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## Three Essays in Labour Macroeconomics

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The curious task of economics is to demonstrate to men how little they really know about what they imagine they can design.

FRIEDRICH A. HAYEK
The Fatal Conceit

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## Contents

1 Choosing Employment Protection ..... 9
1.1 Introduction ..... 9
1.2 Related literature and Contribution ..... 14
1.3 Descriptive Evidence ..... 19
1.3.1 FTCs and OECs among workers and firms ..... 19
1.3.2 Duration of the Contract ..... 22
1.3.3 Persistence of Type of Contract ..... 24
1.3.4 On the job search ..... 24
1.4 Toy Model ..... 29
1.4.1 Search and Matching ..... 29
1.4.2 Timing of Events ..... 30
1.4.3 Value Functions ..... 33
1.4.4 Optimal Search Intensity ..... 34
1.4.5 Optimal Labour Contract in the presence of on-the-job search ..... 36
1.5 Full Model ..... 40
1.5.1 Search and Matching ..... 41
1.5.2 Production, Timing and Bayesian Updating ..... 42
1.5.3 Value Function ..... 45
1.5.4 Firing Thresholds ..... 50
1.5.5 Optimal search intensity ..... 51
1.5.6 Surplus ..... 53
1.5.7 Optimal Contractual decision ..... 54
1.5.8 Steady-State Equilibrium ..... 57
1.6 Calibration ..... 60
1.6.1 Externally Calibrated Parameters ..... 60
1.6.2 Estimated Parameters ..... 62
1.6.3 Results ..... 67
1.7 Counterfactual scenarios ..... 71
1.7.1 Cut in the firing costs ..... 71
1.7.2 Lump-sum Tax on FTC ..... 71
1.7.3 Counterfactual results and discussion ..... 72
1.8 Further Research and Conclusion ..... 75
1.8.1 INPS Dataset ..... 77
1.8.2 Nash-Bargaining without Employment Protection ..... 94
1.8.3 Nash-Bargaining with Firing Costs as a pure waste ..... 95
1.8.4 Nash-Bargaining with Severance Payment ..... 97
1.8.5 Higher firing threshold for FTC $\left(\hat{y}_{f}>\hat{y}_{p}\right)$ ..... 97
2 The Drivers of EU Unemployment during the Great Recession ..... 107
2.1 Introduction ..... 107
2.2 Methodology ..... 110
2.2.1 Inference of SDF shocks ..... 110
2.2.2 Inference of aggregate productivity shocks ..... 113
2.3 Model with Representative Agent ..... 115
2.3.1 Calibration ..... 118
2.3.2 Inspecting the mechanisms ..... 125
2.3.3 The effects of SDF shocks vs productivity shocks ..... 126
2.3.4 The role of Labor Market Institutions ..... 127
2.4 Model with Heterogeneous Agents ..... 131
2.4.1 Description of the model and main mechanisms ..... 131
2.4.2 Timing ..... 133
2.4.3 The model ..... 134
2.5 Results ..... 145
2.5.1 Model with Representative Agent ..... 145
2.6 Conclusion ..... 149
2.7 Equations of the RA model ..... 153
2.7.1 System of Equations ..... 153
2.7.2 System of Log-Linear Equations ..... 156
2.7.3 Steady State ..... 158
2.8 Equations of the HA model ..... 160
2.8.1 Signals and abilities ..... 160
2.9 Robustness Checks in Inference of SDF Shocks ..... 161
3 Labor Mobility of Heterogeneous Workers in the EU ..... 165
3.1 Introduction ..... 165
3.2 Skills, complementarity and moving costs ..... 170
3.3 Skill bias in European mobility ..... 175
3.4 A model of worker mobility in Europe ..... 177
3.4.1 Consumption ..... 177
3.4.2 Production ..... 179
3.4.3 Labor Markets ..... 180
3.4.4 Migration decision ..... 184
3.4.5 Goods Markets ..... 187
3.4.6 Steady State Equilibrium ..... 188
3.4.7 Currency Devaluation and Monetary Policy ..... 188
3.4.8 Wage Rigidity ..... 190
3.5 Data ..... 191
3.6 Calibration ..... 192
3.6.1 Externally Set Parameters ..... 193
3.6.2 Estimated Parameters and Targeted Moments ..... 194
3.6.3 Estimation protocol ..... 195
3.6.4 National-specific wages ..... 196
3.7 Results and Counterfactual Scenarios ..... 196
3.7.1 Productivity and Demand Shocks ..... 197
3.7.2 Symmetric Productivity Shock ..... 199
3.7.3 Non-permanent Shock ..... 199
3.7.4 Welfare Considerations ..... 200
3.8 Conclusion ..... 201

## Chapter 1

## Choosing Employment Protection: <br> the role of On-the-Job Search and

## Ability Learning

### 1.1 Introduction

A labour contract often entails some protection for the worker in case of involuntary separation. In continental Europe, institutional constraints play a central role in the regulation of these contractual aspects. These constraints divide labour contracts into two broad categories: Open-Ended Contracts (OECs) and Fixed-Term Contracts (FTCs). The former are the traditional labour contracts which generally involve relatively high separation costs. The latter are flexible and less protected contracts, where the two parties already fix an expiration date before starting the working relationship. The economic literature has extensively studied the effects of the introduction of FTCs in Europe, but it has paid little attention to the agents' choice between the two types of contracts. In this regard, while it is quite intuitive to understand the joint advantage of opting for a FTC, that of avoiding the separation costs associated with the OEC, it is less obvious why workers and firms should sign
a contract associated with employment protection. Nevertheless, the OEC is still the most common labour contract among the workforce, suggesting that it entails some benefits for both agents.

The first contribution of this paper is to show that, in presence of on-the-job search and heterogeneity in match productivity, agents could optimally choose employment protection to reduce an otherwise excessive search of the worker while employed, hence increasing the joint surplus to share. In the vast majority of the literature with FTCs, the contractual choice is modelled by postulating an exogenous probability of receiving each contract or by assuming an exogenous difference in the duration or the productivity of the two contracts. ${ }^{1}$ Instead, in my model, the choice of the type of contract becomes endogenous to the characteristics of workers and matches. Indeed, it is well-documented that FTCs are unevenly distributed among workers and jobs ${ }^{2}$, suggesting that their characteristics play an important role in the choice of the type of labour contract. Three facts in particular clearly emerge from an analysis of the distribution of FTCs: (i) FTCs are disproportionately widespread among young workers; (ii) they are correlated with low-wage positions; and (iii) they are persistent, that is, a worker employed with a FTC has a higher probability than his peers of being employed with the same type of contract years later. The second contribution of the paper is then to explain these facts by introducing in the model an additional dimension of heterogeneity, namely ex-ante unknown workers' ability. Specifically, agents discover the worker-specific ability over time, through the observation of period-by-period production, which in turn acts as a signal of the underlying worker ability. Experience then naturally emerges as a third dimension of heterogeneity, as it reduces the uncertainty about the worker-specific ability. The third contribution of this paper is to perform welfare comparisons, analyzing costs and benefits of alternative policy interventions for different types of agents. The heterogeneity of welfare effects gives rise to policy trade-offs, as it was highlighted in previous papers that

[^0]examine the support for specific labour market policies. ${ }^{3}$ However, I perform these comparisons in an environment where agents also optimize over the type of labour contract. This is particularly important in presence of heterogeneity, given that the type of contract is not independent of jobs' and workers' characteristics. Indeed, I show that different labour market policies with the same objective of a reduction in the share of FTCs, such as a tax on FTCs or a firing cost cut, can have opposite effects on different types of workers. Moreover, these outcomes are due to the endogenous contractual choices of the agents, that create persistence in the type of contract a worker receives. As a result, compared to a scenario in which the contract type is exogenously given, the burden of different policies is more concentrated on particular segments of the workforce.

The mechanism underlying the contractual choice is illustrated in a simplified version of the full model that will be developed. The simpler model is a discrete-time search and matching Diamond-Mortensen-Pissarides model with on-the-job search and heterogeneity in the match-specific productivity. When agents meet, they observe the match-specific productivity draw and bargain over the wage, which will be re-bargained at every period. Then, the worker chooses how intensively to search for another job while employed, balancing search costs and benefits in terms of the probability to receive a new offer. Importantly, search intensity cannot be bargained with the firm as the worker keeps the "right-to-manage" his on-the-job search activity. In this setting, the worker chooses an excessively high search intensity from the joint firm-worker perspective, since he is not internalizing the damage to the firm in case he actually quits. Indeed, in this event, the firm will lose its part of the match surplus and will have to open a new vacancy. In other words, the worker's private marginal benefit from searching on-the-job is generally higher than the joint marginal benefit of the firm-worker pair.

One can show that in this environment an optimal labour contract involves a backload of the wage, that is, a commitment to future higher wages against lower wages today. This

[^1]raises the continuation value of the worker and hence reduces incentives to search. I then show that employment protection, in terms of either firing costs or severance payments, provides the agents with an instrument to commit to this outcome. In fact, via the introduction of separation costs, which credibly grant a higher wage to the worker in the next periods, employment protection shifts worker's utility from the present into the future. This result is in line with the literature on hidden on-the-job search. In a similar environment, Lentz (2014) shows that the optimal labour contract would involve an increasing wage path. ${ }^{4}$ This solution requires the two parties to commit to a given wage profile, without subsequent bargaining. This assumption is clearly not realistic with a FTC, as both parties know that continuation beyond the original contract duration, whether with a renewal or with a transformation, will necessarily involve agreement to a new contract and thus a new bargaining. An OEC solves this commitment problem, as employment protection raises future wages by reducing the future outside option of the firm, even in an environment with period-by-period bargaining.

The full model adds to on-the-job search with variable intensity and match-specific productivity, the additional element of ex-ante unknown worker-specific ability, which is learned over time. Further, to focus on the European setting, it restricts the choice of labour contracts to FTCs and OECs. Specifically, a FTC can be terminated at the end of every period with no additional cost, while an OEC features employment protection in the form of firing costs that the firm has to pay in case of endogenous termination of the match. In making this choice, agents maximize the surplus, balancing the benefits of FTCs in terms of lower firing costs, with the benefits of OECs in terms of lower on-the-job search intensity.

I calibrate the structural model using Italian microdata contained in two rich administrative datasets: (i) "Mercurio", from "Veneto Lavoro", collecting all the job histories of workers of Veneto, one of the largest Italian regions; and (ii) a sample of social security records from INPS, the Italian social security agency. These datasets contain the entire working careers of millions of employed workers, covering several decades. However, I restrict my analysis

[^2]to the years after 2000, given that FTCs were introduced in Italy in 1997. I also use the Italian Labour Force Survey ("Rilevazione Continua delle Forze di Lavoro", RCFL), a crosssectional dataset collected every quarter, which contains information related to on-the-job search intensities and wages.

I estimate the parameters of the model performing a Monte Carlo Markov Chain estimation. The large number of observations and the detailed sequence of labour contracts contained in the datasets allow me to estimate the key parameters of the model, using as targets several aggregate moments of the Italian labour market. More specifically, I use: (i) the unemployment rate; (ii) the share of FTCs; (iii) the job-finding, separation, quitting and contract transformation rates; and (iv) selected moments of the wage distribution. The calibrated model can reproduce the (untargeted) three facts mentioned above: FTCs are mostly used by inexperienced and low-skilled workers, and in low-pay jobs. In addition, the model can explain the emergence of a dual labour market, in which some workers experience long sequences of fixed-term contracts and unemployment, while others are able to keep their job position for a long period of time.

Finally, I analyze the effects of some counterfactual scenarios that resemble real policy interventions: (i) a $25 \%$ cut in the firing costs; (ii) a lump-sum tax on all FTCs, equivalent to $1 \%$ of the average wage and rebated back to all workers. ${ }^{5}$

The richness of the model allows evaluating policies along several dimensions. First, the effects of these two policies are indeed highly heterogeneous across workers. In particular, while both these reforms aim to reduce the share of fixed-term contracts, they have opposite effects on the workers' welfare among low and high expected ability workers: a lump-sum FTC tax is effectively a low-ability tax, reducing the wage of low-ability workers and their job-finding probability; conversely, a firing costs cut reduces the welfare of the "insiders" with an OEC, while it helps low-ability workers to enter the labour market. Secondly, the fact that the contract is chosen endogenously is crucial to assess these heterogeneous effects correctly.

[^3]For instance, if the type of the contract were to be assigned randomly, a policy such as a lump-sum FTC-tax would fall randomly on the labour force, with a negative impact only on current holders of FTCs. Instead, the active choices of the contract types made by agents, transform this policy in a tax on low-ability workers. Finally, the general equilibrium effects of these policies play an important role in determining the final labour market outcomes, the final share of FTCs and the economy's overall efficiency.

The remainder of the paper proceeds as follows. In Section 2, I describe the related literature and the contribution of this paper. In Section 3, I present some descriptive statistics about the distribution of FTCs in Italy. In Section 4, I illustrate a simplified version of the model that explains the role of employment protection in the presence of on-the-job search. In Section 5, I describe the full model, allowing for unknown worker-specific ability. In Section 6, I calibrate the model on the Italian data. In Section 7, I evaluate two labour market policies and conclude.

### 1.2 Related literature and Contribution

As already anticipated in the introduction, the previous literature that introduced the choice of the contract generally assumed that an exogenous fraction of workers receives an OEC: see for example Cahuc and Postel-Vinay (2002) in which to the standard search and matching model, the authors added the assumption of an exogenous fraction $p$ of the new matches as fixed-term contracts.

Faccini (2014) uses a similar model in which agents are forced to transform the contract into an OECs with some exogenous probability. This paper highlights an important advantage of FTCs: the possibility to test the productivity of the match. Indeed, there is ex-ante unknown match-specific productivity that is revealed with a certain probability at the end of every period. My work shares with this paper the idea of productivity as an experience good that is discovered over time. However, in my model, the worker's ability is the unknown
variable that the agents will learn over time. This fact has important consequences in the final distribution of FTCs among the population, as it will be clear later.

This representation of the choice of the contract would be consistent with the idea that the presence of permanent contracts is entirely due to imposed institutional constraints. Indeed, it is certainly true that labour market regulation set limits to the use of Fixed-Term Contracts. ${ }^{6}$ I will capture these restrictions with an exogenous "transformation" shock, which forces agents to transform a FTC into an OEC with a certain probability. However, there is evidence that the choice in favour of employment protection is not fully due to legislation. Indeed, the OEC is commonly employed even at the beginning of a working relationship, when possible institutional constraints are less binding. Moreover, we can observe that some forms of employment protection are common even in countries in which they are not mandatory. For instance, in the USA severance payments are often included in labour contracts, as part of workers' benefits. ${ }^{7}$

Other papers took a reduced-form approach to the choice between OECs and FTCs, assuming some exogenous differences between contracts. For instance, Caggese and Cuñat (2008) assumes that an OEC assures higher productivity to the match. Instead, Garibaldi (2006) and Cao et al. (2010) assume that FTCs have a lower expected duration. In particular, Cao et al. (2010) assumes that only workers with a FTC can search on-the-job. This could be thought of as a shortcut for this work's main mechanism, allowing for a tractable model. However, in this way, we do not consider the endogenous nature of the searching decision, which could have important consequences in aggregate terms and for the type of workers that receive the FTC. Indeed, these papers abstract from heterogeneity in workers' ability, limiting the persistence in the type of the contract a worker receives.

One paper focused on the explanation of the coexistence of OECs and FTCs is Cahuc et

[^4]al. (2016). The authors noticed that in reality, FTCs are highly expensive to terminate until the expiration rate. With the additional assumption of a small cost of writing a contract, they can explain the spread of FTCs among the jobs with a limited expected duration, while OECs becomes optimal for jobs with an expected duration long enough. This framework is undoubtedly suited to explain the use of FTCs among very short jobs with a predetermined duration. Instead, my model can explain the use of FTCs even when there is no heterogeneity in the exogenous expected duration of the match. Indeed, in labour surveys ${ }^{8}$ a considerable share (between 25 and $33 \%$ ) of workers claim to be employed with a FTC because they are in a trial period, supporting the idea that the FTCs can be used as a stepping stone towards the OECs or more generally to a long-lasting employment relationship.

Another relevant explanation for the coexistence of FTCs and OECs is described in Crechet (2018), where the author assumes differences in the risk-aversion between employer and employee to explain the surplus gain that an OEC can create by providing some insurance to the worker. This is in line with a more general rationalization for the use of employment protection, described in Pissarides (2010). In my model, I offer an alternative explanation for the use of employment protection, which works even when there is no difference in the agents' risk-aversion. Moreover, my model could explain the counter-intuitive finding in Lalé (2019), that in the presence of risk-averse agents and Nash-bargaining, severance payments actually reduce agents' welfare by inducing an increasing wage path over time, that runs counter to having a smooth consumption path. The increasing wage path following the introduction of employment protection is present in my model, but it is a desirable feature for the agents, since it reduces the otherwise excessively high on-the-jobs search intensity.

This work contributes to this literature, explaining both the coexistence of FTCs and OECs and the peculiar distribution of FTCs in the workforce, adding three relevant heterogeneity dimensions: match-specific productivity, worker-specific ability and experience. These heterogeneity dimensions allow me to explain the three facts I mentioned in the in-

[^5]troduction: the spread of FTCs among young workers, the high correlation of FTCs with low wages and the persistence in the type of contract. Regarding the latter, there is in the literature a shared concern regarding FTCs when they become a dead-end for workers, see for example Ichino et al. (2008) and Gagliarducci (2005). However, in these works, they generally found evidence that temporary jobs are stepping stones to reach a permanent position and not a never-ending trap for the worker, meaning that they are still a better option than unemployment. In this work, I investigate these aspects in the evaluation of labour market policies that reduce the share of FTCs, taking into account the possible negative effects on "outsides", meaning unemployed or people employed with FTCs.

My work also contributes to the vast literature focused on the European dual labour market. This strand of literature started right after the liberalization of the temporary labour contracts in Spain, the first country that largely allowed for this kind of contracts. A review can be found in Dolado et al. (2002). This literature has always focused on the aggregate impacts of these labour market reforms on unemployment, productivity and labour market flows. One main finding of these works is that an increase in the share of temporary contracts leads to higher volatility in the labour market. García-Serrano and Jimeno (1999) uses a pooled cross-section data from 17 sectors in 17 Spanish regions to estimate that an increase in the percentage of fixed-term contracts leads to an increase in overall labour mobility. However, FTCs could increase the on-the-job search of workers, counteracting the beneficial effects on the average unemployment duration, with an increase in job-to-job transitions. This mechanism is captured in my model by the presence of endogenous on-thejob search and the same mechanism was present in Boeri (1999). In the latter, the author focused on a specific case of temporary jobs, activated after separation in some European countries, finding that the increased job-to-job transitions of these workers crowd out the job-finding probability of the unemployed. The long-term effect of this increased volatility on the unemployment rate and the employment level is ambiguous, but in the short-term, the simple introduction of fixed-term contracts seems to generate a honey moon effect, as
shown in Boeri and Garibaldi (2007). This work also provides evidence that, in the long-run, the increase in fixed-term contracts could lead to a reduction in productivity. Moreover, the combination of widespread use of fixed-term contracts and high firing costs on permanent contracts leads to an amplification of the unemployment fluctuations along the business cycle, as shown in Bentolila et al. (2012). There is instead some evidence of less training and investment in human capital for workers with temporary contracts (as documented in Booth et al. (2002), Lucidi and Kleinknecht (2009) and Albert et al. (2005)). However, there is also some evidence of a boost in productivity, at least in the short term, deriving from the use of the same temporary contracts through a reduction in the absenteeism rate (Ichino and Riphahn (2005) and Engellandt and Riphahn (2005)). Overall, it seems that FTCs increase productivity in the short-term, but they could potentially lower long-term productivity.

Finally, I briefly summarize the empirical literature on the relationship between contract type and worker and firm characteristics. Portugal and Varejão (2009) shows that human capital is an important determinant for the contractual choice, both on worker's and firm's side. They find that highly-skilled vacancies are more likely to be filled with a permanent contract, given that the hiring process is harder, that the screening process is faster and that firing a skilled-worker, on which the firm has invested, is relatively more expensive. For these reasons, firms that employed highly-skilled workers are more likely to have OEC with them. They also provide evidence of the FTC as a screening tool, given the correlation between the destruction of permanent positions and the creation of new fixed-term vacancies. This is in line with the observed correlation between OEC and high wages. On the workers' side, they provide evidence, consistent with other works, that young, female and low-educated workers are more likely to be employed by fixed-term contracts. In Booth et al. (2002), the authors give a similar picture in describing the characteristics of workers employed through temporary contracts, with a striking difference that more educated men seem to be more likely employed with these kinds of contracts.

Finally, the recent Italian labour market reform has not yet been studied extensively, but
some results can be found in Sestito and Viviano (2016). Using the same administrative source of my work, the authors use a difference-in-difference estimation that gives us the impact of the reduction in firing costs and the tax-incentive for OEC on the number of permanent contracts signed in the first half of 2015. They find that almost half of the total amount of the new contracts can be attributed to the two reforms. Relevant for my work, they found that most of the effects on the share of FTCs signed are due to the tax-incentive, while the cut in the firing costs has a more limited role. This is in line with the results in my counterfactual simulations, where I show that a tax-incentive for OECs is more effective in raising their share. However, this type of policy is at risk of harming low-income workers.

### 1.3 Descriptive Evidence

As I have already anticipated, the sources of information of this section are mainly three: the Italian Labour Survey (Rilevazione sulle Forze di Lavoro), elaborated by the Italian Statistical Agency (ISTAT), a sample of working histories from the social security data collected by INPS and the dataset Mercurio, collecting all working histories of workers in Veneto, from the regional office Veneto Lavoro.

Using these datasets, I provide some evidence about the diffusion of the two types of labour contracts and on-the-job search in Italy. In particular, I will document the three important facts about the FTC distribution that I will use as a validation of the model in the empirical section.

### 1.3.1 FTCs and OECs among workers and firms

I derive a characterization of the labour force employed with the two possible types of labour contracts from the quarterly Labour Survey of ISTAT in $2013 .{ }^{9}$ The population of reference

[^6]

Figure 1.1: FTC and age. Source: ISTAT RCFL, 2013. These scatter plots pictures FTC share among employees in different age classes. The left plot shows the unconditional data, while the right plots the same data after conditioning for sex, income, educational level, foreigner dummy, years of tenure, firm size, sector (2 digits), occupation (3 digits), part-time dummy, geographical area.
of the survey are the legal workers excluding self-employed, which in Italy are around $25 \%$ of the total legal workers.

The detailed description is presented in Appendix B, where I reported the share of FTCs in different sectors, profession and regions. Moreover, I run a probit estimation on all observable, in order to have more robust evidence of the characteristics of the jobs and workers employed with a FTC. Here I summarize the main points for the theoretical and empirical analysis.

FTCs and OECs are both widely spread in the Italian labour market, across all sectors, jobs and workers categories. However, as I already anticipated, there are prominent differences in the share of FTCs in different segments of the labour market.

The percentage of workers employed with a FTC in 2013 was $13.3 \%$, but FTCs are much more diffused among the youngest cohorts, where they reach the majority of the total workers. This is the first fact that I will use as a validation of the empirical model later in the paper.

Also, the number of years in the same firm is highly correlated with the probability of having an OECs. Indeed, among the matches created in 1 year or less, the FTC share raises to $55.6 \%$. On the contrary, the difference between male and female is not high, nor we have a clear relationship with education.


Figure 1.2: FTC and income. Source: ISTAT RCFL, 2013. These scatter plots pictures FTC share over monthly income. Data are binned in 20 dots. The left plot shows the unconditional data, while the right plots the same data after conditioning for age, age squared, sex, educational level, foreigner dummy, years of tenure, firm size, sector (2 digits), occupation (3 digits), part-time dummy, geographical area.

The second fact that I will explain with my model is that FTCs are generally associated with lower wages, even considering the observable characteristics of the workers (Booth et al. (2002)). It is hard to claim that there is a causal link in this relationship since many unobservable characteristics could bias the results, and there are clear problems of selfselection of both workers and job positions. Indeed, in Berton et al. (2015), performing a lab experiment, workers ask for a higher salary in order to accept contracts with a lower expected duration, in particular, they asked for compensation to accept FTCs with a duration of fewer than three years. There are no conclusive studies on the possible "wage premium" earned by workers employed through less costly fixed-term jobs to the best of my knowledge.

This close relation between FTCs and income can be observed in figure 1.2. FTC shares are higher among low-income jobs, even controlling for other characteristics.

In term of sectors, FTCs are most common in agriculture, hotel and restaurants, where seasonal jobs are the standard. In agriculture, FTCs cover more than half of the overall working force. In some sectors, generally skill-intense, they are less common: for instance, they reach just a share of $3.7 \%$ in finance.

If we analyzed the job occupations instead, we could see that the highest percentage of

FTC is present in unskilled jobs, while the lowest in managerial positions.
Controlling for the other observables, it appears that the highest share of OECs is present in the army and in jobs that require technical specialization, while this is not the case for professional jobs and scientific research, where FTC are quite diffuse.

Overall, we can already confirm that both contractual forms are used in Italy. Also, this choice is partially related firm and sector characteristics, but partially also on the job characteristics and requirements, that translate into workers characteristics.

In the survey, they also asked workers if they were satisfied with the FTCs. The percentage of workers that answered positively is extremely low. More than $95 \%$ of workers with a FTC said they accepted it just because they could not find an OEC, not because they prefer it.

### 1.3.2 Duration of the Contract

The distribution of the duration of the FTC is reported in figure 1.3.
The average duration is 12.5 months, but the median is at 9 months. Therefore most of the contracts have a duration of less of one year, but there is a relevant share with a duration exceeding 2 or even 3 years. From 2012 the law set a limit of 3 years for FTCs. However, there were exceptions for staff leasing agencies or if there were specific agreements with the labour unions.

The picture is different if we look at new contracts (figure 1.5), here the share of FTCs and particularly short temporary contracts is much higher. In this case, I rely on the data coming from the dataset "Mercurio" of Veneto Lavoro, that collects all administrative data about hirings and firings in Veneto, a large region in the North-East of Italy. ${ }^{10}$

Differently from France, where Cahuc et al. (2016) pointed out that temporary contracts shorter than 1 month are more of $60 \%$ of all new contracts, FTC contracts in Italy are generally longer than 1 month. Moreover, most of the very short contracts are consecutive

[^7]

Figure 1.3: Duration of FTC. Source: ISTAT, 2013, RCFL. Contracts with duration above 50 months are reported at 50 .


Figure 1.4: Duration of FTC among new contracts (left) and new hirings (right) Source: Veneto Lavoro, 2013, Mercurio. Population: non-seasonal new contracts in Veneto. Contracts with duration above 24 months are reported at 24 .
contracts between the same firm-worker pair. If we consider only new hires (defined as a contract between a new firm-worker pair in 2013), FTCs with a duration shorter then 1 months are less than $20 \%$ of the total.

OECs constitute $24.1 \%$ of new contracts in Veneto in 2013, and they reach $29 \%$ among new hires. This data are consistent with the previous one reporting a high share of FTCs among newly formed matches, and it depicts FTC as a standard practice to start a career in a firm in the Italian labour market.


Figure 1.5: Duration of FTC among new contracts (left) and new hirings (right) Source: Veneto Lavoro, 2013, Mercurio. Population: non-seasonal new labour contracts in Veneto.

### 1.3.3 Persistence of Type of Contract

As shown, FTCs are probably related to worker characteristics and occupations. From this, it should not surprise that workers are somehow persistently employed with the same type of labour contract. This is a fact that has been already found and discussed in the literature ${ }^{11}$, as mentioned in the previous section. However, until now, there is no conclusive evidence about the role of FTCs in shortening or lengthening the process for "outsiders", people in unemployment or outside the labour force, to obtain a standard OEC.

In figure 1.6 I show the share of FCTs among a sample of workers that started a job in January 2005, respectively with a FTC or an OEC. Even after 10 years, the share of FTCs among the one that started with a FTC in 2005 was significantly higher.

### 1.3.4 On the job search

The Labour Survey asks workers if they are searching for another job and why they are doing so. I can use this information to provide some evidence that on-the-job search is related to the choice of the contract.

In 2013 the percentage of workforce searching on-the-job was $4.3 \%$. However, it was $12.1 \%$ among workers with a FTC and only $3.3 \%$ among the others. Among all, other important

[^8]

Figure 1.6: FTCs share among sample of workers starting a job in January 2005, with a FTC (blue) or an OEC (red). INPS data


Figure 1.7: On-the-job searching workers by age (left) and income (right), Italy, 2013. Source: ISTAT
determinants of the probability of searching are age and income. In figure 1.7, I show the percentage of searching workers divided by income and age and the difference between FTC and OEC workers is still large.

In appendix B, I report the predictive margins derived from a probit estimation of the probability of performing on-the-job search on all the observables. If we keep all other observable at their mean levels in the populations, an employee with an OEC has a $6.7 \%$ probability of being searching on-the-job; however, the probability raises to $11.8 \%$ if the employee has a FTC.

These results cannot be taken as conclusive evidence that the type of contracts causes different searching intensity in the worker since other variables are possibly driving both the search intensity and the choice of the contract. However, they at least confirm that in the population workers with a FTC search more while on-the-job.

The difference in search intensity also translates to a difference in the job-to-job transition rates. This observation comes from the dataset "Mercurio" of Veneto Lavoro, that collects all working histories of workers from around 2000 till 2018 in Veneto, a region in the North-East of Italy ${ }^{12}$.

In Appendix B, I plot the working histories of all workers that started a new contract

[^9]in Veneto January 2007 and 2013. The interesting evidence is that transitions from FTC to OEC, within the same firm or not, are relatively diffuse. For some workers, it seems that FTCs are the access door for a stable job.

Instead, using the same population of workers, figures 1.8 and 1.9 report the percentage of workers performing job-to-job transitions every month. I identify a job-to-job transition by two consecutive job-contracts interrupted by an unemployment spell of at most 30 days. The labour force that is interested in this job-to-job transition is around $2 \%$ of the overall labour force every month, and it is higher for FTCs where it reaches the $5 \%$ in the peak 1 year after the beginning of a contract.

It is important to notice that we are considering a selected population of workers since they are starting a new contract in January. It is reasonable to believe that workers decrease the search intensity as the match continues. Therefore the job-to-job transitions are probably lower among the entire workforce. However, from these graphs, it seems that the higher on-the-job search that is correlated with FTC also translate in a higher propensity to perform job-to-job transitions.


Figure 1.8: Monthly J2J transitions over Total Employees, for workers starting in January 2007


Figure 1.9: Monthly J2J transitions over Total Employees, for workers starting in January 2013

### 1.4 Toy Model

In this section, I introduce a variant of the Diamond-Mortensen-Pissarides (DMP) model with on-the-job search to explain the agents' choice about employment protection. I called it the "Toy model" since it will be developed in the next section with the addition of the uncertainty about workers' ability and endogenous firings.

The model is a search and matching model in discrete time. It is characterized by costly searching activity, that can be performed by both employed and unemployed workers, and by heterogeneity in the match-specific productivity $x$, that is drawn from a uniform distribution $\mathcal{U}(\underline{\mathrm{x}}, \bar{x})$ at the beginning of a match. I am assuming that even at $x=\underline{\mathrm{x}}$ the match is productive enough to generate a positive surplus. Therefore, the agents will not be willing to separate.

### 1.4.1 Search and Matching

There is a continuum of firms and workers, normalized to unity. Workers can be either employed or unemployed. Firms employ just one worker and they post costly vacancies that can be filled by unemployed or searching workers.

The number of matches formed every period is determined by a matching function

$$
m(\varsigma, v)=A \varsigma^{\eta} v^{1-\eta}
$$

where $\varsigma$ is a search-adjusted unemployment measure. Agents optimize the search intensity as I described in details later on.

Then, I call $\theta$ the search-adjusted labour market tightness

$$
\theta=\frac{v}{\varsigma}
$$

and $p(\theta)=m(1, \theta)$ will refer to the probability of being matched per unit of search.

From the firm side, the probability of filling a vacancy is $q$ and it is given by:

$$
q(\theta)=(1-\xi) \frac{m(\sigma, v)}{v}=(1-\xi) m\left(\frac{1}{\theta}, 1\right)
$$

where $\xi$ is the endogenous fraction of matches that are discarded by the workers. This happens because some on-the-job searching workers discard the new matches and they continue their current relationship.

### 1.4.2 Timing of Events

As anticipated, every match has a match-specific component $x$ that it is drawn from a uniform distribution $U(\underline{\mathrm{x}}, \bar{x})$ and it is observed when the agents meet.

Agents Nash-bargain over the wage at the beginning of every period, resulting in a split of the match surplus according to their bargaining power. ${ }^{13}$

After that, the worker performs his on-the-job searching activity according to a certain search intensity, that affects the subsequent probability of receiving a new offer. In the next section, I will describe this choice in details.

Then, the production realizes and the agents receive their payoffs.
After, a possible separation shock can materialize with a certain exogenous probability $\lambda$. In this case, the match is broken and the worker returns to unemployment. However, the worker is still kept until the end of the period, so that he has the possibility to perform a job-to-job transition if he receives a new offer.

Indeed, with a certain probability that depends on the worker's previous search intensity, a new offer can materialize at the end of the period. In this case, the worker compares the new offer in hand with the current match and either quit or discard the new offer.

Finally, a new period begins and the agents start from bargaining again over the wage.

[^10]
## Timing



Figure 1.11 summarizes the events described.

## Search intensity decision and Quitting

Searching for a job is a costly activity for workers. Nevertheless, they undertake it even while employed. I have already shown some evidence of on-the-job search in Italy; other papers confirmed that an important share of employed workers performs at least some search activity ${ }^{14}$.

To capture this empirical evidence, workers spend some effort searching for a job both during unemployment and on-the-job. Following the literature, for example, Postel-Vinay and Robin (2004) or Faberman et al. (2017) the cost of the worker is a convex function, such as:

$$
h_{i} s^{\nu}
$$

with $\nu>1$ and $i=e, u$ depending on the employment status of the worker. In this way, I am allowing for a difference in the magnitude of the searching costs for employed and unemployed workers. I interpret $s \geq 0$ as a measure of search intensity.

[^11]Then, $p(\theta)$ will be the probability of finding a job per unit of search intensity $s$.

Importantly, during the Nash-bargaining, the firm and worker cannot bargain over $s$, since the worker keeps the right to manage his search intensity. This natural assumption ${ }^{15}$ has important consequences, as we will see.

For this reason, search intensity $s$ is chosen by the worker, that compares the benefits and costs of searching. I call $s^{*}$ the result of this maximization problem.

The worker's benefits are represented by the possible worker's welfare increase coming from a new offer. In computing these benefits, we need to define the bargaining protocol with the old and the new firm. In order to keep the model simple, I make the questionable assumption that workers receive the same offers of unemployed agents.

This is in line with some literature that assumes that workers bargain over the wage with the new firm in a subsequent period when they do not have anymore the opportunity to return back to the previous firm. ${ }^{16}$

Given this assumption ${ }^{17}$ and using the fact that the agents split the surplus using the Nash-bargaining protocol, the worker decides to perform a job-to-job transition if the surplus of the new match is higher the surplus of the old one, or equivalently if the new $x^{\prime}$ drawn for the new match is higher than the $x$ of the old match or equivalently.

Therefore, we can already compute the quitting probability of the worker $Q(x)$ :

$$
\begin{equation*}
Q(x)=\overbrace{p(\theta) s^{*}(x)}^{\text {new offer prob }} \frac{\bar{x}-x}{\bar{x}-\underline{\mathrm{x}}} \tag{1.1}
\end{equation*}
$$

[^12]
### 1.4.3 Value Functions

We can now analyze the value functions of this Toy model. In all the paper the discount factor is indicated by $\beta$, $\gamma$ refers to the bargaining power of the worker, $J$ indicates firm's value function of a filled job, $V$ is the value of a vacancy, $W$ is worker's value function when employed, while $U$ is the value of unemployment. Finally, $Z$ indicates the joint welfare of a match $(W+J)$, while $S$ indicates the joint surplus. This surplus is split according to the Nash-bargaining rule. The details of the bargaining in different set-ups are described in Appendix D.

## Firms

$$
\begin{gather*}
J(x)=x-w(x)+\beta(1-\lambda)(1-Q(x)) J(x)+\beta(1-\lambda) Q(x) V+\beta \lambda V \\
J(x)=\frac{x-w(x)+\beta(1-\lambda) Q(x) V+\beta \lambda V}{1-\beta(1-\lambda)(1-Q(x))} \tag{1.2}
\end{gather*}
$$

The firm gets the production and it pays the wage, then it gets the discounted continuation value of the filled vacancy if the worker does not separate and he does not quit. Otherwise, it gets the value of the empty vacancy that we now compute.

$$
\begin{gather*}
V=-\kappa+q(\theta) \beta \mathbb{E}(J(x))+\beta(1-q(\theta)) V \\
V=\frac{-\kappa+q(\theta) \beta \mathbb{E}(J(x))}{1-\beta(1-q(\theta))} \tag{1.3}
\end{gather*}
$$

where $\mathbb{E} J(x)$ is the expect continuation value from a new match. It is important to notice that this is not simply the expected value of $J(x)$ over the distribution of $x$, given the presence of workers searching on-the-job and discarding offers. ${ }^{18}$

Using the free-entry condition, I can set $V=0$. In the remaining of the section, I will exploit this in the computations.

[^13]
## Workers

$W(x)=w(x)-\overbrace{h_{e}\left(s^{*}\right)^{\nu}}^{\text {effort cost }}+\beta(1-\lambda) Q(x) \bar{W}^{N}(x)+\beta(1-\lambda)(1-Q(x)) W(x)+\beta \lambda\left(1-p s^{*}\right) U+\beta \lambda p s^{*} \bar{W}^{N}(\underline{\mathrm{x}})$

The worker receives the wage and he pays the cost of the on-the-job search, then he gets the continuation value if he does not separate and he does not quit. If he quits, he receives $W^{N}(x)$ that is the expected value of the new offer given that accepted it. If he separates, he returns to unemployment if he has not received any new offer. Instead, if he has received a new offer, he accepts it for sure. For this reason, I indicated this continuation value as $\bar{W}^{N}(\underline{\mathrm{x}})$, since it is as if the worker is situated in the worst possible match, accepting any other new offer. Similarly, I use this notation for the unemployment value.

$$
\begin{equation*}
U=b-h_{u}\left(s_{U}^{*}\right)^{\nu}+\beta p s_{U}^{*} \bar{W}^{N}(\underline{\mathrm{x}})+\beta\left(1-p s_{U}^{*}\right) U \tag{1.5}
\end{equation*}
$$

The unemployed worker spends costly effort searching for a job, and he gets a flow payment $b$, that represents government unemployment benefits. Then with a probability depending on $s_{U}^{*}$, the search intensity, he gets the expected continuation value of a new job. Otherwise, he remains in unemployment.

### 1.4.4 Optimal Search Intensity

The worker decides the optimal amount of search intensity equating marginal costs and marginal benefits. The marginal costs are increasing in $s$, given the convexity assumption:

$$
M C=\nu h s^{\nu-1}
$$

From the value function of the worker, we can recover the marginal benefits of $s$

$$
\begin{equation*}
M B=\beta p\left[(1-\lambda)\left(\frac{\bar{x}-x}{\bar{x}-\underline{\mathrm{x}}}\right)\left(\bar{W}^{N}(x)-W(x)\right)+\lambda\left(\bar{W}^{N}(\underline{\mathrm{x}})-U\right)\right] \tag{1.6}
\end{equation*}
$$

They are constant with respect to $s$, then it is easy to compute the jointly optimal search intensity $s^{J}$ :

$$
s^{*}=\left(\frac{M B}{\nu h}\right)^{\frac{1}{\nu-1}}
$$

## A benchmark: jointly optimal solution

As we mentioned, the worker keeps the right to manage the search intensity. However, suppose that the agents could bilaterally bargain search intensity at the moment in which they also fix the wage.

The two agents could set the search intensity in order to maximize the joint welfare $Z(x)=W(x)+J(x)$, then they would still use the wage as a transfer to split the surplus according to the Nash rule.

The solution to this problem comes from the equality of marginal costs and joint marginal benefits of searching.

From the value functions, we can recover the joint welfare (using the free entry condition).

$$
\begin{gather*}
Z(x)=x-h\left(s^{*}\right)^{\nu}+\beta(1-\lambda) Q(x) \bar{W}^{N}(x)+\beta(1-\lambda)(1-Q(x)) Z(x)+\beta \lambda\left(1-p s^{*}\right) U+\beta \lambda p s^{*} \bar{W}^{N}(\underline{\mathrm{x}}) \\
Z(x)=\frac{x-h\left(s^{*}\right)^{\nu}+\beta(1-\lambda) Q(x) \bar{W}^{N}(x)+\beta \lambda\left(1-p s^{*}\right) U+\beta \lambda p s^{*} \bar{W}^{N}(\underline{\mathrm{x}})}{1-\beta(1-\lambda)(1-Q(x))} \tag{1.7}
\end{gather*}
$$

Then the joint marginal benefits of searching are the following:

$$
\begin{equation*}
J M B=\beta p\left[(1-\lambda)\left(\frac{\bar{x}-x}{\bar{x}-\underline{\mathrm{x}}}\right)\left(\bar{W}^{N}(x)-W(x)-J(x)\right)+\lambda\left(\bar{W}^{N}(\underline{\mathrm{x}})-U\right)\right] \tag{1.8}
\end{equation*}
$$

It can be noticed that

$$
J M B=M B-\beta p(1-\lambda)\left(\frac{\bar{x}-x}{\bar{x}-\underline{\mathrm{x}}}\right) J(x)
$$

Then, since the jointly optimal search intensity $s^{J}=\left(\frac{J M B}{\nu h}\right)^{\frac{1}{\nu-1}}$, this implies

$$
s^{J} \leq s^{*}
$$

In particular, if $J(x)>0$, then the search intensity of chosen by the worker is higher than the jointly optimal search intensity.

It is important to notice that both agents would benefit (at least weakly) if they could jointly set the search intensity. The repeated Nash-bargaining assumption assures this. ${ }^{19}$ The assumption of the repeated bargain is important also to avoid the problem raised in Shimer (2006) about the non-convexity of the bargaining set in the presence of on-the-job search. Indeed, in my case, the bargaining does not involve future wages, so it does not change the marginal benefits of searching.

### 1.4.5 Optimal Labour Contract in the presence of on-the-job search

The result that, in the presence of on-the-job search, the worker could exert an excess of search effort compared to the jointly optimal one ${ }^{20}$ is not new to the literature. It has been highlighted in Lentz (2014), that analyzes the optimal labour contract in the presence of hidden on-the-job search and Stevens (2004). In both cases, they claimed that the immediate optimal solution of the problem is for the worker to "buy" his job, with an upfront payment that guarantees him all the future surpluses of the match. In the absence of this possibility, the optimal contract is backloaded: the firm should commit to an increasing utility path for the worker.

Even in my case, it is straightforward to see that if $J(x)=0$, the optimal joint search intensity $s^{J}$ coincides with the choice of the worker $s^{*}$. Therefore, if the firm could credi-

[^14]bly commit, the optimal contract would promise all the joint welfare to the worker in the subsequent periods, while using an upfront payment to split the surplus according to the Nash-bargaining rule.

Formally, I call $W^{\prime}(x)$ and $J^{\prime}(x)$ the continuation values, respectively of worker and firm, that the agents set at the beginning of the match when they first bargain, if they could commit not to change them in any subsequent period.

Then, the optimal contract would state

$$
\begin{gathered}
W^{\prime}(x)=x-h\left(s^{*}\right)^{\nu}+\beta(1-\lambda) Q(x) \bar{W}^{N}(x)+\beta(1-\lambda)(1-Q(x)) W^{\prime}(x)+\beta \lambda\left(1-p s^{*}\right) U+\beta \lambda p s^{*} \bar{W}^{N}(\underline{\mathrm{x}}) \\
J^{\prime}(x)=0
\end{gathered}
$$

The worker receives a wage equal to the entire production at every period. Only in the first period, he receives $w(x)^{21}$ such that

$$
(1-\gamma)(W(x)-U)=\gamma(J(x)-V)
$$

where $W(x)$ and $J(x)$ have the same initial expression, but $W^{\prime}(x)$ and $J^{\prime}(x)$ as continuation values in the match.

## The role of Employment Protection

Suppose that the firm is not able to commit not to re-bargain again in the future. ${ }^{22}$ Then, employment protection could be chosen by agents in order to obtain effectively the same wage schedule previously described, but through a standard Nash-bargaining procedure.

In practice, when agents meet for the first time, they set the employment protection in

[^15]order to maximize the surplus of the match and they contemporaneously bargain over the wage.

In this way, they can still use the wage to obtain the usual surplus split. Notice that the outside option of the firm is still the empty vacancy since it has not signed any contract with the worker.

$$
(1-\gamma)(W(x)-U)=\gamma(J(x)-V)
$$

Here I model employment protection as a pure waste $(f c)$ paid by the firm only in the case of a voluntary separation initiated by the firm. ${ }^{23}$ At the end of the section, I will show that the same results carry over if we model the firing costs as severance payment from the firm to the worker.

## Proposition

The optimal level of $f c$ will be the following:

$$
\begin{equation*}
f c(x)=\frac{x-w^{*}(x)}{\gamma[1-\beta(1-\lambda)(1-Q(x))]} \tag{1.9}
\end{equation*}
$$

where $w^{*}(x)$ is the wage that would have split the surplus according to the Nash-Bargaining rule in the absence of the firings costs, but still keeping the search intensity at the optimal level $s^{J}$.

The presence of the firing costs determines a rise in the wage of the worker starting from the second period of the match. This is due to the lower outside option of the firm, that incorporates the firing costs: the Nash-Bargaining from the second period onward split the surplus such that

$$
(1-\gamma)\left(W^{\prime}(x)-U\right)=\gamma\left(J^{\prime}(x)-V+f c\right)
$$

If $f c$ takes the value stated in equation 1.9, then $J^{\prime}(x)=0$ and $\left.W^{\prime}(x)-U\right)=Z(x)$ as in the

[^16]optimal contract with commitment.

## Proof

Notice that absent any firing cost, $w^{*}(x)$ is such that

$$
J(x)-V=(1-\gamma) S(x)=(1-\gamma)(Z(x)-U-V)
$$

recalling the value of $J(x)$ from equation 1.2 , and setting $V=0$ from the free entry condition, we get

$$
\begin{equation*}
\frac{x-w^{*}(x)}{1-\beta(1-\lambda)(1-Q(x))}=(1-\gamma)(Z(x)-U) \tag{1.10}
\end{equation*}
$$

If we introduce the firing costs, keeping constant the search intensity $s^{J}$, then the value $Z(x)=Z\left(x^{\prime}\right)$ does not change, since the firing costs are never actually paid. However, they affect the welfare of the firm and the worker, since the wage must adapt to guarantee the same share $1-\gamma$ of the surplus to the firm.

$$
\begin{gather*}
\frac{x-w(x)}{1-\beta(1-\lambda)(1-Q(x))}-(-f c)=(1-\gamma)(Z(x)-U-(-f c)) \\
\frac{x-w(x)}{1-\beta(1-\lambda)(1-Q(x))}=(1-\gamma)\left(Z^{\prime}(x)-U\right)-\gamma f c \tag{1.11}
\end{gather*}
$$

Subtracting equation 1.12 from equation 1.10, we get

$$
\frac{w(x)-w^{*}(x)}{1-\beta(1-\lambda)(1-Q(x))}=\gamma f c
$$

From this, we can conclude that if to conclude that if $f c$ is set to $\frac{x-w^{*}(x)}{\gamma(1-\beta(1-\lambda)(1-Q(x)))}$, then the wage would increase by $x-w^{*}(x)$ reducing the firm's profits exactly to zero.

We can notice that the optimal $f c$ are increasing in the firm's profit so in the matchspecific productivity $x$. This result is in line with the fact that labour contracts with stronger employment protection are more diffuse among high-paid jobs.

It is important to highlight the fact that in this toy model firing costs are just a threat to the firm, used to ensure credibility to the wage schedule. If endogenous firings were to be added to the picture, then a trade-off would emerge for the choice of employment protection: the gains from the reduction in on-the-job search and the costs in case of voluntary separation. The full-model will address this issue.

## Employment Protection as Severance Payment

If we model employment protection not as pure waste, but a compulsory transfer from the firm to the worker in case of a separation initiated by the firm, then we have to modify the result of the Nash-Bargaining. The reason is that in this case, $f c$ not only decreases the outside option of the firm, but it increases the outside option of the worker.

$$
\begin{equation*}
\frac{x-w(x)}{1-\beta(1-\lambda)(1-Q(x))}-(V-f c)=(1-\gamma)(Z(x)-(U+f c)-(-f c)) \tag{1.12}
\end{equation*}
$$

Rearranging and using the free-entry condition:

$$
\frac{x-w(x)}{1-\beta(1-\lambda)(1-Q(x))}=(1-\gamma)(Z(x)-U)+f c
$$

Therefore we can obtain the optimal labour contract again by setting

$$
f_{c}=\frac{w(x)-w^{*}(x)}{1-\beta(1-\lambda)(1-Q(x))}
$$

The severance payment can be set at a lower level than before, as they affect both the outside option of the agents.

### 1.5 Full Model

The full model has the same features of the Toy model with two additional elements. Firstly, workers have a subjective ability that is unknown, so that they are ex-ante homogeneous, but
they reveal their type over time through a Bayesian updating process. Secondly, the presence of institutional constraints on the type of labour contracts available: agents can choose only between Fixed-Term Contracts and Open-Ended Contracts.

FTCs involve no firing costs, while OECs are associated with a firing cost $f c$ to be paid by the firm in the form of pure waste, only in case of voluntary separation. Moreover, I assume that for legal limitations, a FTC has a probability $\varphi$ to experience a "transformation shock", in which case it has to be transformed into a permanent contract or terminated. ${ }^{24}$ Importantly, once an OEC is signed, it cannot be reverted back to a FTC in subsequent periods by the same firm. This indeed is generally forbidden and rarely observed in data.

### 1.5.1 Search and Matching

There is a continuum of firms and workers, normalized to unity. Workers have an exogenous probability of dying every period called $\lambda_{d}$, and they are immediately replaced by the same amount of newborn unemployed workers. Workers can be either employed (with one of the two kind contracts) or unemployed. Firms employ just one worker and they post costly vacancies that can be filled by searching workers.

As in the Toy model, we have a Cobb-Douglas matching function

$$
m(\varsigma, v)=A \varsigma^{\eta} v^{1-\eta}
$$

where $\varsigma$ is a search-adjusted unemployment measure. Similarly we define $q, \theta$ and $p$ as in section 1.4.1. However, in this case, the match is allowed to have a negative expected surplus. In this case, the match is broken before starting, the worker returns to the unemployment pool and no costs are paid.

[^17]
### 1.5.2 Production, Timing and Bayesian Updating

Every worker $i$ has inner productivity $\alpha_{i}$ that is unobservable and it is drawn from a known continuous and smooth distribution with $\operatorname{cdf} F(\alpha)$. In addition, every match $i, j$ has a matchspecific component $x_{i, j}$ that it is drawn from a known distribution $G(x)$ and it is observed when the agents meet.

The output of the match is the sum of these components and a noisy term. ${ }^{25}$. Therefore, it can be used as a noisy signal of the worker's ability. At every period $t$ the production is :

$$
y_{i, j, t}=\alpha_{i}+x_{i, j}+\varepsilon_{i, j, t}
$$

where $\varepsilon_{i, j, t}$ is white noise, so it is an i.i.d. shock with mean $0 .{ }^{26}$
All the agents ${ }^{27}$ observe the realized production at every period and they perform a Bayesian updating of the distribution of the worker's productivity $\alpha_{i}$ and therefore they update also the expected production of the future. In particular, I call $\phi$ the set of moments about the ability distribution of the worker that is relevant for the agents. $\phi$ will be the moments at the beginning of every period, while $\phi^{\prime}$ will be the updated moments after the production realization.

## Example

Even if the theoretical section does not assume a specific distribution, it is worth describing the Bayesian Updating procedure I will assume in the empirical session.

In that section, I assume that the prior distribution for the worker-specific productivity $\alpha$ is a normal distribution with parameters $\phi_{0}=\left(\mu_{0}, \sigma_{0}^{2}\right)$ and the error term is normally

[^18]
## Timing


distributed as well, with parameters $\left(0, \sigma_{\varepsilon}^{2}\right)$.
At every realization of the production $y_{t}$, the agents will update their prior beliefs about productivity.

Given the choices about the distributions, the posterior distribution of $\alpha$ will be a normal with the following parameters $\phi_{i}^{\prime}$ :

$$
\begin{gathered}
\mu_{i}^{\prime}=\frac{\sigma_{\varepsilon}^{2} \mu_{i}+\sigma_{i}^{2}\left(y_{i, j, t}-x_{i, j}\right)}{\sigma_{\varepsilon}^{2}+\sigma_{i}^{2}} \\
\left(\sigma_{i}^{2}\right)^{\prime}=\frac{\sigma_{\varepsilon}^{2} \sigma_{i}^{2}}{\sigma_{\varepsilon}^{2}+\sigma_{i}^{2}}
\end{gathered}
$$

The parameter $\sigma_{i}^{2}$ converges to zero as the number of observed productions growths to infinity: the agents have a better knowledge of the true productivity over time. Consequently, the last observed production is less and less informative as the match continues, as we can observe from the fact that $y_{i, j, t}$ "weights" less in the updating of $\mu_{i}$ as $\sigma_{i}^{2}$ declines.

## Timing

In figure 1.13 I represent the orders of the events in a period.
As in the Toy model, when a match is formed there is the draw of the productivity
parameter $x_{i, j}$. Then, at the beginning of every period agents sign a new contract and they contemporaneously bargain over the wage, that must be the same for every realized production in that specific period. ${ }^{28}$. The wage is Nash-bargained and it is a function of the contractual choices of the agents, the productivity parameter and the beliefs on the moments of the $\alpha_{i}$ distribution. Both present and past chosen contracts will matter for the wage, as we will see later.

Subsequently, the worker has to choose the optimal amount of search intensity. As in the Toy model, he keeps the right-to-manage this variable.

Then, the production realizes and the agents perform the Bayesian updating described.
After, two possible exogenous shocks can materialize: the first one is a transformation shock, with probability $\varphi$, that forces a transformation of a FTC into a OEC or a separation. The second one is the exogenous separation shock that happens with an exogenous probability $\lambda$. In this case, the match is broken, regardless of the type of contract. In both cases, the worker will still be kept until the end of the period, so that he has the possibility of performing a job-to-job transition.

The process of offer realization and quitting decision is the same as in the Toy model.
After this, the firm itself has the possibility to terminate the match if the surplus of the match is negative. I call this firing of the worker, but it should be noticed that given the repeated bargaining assumption, it is a jointly optimal decision.

In this last case of an endogenous separation, the firm pays a firing cost $f c$ only if she signed an OEC contract. That $f c$ is a pure waste. ${ }^{29}$ This $f c$ can be thought as the legal and bureaucratic costs related to the firing procedure. If the contract is a FTC instead, $f c$ is standardized to zero.

Finally, a new period begins and the two agents start from re-bargaining again over the

[^19]wage and the contract.

### 1.5.3 Value Function

I keep the following notation through all the rest of this paper: for clarity, I am not using the individual or match-specific indexes. Subscripts indicate the previous and the actual contract between agents, for example in $W_{f, p}$ the $f$ refers to a previous FTC, while $p$ indicates a present OEC. The past contract is important as well since it determines the outside option of the firm. New matches do not have a previous contract; however, I will still indicate them as if they had a previous FTC contract. Indeed, since FTC does not involve firing costs, a new match is in this sense equivalent to a re-bargained contract after a period of a fixed-term contract.

For wages, we will have $w_{i, j}(\phi, x)$ as the results of a Nash-Bargaining of a worker with a prior distribution of the productivity determined by $\phi$ and the match productivity draw $x$. The indexes $i, j$ will be $p$ or $f$ following the rules just stated.

It is also convenient to indicate with $\beta$ the discount factor incorporating the risk of dying. Formally:

$$
\beta=\left(1-\lambda_{d}\right) \beta^{*}
$$

where $\beta^{*}$ is the true discount factor and $\left(1-\lambda_{d}\right)$ is the surviving probability.

## Firms

Differently from the Toy model, firms are matched with workers that are heterogeneous in term of expected ability and previous employment condition. The probability of being matched with a worker of one specific type depends on their searching intensity and their distribution in the population.

In particular, conditioned of being matched with someone, the probability of finding an unemployed worker with an ability distribution characterized by $\phi$ is the following:

$$
\begin{equation*}
q_{u}(\phi)=\frac{s_{u}^{*}(\phi) u(\phi)}{\int_{-\infty}^{\infty}\left[s_{u}(\phi) u(\phi)+\int_{-\infty}^{\infty}\left(s_{p}^{*}(\phi, x) e_{p}(\phi, x)+s_{f}^{*}(\phi, x) e_{f}(\phi, x)\right) d G(x)\right] d F(\phi)} \tag{1.13}
\end{equation*}
$$

where I indicate as $u(\phi)$ the measure of unemployed characterized by the moments $\phi$ and similarly, I call $e_{p}(\phi, x)$ and $e_{f}(\phi, x)$ the measure of employed workers in a match with productivity $x$, characterized by moments $\phi$ and with a OEC or a FTC respectively.

In the same way, we can get the probability of being matched with a worker characterized by the moments of the ability distribution $\phi$ and employed in a match with a specific $x$.

The value of the filled vacancy depends crucially on all this information. For this reason, I introduce here the following average filled vacancy values:

$$
\begin{gathered}
J_{u}(\phi)=\int_{-\infty}^{\infty} \max _{i \in\{f, p\}}\left(J_{f, i}(\phi, x) ; V\right) d G(x) \\
J_{f}(\phi, x)=\int_{-\infty}^{\hat{x}_{f}} V d G\left(x^{\prime}\right)+\int_{\hat{x}_{f}}^{\infty} \max _{i \in\{f, p\}}\left(J_{f, i}\left(\phi, x^{\prime}\right)\right) d G\left(x^{\prime}\right) \\
J_{p}(\phi, x)=\int_{-\infty}^{\hat{x}_{p}} V d G\left(x^{\prime}\right)+\int_{\hat{x}_{p}}^{\infty} \max _{i \in\{f, p\}}\left(J_{f, i}\left(\phi, x^{\prime}\right)\right) d G\left(x^{\prime}\right)
\end{gathered}
$$

The first value $J_{u}(\phi)$ is the average value of a match with an unemployed worker with ability $\phi . \quad J_{p}$ and $J_{f}$ are the average values of a match with a permanent or temporary contract respectively, characterized by $\phi$ and $x$. In these cases, $\hat{x}_{i}$ represents the level of the productivity that is necessary to convince the worker to change job, as we will see later in details.

Taking all this into account, the value of an open vacancy is equal to:

$$
\begin{array}{r}
V=-\kappa+\beta^{*}(1-q(\theta)) V+ \\
+\beta^{*} q(\theta) \frac{\int_{-\infty}^{\infty}\left(s_{u} u(\phi) J_{u}(\phi)+\int_{-\infty}^{\infty}\left[s_{f} e_{f}(\phi, x) J_{f}(\phi, x)+s_{p} e_{p}(\phi, x) J_{p}(\phi, x)\right] d G(x)\right) d F(\phi)}{\int_{-\infty}^{\infty}\left[s_{u}(\phi) u(\phi)+\int_{-\infty}^{\infty}\left(s_{p}(\phi, x) e_{p}(\phi, x)+s_{f}(\phi, x) e_{f}(\phi, x)\right) d G(x)\right] d F(\phi)} \tag{1.14}
\end{array}
$$

As usual, using the free entry condition, we can set the value of the open vacancy equal to zero.

$$
V=0
$$

## Permanent contract

$$
\begin{align*}
J_{i, p}(\phi, x) & =\mathbb{E}(y)-w_{i, p}(\phi, x)+\beta \lambda V+\beta^{*} \lambda_{d} V+ \\
& +\beta(1-\lambda) p(\theta) s_{p}^{*}(\phi, x) \int_{-\infty}^{+\infty}\left[\mathbb{1}_{W} \operatorname{Max}\left(J_{p, p}\left(\phi^{\prime}, x\right), V-f c\right)+\left(1-\mathbb{1}_{W}\right) V\right] d F(y) \\
& +\beta(1-\lambda)\left(1-p(\theta) s_{p}^{*}(\phi, x)\right) \int_{-\infty}^{+\infty} \operatorname{Max}\left(J_{p, p}^{t+1}\left(\phi^{\prime}, x\right), V-f c\right) d F(y) \tag{1.15}
\end{align*}
$$

where $i \in\{f, p\}$ indicate the previous contract and $\mathbb{1}_{W}$ indicates worker's choice to continue the relationship.

In addition to the Toy model, we have the probability for the worker to die and terminate the match, but more importantly, the uncertainty about the worker's ability. This feature creates the possibility for the worker to be fired. However, the permanent contract implies firing costs in case of an endogenous separation. Therefore agents separate only if the loss for the firm from continuing the match is higher then the firing costs.

## Temporary contract

$$
\begin{align*}
J_{f, f}(\phi, x) & =\mathbb{E}(y)-w_{f, f}(\phi, x)+\beta \lambda V+\beta^{*} \lambda_{d} V+ \\
& +\beta(1-\lambda)\left[\varphi \left(p(\theta) s_{f}^{*}(\phi, x) \int_{-\infty}^{+\infty}\left[\mathbb{1}_{W} \operatorname{Max}\left(J_{f, p}\left(\phi^{\prime}, x\right), V\right)+\left(1-\mathbb{1}_{W}\right) V\right] d F(y)\right.\right. \\
& \left.+\left(1-p(\theta) s_{f}^{*}(\phi, x)\right) \int_{-\infty}^{+\infty} \operatorname{Max}\left(J_{f, p}\left(\phi^{\prime}, x\right), V\right) d F(y)\right)+ \\
& +(1-\varphi)\left(p(\theta) s_{f}^{*}(\phi, x) \int_{-\infty}^{+\infty}\left[\mathbb{1}_{W} \operatorname{Max}\left(\max _{i \in\{p, f\}} J_{f, i}\left(\phi^{\prime}, x\right), V\right)+\left(1-\mathbb{1}_{W}\right) V\right] d F(y)\right. \\
& \left.\left.+\left(1-p(\theta) s_{f}^{*}(\phi, x)\right) \int_{-\infty}^{+\infty} \operatorname{Max}\left(\max _{i \in\{p, f\}} J_{f, i}\left(\phi^{\prime}, x\right), V\right) d F(y)\right)\right] \tag{1.16}
\end{align*}
$$

The equation is similar to the previous one, but the firms do not have to pay the firing costs anymore in case of termination. Also, with probability $1-\varphi$ it has the option to continue with another period of FTC (last two lines), while with probability $\varphi$ the firm is forced to transform the contract or to terminate. Notice that the choice of the worker also depends on this realization of the "transformation shock", since when considering the new offer, the worker already knows if the next period he will be employed with a FTC or an OEC or if he is going to be fired.

## Workers

Workers decide the search intensity as in the Toy model. Now, it is a function of both job and worker characteristics, so it should be indicate as $s_{i}^{*}(\phi, x)$ and $i \in\{f, p\}$. For notational convenience, I indicate it only as a function of the labour contract.

I refer to $x^{n}$ to indicate the new productivity draw for a new match. Finally, to simplify further the notation, I indicate as $W(\phi, x)=\max _{i \in\{p, f\}} W_{f, i}(\phi, x)$, that is the worker's welfare maximized by the best labour contract.

## Permanent contract

$$
\begin{align*}
W_{i, p}(\phi, x) & =w_{i, p}(\phi, x)-h_{e}\left(s_{p}^{*}\right)^{\nu}+ \\
& +\beta \lambda \int_{-\infty}^{+\infty}\left[\left(1-p(\theta) s_{p}^{*}\right) U\left(\phi^{\prime}\right)+p(\theta) s_{p}^{*} \int_{-\infty}^{+\infty} \operatorname{Max}\left(W\left(\phi^{\prime}, x^{n}\right), U\left(\phi^{\prime}\right) d G(x)\right] d F(y)+\right. \\
& +\beta(1-\lambda) \int_{-\infty}^{+\infty}\left[p(\theta) s_{p}^{*} \int_{-\infty}^{+\infty} \operatorname{Max}\left(W_{p, p}\left(\phi^{\prime}, x\right), W\left(\phi^{\prime}, x^{n}\right), U\left(\phi^{\prime}\right)\right) d G(x)+\right. \\
& +\left(1-p(\theta) s_{p}^{*}\right)\left(\operatorname{Max}\left(W_{p, p}\left(\phi^{\prime}, x\right), U\left(\phi^{\prime}\right)\right)\right] d F(y) \tag{1.17}
\end{align*}
$$

where $i \in\{p, f\} .{ }^{30}$
Again, the only important difference with the Toy model is the possibility of firings. I recall that the Nash-bargaining assures that any endogenous separation is beneficial to both agents. This explains the presence of the maximization in the continuation values: the decision about the endogenous firing can also be modelled as if the worker were comparing the value of continuing the match with the value of unemployment.

## Fixed-term contract

$$
\begin{align*}
W_{f, f}(\phi, x) & =w_{f, f}(\phi, x)-h_{e}\left(s_{f}^{*}\right)^{\nu}+ \\
& +\beta \lambda \int_{-\infty}^{+\infty}\left[\left(1-p(\theta) s_{f}^{*}\right) U\left(\phi^{\prime}, x\right)+p(\theta) s_{f}^{*} \int_{-\infty}^{+\infty} \operatorname{Max}\left(W\left(\phi^{\prime}, x^{n}\right), U\left(\phi^{\prime}\right)\right) d G\left(x^{n}\right)\right] d F(y)+ \\
& +\beta(1-\lambda)(1-\varphi) \int_{-\infty}^{+\infty}\left[p(\theta) s_{f}^{*} \int_{-\infty}^{+\infty}\left[\operatorname{Max}\left(W\left(\phi^{\prime}, x\right), W\left(\phi^{\prime}, x^{n}\right), U\left(\phi^{\prime}\right)\right)\right] d G\left(x^{n}\right)+\right. \\
& +\left(1-p(\theta) s_{f}^{*}\right)\left(\operatorname{Max}\left(W\left(\phi^{\prime}, x\right), U\left(\phi^{\prime}\right)\right)\right] d F(y)+ \\
& +\beta(1-\lambda) \varphi \int_{-\infty}^{+\infty}\left[p(\theta) s_{f}^{*} \int_{-\infty}^{+\infty}\left[\operatorname{Max}\left(W_{f, p}\left(\phi^{\prime}, x\right), W\left(\phi^{\prime}, x^{n}\right), U\left(\phi^{\prime}\right)\right)\right] d G\left(x^{n}\right)+\right. \\
& +\left(1-p(\theta) s_{f}^{*}\right)\left(\operatorname{Max}\left(W_{f, p}\left(\phi^{\prime}, x\right), U\left(\phi^{\prime}\right)\right)\right] d F(y) \tag{1.18}
\end{align*}
$$

FTC that can be renewed or voluntarily transformed into an OEC with probability 1 -

[^20]$\varphi$, otherwise in the continuation value the agents will simply have to decide between the endogenous separation and the permanent contract (last two lines of the value function).

Comparing this with the OEC value function, there are differences in wage, different optimal search intensity, different continuation values due to the absence of firing costs and the option to choose for a FTC in the future, that is possible only in this case.

## Unemployed

The unemployed worker gains the unemployment benefit, pays the searching costs and then he gets the expected discounted continuation value.

$$
\begin{gather*}
U(\phi)=b-h_{u}\left(s_{u}^{*}(\phi)\right)^{\nu}+\beta p(\theta) s_{u}^{*} \int_{-\infty}^{+\infty} \operatorname{Max}\left(W\left(\phi^{\prime}, x^{n}\right), U(\phi)\right) d G(x)+\beta\left(1-p(\theta) s_{u}^{*}\right) U(\phi) \\
U(\phi)=\frac{b-h_{u}\left(s_{u}^{*}(\phi)\right)^{\nu}+\beta p(\theta) s_{u}^{*} \int_{-\infty}^{+\infty} \operatorname{Max}\left(W\left(\phi^{\prime}, x^{n}\right), U(\phi)\right) d G(x)}{1-\beta\left(1-p(\theta) s_{u}^{*}\right)} \tag{1.19}
\end{gather*}
$$

### 1.5.4 Firing Thresholds

The endogenous separation happens when the surplus of the match becomes negative. Given the Nash-bargaining protocol, we can analyze the case as if the firm decides to fire the worker.

This decision has a reservation property: there exists a level of production below which the match is broken and above which the match is continued.

To see this it is sufficient to notice that a separation occurs when $J_{p, p}\left(\phi^{\prime}, x\right)<f c$ or $\left.\max \left(J_{f, f}\left(\phi^{\prime}, x\right), J_{f, p}\left(\phi^{\prime}, x\right)\right)<0\right)$ and the firm's value function is monotonically increasing in the expected production. The expected production depends on the updated ability distribution, and this is monotonically increasing in the realized production. Therefore, given our assumptions about the ability distribution, we can conclude that there exists a level of production that acts as a threshold, below which the match is destroyed.

However, we will have different thresholds for every $\phi, x$ and two different thresholds for each of them, depending on the type of contract.

Given the firing costs, the outside option is lower for a permanent contract compared to a FTC. A firm has to pay $f c$ to terminate an OEC, while the outside option for a FTC is the vacancy $V=0$. For this reason, the threshold will be higher for the FTC.

Therefore, I call $\bar{y}_{p}(\phi, x)$ and $\bar{y}_{f}(\phi, x)$ respectively, the level of production such that:

$$
J_{p, p}\left(\phi^{\prime}, x\right)=-f c
$$

$$
\max _{i \in\{p, f\}} J_{f, i}\left(\phi^{\prime}, x\right)=0
$$

In Appendix E I show that indeed the threshold for the FTC is at least as high as the one for OEC. ${ }^{31}$

I also need a third threshold, $\bar{y}_{f^{\prime}}(\phi, x)$ for the case in which the agents are forced to transform a FTC in an OEC. In this case, the continuation surplus of the match is weakly decreased ${ }^{32}$.

### 1.5.5 Optimal search intensity

The quitting decision of the worker is similar to the Toy model: the worker leaves if his welfare is higher with the new firm rather than with the old one. However, now the welfare depends on the ability of the worker and the type of contract. Given the presence of employment protection, the simple fact that the match-specific draw is higher for the new match does not assure that the worker decides to quit.

To see this, suppose a worker employed with an OEC receives a new offer by a firm with the exact match-specific productivity. However, a FTC in the new match allows increasing the total surplus by an infinitesimal amount. In this case, the worker discards the new offer, since his welfare is higher in the old firm ${ }^{33}$.

[^21]I call $\hat{x}_{i}(\phi, x)$ the quitting thresholds, with $i \in\left\{f, p, f^{\prime}\right\}$ where we will have

$$
\begin{gathered}
W_{p, p}\left(\phi^{\prime}, x\right)=\max _{i \in\{p, f\}} W_{f, i}\left(\phi^{\prime}, \hat{x}_{p}\right) \\
\max _{i \in\{p, f\}} W_{f, i}\left(\phi^{\prime}, x\right)=\max _{i \in\{p, f\}} W_{f, i}\left(\phi^{\prime}, \hat{x}_{f}\right) \\
W_{f, p}\left(\phi^{\prime}, x\right)=\max _{i \in\{p, f\}} W_{f, i}\left(\phi^{\prime}, \hat{x}_{f^{\prime}}\right)
\end{gathered}
$$

As for the firing threshold, I need this third case $f^{\prime}$ to describe the situation in which a transformation shock forces the agents to transform the FTC contract in an OEC.

Given the thresholds, we can compute the probability $Q_{i}\left(\phi^{\prime}, x\right)$ that a worker quits for every possible updated worker's ability distribution and therefore for every observed production. Given the distribution of the match-specific component, it will be:

$$
Q\left(\phi^{\prime}, x\right)_{i}=p(\theta) s_{i}^{*}(\phi, x)\left(1-G\left(\hat{x}_{i}\right)\right)
$$

Even the optimal search intensity $s_{i}^{*}(\phi, x)$ depends on the labour contract, and it is obtained as in the Toy model, by equating the marginal benefits of the worker (MB) to the marginal costs $M C=\nu h_{e} s^{\nu-1}$.

$$
\begin{aligned}
M B_{f} & =\beta p(\theta)\left[\lambda \int _ { - \infty } ^ { \infty } \int _ { - \infty } ^ { \infty } \left(\max \left(\left(W\left(\phi^{\prime}, x\right)-U\left(\phi^{\prime}\right), 0\right)\right) d G(x) d F(y)+\right.\right. \\
& +(1-\lambda)(1-\varphi) \int_{-\infty}^{\infty} \int_{\bar{x}_{f}(\phi, x)}^{\infty}\left(\max \left(W\left(\phi^{\prime}, x^{n}\right)-\max _{i \in\{p, f\}}\left(W_{f, i}\left(\phi^{\prime}, x\right), U\left(\phi^{\prime}\right)\right), 0\right)\right) d G(x) d F(y)+ \\
& \left.+(1-\lambda) \varphi \int_{-\infty}^{\infty} \int_{\bar{x}_{p^{\prime}}(\phi, x)}^{\infty}\left(\max \left(W\left(\phi^{\prime}, x^{n}\right)-\operatorname{Max}\left(W_{f, p}\left(\phi^{\prime}, x\right), U\left(\phi^{\prime}\right)\right), 0\right)\right) d G(x) d F(y)\right]
\end{aligned}
$$

Equalizing it to $M C$, we get the expression for the optimal amount of search intensity.

Similarly for the OEC

$$
\begin{aligned}
M B_{p} & =\beta p(\theta)\left[\lambda \int _ { - \infty } ^ { \infty } \int _ { - \infty } ^ { \infty } \left(\max \left(\left(W\left(\phi^{\prime}, x\right)-U\left(\phi^{\prime}\right), 0\right)\right) d G(x) d F(y)+\right.\right. \\
& \left.+(1-\lambda) \int_{-\infty}^{\infty} \int_{\bar{x}_{p}(\phi, x)}^{\infty}\left(\max \left(W\left(\phi^{\prime}, x^{n}\right)-\operatorname{Max}\left(W_{p, p}\left(\phi^{\prime}, x\right), U\left(\phi^{\prime}\right)\right), 0\right)\right) d G(x) d F(y)\right]
\end{aligned}
$$

The marginal benefits of increasing $s$ are generally higher for the FTC, since in the OEC the employment protection raises the wage of the worker, increasing his continuation value in the match and the probability of staying. ${ }^{34}$

### 1.5.6 Surplus

$$
\begin{align*}
& S_{f, p}(\phi, x)=S_{p, p}(\phi, x)-f c=J_{f, p}(\phi, x)-V+W_{f, p}(\phi, x)-U(\phi)= \\
& =\mathbb{E}(y)-h_{e}\left(s_{p}^{*}\right)^{\nu}+ \\
& +\beta \lambda \int_{-\infty}^{\infty}\left[\left(1-p(\theta) s_{p}^{*}\right)\left(U\left(\phi^{\prime}\right)\right)+p(\theta) s_{p}^{*} \int_{-\infty}^{\infty} \max \left(W\left(\phi^{\prime}, x^{n}\right), U\left(\phi^{\prime}\right)\right) d G(x)\right] d F(y)+ \\
& +\beta(1-\lambda)\left[\int_{-\infty}^{\bar{y}_{p}}\left[-f c\left(1-Q_{p}\right)+\left(1-p(\theta) s_{p}^{*}\right) U\left(\phi^{\prime}\right)+p(\theta) s_{p}^{*} \int_{-\infty}^{\infty} \max \left(W\left(\phi^{\prime}, x^{n}\right), U\left(\phi^{\prime}\right)\right) d G(x)\right] d F(y)+\right. \\
& \left.+\int_{\bar{y}_{p}(\phi, x)}^{\infty}\left[\left(1-p(\theta) s_{p}^{*}\right)\left(W_{p, p}\left(\phi^{\prime}, x\right)+J_{p, p}\left(\phi^{\prime}, x\right)\right)+p(\theta) s_{p}^{*} \int_{\hat{x}_{p}}^{\infty} W\left(\phi^{\prime}, x^{n}\right) d G(x)\right] d F(y)\right]-U(\phi) \tag{1.20}
\end{align*}
$$

Now, we can write the surplus of a match and look at the jointly optimal decision about the labour contract.

The surplus equation is composed of different lines: in the first one we have the productivity minus the searching costs, therefore the immediate component of the surplus. The second line is the continuation value when there is an exogenous separation. The third line is continuation value when the observed production is so low that the worker will be fired. Finally, we have the continuation value when the expected productivity of the worker is high enough to be kept in the match.

[^22]\[

$$
\begin{align*}
& S_{f, f}(\phi, x)=J_{f, f}(\phi, x)-V+W_{f, f}(\phi, x)-U(\phi)= \\
& =\mathbb{E}(y)-h_{e}\left(s_{f}^{*}\right)^{\nu}+ \\
& +\beta \lambda \int_{-\infty}^{\infty}\left[\left(1-p(\theta) s_{f}^{*}\right)\left(U\left(\phi^{\prime}\right)\right)+p(\theta) s_{f}^{*} \int_{-\infty}^{\infty} \max \left(W\left(\phi^{\prime}, x^{n}\right), U\left(\phi^{\prime}\right)\right) d G(x)\right] d F(y)+ \\
& +\beta(1-\lambda)(1-\varphi)\left[\int_{-\infty}^{\bar{y}_{f}}\left[\left(1-p(\theta) s_{f}^{*}\right) U\left(\phi^{\prime}\right)+p(\theta) s_{f}^{*} \int_{-\infty}^{\infty} \max \left(W\left(\phi^{\prime}, x^{n}\right), U\left(\phi^{\prime}\right)\right) d G(x)\right] d F(y)+\right. \\
& \left.+\int_{\bar{y}_{p}(\phi, x)}^{\infty}\left[\left(1-p(\theta) s_{f}^{*}\right) \max _{i \in\{p, f\}}\left(W_{f, i}\left(\phi^{\prime}, x\right)+J_{f, i}\left(\phi^{\prime}, x\right)\right)+p(\theta) s_{f}^{*} \int_{\hat{x}_{f}}^{\infty} W\left(\phi^{\prime}, x^{n}\right) d G(x)\right] d F(y)\right]+ \\
& +\beta(1-\lambda) \varphi\left[\int_{-\infty}^{\bar{y}_{f^{\prime}}}\left[\left(1-p(\theta) s_{f}^{*}\right) U\left(\phi^{\prime}\right)+p(\theta) s_{f}^{*} \int_{-\infty}^{\infty} \max \left(W\left(\phi^{\prime}, x^{n}\right), U\left(\phi^{\prime}\right)\right) d G(x)\right] d F(y)+\right. \\
& \left.+\int_{\bar{y}_{f^{\prime}}(\phi, x)}^{\infty}\left[\left(1-p(\theta) s_{f}^{*}\right)\left(W_{f, p}\left(\phi^{\prime}, x\right)+J_{f, p}\left(\phi^{\prime}, x\right)\right)+p(\theta) s_{f}^{*} \int_{\hat{x}_{f^{\prime}}}^{\infty} W\left(\phi^{\prime}, x^{n}\right) d G(x)\right] d F(y)\right]-U(\phi) \tag{1.21}
\end{align*}
$$
\]

For FTC the expression is similar, but there are no firing costs and we have to separate the two cases in which a transformation shock realizes or not.

### 1.5.7 Optimal Contractual decision

We can finally look at the contractual decision made by the agent at the beginning of a period.

In the Toy model, we have already shown the fact that the agents can increase the joint welfare by reducing the searching intensity of the worker. Employment protection is a way to achieve this reduction in search intensity. This fact is true even in the full model.

However, if we take the difference of the two surpluses $S_{f, p}(\phi, x)-S_{f, f}(\phi, x)$, we notice that employment protection comes at a cost in the full model. The costs are composed by the firing costs in case of an endogenous separation, by the inefficient retention of unproductive matches and by the reduction in the continuation surplus.

$$
\begin{align*}
& S_{f, p}(\phi, x)-S_{f, f}(\phi, x)=\beta(1-\lambda)(F C+I R+C O N T)+  \tag{1.22}\\
& +\Delta E \text { ffort }+\beta \Delta Q u i t
\end{align*}
$$

In details

$$
F C=-f c\left(\int_{-\infty}^{\bar{y}_{p}}\left(1-Q_{p}\left(\phi^{\prime}, x\right)\right) d F(y)\right) \leq 0
$$

FC represents the expected firing costs actually paid by the firm.

$$
I R=\int_{\bar{y}_{p}}^{\bar{y}_{f^{\prime}}}\left(1-Q_{p}\left(\phi^{\prime}, x\right)\right)\left(J_{p, p}\left(\phi^{\prime}, x\right)+W_{p, p}\left(\phi^{\prime}, x\right)-U\left(\phi^{\prime}\right)\right) d F(y) \leq 0
$$

IR are the costs of the unproductive match kept only for the presence of the employment protection.
$C O N T=(1-\varphi) \int_{\bar{y}_{f}}^{\infty}\left[W_{p, p}\left(\phi^{\prime}, x\right)+J_{p, p}\left(\phi^{\prime}, x\right)-\max _{i \in\{f, p\}}\left(J_{p, i}\left(\phi^{\prime}, x\right)+W_{p, i}\left(\phi^{\prime}, x\right)\right)\right] d F(y) \leq 0$

Finally, these are the costs link with the absence of choice in the next period, when a FTC can allow selecting the best labour contract again, while the OEC forces the agents to continue with an OEC.

The red components are the ones that we have already encountered in the Toy model: they are due to the change in search intensity depending on the amount of employment protection.

They are composed of the difference in search costs

$$
\Delta E \text { ffort }=h_{e}\left(s_{f}^{*}(\phi, x)^{\nu}-s_{p}^{*}(\phi, x)^{\nu}\right)
$$

and the difference in the results of the search effort

$$
\begin{aligned}
& \Delta \text { Quit }=p\left(s_{f}^{*}-s_{p}^{*}\right)\left(\lambda\left(U(\phi)-W\left(\phi^{\prime}, x^{n}\right)\right)+\right. \\
& \quad+(1-\lambda) \int_{-\infty}^{\infty} \int_{\hat{x}}^{\infty}\left[\max \left(W_{p, p}\left(\phi^{\prime}, x\right)+J_{p, p}\left(\phi^{\prime}, x\right),-f c\right)-W\left(\phi^{\prime}, x^{n}\right)\right] d G(x) d F(y)
\end{aligned}
$$

However, there is an important difference with respect to the Toy model: here, there is no assurance that this red component has a positive effect on the overall surplus. This is the consequence of the fact that now matches can have a negative surplus, on top of the assumption that the agents do not choose employment protection, but it is set exogenously.

An example of this is the case in which for every realized productivity, the continuation surplus is low enough that the match would be terminated (negative continuation surplus), but it is not low enough to lead the firm's profit below the firing costs.

In this situation, the jointly marginal benefits of search intensity are higher than the worker's marginal benefits, since he is not internalizing the negative welfare of the firm in continuing the match. This leads to an inefficiently low search intensity of the worker. ${ }^{35}$ In these circumstances, these red components are negative.

Nevertheless, if the expected surplus of the match is high enough, the worker will tend to over-search as in the Toy model. ${ }^{36}$ This generates a trade-off between the costs of employment protection and the gains in terms of lower search intensity.

It can be noted that employment protection costs are decreasing in the worker's ability and the match-productivity draw, since the higher the expected match surplus, the lower the possibility of endogenous firings. However, they are also decreasing in the quitting probability, since the higher the chance that the worker quits, the lower the disbursement by the firm.

Similarly, the benefits of an OEC are lower for low-quality matches, since the difference

[^23]between the $M B$ of search intensity and the $J M B$ decreases (eventually becoming negative) as the match surplus decreases.

In the calibration section, I will show how the optimal labour contract varies with $x$ and the expected worker's ability.

### 1.5.8 Steady-State Equilibrium

In order to solve for the steady-state equilibrium, I need to compute all the flows that characterize it. Indeed, we have many different possibilities, for every employment status, $\phi$, $x$ and type of contract.

Also, the matching function will take into account the presence of on-the-job search.
In particular, it is now possible to give a formal description of the searching-adjusted unemployment:

$$
\varsigma=\int_{\phi}\left[s_{u}^{*}(\phi) u(\phi)+\int_{x} \sum_{i=p, f} s_{i}^{*}(\phi, x) e_{i}(\phi, x) d(x)\right] d(\phi)
$$

Searching workers will decrease the effective market tightness.
We can now formally compute the parameter $\xi$ that indicates the shares of new job offers that are discarded by agents.

To do so, we need the amount of discarded offers (Disc):

$$
\begin{gathered}
\text { Disc }=\int_{\phi}\left[s_{u}^{*} u(\phi) G\left(\hat{x}_{u}\right)+\int_{x} s_{p}^{*} e_{p}^{*}(\phi, x)\left(\lambda G\left(\hat{x}_{u}\right)+(1-\lambda) G\left(\hat{x}_{p}\right)\right) d x+\right. \\
\left.+\int_{x} s_{f}^{*} e_{f}^{*}(\phi, x)\left(\varphi\left(\lambda G\left(\hat{x}_{u}\right)+(1-\lambda) G\left(\hat{x}_{f^{\prime}}\right)\right)+(1-\varphi)\left(\lambda G\left(\hat{x}_{u}\right)+(1-\lambda) G\left(\hat{x}_{f}\right)\right)\right) d x\right] d \phi
\end{gathered}
$$

The share of searching units performed by workers that will not accept the new offers will be:

$$
\xi=\frac{D i s c}{\varsigma}
$$

## Inflows and Outflows

The inflow of workers into unemployment every period is given by both endogenous and exogenous separations:

$$
\begin{aligned}
& \operatorname{In}_{u}(\phi)=\left(1-\lambda_{d}\right) \int_{y}\left(\int _ { \phi } \mathbb { 1 } _ { \phi ^ { \prime } = \phi } \left[\int _ { x } \left[\lambda+(1-\lambda) \mathbb{1}_{y<y_{p}}\left(1-p(\theta) s_{p}^{*}\left(1-G\left(\hat{x_{p}}\right)\right)\right] e_{p}(\phi, x) d x+\right.\right.\right. \\
& \quad+(1-\varphi) \int_{x}\left[\lambda+(1-\lambda) \mathbb{1}_{y<\bar{y}_{f}}\left(1-p(\theta) s_{f}^{*}\left(1-G\left(\hat{x_{f}}\right)\right)\right] e_{f}(\phi, x) d x+\right. \\
& \left.\quad+\varphi \int_{\phi} \int_{x}\left[\lambda+(1-\lambda) \mathbb{1}_{y<\bar{y}_{f^{\prime}}}\left(1-p(\theta) s_{f}^{*}\left(1-G\left(\hat{x_{f^{\prime}}}\right)\right)\right] e_{f}(\phi, x) d x\right] d \phi\right) d F(y)
\end{aligned}
$$

The outflows from unemployment are instead given by the match with unemployed:

$$
\operatorname{Out}_{u}(\phi)=\left(1-\lambda_{d}\right) p(\theta) s_{u}^{*}(\phi) u(\phi)\left(1-G\left(\hat{x}_{u}\right)\right)
$$

Then we have inflows to employment, coming by either unemployed and employed workers.

$$
I n_{e_{i}}(\phi, x)=\text { Out }+J t J=\left(1-\lambda_{d}\right) p(\theta) \int_{\phi} s_{u}^{*}(\phi) u(\phi) f(x) \mathbb{1}_{x \geq \hat{x}_{u}} d \phi+J t J
$$

where Job-to-Job transitions:

$$
\begin{aligned}
J t J\left(\phi, x^{n}\right) & =\left(1-\lambda_{d}\right) p(\theta) \int_{y} \int_{\phi} \mathbb{1}_{\phi^{\prime}=\phi}\left(\int_{-\infty}^{+\infty} s_{p}^{*} e_{p}(\phi, x)\left[\lambda f\left(x^{n}\right) \mathbb{1}_{x^{n}>\hat{x}_{u}}+(1-\lambda) \mathbb{1}_{x^{n}>\hat{x}_{p}} d x\right]+\right. \\
& +(1-\varphi) \int_{-\infty}^{+\infty} s_{f}^{*} e_{f}(\phi, x)\left[\lambda f\left(x^{n}\right) \mathbb{1}_{x^{n}>\hat{x}_{u}}+(1-\lambda) \mathbb{1}_{x^{n}>\hat{x}_{f}} d x\right]+ \\
& \left.+\varphi \int_{-\infty}^{+\infty} s_{f}^{*} e_{f}(\phi, x)\left[\lambda f\left(x^{n}\right) \mathbb{1}_{x^{n}>\hat{x}_{u}}+(1-\lambda) \mathbb{1}_{x^{n}>\hat{x}_{f^{\prime}}} d x\right]\right) d \phi d F(y)
\end{aligned}
$$

Finally, at every period, a fraction $\lambda_{d}$ of the population is created. In the calibration section, I describe which prior distribution $\phi$ they are endowed with. I indicate them as $n(\phi)$ and they have an immediate probability $p(\theta) s_{u}(\phi)$ to be employed. Otherwise, they enter in unemployment.

The equilibrium conditions that close the model are then:

$$
I n_{u}(\phi)=O u t_{u}(\phi)+\int_{\phi}\left(1-p(\theta) s_{u}^{*}(\phi)\right) n(\phi) d \phi
$$

For every $\phi, x$ and $t$

$$
\operatorname{In}_{e_{i}(\phi, x)}=e_{i}(\phi, x)
$$

This last condition comes from the fact that at every period all employed workers change condition, either by separating or by updating $\phi$.

## Equilibrium conditions

I summarize the equilibrium conditions of a steady-state of the model:

1. Matching function

$$
m(\varsigma, v)=A \varsigma^{\eta} v^{1-\eta}
$$

whit the appropriate definition of search-adjusted unemployment;
2. From the free entry condition $V=0$, we get

$$
\frac{\kappa}{\beta q(\theta)}=\bar{J}
$$

where $q(\theta)=(1-\xi) m\left(\frac{1}{\theta}, 1\right)$ and $\bar{J}$ is the average profit of the firm, as described by equation 1.15.
3. Flows equilibrium

$$
I n_{u}(\phi)=O u t_{u}(\phi)
$$

For every $\phi, x$ and $t$

$$
I n_{e_{i}(\phi, x)}=e_{i}(\phi, x)
$$

4. Firms and Workers maximize their value functions;
5. Wages are Nash-bargained
6. Search-effort is chosen such that the FOC conditions hold.

### 1.6 Calibration

In this section, I provide a calibration of the model using the dataset I already described: the Labour Force Surveys by ISTAT, the microdata from INPS collecting a sample of Italian working histories from 2000 until 2015 and the dataset "Mercurio" from Veneto Lavoro, collecting all the working histories from the region Veneto. ${ }^{37}$

Some parameters are taken from the literature or externally calibrated, while others are estimated using a Monte Carlo Markov Chain estimation.

### 1.6.1 Externally Calibrated Parameters

In my model, one period is equivalent to 6 months.

## Discount Factor

I set the discount factor $\beta^{*}=0.976$, that is equivalent to an annual discount rate of $5 \%$.

## Matching Function and Bargaining Power

I assume that the unemployment coefficient in the matching function is the same estimated in Shimer (2005a), $\eta=0.72$. This is a standard value in the search and matching literature. Then, I set the matching efficiency parameter $\varsigma=0.5$. It is known that in a standard search and matching model, the vacancy cost and the matching efficiency parameter are only jointly identified, if there is no information on the measure of vacancies, but only on the unemployment level. Therefore, I set the matching efficiency arbitrarily, using the observed unemployment levels to calibrate the correct vacancy cost associated. I set the bargaining

[^24]power of the worker equal to the elasticity of the matching function to unemployment, $\gamma=$ 0.72 , that is the standard Hosios condition for optimality. ${ }^{38}$

## Unemployment Benefit

The OECD collects the net replacement ratio of the unemployment benefits for different categories of workers across the developed countries. In my work, I use the replacement rate that was in place in Italy in 2013, taking as a benchmark a single worker without children, into unemployment for 6 months and with a previous wage equal to the average wage. Therefore I set the unemployment benefit to $59 \%$ of the average wage.

## Transitory Shock

Since 2012, in Italy the FTC cannot last more than 3 years, summing up all subsequent FTC between the same employee-employer couple. However, there were several exceptions to this law, for instance, for employment agencies or whenever there were special agreements between the firm and the trade unions.

To capture these institutional constraints, I set the transformation shock probability $\varphi=0.16$, meaning that on average, a FTC experience a transformation shock after about 6 periods (three years).

## Death Probability

I set the probability of workers to drop out from the model equal to the probability from which the workers dropped out from the INPS dataset from 2000 till 2010 and that did not return back in the dataset. This happened with probability $\lambda_{d}=3 \%$. ${ }^{39}$

[^25]This probability included all possible motivations for which an employee could exit the labour force, including the decision to become self-employed or inactive.

I assume that when workers are born, they have an initial draw of the production observed by everyone, based on their true ability and setting $x=0$. In this way, they enter the labour force with a slightly heterogeneous prior distribution.

### 1.6.2 Estimated Parameters

The parameters that remain to estimate using a set of the moments are the following: the average ability of the worker $\mu_{\alpha}$, the variance in the ability of worker $\sigma_{\alpha}^{2}$, the variance in the match-specific component $\sigma_{x}^{2}$, the variance in the white noise in the production $\sigma_{\varepsilon}^{2}$, the parameters for searching costs $\nu, h_{e}$ and $h_{u}$, the firing costs $f c$, the vacancy cost $\kappa$, the exogenous separation rate $\lambda$.

For the estimation, the MCMC procedure starts from an initial guess of the parameters.
For every iteration, I draw a new set of parameters from a normal distribution centred on the parameters last accepted in the iteration process. Then, I estimate a loss function taking the square differences of some realized moments with respect to the targets from the data. The ratio of the value of this loss function to the one of the previous step determines the probability to accept or reject the new draw of parameters.

The process continues until converging to a distribution of parameters.
In this procedure, all parameters are jointly estimated; however, the chosen moments are thought to target specific parameters in the set. Here I describe all the chosen targets.

## Maximum wages percentiles

From the INPS dataset, I considered the period from 2005 until 2015. I considered a worker actually employed in a firm in the 6 months window if he worked at least 13 weeks in the period. As a preliminary step, I regressed the income on the following observables: sex, geographical area, profession (6 categories), time fixed effect (semester of the year). I
considered the constant plus the residuals of this regression as the wage of the worker for the period.

I performed this preliminary step in order to eliminate some heterogeneity in the data that are strongly influencing the distribution of wages in the sample, but that I am not capturing in the model. The underlying assumption is that the model can be applied within each category.

Then, for every worker, I compute the maximum wage earned in the dataset and I build the distribution of the maximum wages. As moments, I am using the following percentiles: $20 \%, 40 \%, 50 \%, 60 \%, 80 \%$. Using the maximum wage avoids one issue of the important role of working experience in the ability of the worker, that is absent in the model. ${ }^{40}$

## Standard Deviation in wages for the same worker

From the same data, cleaned as described previously, I take as moments the standard deviation in the wages between firms for the same worker. In practice, I estimate the average wage for the same worker-firm pair. Then I compute the standard deviation of this measure among the life-time of the worker. This measure should provide important information for the estimation of the variance in match-specific productivity.

## Separation Rates

The same dataset is useful also to compute the separation rates. I called a separation only a labour contract that is followed by a contract in a firm different from the previous one unless the time between the two contracts is shorter than one month (then it is a Job-toJob transition). I consider a separation even a contract followed more than 1 year later by another contract with the same worker-firm pair. I compute the separation rates for the two types of labour contracts.

[^26]
## Job-to-Job transition rates

I have already shown some data about the job-to-job transition rates from the Veneto Lavoro dataset. Here, I compute the job-to-job transition rates inside from the INPS dataset, in order to keep consistency with the other targeted moments. I define a job-to-job transition a new hire that happens less than 1 month after the termination of the previous job. I computed the rates for both types of labour contracts.

## Job Finding Rate for Unemployed

I compute the job-finding rate for unemployed in 2013 starting from a subsample of the workers in the INPS dataset that satisfied the following conditions: (i) individuals that worked at some point between 2000 and 2012, (ii) they were not employed in the second semester of 2012 (iii) they worked at a certain point from 2013 to 2016. ${ }^{41}$. Then, I compute the share of these workers that found a job in the first semester of 2013 and I compute the job-finding probability of the first semester. I repeated the same measurement for the second semester, with the corrected conditions (i), (ii) and (iii).

## FTC share

I compute the FTC in the INPS dataset in order to have a useful moment for the determination of the firing costs.

## FTC transformation rate

From the INPS dataset, I compute the share of FTC that becomes OEC in the next period between the same firm-worker pair.

[^27]| Name | Parameter | Median | 90 confidence interval |
| :---: | :---: | :---: | :---: |
| Vacancy Cost | $\kappa$ | 0.0558 | $0.0365-0.1004$ |
| Firing Costs | $f_{c}$ | 1.189 | $0.5962-1.759$ |
| Average Workers Productivity | $\mu_{\alpha}$ | 1.893 | $1.607-2.179$ |
| Workers Productivity SD | $\sigma_{\alpha}$ | 1.499 | $1.017-2.005$ |
| Match Productivity SD | $\sigma_{x}$ | 0.6862 | $0.5009-0.8780$ |
| Production Shock SD | $\sigma_{\varepsilon}$ | 2.742 | $1.9506-4.109$ |
| Searching cost Employed | $h_{e}$ | 4.931 | $4.513-8.4606$ |
| Searching cost Unemployed | $h_{u}$ | 0.2646 | $0.0838-0.6177$ |
| Searching cost Convexity | $\nu$ | 2.659 | $2.219-3.130$ |
| Exogenous Separation Probability | $\lambda$ | 0.0221 | $0.0171-0.0288$ |

Table 1.1: Estimated Parameters. Median indicates the median values taken by the parameters in the simulations of the MCMC estimation. 90 confidence interval gives the interval in which lies 90 percent of the MC simulations.

## Unemployment rate

I take the unemployment rate from the ISTAT dataset in 2013.
In the following tables, I reported the estimated parameters and the ability of the model to target the chosen moments. I chose the parameters that minimize the loss function and I report their 90 -per cent confidence interval.

| Moments | Targets | Model |
| :---: | :---: | :---: |
| Unemployment rate | $8.65 \%$ | $13.5 \%$ |
| FTC share | $12.9 \%$ | $12.2 \%$ |
| Separation Rate FTC | $9.1 \%$ | $8.27 \%$ |
| Separation Rate OEC | $2.8 \%$ | $2.0 \%$ |
| Transformation Rate (FTC-OEC) | $10.5 \%$ | $14.4 \%$ |
| Job-to-Job Rate FTC | $3.0 \%$ | $3.4 \%$ |
| Job-to-Job Rate OEC | $1.0 \%$ | $0.81 \%$ |
| Max Wage 20th percentile | 1.954 | 2.423 |
| Max Wage 40th percentile | 2.889 | 3.032 |
| Max Wage 50th percentile | 3.04 | 3.338 |
| Max Wage 60th percentile | 3.613 | 3.649 |
| Max Wage 80th percentile | 4.652 | 4.413 |
| SD Wage across Firms | 0.5463 | 0.3605 |
| Job finding Rate of Unemployed | 0.265 | 0.268 |
| Mat |  |  |

Table 1.2: Targeted Moments and Model Simulated Moments. Parameters set at the median level in the MCMC estimation


Figure 1.14: Contractual decision by match-specific productivity draw and expected worker ability. In yellow the agents choose an OEC, in blue a FTC. Other colors are mixed areas due to interpolations between grid points.

### 1.6.3 Results

First of all, I describe the optimal contract for new workers just entering the labour market. In figure 1.15, we can see the choice of the contract for workers with no experience, varying their expected productivity and match-specific draw. Along the horizontal axes, I vary the match-specific productivity, while the expected ability is on the vertical axes. It can be noted that low-ability workers and the low-productivity match draws are the ones receiving the FTC, while the opposite is true for high-ability and high-match draws. Different colours are present because the grid-points are not enough to find the exact discontinuity, where the optimal contract switch from FTC to OEC. To roughly determines what kind of contract is present in a determined region, I rely on linear interpolation.

The results are in line with the observed fact that OEC is correlated with high wages, while FTCs dominate all low-wages regions.

Similarly, I plot the preferred contract for a fixed expected worker ability $(\alpha=3.158)$ and for a fixed match-specific productivity $(x=0.28)$, varying the experience and the matchspecific productivity and worker ability respectively.


Figure 1.15: Contractual decision by experience and match-specific productivity (left) or expected worker ability (right). In yellow the agents choose an OEC, in blue a FTC. Other colors are mixed areas due to interpolations between grid points. On the left the matchspecific productivity is fixed at 0.28 , while on the right the expected worker ability is fixed at 3.158.


Figure 1.16: Share of FTC employees among workers with different working experience. Data from INPS Dataset, 2013.


Figure 1.17: Share of FTC among Wage Quintiles

In figure 1.16, instead, I report the share of FTC among workers with different working experience. My model can reproduce the observed increase in the share of OEC among more experienced workers.

If we look at the share of FTC per income quintile (figure 1.17), we see that the share is decreasing in the income, with an exception: the rise in the share in the second quintile. The raise is explained by the large use of FTC when the worker enters the labour market for the first time. Since his ability at the peak of uncertainty, his expected productivity is close to the average of the entire population. This raises the share of FTCs in that section of the wage distribution, given that in my model there is no direct impact of training or learning-by-doing and therefore new-born workers are in expectations as productive as the rest of workers. In the real world, it is reasonable to assume that young workers start with lower productivity that would translate in lower initial wages.

Until now, we have just looked at the cross-section description, but it is interesting to look at some aspects of the working histories of agents. In figure 1.18, we have the average unemployment duration for workers with 5 years of working experience, according to their expected ability, following them for 10 years after an unemployment episode.

It can be noted that there is a substantial rise in the unemployment duration of low-


Figure 1.18: Unemployment Duration for workers with 5 years of Working Experience. Average Expected Productivity on the left and 20th Percentile on the right


Figure 1.19: Share of FTC over time among newly employed workers. Circles are simulated working histories, while diamonds are real data.
ability workers. Moreover, in figure 1.19, I show how the model is able to reproduce the third descriptive fact that I previously documented in figure 1.6, that described the persistence in the type of contract that we observe in the real data.

### 1.7 Counterfactual scenarios

In this section, I run some simulation of possible policy interventions that resemble the ones implemented or discussed by governments over the last decades.

### 1.7.1 Cut in the firing costs

The first policy change is a cut in the firing costs. A reform that has been generally invoked by international organizations to increase the labour market flexibility, labour mobility and to reduce the duality between FTCs and OECs ${ }^{42}$. This policy was enacted in Spain in 2012 during the sovereign debt crisis when the government imposes a cap on the severance payments. Similar policies were passed in Italy in 2014 (the so-called Jobs Act) and more recently in France in 2017. However, in these two cases, the governments mostly tried to reduce the uncertainty associated with firing costs, limiting the role of judges in the decision about unfair dismissals.

I assume that the government implements a large cut of $25 \%$ of the firing costs. Then, I let the model reach the new steady-state, and I compare it with the previous steady-state obtained from the calibration. I keep all the other parameters of the model fixed.

### 1.7.2 Lump-sum Tax on FTC

The second policy I analyze is a lump-sum tax on all fixed-term contracts, rebated to employed workers. In this way, it could be seen equivalently as a lump-sum subside to openended contracts. Similar policies have been enacted by the Italian government in 2014 and

[^28]| Macro Indicator | Initial Steady State | Cut in Firing costs | Change | Tax on FTCs | Change |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Unemployment rate | $13.5 \%$ | $13.5 \%$ | $-0.0 \%$ | $13.8 \%$ | $+0.3 \%$ |
| Output | 2.746 | 2.737 | $-0.35 \%$ | 2.730 | $-0.6 \%$ |
| Share of FTCs | $12.2 \%$ | $11.8 \%$ | $-0.4 \%$ | $5.1 \%$ | $-7.1 \%$ |
| Job-find of Unempl. | 0.268 | 0.267 | $-0.1 \%$ | 0.261 | $-0.7 \%$ |
| Average Productivity | 3.175 | 3.165 | $-0.3 \%$ | 3.167 | $-0.2 \%$ |
| Average Wage | 3.135 | 3.123 | $-0.4 \%$ | 3.101 | $-1.1 \%$ |
| Average Welfare | 64.86 | 64.57 | $-0.5 \%$ | 64.74 | $-0.2 \%$ |
| $\%$ of unprod. matches | $17.9 \%$ | $12.2 \%$ | $-5.7 \%$ | $24.9 \%$ | $+7.0 \%$ |

Table 1.3: Change in main indicators at the steady-state, after a $25 \%$ cut in firing costs or a FTC-only lump-sum tax equal to $1 \%$ of the average wage. Unproductive matches are defined as matches that are kept only for the presence of firing costs, but that would be terminated otherwise.
subsequently in 2018. In the first case, the government introduces a three-year social security exemption for all newly signed OECs (including transformations from existing FTCs), in 2018 the government increases the social security contribution at every subsequent FTC between the same firm-worker couple. In my exercise, I am not able to reproduce exactly these policies, since they would require to insert the tenure in the model. However, I am able to simulate the effect of a permanent increase in taxation on one type of contract.

I assume that at every period, the government charges all FTC of a lump-sum tax equivalent to $1 \%$ of the average wage. The collected sum is then rebated back in equal shares to all workers.

### 1.7.3 Counterfactual results and discussion

In the following table, I report the change in the most important economic and labour market indicators following the two policy interventions.

Both the two policy changes lead to a decrease in the share of FTCs since they reduce the upper productivity thresholds above which an OEC is preferred. However, while the tax on FTC has considerable effects in shifting the choice of agents towards the OEC, the cut in the firing cost has a much weaker effect. This is due to the change in the bottom productivity thresholds for firms, that are now accepting both workers with lower ability and lower match-specific draws. These low-productive workers are generally employed with FTCs, therefore while the cut in the firing costs lower the threshold for which an OEC is optimal, it contemporaneously allows for more job-creation of low-quality matches with FTCs. The opposite is true with a FTC-tax. This tax raises the lower threshold for the job-creation since firms have to sustain the tax for FTCs. The region where a FTC is optimal is therefore squeezed from both sides, leading to a considerable reduction in the share of FTCs.

It can be noted that both policies have negative effects on production and average welfare, but similar aggregate results hide strongly different effects on different types of workers. In Appendix F, I collect the figures describing the change in the welfare of different groups of workers. Here, I described the most important effects of these policies.

The firing costs reduction reduces the wages of all workers with an OEC, by raising the firm outside option, so the reform reduces the welfare of all workers already protected by the OEC. Moreover, even considering the symmetric gains of firms, the policy induces a net loss in the surplus for matches that seek employment protection to reduce the excessive worker on-the-job searching intensity. ${ }^{43}$ These considerations particularly harms high-ability workers and low-ability workers in a highly productive match with OECs, as highlighted in figure 1.28. After the reform, there is a reduction in the share of inefficient matches, that are matches with a negative surplus, kept only because of the firing costs. Before the reform, they were around $15 \%$ of all matches, while this share is reduced to $10 \%$ after the reform. However, this cleansing effect is not enough to compensate the reduction in the average productivity, due

[^29]to a lower upfront selectivity of firms. The endogeneity in the contractual choice plays an important role in reducing the magnitude of this cleansing effect. Indeed, the fact that OECs are used for high-quality matches, both in terms of worker-specific ability and match-specific productivity, reduces both the share and the negative burden of these unproductive matches, compared to a scenario in which the contract was exogenously assigned. The policy change has mixed effects on unemployed and workers with FTCs (figures 1.26 and 1.27). In these cases, workers with medium-low ability benefit from firms being less selective and having a higher chance of obtaining a longer employment spell. High-productive workers instead pay the cost of a lower surplus, without particular gains. Another remark is that the policy does not considerably increase the job-finding rate of unemployed. This is a consequence of the crowding-out effect of employed workers, that increase the on-the-job search intensity. Before the reform, unemployed constituted $65 \%$ of the aggregate search-intensity, then this share decreases to $60 \%$, compensated by employed workers.

The effects of FTC-tax are different. They predominantly hurt low-ability workers, by raising the minimum productivity draw to be accepted with a FTC, leading to longer unemployment spells. Instead, the high ability workers benefit from the tax-rebate and from the higher probability of receiving a permanent contract, even in a context of higher unemployment rate and lower average productivity. Firms use fewer FTCs and tend to transform them earlier, reducing the role of FTCs as a screening device for workers. As a consequence, the share of inefficient matches increases considerably from $15 \%$ to $20 \%$ of the total, leading to a reduction of the average productivity, even in a context in which firms are more selective at the recruitment stage. It is useful to compare the obtained welfare changes depicted in figure 1.29, 1.30 and 1.31, with the same welfare changes in the case we were to assume that the agents could not choose the type of contract and the only possibility to obtain the OEC were through the exogenous "transformation" shock. This case is illustrated in figures from 1.32 to 1.37 . It is apparent that the welfare implications of the policies are largely modified by the absence of an endogenous choice. The intuition is the following: the endogenous choice of
the contract creates a persistence in the type of contract a worker receives, since it is linked to his expected ability. For this reason, a policy that reduces that for example reduces the firing costs harm predominantly workers that are expecting to be employed with this type of contracts, that are high-ability workers. Even more clear is the fact that a FTC-tax will be paid exclusively by workers that are expecting to experience some periods of temporary contracts. The correlation between the contract type and the worker ability determines the winner and loser in welfare terms of these policies.

### 1.8 Further Research and Conclusion

In conclusion, in this work, I developed a theoretical model that is able to explain the choice about the juridical form of a labour contract and in particular that could justify the choice in favour of employment protection. A temporary contract has the advantage that it saves the firing costs in case the overall match-productivity is too low and does not force the firm to keep unproductive matches. However, in the presence of on-the-job search, there is an incentive for the agents to choose for some employment protection. This is due to an excess of on-the-job search performed by the worker from a joint firm-worker perspective since, in his optimal effort choice, he does not take into account the welfare of the firm. The employment protection can credibly shift some utility of the worker from the present to the future and this reduces the searching incentive, increasing the joint surplus. Descriptive evidence confirms the fact that workers with a FTC indeed are more likely searching on-the-job and they perform more job-to-job transitions.

Adding in the model the fact that the ability of workers is discovered over time, the calibrated model reproduces the observation that FTCs are used mainly by young workers, that they correlate with low-wages and that they are persistent, meaning that workers with a FTC tend to be employed with a similar contract in the future.

This reveals the importance of taking into account the endogenous nature of the choice
of the contract, that leads to differences in the impact of labour market reforms on different workers. Popular policies to limit the spread of fixed-term contracts, such as a cut in firing costs and a tax on FTCs, can have a different impact among workers. In particular, a reduction in the firing costs hit workers with an OECs and high-ability workers, that experience a reduction in wages and they react by increasing their on-the-job search, crowding-out unemployed workers. Low-ability workers among unemployed and temporary workers can benefit from such a policy. On the contrary, a tax on FTC hurts, particularly low-ability workers, while it increases the welfare of high-ability wor-kers in good matches. Overall, a tax on FTC has a considerable negative impact on the average productivity, by forcing firms to limit to the role of FTCs as a screening device for unproductive matches.

In 2015 the government of Italy decided to modify the firing rules for new OECs, introducing a fixed severance payment to be paid from the firm to the worker, proportional to worker's number of years in the firm and wage. For future research, it would be possible to compare the realized effect of this reform with the predictions of the model, evaluating the welfare effects on different categories of workers.

## Appendix A. Datasets

### 1.8.1 INPS Dataset

This dataset contains the annual records of a sample (based on workers) of Italian labour contracts from 1985 until 2016. It is based on the mandatory declarations of employers for social contributions. The dataset has a total of more than 35 millions entries. However, an institutional framework for FTCs was introduced in Italy only in 1997. For this reason, in this paper, I generally use only information from 2000 onward.

On top of the year, the worker and firm identifiers, the dataset give the following information about the labour contracts: type of contact (OEC, FTC, seasonal), annual wage, profession (6 categories), Time (full or part-time), number of days paid, starting and ending date (if in the year), the reason for separation. Concerning workers, the dataset gives information about gender, age, region of residence. About firms, the dataset provides sector (100 sectors) and dimensional class (14 classes).

## Mercurio from Veneto Lavoro

This dataset of working histories is collected by the regional agency in Veneto, one region of the North-East of Italy.

The Institution collects all the mandatory communications of working relationships in Veneto. The panel can reconstruct all the working histories of the inhabitants after 2000, even if they migrate outside the region, as long as workers are in Italy and they declare their change of living place.

The dataset gives access to many characteristics of the labour contracts, of workers and firms: age, sex, education, living place, occupation, firm sector. Importantly, it provides information about the starting date, type, duration of the hiring contracts, transformation and also job-destruction exact date and motivations.

However, the dataset has two main drawbacks: it does not provide the wage of the contract
and it can miss the share of employed workers that started their job before the dataset was created and they never quit or had a change in their labour contract. For this reason, for this paper, I relied mostly on data from the previously described INPS dataset, using this rich dataset only when other sources are missing.

In terms of numbers, the dataset covers the working outcomes of more than 3 million workers, 8 hundred employer and more than 15 millions of job relationships. The dataset starts before 2000 and is updated until 2018. From this large dataset, I extracted a random sub-sample based on the day of birth of the worker.

## Rilevazione Continua delle Forze di Lavoro

This dataset is the quarterly Labour Force Survey organized by the Italian Statistical Office. The survey uses a questionnaire standardized at European level. Annually, approximately 250 thousand households are interviewed, selected as representative of the Italian population. Every household is interviewed 4 times in 15 months: in two consecutive quarters and then for the other 2 quarters after a break of 3 months.

I use this dataset in this paper for the descriptive evidence section and the moments targeted in the calibration. I use the 2013 data in order to avoid the important labour market reforms that the Italian government started in 2014. However, I performed some robustness checks using the years from 2010 to 2014, and the results do not change significantly.

Restricting the data to the year 2013, I have 611255 observations across 4 quarters, of which 410750 in the working-age 15-65. As shown in figure 1.20 , among people 15-65 years old, workers are $49.7 \%$ per cent. Among these, $24.9 \%$ are self-employed. Therefore most of the data used in this paper come from the remaining 153417 employed workers.


Figure 1.20: Source: ISTAT RCFL, 2013. Employment Condition among 15-65 years old.

## Appendix B. Descriptive Statistics

The following tables give some descriptive statistics about FTC diffusion in different sectors, professions and Italian areas. Then I perform a probit estimation of having a FTC contract on all possible observables.

|  | Managerial and lawmaking | Professional and scientific activities |  | Highly Technical activity | Administrative |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Agriculture | 0\% | 5.0\% |  | 11.3\% |  | 9.3\% |
| Industry | 0.8\% | 5.7\% |  | 5.5\% |  | 7.2\% |
| Constructions | 2.8\% | 8.6\% |  | 8.9\% |  | 8.5\% |
| Commerce | 6.7\% | 8.0\% |  | 5.6\% |  | 8.8\% |
| Hotels \& Restaurants | 12.8\% | 23.8\% |  | 18.9\% |  | 34.6\% |
| Stock \& Transports | 3.2\% | 4.1\% |  | 6.7\% |  | 5.1\% |
| Inform. \& Commun. | 3.3\% | 8.3\% |  | 10.28\% |  | 9.8\% |
| Finance \& Insurance | $0 \%$ | 1.2\% |  | 3.3\% |  | 5.3\% |
| Real est., firm services | 2.7\% | 16.2\% |  | 13.3\% |  | 15.0\% |
| Public Administration | 13.9\% | 4.8\% |  | 4.1\% |  | 6.9\% |
| Education \& Health | 4.3\% | 15.3\% |  | 7.0\% |  | 8.8\% |
| Other services | 10.9\% | 12.8\% |  | 20.7\% |  | 14.1\% |
| OVERALL | 5.7\% | 12.8\% |  | 7.3\% |  | 9.2\% |
|  | Skilled activity in services | Skilled activity in industries | Transport | $\begin{array}{l\|c} \text { t } & \text { Unskilled } \\ \text { activity } \end{array}$ | Army | OVERALL |
| Agriculture | 34.0\% | 41.0\% | 46.9\% | 71.2\% |  | 58.6\% |
| Industry | 7.9\% | 10.3\% | 9.6\% | 14.4\% |  | 8.9\% |
| Constructions | 27.3\% | 16.2\% | 10.3\% | 22.2\% |  | 14.9\% |
| Commerce | 16.1\% | 13.8\% | 15.9\% | 19.4\% |  | 13.5\% |
| Hotels \& Restaurants | 32.6\% | 25.3\% | 25.0\% | 35.8\% |  | 32.4\% |
| Stock \& Transports | 18.8\% | 10.4\% | 10.1\% | 17.2\% |  | 9.0\% |
| Inform. \& Commun. | 17.7\% | 4.3\% | 7.1\% | 15.8\% |  | 9.4\% |
| Finance \& Insurance | 12.5\% | 0\% | 0\% | 0\% |  | 3.7\% |
| Real est., firm services | 19.2\% | 24.2\% | 23.3\% | 15.3\% |  | 15.6\% |
| Public Administration | 3.6\% | 13.9\% | 13.8\% | 19.7\% | 5.1\% | 6.2\% |
| Education \& Health | 13.2\% | 12.6\% | 13.7\% | 11.5\% |  | 12.0\% |
| Other services | 17.2\% | 19.8\% | 22.6\% | 7.1\% |  | 13.2\% |
| OVERALL | 18.7\% | 14.4\% | 11.0\% | 21.4\% | 5.1\% | 13.3\% |

Table 1.4: Percentage of FTCs over the total in different sectors and occupations in Italy, 2013

|  | North-West | North-East | Centre | South | Islands |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Agriculture | $34.5 \%$ | $47.0 \%$ | $48.9 \%$ | $70.4 \%$ | $64.2 \%$ |
| Industry | $7.2 \%$ | $9.5 \%$ | $10.8 \%$ | $9.4 \%$ | $10.6 \%$ |
| Constructions | $13.9 \%$ | $12.4 \%$ | $12.8 \%$ | $18.6 \%$ | $19.6 \%$ |
| Commerce | $11.7 \%$ | $14.9 \%$ | $14.2 \%$ | $14.2 \%$ | $12.3 \%$ |
| Hotels \& Restaurants | $27.4 \%$ | $38.7 \%$ | $29.4 \%$ | $33.9 \%$ | $31.6 \%$ |
| Stock \& Transports | $8.0 \%$ | $10.0 \%$ | $7.5 \%$ | $10.7 \%$ | $9.4 \%$ |
| Inform. \& Commun. | $9.5 \%$ | $9.5 \%$ | $10.2 \%$ | $7.7 \%$ | $8.2 \%$ |
| Finance \& Insurance | $3.1 \%$ | $5.1 \%$ | $3.7 \%$ | $2.9 \%$ | $2.9 \%$ |
| Real est., firm services | $13.7 \%$ | $16.5 \%$ | $15.0 \%$ | $18.9 \%$ | $16.2 \%$ |
| Public Administration | $3.7 \%$ | $4.8 \%$ | $2.8 \%$ | $6.5 \%$ | $14.6 \%$ |
| Education \& Healthcare | $11.2 \%$ | $13.5 \%$ | $12.2 \%$ | $11.5 \%$ | $12.0 \%$ |
| Other services | $9.9 \%$ | $15.1 \%$ | $11.4 \%$ | $18.6 \%$ | $16.1 \%$ |
| OVERALL | $10.5 \%$ | $13.6 \%$ | $12.6 \%$ | $16.3 \%$ | $17.0 \%$ |

Table 1.5: Caption

Margins of Probit Estimation: probability of having a FTC
Predictive margins of probit Estimation: probability of performing on-the-job search

| Variables | FTC | Variable | FTC |
| :---: | :---: | :---: | :---: |
|  |  | 1st income decile | 0.251*** |
| Age 15-19 | $\begin{gathered} 0.525^{* * *} \\ (0.0189) \end{gathered}$ | 2nd income decile | (0.00428) |
|  |  |  | 0.204*** |
|  |  |  | (0.00347) |
| Age 20-24 | $0.397^{* * *}$ | 3rdincome decile | 0.175*** |
|  | (0.00609) |  | (0.00276) |
| Age 25-29 | 0.253*** | 4th income decile | $0.145^{* * *}$ |
|  | (0.00402) |  | (0.00287) |
| Age 30-34 | $0.157^{* * *}$ | 5th income decile | $0.127^{* * *}$ |
|  | (0.00277) |  | (0.00246) |
| Age 35-39 | 0.123*** | 6th income decile | 0.105*** |
|  | (0.00214) |  | (0.00245) |
| Age 40-44 | $0.105^{* * *}$ | 7th income decile | 0.0853*** |
|  | (0.00186) |  | (0.00225) |
| Age 45-49 | 0.0926*** | 8th income decile | 0.0676*** |
|  | (0.00172) |  | (0.00204) |
| Age 50-54 | $0.0787^{* * *}$ | 9th income decile | 0.0545*** |
|  | (0.00172) |  | (0.00198) |
| Age 55-59 | 0.0619*** | 10th income decile | 0.0431*** |
|  | (0.00175) |  | (0.00213) |
| Age 60-64 | 0.0639*** | $<10$ employees | $0.127^{* * *}$ |
|  | (0.00284) |  | (0.00140) |
| Age 65-69 | $0.0827 * * *$ | 10-15 employees | 0.137*** |
|  | (0.00820) |  | (0.00231) |
| Male | $0.128^{* * *}$ | 16-19 employees | $0.143^{* * *}$ |
|  | (0.00122) |  | (0.00327) |
| Female | $0.134^{* * *}$ | 20-49 employees | 0.141*** |
|  | (0.00135) |  | (0.00198) |
| Italian | 0.134*** | 50-249 employees | 0.130*** |
|  | (0.000872) |  | (0.00190) |
| Foreigner-European | 0.127*** | 250+ employees | 0.116*** |
|  | (0.00380) |  | (0.00258) |
| Foreigner-Non European | 0.104*** | North-West | 0.121*** |
|  | (0.00251) |  | (0.00143) |
| Primary School or lower | 0.152*** | North-East | 0.145*** |
|  | (0.00412) |  | (0.00165) |
| Middle School | 0.122*** | Centre | $0.127^{* * *}$ |
|  | (0.00132) |  | (0.00175) |
| High-School | 0.128*** | South | 0.129*** |
|  | (0.00125) |  | (0.00179) |
| Degree or more | $0.164^{* * *}$ | Islands | $0.136^{* * *}$ |
|  | (0.00323) |  | (0.00251) |
| Full time | 0.129*** | Sectors (2 digits) dummies | YES |
|  | (0.000990) | Occupations (3 digits) dummies | YES |
| Part time | 0.135*** | Observations | 146,475 |
|  | (0.00193) | Standard errors in parentheses *** $\mathrm{p}<0.01,{ }^{* *} \mathrm{p}<0.05,{ }^{*} \mathrm{p}<0.1$ |  |
|  |  |  |  |

Table 1.6: Source: ISTAT RCFL, 2013. Probit estimation of having a FTC on observables.

| Variables | On-the-Job search |
| :---: | :---: |
| OEC | $\begin{gathered} 0.06696^{* * *} \\ (0.00476) \end{gathered}$ |
| FTC | $\begin{gathered} 0.11828 \text { *** } \\ (0.006893) \end{gathered}$ |
| Male | $\begin{gathered} 0.09212 \text { *** } \\ (0.00575) \end{gathered}$ |
| Female | $\begin{gathered} 0.06434 * * * \\ (0.00470) \end{gathered}$ |
| Full time | $\begin{aligned} & 0.0710 \text { *** } \\ & (0.00502) \end{aligned}$ |
| Part time | $\begin{gathered} 0.10317 \text { *** } \\ (0.00609) \end{gathered}$ |
| Educational level dummies | Yes |
| Firm class size | Yes |
| Italian dummy | Yes |
| Sector dummies (2 digits) | Yes |
| Occupation dummies (3 digits) | Yes |
| Observations | 143,328 |
| Standard errors in parentheses *** $\mathrm{p}<0.01,{ }^{* *} \mathrm{p}<0.05,{ }^{*} \mathrm{p}<0.1$ |  |

Table 1.7: Source: ISTAT RCFL, 2013. Probit estimation, predictive margins of performing on-the-job search. Age and income at their means.


Figure 1.21: Workers histories, January 2007. Source: Veneto Lavoro

## Veneto Lavoro: "Mercurio"

The figures represent workers' job histories of a random sample of employees (based on the date of birth) hired in Veneto in January 2007 (Figure 1.21) and 2013 (Figure 1.22). I excluded all workers that were employed for less than 120 days in the time-span considered (1209 days). In addition, I excluded workers that spent more than half of their working days in the following sectors: agriculture, tourism and domestic. Indeed, workers in these sectors are subject to special regulations and they are mostly influenced by seasonality.

The colours indicate different labour contacts and different employers. In particular the dark blue and red are the workers that keep a job in the same firm respectively with an OEC and a FTC. Therefore the graph indicate that in 2007 around $65 \%$ of the hired workers sign a FTC, while around $35 \%$ sign an OEC. Half of this $35 \%$ will leave the firm in the following 1200 days, but the percentage of "leavers" is much higher for FTC.

The purple area is composed of workers that had their FTC transformed into a OEC in the same firm. They are a considerable but not extremely high percentage of the labour


Figure 1.22: Workers histories, January 2013. Source: Veneto Lavoro
force.
The lighter blue and red are the workers that move to another firm (intermediate colour) or a third or more (the lightest colour). An important share of workers performs these transitions, keeping the same type of contract from one firm to another.

The yellow area instead represents workers moving from an FTC to an OEC but in a second (intermediate colour) or third or more (the lightest colour) firm. It is interesting to notice that an important share of workers can arrive at an OEC from an initial FTC.

Green areas instead indicate the opposite case: workers that leave an OEC for a FTC in another firm. The percentage of workers in this situation is quite small, especially in 2013.

Finally, the grey area is workers that are not employed any more. I cannot distinguish between unemployed, self-employed or out of the labour force. The lighter area indicates workers whose last contract was an FTC, the opposite for the darker one. It is not surprising that there is a large share of workers that use the FTC just for some occasional jobs.

The comparison between the two years can reveal two different situations in the labour


Figure 1.23: Job positions histories, January 2007. Source: Veneto Lavoro
market. In 2007 Veneto was in a situation of full employment, entering the Great Recession, while in 2013 Italy and Veneto hit the bottom of the second recession and a slow recovery started from 2014. In addition, in 2015 fiscal incentive for OEC and the labour market reform took place.

Instead, figures 1.23 and 1.24 refer to job positions histories of a sample of firms, opened in January 2007 and 2013. I defined a job position as a sequence of job-contracts in the same firm for the same job-title, signed with the same or ever different workers. Job-titles are divided into 3-digits categories, allowing for quite precise identification.

The sequence of contracts could also partially overlap or with a short break, depending on the duration of the contracts itself. I choose the following "interval", that is the number of days between the end of the previous contract and the start of the new one:

$$
-63<\text { interval }<\max \{15, \min \{\text { duration }, 360\}\}
$$

Where the duration is the length of the previous contract.
The graphs are similar to the previous ones: lighter colours indicate a change in the


Figure 1.24: Job positions histories, January 2013. Source: Veneto Lavoro
worker. Red areas indicate FTCs, while the blue and yellow are OECs. Black areas are vacancy waiting to be filled, while grey areas are job-positions destroyed and never re-filled. The graphs show that a large number of job-positions (almost 50\%) are expiring within the 1200 days interval. In addition, it can be noticed that a considerable share of job-positions is filled by workers at first "tested" with a FTC. Moreover, job-positions covered with FTC are not usually filled by a sequence of different workers, but they are most likely be covered by an OEC or be destroyed.

Finally, in figure 1.25, I report the fraction of the labour force employed with a FTC in the main European countries in 2017. It is apparent that FTCs are now widely used in all continental Europe and if we restrict the attention to people under30, we can see that in Italy and Spain (but also in others like Portugal and Poland) ${ }^{44}$ more than $40 \%$ of the young labour force is employed with this kind of contracts.

[^30]

Figure 1.25: Share of labor force using FTC

## APPENDIX C

## Toy model with Sequential Auction

In this section, I rewrite the Toy model assuming that the wage is determined not by Nashbargaining, but by a sequential auction framework as in Postel-Vinay and Robin (2002).

I summarize here the main assumptions of this protocol. Firms have all the bargaining power, and they decide the wage in order to maximize their profit under the participation constraint of the worker. For the moment, I assume wages are fixed at the beginning for the entire duration of the match unless workers receive new offers. In this case, workers use the new offer to make the firms competing à la Bertrand: the new firm make a take-it-or-leave-it offer to the worker, and the old firm can counter the offer-

Formally, I call $W(w, x)$ the value function of a worker in a match of productivity $x$ whose wage is $w$, similarly for the value function $J(w, x)$.

Suppose the worker is in a match with productivity $x$ and he obtains a new offer from a firm whose productivity is $x^{\prime}$. I call $\delta\left(x, x^{\prime}\right)$ the minimum wage for which the worker would
switch from a match with productivity $x$ to a firm whose productivity is $x^{\prime}$.

$$
W\left(\delta\left(x, x^{\prime}\right), x^{\prime}\right)=W(x, x)
$$

Indeed, the old firm can at most offer a wage equal to the match-productivity and the welfare associated with it is the maximum the worker can obtain remaining in the old match.

When a worker obtains a new offer, three different situations can emerge: the workers discard the new offer, the worker remains in the old firm, but he uses the new offer to raise his wage or the worker quits and join the new firm.

The first situation happens if $x>x^{\prime}$ and $W(w, x)>W\left(x^{\prime}, x^{\prime}\right)$ so that even the best offer the new firm can make is no match for the current worker's welfare. The second situation realizes if $x$ is still larger than $x^{\prime}$, but $W(w, x)<W\left(x^{\prime}, x^{\prime}\right)$. In this case, the old firm raises the wage of the worker in order to at least match the best outside offer and keep the worker. Finally, if $x^{\prime}>x$, then the worker quits and the new firm offers exactly the wage to at least match the best offer of the old firm $\delta\left(x, x^{\prime}\right)^{45}$

## Value Functions

## Firms

$$
\begin{gathered}
J(w, x)=x-w+\beta(1-\lambda)\left(1-p s^{*}\left(\frac{\bar{x}-\hat{x}}{\bar{x}-\underline{\mathrm{x}}}\right)\right) J(w, x)+\beta(1-\lambda) p s^{*}\left(\frac{x-\hat{x}}{\bar{x}-\underline{\mathrm{x}}}\right) \hat{J}(x)+ \\
+\beta(1-\lambda) Q(x) V+\beta \lambda V
\end{gathered}
$$

I called $\hat{x}$ the minimum level of productivity for which the firm has at least to raise the wage of the worker to match the outside offer and I called $\hat{J}$ the average continuation value for the firm in the interval from $\hat{x}$ till $x$, at which point the worker quits.

$$
\hat{J}(x)=\frac{\int_{\hat{x}}^{x} J\left(\delta\left(x, x^{\prime}\right), x\right) d x^{\prime}}{x-\hat{x}}
$$

[^31]The quitting probability is unchanged with respect to the main model

$$
Q(x)=p s^{*}\left(\frac{\bar{x}-x}{\bar{x}-\underline{\mathrm{x}}}\right)
$$

Nevertheless, the firm has a generally lower probability of remaining with the same welfare in the subsequent period, because of the possible renegotiation.

It is also important to notice that $s^{*}$ this time is a function of both $x$ and $w$.
Using the free-entry condition, I can set directly $V=0$.

## Workers

$$
\begin{aligned}
W(w, x)=w & -h\left(s^{*}\right)^{\nu}+\beta(1-\lambda)\left(1-p s^{*}\left(\frac{\bar{x}-\hat{x}}{\bar{x}-\underline{\mathrm{x}}}\right)\right) W(w, x)+\beta(1-\lambda) Q(x) \bar{W}^{N}(x)+ \\
& +\beta(1-\lambda) p s^{*}\left(\frac{x-\hat{x}}{\bar{x}-\underline{\mathrm{x}}}\right) \hat{W}(x)+\beta \lambda\left(1-p s^{*}\right) U+\beta \lambda p s^{*} \bar{W}^{N}(U)
\end{aligned}
$$

As before, the probability that the workers keeps the same welfare in the subsequent period is reduced by the probability of renegotiation, that happens if the new offer is between $\hat{x}$ and $x$. I called $\hat{W}$ the average value of this continuation value if the new offer is in that range.

$$
\hat{W}(x)=\frac{\int_{\hat{x}}^{x} W\left(\delta\left(x, x^{\prime}\right), x\right) d x^{\prime}}{x-\hat{x}}
$$

Then, I called $\bar{W}^{N}(U)$ the average welfare of a new job coming from unemployment.

## Optimal search intensity

As usual, I assume the right to manage of the worker regarding the search intensity. Then the marginal benefit of search are:

$$
\begin{gathered}
M B=\beta p\left[(1-\lambda)\left(\frac{\bar{x}-x}{\bar{x}-\underline{\mathrm{x}}}\right)\left(\bar{W}^{N}(x)-W(w, x)\right)+\lambda\left(\bar{W}^{N}(U)-U\right)+\right. \\
\left.+(1-\lambda)\left(\frac{x-\hat{x}}{\bar{x}-\underline{\mathrm{x}}}\right)(\hat{W}(x)-W(w, x))\right]
\end{gathered}
$$

Again, the search intensity is obtained by equating $M B$ with the marginal costs

$$
s^{*}=\left(\frac{M B}{\nu h}\right)^{\frac{1}{\nu-1}}
$$

In comparison with equation 1.6, the marginal benefits have an additional last term in the second line: the worker has an extra-incentive to search in order to increase his continuation value with the incumbent firm. This suggests another fact: the firm can actively influence the optimal searching effort by choosing the wage. For this reason, the optimal wage offered by the firm can potentially be higher than the reservation wage of the worker, as I show later.

## Optimal joint search intensity

As in the main model, we compute the joint welfare to obtain the jointly optimal searching effort. However, it is important to notice that while in the main model the Nash-bargaining assured that moving from $s^{*}$ to the optimal joint search intensity is beneficial for both agents, this is not the case in this framework, since the firms can seize the entire surplus. Therefore, moving from $s^{*}$ to $s^{J}$ the worker is at most indifferent.

$$
J M B=M B-\beta(1-\lambda) p\left[\left(\frac{\bar{x}-x}{\bar{x}-\underline{\mathrm{x}}}\right) J(w, x)+\left(\frac{x-\hat{x}}{\bar{x}-\underline{\mathrm{x}}}\right)(J(x)-\hat{J}(w, x))\right]
$$

The difference between the joint marginal benefits and the marginal benefits of the worker is now larger because now higher search intensity leads to a higher probability of renegotiation, that is a loss for the firm not internalized by the worker.

From the equation of JMB and the marginal cost we obtain again that

$$
s^{J} \leq s^{*}
$$

Therefore, even in this scenario, the worker performs an excess of on-the-job search compared to what would maximize the joint surplus of the match.

## Optimal contract discussion

The optimal contract in this environment is well analyzed in the paper Lentz (2014). What is important in my case is that the perfect solution would be to allow the worker to "buy" his job: his wage would coincide with the production for all the duration of the match in exchange of a payment upfront that would leave him at the reservation utility.

If we restrict the model not to allow negative wages, but we allow the possibility to commit to future wages, the solution exists, it is unique, and it consists of an increasing wage path.

If instead, we force the firm to commit to a flat wage, then we can notice that the firm could potentially decide to raise the wage above the reservation wage, even if this would give a part of the surplus to the worker.

To see this, notice that the maximization problem of the firm would be

$$
\begin{gathered}
\max _{w} J(w, x) \text { s.t } \\
\quad w \geq \delta\left(x, x^{\prime}\right) \\
s^{*}=\left(\frac{M B}{\nu h}\right)^{\frac{1}{\nu-1}}
\end{gathered}
$$

where $x^{\prime}$ could be the unemployment benefit if the worker is not employed.
The FOC of $J(w, x)$ with respect to $w$ is a complicated object because it depends on both the derivative of the searching intensity and the change in the probability and expected value of a renegotiation. Indeed, a higher $w$ reduced the search intensity raises the value of $\hat{x}$ and it, therefore, it changes also $\hat{J}(x)$.

I call $\hat{J}^{\prime}$ the following derivative

$$
\hat{J}^{\prime}(w, x)=\frac{\left(\partial \frac{x-\hat{x}}{\bar{x}-\underline{\underline{x}}} \hat{J}(x)\right)}{\partial w}<0
$$

This derivative is smaller than zero because a raise in $w$ determines a higher value of $\hat{x}$ and a reduction in the profit flows.

$$
\begin{aligned}
& F O C= \frac{-\left[1-\beta(1-\lambda)\left(1-p s^{*}\left(\frac{\bar{x}-\hat{x}}{\bar{x}-\underline{\mathbf{x}}}\right)\right)\right]-(x-w) \beta(1-\lambda) p \overbrace{\left[\left(\frac{\bar{x}-\hat{x}}{\bar{x}-\underline{\mathrm{x}}}\right) \frac{\partial s^{*}}{\partial w}-\frac{s^{*}}{\bar{x}-\underline{\mathrm{x}}} \frac{\partial \hat{x}}{\partial w}\right]}^{<0}}{\left(1-\beta(1-\lambda)\left(1-p s^{*}\left(\frac{\bar{x}-\hat{x}}{\bar{x}-\underline{\underline{x}}}\right)\right)\right)^{2}}+ \\
&+\beta(1-\lambda) p(\frac{\overbrace{\left[\frac{\partial s^{*}}{\partial w}\left(\frac{x-\hat{x}}{\bar{x}-\underline{\mathrm{x}}}\right) \hat{J}(x)+s^{*} \frac{\partial \hat{J}(x)}{\partial w}\right]}^{<0}\left(1-\beta(1-\lambda)\left(1-p s^{*}\left(\frac{\bar{x}-\hat{x}}{\bar{x}-\underline{\mathrm{x}}}\right)\right)\right)}{\left(1-\beta(1-\lambda)\left(1-p s^{*}\left(\frac{\bar{x}-\hat{x}}{\bar{x}-\underline{\mathbf{x}}}\right)\right)\right)^{2}}- \\
& \frac{\left(s^{*}\left(\frac{x-\hat{x}}{\bar{x}-\underline{\mathbf{x}}}\right) \hat{J}(x)\right) \beta(1-\lambda) p\left[\left(\frac{\bar{x}-\hat{x}}{\bar{x}-\underline{\mathbf{x}}}\right) \frac{\partial s^{*}}{\partial w}-\frac{s^{*}}{\bar{x}-\underline{\mathrm{x}}} \frac{\partial \hat{x}}{\partial w}\right]}{\left.\left(1-\beta(1-\lambda)\left(1-p s^{*}\left(\frac{\bar{x}-\hat{x}}{\bar{x}-\underline{\mathbf{x}}}\right)\right)\right)^{2}\right)}
\end{aligned}
$$

Overall, the FOC has an ambiguous sign, since by raising the wage the firm loses part of the profit flows, but it increases the expected duration of them, by diminishing the search intensity and the probability of a future re-bargaining.

We can see this more clearly rewriting the numerator as

$$
\begin{aligned}
& F O C_{n u m}=-\left(x-w+\beta(1-\lambda) p s^{*}\left(\frac{x-\hat{x}}{\bar{x}-\underline{\mathrm{x}}}\right) \hat{J}(x)\right) \overbrace{\left[\left(\frac{\bar{x}-\hat{x}}{\bar{x}-\underline{\mathrm{x}}}\right) \frac{\partial s^{*}}{\partial w}-\frac{s^{*}}{\bar{x}-\underline{\mathrm{x}}} \frac{\partial \hat{x}}{\partial w}\right]}^{<0}+ \\
& +\overbrace{\left(\beta(1-\lambda) p\left[\frac{\partial s^{*}}{\partial w}\left(\frac{x-\hat{x}}{\bar{x}-\underline{\mathrm{x}}}\right) \hat{J}(x)+s^{*} \frac{\partial \hat{J}(x)}{\partial w}\right]-1\right)}^{<0}\left[1-\beta(1-\lambda)\left(1-p s^{*}\left(\frac{\bar{x}-\hat{x}}{\bar{x}-\underline{\mathrm{x}}}\right)\right)\right]
\end{aligned}
$$

The first line represents the gains from a higher wage, due to the higher duration, while the second line is the costs. The FOC does not have to hold with equality, since there is a
reservation wage for the worker. ${ }^{46}$ If at the reservation wage the FOC is negative, then the optimal solution is for the firm to pay the reservation wage and not to try to reduce the search intensity. However, if the FOC holds with equality, the firm pays a higher wage than the reservation wage in an attempt to reduce $s^{*}$.

## Optimal contract

Finally, relying on the paper of Lentz (2014), we can show that the previous contract with a fixed wage is inefficient and that the firms can improve the joint welfare by offering a contract that is backloaded.

## APPENDIX D. Nash-Bargaining

### 1.8.2 Nash-Bargaining without Employment Protection

The wage is set using the standard result of Nash bargaining, where $\gamma \in[0,1]$ represents the contractual power of the worker.

In the general case without employment protection, the wage maximize the following expression

$$
\max _{w}(W-U)^{\gamma}(J-V)^{1-\gamma}
$$

This result in the surplus being split according to this rule

$$
(1-\gamma)(W-U)=\gamma(J-V)
$$

or in other terms

$$
\begin{gathered}
W-U=\gamma S \\
J-V=(1-\gamma) S
\end{gathered}
$$

[^32]In the Toy model, all the value functions are dependent on $x$, therefore even the wage is depending on $x$. In the full model, in the case of the FTC, the wage depends on both $x$ and the worker prior distribution, characterized by $\phi$.

### 1.8.3 Nash-Bargaining with Firing Costs as a pure waste

If the two agents are already in a contract with firing costs, the outside option of the firm is to pay the firing costs. Therefore the surplus of the match is

$$
S=W-U+J-(V-f c)
$$

The wage maximizes

$$
\max _{w}(W-U)^{\gamma}(J-V+f c)^{1-\gamma}
$$

and it is generally higher than without firing costs, everything else equal.
However, this is not the case when two agents have just met and they are bargaining for a new contract. In this last case, the outside option of the firm is the empty vacancy, since it can terminate the match immediately without any further cost.

To give an example, I show the specific case of the OEC in the full model. We have to separately consider the case in which the two agents have just met (or equivalently, they are transforming a FTC into an OEC) and when they are bargaining after an entire period with an OEC.

In the former case, the wage is the one that maximizes

$$
\max _{w_{f, p}(\phi, x)}\left(W_{f, p}(\phi, x)-U(\phi)\right)^{\gamma}\left(J_{g, p}(\phi, x)-V\right)^{1-\gamma}
$$

so, the outside option of firm is only the empty vacancy and the resulting wage split the
surplus in the usual way:

$$
(1-\gamma)\left(W_{f, p}(\phi, x)-U(\phi)\right)=\gamma\left(J_{f, p}(\phi, x)-V\right)
$$

In the second case, the outside option becomes the empty vacancy minus the firing costs. Then the wage maximizes

$$
\max _{w_{p, p}(\phi, x)}\left(W_{p, p}(\phi, x)-U(\phi)\right)^{\gamma}\left(J_{p, p}(\phi, x)-V+f c\right)^{1-\gamma}
$$

and the two shares are

$$
\begin{gathered}
J_{p, p}(\phi, x)-V+f c=(1-\gamma) S_{p, p}(\phi, x)=(1-\gamma)\left(W_{p, p}(\phi, x)-U+J_{p, p}(\phi, x)-V\right) \\
W_{p, p}(\phi, x)-U(\phi)=\gamma S_{p, p}(\phi, x)=\gamma\left(W_{p, p}(\phi, x)-U+J_{p, p}(\phi, x)-V\right)
\end{gathered}
$$

Notice that $W_{p, p}+J_{p, p}=W_{f, p}+J_{f, p}$, since the two contracts are identical, but from the initial wage, that is just a transfer between the two agents. From this, we can verify that

$$
J_{p, p}(\phi, x)-V+f c=(1-\gamma) S_{p, p}(\phi, x)=(1-\gamma)\left(S_{f, p}(\phi, x)+f c\right)
$$

Therefore

$$
\begin{gathered}
J_{p, p}(\phi, x)-V+f c=J_{f, p}(\phi, x)-V+(1-\gamma) f c \\
J_{p, p}(\phi, x)=J_{f, p}(\phi, x)-V-\gamma f c
\end{gathered}
$$

Similarly

$$
W_{p, p}(\phi, x)=W_{f, p}(\phi, x)+\gamma f c
$$

### 1.8.4 Nash-Bargaining with Severance Payment

This case is similar to the previous one, with the difference that also the outside option of the worker is affected by employment protection. Indeed, the firing cost for the firm is transferred to the worker in case of separation. In this case, the wage is determined through this maximization:

$$
\max _{w}(W-U-f c)^{\gamma}(J-V+f c)^{1-\gamma}
$$

and the two shares are

$$
\begin{gathered}
J-V+f c=(1-\gamma) S=(1-\gamma)(W-U+J-V) \\
W-f c-U=\gamma S=\gamma(W-U+J-V)
\end{gathered}
$$

## APPENDIX E. Thresholds Proof

### 1.8.5 Higher firing threshold for FTC $\left(\hat{y}_{f}>\hat{y}_{p}\right)$

I start from observing (as shown in Appendix D) that

$$
J_{p, p}\left(\phi^{\prime}, x\right)=J_{f, p}\left(\phi^{\prime}, x\right)-\gamma f c
$$

Then if $\max _{i \in\{p, f\}} J_{f, i}\left(\phi^{\prime}, x\right)=J_{f, p}\left(\phi^{\prime}, x\right)$, it is immediate that at $y=\hat{y}_{f}\left(\right.$ implying $\left.J_{f, p}\left(\phi^{\prime}, x\right)=0\right)$, $J_{p, p}\left(\phi^{\prime}, x\right)=-\gamma f c>-f c$.

Then, $\hat{y}_{p}$ must be lower in order to reach the threshold where $J_{p, p}\left(\phi^{\prime}, x\right)=-f c$.
If instead $\max _{i \in\{p, f\}} J_{f, i}\left(\phi^{\prime}, x\right)=J_{f, f}\left(\phi^{\prime}, x\right)$, then consider the highest possible difference between the surplus of the two contracts in favor of FTC. This happens when the contract is going to be terminated by the firm in any case for a too low productivity of the match. In this case the worker is going to choose the exact same amount of search intensity, since in both cases he is going to be fired. The only difference is the presence of the firing costs.


Figure 1.26: Change in welfare of unemployed workers at the steady state with a cut of $25 \%$ of firing costs.

This costs are going to be paid with probability $(1-\lambda)$. Therefore, the maximum difference between the two contracts can be $S_{f, f}\left(\phi^{\prime}, x\right)-S_{f, p}\left(\phi^{\prime}, x\right) \leq \beta(1-\lambda) f c<f c$. From this we can claim that $J_{f, f}\left(\phi^{\prime}, x\right)-J_{f, p}\left(\phi^{\prime}, x\right) \leq(1-\gamma) \beta(1-\lambda) f c$. Then we can reach the conclusion that even if $J_{f, f}\left(\phi^{\prime}, x\right)$ can be higher than $J_{f, p}\left(\phi^{\prime}, x\right)$, at most it can be

$$
J_{f, f}\left(\phi^{\prime}, x\right)=J_{f, p}\left(\phi^{\prime}, x\right)+(1-\gamma) \beta(1-\lambda) f c
$$

but then we can finally arrive at

$$
J_{f, f}\left(\phi^{\prime}, x\right)-J_{p, p}\left(\phi^{\prime}, x\right)=(1-\gamma) \beta(1-\lambda) f c+\gamma f c<f c
$$

## APPENDIX F. Welfare Comparisons

The first three graphs report the changes in welfare after a cut in firing costs for workers, depending on their employment status. The second three graphs report the same changes after a FTC-tax.


Figure 1.27: Change in welfare of FTC employed workers at the steady state with a cut of $25 \%$ of firing costs.


Figure 1.28: Change in welfare of OEC employed workers at the steady state with a cut of $25 \%$ of firing costs.


Figure 1.29: Change in welfare of unemployed workers at the steady state with a lump-sum on FTCs equal $1 \%$ of average wage.


Figure 1.30: Change in welfare of FTC employed workers at the steady state with a lump-sum on FTCs equal $1 \%$ of average wage.


Figure 1.31: Change in welfare of OEC employed workers at the steady state with a lump-sum on FTCs equal $1 \%$ of average wage.

## Absence of Endogenous Choice

In this section the table with the aggregate indicators and the graphs with the welfare changes, after the implementation of policy changes, in an environment in which agents cannot choose the type of contract. More precisely, I am keeping the same structural parameters estimated in the main model. However, I am assuming that the agents cannot choose to sign an OEC. The only way in which the agents can obtain an OEC is through the "transformation" shock. I am assuming that this exogenous shock realizes at the beginning of every period (including the initial match-formation stage). Also, I am calibrating this parameter in order to match the observed share of FTC in Italy.

| Macro Indicator | Initial Steady State | Cut in Firing costs | Change | Tax on FTCs | Change |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Unemployment rate | $13.2 \%$ | $13.0 \%$ | $-0.2 \%$ | $13.3 \%$ | $+0.1 \%$ |
| Output | 2.741 | 2.739 | $-0.0 \%$ | 2.730 | $-0.2 \%$ |
| Share of FTCs | $14.5 \%$ | $14.4 \%$ | $-0.1 \%$ | $14.2 \%$ | $-0.3 \%$ |
| Job-find. of Unempl. | 0.263 | 0.266 | $+0.3 \%$ | 0.262 | $-0.1 \%$ |
| Average Productivity | 3.160 | 3.150 | $-0.3 \%$ | 3.155 | $-0.2 \%$ |
| Average Wage | 3.122 | 3.111 | $-0.4 \%$ | 3.101 | $-0.7 \%$ |
| Average Welfare | 64.73 | 64.50 | $-0.4 \%$ | 64.67 | $-0.1 \%$ |
| $\%$ of unprod. matches | $23.5 \%$ | $16.9 \%$ | $-6.6 \%$ | $23.9 \%$ | $+0.4 \%$ |

Table 1.8: Change in main indicators at the steady-state, after a $25 \%$ cut in firing costs or a FTC-only lump-sum tax equal to $1 \%$ of the average wage. Unproductive matches are defined as matches that are kept only for the presence of firing costs, but that would be terminated otherwise.


Figure 1.32: Change in welfare of unemployed workers at the steady state with a cut of $25 \%$ of firing costs. Choice of the contract NOT allowed.


Figure 1.33: Change in welfare of FTC employed workers at the steady state with a cut of $25 \%$ of firing costs. Choice of the contract NOT allowed.


Figure 1.34: Change in welfare of OEC employed workers at the steady state with a cut of $25 \%$ of firing costs. Choice of the contract NOT allowed.


Figure 1.35: Change in welfare of unemployed workers at the steady state with a lump-sum on FTCs equal $1 \%$ of average wage. Choice of the contract NOT allowed.


Figure 1.36: Change in welfare of FTC employed workers at the steady state with a lump-sum on FTCs equal $1 \%$ of average wage. Choice of the contract NOT allowed.


Figure 1.37: Change in welfare of OEC employed workers at the steady state with a lump-sum on FTCs equal $1 \%$ of average wage. Choice of the contract NOT allowed.

## Chapter 2

# The Drivers of EU Unemployment during the Great Recession 

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### 2.1 Introduction

Aggregate data on labor market outcomes reveal a significant amount of differences across European countries. Notably, unemployment rates differ both in levels and in volatility. We seek to explain the differences across Germany, France, Spain and Italy in terms of unemployment dynamics, with particular focus around the Great Recession.

We start with a standard, representative agent Diamond-Mortensen-Pissarides (DMP) model of labor market with search and matching frictions. In addition to the more traditional productivity shock, we augment the model with a discount factor shock. The role of discount factors in labor market outcomes is a recent addition to the literature. Discounts are considered a possible explanation of observed unemployment fluctuations. ${ }^{1}$ We also briefly study the effect of a possible separation shock.

[^33]We first provide evidence that returns on European financial assets are highly correlated with unemployment, possibly more than labor productivity. We then assess the ability of discount factors and workers' productivity to generate variation in unemployment. We find that discount factors are a promising source of variation to explain fluctuations in European unemployment.

We proceed by analyzing the predictions of the model through impulse-response functions. We consider how these predictions vary after changes in the calibration, which reflect changes in Labor Market Institutions (LMI). Changes to the average job-finding rate, the average separation rate and the extent of wage rigidity account for many differences between the US and EU labor markets. However, they do not account for the differences across EU markets.

We estimate exogenous shocks to the aggregate discount factor and aggregate productivity directly from the data. We input the shocks into the model and obtain simulations. We then compare the simulations to the data to assess the performance of the representative agent model in explaining observed data. We find that discount factor shocks can explain a significant part of the variation in unemployment for all European countries, contrary to productivity shocks, even without wage rigidity. However, no exogenous shock can account for the differences across EU countries.

Motivated by these attempts, we extend the model with in order to allow for dual labor markets. The extended model features heterogeneous agents that can write Fixed-Term Contracts (FTC) and Open-Ended Contracts (OEC). Firms and workers choose between the two contracts when matched. The FTC is more flexible relative to the OEC in that firms can terminate it without any firing cost. A FTC lasts only one period, but agents may choose to renew it. On the other hand, an OEC features a lower exogenous job-destruction rate relative to a FTC. We interpret the difference between the two exogenous job-destruction rates as different probabilities with which a worker quits a job for reasons not directly related to labor market outcomes.

Matches between firms and workers are heterogeneous because of idiosyncratic productiv-
ity, which is drawn at the match. However, matched agents initially observe only one signal about the match-specific productivity. They fully observe it only after some periods with some exogenous arrival rate.

Future work consists of replicating the methodology applied to the representative agent model with the heterogeneous agents, dual labor market model. The goal is to understand whether different implementations of this duality can account for differences in unemployment dynamics across EU countries.

This paper contributes to several branches of the literature. First, we document that discount factor shocks are a promising explanation for the volatility in European unemployment, similarly to Hall (2017) and Borovička and Borovičková (2018). We additionally propose a way to estimate discount factor shocks from stock market data, in a similar spirit to Borovička and Borovičková (2018).

Second, we find that the representative agents DMP model with country-specific productivity and discount factor shocks cannot fully account for the differences in labor market outcomes across EU countries.

Third, we contribute to the dual labor market literature by providing a model of business cycles with a possible choice for agents between OEC and FTC. The choice that agents faces is similar to Garibaldi and Violante (2005), but with the addition of a learning process, that is instead present in Faccini (2014). These aspects become fundamental in presence of shocks to productivity or to the discount factor, since different contracts can induce different responses of agents. FTCs for example are easy to terminate and they can be adjusted quickly, as noted in Caggese and Cuñat (2008), increasing the unemployment level. Finally, these aspects seems relevant in determining the overall performance of a labor market after a shock, as noted in Bentolila et al. (2012).

The rest of this document is organized as follows. Section 2.2 presents the overall methodology we use. Section 2.3 explains the representative-agent search and matching model and inspects the main mechanisms with impulse-response functions. Section 2.4 elaborates on
the heterogeneous-agents model. Section 2.5 presents the results we obtain with the representative agent model. Section 2.6 concludes.

### 2.2 Methodology

In this section we present the methodology we use to assess the effectiveness of a model in explaining variation in the data. The overall procedure consists of simulating the model by "feeding in" exogenous shocks we estimate externally. This gives us simulated time series for labor market variables. We compare the simulations against the data. Here, we proceed to explain how we obtain the time series estimates of the exogenous shocks.

### 2.2.1 Inference of SDF shocks

We rely on stock market data to obtain a time series for the discount factor, $\beta_{t}$. As the steps we take are applied to each national series independently, we omit country-specific indices in the notation that follows. It is important to note that we abstract from any microfoundation of the SDF and we are silent about the causes that move discounts. Our goal here is to find an observable proxy for the SDF.

Consider the following asset-pricing equation:

$$
\begin{equation*}
\mathbf{E}_{t}\left(\beta_{t+1} R_{t+1}\right)=1 \tag{2.1}
\end{equation*}
$$

where $t$ denotes a month, $\beta_{t+1}$ is the SDF and $R_{t+1}$ is the gross return of a given financial asset from $t$ to $t+1$. Log-linearizing (2.1) we obtain the relationship $\mathbf{E}_{t}\left(\hat{\beta}_{t+1}\right)=-\mathbf{E}_{t}\left(\hat{R}_{t+1}\right)$, where the hat denotes that the variable is expressed in log-deviations from the steady state. By log-linearizing around the deterministic steady state, we are dropping any moment higher than the first. In the implementation that follows, we assume $\hat{\beta}_{t+1}=-\hat{R}_{t+1}$, making stronger assumptions about the relationship between the unobservable SDF and the observable returns.

As stock market returns exhibit much high-frequency variation, we smooth them by compounding returns in the following way:

$$
1+\bar{r}_{t} \equiv \sqrt[12]{\prod_{s=0}^{11}\left(1+r_{t+s}\right)}
$$

where $r_{t}$ is the monthly data point provided by WRDS. In words, we are taking the geometric average of a year of returns in a forward-looking way. Compounding returns forward reduces our sample size by one year at the end of the sample.

Because we solve the representative agent model by log-linearing it, we do not relate levels of financial returns to the levels of the SDF. Instead, we relate their log-deviations from the steady-state. To this end, we construct the measure $\tilde{r}_{t}$ as

$$
\tilde{r}_{t}=\log \left(1+\bar{r}_{t}-r_{t}^{f}\right),
$$

where we normalize the stock return of the financial asset by a risk-free rate, and we compute its trend-cycle decomposition using the Hodrick-Prescott filter with smoothing parameter $1600 \cdot 3^{4}$. Because we take logs, the resulting cycle can be interpreted as a log-deviation from the trend. Figure 2.1 plots the measures $\tilde{r}_{t}$ together with observed de-trended unemployment for each of the four countries. The two series feature strongly correlated co-movements in each of the countries.

As we are constrained by data on productivity, which is available at quarterly frequency, we aggregate returns from monthly to quarterly. To compute the gross return for a given quarter, we compound the gross monthly returns observed within the quarter. The result scales to percent per quarter. Because of this transformation, we use the subscript $t$ to indicate a quarter in the remainder of the paper.

In line with the asset pricing literature, ${ }^{2}$ one may be worried that the risk premia we compute are not only driven by variations in discounts, but also in expected future cash-

[^34]

Figure 2.1: Unemployment (orange solid line) and the spread between stock market returns and the EONIA (blue dashed line), expressed as percent per month.
flows. In order to isolate variation in returns that we can attribute to discounts, we control for a measure of future economic conditions. With US data, we could do so by controlling for dividend growth and/or variations in dividend-price ratios. However, as dividends in European markets do not play the same important role they do in US markets, ${ }^{3}$ we use a different variable. The control variable we consider is the Leading Economic Indicator (LEI) by OECD, which provides qualitative forward-looking information about the state of the business cycle. This justifies the following specification for the identification of SDF shocks:

$$
\begin{equation*}
\tilde{r}_{t}=\alpha+\rho_{\beta} \tilde{r}_{t-1}+\delta L E I_{t-1}+\eta_{t} . \tag{2.2}
\end{equation*}
$$

By construction, the innovations $\eta_{t}$ will not be systematically correlated with the Leading Economic Indicator. Hence we attribute the variation in these shocks to variation in discounts. We specify an $\operatorname{AR}(1)$ component in order to account for the dynamics we specify in the model. We use the estimates of the persistency $\rho$ and the volatility of $\eta_{t}$ to calibrate

[^35]Table 2.1: Parameters for the quarterly process on $\beta_{t}$ inferred from output per worker data. The steady state value $\tilde{\beta}$ is set and not estimated.

| Parameter | Germany | France | Spain | Italy |
| :---: | :---: | :---: | :---: | :---: |
| $2-5 \tilde{\beta}$ | 0.9901 | 0.9883 | 0.9883 | 0.9955 |
| $\rho_{\beta}$ | 0.74398 | 0.79455 | 0.7912 | 0.79725 |
| $\sigma_{\beta}$ | 0.02733 | 0.02305 | 0.02371 | 0.02391 |

the parameters in Equation (2.17). We set the steady state value of the discount factor such that the associated discount rate equals the historical average of gross returns in the sample period. In order to simulate unemployment from the model, we feed $-\eta_{t}$ in place of $\sigma^{\beta} \varepsilon_{t}^{\beta}$ in Equation (2.17). The summary statistics of the regression are presented in Table 2.1.

In addition to the steps detailed above, we compute other measure of monthly SDF to assess the robustness of the methodology. We consider an alternative, Euro Area-wide measure of LEI, as opposed to the country-specific one. We infer the process directly from the data, without accounting for the Leading Economic Indicators. We considered the part of variation of returns that could be predicted by dividend-price ratios or the LEIs. We also verified that European dividend-price ratios have low predictive power with respect to stock market returns.

### 2.2.2 Inference of aggregate productivity shocks

We employ a simpler, but similar, approach to obtain a series of productivity shocks to feed in the model. We use quarterly data on real GDP and on the number of employed people in each country to compute our measure of output per worker. We express the result as an index number, where the base period is the first quarter of 2010.

Similarly to before, we obtain log-deviations by computing the logarithm of productivity and then applying the HP filter with smoothing parameter equal to 1600. Figure 2.2 already showed the resulting series We finally fit an $\mathrm{AR}(1)$ process on the cycle component of the


Figure 2.2: Detrended log of output per worker (blue dashed line, right axis) and detrended unemployment (orange solid line, left axis).
decomposition:

$$
\begin{equation*}
\tilde{z}_{t}=\omega+\rho_{z} \tilde{z}_{t-1}+\nu_{t} . \tag{2.3}
\end{equation*}
$$

In order to simulate unemployment from the model, we feed $\nu_{t}$ in place of $\sigma^{z} \varepsilon_{t}^{z}$ in Equation (2.18). The summary statistics of the regression are presented in Table 2.2.

Table 2.2: Parameters for the quarterly process on $z_{t}$ inferred from output per worker data. The steady state value $\tilde{z}$ is set and not estimated.

| Parameter | Germany | France | Spain | Italy |
| :---: | :---: | :---: | :---: | :---: |
| $2-5 \tilde{z}$ | 1 | 1 | 1 | 1 |
| $\rho_{z}$ | 0.82428 | 0.92073 | 0.96618 | 0.8597 |
| $\sigma_{z}$ | 0.00850 | 0.00468 | 0.00371 | 0.0066 |

### 2.3 Model with Representative Agent

The model we use is a standard version of the Diamond, Mortensen and Pissarides (DMP) labor market model with search and matching frictions, whereby jobs are created according to the expected discounted profits over the match duration and exogenously destroyed at a given rate. We adjust our formulation to include three exogenous sources of variation: workers' productivity, an exogenous job destruction rate and a stochastic discount factor (SDF). In most of the analysis we focus on productivity and SDF shocks, but also briefly discuss separation shocks, as their impact is in part similar to SDF shocks.

While productivity and the separation rate are standard driving forces in the literature, the stochastic discounter only recently appeared in labor market models. We denote the SDF with $\beta_{t+1}$. We think of $\beta_{t+1}$ simply as a random variable that allows agents to discount the future. In the consumption-based capital asset pricing model, the SDF is defined as the ratio of subsequent marginal utilities in consumption. In the financial economics literature, instead, the SDF is any random variable that prices a given asset. In line with Hall (2017), we abstract from any microfoundation, as we prefer to be agnostic about the microeconomic interpretation of a stochastic discounter. We let the SDF be time-varying to allow agents in our model to discount the future depending on the current aggregate state of the economy. We finally assume that the SDF is common across workers and firms. In what follows, we infer a sequence of realizations for the SDF to feed in the model. We do so by relating it to financial returns observed on the stock market.

Workers can be employed or unemployed and we abstract from labor force participation decisions. If unemployed, workers collect the unemployment benefit $b$ and expect a future payoff stream by considering the probability $p_{t}$ of finding a job. Such future payoff stream is discounted at the time-varying rate $\beta_{t+1}$. The sum of current and future payoffs gives the unemployment value, $U_{t}$ :

$$
\begin{equation*}
U_{t}=b+\mathbf{E}_{t}\left\{\beta_{t+1}\left(p_{t} W_{t+1}+\left(1-p_{t}\right) U_{t+1}\right)\right\} . \tag{2.4}
\end{equation*}
$$

If employed, workers earn the wage $w_{t}$ and a future stream of wages that is discounted by $\beta_{t+1}$ and consider the probability of job destruction $s_{t}$. The value of working is denoted with $W_{t}$ and is given by:

$$
\begin{equation*}
W_{t}=w_{t}+\mathbf{E}_{t}\left\{\beta_{t+1}\left(\left(1-s_{t}\right) W_{t+1}+s_{t} U_{t+1}\right)\right\} \tag{2.5}
\end{equation*}
$$

The difference between the value of working and the value of unemployment is the workers' surplus from employment:

$$
\begin{equation*}
W_{t}-U_{t}=w_{t}-b+\mathbf{E}_{t}\left\{\beta_{t+1}\left(1-s_{t}-p_{t}\right)\left(W_{t+1}-U_{t+1}\right)\right\} \tag{2.6}
\end{equation*}
$$

Firms hire workers by posting vacancies. If a firm hires, then it collects the value $J_{t}$, which is composed of the current profit, productivity minus wage, and the discounted future expected stream of profits:

$$
\begin{equation*}
J_{t}=z_{t}-w_{t}+\mathbf{E}_{t}\left\{\beta_{t+1}\left(\left(1-s_{t}\right) J_{t+1}+s_{t} V_{t+1}\right)\right\} \tag{2.7}
\end{equation*}
$$

Posting a vacancy costs $\kappa$ per period, but allows a firm to hire. The value of an open vacancy is given by:

$$
\begin{equation*}
V_{t}=-\kappa+\mathbf{E}_{t}\left\{\beta_{t+1}\left(q_{t} J_{t+1}+\left(1-q_{t}\right) V_{t+1}\right)\right\} \tag{2.8}
\end{equation*}
$$

where $q_{t}$ is the vacancy-filling rate. Free entry drives the value of a vacancy to zero:

$$
\begin{align*}
-\kappa+\mathbf{E}_{t}\left\{\beta_{t+1} q_{t} J_{t+1}\right\} & =0  \tag{2.9}\\
\frac{\kappa}{q_{t}} & =\mathbf{E}_{t}\left\{\beta_{t+1} J_{t+1}\right\} . \tag{2.10}
\end{align*}
$$

By combining the value of a job $J_{t}$ and the free-entry condition, we obtain:

$$
\begin{equation*}
J_{t}=z_{t}-w_{t}+\mathbf{E}_{t}\left\{\beta_{t+1}\left(1-s_{t}\right) J_{t+1}\right\} . \tag{2.11}
\end{equation*}
$$

Workers and firms are matched according to a matching function $m_{t}$ that we assume to be Cobb-Douglas:

$$
\begin{equation*}
m_{t}=\sigma^{m} u_{t}^{\sigma} v_{t}^{1-\sigma} \tag{2.12}
\end{equation*}
$$

where $\sigma^{m}$ denotes the efficiency of the matching process, $u_{t}$ is the unemployment rate and $v_{t}$ is the vacancy rate. Unemployment at date $t+1$ equals date $t$ unemployment plus exogenous layoffs, minus new matches:

$$
\begin{equation*}
u_{t+1}=u_{t}+s_{t}\left(1-u_{t}\right)-m_{t} \tag{2.13}
\end{equation*}
$$

The probability for a worker to find a job must equal the number of new matches relative to the mass of unemployed workers, $p_{t}=m_{t} / u_{t}$; similarly, the probability for a firm to fill a vacancy is $q_{t}=m_{t} / v_{t}$.

The wage in this model is set according to the Nash bargaining protocol, whereby workers and firms agree on a wage that maximizes a function of the parties' surpluses:

$$
\begin{equation*}
w_{t}^{N B}=\arg \max _{w_{t}}\left(W_{t}-U_{t}\right)^{\eta}\left(J_{t}\right)^{1-\eta} \tag{2.14}
\end{equation*}
$$

The first-order condition for this problem gives the equilibrium wage, which is determined by a surplus sharing rule:

$$
\begin{equation*}
w_{t}^{N B}=\eta\left(z_{t}+p_{t} \frac{\kappa}{q_{t}}\right)+(1-\eta) b \tag{2.15}
\end{equation*}
$$

When we consider wage rigidity, we impose a rule such that

$$
\begin{equation*}
w_{t}=(1-\gamma) w_{t}^{N B}+\gamma \bar{w} \tag{2.16}
\end{equation*}
$$

where $\bar{w}$ is the steady state value of the wage and $\gamma$ is a parameter governing the degree of wage rigidity.

We close the model by introducing the stochastic processes for the exogenous variables. We specify $\operatorname{AR}(1)$ processes for each of them, which is common practice in the literature in order to introduce persistency effects in agents' expectations.

$$
\begin{align*}
& \log \left(\beta_{t}\right)=\left(1-\rho^{\beta}\right) \log (\tilde{\beta})+\rho^{\beta} \log \left(\beta_{t-1}\right)+\sigma^{\beta} \varepsilon_{t}^{\beta}  \tag{2.17}\\
& \log \left(z_{t}\right)=\left(1-\rho^{z}\right) \log (\tilde{z})+\rho^{z} \log \left(z_{t-1}\right)+\sigma^{z} \varepsilon_{t}^{z}  \tag{2.18}\\
& \log \left(s_{t}\right)=\left(1-\rho^{s}\right) \log (\tilde{s})+\rho^{z} \log \left(s_{t-1}\right)+\sigma^{s} \varepsilon_{t}^{s} \tag{2.19}
\end{align*}
$$

where each of the shocks $\varepsilon_{t}^{i}$, with $i \in\{\beta, z, s\}$, is independently and identically distributed according to standard Gaussian distributions.

### 2.3.1 Calibration

As anticipated above, we start our analysis with a baseline monthly calibration that targets US labor market moments. We pick this baseline to be the same as in Shimer (2005b), which represents a widely known benchmark for the literature. Table 2.3 presents the calibration. We normalize the average labor productivity to one. The unemployment benefit $b$ is set to 0.4 : this means that the unemployment benefit is roughly 40 percent of the average labor income, which amounts to approximately 0.96 with this calibration. We set the average separation rate $s$ to 0.03 , so that employment lasts roughly 2.7 years on average ( 33 months). We let the vacancy cost $\kappa$ vary to target an average job-finding rate of 0.45 in US data and normalize the matching efficiency $\sigma^{m}$ to one. We set the elasticity of matches to unemployment $\sigma$ to 0.5 , a

Table 2.3: Values of calibrated parameters expressed in monthly terms.

| Target/Parameter | Meaning | Values |
| :---: | :--- | :--- |
| $\tilde{z}$ | Steady-state value of productivity | 1 (normalization) |
| $b$ | Unemployment benefit | 0.4 |
| $\eta$ | Workers' bargaining power | 0.5 |
| $\tilde{p}$ | Target job-finding rate | 0.45 |
| $\tilde{q}$ | Target vacancy-filling rate | 0.7 |
| $\sigma^{m}$ | Matching efficiency | 1 (normalization) |
| $\sigma$ | Elasticity of matching to unemployment | 0.5 |
| $\tilde{s}$ | Average job destruction rate | 0.03 |
| $\rho^{\beta}$ | Persistence of SDF process | $0.95^{1 / 3}$ |
| $\rho^{z}$ | Persistence of productivity process | $0.95^{1 / 3}$ |
| $\rho^{s}$ | Persistence of separation rate | $0.95^{1 / 3}$ |
| $\sigma^{\beta}$ | Volatility of shocks to SDF | 0.1527 |
| $\sigma^{z}$ | Volatility of shocks to productivity | 0.015 |
| $\sigma^{s}$ | Volatility of shocks to separation rate | 0.2887 |

midpoint of the estimates in the literature. ${ }^{4}$ We set the worker's bargaining power $\eta$ to 0.5 assigning equal power to both parties and satisfying the Hosios (1990) efficiency condition. The persistencies of the exogenous processes $\rho_{\beta}, \rho_{z}$ and $\rho_{s}$ are set equal in order to compare the Impulse-Response Functions that follow. Finally, we set the volatilities for the exogenous shocks $\sigma^{\beta}, \sigma^{z}$ and $\sigma^{s}$ so that the implied volatility of output, with each of those shocks alone, matches the observed volatility in the data. This implies that the Impulse-Response Functions should be interpreted relative to output.

We then develop our own calibration in order to assess the role of Labor Market Institutions. We do so by using the baseline calibration and changing the unemployment benefit $b$, the job-finding probability $\tilde{p}$ and the separation rate $\tilde{s}$ on a country by country basis.

To set a value of $b$, we use annual data on Net Replacement Rates (NRRs) by OECD. These measure the fraction of the average income that a household retains after a transition from employment to unemployment. The available data is rich in terms of slicing the reference population. We consider the NRRs for households composed of two adults with two children

[^36]and where the second adult is inactive. We further narrow the choice of the value to those households that are two months into unemployment. As OECD provides an annual time series for the NRRs, we compute the historical average on the sample period we consider and we set this value to $b$ in the calibration. We do not choose NRRs for households where the second adult is employed as the NRR, by definition, is considerably driven up by his/her income earnings. ${ }^{5}$ This is documented by Figure 2.3, where we also observe that, in general, the US provide lower benefit and assistance to unemployed households. Figure 2.4 shows the rates for the household composition we choose, by unemployment duration. We note that in general, the levels of the NRRs drop considerably in the long term (5 years). Given that the average duration of unemployment in European countries is roughly between 11 and 19 months, ${ }^{6}$ and thus closer to two months than five years, we restrict our attention to the NRR measured at the second month of unemployment. We also see that the speed of the drop varies significantly across countries. Further motivating our choice of NRR is the fact that OECD only includes cash flows in the calculation of the NRRs, we choose the higher values. In the model, $b$ represents any benefit a household might collect every period, including any non-monetary flow (e.g., home production, leisure). We therefore prefer picking the higher values of NRR.

We estimate the values of the steady state job-finding probability $\tilde{p}$ and the separation rate $\tilde{s}$ by partially replicating Elsby et al. (2013). The replication is necessary to extend their methodology to our sample period. In fact, their results stop at 2009, while our sample period ends in August 2017. Following their steps, we compute the job-finding probabilities conditional on the duration of unemployment (less than a month, less than three months,

[^37]

Figure 2.3: Net Replacement Rates by household composition. The values are averages of the yearly observations.


Indicator2 $\quad 7$

- Initial (Previous Earnings 100\%)

■ 5 Year (Previous Earnings 100\%)

- Long Term (Previous Earnings 100\%)

Figure 2.4: Net Replacement Rates by unemployment duration for married couples with two children and inactive spouse. The values are averages of the yearly observations. The data labeled with " 5 year" are averages of the NRRs reported across durations. The data labeled with "long term" refer to households who have been unemployed for five years.


Figure 2.5: Job-finding and separation probabilities using the methodology in Elsby et al. (2013) on our sample period.
less than six months and less than a year). Elsby et al. proceed to compute a set of optimal weights to average out the conditioning of each measure. In our replication exercise, we observe that their results are almost entirely driven by the job-finding probability for those who have been unemployed by less than a year. We therefore pick this duration of unemployment as representative of the unconditional job-finding probability. With such probability and with data on unemployment, Elsby et al. invert the continuous time-based law of motion of unemployment to recover the separation probability. We do the same here. Figure 2.5 shows the results we obtain by replicating Elsby et al. (2013) on our sample period. We verify that our results largely coincide with theirs where the sample periods intersect. As their methodology gives annual estimates of the two probabilities, we take historical averages to set the steady state values $\tilde{p}$ and $\tilde{s}$.

With given values of the steady state transition probabilities, our model pins down the steady state values of unemployment through the steady state version of the law of motion

Table 2.4: Country-specific calibration.

| Target | United States | Germany | France | Spain | Italy |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $(\mathrm{lr}) 2-2(\mathrm{l}) 3-6 b$ | 0.4791 | 0.7013 | 0.6904 | 0.7351 | 0.6858 |
| $\tilde{p}$ | 0.3559 | 0.0647 | 0.0740 | 0.0885 | 0.0519 |
| $\tilde{s}$ | 0.0338 | 0.0045 | 0.0074 | 0.0164 | 0.0052 |
| $\tilde{u}$ | 0.0603 | 0.0657 | 0.0906 | 0.1563 | 0.0908 |

of unemployment:

$$
\begin{equation*}
\tilde{u}=\frac{\tilde{s}}{\tilde{s}+\tilde{p}} . \tag{2.20}
\end{equation*}
$$

Table 2.4 summarizes the values we set in our calibration. As we apply this calibration methodology also to US data, we can compare US steady state values with the corresponding European ones. Both the job-finding and the separation rates are significantly lower in the European countries we consider relative to the US. This implies both a longer average duration of unemployment (through lower $\tilde{p}$ ) and a longer average duration of employment (through lower $\tilde{s}$ ). Because of these differences, we refer to the US as a fluid labor market and to the European ones as sclerotic. In other words, fluid environments feature more faster transitions into and from unemployment relative to sclerotic ones.

We also observe that the unemployment benefits differ from the baseline calibration. On average, European countries provide higher transfers to unemployed households than the US. As is known in the literature, unemployment benefits may play an important role in explaining unemployment fluctuations. For example, Hagedorn and Manovskii (2008) show that with high enough benefits and for particular values of the workers' bargaining power, a DMP model may not need wage rigidity to explain unemployment only through variation in workers' productivity.

Finally, we change the degree of wage rigidity. As mentioned above, we do so by setting values of $\gamma$ in Equation (2.16). Setting $\gamma=1$ means allowing for full flexibility in the wage bargaining protocol, while imposing $\gamma=0$ pins down wages to their steady state value forever.

While we do not calibrate the degree of wage rigidity, we change its value to arbitrary values to show how exogenous shocks differently propagate throughout the labor market.

As we anticipated above, we produce quarterly simulations. Therefore we also convert the monthly calibration to a quarterly one, specifically the average job finding and job separation rates.

### 2.3.2 Inspecting the mechanisms

We explore the qualitative predictions of our model using Impulse-Response Functions (IRFs). Figure 2.6 shows the Impulse-Response Functions of our model to shocks to the three exogenous variables of one standard deviation size. In particular, as mentioned above, the calibration of those standard deviations are such that a standard deviation of output simulated with each shock alone matches the data. The qualitative implications of the model are standard when compared to the literature. As already pointed out in Shimer (2005b), productivity shocks cannot produce amplification of unemployment and number of vacancies relative to output. Consistently with the literature, shocks to the separation rate do not generate the negative correlation between unemployment and vacancies (also known as the Beveridge Curve).

Note that the impulse responses of output and unemployment are exactly the same in case of separation and discount factor shocks. This is the case for two reasons. First, the processes are calibrated in such a way that the volatility of output is the same after each shock, separately, hits the economy. At the same time, the model assumes that output is unaffected by the two shocks upon impact and that it reacts only in subsequent periods. Second, both shocks enter discounting the same way- $\left(1-s_{t+1}\right) \beta_{t+1}$-hence the impact of these shocks on the value functions is similar. The difference is that only discount factor shocks enter the job creation condition while only separation shocks enter the law of motion of unemployment. This also explains the different responses in the evolution of vacancies.


Figure 2.6: Impulse-Response Functions (IRFs) under the baseline calibration.

### 2.3.3 The effects of SDF shocks vs productivity shocks

A positive shock to the discount factor enters the model through the firms' incentive to hire by making them more forward-looking. In other words, payoffs further ahead in the future are discounted less. This incentivizes firms to hire, raising vacancies and reducing unemployment. As more firms enter the market, total production increases, but only after one period (that is, not on impact). This happens because the model's timing implies that it takes one period for a new match to start producing. Unemployed workers find jobs more easily because of increased opportunities. At the same time, higher entry by firms makes it more difficult for each firm to find a worker. As the total surplus in the economy rises, wages rise. Compared to the shocks to productivity, shocks to the discount factor cause larger movements in labor market activity (vacancies, unemployment, job finding and job filling rates) relative to output. Moreover, movements in discounts can generate the Beveridge curve.

A positive shock to workers' productivity generates the same fluctuations in terms of sign of the discount shock. More firms enter the market and, as the overall surplus increases, wages rise. Job filling rates decrease for firms, while unemployed workers have better chances to find a job. The intuition for the effects is similar as the one for SDF shocks. A positive increase in workers' productivity also increases the firms' value of a job. However, this occurs because of higher current and future expected cash flows $z_{t+s}-w_{t+s}$ from the match, as opposed to higher valuation of future cash flows. Because of increased time $t$ productivity, output responds on impact.

The amplification of SDF shocks largely depends on the persistence of the SDF shocks and the extent of wage rigidity. The left panel of Figure 2.7 illustrates the point. For a given degree of wage rigidity, a decrease in the persistence of the SDF shock makes unemployment react in a much less volatile manner. Moreover, the role of wage rigidity in the amplification of the shocks changes depending on the persistence. We draw similar conclusions about productivity shocks, as illustrated on the right panel of Figure 2.7. It remains true, however, that productivity shocks generate variation of unemployment (relative to output) one order of magnitude lower than SDF shocks (as illustrated by the different scale of the two panels).

### 2.3.4 The role of Labor Market Institutions

We begin analyzing the role of Labor Market Institutions by comparing the IRFs to the different shocks under different calibrations. As we are ultimately interested in the dynamics of unemployment, we focus on the response of unemployment to the different shocks and we provide the intuition for the changes by looking at the equations of the model.

We clarify here that we use the term "Labor Market Institution" in a broad sense. Through the lens of our model, a direct way a policy maker may influence labor markets is to change the policies to allocate unemployment insurance. However, we also think of LMIs as the environment in which the labor market exists. This includes, for example, the laws that define and regulate labor contracts. In this sense, LMIs also have an effect on


Figure 2.7: IRF of unemployment to a SDF shock (left) and to a productivity shock (right), for different wage rigidity $(\gamma)$ and persistence of each shock $\left(\rho^{\beta}, \rho^{z}\right)$.
how dynamic a market is, particularly in terms of the average durations of employment and unemployment.

The left panel of Figure 2.8 shows the response of unemployment to a positive discount factor shock calibrated with the $\operatorname{AR}(1)$ properties as in Table 2.3. However, the unemployment benefit, the job-finding probability and the separation rate are changed to capture a fluid labor market (the US) and a sclerotic labor market (European countries). In particular, the "fluid" calibration has $b=0.4, \tilde{p}=0.45$ and $\tilde{s}=0.03$, which are the baseline values. The "fluid (high b)" calibration has $b=0.7$ and $\tilde{p}$ and $\tilde{s}$ as above (where 0.7 approximates the values in European countries from Table 2.4). The "sclerotic" calibration has $b=0.7$, $\tilde{p}=0.07$ and $\tilde{s}=0.008$ (again see Table 2.4).

We make two observations. First, unemployment benefits do not impact the transmission or amplification of SDF shocks, while they significantly amplify productivity shocks. The relative average value of non work to work activities - $b$ in the model (with $z$ normal-
ized to 1)—has received a lot of attention in the literature. ${ }^{7}$ This because the literature on unemployment dynamics within search and matching models has so far focused on productivity shocks as a driving force. Productivity shocks impact hiring by changing current and future cash flows, whose response is in turn largely determined by the relative value of $b$ to productivity (via its effects on the relative values of productivity and wages). Discount shocks instead affect hiring by changing the valuation of given cash flows, in multiplicative manner, and their impact is thus unaffected by the relative average values of the cash flows components. Second, sclerotic labor markets exacerbate the effects of discount factor shocks on unemployment relative to fluid (with high $b$ ) markets: the response of unemployment is larger and more persistent. To understand why this is the case, consider the law of motion for unemployment (2.13) rearranged and log-linearized (assume the separation rate is constant):

$$
\hat{u}_{t+1}=(1-\tilde{s}-\tilde{p}) \hat{u}_{t}-\tilde{p} \hat{p}_{t} .
$$

Now, in fluid labor markets both $\tilde{p}$ and $\tilde{s}$ tend to be high, so that $1-\tilde{s}-\tilde{p}$ tends to be low. This means that the variation in unemployment is primarily driven by the job-finding rate. Conversely, in sclerotic labor markets, $\tilde{p}$ and $\tilde{s}$ are low, so that $1-\tilde{s}-\tilde{p}$ is high. This means that it is the variation in unemployment growth that is primarily driven by $\hat{p}_{t}$, which generate more persistent dynamics for unemployment.

The right panel of Figure 2.8 plots the response of unemployment to a positive productivity shock. Setting a high unemployment benefit in a fluid labor market amplifies the response of unemployment to a productivity shock, as discussed above. On the other hand, sclerotic markets increase the average duration of employment and unemployment, increasing the persistence of the response of unemployment to productivity shocks, but decreasing amplification.

We finally observe that the effects of Labor Market Institutions depend on how persistent the shocks are. The effect of the interaction between LMIs and the persistency of the shocks

[^38]

Figure 2.8: IRF of unemployment to a SDF shock (left) and to a productivity shock (right), for different calibrations.
is different for SDF and for productivity impulses. Figure 2.9 documents this fact. Again, the two calibrations only differ because of different values of the transition probabilities $\tilde{p}$ and $\tilde{s}$. In the left panel we see that a persistent discount factor shock is greatly amplified by sclerotic environments relative to fluid ones, although the effect relies on the persistence of the shock. With less persistent shocks, discount factors are less amplified. In this case, the magnitude of the response of unemployment is roughly unchanged across calibrations, although its persistence is higher in sclerotic environments. The effect travels through the increased average duration of both employment and unemployment. Conversely, the persistency of productivity shocks is less crucial than the fluidity of the market for the amplification mechanism.


Figure 2.9: IRF of unemployment to a SDF shock (left) and to a productivity shock (right), for different calibrations and persistency of the shocks.

### 2.4 Model with Heterogeneous Agents

### 2.4.1 Description of the model and main mechanisms

This model takes a standard Diamond-Mortensen-Pissarides model and adds a major departure: the presence of both temporary and permanent contracts. Temporary contracts can be endogenously destroyed at no cost, while permanent contracts can be destroyed subject to a firing cost. On the other hand, permanent contracts benefit from a lower probability of exogenous separation. To generate a trade-off between temporary and permanent contracts, we consider a distribution of match-specific productivities, which are first signalled and then randomly revealed at a later stage. When an unemployed worker and a firm are matched, a match-specific productivity $a$ is drawn, but only a noisy signal $s$ is observed by agents. Depending on the signal, agents decide to discard the match, sign a temporary contract or sign a permanent contract. After the first period of a contract, the productivity $a$ is revealed
with probability $\xi$ through a Calvo lottery. Depending on the information available to them, agents decide whether to terminate an existing contract, or to renew it. If a contract is temporary, then it can be converted to a permanent one. If a contract is permanent, then it can either be destroyed or renewed. However, if a contract is temporary, the option to renew it as such is unavailable to agents with probability $\phi$, in which case they either destroy the contract or convert it into a permanent one.

Temporary contracts provide a way for agents to insure against the risk of low productivity $a$ in face of a high signal $s$. Once such uncertainty is resolved, agents have no incentive to opt for a temporary contract other than the absence of a firing cost. The advantage of a permanent contract is the reduced risk of exogenous separation.

To understand better the model, let us consider a newly formed match and follow it through time. Matches that take place in period $t-1$ become productive only at period $t$. At the match, a signal $s$ over the the match-specific productivity is known. At the beginning of period $t$, the pair $\left(\beta_{t}, z_{t}\right)$ becomes common knowledge. As soon as aggregate uncertainty resolves, the worker and the firm involved in the new match bargain. Based on $s$ and the aggregate state, firms and workers bargain on wages and they decide whether to reject the match, write a temporary contract or write a permanent contract. After the contract has been written, production happens. With some exogenous probabilities $\lambda^{T}>\lambda^{P}$, the match is broken: the worker will go back to unemployment and the firm will post a new vacancy in period $t+1$. If the match is not broken, then agents learn the true value of $a$ with probability $\xi$. If they do not learn $a$, they retain their knowledge of $s$ and $a \mid s$. Period $t$ ends.

Period $t+1$ begins and the pair $\left(\beta_{t+2}, z_{t+1}\right)$ becomes common knowledge. Based on either $s$ or $a$ and the aggregate state, the worker and the firm bargain on the wage for period $t+1$ and decide whether to separate or continue the contract. If a contract continues, then it can be again either temporary or permanent. However, with probability $\phi$, the option of keeping a temporary contract is unavailable to agents. As the wage at $t+1$ is bargained before the new contract is written and before production takes place, wages for the second period of
a contract are going to depend on the type of contract that was in place in period $t$. In particular, the wage for a permanent contract at $t+1$ that was temporary at $t$ will differ from the wage for a permanent contract at $t+1$ that was permanent at $t$. This happens because the firm has different outside options. If the first contract was temporary, then the firm may decide to fire the worker at no cost. If the first contract was permanent, then the firm may decide to fire the worker at a fixed firing cost $f$. After the contract for period $t+1$ has been written production takes place. With some exogenous probability $\lambda^{T}$ or $\lambda^{P}$, the match is broken. If agents did not learn $a$ at the end of the previous period, then they will have the chance to learn it with probability $\xi$. Period $t+1$ ends.

If a match survives this far, then the dynamics of period $t+1$ will repeat in subsequent periods.

### 2.4.2 Timing

Within each period $t$, the following happens, in this order.

1. Aggregate uncertainty $\left(\beta_{t}, z_{t}\right)$ resolves.
2. New matches are formed: both $a$ and $s$ are drawn, but only $s$ is observed.
3. All agents in a match (newly formed or not) bargain over wages and contracts on the basis of $s$ or $a$ depending on their information set. Agents in an old match that are bound by a temporary contract are kept from renewing it as temporary with probability $\phi$ (and are so left with either endogenously breaking the contract or transorming it to a permanent).
4. Production takes place.
5. Matches are exogenously destroyed with probability $\lambda^{T}$ or $\lambda^{P}$, depending on which contract is in place.
6. Agents that know $s$ but not $a$ gain knowledge of $a$ with probability $\xi$. Those who do not learn $a$ retain their knowledge of $s$ and $a \mid s$.

### 2.4.3 The model

The choice of endogenously terminating a contract is a bilateral decision from both the firm and the worker. In other words, if the firm finds it convenient to fire the worker, also the worker will find it convenient to return to unemployment. This assumption simplifies the exposition and the solution of the model. We refer to the endogenous separations as the firm deciding to fire the worker. We interpret exogeonus separations as matches that break for reasons we cannot capture in the model (e.g., a firm ceasing its activity for reasons not related to the labor market). In case of an exogenous separation, the firm never incurs in any firing cost.

Idiosyncratic (match-specific) productivity

$$
a \stackrel{i i d}{\sim} \mathcal{N}\left(\mu_{a}, \sigma_{a}\right)
$$

Signal to employer at the match

$$
s=a+\sigma_{s} \varepsilon_{t}^{s} \quad \varepsilon_{t}^{s} \stackrel{i i d}{\sim} \mathcal{N}(0,1)
$$

The prior is $a \sim \mathcal{N}\left(\mu_{a}, \sigma_{a}^{2}\right)$, likelihood is $s \mid a \sim \mathcal{N}\left(a, \sigma_{s}^{2}\right)$. The posterior is

$$
a \left\lvert\, s \sim \mathcal{N}\left(\frac{s \sigma_{a}^{2}+\mu_{a} \sigma_{s}^{2}}{\sigma_{a}^{2}+\sigma_{s}^{2}}, \frac{\sigma_{a}^{2} \sigma_{s}^{2}}{\sigma_{a}^{2}+\sigma_{s}^{2}}\right) .\right.
$$

Let $F_{a \mid s}(\cdot)$ denote the CDF of $a \mid s$.
The exogenous processes are the aggregate discount factor $\beta$ and the aggregate produc-
tivity $z$. They evolve according to $\mathrm{AR}(1)$ processes:

$$
\begin{array}{ll}
z_{t}=\left(1-\rho_{z}\right) \bar{z}+\rho_{z} z_{t-1}+\sigma_{z} \varepsilon_{t}^{z} & \varepsilon_{t}^{z} \stackrel{i i d}{\sim} \mathcal{N}(0,1) \\
\beta_{t}=\left(1-\rho_{\beta}\right) \bar{\beta}+\rho_{\beta} \beta_{t-1}+\sigma_{\beta} \varepsilon_{t}^{\beta} & \varepsilon_{t}^{\beta} \stackrel{i i d}{\sim} \mathcal{N}(0,1)
\end{array}
$$

Note that $\beta_{t}$ is subject to the shock $\varepsilon_{t}^{\beta}$. In other words, the value $\beta_{t}$ is known at the beginning of period $t$.

In the following, the expectation operator ${ }_{t}(\cdot)$ is taken with respect to aggregate uncertainty, which is here given by productivity $z_{t}$ and discounts $\beta_{t}$. The expectation with respect to idiosyncratic uncertainty is explicitly written with an integral.

As anticipated above, we have two relevant categories of periods. The first one is about the first periods of a contract, from the match up until the learning of the match-specific productivity (unless the match is broken before this moment). In these periods, agents take decisions based on their knowledge of $s$ and the aggregate state. Each contract can be either permanent or temporary. The values and the wages in this category of periods present the superscript $T$ or $P$ to reflect the choice of the contract. After the first period of a match, the type of the contract in the previous period is relevant for wage bargaining as mentioned above. We keep track of this by attaching the superscripts $\{T, T\},\{T, P\}$ and $\{P, P\}$ to the value functions and the wages. Values and wages in this category of periods are function of $s$ and not of $a$.

The second type of periods is about the later periods of a contract starting from the moment where agents learn about $a$. Again, each contract can be either temporary or permanent and the type of the previous contract is relevant. Values and wages in this category of periods are function of $a$ and not of $s$. We group the exposition of the value functions, surpluses and wages by these periods.

## Periods of a match where only $s$ is known

Let $J_{t}^{i}$ denote the value of a temporary $(i=T)$ or a permanent $(i=P)$ job at the first period after a match. Let the value of a worker $W_{t}$ be superscripted similarly.

At the match, a match-specific productivity $a$ is drawn. However, at this stage workers and firms only observe the noisy signal $s=a+\varepsilon$. At the match, workers and firms bargain a wage and a contract type given their knowledge of $s$. Agents can decide to reject the match if the signal $s$ is too low. Otherwise they decide whether to start a temporary contract or a permanent one. The firm has the following values for a job at this stage and at this first period of a match, $J_{t}^{T}(s)$ and $J_{t}^{P}(s)$ :

$$
\begin{aligned}
J_{t}^{T}(s)= & \int_{-\infty}^{\infty} a F_{a \mid s}(a)+z_{t}-w_{t}^{T}(s)+\beta_{t t}\left(\lambda^{T} V_{t+1}+\left(1-\lambda^{T}\right) \xi \times\right. \\
& \times\left[(1-\phi) \int_{-\infty}^{\infty} \max \left\{V_{t+1} ; J_{t+1}^{T, T}(a) ; J_{t+1}^{T, P}(a)\right\} F_{a \mid s}(a)+\phi \int_{-\infty}^{\infty} \max \left\{V_{t+1} ; J_{t+1}^{T, P}(a)\right\} F_{a \mid s}(a)\right]+ \\
& \left.+\left(1-\lambda^{T}\right)(1-\xi)\left[(1-\phi) \max \left\{V_{t+1} ; J_{t+1}^{T, T}(s) ; J_{t+1}^{T, P}(s)\right\}+\phi \max \left\{V_{t+1} ; J_{t+1}^{T, P}(s)\right\}\right]\right) \\
J_{t}^{P}(s)= & \int_{-\infty}^{\infty} a F_{a \mid s}(a)+z_{t}-w_{t}^{P}(s)+\beta_{t t}\left(\lambda^{P} V_{t+1}+\left(1-\lambda^{P}\right) \times\right. \\
& \left.\times\left[\xi \int_{-\infty}^{\infty} \max \left\{V_{t+1}-f ; J_{t+1}^{P, P}(a)\right\} F_{a \mid s}(a)+(1-\xi) \max \left\{V_{t+1}-f ; J_{t+1}^{P, P}(s)\right\}\right]\right) .
\end{aligned}
$$

The firm observes the signal $s$ and computes the expected match-specific productivity given the posterior distribution of $a, F_{a \mid s}(a)$. It also collects the aggregate productivity $z_{t}$. It pays the wage, which depends on the type of contract. With probability $\lambda^{T}\left(\lambda^{P}\right)$, the temporary (permanent) contract is exogenously destroyed and the firm posts a new vacancy. If the match is not exogenously destroyed, then the firm decides whether to endogenously fire the worker and collect $V$ or to keep the contract. If the existing contract is temporary, it can either be renewed as such or converted to a permanent contract. The choice of keeping the temporary contract is unavailable to agents with probability $\phi$. If the existing contract is permanent, the firm will have to pay a fixed firing cost $f$ in order to fire the worker. With probability $\xi$, agents learn the true match-specific productivity $a$. If they do not learn it,
then they retain the knowledge of $s$ as drawn at the beginning of the match, together with the posterior distribution $a \mid s$.

On the other hand the worker collects the values of working $W_{t}^{T}(s)$ and $W_{t}^{P}(s)$ :

$$
\begin{aligned}
& W_{t}^{T}(s)= w_{t}^{T}(s)+\beta_{t t}\left(\lambda^{T} U_{t+1}+\left(1-\lambda^{T}\right) \xi\left[(1-\phi) \int_{-\infty}^{\infty} \max \left\{U_{t+1} ; W_{t+1}^{T, T}(a) ; W_{t+1}^{T, P}(a)\right\} F_{a \mid s}(a)+\right.\right. \\
&\left.+\phi \int_{-\infty}^{\infty} \max \left\{U_{t+1} ; W_{t+1}^{T, P}(a)\right\} F_{a \mid s}(a)\right]+ \\
&\left.+\left(1-\lambda^{T}\right)(1-\xi)\left[(1-\phi) \max \left\{U_{t+1} ; W_{t+1}^{T, T}(s) ; W_{t+1}^{T, P}(s)\right\}+\phi \max \left\{U_{t+1} ; W_{t+1}^{T, P}(s)\right\}\right]\right) \\
& W_{t}^{P}(s)=w_{t}^{P}(s)+\beta_{t t}\left(\lambda^{P} U_{t+1}+\left(1-\lambda^{P}\right)\left[\xi \int_{-\infty}^{\infty} \max \left\{U_{t+1} ; W_{t+1}^{P, P}(a)\right\} F_{a \mid s}(a)+\right.\right. \\
&\left.\left.+(1-\xi) \max \left\{U_{t+1} ; W_{t+1}^{P, P}(s)\right\}\right]\right) .
\end{aligned}
$$

Given the signal, the worker collects the wage, which again differs according to the type of contract. If the match is exogenously destroyed with probability $\lambda^{T}$ or $\lambda^{P}$, the worker collects the expected value of unemployment. If the match is not exogenously destroyed, then the worker either continues working or goes back to unemployment if the firm fires her.

After the first period of the match, agents may still ignore the true value of $a$. In this case, the value functions for the firm are

$$
\begin{aligned}
J_{t}^{T, T}(s)= & \int_{-\infty}^{\infty} a F_{a \mid s}(a)+z_{t}-w_{t}^{T, T}(s)+\beta_{t t}\left(\lambda^{T} V_{t+1}+\left(1-\lambda^{T}\right) \xi[(1-\phi) \times\right. \\
& \left.\times \int_{-\infty}^{\infty} \max \left\{V_{t+1} ; J_{t+1}^{T, T}(a) ; J_{t+1}^{T, P}(a)\right\} F_{a \mid s}(a)+\phi \int_{-\infty}^{\infty} \max \left\{V_{t+1} ; J_{t+1}^{T, P}(a)\right\} F_{a \mid s}(a)\right]+ \\
& \left.+\left(1-\lambda^{T}\right)(1-\xi)\left[(1-\phi) \max \left\{V_{t+1} ; J_{t+1}^{T, T}(s) ; J_{t+1}^{T, P}(s)\right\}+\phi \max \left\{V_{t+1} ; J_{t+1}^{T, P}(s)\right\}\right]\right) \\
J_{t}^{T, P}(s)= & \int_{-\infty}^{\infty} a F_{a \mid s}(a)+z_{t}-w_{t}^{T, P}(s)+\beta_{t t}\left(\lambda^{P} V_{t+1}+\left(1-\lambda^{P}\right) \xi \times\right. \\
& \left.\times \int_{-\infty}^{\infty} \max \left\{V_{t+1}-f ; J_{t+1}^{P, P}(a)\right\} F_{a \mid s}(a)+\left(1-\lambda^{P}\right)(1-\xi) \max \left\{V_{t+1}-f ; J_{t+1}^{P, P}(s)\right\}\right) \\
J_{t}^{P, P}(s)= & \int_{-\infty}^{\infty} a F_{a \mid s}(a)+z_{t}-w_{t}^{P, P}(s)+\beta_{t t}\left(\lambda^{P} V_{t+1}+\left(1-\lambda^{P}\right) \xi \times\right. \\
& \left.\times \int_{-\infty}^{\infty} \max \left\{V_{t+1}-f ; J_{t+1}^{P, P}(a)\right\} F_{a \mid s}(a)+\left(1-\lambda^{P}\right)(1-\xi) \max \left\{V_{t+1}-f ; J_{t+1}^{P, P}(s)\right\}\right)
\end{aligned}
$$

and for the worker are

$$
\begin{aligned}
& W_{t}^{T, T}(s)= w_{t}^{T, T}(s)+\beta_{t t}\left(\lambda^{T} U_{t+1}+\left(1-\lambda^{T}\right) \xi\left[(1-\phi) \int_{-\infty}^{\infty} \max \left\{U_{t+1} ; W_{t+1}^{T, T}(a) ; W_{t+1}^{T, P}(a)\right\} F_{a \mid s}(a)+\right.\right. \\
&\left.+\phi \int_{-\infty}^{\infty} \max \left\{U_{t+1} ; W_{t+1}^{T, P}(a)\right\} F_{a \mid s}(a)\right]+ \\
&\left.+\left(1-\lambda^{T}\right)(1-\xi)\left[(1-\phi) \max \left\{U_{t+1} ; W_{t+1}^{T, T}(s) ; W_{t+1}^{T, P}(s)\right\}+\phi \max \left\{U_{t+1} ; W_{t+1}^{T, P}(s)\right\}\right]\right) \\
& W_{t}^{T, P}(s)=w_{t}^{T, P}(s)+\beta_{t t}\left(\lambda^{P} U_{t+1}+\left(1-\lambda^{P}\right)\left[\xi \int_{-\infty}^{\infty} \max \left\{U_{t+1} ; W_{t+1}^{P, P}(a)\right\} F_{a \mid s}(a)+\right.\right. \\
&\left.\left.+(1-\xi) \max \left\{U_{t+1} ; W_{t+1}^{P, P}(s)\right\}\right]\right) \\
& W_{t}^{P, P}(s)=w_{t}^{P, P}(s)+\beta_{t t}\left(\lambda^{P} U_{t+1}+\left(1-\lambda^{P}\right)\left[\xi \int_{-\infty}^{\infty} \max \left\{U_{t+1} ; W_{t+1}^{P, P}(a)\right\} F_{a \mid s}(a)+\right.\right. \\
&\left.\left.\quad+(1-\xi) \max \left\{U_{t+1} ; W_{t+1}^{P, P}(s)\right\}\right]\right) .
\end{aligned}
$$

The meaning of these values is analogous to the Bellman equations above. The main difference lies in the wages, which depend on the type of the previous contract because of different outside options for the firm.

The surpluses from temporary and permanent contracts before such contracts are signed are

$$
\begin{aligned}
S_{t}^{T}(s) & =\left[J_{t}^{T}(s)-V_{t}\right]+\left[W_{t}^{T}(s)-U_{t}\right] \\
S_{t}^{P}(s) & =\left[J_{t}^{P}(s)-V_{t}\right]+\left[W_{t}^{P}(s)-U_{t}\right] \\
S_{t}^{T, T}(s) & =\left[J_{t}^{T, T}(s)-V_{t}\right]+\left[W_{t}^{T, T}(s)-U_{t}\right] \\
S_{t}^{T, P}(s) & =\left[J_{t}^{T, P}(s)-V_{t}\right]+\left[W_{t}^{T, P}(s)-U_{t}\right] \\
S_{t}^{P, P}(s) & =\left[J_{t}^{P, P}(s)-V_{t}\right]+\left[W_{t}^{P, P}(s)-U_{t}\right]
\end{aligned}
$$

The surpluses of the firm $J_{t}^{P}(s)$ and $J_{t}^{T, P}$ do not include the firing cost because the surpluses are measured before the contract in period $t$ is signed, so that the firm may simply decide not to sign a contract if the permanent one is not convenient.

Wages are set at the match according to Nash bargaining.

$$
\begin{aligned}
w_{t}^{T}(s) & =\arg \max _{w_{t}^{T}(s)}\left[J_{t}^{T}(s)-V_{t}\right]^{\eta}\left[W_{t}^{T}(s)-U_{t}\right]^{1-\eta} \\
w_{t}^{P}(s) & =\arg \max _{w_{t}^{P}(s)}\left[J_{t}^{P}(s)-V_{t}\right]^{\eta}\left[W_{t}^{P}(s)-U_{t}\right]^{1-\eta} \\
w_{t}^{T, T}(s) & =\arg \max _{w_{t}^{T, T}(s)}\left[J_{t}^{T, T}(s)-V_{t}\right]^{\eta}\left[W_{t}^{T, T}(s)-U_{t}\right]^{1-\eta} \\
w_{t}^{T, P}(s) & =\arg \max _{w_{t}^{T, P}(s)}\left[J_{t}^{T, P}(s)-V_{t}\right]^{\eta}\left[W_{t}^{T, P}(s)-U_{t}\right]^{1-\eta} \\
w_{t}^{P, P}(s) & =\arg \max _{w_{t}^{P, P}(s)}\left[J_{t}^{P, P}(s)-\left(V_{t}-f\right)\right]^{\eta}\left[W_{t}^{P, P}(s)-U_{t}\right]^{1-\eta} .
\end{aligned}
$$

Note that the relevant surplus of the firm in determining the wage of a permanent contract, $w_{t}^{P}(s)$, is simply $J_{t}^{P}(s)-V_{t}$ and not $J_{t}^{P}(s)-V_{t}-f$. The argument is analogous in the case of $w_{t}^{T, P}(s)$.

## Periods of a match where $a$ is known

Aggregate uncertainty resolves at the beginning of the period. Before production, the parties renegotiate the wage on the basis of $a$. Contracts that were previously temporary may be kept temporary or converted to permanent, or they can be rescinded. The option of keeping a temporary contract is unavailable to agents with probability $\phi$. Contracts that were permanent can only be kept permanent or rescinded. If the previous contract was temporary, and because such contract is in place at the moment of the renegotiation, the firm does not consider the firing cost when bargaining the wage for the second period. For this reason we need to keep track of the contract type in the previous period.

The firm's value of a job are the following:

$$
\begin{aligned}
& J_{t}^{T, T}(a)=a+z_{t}-w_{t}^{T, T}(a)+\beta_{t t}\left(\lambda^{T} V_{t+1}+\left(1-\lambda^{T}\right)\left[(1-\phi) \max \left\{V_{t+1} ; J_{t+1}^{T, T}(a) ; J_{t+1}^{T, P}(a)\right\}+\right.\right. \\
&\left.\left.+\phi \max \left\{V_{t+1} ; J_{t+1}^{T, P}(a)\right\}\right]\right) \\
& J_{t}^{T, P}(a)=a+z_{t}-w_{t}^{T, P}(a)+\beta_{t t}\left(\lambda^{P} V_{t+1}+\left(1-\lambda^{P}\right) \max \left\{V_{t+1}-f ; J_{t+1}^{P, P}(a)\right\}\right) \\
& J_{t}^{P, P}(a)=a+z_{t}-w_{t}^{P, P}(a)+\beta_{t t}\left(\lambda^{P} V_{t+1}+\left(1-\lambda^{P}\right) \max \left\{V_{t+1}-f ; J_{t+1}^{P, P}(a)\right\}\right) .
\end{aligned}
$$

Firms collect production $a+z_{t}$ and pay wages. If the contract is not exogenously destroyed with probability $\lambda^{P}$, then the firm decides whether to keep the worker or to fire her.

Workers' value of working is similar as above, with the exception of the wage.

$$
\begin{aligned}
& \begin{array}{l}
W_{t}^{T, T}(a)=w_{t}^{T, T}(a)+\beta_{t t}\left(\lambda^{T} U_{t+1}+\left(1-\lambda^{T}\right)\left[(1-\phi) \max \left\{U_{t+1} ; W_{t+1}^{T, T}(a) ; W_{t+1}^{T, P}(a)\right\}+\right.\right. \\
\left.\left.\quad+\phi \max \left\{U_{t+1} ; W_{t+1}^{T, P}(a)\right\}\right]\right) \\
W_{t}^{T, P}(a)=w_{t}^{T, P}(a)+\beta_{t t}\left(\lambda^{P} U_{t+1}+\left(1-\lambda^{P}\right) \max \left\{U_{t+1} ; W_{t+1}^{P, P}(a)\right\}\right) \\
W_{t}^{P, P}(a)=w_{t}^{P, P}(a)+\beta_{t t}\left(\lambda^{P} U_{t+1}+\left(1-\lambda^{P}\right) \max \left\{U_{t+1} ; W_{t+1}^{P, P}(a)\right\}\right) .
\end{array} .
\end{aligned}
$$

Surpluses after the first period of a contract before the new temporary/permanent contract is signed are

$$
\begin{aligned}
S_{t}^{T, T}(a) & =\left[J_{t}^{T, T}(a)-V_{t}\right]+\left[W_{t}^{T, T}(a)-U_{t}\right] \\
S_{t}^{T, P}(a) & =\left[J_{t}^{T, P}(a)-V_{t}\right]+\left[W_{t}^{T, P}(a)-U_{t}\right] \\
S_{t}^{P, P}(a) & =\left[J_{t}^{P, P}(a)-\left(V_{t}-f\right)\right]+\left[W_{t}^{P, P}(a)-U_{t}\right]
\end{aligned}
$$

Wages in this period are again Nash-bargained:

$$
\begin{aligned}
& w_{t}^{T, T}(a)=\arg \max _{w_{t}^{T, T}(a)}\left[J_{t}^{T, T}(a)-V_{t}\right]^{\eta}\left[W_{t}^{T, T}(a)-U_{t}\right]^{1-\eta} \\
& w_{t}^{T, P}(a)=\arg \max _{w_{t}^{T, P}(a)}\left[J_{t}^{T, P}(a)-V_{t}\right]^{\eta}\left[W_{t}^{T, P}(a)-U_{t}\right]^{1-\eta} \\
& w_{t}^{P, P}(a)=\arg \max _{w_{t}^{P, P}(a)}\left[J_{t}^{P, P}(a)-\left(V_{t}-f\right)\right]^{\eta}\left[W_{t}^{P, P}(a)-U_{t}\right]^{1-\eta}
\end{aligned}
$$

Again, the firm does not consider firing costs if the previous contract was temporary, as this renegotiation happens with the previous contract in place.

## Other equations of the model

Let $F_{s}(\cdot)$ denote the CDF of the signals $s$. The value of unemployment for a worker is:

$$
U_{t}=b+\beta_{t t}\left(\left(1-p_{t}\right) U_{t+1}+p_{t} \int_{-\infty}^{\infty} \max \left\{U_{t+1} ; W_{t+1}^{T}(s) ; W_{t+1}^{P}(s)\right\} F_{s}(s)\right)
$$

The unemployed worker collects the unemployment benefit $b$. She finds a job with probability $p_{t}$ If she finds a job, then she will complete the unemployment spell for period $t$ and the match will be effective at time $t+1$, at which point she may either get a temporary contract or a permanent one depending on the (unknown at this stage) draw of the signal $s$. If she does not find a job, she continues collecting the value of unemployment.

Firms need to post vacancies before hiring. The value of opening a vacancy is

$$
V_{t}=-\kappa+\beta_{t t}\left(\left(1-q_{t}\right) V_{t+1}+q_{t} \int_{-\infty}^{\infty} \max \left\{V_{t+1} ; J_{t+1}^{T}(s) ; J_{t+1}^{P}(s)\right\} F_{s}(s)\right)
$$

The firm faces a fixed cost $\kappa$ to open a vacancy. It will find a worker with probability $q_{t}$, but the match is assumed to be effective starting in period $t+1$. In this case, the firm will decide whether to sign a temporary or permanent contract on the basis of the (unknown at this stage) signal $s$. If the firm does not find a worker, then it collects the future value of a
vacancy.

We assume free-entry for firms, which drives the value of a vacancy to zero. By setting $V_{t}=0$ at all periods $t$, we have

$$
\frac{\kappa}{q_{t}}=\beta_{t t}\left(\int_{-\infty}^{\infty} \max \left\{0 ; J_{t+1}^{T}(s) ; J_{t+1}^{P}(s)\right\} F_{s}(s)\right)
$$

Matches depend on the stock of unemployed people and the number of open vacancies:

$$
m_{t}=\sigma_{m} u_{t}^{\sigma} v_{t}^{1-\sigma}
$$

The transition probabilities are:

$$
p_{t}=\frac{m_{t}}{u_{t}} \quad q_{t}=\frac{m_{t}}{v_{t}}
$$

Let $C_{t}(\cdot)$ denote the contract that is chosen by firms and workers. Formally,

$$
\begin{aligned}
& C_{t}^{0}(s) \equiv \arg \max \left\{V_{t} ; J_{t}^{T}(s) ; J_{t}^{P}(s)\right\} \in\{N, T, P\} \\
& C_{t}^{T}(s) \equiv \arg \max \left\{V_{t} ; J_{t}^{T, T}(s) ; J_{t}^{T, P}(s)\right\} \in\{N, T, P\} \\
& C_{t}^{P}(s) \equiv \arg \max \left\{V_{t}-f ; J_{t}^{P, P}(s)\right\} \in\{N, P\} \\
& C_{t}^{\phi}(s) \equiv \arg \max \left\{V_{t} ; J_{t}^{T, P}(s)\right\} \in\{N, P\} \\
& C_{t}^{T}(a) \equiv \arg \max \left\{V_{t} ; J_{t}^{T, T}(a) ; J_{t}^{T, P}(a)\right\} \in\{N, T, P\} \\
& C_{t}^{P}(a) \equiv \arg \max \left\{V_{t}-f ; J_{t}^{P, P}(a)\right\} \in\{N, P\} \\
& C_{t}^{\phi}(a) \equiv \arg \max \left\{V_{t} ; J_{t}^{T, P}(a)\right\} \in\{N, P\},
\end{aligned}
$$

where $N$ stands for none, $T$ for temporary and $P$ for permanent.

With these definitions, we start the following accounting exercise. The probabilities to
sign no/a temporary/a permanent contract in the first period of the matches are

$$
\begin{aligned}
N_{t}^{0} & \equiv \int_{-\infty}^{\infty} C_{t}^{0}(s)=N F_{s}(s) \\
T_{t}^{0} & \equiv \int_{-\infty}^{\infty} C_{t}^{0}(s)=T F_{s}(s) \\
P_{t}^{0} & \equiv \int_{-\infty}^{\infty} C_{t}^{0}(s)=P F_{s}(s) .
\end{aligned}
$$

In words, $N_{t}^{0}$ is the probability that new matches that are rejected, while $T_{t}^{0}$ and $P_{t}^{0}$ are the probabilities that temporary and permanent contracts are signed right after a match. The probabilities for contracts after the first period of a match that were previously temporary contracts are

$$
\begin{array}{rlr}
N_{t}^{T ; s} \equiv \int_{-\infty}^{\infty} C_{t}^{T}(s)=N F_{s}(s) & N_{t}^{T ; a} \equiv \int_{-\infty}^{\infty} C_{t}^{T}(a)=N F_{a}(a) \\
T_{t}^{T ; s} \equiv \int_{-\infty}^{\infty} C_{t}^{T}(s)=T F_{s}(s) & T_{t}^{T ; a} \equiv \int_{-\infty}^{\infty} C_{t}^{T}(a)=T F_{a}(a) \\
P_{t}^{T ; s} \equiv \int_{-\infty}^{\infty} C_{t}^{T}(s)=P F_{s}(s) & P_{t}^{T ; a} \equiv \int_{-\infty}^{\infty} C_{t}^{T}(a)=P F_{a}(a),
\end{array}
$$

those that were previously permanent are

$$
\begin{array}{ll}
N_{t}^{P ; s} \equiv \int_{-\infty}^{\infty} C_{t}^{P}(s)=N F_{s}(s) & N_{t}^{P ; a} \equiv \int_{-\infty}^{\infty} C_{t}^{P}(a)=N F_{a}(a) \\
P_{t}^{P ; s} \equiv \int_{-\infty}^{\infty} C_{t}^{P}(s)=P F_{s}(s) & P_{t}^{P ; a} \equiv \int_{-\infty}^{\infty} C_{t}^{P}(a)=P F_{a}(a),
\end{array}
$$

and those that were temporary and cannot be renewed (because of the lottery with probability $\phi)$

$$
\begin{array}{rlrl}
N_{t}^{\phi ; s} & \equiv \int_{-\infty}^{\infty} C_{t}^{\phi}(s)=N F_{s}(s) & N_{t}^{\phi ; a} \equiv \int_{-\infty}^{\infty} C_{t}^{\phi}(a)=N F_{a}(a) \\
P_{t}^{\phi ; s} \equiv \int_{-\infty}^{\infty} C_{t}^{\phi}(s)=P F_{s}(s) & P_{t}^{\phi ; a} \equiv \int_{-\infty}^{\infty} C_{t}^{\phi}(a)=P F_{a}(a)
\end{array}
$$

In words (and taking an example), $P_{t}^{T, a}$ is the probability that a worker signs a permanent
contract after coming from a temporary and given that she observed the match-specific productivity $a$.

We measure the stocks of workers that are employed with a temporary/permanent contract depending on whether they only know $s$ or they already know $a$. The mass of workers that obtain a job is:

$$
\begin{aligned}
& e_{t+1}^{T ; s}=p_{t} u_{t} T_{t}^{0} \\
& e_{t+1}^{P ; s}=p_{t} u_{t} P_{t}^{0}
\end{aligned}
$$

In essence, this is considering workers that find a job from unemployment $\left(p_{t} u_{t}\right)$ and that sign a temporary/permanent contract. There cannot be stocks $e_{t}^{; a}$ because nobody learns $a$ right after the match. The stocks of workers that are employed after the first period of a contract given they were employed under a temporary/permanent are:

$$
\begin{aligned}
e_{t+1}^{T, T} & =e_{t}^{T, T}+e_{t}^{T}\left(1-\lambda^{T}\right) T_{t}^{T}-e_{t}^{T, T}\left[\lambda^{T}+\left(1-\lambda^{T}\right)\left[\phi N_{t}^{\phi}+(1-\phi) N_{t}^{T}\right]\right] \\
e_{t+1}^{T, P} & =e_{t}^{T}\left(1-\lambda^{T}\right) P_{t}^{T} \\
e_{t+1}^{P, P} & =e_{t}^{P, P}+e_{t}^{P}\left(1-\lambda^{P}\right) P_{t}^{P}-e_{t}^{P, P}\left[\lambda^{P}+\left(1-\lambda^{P}\right) N_{t}^{P}\right] .
\end{aligned}
$$

Each worker transitions to a permanent contract provided that the match is not exogenously broken and provided that both the firm and the worker found it profitable to sign the new permanent contract. The transitions from permanent to permanent need to account for workers whose contract lasted for three periods or more, together with those that will become unemployed for either exogenous or endogenous reasons. Note that $e_{t}^{T, P}$ is reset at every period (i.e., it does not depend on its past value).

Unemployment evolves according to the following law of motion

$$
\begin{aligned}
u_{t+1}=u_{t} & -m_{t}\left[T_{t}^{0}+P_{t}^{0}\right]+\lambda^{T}\left[e_{t}^{T}+e_{t}^{T, T}\right]+\lambda^{P}\left[e_{t}^{P}+e_{t}^{T, P}+e_{t}^{P, P}\right]+ \\
& +\left(1-\lambda^{T}\right) N_{t}^{T}\left[e_{t}^{T}+e_{t}^{T, T}\right]+\left(1-\lambda^{P}\right) N_{t}^{P}\left[e_{t}^{P}+e_{t}^{T, P}+e_{t}^{P, P}\right] .
\end{aligned}
$$

### 2.5 Results

In this section we illustrate the results we obtain with the methodology and the models presented above. While we do have results with the representative agent model, we are still working on the model with heterogeneous agents. At the current stage, we only present the results we obtain with the representative agent model, which justify our attention to dual labor markets.

### 2.5.1 Model with Representative Agent

We finally turn to generating the series of simulated unemployment. We obtain the simulations by feeding in the shocks as estimated in (2.2) (changing the sign) and in (2.3) into (2.17) and (2.18) respectively. We produce quarterly simulations.

First we show the simulations by only feeding in SDF shocks. We show how the simulations are affected by different degrees of wage rigidity and by sclerotic labor markets. We repeat the analysis with simulations obtained by only using productivity shocks. For these specific simulations, where we comment on the differences between fluid and sclerotic environments, we only vary the transition probabilities $\tilde{p}$ and $\tilde{s}$. This means we keep the relative value of non-work to work activity, $b$, pinned down to 0.4 . We do so because we want to focus on the effect of slower transitions to and from unemployment and we want to abstract from different values of $b$. We also allow full wage flexibility. We finally allow both shocks into the final simulation, where we assess the relative contribution of each source of variation. As a way to numerically assess the "fit" of the simulations to the data, we regress simulated

Table 2.5: Covariance between simulations (by wage rigidity) and data relative to the volatility of observed (HP-filtered) unemployment. Only SDF shocks

| Wage rigidity | Germany | France | Spain | Italy |
| :--- | :---: | :---: | :---: | :---: |
| 2-5 Flexible $(\gamma=1)$ | 0.3415 | 0.7627 | 0.1523 | 0.3087 |
| Semi-rigid $(\gamma=0.5)$ | 0.4284 | 1.0123 | 0.2071 | 0.3893 |
| Rigid $(\gamma=0)$ | 0.5571 | 1.4315 | 0.3020 | 0.5133 |

unemployment on observed (HP-filtered) unemployment. If the model is able to perfectly match the data, the slope of the regression will be unity. If the simulated variation is less than observed volatility, then the absolute value of the slope will be between zero and one. If the simulations are more volatile then the data, the absolute value of the slope coefficient will be greater than one. If the sign of the slope is negative, then positive variation in the data is associated with a negative variation in the simulations.

Figure 2.10 shows the simulations obtained by only using SDF shocks by degree of wage rigidity. In doing this, we completely shut down productivity shocks. We observe that wage rigidity amplifies the variation of unemployment, although the effect is different across countries. This is not surprising, as we verified with the IRF in Figure 2.7 that the effect of wage rigidity varies with the persistence of discounts. The persistence of discounts in our data is between 0.7 and 0.8 . In particular, the persistence in Germany is lower than in other countries, explaining why the effect of wage rigidity in Germany is weaker. Table 2.5 accompanies these findings. We observe that introducing wage rigidity increases the correlation between the simulations and the data, with the effect being weaker in Germany. In the case of France, full wage rigidity makes the simulations more volatile relative to the data.

Figure 2.11 shows the simulated unemployment using only SDF shocks, by fluidity of the labor market. Here, the unemployment benefit $b$ is set to 0.4 to focus on the differences caused by the variation in transition probabilities. As we observed in the Impulse-Response Functions in Figure 2.8, fluid labor markets allow for similarly volatile but less persistent

Table 2.6: Covariance between simulations and data (by LMI), relative to the volatility in observed (HP-filtered) unemployment. Fully flexible wages. Unemployment benefit set to $b=0.4$.

| LMIs | Germany | France | Spain | Italy |
| :--- | :---: | :---: | :---: | :---: |
| Only $\beta$-shocks |  |  |  |  |
| Fluid | 0.2841 | 0.4057 | 0.0889 | 0.2570 |
| Sclerotic | 0.3415 | 0.7627 | 0.1523 | 0.3087 |
| Only z-shocks |  |  |  |  |
| Fluid | 0.0729 | 0.0422 | -0.0947 | 0.1319 |
| Sclerotic | 0.0171 | 0.0592 | -0.1181 | 0.0406 |

responses of unemployment relative to sclerotic environments. Moreover, the (small) differences between fluid and sclerotic environments are consistent with the finding in the left panel of Figure 2.9, where we showed that SDF shocks with lower persistence are less amplified in sclerotic markets than highly persistent ones. Yet, for all countries more sclerotic labor markets generate higher volatility than fluid ones conditional on discount shocks. The top panel of Table 2.6 shows that sclerotic markets are more important in France than in other countries in amplifying the variation in discounts.

We assess the role of LMIs on the propagation of productivity shocks with Figure 2.12. Consistently with the literature, our model with productivity shocks does not generate enough unemployment volatility to explain the data. Productivity does a worse job under sclerotic labor markets relative to fluid ones: as we observed in the right panel of Figure 2.8, productivity shocks cause more persistent but less volatile movements in unemployment. The bottom panel of Table 2.6 summarizes the results. The measure of "fit" for France slightly increases with the sclerotic calibration relative to the fluid one, but a closer inspection of the corresponding plot reveals that this is due to a better timing of the variations rather than to increased volatility. In Spain, productivity shocks cause the wrong signs in the variation of simulated unemployment. The joint dynamics of productivity and unemployment in Spain constitute a long-standing puzzle. As argued in Comin et al. (2019), this may be due to the reliance in Spain on fixed-term contracts in recent decades. During the Great Recession,


Figure 2.10: Simulated unemployment feeding in only SDF shocks, by degree of wage rigidity. Country specific calibration.


Figure 2.11: Simulated unemployment feeding in only SDF shocks, by fluidity of labor markets. Wages are fully flexible.

Table 2.7: Covariance between simulations and data, relative to the volatility in observed (HP-filtered) unemployment.

| Source of variation | Germany | France | Spain | Italy |
| :--- | :---: | :---: | :---: | :---: |
| 2-5 Only $z$-shocks | 0.0344 | 0.1147 | -0.2715 | 0.0776 |
| Only $\beta$-shocks | 0.3415 | 0.7627 | 0.1523 | 0.3087 |
| Both shocks | 0.3748 | 0.8781 | -0.1191 | 0.3882 |

workers in fixed-terms contracts, likely working lower hours and at lower productivity than workers in fixed-term contracts, have been the first to lose employment. This may explain why both output per worker and unemployment have increased.

We conclude by comparing the relative contribution of SDF shocks to productivity shocks. Figure 2.13 shows simulated unemployment as predicted by both shocks fed in the model. Table 2.7 provides a numerical representation of the results. It also shows the simulations where one shock is shut down, in order to provide a sense of the decomposition of the overall effects. We see that the simulations with both shocks are predominantly driven by SDF shocks as opposed to productivity shocks. Quantitatively, our model fits France better than Germany and Italy, while we predict the wrong variations in Spain due to productivity.

### 2.6 Conclusion

In this paper, we wrote a representative agent model of the labor market with search and matching frictions, augmented with a discount factor shock. We estimate discount factor and aggregate productivity shocks externally relative to the model. We simulated the model with the estimated shocks and compared the simulations to the data. We found that discount factor shocks can explain a significant portion of the volatility of unemployment, contrary to productivity shocks. However, neither the discount factor nor the productivity shocks can explain the differences in unemployment dynamics that we observe across European countries.

We are writing a heterogeneous agents model with match-specific productivities to intro-


Figure 2.12: Simulated unemployment feeding in only productivity shocks, by fluidity of labor markets. Wages are fully flexible.


Figure 2.13: Decomposition of simulated unemployment feeding in both shocks. Wages are fully flexible. Country-specific calibration.
duce Fixed-Term Contracts and Open-Ended Contracts.

## Appendix

### 2.7 Equations of the RA model

Here we list all equations of the model with a representative agent.

### 2.7.1 System of Equations

## Workers

Value of unemployment:

$$
U_{t}=b+\mathbf{E}_{t}\left\{\beta_{t+1}\left(p_{t} W_{t+1}+\left(1-p_{t}\right) U_{t+1}\right)\right\}
$$

Value of work:

$$
W_{t}=w_{t}+\mathbf{E}_{t}\left\{\beta_{t+1}\left(\left(1-s_{t}\right) W_{t+1}+s_{t} U_{t+1}\right)\right\} .
$$

Surplus:

$$
\begin{aligned}
W_{t}-U_{t} & =w_{t}+\mathbf{E}_{t}\left\{\beta_{t+1}\left(\left(1-s_{t}\right) W_{t+1}+s_{t} U_{t+1}\right)\right\} \\
& =-b-\mathbf{E}_{t}\left\{\beta_{t+1}\left(p_{t} W_{t+1}+\left(1-p_{t}\right) U_{t+1}\right)\right\} \\
& =w_{t}-b+\mathbf{E}_{t}\left\{\beta_{t+1}\left(1-s_{t}-p_{t}\right)\left(W_{t+1}-U_{t+1}\right)\right\} .
\end{aligned}
$$

## Firms

Value of a job:

$$
J_{t}=z_{t}-w_{t}+\mathbf{E}_{t}\left\{\beta_{t+1}\left(\left(1-s_{t}\right) J_{t+1}+s_{t} V_{t+1}\right)\right\}
$$

Value of a vacancy:

$$
V_{t}=-\kappa+\mathbf{E}_{t}\left\{\beta_{t+1}\left(q_{t} J_{t+1}+\left(1-q_{t}\right) V_{t+1}\right)\right\} .
$$

Free-entry condition:

$$
\begin{aligned}
-\kappa+\mathbf{E}_{t}\left\{\beta_{t+1} q_{t} J_{t+1}\right\} & =0 \\
\frac{\kappa}{q_{t}} & =\mathbf{E}_{t}\left\{\beta_{t+1} J_{t+1}\right\} .
\end{aligned}
$$

Output:

$$
y_{t}=z_{t}\left(1-u_{t}\right) .
$$

The previous equations give:

$$
J_{t}=z_{t}-w_{t}+\mathbf{E}_{t}\left\{\beta_{t+1}\left(1-s_{t}\right) J_{t+1}\right\}
$$

## Matching

Matching technology:

$$
m_{t}=\sigma^{m} u_{t}^{\sigma} v_{t}^{1-\sigma} .
$$

Law of motion of unemployment:

$$
u_{t+1}=u_{t}+s_{t}\left(1-u_{t}\right)-m_{t}
$$

Job-finding rate:

$$
p_{t}=\frac{m_{t}}{u_{t}} .
$$

Job-filling rate:

$$
q_{t}=\frac{m_{t}}{v_{t}}
$$

## Wage Bargaining

Nash problem:

$$
w_{t}=\arg \max _{w_{t}}\left(W_{t}-U_{t}\right)^{\eta}\left(J_{t}\right)^{1-\eta}
$$

Sharing rule:

$$
\begin{aligned}
\eta J_{t} & =(1-\eta)\left(W_{t}-U_{t}\right) \\
\eta\left(z_{t}-w_{t}+\mathbf{E}_{t}\left\{\beta_{t+1}\left(1-s_{t}\right) J_{t+1}\right\}\right) & =(1-\eta)\left(w_{t}-b+\mathbf{E}_{t}\left\{\beta_{t+1}\left(1-s_{t}-p_{t}\right)\left(W_{t+1}-U_{t+1}\right)\right\}\right) \\
\eta\left(z_{t}-w_{t}+\left(1-s_{t}\right) \frac{\kappa}{q_{t}}\right) & =(1-\eta)\left(w_{t}-b+\left(1-s_{t}-p_{t}\right) \frac{\eta}{1-\eta} \frac{\kappa}{q_{t}}\right) \\
w_{t} & =\eta\left(z_{t}+p_{t} \frac{\kappa}{q_{t}}\right)+(1-\eta) b .
\end{aligned}
$$

## Exogenous Processes

Discount factor:

$$
\log \left(\beta_{t}\right)=\left(1-\rho^{\beta}\right) \log (\tilde{\beta})+\rho^{\beta} \log \left(\beta_{t-1}\right)+\sigma^{\beta} \varepsilon_{t}^{\beta}, \quad \quad \varepsilon_{t}^{\beta} \sim \mathcal{N}(0,1)
$$

Workers' productivity:

$$
\log \left(z_{t}\right)=\left(1-\rho^{z}\right) \log (\tilde{z})+\rho^{z} \log \left(z_{t-1}\right)+\sigma^{z} \varepsilon_{t}^{z}, \quad \varepsilon_{t}^{z} \sim \mathcal{N}(0,1)
$$

Separation rate:

$$
\log \left(s_{t}\right)=\left(1-\rho^{s}\right) \log (\tilde{s})+\rho^{z} \log \left(s_{t-1}\right)+\sigma^{s} \varepsilon_{t}^{s}, \quad \varepsilon_{t}^{s} \sim \mathcal{N}(0,1)
$$

### 2.7.2 System of Log-Linear Equations

Matching

$$
\tilde{m}_{t}=\sigma \hat{u}_{t}+(1-\sigma) \hat{v}_{t}
$$

Unemployment

$$
\begin{aligned}
u_{t+1} & =u_{t}+s_{t}\left(1-u_{t}\right)-m_{t} \\
\tilde{u} \hat{u}_{t+1} & =\tilde{u} \hat{u}_{t}+\tilde{s}(1-\tilde{u}) \hat{s}_{t}-\tilde{s} \tilde{u} \hat{u}_{t}-\tilde{m} \hat{m}_{t} \\
\hat{u}_{t+1} & =\hat{u}_{t}+\frac{\tilde{s}(1-\tilde{u})}{\tilde{u}} \hat{s}_{t}-\tilde{s} \hat{u}_{t}-\tilde{p} \hat{m}_{t}
\end{aligned}
$$

Job-finding rate

$$
\hat{p}_{t}=\hat{m}_{t}-\hat{u}_{t}
$$

Job-filling rate

$$
\hat{q}_{t}=\hat{m}_{t}-\hat{v}_{t}
$$

Wage

$$
\tilde{w} \hat{w}_{t}=\eta \tilde{z} \hat{z}_{t}+\eta \tilde{p} \frac{\kappa}{\tilde{q}}\left(\hat{p}_{t}-\hat{q}_{t}\right)
$$

Free entry

$$
-\hat{q}_{t}=\mathbf{E}_{t}\left\{\hat{\beta}_{t+1}+\hat{J}_{t+1}\right\}
$$

Value of a job

$$
\tilde{J} \hat{J}_{t}=\tilde{z} \hat{z}_{t}-\tilde{w} \hat{w}_{t}+(1-\tilde{s}) \mathbf{E}_{t}\left\{\tilde{\beta} \tilde{J}\left(\hat{\beta}_{t+1}+\hat{J}_{t+1}\right)\right\}-\tilde{\beta} \tilde{J} \tilde{s} \hat{s}_{t}
$$

Value of unemployment

$$
\begin{aligned}
\tilde{U} \hat{U}_{t} & =\mathbf{E}_{t}\left\{\tilde{\beta} \tilde{p} \tilde{W}\left(\hat{\beta}_{t+1}+\hat{p}_{t}+\hat{W}_{t+1}\right)\right\} \\
& +\mathbf{E}_{t}\left\{\tilde{\beta} \tilde{U}\left(\hat{\beta}_{t+1}+\hat{U}_{t+1}\right)-\tilde{p} \tilde{\beta} \tilde{U}\left(\hat{\beta}_{t+1}+\hat{p}_{t}+\hat{U}_{t+1}\right)\right\}
\end{aligned}
$$

Value of work

$$
\begin{aligned}
\tilde{W} \hat{W}_{t} & =\tilde{w} \hat{w}_{t}+\mathbf{E}_{t}\left\{\tilde{\beta}(1-\tilde{s}) \tilde{W}\left(\hat{\beta}_{t+1}+\hat{W}_{t+1}\right)\right\} \\
& +\mathbf{E}_{t}\left\{\tilde{\beta} \tilde{s} \tilde{U}\left(\hat{\beta}_{t+1}+\hat{U}_{t+1}\right)\right\}-\tilde{\beta}(\tilde{W}-\tilde{U}) \tilde{s} \hat{s}_{t}
\end{aligned}
$$

Output

$$
\hat{y}_{t}=\tilde{z}_{t}-\frac{\tilde{u}}{1-\tilde{u}} \hat{u}_{t}
$$

Market tightness

$$
\hat{\theta}_{t}=\hat{v}_{t}-\hat{u}_{t}
$$

Discount factor shock

$$
\begin{aligned}
\hat{\beta}_{t} & =\rho^{\beta} \hat{\beta}_{t-1}+\sigma^{\beta} \varepsilon_{t}^{\beta} \\
\varepsilon_{t}^{\beta} & \sim \mathcal{N}(0,1)
\end{aligned}
$$

Productivity shock

$$
\begin{aligned}
& \hat{z}_{t}=\rho^{z} \hat{z}_{t-1}+\sigma^{z} \varepsilon_{t}^{z} \\
& \varepsilon_{t}^{z} \sim \mathcal{N}(0,1)
\end{aligned}
$$

Separation shock

$$
\begin{aligned}
& \hat{s}_{t}=\rho^{s} \hat{s}_{t-1}+\sigma^{s} \varepsilon_{t}^{s} \\
& \varepsilon_{t}^{s} \sim \mathcal{N}(0,1)
\end{aligned}
$$

### 2.7.3 Steady State

Matching

$$
\tilde{m}=\sigma^{m} \tilde{u}^{\sigma} \tilde{v}^{1-\sigma}
$$

Unemployment

$$
\begin{aligned}
& 0=\tilde{s}(1-\tilde{u})-\tilde{p} \tilde{u} \\
& \tilde{u}=\frac{\tilde{s}}{\tilde{s}+\tilde{p}}
\end{aligned}
$$

Job-finding rate

$$
\tilde{p}=\frac{\tilde{m}}{\tilde{u}}
$$

Job-filling rate

$$
\tilde{q}=\frac{\tilde{m}}{\tilde{v}}
$$

Wage

$$
\tilde{w}=\eta\left(\tilde{z}+\tilde{p} \frac{\kappa}{\tilde{q}}\right)+(1-\eta) b
$$

Free entry

$$
\frac{\kappa}{\tilde{q}}=\tilde{\beta} \tilde{J}
$$

Value of a job

$$
\tilde{J}=\tilde{z}-\tilde{w}+\tilde{\beta}(1-\tilde{s}) \tilde{J}
$$

Value of unemployment

$$
\tilde{U}=b+\tilde{\beta}(\tilde{p} \tilde{W}+(1-\tilde{p}) \tilde{U})
$$

Value of work

$$
\tilde{W}=\tilde{w}+\tilde{\beta}((1-\tilde{s}) \tilde{W}+\tilde{s} \tilde{U})
$$

Output

$$
\tilde{y}=\tilde{z}(1-\tilde{u})
$$

Market tightness

$$
\tilde{\theta}=\frac{\tilde{v}}{\tilde{u}}
$$

### 2.8 Equations of the HA model

### 2.8.1 Signals and abilities

The prior and the likelihood are

$$
f_{a}(a)=\frac{1}{\sqrt{2 \pi \sigma_{a}^{2}}} \cdot \exp \left(-\frac{1}{2}\left[\frac{a-\mu_{a}}{\sigma_{a}}\right]^{2}\right), \quad f_{s \mid a}(s)=\frac{1}{\sqrt{2 \pi \sigma_{s}^{2}}} \cdot \exp \left(-\frac{1}{2}\left[\frac{s-a}{\sigma_{s}}\right]^{2}\right)
$$

The posterior is

$$
\begin{aligned}
f_{a \mid s}(a) & \equiv \frac{f_{a}(a) \cdot f_{s \mid a}(s)}{f_{s}(s)} \\
& \propto f_{a}(a) \cdot f_{s \mid a}(s) \\
& \propto \exp \left(-\frac{1}{2}\left[\frac{a-\mu_{a}}{\sigma_{a}}\right]^{2}\right) \cdot \exp \left(-\frac{1}{2}\left[\frac{s-a}{\sigma_{s}}\right]^{2}\right) \\
& \propto \exp \left(-\frac{1}{2}\left[\frac{a^{2}-2 a \mu_{a}+\mu_{a}^{2}}{\sigma_{a}^{2}}+\frac{s^{2}-2 s a+a^{2}}{\sigma_{s}^{2}}\right]\right) \\
& \propto \exp \left(-\frac{1}{2}\left[\frac{a^{2}-2 a \mu_{a}}{\sigma_{a}^{2}}+\frac{a^{2}-2 s a}{\sigma_{s}^{2}}\right]\right) \\
& \propto \exp \left(-\frac{1}{2}\left[\frac{a^{2}\left(\sigma_{a}^{2}+\sigma_{s}^{2}\right)-2 a\left(s \sigma_{a}^{2}+\mu_{a} \sigma_{s}^{2}\right)}{\sigma_{a}^{2} \cdot \sigma_{s}^{2}}\right]\right) \\
& \propto \exp \left(-\frac{1}{2}\left[a^{2} \cdot \frac{\sigma_{a}^{2}+\sigma_{s}^{2}}{\sigma_{a}^{2} \sigma_{s}^{2}}-2 a \cdot \frac{s \sigma_{a}^{2}+\mu_{a} \sigma_{s}^{2}}{\sigma_{a}^{2} \sigma_{s}^{2}}\right]\right) .
\end{aligned}
$$

This is the kernel of a univariate Gaussian distribution. The term that multiplies $a^{2}$ is the inverse of the variance, while the term that multiplies $-2 a$ is the mean divided by the
variance. ${ }^{8}$ Hence

$$
(a \mid s)=\frac{s \sigma_{a}^{2}+\mu_{a} \sigma_{s}^{2}}{\sigma_{a}^{2} \sigma_{s}^{2}} \cdot(a \mid s), \quad(a \mid s)=\left[\frac{\sigma_{a}^{2}+\sigma_{s}^{2}}{\sigma_{a}^{2} \sigma_{s}^{2}}\right]^{-1}
$$

from which we get

$$
(a \mid s)=\frac{s \sigma_{a}^{2}+\mu_{a} \sigma_{s}^{2}}{\sigma_{a}^{2}+\sigma_{s}^{2}}, \quad(a \mid s)=\frac{\sigma_{a}^{2} \sigma_{s}^{2}}{\sigma_{a}^{2}+\sigma_{s}^{2}}
$$

### 2.9 Robustness Checks in Inference of SDF Shocks

We have inferred shocks to the SDF in several ways

- Using data on government yields, both "as is" and net of EONIA. Simulations exhibit volatility, but not as much as those coming from stock market data. We decided to keep these data as alternative, and to focus on stock market data.
- Using stock market returns, both "as is" and net of EONIA. We did this in a number of ways: using ex-post (realized and observed) data and inferring ex-ante returns. We define ex-ante returns the fraction of observed returns that can be predicted by another variable one period in advance. As this explicitly involves (rational) expectations, we narrow our attention on the variations that can be due to differences in future expectations. With US data, a natural choice here would be the price-dividend ratio, which is known for its predictive power on stock prices. However, the ratio does not share this desirable property in our data.

[^39]Table 2.8: Volatility of monthly simulations relative to volatility of data. Country-specific calibration. Fully flexible wages.

| Fed-in shock | Germany | France | Spain | Italy |
| :--- | :---: | :---: | :---: | :---: |
| 2-5 Ex-post stock mkt data | 0.54726 | 0.89685 | 0.3795 | 0.63203 |
| Ex-ante - dp ratios | 0.075022 | 0.02551 | 0.04048 | 0.015882 |
| Ex-ante - CLEI | 0.32045 | 0.78644 | 0.28654 | 0.83183 |
| Ex-ante - ELEI | 0.45528 | 0.76674 | 0.34899 | 0.68384 |

- Ex-post data: simulations exhibit the magnitude of volatility we expected, especially after seeing the impulse-reponse functions of the model. There is a concern about identification of shocks: using realized data, we capture also shocks to dividend growth and other variables affecting returns that do not enter our model.
- Ex-ante data, using the dividend-price ratio as predictor: simulations of unemployment exhibit very little variation, even lower than simulations using gov't bond yields. The reason is that the dividend-price ratio is a poor predictor of European stock market returns. This fact is documented in very few papers in the literature.
- Ex-ante data, using the ELEI (European Leading Economic Indicator) as predictor: simulations are almost as volatile as those obtained with ex-post data. This is in line with the findings of Zhu and Zhu (2014), who show that LEIs are good predictors of European stock market returns.
- Ex-ante data, using the CLEI (Country-specific Leading Economic Indicator) as predictor: simulations are almost identical to those found with ex-ante returns on ELEI. Differences are most noticeable in the period 2010-2012 and for Spain and Italy.

Table 2.9: Volatility of monthly simulations relative to volatility of data. US calibration. Fully flexible wages.

| Fed-in shock | Germany | France | Spain | Italy |
| :--- | :---: | :---: | :---: | :---: |
| 2-5 Ex-post stock mkt data | 0.47964 | 0.51982 | 0.14959 | 0.41885 |
| Ex-ante - dp ratios | 0.094329 | 0.028839 | 0.018192 | 0.020127 |
| Ex-ante - CLEI | 0.18993 | 0.3105 | 0.076425 | 0.35919 |
| Ex-ante - ELEI | 0.25981 | 0.32699 | 0.097434 | 0.28228 |

## Chapter 3

## Labor Mobility of Heterogeneous Workers in the European Union <br> Riccardo Franceschin \& Simon Görlach

### 3.1 Introduction

Labor mobility is a corner stone in research on optimal currency areas, pioneered by Mundell (1961). In the presence of price rigidities, labor mobility generally is expected to help mitigate the burden of asymmetric shocks to demand or to productivity when exchange rate adjustments are prevented by the common currency (Blanchard and Katz, 1992; Boeri and Brücker, 2005; Lane, 2006). Indeed, the European monetary union has seen an increase in south-north migration in the wake of the euro crisis, which hit member countries of the currency union asymmetrically. Yet, if workers are heterogeneous in both their productivity and their propensity to migrate, any complementary across skill groups may thwart the potential of labor mobility to help absorbing asymmetric shocks. Absent any inter-regional fiscal compensations, selective labor flows can even aggravate-rather than absorb-macroeconomic shocks. This is not a theoretical curiosity, as mobility has repeatedly been shown to rise with workers' skill and income level (Grogger and Hanson, 2011; Docquier and Rapoport, 2012).

In this paper, we first show that Europe is no exception to this empirical fact, by documenting that within Europe highly educated workers are considerably more mobile. We then develop a dynamic equilibrium model with search frictions and different types of workers to evaluate the contribution of labor mobility to the absorption of asymmetric shocks within a currency union. Skilled and unskilled workers are not perfect substitutes in production, and are heterogeneous in their preference for different countries. National markets are integrated and different countries' goods are not perfect substitutes to consumers, so that each individual consumes both domestic and foreign goods. To make the model applicable to the European context, where some but not all countries of the Single Market share the same currency, our model features nominal wage rigidity, which gives a role to monetary policy, and which allows us to reproduce realistic fluctuations in employment.

We calibrate this model to data from the European Union Labour Force Survey and macroeconomic data for several European countries. From these data, we can recover migration flows as well as labor market outcomes. The calibration provides us with preference parameters, which together with international wage differentials determine migration flows at the steady state. Our calibration confirms that even conditional on observed employment and wage differentials across different labor markets, it is generally less costly for skilled individuals to live and work in a foreign country. The model allows us to evaluate the degree to which the effect of asymmetric demand and productivity shocks on unemployment and earnings levels across countries is altered by international migration, accounting for the empirical fact that high skilled workers are generally more mobile.

In a theoretical analysis Farhi and Werning (2014) highlight the importance of distinguishing asymmetric shocks to tradable and non-tradable sectors. They show that migration may leave the effect on per-capita outcomes unchanged in case of an asymmetric demand shock on the non-tradable sector, since the outflows of migrants would further reduce the demand for goods and services of the sector hit. On the other hand, migration out of a region that suffers a negative shock to its tradable sector is predicted to improve outcomes for stay-
ers. Relative to their setup, international mobility in our model with worker heterogeneity has a less straightforward effect on the welfare of different groups of stayers, even when goods are tradable. The less positive effects of labor mobility may derive both from a worsening of the worker pool in the country hit by the shock if more productive workers emigrate, and due to increased scarcity of an essential input factor if different worker types are complements in production. A net assessment of the benefits of labor mobility in a currency union thus becomes an empirical question.

We examine both the effects of permanent changes in productivity and demand parameters on the steady state, as well as the transition between equilibria. Furthermore, we evaluate the effects of a non-permanent shock to the productivity of one country. Our counterfactual analysis is similar to that by Di Giovanni et al. (2015) in that we compare outcomes under the baseline with migration costs estimated to match observed migration levels to outcomes when migration is prohibited. As a first scenario, we consider the case in which a negative TFP or demand shock hits one economy in the monetary union (Italy, in our example), and we compare the adjustments that follow to a counterfactual scenario in which workers have no possibility to migrate. Both permanent demand and productivity shocks lead to an increase in unemployment and lower wages for all skill groups in the country hit by the shock. While this triggers a rise in net outflows across skill groups from the country hit by the shock, emigration flows increase relatively more among the high skilled.

Compared to the non-migration scenario, high skilled stayers considerably benefit from this outflow as both unemployment and wages revert back towards the initial steady state. For low skilled stayers, in contrast, migration leads to an only small reduction in unemployment. The latter results from the lower propensity of low skilled workers to migrate. Together with a complementarity in production, this "brain drain" reduces the productivity of stayers which counteracts the otherwise positive effects of emigration in response to a negative shock that operates through an increase in labor market tightness.

The difference in the ability of migration to mitigate the negative effects of a permanent
asymmetric shock extends to workers' welfare. ${ }^{1}$ Whereas the welfare loss of high skilled workers is almost halved by the possibility to migrate out, migration reduces the loss in welfare for low skilled workers by only about $20 \%$. The magnitude of this loss reduction is similar to that resulting from a complete removal of wage rigidities.

Overall, these results suggest that labor mobility does play a positive role in mitigating the effects of an asymmetric shock. Nonetheless, it cannot fully substitute for other equilibrating policies, in particular since its benefits are highly heterogeneous across the workforce. We also compare our findings to a situation in which the asymmetric shock is non-permanent, and find that the mitigating effects of migration are more pronounced for labor outcomes of high skilled workers, whereas migration leaves the effects on unskilled worker virtually unchanged. Overall, the impact of migration becomes much less relevant for a non-persistent shock. The reason is that migration rates are generally low, so that despite a short-term increase in emigration, effects on GDP per capita or unemployment are small. At current levels of migration flows in Europe, migration thus only can have a limited role in the mitigation of transitory asymmetric shocks.

Our paper contributes to the literature that studies the role of labor mobility in mitigating regional differences in business cycles within a currency union, an area that has received renewed attention since the introduction of the euro. Arpaia et al. (2016), and Basso et al. (2018) investigate the co-movement of labor demand and migration in Europe, and broadly find that migration is both cyclical and cushions asymmetric shocks. In a related strand of papers, House et al. (2018), Hart and Clemens (2019), and Hauser and Seneca (2019) formulate dynamic equilibrium models with an explicit role for monetary policy. One of the central results in these papers is the prediction of a positive effect of international mobility when parts of a currency area are exposed to asymmetric shocks. A common feature of the models used, however, is the homogeneity of workers, who respond to macroeconomic conditions, and in turn shape equilibrium outcomes. We build on this research, but argue that results may

[^40]require a qualification if heterogeneous workers are both complementary in production and differ in their preference for migration. Earlier papers analyzing macroeconomic adjustments in the euro zone, like Smets and Wouters (2003), have abstracted from labor mobility.

Spatial equilibrium models featuring labor mobility have also been used to evaluate the effects of trade exposure and factor market integration (Kovak, 2013; Caliendo et al., 2017, 2019; Dix-Carneiro and Kovak, 2017, 2019), though without a role for monetary policy. ${ }^{2}$ The fact that migration from Southern to Northern Europe is predominantly high skilled has also been documented in a recent paper by Schivardi and Schmitz (2018), who evaluate the contribution of the IT revolution to the divergence across European countries. The authors use an equilibrium model in which high skilled workers may migrate to Northern European countries, where IT adoption has advanced further.

Finally, our paper contributes to literature evaluating dynamic responses in labor market outcomes for non-migrants following an increase in immigration. While most studies examine short-run effects of immigration in static settings, Braun and Weber (2016) and Monras (2020), for instance, examine labor market adjustments over time following the post-World War II refugee migration shock in Germany and the Mexican peso crisis, respectively. (Llull, 2018) examines the effect of native's human capital decisions. More similar to our framework are studies by Chassamboulli and Palivos (2014), Battisti et al. (2018) and Zaharieva and Iftikhar (2019), who use equilibrium search models to evaluate the economic and welfare effects of immigration in different sets of countries. Our paper differs in several dimensions, most importantly in that we endogenize migration, in that migration can be temporary, and in that we explicitly account for a subset of countries belonging to a currency union.

In what follows we first sort ideas about the conditions under which international mobility of labor may have adverse effects on less productive countries. In Section 3.3 we then provide empirical evidence for the importance of accounting for worker heterogeneity in our setting, which motivates the model presented in Section 3.4. We describe the data used, and the

[^41]model's calibration in Sections 3.5 and 3.6, and finally present the results of our analysis in Section 3.7.

### 3.2 Skills, complementarity and moving costs

To demonstrate the theoretical possibility of a negative effect of migration on non-migrants in a country suffering from decreased productivity, and to illustrate how this effect depends on some core parameters, we setup a simple two country model with two types of workers. In Section 3.4 , we bring a fully fledged dynamic general equilibrium model with search friction and a role for monetary policy to the data in order to evaluate the degree to which migration within the euro zone can help mitigate asymmetric shocks.

For now, suppose that there are only two countries $j \in\{1,2\}$, whose national production derives from the input of $H_{j}$ high skilled and $L_{j}$ low skilled workers according to a production function

$$
Y_{j}=A_{j}\left(\alpha L_{j}^{\rho}+(1-\alpha) H_{j}^{\rho}\right)^{1 / \rho}
$$

with total factor productivity $A_{j}$, income shares $\alpha$ and $(1-\alpha)$ for low and high skilled workers, and an elasticity of substitution $1 /(1-\rho)$ between the two inputs. In the simplified model presented here, suppose that workers are paid their marginal product and that location choices only depend on wages and local amenities. These amenities, however, are valued differently across individuals, with the payoff for individual $i$ of nationality $n$ from working in country $j$ given by

$$
u_{n, j}^{L}=w_{j}^{L}+\epsilon_{i, j_{i}}=\alpha A_{j}^{\rho}\left(Y_{j} / L_{j}\right)^{1-\rho}+\epsilon_{i, j_{i}}
$$

if the individual is low skilled, and by

$$
u_{n, j}^{H}=w_{j}^{H}+\epsilon_{i, j_{i}}=(1-\alpha) A_{j}^{\rho}\left(Y_{j} / H_{j}\right)^{1-\rho}+\epsilon_{i, j_{i}},
$$

if not, where $\epsilon_{i, j_{i}}$ is drawn from a nationality and skill-specific distribution. In Section 3.4, a dynamic model with search frictions and country specific income shares will be matched to the distribution of wages and unemployment rates across countries and skill types. If the idiosyncratic valuations for location amenities are drawn from a type I extreme value distribution with skill-specific mean $\mu_{n, j}^{s}$ for workers with skill $s \in\{H, L\}$ and origin nationality $n \in\{1,2\}$, a fraction

$$
\left(u_{j}^{s}>u_{n}^{s}\right)=\frac{\exp \left(w_{j}^{s}+\mu_{n, j}^{s}\right)}{\left(\exp \left(w_{j}^{s}+\mu_{n, j}^{s}\right)+\exp \left(w_{n}^{s}+\mu_{n, n}^{s}\right)\right)}
$$

would derive higher utility from moving to $j$ than from staying in $n$. Yet, migration of workers of either skill type will affect wages. If allowed, migration will thus continue until payoffs within each skill type are equalized across countries.

For $\alpha<0.5$ and $A_{1}<A_{2}$, two fundamentals in this simple model determine the effect which allowing for migration has on the wages of non-migrants in the less productive country: the degree of complementarity between high and low skilled workers, $\rho$, and the average relative preference for different locations, $\mu_{n, j}^{s}$. To make a case, suppose $\mu_{n, j}^{L}<\mu_{n, n}^{L}=$ $\mu_{n, j}^{H}=\mu_{n, n}^{H}$, so that on average low skilled worker suffer a moving cost relative to high skilled workers and relative to staying in the country of their nationality $n$. Then integrated labor markets depress (raise) the wage of low (high) skilled workers the more, the more negative $\mu_{n, j}^{L}$ and the lower the substitutability $\rho$ between the different types of workers is. Figure 3.1 illustrates this for a set of baseline parameters $\alpha=0.4, A_{1}=1, A_{2}=2, \rho=0.75, \mu_{n, j}^{L}=-1$ and $\mu_{n, n}^{L}=\mu_{n, j}^{H}=\mu_{n, n}^{H}=0$. Specifically, the figure shows-for varying values of $\rho$ and $\mu_{n, j}^{L}$-the percentage changes in skill-specific equilibrium wages in low productivity country 1 when switching from autarky to internationally integrated labor markets. Panel (a) shows
a marked decrease in the wage of low skilled non-migrants in country 1 when skill types are complement (low levels of $\rho$ ). For the given set of parameters, this wage loss amounts to 4 percent even for a value of $\rho \approx 0$. In this simple frictionless model the loss only vanishes as workers become perfect substitutes ( $\rho=1$ ). Panel (b) shows the corresponding changes for different moving costs of low skilled workers, keeping $\rho$ at the value 0.75 common in the literature (see e.g. Caliendo et al., 2017). Again, as high skilled workers leave for the more productive country, low skilled workers suffer a wage loss, whereas high skilled workers staying in country 1 gain. Across skill groups, the origin country suffers an average output per capita loss of 0.2 percent when $\mu_{n, j}^{L}=-3$.


Figure 3.1: Percentage change in equilibrium wages in low productivity country 1 when moving from autarky to internationally integrated labor markets; (a) for different degrees of substitutability, and (b) different levels of moving costs for low skilled workers. Baseline parameterization: $\alpha=0.4, A_{1}=1, A_{2}=2, \rho=0.75, \mu_{n, j}^{L}=-1$ and $\mu_{n, n}^{L}=\mu_{n, j}^{H}=\mu_{n, n}^{H}=0$.

However, the migration scenario can be a Pareto improvement over the autarky scenario if lump-sum transfers conditional on nationality and skill-type were allowed. Indeed, suppose that a benevolent social planner could decide to switch from autarky to open borders, implementing also the following transfers: (1) a lump-sum transfer to less-mobile workers (low-skilled in our example) in the less productive country and (2) a lump-sum transfer to
more-mobile workers (high-skilled in our case) in the more productive country. The transfers are financed by lump-sum taxes paid by all workers. These transfers are set so as to keep the wage of all workers at least at the level that would have prevailed under autarky. In this way, we obtain the result showed in figure 3.2: low-skilled workers in country 1 are kept at the same level of utility, while high-skilled workers enjoy the benefit of migration. In Appendix C, we plot the amount of tax and net benefits (transfers-tax) that are necessary to achieve these outcomes for different parameter values $\rho$ and $\mu_{n, j}^{L}$.


Figure 3.2: Percentage change in equilibrium wages in low productivity country 1 when moving from autarky to internationally integrated labor markets with transfers compensating "losers"; (a) for different degrees of substitutability, and (b) different levels of moving costs for low skilled workers. Baseline parameterization: $\alpha=0.4, A_{1}=1, A_{2}=2, \rho=0.75$, $\mu_{n, j}^{L}=-1$ and $\mu_{n, n}^{L}=\mu_{n, j}^{H}=\mu_{n, n}^{H}=0$.

If we constrained the set of possible fiscal instrument of the social planner to countryspecific tax and transfers, this result of Pareto-optimality of the open border policy does not hold anymore. Indeed, the social planner would be able to compensate losing workers only in one of the two countries. However, losses are suffered by both low-skilled workers in the lowproductivity country and high-skilled workers in the high-productivity country. A realistic policy that redistributes the gains from migration from the receiving country to the sending
country would not be able to compensate the losses of high-skilled workers in the receiving country, as is shown in Figure 3.3. In this case, the social planner implements a tax on all workers in country 2 distributed equally to all workers in country 1 . The total amount of the transfer is calculated in order to keep low-skilled workers in the low-productivity country at the same utility level as in the autarky case.


Figure 3.3: Percentage change in equilibrium wages in low productivity country 2 when moving from autarky to internationally integrated labor markets with transfers from country 2 to country 1 ; (a) for different degrees of substitutability, and (b) different levels of moving costs for low skilled workers. Baseline parameterization: $\alpha=0.4, A_{1}=1, A_{2}=2, \rho=0.75$, $\mu_{n, j}^{L}=-1$ and $\mu_{n, n}^{L}=\mu_{n, j}^{H}=\mu_{n, n}^{H}=0$.

Note that without frictions and with constant returns to scale, less mobile workers who are less than perfect substitutes to other input factors are bound to lose from integrated labor markets, if not compensated. This is does not necessarily hold in the more realistic and dynamic model that we bring to the data in Sections 3.4-3.7. In particular, search frictions imply that emigration may be a means to reduce unemployment in the emigration country. Accordingly, the actual capability of internationally integrated labor markets to buffer asymmetric shocks is an empirical question, which this paper is set to answer. The global welfare effect of facilitating international mobility of labor, that is a removal of barriers,
is necessarily positive in either model. Hence, there is in principal scope for welfare improving redistribution even in case labor market integration has a negative effect on some types of workers. Yet, feasible policies that are constrained to not discriminate workers on basis of their nationality may limit this possibility.

### 3.3 Skill bias in European mobility

Migrants are positively selected on marketable skills in many contexts. Docquier et al. (2009) shows that emigrants from virtually any region in the world to the OECD are positively selected on education. Migration within Europe today is no exception. While migrant populations that have their origins in the guest worker agreements of the 1950s and 60s were predominantly low skilled, more recent migrant flows have changed the picture. Using aggregate data from Eurostat on foreign-born populations, Figure 3.4 reveals that until the early 2000s, foreign-born Europeans were slightly less educated than the overall population in their respective European host country. The higher mobility of college-educated individuals has led to resident migrant populations on average surpassing native host country populations in terms of education. This gap continuously widens, indicating the positive selection of more recent migrants.

To illustrate the difference in high and low skilled migration flows more directly, we use individual level data from the European Union Labour Force Survey (EU-LFS). We draw heavily on this dataset also for the calibration of the equilibrium model presented below, and explain the features of the EU-LFS in more detail in Section 3.5. The EU-LFS reports both individuals' current and previous year's country of residence, we use this information to compute the share of recent migrants in each country, separately for individuals with and without tertiary education. ${ }^{3}$ Figure 3.5 shows a considerably higher migrant share among highly skilled individuals, irrespective of whether all or only European nationals are considered.

[^42]

Figure 3.4: Percentage of college-degree workers among working-age population, separately for native and European foreign-born populations. Source: Eurostat
(a) All

(b) European nationals


Figure 3.5: Shares of individuals arrived during previous year among individuals with and without tertiary education, considering (a) individuals of any origin and (b) only European nationals. Source: European Union Labour Force Survey

In light of these facts, an analysis that is set to evaluate the impact of migration on macroeconomic outcomes ought to account for worker heterogeneity in the propensity to migrate. Part of the skill bias in migration may stem from international differences in the returns to education, and hence in the benefits from migration in terms of wage gains. The model described in the next section explicitly accounts for labor market outcomes in
an equilibrium framework, so that heterogeneity in the preference for migration above and beyond economic outcomes can be isolated.

### 3.4 A model of worker mobility in Europe

The question regarding the role of migration in a currency union, together with the above evidence on the mobility of different groups of workers, motivates our modeling choice: a spatial equilibrium model in which search frictions and sticky wages generate both unemployment and a role for monetary policy. Monetary policy within and outside the monetary union may respond differently to any given shock. Importantly, the model distinguishes workers of different skill levels, who are geographically mobile, but heterogeneous in their preference for different locations. Time is discrete and a period represents one year.

### 3.4.1 Consumption

Each period, an individual $i$ consumes a basket of goods from $N$ different countries that we indicate with subscript $j \in\{1, \ldots, N\}$. The individual's current country of residence is indexed as $j_{i}$, and its amenities are valued differently across individuals. The consumption vector $\left(c_{i, 1}, \ldots, c_{i, N}\right)$ and location amenities $\epsilon_{i, j_{i}}$ generate a utility flow

$$
\begin{equation*}
\mathcal{U}\left(c_{i, 1}, \ldots, c_{i, N} ; j_{i}\right)=\left(\sum_{j} \psi_{j} c_{i, j}^{\xi}\right)^{\frac{1}{\xi}}+\epsilon_{i, j_{i}} \tag{3.1}
\end{equation*}
$$

where $\xi$ determines the substitutability across goods of different origin, and $\psi_{j}$ determines relative levels of demand. The sum across all $\psi_{j}$ is normalized to one, so that $\sum_{j} \psi_{j}=1$. In addition to utility from the consumption of goods, the individuals derives utility from the location amenities of the country $j_{i}$ he or she resides in. The valuation of these amenities is individual-specific, and each period a vector $\boldsymbol{\epsilon}_{i}$ with realizations for each country is drawn ${ }^{4}$ from a distribution with mean $\mu_{j_{i}, s_{i}, n_{i}}$, which varies across individuals of different skill levels

[^43]$s$ and nationality $n$. For an individual's country of origin $j_{i}=n_{i}$, we normalize this mean to $\mu_{n_{i}, s_{i}, n_{i}}=0$ (more on this below). This idiosyncratic preference component implies that individuals differ in the propensity to migrate conditional on labor market outcomes.

Individuals receive labor income if employed, and benefits if unemployed. Given the labor market frictions detailed in Section 3.4.3, firms may generate positive profits that are redistributed across individuals in the same country. Hence, the latter further receive capital income in form of a lump-sum transfer. Finally, individuals pay lump-sum taxes to the government.

Individuals consume their income, so that the budget constraint for an individual of skill type $s_{i}$ who currently resides in country $j_{i}$ is given by

$$
\sum_{j} P_{j} c_{i, j}=b_{j_{i}}+\mathbb{1}_{e_{i}}\left(w_{j_{i}, s_{i}}-b_{j_{i}}\right)+d_{j_{i}}-T_{j_{i}} \equiv I_{i}
$$

where $P_{j}$ denotes the price of goods from country $j$, and $\mathbb{1}_{e_{i}}$ indicates individual $i$ 's employment status, $b_{j_{i}}$ is the unemployment benefit level in country $j_{i}$, and labor income is given by $w_{j_{i}, s_{i}}$. Finally, $d_{j_{i}}$ and $T_{j_{i}}$ are the lump-sum dividends received and taxes paid by the individual, both of which depend on the individual's country of residence (and if employed on his or her work place). In what follows, we denote individual $i$ 's income as $I_{i}=I\left(s_{i}, e_{i}, j_{i}\right)$, which depends on the individual's skill level, employment status and country of residence.

Given the above utility function, the optimal quantity of good $c_{j}$ consumed by individual $i$ will be

$$
c_{i, j}=I_{i}\left(\frac{\psi_{j}}{P_{j}}\right)^{\frac{1}{1-\xi}}
$$

These preferences further yield a convenient price index $P^{u}$, which measures the price per
unit of utility:

$$
\begin{equation*}
P^{u}=\left(\sum_{j} P_{j}^{(1-\xi)} \psi_{j}^{\xi}\right)^{\frac{1}{1-\xi}} \tag{3.2}
\end{equation*}
$$

### 3.4.2 Production

Each country produces a final goods variety that is internationally tradable. Each final good is produced using national intermediate inputs $H_{j}$ and $L_{j}$ from two sectors that employ high and lower skilled workers, respectively. Firms in country $j$ have a production function

$$
\begin{equation*}
Y_{j}=A_{j}\left(\alpha_{j} L_{j}^{\rho}+\left(1-\alpha_{j}\right) H_{j}^{\rho}\right)^{\frac{1}{\rho}} \tag{3.3}
\end{equation*}
$$

with country-specific total factor productivity, and a country-specific relative efficiency of the intermediate inputs. Given this technology, demand functions for the intermediate goods in each country $j$ are

$$
\begin{gather*}
L_{j}=Y_{j}\left(\frac{P_{j} \alpha_{j} A_{j}^{\rho}}{p_{j, L}}\right)^{\frac{1}{1-\rho}}  \tag{3.4}\\
H_{j}=Y_{j}\left(\frac{P_{j}\left(1-\alpha_{j}\right) A_{j}^{\rho}}{p_{j, H}}\right)^{\frac{1}{1-\rho}}, \tag{3.5}
\end{gather*}
$$

where $p_{j, H}$ and $p_{j, L}$ denote intermediated input prices. Hence, the higher the price of the final good, and the lower the price of an intermediate good, the higher the demand for the intermediate good. Whereas final goods are assembled by competitive firms, intermediate goods production is subject to labor market frictions, as detail below.

### 3.4.3 Labor Markets

Labor markets in each intermediate goods sector operate in a Diamond-Mortensen-Pissarides search and matching framework. Depending on his or her skill type, a worker can be employed in either the high or the low skilled sector, where each worker produces one unit of the respective intermediate good. The respective values $p_{j, H}$ and $p_{j, L}$ of intermediate goods in country $j$ are determined in equilibrium.

Nominal wages are determined by Nash-bargaining and vary across countries and sectors. Wages can be re-bargained every period, and in steady state are fully flexible. Later on, we will introduce rigidity in wage adjustment.

We assume that firms cannot discriminate based on a worker's nationality, so that there will be one wage per country and skill level. ${ }^{5}$ This wage is determined-via the Nash bargaining described below-by computing the mean surplus of a job match, weighted by the prevalence of each nationality in the country for a given skill group.

The timing in the model is as follows: at the beginning of each period, workers draw the vector of preference shock over locations and, based on this shock and their employment status, choose a location. If previously unemployed, individuals are matched with an empty vacancy with the prevailing equilibrium job-finding probability, whereas employed workers face a country- and skill-specific probability of losing their job. Finally, depending on their employment status, individuals receive either labor income or unemployment benefits that are spent on consumption goods as described above. Figure 3.6 illustrates this timing within each period.

We next describe the values associated with different employment states. These values are similar to those in a standard search and matching model, however augmented with locational preferences and the possibility to migrate. Specifically, the values attributed to working and unemployment in country $j$ by an individual of skill type $s$ and nationality $n$

[^44]

Figure 3.6: Assumed timing within each period in the model.
are given by

$$
\begin{align*}
W_{j, s, n} & =\frac{w_{j, s}+d_{j}-T_{j}}{P^{u}}+\varepsilon_{j}+\frac{\mathbb{W}_{j, s, n}}{1+r} \\
U_{j, s, n} & =\frac{b_{j}+d_{j}-T_{j}}{P^{u}}+\varepsilon_{j}+\frac{\mathbb{U}_{j, s, n}}{1+r} \tag{3.6}
\end{align*}
$$

where the expected continuation values are respectively denoted by $\mathbb{W}_{j, s, n}$ and $\mathbb{U}_{j, s, n}{ }^{6}$, and are discounted with real interest rate $r$. Prices are endogenous, and income is expressed in nominal terms. Dividing income by the price index $P^{u}$ defined in equation 3.2 yields utility flows.

Values are indexed by nationality, since the distribution of preferences over different locations is nationality-specific, whereas $\varepsilon_{j}$ is the individual realization of that preference for a specific country $j$. Continuation values are based on individuals' expectations about future location choices and employment transitions. We denote with $f_{j, s}$ the probability that an unemployed individual of type $s$ in country $j$ is matched to a job, and with $x_{j, s}$ the probably for an employed worker of losing the job.

Separations can occur either exogenously with probability $x_{j, s}$, or because a worker decides to not stay in the current country of residence. The latter occurs with an endogenous probability $\left(1-\pi_{j, s, n}^{j}\right)$, where we denote with $\pi_{j, s, n}^{d}$ the probability that a worker of skill $s$ and nationality $n$ in country $j$ chooses to locate in country $d$. We specify this probability in Section 3.4.4 below. Hence, the value of a filled vacancy for an intermediate goods producer

[^45]in sector $s \in\{H, L\}$ is given by
$$
J_{j, s, n}=p_{j, s}-w_{j, s}+\frac{\left(x_{j, s}+\left(1-x_{j, s}\right)\left(1-\pi_{j, s, n}^{j}\right)\right) V a c_{j, s}+\left(1-x_{j, s}\right) \pi_{j, s, n}^{j} J_{j, s, n}}{1+r}
$$

Open vacancies involve a cost $\kappa_{j, s}$, so that the value of an open vacancy is

$$
V a c_{j, s}=-\kappa_{j, s}+\frac{q_{j, s} \Pi_{j, s}+\left(1-q_{j, s}\right) V a c_{j, s}}{1+r}
$$

where $q_{j, s}$ is the probability that the vacancy is filled. If matched, the firm generates an expected profit $\Pi_{j, s}$, where the expectation is over the nationalities composition within the unemployment pool.

Free firm entry reduces the value of a vacancy to zero for every country and skill, so that

$$
\begin{equation*}
(1+r) \kappa_{j, s}=q_{j, s} \Pi_{j, s} . \tag{3.7}
\end{equation*}
$$

We assume Nash bargaining, which shares the surplus

$$
S_{j, s, n}=J_{j, s, n}-V a c_{j, s}+W_{j, s, n}-U_{j, s, n}
$$

of a match between a firm and a worker. The resulting wage $w_{j, s}$ is the transfer between firm and worker that satisfies the condition

$$
\begin{equation*}
\beta \sum_{n} J_{j, s, n} e_{j, s, n}=(1-\beta) \sum_{n}\left(W_{j, s, n}-U_{j, s, n}\right) e_{j, s, n}, \tag{3.8}
\end{equation*}
$$

where $\beta$ is the bargaining power of workers, and $e_{j, s, n}$ is the share of nationality $n$ among workers of skill $s$ in country $j$.

Since firms cannot discriminate based on workers' nationality, searching workers are matched with empty vacancies at random within country and skill sectors. We assume a
matching function

$$
\begin{equation*}
m\left(u_{j, s}, v_{j, s}\right)=\varsigma u_{j, s}^{\eta} v_{j, s}^{(1-\eta)} \tag{3.9}
\end{equation*}
$$

where $\varsigma$ is the matching efficiency, and $\eta$ the elasticity with respect to the unemployment pool. The unemployment rate $u_{j, s}$ includes both unemployed individuals present in the country at the beginning of the period, and the newcomers from abroad.

New empty vacancies are posted by firms until the free entry condition (3.7) is satisfied. Defining market tightness as

$$
\theta_{j, s}=\frac{v_{j, s}}{u_{j, s}},
$$

the job finding probability can be written as

$$
f_{j, s}=\varsigma \theta_{j, s}^{1-\eta},
$$

whereas the vacancy filling probability is given by

$$
q_{j, s}=\varsigma \theta_{j, s}^{-\eta}=f_{j, s} \theta_{j, s}
$$

The equilibrium employment level in each sector within a country then determines supply of the respective intermediate inputs

$$
\begin{equation*}
H_{j}=\sum_{n} e_{j, H, n} \tag{3.10}
\end{equation*}
$$

and

$$
\begin{equation*}
L_{j}=\sum_{n} e_{j, L, n} . \tag{3.11}
\end{equation*}
$$

### 3.4.4 Migration decision

Each period, individuals decide on a country of residence, whose amenities they will enjoy, and on whose labor market they will work or search for a job. From their respective country of origin, individuals can either stay or move to any of the other $N-1$ countries. For individuals already residing abroad, we restricted the choice set to either staying in the current location or returning to the country of origin. ${ }^{7}$ When moving from country $j$ to country $d$, individuals pay a cost $k_{j, d}$. This cost is pairwise specific and symmetric, based on the distance between the two countries. ${ }^{8}$

Under the assumption that preference realizations $\boldsymbol{\varepsilon}$ are drawn from a type I extreme value distribution with mean $\mu_{j, s, n}$, the probability that an individual in country $j$ chooses to move to any given destination $d$ has the following closed form solution. ${ }^{9}$

$$
\pi_{j, s, n}^{d}(W)=\frac{\exp \left(\mu_{d, s, n}+f_{d, s} W_{d, s, n}+\left(1-f_{d, s}\right) U_{d, s, n}-k_{j, d}\right)}{\Xi_{j, s, n}^{W}}
$$

if an individual is currently employed, and

$$
\pi_{j_{i}, s, n}^{d}(U)=\frac{\exp \left(\mu_{d, s, n}+f_{d, s} W_{d, s, n}+\left(1-f_{d, s}\right) U_{d, s, n}-k_{j, d}\right)}{\Xi_{j, s, n}^{U}}
$$

if not.
To simplify the notation, in the previous formulas we indicated as $\Xi_{j, s, n}^{W}$ the sum of the exponential of all possible continuation values depending on the chosen destination for an employed worker living in country $j$, with skill $s$ and nationality $n$ and similarly $\Xi_{j, s, n}^{U}$ for unemployed agents.

[^46]\[

$$
\begin{aligned}
& \Xi_{j, s, n}^{W}=\exp \left(\mu_{j, s, n}+\left(1-x_{j, s}\right) W_{j, s, n}+x_{j, s} U_{j, s, n}\right)+\sum_{h \neq j} \exp \left(\mu_{h, s, n}+f_{h, s} W_{h, s, n}+\left(1-f_{h, s}\right) U_{h, s, n}-k_{j, h}\right) \\
& \Xi_{j, s, n}^{U}=\exp \left(\mu_{j, s, n}+\left(1-f_{j, s}\right) W_{j, s, n}+f_{j, s} U_{j, s, n}\right)+\sum_{h \neq j} \exp \left(\mu_{h, s, n}+f_{h, s} W_{h, s, n}+\left(1-f_{h, s}\right) U_{h, s, n}-k_{j, h}\right)
\end{aligned}
$$
\]

Staying in the current country of residence does not involve a moving cost, and thus the corresponding choice probabilities are given by

$$
\pi_{j_{i}, s, n}^{j_{i}}(W)=\frac{\exp \left(\mu_{j_{i}, s, n}+\left(1-x_{j_{i}, s}\right) W_{j_{i}, s, n}+x_{j_{i}, s} U_{j_{i}, s, n}\right)}{\Xi_{j, s, n}^{W}}
$$

and

$$
\pi_{j_{i}, s, n}^{j_{i}}(U)=\frac{\exp \left(\mu_{j_{i}, s, n}+f_{j_{i}, s} W_{j_{i}, s, n}+\left(1-f_{j_{i}, s}\right) U_{j_{i}, s, n}\right)}{\Xi_{j, s, n}^{U}}
$$

An important parameter in our framework is $\mu_{j, s, n}$, the average preference of individuals of nationality $n$ and skill $s$ towards any given country $j$. Besides economic differences, this parameter will determine migration flows, and account for heterogeneity in individual mobility across skill groups, conditional on labor market outcomes.

Given the distributional assumption on $\boldsymbol{\varepsilon}$, the expected continuation values in the two employment states are
$\mathbb{W}_{j_{i}, s, n}=\log \left[\exp \left(\left(1-x_{j_{i}, s}\right) W_{j_{i}, s, n}+x_{j_{i}, s} U_{j_{i}, s, n}\right)+\sum_{j \neq j_{i}} \exp \left(f_{j, s} W_{j, s, n}+\left(1-f_{j, s}\right) U_{j, s, n}-k_{j, j_{i}}\right)\right]+\gamma$
and
$\mathbb{U}_{j_{i}, s, n}=\log \left[\exp \left(f_{j_{i}, s} W_{j_{i}, s, n}+\left(1-f_{j_{i}, s}\right) U_{j_{i}, s, n}\right)+\sum_{j \neq j_{i}} \exp \left(f_{j, s} W_{j, s, n}+\left(1-f_{j, s}\right) U_{j, s, n}-k_{j, j_{i}}\right)\right]+\gamma$,
where $\gamma \approx 0.577$ is the Euler constant. That is, workers migrating to another country are subject to the destination country's job finding rate.

We now can describe the different flows that characterize this labor market. The stocks of employed and unemployed workers are measure at the end of each period, before the new preference shocks are drawn and individuals start to relocate. Indeed, migration is part of the overall worker flows in this model. The flow into the unemployed population of skill $s$ and nationality $n$ in a country $j$ equals

$$
\begin{equation*}
i n f l_{j, s, n}^{u}=x_{j, s} \pi_{j, s, n}^{j}(W) e_{j, s, n}+\left(1-f_{j, s}\right)\left(\sum_{\iota \neq j} \pi_{\iota, s, n}^{j}(W) e_{\iota, s, n}+\sum_{\iota \neq j} \pi_{\iota, s, n}^{j}(U) u_{\iota, s, n}\right), \tag{3.12}
\end{equation*}
$$

where $e_{j, s, n}$ is the number of employed workers for a given country-skill-nationality combination. The first term in equation (3.12) represents workers who decide to stay in the country, but lose their job. The second and third terms are immigrants who have left unemployment or employment in their previous country of residence, and fail to find a job right away.

The flow into employment in turn is

$$
\begin{equation*}
\operatorname{infl} l_{j, s, n}^{e}=f_{j, s} \pi_{j, s, n}^{j}(U) u_{j, s, n}+f_{j, s}\left(\sum_{\iota \neq j} \pi_{\iota, s, n}^{j}(W) e_{\iota, s, n}+\sum_{\iota \neq j} \pi_{\iota, s, n}^{j}(U) u_{\iota, s, n}\right) \tag{3.13}
\end{equation*}
$$

Again, the first term in (3.13) represents the unemployed workers who find a job after deciding to stay in the country. The second and third terms are immigrants who have left unemployment or employment in their previous country of residence, and find a job immediately.

Outflows from unemployment and employment, repsectively, equal

$$
\text { outfl } l_{j, s, n}^{u}=\left(1-\pi_{j, s, n}^{j}(U)+f_{j, s} \pi_{j, s, n}^{j}(U)\right) u_{j, s, n}
$$

and

$$
\text { outfle } l_{j, s, n}^{e}=\left(1-\pi_{j, s, n}^{j}(W)+x_{j, s} \pi_{j, s, n}^{j}(W)\right) e_{j, s, n}
$$

In a steady state equilibrium, outflows and inflows for all different countries, skills and nationalities are balanced, yielding the conditions

$$
\begin{align*}
& \text { out } l_{j, s, n}^{u}=\text { infl }_{j, s, n}^{u}  \tag{3.14}\\
& \text { outfl } l_{j, s, n}^{e}=\text { infl } l_{j, s, n}^{e} . \tag{3.15}
\end{align*}
$$

### 3.4.5 Goods Markets

Given our assumptions about preferences, all individuals consume the same proportions of the goods produced in each country, using all their income. The relative proportions are determined by the taste parameters $\phi_{j}$, final goods' prices $P_{j}$, and the elasticity of substitution $1 /(1-\xi)$ across goods (see equation (3.1)). The overall scale of consumption is given by an individual's income. Note first that under the assumption of a balanced government budget payments to unemployed individuals and tax payments in each country offset each other:

$$
\begin{equation*}
\forall j: \quad b_{j} \sum_{s} \sum_{n} u_{j, s, n}=T_{j} \sum_{s} \sum_{n}\left(e_{j, s, n}+u_{j, s, n}\right) . \tag{3.16}
\end{equation*}
$$

Hence, total aggregate demand is given by the sum of labor income $w_{j, s}$ and dividends $d_{j}$ across all countries:

$$
I=\int_{i} I_{i} \mathrm{~d} i=\sum_{j} \sum_{s} \sum_{n} d_{j} u_{j, s, n}+\sum_{j} \sum_{s} \sum_{n}\left(w_{j, s}+d_{j}\right) e_{j, s, n} .
$$

Equilibrium on the market of each country's final good then requires that

$$
\begin{equation*}
Y_{j}=I\left(\frac{\psi_{j}}{P_{j}}\right)^{\frac{1}{1-\xi}} \tag{3.17}
\end{equation*}
$$

where $\psi_{j}$ is the taste parameter of consumers for good $j$.

### 3.4.6 Steady State Equilibrium

A steady state equilibrium is define as the vector of prices $P_{j}, p_{j, H}, p_{j, L}$, wages $w_{j, s}$, intermediate and final goods quantities $H_{j}, L_{j}, Y_{j}$, and agents' distribution $u_{j, s, n}, e_{j, s, n}$ over employment states for each country, skill level and nationality, such that the following conditions are satisfied:

1. Demand (3.17) for the final good produced in each country equals supply (3.3);
2. Demand for intermediate goods (3.5) and (3.4) equals supply (3.10) and (3.11), for each country;
3. Government budged is balanced (3.16);
4. Flows into and out of employment and unemployment in each country and sector are balanced, (3.14) and (3.15);
5. Free entry conditions (3.7) hold in each country and sector;
6. Wages in each country and sector share the surpluses according to (3.8);
7. Individuals maximize their utility in choosing the basket of goods and the country of residence;
8. Matches are formed according to the matching function (3.9).

### 3.4.7 Currency Devaluation and Monetary Policy

To adapt our model to the European context, the empirical implementation will investigate not only economic outcomes within a monetary union, but also allow for different currency areas that are interlinked via goods and labor markets. That is, while a number of European Union countries share the euro as a currency subject to a common monetary policy, other countries like have maintained national currencies and independent central banks. Yet, all these countries are part of the European Single Market that has integrated goods and factor
markets. Some of those non-euro countries, like Poland, are major migrant sending countries to the euro zone, whereas other, like the United Kingdom have been major receiving countries of labor migrants from other parts of the European Union.

We thus allow for different monetary policies in countries inside and outside the monetary union. In particular, central banks are allowed to intervene in setting the interest rate andthrough a no-arbitrage condition - to increase or decrease the value of their currency, affecting the prices of traded goods. To be explicit, we rewrite the price of a country's final good as

$$
P_{j}=E_{j} P_{j}^{*},
$$

where $E_{j}$ is the value of the currency of country $j$ and $P_{j}^{*}$ is the price of national good in local currency. In our empirical counterpart, we will normalize the value of the euro to 1 .

From the no-arbitrage condition between two country $j$ and $j^{\prime}$, we have

$$
\begin{equation*}
\left(1+i n t_{j}\right)=\left(1+i n t_{j^{\prime}}\right) \mathbb{E}\left(\frac{E_{j^{\prime}, t+1}}{E_{j^{\prime}, t}} \cdot \frac{E_{j, t}}{E_{j, t+1}}\right), . \tag{3.18}
\end{equation*}
$$

where $i n t_{j}$ and $i n t_{j^{\prime}}$ denote the nominal interest rates in the two countries.

The monetary policy that we assume is a simple Taylor-rule, as in House et al. (2018) and Nakamura and Steinsson (2014). A national central bank reacts to deviations in output and inflation from the steady state, and sets nominal interest rates as

$$
\begin{equation*}
i n t_{j, t}=i \bar{n} t+\phi i n t_{j, t-1}+(1-\phi)\left(\phi_{y} \Delta \text { output }_{j, t}+\phi_{p} \Delta \text { inflation }_{j, t}\right) \tag{3.19}
\end{equation*}
$$

For the monetary union, we assume that the central bank considers the weighted average of these deviations for the countries that are part of the union. The weights will be the country's GDP. We assume that the reactions parameters $\phi, \phi_{Q}, \phi_{\pi}^{p}$ are common to all countries.

### 3.4.8 Wage Rigidity

We want to study how integrated labor markets affect not only long-run equilibria, but also the ability of migrant flows to absorb asymmetric shocks to different countries within a monetary union.

As noticed by Shimer (2005a), a standard search and matching model with flexible wages give rise to an elasticity of the market tightness to productivity shocks that is low relative to what is commonly observed in the data. The reason is that a productivity shock is largely absorbed by a change in the wages. A certain degree of wage rigidity on the other hand can, for a given shock level, give rise to a more realistic response in unemployment (?). Nominal rigidities in wages have also been documented in empirical work (see, for instance, Barattieri et al. (2014) for evidence in the US, and Fehr and Goette (2005) for Switzerland). Finally, nominal rigidities are at the very core of monetary policy analysis, the focus of this study.

For these reasons, we are embedding a tractable form of wage rigidity into the model. Specifically, we assumed that wages in each country follow a path

$$
w_{j, s, n, t}=\frac{E_{j, t}}{E_{j, t+1}} \omega w_{j, s, n, t-1}+(1-\omega) w_{j, s, n, t}^{*}
$$

where $w_{j, s, n, t}^{*}$ is the wage that would have split the surplus according the Nash-Bargaining rule. The wage that actually paid by firms is a weighted average of this hypothetical wage and the wage prevailing in the previous period. $\omega$ is the degree of rigidity, where $\omega=0$ implies that wages are fully flexible, whereas $\omega=1$ corresponds to a model in which wages never deviate from an initial steady state level.

The rigidity applies to the nominal wage in the local currencies, therefore the monetary policy can act in order to accelerate the adjustment process, reducing the real rigidity of wages.

The choice of the wage rigidity is completely irrelevant for the estimation of the parameters of the models, since they are estimated at the steady state, where wage rigidity does not play
a role. However, it becomes crucial for the welfare implications of a shock. In the result section, we will show the different results both in presence or in absence of wage rigidity.

### 3.5 Data

Our aim is to evaluate the role of labor mobility in the euro zone. The European context is one where labor markets are integrated beyond the monetary union, since the Single Market include countries that have not adopted the euro. We thus calibrate the above model to data on migration and labor market outcomes in both euro zone and non-euro zone countries. The dataset that fits our purpose best is the micro-level European Union Labour Force Survey (EU-LFS). The EU-LFS collects harmonized data from national surveys of the labor force in member countries of the European Union, as well as a number of other countries in Europe.

From these surveys we recover information about population stocks, migration flows, labor transitions, wages, education level and education premium. More specifically, we refer to high skilled labor as workers with at least a college degree, while low skilled labor is includes all other workers.

Migration flows are generally difficult to measure since different countries have different rules with respect to the mandatory communications (if any) for moving in and out of a country. The EU-LFS does, however, inquire about respondents' country of residence in the previous year. For privacy concerns, we cannot observe the nationality of foreign citizens at a country level. Nevertheless, we do know whether a respondent is a native citizen or not. Under the assumption that agents only move (in either direction) between their country of citizenship and some foreign country rather than between third countries, this is sufficient for us to construct a migration flow matrix. To avoid too small numbers of observed migrants in some single waves, we pool data for the years 2012-2017. This allows more precisely measured migration rates, at the cost of some variation in the economic conditions behind the observed migrations.

As for some smaller countries this pooling over time is not enough, the calibration will be based on information from a total of 17 countries. For the euro-area, these include Austria, Belgium and Luxembourg ${ }^{10}$, Denmark, France, Germany, Italy, Netherlands, Portugal and Spain. To represent the non-euro zone location of the model more realistically, we account for the large economic disparity among countries outside the monetary union, which on the one hand include high income countries like Sweden and Switzerland, and on the hand the lower income and more recent member states of the European Union. Rather than pooling all these into one residual location, the calibration thus considers two distinct outside locations, which comprise the UK, Sweden and Switzerland on the one hand, and Poland, Romania, Czech Republic and Hungary on the other. In our context, this distinction not only reflects the different productivity levels across those groups, but also the fact that the former group are net immigration countries, whereas the latter are predominantly migrant sending countries. We complement our micro data with macroeconomic measures from EUROSTAT, such as countries' GDP, and we take unemployment benefit replacement ratios from the OECD. ${ }^{11}$

To be consistent with the choice of merging migration data from 2012 to 2017, we use average GDP of the countries analyzed for the same years, the average wage for high and low educated workers, and the average stocks of natives in each country. We also compute the average stocks of non-citizens living in a country, but exclude non-European workers, which are not part of our analysis.

### 3.6 Calibration

Since we group a number of countries as described in the previous section, the calibrated model features eleven locations: nine euro zone locations plus two locations outside the monetary union. We jointly calibrate a total of 319 parameters to moments from the data, and take others from the literature. We first describe the latter set of parameters, before

[^47]discussing the identification of the calibrated parameters.

### 3.6.1 Externally Set Parameters

Discount Factor. In the calibration, a time period is taken to be one year, and we assume an annual interest rate of $5 \%$, which implies a discount factor of 0.9524 .

Elasticity of consumption goods. We set the elasticity parameter in the CES part of consumers' utility function (3.1) to $\xi=0.75$. This yields an elasticity of substitution between national goods of $4 .{ }^{12}$

Elasticity of substitution between high and low educated workers. The elasticity parameter in the CES production function of intermediate goods (3.3) is set to $\rho=0.75$, implying an elasticity of substitution between workers of different types equal to 4 . This is the value reported by Caliendo et al. (2017), which they estimate on Portuguese matched employer-employee data. Other estimates in the literature range between 1.5 (Ottaviano et al., 2012) and 5 (Dustmann et al., 2009). ${ }^{13}$

Unemployment benefit. We use the net replacement ratio of unemployment benefits collected by the OECD for different workers categories. For our purpose, we take the average replacement ratio during the years considered (2012-2017) for each country in our sample, using as benchmark a single worker without children, who has been unemployed for 6 months and earned a previous wage equal to $2 / 3$ the average wage. We then set the unemployment benefit in each country to this replacement ratio times the equilibrium wage for low skilled workers.

[^48]Matching function and bargaining power. We take the unemployment coefficient in the matching function from the estimation by Shimer (2005a), borrowed also by House et al. (2018), hence $\eta=0.72$. Then, we set the matching efficiency parameter to $\varsigma=0.25$. As is well-known, the unemployment level only jointly identifies the vacancy cost and matching efficiency parameters. We thus set the matching efficiency parameter to $\varsigma=0.25$, and calibrate the corresponding vacancy cost to match unemployment rates. As it common in the literature, we set the bargaining power of the worker equal to the elasticity of the matching function to unemployment, $\beta=0.72$.

Separation rates. We use the European Labour Force Survey to compute yearly employment-to-unemployment transition rates separately for high and low-educated workers, based on respondents' working condition in the previous year. For each skill level and country analyzed, we compute the average of these transition rates for the years considered, and use these as the exogenous separation rates in our model.

### 3.6.2 Estimated Parameters and Targeted Moments

We next describe the parameters that we estimate by targeting a set of specific data moments: the average preference over a location $\mu_{j, s, n}$ for every country, skill and nationality, the moving cost $k$, total factor productivity $A_{j}$ for each country, the vacancy cost $\kappa_{j, s}$ for each country and skill group, the relative efficiency $\alpha_{j}$ of the two labor inputs for each country, and the preference parameter $\varphi_{j}$ for each country's final good.

While different moments jointly identify the model's parameters, Table 3.1 systematically lists the different groups of parameters and the moments most directly contributing to their identification. Specifically, we use as identifying moments means across the years 2012-2017 of the following variables: the number of European non-nationals in each of the countries considered identifies the preference parameters $\mu_{j, s, n}$, return migration flows for every country and skill, as described in Section 3.5, identifies moving costs (more on this below), the
unemployment rate in each country and skill group identifies vacancy cost $\kappa_{j, s}$, whereas the skill-premium in each country ${ }^{14}$ identifies the production function parameters $\alpha_{j}$. Finally, the price level in each country and each country's GDP identify country-specific TFP $A_{j}$ and consumers' preferences $\varphi_{j}$ for different countries' goods.

We let moving cost be a linear function of the distance between countries. In other words, the cost of moving from country $i$ to country $j$ and vice-versa is given by

$$
k=k_{0}+k_{1} D_{i, j}
$$

where $k_{0}$ and $k_{1}$ are estimated parameters identified from migrant flows observed in the Labour Force Survey, and $D_{i, j}$ denotes the distance between the capital cities of the two countries, as measured in the GeoDist database, provided by CEPII.

### 3.6.3 Estimation protocol

The model's parameters are calibrated by minimizing the distance between observed data moments and their counterparts simulated from the model. Given the large number of parameters, we use a blocking estimation strategy. In particular, we first estimate all parameters keeping the moving cost parameters $k_{0}, k_{1}$ and the average preferences over a location $\mu$ fixed. Then, we estimate the preference parameters keeping $k_{0}$ and $k_{1}$ fixed. Finally, we estimate the parameters $k_{0}$ and $k_{1}$, keeping all the rest fixed. We iterate these three steps until convergence.

The table 3.1 indicates which of the targeted moments contribute primarily to identification on any given set of parameters in the estimation procedure previously described.

[^49]| Name | Parameter | Target | Mean | SD |
| :---: | :---: | :---: | :---: | :---: |
| TFP | A | GDP | 153.7 | 49.0 |
| Location Shock Distrib. | $\mu$ | Migrations flows | -0.985 | 4.816 |
| Taste national goods | $\psi$ | Price level | 0.0909 | 0.0272 |
| Migration cost | $k_{0}+k_{1} D_{i, j}$ | Foreign population | 4.69 | 0.72 |
| Vacancy cost | $\kappa$ | Unemployment | 0.045 | 0.0536 |
| Low-Educated efficiency | $\alpha$ | Skill premium | 0.4447 | 0.0326 |

Table 3.1: Parameters and Moments. Mean indicates the mean value across of the parameter across the 11 countries and SD its standard deviation

### 3.6.4 National-specific wages

The model assumes one wage by country of residence and skill instead of allowing for additional heterogeneity depending on the nationality of workers. The reason for this assumption is the following: A national-specific wage would be subject to the expected duration of a worker-firm match, which strongly depends on the share of workers that emigrate from a country. The share of workers emigrating, in turn, depends on expected future wages. This implies that equilibria are very unstable with respect to small variations in parameter values. Our maintained assumption drastically drastically the instability of the resulting equilibrium, the time to compute it, and the estimated parameter values. While this is a simplification, the institutional environment in the European Union is indeed such that wages for a large share of the labor force are bargained by unions at firm, sector or even country level.

### 3.7 Results and Counterfactual Scenarios

The calibration provide us with parameter values that make the model's steady state predictions consistent with the data. In particular, we recover the preference parameters of the agents. The estimates indicate that, on average, high skilled workers find living abroad less
costly. On average, for the same nationality and the same destination country, unskilled workers suffer a higher utility cost of living abroad equal to 0.63 , equivalent to $3.2 \%$ of their wage. This cost, however, is heterogeneous across countries and nationalities.

### 3.7.1 Productivity and Demand Shocks

We analyze the response of the economy to two types of asymmetric permanent shocks: ${ }^{15}$ a total factor productivity (TFP) shock that hits one country (Italy, in our example), lowering permanently its country-specific productivity $A_{j}$ by 1 percent, and a similarly asymmetric demand shock. To simulate the latter, we permanently lower the preference parameter $\varphi_{j}$ governing demand for a country's national good by 1 percent. ${ }^{16}$ These are "MIT shocks"shocks that are permanent, unexpected for the agents, and which lead to a deterministic aggregate path towards the new steady state. We therefore not only compare the two steady states, but also examine the transition period between equilibria. ${ }^{17}$

Our aim is to compare the effects of these shocks in the baseline scenarios with spatially integrated labor market to the same outcomes if migration was prohibited. Figures 3.7 and ?? show the adjustment paths. In each graph, the baseline is labeled "MIG", while the counterfactual no-migration scenario is called "No MIG".

The simulations show that the effects of negative demand and productivity shocks are similar. Both increase unemployment in both groups of workers for some years, and depress wages and GDP per capita in the country hit by the shock (Italy). We also show the effect on migration flows (in thousands) for Italy, and for the two largest receiving countries France and Germany. Both shocks cause a sizable surge in the number of emigrants from Italy, with the impact being stronger for skilled workers. This difference between skill groups is

[^50]important for the comparison between migration and non-migration scenarios: skilled nonmigrants in the country hit by the shock benefit from emigration and the ensuing decrease in labor supply. Taken together, emigration and the decrease in wages leads to a re-bounce in the unemployment rate of skilled workers remaining in the country.

Low skilled workers, on the other hand, emigrate to a lesser extent, and are negatively affected by the emigration of complementary worker types. Our calibrated model shows that the emigration of low skilled workers is not sufficient to compensate for this effect. Low skilled worker thus transit to a worse steady state, characterized by a higher unemployment rate and lower wage than would be the case if migration was prohibited. As a result, the average skill level in the country experiencing the negative shock deteriorates, shown also by the larger decrease of GDP per capita in the migration scenario.

There are only two relevant differences between demand and productivity shocks: first, while the negative productivity shock raises the price of the final national good, the taste shocks affecting demand lead to a decrease in the price. Second, the impact of the shock on the other countries differs: whereas the TFP shock has a general equilibrium effect that decrease the production in all countries, resulting from the decline in income in Italy, the "taste" shock increases the demand for the national goods of other countries. In the latter case, labor market outcomes therefore improve in other countries. Quantitatively, a shock to the Italian economy has relatively small effects on other locations, as we show when discussing the welfare implications of different shocks in Section 3.7.4. However, we can notice in the graphs the increase of immigrants and slight decrease of emigrants from the other two main countries of the Union, France and Germany.

We compare these results from the full model with wage rigidity to a model with fully flexible wages. Results in both cases are qualitatively similar, but the presence of wage rigidity amplifies the effects of a shock on the labor market. In particular, in the model with flexible wages, shocks are mostly absorbed by the immediate change in wages. Figure 3.9 displays the effects of a TFP shock in a model with flexible wages, and shows the smaller
changes in unemployment absent any form of wage rigidity.

### 3.7.2 Symmetric Productivity Shock

For comparison, we also simulate a symmetric shock that hits all countries simultaneously, reducing permanently the TFP by 1 percent. As expected, the effects on unemployment and wages are more pronounced. Since emigration now has become a less attractive outside option for all workers, effects further are more similar across the different types of worker. Yet, since high skilled workers are still more likely to move after a job loss than low skilled workers, the qualitative pattern that integrated labor market help mitigate shocks for primarily for the high skilled persists: As shown in Figure ??, the increase in unemployment among high skilled workers is less severe in the baseline with migration than in the non-migration counterfactual. The opposite is true for low skilled workers. Similarly, the decline in high skilled wages is smaller with migration, while the wage loss is virtually unchanged for low skilled workers.

While low and high skilled workers may emigrate differentially in response to a job loss, total emigration from Italy in this scenario is similar for low and high skilled workers. The role of heterogeneity thus is particularly important in case of asymmetric shocks discussed above. Indeed, it is primarily asymmetric negative shocks that make selective emigration more pronounced, thus aggravating brain drain from the country experiencing the negative shock.

### 3.7.3 Non-permanent Shock

We further contrast the effects of a permanent shock as described above to those of a more temporary shock that initially reduces Italian TFP by 1 percent. Productivity then gradually reverts back to the initial level, following the simple process

$$
A_{j, t}=\rho_{A} A_{j, t-1}+\left(1-\rho_{A}\right) A_{j, 0}
$$

where $A_{j, 0}$ was country $j$ 's (Italy's) the initial productivity level. We examine outcomes for a value of the persistence parameter $\rho_{A}$ of $0.8 .{ }^{18}$

Effects on the labor market are weaker than the effects of a permanent shock. In particular, the shock leads to a lower migration reaction. ${ }^{19}$

In our model agents suffer a moving cost in addition to the transitory location taste shocks. Anticipating that productivity ultimately reverts to its initial level, this cost prevents a strong response in migration. This further implies a rather mild reduction in welfare due to the shock, as we show in the next section.

### 3.7.4 Welfare Considerations

As a last part of the analysis, we investigate the welfare effects of different types of shocks for low and high skilled workers. In Figure 3.12, we visualize these effects, with numbers in percentage deviation from initial Welfare.

The first four graphs show the effects of the permanent negative TFP shock (of 1 percent) on Italy, with and without wage rigidity. In the absence of the option to migrate (red bars), welfare losses are generally higher than in the baseline case of migration (blue bars). However, it can be seen that this reduction in the welfare loss through migration is more pronounced for skilled workers, who are more prone to choose the outside option of migration.

The same applies to a demand shock, although in this case we find that the welfare of workers in other countries increases as a result of the rise in demand for goods produced in those countries. Finally, the symmetric shock affects the welfare of all countries and the migration possibility does not affect the overall welfare loss.

In sum, these results show that accounting for worker heterogeneity does not overthrow the idea of labor mobility as an element of stabilization in a monetary union. Nonetheless, the gains from migration are distributed unevenly, and conclusions may well reverse for labor

[^51]market outcomes. Note also that even if labor mobility was not able to improve the welfare of stayers at all, this would not imply a claim that labor mobility is welfare decreasing. Rather, once the gains for migrants and for receiving countries are accounted for, migration would generate a net benefit. Our results ultimately point towards a role for other policies, including re-distribution across national borders, which may be required to absorb asymmetric shocks that cannot fully be mitigated through trade or factor movements in a currency union.

### 3.8 Conclusion

In this paper we revisit the notion that labor mobility plays an important role in reducing the impact of a shock in a currency union characterized by price rigidities. We formulate a spatial integrated equilibrium model with search frictions in which we acknowledge the heterogeneity in skill levels among workers. We assume that some degree of complementarity exists between different types of workers, and allow worker types to differ in terms of their preference distributions over locations. Data from the European Union Labour Force Survey show recent migration in Europe to be predominantly positively selected. On a theoretical level, labor mobility may thus aggravate the condition for a country hit by a negative shock to demand or productivity, if this reinforces selective migration.

Calibrating the model to EU-LFS data on labor market outcomes and migrant flows confirms that labor mobility reduces the welfare loss following economic shocks. However, given the higher relative mobility of high skilled workers, this shock mitigation is distributed very unevenly, with little loss reduction for low skilled workers. This shows that labor mobility is an important factor for the stability of a currency union, but that it is not a substitute of other compensating policies that concur in the debate, such as a more coordinated fiscal policy. Our results thus confirm concerns voiced for instance by Carlin (2013) and Krugman (2013), not least to maintain political support for economic unions like the European (cf Gancia et al., 2020).

(a) Unemployment Rate Italy, Unskilled (left) and Skilled(right)


(b) Wages Italy relative to Steady State, Unskilled (left) and Skilled(right)


(c) GDP per capita (left) and Prices (right) relative to steady state, Italy


(d) Immigrants to selected countries, Unkilled (left) and Skilled (right)


(e) Emigrants from selected countries, Unkilled (left) and Skilled (right)

Figure 3.7: Permanent Negative TFP Shock in Italy ( $-1 \%$ ), 0.9 wage rigidity

(a) Unemployment Rate Italy, Unskilled (left) and Skilled(right)

(b) Wages Italy relative to Steady State, Unskilled (left) and Skilled(right)

(c) GDP per capita (left) and Prices (right) relative to steady state, Italy


(d) Immigrants to selected countries, Unskilled (left) and Skilled (right)


(e) Emigrants from selected countries, Unskilled (left) and Skilled (right)

Figure 3.8: Permanent Negative Taste Shock in Italy (-1\%), 0.9 wage rigidity

(a) Unemployment Rate Italy, Unskilled (left) and Skilled(right)

(b) Changes in Wages Italy relative to Steady State, Unskilled (left) and Skilled(right)

(c) Changes in GDP per capita (left) and Prices (right) relative to steady state, Italy

(d) Immigrants to selected countries, Unkilled (left) and Skilled (right)


(e) Emigrants from selected countries, Unkilled (left) and Skilled (right)

Figure 3.9: Permanent Negative TFP Shock in Italy (-1\%), fully-flexible wage

(a) Unemployment Rate Italy, Unskilled (left) and Skilled(right)

(b) Change in Wages Italy relative to Steady State, Unskilled (left) and Skilled(right)

(c) Change in GDP per capita (left) and Prices (right) relative to steady state, Italy


(d) Immigrants to selected countries, Unskilled (left) and Skilled (right)


(e) Emigrants from selected countries, Unskilled (left) and Skilled (right)

Figure 3.10: Permanent Negative TFP Shock in All countries (-1\%), 0.9 wage rigidity

(a) Unemployment Rate Italy, Unskilled (left) and Skilled(right)


(b) Wages Italy relative to Steady State, Unskilled (left) and Skilled(right)


(c) GDP per capita (left) and Prices (right) relative to steady state, Italy


(d) Immigrants to selected countries, Unskilled (left) and Skilled (right)


(e) Emigrants from selected countries, Unskilled (left) and Skilled (right)

Figure 3.11: Non-permanent Negative TFP Shock in Italy (-1\%), 0.9 wage rigidity

(a) Permanent TFP shock (-1\%) to Italy, no wage rigidity

(b) Permanent TFP shock ( $-1 \%$ ) to Italy, 0.9 wage rigidity

(c) Permanent Taste shock ( $-1 \%$ ) to Italy, 0.9 wage rigidity

(d) Permanent TFP shock ( $-1 \%$ ) to all countries, 0.9 wage rigidity

Figure 3.12: Welfare losses in \% of the initial Welfare

## Appendix A

In this appendix are the data sources we used in the calibration:

- European Union Labour Force Survey from 2012 to 2017 for labor and migration data;
- Eurostat Annual National Account for GDP data;
- Eurostat Harmonised index of consumer prices for the price level;
- OECD net replacment rates in unemployment;
- GeoDist Database by CEPII for country distances.

|  | Target | Model |
| :--- | :--- | :--- |
| GDP AT | 313276.3833 | 313187.2298 |
| GDP BE | 382537.1 | 382670.8999 |
| GDP DE | 2793108.4667 | 2791300.1968 |
| GDP DK | 257628.0667 | 257665.5811 |
| GDP ES | 1069143.3333 | 1068303.2522 |
| GDP FR | 2095770.7333 | 2096399.8329 |
| GDP IT | 1564054.0333 | 1562948.3629 |
| GDP NL | 662270.6667 | 662248.9934 |
| GDP PT | 172392.1 | 172440.7261 |
| GDP Rich | 3262022 | 3261453.2832 |
| GDP Poor | 837226 | 838402.2371 |

Table 3.2: GDP in the Data and in the Model

|  | Target | Model |
| :--- | :--- | :--- |
| Skill Premium AT | 0.34047 | 0.34034 |
| Skill Premium BE | 0.43172 | 0.43203 |
| Skill Premium DE | 0.51006 | 0.50981 |
| Skill Premium DK | 0.34775 | 0.34785 |
| Skill Premium ES | 0.54855 | 0.54865 |
| Skill Premium FR | 0.4223 | 0.42223 |
| Skill Premium IT | 0.41621 | 0.41666 |
| Skill Premium NL | 0.45457 | 0.45455 |
| Skill Premium PT | 0.38082 | 0.38047 |
| Skill Premium Rich | 0.38616 | 0.38579 |
| Skill Premium Poor | 0.81952 | 0.81978 |

Table 3.3: Skill premium in the Data and in the Model

## Appendix B

Sample of Data Moments and Model Results

|  | AT | BE | DE | DK | ES | FR | IT | NL | PT | Rich | Poor |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| AT | 0 | 0.001 | 3.054 | 0.148 | 0 | 0.033 | 0.013 | 0.117 | 0 | 0.227 | 0.018 |
|  | $(0)$ | $(0.014)$ | $(3.014)$ | $(0.154)$ | $(0)$ | $(0.047)$ | $(0.029)$ | $(0.133)$ | $(0)$ | $(0.215)$ | $(0.033$ |
| BE | 0.008 | 0 | 0.578 | 0.057 | 0.199 | 1.717 | 0.118 | 3.230 | 0.142 | 0.0320 | 0.007 |
|  | $(0.024)$ | $(0)$ | $(0.581)$ | $(0.0717)$ | $(0.181)$ | $(1.686)$ | $(0.111)$ | $(3.242)$ | $(0.147)$ | $(0.047)$ | $(0.023$ |
| DE | 0.816 | 0.274 | 0 | 0.331 | 0.894 | 2.077 | 0.275 | 3.228 | 0.385 | 3.853 | 1.275 |
|  | $(0.822)$ | $(0.287)$ | $(0)$ | $(0.345)$ | $(0.910)$ | $(2.087)$ | $(0.287)$ | $(3.227)$ | $(0.384)$ | $(3.838)$ | $(1.279$ |
| DK | 0.001 | 0.000 | 0.620 | 0 | 0.230 | 0.233 | 0 | 0.012 | 0 | 0.870 | 0.024 |
|  | $(0.012)$ | $(0.011)$ | $(0.604)$ | $(0)$ | $(0.211)$ | $(0.219)$ | $(0)$ | $(0.0275)$ | $(0.007)$ | $(0.842)$ | $(0.039$ |
| ES | 0.034 | 0.579 | 3.271 | 0.422 | 0 | 0.788 | 0.084 | 0.006 | 0.470 | 2.000 | 0.075 |
|  | $(0.048)$ | $(0.590)$ | $(3.222)$ | $(0.440)$ | $(0)$ | $(0.800)$ | $(0.010)$ | $(0)$ | $(0.471)$ | $(1.969)$ | $(0.086$ |
| FR | 0.067 | 1.564 | 2.912 | 0.252 | 0.806 | 0 | 0.194 | 0.131 | 2.774 | 4.897 | 0.003 |
|  | $(0.056)$ | $(1.574)$ | $(2.882)$ | $(0.259)$ | $(0.811)$ | $(0)$ | $(0.194)$ | $(0.144)$ | $(2.74)$ | $(4.806)$ | $(0)$ |
| IT | 0.064 | 0.332 | 1.407 | 0.0728 | 0.342 | 0.723 | 0 | 0.021 | 0.003 | 0.220 | 0.001 |
|  | $(0.079)$ | $(0.340)$ | $(1.391)$ | $(0.069)$ | $(0.327)$ | $(0.714)$ | $(0)$ | $(0.035)$ | $(0.018)$ | $(0.220)$ | $(0.013$ |
| NL | 0.006 | 0.501 | 2.310 | 0.049 | 0.160 | 0.247 | 0.00389 | 0 | 0.574 | 0.001 | 0.002 |
|  | $(0.014)$ | $(0.510)$ | $(2.286)$ | $(0.065)$ | $(0.145)$ | $(0.260)$ | $(0.019)$ | $(0)$ | $(0.557)$ | $(0)$ | $(0)$ |
| PT | 0.001 | 0.199 | 0.466 | 0.041 | 0.232 | 0.491 | 0.000 | 0.001 | 0 | 1.207 | 0.001 |
|  | $(0)$ | $(0.201)$ | $(0.472)$ | $(0.036)$ | $(0.243)$ | $(0.484)$ | $(0)$ | $(0.005)$ | $(0)$ | $(1.197)$ | $(0.006$ |
| Rich | 0.138 | 0.010 | 2.643 | 0.140 | 3.379 | 1.837 | 0.8812 | 0.304 | 0.448 | 0 | 0.001 |
|  | $(0.152)$ | $(0.116)$ | $(2.630)$ | $(0.155)$ | $(3.40)$ | $(1.837)$ | $(0.894)$ | $(0.306)$ | $(0.444)$ | $(0)$ | $(0.010$ |
| Poor | 2.653 | 0.714 | 8.629 | 0.302 | 0.993 | 0.954 | 2.269 | 2.407 | 0.0677 | 7.418 | 0 |
|  | $(2.640)$ | $(0.712)$ | $(8.48)$ | $(0.313)$ | $(0.992)$ | $(0.959)$ | $(2.245)$ | $(2.398)$ | $(0.061)$ | $(7.286)$ | $(0)$ |

Table 3.4: Annual low-educate emigrants in thousands, model results, (targets) in parentheses. Rows are sending countries, Columns are receiving countries.

|  | AT | BE | DE | DK | ES | FR | IT | NL | PT | Rich | Poor |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| AT | 0 | 0.054 | 2.269 | 0.053 | 0.426 | 0.260 | 0.084 | 0.248 | 0 | 0.272 | 0.000 |
|  | $(0)$ | $(0.064)$ | $(2.244)$ | $(0.044)$ | $(0.428)$ | $(0.274)$ | $(0.096)$ | $(0.255)$ | $(0)$ | $(0.286)$ | $(0.014)$ |
| BE | 0.031 | 0 | 0.423 | 0.108 | 0.360 | 5.257 | 0.001 | 1.622 | 0 | 0.205 | 0.002 |
|  | $(0.046)$ | $(0)$ | $(0.428)$ | $(0.108)$ | $(0.356)$ | $(5.238)$ | $(0)$ | $(1.603)$ | $(0.015)$ | $(0.218)$ | $(0.018)$ |
| DE | 0.770 | 0.542 | 0 | 0.169 | 1.730 | 3.518 | 0.306 | 2.004 | 0.221 | 5.173 | 0.711 |
|  | $(0.753)$ | $(0.527)$ | $(0)$ | $(0.157)$ | $(1.686)$ | $(3.593)$ | $(0.286)$ | $(1.996)$ | $(0.214)$ | $(5.296)$ | $(0.700)$ |
| DK | 0.003 | 0.036 | 0.392 | 0 | 0.007 | 0.354 | 0 | 0.020 | 0.004 | 0.512 | 0.067 |
|  | $(0.019)$ | $(0.051)$ | $(0.389)$ | $(0)$ | $(0.020)$ | $(0.355)$ | $(0.015)$ | $(0.035)$ | $(0.018)$ | $(0.510)$ | $(0.0768)$ |
| ES | 0.062 | 0.255 | 1.091 | 0.127 | 0 | 2.060 | 0.034 | 0.225 | 0.290 | 4.439 | 0.532 |
|  | $(0.076)$ | $(0.248)$ | $(1.115)$ | $(0.133)$ | $(0)$ | $(2.143)$ | $(0.047)$ | $(0.215)$ | $(0.292)$ | $(4.652)$ | $(0.540)$ |
| FR | 0.010 | 1.093 | 1.447 | 0.154 | 1.156 | 0 | 0.039 | 0.222 | 0.307 | 7.421 | 0.310 |
|  | $(0.024)$ | $(1.081)$ | $(1.423)$ | $(0.142)$ | $(1.122)$ | $(0)$ | $(0.042)$ | $(0.210)$ | $(0.296)$ | $(7.413)$ | $(0.295)$ |
| IT | 0.042 | 0.046 | 0.595 | 0.100 | 0.438 | 2.128 | 0 | 0.000 | 0.102 | 0.138 | 0.000 |
|  | $(0.057)$ | $(0.050)$ | $(0.642)$ | $(0.105)$ | $(0.472)$ | $(2.324)$ | $(0)$ | $(0.011)$ | $(0.101)$ | $(0.158)$ | $(0)$ |
| NL | 0.040 | 0.301 | 1.393 | 0.145 | 0.043 | 0.370 | 0.033 | 0 | 0.224 | 0.000 | 0.001 |
|  | $(0.050)$ | $(0.302)$ | $(1.377)$ | $(0.141)$ | $(0.035)$ | $(0.375)$ | $(0.029)$ | $(0)$ | $(0.224)$ | $(0)$ | $(0)$ |
| PT | 0.001 | 0.004 | 0.164 | 0.005 | 0.263 | 0.001 | 0.004 | 0.002 | 0 | 0.566 | 0.0188 |
|  | $(0)$ | $(0)$ | $(0.171)$ | $(0)$ | $(0.270)$ | $(0)$ | $(0.020)$ | $(0.007)$ | $(0)$ | $(0.561)$ | $(0.033)$ |
| Rich | 0.085 | 0.37539 | 2.890 | 0.274 | 2.162 | 2.222 | 0.769 | 0.625 | 0.625 | 0 | 0.265 |
|  | $(0.097)$ | $(0.371)$ | $(2.825)$ | $(0.271)$ | $(2.091)$ | $(2.210)$ | $(0.715)$ | $(0.618)$ | $(0.628)$ | $(0)$ | $(0.250)$ |
| Poor | 0.326 | 0.579 | 1.794 | 0.001 | 0.251 | 0.565 | 0.338 | 0.646 | 0.135 | 4.447 | 0 |
|  | $(0.339)$ | $(0.591)$ | $(1.753)$ | $(0.007)$ | $(0.256)$ | $(0.568)$ | $(0.334)$ | $(0.653)$ | $(0.121)$ | $(4.448)$ | $(0)$ |

Table 3.5: Annual high-educated emigrants in thousands, model results, (targets) in parentheses. Rows are sending countries, Columns are receiving countries.

|  | Target | Model |
| :--- | :--- | :--- |
| AT | 0.051002 | 0.051095 |
| BE | 0.085448 | 0.085515 |
| DE | 0.05066 | 0.050788 |
| DK | 0.065925 | 0.065923 |
| ES | 0.2634 | 0.26341 |
| FR | 0.11506 | 0.1151 |
| IT | 0.12373 | 0.12373 |
| NL | 0.069583 | 0.069598 |
| PT | 0.13566 | 0.13568 |
| Rich | 0.07362 | 0.073636 |
| Poor | 0.082724 | 0.082752 |

Table 3.6: Unemployment Rate unskilled workers

|  | Target | Model |
| :--- | :--- | :--- |
| AT | 0.025523 | 0.025484 |
| BE | 0.033155 | 0.033124 |
| DE | 0.018814 | 0.018821 |
| DK | 0.040348 | 0.040297 |
| ES | 0.12482 | 0.1248 |
| FR | 0.054099 | 0.054088 |
| IT | 0.064918 | 0.064885 |
| NL | 0.032043 | 0.03202 |
| PT | 0.095195 | 0.095216 |
| Rich | 0.030897 | 0.030934 |
| Poor | 0.03869 | 0.038654 |

Table 3.7: Unemployment Rate Skilled workers

|  | Target | Model |
| :--- | :---: | :---: |
| AT | 0.6402 | 0.0241 |
| BE | 0.7662 | 0.0375 |
| DE | 3.4685 | 4.5016 |
| DK | 0.2301 | 0.0334 |
| ES | 1.7528 | 2.0756 |
| FR | 1.9815 | 3.6217 |
| IT | 1.9789 | 0.1281 |
| NL | 0.4219 | 0.2077 |
| PT | 0.1953 | 0.2403 |
| Rich | 4.7963 | 5.4295 |
| Poor | 0.3368 | 0.1021 |

Table 3.8: Total stock of migrants in millions

## Appendix C

Transfers for Pareto-Optimal Solution 2-countries model. Country 2 has a higher TFP with respect to Country 1 and low-skilled workers pay a higher cost for moving with respect to high-skilled workers.


Figure 3.13: Net Benefits and Taxes necessary to obtain a Pareto-Optimal improvement from autarky to internationally integrated labor markets;; (a) for different degrees of substitutability, and (b) different levels of moving costs for low skilled workers. Baseline parameterization: $\alpha=0.4, A_{1}=1, A_{2}=2, \rho=0.75, \mu_{n, j}^{L}=-1$ and $\mu_{n, n}^{L}=\mu_{n, j}^{H}=\mu_{n, n}^{H}=0$.

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[^0]:    ${ }^{1}$ See among others Postel-Vinay and Robin (2002), Boeri and Garibaldi (2007), Faccini (2014).
    ${ }^{2}$ A complete characterization can be found in Booth et al. (2002), which focuses on the UK, but a similar characterization is also present in Portugal and Varejão (2009) focusing on Portugal.

[^1]:    ${ }^{3}$ Cahuc and Postel-Vinay (2002) and Boeri et al. (2012), among others

[^2]:    ${ }^{4}$ See also Stevens (2004).

[^3]:    ${ }^{5}$ As I will describe in Section 7, similar reforms were enacted in Spain, Italy and France.

[^4]:    ${ }^{6}$ In Italy, they varied over time, but they generally imposed a cap on the ratio between FTCs and OECs in the same firm (with several exceptions), they limit the number of renewals and the overall maximum duration.
    ${ }^{7}$ Unfortunately, precise data about the extension of this type of agreements are lacking. Using available surveys, Parsons (2017) shows that in $200023 \%$ of the American labour force had access to severance pay in case of separations, a percentage that rose to $34 \%$ among workers in medium-large firms.

[^5]:    ${ }^{8}$ For example, the Italian Labour Survey, RIL provided by ISTAT.

[^6]:    ${ }^{9}$ I described the dataset in appendix A. I use the 2013 data to show the situation before the labour market reforms of the followings years that had some effects on the choice of the contract, as shown in Sestito and Viviano (2016). However, the distribution of the contracts in the population is similar even today.

[^7]:    ${ }^{10}$ The findings are therefore valid only for that specific region since the socio-economic characteristics are different from the rest of Italy. However, there are reasons to believe that the picture in the rest of the country is similar.

[^8]:    ${ }^{11}$ Gagliarducci (2005), for example.

[^9]:    ${ }^{12}$ The dataset is described more in details in Appendix A.

[^10]:    ${ }^{13}$ Later, I analyze the implications of this assumption in presence of on-the-job search. In particular, the fact that the Nash-bargain is repeated at every period solves a problem of non-convex bargaining sets raised in Shimer (2006).

[^11]:    ${ }^{14}$ For example in Faberman et al. (2017), the authors report the result of a relevant survey done in the US on this topic. They show that $20 \%$ of employed workers can be classified as "searchers", with $23 \%$ of workers looking for a new job in the last 4 weeks and almost $20 \%$ actively applying for a vacancy.

[^12]:    ${ }^{15}$ It can be rationalized by the fact that the firm cannot monitor the true search intensity performed by the worker.
    ${ }^{16}$ This assumption is very helpful in simplifying the empirical estimation, given that it allows not to keep track of past wages of the agents. However, it is not qualitatively essential for the results of this section. Indeed, I show in Appendix C that using the sequential auctions protocol, we arrive at similar conclusions.
    ${ }^{17}$ Another implicit assumption is that workers are not paying any other cost related to the job-to-job transition. In reality, this is hardly the case, leading to the possibility of rejections of more productive matches and an overall reduction in the benefits of on-the-job search.

[^13]:    ${ }^{18}$ I will formally define and numerically compute this value later in the full model.

[^14]:    ${ }^{19}$ In Appendix C, I show that this is not the case if we assume that the firm fixes the wage at the beginning of a match and it has all the bargaining power. Then, the possibility to jointly decide $s$ still increases the joint welfare, but it could reduce the worker's welfare.
    ${ }^{20}$ The jointly optimal search intensity is different from the socially optimal one since we are not considering search frictions and most importantly the welfare benefits of the new firms matched with the worker.

[^15]:    ${ }^{21}$ It is allowed to be negative so that the worker actually pays the firm.
    ${ }^{22}$ This assumption can be justified by the acknowledgement that an external judge must sanction a contract violation. This option is generally costly in the first place, and specifically for labour contracts, a violation can also be substantially hard to prove, given the multiplicity of reasons that could justify a contract modification or interruption.

[^16]:    ${ }^{23}$ There is no cost for quitting, nor in the case of the exogenous separation.

[^17]:    ${ }^{24}$ This assumption capture legal limitations on FTC present in Italy. For instance, since 2012, FTC has a maximum duration of 3 years, including renewals, recently reduced to 2 years.

[^18]:    ${ }^{25}$ It would be interesting to study a generalization of this assumption, with possible complementarities between worker's ability and match productivity. This would affect the search intensity of different type of workers, possibly leading to a larger distance between $s^{*}$ and $s^{J}$ of good workers.
    ${ }^{26}$ This assumption of a pure transitory shock is used mainly for tractability. Ideally, it would be interesting to allow for some form of persistence and check the robustness of the results. Following the example of Postel-Vinay and Turon (2010), I could introduce a probability of experiencing a shock at every period.
    ${ }^{27}$ Not only the agents in the match but also all other employers.

[^19]:    ${ }^{28}$ There are several possible justifications for the absence of state-contingent wages in the short term, for example, a cost of writing contracts or incentives related to the efficiency wages theory.
    ${ }^{29}$ In the Toy model, I showed that this assumption is not fundamental. Similarly, in the full-model, a severance payment has similar advantages in terms of lower search intensity and it is still costly to the agents, given the uncertainty over the worker's ability. A OEC imposes inefficient retention of matches with a negative surplus that would have been jointly terminated, absent any severance pay.

[^20]:    ${ }^{30}$ The previous contract just determines the current wage of the worker, as shown in Appendix D.

[^21]:    ${ }^{31}$ The proof starts form $J_{p, p}\left(\phi^{\prime}, x\right)=J_{f, p}\left(\phi^{\prime}, x\right)-\gamma f c$ and uses the fact that $S_{f, p}\left(\phi^{\prime}, x\right)-S_{f, f}\left(\phi^{\prime}, x\right)<f c$
    ${ }^{32}$ This comes directly from the fact that they could have chosen to voluntarily transform the labour contract if that was optimal). Therefore we have $\bar{y}_{f}(\phi, x) \leq \bar{y}_{f^{\prime}}(\phi, x)$
    ${ }^{33}$ More precisely, his wage is higher by $\gamma f c$.

[^22]:    ${ }^{34}$ This is not true for all possible value of $x$ and $\phi$, since the FTC has the advantage that it provides the possibility to continue with a FTC, possibly increasing the welfare of the worker.

[^23]:    ${ }^{35}$ This can happen if the match is discovered unproductive later on when the firing costs limit the possibility to terminate it.
    ${ }^{36}$ I underline once again the fact that this is not the benevolent social planner point of view, since we are not considering the surplus of the new firm.

[^24]:    ${ }^{37}$ A more detailed description is presented in Appendix A.

[^25]:    ${ }^{38}$ Note that in my model with endogenous search intensity, this condition does not assure social optimality of the search and matching process. Nevertheless, it is still useful to use this condition in order to compare the results with other papers in the literature.
    ${ }^{39}$ I am probably slightly overestimating the dropping out probability since some of these workers could come back in the dataset in the following years. However, this rarely occurred in the past.

[^26]:    ${ }^{40} \mathrm{I}$ am considering adding a deterministic growth in the worker's ability to capture the main trend. This should also increase the incentive for the agents to provide employment protection in order to reduce the search intensity.

[^27]:    ${ }^{41}$ Therefore, I am not including the workers who are not returning back into the dataset after 2012, since I am classifying them as out of labour force. For this reason, this job-finding probability is probably an upper bound, since some of these individuals were unemployed failing in finding a job.

[^28]:    ${ }^{42}$ See for example for Economic Co-operation et al. (2013), where they investigate the success of this policy if coupled with an investment in active labour market policies.

[^29]:    ${ }^{43} \mathrm{~A}$ limit of this counterfactual is the assumption that agents cannot agree to stipulate a private agreement regarding severance payments. It is rarely observed in Italy but could become a possibility if indeed firing costs were to be consistently lowered. This would also allow a comparison with a laissez-fair scenario, resembling the US labour market. This interesting possibility is left for future research.

[^30]:    ${ }^{44}$ Source Eurostat, 2017

[^31]:    ${ }^{45}$ This wage can be lower than $w$, since there are expectations of future wage increases, as shown in details in Postel-Vinay and Robin (2002).

[^32]:    ${ }^{46}$ Notice that due to the envelope theorem, it is straightforward to see that the worker always benefits from higher wages.

[^33]:    ${ }^{1}$ See Hall (2017) and Borovička and Borovičková (2018).

[^34]:    ${ }^{2}$ Importantly, Campbell and Shiller (1988).

[^35]:    ${ }^{3}$ We verify this with our data.

[^36]:    ${ }^{4}$ See Blanchard and Diamond (1989) and Petrongolo and Pissarides (2001).

[^37]:    ${ }^{5}$ In fact, for any given year in the OECD' dataset,

    $$
    \mathrm{NRR} \equiv \frac{y_{O W}}{y_{I W}},
    $$

    where $y_{O W}$ is out-of-work net household earnings and $y_{I W}$ is in-work net household earnings. The two measures are taken after and before (respectively) the transition to unemployment. As both measures are net household earnings, both include any labor income earning that is got by the adult that does not transition to unemployment.
    ${ }^{6}$ See Table 2.4 below. In particular, the average duration of unemployment is given by $1 / \tilde{p}$. As we calibrate by targeting monthly moments, the average duration is expressed in months.

[^38]:    ${ }^{7}$ See in particular Hagedorn and Manovskii (2008) and Chodorow-Reich and Karabarbounis (2016).

[^39]:    ${ }^{8}$ To see this, consider a generic univariate Gaussian $x \sim \mathcal{N}\left(\mu, \sigma^{2}\right)$ and write its density function:

    $$
    \begin{aligned}
    f_{x}(x) & =\frac{1}{\sqrt{2 \pi \sigma^{2}}} \exp \left(-\frac{1}{2}\left[\frac{x^{2}-2 x \mu+\mu^{2}}{\sigma^{2}}\right]\right) \\
    & \propto \exp \left(-\frac{1}{2}\left[\frac{x^{2}-2 x \mu}{\sigma^{2}}\right]\right) \\
    & \propto \exp \left(-\frac{1}{2}\left[x^{2} \cdot \frac{1}{\sigma^{2}}-2 x \cdot \frac{\mu}{\sigma^{2}}\right]\right)
    \end{aligned}
    $$

[^40]:    ${ }^{1}$ Since the ability to migrate always is welfare improving, we compare welfare changes in response to economic shocks within scenarios allowing or ruling out migration.

[^41]:    ${ }^{2}$ On a theoretical level, international convergence through migration as been discussed, for instance by Ottaviano (1999) and Felbermayr et al. (2015).

[^42]:    ${ }^{3}$ We considered a worker "skilled" if he or she possesses at least a Bachelor Degree.

[^43]:    ${ }^{4}$ To ease notation, we omit time subscripts through out the presentation of the model, thus also form $\boldsymbol{\epsilon}$.

[^44]:    ${ }^{5}$ We solved the model numerically, and this assumption is important for a stable unique equilibrium. We discuss this issue in the empirical section below.

[^45]:    ${ }^{6}$ The expected continuation values are not the standard of a DMP model, since the agents have to choose their destination country at every period. In particular, $j$ indicates the starting country of residence, but the final one will depend on the realized idiosyncratic preference shock. We are using the results of Rust (1987) to solve for this problem.

[^46]:    ${ }^{7}$ We maintain this assumption due to the small number of migrations between different foreign countries observed in the data. The assumption is not required on a theoretical level.
    ${ }^{8}$ We clarify the definition of distance and the relationship with the migration cost in Section 3.6.2.
    ${ }^{9}$ These results come from Rust (1987).

[^47]:    ${ }^{10}$ We decided to merge these two state rather than excluding Luxembourg.
    ${ }^{11}$ [Appendix A lists these sources in more detail.

[^48]:    ${ }^{12}$ We performed robustness checks for elasticities of substitution ranging between 2 and 6 , and found no qualitative changes.
    ${ }^{13}$ Robustness checks varying the elasticity of substitution between 2 and 6 showed to affect mostly the labor market response to a shock, but our main results are qualitatively robust to different values.

[^49]:    ${ }^{14}$ This is computed as the ratio of the average wage for workers with a college degree over the average wage of workers without a degree.

[^50]:    ${ }^{15}$ We contrast this to the effects of non-permanent shock in Section 3.7.3 and to a symmetric shock in Section 3.7.2.
    ${ }^{16} \mathrm{We}$ readjust $\varphi_{j}$ for all other countries in order to preserve the property $\sum_{j} \varphi_{j}=1$.
    ${ }^{17}$ Note that in a setup in which agents expect TFP or demand to follow a persistent stochastic processes, the dimensionality of the problem would rapidly explode, since our model features 11 locations between which agents can migrate. The model thus would have 11 (possibly correlated) stochastic processes, which renders computation of the agents' value functions infeasible.

[^51]:    ${ }^{18}$ For $\rho_{A}=1$, results are as for the permanent shock discussed above, whereas the effects of the shock become ever smaller as $\rho_{A}$ approaches 0 .
    ${ }^{19}$ For this reason, and to still compare the different countries, we show changes in relative terms with respect to the steady state in Figure 3.11.

