

# Self-Organized Criticality in Economic Fluctuations: The Age of Maturity

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#### Abstract

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Bak, Chen, Scheinkman and Woodford (1993) and Scheinkman and Woodford (1994), hypothesize that emergence of macro-financial fluctuations reflects an underlying self-organized critical state. The wave of innovative thinking sparked by their proposal continues to produce interesting contributions in many areas of economics, ranging from macroeconomics to finance. In this review, we propose a guided tour to these achievements highlighting that analysis of self-organized critical equilibria is a promising avenue to establish a nexus between i) a statistical equilibrium characterized by the spontaneous emergence of dynamic critical fluctuations and ii) a strategic equilibrium concept modeling a large number of interacting players, like firms, traders, or consumers, where the critical state is the stable outcome arising from a tradeoff between cooperation and competition.

#### Contribution to the field

A mini-review where I try to clarify the relevance of the notion of Self-Organized Criticality in the fields of Macroeconomics and Finance

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## Self-Organized Criticality in Economic Fluctuations: The Age of Maturity.

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#### Abstract

Bak, Chen, Scheinkman and Woodford (1993) and Scheinkman and Woodford (1994), hypothesize that emergence of macro-financial fluctuations reflects an underlying self organized critical state. The wave of innovative thinking sparked by their proposal continues to produce of interesting contributions in many areas of economics, ranging from macroeconomics to finance. In this review, we propose a guided tour to these achievements highlighting that analysis of self-organized critical equilibria is a promising avenue to establish a nexus between: i) a statistical equilibrium characterized by the spontaneous emergence of dynamic critical fluctuations and ii) a strategic equilibrium concept modeling a large number of interacting players. The critical state is the stable outcome arising from a tradeoff between cooperation and competition.

## 1 Introduction

The interaction between economic and physical sciences is so rich that a new interdisciplinary field has emerged, Econophysics. While its cultural appealing is undisputable, contributions that are considered relevant and welldisciplined both by economists and physicists are rare. It is fair to say that the SOC paradigm for economic and financial fluctuations put forward in the seminal contributions Bak et al. [1987], Scheinkman and Woodford [1994], Bak et al. [1993] is one of them.

While many are sympathetic with the proposal that a Self Organized Critical (SOC) underlies financial and macroeconomic fluctuations, the introduction of a common language and of analytic tools useful to make this proposal effective both at descriptive and normative level are still under way. In fact, the virtue and the sin of the early SOC proposal lies in the use of a simple toy model to exemplify a number of characteristics that are expected to play a paradigmatic role.

It is virtually impossible to produce a systematic discussion of the overall state of the art, given the relevance of the topic and the number of research contribution in this area. Hence, in this short review I propose a guided tour through the research spillover originated from those seminal proposals and argue that SOC is an important building block of an emerging interdisciplinary paradigm suitable to frame a notion of statistical equilibrium relevant for both social and natural sciences.

In writing this contribution, I will take a purely subjective point of view with the deliberate goal of highlighting the strengths but also the open issues that are to be clarified to make the notion of SOC state a more disciplined and useful instrument of economic analysis.

The review is organized as follows: we will start focusing in the next Section 2 on those common misconceptions and cultural differences that have so far limited the interaction among researchers with such different backgrounds. Then we dedicate a section to each one of the following three themes that we consider SOC 'identitarian'. In Section 3 we revisit the original model trying to clarify why the notion of 'spontaneous emergence of a critical state' fits well in the traditional economic debate about the origin of macroeconomic fluctuations. In Section 4 we review the interdisciplinary attempts to improve the econometric description of financial and economic fluctuations. In particular, we focus on those that analyze financial time series borrowing the conventional tools used to analyze critical phenomena in physics: scaling, universality and renormalization. In Section 5, we review the growing body of literature focusing on the notion of 'avalanche' that we interpret as correlated sequence of spatiotemporal

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events. We conclude with a focus on those research directions that in our opinion should be targeted by the coming research efforts in order to complete our understanding of the SOC paradigm with the provision of effective policy instruments.

## 2 Economic equilibrium vs Statistical (Non)-Equilibrium.

While the use of probability and statistics is common to both natural and social sciences, its theoretical foundations are radically different. Hansen [2014], the contribution written in occasion of the attribution of the Nobel Prize for economic sciences offers a key insight that clarifies the critical challenges underlying the quantification of uncertainty in a dynamic stochastic economy. The line of attack to macro-financial modeling proposed in Hansen [2014] fits nicely within a non-equilibrium statistical mechanics analysis of open systems: 'Exogenous shocks repeatedly perturb a dynamic equilibrium through the model's endogenous transmission mechanisms' (Hansen [2014] page 946). As Hansen points out, macro-financial modeling poses two challenges. The first one has a descriptive focus: to describe what is called '*uncertainty outside the model*', i.e. to best reproduce statistical features and impulse response of measured real-world time series. This challenge fits well within a conventional statistical mechanical approach. The second challenge faced in economic modeling is that the model describes also *inside uncertainty: '... agents* inside our model, be it consumers, entrepreneurs, or policy makers, must also confront uncertainty as they make decisions. I refer to this as inside uncertainty ...'. This second aspect is intimately related with the normative focus of economics: 'The modeler's choice regarding insiders' perspectives on an uncertain future can have significant consequences for each model's equilibrium outcomes'. It is this second objective that poses a formidable hurdle to the statistical mechanical description of economic fluctuations. This second requirement changes qualitatively the description at micro-level of the system: description of individual economic behavior requires the introduction of a set of control variables to describe actions at individual and collective level. So the conventional mechanical description of a particle must be replaced by the micro description of individual decision making. While the emphasis in physics is more focused on the descriptive content, these considerations explain why economists often consider acceptable a poor description of *outside uncertainty* as an acceptable price to pay in order to achieve a better description of *inside uncertainty* that is necessary to account for the policy reaction of individuals.

At first sight, these fundamental issues look completely unrelated and far from the SOC paradigm. On the contrary they have a strict connection. Intuitively, the SOC automata microscopic rules are those that describe the rational behavior of individuals. The choice of an individual depends crucially on the inference process that the decision maker adopts to frame observed data in a model. Mastromatteo and Marsili [2011] provide a thorough information theoretic analysis of the relationship between inference and emergence of critical behavior within an abstract statistical information theory framework. They show that inference procedures are likely to yield models which are close to singular values of parameters, akin to critical points in physics where phase transitions occur. Hence, following this argument, when inside uncertainty is taken into account, emergence of critical behavior will look like the rule rather than the exception.

A similar, yet simpler example of a micro-funded process is proposed in Blume et al. [2011]. They model a game of strategic network formation where agents must collectively form a network in the face of the following trade-off: each agent receives benefits from the direct links it forms to others, but these links expose it to the risk of being hit by a cascading failure that might spread over multi-step paths. They prove that the resulting optimal networks are, in a precise sense, situated just beyond a (percolation type) phase transition in the behavior of the cascading failures.

It is worth observing that in this economic detour, we touched statistical equilibrium physics in relation to the definition of a second-order critical point corresponding to the critical point for percolation but also the notion of a cascading failure that intrinsically requires the non-equilibrium dynamics. An economy is fundamentally an open system where the *endogenous* reaction is induced from the agents that re-optimize their decisions. This makes the overall response highly non-linear and certainly not amenable of a standard equilibrium statistical physics treatment.

In light of these considerations, it is striking and puzzling that the SOC paradigm may often provide an effective and accurate description of many empirical stylized properties of a competitive (economic) equilibrium.

## 3 Self-organization and the 'invisible hand'.

During the early 90's many authors have emphasized the role of complexity and non-linearity in economics and the potential interaction with the analysis of complex phenomena in natural sciences, for a review see e.g. Anderson

[2018]. It is within this framework that Bak et al. [1993] and Scheinkman and Woodford [1994] formulate the proposal that a SOC state may explain business cycle fluctuations in the level of economic activity indicators. In the original model, sharing many characteristics of the Dhar and Ramaswamy [1989] directed sandpile and the stochastic failure sharing model Manna [1991] models, critical fluctuations are driven by local interaction between customers and suppliers forming a network of producers with non-convex technologies. One key original aspect of the SOC approach to the analysis of macro-financial fluctuations is the role played by what is called the 'large economy limit', see e.g. Scheinkman and Woodford [1994] '...we argue that aggregate fluctuations in production continue to occur in the large-economy limit ...'

From a statistical mechanical point of view, this statement may seem quite natural. It mimics the necessity to consider the so called 'thermodynamic limit' to match a model with the relevant observables. As discussed in the previous section, the extension of this argument to the analysis of macro-financial fluctuations is far from trivial, it is the problem of aggregation that has to be analyzed more carefully. A more recent strand of research has highlighted the main role that the network of firm interaction plays in the propagation of shocks in production economies, see e.g. Carvalho and Gabaix [2013] and Acemoglu et al. [2016]. Taschereau-Dumouchel [2019] provides an extensive review and proposes a model calibrated on real data. An undisputable merit of these approaches is their compliance both at micro and macro-level with the basic economic principles that regulate the firm and consumer actions.

From the point of view of statistical mechanics they are not completely satisfactory: a more disciplined approach to the analysis of the large economy limit is proposed by Moran and Bouchaud [2019] relying on random matrix theory. In this case the authors show explicitly that a network economy triggered toward the critical point generates power-tail size distributions of firms. The authors conjecture that evolutionary and behavioral forces conspire to drive the economy towards marginal stability.

In a different economic context Nirei et al. [2019] offer an important example, to our knowledge the first one, of a properly micro-funded 'snow ball effect' that drives the spontaneous emergence of critical behavior in the large economy limit. Nirei et al. [2019] present a state-dependent pricing model that describes the inflation fluctuations driven by idiosyncratic shocks hitting the cost of price changes of individual firms. Firms' pricing behavior in equilibrium exhibits complementarity: the critical cascading effect is sparked by firm's repricing that reduces all the competitors' relative prices, thus inducing more of them to reprice. As most of the mean field approaches to self-organized critical models, see also Zapperi et al. [1995], they model the cascade process relying on the classical theory of critical branching processes.

## 4 SOC and Econometrics

One of the strongest arguments in favor of the approach pursued by Econophysics is the inadequacy of linear stationary models in providing a reliable description of the basic properties of macro-financial time series models.

From an empirical point of view, a 'minimalist' characterization of SOC Critical models, requires the spontaneous emergence of power law distributions and scaling relationships. In particular, see e.g. Manna [1991] and Tebaldi et al. [1999], it postulates that the size of the response (the avalanche) to an exogenous idiosyncratic shock (the addition of a grain in the sandpile) follows a law satisfying a finite-size scaling ansatz:

$$P_s(s,L) := s^{-\tau_s} F_s\left(\frac{s}{L^{D_s}}\right).$$

Then, assuming that  $F_s(x)$  has an exponential decay for  $x \gg 1$ , L is a fundamental cutoff to scale invariant behavior. Furthermore scale invariant critical behavior requires also that the size of the response is related to its spatial extension a and to its lifetime T by homogeneous relations i.e.  $s \sim a^{\gamma}$  and  $s \sim T^{\beta}$ . This implies that the finite size scaling relation holds also for the distributions of a and T:

$$P_{a}(a,L) := a^{-\tau_{a}}F_{a}\left(\frac{a}{L^{D_{a}}}\right), P_{T}(T,L) := T^{-\tau_{T}}F_{T}\left(\frac{T}{L^{D_{T}}}\right)$$

$$\frac{D_{s}}{D_{a}} = \frac{\tau_{s}-1}{\tau_{a}-1} = \gamma, \qquad \frac{D_{s}}{D_{T}} = \frac{\tau_{s}-1}{\tau_{T}-1} = \beta$$

$$(1)$$

These relations hardwire scaling invariance in the parametric description of the impulse response and are difficult if not impossible to derive within the class of linear stationary models. Reference to SOC appears in all the early contributions Mantegna and Stanley [1999], Gabaix et al. [2003], Gabaix [2009] and Cont and Bouchaud [1997]. One important debatable question is whether, beyond power laws, macro-financial fluctuations show consistency with the full articulated set of observable implications that characterize dynamic critical phenomena in physics. In fact, a joint test of power law behavior, finite size-scaling and universality within a properly defined renormalization group fixed point, is still missing. In the following we review the interesting results that are suggestive of the possibility that an attracting fixed point may underlie the SOC properties of financial time series.

#### 4.1 Universality

Emergence of universal scaling relations in financial markets has a relatively long history in relation to return data. It is a relatively more recent acknowledgment in relation to the market microstructure behavior. Kyle and Obizhaeva [2016] formulate the so called market microstructure invariance hypothesis. It claims ad verifies the existence of a universal invariant quantity I representing the average cost of a single bet. It is expressed in dollars, independent of the asset and constant over time. Dimensional analysis suggests a relation of the form:

$$\frac{PQ}{I} = f\left(\sigma_{\rm d}^2 \frac{Q}{V}\right) \,, \tag{2}$$

where, P is the share price in dollars,  $\sigma_d^2$  denotes the square daily volatility, V the total daily amount traded with *bets* and Q the average volume of an individual *bet*. Invoking the Modigliani-Miller capital structure irrelevance principle yields  $f(x) \sim x^{-1/2}$ , which implies up to a numerical factor that:

$$I = \frac{\sigma_{\rm d} P Q^{3/2}}{V^{1/2}} := \frac{\mathcal{R}}{N^{3/2}},\tag{3}$$

where  $\mathcal{R} := \sigma_d PV$  measures the total dollar amount of risk traded per day (also referred to as total exchanged risk or trading activity) while N := V/Q represents the number of daily *bets* for a given contract. Bucci et al. [2020] propose a reformulation of the hypothesis that improves the finite size scaling collapse. A thorough discussion of the potential role played by SOC in explaining market microstructure liquidity dynamics is given in Fosset et al. [2020]. In classical critical phenomena, universality finds its explanation in the simple observation that macroscopic (aggregate) properties will depend only on the stochastic limiting properties of a model that are not removed by a progressive integration (averaging) of micro-fluctuations. The progressive averaging procedure is part of the so called renormalization group transformation.

#### 4.2 Renormalization Group

A natural bridge between critical phenomena and stochastic modeling of financial time series is offered by the probabilistic interpretation of the Renormalization Group transformation proposed in Jona-Lasinio [2001]. In this formulation, the RG theory is nothing else than a stochastic limit theory for possibly correlated random variables. Fixed points for the semigroup genearted iterating the RG transformation represent the achievable limit distributions. Only properties that survive to the iteration and are relevant to define the fixed point are macroscopically observable. Ortu et al. [2020] includes an 'econometrics friendly' introduction to RG transformations in time series analysis. They formalize an extension of the (real space) RG transformation introducing an RG operator, the  $\mathcal{R}$  operator in paper's notation, that acts on the classical space of stationary square integrable time series.

The most flexible operational approach to stochastic modeling of financial return time series relying on the Renormalization Group approach is originally proposed in Baldovin and Stella [2007] and systematically exposed in Zamparo et al. [2013]. It is grounded on a 'fine-graining' procedure obtained 'inverting' the real space renormalization group (RG hereafter) flow in the space of return probability distributions. In this case the starting point of the procedure is the fixed point equation that characterizes the scale invariant distribution. Recovery of the observed ones occurs considering a 'cascading procedure' that tracks backwards the conventional RG coarse graining procedure. Merits and challenges of this approach are extensively discussed in Zamparo et al. [2013]. This approach seems to offer the best setup to frame the probabilistic analysis of (possibly self-organized) critical behavior in time series. The key ingredients that in our opinion are necessary to characterize a SOC ensemble (a set of counterfactuals) are essentially two: (i) an endogenous propagation dynamics that preserves scale invariance of conditional moments and (ii) a random rescaling factor designed to embody also exogenous influences. The fine graining procedure has found application in a number of interesting declinations, see Andreoli et al. [2012], Baldovin et al. [2015], Peirano and Challet [2012] that witness the applicative content of the theory.

One crucial observation, noticed in Tebaldi et al. [1999] for the SOC abelian sandpile and explored also in relation to the fine graining by Zamparo et al. [2013], is that statistical description of impulse-response functions may require a multiscaling framework, an extension of the conventional finite-size scaling framework. In this case, the probability distribution can be seen as a superposition of scale-invariant clusters with heterogeneous fractal properties. Multifractal formalism requires that the scaling properties of  $P_s(s, L)$  are described by the spectrum:

$$f(\alpha) := \frac{\log\left(\int_{L^{\alpha}}^{+\infty} P_s(x, L) \, dx\right)}{\log\left(L\right)}, \ \alpha := \frac{\log\left(s\right)}{\log\left(L\right)} \text{ for } \quad L \to +\infty.$$

Then, in the limit  $L \to +\infty$ , asymptotic moment scaling functions are determined by:

$$\langle s^{q} \rangle \stackrel{L \to +\infty}{\approx} L^{\sigma(q)}, \, \sigma(q) := \sup_{\alpha} \left[ \alpha q + f(\alpha) \right]$$

These equations are consistent with a multiplicative decomposition of the random propagation effects and cast a direct connection with the analysis of financial markets building on the similarity of volatility fluctuations with energy dissipation cascades in turbulence. Early contributions by Mandelbrot et al. [1997], Ghashghaie et al. [1996] established a strong analogy between turbulent cascades and volatility clustering. A new generation of refined multiscale models like for example the multifractal random walk model Bacry et al. [2001] are capable to match empirical properties of observed time series volatility.

Beyond scale invariance, characterization of SOC behavior relies also on property (ii), i.e. the existence of random correlated sequences of micro events (an avalanche in the SOC dictionary) that generate a breakdown of time translation invariance. Statistical description of dynamic clustering within a scale invariant model is analyzed by Baldovin et al. [2013]. This class of models is able to reproduce the stylized behavior of volatility intermittent decay after a main financial shock. As observed in Lillo and Mantegna [2003], it parallels the Omori law in geophysics for the seismic activity after an earthquake of exceptional magnitude.

## 5 Avalanche dynamics

We dedicate the last concluding section to the paradigmatic role played by the concept of avalanche that is as intuitive as difficult to formalize within a macro-financial model. Along this review we encountered already many notions that are close friends of the notion of avalanche: the reorganization of the supply chain economy in a competitive equilibrium that is hit by a shock, the sequence of correlated orders that are determined by a single trade decision, the random rescaling factor in the 'fine grained' model. Last but not least, recent macro-financial research has shown that propagation of financial shocks across financial institutions interconnected by the web of financial claims, plays a major role in the unfolding of financial crises, see e.g. Haldane and May [2011]. Consideration of the financial system as a SOC state has provided a great insight into the economic collapse following the sub-prime crises of 2008 and the following sovereign credit crisis in Europe. Roukny et al. [2013] provide an extensive and interdisciplinary review.

Despite the large amount of interesting empirical evidence and modelization efforts, research has not yet reached a consensus on a precise, empirically testable definition of avalanche dynamics. Intuitively, it is meant to be a dynamic counterpart to the self-similar ensemble of clusters that characterizes fluctuations in an equilibrium system close to the critical point, where susceptibility diverges. Here's where the tension between the equilibrium and non-equilibrium description of avalanche statistics is more evident. Consider for example the 'Ising Model' of SOC, the Abelian Sandipile. In this case the steady state model has an exact characterization in terms of the  $q \rightarrow 0$  limit of the equilibrium Potts model and critical properties can be derived from its conformal invariance, see Dhar [2006]. Attempts to extend the mapping and characterize the dynamic (non-equilibrium) scaling properties of avalanche clusters have so far not been successfull.

In conclusion, the previous considerations highlighted that the search of a satisfactory self-organized critical model for macro-financial fluctuations after thirty years is far from the end. Indeed SOC is still a promising avenue toward a truly effective interdisciplinary paradigm to characterize critical statistical fluctuations arising in strategic equilibria of interacting rational individuals.

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