega_p^t))\mathbb{E}\left[\frac{U'(C_{t+1}(\bar{s}^{t+1}))}{P_{t+1}(\bar{s}^{t+1})}\middle|\, \bar{s}^t\right], \qquad \forall \bar{s}^t \in \bar{S}^t, \tag{16}$$

along with the transversality condition:

$$\lim_{t\to\infty} \beta^t \mathbb{E}\left[U'(C_t(\bar{s}^t))B_t(\bar{s}^t)\right] = 0; \tag{17}$$

(ii) the following firm optimality condition is satisfied:

$$p_{it}(\omega_i^t) = \chi \mathbb{E}\left[q_{it}(\omega_i^t, \bar{s}^t)\left\{\frac{W_t(\bar{s}^t)}{A(s_t)}\right\}\middle|\, \omega_i^t\right] \tag{18}$$

with $q_{it}(\omega_i^t, \bar{s}^t)$ defined in (14), along with the intermediate-good demand condition, namely, (5);

(iii) the household and government budget sets are satisfied;

(iv) all markets clear, namely, conditions (6)-(8) are satisfied.

Proof. See the appendix, Section A.4. □

The household optimality conditions are given by equations (15)-(17). Condition (15) is the household's intratemporal optimality condition, condition (16) is the household's intertemporal Euler equation, and condition (17) is the transversality condition. Note that in the household's Euler equation the expectation is taken conditional on $\bar{s}^t$, the household's information set at time t.

On the other hand, intermediate good firms make their nominal pricing decisions under incomplete information about nominal marginal costs. Dispersion in information, ω_{it}, may lead to dispersion in nominal prices. Dispersion in prices, in turn, translates into dispersion in production across firms, as output is determined according to the CES demand function (5).

Finally, recall that under flexible prices the monetary authority has no power to vary the allocation. Under sticky prices, this is no more the case: the monetary authority has some power to control real allocations via the nominal interest rate ι_t, which enters the household's Euler equation (16). This power, however, may be limited by the measurability constraint on the central banker's policy tool.

5 Optimal Monetary Policy

In this section we consider the question of optimal monetary policy. Throughout this section we will maintain an assumption that the central bank does not share its private information with the public; we will relax this assumption in the following section, Section 6, when we consider central bank disclosure.

Recall from Lemma 2 that the first best allocation is implementable under flexible prices. In this section we will show that all flexible-price allocations are implementable under sticky prices. As a result, an "optimal" policy is any monetary policy that implements flexible-price allocations coupled with a subsidy that offsets the monopolistic markup—such a policy implements the first best.

We focus our discussion on two classes of implementations. The first class of implementations are ones in which the nominal interest rate does not condition on the central banker's private signal at time t. The second class of implementations are ones in which nominal interest rates do condition on the central banker's private signal at time t.

5.1 Implementations that ignore central bank information

The following result shows that the set of flexible-price allocations can be implemented under sticky prices.

Proposition 1. *Take any flexible-price allocation $\xi \in X^f$ with corresponding functions $\{\mathcal{C}(\cdot), \mathcal{L}(\cdot)\}$ and let $g : S \to \mathbb{R}_+$ be the function defined by $g(s_t) \equiv \sum_{s_{t+1}\in S} U'(\mathcal{C}(s_{t+1}))\mu(s_{t+1}|s_t)$.*

The following paths of nominal interest rates and aggregate prices implement ξ under sticky prices:

$$1 + \iota_t(\omega_p^t) = \mathcal{I}(s_{t-1}) \qquad \textit{and} \qquad P_t(\bar{s}^t) = \mathcal{P}(s_{t-1}), \tag{19}$$

where $\mathcal{I} : S \to \mathbb{R}_+$ and $\mathcal{P} : S \to \mathbb{R}_+$ are two functions defined by:

$$\mathcal{I}(s_{t-1}) \equiv \frac{1}{\beta} U'(\mathcal{C}(s_{t-1}))g(s_{t-1})^{-1}, \tag{20}$$

$$\mathcal{P}(s_{t-1}) \equiv [U'(\mathcal{C}(s_{t-1}))]^{-1}g(s_{t-1}). \tag{21}$$

Proof. The interest rate and path of prices must satisfy the Euler equation of the household under sticky prices at the flexible-price allocation:

$$\frac{U'(\mathcal{C}(s_t))}{P_t(\bar{s}^t)} = \beta(1 + \iota_t(\omega_p^t))\mathbb{E}\left[\left. \frac{U'(\mathcal{C}(s_{t+1}))}{P_{t+1}(\bar{s}^{t+1})} \right| \bar{s}^t \right]. \tag{22}$$

With the nominal interest rate and the path of nominal prices proposed in (19), we may rewrite the Euler equation as follows:

$$U'(\mathcal{C}(s_t)) = \beta\mathcal{I}(s_{t-1})\frac{\mathcal{P}(s_{t-1})}{\mathcal{P}(s_t)} \sum_{s_{t+1}\in S} U'(\mathcal{C}(s_{t+1}))\mu(s_{t+1}|s_t).$$

The above equation is satisfied by the functions $\mathcal{I}$ and $\mathcal{P}$ defined in (20) and (21).

Finally, it is straightforward to verify that there is no dispersion in intermediate-good firm prices: $p_{it}(\omega_i^t) = \mathcal{P}(s_{t-1})$ for all $\omega_i^t \in \Omega^t$. See the appendix, Section A.5, for details. □

Proposition 1 establishes that any flexible-price allocation can be implemented under sticky prices. Recall that under flexible prices, there is no dispersion in output across firms. Furthermore, the allocation under flexible prices varies with TFP, $A(s_t)$. How can such properties be preserved when both the central banker and all firms make their respective decisions under incomplete information about the fundamental state?

Let us tackle the first property: that there is no dispersion in production across firms. In order for this to hold under sticky prices, all firms must set the same nominal price—this follows from the CES demand function. However, recall that it is individually optimal for each firm to set a nominal price equal to its expected nominal marginal cost:

$$p_{it}(\omega_i^t) = \chi \mathbb{E}\left[q_{it}(\omega_i^t, \bar{s}^t) \left\{ \frac{W_t(\bar{s}^t)}{A(s_t)} \right\} \middle| \omega_i^t \right].$$

Given that private signals ω_{it} differ across firms, the only way to ensure that all firms set the same nominal price is to make the nominal marginal cost, $W_t(\bar{s}^t)/A(s_t)$, *measurable* in each and every firm's information set.

The aggregate price level presented in Proposition 1 accomplishes this task. In particular, at the proposed implementation, $W_t(\bar{s}^t)/A(s_t) = P_t(\bar{s}^t) = \mathcal{P}(s_{t-1})$. The aggregate price level at time t depends only on the past fundamental state, s_{t-1}. Because this state is common knowledge, firms can optimally disregard their private signal ω_{it} and set prices according to:

$$p_{it}(\omega_i^t) = \mathcal{P}(s_{t-1}), \qquad \forall \omega_i^t \in \Omega^t.$$

In sum, when the firm's optimal price is common knowledge, there is no price dispersion across firms and, consequently, no output dispersion.

While this argument explains how all firms set the same nominal price, it does not yet explain how the aggregate allocation can vary in the appropriate way with the fundamental state s_t. Note that at the flexible-price allocation, the Euler equation satisfies:

$$U'(\mathcal{C}(s_t)) = \beta R(s_t) \sum_{s_{t+1} \in S} U'(\mathcal{C}(s_{t+1}))\mu(s_{t+1}|s_t), \tag{23}$$

where $R(s_t)$ denotes the (gross) real interest rate. From this equation, it is clear that the real risk-free rate at the flexible-price allocation is contingent on the current fundamental, s_t, and the current fundamental alone.

In our framework, the nominal interest rate cannot be measurable in the fundamental s_t at time t—it can be contingent only on the central banker's incomplete information set, ω_p^t.

Proposition 1 demonstrates that this measurability constraint on the central banker can, in fact, be circumvented. The real interest rate at any flexible-price allocation can be replicated with a nominal interest rate that is contingent only on the past fundamental state, s_{t-1}, which is known to the central banker at time t, and the path of prices described in (19).

With the nominal interest rate and the path of prices proposed in (19), the real interest rate between periods t and $t+1$ is given by:

$$R(s_t) = \mathcal{I}(s_{t-1}) \frac{\mathcal{P}(s_{t-1})}{\mathcal{P}(s_t)}.$$

Therefore, in order to generate the appropriate contingency of the real interest rate on the current fundamental, s_t, the *future* price must react to today's state. This is possible, as we have proposed a path of prices such that the following period's price is contingent on the current fundamental: $P_{t+1}(\bar{s}^{t+1}) = \mathcal{P}(s_t)$. Furthermore, by the following period all firms will have learned today's state and, hence, will be able to set the "correct" nominal price.

In sum, the interest rate and price path proposed in Proposition 1 implements flexible-price allocations under sticky prices despite the measurability constraints on both the firms' nominal pricing decisions and the monetary policy tool. The following result is then immediate.

Theorem 2. *Let $\mathcal{X}^s$ denote the set of allocations that can be implemented as an equilibrium under sticky prices;*

$$\xi^* \in \mathcal{X}^f \subset \mathcal{X}^s.$$

Therefore, an optimal policy is a monetary policy that implements flexible-price allocations and a constant subsidy that offsets the markup.

Proof. The statement that $\mathcal{X}^f \subset \mathcal{X}^s$ follows from Proposition 1. Combining this with Theorem 1 provides the result. □

Proposition 1 provides only one possible implementation of flexible-price allocations—it is not a unique implementation. That said, the implementation presented in Proposition 1 may be desirable for its simplicity.

First and foremost, the proposed implementation features a nominal interest rate that is not contingent on the private information of the central banker, ω_{pt}. The central banker commits to set its interest rate based solely on past states that are, as of time t, common knowledge. As a result, from the point of view of firms at time t, there are no monetary policy "shocks."

Second, the functions $\{\mathcal{I}(s_{t-1}), \mathcal{P}(s_{t-1})\}$ that characterize the nominal interest rate and the aggregate price level inherit two arguably convenient properties of the functions $\{\mathcal{C}(s_t), \mathcal{L}(s_t)\}$. In particular, these functions are time-invariant and history-independent. The nominal interest rate and aggregate price level at time t depend only on the past fundamental s_{t-1} and not on the entire history of previous shocks; furthermore, this relationship does not change over time.

The full set of implementations that condition only on past fundamentals. The aggregate price function $\mathcal{P}(\cdot)$ in (21) is contingent on the minimal number of state variables needed to implement flexible price allocations: one.[9] As noted above, in order to generate the appropriate contingencies of the real interest rate on current fundamentals, the future price level must vary with these fundamentals. It is therefore necessary that the price level be contingent on at least one state variable—the previous period's fundamental. Proposition 1 demonstrates that it need not be contingent on more states.

However, one could allow for the aggregate price level and the nominal interest rate at time t to be measurable in more fundamental states, e.g. $s_{t-2}, s_{t-3}, \ldots$, without affecting their ability to implement flexible price allocations. This is simply because all past states are common knowledge and can thereby be incorporated into prices without any real effects. We demonstrate this with the following proposition.

Proposition 2. *Take any flexible-price allocation $\xi \in X^f$ with corresponding functions $\{\mathcal{C}(\cdot), \mathcal{L}(\cdot)\}$ and let $g : S \to \mathbb{R}_+$ be the function defined by $g(s_t) \equiv \sum_{s_{t+1} \in S} U'(\mathcal{C}(s_{t+1}))\mu(s_{t+1}|s_t)$.*

The following paths of nominal interest rates and aggregate prices implement ξ under sticky prices:

$$1 + \iota_t(\omega_p^t) = \mathcal{I}_t(s^{t-1}) \qquad \text{and} \qquad P_t(\bar{s}^t) = \mathcal{P}_t(s^{t-1}), \tag{24}$$

where $\mathcal{I}_t : S^{t-1} \to \mathbb{R}_+$ is a sequence of positive-valued functions defined on S^{t-1}, and $\mathcal{P}_t : S^{t-1} \to \mathbb{R}_+$ is a sequence of positive-valued functions defined recursively by:

$$\mathcal{P}_{t+1}(s^t) = \beta \mathcal{I}_t(s^{t-1})\mathcal{P}_t(s^{t-1})\frac{g(s_t)}{U'(\mathcal{C}(s_t))}. \tag{25}$$

where $\mathcal{P}_0 > 0$ is a known constant.

Proof. The interest rate and path of prices must satisfy the Euler equation of the household under sticky prices at the flexible-price allocation: (22). With the nominal interest rate and the path of nominal prices proposed in (24), we may rewrite this Euler equation as follows:

$$U'(\mathcal{C}(s_t)) = \beta \mathcal{I}_t(s^{t-1})\frac{\mathcal{P}_t(s^{t-1})}{\mathcal{P}_{t+1}(s^t)}g(s_t).$$

For any sequence of positive-valued functions for the nominal interest rate, $\mathcal{I}_t(\cdot)$, the above equation is satisfied by the sequence of functions $\mathcal{P}_t(\cdot)$ defined in (25).

It is straightforward to verify that there is no dispersion in intermediate-good firm prices due to the fact that $s^{t-1} \in \omega_i^t$ for all $\omega_i^t \in \Omega^t$. See the appendix, Section A.6, for details. □

Proposition 2 provides a characterization of all possible implementations of flexible price allocations such that the nominal interest rate and the aggregate price level condition only on

[9] We formalize this statement in Theorem 3 below.

past fundamentals. The implementation provided in Proposition 1 is thereby nested in this class. Proposition 2 is more general in that it only requires that the sequence of functions $\{\mathcal{I}_t(\cdot), \mathcal{P}_t(\cdot)\}$ describing the nominal interest rate and price level satisfy condition (25) at all dates and histories.

Proposition 2 demonstrates that the price level and nominal interest rate at time t can depend on the entire history of fundamental shocks, s^{t-1}, yet still implement flexible price allocations. This is due to the simple yet powerful property that in this class of models, common knowledge states can be incorporated into nominal variables without affecting real economic outcomes.

5.2 Implementations that incorporate central bank information

Section 5.1 restricts attention to implementations that do not rely on the central banker's private information. We now explore implementations in which the nominal interest rate is contingent on the private signal of the central banker, ω_{pt}. From the point of view of firms at time t, these interest rates feature monetary policy "shocks." Nevertheless, flexible-price allocations can still be implemented.[10]

We begin by presenting an implementation that is similar to the one presented in Proposition 3 in that the nominal interest rate and the aggregate price level can be represented by time-invariant, history-independent functions, $\mathcal{I}(\cdot)$ and $\mathcal{P}(\cdot)$.

Proposition 3. *Take any flexible-price allocation $\xi \in X^f$ with corresponding functions $\{\mathcal{C}(\cdot), \mathcal{L}(\cdot)\}$ and let $g : S \to \mathbb{R}_+$ be the function defined by $g(s_t) \equiv \sum_{s_{t+1} \in S} U'(\mathcal{C}(s_{t+1}))\mu(s_{t+1}|s_t)$.*

Let $f : \Omega_p \to \mathbb{R}_+$ be a positive-valued function defined on Ω_p. The following paths of nominal interest rates and aggregate prices implement ξ under sticky prices:

$$1 + \iota_t(\omega_p^t) = \mathcal{I}(\omega_{pt}, \omega_{pt-1}, s_{t-1}) \qquad \textit{and} \qquad P_t(\bar{s}^t) = \mathcal{P}(\omega_{pt-1}, s_{t-1}), \tag{26}$$

where $\mathcal{I} : \Omega_p^2 \times S \to \mathbb{R}_+$ and $\mathcal{P} : \Omega_p \times S \to \mathbb{R}_+$ are the two functions defined by:

$$\mathcal{I}(\omega_{pt}, \omega_{pt-1}, s_{t-1}) \equiv \frac{1}{\beta} U'(\mathcal{C}(s_{t-1}))g(s_{t-1})^{-1} f(\omega_{pt}) f(\omega_{pt-1})^{-1}, \tag{27}$$

$$\mathcal{P}(\omega_{pt-1}, s_{t-1}) \equiv [U'(\mathcal{C}(s_{t-1}))]^{-1} g(s_{t-1}) f(\omega_{pt-1}). \tag{28}$$

Proof. The interest rate and path of prices must satisfy the Euler equation of the household under sticky prices at the flexible-price allocation, as in (22). Substituting in the nominal interest

[10] We are deeply indebted to V.V. Chari and Luis Perez for making us aware of this possibility and helping us to derive these results. Our understanding of issues of implementation has benefited greatly from our discussions with them.

rate and the path of nominal prices proposed in (26), we may rewrite the Euler equation as follows:

$$U'(\mathcal{C}(s_t)) = \beta \mathcal{I}(\omega_{pt}, \omega_{pt-1}, s_{t-1}) \frac{\mathcal{P}(\omega_{pt-1}, s_{t-1})}{\mathcal{P}(\omega_{pt}, s_t)} g(s_t).$$

The above equation is satisfied by the functions $\mathcal{I}(\cdot)$ and $\mathcal{P}(\cdot)$ defined in (27) and (28).

It is straightforward to verify that there is no dispersion in intermediate-good firm prices due to the fact that $(\omega_{pt-1}, s_{t-1}) \in \omega_i^t$ for all $\omega_i^t \in \Omega^t$. See the appendix, Section A.6, for details. □

Proposition 3 demonstrates how one can augment the implementation in Proposition 1 to include arbitrary contingencies of the nominal interest rate on the central banker's private signal, ω_{pt}. These contingencies are summarized by the positive-valued function $f(\omega_{pt})$.

Recall that the real interest rate at the flexible-price allocation is contingent on the current fundamental, s_t, and the current fundamental alone; see equation (23). The flexible-price real interest rate can be replicated with the nominal interest rate and path of prices defined in (26) as follows:

$$R(s_t) = \mathcal{I}(\omega_{pt}, \omega_{pt-1}, s_{t-1}) \frac{\mathcal{P}(\omega_{pt-1}, s_{t-1})}{\mathcal{P}(\omega_{pt}, s_t)} = \frac{1}{\beta} U'(\mathcal{C}(s_t)) g(s_t)^{-1},$$

where the second equality follows from (27) and (28).

As with the implementation in Proposition 1, in order to generate the appropriate contingency of the real interest rate on the current fundamental, the future price, P_{t+1}, must react to today's fundamental, s_t. Under this implementation the future price must not only vary with the current fundamental in the appropriate way, but it must also vary with ω_{pt}, the private signal of the central banker. Specifically, the future price must "correct" for the contingency on ω_{pt} introduced through the nominal interest rate. This is possible without introducing price dispersion: by period $t+1$, all agents will have learned not only the fundamental at time t but also the private signal of the central banker. It follows that all firms will be able to set the "correct" nominal price.

The persistence of central bank signal "errors." Proposition 3 provides an implementation in which the nominal interest rate and the aggregate price level are represented by time-invariant, history-independent functions, $\mathcal{I}(\cdot)$ and $\mathcal{P}(\cdot)$, and the nominal interest rate is measurable in the central banker's private signal.

The nominal interest rate in (27), however, has a peculiar feature. In particular, it is contingent not only on the current realization of the central banker's private signal, ω_{pt}, but also on the *past* realization of this signal, ω_{pt-1}.

The latter contingency arises for the following reason. As already noted, in order to implement flexible-price allocations, the following period price level must vary with ω_{pt}. However, in order for the price level to not inherit the effect of this signal forever—and hence remain history-independent—the following period's nominal interest rate must also vary with ω_{pt} in order to "correct" for the added contingency introduced by P_{t+1}. It follows that, in any period t, the nominal interest rate is contingent on ω_{pt-1}.

This particular property may or may not be desirable. It implies that the nominal interest rate must vary with the "noise" or "error" inherent in the central banker's past private signal even after the past fundamental state s_{t-1} has become common knowledge.

One might find it preferable to eliminate this feature. In what follows, we restrict attention to implementations in which the interest rate does not exhibit this property. Specifically, we let the nominal interest rate at time t be measurable in the current realization of the central banker's private signal, ω_{pt}, but not on past signal realizations, $\omega_{pt-1}, \omega_{pt-2}, \ldots$.

Proposition 4. *Take any flexible-price allocation $\xi \in X^f$ with corresponding functions $\{\mathcal{C}(\cdot), \mathcal{L}(\cdot)\}$ and let $g : S \to \mathbb{R}_+$ be the function defined by $g(s_t) \equiv \sum_{s_{t+1} \in S} U'(\mathcal{C}(s_{t+1}))\mu(s_{t+1}|s_t)$.*

The following paths of nominal interest rates and aggregate prices implement ξ under sticky prices:

$$1 + \iota_t(\omega_p^t) = \mathcal{I}_t(\omega_{pt}, s^{t-1}) \qquad \textit{and} \qquad P_t(\bar{s}^t) = \mathcal{P}_t(\bar{s}^{t-1}), \tag{29}$$

where $\mathcal{I}_t : \Omega_p \times S^{t-1} \to \mathbb{R}_+$ is a sequence of positive-valued functions defined on $\Omega_p \times S^{t-1}$, and $\mathcal{P}_t : \bar{S}^{t-1} \to \mathbb{R}_+$ is a sequence of positive-valued functions defined recursively by:

$$\mathcal{P}_{t+1}(\bar{s}^t) = \beta \mathcal{I}_t(\omega_{pt}, s^{t-1})\mathcal{P}_t(\bar{s}^{t-1})\frac{g(s_t)}{U'(\mathcal{C}(s_t))}. \tag{30}$$

where $\mathcal{P}_0 > 0$ is a known constant.

Proof. The interest rate and path of prices must satisfy the Euler equation of the household under sticky prices at the flexible-price allocation, as in (22). Substituting in for the nominal interest rate and for the path of prices proposed in (29), we may write the Euler equation as follows:

$$U'(\mathcal{C}(s_t)) = \beta \mathcal{I}_t(\omega_{pt}, s^{t-1})\frac{\mathcal{P}_t(\bar{s}^{t-1})}{\mathcal{P}_{t+1}(\bar{s}^t)}g(s_t).$$

For any sequence of positive-valued functions for the nominal interest rate, $\mathcal{I}_t(\cdot)$, the above equation is satisfied by the sequence of functions $\mathcal{P}_t$ defined in (30).

It is straightforward to verify that there is no dispersion in intermediate-good firm prices due to the fact that $\bar{s}^{t-1} \in \omega_i^t$ for all $\omega_i^t \in \Omega^t$. See the appendix, Section A.6, for details. □

Proposition 4 characterizes paths for the nominal interest rate and the aggregate price level that together implement flexible-price allocations and in which the nominal interest rate is measurable only in the central banker's current private signal, ω_{pt}, and in past fundamentals, s^{t-1}. Importantly, we restrict the nominal interest rate to not condition on past private signal realizations, $\omega_{pt-1}, \omega_{pt-2}, \ldots$.

When we restrict the nominal interest rate in this fashion, past central bank signal realizations instead pop up in the nominal price level. The path of the aggregate price level, defined in

(30), is history-dependent. In particular, the price level at time $t+1$ depends on the entire history of central bank signals $(\omega_{pt}, \omega_{pt-1}, \omega_{pt-2}, \ldots)$. Therefore, with this restriction on the nominal interest rate, the price level inherits the effects of the central bank's signal noise forever.

The bottom line. The main lesson here relative to Section 5.1 is that there exist implementations of flexible-price allocations which feature nominal interest rates that condition on the central banker's private signal. However, the central banker's private signal by definition features noise, or "errors," that are clearly not present in real allocations (under flexible prices).

In order to implement flexible price allocations, then, central bank signal noise must show up in future nominal variables. The implementations presented in this section feature a persistent effect of central bank signal noise on either future interest rates or future price levels—in some cases long after current fundamentals have become common knowledge. Whether or not this is an undesirable property of these implementations we leave up to the reader.

5.3 The full set of implementations and remarks

We have thus far focused on two classes of implementations. The implementations in Section 5.1 feature a nominal interest rate that is contingent only on past fundamentals, but not on the central banker's private signal; in contrast, the implementations presented in Section 5.2 allow the nominal interest rate to vary with the central banker's private signal.

We close our discussion with a characterization of the full set of flexible-price implementations. This is followed by a set of remarks.

Proposition 5. *Take any flexible-price allocation $\xi \in X^f$ with corresponding functions $\{\mathcal{C}(\cdot), \mathcal{L}(\cdot)\}$ and let $g : S \to \mathbb{R}_+$ be the function defined by $g(s_t) \equiv \sum_{s_{t+1} \in S} U'(\mathcal{C}(s_{t+1}))\mu(s_{t+1}|s_t)$.*

The following paths of nominal interest rates and aggregate prices implement ξ under sticky prices:

$$1 + \iota_t(\omega_p^t) = \mathcal{I}_t(\omega_p^t) \qquad \text{and} \qquad P_t(\bar{s}^t) = \mathcal{P}_t(\bar{s}^{t-1}), \tag{31}$$

where $\mathcal{I}_t : \Omega_p^t \to \mathbb{R}_+$ is a sequence of positive-valued functions defined on Ω_p^t, and $\mathcal{P}_t : \bar{S}^{t-1} \to \mathbb{R}_+$ is a sequence of positive-valued functions defined recursively by:

$$\mathcal{P}_{t+1}(\bar{s}^t) = \beta \mathcal{I}_t(\omega_p^t) \mathcal{P}_t(\bar{s}^{t-1}) \frac{g(s_t)}{U'(\mathcal{C}(s_t))}, \tag{32}$$

where $\mathcal{P}_0 > 0$ is a known constant.

Proof. The interest rate and path of prices must satisfy the Euler equation of the household under sticky prices at the flexible-price allocation: (22). With the paths for the nominal interest rate and prices proposed in (31), we may rewrite the Euler equation as follows:

$$U'(\mathcal{C}(s_t)) = \beta \mathcal{I}_t(\omega_p^t) \frac{\mathcal{P}_t(\bar{s}^{t-1})}{\mathcal{P}_{t+1}(\bar{s}^t)} g(s_t)$$

For any given sequence of functions, $\mathcal{I}_t(\cdot)$ and $h_t(\cdot)$, the above equation is satisfied at all dates and histories by the sequence of functions $\mathcal{P}_t(\cdot)$ defined in (32).

It is straightforward to verify that there is no dispersion in intermediate-good firm prices due to the fact that $\bar{s}^{t-1} \in \omega_i^t$ for all $\omega_i^t \in \Omega^t$. See the appendix, Section A.6, for details. $\square$

Proposition 5 characterizes the full set of flexible-price implementations in our setting; it thereby nests the implementations presented in Propositions 1-4.

Proposition 5 places no restrictions on the nominal interest rate aside from the measurability constraint on the central banker. It likewise places almost no restrictions on the aggregate price level: the price level at time t is allowed to be contingent on the largest information set that is common knowledge to all firms at time t—specifically, $\bar{s}^{t-1}$. By restricting the aggregate price to be contingent on at most $\bar{s}^{t-1}$, we ensure that all firms can set the "correct" nominal price at every date and history.

Finally, these implementations only require that the sequence of nominal interest rates and prices satisfy condition (32). This condition ensures that the Euler equation at the flexible-price allocation holds at all dates and histories. We conclude this section with a few remarks.

The zero persistence special case. None of the implementations presented in this section rely on the assumption that productivity is persistent. In the special case that TFP is i.i.d., the function $g(s_t)$ is equal to a constant and Propositions 1-5 continue to hold.

Public signals. The largest information set that is common knowledge to all firms at time t is $\bar{s}^{t-1}$. This is because we have defined all firm signals ω_{it} about current fundamentals to be purely private signals.[11]

One could instead imagine a setting in which firms observe not only their private signals but also what are known as "public signals." Public signals are signals about the fundamental s_t that are observed by all firms at time t and are therefore common knowledge (in addition to the past history of aggregate states).

Proposition 5 can readily be extended to allow for public signals; we provide this extension in Appendix B. We show how the price level can vary with not only the past history of states, but also the current realization of public signals, and yet remain compatible with flexible-price allocations. This echoes our earlier point that in this class of models, common knowledge variables—including public signals—can be incorporated into prices without any real effects.

Drivers of the business cycle. Our setting assumes that the business cycle is driven entirely by productivity shocks. By implication, flexible-price allocations are efficient. The best that

[11]Specifically, our setting assumes that the draws of ω_{it} are i.i.d. across firms conditional on (s_t, ω_{pt}). This assumption was made for simplicity as well as to ensure that a law of large number applies.

monetary policy can do is maintain productive efficiency and replicate flexible price allocations (Correia, Nicolini, and Teles, 2008). We expect the lessons delivered in this section—specifically, that implementation of flexible price allocations is both desirable and feasible—to be robust to alternative specifications of the economic environment in which flexible price allocations remain efficient.[12]

These lessons, however, hinge on the desirability of implementing flexible price outcomes. If instead the business cycle were driven by cost-push, or mark-up, shocks, these lessons would no longer apply. In the absence of appropriate state-contingent taxes, mark-up shocks render flexible price allocations inefficient. Unconstrained monetary policy then faces a trade-off between maintaining productive efficiency and substituting for missing tax contingencies.

In our setting, however, there is an additional constraint: the informational constraint on the nominal interest rate. How this constraint interacts with the assumed absence of state-contingent taxes—another form of measurability constraint on a policy instrument—is a question that we leave open for future research.

Relation to the divine coincidence. If one interprets the "divine coincidence" to mean that monetary policy should maintain productive efficiency and implement flexible price allocations as in Correia, Nicolini, and Teles (2008), then divine coincidence holds in our model.

On the other hand, there are some who interpret the "divine coincidence" to mean that price stability is optimal: price stability minimizes both inflation and the output gap in certain models. This particular interpretation of divine coincidence does not hold in our model: a stable price level is never optimal. We state this formally as follows.

Theorem 3. *In any sticky-price equilibrium that implements a flexible-price allocation, the aggregate price level at time t varies with the time $t-1$ fundamental, s_{t-1}.*

Proof. Take any sticky-price equilibrium that implements a flexible-price allocation. By Proposition 5, prices must satisfy equation (32). For any t, the functions $\mathcal{I}_t(\cdot)$ and $\mathcal{P}_t(\cdot)$ cannot be measurable in s_t. It is therefore necessary that the function $\mathcal{P}_{t+1}(\cdot)$ is contingent on s_t in order for (32) to hold. □

Theorem 3 formalizes what we had previously asserted: that in order to implement flexible-price allocations, the aggregate price level must be contingent on at least one state variable—the previous period's fundamental. Proposition 1 demonstrates that the contingency of prices on that one state is also sufficient.

[12] For example, nothing of substance would change if we were to let s_t map to shocks to the household's discount factor, $\beta_t = \beta(s_t)$, or shocks to the household's marginal rate of substitution between consumption and labor. The first-best allocation would remain implementable under flexible prices, and all flexible price allocations would remain implementable under sticky prices (Theorems 1 and 2).

A direct implication of Theorem 3 is that a stable price path—a price level that is invariant to current and past states—can never be optimal in our context.

The monetary policy tool. Throughout we have assumed that the monetary policy instrument is the nominal interest rate. In contrast, Kohlhas (2020) studies a model in which the central banker's tool is constrained to be measurable in the central banker's information set, but the tool is assumed to be the money supply.

We now ask whether the choice of the monetary policy tool is relevant. Let $M_t(\bar{s}^t) \equiv P_t(\bar{s}^t)C_t(\bar{s}^t)$ denote aggregate nominal demand, or money supply, at time t in history $\bar{s}^t$. We consider the equilibrium path of money supply along any flexible-price implementation.

Theorem 4. *In any sticky-price equilibrium that implements a flexible-price allocation, the money supply at time t is contingent on the time t fundamental, s_t.*

Proof. Take any sticky-price equilibrium that implements a flexible-price allocation. By Lemma 1, aggregate real output at time t is contingent only on the time t fundamental: $C_t(s^t) = \mathcal{C}(s_t)$. By Proposition 5, the aggregate price level is contingent on at most $\bar{s}^{t-1}$: $P_t(\bar{s}^t) = \mathcal{P}_t(\bar{s}^{t-1})$. Therefore, money supply at time t is contingent on the time t fundamental: $M_t(\bar{s}^t) = \mathcal{P}_t(\bar{s}^{t-1})\mathcal{C}(s_t)$. □

For all possible implementations of flexible-price allocations, the money supply in our context varies with the current economic fundamental. It therefore violates the measurability restriction in Kohlhas (2020). This explains why the first best cannot be achieved in their setting, and optimal policy implements only a second best.

The time-consistency of forward guidance. The implementations presented in this section feature nominal interest rates and price levels that respond to past states. One can think of these contingencies as a form of forward guidance in the sense that the central banker commits to future interest rates that are contingent on current economic fundamentals. These future contingencies are essential for circumventing measurability constraints and implementing flexible-price allocations.

In typical forward-guidance settings, a time-inconsistency problem emerges. While the policymaker today would like to use future interest rates to, say, escape the zero lower bound, forward guidance comes at the cost of distorting future allocations. Therefore, if given the opportunity, the future central banker would be tempted to renege on past promises.

In our setting there exists no such trade-off. The paths for interest rates and prices described in Propositions 1-5, when combined with an appropriate subsidy, implement the first-best. They are, therefore, time-consistent: future central bankers would have no incentive to deviate.

The reason for time-consistency in our model lies in the form of nominal rigidity we assume: an informational friction. Firms are free to set prices every period, but they do so under incomplete information about the current aggregate state. However, past states are common knowledge. Thus, if the price level varies only with past states, firms can perfectly adjust their prices to reflect such states. As a result, contingencies of policy or prices on past states have no real effect on current allocations. At the same time, it is precisely these contingencies of prices and interest rates *in the future* that implement flexible-price allocations today.

Indeterminacy. Our analysis has followed the classic Ramsey approach by specifying the policy instruments as functions of the exogenous states. We have provided multiple paths for the nominal interest rate and prices that are consistent with flexible-price allocations. However, we have said nothing about determinacy and unique implementation.

It is well known that in monetary models in which the interest rate is the policy instrument, multiple equilibrium paths for inflation and output can be consistent with the same path for the nominal interest rate (Sargent and Wallace, 1975). Eliminating indeterminacy and achieving unique implementation of flexible-price allocations in our setting is of course desirable, but to do so one would need to specify more sophisticated monetary policy, for example along the lines of Atkeson, Chari, and Kehoe (2010). We leave this analysis for future work.

6 The Welfare Effects of Central Bank Disclosure

Is it socially desirable for the central bank to publicly disclose its private information? In this section we consider the welfare implications of central bank information disclosure. Formally, we compare equilibrium welfare in the case in which the central banker publicly discloses its private signal in every period, i.e.

$$\omega_p^t \in \omega_i^t, \qquad \forall i \in I, t \in \{0, 1, \ldots\}, \tag{33}$$

to equilibrium welfare in the case in which the central banker never discloses its private signal.

This question cannot be answered without taking a stand on monetary policy. In the previous section, however, we demonstrated that optimal monetary policy implements flexible-price allocations. But note that, at any flexible price allocation, central bank disclosure has no real effects: it is already *as if* firms have complete information about the underlying fundamental. Therefore, in order to make this question meaningful in our context, we will assume that for some (unmodeled) reason, the central bank follows a sub-optimal interest rate rule. In this case, the equilibrium moves away from flexible-price allocations and central bank information disclosure can have real effects.

6.1 The log-linear Gaussian setting

In order to answer this question, we will rely on an explicit, closed-form solution of equilibrium prices and allocations. This requires making some functional form assumptions.

First, we let household preferences be homothetic and given by:

$$U(C) = \frac{C^{1-\gamma}}{1-\gamma} \qquad \text{and} \qquad V(L) = \frac{L^{1+1/\eta}}{1+1/\eta}, \tag{34}$$

where $\gamma > 0$ is the inverse elasticity of intertemporal substitution and $\eta > 0$ is the Frisch elasticity of labor supply.

Second, we modify our earlier specification and assume that all states and signals are continuous random variables. In particular, we model these shocks as Gaussian. We assume that log productivity follows an AR(1) process given by:

$$\log A_t = \rho \log A_{t-1} + u_t,$$

where $\rho \in (0,1)$ is the persistence parameter and $u_t \sim \mathcal{N}(0, \sigma_0^2)$ is the period t innovation; we let $\kappa_0 \equiv 1/\sigma_0^2$. For shorthand we will use $a_t \equiv \log A_t$ to denote the log of aggregate productivity.

Signals and Information Sets. We model the private information of firms and the central banker as follows. In every period t, each firm i observes a noisy private signal x_{it} about current productivity given by:

$$x_{it} = a_t + \zeta_{it}^a,$$

where $\zeta_{it}^a \sim \mathcal{N}(0, \sigma_x^2)$ is the idiosyncratic noise in the firm's private signal. Similarly, in period t, the central banker observes a noisy private signal x_{pt} about current productivity given by:

$$x_{pt} = a_t + \epsilon_{pt},$$

where $\epsilon_{pt} \sim \mathcal{N}(0, \sigma_p^2)$ is the noise in the policymaker's signal. For future reference, we let $\kappa_x \equiv 1/\sigma_x^2$ denote the precision of the firm's private signal and $\kappa_p \equiv 1/\sigma_p^2$ denote the precision of the policymaker's signal.

In addition, we assume that in every period t, each firm i observes a noisy private signal z_{it} about the central banker's signal error given by:

$$z_{it} = \epsilon_{pt} + \zeta_{it}^z,$$

where $\zeta_{it}^z \sim \mathcal{N}(0, \sigma_z^2)$ is the idiosyncratic noise in the firm's signal. For future reference, we let $\kappa_z \equiv 1/\sigma_z^2$.

In terms of our earlier notation, the central bank and the firm's private signals at time t are denoted by $\omega_{pt} = \{x_{pt}\}$ and $\omega_{it} = \{x_{it}, z_{it}\}$, respectively. It follows that the central banker's information set at time t is given by $\omega_p^t = \{\omega_{pt}, \bar{s}^{t-1}\}$.

For firms, the information set of firm i at time t under the "no disclosure" policy is given by $\omega_i^t = \{\omega_{it}, \bar{s}^{t-1}\}$. Under the "central bank public disclosure" policy, the information set of firm i at time t is instead given by $\omega_i^t = \{\omega_{pt}, \omega_{it}, \bar{s}^{t-1}\}$.

6.2 Monetary Policy

We focus on log-linear nominal interest rates. We restrict attention to equilibria that satisfy the following structure for the nominal interest rate and the household's marginal value of wealth $\Lambda(\bar{s}^t)$.

Lemma 4. *Let the nominal interest rate satisfy:*

$$\log(1 + \iota_t(\omega_p^t)) = \log \psi(\omega_p^t) = \psi_0 a_{t-1} + \psi_p x_{pt}, \tag{35}$$

with arbitrary coefficients $(\psi_0, \psi_p) \in \mathbb{R}^2$*, and let the household's marginal value of wealth at time* t *satisfy:*

$$\log \Lambda(\bar{s}^t) = \psi_p x_{pt} + \psi_0 a_{t-1} + \psi_a a_t. \tag{36}$$

with $\psi_a \in \mathbb{R}$*. The household's Euler equation implies:*

$$\psi_a = \frac{1}{1-\rho}(\psi_0 + \rho\psi_p). \tag{37}$$

Proof. See the appendix, Section A.8. □

Condition (35) specifies a log-linear nominal interest rate that is measurable with respect to the central bank's information set. The monetary authority can freely choose the interest rate coefficients $(\psi_0, \psi_p) \in \mathbb{R}^2$. The coefficient ψ_0 denotes the loading of the nominal interest rate on last period's productivity, while ψ_p denotes the loading of the interest rate on the central banker's noisy private signal of current productivity, x_{pt}.

Lemma 4 furthermore provides an explicit characterization for the household's equilibrium marginal value of wealth. We restrict attention to the particular log-linear, time-invariant, and history-independent specification for the marginal value of wealth $\Lambda(\bar{s}^t)$ given by (36). For any combination of $(\psi_0, \psi_p) \in \mathbb{R}^2$, the household's marginal value of wealth $\Lambda(\bar{s}^t)$ satisfies a fixed point in the Euler equation; this fixed point pins down the coefficient ψ_a according to (37).

These restrictions, along with the homotheticity and Gaussian specification, allow us to obtain a log-linear, closed-form solution for the sticky-price equilibrium given any policy. The following proposition furthermore demonstrates that the class of interest rate rules described by (35) is sufficiently rich to implement the efficient allocation.

Proposition 6. *Within the class of equilibria that satisfy the conditions stated in Lemma 4, the first-best allocation* ξ^* *can be implemented with a subsidy that offsets the monopolistic markup and a unique log-linear nominal interest rate given by*

$$\psi_0^* = -\frac{\gamma(1/\eta + 1)}{1/\eta + \gamma}(1-\rho) \qquad \textit{and} \qquad \psi_p^* = 0. \tag{38}$$

Proof. See the appendix, Section A.9. □

Within the class of equilibria that satisfy the conditions stated in Lemma 4, there is a unique nominal interest rate that implements flexible-price allocations. The implementation provided in (38) corresponds directly to the simple implementation proposed in Proposition 1; in particular, zero weight is placed on the private signal of the central banker: $\psi_p^* = 0$. The reason this policy is unique is that we restrict attention to the class of equilibria described in Lemma 4. This class, for example, directly rules out the larger classes of implementations proposed in Propositions 2-4. We make this restriction mainly for tractability—we want a unique sticky-price equilibrium for any interest rate rule within this class so that we may study how equilibrium allocations change as monetary policy moves away from its optimum (ψ_0^*, ψ_p^*).

6.3 Welfare loss decomposition

To determine the welfare effects of central bank information disclosure, we begin with a general characterization of equilibrium welfare. We define welfare to be the unconditional average of the representative household's utility:

$$\mathcal{W} \equiv \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \left\{ \frac{C(\bar{s}^t)^{1-\gamma}}{1-\gamma} - \frac{L(\bar{s}^t)^{1+1/\eta}}{1+1/\eta} \right\}\right].$$

The following proposition provides a decomposition of equilibrium welfare relative to its first-best level.

Proposition 7. *Let $\mathcal{W}^*$ denote the level of welfare at the first-best allocation ξ^* and suppose the constant tax τ is chosen optimally. Then, welfare at any equilibrium allocation is given by*

$$\mathcal{W} = \mathcal{W}^* \exp\left\{ -\frac{1}{2} \cdot \frac{(1-\gamma)(1+1/\eta)}{1/\eta+\gamma} \mathcal{L} \right\}, \tag{39}$$

where $\mathcal{L}$ denotes the welfare loss from first best. Welfare loss may be decomposed as follows:

$$\mathcal{L} \equiv \mathcal{D} + (1/\eta + \gamma)\mathcal{V},$$

where

$$\mathcal{D} \equiv \theta \mathrm{var}_\omega \left[\log p_{it} | \bar{s}^t\right] \qquad \textit{and} \qquad \mathcal{V} \equiv \mathrm{var}(\log Y_t - \log Y_t^*), \tag{40}$$

where $\mathrm{var}_\omega\left[\cdot|\bar{s}^t\right]$ denotes cross-sectional dispersion and $\mathrm{var}[\cdot]$ denotes the unconditional variance.

Proof. See the appendix, Section A.11. □

Before discussing this welfare decomposition, let us comment on the optimal tax τ. In the first best, this tax is set to offset firms' monopoly power which requires $\chi = 1$. Away from this benchmark, informational frictions are an additional source of distortions affecting firms' price-setting decisions. They result in an aggregate labor wedge that covaries with aggregate productivity. At the optimum, the non-state-contingent tax τ must counteract both types of distortions—the ones coming from monopoly power and the ones due to informational frictions. We characterize the optimal tax in the proof of the proposition.

With this constant tax in place, Proposition 7 shows that the welfare loss relative to the first-best benchmark is summarized by two components, $\mathcal{D}$ and $\mathcal{V}$. The former represents productive inefficiency due to dispersion in prices, while the latter represents volatility of the aggregate labor wedge, or volatility of the output gap.

Consider first productive inefficiency. Cross-sectional dispersion in prices leads to misallocation of inputs and production across intermediate-good firms. This misallocation manifests as a loss in total factor productivity relative to the first best, also known as the "efficiency wedge". Lower TFP has a direct, negative effect on equilibrium welfare. Furthermore, $\mathcal{D}$ is an increasing function of θ, the elasticity of substitution across intermediate goods. When intermediate goods are more substitutable, a given level of price dispersion translates into greater misallocation of production across firms and, hence, greater welfare loss.

Consider now the volatility of the output gap $\mathcal{V}$. Away from the first best, this term is strictly positive since the economy features a state-dependent average pricing error. That is, in equilibrium there exists a state-dependent wedge between the real wage $W_t(\bar{s}^t)/P(\bar{s}^t)$ and the marginal product of labor, i.e. a labor wedge.[13] What is more, the proof of Proposition 7 shows that this labor wedge is proportional to the difference between equilibrium (log) output and its first-best counterpart, that is, the "output gap". While the constant tax τ removes the *average* output gap, the *volatility* of the output gap contributes to equilibrium welfare loss.

Finally, note that the size of welfare loss depends on the parameters γ and η. These parameters govern the curvature of household's utility with respect to movements in consumption and labor, respectively, and therefore determine how such movements in the labor wedge translate into welfare loss.

6.4 The social value of disclosure

We now consider the question of whether central bank disclosure is welfare improving. As noted previously, the answer to this question depends on the interest rate policy set by the central bank; we will consider interest rate policies that satisfy condition (35) but deviate from the optimum (ψ_0^*, ψ_p^*). Specifically, we use the characterization in Proposition 7 to compare two scenarios: welfare loss under no disclosure and welfare loss under public disclosure. With a slight abuse of notation, we let $\mathcal{L}_0(\psi_0, \psi_p)$ and $\mathcal{L}_d(\psi_0, \psi_p)$ denote, respectively, the welfare loss under

[13] The household in our model is always on its intratemporal condition, so that the marginal rate of substitution between labor and consumption is equal to the real wage. Therefore, the labor wedge manifests only on the production side.

no disclosure and welfare loss under disclosure, as a function of policy parameters (ψ_0, ψ_p).

Lemma 5. *Take any interest rate policy* $(\psi_0, \psi_p) \in \mathbb{R}^2$ *and define*

$$\Delta(\psi_0, \psi_p) \equiv \mathcal{L}_0(\psi_0, \psi_p) - \mathcal{L}_d(\psi_0, \psi_p)$$

to be the difference in welfare loss across the two disclosure policies. This difference satisfies

$$\Delta(\psi_0, \psi_p) = a_0(\psi_0 - \psi_0^* + \psi_p)^2 + b_0\psi_p^2 + c_0(\psi_0 - \psi_0^* + \rho\psi_p)^2,$$

where $a_0 > 0$*,* $b_0 > 0$*, and* $c_0 < 0$ *are combinations of model parameters.*

Proof. See the online appendix, Section 5. □

First, note that at the optimal interest rate policy, $\Delta(\psi_0^*, \psi_p^*) = 0$. This is a direct consequence of the fact that the optimal policy implements the complete-information first-best. At the optimal allocation, any additional information provided to the firm has no effect on allocations and, hence, welfare whatsoever.

Suppose now that the interest rate rule moves away from the optimum. The difference $\Delta(\psi_0, \psi_p)$ can be either positive or negative. When it is positive, central bank disclosure is welfare-improving. On the contrary, when $\Delta(\psi_0, \psi_p) < 0$, central bank disclosure reduces welfare.

The following theorem uses the decomposition in Lemma 5 to provide a characterization of the welfare effects of central bank disclosure.

Theorem 5. *There exists a constant* $\overline{K}_x > 0$ *and a decreasing positive function* $\overline{K}_p(\kappa_x)$ *such that* $\Delta(\psi_0, \psi_p^*) > 0$ *for all* $\psi_0 \neq \psi_0^*$*, and* $\Delta(\psi_0^*, \psi_p) > 0$ *for all* $\psi_p \neq \psi_p^*$*, if*

(i) $1/2 \leq \theta\gamma \leq 2$*; or if*

(ii) $\theta\gamma > 2$ *and* $\kappa_x \geq \overline{K}_x$*; or if*

(iii) $\theta\gamma > 2$ *,* $\kappa_x < \overline{K}_x$ *and* $\kappa_p > \overline{K}_p(\kappa_x)$*.*

Proof. See the appendix, Section A.13. □

Theorem 5 provides sufficient conditions for which central bank disclosure is welfare-improving. In Theorem 5, we let the interest rate rule deviate from its optimum along each of its dimensions. That is, we set $\psi_p = \psi_p^*$ and allow the loading on past productivity to vary. We then set $\psi_0 = \psi_0^*$, and allow the interest rate sensitivity to the central banker's signal to vary. In both cases, public disclosure of the central banker's signal is welfare-improving if parameters satisfy the conditions stated in the theorem: these are joint conditions on the preferences of

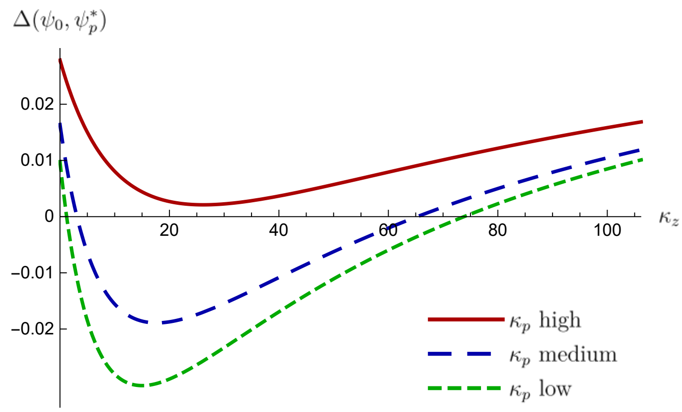

Figure 1. The welfare effects of central bank disclosure. This figure plots $\Delta(\psi_0, \psi_p^*)$ as a function of κ_z, for different values of κ_p and for an arbitrary $\psi_0 \neq \psi_0^*$. Parameters are such that $\gamma\theta > 2$ and $\kappa_x < \overline{K}_x$.

the household, the elasticity of substitution across goods, and the precision of firm and central bank private information.

Consider first part (i). The typical range for the value of the elasticity of substitution used in the New Keynesian literature is $\theta \in (4, 8)$.[14] Even setting θ to the lowest value in this range, 4, the condition $\gamma\theta \geq 1/2$ is met as long as $\gamma > 1/8$, which holds for typical parameter estimates of the inverse elasticity of substitution and of the coefficient of relative risk aversion. For the same reason, however, it is unlikely that the condition $\gamma\theta \leq 2$ is met.

We thereby conclude that whether or not central bank disclosure is welfare-improving depends on the precision of information available to the firms and the central banker (parts ii and iii). More precisely, Theorem 5 states that as long as either firms' private information (part ii) or the central banker's information (part iii) about productivity is sufficiently precise, then central bank disclosure improves welfare. It follows that for central bank information disclosure to lower welfare, it must be the case that *both* firm and central bank information about aggregate productivity are sufficiently noisy.

Figure 1 provides a numerical illustration of this last point. Specifically, we set $\psi_p = \psi_p^*$ and consider an arbitrary ψ_0; Figure 1 plots $\Delta(\psi_0, \psi_p^*)$ as a function of the precision of firm private information, κ_z, for different values of the precision of central bank information, κ_p. We choose parameters so that $\gamma\theta > 2$ and $\kappa_x < \overline{K}_x$, thus, the relevant case is part (iii) of Theorem 5. For central bank disclosure to be detrimental to welfare, the precision κ_p must be sufficiently low (below the threshold $\overline{K}_p(\kappa_x)$ of Theorem 5). The figure shows that for sufficiently low values of κ_p, there exists an interval for κ_z for which disclosure reduces welfare.[15]

[14]The modal values used in New Keynesian literature appear to be $\theta = 6$ and $\theta = 7$; see e.g. McKay, Nakamura, and Steinsson (2016) and Christiano, Eichenbaum, and Rebelo (2011).

[15]This statement can be proved formally: in the proof of case (iii) of Theorem 5, we show that, if $\kappa_p < \overline{K}_p(\kappa_x)$, there exists a subset of (κ_z, κ_p) for which $\Delta(\psi_0, \psi_p^*) < 0$ for all $\psi_0 \neq \psi_0^*$.

Together, Theorem 5 and Figure 1 indicate that the welfare effects of central bank disclosure are, in general, ambiguous. However, as long as either firm or central bank information is sufficiently precise, central bank disclosure is socially desirable.

Intuition. Central bank disclosure has two opposing effects on welfare. On the one hand, by sharing its private information, central bank disclosure provides all firms with more information about current (and future) productivity. Central bank disclosure also provides firms with the *specific* information of the central bank. It follows that firms will be able to anticipate the current period's interest rate with certainty: even under a sub-optimal interest rate policy, central bank disclosure eliminates all monetary policy "shocks." This reduction in fundamental and interest rate uncertainty has a negative effect on both the dispersion of prices and output gap volatility—the two components of welfare loss.

On the other hand, central bank disclosure leads to greater correlation in firm pricing errors. This is because the central banker's signal acts as a noisy public signal on which firms can now coordinate. Greater correlation of prices has a positive effect on output gap volatility—a force that pushes against the previous effects. This constitutes the detrimental welfare effect of central bank disclosure.

Depending on the relative strength of these two opposing effects, disclosure can either increase or reduce welfare. If firms possess very precise information about the aggregate fundamental, they will rely less on the public signal disclosed by the central bank. As a result, the benefits of central bank disclosure will outweigh the costs. Similarly, if the central banker's information is sufficiently precise, its disclosure will not have a large impact on output gap volatility. In this case, too, central bank disclosure will be socially desirable.

Limit case. We conclude this section with a discussion of the particular limit in which the central bank's signal becomes infinitely precise. Formally, when $\kappa_p \to \infty$, the coefficients in Lemma 5 are such that $b_0 \to 0$ and $c_0 \to 0$ and a_0 is independent of κ_p. Therefore,

$$\Delta(\psi_0, \psi_p) \to a_0 \left(\psi_0 - \psi_0^* + \psi_p\right)^2 .$$

Since $a_0 > 0$, it follows that central bank disclosure is always desirable if the central bank's information is infinitely precise. This is simply due to the fact that when the central bank discloses an infinitely precise signal, the information of firms becomes infinitely precise. At this limit, disclosure effectively implements the flexible price allocation (under any monetary policy) which we know to be efficient.

7 Conclusion

In this paper we study a relatively standard macro model of nominal price rigidities that originate in informational frictions. We depart from much of the previous literature on this topic by imposing an additional measurability constraint on the central banker. In particular, the central banker sets the nominal interest rate under incomplete information about current economic conditions. We furthermore allow firms to have some information about policy that is orthogonal to their beliefs of economic fundamentals.

Our first set of results concerns optimal monetary policy in this context. We find that there exists paths for nominal interest rates and prices that implement flexible-price allocations and, by implication, the complete-information first best. We characterize the full set of flexible-price implementations. All implementations share a common feature: the aggregate price level must respond to the past economic fundamental. It follows that price stability can never be optimal in our context.

Our second set of results analyzes the welfare effects of central bank disclosure when monetary policy is assumed to be sub-optimal. We focus on a particular class of equilibria and provide sufficient conditions under which, for certain deviations of the interest rate away from the optimal one, central bank disclosure is welfare-improving.

Our model excludes some features that many feel to be relevant for business cycle fluctuations and monetary policy considerations. First, we abstract from the zero lower bound on the nominal interest rate. Second, we abstract from what is known as the "forward guidance puzzle" (Del Negro, Giannoni, and Patterson, 2015) by ruling out all frictions that are known to mitigate the power of future contingencies of monetary policy. This includes forms of bounded rationality (Farhi and Werning, 2017; Woodford, 2018; Gabaix, 2020), dispersion of household beliefs (Angeletos and Lian, 2018), and household liquidity constraints (McKay, Nakamura, and Steinsson, 2016).

Finally, we say nothing about price level determinacy. In order to rule out indeterminacy and achieve unique implementation, one might consider the strategies proposed by Atkeson, Chari, and Kehoe (2010) and Bassetto (2002).

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A Appendix: Proofs

A.1 The Household's problem

We first present and solve the household's problem which will be used in later proofs. The household's problem chooses a plan for consumption and labor in order to solve the following problem

$$\max_{C,L,B} \sum_{t=0}^{\infty} \beta^t \sum_{\bar{s}^t} \mu(\bar{s}^t) \left[U(C_t(\bar{s}^t)) - V(L_t(\bar{s}^t)) \right],$$

subject to the budget constraint:

$$P_t(\bar{s}^t)C_t(\bar{s}^t) + B_t(\bar{s}^t) \leq W_t(\bar{s}^t)L_t(\bar{s}^t) + (1 + \iota_t(\omega_{pt}, \bar{s}^{t-1}))B_{t-1}(\bar{s}^{t-1}) + \Pi_t(\bar{s}^t) + T_t(\bar{s}^t).$$

where $\Pi_t(\bar{s}^t) = \sum_{\omega_{it} \in \Omega} \pi_{it}(\omega_{it}, \bar{s}^t)\varphi(\omega_{it}|\bar{s}^t)$ are nominal profits.

We write the Lagrangian as follows:

$$\begin{aligned}
\mathcal{L} = & \sum_{t=0}^{\infty} \beta^t \sum_{\bar{s}^t} \mu(\bar{s}^t) \left[U(C_t(\bar{s}^t)) - V(L_t(\bar{s}^t)) \right] \\
& - \sum_{t=0}^{\infty} \beta^t \sum_{\bar{s}^t} \mu(\bar{s}^t)\Lambda_t(\bar{s}^t)(P_t(\bar{s}^t)C_t(\bar{s}^t) + B_t(\bar{s}^t) - W_t(\bar{s}^t)L_t(\bar{s}^t) - (1 + \iota_t(\omega_{pt}, \bar{s}^{t-1}))B_{t-1}(\bar{s}^{t-1})) \\
& + \sum_{t=0}^{\infty} \beta^t \sum_{\bar{s}^t} \mu(\bar{s}^t)\Lambda_t(\bar{s}^t)(\Pi_t(\bar{s}^t) + T_t(\bar{s}^t)).
\end{aligned}$$

The household's first-order conditions with respect to consumption and labor are, respectively, given by

$$\mu(\bar{s}^t)U'(C_t(\bar{s}^t)) - \mu(\bar{s}^t)\Lambda_t(\bar{s}^t)P_t(\bar{s}^t) = 0, \tag{41}$$

$$-\mu(\bar{s}^t)V'(L_t(\bar{s}^t)) + \mu(\bar{s}^t)\Lambda_t(\bar{s}^t)W_t(\bar{s}^t) = 0, \tag{42}$$

the household's first-order condition with respect to bonds is given by

$$\beta^t \mu(\bar{s}^t)\Lambda_t(\bar{s}^t) - \beta^{t+1} \sum_{\bar{s}^{t+1}|\bar{s}^t} \mu(\bar{s}^{t+1})\Lambda_t(\bar{s}^{t+1})(1 + \iota_t(\omega_{pt}, \bar{s}^{t-1})) = 0.$$

and the transversality condition is given by

$$\lim_{t\to\infty} \sum_{\bar{s}^t} \beta^t \mu(\bar{s}^t)U'(C_t(\bar{s}^t))B_t(\bar{s}^t) = 0. \tag{43}$$

Therefore, the household's intratemporal condition is given by

$$\frac{V'(L_t(\bar{s}^t))}{U'(C_t(\bar{s}^t))} = \frac{W_t(\bar{s}^t)}{P_t(\bar{s}^t)} \tag{44}$$

and the household's Euler equation is given by

$$\Lambda_t(\bar{s}^t) = \beta(1 + \iota_t(\omega_{pt}, \bar{s}^{t-1})) \sum_{\bar{s}^{t+1}} \Lambda_t(\bar{s}^{t+1})\mu(\bar{s}^{t+1}|\bar{s}^t).$$

Using the fact that $U'(C_t(\bar{s}^t)) = \Lambda_t(\bar{s}^t)P_t(\bar{s}^t)$, we may rewrite the Euler equation as

$$\frac{U'(C_t(\bar{s}^t))}{P_t(\bar{s}^t)} = \beta(1 + \iota_t(\omega_{pt}, \bar{s}^{t-1})) \sum_{\bar{s}^{t+1}} \frac{U'(C_t(\bar{s}^{t+1}))}{P_t(\bar{s}^{t+1})}\mu(\bar{s}^{t+1}|\bar{s}^t),$$

or alternatively as

$$\frac{U'(C_t(\bar{s}^t))}{P_t(\bar{s}^t)} = \beta(1 + \iota_t(\omega_{pt}, \bar{s}^{t-1}))\mathbb{E}\left[\left.\frac{U'(C_t(\bar{s}^{t+1}))}{P_t(\bar{s}^{t+1})}\right| \bar{s}^t\right]. \tag{45}$$

A.2 Proof of Lemma 1.

Under flexible prices, the firm's problem at time t is given by

$$\max_{p_i'} (1 - \tau)p_i' y_{it}(s^t) - W_t(s^t)\frac{y_{it}(s^t)}{A(s_t)},$$

subject to CES demand function (5). The firm's first-order condition is given by

$$(\theta - 1)(1 - \tau)(p_{it}(s^t))^{-\theta} - \theta\frac{W_t(s^t)}{A(s_t)}(p_{it}(s^t))^{-\theta-1} = 0.$$

Solving the latter for $p_{it}(s^t)$ gives

$$p_{it}(s^t) = \left(\frac{\theta - 1}{\theta}\right)^{-1} \frac{1}{1 - \tau} \cdot \frac{W_t(s^t)}{A(s_t)}, \tag{46}$$

as in (10).

Therefore, an allocation ξ, a policy ϑ, and price system ϱ, are part of a flexible-price equilibrium if and only if the following four properties hold: (i) the following household optimality conditions are satisfied:

$$\frac{V'(L_t(s^t))}{U'(C_t(s^t))} = \frac{W_t(s^t)}{P_t(s^t)}, \qquad \forall s^t \in S^t, \tag{47}$$

$$\frac{U'(C_t(s^t))}{P_t(s^t)} = \beta(1 + \iota(s^t))\mathbb{E}\left[\left.\frac{U'(C_{t+1}(s^{t+1}))}{P_{t+1}(s^{t+1})}\right| s^t\right], \qquad \forall s^t \in S^t, \tag{48}$$

along with the transversality condition:

$$\lim_{t\to\infty} \beta^t \mathbb{E}\left[U'(C_t(s^t))B_t(s^t)\right] = 0.$$

(ii) the following firm optimality condition is satisfied:

$$p_{it}(s^t) = \left(\frac{\theta-1}{\theta}\right)^{-1} \frac{1}{1-\tau} \frac{W_t(s^t)}{A(s_t)}, \qquad \forall s^t \in S^t,$$

along with the intermediate-good demand condition, namely, (5), (iii) the household's and government budget sets are satisfied, (iv) and all markets clear, namely, conditions (6)-(8) are satisfied.

We will now use this equilibrium characterization to prove necessity and sufficiency of the conditions stated in Lemma 1.

Necessity. Firm's optimality condition (46) implies that all firms set the same nominal price, $p_{it}(s^t) = P_t(s^t)$ for all $i \in I$. We may therefore rewrite (46) as follows:

$$1 = \left(\frac{\theta-1}{\theta}\right)^{-1} \frac{1}{1-\tau} \cdot \frac{1}{A(s_t)} \cdot \frac{W_t(s^t)}{P_t(s^t)}.$$

Combining this with the household's intratemporal optimality condition in (9) yields

$$1 = \left(\frac{\theta-1}{\theta}\right)^{-1} \frac{1}{1-\tau} \cdot \frac{1}{A(s_t)} \cdot \frac{V'(L_t(s^t))}{U'(C_t(s^t))}.$$

This proves necessity of the intratemporal condition in (12), with $\chi \equiv \left(\frac{\theta-1}{\theta}\right)^{-1} \frac{1}{1-\tau}$.

Finally, note that if $p_{it}(s^t) = P_t(s^t)$ for all $i \in I$, then by (5) it follows that production is identical across all intermediate good producers: $y_{it}(s^t) = Y_t(s^t)$ for all $i \in I$, and similarly by the production function (6), that labor is identical across producers: $\ell_{it}(s^t) = L_t(s^t)$ for all $i \in I$. This proves necessity of the conditions in (11) as well as the aggregate production function in (12).

Sufficiency. We now prove that the conditions stated in Lemma 1 are furthermore sufficient.

To do so, we take any strictly positive constant $\chi \in \mathbb{R}_+$ and allocation ξ that satisfies conditions (11)-(12). We now show that there exists a price system ϱ and a policy ϑ that supports this allocation as a flexible-price equilibrium.

First, set the tax rate so that $1-\tau = \left(\frac{\theta-1}{\theta}\right)^{-1} \chi^{-1}$. For any strictly positive χ, such a tax rate exists. This implies that condition (12) may be rewritten as

$$U'(C_t(s^t)) = \left(\frac{\theta-1}{\theta}\right)^{-1} \frac{1}{1-\tau} V'(L_t(s^t)) \frac{1}{A(s_t)}. \tag{49}$$

Next, we set the real wage $W_t(s^t)/P_t(s^t)$ in order to satisfy the household's intratemporal condition:

$$\frac{W_t(s^t)}{P_t(s^t)} = \frac{V'(L_t(s^t))}{U'(C_t(s^t))}.$$

Substituting the real wage into (49) gives us

$$1 = \left(\frac{\theta - 1}{\theta}\right)^{-1} \frac{1}{1-\tau} \cdot \frac{W_t(s^t)}{P_t(s^t)} \cdot \frac{1}{A(s_t)}. \tag{50}$$

Next, note that zero dispersion in output (11) along with the CES demand function (5) implies that there must be zero price dispersion in equilibrium: $p_{it}(s^t) = P(s^t)$ for all $i \in I$. Therefore, equation (50) must hold for at every price $p_{it}(s^t)$:

$$1 = \left(\frac{\theta - 1}{\theta}\right)^{-1} \frac{1}{1-\tau} \cdot \frac{W_t(s^t)}{p_{it}(s^t)} \cdot \frac{1}{A(s_t)},$$

and, as a result, individual firm's optimality condition (46) is satisfied. Finally, the nominal interest rate and the sequence of prices must jointly satisfy the household's Euler equation for bond holdings in (9). The transversality condition holds trivially since, in equilibrium, $B_t(s^t) = 0$ and $C_t(s^t) > 0$ in all states and periods.

What remains to be shown is that the household's budget constraint and the government's budget constrained are satisfied at this allocation. First, note that the government's budget set holds by the assumption that $T_t(s^t) = \tau \int p_{it}(s^t) y_{it}(s^t) di$ and that bonds are in zero net supply: $B_t(s^t) = 0$ in all states and periods. The household's budget set is given by

$$P_t(s^t)C_t(s^t) \le W_t(s^t)L_t(s^t) + \int \pi_{it}(s^t)di + T_t(s^t).$$

Substituting in for profits and lump-sum taxes, this constraint becomes

$$P_t(s^t)C_t(s^t) \le W_t(s^t)L_t(s^t) + \int \left[(1-\tau)p_{it}(s^t)y_{it}(s^t) - W_t(s^t)\ell_{it}(s^t)\right] di + \tau \int p_{it}(s^t)y_{it}(s^t)di,$$

which is automatically satisfied via the resource constraints in (7) and (8). QED.

A.3 Proof of Lemma 2.

The planner's problem is to maximize utility

$$\sum_{t=0}^{\infty} \beta^t \sum_{s^t} \mu(s^t) \left[U(C_t(s^t)) - V(L_t(s^t))\right],$$

subject to resource constraints (7)-(8). From the CES technology:

$$y_{it}(s^t) = Y_t(s^t) = C_t(s^t), \qquad \text{and} \qquad \ell_{it}(s^t) = L_t(s^t), \qquad \forall i, s^t,$$

Note that there is no capital in this model, nor any other type of dynamic consideration in terms of allocations. Therefore, this problem can be solved both period-by-period and state-by-state:

$$\max_{C,L} U(C_t(s^t)) - V(L_t(s^t)),$$

subject to

$$C_t(s^t) = A(s_t)L_t(s^t). \tag{51}$$

The planner's first-order conditions are given by

$$U'(C_t(s^t)) - \Gamma_t(s^t) = 0,$$
$$-V'(L_t(s^t)) + \Gamma_t(s^t)A(s_t) = 0,$$

where $\Gamma_t(s^t)$ is the Lagrange multiplier on the resource constraint (51) in history s^t. It follows that the efficient allocation ξ^* can be implemented under flexible prices with $\chi = 1$. QED.

A.4 Proof of Lemma 3.

Under sticky prices, the firm's problem at time t is given by

$$\max_{p_i'} \mathbb{E}\left[\Lambda_t(\bar{s}^t)\left\{(1-\tau)p_i' y_{it}(\omega_i^t,\bar{s}^t) - W_t(\bar{s}^t)\frac{y_{it}(\omega_i^t,\bar{s}^t)}{A(s_t)}\right\}\middle|\,\omega_i^t\right],$$

subject to CES demand function (5). Substituting in the demand function gives us the following objective function:

$$\max_{p_i'} \mathbb{E}\left[\Lambda_t(\bar{s}^t)\left\{(1-\tau)(p_i')^{1-\theta}P_t(\bar{s}^t)^\theta Y_t(\bar{s}^t) - \frac{W_t(\bar{s}^t)}{A(s_t)}(p_i')^{-\theta}P_t(\bar{s}^t)^\theta Y_t(\bar{s}^t)\right\}\middle|\,\omega_i^t\right].$$

The firm's first order condition is given by

$$\mathbb{E}\left[\Lambda_t(\bar{s}^t)\left\{(\theta-1)(1-\tau)(p_{it}(\omega_i^t))^{-\theta}P_t(\bar{s}^t)^\theta Y_t(\bar{s}^t) - \theta\frac{W_t(\bar{s}^t)}{A(s_t)}(p_{it}(\omega_i^t))^{-\theta-1}P_t(\bar{s}^t)^\theta Y_t(\bar{s}^t)\right\}\middle|\,\omega_i^t\right] = 0,$$

which, using (5), may be rewritten as

$$\mathbb{E}\left[\Lambda_t(\bar{s}^t)\left\{(\theta-1)(1-\tau)y_{it}(\omega_i^t,\bar{s}^t) - \theta\frac{W_t(\bar{s}^t)}{A(s_t)}p_{it}(\omega_i^t)^{-1}y_{it}(\omega_i^t,\bar{s}^t)\right\}\middle|\,\omega_i^t\right] = 0.$$

Therefore, the firm's optimal price satisfies

$$\mathbb{E}\left[\Lambda_t(\bar{s}^t)y_{it}(\omega_i^t,\bar{s}^t)\left\{p_{it}(\omega_i^t) - \left(\frac{\theta-1}{\theta}\right)^{-1}\frac{1}{1-\tau}\frac{W_t(\bar{s}^t)}{A(s_t)}\right\}\middle|\,\omega_i^t\right] = 0.$$

Solving the latter for $p_{it}(\omega_i^t)$ provides the following expression:

$$p_{it}(\omega_i^t) = \left(\frac{\theta-1}{\theta}\right)^{-1}\frac{1}{1-\tau}\mathbb{E}\left[\frac{\Lambda_t(\bar{s}^t)y_{it}(\omega_i^t,\bar{s}^t)}{\mathbb{E}\left[\Lambda_t(\bar{s}^t)y_{it}(\omega_i^t,\bar{s}^t)\middle|\,\omega_i^t\right]}\left\{\frac{W_t(\bar{s}^t)}{A(s_t)}\right\}\middle|\,\omega_i^t\right]. \tag{52}$$

This coincides with the equation presented in (18), with the function $q_{it}(\omega_i^t,\bar{s}^t)$ defined in (14).

The household's optimality conditions are given by (44) and (45), along with the transversality condition (43). QED.

A.5 Proof of Proposition 1.

As already demonstrated in the main text, the interest rate and path of prices proposed in Proposition 1 satisfy the Euler equation under sticky prices at the flexible-price allocation.

What remains to be shown is that there is no dispersion in nominal prices. From the household intratemporal equation (15), the nominal wage at the flexible-price allocation satisfies

$$W_t(\bar{s}^t) = \frac{V'(\mathcal{L}(s_t))}{U'(\mathcal{C}(s_t))} P_t(\bar{s}^t).$$

From Lemma 1, the following condition holds at the flexible-price allocation:

$$\frac{V'(\mathcal{L}(s_t))}{U'(\mathcal{C}(s_t))} = A(s_t)\chi^{-1}.$$

This implies that nominal wage under sticky prices at the flexible-price allocation satisfies

$$W_t(\bar{s}^t) = A(s_t)\chi^{-1} P_t(\bar{s}^t). \tag{53}$$

The firm's optimality condition under sticky prices is given by (18). Substituting in for the nominal wage in (53) yields

$$p_{it}(\omega_i^t) = \mathbb{E}\left[q_{it}(\omega_i^t, \bar{s}^t) P_t(\bar{s}^t) \middle| \omega_i^t \right],$$

where, under the proposed implementation, $P_t(\bar{s}^t) = \mathcal{P}(s_{t-1})$. It follows that $p_{it}(\omega_i^t) = \mathbb{E}\left[q_{it}(\omega_i^t, \bar{s}^t)\mathcal{P}(s_{t-1}) \middle| \omega_i^t \right] = \mathcal{P}(s_{t-1})$ for all $\omega_i^t \in \Omega_i^t$. QED.

A.6 Proof of Propositions 2-5.

For each of these propositions, we demonstrate in the main text that the nominal interest rate and path of prices satisfy the Euler equation at the flexible-price allocation. What remains to be shown is that there is no dispersion in nominal prices across intermediate-good firms. The proof of this statement follows the exact same steps as those found in the proof of Proposition 1. QED.

A.7 Auxiliary Lemma for Section 6

The following lemma characterizes the set of implementable allocations in the log-linear Gaussian setting of Section 6.

Lemma 6. *An allocation is implementable as a sticky-price equilibrium if and only if there exists a strictly positive constant $\chi \in \mathbb{R}_+$ and strictly positive-valued function $\psi : \Omega \to \mathbb{R}_+$ such that the allocation together with functions ε_i and Λ satisfies:*

$$U'(C_t(\bar{s}^t)) = \chi \frac{V'(L_t(\bar{s}^t))}{A(s_t)} \left(\frac{y_{it}(\omega_i^t, \bar{s}^t)}{Y_t(\bar{s}^t)} \right)^{\frac{1}{\theta}} \varepsilon_{it}(\omega_{it}, \bar{s}^t)^{-1}, \tag{54}$$

with

$$\varepsilon_{it}(\omega_{it}, \bar{s}^t) \equiv \frac{V'(L_t(\bar{s}^t))A(s_t)^{-1}\Lambda_t(\bar{s}^t)^{-1}}{\mathbb{E}\left[q_{it}(\omega_i^t, \bar{s}^t)\left\{V'(L_t(\bar{s}^t))A(s_t)^{-1}\Lambda_t(\bar{s}^t)^{-1}\right\}\middle|\, \omega_i^t\right]} > 0, \tag{55}$$

where

$$\Lambda_t(\bar{s}^t) = \beta\psi(\omega_{pt}, \bar{s}^{t-1})\mathbb{E}[\Lambda_t(\bar{s}^{t+1})|\bar{s}^t], \tag{56}$$

along with technology and resource constraints, (6)-(8).

Proof. See the online appendix, Section 1. □

A.8 Proof of Lemma 4.

We conjecture that the Lagrange multiplier is log-linear as in (36). The Euler equation in (56) implies

$$\begin{aligned}\psi_p x_{pt} + \psi_0 a_{t-1} + \psi_a a_t =& \psi_p x_{pt} + \psi_0 a_{t-1} \\ &+ \mathbb{E}[\psi_p x_{p,t+1} + \psi_0 a_t + \psi_a a_{t+1}|\bar{s}^t].\end{aligned}$$

Note that $a_t \in \bar{s}^t$. Furthermore, recall that the planner's signal is given by $x_{pt} = a_t + \epsilon_{pt}$. The above condition thereby reduces to

$$\psi_a a_t = \psi_0 a_t + (\psi_a + \psi_p)\mathbb{E}[a_{t+1}|\bar{s}^t].$$

Household expectations are given by

$$\mathbb{E}[a_{t+1}|\bar{s}^t] = \rho a_t.$$

Plugging the latter into the above condition gives

$$\psi_a a_t = \psi_0 a_t + (\psi_a + \psi_p)\rho a_t.$$

We therefore obtain

$$\psi_a = \psi_0 + (\psi_a + \psi_p)\rho.$$

Solving the latter for ψ_a yields the expression in (37). QED.

A.9 Proof of Proposition 6.

In order to implement flexible-price allocations under sticky prices, the following conditions must hold:

$$\varepsilon_{it}(\omega_{it}, \bar{s}^t) = 1, \qquad \forall \omega_{it}, \bar{s}^t. \tag{57}$$

That is, firms must not make any pricing mistakes. From (55), conditions (57) are equivalent to the following conditions:

$$\mathbb{E}\left[q_{it}(\omega_i^t, \bar{s}^t)\left\{V'(L_t(\bar{s}^t))A(s_t)^{-1}\Lambda_t(\bar{s}^t)^{-1}\right\}\middle|\, \omega_i^t\right] = V'(L_t(\bar{s}^t))A(s_t)^{-1}\Lambda_t(\bar{s}^t)^{-1}, \quad \forall \omega_i^t, \bar{s}^t. \tag{58}$$

In addition, from Lemma 4, for a given interest rate (35), the Lagrange multiplier associated to the household's problem satisfies

$$\log \Lambda_t(\bar{s}^t)^{-1} = -\psi_p x_{pt} - \psi_0 a_{t-1} - \psi_a a_t.$$

Plugging this expression into (58) gives

$$\mathbb{E}\left[q_{it}(\omega_i^t, \bar{s}^t) \left\{ V'(L_t(\bar{s}^t)) A(s_t)^{-1} \exp(-\psi_p x_{pt} - \psi_a a_t) A(s_{t-1})^{-\psi_0} \right\} \Big| \, \omega_i^t \right]$$
$$= V'(L_t(\bar{s}^t)) A(s_t)^{-1} \exp(-\psi_p x_{pt} - \psi_a a_t) A(s_{t-1})^{-\psi_0}, \quad \forall \omega_i^t, \bar{s}^t.$$

Next, note that $\bar{s}^{t-1}$ is known at time t. Therefore, the term $A(s_{t-1})$ can be taken out of the expectation operator, hence, the above condition reduces to

$$\mathbb{E}\left[q_{it}(\omega_i^t, \bar{s}^t) g_t(\bar{s}^t) \big| \, \omega_i^t \right] = g_t(\bar{s}^t), \quad \forall \omega_i^t, \bar{s}^t, \tag{59}$$

where we let

$$g_t(\bar{s}^t) \equiv V'(L_t(\bar{s}^t)) A(s_t)^{-1} \exp(-\psi_p x_{pt} - \psi_a a_t). \tag{60}$$

In order for (59) to hold for all ω_{it}, it must be the case that

$$\mathbb{E}[q_{it}(\omega_i^t, \bar{s}^t) g_t(\bar{s}^t) | (\omega_{it}, \bar{s}^{t-1})] = \mathbb{E}[q_{it}(\omega_j^t, \bar{s}^t) g(\bar{s}^t) | (\omega_{jt}, \bar{s}^{t-1})], \quad \forall \omega_{it}, \omega_{jt}.$$

Note, however, that

$$\mathbb{E}[\exp x_{pt} | \omega_{it}] \neq \mathbb{E}[\exp x_{pt} | \omega_{jt}],$$

due to the fact that $z_{it} \neq z_{jt}$. Therefore, (59) holds for all ω_{it} if and only if $\psi_p^* = 0$.

With $\psi_p^* = 0$, function $g_t(\bar{s}^t)$ defined in (60) reduces to

$$g_t(\bar{s}^t) = V'(L_t(\bar{s}^t)) A(s_t)^{-1} \exp(-\psi_a a_t) = V'(L_t(\bar{s}^t)) A(s_t)^{-1} A(s_t)^{-\psi_a}.$$

In order for (59) to hold for all ω_{it}, it must be the case that $g_t(\bar{s}^t) \in (\omega_{it}, \bar{s}^{t-1})$, for all $\omega_{it}, \bar{s}^{t-1}$. This is true if and only if

$$V'(L_t(\bar{s}^t)) A(s_t)^{-1} A(s_t)^{-\psi_a} = G, \quad \forall \bar{s}^t,$$

where G is an arbitrary constant. Homothetic preferences as in (34) imply $V'(L_t(\bar{s}^t)) = L_t(\bar{s}^t)^{1/\eta}$. Furthermore, under flexible-price allocations, $L_t(\bar{s}^t) = A(s_t)^{\frac{1-\gamma}{1/\eta+\gamma}}$. We can therefore rewrite the above equation as

$$A(s_t)^{\frac{1-\gamma}{1/\eta+\gamma}(1/\eta)} A(s_t)^{-1} A(s_t)^{-\psi_a} = G.$$

The unique ψ_a that satisfies this condition is given by

$$\psi_a = -\gamma \frac{1/\eta + 1}{1/\eta + \gamma}.$$

Finally, ψ_a is the coefficient in the Lagrange multiplier in the household's problem associated to the interest-rate policy. From Lemma 4, the interest rate in (35) must then satisfy

$$\psi_a = \frac{1}{1-\rho}\psi_0,$$

where we have used the fact that $\psi_p^* = 0$. Therefore,

$$\psi_0^* = -\gamma \frac{1/\eta + 1}{1/\eta + \gamma}(1-\rho),$$

as was to be shown. QED.

A.10 Auxiliary Lemma: Aggregate Consumption and Labor

Lemma 7. *Equilibrium aggregate output and aggregate labor satisfy the following system of two equations:*

$$\chi V'(L_t(\bar{s}^t)) = \bar{\varepsilon}_t(\bar{s}^t)U'(Y_t(\bar{s}^t))\frac{Y_t(\bar{s}^t)}{L_t(\bar{s}^t)} \qquad \textit{and} \qquad Y_t(\bar{s}^t) = \delta A(s_t)L_t(\bar{s}^t), \tag{61}$$

where we let $\bar{\varepsilon}_t(\bar{s}^t) \equiv \kappa \exp\left\{\int_\omega \log \varepsilon_{it}(\omega, \bar{s}^t)\varphi(\omega|\bar{s}^t)d\omega\right\}$*, for some* $\kappa > 0$ *defined in the proof, be the aggregate labor wedge in state* $\bar{s}^t$ *and*

$$\delta \equiv \exp\left\{-\frac{\theta}{2}\text{var}_\omega\left[\log \varepsilon_{it}(\omega_{it}, \bar{s}^t)|\bar{s}^t\right]\right\} \tag{62}$$

be the aggregate efficiency wedge.

Proof. See the online appendix, Section 2. □

A.11 Proof of Proposition 7.

We first use Lemma 7 to solve for equilibrium aggregate output and labor as functions of TFP and the labor wedge. Homothetic preferences as in (34) imply the following solution for equilibrium output and labor:

$$Y_t(\bar{s}^t) = (\chi^{-1}\bar{\varepsilon}_t(\bar{s}^t))^{\frac{1}{1/\eta+\gamma}}(\delta A(s_t))^{\frac{1/\eta+1}{1/\eta+\gamma}} \qquad \text{and} \qquad L_t(\bar{s}^t) = (\chi^{-1}\bar{\varepsilon}_t(\bar{s}^t))^{\frac{1}{1/\eta+\gamma}}(\delta A(s_t))^{\frac{1-\gamma}{1/\eta+\gamma}}. \tag{63}$$

Furthermore, at the first best,

$$Y_t^*(\bar{s}^t) = A(s_t)^{\frac{1/\eta+1}{1/\eta+\gamma}} \qquad \text{and} \qquad L_t^*(\bar{s}^t) = A(s_t)^{\frac{1-\gamma}{1/\eta+\gamma}}.$$

We use the latter expressions to first characterize first-best realized utility. Realized per-period utility in state $\bar{s}^t$ is given by

$$\mathcal{U}_t(\bar{s}^t) \equiv \frac{Y_t(\bar{s}^t)^{1-\gamma}}{1-\gamma} - \frac{L_t(\bar{s}^t)^{1+1/\eta}}{1+1/\eta}. \tag{64}$$

Thus, first-best realized per-period utility $\mathcal{U}^*(s^t)$ is given by

$$\mathcal{U}^*(s^t) = \frac{1}{1-\gamma} A(s_t)^{\frac{1/\eta+1}{1/\eta+\gamma}(1-\gamma)} - \frac{1}{1+1/\eta} A(s_t)^{\frac{1-\gamma}{1/\eta+\gamma}(1+1/\eta)}$$

or

$$\mathcal{U}^*(s^t) = \frac{1/\eta+\gamma}{(1-\gamma)(1+1/\eta)} A(s_t)^{\frac{(1-\gamma)(1+1/\eta)}{1/\eta+\gamma}}. \tag{65}$$

Now, consider realized utility away from the first best. Substituting equilibrium output and labor from (63) into (64), we get

$$\mathcal{U}_t(\bar{s}^t) = \frac{1}{1-\gamma}(\chi^{-1}\bar{\varepsilon}_t(\bar{s}^t))^{\frac{1-\gamma}{1/\eta+\gamma}}(\delta A(s_t))^{\frac{(1/\eta+1)(1-\gamma)}{1/\eta+\gamma}} - \frac{1}{1+1/\eta}(\chi^{-1}\bar{\varepsilon}_t(\bar{s}^t))^{\frac{1+1/\eta}{1/\eta+\gamma}}(\delta A(s_t))^{\frac{(1/\eta+1)(1-\gamma)}{1/\eta+\gamma}}.$$

Taking the unconditional expectation of both sides and summing over time yields expected welfare:

$$\mathcal{W} = \sum_{t=0}^{\infty} \beta^t \delta^{\frac{(1/\eta+1)(1-\gamma)}{1/\eta+\gamma}} \mathbb{E}\left[\frac{1}{1-\gamma}(\chi^{-1}\bar{\varepsilon}_t(\bar{s}^t))^{\frac{1-\gamma}{1/\eta+\gamma}} A(s_t)^{\frac{(1/\eta+1)(1-\gamma)}{1/\eta+\gamma}} - \frac{1}{1+1/\eta}(\chi^{-1}\bar{\varepsilon}_t(\bar{s}^t))^{\frac{1+1/\eta}{1/\eta+\gamma}} A(s_t)^{\frac{(1/\eta+1)(1-\gamma)}{1/\eta+\gamma}}\right].$$

Consider now the expectation $\mathbb{E}[\bar{\varepsilon}_t(\bar{s}^t)^{\kappa_1} A(s_t)^{\kappa_2}]$, for some scalars κ_1, κ_2. Properties of log-Normal distributions yield

$$\mathbb{E}[\bar{\varepsilon}_t(\bar{s}^t)^{\kappa_1} A(s_t)^{\kappa_2}] = \exp\begin{pmatrix} \kappa_1 \mathbb{E}[\log \bar{\varepsilon}_t(\bar{s}^t)] + \kappa_2 \mathbb{E}[\log A(s_t)] + \frac{1}{2}\kappa_1^2 \mathrm{var}(\log \bar{\varepsilon}_t(\bar{s}^t)) \\ +\frac{1}{2}\kappa_2^2 \mathrm{var}(\log A(s_t)) + \kappa_1\kappa_2 \mathrm{cov}(\log \bar{\varepsilon}_t(\bar{s}^t), \log A(s_t)) \end{pmatrix},$$

hence,

$$\mathbb{E}[\bar{\varepsilon}_t(\bar{s}^t)^{\kappa_1} A(s_t)^{\kappa_2}] = \mathbb{E}[\bar{\varepsilon}_t(\bar{s}^t)]^{\kappa_1}\mathbb{E}[A(s_t)]^{\kappa_2} \exp\begin{pmatrix} \frac{1}{2}\kappa_1(\kappa_1-1)\mathrm{var}(\log \bar{\varepsilon}_t(\bar{s}^t)) + \frac{1}{2}\kappa_2(\kappa_2-1)\mathrm{var}(\log A(s_t)) \\ +\kappa_1\kappa_2 \mathrm{cov}(\log \bar{\varepsilon}_t(\bar{s}^t), \log A(s_t)) \end{pmatrix}.$$

Applying the latter to our case yields

$$\begin{aligned}
\mathcal{W}\delta^{-\frac{(1/\eta+1)(1-\gamma)}{1/\eta+\gamma}} &= \sum_{t=0}^{\infty}\beta^t\left(\frac{1}{1-\gamma}\mathbb{E}\left[(\chi^{-1}\bar{\varepsilon}_t(\bar{s}^t))^{\frac{1-\gamma}{1/\eta+\gamma}} A(s_t)^{\frac{(1/\eta+1)(1-\gamma)}{1/\eta+\gamma}}\right] - \frac{1}{1+1/\eta}\mathbb{E}\left[(\chi^{-1}\bar{\varepsilon}_t(\bar{s}^t))^{\frac{1+1/\eta}{1/\eta+\gamma}} A(s_t)^{\frac{(1/\eta+1)(1-\gamma)}{1/\eta+\gamma}}\right]\right) \\
&= \sum_{t=0}^{\infty}\beta^t\left(\frac{1}{1-\gamma}\mathbb{E}[\chi^{-1}\bar{\varepsilon}_t(\bar{s}^t)]^{\frac{1-\gamma}{1/\eta+\gamma}}\mathbb{E}[A(s_t)]^{\frac{(1/\eta+1)(1-\gamma)}{1/\eta+\gamma}} \exp G_1\right. \\
&\quad \left. - \frac{1}{1+1/\eta}\mathbb{E}[\chi^{-1}\bar{\varepsilon}_t(\bar{s}^t)]^{\frac{1+1/\eta}{1/\eta+\gamma}}\mathbb{E}[A(s_t)]^{\frac{(1/\eta+1)(1-\gamma)}{1/\eta+\gamma}} \exp G_2\right),
\end{aligned}$$

where we have defined

$$\begin{aligned}
G_1 &\equiv \frac{1}{2}\frac{1-\gamma}{1/\eta+\gamma}\left(\frac{1-\gamma}{1/\eta+\gamma}-1\right)\mathrm{var}(\log \bar{\varepsilon}_t(\bar{s}^t)) + \Phi + \frac{1-\gamma}{1/\eta+\gamma}\frac{(1/\eta+1)(1-\gamma)}{1/\eta+\gamma}\mathrm{cov}(\log \bar{\varepsilon}_t(\bar{s}^t), \log A(s_t)), \\
G_2 &\equiv \frac{1}{2}\frac{1+1/\eta}{1/\eta+\gamma}\left(\frac{1+1/\eta}{1/\eta+\gamma}-1\right)\mathrm{var}(\log \bar{\varepsilon}_t(\bar{s}^t)) + \Phi + \frac{1+1/\eta}{1/\eta+\gamma}\frac{(1/\eta+1)(1-\gamma)}{1/\eta+\gamma}\mathrm{cov}(\log \bar{\varepsilon}_t(\bar{s}^t), \log A(s_t)), \\
\Phi &\equiv \frac{1}{2}\frac{(1/\eta+1)(1-\gamma)}{1/\eta+\gamma}\left(\frac{(1/\eta+1)(1-\gamma)}{1/\eta+\gamma}-1\right)\mathrm{var}(\log A(s_t)).
\end{aligned}$$

Furthermore, in the online appendix (Lemma A.2 and Lemma A.4) we compute the log-linear equilibrium in closed-form and show that the moments of $\bar{\varepsilon}_t(\bar{s}^t)$ are independent of time.

To further simplify the expression above, consider the value $\mathbb{E}[\chi^{-1}\bar{\varepsilon}_t(\bar{s}^t)]$ that maximizes $\mathcal{W}$. This value, which we denote this value with $\bar{\varepsilon}^*$, satisfies the first-order condition

$$(\bar{\varepsilon}^*)^{\frac{1-\gamma}{1/\eta+\gamma}} \exp G_1 - (\bar{\varepsilon}^*)^{\frac{1+1/\eta}{1/\eta+\gamma}} \exp G_2 = 0. \tag{66}$$

Also, let $\widehat{\mathcal{W}}$ denote the value of $\mathcal{W}$ when $\mathbb{E}[\chi^{-1}\bar{\varepsilon}_t(\bar{s}^t)] = \bar{\varepsilon}^*$. Then,

$$\widehat{\mathcal{W}} = \delta^{\frac{(1/\eta+1)(1-\gamma)}{1/\eta+\gamma}} \left(\frac{1}{1-\gamma} - \frac{1}{1+1/\eta}\right) \mathbb{E}[A(s_t)]^{\frac{(1/\eta+1)(1-\gamma)}{1/\eta+\gamma}} (\bar{\varepsilon}^*)^{\frac{1-\gamma}{1/\eta+\gamma}} \exp G_1$$

and, therefore,

$$\mathcal{W} = \frac{1}{1-\beta} \cdot \frac{1}{\frac{1}{1-\gamma} - \frac{1}{1+1/\eta}} \left(\frac{1}{1-\gamma} \left(\frac{\mathbb{E}[\chi^{-1}\bar{\varepsilon}_t(\bar{s}^t)]}{\bar{\varepsilon}^*} \right)^{\frac{1-\gamma}{1/\eta+\gamma}} - \frac{1}{1+1/\eta} \left(\frac{\mathbb{E}[\chi^{-1}\bar{\varepsilon}_t(\bar{s}^t)]}{\bar{\varepsilon}^*} \right)^{\frac{1+1/\eta}{1/\eta+\gamma}} \right) \widehat{\mathcal{W}}.$$

In addition, from equation (66),

$$\bar{\varepsilon}^* = \exp(G_1 - G_2),$$

thus,

$$\widehat{\mathcal{W}} = \delta^{\frac{(1/\eta+1)(1-\gamma)}{1/\eta+\gamma}} \frac{\gamma+1/\eta}{(1-\gamma)(1+1/\eta)} \mathbb{E}[A(s_t)]^{\frac{(1/\eta+1)(1-\gamma)}{1/\eta+\gamma}} \exp\left\{ \frac{1+1/\eta}{1/\eta+\gamma} G_1 - \frac{1-\gamma}{1/\eta+\gamma} G_2 \right\}.$$

The function

$$w(x) \equiv \frac{(1-\gamma)(1+1/\eta)}{\gamma+1/\eta} \left(\frac{1}{1-\gamma} x^{\frac{1-\gamma}{1/\eta+\gamma}} - \frac{1}{1+1/\eta} x^{\frac{1+1/\eta}{1/\eta+\gamma}} \right)$$

is strictly concave and achieves its maximum at $x = 1$ when $\gamma < 1$, and it is strictly convex and achieves its minimum at $x = 1$ when $\gamma > 1$. Since $\widehat{\mathcal{W}}$ is strictly positive when $\gamma < 1$, while it is strictly negative when $\gamma > 1$, it follows that $\mathcal{W}$ is maximized when the tax τ satisfies $\mathbb{E}[\chi^{-1}\bar{\varepsilon}_t(\bar{s}^t)] = \bar{\varepsilon}^*$.

Finally, using the definition of G_1 and G_2 above together with (62),

$$\widehat{\mathcal{W}} = \left(\frac{1}{1-\gamma} - \frac{1}{1+1/\eta}\right) \mathbb{E}[A(s_t)]^{\frac{(1/\eta+1)(1-\gamma)}{1/\eta+\gamma}} \exp(\Phi) \exp\left\{ -\frac{1}{2} \frac{(1-\gamma)(1+1/\eta)}{1/\eta+\gamma} \mathcal{L} \right\}.$$

The proof of the statement follows from the fact that

$$\mathcal{W}^* \equiv \frac{1}{1-\beta} \cdot \frac{1/\eta+\gamma}{(1-\gamma)(1+1/\eta)} \mathbb{E}\left[A(s_t)^{\frac{(1-\gamma)(1+1/\eta)}{1/\eta+\gamma}} \right]$$

and, using properties of log-Normal distributions,

$$\begin{aligned} \frac{1/\eta+\gamma}{(1-\gamma)(1+1/\eta)} \mathbb{E}\left[A(s_t)^{\frac{(1-\gamma)(1+1/\eta)}{1/\eta+\gamma}} \right] &= \frac{1/\eta+\gamma}{(1-\gamma)(1+1/\eta)} \mathbb{E}\left[A(s_t)^{\frac{(1-\gamma)(1+1/\eta)}{1/\eta+\gamma}} \right] \\ &= \frac{1/\eta+\gamma}{(1-\gamma)(1+1/\eta)} \mathbb{E}[A(s_t)]^{\frac{(1-\gamma)(1+1/\eta)}{1/\eta+\gamma}} \exp(\Phi). \end{aligned}$$

QED.

A.12 Proof of Lemma 5.

The proof of the lemma is in the online appendix, Section 5. For convenience, here we report the expressions for the coefficients in the statement of the lemma:

$$
\begin{aligned}
a_0 \equiv & \frac{\theta(\gamma\eta+1)\kappa_x+\eta\kappa_0}{(1-\rho)^2\left(\gamma\eta\left(\kappa_0+\kappa_x\right)+\kappa_x\right)^2}(\gamma\eta+1), \\
b_0 \equiv & \frac{\theta(\gamma\eta+1)\kappa_z+\eta\kappa_p}{\left(\gamma\eta\left(\kappa_p+\kappa_z\right)+\kappa_z\right)^2}(\gamma\eta+1), \\
c_0 \equiv & -\frac{\theta(\gamma\eta+1)(\kappa_x+\kappa_z)+\eta(\kappa_0+\kappa_p)}{(1-\rho)^2\left(\gamma\eta\left(\kappa_0+\kappa_p+\kappa_x+\kappa_z\right)+\kappa_x+\kappa_z\right)^2}(\gamma\eta+1).
\end{aligned}
$$

A.13 Proof of Theorem 5.

We begin with $\Delta(\psi_0,\psi_p^*)$, for all ψ_0. From Lemma 5, we have

$$
\begin{aligned}
\Delta(\psi_0,\psi_p^*) &= (a_0+c_0)\left(\psi_0-\psi_0^*\right)^2 \\
&= \frac{\gamma\eta+1}{(1-\rho)^2}\left[\frac{\theta(\gamma\eta+1)\kappa_x+\eta\kappa_0}{(\gamma\eta(\kappa_0+\kappa_x)+\kappa_x)^2}-\frac{\theta(\gamma\eta+1)(\kappa_x+\kappa_z)+\eta(\kappa_0+\kappa_p)}{(\gamma\eta(\kappa_0+\kappa_p+\kappa_x+\kappa_z)+\kappa_x+\kappa_z)^2}\right]\left(\psi_0-\psi_0^*\right)^2 \\
&\equiv \frac{\gamma\eta+1}{(1-\rho)^2}\Gamma(\kappa_z,\kappa_p)\left(\psi_0-\psi_0^*\right)^2.
\end{aligned}
$$

The sign of $\Delta(\psi_0,\psi_p^*)$ is the same as the sign of $\Gamma(\kappa_z,\kappa_p)$. Take any $\kappa_p>0$ and consider the derivative

$$
\frac{\partial}{\partial\kappa_z}\Gamma(\kappa_z,\kappa_p)=(\gamma\eta+1)\frac{\eta\kappa_0(2-\gamma\theta)+\eta(2-\gamma\theta)\kappa_p+\theta(\gamma\eta+1)(\kappa_x+\kappa_z)}{[\gamma\eta\kappa_0+\gamma\eta\kappa_p+(\gamma\eta+1)(\kappa_x+\kappa_z)]^3}. \tag{67}
$$

Suppose first that $\gamma\theta\leq 2$. In this case, (67) is always positive, thus, $\kappa_z=0$ is a global minimum for $\Gamma(\cdot,\kappa_p)$. Letting $\widehat{\Gamma}(\kappa_p)$ denote the minimum value of $\Gamma(\cdot,\kappa_p)$, we have

$$
\Gamma(\kappa_z,\kappa_p)\geq\widehat{\Gamma}(\kappa_p)=\Gamma(0,\kappa_p)=\frac{\theta(\gamma\eta+1)\kappa_x+\eta\kappa_0}{[\gamma\eta(\kappa_0+\kappa_x)+\kappa_x]^2}-\frac{\theta(\gamma\eta+1)\kappa_x+\eta(\kappa_0+\kappa_p)}{[\gamma\eta(\kappa_0+\kappa_p+\kappa_x)+\kappa_x]^2}. \tag{68}
$$

Also,

$$
\widehat{\Gamma}'(\kappa_p)=\eta\frac{\gamma\eta(\kappa_0+\kappa_p)+(\gamma\eta+1)(2\gamma\theta-1)\kappa_x}{[\gamma\eta(\kappa_0+\kappa_p+\kappa_x)+\kappa_x]^3}>0.
$$

We conclude that $\Gamma(\kappa_z,\kappa_p)\geq\widehat{\Gamma}(\kappa_p)>\widehat{\Gamma}(0)=0$.

Suppose now that $\gamma\theta>2$. From (67), we have $\max\{0,K_z(\kappa_p)\}=\arg\min_{\kappa_z}\Gamma(\kappa_z,\kappa_p)$, where

$$
K_z(\kappa_p)\equiv\frac{\eta(\gamma\theta-2)}{\theta(1+\gamma\eta)}(\kappa_0+\kappa_p)-\kappa_x.
$$

Note that $K_z(\kappa_p)\leq 0$ only if $\kappa_x\geq\overline{K}_x\equiv\kappa_0\eta(\gamma\theta-2)/[\theta(1+\gamma\eta)]$.

Consider the case $\kappa_x < \overline{K}_x$. Then $K_z(\kappa_p) = \arg\min_{\kappa_z} \Gamma(\kappa_z, \kappa_p)$, for all κ_p. The latter is a global minimum for $\Gamma(\cdot, \kappa_p)$ since $\partial\Gamma(\kappa_z, \kappa_p)/\partial\kappa_z$ is negative for $\kappa_z < K_z(\kappa_p)$ and positive otherwise. Thus,

$$\Gamma(\kappa_z, \kappa_p) \geq \widehat{\Gamma}(\kappa_p) = \Gamma(K_z(\kappa_p), \kappa_p) = \frac{\eta\kappa_0 + \theta(\gamma\eta + 1)\kappa_x}{[\gamma\eta(\kappa_0 + \kappa_x) + \kappa_x]^2} - \frac{\theta^2}{4\eta(\gamma\theta - 1)(\kappa_0 + \kappa_p)}. \tag{69}$$

We have,

$$\widehat{\Gamma}'(\kappa_p) = \frac{\theta^2}{4\eta(\gamma\theta - 1)(\kappa_0 + \kappa_p)^2} > 0.$$

Also,

$$\widehat{\Gamma}(0) = -\frac{[\eta\kappa_0(2 - \gamma\theta) + \theta(\gamma\eta + 1)\kappa_x]^2}{4\eta\kappa_0(\gamma\theta - 1)[\gamma\eta(\kappa_0 + \kappa_x) + \kappa_x]^2} < 0$$

and

$$\lim_{\kappa_p \to \infty} \widehat{\Gamma}(\kappa_p) = \frac{\eta\kappa_0 + \theta(\gamma\eta + 1)\kappa_x}{[\gamma\eta(\kappa_0 + \kappa_x) + \kappa_x]^2} > 0.$$

It follows that there exists a threshold $\overline{K}_p(\kappa_x)$, defined by $\Gamma(K_z(\overline{K}_p(\kappa_x)), \overline{K}_p(\kappa_x)) = 0$ or

$$\overline{K}_p(\kappa_x) = \frac{(\eta\kappa_0(\gamma\theta - 2) - \theta(\gamma\eta + 1)\kappa_x)^2}{4\eta(\gamma\theta - 1)(\eta\kappa_0 + \theta(\gamma\eta + 1)\kappa_x)},$$

such that $\Gamma(\kappa_z, \kappa_p) > 0$ for $\kappa_p > \overline{K}_p(\kappa_x)$. Moreover, if $\kappa_p < \overline{K}_p(\kappa_x)$, there is an neighborhood of $(K_z(\overline{K}_p(\kappa_x)), \overline{K}_p(\kappa_x))$ such that $\Gamma(\kappa_z, \kappa_p) < 0$. The derivative of $\overline{K}_p(\kappa_x)$ is

$$\overline{K}_p'(\kappa_x) = \frac{\theta(\gamma\eta + 1)}{4\eta(\gamma\theta - 1)} \left(1 - \frac{\eta^2\kappa_0^2(\gamma\theta - 1)^2}{(\eta\kappa_0 + \theta(\gamma\eta + 1)\kappa_x)^2}\right),$$

which is negative since $\kappa_x < \overline{K}_x$.

Finally, suppose $\kappa_x \geq \overline{K}_x$. We have to consider two cases, depending on whether $0 = \arg\min_{\kappa_z} \Gamma(\kappa_z, \kappa_p)$ or $K_z(\kappa_p) = \arg\min_{\kappa_z} \Gamma(\kappa_z, \kappa_p)$. The former case occurs for all values of κ_p such that $K_z(\kappa_p) \leq 0$, i.e. $\kappa_p \leq (\kappa_x - \overline{K}_x)[\theta(1 + \gamma\eta)]/\eta(\gamma\theta - 2)$. For such values, $\Gamma(\kappa_z, \kappa_p) \geq \widehat{\Gamma}(\kappa_p)$, where $\widehat{\Gamma}(\cdot)$ is given by (68), which is positive and increasing in κ_p. The latter case occurs when $\kappa_p > (\kappa_x - \overline{K}_x)[\theta(1 + \gamma\eta)]/\eta(\gamma\theta - 2)$. For such values, $\Gamma(\kappa_z, \kappa_p) \geq \widehat{\Gamma}(\kappa_p)$, where $\widehat{\Gamma}(\cdot)$ is given by (69), which is positive and increasing in κ_p for $\kappa_p \geq \overline{K}_p(\kappa_x)$. Simple steps of algebra prove that $(\kappa_x - \overline{K}_x)[\theta(1 + \gamma\eta)]/\eta(\gamma\theta - 2) > \overline{K}_p(\kappa_x)$ (as long as $\kappa_x \geq \overline{K}_x$ and $\gamma\theta > 2$), thus, the inequality $\kappa_p \geq \overline{K}_p(\kappa_x)$ is implied by $\kappa_p > (\kappa_x - \overline{K}_x)[\theta(1 + \gamma\eta)]/\eta(\gamma\theta - 2)$. We conclude that $\Gamma(\kappa_z, \kappa_p) \geq \widehat{\Gamma}(\kappa_p) > 0$ for all κ_p.

Consider now $\Delta(\psi_0^*, \psi_p)$, for all ψ_0^*. From Lemma 5, we have

$$\begin{aligned}\Delta(\psi_0^*, \psi_p) &= \left(a_0 + b_0 + c_0\rho^2\right)\psi_p^2 \\ &= \frac{\gamma\eta+1}{(1-\rho)^2}\left[\frac{\theta(\gamma\eta+1)\kappa_x + \eta\kappa_0}{(\gamma\eta(\kappa_0+\kappa_x)+\kappa_x)^2} - \rho^2\frac{\theta(\gamma\eta+1)(\kappa_x+\kappa_z)+\eta(\kappa_0+\kappa_p)}{(\gamma\eta(\kappa_0+\kappa_p+\kappa_x+\kappa_z)+\kappa_x+\kappa_z)^2}\right]\psi_p^2 + b_0\psi_p^2 \\ &> \frac{\gamma\eta+1}{(1-\rho)^2}\left[\frac{\theta(\gamma\eta+1)\kappa_x + \eta\kappa_0}{(\gamma\eta(\kappa_0+\kappa_x)+\kappa_x)^2} - \frac{\theta(\gamma\eta+1)(\kappa_x+\kappa_z)+\eta(\kappa_0+\kappa_p)}{(\gamma\eta(\kappa_0+\kappa_p+\kappa_x+\kappa_z)+\kappa_x+\kappa_z)^2}\right]\psi_p^2 + b_0\psi_p^2 \\ &= \frac{\gamma\eta+1}{(1-\rho)^2}\Gamma(\kappa_z,\kappa_p)\psi_p^2 + b_0\psi_p^2.\end{aligned}$$

Since $b_0 > 0$, a sufficient condition for $\Delta(\psi_0^*, \psi_p) > 0$, for all $\psi_p \neq 0$, is $\Gamma(\kappa_z, \kappa_p) \geq 0$ and the proof follows from the arguments above.

Finally, from the expressions for the coefficients a_0, b_0 and c_0 in Lemma 5, it is immediate to see that $b_0 \to 0$ and $c_0 \to 0$ as $\kappa_p \to \infty$ and, therefore,

$$\Delta(\psi_0, \psi_p) \to a_0\left(\psi_0 - \psi_0^* + \rho\psi_p\right)^2.$$

QED.

B Appendix: Implementations with Public Signals

In this appendix we allow for the existence of public signals. The central banker's information structure described in Section 2 remains unchanged. In contrast, on the production side we assume that firms in every period observe their private signal ω_{it} (modeled as before) as well as a public signal. We model the public signal as follows.

In each period t, Nature draws a random variable ς_t from a finite set Ω_ς according to a probability distribution φ_ς. We let $\varphi(\varsigma_t|s_t, \omega_{pt})$ denote the probability of ς_t conditional on (s_t, ω_{pt}). All firms observe ς_t in addition to their private signal; ς_t thus represents public information in period t. Note that we allow the public signal to contain information about both the fundamental state, s_t, as well as the signal of the central banker, ω_{pt}.

The information set of firm i at time t is thus given by $\omega_i^t = (\omega_{it}, \varsigma_t, \bar{s}^{t-1})$. We furthermore augment our definition of the aggregate state to include the realization of the public signal:

$$\bar{s}_t = \{s_t, \omega_{pt}, \varsigma_t, \varphi(\omega_{it}|s_t, \omega_{pt})\}.$$

We these slight modifications, all other definitions in Section 2 remain unchanged.

The set of flexible-price allocations is unaltered. This is due to the fact that all flexible price allocations are functions only of the fundamental state, s_t, and are therefore invariant to the firms' information structure. With this in mind, the following proposition demonstrates how one may extend the implementations presented in Proposition 5 to settings with public signals.

Proposition 8. *Take any flexible-price allocation* $\xi \in X^f$ *with corresponding functions* $\{\mathcal{C}(\cdot), \mathcal{L}(\cdot)\}$.

The following paths for nominal interest rates and aggregate prices implement ξ *under sticky prices:*

$$1 + \iota_t(\omega_p^t) = \mathcal{I}_t(\omega_p^t) \qquad \text{and} \qquad P_t(\bar{s}^t) = h_t(\varsigma_t)\mathcal{P}_t(\bar{s}^{t-1}), \tag{70}$$

where $\mathcal{I}_t : \Omega_p^t \to \mathbb{R}_+$ *is a sequence of positive-valued functions defined on* Ω_p^t, $h_t : \Omega_\varsigma \to \mathbb{R}_+$ *is a sequence of positive-valued functions defined on* Ω_ς, *and* $\mathcal{P}_t : \bar{S}^{t-1} \to \mathbb{R}_+$ *is a sequence of positive-valued functions defined recursively by:*

$$\mathcal{P}_{t+1}(\bar{s}^t) = \beta\mathcal{I}_t(\omega_p^t)\frac{h_t(\varsigma_t)\mathcal{P}_t(\bar{s}^{t-1})}{U'(\mathcal{C}(s_t))}\mathbb{E}\left[\left.\frac{U'(\mathcal{C}(s_{t+1}))}{h_{t+1}(\varsigma_{t+1})}\right| s_t\right], \tag{71}$$

where $\mathcal{P}_0 > 0$ *is a known constant.*

Proof. The interest rate and path of prices must satisfy the Euler equation of the household under sticky prices at the flexible-price allocation: (22). With the paths for the nominal interest rate and prices proposed in (70), we may rewrite the Euler equation as follows:

$$\frac{U'(\mathcal{C}(s_t))}{h_t(\varsigma_t)\mathcal{P}_t(\bar{s}^{t-1})} = \beta\mathcal{I}_t(\omega_p^t)\frac{1}{\mathcal{P}_{t+1}(\bar{s}^t)}\mathbb{E}\left[\left.\frac{U'(\mathcal{C}(s_{t+1}))}{h_{t+1}(\varsigma_{t+1})}\right| s_t\right] \tag{72}$$

For any given sequence of functions, $\mathcal{I}_t(\cdot)$ and $h_t(\cdot)$, the above equation is satisfied at all dates and histories by the sequence of functions $\mathcal{P}_t(\cdot)$ defined in (71).

It is straightforward to verify that there is no dispersion in intermediate-good firm prices due to the fact that $(\varsigma_t, \bar{s}^{t-1}) \in \omega_i^t$ for all $\omega_i^t \in \Omega^t$. The proof of this statement follows the exact same steps as those found in the proof of Proposition 1. □

Proposition 8 augments Proposition 5 in that it characterizes a large set of implementations of flexible-price allocations in the new setting with public signals.

This set is "large" in the following sense. First, it places no restrictions on the nominal interest rate aside from the measurability constraint imposed in part (ii) of Assumption 1. It likewise places *almost* no restrictions on the aggregate price level. The price level at time t is allowed to be contingent on the largest set that is common knowledge to firms at time t, specifically $(\varsigma_t, \bar{s}^{t-1})$. Note that this set includes not only past aggregate states, but also public signals at time t. By restricting the aggregate price level to be contingent on at most $(\varsigma_t, \bar{s}^{t-1})$, we ensure that all firms can set the "correct" nominal price at every date and history.

Finally, Proposition 8 requires that the sequence of nominal interest rates and prices satisfy condition (71); this ensures that the Euler equation at the flexible-price allocation holds at all dates and histories.

Why then is this set not the full set of flexible-price implementations in this new setting with public signals? The reason we cannot be fully sure that this is the entire set of flexible-price implementations is that there is one more restriction placed on the aggregate price level that may or may not be innocuous. In particular, in (70) we impose that the price level is log-separable in $(\varsigma_t, \bar{s}^{t-1})$; that is:

$$\log P_t(\bar{s}^t) = \log h_t(\varsigma_t) + \log \mathcal{P}_t(\bar{s}^{t-1}) \tag{73}$$

where $h_t(\cdot)$ is the component measurable in ς_t and $\mathcal{P}_t(\cdot)$ is the component measurable in $\bar{s}^{t-1}$. This log-separability of the aggregate price level in past states, $\bar{s}^{t-1}$, and current public signals, ς_t, allows for a relatively clean characterization of the price level: the component of the future price level that is contingent on the past history, $\mathcal{P}_{t+1}(\bar{s}^t)$, can be taken out of the expectation in the household's Euler equation, as seen in (72), and be given an explicit recursive definition in (71).

Therefore, Proposition 8 provides the full set of flexible-price implementations in which the aggregate price level is log-separable (73). There could in theory be more implementations of flexible price allocations in which the price level does not satisfy the log-separability property; whether or not these implementations exist is beyond the scope of this paper.[16]

[16]Note that the vast majority of the New Keynesian literature focuses on log-linearized equilibrium solutions. In these equilibria, the log-separability property (73) holds by construction (or one might say, by brute force).