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The undersigned

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Thesis title:

FINANCIAL MARKETS AND DEMOGRAPHICS

PhD in	**FINANCE**
Cycle	**XXII**
Candidate's tutor	**CARLO A. FAVERO**
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FINANCIAL MARKETS AND DEMOGRAPHICS

Dissertation in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Finance (XXII Cycle)

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Acknowledgments

I write this section while flying over the clouds to the job market in Denver, Colorado. As the first JM candidate from the Finance Department, I feel like jumping from the airplane with a backpack that includes all the material that will be useful for my future research career conditinal on the fact that the parashute given to me helps me embark smoothly upon an academic position. Even though slightly disturbed by the uncertainty involved, I feel calm because I am not alone in this plane.

There is Hannes Wagner with all his enthusiasm and endless psychological support, there is Barbara Rindi, a great advisor who is always caring, willing to share ideas and motivates me for high-quality research. I can also hear the potent voice of Fulvio Ortu ringing in my ears who kindly accepted to be in my thesis committe and supported me throughout the entire process. But I feel confident particularly because there is the captain in the plain, Carlo A. Favero, my academic coach, a perfect advisor (always available for advice despite the long queue in front of his office), a prolific coauthor and a great role model. Probably without him I would not have been on this trip and this travel book would not have been written. I really feel privileged to have had the chance to work in such a great environment.

I should mention others who made this trip such an enjoyable experience; my classmates (Annaig, Clara , Luna, Federico and Felix) for sharing all the good memories, my coauthors (Andrea, Alessandro, Haoxi and Yuanji) both for their personal and professional contribution and Paola for her continuous support.

Finally, special thanks to the organizers, my family, without their support and trust in me, I would have never taken off in the first place.

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Preface

My dissertation titled "Financial Markets and Demographics" consists of three chapters. In broader terms, it analyses the role of demographic fluctuations in determing equilibrium asset prices and predicting future asset returns. It documents the strong impact of changing population age structure on financial markets through the life-cycle investment behavior.

The first chapter written jointly with Prof. Carlo A. Favero and Andrea Tamoni focuses on the role of demographic factors in the stock market and the use of demographic information for long-horizon forecasting of market returns. This chapterdocuments the existence of a slowly evolving trend in the log dividend-price ratio, dp, determined by a demographic variable, MY: the proportion of middle-aged to young population. Deviations of dp from this long-run component explain transitory but persistent fluctuationsin stock market returns. The relation between MY and dp is a prediction of an overlapping generation model. The joint significance of MY and dp in long-horizon forecasting regressions for market returns explain the mixed evidence on the ability of dp to predict stock returns and provide a model-based interpretation of statistical corrections for breaks in the mean of this financial ratio. We presented the paper in several conferences including European Finance Association (EFA 2009) and NBER Summer Institute (July, 2010). Recently it has also been accepted for publication in Journal of Financial and Quantitative Analysis.

Tthe second chapter written jointly with Prof. Carlo A. Favero and Haoxi Yang, studies role of demographics in bond markets. In this chapter, we relate the very persistent component of interest rates to a specific demographic variable, MY. We first reconsider the results in Fama (2006) to document how MY captures the long run component identified by Fama in his analysis of the one-year spot rate. Using MY to model this low frequency component of interest rates is particularly useful for forecasting the term structure as the demographic variable is exogenous and highly predictable, even at very long horizons. We then study the forecasting performance of a no-arbitrage affine term structure model that allows for the presence of a persistent component driven by demographics. This performance is is superior to that of a traditional affine term structure model with macroeconomic factors (e.g. Ang, Dong and Piazzesi, 2005).

The last chapter addresses the issue of strong comovement between equity and bond markets, two main asset classes for investors. This chapter explains this comovement based on the relation between the changing population age structure and financial mar-

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kets. In particular, based on the evidence from out-of sample forecasts, VAR simulations and predictive regressions, I argue that equity and bond yields share a common demographic component. Identification of this common source through a model-based demographic variables explains several emprical puzzles; in the first place it explains the strong correlation between equity and bond yields, but it also shows that models that include the demographic variable are able to show both equity and bond return predictability consistent with the conjecture that the equilibrium relation is driven by demographic fluctuations. International evidence also supports this claim.

I believe that the importance of such an important channel has not attracted enough attention in finance despite its immense implications. This is partly due to the confounding results provided in the previous literature. I hope that the evidence provided in this dissertation contributes to a change in this attitude.

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1 Demographic Trends and the Predictability of Stock Returns

Introduction

This chapter[1] characterizes the relationship between the dividend-price ratio and stock market returns in a model where a demographic variable, MY_t, the middle-aged to young ratio, captures the slowly evolving component in the dividend-price ratio. Interest in this model is partly motivated by the very high persistence of the dividend-price ratio that makes long-horizon regressions hard to interpret MY_t allows to extract from the log dividend-price a stationary variable capturing time-variation in the investment opportunity set and to specify a more reliable forecasting model for long-horizon stock market returns. Demographics are a very natural input into a forecasting model of long-horizon returns, and, consequently, into the optimal asset allocation decision of a long-horizon investor. We interpret MY_t as the information component that drives long-horizon stock market fluctuations after the noise in short-horizon stock market fluctuations subsides.

The empirical relevance of the dividend-price ratio for predicting long-run stock market returns is one of the most debated issues in financial econometrics. In fact, this variable regularly plays an important role in recent empirical literature that has replaced the long tradition of the efficient market hypothesis (Fama, 1970) with a view of predictability of returns (see, for example, Cochrane, 2007). However, there is an ongoing debate on the robustness of return predictability and its potential use from a portfolio allocation perspective (Boudoukh et al., 2008; Goyal and Welch, 2008).

Most of the available evidence on predictability can be framed within the dynamic dividend growth model proposed by Campbell and Shiller (1988). This model relies on a

[1] Written jointly with Carlo A. Favero and Andrea Tamoni.

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loglinearized version of one-period returns on the stock portfolio. Under the assumption of its stationarity and of the validity of a standard transversality condition, the log of the price-dividend ratio, dp_t, is expressed as a linear function of the future discounted dividend growth, $\Delta \mathrm{d}_{t+j}$ and of future returns, h^s_{t+j} :

$$\mathrm{dp}_t = \overline{\mathrm{dp}} + \sum_{j=1}^{\infty} \rho^{j-1} \mathrm{E}_t[(\mathrm{h}^s_{t+j} - \bar{\mathrm{h}}) - (\Delta \mathrm{d}_{t+j} - \bar{\mathrm{d}})] \quad (1.1)$$

where $\overline{\mathrm{dp}}$, the mean of the dividend-price ratio, $\bar{\mathrm{d}}$, the mean of dividend growth rate, $\bar{\mathrm{h}}$, the mean of log return and ρ are constants.

Under the maintained hypothesis that stock market returns, and dividend-growth are covariance-stationary, Eq.(1.1) says that the log dividend-price ratio is stationary, i.e. the (log) price and the (log) dividend are cointegrated with a (-1,1) cointegrating vector, and that deviations of (log) prices from the common trend in (log) dividends summarize expectations of either stock market returns, or dividend growth or some combination of the two.

The empirical investigation of the dynamic dividend growth model has established a few empirical results:

(i) dp_t is a very persistent time-series and forecasts stock market returns and excess returns over horizons of many years (Fama and French, 1988; Campbell and Shiller (1988), Cochrane (2005, 2007).

(ii) dp_t does not have important long-horizon forecasting power for future discounted dividend-growth (Campbell, 1991; Campbell, Lo and McKinlay, 1997; Cochrane, 2001).

(iii) the very high persistence of dp_t has led some researchers to question the evidence of its forecasting power for returns, especially at short-horizons. Careful statistical analysis that takes full account of the persistence in dp_t provides little evidence in favour of the stock-market return predictability based on this financial ratio (Nelson and Kim, 1993; Stambaugh, 1999; Ang and Bekaert, 2007; Valkanov, 2003; Goyal and Welch, 2003; Goyal and Welch, 2008). Structural breaks have also been found in the relation between dp_t and future returns (Neely and Weller, 2000; Paye and Timmermann, 2006; Rapach and Wohar, 2006).

(iv) More recently, Lettau and Ludvigson (2001, 2005) have found that dividend growth and stock returns are predictable by long-run equilibrium relationships derived from a linearized version of the consumer’s intertemporal budget constraint. The excess consumption with respect to its long run equilibrium value is defined by the authors alterna-

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tively as a linear combination of labour income and financial wealth, cay_t, or as a linear combination of aggregate dividend payments on human and non-human wealth, cdy. cay_t and cdy_t are much less persistent than dp_t, they are predictors of stock market returns and dividend-growth, and, when included in a predictive regression relating stock market returns to dp_t, they swamp the significance of this variable. Lettau and Ludvigson (2005) interpret this evidence in the light of the presence of a common component in dividend growth and stock market returns. This component cancels out from (1.1), cay_t and cdy_t are instead able to capture it as the linearized intertemporal consumer budget constraint delivers a relationship between excess consumption and expected dividend growth or future stock market returns that is independent from their difference.

A recent strand of the empirical literature has related the contradictory evidence on the dynamic dividend growth model to the potential weakness of its fundamental hypothesis that the log dividend-price ratio is a stationary process (Lettau and Van Nieuwerburgh, 2008, LVN henceforth). LVN use a century of US data to show evidence on the breaks in the constant mean $\overline{\text{dp}}$. We report the time series of US data on dp_t over the last century in Figure 1. As a matter of fact, the evidence from univariate test for non-stationarity and bivariate cointegration tests does not lead to the rejection of the null of the presence of a unit-root in dp_t[2].

As shown in Figure 1, LVN identify two statistically significant breaks in the mean of dp_t in 1954 and 1991. They then provide evidence that deviations of dp_t from its time-varying mean have a much stronger forecasting power for stock market returns than deviations of dp_t from a constant mean[3]. This evidence for time-variation in the mean of the dividend-price ratio has been also confirmed by Johannes et al. (2008), who estimate the process for log dividend-price ratio within a particle filtering framework.

So far the evidence towards a slowly evolving mean in dp_t has been reported as a pure statistical fact. LVN give some hints on possible causes for the breaks arising from economic fundamentals due to technological innovation, changes in expected return, etc. but do not explore the possible effects of fundamentals any further. The idea of correcting dp_t to reduce its persistence has been also pursued by an alternative strand of

[2] The Dickey-Fuller test for the null of non-stationarity delivers an observed statistics of -2.34 when computed over the full sample 1911-2008 and a value of -1.72 when computed over the sample 1955-2008. This evidence is confirmed by the implementation of the Johansen (1991) test on a bivariate VAR for p_t and d_t, that does not reject the null hypothesis of at most zero cointegrating vectors over the full-sample and the post-war subsample.

[3] These results are confirmed by the search for possible structural breaks in the cointegrating relationship based on the application of the recursive test based on the non zero-eigenvalues suggested in Hansen and Johansen (1999). The eigenvalue shows a remarkable amount of variability over the examination period with indication of three break points around 1950, 1980, 2000.

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research that relates the apparent non-stationarity of this variable to a shift in corporate payout policies. Boudoukh et al. (2007) provide a new measure of the cash flow based net payout yield (dividends plus repurchases minus issuances are used instead of dividends to construct the relevant ratio) that is more quickly mean reverting than the dividend-price ratio. Yet, this suggested measure is unlikely to explain the full decrease in this financial ratio as argued by LVN. Moreover other financial ratios such as earning-price ratio witness similar declines.

The aim of our chapter is to investigate the possibility that the slowly evolving mean in the log dividend-price is related to demographic trends. We first illustrate how the theoretical model by Geanakoplos, Magill and Quinzii (2004, henceforth, GMQ) implies that a specific demographic variable, MY_t, the proportion of middle-aged to young population, explains fluctuations in the dividend yield.

GMQ consider an overlapping generation model where the demographic structure mimics the pattern of live births in the US, which have featured alternating twenty-year periods of boom and busts. They conjecture that the life-cycle portfolio behavior (Bakshi and Chen, 1994), which suggests that agents should borrow when young, invest for retirement when middle-aged, and live off from their investment once they are retired, plays an important role in determining equilibrium asset prices. In their model, the demographic structure requires that when the MY ratio is small (large), there will be excess demand for consumption (saving) by a large cohort of retirees (middle-aged). For the market to clear, equilibrium prices of financial assets and therefore the dividend-price ratio should adjust, i.e. decrease (increase), so that saving (consumption) is encouraged for the middle-aged. We take the GMQ model to the data via the conjecture that fluctuations in MY_t could capture a slowly evolving mean in dp_t within the dynamic dividend growth model. Demographic trends should capture the slowly evolving mean in dp_t and then, deviations of dp_t from $\overline{\mathrm{dp}_t}$ could be used as a potential predictor for long-term stock market returns and dividend growth. Our empirical strategy has the potential for identifying separately the importance of demographic variables for high-frequency and low-frequency fluctuations in asset prices. Investigations conducted in the literature on the interaction between asset prices and demographic variables have traditionally concentrated either on high-frequency or low frequency fluctuations but have never considered an empirical framework based on the dynamic dividend growth model, capable of accommodating both of them, with a different role (see Erb et al., 1996; Poterba, 2001; Goyal, 2004; Ang and Maddaloni, 2005; DellaVigna and Pollet, 2006).

We first use long-run predictive regressions and cointegration analysis to assess the sta-

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tistical significance of MY_t in a dynamic dividend growth model. The robustness of our results is evaluated by comparing the predictive power of the dividend-price ratio corrected for demographics with that of the dividend-price ratio, the dividend-price ratio corrected for breaks in mean (LVN) and the cash flow based net payout yield (Boudoukh et al., 2007). The role of MY_t is then further investigated against different alternative specifications, in particular those based on cay_t and cdy_t. Finally, the availability of long-run projections for MY_t is exploited to derive predictions of long-run equity returns up to 2050.

Demography and the Dividend-Price Ratio: The GMQ Model

GMQ analyze an overlapping generation model in which the demographic structure mimics the pattern of live births in the US. Live births in the US have featured alternating twenty-year periods of boom and busts. They consider an OLG exchange economy with a single good (income) and three periods; young, middle-aged, retired. Each agent (except retirees) has an endowment and labor income w= (w^y,w^m,0). There are two types of financial instruments, a riskless bond and a risky asset, which allow agents to redistribute income over time. In the simplest version of the model, dividends and wages are deterministic, hence the bond and the risky asset are perfect substitutes. GMQ assume that in *odd (even)* periods a large (small) cohort N(n) enters the economy, therefore in every odd (even) period there will be {N,n,N} ({n,N,n}) cohorts living. They conjecture that the life-cycle portfolio behavior (Bakshi and Chen, 1994) which suggests that agents should borrow when young, invest for retirement when middle-aged, and live off from their investment once they are retired, plays important role in determining equilibrium asset prices.

Let q_o (q_e) be the bond price and $\{\mathrm{c}_y^o,\mathrm{c}_m^o,\mathrm{c}_r^o\}$ ($\{\mathrm{c}_y^e,\mathrm{c}_m^e,\mathrm{c}_r^e\}$) the consumption stream in the odd (even) period. The agent born in an odd period then faces the following budget constraint

$$\mathrm{c}_y^o + \mathrm{q}_o\mathrm{c}_m^o + \mathrm{q}_o\mathrm{q}_e\mathrm{c}_r^o = \mathrm{w}^y + \mathrm{q}_o\mathrm{w}^m \tag{1.2}$$

and in an even period

$$\mathrm{c}_y^e + \mathrm{q}_e\mathrm{c}_m^e + \mathrm{q}_o\mathrm{q}_e\mathrm{c}_r^e = \mathrm{w}^y + \mathrm{q}_e\mathrm{w}^m \tag{1.3}$$

Moreover, in equilibrium the following resource constraint must be satisfied

$$\mathrm{Nc}_y^o + \mathrm{nc}_m^e + \mathrm{Nc}_r^o = \mathrm{Nw}^y + \mathrm{nw}^m + \mathrm{D} \tag{1.4}$$

$$\mathrm{nc}_y^e + \mathrm{Nc}_m^o + \mathrm{nc}_r^e = \mathrm{nw}^y + \mathrm{Nw}^m + \mathrm{D} \tag{1.5}$$

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where D is the aggregate dividend for the investment in financial markets. If q_o were equal to q_e, the agents would choose to smooth their consumption, i.e. $c_y^i = c_m^i = c_r^i$ for i = o, e, but then for values of wages and aggregate dividend calibrated from US data the equilibrium condition above would be violated leading to excess demand either for consumption or saving. To illustrate this point we refer to the calibration provided by GMQ; take $N = 79$, $n = 69$ as the size (in millions) of Baby Boom (1945-64) and Baby Bust (1965-84) generations (thus, we obtain in an even period a high MY ratio of $MY_t = \frac{N}{n} = 1.15$, and in odd period $MY_t = \frac{n}{N} = 0.87$ (see Figure 2)) and $w^y = 2$, $w^m = 3$ to match the ratio (middle to young cohort) of the average annual real income in US. We can calculate the total wage in even and odd periods using $Nw^y + nw^m$ for odd periods and $nw^y + Nw^m$ for even periods, and then given the average ratio (0.19) of dividend to wages we compute the aggregate dividends. Assuming an annual discount factor of 0.97, which translates to a discount of 0.5 in the model of 20-year periods, if $q_o = q_e = 0.5$ were to hold and agents smooth their consumption, from the budget constraint (eq. 6-7) we obtain $c_y^i = c_m^i = c_r^i = \bar{c} = 2$, but then the resource constraint (eq. 8-9) above would have been violated. For instance, an agent from the Baby Bust generation would enter in an even period in the model, i.e. (n, N, n) and high MY ratio, and faces the following aggregate resource constraint: $n(c_y^e - w^y) + N(c_m^e - w^m) + nc_r^e - D = 69 \times (2-2) + 79(2-3) + 69 \times 2 - 70 = -11$, where $D = 0.19(\frac{375+365}{2}) = 70$. This leads to excess saving in the economy. For equilibrium conditions to hold, the model implies that asset prices should increase and hence discourage saving in the economy (the experience we observed during the 90's in US). When the MY ratio is small (large), i.e. an odd (even) period, there will be excess demand for consumption (saving) by a large cohort of retirees (middle-aged) and for the market to clear, equilibrium prices of financial assets should adjust, i.e. decrease (increase), so that saving (consumption) is encouraged for the middle-aged. Thus, letting q_t^b be the price of the bond at time t, in a stationary equilibrium, the following holds

$$\begin{aligned} q_t^b &= q_o \text{ when period odd} \\ q_t^b &= q_e \text{ when period even} \end{aligned}$$

together with the condition $q_o < q_e$. Moreover the model predicts a positive correlation between MY and market prices, consequently a negative correlation with the dividend yield.

So, since the bond prices alternate between q_o and q_e, then the price of equity must also

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alternate between q_t^e and q_t^e as follows

$$q_o^{eq} = Dq_o + Dq_oq_e + Dq_oq_eq_o + ...$$
$$q_e^{eq} = Dq_e + Dq_eq_o + Dq_eq_oq_e + ...$$

which implies

$$DP_o = \frac{D}{q_o^{eq}} = \frac{1 - q_oq_e}{q_oq_e + q_o}$$
$$DP_e = \frac{D}{q_e^{eq}} = \frac{1 - q_oq_e}{q_oq_e + q_e}$$

where DP_o (DP_e) is the dividend-price ratio implied by low (high) MY in the model for odd (even) periods.

The Empirical Evidence

The GMQ model provides a foundation for a long-run negative relationship between the dividend-price ratio and demography. GMQ define the empirical counterpart of the MY_t ratio as the proportion of the number of agents aged 40-49 to the number of agents aged 20-29, which serves as a sufficient statistic for the whole population pyramid. We report the MY_t ratio in Figure 2. Interestingly, this variable shows highly persistent dynamics and a twin peaked behavior, with peaks and troughs around 1950, 1980, 2000, close to the break points in $\overline{dp_t}$ identified by LVN.

To combine the GMQ model with the dynamic dividend growth we consider the derivation of LVN, who allow for a time varying mean in the linearization and consider MY_t as the potential determinant of this slowly evolving process.

$$dp_t = \overline{dp_t} + \sum_{j=1}^{\infty} \rho^{j-1} E_t[(h_{t+j}^s - \bar{h}) - (\Delta d_{t+j} - \bar{d})] \quad (1.6)$$
$$\overline{dp_t} = \beta_0 + \beta_1 MY_t + u_t$$

Inserting GMQ into the dynamic dividend growth model leads to the prediction that the (log) dividend-price adjusted for demographics should be significant in the long-horizon forecasting regression for real stock market returns, the real dividend growth, and their difference. MY_t should also be significant in explaining the persistence of the dividend-price, and the variable predicted to be stationary in this extended model is not the dividend-price but a combination between price, dividends and MY_t. We investigate the hypothetical cointegrating relation between dividend, prices and MY_t, by running the Johansen (1988) procedure on a cointegrating system based on the vector of variables

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$$\mathbf{y}_t' = \left[\begin{array}{ccc} \mathrm{d}_t & \mathrm{p}_t & \mathrm{MY}_t \end{array} \right].$$

Long-Horizon Forecasting Regressions

We report in Table 1, 2 and 3 the evidence from the long-horizon forecasting regression. To make our evidence directly comparable with that reported in Lettau and Ludvigson (2005) we consider predictive regressions for annual data with horizons ranging from one to six years. We consider the annual data for the S&P 500 index from 1909 to 2008 taken from Robert Shiller's website, dividends are twelve-month moving sums of dividends paid on the S&P 500 index. These series coincide with those used in Goyal and Welch (2008), and made available at Amit Goyal's website. A full description of all data used in our empirical analysis is provided in the Data Appendix.

Table 1, 2 and 3 report the evidence for forecasting returns, dividend growth, returns adjusted for dividend growth, based on the following three models:

$$\begin{aligned} \sum_{j=1}^{k} (\mathrm{h}_{t+j}^{s}) &= \beta_0 + \beta_1 \mathrm{p}_t + \beta_2 \mathrm{d}_t + \beta_3 \mathrm{MY}_t + \varepsilon_{t,t+j} \\ \sum_{j=1}^{k} (\Delta \mathrm{d}_{t+j}) &= \beta_0 + \beta_1 \mathrm{p}_t + \beta_2 \mathrm{d}_t + \beta_3 \mathrm{MY}_t + \varepsilon_{t,t+j} \\ \sum_{j=1}^{k} (\mathrm{h}_{t+j}^{s} - \Delta \mathrm{d}_{t+j}) &= \beta_0 + \beta_1 \mathrm{p}_t + \beta_2 \mathrm{d}_t + \beta_3 \mathrm{MY}_t + \varepsilon_{t,t+j} \\ \mathrm{k} = & \quad 1, \ldots, 6 \end{aligned}$$

In each forecasting regression MY_t is measured at the end of the forecasting period. We report heteroskedastic and autocorrelated consistent (HAC) covariance matrix estimators using Bartlett kernel weights as described in Newey and West (1987) to account for overlapping observations where the bandwidth has been selected following the procedure described in Newey and West (1994). Alternatively, we also conduct a (wild) bootstrap exercise (Davidson and Flachaire, 2008) to compute p-values. To take care of the potential effect on statistical inference in finite sample of the use of overlapping data we also report the rescaled t-statistic recommended by Valkanov (2003) for the hypothesis that the regression coefficient on the dividend-price adjusted for the effect of demographics is zero. We report test of predictability at each horizon but we also compute joint tests across horizons based on SUR estimation and report in the last row the relevant χ^2 statistics with associated p-values.

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The evidence can be summarized as follows:

i) MY_t is always significant along withp_t and d_t in all the forecasting regressions for real stock market returns (Panel A). The adjusted R^2 of the predictive regression increases with the horizon from 0.09 at the 1-year horizon to 0.54 at the 6-year horizon. Consistently with the prediction of the GMQ model, the effect of MY is negative on the slowly evolving mean of the dividend-price and hence positive for expected returns at all horizons.

ii) MY_t is never significant in the forecasting regressions for real dividend growth (Panel B). The adjusted R^2 of the predictive regression declines with the horizon from 0.15 at the 1-year horizon to 0.06 at the 6-year horizon.

iii) MY_t is always significant along with p_t and d_t in all the forecasting regressions for real stock market returns adjusted for real dividend growth (Panel C). The adjusted R^2 of the predictive regression increases with the horizon from 0.26 at the 1-year horizon to 0.67 at the 6-year horizon. The evidence of the strongest predictability of $\sum_{j=1}^{k}(h^s_{t+j} - \Delta d_{t+j})$ is fully consistent with the dynamic dividend growth model. Such evidence, paired with that on the different forecastability of the two components of stock market returns adjusted for dividend growth, rules out the dominance of a common stochastic component for the determination of the dynamics of dividend growth and stock market returns.

iv) MY_t dominates alternative approaches proposed in the literature to capture an evolving mean in the dividend-price ratio. In the last rows of each panel of the Tables 1, 2 and 3 we report the results of augmenting the long-run forecasting regressions based on the GMQ model with alternative filtered dividend-price series. In particular we consider, dp_t^{LVN}, the (log) dividend-price corrected for breaks in LVN and dp_t^{BMRR}, the cash flow based net payout yield (dividends plus repurchases minus issuances) proposed by Boudoukh et al. (2007).

Overall the long-run forecasting regressions lend strong support to the inclusion of MY_t in the traditional dynamic dividend growth model. The dividend-price corrected for a slowly long-run mean, determined by MY_t, predicts long-run stock market returns and long-run stock market returns adjusted for dividend growth, but it does not predict long-run dividend growth. The R^2 associated to the relevant predictive regressions increases with the horizon. This evidence of a positive relation between predictability and the forecasting horizon is interesting, in that both the dynamic dividend growth model and

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the GMQ model establish a predictive relation for long-run returns. In fact, the most natural horizon for the GMQ model is one generation, i.e. about twenty years. Of course, it is difficult to establish some evidence via predictive regressions for twenty years returns, as we have only one century of data. To give the reader a visual impression on the relationship between real stock market returns and MY_t at a frequency as close as possible to that implied by the relevant models, we report in Figure 3 MY_t and 20-year real stock market returns. We find the graphical evidence interesting and fully consistent with the statistical evidence from the long-run regressions at higher frequencies.

Cointegration

The evidence of forecasting power of a linear combination of dividend, prices and MY_t for forecasting long-run returns and long-run returns adjusted for dividend growth, provides indirect evidence of stationarity of such a combination. The validity of this hypothesis can be further investigated by running the Johansen (1988, 1991) procedure on a cointegrating system based on the vector of variables $\mathbf{y}_t' = \begin{bmatrix} \mathrm{d}_t & \mathrm{p}_t & \mathrm{MY}_t \end{bmatrix}$. We then test for cointegration within a three-variate VAR[4].

We report in Table 4 and 5 the evidence for the full-sample and for the sub-sample 1955-2008. The results lead to the rejection of the null of at most zero cointegrating vectors, while the null of at most one cointegration vector cannot be rejected. The evidence in favour of one cointegrating vector in which all variables are always significant confirms that the high persistence of the dividend-price is matched by the high persistence of MY_t. Using the augmented Dickey-Fuller test, the null of a unit root in MY cannot be rejected. The coefficients determining the adjustment in presence of disequilibrium in the Vector Error Correction model confirm the evidence from the forecasting regressions reported in the previous section: stock market returns adjust in presence of disequilibrium. The significance of MY_t increases in the second sub-sample, where LVN found the two breaks in dp_t[5].

[4] See Appendix A for the details of the specification of our statistical model. In a previous version of this paper we allow for a presence of a technology-driven trend (GMQ, p.6), proxied by Total Factor Productivity, in the long-run equilibium relationship. We have decided to exclude TFP from the cointegrating relationship on the basis of two arguments i) the presence of a technology driven trend in the dividend pice ratio is very hard to justify theoretically ii) the TFP trend does not attract any significance when included in the long-run forecasting regressions discussed in the previous section. We are grateful to an anonymos referee for attracting our attention on this point.

[5] We have also investigated the stability of the cointegrating relationship by using the recursively calculated eigenvalues and the tests for constancy of the parameters in the cointegrating space proposed by Nyblom (1989), Hansen and Johansen (1999) and Warne et al. (2003). The results, available upon request, show no evidence of instability.

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Figures 4.A - 4.B provide a graphical assessment of the capability of MY_t of capturing the slowly evolving mean of dp_t. Figure 4.A reports the residuals from our cointegrating vector, along with dp_t, and the deviations of dp_t from $\overline{\mathrm{dp}}^{LVN}$, the shifting mean identified by LVN. Figure 4.B reports residuals from our cointegrating vector with the cycle of dp_t, obtained by applying an Hodrick-Prescott filter to the original series. The graphical evidence illustrates how the cointegration based correction matches the break-based correction in LVN (2008) and the cycle obtained by applying the HP filter. It is important to note that while the cointegration based analysis can be promptly used for forecasting, the same does not apply to both the HP filter and the correction for breaks.

Overall we take the evidence of long-run forecasting regressions and cointegration analysis as consistently supportive of the GMQ model. Two more remarks are in order before we move forward.

First, in the GMQ model, bond and stock are perfect substitutes, therefore the evaluation of the performance of MY_t in forecasting yields to maturity of long-term bonds seems a natural extension of our empirical investigation. In fact, the debate on the so-called FED model (Lander et al., 1997) of the stock market, based on a long-run relation between the price-earning ratio and the long-term bond yield, brings some interesting evidence on this issue. The FED model is based on the equalization, up to a constant, between long-run stock and bond market returns. This feature is shared by the GMQ framework, and it requires a constant relation between the risk premium on long-term bonds and stocks. It has been shown that, although the FED model performs well in periods where the stock and bond market risk premia are strongly correlated, some measure of the fluctuations in their relative premium is necessary to model periods in which volatilities in the two markets have been different (see, for example, Asness, 2003). As a consequence, to put MY_t at work to explain bond yields, some modelling of the relative bond/stock risk premia is also in order. We consider this as an interesting extension beyond the scope of this chapter which is investigated in the last chapter.

Second, although MY_t is the GMQ model consistent measure of demographics, there are a number of different potential measures for demographic trends. We have therefore conducted robustness analysis of our cointegration results to the introduction of different measures of demographic structure of the population and productivity trends. The results, discussed in Appendix B, are supportive of our preferred specification.

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MY, CAY and CDY

In the light of the evidence reported in the previous section it is interesting to reconsider point iv) in the introduction and evaluate the significance of the introduction of MY_t in the dynamic dividend growth model against cay_t and cdy_t. As stated in the introduction, Lettau and Ludvigson (2001, 2005) have found that dividend growth and stock returns are predictable by long-run equilibrium relationships derived from a linearized version of the consumer's intertemporal budget constraint. The excess consumption with respect to its long run equilibrium value is defined by Lettau and Ludvigson (2001) alternatively as a linear combination of labour income and financial wealth, cay_t, or as a linear combination of aggregate dividend payments on human and non-human wealth, cdy_t. cay_t and cdy_t are much less persistent time-series than dp_t, they are predictors of both stock returns and dividend-growth, and when included in a predictive regression relating stock market returns to dp_t, they swamp the significance of this variable.

Evaluating the effect of the inclusion of cay_t and cdy_t in the long-run forecasting regressions that also include MY_t is important for a number of reasons. First, it is a parsimonious way of evaluating the model with MY_t against all financial ratios traditionally adopted to predict returns. In fact, Lettau and Ludvigson (2001, 2005) show the superior performance in predicting long-run returns of cay_t and cdy_t with respect to all the traditionally adopted financial ratios, such as the detrended short term interest rate (Campbell, 1991; Hodrick, 1992), the log dividend earnings ratio and the log price earning ratio (Lamont, 1998), the spread of long term bond yield (10Y) over 3M Treasury bill, and the spread between the BAA and the AAA corporate bond rates. Second, it would allow further investigation on the presence of a common component in dividend and stock market returns suggested by Lettau and Ludvigson (2005) but not consistent with our findings in Table 3, that witness the significance of MY_t for predicting long-run returns and long-run returns adjusted for dividend growth. Third, it could shed further light on the relative importance of cay_t and cdy_t and MY_t for predicting returns and dividend growth in the dynamic dividend growth model. Note that a joint significance of cay_t or cdy_t and MY_t in long-run forecasting regressions for real stock market returns is fully consistent with the GMQ model if the significance of MY_t is interpreted in the light of its role as a predictor for $\overline{\mathrm{dp}_t}$ while cay_t or cdy_t are taken as predictors of long term expectations of real returns and dividend growth.

We report the relevant evidence in Tables 6, 7 and 8. cay_t and cdy_t are estimated by Lettau and Ludvigson (2001,2005) as cointegrating residuals for the systems (c_t, a_t, y_t) and (c_t, d_t, y_t), where c_t is log consumption, y_t is log labor income, a_t is log asset wealth

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(net worth), d_t is log stock market dividends. We have taken the cointegrating relationship directly from Lettau and Ludvigson (2005): $cay_t = c_t - 0.33a_t - 0.57y_t$, $cdy_t = c_t - 0.13d_t - 0.68y_t$. The evidence clearly indicates that the significance of dp_t corrected for MY_t in the long-horizon regressions is not reduced by the augmentation of the model with cay_t and cdy_t. These two variables, and in particular cdy_t, have strong predictive power for dividend growth. Therefore, the evidence that the best predictive model for long-horizon stock returns is the one combining dividend-price with the demographic variable and cdy_t is indeed fully consistent with an interpretation based on the Dynamic Dividend Growth model where MY_t explains the slowly evolving component of the mean of the dividend-price and cdy_t acts as a predictor of dividend-growth. Such an interpretation is supported by the long-horizon regressions for stock returns adjusted for dividend growth, in which both MY_t and cdy_t enter with highly significant coefficients of the opposite sign, positive for MY_t and negative for cdy_t.

In Figure 5.A and 5.B we plot $dp_t - \overline{dp_t}$ against cay_t and cdy_t, respectively. We derive $\overline{dp_t}$ by using the coefficients from Table 1 (k=3)[6], while cay_t and cdy_t series are taken from Lettau and Ludvigson (2005). The graph shows positive but not too strong correlation between $dp_t - \overline{dp_t}$ and cay_t (cdy_t), of 0.57 (0.18). This evidence is consistent with our inclusion of MY_t in the dynamic dividend growth model and the derivation of cay_t and cdy_t from the consumer's intertemporal budget constraint. Consider for example $dp_t - \overline{dp_t}$ and cay_t, they have a common component, which is the weighted sum of future returns, but they are also determined by idiosyncratic components: future dividend growth and future consumption growth, respectively.

Out-of-Sample Evidence

In this section we follow Goyal and Welch (2008), and analyze the performance of cdy_t and MY_t adjusted dividend-price ratio from the perspective of a real-time investor. We therefore consider out-of-sample evidence, for the 1-year, 2-year, and 3-year horizons, and we compare the performance of the bivariate model based on the combination of the two predictors with that of the two univariate models based on each predictor and the univariate models based on dp_t and dp_t^{LVN}.

We run rolling forecasting regressions for the one, three and five years ahead horizon by using 1955-1981 as an initialization sample. The forecasting period beginning in 1982

[6] We take 3-year horizon as a representative example, results for other horizons remain qualitatively similar (available upon request). We use the coefficients from table Table 1, the results are very similar when Table 6 is used instead.

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includes the anomalous period of late 90's where the sharp increase in the stock market index weakens the forecasting power of financial ratios. In particular, we consider both the univariate models and the bivariate encompassing model; we compare the forecasting performance with the historical mean benchmark. In the first two columns of Table 9 we report the adjusted $\bar{R}^2$ and t-statistics using the full sample 1955-2008. Then we report mean absolute error (MAE) and root mean square error (RMSE) calculated based on the residuals in the forecasting period, namely 1982-2008. The first column of the out-of- sample panel reports the out-of-sample R^2 statistics (Campbell and Thomson, 2008) which is computed as

$$R^2_{OS} = 1 - \frac{\sum_{t=t_0}^{T}(r_t - \hat{r}_t)^2}{\sum_{t=t_0}^{T}(r_t - \bar{r}_t)^2}$$

where $\hat{r}_t$ is the forecast at t -1 and $\bar{r}_t$ is the historical average estimated until t -1. In our exercise, $t_0 = 1982$ and T$=$ 2008. If R^2_{OS} is positive, it means that the predictive regression has a lower mean square error than the prevailing historical mean. In the last column, we report the Diebold-Mariano (DM) t-test for checking equal-forecast accuracy from two nested models for forecasting h-step ahead excess returns.

$$DM = \sqrt{\frac{(T + 1 - 2 * h + h * (h - 1))}{T}} * \left[\frac{\bar{d}}{\widehat{se}(\bar{d})}\right]$$

where we define e^2_{1t} as the squared forecasting error of prevailing mean, and e^2_{2t} as the squared forecasting error of the predictive variables, $d_t = e^2_{1t} - e^2_{2t}$, i.e. the difference between the two forecast errors, $\bar{d} = \frac{1}{T}\sum_{t=t_0}^{T} d_t$ and $\widehat{se}(\bar{d}) = \frac{1}{T}\sum_{\tau=-(h-1)}^{h-1}\sum_{t=|\tau|+1}^{T}(d_t - \bar{d}) * (d_{t-|\tau|} - \bar{d})$. A positive DM t-test statistic indicates that the predictive regression model performs better than the historical mean.

We report in Figure 6 the cumulative squared prediction errors of the historical mean minus the cumulative squared prediction error of our best forecasting model. We use all available data from 1910 until 1954 for initial estimation and then recursively calculate the cumulative squared prediction errors until the sample end, namely 2008.

Overall, the results reported in Table 9 and Figure 6 confirm the evidence from the forecasting regressions, with a clear indication that the model combining cdy_t and MY_t adjusted dividend-price ratio dominates all alternative specifications, both within-sample and out-of-sample.

Long-Run Equity Premium Projections

An interesting feature of MY_t is that long-run forecasts for this variable are readily available. In fact, the Bureau of Census (BoC) provide projections up to 2050 for MY_t.

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In this section we combine a long-run horizon regression with the cointegrating system estimated in section 2 to construct a model that can be simulated to generate long-run equity premium projections.

We concentrate on 5-year excess returns and estimate the following model:

$$\sum_{j=1}^{5}(\mathrm{h}^s_{t+j} - \mathrm{r}_{f,t+j}) = c_1 + c_2\left(\mathrm{p}_t - c_3\mathrm{d}_t - c_4\mathrm{MY}_t\right) + \mathrm{u}_{1t} \quad (1.7)$$

$$\begin{bmatrix} \Delta\mathrm{p}_{t+1} \\ \Delta\mathrm{d}_{t+1} \end{bmatrix} = \begin{bmatrix} c_5 \\ c_{10} \end{bmatrix} + \begin{bmatrix} c_6 \\ c_{11} \end{bmatrix} \begin{bmatrix} 1 & -c_3 & -c_4 \end{bmatrix} \begin{bmatrix} \mathrm{p}_t \\ \mathrm{d}_t \\ \mathrm{MY}_t \end{bmatrix} + \begin{bmatrix} c_7 & c_8 & c_9 \\ c_{12} & c_{13} & c_{14} \end{bmatrix} \begin{bmatrix} \Delta\mathrm{p}_t \\ \Delta\mathrm{d}_t \\ \Delta\mathrm{MY}_t \end{bmatrix} + \begin{bmatrix} \mathrm{u}_{2t} \\ \mathrm{u}_{2t} \end{bmatrix},$$

(1.7) Combines a long-run forecasting regression for five year excess returns, defined as the difference between returns on the S&P500 and the risk-free rate[7], with the equations for $\Delta\mathrm{p}_{t+1}$, $\Delta\mathrm{d}_{t+1}$ in the cointegrated VAR estimated in Section 2. Equity premium projections are obtained by forward simulation of the first equation. This requires projections for the three right-hand side variables. We obtain them directly from the BoC for MY_t and by forward simulation of the CVAR estimated in Section 2 for p_t and d_t.Three comments are in order on the specification of (1.7). First, omitting an equation for MY_t from the model used for projections requires (strong) exogeneity of this variable: we believe in the validity of such an assumption. Second, we impose cross-equation restrictions in order to have the same estimates of the coefficients determining the long-run equilibrium of the system in the equation for excess-returns and in the equation for 1-year returns and dividend growth. Third, we did not report the results based on the inclusion of cdy_t in our forecasting model. In fact, the long-horizon forecast for this variable do rapidly converge to its historical mean to leave the variability of projections of the risk-premium to be dominated by projections for MY_t. Moreover, as pointed out by Goyal and Welch (2008), this variable might suffer from look-ahead bias, as the cointegrating coefficients are computed using full-sample estimates.

The estimates are fully consistent with those reported in Table 1 and 4[8]. Figure 7 illus-

[7] See the Data Appendix for a detailed description of the construction of our risk-free rate.
[8] All the evidence reported for the long-run forecasting regressions are based on real equity returns, the dependent variable consistent with the dynamic dividend growth model. Results are robust when excess returns are used as a dependent variable instead of real returns.

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trates the results from the projection of the model.

Over the sample up to 2008 we report (pseudo) out-of-sample 5-year annualized equity premium forecasts and its realizations. The model consistently performs very well with only two exceptions: the 1929 crisis and the boom market at the end of the millennium. We then conduct the out-of-sample exercise by estimating the model with data up to 2008, and then by solving it forward stochastically to obtain out-of-sample forecasts until 2050. Our simulation predicts a rapid stock market recovery for the next two years followed by fluctuations of the risk premium around a mean of 5.02 per cent, just below the historical average. The width of the 95 per cent confidence intervals points to the existence of a sizeable amount of uncertainty around point estimates. Interestingly, the model does not foresee a dramatic market meltdown, a "doomsday" scenario, due to a collective exit from the stock market by the retired baby boomers. This evidence is a natural outcome of the GMQ model which relies on the cyclicality of U.S. age structure.

Conclusions

This chapter has documented the existence of a slowly evolving trend in the mean dividend-price ratio determined by a demographic variable, MY_t, the proportion of middle-age to young population. We have shown that MY_t captures well a slowly evolving component in the mean dividend-price ratio and it is strongly significant in long-horizon regressions for real stock market returns.

A model including MY_t overperforms all alternative models for forecasting returns. The best forecasting model for real stock market returns found in our work is the one combining MY_t with cdy_t, a variable constructed by Lettau and Ludvigson (2005) to capture excess consumption with respect to its long run equilibrium value. We take this evidence as strongly supportive of the Dynamic Dividend Growth model with an evolving mean, determined by MY_t. In fact, the model predicts that long-horizon returns should depend on the deviations of the dividend-price ratio from its mean and on long-run dividend growth. We show that MY_t models the mean of the dividend-price ratio while cdy_t is a predictor of long-horizon dividend-growth, confirming the evidence in Lettau and Ludvigson (2005). We provide evidence that an important component of time-varying expected returns is captured by allowing the mean of the dividend-price ratio to fluctuate MY_t. The importance of such a component increases with the forecasting horizon.

The empirical results we have reported should be of special relevance to the strategic asset allocation literature (e.g. Campbell and Viceira, 2002), in which the log dividend-

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price ratio is often used in VAR models as a stationary variable capturing time-variation in the investment opportunity set, and as an input into the optimal asset allocation decision of a long-horizon investor. In a companion paper (Favero and Tamoni, 2010) we show that allowing for the presence of MY_t in the VAR models that are used to estimate the time profile of stock market return and its volatility does cast new light on the hot debate on the safety of stock market investment for the long-run (Pastor and Stambaugh, 2009).

Finally, by exploiting the exogeneity and the predictability of MY_t, we have also provided projections for equity risk premia up to 2050. Our simulations point to an average equity risk premium of about five per cent for the period 2010-2050.

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Tables

TABLE 1

Long-horizon regression. Sample 1910 -2008. Annual data.

Panel A. k-period regressions for real stock returns

$$\sum_{j=1}^{k}(h^{s}_{t+j}) = \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \varepsilon_{t,t+j}$$

$\chi^2_{(t-stat)}$ 15.63 (0.000) horizon k in years

	1	2	3	4	5	6
β_1	−0.27	−0.28	−0.23	−0.21	−0.19	−0.16
(t-stat)	(−4.000)	(−6.287)	(−6.563)	(−9.077)	(−9.352)	(−7.824)
β_2	0.30	0.32	0.26	0.24	0.21	0.17
(t-stat)	(3.679)	(5.748)	(5.801)	(7.974)	(8.291)	(7.065)
β_3	0.44	0.45	0.37	0.34	0.29	0.26
(t-stat)	(4.065)	(5.875)	(6.509)	(7.627)	(7.865)	(7.710)
$\beta_1 = -\beta_2$	−0.19	−0.20	−0.16	−0.16	−0.14	−0.13
(t-stat / w.b.p-values)	(−3.645 / 0.002)	(−5.439 / 0.000)	(−6.016 / 0.000)	(−7.652 / 0.000)	(−8.584 / 0.002)	(−7.740 / 0.008)
$t/\sqrt{T}$ - test	{0.30*}	{0.49**}	{0.59**}	{0.75**}	{0.89**}	{0.95**}
$\beta_3 \mid (\beta_1 = -\beta_2)$	0.42	0.44	0.38	0.36	0.33	0.29
(t-stat)	(3.447)	(4.636)	(5.059)	(6.006)	(6.578)	(6.915)
Adj. R^2	0.09	0.25	0.33	0.44	0.52	0.54
Adj. $R^2(\beta_1 = -\beta_2)$	0.07	0.20	0.27	0.37	0.46	0.50
F-statistic	4.10	12.18	16.74	26.62	36.74	39.48

Panel B. Testing MY against alternative models

$$\sum_{j=1}^{k}(h^{s}_{t+j}) = \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \beta_4 Z_t + \varepsilon_{t,t+j}$$

$Z_t =$		1	2	3	4	5	6
dp_t^{LVN}	β_3	0.44	0.45	0.39	0.36	0.32	0.28
	$(t-stat)$	(4.149)	(6.138)	(7.242)	(9.009)	(10.383)	(10.449)
	β_4	0.07	0.09	0.05	0.03	0.01	−0.03
	$(t-stat)$	(0.665)	(1.128)	(0.665)	(0.626)	(0.140)	(−0.636)
dp_t^{BMRR}	β_3	0.44	0.46	0.39	0.35	0.31	0.27
	$(t-stat)$	(2.850)	(4.849)	(6.229)	(8.561)	(10.286)	(10.925)
	β_4	0.51	0.37	0.21	0.09	0.02	−0.03
	$(t-stat)$	(3.602)	(2.945)	(2.443)	(1.496)	(0.407)	(−1.103)

Table 1 reports in parentheses t-statistic based on heteroskedastic and autocorrelated consistent (HAC) covariance matrix estimators using Bartlett kernel weights as described in Newey and West (1987) where the bandwidth has been selected following the procedure described in Newey and West (1994). For the univariate model with the restriction $(\beta_1 = -\beta_2)$, $\sum_{j=1}^{k}(h^{s}_{t+j}) = \beta_0 + \beta_1(p_t - d_t + \frac{\beta_3}{\beta_1}MY_t) + \varepsilon_{t,t+j}$, we also conduct a (wild) bootstrap exercise (Davidson and Flachaire (2008)) to compute p-values and report for β_1 $t/\sqrt{T}$-test suggested in Valkanov (2003) in curly brackets. Significance at the 5% and 1% level of the $t/\sqrt{T}$ test using Valkanov's (2003) critical values is indicated by * and **, respectively. The χ^2 statistics with associated p-values is for the joint tests of significance across all different horizon within a SUR estimation framework.

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TABLE 2

Long-horizon regressions. Sample 1910 - 2008. Annual data.

Panel A. k-period regressions for real dividend-growth

$$\sum_{j=1}^{k}(\Delta d_{t+j}) = \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \varepsilon_{t,t+j}$$

χ^2 (t-stat): 2.66 (0.015) — horizon k in years

	1	2	3	4	5	6
β_1 (t-stat)	0.19 (2.097)	0.10 (1.827)	0.05 (1.400)	0.03 (1.222)	0.02 (1.006)	0.01 (0.803)
β_2 (t-stat)	−0.23 (−2.196)	−0.13 (−1.996)	−0.07 (−1.585)	−0.04 (−1.441)	−0.03 (−1.308)	−0.02 (−1.153)
β_3 (t-stat)	−0.15 (−1.250)	−0.03 (−0.371)	0.04 (0.780)	0.07 (1.863)	0.08 (2.428)	0.08 (2.755)
$\beta_1 = -\beta_2$ (t-stat / w.b.p-values)	0.10 (1.580 / 0.240)	0.05 (1.118 / 0.433)	0.02 (0.638 / 0.353)	0.01 (0.298 / 0.171)	−0.00 (−0.018 / 0.096)	−0.00 (−0.234 / 0.106)
$\beta_3 \mid (\beta_1 = -\beta_2)$ (t-stat)	−0.12 (−1.118)	−0.03 (−0.366)	0.03 (0.714)	0.06 (1.700)	0.06 (2.067)	0.06 (2.134)
Adj. R^2	0.15	0.09	0.05	0.04	0.05	0.06
Adj. $R^2(\beta_1 = -\beta_2)$	0.06	0.03	0.02	0.02	0.03	0.03
F-statistic	6.87	4.05	2.73	2.52	2.66	3.04

Panel B. Testing MY against alternative models

$$\sum_{j=1}^{k}(\Delta d_{t+j}) = \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \beta_4 Z_t + \varepsilon_{t,t+j}$$

$Z_t =$		1	2	3	4	5	6
dp_t^{LVN}	β_3 (t-stat)	−0.09 (−0.877)	−0.00 (−0.033)	0.05 (0.993)	0.07 (1.886)	0.08 (2.511)	0.09 (2.886)
	β_4 (t-stat)	−0.24 (−2.412)	−0.14 (−2.094)	−0.08 (−1.520)	−0.05 (−0.942)	−0.02 (−0.321)	0.00 (0.024)
dp_t^{BMRR}	β_3 (t-stat)	−0.18 (−1.304)	−0.05 (−0.697)	−0.00 (−0.001)	0.03 (0.772)	0.04 (1.752)	0.06 (2.509)
	β_4 (t-stat)	0.09 (0.879)	0.14 (1.981)	0.16 (2.122)	0.16 (2.206)	0.15 (2.977)	0.08 (2.304)

Table 2 reports in parentheses t-statistic based on heteroskedastic and autocorrelated consistent (HAC) covariance matrix estimators using Bartlett kernel weights as described in Newey and West (1987) where the bandwidth has been selected following the procedure described in Newey and West (1994). For the univariate model with the restriction $(\beta_1 = -\beta_2)$, $\sum_{j=1}^{k}(\Delta d_{t+j}) = \beta_0 + \beta_1(p_t - d_t + \frac{\beta_3}{\beta_1}MY_t) + \varepsilon_{t,t+j}$, we also conduct a (wild) bootstrap exercise (Davidson and Flachaire (2008)) to compute p-values. The χ^2 statistics with associated p-values is for the joint tests of significance across all different horizon within a SUR estimation framework.

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TABLE 3

Long-horizon regressions. Sample 1910 - 2008. Annual data.

Panel A. k-period regressions for real stock returns adjusted for dividend growth

$$\sum_{j=1}^{k}(h^s_{t+j} - \Delta d_{t+j}) = \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \varepsilon_{t,t+j}$$

χ^2 (t–stat) 27.94 (0.000) horizon k in years

	1	2	3	4	5	6
β_1	−0.46	−0.39	−0.28	−0.24	−0.20	−0.17
(t-stat)	(−6.159)	(−8.285)	(−9.393)	(−10.344)	(−14.944)	(−12.523)
β_2	0.53	0.45	0.33	0.28	0.24	0.20
(t-stat)	(6.127)	(8.234)	(9.157)	(10.022)	(13.150)	(10.940)
β_3	0.59	0.47	0.33	0.27	0.22	0.18
(t-stat)	(4.631)	(5.313)	(5.297)	(5.697)	(6.202)	(6.361)
$\beta_1 = -\beta_2$	−0.28	−0.24	−0.18	−0.16	−0.14	−0.12
(t-stat / w.b.p-values)	(−4.961/0.001)	−6.603 / 0.000)	(−7.345 / 0.000)	(−8.516 / 0.000)	(−13.217 / 0.000)	(−12.976 / 0.002)
$t/\sqrt{T}$ - test	{0.47**}	{0.67**}	{0.77**}	{0.91**}	{1.04**}	{1.12**}
$\beta_3 \mid (\beta_1 = -\beta_2)$	0.55	0.47	0.35	0.31	0.27	0.23
(t-stat)	(3.610)	(4.206)	(4.204)	(4.646)	(5.160)	(5.406)
Adj. R^2	0.24	0.45	0.51	0.60	0.66	0.67
Adj. $R^2(\beta_1 = -\beta_2)$	0.16	0.29	0.35	0.43	0.50	0.53
F-statistic	11.49	27.74	35.07	50.25	65.08	67.56

Panel B. Testing MY against alternative models

$$\sum_{j=1}^{k}(h^s_{t+j} - \Delta d_{t+j}) = \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \beta_4 Z_t + \varepsilon_{t,t+j}$$

$Z_t =$		1	2	3	4	5	6
dp_t^{IVN}	β_3	0.53	0.45	0.34	0.29	0.24	0.19
	(t–stat)	(4.111)	(5.041)	(5.453)	(6.172)	(7.314)	(8.584)
	β_4	0.31	0.23	0.13	0.08	0.02	−0.03
	(t–stat)	(2.618)	(2.739)	(1.915)	(1.512)	(0.513)	(−0.719)
dp_t^{BMRR}	β_3	0.62	0.51	0.39	0.32	0.27	0.21
	(t–stat)	(4.100)	(6.105)	(6.490)	(7.313)	(8.837)	(9.256)
	β_4	0.42	0.22	0.05	−0.07	−0.13	−0.11
	(t–stat)	(2.294)	(1.881)	(0.704)	(−1.734)	(−4.349)	(−4.035)

Table 3 reports in parentheses t-statistic based on heteroskedastic and autocorrelated consistent (HAC) covariance matrix estimators using Bartlett kernel weights as described in Newey and West (1987) where the bandwidth has been selected following the procedure described in Newey and West (1994). For the univariate model with the restriction $(\beta_1 = -\beta_2)$, $\sum_{j=1}^{k}(h^s_{t+j} - \Delta d_{t+j}) = \beta_0 + \beta_1(p_t - d_t + \frac{\beta_3}{\beta_1}MY_t) + \varepsilon_{t,t+j}$, we also conduct a (wild) bootstrap exercise (Davidson and Flachaire (2008)) to compute p-values and report for β_1 $t/\sqrt{T} - test$ suggested in Valkanov (2003) in curly brackets. Significance at the 5% and 1% level of the $t/\sqrt{T}$ test using Valkanov's (2003) critical values is indicated by * and **, respectively. The χ^2 statistics with associated p-values is for the joint tests of significance across all different horizon within a SUR estimation framework.

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TABLE 4

Cointegrated VAR Estimates. Sample 1910-2008. Annual Data.

Panel A. Cointegrating vector	p_t	d_t	MY_t	C
$\boldsymbol{\beta}$ $(s.e.)$	-1.00	1.21 (0.035)	1.107 (0.25)	2.16
Panel B. Error Correction Model	Δp_t	Δd_t	ΔMY_t	
$\boldsymbol{\alpha}$ $(s.e.)$	0.29 (0.096)	-0.12 (0.046)	0.007 (0.007)	
Adj. R^2	0.126	0.43	0.63	
Panel C. Cointegration Test	Trace	p-value	Max eigen	p-value
Hypothesized No of CE(s)				
None	29.68	0.05	22.86	0.028
At Most 1	6.82	0.59	6.75	0.51

TABLE 5

Cointegrated VAR Estimates. Sample 1955-2008. Annual Data.

Panel A. Cointegrating vector	p_t	d_t	MY_t	C
$\boldsymbol{\beta}$ $(s.e.)$	-1.00	1.248 (0.035)	1.14 (0.14)	2.07
Panel B. Error Correction Model	Δp_t	Δd_t	ΔMY_t	
$\boldsymbol{\alpha}$ $(s.e.)$	0.63 (0.16)	-0.02 (0.035)	0.03 (0.015)	
Adj.R^2	0.22	0.40	0.75	
Panel C. Cointegration Test	Trace	p-value.	Max eigen	p-value
Hypothesized No of CE(s)				
None	28.71	0.06	19.48	0.08
At Most 1	9.22	0.34	8.81	0.30

Table 4 and 5 report the trace and max eigenvalue statistics obtained from Johansen cointegration test with linear trend in the data. We report the coefficients of the cointegrating vector β and the weights α (see Appendix A) for the whole sample (Table 4) and for the post-war sample (Table 5). The reported p-values for the relevant null to test for cointegration are McKinnon-Haugh-Michelis (1999) p-values. The lag length in the VAR model is chosen according to optimal information criteria, i.e. sequential LR test, Akaike (AIC), Schwarz (SIC), Hannan-Quinn (HQ) information criterion.

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TABLE 6

cay and cdy. Sample 1948-2008. Annual Data.

k-period regressions for real stock returns

$$\sum_{j=1}^{k}(\mathrm{h}^s_{t+j}) = \beta_0 + \beta_1 \mathrm{p}_t + \beta_2 \mathrm{d}_t + \beta_3 \mathrm{MY}_t + \beta_4 \mathrm{z}_t + \varepsilon_{t,t+j}$$

horizon k in years

	z_t	1	2	3	4	5	6
β_1 $(t-stat)$	cay_t	-0.41 (-4.050)	-0.29 (-4.694)	-0.21 (-5.570)	-0.20 (-7.222)	-0.19 (-8.257)	-0.15 (-8.790)
	cdy_t	-0.51 (-6.487)	-0.40 (-6.442)	-0.29 (-6.840)	-0.24 (-8.592)	-0.21 (-11.549)	0.19 (-12.403)
β_2 $(t-stat)$	cay_t	0.50 (3.994)	0.36 (4.492)	0.25 (5.230)	0.24 (6.373)	0.22 (7.067)	0.17 (7.384)
	cdy_t	0.63 (6.210)	0.48 (6.134)	0.34 (6.287)	0.28 (7.143)	0.25 (9.064)	0.22 (9.832)
β_3 $(t-stat)$	cay_t	0.66 (4.194)	0.48 (5.077)	0.36 (6.327)	0.32 (8.823)	0.29 (10.967)	0.24 (13.000)
	cdy_t	0.781 (5.467)	0.57 (5.774)	0.42 (6.146)	0.35 (7.756)	0.31 (9.394)	0.26 (10.067)
β_4 $(t-stat)$	cay_t	2.06 (1.336)	2.41 (3.329)	1.94 (3.111)	0.87 (1.192)	0.71 (1.471)	1.15 (3.739)
	cdy_t	-0.51 (-0.718)	0.25 (0.490)	0.27 (1.102)	0.04 (0.225)	0.06 (0.498)	0.44 (3.246)
Adj. R^2	cay_t	0.35	0.61	0.70	0.75	0.82	0.87
	cdy_t	0.34	0.56	0.65	0.74	0.81	0.86

Table 6 reports the OLS estimates from k-period regressions for real stock returns. Each column reports a different horizon, odd (even) rows refer to $\mathrm{z}_t = \mathrm{cay}_t$ (cdy_t). The reported t-statistics are based on heteroskedastic and autocorrelated consistent (HAC) covariance matrix estimators using Bartlett kernel weights as described in Newey and West (1987) where the bandwidth has been selected following the procedure described in Newey and West (1994).

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TABLE 7

cay and cdy. Sample 1948-2008. Annual data.

k-period regressions for real dividend-growth

$$\sum_{j=1}^{k}(\Delta d_{t+j}) = \beta_0 + \beta_1 \mathrm{p}_t + \beta_2 \mathrm{d}_t + \beta_3 \mathrm{MY}_t + \beta_4 \mathrm{z}_t + \varepsilon_{t,t+j}$$

		horizon k in years					
		1	2	3	4	5	6
β_1 $(t-stat)$	cay_t	0.10 (1.649)	0.06 (1.481)	0.03 (0.977)	0.01 (0.451)	0.01 (0.268)	0.01 (0.251)
	cdy_t	−0.03 (−0.528)	−0.04 (−0.981)	−0.03 (−1.392)	−0.02 (−1.353)	−0.01 (−1.353)	−0.01 (−0.608)
β_2 $(t-stat)$	cay_t	−0.14 (−1.733)	−0.09 (−1.574)	−0.04 (−1.109)	−0.02 (−0.626)	−0.01 (−0.466)	−0.01 (−0.463)
	cdy_t	0.01 (0.233)	0.03 (0.698)	0.03 (1.071)	0.02 (0.940)	0.01 (0.586)	0.01 (0.284)
β_3 $(t-stat)$	cay_t	−0.07 (−0.838)	−0.02 (−0.363)	0.02 (0.542)	0.04 (1.286)	0.05 (1.806)	0.05 (2.406)
	cdy_t	0.02 (0.188)	0.04 (0.669)	0.05 (1.479)	0.05 (2.033)	0.04 (2.238)	0.04 (2.416)
β_4 $(t-stat)$	cay_t	2.92 (2.503)	2.49 (3.176)	1.62 (4.377)	0.99 (3.359)	0.61 (2.084)	0.39 (1.244)
	cdy_t	1.16 (1.561)	1.00 (2.115)	0.69 (4.500)	0.59 (4.853)	0.56 (4.272)	0.52 (3.670)
Adj. R^2	cay_t	0.33	0.42	0.40	0.28	0.20	0.19
	cdy_t	0.20	0.27	0.30	0.31	0.33	0.35

Table 7 reports the OLS estimates from k-period regressions for real dividend-growth. Each column reports a different horizon, odd (even) rows refer to $\mathrm{z}_t = \mathrm{cay}_t$ (cdy_t). The Table reports in parentheses t-statistic based on heteroskedastic and autocorrelated consistent (HAC) covariance matrix estimators using Bartlett kernel weights as described in Newey and West (1987) where the bandwidth has been selected following the procedure described in Newey and West (1994).

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TABLE 8

cay and cdy. Sample 1948-2008. Annual data.

k-period regressions for real stock returns adjusted for dividend growth

$$\sum_{j=1}^{k}(\mathrm{h}^{s}_{t+j} - \Delta \mathrm{d}_{t+j}) = \beta_0 + \beta_1 \mathrm{p}_t + \beta_2 \mathrm{d}_t + \beta_3 \mathrm{MY}_t + \beta_4 \mathrm{z}_t + \varepsilon_{t,t+j}$$

		horizon k in years					
		1	2	3	4	5	6
β_1 $(t\text{−}stat)$	cay_t	-0.51 (-4.796)	-0.36 (-5.417)	-0.24 (-7.799)	-0.22 (-9.769)	-0.19 (-9.506)	-0.15 (-10.203)
	cdy_t	-0.49 (-6.547)	-0.36 (-8.500)	-0.25 (-9.022)	-0.21 (-9.930)	-0.20 (-13.824)	-0.18 (-15.141)
β_2 $(t\text{−}stat)$	cay_t	0.65 (4.713)	0.45 (5.209)	0.29 (7.635)	0.26 (9.596)	0.23 (8.871)	0.18 (9.070)
	cdy_t	0.62 (6.220)	0.45 (7.653)	0.31 (7.887)	0.26 (8.109)	0.24 (10.452)	0.21 (11.669)
β_3 $(t\text{−}stat)$	cay_t	0.72 (4.487)	0.50 (5.255)	0.34 (6.670)	0.29 (9.758)	0.24 (10.237)	0.19 (10.029)
	cdy_t	0.77 (6.020)	0.53 (6.801)	0.37 (6.419)	0.31 (7.152)	0.27 (8.580)	0.21 (8.883)
β_4 $(t\text{−}stat)$	cay_t	-0.86 (-0.508)	-0.08 (-0.079)	0.329 (0.501)	-0.12 (-0.158)	0.10 (0.169)	0.757 (2.278)
	cdy_t	-1.67 (-2.428)	-0.74 (-1.295)	-0.422 (-2.141)	-0.55 (-3.636)	-0.50 (3.818)	-0.08 $-0.722)$
Adj. R^2	cay_t	0.28	0.50	0.63	0.72	0.79	0.87
	cdy_t	0.32	0.52	0.64	0.74	0.81	0.85

Table 8 reports the OLS estimates from k-period regressions for real stock returns adjusted for dividend-growth. Each column reports a different horizon, odd (even) rows refer to $\mathrm{z}_t = \mathrm{cay}_t$ (cdy_t). The Table reports in parentheses t-statistic based on heteroskedastic and autocorrelated consistent (HAC) covariance matrix estimators using Bartlett kernel weights as described in Newey and West (1987) where the bandwidth has been selected following the procedure described in Newey and West (1994).

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TABLE 9

Predictive Performance. Sample 1948-2008. Annual data.

	In-Sample				Out-of-Sample			
Panel A (k=1)	R^2	t-stat	MAE	RMSE	R^2_{OS}	MAE	RMSE	DM
dp_t	3.03	1.64	12.92	16.17	−11.22	14.54	18.60	−17.43
dp_t^{LVN}	6.36	2.64	11.93	16.20	−5.25	13.58	18.09	−8.09
cdy_t	−1.47	0.48	12.03	14.00	−16.11	13.24	15.12	−19.97
dp_t^{DT}	19.48	4.37	10.08	15.32	11.20	10.91	16.27	4.68
$dp_t^{DT} + cdy_t$	38.64	5.97/ − 0.32	8.71	10.97	28.61	9.83	11.86	14.36
Hist.Mean	–	–	12.92	16.70	–	13.40	17.63	–
Panel B (k= 2)	R^2	t-stat	MAE	RMSE	R^2_{OS}	MAE	RMSE	DM
dp_t	5.46	1.70	15.72	20.71	−55.77	24.21	30.94	−4.01
dp_t^{LVN}	15.74	2.45	14.20	19.91	−11.72	19.99	26.20	−3.30
cdy_t	2.92	1.13	16.00	21.93	−55.19	20.04	27.45	−1.33
dp_t^{DT}	48.32	7.88	12.20	17.52	35.52	14.39	19.90	3.11
$dp_t^{DT} + cdy_t$	62.63	6.32/1.40	10.58	14.17	40.85	13.16	16.94	6.08
Hist. Mean	–	–	16.19	21.91	–	18.65	24.79	–
Panel C (k= 3)	R^2	t-stat	MAE	RMSE	R^2_{OS}	MAE	RMSE	DM
dp_t	6.34	1.99	18.26	24.87	−88.56	33.98	43.59	1.32
dp_t^{LVN}	10.76	1.70	17.91	24.81	−25.89	28.40	35.61	−4.41
cdy_t	5.91	1.41	19.45	26.94	−41.43	26.64	33.70	−1.97
dp_t^{DT}	49.73	6.70	13.22	19.00	43.95	18.15	23.32	2.90
$dp_t^{DT} + cdy_t$	64.89	7.24/2.99	13.67	16.99	48.54	17.06	20.33	2.50
Hist. Mean	–	–	19.38	26.65	–	25.26	31.74	–

Table 9 presents statistics on k-year ahead forecast errors (in-sample and out-of-sample) for stock returns. The first column lists the regressors in both univariate and bivariate predictive regressionsdp_t, log dividend-price ratio, dp_t^{LVN}, dp_t corrected for breaks in mean (LVN), dp_t^{DT}, dp_t adjusted for MY_t and cdy_t, cointegrated vector suggested by Lettau and Ludvigson (2005). The sample starts in 1948 and we construct first forecast in 1982. All numbers are in percent. RMSE is the root mean square error, MAE is the mean absolute error. DM is the Diebold and Mariano (1995) t-statistic for difference in MSE of the unconditional forecast and the conditional forecast. The out-of-sample R^2_{OS} compares the forecast error of the historical mean with the forecast from predictive regressions.

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Figures

Figure 1. Time Series of Log Dividend-Price Ratio

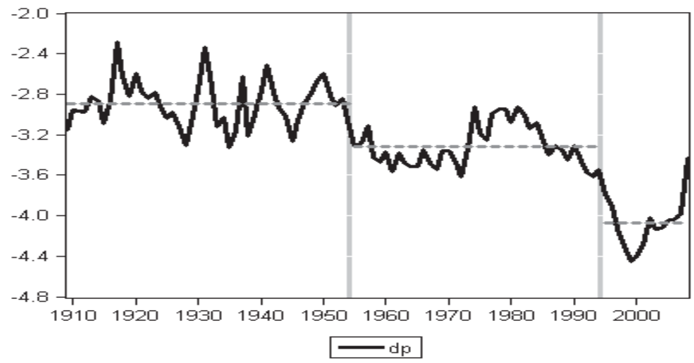

Figure 1 plots the time series of log dividend-price ratio (dp_t). Two vertical lines indicate the breaks in 1954 and 1994 identified by LVN. Sample 1909 - 2008. Annual data.

Figure 2. Time Series of Middle-Young (MY) Ratio

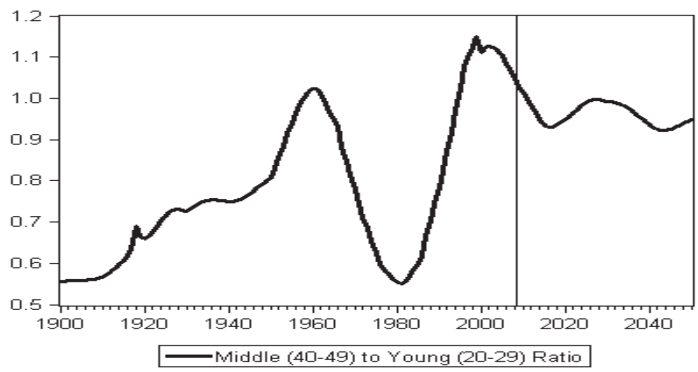

Figure 2 plots the time series of middle-young (MY) ratio. The vertical line in 2008 indicates the end of in-sample data and the start of Bureau of Census projections. Sample 1909-2050. Annual data.

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Figure 3. MY and 20-year Real US Stock Market Returns

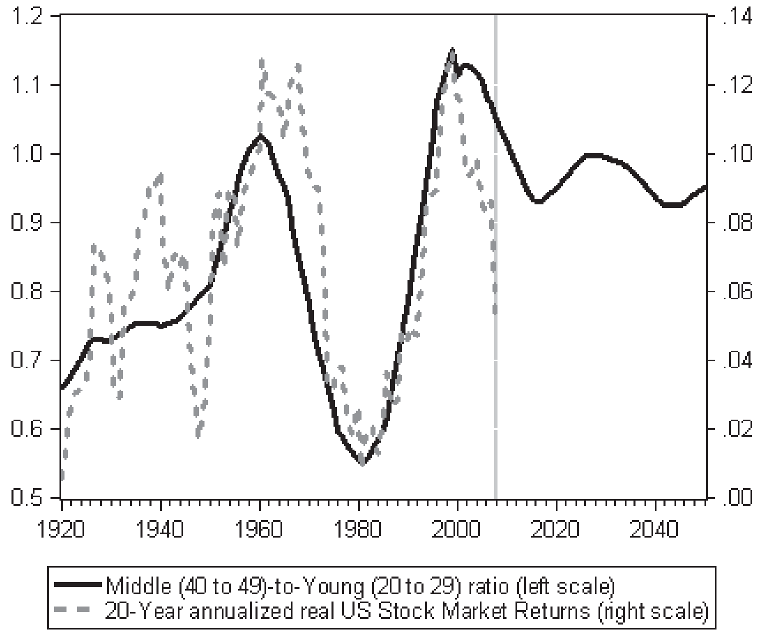

Figure 3 plots the middle-young ratio (MY) and the annualized real US stock market returns. The vertical line in 2008 indicates the end of in-sample data and the start of Bureau of Census projections. Sample 1920-2050. Annual data.

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Figure 4.A. Alternative Measures of the Cycle in dp$_t$

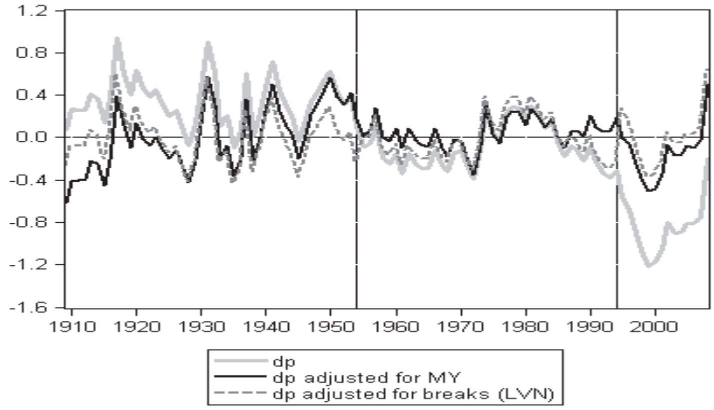

Figure 4.B. Hodrick and Prescott (HP) Filtered Cycle in dp$_t$

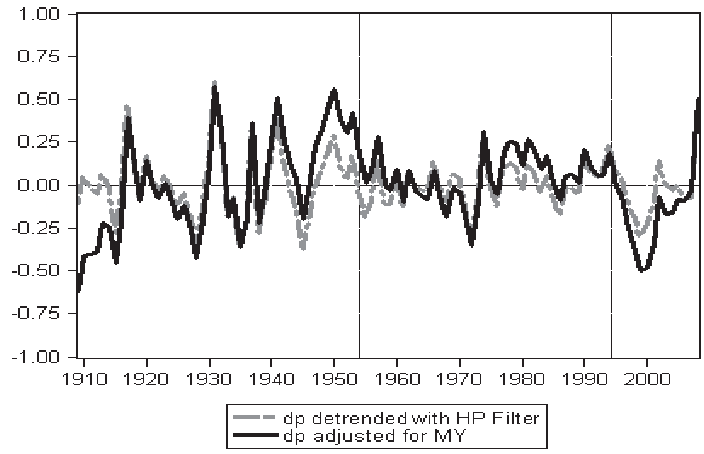

Figure 4.A plots dividend-price ratio, dp$_t$, dp$_t$ adjusted for breaks (LVN) and fluctuations of dp$_t$ around a time-varying mean determined by MY$_t$. We estimate a vector error correction model following Johansen procedure to determine the cointegrating vector between dp$_t$ and MY$_t$ (see Table 4 Panel A). Figure 4.B illustrates an alternative measure of the cycle in dp$_t$ using Hodrick and Prescott (HP) filter with a smoothing parameter equal to 100 (Jaimovich and Siu (2008)). Two vertical lines indicate the breaks in 1954 and 1994 identified by LVN. Sample 1910-2008. Annual data.

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Figure 5.A. cay$_t$ vs. dp$_t$ adjusted for MY$_t$

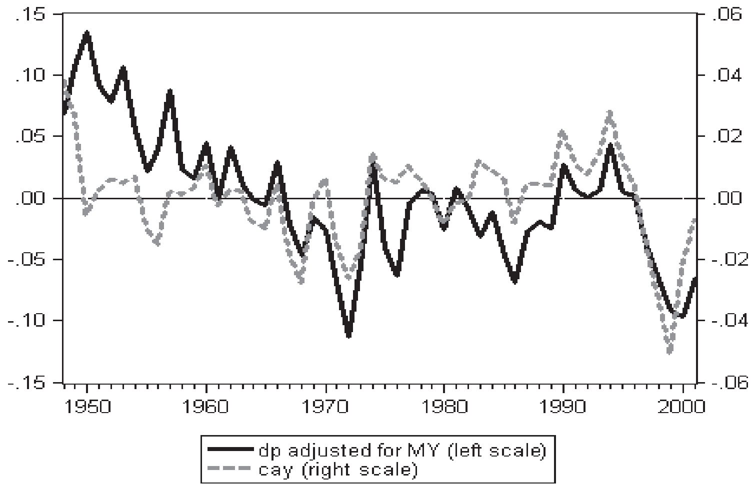

Figure 5.B. cdy$_t$ vs. dp$_t$ adjusted for MY$_t$

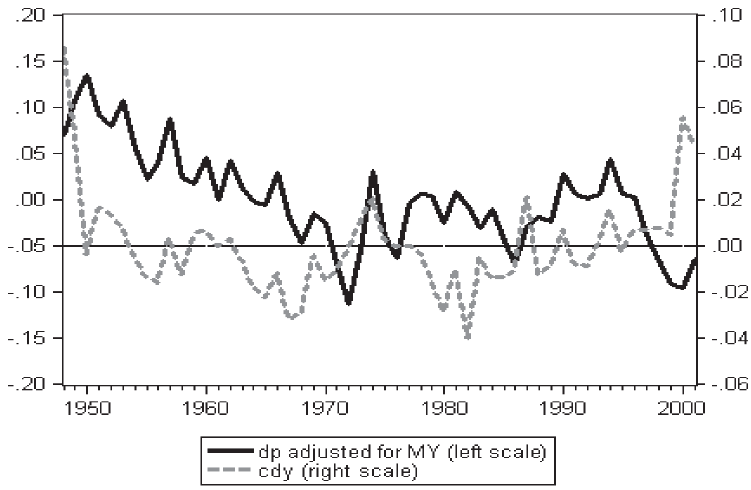

Figure 5.A plots cay$_t$ and dp$_t$ adjusted for MY$_t$. Figure 5.B plots cdy$_t$ and dp$_t$ adjusted for MY$_t$. cay$_t$ and cdy$_t$ are the annual series taken from Martin Lettau's website, dp$_t$ is adjusted for MY$_t$ using the coefficients estimated from predictive regressions reported in Table 1 (k=3). Sample 1910-2008. Annual data.

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Figure 6. Out-of-Sample Predictive Performance

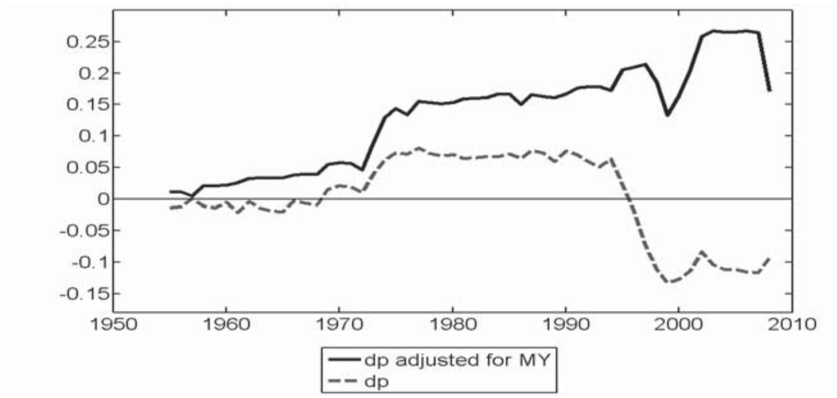

Figure 6 plots the difference between the cumulative RMSE of forecasts based on the historical prevaling mean and forecasting models based on either dp_t (dashed line) or dp_t adjusted for MY (solid line). The estimation sample is 1910-1954 and the forecasts cover the period 1955-2008. Annual data.

Figure 7. Long-Run Equilibrium Projections

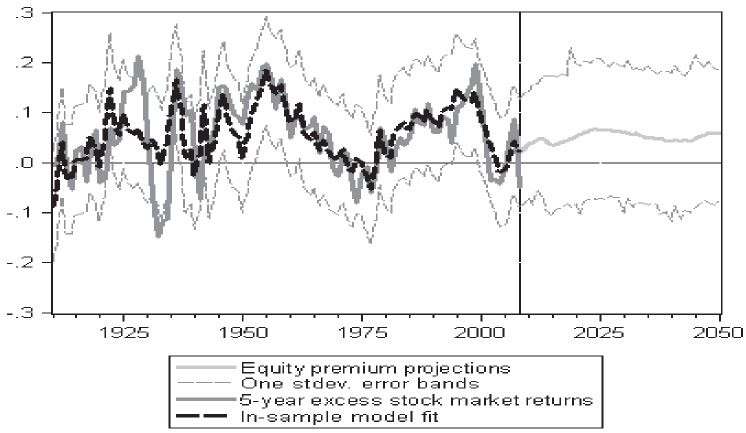

Figure 7 plots 5-year stock market return (solid dark gray line), in-sample prediction (dashed black line) and out-of-sample projections for excess returns (solid gray line) along with 95% confidence intervals (dashed gray lines). The vertical line in 2008 indicates the end of in-sample data and the start of the projections. Sample 1910-2050. Annual data.

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A APPENDIX A

The Statistical Model for Cointegration Analysis

We consider the following statistical model:

$$\begin{aligned} \mathbf{y}_t &= \sum_{i=1}^{n} \mathbf{A}_i \mathbf{y}_{t-i} + \mathbf{u}_t \\ & \mathbf{y}_t \text{ is a } m \times 1 \text{ vector of variables} \end{aligned}$$

This model can be re-written as follows

$$\begin{aligned} \Delta \mathbf{y}_t &= \mathbf{\Pi}_1 \Delta \mathbf{y}_{t-1} + \mathbf{\Pi}_2 \Delta \mathbf{y}_{t-2} + ...\mathbf{\Pi}_{n-1} \Delta \mathbf{y}_{t-n+1} + \mathbf{\Pi} \mathbf{y}_{t-1} + \mathbf{u}_t \\ &= \sum_{i=1}^{n-1} \mathbf{\Pi}_i \Delta \mathbf{y}_{t-i} + \mathbf{\Pi} \mathbf{y}_{t-1} + \mathbf{u}_t, \end{aligned}$$

where:

$$\begin{aligned} \mathbf{\Pi}_i &= -\left(I - \sum_{j=1}^{i} \mathbf{A}_j \right), \\ \mathbf{\Pi} &= -\left(I - \sum_{i=1}^{n} \mathbf{A}_i \right). \end{aligned}$$

Clearly the long-run properties of the system are described by the properties of the matrix Π. There are three cases of interest:

1. rank $(\Pi) = 0$. The system is non-stationary, with no cointegration between the variables considered. This is the only case in which non-stationarity is correctly removed simply by taking the first differences of the variables;

2. rank $(\Pi) =$ m, full. The system is stationary;

3. rank (Π) $=$ k $<$ m. The system is non-stationary but there are k cointegrating relationships among the considered variables. In this case $\mathbf{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}'$, where α is an (m $\times$ k) matrix of weights and $\boldsymbol{\beta}$ is an (k $\times$ m) matrix of parameters determining the cointegrating relationships.

Therefore, the rank of Π is crucial in determining the number of cointegrating vectors. The Johansen procedure is based on the fact that the rank of a matrix equals the number

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of its characteristic roots that differ from zero. The Johansen test for cointegration is based on the estimates of the two characteristic roots of $\mathbf{\Pi}$ matrix. Having obtained estimates for the parameters in the $\mathbf{\Pi}$ matrix, we associate with them estimates for the m characteristic roots and we order them as follows $\lambda_1 > \lambda_2 > ...\lambda_m$. If the variables are not cointegrated, then the rank of Π is zero and all the characteristic roots equal zero. In this case each of the expression $\ln(1-\lambda_i)$ equals zero, too. If, instead, the rank of Π is one, and $0 < \lambda_1 < 1$, then $\ln(1-\lambda_1)$ is negative and $\ln(1-\lambda_2) = \ln(1-\lambda_3) = ... = \ln(1-\lambda_m) = 0$. The Johansen test for cointegration in our bivariate VAR is based on the two following statistics that Johansen derives based on the number of characteristic roots that are different from zero:

$$\begin{aligned} \lambda_{\text{trace}}(\mathrm{k}) &= -\mathrm{T}\sum_{i=k+1}^{m} \ln\left(1-\widehat{\lambda}_i\right), \\ \lambda_{\max}(\mathrm{k}, \mathrm{k}+1) &= -\mathrm{T}\ln\left(1-\widehat{\lambda}_{k+1}\right), \end{aligned}$$

where T is the number of observations used to estimate the VAR. The first statistic tests the null of at most k cointegrating vectors against a generic alternative. The test should be run in sequence starting from the null of at most zero cointegrating vectors up to the case of at most m cointegrating vectors. The second statistic tests the null of at most k cointegrating vectors against the alternative of at most k $+1$ cointegrating vectors. Both statistics are small under the null hypothesis. Critical values are tabulated by Johansen (1991) and they depend on the number of non-stationary components under the null and on the specification of the deterministic component of the VAR.

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B APPENDIX B

Robustness Analysis for the Cointegrating Evidence

To assess the robustness of our cointegrating relationship in identifying the low frequency relation between stock market and demographics, we evaluate the effect of augmenting our baseline relation with an alternative demographic factor. Research in demography has recently concentrated on the economic impact of the *demographic dividend* (Bloom et al., 2003; Mason&Lee, 2005). The demographic dividend depends on a peculiar period in the demographic transition phase of modern population in which the lack of synchronicity between the decline in fertility and the decline in mortality typical of advanced economies has an impact on the age structure of population. In particular a high support ratio is generated, i.e. a high ratio between the share of the population in working age and the share of population economically dependent. Empirical evidence has shown that the explicit consideration of the fluctuations in the support ratio delivers significant results in explaining economic performance (see Bloom et al., 2003). The concept of *Support Ratio (SR)* has been precisely defined by Mason and Lee (2005) as the ratio between the number of effective number of producers, L_t, over the effective number of consumers, N_t (Mason&Lee, 2005). In practice we adopt the following empirical proxy:

$$\mathrm{SR} = \mathrm{a2064}/(\mathrm{a019} + \mathrm{a65ov})$$

where $\mathrm{a2064}$: Share of population between age 20-64, $\mathrm{a019}$: Share of population between age 0-19, a65ov: Share of population age 65+[9].

SR did not attract a significant coefficient when we augmented our cointegrating specification with this variable.

[9] We have checked robustness of our results by shifting the upper limit of the producers to the age of 75. This is consistent with the evidence on the cross-sectional age-wealth profile from Survey of Consumer Finances, provided in Table 1 of Poterba (2001), which shows that the population share between 64-74 still holds considerable amount of common stocks. Results are available upon request.

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C APPENDIX C

Description of all Time-series used in our Empirical Investigation

Stock Market Prices: S&P 500 index yearly prices from 1909 to 2008 are from Robert Shiller's website, we take december observations.

Stock Market Dividends: Dividends are twelve-month moving sums of dividends paid on the S&P 500 index. They are from the Robert Shiller website for the period 1900-2008.

These series coincide with those used in Goyal and Welch (2008), and made available at http://www.bus.emory.edu/AGoyal/Research.html.

Stock Market Returns: For S&P 500 index, to construct the continuously compounded return r_t, we take the ex-dividend-price P_t add dividend D_t over P_{t-1} and take the natural logarithm of the ratio.

Risk-free Rate: We download secondary market 3-Month Treasury Bill rate from St.Louis (FRED) from 1934-2008. The risk-free rate for the period 1920 to 1933 is from New York City from NBER's Macrohistory data base. Since there was no risk-free short-term debt prior to the 1920's, we estimate it following Goyal and Welch (2008). We obtain commercial paper rates for New York City from NBER's Macrohistory data base. These are available for the period 1871 to 1970. We estimate a regression for the period 1920 to 1971, which yielded

$$\text{T} - \text{billRate} = -0.004 + 0.886 \times \text{ Commercial Paper Rate.}$$

Therefore, we instrument the risk-free rate for the period 1909 to 1919 with the predicted regression equation.

Hence we build our dependent variable which is the equity premium ($r_{m.t} - r_{f,t}$), i.e., the rate of return on the stock market minus the prevailing short-term interest rate in the year t -1 to t.

Second, we construct the independent variables commonly used in the long horizon

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stock market prediction literature; namely

Log Dividend-Price Ratio (dp_t): the difference between the log of dividends and the log of prices.

Consumption, wealth, income ratio (cay): The series is taken from Lettau and Ludvigson (2001). Data are available from Martin Lettau's website at annual frequency from 1948 to 2001.

Consumption, dividend, income ratio (cdy): The series is taken from Lettau and Ludvigson (2005). Data are available from Martin Lettau's website at annual frequency from 1948 to 2001.

Demographic Variables

The U.S annual population estimates series are collected from U.S Census Bureau and the sample covers estimates from 1900-2050.

DATA SOURCES

Amit Goyal's Website: http://www.bus.emory.edu/AGoyal/Research.html

Martin Lettau's Website: http://faculty.haas.berkeley.edu/lettau/

Andrew Mason's Website: http://www2.hawaii.edu/~amason/

Michael R. Roberts' Website: http://finance.wharton.upenn.edu/~mrrobert/

Robert Shiller's Website: http://www.econ.yale.edu/~shiller/

Bureau of Labor Statistics Webpage: http://www.bls.gov/data/

FRED: http://research.stlouisfed.org/fred2/

NBER Macrohistory Data Base:

http://www.nber.org/databases/macrohistory/contents/chapter13.html.

US Census Bureau: http://www.census.gov/popest/archives/

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2 The Behavior of Interest Rates and Demographics

Introduction

The behavior of interest rates (Fama, 2006) reveals that spot rates are the sum of two processes, (i) a very persistent long term expected value and (ii) a mean-reverting component. In this chapter[10] we relate the very persistent component of interest rates to a specific demographic variable, i.e. the proportion of middle-aged to young population. The presence of the very persistent long-term expected value has important implications for affine term structure models in which factors driving the term structure are modelled via stationary VAR representations. In fact, the stationary VAR representations cannot capture the persistent component in the term structure and such omission might explain the unsatisfactory forecasting performance of affine term structure models noted in the literature (see, Duffee (2002), Moench (2008)). Using MY_t to model the persistent component of interest rates is particularly useful for forecasting the term structure as the demographic variable is exogenous and highly predictable even for very long-horizons (the Bureau of Census currently publishes on its website projections for the age structure of the population with a forecasting horizon up to fifty years ahead).

Figure 1 provides some prima-facie visual evidence on our empirical approach. The Figure shows, for US post-war data, the relationship between long-term interest rates, short-term interest rates, mid-term inflation (3-year) and the demographic variable, MY_t, defined as the ratio of middle-aged (40-49) to young (20-29) population. Demographic trends are a slow-moving information variable, whose forecasting power for interest rates is low at high frequency but becomes high at low frequencies, when the effect of the fluctuations of stationary component of yield curve fluctuations subsides. Intuitive reasoning and formal modeling hints at demography as an important

[10] Written jointly with Carlo A. Favero and Haoxi Yang.

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variable to determine the long-run behavior of the financial markets. In fact, abundant evidence is available on the impact of the demographic structure of the population on long-run stock-market returns (Ang and Maddaloni (2005), Bakshi and Chen (1994), Goyal (2004), Della Vigna and Pollet (2007) and Favero, Gozluklu and Tamoni (2010)). To our knowledge the empirical study on the empirical relation between the bond market and demographics is much more limited (the only study we are aware of is McMillan, H. and J. B. Baesel, 1988), despite the strong interest for co-movements between the stock and the bond markets (Lander et al.(1997), Campbell and Vuoltenaho (2004), Bekaert and Engstrom (2010)). This chapter fills this gap by presenting evidence on the relation between demographics and the persistent component of the yield curve and on the importance of using this information to de-trend the term structure before producing VAR-based forecasts.

The rest of this chapter is organized as follows. In the next section we place our contribution in the literature and discuss the rationale for the relationship between the demographic variable MY_t and the persistent component in interest rates. The dataset and the results in Fama (2006) are then reconsidered to document how MY_t captures the long-run component identified by Fama in his analysis of the one-year spot rate. The evidence naturally leads to pursue the consequences of the permanent-transitory decomposition for forecasting the term structure. In particular, we propose a no-arbitrage affine term structure model that allows for the presence of a low-frequency component driven by demographics. The forecasting performance of our model is then examined and compared with that of a traditional affine term structure model with macroeconomic factors (e.g. Ang, Dong and Piazzesi, 2005). We then devote a section to exploit the peculiar feature of an affine term structure with exogenous demographic factors to produce long-term forecast for the yield curve up to 2040. The last section concludes.

Related Literature

This chapter brings together three different strands of the literature: i) the one analyzing the implication of the presence of a persistent component for spot rates predictability, ii) the affine no-arbitrage term structure models with observable macro factors and latent variables and iii) the one linking demographic fluctuations with asset prices.

The literature on spot rates predictability has emerged from a view in which forecastability is determined by the slowly mean-reverting nature of the relevant process. Recently, it moved to a consensus that modelling a persistent component is a necessary

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requirement for a good predictive performance.

Traditionally, yield curve modelling in finance is governed by parsimony principle; all the relevant information to price bonds at any given point in time is summarized by a small number of stationary factors (Litterman and Scheinkman, 1991). Both macro and finance term structure models agree on the role of at least two factors, i.e. the level and the slope, that capture a slow mean-reverting component and a more rapid (business-cycle-length) mean-reverting component. Besides providing a structural interpretation, this literature also documents the role of forward interest rates in forecasting future spot rates for longer horizons (e.g. Fama and Bliss, 1979). Particularly linear combinations of forward rates are successful in predicting term premia (Cochrane and Piazzesi, 2005). Early literature attributes this predictability to the mean reversion of the spot rate toward a constant expected value. This view has been recently challenged; the predictability of the spot rate captured by forward rates is either attributed to a slowly moving, yet still stationary, mean (Balduzzi, Das, and Foresi, 1998) or a time-varying very persistent long-term expected value (Fama, 2006). Following Hamilton (1988), other papers estimate regime-switching models for the spot rate (for example, Gray (1996), Ang and Bekaert (2002)) to capture the time variation of the long run mean.

The importance of such a persistent component for modelling and forecasting the term structure is our main focus. The existence of such a component raises two immediate issues. First, the validity of all forecasting models based on a stationary representation of factors is not warranted. Second, some investigation is needed on the determinants of persistence.

The affine no-arbitrage term structure models with observable factors are commonly based on a stationary representation of factors, where the following specification is generally adopted:

$$y_{t,t+n} = \frac{-1}{n}\left(A_n + B_n' X_t\right) + \varepsilon_{t,t+n} \qquad \varepsilon_{t,t+n} \sim i.i.d.N(0,\sigma_n^2) \quad (2.1)$$

$$X_t = \mu + \Phi X_{t-1} + v_t \qquad v_t \sim i.i.d.N(0,\Omega) \quad (2.2)$$

where $y_{t,t+n}$ denotes the yield at time t of a zero-coupon government bond maturing at time $t+n$, bonds are priced via a vector X_t of stationary state variables, that can be observable or unobserved. The no-arbitrage assumption imposes the following structure on the coefficients of the measurement equation (for $n \geq 1$):

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$$A_{n+1} = A_n + B_n' \left(\mu - \Omega\lambda_0\right) + \tfrac{1}{2} B_n' \Omega B_n + A_1$$
$$B_{n+1}' = B_n' \left(\Phi - \Omega\lambda_1\right) + B_1'$$

The potential problem with this general structure is that while yields contain a persistent component, the state evolves as a stationary VAR which, independently from imposing the no-arbitrage restrictions, is designed to model a mean-reverting process and cannot capture the time series behavior of persistent variables.

In fact, several papers indicate that macroeconomic variables have strong effects on future movements of the yield curve (among others, Ang and Piazzesi (2003), Diebold, Rudebusch, and Aruoba (2005) and Rudebusch and Wu (2008)). Ang and Piazzesi (2003) show that a mixed factor model (with three latent financial factors plus output and inflation) performs better than a yields-only model in terms of one step ahead forecast at quarterly frequency. Others (e.g. Bekaert, Cho, and Moreno (2003), Gallmeyer, Hollifield, and Zin (2005), Rudebusch and Wu (2008), and Hordahl, Tristani, and Vestin (2006)) estimate structural models with interest rates and macro variables. Ang, Dong and Piazzesi (2005), and Dewachter and Lyrio (2006), investigate how no-arbitrage restrictions can help estimate different policy rules. In all these macro-finance models the process determining the macro variables is taken to be stationary. This assumption is not consistent with the presence of a low-frequency component in determining the behavior of interest rates and might therefore explain the, somewhat disappointingly, mixed results from the forecasting performance of affine term structure models (Duffee (2002), Favero, Niu and Sala (2010)). Forecasting interest rates in the presence of a highly persistent component in rates requires the existence of a factor capable of modelling the persistence. Inflation is an obvious candidate as far as the persistence is concerned, but it is an endogenous factor, since it is influenced by monetary policy, and its turning points are very difficult to predict.

Any factor built on the basis of demographic information is instead exogenous and predictable, given the nature of demographic variable. The interesting question here is why should demographics influence the behavior of interest rates and why the relevant effect of demographics can be captured by the relative size of the middle aged to young population.

Fluctuations in nominal interest rates must be due to either movements in real interest rates, expected inflation, or the inflation risk premium. There is available literature relating demographic factors to the first two components.

Geanakoplos, Magill and Quinzii (2004, henceforth, GMQ) consider an overlap-

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ping generation model in which the demographic structure mimics the pattern of live births in the U.S., that have featured alternating twenty-year periods of boom and busts. They conjecture that the life-cycle portfolio behavior (Bakshi and Chen, 1994), which suggests that agents should borrow when young, invest for retirement when middle-aged, and live off their investment once they are retired, plays an important role in determining equilibrium asset prices. Consumption smoothing by the agents, given the assumed demographic structure requires that when the MY_t ratio is small (large), there will be excess demand for consumption (saving) by a large cohort of retirees (middle-aged) and for the market to clear, equilibrium prices of financial assets should adjust, i.e. decrease (increase), so that saving (consumption) is encouraged for the middle-aged (young). The model predicts that the price of all financial assets should be positively related to MY_t and it therefore also predicts the negative correlations between yields and MY_t exhibited in Figure 1. In the first chapter we have illustrated how the persistent component in the dividend/price ratio is captured by MY_t. In the GMQ model bond and stock are perfect substitutes, therefore the evaluation of the performance of MY_t in determining the persistent component of interest rates is a natural step within this framework[11]. The GMQ model provides an explicit foundation for the relationship between MY_t and real interest rates. In a separate strand of literature, Lindh and Malberg (2000) investigate the hypothesis that inflation pressures covary with the age distribution unless accommodated by monetary policy. The results of the estimation of a relation between inflation and age structure on annual OECD data covering 1960–1994 for 20 countries suggest the existence of an age pattern of inflation effects. It is consistent with the hypothesis that increases in the population of net savers dampen inflation, whereas especially the younger retirees fan inflation as they start consuming out of accumulated pension claims. This confirms the positive correlation between the young-to-middle aged ratio and inflation illustrated in Figure 1, which is implied by the life-cycle saving behavior (Bakshi and Chen, 1994). The importance of demographic projections is also documented in the last chapter, where the role of time variation in the age structure in shaping the long-run co-movement between inflation and financial markets is analyzed.

We merge these three strands of the literature by using MY_t to model the persistent component in spot rates and by then proposing a no-arbitrage affine term structure model capable of accommodating a long run component in the term structure via the low frequency time-series properties of the age structure of population.

[11] Note that the existence of a relation between the dividend price ratio and MY implies a similar relation between MY and interests rates also when the perfect substitutability assumption is relaxed by maintaining the hypothesis that the relative risk premium on stock and bonds is stationary. In fact a stationary relative risk term premium cannot determine the persistent components of interest rates and the dividend/price ratio.

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Demographics and the Permanent Component of Spot Rates

Fama (2006) explains the evidence that forward rates forecast future spot rates in terms of a mean reversion of spot rates towards a non-stationary long-term mean which is subject to sequence of permanent shocks. It is observed that these shocks are on balance positive from the beginning of the fifties to mid 1981 and negative afterwards. Hence Fama proposes a decomposition of the spot interest rates, $y_{t,t+n}$[12] in two processes: a long term expected value $K_{t,t+n}$, that is subject to permanent shocks, and a mean reverting component $X_{t,t+n}$.

$$y_{t,t+n} = K_{t,t+n} + X_{t,t+n}$$

On the basis of this decomposition Fama addresses the question of predicting changes in the one-year spot rate at different horizons (from 1-year to 4-year ahead) using monthly data. The predictive model is designed to evaluate the forecasting ability of the spread between forward and spot rates and the deviations of the spot rates from its long term expected value.

The following model is estimated on a sample of monthly data 1958:6-1984:12:

$$y_{t+12x,t+12x+12} - y_{t,t+12} = a^x + b^x D_t + c^x[f_{t,t+12x,t+12x+12} - y_{t,t+12}] + d^x[y_{t,t+12} - K_{t,t+12}] + \varepsilon_{t+12x}$$

$$K_{t,t+12} = \frac{1}{60}\sum_{i=1}^{60} y_{t-i-1,t+12-i-1} \qquad (2.3)$$

where $f_{t,t+12x,t+12x+12}$ is the one-year forward rate at time t of an investment with settlement after x years and maturity in $x+1$ years, $y_{t,t+12}$ is the one-year spot interest rate and $K_{t,t+12}$ is measured as the moving average of the most recent past five years of the spot rates.

Note that the model also includes a dummy variable D_t that is equal to one for the first part of the sample up to August 1981 and zero otherwise. This variable captures the turning point in the behavior of interest rates from a positive upward trend to a negative

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12 We adopt Cochrane and Piazzesi (2005) notation for log bond prices:

$$p_{t,t+n} = \log \text{ price of n-year discount bond at time t}$$

where the parenthesis in the superscript refers to time to maturity. Then the continuously compunded spot rate is

$$y_{t,t+n} \equiv -\frac{1}{n} p_{t,t+n}$$

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upward trend occurred in mid 1981 and clearly detectable from Figure 1.[13]

A model with the restriction $d^x = 0$ is first estimated to obtain a positive and significant estimate of c^x with a significance increasing with the horizon x. However, when the restriction $d^x = 0$ is relaxed, then the null hypothesis that c^x is not statistically different from zero cannot be rejected. d^x is estimated significantly negative increasing with the horizon. All results are crucially affected by the inclusion of the dummy, in that when the b^x is restricted to zero the predictive power of the regression is dramatically reduced. In the light of these results Fama concludes that there is evidence of mean reversion of the spot rates toward a time varying expected value and that the predictability of the spot rate captured by the forward rate is exclusively related to this phenomenon.

In Table 1 below we replicate the empirical results in Fama (2006) both using the same sample with monthly observations and by considering quarterly data over a sample extended to the last quarter of 2008[14].

Our results fully replicate those reported by Fama (2006) on monthly data and are robust when the sample is extended to 2008 and quarterly data are considered.

We then use the Fama (2006) setup to assess the importance of the demographic variable MY_t in capturing the long term expected value $K_{t,t+n}$. We consider the following alternative specification to (2.3):

$$y_{t+12x,t+12x+12} - y_{t,t+12} = a^x + b^x D_t + c^x [f_{t,t+12x,t+12x+12} - y_{t,t+12}] + d^x [y_{t,t+12} - K_{t,t+12}] + \varepsilon_{t+12x}$$

$$K_{t,t+12} = e^x MY_t \tag{2.4}$$

The results from the estimation, reported in Panel B of Table 1, show that the coefficient on MY_t is strongly significant in all regressions at the different frequencies and horizons. The performance of the forecasting model improves as the forecasting horizon increases (with an R^2 going from 0.24 at the one-year horizon to 0.64 at the four-year horizon). Importantly, when $K_{t,t+12}$ is modelled via demographics the coefficients on the dummy variable capturing the turning points in the underlying trend is not significant anymore, witnessing the capability of demographics of capturing the change in the underlying trend affecting spot rates.

[13] A possible economic rationale for this turning point is introduced with reference to the experience of Federal Reserve introducing a fiduciary currency in 1971 that created permanently high inflation expectations up to mid-1981, until Federal Reserve gained enough experience with the new system and learned how to manage inflation using a fiduciary currency.

[14] 1-year Treasury bond yields are computed from the (monthly and quarterly) price series from the Fama CRSP zero coupon files. Middle-young ratio data is available at annual frequencies from Bureau of Census (BoC) and it has been interpolated to obtain monthly and quarterly series.

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To further test the forecasting performance of the model, we conduct a (pseudo) out-of sample forecasting comparison of the models (2.3) and (2.4). We generate out-of-sample forecasts for the period 1994Q1-2008Q4, based on a model estimated with a rolling window of 140 observations.

The results are reported in Table 2. The first column reports the out-of-sample R^2 statistics (Campbell and Thomson, 2008) which is computed as

$$R^2_{OS,x} = 1 - \frac{\sum_{t=t_0}^{T}\left[(y_{t+12x,t+12x+12} - y_{t,t+12}) - (y_{t+12x,t+12x+12} - y_{t,t+12})^{pr}\right]^2}{\sum_{t=t_0}^{T}\left[(y_{t+12x,t+12x+12} - y_{t,t+12}) - (y_{t+12x,t+12x+12} - y_{t,t+12})^{av}\right]^2}$$

where $(y_{t+12x,t+12x+12} - y_{t,t+12})^{pr}$ are the model-based projections at horizon x and $(y_{t+12x,t+12x+12} - y_{t,t+12})^{av}$ is the historical average change computed with data available at the time of forecasting. In our exercise, $t_0 = 1994$ and $T = 2008$. If R^2_{OS} is positive, it means that the predictive regression has a lower mean square error than the prevailing historical mean. Then we report mean absolute error (MAE) and root mean square error (RMSE) calculated based on the residuals in the forecasting period, namely 1994-2008. In the last column, we report the Diebold-Mariano (DM) t-test for checking equal-forecast accuracy from two nested models for forecasting h-step ahead spot rate changes.

$$\mathrm{DM} = \sqrt{\frac{(\mathrm{T}+1-2*\mathrm{h}+\mathrm{h}*(\mathrm{h}-1))}{\mathrm{T}}} * \left[\frac{\bar{\mathrm{d}}}{\widehat{\mathrm{se}}(\bar{\mathrm{d}})}\right]$$

where we define e^2_{1t} as the squared forecasting error of prevailing mean, and e^2_{2t} as the squared forecasting error of the predictive variables, $d_t = e^2_{1t} - e^2_{2t}$, i.e. the difference between the two forecast errors, $\bar{d} = \frac{1}{T}\sum_{t=t_0}^{T} d_t$ and $\widehat{se}(\bar{d}) = \frac{1}{T}\sum_{\tau=-(h-1)}^{h-1}\sum_{t=|\tau|+1}^{T}(d_t - \bar{d}) * (d_{t-|\tau|} - \bar{d})$. A positive DM t-test statistic indicates that the predictive regression model performs better than the historical mean.

We note that the forecasting performance of the two alternative models is similar (although no dummy is included in the model with demographics) and that the relative performance of the model based on demographics improves with the forecasting horizon. We interpret these results as supportive of the view that the persistent component in the 1-year interest rates can be related to demographic factors that are therefore very useful predictors, especially, at long-horizons.

An affine No-Arbitrage Term Structure Model with Demographics

The evidence reported in the previous section supports the hypothesis of the existence of a permanent component in 1-year spot interest rates and its relation with the demographic variable MY_t. As discussed in the introduction, in affine term structure models

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stationarity of the state space is assumed to justify its VAR representation and to derive long-term VAR-based projections. Therefore, the presence of a permanent component in spot-rates poses an important econometric challenge to these models. Consider, for example, a typical affine model with macroeconomic factors in a data-rich environment (Bernanke and Boivin (2003), Ang, Dong and Piazzesi (2005), Ludvigson and Ng (2009)). The term structure is described as follows:

$$\begin{aligned} y_{t,t+n} &= \frac{-1}{n}\left(A_n + B_n' X_t\right) + \varepsilon_{t,t+n} & \varepsilon_{t,t+n} &\sim i.i.d.N(0,\sigma_n^2) \\ X_t &= \mu + \Phi X_{t-1} + v_t & v_t &\sim i.i.d.N(0,\Omega) \\ y_{t,t+1} &= \delta_0 + \boldsymbol{\delta}_1' X_t \end{aligned} \tag{2.5}$$

where $y_{t,t+n}$ denotes the yield at time t of a zero-coupon government bond maturing at time $t+n$, the vector of the states $X_t = [f_t^o,\, f_t^u]$, where f_t^o are two observable factors extracted from large-data sets to capture all the relevant "real" and inflation information which the Fed uses to set the monetary policy rate in a data-rich environment, while f_t^u contains unobservable factor(s) needed to capture the persistent component in interest rates. Note that the one-period yield is typically the monetary policy rate which is modelled as function of the relevant information in the central bank reaction function as in a Taylor-type rule (Ang, Dong and Piazzesi, 2005). We consider the two factors estimated by Ludvigson and Ng (2009) to capture "real" and inflation information. This model is completed by assuming a linear (affine) relation between the price of risk, Λ_t,and the states X_t by specifying the pricing kernel, m_{t+1}, consistently and by imposing no-arbitrage restrictions (see, for example, Duffie and Kan (1996), Ang and Piazzesi (2003)):

$$\begin{aligned} \Lambda_t &= \lambda_0 + \lambda_1 X_t \\ m_{t+1} &= \exp(-y_{t,t+1} - \tfrac{1}{2}\Lambda_t'\Omega\Lambda_t - \Lambda_t\varepsilon_{t+1}) \\ A_{n+1} &= A_n + B_n'\left(\mu - \Omega\lambda_0\right) + \tfrac{1}{2}B_n'\Omega B_n + A_1 \\ B_{n+1}' &= B_n'\left(\Phi - \Omega\lambda_1\right) + B_1' \end{aligned}$$

This structure requires a stationary representation of the states that makes the model "incongruent" in the case of the presence of a persistent component in spot rates. The "incogruency" comes from the fact that highly persistent time series are linearly related to a stationary process. Table 3, which illustrates the time series properties of spot rates with different maturity and the two Ludvigson and Ng factors, highlights the empirical relevance of this problem.

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The evidence on the time series properties of MY_t also confirms the high persistence and it is consistent with the evidence provided in the previous section of the potential of this demographic factor for modelling the permanent component in spot rates. In light of this evidence, we propose the following affine term structure model with demographics:

$$\begin{aligned} y_{t,t+n} &= -\frac{1}{n}\left(A_n + B_n' X_t + \Gamma_n \mathrm{MY}_t^n\right) + \varepsilon_{t,t+1} \qquad \varepsilon_{t,t+n} \sim N(0,\sigma_n^2) \\ X_t &= \mu + \Phi X_{t-1} + \nu_t \quad \nu_t \sim i.i.d.N(0,\Omega) \\ y_{t,t+1} &= \delta_0 + \boldsymbol{\delta}_1' X_t + \delta_2 \mathrm{MY}_t \end{aligned}$$

where $\Gamma_n = \left[\gamma_0^n, \gamma_1^n \cdots, \gamma_{n-1}^n\right]$, and $\mathrm{MY}_t^n = \left[\mathrm{MY}_t, \mathrm{MY}_{t+1} \cdots, \mathrm{MY}_{t+n-1}\right]'$ and $X_t = [f_t^o, f_t^u]$.

By using the traditional relation between the price of risk and the states and by imposing the no-arbitrage restrictions we obtain (see appendix)

$$\begin{aligned} A_{n+1} &= A_1 + A_n + B_n'(\mu - \Omega\lambda_0) + \frac{1}{2}B_n'\Omega B_n \\ B_{n+1} &= B_n'\Phi - B_n'\Omega\lambda_1 + B_1' \\ \Gamma_{n+1} &= [-\delta_2, \Gamma_n] \end{aligned}$$

As we include the demographic variable in the process for monetary policy rates, yields with maturity $t+n$ are function of the demographic variable between time t and time $t+n$ as they reflect the expected path of future monetary policy. MY_t is included into the model as an exogenous variable, its high degree of predictability (the Bureau of Census posts on its website projections for the composition of population with a fifty year-ahead horizon) allows to model empirically the relation between $y_{t,t+n}$ and MY_t^n without specifying a model to predict future values of MY_t.

We propose a model where the demographic variable MY_t enters linearly in the monetary policy rule, to capture the permanent component in spot rates. The price of risk is independent from the demographic variable and it is affine in the vector of state factors. The empirical evidence discussed in the previous section points to demographics as an observable factor capable of capturing the permanent component of 1-year spot rates. This evidence can be extended to the 3-month rates (the rate with the shortest maturity in the sample of quarterly data we analyze in this section). We report the results in Table 4 the results. The estimated coefficient on MY_t is always significant

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with stable loadings.

The no-arbitrage restrictions, appropriately modified from the baseline case with no demographics, are imposed on the system (see Appendix for a full derivation). The entire term structure is affected by the permanent component of spot rates as long-term spot rates are affected by expected monetary policy rates. Note that in our specification we do not make the price of risk function of the demographic variable consistently with the idea that risk premia are stationary and cannot explain the presence of a permanent component in the term structure. However, as the entire term structure depends on future expected monetary policy, then the future values of the demographic variable affect yields at all maturities.

Model Specification and Estimation

We consider as a benchmark model the discrete-time no-arbitrage term structure model with both observable and unobservable variables, first suggested by Ang and Piazzesi (2003). Following the specification analysis of Pericoli and Taboga (2008), we focus on a parsimonious model including three latent factors and only contemporaneous values of the macro variables. We use the Chen and Scott's (1993) methodology; given the set of parameters and observed yields latent variables are extracted by assuming that number of bonds which are priced exactly is equal to the number of unobserved variables. Hence we assume that 3-month, 2-year and 5-year bond prices are measured without error and estimate the model with maximum likelihood. We assume the state dynamics to follow a VAR(1). We do not impose any restriction on the VAR coefficient matrix, while we make sure that the following conditions are met in our estimation (Favero, Niu and Sala, 2010):

i) the covariance matrix Ω is block diagonal with the block corresponding to the unobservable yield factor being identity, and the block corresponding to the observable factors being unrestricted, i.e.

$$\Omega = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \Omega^o \end{bmatrix}$$

ii) the loadings on the factors in the short rate equation are positive, $0 \leq -A_1$

iii) $X_0^u = 0$.

Despite its heavier parametrization, this specification allows interaction between yield factors and macro factors without sacrificing forecasting power (Favero, Niu and

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Sala, 2010).

Out-of Sample Forecasts

An interesting feature of MY_t is that long-run forecasts for this variable are readily available. In fact, the Bureau of Census (BoC) provide projections up to 2050 for MY_t. Therefore we focus on the properties of out-of-sample forecasts of our model at different horizons. In our multi-period ahead forecast, we choose iterated forecast procedure, where multiple step ahead states are obtained by iterating the one-step model forward

$$\begin{aligned}\widehat{y}_{t+h,t+h+n|t} &= a_n + b_n \hat{X}_{t+h|t} + \Gamma_n \mathrm{MY}^n_{t+h} \\ \hat{X}_{t+h|t} &= \sum_{i=0}^{h} \hat{\Phi}^i \mu + \hat{\Phi}^h \hat{X}_t\end{aligned}$$

where $a_n = -\frac{1}{n} A_n, b_n = -\frac{1}{n} B_n$ are obtained by no-arbitrage restrictions. Forecast are produced on the basis of rolling estimation with a rolling window of one hundred observation, the first sample used for estimation is 1964Q1-1988Q4. We consider 6 forecasting horizons (denoted by h): one quarter, one year, two years, three years, four years and five years. For the one quarter ahead forecasting horizon, we conduct our exercise for all dates in the period 1989Q1 - 2007Q4, a total of 76 periods; for the 1-year ahead forecast, we end up with a total of 73 forecasts, and so on, up to the 5-year ahead forecast, for which we end up with 57 forecasts.

Forecasting performance is measured by the ratio of the root mean squared forecast error (RMSFE) of each the affine model with demographics to the RMSFE of a random walk forecast and to the RMSFE of the benchmark yield-macro model without the demographic variable. Forecasting results from different models are reported in Table 6 and Table 7. Bold characters are used when the ratio of the model's RMSFE to that of the random walk is in the range [0.9, 1) while bold and underline characters are used for rations smaller than 0.9. Two specifications for risk prices are estimated

i) Constant prices of risk: $\lambda_0 \neq 0, \lambda_1 = 0$.(Table 6)

ii) Time-varying prices of risk, $\lambda_1 \neq 0, \lambda_1$ diagonal. (Table 7)

Table 6 reports the RMSFE of the model with demographics relative to the random walk and benchmark yield-macro model in the case of constant risk pricing ($\lambda_1 = 0$). The evidence shows clearly that the inclusion of the demographic variable in the model

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improves the forecasting performance for the entire term structure as the forecasting horizon gets larger. The results become much stronger when time-varying risk prices are allowed.

The intuition behind the superior forecasting performance of the model augmented with demographics is illustrated in Figure 4 , where we report the actual time-series for different yields along with the dynamically simulated values for the same yields based on the benchmark affine model with macro factors and the model augmented with demographics. Figure 4 reports the results of dynamic simulation from the beginning of the sample given estimation over the full-sample. The dynamic simulation of the benchmark model converges rather rapidly to the unconditional mean and it is not capable of capturing the persistent component in the term structure. The picture is very different for the simulated values from the model with demographics where the middle-to-young ratio allows keeps the simulated value remarkably close to the long-run trend of the term structure.

Long-Term Projections

One of the appealing features of an affine term structure model with demographics factors is that the availability of long-term projections for the age-structure of the population which can be exploited to produce long-term projections for the yield curve. In our specification yields at time $t+j$ with maturities $t+j+n$ are functions of all realization of MY between $t+j$ and $t+j+n$. The exogeneity of the demographic variable and the availability of long term projections is combined in the affine model with a parsimonious parameterization generated by the no-arbitrage restrictions that allow to weight properly all future values of MY with the estimation of few coefficients. As a result future paths up to 2045 can be generated for the entire term structure, given the availability of demographic projections up to 2050 (the Bureau of Census websites provides projections for demographics variable up to 2050 and the current 5-year yield depends on the values of MY over the next five years). Note that an affine model with demographics allow for a very different out-of-sample dynamics from those that can be generated by a Fama (2006) type model based on the decomposition of interest rates in long-term expected mean and mean-reverting component and on the measurement of the long-term expected mean as the moving average of the most recent past five years of the spot rates. In fact, long-term projections from the Fama model would not differ from those of an autoregressive process, they would just converge more slowly to the unconditional mean. To illustrate this feature of the model we report in Figure 5 all yields reported in Figure

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2 by also drawing their predicted out-of-sample path up to 2045 .

The predicted path of the age structure of the population drives the forecast of the term structure up in the range between six per cent and eight per cent over the next twenty years. An affine model with stationary macro factors estimated over the last twenty years could never produce such forecasts, as they are clearly far away from the unconditional sample mean.

Conclusions

There is a general agreement on the existence of a common persistent component in the entire term structure of interest rates. In this chapter we have shown that it is possible to model such a persistent component using a demographic variable, to ratio of middle aged to young population, MY_t. We have first provided evidence that MY_t is a better measure of the persistent component of the 1-year rate than the moving-average of the most recent past five years of the spot rates used by Fama (2006). We have then explored the implication of this evidence for the affine model of the term structure of interest rates. Affine models are built by assuming that the relevant state variable to price bonds evolve as a stationary VAR. The problem arises form the fact that a stationary VAR, independently from imposing the no-arbitrage restrictions, is designed to model a mean-reverting process and cannot capture the time series behavior of persistent variables. We propose to include MY_t in affine model by considering it as a determinant of the trend in the short-term rate. As the entire term structure is function of the short-term rate this allows to have a common persistent component in all yields. Importantly this happens without changing the traditional model for the price of risk, where no persistent component is introduced. Our results show that the forecasting performance of an affine model with demographics uniformly outperforms that of traditional affine models with stationary factors. We believe that our evidence is important both for long-term projections of the yield curve and for modelling its fluctuations around the persistent component.

To illustrate this point we have produced model-based forecast for the term structure up to 2045. The demographics bases forecast of the yield curve over the next twenty year are significantly higher then the sample mean observed over the course of the last twenty years.

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Tables

TABLE 1. Regressions to explain one-year to four-year changes in the spot rate

Predictive Model: $y_{t+px,t+px+p} - y_{t,t+p} = a^x + b^x D_t + c^x [f_{t,t+px,t+px+p} - y_{t,t+p}] + d^x [y_{t,t+p} - K_{t,t+p}] + \varepsilon_{t+px}$

Panel A. Fama's Model

$$K_{t,t+p} = \frac{1}{5p} \sum_{i=1}^{5p} y_{t-i-1,t+p-i-1}$$, p=12(4), if monthly (quarterly) data

	a^x (t–stat)	b^x (t–stat)	c^x (t–stat)	d^x (t–stat)	R^2	a^x (t–stat)	b^x (t–stat)	c^x (t–stat)	d^x (t–stat)	R^2
Exclude Dummy	June,1958-December 2004					1958Q2-2008Q4				
$x = 2$	0.178		−0.460	−0.283	0.05	0.156		−0.380	−0.286	0.05
	(0.68)		(−1.18)	(−1.52)		(0.57)		(−0.98)	(−1.82)	
$x = 3$	0.380		−0.551	−0.583	0.09	0.437		−0.561	−0.644	0.12
	(0.63)		(−1.27)	(−3.27)		(0.78)		(−1.32)	(−4.09)	
$x = 4$	−0.172		0.133	−0.385	0.10	−0.180		0.218	−0.426	0.14
	(−0.25)		(0.29)	(−1.80)		(−0.26)		(0.54)	(−2.41)	
$x = 5$	−0.506		0.584	−0.205	0.12	−0.401		0.484	−0.282	0.15
	(−0.84)		(1.11)	(−0.64)		(−0.66)		(0.95)	(−0.90)	
Include Dummy	June,1958-December 2004					1958Q2-2008Q4				
$x = 2$	−0.743	1.576	−0.216	−0.417	0.21	−0.574	1.405	−0.216	−0.398	0.18
	(−1.90)	(3.07)	(−0.65)	(−2.86)		(−1.54)	(2.91)	(−0.65)	(−2.96)	
$x = 3$	−1.207	2.808	−0.391	−0.859	0.36	−0.886	2.479	−0.393	−0.854	0.34
	(−1.82)	(3.43)	(−0.97)	(3.45)		(−1.44)	(3.61)	(−1.04)	(−6.68)	
$x = 4$	−2.519	3.969	0.413	−0.716	0.51	−2.289	3.814	0.439	−0.787	0.50
	(−3.33)	(4.29)	(1.11)	(−2.73)		(−3.79)	(4.94)	(1.49)	(−5.57)	
$x = 5$	−3.076	4.730	0.632	−0.729	0.62	−3.047	4.746	0.714	−0.752	0.62
	(−4.51)	(5.40)	(1.86)	(−2.63)		(−5.85)	(5.47)	(2.75)	(−4.35)	

Panel B. Demographic Model

$K_{t,t+p} = e^x \mathrm{MY}_t$

	a^x (t–stat)	b^x (t–stat)	c^x (t–stat)	d^x (t–stat)	e^x (t–stat)	R^2	a^x (t–stat)	b^x (t–stat)	c^x (t–stat)	d^x (t–stat)	e^x (t–stat)	R^2
	June,1958-December 2004						1958Q2-2008Q4					
$x = 2$	7.388		−0.335	−0.475	0.109	0.25	7.576		−0.255	−0.493	0.109	0.24
	(4.08)		(−1.39)	(−4.88)	(3.23)		(4.37)		(−1.00)	(−5.51)	(3.41)	
$x = 3$	14.261		−0.440	−0.938	0.106	0.47	14.617		−0.375	−0.974	0.104	0.47
	(4.63)		(−1.26)	(−6.43)	(3.83)		(4.73)		(−1.11)	(−6.42)	(3.97)	
$x = 4$	16.947		−0.142	−1.117	0.108	0.56	17.004		−0.003	−1.138	0.106	0.58
	(4.51)		(−0.53)	(−5.99)	(3.93)		(4.61)		(−0.01)	(−5.83)	(4.12)	
$x = 5$	19.375		−0.134	−1.295	0.106	0.64	19.539		−0.127	1.320	0.103	0.64
	(4.79)		(−0.70)	(−6.39)	(4.17)		(4.94)		(−0.69)	(−6.50)	(4.32)	

Table 1. Fama Regressions. r(t) is the one-year spot rate observed at time t. f(x:t) is the forward rate observed at t for the year from t+x-1 to t+x. D is a dummy variable that is 1 for June 1958 to August 1981. K(t) is the average value of the spot rate for the 60 months ending in month t-1. In the left panel we replicate Fama's results using the same monthly sample, June 1958- December 2004. The right panel updates the sample up to December 2008 using quarterly data. The standard errors of the regression coefficients are adjusted for autocorrelation due to overlap of monthly observations with the method of Hansen and Hodrick (1980. The t-statistics are reported in parentheses. The regression R^2 are adjusted for degrees of freedom.

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TABLE 2. Predictive Performance (Pseudo) Out-of-sample 1994Q1-2008Q4

	FAMA				MY			
Panel A	Ros^2	MAE	$RMSE$	DM	Ros^2	MAE	$RMSE$	DM
$x=2$	−0.4383	1.6004	1.8853	−0.0242	−0.5623	1.6352	1.9649	−0.0490
$x=3$	−0.2019	2.1945	2.5085	−0.0017	**−0.1751**	2.2370	**2.4804**	**−0.0002**
$x=4$	−0.0531	2.4416	2.7362	0.0010	**0.1115**	**2.1563**	**2.5133**	**0.0024**
$x=5$	0.2303	1.9553	2.4541	0.0029	**0.3102**	**1.8226**	**2.3234**	**0.0035**
	MAE_{MY}/MAE_{FAMA}				$RMSE_{MY}/RMSE_{FAMA}$			
Panel B	$x=2$	$x=3$	$x=4$	$x=5$	$x=2$	$x=3$	$x=4$	$x=5$
	1.0217	1.0194	**0.8832**	**0.9321**	1.0422	**0.9888**	**0.9186**	**0.9467**

Table 2 presents statistics on x-year ahead out-of-sample forecast errors for changes in spot rates. The first column lists the out-of-sample R^2_{OS} which compares the forecast error of the historical mean with the forecast from predictive regressions. RMSE is the root mean square error, MAE is the mean absolute error. DM is the Diebold and Mariano (1995) t-statistic for difference in MSE of the unconditional forecast and the conditional forecast. The sample starts in 1958Q2 and we construct first forecast in 1994Q1.

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TABLE 3. Summary Statistics of the Data in Affine Term Structure Models							
	Central Moments				Autocorrelations		
	mean	Stdev	Skew	Kurt	Lag 1	Lag 2	Lag 3
3-month	5.850	2.778	1.061	4.775	0.917	0.866	0.843
1-year	6.282	2.748	0.817	4.107	0.925	0.881	0.841
2-year	6.483	2.698	0.853	3.980	0.935	0.891	0.852
3-year	6.643	2.615	0.876	3.915	0.941	0.899	0.860
4-year	6.774	2.571	0.928	3.950	0.945	0.904	0.865
5-year	6.851	2.521	0.910	3.680	0.949	0.910	0.875
inflation factor	0.116	0.922	2.149	16.463	0.337	0.100	0.170
real factor	−0.026	0.965	1.225	6.631	0.681	0.450	0.248
middle-young	0.836	0.211	0.102	1.500	0.995	0.989	0.981

Table 3. Summary Statistics of data. We report the mean, standard deviation, skewness and kurtosis together with the autocorrelation coefficients up to 3 lags. The 1, 4, 8, 12, 16, 20 quarter yields are annualized (in percentage) zero coupon bond yields from the Fama-Bliss CRSP bond files. Inflation and real activity refer to the price and output factors extracted from large dataset provided by Ludvigson and Ng (2009). Sample 1964Q1-2007Q4. Quarterly data.

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TABLE 4. The Short-rate and Demographic Variable

$y^{3m}_{t+4x,t+4x+4} - y^{3m}_{t,t+4} = a^x + b^x[y^{3m}_{t,t+4} - K_{t,t+4}] + \varepsilon_{t+4x}$, $K_{t,t+4} = c^x MY_t$

	a^x (t–stat)	b^x (t–stat)	c^x (t–stat)	R^2
Panel A. Sample 1958Q2-2008Q4				
$x=2$	6.451 (3.72)	−0.451 (−4.50)	0.105 (2.93)	0.23
$x=3$	12.573 (5.05)	−0.888 (−6.78)	0.102 (3.86)	0.45
$x=4$	16.501 (5.06)	−1.167 (−7.02)	0.102 (4.11)	0.59
$x=5$	17.895 (4.64)	−1.280 (−6.73)	0.100 (3.92)	0.64
Panel B. Sample 1971Q1-2008Q4				
$x=2$	6.154 (3.48)	−0.433 (−3.95)	0.102 (4.08)	0.21
$x=3$	12.555 (5.08)	−0.897 (−6.62)	0.098 (4.08)	0.45
$x=4$	16.840 (5.14)	−1.204 (−6.89)	0.098 (4.08)	0.61
$x=5$	18.379 (4.61)	−1.326 (−6.32)	0.098 (4.08)	0.66
Panel C. Sample 1982Q1-2008Q4				
$x=2$	5.941 (2.45)	−0.538 (−4.61)	0.070 (1.85)	0.35
$x=3$	10.700 (3.59)	−0.923 (−5.64)	0.074 (2.67)	0.56
$x=4$	12.906 (3.91)	−1.126 (−5.22)	0.074 (3.02)	0.68
$x=5$	13.147 (4.60)	−1.205 (−8.10)	0.069 (3.34)	0.75

Table 4. We regress three-month T-bill, y^{3m}_t , on a constant and deviations of the short rate from its long-run mean captured by middle-aged to young ratio. In each panel, we provide evidence for the longest sample (post-war period) and other sub-samples representing different monetary policy regimes. We report the Newey-West corrected t-statistics in parentheses. Quarterly data.

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TABLE 5.A. Comparison of Parameter Estimates (constant risk price), Sample 1969Q1-2007Q4

	Macro Model with MY_t					Macro Model				
	π_t	g_t	$f^u_{1,t}$	$f^u_{2,t}$	$f^u_{3,t}$	π_t	g_t	$f^u_{1,t}$	$f^u_{2,t}$	$f^u_{3,t}$
π_{t-1}	0.286	–0.037	–0.079	–0.019	0.068	0.270	–0.020	–0.073	0.042	0.055
	(0.068)	(0.046)	(0.056)	(0.164)	(0.034)	(0.338)	(0.125)	(0.036)	(0.037)	(0.028)
g_{t-1}	0.190	0.531	–0.117	–0.020	0.020	0.162	0.588	–0.117	0.059	–0.019
	(0.085)	(0.120)	(0042)	(0.149)	(0.048)	(0.080)	(0.205)	(0.122)	(0.063)	(0.117)
$f^u_{1,t-1}$	0.269	0.419	0.832	0.140	–0.194	0.292	0.397	0.977	–0.064	–0.128
	(0.140)	(0.215)	(0.614)	(0.286)	(0.088)	(0.290)	(0.462)	(0.180)	(0.158)	(0.420)
$f^u_{2,t-1}$	–0.000	0.187	0.300	0.789	0.110	–0.005	0.160	0.116	0.862	0.164
	(0.167)	(0.356)	(0.251)	(0.357)	(0.397)	(0.079)	(0.190)	(0.037)	(0.044)	(0.046)
$f^u_{3,t-1}$	0.186	0.141	–0.143	0.042	0.857	0.223	0.141	–0.222	0.111	0.636
	(0.167)	(0.070)	(0.302)	(0.285)	(0.311)	(0.290)	(0.530)	(0.038)	(0.152)	(0.214)
$y_{t,t+1}$										
δ_1	–0.156	–0.519	2.621	1.039	0.000	–0.141	–0.543	2.815	0.000	0.000
	(0.087)	(0.226)	(1.271)	(2.890)	(0.000)	(0.091)	(0.677)	(1.942)	(0.000)	(0.000)
δ_2			–0.0208							
			(0.0036)							

Table 5.A. Parameters estimates. This table reports the parameter estimates of maximum likelihood estimation of the no-arbitrage models, including macro factors (Macro Model) and middle-aged-young ratio (Macro Model with MY_t). The model is estimated assumingh constant risk price $\lambda_1 = 0$, i.e. Sample 1969Q1-2007Q4.

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TABLE 5.B. Comparison of Parameter Estimates (tv risk price), Sample 1969Q1-2007Q4

	Macro Model with MY_t					Macro Model				
	π_t	g_t	$f^u_{1,t}$	$f^u_{2,t}$	$f^u_{3,t}$	π_t	g_t	$f^u_{1,t}$	$f^u_{2,t}$	$f^u_{3,t}$
π_{t-1}	0.222	–0.118	–0.043	0.007	–0.038	0.213	–0.095	–0.044	0.011	–0.049
	(0.090)	(0.080)	(0.050)	(0.092)	(0.157)	(0.791)	(0.251)	(0.036)	(0.012)	(0.014)
g_{t-1}	0.207	0.521	–0.102	0.021	–0.054	0.179	0.583	–0.083	0.039	–0.105
	(0.086)	(0.111)	(0.115)	(0.063)	(0.364)	(0.063)	(0.147)	(0.030)	(0.010)	(0.083)
$f^u_{1,t-1}$	0.223	0.500	0.907	–0.083	0.063	–0.010	–0.047	0.962	0.002	0.027
	(0.115)	(0.129)	(0.239)	(0.335)	(0.864)	(0.094)	(0.247)	(0.012)	(0.031)	(0.101)
$f^u_{2,t-1}$	0.186	0.114	–0.103	0.766	–0.153	–0.212	–0.018	0.010	0.554	0.258
	(0.363)	(0.374)	(0.435)	(1.229)	(0.699)	(0.505)	(0.156)	(0.144)	(0.170)	(0.082)
$f^u_{3,t-1}$	0.144	0.116	–0.091	–0.234	0.679	0.316	0.482	0.137	0.081	0.805
	(0.602)	(0.446)	(0.402)	(0.476)	(1.243)	(0.295)	(0.801)	(0.144)	(0.089)	(0.340)
$y_{t,t+1}$										
δ_1	–0.240	–0.542	2.565	0.000	0.901	–0.255	–0.569	0.981	0.000	2.520
	(0.109)	(0.602)	(1.057)	(0.000)	(2.562)	(0.017)	(0.034)	(0.059)	(0.000)	(0.141)
δ_2			–0.022							
			(0.017)							

Table 5.B. Parameters estimates. This table reports the parameter estimates of maximum likelihood estimation of the no-arbitrage models, including macro factors (Macro Model) and middle-aged-young ratio (Macro Model with MY_t). The model is estimated assumingh constant risk price $\lambda_1 \neq 0$, i.e. Sample 1969Q1-2007Q4.

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TABLE 6. Yield Forecasts (constant risk price)

Panel A. FRMSE (Random-walk)

horz	1	4	8	12	16	20
$y_{t,t+1}$	1.1378	**0.9501**	**0.9321**	**<u>0.8209</u>**	**<u>0.7822</u>**	**<u>0.8355</u>**
$y_{t,t+4}$	1.1591	1.0334	**0.9339**	**<u>0.8214</u>**	**<u>0.7960</u>**	**<u>0.8717</u>**
$y_{t,t+8}$	1.1113	1.0482	**0.9444**	**<u>0.8506</u>**	**<u>0.8489</u>**	**0.9481**
$y_{t,t+12}$	1.0788	1.0400	**0.9509**	**<u>0.8763</u>**	**<u>0.8886</u>**	1.0052
$y_{t,t+16}$	1.0910	1.0392	**0.9626**	**0.9073**	**0.9293**	1.0506
$y_{t,t+20}$	1.0978	1.0409	**0.9815**	**0.9414**	**0.9720**	1.0956

Panel B. FRMSE (Yield-Macro Model)

horz	1	4	8	12	16	20
$y_{t,t+1}$	1.2607	1.0192	1.0016	**0.9990**	**0.9971**	1.0079
$y_{t,t+4}$	1.1582	1.0133	1.0006	**0.9968**	**0.9969**	1.0079
$y_{t,t+8}$	1.0980	1.0079	**0.9975**	**0.9940**	**0.9968**	1.0072
$y_{t,t+12}$	1.0861	1.0033	**0.9909**	**0.9882**	**0.9901**	**0.9980**
$y_{t,t+16}$	1.0877	**0.9980**	**0.9831**	**0.9810**	**0.9801**	**0.9835**
$y_{t,t+20}$	1.0842	**0.9881**	**0.9704**	**0.9705**	**0.9660**	**0.9648**

Table 6. Out-of Sample Yield Forecasts ($\lambda_1 = 0$). h indicates 1, 4, 8, 12, 16, 20 quarter out-of-sample forecasts. We measure forecasting performance as the ratio of the root mean squared forecast error (RMSFE) of our model to the RMSFE of a random walk forecast and the benchmark yield-macro model. We show the comparison of forecasting results from different models in Table 3. The table shows better forecasts with respect to the random walk and yield-macro model with bold characters for the range of [0.9, 1,) and with added underline for ratios smaller than 0.9. Sample 1964Q1-2007Q4. Quarterly data.

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TABLE 7. Yield Forecasts (time-varying risk price)						
Panel A. FRMSE (Random-walk)						
horz	1	4	8	12	16	20
$y_{t,t+1}$	1.1514	1.0462	**0.8924**	**<u>0.7958</u>**	**<u>0.7871</u>**	**<u>0.8538</u>**
$y_{t,t+4}$	1.2718	1.1253	**0.9178**	**<u>0.8281</u>**	**<u>0.8264</u>**	**0.9158**
$y_{t,t+8}$	1.2032	1.1173	**0.9487**	**<u>0.8833</u>**	**<u>0.8974</u>**	1.0131
$y_{t,t+12}$	1.1401	1.0890	**0.9711**	**0.9234**	**0.9486**	1.0790
$y_{t,t+16}$	1.1234	1.0699	**0.9953**	**0.9706**	**0.9982**	1.1318
$y_{t,t+20}$	1.1188	1.0551	1.0246	1.0142	1.0424	1.1756
Panel B. FRMSE (Yield-Macro Model)						
horz	1	4	8	12	16	20
$y_{t,t+1}$	1.1649	**0.9494**	**<u>0.8880</u>**	**<u>0.8706</u>**	**<u>0.8429</u>**	**<u>0.7979</u>**
$y_{t,t+4}$	1.0726	**0.9534**	**<u>0.8937</u>**	**<u>0.8770</u>**	**<u>0.8440</u>**	**<u>0.7986</u>**
$y_{t,t+8}$	1.0528	**0.9504**	**<u>0.8931</u>**	**<u>0.8785</u>**	**<u>0.8435</u>**	**<u>0.7967</u>**
$y_{t,t+12}$	1.0541	**0.9479**	**<u>0.8915</u>**	**<u>0.8753</u>**	**<u>0.8374</u>**	**<u>0.7899</u>**
$y_{t,t+16}$	1.0553	**0.9410**	**<u>0.8861</u>**	**<u>0.8677</u>**	**<u>0.8277</u>**	**<u>0.7789</u>**
$y_{t,t+20}$	1.0431	**0.9280**	**<u>0.8761</u>**	**<u>0.8575</u>**	**<u>0.8142</u>**	**<u>0.7663</u>**

Table 7. Out-of Sample Yield Forecasts ($\lambda_1 \neq 0$). h indicates 1, 4, 8, 12, 16, 20 quarter out-of-sample forecasts. We measure forecasting performance as the ratio of the root mean squared forecast error (RMSFE) of our model to the RMSFE of a random walk forecast and the benchmark yield-macro model. We show the comparison of forecasting results from different models in Table 3. The table shows better forecasts with respect to the random walk and yield-macro model with bold characters for the range of [0.9, 1,) and with added underline for ratios smaller than 0.9. Sample 1964Q1-2007Q4. Quarterly data.

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Figures

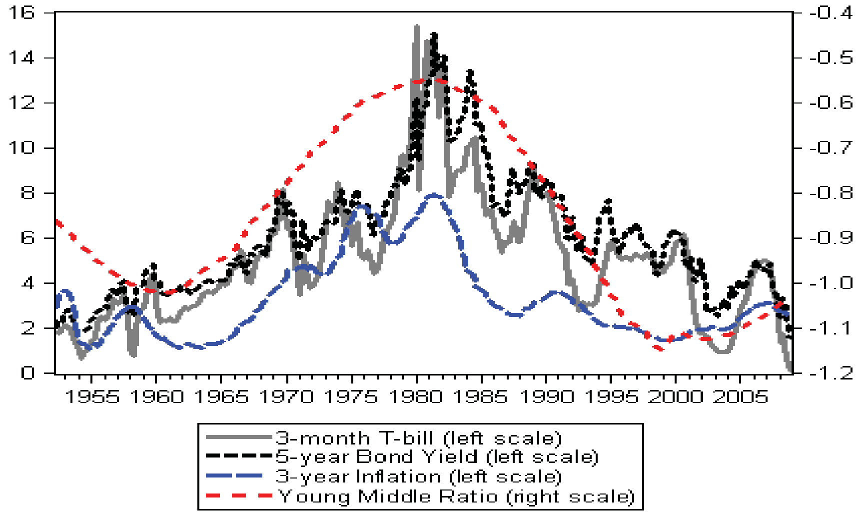

Figure 1. Nominal Bond Yields, 3-year Inflation and Young-Middle Ratio. The historical yield series are taken from Goyal and Welch (2008) database. Young-Middle ratio is interpolated from annual series used in Favero et al. (2010). Sample 1952Q2-2008Q4.

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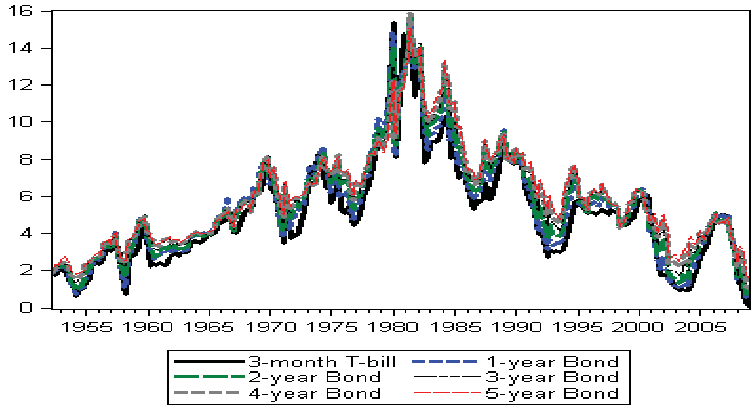

Figure 2. Bond yields for different maturities. The figure plots the quarterly (annualized) zero coupon bond yields of maturity 3-month, 1-year, 2-year, 3-year, 4-year and 5-year. Sample 1952Q2-2008Q4.

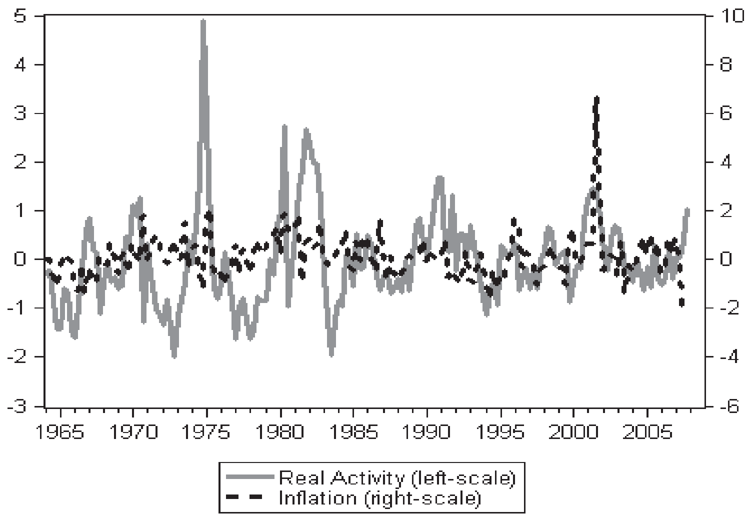

Figure 3. Nominal and Real factors. These are the factors extracted from large dataset provided by Ludvigson and Ng (2009). Sample 1964Q1-2007Q4.

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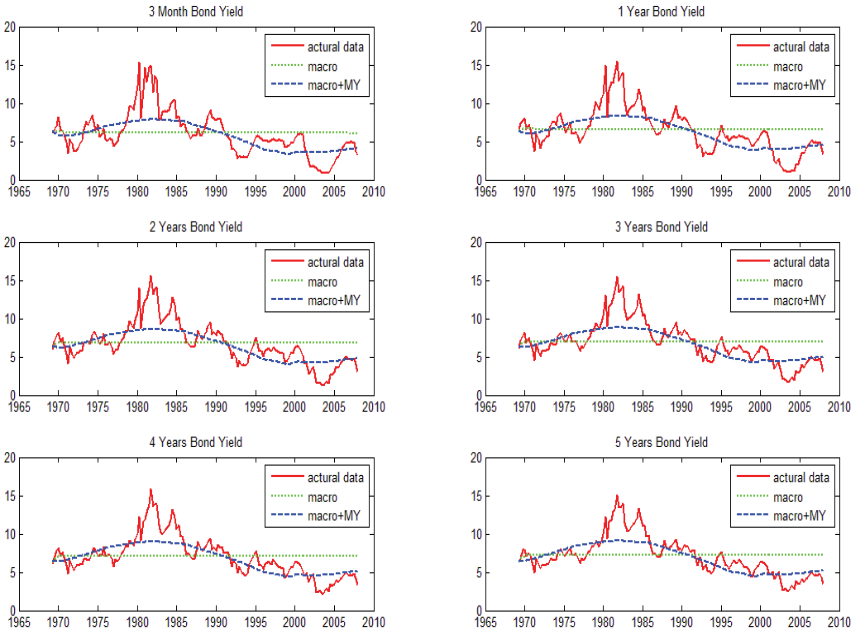

Figure 4. Dynamic Simulations. This figure plots the historical time series for bond yields (maturity: 3m, 1y, 2y, 3y, 4y, 5y) alongwith those dynamically simulated from the benchmark affine model with macro factors (dotted green line) and that augmented with demographics (dashed blue line). The no-arbitrage term structure models are estimated over the whole sample 1969Q1-2007q4. Using the estimated model parameters, models are solved dynamically forward from 1968Q4.

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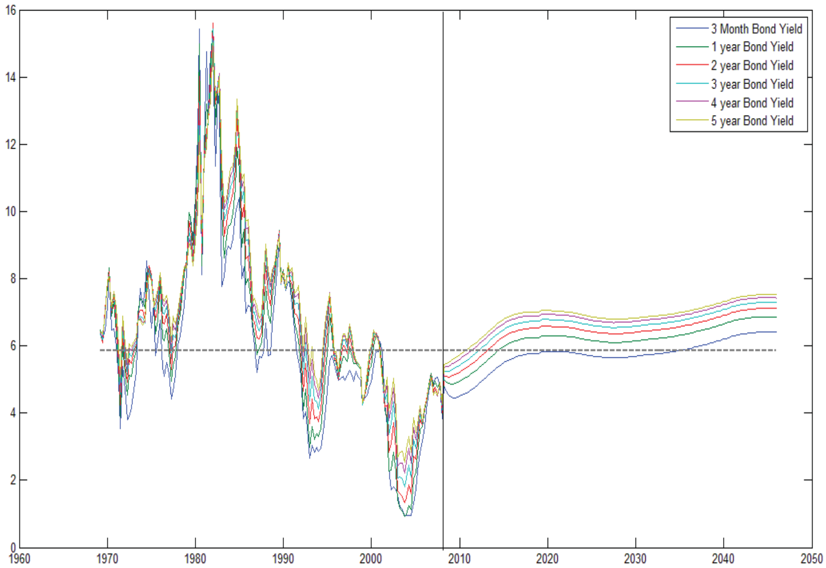

Figure 5. In-Sample Estimation and out of Sample Prediction. Here we use no arbitrage term structure model with demographics. This figure plots the in sample estimated value 1969Q1-2007Q4 and out-of-sample prediction path 2008Q1-2045Q4 with all maturities: 3m, 1y, 2y, 3y, 4y, 5y. The no-arbitrage term structure models are estimated over the whole sample 1969Q1-2007q4. Using the estimated model parameters, models are solved dynamically forward from 1968Q4. The gray dashed line is in sample average of short rate (3 month), and the grey solid line shows end of in sample estimation.

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D APPENDIX

$$
\begin{aligned}
y_{t,t+n} &= -\frac{1}{n}\left(A_n + B_n' X_t + \Gamma_n \mathrm{MY}_t^n\right) + \varepsilon_{t,t+1} \qquad \varepsilon_{t,t+n} \sim N(0, \sigma_n^2) \\
X_t &= \mu + \Phi X_{t-1} + \nu_t \quad \nu_t \sim i.i.d.N(0, \Omega) \\
y_{t,t+1} &= \delta_0 + \boldsymbol{\delta}_1' X_t + \delta_2 \mathrm{MY}_t \\
\Lambda_t &= \lambda_0 + \lambda_1 X_t \\
m_{t+1} &= \exp(-y_{t,t+1} - \frac{1}{2}\Lambda_t' \Omega \Lambda_t - \Lambda_t \varepsilon_{t+1}) \\
P_t^{(n)} &\equiv \left[\frac{1}{1+Y_{t,t+n}}\right]^n, \; y_{t,t+n} \equiv \ln\left(1 + Y_{t,t+n}\right) \\
\Gamma_n \mathrm{MY}_t^n &\equiv \left[\gamma_0^n, \gamma_1^n \cdots, \gamma_{n-1}^n\right] \begin{bmatrix} \mathrm{MY}_t \\ \mathrm{MY}_{t+1} \\ \vdots \\ \mathrm{MY}_{t+n-1} \end{bmatrix}
\end{aligned}
$$

Bond prices can be recursively computed as:

$$
\begin{aligned}
P_t^{(n)} &= E_t[m_{t+1} P_{t+1}^{(n-1)}] \\
&= E_t[m_{t+1} m_{t+2} P_{t+2}^{(n-2)}] \\
&= E_t[m_{t+1} m_{t+2} \cdots m_{t+n} P_{t+n}^{(0)}] \\
&= E_t[m_{t+1} m_{t+2} \cdots m_{t+n} 1] \\
&= E_t[\exp(\sum_{i=0}^{n-1}(-y_{t+i,t+i+1} - \frac{1}{2}\Lambda_{t+i}' \Omega \Lambda_{t+i} - \Lambda_{t+i}' \nu_{t+i+1}))] \\
&= E_t[\exp(A_n + B_n' X_t + \Gamma_n' \mathrm{MY}_t^n)] \\
&= E_t[\exp(-n y_{t,t+n})] \\
&= E_t^Q[\exp(-\sum_{i=0}^{n-1} y_{t+i,t+i+1})]
\end{aligned}
$$

where E_t^Q denotes the expectation under the risk-neutral probability measure, under

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which the dynamics of the state vector X_t are characterized by the risk neutral vector of constants μ^Q and by the autoregressive matrix Φ^Q

$$\begin{aligned} \mu^Q &= \mu - \Omega\lambda_0 \\ \Phi^Q &= \Phi - \Omega\lambda_1 \end{aligned}$$

To derive the coefficients of the model, let us start with $n = 1$:

$$\begin{aligned} P_t^{(1)} &= \exp(-y_{t,t+1}) \\ &= \exp(-\delta_0 - \delta_1' X_t - \delta_2 \mathrm{MY}_t) \end{aligned}$$

$A_1 = -\delta_0$, $B_1 = -\delta_1$ and $\Gamma_1 = \gamma_0^1 = -\delta_2$,

Then for the general case $n + 1$,we have:

$$\begin{aligned} P_t^{(n+1)} &= E_t[m_{t+1} P_{t+1}^{(n)}] \\ &= E_t[\exp(-y_{t,t+1} - \frac{1}{2}\Lambda_t'\Omega\Lambda_t - \Lambda_t'\nu_{t+1}) \exp(A_n + B_n' X_{t+1} + \Gamma_n \mathrm{MY}_{t+1}^n)] \\ &= \exp(-y_{t,t+1} - \frac{1}{2}\Lambda_t'\Omega\Lambda_t + A_n) E_t[\exp(-\Lambda_t'\nu_{t+1} + B_n' X_{t+1} + \Gamma_n \mathrm{MY}_{t+1}^n)] \\ &= \exp(-y_{t,t+1} - \frac{1}{2}\Lambda_t'\Omega\Lambda_t + A_n + \Gamma_n \mathrm{MY}_{t+1}^n) \\ &\quad E_t[\exp(-\Lambda_t'\nu_{t+1} + B_n'(\mu + \Phi X_t + \nu_{t+1}))] \\ &= \exp[-\delta_0 - \delta_1' X_t - \delta_2 MY_t - \frac{1}{2}\Lambda_t'\Omega\Lambda_t + A_n + \Gamma_n \mathrm{MY}_{t+1}^n + B_n'(\mu + \Phi X_t)] \\ &\quad E_t[\exp(-\Lambda_t'\nu_{t+1} + B_n'\nu_{t+1})] \\ &= \exp[-\delta_0 - \delta_1' X_t - \frac{1}{2}\Lambda_t'\Omega\Lambda_t + A_n - \delta_2 \mathrm{MY}_t + B_n'(\mu + \Phi X_t) \\ &\quad + \Gamma_n \mathrm{MY}_{t+1}^n] \exp\{E_t[(-\Lambda_t' + B_n')\nu_{t+1}] + \frac{1}{2} var[(-\Lambda_t' + B_n')\nu_{t+1}]\} \\ &= \exp[-\delta_0 - \delta_1' X_t - \frac{1}{2}\Lambda_t'\Omega\Lambda_t + A_n + B_n'(\mu + \Phi X_t) \\ &\quad + \left[-\delta_2, \gamma_0^n, \gamma_1^n \cdots, \gamma_{n-1}^n\right] \mathrm{MY}_t^{n+1}] \exp\{\frac{1}{2} var[(-\Lambda_t' + B_n')\nu_{t+1}]\} \end{aligned}$$

To simplify the notation we define

$$[-\delta_2, \Gamma_n] \equiv \left[-\delta_2, \gamma_0^n, \gamma_1^n \cdots, \gamma_{n-1}^n\right]$$

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$$
\begin{aligned}
&= \exp[-\delta_0 - \delta_1' X_t - \frac{1}{2}\Lambda_t'\Omega\Lambda_t + A_n + B_n'(\mu + \Phi X_t) \\
&\quad +[-\delta_2, \Gamma_n]\mathrm{MY}_t^{n+1}] \exp\{\frac{1}{2}E_t[(-\Lambda_t' + B_n')\nu_{t+1}\nu_{t+1}^{'}(-\Lambda_t + B_n)]\} \\
&= \exp[-\delta_0 - \delta_1' X_t - \frac{1}{2}\Lambda_t'\Omega\Lambda_t + A_n + B_n'(\mu + \Phi X_t) \\
&\quad +[-\delta_2, \Gamma_n]\mathrm{MY}_t^{n+1}] \exp\{\frac{1}{2}[\Lambda_t'\Omega\Lambda_t - 2B_n'\Omega\Lambda_t + B_n'\Omega B_n)]\} \\
&= \exp[-\delta_0 + A_n + B_n'\mu + (B_n'\Phi - \delta_1')X_t - B_n'\Omega\Lambda_t + \frac{1}{2}B_n'\Omega B_n \\
&\quad +[-\delta_2, \Gamma_n]\mathrm{MY}_t^{n+1}] \\
&= \exp[-\delta_0 + A_n + B_n'\mu + (B_n'\Phi - \delta_1')X_t - B_n'\Omega(\lambda_0 + \lambda_1 X_t) + \frac{1}{2}B_n'\Omega B_n \\
&\quad +[-\delta_2, \Gamma_n]\mathrm{MY}_t^{n+1}] \\
&= \exp[A_1 + A_n + B_n'(\mu - \Omega\lambda_0) + \frac{1}{2}B_n'\Omega B_n + (B_n'\Phi - B_n'\Omega\lambda_1 + B_1')X_t \\
&\quad +[-\delta_2, \Gamma_n]\mathrm{MY}_t^{n+1}]
\end{aligned}
$$

Then we can find the coefficients following the difference equations

$$
\begin{aligned}
A_{n+1} &= A_1 + A_n + B_n'(\mu - \Omega\lambda_0) + \frac{1}{2}B_n'\Omega B_n \\
B_{n+1} &= B_n'\Phi - B_n'\Omega\lambda_1 + B_1' \\
\Gamma_{n+1} &= [-\delta_2, \Gamma_n]
\end{aligned}
$$

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3 Inflation, Stock Market and Long-Term Investors

This chapter shows the common demographic component shared by equity and bond yields to explain the slow-evolving time-series covariation. Yields to U.S. equity and bond markets follow surprisingly similar paths in the post-war period (e.g. Thomas and Zhang, 2008; Bekaert and Engstrom, 2010). This evidence incites debate on the validity of rational valuation models that rely on relative pricing of equity and bond markets (e.g. Asness, 2003; Estrada, 2009). At the same time, yields to the aggregate stock market are positively correlated with inflation (e.g. Fama and Schwert, 1977; Wei, 2010)[15]. This observation is puzzling, since conventional wisdom suggests that the stock market represents real assets and hence should be a good hedge against inflation. Behavioral explanations such as the "inflation illusion" (Modigliani-Cohn, 1979; Campbell and Vuolteenaho, 2004) or risk-based stories (Brant and Wang, 2003; Bekaert and Engstrom, 2010) have been suggested to reconcile this evidence.

In this chapter I provide an alternative explanation based on the relation between the population age structure and financial markets. First, I show that a common demographic factor drives the persistent, long-run components of both financial yields and inflation. This link is crucial to identify the sources of low-frequency covariation between equity and bond yields, since inflation is the main driver of bond yields and surprisingly correlated with equity yields. Then I test the relevance of demographic fluctuations for long-term investors in the context of return predictability within a present-value framework. Specifications including both financial yields and a model-based demographic variable improve the performance of return forecasting models; current deviations from the equilibrium relations carry useful information for the future return paths. However, return predictability does not imply a good hedge against unexpected inflation. Finally, I use a cross-country panel to document cross-sectional variation of the demographic effect. This evidence provides out-of-sample support for the importance of a common

[15] Such a relation holds in general for equity yields either measured as dividend or earnings yield. A similar (negative) correlation is observed between real stock market returns and inflation.

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demographic component.

The U.S. population age structure evolves over time and features twenty-year boom and bust cycles (see Figure 1). This life-cycle pattern appears as predictable components in financial yields and inflation once we account for the gradual change in these series using a specific demographic variable; the proportion of the middle-aged to young population, MY_t. This variable is a proxy for the net savers in the economy. Figure 2 plots earnings yield to aggregate U.S. stock market[16] and annual CPI inflation over the post gold-standard sample (1933-2009) together with the long-run components accounted for by MY_t. The figure shows that when middle-aged population from the silent generation started to save for retirement, they faced more than two times higher earning yields and three times higher inflation in the eighties compared to their parents around the sixties. On the contrary, the savers from the baby-boom generation were exposed to 66% lower earning yields and 77% lower inflation around the millenium.

The interest in MY_t as an empirical proxy to measure the change in the U.S. age structure is not arbitrary; it is derived from an overlapping generations (OLG) model by Geanakoplos, Magill and Quinzii (2004, henceforth GMQ). GMQ conjecture that the life-cycle portfolio behavior (Bakshi and Chen, 1994) plays an important role in determining equilibrium asset prices; given the assumed demographic structure, consumption smoothing requires that when MY_t is large, there is excess demand for saving by a large cohort of middle-aged population. For the market to clear, equilibrium prices of financial assets and therefore the yields should adjust. GMQ mainly focus on the real economy and hence on real prices of financial assets[17], so the model provides no predictions on inflation, an important component of nominal bond yields. This chapter extends the GMQ model by introducing money as an additional asset in their framework; households do not only make consumption and savings decision, but they also decide upon the composition of saving vehicles; productive capital versus real money balances. This model shows how in equilibrium both real returns to productive capital and money are determined by the age structure of the population.

The GMQ model is part of a strand of literature aimed at explaining market fluctuations with demographic factors. Bakshi and Chen (1994) develop the life-cycle investment hypothesis which asserts that an investor in an early stage of her life allocates more wealth on housing and switches to financial assets at a later stage. Starting from

[16] This series is the cyclically adjusted earning price ratio (using a 10-year window of earnings) taken from Robert Shiller's webpage.
[17] Similarly, based on an OLG model Piazzesi and Schneider (2010) analyze the relation between inflation and prices of real assets.

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this literature Erb, Harvey, and Viskanta (1996) and Ang and Maddaloni (2005) study the effects of demographics in an international context and document the link between demographics and risk premia. However, the evidence is not conclusive (Poterba, 2001; Goyal, 2004). While theoretical models suggest strong demographic effects on financial markets, empirical studies face difficulties documenting these effects. This is mainly because ad-hoc demographic variables are not successful in isolating the relevant low-frequency information from the noise in financial markets. This chapter contributes to this literature by analyzing jointly stock and bond markets using a demographic variable that is justified by economic theory.

The study of the comovement between equity and bond markets is a well-established approach; equity and bonds are the two main asset classes considered in portfolio allocation (e.g. Keim and Stambaugh, 1986). The previous empirical literature provides evidence on stock and bond return correlation (e.g. Fama and French, 1993, Li, 2002)[18], but this evidence is hard to justify within the present value models assuming constant risk premia (e.g. Barsky, 1989; Shiller and Beltratti, 1992; Campbell and Ammer; 1993). Thus two diverse strands of literature focus on predictability of stock and bond market returns separately. Despite the critical view (see, for example,Welch and Goyal, 2008), previous literature shows predictive ability of both financial ratios (e.g. Ang and Bekaert, 2007; Cochrane, 2008; Ferreira and Santa-Clara, 2010) and macro time series such as consumption-wealth ratio (Lettau and Ludvigson, 2001) and output gap (Cooper and Priestley, 2009). In particular, previous studies document the role of dividend yield in forecasting long term returns (e.g. Cochrane, 2008) and justify its use within the dynamic dividend growth model proposed by Campbell and Shiller (1988). This model relies on a log-linearized version of one-period returns on a stock portfolio. Yet, the derivation of the model and hence its forecasting performance crucially relies on the stationarity of dividend yields. Lettau and Van Nieuwerburgh (2008) challenge this point and find breaks in the long-run mean of dividend yield, while Favero, Gozluklu and Tamoni (2009, henceforth FGT) show the strong empirical link between the dividend yield persistence and demographic fluctuations.

Similarly, the bond market literature highlights the role of forward interest rates in forecasting future spot interest rates for longer horizons (e.g. Fama and Bliss, 1987). In particular, linear combinations of forward rates are successful in predicting term premia (Cochrane and Piazzesi, 2005). An early literature attributes bond yield predictabil-

[18] A recent literature also analyzes both markets from a high frequency perspective (i.e. daily returns) and show strong linkages (Marquering and De Goeu, 2004; Connolly et al., 2005; Ranaldo and Christiansen, 2007).

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ity to mean reversion of the spot rate towards a constant expected value. Yet, recent literature argues that the predictability of the spot rate captured by forward rates is either due to a moving, yet still stationary, mean (Balduzzi, Das, and Foresi, 1998) or a time-varying, but non-stationary long-term expected value (Fama, 2006). In the second chapter, we develop a no-arbitrage affine term structure model based on the idea that the slow mean-reverting component of the spot rate is driven by demographic fluctuations, while the more rapid (business-cycle-length) mean-reverting component is captured by macroeconomic factors. This paper argues that stock and bond return predictability evidence is tightly linked to the slow evolution of population age structure and proposes a trend/cycle decomposition for financial yields and inflation (Stock and Watson, 2007), where the trend components are driven by a shared demographic factor[19].

A growing body of literature focuses on the joint dynamics of stock and bond markets[20]. Several recent papers are similar to this one in their focus. For instance, Campbell, Sunderan and Viceira (2010) develop a model based on five state variables to explain the covariance between stock and bond returns and find that stock-bond covariance is driven by the covariance between nominal variables and the real economy. Koijen, Lustig and Van Nieuwerburgh (2010) propose a arbitrage-free stochastic discount factor (SDF) model where the pricing factors are motivated by a permanent/transitory decomposition of the pricing kernel and price cross-section of returns. However, none of these papers explicitly consider time-series variation in demographics as the source of a persistent, slow-moving trend component.

The remainder of the chapter is organized as follows; section II introduces the GMQ model and its extension. Section III provides empirical results on the demographic factor driving the long-run component of financial yields and inflation. Section IV introduces the present-value framework and report empirical results on return predictability. Section V analyzes inflation hedging using demographic information. Section VI provides cross-country evidence while section VII concludes.

[19] While the channel between bond yields and inflation is obvious, other studies (e.g. Pilotte, 2003; Boucher, 2006) also analyze the comovement between dividend (earnings) yields and inflation.

[20] The list is long, but some important contributions are Bakshi and Chen (1996), Mamasky (2002), Bekaert, Engstrom and Granadier (2006), Lettau and Wachter (2008), Baele, Bekaert and Inghelbrecht (2010).

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The Model

Benchmark Model[21]

GMQ propose an OLG exchange economy with a single consumption good (income) and three periods; young, middle-aged, retired. Each agent (except retirees) has an endowment, labor income, w= (w_y,w_m,0) and there are two types of financial instruments, riskless bond and risky equity which allow agents to redistribute income over time. In their simple benchmark model, dividends and wages are deterministic, hence bond and equities are perfect substitutes. GMQ assume that in *odd (even)* periods a large (small) cohort N(n) enters the economy, therefore in every odd (even) period there will be {N,n,N}({n,N,n}) cohorts living.

The model is based on the life-cycle portfolio behavior (Bakshi and Chen, 1994) which suggests that young agents should borrow, invest for retirement when middle-aged, and dissave during retirement. This mechanism plays an important role in determining equilibrium asset prices.

Let q^j be the bond price for period j={odd,even} and $\{c_y^j, c_m^j, c_r^j\}$ the consumption stream for the agent born in period *j*. An agent born in the odd period then faces the following budget constraint

$$c_y^o + q^o c_m^o + q^o q^e c_r^o = w_y + q^o w_m \tag{3.1}$$

and in the even period

$$c_y^e + q^e c_m^e + q^e q^o c_r^e = w_y + q^e w_m \tag{3.2}$$

Moreover, in equilibrium the following resource constraint must be satisfied

$$Nc_y^o + nc_m^e + Nc_r^o = Nw_y + nw_m + D \tag{3.3}$$

$$nc_y^e + Nc_m^o + nc_r^e = nw_y + Nw_m + D \tag{3.4}$$

where D is the aggregate dividend for the investment in financial markets. If q^o were equal to q^e, the agents would choose to smooth their consumption, i.e. $c_y^j = c_m^j = c_r^j$ for each *j,* but then for values of wages and aggregate dividend calibrated from US data the equilibrium condition above would be violated leading to excess demand either for consumption or saving. To illustrate this point I refer to the calibration provided by GMQ; take $N = 79, n = 69$ as the size (in millions) of Baby Boom (1945-64) and

[21] The same model has been introduced in chapter one, I repeat here the section to facilitate the comparison with the alternative model.

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Baby Bust (1965-84) generations[22] and $w_y = 2$,$w_m = 3$ to match the ratio (middle to young cohort) of the average annual real income in the U.S. We can calculate the total wage in even and odd periods using Nw_y+nw_m for odd periods and nw_y+Nw_m for even periods, and then given the average ratio (0.19) of dividend to wages we compute the aggregate dividends. Assuming an annual discount factor of 0.97, which translates to a discount of 0.5 in the model of 20-year periods, if q^o=q^e=0.5 were to hold and agents smooth their consumption, from the budget constraint (eq. 1-2) we obtain c^i_y=c^i_m=c^i_r=$\bar{c}$=2, but then the resource constraint (eq. 3-4) above would have been violated. For instance, an agent from Baby Bust generation would enter in an even period in the model, i.e. (n, N, n) and high MY ratio, and faces the following aggregate resource constraint: $n(c^e_y-w_y)+N(c^e_m-w_m)+nc^e_r-D= 69\times(2-2)+79(2-3)+69\times2-70 = -11$, where D= $0.19(\frac{375+365}{2}) = 70$. This leads to excess saving in the economy. For equilibrium conditions to hold, the model implies that asset prices should increase and hence discourage saving in the economy (the experience we observed during 90's in US). When the MY ratio is small (large), i.e. an odd (even) period, there will be excess demand for consumption (saving) by a large cohort of retirees (middle-aged) and for the market to clear, equilibrium prices of financial assets should adjust, i.e. decrease (increase), so that saving (consumption) is encouraged for the middle-aged. Thus, letting q^b_t be the price of the bond at time t, in a stationary equilibrium, the following holds

$$q^b_t=q^o=q^o\left(\frac{c^o_y}{c^o_m}\right) \text{ when period is odd}$$
$$q^b_t=q^e=q^e\left(\frac{c^e_y}{c^e_m}\right) \text{ when period is even}$$

together with the condition $q^o<q^e$. Hence the model predicts a positive correlation between MY and market prices.

Since bond prices alternate between q^o and q^e,the price of equity must also alternate between q^o_{eq} and q^e_{eq} as follows

$$\begin{aligned} q^o_{eq} &= Dq^o+Dq^oq^e+Dq^oq^eq^o+.... \\ q^e_{eq} &= Dq^e+Dq^eq^o+Dq^eq^oq^e+.... \end{aligned}$$

which implies

$$\begin{aligned} DP^o &= \frac{D}{q^o_{eq}} = \frac{1-q^oq^e}{q^oq^e+q^o} \\ DP^e &= \frac{D}{q^e_{eq}} = \frac{1-q^oq^e}{q^oq^e+q^e} \end{aligned}$$

where DP^j is the dividend price ratio implied by low (high) MY in the model for period

[22] Thus, we obtain in the even period a high middle-aged young ratio of MY=$\frac{N}{n}$=1.15, and in the odd period a low MY=$\frac{n}{N}$=0.87 (See Figure 1).

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j. Calibration results using post-war U.S. data confirm these predictions[23]; a switch from high MY ratio ($\frac{\mathrm{N}}{\mathrm{n}} = \frac{\text{Baby Boom}}{\text{Baby Bust}} = 1.15$) to low MY ratio ($\frac{\text{Baby Bust}}{\text{Echo Boomers}} = 0.87$) predicts a 30% increase in dividend yields.

Model with Money Demand

The benchmark model describes an economy without money. Households can transfer their income across different periods of life by saving, i.e. investing in productive capital via bonds or equity. This is done by setting the present value of consumption to the present value of current and future income. In frictionless OLG models money is only valued if it is not dominated by other assets, otherwise households would only save and would not demand real money holdings (Blanchard and Fisher, 1989)[24]. To justify the demand for money and hence allow for inflation in this simple economy, the following feature is introduced to the model; households choose; i) optimal consumption/savings given the lifetime resources, ii) how to allocate savings across different assets given prices. Preferences on different assets are represented by the Dixit-Stiglitz specification

$$\mathrm{s}_i^j = \left(\sum_{k=1}^{N} (\mathrm{a}_{i,k}^j)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

where s_i^j is the real saving by agent i=$\{$y,m$\}$ born in period *j*, N is the number of assets available for saving, $a_{i,k}^j$ is the amount allocated to asset *k* by agent *i* born in period *j* and σ measures the constant elasticity of substitution (CES) between different assets[25]. Note that money is introduced as an additional asset (Danthine and Donaldson, 1986; Stulz, 1986). The single consumption good remains to be the numeraire. In the following analysis, I restrict the number of assets to k=2, $\mathrm{a}_1 = m$ (money) and $\mathrm{a}_2 = z$ (investment in productive capital). The two assets differ in terms of their real returns determined endogenously in the model, but the key difference is that agents have exogenous preferences over real money balances and investments in capital markets. This exogenous preference, say for convenience or diversification, is captured by the parameter σ. Households do not derive direct utility from holding real money balances, but make their asset allocation decision based on their relative preference over assets given the budget for savings[26].

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[23] Adding more features such as children, bequest motives, social security or production sector with convex adjustment costs does not change the qualitative results of the model.
[24] In the macro literature, money enters in the models under different specifications, i.e. in the utility functions, shopping time models and as cash-in advance constraints (Walsh, 2003).
[25] If $\sigma = 0$, two assets are perfect complements, in the limit if $\sigma = \infty$, they are perfect substitutes.
[26] As an illustrative example, one can think of this exogenous preference as a "x % liquid asset rule" suggested by fund manager.

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First, I define an asset price index Q_I^j for period j which is determined both by the consumption-savings decision and the optimal choice for savings given the preferences over assets. This price index allows households to allocate their income across time and hence the previous budget (eq. 1-2) and resource constraints (eq. 3-4) remain the same with a slight change in the aggregate dividends[27]. The agent born in the odd period then faces the following budget constraint

$$c_y^o + Q_I^o c_m^o + Q_I^o Q_I^e c_r^o = w_y + Q_I^o w_m \tag{3.5}$$

and in the even period

$$c_y^e + Q_I^e c_m^e + Q_I^e Q_I^o c_r^e = w_y + Q_I^e w_m \tag{3.6}$$

Moreover, in equilibrium the following resource constraint must be satisfied

$$N c_y^o + n c_m^e + N c_r^o = N w_y + n w_m + D^* \tag{3.7}$$

$$n c_y^e + N c_m^o + n c_r^e = n w_y + N w_m + D^* \tag{3.8}$$

where D^* is the aggregate dividend for the investment in financial markets and $D > D^*$.

Then utility maximization with Dixit-Stiglitz preferences for asset allocation (under the given savings budget) delivers

$$\frac{m_y^j}{z_y^j} = \left(\frac{q_m^j}{q_z^j}\right)^{-\sigma}, \frac{m_m^j}{z_m^j} = \left(\frac{q_m^{j+1}}{q_z^{j+1}}\right)^{-\sigma}$$

where $q_m^j (q_z^j)$ is the price of real money holdings (bonds) in period j. The relative demand for assets is determined both by relative prices and the constant elasticity of substitution. By combining the relative demand for assets and the budget allocated for savings, we can write the price index as

$$Q_I^j = \left[\sum_{i=1}^{2} (q_i^j)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

Hence we obtain the optimal demand for assets in terms of prices and aggregate demand for savings

$$m_y^{j^*} = \left(\frac{q_m^j}{Q_I^j}\right)^{-\sigma} s_y^{j^*},\ z_y^{j^*} = \left(\frac{q_z^j}{Q_I^j}\right)^{-\sigma} s_y^{j^*}$$

$$m_m^{j^*} = \left(\frac{q_m^{j+1}}{Q_I^{j+1}}\right)^{-\sigma} s_m^{j^*},\ z_m^{j^*} = \left(\frac{q_z^{j+1}}{Q_I^{j+1}}\right)^{-\sigma} s_m^{j^*}$$

In the following analysis, I assume that there is a fixed supply of real money bal-

[27] Some resources will be diverted from productive sector and hence dilute profits.

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ances $\overline{M}^S$ in each period[28] and the money market clears. Hence, we have the following market clearing conditions

$$\mathrm{Nm}_y^o + \mathrm{nm}_m^e = \overline{M}^S \quad (3.9)$$

$$\mathrm{nm}_y^e + \mathrm{Nm}_m^o = \overline{M}^S \quad (3.10)$$

The optimization over savings demand implies that[29]

$$\mathrm{N}\left(\frac{\mathrm{q}_m^o}{\mathrm{Q}_I^o}\right)^{-\sigma} \mathrm{s}_y^o + \mathrm{n}\left(\frac{\mathrm{q}_m^o}{\mathrm{Q}_I^o}\right)^{-\sigma} \mathrm{s}_m^e = \overline{M}^S$$

$$\mathrm{n}\left(\frac{\mathrm{q}_m^e}{\mathrm{Q}_I^e}\right)^{-\sigma} \mathrm{s}_y^e + \mathrm{N}\left(\frac{\mathrm{q}_m^e}{\mathrm{Q}_I^e}\right)^{-\sigma} \mathrm{s}_m^o = \overline{M}^S$$

$$\left(\frac{\overline{M}^S}{(\mathrm{Ns}_y^o + \mathrm{ns}_m^e)}\right)^{-\frac{1}{\sigma}} \mathrm{Q}_I^o = \mathrm{q}_m^o$$

$$\left(\frac{\overline{M}^S}{(\mathrm{ns}_y^e + \mathrm{Ns}_m^o)}\right)^{-\frac{1}{\sigma}} \mathrm{Q}_I^e = \mathrm{q}_m^e$$

The last two equations show the functional link between the price of money, q_m, and the asset price index, Q_I^j. Note that inflation is not explicitly modelled, but the return of the second asset (money) is interpreted as the inverse of price inflation. A large cohort saving for retirement who chooses to include money in their portfolio increases the value of money and reduces inflation. Thus in this model, demand for money creates deflationary pressures and predicts a positive correlation between the price of money, q_m, and the net savers in the economy, MY_t ($\mathrm{q}_m^e > \mathrm{q}_m^o$). Model calibration confirms this prediction for different choices of σ, but the results can only be interpreted qualitatively[30].

One important feature of the model is that it does not assume independence between real rates and money returns. This is in contrast to the Fisher (1930) hypothesis which suggests a decomposition of nominal (observable) values into orthogonal real rate and inflation components. The model itself does not refute the Fisher hypothesis, since it allows dependence by construction to incorporate demographic effects. But it illustrates why the theory has faced important empirical challenges (e.g. Fama and Schwert, 1977; Gultekin, 1983) and caused controversy in the literature (e.g. Barsky, 1987; Boudukh and Richardson, 1993). In the following sections I show why the independence hypothesis is not supported by U.S. data.

28 Proportion of the aggregate dividends that could have been generated if invested in productive capital.
29 Note that I assume a constant σ for all cohort.
30 Many important features are excluded from the model, e.g. government who creates fiat money (hence long-run inflation).

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The Demographic Factor

In this section, I test the conjecture that a common persistent component caused by time-variation in population age structure drives both key financial variables and inflation. First I provide an out-of sample evaluation to see whether the long-run means of financial yields and inflation are driven by a common demographic factor. Throughout the chapter I use the same empirical proxy to capture this demographic effect; I adopt the empirical counterpart (GMQ, 1994) of the model-implied MY_t which is the proportion of the number of agents aged 40-49 to the number of agents aged 20-29.

Autoregressive Benchmark

The first out-of-sample test compares the forecasts of an autoregressive, AR(1) benchmark with a specification that augments the autoregressive process with MY_t. I implement a recursive estimation, i.e. move the sample forward one observation at a time and re-estimate the model at every step, starting from the period 1933-1968. The annual sample starts in 1933 when the gold standard had been abolished and the fiat money system commenced[31]. I generate out-of sample forecasts for different forecasting horizons (h=1,3,5 years) by solving the autoregressive model forward and incorporating the actual values of MY_t in the forward solution. In particular, I estimate the following models

$$\begin{aligned} \text{Benchmark model} &: \quad x_t = \alpha_0 + \alpha_1 x_{t-1} + \varepsilon_t \\ \text{Augmented model} &: \quad x_t = \beta_0 + \beta_1 x_{t-1} + \beta_2 \mathrm{MY}_t + \varepsilon_t \\ x_t &\in \{\pi_t, R_{lt}, dp_t\} \end{aligned}$$

where π_t is the annual CPI inflation, dp_t is the log dividend yield and R_{lt} is the long term bond yield.

The inclusion of MY_t in the financial yield equations is a direct implication of the GMQ model; a higher MY_t implies higher prices and lower yields for financial assets. Hence, we should expect negative values for β_2 in the financial yield equations. Similarly, life-cycle patterns in consumption and saving behavior predicted by the OLG model suggest negative loadings on MY_t in inflation equations. Panel A of Table 1 reports the median parameter estimates for the augmented model over different forecasting horizons. We note that autoregressive parameters β_1 are far below one, and all

[31] One can argue that the move from fixed to floating rate regime around 1971 can be considered as another reqime switch. I adress this point by including another subsample in my analysis in later sections.

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β_2 estimates are negative as expected.

Panel B shows the relative out-of sample forecasting performance of both models including inflation, log dividend yield and bond yield. I report the ratio of the root mean square error and mean absolute error of two models for different forecasting horizons h=1,3,5 years. The first out-of-sample forecasts are made for 1969 (h=1), 1972 (h=3) and 1974 (h=5). Results are reported for the period up to 2009 with using the rolling windows technique. A ratio lower than 1 indicates a better forecasting performance for the augmented model with MY_t. With the one year forecasting horizon, both models are comparable in terms of both criteria, but once we move to 3-year and 5-year forecasting horizon, the augmented model using demographic information dominates substantially, indicating the role of MY_t in determining the long-run path of financial yields and inflation[32].

A VAR Comparison

The second out-of sample test focuses on inflation and compares a benchmark VAR model that includes annual inflation, HP-filtered log real GDP, 3-month T-bill rate and commodity price inflation[33] as endogenous variables with an alternative VAR that includes inflation, equity and bond yields as endogenous variables and MY_t as exogenous. These models are again estimated using the rolling windows technique starting from the period 1933-1968. I generate out-of sample forecasts for the same (h=1,3,5 years) forecasting horizons by solving the VAR model forward and plugging in the actual values of MY_t in the forward solution.

$$
\begin{aligned}
\text{Benchmark VAR} &: \mathbf{Y}_t = \mu + \Phi \mathbf{Y}_{t-1} + \Sigma\varepsilon_t, \;\; \varepsilon_t \sim N(0,1) \\
\mathbf{Y}_t &= [\pi_t, ggap_t, R_{st}, \pi_t^{com}]' \\
\text{Exogenous VAR} &: \mathbf{Y}_t = \mu + \Theta \mathbf{Y}_{t-1} + \gamma \mathbf{X}_t + \Sigma\varepsilon_t \\
\mathbf{Y}_t &= [\pi_t, R_{lt}, dp_t]' \\
X_t &= \mathrm{MY}_t
\end{aligned}
$$

where π_t is the annual CPI inflation, $ggap_t$ is the HP-filtered output gap, R_{st} short term interest rate and π_t^{com} is the annual commodity price inflation.

The first row in Panel C of Table 1 compares the out-of-sample inflation forecasting performance of a VAR including only annual inflation and financial yields against the

32 In the table I report 5-year bond yields using post-war data which starts in 1952.

33 These variables are commonly used in the monetary VAR literature (e.g. Christiano, Eichenbaum and Evans, 1996; Bernanke and Mihov, 1998; Sims and Zha, 2006).

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benchmark VAR model. This model performs worse in terms of both criteria against the benchmark model in all the forecasting horizons. But the VAR model augmented with the exogenous MY_t, provides improved forecasting performance relative to the benchmark model, in particular in longer horizons. The key message of this section is not that the exogenous VAR suggested here is the best forecasting model for inflation (a more parsimonious AR(1) model augmented with MY_t performs better), yet the demographic variable captures an important common long-run component that can be useful for long horizon inference, so forecasting models should be fine-tuned to incorporate slow-moving predictable demographic fluctuations.

To highlight this important point, I first estimate the VAR model[34] for the whole sample, and use the coefficient estimates to solve the model forward staring in year 1968. Hence, I generate endogenous variables using only the ex-ante, i.e. model implied, values up to 2020. Figure 3 plots the historical series of endogenous variables in the model, with the model implied values, both including and excluding the exogenous MY_t. This historical simulation allows us to observe the level effect introduced by the demographic variable which captures the exogenous time variation in the population age structure. Such an effect is absent in a standard VAR with only endogenous variables. The model simulations also suggest that bond yields will be lower than the historical average over the next decade, while inflation and equity yields are expected to converge to their unconditional means.

Inflation and Valuation Models

Two empirical findings regarding the role of inflation in financial markets long perplexed both researchers and practitioners; i) strong comovement between equity yields, a real variable, and nominal bond yields in the post-war period, ii) strong positive correlation between equity yields and inflation.

The first empirical fact, also formalized under the Fed model, is used among investor professionals to detect stock market mispricing relative to bond market (e.g. Lander, Orphanides and Douvogiannis, 1997), but it is not clear to academics how to reconcile this empirical "anomaly" with rational explanations of stock pricing (Ritter and Warr, 2002; Asness, 2003; Estrada, 2009). The related second fact is in contrast with the Fisher hypothesis (1930) which posits a one-to-one relationship between inflation and nominal rates in a world of perfect foresight and casts serious doubt on inflation hedging

[34] I set the lag length to 2 following AIC(Akaike) and BIC(Bayesian) information criteria.

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ability of stocks for long term investors.

One possible explanation (Campbell and Vuolteenaho, 2004; Feinman, 2005) relies on behavioral biases, so-called Inflation (money) illusion formalized under Modigliani-Cohn hypothesis (1979). Such behavioral arguments suggest that stock market investors fail to increase the expected nominal cash payouts in response to increases in expected inflation, so that one can explain the comovement of dividend yield with nominal bond yields and negative relation between inflation and stock market returns. Yet, recent literature questions whether such a behavioral bias shared by only stock market participants can explain these facts (Thomas and Zhang, 2008; Wei, 2010). Another explanation is provided by Fama's Proxy Hypothesis (1981) which suggests that negative relation between stock returns and inflation is proxying for the positive relation between stock returns and expected real activity. My approach in this chapter differs from these explanations in terms of identifying the source of the comovement.

In the next subsections, I show how to reconcile the demographic factor with asset yields and inflation within a present-value approach. Provided that valuation ratios such as dividend yields are accurate anchors for long term stock return expectations as the dynamic dividend growth model suggests (Campbell and Shiller, 1988; Cochrane, 2008), these financial ratios are tightly linked to long term bond yields when the equity and bond market risk premia are strongly correlated and a relative premium is necessary to model periods in which volatilities in the two markets have been different (Bernstein, 1997; Asness, 2003).

Present Value Models and Demographics

Based on the theoretical model by GMQ (2004), FGT (2009) establish the empirical link between the slowly evolving mean in the log dividend-price ratio and demographic trends. FGT show how the GMQ model implies that a specific demographic variable, MY_t, the proportion of the middle-aged to the young population, explains fluctuations in the dividend yield within the dynamic dividend growth model proposed by Campbell and Shiller (1988). Below I summarize this relationship:

$$\mathrm{dp}_t = \overline{\mathrm{dp}_t} + \sum_{j=1}^{\infty} \rho_E^{j-1} \mathrm{E}_t[(\mathrm{r}^E_{t+j} - \bar{\mathrm{r}}) - (\Delta \mathrm{d}_{t+j} - \bar{\mathrm{d}})] \quad (3.11)$$

$$\overline{\mathrm{dp}_t} = \alpha_0 + \alpha_1 \mathrm{MY}_t + \mathrm{u}_t^{dp} \quad (3.12)$$

$$E_t(r^E_{t,t+h}) = \beta_0 + \beta_1 \mathrm{dp}_t + \beta_2 \mathrm{MY}_t + \varepsilon_{t,t+h} \quad (3.13)$$

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where dp_t is the log dividend-price ratio, $\overline{\mathrm{dp}_t}$ is the long-run time-varying mean of log dividend-price ratio, $r^E_{t,t+h}$ is the real holding-period return from the equity market for time t to t+h, $\Delta \mathrm{d}_t$ is the dividend growth and ρ_E is the loglinearization constant. The first equation is derived from the definition of returns applying a log-linear approximation under the assumption of stationary dividend yields. While Lettau and Van Nieuwerburgh (2008) challenge this point documenting breaks in the long-run mean of dividend yield, FGT (2009) argue in favor of a slow-evolving mean driven by demographic fluctuations based on the GMQ model (eq. 8). The last equation is the manifestation of this long-run equilibrium between dividend yields and the demographic factor, while deviations from the long-run relation predict expected future returns given the lack of dividend growth predictability (Cochrane, 2008). The life-cycle investment hypothesis formalized in the GMQ model (GMQ, 2004) suggests a negative α_1 and positive β_2 and these predictions are confirmed within the present-value framework (FGT, 2009).

Even though equity and bonds are perfect substitutes in the baseline (risklesss) GMQ model, the empirical evidence provided in the chapter is much stronger for equity markets and less convincing for riskless bonds. This is primarily because GMQ do not explicitly model the long term inflation which drives the long term interest rates. Hence the main prediction of the model is about the real rates, but not the nominal counterparts. FGY (2010) on the other hand focus on the nominal yields and decompose the short rate into permanent and temporary components following Fama (2006) and model the permanent component as a linear function of the demographic factor in an affine no-arbitrage term structure framework where the temporary component is driven by stationary state variables. Yet, in this model inflation is included as one of the state variables in the system and assumed to follow an exogenous AR process. Here based on the evidence in the previous sections, I relax this assumption and conjecture that inflation is not immune to exogenous demographic fluctuations.

We can derive an (exact) present value relation for the bond market based on the Campbell and Ammer (1993) decomposition for n-period zero-coupon bond returns. They decompose the expected excess bond returns into expected inflation, real rates and risk premia (Viceira, 2010). I write the log annual return from buying an k-year bond at time t and selling it as (k-1)-year bond at time t+1

$$r^B_{k,t+1} \equiv p_{k-1,t+1} - p_{k,t}$$

Solving the above equation forward and using the terminal condition $p_{0,t+k} = 0$, we

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obtain the ex-post relation

$$p_{k,t} = -\sum_{j=0}^{k-1} r^B_{k-j,t+1+j}$$

Defining the log excess bond returns as

$$xr^B_{k,t+1} = r^B_{k,t+1} - y_{1,t+1}$$

where $y_{1,t+1}$ is the nominal yield on the short rate from t to t+1 and the unexpected component as

$$\widetilde{xr}^B_{k,t+1} = xr^B_{k,t+1} - E_t xr^B_{k,t+1}$$

By taking expectations, the decomposition of expected excess bond returns is obtained from the following equation

$$\widetilde{xr}^B_{k,t+1} = (E_{t+1} - E_t)\left\{-\sum_{j=1}^{K-1}(y^r_{k,t+1+j} + \pi_{t+1+j} + xr^B_{k-j,t+1+j})\right\} \quad (3.14)$$

$$\overline{y}^r_t = \gamma_0 + \gamma_1 \mathrm{MY}_t + \mathrm{u}^y_t \quad (3.15)$$

$$\overline{\pi}_t = \delta_0 + \delta_1 \mathrm{MY}_t + \mathrm{u}^\pi_t \quad (3.16)$$

Equation (14) makes the role of long term expectations on excess bond returns explicit. In principle, any variable that proxies long term expectations of inflation, real risk-free rate or risk premia should also predict expected excess bond returns. Equations (15&16) are the empirical counterparts of equilibrium relations predicted by the theoretical models presented in the previous sections, where variables with bar $(\overline{y}_t, \overline{\pi}_t\}$ denote the time-varying long run means of real returns and inflation, respectively. The GMQ model suggests a negative loading γ_1 of the real component $\overline{y}^r_{,t}$ on MY_t[35] while the model with money demand suggests a similar (negative) loading of δ_1 on the demographic variable. In the empirical analysis, I will exploit these equilibrium relations between real rates, inflation and MY to test the role of demographic fluctuations on the predictability of bond returns.

In the following two sections, I first test different equity yield specifications implied by the Fed model to evaluate the role of demographics in determining the low-frequency comovement between equity and bond yields. Then, I assess the implications implied by the present value relations on return predictability by taking into account other sources related to time-varying risk premia.

[35] Notice that the real risk free-rate is assumed to have the same maturity as the long term nominal bond. We do not know empirically the loading of the demographic variable on the long term real rate, since the real component is unobservable. Yet, as a preliminary test I construct the real rate using the data provided by Bekaert and Engstrom (2010) extending it to the recent sample using the Kalman filter procedure suggested in their paper. An OLS regression of the real rate on MY_t delivers significant results with expected sign.

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Long-run Relation

Do we really understand the time-series relation between equity and bond yields, can we believe in a valuation model that relies on a joint mechanism that ties stock and the bond markets? How can we reconcile the stock market yield, a real variable, with nominal bond yields? In a recent paper, Bekaert and Engstrom (2010) attempt to answer these questions and suggest a mechanism where expected inflation coincides with periods of high uncertainty and risk aversion, hence the strong comovement between equity and bond yields (i.e. the Fed model) is rationalized. However, there are still some concerns that have to be addressed: i) the Fed model works perfectly in their subsample, but less so in the recent crises period and in the pre-war sample (e.g. Asness, 2003), ii) it might be conceivable to believe that short-term (e.g. one-year) inflation expectations are counter-cyclical, but it is less clear why we should expect a similar cyclical pattern for long-term inflation expectations which is the relevant concern for long term investors.

In this section, I first test different equity yield models in the post gold-standard period using annual data. In particular, I run standard OLS regressions of the equity yield on the long term (10-year) nominal bond yield and on the long term real bond yield. I control for relative stock-bond volatility, σ_{EB} (Asness, 2003) and time-varying habit based-risk aversion (Campbell and Cochrane, 1999), ra_t and test the role of demographic fluctutations in the equity-bond yield relation. Relative stock-bond volatility is logarithm of the realized volatility (10-year window of monthly observations). I construct the surplus ratio, $\mathrm{S}_t = \frac{\mathrm{C}_t - \mathrm{H}_t}{\mathrm{C}_t}$ to proxy the time-varying risk aversion, where C_t is the real personal consumption, and H_t is the 'habit stock', a 10 year moving average of past consumption levels. The dependent variable, equity yield, is either the dividend price ratio for the SP500 index (Panel A) or the cyclically adjusted earnings price ratio (Panel B) collected from Robert Shiller's website.

The first model (1) in Table 3 is the baseline Fed model including the correction for the relative volatility (Asness, 2003) between equity and bond markets; both variables enter significantly with an adjusted R^2 of 32% (19%) in Panel A(B). Once I replace the nominal bond yield with the real counterpart (contructed either by subtracting long term (annualized) realized inflation $\pi_{t,t+h}$ over the life of the bond (2) or the lagged value of inflation proxying for inflation expectations $\tilde{\pi}_{t,t+h}$(3)), both variables become insignificant and the adjusted R^2 drops significantly. This result hints at the importance of inflation expectations in the equity-bond yield relation, yet the proxy for inflation expectations is far from being perfect in the long annual sample. Once I include the demographic variable middle-aged to young ratio, MY_t, this variable enters very strongly

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with expected negative sign and increases the adjusted R^2 substantially (up to 70%) in both panels. This result remains virtually untouched once I include a risk aversion variable. This evidence suggests that the omitted demographic component plays an important role in determining the long-run level of both equity and bond yields.

The long annual sample suffers from the lack of both high frequency and survey-based data. I repeat the same analysis using the Bekaert and Engtrom sample extending into the crisis period up to 2009Q4. I construct realized second moment measures (e.g. Viceira, 2010) for equity and (5-year) bond yields using daily data (5-year window of daily observations from Gurkaynak, Wright and Sack (2006) database). I use survey-based long term SPF inflation expectations which is only available starting in 1980, but following Bekaert and Engstrom I obtain the early sample values applying a Kalman filter[36]. As an additional control, I also construct an inflation uncertainty variable, σ_t^{inf}, obtained from the individual SPF four-quarter inflation (GDP deflator) forecast dispersion (David and Veronesi, 2009).

In Table 4, we first notice the increasing adjusted R^2 supporting the success of the Fed model. The performance of the baseline model improves to 77% (76%). In the post-war sample, the real long term bond yield as well enters significantly with an adjusted R^2 of 60% (56%) in a specification without the demographic variable. Once MY_t is included in the model, it subsumes the significance of the long term real rate with highly significant (and stable across equations) coefficients supporting the predictions of the GMQ model. The inclusion of additional risk aversion and uncertainty controls does not change the picture pointing out the importance of demographic fluctuations.

Return Predictability, Is it There?

The present-value framework of Campbell and Shiller (1988) explains the link between equity yields and stock returns and justifies the use of dividend yields as a predictor of market returns (e.g. Ang and Bekaert, 2007). Consistently, FGT (2009) use this model to show that the GMQ model implies equity return predictability. Equation (11) above summarizes this relation. Hence, we expect that the close contemporaneous link between equity and bond yields implies that long term bond yields together with the demographic factor can be used to forecast stock market returns. The Campbell and Ammer (1993) model on the other hand allows us to test whether stock yields together

[36] I assume that four-quarter inflation expectations, long-term inflation expectations, nominal and real bond yields and inflation uncertainty evolve according to a stable VAR. Real rates are provided from Bekaert and Engstrom and extended using their suggested Kalman filter to the current period.

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with the demographic variable imply bond return predictability within the present value framework. Next, I test these implications for different investment horizons.

I run return forecasting regressions for real equity returns and excess one-year bond returns using the following specifications which include controls such as the consumption-wealth ratio, cay (Lettau and Ludvigson, 2001) and the tent-shape bond factor (Cochrane and Piazzesi, 2005). I restrict the sample to the one employed in the previous analysis, i.e. 1968q4-2009q4.

$$\begin{aligned} r^{E}_{t,t+h} &= \alpha_0 + \alpha_1 y_{5,t} + \alpha_2 dp_t + \alpha_3 \sigma_{SB} + \alpha_4 MY_t + e_{t,t+h}, h = 1,3,5 \\ xr^{B}_{k,t+1} &= \alpha_0 + \alpha_1 y_{5,t} + \alpha_2 dp_t + \alpha_3 \sigma_{SB} + \alpha_4 MY_t + e_{t,t+1}, k = 2,3,5 \end{aligned}$$

The return predictability results are reported in Table 5a and Table 5b. Table 5a reports the results for the long run (1-year, 3-year, 5-year) equity return predictability implied by the Fed model, an augmented model with demographic variable MY_t and the control variable, consumption-wealth ratio, cay_t. The Fed model performs poorly when the demographic variable is excluded. At a 1-year horizon, only the relative volatility variable enters significantly, while stock return predictability improves in longer horizons consistent with the prediction of the Fed model. The inclusion of the demographic variable substantially increases the adjusted R^2 at all horizons highlighting the role of persistent component in returns originating from demographic fluctuations, in line with the finding of FGT (2009). The consumption-wealth ratio, cay_t, further improves stock return predictability while the role of demographics remains significant in the presence of time varying risk premia.

Table 5b repeats the same analysis for the bond market; the dependent variable is the annual excess zero-coupon bond return for different maturities (2-year, 3-year, 5-year). I replace the consumption-wealth ratio with the tent-shaped Cochrane-Piazzesi factor as an additional control. The predictive power for bond returns in the baseline model (without demographics and control variable) is higher for 2-year bonds at 20% decreasing to 16% for 5-year bonds. The equity yield never enters significantly in the predictive regressions while long term nominal bond yields are highly significant. The role of relative volatility diminishes once we control for time varying risk premia via Cochrane-Piazzesi factor while MY_t consistently improves predictive power across bond returns with different maturities passing the test against the control variable. Overall, consistent with the predictions of the present-value models predictive regressions confirm the conjecture that deviations from the equilibrium relations both in equity and

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bond markets contain information regarding future returns.

Inflation Hedging and Long Term Investors

The time-series behavior of interest rates and inflation is not only essential for finance and macroeconomic theory, but also has practical relevance for long term investors for strategic and tactical asset allocations. Investors would like to know which financial assets are effective hedges against inflation to protect their wealth from erosion due to price inflation. In this section, I address this question in the presence of time variation in population age structure that cause persistent variation both in real interest rates and expected inflation. In particular, I follow two different approaches to build anticipated inflation and expected returns and test whether equities and bonds can provide effective hedges against unexpected inflation.

Direct Regressions

First, I estimate a system of equations that is consistent with the previous literature on return predictability (e.g. FGT, 2009; FGY, 2010) and with the evidence provided in this paper. The method of direct system estimation via generalized methods of moments (GMM) has the advantage of milder assumptions regarding the data-generating process error terms compared to a VAR-based multi-period prediction, but suffers from the lack of a large sample of independent observations due to overlapping observations. Hence, I also develop a VAR-based estimation to compare the results.

To establish the link with the Fed model, I estimate an eight-equation system by GMM, where the persistent component of each series is modelled with the middle-aged to young ratio,MY_t, while another variable (e.g. output gap or industrial production growth) proxies for the overall real activity:

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$$
\begin{aligned}
\pi_{t,t+1} &= \alpha_0 + \alpha_1 \pi_{t-1,t} + \alpha_2 \xi_t + \alpha_3 MY_{t+1} + e^1_{t,t+1} \\
\pi_{t,t+5} &= \alpha_4 + \alpha_5 \pi_{t-1,t} + \alpha_6 \xi_t + \alpha_7 MY_{t,t+5} + e^2_{t,t+5} \\
r_t &= \alpha_8 + \alpha_9 r_{t-1} + \alpha_{10} \pi_{t-1,t} + \alpha_{11} \xi_t + \alpha_{12} MY_t + e^3_t \\
y_{t,t+1} &= \alpha_{13} + \alpha_{14} r_t + \alpha_{15} \xi_t + \alpha_{12} MY_{t+1} + e^4_{t,t+1} \\
y_{t,t+5} &= \alpha_{16} + \alpha_{17} r_t + \alpha_{18} \xi_t + \alpha_{12} MY_{t,t+5} + e^5_{t,t+5} \\
dp_{t+1} &= \alpha_{19} + \alpha_{20} dp_t + \alpha_{21} MY_{t+1} + e^6_{t,t+1} \\
r^E_{t,t+1} &= \alpha_{22} + (1-\alpha_{20}) dp_t + \alpha_{23} r_t - \alpha_{21} MY_{t+1} + e^7_{t,t+1} \\
r^E_{t,t+5} &= \delta_{24} + \frac{(1-\alpha_{20})}{5} dp_t + \alpha_{25} r_t - \alpha_{21} (\sum_{j=1}^{5} \alpha^j_{20} MY_{t,t+5+1-j}) + e^8_{t,t+5}
\end{aligned}
$$

where $\pi_{t,t+1}$ is the inflation rate (GDP deflator), ξ_t is the real activity variable, i.e. industrial production growth[37], dp_t is the log equity yield, r_t is the short rate, $y_{t,t+k}$ is the bond yield of a K-year discount bond at time t (e.g. K=5: 5-year bond yield)[38] and $r^E_{t,t+h}$ is the holding period (h) market return.

The first two equations are consistent with the evidence provided in this paper. I impose restrictions in Equation 3-5 to allow for a Taylor-type policy rule consistent with no-arbitrage term structure of the interest rate that accounts for demographic fluctuations (FGY, 2010). The last three equations are taken from the specification suggested by Favero and Tamoni (2010) where the authors model the term structure of stock market risk. Expected values of each variable are the fitted values of each equation and residuals are interpreted as unexpected shocks. I estimate the system by GMM for the fiat-money sample, 1933-2009.

A few observations are worth noting regarding the estimation results: coefficients on the demographic variable enter significantly in each equation, while industrial production growth only enters significantly in bond yield equations. The adjusted R^2 for inflation (around 35%)[39] and bond yields (above 85%) are high[40], while the predictive ability for stock market returns is modest around 18(22)% for 1-year (5-year) returns, consistent with previous evidence in the literature (see appendix B for estimation results).

Insert Table 6 here

[37] The choice of the real activity variable is not consequential. Results are similar once output gap or unemployment rate is used instead.

[38] I take 10-year bond yields from Robert Shiller's database to proxy for the long term bond yield.

[39] The decrease in $\bar{R}^2$ can be partly attributed to the inclusion of the pre-war sample and partly to the fact that inflation is measured by the consumer price index (noisier) rather than through the implicit GDP deflator.

[40] The high explanatory power is largely due to the autoregressive term.

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In Panel A of Table 6, I report univariate regression coefficients of ex-post stock returns and bond yields on unexpected Inflation, where the latter is defined as the difference between actual and model implied inflation rates. An asset with significant and positive β loading indicates good hedge for unexpected inflation. I also report Pearson correlation coefficients between model implied returns (yields) and expected inflation for one-year and five-year horizon. β coefficients are both small and insignificant for both equity and bond returns either for one-year or five year investment horizon. These results cast serious doubt on the hedging ability of traditional financial assets for long-term investors against unexpected inflation.

VAR Approach

As I stressed above, the major limitation of the GMM approach is the overlapping observations that are used in direct regressions. To overcome this problem, I also estimate an exogenous VAR(p) system following Engsted and Tanggaard (2002) where the model includes the one-year stock return, the one-year bond return for 10-year treasury bond, the short rate (3-month T-bill), the inflation rate, the log dividend price ratio and the industrial production growth as endogenous variables and MY$_t$ as an exogenous variable. I set the lag length to one following the Schwarz Information Criterion (SIC)[41] and estimate the following model

$$\begin{aligned}
\mathbf{Y}_t &= \mu + \alpha_1 \mathbf{Y}_{t-1} + \beta \mathbf{X}_t + \Sigma \varepsilon_t \\
\mathbf{Y}_t &= [r^B_{t,t+1}, r_t, r^E_{t,t+1}, \pi_t, , dp_t, \xi_t]' \\
X_t &= MY_t \\
\hat{\pi}_{t,t+h} &= \pi_{t,t+h} - \tilde{\pi}_{t,t+h} = \pi_{t,t+h} - E(\sum_{i=0}^{h} \pi_{t+i,t+i+1} | \Theta_t) \\
\hat{\pi}_{t,t+h} &= \pi_{t,t+h} - e'_\pi \sum_{i=1}^{h} (\Phi^i \mathbf{Y}_{t-1} + \beta^i MY^h_t) \\
r^i_{t,t+h} &= e'_{r^i} \sum_{i=1}^{h} (\Phi^i \mathbf{Y}_{t-1} + \beta^i MY^h_t), i = E, B
\end{aligned}$$

where $r^i_{t,t+h}$,are h-period returns for equity and bonds, and MY^h_t is the time path of the middle-aged young ratio over time t to t+h. $\tilde{\pi}_{t,t+h}$, the h-period expected inflation, is obtained from the exogenous VAR projections. $\hat{\pi}_{t,t+h}$, the unexpected inflation is extracted by subtracting $\tilde{\pi}_{t,t+h}$ from the actual cumulative inflation $\pi_{t,t+h}$ over the

[41] I consider several other information criteria such as the sequential modified LR test, the Akaike information criterion and the Hannan-Quinn information criterion and chose the minimum lag suggested for parsimony.

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investment horizon.

Panel B of Table 6 reports the univariate regression coefficients of ex-post equity returns and bond yields on unexpected inflation extracted from the VAR system. The results are in line with the GMM estimations; the coefficient estimates are similar and insignificant for both asset types and investment horizon. Therefore, in light of these results, long-term investors should not be deceived by improved predictability in stock and bond markets which does not necessarily imply better hedging against inflation.

Cross-Country Evidence

The evidence so far is limited to the U.S. time series. Note that the OLG model which I use to motivate the empirical analysis makes strong simplifying assumptions: i) the model is designed for closed economies. ii) it is calibrated using U.S. data, iii) it assumes close to stationary population structure featuring boom and bust cycles. Given the low-frequency nature of the data and relatively short samples, it is hard to generalize the results and exclude alternative explanations between the tight link between the financial yields and inflation. One way to overcome this problem is proving evidence from other countries (Estrada, 2009; Bekaert and Engstrom, 2010). Cross-country analysis provides supporting evidence to the extent that simplifying assumptions are not largely violated. I consider the same panel of 20 countries in previous studies.

Although most countries in the sample exhibit similar cycles in their demographic structure, there is vast heterogeneity among countries in terms of the importance of stock markets as a channel for aggregate saving. This heterogeneity is evident in different stock market participation patterns (see Figure 4, Panel A).[42] We would expect higher demographic effects on financial markets in countries where equity markets play an important role for savings. To test this conjecture, I construct the demographic variable, middle-aged to young ratio for each country and run OLS regressions of stock market participation on the demographic variable. We obtain a positive slope coefficient of 1.83 (t-stat=2.46)) indicating a stronger demographic effect on financial markets in those countries with higher stock market participation rate.

Next, I run pooled OLS regressions to test whether the U.S. evidence on a shared demographic component in financial yields and inflation is also confirmed for the same panel of 20 countries. Table 7 reports regression coefficients for both the autoregressive

[42] Data are from Giannetti and Koskinen (2005) collected from several sources around the millenium. Hence it does not take into account the time-series variation in participation patterns.

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specification (log dividend yield, 10 year bond yield and inflation) and the augmented model including MY_t. Inflation and long term bond yield regressions are run on a balanced panel with 980 observations for the time period 1960-2009, while equity yield regressions are run on an unbalanced panel limited by yield data availability for each country.

Insert Table 7 here

Panel A shows the regression coefficients for the inflation time series in the panel; the coefficient on MY_t is -0.025 (t-stat=-5.13), while the autoregressive coefficient remains highly significant. The long term bond yield regressions exhibit a similar picture. MY_t is significant with a coefficient of -0.016 (t-stat= -6.21). In Panel C, MY_t enters significantly in the autoregressive specification for the log dividend yield series with a coefficient of -0.001 (t-stat=-2.63). We notice that the inclusion of the demographic variable does not change substantially the adjusted R^2 of the regressions, yet enters significantly despite the autoregressive term. The sign and the significance of these coefficients confirm the US evidence and provide out-of-sample support for the presence of a common demographic component proxied by MY_t[43].

Finally, I conduct a similar heuristic analysis suggested Bekaert and Engstrom (2010): The demographic effect on the equity yield is captured by the (negative) correlation between equity yield and MY_t for each country covering the annual sample from 1960 to 2009[44]. The cross-sectional equity-bond yield correlations (y-axis) and demographic effect on equity yield (x-axis) are plotted in Panel B of Figure 4[45]. The strong relationship is evident; the higher the effect of the demographic variable on financial markets (stronger negative correlation), the higher is the link between stock and bond yields. A cross-sectional OLS regression of the equity-bond correlation on demographic effect delivers a slope coefficient of 0.578 (t-stat=3.68) significant at the 1 percent level ($R^2 = 40\%$). The results are robust to outliers; a robust regression analysis delivers similar results; a slope coefficient of 0.590 (t-stat=3.43)[46]. Similarly, in Panel C of Figure 5, the equity-bond yield correlation on the y-axis is replaced with equity

[43] The results are robust to the inclusion of country-specific fixed effects.
[44] MY_t data for individual countries are obtained from World Bank projections and start in 1960. The Dividend yield series (Global Financial Data) of some countries is shorter (see appendix). For time series less than 30 years of data, I compute non-parametric (Spearman) correlations.
[45] There are some differences from the figure provided by Bekeart and Engstrom (2010). One notable difference is for Japan, where the correlation is positive in my sample. Note that I use a longer, but lower frequency sample and a different dataset. However, my results are consistent with the ones provided by Estrada (2009) who uses the same data source.
[46] Italy is the only country with a significant positive correlation between equity yields and MY_t which is not consistent with the predictions of GMQ model. Excluding this observation from the analysis does not change the results.

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yield-inflation correlation. Again, the demographic effect on the equity yield strongly drives the equity yield-inflation correlation ($R^2 = 53\%$). The regression line has a slope of 0.608 (t-stat=4.70)[47].

Overall, the effect of a time varying age-structure on financial markets varies substantially across countries covered in the panel and this demographic effect provides a consistent explanation for the joint path of equity and bond yields and the positive correlation between equity yields and inflation. This is an important channel which has not been explored in the previous literature in the context of the Fed Model and the Fisher hypothesis.

Concluding Remarks

This chapter documents the role of time variation in the population age structure in shaping financial markets through both real and nominal effects. This is not the first study that stresses the importance of demographics for financial markets. The net demand for financial assets by various age groups does provide important information on the aggregate demand for financial assets as the population structure changes. But the very same change has also important ramifications for inflation dynamics. Hence the novel approach in this chapter suggests a channel through which demography shapes the puzzling time series behavior of both key financial variables and inflation. Demographics cannot explain all the time-variation in these variables, neither it should. Yet, I argue that the first-order effects of the population age structure on financial markets are too important to be dismissed. This chapter is a further attempt to highlight this point.

Relying on a long U.S. time-series I show that the time variation in age structure proxied by a demographic variable, the middle-aged to young ratio, explains the puzzling high correlation between stock yields, a real variable, and nominal bond yields. The empirical proxy used in this study is derived from an OLG model calibrated to U.S. data, hence the results for international panel shall be taken with caution. A more detailed analysis with country specific models will be essential to fully understand the picture drawn in this paper.

This analysis has important implications for long term investors with stylized portfolio choice. Time series patterns of financial series and inflation not only drive the

[47] Because the variables included in the regression analysis are correlations and thus limited to the interval [-1,1], as a robustness check, I also transformed the variables applying ln(1+corr)/ln(2-corr), which maps the variables to the $[-\infty, \infty]$ interval. The t-stats and R^2 are lower, but still highly significant.

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predictability of returns, but are also crucial for portfolio allocations (Schotman and Schweitzer, 2000). In this paper, I address one aspect of the problem, namely inflation hedging. Following two diverse models, I show that neither bonds (as expected) nor stocks (in contrast to the conventional belief) provide a good hedge for unexpected inflation. A more elaborate analysis of portfolio weights in the context of long term investment is not addressed in this paper. A more comprehensive study for portfolio choice is certainly interesting and on my agenda for future research.

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Tables

TABLE 1. Out-of-sample Performance (1969-2009)								
Panel A		π_t	dp_t	R_{lt}		π_t	dp_t	R_{lt}
Coefficients (std. dev)	β_1	0.489 (0.02)	0.640 (0.07)	0.684 (0.06)	β_2	-0.063 (0.01)	-0.005 (0.00)	-0.036 (0.01)
Panel B		$RMSE_{AR(1)+MY}/RMSE_{AR(1)}$				$MAE_{AR(1)+MY}/MAE_{AR(1)}$		
Horizon		$h=1$	$h=3$	$h=5$		$h=1$	$h=3$	$h=5$
$\pi_{t,t+h}$		0.92	0.84	0.74		0.99	0.96	0.89
$dp_{t,t+h}$		1.01	0.96	0.85		1.03	1.00	0.91
$R_{lt,t+h}$		0.95	0.89	0.84		0.91	0.86	0.79
Panel C		$RMSE_{VAR^{MY}}/RMSE_{VAR^{M}}$				$MAE_{VAR^{MY}}/MAE_{VAR^{M}}$		
Horizon		$h=1$	$h=3$	$h=5$		$h=1$	$h=3$	$h=5$
$\pi^{En}_{t,t+h}$		1.07	1.15	1.22		1.03	1.10	1.15
$\pi^{Ex}_{t,t+h}$		0.93	0.84	0.76		0.94	0.87	0.79

Table 1. Out-of Sample Performance. Panel A shows the median coefficient estimates (standard deviations in parentheses over the sample 1969-2009) of β_1 and β_2 in the augmented autoregressive specification of π_t, inflation, dp_t,log dividend yield (scaled by 1/100) and R_{lt},5-year bond yield (start in 1952). Panel B compares the out-of sample forecasting performance of the autoregressive AR (1) specification with an alternative specification including MY_t. The table reports both the ratio of the root mean square error and mean absolute error of both models for different forecasting horizons h=1,3,5 years. The model is first estimated for the period 1933-1968 and first out of sample forecast is made for 1969 (h=1), 1972 (h=3), 1974 (h=5). Results from the rolling window estimations are reported for the period 1969-2009. Panel C compares the out-of-sample forecasting performance of both an endogenous and an exogenous VAR(1) model including annual inflation (CPI), log dividend-price ratio, long term bond yield (10-year) as endogenous variables and the demographic variable MY_t as an exogenous variable in the latter model. The benchmark multivariate model is a monetary VAR(1) including annual inflation, HP-filtered log real GDP, 3-month risk free rate and commodity price inflation. Annual sample.

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TABLE 2. VAR model. Sample 1933-2009.			
Model: $\mathbf{Y}_t = \mu + \Theta\mathbf{Y}_{t-1} + \gamma\mathbf{X}_t + \Sigma\varepsilon_t$			
$\mathbf{Y}_t = [\pi_t, R_{lt}, dp_t]'$, $\mathbf{X}_t$ =MY$_t$			
Variables	R$_{lt}$	π_t	dp$_t$
constant	0.007 (0.88)	0.038 (1.77)	−0.003 (−1.90)
R_{lt-1}	0.777 (6.16)	0.529 (1.62)	0.013 (0.49)
R_{lt-2}	0.135 (1.09)	−0.620 (−1.95)	−0.062 (−2.34)
π_{t-1}	0.064 (1.44)	0.640 (5.57)	−0.017 (−1.75)
π_{t-2}	−0.040 (−0.99)	−0.343 (−3.27)	0.014 (1.67)
dp$_{t-1}$	−0.650 (−1.16)	−1.438 (−0.99)	0.485 (4.02)
dp$_{t-2}$	0.376 (0.74)	−0.993 (−0.76)	0.025 (0.23)
MY$_t$	−0.015 (−1.32)	−0.105 (−3.58)	−0.012 (−5.06)
$\bar{R}^2$	0.92	0.54	0.86

Table 2. Alternative VAR. The VAR model includes long-term (10-year) bond yield, annual CPI inflation, log dividend yield (scaled by 1/100) in a VAR(2) system as endogenous variables, and middle-aged to young ratio (together with a constant) as exogenous variable. The lag length is set p=2 according to AICand BIC information criteria. The model is estimated for the period 1933-2009. Annual data.

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TABLE 3. Long-run Relation. Sample 1933-2009. Annual Data.

Dependent Variables	α	R_{lt}	R_{lt}^{r1}	R_{lt}^{r2}	σ_{EB}	ra_t	MY_t	$\bar{R}^2$	DW
Panel A. Dividend Yield									
(1)	0.021	0.306			0.015			0.32	0.60
	(3.15)	(2.90)			(4.22)				
(2)	0.037		0.033		0.008			0.21	0.49
	(8.00)		(0.27)		(2.82)				
(3)	0.034			0.113	0.009			0.22	0.49
	(7.06)			(0.84)	(3.15)				
(4)	0.099	–0.095			0.007		–0.067	0.70	1.30
	(9.04)	(–1.12)			(3.82)		(–8.96)		
(5)	0.096	–0.093			0.007	0.029	–0.067	0.71	1.29
	(9.87)	(–1.07)			(3.69)	(1.10)	(–9.26)		
Panel B. Earnings Yield									
(1)	0.025	0.547			0.016			0.19	0.33
	(1.64)	(2.71)			(2.69)				
(2)	0.062		0.061		0.004			-0.01	0.26
	(5.68)		(0.25)		(0.75)				
(3)	0.057			0.189	0.006			0.01	0.25
	(5.25)			(0.77)	(1.08)				
(4)	0.165	0.011			0.005		–0.121	0.70	0.84
	(9.12)	(0.08)			(1.50)		(–9.67)		
(5)	0.164	0.087			0.006	–0.014	–0.118	0.70	0.85
	(7.94)	(0.47)			(1.44)	(–0.34)	(–8.85)		

Table 3. Long-run Relation. This table reports the estimates of different equity yield models including R_{lt},nominal long term goverment bond yield, R_{lt}^{r1},real long term government bond yield obtained substracting either realized long term inflation ($\pi_{t,t+h}$) over the life of the bond or R_{lt}^{r2},expected (lagged) long term inflation $\tilde{\pi}_{t,t+h}$ controlling for σ_{EB},relative equity-bond volatility, ra_t, time-varying habit based-risk aversion (Campbell and Cochrane, 1999) and demographic fluctutations proxied by MY_t. Relative equity-bond volatility is logarithm of the ratio of the realized volatilities (10-year window of monthly observations). The dependent variable, equity yield is measured either by the (twelve-month) dividend price ratio to SP500 index or the cyclically adjusted earnings price ratio collected from Robert Shiller's website. Newey-West (1994) HAC corrected t-statistics are provided in parentheses to account for potential autocorrelation and heteroscedasticity in residuals. The last two columns report adjusted R^2 and Durbin-Watson statistic. Annual data.

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TABLE 4. Long-run Relation. Sample 1968q4-2009q4. Quarterly Data.									
Dependent Variables	α	R_{lt}	R^r_{lt}	σ_{EB}	ra_t	$\sigma_t^{\inf}$	MY_t	$\bar{R}^2$	DW
Panel A. Dividend Yield									
(1)	0.032	0.292		0.013				0.77	0.30
	(6.26)	(9.86)		(5.12)					
(2)	0.054		0.292	0.019				0.60	0.18
	(8.91)		(4.59)	(5.74)					
(3)	0.061	0.120		0.006			−0.034	0.86	0.39
	(8.05)	(2.52)		(2.88)			(−5.89)		
(4)	0.074		0.071	0.005			−0.044	0.84	0.35
	(16.99)		(1.23)	(2.60)			(−10.19)		
(5)	0.028	0.086		0.003	0.012	0.304	−0.037	0.89	0.57
	(2.44)	(2.75)		(1.30)	(3.46)	(2.71)	(−6.94)		
Panel B. Earnings Yield									
(1)	0.016	0.737		0.028				0.76	0.26
	(2.33)	(9.06)		(4.76)					
(2)	0.045		0.796	0.043				0.56	0.16
	(7.16)		(4.90)	(5.44)					
(3)	0.106	0.323		0.016			−0.074	0.85	0.29
	(5.86)	(2.64)		(3.08)			(−5.85)		
(4)	0.140		0.227	0.017			−0.097	0.83	0.24
	(11.84)		(1.290)	(2.60)			(−9.60)		
(5)	0.032	0.241		0.008	0.027	1.098	−0.075	0.90	0.50
	(1.48)	(3.17)		(1.66)	(4.15)	(2.56)	(−7.29)		

Table 4. Long-run Relation. This table reports the estimates of different equity yield models including R_{lt} nominal long term goverment bond yield, R^r_{lt}, real long term government bond yield obtained substracting survey based long term SPF inflation expectations, controlling for σ_{EB},relative equity-bond volatility, ra_t time-varying habit based-risk aversion (Campbell and Cochrane, 1999), $\sigma_t^{\inf}$, inflation uncertainty (David and Veronesi, 2009) and demographic fluctutations proxied by MY_t. Relative equity-bond volatility is the logarithm of the ratio of realized volatilities (5-year window of daily observations). The dependent variable, equity yield is measured either by the (twelve-month) dividend price ratio to SP500 index or the cyclically adjusted earnings price ratio collected from Robert Shiller's website. Newey-West (1994) HAC corrected t-statistics are provided in parentheses to account for potential autocorrelation and heteroscedasticity in residuals. The last two columns report adjusted R^2 and Durbin-Watson statistic. Quarterly data.

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TABLE 5.A. Equity Return Predictability. Sample 1968q4-2009q4. Quarterly Data.

Predictive Model: $r^{E}_{t,t+h} = \alpha_0 + \alpha_1 \mathrm{R}_{lt} + \alpha_2 \mathrm{dp}_t + \alpha_3 \sigma_{SB} + \alpha_4 \mathrm{MY}_t + e_{t,t+h},\ h = 1,3,5$

Dependent Variables	α	R_{lt}	dp_t	σ_{EB}	MY_t	cay_t	$\bar{\mathrm{R}}^2$
	0.081	0.084	0.044	−0.045			0.08
	(0.73/0.468)	(0.23/0.818)	(1.39/0.167)	(−1.95/0.054)			
$r^{E}_{t,t+1}$	0.032	0.564	0.158	−0.040	0.304		0.31
(t–$stat$/p-$value$)	(2.44/0.016)	(1.42/0.159)	(5.51/0.000)	(−1.76/0.080)	(4.22/0.000)		
	0.138	0.457	0.111	−0.026	0.225	0.783	0.36
	(1.49/0.138)	(1.29/0.198)	(3.109/0.002)	(−1.24/0.216)	(3.04/0.003)	(2.24/0.027)	
	0.161	0.292	0.033	−0.042			0.22
	(2.81/0.006)	(1.22/0.223)	(1.43/0.156)	(−3.06/0.003)			
$r^{E}_{t,t+3}$	0.0356	0.587	0.129	−0.037	0.247		0.61
(t–$stat$/p-$value$)	(0.43/0.670)	(2.87/0.005)	(6.34/0.000)	(−3.51/0.001)	(5.11/0.000)		
	0.089	0.491	0.083	−0.025	0.165	0.787	0.72
	(1.58/0.116)	(3.28/0.001)	(3.95/0.000)	(−3.03/0.003)	(4.41/0.000)	(4.89/0.000)	
	0.033	0.365	0.035	−0.043			0.36
	(0.68/0.495)	(2.25/0.026)	(2.65/0.009)	(−4.45/0.000)			
$r^{E}_{t,t+5}$	0.118	0.570	0.101	−0.038	0.176		0.63
(t–$stat$/p-$value$)	(2.79/0.006)	(4.42/0.000)	(6.03/0.000)	(−4.26/0.000)	(4.32/0.000)		
	0.052	0.477	0.056	−0.031	0.083	0.810	0.76
	(1.71/0.09)	(4.73/0.000)	(4.50/0.000)	(−5.11/0.000)	(2.70/0.008)	(5.34/0.000)	

Table 5.A. Equity Return Predictability. This table reports the results of the long run (1-year, 3-year, 5-year) equity return predictability tests on the variables implied by the Fed model. In particular, I\ regress cumulative real stock market (SP500) returns on nominal long term goverment bond yield, R_{lt},equity yield dp_t (measured as log dividend price ratio), relative equity-bond volatility, σ_{EB}, and middle aged-young ratio MY_t controlling for consumption-wealth ratio, cay_t (from Martin Lettau's website). The last column reports adjusted R^2 and Newey-West (1994) HAC corrected t-statistics and p-values are provided in parentheses to account for potential autocorrelation and heteroscedasticity in residuals. Quarterly data.

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TABLE 5.B. Bond Return Predictability. Sample 1968q4-2009q4. Quarterly Data.

Predictive Model: $xr^B_{k,t+1} = \alpha_0 + \alpha_1 R_{lt} + \alpha_2 dp_t + \alpha_3 \sigma_{SB} + \alpha_4 MY_t + e_{t,t+1}, k = 2,3,5$

Dependent Variables	α	R_{lt}	dp_t	σ_{EB}	MY_t	CP_t	$\bar{R}^2$
$xr^B_{2,t+1}$ (*t–stat/p-value*)	−0.082 (−2.51/0.01)	0.428 (3.32/0.001)	−0.010 (−1.17/0.246)	−0.014 (−1.93/0.055)			0.20
	−0.057 (−1.79/0.076)	0.501 (3.97/0.000)	0.011 (1.05/0.297)	−0.013 (−1.87/0.063)	0.054 (3.06/0.003)		0.24
	−0.062 (−1.83/0.07)	0.402 (3.59/0.000)	0.002 (0.20/0.84)	−0.004 (−0.72/0.472)	0.046 (2.52/0.013)	0.307 (3.65/0.000)	0.34
$xr^B_{3,t+1}$ (*t–stat/p-value*)	−0.158 (−2.72/0.007)	0.730 (3.01/0.003)	−0.021 (−1.33/0.187)	−0.025 (−1.85/0.067)			0.18
	−0.113 (−2.01/0.046)	0.858 (3.60/0.000)	0.015 (0.82/0.411)	−0.024 (−1.75/0.082)	0.094 (2.88/0.005)		0.22
	−0.124 (−2.11/0.036)	0.643 (3.24/0.002)	−0.003 (−0.15/0.883)	−0.006 (−0.49/0.627)	0.077 (2.38/0.019)	0.670 (4.72/0.000)	0.35
$xr^B_{5,t+1}$ (*t–stat/p-value*)	−0.247 (−2.63/0.010)	1.111 (2.74/0.007)	−0.031 (−1.17/0.244)	−0.045 (−1.93/0.055)			0.16
	−0.175 (−1.98/0.049)	1.316 (3.29/0.001)	0.027 (0.90/0.368)	−0.042 (−1.82/0.070)	0.150 (2.67/0.009)		0.19
	−0.197 (−2.23/0.027)	0.911 (2.91/0.004)	−0.007 (−0.22/0.823)	−0.008 (−0.46/0.649)	0.119 (2.28/0.024)	1.258 (5.50/0.000)	0.35

Table 5.B. Bond Return Predictability. Panel A reports the results of the annual return to 2-year, 3-year and 5-year bond return predictability tests on thevariables implied by the Fed model. In particular, I\ regress cumulativereal stock market (SP500) returns on nominal long term goverment bond yield, R_{lt},equity yield dp_t (measured as log dividend price ratio), relative equity-bond volatility, σ_{EB}, and middle aged-young ratio MY_t controlling for tent-shaped Cochrane-Piazzesi factor. I report both adjusted R^2 and Newey-West (1994) HAC corrected t-statistics and p-values in parentheses to account for potential autocorrelation and heteroscedasticity in residuals. Quarterly data.

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TABLE 6. Univariate Regression of Ex-post Returns on Unexpected Inflation

Univariate Model: $r^i_{t,t+h} = \alpha + \beta\hat{\pi}_{t,t+h} + \varepsilon_t$, i=$\{E, B\}$, h=$\{1, 5\}$								
Horizon	1-Year				5-Year			
GMM	α	β	$\bar{R}^2$	$\rho(\tilde{r}_t, \tilde{\pi}_t)$	α	β	$\bar{R}^2$	$\rho(\tilde{r}_{t,t+5}, \tilde{\pi}_{t,t+5})$
stocks	0.128	-0.584	-0.01	0.02	0.121	0.359	0.00	0.45
$(t-stat)$	(6.23)	(-1.45)		(0.17)	(8.60)	(0.51)		(4.41)
bonds	0.056	-0.151	-0.01	0.47	0.055	-0.165	-0.01	0.53
$(t-stat)$	(5.31)	(-0.97)		(4.65)	(6.06)	(-0.61)		(5.45)
VAR	α	β	$\bar{R}^2$	$\rho(\tilde{r}_t, \tilde{\pi}_t)$	α	β	$\bar{R}^2$	$\rho(\tilde{r}_{t,t+5}, \tilde{\pi}_{t,t+5})$
stocks	0.137	-0.249	-0.01	0.00	0.104	0.510	0.01	0.57
$(t-stat)$	(3.45)	(-0.30)		(0.00)	(4.03)	(0.94)		(5.96)
bonds	0.081	-0.704	0.03	0.37	0.056	-0.034	-0.01	0.50
$(t-stat)$	(4.30)	(-2.08)		(3.47)	(5.03)	(-0.21)		(5.04)

Table 6. Inflation Hedging. This table reports univariate regression coefficients of ex-post returns on unexpected Inflation both for the period 1933-2009. $\backslash$ Expected inflation, expected stock returns and bond yields are the fitted values from a system of equations estimated by general methods of moments (GMM) and an unrestricted exonegous VAR including short rate, 1-year and 5-year bond yields, inflation, log dividend-price ratio and output gap as endogenous variables and middle-aged to young ratio as exogenous variable. $\rho(\tilde{r}_t, \tilde{\pi}_t)$ indicate the Pearson correlation coefficient (associated t-statistics) of the model implied returns (yields) and inflation. The table reports adjusted R^2 and Newey-West (1994) HAC corrected t-statistics in parentheses to account for potential autocorrelation and heteroscedasticity in residuals. Sample 1933-2009. Annual data.

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TABLE 7. Pooled OLS Regressions					
Benchmark model: $x_t = \alpha_0 + \alpha_1 x_{t-1} + \varepsilon_t$					
Augmented model: $x_t = \beta_0 + \beta_1 x_{t-1} + \beta_2 \mathrm{MY}_t + \varepsilon_t$					
Specification	π_{t-1}	R_{lt-1}	DP_{t-1}	MY_t	$\bar{\mathrm{R}}^2$
Panel A. Annual Inflation, π_t					
(1)	0.820 (44.29)				0.67
(2)	0.767 (36.48)			−0.025 (−5.13)	0.68
Panel B. Bond Yield, R_{lt}					
(1)		0.939 (81.44)			0.87
(2)		0.892 (65.86)		−0.016 (−6.21)	0.87
Panel C. Equity Yield, DP_t					
(1)			0.849 (47.75)		0.73
(2)			0.825 (37.72)	−0.001 (−2.63)	0.73

Table 7. Pooled OLS Regressions. This table reports the pooled OLS regression coefficients (t-statistics in parentheses) for eachspecification (1) benchmark model and (2) augmented model with MY_t for annual inflation π_t, long term bond yield R_{lt} and log dividend yield dp_t. The last column reports the adjusted R^2. In Panel A and Panel B, the regressions are run in a balanced panel of 980 observation, while Panel C regressions are run in an unbalanced panel due to the lack of longer historical equity yield series.

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Figures

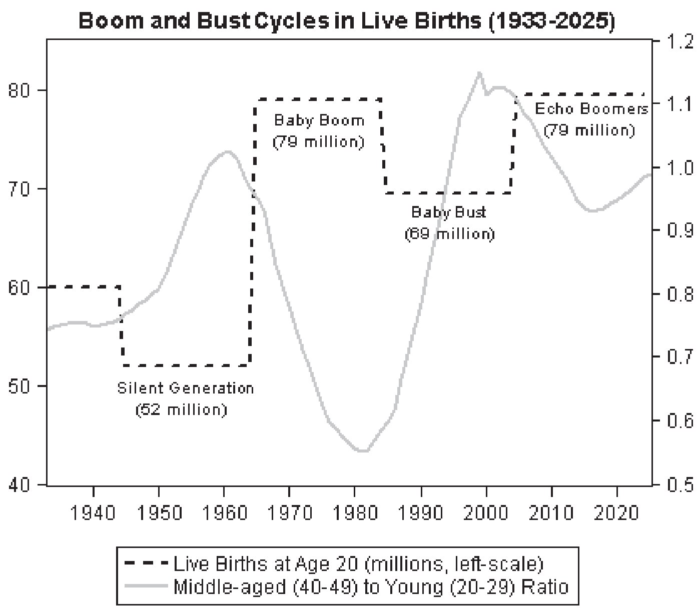

Figure 1. Boom Bust Cycles in Live Births. This figure plots the total number of life births (bar graph with dashed-line) at age 20 (the start of economic life) and the demographic variable, MY_t(solid line), measured as the proportion of middle-aged (40-49) to young (20-29) population. Sample 1933-2024. Annual data.

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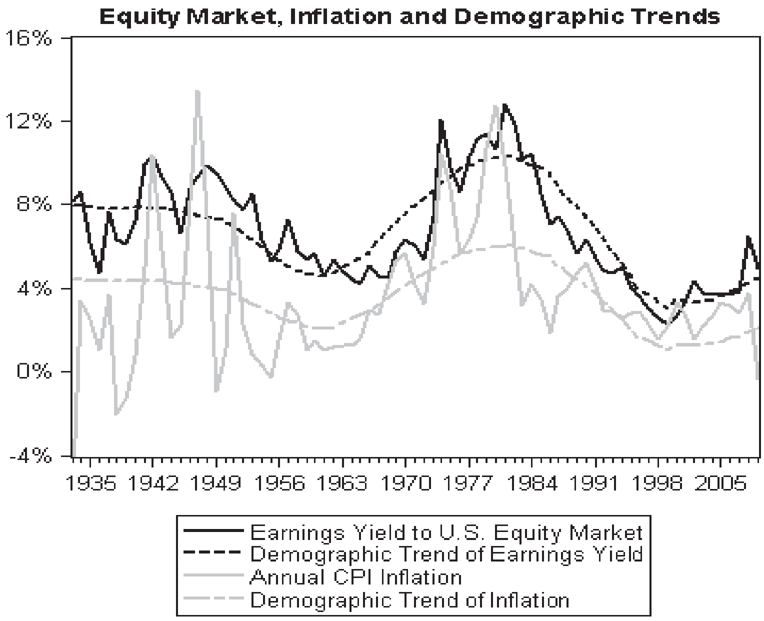

Figure 2. Equity Market, Inflation and Demographic Trends. This figure plots the earnings yield collected from Robert Shiller's database (using a 10-year window of earnings) and the annual CPI inflation together with the trend components obtained by fitting each variable on the demographic variable, MY_t, the proportion of middle-aged to young population. Sample 1933-2009. Annual data.

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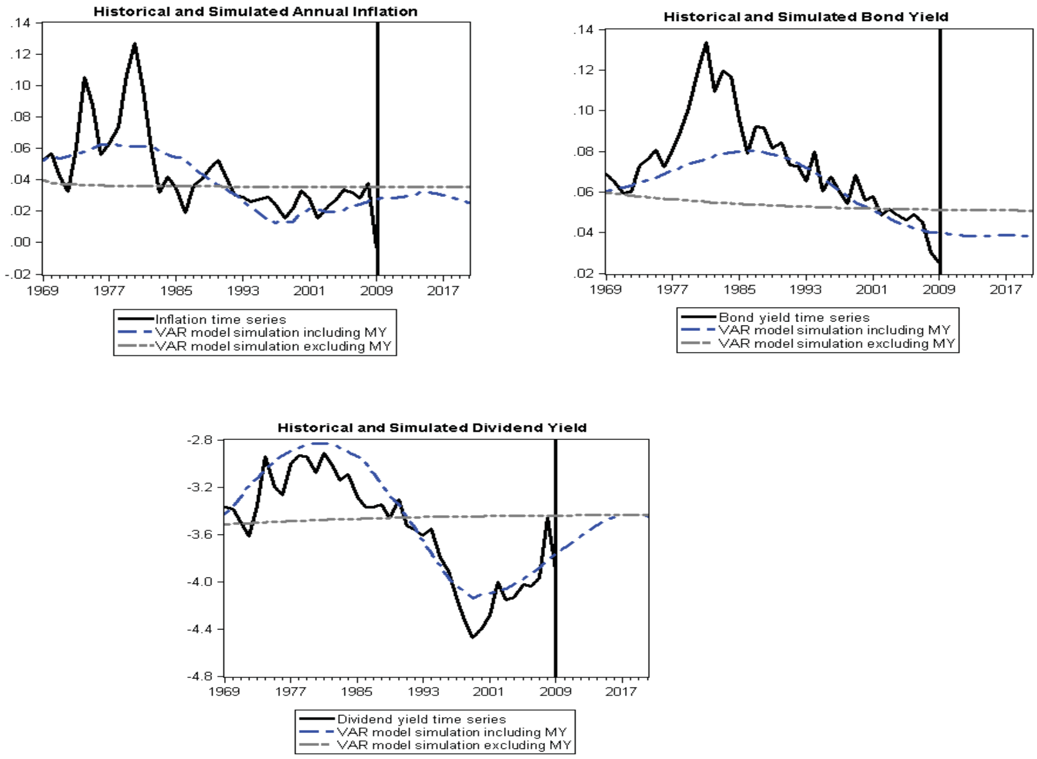

Figure 3. VAR Model Simulations. This figure plotsthe historical time series for long-term (10-year) bond yield, annual CPIinflation and dividend yield). VAR model simulation of a system consistingof these three time series and an exonegous VAR system including middle-aged to young ratio (MY_t) as an exogenous variable. The models are estimated for the period 1933-2009; I take as initial point 1968 and solve forward the models up to 2020 to obtain model-implied series. Vertical line at 2009 indicates the end of the in-sample data and start of future projections. Annual data.

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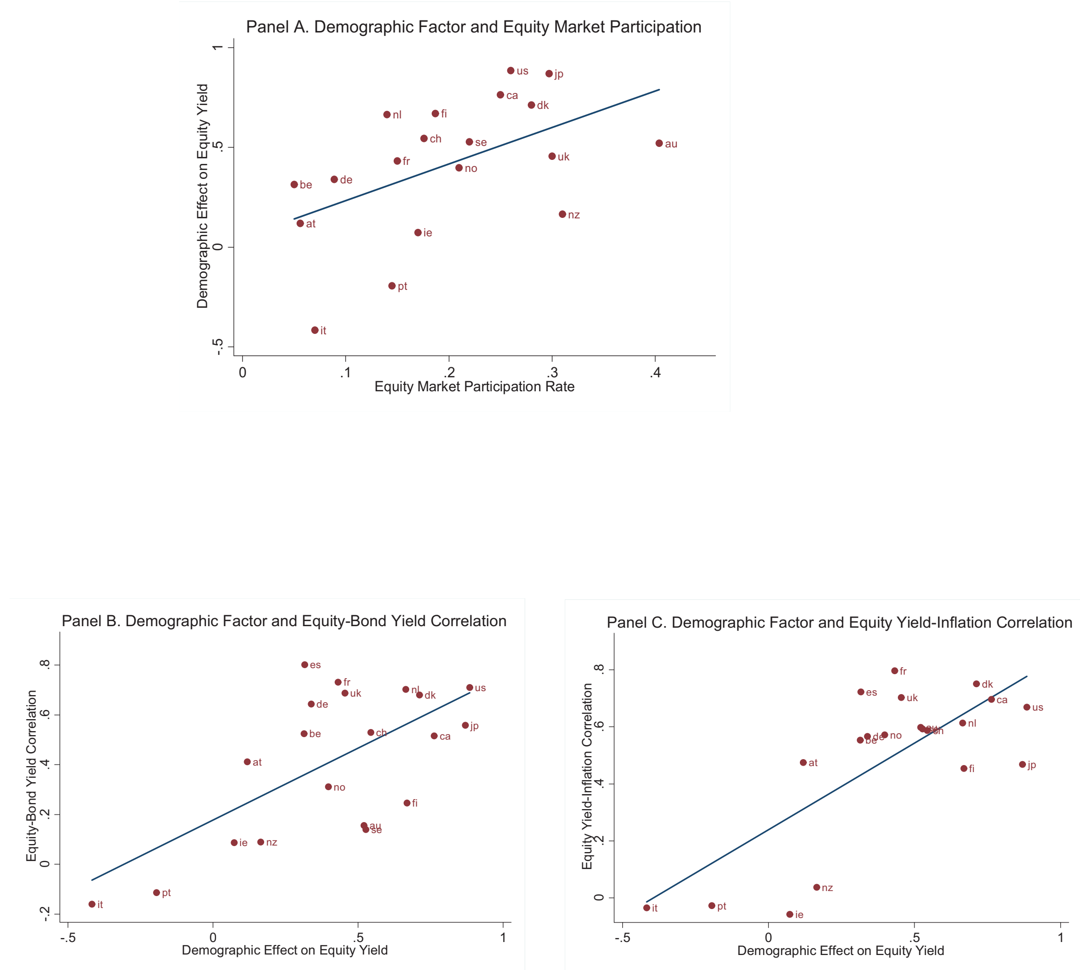

Figure 4. Cross-country Relationship. Panel A provides a scatterplot along demographic effect on equity yield (y-axis, measured by the negative correlation between dividend yield and MY_t) and the stock market participation rates (x-axis, obtained by Giannetti and Koskinen, 2005). The slope of the regression line is 1.83 (t-stat=2.46). Panel B plots countries in the panel set along the the country specific time-series correlation between equity yield and the long-term nominal bond yield (y-axis) and the demographic effect on equity yield (x-axis). The slope of the regression line is 0.578 (t-stat=3.68). Panel C plots countries in the panel set along b) the country specific time-series correlation between dividend yield and inflation (y-axis) and the demographic effect on equity yield. The slope of the regression line is 0.608 (t-stat=4.70). The sample is annual (details are provided in Appendix C).

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discussa presso Università Commerciale Luigi Bocconi-Milano nell'anno 2011
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E APPENDIX A

Description of Time-series and Data Sources

Stock Market Prices: S&P 500 index yearly prices from 1933 to 2009 are from Robert Shiller's website, I take december observations.

Stock Market Dividends: Dividends are twelve-month moving sums of dividends paid on the S&P 500 index. They are from the Robert Shiller website for the period 1900-2008. These series coincide with those used in Goyal and Welch (2008).

Stock Market Returns: For S&P 500 index, to construct the continuously compounded return r_t, I take the ex-dividend-price P_t add dividend D_t over P_{t-1} and take the natural logarithm of the ratio. I also use the value-weighted S&P 500 index returns collected from WRDS database.

Risk-free Rate: 3-Month Treasury Bill rate is taken from Goyal and Welch (2008) extended collecting data from St.Louis (FRED).

Log Dividend-Price Ratio (dp_t): the difference between the log of dividends (one-year trailing) and the log of prices.

Inflation: Long time-series of CPI inflation data is collected from Robert Shiller's website, while GDP implicit price deflator or chain price index (collected from St.Louis (FRED)) is used to calculate post-war inflation.

Bond yields: Long-term government bond yields are both from Ibbotson's Stocks, Bonds, Bills and Inflation Yearbook taken from Goyal and Welch (2008) and from Robert Shiller's website (annual one-year treasury bond-yield series). Daily 5-year bond yields are collected from Gurkaynak, Wright and Sack (2007) database. I also use Fama-Bliss Discount Bond Prices from CRSP database to calculate annual bond returns.

Long Term Rate of Return: Long-term government bond returns for the period are from Ibbotson's Stocks, Bonds, Bills and Inflation Yearbook taken from Goyal and Welch (2008).

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Demographic Variable: The U.S. annual population estimates series are collected from U.S. Census Bureau and the sample covers estimates from 1900-2050. Middle-aged to young ratio,MY_t is calculated as the ratio of the age group 40-49 to age group 20-29.

Output gap: is calculated as the difference of real GDP at 2005 prices from the potential GDP (over potential GDP) taken from St.Louis (FRED). Alternatively we also calculate output gap as HP filtered series of real GDP with smoothing parameter $\lambda = 100(14400)$ for annual (quarterly) data.

Industrial production: Seasonally adjusted montly industrial production index series is taken from St.Louis (FRED). Quarterly and yearly series are obtained from montly averages.

Commodity price inflation: The annual log difference of Dow Jones-AIG commodity price index (Global Financial data).

UK Data: UK inflation data is computed from the composite price index (Office for National Statistics). One-year and long-horizon (5-year) UK bond yields are collected from datastream. UK demographic data is collected from UN Demographic Yearbook for the period 1948-1990 and from International database (US Census Bureau) for 1991-2009.

One-year inflation expectations: One-year inflation expectations collected both from University of Michigan Survey provided by St.Louis (FRED) and Survey of Professional Forecasters provided by Philadelphia FED.

Inflation Uncertainty: Following David and Veronesi (2009), I obtain survey data from the Survey of Professional Forecasters, available at the Federal

Reserve Bank of Philadelphia. The data are available since 1968 with several missing entries. I use the cross-sectional dispersion of the percentage inflation growth (four-quarter, column2 and column6, see SPF documentation) of individual forecasts as a measure of forecasters' uncertainty. Formally, for each quarter t, let $F_{i,t+4}$ be the forecast of individual i of the price index level at time t + 4; and let I_t be its current level. If n_t is the number of individuals at time t, I define the time t "uncertainty" on the inflation at time t +4 as

$$\sigma_{t,4}^{\inf} = \sqrt{\frac{1}{n_t}\sum_{i=1}^{n_t}(\frac{F_{i,t+4}}{I_t} - \frac{1}{n_t}\sum_{i=1}^{n_t}\frac{F_{i,t+4}}{I_t})^2}$$

to avoid typos and mistakes, I delete observations for $\frac{F_{i,t+4}}{I_t}$ that are four standard deviations away from the mean forecast.

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Realized Volatility: To calculate the realized volatility from month t to month t+n, I use the measure of integrated instantaneous volatility $\widehat{\sigma}^2_{t,t+n} = \frac{1}{22n}\sum_{d=t_1}^{t_D} y^2_{i,d}$, $i = E, B$, where the number of days in a month is normalized to 22. Daily (annualized) yields are obtained from GWS (2007), and daily equity yields are calculated by dividing the monthly cumulative (12-month) dividend (earnings) data over daily SP500 index collected from CRSP database.

International Database: Cross-country CPI inflation, stock and bond yields are collected from Global Financial Data. Stock yield is the dividend yield to the benchmark index and bond yield is the 10-year constant maturity government bond yields. For Finland and Japan, I use shorter maturity bonds, 5-year and 7-year, respectively, since a longer time-series is available. MY_t estimates for 1960-2008 are from World Bank Population estimates and projections from 2009-2050 are collected from International database (US CensUS Bureau).

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F APPENDIX B

Estimation Results for Direct Regressions via GMM and VARX

In this appendix, I provide the estimation results for the model introduced in section IV. The eigth-equation system is estimated via generalized methods of moments (GMM) where the moments are the OLS normal conditions and the first stage weighting matrix is the identity matrix. Below, the coefficient estimates (associated HAC corrected t-statistics in parenthesis) and adjusted R^2 are reported for each equation. The model is estimated with annual data for the fiat-money (after the gold standard) period, 1933-2009. The real activity is proxied by annual industrial production growth.

$$
\begin{aligned}
\pi_{t,t+1} &= \underset{(2.91)}{0.053} + \underset{(5.11)}{0.490}\pi_{t-1,t} + \underset{(1.96)}{0.080}X_t - \underset{(-2.55)}{0.044}\,MY_{t+1} + e^1_{t,t+1} \quad, \bar{R}^2 = 0.39 \\
\pi_{t,t+5} &= \underset{(2.67)}{0.066} + \underset{(3.00)}{0.302}\pi_{t-1,t} + \underset{(1.26)}{0.048}X_t - \underset{(-2.26)}{0.053}\,MY_{t,t+5} + e^2_{t,t+5}, \bar{R}^2 = 0.36 \\
r_t &= \underset{(2.38)}{0.014} + \underset{(15.85)}{0.872}r_{t-1} + \underset{(1.18)}{0.050}\pi_{t-1,t} + \underset{(0.33)}{0.003}X_t - \underset{(-1.87)}{0.013}\,MY_t + e^3_t, \bar{R}^2 = 0.84 \\
y_{t,t+1} &= \underset{(2.75)}{0.016} + \underset{(19.31)}{1.062}r_t + \underset{(2.20)}{0.031}X_t - \underset{(-1.87)}{0.013}\,MY_{t+1} + e^4_{t,t+1}, \bar{R}^2 = 0.87 \\
y_{t,t+5} &= \underset{(5.32)}{0.033} + \underset{(20.34)}{0.827}r_t + \underset{(1.98)}{0.015}X_t - \underset{(-1.87)}{0.013}\,MY_{t,t+5} + e^5_{t,t+5}, \bar{R}^2 = 0.85 \\
dp_{t+1} &= \underset{(-3.43)}{-0.586} + \underset{(11.06)}{0.701}\,dp_t - \underset{(-3.28)}{0.510}\,MY_{t+1} + e^6_{t,t+1}, \bar{R}^2 = 0.79 \\
r^E_{t,t+1} &= \underset{(3.57)}{0.670} + (1 - \underset{(11.06)}{0.701})dp_t + \underset{(1.19)}{1.007}r_t + \underset{(-3.28)}{0.510}\,MY_{t+1} + e^7_{t,t+1}, \bar{R}^2 = 0.19 \\
r^E_{t,t+5} &= \underset{(0.81)}{0.059} + \frac{(1 - \underset{(11.06)}{0.701})}{5}dp_t + \underset{(0.95)}{0.568}r_t + \underset{(-3.28)}{0.510}\,(\sum_{j=1}^{5}\underset{(11.06)}{0.701}^j MY_{t,t+5+1-j}) + e^8_{t,t+5} \,, \bar{R}^2 = 0.22
\end{aligned}
$$

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VAR model

Below the estimation results for the following exogenous VAR(1) model are reported

$$
\begin{aligned}
\mathbf{Y}_t &= \mu + \alpha_1 \mathbf{Y}_{t-1} + \beta \mathbf{X}_t + \Sigma \varepsilon_t \\
\mathbf{Y}_t &= [r^B_{t,t+1}, r_t, r^E_t, \pi_t, dp_t, \Delta ip_t]' \\
X_t &= MY_t
\end{aligned}
$$

wh ich includes $r^B_{t,t+1}$,one-year bond return for 10-year treasury bond, r_t,short rate (3-month T-bill), $r^E_{t,t+1}$,one-year stock return, π_t inflation rate (CPI inflation), dp_t,log dividend price ratio and Δip_t,industrial production growth as endogenous variables and MY$_t$,middle-aged to young ratio as an exogenous variable in the system. Table B1 reports both the coefficient estimates (t-statistics in parentheses) and in the last row adjusted R^2 for each equation. The lag lenght is chosen to be one according to Schwartz information criterion.

Table B1. exogenous VAR model. Sample 1933-2009.

Variables	$r^B_{t,t+1}$	r_t	$r^E_{t,t+1}$	π_t	dp_t	Δip_t
$r^B_{t-1,t}$	-0.137 (-1.17)	-0.068 (-4.62)	0.356 (1.49)	-0.080 (-2.19)	-0.365 (-1.45)	0.144 (1.17)
r_{t-1}	0.930 (2.09)	0.884 (15.73)	1.716 (1.89)	0.259 (1.87)	-2.189 (-2.29)	-0.714 (-1.53)
$r^E_{t-1,t}$	-0.038 (-0.70)	0.012 (1.78)	0.040 (0.37)	-0.040 (-2.42)	0.112 (0.97)	0.028 (0.50)
π_{t-1}	-0.075 (-0.24)	0.001 (0.01)	0.437 (0.70)	0.290 (3.05)	0.059 (0.09)	-0.287 (-0.89)
dp_{t-1}	-0.034 (-0.88)	-0.010 (-2.02)	0.332 (4.23)	-0.001 (-0.09)	0.635 (7.68)	0.025 (0.62)
Δip_{t-1}	-0.006 (-0.05)	0.009 (0.55)	-0.173 (-0.69)	0.074 (1.93)	0.497 (1.88)	-0.056 (-0.44)
MY$_t$	-0.030 (-0.29)	-0.029 (-2.19)	0.694 (3.27)	-0.050 (-1.54)	-0.834 (3.72)	-0.047 (-0.43)
$\bar{R}^2$	0.08	0.89	0.18	0.42	0.84	0.04

Table B1. reports the estimation results of the exogenous VAR(1). Annual Sample.

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G APPENDIX C

Cross-Country Database

The table below summarizes the cross-country information used in the paper.

Table C1. Cross-country Panel. Sample 1960-2009. Annual Data.

Country	Code	Avg. MY	Range MY	$\rho(DP_t,R_{lt})$	$\rho(DP_t,\pi_t)$	$\rho(DP_t,MY)$	$\rho(R_{lt}, MY)$	$\rho(\pi_t,MY)$	Start Year
Australia	au	0.85	[0.63,1.08]	0.156 (1.09)	0.598 (5.17)	−0.521 (−4.22)	−0.784 (−8.76)	−0.772 (−8.42)	1960
Austria	at	0.92	[0.84,1.40]	0.412 (2.83)	0.475 (3.37)	−0.119 (0.75)	−0.822 (−9.00)	−0.411 (−2.82)	1969
Belgium	be	0.96	[0.69,1.25]	0.525 (4.28)	0.554 (4.62)	−0.314 (−2.29)	−0.839 (−10.70)	−0.391 (−2.94)	1960
Canada	ca	0.86	[0.56,1.23]	0.516 (4.17)	0.696 (6.71)	−0.763 (−8.18)	−0.821 (−9.96)	−0.716 (−7.10)	1960
Denmark	dk	0.94	[0.71,1.33]	0.680 (5.79)	0.751 (7.10)	−0.712 (−6.34)	−0.827 (−9.19)	−0.747 (−7.03)	1969
Finland	fi	0.93	[0.65,1.30]	0.246 (1.72)	0.454 (3.45)	−0.669 (−6.11)	−0.522 (−4.50)	−0.792 (−8.80)	1962
France	fr	0.90	[0.69,1.11]	0.732 (7.38)	0.797 (9.15)	−0.432 (−3.32)	−0.811 (−9.59)	−0.677 (−6.37)	1960
Germany	de	0.98	[0.68,1.45]	0.644 (5.89)	0.567 (4.77)	−0.339 (−2.49)	−0.646 (−5.86)	−0.232 (−1.65)	1960
Ireland	ie	0.80	[0.60,1.13]	0.087 (0.37)	−0.058 (−0.24)	−0.073 (−0.31)	0.345 (1.56)	−0.574 (−2.97)	1990
Italy	it	0.95	[0.78,1.54]	−0.160 (−1.16)	−0.035 (−0.24)	0.417 (3.18)	−0.402 (−3.04)	−0.175 (−1.23)	1960
Japan	jp	0.90	[0.59,1.17]	0.558 (4.65)	0.468 (3.67)	−0.870 (−12.22)	−0.594 (−5.12)	−0.571 (−4.81)	1960
Holland	nl	0.88	[0.65,1.33]	0.703 (6.18)	0.614 (4.86)	−0.665 (−5.55)	−0.875 (−11.30)	−0.543 (−4.04)	1969
New Zealand	nz	0.83	[0.61,1.13]	0.089 (0.44)	0.037 (0.18)	−0.165 (−0.82)	−0.848 (−7.85)	−0.441 (−2.41)	1984
Norway	no	0.92	[0.65,1.23]	0.312 (2.05)	0.572 (4.35)	−0.398 (−2.71)	−0.725 (−6.58)	−0.776 (−7.69)	1969
Portugal	pt	0.84	[0.71,1.09]	−0.113 (−0.51)	−0.028 (−0.12)	0.194 (0.886)	−0.922 (−10.65)	−0.817 (−6.34)	1988
Spain	es	0.84	[0.70,1.20]	0.802 (8.06)	0.723 (6.29)	−0.316 (−2.00)	−0.647 (−5.09)	−0.360 (−2.32)	1960
Sweden	se	0.97	[0.74,1.19]	0.140 (0.98)	0.593 (5.10)	−0.527 (−4.29)	−0.518 (−4.19)	−0.717 (−7.17)	1960
Switzerland	ch	0.95	[0.75,1.33]	0.530 (4.05)	0.587 (4.69)	−0.544 (−4.19)	−0.767 (−7.74)	−0.583 (−4.65)	1966
UK	uk	0.93	[0.74,1.18]	0.688 (6.59)	0.703 (6.86)	−0.455 (−3.54)	−0.877 (−12.62)	−0.575 (−4.87)	1960
USA	us	0.86	[0.55,1.15]	0.710 (6.98)	0.669 (6.23)	−0.885 (−13.19)	−0.820 (−9.91)	−0.666 (−6.02)	1960

Table C1. Cross-country Panel. This table reports the country specific average MY_t, the middle-aged to young ratio (over the sample 1960-2009), the range of MY_t, Pearson correlations between the equity and bond yields, $\rho(DP_t,R_{lt})$, the equity yield and inflation, $\rho(DP_t,\pi_{lt})$, the equity yield and MY_t, $\rho(DP_t,MY_t)$, the bond yield and MY_t, $\rho(R_{lt},MY_t)$ and inflation and MY_t, $\rho(\pi_t,MY_t)$. For countries with less than 30 observations, non-parametric spearman correlation is reported. Last column indicates the first year of data.

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