Online Appendix for "Optimal Monetary Policy and Disclosure with an Informationally-Constrained Central Banker"

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Abstract

This is the online appendix for "Optimal Monetary Policy and Disclosure with an Informationally-Constrained Central Banker." In this document we provide the proofs for Lemma 5, 6 and 7 in the main text and characterize, in closed-form, the equilibrium for the log-linear economy presented in Section 6 of the main text.

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1 Proof of Lemma 6

By Lemma 3 in the paper, a sticky-price equilibrium therefore satisfies firm optimality condition (18), CES demand function (5), household intratemporal and intertemporal optimality conditions (15) and (16), the transversality condition (17), the household and government budget constraints, technology (6), and goods and labor market clearing conditions (7) and (8), respectively.

We will now use this equilibrium characterization to prove necessity and sufficiency of the conditions stated in the lemma.

Necessity. First, combining (55) with (14), (15) and the definition of $\Lambda_t(\bar{s}^t)$, we can rewrite ε_{it} as follows:

$$\varepsilon_{it}(\omega_{it}, \bar{s}^t) = \frac{W_t(\bar{s}^t)A(s_t)^{-1}}{\mathbb{E}\left[q_{it}(\omega_i^t, \bar{s}^t)\left\{W_t(\bar{s}^t)A(s_t)^{-1}\right\}\middle|\,\omega_i^t\right]}. \tag{A.1}$$

Multiplying both sides of firm's optimality condition (18) by $\varepsilon_{it}(\omega_{it}, \bar{s}^t)$ yields

$$p_{it}(\omega_i^t)\varepsilon_{it}(\omega_{it}, \bar{s}^t) = \left(\frac{\theta-1}{\theta}\right)^{-1}\frac{1}{1-\tau}\cdot\frac{W_t(\bar{s}^t)}{A(s_t)}.$$

Next, using the household's intratemporal optimality condition (15), we can rewrite the above equation as follows:

$$\frac{p_{it}(\omega_i^t)}{P_t(\bar{s}^t)}\varepsilon_{it}(\omega_{it}, \bar{s}^t) = \left(\frac{\theta-1}{\theta}\right)^{-1}\frac{1}{1-\tau}\cdot\frac{1}{A(s_t)}\cdot\frac{V'(L_t(\bar{s}^t))}{U'(C_t(\bar{s}^t))}.$$

Finally, using the CES demand function (5), the above condition can be written in terms of allocations and wedges alone:

$$\varepsilon_{it}(\omega_{it}, \bar{s}^t) = \left(\frac{\theta-1}{\theta}\right)^{-1}\frac{1}{1-\tau}\cdot\frac{1}{A(s_t)}\cdot\frac{V'(L_t(\bar{s}^t))}{U'(C_t(\bar{s}^t))}\left(\frac{y_{it}(\omega_i^t, \bar{s}^t)}{Y_t(\bar{s}^t)}\right)^{\frac{1}{\theta}}.$$

This proves necessity of condition (54), with $\chi \equiv \left(\frac{\theta-1}{\theta}\right)^{-1}\frac{1}{1-\tau}$.

Finally, the household's intertemporal Euler equation (16) coincides with equilibrium condition (56), with $\psi(\omega_{pt}, \bar{s}^{t-1}) \equiv 1 + \iota_t(\omega_{pt}, \bar{s}^{t-1})$.

Sufficiency. We now prove that the conditions stated in the lemma are also sufficient.

To do so, we take any strictly positive constant $\chi \in \mathbb{R}_+$, function $\psi : \Omega \to \mathbb{R}_+$, and allocation ξ that satisfies conditions (54)-(56) along with technology and resource constraints (6)-(8), and show that there exists a price system ϱ and a policy ϑ that supports this allocation as a sticky-price equilibrium.

First, we set the nominal wage as follows:

$$W_t(\bar{s}^t) = \frac{V'(L_t(\bar{s}^t))}{\Lambda_t(\bar{s}^t)}. \tag{A.2}$$

Letting the function $\Lambda_t(\bar{s}^t)$ denote the household's marginal utility of wealth, the first-order condition of the household with respect to labor supply (15) is satisfied. Second, we set the nominal interest rate equal to $1+\iota_t(\omega_{pt}, \bar{s}^{t-1}) = \psi(\omega_{pt}, \bar{s}^{t-1})$. Condition (56) thereby ensures that the household's intertemporal Euler equation (16) is satisfied. Third, we define two functions

$$\begin{aligned} \varepsilon^{\omega}_{it}(\omega_{it}, \bar{s}^{t-1}) &\equiv \mathbb{E}\left[q_{it}(\omega^t_i, \bar{s}^t) \left\{ V'(L_t(\bar{s}^t)) A(s_t)^{-1} \Lambda_t(\bar{s}^t)^{-1} \right\} \middle| \, \omega^t_i \right], \\ \varepsilon^{s}_{t}(\bar{s}^t) &\equiv V'(L_t(\bar{s}^t)) A(s_t)^{-1} \Lambda(\bar{s}^t)^{-1}, \end{aligned} \tag{A.3}$$

so that we may decompose the wedge defined in (55) as follows:

$$\varepsilon_{it}(\omega_{it}, \bar{s}^t) = \varepsilon^{\omega}_{it}(\omega_{it}, \bar{s}^{t-1})^{-1} \varepsilon^{s}_{t}(\bar{s}^t). \tag{A.4}$$

We then set the nominal price of each firm to the part that is measurable only in ω^t_i:

$$p_{it}(\omega^t_i) = \chi \varepsilon^{\omega}_{it}(\omega_{it}, \bar{s}^{t-1}). \tag{A.5}$$

Thus, the aggregate price level is given by

$$P_t(\bar{s}^t) = \chi \left[\int \varepsilon^{\omega}_{it}(\omega, \bar{s}^{t-1})^{\theta-1} \varphi(\omega | \bar{s}^t) d\omega \right]^{\frac{1}{\theta-1}}.$$

Fourth, we set the tax rate so that $1-\tau = \left(\frac{\theta-1}{\theta}\right)^{-1} \chi^{-1}$. For any strictly positive χ, such a tax rate exists. Plugging this tax rate and the nominal wage (A.2) into our expression for the nominal price, (A.5), we get

$$p_{it}(\omega^t_i) = \left(\frac{\theta-1}{\theta} \right)^{-1} \frac{1}{1-\tau} \mathbb{E}\left[q_{it}(\omega^t_i, \bar{s}^t) \left\{ W_t(\bar{s}^t) A(s_t)^{-1} \right\} \middle| \, \omega^t_i \right].$$

The firm's optimality condition (18) is therefore satisfied at these nominal prices.

Finally, we take condition (54). Using the decomposition of the wedge in (A.4) and the CES demand function (5), we rewrite condition (54) as follows:

$$U'(C_t(\bar{s}^t))\frac{\varepsilon_t^s(\bar{s}^t)}{\varepsilon_{it}^{\omega}(\omega_{it},\bar{s}^{t-1})}\frac{p_{it}(\omega_i^t)}{P_t(\bar{s}^t)}=\chi\frac{V'(L_t(\bar{s}^t))}{A(s_t)}.$$

Using the nominal price set in (A.5), the above condition can be rewritten as

$$U'(C_t(\bar{s}^t))\frac{\varepsilon_t^s(\bar{s}^t)}{P_t(\bar{s}^t)}=\frac{V'(L_t(\bar{s}^t))}{A(s_t)}. \tag{A.6}$$

Substituting the definition of $\varepsilon_t^s(\cdot)$ from (A.3) into (A.6), we get that $U'(C_t(\bar{s}^t))/P_t(\bar{s}^t)=\Lambda_t(\bar{s}^t)$.

To conclude our proof of sufficiency, note that it is trivial to show that both the household's budget constraint and the government's budget constrained are satisfied at the proposed allocation. The proof follows the same logic as the one in the sufficiency part of Lemma 1 in the main appendix. QED.

2 Proof of Lemma 7

First, with a slight abuse of notation we rewrite $y_{it}(\omega_i^t,\bar{s}^t)$ and $\ell_{it}(\omega_i^t,\bar{s}^t)$ as $y_{it}(\omega_{it},\bar{s}^t)$ and $\ell_{it}(\omega_{it},\bar{s}^t)$, respectively. Aggregate output is then given by

$$Y_t(\bar{s}^t)=\left[\int_\omega y_{it}(\omega_{it},\bar{s}^t)^{\frac{\theta-1}{\theta}}\varphi(\omega|\bar{s}^t)d\omega\right]^{\frac{\theta}{\theta-1}}. \tag{A.7}$$

By log-Normality of $y_{it}(\omega_{it},\bar{s}^t)$ and using the moment-generating function for the Normal distribution, we have that

$$\int_\omega y_{it}(\omega,\bar{s}^t)^{\frac{\theta-1}{\theta}}\varphi(\omega|\bar{s}^t)d\omega=\exp\left\{\left(\frac{\theta-1}{\theta}\right)\int_\omega \log y_{it}(\omega,\bar{s}^t)\varphi(\omega|\bar{s}^t)d\omega+\frac{1}{2}\left(\frac{\theta-1}{\theta}\right)^2\mathrm{var}_\omega\left[\log y_{it}(\omega_{it},\bar{s}^t)|\bar{s}^t\right]\right\}.$$

Rearranging, the latter becomes

$$\begin{aligned}\int_\omega y_{it}(\omega,\bar{s}^t)^{\frac{\theta-1}{\theta}}\varphi(\omega|\bar{s}^t)d\omega=&\exp\left\{\left(\frac{\theta-1}{\theta}\right)\left[\int_\omega \log y_{it}(\omega,\bar{s}^t)\varphi(\omega|\bar{s}^t)d\omega+\frac{1}{2}\mathrm{var}_\omega\left[\log y_{it}(\omega_{it},\bar{s}^t)|\bar{s}^t\right]\right]\right\}\\&\times\exp\left\{\frac{1}{2}\left(\frac{\theta-1}{\theta}\right)\left(-\frac{1}{\theta}\right)\mathrm{var}_\omega\left[y_{it}(\omega_{it},\bar{s}^t)|\bar{s}^t\right]\right\},\end{aligned}$$

which may be rewritten as

$$\int_\omega y_{it}(\omega,\bar{s}^t)^{\frac{\theta-1}{\theta}}\varphi(\omega|\bar{s}^t)d\omega=\left[\int_\omega y_{it}(\omega,\bar{s}^t)^{\frac{\theta-1}{\theta}}\varphi(\omega|\bar{s}^t)d\omega\right]^{\frac{\theta-1}{\theta}}\exp\left\{-\frac{1}{2}\left(\frac{\theta-1}{\theta}\right)\left(\frac{1}{\theta}\right)\mathrm{var}_\omega\left[y_{it}(\omega_{it},\bar{s}^t)|\bar{s}^t\right]\right\}.$$

Therefore, aggregate output can be written as

$$Y_t(\bar{s}^t)=\left[\int_\omega y_{it}(\omega,\bar{s}^t)^{\frac{\theta-1}{\theta}}\varphi(\omega|\bar{s}^t)d\omega\right]\exp\left\{-\frac{1}{2}\frac{1}{\theta}\mathrm{var}_\omega\left[\log y_{it}(\omega_{it},\bar{s}^t)|\bar{s}^t\right]\right\},$$

where $y_{it}(\omega_{it}, \bar{s}^t) = A(s_t)\ell_{it}(\omega_{it}, \bar{s}^t)$. Recall that aggregate labor is given by $L_t(\bar{s}^t) = \int_\omega \ell_{it}(\omega, \bar{s}^t)\varphi(\omega|\bar{s}^t)d\omega$. As a result,

$$Y_t(\bar{s}^t) = A(s_t)L_t(\bar{s}^t)\exp\left\{-\frac{1}{2}\frac{1}{\theta}\text{var}_\omega\left[\log y_{it}(\omega_{it}, \bar{s}^t)|\bar{s}^t\right]\right\}.$$

Finally, from equilibrium conditions (49), it must be the case that cross-sectional dispersion in output satisfies

$$\text{var}_\omega\left[\log y_{it}(\omega_{it}, \bar{s}^t)|\bar{s}^t\right] = \text{var}_\omega\left[\theta\log\varepsilon_{it}(\omega_{it}, \bar{s}^t)|\bar{s}^t\right] = \theta^2\text{var}_\omega\left[\log\varepsilon_{it}(\omega_{it}, \bar{s}^t)|\bar{s}^t\right].$$

Therefore, the aggregate production function may be written as follows:

$$Y_t(\bar{s}^t) = A(s_t)L_t(\bar{s}^t)\exp\left\{-\frac{\theta}{2}\text{var}_\omega\left[\log\varepsilon_{it}(\omega_{it}, \bar{s}^t)|\bar{s}^t\right]\right\},$$

as was to be shown.

Next, we take the equilibrium intratemporal condition in (54) and rewrite it as

$$y_{it}(\omega_{it}, \bar{s}^t)^{\frac{1}{\theta}} = \varepsilon_{it}(\omega_{it}, \bar{s}^t)\frac{U'(C_t(\bar{s}^t))}{\chi V'(L_t(\bar{s}^t))}A(s_t)Y_t(\bar{s}^t)^{\frac{1}{\theta}}.$$

Raising both sides to the power $\theta - 1$ yields

$$y_{it}(\omega_{it}, \bar{s}^t)^{\frac{\theta-1}{\theta}} = \varepsilon_{it}(\omega_{it}, \bar{s}^t)^{\theta-1}\left[\frac{U'(C_t(\bar{s}^t))}{\chi V'(L_t(\bar{s}^t))}A(s_t)\right]^{\theta-1}Y_t(\bar{s}^t)^{\frac{\theta-1}{\theta}}.$$

Integrating the latter over ω_{it} gives

$$\int_\omega y_{it}(\omega, \bar{s}^t)^{\frac{\theta-1}{\theta}}\varphi(\omega|\bar{s}^t)d\omega = \int\varepsilon_{it}(\omega, \bar{s}^t)^{\theta-1}\varphi(\omega|\bar{s}^t)d\omega\left[\frac{U'(C_t(\bar{s}^t))}{\chi V'(L_t(\bar{s}^t))}A(s_t)\right]^{\theta-1}Y_t(\bar{s}^t)^{\frac{\theta-1}{\theta}},$$

which, together with (A.7), implies

$$1 = \left[\int\varepsilon_{it}(\omega, \bar{s}^t)^{\theta-1}\varphi(\omega|\bar{s}^t)d\omega\right]^{\frac{1}{\theta-1}}\frac{U'(C_t(\bar{s}^t))}{\chi V'(L_t(\bar{s}^t))}A(s_t).$$

Therefore, the intratemporal condition in (61) is satisfied with the labor wedge defined as follows:

$$\bar{\varepsilon}_t(\bar{s}^t) \equiv \left[\int\varepsilon_{it}(\omega, \bar{s}^t)^{\theta-1}\varphi(\omega|\bar{s}^t)d\omega\right]^{\frac{1}{\theta-1}}\delta^{-1}.$$

Again, using the log-Normality of $\varepsilon_{it}(\omega_{it}, \bar{s}^t)$, we can rewrite the latter as

$$\begin{aligned}\log\bar{\varepsilon}_t(\bar{s}^t) =& \frac{1}{\theta-1}\left\{(\theta-1)\int_\omega\log\varepsilon_{it}(\omega, \bar{s}^t)\varphi(\omega|\bar{s}^t)d\omega + \frac{1}{2}(\theta-1)^2\,\text{var}_\omega\left[\log\varepsilon_{it}(\omega_{it}, \bar{s}^t)|\bar{s}^t\right]\right\}\\ &+\frac{\theta}{2}\text{var}_\omega\left[\log\varepsilon_{it}(\omega_i, \bar{s}^t)|\bar{s}^t\right].\end{aligned}$$

Therefore,

$$\log \bar{\varepsilon}_t(\bar{s}^t) = \int_\omega \log \varepsilon_{it}(\omega, \bar{s}^t)\varphi(\omega|\bar{s}^t)d\omega + \frac{1}{2}(\theta - 1)\,\mathrm{var}_\omega\left[\log \varepsilon_{it}(\omega_{it}, \bar{s}^t)|\bar{s}^t\right] + \frac{\theta}{2}\mathrm{var}_\omega\left[\log \varepsilon_{it}(\omega_i, \bar{s}^t)|\bar{s}^t\right]$$

and, as a result,

$$\bar{\varepsilon}_t(\bar{s}^t) = \kappa \exp\left\{\int_\omega \log \varepsilon_{it}(\omega, \bar{s}^t)\varphi(\omega|\bar{s}^t)d\omega\right\},$$

where $\kappa \equiv \exp\left\{\left(\theta - \frac{1}{2}\right)\mathrm{var}_\omega\left[\log \varepsilon_{it}(\omega_{it}, \bar{s}^t)|\bar{s}^t\right]\right\}$ is a constant, which coincides with the definition given in the statement of the lemma. QED.

3 Equilibrium under no disclosure

Lemma A.1. *Equilibrium firm's output strategy in the economy without disclosure is log-linear and given by*

$$\log y_{it}(\omega_i^t, \bar{s}^t) = \phi_a' \rho a_{t-1} + \phi_x x_{it} + \phi_a a_t + \phi_p x_{pt} + \phi_z z_{it}, \tag{A.8}$$

where

$$\begin{aligned}
\phi_a' &= \frac{\eta}{1+\gamma\eta} \cdot \frac{\kappa_0\left[(\psi_a + \psi_p)(1+\gamma\eta) + \gamma(1+\eta)\right]}{\kappa_x(1+\gamma\eta) + \kappa_0\gamma\eta}, \\
\phi_x &= \frac{\theta\kappa_x\left[(\psi_a + \psi_p)(1+\gamma\eta) + \gamma(1+\eta)\right]}{\kappa_x(1+\gamma\eta) + \kappa_0\gamma\eta}, \\
\phi_a &= -\frac{(\eta+1)(\gamma\theta-1)\kappa_x}{\gamma\eta\left(\kappa_0+\kappa_x\right)+\kappa_x} + \frac{\eta(\gamma\eta+1)(\gamma\theta-1)\left(\kappa_0\kappa_z - \kappa_p\kappa_x\right)}{\left(\gamma\eta\left(\kappa_0+\kappa_x\right)+\kappa_x\right)\left(\gamma\eta\left(\kappa_p+\kappa_z\right)+\kappa_z\right)}\psi_p - \frac{\eta\kappa_0 + \theta(\gamma\eta+1)\kappa_x}{\gamma\eta\left(\kappa_0+\kappa_x\right)+\kappa_x}\psi_a, \\
\phi_p &= -\frac{\theta\kappa_z(1+\gamma\eta) + \kappa_p\eta}{\kappa_z(1+\gamma\eta) + \kappa_p\gamma\eta}\psi_p, \\
\phi_z &= \frac{\theta\kappa_z(1+\gamma\eta)}{\kappa_z(1+\gamma\eta) + \kappa_p\gamma\eta}\psi_p.
\end{aligned}$$

Proof. We guess and verify that the equilibrium is log-linear. We propose the following log-linear strategy for the price:

$$\log p_{it}(\omega_i^t) = g_a' \rho a_{t-1} + g_x x_{it} + g_z z_{it}, \tag{A.9}$$

which implies that the aggregate price must satisfy

$$\log P_t(\bar{s}^t) = g_a' \rho a_{t-1} + g_x a_t + g_z \epsilon_{pt}. \tag{A.10}$$

We propose the following log-linear strategy for individual firm output:

$$\log y_{it}(\omega_i^t, \bar{s}^t) = \phi_a' \rho a_{t-1} + \phi_x x_{it} + \phi_a a_t + \phi_p x_{pt} + \phi_z z_{it}, \tag{A.11}$$

which implies that aggregate output must satisfy

$$\log Y_t(\bar{s}^t) = \phi_a' \rho a_{t-1} + (\phi_x + \phi_a) a_t + \phi_p x_{pt} + \phi_z \epsilon_{pt}. \tag{A.12}$$

Therefore, there are eight unknown coefficients:

$$\{g'_a, g_x, g_z, \phi'_a, \phi_x, \phi_a, \phi_p, \phi_z\}.$$

First, we use the CES demand function

$$\frac{y_{it}(\omega_i^t, \bar{s}^t)}{Y_t(\bar{s}^t)} = \left(\frac{p_{it}(\omega_i^t)}{P_t(\bar{s}^t)}\right)^{-\theta}.$$

This implies

$$\log y_{it}(\omega_i^t, \bar{s}^t) - \log Y_t(\bar{s}^t) = -\theta(\log p_{it}(\omega_i^t) - \log P_t(\bar{s}^t)).$$

Using (A.9)-(A.12), the difference can be written as

$$\log y_{it}(\omega_i^t, \bar{s}^t) - \log Y_t(\bar{s}^t) = \phi_x(x_{it} - a_t) + \phi_z(z_{it} - \epsilon_{pt})$$

and, similarly,

$$\log p_{it}(\omega_i^t) - \log P_t(\bar{s}^t) = g_x(x_{it} - a_t) + g_z(z_{it} - \epsilon_{pt}).$$

Therefore, this gives us the following three coefficient restrictions:

$$\phi_x = -\theta g_x,$$
$$\phi_z = -\theta g_z.$$

From Proposition 2 in the main text,

$$p_{it}(\omega_i^t) = \chi \mathbb{E}\left[q_{it}(\omega_i^t, \bar{s}^t) \left\{ \frac{W_t(\bar{s}^t)}{A(s_t)} \right\} \middle| \omega_{it}, \bar{s}^{t-1} \right],$$

where $q_{it}(\omega_i^t, \bar{s}^t)$ is defined in equation (14). Taking logs of both sides, using equation (15), the definition of $\Lambda_t(\bar{s}^t)$, and the properties of log-normal distributions, we have that, up to a constant, the optimal price satisfies the following condition:

$$\log p_{it}(\omega_i^t) = \mathbb{E}\left[\log\left\{ V'(L_t(\bar{s}^t)) A(s_t)^{-1} \Lambda_t(\bar{s}^t)^{-1} \right\} \middle| \omega_i^t \right],$$

where $V'(L_t(\bar{s}^t)) = L_t(\bar{s}^t)^{1/\eta}$. We can thus rewrite the latter as

$$\log p_{it}(\omega_i^t) = \mathbb{E}\left[\frac{1}{\eta} \log L_t(\bar{s}^t) - \log A(s_t) - \log \Lambda_t(\bar{s}^t) \middle| \omega_i^t \right].$$

Next, note that, from Lemma 7 in the main appendix, $\log Y_t(s^t) = \log A(s_t) + \log L_t(\bar{s}^t) + const$. Therefore,

$$\log p_{it}(\omega_i^t) = \mathbb{E}\left[\frac{1}{\eta} \log Y_t(\bar{s}^t) - \left(1 + \frac{1}{\eta}\right) \log A(s_t) - \log \Lambda_t(\bar{s}^t) \middle| \omega_i^t \right]. \tag{A.13}$$

Substituting in for output and the policy variable and using Lemma 4 in the paper,

$$\log p_{it}(\omega_i^t) = \mathbb{E}\left[\frac{1}{\eta}(\phi_a'\rho a_{t-1} + (\phi_x+\phi_a)a_t + \phi_p x_{pt} + \phi_z\epsilon_{pt}) - \left(1+\frac{1}{\eta}\right)a_t - (\psi_p x_{pt} + \psi_0 a_{t-1} + \psi_a a_t)\middle|\, \omega_i^t\right].$$

This can be written as

$$\log p_{it}(\omega_i^t) = \left(\frac{1}{\eta}\rho\phi_a' - \psi_0\right)a_{t-1} + \left(\frac{1}{\eta}\phi_p - \psi_p\right)\mathbb{E}\left[x_{pt}|\,\omega_i^t\right] + \left[\frac{1}{\eta}(\phi_x+\phi_a) - \left(1+\frac{1}{\eta}\right) - \psi_a\right]\mathbb{E}\left[a_t|\,\omega_i^t\right] + \frac{1}{\eta}\phi_z\mathbb{E}\left[\epsilon_{pt}|\,\omega_i^t\right].$$

Note that $x_{pt} = a_t + \epsilon_{pt}$, thus,

$$\log p_{it}(\omega_i^t) = \left(\frac{1}{\eta}\rho\phi_a' - \psi_0\right)a_{t-1} + \left[\frac{1}{\eta}(\phi_p+\phi_z) - \psi_p\right]\mathbb{E}\left[\epsilon_{pt}|\,\omega_i^t\right] + \left[\frac{1}{\eta}\phi_p - \psi_p + \frac{1}{\eta}(\phi_x+\phi_a) - \left(1+\frac{1}{\eta}\right) - \psi_a\right]\mathbb{E}\left[a_t|\,\omega_i^t\right].$$

Standard properties of Normal distributions imply

$$\mathbb{E}\left[a_t|\,\omega_i^t\right] = \frac{\kappa_0}{\kappa_0+\kappa_x}\rho a_{t-1} + \frac{\kappa_x}{\kappa_0+\kappa_x}x_{it},$$

$$\mathbb{E}\left[\epsilon_{pt}|\,\omega_i^t\right] = \frac{\kappa_z}{\kappa_p+\kappa_z}z_{it}.$$

As a result,

$$\log p_{it}(\omega_i^t) = \left(\frac{1}{\eta}\rho\phi_a' - \psi_0\right)a_{t-1} + \left[\frac{1}{\eta}(\phi_p+\phi_z) - \psi_p\right]\frac{\kappa_z}{\kappa_p+\kappa_z}z_{it} + \left[\frac{1}{\eta}\phi_p - \psi_p + \frac{1}{\eta}(\phi_x+\phi_a) - \left(1+\frac{1}{\eta}\right) - \psi_a\right]\left(\frac{\kappa_0}{\kappa_0+\kappa_x}\rho a_{t-1} + \frac{\kappa_x}{\kappa_0+\kappa_x}x_{it}\right).$$

The latter needs to match our conjecture (A.9). Therefore,

$$\begin{aligned} g_a' &= \left(\frac{1}{\eta}\phi_a' - \psi_0\rho^{-1}\right) + \left[\frac{1}{\eta}\phi_p - \psi_p + \frac{1}{\eta}(\phi_x+\phi_a) - \left(1+\frac{1}{\eta}\right) - \psi_a\right]\frac{\kappa_0}{\kappa_0+\kappa_x}, \\ g_x &= \left[\frac{1}{\eta}\phi_p - \psi_p + \frac{1}{\eta}(\phi_x+\phi_a) - \left(1+\frac{1}{\eta}\right) - \psi_a\right]\frac{\kappa_x}{\kappa_0+\kappa_x}, \\ g_z &= \left[\frac{1}{\eta}(\phi_p+\phi_z) - \psi_p\right]\frac{\kappa_z}{\kappa_p+\kappa_z}. \end{aligned}$$

Since, by definition,

$$\Lambda_t(\bar{s}^t) \equiv U'(C_t(\bar{s}^t))/P_t(\bar{s}^t)$$

or

$$\log\Lambda_t(\bar{s}^t) = -\gamma\log Y_t(\bar{s}^t) - \log P_t(\bar{s}^t),$$

using (A.10) and (A.12), we must have

$$\begin{aligned}&\psi_p x_{pt} + \psi_0 a_{t-1} + \psi_a a_t + g_a' \rho a_{t-1} + g_x a_t + g_z \epsilon_{pt} \\ &= -\gamma\left(\phi_a' \rho a_{t-1} + (\phi_x + \phi_a) a_t + \phi_p x_{pt} + \phi_z \epsilon_{pt}\right).\end{aligned}$$

Therefore, the following equations must hold:

$$\begin{aligned}\psi_0 \rho^{-1} + g_a' &= -\gamma \phi_a', \\ \psi_p + \psi_a + g_x &= -\gamma(\phi_x + \phi_a + \phi_p), \\ \psi_p + g_z &= -\gamma(\phi_p + \phi_z).\end{aligned}$$

To sum up, log-linear equilibrium is the solution to the following system of eight equations in eight unknowns:

$$\begin{aligned}\phi_x &= -\theta g_x, \\ \phi_z &= -\theta g_z, \\ g_a' &= \left(\frac{1}{\eta}\phi_a' - \psi_0 \rho^{-1}\right) + \left[\frac{1}{\eta}\phi_p - \psi_p + \frac{1}{\eta}(\phi_x + \phi_a) - \left(1 + \frac{1}{\eta}\right) - \psi_a\right] \frac{\kappa_0}{\kappa_0 + \kappa_x}, \\ g_x &= \left[\frac{1}{\eta}\phi_p - \psi_p + \frac{1}{\eta}(\phi_x + \phi_a) - \left(1 + \frac{1}{\eta}\right) - \psi_a\right] \frac{\kappa_x}{\kappa_0 + \kappa_x}, \\ g_z &= \left[\frac{1}{\eta}(\phi_p + \phi_z) - \psi_p\right] \frac{\kappa_z}{\kappa_p + \kappa_z}. \\ \psi_0 \rho^{-1} + g_a' &= -\gamma \phi_a', \\ \psi_p + \psi_a + g_x &= -\gamma(\phi_x + \phi_a + \phi_p), \\ \psi_p + g_z &= -\gamma(\phi_p + \phi_z).\end{aligned}$$

Note that we can partition the above equations into three groups. The equations in the first group are

$$\begin{aligned}\phi_z &= -\theta g_z, \\ \psi_p + g_z &= -\gamma(\phi_p + \phi_z), \\ g_z &= \left[\frac{1}{\eta}(\phi_p + \phi_z) - \psi_p\right] \frac{\kappa_z}{\kappa_p + \kappa_z}.\end{aligned}$$

By replacing the second equation into the third, we obtain an equation for g_z:

$$g_z = \left(-\frac{1}{\gamma\eta}(\psi_p + g_z) - \psi_p\right) \frac{\kappa_z}{\kappa_p + \kappa_z},$$

whose solution is given by

$$g_z = -\frac{\kappa_z(1 + \gamma\eta)}{\kappa_z(1 + \gamma\eta) + \kappa_p \gamma\eta} \psi_p.$$

Replacing this back into the other two equations, we obtain solutions for $\phi_z$and ϕ_h:

$$\phi_z = \frac{\theta\kappa_z(1+\gamma\eta)}{\kappa_z(1+\gamma\eta)+\kappa_p\gamma\eta}\psi_p,$$
$$\phi_p = -\frac{\theta\kappa_z(1+\gamma\eta)+\kappa_p\eta}{\kappa_z(1+\gamma\eta)+\kappa_p\gamma\eta}\psi_p.$$

The second group of equations is given by

$$\begin{aligned}\phi_x &= -\theta g_x,\\ \psi_p+\psi_a+g_x &= -\gamma(\phi_x+\phi_a+\phi_p),\\ g_x &= \left[\frac{1}{\eta}\phi_p-\psi_p+\frac{1}{\eta}(\phi_x+\phi_a)-\left(1+\frac{1}{\eta}\right)-\psi_a\right]\frac{\kappa_x}{\kappa_0+\kappa_x},\end{aligned}$$

Simple steps of algebra give the following solutions for g_x, ϕ_x,ϕ_a:

$$\begin{aligned}g_x &= -\frac{\kappa_x\left[(\psi_a+\psi_p)(1+\gamma\eta)+\gamma(1+\eta)\right]}{\kappa_x(1+\gamma\eta)+\kappa_0\gamma\eta},\\ \phi_x &= \frac{\theta\kappa_x\left[(\psi_a+\psi_p)(1+\gamma\eta)+\gamma(1+\eta)\right]}{\kappa_x(1+\gamma\eta)+\kappa_0\gamma\eta},\\ \phi_a &= -\frac{(\eta+1)(\gamma\theta-1)\kappa_x}{\gamma\eta\left(\kappa_0+\kappa_x\right)+\kappa_x}+\frac{\eta(\gamma\eta+1)(\gamma\theta-1)\left(\kappa_0\kappa_z-\kappa_p\kappa_x\right)}{\left(\gamma\eta\left(\kappa_0+\kappa_x\right)+\kappa_x\right)\left(\gamma\eta\left(\kappa_p+\kappa_z\right)+\kappa_z\right)}\psi_p-\frac{\eta\kappa_0+\theta(\gamma\eta+1)\kappa_x}{\gamma\eta\left(\kappa_0+\kappa_x\right)+\kappa_x}\psi_a.\end{aligned}$$

The remaining two equations are

$$\begin{aligned}\psi_0\rho^{-1}+g_a' &= -\gamma\phi_a',\\ g_a' &= \left(\frac{1}{\eta}\phi_a'-\psi_0\rho^{-1}\right)+\left[\frac{1}{\eta}\phi_p-\psi_p+\frac{1}{\eta}(\phi_x+\phi_a)-\left(1+\frac{1}{\eta}\right)-\psi_a\right]\frac{\kappa_0}{\kappa_0+\kappa_x},\end{aligned}$$

To solve this, note that we can write

$$g_a' = \frac{\kappa_0}{\kappa_x}g_x\frac{\gamma\eta}{1+\gamma\eta}-\psi_0\rho^{-1}.$$

Using the previous solution for g_x, we obtain the following solutions for g_a' and ϕ_a':

$$\begin{aligned}g_a' &= -\frac{\gamma\eta}{1+\gamma\eta}\cdot\frac{\kappa_0\left[(\psi_a+\psi_p)(1+\gamma\eta)+\gamma(1+\eta)\right]}{\kappa_x(1+\gamma\eta)+\kappa_0\gamma\eta}-\psi_0\rho^{-1},\\ \phi_a' &= \frac{\eta}{1+\gamma\eta}\cdot\frac{\kappa_0\left[(\psi_a+\psi_p)(1+\gamma\eta)+\gamma(1+\eta)\right]}{\kappa_x(1+\gamma\eta)+\kappa_0\gamma\eta}.\end{aligned}$$

Therefore, the solution to the system of equations is

$$
\begin{aligned}
g_a' &= -\frac{\gamma\eta}{1+\gamma\eta} \cdot \frac{\kappa_0\left[(\psi_a+\psi_p)(1+\gamma\eta)+\gamma(1+\eta)\right]}{\kappa_x(1+\gamma\eta)+\kappa_0\gamma\eta} - \psi_0\rho^{-1}, \\
g_x &= -\frac{\kappa_x\left[(\psi_a+\psi_p)(1+\gamma\eta)+\gamma(1+\eta)\right]}{\kappa_x(1+\gamma\eta)+\kappa_0\gamma\eta}, \\
g_z &= -\frac{\kappa_z(1+\gamma\eta)}{\kappa_z(1+\gamma\eta)+\kappa_p\gamma\eta}\psi_p, \\
\phi_a' &= \frac{\eta}{1+\gamma\eta} \cdot \frac{\kappa_0\left[(\psi_a+\psi_p)(1+\gamma\eta)+\gamma(1+\eta)\right]}{\kappa_x(1+\gamma\eta)+\kappa_0\gamma\eta}, \\
\phi_x &= \frac{\theta\kappa_x\left[(\psi_a+\psi_p)(1+\gamma\eta)+\gamma(1+\eta)\right]}{\kappa_x(1+\gamma\eta)+\kappa_0\gamma\eta}, \\
\phi_a &= -\frac{(\eta+1)(\gamma\theta-1)\kappa_x}{\gamma\eta\left(\kappa_0+\kappa_x\right)+\kappa_x} + \frac{\eta(\gamma\eta+1)(\gamma\theta-1)\left(\kappa_0\kappa_z-\kappa_p\kappa_x\right)}{\left(\gamma\eta\left(\kappa_0+\kappa_x\right)+\kappa_x\right)\left(\gamma\eta\left(\kappa_p+\kappa_z\right)+\kappa_z\right)}\psi_p - \frac{\eta\kappa_0+\theta(\gamma\eta+1)\kappa_x}{\gamma\eta\left(\kappa_0+\kappa_x\right)+\kappa_x}\psi_a, \\
\phi_p &= -\frac{\theta\kappa_z(1+\gamma\eta)+\kappa_p\eta}{\kappa_z(1+\gamma\eta)+\kappa_p\gamma\eta}\psi_p, \\
\phi_z &= \frac{\theta\kappa_z(1+\gamma\eta)}{\kappa_z(1+\gamma\eta)+\kappa_p\gamma\eta}\psi_p.
\end{aligned}
$$

□

Lemma A.2. *(i) Equilibrium aggregate output in the economy without disclosure satisfies*

$$\log Y_t(\bar{s}^t) = \phi_a'\rho a_{t-1} + \Phi_a a_t + \Phi_\epsilon \epsilon_{pt}, \tag{A.14}$$

where

$$
\begin{aligned}
\Phi_a &= \frac{(\eta+1)\kappa_x - \eta\kappa_0\left(\psi_a+\psi_p\right)}{\gamma\eta\left(\kappa_0+\kappa_x\right)+\kappa_x}, \\
\Phi_\epsilon &= -\frac{\eta\kappa_p}{\gamma\eta\left(\kappa_p+\kappa_z\right)+\kappa_z}\psi_p.
\end{aligned}
$$

(ii) The equilibrium labor wedge satisfies

$$\log \bar{\varepsilon}_t(\bar{s}^t) = M_a'\rho a_{t-1} + M_a a_t + M_\epsilon \epsilon_{pt}, \tag{A.15}$$

where

$$
\begin{aligned}
M_a &= \frac{1}{\eta}\left[(1+\gamma\eta)\Phi_a - (1+\eta)\right], \\
M_\epsilon &= \frac{1}{\eta}(1+\gamma\eta)\Phi_\epsilon, \\
M_a' &= \frac{1}{\eta}(1+\gamma\eta)\phi_a'.
\end{aligned}
$$

Proof. Part *(i)*. Individual firm's output is given in (A.8). Aggregating across firms, we obtain the following expression for aggregate output:

$$\log Y_t(\bar{s}^t) = \phi_a'\rho a_{t-1} + (\phi_x+\phi_a)a_t + \phi_p x_{pt} + \phi_z \epsilon_{pt}.$$

Noting that the central bank's signal is given by $x_{pt} = a_t + \epsilon_{pt}$, we can rewrite the latter as

$$\log Y_t(\bar{s}^t) = \phi_a' \rho a_{t-1} + (\phi_x + \phi_a + \phi_p) a_t + (\phi_p + \phi_z)\epsilon_{pt}.$$

which allows us to write aggregate output as in (A.14), with

$$\Phi_a = \phi_x + \phi_a + \phi_p,$$
$$\Phi_\epsilon = \phi_p + \phi_z.$$

The proof then follows from Lemma A.1.

Part *(ii)*. First, from Lemma 7 in the main appendix,

$$Y_t(\bar{s}^t) = (\chi^{-1}\bar{\varepsilon}_t(\bar{s}^t))^{\frac{1}{1/\eta+\gamma}} (\delta A(s_t))^{\frac{1/\eta+1}{1/\eta+\gamma}}.$$

Writing the latter in logs, we have

$$\log \bar{\varepsilon}_t(\bar{s}^t) = \frac{1}{\eta}\left\{(1+\gamma\eta)\log Y_t(\bar{s}^t) - (1+\eta)\log A(s_t)\right\},$$

where we abstract from the constant. Therefore,

$$\log \bar{\varepsilon}_t(\bar{s}^t) = \frac{1}{\eta}(1+\gamma\eta)\phi_a'\rho a_{t-1} + \frac{1}{\eta}\left[(1+\gamma\eta)\Phi_a - (1+\eta)\right] a_t + \frac{1}{\eta}(1+\gamma\eta)\Phi_\epsilon \epsilon_{pt}$$

and the proof follows by matching coefficients.

□

4 Equilibrium under disclosure

Lemma A.3. *Equilibrium firm's output strategy in the economy with disclosure is log-linear and given by*

$$\log y_{it}(\omega_i^t, \bar{s}^t) = \phi_a'\rho a_{t-1} + \phi_x x_{it} + \phi_a a_t + \phi_p x_{pt} + \phi_z z_{it}, \tag{A.16}$$

where

$$\phi_a' = \frac{\eta}{1+\gamma\eta} \cdot \frac{\kappa_0[\psi_a(1+\gamma\eta) + \gamma(1+\eta)]}{(1+\gamma\eta)(\kappa_x+\kappa_z) + (\kappa_p+\kappa_0)\gamma\eta},$$
$$\phi_x = \frac{\theta\kappa_x[\psi_a(1+\gamma\eta) + \gamma(1+\eta)]}{(1+\gamma\eta)(\kappa_x+\kappa_z) + (\kappa_p+\kappa_0)\gamma\eta},$$
$$\phi_a = -\frac{\psi_a[\eta\kappa_0 + \eta\kappa_p + \theta(1+\gamma\eta)(\kappa_x+\kappa_z)] + (\eta+1)(\gamma\theta-1)(\kappa_x+\kappa_z)}{(1+\gamma\eta)(\kappa_x+\kappa_z) + (\kappa_p+\kappa_0)\gamma\eta},$$
$$\phi_p = \frac{\eta\kappa_p + \theta(\gamma\eta+1)\kappa_z}{1+\gamma\eta} \cdot \frac{\psi_a(1+\gamma\eta) + \gamma(1+\eta)}{(1+\gamma\eta)(\kappa_x+\kappa_z) + (\kappa_p+\kappa_0)\gamma\eta},$$
$$\phi_z = -\frac{\theta\kappa_z[\psi_a(1+\gamma\eta) + \gamma(1+\eta)]}{(1+\gamma\eta)(\kappa_x+\kappa_z) + (\kappa_p+\kappa_0)\gamma\eta}.$$

Proof. We guess and verify that equilibrium variables are log-linear. We propose the following log-linear strategy for the price:

$$\log p_{it}(\omega_i^t) = g_a' \rho a_{t-1} + g_x x_{it} + g_z z_{it} + g_p x_{pt}, \tag{A.17}$$

which implies that the aggregate price must satisfy

$$\log P_t(\bar{s}^t) = g_a' \rho a_{t-1} + g_x a_t + g_z \epsilon_{pt} + g_p x_{pt}, \tag{A.18}$$

We propose the following log-linear strategy for individual firm output:

$$\log y_{it}(\omega_i^t, \bar{s}^t) = \phi_a' \rho a_{t-1} + \phi_x x_{it} + \phi_a a_t + \phi_p x_{pt} + \phi_z z_{it}, \tag{A.19}$$

which implies that aggregate ouput must satisfy

$$\log Y_t(\bar{s}^t) = \phi_a' \rho a_{t-1} + (\phi_x + \phi_a) a_t + \phi_p x_{pt} + \phi_z \epsilon_{pt}. \tag{A.20}$$

There are nine unknown coefficients:

$$\{g_a', g_x, g_z, g_p, \phi_a', \phi_x, \phi_a, \phi_p, \phi_z\}.$$

First, we use the CES demand function

$$\frac{y_{it}(\omega_i^t, \bar{s}^t)}{Y_t(\bar{s}^t)} = \left(\frac{p_{it}(\omega_i^t)}{P_t(\bar{s}^t)} \right)^{-\theta}.$$

This implies

$$\log y_{it}(\omega_i^t, \bar{s}^t) - \log Y_t(\bar{s}^t) = -\theta(\log p_{it}(\omega_i^t) - \log P_t(\bar{s}^t))$$

Using (A.17)-(A.20), the difference can be written as:

$$\log y_{it}(\omega_i^t, \bar{s}^t) - \log Y_t(\bar{s}^t) = \phi_x(x_{it} - a_t) + \phi_z(z_{it} - \epsilon_{pt})$$

and, similarly,

$$\log p_{it}(\omega_i^t) - \log P_t(\bar{s}^t) = g_x(x_{it} - a_t) + g_z(z_{it} - \epsilon_{pt}).$$

Therefore, this gives us the following two coefficient restrictions:

$$\phi_x = -\theta g_x,$$
$$\phi_z = -\theta g_z.$$

The price is given by equation (A.13).

$$\log p_{it}(\omega_i^t) = \mathbb{E}\left[\frac{1}{\eta} \log Y_t(\bar{s}^t) - \left(1 + \frac{1}{\eta}\right) \log A(s_t) - \log \Lambda_t(\bar{s}^t) \middle| \omega_i^t \right].$$

Substituting in for output and the policy variable and using Lemma 3 in the paper,

$$\log p_{it}(\omega_i^t) = \mathbb{E}\left[\frac{1}{\eta}(\phi_a'\rho a_{t-1} + (\phi_x + \phi_a)a_t + \phi_p x_{pt} + \phi_z \epsilon_{pt}) - \left(1 + \frac{1}{\eta}\right) a_t - (\psi_p x_{pt} + \psi_0 a_{t-1} + \psi_a a_t) \middle| \omega_i^t\right].$$

Since $x_{pt} = a_t + \epsilon_{pt}$, the latter can be rewritten as

$$\log p_{it}(\omega_i^t) = \left(\frac{1}{\eta}\rho\phi_a' - \psi_0\right) a_{t-1} + \left[\frac{1}{\eta}(\phi_p + \phi_z) - \psi_p\right] x_{pt} + \left[\frac{1}{\eta}(\phi_x + \phi_a - \phi_z) - \left(1 + \frac{1}{\eta}\right) - \psi_a\right] \mathbb{E}\left[a_t | \, \omega_i^t\right].$$

Standard properties of Normal distributions imply

$$\mathbb{E}\left[a_t | \, \omega_i^t\right] = \frac{\kappa_0}{\kappa}\rho a_{t-1} + \frac{\kappa_x}{\kappa} x_{it} - \frac{\kappa_z}{\kappa} z_{it} + \frac{\kappa_p + \kappa_z}{\kappa} x_{pt},$$

where $\kappa \equiv \kappa_0 + \kappa_x + \kappa_p + \kappa_z$. As a result,

$$\log p_{it}(\omega_i^t) = \left(\frac{1}{\eta}\rho\phi_a' - \psi_0\right) a_{t-1} + \left[\frac{1}{\eta}(\phi_p + \phi_z) - \psi_p\right] x_{pt} + \left[\frac{1}{\eta}(\phi_x + \phi_a - \phi_z) - \left(1 + \frac{1}{\eta}\right) - \psi_a\right] \left(\frac{\kappa_0}{\kappa}\rho a_{t-1} + \frac{\kappa_x}{\kappa} x_{it} - \frac{\kappa_z}{\kappa} z_{it} + \frac{\kappa_p + \kappa_z}{\kappa} x_{pt}\right).$$

The latter needs to match our conjecture (A.17). Therefore,

$$g_z = -\left[\frac{1}{\eta}(\phi_x + \phi_a - \phi_z) - \left(1 + \frac{1}{\eta}\right) - \psi_a\right] \frac{\kappa_z}{\kappa},$$

$$g_a' = \left(\frac{1}{\eta}\phi_a' - \psi_0\rho^{-1}\right) + \left[\frac{1}{\eta}(\phi_x + \phi_a - \phi_z) - \left(1 + \frac{1}{\eta}\right) - \psi_a\right] \frac{\kappa_0}{\kappa},$$

$$g_p = \left[\frac{1}{\eta}(\phi_p + \phi_z) - \psi_p\right] + \left[\frac{1}{\eta}(\phi_x + \phi_a - \phi_z) - \left(1 + \frac{1}{\eta}\right) - \psi_a\right] \frac{\kappa_p + \kappa_z}{\kappa},$$

$$g_x = \left[\frac{1}{\eta}(\phi_x + \phi_a - \phi_z) - \left(1 + \frac{1}{\eta}\right) - \psi_a\right] \frac{\kappa_x}{\kappa}.$$

Since, by definition,

$$\Lambda_t(\bar{s}^t) \equiv U'(C_t(\bar{s}^t))/P_t(\bar{s}^t)$$

or

$$\log \Lambda_t(\bar{s}^t) = -\gamma \log Y_t(\bar{s}^t) - \log P_t(\bar{s}^t),$$

using (A.18) and (A.20), we must have

$$\psi_p x_{pt} + \psi_0 a_{t-1} + \psi_a a_t + g_a'\rho a_{t-1} + g_x a_t + g_z \epsilon_{pt} + g_p x_{pt} = -\gamma\left(\phi_a'\rho a_{t-1} + (\phi_x + \phi_a)a_t + \phi_p x_{pt} + \phi_z \epsilon_{pt}\right)$$

or

$$\begin{aligned}
&\psi_p(a_t+\epsilon_{pt})+\psi_0 a_{t-1}+\psi_a a_t+g_a'\rho a_{t-1}+g_x a_t+g_z\epsilon_{pt}+g_p(a_t+\epsilon_{pt})\\
=&-\gamma\left(\phi_a'\rho a_{t-1}+(\phi_x+\phi_a)a_t+\phi_p(a_t+\epsilon_{pt})+\phi_z\epsilon_{pt}\right)
\end{aligned}$$

Therefore, the following equations must hold:

$$\begin{aligned}
\psi_p+g_z+g_p&=-\gamma(\phi_p+\phi_z),\\
\psi_0\rho^{-1}+g_a'&=-\gamma\phi_a',\\
\psi_p+\psi_a+g_x+g_p&=-\gamma(\phi_x+\phi_a+\phi_p).
\end{aligned}$$

To sum up, log-linear equilibrium is the solution to the following system of nine equations in nine unknowns:

$$\begin{aligned}
\phi_x=&-\theta g_x,\\
\phi_z=&-\theta g_z,\\
g_z=&-\left[\frac{1}{\eta}(\phi_x+\phi_a-\phi_z)-\left(1+\frac{1}{\eta}\right)-\psi_a\right]\frac{\kappa_z}{\kappa},\\
g_a'=&\left(\frac{1}{\eta}\phi_a'-\psi_0\rho^{-1}\right)+\left[\frac{1}{\eta}(\phi_x+\phi_a-\phi_z)-\left(1+\frac{1}{\eta}\right)-\psi_a\right]\frac{\kappa_0}{\kappa},\\
g_p=&\left[\frac{1}{\eta}(\phi_p+\phi_z)-\psi_p\right]+\left[\frac{1}{\eta}(\phi_x+\phi_a-\phi_z)-\left(1+\frac{1}{\eta}\right)-\psi_a\right]\frac{\kappa_p+\kappa_z}{\kappa},\\
g_x=&\left[\frac{1}{\eta}(\phi_x+\phi_a-\phi_z)-\left(1+\frac{1}{\eta}\right)-\psi_a\right]\frac{\kappa_x}{\kappa},\\
\psi_p+g_z+g_p=&-\gamma(\phi_p+\phi_z),\\
\psi_0\rho^{-1}+g_a'=&-\gamma\phi_a',\\
\psi_p+\psi_a+g_x+g_p=&-\gamma(\phi_x+\phi_a+\phi_p).
\end{aligned}$$

Steps of algebra analogous to those in the no-disclosure case yield the solution:

$$
\begin{aligned}
g'_a &= -\frac{\gamma\eta}{1+\gamma\eta}\cdot\frac{\kappa_0[\psi_a(1+\gamma\eta)+\gamma(1+\eta)]}{(1+\gamma\eta)(\kappa_x+\kappa_z)+\gamma\eta(\kappa_p+\kappa_0)} - \psi_0\rho^{-1},\\
g_x &= -\frac{\kappa_x[\psi_a(1+\gamma\eta)+\gamma(1+\eta)]}{(1+\gamma\eta)(\kappa_x+\kappa_z)+\gamma\eta(\kappa_0+\kappa_p)},\\
g_z &= \frac{\kappa_z[\psi_a(1+\gamma\eta)+\gamma(1+\eta)]}{(1+\gamma\eta)(\kappa_x+\kappa_z)+\gamma\eta(\kappa_0+\kappa_p)},\\
g_p &= -\frac{\gamma\eta\,(\kappa_p+\kappa_z)+\kappa_z}{1+\gamma\eta}\cdot\frac{\psi_a(1+\gamma\eta)+\gamma(1+\eta)}{(1+\gamma\eta)(\kappa_x+\kappa_z)+(\kappa_p+\kappa_0)\gamma\eta} - \psi_p,\\
\phi'_a &= \frac{\eta}{1+\gamma\eta}\cdot\frac{\kappa_0[\psi_a(1+\gamma\eta)+\gamma(1+\eta)]}{(1+\gamma\eta)(\kappa_x+\kappa_z)+(\kappa_p+\kappa_0)\gamma\eta},\\
\phi_x &= \frac{\theta\kappa_x[\psi_a(1+\gamma\eta)+\gamma(1+\eta)]}{(1+\gamma\eta)(\kappa_x+\kappa_z)+(\kappa_p+\kappa_0)\gamma\eta},\\
\phi_a &= -\frac{\psi_a[\eta\kappa_0+\eta\kappa_p+\theta(1+\gamma\eta)(\kappa_x+\kappa_z)]+(\eta+1)(\gamma\theta-1)(\kappa_x+\kappa_z)}{(1+\gamma\eta)(\kappa_x+\kappa_z)+(\kappa_p+\kappa_0)\gamma\eta},\\
\phi_p &= \frac{\eta\kappa_p+\theta(\gamma\eta+1)\kappa_z}{1+\gamma\eta}\cdot\frac{\psi_a(1+\gamma\eta)+\gamma(1+\eta)}{(1+\gamma\eta)(\kappa_x+\kappa_z)+(\kappa_p+\kappa_0)\gamma\eta},\\
\phi_z &= -\frac{\theta\kappa_z[\psi_a(1+\gamma\eta)+\gamma(1+\eta)]}{(1+\gamma\eta)(\kappa_x+\kappa_z)+(\kappa_p+\kappa_0)\gamma\eta}.
\end{aligned}
$$

□

Lemma A.4. *(i) Equilibrium aggregate output in the economy with disclosure satisfies*

$$\log Y_t(\bar{s}^t) = \phi'_a\rho a_{t-1} + \Phi_a a_t + \Phi_\epsilon \epsilon_{pt}, \tag{A.21}$$

where

$$
\begin{aligned}
\Phi_a &= \frac{1}{(1+\gamma\eta)(\kappa_x+\kappa_z)+(\kappa_p+\kappa_0)\gamma\eta}\left\{(\eta+1)(\kappa_x+\kappa_z) - \eta\kappa_0\psi_a + \gamma\eta\frac{1+\eta}{1+\gamma\eta}\kappa_p\right\},\\
\Phi_\epsilon &= \frac{1}{(1+\gamma\eta)(\kappa_x+\kappa_z)+(\kappa_p+\kappa_0)\gamma\eta}\left\{\eta\kappa_p\psi_a + \gamma\frac{1+\eta}{1+\gamma\eta}\eta\kappa_p\right\},
\end{aligned}
$$

(ii) The equilibrium labor wedge satisfies

$$\log\bar{\varepsilon}_t(\bar{s}^t) = M'_a\rho a_{t-1} + M_a a_t + M_\epsilon \epsilon_{pt}, \tag{A.22}$$

where

$$
\begin{aligned}
M_a &= \frac{1}{\eta}\left[(1+\gamma\eta)\Phi_a - (1+\eta)\right],\\
M_\epsilon &= \frac{1}{\eta}(1+\gamma\eta)\Phi_\epsilon,\\
M'_a &= \frac{1}{\eta}(1+\gamma\eta)\phi'_a.
\end{aligned}
$$

Proof. Part *(i)*. Individual firm's output is given in (A.16). Aggregating across firms, we obtain the following expression for aggregate output:

$$\log Y_t(\bar{s}^t) = \phi_a' \rho a_{t-1} + (\phi_x + \phi_a) a_t + \phi_p x_{pt} + \phi_z \epsilon_{pt}.$$

We can rewrite this as

$$\log Y_t(\bar{s}^t) = \phi_a' \rho a_{t-1} + (\phi_x + \phi_a + \phi_p) a_t + (\phi_p + \phi_z) \epsilon_{pt},$$

which allows us to write aggregate output as in (A.21), with

$$\Phi_a = \phi_x + \phi_a + \phi_p,$$
$$\Phi_\epsilon = \phi_p + \phi_z.$$

The proof then follows from Lemma A.3.

Part *(ii)*. As in the proof of Lemma A.2,

$$\log \bar{\varepsilon}_t(\bar{s}^t) = \frac{1}{\eta} \left\{ (1+\gamma\eta) \log Y(\bar{s}^t) - (1+\eta) \log A(s^t) \right\},$$

where we abstract from the constant. Therefore,

$$\log \bar{\varepsilon}_t(\bar{s}^t) = \frac{1}{\eta}(1+\gamma\eta)\phi_a' \rho a_{t-1} + \frac{1}{\eta}\left[(1+\gamma\eta)\Phi_a - (1+\eta)\right] a_t + \frac{1}{\eta}(1+\gamma\eta)\Phi_\epsilon \epsilon_{pt}$$

and the proof follows by matching coefficients.

□

5 Proof of Lemma 5

We begin with the no-disclosure case. We compute each term of $\mathcal{L}$ separately. First, from the proof of Lemma 7, we have that

$$\text{var}_\omega \left[\log \varepsilon_{it}(\omega_{it}, \bar{s}^t) | \bar{s}^t\right] = \text{var}_\omega \left[\frac{1}{\theta} \log y_{it}(\omega_i^t, \bar{s}^t) | \bar{s}^t\right].$$

Thus,

$$\mathcal{D}_0 \equiv \theta \text{var}_\omega \left[\log \varepsilon_{it}(\omega_{it}, \bar{s}^t) | \bar{s}^t\right] = \frac{1}{\theta} \text{var}_\omega \left[\log y_{it}(\omega_i^t, \bar{s}^t) | \bar{s}^t\right],$$

where we use the subscript "0" to denote variables under no disclosure. Using (A.8),

$$\mathcal{D}_0 = \frac{1}{\theta}\phi_{x0}^2 \frac{1}{\kappa_x} + \frac{1}{\theta}\phi_{z0}^2 \frac{1}{\kappa_z}.$$

Straightforward steps of algebra then imply

$$\mathcal{D}_0 = \frac{\theta \kappa_x (\gamma\eta + 1)^2}{(1-\rho)^2 \left(\gamma\eta\left(\kappa_0 + \kappa_x\right) + \kappa_x\right)^2} \left(\psi_0 - \psi_0^* + \psi_p\right)^2 + \frac{\theta(\gamma\eta+1)^2 \kappa_z}{\left(\gamma\eta\left(\kappa_p + \kappa_z\right) + \kappa_z\right)^2} \psi_p^2.$$

Similarly, using (A.15),

$$\begin{aligned}\mathcal{V}_0 &\equiv \text{var}\left[\log \bar{\varepsilon}(\bar{s}^t)\right] \\ &= \left(M'_{a0}+M_{a0}\right)^2 \frac{\rho^2}{1-\rho^2}\cdot\frac{1}{\kappa_0}+M_{a0}^2\frac{1}{\kappa_0}+M_{\epsilon 0}^2\frac{1}{\kappa_p}.\end{aligned}$$

It follows that

$$\mathcal{V}_0 = \frac{\kappa_0(\gamma\eta+1)^2}{(1-\rho)^2\left(\gamma\eta\left(\kappa_0+\kappa_x\right)+\kappa_x\right)^2}\left(\psi_0-\psi_0^*+\psi_p\right)^2+\frac{(\gamma\eta+1)^2\kappa_p}{\left(\gamma\eta\left(\kappa_p+\kappa_z\right)+\kappa_z\right)^2}\psi_p^2.$$

Consider now the case with disclosure. First, using (A.16),

$$\mathcal{D}_d = \frac{1}{\theta}\phi_{xd}^2\frac{1}{\kappa_x}+\frac{1}{\theta}\phi_{zd}^2\frac{1}{\kappa_z}.$$

Straightforward steps of algebra then imply

$$\mathcal{D}_d = \frac{\theta\left(\kappa_x+\kappa_z\right)(\gamma\eta+1)^2}{(1-\rho)^2\left(\gamma\eta\left(\kappa_0+\kappa_p+\kappa_x+\kappa_z\right)+\kappa_x+\kappa_z\right)^2}\left(\psi_0-\psi_0^*+\rho\psi_p\right)^2.$$

Similarly, using (A.22),

$$\mathcal{V}_d = \left(M'_{ad}+M_{ad}\right)^2\frac{\rho^2}{1-\rho^2}\cdot\frac{1}{\kappa_0}+M_{ad}^2\frac{1}{\kappa_0}+M_{\epsilon d}^2\frac{1}{\kappa_p}.$$

It follows that

$$\mathcal{V}_d = \frac{\left(\kappa_0+\kappa_p\right)(\gamma\eta+1)^2}{(1-\rho)^2\kappa_p\left(\gamma\eta\left(\kappa_0+\kappa_p+\kappa_x+\kappa_z\right)+\kappa_x+\kappa_z\right)^2}\left(\psi_0-\psi_0^*+\rho\psi_p\right)^2.$$

To complete the proof, we compute:

$$\Delta(\psi_0,\psi_p)\equiv \mathcal{D}_0-\mathcal{D}_d+\frac{1}{1/\eta+\gamma}\left(\mathcal{V}_0-\mathcal{V}_d\right).$$

We have

$$\begin{aligned}\mathcal{D}_0-\mathcal{D}_d =&\frac{\theta\kappa_x(\gamma\eta+1)^2}{(1-\rho)^2\left(\gamma\eta\left(\kappa_0+\kappa_x\right)+\kappa_x\right)^2}\left(\psi_0-\psi_0^*+\psi_p\right)^2+\frac{\theta(\gamma\eta+1)^2\kappa_z}{\left(\gamma\eta\left(\kappa_p+\kappa_z\right)+\kappa_z\right)^2}\psi_p^2 \\ &-\frac{\theta\left(\kappa_x+\kappa_z\right)(\gamma\eta+1)^2}{(1-\rho)^2\left(\gamma\eta\left(\kappa_0+\kappa_p+\kappa_x+\kappa_z\right)+\kappa_x+\kappa_z\right)^2}\left(\psi_0-\psi_0^*+\rho\psi_p\right)^2\end{aligned}$$

and

$$\begin{aligned}\mathcal{V}_0-\mathcal{V}_d =&\frac{\kappa_0(\gamma\eta+1)^2}{(1-\rho)^2\left(\gamma\eta\left(\kappa_0+\kappa_x\right)+\kappa_x\right)^2}\left(\psi_0-\psi_0^*+\psi_p\right)^2+\frac{(\gamma\eta+1)^2\kappa_p}{\left(\gamma\eta\left(\kappa_p+\kappa_z\right)+\kappa_z\right)^2}\psi_p^2 \\ &-\frac{\left(\kappa_0+\kappa_p\right)(\gamma\eta+1)^2}{(1-\rho)^2\kappa_p\left(\gamma\eta\left(\kappa_0+\kappa_p+\kappa_x+\kappa_z\right)+\kappa_x+\kappa_z\right)^2}\left(\psi_0-\psi_0^*+\rho\psi_p\right)^2.\end{aligned}$$

Therefore,

$$
\begin{aligned}
a_0 \equiv & \frac{\theta(\gamma\eta+1)\kappa_x+\eta\kappa_0}{(1-\rho)^2\left(\gamma\eta\left(\kappa_0+\kappa_x\right)+\kappa_x\right)^2}(\gamma\eta+1), \\
b_0 \equiv & \frac{\theta(\gamma\eta+1)\kappa_z+\eta\kappa_p}{\left(\gamma\eta\left(\kappa_p+\kappa_z\right)+\kappa_z\right)^2}(\gamma\eta+1), \\
c_0 \equiv & -\frac{\theta(\gamma\eta+1)(\kappa_x+\kappa_z)+\eta(\kappa_0+\kappa_p)}{(1-\rho)^2\left(\gamma\eta\left(\kappa_0+\kappa_p+\kappa_x+\kappa_z\right)+\kappa_x+\kappa_z\right)^2}(\gamma\eta+1).
\end{aligned}
$$

QED.