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Abstract:

Ambiguity may be high when there exists conflicting information regarding the probability model characterizing the randomness of a phenomenon. In this paper, we report the results of an experiment jointly eliciting the attitudes individuals exhibit towards risk and model uncertainty. Using a joint elicitation procedure, we are then able to quantify precisely the strength of individuals' attitude towards ambiguity and to characterize its main properties. Our results provide empirical evidence of decreasing absolute ambiguity aversion (DAAA) and constant relative ambiguity aversion (CRAA), and shed new light on the way ambiguity attitude may affect important decisions, such as the choice of health insurance policies or the optimal investment strategy in the face of climate change.

**Submission to the Special Issue on Behavioral Insurance in the *Journal of Economic Behavior & Organization***

Dear François, Dear Prof. Corcos, Dear Prof. Montmarquette

Please find enclosed our article "*Characterizing ambiguity attitudes using conflicting information*".

This paper provides a methodological contribution to the experimental literature on ambiguity aversion, which could be of great interest for questions related, for example, to health insurance and to self-insurance/self-protection investments in the context of climate change.

Specifically, we propose a new experimental method to characterize more precisely the extent to which ambiguity aversion exists and to characterize its main properties.

Our approach explicitly makes use of the information provided by different *experts* regarding the probabilistic model to be used for characterizing a risk.

We contribute to the literature in several ways:

- 1) We find that the epistemic uncertainty concerning who is the "correct" expert to follow is not considered the same way as the aleatory uncertainty concerning the outcome of well-defined risk. More specifically, our subjects exhibit a stronger aversion towards epistemic uncertainty (model uncertainty) than towards aleatory uncertainty (risk).
- 2) The former result is interpreted as evidence of ambiguity aversion, which we are able to quantify precisely. In particular, our design enables us to elicit the degree of ambiguity aversion, which we estimate to be around 0.5 when considered in relative terms.
- 3) We find evidence of decreasing absolute ambiguity aversion (DAAA) and constant relative ambiguity aversion (CRAA).

As the properties of the ambiguity aversion function have been shown to be of great importance for determining the effect of ambiguity on optimal decision in insurance, self-insurance and self-protection problems (Alary, Gollier, Treich, 2013; Berger, 2016), we believe that our paper is particularly suitable for the Special Issue on Behavioral Insurance in the *Journal of Economic Behavior & Organization*.

We thank you very much for your time and effort and are looking forward to the evaluation.

Sincerely,

Loïc Berger and Valentina Bosetti

# Characterizing ambiguity attitudes using conflicting information

Loïc Berger

Valentina Bosetti\*

## Abstract

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# Characterizing ambiguity attitudes using conflicting information

Loïc Berger      Valentina Bosetti\*

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1 “Ambiguity may be high [...] when there are questions of reliability  
2 and relevance of information, and particularly where there is *conflicting*  
3 opinion and evidence.”  
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7 Ellsberg (1961, p. 659)  
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## 10 **1 Introduction**

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14 Almost sixty years ago, Ellsberg (1961) proposed a series of experiments whose  
15 results suggest that people generally prefer situations in which they know the  
16 probabilities of events occurrence to situations in which these probabilities are  
17 unknown. This behavior –which violates a key axiom of the classical model of  
18 choice under uncertainty– is known as *ambiguity aversion*. It has been the focus  
19 of a large body of literature, which has explored both the theoretical and experi-  
20 mental sides of the topic. According to Ellsberg himself, this preference is neither  
21 due to the relative desirability of the possible payoffs, nor to the relative likeli-  
22 hood of the events affecting them, but rather to “*the nature of one’s information*  
23 *concerning the relative likelihood of events*”. In particular, Ellsberg (1961) wrote:  
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30 “What is at issue might be called the ambiguity of this information,  
31 a quality depending on the amount, type, reliability and “unanimity”  
32 of information, and giving rise to one’s degree of “confidence in an  
33 estimation of relative likelihoods.”  
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37 Ambiguity aversion thus results from a preference for situations in which all events  
38 relevant to decision making are associated with obvious probability assignments,  
39 on which everyone agrees (i.e. what is generally called “risk” in economics) to  
40 situations in which some events do not have an obvious, unanimously agreeable,  
41 probability assignment (Ghirardato, 2004).  
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45 Ambiguity is present in virtually all real-life situations. It plays a major role  
46 in many problems policymakers and public authorities may face and thus directly  
47 affects their decision making processes. Ambiguity may, for example, result from  
48 the existence of conflicting expert opinions about the probability of a particular  
49 event (Cabantous, 2007; Cabantous et al., 2011). Or, it may arise from the ex-  
50 istence of multiple, plausible and conflicting models predicting the probabilistic  
51 effects of alternative policies.  
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56 *Health Insurance.* As in Gilboa and Marinacci (2013); Marinacci (2015), con-  
57 sider the problem of a decision maker (DM) who has to choose whether to buy  
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1 insurance against the risk of a heart disease. By browsing the internet and provid-  
2 ing some personal information about herself (age, cholesterol level, blood pressure,  
3 smoking habits), the DM is confronted with different probability assessments of  
4 heart disease provided by the risk calculators of different hospitals. Thus different  
5 “experts”<sup>1</sup> provide different *probability models* for the same event, i.e. “suffering  
6 a heart attack in the next years”. As illustrated in Gilboa and Marinacci (2013),  
7 the estimates may vary substantially from one expert to the other. The reason  
8 for this being that simple relative frequencies cannot be used as a definition of the  
9 probability of a heart disease since no two human bodies are perfectly identical.  
10 Instead, the calculators assess the probabilities for different individuals depend-  
11 ing on their characteristics using more complex methods. As a consequence, the  
12 assessed probabilities are not perfectly objective: the team in charge of the cal-  
13 culator in each hospital has to choose the individual characteristics to be used,  
14 the database, as well as the estimation technique. Altogether, this results in the  
15 existence of a set of possible probability models rather than a unique probability  
16 distribution.

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26 *Environmental Disasters.* A policymaker may have to design an investment  
27 strategy, entailing both mitigation and adaptation efforts, in response to climate  
28 change.<sup>2</sup> In this context, different probability distributions may exist to charac-  
29 terize the same event (Heal and Millner, 2014). This happens either because there  
30 is not enough information available; because different climate predictions exist  
31 –which depend on different data sets, different estimation techniques or numerical  
32 models– or because different experts provide different probabilities assessments  
33 for the same event occurrence. As a consequence, we do not know, for example,  
34 how global warming will affect the probability of being confronted to a climate  
35 catastrophe (Lenton et al., 2008). Since these events have not been encountered  
36 in recent history, their likelihood of occurrence is extremely difficult to quantify  
37 precisely. Decisions are therefore generally made based on the advices of scientific  
38 experts –as those reviewing the available literature in the IPCC reports– whose  
39 probability assessments may vary significantly (Kriegler et al., 2009; Berger et al.,  
40 2017). In this case too, different probability models therefore exist to characterize  
41 the same event.  
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51 <sup>1</sup>Throughout the paper, we refer to *experts* as those individuals or entities who presumably  
52 have more information and/or expertise than the decision maker, and who are acting as advisors  
53 by providing information (Budescu et al., 2003).

54 <sup>2</sup>*Adaptation* is the process of “adjustment to actual or expected climate and its effects. In  
55 human systems, adaptation seeks to moderate harm or exploit beneficial opportunities” (IPCC,  
56 2014a), while *mitigation* is “a human intervention to reduce the sources or enhance the sinks of  
57 greenhouse gases” (IPCC, 2014b).  
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1 Whether concerning health related issues or environmental questions, what is  
2 common to the above-mentioned examples is the idea that ambiguity and elements  
3 of insurance of different types are tightly intertwined. Specifically, while the first  
4 example concerns the provision of market insurance in the presence of ambiguity,  
5 the second example deals with ambiguity and self-insurance/self-protection in the  
6 sense of Ehrlich and Becker (1972). These risk management tools are generally  
7 used to deal with the risk of a monetary loss when market insurance is not avail-  
8 able. In both situations, the DM has the opportunity to undertake an effort to  
9 modify the distribution of a risk. In the case of self-insurance (SI), the effort is  
10 aimed at reducing the size of the potential loss, while in the case of self-protection  
11 (SP), the effort aims to reduce the probability of suffering from the loss. In that  
12 sense, and as mentioned by Berger (2016), investments in adaptation efforts in  
13 the face of climate change may seen as an example of SI, while mitigation efforts  
14 correspond to SP.

15 To what extent do ambiguity preferences affect the decisions to purchase health  
16 insurance policies, or to undertake SI/SP efforts in the presence of climate change?  
17 The answer to this question fundamentally depends on the strenght of ambiguity  
18 aversion, as well as on some behavioral properties of the function characteriz-  
19 ing ambiguity preferences. In theoretical works, Snow (2011) and Alary et al.  
20 (2013) have for example shown, using single period models, that ambiguity aver-  
21 sion increases the insurance coverage rate and the optimal level of SI, while it may  
22 increase or decrease the optimal level of SP. In a two-period setting, Berger (2016)  
23 and Berger et al. (2017) have shown that ambiguity aversion alone is not sufficient  
24 to sign the effect ambiguity has on the decision to (self-)insure or self-protect. An  
25 additional condition of non-increasing absolute ambiguity aversion is also required  
26 in most usual situations where the degree of model disagreement among experts  
27 decreases with the effort. Yet, at an empirical level, the question has not been  
28 tackled in much details.

29 The aim of this paper is to provide a methodological contribution to this litera-  
30 ture. Specifically, what we propose in this study is an experimental method able to  
31 quantify more precisely the extent to which ambiguity aversion exists and to char-  
32 acterize its main properties.<sup>3</sup> To do so, we decompose ambiguity into two distinct  
33 layers of analysis, following an approach proposed by Hansen (2014), Marinacci  
34 (2015) and Hansen et al. (2016). The first layer, commonly referred to *risk* (or  
35 *aleatory uncertainty*), features the probability measure associated with the ran-

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36 <sup>3</sup>Few studies have actually attempted to quantify the strength of the ambiguity aversion  
37 effect. Notable exceptions, using various models, are Abdellaoui et al. (2011); Ahn et al. (2014);  
38 Dimmock et al. (2015).

1 domness of an event. It refers to the physical quantification of uncertainty by  
2 means of a probability model that specifies fully the outcome probabilities. This  
3 layer of *risk within a model* is the one typically studied by economists in problems  
4 involving decision making under uncertainty (see for example Dionne and Eeck-  
5 houdt (1985); Jullien et al. (1999); Eeckhoudt and Gollier (2005) in the context  
6 of SI-SP). The second layer, referred to as *model uncertainty* (or *epistemic un-*  
7 *certainty*), represents the uncertainty about which alternative probability model  
8 should be used to assign the probabilities.<sup>4</sup> This layer of analysis appears when the  
9 correct risk is itself unknown. This is for example the case when multiple expert  
10 assessments appear plausible in describing the randomness of a phenomenon.

11 In a controlled experimental environment which extends Holt and Laury's  
12 (2002) and Andersen et al.'s (2008) setups, we confront our subjects with var-  
13 ious risk and model uncertainty situations. Using the distinction between the  
14 two, our design enables us to quantify precisely the degree of ambiguity aversion  
15 and to study its main properties by means of non-parametric statistics as well as  
16 structural econometric analyses of choice patterns.

17 There are two main findings emerging from our analysis. First, subjects tend to  
18 be both risk and model uncertainty averse, but exhibit stronger aversion towards  
19 model uncertainty than towards risk. This behavioral characteristic is interpreted  
20 as evidence of ambiguity aversion, which is then elicited via a joint estimation  
21 procedure. Second, and analogously to what has been previously reported for risk  
22 aversion (Holt and Laury, 2002, 2005), we find that model uncertainty aversion  
23 is decreasing when considered in absolute terms, and increasing when considered  
24 in relative terms. In terms of ambiguity attitude, we find evidence of decreasing  
25 absolute ambiguity aversion (DAAA) and constant relative ambiguity aversion  
26 (CRAA).

## 243 2 Experimental procedures

244 This section presents the procedures followed in the experiment. We study  
245 two different types of uncertain situations, represented by urns filled with balls  
246 that may be either red or black. The first situation is a simple risk situation in  
247 which the number of red and black balls –and therefore the probability model– is  
248 objectively known. The second situation is a model uncertainty situation where  
249 ignorance is first achieved by letting both the number and the color of the balls in  
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251 <sup>4</sup>Note that the term “epistemic” derives from the Ancient Greek ἐπιστήμη, which means  
252 “knowledge”, while the term “aleatory”, which originates from the Latin *alea*, refers to any  
253 game of chance involving dice.



the urn to be unknown before two experts provide their assessment –or probability models– of the urn composition. This situation presents uncertainty in two layers: two possible models (the two experts’ assessments) representing two distinct risks are provided to the subjects. The experiment consists of a sequence of eight tasks. In each task, subjects are confronted with a series of binary choices, presented in the form of ordered tables, as popularized by Holt and Laury (2002).

## 2.1 The design

The two types of uncertain situation are represented by urns containing red and black balls. Each urn describes a particular type of uncertainty that we distinguish as follows:

- *Risk*: the proportion of red and black balls in the urn is known;
- *Model uncertainty*: the proportion of red, black and the total number of balls in the urn are unknown. However, information is provided by two “experts”: each giving her own assessment of the composition of the urn.

The probability of drawing a red (black) ball  $P(r)$  ( $P(b)$ ) is thus objectively known in the case of risk, but not in the case of model uncertain. In this latter case however, subjects are still given information regarding the possible compositions of the urn, via the experts’ assessments.

**CE tasks** The first set of tasks takes the form of certainty equivalent (CE) tasks. In these tasks, subjects are asked to make a series of choices between an uncertain prospect and sure amounts of money. The uncertain prospect is either represented by a risky lottery or by a situation of model uncertainty. Specifically, by letting  $O$  denote the set of monetary outcomes, and  $\bar{o}_p \underline{o}$  the binary lottery yielding  $\bar{o} \geq \underline{o} \in O$  with probability  $p$  and  $\underline{o} \in O$  otherwise, subjects are asked to make a series of ten choices between  $\bar{o}_p \underline{o}$  and different values of  $o \in O$  ordered from  $\bar{o}$  to  $\underline{o}$ .<sup>5</sup> In the case of model uncertainty, the lottery  $\bar{o}_p \underline{o}$  is replaced by the uncertain situation denoted  $\bar{o}_{\hat{p}_1 \hat{p}_2} \underline{o}$ , in which the subject is only given the two experts’ assessed probabilities of winning  $\hat{p}_1$  and  $\hat{p}_2$ . The design of the CE tasks is standard. It enables us to characterize the payoff that would leave the subject indifferent between the prospect and the sure amount of money in both situations of risk and model uncertainty. Ultimately, the CE tasks provide a direct measure of the strength of model uncertainty aversion relative to risk aversion. To ease

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<sup>5</sup>We used the following values in our CE tasks:  $\bar{o} = 25$ ,  $\underline{o} = 4$ ,  $p = 0.5$ , and  $o \in \{25, 18, 15, 14, 13, 12, 10, 8, 6, 4\}$ .

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2 comparisons, we only consider the case of *dogmatic experts* in the CE tasks: each  
3 expert’s assessment is degenerate in the sense that a probability 100% is associated  
4 with a particular event. This enables us to isolate the effect of model uncertainty  
5 alone and to capture model uncertainty preferences (see hereafter).  
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9 **PL tasks** In the remaining tasks of the experiment, we use a double price list  
10 (PL) procedure to jointly elicit risk and model uncertainty attitudes. Two risky  
11 tasks are designed following Holt and Laury’s (2002) PL standard procedure,<sup>6</sup>  
12 while in the four remaining uncertain tasks, subjects are asked to make a series  
13 of choices between risky situations  $\bar{o}_p \underline{o}$  and situations of model uncertainty  $\bar{o}_{\hat{p}_1 \hat{p}_2} \underline{o}$ .  
14 The PL procedure is one of the most commonly employed elicitation methods to  
15 represent choices between gambles (Andersen et al., 2006). It is widely considered  
16 as a transparent procedure that rarely confuses subjects about the incentives to  
17 respond truthfully (Harrison and Rutström, 2008). However, one of the main  
18 disadvantages of this method is that subjects typically have the possibility to  
19 switch freely between the two options as they progress down the decision tables.  
20 They may therefore make inconsistent choices either by switching more than once,  
21 or by making reverse choices (Charness, Gneezy, and Imas, 2013). While we  
22 recognize these inconsistent behaviors raise additional difficulties –given that they  
23 are difficult to rationalize under standard assumptions on preferences, and that  
24 the estimation technique and inference of risk and model uncertainty attitudes  
25 require a unique switching point– we decided not to enforce consistent choices  
26 in this experiment.<sup>7</sup> Rather, we view such behavior as indicative of failing to  
27 understand the instructions correctly, or of confusion on the part of the subjects,  
28 and discard this inconsistent data from the analysis of our results.  
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41 **Discussion** We aimed at a design that, at the same time, remains simple and  
42 yet emphasizes the difference between risk and model uncertainty. The design  
43 presents few caveats, that we discuss here. While two expert’s assessments were  
44 provided in the model uncertainty tasks, it should be understood that these ex-  
45 perts were not physically present in the lab during the experiment. Although this  
46 situation is close to many real-life situations in which expert opinions are presented  
47 without the physical presence of the experts themselves, this feature may have led  
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52 <sup>6</sup>In a nutshell, in each task subjects are asked to make a series of ten choices between two  
53 options (Option A and Option B), each presenting a different lottery (the full set of tasks can  
54 be found in the online supplemental Appendix).  
55

56 <sup>7</sup>Several techniques have been proposed in the literature to enforce consistency in the subjects’  
57 choices (see for example Andersen et al. (2006)), but with the major drawback that they may  
58 significantly bias the results (Charness, Gneezy, and Imas, 2013).  
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1 subjects to downplay the experts' role or to partially ignore them (making the sit-  
2 uation closer to a case of full ignorance). To minimize the potential bias, we gave  
3 particular attention to the way we presented the experts and their assessments.<sup>8</sup>  
4 We are confident that most of our subjects incorporated the information provided  
5 when making their choices. We are indeed able to show that subjects' choices  
6 monotonically follow the stochastic dominance criteria induced by the changes in  
7 assessments provided by the two experts (see Section 5.2). Relatedly, one could  
8 fear that the absence of physical experts in the room could have deceived our  
9 subjects, who could then have been tempted to suspect or mistrust the whole  
10 experiment.<sup>9</sup> Once again, we are able to show that this suspicion argument is  
11 inconsistent with the data we collected. However, to make sure we were on the  
12 safe side, we explicitly indicated in the instructions that were read aloud at the  
13 beginning of the experiment: "This experiment is about decision-making. There  
14 will be no deception in the experiment."  
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## 25 2.2 The randomness device

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27 Since one of our main goals in the experiment is to characterize the way indi-  
28 viduals behave in the presence of model uncertainty we need to make sure that,  
29 in the absence of experts' information, subjects are in a situation of perfect ig-  
30 norance.<sup>10</sup> To mimic the situation of perfect ignorance, we construct the model  
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34 <sup>8</sup>For that purpose, we specifically mentioned the following in the instructions: "These experts  
35 are the best we could find for this situation. They are both experienced and both have excellent  
36 track records".

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38 <sup>9</sup>While most experimental economists agree that deception should be avoided, nobody knows  
39 exactly and with certainty what deception is and is not (Krawczyk et al., 2013). Yet, a consensus  
40 has emerged across disciplinary borders that deception involves *intentional and explicit provision*  
41 *of misinformation* (Hertwig and Ortmann, 2008). Adair, Dushenko, and Lindsay (1985) for  
42 example define deception as "the provision of information that actively misled subjects regarding  
43 some aspect of the study", while Menges (1973) talks about instances where "the subject is given  
44 misleading or erroneous information". The main argument against the use of deception is that it  
45 jeopardizes future experiments if the subjects ever find out that they were deceived and report  
46 this information to their friends. Banning deception therefore ensures negative reputational  
47 spillover effects are avoided and maintains a "reputation among the student population for  
48 honesty in order to ensure that subject actions are motivated by the induced monetary rewards  
49 rather than by psychological reactions to suspected manipulation" (Davis and Holt, 1993). In  
50 that sense, our design was not subject to deception. In practice indeed, the way we constructed  
51 the model uncertainty urn is by randomly selecting one of the two models introduced by the  
52 experts in the design, and implementing her assessment.

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54 <sup>10</sup>In other words, they could a priori consider any probability in the continuum  $[0, 1]$ . Remark  
55 that with a device like an urn, this situation would correspond to an urn containing an infinite  
56 number of balls. This is different from the canonical Ellsberg examples in which (objective)  
57 information is given concerning the total number of balls, thus enabling the decision maker to  
58 posit a restricted set of possible objective models  $M$ , making the Ellsberg case itself a situation  
59 of model uncertainty.  
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uncertainty urn in such a way that the total number of balls in the urn is itself unknown, and comprised between 1 and 100. We call this modification of Ellsberg’s canonical example, that reduces the information bias due to the peculiarity of the urn representation, the *randomness device*. In such a situation, the total number of potential objective models is 3045, which is the cardinality of the Farey sequence of order 100.<sup>11</sup> To see this, consider Table 1 below. It presents the sets of potential models when the *maximum* number of balls  $N$  is known to be between 1 (first row) and 8 (last row). As can be seen, when the maximum number of balls is  $N = 1$ ,

Table 1: Sets of models and their corresponding cardinality when the maximum number of balls in the urn is  $N$

$N$	Set of possible models: $M_N = \{P(r)\}$																$ M_N $							
1	$\frac{0}{1}$																$\frac{1}{1}$	2						
2	$\frac{0}{1}$									$\frac{1}{2}$							$\frac{1}{1}$	3						
3	$\frac{0}{1}$						$\frac{1}{3}$			$\frac{1}{2}$			$\frac{2}{3}$				$\frac{1}{1}$	5						
4	$\frac{0}{1}$			$\frac{1}{4}$		$\frac{1}{3}$				$\frac{1}{2}$			$\frac{2}{3}$	$\frac{3}{4}$			$\frac{1}{1}$	7						
5	$\frac{0}{1}$			$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$		$\frac{2}{5}$		$\frac{1}{2}$		$\frac{3}{5}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$		$\frac{1}{1}$	11						
6	$\frac{0}{1}$		$\frac{1}{6}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$		$\frac{2}{5}$	$\frac{1}{2}$	$\frac{3}{5}$		$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{6}$		$\frac{1}{1}$	13						
7	$\frac{0}{1}$	$\frac{1}{7}$	$\frac{1}{6}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{2}{7}$	$\frac{1}{3}$	$\frac{2}{5}$	$\frac{3}{7}$	$\frac{1}{2}$	$\frac{4}{7}$	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{5}{7}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{6}$	$\frac{6}{7}$	$\frac{1}{1}$	19				
8	$\frac{0}{1}$	$\frac{1}{8}$	$\frac{1}{7}$	$\frac{1}{6}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{2}{7}$	$\frac{1}{3}$	$\frac{3}{8}$	$\frac{2}{5}$	$\frac{3}{7}$	$\frac{1}{2}$	$\frac{4}{7}$	$\frac{3}{5}$	$\frac{5}{8}$	$\frac{2}{3}$	$\frac{5}{7}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{6}$	$\frac{6}{7}$	$\frac{7}{8}$	$\frac{1}{1}$	23

the set of possible models  $M$  consists of two elements:  $M_1 = \{P(r) \in \{0, 1\}\}$ , where  $P(r)$  denotes the probability of drawing a red ball. When  $N = 2$ , the cardinality of  $M$  increases to  $|M| = 3$ , such that  $M_2 = \{P(r) \in \{0, \frac{1}{2}, 1\}\}$ , and so on. We view this device sufficiently complicated to prevent subjects from doing any calculation of probability distribution over the possible compositions of the urn and their corresponding weights.<sup>12</sup> In that sense, absent of any additional information from the experts, subjects will –most likely– be unable to compute the set of possible objective models, and end up in a situation close to one of perfect ignorance :

$$M_{100} = \{P(r) \in \{\mathfrak{F}_{100}\}\} \sim \{P(r) \in [0, 1]\}. \quad (1)$$

<sup>11</sup>A Farey sequence of order  $N$ , denoted  $\mathfrak{F}_N$ , is the ascending series of irreducible fractions between 0 and 1 whose denominators do not exceed  $N$  (Hardy et al., 1979).

<sup>12</sup>Note for example that even in the case in which all the possible total numbers of balls, and all their possible compositions are assumed to be equally probable, the possible models are not weighted uniformly. E.g., for a total number of balls comprised between 1 and 3, five different models exist:  $M_3 = \{P(r) \in \{0, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, 1\}\}$ . Assuming the number of balls in the urn is uniformly distributed between 1 and 3, and that for each case the different models are weighted equally, we end up with weights attached to the possible models that are respectively  $q = \{\frac{13}{36}, \frac{1}{12}, \frac{1}{9}, \frac{1}{12}, \frac{13}{36}\}$ .

1 We feel that such a setup, that emphasizes Frank Knight’s (1921) original dis-  
2 tinction between “measurable” and “unmeasurable” uncertainty (which cannot  
3 be represented by numerical probabilities) better reflects the actual state of indi-  
4 viduals facing complex problems. Such is the case in a large fraction of modern  
5 science problems for example, where the level of abstraction and mathematical re-  
6 quirement to understand processes are such that individuals cannot have a mental  
7 construct of the problem they are facing.  
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## 10 **2.3 Recruitment and administration**

11 The experiment took place in the laboratory of Bocconi University (Italy). 189  
12 subjects were recruited through an internal experimental economics recruitment  
13 system. Each subject was authorized to participate only once and had to sign up in  
14 advance for a particular time slot. The experiment was organized into 12 sessions  
15 taking place over four days. Each session lasted approximately 75 minutes, and  
16 comprised of 13 to 19 subjects. Subjects were provided with paper, pen and a  
17 calculator. A session typically started with silent reading of general instructions  
18 which were printed and provided to each subject in the cubicle to which she/he was  
19 assigned. The experimenter then read once more the instructions aloud and made  
20 sure everything was clear, before the subjects started a computerized training  
21 session that introduced them to the concepts of risky and model uncertain urns,  
22 and decision tables. The experiment was then performed on computers, with the  
23 order of tasks being randomized. Overall, the different tasks constituting our  
24 experiment were associated with a random incentive system to determine the final  
25 payoff. Once all subjects had answered all the questions, they were asked to fill in  
26 a short socio-economic questionnaire before being told their payoffs (i.e. which of  
27 their decisions had been randomly selected, what was the color of the ball drawn  
28 from the urn they chose (if any), and what was the corresponding amount they  
29 won). Subjects were then paid in cash a €5 participation fee, and the additional  
30 amount (up to €35) won on the basis of the choices they made. The average  
31 gain was about €18.50 per subject. The lab experiment was programmed and  
32 conducted with the experiment software z-Tree (Fischbacher, 2007). Details of  
33 the experimental procedure and instructions are provided in the Supplemental  
34 Material available online.  
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### 3 Theoretical predictions

In this section, we present our theoretical predictions. We use a specific model of choice which has been proposed by Marinacci (2015) building on Klibanoff et al. (2005). This model, known as the *smooth model*, is fairly general. It enriches and encompasses many of the recent theories of choice under uncertainty in cases where information is incorporated in the decision problem, while allowing for a distinction between the notions of aleatory and epistemic uncertainty.

#### 3.1 The smooth model

The decision maker evaluates acts  $f$  whose outcome depends on the realization of an observable state. For each decision, the state may be associated with the color  $c \in \{r, b\}$  of the ball (red  $r$ , or black  $b$ ) drawn from the urn under consideration. In other words, each draw may be seen as the realization of a random variable that is characterized by an objective probability distribution, or *model*, corresponding to a specific composition of the urn. For a given model, the uncertainty on the outcome is of the *aleatory* type and is generally called *risk* in economics.<sup>13</sup> Conditionally on the model, the probability of drawing a red (black) ball is therefore *objective* and refers to a physical concept (in this case a specific composition of the urn).

As in many real-life decision problems, it might however be the case that multiple possible compositions of the urn exists, and that the decision maker (DM) is uncertain about which is the correct one. If this is the case, the probability model generating the observations is itself uncertain. We suppose that, following the ex-ante information, the DM is able to posit a set of potential models describing the likelihoods of the different states. This set  $M$  of possible compositions of the urn is assumed to be consistent with the available information, which is provided by the experts. In accordance with Wald (1950), it is taken as a datum of the decision problem.<sup>14</sup> In other words, the DM behaves as if she knows that states are generated by a probability model that belongs to the collection  $M$ . This therefore means that a second layer of uncertainty may add onto the first layer of risk. This additional layer of uncertainty is however not of the aleatory type. It has rather an *epistemic* nature: the DM does not know which is the most accurate model among the two, and the probability that may be attached to each of them

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<sup>13</sup>Other denominations such as *physical*, *objective*, *inside*, or *measurable* uncertainty may also be found in the literature. They all refer to situations in which the probability distribution is are known.

<sup>14</sup>This set of models is analogous to what Ellsberg (1961) calls “reasonable” distributions in his subjective setting. In general, the set  $M$  is non-singleton, and the true model is assumed to belong to  $M$  (no misspecification issues).

is therefore nothing but a representation of the DM’s degrees of belief. Altogether this makes the situation uncertain (or ambiguous) rather than risky. We will refer to this situation as *model uncertainty*. The smooth representation of preferences under model uncertainty assumes that epistemic uncertainty is quantified through a single prior probability measure over the set  $M$  of models (Marinacci, 2015). The different possible models –which may be fully characterized by their probability of drawing a red ball– are indexed by a parameter  $\theta$  and noted  $P_\theta(r)$ . The DM attaches a probability  $\mu(\theta)$  to each of them, reflecting the personal information she has on their likelihood.<sup>15</sup> The DM then chooses the act that maximizes her utility given by the two-stage criterion:

$$U(f) = E_\theta(v \circ u^{-1}) \left( \sum_{c \in \{r,b\}} \tilde{P}_\theta(c) u(f(c)) \right). \quad (2)$$

In this expression,  $E_\theta$  is the expectation operator taken over the prior distribution:  $E_\theta \tilde{X}_\theta = \int_M X(\theta) d\mu(\theta)$ ,  $u$  is the von Neumann and Morgenstern (1944) utility function capturing the DM’s risk attitude (i.e. towards aleatory uncertainty), and  $v$  captures the attitude towards model uncertainty (i.e. towards epistemic uncertainty). These functions are assumed to be strictly increasing, continuous, and cardinally unique. Criterion (2) may be interpreted as follows. In the first stage, the DM evaluates the expected payoff per each possible model  $P_\theta(r)$  and expresses it in monetary terms through a certainty equivalent  $c_\theta \equiv u^{-1} \left( \sum_{c \in \{r,b\}} \tilde{P}_\theta(c) u(f(c)) \right)$ . These certainty equivalents  $c_\theta$  represent the amount of money which would make the DM indifferent between getting such amount for sure and facing the risk associated with the model. One certainty equivalent  $c_\theta$  may be computed for each model. It only depends on risk attitude via the function  $u$ : the more risk averse the DM is, the lower  $c_\theta$ . In the second stage, the DM addresses model uncertainty and summarizes the utility of the act by evaluating a global expected payoff  $E_\theta v(\tilde{c}_\theta)$  across the certainty equivalents using her attitude towards model uncertainty  $v$  and her prior belief  $\mu$ . Remark that in the case where both attitudes towards the different types of uncertainty are identical (i.e. whenever  $v$  is equal –up to an affine transformation– to  $u$ ), we recover the classical subjective expected utility model of Cerreia-Vioglio et al. (2013), which encompasses Savage’s (1954) model. Representation 2 therefore encompasses both the Savagian subjective expected utility and the classical von Neumann-Morgenstern (when  $M$  is a singleton) representations.

<sup>15</sup>In that sense, the prior probability measure  $\mu$  taken over the set of possible models is subjective: it reflects “personal information on models that the DM may have, in addition to objective information  $M$ ” (Cerreia-Vioglio et al., 2013).

When attitudes towards risk and model uncertainty are different, representation (2) consists of a version of smooth ambiguity aversion (Klibanoff et al., 2005) where the ambiguity aversion function is recovered by letting  $\phi \equiv v \circ u^{-1}$ . In that sense, the DM is ambiguity averse if she is more averse to model uncertainty than to risk (i.e.  $v$  more concave than  $u$ ).

### 3.2 Predictions

We now present the theoretical predictions for individual choices under risk and model uncertainty. We are particularly interested in characterizing the properties of the two functions  $u$  and  $v$  representing attitudes towards different types of uncertainty, and in confronting the smooth model uncertainty framework to previous results obtained in the literature.

**Hypothesis 1.** We expect subjects to be both risk and model uncertainty averse, in the sense that they generally prefer the degenerate lottery, giving  $\sum_{c \in \{r, b\}} \bar{P}(c) f(c)$  with certainty, to any uncertain situation in which an act  $f$  yields  $f(c)$  with (expected) probability  $\bar{P}(c) \forall c \in \{r, b\}$ . By letting  $C^R$  and  $C^{MU}$  denote the certainty equivalents under risk and model uncertainty, respectively defined as

$$C^R \equiv u^{-1} \left( \sum_{c \in \{r, b\}} P(c) f(c) \right) \quad (3)$$

$$C^{MU} \equiv v^{-1} \left( E_{\theta}(v \circ u^{-1}) \left( \sum_{c \in \{r, b\}} \tilde{P}_{\theta}(c) u(f(c)) \right) \right), \quad (4)$$

and by letting  $\bar{C}$  be the sure amount corresponding to the expected gain of the prospect, we expect to observe:

$$\bar{C} \geq C^R \quad (5)$$

$$\bar{C} \geq C^{MU}. \quad (6)$$

Expressed in terms of functional properties, these hypotheses simply become  $u'' \leq 0$  and  $v'' \leq 0$ . While the first hypothesis is trivial and has been extensively documented (Holt and Laury, 2002, 2005; Andersen et al., 2008), it is useful for what we are testing next, which is whether ambiguity aversion may result from a stronger aversion towards model uncertainty than towards risk.



**Hypothesis 2.** Considering the decomposition of ambiguity into risk and model uncertainty, we expect our subjects to prefer risk to model uncertainty, in accordance with the results emphasized by Ellsberg (1961). Furthermore, we also predict that the degree of model uncertainty aversion (and of ambiguity aversion) are finite. In other words, we expect subjects not to behave according to a maxmin decision criterion in the sense of Wald (1950). In terms of certainty equivalents, these predictions may be written (under the assumption of equal expected values) as:

$$C^R \geq C^{MU} > \underline{C}, \quad (7)$$

where  $\underline{C}$  refers to the certainty equivalent obtained under the worst possible model. In terms of functions  $u$  and  $v$ , the first part of (7) simply turns out to be  $v$  being more concave than  $u$ , in the sense of Arrow-Pratt (i.e.  $-\frac{v''}{v'} \geq -\frac{u''}{u'}$ ), while the second part translates to  $-\frac{v''}{v'} < \infty$ .<sup>16</sup>

**Hypothesis 3.** Analogously to what is widely accepted in the risk theory literature, and given the similarity of our procedure with the one used in Holt and Laury (2002, 2005), we expect to observe decreasing absolute risk aversion (DARA) and increasing relative risk aversion (IRRA) for both functions  $u$  and  $v$ .<sup>17</sup> Specifically, by changing the values of the gains proposed and the probabilities that are associated with these gains, we expect to observe:

$$\frac{\partial}{\partial w_0} \left[ -\frac{u''(w_0)}{u'(w_0)} \right] \leq 0 \quad \text{and} \quad \frac{\partial}{\partial w_0} \left[ -\frac{u''(w_0)}{u'(w_0)} w_0 \right] \geq 0, \quad (8)$$

where  $w_0$  denotes the individual's wealth level, which is composed of the individual's background wealth  $\omega$ , and the expected gain in each lottery  $\bar{p}\underline{c}$ :  $w_0 = \omega + p\bar{c} + (1-p)\underline{c}$ . Similarly, we expect the DARA and IRRA properties of function  $v$  to be translated to :

$$\frac{\partial}{\partial w_1} \left[ -\frac{v''(w_1)}{v'(w_1)} \right] \leq 0 \quad \text{and} \quad \frac{\partial}{\partial w_1} \left[ -\frac{v''(w_1)}{v'(w_1)} w_1 \right] \geq 0, \quad (9)$$

<sup>16</sup>Or, equivalently  $v^{-1} \left( E_{\theta}(v \circ u^{-1}) \left( \sum_{c \in \{r_i, b_i\}} \tilde{P}_{\theta}(c) u(f_i(c)) \right) \right) > u^{-1} \left( \min_{\theta} \sum_{c \in \{r_i, b_i\}} P_{\theta}(c) u(f_i(c)) \right)$ .

<sup>17</sup>To be completely precise, we should talk about “decreasing absolute model uncertainty aversion” and “increasing relative model uncertainty aversion” in the case of function  $v$ , but for the sake of simplicity we prefer to refer to the widely used acronyms DARA and IRRA for the  $v$  function as well. While the DARA property seems well accepted in the literature, note however that the IRRA property is subject to debate when investigated outside of the lab environment (Harrison et al., 2007; Brunnermeier and Nagel, 2008; Chiappori and Paiella, 2011).

1 where the individual's wealth level  $w_1$ , in situations of model uncertainty, is an  
2 average of certainty equivalent wealth levels under the two expert's models.<sup>18</sup>  
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6 **Hypothesis 4.** Since ambiguity aversion in this setup results from the combi-  
7 nation of attitudes towards risk and model uncertainty, we are able to indirectly  
8 characterize the properties of the ambiguity function. In particular, we are inter-  
9 ested in knowing whether the absolute ambiguity aversion is constant or whether  
10 it is increasing or decreasing, in the sense that agents are willing to pay more or  
11 less to remove all source of uncertainty as their level of expected utility increases.  
12 Constant absolute ambiguity aversion (CAAA), as argued by Grant and Polak  
13 (2013), is an implicit characteristic of many of the ambiguity models proposed  
14 in the theoretical literature. It is for example implicitly assumed in the models  
15 of Gilboa and Schmeidler (1989); Hansen and Sargent (2001); Maccheroni et al.  
16 (2006). On the contrary, decreasing absolute ambiguity aversion (DAAA) is a  
17 condition that has been shown to play an important role in the determination of  
18 the precautionary saving motive under ambiguity (Gierlinger and Gollier, 2008;  
19 Berger, 2014), in the chances of survival of ambiguity averse investors (Guerd-  
20 jikova and Sciubba, 2015), or in the choice of optimal abatement policies under  
21 scientific model uncertainty (Berger et al., 2017). We therefore expect to observe  
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$$31 \quad \frac{\partial}{\partial U} \left[ -\frac{\phi''(U)}{\phi'(U)} \right] \leq 0, \quad (10)$$

32 where  $U$  is the individual's expected utility level when the probabilities given by  
33 experts are averaged.<sup>19</sup> Note that the domain of the ambiguity function  $\phi = v \circ u^{-1}$   
34 is different than that of  $u$  and  $v$ , which are defined over monetary outcomes, while  
35  $\phi$  takes arguments that belong to a set of expected utilities.  
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## 43 4 General results

44 In this section, we report the results from the CE tasks. These results char-  
45 acterize attitudes towards risk and model uncertainty directly from the choices  
46 made by the subjects, and do not necessarily rely on the smooth representation  
47 of preferences. To facilitate the derivation of our results, we impose the following  
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52 <sup>18</sup>More precisely, in the uncertain situation characterized by  $\bar{o}_{\hat{p}_1 \hat{p}_2 \underline{o}}$ ,  $w_1$  is computed as  $w_1 =$   
53  $\mu(1)u^{-1}(\hat{p}_1 u(\omega + \bar{o}) + (1 - \hat{p}_1)u(\omega + \underline{o})) + \mu(2)u^{-1}(\hat{p}_2 u(\omega + \bar{o}) + (1 - \hat{p}_2)u(\omega + \underline{o}))$ . It is then  
54 obviously increasing in the background wealth  $\omega$ , in the payoff outcomes  $\bar{o}$  and  $\underline{o}$ , and in the  
55 expert's probabilities  $\hat{p}_1$  and  $\hat{p}_2$ .  
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57 <sup>19</sup>Note that in a recent contribution, Baillon and Placido (2016) also tested the CAAA and  
58 DAAA hypotheses using a framework different from ours and found evidence of DAAA under  
59 Ellsberg's type of uncertainty.  
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1 assumption of symmetry that we briefly discuss.

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3 *Symmetry*: For each model uncertainty situation, the decision maker assigns sym-  
4 metric weights to the two experts.  
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6 This assumption is akin the one generally imposed in Ellsberg’s type of exper-  
7 iments, stating that DMs are indifferent between betting on red or black balls.  
8 Its empirical validity has for example been supported by Abdellaoui et al. (2011)  
9 and Chew et al. (2017). The assumption relies on a symmetry of information  
10 argument. Given that the information on the two experts is perfectly symmetric  
11 (the experts only differentiate themselves by their names: “Expert 1” and “Expert  
12 2” and their assessments of the urn composition), there should be no reason to  
13 attach more weight to one over the other. The prior distribution over the models  
14 in consequence reflects this symmetry, so that we have  $\mu(1) = \mu(2) = 1/2$ .<sup>20</sup>  
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20 The CE tasks enable us to obtain a direct measure of the strength of model  
21 uncertainty aversion relative to risk aversion. By considering *dogmatic experts* (i.e.  
22 experts who assess a probability 100% or 0% or winning), we are able to isolate  
23 the effect of model uncertainty alone from risk aversion and to capture directly  
24 model uncertainty preferences.<sup>21</sup> Table 2 reports the descriptive statistics of the  
25 certainty equivalents associated with the risky and model uncertainty prospects  
26 (i.e.  $25_{.5}4$  and  $25_{10}4$ , respectively). We use the decision item at which the subject  
27 switches from the sure outcome to the uncertain prospect to approximate her  
28 certainty equivalents. Formally, for a subject who switches from the sure outcome  
29  $o_i$  after decision  $i \in \{1, 2, \dots, 10\}$ , the certainty equivalent  $C^J$ , for  $J \in \{R, MU\}$ ,  
30 is given by  
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$$37 \quad C^J = \begin{cases} o_1 + \frac{1}{2}(o_2 - o_1) & \text{if } i < 1 \\ 38 \quad \frac{1}{2}(o_{i+1} + o_i) & \text{if } i \in \{1, \dots, 9\} \\ 39 \quad o_{10} - \frac{1}{2}(o_{10} - o_9) & \text{if } i > 9 \end{cases} \quad (11)$$

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43 The reported certainty equivalents therefore correspond to the midpoint between  
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45 <sup>20</sup>Note that if it was not the case, we would observe subjects’ choices to be biased in favor of  
46 model uncertainty situations. The results we obtain in this case would only represent a lower  
47 bound of what are individuals’ exact preferences.

48 <sup>21</sup>To see this, remark that in the case of model uncertainty with dogmatic experts, the cer-  
49 tainty equivalent collapses to  $C^{MU} = v^{-1} \left( \sum_{c \in \{r,b\}} \frac{1}{2} v(f(c)) \right)$ , while the certainty equivalent  
50 of the 50-50 risky situation is  $C^R = u^{-1} \left( \sum_{c \in \{r,b\}} \frac{1}{2} u(f(c)) \right)$ . Furthermore, the presence of  
51 dogmatic experts ensures that the results we obtain concerning model uncertainty aversion are,  
52 at worst, underestimated. Indeed, if subjects were to consider any other alternative model,  
53 the preferences associated with the observed choices would have to reflect a higher aversion to  
54 model uncertainty than what is presented in our results (since any other symmetric distribution  
55 in the space of expected utilities would consist in a mean preserving contraction of the dogmatic  
56 experts’ distribution).  
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the highest outcome for which the uncertain prospect is preferred and the lowest outcome which is preferred to the uncertain prospect. The consistent sub-sample is made of 169 subjects. As can be observed from Table 2, the results confirm

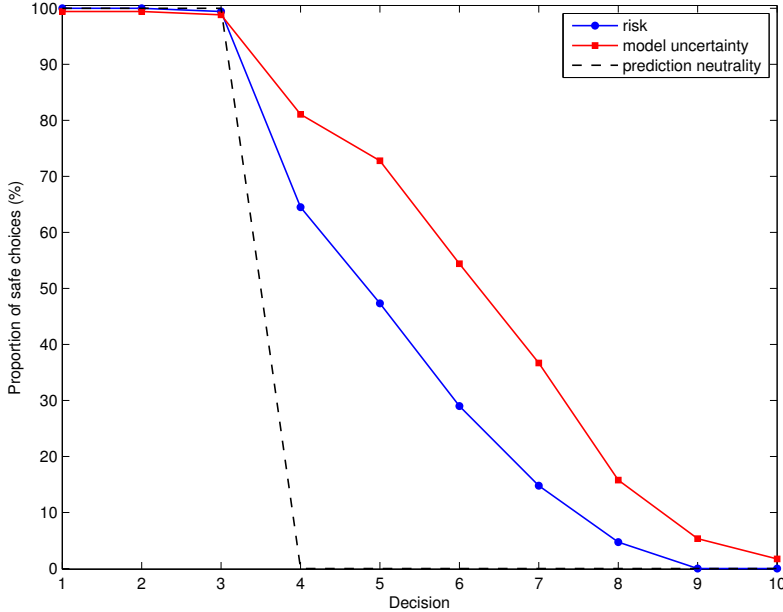
Table 2: Descriptive Statistics of the certainty equivalents

	Mean	Std. Error	Median	Min	Max	Obs.
$C^R$	12.57	0.17	13.5	7	16.5	169
$C^{MU}$	11.04	0.24	11	3	28.5	169

our first two predictions: subjects are on average ready to pay a higher premium –measured as the difference between the expected gain and the certainty equivalent ( $\bar{C} - C^J$ )– to avoid a situation characterized by epistemic uncertainty than to avoid a situation characterized by aleatory uncertainty. In particular, for an expected outcome  $\bar{C} = 14.50$ , the mean amount that our subjects deem equivalent to the risky situation is  $C^R = 12.57$ , while under model uncertainty it is only  $C^{MU} = 11.04$ . The difference between the certainty equivalents ( $C^R - C^{MU}$ ) is significantly positive ( $p$ -value=8.9e-11), and both are higher than the certainty equivalent obtained under the worst possible model  $\underline{C} = 4$ .

The distribution of  $C^R$  second order stochastically dominates that of  $C^{MU}$ . While  $C^R$  does not first order stochastically dominate (FOSD)  $C^{MU}$ , this is only because there is one subject who systematically preferred the model uncertainty situation to the sure outcome (even when the choice was made between €25 for sure, and a situation in which one expert expressed a 100% probability the gain is €25, while the other expert expressed a 100% probability the gain is €4). We can only speculate what the preferences of this subject are. He/she could be a very optimistic subject who always trusts the expert predicting the highest outcome. In that sense, his/her first choice would express indifference between two situations yielding €25. Once we remove this subject from the sample, we recover the result that  $C^R$  FOSD  $C^{MU}$ . This result is illustrated in Figure 1 and Table 3, which display the proportions of safe choices –expressed by preference for the sure amount– for each of the ten decisions in the risky and model uncertainty situations, respectively. The dashed line represents the prediction under the assumption of neutrality (towards either risk or model uncertainty). In this case, the certainty equivalents of both situations are the same, and the probability that the sure outcome is chosen is 1 for the first three decisions, and 0 for the remaining ones. The blue line presents the observed choice frequency of the

Figure 1: Proportion of safe choices in the CE tasks



sure outcome option in the risk situation. As can be observed, it is at the right of the risk neutral prediction, indicating a tendency for risk aversion among subjects ( $C^R \leq \bar{C}$ ). The red line represents the observed frequency in the case of model uncertainty. It lies to the right of the blue line, suggesting our subjects manifest a stronger aversion to model uncertainty than to risk ( $C^{MU} \leq C^R \leq \bar{C}$ ). We finally note that our subjects did not express extreme model uncertainty aversion (which would have consisted in a proportion safe choice of 100% for each decision). In fact, only two subjects (i.e. 1.2% of the sample) expressed an extreme form of pessimism by systematically selecting the certain outcome when confronted to model uncertainty.<sup>22</sup> We can therefore confidently reject the maxmin expected utility hypothesis in which subjects only consider the worst possible model (Wald, 1950). A Wilcoxon signed-rank test statistically confirms ( $p$ -value=3.2e-12) that the risky alternative is valued differently than the corresponding model uncertainty prospect.<sup>23</sup> Table 3 provides the implied interval for the parameters of risk or model uncertainty aversion in the context of the smooth model. For this exercise, we consider the special case of power (or CRRA<sup>24</sup>) functions. Using an

<sup>22</sup>Note that a looser definition of extreme pessimism which considers those subjects who expressed nine safe choices before switching to the model uncertainty situation as being indifferent between the two options, leads to 5.3% of extreme model uncertainty averse individuals.

<sup>23</sup>The significance of the one-sided test, where the alternative hypothesis is that the median of the switching point in the model uncertainty task is greater than in the risk aversion task, is 4.2e-12.

<sup>24</sup>A utility function has the CRRA property if it takes the form  $u(x) = \frac{x^{1-r}}{1-r}$ , where  $r$  is the coefficient of relative risk aversion (when  $r = 1$ , this collapses to  $u(x) = \ln x$ ).

Table 3: Classification of choices (CE tasks)

Number of safe choices	Range of relative risk or model uncertainty aversion: $u(x)$ or $v(x) = x^{1-r}/1 - r$	Proportion of choices	
		Risk	Model Uncertainty
0-1	$r < -1.04$	0.00 %	0.59 %
2	$-1.04 < r < -0.12$	0.56 %	0.59 %
3	$-0.12 < r < 0.12$	34.91 %	17.75 %
4	$0.12 < r < 0.34$	17.16 %	8.28 %
5	$0.34 < r < 0.55$	18.34 %	15.38 %
6	$0.55 < r < 1$	14.20 %	20.71 %
7	$1 < r < 1.55$	10.06 %	21.89 %
8	$1.55 < r < 2.58$	4.73 %	9.47 %
9-10	$2.58 < r$	0.00 %	5.33 %

estimation procedure that will be described in the next section, we also find the best estimates for the coefficients of relative risk and model uncertainty aversion when both  $u$  and  $v$  are of CRRA type to be respectively  $r_u = 0.42$  and  $r_v = 0.83$  in the CE tasks.

At the individual level, we finally note that 88 (resp. 62, 19) subjects, representing 52% (resp. 37%, 11%) of the sample expressed a stronger (resp. equal, weaker) aversion towards model uncertainty than towards risk. Following the theoretical model presented above, these subjects may be considered as exhibiting ambiguity aversion, neutrality and loving, respectively.

## 5 Characterizing smooth preferences

In this section, we also use the choices made in the various PL tasks to further characterize preferences under uncertainty in the context of the smooth model. In particular, we use the 80 binary choices each subject typically provided to infer attitudes towards risk and model uncertainty, and then use this information to quantify the degree of ambiguity aversion. In total, 14% of the choices made in the eight PL tasks were deemed inconsistent (reverse choices or multiple switching points) and were discarded from the analysis. This is in line with what has been previously reported in other lab experiments (Holt and Laury, 2002).

### 5.1 Preliminary remark

Since the smooth model we study involves two distinct behavioral characteristics (i.e. risk and model uncertainty attitudes), the experimental procedure has

to be designed such that it generates data that are rich enough to disentangle the different components and ultimately allows for estimating the subjects' attitude towards ambiguity. The double PL procedure, which presents choices in the presence of both aleatory and epistemic uncertainty, is designed for this purpose. It enables us to jointly elicit risk and model uncertainty attitudes. To see the importance of using a joint procedure, consider the identification of risk and model uncertainty under the smooth model. Assuming expression (2) correctly describes preferences over uncertain alternatives, a subject is indifferent between two options  $\bar{o}_p \underline{o}$  and  $\bar{o}_{\hat{p}_1 \hat{p}_2} \underline{o}$  if and only if:

$$(v \circ u^{-1})\left(pu(\omega + \bar{o}) + (1 - p)u(\omega + \underline{o})\right) = \frac{1}{2}(v \circ u^{-1})\left(\hat{p}_1 u(\omega + \bar{o}) + (1 - \hat{p}_1)u(\omega + \underline{o})\right) + \frac{1}{2}(v \circ u^{-1})\left(\hat{p}_2 u(\omega + \bar{o}) + (1 - \hat{p}_2)u(\omega + \underline{o})\right), \quad (12)$$

where  $\omega$  represents background wealth. When considered in terms of attitude towards ambiguity, the identity  $\phi = v \circ u^{-1}$  enables us to rewrite (12) as

$$\phi\left(pu(\omega + \bar{o}) + (1 - p)u(\omega + \underline{o})\right) = \frac{1}{2}\phi\left(\hat{p}_1 u(\omega + \bar{o}) + (1 - \hat{p}_1)u(\omega + \underline{o})\right) + \frac{1}{2}\phi\left(\hat{p}_2 u(\omega + \bar{o}) + (1 - \hat{p}_2)u(\omega + \underline{o})\right). \quad (13)$$

From (12) and (13), it is clear that estimating model uncertainty aversion ( $v$ ) or ambiguity aversion ( $\phi$ ) under the assumption of risk neutrality ( $u$  linear) may yield the same result. Ambiguity aversion is therefore significantly overestimated when risk neutrality is assumed.<sup>25</sup> If we relax the assumption of risk neutrality and let risk aversion  $-u''/u'$  be positive, it becomes clear from the relationship  $-\phi''/\phi' = (-v''/v' + u''/u')/u'$  that the implied degree of absolute ambiguity aversion is lower. One can therefore not capture the distinction between model uncertainty and ambiguity aversion without estimating the level of risk aversion, for which separated risky tasks also need to be performed.

## 5.2 The double PL tasks

In the double PL tasks, we exploit the comparisons between risk and model uncertainty prospects with experts that are no longer dogmatic. The purely risky tasks are based on Holt and Laury's (2002) mechanism, which has become a standard for elicitation of risk aversion. The model uncertain tasks are constructed analogously. Table 4 illustrates the type of choices proposed in this part of the

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<sup>25</sup>We discuss the estimation results of this particular case in Appendix C.

experiment. In this example, Option A offers either €35 or €1 with equal probability, while Option B offers the same outcomes with unknown probabilities. In the latter case, respondents are given information in the form of the two experts' assessments. In the first decision for example, Expert 1 assesses the probability of obtaining €35 to be 50%, while Expert 2 is 100% sure the outcome will be €1. The expected value of Option A ( $EV^A$ ), the expected value of Option B if either Expert 1 or Expert 2 is correct (respectively  $EV_1^B$  and  $EV_2^B$ ), the average expected value of Option B ( $EV^B$ ) under the assumption of equal weights attached to each expert, and its standard deviation ( $SD^B$ ), are also provided in Table 4, but were not given to subjects during the experiment. While the expected value of Option

Table 4: Payoff table in the model uncertainty aversion tasks

Option A			Option B				$EV^A$	$EV_1^B$	$EV_2^B$	$EV^B$	$SD^B$
$\bar{o}$	$p$	$\underline{o}$	$\bar{o}$	$\hat{p}_1$	$\hat{p}_2$	$\underline{o}$	(€)	(€)	(€)	(€)	(€)
35	0.5	1	35	0.5	0	1	18	18	1.0	9.5	8.5
35	0.5	1	35	0.9	0	1	18	31.6	1.0	16.3	15.3
35	0.5	1	35	0.9	0.09	1	18	31.6	4.1	17.8	13.8
35	0.5	1	35	0.8	0.19	1	18	28.2	7.5	17.8	10.4
35	0.5	1	35	0.8	0.21	1	18	28.2	8.1	18.2	10.0
35	0.5	1	35	0.7	0.31	1	18	24.8	11.5	18.2	6.6
35	0.5	1	35	0.6	0.41	1	18	21.4	14.9	18.2	3.2
35	0.5	1	35	0.55	0.46	1	18	19.7	16.6	18.2	1.5
35	0.5	1	35	0.51	0.50	1	18	18.3	18.0	18.2	0.2
35	0.5	1	35	0.61	0.60	1	18	21.7	21.4	21.6	0.2

Notes: Probabilities always refer to the outcome  $\bar{o} \geq \underline{o} \in O$ .  $EV^B = \frac{1}{2}EV_1^B + \frac{1}{2}EV_2^B$ ;  $SD^B = \left(\frac{1}{2}(EV_1^B - EV^B)^2 + \frac{1}{2}(EV_2^B - EV^B)^2\right)^{0.5}$

A is kept constant throughout the various choices, the expected value of Option B is increasing as one proceeds down the table. The standard deviation, on the other hand, is decreasing (except between the first and second decision). Overall, the decision table is constructed in such a way that, for any increasing utility function, Option B always stochastically dominates (in the first or second order sense) the previous decision as one proceeds down the table.<sup>26</sup> Under the smooth model, this feature should induce subject to switch only once, from Option A to Option B, while progressing down the table. The subjects went through four tasks similar to the one illustrated in Table 4, which vary in the proposed payoffs and probabilities. The set of payoffs and probabilities is designed in a way that the final payoffs span the range of income over which we are estimating model uncertainty aversion, which is the same as the one over which risk aversion is estimated.<sup>27</sup>

<sup>26</sup>It is for example easy to see that Option B in the second decision first order stochastically dominates Option B in the first decision, and that Option B in the fourth decision second order stochastically dominates Option B in the third decision.

<sup>27</sup>In particular, Option A in the three other uncertain tasks takes values as follows:  $3.5_{0.5}0.1$ ,  $35_{0.1}1$ ,  $35_{0.9}1$ , and Option B covers the space around these values analogously to what is pre-



### 5.3 Eliciting risk and model uncertainty attitudes

We use each of the subjects' binary choices to estimate the parameters of two latent utility functions that explain these choices. We allow for a stochastic error structure, as opposed to a strictly deterministic structural estimation procedure, as we want to allow for subjects to make some errors and, at the same time, to account for the panel structure of the data. Given the important support for the CRRA hypothesis in the empirical literature on risk aversion (Harrison, Lau, and Rutström, 2007; Brunnermeier and Nagel, 2008; Chiappori and Paiella, 2011), but at the same time the experimental evidence found in favor of increasing relative risk aversion (IRRA) (Holt and Laury, 2002), we maintain a generic parametric structure for the identification problem. We let both utility functions representing risk and model uncertainty attitudes be of the expo-power (EP) form (Saha, 1993). In the case of risk, this means that the utility function takes the following form:

$$u(x) = \frac{1 - \exp(-a_u(\omega + x)^{1-r_u})}{a_u}. \quad (14)$$

This representation includes CRRA and constant absolute risk aversion (CARA) as special cases, and exhibits the desirable properties of decreasing absolute risk aversion and increasing relative risk aversion for positive values of the parameters  $a_u$  and  $r_u$  (Abdellaoui, Barrios, and Wakker, 2007).<sup>28</sup> Note the presence of  $\omega$ , representing background wealth in expression (14). As is generally the case in the experimental literature, we assume  $\omega = 0$ . It should however be clear that in a situation in which  $\omega > 0$ , the same observed choices would imply higher risk aversion. Using the procedure proposed by Andersen et al. (2008), we then construct the expected utility of the two options comprising each decision by using candidate values of parameters  $a_u$  and  $r_u$ , and a linking index in order to infer the likelihood of the observed choice. The parameters of the latent utility function (14) are then chosen in order to maximize the likelihood of getting the observed ranking of the different options, taking into account a Luce (1959) error specification with a structural noise parameter.<sup>29</sup> The first part of Table 5 presents the estimates obtained from the risky tasks. Given the prominent position

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sented in Table 4. For a detailed description of our experimental design, please refer to the Online supplemental Appendix.

<sup>28</sup>As is well known, the Arrow-Pratt index of relative risk aversion of the EP function is  $-u''(x)(\omega + x)/u'(x) = r_u + a_u(1 - r_u)(\omega + x)^{1-r_u}$ . It is then easy to see that this function exhibits CRRA of value  $r_u$  when  $a_u = 0$ , and CARA of  $a_u$  when  $r_u = 0$ .

<sup>29</sup>The statistical specification we use allows for taking into account the correlation between responses given by the same subject. Robust estimates considering clustering corrections are provided in Appendix B. There is essentially no difference in the significance of our estimates in this case.

CRRA has achieved in the theoretical and empirical literature, we provide both the estimates for the cases in which  $u$  is of the CRRA and EP type. The estimate

Table 5: Estimates of risk, model uncertainty and ambiguity preferences

	$u$		$v$		$\phi$	
	CRRA	EP	CRRA	EP	CRAA	EP
$a$		0.0294*** (0.00215)		0.152*** (0.0542)		-1.802 (0.9655)
$r$	0.279*** (0.0119)	0.135*** (0.0193)	0.738*** (0.0210)	0.467*** (0.0542)	0.534*** (0.0261)	0.86*** (0.0452)
noise parameter	0.103*** (0.00327)	0.105*** (0.00330)	0.0358*** (0.00237)	0.0534*** (0.00343)	0.0476*** (0.00213)	0.0363*** (0.00184)
Observations	5320	5320	7570	7570	7570	7570
Loglikelihood	-1550.3	-1516.8	-3682.5	-3682.1	-3680.6	-3675.1

Notes: Luce error specification is used in the estimation. Standard errors in parentheses. The EP risk specification is used to estimate  $v$  and  $\phi$ . \*  $p$ -value < 0.05, \*\*  $p$ -value < 0.01, \*\*\*  $p$ -value < 0.001

of the CRRA parameter we obtain is 0.28, which is lower than the one we found using the CE task only. When the EP specification is considered, we estimate  $r_u = 0.135$  and  $a_u = 0.029$ , which implies IRRA. While the focus of our analysis is on comparing these estimates with the ones obtained for the model uncertainty function  $v$ , we note that their absolute magnitudes are consistent with the results obtained by Holt and Laury (2002); Andersen et al. (2008). We however recognize that the estimates we obtain only hold locally over the domain of stakes offered in our experiment. The last two rows of Table 5 present information about the data used (30 risk aversion choices for each of the 189 subjects, minus the inconsistent choices that are discarded) and the resulting log-likelihood values. As can be seen, the log-likelihood of the EP specification is slightly better than the CRRA one, but this should not be surprising given that the estimates are all significant, and the hypothesis  $a_u = 0$  is therefore rejected. Given the superiority of the EP specification in explaining the observed choices in the risky tasks, this is the specification we consider in the remaining part of the estimation procedure. We then estimate the model uncertainty aversion function  $v$ , which takes the general EP form:

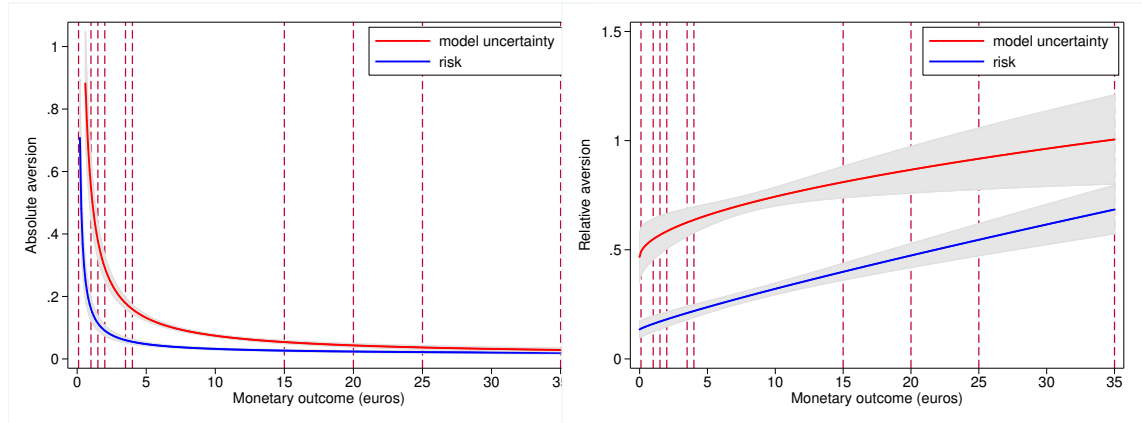
$$v(CE) = \frac{1 - \exp(-a_v(CE)^{1-r_v})}{a_v}, \quad (15)$$

where  $CE$  represents the certainty equivalent wealth for a given model  $\theta$ :  $CE \equiv u^{-1}(\hat{p}_\theta u(\bar{o}) + (1 - \hat{p}_\theta)u(\underline{o}))$ . The second part of Table 5 presents the estimates obtained from our five uncertain PL tasks. Estimates for the special cases of  $v$  being of the CRRA type ( $a_v = 0$ ) are also provided for indicative purposes. In that

case, the coefficient estimated is significantly higher than the one obtained in the case of risk. It should however be noted that this specification leads to a smaller log-likelihood value than the general expo-power formulation (15). Focusing on the EP specification, we remark that the estimates we obtain ( $a_v = 0.152$  and  $r_v = 0.467$ ) with the joint identification procedure are both significantly positive. This implies our subjects exhibit both decreasing absolute model uncertainty aversion and increasing relative model uncertainty aversion.

Our interest lies in comparing the estimates obtained for model uncertainty with the ones obtained for risk aversion. In Figure 2, we provide the paths of estimated absolute and relative aversion indexes for both risk and model uncertainty over the experimental prize domain. As predicted, we observe that the indexes are both decreasing in the monetary outcome when considered in absolute terms and increasing in relative terms (DARA and IRRRA). Interestingly, we

Figure 2: Absolute (left) and relative (right) risk and model uncertainty aversion using EP estimates (95% confidence in grey).



also directly observe from Figure 2 that the degree of model uncertainty aversion is significantly higher (in both absolute and relative terms) than the one of risk aversion. This result confirms our main hypothesis that subjects are more averse to model uncertainty than to risk. Specifically, while the index of relative risk aversion is respectively 0.32 and 0.62 when the monetary outcome considered is either  $x = 10$  or  $x = 30$ , the index of relative model uncertainty aversion takes values of 0.74 and 0.96 for the corresponding outcomes. Note that in the special case where both  $u$  and  $v$  are of the CRRA type, the indices of relative aversion to risk and model uncertainty are  $r_u = 0.28$  and  $r_v = 0.73$  when jointly estimated.<sup>30</sup> These differences observed between the attitudes towards objective and

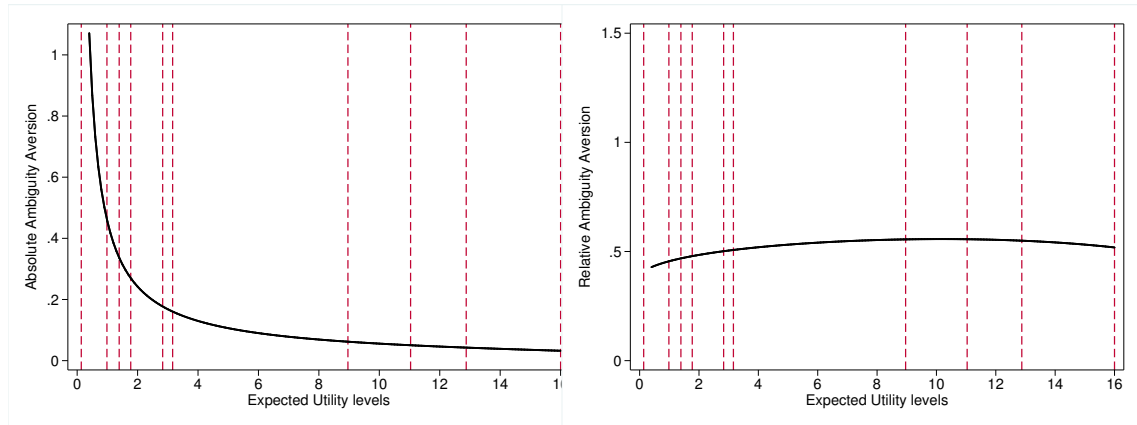
<sup>30</sup>In order to assess the sensitivity of the model uncertainty aversion index to variations in relative risk aversion, we also used the maximum likelihood procedure to estimate  $r_v$  using

subjective probabilities now enable us to quantify the attitude subjects manifest towards ambiguity.

## 5.4 The implications for ambiguity attitude

The joint characterization of functions  $u$  and  $v$  representing the subjects' attitudes towards two different types of uncertainty has an important direct implication for ambiguity aversion. Using the identity  $\phi \equiv v \circ u^{-1}$  and the results obtained in the previous section, we are now able to characterize directly the attitude subjects manifest towards ambiguity, and to compute the parameters of absolute and relative ambiguity aversion (see Appendix A for the detailed analytical computations under the double EP specification). These parameters are represented in Figure 3. While we observe a clear decreasing trend in the degree of absolute am-

Figure 3: Absolute (left) and relative (right) ambiguity aversion obtained with EP function estimates



biguity aversion, we remark that the degree of relative ambiguity aversion seems to be fairly constant over the domain considered. As already mentioned, it should however be clear that the domain of the ambiguity function  $\phi$  is not the same monetary outcome domain as the one used in the study of  $u$  and  $v$ . Instead,  $\phi$  is defined over expected utility levels  $U$ . In this sense, the vertical dashed red lines in Figure 3 represent the levels of utility obtained for the corresponding monetary outcomes in Figure 2, when the utility function  $u$  is of the EP type and coefficients are the ones reported in Table 5.

To assess the robustness of the constant relative ambiguity aversion (CRAA) result featured in Figure 3, we apply the joint estimation procedure directly to  $u$  different (exogenously given) values of  $r_u$ . These additional results are presented in Appendix C.

and  $\phi$  (both of which are of the EP type). In other words, we let the ambiguity aversion function be:

$$\phi(U) = \frac{1 - \exp(-a_\phi(U)^{1-r_\phi})}{a_\phi}, \quad (16)$$

where  $U$  represents the expected utility obtained under a given model  $\theta$ :  $U \equiv \hat{p}_\theta u(\bar{o}) + (1 - \hat{p}_\theta)u(\underline{o})$ , and  $u$  is defined as in equation (14). The estimated results are provided in the last two columns of Table 5. In this case, the coefficient  $a_\phi$  of the EP formulation is not significant at the 5% level ( $p - value = 0.062$ ). The function describing preferences towards ambiguity is therefore instead better represented by a CRAA function. Under this particular specification, the constant relative ambiguity aversion parameter is estimated to be 0.53. It does not perfectly match the value observed in Figure 3, but this should not be surprising given that the ambiguity functions do not share the same specifications in the two cases. If we instead consider the case of  $u$  being CRRA, we also obtain a non significant coefficient  $a_\phi$  ( $p - value = 0.093$ ) under the EP specification, and estimate the coefficient  $r_\phi = 0.62$  under constant relative ambiguity aversion.<sup>31</sup>

## 6 Conclusion

Uncertainty is pervasive in collective as well as in individual decision making. During the past few years, a vast literature aiming at better formalizing the decision process in the face of uncertainty has been growing and encompassing multiple academic fields. This body of research investigates how individuals integrate available information in the process of decision making through the development of theoretical frameworks and experimental analyses. In particular, multiple decision models have been developed to account for attitudes towards ambiguity. These models have been adopted to explain individuals' behavior in multiple contexts and are increasingly applied to prescribe optimal strategies in the face of uncertainty.

The growing application of ambiguity aversion models calls for the development of experimental efforts enabling a better understanding of the underlying mechanisms at play and a more precise quantification of ambiguity preferences, similar to what has been done in the study of risk. In this paper, we provide new experimental evidence on ambiguity attitudes by means of attitudes towards

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<sup>31</sup>In this case, the result could have been obtained directly from the twofold CRRA estimation results provided in Appendix A, given that  $\phi$  is of the CRAA type with  $r_\phi = \frac{r_v - r_u}{1 - r_u}$ , when both  $u$  and  $v$  are CRRA (Berger et al., 2017).

1 model uncertainty in relation to risk. In particular, our design enables us to dis-  
2 entangle the role played by aleatory and epistemic uncertainty in determining  
3 individuals' ambiguity attitudes. We are then able to quantify, through a joint  
4 elicitation procedure, the extent to which ambiguity aversion exists as well as the  
5 properties of the ambiguity aversion function in the context of the smooth model  
6 (Klibanoff et al., 2005; Marinacci, 2015).  
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9  
10 There are two main findings emerging from our analysis. First, we show that  
11 subjects tend to be both risk and model uncertainty averse, but exhibit stronger  
12 aversion to model uncertainty than to risk. Following the smooth model of choice  
13 under uncertainty, we interpret this behavioral characteristic as evidence of am-  
14 biguity aversion. Using a joint estimation procedure, we elicit the degree of am-  
15 biguity aversion, which we estimate to be around 0.5 when considered in relative  
16 terms. Second, investigating in more detail attitude towards model uncertainty,  
17 we find that model uncertainty aversion is decreasing in wealth when considered  
18 in absolute terms, and increasing when considered in relative terms. In regards  
19 to ambiguity attitude, we find evidence of decreasing absolute ambiguity aversion  
20 (DAAA) and constant relative ambiguity aversion (CRAA).  
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# Appendix

## A Absolute and relative ambiguity aversion

When the functions characterizing risk and model uncertainty preferences are both of the expo-power type, and are respectively defined as  $u(x) = \frac{1 - \exp(-a_u x^{1-r_u})}{a_u}$  and  $v(x) = \frac{1 - \exp(-a_v x^{1-r_v})}{a_v}$ , the ambiguity function  $\phi \equiv v \circ u^{-1}$  may be written as:

$$\phi(U) = \frac{1 - \exp\left(-a_v \left(-\frac{\ln(1-a_u U)}{a_u}\right)^{\frac{1-r_v}{1-r_u}}\right)}{a_v},$$

where  $U$  belongs to the space of expected utilities. In that case, it may be shown that the Arrow-Pratt absolute ambiguity aversion index is:

$$-\frac{\phi''(U)}{\phi'(U)} = \frac{a_v \left(\frac{1-r_v}{1-r_u}\right) \left(\frac{-\ln(1-a_u U)}{a_u}\right)^{\frac{r_u-r_v}{1-r_u}} + \left(\frac{a_u}{\ln(1-a_u U)}\right) \frac{r_u-r_v}{1-r_u} - a_u}{1 - a_u U}. \quad (\text{A.1})$$

It is then easy to see that in the special case in which  $u$  and  $v$  are both of the CARA type (i.e. when  $r_u = r_v = 0$ ), this index collapses to:

$$-\frac{\phi''(U)}{\phi'(U)} = \frac{a_v - a_u}{1 - a_u U}. \quad (\text{A.2})$$

This index is positive whenever  $a_v > a_u$ , so that absolute ambiguity aversion results from higher absolute model uncertainty aversion than absolute risk aversion. Similarly, in the special case in which  $u$  and  $v$  are both of the CRRA type (i.e. when  $a_u = a_v = 0$ ), the absolute ambiguity aversion becomes:

$$-\frac{\phi''(U)}{\phi'(U)} = \frac{r_v - r_u}{(1 - r_u)U}, \quad (\text{A.3})$$

and is positive whenever  $r_v > r_u$ .

## B Robust estimates of risk and model uncertainty preferences

Since each of our subjects provided multiple choices in the experiment, we may want to correct for the possible correlation of errors associated with a given subject (which may for example be due to unobserved individual effects). In this

case, the residuals from the same subject are treated as potentially correlated, and the correction is made when calculating standard errors of estimates. As argued in Andersen et al. (2008), this procedure allows heteroskedasticity between and within clusters, as well as autocorrelation within clusters, and generalizes the “robust standard errors” approach popular in econometrics. The estimates of risk and model uncertainty aversion with robust standard errors are presented in Table B.1.

Table B.1: Robust estimates of risk, model uncertainty and ambiguity preferences

	$u$		$v$	
	CRRA	EP	CRRA	EP
$a$		0.0294*** (0.00398)		0.152*** (0.0252)
$r$	0.279*** (0.0281)	0.135*** (0.0278)	0.738*** (0.0444)	0.467*** (0.0842)
noise parameter	0.103*** (0.00378)	0.105*** (0.00384)	0.0358*** (0.00492)	0.0534*** (0.00643)
Observations	5320	5320	7570	7570 0
Loglikelihood	-1550.3	-1516.8	-3682.5	-3682.1

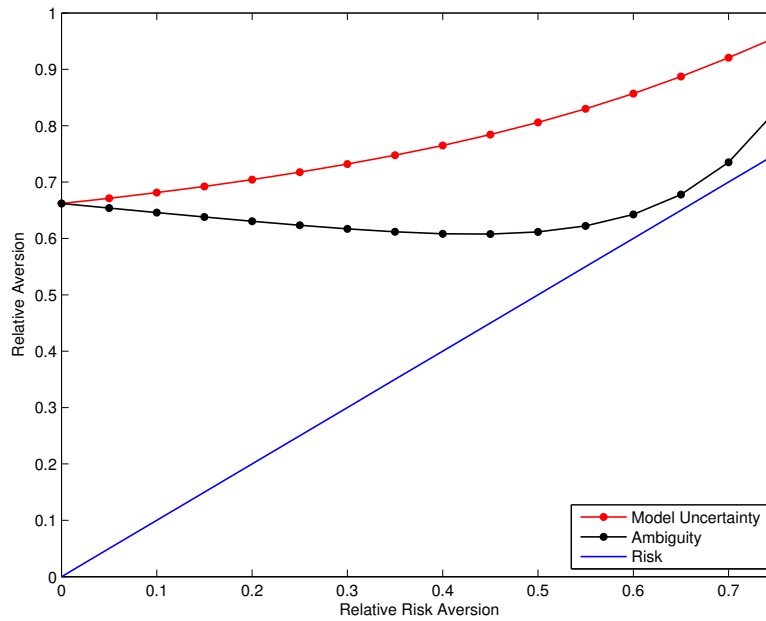
Notes: Estimation uses Luce error specification. Standard errors in parentheses. The EP risk specification is used to estimate  $v$ . \*  $p$ -value < 0.05, \*\*  $p$ -value < 0.01, \*\*\*  $p$ -value < 0.001

## C Sensitivity of model uncertainty aversion

In order to get further insight regarding the sensitivity of model uncertainty aversion to risk aversion, we used the results from our PL tasks to estimate the degree of relative model uncertainty aversion alone, for values of relative risk aversion ranging from 0 to 0.75. It should be clear that in this situation the estimation procedure is not anymore jointly realized (given the exogenously chosen values of risk aversion considered). In Figure C.1, we present the results of these estimations in the particular case where both  $u$  and  $v$  are of the CRRA type. For any value of relative risk aversion, we remark that the relative model uncertainty parameter estimated (represented in red) is significantly higher than the 45 degree line (in blue) representing relative risk aversion. In particular, it goes from  $r_v = 0.66$  when  $r_u = 0$  to  $r_v = 0.96$  when  $r_u = 0.75$ . We also present in black the degree of relative ambiguity aversion implied by the twofold CRRA specification. Following Berger et al. (2017), the index of relative ambiguity aversion in this situation is given by  $r_\phi = \frac{r_v - r_u}{1 - r_u}$ . As can be observed,  $r_v$  and  $r_\phi$  both coincide



Figure C.1: Relative model uncertainty aversion for different values of relative risk aversion when both  $u$  and  $v$  are of the CRRA type



when risk neutrality is considered (i.e.  $r_u = 0$ ). Following the discussion provided in the main body of the paper, we claim that the level of ambiguity aversion is overestimated when measured in situations of risk neutrality, and remark that for positive values of  $r_u$  the index of relative ambiguity aversion is always lower than the one of relative model uncertainty aversion.

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