The Term Structure of Interest Rates and Macroeconomic Dynamics

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Introduction

The main motivation of this dissertation is to link the term structure of interest rates to macroeconomic dynamics.

Over the last decade, policy-makers and practitioners have been using different term structure models. Canonical models explain yield curves by a limited number of yield factors. However, there is an obvious potential misspecification of such yields-only framework: the omission of macroeconomic variables to which the monetary policy maker reacts. All three chapters of my thesis try to overcome this shortcoming and present different joint models for the term structure of interest rates and the macroeconomy

The first chapter develops on the VAR framework, originally proposed by Campbell and Shiller (1987) to evaluate the Expectations Theory, along three dimensions: the use of a testing method based on a real-time procedure in which the econometrician is given the same information available to market participants, the measurement of the risk premium, the specification of the implicit monetary policy maker's reaction function. We use financial factors and macroeconomic information to construct a test of the theory based on simulating investors' effort to use the model in 'real time' to forecast future monetary policy rates. The application of our approach to a monthly sample of US data from the eighties onward delivers an explicit estimate for risk premia with an associated confidence interval. The observation of such variables leads us to conclude that, even if estimates of the term premium are time varying, the deviation from the ET are not always significant and that fluctuations of risk premia are not large when some forecasting model for short-term rates is adopted and a proper evaluation of uncertainty associated to policy rates forecast is considered.

While in the first chapter we abstract from the modelling of the pricing kernel, the second chapter of my thesis develops a term structure model based on No-Arbitrage assumption, which allows for explicit time varying term premia. In particular, the model combines a Structural Vector Autoregression (SVAR) with a no-arbitrage approach to build a multifactor

Affine Term Structure Model (ATSM). The resulting No-Arbitrage Structural Vector Autoregressive (NA-SVAR) model contains a pricing kernel, which implies that expected excess returns are driven by the structural macroeconomic shocks. As a simple application of NA-SVAR model, we study the effects of supply, demand and monetary policy shocks on the UK yield curve. We show that all shocks affect the slope of the yield curve, with demand and supply shocks accounting for a large part of the time variation in bond yields. The short end of the yield curve is driven mainly by the expectations component, while the term premium matters for the dynamics of the long end of the yield curve.

As a summary of different approaches, the third chapter of my thesis presents different term structure models to proxy the term premia. In particular, we consider discrete Term Structure Models (TSM), which specify the driving stochastic process for the yield curve by Gaussian Vector Autoregression (VAR). First, we refer to No-Arbitrage Affine TSM. Second, we estimate term premium by two-step procedure: we fit the yield curve by Nelson-Siegel model and then we add the assumption about the dynamics of the factors. Finally, the results are compared to the estimates implied by simple unrestricted VAR. We find that differences in term premia estimates among alternative specifications of Term Structure VAR Models are small.

Chapter 1

Financial Factors, Macroeconomic Information and the Expectations Theory of the Term Structure of Interest Rates

1.1 Introduction

The objective of this paper is to provide new evidence on the expectations theory (ET) of the term structure of interest rates.

How is this possible?

Our starting point is the widely cited work by Campbell and Shiller(1987)(CS), where they implement a bivariate vector autoregressive (VAR), which is different from the bulk of the available literature which rejects the ET within a single-equation, limited information approach (see, for example, Campbell,1995, Fama and Bliss,1987, and Cochrane,2001). CS implement a test which still rejects the ET but their analysis of the data leads them to conclude that there is an important element of truth to the expectations theory of the term structure.

We develop on the CS framework along three dimensions: the use of a testing method based on a real-time procedure in which the econometrician is given the same information available to market participants when they make their decisions on portfolio allocation, the specification of the implicit monetary policy maker's reaction function, the measurement of the risk premium in case of rejection of the null of the ET.

First, CS test the restrictions imposed by the ET on a VAR model in the spread between long and short term interest rates and the change of short-term interest rates and by using only in-sample information. Such procedure cannot simulate the investors' effort to use the model in 'real time' to forecast future monetary policy rates: the information from the whole sample is used to estimate parameters while investors can use only historically available information to generate (up to n-period ahead) predictions of policy rates. Moreover, the within sample test understates the uncertainty of agents who forecast policy rates by out-of-sample projections. In this paper we use the present value framework to generate real time forecast for future policy rates. At each point in time we estimate, using the historically available information, a model and then we use it to project out-of-sample policy rates up to the nth-period ahead. Given the path of simulated future policy rates, we can construct yield to maturities consistent with the Expectations Theory. Using the historically available information on uncertainty we perform dynamic stochastic simulations and construct confidence bounds around the ET-consistent long-term rates. These bounds reflect explicitly the uncertainty associated with out-of-sample projections. Then we test the ET by checking if the observed long-term rates fluctuate within the bounds.

Second, by having an explicit model for the short-rate in their testing framework CS circumvent one of the main assumptions of the single-equation approach to the ET, namely the use of ex-post realized returns as a proxy for ex-ante expected returns. In a recent paper, Elton (1999) clearly asserts that there is ample evidence against the belief that information surprises tend to cancel out over time and hence realized returns cannot be considered as an appropriate proxy for expected returns. Interestingly, Campbell (2001) finds strong effects of expectations errors on the single-equation tests, which are confirmed by a number of papers which concentrates on expectations errors by relating them to peso problems or to the very low predictability of short term interest rates. In a famous study Mankiw and Miron, 1986, using data on a three and six month maturity, found evidence in favor of the expectation theory prior to the founding of the Federal Reserve System in 1915. They show that the shift in regime occurred with the founding of the Fed led to a remarkable decrease in the predictability of short-term interest rates. Rudebusch, 1995, and Balduzzi et al., 1997, expand on this evidence by looking at more recent data. As a consequence of the use of ex-post realized returns as a proxy for ex-ante expected returns the single-equation approach cannot identify if the empirical failure of the model is due to systematic expectations errors, or to shifts in the risk premia. CS have an implicit model to construct expectations, they find much milder evidence against the ET but they do not exploit their model to construct a measure of risk premium.

By implementing our simulation based procedure we can explicitly measure deviations from the ET and, under the null that our proposed model delivers expected future policy rates not different from those expected by the market, interpret them as a measure of risk premium.

Third, on a different, but clearly related, ground McCallum(1994) is the first to argue that the limited information approach might cause bias in the estimates due to simultaneity. He shows that the anomalous empirical findings based on a single equation evidence can be rationalized with the expectations theory by a recognition of an exogenous term premium plus the assumption that monetary policy involves smoothing of the policy rates together with the responses to the prevailing level of the spread. Interestingly, the bi-variate framework considered by CS matches exactly the scenario used by McCallum to illustrate the simultaneity bias in the single-equation approach. However, McCallum himself notes that a reaction function according to which the Fed reacts to the spread only represents a simplification relative to the actual behaviour of the Fed, which almost certainly responds to recent inflation and output or employment movements, as well as to the spread. In fact, both the financial literature and the macroeconomic literature point to potential mis-specification of the simple reaction function used by CS.

There is ample empirical evidence that a three-factor model is needed to accurately describe the term structure and that the use of term structure related factors is of considerable help in modelling monetary policy rates (see, for example, Ang and Piazzesi(2003)), it is easy to see that in the CS approach only two factors are considered. The success of Taylor rules (Taylor,1993, Clarida, Gali and Gertler, 1998, 1999, 2000) points out an obvious potential misspecification of the original Campbell-Shiller framework: the omission of macroeconomic variables to which the monetary policy maker reacts. We shall assess potential mis-specification effects by using an extended VAR which includes three factors for the term structure and macroeconomic variables used in Taylor rules.

The paper is organized as follows. Section 1 illustrates the testing framework by contrasting the Present Value approach with our simulation based alternative. Section 2 illustrates our testing framework and our extension of the information set. Section 3 presents the empirical evidence. Section

4 contains an assessment of the robustness of our results to the use of a different sample and of a different method for updating parameter estimates upon accrual of new information. Section 5 concludes.

1.2 Testing framework

We introduce our testing framework by comparative evaluation of the traditional present value approach and of proposed simulation based approach.

1.2.1 The Present Value approach

We describe the Present Value approach by adopting the linearized expectations model of Shiller (1979) in the bi-variate framework proposed by CS.

We start by imposing a no-arbitrage condition, according to which the expected one-period holding returns from long-term bonds must be equal the risk-free short term interest rate plus a term premium. For long term bonds bearing a coupon C, $H_{t,T}$ is a non-linear function of the yield to maturity $R_{t,T}$. Shiller (1979) proposes a linearization which takes the approximation in the neighborhood $R_{t,T} = R_{t+1,T} = \bar{R} = C$, in which case we have:

$$E[H_{t,T} \mid I_t] = E\left[\frac{R_{t,T} - \gamma_T R_{t+1,T}}{1-\gamma_T} \mid I_t\right] = r_t + \phi_{t,T} \tag{1.1}$$

where $H_{t,T}$ is the one-period holding return of a bond with maturity date T, I_t is the information set available to agents at time t, r_t is the short term interest rate,γ_T is a constant of linearization which depends on the maturity of the bond and $\phi_{t,T}$ is a term premium defined over a one-period horizon for holding the bond with residual maturity $T-t$. Consider the above expression for a very long term bond , by recursive substitution, under the terminal condition that at maturity the price equals the principal,we obtain:

$$R_{t,T} = R^*_{t,T} + E[\Phi_{t,T} \mid I_t] = \frac{1-\gamma}{1-\gamma^{T-t}} \sum_{j=0}^{T-t-1} \gamma^j E[r_{t+j} \mid I_t] + E[\Phi_{t,T} \mid I_t] \tag{1.2}$$

where $\lim_{T\longrightarrow\infty} \gamma_T = \gamma = 1/(1+\bar{R})$ and $\Phi_{t,T}$ is the term premium over the whole life of the bond:

$$\Phi_{t,T} = \frac{1-\gamma}{1-\gamma^{T-t}} \sum_{j=0}^{T-t-1} \gamma^j \phi_{t+j,T}$$

CS tests the ET[1] by using equation (1.2) in considering the case of the risk free rate and a very long term bond. In such case, the null of the ET is imposed in strong form by imposing that $E[\Phi_{t,T} \mid I_t]$ is zero and in weak form by imposing that $E[\Phi_{t,T} \mid I_t]$ is captured by a constant. CS consider de-meaned variables, and hence test a weak form of the ET by considering the following restriction:

$$R_{t,T} = R^*_{t,T} \approx (1-\gamma) \sum_{j=0}^{T-t-1} \gamma^j E[r_{t+j} \mid I_t] \tag{1.3}$$

which could be re-written in terms of spread between long and short-term rates, $S_{t,T} = R_{t,T} - r_t$:

$$S_{t,T} = S^*_{t,T} = \sum_{j=1}^{T-t-1} \gamma^j E[\Delta r_{t+j} \mid I_t] \tag{1.4}$$

(1.4) shows that a necessary condition for the ET to hold puts constraints on the long-run dynamics of the spread. In fact, the spread should be stationary being a weighted sum of stationary variables. Obviously, stationarity of the spread implies that, if yields are non-stationary, they should be cointegrated with a cointegrating vector (1,-1). However, the necessary and sufficient conditions for the validity of the ET impose restrictions both on the long-run and the short run dynamics.

Assuming[2] that $R_{t,T}$ and r_t are cointegrated with a cointegrating vector (1,-1), CS construct a bivariate stationary VAR in the first difference of the short-term rate and the spread :

$$\begin{aligned} \Delta r_t &= a(L)\Delta r_{t-1} + b(L)S_{t-1} + u_{1t} \\ S_t &= c(L)\Delta r_{t-1} + d(L)S_{t-1} + u_{2t} \end{aligned} \tag{1.5}$$

Stack the VAR as:

[1] In fact CS use de-meaned-variables, that is equivalent to test a weak form of the Expectations Theory, in the sense that de-meaning eliminates a constant risk premium.

[2] In fact, the evidence for the restricted cointegrating vector which constitutes a necessary condition for the ET to hold is not found to be particularly strong in the original CS work.

$$\begin{bmatrix} \Delta r_t \\ . \\ . \\ \Delta r_{t-p+1} \\ S_t \\ . \\ . \\ S_{t-p+1} \end{bmatrix} = \begin{bmatrix} a_1 & . & . & a_p & b_1 & . & . & b_p \\ 1 & . & . & 0 & 0 & . & . & 0 \\ 0 & . & . & 0 & 0 & . & . & 0 \\ 0 & . & 1 & 0 & 0 & . & . & 0 \\ c_1 & . & . & c_p & d_1 & . & . & d_p \\ 0 & . & . & 0 & 1 & . & . & 0 \\ 0 & . & . & 0 & 0 & . & . & 0 \\ 0 & . & . & 0 & 0 & . & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta r_{t-1} \\ . \\ . \\ \Delta r_{t-p} \\ S_{t-1} \\ . \\ . \\ S_{t-p} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ . \\ . \\ 0 \\ u_{2t} \\ . \\ . \\ 0 \end{bmatrix} \tag{1.6}$$

This can be written more succinctly as:

$$z_t = Az_{t-1} + v_t \tag{1.7}$$

The ET null puts a set of restrictions which can be written as :

$$g'z_t = \sum_{j=1}^{T-1} \gamma^j h' A^{j\prime} z_t \tag{1.8}$$

where $g\prime$ and $h\prime$ are selector vectors for S and Δr correspondingly (i.e. row vectors with 2p elements, all of which are zero except for the p+1st element of $g\prime$ and the first element of $h\prime$ which are unity). Since the above expression has to hold for general z_t, and, for large T, the sum converges under the null of the validity of the ET, it must be the case that:

$$g' = h'\gamma A(I - \gamma A)^{-1} \tag{1.9}$$

which implies:

$$g'(I - \gamma A) = h'\gamma A \tag{1.10}$$

and we have the following constraints on the individual coefficients of VAR(1.5):

$$\{c_i = -a_i, \forall i\}, \{d_1 = -b_1 + 1/\gamma\}, \{d_i = -b_i, \forall i \neq 1\} \tag{1.11}$$

The above restrictions are testable with a Wald test. By doing so using US data between the fifties and the eighties Campbell and Shiller (1987) rejected the null of the ET. However, when CS construct a theoretical spread $S^*_{t,T}$, by imposing the (rejected) ET restrictions on the VAR they find that, despite the statistical rejection of the ET, $S^*_{t,T}$ and $S_{t,T}$ are strongly correlated.

1.2.2 A new testing framework with an extended information set

We extend the CS approach along two dimensions: the specification of the VAR and the construction of a test based on information available in real time.

Both the financial and the macroeconomic empirical literature suggest that the parsimonious model consisting of the spread and the change in the short-term rate could be in fact too parsimonious to fit the data. The financial literature has shown that the construction of a satisfactory model of the term structure requires at least three factors, usually labelled as level, slope and curvature. Rethinking the CS empirical work in this framework makes clear that they have included in their bivariate VAR some proxy for the level and the slope of the term structure, but they have omitted the curvature. Interest rate rules, which feature (very) persistent of policy rates responding to central bank's perceptions of (expected) inflation and output gaps (Taylor,1993, Clarida, Gali and Gertler, 1998, 1999, 2000) not only track the data well but are also capable of explaining the high inflation in the seventies in terms of an accommodating behaviour towards inflation in the pre-Volcker era.

The success of Taylor rules points out an obvious potential misspecification of the original Campbell-Shiller framework: the omission of macroeconomic variables to which the monetary policy maker reacts. Interestingly, Fuhrer(1996) uses a simple Taylor-rule type reaction function, the expectations model and reduced-form equations for output and inflation to solve for the reaction function coefficients that delivers long-term rates consistent with the Expectations Theory. He finds that modest and smoothly evolving time-variation in the reaction functions parameters is sufficient to reconcile the expectations model with the long-bond data. Favero(2002) extends Fuhrer framework to derive standard errors for long-term rates consistent with the ET. Our approach of extending the VAR framework is closely related, but very different, from recent work by Roush(2003). Roush considers a VAR model with macro and financial variables to show that the expectations theory of the term structure holds conditional on an exogenous change in monetary policy. The paper adds to the picture the important issue of identification but it does not provide evidence on the impact of the extension of the original CS information set on the outcome of the test for the unconditional validity of cross equation restrictions; moreover, the attention is limited to the within-sample evidence.

The bivariate CS approach has an implicit reaction function according to which the only determinant of policy rates are long-term rates, therefore we have a potential mis-specification due to the omission of macroeconomic factors.

However, we think that our main contribution is not the augmentation of the original dimension of the VAR but the proposal of a new approach to test the ET based on information available in real time. To show our point, consider a cointegrated VAR framework, in which the original set of variables used by CS is extended by including a vector of variables **X**. Such vector includes financial factors and macroeconomic variables. At each point in time we estimate, using the historically available information, the following model:

$$\begin{aligned} \Delta r_t &= a_0 + a_1(L)\Delta r_{t-1} + a_2(L)S_{t-1} + a_3(L)\mathbf{X}_{t-1} + u_{1t} \\ S_t &= b_0 + b_1(L)\Delta r_{t-1} + b_2(L)S_{t-1} + b_3(L)\mathbf{X}_{t-1} + u_{2t} \\ \mathbf{X}_t &= c_0 + c_1(L)\Delta r_{t-1} + c_2(L)S_{t-1} + c_3(L)\mathbf{X}_{t-1} + \mathbf{u}_{3t} \end{aligned}$$

$$\begin{bmatrix} u_{1t} \\ u_{2t} \\ \mathbf{u}_{3t} \end{bmatrix} \backsim N\left[0, \Sigma\right]$$

We then simulate the estimated model forward, to obtain projection for all the relevant policy rates and to construct ET-consistent spread[3] as follows:

$$\hat{S}^{*}_{t,T} = \sum_{j=1}^{T-t-1} \gamma^j E[\Delta r_{t+j} \mid \Omega_t] \tag{1.12}$$

where, $E[\Delta r_{t+j} \mid \Omega_t]$ are the VAR-based projections for the future changes in policy rates, hence Ω_t is the information set used by the econometrician to predict on the basis of the estimated VAR model . Given this simulation based version of the ET consistent spread we can also construct a confidence interval around it. Confidence intervals around simulated series are usually constructed by adopting stochastic simulation techniques. In a standard stochastic simulation the model is simulated forward repeatedly for N draws of its stochastic components. In general, there are two sources of uncertainty: residuals and coefficient uncertainty. At each repetition, errors are generated for each observation in accordance with the residual uncertainty in the model. Residuals are drawn from a multivariate normal distribution $N\left(0, \hat{\Sigma}\right)$ where $\hat{\Sigma}$ is the estimated variance-covariance matrix of

[3] For consistency with CS, we simulate the model forward after de-meaning.

residuals of our VAR. Similarly, VAR coefficients are drawn from a multivariate normal distribution with the vector mean given by the point estimates of coefficient and the variance-covariance matrix given by the parameters' variance-covariance matrix. However, the confidence interval constructed by allowing for residuals and coefficient uncertainty will be a confidence interval for the evolution of $\sum_{j=1}^{T-t-1} \gamma^j [\Delta r_{t+j} \mid \Omega_t]$ which is very different, and certainly larger, than a confidence interval for $\hat{S}^*_{t,T} = \sum_{j=1}^{T-t-1} \gamma^j E[\Delta r_{t+j} \mid \Omega_t]$[4] However, it is immediate to construct bounds for $\hat{S}^*_{t,T}$ by performing the stochastic simulation allowing only for coefficients uncertainty. While future realized policy rates are affected both by parameters uncertainty and shocks, future expected policy rates are not affected by shocks, hence the only source of uncertainty for the ET consistent spread is parameters' uncertainty. ET consistent yields are calculated applying equation (1.12) to each of the N simulated paths of future expected short-term rates: among these, the 0.5th, 0.05th, and 0.95th quantiles represent respectively the "theoretical" ET-consistent yield and its 95% confidence bounds. The estimation window is then enlarged by one observation and simulation horizon is shifted one period ahead and the same steps are repeated.

Importantly, in implementing our procedure the econometrician uses the same information available to market participants in real-time. Future policy rates at time t are constructed using information available in real time for parameters estimation and forward projection of the model. Point forecasts and their confidence bounds define a region inside which the actual long term rates should lie if the ET holds. By combining (1.4) and (1.12), we have:

[4] We thank a referee for making us note this point. In fact, bounds constructed by allowing both for residuals and coefficients uncertainty could be thought of as a simulation equivalent of the volatility bounds proposed by Shiller(1979).

$$
\begin{aligned}
S_{t,T} &= \sum_{j=1}^{T-t-1} \gamma^j E_t[\Delta r_{t+j} \mid I_t] + E[\Phi_{t,T} \mid I_t] && (1.13) \\
\widehat{S^*_{t,T}} &= \sum_{j=1}^{T-t-1} \gamma^j E_t[\Delta r_{t+j} \mid \Omega_t] && (1.14) \\
S_{t,T} - \widehat{S^*_{t,T}} &= \left(\sum_{j=1}^{T-t-1} \gamma^j E_t[\Delta r_{t+j} \mid I_t] - \sum_{j=1}^{T-t-1} \gamma^j E_t[\Delta r_{t+j} \mid \Omega_t] \right) + E[\Phi_{t,T} \mid I_t] \\
S_{t,T} - \widehat{S^*_{t,T}} &= \xi_t + E[\Phi_{t,T} \mid I_t] && (1.15)
\end{aligned}
$$

Equation (1.15) makes clear that deviation of $S_{t,T}$ from $\widehat{S^*_{t,T}}$ can be explained by the effect of the risk premia or by differences between model based forecasts, which are derived by using the information set used by the econometrician Ω_t, and agents' expectations, which are formed given the information set I_t , unknown to the econometrician. Under the assumption that the first term is negligible, (statistically) significant deviations of $S_{t,T}$ from $\widehat{S^*_{t,T}}$ do offer a measurable counterpart of the risk premium.

1.3 The Empirical Evidence

We shall present our empirical evidence in three sub-sections. The first sections discusses our data-set, and our choice of sample for estimation and simulation, the second section presents the replica of the CS procedure on our data-set and an application of our simulation based procedure on the CS specification, while the third section illustrates the extension of the original specification to include financial factors and macroeconomic variables.

1.3.1 The data-set

Our basic data set consists of a set of zero-coupon equivalent US yields, provided by Brousseau, V. and B. Sahel (1999). In particular, we consider data on zero-coupon equivalent yields for US data measured at the following maturities[5]:

[5] The data were indly made available by the ECB, and they are posted on Favero's website at the following address: http:/www.igier.uni-bocconi.it/personal/favero in the section working papers

1-month, 2-month, 3-month, 6-month, 9-month, 1-year, 2-year, 3-year, 5-year, 7-year, 10-year.

From this data set we construct financial factors by estimating at each point of our time series t, by non-linear least squares, on the cross-section of eleven yields, the following Nelson-Siegel model:

$$y_{t,t+k} = L_t + SL_t \frac{1-\exp\left(-\frac{k}{\tau_1}\right)}{\frac{k}{\tau_1}} + C_t\left(\frac{1-\exp\left(-\frac{k}{\tau_1}\right)}{\frac{k}{\tau_1}} - \exp\left(-\frac{k}{\tau_1}\right)\right) \tag{1.16}$$

which is implicit in the instantaneous forward curve:

$$f_{tk} = L_t + SL_t \exp\left(-\frac{k}{\tau_1}\right) + C_t \frac{k}{\tau_1}\exp\left(-\frac{k}{\tau_1}\right) \tag{1.17}$$

The parameter τ_1 is kept constant over time[6], as this restriction decreases the volatility of the β parameters, making them more predictable in time. As discussed in Diebold and Li (2002) the above interpolant is very flexible and capable of accommodating several stylized facts on the term structure and its dynamics. In particular, L_t, SL_t, C_t, which are estimated as parameters in a cross-section of yields, can be interpreted as latent factors. L_t has a loading that does not decay to zero in the limit, while the loading on all the other parameters do so, therefore this parameter can be interpreted as the long-term factor, the level of the term-structure. The loading on SL_t is a function that starts at 1 and decays monotonically towards zero; it may be viewed a short-term factor, the slope of the term structure. In fact, $r_t^{rf} = L_t + SL_t$ is the limit when k goes to zero of the spot and the forward interpolant. We naturally interpret r_t^{rf} as the risk-free rate. Obviously SL_t, the slope of the yield curve, is nothing else than the minus the spread in Campbell-Shiller. C_t is a medium term factor, in the sense that their loading start at zero, increase and then decay to zero (at different speed). Such factor captures the curvature of the yield curve. In fact, Diebold and Li show that it tracks very well the difference between the sum of the shortest and the longest yield and twice the yield at a mid range (2-year maturity). The repeated estimation of loadings using a cross-section of yields at different maturities allows to construct a time-series for our factors. We report in Figure 1 the three factors, while Figure 2 shows the goodness of fit of the

[6]We restrict τ_1 at the value of 0.87, which is the median, over the time series, of the estimated value of τ_1 in a four parameter version of the Nelson-Siegel interpolant.

Nelson and Siegel interpolation for all yields considered in our sample. The extreme good performance of the Nelson-Siegel interpolant for our observed data shows that the fact that we have fitted the Nelson-Siegel model to zero coupon equivalent yields rather than to individual yields should not be a cause of concern for the problem at hand.

Note that the fact that we use zero-coupon equivalent yields has a relevant implication for the CS linearization, which should be applied taking the limit of the relevant formuale when γ approaches 1. In particular, we have:

$$\begin{aligned} R_{t,T} &= R^*_{t,T} + E[\Phi_{t,T} \mid I_t] = \lim_{\gamma \to 1} \frac{1-\gamma}{1-\gamma^{T-t}} \sum_{j=0}^{T-t-1} \gamma^j E[r_{t+j} \mid I_t] + E[\Phi_{t,T} \mid I_t] \\ &= \frac{1}{T-t} \sum_{j=0}^{T-t-1} E[r_{t+j} \mid I_t] + E[\Phi_{t,T} \mid I_t] \end{aligned} \tag{1.18}$$

and, given that $R^*_{t,T} = \frac{1}{T-t} \sum_{j=0}^{T-t-1} E[r_{t+j} \mid I_t]$, we then have

$$S^*_{t,T} = R^*_{t,T} - r_t = \sum_{j=1}^{T-t} \left(1 - \frac{j}{T-t}\right) E\Delta[r_{t+j} \mid I_t]$$

Our empirical analysis will be based on a simulation sample starting at the beginning of the eighties. One of the main points of our paper is to construct expected future policy rates by considering explicitly the central bank reaction functions, so we have chosen the initial date of the sample to concentrate on an era of homogenous monetary policy, i.e. the Volcker-Greenspan era. In fact, there is ample empirical evidence that, from the beginning of the eighties onward, the Fed engaged in interest rate targeting and that the behaviour of policy rates can be successfully described by a Taylor rule. The traditional argument of a Taylor rule are expected inflation and some measure of the output gap. Our framework for simulating policy rates is geared to mimic the decisions of agents in real-time. Orphanides (2001) has shown that data revisions could generate misleading inference. For this reason, as suggested by Evans(2003), we consider as macroeconomic factors variables which are not subject to revision: the CPI inflation and unemployment rate.

We presents our empirical evidence in three parts: a replica on our data-set of the original Campbell-Shiller results, an application of our simulation based procedure on the CS specification, the extension of the original specification to include financial factors and macroeconomic variables.

1.3.2 Testing the ET with a bivariate VAR

The discussion of the measurement of financial factors makes clear that the closest model to CS original specification in our framework is the following:

$$\begin{bmatrix} \Delta r_t^{rf} \\ S_t \end{bmatrix} = A(L) \begin{bmatrix} \Delta r_{t-1}^{rf} \\ S_{t-1} \end{bmatrix} + u_t \tag{1.19}$$

where $r_t^{rf} = L_t + SL_t$,and $S_t = -SL_t$.Our specification differs from CS in that they take the one-month rate as the short term rate and the yield to maturity on 10-year bonds as the long term rate. Interestingly the level factor,L_t, is the asymptote of the term structure, hence cross equation restrictions on the VAR hold exactly for the spread constructed by using this factor while they are just approximate for the spread constructed using a 10-year yield [7] We also estimate our model recursively, allowing for a smooth evolving path in the estimated coefficients. This procedure might capture historical shifts in market perceptions of the policy target for inflation, which have been shown (Kozicki and Tinsley, 2001) to be important to achieve a satisfactory specification of agents' expectations. We report the results of the application of the CS testing methodology, based on a four-lag VAR, in Figure 3. Figure 3 reports the results of the test for the ET cross-equation restrictions, which is conducted recursively after using the sample 1974:6 1991:12 for initialization. The ET restrictions are consistently rejected, however,as in the original work of CS the actual spread has a correlation of with the spread obtained by imposing the invalid restrictions of .96. this is the evidence that leads Campbell and Shiller to conclude that "... deviations from the present value model for bonds are transitory...", however no measurement of the risk premium is explicitly provided by the two authors.

We report in Figure 4 the results of our simulation based test of the ET. We use our model to simulate ET consistent 10-year yields to maturity and their associated confidence intervals. Figure 4 ET consistent yields to maturity along with their associated confidence interval and the actual yields. Under the null of the ET the observed yields should fall within the bounds. In fact, the actual yields lie consistently above the simulated ones,

[7] As a matter of fact we have tested that for simulation based on our VAR specification ten year is sufficiently far in the future to give a good approximation of infinite.

but they are outside the 95 per cent confidence intervals, constructed under the null of the ET, only in a short subsample covering the period 1991-1994. Interestingly, a positive risk difference between actual and simulated yields is what we should observe in the presence of risk premium, when the impact of the difference between the information sets used by the agents and the econometrician is negligible. Overall, we attribute the difference between the results of our simulation based methodology and the traditional CS to the fact that the tests for the cross-equations restrictions understates the uncertainty faced by the agents in real time and therefore uses a too tight statistical criterion. Our evidence of non-rejection of the EH is consistent with the evidence proposed by CS of the very high correlation between the actual spread and the spread obtained by simulating imposing the restrictions (rejected by the Wald test). Our results confirm the much less strong evidence against generated by model in which expectations are explicitly derived rather than taking the ex-post realized returns as a proxy for ex-ante expected returns. This is not new, in fact Bekaert and Hodrick (2000) find the same results from a different perspective:use of the small sample distribution of the relevant tests in VAR models leads to much less strong evidence against the ET.

We believe that it is important to assess this first set of results against those obtained by enlarging the information set of the VAR following the available empirical evidence form studies on the term structure and on the empirical analysis of monetary policy. In particular, the difference between actual and simulated rates is sizeable when significant and we think that it would be interesting to see how this distance is affected by the enlargement of the information set which we shall implement in the next section.

1.4 Testing the ET with a model with financial factors and macroeconomic variables

Our VAR with financial factor and macroeconomic variables takes the following specification:

$$\begin{bmatrix} \Delta r_t^{rf} \\ -S_t \\ C_t \\ \pi_t \\ UN_t \end{bmatrix} = A(L) \begin{bmatrix} \Delta r_{t-1}^{rf} \\ -S_{t-1} \\ C_{t-1} \\ \pi_{t-1} \\ UN_{t-1} \end{bmatrix} + u_t \qquad (1.20)$$

We consider the three factors obtained via the application of the Nelson-

Siegel interpolant together with CPI inflation, π_t, and the unemployment rate,

UN_t, which are our proxies for the variables normally entered as arguments of Taylor rules. Importantly, our macroeconomic variables are not subject to revision, consistently with our intention of using the model to replicate the decision process of agents in real time. As in the VAR with financial factors our representation is stationary and it allows for the cointegrating relationship which constitute a necessary condition for the ET to hold, being also consistent with the presence of a stationary risk premium[8]. Estimation is conducted on the same sample with the two variables VAR and, on the basis of the traditional lag selection criteria, we adopt a VAR of length two.[9] The results of the recursive within sample test and of the simulation based out-of-sample procedure are reported respectively in Figure 5 and Figure 6. The results of the Wald tests are very similar to those obtained in the basic model. However, the enlargement of the information set generates some notable modification in the simulation based procedure. The 95 per confidence interval constructed around the ET consistent 10-year yields become much narrower that in the case of the two variables specification adopted in the previous section. Moreover, the difference simulated yields get much closer to actual yields and most of the evidence of violation of the ET comes from 1994, a period which has been widely cited in the literature as featuring an episode of "inflation scare" (see, for example, Rudebusch,1998). We interpret these results as evidence for the importance of the VAR enlargement to achieve a better identification of the expectations for the future path of the financial and macroeconomic variables relevant to determine monetary policy.

1.5 Robustness

The results on the size and the significance of risk premium delivered by our simulation based approach call for some robustness analysis. In particular, we want to make sure that our sample initialization is not inappropriate

[8] The trace statistics for the null of at most four cointegrating vectors yieded an observed values of 6.35, for the estimation on the full sample and of 5.2 for the estimation on the shortest sample used in the recursive approach, while the five per cent critical value is 3.76 (We allowed for a constant restricted to belong to the cointegrating vector)

[9] The lag length criteria do not uniformly favour two lags for all possible sample splits. So we have analyzed the robustness of our results to the adoption of a four-lags VAR. The evidence, available upon request, shows that moving from a lag length of two to a lag length of four leaves our results unaltered.

in that our initial VAR estimates are not contaminated by large residuals. In fact, after the Volcker disinflation, the volatility of macroeconomic variables has decreased remarkably in the eighties. We conduct our robustness check by concentrating on our five variable VAR specification, by considering as a benchmark the recursive estimation approach with initial sample 1974:6-1991:12 discussed in the previous section and by considering as an alternative estimation strategy a rolling estimation with initialization 1974:6 1991:12 and a fixed window of 210 observations. The alternative estimation method is chosen to evaluate the impact of our choice of initialization for the recursive estimation. In fact, the last sample for our rolling estimation approach is 1984:6-2001:12 and covers a very different period from the initial one in terms of (unconditional) volatility of all variables included in the VAR. Moreover, our rolling estimation could also provide evidence against the potential objection that some estimates (see, for example, Bernanke-Mihov(1998)) suggest that the starting period of the Volcker Greenspan era should be located at the beginning of the 1984, and simulation and tests based on post 1984 data could be different from those based on pre 1984 data.

We find the results of the application of the Wald tests and of the simulation based procedure, reported in Figures 7 and 8, interesting.

The uniform rejection of the theory obtained by the recursive approach based on the initialization on the large sample is not confirmed by the rolling approach, which does not lead to rejection of the theory for an estimation sample of 210 observations ending after the end of 1999. Very differently, the results of the simulation based approach in the five variables VAR are very robust to the two different estimation strategies. We report in Figure 8 the difference between actual 10-year yields and 10-year yields simulated under the null of the ET, obtained by projections based on rolling and recursive estimation for the five variables VAR and the two variables VAR. The results derived using the five factor models are very robust to the choice of the rolling and recursive estimation techniques, delivering differences which are positive when significantly different from zero and reaching their peaks during the inflation scare of 1994. The results from the two variables VAR are instead sensitive to the estimation technique. In this case the rolling method delivers series which fluctuate at a level consistently lower than the recursive technique and closer to the series obtained from the five variables VAR. This evidence can be naturally interpreted as indicating mis-specification caused by omitted variables in the more parsimonious model. Interestingly, the results from the five variables VAR are consistent with the evidence, originally reported in CS, that the correlation between the actual spread and the spread obtained under the null of EH is very high even when the null is rejected. Our interpretation of these facts is that the uncertainty faced by the agents in simulating the model to obtain path for the relevant variables to forecast monetary policy is rather stable in a sufficiently parameterized model, even if the coefficients in the estimated VAR do vary over time.

1.6 What have we learned? A discussions of our results and their relation to the literature

In this paper we have simulated the real time decision of agents who forecast policy rates by projecting forward a model including financial factors and macro variables to generate long-term rates consistent with the expectations theory along with a confidence interval reflecting the uncertainty associated to out-of-sample forecasting. Our evidence shows that, for different specifications of the information set, the observed long-term yields are, with very few exceptions, contained in the confidence interval generated by our model. Our procedure delivers an observable counterpart of the deviation of the long-term rates from those consistent with the ET. Upon significance of such deviations we can interpret this variable as a proxy for risk premium under the null hypothesis that model based forecasts are not different from agents' expectations. Our empirical results show that a better specification of the VAR used to forecast future monetary policy delivers more credible estimates of the risk premium.

The standard response in finance to the empirical rejection of the Expectations Theory has been modelling the term structure based on the assumption that there are no riskless arbitrage opportunities among bonds of various maturities. The standard model is based on three components: a transition equation for the state vector relevant for pricing bonds, made traditionally of latent factors, an equation which defines the process for the risk-free one-period rate and a relation which associates the risk premium with shocks to the state vector, defined as a linear function of the state of the economy. In such structure, the price of a j-period nominal bond is a linear function of the factors. Unobservable factors and coefficients in the bond pricing functions are jointly estimated by maximum likelihood methods (see, for example, Chen and Scott(1993)). This type of models usually provides a very good within sample fit of different yields but do not perform well in forecasting. Duffee(2002) shows that the forecasts produce by no-arbitrage models with latent factors do not outperform the random walk model.

Recently the no-arbitrage approach has been extended to include some observable macroeconomic factors in the state vector and to explicit allow for a Taylor-rule type of specification for the risk-free one period rate. Ang and Piazzesi(2002) and Ang, Piazzesi and Wei(2003) show that the forecasting performance of a VAR improves when no-arbitrage restrictions are imposed and that augmenting non-observable factors models with observable macroeconomic factors clearly improves the forecasting performance. Hordahl et al.(2003) and Rudebusch and Wu(2003) use a small scale macro model to interpret and parameterize the state vector; forecasting performance is improved and models have also some success in accounting for the empirical failure of the Expectations Theory.

No-arbitrage models with observable factors feature a complicated parameterization and cannot accommodate time variation in the parameters of the state vector relevant for pricing bonds. Within this approach, the failure of ET is entirely abscribed to the presence of a time-varying risk premium, which is modelled as a linear function of the state of the economy. There is a lot in common between the latest developments of the no-arbitrage approach and the approach to the term structure proposed in our paper. We share the view on the importance of augmenting the information set with macroeconomic and financial factors to model the yield curve but we concentrate directly on a VAR model for all the relevant factors and we derive risk premium as a residual. The main cost of our approach is that our derived proxy for the risk premium is valid only under the assumption that the difference between the agents information set and the econometrician' information set does not lead to different future projected short-term rates. The main advantage is a much more parsimonious (and linear) parameterization, which easily accommodates time-variation in the parameters describing the state vector relevant for pricing bonds. Our findings suggests that the importance of fluctuations of risk premia in explaining the deviation from the ET might be reduced when some forecasting model for short-term rates is adopted and a proper evaluation of uncertainty associated to policy rates forecast is considered. We believe that improving the forecasting model for policy rates within a no-arbitrage approach is an important step to assess the relative weight of forecasting errors and risk premia in explaining deviations from the Expectations Theory. This is on our agenda for future research.

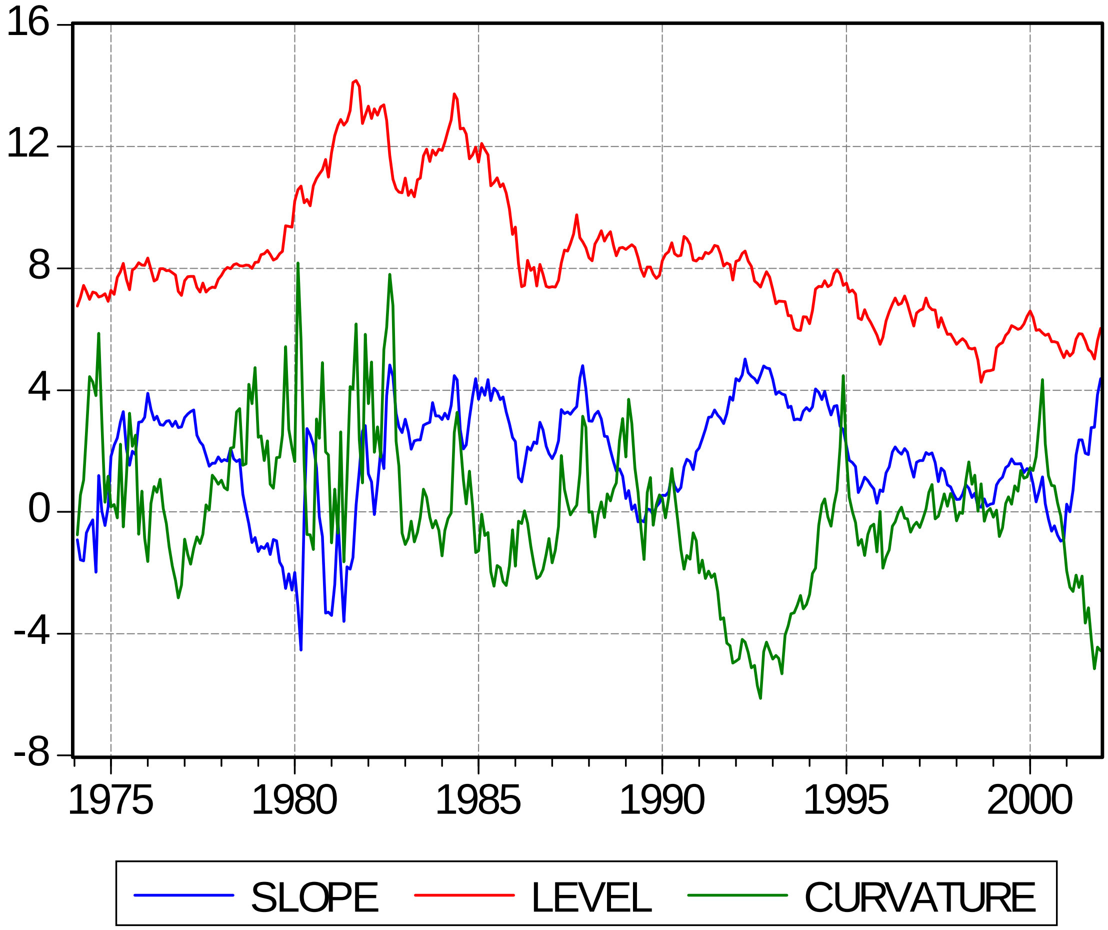

Figure 1: the time series of the three Nelson-Siegel factors for the US yield curve

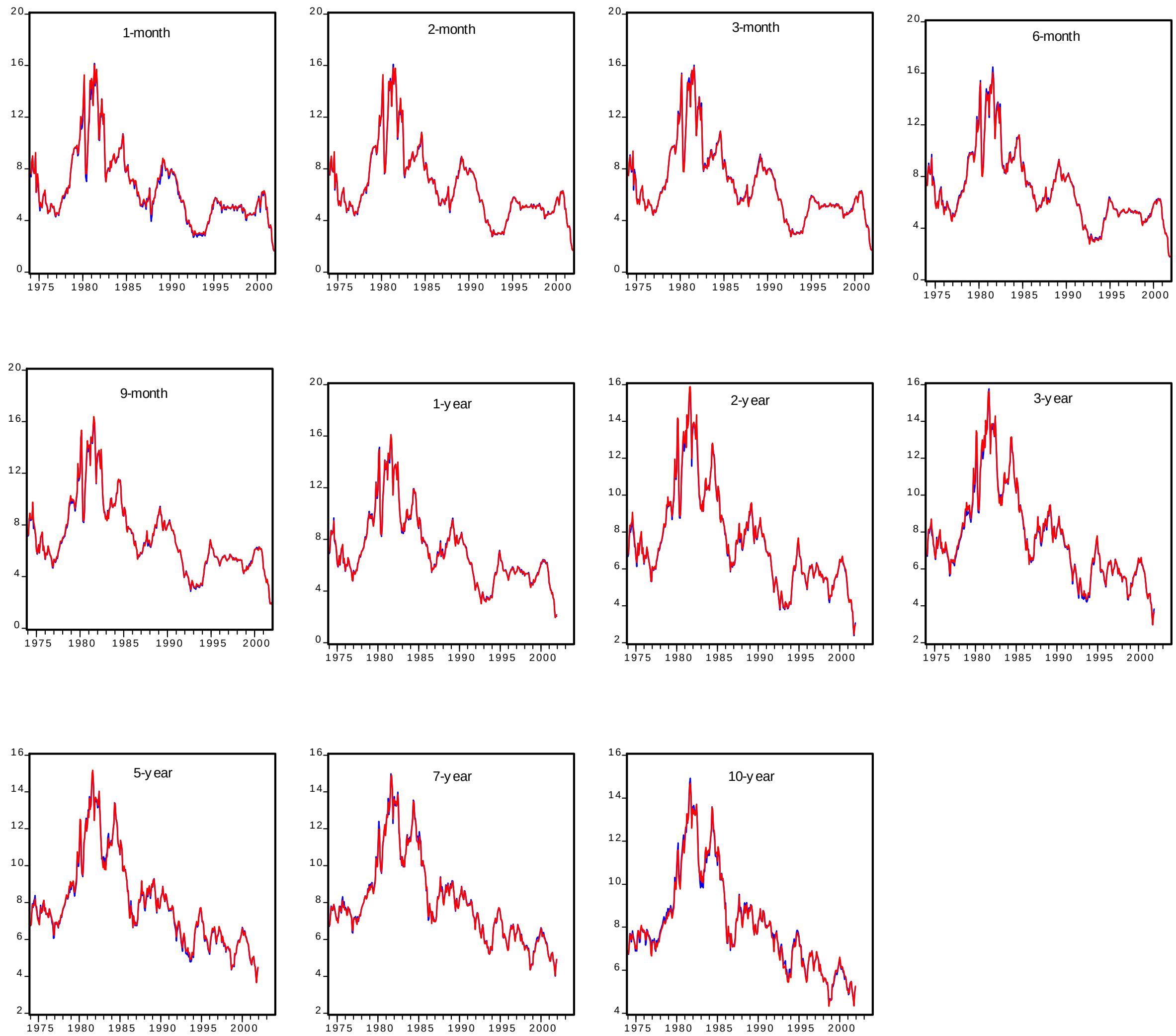

Figure 2: the time series of yields at different maturities and the Nelson-Siegel interpolants

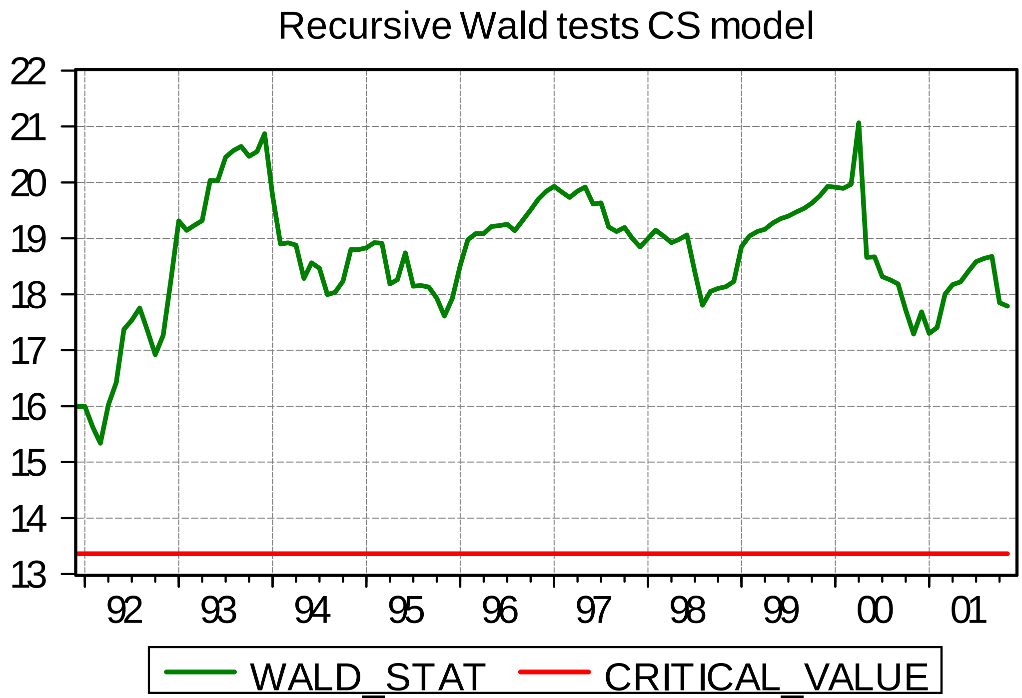

Figure 3: Recursive tests (and five per cent critical value) for the validity of the cross-equation restrictions implied by the Expectations Theory in a four-lags VAR with two financial factors (change in policy rates and slope of the yield curve, as in CS).

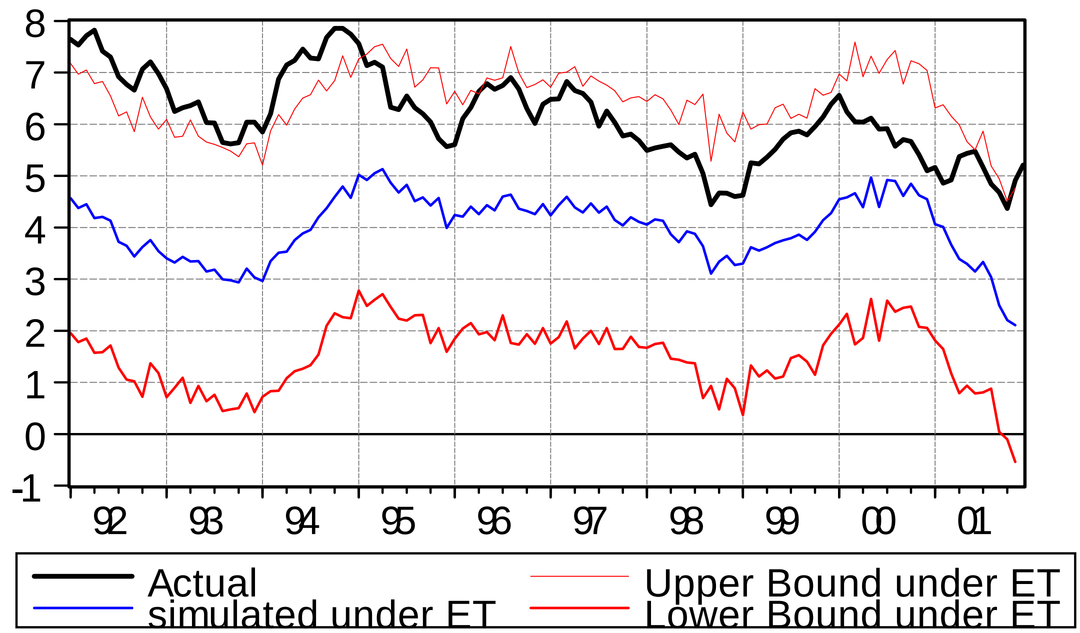

Figure 4: Simulated ET-consistent 10-year yields to maturity based on the CS model, with lower and upper bond of its 95% Confidence Interval

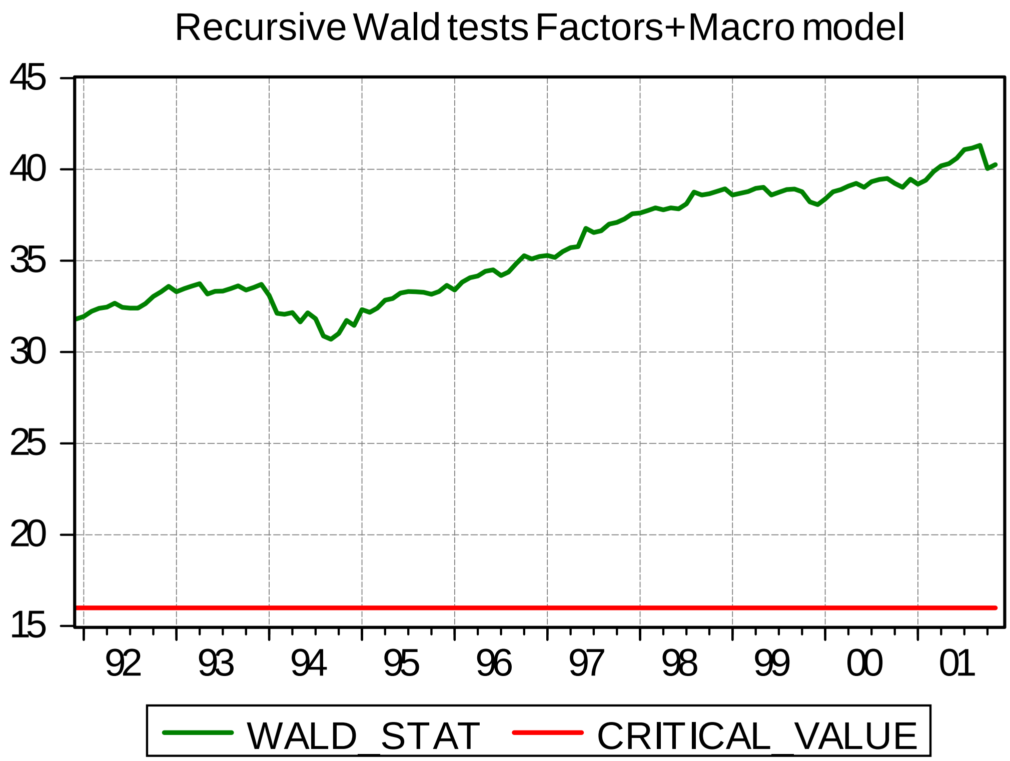

Figure 5: Recursive tests (and five per cent critical value) for the validity of the cross-equation restrictions implied by the Expectations Theory in a VAR with three financial factors and two macroeconomic variables.

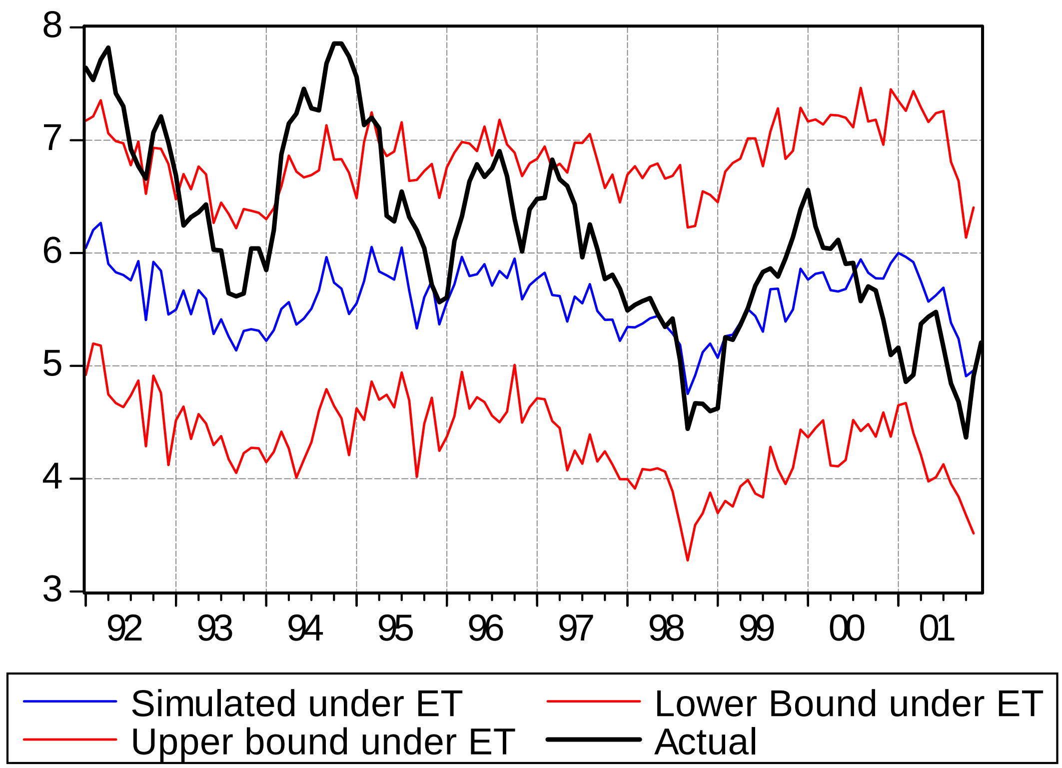

Figure 6: Simulated ET-consistent 10-year yields to maturity based on the model with financial factors and macroeconomic variables, with lower and upper bond of its 95% Confidence Interval

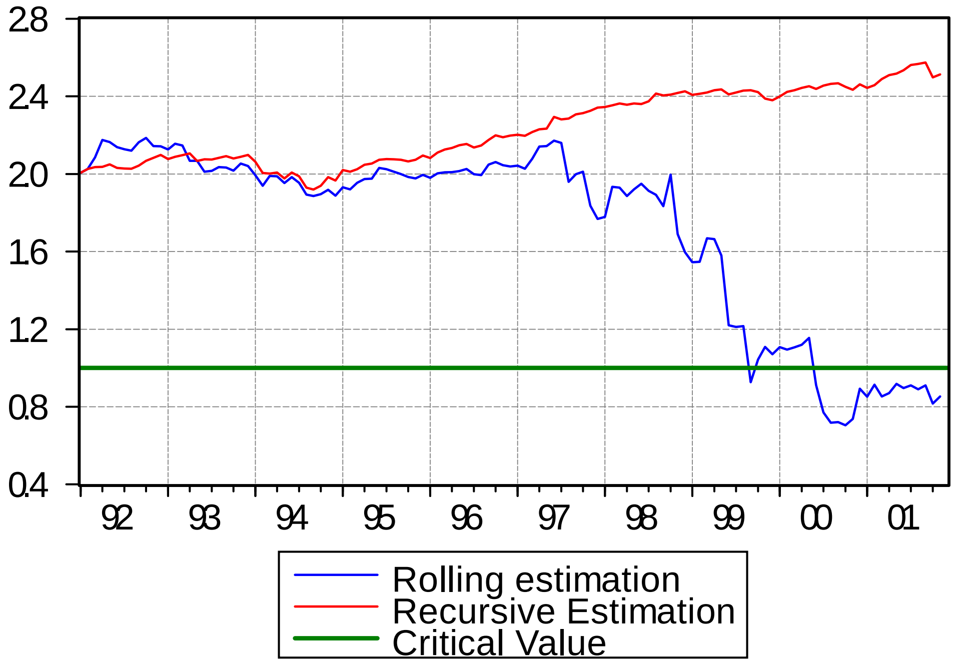

Figure 7: Wald tests for the EH restrictions on the VAR with financial factors and macroeconomic variables. The reported tests, scaled by their 95 per cent critical value, are recursively computed for all end sample points from 1992:1 to 2001:12. Initial sample points are chosen by twodifferent methods: Recursive estimation is based on anchoring the first observation to 1974:6 , Rollling estimation is results based on a rolling estimation with initialization 1974:6 1991:12 and a fixed window of 210 observations.

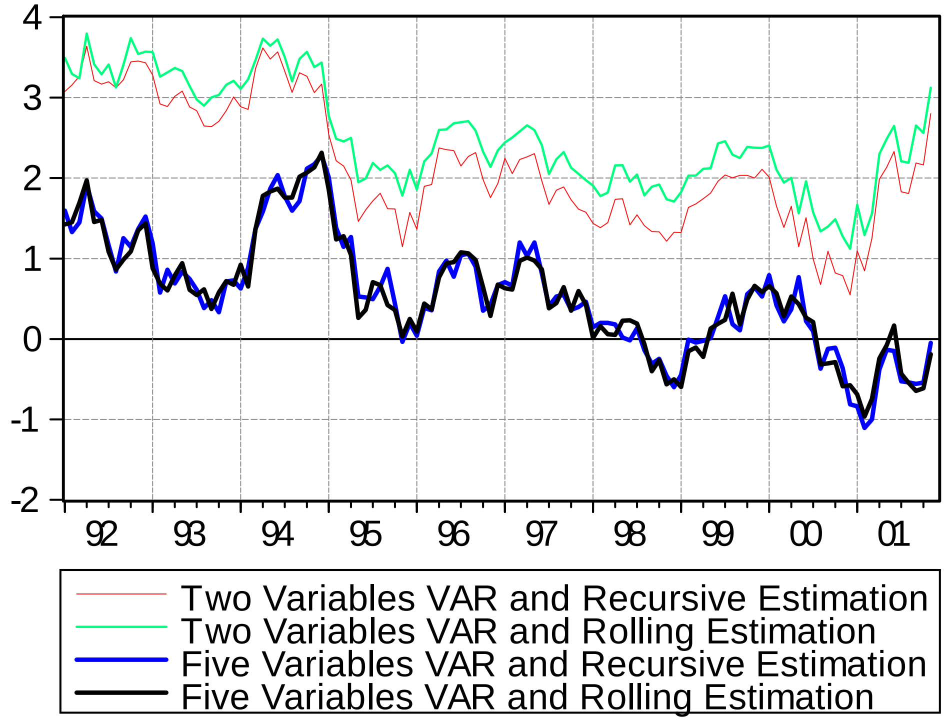

Figure 8: Time-series of the difference between actual and simulated yields under ET . Yields are simulated based respectively on recursive and rolling estimation of a five-variables VAR, and of a two-variables VAR

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Chapter 2

A No-Arbitrage Structural Vector Autoregressive Model of the UK Yield Curve

2.1 Introduction

The main motivation of the paper is to try to understand how the term structure of nominal interest rates responds to the fundamental shocks of the economy.

Our paper builds upon and extends two strands of research. The first is the no-arbitrage term structure literature, and the second is connected to empirical macroeconomic SVAR models. At the nexus of these two strands, we present a yield curve model that relates fundamental macroeconomic shocks to the bond pricing behavior of the economic agents.

As suggested by theoretical and empirical research, the dynamics of the term structure can be explained by a limited number of factors. But what is the nature of the factors driving the yield curve and how are they related to the economy? In canonical arbitrage-free term structure models, the factors driving the term structure are attributed to pure latent factors (see, for example, Duffie and Kan (1996), and Dai and Singleton (2000)). By contrast, a growing macro-finance literature links the dynamics of the term structure to macroeconomic variables: deviating from the pure latent structure, macroeconomists show that the shocks to macroeconomic factors account for a large part of the time variation in bond yields. For example, in pioneering work introducing macroeconomic variables into a term structure model, Ang and Piazzesi (2003) claim that macro-factors explain up to 85%

of the variation in US bond yields. More recently, for the UK, Lildholdt, Peacock and Panigirtzoglou (2007) confirm the importance of macroeconomic factors for the yield curve. They find that inflation and output gap drive the short end of the yield curve, whereas long-run inflation dominates the long end. However, the shocks in these models are not fully structural. A simple structural model of macroeconomy based on the Euler equation for consumption and pricing equation for firms is developed by Rudebusch and Wu (2004), who interpret the yield dynamics in terms of a structural macroeconomic model. Nevertheless, they employ a reduced-form pricing kernel, which conflicts with the specification of the marginal rate of substitution in their Euler equation.

From the macroeconomic literature side, there are a few empirical studies using a SVAR framework to describe the joint dynamics of the macroeconomy and the yield curve. For instance, Evans and Marshall (1998) were among the first to study the effects of monetary policy shocks on the yield curve in joint macroeconomic-term structure SVAR framework. By including single yields into standard macroeconomic SVARs, they describe the dynamics of the yield curve by studying the impulse responses of single yields, devoid of a dynamic term structure model of interest rates. Obviously, the inclusion of separate yields in a macroeconomic VAR is not an efficient approach, since it omits information contained in the whole yield curve. The impulse responses can be modelled only for the yields on observed bonds, while the method has no implications for the yields on bonds with non-traded or intermediate maturities. Moreover, Evans and Marshall (1998) do not rule out the arbitrage opportunities and cannot explain whether the changes in yields are due to the revision of expectations or due to the changes in risk premia.

Taking this inefficiency in consideration, we contribute to the macro-finance term structure literature by combining an arbitrage-free term structure model with a SVAR approach. The combination of the SVAR and the ATSM helps us to achieve several goals. Primarily, we are able to model the yield curve across maturities and across time jointly with the macroeconomic dynamics. In particular, we relate the yield curve to the fundamental structural macroeconomic shocks and impose cross-equation no-arbitrage restrictions into the parameters of yields.

The model is estimated in two steps. First, we identify structural fundamental shocks from an SVAR based on macroeconomic variables and the policy rate. Second, we estimate the yield parameters (restricted due to no-arbitrage) by minimizing the sum of squared fitting errors of the model following the nonlinear least squares approach, as in Ang, Piazzesi, and Wei

(2006). The two-step procedure simplifies the estimation of highly parametrized ATSM considerably.

As an application, we study fundamental shocks of the UK economy and establish their role in determining the term structure of UK nominal interest rates. A structural VAR is used to identify aggregate supply, aggregate demand, and monetary policy shocks. We then analyze the shocks' effects on the nominal yield curve with the help of the term structure model of interest rates. More specifically, we estimate a three-variable SVAR model based on the output gap, inflation and short-term interest rate. Following Blanchard and Quah (1989), supply shocks are identified via long-run restrictions, assuming that supply shocks alone have a long-run impact on the level of output. Additionally, we impose short-run restrictions on the variance-covariance matrix of residuals, which allows us to identify monetary policy shocks. Thus, in the context of the term structure model, we assume that agents are concerned with fundamental risks of the economy when they price bonds and, combining the SVAR and no-arbitrage approaches, we build a three-factor ATSM for the UK yield curve.

The results can be summarized as follows. We show that demand and supply shocks have different effects on the yield curve. Supply shocks, together with demand shocks, drive the short end of the yield curve, whereas demand shocks dominate the long end of the yield curve. Both, demand and supply shocks, affect the slope of the yield curve positively on impact. This result confirms the finding that the slope of the yield curve and the economic activity are linked together (see, for example, Estrella and Hardouvelis (1991), and Harvey (1988), who first documented the leading indicator properties of the yield curve slope for future economic activity). Finally, the monetary policy shock affects the whole yield curve, with the effect decreasing with maturity. We also show that the short end of the yield curve moves due to the changes in expectations, while the long end of the yield curve movements are due to the risk premia dynamics.

Although the model performs well overall, it does not fit the long end of the yield curve well, which suggests that including more factors might be an improving extension of my example model.

The rest of the paper is structured as follows. In Section 2 we outline the basic assumptions and implications of the canonical ATSM. We also show how to modify the model and impose a macroeconomic structure. Section 3 shows how to combine the SVAR and ATSM approaches. The details of the estimation method follow in Section 4. Our results are then presented in Section 5. Finally, some conclusions and possible extensions are provided in Section 6.

2.2 Related Literature

Our paper builds upon and extends two strands of literature. The first is the macro-financial no-arbitrage term structure literature, and the second is related to empirical macroeconomic SVAR models. At the nexus of these two strands, this paper identifies the fundamental macroeconomic shocks and associates them with the bond pricing behavior of the economic agents. In this section, we outline the basic concepts of the related models and briefly highlight their main features.

2.2.1 No-arbitrage term structure models

The fundamental assumption of the no-arbitrage term structure models is the statement that there are only a few variables, X_t, relevant for bond pricing (Duffie and Kan (1996), Dai and Singleton (2000)). These models price all bonds in economy by specifying the dynamics of the state vector, setting initial conditions for the bond pricing rule, and specifying the market prices of risk, Λ_t. In a discrete-time setup the three basic assumptions of ATSM take the following form:

1) The transition equation for the state vector relevant for pricing bonds follows the Gaussian VAR:

$$X_t = \mu + \Phi X_{t-1} + \Sigma\varepsilon_t, \tag{2.1}$$

where X_t is an $(n \times 1)$ -vector of state variables, ε_t is an $(n \times 1)$ -vector of i.i.d. shocks with zero mean and identity covariance matrix; μ is $(n \times 1)$, Φ is $(n \times n)$, Σ and is $(n \times n)$.

2) The one-period interest rate is a linear function of the state variables:

$$r_t = \delta_0 + \delta_1 X_t, \tag{2.2}$$

where δ_0 is a scalar, and δ_1 is an $(1 \times n)-$vector.

3) The prices of risk associated with shocks ε_t, denoted by Λ_t, are an affine function of the state of the economy (see Duffee (2002)):

$$\Lambda_t = \lambda_0 + \lambda_1 X_t \tag{2.3}$$

for the $(n \times 1)$ vector λ_0, and the $(n \times n)$ matrix λ_1. We use market prices of risk to specify a stochastic discount factor that transforms the physical distribution of bond prices into its risk-neutral equivalent.[1] In the case of the ATSM, the stochastic discount factor takes the form:

$$M_{t+1} \equiv \exp(-r_t)\exp(-\frac{1}{2}\Lambda_t'\Lambda_t - \Lambda_t'\Sigma_t^{-1}\varepsilon_{t+1}) \tag{2.4}$$

Under these assumptions, the price and yield of any maturity are affine functions of the state variables:

$$p_{t,n} = A_n + B_n'X_t, \tag{2.5}$$

$$y_{t,n} = -\frac{1}{n}\left(A_n + B_n'X_t\right), \tag{2.6}$$

where $y_{t,n}$ is a continuously compounded yield on the bond of maturity n at time t; scalar A_n and the $(n \times 1)$ vector B_n depend on the parameters $\{\lambda_0, \lambda_1, \delta_0, \delta_1, \mu, \Phi, \Sigma\}$ and, as shown in the Appendix, they are the solutions of the system of difference equations:

$$\begin{aligned} A_{n+1} &= A_n + B_n'\mu - B_n'\Sigma\lambda_o + \tfrac{1}{2}B_n'\Sigma\Sigma' B_n - \delta_0 \\ B_{n+1} &= (\Phi - \Sigma\lambda_1)' B_n - \delta_1' \end{aligned} \tag{2.7}$$

The no-arbitrage ATSM (NA-ATSM) has gained huge popularity in the finance literature due to the fact that the implied affine functions of a few unobservable (latent) factors explain almost all movements of the yield curve (see Duffie and Kan (1996), or Dai and Singleton (2000)). Nevertheless, the pure ATSM has not gained the same popularity among economists since it is not suitable for many macroeconomic policy applications. There is no theory behind the NA-ATSM apart from the no-arbitrage assumption and the economic nature of the latent factors is unclear. Observing that the short-term rate is an instrument for the Monetary Policy, macroeconomists have proposed a possible solution: to combine No-Arbitrage ATSM models with macroeconomic models.

The attempts to incorporate macroeconomic theory into the no-arbitrage models can be divided into three groups. First, one can assume that the state vector is completely unobservable and after that can search for its macroeconomic interpretation by Taylor rules, or by other typical macroeconomic relations (see Rudebusch and Wu (2004) who interpret latebt factors as the

[1] If investors are risk neutral, the assumption of no-arbitrage implies that $P_{t,n} = E_t(e^{-r_t}P_{t+1,n-1})$. However, if investors are risk averse, then their pricing behavior differs from that of risk neutral agents, since the former take into account the amount of risk affecting the future prices. In this case, a stochastic discount factor, M_t, transforms the observed distribution of prices into a risk-neutral equivalent distribution: $P_{t,n} = E_t(M_{t+1}P_{t+1,n-1})$.

perceived inflation target and output gap, and therefore enrich the state dynamics with variables related to inflation and output). Second, certain authors argue that all factors related to bond pricing are observable. One example of this method is the paper by Ang, Piazzesi and Wei (2006) who combine empirical unrestricted VAR macro-model with a No-Arbitrage representation of yields. The paper by Ang and Bekaert (2004) belongs to the third group, which is a combination of the first two approaches. The authors suppose that the state vector relevant for the bond pricing consists of both, latent and observable, factors. This approach is very popular and there are a number of papers which adopt it: Hördhal, Tristani, and Vestin (2005), Ang and Piazzesi (2003), Dai and Philippon (2004) among others. Our model also belongs to the third, "mixed", group of ATSMs, as we assume that the state vector consists of both, latent and observable factors.

2.2.2 Structural VAR models

Returning to the second strand of the literature, our paper belongs to a class of macroeconomic Structural VAR models that allow a researcher to transform the reduced-form VAR model into a system of structural equations of the economy. The identification of structural shocks is an extremely controversial venture since, imposing different identifying assumptions, it is possible to derive dissimilar conclusions about important economic questions. Restrictions depend on the variables included and on the shocks to be identified. Standard restrictions employed in the literature impose constraints on the short run or long-run impact of particular shocks on variables or informational gaps. For example, an assumption, in which output is not observed by central banks when making decisions on interest rates, results in a *short-run zero restriction.* Instead, *short-run sign restriction* is necessary to make sure that the impact of positive monetary policy shock is non-negative on the interest rate, and non-positive on real GDP growth and inflation. If only technology shocks had a permanent impact on the output and no other shock affected real activity in the long-run, then an econometrician would employ a *long-run zero restriction.*

Different empirical studies have used a SVAR framework to describe the joint dynamics of the macroeconomy and the yield curve. For instance, Evans and Marshall (1998) were among the first to study the effects of monetary policy shocks on the yield curve in a joint macroeconomic-term structure

SVAR framework.[2] They use the following vector autoregression:

$$\begin{bmatrix} Z_t \\ y_{t,n} \end{bmatrix} = \begin{bmatrix} A(L) & 0 \\ C(L) & D(L) \end{bmatrix} \begin{bmatrix} Z_{t-1} \\ y_{t-1,n} \end{bmatrix} + \begin{bmatrix} a & 0 \\ c & b \end{bmatrix} \begin{bmatrix} \varepsilon_t^Z \\ \varepsilon_t^n \end{bmatrix} \tag{2.8}$$

In their structural VARs, only macroeconomic variables, Z_t, affect the term structure.[3] However, such SVARs describe the dynamics of the yield curve studying the impulse responses of single yields, devoid of a dynamic term structure model of interest rates.

The term structure literature, instead, has been operating with rather arbitrary identification schemes. For example, in the work by Ang and Piazzesi (2003), there are 5 factors (3 latent and 2 principal components for real activity and inflation) and the variance-covariance matrix of residuals is assumed to be block-diagonal: macro-factors are recursively identified, latent factors are orthogonal. Unfortunately, it is unclear from the model how these shocks should be interpreted. Rudebusch and Wu (2004) work with two latent factors and VAR dynamics enriched by inflation and the output gap. Again, the interpretation of their shocks is vague. Lildholdt, Panigirtzoglou and Peacock (2007) introduce 3 factors (two unobserved factors and inflation as observed factor) and assume a diagonal variance-covariance matrix of residuals. Therefore, their shocks are orthogonal, and hence not fully structural.

There are two recent papers that work with no-arbitrage Structural VAR models. These are the papers by Ang, Dong and Piazzesi (2005) and Dai and Philippon (2005). Dai and Philippon (2005) were the first to estimate an ATSM based on Structural VAR dynamics. They present a macro-finance model that consider the pricing kernel as driven by several shocks, one of which is a structural fiscal policy shock identified using the long-run identification strategy by Blanchard and Perotti (2002). The approach of Ang, Dong and Piazzesi (2005) is very different: they are able to identify monetary policy shocks without imposing structural restrictions on the variance-covariance matrix of residuals. The authors show that the same ATSM accommodates several types of Taylor Rules: a benchmark Taylor Rule, a backward-looking Taylor Rule, and a forward-looking Taylor Rule.

[2] See also Bagliano and Favero (1998), who estimate VARs with macroeconomic variables, short- and long-term yields.

[3] Neither contemporaneous nor lagged values of the bond yields, y_t^j, enter into the equations for the macroeconomic variables. These assumptions ensure that the shocks to macroeconomic variables are invariant to bond maturity.

2.3 The Model

In contrast to previous no-arbitrage studies, our paper identifies all fundamental macroeconomic shocks as in the SVAR literature, and then uses ATSM to price bonds with respect to the identified shocks.

In our model, we choose n observable macroeconomic factors and one latent factor, f_t, to represent the state vector, X_t, of the yield curve. We denote Z_{0t} as a $(n \times 1)-$ vector of observed macroeconomic variables and describe the short term interest rate by the relationship:

$$r_t = \delta_0 + \delta_1 X_t \equiv \delta_0 + \boldsymbol{\delta}_z Z_{0t} + \delta_f \ f_t, \tag{2.9}$$

where $X_t \equiv \begin{bmatrix} Z_{0t} \\ f_t \end{bmatrix}$.

While number of observable macroeconomic factors could be arbitrary, it is important to only have one latent factor, since in this case we can map the state vector into the vector of observable variables and therefore to simplify the standard estimation method a lot[4]. Given (2.9), the state vector X_t is mapped to Z_t by the relation:

$$\begin{aligned} Z_t &= \begin{bmatrix} \mathbf{0}_{(n\times1)} \\ \delta_0 \end{bmatrix} + \begin{bmatrix} I_{(n\times n)} & \mathbf{0}_{(n\times1)} \\ \boldsymbol{\delta}_{z(1\times n)} & \delta_f \end{bmatrix} X_t & (2.10) \\ &\equiv M_{0((n+1)\times1)} + M_{((n+1)\times(n+1))} \cdot X_t & (2.11) \end{aligned}$$

Observed macroeconomic variables $Z_t = (Z_{0,t}, \ r_t)'$ are assumed to follow the SVAR process:

$$AZ_t \ = \alpha + B(L)Z_{t-1} + \varepsilon_t, \tag{2.12}$$

with $A = A_{(n+1)\times(n+1)}$, $\alpha = \alpha_{(n+1)\times1}$, $B = B_{(n+1)\times(n+1)}$, or, in a reduced form representation:

$$Z_t \ = \tilde{\mu} + \tilde{\Phi}(L)Z_{t-1} + \tilde{\Sigma}\varepsilon_t \tag{2.13}$$

where $\tilde{\Phi}(L)$ is a polynomial in the lag operator, and the maroeonomic shocks to be identified are given by $\varepsilon_t, \ \varepsilon_t \sim N(0, I), \ \tilde{\Sigma}\tilde{\Sigma}' = V$.

Interestingly, the dynamics of the vector Z_t are not affected by the no-arbitrage restrictions, since the short-term rate, r_t, is a risk-free rate and hence has no need to be adjusted for risk. Once the shocks are identified in

[4] According to Dai and Singleton (2000), affine term structure models require three latent factors to match yield curve dynamics.

the macroeconomic SVAR (2.13), equations (2.9) and (2.12) imply that the state vector follows the VAR process:

$$X_t = \mu + \Phi X_{t-1} + \Sigma\varepsilon_t, \tag{2.14}$$

where

$$\mu = M^{-1}\left(\tilde{\mu} - (I - \tilde{\Phi})\ M_0\right) \tag{2.15}$$

$$\Phi = M^{-1}\tilde{\Phi}M, \tag{2.16}$$

$$\Sigma = M^{-1}\tilde{\Sigma} \tag{2.17}$$

This structure is a standard assumption about the dynamics of the state vector in affine term structure models.

The market prices of risk are assumed to be affine functions of the state of economy (see Duffee (2002))

$$\Lambda_t = \lambda_0 + \lambda_1 X_t, \tag{2.18}$$

and the pricing kernel is given by

$$m_{t+1} \equiv -\frac{1}{2}\Lambda_t'\Lambda_t - \Lambda_t'\tilde{\Sigma}^{-1}M\varepsilon_{t+1} - r_t, \tag{2.19}$$

where $\tilde{\Sigma}$ is a variance-covariance factor from the SVAR of observed macroeconomic variables, and M is a matrix mapping observed variables into the state vector.

Equations (2.14), (2.18), and (2.9) constitute a standard affine term structure model with one latent and n observable factors. The main feature of the model, however, is that the pricing kernel is a function of *structural macroeconomic shocks*, ε_{t+1}, identified by the restrictions on SVAR. In general, our approach allows us to explain the yield curve with any kind of SVAR model based on the macroeconomic variables and the short-term interest rate. We call the approach a No-Arbitrage Structural Vector Autoregressive (NA-SVAR) Model of the Yield Curve and, in what follows, we propose a simple three-variable SVAR model as an application of our approach.

2.3.1 An Example

We want to explain the dynamics of the state vector by supply, demand and monetary policy shocks, $\varepsilon_t = \left(\varepsilon_t^S, \varepsilon_t^D, \varepsilon_t^{MP}\right)'$. As the model defines the short-term rate as a linear combination of the state variables in X_t, a natural assumption is to assume that the observable state variables are the rate of inflation, π_t, and a real variable. Due to the unobservability of potential output and the output gap, we use annual GDP growth, g_t, as a proxy for real activity. Thus $X_t = (g_t,\ \pi_t,\ f_t)'$ similar to Ang, Dong, Piazzesi (2005) and we describe a short-term interest rate by Taylor Rule:

$$r_t = \delta_0 + \delta_1 X_t \equiv \delta_0 + \delta_g g_t + \delta_\pi \pi_t + \delta_f\ f_t, \tag{2.20}$$

However, we restrain ourselves from interpreting the latent factor as a monetary policy shock, since the simple benchmark Taylor Rule (2.20) is likely to be misspecified.[5] Instead, we identify monetary policy shocks from our Structural VAR framework.

The identification scheme is based on the approach by Blanchard and Quah (1989), who use a long-run restriction to identify aggregate supply and aggregate demand shocks in a bivariate model of GNP and unemployment. Blanchard and Quah (1989) attain identification by limiting aggregate demand not to have a permanent effect on the level of GNP. Here we also use the Blanchard-Quah identifying strategy to separate supply shocks from other economic surprises. We interpret the fluctuations in the state vector as due to two types of shocks: shocks that have permanent effect on output and shocks that do not. The first type of shock is interpreted as a supply shock. In addition, given that the one-period interest rate is among the SVAR variables, one of the non-permanent fundamental shocks could be interpreted as a monetary policy shock.

In our example, we identify the supply shock by constraining other disturbances to have a zero long-run effect on output. Technically, we impose long-run restrictions, assuming that supply shock ε_t^S is the only shock that has a permanent effect on output. Formally, this exclusion constraint is specified by:

$$\lim_{j \longrightarrow \infty} [E_t g_{t+j} - E_{t-1} g_{t+j}] = f(\varepsilon_t^S).$$

It is straightforward to show that

$$\lim_{j \longrightarrow \infty} [E_t g_{t+j} - E_{t-1} g_{t+j}] = e1_{(1\times3)} \cdot \left((I_{(3\times3)} - \tilde{\Phi})^{-1} \tilde{\Sigma} \varepsilon_t \right), \tag{2.21}$$

[5] For instance, empirical applications find that the lagged interest rate is an omitted variable from the benchmark Taylor rule, a result that is interpreted as interest rate smoothing by the monetary policy authorities (see, for example, Clarida, Galì and Gertler (2000)).

where $e1_{(1\times 3)} = [1\ 0\ 0]$ is a selection row vector. Indeed,

$$\begin{aligned}
E_t X_{t+j} &= \tilde{\Phi}\ X_{t+j-1} + \tilde{\Sigma}\varepsilon_t = \tilde{\Phi}(\tilde{\Phi} X_{t+j-2} + \tilde{\Sigma}\varepsilon_t) + \tilde{\Sigma}\varepsilon_t \\
&= \tilde{\Phi}^2 X_{t+j-2} + \tilde{\Phi}\tilde{\Sigma}\varepsilon_t + \tilde{\Sigma}\varepsilon_t = ... \\
&= \tilde{\Phi}^{j+1} X_{t+j-1} + \tilde{\Phi}^j \tilde{\Sigma}\varepsilon_t + \tilde{\Phi}^{j-1}\tilde{\Sigma}\varepsilon_t + ... + \tilde{\Sigma}\varepsilon_t; \\
E_{t-1} X_{t+j} &= \tilde{\Phi}^{j+1} X_{t+j-1},
\end{aligned}$$

implying that

$$\begin{aligned}
\lim_{j\longrightarrow\infty} [E_t X_{t+j} - E_{t-1} X_{t+j}] &= \lim_{j\longrightarrow\infty} (\tilde{\Phi}^j + \tilde{\Phi}^{j-1} + ... + I_{(3\times 3)})\tilde{\Sigma}\varepsilon_t \\
&= (I_{(3\times 3)} - \tilde{\Phi})^{-1}\tilde{\Sigma}\varepsilon_t,
\end{aligned}$$

Adding the long-run exclusion restriction is then equivalent to imposing the following constraint on the dynamics of the state vector:

$$D \equiv (I_{(3\times 3)} - \tilde{\Phi})^{-1}\tilde{\Sigma} = \begin{bmatrix} \cdot & 0 & 0 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}, \tag{2.22}$$

where $\cdot$ denotes a free parameter.

To calculate the dynamic effects of the supply shock, we require estimates of $\tilde{\Phi}$, which could be obtained by ordinary least squares estimation, and the first column of $\tilde{\Sigma}$, which in turn can be obtained once we know the matrix D. Note that

$$\begin{aligned}
DD' &= (I_{(3\times 3)} - \tilde{\Phi})^{-1}\tilde{\Sigma}\tilde{\Sigma}'(I_{(3\times 3)} - \tilde{\Phi}')^{-1} \qquad (2.23) \\
&= (I_{(3\times 3)} - \tilde{\Phi})^{-1} V_0 (I_{(3\times 3)} - \tilde{\Phi}')^{-1} \\
&= \begin{bmatrix} d_{11} & 0 & 0 \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix} \begin{bmatrix} d_{11} & d_{21} & d_{31} \\ 0 & d_{22} & d_{32} \\ 0 & d_{23} & d_{33} \end{bmatrix}' \\
&= \begin{bmatrix} d_{11}^2 & d_{21}d_{11} & d_{31}d_{11} \\ d_{21}d_{11} & d_1 & d_2 \\ d_{31}d_{11} & d_2 & d_3 \end{bmatrix}
\end{aligned}$$

Thus, the first element of D, d_{11}, can be obtained given $\tilde{\Phi}$ and the variance-covariance matrix of residuals V_0, assuming that $d_{11} > 0$. Given d_{11}, the elements d_{21}, d_{31} can be easily estimated together with the first column of $\tilde{\Sigma}$.

Additionally, we impose short-run restrictions to distinguish between the two remaining shocks. We impose standard zero restrictions, under which the monetary policy actions have no immediate effect on both inflation and real output[6]:

$$\tilde{\Sigma} = \begin{bmatrix} \cdot & \cdot & 0 \\ \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot \end{bmatrix}. \tag{2.24}$$

2.4 Estimation

2.4.1 Data

We use monthly data on continuously compounded nominal spot yields from the Bank of England's dataset, assuming them to be default-risk-free. To estimate the model we work with bonds of a wide range of maturities: policy rate, 9 months, 12 months, 3 years, 5 years, 7 years, and 10 years.

The sample is limited to 1992:10-2006:12. While the relationship between yields might be stable over time, the relationship between interest rates and macroeconomic variables is likely to have changed over time. Thus we limit our analysis to the recent monetary policy regime and focus on the period of inflation targeting.

As a proxy for inflation, we use annual CPI inflation[7]. Annual monthly GDP growth estimates are taken from NIESR dataset. The series are displayed in Figure 2.1.

2.4.2 Model Identification

For our particular sample and variables, we specify the dynamics of the state vector by a VAR(1):

$$Z_t = \tilde{\mu} + \tilde{\Phi}_1 Z_{t-1} + \tilde{\Sigma}\varepsilon_t. \tag{2.25}$$

It is easy to verify that in our case the exclusion restriction (2.22) for the identification takes the form:

$$\left(I_{(3\times 3)} - \tilde{\Phi}_1\right)^{-1} \tilde{\Sigma} = \begin{bmatrix} \cdot & 0 & 0 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}. \tag{2.26}$$

[6] Since in this case the model would be overidentified, we alternatively realx the assumption that monetary policy shock has no effect on output in the short run and thus let $\tilde{\Sigma}(1,3)$ as a free parameters. The results do not change significantly.

[7] The results are similar for RPIX inflation, which was the UK's target measure during 1992-2003.

The impulse response γ_h of Z_{t+h} to a unit shock in ε_t can then be computed as

$$\gamma_h = \tilde{\gamma}_h \tilde{\Sigma} \tag{2.27}$$
$$\tilde{\gamma}_h = \tilde{\Phi}_1 \tilde{\gamma}_{h-1} \tag{2.28}$$

with initial conditions $\tilde{\gamma}_{-1} = 0,\ \tilde{\gamma}_0 = I_{3\times3}$. Here the (j,l) element of γ_h represents the response of jth element in Z_{t+h} to a unit shock in the lth element of ε_t. (See, for example, Christiano, Eichenbaum and Evans (1999)).

In constructing confidence intervals for impulse responses, we use bootstrap simulated distribution percentiles, i.e extract the relevant bands directly from the ordered replications at each horizon. Since the simple bootstrap method suffers from the small sample biasedness and the lack of scale invariance, we use a bootstrap-after-the-bootstrap procedure suggested by Kilian (1998). The approach is summarized in Canova (2005), Chapter 4.

Let $\alpha = \{\lambda_0, \lambda_1, \delta_0, \delta_1, \mu, \Phi, \Sigma\}$ be the set of free parameters in the model. To estimate yield curve parameters and extract the latent factor, we use the approach attributed to Chen and Scott (1993). In this setting, N unobservable factors are computed by assuming that N bond yields are measured without error. Other interest rates are assumed to be measured with error and provide over-identifying restrictions for term structure models. In our joint macro-finance model, there is only one latent factor, and thus we assume that only one yield, a 1-month rate, r_t, is observed exactly. Although in the approach by Chen and Scott (1993) the choice of the yields measured without errors is arbitrary, in our case it is natural to assume that the one-month rate is observed without error, as it is closely linked to the perfectly observed policy rate. The vector Y_t^{Er} of the remaining bond yields is instead observed with independently distributed zero-mean errors. This assumption has the additional important advantage that we can implement a two-step estimation procedure, which decreases the number of parameters to be estimated drastically compared to a one-step maximum likelihood approach.

Since the SVAR of the observed variables (2.13) is not affected by no-arbitrage restrictions, we can estimate the VAR parameters $\tilde{\mu}, \tilde{\Phi}, \tilde{\Sigma}$ by ordinary least squares (OLS) in the first step. Then, in the second step, given the VAR estimates, we map the observed variables into the state vector by equation (2.29) and get the estimates for the state dynamics and the risk price parameters by the GMM method proposed by Ang, Piazzesi and Wei

(2006). Namely, we minimize the sum of squared fitting errors of the model

$$\min_{\{\lambda_0,\lambda_1,\delta_0,\delta_1\}} \sum_{n=1}^{N}\sum_{t=1}^{T} \left(y_{t,n} - \hat{y}_{t,n}\right)^2 ,$$

where we compute model-implied yields, $\hat{y}_{t,n} = -\frac{1}{n}\left(A_n + B_n' X_t\right)$, with the factors, X_t, extracted from the inverted state relation (2.10) of the model:

$$X_t = M^{-1}[Z_t - M_0], \tag{2.29}$$

with

$$\begin{aligned} M_0 &= \begin{bmatrix} 0 & 0 & \delta_0 \end{bmatrix}', \\ M &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \delta_g & \delta_\pi & \delta_f \end{bmatrix} \end{aligned}$$

The standard errors are calculated using two-step GMM, with moments from each stage of the procedure as in Ang, Piazzesi, and Wei (2006).

Due to the presence of the latent factor, the model is not identified: the latent factor could be arbitrarily scaled and shifted, producing observationally equivalent systems. We therefore normalize the model by imposing the loading of the short rate on the latent factor $\delta_f = 1$, and by constraining the mean of the latent factor to be zero. We further assume that the market prices of risk affecting the time variation of the risk premia are represented by matrix λ. Since several elements were insignificant when we estimated a full matrix λ_1, we restrict them to be zeros:

$$\lambda_1 = \begin{pmatrix} \lambda_{11} & 0 & 0 \\ \lambda_{21} & \lambda_{22} & 0 \\ 0 & 0 & \lambda_{33} \end{pmatrix}.$$

2.5 Results

To explain the dynamics of the yield curve, we first apply OLS to estimate the reduced form VAR for GDP growth, CPI inflation and the short-term interest rate. The structural residuals are then retrieved by the help of long-run and short-run restrictions. The responses of the observed factors to the fundamental shocks are shown in Figure 2.2. The impulse response functions of the VAR confirm the findings of Blanchard and Quah (1989): a supply shock permanently affects the level of output due to its strictly positive effect on output growth and negatively affects inflation on impact. After a negative response to the positive supply shock, inflation increases and comes back to zero after approximately four years. A positive supply shock has a significant positive and persistent effect on the short-term interest rate. A positive demand shock increases both prices and output growth, but the effect of demand on the output seems to be statistically insignificant.

The estimated parameters determining market prices of risk are reported in Table 1. The parameters in λ_0, affecting only the constant yield coefficient $\left(-\frac{1}{n}A_n\right)$ are significantly different from zero, which means that the fundamental risks affect the average term spreads and expected returns. The parameters in λ_1, affecting both the time variation in the yields and indirectly the constant yield coefficient, are significantly different from zero as well. Thus the hypothesis: $\lambda_{0i} = \lambda_{1i} = 0$, for all i, under which the risk premium is zero and investors are risk neutral, is rejected by the model.

Table 2 reports first and second moments of the observed yields and those predicted by the estimated model. We see that the model fits the short end of the yield curve reasonably well, but the long end is not fully captured: long term yields implied by the model are less volatile than in the data. This unexplained variance is attributed to the measurement errors. The issue of a large "excess volatility" in long-term interest rates is not new in the literature. It was raised more than twenty-five years ago by Shiller (1979) and it is still relevant today (see, for instance, Elingsen and Söderstrom (2005), who confirm that observed long-term interest rates seem to be excessively sensitive to fundamental innovations.).

Finally, Figure 2.3 depicts the demeaned latent variable, f_t, and the residual from estimating the benchmark Taylor Rule

$$r_t = \alpha + \beta\pi_t + \gamma g_t + \epsilon_t$$

using OLS. We see that the latent factor and OLS residual follow the same pattern (the correlation coefficient is 0.644). However, we would not interpret the latent factor as a monetary policy shock: the variable is not a white noise and is quite persistent. Moreover, the monetary authorities react to the dynamics of the macroeconomic conditions, which means that, in principle, the monetary policy shock implied by SVAR is a more appropriate measure.

2.5.1 Model Dynamics and Fundamental Shocks

Figures 2.4-2.6 show how our identified fundamental shocks affect the yield curve. More precisely, each plot shows the reaction of all maturities to a one unit standard deviation impulse to each fundamental shock. The monetary policy shock shifts the level of the yield curve upward with an effect decreasing with maturity, as reported by Ang and Piazzesi (2003). Supply shocks by contrast have a small effect on short-term interest rates, but a large effect on long rates, thus increasing the slope of the yield curve on the impact, which is the same pattern Evans and Marshall (2001) present for an unrestricted SVAR of the US yield curve. In our model, demand shocks on average increase long term interest rates, with the effect increasing with maturity. These results confirm the well-known empirical finding that the slope of the yield curve and economic activity are interrelated: an increase in the slope tends to indicate a higher GDP growth in the future (see Estrella and Hardouvelis (1991), and Harvey (1988)).

In addition, we would like to compare the impulse responses of yields produced by our model and by those produced without imposing No-Arbitrage, using the approach by Evans and Marshall (1998). However, we have met several difficulties in implementing a direct comparison. For example, the lag length optimal for the VAR of the state vector produces very imprecise estimates of the yield equation in (2.8), or, for certain cases, the companion matrix in (2.8) could have the eigenvalues outside the unit circle. Thus, in order to confront the most appropriate impulse responses, we impose additional zero restrictions on the yield parameters in VAR (2.8) and estimate the following form of VAR:

$$\begin{bmatrix} Z_t \\ y_{t,n} \end{bmatrix} = \begin{bmatrix} \tilde{\mu} \\ c_{0n} \end{bmatrix} + \begin{bmatrix} \tilde{\Phi}(L) & 0 \\ C^n(L) & 0 \end{bmatrix} \begin{bmatrix} Z_{t-1} \\ y_{t-1}^n \end{bmatrix} + \begin{bmatrix} \tilde{\Sigma} & 0 \\ c_n & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_t^Z \\ \varepsilon_t^n \end{bmatrix} \quad (2.30)$$

These restrictions assure that the yields implied by both models are functions of the same state variables, since our model explains the dynamic of yields by the following state-space model:

$$\begin{cases} Z_t = \tilde{\mu} + \tilde{\Phi}(L) Z_{t-1} + \tilde{\Sigma}\varepsilon_t \\ y_{t,n} = \alpha_n + \beta_n(L) Z_t + \zeta_t^n \ , \end{cases} \quad (2.31)$$

where ζ_t^n is a measurement error of the $n-$maturity yield. The difference between (2.30) and (2.31) is the No-Arbitrage restrictions imposed on the coefficients α_n, β_n. Figure 2.7 shows the impulse responses of the yields implied by our model and by Evans and Marshall (1998)'s approach. The only inconsistency is seen when comparing the impact of monetary policy shocks on the long end of the yield curve. While both models imply that the long-term yields should increase on impact, Evans and Marshall (1998)'s model suggests that the effect of the monetary policy shock does not vanish even after ten years, which is rather counterintuitive. In general, both models produce similar short-term yield impulse responses to the whole range of the shocks.

While the approach by Evans and Marshall (1998) describes the dynamics of observed yields consistently with the NA-SVAR approach, it cannot explain whether changes are due to the revision of expectations or due to changes in risk premia. The advantage of our approach is that we have a complete model of the yield curve with its decomposition into expectations of risk free rate and risk premia. Figure 2.8 explains which of the yield components, expectations or risk premium, has a major impact on the dynamics of the yield curve. We find that the expectations component, $\sum_{i=0}^{n-1} E_t(r_{t+i})$, explains almost all movements of the short end of the yield curve, while it has little explanatory power for the long end of the yield curve.

Finally, a useful supplementary description of yield curve dynamics can be obtained from the variance decomposition shown in Table 3. The 1-month yield is driven by all three shocks, but predominantly by monetary policy. The 12-month yield is driven by supply and monetary policy shocks and, to a lesser extent, by demand shocks. Movements in the 10-year yield can be attributed to shocks to demand. Interestingly, monetary policy shocks have very little explanatory power for the variance of the long end of the yield curve (less than 7%). This implication is consistent with the "conundrum" recently observed in the data: UK long-term interest rates have not responded to the tighter monetary policy in 2006 and have remained at low levels.

2.6 Conclusions

This paper is only a first approach to identify fundamental economic shocks in the ATSM framework. We show how to price bonds with respect to the fundamental shocks of the economy. As a simple example of the NA-SVAR approach, we have chosen a three factor model of the UK yield curve. We interpret fluctuations in the state vector as due to two types of shocks: shocks that have permanent effects on output and shocks that do not. The first type of shock is interpreted as a supply shock, whereas the other shocks are related to demand and monetary policy.

Under this interpretation we show that positive supply shocks affect the whole yield curve, while positive demand shocks increase mostly the long end of the yield curve on impact and thus increase the slope of the yield curve. Demand and supply shocks account for a large part of the time variation in bond yields. In particular, demand shocks explain more than 80% of the variation in the long end of the yield curve in the UK. Moreover, we find that the short end of the yield curve is driven mostly by expectations, while the long end of the curve moves mostly due to changes in risk premia.

More elaborate models with more plausibly identified shocks could be employed to study the behavior of the yield curve, which is a promising direction on the nexus of the SVAR and no-arbitrage term structure literatures. In addition, the NA-SVAR model incorporates nominal yields and the inflation expectations and hence has implications for the real yield curve. That would be an additional paper to write.

2.7 Appendix

2.7.1 Bond Prices under No-Arbitrage

In this part we specify the recursive structure of the coefficients in the bond pricing equation. Putting together all assumptions made above, we get

$$p_t(n+1) = E_t\{m_{t+1} + p_{t+1}(n)\} + \frac{1}{2}Var_t\{m_{t+1} + p_{t+1}(n)\}$$

$$= E_t\left\{-R_{1,t} - \frac{\lambda_t'\lambda_t}{2} - \lambda_t'\varepsilon_{t+1} + A_n + B_n'X_{t+1}\right\} + \frac{1}{2}Var_t\{m_{t+1} + p_{t+1}(n)\}$$

$$= -R_{1,t} - \frac{\lambda_t'\lambda_t}{2} + A_n + E_t[B_n'(\mu + \Phi X_t + \Sigma\varepsilon_{t+1})] + \frac{1}{2}Var_t\left\{-\lambda_t'\varepsilon_{t+1} + B_n'\Sigma\varepsilon_{t+1}\right\}$$

$$= -\delta_0 - \delta_1'X_t - \frac{\lambda_t'\lambda_t}{2} + A_n + B_n'(\mu + \Phi X_t) + \frac{1}{2}Var_t\left\{\left(B_n'\Sigma - \lambda_t'\right)\varepsilon_{t+1}\right\}$$

$$= -\delta_0 - \delta_1'X_t - \frac{\lambda_t'\lambda_t}{2} + A_n + B_n'(\mu + \Phi X_t) + \frac{\left(-\lambda_t' + B_n'\Sigma\right)\left(-\lambda_t' + B_n'\Sigma\right)'}{2}$$

$$= \left(-\delta_0 + A_n + B_n'\mu + \frac{1}{2}B_n'\Sigma\Sigma'B_n - B_n'\Sigma\lambda_0\right) + \left(-\delta_1' - B_n'\Sigma\lambda_1 + B_n'\Phi\right)X_t$$

We get A_{n+1}, B_{n+1} as a solution of the system of difference equations with initial condition $A_1 = \delta_0,\ B_1 = -\delta_1'$:

$$\left\{\begin{array}{l} A_{n+1} = A_n + B_n'\mu - B_n'\Sigma\lambda_o + \frac{1}{2}B_n'\Sigma\Sigma'B_n - \delta_0 \\ B_{n+1} = (\Phi - \Sigma\lambda_1)'B_n - \delta_1' \end{array}\right\} \quad \text{(A4)}$$

Thus, the continuously compounded yield on a zero-coupon bond of maturity n, is an affine structure of the state:

$$y_t(n) = -\frac{1}{n}(A_n + B\prime_n X_t) \equiv a_n + b_n'X_t \quad \text{(A5)}$$

Table 1: Market prices of risk estimates

Prices of risk, λ_0			Prices of risk, λ_1		
λ_{0g}	0.487	(0.003)	λ_{11}	-343.64	(921.99)
$\lambda_{0\pi}$	1.183	(0.031)	λ_{21}	-381.5	(324.6)
λ_{0f}	0.123	(0.001)	λ_{22}	-200.75	(102.01)
			λ_{33}	-235.02	(232.33)

Note: Estimated Standard errors in parenthesis

Table 2: Model implied and observed moments

	1st moments		2nd moments	
	Data	Model	Data	Model
1-month	5.42	5.42	1.09	1.09
9-month	5.34	5.37	1.11	1.05
1-year	5.39	5.39	1.12	1.04
3-year	5.69	5.66	1.26	1.00
5 - year	5.87	5.90	1.40	1.02
7- year	5.92	6.02	1.48	1.07
10 - year	5.95	5.99	1.56	1.20

Table 3: Model Variance Decomposition

Forecast Horizon	Supply	Demand	MP
	1-month yield		
1 month	20.7	2.1	76.9
12 months	46.3	6.2	47.5
60 month	48	8.2	43.8
	12-month yield		
1 month	44.6	13.1	42.3
12 months	47.1	14.7	38.2
60 month	46.3	16.3	37.4
	10-year yield		
1 month	10.8	82.6	6.5
12 months	4.2	87.6	8.2
60 month	3.4	87.9	8.7

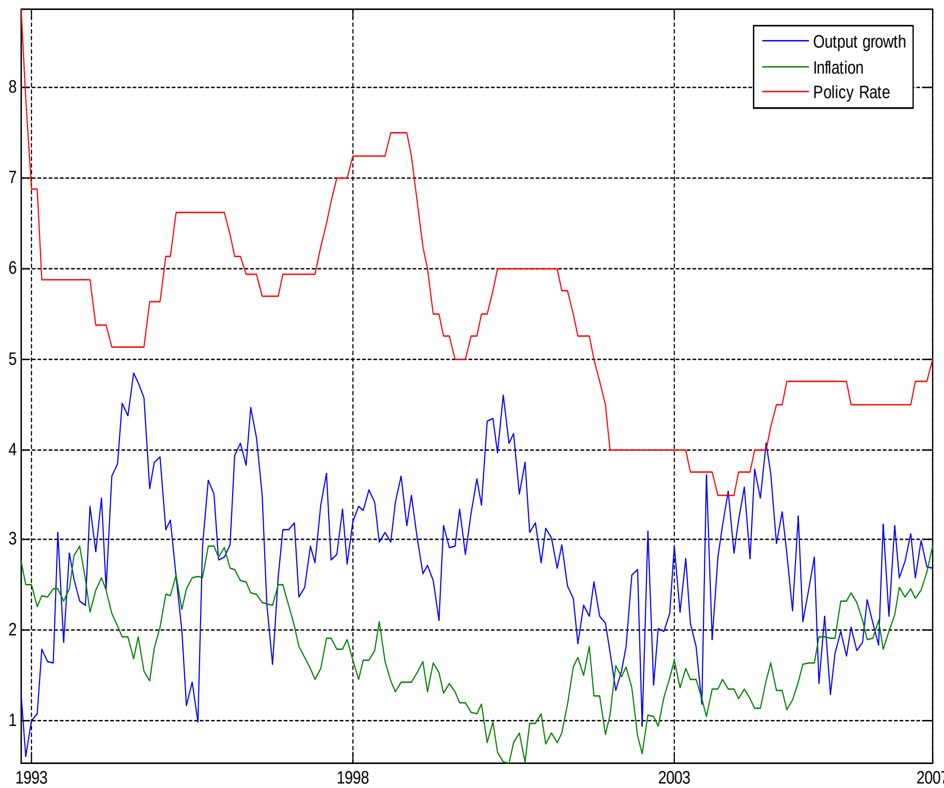

Figure 2.1: Observed variables at monthly frequency.

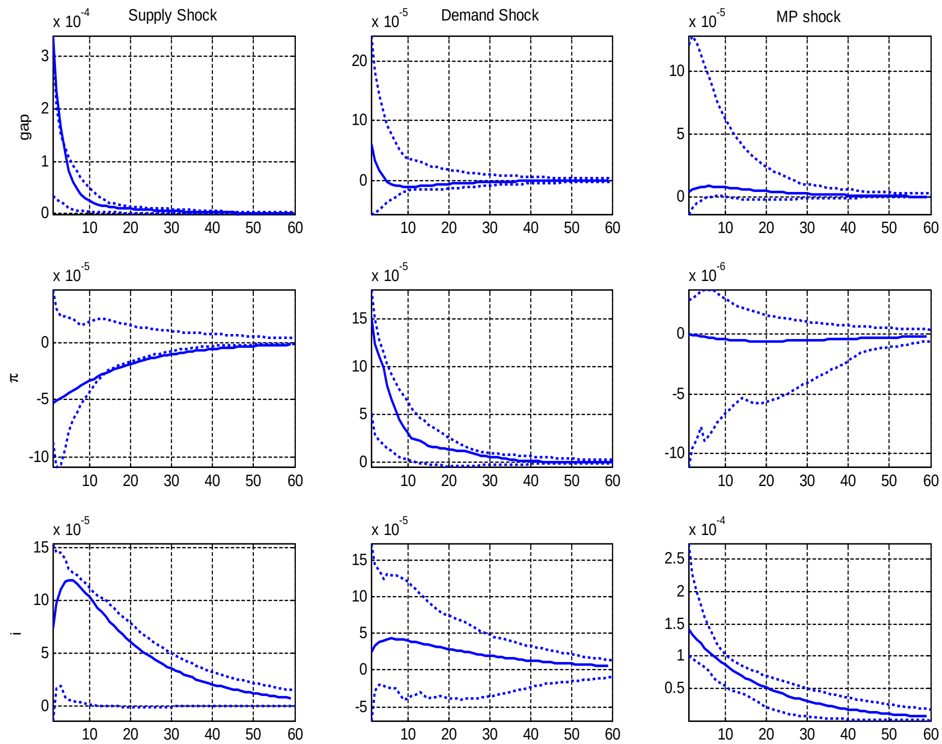

Figure 2.2: Impulse response functions of the observed macroeconomic variables to the fundamental shocks. Monetary policy shock is identified by zero short-run restrictions on inflation and output. 90% error bands are calculated using bootstrap-after-bootstrap method.

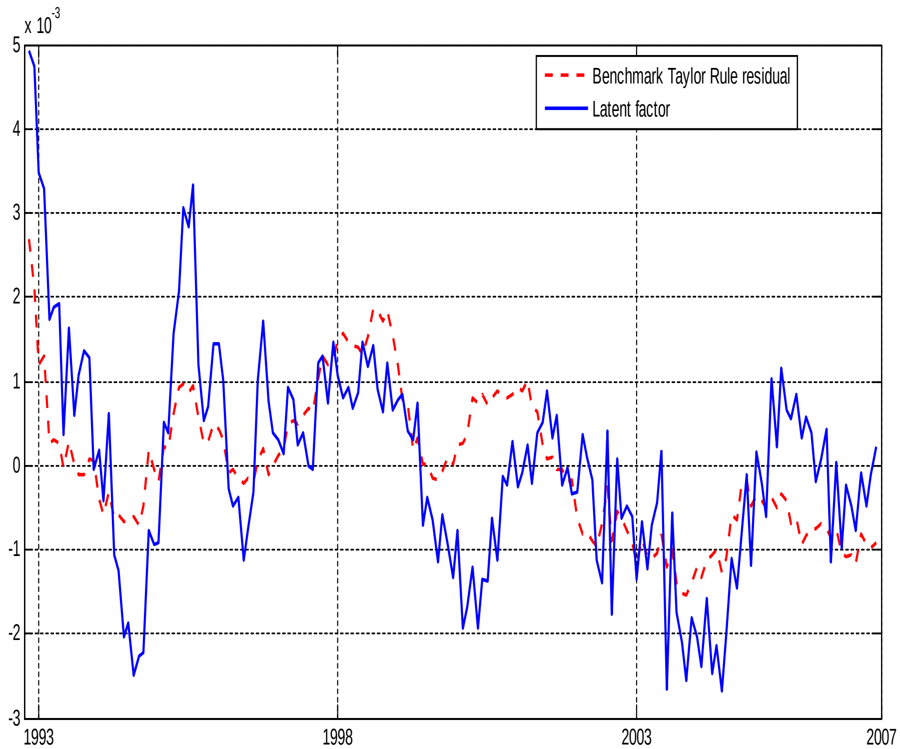

Figure 2.3: We plot the estimate of the latent variable, f_t, (solid line) and the residual from the benchmark Taylor Rule estimated by OLS regression of short term interest rate on output gap and CPI inflation (dashed line). The correlation between two time series is high: corr=0.64

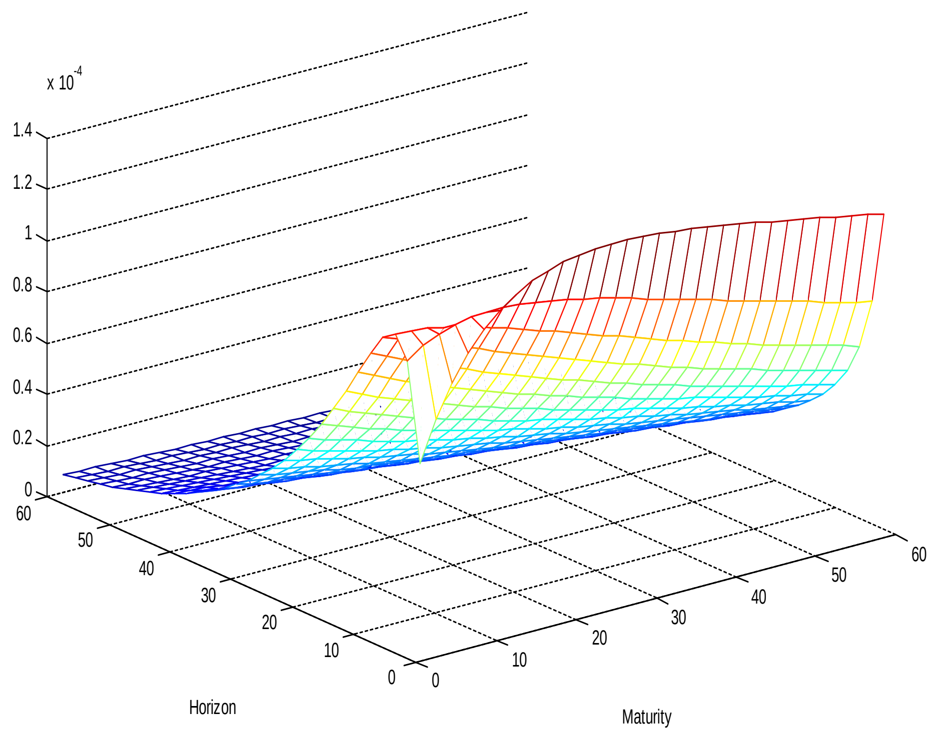

Figure 2.4: The impulse response function of the yield curve with respect to a one standard deviation supply shock. X-axis: maturity in months. Y-axis: projection horizon, months.

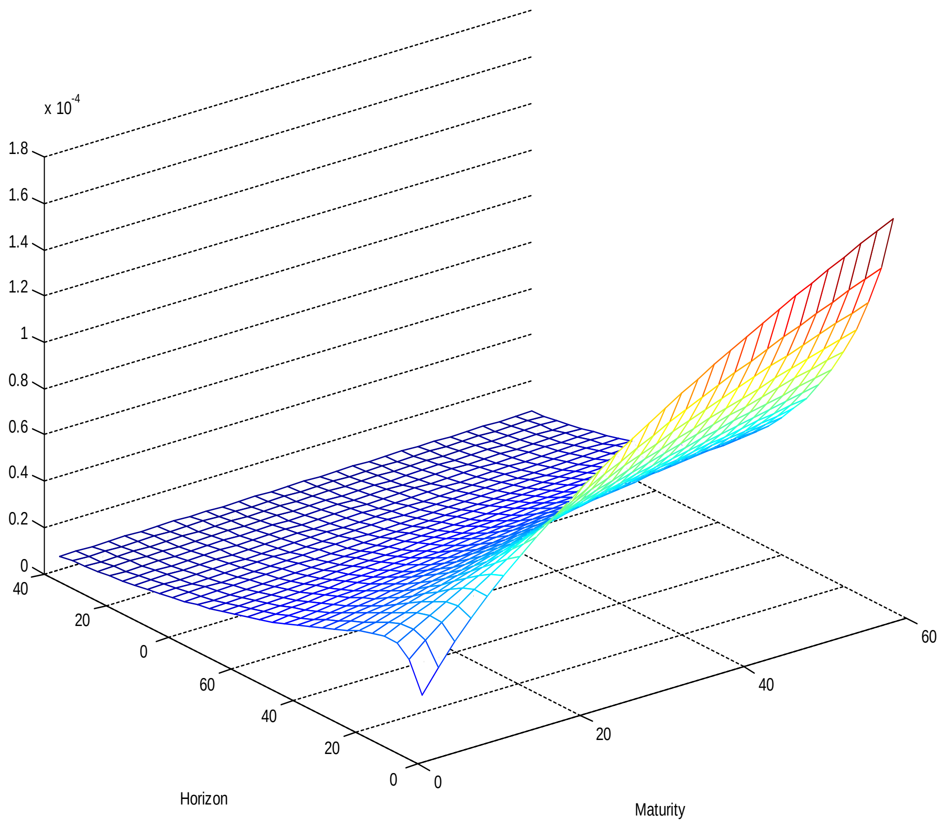

Figure 2.5: The impulse response function of the yield curve with respect to a one standard deviation demand shock. X-axis: maturity in months. Y-axis: projection horizon, months.

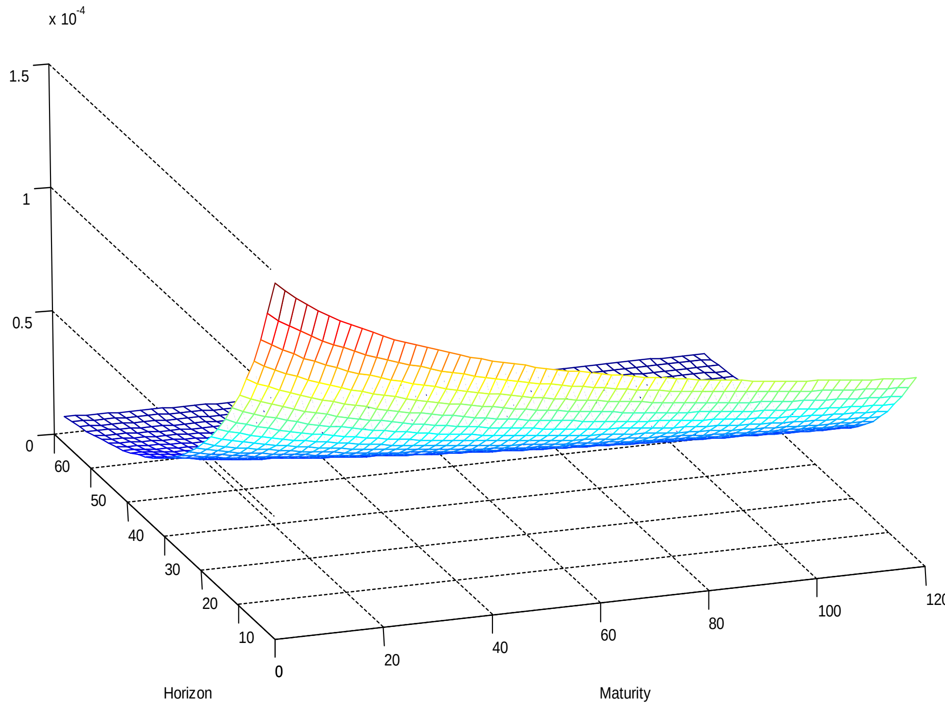

Figure 2.6: The impulse response function of the yield curve with respect to a one standard deviation monetary policy shock. X-axis: maturity in months. Y-axis: projection horizon, months.

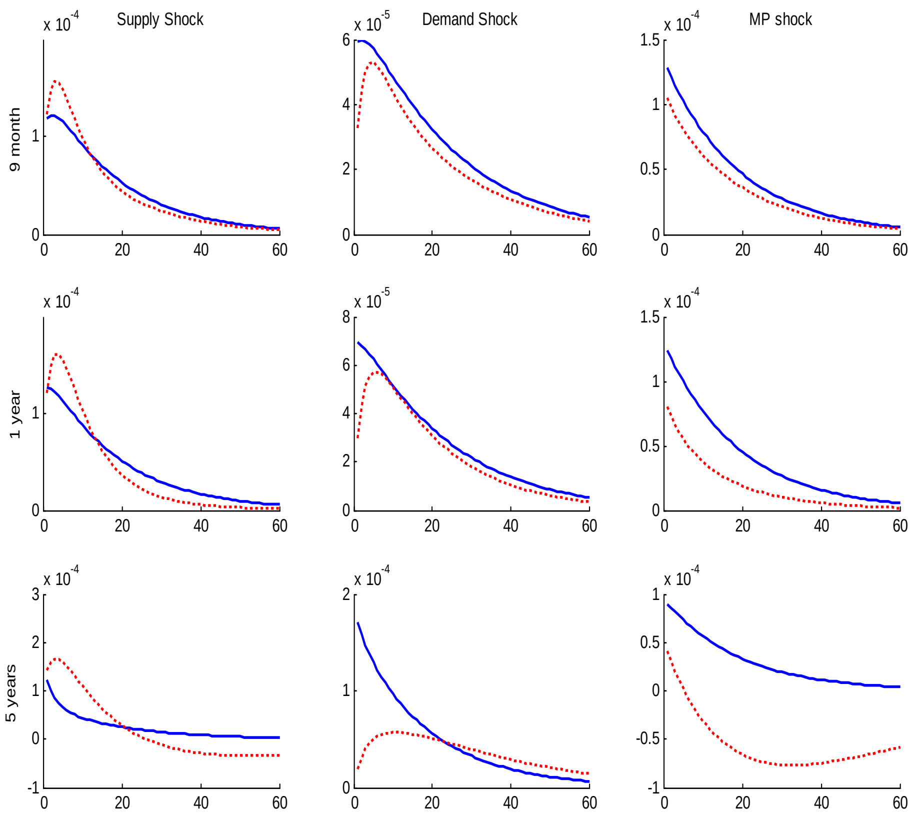

Figure 2.7: Solid line: model implied responses of the yield curve to the fundamental shocks. Dotted line: impulse responses of the yield curve according to the model by Evans and Marshall (1998) (see equation (2.30)).

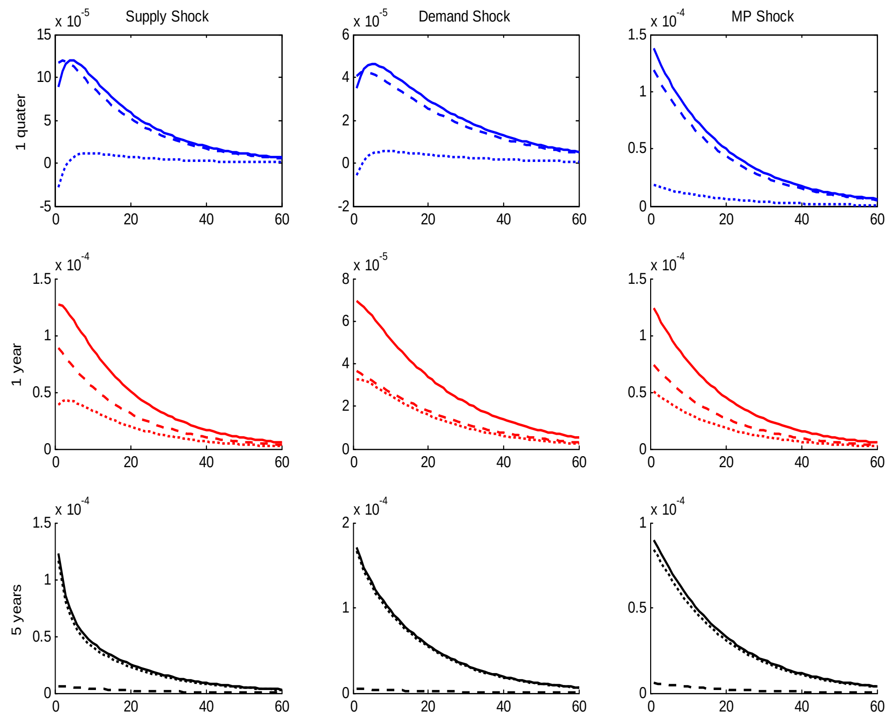

Figure 2.8: The impulse response functions of the selected yields with respect to a one standard deviation fundamental shock (solid line). The responses of the expectations component are given by the dashed line, while the responses of the risk premium component are given by the dotted line.

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Chapter 3

Measuring Term Premium. Robustness across Alternative Dynamic Term Structure Models

3.1 Introduction

The key objective of this paper is to examine how different specifications of yield curve models affect the estimates of the term premium, i.e. the difference between the yield to maturity of long-term bond and the average of expected future short-term bond yields:

$$TP_{n,t} = y_t^n - \frac{1}{n}\sum_{j=0}^{n-1} E_t[i_{t+j}]. \tag{3.1}$$

Over the last decade, different term structure models have been used for the analysis of the term premium. The traditional expectations hypothesis, which states that the term premium is constant, is widely rejected by the empirical research. The term premium is time varying and, moreover, it appears to be important variable in finance and macroeconomic literature. Nevertheless, a suitable model for term premium is still required.

Despite the simplicity of the concept, there are severe challenges for the term premium estimation. First, the market expectations are not observable and there is neither a commonly accepted theoretical model nor an agreed method to proxy these expectations. Second, in order to estimate term premium for any maturity we have to work with the whole yield curve, which is also unobservable. Thus, only when the term structure model provides both, the framework for the shape of the yield curve and the proxy for the expectations, it could be considered as a flexible tool for the term premium analysis.

While it is not a problem to find a good yield curve fitting model, modeling the market expectations is a difficult task. The common approach is to use ex-post observed returns as a valid proxy for ex-ante expected returns. However, the approach has been questioned by Elton (1999), who provided ample evidence against the belief that information surprises tend to cancel out over time. Hence, realized returns cannot be considered as an appropriate proxy for expected returns. Campbell and Shiller (1987) circumvent this problem and propose the VAR framework as an explicit model for the expected future rates. Given the path of VAR-projected future short rates, it is possible to construct yields to maturity consistent with the expectations theory and, as a residual, the term premium.

A VAR framework is still too general to be a final solution to the problem of term premium estimation. Two further questions remain open: which variables to include into a VAR?, and which restrictions to impose on VAR coefficients?

In the most simple case, Campbell and Shiller (1987) use bivariate yields-only VAR model, according to which a unique determinant of policy rates is a long-term rate. However, the success of Taylor rules (Taylor, 1993) points out an obvious potential mis-specification of the yields-only framework: the omission of macroeconomic variables to which the monetary policy maker reacts. Thus, focusing on the estimation of the expected future short-term rates, it is natural to enrich the VAR with variables related to inflation and output.

Including observable yields and macroeconomic variables into unrestricted VAR model produces a large number of coefficients. To reduce the number of estimated parameters, alternative assumptions coming from finance or marcoeconomic theory could be imposed. Among the most popular finance yield curve models are Affine Term Structure Models (ATSM) (see Duffie and Kan (1996); Dai and Singleton (2000)), which, in discrete time, imply VAR of yields with complex cross-equation restrictions due to the no-arbitrage assumption. Despite the high dimensionality and extreme non-linearity, many authors (see e.g. Ang, Piazzesi and Wei (2005), Hördahl, Tristani and Vestin (2005)) use this type of models to estimate term premium (1). In contrast, there are less computationally demanding ways to measure term premia by factor models. For example, it is possible to estimate term premium for any maturity using a specific parametric form for the yield curve and specifying additionally the dynamics for the factors. An example of this approach could be found in Diebold and Li (2005), and Carriero, Favero and Kaminska (2006), who assume Nelson and Siegel (1987)'s relationship between yields of different maturities and therefore impose restrictions on the VAR coefficients.

The natural question is: What is the impact of alternative restrictions on VAR-s for modeling the term premium?

In this paper, we seek to answer this question by studying different discrete TSM, which specify the driving stochastic process for the yield curve by Gaussian VAR. First, we consider unrestricted VAR models. Second, to provide estimates of term premia, we estimate two types of ATSM: following the recent tendency, together with standard ATSM, we consider also joint macro-finance ATSM (as Ang and Bekaert (2003), Rudebusch and Wu (2004) etc.). Finally, we explore VARs implied by Nelson-Siegel dynamic factor model.

We find that differences in term premia estimates among alternative specifications of discrete Term Structure Models are small.

The paper is organized as follows. Section 2 defines the term premium from the perspective of the expectations hypothesis. Section 3 discusses VAR, dynamic Nelson-Siegel and No-Arbitrage Affine TSM approaches. Section 4 summarizes estimation details, while Section 5 compares the term premium estimates from different approaches. The last section concludes.

3.2 Expectations Hypothesis and Term Premium

The Expectations Hypothesis (EH) can be represented in several forms (see Cox, Ingersoll, and Ross (1985)). We work here with the Yield to Maturity Expectations Hypothesis in its logarithmic form[1].

Let y_t^n, i_t denote n-period yield and one-period interest rate respectively. Then logarithmic form of the Expectation Hypothesis states that

$$y_t^n = \frac{1}{n}\sum_{j=0}^{n-1} E_t[i_{t+j}] + TP_n, \tag{3.2}$$

where $E_t[i_{t+j}]$ denotes the market's expectations at time t of the one-period interest rate at time $t+j$. The term premium TP_n could be viewed as a

[1] The log form is only the approximation of the EH and is not appropriate for the periods when the rates of returns take high values (like 1980-1983).

sum of the risk premium and Jensen inequality term (see Appendix for the details).

The traditional form of EH assumes that a term premium is constant (zero in the case of Pure EH). Nevertheless the assumption of constant premium is merely a technical simplification of the theory. We follow Longstaff (1990) and Hamilton and Kim (2002) and from here and below consider a variable term premium[2]. Rearranging terms, we find an expression for the term premium:

$$TP_{n,t} = y_t^n - \frac{1}{n}\sum_{j=0}^{n-1} E[i_{t+j} \mid I_t] \tag{3.3}$$

The estimation of the term premium is difficult in practice as it involves expectations about the future path of the short-term interest rate, and alternative decompositions may differ substantially depending on how expectations are modelled.

3.3 VAR-based Models for Expectations

3.3.1 Unrestricted VAR Models

Our VAR-based approach is closely related to the paper by Campbell and Shiller (1987). Using the representation of the short-term rate from VAR model, they do not employ ex-post realized returns as a proxy for ex-ante expected returns. The bivariate CS approach has an implicit reaction function according to which the only determinant of policy rates are long-term rates. In general, standard VAR models include interest rates and inflation in levels, alternative specifications include the measures of the real activity as well (i.e. Kozicky and Tinsley (2001)).

Ang, Piazzesi, and Wei (2005) also derive expectations for future policy rates considering a vector of state variables that follows a Gaussian Vector Autoregression with one lag:

$$Y_t = \mu^U + \Phi^U Y_{t-1} + \Sigma\epsilon_t \tag{3.4}$$

In their case, the vector Y_t contains two observed factors from the yield curve, the 3-month rate, i_t^1, expressed at a quarterly frequency, to proxy for the level of the yield curve, and the 5-year term spread, $i_t^{20} - i_t^1$, to proxy for the slope of the yield curve, the last factor is the quarterly real GDP

[2] With time varying term premium, the EH still holds if the term premium is restricted to be orthogonal to the spread.

growth, $\Delta_4 y_t$. Expected risk-free rates are derived by simulating the VAR from (3.4) forward.

In this paper, working with unrestricted VAR, we consider two alternative measures of TP_t. The first one is obtained by applying unrestricted VAR(1) model to the vector Y_t, which contains the short yield, y_t^3, expressed at a monthly frequency, and the 5-year yield, y_t^{60} : $Y_t' = \left[y_t^3, y_t^{60}\right]$. We shall assess potential mis-specification effects of yields-only VAR model by using an extended VAR, so that in the second case the state vector contains one-quarter yield, y_t^3, the 5-year yield, y_t^{60}, and inflation rate π_t : $Y_t' = \left[y_t^3, y_t^{60}, \pi_t\right]$.

By implementing the simulation based procedure it is possible to measure deviations from the EH and, under the null that the proposed model delivers expected future policy rates not different from those expected by the market, interpret them as a measure of risk premium. In each case, we simulate the estimated model (3.4) forward, to obtain projection for all relevant policy rates and to construct $\hat{ET}_t$, which stands for the EH consistent spreads, as follows:

$$\hat{ET}_t = \frac{1}{20}\sum_{j=0}^{19} E[y_{t+j}^3 \mid \Omega_t] \tag{3.5}$$

where, $E[y_{t+j}^3 \mid \Omega_t]$ are the VAR-based projections for the future changes in policy rates, hence Ω_t is the information set used by the econometrician to predict on the basis of the estimated unrestricted VAR model. The unrestricted VAR-based measure of term premium is then given by

$$TP_{3,60,t} = y_t^{60} - \frac{1}{20}\sum_{j=0}^{19} E[y_{t+j}^3 \mid \Omega_t] \tag{3.6}$$

The information set Ω_t depends on the current and lagged values of a state vector and does not include any theoretical restriction. However, when working with term structure models, it is natural to include information on the absence of arbitrage opportunities into the information set of the econometrician. We address how to impose the No-Arbitrage restrictions into the VAR model in the next section.

3.3.2 No-Arbitrage Term Structure Models

According to financial No-Arbitrage TSM, there are only few factors, X_t, relevant for pricing risk in the bond sector. If investors are risk-neutral, i.e. they care only about expected return and do not atke associated risk into consideration, then No-Arbitrage implies that any bond is priced by the following rule:

$$P_{n+1,t} = E_t(e^{-r_t} P_{n,t+1}), \tag{3.7}$$

with bond's price $P_{n,t} = P_n(X_t),\ X_{t+1}|X_t \sim N(\mu_t, \Sigma_t\Sigma_t'),\ \mu_t = \mu(X_t),\ \Sigma_t = \Sigma(X_t)$. If investors are not risk-neutral, then their behavior can be represented as that of risk-neutral with "distorted beliefs" about the distribution of $X_t : X_{t+1}|X_t \sim N(\mu_t^Q, \Sigma_t\Sigma_t')$, where $\mu_t^Q = \mu_t - \Sigma_t\Lambda_t$. Assuming that market uses these distorted beliefs to evaluate $P_t = E_t^Q(e^{-r_t}P_{t+1})$, the density to find these expectations would be

$$\begin{aligned}
f_t^Q(X_{t+1}) &= (2\pi)^{-\frac{N}{2}} |\Sigma_t|^{-1} \exp(-\frac{1}{2}(X_{t+1} - \mu_{t+1}^Q)'(\Sigma_t\Sigma_t')^{-1}(X_{t+1} - \mu_{t+1}^Q)) = \\
&= (2\pi)^{-\frac{N}{2}} |\Sigma_t|^{-1} \exp(\frac{(X_{t+1} - \mu_t + \Sigma_t\Lambda_t)'(\Sigma_t\Sigma_t')^{-1}(X_{t+1} - \mu_t + \Sigma_t\Lambda_t)}{-2}) \\
&= f_t(X_{t+1}) \exp(-\frac{1}{2}\Lambda_t'\Lambda_t - \Lambda_t'\Sigma_t^{-1}(X_{t+1} - \mu_t)) \\
&\equiv f_t(X_{t+1}) \exp(-\frac{1}{2}\Lambda_t'\Lambda_t - \Lambda_t'\Sigma_t^{-1}\varepsilon_{t+1})
\end{aligned}$$

The pricing kernel, M_{t+1}, is given by

$$M_{t+1} \equiv e^{-r_t} e^{-\frac{1}{2}\Lambda_t'\Lambda_t - \Lambda_t'\Sigma_t^{-1}\varepsilon_{t+1}} \tag{3.8}$$

$$P_t = E_t^Q(e^{-r_t}P_{t+1}) = E_t(M_{t+1}P_{t+1}) \tag{3.9}$$

Estimating market prices of risk econometrician faces numerous challenges. Importantly, the presence of unobservable (latent) variables and the absence of the closed-form solution of the system of stochastic difference equations for bond prices prevents the use of the maximum likelihood estimation. A closed-form solution for bond prices could be obtained by imposing the affine structure into the model. Therefore, the majority of empirical studies adopt a specification which is affine. We follow this direction and consider only Affine Term Structure Models.

Canonical ATSM contains 3 basic equations:

1) Transition equation for the state vector relevant for pricing bonds (Gaussian VAR):

$$X_t = \mu + \Phi X_{t-1} + \Sigma\varepsilon_t, \tag{3.10}$$

where X_t is an $(n \times 1)$ -vector of state variables, ε_t is an $(n \times 1)$ -vector of i.i.d. shocks with zero mean and identity covariance matrix; μ is $(n \times 1)$, Φ is $(n \times n)$, and Σ is $(n \times n)$.

2) Definition of one period rate as a linear function of the state variables,

$$r_t = \delta_0 + \delta_1 X_t, \tag{3.11}$$

where δ_0 is a scalar; and δ_1 is an $(1 \times n)$.

3) The price of risk, Λ_t, is associated with shocks ε_t and is identified as an affine function of the state of economy (see Duffee (2002)).

$$\Lambda_t = \lambda_0 + \lambda_1 X_t, \tag{3.12}$$

where λ_0 is $(n \times 1)$; and λ_1 is $(n \times n)$.

Under these assumptions, as we show in the Appendix, the price of a bond of any maturity is an affine function of the state variables:

$$p_{t,n} = A_n + B_n' X_t,$$

where A_n, B_n are the functions of the parameters $\{\lambda_0, \lambda_1, \delta_0, \delta_1, \mu, \Phi, \Sigma\}$:

$$\begin{cases} A_{n+1} = A_n + B_n'\mu - B_n'\Sigma\lambda_o + \frac{1}{2}B_n'\Sigma\Sigma' B_n - \delta_0 \\ B_{n+1}' = B_n'\Phi - B_n'\Sigma\lambda_1 - \delta_1' \end{cases} \tag{3.13}$$

The yield on a zero-coupon bond of maturity n is also affine function of the state:

$$\begin{aligned} y_t^n &= -\frac{1}{n}(A_n + B\prime_n X_t) && (3.14) \\ &\equiv a_n + b_n' X_t && (3.15) \end{aligned}$$

No-Arbitrage VAR

In this subsection we are going to show how the No-Arbitrage assumption results in a set of restrictions on VAR from (3.4).

To estimate parameters and obtain the factors in ATSM we employ the method by Chen and Scott (1993). In this setting, N unobservable factors are calculated by assuming that N bonds in the cross section are priced with no error, i.e. there are as many yields, Y_{1t}, measured without error as there are latent factors in X_t. The same methodology was used by Ang and Bekaert (05), and Rudebusch and Wu (03).

We denote the set of yields measured without error as $Y_{1t\ (N_1 \times 1)}$ and the yields measures with errors as $Y_{2t\ (N_2 \times 1)}$. The whole set of yields is denoted as $Y_t = (Y_{1t}, Y_{2t})$ with dimension $(N_Y \times 1)$, where $N_Y = N_1 + N_2$. Given the expression (3.14), the yields observed without error take the form:

$$Y_{1t} = \mathbf{A1} + \mathbf{B1} X_t. \tag{3.16}$$

where $\mathbf{A1}$ is the $(N_1 \times 1)$ vector stacking the $(-\frac{1}{n}A_n)$ terms for the Y_{1t} yields observed without error, $\mathbf{B1}$ is a $(N_1 \times N)$ matrix which stacks the vectors $(-\frac{1}{n}B\prime_n)$ for the yields observed without error. We estimate model parameters by MLE (see details in Appendix). Once all parameters of the model are estimated, the unobservable factors, X_t, could be extracted by inverting the pricing relationship (3.16) of the model:

$$X_t = \mathbf{B1}^{-1}[Y_{1t} - \mathbf{A1}] \tag{3.17}$$

Given the dynamics of the latent factors (3.10), the dynamics of the observed bond yields can be retrieved by combining equations, (3.16), (3.17) in the following way:

$$\begin{aligned} Y_{1t} &= \mathbf{A1} + \mathbf{B1}\left(\mu + \Phi X_{t-1} + \Sigma\varepsilon_t\right) \\ &= \left(\mathbf{A1} + \mathbf{B1}\mu - \mathbf{B1}\Phi\mathbf{B1}^{-1}\mathbf{A1}\right) + \mathbf{B1}\Phi\mathbf{B1}^{-1}Y_{1t-1} + \mathbf{B1}\Sigma\varepsilon_t \end{aligned} \tag{3.18}$$

Note that the No-Arbitrage assumption implies the VAR for the observable variables with complex cross equation restrictions. We denote this VAR as

$$Y_{t-1} = \mu^{NA} + \Phi^{NA} Y_{t-1} + \Sigma^{NA}\epsilon_t, \tag{3.19}$$

where Y_t stands for the observed yields (in our case $Y_t = [y_t^3, y_t^{60}]$),

$$\mu^{NA} \equiv \mathbf{A1} + \mathbf{B1}\mu - \mathbf{B1}\Phi\mathbf{B1}^{-1}\mathbf{A1} \tag{3.20}$$

$$\Phi^{NA} \equiv \mathbf{B1}\Phi\mathbf{B1}^{-1} \tag{3.21}$$

The No-Arbitrage based measure of term premium is then

$$TP^{NA}_{3,60,t} = y_t^{60} - \frac{1}{20}\sum_{j=0}^{19} E[y^3_{t+j} \mid \Omega_t^{NA}], \tag{3.22}$$

where information set of the econometrician, Ω_t^{NA}, includes the theoretical assumption of No-Arbitrage.

In general, the restrictions are imposed on all yields, those measured with and without error. Indeed, a VAR for exactly observable variables and an additional yield measured with error takes form:

$$\begin{bmatrix} Y_{1t} \\ Y_{2t} \end{bmatrix} = \begin{bmatrix} \mu^{NA} \\ \mathbf{A2} + \mathbf{B2}\ \mathbf{B1^{-1}}(\mu^{NA} - \mathbf{A1}) \end{bmatrix} + \tag{3.23}$$

$$+ \begin{bmatrix} \Phi^{NA} & 0 \\ \mathbf{B2B1^{-1}}\Phi^{NA} & 0 \end{bmatrix} \begin{bmatrix} Y_{1t-1} \\ Y_{2t-1} \end{bmatrix} + \begin{bmatrix} \Sigma^{NA} & 0 \\ \mathbf{B2B1}^{-1}\Sigma^{NA} & \Upsilon \end{bmatrix} \begin{bmatrix} \epsilon_t \\ \zeta_t \end{bmatrix}.$$

The huge popularity of the No-Arbitrage ATSM in finance is due to the fact that implied affine functions of few unobservable (latent) factors could explain almost all movements of the yield curve (see Duffie, Kan (96); Dai, Singleton (00))[3]. Nevertheless, the pure ATSM has not gained the same popularity among economists since the model is not useful for Macroeconomic Policy. There is no any theory behind the NA-ATSM apart from the No-Arbitrage assumption and the economic nature of the latent factors is unknown. Observing that short term rate is a Policy Rate, macroeconomists have proposed a possible solution: to combine ATSM No-Arbitrage models with Macroeconomic models.

ATSM joint with Macroeconomic Models

Table 1 summarizes the recent work on the affine term structure models enriched by macroeconomic information.

The empirical studies incorporating the macroeconomic theory into the No-Arbitrage models reported in Table 1 employ quite a variety of model specifications and data. Consequently, the results are difficult to compare directly. Nevertheless, there is a robust result: all models confirm the importance of the inflation for the pricing of bonds. For instance, in the model of Rudebusch and Wu (2004), the level factor reflects market participants' views about the inflation target of the central bank. Diebold, Rudebusch, and Aruoba (2005) also find that the level factor is highly correlated with inflation. Inflation turns to be a priced risk factor in the models by Buraschi, Jiltsov (04), Ang, Piazzesi(03) etc.

To apply no-arbitrage macro-finance framework to the analysis of the term premium, we concentrate on the approach of Ang and Bekaert (04) and include inflation in ATSM as an observable factor. In this case the measure of the term premium is derived from the simulation of VAR given by equation (3.19), with $Y_t = [y_t^3, y_t^{60}, \pi_t]$

[3] just 2 factors could explain more than 99%

3.3.3 Nelson-Siegel Approach

In this section we use an alternative method to extract latent factors driving the yield curve. We estimate the yield curve at each point in time by the help of the simple term structure model proposed by Nelson and Siegel (1987). At each point of time, we construct financial factors by estimating (by non-linear least squares, on the cross-section of observed yields) the following Nelson-Siegel model :

$$y_t^k = L_t + SL_t \frac{1 - \exp\left(-\frac{k}{\tau_1}\right)}{\frac{k}{\tau_1}} \tag{3.24}$$

The parameter τ_1 is kept constant over time[4], as this restriction decreases the volatility of the parameters $X_t^{NS} = (L_t, SL_t)'$, making them more predictable in time. As discussed in Diebold and Li (2002) the above interpolant is very flexible and capable of accommodating several stylized facts on the term structure and its dynamics. In particular, L_t, SL_t, which are estimated as parameters in a cross-section of yields, can be interpreted as latent factors. L_t has a loading that does not decay to zero in the limit, while the loadings on all the other parameters do so, therefore this parameter can be interpreted as the long-term factor, the level of the term-structure. The loading on SL_t is a function that starts at 1 and decays monotonically towards zero; it may be viewed a short-term factor, the slope of the term structure. In fact, $r_t^{rf} = L_t + SL_t$ in the limit when k goes to zero of the spot and the forward interpolant. We naturally interpret r_t^{rf} as the risk-free rate. Obviously, SL_t is the slope of the yield curve. The repeated estimation of loadings using a cross-section of yields at different maturities allows to construct a time-series for our factors.

Interestingly, Nelson-Siegel model of the term structure is consistent with implications of the No-Arbitrage ATSM presented in previous subsection: As before, the yields are affine in state factors:

$$y_t^k = a^{NS,k} + b^{NS,k} X_t^{NS}, \tag{3.25}$$

where the following restriction holds:

[4] We restrict τ_1 at the value of 1.8.

$$a^{NS,k} = 0 \quad (3.26)$$

$$b^{NS,k} = \left(1, \frac{1-\exp\left(-\frac{k}{\tau_1}\right)}{\frac{k}{\tau_1}}\right) \quad (3.27)$$

The often-quoted shortcoming of the Nelson-Siegel model is its static nature: the factors are extracted from the current yield curve and the information about the shapes of the past yield curves is omitted. To overcome this drawback, it is naturally to add the assumptions on the dynamics of the Nelson-Siegel factors. Following Diebold, Li (2005) and Carriero-Favero-Kamisnka (2006), we assume that the factors follow the Gaussian VAR process:

$$X_t^{NS} = \mu^{NS} + \Phi^{NS} X_{t-1}^{NS} + \Sigma^{NS} \epsilon_t. \quad (3.28)$$

Finally, given the factor dynamics (3.28) and linear relationship between yields and factors (3.25), it can be easily shown that the corresponding dynamics of the Nelson-Siegel yields is also described by VAR:

$$Y_{K,J,t}^{NS} = \mu_{K,J}^{NS} + \Phi_{K,J}^{NS} Y_{K,J,t-1}^{NS} + \Sigma_{K,J}^{NS} \epsilon_t, \quad (3.29)$$

$$\mu_{K,J}^{NS} \equiv B_{K,J}^{NS} \mu^{NS} \quad (3.30)$$

$$\Phi_{K,J}^{NS} \equiv B_{K,J}^{NS} \Phi^{NS} \left(B_{K,J}^{NS}\right)^{-1} \quad (3.31)$$

$$\Sigma_{K,J}^{NS} \equiv B_{K,J}^{NS} \Sigma^{NS} \quad (3.32)$$

$$B_{K,J}^{NS} \equiv \left\| \begin{matrix} 1 & \tau_1 \frac{1-\exp\left(-\frac{k}{\tau_1}\right)}{k} \\ 1 & \tau_1 \frac{1-\exp\left(-\frac{j}{\tau_1}\right)}{j} \end{matrix} \right\| \quad (3.33)$$

where $Y_{K,J,t}^{NS}$ denotes the vector of K- and J- maturity yields implied by Nelson-Siegel model (3.24). In fact, the model implies that model-implied yields follow VAR process, that is, Nelson-Siegel parametric restrictions are imposed on the VAR coefficients for yields of different maturities.

The Nelson-Siegel VAR-based measure of term premium $TP_{60,t}^{NS}$ is then

$$TP_{3,60,t}^{NS} = y_t^{NS,60} - \frac{1}{20} \sum_{j=0}^{19} E[y_{t+j}^{NS,3} \mid \Omega_t^{NS}]. \quad (3.34)$$

3.4 Estimation

We consider monthly data on bond yields. In particular, we focus on the US zero-coupon bonds, assuming them to be default-risk-free. The data is available on the G. Duffee's home page.[5] and consists of time series of six yields,. $[y^3, y^6, y^{12}, y^{24}, y^{60}, y^{120}]$.

First, we estimate two-factor models: standard ATSM ($ATSM(2,0)$), dynamic Nelson-Siegel model[6], and unrestricted VAR of observed yields.

Second, to apply our framework to the analysis of the US term structure we use a standard approach by including the annual US CPI inflation at time t, π_t, into the model. Again, we estimate a one-lag VAR model for three cases: ATSM with 2 latent factors and inflation as an observable factor($ATSM(2,1)$); VAR of Nelson-Siegel-implied yields and inflation; and unrestricted VAR of observed yields and inflation.

We limit the sample to 1988:1-1998:12 for all models under consideration. The choice of the sample was influenced by the question of the stability of the estimates. While the relationship between yields might remain stable over time, the relationship between interest rates and macroeconomic variables has changed over time. Thus, including observable macrofactors into ATSM, we have to limit samples to short intervals of plausible stability in MP regime (as Rudebusch, Wu (04)).[7]

For every VAR model under consideration, given the results of the estimation, the companion matrix is retrieved. For each point of our sample, the VARs are then projected for an horizon up to twenty observations to generate observation of the ET-consistent five year yield. The ET-consistent yields are then to be compared with observed yields, and the difference is interpreted as a term premium. The procedure is repeated for a total of 120 simulations of each model.

[5] Duffee uses mixed data sourses. The data through February 1991 are from McCulloch's home page, After February 1991, the data are from Rob Bliss.

[6] The Nelson-Siegel term structure approximation is based on five yields from the Duffe's data set, $[y^3, y^6, y^{12}, y^{24}, y^{60}]$.

[7] The ATSM with Regime Switching (RS) could be an alternative solution. Dai, Singleton, Yang (03) and Ang and Bekaert (04) develop and empirically implement an arbitrage-free, dynamic term structure model (DTSM) with regime-shifts. However, the method is computationalyy demanding and, in order to obtain closed form solution, a number of strong restrictions must be imposed on RS process.

3.4.1 Estimation of ATSM

The most computationally demanding model is ATSM. We detail how to compute the likelihood function in Appendix.

The Chenn-Scott estimation approach requires to assume that, in two latent factors models, there are exactly two reference yields specified without errors. The likelihood function is the likelihood of the yields measured without error multiplied by the likelihood of the measurement errors. The well-known problem of the Chen-Scott approach is that different choices of the reference bonds imply different state variable realizations. In order to choose the reference yields, we estimate the model for all possible combinations of the pairs of reference yields and compare their fit and stability (i.e. the model should produce good fit for any out-of sample long-term yield, which is y^{120} for our case). The best performance of the two-factor $ATSM$ model is achieved when the reference yields are chosen to be $[y^3, y^{60}]$.

Imposing No-Arbitrage assumption into VAR by itself creates significant computational problems, which become really huge with additional macro-factors included. The trade-off is to limit certain parameters in $\{\lambda_0, \lambda_1, \delta_0, \delta_1, \mu, \Phi, \Sigma\}$, which is, of course, not the first best solution. In order to limit the number of parameters $\{\lambda_0, \lambda_1, \delta_0, \delta_1, \mu, \Phi, \Sigma\}$ to be estimated, we de-mean the values of the variables for all models (the procedure should not distort the results, since we limit the sample to the stable interval). De-meaning allows us to set $\lambda_0 = \delta_0 = \mu = 0$, and thus, to significantly decrease a number of parameters to be estimated. For the sake of the factors identification, we assume that Σ is diagonal, while Φ is lower trigonal. The parameters in δ_1 are normalized to be 1 in the case of the two-factor model, while in the case of the joint macro-finance model, the inflation factor loading is unconstrained, i.e. $\delta_1 = [1\ 1\ \delta_\pi]$.

We solve the nonlinear optimization problem of maximizing log-likelihood function by using the MATLAB 6.5. routine $fminsearch$ which represents a generalization of the Nelder-Mead simplex algorithm. Finally, we compute the standard errors for the estimated parameters using an approximation of the parameter covariance matrix based on the inverse of the Hessian matrix evaluated numerically.

3.5 Empirical Results

3.5.1 Parameter Estimates for ATSM

We report the parameter estimates for both ATS models in Table 2.

All factors appear to be higly persistent. Interestingly, our estimates confirm the welknown results by Clarida, Gali, Gertler (2000) of Fed "active" policy since 1980s, since coefficient on inflation in policy rule is larger than one, $\delta_\pi = 1.14$.

3.5.2 Two factor models and implied VARs of yields

As described above, we consider two modelling strategies for the latent factors: ATSM and Nelson-Siegel approaches. Additionally, we employ simple unrestricted VAR framework based on the observable yields, so that we end up with three different estimates of the companion matrix for $[y^3, y^{60}]$. The estimates of the companion matrices Φ^U, Φ^{NA}, Φ^{NS} from VARs (3.4), (3.19), (3.29) are provided in Table 3.

An important implication of the estimation is that the corresponding coefficients from three alternative models do not differ significantly. The results clearly show that different VAR-based models will imply similar ET-consistent yields. As a conclusion, Figure 3.1 shows the striking similarity between the term premia obtained by alternative models.

All measurs of the term premium has declined substantially since 1995, which is consistent with the results of other studies (see, for instance, Rudebusch and Wu (2004), Kim and Wright (2005)). However, the term premium implied by ATSM is less volotile.

3.5.3 Joint macro-finance models

In this section we report the results of estimations using alternative macro-finance specifications of VAR and show that our results are robust. We provide the relevant evidence in Table 4, where we report the results of estimating all our models with inflation included in VAR.

Table 5 summarizes the key results of our analysis. The results show that our estimates of term premium are robust both to the choice of the model and to the inclusion of macroeconomic information into the model.

3.5.4 Time Consistent Estimation of Term Premium

The VAR-based projections described in the previous sections have some limitations. For all considered models, the VAR is estimated only once on the full-sample and therefore VAR based projections are not based on the information available in real time to agents. Such procedure cannot simulate the investors' effort to use the model in 'real time' to forecast future monetary policy rates, as the information from the whole sample is used to estimate parameters while investors can use only historically available information to generate (up to n-period ahead) predictions of policy rates. In this section at each point in time we estimate, using the historically available information, a model on and then we use it to project out-of-sample policy rates up to the nth-period ahead. Given the path of simulated future policy rates, we can construct yield to maturities consistent with the Expectations Theory and, as a residual, the term premium.

In this section we simulate the real time decision of agents who forecast policy rates by projecting forward a model to generate long-term yields consistent with the expectations theory. We propose measures for ER_t and TP_t.To construct such measures we estimate at each point in time, using the historically available information, the following model:

$$\begin{aligned} X_t &= \mu + \Phi(L)X_{t-1} + \Sigma\epsilon_t \\ X_t' &= \left[y_t^3, y_t^{60}, \pi_t\right] \end{aligned}$$

We then simulate the estimated model forward, to obtain projection for all the relevant policy rates and to construct ET, which is the ET-consistent long term yield, as follows:

$$\hat{ER}_t = \frac{1}{20}\sum_{j=1}^{19} E[y_{t+j}^3 \mid \Omega_t] \tag{3.35}$$

where, $E[y_{t+j}^3 \mid \Omega_t]$ are the VAR-based projections for the future changes in policy rates, hence Ω_t is the information set used by the econometrician to predict on the basis of the estimated VAR model .

Importantly, in implementing our procedure the econometrician uses the same information available to market participants in real-time. Future policy rates at time t are constructed using information available in real time for parameters estimation and forward projection of the model.

Unrestricted VAR and dynamic Nelson-Siegel procedures can easily accommodate the time consistent estimation of the term premium. The time varying parameters of the VAR can be obtained by estimating the model on one sample and then re-estimating it on the next.[8]

However, this is not the case for the ATSM, since the theoretical structure of the model assumes constant parameters.

[8] For the proof of the importance of the time varying coefficients see Figure 3.2.

Ang and Bekaert (04) propose an alternative solution for time varying parameters of the ATSM model, that is ATSM with Regime Switching (RS). The RS - approach to the ATSM is not new. Dai,Singleton, Yang (03) also develop and empirically implement an arbitrage-free, dynamic term structure model (DTSM) with regime-shifts. With RS, the ATSM become more computationally difficult, nevertheless both models provide considerable limitations. Dai, Singleton and Yang use only finance information in their model. Ang and Bekaert (04), in order to receive a solution in the closed form, assume that only mean and volatility of the variables change across regimes, while the mean-reversion of all variables is not regime-dependent. It is very restrictive assumption and is in odds with the results established by the authors in their previous works.[9] Moreover, Dai, Singleton, and Yang (03) strongly reject the assumption of RS with constant transition probability, which is the case of the model by Ang and Bekaert (04).

3.6 Conclusion

In this paper we employ alternative discrete term structure models in order to estimate the term premium. In particular, we consider the models, which characterize the expectations of the future yields by VAR framework. The simple VAR of the observed yields is too limited framework since it does not allow to estimate the term premia for any particular maturity. On the other hand, the dynamic term structure models produce information about the whole yield curve and thus provide a flexible tool for term premium analysis.

Among the most popular dynamic term structure models are the No-Arbitrage Affine Term Structure Models (ATSM). Following the recent tendency, together with standard two-factor ATSM, we consider also joint macro-finance ATSM and enrich the model with macroeconomic information. All types of ATSM impose on the VAR complex cross-equation restrictions due to the no-arbitrage assumption. The model is high dimensional and extremely non-linear. This produces the maximum likelihood function with numerous local optima and implies a difficult optimization problem for ATSM estimation.

On the other hand, we estimate term premium for any maturity by less computationally demanding model (Nelson-Siegel approach) specifying

[9] Bekaert et al (01) and Ang and Bekaert (02) reported evidence on state-dependent mean-reversion in short rates. Evidence on state-dependent mean-reversion in inflation is reported in Evans and Lewis (95).

additionally only dynamics of the state vector. We chose the input variables to be identical for the considered TSM, therefore the premia estimates are the functions of the same variables and could be compared directly. Another advantage of the Nelson-Siegel procedures is that it can easily accommodate the time consistent estimation of the term premium, while ATSM does not permit time variability of the parameters.

The main result of our study is that alternative approaches produces the strongly correlated term premia, and thus the less computationally demanding and more flexible method could be used in order to obtain a proxy for term premia.

3.7 Appendix

3.7.1 Term Premium

Since the pricing kernel, M_{t+1}, prices all assets in the economy, for return of any asset

$$E_t(M_{t+1}(1 + R_{t+1})) = 1,$$

Then the above equation allows bond prices to be computed recursively:

$$P_t\left(n\right) = E_t\left\{M_{t+1}P_{t+1}\left(n-1\right)\right\} \tag{A1}$$

To keep matters simple, we assume that bond prices are exponential affine functions of X_t', $P_t(n) = \exp(A_n + B\prime_n X_t')$, so that the log prices of bonds with maturity n are given by:

$$p_t(n) = A_n + B\prime_n X_t' \tag{A2}$$

Under the assumption that M_{t+1} is conditionally lognormally distributed and $X_{t+1}$is normally distributed, we can take logs of the Pricing Kernel to obtain

$$p_t\left(n\right) = E_t\left\{m_{t+1} + p_{t+1}\left(n-1\right)\right\} + \frac{1}{2}Var_t\left\{m_{t+1} + p_{t+1}\left(n-1\right)\right\} \tag{A3}$$

The difference equation for prices (A3) imply the difference equation for the log yields to maturity, $y_t(n) = -\frac{p_t(n)}{n}$, which provides the explicit form for the term premium, and, moreover, it imposes the No-Arbitrage restrictions into the model parameters. Given the initial condition, $n = 1$:

$$r_t = E_t(m_{t+1}) + \frac{1}{2}Vart[m_{t+1}],$$

the difference equation in yields could be solved recursively:

$$y_t\left(n\right)=\frac{1}{n}E_t(\sum_{i=0}^{n-1}r_{t+i})+TP_{n,t},$$

where

$$TP_{n,t}=-\frac{1}{2n}\sum_{i=0}^{n-1}Var_t[(n-i)y_{t+i}\left(n-i\right)]+\frac{1}{n}\sum_{i=0}^{n-1}Cov_t(m_{t+i},(n-i)y_{t+i}\left(n-i\right))$$

3.7.2 Bond Prices under No-Arbitrage

In this part we specify the recursive structure of the coefficients in the bond pricing equation (A2). Putting together all assumptions made above, we get

$$\begin{aligned}
p_t\left(n+1\right)&=E_t\left\{m_{t+1}+p_{t+1}\left(n\right)\right\}+\frac{1}{2}Var_t\left\{m_{t+1}+p_{t+1}\left(n\right)\right\}=\\
&=E_t\left\{-R_{1,t}-\frac{\lambda_t'\lambda_t}{2}-\lambda_t'\varepsilon_{t+1}+A_n+B_n'X_{t+1}\right\}+\frac{1}{2}Var_t\left\{m_{t+1}+p_{t+1}\left(n\right)\right\}=\\
&=-R_{1,t}-\frac{\lambda_t'\lambda_t}{2}+A_n+E_t[B_n'\left(\mu+\Phi X_t+\Sigma\varepsilon_{t+1}\right)]+\frac{1}{2}Var_t\left\{-\lambda_t'\varepsilon_{t+1}+B_n'\Sigma\varepsilon_{t+1}\right\}=\\
&=-\delta_0-\delta_1'X_t-\frac{\lambda_t'\lambda_t}{2}+A_n+B_n'\left(\mu+\Phi X_t\right)+\frac{1}{2}Var_t\left\{\left(B_n'\Sigma-\lambda_t'\right)\varepsilon_{t+1}\right\}=\\
&=-\delta_0-\delta_1'X_t-\frac{\lambda_t'\lambda_t}{2}+A_n+B_n'\left(\mu+\Phi X_t\right)+\frac{\left(-\lambda_t'+B_n'\Sigma\right)\left(-\lambda_t'+B_n'\Sigma\right)'}{2}=\\
&=\left(\delta_0+A_n+B_n'\mu+\frac{1}{2}B_n'\Sigma\Sigma'B_n-\lambda_0'B_n'\Sigma\right)+\left(-\delta_1'-B_n'\Sigma\lambda_1'+B_n'\Phi\right)X_t
\end{aligned}$$

We get A_{n+1},B_{n+1} as a solution of the system of difference equations with initial condition $A_1=\delta_0,\ B_1=-\delta_1'$:

$$\left\{\begin{array}{c}A_{n+1}=A_n+B_n'\mu-B_n'\Sigma\lambda_o+\frac{1}{2}B_n'\Sigma\Sigma'B_n\\ B_{n+1}'=B_n'\Phi-\delta_1'-B_n'\Sigma\lambda_1\end{array}\right\} \quad \text{(A4)}$$

The yield on a zero-coupon bond of maturity n, is affine structure of the state:

$$y_t(n)=-\frac{1}{n}(A_n+B\prime_nX_t)\equiv a_n+b_n'X_t \quad \text{(A5)}$$

3.7.3 Maximum Likelihood estimation

We follow the Chen-Scott method and assume that there are as many yields measured without error as there are latent factors in X_t. The same methodology was used by Ang, Bekaert (05), and Rudebusch, Wu (03).

We denote the set of yields measured without error as $Y_{1t}(N_1 \times 1)$ and the yields measures with errors as $Y_{2t}(N_2 \times 1)$. In the case of the pure yields framework, the total number of factors is N_1,while in the case of the joint macro-finance framework, the total number of factors in X_t is $N = N_1 + 1$, because the inflation is observable factor. Given the expression (3.14), the yields observed without error take the form:

$$Y_{1t} = \mathbf{A1} + \mathbf{B1}X_t. \tag{3.36}$$

where $\mathbf{A1}$ is the $(N_1 \times 1)$ vector stacking the $(-\frac{1}{n}A_n)$ terms for the Y_{1t} yields observed without error, $\mathbf{B1}$ is a $(N_1 \times N)$ matrix which stacks the vectors $(-\frac{1}{n}B\prime_n)$ for the yields observed without error.

Only latent factors model

The unobservable factors, X_t, then can be extracted by inverting the pricing relationship (3.16) of the model:

$$X_t = \mathbf{B1}^{-1}[Y_{1t} - \mathbf{A1}] \tag{3.37}$$

and, as we have shown in the section 3, the dynamics of the yields measured exactly is described by the VAR with cross-equation restrictions due to the No-Arbitrage assumption:

$$Y_{1,t} = \mu^{NA} + \Phi^{NA}Y_{1,t-1} + \Sigma^{NA}\epsilon_t, \tag{3.38}$$

The yields observed with errors, Y_{2t}, take the form

$$Y_{2t} = \mathbf{A2} + \mathbf{B2}X_t + \zeta_t, \tag{3.39}$$

where $\mathbf{A2}, \mathbf{B2}$ are $(N_2 \times 1)-$vector and $(N_2 \times N)-$matrix stacking correspondingly the terms $(-\frac{1}{n}A_n)$ and $(-\frac{1}{n}B\prime_n)$ for the vectors observed with errors, and $\zeta_t \sim N(0, \Upsilon)$ is the measurment error. Given the factors and the parametrs of the model, equation (3.39) provides solution for the errors, ζ_t.

Given the system

$$\begin{cases} Y_{1,t} = \mu^{NA} + \Phi^{NA}Y_{1,t-1} + \Sigma^{NA}\epsilon_t \\ Y_{2t} = \mathbf{A2} + \mathbf{B2}\mathbf{B1}^{-1}[Y_{1t} - \mathbf{A1}] + \zeta_t \end{cases}$$

we estimate the likelihood function as

$$L = \prod_t f(Y_{1,t}|I_{t-1}) f(Y_{2,t}|Y_{1,t}, I_{t-1})$$

Model with inflation as observable factor

The unobservable factors, X_t, can be extracted by inverting the pricing relationship (3.16) of the model:

$$X_t = \tilde{\mathbf{B1}}^{-1}[Z_t - \tilde{\mathbf{A1}}], \qquad (3.40)$$

where $Z_t = (Y_{1t}, \pi_t)$, and,

$$\begin{aligned} \tilde{\mathbf{A1}} &= \begin{bmatrix} \mathbf{A1} \\ 0 \end{bmatrix} \\ \tilde{\mathbf{B1}} &= \begin{bmatrix} \mathbf{B1} \\ e_3' \end{bmatrix}. \end{aligned}$$

Given the factor representation (3.40), the likelihood function is obtained following the same alghorithm as in the case of the ATSM with latent factors only.

Table 1. Different types of joint macro-finance models

Paper	State vector	Frequency	Dynamics	Method	Yields exact	Yields with error	Sample
Ang, Bekaert (04)	2 latent+π	Q	RS-VAR(1)	MLE	1q, 5y	1y, 3y	1952-2000
Ang, Piazzesi (03)	3 latent +"π"+"y"	M	VAR(12)	2-step LS	1m, 1y, 5y	1q, 3y	1952-2000
Ang et al (05)	1q+5y+growth	Q	VAR(1)	2-step LS	1q, 5y	-	1964-2001
Dai et al (03)	3 latent	M	RS-VAR(1)	MLE	2q, 2y, 10y	5y	1970-1995
Hordhal et al (05)	1 latent +r+π+y	M	VAR(3)	MLE	1m, 3y	1q, 2q, 1y, 7y	1975-1998
Rudebusch, Wu (04)	2 latent	M	VAR(1)	MLE	1m, 5y	1q,1y, 3y	1988-2000

Table 2 . Parameter estimates for ATSM.

	2-factor ATSM	3-factor ATSM
λ_1	$\begin{bmatrix} -1.0585 & 6.7534 \\ 1.8218 & -4.9356 \end{bmatrix}$	$\begin{bmatrix} 2.5627 & -18.1559 & -3.0091 \\ 0.0311 & -1.9476 & 0.7869 \\ -4.3771 & -0.0551 & 1.7464 \end{bmatrix}$
δ_1	-	1.1476
Φ	$\begin{bmatrix} 0.98507 & 0 \\ 0.027779 & 0.90766 \end{bmatrix}$	$\begin{bmatrix} 0.9279 & 0 & 0 \\ 0.0034305 & 0.98587 & 0 \\ -0.053238 & -1.2307 & 0.8839 \end{bmatrix}$
Σ	$\begin{bmatrix} 0.011451 & 0 \\ 0 & 0.010427 \end{bmatrix}$	$\begin{bmatrix} 0.025531 & 0 & 0 \\ 0 & 0.0014736 & 0 \\ 0 & 0 & 0.015463 \end{bmatrix}$

Table 3. Alternative estimates of companion matrix $\Phi = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix}$,
describing bivariate VAR(1) dynamics of the reference yields $[y^3, y^{60}]$.

	Φ^U	Φ^{NA}	Φ^{NS}
ϕ_{11}	0.9555 (0.0236)	0.9564 (0.0056)	0.9596 (0.0256)
ϕ_{12}	0.0601 (0.0339)	0.0588 (0.0152)	0.0507 (0.0367)
ϕ_{21}	0.0061 (0.0322)	0.0238 (0.0066)	0.0088 (0.0357)
ϕ_{22}	0.9652 (0.0463)	0.9363 (0.0177)	0.9564 (0.051)

Note: De-meaned variables. Sample 1988.01-1998.12. Standard errors in parentheses.

Table 4. Alternative estimates of companion matrix $\Phi = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{bmatrix}$, describing bivariate VAR(1) dynamics of the reference vector $[y^3,\ y^{60},\ \pi]$

	Φ^U	Φ^{NA}	Φ^{NS}
ϕ_{11}	0.9696 (0.023)	0.88390 (0.0180)	0.9712 (0.0251)
ϕ_{12}	0.1044 (0.0352)	0.1595 (0.1591)	0.0879 (0.0377)
ϕ_{13}	-0.0903 (0.0322)	-0.02290 (0.0134)	-0.0821 (0.0284)
ϕ_{21}	0.0275 (0.0327)	0.000 (0.00)	0.0050 (0.0361)
ϕ_{22}	0.9533 (0.0500)	0.9852 (0.0088)	0.9443 (0.0542)
ϕ_{23}	0.0243 (0.0391)	-0.0283 (0.0007)	0.0265 (0.0409)
ϕ_{31}	0.0216 (0.0248)	0.000 (0.000)	0.0209 (0.0248)
ϕ_{32}	0.0444 (0.0379)	-0.0014 (0.2339)	0.0361 (0.0373)
ϕ_{33}	0.9281 (0.0296)	0.9286 (0.0197)	0.9360 (0.0281)

Note: Sample is 1988.01-1998.12. Standard errors are in parentheses.

Table 5. Correlations across alternative term premium estimates

	ATSM(2)	ATSM(3)	NS(2)	NS(2)+π	VAR(2)	VAR(3)
ATSM(2)	1.000	0.879	0.903	0.931	0.754	0.867
ATSM(3)	0.879	1.000	0.932	0.887	0.824	0.827
NS(2)	0.903	0.932	1.000	0.989	0.946	0.965
NS(2)+π	0.931	0.887	0.989	1.000	0.930	0.979
VAR(2)	0.754	0.824	0.946	0.930	1.000	0.970
VAR(3)	0.867	0.827	0.965	0.979	0.970	1.000

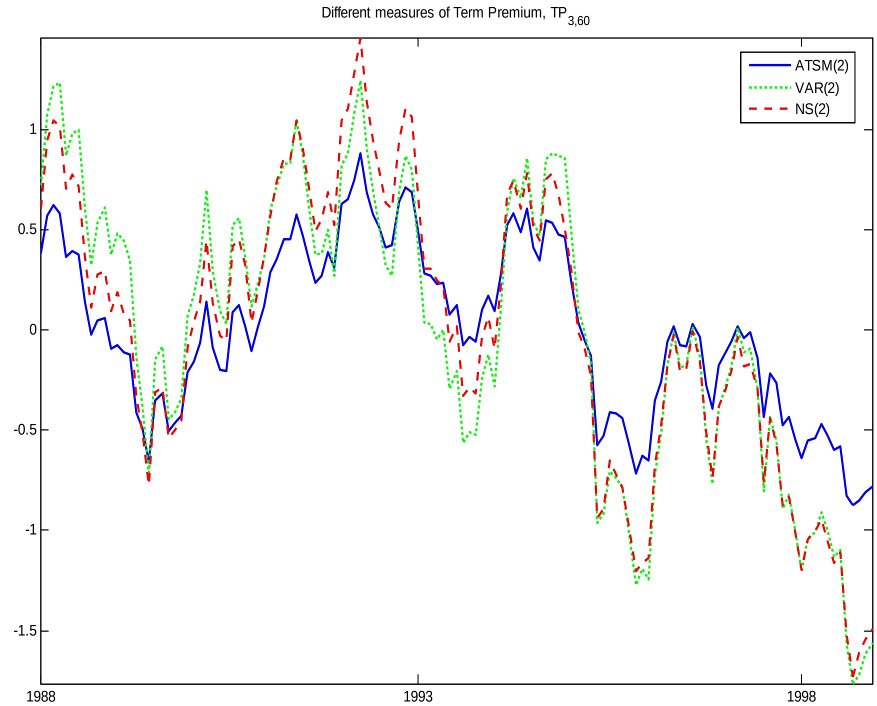

Figure 3.1: Term premia impied by alternative two-factor models.

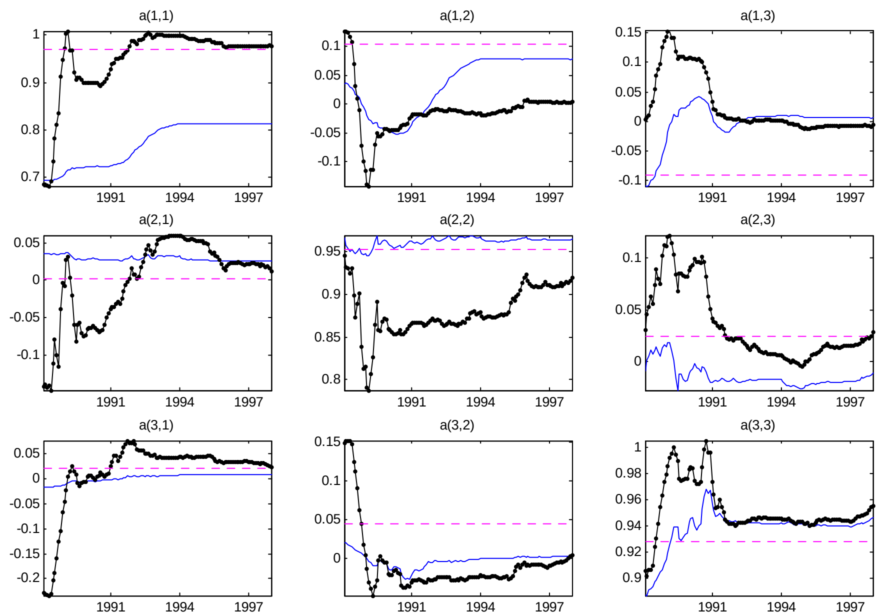

Figure 3.2: Time-varying VAR coefficients.

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