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ESSAYS ON MONETARY POLICY IN OPEN ECONOMIES

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Luca Martino Francesco Colantoni
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Essays on Monetary policy in Open Economies

Luca M. F. Colantoni

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Introduction

The object of analysis of this thesis is monetary policy in small open economies. The first three chapters are devoted to the analysis of the so-called phenomenon of dollarization while the last chapter focuses on some aspects of Canadian economy and how monetary policy makers changed their behaviour in passing to an inflation targeting regime.

Broadly speaking *dollarization* is the process of substituting a foreign currency for a domestic currency to fulfill the essential functions of money as a medium of exchange (*currency substitution* or *transaction dollarization*), store of value (*financial dollarization* which, according to its sign, is also known as *asset/liability dollarization*) and/or unit of account (*price dollarization*). It is a distinguishing feature of developing and transition economies and can play an important role in such policy decisions as conducting the monetary policy, implementing a stabilization program, determining an appropriate exchange rate regime and financing government deficit.

We start our analysis by reviewing some recent analytical and empirical issues on dollarization, mainly in its forms of currency substitution and financial dollarization. Despite the data availability problem, there is evidence that such phenomenon is still relevant and widespread even in those economies that have achieved a successful stabilization of the economy.

Having assessed the empirical and theoretical relevance of this phenomenon, we proceed with our analysis trying to understand what are the economic policy implications of currency substitution and assess the welfare costs associated with different policies when operating in a dual-currency environment. To this purpose, in the second and third chapters in the stream of *New Open Macroeconomic* we exploit recently developed instruments of macroeconomic analysis, namely *dynamic stochastic general equilibrium models* (DSGE). These are micro founded models based on optimizing agents, with nominal rigidities and market imperfections. They are a quite powerful tool for macroeconomic analysis since they can be solved numerically in order to compute steady state values, simulate the dynamics of variables of interest and perform welfare analysis.

In the second chapter we present a money in the utility function where liquidity services are generated by two currencies (domestic and foreign). An important assumption is the non separability between consumption and liquidity services in the utility function. More precisely, we work on a model very similar to the one by Felices and Tuesta (2006) extending their analysis to a numerical solution of the whole model up to the second order of approximation. This enables us to extend previous analysis in two respects. Firstly, we provide a description of the behaviour (impulse responses and volatilities) of the whole economy under two sources of shocks, a shock to technology and a shock to foreign interest rate. Secondly, we perform welfare analysis, for which second order approximation is necessary. We show that, dollarization may play a marginal role even under non separability when, instead of considering a shock to foreign interest rate, we consider a shock to technology. Overall, impulse responses are larger the greater the degree of dollarization and macro volatility increases with it. When coming to welfare implications, dollarization is generally welfare decreasing when consumption and money services are complements, while some degree of dollarization is preferred under substitutability. Finally, a fix exchange rate is preferred for all the considered degrees of currency dollarization under the substitutability case. On the contrary, a flexible exchange rate regime is welfare increasing when we consider complementarity and intermediate degrees of dollarization.

In the third chapter we address the issue of liability dollarization and, more precisely, its interaction with exchange rate variability. Liability dollarization is manly due to financial imperfections and, being monetary policy linked to credit markets' conditions, it can affect the transmission mechanism of monetary policy and make the financial system more vulnerable to exchange rate fluctuations. Following previous literature we account for financial dollarization by assuming imperfect financial markets and allowing agents to borrow foreign denominated assets. By means of a second-order approximation solution we study quantitatively the welfare effects of exchange rate risk in presence of liability dollarization finding that fix exchange rate is associated with higher costs and a lower level of welfare.

Finally, in the fourth chapter we devote our attention to some broad characteristic of monetary policy in Canada over the period from early 1970 till early 2007. To such purpose, we apply vector autoregressive (VAR) techniques which enable us to gain some insight on the transmission mechanism of monetary policy. We then use the VAR results to calibrate monetary policy key parameters in a dynamic stochastic general equilibrium model.

Chapter 1

Dollarization: Some Issues

Luca M. F. Colantoni
Bocconi University

Abstract

This note reviews analytical and empirical issues on dollarization, mainly in its forms of currency substitution and financial dollarization.

Keywords: dollarization, currency substitution, financial dollarization.

JEL codes: E5, F31, F41.

1.1 Introduction

When the environment of a country is uncertain due to changing property rights, political instability, large budget deficits, and high inflation, foreign currency may take on all or at least some of the functions of domestic money to hedge economic agents from these adversities. This is what happened in many emerging and transition economies especially during the eighties and in some cases during the nineties. Some Latin American Countries (LACs) and more recently Central and Eastern European Countries' (CEECs') are among those economies where the domestic currency has been partially replaced in its functions by a foreign currency, usually the US Dollar, a phenomenon known in the literature as *dollarization*.

Broadly speaking *dollarization* is the process of substituting a foreign currency for a domestic currency to fulfill the essential functions of money as a medium of exchange (*currency substitution* or *transaction dollarization*), store of value (*financial dollarization* which, according to its sign, is also known as *asset/liability dollarization*) and/or unit of account (*price dollarization*). Dollarization is *official* when a nation adopts de jure the currency of a foreign nation to wholly replace its domestic currency. This is also known as *full dollarization*. *Partial dollarization*, also defined as *unofficial* or de facto dollarization, occurs when individuals and firms voluntarily choose to use a foreign currency as a substitute for some of the monetary services of the domestic currency.

Partial dollarization is a widespread phenomenon. Moreover it proves to be quite persistent since, because of hysteresis effects and habit persistence, the amount of foreign real money balances rarely falls to negligible levels even after a successful stabilization of the economy. Partial dollarization plays an important role in financing government deficit, conducting the monetary policy, implementing a stabilization program and determining an appropriate exchange rate regime. The fact that partial dollarization makes active exchange rate intervention more dangerous is crucial for candidate countries who plan to join the European Union (EU). In fact, new members of the EU are required not only to meet the Maastricht convergence criteria but also to participate in the ERM-II (Exchange Rate Arrangement between the Euro area and EU members outside the Euro area). At issue is first, whether and when certain CEECs will officially *euroize*, that is adopt the euro de jure as their sole legal tender. Of course this choice and the effects of participating in the ERM-II depend on the degree of partial dollarization. A similar discussion is underway in several LACs on the opportunity of implementing a peg to a stronger foreign currency or the possible adoption of the U.S. dollar as official currency[1]: indeed, Ecuador, El Salvador and Guatemala have all recently done so.

Indeed, full dollarization has been actively debated in a number of emerging market economies. Proponents of full dollarization argue that, it can reduce country risk premia by eliminating devaluation risk and increasing credibility in economic policies, thus lowering aggregate volatility. On the other hand, opponents of full dollarization believe that the costs of such stabilization policy may well exceed its benefits.

Being a feature of emerging economies, policy makers of these economies should take dollarization (in its various forms) into account. Nevertheless, as recently pointed out by Calvo (2006), monetary policy literature is still centered on sophisticated analyses of how to implement monetary policies in developed countries with little or no reference to this phenomenon. Hence, the importance to study dollarization to better assess its policy implications and the welfare costs associated with different policies when operating in a dollarized environment.

An early survey on the problem of currency substitution is presented by Calvo and Vegh (1992) for developing countries. Savastano, M. A. (1992) provides similar insightful studies on Latin America. Surveys of theoretical and empirical problems and developments in the field include a paper by Giovannini, A. and B. Turtelboom (1994) and the book by Mizen, P. and E. J. Pentecost (1996). Many are the works providing evidence for different countries and regions. Among them can be found Ramirez-Rojas (1985) (for some LAC), Sahay and Vegh (1995), Savastano (1999) (for transition economies). One of the first empirical studies of currency substitution in transition economies, the case of Latvia, is presented in Sarajevs (2000), while Feige and Dean (2002) empirically assess currency substitution in many CEECs. Recently, financial dollarization both in developing and transition countries has been empirically analyzed in Arteta (2003, 2005).

In this note we will focus on some analytical and empirical issues raised by partial dollarization mainly in its forms of currency substitution and financial dollarization. As for price dollarization, being set in foreign currency, prices become perfectly indexed to the exchange rate thus determining the effectiveness of monetary policy and eliminating its short-run effects. In the literature, price dollarization has been treated as an invoicing decision of firms, i.e. as the decision of pricing in the exporter's or the importer's currency under international trade[2]. At the empirical level, Honohan and Shi (2002)

[1] Strictly speaking a peg to the dollar is not the same as full dollarization, the two environments differing in terms of who takes the seigniorage (either the domestic or the foreign monetary authority).

[2] See for example, Giovannini (1988), Donnafeld and Zilcha (1991), Johnson and Pick (1997) and Bacchetta and van Wincoop (2001).

measured price dollarization by the short-run level of pass-through of the exchange rate. To our knowledge only Castillo and Montoro (2004) build a general equilibrium model which provides a link between currency substitution, financial dollarization and price dollarization. According to this link, represented by income distribution, financial dollarization positively determines currency substitution. Moreover, income distribution ends up determining also invoicing decisions of firms in such a way that luxury goods endogenously priced in dollars and goods associated to low income customers priced in domestic currency.

The rest of this note is organized as follows: section 2 addresses some recent theoretical aspects of currency substitution and financial dollarization. Section 3 reviews some empirical issues while section 4 discusses macroeconomic and policy implications of dollarization. Section 5 concludes.

1.2 Analytical Issues

1.2.1 Currency substitution

In the literature we can find three ways of generating a demand for foreign currency in order to create a dual or multi currency environment. Correspondingly we can classify three types of models[3]: money in the utility function models, cash-in-advance models and transaction costs models[4].

For a discussion of the last two types of models we refer the interested reader to the survey by Giovannini and Turtleboom (1992). A recent example of a cash in advance model can be found in Uribe (1997) who analyses hysteresis in money velocity.

Most of recent analysis on currency substitution has applied utility function models. In these models, liquidity services directly enter into agents' utility who maximize utility subject to a budget constraint[5]. In such models what is relevant is the functional form used to model such liquidity services, i.e. $h(m, m^*)$ which are a function of both domestic and foreign real money balances[6]. The general approach relies on the well known CES

[3] In general, these are ways to generate a dual currency environment. Thus, they can be used to model not only currency substitution but also other forms of dollarization. Nevertheless, we introduce them in this section since in the literature they have been most used as a way of modeling currency substitution.

[4] Vegh (1989) contains an example of shopping time model.

[5] See among others: Bufman and Leiderman (1992), Imrohoroglu (1994), Cuddington, Garcia and Westbrook (2002), Felices and Tuesta (2006) and Batini, Pearlman and Levine (2006).

[6] Inside a framework with transaction costs, McNelis and Asilis (1992) model liquidity services by means of a Cobb-Douglas technology.

functional form:

$$h\left(\cdot\right)=\left[\alpha\left(\frac{M_t}{P_t}\right)^{\frac{\chi-1}{\chi}}+(1-\alpha)\left(\frac{\varepsilon_t M_t^*}{P_t}\right)^{\frac{\chi-1}{\chi}}\right]^{\frac{1}{\chi-1}} \tag{1.1}$$

where $M,\ M^*,\ P,\ P^*$ and ε are domestic currency, foreign currency, domestic price index, foreign price index and nominal exchange rate, respectively. This is a convenient functional form that separates the elasticity of currency substitution χ, from the share, $(1-\alpha)$ of foreign real balances in the production of domestic liquidity services[7]. It can be proved that the money demands spanned from such functional form lead to the following foreign to domestic currency ratio

$$RF_t\equiv\frac{m_t^*}{m_t}=\left[\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{R_t}{R_t-1}\right)\left(\frac{R_t^*-1}{R_t^*}\right)\right]^{-\chi} \tag{1.2}$$

where R and R^* are gross domestic and foreign interest rates respectively, and $\frac{\partial RF_t}{\partial\alpha}<0$ and $\frac{\partial RF_t}{\partial R_t}>0$. Note that when $R=R^*$ the previous ratio is constant and its value depends on α and χ. The degree of dollarization of the economy can be measured by the following dollarization index:

$$DI_t\equiv\frac{m_t^*}{m_t^*+m_t}=\left\{\left[\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{R_t}{R_t-1}\right)\left(\frac{R_t^*-1}{R_t^*}\right)\right]^{\chi}+1\right\}^{-1} \tag{1.3}$$

with $\frac{\partial DI_t}{\partial\alpha}<0$ and $\frac{\partial DI_t}{\partial R_t}>0$. It is important to note that, depending on R , the degree of dollarization of the economy is affected by monetary policy. More precisely, as the opportunity cost of holding domestic currency, R, increases the demand of domestic money holdings decreases and so the degree of dollarization of the economy increases. On the contrary, DI decreases with the preference for domestic currency (i.e. as α increases). Finally, in a steady state were $R_t=R_t^*$ it will depend only on parameters α and χ.

$$DI_t=\left\{\left(\frac{\alpha}{1-\alpha}\right)^{\chi}+1\right\}^{-1} \tag{1.4}$$

An alternative formulation is the one proposed by Obstfeld and Rogoff (1995)

$$h\left(\cdot\right)=\frac{1}{1-\sigma}\left[\frac{M_t}{P_t}+a_1\frac{\varepsilon_t M_t^*}{P_t}-\frac{a_2}{2}\left(\frac{\varepsilon_t M_t^*}{P_t}\right)^2\right]^{1-\sigma} \tag{1.5}$$

[7] This specification delivers a steady state with positive foreign real money balances as long as $\alpha>0$.

where $a_1 > 1-\beta$ and all parameters are larger than zero[8]. This functional form rationalizes the legal restrictions on foreign currency use whose costs are measured by the quadratic term into the brackets. Notice that two types of money enter the utility function separately. This assumption insures that money holdings do not directly affect marginal rates of intertemporal substitution of consumption. A major argument for sticking to the assumption of money services separability is the desire to maintain some level of analytical tractability of the model. The advantage of such specification is the fact that it delivers a clear foreign currency demand which, for interior equilibria, §(i.e. for $m^* > 0$) is

$$m_t^* \equiv \frac{\varepsilon_t M_t^*}{P_t} = \frac{1}{a_2}\left(a_1 - 1 + \frac{\frac{\varepsilon_{t+1}}{\varepsilon_t} - 1}{r_t}\right) \tag{1.6}$$

with $r = R - 1$ is the net nominal domestic interest rate.

In the case $\frac{1}{a_2}\left(a_1 - 1 + \frac{1}{r_t}\left(\frac{\varepsilon_{t+1}}{\varepsilon_t} - 1\right)\right) < 0$, the foreign currency holding would be equal to zero. Hence, agents will hold the foreign currency if the economy experiences a high enough nominal depreciation. Or, in other words, the demand for the real foreign money balances responds positively to an increase in the rate of depreciation of the exchange rate $\triangle\varepsilon_{t+1} \equiv \frac{\varepsilon_{t+1}}{\varepsilon_t}$. If a_2 is small enough, meaning low costs of holding foreign currency, then very small changes in the exchange rate can induce high demand for foreign currency. Finally, in steady state:

$$m_t^* = \max\left\{\frac{1}{a_2}\left(a_1 - 1 + \frac{\triangle\varepsilon - 1}{r}\right), 0\right\} \tag{1.7}$$

Intuitively, if there are no economic incentives to hold foreign currency, i.e. the rate of depreciation of exchange rate is equal to one, and there are non-zero costs of holding or using foreign currency (e.g. due to foreign exchange market fees, or fines for evading government regulations) no rational economic agent will hold foreign currency balances in such a steady-state (we assume that $a_1 - 1 < 0$).

1.2.2 Financial dollarization

We can distinguish two ways of modeling financial dollarization, i.e. the fact that either the government or the private sector, or both, have foreign-exchange denominated short-term debt obligations. The first one[9], simply consists in allowing households and

[8]Note that with this specification the utility from foreign money is increasing only up to a certain amount, and then decreasing.

[9]A recent example is Schmitt-Grohè and Uribe (2001).

government to buy and issue foreign currency denominated assets which thus enter in their budget constraints[10]. Assuming market imperfections the model is closed by means of a debt-elastic interest rate rule according to which the interest rate paid on foreign currency denominated bonds depends on foreign interest rate plus a risk premium $\Psi(F_t)$ assumed increasing in the net foreign asset position F

$$i_t = i_t^* + \Psi(F_t)$$

$$\Psi(F_t) \equiv \psi_2(e^{F_t - \bar{F}} - 1)$$

where

$$F_t \equiv -\frac{\varepsilon_t}{P_{H,t}} B_t^*$$

investors are required to pay a risk premium ρ

Alternatively, Cèspedes, Chang and Velasco (2001, 2002) consider an economy populated by capitalists as well. At the beginning of each period investors receive the revenues from their previous investment K_{t-1} and repay some foreign debt D_{t-1}. Investors can borrow *dollars* in the world market which, together with their *net worth*, finance purchases of new financial capital. Hence, liabilities are dollarised. as emphasized in Calvo (1999).

The value of investment is

$$P_t K_t = P_{t-1}(R_{t-1}K_{t-1} - \varepsilon_{t-1}D_{t-1}) + P_t \varepsilon_t D_t$$

where the starting period *net worth* is

$$P_{t-1}N_{t-1} = P_{t-1}[R_{t-1}K_{t-1} - (1 + \tilde{\imath}_{t-1})\varepsilon_{t-1}D_{t-1}]$$

and

$$1 + \tilde{\imath}_t = (1 + i_t^*)(1 + \rho_t)$$

The gross safe dollar interest rate is given by $(1 + i_t^*)$. However, because of market imperfections domestic investors are required to pay a risk premium $\rho\left(\frac{P_t K_t}{P_{t-1}N_{t-1}}\right)$. The risk premium is assumed to be increasing in the ratio of the value of investment to net worth.

Note that a real devaluation (an increase in ε) reduces, ceteris paribus, the net

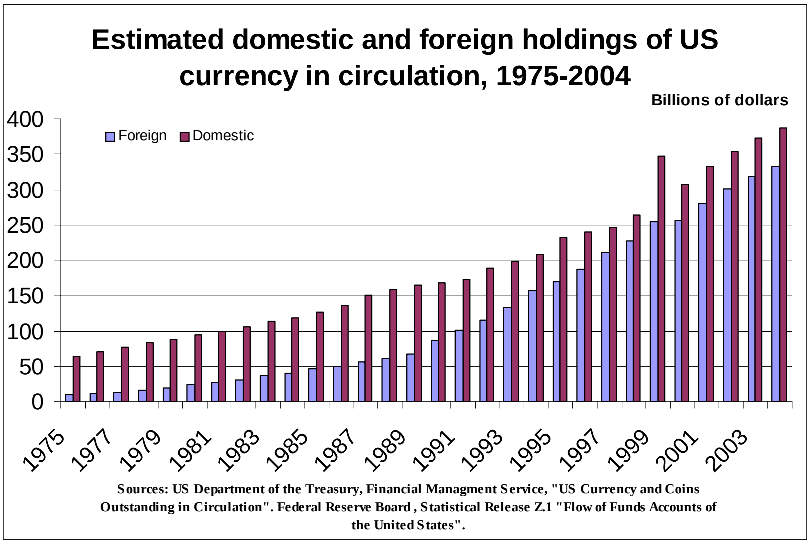
[10] In an extreme version, there are no domestic bonds and only foreign bonds can be issued. This is what Eichengreen and Hausmann (1999) define as *original sin*.

worth, thus increasing the risk premium. So, a real devaluation potentially can have a contractionary effect.

1.3 Empirical Issues

1.3.1 The demand for US Currency

US bank notes[11] are widely used outside the United States and their demand has been increasing steadily over the last two decades. Just to have an idea, in the '90s overall US currency in circulation increased an average of 8 percent per year - from \$268.2 billions to \$601.2 billions. A sizeable share of this growth can be attributed to overseas demand whose amount is estimated to have increased significantly[12] beginning in the late '80s and continued to grow through most of the '90s. In 2004, according to the Federal Reserve more than 50% of US currency in circulation outside banks was estimated to be abroad.

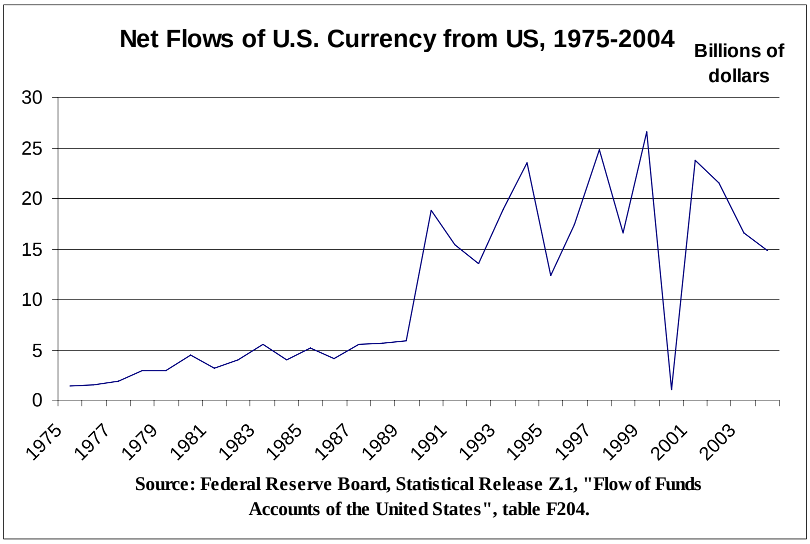


Based on estimates of net payments, international demand for US currency increased 219% in 1990 during the Gulf War and 24% in 1994 during the Mexican peso crisis. The remarkable decrease in net flows in 2000 (end of the uncertainty due to the millennium data change?) was soon followed by an increase in 2001 when international demand for US currency grew by more than 2000% (uncertainty due to the introduction of the

[11] Because of data availability, most of this section is referred to dollarizaiton when the foreign currency is the US dollar.

[12] On average in the last two decades the overseas stock has been growing at about two times the rate of growth of the domestic stock.

Euro? 9/11?).



Given the size and diffusion of the phenomenon[13], it can be interesting to understand what are the determinants of foreign demand of US currency (and, in general, of currency different from the domestic one). Even more interesting can be an answer to what are the economic policy implications of dollarization.

It is not our aim to review the, by now, quite extensive empirical analysis of currency substitution and demand of foreign currency. It is sufficient to say that first approaches relied on the estimation of simple regression equations of demands for domestic and foreign money as functions of the correspondent interest rates[14]. However, a limit of such procedure is that it is based on static models that do not consider potentially relevant intertemporal channels. Accordingly, recent empirical literature has moved in the direction of the estimation of structural models[15].

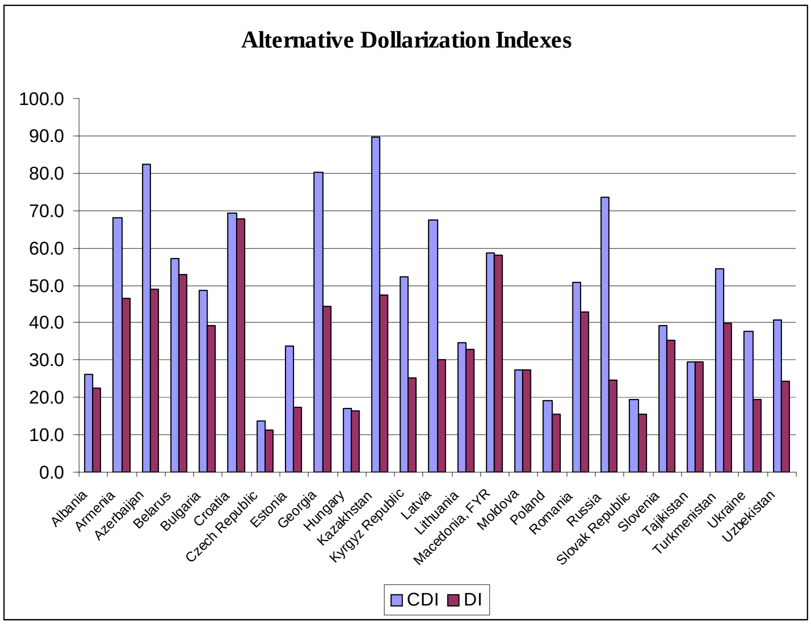
Most of this literature is seriously flawed by the difficulty in retrieving reliable data and there has been a great deal of speculation on the precise amounts of foreign currency held abroad. The following figure shows a country-by-country comparison of the conventional IMF dollarization proxy (DI), mainly based of foreign currency deposit ratios, and of Feige broader unofficial dollarization index (CDI) that takes explicit account of

[13] A recent estimate of the actual extent of dollarization or euroization in transition countries is provided by Dean and Feige [?], who try to solve the major limitation of any analysis of unofficial foreign currency use, i.e. the fact that the amount of foreign cash in circulation (FCC) is typically unknown. Moreinformation on this measure can be found in the next section.

[14] See for example Cuddington (1983).

[15] See for example, Bufman and Leiderman (1993), Imrohoroglu (1994) and Selçuk (2003).

the estimated amount of foreign currency cash in circulation in each nation in 2001.



Source: Feige (2003).

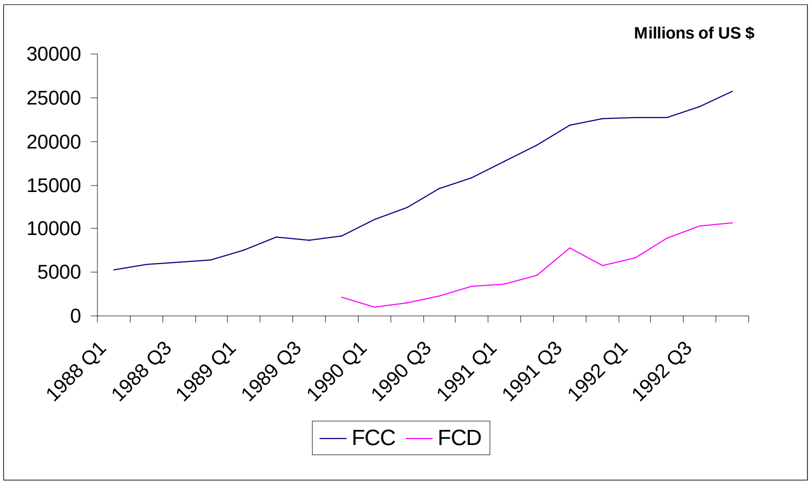
It can be seen that, in some cases, the two measures significantly differ thus leading to different conclusions on the relative importance of the phenomenon in the economy object of analysis. In the next section we will review some issues arising with this topic.

1.3.2 Measuring Currency Substitution

The dollarization literature suffered from an empirical problem when attempting to assess positive issues concerning causes, consequences, costs and benefits of currency and asset substitution. This problem is due to the fact that there is usually no data available on foreign currency circulating in an economy and hence the currency substitution phenomenon is unobservable. In fact, currency movements are difficult to measure (it can be easily concealed and readily carried across borders, even in large quantities) and estimates of the foreign component of currency stocks and flows have been subject to a great deal of speculation.

The traditional literature has adopted the use of foreign currency deposits in domestic banks as the best indicator of dollarization. Severe data shortcomings have repeatedly prevented attempts to construct a reliable measure of partial dollarization that includes estimates of other foreign currency assets including cash holdings of foreign currency. Initially, to cope with this problem, the proportion of foreign currency deposits (FCD) (denominated assets in M2) in the domestic banking system was used to measure unofficial dollarization in general and more precisely as a proxy to currency substitution (see Savastano, 1992). A recent survey of selected developing countries by the IMF found 52 that were highly or moderately dollarized as of 1995 (Baliño et Al., 1999).

But, as pointed out by Feige 2003, asset substitution and currency substitution need not to co-move given the differences between transactions and asset motives for holding currencies. A clear example is Argentine. In the following figure we plot cumulated net shipments of US currency to Argentina (FCC) and foreign currency deposits of the Argentinian private sector (FCD). We can distinguish episodes in which currency substitution and financial dollarization move in opposite directions. This can occur when rising confidence in the domestic banking system and improvements in its provision of transactions services induces a shift from FCC to FCD.



Sources. FCC: CMIR. FCD: BCRA (ent105).

Hence the importance to understand what are the determinants of foreign demand of US currency.

Estimates of the extent to which notes of the U.S. dollar and a few other currencies circulate outside their countries of origin give a rough idea of how widespread unofficial dollarization is. Researchers at the Federal Reserve System estimate that foreigners hold 55 to 70 percent of U.S. dollar notes, mainly as $100 bills[16]. The amount of dollar currency in circulation is currently about $720 billion, which implies that foreigners hold roughly $420 billion[17].

[16] Various estimations of US currency in circulation outside the United States can be found in a series of papers and reports. See, for example, U.S. Treasury (2000, 2003), Judson and Porter (2001, 1996), Porter and Weinbach (1999), Porter (1993).

[17] Seitz (1995) estimated a similar result for the German mark and by analogy one can think that the

Conceptually, the methods to estimate the amount of foreign currency cash holdings (FCC) can be divided into *indirect* and *direct methods*.

The first ones rely on the analysis of domestic variables to project foreign demand and can be distinguished in *denomination displacement method* and *money demand method*. The interested reader can find more information on such methodology in Feige and Dean (2002) and Porter (1996). In what follows we focus our attention to the direct methods.

1.3.3 Direct Measures

Direct methods rely on statistics directly related to the foreign currency demand as represented by, for example, customs reports or direct population surveys.

The Federal Reserve and the Treasury have information on the amounts of US dollars that are held abroad from various sources such as U.S. Customs reports, shipment data from overseas bank note wholesalers and published proxies for those shipments. Moreover, there are estimates based on in-country surveys from dollar–using countries, national surveys of domestic currency holdings, and a variety of empirical models developed by the Federal Reserve and others that estimate[18] overseas flows or holdings based on realistic assumptions concerning international currency usage. Table 1 shows some preliminary results from such estimates[19]. In the first column we show results from currency surveys conducted by the Treasury and Federal Reserve between 1997 and 2002. The other two columns report computations by Feige (2003) based on new data collected by United States Customs Service under the Currency and Foreign Transactions Reporting Act and data from a survey on 5 Europeans countries commissioned by the Austrian National Bank (ONB). As expected, the dollarization degree tends to be higher in economies that have experienced high rates of inflation and/or exchange rate crisis, even when these occurred much earlier. Some of these countries officially dollarized their economies (e.g. Argentina and Ecuador), but an economy can be heavily dollarized even

same holds for the euro even if at the moment there is no study on it.

[18] Various estimations of US currency in circulation outside the United States can be found in a series of papers and reports. See, for example, U.S. Treasury (2000, 2003), Judson and Porter (2001, 1996), Porter and Weinbach (1999), Porter (1993).

[19] Unfortunately, due to lack of data, the table does not include estimates for countries (particularly from Latin America) known to be dollarized economies.

in absence of official dollarization (e.g. Russia, Ukraine).

Table 1: Estimates of Foreign Currency held as Cash

Country	GDP held in the form of U.S. currency (%) 1997-2002	Foregin Currency to Total Currency (%) 2001	$ Per Capita FCC 2001
Albania		14	46
Argentina	17.5		
Armenia		62	55
Azerbaijan		82	169
Belarus	5.8	34	17
Bulgaria	2.8	41	125
Cambodia	25.2		
Chile	0.4		
China,P.R.: Mainland	0.9		
China,P.R.:Hong Kong	1.2		
Croatia		35	117
Czech Republic		21	129
Dominica	3.9		
Ecuador	7.3		
Egypt	0.4		
El Salvador	7.5		
Estonia		59	414
Georgia		79	123
Hungary		6	25
Indonesia	0.3		
Kazakhstan		95	1024
Korea	2.3		
Kyrgyz Republic		48	20
Latvia	5.5	79	1209
Lithuania	3.6	11	25
Macedonia, FYR		5	5
Mexico	0.6		
Paraguay	0.6		
Peru	3.8		
Philippines	1.0		
Poland	0.4	27	93
Romania	0.8	55	61
Russia	10.0	87	903
Slovak Republic		28	123
Slovenia		54	329
South Africa	3.1		
Thailand	0.1		
Turkey	2.6		
Ukraine		64	131
Vietnam	2.7		

Sources. First Column: US Treasury Department (2003). Second and third column: Feige (2003)

Another direct source of information that can be used to determine the approximate amounts of US cash in circulation in different countries is represented by the Reports of International Transportation of Currency or Monetary Instruments (CMIR)[20]. These

[20] An example of use of CMIR data in understanding the implications of currency substitution can

reports, collected by the US Custom Service since 1977, have to be filed by any person or institution importing or exporting currency or other monetary instrument in amounts exceeding \$10,000[21].

Although the CMIR estimates and informal interview estimates for some countries are quite different, both sources confirm the belief that currency substitution is quite widespread in transition economies.

There is anecdotal evidence that many of the Central Eastern European Countries (CEECs) employed national currencies of European nations, in addition to dollars, as co-circulating currencies. The Austrian National Bank (ONB) commissioned Gallup to conduct a series of surveys in five CEECs[22] in order to determine the extent of FCC holdings of various non-local currencies. Each of the 18 surveys conducted between June 1997 and November 2005 involved approximately 1000 persons per country above the age of 14. The main focus is to establish estimates of foreign currency cash holdings in the respective countries. Differently from CMIR data, the ONB data contains also information on the motives for the demand of FCC and characteristics of individuals interviews. The surveys do not include commercial cash holdings (e.g. tourism) and cannot measure criminal money. Therefore, it is likely that the estimated figures understate[23] the true amount of currency circulating abroad.

In the following figure we show the amount of FCC as a percentage of average monthly wage for Czech Republic[24]. As can be seen the currency substitution phenomenon has been quite relevant in late 90s' and early 2000 touching a maximum of 38% in 1999. Then, its relevance decreased to finally fluctuate around a value of 3-4%. Finally, note that with the introduction of the euro it took no more than two years to individuals to completely substitute the dollar with the European currency as their preferred FCC.

ONB data provide valuable information because they allow to assess changes in the behaviour of agents over time (under the assumption that the method bias is constant over time). They can help to understand what are the determinants of foreign demand

be found in Kamin and Ericsson (2003).

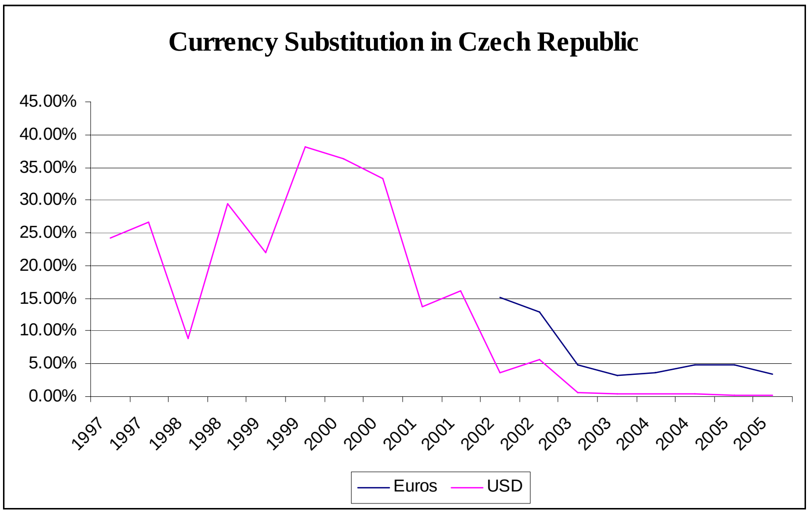
[21] Initially the limit was \$5,000. For a detailed description of the CMIR data base see Feige (1996).

[22] Croatia, Czech Republic, Hungary, Slovakia and Slovenia. For a detailed description of the ONB data set see Stix (2001).

[23] Survey results concerning self-admitted currency holdings are best considered as lower bound estimates of actual currency holdings since surveys are known to suffer from underreporting bias.

[24] The author is indebeted to the Austrian National Bank (ONB) for providing its survey estimates. A description of the database can be found in Stix (2001).

of US currency (and, in general, of currency different from the domestic one).



Source: author's calculations based on Austrian Central Bank Survey data.

1.3.4 Financial Dollarization: some evidence

The literature on financial dollarization has focused on its potential damages since the late '90s' financial crises which placed attention on the role played by liability dollarization, more precisely on balance sheets effects and currency mismatches.

Even if much less serious than those affecting currency substitution. In fact, sectorial data on the foreign currency liabilities of different economic agents and on the linkages across the balance sheets of those agents are not available for the large majority of countries. The few empirical studies on liability dollarization that exist have therefore relied on indirect measures (such as the pass-through from the exchange rate to prices) rather than on quantity-based estimates of external foreign currency liabilities to gather support for their key hypotheses.

In a series of papers Arteta (2003, 2005) exploits a new dataset on both deposit and credit dollarization for 92 emerging economies and, by means of multivariate analysis, finds that floating exchange rates have a positive effect on both deposit and credit dollarization. Another of his findings is that the effect on deposit dollarization is greater thus potentially causing currency mismatches. Notwithstanding this, according to his empirical analysis financial dollarization does not seem to increase crisis risks and crisis costs. In order to avoid these, adequate macroeconomic, financial and exchange rate policies seem more relevant.

As the same Arteta notes, his dataset measures deposit and credit dollarization not financial dollarization. For such purpose it would be necessary to collect additional data on other components of banks' balance sheets.

Reinhart, Rogoff and Savastano (2003) propose a solution to the difficulty of retrieving a comprehensive measure of financial dollarization. To such purpose they build a composite index of financial dollarization based on the following three components: bank deposits in foreign currency as a share of broad money, total external debt as a share of GNP and domestic government debt denominated in a foreign currency as a share of total domestic government debt. Their analysis on a sample of 90 non-industrial economies shows that there has been a large increase in the degree and incidence of dollarization in the countries analyzed.

1.4 Policy Issues

Being dollarization a distinctive feature of emerging economies policy debates on such economies have started considering it. As already noted, dollarization, particularly financial dollarization could affect the transmission mechanism of monetary policy and make financial system more vulnerable to exchange rate fluctuations. The interaction between dollarization and exchange rate volatility has been topic of a long lasting debate.

Part of the literature, both theoretical and empirical, has stressed the fact that partial dollarization increases the cost of exchange rate volatility. This, in turn, induces the central bank to intervene in the foreign exchange markets to prevent fluctuations in the nominal exchange rate. In fact, in presence of financial dollarization with all assets and liabilities denominated in the same currency, a fix exchange can help firms, banks and households to prevent currency mismatches. On the other hand, liability dollarization may be a result of pegging magnified by the overconfidence and moral hazard problems that pegging may bring about. Thus, fear of floating induces more liability dollarization creating a vicious circle. Finally, proponents of hard pegs (e.g. full dollarization, currency boards) observe that in presence of floating exchange rates investors will shy away from foreign-exchange denominated debts because of larger currency risk than under fix. Opponents argue that in many cases fixed exchange rates were a cause of currency and banking crises in emerging countries and that, following Mundell's prescription, floating exchange rates can help in absorbing negative shocks.

At this point proponents of full dollarization have stressed the relevance of another typical problem of emerging economies[25], namely the *sudden stop* problem, i.e. the immediate drying up of access to world financial markets. When capital leaves an emerging market abruptly it can lead to a considerable change in the real exchange rate triggering a crisis. This problem as well as dollarization assumes increasing relevance as the developing economy integrates with the rest of the world. In fact, as this process deepens official capital flows shrink and private capital flows assume an increasingly important role. Hence, the central bank ends up governing limited resources becoming powerless to successfully push away a speculative attack, act as lender of last resort or conduct independent monetary policy. This last one has to accommodate the international capital markets' desiderata in order to prevent sudden stops. So, when the domestic economy is facing an adverse shock, it would most benefit from low interest rates. But at the same time because of the slump a sudden stop problem threatens and in order to avoid it monetary policy will tighten.

Previous analysis is strictly related to another policy issue raised by the presence of dollarization, namely the effectiveness of the lender of last resort. According to part of the literature dollarization detracts from the central bank's ability to operate as lender of last resort. The greater the extent and variability of dollarization, the weaker is the central bank's knowledge and control over the effective money supply. Growing currency substitution reduces the ability of the monetary authority to earn seigniorage from its own currency issue. Unofficial dollarization reflects citizen's perceptions of the stability of the domestic monetary regime, the credibility of monetary policies and the perceived stability of the domestic banking system. Unofficial dollarization not only makes the outcomes of monetary policy less certain, it also has fiscal consequences. Foreign cash transactions rarely leave a paper trail. They therefore reduce the costs of tax evasion and increase the size of the unreported (unofficial) economy. This weakens the government's fiscal ability to command real resources from the private sector and deepens fiscal deficits. The shifting of economic activity toward the underground economy distorts macroeconomic information systems (Feige 1990, 1997), thereby adding to the difficulty of formulating macroeconomic policy.

By obscuring financial transactions, unofficial dollarization also reduces the cost of enterprise theft, and may facilitate greater corruption and rent seeking. Given these extensive ramifications, informed policy decision-making requires better knowledge of the nature, extent, causes and consequences of unofficial dollarization as well as the specific effects of its components, currency substitution and asset substitution.

[25] See for example, Calvo (2006).

1.5 Conclusions

In this note we focused on some analytical and empirical issues raised by dollarization, mainly in its forms of currency substitution and financial dollarization. Despite the data availability problem, there is evidence that such phenomenon is still relevant and widespread in developing and transition economies, even those that have achieved a successful stabilization of the economy.

Being a feature of emerging economies, policy makers of these economies should take dollarization (in its various forms) into account since it can play an important role in such policy decisions as conducting the monetary policy, implementing a stabilization program, determining an appropriate exchange rate regime and financing government deficit.
Indeed, there has been a long debate on full dollarization. Proponents of full dollarization argue that, it can reduce country risk premia by eliminating devaluation risk and increasing credibility in economic policies, thus lowering aggregate volatility. On the other hand, opponents of full dollarization believe that the costs of such stabilization policy may well exceed its benefits.

Nevertheless, monetary policy literature is still centered on monetary policies in developed countries with little or no reference to partial dollarization. Hence, the importance to study models which take it into account. Such research should lead to a better understanding of dollarization's interactions with other features of emerging economies (as hyperinflation and high volatility of the exchange rate) as well as to a better assessment of its welfare costs. Such analysis can help policy makers of developing economies in designing optimal policies.

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Chapter 2

Optimal Exchange Rate Policy in a Partially Dollarized Small Open Economy

Luca M.F. Colantoni[1]
Bocconi University

Abstract

Dollarization is a widespread phenomenon and proves to be quite persistent since, because of hysteresis effects and habit persistence, the amount of foreign real money balances rarely falls to negligible levels even after a successful stabilization of the economy. Hence, it is important to assess the welfare costs associated with different policies when operating in a dual-currency environment. To such purpose we extend Felices and Tuesta (2006) by numerically solving the whole non linear model up to the second order approximation.

Keywords: dollarization, Small Open Economy, welfare analysis.

JEL codes: D58, E52, F31, F41.

[1] I wish to thank Tommaso Monacelli for very useful comments.

2.1 Introduction

When the environment of a country is uncertain due to changing property rights, political instability, large budget deficits, and high inflation, foreign currency may take on all or at least some of the functions of domestic money to hedge economic agents from these adversities. This is what happened in many emerging and transition economies especially during the eighties and in some cases during the nineties. Some Latin American Countries (LACs) and more recently Central and Eastern European Countries' (CEECs') are among those economies where the domestic currency has been partially replaced in its functions by a foreign currency, usually the US Dollar, a phenomenon known in the literature as *dollarization*.

The Federal Reserve and the Treasury have information on these subjects from various sources such as U.S. Customs reports, shipment data from overseas bank note wholesalers and published proxies for those shipments. Moreover, there are estimates based on in-country surveys from dollar–using countries, national surveys of domestic currency holdings, and a variety of empirical models developed by the Federal Reserve and others that estimate[2] overseas flows or holdings based on realistic assumptions concerning international currency usage. Table 1 shows some preliminary results from such estimates[3]. In the first column we show results from currency surveys conducted by the Treasury and Federal Reserve between 1997 and 2002. The other two columns report computations by Feige (2003) based on new data collected by United States Customs Service under the Currency and Foreign Transactions Reporting Act and data from a survey on 5 Europeans countries commissioned by the Austrian National Bank (ONB).

As expected, the dollarization degree tends to be higher in economies that have experienced high rates of inflation and/or exchange rate crisis, even when these occurred much earlier. Some of these countries officially dollarized their economies (e.g. Argentina and Ecuador), but an economy can be heavily dollarized even in absence of official dollarization (e.g. Russia, Ukraine).

Not only the phenomenon is widespread but it also proves to be quite persistent since, because of hysteresis effects and habit persistence[4], the amount of foreign real

[2] Various estimations of US currency in circulation outside the United States can be found in a series of papers and reports. See, for example, U.S. Treasury (2000, 2003), Judson and Porter (2001, 1996), Porter and Weinbach (1999), Porter (1993).

[3] Unfortunately, because of lack of data, the table does not include estimates for countries (particularly from Latin America) known to be dollarized economies.

[4] On this topic see among others Uribe (1997).

money balances rarely falls to negligible levels even after a successful stabilization of

Table 1: Estimates of Foreign Currency held as Cash

Country	GDP held in the form of U.S. currency (%) 1997-2002	Foregin Currency to Total Currency (%) 2001	$ Per Capita FCC 2001
Albania		14	46
Argentina	17.5		
Armenia		62	55
Azerbaijan		82	169
Belarus	5.8	34	17
Bulgaria	2.8	41	125
Cambodia	25.2		
Chile	0.4		
China,P.R.: Mainland	0.9		
China,P.R.:Hong Kong	1.2		
Croatia		35	117
Czech Republic		21	129
Dominica	3.9		
Ecuador	7.3		
Egypt	0.4		
El Salvador	7.5		
Estonia		59	414
Georgia		79	123
Hungary		6	25
Indonesia	0.3		
Kazakhstan		95	1024
Korea	2.3		
Kyrgyz Republic		48	20
Latvia	5.5	79	1209
Lithuania	3.6	11	25
Macedonia, FYR		5	5
Mexico	0.6		
Paraguay	0.6		
Peru	3.8		
Philippines	1.0		
Poland	0.4	27	93
Romania	0.8	55	61
Russia	10.0	87	903
Slovak Republic		28	123
Slovenia		54	329
South Africa	3.1		
Thailand	0.1		
Turkey	2.6		
Ukraine		64	131
Vietnam	2.7		

Sources. First Column: US Treasury Department (2003). Second and third column: Feige (2003)

the economy. It is of particular interest since extensive currency substitution not only makes domestic monetary and fiscal policies less effective, it also makes active exchange rate intervention more dangerous. In this respect, the adoption of new exchange rate regimes is a topic particularly crucial for those countries who wish to join the EU. In fact, accession countries are required not only to meet the Maastricht convergence criteria but also to participate in the ERM-II (Exchange Rate Arrangement between the Euro area and EU members outside the Euro area).

Hence, given the size and diffusion of the phenomenon, it can be interesting to understand what are the economic policy implications of currency substitution and assess the welfare costs associated with different policies when operating in a dual-currency environment.

The aim of this paper is to provide a starting point in such analysis by using recently developed instruments of macroeconomic modelling and welfare analysis. A recent contribution to welfare implications of dollarization can be found in Schmitt-Grohè and Uribe (2001). By means of an optimizing model of a small open economy calibrated to the Mexican economy, the authors compare the welfare costs of economic fluctuations under alternative monetary policies (*full dollarization* in the form of fixed exchange rate[5], inflation targeting, money growth rate pegs, or devaluation rate rules). They find that dollarization is the least successful of the monetary policy rules considered. However, strictly speaking their paper is not about *currency substitution* since only domestic currency is present in the model economy. This is not an omission of small account.

Instead, in this paper we consider a model with two currencies (domestic and foreign) in the economy. More precisely, we work on a model very similar to the one by Felices and Tuesta (2006). They show that transaction dollarization tends to add to the intrinsic volatility of inflation and output, requiring a more aggressive policy response by the monetary policy authority. Closely related is the work by Batini, Levine and Pearlman (2006) which asks whether it is possible to achieve an explicit inflation target when an economy is dollarized. However, such conclusions are drawn from a first order solution of a smaller (reduced to 3 equations) version of their model.

Differently from Felices and Tuesta (2006) we compute a numerical solution of the whole model up to the second order of approximation. This enables us to extend their analysis in two respects. Firstly, we provide a description of the behaviour (impulse responses and volatilities) of the whole economy under two sources of shocks, a shock to technology and a shock to foreign interest rate. Secondly, we perform welfare analysis, for which second order approximation is necessary.

As already pointed out by Felices and Tuesta (2006) in the standard model with

[5] Strictly speaking a peg to the dollar is not the same as full dollarization, the two environments differring in terms of who takes the seigniorage (either the domestic or the foreign monetary authority). But seigniorage is not the subject of their study neither of this work.

complete international markets, dollarization[6] does not play a role when consumption and liquidity services are separable. We show that, dollarization plays a marginal role even under non separability when, instead of considering a shock to foreign interest rate, we consider a shock to technology. Overall, impulse responses are larger the greater the degree of dollarization and macro volatility increases with it.

When coming to welfare implications, dollarization is generally welfare decreasing when consumption and money services are complements, while some degree of dollarization is preferred under substitutability. Finally, a fix exchange rate is preferred for all the considered degrees of currency dollarization under the substitutability case. On the contrary, a flexible exchange rate regime is welfare increasing when we consider complementarity and intermediate degrees of dollarization.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 contains the calibration while in section 4 and 5 we perform dynamics and welfare analysis, respectively. Section 6 concludes.

2.2 A Small Open Economy Model

We consider a small open economy (SOE) whose relative size is n while that of the rest of the world[7]. (ROW) is $(1-n)$ The SOE is composed of infinitely-lived individuals and of a continuum of firms whose shares are owned by the consumers. The distinctive feature of the SOE is that agents have the possibility to use foreign currency. Use of foreign money as a mean of savings (*asset substitution*) and a mean of transaction (*currency substitution*) can be justified for countries with high inflation and unstable economy, or for countries with incomplete financial sector, i.e. in developing or transition economies. We model the *dollarized* economy by allowing two monies in the utility function. Saving is possible by holding domestic/foreign money and bonds.

As for the production part, there are two types of home produced goods: final (X) and intermediate (Y). Final good is nontradable. Intermediate good is used as an input for home (X_H) and foreign (X_H^*) production. Intermediate goods are also produced abroad: imported intermediates are called X_F. The final good sector is perfectly competitive, while the intermediate sector is characterised by nominal rigidities in the form of monopolistic competition and adjustment costs à la Rotemberg (1982). Labor is the

[6] Thorughtout the paper the terms *currency substitution, transaction dollarization and dollariazition,* are used as synonimous.

[7] See for example Faia and Monacelli (2006).

only input in the intermediate sector, while intermediate inputs only are required for the production of final goods.

Finally, we assume that financial markets of our *SOE* are complete in the sense that state-contingent securities are available. More specifically, following Chari et al. (2002), in each period t our economy experiences one of the infinitely many states s_t with s_0 as the initial realization.

In the *ROW* representative household and firms face problems similar[8] to the ones of the small open economy. We assume that the size of the *SOE* is negligible relative to the *ROW* (i.e. $n \to 0$), which allows us to treat the latter as if it was a closed economy.

2.2.1 Households

Our small open economy model is inhabited by a representative household whose instantaneous utility function takes the form[9]

$$u\left(C_t, N_t, m_t, m_t^*\right) = U\left(C_t, m_t, m_t^*\right) - V\left(N_t\right) \tag{2.1}$$

where C_t is a consumption good, N_t denotes hours of labor, ε_t is the nominal exchange rate (the price of foreign currency in terms of home currency), $m_t \equiv \frac{M_t}{P_t}$ stands for real home currency holdings and $m_t^* \equiv \frac{\varepsilon_t M_t^*}{P_t}$ is real foreign currency holdings. Following part of the literature on currency substitution we introduce real money balances in the utility function[10]. Feenstra (1986) demonstrates a functional equivalence between using real balances as an argument of the utility function and entering money into liquidity costs which appear in the budget constraint. Moreover, as in Chari at al. (2002), Felices and Tuesta (2006) and Batini et al. (2006) the utility function is non separable in consumption and money services

$$U\left(C_t, m_t, m_t^*\right) = \frac{1}{1-\sigma}\left\{\left[bC_t^{\frac{\omega-1}{\omega}} + (1-b)\, H\left(m_t, m_t^*\right)^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}}\right\}^{1-\sigma} \tag{2.2}$$

this makes marginal utility of consumption depend on money holdings and, in the end, on interest rate. The assumption of non separability makes the economy to behave differently as its degree of dollarization changes. This gives a role to policy different from the standard one with implications for the optimal policy. In this respect, particularly

[8] Of course in the Rest of the World there is no *dollarization*. Hence, household's utility will depend only on consumption, money of the rest of the world only and leisure $u^*\left(C_t^*, N_t^*, m_t^*\right)$.

[9] Because of market completeness we can drop the household index (j).

[10] See among others Calvo (1985) and Imrohoroglu (1994).

relevant is the parameter ω which indicates whether liquidity services and consumption are complements or substitutes[11].

The representative household seeks to maximize

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[U\left(C_t, m_t, m_t^*\right) - V\left(N_t\right) \right] \right\} \tag{2.3}$$

where E_0 is the expectation operator conditional upon the information available at time 0 and $\beta \in (0,1)$ is the rate at which households discount future utility.

As in Imrohoroglu (1994) money services are produced by using a combination of domestic and foreign real balances in a constant elasticity of substitution (CES) production function

$$H(\cdot) = \left[\alpha \left(\frac{M_t}{P_t} \right)^{\frac{\chi-1}{\chi}} + (1-\alpha) \left(\frac{\varepsilon_t M_t^*}{P_t} \right)^{\frac{\chi-1}{\chi}} \right]^{\frac{1}{\chi-1}} \tag{2.4}$$

where $\alpha \in (0,1)$, and $\chi > 0$. This is a convenient functional form that separates the elasticity of currency substitution χ, from the share, $(1-\alpha)$, of foreign real balances in the production of domestic liquidity services[12].

Finally, we assume that markets are complete both domestically and internationally and that households have access to a complete contingent one period nominal bond denominated in home currency[13].

Hence, the representative household faces a sequence of budget constraints of the form

$$\begin{aligned} & P(s_t)\, C(s_t) + M(s_t) + \varepsilon(s_t)\, M^*(s_t) + \sum_{s_t} Q(s_{t+1}|s_t)\, B(s_{t+1}) \\ & \leqslant B(s_t) + M(s_{t-1}) + \varepsilon(s_t)\, M^*(s_{t-1}) + W(s_t)\, N(s_t) + D(s_t) + T_t \end{aligned} \tag{2.5}$$

$\forall t$, where $Q(s_{t+1}|s_t)$ is the price of the bond in units of home currency in state s_t and $D(s_t)$ is net-profit (net of taxes on revenues, distortionary taxation) from owned domestic intermediate firms and T_t transfers from government. The initial conditions $M(s_{-1})$, $M^*(s_{-1})$ and $B(s_0)$ are given.

The right hand side of the budget constraint gives the available resources as the sum of gross return on the bond holding, initial money holdings, labour income, profits from

[11] For a discussion of this point see Felices and Tuesta (2006).

[12] This specification delivers a steady state with positive foreign real money balances as long as $\alpha > 0$. In Colantoni and Kaminska (2004) we consider another possible specification for the $h(\cdot)$ function, namely, the one proposed by Obstfeld and Rogoff (1995). In this case steady state foreign money holdings are positive according to parameters measuring its costs and gains and the expected devaluation rate.

[13] Under international complete markets it does not matter the currency denomination of the bonds.

intermediates in tradables, less government taxation. These resources are used to cover consumption and to acquire the next period money balances and new bond holdings. Notice that $M(s_{t-1})$ denotes the quantity of nominal money balances acquired during period t and carried over into period $t+1$. All variables are expressed in units of domestic currency.

Control variables are total consumption C_t, domestic nominal money holdings M_t, foreign nominal money holdings M_t^*, working hours N_t, nominal bond holdings denominated in home currency B_t. Money does appear in both the budget constraint and the utility function, so that money holdings can affect the paths of consumption through the path of prices.

In what follows we assume that the disutility of labor takes the form

$$V(N_t) = \frac{N^{1+\sigma_2}}{1+\sigma_2} \tag{2.6}$$

Optimality Conditions

The household chooses the set of stochastic processes $\{C_t, M_t, M_t^*, N_t, B_t, B_t^*\}_{t=0}^{\infty}$ so to maximize (3.1) subject to (3.2) and some borrowing limit that prevents from engaging in Ponzi-type schemes, taking as given the sequences $\{P_t, \varepsilon_t, W_t, r_t\}$. The associated optimality conditions are

(*1*)→Euler equation

$$E_t\{Q(s_{t+1}|s_t)\} = \beta E_t\left\{\frac{U_C(C_{t+1})}{U_C(C_t)}\left(\frac{P_t}{P_{t+1}}\right)\right\} \tag{2.7}$$

optimality condition for the allocation of wealth among state contingent securities which gives the stochastic discount factor $Q(s_{t+1}|s_t)$. Equating (3.3) for each couple of households in the population delivers the risk sharing conditions implied by the assumption of complete markets.

(*2*)→Labor Supply in Intermediate Good Production, N_t; is given by a standard intratemporal optimality condition

$$V_N(N_t) = \frac{W_t}{P_t}U_C(C_t) \tag{2.8}$$

This equation is the labour-leisure trade-off condition that comes from utility maximization with respect to wages. It ensures that marginal disutility of the additional factor supply (due to leisure foregone) on the left hand side is compensated by an extra unit of marginal utility of consumption, such that an extra unit of labour supply can buy at the real factor price.

Since we consider two currencies, money demand equations differ from the standard ones

(*3*)→Domestic Money Demand

$$H_m(m_t) = U_C(C_t) - \beta E_t \left\{ U_C(C_{t+1}) \left(\frac{P_t}{P_{t+1}} \right) \right\} \tag{2.9}$$

where $H_m(m_t)$ is the marginal utility of domestic money holdings[14];

(*4*)→Foreign Money Demand

$$H_{m^*}(m_t^*) = U_C(C_t) - \beta E_t \left\{ U_C(C_{t+1}) \left(\frac{P_t}{P_{t+1}} \right) \left(\frac{\varepsilon_{t+1}}{\varepsilon_t} \right) \right\} \tag{2.10}$$

where $H_{m^*}(m_t^*)$ is the marginal utility of foreign money holdings[15].

The last optimality conditions are the budget constraint (3.2) and the no Ponzi game condition

$$\lim_{k \to \infty} E_t \left\{ Q(s_{t+k+1} | s_t) \left(M_{t+k} + B_{t+k} + \varepsilon_{t+k} M_{t+k}^* + \varepsilon_{t+k} B_{t+k}^* \right) \right\} = 0$$

A first order condition analogous to (3.3) holds for the rest of the world[16], i.e.

$$E_t \{ Q(s_{t+1} | s_t) \} = \beta E_t \left\{ \frac{U_{C^*}(C_{t+1}^*)}{U_{C^*}(C_t^*)} \left(\frac{P_t^*}{P_{t+1}^*} \right) \left(\frac{\varepsilon_t}{\varepsilon_{t+1}} \right) \right\} \tag{2.11}$$

Substituting for the bond price and iterating we get

$$\frac{U_C(C_t)}{U_C(C_0)} \left(\frac{P_0}{P_t} \right) = \frac{U_C^*(C_t^*)}{U_C^*(C_0^*)} \left(\frac{P_0^*}{P_t^*} \right) \left(\frac{\varepsilon_0}{\varepsilon_t} \right) \tag{2.12}$$

Defining the real exchange rate as

$$\mathcal{Q}_t \equiv \frac{\varepsilon_t P_t^*}{P_t} \tag{2.13}$$

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[14] $H_m(m_t) \equiv \alpha (m_t)^{-\frac{1}{\chi}} \left[\alpha (m_t)^{\frac{\chi-1}{\chi}} + (1-\alpha)(m_t^*)^{\frac{\chi-1}{\chi}} \right]^{\frac{1}{\chi-1}}$

[15] $H_{m^*}(m_t^*) \equiv (1-\alpha)(m_t^*)^{-\frac{1}{\chi}} \left[\alpha (m_t)^{\frac{\chi-1}{\chi}} + (1-\alpha)(m_t^*)^{\frac{\chi-1}{\chi}} \right]^{\frac{1}{\chi-1}}$

[16] Here we are implicity assuming the uncovered interest parity 2.18 which holds given the complete asset market structure.

we get the standard risk sharing condition for consumption

$$U_C(C_t) = \vartheta_0 U_C^*(C_t^*) \mathcal{Q}_t^{-1} \tag{2.14}$$

for all t, and where $\vartheta_0 = \left[\frac{U_C(C_0)}{U_C^*(C_0^*)}\mathcal{Q}_0^{-1}\right]$ is a constant depending on initial conditions. This equation delivers the relation between domestic and foreign consumption linked through the real exchange rate.

By no arbitrage assumption

$$R_t E_t \{Q(s_{t+1}|s_t)\} = 1 \tag{2.15}$$

where R_t is the gross return on a riskless one-period discount bond paying off one unit of domestic currency in $t+1$.

Using this, the (3.3) can be rewritten as the usual stochastic Euler equation

$$\beta R_t E_t \left\{ \frac{U_C(C_{t+1})}{U_C(C_t)} \left(\frac{P_t}{P_{t+1}} \right) \right\} = 1 \tag{2.16}$$

and, analogously in the rest of the world

$$\beta R_t^* E_t \left\{ \frac{U_{C^*}(C_{t+1}^*)}{U_{C^*}(C_t^*)} \left(\frac{P_t^*}{P_{t+1}^*} \right) \right\} = 1 \tag{2.17}$$

where, by the uncovered interest parity

$$R_t^* \left(\frac{\varepsilon_{t+1}}{\varepsilon_t} \right) = R_t \tag{2.18}$$

Combining (2.9) with (2.16) the demand for domestic money holdings becomes

$$H_m(m_t) = 1 - R_t^{-1} \tag{2.19}$$

and combining (2.10) with (2.17) the demand for foreign money holdings becomes

$$H_{m^*}(m_t^*) = 1 - (R_t^*)^{-1} \tag{2.20}$$

Using these last two equations we can obtain the relative demand of foreign currency with respect to domestic currency

$$RF_t \equiv \frac{m_t^*}{m_t} = \left[\left(\frac{\alpha}{1-\alpha} \right) \left(\frac{R_t}{R_t - 1} \right) \left(\frac{R_t^* - 1}{R_t^*} \right) \right]^{-\chi} \tag{2.21}$$

with $\frac{\partial RF_t}{\partial \alpha} < 0$ and $\frac{\partial RF_t}{\partial R_t} > 0$. Note that when $R = R^*$ the previous ratio is constant and its value depends on α and χ. The degree of dollarization of the economy can be measured by the following dollarization index:

$$DI_t \equiv \frac{m_t^*}{m_t^* + m_t} = \left\{ \left[\left(\frac{\alpha}{1-\alpha} \right) \left(\frac{R_t}{R_t - 1} \right) \left(\frac{R_t^* - 1}{R_t^*} \right) \right]^{\chi} + 1 \right\}^{-1} \tag{2.22}$$

with $\frac{\partial DI_t}{\partial \alpha} < 0$ and $\frac{\partial DI_t}{\partial R_t} > 0$. It is important to note that, depending on R , the degree of dollarization of the economy is affected by monetary policy. More precisely, as the opportunity cost of holding domestic currency, R, increases the demand of domestic money holdings decreases and so the degree of dollarization of the economy increases. On the contrary, DI decreases with the preference for domestic currency (i.e. as α increases). Finally, as we will see, in a steady state were $R_t = R_t^*$ it will depend only on parameters α and χ.

2.2.2 Firms

For the supply side we adopt a structure similar to the one in Romer (1990). There is a final good sector which is perfectly competitive, while the tradeable intermediate good is characterized by monopolistic competition.

Final goods sector

Because the production function is homogeneous of degree one, final output can be described in terms of the actions of a single, aggregate, price-taking firm. The firms are perfectly competitive and produce final goods from intermediate goods according to the following CES aggregator (Dixit and Stiglitz, 1977)

$$X_t = \left[\gamma^{\frac{1}{\rho}} \left[X_{H,t} \right]^{\frac{\rho-1}{\rho}} + (1-\gamma)^{\frac{1}{\rho}} \left[X_{F,t} \right]^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \tag{2.23}$$

where $X_{H,t}$ and $X_{F,t}$ are intermediate goods produced at home and abroad (imported) respectively. Parameter ρ will determine the elasticity of substitution between home and foreign goods, and γ together with ρ will determine the ratio of imports to output. γ is a function of the relative size, n, of our economy with respect to the rest of the world and the degree of openness λ

$$\gamma \equiv 1 - (1-n)\,\lambda \tag{2.24}$$

Note that the assumption of small economy implies $n \to 0$ and so $\gamma \to 1 - \lambda$.

In turn, each basket of intermediate goods is composed of a continuum of different varieties indexed by j and l respectively. The corresponding *home intermediate good index* and *foreign intermediate good index* are given accordingly as

$$X_{H,t} = \left[\left(\frac{1}{n}\right)^{\frac{1}{\phi}} \int_0^n X_{H,t}(j)^{\frac{\phi-1}{\phi}} dj\right]^{\frac{\phi}{\phi-1}}, \qquad X_{F,t} = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\phi}} \int_n^1 X_{F,t}(l)^{\frac{\phi-1}{\phi}} dl\right]^{\frac{\phi}{\phi-1}} \tag{2.25}$$

where the parameter ϕ will determine the mark-up price over the marginal cost.

We design the final good producer's problem using the budget separation method.

1) Inter-input allocation. The firms choose inputs quantities X_t, $X_{H,t}$ and $X_{F,t}$ to solve the following profit maximization problem

$$max\left[P_t \cdot X_t - \int_0^1 P_{H,t}(j)X_{H,t}(j)dj - \int_0^1 P_{F,t}(l)X_{F,t}(l)dl\right] \tag{2.26}$$

subject to (3.5) and where P_t is price index taken as given (because of perfect competition assumption)

$$P_t = \left\{\gamma\,[P_{H,t}]^{1-\rho} + (1-\gamma)\,[P_{F,t}]^{1-\rho}\right\}^{\frac{1}{1-\rho}} \tag{2.27}$$

where the index of prices domestically produced intermediate good and the index for imported intermediate goods[17] are

$$P_{H,t} = \left\{\frac{1}{n}\int_0^n [P_{H,t}(j)]^{1-\phi} dj\right\}^{\frac{1}{1-\phi}}, \qquad P_{F,t} = \left\{\frac{1}{1-n}\int_n^1 [P_{F,t}(l)]^{1-\phi} dl\right\}^{\frac{1}{1-\phi}} \tag{2.28}$$

Solving the problem in (2.26) we get the demand of intermediate baskets

$$X_{H,t} = \gamma\left(\frac{P_{H,t}}{P_t}\right)^{-\rho} X_t, \qquad X_{F,t} = (1-\gamma)\left(\frac{P_{F,t}}{P_t}\right)^{-\rho} X_t \tag{2.29}$$

[17] Expressed in domestic currency.

2) Intra-basket allocation. Proceeding as in the previous step, the cost minimization gives the following intra-basket demands

- home demand for domestic intermediates

$$X_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\phi} X_{H,t} \tag{2.30}$$

- home demand for imports

$$X_{F,t}(l) = \left(\frac{P_{F,t}(l)}{P_{F,t}}\right)^{-\phi} X_{F,t} \tag{2.31}$$

Foreign sector

In the rest of the world, indexed by F, a representative household, final and intermediate goods firms, face problem identical to the ones outlined above. Allocations and prices are denoted with an asterisk.

Thus, the final good production is

$$X_t^* = \left[(\gamma^*)^{\frac{1}{\rho}} \left[X_{H,t}^*\right]^{\frac{\rho-1}{\rho}} + (1-\gamma^*)^{\frac{1}{\rho}} \left[X_{F,t}^*\right]^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \tag{2.32}$$

with $\gamma^* \equiv n\lambda$ and where the parameters and variables have an interpretation similar to the previous one. The subscripts H and F indicate that the intermediate good is produced in the small open economy and in the rest of the world, respectively. Note that as $n \to 0$ the SOE intermediate good does not enter in the production of the final good of the rest of the world[18].

Following the same lines as above, optimality conditions yield the following demands for intermediate baskets by the rest of the world

$$X_{H,t}^* = (1-\gamma^*)\left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\rho} X_t^*, \qquad X_{F,t}^* = \gamma^* \left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\rho} X_t^* \tag{2.35}$$

And so

[18] By the same argument note that the foreign price index

$$P_t^* = \left\{\gamma^* \left[P_{H,t}^*\right]^{1-\rho} + (1-\gamma^*)\left[P_{F,t}^*\right]^{1-\rho}\right\}^{\frac{1}{1-\rho}} \tag{2.33}$$

as $n \to 0$ becomes

$$P_t^* = P_{F,t}^* \tag{2.34}$$

- F demand for the SOE produced intermediate good (exports):

$$X^*_{H,t}(j) = \left(\frac{P^*_{H,t}(j)}{P^*_{H,t}} \right)^{-\phi} X^*_{H,t} \tag{2.36}$$

- F demand for the intermediate good produced in the rest of the world

$$X^*_{F,t}(l) = \left(\frac{P^*_{F,t}(l)}{P^*_{F,t}} \right)^{-\phi} X^*_{F,t} \tag{2.37}$$

Intermediate goods sector

The market is populated by a continuum of firms acting as monopolistic competitors, since intermediate goods substitute imperfectly for one another as inputs to producing the final good. During period t, the representative intermediate goods-producing firm hires $N_t(j)$ units of labor, in order to produce $Y_t(j)$ units of intermediate good according to the production function given by:

$$Y_{H,t}(j) = A_t N_t(j) \tag{2.38}$$

where A_t is a technology shifter common to all firms and $a_t \equiv \log A_t$ follows the AR(1) process $a_t = \rho_a a_{t-1} + v_{at}$.

During each period t, the representative intermediate goods-producing firm sets a nominal price $P_{H,t}(j)$, subject to requirement that it satisfies (2.63).

The existence of an economy-wide competitive factor market implies that all firms will pay the same rental rate r_t and the same nominal wage W_t. This also implies that all firms face a common nominal marginal cost (in particular, independent of the level of individual output) that we denote by

$$MC^n_t = (1 - \tau) \frac{W_t}{A_t} \tag{2.39}$$

where τ is an employment subsidy introduced to offset the static distortion due to the monopolistic competition in the intermediate goods market[19].

Differently from Felices and Tuesta (2006) where Calvo pricing is assumed, we consider adjustment costs à la Rotemberg (1982), given by[20]

[19] See Galì and Monacelli (2005) for a discussion on this.

[20] This form is mutuated from Ireland (2004). We assume the same adjustment cost for goods sold domestically and goods exported.

$$AC_{P,t}\left(j\right) \equiv \frac{\varphi_P}{2}\left[\frac{1}{\pi_H}\frac{P_{H,t}(j)}{P_{H,t-1}(j)} - 1\right]^2 Y_t(j) \tag{2.40}$$

where π_H is the gross steady state rate of inflation in the intermediate sector. Note that if the price adjustment cost parameter $\varphi_P = 0$ the model collapses to a flexible price specification. Also note that in steady state the price adjustment costs are equal to zero.

The cost of price adjustment makes the firm's problem dynamic. Assuming no price discrimination, each firm chooses price $P_{H,t}(j)$ and outputs $X_{H,t}(j), X^*_{H,t}(j)$ in order to maximize the expected discounted value of profits, i.e.

$$\max_{\bar{P}_{H,t}(j)} E_t \sum_{k=0}^{\infty} \varphi^k Q_{t,t+k} D_{t+k}(j) \tag{2.41}$$

where

$$D_t(j) = P_{H,t}(j)Y_{H,t}(j) - W_t N_t\left(j\right) - P_{H,t}AC_{P,t}(j)$$

subject to production technology (3.25) and final sector demand (2.63).

The first term is sales net of revenues while the last two ones are production costs and adjustment costs, respectively. Since firms are assumed to be owned by the representative household, they value future payoffs according to the household's intertemporal marginal rate of substitution in consumption and so the pricing kernel used to value random date $t+n$ payoffs is

$$Q_{t,t+k} = \beta^k \frac{U_C\left(C_{t+k}\right)}{U_C\left(C_t\right)}\left(\frac{P_t}{P_{t+k}}\right) \tag{2.42}$$

Assuming a symmetric equilibrium, where all firms are identical

$$X_{H,t}\left(j\right) = X_{H,t} \qquad Y_t\left(j\right) = Y_t \qquad N_t\left(j\right) = N_t \qquad P_{H,t}\left(j\right) = P_{H,t} \tag{2.43}$$

The optimization problem implies the following pricing behaviour[21]

$$\begin{aligned}(\phi-1)\frac{Y_t(j)}{P_t} \quad = \quad & \left\{\phi MC_t^n \frac{Y_t(j)}{P_{H,t}(j)P_t} - \varphi_P\left[\frac{1}{\pi_H}\frac{P_{H,t}(j)}{P_{H,t-1}(j)} - 1\right]\left[\frac{Y_t(j)}{P_{H,t-1}(j)\pi_H}\right]\right\} \\ & + \beta\varphi_P E_t\left\{\frac{U_C\left(C_{t+1}\right)}{U_C\left(C_t\right)}\left[\frac{1}{\pi_H}\frac{P_{H,t+1}(j)}{P_{H,t}(j)} - 1\right]\left[\frac{Y_{t+1}(j)}{P_{H,t}(j)\pi_H}\frac{P_{H,t+1}(j)}{P_{H,t}(j)}\right]\right\}\end{aligned} \tag{2.44}$$

[21] See the appendix for the complete derivation.

which can be rewritten as

$$(\phi-1)\,\tilde{P}_{H,t}=\phi MC_t-\varphi_P\left(\frac{\pi_{H,t}}{\pi_H}-1\right)\frac{\pi_{H,t}}{\pi_H}+\beta\varphi_P E_t\left\{\frac{C_{t+1}^{-\sigma_1}}{C_t^{-\sigma_1}}\left(\frac{\pi_{H,t+1}}{\pi_H}-1\right)\frac{\pi_{H,t+1}}{\pi_H}\frac{Y_{t+1}}{Y_t}\right\} \tag{2.45}$$

with $MC_t=\frac{MC_t^n}{P_t}=(1-\tau)\,\frac{W_t}{A_tP_t}$ and where we dropped index j.

As usual if $\varphi_P=0$ (i.e. no price adjustment costs) the above pricing condition boils down to:

$$P_{H,t}(j)=\frac{\phi}{\phi-1}MC_t^n=\mu MC_t^n \tag{2.46}$$

where $\mu=\frac{\phi}{\phi-1}$ denotes the desired (constant) markup value. Hence a representative firm chooses the price for its differentiated product as a constant markup over the marginal cost. This stems from the imperfect competition feature of the market. In fact, as $\phi\to\infty$ in the case of perfectly competitive output markets, $P_{H,t}=MC_t^n$, which is the usual pricing condition of a firm acting as a price taker.

Hence, in presence of price adjustment costs, price-setting will deviate from the simple markup rule by some additional terms: the resource cost of setting a price and a forward looking component reflecting the price that if the firm expects the need to change prices further in the next period, it will tend to change the price more today so to minimize future adjustment costs[22].

2.2.3 Terms of trade and some identities

We define terms of trade as

$$S_t\equiv\frac{P_{F,t}}{P_{H,t}} \tag{2.47}$$

then relative price indices can be rewritten as

$$\tilde{P}_{H,t}\equiv\frac{P_{H,t}}{P_t}=\left\{\gamma+(1-\gamma)\,S_t^{1-\rho}\right\}^{\frac{1}{\rho-1}}\equiv g(S_t),\qquad \frac{P_{F,t}}{P_t}=\left\{\gamma S_t^{\rho-1}+(1-\gamma)\right\}^{\frac{1}{\rho-1}}=S_tg(S_t) \tag{2.48}$$

and, from the definition of *CPI* (eq. 2.27), we get the following relation

$$\pi_t^{1-\rho}=\gamma\left[\pi_{H,t}g(S_{t-1})\right]^{1-\rho}+(1-\gamma)\left[\pi_{F,t}g(S_{t-1})\right]^{1-\rho} \tag{2.49}$$

[22] Rotemberg pricing is, by now, quite common in the literature. Alternative means to introduce price stickiness are the Calvo (1983) and Yun (1996) pricing models.

or, in other terms

$$\pi_t = \frac{\left\{(1-\lambda) + \lambda S_t^{1-\rho}\right\}^{\frac{1}{1-\rho}}}{\left\{(1-\lambda) + \lambda S_{t-1}^{1-\rho}\right\}^{\frac{1}{1-\rho}}} \pi_{H,t} \tag{2.50}$$

Movements in the terms of trade reflect movements in relative prices and, hence, imply demand shifts. In fact, relative demands of intermediate can be expressed as

$$\frac{X_{H,t}}{X_t} = \gamma \left(\left\{ \gamma + (1-\gamma) S_t^{1-\rho} \right\}^{\frac{1}{1-\rho}} \right)^{\rho} = \gamma \left[g(S_t) \right]^{-\rho} \tag{2.51}$$

$$\frac{X_{F,t}}{X_t} = (1-\gamma) \left(\left\{ \gamma S_t^{\rho-1} + (1-\gamma) \right\}^{\frac{1}{1-\rho}} \right)^{\rho} = (1-\gamma) \left[S_t g(S_t) \right]^{-\rho} \tag{2.52}$$

In addition, we assume that there are no barriers to trade such that the *law of one price* holds for each good at all times, implying that the prices of importables and exportables, $P_{F,t}(l)$ and $P_{H,t}(j)$, are linked to the respective world prices, $P_{F,t}^*(l)$ and $P_{H,t}^*(j)$, by the relationships

$$P_{F,t}(l) = \varepsilon_t P_{F,t}^*(l) \qquad \forall l \qquad \text{and} \qquad P_{H,t}(j) = \varepsilon_t P_{H,t}^*(j) \qquad \forall j \tag{2.53}$$

where $P_{F,t}^*(l)$ is the price of foreign good denominated in foreign currency. Integrating over all goods we obtain

$$P_{F,t} = \varepsilon_t P_{F,t}^* \qquad \text{and} \qquad P_{H,t} = \varepsilon_t P_{H,t}^* \tag{2.54}$$

Moreover, since the goods produced in the SOE represent a negligible fraction of the world's consumption basket, we can consider the rest of the world is an approximately closed economy with

$$P_t^* = P_{F,t}^*, \qquad \pi_t^* = \pi_{F,t}^* \tag{2.55}$$

Using the previous result , the law of one price conditions 3.44 and the terms of trade definition 3.37, the real exchange rate can be rewritten as

$$\mathcal{Q}_t \equiv \frac{\varepsilon_t P_t^*}{P_t} = \frac{\varepsilon_t P_{F,t}^*}{P_t} = \frac{P_{F,t}}{P_t} = \frac{P_{H,t}}{P_t} S_t \tag{2.56}$$

2.2.4 Monetary Policy

We will consider two different monetary regimes. In order to compare the welfare effects of exchange rate variability we assume the following open economy version of the Taylor rule[23]

$$\frac{1+i_t}{1+i} = \left(\frac{1+\pi_t}{1+\pi}\right)^{\omega_\pi} \left(\frac{\varepsilon_t}{\varepsilon}\right)^{\omega_\varepsilon} \tag{2.57}$$

where $\omega_\pi \geq$ and $\omega_\varepsilon \in [0,1]$ are the feedback coefficients to inflation and exchange rate, respectively, and i, π and ε are the steady state values of interest rate, inflation and exchange rate. This rule permits a *fixed exchange rate* regime for $\omega_\varepsilon \to \infty$, or alternatively a *flexible exchange rate* regime for $\omega_\varepsilon = 0$. Moreover, it allows to consider the trade off between the objectives of inflation and exchange rate stabilization imposed by EU accession criteria.

It is assumed that the monetary authority can commit to set this parameter at a time invariant value[24]. Finally, policies are specified in such a way that they give rise to the same nonstochastic steady state.

Being interested in monetary policy, for simplicity we assume the following government's budget constraint

$$M_t = M_{t-1} + T_t \tag{2.58}$$

The assumed fiscal policy implies that the government rebates seigniorage revenues to the public through lump-sum transfers. Note that in presence of currency substitution such revenues are smaller[25].

2.2.5 Market Clearing and Equilibrium

World consumption and output

As already pointed out, we assume that in the ROW representative household and firm face a problem similar to the one of the small open economy. However, by the assumption of small economy ($n \to 0$) imported goods do not enter neither in the utility function nor in the production function.

Hence[26], combining 2.29, 2.31, 2.35 and 2.37 we get total demand of foreign produced

[23] See Natalucci and Ravenna (2002). Felices and Tuesta (2006) use a Taylor rule which targets $\pi_{H,t}$ the inflation of domestic intermediate good.

[24] This because we are just interested in comparing steady states under different economic policies and not in characterizing an optimal policy.

[25] In this paper we do not adress the dollarization's implications for seignorage revenue. On this issue see, for example, Schmitt-Grohé and Uribe (1999).

[26] Recall that as $n \to 0$ we have that $\gamma^* \to 0$ and $P^*_t = P^*_{F,t}$.

intermediate good

$$\begin{aligned} X^d_{F,t}(l) &= X_{F,t}(l) + X^*_{F,t}(l) \\ &= \left(\frac{P_{F,t}(l)}{P_{F,t}}\right)^{-\phi}\left(\frac{P_{F,t}}{P_t}\right)^{-\rho}\gamma^* X_t + \left(\frac{P^*_{F,t}(l)}{P^*_{F,t}}\right)^{-\phi}\left(\frac{P^*_{F,t}}{P^*_t}\right)^{-\rho}(1-\gamma^*)X^*_t \\ &= \left(\frac{P^*_{F,t}(l)}{P^*_{F,t}}\right)^{-\phi} X^*_t \end{aligned} \tag{2.59}$$

where, we used 3.43 and 3.44.

By market clearing

$$Y^*_t(l) = X^d_{F,t}(l) = \left(\frac{P^*_{F,t}(l)}{P^*_{F,t}}\right)^{-\phi} X^*_t \tag{2.60}$$

Using the market clearing condition $X^*_t = C^*_t$ into the Euler equation 2.17 yields

$$\beta R^*_t E_t \left\{ \frac{U_{C^*}\left(X^*_{t+1}\right)}{U_{C^*}\left(X^*_t\right)} \left(\frac{P^*_t}{P^*_{t+1}}\right) \right\} = 1 \tag{2.61}$$

This is the so-called *new IS equation* for the rest of the world and is the same as the usual result of closed economy.

Small Open Economy

In this section we describe the demand side of our small open economy. By the same line of reasoning, we obtain total demand of intermediate goods produced in the SOE as

$$\begin{aligned} X^d_{H,t}(j) &= X_{H,t}(j) + X^*_{H,t}(j) \\ &= \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\phi}\left(\frac{P_{H,t}}{P_t}\right)^{-\rho}\gamma X_t + \left(\frac{P^*_{H,t}(j)}{P^*_{H,t}}\right)^{-\phi}\left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\rho}(1-\gamma)X^*_t \end{aligned} \tag{2.62}$$

where we combined (2.29), (2.30), (2.35) and (2.36). Using (3.43), (3.44) and the small open economy assumption[27]

$$X_{H,t}^{d}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\phi} \left(\frac{P_{H,t}}{P_t}\right)^{-\rho} \left[(1-\lambda) X_t + (\mathcal{Q}_t)^{\rho} \lambda X_t^{*}\right] \tag{2.63}$$

with the real exchange rate defined as in (2.13).

For simplicity define

$$\bar{X}_{H,t} = \left(\frac{P_{H,t}}{P_t}\right)^{-\rho} \left[(1-\lambda) X_t + (\mathcal{Q}_t)^{\rho} \lambda X_t^{*}\right] \tag{2.64}$$

which depends on total final goods both produced at home and produced abroad. Note the presence of the real exchange rate and rest of the world consumption due to the fact we deal with an open economy.

By market clearing

$$Y_t(j) = X_{H,t}^{d}(j) \tag{2.65}$$

We now turn to the description of a symmetric equilibrium with an initial level of net foreign assets equal to zero, $B_0 = 0$. In the symmetric equilibrium, all firms behave identically and all households behave identically, therefore, one can work with a single representative household and a single representative firm. We can drop the index notation in future references so that in the symmetric equilibrium we have:

$$X_{H,t}(j) = X_{H,t} \quad Y_t(j) = Y_t \quad N_t(j) = N_t \quad P_{H,t}(j) = P_{H,t} \quad D_t(j) = D_t \tag{2.66}$$

Hence

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\rho} \left[(1-\lambda) X_t + (\mathcal{Q}_t)^{\rho} \lambda X_t^{*}\right] \tag{2.67}$$

which, by market clearing $X_t = C_t$, $X_t^{*} = C_t^{*}$ and the definition of the terms of trade 3.37 becomes

$$Y_t = S_t^{\rho} \left[(1-\lambda) \mathcal{Q}_t^{-\rho} C_t + \lambda C_t^{*}\right] \tag{2.68}$$

Summarizing, the demand side of the economy is described by the risk sharing condition (2.14), the consumption Euler equation (2.16), the demand of intermediate good (2.68), money demands (2.9 and 2.10), the uncovered interest parity (2.18) and the

[27] Recall that as $n \to 0$ we have that $\gamma \to (1-\lambda)$ and $P_t^{*} = P_{F,t}^{*}$.

market clearing condition of final good

$$X_t = C_t + \tilde{P}_{H,t} \frac{\varphi_p}{2} \left(\frac{\pi_{H,t}}{\pi_H} - 1 \right) Y_t \tag{2.69}$$

together with the equilibrium conditions of intermediate goods:

$$X_{H,t} = (1-\lambda) X_t \left\{ (1-\lambda) + \lambda S_t^{1-\rho} \right\}^{\frac{\rho}{1-\rho}} \tag{2.70}$$

$$X_{F,t} = \lambda X_t \left\{ (1-\lambda) + \lambda S_t^{1-\rho} \right\}^{\frac{\rho}{1-\rho}} \tag{2.71}$$

The supply side is summarized by intermediate production function (3.25), the labour Euler equation (2.8) and the Phillips curve (2.45).

Policy rules are those described before (2.57 and 3.36).

A stationary rational expectation equilibrium is a set of stationary stochastic processes $\{C_t, N_t, m_t, m_t^*, \varepsilon_t, i_t, w_t, X_t, Y_t, X_{H,t}, X_{F,t}, S_t, \mathcal{Q}_t, \pi_t, \pi_{H,t}\}_{t=0}^{\infty}$ satisfying previous equations together with prices relations (3.38) and (3.40), given exogenous processes $\{i_t^*, A_t\}_{t=0}^{\infty}$ and initial values for ϑ_0, F_0, S_0. Since the equilibrium of the model is a highly nonlinear system of equations, it is not possible to obtain a solution in its closed form. Thus, the model is solved numerically[28].

2.3 Calibration

We consider a domestic shock, namely a shock to technology, and a foreign shock, namely a shock to foreign interest rate. We use the Effective Federal Funds Rate[29] as proxy of the foreign interest rate and, by using quarterly data over a period ranging from 1954q3 to 2006q3, we fit an AR(1) process to it in order to calibrate its shock. As for the technology shock we calibrate it at standard values[30]. The two shocks are distributed as follow

$$\begin{gathered}
\left[\log(A_t) - \log(\overline{A})\right] = \rho_a \left[\log(A_{t-1}) - \log(\overline{A})\right] + e_{at} \\
i_t^* = \rho_i i_{t-1}^* + e_{i^*,t} \\
e_{at} \sim N(0, 0.01^2) \qquad e_{i^*,t} \sim N(0, 0.03^2) \\
\rho_a = 0.9 \qquad \rho_i = 0.96
\end{gathered} \tag{2.72}$$

The rest of the parameters are calibrated as follows[31].

[28] By means of *Dynare*. See Collard and Juillard (2000), Juillard (2004).
[29] Available at the St. Louis Fed, http://research.stlouisfed.org/fred2/
[30] See Bergin and Tchakarov (2004), for example.
[31] For comparison purpouses most of the parameterization is taken from Felices and Tuesta (2006).

2.3.1 Preferences

The discount factor, β, is set equal to 0.99 and we interpret a period as one quarter. In order to isolate the effects of dollarization elasticity consumption (σ_1) and of labour supply (σ_2) are both set equal to 1. Particularly interesting for us are the money demand parameters: α, χ and ω. Following Felices and Tuesta (2006) the elasticity of substitution between the two currencies (χ) is set equal to 4.1 and the weight (b) of consumption in the utility function is set equal to 0.83. A key parameter is ω which tells whether consumption and liquidity services are complements $(\omega < 1)$ or substitutes $(w > 1)$. For parameter α we use various values so to consider economies with different degrees of dollarization, ranging from *low dollarization* $(DI = 5\%)$ to *high dollarization* $(DI = 95\%)$.

2.3.2 Technology

Following Bergin and Tchakarov (2004) the price adjustment cost, φ_P, is set at 50.

The elasticity of substitution, ρ, between imported intermediate good, X_F, and domestic intermediate good, X_H, is set equal to 1. As already noted, parameter γ, the share of domestic intermediate good, X_H, in the production of final output can serve as a proxy for the openness of the economy. Hence, it describes the level of a small open economy's dependence on the rest of the world. It is set equal to 0.36. The degree of monopolistic competition, ϕ, is set at 7.66 implying a markup of 15%.

2.3.3 Exchange Rate Policy

We set the feedback coefficient to CPI inflation to the value $\omega_\pi = 1.5$ while we let vary $\omega_\varepsilon \in [0, \infty]$ in order to consider different exchange rate regimes ranging from *flexible exchange rate regime* $(\omega_\varepsilon = 0)$ to *fixed exchange rate regime* $(\omega_\varepsilon \to \infty)$.

2.4 Simulations

In this section we perform some sensitivity analysis and stochastic simulations in order to assess the behaviour of the dollarized economy. To such purpose, for given ω and χ, we vary α. More specifically, we let α assume the following values: (1) $\alpha = 0.6722$ which implies a *low degree of dollarization* $(DI = 5\%)$; (2) $\alpha = 0.5$ which implies a *medium degree of dollarization* $(DI = 50\%)$; (3) $\alpha = 0.3278$ which implies a *high dollarization* $(DI = 95\%)$.

This exercise is carried out under the following Taylor rule

$$\frac{1+i_t}{1+i} = \left(\frac{1+\pi_{H,t}}{1+\pi}\right)^{\omega_\pi} \tag{2.73}$$

with $\omega_\pi = 1.5$ and where the central bank targets domestic intermediate inflation[32].

2.4.1 The Non Separability Case

As expected, dynamics differ as the degree of dollarization changes[33] and this difference is greater when considering a shock to foreign interest rate.

The fact that consumption and liquidity services are non separable implies the presence of another channel of transmission of shocks. Now, a change in money services implies a change in the marginal utility of consumption thus affecting consumption choice. Changes in money demands occurs when there is a change in their prices and so this additional channel is mainly triggered by the foreign interest rate shock. It is less relevant when considering a technology shock since it affects money services only indirectly.

In general, the greater the degree of dollarization the greater the response.

Complementarity $(0 < \omega < 1)$

Under a shock to foreign interest rate a lower degree of dollarization is associated with a lower negative response in consumption. In fact a positive shock to foreign interest rate lowers foreign currency in the domestic economy and also consumption, being money and consumption complements.

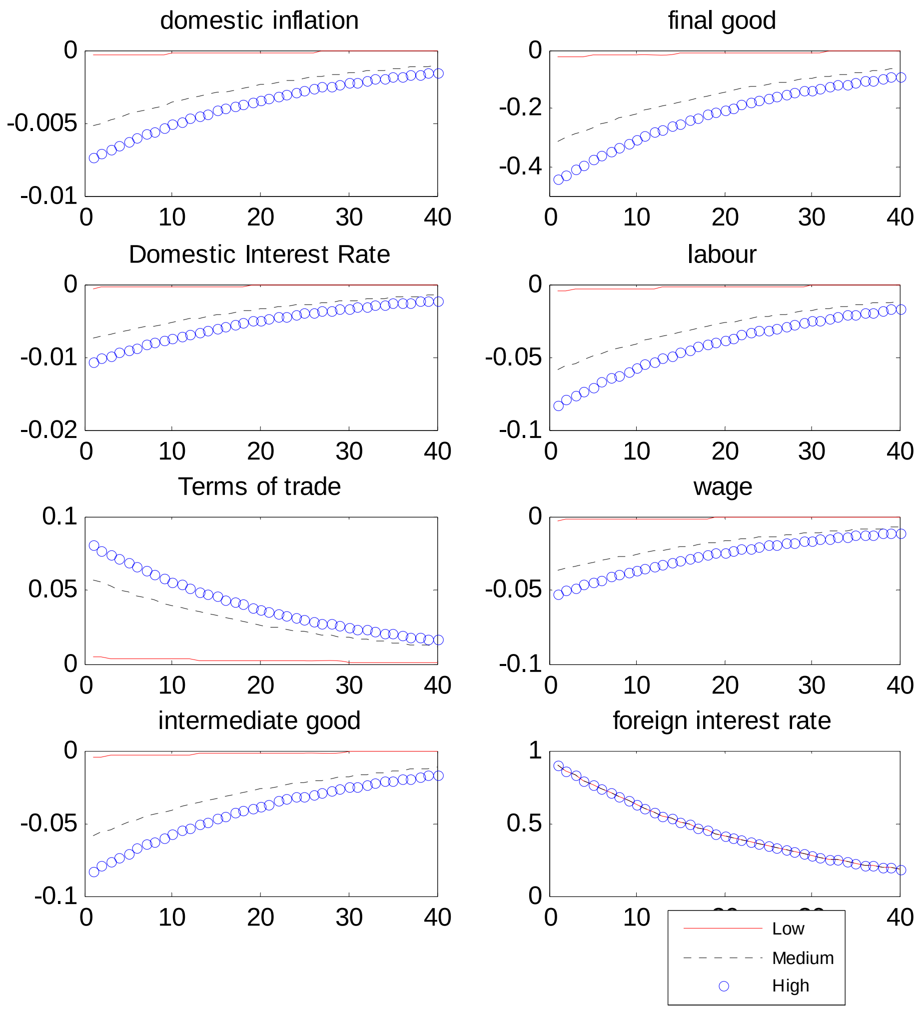
When considering a shock to technology the additional channel works indirectly through the domestic interest rate: an increase in productivity reduces marginal cost and domestic inflation thus triggering a decrease in domestic interest rate, an increase in demand of domestic currency and an increase in consumption.

As for labour, it decreases under both shocks.

[32] The same excercise has been performed for CPI targeting with similar results.

[33] The same exercise has been performed considering the case when consumption and liquidity services are separable. As already pointed out by Felices and Tuesta (2006), economies with different degrees of dollarization have same dynamics. In the proposed set up, it seems that non separability between consumption and liquidity services is necessary in order for dollarization to play a role.

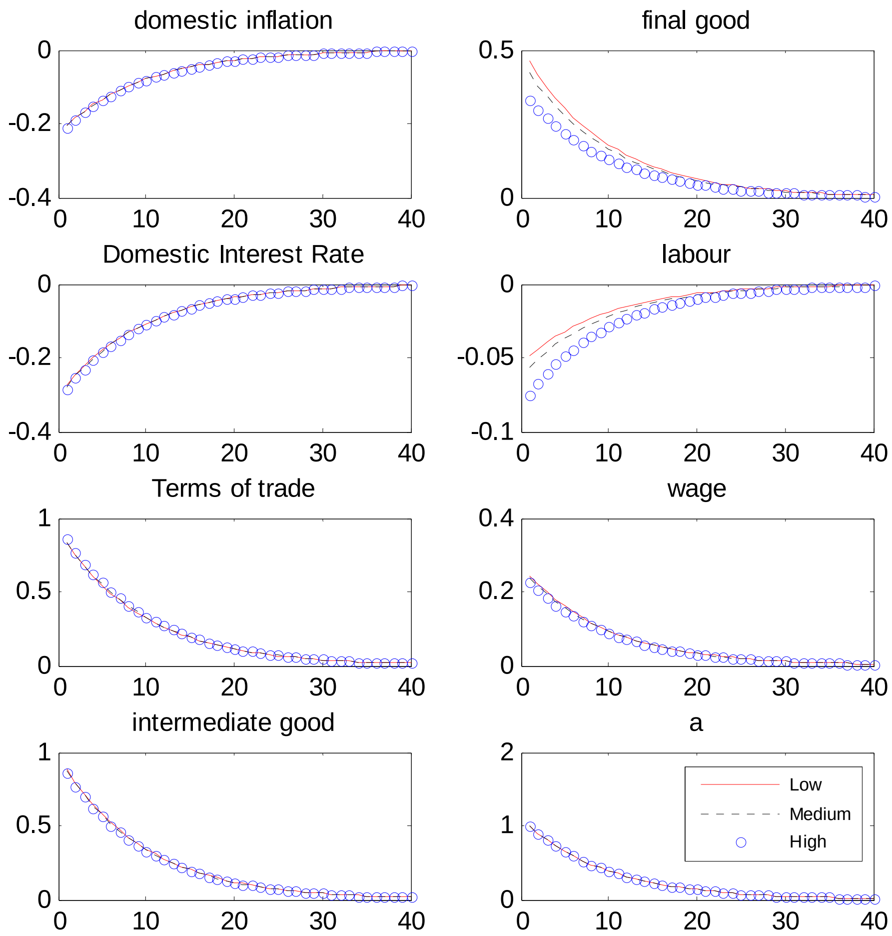
Shock to Foreign Interest Rate



Selected IRs to a Shock to Foreign Interest Rate - complements

Having increased its opportunity cost, namely the foreign interest rate, foreign currency demand decreases (2.10). Being complements, the decrease in foreign currency lowers marginal utility of consumption and so by the euler equation consumption decreases. Then, by market clearing (3.49) final good demand is reduced and so are domestic demand of intermediate good (3.8) and imports of the intermediate good (3.9). By market clearing (2.68) since the demand of domestic intermediate good has decreased, intermediate output decreases and by production function (3.25) labour decreases as well. By the labour Euler equation (2.8), with consumption and labour decreasing wages decrease as well. Hence, marginal cost decreases (2.39) and so does the price of the domestic intermediate (2.45). By the Taylor rule (3.54) domestic interest rate decreases. Finally, by the risk sharing condition (2.14) having marginal utility of consumption decreased, the real exchange rate and the terms of trade increase.

Positive shock to technology



Selected IRs to a Shock to Technology - complements

By the production function (3.25), intermediate good increases. The increase in technology reduces the marginal cost as well (2.39) and so the price of domestic intermediate decreases (2.45). Then, by the Taylor rule (3.54) domestic interest rate decreases leading to a lower real interest rate and to a greater consumption (2.16). Then, by market clearing (3.49) final good demand increases and so do domestic demand of intermediate good

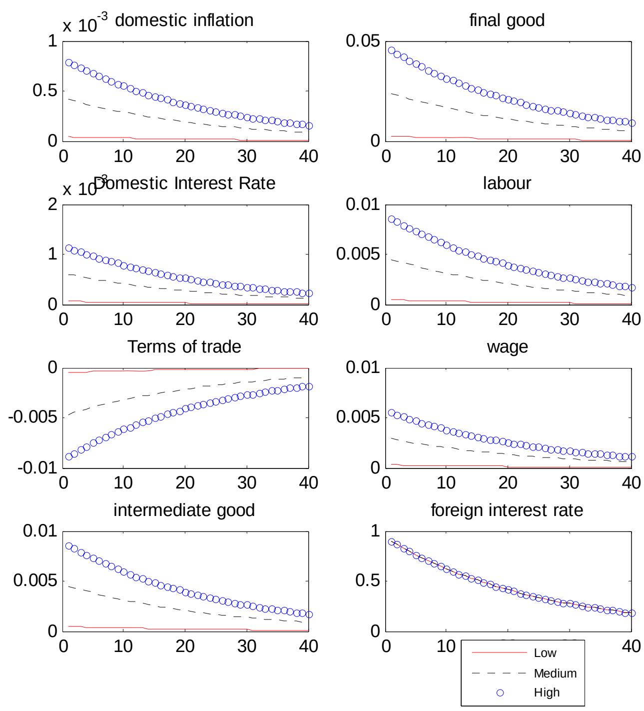
(3.8) and imports of the intermediate good (3.9). Having decreased the price of domestic intermediate, the terms of trade increases (depreciation) (3.37) making domestic goods more competitive (**??**). Because of the high increase in productivity it is necessary a lower amount of labour even to produce a higher amount of intermediate good. Finally, by the labour Euler equation (2.8), wages decrease.

Substitutability ($\omega > 1$)

Under both shocks a higher degree of dollarization is associated with a higher positive response in consumption. In fact a positive shock to foreign interest rate lowers foreign currency in the domestic economy and so consumption increases, being money and consumption substitutes. The same occurs, even if to a much smaller extent, when considering a shock to technology[34]. As for labour, it increases when the economy is hit by a foreign interest rate shock and decreases in presence of a shock to technology.

In the following figures we show the impulse responses. Comments are similar to previous ones.

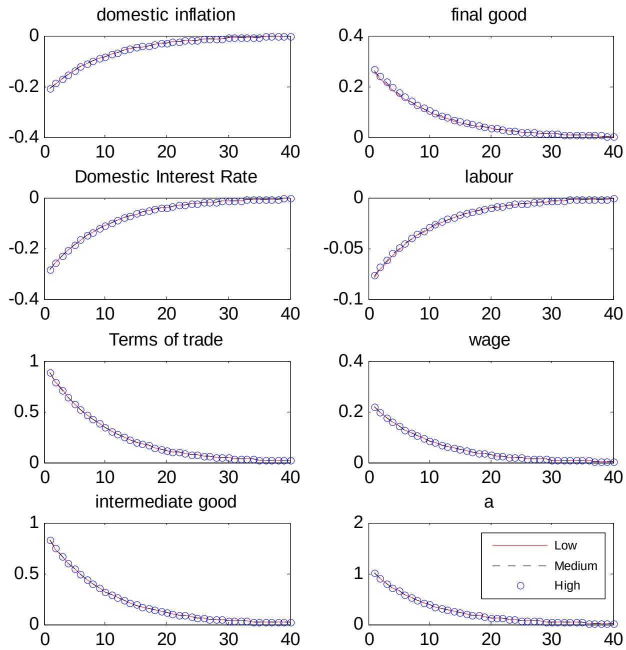
Shock to Foreign Interest Rate



Selected IRs to a Shock to Foreign Interest Rate - substitutes

[34]There is a slight difference. This probably depends on the parameterization used.

Positive shock to technology



Selected IRs to a Shock to Technology - substitutes

Volatilities

Dollarization can make an economy relatively more unstable. In the following table we show the standard deviations (normalized to first value) of some variables of interest (namely, consumption, inflation of intermediate good, domestic interest rate and gross devaluation rate) for different degrees of dollarization under the two cases of complementarity and substitutability.

When money services and consumption are complements a higher degree of dollarization is associated with higher volatility both in output and inflation. This result is in line with Felices and Tuesta (2006) which was found when considering only a shock to foreign interest rate. Note also that as the degree of dollarization increases the exchange rate volatility increases as well.

In the substitutability case, output volatility increases with the degree of dollarization while the volatility of the other shown variables decreases. This difference with Felices and Tuesta (2006) could be due to the fact that here we are considering also a shock to technology.

Table 1: Volatility and Dollarization (both shocks)

	Complements			*Substitutes*		
DI	**0.05**	**0.50**	**0.95**	**0.05**	**0.50**	**0.95**
$\boldsymbol{\sigma}_x$	1.00000	1.28321	1.50684	1.00000	1.04746	1.10713
σ_{π_H}	1.00000	1.01164	1.03234	1.00000	0.99835	0.99671
σ_π	1.00000	1.01392	1.03619	1.00000	0.99827	0.99656
σ_s	1.00000	1.02156	1.04836	1.00000	0.99793	0.99595
$\boldsymbol{\sigma}_{\Delta\varepsilon}$	1.00000	1.00776	1.01214	1.00000	0.99935	0.99870

2.5 Welfare, Currency Substitution and Exchange Rate Policy

In this section we perform some exercises of welfare analysis. As standard in the literature, welfare is measured as the part of the utility of the representative household depending on consumption and labour, i.e.

$$W\left(C_t, N_t\right) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\sigma_2}}{1+\sigma_2}$$

Under both cases of complementarity and substitutability, the movements in consumption and labour depicted in the previous section affect welfare in an ambiguous way. Hence the need to compute welfare.

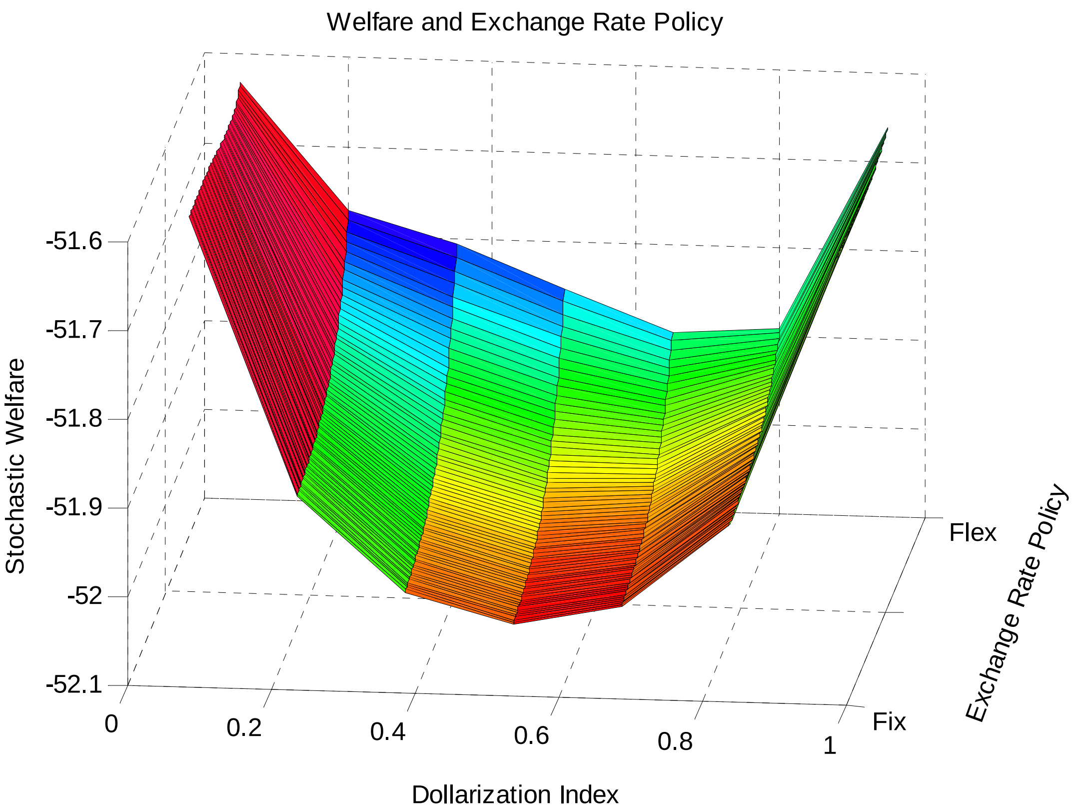
To this purpose we resort to a second order approximation solution. In fact, contrary to the standard methodology which relies upon first order approximations, second order solution enables us to take into account both the direct and indirect effects of variability on welfare. This means that we can compare welfare across policies that do not have first-order effects on the model's deterministic steady state.

Using the following Taylor rule

$$\frac{1+i_t}{1+i} = \left(\frac{1+\pi_t}{1+\pi}\right)^{\omega_\pi} \left(\frac{\varepsilon_t}{\varepsilon}\right)^{\omega_\varepsilon} \tag{2.74}$$

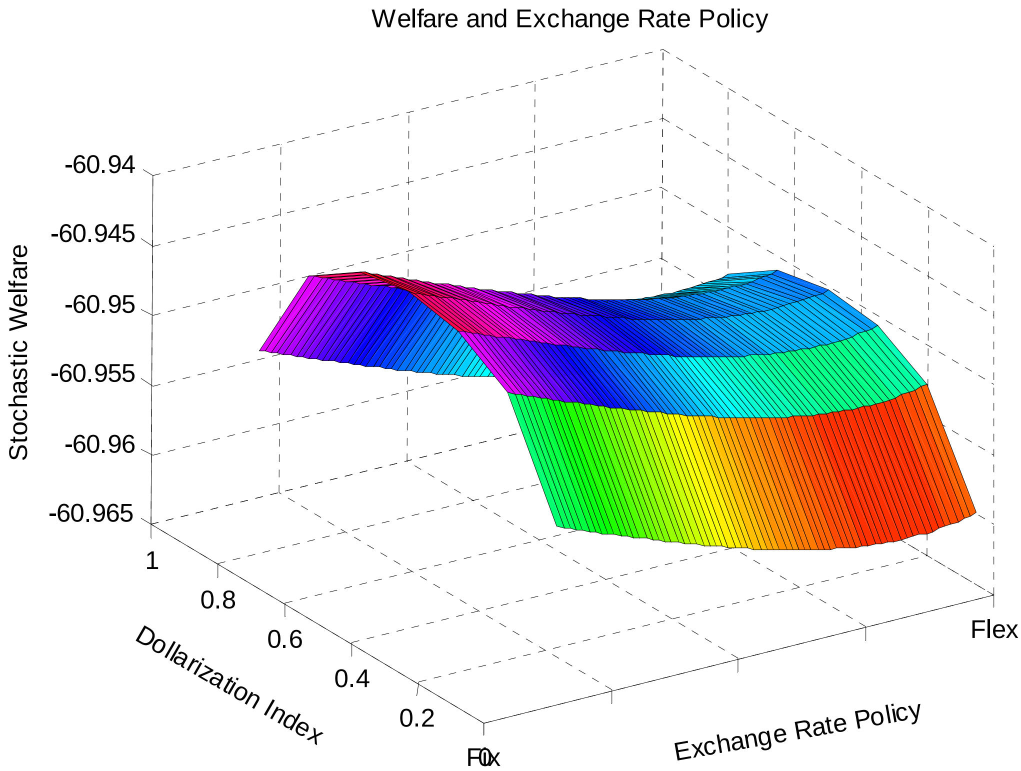
we compute and plot welfare against different values of (ω_ε) the coefficient measuring the weight given to the object of exchange rate stability and different degrees of dollarization (changing values of parameter α). This exercise is performed for the two cases of complementarity ($\omega < 1$) and substitutability ($\omega > 1$).

In the first case a flexible exchange rate is welfare increasing for most degrees of dollarization. A fix exchange rate policy is preferable or at least equal to a flexible exchange rate, in the cases of very low (5%) and very high degrees (95%) of dollarization, i.e. when we almost revert to the case of a one currency environment. As for the welfare implications of dollarization, welfare initially decreases in the degree of dollarization and then as the economy becomes so dollarized that it almost reverts to a one currency environment, welfare increases again. .



Complements

Under substitutability a fix exchange rate is welfare increasing for all the considered degrees of dollarization. Moreover, welfare initially increases in the degree of dollarization and then it decreases again. So, while in the complementarity case the preferable degree of dollarization is zero, when consumption and liquidity services are substitutes it is non zero[35].



Substitutes

2.6 Conclusions

In this work we try to assess the welfare implications of dollarization. To this purpose we extend Felices and Tuesta (2006) work so to compute a numerical solution of the whole model up to the second order of approximation. We show that the relevance of dollarization depends not only on the fact whether there is or not separability between consumption and money services. Its relevance depends also on the nature of the shock. In fact, not surprisingly, dollarization plays a marginal or reduced role when the economy is hit by a real shock as a domestic technology shock. Overall, impulse responses are larger the greater the degree of dollarization.

In line with Felices and Tuesta (2006) when money services and consumption are complements a higher degree of dollarization is associated with higher volatility of output, inflation and exchange rate. In the substitutability case, output volatility increases

[35] At least for the values of ω considered.

with the degree of dollarization while the volatility of the other variables of interest decreases.

When coming to welfare implications, dollarization is generally welfare decreasing when consumption and money services are complements, while some degree of dollarization is preferred under substitutability.

Finally, a fix exchange rate is preferred for all the considered degrees of currency dollarization under the substitutability case. On the contrary, a flexible exchange rate regime is welfare increasing when we consider complementarity and intermediate degrees of dollarization.

2.A Appendix

2.A.1 Computing the Phillips Curve

$\max_{P_{H,t}(j)} E_0 \sum_{t=0}^{\infty} R_t D_t(j),$

where

$$D_t(j) = P_{H,t}(j)Y_t(j) - W_t N_t(j) - P_t AC_{P,t}(j) \tag{2.75}$$

$$AC_{P,t}(j) \equiv \frac{\varphi_P}{2}\left[\frac{1}{\pi_H}\frac{P_{H,t}(j)}{P_{H,t-1}(j)} - 1\right]^2 Y_t \tag{2.76}$$

and subject to

$$Y_t(j) \leq X_{H,t}^d(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\phi}\left(\frac{P_{H,t}}{P_t}\right)^{-\rho}\left[(1-\lambda)X_t + (\mathcal{Q}_t)^{\rho}\lambda X_t^*\right] \tag{2.77}$$

$$Y_t(j) = A_t N_t(j) \tag{2.78}$$

and so

$$W_t N_t(j) = \frac{W_t}{A_t} Y_t(j) \tag{2.79}$$

Finally, using the market clearing condition and the given definition of marginal cost

$$W_t N_t(j) = MC_t^n \cdot Y_t(j) \tag{2.80}$$

where

$$MC_t^n \equiv \frac{W_t}{A_t}$$

So, using 2.77 with equality the problem becomes

$$\max_{P_{H,t}(j)} E_0 \sum_{t=0}^{\infty} R_t \left\{\left[P_{H,t}(j) - MC_t^n\right]\frac{Y_t(j)}{P_t} - AC_{P,t}(j)\right\} \tag{2.81}$$

The first order condition for this problem is

$$R_t\left\{\frac{Y_t(j)}{P_t} - \phi\frac{Y_t(j))}{P_t} + \phi MC_t^n \frac{Y_t(j)}{P_{H,t}(j)P_t} - \varphi_P\left[\frac{1}{\pi_H}\frac{P_{H,t}(j)}{P_{H,t-1}(j)} - 1\right]\left[\frac{Y_t}{P_{H,t-1}(j)\pi_H}\right]\right\} \tag{2.82}$$
$$+E_t\left\{R_{t+1}\varphi_P\left[\frac{1}{\pi_H}\frac{P_{H,t+1}(j)}{P_{H,t}(j)} - 1\right]\left[\frac{Y_{t+1}}{P_{H,t}(j)\pi_H}\frac{P_{H,t+1}(j)}{P_{H,t}(j)}\right]\right\} = 0$$

which can be rewritten as

$$\begin{aligned}(\phi-1)\frac{Y_t(j)}{P_t} &= \left\{\phi MC_t^n \frac{Y_t(j)}{P_{H,t}(j)P_t} - \varphi_P\left[\frac{1}{\pi_H}\frac{P_{H,t}(j)}{P_{H,t-1}(j)}-1\right]\left[\frac{Y_t}{P_{H,t-1}(j)\pi_H}\right]\right\} \\ &+\beta\varphi_P E_t\left\{\frac{U_C(C_{t+1})}{U_C(C_t)}\left[\frac{1}{\pi_H}\frac{P_{H,t+1}(j)}{P_{H,t}(j)}-1\right]\left[\frac{Y_{t+1}}{P_{H,t}(j)\pi_H}\frac{P_{H,t+1}(j)}{P_{H,t}(j)}\right]\right\}\end{aligned} \tag{2.83}$$

and for $\varphi_P = 0$ it collapses to

$$P_{H,t}(j) = \frac{\phi}{(\phi-1)}MC_t^n \tag{2.84}$$

the usual mark up formula with $\mu = \frac{\phi}{(\phi-1)}$ as the mark up. In a symmetric equilibrium, where all firms are identical we have:

$$X_{H,t}(j) = X_{H,t} \qquad N_t(j) = N_t \qquad P_{H,t}(j) = P_{H,t} \tag{2.85}$$

Hence, we can rewrite the Phillips Curve:

$$\begin{aligned}(\phi-1) &= \left\{\phi\frac{W_t}{A_t}\frac{1}{P_{H,t}} - \varphi_P\left[\frac{\pi_{H,t}}{\pi_H}-1\right]\left[\frac{P_t}{P_{H,t-1}\pi_H}\frac{Y_t}{Y_t}\right]\right\}+ \\ &+\beta\varphi_P E_t\left\{\frac{U_C(C_{t+1})}{U_C(C_t)}\left[\frac{\pi_{H,t+1}}{\pi_H}-1\right]\left[\frac{P_t}{P_{H,t}}\frac{\pi_{H,t+1}}{\pi_H}\frac{Y_{t+1}}{Y_t}\right]\right\}\end{aligned} \tag{2.86}$$

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Chapter 3

Exchange Rate Variability in a Small Open Economy with Liability Dollarization

Luca M.F. Colantoni[1]
Bocconi University

Abstract

An important feature of transition economies such as the Central and Eastern European countries is the so-called phenomenon of dollarization. In this paper we study a small open economy model with frictions, whose main distinctive feature is the presence of liability dollarization. This allows for an additional channel through which the exchange rate can affect the behavior of the economy. Hence, active exchange rate intervention can be dangerous. In this respect, the adoption of new exchange rate regimes is a topic particularly crucial for those countries who wish to join the EU. The equilibrium for the economy is presented by a highly non-linear multiequational system solved numerically up to a second order approximation. Fix exchange rate results to be welfare decreasing for most degrees of liability dollarization.

Keywords: dollarization, Small Open Economy, exchange rate regimes.

JEL codes: D58, E52, F31, F41.

[1] I wish to thank participants to CEF 2006 Conference for useful comments.

3.1 Introduction

In this paper we consider an important feature of transition economies, namely *liability dollarization*, together with a feature of international economics, i.e. nominal exchange rate variability. This last one has been blamed for limiting gains from international trade and for lowering welfare. The creation of the euro as well as the adoption of managed exchange rate regimes in many countries are partly due to the desire to reduce the nominal exchange rate risk. Recently, in the stream of New Open Economy Macroeconomics[2] a number of papers begun to improve the literature on optimal currency areas by formalizing Mundell's analysis of the welfare implications of exchange rate risk[3]. Extensive dollarization not only makes domestic monetary and fiscal policies less effective, it also makes active exchange rate intervention more dangerous. In this respect, the adoption of new exchange rate regimes is a topic particularly crucial for those countries who wish to join the EU. In fact, accession countries are required not only to meet the Maastricht convergence criteria but also to participate in the ERM-II (Exchange Rate Arrangement between the Euro area and EU members outside the Euro area). The main aim of this paper is to see how financial dollarization interacts with exchange rate variability and which kind of exchange rate policy is preferable in its presence.

This paper extends previous literature on exchange rate variability by considering an economy characterised by *financial/liability dollarization*, i.e. the fact that either the government or the private sector, or both, have foreign-exchange denominated short-term debt obligations. Such phenomenon is manly due to financial imperfections[4] and, being monetary policy linked to credit markets' conditions, it can affect the transmission mechanism of monetary policy and make the financial system more vulnerable to exchange rate fluctuations. In fact, when such obligations are so large that exceed the stock of international reserves currency crises can occur even under flexible exchange rate arrangements. Moreover, borrowers with foreign exchange denominated debts not matched by foreign exchange assets can be forced into bankruptcy by an unexpected depreciation of the exchange rate. Hence, the presence of such currency mismatches seems to call for some form of exchange rate peg.

Hence, policy makers of these economies should take liability dollarization into account. Nevertheless, as recently pointed out by Calvo (2006), monetary policy literature

[2]See Lane (2001) for a survey of the New Open Economy Macroeconomics.

[3]Among others, Obstfeld and Rogoff (2001), Devereux and Engel (2000), Bacchetta and Van Wincoop (2000) and Bergin and Tchakarov (2004). This last one can be considered as one of the main references for our work.

[4]In this respect see Eichengreen and Hausmann (1999) who define this phenomenon as *original sin*.

is still centred on sophisticated analyses of how to implement monetary policies as inflation targeting in sound economies with little or no reference to the financial imperfections of transition economies[5].

In this paper we account for financial dollarization by assuming imperfect financial markets and allowing agents to borrow foreign denominated assets. This is done following Cespedes, Chang and Velasco (2001, 2002) whose main conclusion is consistent with the conventional Mundell-Fleming's prescription which promotes the implementation of flexible exchange rate regimes. This result holds even when such financial frictions are present in the model. In fact, by allowing the currency to depreciate, a flexible exchange rate can insulate the economy against adverse external shocks more effectively than a fixed exchange rate. However, such conclusions are made comparing impulse response functions under the two alternative regimes of fixed and flexible exchange rates. More precisely, Cespedes, Chang and Velasco (2002) contrast the behavior of different exchange rate regimes when the economy is subject to foreign interest rate and export shocks while Cespedes, Chang and Velasco (2001) recognize the limitations of this approach and construct an ad-hoc loss function. Another related paper is Schmitt-Grohé and Uribe (2001) who have recently contributed to the debate analyzing the costs of *full dollarization*. By means of an optimizing model of a small open economy calibrated to the Mexican economy, the authors compare the welfare costs of economic fluctuations under alternative monetary policies (inflation targeting, money growth rate pegs, devaluation rate rules, or *full dollarization* in the form of fixed exchange rate). Differently from Cèspedes, Chang and Velasco they do not explicitly model the financial sector. Instead, they consider households who buy foreign currency denominated bonds and, in order to account for financial incompleteness, they close the model by a debt-elastic interest rate rule.

This paper extends Cespedes, Chang and Velasco (2001, 2002) by constructing a richer model. This is used to investigate what are the main implications of liability dollarization in terms of welfare by means of a second-order approximation solution.

Differently from Cespedes, Chang and Velasco (2001, 2002) who have wage rigidities, we model nominal rigidities by price adjustment costs as in Rotemberg (1982) while market imperfections are considered through monopolistic competition in the intermediate goods sector. The presence of nominal rigidities allows for non-neutral monetary policy effects, while the presence of market imperfections (namely, monopolistic competition

[5] Exceptions are Caballero and Krishnamurthy (2005), Céspedes, Chang and Velasco (2001) and Chang and Velasco (2001).

in the intermediate goods sector) allows for non trivial pricing decisions and makes the output demand-determined in the short run. Financial market incompleteness is captured by allowing some agents, the so-called capitalists, to borrow foreign denominated assets but at an interest rate greater than the foreign one and which is a measure of the risk premium.

The paper is organized as follows. Section 2 illustrates the model. Section 3 presents the calibration. Section 4 contains results of the numerical solution. Section 5 concludes.

3.2 A Small Open Economy Model

We consider a small open economy (SOE) composed of infinitely-lived individuals and of a continuum of firms whose shares are owned by the consumers. Agents have the possibility to use foreign currency. Use of foreign money as a mean of savings (asset/liability substitution) and a mean of transaction (currency substitution) can be justified for countries with high inflation and unstable economy, or for countries with incomplete financial sector, i.e. in developing or transition economies. We model the 'dollarized' economy by allowing some agents, the so-called capitalists, to borrow foreign denominated assets but at an interest rate greater than the foreign one and which is a measure of the risk premium.

As for the production part, there are two types of home produced goods: final (X) and intermediate (Y). Final good is nontradable. Intermediate good is used as an input for home (X_H) and foreign (X_H^*) production. Intermediate goods are also produced abroad: imported intermediates are called (X_F). The final good sector is perfectly competitive, while the intermediate sector is characterised by nominal rigidities in the form of monopolistic competition and adjustment costs à la Rotemberg (1982). Capital and labor are the inputs in the intermediate sector. However, intermediate inputs only are required for the production of final goods. Finally, we assume that financial markets of our SOE are incomplete in the sense that state-contingent securities are not available.

3.2.1 Households

Our small open economy model is inhabited by a representative household who seeks to maximize[6]

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma_1}}{1-\sigma_1} - \frac{L_t^{1+\sigma 2}}{1+\sigma_2} \right\} \tag{3.1}$$

[6]For simplicity of notation, in wha follows we drop the household index (j).

where C_t is a consumption good, L_t denotes hours of labor, and ε_t is the nominal exchange rate.

We assume that households have access to safe domestic deposits B_t, whose interest rate is described by (3.35), and face a sequence of budget constraints of the form

$$P_t C_t + B_t \leqslant (1 + i_{t-1}) B_{t-1} + W_t L_t + \Pi_t - T_t \tag{3.2}$$

$\forall t$, where T_t denotes lump-sum taxes/transfers and D_t is profit from owned domestic intermediate firms. All variables are expressed in units of domestic currency.

The right hand side of the budget constraint gives the available resources as the sum of gross return on the bond holding, labour income, profits from intermediates in tradeables, revenues from renting capital, less government taxation. These resources are used cover consumption, investment and to acquire the next period money balances and new bond

Control variables are total consumption C_t, working hours L_t, nominal bond holdings B_t.

Optimality Conditions

The household chooses the set of stochastic processes $\{C_t, L_t, B_t\}_{t=0}^{\infty}$ so to maximize 3.1 subject to 3.2 and some borrowing limit that prevents from engaging in Ponzi-type schemes, taking as given the sequences $\{P_t, \varepsilon_t, i_{t-1}, W_t, r_t\}$. The associated optimality conditions are

(*1*)→Euler equation

$$\beta \left(1 + i_t\right) E_t \left\{ \left(\frac{C_t}{C_{t+1}} \right)^{\sigma_1} \left(\frac{P_t}{P_{t+1}} \right) \right\} = 1 \tag{3.3}$$

(*2*)→Labor Supply in Intermediate Good Production, L_t; is given by a standard intratemporal optimality condition

$$\left(L_t\right)^{\sigma_2} = \frac{W_t}{P_t} C_t^{-\sigma_1} \tag{3.4}$$

This equation is the labour-leisure trade-off condition that comes from utility maximization with respect to wages. It ensures that marginal disutility of the additional factor supply (due to leisure foregone) on the left hand side is compensated by an extra unit of marginal utility of consumption, such that an extra unit of labour supply can buy at the real factor price.

In the ROW a representative household faces a problem identical to the one outlined above. We assume that the size of the SOE is negligible relative to the ROW, which allows us to treat the latter as if it was a closed economy.

3.2.2 Firms

For the supply side we adopt a structure similar to the one in Romer (1990). There is a final good sector which is perfectly competitive, while the tradeable intermediate good is characterized by monopolistic competition.

Final goods sector

Because the production function is homogeneous of degree one, final output can be described in terms of the actions of a single, aggregate, price-taking firm. The firms are perfectly competitive, the output is determined as

$$X_t = \left[\gamma^{\frac{1}{\rho}} \left[X_{H,t}\right]^{\frac{\rho-1}{\rho}} + (1-\gamma)^{\frac{1}{\rho}} \left[X_{F,t}\right]^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \tag{3.5}$$

$X_{H,t}$ is a home produced intermediate good, $X_{F,t}$ is imported intermediate good, both used in the production of domestic final good. Parameter ρ will determine the elasticity of substitution between home and foreign goods, while γ will determine the ratio of imports to GDP.

We design the firm's problem using the budget separation method.

1) Inter-input allocation. The firms choose inputs quantity $X_{H,t}$ and $X_{F,t}$ to solve the following PMP (Profit Maximization Problem).

$$\max_{X_{H,t},\, X_{F,t}} \quad P_t \cdot X_t - P_{H,t}X_{H,t} - P_{F,t}X_{F,t} \tag{3.6}$$

subject to 3.5 and where P_t is price index taken as given (perfect competition)

$$P_t = \left\{\gamma \left[P_{H,t}\right]^{1-\rho} + (1-\gamma) \left[P_{F,t}\right]^{1-\rho}\right\}^{\frac{1}{1-\rho}} \tag{3.7}$$

The FOC are the following

$$X_{H,t}: \qquad P_t \left[\frac{\rho}{\rho-1} \left(X_t\right)^{\frac{1}{\rho}} \frac{\rho-1}{\rho} \gamma^{\frac{1}{\rho}} \left(X_{H,t}\right)^{-\frac{1}{\rho}}\right] - P_{H,t} = 0$$

$$X_{F,t}: \qquad P_t \left[\frac{\rho}{\rho-1} (X_t)^{\frac{1}{\rho}} \frac{\rho-1}{\rho} (1-\gamma)^{\frac{1}{\rho}} (X_{F,t})^{-\frac{1}{\rho}} \right] - P_{F,t} = 0$$

Rearranging, we get

$$X_{H,t} = \gamma \left(\frac{P_{H,t}}{P_t} \right)^{-\rho} X_t \tag{3.8}$$

$$X_{F,t} = (1-\gamma) \left(\frac{P_{F,t}}{P_t} \right)^{-\rho} X_t \tag{3.9}$$

2) Intra-basket allocation. In turn, each basket of intermediate goods is composed of a continuum of different varieties indexed by j. The corresponding Home Intermediate Good Index and Foreign Intermediate Good Index are given accordingly as

$$X_{H,t} = \left[\int_0^1 X_{H,t}(j)^{\frac{\phi-1}{\phi}} dj \right]^{\frac{\phi}{\phi-1}} \tag{3.10}$$

$$X_{F,t} = \left[\int_0^1 X_{F,t}(l)^{\frac{\phi-1}{\phi}} dl \right]^{\frac{\phi}{\phi-1}} \tag{3.11}$$

The parameter ϕ will determine the mark-up price over the marginal cost.

The Home and Foreign Intermediate Price indices are

$$P_{H,t} = \left\{ \int_0^1 \left[P_{H,t}(j) \right]^{1-\phi} dj \right\}^{\frac{1}{1-\phi}} \tag{3.12}$$

$$P_{F,t} = \left\{ \int_0^1 \left[P_{F,t}(l) \right]^{1-\phi} dl \right\}^{\frac{1}{1-\phi}} \tag{3.13}$$

Proceeding as in the previous step, the cost minimization gives the following intra-basket demands

- home demand for domestic intermediates

$$X_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\phi} X_{H,t} \tag{3.14}$$

- home demand for imports

$$X_{F,t}(l) = \left(\frac{P_{F,t}(l)}{P_{F,t}}\right)^{-\phi} X_{F,t} \tag{3.15}$$

Foreign sector

In the rest of the world a representative household, final and intermediate good firms, face problem identical to the ones outlined above. Allocations and prices are denoted with an asterisk.

Thus, the final good production is

$$X_t^* = \left[(\gamma^*)^{\frac{1}{\rho}} \left[X_{H,t}^*\right]^{\frac{\rho-1}{\rho}} + (1-\gamma^*)^{\frac{1}{\rho}} \left[X_{F,t}^*\right]^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \tag{3.16}$$

where the parameters and variables have an interpretation similar to the previous one. Note that as $\gamma^* \to 0$ the SOE intermediate good does not enter in the production of the final good of the rest of the world.

Following the same lines as above, optimality conditions yield

- demand for the SOE produced intermediate good (exports)

$$X_{H,t}^* = \gamma^* \left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\rho^*} X_t^* \tag{3.17}$$

- demand for the intermediate good produced in the rest of the world

$$X_{F,t}^* = (1-\gamma^*) \left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\rho^*} X_t^* \tag{3.18}$$

And so

- foreign demand for exports:

$$X_{H,t}^*(j) = \left(\frac{P_{H,t}^*(j)}{P_{H,t}^*}\right)^{-\phi^*} X_{H,t}^* \tag{3.19}$$

- foreign demand for their own goods:

$$X_{F,t}^*(l) = \left(\frac{P_{F,t}^*(l)}{P_{F,t}^*}\right)^{-\phi^*} X_{F,t}^* \tag{3.20}$$

In addition, we assume that there are no barriers to trade such that the Law of One Price (LOP) holds for each good, implying that the prices of importables and exportables, $P_{F,t}(l)$ and $P_{H,t}(j)$,are linked to the respective world prices, $P^{*}_{F,t}(l)$ and $P^{*}_{H,t}(j)$, by the relationships

$$P_{F,t}(l) = \lambda \varepsilon_t P^{*}_{F,t}(l) \tag{3.21}$$

$$P_{H,t}(j) = \lambda \varepsilon_t P^{*}_{H,t}(j) \tag{3.22}$$

for all i, where ε_t is the nominal exchange rate (the price of foreign currency in terms of home currency), and $P^{*}_{F,t}(l)$ is the price of foreign good denominated in foreign currency. Integrating over all goods we obtain

$$P_{F,t} = \lambda \varepsilon_t P^{*}_{F,t} \tag{3.23}$$

$$P_{H,t} = \lambda \varepsilon_t P^{*}_{H,t} \tag{3.24}$$

Intermediate goods sector

The market is populated by a continuum of firms acting as monopolistic competitors, since intermediate goods substitute imperfectly for one another as inputs to producing the final good. During period t, the representative intermediate goods-producing firm rents capital $K_t(j)$ and hires $L_t(j)$ units of labor, in order to produce $Y_t(j)$ units of intermediate good according to the production function given by:

$$Y_t(j) = A_t K^{\alpha}_t(j)\, L^{1-\alpha}_t(j) \tag{3.25}$$

where A_t is a technology shifter common to all firms.

During each period t, the representative intermediate goods-producing firm sets a nominal price $P_{H,t}(j)$, subject to requirement that it satisfies the representative finished goods-producing firm's demand.

The existence of an economy-wide competitive factor market implies that all firms will pay the same rental rate r_t and the same nominal wage W_t. This also implies that all firms face a common nominal marginal cost (in particular, independent of the level of individual output) that we denote by MC_t

$$MC_t = \frac{(r_t P_t)^{\alpha} W^{1-\alpha}_t}{A^{\alpha}_t \alpha (1-\alpha)^{1-\alpha}} \tag{3.26}$$

Each firm faces a quadratic cost of price adjustment as in Rotemberg (1982) and given by[7]

$$AC_{P,t}\left(j\right) \equiv \frac{\varphi_P}{2}\left[\frac{1}{\pi_H}\frac{P_{H,t}(j)}{P_{H,t-1}(j)} - 1\right]^2 Y_t(j) \tag{3.27}$$

where π_H is the gross steady state rate of inflation in the intermediate sector. Note that if the price adjustment cost parameter $\varphi_P = 0$ the model collapses to a flexible price specification. Also note that in steady state the price adjustment costs are equal to zero.

Cost minimization implies the following efficiency condition for the choice of labor input and capital

$$P_t r_t K_t = \frac{\theta}{1-\theta} W_t L_t \tag{3.28}$$

The cost of price adjustment makes the firm's problem dynamic. Assuming no price discrimination, each firm chooses price $P_{H,t}(j)$ and outputs $X_{H,t}(j), X^*_{H,t}(j)$ in order to maximize its total market value, i.e.

$$\max_{P_{H,t}(j)} E\sum_{t=0}^{\infty} R_t \Pi_t(j) \tag{3.29}$$

where

$$\Pi_t(j) = P_{H,t}(j)Y_t(j) - r_t P_t K_t\left(j\right) - W_t L_t\left(j\right) - P_{H,t} AC_{P,t}(j),$$

subject to production technology 3.25 and final sector demands 3.14 and 3.19.

Since firms are assumed to be owned by the representative household, they value future payoffs according to the household's intertemporal marginal rate of substitution in consumption and so the pricing kernel used to value random date $t+n$ payoffs is

$$R_t = \beta^t C_t^{-\sigma_1} \tag{3.30}$$

Assuming a symmetric equilibrium, where all firms are identical

$$X_{H,t}\left(j\right) = X_{H,t} \qquad K_t\left(j\right) = K_t \qquad L_t\left(j\right) = L_t \qquad P_{H,t}\left(j\right) = P_{H,t} \tag{3.31}$$

[7] This form is mutuated from Ireland (2004). Following Bergin and Tchakarov (2004) we assume the same adjustment cost for goods sold domestically and goods exported. Bergin (2003) has different adjustment costs.

the optimization problem implies the following pricing behaviour[8]

$$\begin{aligned}(\phi-1)\frac{Y_t(j)}{P_t} &= \left\{\phi MC_t^n \frac{Y_t(j)}{P_{H,t}(j)P_t} - \varphi_P\left[\frac{1}{\pi_H}\frac{P_{H,t}(j)}{P_{H,t-1}(j)}-1\right]\left[\frac{Y_t(j)}{P_{H,t-1}(j)\pi_H}\right]\right\} \\ &+\beta\varphi_P E_t\left\{\frac{U_C(C_{t+1})}{U_C(C_t)}\left[\frac{1}{\pi_H}\frac{P_{H,t+1}(j)}{P_{H,t}(j)}-1\right]\left[\frac{Y_{t+1}(j)}{P_{H,t}(j)\pi_H}\frac{P_{H,t+1}(j)}{P_{H,t}(j)}\right]\right\}\end{aligned} \quad (3.32)$$

where

$$MC_t^n \equiv \frac{(P_t r_t)^\alpha (W_t)^{1-\alpha}}{A_t^\alpha \alpha (1-\alpha)^{1-\alpha}} \quad (3.33)$$

As usual if $\varphi_P = 0$ (i.e. no price adjustment costs) the above pricing condition boils down to:

$$P_{H,t} = \frac{\phi}{\phi-1}MC_t^n = \upsilon MC_t^n \quad (3.34)$$

where $\upsilon = \frac{\phi}{\phi-1}$ denotes the desired (constant) markup value. Hence a representative firm chooses the price for its differentiated product as a constant markup over the marginal cost[9]. This stems from the imperfect competition feature of the market. In fact, as $\phi \to \infty$ in the case of perfectly competitive output markets, $P_{H,t} = MC_t^n$, which is the usual pricing condition of a firm acting as a price taker.

Capitalists

In this subsection we briefly describe the structure introduced by Cespedes, Chang, and Velasco (2002) in order to take into account the phenomenon of *liability dollarization*. At the beginning of each period t capitalists, who are risk neutral, use their net worth to finance investment (K_t). If their net worth is not enough, capitalists finance the remainder with foreign loans $\left(B_{t+1}^*\right)$ which are subject to financial frictions[10]. Hence, the budget constrain in real terms is:

$$N_t + D_t = K_t$$

where N_t is net worth and $D_t \equiv \frac{\varepsilon_t B_t^*}{P_t}$ is the domestic currency denominated value of foreign debt. In presence of imperfections the optimal choice rule for investment is to

[8] See the appendix for the complete derivation.

[9] Rotemberg pricing is, by now, quite common in the literature. Alternative means to introduce price stickiness are the Calvo (1983) and Yun (1996) pricing models.

[10] Such frictions can be due to informational or enforcement problems. In what follows we limit to we limit to describe the aggregate behaviour of capitalists. See Cespedes, Chang, and Velasco (2002), and Bernanke, Gertler, and Gilchrist (2000) for further details.

equalize the expected yield on capital (in dollars) to the world interest rate, i.e.

$$E_t[r_t] = E_t\left[(1+i_t^*)(1+\eta_t)\frac{\varepsilon_{t+1}}{\varepsilon_t}\right]$$

where $\left(1+\eta_{t+1}\right)$ is the risk premium which is given by

$$1+\eta_t = \left(\frac{K_t}{N_t}\right)^{\mu}$$

At the beginning of each period capitalists collect the income from capital, repay foreign debt and consume a fraction $(1-\delta)$ of the remainder. Hence, the law of motion of net worth is[11]

$$N_t = \delta\left\{K_{t-1} - \left(1+i_{t-1}^*\right)\left(1+\eta_{t-1}\right)\frac{\varepsilon_t}{\varepsilon_{t-1}}\frac{P_{t-1}}{P_t}D_{t-1}\right\}$$

Note that, affecting the peso value of foreign debt, the exchange rate can affect capitalist's net worth. In fact, ceteris paribus, a real depreciation will reduce net worth and so increase the risk premium. It is through this channel that the exchange rate can have additional effects on the variables.

Finally, in steady state we have

$$N + D = K$$

$$1+\eta = \left(\frac{K}{N}\right)^{\mu}$$

which together deliver

$$1+\eta = \left(\frac{K}{K-D}\right)^{\mu}$$

from which we can derive the debt to investment ratio[12] of steady state

$$\frac{D}{K} = 1-(1+\eta)^{-\frac{1}{\mu}} = 1-\{\delta(1+i_t^*)\}^{\frac{1}{\mu}}$$

---

[11] For simplicity we are assuming that capital depreciates completely during production.

[12] It can be easily shown that in a steady state with $\sigma_1 = \sigma_2 = 1$

$$K = \frac{\alpha\delta l^2}{\alpha\delta l^2 + \alpha - 1}X$$

and so

$$\frac{D}{X} = \frac{\alpha\delta l^2}{\alpha\delta l^2 + \alpha - 1}\left\{1-\left[\delta(1+i_t^*)\right]^{\frac{1}{\mu}}\right\}$$

which can be interpreted as debt to GDP ratio.

Hence, the steady state level of indebtedness of the economy will depend on two key parameters, namely δ and μ. In the next section, once fixed δ to a value, we will vary μ so to consider different levels of indebtedness.

3.2.3 Monetary Policy

We will consider two different monetary regimes. In order to compare the welfare effects of exchange rate variability we assume the following open economy version of the Taylor rule[13]

$$\frac{1+i_t}{1+i} = \left(\frac{1+\pi_t}{1+\pi}\right)^{\omega_\pi} \left(\frac{\varepsilon_t}{\varepsilon}\right)^{\omega_\varepsilon} \tag{3.35}$$

where $\omega_\pi \geq$ and $\omega_\varepsilon \in [0, 1]$ are the feedback coefficients to inflation and exchange rate, respectively, and i, π and ε are the steady state values of interest rate, inflation and exchange rate. This rule permits a *fixed exchange rate* regime for $\omega_\varepsilon \to \infty$, or alternatively a *flexible exchange rate* regime for $\omega_\varepsilon = 0$. Moreover, it allows to consider the trade off between the objectives of inflation and exchange rate stabilization imposed by EU accession criteria.

It is assumed that the monetary authority can commit to set this parameter at a time invariant value[14]. Finally, policies are specified in such a way that they give rise to the same nonstochastic steady state.

Being interested in monetary policy, for simplicity we assume the following government's budget constraint

$$M_t = M_{t-1} + T_t \tag{3.36}$$

The assumed fiscal policy implies that the government rebates seigniorage revenues to the public through lump-sum transfers. Note that in presence of currency substitution such revenues are smaller[15].

3.2.4 Terms of trade and some identities

We define terms of trade as

$$S_t \equiv \frac{P_{F,t}}{P_{H,t}} \tag{3.37}$$

[13] See Natalucci and Ravenna (2002). Felices and Tuesta (2006) use a Taylor rule which targets $\pi_{H,t}$ the inflation of domestic intermediate good.

[14] This because we are just interested in comparing steady states under different economic policies and not in characterizing an optimal policy.

[15] In this paper we do not adress the dollarization's implications for seignorage revenue. On this issue see, for example, Schmitt-Grohé and Uribe (1999).

then relative price indices can be rewritten as

$$\tilde{P}_{H,t} \equiv \frac{P_{H,t}}{P_t} = \left\{\gamma + (1-\gamma)\, S_t^{1-\rho}\right\}^{\frac{1}{\rho-1}} \equiv g(S_t), \quad \frac{P_{F,t}}{P_t} = \left\{\gamma S_t^{\rho-1} + (1-\gamma)\right\}^{\frac{1}{\rho-1}} = S_t g(S_t) \tag{3.38}$$

and, from the definition of *CPI* (eq. 2.27), we get the following relation

$$\pi_t^{1-\rho} = \gamma \left[\pi_{H,t} g(S_{t-1})\right]^{1-\rho} + (1-\gamma)\left[\pi_{F,t} g(S_{t-1})\right]^{1-\rho} \tag{3.39}$$

or, in other terms

$$\pi_t = \frac{\left\{(1-\lambda) + \lambda S_t^{1-\rho}\right\}^{\frac{1}{1-\rho}}}{\left\{(1-\lambda) + \lambda S_{t-1}^{1-\rho}\right\}^{\frac{1}{1-\rho}}} \pi_{H,t} \tag{3.40}$$

Movements in the terms of trade reflect movements in relative prices and, hence, imply demand shifts. In fact, relative demands of intermediate can be expressed as

$$\frac{X_{H,t}}{X_t} = \gamma \left(\left\{\gamma + (1-\gamma)\, S_t^{1-\rho}\right\}^{\frac{1}{1-\rho}}\right)^{\rho} = \gamma \left[g(S_t)\right]^{-\rho} \tag{3.41}$$

$$\frac{X_{F,t}}{X_t} = (1-\gamma) \left(\left\{\gamma S_t^{\rho-1} + (1-\gamma)\right\}^{\frac{1}{1-\rho}}\right)^{\rho} = (1-\gamma)\left[S_t g(S_t)\right]^{-\rho} \tag{3.42}$$

In addition, we assume that there are no barriers to trade such that the *law of one price* holds for each good at all times, implying that the prices of importables and exportables, $P_{F,t}(l)$ and $P_{H,t}(j)$, are linked to the respective world prices, $P^*_{F,t}(l)$ and $P^*_{H,t}(j)$, by the relationships

$$P_{F,t}(l) = \varepsilon_t P^*_{F,t}(l) \qquad \forall l \qquad \text{and} \qquad P_{H,t}(j) = \varepsilon_t P^*_{H,t}(j) \qquad \forall j \tag{3.43}$$

where $P^*_{F,t}(l)$ is the price of foreign good denominated in foreign currency. Integrating over all goods we obtain

$$P_{F,t} = \varepsilon_t P^*_{F,t} \qquad \text{and} \qquad P_{H,t} = \varepsilon_t P^*_{H,t} \tag{3.44}$$

Moreover, since the goods produced in the SOE represent a negligible fraction of the world's consumption basket, we can consider the rest of the world is an approximately closed economy with

$$P^*_t = P^*_{F,t}, \qquad \pi^*_t = \pi^*_{F,t} \tag{3.45}$$

Using the previous result , the law of one price conditions 3.44 and the terms of trade definition 3.37, the real exchange rate can be rewritten as

$$\mathcal{Q}_t \equiv \frac{\varepsilon_t P_t^*}{P_t} = \frac{\varepsilon_t P_{F,t}^*}{P_t} = \frac{P_{F,t}}{P_t} = \frac{P_{H,t}}{P_t} S_t \tag{3.46}$$

3.2.5 Market Clearing and Equilibrium

We now turn to the description of a symmetric equilibrium with an initial level of net foreign assets equal to zero, $B_0 = 0$. In the symmetric equilibrium, all firms behave identically and all households behave identically, therefore, one can work with a single representative household and a single representative firm. We can drop the superscript notation in future references so that in the symmetric equilibrium we have:

$$X_{H,t}(j) = X_{H,t} \quad Y_t(j) = Y_t \quad L_t(j) = L_t \quad P_{H,t}(j) = P_{H,t} \quad \Pi_t(j) = \Pi_t \tag{3.47}$$

We assume that there are no government and domestic bonds. Since the goods produced in the SOE represent a negligible fraction of the world's consumption basket, we can consider the rest of the world is an approximately closed economy with

$$P_t^* \approx P_{F,t}^*, \qquad \pi_t^* \approx \pi_{F,t}^* \tag{3.48}$$

In equilibrium aggregate supply is equal aggregate demand, therefore

$$X_t = \{C_t + [K_{t+1} - (1-\delta) K_t] + AC_{I,t}\} + \tilde{P}_{H,t} \frac{\varphi_P}{2} \left[\frac{1}{\pi_H} \frac{P_{H,t}}{P_{H,t-1}} - 1 \right]^2 \cdot Y_t \tag{3.49}$$

$$Y_t = X_{H,t} + X_{H,t}^* \tag{3.50}$$

$$L_t^s = L_t^d \tag{3.51}$$

$$M_t = M_{t-1} + T_t \tag{3.52}$$

To deal with the non stationary nominal variables in the system, we consider stationary variables expressed in real terms such as $D_t \equiv -\frac{\varepsilon_t B_t^*}{P_t}$ (net foreign asset position),and $w \equiv \frac{W_t}{P_{H,t}}$ (real wages).

A stationary rational expectation equilibrium is a set of stationary stochastic processes $\{C_t, L_t, K_{t+1}, D_t, \varepsilon_t, i_t, r_t, w_t, X_t, Y_t, X_{H,t}, X_{F,t}, S_t, \pi_t, \pi_{H,t}\}_{t=0}^{\infty}$ satisfying 3.3-3.4, 3.8-3.9, 3.24-3.28, 3.32-3.50, and 3.49 given exogenous processes $\left\{i_t^*, X_{H,t}^*, A_t\right\}_{t=0}^{\infty}$ and initial values for K_0, F_0, S_0.

3.3 Calibration

We consider a domestic shock, namely a shock to technology, and two foreign shock, namely a shock to foreign interest rate and a shock to exports. We use the Effective Federal Funds Rate[16] as proxy of the foreign interest rate and, by using quarterly data over a period ranging from 1954q3 to 2006q3, we fit an AR(1) process to it in order to calibrate its shock. As for the technology shock we calibrate it at standard values[17]. The three shocks are distributed as follow

$$\begin{gathered}
\left[\log\left(A_t\right) - \log\left(\overline{A}\right)\right] = \rho_a \left[\log\left(A_{t-1}\right) - \log\left(\overline{A}\right)\right] + e_{a,t} \\
i_t^* = \rho_i i_{t-1}^* + e_{i^*,t} \\
\log\left(X_{H,t}^*\right) = \rho_x \log\left(X_{H,t-1}^*\right) + e_{x,t} \\
e_{a,t} \sim N\left(0, 0.01^2\right) \qquad e_{i^*,t} \sim N\left(0, 0.03^2\right) \qquad e_{x,t} \sim N\left(0, 0.08^2\right) \\
\rho_a = 0.9 \qquad \rho_i = 0.96 \qquad \rho_x = 0.5
\end{gathered} \tag{3.53}$$

The rest of the parameters are calibrated as follows[18].

3.3.1 Preferences

The discount factor, β, is set equal to 0.99 and we interpret a period as one quarter. In order to isolate the effects of liability dollarization elasticity consumption (σ_1) and of labour supply (σ_2) are both set equal to 1. Particularly interesting for us are the parameters determining the steady state values of risk premium and debt to investment ratio: δ and μ.. Following Cespedes, Chang and Velasco (2002) we set $\delta = 0.9707$ in order to have a risk premium $\eta = 0.02$. For parameter μ we use various values so to consider economies with different degrees of indebtedness, ranging from *low* $\left(\frac{D}{K} = 5\%\right)$ to *high* $\left(\frac{D}{K} = 95\%\right)$.

3.3.2 Technology

Following Bergin and Tchakarov (2004) the price adjustment cost, φ_P, is set at 50.

The elasticity of substitution, ρ, between imported intermediate good, X_F, and domestic intermediate good, X_H, is set equal to 1. As already noted, parameter γ, the share of domestic intermediate good, X_H, in the production of final output can serve

[16] Available at the St. Louis Fed, http://research.stlouisfed.org/fred2/

[17] See Bergin and Tchakarov (2004), for example.

[18] For comparison purpouses most of the parameterization is taken from CEspedes, Chang and Velasco (2002).

as a proxy for the openness of the economy. Hence, it describes the level of a small open economy's dependence on the rest of the world. It is set equal to 0.36. The degree of monopolistic competition, ϕ, is set at 11 implying a markup of 10%. Assuming the tradable sector to be capital-intensive, the capital share in production, α, is set at 0.67.

3.3.3 Exchange Rate Policy

Being interested in the effects of exchange rate variability under financial dollarization we consider the following alternative exchange rate regimes: (1) *Fixed exchange rate* with *CPI inflation targeting*, by setting[19] $\omega_\varepsilon \to \infty$ and $\omega_\pi = 1.5$; (2) *Flexible exchange rate regime* with *CPI inflation targeting*, by setting $\omega_\varepsilon = 0$ and $\omega_\pi = 1.5$.

3.4 Sensitivity and Welfare

Since the equilibrium is a highly nonlinear system of equations, it is not possible to obtain a solution in its closed form. Hence the model is solved numerically applying Matlab codes (more precisely, DYNARE package[20]). Being interested in welfare implications of different monetary policies, the model is solved numerically up to the second order of approximation. In fact, contrary to the standard methodology which relies upon first order approximations, second order solution enables us to take into account both the direct and indirect effects of variability on welfare. This means that we can compare welfare across policies that do not have first-order effects on the model's deterministic steady state.

In this section we review the calibration of the model, the solution of the steady-state values of the variables, and the accurate second-order approximation. We follow Cespedes, Chang and Velasco (2002) when calibrating the model. The parameters are depicted in Table 1. Here we provide a succinct review of a few parameters.

3.4.1 Simulations

Before computing welfare costs, in this section we perform some sensitivity analysis and stochastic simulations in order to assess the behaviour of the model economy.

In particular, we show responses of selected variables to two foreign shocks: a foreign interest rate shock and an export shock. This enables us to see how the net worth effect

[19] Actually, in the matlab code we normalize the coefficient and let vary $\theta \equiv \frac{\omega_\varepsilon}{1-\omega_\varepsilon} \in [0,1]$.
[20] See Collard and Juillard (2000), Juillard (2004).

works and how sensitive are dynamics to the level of indebtedness (in the case of flexible exchange rate) and to the exchange rate policy (for a given level of indebtedness).

3.4.2 Different levels of indebtedness

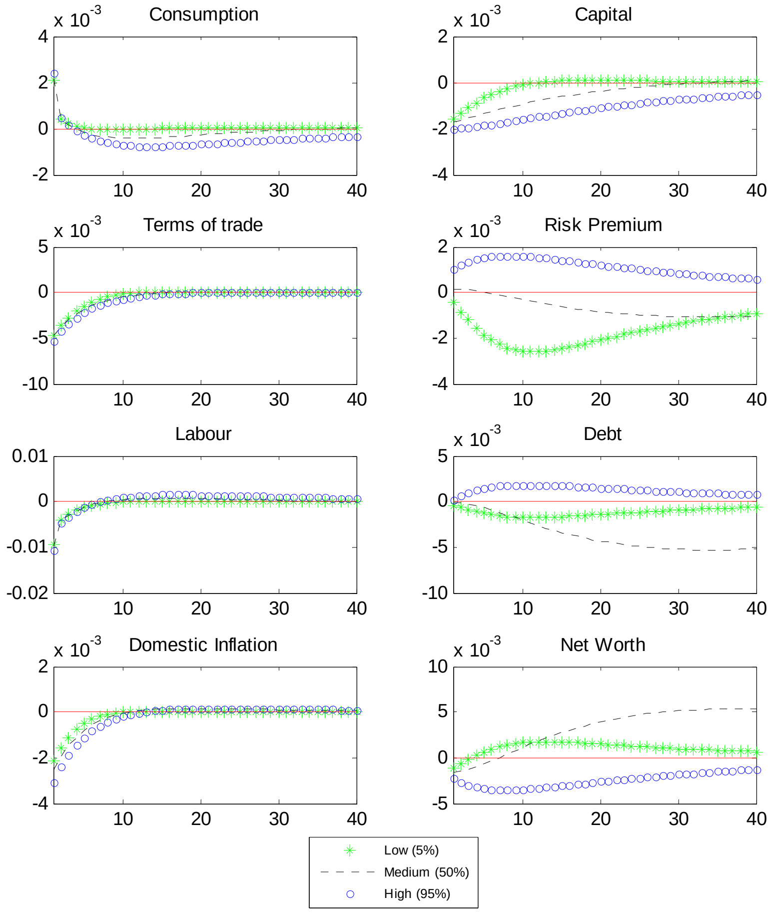
Firstly, we vary parameter μ which, as previously noted, together with δ determines the steady state debt to investment ratio. More specifically, we will consider three different debt to investment ratios: low (5%), medium (50%) and high (95%). This exercise is carried out under the following Taylor rule

$$\frac{1+i_t}{1+i} = \left(\frac{1+\pi_t}{1+\pi}\right)^{\omega_\pi} \tag{3.54}$$

with $\omega_\pi = 1.5$ and where the central bank targets CPI inflation.

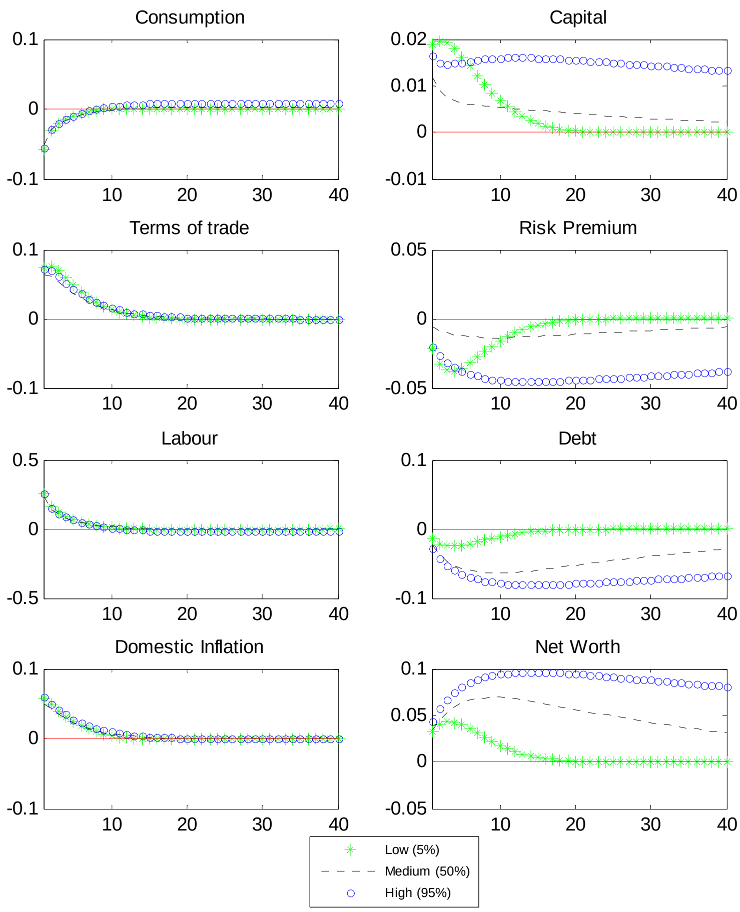
As expected the variables whose dynamics differ most across various levels of indebtedness are capital, net worth, risk premium and debt. It's more a 'financial' phenomenon. As for the other variables shown, the dynamics do not differ so much, at least in the first periods, when considering different debt to investment ratios. This is because movements in the net worth indirectly affect such variables and its effects are smaller in size and take longer.

The increase in the foreign interest rate reduces net worth and thus capital. When the level of indebtedness is high the negative effect on net worth is greater implying an increase in debt and an increase in the risk premium. The size of the net worth effect is lower for lower levels of indebtedness: in such cases debt and the risk premium decrease. The reduction in capital causes a reduction in domestic intermediate, labour and marginal cost, implying an increase in the relative price of the intermediate good and so a decrease in the terms of trade (depreciation). This implies a reduction in inflation and so, by the Taylor rule, in domestic interest rate causing an increase in consumption. Note that for sufficiently high levels of indebtedness consumption, after an initial increase, decreases and then reverts to its steady state value.



Selected IRs to a Shock to Foreign Interest Rate

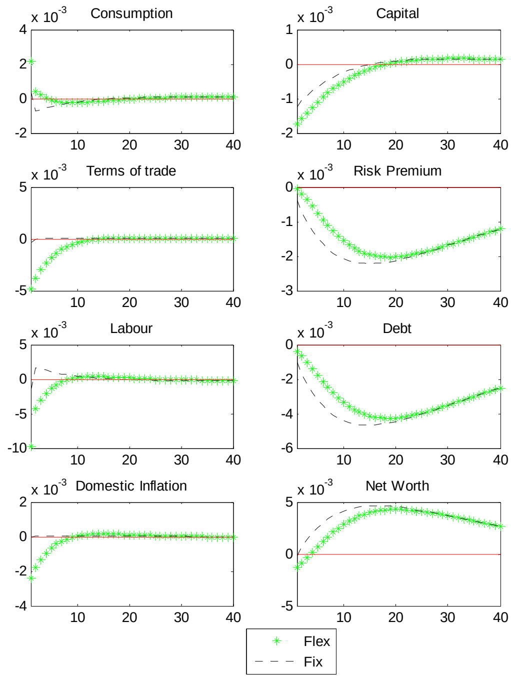
The effects of an increase in exports are almost similar to the positive foreign interest rate shock case. Summarising, an increase in exports increases, by market clearing, production of intermediate good and so both labor and capital increase. Hence the risk premium decreases, net worth increases and debt decreases. These effects are greater and more persistent the higher the debt to investment ratio. Note that this time consumption, after an initial reduction, increases for sufficiently high levels of indebtedness.



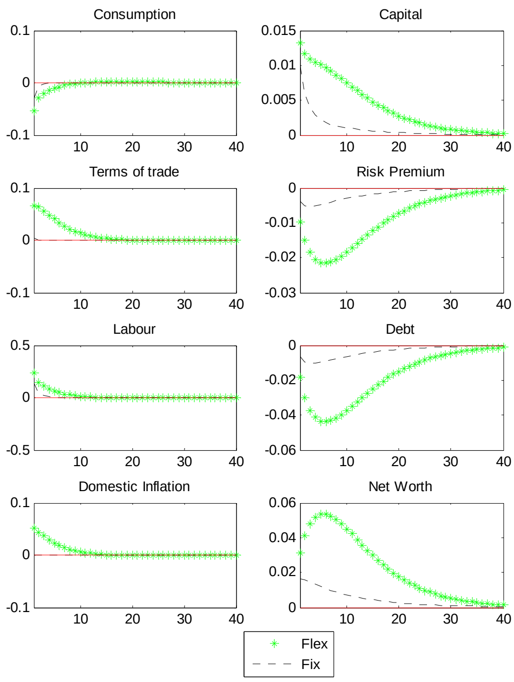
Selected IRs to a Positive Shock to Exports

3.4.3 Flexible vs Fix Exchange Rate

Then we fix a low level of indebtedness (i.e. 16%) and consider the responses to the same shocks under two alternative exchange rate regimes: *flexible* (for $\omega = 0$) and *fix* (for $\omega \rightarrow \infty$). The signs of the responses are consistent with those of the previous exercise. Nevertheless, the responses are somewhat larger in the flexible exchange rate regime. This confirms the idea that net worth effects can be amplified by a flexible exchange rate, making it a potentially welfare decreasing policy for those economies with a high level of indebtedness.



Selected IRs to a Shock to Foreing Interest Rate



Selected IRs to a Positive Shock to Exports

3.4.4 Welfare

Now, we turn to the welfare analysis. As standard in the literature, welfare is measured as the part of the utility of the representative household depending on consumption and labour, i.e.

$$W\left(C_t, N_t\right) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\sigma_2}}{1+\sigma_2}$$

The movements in consumption and labour depicted in the previous section affect welfare in an ambiguous way. Hence the need to compute welfare.

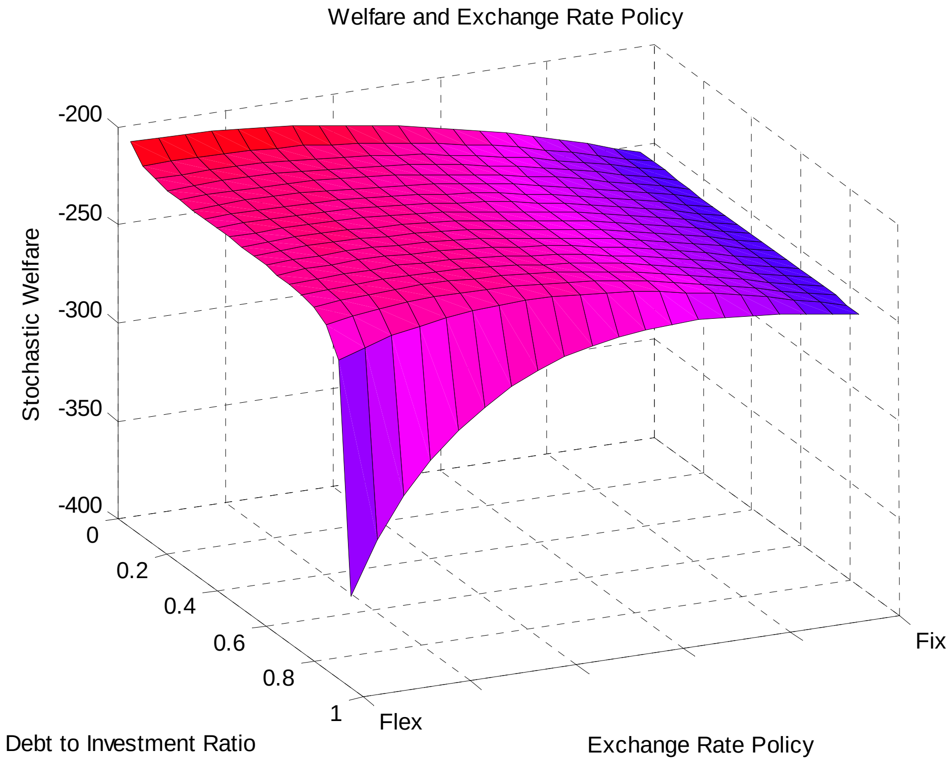
To this purpose we resort to a second order approximation solution. In fact, contrary to the standard methodology which relies upon first order approximations, second order solution enables us to take into account both the direct and indirect effects of variability on welfare. This means that we can compare welfare across policies that do not have first-order effects on the model's deterministic steady state.

Using the following Taylor rule

$$\frac{1+i_t}{1+i} = \left(\frac{1+\pi_t}{1+\pi}\right)^{\omega_\pi} \left(\frac{\varepsilon_t}{\varepsilon}\right)^{\omega_\varepsilon} \tag{3.55}$$

we compute and plot welfare against different values of (ω_ε) the coefficient measuring the weight given to the object of exchange rate stability and different degrees of liability dollarization (changing values of parameter μ).

Our results confirm Cespedes, Chang and Velasco (2002) findings: a flexible exchange rate is welfare increasing for most values of debt to investment ratio. A fix exchange rate policy is preferable only for very highly financially dollarized economies, i.e. when almost all the investment is obtained borrowing foreign assets. As for welfare implications of liability dollarization, for a given exchange rate policy welfare decreases in the debt to investment ratio.



3.5 Conclusions

The paper has built a small open economy model with frictions in order to examine the interaction between exchange rate variability and liability dollarization. In particular we examined quantitatively the welfare effects of exchange rate risk in presence of such phenomenon. In order to do so the model has been solved numerically up to the second order approximation. That is a novelty of this work. In fact, standard methodology relies upon log-linear approximations, which would miss many of the indirect implications of risk on welfare. Our measure of welfare is conditional since takes into account the transition dynamics due to the implementation of the policy rule.

As in Cespedes, Chang and Velasco (2002) fix exchange rate is associated with higher costs and a lower level of welfare. If we interpret such policy as *full dollarization* then we have to conclude that also in presence of currency substitution flexible exchange rates are preferable to full dollarization.

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Chapter 4

Monetary Policy in Canada

Luca M.F. Colantoni[1]
Bocconi University

Abstract

Using a DSGE model for a small open economy we calibrate monetary policy key parameters to match impulse responses generated by a VAR estimation of Canadian data. In this exercise we distinguish two periods: a pre inflation targeting one (from 1970 to 1990) and an inflation targeting regime (from 1991 to 2007). The obtained calibrations confirm a major concern for inflation during the second period and a decreased attention in interest rate smoothing.

Keywords: monetary policy, Small Open Economy, calibration.

JEL codes: D58, E52, F41.

[1] I am grateful to Paolo Bianchi, Andrea Carriero, Efrem Castelnuovo.

4.1 Introduction

A first objective of this work is to describe some broad characteristic of the transmission mechanism of monetary policy in Canada. To this purpose we will focus on the conventional *interest rate channel*[2]. We focus on it because this is the usual way in which monetary policy is presumed to operate, at least in economies with a fairly developed financial system. Moreover, studying the interest rate channel can provide a first step in researching alternative explanations such as, for example, the *credit channels of monetary policy*[3]. Of course, the behaviour of such variables as inflation, for example, cannot be a completely idiosyncratic Canadian feature. However, policy authorities can still have played an important role and we propose to shed some light on their actions.

Our analysis will cover a quite long period from early 1970 till early 2007. As highlighted in a recent review (see Bordo and Redish 2005) when coming to monetary policy we can consider two sub periods: what they call the *explicit monetarism of the mid 70s* and the *adoption of inflation targets in the early 90s.*

Inflation targeting was officially introduced in Canada in February 1991 with an agreement between the Bank of Canada and the federal government that set out a series of targets for reducing inflation[4]. Accordingly we will divide our analysis over the two sub samples in order to see if and in what terms this change has affected the monetary mechanism.

A second objective of the paper is to 'estimate' monetary policy key parameters in order to assess how the behaviour of the authority has effectively changed over two sub samples. This is done by means of a calibration exercise.

More precisely, following the literature in macroeconomic analysis, we will rely on two approaches: vector autoregressive (VAR) techniques and dynamic stochastic general equilibrium (DSGE) models. We think of these approaches as complementary.

[2] The interest rate channel has to do with the effects of changes in policy controlled interest rates on aggregate demand components, output and prices not due to financial market imperfections. Simplifying, within an IS-LM framework changes in interest rates are transmitted to the real cost of capital thus changing the optimal capital output ratio and the required return from investment. Thus, higher interest rates lead to a reduction in household consumption because saving becomes more attractive, payments on existing loans rise and consumers are less inclined to take new loans. A rise in interest rates also makes it more expensive for firms to finance investment thus leading to a drop in firms' demand as well. Lower aggregate demand can result in lower resource utilization and, finally, in a decrease in inflation as predicted by the Phillips curve.

[3] A recent work on the credit channel in Canada can be found in Roldos (2007). For an overview of the literature on this topic see Gertler and Gilchrist (1993), Ramey (1993), Dimsdale (1994) and Bernanke and Gertler (1995).

[4] Since the end of 1995, the target for the annual rate of total consumer price inflation (CPI) has been the 2%.

First of all, we will resort to Vector Autoregressions being them the most widely used empirical methodology to anlayse the transmission mechanism. This stream of literature, which stems from the seminal work of Sims (1980), uses VARs estimated on a relatively small number of series to understand the transmission of economic policies, mainly monetary. Among recent works, Christiano, Eichenbaum and Evans (1999) and Leeper, Sims and Zha (1998) review the results for the monetary transmission mechanism in the USA. Results show that contractionary monetary policy shocks lead to a temporary decrease in output and to a gradual decline in inflation. More recently, similar results were obtained also for the Euro area as a whole and for its individual members[5].

In section 2, we present a VAR model for the Canadian economy and describe results from its estimation on data from 1970 to 2007. We follow the standard identification scheme (see for example, Christiano, Eichenbaum and Evans (1999) and Eichenbaum and Evans (1995)). The technique can be compared to Kim (2001) who, among others[6], addresses the specific issue of international monetary policy transmission by means of VARs in order to examine how US monetary policy affects a smaller G-6 economy[7].

Being an oversimplification of reality VARs often fail in uncovering truly structural parameters and other links in transmission. To partly solve such problem, in section 3 VAR estimates are used to calibrate a DSGE model of the Canadian economy. This implies building a micro founded model of the whole economy that can then be estimated and/or simulated in order to retrieve the impulse response functions of the variables of interest. Among such works, it is worth to mention Smets and Wouters[8] (2003) who launched a project on the modelization of the euro area and estimated with Bayesian techniques a quite rich version of the standard DSGE model of a closed economy with sticky prices and wages subject to many structural shocks (such as, for example, productivity, monetary policy shocks and preference shocks). In this stream, Adjémian, Darracq Pariès and Smets (2006) explicitly considers the international dimension of the transmission mechanism by estimating a two-country model for the US-EA. Bouakez and Rebei (2005) estimate a small open economy model over the period 1973-2003 by means of maximum-likelihood methods in order to test the hypothesis that exchange rate pass-through has decreased in Canada. They find that pass-through to Canadian import prices has been stable while pass-through to Canadian consumer prices has declined in

[5] A recent review on the Euro area evidence is contained in Angeloni, Kashyap and Mojon (2003).

[6] A rapid review of the literature on this topic can be found in the same Kim (2001).

[7] Most of the series are constructed as an aggregate measure of non-US G-6 countries by geometric average of individual country measures taking out Canada because of endogenity issues.

[8] By now the number of works applying this approach is quite relevant.

recent years. Their model uses a monetary policy rule which nests the two cases of pure inflation targeting and exogenous money supply. They parametrize this rule so to impose the purely exogenous process for money growth in the first period of analysis (from 1973 to 1987) while leaving the policy parameters free to move in the second period (from 1988 onwards). However, they do not allow neither for interest rate smoothing nor for targeting of the real exchange rate. Instead, our policy rule allows for this and the parameters will be calibrated on both periods without imposing, ex ante, a shift in the policy regime. More precisely, we will use the small open economy model, namely the one by Adolfson, Laséen, Lindé and Villani (2007), and consider the Canadian small open economy against the exogenously given US economy. In performing the calibration exercise we will follow, among others, Christiano, Eichenbaum and Evans (2005) by matching DSGE simulated impulse responses with the empirical ones computed from the VAR estimation.

Section 4 reports the results of the calibration exercise which performs well in matching the impulse response of the interest rate, while does not perform so well in matching other targeted responses, especially under the pre inflation targeting regime. Over all, the obtained calibrations confirm a major concern for inflation during the second period and a decreased attention in interest rate smoothing. Finally, section 5 concludes.

4.2 VAR

4.2.1 The Empirical Model

This section presents a VAR estimate used to establish some broad characteristic of the transmission mechanism of monetary policy in Canada. The empirical impulse response will serve to calibrate the theoretical DSGE model in Section 3. In order to be more coherent with the DSGE modeling assumption of a SOE, in this section the Canadian VAR block will be estimated together with a VAR block for the big 'closed' Rest of the World whose variables are approximated by the US ones.

The empirical model is a variant of Kim (2001) which is a version of Eichenbaum and Evans (1995) where the set of variables is extended to include also the foreign country variables that can help identifying the international transmission channels. As in Kim (2001), identification of the monetary policy shock is obtained by a simple recursiveness assumption. As in Christiano, Eichenbaum and Evans (1999) the policy rule of the Bank of Canada is specified as follows

$$S_t = f(\Omega_t) + \sigma_s \varepsilon_t^s \tag{4.1}$$

where S_t is the monetary policy instrument and Ω_t is the monetary authority's information set. It contains decision variables which can be divided in two groups: those which present both contemporaneous and lagged realizations (X_{1t}) and those which enter only with their lags (X_{2t}). The random variable $\sigma_s \varepsilon_t^s$ is a monetary policy shock which, by the recursiveness assumption, is orthogonal to Ω_t. As usual, ε_t^s is normalized to have unit variance and σ_s is the standard deviation of the monetary policy shock.

If the policy rule 4.1 is a linear function and provided that its variables in Ω_t have a linear relationship with their contemporaneous and lagged values, the structural system is given by

$$G(L) Y_t = \sigma \varepsilon_t \tag{4.2}$$

where $G(L)$ is a matrix of parameters with lag operator L, Y_t is an $n \times 1$ vector of macroeconomic variables (data) and ε_t is an $n \times 1$ vector of structural shocks including the monetary policy shock ε_t^s. Finally, σ is an $n \times n$ matrix with all non-diagonal elements equal to zero. All structural shocks are normalized to have unit variance and the diagonal elements of σ are their corresponding standard deviations.

In order to extract the shock we have first to estimate the reduced form-equation (VAR)

$$Y_t = B(L) Y_{t-1} + u_t \tag{4.3}$$

where $B(L)$ is a matrix of parameters with lag operator L and is an $n \times 1$ vector of reduced disturbances.

To recover the structural parameters from the estimated parameters in the reduced form decompose $G(L)$ as

$$G(L) = G_0 + G_1(L)$$

where G_0 is a matrix with only contemporaneous parameters and $G_1(L)$ is a matrix without contemporaneous parameters. This is done because the identification scheme under consideration imposes restrictions on contemporaneous structural parameters only. Then, the structural disturbances are

$$\varepsilon_t = A_0 u_t$$
$$with\ A_0 \equiv \sigma^{-1} G_0$$

If u_t is estimated and $\sigma^{-1}G_0$ is known we can compute the structural shocks and thus the monetary policy shock ε_t^s. However, A_0 is generally unknown and, following Sims (1980), it is identified by Cholesky decomposition of the estimated reduced form residuals. Defining the variance-covariance matrices for u_t and ε_t as Σ and Λ respectively, this will result in the following relationship

$$\Sigma = A_0^{-1}\Lambda A_0^{-1}$$

where Λ is an identity matrix.

So far, the discussion has followed Kim (2001). However, since we are considering a small open economy (SOE) against the Rest of the World, it is reasonable to assume that variables of Canada do not affect the monetary policy decision and the variables of the US. Hence, following Ming Chien Lo (2003), we resort to a block-exogenous reduced form system as in Hamilton (1994) and impose this restriction. Strictly speaking, the model consists of two sub-systems: one for the Rest of the World and one for Canada. In this bilateral relationship, the US sub-system behaves as if it were a closed economy since the SOE cannot affect it, while the US variables, as the monetary policy instrument and output, can affect those of Canada. The structural system 4.2 and the reduced form system 4.3 remain true for the US and can be rewritten as

$$G^{US}(L)\,Y_t^{US} = \sigma\varepsilon_t \tag{4.4}$$

and

$$Y_t^{US} = B^{US}(L)\,Y_{t-1}^{US} + u_t^{US} \tag{4.5}$$

The reduced form sub-system of the SOE is given by

$$Y_t^{CN} = B_{US}^{CN}(L)\,Y_t^{US} + B^{CN}(L)\,Y_{t-1}^{CN} + u_t^{CN} \tag{4.6}$$

where $B_{US}^{CN}(L)$ is a matrix that includes contemporaneous coefficients while $B^{CN}(L)$ is one that does not[9].

Systems 4.5 and 4.6 show how the US monetary policy shock ε_t^s can affect the SOE via Y_t^{CN}. Since by assumption SOE variables do not affect the US variables, estimates of ε_t^s are extracted as previously outlined.

Note that, since 4.6 is in a reduced form and structural decomposition is absent, the term $B_{US}^{CN}(L)\,Y_t^{US}$ not only records how the Canadian monetary authority responds to

[9] In terms of Ω_t matrix, the foreign variables enter only in (X_{2t}) component.

the monetary policy shock of the Rest of the World, but also how the market responds to the shock. To see this, note that using 4.5 into 4.6 we have

$$Y_t^{CN} = C(L)\,Y_{t-1}^{US} + B^{CN}(L)\,Y_{t-1}^{CN} + v_t^{CN}$$

where $C(L) = B_{US}^{CN}(L)\,B^{US}(L)$ and $v_t^{CN} = u_t^{CN} + B_{US}^{CN}(L)\,A_0\varepsilon_t$. Contemporaneous and lagged policy shocks of ε_t^s affect elements in Y_t^{CN} directly via v_t^{CN} and indirectly via Y_{t-1}^{US}.

4.2.2 Description of the data

We use quarterly data for the period from 1970q1 to 2007q1 taken from IFS database from IMF. Inflation targeting has been used in Canada since February 1991. Hence, we can split the observations into two samples: a pre inflation targeting regime (before 1991q1) and the inflation targeting regime (from 1991q1 onwards).

Canadian data include Real GDP, the annualized quarterly GDP deflator, the 3 month T-bill rate[10] and the Real Effective Exchange Rate[11]. As already said, the Rest of the World data is approximated by US data which include real GDP, GDP deflator and Fed Funds rate.

As we look at economic growth (*figure A1*) in Canada since 1991, it is quite clear that the benefits expected from inflation targeting have materialized delivering a more stable environment. The business cycle is still present, but output volatility (see *Table 1*) has diminished corroborating the hypothesis that inflation targeting has also been successful as a macroeconomic stabilizer. Now, we proceed with the VAR estimation and computation of impulse responses.

4.2.3 Impulse Responses for the Canadian Economy

Being interested in the monetary transmission mechanism which is a short-run phenomenon the VAR models are estimated in levels[12].

Figures *1* and *2* report the results for a shock to interest rate for each of the two

[10] The overnight rate, which is the monetary policy instrument of the Bank of Canada, turns out to be unstable during the first period. Hence we use the T-bil rate which is highly correlated with the overnight rate.

[11] All data are seasonally adjusted.

[12] By doing so we allow for implicit cointegrsation relationships in the data without explicitly imposing cointegration. This may come at the cost of smaller efficiency in the estimation but avoids the cost of potential incosistencies if the incorrect identifications are imposed.

monetary policy regimes[13]. The dynamics of the variables are consistent with theory predictions and results are quite similar to those by Roldos[14] (2006) who, over the period from 1971 to 2005, estimated a series of vector autoregression models to characterize the dynamics of output and prices after a monetary shock in presence of financial disintermediation. We can see that the decline in output is smoother and more persistent in the inflation targeting regime (but not so significant) while it is deeper and shorter before the adoption of inflation targeting. The response in inflation is more persistent in the first period and in both periods the response of GDP deflator is even not significantly different from zero. The standard deviation of the monetary policy shocks in the second period is just 72% of the one in the first period. This decline in the importance of monetary shocks, the unexpected part of monetary policy captured by the VAR shocks is consistent with the findings of Roldos.

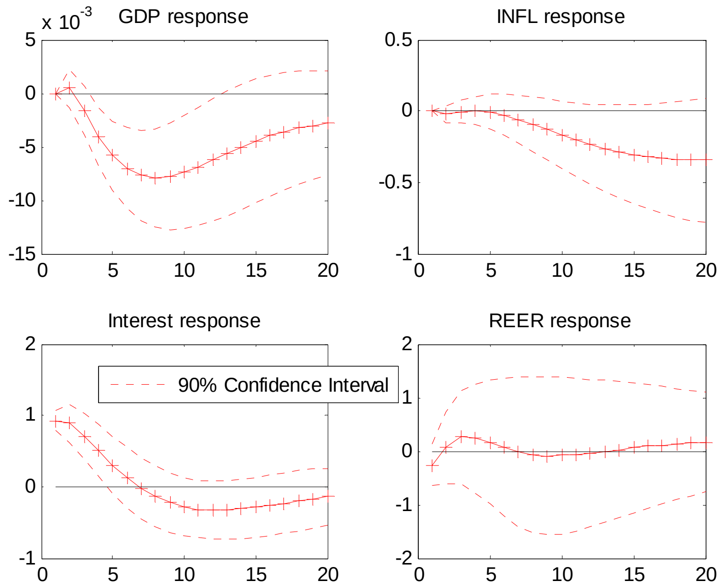

Figure 1. Canada: Impulse responses - pre inflation targeting period, 90 per cent confidence bands

[13]Impulse responses are shown together with 90% confidence interval computed by bootstrapping procedure.

[14]For his VAR, Roldos employs a measure of the exchange rate while we use real effective exchange rate.

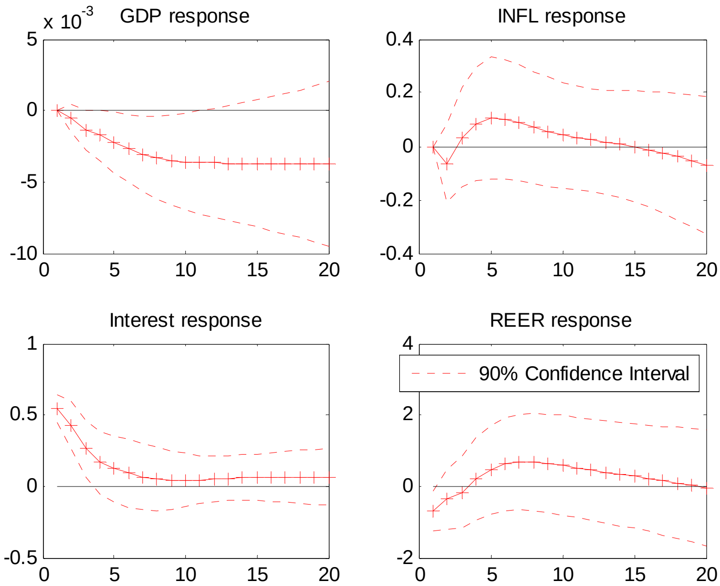

Figure 2. Canada: Impulse responses - inflation targeting period, 90 per cent confidence bands

4.3 A Small Open Economy Model

In this section we describe the DSGE model used for the Canadian economy. The model is a simplification of the one by Adolfson, Laséen, Lindé and Villani (2007). It is a SOE model where the Rest of the World is modeled as an exogenous VAR previously estimated.

The model is simplified in many aspects. First of all, since we are just interested in monetary policy and the transmission of an interest rate shock we do not include a permanent technology shock as well as all other shocks the authors introduce in order to allow for the Bayesian estimation. Another simplification is the departure from the assumption of distortionary taxation substituted with lump sum taxation.

The description of the full non linear model can be found in the above mentioned paper while in the following we mostly report its linearized version with foreign variables and parameters denoted by an asterisk.

4.3.1 Households

Our small open economy model is inhabited by a representative household whose utility depends on consumption C_t, cash holdings Q_t and leisure $(1 - H_t)$. Moreover it owns capital, $\bar{K}_t$, which rents to firms at a rental rate r_t^k and which is subject to capital utilization costs. Other rigidities are represented by internal habit persistence in consumption and investment adjustment costs. The household supplies labor monopolistically to a continuum of labor markets of measure 1 and has access to complete financial markets to allocate consumption over time. Par of its behaviour is summarized by the following equations[15]:

$$\begin{aligned} \left(1 + b^2\beta\right) c_t &= b\beta c_{t+1} + bc_{t-1} - (1 - b\beta)(1 - b)\left(\psi_t + \gamma_t^{c,d}\right) && (4.7) \\ r_t &= \pi_{t+1} - \left(\psi_{t+1} - \psi_t\right) && (4.8) \\ q_t &= -\frac{1}{\sigma_q}\left[\psi_t + \frac{R}{R-1}\right] r_{t-1} && (4.9) \end{aligned}$$

which are the log-linearised versions of the first order conditions with respect to consumption, domestic assets and cash holdings, respectively. As standard in notation, β is the subjective discount factor, c_t is 'the deviation of' consumption, $\gamma_t^{c,d}$ is the relative price of the consumption basket, b is the habit persistence parameter, ψ_t is the lagrange multiplier, r_t is the nominal interest rate on domestic bonds and π_t is the inflation in terms of domestically produced goods.

We assume a risk premium function[16] of net foreign asset holdings a_t. Calling s_t the nominal exchange rate, with Δs_t as the gross devaluation rate of the domestic currency, we have the arbitrage condition

$$\Delta s_{t+1} = r_t - r_t^* + \phi_a a_t \qquad (4.10)$$

where r_t^* is the nominal interest rate on foreign bonds. It is an uncovered interest parity augmented by the risk premium component $\phi_a a_t$.

Each household is a monopolist supplier of a differentiated labour service and faces a random probability $(1 - \zeta_w)$ of reoptimizing its wage. When households cannot reoptimize their next period wage is indexed to past CPI inflation, π_t^c, and to current inflation target, $\bar{\pi}_{t+1}^c$. Wage setting is given by

[15] As usual, variables are in terms of logarithmic deviations from steady state.

[16] For a rationalization and example of such functions, see Schmitt-Grohè and Uribe or Bergin.

$$\eta_0 \left[w_{t-1} - (\pi_{t-1} - \bar{\pi}_t^c)\right] + \eta_1 w_t + \eta_2 \left[w_{t+1} + (\pi_{t+1} - \rho_\pi \bar{\pi}_t^c)\right] + \eta_3 \left(\pi_{t-1}^c - \bar{\pi}_t^c\right) + \eta_4 \left(\pi_t^c - \rho_\pi \bar{\pi}_t^c\right) + \eta_5 \left(\psi_t - h_t\right) = 0 \tag{4.11}$$

with

$$\begin{aligned}
b_w &= \frac{\lambda_w - (1-\lambda_w)}{(1-\beta\zeta_w)(1-\zeta_w)} \\
\eta_0 &= b_w \zeta_w \\
\eta_1 &= \lambda_w - b_w \left(1 + \beta\zeta_w^2\right) \\
\eta_2 &= \beta\eta_0 \\
\eta_3 &= \eta_0 \kappa_w \\
\eta_4 &= -\beta\eta_3 \\
\eta_5 &= (1-\lambda_w)
\end{aligned}$$

and where w_t is the wage, λ_w is the constant wage mark-up and κ_w captures wage indexation.

Investment decisions are subject to adjustment costs assumed to be zero in steady state. Alternatively to investment, the household can increase the stock of capital by changing its utilization rate. Also this action is assumed to be subject to capacity utilization costs. The choices of the capital stock, investment and the capacity utilization rate are described by

$$\bar{k}_{t+1} = (1-\delta)\,\bar{k}_t + \delta i_t \tag{4.12}$$

$$P_{k,t} = \gamma_t^{i,d} + S\text{''}\left[(i_t - i_{t-1}) - \beta\,(i_{t+1} - i_t)\right] \tag{4.13}$$

$$P_{k,t} = \beta\,(1-\delta)\,P_{k,t+1} + \left[1 - \beta\,(1-\delta)\right] r_{t+1}^k + \left(\psi_{t+1} - \psi_t\right) \tag{4.14}$$

where δ is the depreciation rate, S" is the investment adjustment cost„ $P_{k,t}$ is the price of capital, $\gamma_t^{i,d}$ is the relative price of investment i_t. The first equation is the law of motion of capital, the other two come from the balancing of the costs and benefits of investment i_t.

4.3.2 Firms

The supply side has the following structure: the domestic firms produce a differentiated good with capital and labour inputs. This good is the intermediate good for the production of a final homogeneous good which is used for consumption and investment. Then, in the economy operate also exporting firms and importing firms (which import both consumption and investment goods)[17].

Prices are set à la Calvo. Each firm in each sector $(i = d, x, mc, mi)$ faces a random probability $(1 - \zeta_i)$ that it can reoptimize its price in any period. When it is not allowed to reoptimize its price the next period price is indexed to last period inflation π_t and to current inflation target $\bar{\pi}^c_{t+1}$. For example, in the case of domestic firms the price equation is

$$\begin{aligned} \pi_t - \bar{\pi}^c_t &= \frac{\beta}{1+\kappa_d\beta}\left(\pi_{t+1} - \rho_\pi \bar{\pi}^c_t\right) + \frac{\kappa_d}{1+\kappa_d\beta}\left(\pi_{t-1} - \bar{\pi}^c_t\right) - \frac{\kappa_d\beta\left(1-\rho_\pi\right)}{1+\beta}\bar{\pi}^c_t \\ &\quad - \frac{\left(1-\beta\zeta_d\right)\left(1-\zeta_d\right)}{\zeta_d\left(1+\kappa_d\beta\right)}\left(mc_t + \lambda_{d,t}\right) \end{aligned} \tag{4.15}$$

where mc_t is the marginal cost, κ_d is the indexation parameter, $\lambda_{d,t}$ is the time-varying mark up which varies over time around a constant mean and the inflation target $\bar{\pi}^c_t$ is assumed to follow an AR(1) process with persistence coefficient ρ_π. Analogous price equations can be built for the prices set by importing firms, of consumption goods $(\pi^{m,c}_t)$ and investment $\left(\pi^{m,i}_t\right)$, and exporting firms (π^x_t)

The production function is quite standard

$$y_t = \lambda_d\left[\alpha k_t + (1-\alpha)\, h_t\right] \tag{4.16}$$

where α is the usual Cobb-Douglas capital share. Accordingly, the marginal cost is

$$mc_t = \alpha r^k_t + (1-\alpha)\left(w_t + r^f_t\right) \tag{4.17}$$

where r^f_t is the gross effective rate of interest paid by firms to produce domestic good y_t reflecting the assumption that part of the intermediate firms' wage has to be financed in advance.

[17] For more details on the supply side, see Adolfson et al (2007).

Rest of the world

Following Adolfson et al. (2007) the rest of the world is modeled as a VAR on output, inflation and interest rate. For this VAR we used the same US data employed in the previous section.

4.3.3 Public authorities

Public expenditure g is assumed to follow an AR(1) process $g_t = \rho_g g_{t-1} + \varepsilon_t^g$. The government finances public spending with lump sum taxes ($g_t = T_t$) consequently there is no government debt.

The central bank adjusts the short term interest rate in response to deviations of CPI inflation from an inflation target $\left(\pi_{t-1}^c - \bar{\pi}_t^c\right)$, the output gap y_t and the real exchange rate x_t. Hence, the policy rule is modeled as

$$\begin{aligned} r_t &= \rho_r r_{t-1} + (1-\rho_r)\left[\bar{\pi}_t^c + \rho_\pi\left(\pi_{t-1}^c - \bar{\pi}_t^c\right) + \rho_y y_{t-1} + \rho_x x_{t-1}\right] \\ &\quad + \rho_{\Delta\pi}\left(\pi_t^c - \pi_{t-1}^c\right) + \rho_{\Delta y}\left(y_t - y_{t-1}\right) + \varepsilon_t^r \end{aligned} \tag{4.18}$$

where interest rate smoothing is allowed for and the time varying inflation target is assumed to follow an AR(1) process

$$\bar{\pi}_t^c = \rho_\pi \bar{\pi}_{t-1}^c + \varepsilon_t^\pi, \quad \varepsilon_t^\pi \sim (0, \sigma_\pi) \tag{4.19}$$

The log-linearized version of the real exchange rate is given by

$$\begin{aligned} x_t &= s_t + P_t^* - P_t^c \\ &= -\omega_c\left[\left(\gamma^{c,mc}\right)^{(\eta_c-1)}\right]\gamma_t^{mc,d} - \gamma_t^{x,*} - mc_t^x \end{aligned} \tag{4.20}$$

where ω_c and ω_i are the shares of imports in consumption and of imports in investment, respectively.

Then, the measure of CPI inflation rate is

$$\pi_t^c = \left[(1-\omega_c)\left(\gamma_t^{d,c}\right)^{1-\eta_c}\right]\pi_t + \left[\omega_c\left(\gamma_t^{mc,c}\right)^{1-\eta_c}\right]\pi_t^{m,c} \tag{4.21}$$

Finally, ε_t^r is an i.i.d. shock to monetary rule.

4.3.4 Market Clearing and Equilibrium

The linearized aggregate resource constraint is

$$\begin{gathered}(1-\omega_c)\left(\gamma^{c,d}\right)^{\eta_c} c_y\left(c_t+\eta_c\gamma_t^{c,d}\right)+(1-\omega_i)\left(\gamma^{i,d}\right)^{\eta_i} i_y\left(i_t+\eta_i\gamma_t^{i,d}\right) \\ +g_y g_t+y_y^*\left(y_t^*-\eta_f\gamma_t^{x,*}\right)=y_t-r^k\bar{k}_y u_t\end{gathered} \tag{4.22}$$

where η_c (η_i) is the elasticity of substitution between domestic and foreign consumption (investment) goods, $c_y, i_{y,}g_y, y_y^*$ and k_y are steady state shares of consumption, investment, government spending, foreign output and capital stock to output and the last term counts for capital utilization costs which are given by

$$\begin{aligned} u_t &\equiv \left(k_t-\bar{k}_t\right) \\ &= \frac{1}{\sigma_a} r_t^k \end{aligned} \tag{4.23}$$

where σ_a is the capital utilization cost parameter. The loan market clearing condition is

$$vwh\left(v_t+w_t+h_t\right)=\frac{\mu m}{\pi}\left(\mu_t+m_t-\pi_t\right) \tag{4.24}$$

where v_t and μ_t are the fraction of intermediate firms' wage bill that has to be financed in advance and the money growth rate which is given by

$$\mu_t=m_{t+1}+\pi_t-m_t \tag{4.25}$$

Finally net foreign assets dynamics is described by

$$\begin{aligned} a_t &= \left(c^m+i^m\right)\gamma_t^f-y^*\left(mc_t^x+\eta_f\gamma_t^{x,*}\right)-c^m\left(c_t-\eta_c\left(1-\omega_c\right)\left(\gamma^{c,d}\right)^{(\eta_c-1)}\gamma_t^{mc,d}\right) \\ &\quad -i^m\left(i_t-\eta_i\left(1-\omega_i\right)\left(\gamma^{i,d}\right)^{(\eta_i-1)}\gamma_t^{mi,d}\right)+\frac{R}{\pi}a_{t-1}\end{aligned} \tag{4.26}$$

A stationary rational expectation equilibrium is a set of stationary stochastic processes $\{c_t,\ m_{t+1},\mu_t,a_t,h_t,\bar{k}_{t+1},k_t,i_t,q_{t,}y_t,y_t^*,\pi_t,\bar{\pi}_t^c,\pi_t^c,\pi_t^{m,c},\pi_t^{m,i},\pi_t^x,w_t,\psi_t,P_{k,t},\Delta s_t,x_t,\gamma_t^{mc,d},\gamma_t^{mi,d},$ $\gamma_t^{x,*},mc_t^x,mc_t,\lambda_{d,t},\lambda_t^{m,c},\lambda_t^{m,i},\lambda_t^x\}_{t=0}^{\infty}$ satisfying previous equations given initial conditions and exogenous processes.

4.4 Calibration

In this section we first describe how we calibrate the model and then we report the results of the calibration exercise.

Being interested in the monetary policy rule we let vary only its parameters $\left\{\rho_r, \rho_\pi, \rho_y, \rho_{\Delta y}, \rho_{\Delta\pi}, \rho_x\right\}$ while pre-setting the other structural parameter to sensible values. We start setting the parameters on which the steady state depends, in such a way to match means of series of relevance. The money growth rate μ is set to 1.005 implying a steady state inflation of 2.02. The discount factor β is set to 0.99 in order to have an annual steady state real interest rate of 6.2%. The quarterly depreciation rate δ is set to 0.022 while the share of capital α is set to 0.33 delivering an investment output ratio of 20.05%. The constant in the labour disutility is set to 10.5 in order to have agents devoting 31% of their time to labour. Finally, the share of government spending on output is set to the mean value of the ratio between the two observed series, i.e. 18.16%.

The rest of the structural parameters are set to the values of the benchmark parameterization in Adolfson et al. (2007)[18].

4.4.1 Methodology

The policy parameters $\theta = \left\{\rho_r, \rho_\pi, \rho_y, \rho_{\Delta y}, \rho_{\Delta\pi}, \rho_x\right\}$ are object of our calibration exercise. In this, we follow Christiano, Eichenbaum and Evans (2005) and estimate the parameters by minimizing the square distance between selected impulse responses $irf(\theta)$ generated by the DSGE model and the VAR empirical ones irf. More precisely, we target impulse responses of output, inflation and interest rate to a shock to the interest rate and the estimator $\tilde{\theta}$ is given by

$$\tilde{\theta} \equiv \arg\min_{\theta} \left[irf - irf(\theta)\right]' W \left[irf - irf(\theta)\right] \qquad (4.27)$$

The weighting matrix W is a diagonal matrix with the inverse of sample variance along the diagonal. These variances are the same ones used to build the confidence intervals in the VAR estimation[19]. By this procedure, the estimation will try to pick parameters values such that simulated impulse responses lie inside these confidence intervals as much as possible. As already said we perform this exercise separately for each

[18] See table 3 for the principal ones.

[19] The confidence intervals were compute by bootstrapping procedure. This generates a set of possible impulse response per each pair impulse-responding variable. Variances are computed at each horizon on each of these sets of bootstrapped impulse responses.

of the two monetary regimes.

To this purpose we apply Dynare package to solve the DSGE model and compute impulse responses. We then solve the minimization problem impodsing some constraints on the values that these parameters can assume. More precisely, we let vary the parameters over the following regions:

$$\begin{aligned}
\rho_r &= [0.5, 0.99] \\
\rho_\pi &= [1.1, 2.1] \\
\rho_y &= [0, 0.3] \\
\rho_{\Delta\pi} &= [0, 0.7] \\
\rho_{\Delta y} &= [-0.05, 0.7] \\
\rho_x &= [-0.05, 0.15]
\end{aligned}$$

These regions are set in order to include most of sensible values[20] and allow for a solution of the model (i.e., existence and stationary of the steady state).

4.4.2 Results

The results of the calibration exercise are reported in *table 3*, while in *figures A5* and *A6* we plot the calibrated impulse responses together with the VAR estimated ones. The obtained calibration performs well in matching the impulse response of the interest rate in both regimes (the pre inflation targeting period and the inflation targeting regime). More problematic the matching of the impulse responses of inflation and output. In fact, a part from the case of inflation under the inflation targeting regime, the other calibrated impulse responses do not entirely lie inside the confidence interval.

In general, the calibration exercise performs better under the inflation targeting regime. This difference in performance, as well as the problem in matching output response, does not seem to be sensible to the monetary policy regime: in fact, the calibrated impulse responses of the interest rate are satisfactory under both regimes. The problem seems to lie in the lack of more inertia meaning that to improve the matching we should move other parameters.

Looking at the calibrated values (*table 3*), one may infer a stronger preference for inflation stability in the second period, a lower attention to interest rate smoothing and

[20] They turn out to be also those with most probability in the standard priors used in Bayesian methods.

almost no concern in output gap. All results are in line with what expected and with previous literature.

4.5 Conclusions

In this work we, first, described some broad characteristic of the transmission mechanism of monetary policy in Canada focusing our analysis on the conventional *interest rate channel*. The analysis covered the period from early 1970 till early 2007 which was splitted into two sub-samples to take into account of the policy change in 1991 with the official adoption of inflation targeting.

Vector autoregressive analysis shows that the decline in output is smoother and more persistent during the inflation targeting regime (but not so significant) while it is deeper and shorter before the adoption of inflation targeting. The response in inflation is more persistent in the first period and in both periods the response of GDP deflator is even not significantly different from zero.

In section 3 we preceded with estimating monetary policy key parameters in order to assess how the behaviour of the authority has effectively changed over the two sub samples. Following previous literature, see for example Christiano, Eichenbaum and Evans (2005), the estimation has been carried out by matching DSGE simulated impulse responses with the VAR estimated ones. In general, the calibration exercise performs better under the inflation targeting regime. This difference in performance does not seem to be sensible to the monetary policy regime but to a lack of inertia sufficient to match the more persistent environment of the pre inflation targeting regime. As expected, from the calibrated values one may infer a stronger preference for inflation stability in the second period, a lower attention to interest rate smoothing and almost no concern in output gap. All results are in line with what expected and with previous literature.

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4.6 Appendix

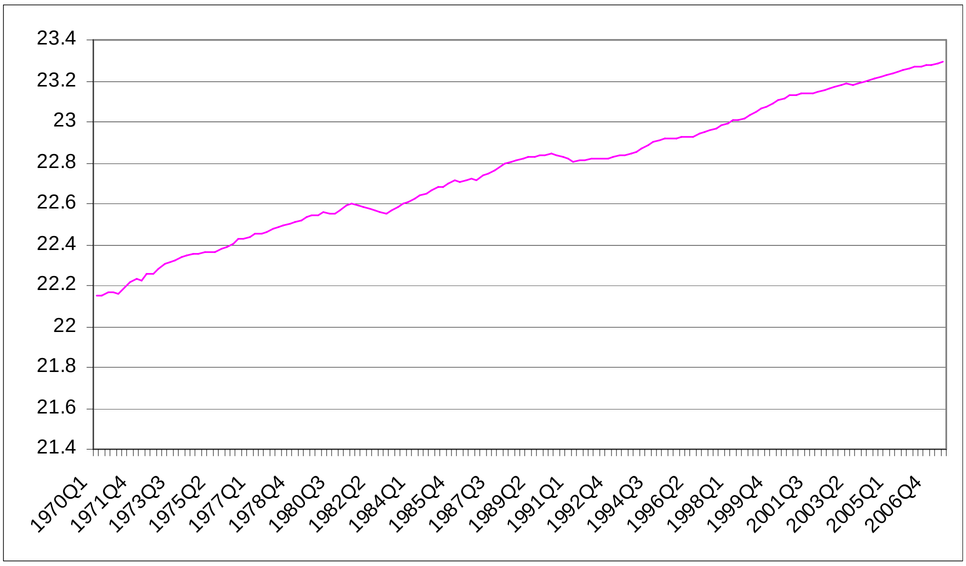

Figure A1: Canada - Log of Real GDP

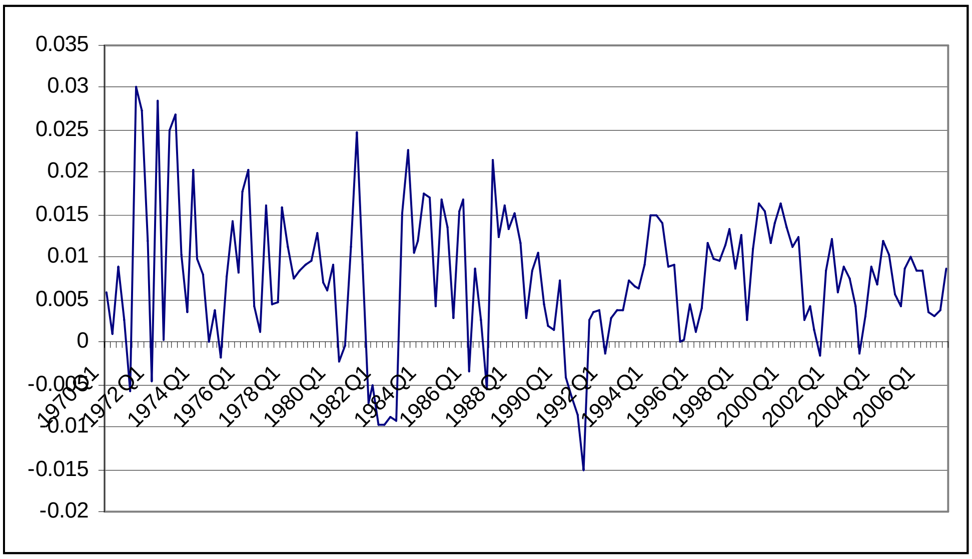

Figure A2: Canada - Real Growth

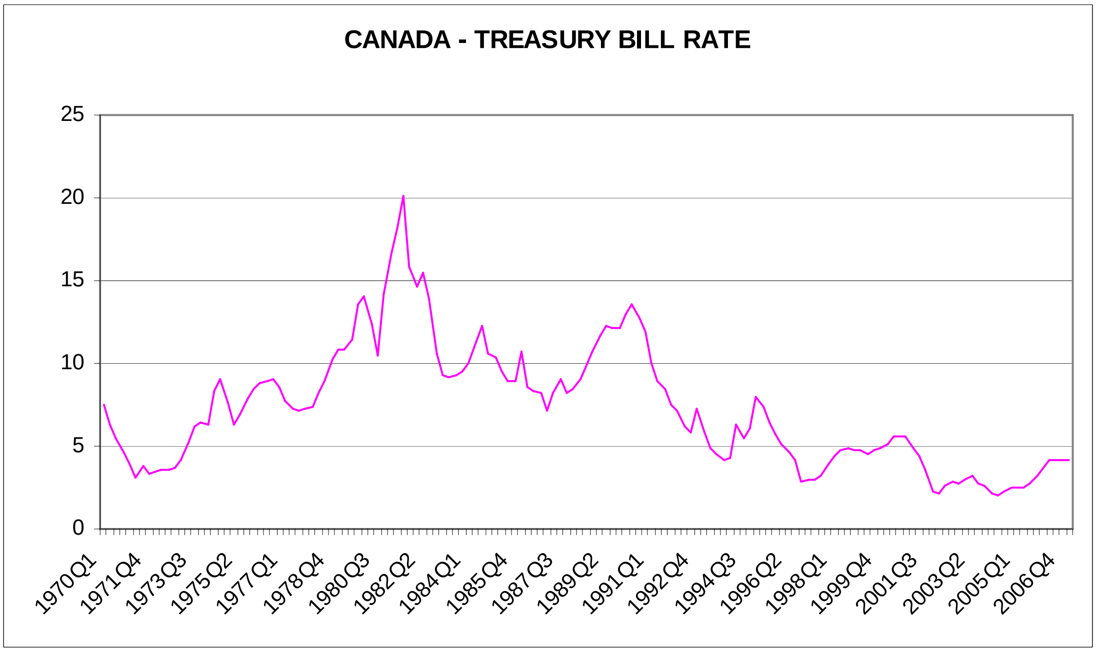

Figure A3: Canadian 3 month T-bill rate

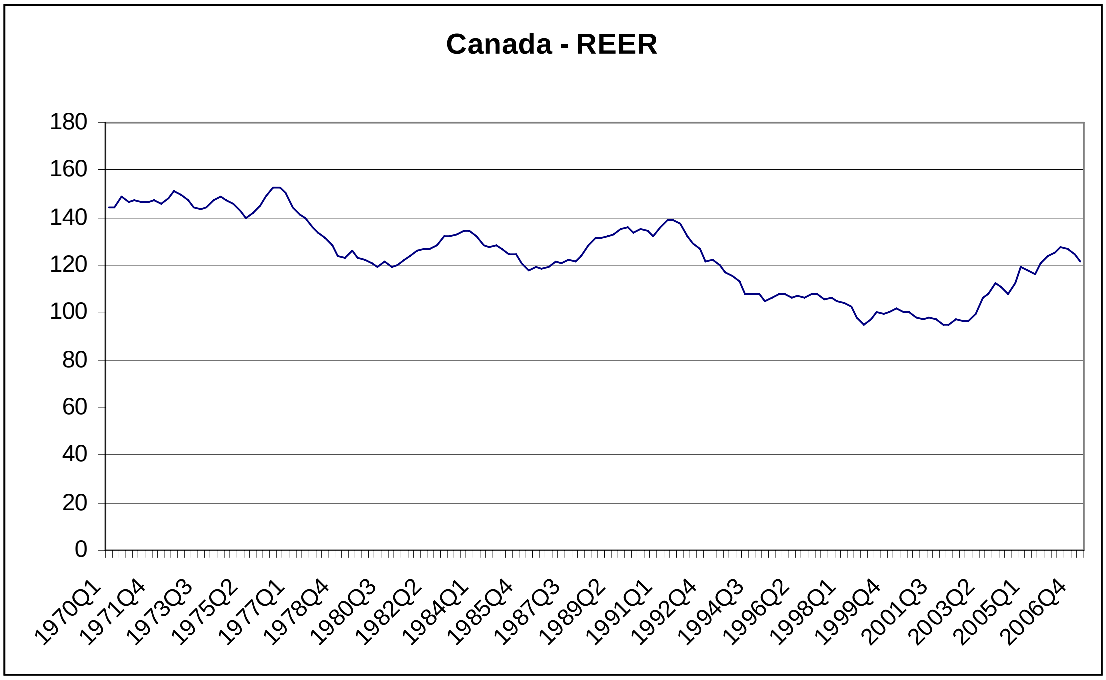

Figure A4: Canada Real Effective Exchange Rate

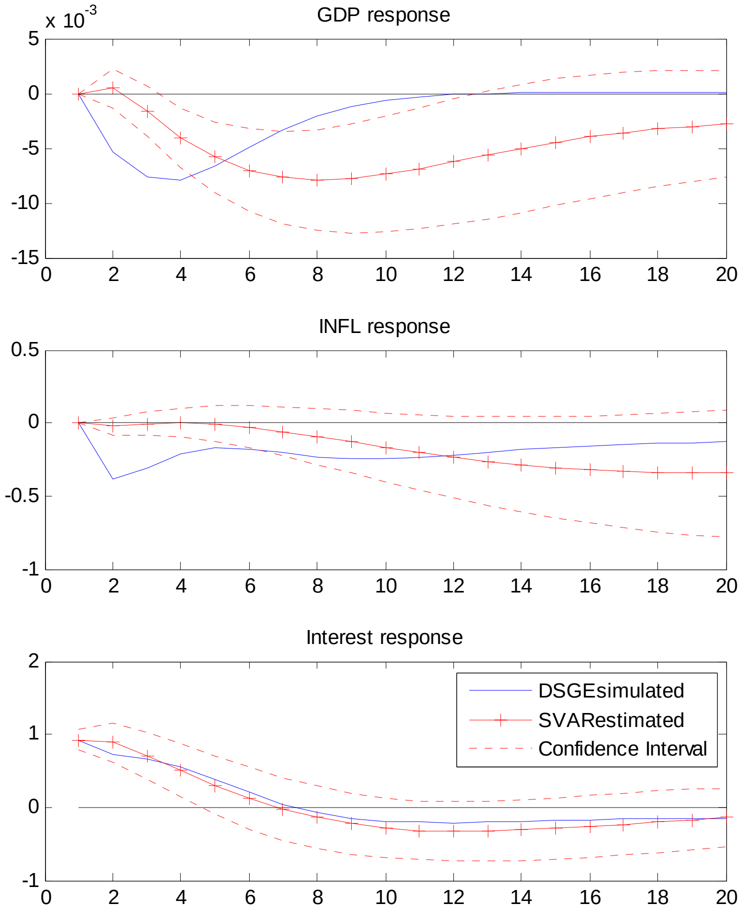

Figure A5: Selected impulse responses to a monetary shock - pre inflation targeting period

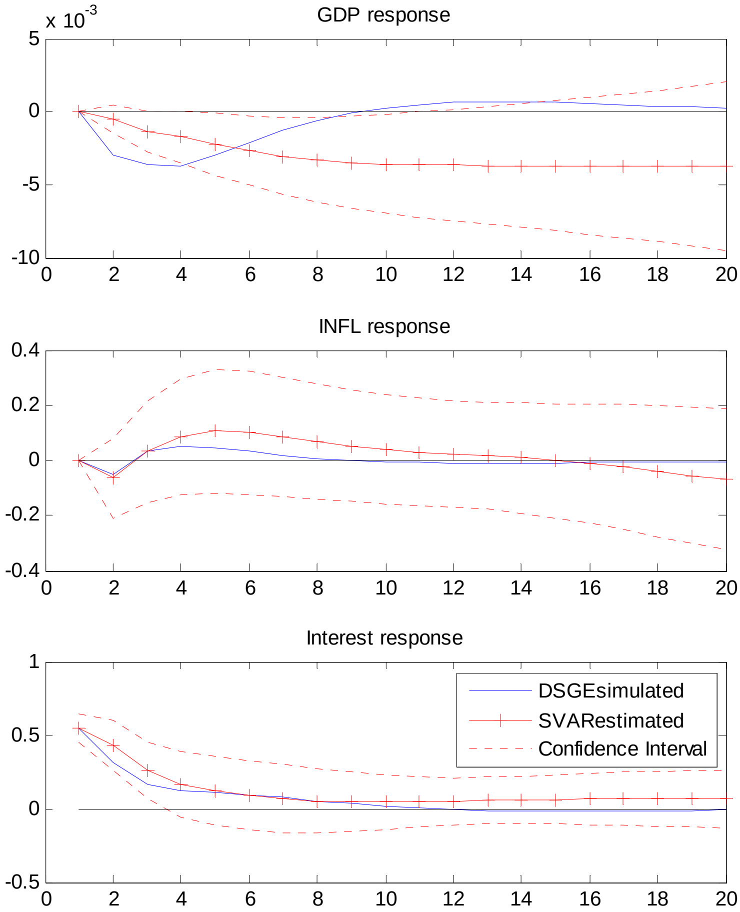

Figure A6: Selected impulse responses to a monetary shock - inflation targeting regime

Table 1: Variances and Covariances

	1970q1_1990q4					1991q1_2007q1			
	π	r	y	x		π	r	y	x
π	398.3475				π	75.248			
r	41.585	11.930			r	−10.311	3.204		
y	3.898	0.427	0.040		y	1.295	−0.197	0.024	
x	−146.656	−22.946	−1.496	110.277	x	−6.048	9.143	−0.517	144.621

Table 2: DSGE Fixed Parameters

Parameter	Assigned Value	Parameter	Assigned Value
β	0.99	$\kappa_{m,c}$	0.5
α	0.33	$\kappa_{m,i}$	0.5
μ	1.005	κ_x	0.5
δ	0.022	λ_w	1.05
b	0.650	λ_d	1.2
g_y	0.1816	$\lambda_{m,c}$	1.2
A_l	10.5	$\lambda_{m,i}$	1.2
ζ_w	0.675	λ_x	1
ζ_d	0.675	S"	7.64
$\zeta_{m,c}$	0.5	η_c	5
$\zeta_{m,i}$	0.5	η_i	1.5
ζ_x	0.5	η_f	1.5
κ_w	0.5	$\widetilde{\phi}$	0.05
κ_d	0.5	ρ_{π^c}	0.975

Table 3: DSGE Calibrated Parameters

	$1970q1\_1990q4$	$1991q1\_2007q1$
ρ_r	0.91	0.55
ρ_π	1.1	2..05
ρ_y	0.3	0
$\rho_{\Delta\pi}$	0.75	0.2
$\rho_{\Delta y}$	-0.05	0.01
ρ_x	-0.05	-0.05
value obj. fct.	0.0151255	0.00104377