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Essays in Macroeconomics

PhD in	Economics and Finance
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Acknowledgement

In short space but with full heart of gratitude, I want to express my sincere thanks for what I have received from Bocconi University. At this moment, I want to thank all professors of Bocconi University who have educated me and helped me in my studies and research.

Importantly, I would thank several important professors more during my studies at Bocconi University. Firstly, I must thank my supervisor, Professor Francesco Giavazzi, for his great advising, helps, and encouragement. Professor Giavazzi corrected me from wrong research directions for several times and motivated my works as a great master. Professor Giavazzi advised me on how to lay out an academic paper and how to improve my writting. When he was correcting my bad writings, I owed him so much. I can not express more thanks to him than enough. I will continue to benefit from his talks to me in the my rest life.

I owe specially to Professor Salvatore Nunnari for his important advising. Professor Nunnari advised me in my first paper, *Anti-Corruption and Political Sustainability in China.* At that time, I almost knew nothing on how to write an academic paper and on which aspects I must improve. He patiently advised me and I made my first step in academic researches.

I also owe a lot to Professor Dmitriy Sergeyev for his special advising. I still remember clearly what he said in our first meeting three years ago. In that meeting, he told me three ways of making innovations in economics (New Data, New Methods, New Theories). This three-new classification of innovations immediately cleared confusions in my mind. Chapter 2 on liquidity was developed during his great teaching on macro-finance and benefited a lot from his talks. I think I will always benefit from his advising in the rest of my academic career.

Last but not least, I need to thank Professor Nicola Pavoni. Professor Pavoni taught my first course on macroeconomics. When I entered the Phd program of Bocconi University, I knew very little of macroeconomics. I listened his course for the second-year PhD students twice. His teaching inspired my interests in macroeconomics and I decided worked on macroeconomics as my main field. As the director of the Ph.D program, Professor Pavoni also helped me a lot on how to survive and succeed in this program.

I must also thank Mrs Angela Baldassarre for her everyday helps during my studies. In the last sentence, I want write thanks to my referees in job markets, Professors Giavazzi, Nunnari, and Sergeyev.

Contents

1 Thesis Introduction

My thesis has three parts, corresponding to three papers I did during my studies. My fields are macroeconomics and other related topics, for example finance and politics. Below are the summaries of my three papers, which are organized into three chapters.

Chapter 1: Regional Fiscal Policy with Factor Mobility: A Tale of Two Heterogeneous Regions

Abstract: Facing asymmetric shocks, regional economies can be heterogeneous in business cycles and inefficient in factor allocation within a large country or economic area. Production factors (labor and capital) are partially mobile across regions. This paper study two aspects of regional economies:(1) how does labor mobility respond to asymmetric shocks and alleviate regional heterogeneity; (2) the interactive effects between government investment and factor mobility. This paper builds a two-region economy with one central government that decides a uniform monetary policy and regional fiscal policies. I characterize inefficiencies arising from price stickiness in the short term and productivity disparities in the long term without price frictions, both of which call for government interventions. The first inefficiency causes an asymmetric regional boom or recession; the second inefficiency originates from the naturally separated non-traded sector and leads to a sub-optimal trap. Factor mobility plays a natural, but insufficient, role in risk sharing and economic synchronization. To eliminate regional difference, I designate the government spending and mobility subsidy policy at the regional level. In calibration, I perform quantitative experiments under two regimes of factor markets (autarky and mobile) in the short and medium term. In the short-term model with price frictions, the regional fiscal policy can target regional economic adversities. In the long-term, fiscal investments to improve regional productivity can promote economy-wide production and benefit both regions if factors are mobile.

Chapter 2: Endogenous Liquidity and Macroeconomic Implications

Abstract: One decade after the financial crisis of 2007-2008, the cause of this crisis is still in debate. This paper studies the endogenous liquidity of assets in a closed economy and characterizes a general, non-parametric mechanism of economic fluctuations, including severe crises. I endogenize liquidity in terms of following aspects: (i) a new construction of the liquidity property of assets; (ii) liquidity-augmented asset pricing; (iii) liquidity creation and evolution in the financial market. I derive asset pricing with consideration of liquidity and show that asset prices, augmented by liquidity service, inflate with liquidity premium and induce distorted investments in the real economy.

Securities, which are widely used to facilitate transactions, induce new issuance and inevitably lower the pecuniary yields of the physical capital that backs them. The consequence is that asset prices and privately created liquidity become fragile, in the sense that small shocks can lead to large drops in asset prices and damage balance sheets of financial intermediaries. According to this theory, asset prices and liquidity play a central role; this points to the importance of stabilizing asset prices, not only commodity prices. To understand macroeconomy better, I present the role of liquidity in a macroeconomic environment with nominal frictions and financial intermediaries. When facing liquidity disruptions, the government has a role as an accountable liquidity planner. I analyze the associated policies in recessions that can be conducted by fiscal and monetary authorities. The present theory is consistent with the classic wisdom before the second world war.

Chapter3: Anti-Corruption and Political Sustainability in China

Abstract: What is happening underneath China's anti-corruption campaigns and how does a political regime secure itself from revolutions? This paper proposes a model studying the dynamic conflicts between two types of politicians, *Good* and *Bad*, in a non-elective regime facing a threat of revolution, a Chinese feature of political regimes. With corruption as the extra revenue extraction from citizens, citizens may overthrow the regime and replace all incumbent politicians (*Good* and *Bad*) through a revolution. Anti-corruption campaigns reduce the total political extraction and secure the regime. Corrupt or *Bad* politicians would attempt to stop the anti-corruption campaign from inside. Another type of politician, the *Good* politician, needs to balance the inside and outside risks when he decides the intensity of the anti-corruption campaign. My results show that an anti-corruption campaign not only obtains support from the *Good* politician, but also partially from the *Bad* politician. In e internal conflicts will not happen in cases with very high or low probabilities of revolution. However, circumstances in the middle range of the percentage of *Bad* politicians has a relatively higher risk of internal conflicts. Regarding one more specific characteristic of Chinese economy, a large and valuable sector of *SOEs* (State-Owned Enterprises) actually makes the regime weaker from inside, which is counter intuitive. I also extend my model to incorporate productivity and myopic shocks in the dynamic environment. Negative productivity shocks cause more intensive anti-corruption and myopic behaviors leads to more toleration and increases of corruption.

These three chapters are relatively separate so readers can read each of them without knowing other two.

Part I

Chapter 1: Regional Fiscal Policy with Factor Mobility: A Tale of Two Heterogeneous Regions

Abstract: Facing asymmetric shocks, regional economies can be heterogeneous in business cycles and inefficient in factor allocation within a large country or economic area. Production factors (labor and capital) are partially mobile across regions. This paper study two aspects of regional economies:(1) how does labor mobility respond to asymmetric shocks and alleviate regional heterogeneity; (2) the interactive effects between government investment and factor mobility. This paper builds a two-region economy with one central government that decides a uniform monetary policy and regional fiscal policies. I characterize inefficiencies arising from price stickiness in the short term and productivity disparities in the long term without price frictions, both of which call for government interventions. The first inefficiency causes an asymmetric regional boom or recession; the second inefficiency originates from the naturally separated non-traded sector and leads to a sub-optimal trap. Factor mobility plays a natural, but insufficient, role in risk sharing and economic synchronization. To eliminate regional difference, I designate the government spending and mobility subsidy policy at the regional level. In calibration, I perform quantitative experiments under two regimes of factor markets (autarky and mobile) in the short and medium term. In the short-term model with price frictions, the regional fiscal policy can target regional economic adversities. In the long-term, fiscal investments to improve regional productivity can promote economy-wide production and benefit both regions if factors are mobile.

Key Words: Regional Economies, Factor Mobility, Fiscal Policies

Mundell (1961) is famously cited for his emphasis on labor mobility as the precondition for optimal currency areas (OCAs). On commodity trade and factor mobility, Mundell (1957) says the following:

> *"Commodity movements are at least to some extent a substitute for factor movements. The absence of trade impediments implies commodity-price equalization and, even when factors are immobile, a tendency toward factor-price equalization. It is equally true that perfect mobility of factors results in factor-price equalization and, even when commodity movements cannot take place, in a tendency toward commodity-price equalization.*
> *There are two extreme cases between which are to be found the conditions in the real world: there may be perfect factor mobility but no trade, or factor immobility with unrestricted trade. The classical economist generally chose the special case where factor of production were (internationally) immobile."*

When studying economic fluctuations a large country or a monetary union, factor autarky is the standard assumption; this is to say, factor mobility is largely ignored. The literature on new open macroeconomics and monetary unions has not incorporated the impact of factor market integration (see, e.g., Obstfeld and Rogoff (1995), Corsetti and Pesenti (1997), Obstfeld and Rogoff (1999), Benigno and Benigno (2003), Gali and Monacelli (2005), and Farhi and Werning (2017a)).

Regions in large countries, such as the United States and China, and special economic zones, such as the European Monetary Union, share domestically integrated financial markets and common monetary policies. However, the real economic status and technology of any given region within these economic zones are likely to be different and are not synchronized automatically. Regarding complete financial markets, Farhi and Werning (2017a) studies one solution to the regional differences (i.e., portfolio taxes on financial assets) but also discusses the limitations when implementing portfolio taxes.

When studying regional economies, I pay extra attention to fiscal policy for several reasons. First, the output effect of government spending has been a classic issue since the emergence of macroeconomics, an issue revived recently because of large government stimulus plans, such as the American Recovery and Reinvestment Act (ARRA) in the United States and the 4 trillion RMB (aproximate 586 billion USD) stimulus package in China, after the financial crisis in 2008. Government spending must be implemented in one specific region, instead of evenly throughout the area, and the spending in one region can cause asymmetric output effects for other regions. Second, regional disparities and inequalities do exist, and one important object of fiscal policy is to mitigate regional disparities in the short and long term. Moreover, within currency unions, fiscal relationships between regions are an essential issue when studying policy effects. For example, government spending in one region is financed by nationwide factor taxes. Hence, fiscal burdens are shared among regions.

To synchronize regional economies, factor mobility and fiscal policy are indispensable. The goal of this paper is to investigate the effects of factor mobility and the associated policies in a country or an economic zone, as emphasized by Mundell in the early literature on OCAs. The present work focuses on two aspects of regional economies. First, how labor mobility responds to asymmetric shocks and alleviate the differences in regional business cycles; second, when the central government makes public investment in one region how factor mobility interacts with fiscal policy and influence the policy effects in all regions.

This paper studies a regional economy in a monetary and fiscal union, such as the United States or China. The framework and conclusions that I provide can also apply to a monetary union, such as the European Currency Union, without explicit fiscal interventions across regions; nevertheless, they can have implicit interventions across regions through the monetary channel (Reis (2016a)). To characterize economic interactions across regions, I build the model of a two-region economy, comprising a *Home* and a *Foreign* region, which are subjected to the same monetary policy and share an integrated fiscal regime. Here, an integrated fiscal regime means that the two regions share identical tax rates and there is a government deciding fiscal policies in all regions. As emphasized above, fiscal spending does not happen evenly across the two regions. To facilitate the exposition of the general framework, I present the standard model in Section 3, which is a two-sector (i.e., traded and non-traded) and two-region (i.e., *Home* and *Foreign*) model. In the standard model, all the theoretical components are present, which will be used for the following analysis.

My analysis includes two parts: the short-term model with price frictions and the long-term one, with capital but without nominal frictions. Before jumping to the consequences of nominal frictions and policy effects, I analytically characterize the first-best outcome, which are achieved with a hypothetical social planner. The first-best allocation satisfies the condition that marginal productivities are equalized across regions and sectors. The presence of price rigidities causes the first inefficiency as the result of an insufficient price adjustment in the commodity market. With the presence of a non-traded sector and flexible price, however, the perfectly decentralized allocation is not necessarily the first-best outcome. This second inefficiency originates in the non-traded sector, whose products are only produced and consumed by local households. After incorporating the non-traded sector, the market equilibrium only requires that the composite productivity of the traded and non-traded sectors equalize across regions. As a result, there is one possible combination of a low-productivity non-traded sector and a high-productivity traded sector in one region, and in another region, the combination is the opposite.

In the short term, after temporary shocks hit a region, exogenous restrictions constrain prices from flexible changes. Price rigidities cause labor market wedges in the short term. One novel result is that economic responses of the two regions to a local shock are asymmetric. I illustrate this point with technology shocks and government spending shocks. After each shock, especially a technology shock, labor wedge responses are asymmetric, which leads to opposite business cycles in two regions (i.e., one region experiences a boom and the other experiences a recession). With labor mobility, asymmetric situations are alleviated though still exist. With the connection in the labor market, labor wedges in the two regions respond in the same direction to a technology shock (i.e., boom or recession) rather than in a opposite direction; after a demand shock, the two regions also have parallel responses. When facing different economic conditions in different regions, a monetary policy can only target the average output gaps or wedges instead of targeting each regional wedge. To achieve economic synchronization across regions, I characterize structural policies through regional government spending or mobility subsidies.

Government spending, as a form of demand management, also works at the regional level. Government expenditure in one region has spillover effects not only through goods markets but also through factor markets, because higher demand and wages induce workers to work in the region where government spending occurs. By managing the demand for local products, the government policy can influence regional labor markets. This policy can also facilitate mobility across regions and achieve the same degree of labor wedges across regions. A fiscal policy that targets economic differences across regions does not solve all temporary difficulties, but it can forge symmetric regional economies for monetary policy to work on, and it can avoid the problem of monetary policy only dealing with the average economic condition.

Another problem of regional economies is the potential productivity loss arising from non-traded sectors. One goal of fiscal policy at the regional level is to eliminate the asymmetric productivity of sectors (Traded v.s. Non-traded). In this paper, I characterize one trap of factor allocation with trade balance between regions without price frictions. In this trap, the factor allocation between regions cannot automatically be the first-best outcome and fiscal interventions are desirable. Hence, in the second part, I examine the effects of fiscal spending, which has a productivity-improving effect on a targeting sector in *Home* region. To examine the adjustment effects of factor mobility, I divide policy experiments into four factor mobility contexts: $\{\text{Capital, Labor}\} \times \{\text{Mobile, Immobile}\}$. In the policy experiments based on the US economy, the government makes a public investment in the non-traded sector in *Home* region with economy-wide tax revenues. The quantitative results mark the importance of factor mobility (labor and capital) when the public investment is implemented in one region but financed by economy-wide tax revenues. When both capital and labor are mobile, government investment in one region improves the economy-wide welfare, and the improvement is larger than in other three contexts; when both factors are immobile, public investment across regions leads to negative welfare effects in all regions. This result highlights the importance of factor mobility in deciding growth and welfare effects of a local public investment.

The paper is constructed as follows. Next section summarizes the existing literature and the empirical results on labor mobility in the United States, China, and the European Union. Section 3 presents the full-fledged model with two sectors and two regions. Section 4 shows the first-best outcome and the asymmetric economic status under pricing constraints after a shock. The quantitative section, 5, demonstrates the economic sense obtained in Section 4 with numerical exercises. The extension model in Section 6 examines the allocation of two factors, capital and labor, in long-term equilibrium and also presents the nationwide effects of government investment in one region.

2 Literature Review and Background

2.1 Related Literature

There are two literature streams preceding this paper. The first one is on government spending and fiscal multipliers; the second is the literature on spatial economics with factor mobility.

Regional Fiscal Policy

Government spending and fiscal multipliers have regained attention since the financial crisis in 2008. Baxter and King (1993) is a seminal paper on government spending in general equilibrium and has some features like mine in which the government expenditures have a productivity-improving function. Woodford (2011) and Christiano, Eichenbaum, and Rebelo (2011) study the output multiplier of government spending and emphasize the role of zero lower bound in deciding the magnitude of government spending multiplier . On regional fiscal multipliers, Nakamura and Steinsson (2014) is a remarkable paper with an intrinsic framework of the international trade, assuming that there is no factor mobility across regions.

In the theoretical literature of fiscal multipliers, some add a utility term in the household utility function when they assume that government spending has welfare effects, like Christiano, Eichenbaum, and Rebelo (2011). There is one more effect of government purchases, productivity improvement, which was emphasized in papers based on the Neoclassical framework, like Barro (1990), Aschauer (1989), and Baxter and King (1993). In the second half of this paper, government expenditure is a public investment and has a productivity effect on its target sector and region. I made this setup to examine the effects and how regional economies change if the public investments and finances are across regions.

Research on fiscal unions has been prominent since the establishment of a currency union, in which the scopes of monetary and fiscal policies are separate. Alesina and Perotti (1998) studies the economic and political risks in fiscal unions and argues that a centralized fiscal policy, when it reduces the uncertainty on the tax base, may create additional uncertainty on the tax rate, named political risks. With the explicit title *Fiscal Unions*, Farhi and Werning (2017a) study cross-country risk sharing within a monetary union which is subject to nominal rigidities. Farhi and Werning (2014) investigate the labor mobility within currency union with special consideration of what causes the demand shortfalls, internal and external; they show that with a shortfall of internal demand, migration may help regional adjustment and with an external shortfall of demand, migration out of the depressed regions may produce a positive spillover for people who stay.

Factor Mobility

Literature of spatial economics, like Redding (2016) and Redding and Rossi-Hansberg (2016), and the early OCA literature (e.g., Mundell (1957) and Mundell (1961), and McKinnon (1963)) assumes factor mobility when they examine the distribution of economic activity.

Migration, i.e., labor mobility, has significant effects on an economy's long-term performance. For example, Blanchard and Katz (1992) investigate the general features of regional booms and slumps by studying the behavior of the US states over 40 years. Kennan and Walker (2011) offer a recent, detailed empirical study of migration across US states. Meanwhile, Tombe and Zhu (2017) studies the effects of migration costs in China on aggregate productivity, and finds that the decline of migration costs accounts for roughly two-fifths of aggregate labor productivity growth in China between 2000 and 2005. Finally, Perotti (2001) studies the issue of factor mobility under two fiscal regimes, centralized and decentralized, from the viewpoint of redistribution.

On the short term responses of factor market to local economic conditions, Hauser (2014) provides evidence on the dynamic behavior of net labor flows across US states after a positive technology shock and builds a two-region dynamic stochastic general equilibrium model with endogenous labor mobility and region-specific shocks. Howard (2017) documents two facts about migration and local economy: (i) within the United States, migration causes a large reduction of the unemployment rate of the receiving cities; (ii) the increasing demand for housing, typical non-traded and durable goods, explains the regional boom. To summarize, regional economic status causes factors to move not only in the long term but also in the short term, as argued in Mundell (1957) that factors, like commodities, move[1]. Farhi and Werning (2014) examine the effectiveness of labor mobility in helping macroeconomic adjustments in currency unions plagued with nominal rigidity, emphasizing the origins (internal or external) of demand imbalances, and analyze the impact on welfare. My study differs from Farhi and Werning (2014) in the analysis focuses and the explicit setup of a mobility mechanism. This paper focuses on demonstrating the connection between regions (with and without mobility) and addressing the asymmetric problem with fiscal instruments.

Fiscal actions can directly cause mobility or induce mobility by changing regional economic conditions. For example, Ellis, Barff, and Markusen (1993) find that defense-dependent scientists and engineers move farther than their non-defense-dependent counterparts and that defense-dependent blue-collar workers are also drawn toward defense-dependent locations when they do move.

On mobility, I consider region-to-region rather than rural-to-urban movement. This paper considers relative economic status between regions and the associated desirability of working or investing outside of one's home region. In the study, physical variations in mobility costs or connectivity are taken as given.

2.2 Empirical Results on Mobility

In the following paragraphs, I will briefly discuss the situations of labor mobility in three major economies, the United States, China, and the European Union, and transfer the associated empirical results from the literature to illustrate the situation in major economies. Among these three major economies, general observations say that there is significant factor mobility across regions, especially in the United States and China.

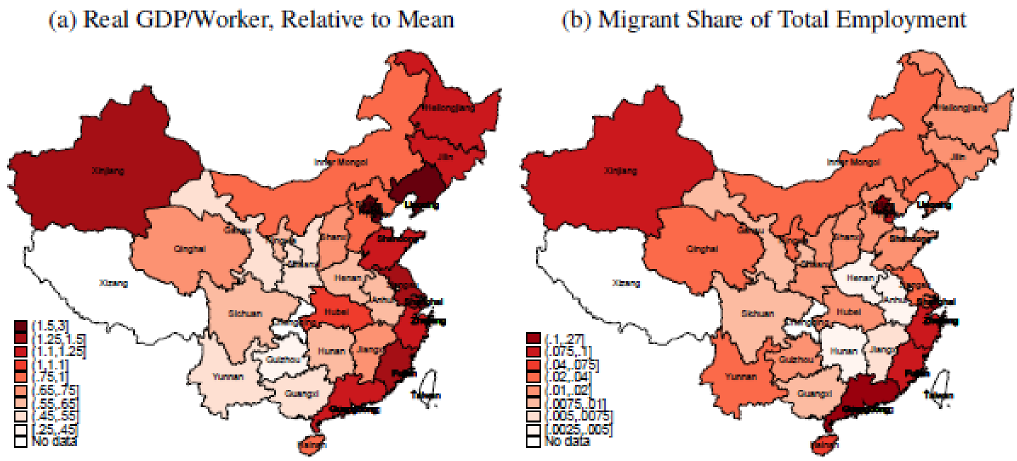
There are two empirical works of importance on labor mobility in the United States, Kennan and Walker (2011) and Hauser (2014). Hauser (2014) documents the response of labor mobility across US states to a positive technological shock. Hauser (2014) identifies technology shocks that increase relative state productivity in the long run for 226 state pairs, encompassing 80% of labor flow across US states during the period 1976–2008 and suggests that heterogeneous responses (repelling, magnetic, and hybrid) of both employment and net labor flows across states are conditional on a positive technology shock. Figure 2.2 is directly from Hauser (2014), showing that labor responds to a technology shock. Kennan and Walker (2011) develops an econometric model of optimal migration in response to income differentials across US states and shows a significant effect of expected income differences on interstate migration; in their simu-

[1]One story, that I experienced, about mobility is that when Milan was holding Expo in 2015 many temporary employees came from other cities of Italy, one of whom I knew when I used Uber and took his ride from Turin to Milan. Hence, the mobility within countries is self-evident.

lations, the elasticity of the relationship between wage and migration is roughly 0.5.

In **China**, people moving for jobs is a widespread phenomenon. According to Tombe and Zhu (2017), the decline of trade and migration costs account for roughly two-fifths of aggregate labor productivity growth in China between 2000 and 2005. Figure 2.1, from Tombe and Zhu (2017), shows the positive correlations between real, per capita GDP and the share of migrant employment across provinces.

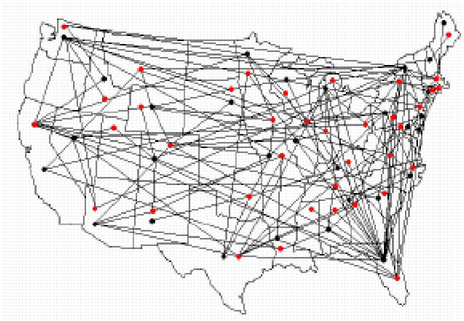
Figure 2.1: Spatial Distribution of Real Incomes and Migration in 2000



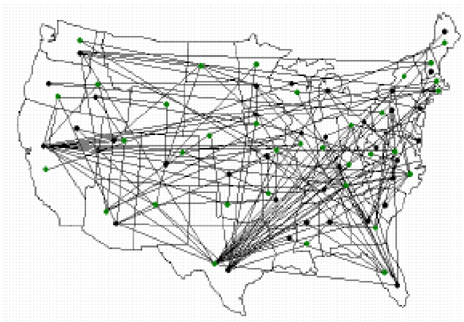
Note: Displays choropleths of relative real income levels for each of China's provinces and the migrant share of total employment. Dark reds indicate both high relative real incomes and large migrant shares of employment.

Figure 2.2: Hauser (2014) Figure 3

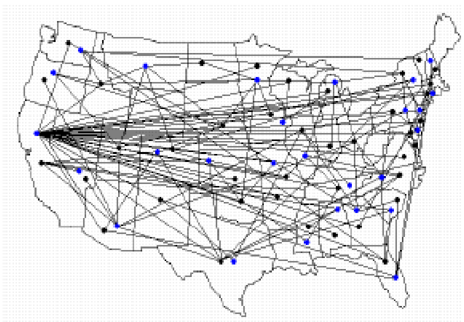
Figure 3: Graphical representation of all state pairs belonging to the three groups. Coloured dots (red, green or blue) represent the state where the productivity shock is identified, and black dots denote the respective partner state for a given pair.



(a) Repelling States



(b) Magnet States



(c) Hybrid States

Note: Hauser (2014) distinguishes three patterns for the estimated impact behavior of both net flows and employment in the state hit by a positive technology shock: Repelling states (negative impact on net flows and employment), Magnet states (positive impact), and Hybrid states (positive impact on net flows, negative impact on employment).

Countries in **the European Union** have large and significant differences in cultures, languages, climates, and institutions across countries and regions. According to a report by Bonin et al. (2008), cross-border mobility rates in the European Union have been relatively small, and mobility between regions within countries is much more pronounced. Table 2.1 gives a brief summary of the current state of geographical mobility in the European Union with the percentages of different movers. The latest research on labor mobility in the Euro Area, Basso, D'Amuri, and Peri (2018) finds that foreign-born individuals' mobility is strongly cyclical; after a comparison between the Euro Area and the United States, they found the migration response to employment shocks in the United States is definitively larger than the Euro Area.

Table 2.1: Mobility to Another Country (in the EU) over Past Ten Years-Frequency and Duration

		thereof:			
	Movers	Short Term	Medium Term	Long Term	Still in Country
EU 27	9.1	20.1	31.6	17.3	30.9
Region					
EU 15	10.2	18.2	30.3	18.0	33.5
NMS 12	5.4	37.0	43.1	12.0	8.0
Gender					
Women	8.6	21.0	31.7	16.5	30.8
Men	9.7	19.3	31.6	18.1	31.0
Age					
12-29	10.8	26.4	28.7	13.8	31.1
30-39	12.3	16.5	30.8	15.5	37.2
40-49	8.4	21.8	40.5	14.4	23.3
50-59	7.4	14.4	35.7	18.1	31.8
59+	6.9	16.7	26.0	30.8	26.5
Years of Education					
15-	7.5	19.3	22.7	20.5	37.6
16-19	7.9	21.1	31.7	17.8	29.4
20+	11.7	19.6	37.5	15.6	27.3
In Education	11.5	20.1	29.6	15.5	34.8

Notes:Weighted averages. Source: Eurobarometer 67.1 own calculation. Column 'Movers' reports population share of individuals who have lived in another country at least once during the past 10 years. Duration of moves refers to last move. Short term moves refer to periods of living in another country of less than one year, medium term moves to periods of 1-5 years, long term moves to periods of 5-10 years.

Putting mobility in an international comparison is difficult and makes little sense, because each economy, as discussed, is unique in its geographical, institutional, and cultural features. Nevertheless, one common pattern that can be inferred is that mobility within an economic union exists and creates responses to relative economic conditions between regions, especially the income differentials. This observation is intuitive and robust, which offers the empirical bedrock for the following theoretical works.

3 The Standard Model

In this section, I present the full framework of an economic union, which can be a country or a currency union, with factor mobility. When I develop this framework, I draw from existing literature on monetary unions and open macroeconomics, like Gali and Monacelli (2005), Gali and Monacelli (2008), Benigno and Benigno (2003) and Farhi and Werning (2017a).

I consider an open economy of two regions, *Home* (H) and *Foreign* (F), and two sectors, traded (T) and non-traded (N). Assume, without loss of generality, that shocks happen *Home* region, and *Foreign* represents the rest of the world. The population of each economy is normalized to one unit, but can change between *Home* region and *Foreign* region. Below I will describe the details of *Home* region. Nevertheless, the results and conclusions, that I derive in *Home* region, are applicable to *Foreign* region. For concise exposition, I use $i \in \{H, F\}$ to index regions. Variables without an asterisk superscript (*) indicate $i = H$ and with asterisk indicate $i = F$. I use $s \in \{N, T\}$ to index non-traded and traded sectors. I called the model in this section standard, because the models in Sections 3, 5, 6, and 6.4, which examine different aspects of this two-region economy, are subsets of this model. In each section, I will present the justification and limitation of the simplification from this full-fledged model. In this economy, economic shocks include preference shocks, technology shocks, and government spending shocks. To focus on the symmetry of regional shocks, I assume that these shocks occur only in *Home* region.

In contrast to the simplifying assumption of complete markets, I assume that interregional financial markets are incomplete. No risk sharing between regions is allowed, only risk-free borrowing and lending with government bonds. Given this assumption, to keep the analysis tractable, the attention is limited to one-time unanticipated shocks to the economy.

3.1 Goods Composition

There are two types of goods, traded and non-traded. Traded goods can be traded and consumed across regions; non-traded goods are supplied only by local monopolistic firms and consumed by local households. The composite consumption of traded goods and non-traded goods are aggregated by a Cobb-Douglas function as below,

$$C_t \equiv \frac{(C_{T,t})^{1-\gamma}(C_{N,t})^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}$$

where $C_{T,t}$ is an index of traded goods, and $C_{N,t}$ is the quantity consumed of a composite of non-traded varieties[2], respectively defined as

$$C_{s,t} \equiv \left[\int_0^1 C_{N,t}(j)di\right]^{\frac{\varepsilon}{\varepsilon-1}}, s \in \{T, N\}.$$

The parameter ε denotes the elasticity of substitution between varieties, and $\gamma \in (0,1)$ is interpreted as household's preferences over traded and non-traded goods. The consumption-based price indexes that correspond to the above preference specification are

$$\bar{P}_t \equiv \bar{P}_{T,t}^{1-\gamma} P_{N,t}^{\gamma}$$

with

$$P_{s,t} \equiv \left(\int_0^1 P_{s,t}(i)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}, s \in \{T, N\}.$$

The traded goods are produced in different regions and traded across regions. *Home* households combine traded goods following the formula

$$C_{T,t} \equiv \left[(1-\eta)^{\frac{1}{\mu}} C_{HT,t}^{\frac{\mu-1}{\mu}} + \eta^{\frac{1}{\mu}} C_{FT,t}^{\frac{\mu-1}{\mu}}\right]^{\frac{\mu}{\mu-1}}$$

and then

$$C_{HT,t} = (1-\eta)\left(\frac{P_{T,t}}{\bar{P}_T}\right)^{-\mu} C_{T,t} \text{ and } \quad C_{FT,t} = \eta\left(\frac{P_{T,t}^*}{\bar{P}_T}\right)^{-\mu} C_{T,t}$$

with $\bar{P}_{T,t} = \left[(1-\eta)P_{T,t}^{1-\mu} + \eta P_{T,t}^{*1-\mu}\right]^{\frac{1}{1-\mu}}$ representing the Consumption Price Index (CPI) of traded goods in *Home* region. The parameter η indexes the degree of home bias and can interpreted as a measure of openness. As η approaches zero, the share *Foreign* traded goods vanishes; as η approaches one, the purchase of local traded goods vanishes. $P_{T,t}$ and $P_{T,t}^*$ represent the Producer Price Index (PPI) of traded goods respec-

[2] A general form of CES aggregator is $C_t \equiv \left[(1-\gamma)^{\frac{1}{\mu}}(C_{T,t})^{\frac{\mu-1}{\mu}} + \gamma^{\frac{1}{\eta}}(C_{N,t})^{\frac{\mu-1}{\mu}}\right]^{\frac{\mu}{\mu-1}}$, where μ is a measure of the substitutability between traded and non-traded goods, with the corresponding CPI taking the form of $P_t \equiv \left[(1-\gamma)P_{T,t}^{1-\mu} + \gamma P_{N,t}^{1-\mu}\right]^{\frac{1}{1-\mu}}$. When $\mu = 1$, this general form reduce to a Cobb-Douglas form of aggregation.

tively from *Home* and *Foreign* regions, with the definition $P_t^i = \left[\int_0^1 P_t^i(j)^{1-\varepsilon}dj\right]^{\frac{1}{1-\varepsilon}}, i \in \{H, F\}$, where $P_t^i(j)$ represents the traded variety j produced in region i.

Households optimally minimize the expenditure on non-traded goods and traded goods produced at *Home* and *Foreign* region. This behavior implies that the demand for non-traded goods and traded goods are functions of their relative prices:

$$C_{N,t} = \gamma C_t \left(\frac{P_{N,t}}{P_t}\right)^{-1} \text{ and } \quad C_{T,t} = (1-\gamma)\, C_t \left(\frac{\bar{P}_{T,t}}{P_t}\right)^{-1},$$

$$C_{HT,t}(i) = (1-\eta)C_{T,t}\left(\frac{P_{T,t}(i)}{\bar{P}_{T,t}}\right)^{-\mu} \text{and} \quad C_{FT,t}(i) = \eta C_{T,t}\left(\frac{P^*_{T,t}(i)}{\bar{P}_{T,t}}\right)^{-\mu}.$$

$C_{iT,t}$ denotes the traded goods produced in region $i \in \{H, F\}$ and consumed by *Home* households and, analogously, $C^*_{iT,t}$ denotes the consumption of traded goods by *Foreign* households produced in the region $i \in \{H, F\}$.

For the *Foreign* households, the composite consumption is analogously written as

$$C_t^* \equiv \frac{\left(C^*_{T,t}\right)^{1-\gamma}\left(C^*_{N,t}\right)^{\gamma}}{(1-\gamma)^{1-\gamma}\,\gamma^{\gamma}}.$$

The CPI of *Foreign* is $P_t \equiv \bar{P}_{T,t}^{1-\gamma} P_{N,t}^{\gamma}$. The consumption of traded goods of *Foreign* households are combined by

$$C^*_{T,t} \equiv \left[(1-\eta)^{\frac{1}{\mu}}\, C^{*\frac{\mu-1}{\mu}}_{FT,t} + \eta^{\frac{1}{\mu}} C^{*\frac{\mu-1}{\mu}}_{HT,t}\right]^{\frac{\mu}{\mu-1}}$$

and

$$C^*_{FT,t} = (1-\eta)\left(\frac{P^*_T}{\bar{P}^*_T}\right)^{-\mu} C^*_{T,t} \text{ and } \quad C_{HT,t} = \eta\left(\frac{P_T}{\bar{P}^*_T}\right)^{-\mu} C^*_{T,t}$$

with $\bar{P}^*_{T,t}$=$\left[(1-\eta)P^{*1-\mu}_{T,t} + \eta P^{1-\mu}_{T,t}\right]^{\frac{1}{1-\mu}}$ representing the price of traded goods consumed in *Home* region. The CPI of traded goods in two regions are different because of home bias. Firms make prices of their products without discriminating against consumers in different regions.

3.2 Households

The population in each of the two regions consists of identical households. The *Home* region is inhabited by a continuum $[0, 1]$ of family members, who are identical and infinitely-lived, seeking to maximize the utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, L_t\right) \tag{3.1}$$

where β denotes the household's subjective discount factor, C_t is private consumption of a composite good, and L_t denotes labor supply or working hours. The utility function, $U(\cdot)$ satisfies $U_C^{'}(\cdot) > 0$, $U_C^{''}(\cdot) < 0$, $U\left(0, L\right) = -\infty$, $U_L^{'}(\cdot) < 0$ and $U_L^{''}(\cdot) > 0$.

Home households face a sequence of budget constraints given by

$$\bar{P}_{T,t}C_{T,t} + P_{N,t}C_{N,t} + \mathbb{E}_t\left\{Q_{t,t+1}B_{t+1}\right\} \leq B_t + W_tL_t - T_t$$

for $t = 0, 1, 2, \ldots$, where W_t is the wage in *Home* region, B_{t+1} is the nominal payoff in period $t+1$ of the portfolio at the end of period t, $Q_{t,t+1}$ is the stochastic discount factor for one period ahead nominal payoffs in period t, and T_t denotes the lump sum tax in period t. To rule out Ponzi games, the household's debt cannot exceed the present value of future income in any state of world.

The household's first order conditions are written as

$$\frac{U_C\left(C_t, L_t\right)}{\bar{P}_{T,t}} \frac{1-\gamma}{C_{T,t}} C_t = \frac{U_C\left(C_t, L_t\right)}{P_{N,t}} \frac{\gamma}{C_{N,t}} C_t = \frac{U_C\left(C_t, L_t\right)}{\bar{P}_t}, \tag{3.2}$$

$$\frac{\bar{P}_{T,t}C_{T,t}}{P_{N,t}C_{N,t}} = \frac{1-\gamma}{\gamma}, \tag{3.3}$$

$$\frac{U_C\left(C_t, L_t\right)}{U_C\left(C_{t+1}, L_{t+1}\right)} = \frac{1}{\mathbb{E}_t\left[Q_{t,t+1}\right]} \cdot \frac{\beta \bar{P}_t}{\bar{P}_{t+1}}, \tag{3.4}$$

$$\frac{U_L\left(C_t, L_t\right)}{U_C\left(C_t, L_t\right)} = \frac{W_t}{\bar{P}_t}. \tag{3.5}$$

*Foreign*households face an analogous problem, and the variables with asterisks represent counterparts in *Foreign* region. I assume that the two regions share an common and incomplete financial market. The intertemporal risk sharing leads to

$$\frac{U_C\left(C_t, L_t\right)}{U_C\left(C_{t+1}, L_{t+1}\right)} \cdot \frac{\bar{P}_{t+1}}{\bar{P}_t} = \frac{U_C\left(C_t^*, L_t^*\right)}{U_C\left(C_{t+1}^*, L_{t+1}^*\right)} \frac{\bar{P}_{t+1}^*}{\bar{P}_t^*} = \frac{1}{\mathbb{E}_t\left\{Q_{t,t+1}\right\}}. \tag{3.6}$$

$\bar{P}^*$ represents the consumption price level of *Foreign* economy. The risk sharing condition states that *Home* and *Foreign* stochastic discount factors agree in asset pricing in the common financial market. Given this risk-sharing across regions, the intratemporal consumption of households in different regions has the relation

$$\frac{U_C\left(C_t\right)}{\bar{P}_t} = \nu_F \frac{U_C\left(C_t^*\right)}{\bar{P}_t^*},$$

for all t, and where ν is a constant that generally depends on initial conditions regarding relative net asset positions. Henceforth, and without loss of generality, symmetric initial conditions are assumed (i.e., zero net foreign asset holdings and ex-ante identical environment), in which $\nu_F = 1$. The derivation of first-order conditions is in Appendix A.

3.2.1 Labor Autarky and Mobility

There are two senses of factor mobility: (i) geographic mobility among regions and (ii) factor mobility among industries (McKinnon (1963)). The factor mobility is taken in the first sense. I consider two cases of the labor market: labor autarky and imperfect mobility. In labor autarky, the production in one region only uses the labor in that region, with the supply conditions presented by equation (3.5). Given that in this economy with capital being suppressed, the labor autarky economy is identical to the two-country open economy connected only with commodity trading.

Labor mobility means that workers, as members of households, work in a region, which does not have to be their home, and earn incomes but spend little or nothing in their workplaces. In reality, there exist at least two cases under this setup. The first one is that workers commute between regions; the second one is that workers consume only the substance and save a large part of their income for the family to spend in their home region. The latter is important in a family society. Hence, households have two potential income regions. To focus on the fact that households can have incomes from working in another region, I disregard the substance consumption in working places and assume that households consume at their home regions and can have two workplaces, though with different working dis-utility.

I allow for labor mobility in the sense that some household members offer their labor services to domestic firms while others are sent to work in the foreign labor market. Aggregate labor supply is a CES-aggregator of family members' labor supply in *Home* and *Foreign* labor markets, defined as L_t and L_t^* respectively:

$$L_t = \left[(1-\eta_l)^{\frac{1}{\mu_l}} \left(L_{H,t}\right)^{\frac{\mu_l - 1}{\mu_l}} + \eta_l^{\frac{1}{\mu_l}} \left(L_{F,t}\right)^{\frac{\mu_l - 1}{\mu_l}} \right]^{\frac{\mu_l}{\mu_l - 1}}$$

$$L_t^* = \left[(1-\eta_l)^{\frac{1}{\mu_l}} \left(L_{F,t}^*\right)^{\frac{\mu_l - 1}{\mu_l}} + \eta_l^{\frac{1}{\mu_l}} \left(L_{H,t}^*\right)^{\frac{\mu_l - 1}{\mu_l}} \right]^{\frac{\mu_l}{\mu_l - 1}}$$

where $\mu_l < 0$ is a measure of the elasticity of substitution of labor services in *Home* labor market and working in *Foreign* labor market, and η_l captures the differences in the disutility attached to working in domestic versus abroad labor markets.

With the two sources of wage income, the budget constraint of households in *Home* region is written as

$$P_{T,t}C_{T,t} + P_{N,t}C_{N,t} + \mathbb{E}_t \left\{Q_{t,t+1}B_{t+1}\right\} \leq B_t + \left[W_t L_{H,t} + W_t^* L_{F,t}\right] - T_t,$$

where $W_t L_{H,t}$ is the nominal wage income from *Home* market, $W_t^* L_{F,t}$ is the nominal wage income from *Foreign* labor market with W_t^* denoting the wage in *Foreign* economy. A household's optimal labor supply conditions in the *Home* and *Foreign* labor markets are given by:

$$\frac{W_t}{\bar{P}_t} = -\frac{U_{L,t}}{U_{C,t}}\frac{\partial L_t}{\partial L_{H,t}} = -\frac{U_{L,t}}{U_{C,t}} \cdot (1-\eta_l)^{\frac{1}{\mu_l}} \left(\frac{L_{H,t}}{L_t}\right)^{-\frac{1}{\mu_l}}, \tag{3.7}$$

$$\frac{W_t^*}{\bar{P}_t} = -\frac{U_{L,t}}{U_{C,t}}\frac{\partial L_t}{\partial L_{F,t}} = -\frac{U_{L,t}}{U_{C,t}} \cdot \eta_l^{\frac{1}{\mu_l}} \left(\frac{L_{F,t}}{L_t}\right)^{-\frac{1}{\mu_l}}. \tag{3.8}$$

For *Foreign* households, the budget constraints and labor allocation among the two regions can be derived analogously. Compared to the standard labor supply condition, labor mobility across regions implies that, for a given aggregate labor supply, households optimally choose the composition, i.e., how many members should be sent to work in domestic or foreign labor markets, according to wage differentials, W_t and W_t^*.

3.3 Firms

There are two sectors in each region, producing traded and non-traded goods, respectively, with different technologies. Adjustment across sectors is in the present research scope; hence, I assume the adjustment between the two sectors within each region is perfect. In each sector, traded and non-traded, a continuum of firms, indexed by $i \in [0,1]$, is operated in *Home* economy and produce differentiated goods with the same technology

$$Y_{N,t}(i) = A_{N,t}L_{N,t}(i)^{1-\alpha},$$
$$Y_{T,t}(j) = A_{T,t}L_{T,t}(j)^{1-\alpha},$$

where $Y_{N,t}(i)$ is the output of the non-traded variety i and $Y_{T,t}(j)$ is the output of the traded variety j, $L_{s,t}, s \in \{N, T\}$ is homogeneous in types and productivity. The production function and construction in *Foreign* region are written symmetrically with an asterisk (*).

Flexible Prices Each variety faces monopolistic competition with constant substitution between varieties, as stated. The optimal pricing of each variety, whether traded or non-traded, follows the rule of marginal cost multiplying the constant markup. The aggregate consumption is denoted by $C_{s,t} = \left[\int_{i\in\mathcal{I}_l} C_{s,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}$, for $s \in \{N, T\}$. Hence, the flexible price of variety j is given by

$$P_{s,t}(j) = \frac{\varepsilon}{\varepsilon-1}\frac{W_t}{(1-\alpha)A_{s,t}L_{s,t}^{-\alpha}}, \text{ for } s \in \{N, T\}. \tag{3.9}$$

Combining the optimal condition of production with the labor supply condition (3.5), we have the equilibrium equations for the two sectors,

$$\frac{W_t}{P_t} = \frac{P_{s,t}}{P_t}\left[A_{s,t}(1-\alpha)L_{s,t}^{-\alpha}\right] = -\frac{U_L(C_t, L_t)}{U_C(C_t, L_t)}, s \in \{N, T\}$$

Recall the consumption-based price composition in *Home* region

$$P_t = \bar{P}_{T,t}^{1-\gamma} P_{N,t}^{\gamma},$$
$$\bar{P}_{T,t} = \left[(1-\eta)P_{HT,t}^{1-\mu} + \eta P_{FT,t}^{1-\mu}\right]^{\frac{1}{1-\mu}}.$$

For exposition convenience, denote $P_t \equiv \mathcal{P}(P_{HT,t}, P_{FT,t}, P_{N,t})$, describing that the consumption based price level in *Home* region is a homothetic aggregator of traded-goods prices of two regions and non-traded goods prices. For *Foreign* region, it is written as $P_t^* \equiv \mathcal{P}^*\left(P_{HT,t}, P_{FT,t}, P_{N,t}^*\right)$.

In these equations, friction-less adjustments between two sectors, traded and non-traded, are assumed to keep the two equations compatible. The perfect adjustment among sectors within regions implies that

$$\frac{P_{N,t}}{P_{T,t}} = \frac{A_{T,t}L_{T,t}^{-\alpha}}{A_{N,t}L_{N,t}^{-\alpha}}.$$

If in a closed economy and with flexible prices, the labor allocation among the two sectors would keep the relationship

$$\frac{L_{N,t}}{L_{T,t}} = \frac{\gamma}{1-\gamma}. \tag{3.10}$$

Although the labor allocation described by equation (3.10) does not directly apply to the present open economy, it offers a benchmark to examine the changes caused by trading.

Sticky Prices Firms in both sectors set prices with Calvo (1983) frictions. In every period, a randomly selected fraction $(1-\theta)$ of firms can reset their prices and the rest of firms sell their products with existing prices. Those firms that reset their prices choose a re-optimizing price to solve

$$\max_{P_{s,t}} \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left[P_{s,t}(j) Y_{s,t+k} - W_{t+k} L_{s,t+k} \right] \text{ for } s \in \{N, T\}.$$

The optimal choice of prices by a firm when it can reset the price is given by

$$\tilde{P}^i_{s,t}(j) = \frac{\varepsilon}{\varepsilon - 1} \cdot \frac{\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} MC_{t+k} Y_{t+k}(j) \right\}}{\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \delta^k Q_{t,t+k} Y_{t+k}(i) \right\}}, s \in \{N, T\} \text{ and } i \in \{H, F\}$$

which intuitively implies that the firm, having the chance to reoptimize its prices, sets the price equal to a constant markup over a weighted average of current and future marginal costs. In this environment, the aggregate price dynamics in each sector of the two regions are described by the equation

$$\left(P^i_{s,t} \right)^{1-\varepsilon} = \theta \left(P^i_{s,t-1} \right)^{1-\varepsilon} + (1-\theta) \left(\tilde{P}^i_{s,t} \right)^{1-\varepsilon}, s \in \{N, T\} \text{ and } i \in \{H, F\}$$

where $\tilde{P}^i_{s,t}$ is the optimal resetting price in period t by firms of sector $s \in \{N, T\}$ in region $i \in \{H, F\}$. For details of price derivation, please refer to Appendix B.

3.4 Government

The economy has a central government that conducts fiscal and monetary policy. The central government arranges expenditures in all regions and operates monetary policy in the integrated financial market. To simplify the analysis and without losing generality, active changes in government policy happen in *Home* region.

About the allocation of local government purchases, Gali and Monacelli (2008) assume that local public purchases are fully allocated to domestically produced goods. Nakamura and Steinsson (2014) assume the same, i.e., that the government only demands the differentiated products in its region. Their assumption implies that government purchases are only open to local firms. However, this paper is studying the regional economies within a country or an economic union, and the discrimination based on production places is limited by law. Hence, I make a slightly different assumption that regional governments' purchases have the same structure as local households, which means that government purchases are partially open to firms from other regions. Given the presence of home (or local) bias, government spending in one region increases the demand for domestic products more than for the products abroad.

For a given level of government spending, G_t, *Home* government allocates expenditures across varieties to minimize the total cost. This optimality implies

$$G_t(i) = \left[\frac{P_{s,t}(i)}{P_{s,t}}\right]^{-\varepsilon} \left[\frac{P_{s,t}}{P_t}\right]^{-1} G_t, s \in \{N, T\}.$$

The budget constraint of the national government is

$$P_t G_t = \tau^w \left(W_t L_t + W_t^* L_t^*\right) + T_t,$$

where T_t is lump-sump tax to households in two regions, and $\tau^\varepsilon = -\frac{1}{\varepsilon}$ is designated to correct monopolistic distortions. The monetary policy follows the classical Taylor rule, described as

$$i_{t+1} = \phi_\pi \pi_t + \phi_y y_t,$$

where π_t and y_t are the economy-wide CPI inflation and output gap.

3.5 Equilibrium

Definition 1. *Given fundamental parameters, productivities, and government spending, a competitive equilibrium for this economy maximizes the intertemporal welfare 3.1, with a set of factor prices in each region,* $\{W_t^i\}_{t=0}^{\infty}$, $i \in \{H, F\}$, *a set of labor allocations, consumption of traded and non-traded goods, and prices,* $\left\{L_t^i, C_{T,t}^i, C_{NT,t}^i, P_{T,t}^i, P_{NT,t}^i\right\}_{t=0}^{\infty}$, $i \in \{H, F\}$, *such that the optimization conditions for consumers and producers hold any and all markets clear.*

The economy-wide traded goods clearing condition:

$$Y_{T,t} = (1-\eta)\left(C_{T,t} + G_{T,t}\right)\left(\frac{P_{T,t}}{\bar{P}_{T,t}}\right)^{-\mu} + \eta C_{T,t}^*\left(\frac{P_{T,t}}{\bar{P}_{T,t}^*}\right)^{-\mu},$$

$$Y_{T,t}^* = \eta\left(C_{T,t} + G_{T,t}\right)\left(\frac{P_{T,t}^*}{\bar{P}_{T,t}}\right)^{-\mu} + (1-\eta) C_{T,t}^*\left(\frac{P_{T,t}^*}{\bar{P}_{T,t}^*}\right)^{-\mu};$$

Regional non-traded goods market clearing condition:

$$Y_{N,t} = C_{N,t} + G_{N,t},$$

$$Y_{N,t}^* = C_{N,t}^*;$$

Labor market clearing condition:

$$L_{N,t} + L_{T,t} + L_{N,t}^* + L_{T,t}^* \equiv \bar{L}_t.$$

In contrast to the standard models of open economies, the first feature of my model is labor mobility. When labor is allowed to supply in another region, the two regional economies are connected not only through commodity trade but also the allocation of labor, though with extra dis-utility. Regional government purchases are open to products from other regions, instead of only to the local region, which is practically assumed in the literature of open economies.

4 Characterization of Equilibria

This section studies the previous model from a positive and normative perspective by presenting the inefficiencies with and without nominal frictions. Through defining the first-best regional allocation, we will harvest convenience in the sequel when we discuss how two forces (nominal frictions and non-traded sector) cause deviations from the ideal allocation.

The first subsection defines the first-best outcome with an imaginary social planner and shows that the decentralized market equilibrium is not necessarily the first-best allocation even without nominal frictions; the second subsection shows that after an unanticipated shock, wedges emerge because of price stickiness. The short-run inefficiency calls for fiscal actions for stabilization. Two possible interventions to minimize regional disparities are proposed with simple formulas.

4.1 The First-Best Regional Allocation

Suppose that there exists a social planner who decides all labor supply and consumption across sectors and regions. The planning problem is indexed by Pareto weights $\lambda^i, i \in \{H, F\}$ for two regions. By varying these Pareto weights, I can trace out the entire constrained Pareto frontier. It maximizes the social production problem:

$$\max_{\left\{L^i_{l,t}\right\}} \sum_{i\in\{H,F\}} \lambda^i V\left(Y^i_{N,t}, Y^i_{T,t}\right)$$

subject to

$$\begin{aligned} &\underbrace{L_{T,t} + L_{N,t}}_{=L_t} + \underbrace{L^*_{T,t} + L^*_{N,t}}_{=L^*_t} = \bar{L}_t \\ &\frac{U_L\left(C^i_t, L^i_{s,t}\right)}{U_C\left(C^i_t, L^i_{s,t}\right)} = \frac{Y_t}{\bar{L}_t} \\ &\text{with } A^i_{s,t} L^i_{s,t} = Y^i_{s,t}, i \in \{H, F\} \text{ and } s \in \{N, T\}. \end{aligned}$$

$V(Y_N, Y_T)$ represents the social planner's preference to goods production. The first constraint is the economy-wide labor resource constraint and the second one is the behavioral condition of labor supply embodied in household's trade-off between working and consumption, with Y_t indicating a composite production of goods.

I assume that the social planner has the same consumption preference with households, i.e., $V(C) = U(C)$. With this assumption, the first-best equilibrium satisfies the conditions:

$$\frac{\lambda}{\lambda^*} = \frac{A^*_{N,t}}{A_{N,t}} \frac{(L_{N,t})^\alpha}{\left(L^*_{N,t}\right)^\alpha} = \frac{A^*_{T,t}}{A_{T,t}} \frac{(L_{T,t})^\alpha}{\left(L^*_{T,t}\right)^\alpha}; \tag{4.1}$$

$$\frac{\frac{\partial C_t}{\partial C_{N,t}}}{\frac{\partial C_t}{\partial C_{T,t}}} = \frac{A_{T,t}}{A_{N,t}} \frac{(L_{N,t})^\alpha}{(L_{T,t})^\alpha} = \frac{A^*_{T,t}}{A^*_{N,t}} \frac{\left(L^*_{N,t}\right)^\alpha}{\left(L^*_{T,t}\right)^\alpha}. \tag{4.2}$$

See Appendix A for the detailed derivation. The conditions given by 4.1 and 4.2 is describing the ideal allocation of labor supply at the regional level, which maximize the economy-wide welfare and individual welfare for identical households. To see the intuition, we can set $\lambda = \lambda^*$ without loss of generality. From (4.1), we see that the marginal productivity of two regions should be equal. In the extreme case where all products are homogeneous and all regions have the same Pareto weights, the marginal productivity of each sector is equal across regions.

However, in a frictionless environment, i.e., with flexible prices and balanced payments, the decentralized allocation is not necessarily the first-best equilibrium. The decentralized equilibrium can be summarized as:

$$\begin{aligned} \bar{P}^i_t &\equiv \mathcal{P}^i \left(P_{HT,t}, P_{FT,t}, P^i_{N,t}\right) \\ \frac{W^i_t}{P^i_t} &= -\frac{U_L\left(L^i_t\right)}{U_C\left(C^i_t\right)}, i \in \{H, F\}; \\ P^i_{s,t} &= \frac{W^i_t}{A_{s,t}} L^\alpha_{s,t}, i\{H, F\}, s \in \{N, T\} \end{aligned}$$

The equilibrium allocation across regions only needs to keep the conditions

$$W^i_t = -\frac{U_L\left(L^i_t\right)}{U_C\left(C^i_t\right)} \left(\bar{P}^i_{T.t}\right)^{1-\gamma} \left(P^i_{N,t}\right)^\gamma = -\frac{U_L\left(L^i_t\right)}{U_C\left(C^i_t\right)} \left[\frac{P^i_{N,t}}{P^i_{T,t}}\right]^\gamma \bar{P}_{T,t}, i \in \{H, F\}$$

$$\frac{W_t}{W^*_t} = \frac{U_L/U_C}{U^*_L/U^*_C} \cdot \left[\frac{P_{N,t}}{P_{T,t}} \times \frac{P^*_{T,t}}{P^*_{N,t}}\right]^\gamma \frac{P_{T,t}}{P^*_{T,t}} = \frac{U_L/U_C}{U^*_L/U^*_C} \cdot \left[\frac{A_{T,t}}{A_{N,t}} \times \frac{A^*_{N,t}}{A^*_{T,t}}\right]^\gamma \frac{P_{T,t}}{P^*_{T,t}}. \tag{4.3}$$

To simplify the above equations, I use the conditions of symmetric equilibrium: (i) individuals have equal ratios between consumption and working, i.e., $\frac{U_L}{U_C} = \frac{U_L^*}{U_C^*}$, and, hence, have no intention to move; (ii) there are no regional deficits, i.e., balanced payments, and $P_T = P_T^*$ in the extreme case. The equation (4.3) can be rearranged as

$$\frac{W_t}{W_t^*} = \left[\frac{A_{T,t}}{A_{N,t}} \cdot \frac{A_{N,t}^*}{A_{T,t}^*}\right]^{\gamma}.$$

In this status, individuals have no incentive to move across regions, and the balance between regions is kept. However, this status is second best because marginal productivities of sectors are not necessarily equalized.

Definition 2. The first-best equilibrium across region satisfies conditions:

$$\frac{\lambda}{\lambda^*} = \frac{A_{N,t}^*}{A_{N,t}} \frac{(L_{N,t})^{\alpha}}{\left(L_{N,t}^*\right)^{\alpha}} = \frac{A_{T,t}^*}{A_{T,t}} \frac{(L_{T,t})^{\alpha}}{\left(L_{T,t}^*\right)^{\alpha}}; \tag{4.4}$$

$$\frac{\frac{\partial C_t}{\partial C_{N,t}}}{\frac{\partial C_t}{\partial C_{T,t}}} = \frac{A_{T,t}}{A_{N,t}} \frac{(L_{N,t})^{\alpha}}{(L_{T,t})^{\alpha}} = \frac{A_{T,t}^*}{A_{N,t}^*} \frac{\left(L_{N,t}^*\right)^{\alpha}}{\left(L_{T,t}^*\right)^{\alpha}}. \tag{4.5}$$

In decentralized equilibrium, the market satisfied the condition

$$\left(A_{T,t}^* L_{T,t}^{*-\alpha}\right)^{1-\gamma} \left(A_{N,t}^* L_{N,t}^{*-\alpha}\right)^{\gamma} = \left(A_{T,t} L_{T,t}^{-\alpha}\right)^{1-\gamma} \left(A_{N,t}^{\gamma} L_{N,t}^{-\alpha}\right)^{\gamma}. \tag{4.6}$$

If the two-region economy has a symmetric distribution of labor, the market outcome in Proposition 2 implies further that

$$\left(A_{T,t}^*\right)^{1-\gamma} \left(A_{N,t}^*\right)^{\gamma} = A_{T,t}^{1-\gamma} A_{N,t}^{\gamma}.$$

Table 4.1 gives one descriptive example of the potential inefficiency of decentralized equilibrium. The productivity combination of the two regions is opposite. However, the aggregate productivity of each region and real wages are equalized. This outcome originates from the geographical property of non-traded goods and their associated market segmentation.

Table 4.1: The Second-Best Equilibrium

	Home	*Foreign*
Traded	High Productivity	Low Productivity
Non-traded	Low Productivity	High Productivity

In the decentralized equilibrium, there is one combination of a low-productivity non-traded sector with a high-productivity traded sector in one region, and in the other region, there is an opposite combination of a non-traded and a traded sector. This composition forms a stable equilibrium with balanced productivities among regions. However, this composition is sub-optimal, which I call second-best equilibrium, as shown in Table 4.1. In this second-best equilibrium, the high productivity of traded sector is absorbed by the low productivity of non-traded sector, because households need two types of goods and ask high wages when they are exposed to the high cost of non-traded goods, for example, housing and labor service; the other region has the opposite condition. Accompanying this second-best equilibrium are the different wages in two regions. When the traded goods from two regions meet in the market, they can have the same competition and lead to balanced payments across regions. In this sub-optimal situation, policy interventions are desired to induce household mobility across regions. The potential policy includes direct subsidies to mobile workers and government expenditures improving the living conditions or productivity of one region. In section 6.4, I will evaluate the effects of government purchases in one region that bring benefits to productivity conditions as one way to eliminate this inefficiency.

In the literature of open macroeconomics, the standard setup is having only one endogenous sector, like Benigno and Benigno (2003) and Gali and Monacelli (2008); if there are two sectors, the traded goods would be given as exogenous endowments. For example, Obstfeld and Rogoff (1995), Schmitt-Grohe and Uribe (2017), Farhi and Werning (2017b), and Na et al. (2018), all make the setup that the non-traded goods are endogenously produced by local firms and the traded goods are given as exogenous endowments. With Proposition 2, I provide the equilibrium condition in open economies with two endogenous sectors and point out that there exists a potential inefficiency because the two sectors are complementary to consumers, and the non-traded goods must be produced with local labor.

In the following subsections, I will investigate two possible forces that cause this regional economy to deviate from the first-best allocation. The first force is nominal frictions, which slow regional adjustments and generate regional boom and bust; the second force is non-traded sectors because they take a portion in consumption combination and are spatially separate. The position of non-traded goods makes it unique to regions and causes spatial differences in productivity even we assume away nominal frictions.

4.2 Regional Wedges in the Short Run

In the analysis of short-run market equilibrium, I express the optimum following the literature of Optimum Currency Area. Adapted from McKinnon (1963), "Optimum" describes a region within which exchange rates are pegged at one in a currency union, and monetary-fiscal policies are used to give the best resolution of three objectives: (i) the maintenance of full employment; (ii) the maintenance of balanced inter-regional payments; (iii) the maintenance of a stable internal average price level. In this subsection, from a steady and balanced regional economy, insurable and asymmetric shocks hit one or both regions and generate wedges with the presence of price frictions. The shock is idiosyncratic, insurable, and temporary.

From setups of the economic environment, we know that consumption-based price indexes are composed as

$$\begin{aligned}\bar{P}_{T,t} &= \left[(1-\eta)P_{T,t}^{1-\mu} + \eta P_{T,t}^{*1-\mu}\right]^{\frac{1}{1-\mu}}, \\ \bar{P}_t &= \left(\bar{P}_{T,t}\right)^{1-\gamma}\left(P_{N,t}\right)^{\gamma}.\end{aligned}$$

As stated previously, $\bar{P}_t \equiv \mathcal{P}\left(P_{T,t}, P_{T,t}^*, P_{N,t}\right)$ and $\bar{P}_t^* \equiv \mathcal{P}^*\left(P_{T,t}, P_{T,t}^*, P_{N,t}^*\right)$ have two common parts, the prices of traded goods from two regions, but with different weights. The regional economies are connected in the traded goods market.

Define the marginal cost of *Home* as MC_t and of *Foreign* as MC_t^*, and the marginal revenues as MR_t and MR_t^*, respectively, for *Home* and *Foreign*. Under the assumption of perfect adjustment among sectors and within the region, the marginal revenue in the traded and non-traded sector are equalized,

$$MC_t = MR_t \equiv P_{N,t}\left[A_{N,t}(1-\alpha)L_{N,t}^{-\alpha}\right] = P_{T,t}\left[A_{T,t}(1-\alpha)L_{T,t}^{-\alpha}\right].$$

The optimal allocation in each sector is written as

$$\frac{W_t}{\bar{P}_t} = \frac{P_{s,t}}{\bar{P}_t}\left[A_{s,t}(1-\alpha)L_{s,t}^{-\alpha}\right] = -\frac{U_L\left(C_t, L_t\right)}{U_C\left(C_t, L_t\right)}, s \in \{N, T\}.$$

The composite optimal allocation in *Home* region is written as

$$\frac{W_t}{\bar{P}_t} = A_{T,t}^{1-\gamma}A_{N,t}^{\gamma}\left(L_{T,t}^{1-\gamma}L_{N,t}^{\gamma}\right)^{-\alpha} = -\frac{U_L\left(C_t, H_t\right)}{U_C\left(C_t, H_t\right)}.$$

Define the wedge between nominal and real value as

$$\mathcal{W}_{s,t}^i \equiv \frac{P_{s,t}^i}{\bar{P}_t} + \frac{1}{MP_{s,t}}\frac{U_L\left(C_t^i, L_t^i\right)}{U_C\left(C_t^i, L_t^i\right)}, i \in \{H, F\}, s \in \{N, T\}.$$

The regional economy is experiencing a recession, when $\mathcal{W}_s^i > 0, i \in \{H, F\}, s \in \{N, T\}$; conversely, the economy is booming, when $\mathcal{W}_s^i < 0$; $\mathcal{W}_s^i = 0$ means no distortion. The deviation form flexible allocations and the emerging of non-zero wedges can appear with two sources: technology shock to any sector in any region and preference shock to any household in any region. The feature that I would like to emphasize is the connection of regional economy. The channel that spillover local shocks to neighbors is the composite price index, $P_t^i \left[\equiv \mathcal{P}^i \left(P_{T,t}, P^*_{T,t}, P_{N,t} \right) \right], i \in \{H, F\}$. The CPI, P_t^i, is a composition of three prices. Any decrease of one of the three prices would have economy-wide deflation of consumption goods and make the wedge of the other deviate from zero if the other two prices are fixed or sticky. The price changes of traded goods in one region have two opposite effects in another region: (i) the change of real income; and (ii) the competition of traded goods.

Under flexible prices, any asymmetric shock to the steady state of a regional economy will cause a surplus or deficit in the market of traded goods. Denote S_t as trade surpluses $(S_t > 0)$ or deficits $(S_t < 0)$ of *Home* and correspondingly the surplus or deficit of *Foreign* region is $-S_t$. The formula of S_t is given as

$$S_t = \underbrace{\eta C^*_{T,t} \left(\frac{P_{T,t}}{\bar{P}^*_{T,t}} \right)^{-\mu} P_{T,t}}_{\text{Sale to Foreign}} - \underbrace{\eta C_{T,t} \left(\frac{P^*_{T,t}}{\bar{P}_{T,t}} \right)^{-\mu} P^*_{T,t}}_{\text{Purchase from Foreign}}.$$

The prices of traded goods without nominal frictions are given by equation (3.9). Price changes would generate surplus for one region and deficit for another. However, price stickiness obstructs this process.

To examine policy facing labor wedges in different sectors and regions, I express households' indirect utility function in term of wedges. Define the indirect utility function of households in *Home* region

$$\mathcal{V}_t = U \left(C_{T,t}, C_{N,t}, L_{T,t} + L_{N,t} \right).$$

This indirect utility function can be rewritten as

$$\mathcal{V}_{C,t} = U_C \left[\mathcal{W}_{T,t} + \mathcal{W}_{N,t} \right].$$

For households in *Foreign* region, $\mathcal{V}^*_C = U^*_C \left[\mathcal{W}^*_{T,t} + \mathcal{W}^*_{N,t} \right]$. Now, consider a planning problem to maximize the welfare of economy-wide households with Pareto weight to each regions. The planning problem is characterized as

$$\max_{P^i_{s,t}} \mathbb{E}_t \sum_{k=0}^{\infty} \sum_{i \in \{H,F\}, s \in \{N,T\}} \beta^k \lambda^i \mathcal{V}^i_{t+k} \tag{4.7}$$

subject to the Euler equation of the individual household,

$$U_C \left(C^i_t, L^i_t \right) = \beta \mathbb{E}_t \left\{ \frac{U_C \left(C^i_{t+1}, L^i_{t+1} \right)}{Q_{t,t+1}} \frac{1}{\Pi^i_{t,t+k}} \right\}, i \in \{H, F\}, s \in \{N, T\} \tag{4.8}$$

where $\Pi^i_{t,t+k} \equiv \frac{\bar{P}^i_{t+k}}{\bar{P}^i_t}, i \in \{H, F\}$ is the CPI inflation between period t and $t+k$ in region i. The optimal decision of the social planner is given by the equations below:

$$\sum_{i\in\{H,F\}} \lambda^i \mathcal{V}^i_{C,t} = \mathbb{E}_t \left\{ \beta \sum_{i\in\{H,F\}} \lambda^i \mathcal{V}^i_{C,t+1} \right\}$$

$$\sum_{i\in\{H,F\}} \lambda^i U^i_{C,t} \left[\mathcal{W}^i_{T,t} + \mathcal{W}^i_{N,t} \right] = \mathbb{E}_t \left\{ \beta \sum_{i\in\{H,F\}} \lambda^i U^i_{C,t+1} \left[\mathcal{W}^i_{T,t+1} + \mathcal{W}^i_{N,t+1} \right] \right\}.$$

After the social planner incorporates the optimal trade-off of households between consumption and working, the planner smooth households' indirect utility function intertemporally. The Euler equation specifies the optimal consumption of individual households who have already incorporated the policy variable, $Q_{t,t+1}$. After merging the equation into the optimal decision of the social planner, we obtain the proposition (1) on monetary policy. When the two regions have symmetric economy at t ($U^i_{C,t} = U^{-i}_{C,t}$), the formula of $Q_{t,t+1}$ can be simplified as (4.10).

Proposition 1. *With price stickiness, the optimal monetary policy satisfies*

$$(Q_{t,t+1})^{-1} = \frac{\mathbb{E}_t \left\{ \sum_{i\in\{H,F\},s\in\{N,T\}} \Pi^i_{t,t+1} U^i_{C,t} \mathcal{W}^i_{s,t+1} \right\}}{\sum_{i\in\{H,F\},s\in\{N,T\}} \lambda^i U^i_{C,t} \mathcal{W}^i_{s,t}}; \tag{4.9}$$

if the two regions start from the symmetric economic conditions, the optimal monetary policy, (4.9), reduces to

$$(Q_{t,t+1})^{-1} = \frac{\mathbb{E}_t \left\{ \sum_{i\in\{H,F\},s\{N,T\}} \lambda^i \Pi^i_{t,t+1} \mathcal{W}^i_{s,t+1} \right\}}{\sum_{i\in\{H,F\},s\in\{N,T\}} \lambda^i \mathcal{W}^i_{s,t}}. \tag{4.10}$$

Proof. See Appendix 8.6. □

After a shock hits the steady state of this regional economy, asymmetric wedges appears. Economy-wide monetary policy can work on the weighted-average wedges but cannot eliminate the wedges of specific regions or sectors. If labor wedges are heterogeneous across regions, monetary policy is not optimal to any region. The necessary outcome is that monetary policy is not enough to one region and too much another one.

Some points of wedges are worth clarifying. First, in the economy with nominal frictions, zero is the jump point of wedges between booms and busts. In an economy without nominal frictions, the zero wedges now and in the future just require $Q = 1$, which corresponds to the Friedman rule. Second, wedges can be positive (bust) or negative (boom) and Q is always positive. How can the optimal policy keep the identity of (4.9)? When implementing the optimal policy, a case when labor wedges are positive can be avoided by increasing the nominal returns of riskless bonds since the government has restrictions on increasing the returns of debts. When implementing the optimal monetary policy, the labor wedge can always be changed to zero if its value is negative. When considering an economy with heterogeneous regions, the monetary policy can change the average wedges to zero from a negative value. Nevertheless, when labor wedges are negative it has restrictions on reducing the returns of its debts, i.e., zero lower bound.

Gali and Monacelli (2008) and Farhi and Werning (2017a) have results that monetary policy targets average wedges or output gaps. Differentiating from Gali and Monacelli (2008), my result focuses on the economy-wide optimal policy instead of small economy; my result differentiates from Farhi and Werning (2017a) with its dynamic setup of optimal policy.

4.3 Risk Sharing and Synchronization from Labor Mobility

As stated, there are insurable risks and symmetric shocks to regions. The integrated financial market does not eliminate the asymmetric economic conditions of different regions, and the monetary policy cannot be optimal to all regions at the same time. Labor mobility can alleviate this difficulty in two aspects: Ex-ante risk sharing and Ex post wedges.

For a generic region $i \in \{H, F\}$, denote L^i_i as the labor supply of workers from region i in region i (i.e., workers work domestic), and denote L^i_{-i} labor supply from region i to region $-i$ (i.e., working abroad). After multiplying the labor supply conditions (i.e., 3.7 and 3.8) with the corresponding labor share, we obtain

$$\frac{U^i_{C,t+1}}{\bar{P}^i_{t+1}} \cdot \frac{L^i_{i,t+1}}{L^i_{t+1}} = -\frac{U^i_{L,t+1}}{W^i_{t+1}} \cdot (1-\eta_l)^{\frac{1}{\mu_l}} \left(\frac{L^i_{i,t+1}}{L^i_{t+1}}\right)^{-\frac{1}{\mu_l}} \cdot \frac{L^i_{i,t+1}}{L^i_{t+1}}$$
$$\frac{U^i_{C,t+1}}{\bar{P}^i_{t+1}} \cdot \frac{L^i_{-i,t+1}}{L^i_{t+1}} = -\frac{U^i_{L,t+1}}{W^{-i}_{t+1}} \cdot \eta_l^{\frac{1}{\mu_l}} \left(\frac{L^i_{-i,t+1}}{L^i_{t+1}}\right)^{-\frac{1}{\mu_l}} \cdot \frac{L^i_{-i,t+1}}{L^i_{t+1}}.$$

Sum the two sides of the above equations and use the condition, $\frac{L^i_{i,t+1}}{L^i_{t+1}} + \frac{L^i_{-i,t+1}}{L^i_{t+1}} = 1$. We obtain the labor supply condition in period $t+1$,

$$\mathbb{E}_t \left\{ \frac{U^i_{C,t+1}}{\bar{P}^i_{t+1}} \right\} = \mathbb{E}_t \left\{ -U^i_{L,t+1} \left[\frac{(1-\eta_l)^{\frac{1}{\mu_l}}}{W^i_{t+1}} \left(\frac{L^i_{i,t+1}}{L^i_{t+1}}\right)^{-\frac{1}{\mu_l}+1} + \frac{\eta_l^{\frac{1}{\mu_l}}}{W^{-i}_{t+1}} \left(\frac{L^i_{-i,t+1}}{L^i_{t+1}}\right)^{-\frac{1}{\mu_l}+1} \right] \right\}.$$

Substitute it in (3.4), we have

$$\frac{U^i_{C,t}}{\bar{P}^i_t} = \beta\mathbb{E}_t\left\{-\frac{U^i_{L,t+1}}{\bar{P}^i_{t+1}Q_{t,t+1}}\left[\frac{(1-\eta_l)}{W^i_{t+1}}\left(\frac{L^i_{i,t+1}}{L^i_{t+1}}\right)^{-\frac{1}{\mu_l}+1}+\frac{\eta_l}{W^{-i}_{t+1}}\left(\frac{L^i_{-i,t+1}}{L^i_{t+1}}\right)^{-\frac{1}{\mu_l}+1}\right]\right\}.$$

The expectation term in the right side has the interpretation of income risk sharing or shared future. The expected incomes are not exclusively from one region. Households expect diversified income, though with more disutility of working in another region.

The labor moving across regions synchronizes the economic status between regions and reduces human-made distortions subject to the common monetary policy. Suppose a positive technology shock or a negative preference shock hits *Home* economy and prices are fixed and wages are flexible, then unemployed *Home* workers would go to *Foreign* region for jobs, and a fraction of the working hours, which were filled by *Foreign* workers, would be taken by the new migrants from *Home* region. The new supply of labor will drive down wages and lead to some degree of unemployment.

With price frictions, the market outcome is that workers have the motivation to work more with lower wage, but the firm does not want to hire more because of the fixed demand. With labor mobility across regions, unemployed workers move to its neighboring regions and drive down the wage there. Hence, labor wedges are connected through the moving of workers. The labor supply conditions and labor wedges with worker moving are given below.

For workers from an arbitrary region $i \in \{H, F\}$, define $\mathcal{W}^i_{i,t}$ and $\mathcal{W}^i_{-i,t}$ as the respective labor wedges in domestic market (i) and in abroad market ($-i$). In the region $i \in \{H, F\}$, the labor supply in two regions and the marginal productivity in nominal terms of each regions are given below. We see that the marginal productivity and wages are determined by the labor inputs from two regions.

$$\begin{aligned}
\frac{W^i_t}{\bar{P}^i_t} &= -\frac{U_L\left(C^i_t, L^i_t\right)}{U_C\left(C^i_t, L^i_t\right)}(1-\eta_l)^{\frac{1}{\mu_l}}\left(\frac{L^i_{i,t}}{L^i_t}\right)^{-\frac{1}{\mu_l}}\\
W^i_t &= P^i_t MP\left(L^i_{i,t}+L^{-i}_{i,t}\right)\\
\frac{W^{-i}_t}{\bar{P}^i_t} &= -\frac{U_L\left(C^i_t, L^i_t\right)}{U_C\left(C^i_t, L^i_t\right)}\eta_l^{\frac{1}{\mu_l}}\left(\frac{L^i_{-i}}{L^i_t}\right)^{-\frac{1}{\mu_l}}\\
W^{-i}_t &= P^{-i}_t MP\left(L^{-i}_{-i,t}+L^i_{-i,t}\right).
\end{aligned}$$

For each region, the domestic and abroad wedges are given below:

$$\mathcal{W}_{i,t}^{i} = \frac{P_t^i}{\bar{P}_t^i} + \frac{1}{MP\left(L_{i,t}^{i} + L_{i,t}^{-i}\right)} \frac{U_L\left(C_t^i, L_t^i\right)}{U_C\left(C_t^i, L_t^i\right)} (1-\eta_l)^{\frac{1}{\mu_l}} \left(\frac{L_{i,t}^{i}}{L_t^i}\right)^{-\frac{1}{\mu_l}} \tag{4.11}$$

$$\mathcal{W}_{-i}^{i} = \frac{P_t^{-i}}{\bar{P}_t^i} + \frac{1}{MP\left(L_{-i,t}^{-i} + L_{-i,t}^{i}\right)} \frac{U_L\left(C_t^i, L_t^i\right)}{U_C\left(C_t^i, L_t^i\right)} \eta_l^{\frac{1}{\mu_l}} \left(\frac{L_{-i}^{i}}{L_t^i}\right)^{-\frac{1}{\mu_l}}. \tag{4.12}$$

For region $i \in \{H, F\}$, equation (4.11) represents the labor supply condition of workers who like to work domestic (i.e., L_H households in *Home* region and L_F^* households in *Foreign* region); equation (4.12) represents the labor supply condition of who like to work abroad (i.e., L_F in *Home* regions and L_H^* in *Foreign* region). With labor mobility, the labor wedges in each region are connected through adjustments to labor supplies between L_i^i and L_{-i}^i, for $i \in \{H, F\}$. The intuition is straightforward that workers can move to other regions and households have incomes from employment in other regions. This is the mechanism of labor mobility, through which one regional economic status spreads to another region.

4.4 The Coordinated Fiscal Policy

Facing heterogeneous regional economies, coordination between monetary and fiscal policy is compulsory. As stated in Proposition 1, the optimal monetary policy targets the average labor wedges across regions, which is not necessarily optimal to an arbitrary region unless that labor wedges are equalized across regions. The fiscal policy coordinated with the optimal monetary policy aims to eliminate the wedge difference across regions. When facing asymmetric shocks on regional level, a stabilizing fiscal policy should eliminate the wedge differences generated by the uninsurable realizations of shocks and price stickiness; if both regions are facing uninsurable shocks, the stabilizing fiscal policy should eliminate the difference of wedges of regions. Then symmetric wedges, after fiscal policy, would be dealt with by monetary policy.

This subsection will analyze how fiscal policies can eliminate wedge difference across. Two fiscal policies are under investigation: regional government spending and mobility facilitation. To achieve perfect synchronization at the regional level, natural mobility is not enough because there is a utility cost for immigrant workers. Perfect synchronization can be achieved by government purchases or subsidy to mobility.

(1) Regional Government Purchases Facing regional shock, the government can correspondingly adjust purchases in the shock-hit region and achieve economic synchronization. Suppose there exists government spending in both regions and the government can freely adjust them,

$$\mathcal{W}_t^i = \frac{P_t^i}{\bar{P}_t^i} + \frac{1}{(1-\alpha)A_t^i\left(L_t^i\right)^{-\alpha}}\frac{U_L^i}{U_C^i}, i \in \{H, F\}$$
$$L_t^i = \left[D\left(\frac{C_t^i + G_t^i}{A_t^i}\right)\right]^{\frac{1}{1-\alpha}}. \tag{4.13}$$

$D(\cdot)$ is an increasing function defined by the elasticity of substitution among goods and the home bias, which are given in subsections 3.1 and 3.4. Government spending in one region does not transfer one-by-one to the demand for local production, and the specific functional form of $D(\cdot)$ depends on market parameters, i.e., market openness and substitution between local and foreign products, and what the government purchases, like traded and non-traded goods. However, local production and employment is an increasing function of government purchases.

When fiscal policy targets wedge differences, government spending in region i, G_t^i, satisfies the formula:

$$\max\left\{0, \mathcal{W}_t^i - \mathcal{W}_t^{-i}\right\} = 0.$$

When considering labor mobility, the regional spending is written as

$$\max\left\{0, \mathcal{W}_{i,t}^i - \mathcal{W}_{i,t}^{-i}\right\} = 0.$$

The government purchases aiming to equalize wedges at the regional level are defined by equation

$$\underbrace{\frac{P_t^{-i}}{\bar{P}_t^{-i}} - \frac{P_t^i}{\bar{P}_t^i}}_{\text{Difference in Goods Demand}} = \underbrace{\frac{U_L\left(C_t^i, L_t^i\right)}{MPL_t^i} \cdot \frac{1}{U\left(C_t^i, L_t^i\right)} - \frac{U_L\left(C_t^{-i}, L_t^{-i}\right)}{MPL_t^{-i}} \cdot \frac{1}{U\left(C_t^{-i}, L_t^{-i}\right)}}_{\text{Difference in Labor Spply}}. \tag{4.14}$$

The channel, through which government purchases affect regional demand, has been given by equation (4.13). The right-hand side of equation (4.14) is describing the difference in goods demands which are governed by prices and the left-hand side is describing labor supply given the demand distribution across regions.

The difference of regional government purchases with labor mobility lies in that the target is labor wedges of working domestic. This target also automatically eliminate the difference of labor wedges of working abroad, $\mathcal{W}_{i,t}^i - \mathcal{W}_{i,t}^{-i}$. These equations says that government purchases should arranged in relatively bust regions $(\mathcal{W}_t^i - \mathcal{W}_t^{-i} > 0)$ and the achieve the outcome that the relative recessions disappear. The unique magnitude of government purchases can be calculated for each circumstance because labor inputs are strictly increasing function of government purchases.

Remark 1. Regional purchases are smaller when considering mobility than not considering mobility.

Remark 1 is the result derived by one step further from the intuition that with labor mobility the local labor wedges are connected and closer between regions than labor autarky. For policy implication, it is worthy to point out that the smaller government purchases implied by labor mobility is desirable.

(2) Mobility Facilitation The second way to achieve economic synchronization among regions is to facilitate mobility. Economic recessions will not be trapped in the origin region, and economic booms will be accessible to other regions within the country or economic union. Suppose the government offers mobility subsidy (*ms*) to migrant workers who are from region i and working in $-i$ region, the labor

$$\frac{W_t^{-i}}{\bar{P}_t^i} = -\frac{U_{L,t}^i}{U_{C,t}^i} \cdot (ms_t \cdot \eta_l)^{\frac{1}{\mu_l}} \left(\frac{L_{-i,t}^i}{L_t^i} \right)^{-\frac{1}{\mu_l}} \text{ for } i \in \{H, F\}. \tag{4.15}$$

The labor wedges of working abroad is rewritten with

$$\mathcal{W}_{-i}^i = \frac{P_t^{-i}}{\bar{P}_t^i} + \frac{1}{MP\left(L_{-i,t}^{-i} + L_{-i,t}^i\right)} \frac{U_L\left(C_t^i, L_t^i\right)}{U_C\left(C_t^i, L_t^i\right)} \cdot (ms_t \cdot \eta_l)^{\frac{1}{\mu_l}} \left(\frac{L_{-i,t}^i}{L_t^i} \right)^{-\frac{1}{\mu_l}}.$$

The optimal fiscal policy impose the condition $\mathcal{W}_{-i,t}^i = \mathcal{W}_{i,t}^i$, which means that households in each regions have the intensity of motivation of working abroad. In each period, the unique *ms* can be derived numerically by comparing $\mathcal{W}_{-i}^i$ and $\mathcal{W}_i^{-i}$.

5 Model Simulations: Short Term Effects

The short-term model is limited to have only one time-varying factor, labor, and sticky prices. In this section, I quantitatively investigate the effects of shock and policy in the short term. To be clear, the exercises are meant to demonstrate the model mechanisms and implications based on parameter values, which are chosen to be as close as possible to a real economy. The real economy chosen is the US economy, as the parameter values estimated of the United States are most available. I choose parameters as realistic as possible and point out the literature sources.

5.1 Specifications and Parameterization

I use the standard form of the utility function, CRRA. In this utility specification, consumption and labor are separable. They are therefore neither complements nor substitutes. The period utility function takes the form

$$U\left(C_t, L_t\right) = \frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{L_t^{1+\varphi}}{1+\varphi}.$$

The intertemporal optimal choice of consumption leads to the Euler equation:

$$\frac{C_t^{-\sigma}}{\bar{P}_t} = \beta \mathbb{E}_t \left\{ \left(1+i_t\right) \frac{\left(C_{t+1}\right)^{-\sigma}}{\bar{P}_{t+1}} \right\};$$

for Households in *Foreign* region, the Euler equation is

$$\frac{C_t^{*-\sigma}}{\bar{P}_t^*} = \beta \mathbb{E}_t \left\{ \left(1+i_t\right) \frac{\left(C_{t+1}^*\right)^{-\sigma}}{\bar{P}_{t+1}^*} \right\},$$

where $1 + i_t = \left[\mathbb{E}_t \left\{Q_{t,t+1}\right\}\right]^{-1}$ is the riskless one-period nominal interest rate. For tractability, I reduce the model to one sector in each region and aggregate the consumption of non-traded goods and home bias to locally-produced traded goods into an enlarged home bias. This simplification does not change the structure of the model or lose the essence of the regional economy[3]. The dynamic model of Farhi and Werning (2017a) also aggregated the non-traded sector into home bias.

Without labor mobility, households only work and collect income from the local labor market. The optimal choice regarding the intratemporal trade-off between consumption and labor supply yields the labor supply equations:

$$\chi L_t^{\varphi} C_t^{\sigma} = \frac{W_t}{\bar{P}_t}$$
$$\chi C_t^{*\sigma} L_t^{*\varphi} = \frac{W_t^*}{\bar{P}_t^*}.$$

In the circumstance of labor mobility, the labor supply and allocation conditions of

[3]On this point, I owe special thanks to Prof. Tommaso Monacelli for telling me that the existence of non-traded goods and home bias have the similar nature in modeling.

residents in two regions are given as follows,

$$\text{for Home Residents}\begin{cases}\frac{W_t}{P_t}=\chi L_t^{\varphi}C_t^{\sigma}\cdot(1-\eta_l)^{\frac{1}{\mu_l}}\left(\frac{L_{H,t}}{L_t}\right)^{-\frac{1}{\mu_l}} & \text{working local (in Home region)}\\ \frac{W_t^*}{P_t}=\chi L_t^{\varphi}C_t^{\sigma}\cdot\eta_l^{\frac{1}{\mu_l}}\left(\frac{L_{F,t}}{L_t}\right)^{-\frac{1}{\mu_l}} & \text{working abroad (in Foreign region)}\end{cases},$$

$$\text{for Foreign Residents}\begin{cases}\frac{W_t^*}{P_t^*}=\chi L_t^{*\varphi}C_t^{*\sigma}\cdot(1-\eta_l)^{\frac{1}{\mu_l}}\left(\frac{L_{F,t}^*}{L_t^*}\right)^{-\frac{1}{\mu_l}} & \text{working local (in Foreign region)}\\ \frac{W_t}{P_t^*}==\chi L_t^{*\varphi}C_t^{*\sigma}\cdot\eta_l^{\frac{1}{\mu_l}}\left(\frac{L_{H,t}^*}{L_t}\right)^{-\frac{1}{\mu_l}} & \text{working abroad (in Home region)}\end{cases}.$$

The calibration and policy experiment are based on the US economy for the availability and consistency of parameter values from existing literature and rich references to contrast my results. The parameter values follows standards of New Keynesian economics (Gali (2008)) or existing literature about the US economy, mainly Christiano and Eichenbaum (2005) and Altig et al. (2010); the parameter values associated with labor mobility are from Hauser (2014). The parameters (σ, χ, and φ) that govern the utility function homogeneous across residents in all regions and their values are taken from Nakamura and Steinsson (2014) and Gali and Monacelli (2016). I choose $\sigma = 1$ to allow balanced growth path for the model with separable preferences. The initial sizes of the two regions are symmetric, 1. I set the subjective discounting factor equal to 0.98 according to standard practice.

The parameters of labor mobility from Hauser (2014) are calibrated based on the US state-level employment and net labor flows during 1976–2008. In Hauser (2014), the utility cost of working in the other region, η_l, is chosen to match the average share of migrant workers to total labor input, which, according to the US state-level data, is 10.7%. The value capturing the elasticity of substitution between domestic and foreign labor supply, μ_l, is chosen to match the average volatility of net flows overall state pairs. Davis and Ortalo-Magne (2011) calibrates the U.S. Decennial Census of Housing and indicates that household expenditure shares (0.24) on housing are constant over time and across US metropolitan areas. I choose an enlarged value of home bias (0.34), which covers the consumption of non-traded goods and the proper home bias of traded goods. I take housing as one typical good and the most important non-traded good; but non-traded goods also include others, for example, labor service. I set the elasticity of substitution between *Home* and *Foreign* products to $\mu = 1$[4].

I set $\alpha = 0.36$, which corresponds to a steady-state share of capital income roughly equal to 36%, as Christiano and Eichenbaum (2005). In the short-term model, capital is omitted and only labor is left in the production function with $1-\alpha$ in the power position. Following Christiano and Eichenbaum (2005), I set $\theta = 0.60$ implying that price contracts last, on average, 2.5 quarters. Finally, we pick the coefficient in Taylor

[4]This is the same value used byNakamura and Steinsson (2014) and Obstfeld and Rogoff (2005).

rule to be $\phi_\pi = 1.5$ and $\phi_y = 0.125$, following the common practice in New Keynesian literature.

Table 5.1: Parameterization

Parameters	Description	Value	Source
		Baseline Model	
β	Discount factor	0.98	Standard Practice
χ	Curvature of labor disutility	2.2	Gali and Monacelli (2016)
σ	Intertemporal substitution rate	1	Gali (2008)
φ	Frisch elasticity of labor supply	1	Gali (2008)
α	The share of labor incomes	0.36	Christiano and Eichenbaum (2005)
ε	Elasticity of substitution (goods)	6.00	Christiano and Eichenbaum (2005)
η	Economy openness	0.69	Nakamura and Steinsson (2014)
μ	Elasticity of between products	2.00	Nakamura and Steinsson (2014)
γ	Share of non-traded goods	0.24,0.34	Davis and Ortalo-Magne (2011)
		Labor Mobility	
μ_l	Elasticity of mobility	-11,-2,-15	Hauser (2014)
η_l	Disutility of working in abroad	0.1,0.04 0.107	Hauser (2014)
		The Government Policy	
Taylor Rule	$i^T_{t+1} = \phi_\pi \pi_t + \phi_y y_t$	$\phi_\pi = 1.5$ $\phi_y = 0.125$	Gali (2008)
$\bar{g}$	Government in steady state (G/Y)	0.2	Baxter and King (1993)
Δg	Spending Shocks $(\Delta G/Y)$	1%	Baxter and King (1993)
ρ_g	The persistence of shocks	0.5 or 0.933	Nakamura and Steinsson (2014)

5.2 Numerical Results

5.2.1 Dynamic Response

To solve the model, I used Dynare [5]. Dynare codes are in Appendix F. I present the dynamic responses to technology shocks and demand shocks. Demand shocks come

[5]For more information about Dynare, please refer to http://www.dynare.org/s.

from surprising government spending in *Home* region. The focus lies on the asymmetric wedge differences of the two regions after the shock in *Home* region, especially how the presence of mobility changes the wedge differences.

1. Dynamic Response to a technology shock

Figure 5.1 and 5.2 illustrate this point by displaying the responses of output, PPI, CPI, and wage inflation. Under labor autarky, illustrated by Figure 5.1, and following a positive technology shock in *Home* region, the *Home* economy has a big jump in output and the *Foreign* economy has a mild rise in output, and both return to a symmetric steady state as the effects of shock vanish. Under labor autarky, the output response shows the opposite direction, the deviation of *Home*'s output is above its steady state, and *Foreign*'s output deviates downward from the steady state after the initial jump of output. With labor mobility, shown in 5.2, not only the output response but also the PPI, CPI, and wage inflation have been synchronized more than their counterparts under labor autarky.

Figure 5.1: Dynamic Response to Technology Shock under Labor Autarky

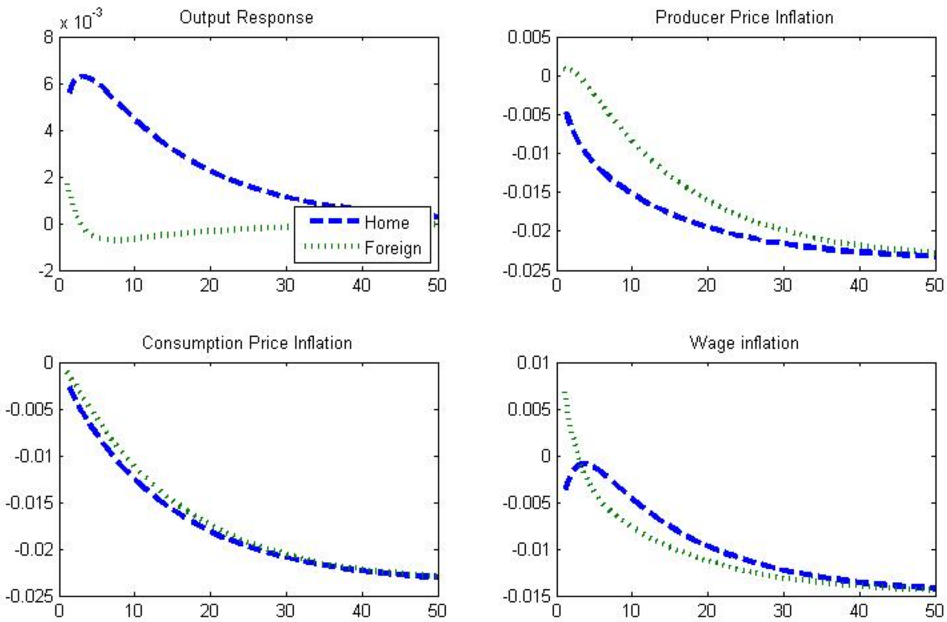
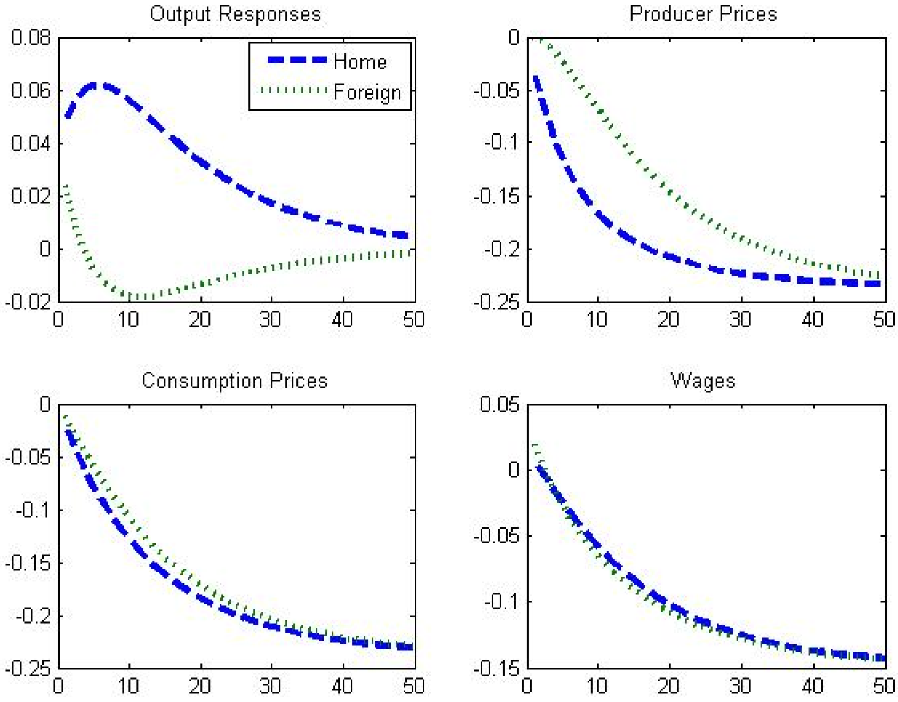


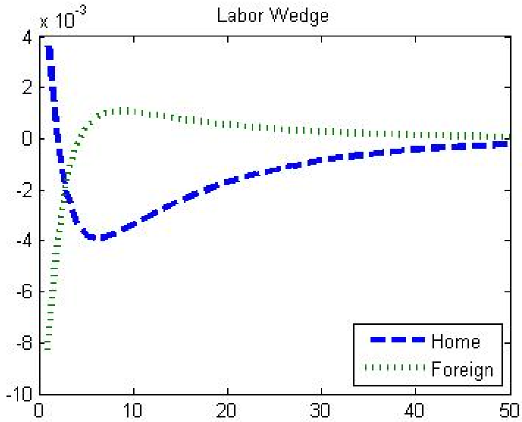
Figure 5.2: Dynamic Response (level) to Technology Shock with Labor Mobility



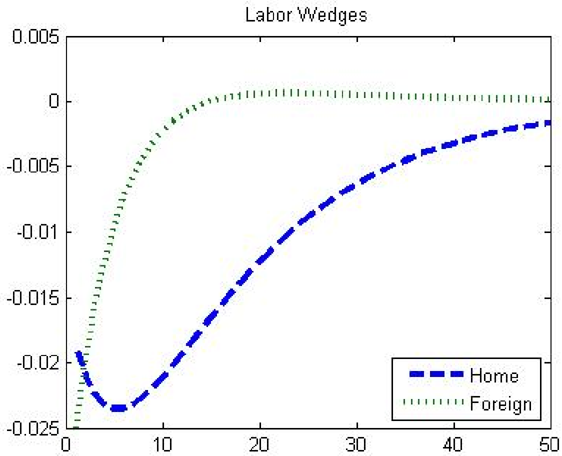
The wedges of two regions after a technology shock are illustrated by Figure 5.3. The comparison of the wedge responses of *Home* and *Foreign* economy shows a striking pattern that, after a technology shock in *Home* region, (i) under labor autarky *Home* economy firstly experiences a mild recession ($\mathcal{W} > 0$) because of price stickiness and then a boom ($\mathcal{W} < 0$) because of trade surpluses, while *Foreign* economy has the opposite experience from a boom to a bust before converging to the steady state; (ii) with labor mobility, however, the wedges of the two regional economies respond in the same direction, both experiencing a sudden drop and gradual rise, though to different extents. The contrast of wedge responses to a supply-side shock (a technology shock here) shows that with labor mobility regional economies achieve some extent of synchronization, though *Home* and *Foreign* wedges are far from being in the same pace.

The different responses of regional economies also strengthen the point that facing an asymmetric shock the monetary policy, which impacts two regions with the same policy instrument, can only achieve a sub-optimal stabilization. Monetary policy can only work on the averaged inefficiencies, which coincide in a single wedge in the closed economy when regional economies are perfectly synchronized. This incapacity of monetary policy calls for structural fiscal policy at the regional level.

Figure 5.3: Wedges caused by a Technology Shock



(a) Wedges under Labor Autarky



(b) Wedges with Labor Mobility

2. Dynamic responses to a regional government spending shock

With nominal frictions, government spending has a positive output effect, which is consistent with the New Keynesian literature on government spending multipliers. In the present study, the main interest is not the magnitude of this spending multiplier, which depends more on individual setup and parameterization. Interestingly, we see different response patterns of two regional economies after a demand shock in *Home* region. With labor mobility, the output of *Home* economy is larger than labor autarky, because of the abundance of labor supply and the reduction of crowding out effects in *Home* region, as shown in a comparison of Figure 5.4 and 5.5.

Figure 5.4: Dynamic Responses to A Regional Spending Shock under Labor Autarky

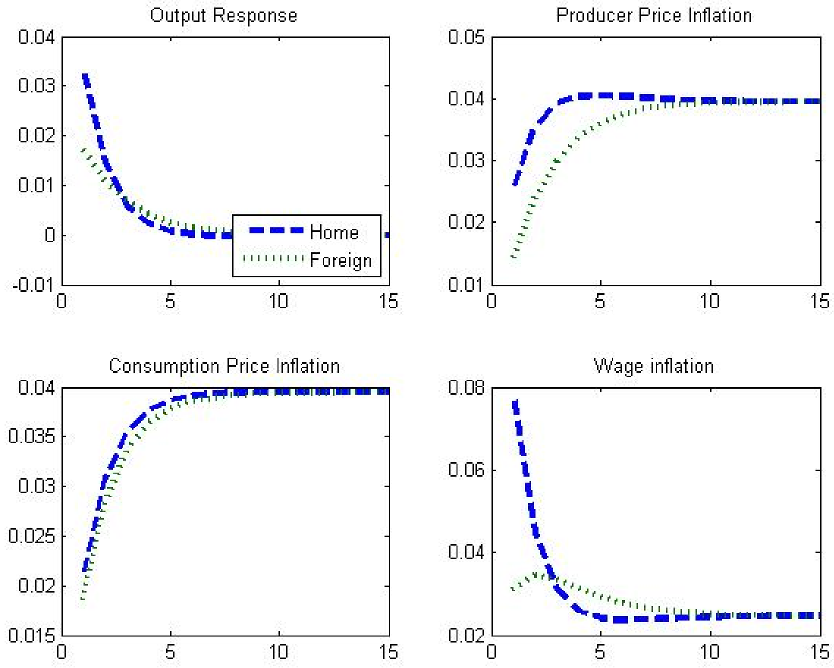


Figure 5.5: Dynamic Responses to A Regional Spending Shock with Labor Mobility

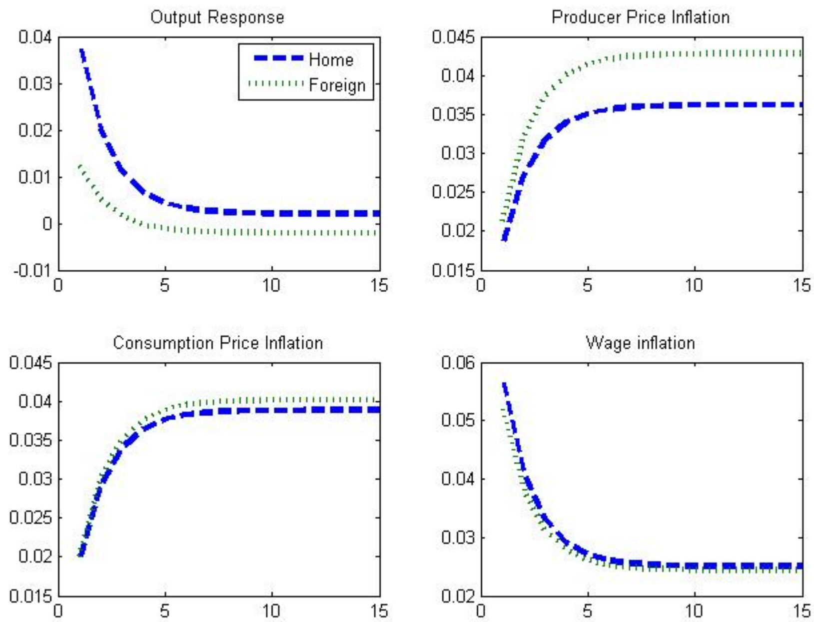
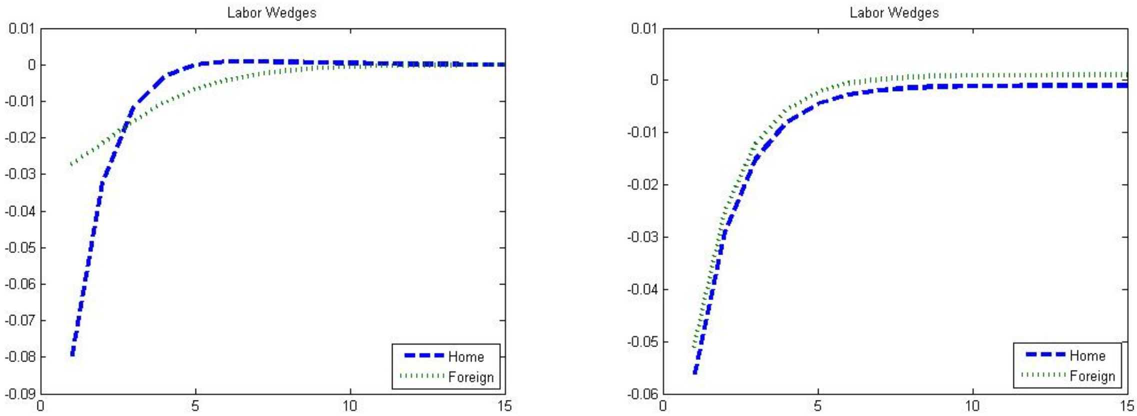


Figure 5.6 illustrates the effect of a demand shock, unexpected government spending in *Home* region. Wedges of two regions indicate that both regions experience a boom ($\mathcal{W} < 0$), but to different extents under labor autarky (Figure 5.6a). The formation of this difference originates from the limited openness or home bias because the demand changes in one region spillover only partially to another region. However, with labor mobility, the economic statuses of the two region's economies are almost synchronized, which is illustrated clearly by Figure 5.6b.

Figure 5.6: Wedges caused by a Regional Spending Shock

(a) Wedges under Labor Autarky (b) with Labor Mobility (I)



I make a sensitivity analysis to see the effects of parameter magnitude. The magnitude changes of mobility parameters can make a big change in output responses. According to my sensitivity analysis (in Appendix E), the parameter η_n, which governs the disutility of working abroad or labor market openness, can change the responses of two regional economies to a technology shock (Figure 8.1 in Appendix E1); however, the responses to a demand shock only change slightly after mobility parameter changes (Figure 8.2 in Appendix E).

5.2.2 Welfare Loss and Optimal Policy

1. Welfare Loss

The economy-wide welfare loss function with labor mobility is different from a simple sum of two welfare functions based on labor autarky. The welfare loss of each region is written as

$$\mathbb{W} = -\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \left\{ \begin{array}{c} \frac{\varepsilon}{\Theta}\left(\pi_{H,t}^2 + \pi_{F,t}^2\right) + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\left(\tilde{y}_t^2 + \tilde{y}_t^{*2}\right) \\ +(1-\alpha)(1+\varphi)\cdot 2(1-\eta_l)\eta_l \left(l_{H,t} - l_{F,t}^*\right)\left(l_{F,t} - l_{H,t}^*\right) \end{array} \right\} \tag{5.1}$$

with $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\varepsilon}$. $\tilde{y}_i$ and $\tilde{y}_t^*$ represent output gaps in *Home* and *Foreign* region, $\pi_{i,t}, i \in \{H, F\}$ is the PPI of regions i. The derivation of this welfare loss function is in Appendix C. The first two terms in the loss function are familiar in New Keynesian models and are identical to the form in Gali and Monacelli (2008). The third term,

$$\left(l_{H,t} - l_{F,t}^*\right)\left(l_{F,t} - l_{H,t}^*\right)$$

, is defined by labor mobility across regions, where $\left(l_{H,t} - l_{F,t}^*\right)$ is interpreted as the relative size change of workers who choose to work locally, and $\left(l_{F,t} - l_{H,t}^*\right)$ is the relative change of workers who work abroad. This term captures the welfare effects of labor mobility. Only when regional economies are symmetric, this item originating from labor mobility is zero and labor mobility across regions is muted. This term also disappears in the labor-autarky limit as $\eta_l \to 0$.

The third term, $\left(l_{H,t} - l_{F,t}^*\right)\left(l_{F,t} - l_{H,t}^*\right)$, in 5.1 captures the connection of labor markets and the correlation of labor supply in the log-linearized variables. Suppose that *Home* region is in a boom and *Foreign* region is in recession or normal, when $\left(l_{H,t} - l_{F,t}^*\right)$ is positive, more workers in *Home* region choose to work locally rather than seeking outside employment compared to the workers in *Foreign* region; at the same time, $\left(l_{F,t} - l_{H,t}^*\right)$ is negative because the workers from *Home* to *Foreign* region become less than the workers from *Foreign* to *Home* region. The product,

$$\left(l_{H,t} - l_{F,t}^*\right)\left(l_{F,t} - l_{H,t}^*\right)$$

, is a negative term in welfare loss function. The composite effect of labor mobility when the two regions have heterogeneous economic conditions is a reduction in welfare loss, i.e., a welfare-improving effect. This welfare-improving outcome comes from the risk sharing and the resulting reduced volatility of labor supply, when the two regions face asymmetric shocks and have heterogeneous economic statuses.

Table 5.2 reports the welfare losses after spending and technology shocks with or without labor mobility. We can see after each shock labor mobility improves welfare gains. No matter which kind of shocks, the welfare-improving effect of mobility is significant.

Table 5.2: Economy-Wide Welfare Loss

	Mobility	Immobility	Welfare Gain (%)
Demand Shock	2.12	2.26	6.2%
Technology Shock	6.47	8.78	26.3%

Note: Standard Deviations expressed in percent

The optimal policy is to minimize the economy-wide welfare loss, given by (5.1), subject to Dynamics IS equation and New Keynesian Phillips curve of two regions. These constraints are summarized as

$$
\begin{aligned}
c_t =& \mathbb{E}_t \left\{ c_{t+1} \right\} - \sigma^{-1} \left(i_t - \mathbb{E}_t \left\{ \bar{\pi}_{t+1} \right\} - \rho \right) \\
c_t^* =& \mathbb{E}_t \left\{ c_{t+1}^* \right\} - \sigma^{-1} \left(i_t - \mathbb{E}_t \left\{ \bar{\pi}_{t+1}^* \right\} - \rho \right) \\
\pi_{H,t} =& \frac{(1-\eta)\bar{\pi}_t - \eta \bar{\pi}_t^*}{1-2\eta} \\
\pi_{F,t} =& \frac{(1-\eta)\bar{\pi}_t^* - \eta \bar{\pi}_t}{1-2\eta} \\
y_t =& (1-\eta) \left(-\mu p_{H,t} + \mu \bar{p}_t \right) c_t + \eta \left(-\mu p_{H,t} + \mu \bar{p}_t^* \right) c_t^* \\
y_t^* =& (1-\eta) \left(-\mu p_{F,t} + \mu \bar{p}_t^* \right) c_t^* + \eta \left(-\mu p_{F,t} + \mu \bar{p}_t \right) c_t \\
\pi_{H,t} =& \beta \mathbb{E}_t \left\{ \pi_{H,t+1} \right\} + \kappa \tilde{y}_t \\
\pi_{F,t} =& \beta \mathbb{E}_t \left\{ \pi_{F,t+1} \right\} + \kappa \tilde{y}_t^* .
\end{aligned}
,
$$

where $\rho = -\log \beta$ is the time discount rate, $\bar{\pi}$ and $\bar{\pi}^*$ are the CPI inflation of the *Home* and *Foreign* economies, $\pi_{H,t}$ and $\pi_{F,t}$ are the *Home* and *Foreign* PPI inflation rates, respectively.

Table 5.3 summarizes the cyclical properties of the model with and without labor mobility. There are several patterns that worth to emphasis.: (i) with labor mobility, the wage correlation of two regions is larger than without; (ii) the output correlation after a demand shock is opposite to that after technology shock. Generally, labor mobility strengthens the economic connection and improves the economic synchronization of regional economies.

Table 5.3: Cyclical Properties of Alternative Mobility Regimes

	Mobility		Immobility		Mobility		Immobility	
	Home	*Foreign*	*Home*	*Foreign*	*Home*	*Foreign*	*Home*	*Foreign*
Output	4.21	1.87	3.81	2.30	24.04	7.68	24.33	3.32
Output Cor.	96.73		95.90		-70.11		-59.30	
CPI Inflation	2.39	2.33	2.44	2.28	4.64	4.14	5.28	4.91
PPI Inflation	2.52	2.20	2.81	1.99	5.98	3.75	6.63	5.04
Term of Trade	1.17		2.17		39.74		35.09	
Wage	26.96	26.89	27.34	26.73	231.67	231.89	157.86	158.62
Wage Cor.	99.98		98.36		99.99		99.41	
	After a Demand Shock (1% GDP)				After a Technology Shock (1%)			

Note: Standard Deviations expressed in percent

To summarize the quantitative results, I emphasize two points. First, after a shock shock happens exclusively in *Home* region (i.e., asymmetric shocks), the presence of labor mobility synchronize regional labor wedges in significant extents; second, after labor mobility improves economy-wide welfare in substantial proportions, by 6.2% after a demand shock and 26.3% after an asymmetric technology shock. These findings point to the importance of social multipliers in facilitating mobility across regions.

6 The Extended Model with Capital and Flexible Prices

As characterized by equation (4.6) and Table 4.1, even perfect nominal prices the decentralized equilibrium is necessarily the first best outcome defined by (2). Government interventions have to change the relative productivity in different regions and sectors. This section aim to investigate the effects of fiscal investments, which can change the regional productivity, with a inter-regional fiscal regime and under different mobility context.

This section extends previous short-term model, which has only labor in production and price frictions, to a medium-scale model with capital and focuses on the long-term effects of regional fiscal policy with nominal flexibility or converged nominal variables. In this long-term economy, there are two sectors (traded and non-traded), two factors (capital and labor), and two regions (*Home* and *Foreign*). Factors are perfectly mobile or totally blocked across regions; I change government financing to factor taxes. To simplify the analysis, I divide factor mobility into four contexts according to each factor's mobility: $\{\text{Capital, Labor}\} \times \{\text{Mobile, Immobile}\}$. The consumption goods market is the same as Subsection 3.1.

6.1 Households

Households live and work in one region, since temporarily working abroad is not allowed. They have capital and collect rent. The household problem is written as

$$\max_{C_t, I_t, K_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C^{1-\sigma}}{1-\sigma} - \chi \frac{L_t^{1+\varphi}}{1+\varphi} \right\}$$

subject to

$$(1+\tau_t^c)\, \bar{P}_t C_t + \bar{P}_t I_t \leq \left(1-\tau_t^k\right) R_t K_t + (1-\tau_t^w)\, W_t L_t - T_t,$$

where consumption is represented by C_t as before, physical capital by K_t, and investment in new physical capital by I_t. Households lease capital services to firms at a competitive rental rate R_t. Profits are taxed by a correction tax rate and rebated to households. The variables, τ_t^c, τ_t^k, and τ_t^w, are consumption, capital, and labor taxes, and potentially vary over time.

The law of motion of capital is written as

$$K_t = (1-\delta) K_{t-1} + I_{t-1}$$

where δ is the capital depreciation rate and I_t is the investment in period t. The investment is spontaneously transferred into capital from consumption goods without adjustment costs. The Euler equation is given as

$$\frac{U_C(C_t)}{(1+\tau_t^c)\bar{P}_t} = \mathbb{E}_t \beta \left\{ \frac{U_C(C_{t+1})}{(1+\tau_{t+1}^c)\bar{P}_{t+1}} \left[\left(1-\tau_{t+1}^k\right) R_{t+1} + 1 - \delta \right] \right\},$$

where $\bar{P}_t$ is the consumption index. Although this section is based on neoclassical framework and prices are defined based on marginal production, prices are useful for indicating the relative productivity of different sectors in different regions. Capital rents in the next periods indirectly depend on the aggregation of productivity changes in two sectors of each region.

Consumption goods are combined by two types of goods, traded and non-traded, with the formula

$$C_t = \frac{(C_{T,t})^{1-\gamma} (C_{N,t})^{\gamma}}{(1-\gamma)^{1-\gamma} \gamma^{\gamma}}.$$

The allocation of production factors satisfies

$$\frac{P_{N,t} Y_{N,t}}{P_{T,t} Y_{T,t}} = \frac{\gamma}{1-\gamma}. \tag{6.1}$$

The nominal price of labor and capital satisfies

$$\frac{W_t}{\bar{P}_t} = -\frac{U_L}{U_C}$$

where $\bar{P}_t$ is the CPI, defined, as previously, $P_{T,t}^{1-\gamma} P_{N,t}^{\gamma}$.

In the context with perfect mobility of capital, the consumption of *Home* and *Foreign* residents satisfies

$$\frac{U_C\left(C_t\right)}{\bar{P}_t} = \nu_F \frac{U_C\left(C_t^*\right)}{\bar{P}_t^*},$$

for all t and where ν_F is a constant, which generally depends on initial conditions regarding the two-region's relative net asset positions. Henceforth, and without loss of generality, symmetric initial conditions are assumed (i.e., zero net foreign asset holdings and ex-ante identical environments), in which $\nu_F = 1$.

6.2 Firms

As in the baseline model without capital, production is broken into two sectors, with a representative final goods producer and a continuum of intermediate producers indexed by $j \in [0,1]$. The final goods sector is identical to the simpler model, generating the same downward-sloping demand for each variety of intermediate goods and the price index. The demands for each variety are the same as defined in Subsection (3.1).

In each region, the production functions, which are of Cobb-Douglas form, are written as

$$Y_{N,t}(i) = A_{N,t} K(i)_{N,t}^{\alpha} L(i)_{N,t}^{1-\alpha}, \tag{6.2}$$

$$Y_{T,t}(j) = A_{T,t} K(j)_{T,t}^{\alpha} L(j)_{T,t}^{1-\alpha}, \tag{6.3}$$

where i is one variety of non-traded goods and j is one variety of traded goods,$A_{s,t}, s \in \{N, T\}$ is the stochastic technology to traded and non-traded sectors in *Home* region. In this section, by focusing on long-term allocation, price stickiness is assumed away. In the symmetric equilibrium, variety subscripts can be suppressed for simplicity. In the following parts, production functions of the two sectors are written as

$$Y_{s,t} = A_{s,t} K_{s,t}^{\alpha} L_{s,t}^{1-\alpha}, s \in \{N, T\}$$

. The heterogeneity of production is only kept across sectors. The production of *Foreign* region is analogous and denoted with an asterisk.

Intermediate firms rent capital and hire labor in perfectly competitive factor markets. The firm's optimal decision decides the prices of factors as

$$\begin{aligned} R_t &= A_{s,t}\alpha K_{s,t}^{\alpha-1}L_{s,t}^{1-\alpha} \\ W_t &= A_{s,t}(1-\alpha)K_{s,t}^{\alpha}L_{s,t}^{-\alpha} \end{aligned} \text{ for } s \in \{N, T\}\,.$$

The above conditions implies [6]

$$P_{s,t} = \frac{W_t^{1-\alpha}R_t^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}A_{s,t}}, s \in \{N, T\}\,.$$

The production sector in *Foreign* region is defined analogously.

6.3 Factor Markets

In a factor-autarky economy, firms only use local factors, labor and capital, in production. The two regions are linked only through trade integration. The factor autarky context will be examined and used as the benchmark. In the circumstance with factor mobility, capital moves freely across regions, and labor only has long-term relocation, since this section investigates long-term effects. The long-term location of households means that workers only have wages from local employment and temporary wages (real) from another region ($\frac{W^*}{P}$ and $\frac{W}{P^*}$) are neglected.

The four contexts of factor mobility are described by Table 6.1. When both factors, capital and labor, are perfectly mobile across regions, the regional economy degenerates into a closed economy (Context I); when capital mobility is blocked and the labor market is perfectly open, this context is indexed II, which is rare in reality; Context III has an integrated capital market, but labor mobility is blocked, which is close to the international economy with the significant presence of multinational corporations; and Context IV is an extreme case where economies are separated in factor markets and connected only through commodity trade. It would be difficult to find a real economy which is exactly the same an one of the four contexts. However, the four contexts define the factor mobility map of any economies; hence, a generic regional economy, even the international economy, can fit into this map.

Table 6.1: Factor Mobility Contexts

	Capital Mobile	Capital Immobile
Labor Mobile	I: Closed Economy	II: Capital Blocked Labor Open
Labor Immobile	III: Capital Open Labor Blocked	IV: Trade Only

[6] For a derivation, please refer to Appendix D.

Capital mobility is captured by the difference of return when capital is hired abroad. Because of the ease of capital moving, especially within a country, it is plausible to assume the polar case of free capital mobility, meaning capital from other regions receives the same returns as from local. Free mobility of capital intergates the capital market into one market. The capital location among two regions and two sectors satisfies the conditions

$$\begin{aligned} R_t &= \alpha A_{N,t}\bar{K}^{\alpha-1}L_{N,t}^{1-\alpha} = \alpha A_{T,t}\bar{K}_t^{\alpha-1}L_{T,t}^{1-\alpha} \\ &= \alpha A_{N,t}^{*}\bar{K}_t^{\alpha-1}L_{N,t}^{*1-\alpha} = \alpha A_{T,t}^{*}\bar{K}_t^{\alpha-1}L_{T,t}^{*1-\alpha} \end{aligned}$$

where $\bar{K} = K_{T,t} + K_{N,t} + K_{T,t}^{*} + K_{N,t}^{*}$. The factor proportion among two sectors (traded v.s. non-traded) in each region satisfies the condition (6.1), which indicates a constant proportion originating from the households' preference over two types of goods. After assuming perfect allocation across sectors in each region, we can focus on regional differences.

Labor is perfectly mobile between sectors within regions. Since we focus on the long-term location of households, the case that households send members to work temporarily in another region is negligible and households are presumed to have only local incomes.

The factor market clearing conditions require

$$\begin{aligned} K_{T,t} + K_{N,t} + K_{T,t}^{*} + K_{N,t}^{*} &= \bar{K}_t \\ L_{T,t}^{*} + L_{N,t}^{*} &= L_t^{*}. \\ L_{T,t} + L_{N,t} &= L_t \end{aligned}$$

Define m_L and m_K as binary variables controlling the openness of factor markets across region. When m=0, the factor markets are separate; when $m = 1$, the factor markets are open. The factor price with mobility variable is given

$$\begin{aligned} W_t &= (1-\alpha)\left(K_t + m_K K_t^{*}\right)^{\alpha}\left(L_t + m_L L_t^{*}\right)^{-\alpha} \\ R_t &= \alpha\left(K_t + m_K K_t^{*}\right)^{\alpha-1}\left(L_t + m_L L_t^{*}\right)^{1-\alpha} \end{aligned}.$$

The analogous forms can be written for *Foreign* economy.

	Capital Mobile	Capital Immobile
Labor Mobile	I: One Closed Economy $\left(\frac{W_I^i}{W_I^{-i}} = \frac{R_I^i}{R_I^{-i}} = 1\right)$	II: Labor Open $\left(\frac{W_{II}^i}{R_{II}^i} = \frac{W_{II}^{-i}}{R_{II}^{-i}} < \frac{W_I^i}{R_I^i}\right)$
Labor Immobile	III: Capital Open $\left(\frac{W_{III}^i}{R_{III}^i} = \frac{W_{III}^{-i}}{R_{III}^{-i}} > \frac{W_I^i}{R_I^i}\right)$	IV: Trade Only $\left(\frac{W_{IV}^i}{W_I^i} > 1, \frac{R_{IV}^i}{R_I^i} > 1\right)$

.

Proposition 2. *Increase of factor mobility, capital or labor, is output- and welfare-improving to both regions.*

Proof. Refer to Appendix D2. □

The output- and welfare- improving effects are the results of lower factor prices with integrated factor markets and the expanded output. The price of products, $P = \frac{W^{1-\alpha}R^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}A}$, declines as factor price declines. As a result, factor market openness has positive output and welfare effects.

One Special Case One special case is when the market for traded goods is perfectly competitive and the real wages are equal across regions. The perfect competition of traded goods market implies that the prices and marginal production costs of traded goods are equal,

$$\begin{aligned} \frac{W_t^{1-\alpha}R_t^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}A_{T,t}} &= \frac{W_t^{*1-\alpha}R_t^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}A_{T,t}^{*}} \\ \left(\frac{W_t}{W_t^*}\right)^{1-\alpha} &= \frac{A_{T,t}}{A_{T,t}^*}. \end{aligned} \tag{6.4}$$

The equalization of real wage implies

$$\begin{aligned} \frac{W_t}{\bar{P}_t} &= \frac{W_t^*}{\bar{P}_t^*} \\ W_t^{\alpha} \times A_{T,t}^{1-\gamma} A_{N,t}^{\gamma} &= W_t^* A_{T,t}^{*1-\gamma} A_{N,t}^{*\gamma}. \end{aligned} \tag{6.5}$$

Combine equation (6.4) and (6.5), and we have

$$\begin{aligned} \frac{W_t}{W_t^*} &= \left(\frac{A_{T,t}}{A_{T,t}^*} \cdot \frac{A_{N,t}^*}{A_{N,t}}\right)^{\gamma} \\ &= \left(\frac{A_{N,t}^*}{A_{N,t}}\right)^{\frac{\gamma}{1-\gamma+\gamma\alpha}}. \end{aligned}$$

The intuition of the relation between the wages required by residents and the productivity of non-traded goods is that low productivity and the implied high prices of non-traded goods force residents to ask for higher wages, for example,where high housing prices and service prices are high. This result highlights again the same feature of regional economy as stated in Proposition 2; the presence of a non-traded sector causes regional disparities, even when the traded goods market is perfectly open and financial markets are fully integrated.

6.4 The Effects of Government Purchases

Many economists disaggregate fiscal plans and investigate their effects separately, like Alesina et al. (2017), which dis-aggregates fiscal plans into three aspects, (i) government spending and investments, (ii) transfers, and (iii) direct and indirect taxes. In this section, I consider one type of government purchases: productivity-augmenting. If public services impact the productivity of production sectors, then the government purchases enter the production function of private firms. The potential benefits of regional government spending can work as an accelerator of factor mobility across regions and as a potential engine of growth.

6.4.1 Government Purchases in Production Function

Government spending can form public capital, which will augment private productivity. In this subsection, I take regional government spending as an investment of public capital and analyze the potential output and welfare effects with factor mobility. My setup and parameter choices are close to Baxter and King (1993).

Output at date t is the result of private capital, public capital, and labor applied in a Cobb-Douglas production function:

$$Y_{s,t} = F\left(K_t, K_t^g, L_t\right) = A_{s,t} K_{s,t}^{\alpha} L_{s,t}^{1-\alpha} \left(K_{g,t}\right)^{\alpha_g}, \text{ for } s \in \{N, T\}$$

where $K_{s,t}$ is the private capital stock in sector s, $K_{g,t}$ is the stock of publicly provided capital in *Home* region, and $L_{s,t}$ is the quantity of labor input. The public capital in *Foreign* region is denoted as $K_{g,t}^*$. Given $K_{g,0}^i$, public capital similarly evolves according to:

$$K_{g,t+1}^i = \left(1 - \delta_k\right) K_{g,t}^i + I_{g,t}^i, i \in \{H, F\}.$$

All the specifications for *Home* region also apply to the economy of *Foreign* region.

Table 6.2: Parameters of the Benefits of Government Purchases

Terms	Description	Values	
s_I^g	Share of Public Investments$s_I^g = \frac{I^g}{Y} = 0.05$	0.05	Baxter and King (1993)
α_g	$\alpha_g = 0.05$ as benchmark	0.05	Baxter and King (1993)
	$\alpha_g = 0.24$ the largest value from	0.24	Aschauer (1989)
δ_k	Depreciation of Public Capital	0.10	Baxter and King (1993)

To see the effects of a government's investment on public capital, I run policy experiments with these parameter values from Aschauer (1989) and Baxter and King (1993).

The government expenditure in *Home* region will improve its productivity in the non-traded sector. The theoretical implication is that the improved productivity will attract factors from another region and generate incomes for all households.

6.4.2 Government Financing

As argued in Baxter and King (1993), how government spending is financed has important effects on the output effects. In this medium-term DSGE model, the government finances its spending with tax revenues. The government can change tax rates to cover its expenditure. I consider four types of taxes as in McKay and Reis (2016), including wage tax, consumption tax, and capital gains tax; lump-sum tax is used to illustrate the distortion effects arising from factor taxes.

The tax policy of the US, calibrated by McKay and Reis (2016) over the period 1960–2011, is shown in Table 6.3. The tax rate on consumption is directly from McKay and Reis (2016), and the tax rate on capital rent is implied from tax rates on profits and capital cost deduction rates, which is 0.680. In the calibration of McKay and Reis (2016), the personal income tax is progressive and not tractable in this research. Hence, I use the average federal tax rate on wage over the same period, 1960–2011, 23.69%. Looking through this period, the maximum tax rate on wages is 29.85%, the minimum 20.01%; and the average number, 23.69% is close to the latest number in 2011, 22.9%. According to McKay and Reis (2016), the tax revenues of these three tax types account for 17.62% of the US GDP, which is close the standard government expenditure and GDP ratio, 20%, in literature.

Table 6.3: US Tax Rates

Tax Type	Parameter	Values	Percent of GDP	Source
Consumption	τ^c	0.054	3.85	McKay and Reis (2016)
Capital rents	τ^k	0.112	2.79	McKay and Reis (2016)
Labor incomes	τ^w	0.2369	10.98	Feenberg and Coutts (1993) and TAXSIM[7]

[7]TAXSIM: http://users.nber.org/~taxsim/

The government budget constraint is:

$$
\begin{aligned}
P_t G_t = & \tau_t \underbrace{\left[\tau^c P_t C_t + \tau^w W_t L_t + \tau^k R_t K_t \right]}_{\text{Fiscal Revenue from Home Region}} + T_t \\
& + \tau_t \underbrace{\left[\tau^c P_t^* C_t^* + \tau^w W_t^* L_t^* + \tau^k R_t^* K_t^* \right]}_{\text{Fiscal Revenues from Foreign Region}} .
\end{aligned}
$$

B_{t-1} is the stock of nominal debt with which the government enters period t. Government expenditure plus interest payments on outstanding debt must equal tax collections plus the issuance of new debt. With an increase in government purchases, there must be a corresponding increase in tax rates to satisfy the government budget identity. The variable τ_t captures the change of tax rates because of government spending. With time-varying τ_t, it is possible to analyze the effects of the corresponding nationwide tax changes to finance new government investments in one region. Tax changes lead to a shrinking tax base and the increase of tax rate. The distortionary taxes cause a larger increase of tax rates than that of government expenditures.

6.5 Productivity in Traded and Non-Traded Sectors

The benchmark productivity levels of traded and non-traded sectors are chosen from Mano and Castillo (2015), which estimate the productivity levels of traded and non-traded sectors for a large panel of countries, including the United States, from the STAN database[8]. I use the US productivity of the two sectors as the benchmark for *Foreign* region, because this studies fiscal policy which happens *Home* region, with the rest of country unchanged. The methodology of Mano and Castillo (2015) is like this. Consider an economy that is divided into multiple industries. An industry, i, is either said to be traded $(j \in T)$ if it produces traded goods, or non-traded $(j \in N)$, if it produces non-traded goods. Let labor productivity at time t in the traded sector and in the non-traded sector be $A_{T,t}$ and $A_{T,t}$, respectively, such that

$$
\begin{aligned}
A_{T,t} &= \sum_{j \in T} \frac{VA_{j,t}}{PVA_{j,t}} \Big/ \sum_{j \in T} L_{i,t} \\
A_{N,t} &= \sum_{j \in N} \frac{VA_{j,t}}{PVA_{j,t}} \Big/ \sum_{j \in N} L_{j,t}
\end{aligned}
$$

[8]The STAN database is a comprehensive tool for analyzing industrial performance at a relatively detailed level of activity across countries. It includes annual measures of output, value added and its components, labor input, investment, and capital stock, from 1970 onward, which allow users to construct a wide range of indicators to focus on areas such as productivity growth, competitiveness and general structural change (for details, STAN Database).

where $VA_{j,t}$ is the gross value added at time t for each industry j, $PVA_{j,t}$ is the price index of gross added at time t for each industry, and $L_{j,t}$ is the total employment at time t for each industry j.

The US traded sector productivity in 2005 is 125 thousand USD per worker, and the level of the non-traded sector is 63 thousand USD. For illustrative purposes, it is sufficient to take the relative productivity of two sectors. Hence, in this quantitative framework, the productivity ratio (A_T^*/A_N^*) in *Foreign* region is set to 2. To illustrate the interesting implications of asymmetric states of the two-region economy, I set the productivity of the non-traded sector in *Home* region equal to 2, and to keep productivity balanced between regions, the implied productivity of the traded sector is $2^{\frac{1-2\gamma}{1-\gamma}} (> 1)$[9].

6.6 Policy Experiments

I make numerical exercises with parameter values from literature, parts of which have been presented earlier. The parameter values related to capital are from Christiano and Eichenbaum (2005) and Altig et al. (2010); the tax rate of the United States is from McKay and Reis (2016). I set $\alpha = 0.36$, which corresponds to the steady state share of capital income, and set $\delta = 0.025$, which implies an annual rate of depreciation on capital equal to 10%. I calculate the corresponding increase of taxes from the steady state of the tax-GDP ratio shown in Table 6.3. Assume that government consumption and investment obey an independent stationary AR(1) process:

$$\ln G_t = \rho_g \ln G_{t-1} + s_g \epsilon_{g,t}.$$

To see the effects of mobility context, I make the same policy experiment in each mobility context: $\{$Capital, Labor$\} \times \{$Mobile, Immobile$\}$, described by Table 6.1. The numerical analysis in this section would compare the economic dynamics under these four contexts.

6.6.1 Output Effects

The output effects of the public investments in both regions are significant and have different patterns in different factor mobility contexts. Figure 6.1 presents the composite output $(Y_{T,t}^{1-\gamma} Y_{N,t}^{\gamma})$ after public investment in *Home* region, which is financed with economy-wide factor tax revenues. Figure 6.2 presents the aggregated output in four factor-mobility contexts, and Figure 6.3 presents the output of two types of goods in each region and four contexts. Both regional economies respond to the government in-

[9]Obviously, the symmetric case $\left(\frac{A_N^*}{A_T^*} = \frac{A_N}{A_T}\right)$ is also balanced and leads the homogeneous structure of regions. Nevertheless, it bears less interests.

vestment with a drop in output right after the shock because the increased factor taxes depressed the motivation for production. Like the formation of public capital and the induced private capital formation, output in *Home* region rebounds quickly. Across the four mobility contexts, the increase of economy-wide output is largest when both capital and labor are mobile; in the opposite context, when neither capital and labor are mobile, the economy-wide output has the least peak point.

Figure 6.1: Economy-wide Composite Output Responses in 4 Mobility Contexts

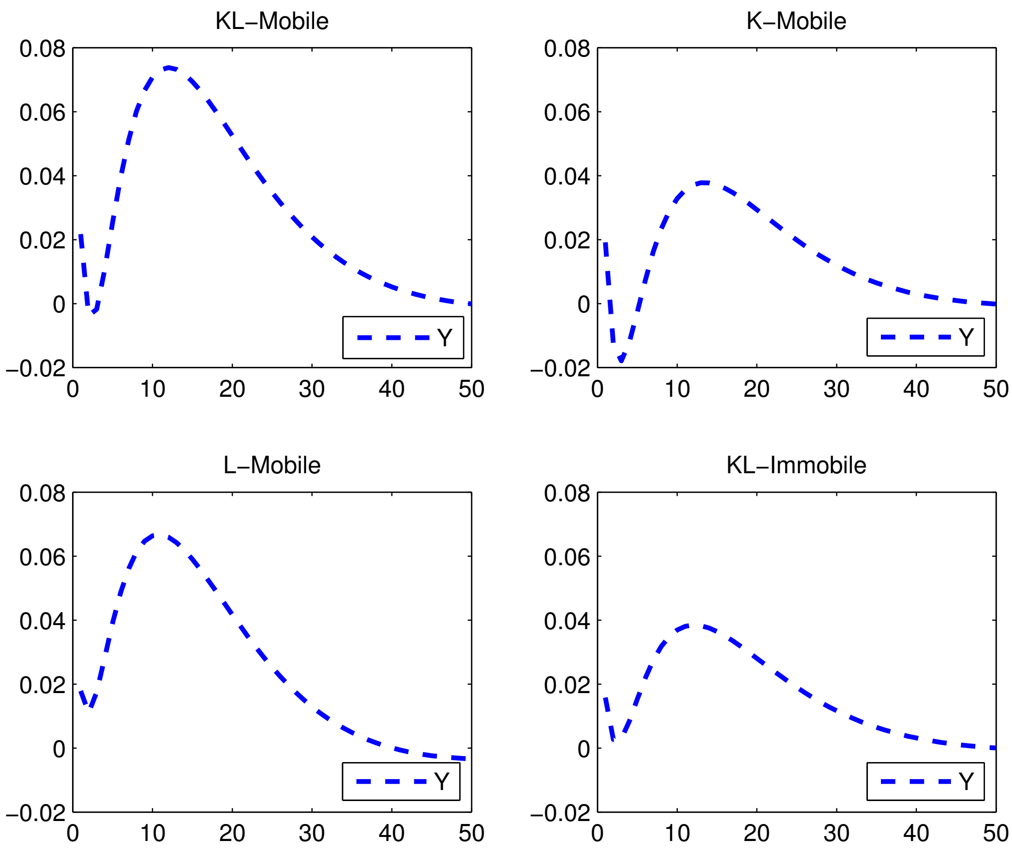


Figure 6.2: Regional Aggregated Output (N&T) in 4 Contexts

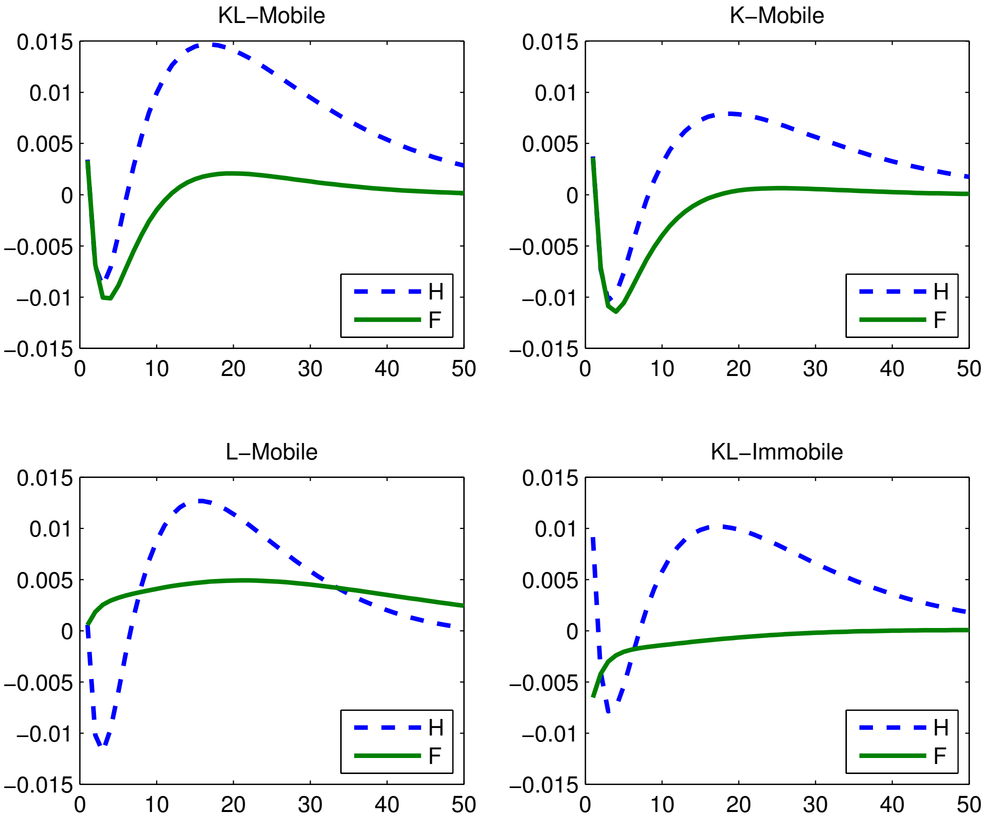
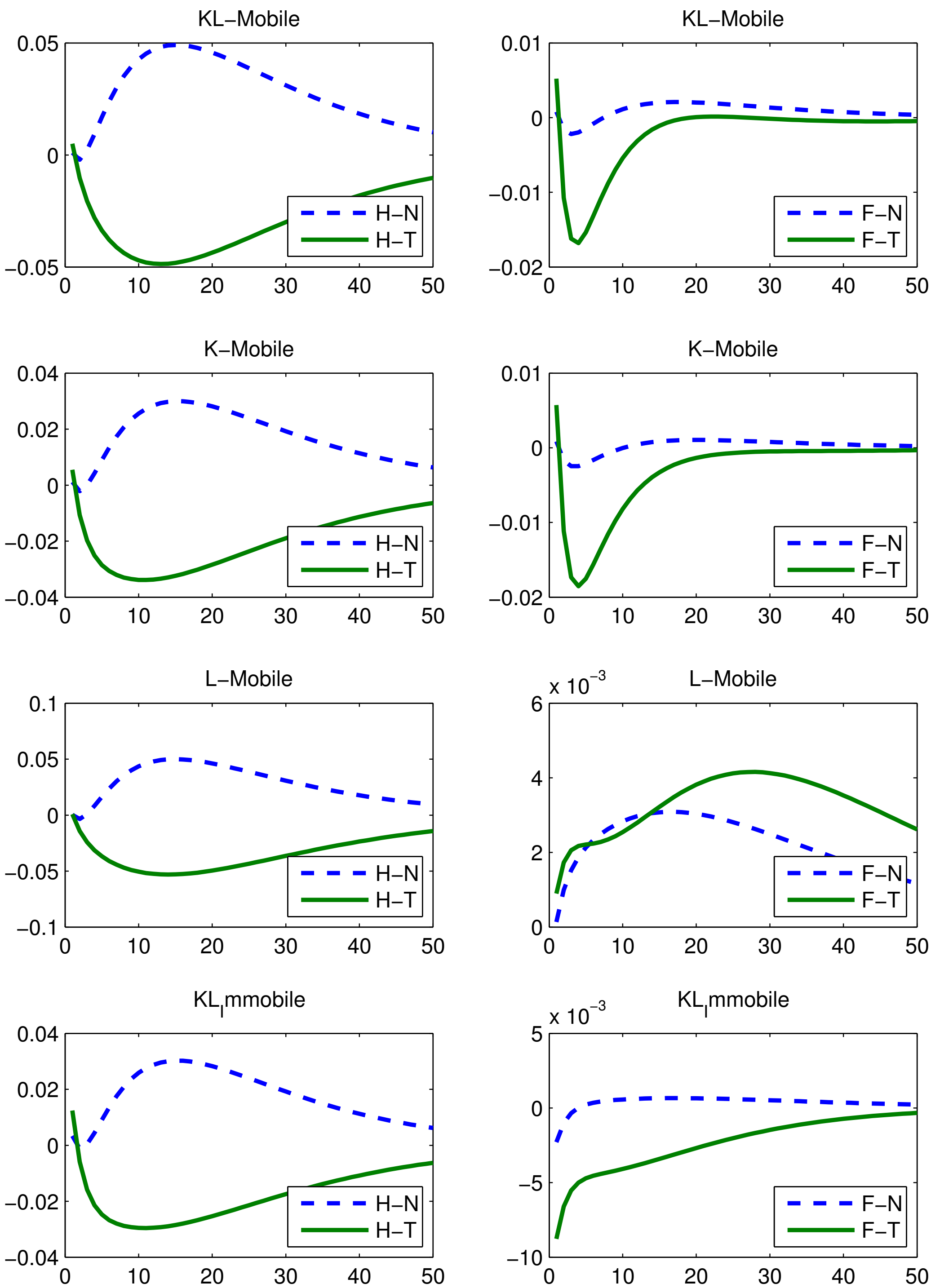


Figure 6.3: Output Responses of 4 Contexts



6.6.2 Welfare Effects

The welfare in steady states is given by Table 6.4 and the welfare responses in four context are shown by Figure 6.4. One can immediately see from Table 6.4 that in mobility Context I, households of two regions have excellent welfare, and in Context IV households have the worst welfare. Moving from Context VI (KL-Immobile) to Context III (only capital mobile) does not improve welfare greatly; however, when moving from Context IV (KL-Immobile) to Context II (only labor mobile) the welfare improving effect is sharp, which is not a strictly worse than Context I. In the long-run, labor mobility makes a bigger change to welfare than capital, because capital can be accumulated and converge over time. The effect of labor immobility can not vanish over time.

Table 6.4: Steady State Welfare after A Public Investment in *Home* Region

Welfare(H, F)	Capital Mobile	Capital Immobile
Labor Mobile	I: $(12.29,\ 14.55)$	II: $(14.08,\ 8.92)$
Labor Immobile	III: $(-0.78,\ 1.48322)$	IV: $(-0.31,\ -3.48)$

Without capital mobility, the responses of welfare and utility of households in *Foreign* region are positive and decline slowly, after the public investment in *Home* region. Without capital mobility, the welfare and utility of households in regions are negative. Without labor mobility, the response direction does not change across the contexts of labor mobility, KL-Mobile vs. K-Mobile and L-Mobile vs. KL-Immobile, but the response ranges change slightly. To contrast KL-Mobile and K-Mobile, without labor mobility the drop of welfare and utility of *Foreign* households is deeper because of the spending and tax shock; according to the contrast between L-Mobile and KL-Immobile, without labor mobility, the welfare and utility drop deeper than the context with labor mobility (L-Mobile).

Figure 6.4: Welfare Effects of Local Public Investment in *Home* Region

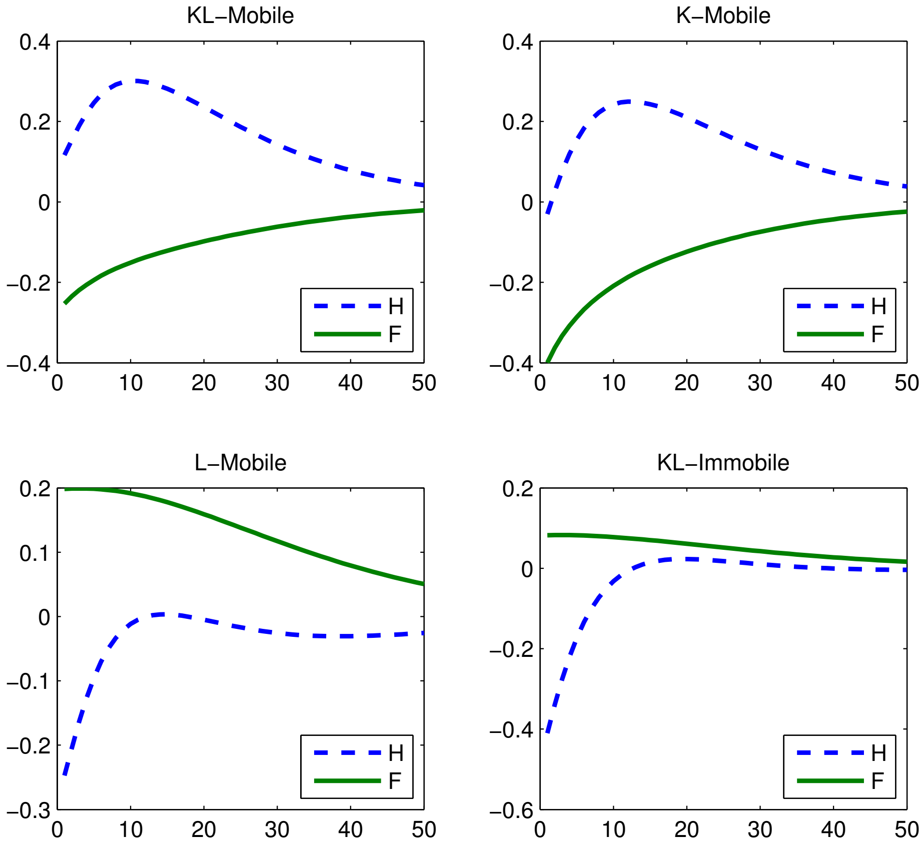
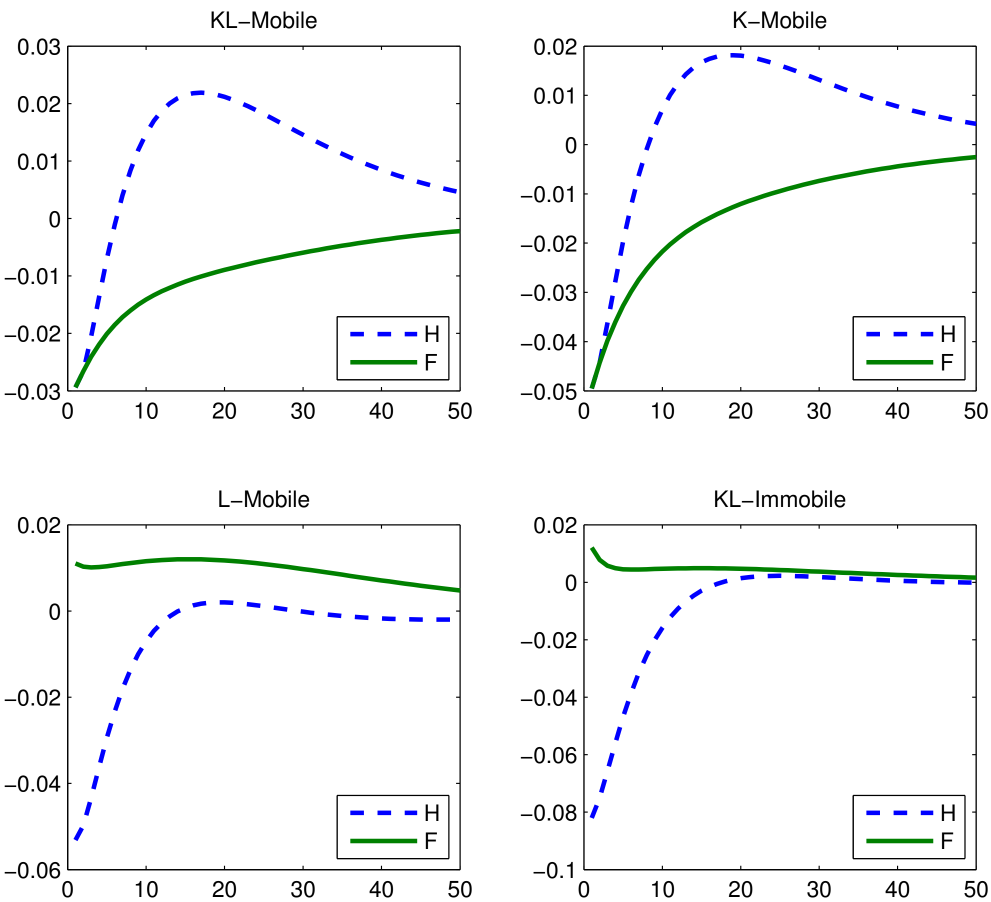


Figure 6.5: Utility Effects of Local Public Investment in *Home* Region



7 Concluding Remarks

This paper examines a two-region economy in the short term and long term and characterizes the fiscal policies to alleviate potential inefficiencies at the regional level. I characterize two types of inefficiencies: the asymmetric labor wedges with pricing frictions and the second-best factor allocation without pricing frictions. The short-term inefficiency arises from the insufficient adjustments of prices after an asymmetric shock, which hits one region. The asymmetric statuses of regional economies limit the effectiveness of monetary policy because the shared monetary policy can only work on the averaged wedges across regions. In the calibration, I illustrate the economic conditions after a technology shock and a demand shock in *Home* region. Both shocks cause asymmetric responses with and without labor mobility, but labor mobility can significantly alleviate the asymmetric economic conditions. These asymmetric economic statuses call for structural fiscal policy at the regional level to achieve economic synchronization between regions. I present two policy designs (regional government spending and mobility subsidies) aiming to synchronize economic situations across regions.

Labor mobility changes the interactions between regions significantly and its role should be emphasized more in research on open economies. When facing heterogeneous economic conditions across regions, monetary policy can only work on the average wedges (or output gaps) across regions and faces a dilemma that with the only policy instrument of nominal interest rates monetary policy necessarily cause welfare costs to some regions. To deal with regional economies with difficulties, government policy can also work on labor mobility, which provides one more channel to reduce the regional difference.

In the second part, I extend the baseline model to incorporate capital and study the long-term effects in different mobility contexts. I focus on the effects of public investment in the non-traded sector in *Home* region. With this public investment, financed by economy-wide factor taxes, factor mobility contexts play a critical role in adjusting the output and welfare effects. According to policy experiments, the full mobility of two factors dominates other contexts. This result clearly demonstrates the importance of factor market integration when fiscal policy plays an important role across regions, especially in a fiscal union or an economic zone with some level of fiscal integration.

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8 Appendix

A. The Model without Capital

A.1 The Consumption Problem and F.O.C.

The consumption problem of the *Foreign* households are analogous to the *Home* household's. I use superscript * to indicate the *Foreign* variables. I use ϕ_F to denote the openness of *Foreign* region. The utility function of the *Foreign* households are the same as the *Home* households.

The composite consumption goods is given by

$$C_t^* = \frac{\left(C_{T,t}^*\right)^{1-\gamma}\left(C_{N,t}^*\right)^{\gamma}}{(1-\gamma)^{(1-\gamma)}\gamma^{\gamma}}.$$

The consumption of traded goods is given

$$C_{T,t}^* = \left[(1-\phi^*)^{\frac{1}{\eta}} C_{HT,t}^{*\frac{\eta-1}{\eta}} + \phi^{\frac{1}{\eta}} C_{FT,t}^{*\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

where

$$C_{HT,t} \equiv \left[\int_0^1 c_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}} \text{ and } C_{FT,t} \equiv \left[\int_0^1 c_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

The amounts of traded and non-traded goods have the structural relation

$$\frac{U_{C_T^*}\left(C_t^*, L_t\right)}{U_{C_N^*}\left(C_t^*, L_t\right)} = \frac{P_{T,t}}{P_{N,t}^*}.$$

Foreign households optimally minimize the expenditure on *Home* traded goods and *Foreign* traded goods. This implies the demand for *Home* traded goods and *Foreign* traded goods and for each of the differentiated products in the economy:

$$C_{HT,t}^* = (1-\phi^*)\, C_{T,t}\left(\frac{P_{HT,t}}{P_{T,t}}\right)^{-\eta} \text{ and } \quad C_{FT,t}^* = \phi^* C_{T,t}^*\left(\frac{P_{FT,t}}{P_{T,t}}\right)^{-\eta},$$
$$c_{HT,t}^*(z) = C_{HT,t}\left(\frac{p_{HT,t}(j)}{P_{HT,t}}\right)^{-\epsilon} \text{ and } \quad c_{FT,t}^*(z) = C_{FT,t}^*\left(\frac{p_{FT,t}(j)}{P_{FT,t}}\right),$$

where

$$P_{HT,t} = \left[\int_0^1 p_{HT,t}(j)dj\right]^{\frac{1}{\varepsilon-1}} \text{and} P_{FT,t} = \left[\int_0^1 p_{FT,t}(z)dj\right]^{\frac{1}{\varepsilon-1}},$$

and

$$P_{T,t}^* = \left[(1-\phi)\, P_{HT,t}^{1-\eta} + \phi P_{FT,t}^{1-\eta}\right]^{\frac{1}{1-\eta}},$$
$$P_t^* = \left(P_{T,t}^*\right)^{1-\gamma}\left(P_{N,t}^*\right)^{\gamma}.$$

The Lagrangian problem is given as

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(C_T, C_N, L_t\right) + \lambda_t \left\{B_t + \left(1 - \tau_t^w\right) W_t L_t - \left[P_{N,t} C_{N,t} + P_{T,t} C_t + \mathbb{E}_t \left\{Q_{t,t+1} B_{t+1}\right\}\right]\right\}$$

F.O.C.

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_{NT,t}} =& \beta^t U_C \cdot \frac{\partial C_t}{\partial C_{N,t}} - \lambda_t P_{N,t} = 0, \\
\frac{\partial \mathcal{L}}{\partial C_{T,t}} =& \beta^t U_C \cdot \frac{\partial C_t}{\partial C_{T,t}} - \lambda_t P_{T,t} = 0, \\
\frac{\mathcal{L}}{\partial L_t} =& \beta U_L + \lambda_t \cdot \left(1 - \tau_t^w\right) W_t = 0, \\
\frac{\partial \mathcal{L}}{\partial B_{t+1}} =& \lambda_{t+1} - \lambda_t \mathbb{E}_t \left\{Q_{t,t+1}\right\} = 0.
\end{aligned}$$

Two useful results:

$$\begin{aligned}
\frac{\partial C_t}{\partial C_{N,t}} &= \gamma \cdot \frac{C_t}{C_{NT,t}}, \\
\frac{\partial C_t}{\partial C_{T,t}} &= (1 - \gamma) \cdot \frac{C_t}{C_{T,t}}.
\end{aligned}$$

Rewrite these equations, we have

$$\begin{aligned}
\frac{P_{N,t}}{P_{T,t}} =& \frac{\partial C_t}{\partial C_{N,t}} / \frac{\partial C_t}{\partial C_{T,t}}, \\
-\frac{U_L}{W_t} =& \frac{U_C \cdot \frac{\partial C_t}{\partial C_{T,t}}}{P_{T,t}} = \frac{U_C \cdot \frac{\partial C_t}{\partial C_{N,t}}}{P_{N,t}}, \\
\mathbb{E}_t \left\{Q_{t,t+1}\right\} =& \left\{\beta \frac{U_{C,t+1} \cdot \frac{\partial C_{t+1}}{\partial C_{N,t+1}}}{P_{N,t+1}}\right\} / \left\{\frac{U_{C,t} \cdot \frac{\partial C_t}{\partial C_{N,t}}}{P_{N,t}}\right\}.
\end{aligned}$$

The period utility function takes the form

$$U\left(C_t, L_t\right) = \frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{L_t^{1+\varphi}}{1+\varphi}.$$

After substitute the period utility function into the optimal conditions of households, we have precise description of consumer behavior, given as:

$$\frac{P_{NT,t}}{P_{T,t}} = \frac{\gamma}{1-\gamma} \cdot \frac{C_{T,t}}{C_{N,t}}, \tag{8.1}$$

$$-\chi \frac{L_t^{\varphi}}{W_t} = \frac{C_t^{-\sigma}}{P_{N,t}} \cdot \gamma \frac{C_t}{C_{N,t}} = \frac{C_t^{-\sigma}}{P_{T,t}} \cdot (1-\gamma) \frac{C_t}{C_{T,t}}, \tag{8.2}$$

$$\left[\mathbb{E}_t \left\{Q_{t,t+1}\right\}\right]^{-1} = \mathbb{E}_t \left\{\frac{C_t^{-\sigma}}{C_{t+1}^{-\sigma}} \cdot \frac{C_t}{C_{t+1}} \frac{C_{N,t+1}}{C_{N,t}} \cdot \frac{P_{N,t+1}}{P_{N,t}}\right\}. \tag{8.3}$$

Denote the price inflation of nontraded goods as $\pi_{N,t+1}$, and denote the price of traded goods as $\pi_{T,t+1}$ written as

$$\Pi_{N,t+1} \equiv \frac{P_{N,t+1}}{P_{N,t}},$$
$$\Pi_{T,t} \equiv \frac{P_{T,t+1}}{P_{T,t}}.$$

Substituting into (8.1) and rewriting a little, we the relationship between the price inflation of two types of goods

$$\frac{P_{N,t+1}}{P_{N,t}} \times \left[\frac{P_{T,t}}{P_{T,t+1}}\right] = \frac{C_{T,t+1}}{C_{T,t}} \times \frac{C_{N,t}}{C_{N,t+1}},$$
$$\frac{\Pi_{N,t+1}}{\Pi_{T,t+1}} = \frac{C_{T,t+1}}{C_{T,t}} \times \frac{C_{N,t}}{C_{N,t+1}}.$$

Then, the Euler equation, (8.3), can be rewritten as

$$\left[\mathbb{E}_t \left\{Q_{t,t+1}\right\}\right]^{-1} = \mathbb{E}_t \left\{\frac{C_t^{-\sigma}}{\beta C_{t+1}^{-\sigma}} \cdot \frac{C_t}{C_{t+1}} \frac{C_{N,t+1}}{C_{N,t}} \cdot \Pi_{N,t+1}\right\}.$$

A.2 Price Stickiness and Wedges

The flexible pricing is

$$P_{s,t}^i = \frac{\varepsilon}{\varepsilon - 1} \frac{W_t^i}{MP_{s,t}^i}, i \in \{H, F\}, s \in \{N, T\}. \tag{8.4}$$

The sale of traded goods of region $i \in \{H, F\}$ is

$$\left(C_{T,t} + C_{T,t}^*\right) P_{T,t}^i \left(\frac{P_{T,t}^i}{\bar{P}_{T,t}}\right)^{-\mu}.$$

The surplus (or deficit) of *Home* region is

$$\begin{aligned} S_t &= \left(C_{T,t} + C_{T,t}^*\right) P_{T,t} \left(\frac{P_{T,t}}{\bar{P}_{T,t}}\right)^{-\mu} - \left(C_{T,t} + C_{T,t}^*\right) P_{T,t}^* \left(\frac{P_{T,t}^*}{\bar{P}_{T,t}}\right)^{-\mu} \\ &= \left(C_{T,t} + C_{T,t}^*\right) \left(\bar{P}_{T,t}\right)^{\mu} \left[\frac{\left(P_{T,t}^*\right)^{\mu-1} - \left(P_{T,t}\right)^{\mu-1}}{\left(P_{T,t} P_{T,t}^*\right)^{\mu-1}}\right]. \end{aligned}$$

We immediately see that trade surplus is decreasing function of prices.

The labor supply condition with flexible price is

$$\frac{W_t^i}{P_t^i} = -\frac{U_L\left(C_t^i, L_t^i\right)}{U_C\left(C_t, L_t^i\right)}. \tag{8.5}$$

Combine flexible pricing equation (8.5) with (8.4), we have

$$\frac{W_t^i}{P_t^i} = \frac{P_{s,t}^i}{P_t^i} MP_{s,t}^i = -\frac{U_L\left(C_t^i, L^i\right)}{U_C\left(C_t^i, L_t^i\right)}.$$

Rearrange the equation above, we have

$$\mathcal{W}_{s,t}^i \equiv \frac{P_{s,t}^i}{P_t^i} + \frac{1}{MP_{s,t}^i} \frac{U_L\left(C_t^i, L_t^i\right)}{U_C\left(C_t^i, L_t^i\right)}, i \in \{H, F\}, s \in \{N, T\}. \tag{8.6}$$

$\mathcal{W}_{s,t}^i$ represents the deviation from optimal allocation of labor, named labor wedge. When prices are perfectly flexible, $\mathcal{W}_{s,t}^i = 0$. When prices are fixed or sticky, the firm adjust hiring according to demand which is decided by the fixed price and cause labor wedges. $\mathcal{W}_s^i > 0$ means recessions in sector s region i and $\mathcal{W}_s^i < 0$ means booming.

(1) Labor Wedges with Mobility

For workers from an arbitrary region $i \in \{H, F\}$, define $\mathcal{W}_{i,t}^i$ and $\mathcal{W}_{-i,t}^i$ as the respective labor wedges in domestic market (i) and in abroad market ($-i$).

In the region $i \in \{H, F\}$,

$$\begin{aligned}
\frac{W_t^i}{\bar{P}_t^i} &= -\frac{U_L\left(C_t^i, L_t^i\right)}{U_C\left(C_t^i, L_t^i\right)} (1-\eta_l)^{\frac{1}{\mu_l}} \left(\frac{L_{i,t}^i}{L_t^i}\right)^{-\frac{1}{\mu_l}} \\
W_t^i &= P_t^i MP\left(L_{i,t}^i + L_{i,t}^{-i}\right) \\
\frac{W_t^{-i}}{\bar{P}_t^i} &= -\frac{U_L\left(C_t^i, L_t^i\right)}{U_C\left(C_t^i, L_t^i\right)} \eta_l^{\frac{1}{\mu_l}} \left(\frac{L_{-i}^i}{L_t^i}\right)^{-\frac{1}{\mu_l}} \\
W_t^{-i} &= P_t^{-i} MP\left(L_{-i,t}^{-i} + L_{-i,t}^i\right).
\end{aligned}$$

For each region, the domestic and abroad wedges are given below

$$\begin{aligned}
\mathcal{W}_{i,t}^i = & \quad \frac{P_t^i}{\bar{P}_t^i} + \frac{1}{MP\left(L_{i,t}^i + L_{i,t}^{-i}\right)} \frac{U_L\left(C_t^i, L_t^i\right)}{U_C\left(C_t^i, L_t^i\right)} (1-\eta_l)^{\frac{1}{\mu_l}} \left(\frac{L_{i,t}^i}{L_t^i}\right)^{-\frac{1}{\mu_l}} \\
\mathcal{W}_{-i}^i = & \quad \frac{P_t^{-i}}{\bar{P}_t^i} + \frac{1}{MP\left(L_{-i,t}^{-i} + L_{-i,t}^i\right)} \frac{U_L\left(C_t^i, L_t^i\right)}{U_C\left(C_t^i, L_t^i\right)} \eta_l^{\frac{1}{\mu_l}} \left(\frac{L_{-i}^i}{L_t^i}\right)^{-\frac{1}{\mu_l}}
\end{aligned}.$$

A.3 Indirect Utility

For *Foreign* households this function is analogous. The indirect utility function can be rewritten as

$$
\begin{aligned}
V_t =& U\left(C_{T,t}, C_{N,t}, L_{T,t}, L_{N,t}\right) \\
V_{C,t} =& V_{C_T,t} + V_{C_N,t} \\
=& \left[U_{C_T,t} + U_{L_T,t}\right] + \left[U_{C_N,t} + U_{L_T,t}\right] \\
=& \left[U_C \frac{\partial C_t}{\partial C_{T,t}} + U_L \left(\frac{C_{T,t}}{A_{T,t}}\right)^{\frac{\alpha}{1-\alpha}} \frac{1}{(1-\alpha)A_{T,t}}\right] + \left[U_C \frac{\partial C_t}{\partial C_{N,t}} + U_L \left(\frac{C_{N,t}}{A_{N,t}}\right)^{\frac{\alpha}{1-\alpha}} \frac{1}{(1-\alpha)A_{N,t}}\right] \\
=& U_C \left[\frac{\partial C_t}{C_{T,t}} + \frac{1}{(1-\alpha)A_{T,t}L_{T,t}^{-\alpha}} \frac{U_L}{U_C}\right] + U_C \left[\frac{\partial C_t}{\partial C_{N,t}} + \frac{1}{(1-\alpha)A_{N,t}L_{N,t}^{-\alpha}} \frac{U_L}{U_C}\right] \\
& \text{with } \frac{\partial C_t}{\partial C_{T,t}} = \frac{(1-\gamma)C_t}{C_{T,t}} = \frac{P_{T,t}}{P_t} \text{ and } \frac{\partial C_t}{\partial C_{N,t}} = \frac{\gamma C_t}{C_{N,t}} = \frac{P_{N,t}}{P_t} \\
=& U_C \left[\frac{P_{T,t}}{P_t} + \frac{1}{(1-\alpha)A_{T,t}L_{T,t}^{-\alpha}} \frac{U_L}{U_C}\right] + U_C \left[\frac{P_{N,t}}{P_t} + \frac{1}{(1-\alpha)A_{N,t}L_{N,t}^{-\alpha}} \frac{U_L}{U_C}\right] \\
=& U_C \left[\mathcal{W}_{T,t} + \mathcal{W}_{N,t}\right]
\end{aligned}
$$

In the following paragraphs, I will approximate the indirect utility to the second order with labor wedges. Rearrange the labor wedge specification, (8.6), we have

$$
U_C\left(C_t^i, L_t^i\right) = \left[MP_{s,t}^i \left(\mathcal{W}_{s,t}^i - \frac{P_{s,t}^i}{P_t^i}\right)\right]^{-1} U_L\left(C_t^i, L_t^i\right).
$$

Denote $\omega_{s,t}^i \equiv \left[MP_{s,t}\left(\mathcal{W}_{s,t}^i - \frac{P_{s,t}^i}{P_t^i}\right)\right]$ capturing the relationship between the marginal utility of consumption and the marginal dis-utility of working.

Take derivative with respect to L and C in the two sides, we obtain

$$
\begin{aligned}
\underbrace{\frac{\partial U_C\left(C_t^i, L_t^i\right)}{\partial C_t^i} \frac{\partial C_t^i}{\partial L_t^i}}_{U_{CL}} &= \left[MP_{s,t}^i \left(\mathcal{W}_{s,t}^i - \frac{P_{s,t}^i}{P_t^i}\right)\right]^{-1} U_{LL}\left(C_t^i, L_t^i\right) \\
U_{CC}\left(C_t^i, L_t^i\right) &= \left[MP_{s,t}^i \left(\mathcal{W}_{s,t}^i - \frac{P_{s,t}^i}{P_t^i}\right)\right]^{-1} \underbrace{\frac{\partial U_L\left(C_t^i, L_t^i\right)}{\partial L_t^i} \frac{\partial L_t^i}{\partial C_t^i}}_{U_{LC}} \\
U_{CC}\left(C_t^i, L_t^i\right) &= -\left[MP_{s,t}^i \left(\mathcal{W}_{s,t}^i - \frac{P_{s,t}^i}{P_t^i}\right)\right]^{-2} U_{LL}\left(C_t^i, L_t^i\right)
\end{aligned}
$$

The last is derived using the condition that $U_{CL} = -U_{LC}$.

For exposition convenience, I use U_C as a substitute of $U_C\left(\bar{C}, \bar{L}\right)$ and U_L as a substitute of

$U_L\left(\bar{C},\bar{L}\right)$. The indirect utility function of *Home* households is

$$\begin{aligned}
V_t^i =&U\left(C_t^i,L_t^i\right)\\
=&U\left(\bar{C}^i,\bar{L}^i\right)+\sum_{s\in\{N,T\}}\left\{U_C\frac{\partial\bar{C}^i}{\partial\bar{C}_s^{\ i}}\cdot\left(C_{s,t}^i-\bar{C}_s^{\ i}\right)+U_L\cdot\left(L_{s,t}^i-\bar{L}_s^{\ i}\right)\right\}+\\
&\frac{1}{2}\begin{bmatrix} U_{CC}\frac{\partial^2\bar{C}^i}{\partial\left(\bar{C}_s^{\ i}\right)^2}\cdot\left(C_t^i-\bar{C}^i\right)^2+U_{LL}\cdot\left(L_t^i-\bar{L}^i\right)^2\\ +U_{LC}\cdot\left(C_t^i-\bar{C}^i\right)\left(L_t^i-\bar{L}^i\right)+U_L\cdot\left(C_t^i-\bar{C}^i\right)\left(L_t^i-\bar{L}^i\right)\end{bmatrix}\\
=&U\left(\bar{C}^i,\bar{L}^i\right)+\sum_{s\in\{N,T\}}\left\{U_C\frac{\partial\bar{C}^i}{\partial\bar{C}_s^{\ i}}\cdot\left(C_{s,t}^i-\bar{C}_s^{\ i}\right)+U_L\cdot\left(L_{s,t}^i-\bar{L}_s^{\ i}\right)\right\}+\\
&+\frac{1}{2}\left[U_{CC}\cdot\left(C_t^i-\bar{C}^i\right)^2+U_{LL}\cdot\left(L_t^i-\bar{L}^i\right)^2\right]\\
&+\frac{1}{2}\left[\underbrace{\frac{\partial U_L\cdot(C_t^i-\bar{C}^i)}{\partial C}\left(L_t^i-\bar{L}^i\right)+\frac{\partial U_C\cdot(L_t^i-\bar{L}^i)}{\partial L}\left(C_t^i-\bar{C}^i\right)}_{\approx\partial U_L\cdot\left(L_t^i-\bar{L}^i\right)+\partial U_C\cdot\left(C_t^i-\bar{C}^i\right)=0}\right]\\
=&U\left(\bar{C}^i,\bar{L}^i\right)+U_C\cdot\left(C_t^i-\bar{C}^i\right)\sum_{s\in\{N,T\}}\left[\frac{P_{s,t}^i}{P_t^i}+\frac{U_L\cdot(L_t^i-\bar{L}^i)}{U_C\cdot(C_t^i-\bar{C}^i)}\right]\\
&\frac{1}{2}U_{CC}\cdot\left(C_t^i-\bar{C}^i\right)^2\left[1+\frac{U_{LL}\cdot(L_t^i-\bar{L}^i)^2}{U_{CC}\cdot(C_t^i-\bar{C}^i)^2}\right]
\end{aligned}$$

In the small neighborhood of the steady state, (C_{ss},L_{ss}), without borrowing or lending, the optimal trade-off between consumption and working implies $-\frac{U_L}{U_C}=\frac{W}{P}=\frac{C-C_{ss}}{L-L_{ss}}$, which implies further that $\partial U_L\cdot\left(L_t^i-L_{ss}^i\right)+\partial U_C\cdot\left(C_t^i-C_{ss}^i\right)=0$. After rearranging the equation above, we obtain

$$\begin{aligned}
&V_t^i-U\left(\bar{C}^i,\bar{L}^i\right)\\
=&U_C\cdot\left(C_t^i-\bar{C}^i\right)\left\{1+\frac{U_L\cdot(L_t^i-\bar{L}^i)}{U_C\cdot(C_t^i-\bar{C}^i)}+\frac{1}{2}\frac{U_{CC}\cdot(C_t^i-\bar{C}^i)}{U_C}\left[1+\frac{U_{LL}\cdot(L_t^i-\bar{L}^i)^2}{U_{CC}\cdot(C_t^i-\bar{C}^i)^2}\right]\right\}\\
=&U_C\cdot\left(C_t^i-\bar{C}^i\right)\\
&\left\{1+\left[MP_{s,t}^i\left(\mathcal{W}_{s,t}^i-\frac{P_{s,t}^i}{P_t^i}\right)\cdot\frac{P_t^i}{W_t^i}\right]+\frac{1}{2}\frac{U_{CC}\cdot(C_t^i-\bar{C}^i)}{U_C}\left[1-\left[MP_{s,t}^i\left(\mathcal{W}_{s,t}^i-\frac{P_{s,t}^i}{P_t^i}\right)\cdot\left(\frac{P_t^i}{W_t^i}\right)\right]^2\right]\right\}
\end{aligned}$$

The budget identity is $WL=PC$ with financing condition as given and there is one special functional form of utility (CRRA), $\sigma\equiv-\frac{U_{CC}C}{U_C}$ and $\varphi\equiv\frac{U_{LL}L}{U_L}$.

(1) Indirect Utility with Mobility and the Optimal Policy

The utility with optimal decisions of working and consumption is

$$
\begin{aligned}
V_t^i &= U_C \frac{\partial C_t^i}{\partial C_{i,t}^i} + U_L \frac{\partial L_t^i}{\partial L_{i,t}^i} \cdot \frac{1}{MPL_t^i} + U_C \frac{\partial C_t^i}{\partial C_{-i,t}^i} + U_L \frac{\partial L_t^i}{\partial L_{-i,t}^i} \cdot \frac{1}{MPL_{-i,t}} \\
&= U_C \left[\frac{\partial C_t^i}{\partial C_{i,t}^i} + \frac{1}{MPL_t^i} \frac{U_L}{U_C} \frac{\partial L_t^i}{\partial L_{i,t}^i} + \frac{\partial C_t^i}{\partial C_{-i,t}^i} + \frac{1}{MPL_{-i,t}} \frac{U_L}{U_C} \frac{\partial L_t^i}{\partial L_{-i,t}^i} \right] \\
&\underbrace{=}_{1} U_C \left[\frac{P_{i,t}}{\bar{P}_t^i} + \frac{1}{MPL_t^i} \frac{U_L}{U_C} \frac{\partial L_t^i}{\partial L_{i,t}^i} + \frac{P_{-i,t}}{\bar{P}_t^i} + \frac{1}{MPL_{-i,t}} \frac{U_L}{U_C} \frac{\partial L_t^i}{\partial L_{-i,t}^i} \right] \\
&= U_{C,t} \left[\mathcal{W}_{i,t}^i + \mathcal{W}_{-i,t}^i \right].
\end{aligned}
$$

The equity 1 is the optimal consumption condition $\left(\frac{\partial C_t^i}{\partial C_{i,t}^i} = \frac{P_{i,t}}{\bar{P}_t^i} \right)$ and the implicit resource constraints $\left(\partial C_{i,t}^i = MPL_t^i \cdot \partial L_{i,t}^i \text{ and } \partial C_{-i,t}^i = MPL_{-i,t} \cdot \partial L_{-i,t}^i \right)$.

The optimal policy, which is conducted by the social planner, maximizes the expected sum of indirect utilities in economy-wide

$$
\max_{Q_{t,t+1}, G_t} \mathbb{E}_t \lambda^i V_t^i = \max_{Q_{t,t+1}, G_t} \mathbb{E}_t \sum_{i \in \{H,F\}} \lambda^i U_{C,t}^i \left[\mathcal{W}_{i,t}^i + \mathcal{W}_{-i,t}^i \right]
$$

subject to the Euler equations of households in each regions

$$
\frac{U_{C,t}^i}{\bar{P}_t^i} = \mathbb{E}_t \left\{ \frac{\beta U_{C,t+1}}{\bar{P}_{t+1}^i Q_{t,t+1}} \right\}
$$

A.3 The First-Best Outcome

The social planing problem is

$$
\max_{\{H_{l,t}^i\}} \sum \lambda^i V \left(Y_{N,t}^i, Y_{T,t}^i \right)
$$

subject to

$$
\begin{aligned}
L_{T,t} + L_{N,t} &= L_t \\
L_{T,t} + L_{N,t} &= L_t^* \\
L_t + L_t^* &= \bar{L}_t \\
-\frac{U_L \left(C_t, L_t \right)}{U_C \left(C_t, L_t \right)} &= \frac{C_t}{L_t}, \\
\text{with } A_{s,t}^i L_{s,t}^i &= Y_{s,t}^i, i \in \{H, F\} \text{ and } s \in \{N, T\}.
\end{aligned}
$$

The Lagrangian function is written as

$$\begin{aligned}\mathcal{L} = &\sum \lambda^i V\left(Y_{N,t}, Y_{T,t}\right) \\ &+ \zeta\left[\bar{L}_t - \left(L_{T,t} + L_{N,t} + L_{T,t} + L_{N,t}\right)\right] \\ &+ \omega\left[C_t U_C\left(C_t, L_t\right) + L_t U_L\left(C_t, L_t\right)\right].\end{aligned}$$

F.O.C.:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial L^i_{N,t}} =& \frac{\partial \lambda^i V\left(Y^i_{N,t}, Y^i_{T,t}\right)}{\partial Y^i_{N,t}} \frac{\partial Y^i_{N,t}}{\partial L^i_{N,t}} - \zeta + \omega U_L(C_t, L_t) = 0 \text{ for } i \in \{H, F\} \\ \frac{\partial \mathcal{L}}{\partial L^i_{l,t}} =& \frac{\partial \lambda^i V\left(Y^i_{N,t}, Y_{T,t}\right)}{\partial Y^i_{T,t}} \frac{\partial Y^i_{T,t}}{\partial L^i_{T,t}} - \zeta + \omega U_L\left(C_t, L_t\right) = 0 \text{ for } i \in \{H, F\} \\ \frac{\partial Y^i}{\partial L^i_{N,t}} =& A^i_{N,t}\left(L^i_{N,t}\right)^{-\alpha} \\ \frac{\partial \mathcal{L}}{\partial Y_{T,t}} =& A^i_{T,t}\left(L^i_{T,t}\right)^{-\alpha}.\end{aligned}$$

Within each region, the allocation allocation of labor among traded and non-traded sectors satisfies

$$\begin{aligned}\frac{\partial V\left(Y^i_{N,t}, Y^i_{T,t}\right)}{\partial Y^i_{N,t}} \frac{\partial Y^i_{N,t}}{\partial L^i_{N,t}} &= \frac{\partial V\left(Y^i_{N,t}, Y^i_{T,t}\right)}{\partial Y^i_{T,t}} \frac{\partial Y^i_{T,t}}{\partial L^i_{T,t}} \\ \frac{\frac{\partial C_t}{\partial C_{N,t}}}{\frac{\partial C_t}{\partial C_{T,t}}} &= \frac{A^i_{T,t}}{A^i_{N,t}} \frac{\left(L^i_{N,t}\right)^{\alpha}}{\left(L^i_{T,t}\right)^{\alpha}}, \text{ for } i \in \{H, F\}.\end{aligned}$$

Within each region, given the productivity level of two sectors the optimal allocation of labor is proportional to the marginal utility that each types of goods can bring. This result show that the optimal production and allocation of labor among two sectors are embodied in the utility of households when they consume these goods.

Across regions, the optimal allocation of labor across regions satisfies

$$\begin{aligned}\frac{\partial \lambda V\left(Y_{N,t}, Y_{T,t}\right)}{\partial Y_{N,t}} \frac{\partial Y_{N,t}}{\partial L_{N,t}} =& \frac{\partial \lambda^* V\left(Y^*_{N,t}, Y^*_{T,t}\right)}{\partial Y^*_{N,t}} \frac{\partial Y^*_{N,t}}{\partial L^*_{N,t}}, \\ \frac{\lambda}{\lambda^*} =& \frac{A^*_{N,t}}{A_{N,t}} \frac{\left(L_{N,t}\right)^{\alpha}}{\left(L^*_{N,t}\right)^{\alpha}}; \\ \frac{\partial \lambda V\left(Y_{N,t}, Y_{T,t}\right)}{\partial Y_{T,t}} \frac{\partial Y_{T,t}}{\partial L_{T,t}} =& \frac{\partial \lambda^* V\left(Y^*_{N,t}, Y^*_{T,t}\right)}{\partial Y^*_{T,t}} \frac{\partial Y^*_{T,t}}{\partial L^*_{T,t}}, \\ \frac{\lambda}{\lambda^*} =& \frac{A^*_{T,t}}{A_{T,t}} \frac{\left(L_{T,t}\right)^{\alpha}}{\left(L^*_{T,t}\right)^{\alpha}}.\end{aligned}$$

This result show that the marginal production of the two region is inversely proportional to its Pareto weights. Using the assumption that the utility function of two regions and products from two regions are identical, the allocation of labor among two regions is proportional to its Pareto weight, given their technology level.

To summarize, the first-best equilibrium is stated as:

$$\begin{aligned}\frac{\lambda}{\lambda^*} &= \frac{A^*_{N,t}}{A_{N,t}}\frac{(L_{N,t})^\alpha}{\left(L^*_{N,t}\right)^\alpha} = \frac{A^*_{T,t}}{A_{T,t}}\frac{(L_{T,t})^\alpha}{\left(L^*_{T,t}\right)^\alpha};\\ \frac{\frac{\partial C_t}{\partial C_{N,t}}}{\frac{\partial C_t}{\partial C_{T,t}}} &= \frac{A_{T,t}}{A_{N,t}}\frac{(L_{N,t})^\alpha}{(L_{T,t})^\alpha} = \frac{A^*_{T,t}}{A^*_{N,t}}\frac{\left(L^*_{N,t}\right)^\alpha}{\left(L^*_{T,t}\right)^\alpha}.\end{aligned}$$

B. Steady State

The steady and balanced state of the two-region economy satisfies conditions as follow. There are some assumption held in balanced steady state: (i) no factor moving across regions; (ii) net financial assets holdings are zero, i.e. individuals from different regions have the same financial status; (iii) residents in different regions have the same utility.

Real wage equivalence

$$\frac{W_t}{P_t} = -\frac{U_L(\cdot)}{U_C(\cdot)} = -\frac{U^*_L(\cdot)}{U_C(\cdot)} = \frac{W^*_t}{P^*_t}.$$

The pricing formula implies

$$\begin{aligned}\frac{W_t}{P_t} &= \frac{\varepsilon-1}{\varepsilon}\left[\left(A_{T,t}L^{-\alpha}_{T,t}\right)^{1-\gamma}\left(A_{N,t}L^{-\alpha}_{N.t}\right)^{\gamma}\right](1-\alpha)\\ &= \frac{\varepsilon-1}{\varepsilon}\left[A^{1-\gamma}_{T,t}A^{\gamma}_{N,t}\left(L^{1-\gamma}_{T,t}L^{\gamma}_{N,t}\right)^{-\alpha}\right](1-\alpha)\\ &= \frac{\varepsilon-1}{\varepsilon}\left[A^{1-\gamma}_{T,t}A^{\gamma}_{N,t}\left(L_{T,t}\left(\frac{\gamma}{1-\gamma}\right)^{\gamma}\right)^{-\alpha}\right](1-\alpha);\\ \frac{W^*_t}{P^*_t} &= \frac{\varepsilon-1}{\varepsilon}\left[\left(A^*_{T,t}\right)^{1-\gamma}\left(A^*_{N,t}\right)^{\gamma}\left(L^{*1-\gamma}_{T,t}L^{*\gamma}_{N,t}\right)^{-\alpha}\right](1-\alpha).\end{aligned}$$

The equality of real wage implied that

$$\left(A^*_{T,t}L^{*-\alpha}_{T,t}\right)^{1-\gamma}\left(A^*_{N,t}L^{*-\alpha}_{N,t}\right)^{\gamma} = A^{1-\gamma}_{T,t}L_{T,t}A^{\gamma}_{N,t}L_{N,t}.$$

which represents the equality of composite productivity. There are indeterminacy in the composition of productivities of the two sectors, traded and non-traded. If this two-regional economy has symmetric distribution of labor, this result implies further that

$$\left(A^*_{T,t}\right)^{1-\gamma}\left(A^*_{N,t}\right)^{\gamma} = A^{1-\gamma}_{T,t}A^{\gamma}_{N,t}.$$

B1. The Derivation of Steady State

(1)Steady State with Labor Autarky

In the steady state, i.e., after the effects of shocks vanish totally, the economy system back to the state with constant nominal variables and zero inflation. Normalize the price level of the two region to unit. The consumption and labor supply of the two regions satisfy the conditions as follow

$$\begin{aligned}\chi L^{\varphi} C^{\sigma} =& \frac{\varepsilon-1}{\varepsilon} \cdot (1-\alpha) A L^{-\alpha} = \frac{W}{P} \\ \chi \left(L^*\right)^{\varphi} \left(C^*\right)^{\sigma} =& \frac{\varepsilon-1}{\varepsilon} \cdot (1-\alpha) A^* \left(L^*\right)^{-\alpha} = \frac{W^*}{P^*} \\ Y = C =& A L^{1-\alpha} \\ Y^* = C^* =& A \left(L^*\right)^{1-\alpha}.\end{aligned}$$

The labor supply and consumption in steady state can be expressed as

$$\begin{aligned}L =& \left[\frac{\varepsilon-1}{\varepsilon} \cdot \frac{1-\alpha}{\chi} \cdot A^{1-\sigma}\right]^{\frac{1}{\varphi+\alpha+\sigma-\sigma\alpha}} \\ C =& \left[\frac{\varepsilon-1}{\varepsilon} \cdot \frac{1-\alpha}{\chi} A\right]^{\frac{1}{\sigma}} (L)^{\frac{-\alpha-\varphi}{\sigma}} \\ L^* =& \left[\frac{\varepsilon-1}{\varepsilon} \cdot \frac{1-\alpha}{\chi} \cdot A^{*1-\sigma}\right]^{\frac{1}{\alpha+\varphi+\sigma-\sigma\alpha}} \\ C^* =& \left[\frac{\varepsilon-1}{\varepsilon} \cdot \frac{1-\alpha}{\chi} A\right]^{\frac{1}{\sigma}} \left(L^*\right)^{\frac{-\alpha-\varphi}{\sigma}}.\end{aligned}$$

(2)Steady State with Labor Mobility

Given the economies of two regions are symmetric, I give details of the derivation of *Home* region. Aggregation of Labor Supply is

$$L = \left[(1-\eta_l)^{\frac{1}{\mu_l}} \left(L_H\right)^{\frac{\mu_l-1}{\mu_l}} + \eta_l^{\frac{1}{\mu_l}} \left(L_F\right)^{\frac{\mu_l-1}{\mu_l}}\right]^{\frac{\mu_l}{\mu_l-1}}$$

The labor supply conditions for the two regions are

$$\begin{aligned}\frac{W}{P} =& -\frac{U_L}{U_C} \frac{\partial L}{\partial L_H} = \chi L^{\varphi} C^{\sigma} \cdot (1-\eta_l)^{\frac{1}{\mu_l}} \left(\frac{L_H}{L}\right)^{-\frac{1}{\mu_l}}, \\ \frac{W^*}{P} =& -\frac{U_{L,t}}{U_{C,t}} \frac{\partial L}{\partial L_{F,t}} = \chi L^{\varphi} C^{\sigma} \cdot \eta_l^{\frac{1}{\mu_l}} \left(\frac{L_F}{L}\right)^{-\frac{1}{\mu_l}}.\end{aligned}$$

In steady states, $\bar{P} = \bar{P}^* = 1$ and $W = W^*$, we have

$$\begin{aligned} \left(\frac{1-\eta_l}{\eta_l}\right)^{\frac{1}{\mu_l}} \cdot \left(\frac{L_H}{L_F}\right)^{-\frac{1}{\mu_l}} &= 1 \\ \frac{1-\eta_l}{\eta_l} &= \frac{L_H}{L_F}. \end{aligned}$$

The steady state condition of labor is

$$\begin{aligned} \frac{\varepsilon-1}{\varepsilon} \cdot (1-\alpha) A \left(L_H + L_H^*\right)^{-\alpha} &= \chi L^{\varphi} C^{\sigma} \cdot (1-\eta_l)^{\frac{1}{\mu_l}} \left(\frac{L_H}{L}\right)^{-\frac{1}{\mu_l}} \\ \frac{\varepsilon-1}{\varepsilon} \cdot (1-\alpha) A^* \left(L_F^* + L_F\right)^{-\alpha} &= \chi L^{\varphi} C^{\sigma} \cdot \eta_l^{\frac{1}{\mu_l}} \left(\frac{L_F}{L}\right)^{-\frac{1}{\mu_l}}. \end{aligned}$$

In the steady state, the labor allocations are symmetric for two regions, i.e., $L_{F,t} = L_{H,t}^*$ and $L_{F,t}^* = L_{H,t}$, so we have

$$\underbrace{L_H + L_H^*}_{\text{Employment in H.}} = \underbrace{L_F^* + L_F}_{\text{Employment in F.}} = \underbrace{L_H + L_F}_{\text{Labor Supply from H.}} = \underbrace{L_F^* + L_H^*}_{\text{Labor Supply from F.}}.$$

With these symmetric conditions

$$\begin{aligned} \frac{\varepsilon-1}{\varepsilon} \cdot (1-\alpha) A \left(L_H + L_F\right)^{-\alpha} &= \chi L^{\varphi} C^{\sigma} \cdot (1-\eta_l)^{\frac{1}{\mu_l}} \left(\frac{L_H}{L}\right)^{-\frac{1}{\mu_l}} \\ \frac{\varepsilon-1}{\varepsilon} \cdot (1-\alpha) A^* \left(L_H + L_F\right)^{-\alpha} &= \chi L^{\varphi} C^{\sigma} \cdot \eta_l^{\frac{1}{\mu_l}} \left(\frac{L_F}{L}\right)^{-\frac{1}{\mu_l}}, \end{aligned}$$

and

$$C = A \left(L_H + L_F\right)^{1-\alpha}.$$

In the steady state

$$\begin{aligned} L_F &= \eta_l L \\ L_H &= (1-\eta_l) L \\ L &= \left[\frac{\epsilon-1}{\epsilon} \cdot \frac{1-\alpha}{\chi} \cdot A^{1-\sigma}\right]^{\frac{1}{\varphi+\alpha+\sigma-\sigma\alpha}} \\ C &= \left[\frac{\epsilon-1}{\epsilon} \cdot \frac{1-\alpha}{\chi} A\right]^{\frac{1}{\sigma}} (L)^{\frac{-\alpha-\varphi}{\sigma}}. \end{aligned}$$

Variables of *Foreign* region satisfy

$$\begin{aligned}
L_H^* &= \eta_l L^* \\
L_F^* &= (1-\eta_l)\, L^* \\
L^* &= \left[\frac{\epsilon-1}{\epsilon}\cdot\frac{1-\alpha}{\chi}\cdot A^{*1-\sigma}\right]^{\frac{1}{\varphi+\alpha+\sigma-\sigma\alpha}} \\
C^* &= \left[\frac{\epsilon-1}{\epsilon}\cdot\frac{1-\alpha}{\chi}A^*\right]^{\frac{1}{\sigma}}\left(L^*\right)^{\frac{-\alpha-\varphi}{\sigma}}.
\end{aligned}$$

(3) Steady State with Mobility Subsidy

The mobility subsidy achieve the condition that $W_t L_{H,t} = W_t^* L_{F,t}$ for households in *Home* regions and $W_t L_{H,t}^* = W_t^* L_{F,t}^*$ for households in *Foreign* region. The derivation is illustrated using the *Home* households and the results is also applied to *Foreign* households. In the symmetric steady state, $W = W^*$, hence $L_H = L_F$,$L_H^* = L_F^*$, and $L = L^*$. In steady state, $ms = \frac{1-\eta_l}{\eta_l}$. The total labor supply by *Home* households is

$$\begin{aligned}
L &= \left[(1-\eta_l)^{\frac{1}{\mu_l}}\left(L_H\right)^{\frac{\mu_l-1}{\mu_l}} + (ms\cdot\eta_l)^{\frac{1}{\mu_l}}\left(L_F\right)^{\frac{\mu_l-1}{\mu_l}}\right]^{\frac{\mu_l}{\mu_l-1}} \\
&= L_{H(F)}\left[2\left(1-\eta_l\right)^{\frac{1}{\mu_l}}\right]^{\frac{\mu_l}{\mu_l-1}}, \\
\frac{L_{H(F)}}{L} &= \left[2\left(1-\eta_l\right)^{\frac{1}{\mu_l}}\right]^{\frac{-\mu_l}{\mu_l-1}} \\
L_{H(F)} &= \left[2\left(1-\eta_l\right)^{\frac{1}{\mu_l}}\right]^{\frac{-\mu_l}{\mu_l-1}} L.
\end{aligned}$$

The labor supply condition is written

$$\begin{aligned}
\frac{\varepsilon-1}{\varepsilon}(1-\alpha)\left(L_H+L_H^*\right)^{-\alpha} &= \chi C^\sigma L^\varphi\left[2\left(1-\eta_l\right)\right]^{\frac{1}{\mu_l-1}} \\
\frac{\varepsilon-1}{\varepsilon}(1-\alpha)\left[\left(\frac{L_H}{L}+\frac{L_F}{L}\right)L\right]^{-\alpha} &= \chi\left[\left(\frac{L_H}{L}+\frac{L_F}{L}\right)^{1-\alpha}L^{1-\alpha}\right]^{\sigma}L^\varphi\left[2\left(1-\eta_l\right)\right]^{\frac{1}{\mu_l-1}} \\
\frac{\varepsilon-1}{\varepsilon}(1-\alpha)\left[2\left[2\left(1-\eta_l\right)^{\frac{1}{\mu_l}}\right]^{\frac{-\mu_l}{\mu_l-1}}L\right]^{-\alpha} &= \chi\left[2\left[2\left(1-\eta_l\right)^{\frac{1}{\mu_l}}\right]^{\frac{-\mu_l}{\mu_l-1}}L\right]^{(1-\alpha)\sigma}L^\varphi\left[2\left(1-\eta_l\right)\right]^{\frac{1}{\mu_l-1}} \\
\frac{\varepsilon-1}{\varepsilon}\frac{(1-\alpha)}{\chi}\left[\left(2\left(1-\eta_l\right)\right)^{\frac{-1}{\mu_l-1}}\right]^{-\alpha}L^{-\alpha} &= \left[\left(2\left(1-\eta_l\right)\right)^{\frac{-1}{\mu_l-1}}\right]^{(1-\alpha)\sigma}L^{(1-\alpha)\sigma}L^\varphi\left[2\left(1-\eta_l\right)\right]^{\frac{1}{\mu_l-1}} \\
\frac{\varepsilon-1}{\varepsilon}\frac{(1-\alpha)}{\chi}\left[2\left(1-\eta_l\right)\right]^{\frac{\alpha}{\mu_l-1}+\frac{(1-\alpha)\sigma}{\mu_l-1}-\frac{1}{\mu_l-1}} &= L^{\alpha+(1-\alpha)\sigma+\varphi}
\end{aligned}$$

$$\begin{aligned}
L &= \left\{\left[\frac{\varepsilon-1}{\varepsilon}\frac{1-\alpha}{\chi}\right]\left[2\left(1-\eta_l\right)\right]^{\frac{\alpha+(1-\alpha)\sigma-1}{\mu_l-1}}\right\}^{\frac{1}{\alpha+\sigma-\alpha\sigma+\varphi}} \\
C &= \left[\frac{\varepsilon-1}{\varepsilon}\frac{1-\alpha}{\chi}\right]^{\frac{1}{\sigma}}\left[2\left(1-\eta_l\right)\right]^{\frac{\alpha-1}{\sigma(\mu_l-1)}}L^{\frac{-\alpha-\varphi}{\sigma}} \\
W &= \chi C^\sigma L^\varphi\left[2\left(1-\eta_l\right)\right]^{\frac{1}{\mu_l-1}}.
\end{aligned}$$

C. Log-linearization, Welfare Loss and Optimal Policy

This appendix derive log-linearized two-region economy and a second-order approximation to the utility households in two regions, when the economy remains in a neighborhood of an efficient steady state. This derivation is modified from the Gali and Monacelli (2008) Chapter 4 appendix 1. Frequent use is made of the following second-order approximation of relative deviation in terms of log deviations

$$\frac{Z_t - Z}{Z} \simeq \hat{z}_t + \frac{1}{2}\hat{z}_t$$

where $\hat{z}_t \equiv z_t - z$ is the log deviation from steady state for a generic variable z_t. All along it is assumed that utility is separable in consumption and hours (i.e., $U_{CH} = 0$). In order to lighten the notation, define $U_t \equiv U(C_t, L_t)$, $U \equiv U(C, L)$, and variables with asterisk, for example $U(C_t^*, L_t^*)$, for *Foreign* region.

The second-order Taylor expansion of $\lambda U_t(C_t, L_t) + \lambda^* U(C_t^*, L_t^*)$ around their respective steady states (C, L) and (C^*, L^*) yields

$$\begin{aligned}
U_t - U \simeq & U_C C\left(\frac{C_t - C}{C}\right) + U_H L\left(\frac{L_t - L}{L}\right) + \frac{1}{2}U_{CC}C^2\left(\frac{C_t - C}{C}\right)^2 + \frac{1}{2}U_{LL}L^2\left(\frac{L_t - L}{L}\right)^2 \\
U_t^* - U^* \simeq & U_C C^*\left(\frac{C_t^* - C^*}{C^*}\right) + U_L L^*\left(\frac{L_t^* - L^*}{L^*}\right) \\
& + \frac{1}{2}U_{CC}C^{*2}\left(\frac{C_t^* - C^*}{C^*}\right)^2 + \frac{1}{2}U_{LL}L^{*2}\left(\frac{L_t^* - L^*}{L^*}\right)^2.
\end{aligned}$$

In terms of log deviations,

$$\begin{aligned}
U_t - U \simeq & U_C C\left(y_t + \frac{1-\sigma}{2}y_t^2\right) + U_L L\left(l_t + \frac{1+\varphi}{2}l_t^2\right) \\
U_t^* - U^* \simeq & U_C C^*\left(y_t^* + \frac{1-\sigma}{2}y_t^{*2}\right) + U_H L^*\left(l_t + \frac{1+\varphi}{2}l_t^{*2}\right)
\end{aligned}$$

where $\sigma \equiv -\frac{U_{CC}}{U_C}C$ and $\varphi \equiv \frac{U_{LL}}{U_L}L$.

The log-linearization of *Home* CPI around the same symmetric steady states

$$\begin{aligned}
\bar{P}_t &= \left[(1-\eta)P_t^{1-\mu} + \eta P_t^{*1-\mu}\right]^{\frac{1}{1-\mu}} \\
\bar{p}_t &= (1-\eta)p_t + \eta p_t^*.
\end{aligned}$$

For *Foreign* economy, the counterparts are

$$
\begin{array}{rcl}
 & \text{Initial Form} & \text{Log-linearized} \\
\bar{P}_t^* = \left[(1-\eta)P_t^{*1-\mu} + \eta P_t^{1-\mu}\right] & \Rightarrow & \bar{p}_t^* = (1-\eta)p_t^* + \eta p_t \\
C_{F,t}^* = (1-\eta)\left(\frac{P_{F,t}}{\bar{P}_t^*}\right)^{-\mu} C_t^* & \Rightarrow & c_{F,t}^* = -\mu\left(p_{F,t} - \bar{p}_t^*\right) + c_t^* \\
C_{H,t}^* = \eta\left(\frac{P_{H,t}}{\bar{P}_t^*}\right)^{-\mu} C_t^* & \Rightarrow & c_{H,t}^* = -\mu\left(p_{H,t} - \bar{p}_t^*\right) + c_t^*.
\end{array}
$$

To rewrite l_t in terms of output, we need to use the fact that

$$
\begin{aligned}
L_t &= \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di, \\
(1-\alpha)l_t &= y_t - a_t + d_t
\end{aligned}
$$

where $d_t \equiv (1-\alpha)\log\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di$. Lemma, presented below, shows that d_t is proportional to the cross-sectional variance of relative prices. The log-linearization of labor supply condition without labor mobility is

$$
\begin{aligned}
\frac{W_t}{\bar{P}_t} &= \chi C_t^{\sigma} L_t^{\varphi} \\
w_t - \bar{p}_t &= \sigma c_t + \varphi l_t.
\end{aligned}
$$

For *Foreign* households the peer condition is written as

$$
\begin{aligned}
\frac{W_t^*}{\bar{P}_t^*} &= \chi C_t^{*\sigma} L_t^{*\varphi} \\
w_t^* - \bar{p}_t^* &= \sigma c_t^* + \varphi l_t^*.
\end{aligned}
$$

With labor mobility, the labor supply conditions of two regions are

$$
\begin{array}{rcl}
\frac{W_t}{\bar{P}_t} = \chi C_t^{\sigma} L_t^{\varphi} \cdot (1-\eta_l)^{\frac{1}{\mu_l}} \left(\frac{L_{H,t}}{L_t}\right)^{-\frac{1}{\mu_l}} & \Rightarrow & w_t - \bar{p}_t = \sigma c_t + \varphi l_t - \frac{1}{\mu_l}\left(l_{H,t} - l_t\right) \\
\frac{W_t^*}{\bar{P}_t} = \chi C_t^{\sigma} L_t^{\varphi} \cdot \eta_l^{\frac{1}{\mu_l}} \left(\frac{L_{F,t}}{L_t}\right)^{-\frac{1}{\mu_l}} & \Rightarrow & w_t^* - \bar{p}_t = \sigma c_t + \varphi l_t - \frac{1}{\mu_l}\left(l_{F,t} - l_t\right) \\
\frac{W_t^*}{\bar{P}_t^*} = \chi C_t^{*\sigma} L_t^{*\varphi} \cdot (1-\eta_l)^{\frac{1}{\mu_l}} \left(\frac{L_{F,t}^*}{L_t^*}\right)^{-\frac{1}{\mu_l}} & \Rightarrow & w_t^* - \bar{p}_t^* = \sigma c_t^* + \varphi l_t^* - \frac{1}{\mu_l}\left(l_{F,t}^* - l_t^*\right) \\
\frac{W_t}{\bar{P}_t^*} = \chi C_t^{*\sigma} L_t^{*\varphi} \cdot \eta_l^{\frac{1}{\mu_l}} \left(\frac{L_{H,t}^*}{L_t^*}\right)^{-\frac{1}{\mu_l}} & \Rightarrow & w_t - \bar{p}_t^* = \sigma c_t^* + \varphi l_t^* - \frac{1}{\mu_l}\left(l_{H,t}^* - l_t^*\right)
\end{array}.
$$

The symmetric steady states imply that

$$W = W^* \Rightarrow \quad \frac{1-\eta_l}{L_H} = \frac{\eta_l}{L_F}.$$

Use the fact that $L_t = L_{H,t} + L_{F,t}$ and take log-linearization,

$$\begin{aligned} L\left(1+l_t\right) &= L_H\left(1+l_{H,t}\right) + L_F\left(1+l_F\right) \\ l_t &= \frac{L_H}{L} l_{H,t} + L_F l_{F,t} \\ l_t &= \left(1-\eta_l\right) l_{H,t} + \eta_l l_{F,t}. \end{aligned}$$

The analogous form for *Foreign* region is

$$l_t^* = \left(1-\eta_l\right) l_{F,t}^* + \eta_l l_{H,t}^*.$$

The national-wide market clearing conditions require

$$\begin{aligned} Y_t &= \left(1-\eta\right)\left(\frac{P_{H,t}}{\bar{P}_t}\right)^{-\mu} C_t + \eta\left(\frac{P_{H,t}}{\bar{P}_t^*}\right)^{-\mu} C_t^* \\ Y\left(1+y_t\right) =& \left(1-\eta\right)\left(\frac{P_H}{\bar{P}}\right)^{-\mu} \exp\left[-\mu\log\left(\frac{P_{H,t}}{\bar{P}_t}\right) + \mu\log\left(\frac{P_H}{\bar{P}}\right)\right] C\left(1+c_t\right) \\ &+ \eta\left(\frac{P_H}{\bar{P}^*}\right)^{-\mu} \exp\left[-\mu\log\left(\frac{P_{H,t}}{\bar{P}_t^*}\right) + \mu\log\left(\frac{P_H}{\bar{P}^*}\right)\right] C^*\left(1+c_t^*\right) \\ =& \left(1-\eta\right)\left(\frac{P_H}{\bar{P}}\right)^{-\mu}\left[1-\mu\log\left(\frac{P_{H,t}}{P_H}\right) + \mu\log\left(\frac{\bar{P}_t}{\bar{P}}\right)\right] C\left(1+c_t\right) \\ &+ \eta\left(\frac{P_H}{\bar{P}^*}\right)^{-\mu}\left[1-\mu\log\left(\frac{P_{H,t}}{P_H}\right) + \mu\log\left(\frac{\bar{P}_t^*}{\bar{P}^*}\right)\right] C^*\left(1+c_t^*\right) \\ =& \left(1-\eta\right)\left(\frac{P_H}{\bar{P}}\right)^{-\mu}\left[1-\mu p_{H,t} + \mu\bar{p}_t\right] C\cdot c_t + \eta\left(\frac{P_H}{\bar{P}^*}\right)^{-\mu}\left[1-\mu p_{H,t} + \mu\bar{p}_t^*\right] C^*\cdot c_t^* \\ Y\cdot y_t =& \left(1-\eta\right)\left(\frac{P_H}{\bar{P}}\right)^{-\mu}\left(-\mu p_{H,t} + \mu\bar{p}_t\right) C\cdot c_t + \eta\left(\frac{P_H}{\bar{P}^*}\right)^{-\mu}\left(-\mu p_{H,t} + \mu\bar{p}_t^*\right) C^*\cdot c_t^*. \end{aligned}$$

With the assumption that the steady state is symmetric, the log-linearization of aggregate demand for *Home* products is rewritten as

$$y_t = \left(1-\eta\right)\left(-\mu p_{H,t} + \mu\bar{p}_t\right) c_t + \eta\left(-\mu p_{H,t} + \mu\bar{p}_t^*\right)\cdot c_t^*.$$

The analogous forms for *Foreign* region are

$$\begin{aligned} Y_t^* &= \left(1-\eta\right)\left(\frac{P_{F,t}}{\bar{P}_t^*}\right)^{-\mu} C_t^* + \eta\left(\frac{P_{F,t}}{\bar{P}_t}\right)^{-\mu} C_t \\ y_t^* &= \left(1-\eta\right)\left(-\mu p_{F,t} + \mu\bar{p}_t^*\right) c_t^* + \eta\left(-\mu p_{F,t} + \mu\bar{p}_t\right) c_t. \end{aligned}$$

The economy-wide utility of households rewritten as

$$
\begin{aligned}
&\lambda\left(U_t-U\right)+\lambda^*\left(U_t^*-U^*\right)\\
=&\lambda\left[U_C C\left(y_t+\frac{1-\sigma}{2}\hat{y}_t^2\right)+U_L L\left(l_t+\frac{1+\varphi}{2}l_t^2\right)\right]\\
&\lambda^*\left[U_C C^*\left(y_t^*+\frac{1-\sigma}{2}y_t^{*2}\right)+U_L L^*\left(l_t+\frac{1+\varphi}{2}l_t^{*2}\right)\right]\\
=&\lambda U_C C\left[y_t+\frac{1-\sigma}{2}y_t^2+\frac{U_L L}{U_C C}\left(l_t+\frac{1+\varphi}{2}l_t^2\right)\right]\\
&\lambda^* U_{C^*}C^*\left[\hat{y}_t^*+\frac{1-\sigma}{2}y_t^{*2}+\frac{U_L L^*}{U_C C^*}\left(l_t^*+\frac{1+\varphi}{2}l_t^{*2}\right)\right]\\
=&\lambda U_C C\left[y_t+\frac{1-\sigma}{2}y_t^2+\frac{U_L L}{U_C C}\left(\left[\left(1-\eta_l\right)l_{H,t}+\eta_l l_{F,t}\right]+\frac{1+\varphi}{2}\left[\left(1-\eta_l\right)l_{H,t}+\eta_l l_{F,t}\right]^2\right)\right]\\
&\lambda^* U_{C^*}C^*\left[y_t^*+\frac{1-\sigma}{2}y_t^{*2}+\frac{U_L L^*}{U_{C^*}C^*}\left(\left[\left(1-\eta_l\right)l_{F,t}^*+\eta_l l_{H,t}^*\right]+\frac{1+\varphi}{2}\left[\left(1-\eta_l\right)l_{F,t}^*+\eta_l l_{H,t}^*\right]^2\right)\right]
\end{aligned}
$$

To proceed and without loss of generality, I impose the symmetric steady states, i.e., $\lambda=\lambda^*$, $U_C C=U_C C^*$, and $U_L L=U_{L^*}L^*$. Hence, the economy-wide social welfare function can be rewritten

$$
\begin{aligned}
&\frac{U_t - U}{U_C C} + \frac{U_t^* - U^*}{U_{C^*} C^*} \\
=& y_t + y_t^* + \frac{U_L L}{U_C C}\left[(1-\eta_l)\, l_{H,t} + \eta_l l_{F,t} + (1-\eta_l)\, l_{F,t}^* + \eta_l l_{H,t}^*\right] \\
& \frac{1-\sigma}{2} y_t^2 + \frac{1-\sigma}{2} y_t^{*2} + \frac{U_L L}{U_C C}\frac{1+\varphi}{2}\left(\left[(1-\eta_l)\, l_{H,t} + \eta_l l_{F,t}\right]^2 + \left[(1-\eta_l)\, l_{F,t}^* + \eta_l l_{H,t}^*\right]^2\right) \\
=& -\left[d_t + d_t^*\right] + \frac{1-\sigma}{2}\left(y_t^2 + y_t^{*2}\right) \\
& - \frac{(1-\alpha)(1+\varphi)}{2}\left(\left[(1-\eta_l)\, l_{H,t} + \eta_l l_{F,t}\right]^2 + \left[(1-\eta_l)\, l_{F,t}^* + \eta_l l_{H,t}^*\right]^2\right) + t.i.p. \\
=& -\left[d_t + d_t^*\right] + \frac{1-\sigma}{2}\left(y_t^2 + y_t^{*2}\right) \\
& - \frac{(1-\alpha)(1+\varphi)}{2}\left(\left[(1-\eta_l)\, l_{H,t} + \eta_l l_{H,t}^*\right]^2 + \left[(1-\eta_l)\, l_{F,t}^* + \eta_l l_{F,t}\right]^2\right) \\
& - \frac{(1-\alpha)(1+\varphi)}{2}\cdot 2(1-\eta_l)\eta_l\left[l_{H,t} l_{F,t} + l_{F,t}^* l_{H,t}^* - l_{H,t} l_{H,t}^* - l_{F,t}^* l_{F,t}\right] \\
=& -\left[d_t + d_t^*\right] + \frac{1-\sigma}{2}\left(y_t^2 + y_t^{*2}\right) - \frac{(1-\alpha)(1+\varphi)}{2}\left(\bar{l}_{H,t}^2 + \bar{l}_{H,t}^{*2}\right) \\
& - \frac{(1-\alpha)(1+\varphi)}{2}\cdot 2(1-\eta_l)\eta_l\left[l_{H,t} l_{F,t} + l_{F,t}^* l_{H,t}^* - l_{H,t} l_{H,t}^* - l_{F,t}^* l_{F,t}\right] + t.i.p. \\
=& -\frac{\epsilon}{2\Theta}\left[var_i\left\{p_t(i)\right\} + var_i\left\{p_t^*(i)\right\}\right] \\
& - \frac{1}{2}\left[-(1-\sigma) y_t + \frac{1+\varphi}{1-\alpha}(y_t - a_t)^2 - (1-\sigma) y_t^* + \frac{1+\varphi}{1-\alpha}(y_t^* - a_t^*)^2\right] \\
& - \frac{(1-\alpha)(1+\varphi)}{2}\cdot 2(1-\eta_l)\eta_l\left(l_{H,t} - l_{F,t}^*\right)\left(l_{F,t} - l_{H,t}^*\right) + t.i.p. \\
=& -\frac{\epsilon}{2\Theta}\left[var_i\left\{p_t(i)\right\} + var_i\left\{p_t^*(i)\right\}\right] + \frac{1}{2}\left[\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)(y_t + y_t^*) - 2\left(\frac{1+\varphi}{1-\alpha}\right)(y_t a_t + y_t^* a_t^*)\right] \\
& - \frac{(1-\alpha)(1+\varphi)}{2}\cdot 2(1-\eta_l)\eta_l\left(l_{H,t} - l_{F,t}^*\right)\left(l_{F,t} - l_{H,t}^*\right) + t.i.p \\
=& -\frac{1}{2}\left[\frac{\varepsilon}{\Theta}\left(var_i\left\{p_t(i)\right\} + var_{i^*}\left\{p_t^*(i^*)\right\}\right) + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\left(y_t - 2 y_t y_t^n + y_t^* - 2 y_t^* y_t^{n*}\right)\right] \\
& - \frac{(1-\alpha)(1+\varphi)}{2}\cdot 2(1-\eta_l)\eta_l\left(l_{H,t} - l_{F,t}^*\right)\left(l_{F,t} - l_{H,t}^*\right) + t.i.p. \\
=& -\frac{1}{2}\left[\frac{\varepsilon}{\Theta}\left(var_i\left\{p_t(i)\right\} + var_{i^*}\left\{p_t^*(i^*)\right\}\right) + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\left(\tilde{y}_t + \tilde{y}_t^*\right)\right] \\
& - \frac{(1-\alpha)(1+\varphi)}{2}\cdot 2(1-\eta_l)\eta_l\left(l_{H,t} - l_{F,t}^*\right)\left(l_{F,t} - l_{H,t}^*\right) + t.i.p.
\end{aligned}
$$

where $y_t^n = \frac{1+\varphi}{\alpha(1-\alpha)+\varphi+\alpha} a_t$ and $\tilde{y}_t = y_t - y_t^n$ and the respective terms for *Foreign* economy are $y_t^{n*} = \frac{1+\varphi}{\alpha(1-\alpha)+\varphi+\alpha} a_t^*$ and $\tilde{y}_t^* = y_t^* - y_t^{n*}$.

Accordingly, a second-order approximation to the economy-wide welfare losses can be written and expressed as a fraction of steady state consumption as

$$
\begin{aligned}
\mathbb{W} &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{U_t - U}{U_C C} + \frac{U_t^* - U^*}{U_{C*} C^*} \right) \\
&= -\frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \left\{ \begin{array}{r} \frac{\varepsilon}{\Theta} \left(var_i \left\{ p_t(i) \right\} + var_{i^*} \left\{ p_t^*(i^*) \right\} \right) + \left(\sigma + \frac{\varphi + \alpha}{1-\alpha} \right) \left(\tilde{y}_t + \tilde{y}_t^* \right) \\ (1-\alpha)(1+\varphi) \cdot 2(1-\eta_l)\eta_l \left(l_{H,t} - l_{F,t}^* \right) \left(l_{F,t} - l_{H,t}^* \right) \end{array} \right\} \\
&= -\frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \left\{ \begin{array}{c} \frac{\varepsilon}{\Theta} \left(\pi_t^2 + \pi_t^{*2} \right) + \left(\sigma + \frac{\varphi + \alpha}{1-\alpha} \right) \left(\tilde{y}_t^2 + \tilde{y}_t^{*2} \right) \\ (1-\alpha)(1+\varphi) \cdot 2(1-\eta_l)\eta_l \left(l_{H,t} - l_{F,t}^* \right) \left(l_{F,t} - l_{H,t}^* \right) \end{array} \right\}
\end{aligned}
$$

Two Lemma used from New-Keynesian literature:

- Lemma 1: In a neighborhood of a symmetric steady state, and up to a second-order approximation, $d_t = \frac{\epsilon}{2\Theta} var_i \left\{ p_t(i) \right\}, \Theta \equiv \frac{1-\alpha}{1-\alpha\epsilon}$. For proof, please refer to Gali (2008), Chapter 4.

- Lemma2: $\sum_{t=0}^{\infty} \beta^t var_i \left\{ p_t(i) \right\} = \frac{\theta}{(1-\beta\theta)(1-\theta)} \sum_{t=0}^{\infty} \beta^t \pi_t^2$. For proof, please refer to Woodford (2003), Chapter 6.

D. The Medium Scale Model

D1. The First Order Condition

In the medium scale model, I assume capital is perfectly mobile and labor is perfectly mobile within a region. So nominal rents equal between region and wage equal between sector. In this neoclassical model, rents and wages are determined by their marginal production. These conditions would decide the optimal allocation of factor among regions and sectors.

Capital rental price satisfies the condition

$$
\begin{aligned}
R_t &= \alpha A_{N,t} K_{N,t}^{\alpha-1} L_{N,t}^{1-\alpha} = \alpha A_{T,t} K_{T,t}^{\alpha-1} L_{T,t}^{1-\alpha} \\
&= \alpha A_{N,t}^* K_{N,t}^{*\alpha-1} L_{N,t}^{*1-\alpha} = \alpha A_{T,t}^* K_{T,t}^{*\alpha-1} L_{T,t}^{*1-\alpha}.
\end{aligned}
$$

Wage satisfies the condition

$$
\begin{aligned}
W_t^* &= (1-\alpha) A_{N,t}^* K_{N,t}^{*\alpha} L_{N,t}^{*-\alpha} = (1-\alpha) A_{T,t}^* K_{T,t}^{*\alpha} L_{T,t}^{*-\alpha}, \\
W_t &= (1-\alpha) A_{N,t} K_{N,t}^{\alpha} L_{N,t}^{-\alpha} = (1-\alpha) A_{T,t} K_{T,t}^{\alpha} L_{T,t}^{-\alpha}.
\end{aligned}
$$

Wage and labor supply satisfies the condition

$$
\begin{aligned}
\frac{W_t}{\bar{P}_t} &= -\frac{U_L}{U_C} \\
\frac{W_t^*}{\bar{P}_t^*} &= -\frac{U_{L^*}}{U_{C^*}}
\end{aligned}
$$

with $\bar{P}_t = P_{T,t}^{1-\gamma} P_{N,t}^{\gamma}$ and $\bar{P}_t^* = P_{T,t}^{*1-\gamma} P_{N,t}^{*\gamma}$.

The cost minimization problem of firms for each region and each each sector is

$$\min WL + RK$$

subject to

$$AK^{\alpha} L^{1-\alpha} = Y.$$

Lagrangian problem is written

$$\mathcal{L} = WL + RK + \lambda \left(Y - AK^{\alpha} L^{1-\alpha} \right)$$

with the first order condition

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial L} =& W - \lambda(1-\alpha) AK^{\alpha} L^{-\alpha} = 0 \\
\frac{\partial \mathcal{L}}{\partial K} =& R - \lambda \alpha AK^{\alpha-1} L^{1-\alpha} = 0 \\
\frac{\partial \mathcal{L}}{\partial \lambda} =& Y - AK^{\alpha} L^{1-\alpha} = 0
\end{aligned}$$

which imply

$$\frac{W}{R} = \frac{1-\alpha}{\alpha} \frac{K}{L}.$$

Substitute back into the first conditions

$$\begin{aligned}
Y = AK^{\alpha} L^{1-\alpha} = AK^{\alpha} \left(\frac{1-\alpha}{\alpha} \frac{R}{W} K \right)^{1-\alpha} \Rightarrow \qquad & K = \frac{Y}{A} \left(\frac{\alpha}{1-\alpha} \frac{W}{R} \right)^{1-\alpha} \\
Y = AK^{\alpha} L^{1-\alpha} = A \left(\frac{\alpha}{1-\alpha} \frac{W}{R} L \right)^{\alpha} L^{1-\alpha} \Rightarrow \qquad & H = \frac{Y}{A} \left(\frac{1-\alpha}{\alpha} \frac{R}{W} \right)^{\alpha}.
\end{aligned}$$

The cost function becomes

$$\begin{aligned}
W \cdot \frac{Y}{A} \left(\frac{1-\alpha}{\alpha} \frac{R}{W} \right)^{\alpha} + R \cdot \frac{Y}{A} \left(\frac{\alpha}{1-\alpha} \frac{W}{R} \right)^{1-\alpha} &= \frac{Y}{A} \left[W \left(\frac{1-\alpha}{\alpha} \frac{R}{W} \right)^{\alpha} + R \left(\frac{\alpha}{1-\alpha} \frac{W}{R} \right)^{1-\alpha} \right] \\
&= \frac{Y}{A} \left[\left(\frac{1-\alpha}{\alpha} \right)^{\alpha} W^{1-\alpha} R^{\alpha} + \left(\frac{\alpha}{1-\alpha} \right)^{1-\alpha} W^{1-\alpha} R^{\alpha} \right] \\
&= \frac{Y}{A} W^{1-\alpha} R^{\alpha} \left[\frac{(1-\alpha)^{\alpha} (1-\alpha)^{1-\alpha}}{\alpha^{\alpha} (1-\alpha)^{1-\alpha}} + \frac{\alpha^{1-\alpha} \alpha^{\alpha}}{(1-\alpha)^{1-\alpha} \alpha^{\alpha}} \right] = \frac{Y}{A} \frac{W^{1-\alpha} R^{\alpha}}{(1-\alpha)^{1-\alpha} \alpha^{\alpha}}.
\end{aligned}$$

The nominal marginal cost of each firm equal $\frac{W^{1-\alpha} R^{\alpha}}{(1-\alpha)^{1-\alpha} \alpha^{\alpha} A}$. Under neoclassical framework, nominal price equal nominal cost

$$P_{s,t} = \frac{W_t^{1-\alpha} R_t^{\alpha}}{(1-\alpha)^{1-\alpha} \alpha^{\alpha} A_{s,t}}, s \in \{N, T\}.$$

The market of traded goods is perfect competitive. Impose the condition that $P^*_{T,t} = P_{T,t}$, we have

$$\frac{(W^*_t)^{1-\alpha} R^\alpha_t}{(1-\alpha)^{1-\alpha}\alpha^\alpha A^*_{T,t}} = \frac{W^{1-\alpha}_t R^\alpha_t}{(1-\alpha)^{1-\alpha}\alpha^\alpha A_{T,t}},$$

which implies

$$\left(\frac{W^*_t}{W_t}\right)^{1-\alpha} = \frac{A^*_{T,t}}{A_{T,t}}.$$

D2. Proof of Proposition 2

For households in i region (with $-i$ representing another region), the first order conditions used in the derivation above:

$$\frac{(1-\tau^w_t)(1-\alpha)}{\bar{P}^i_t}\left(\frac{K^i_t + m_K K^{-i}_t}{L^i_t + m_L L^{-i}_t}\right)^\alpha = -\frac{U_L\left(C^i_t, L^i_t\right)}{U_C\left(C^i_t, L^i_t\right)}$$

$$\mathbb{E}_t\beta\left\{\frac{U_C\left(C^i_{t+1}\right)}{\left(1+\tau^c_{t+1}\right)\bar{P}^i_{t+1}}\left[\left(1-\tau^k_{t+1}\right)R^i_{t+1} + 1 - \delta\right]\right\} = \frac{U_C\left(C^i_t\right)}{\left(1+\tau^c_t\right)\bar{P}^i_t}.$$

The factor price equal to marginal production leads to

$$W^i_t = (1-\alpha)\left(K^i_{s,t} + m_K K^{-i}_{s,t}\right)^\alpha \left(L^i_t + m_L L^{-i}_t\right)^{-\alpha} = (1-\alpha)\left(\frac{K^i_{s,t} + m_K K^{-i}_{l,t}}{L^i_t + m_L L^{-i}_t}\right)^\alpha$$

$$R^i_t = \alpha\left(K^i_{l,t} + m_K K^{-i}_t\right)^{\alpha-1}\left(L^i_t + m_L L^{-i}_t\right)^{1-\alpha} = \alpha\left(\frac{L^i_t + m_L L^{-i}_t}{K^i_{s,t} + m_K K^{-i}_t}\right)^{1-\alpha},$$

where m_K and m_L are binary variable controlling the mobility setup in capital and labor. Given factor stocks each period, market conditions, Blocked or Open, change the factor prices.

Given the capital stock of each region, $K^i_t, i \in \{H, F\}$, sum of two are greater with integrated factor market. See the example of capital

$$Y^i_t = \left(K^i_t\right)^\alpha \left(L^i_t + m_L L^{-i}_t\right)^{1-\alpha}$$

$$Y^{-i}_t = \left(K^{-i}_t\right)^\alpha \left(L^i_t + m_L L^{-i}_t\right)^{1-\alpha}$$

$$Y^i_t + Y^{-i}_t = \left(K^{-i}_t + m_K K^i_t\right)^\alpha \left(L^i_t + m_L L^{-i}_t\right)^{1-\alpha}.$$

When integrated production,

$$\left(K^{-i}_t + m_K K^i_t\right)^\alpha \left(L^i_t + m_L L^{-i}_t\right)^{1-\alpha} > \left[\left(K^i_t\right)^\alpha + \left(K^{-i}_t\right)^\alpha\right]\left(L^i_t + m_L L^{-i}_t\right)^{1-\alpha}$$

, because production function is concave with capital. The marginal outputs with separate and integrated capital market are written

$$\begin{aligned}\frac{\partial Y_t^i}{\partial K_t^i} =&\alpha\left(K_t^i\right)^{\alpha-1}\left(L_t^i+m_L L_t^{-i}\right)^{1-\alpha}\\ \frac{\partial Y_t^i}{\partial K_t^i} =&\alpha\left(K_t^{-i}\right)^{\alpha-1}\left(L_t^i+m_L L_t^{-i}\right)^{1-\alpha}\\ \frac{\partial\left[Y_t^i+Y_t^{-i}\right]}{\partial K_t^i} =&\alpha\left(K_t^i+K_t^{-i}\right)^{\alpha-1}\left(L_t^i+m_L L_t^{-i}\right)^{1-\alpha}\end{aligned}.$$

When capital market integrated, the marginal productivity is lower than the sum of separate market.

When one factor market is across regions, factor price decrease and output increase.

	Capital Mobile	Capital Immobile
Labor Mobile	I: One Closed Economy $\left(\frac{W_I^i}{W_I^{-i}}=\frac{R_I^i}{R_I^{-i}}=1\right)$	II: Labor Open $\left(\frac{W_{II}^i}{R_{II}^i}=\frac{W_{II}^{-i}}{R_{II}^{-i}}<\frac{W_I^i}{R_I^i}\right)$
Labor Immobile	III: Capital Open $\left(\frac{W_{III}^i}{R_{III}^i}=\frac{W_{III}^{-i}}{R_{III}^{-i}}>\frac{W_I^i}{R_I^i}\right)$	IV: Trade Only $\left(\frac{W_{IV}^i}{W_I^i}>1,\frac{R_{IV}^i}{R_I^i}>1\right)$

.

Without factor taxes or subsidies, the intertemporal welfare of the whole economy is written as

$$\begin{aligned}\mathbb{W}=\max\sum_{t=0}^{\infty}\beta^t\sum_{i\in\{H,F\}}\lambda^i U\left(C_t^i,L_t^i\right) &=\max\sum_{t=0}^{\infty}\beta^t\sum_{i\in\{H,F\}}\lambda^i\left[\frac{U_C C_t^i}{\bar{P}_t^i}+L_t^i U_L\right]\\ &=\max\sum_{t=0}^{\infty}\beta^t\sum_{i\in\{H,F\}}\lambda^i U\left(C_t^i\right)\left[\frac{C_t^i}{\bar{P}_t^i}+L_t^i\frac{U_L}{U_C}\right]\\ &=\max\sum_{t=0}^{\infty}\beta^t\sum_{i\in\{H,F\}}\lambda^i U\left(C_t^i\right)\left[\frac{Y_t^i-I_t^i}{\bar{P}_t^i}-L_t^i\frac{W_t^i}{\bar{P}_t^i}\right]\\ &=\max\sum_{t=0}^{\infty}\beta^t\sum_{i\in\{H,F\}}\frac{\lambda^i}{\bar{P}_t^i}U\left(C_t^i\right)\left[\alpha Y_t^i-K_{t+1}^i+(1-\delta)K_t^i\right]\end{aligned}$$

Take derivative of this welfare function with respect to K_{t+1}^i and K_{t+1}^{-i},

$$\begin{aligned}\frac{\partial\mathbb{W}}{\partial K_{t+1}^i}&=\beta^t\left[-\lambda^i\frac{U_C\left(C_t^i\right)}{\bar{P}_t^i}+\lambda^i\beta\frac{U_C\left(C_{t+1}^i\right)}{\bar{P}_{t+1}^i}\left(\alpha\frac{\partial Y_{t+1}^i}{\partial K_{t+1}^i}+(1-\delta)\right)+\lambda^{-i}\beta\frac{U_C(C_{t+1}^{-i})}{\bar{P}_{t+1}^{-i}}\alpha\frac{\partial Y_{t+1}^{-i}}{\partial K_{t+1}^i}\right]\\ \frac{\partial\mathbb{W}}{\partial K_{t+1}^{-i}}&=\beta^t\left[-\lambda^{-i}\frac{U_C\left(C_t^{-i}\right)}{\bar{P}_t^{-i}}+\lambda^{-i}\beta\frac{U_C(C_{t+1}^{-i})}{\bar{P}_{t+1}^{-i}}\left(\alpha\frac{\partial Y_{t+1}^{-i}}{\partial K_{t+1}^{-i}}+(1-\delta)\right)+\lambda^i\beta\frac{U_C(C_{t+1}^i)}{\bar{P}_{t+1}^i}\alpha\frac{\partial Y_{t+1}^i}{\partial K_{t+1}^{-i}}\right].\end{aligned}$$

With capital mobility, the investment of one region's households have spillover effects to another region's production, as $\frac{\partial Y_{t+1}^{-i}}{\partial K_{t+1}^i}>0$; without capital mobility, the spillover effect is blocked, then $\frac{\partial Y_{t+1}^{-i}}{\partial K_{t+1}^{i}}=\frac{\partial Y_{t+1}^{i}}{\partial K_{t+1}^{-i}}=0$.

From the perspective of social planner, optimal investment satisfies

$$\begin{aligned}\frac{\partial \mathbb{W}}{\partial K_{t+1}^{i}}=0\Rightarrow \lambda^{i}\frac{U_C\left(C_t^i\right)}{\bar{P}_t^i}&=\lambda^{i}\beta\frac{U_C\left(C_{t+1}^i\right)}{\bar{P}_{t+1}^i}\left(\alpha\frac{\partial Y_{t+1}^{i}}{\partial K_{t+1}^{i}}+(1-\delta)\right)+\lambda^{-i}\beta\frac{U_C\left(C_{t+1}^{-i}\right)}{\bar{P}_{t+1}^{-i}}\alpha\frac{\partial Y_{t+1}^{-i}}{\partial K_{t+1}^{i}}\\ \frac{\partial \mathbb{W}}{\partial K_{t+1}^{-i}}=0\Rightarrow \lambda^{-i}\frac{U_C\left(C_t^{-i}\right)}{\bar{P}_t^{-i}}&=\lambda^{-i}\beta\frac{U_C\left(C_{t+1}^{-i}\right)}{\bar{P}_{t+1}^{-i}}\left(\alpha\frac{\partial Y_{t+1}^{-i}}{\partial K_{t+1}^{-i}}+(1-\delta)\right)+\lambda^{i}\beta\frac{U_C\left(C_{t+1}^i\right)}{\bar{P}_{t+1}^i}\alpha\frac{\partial Y_{t+1}^{i}}{\partial K_{t+1}^{-i}}\end{aligned}.$$

The individual residents of each region value capital following Euler equations

$$\begin{aligned}\lambda^{i}\frac{U_C\left(C_t^i\right)}{\bar{P}_t^i}&=\lambda^{i}\beta\frac{U_C\left(C_{t+1}^i\right)}{\bar{P}_{t+1}^i}\left(\alpha\frac{\partial Y_{t+1}^{i}}{\partial K_{t+1}^{i}}+(1-\delta)\right)\\ \lambda^{-i}\frac{U_C\left(C_t^{-i}\right)}{\bar{P}_t^{-i}}&=\lambda^{-i}\beta\frac{U_C\left(C_{t+1}^{-i}\right)}{\bar{P}_{t+1}^{-i}}\left(\alpha\frac{\partial Y_{t+1}^{-i}}{\partial K_{t+1}^{-i}}+(1-\delta)\right)\end{aligned}.$$

D.3 The Optimal regional taxes, subsidies, or investment

With labor, consumption, and capital taxes or subsidies, the optimal decisions of households are summarized as below,

$$\begin{aligned}\tau_{L,t}^{i}&=1+\frac{1}{\left(1+\tau_{g,t}^{i}\right)A_t^iF_L\left(K_t^i,L_t^i\right)}\frac{U_L\left(C_t^i,L_t^i\right)}{U_C\left(C_t^i,L_t^i\right)}\\ U_C\left(C_t^i,L_t^i\right)&=\mathbb{E}_t\left\{\beta U_C\left(C_{t+1}^i,L_{t+1}^i\right)\left[\left(1+\tau_{g,t}^{i}\right)A_{t+1}^iF_k\left(K_{t+1}^i,L_{t+1}^i\right)+(1-\delta)\left(1+\tau_{k,t}^{i}\right)\right]\right\}\end{aligned}$$

To facilitate exposition, I repeat here the second-order approximation (Appendix A2) of indirect utility based on wedges.

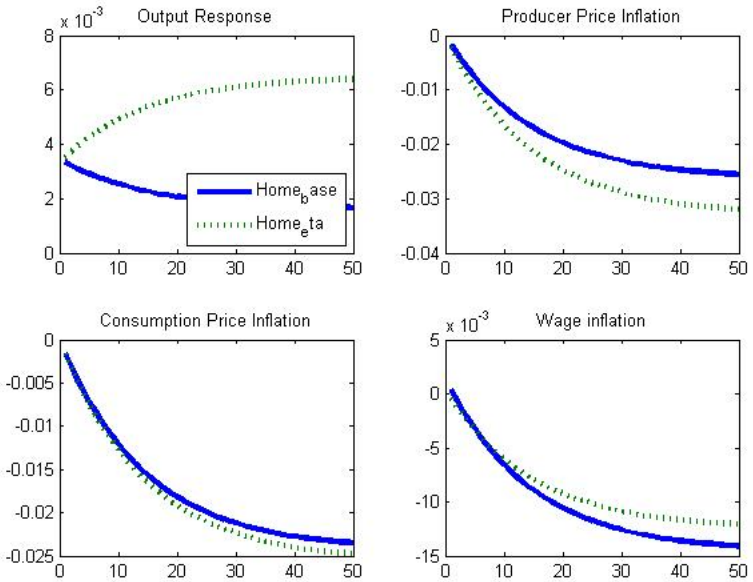
$$\begin{aligned}&V_t^i-U\left(\bar{C}^i,\bar{L}^i\right)\\ =&U_C\cdot\left(C_t^i-\bar{C}^i\right)\left\{1+\frac{U_L\cdot\left(L_t^i-\bar{L}^i\right)}{U_C\cdot\left(C_t^i-\bar{C}^i\right)}+\frac{1}{2}\frac{U_{CC}\cdot\left(C_t^i-\bar{C}^i\right)}{U_C}\left[1+\frac{U_{LL}\cdot\left(L_t^i-\bar{L}^i\right)^2}{U_{CC}\cdot\left(C_t^i-\bar{C}^i\right)^2}\right]\right\}\\ =&U_C\cdot\left(C_t^i-\bar{C}^i\right)\left\{1+\left[MP_{s,t}^i\left(\mathcal{W}_{s,t}^i-\frac{P_{s,t}^i}{P_t^i}\right)\right]\cdot\frac{P_t^i}{W_t^i}\right\}\\ &+\frac{1}{2}\frac{U_{CC}\cdot\left(C_t^i-\bar{C}^i\right)}{U_C}\left[1-\left[MP_{s,t}^i\left(\mathcal{W}_{s,t}^i-\frac{P_{s,t}^i}{P_t^i}\right)\right]^2\cdot\left(\frac{P_t^i}{W_t^i}\right)^2\right]\\ =&U_C\cdot\left(C_t^i-\bar{C}^i\right)\\ &\left\{1+\left[MP_{s,t}^i\left(\mathcal{W}_{s,t}^i-\frac{P_{s,t}^i}{P_t^i}\right)\cdot\frac{P_t^i}{W_t^i}\right]+\frac{1}{2}\frac{U_{CC}\cdot\left(C_t^i-\bar{C}^i\right)}{U_C}\left[1-\left[MP_{s,t}^i\left(\mathcal{W}_{s,t}^i-\frac{P_{s,t}^i}{P_t^i}\right)\cdot\frac{P_t^i}{W_t^i}\right]^2\right]\right\}\end{aligned}$$

$$\frac{V_t^i - U\left(\bar{C}^i, \bar{L}^i\right)}{U_C \cdot \left(C_t^i - \bar{C}^i\right)} = 1 + \left[MP_{s,t}^i \left(\mathcal{W}_{s,t}^i - \frac{P_{s,t}^i}{P_t^i}\right) \cdot \frac{P_t^i}{W_t^i}\right] - \frac{1}{2}\frac{U_{CC} \cdot \left(C_t^i - \bar{C}^i\right)}{U_C}\left[MP_{s,t}^i \left(\mathcal{W}_{s,t}^i - \frac{P_{s,t}^i}{P_t^i}\right) \cdot \frac{P_t^i}{W_t^i}\right]^2 + t.i.p.$$

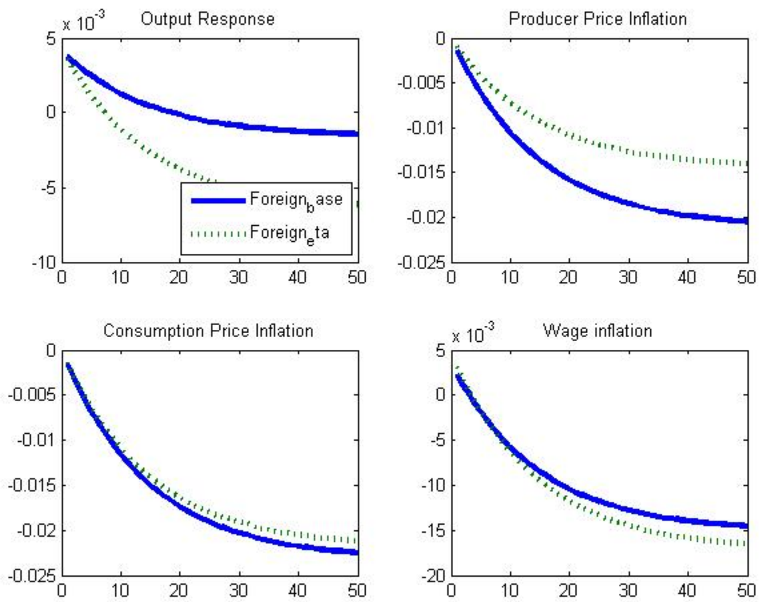
E. Figures

E1. Short Term Model

Figure 8.1: Sensitivity to Mobility Parameters (η_n : 0.107 or 0.005)



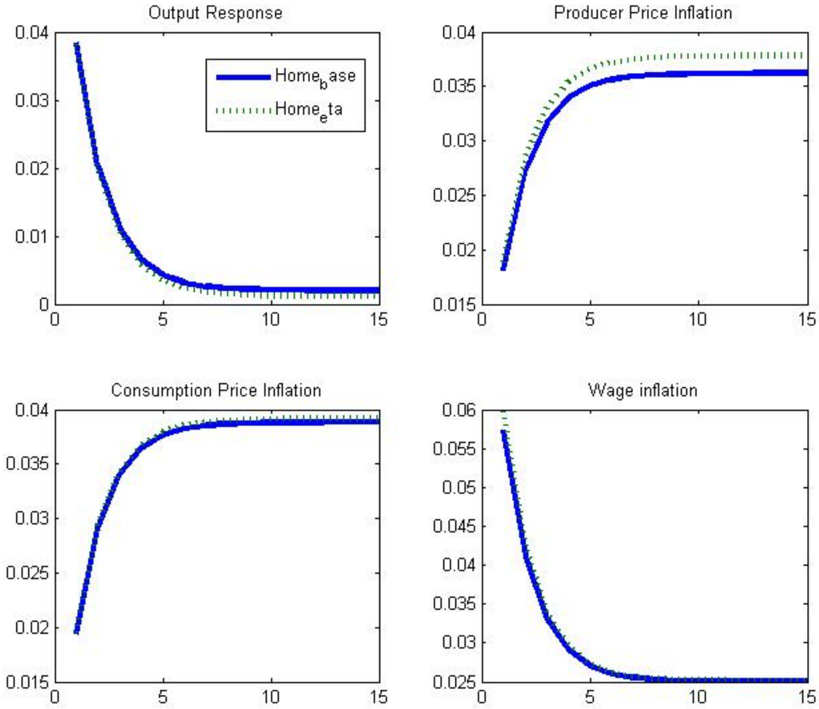
(a) *Home* Economy



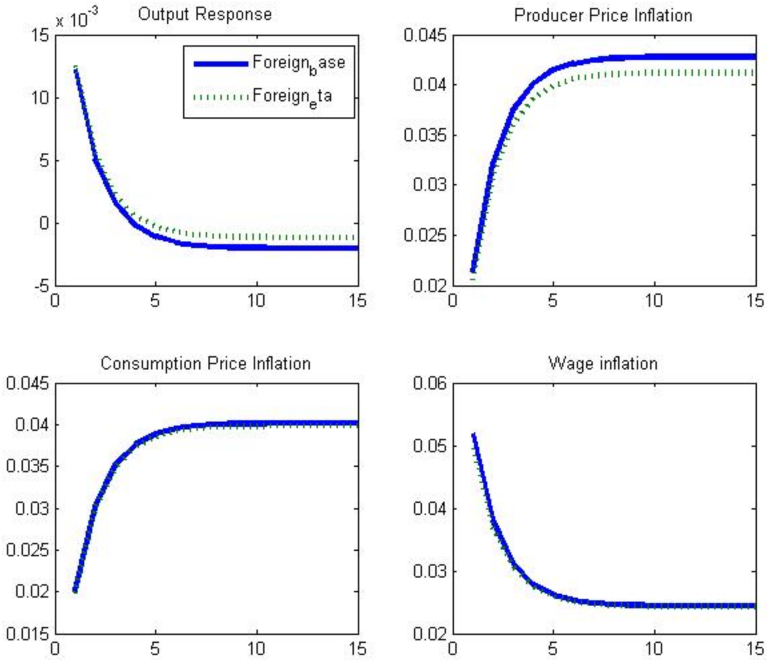
(b) *Foreign* Economy

Notes: The response to a technology shock with two parameter values: the baseline $\eta_n = 0.107$ and the new $\eta_n^{'} = 0.005$.

Figure 8.2: Sensitivity to Mobility Parameters



(a) *Home* Economy

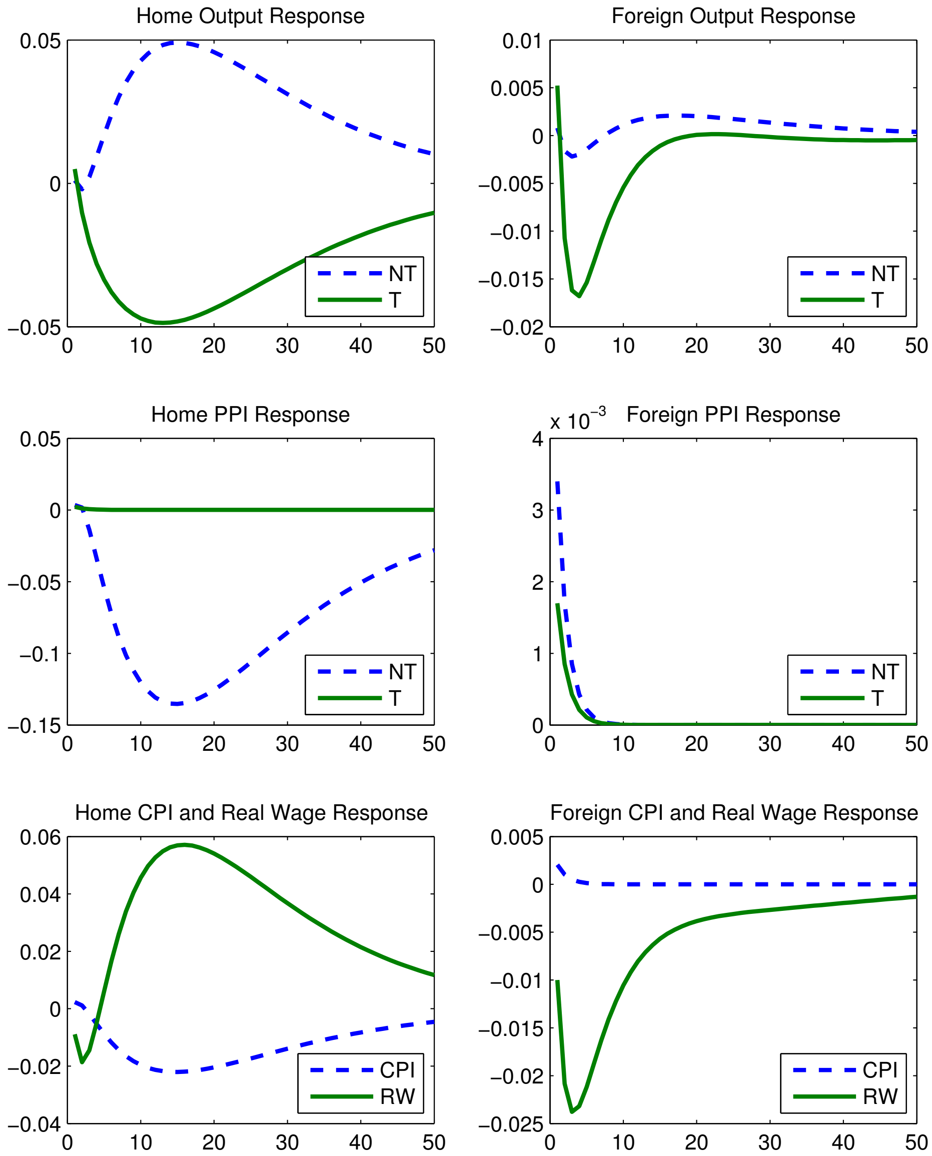


(b) *Foreign* Economy

Notes: The response to a government spending shock with two parameter values: the baseline $\eta_n = 0.107$ and the new $\eta_n^{'} = 0.005$.

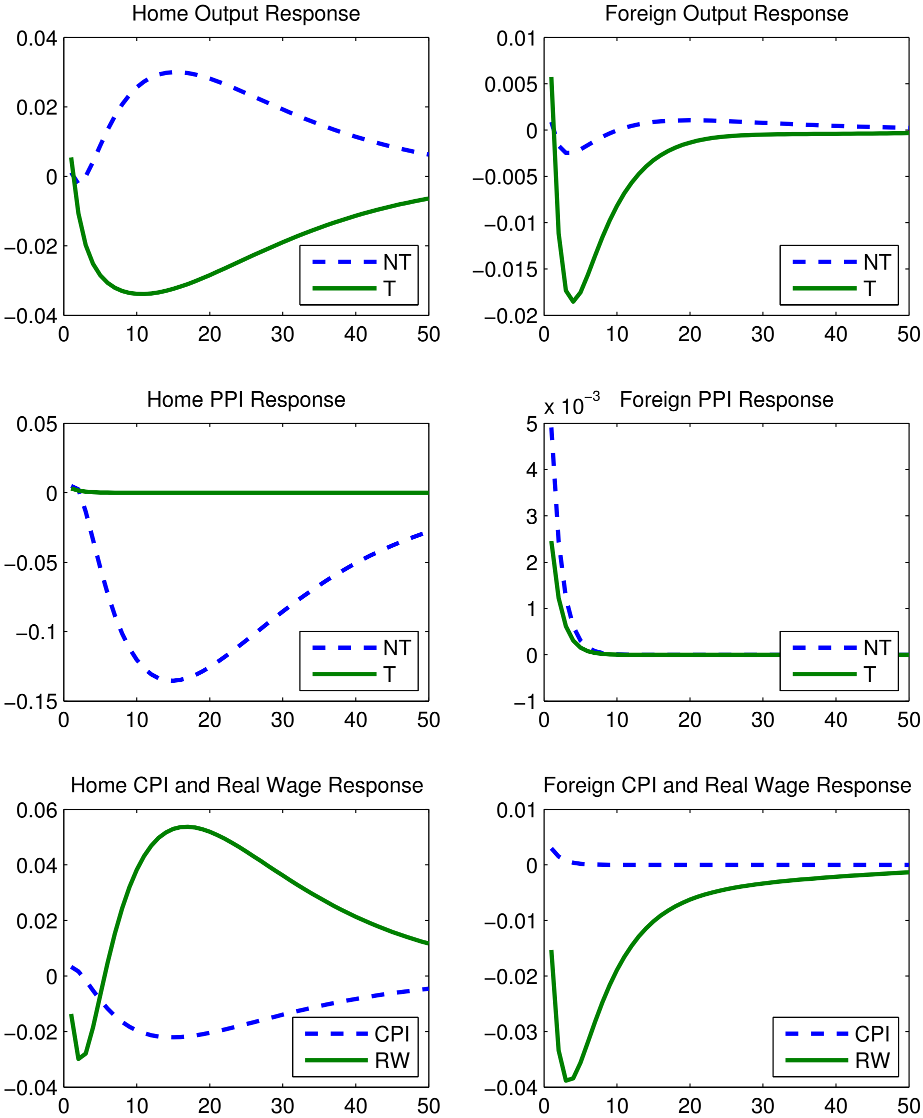
E2. Medium DSGE Model

Figure 8.3: Dynamic Responses of Two Regions with KL_Mobile



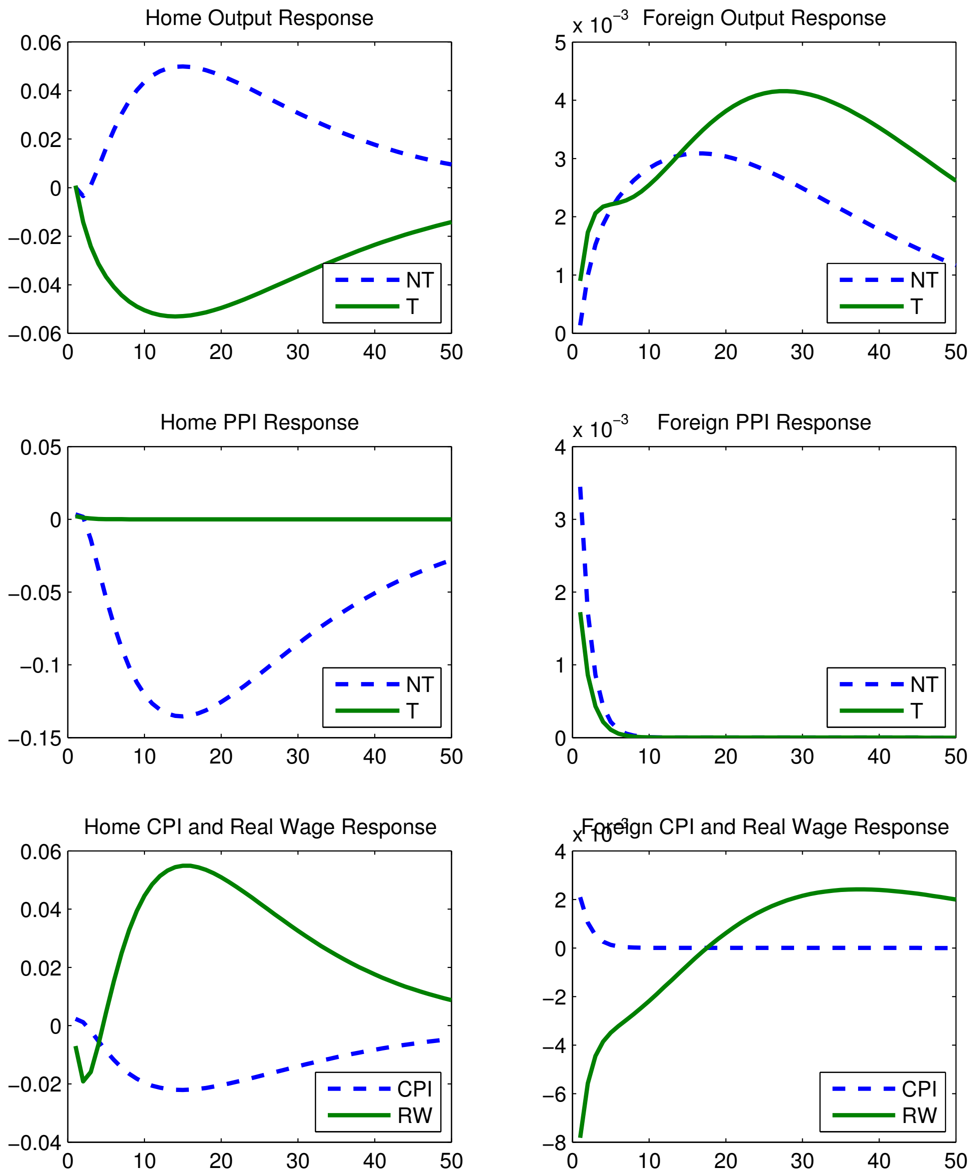
Note: KL_Mobile, both capital and labor are perfectly mobile across regions.

Figure 8.4: Dynamic Response of Two Regions with K_Mobile



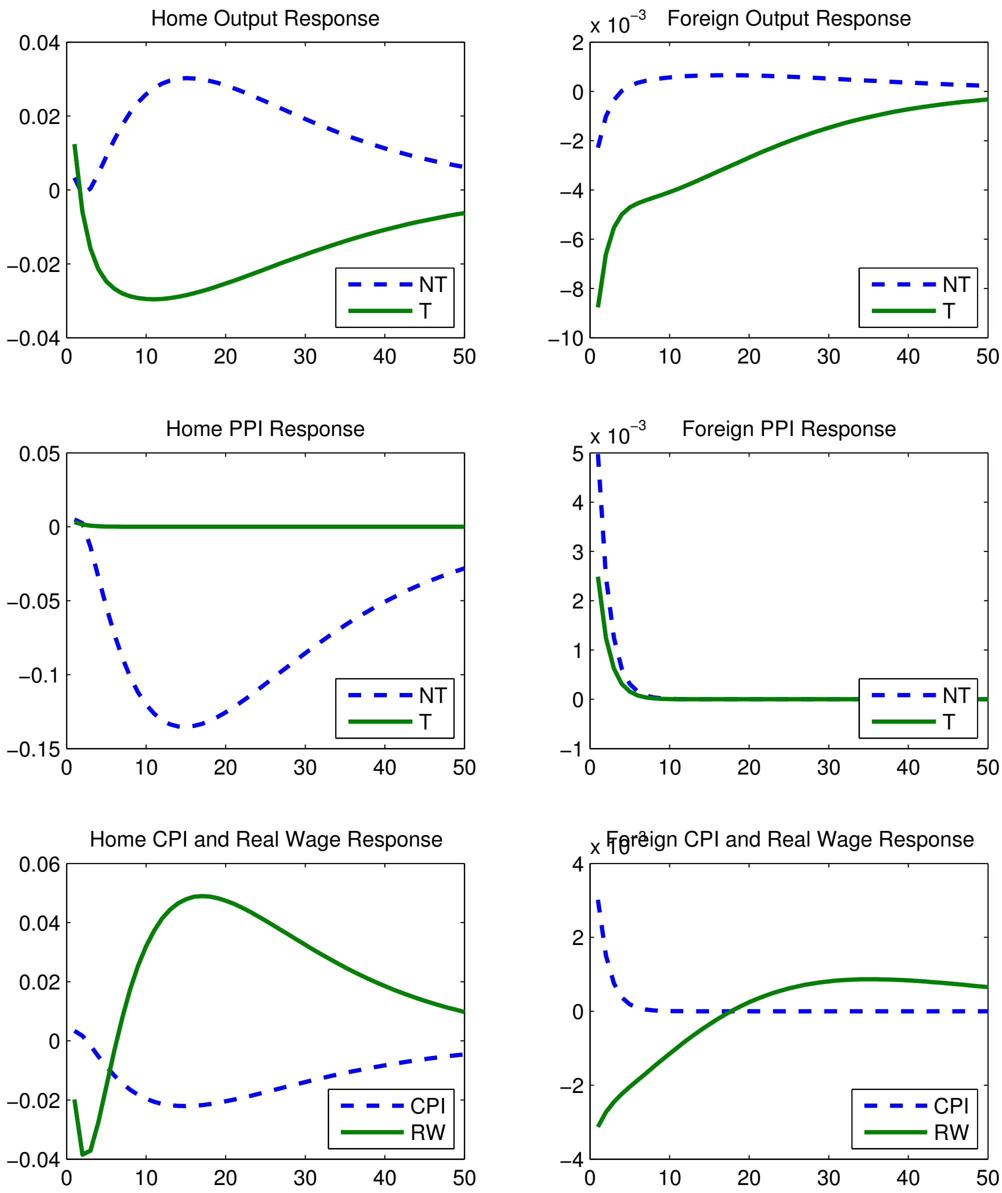
Note: K_Mobile, only capital is mobile across regions.

Figure 8.5: Dynamic Response of Two Regions with L_Mobile



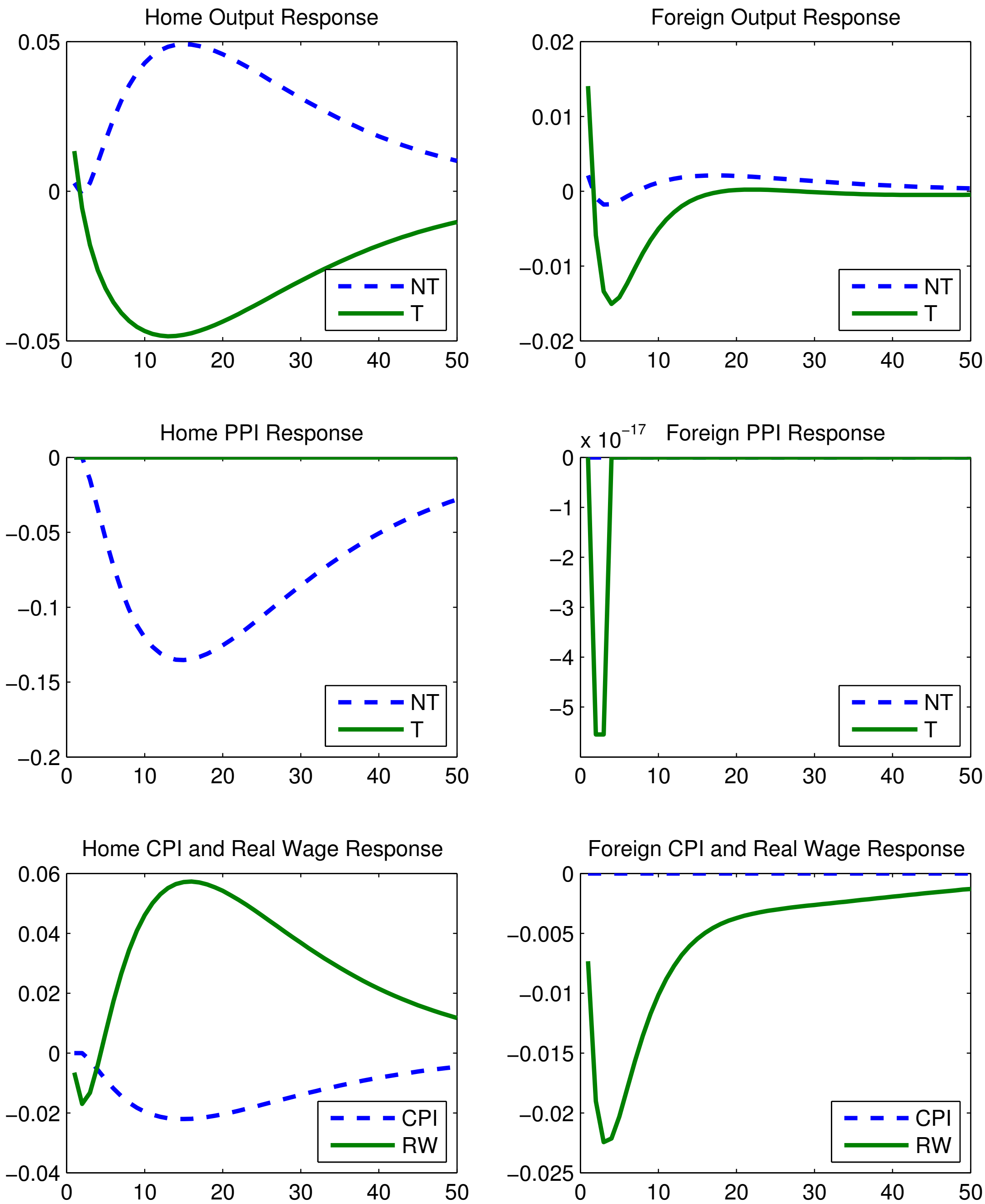
Note: L_Mobile, only labor is mobile across regions.

Figure 8.6: Dynamic Response of Two Regions with KL_Immobile



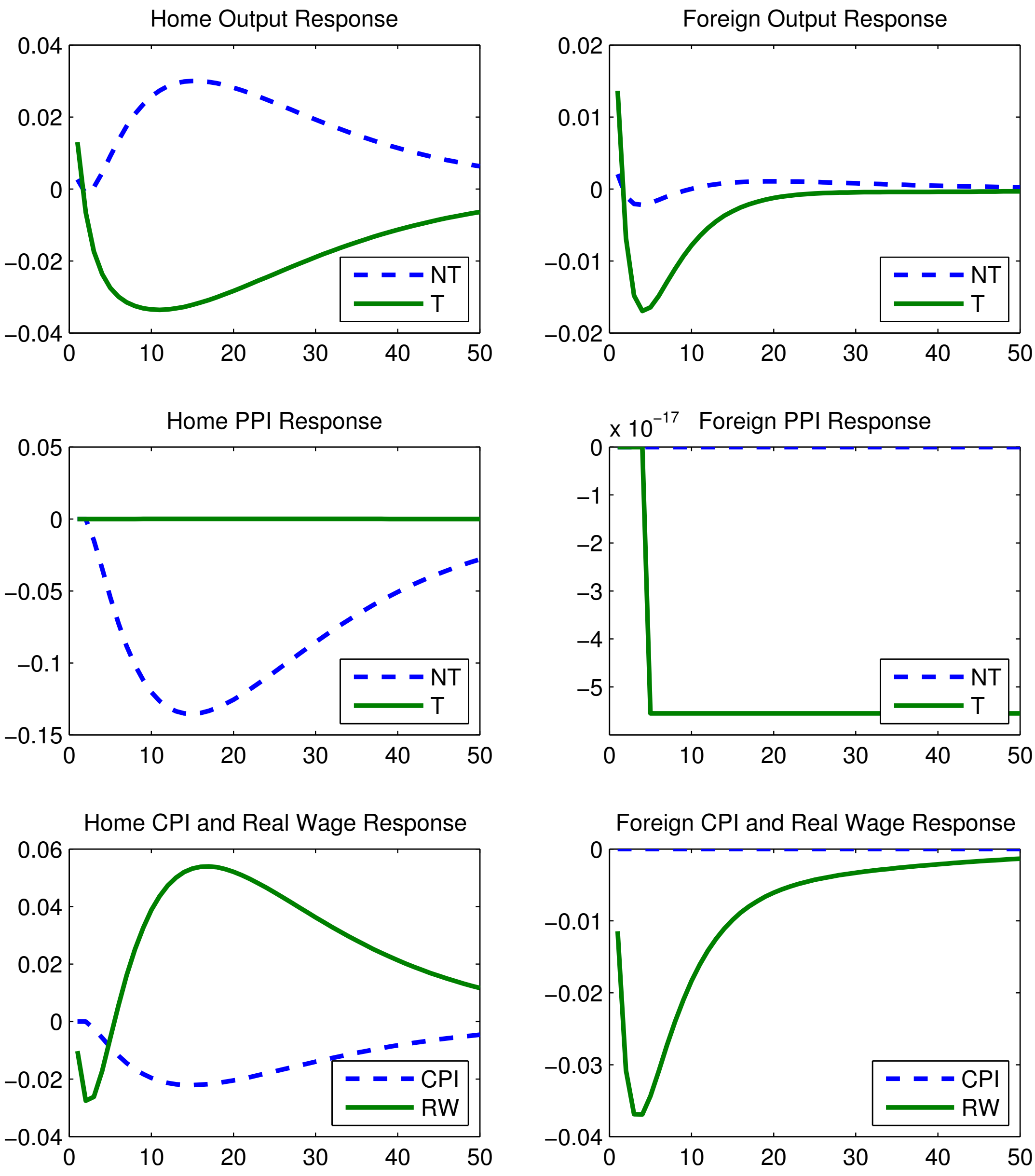
Note: KL_Immobile, neither capital or labor is mobile across regions.

Figure 8.7: Dynamic Response of Two Regions with KL_Mobile and Lump Sum Tax



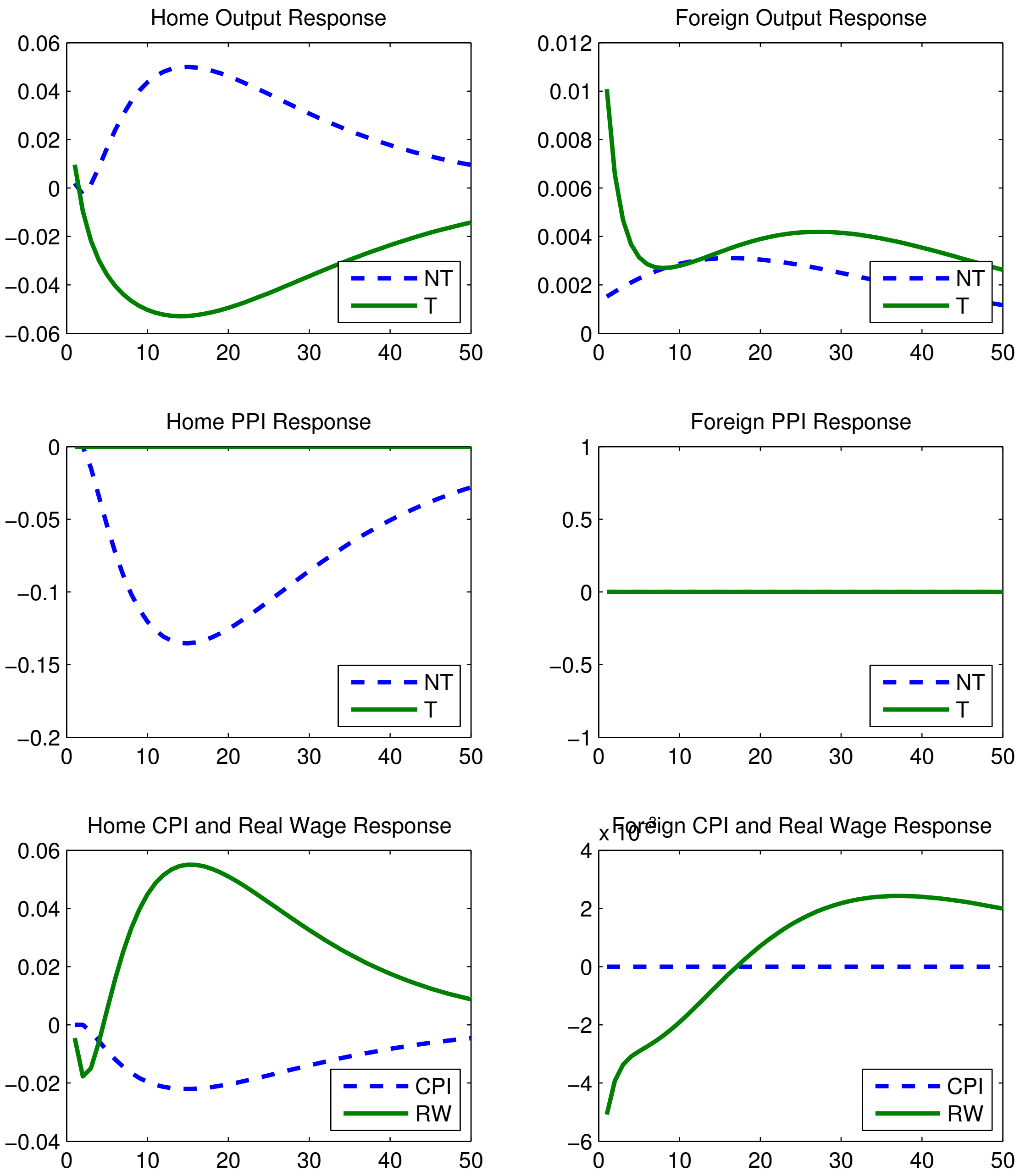
Note: KL_Mobile and Lump Sum Tax mean both capital and labor are mobile and the government spending is financed by lump-sum tax.

Figure 8.8: Dynamic Response of Two Regions with K_Mobile and Lump Sum Tax



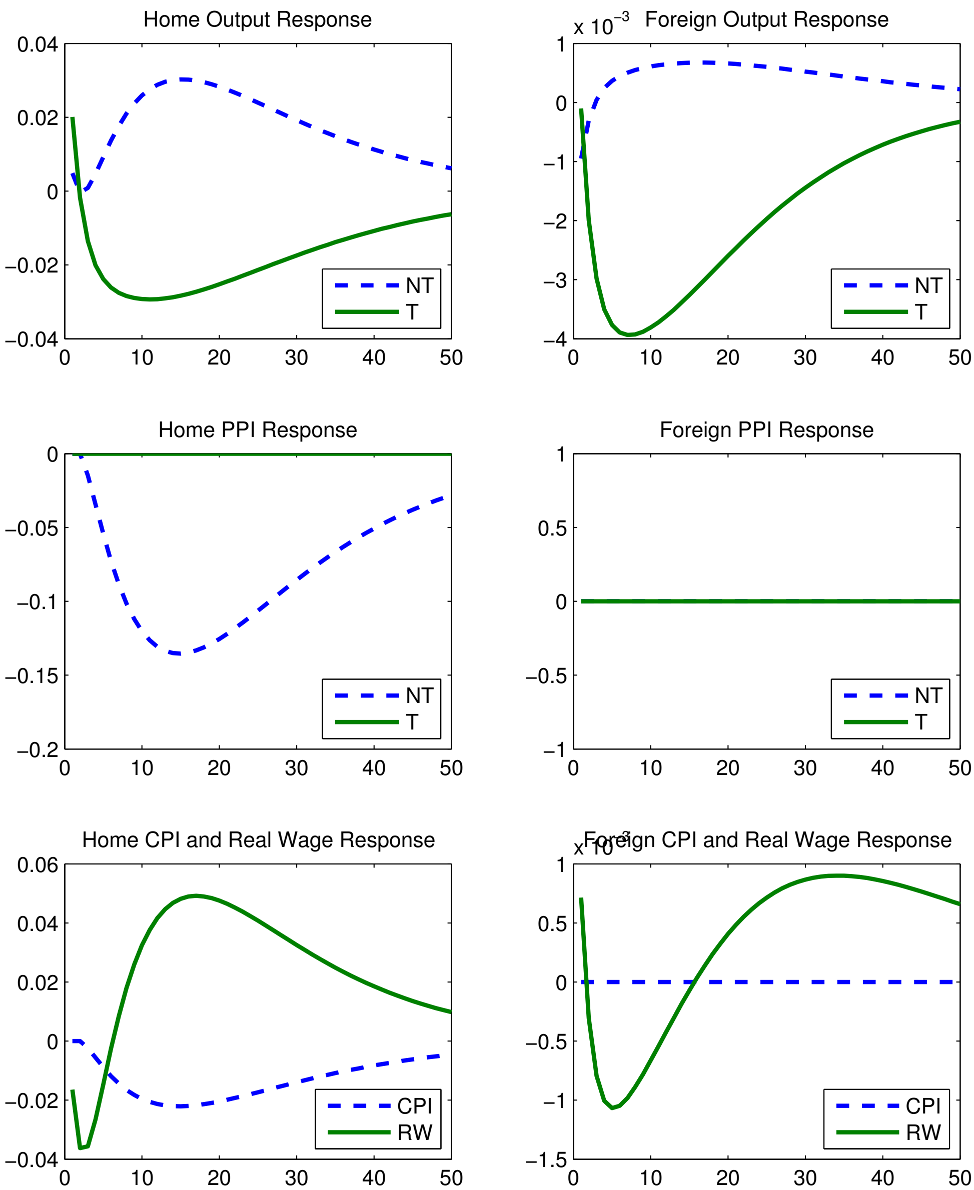
Note: K_Mobile and Lump Sum Tax means only capital is mobile across regions and the government spending is financed by Lump-sum tax.

Figure 8.9: Dynamic Response of Two Regions with L_Mobile and Lump Sum Tax



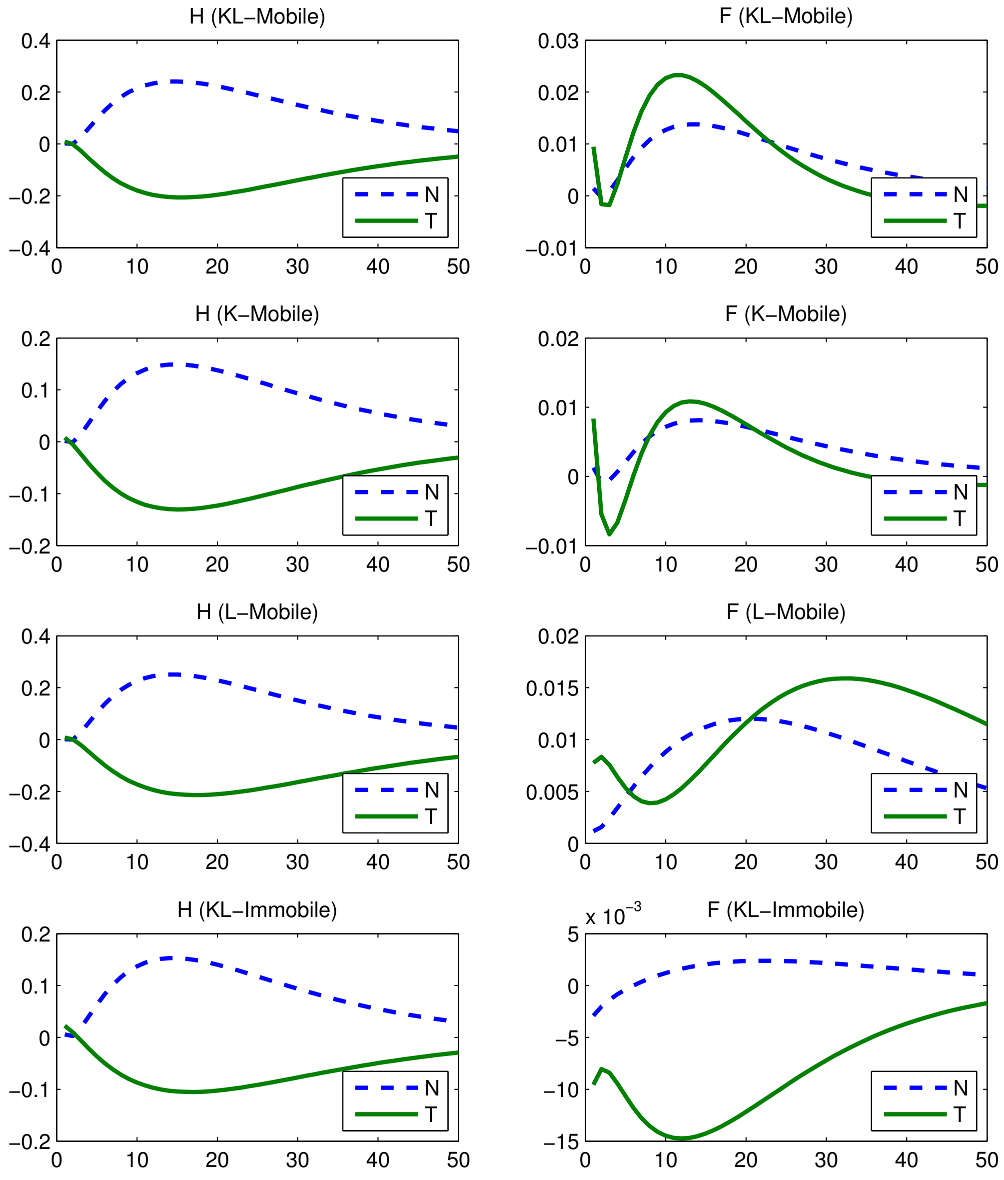
Note: Only labor is mobile across regions and the government spending is financed by lump-sump tax.

Figure 8.10: Dynamic Response of Two Regions with KL_Immobile and Lump Sum Tax



Note: Neither capital or labor is mobile and the government spending in one region is financed by lump-sum tax in the two region.

Figure 8.11: Output Responses with Factor Taxes and $\alpha_g = 0.24$



Note: The government investment is financed by factor taxes in two regions and the productivity parameter is $\alpha_g = 0.24$ (Aschauer (1989)).

Appendix F

Matlab and Dynare Codes are absent here for the length reason but available based on request.

Part II

Chapter 2: Endogenous Liquidity and Macroeconomic Implications

Abstract: One decade after the financial crisis of 2007-2008, the cause of this crisis is still in debate. This paper studies the endogenous liquidity of assets in a closed economy and characterizes a general, non-parametric mechanism of economic fluctuations, including severe crises. I endogenize liquidity in terms of following aspects: (i) a new construction of the liquidity property of assets; (ii) liquidity-augmented asset pricing; (iii) liquidity creation and evolution in the financial market. I derive asset pricing with consideration of liquidity and show that asset prices, augmented by liquidity service, inflate with liquidity premium and induce distorted investments in the real economy. Securities, which are widely used to facilitate transactions, induce new issuance and inevitably lower the pecuniary yields of the physical capital that backs them. The consequence is that asset prices and privately created liquidity become fragile, in the sense that small shocks can lead to large drops in asset prices and damage balance sheets of financial intermediaries. According to this theory, asset prices and liquidity play a central role; this points to the importance of stabilizing asset prices, not only commodity prices. To understand macroeconomy better, I present the role of liquidity in a macroeconomic environment with nominal frictions and financial intermediaries. When facing liquidity disruptions, the government has a role as an accountable liquidity planner. I analyze the associated policies in recessions that can be conducted by fiscal and monetary authorities. The present theory is consistent with the classic wisdom before the second world war.

Key Words: Liquidity, Buisiness Cycles

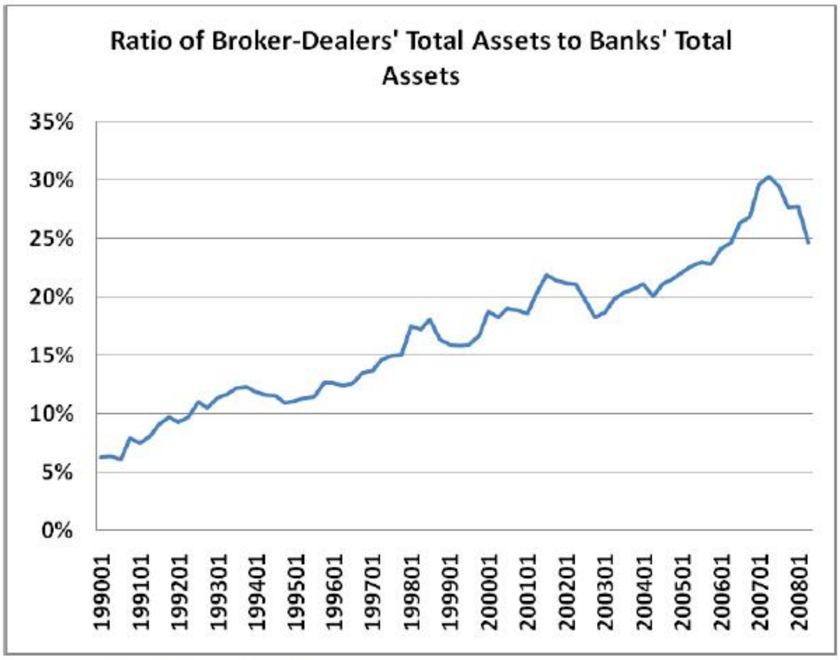
The daily revaluations of the Stock Exchange though they are primarily made to facilitate transfers of old investments between one individual and another, inevitably exert a decisive influence on the rate of current investment. For there is no sense in building up a new enterprise at a cost greater than that at which a similar existing enterprise can be purchased; whilst there is an inducement to spend on a new project what may seem an extravagant sum, if it can be floated off on the Stock Exchange at an immediate profit.

(*John Maynard Keynes (1936), The General Theory of Employment, Interest and Money, Book IV Chapter 12 Section III*)

9 Introduction

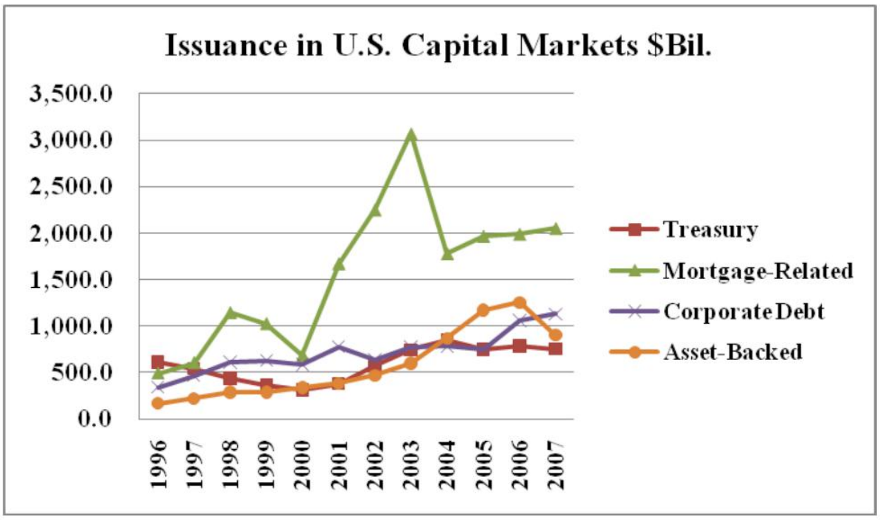
One decade after the financial crisis of 2007-2008, the cause of the crisis is still not clear. This paper studies the mechanism of financial crises and economic recession in general, from the perspective of endogenous liquidity in a broad sense. Much blame has been placed on privately-created securities. During the decade preceding the financial crisis, the rising of collateralized transactions increased and created a demand for collateral with wide acceptance. Securities markets continued to grow, but many major government bond markets expanded only slowly or even contracted. Privately-produced, near-riskless assets, e.g., AAA/Aaa asset-backed securities, were created in response to this shortage (Gorton and Ordonez (2013)). As shown in Figures 9.1 and Figure 9.2 from Gorton and Metrick (2009), the ratio of broker-dealer's total assets to bank's total assets grew steadily from 1990 to 2008, and the new issuance of securities has grown significantly since 2000. The increase of broker-dealer's total assets demonstrates that assets are increasingly used as liquidity tools in financial trading; and we have also seen a greater supply and creation of securities since 2000, as shown in Figure 9.2.

Figure 9.1: Ratio of Broker-Dealer′ Total Assets to Banks′ Total Assets



Source: Federal Flow of Funds.

Figure 9.2: Issuance in U.S. Capital Markets



Sources: U.S. Department of Treasury, Federal Agencies, Thomson Financial, Inside MBS & ABS, Bloomberg.

After the financial crisis of 2007-2008 privately created securities, such as Asset-Backed Security (ABS) and Mortgage-Backed Security (MBS), were blamed; in addition, the unconventional monetary policy also targeted private assets in the financial market. Questions arose, such as: How did privately created securities cause the financial crisis? And how should we interpret the mechanism of unconventional policies. This paper describes a model to answer these questions. In my model, privately created assets are used as financial collateral for funding. The role of funding liquidity affects asset prices and induces issuance of assets in liquidity markets. The new issuance of assets reduces the assets' pecuniary returns and leads to fragility in the liquidity market.

In incomplete markets, corporate treasurers or entrepreneurs need liquid assets in hands for a variety of reasons (Duesenberry (1963)), including everyday expenditures, volatility of cash flow, tax liability, and precautionary reason. This paper models an economy with an incomplete market, in which entrepreneurs can use privately created liquidity tools to facilitate transactions. In financial markets of advanced countries, financial securities are widely used as collateral. In this paper, I consider two kinds of major financial collateral: government bonds and private assets. Government bonds are taken as the benchmark in gauging the liquidity value of private securities. The main interest of this paper is on the production side of the economy in the incomplete market. I consider an economy with a series of representative households, comprising of one consumer and one entrepreneur; the entrepreneur is the monopolistic producer of one intermediate variety or service among a finite measure of goods. The variety and service are interpreted both as one kind of business in the narrow sense, such building houses, and a group or chain of businesses, such as building houses, selling them, and providing financial services to buyers, in a broad sense. Each variety has a constant elasticity of substitution with others. The representative entrepreneur of one industry can invest, using a final composite good as input; in addition, the entrepreneur also has full access to the financial market in which he can sell his equity and buy the equities of other industries. When entrepreneurs invest, collateral (provided by a collection of financial assets with variable liquidity or acceptance) is needed for credit. In this economy, as opposed to Gertler and Kiyotaki (2015), capital accumulation plays a role. The investment opportunities are captured by the relatively higher efficiency of the transformation from composite consumption goods to heterogeneous productive capital.

In the incomplete market, the production of intermediate varieties and services is subject to uninsurable technology shocks, which affect the productivity and investment decisions of the firm. The first step in my analysis is building a model in which firms have require liquidity that can be served by a portfolio of securities, including cash, government bonds, and private assets. In my model, entrepreneurs are unable to pledge their future returns unless these commitments are backed by marketable assets. This assumption is both theoretically and empirically reasonable. Hence, collateral is necessary for the entrepreneur when he is waiting for higher realizations from technology and investment opportunities. I derive the demand for liquidity via assets and money-in-advance constraints, which are analogous to the liquidity constraints in the consumption market associated with Svensson (1985) and Robert E. Lucas and Stokey (1987). The liquidity usage inflates asset prices. For equities, it means higher market prices and greater investment; for debts, it means lower interest rates and credit expansion.

To analyze the liquidity properties of assets, I propose a new construction and measure of liquidity based on two intrinsic properties of assets, $L(A, n)$. The first property is the book amount of an asset (A), which can be interpreted as the depth of the market; the second is the fraction of participants (n), which is interpreted as the width of the market. With the liquidity defined by these two determinants, the entrepreneur, who uses assets as collateral and changes the investment participation (n) of this asset, and the issuer of assets, who produces an asset based on its future income and increases the asset amount (A), are linked in the financial market. To gauge the liquidity value of various assets, government bonds are taken as the benchmark for its large book amount in the market and common acceptance in transactions. In this closed economy with robust public finance, government bonds do not have a default problem; in other words, the sovereign solvency is not in the scope of this closed economy.

The second part of my analysis examines the firm's asset issuance behavior and asset price fragility. The higher price of liquid assets creates room for arbitrage between the financial market and physical investment and induces new issuance of this asset. In the present model, the gradual damage to the fundamentals of assets does not come from moral hazards or the agency problem, such as Gertler and Kiyotaki (2015); instead, it comes from the arbitrage between the physical capital market and the financial market through the financial operations of creating new assets, such as Initial Public Offerings (IPO), Seasoned Equity Offerings (SEO), and securitization. Asset prices, augmented by their liquidity service, are fragile. This process happens over a long period; we admit that severe crises are the consequence of a long process rather than quarterly or yearly evolution.

The essential mechanism is summarized as follows. With the new liquidity construction, I link the asset prices with a liquidity component to the asset price fluctuations. I develop a model in which the asset prices in the financial market are connected with the fundamental investment and asset issuance. Hence, the pricing problem of assets can be the source of fundamental deterioration in the real economy. The liquidity-augmenting asset prices are higher than the assets' fundamental value. With the asset issuance to eliminate the price wedge between the financial market and the physical capital market, the newly issued assets dilute pecuniary income that the backing capital can generate; the asset prices can be sustained without dropping only because of their increased liquidity value. I examine the evolution of the fraction of investors (n) and show that, due to two reasons, the number of holders of privately created liquidity becomes more likely to decrease (i.e., more sellers than buyers) as the liquidity-augmented asset prices induce greater issuance. First, new issuance dilutes the future revenue; second, as the number of holders increases, the probability of selling by holders increases and the number of new buyers decreases. With a larger pool of holders of a liquid asset, the probability is also larger that someone has technology realization that brings better prospects than this liquid asset. To grasp the intuition of participation evolution, it is helpful to keep in mind that the motivation of holding liquid assets is to obtain credit when the holder, who is also an entrepreneur, has better technology to invest in his own firm. The asset price collapse becomes more likely as the issuance accumulates.

The novelty and contribution of my model lie in the endogeneity of liquidity using behavior, private creation of liquidity, and the evolution of assets' liquidity value. Asset price collapses happen eventually due to the issuance of private securities for liquidity usage; it is also an event of liquidity destruction. Some features of this paper need emphasis. First, this paper focuses on the behavior of entrepreneurs, who can invest in physical capital, financial assets, and create assets based on future profits. Preference shocks are absent. However, my framework is flexible enough to incorporate them with easy modification. Second, the private entrepreneur can issue assets in a booming financial market but will not buy back the assets in a bust; that is, private entrepreneurs are not accountable asset price managers of their assets, although they are the initial creators. Third, perfect information is assumed, i.e., the technology realizations and the associated returns of each asset are observable to the financial market participants. With this assumption, this paper rules out information frictions and demonstrates one mechanism of asset price fluctuation with a more general assumption. Deviations from perfect information may modify the mechanism in this paper, but will not eliminate or replace it.

In the baseline model, whose results are stated above, the asset price fluctuations are intrinsic without financial intermediaries. To generate more insights about the common topic in macroeconomics, I add extensions to incorporate nominal rigidity and the financial sector. Price stickiness has two effects. First, in normal times, price stickiness changes the firm's markup and accelerates investment; second, in a recession, price stickiness slows down the appreciation of liquidity and causes a liquidity trap.

Traditionally, there are two ways to understand the role of finance in economic crises. The first asserts that crises result from behavioral issues, such as panics or sentiments of investors; the second asserts that crises arise from fundamental causes. The mechanism presented in this paper assumes the latter and investors become panic for fundamental reasons. In the present paper, the role of the financial sector is professional asset managers who take worker deposits and purchase assets in the financial market. The financial intermediaries channel the workers' savings to the financial market and substantially increase asset liquidity. The intrinsic fluctuations of asset prices can cause insolvency of financial intermediaries and lead to financial disruption. In financial disruption, workers are also liquidity constrained and cannot smooth their consumption.

Policy Implications of Liquidity Stabilization

In this framework, the root of economic crises lies in the creation of private securities and damage to liquidity in markets. After the financial crisis of 2007-2008, an unconventional monetary policy was developed; the balance sheet of central banks in advanced economies became large and exposed to the risk of private assets. Questions about the new policy arose on the mechanisms of unconventional monetary policy and on what are the differences between the unconventional and conventional policies in alleviating economic crisis? With the theory of endogenous liquidity, I investigate macroeconomic policies in recessions from the viewpoint of liquidity stabilization, using the government as as an accountable liquidity planner.

The common feature of the conventional policies and the unconventional one is the creation of government securities to fill the shortage of sound liquidity. The conventional policy, featured in the New Deal in the 1930s, created government bonds and fiat money through the fiscal authority, which was the dominant role; the unconventional monetary policy created interest-bearing reserves for the financial intermediaries to replace the private liquidity and kept the financial sector functioning. The conventional policy, through extending the balance of the fiscal authority during a recession, generates large fiscal revenues after the dysfunction of banks and supports large increase of government expenditure; in contrast, with interest-bearing reserves, the unconventional monetary policy rescued the financial intermediaries and kept the financial sector functioning, which was effective in stabilizing the macroeconomy but had less output effect than the conventional policy implemented by the fiscal authority. In Section 5, I study the government's role as an accountable liquidity planner, which regulates liquidity stocks in markets to fill a liquidity shortage or replace privately created securities with its own securities (government bonds and interest bearing reserves).

9.1 Related Literature

This paper fits into the growing literature that explores the relationship between liquidity and monetary business cycles (Kiyotaki and Moore (2012)) in incomplete markets. The seminal paper, Kiyotaki and Moore (2012), clearly models that "money" should be interpreted broadly to include all financial assets, but consider them with differing liquidity.

In Huggett's economy of incomplete markets (Huggett (1993)), households face uninsured idiosyncratic investment (capital-income) risk or employment risk as well as borrowing (or credit) constraints. I study the incomplete markets following the descriptions in Angeletos and Calvet (2005), Angeletos (2007), and Werning (2015). On incomplete markets, Werning (2015) studies the relationship between aggregate consumption and interest rates. However, Werning (2015) focuses on household labor income risk and aggregate demand block. My study focuses on the entrepreneurial and production risks and the associated effects on asset prices and business cycles. Angeletos, Collard, and Dellas (2016) study the Ramsey policy problem in an economy in which public debts contribute to the supply of assets that private agents can use as buffer stock and collateral, or as the vehicle of liquidity, and find that issuing more debt eases the underlying financial friction.

Investment and production risk are important in shaping the entrepreneur's decisions. Bernanke, Gertler, and Gilchrist (1998) model the dynamic general equilibrium with credit market frictions. Their framework exhibits a "financial accelerator," in that endogenous developments in credit markets work to amplify and propagate shocks to the macroeconomy. Using cross-sectional data, Heaton and Lucas (2000) show that entrepreneurial income risk has a significant influence on portfolio choice and asset prices. In corporate finance literature, Holmstrom and Tirole (1996), Holmstrom and Tirole (1998), and Holmstrom and Tirole (2001) study liquidity needs of firms and how the corporate sector meets its liquidity needs. They show the early insights on liquidity needs in the business sector. Woodford (1990) also models the liquidity in an economy in a similar sense, but he assumes that only public securities can be used as liquidity. Lagos (2006) develops an asset-pricing model in which financial assets are valued for their liquidity as well as for being claims to streams of consumption goods. In contrast to Woodford (1990) and Lagos (2006), my model explores what determines the liquidity of one asset and what are the effects of private liquidity on the macroeconomy. This paper is differentiated from the studies described above in that it examines the endogenous liquidity.

Asset prices and rational bubbles are also studied in the macroeconomic literature. Gali (2014), Martin and Ventura (2012), Martin and Ventura (2016), and Martin and Ventura (2017) study rational asset price bubbles and the conventional monetary policy. Miao and Wang (2017) and Dong, Miao, and Wang (2017) study the effects of rational asset price bubbles on credit constraints and monetary policy. However, literature based on rational bubbles assumes that both the bubbles and financial shocks are exogenous. The endogeneity of liquidity-augmented asset prices is rightly the novelty of my works.

The effects of financial shocks on the macroeconomy have been the focus of macroeconomic research since the financial crisis of 2007-2008. In the existing literature, the usual methods of generating financial crises are the incorporation of a financial shock, such as the asset resaleability shock, credit shocks, or risk shocks. A partial list of papers includes Kiyotaki and Moore (2012), Gertler and Kiyotaki (2015), Del-Negro et al. (2017), He and Krishnamurthy (2017), and Benigno and Nistico (2017). They study the shocks to the financial market which cause severe crises due to nominal rigidities, damages to the balance sheet of financial intermediaries, or liquidity problems. My model needs only technology shocks to heterogeneous firms and can endogenously generate financial fluctuations.

Gorton and Laarits (2018) define the financial crisis as an event in which the holders of short-term debt come to question the collateral backing that debt. With empirical evidence, they show there is a shortage of high-quality collateral by examining the convenience of short-term debt; their results suggest there is a shortage of safe debt, implying that the seeds for a new shadow banking system exist. This study offered the empirical foundation for my theoretical work.

The new policy pattern after the financial crisis caused a large debate on its implications. The series of papers by Reis (Reis (2016c), Reis (2016a), and Reis (2016b) and Hall Hall and Reis (2015), Hall and Reis (2016), and Hall (2016)) analyze the implications of interest-bearing reserves and the potential interaction between fiscal and monetary authorities. Worries about the balance of central banks also arose, as in Negro and Sims (2015), which studies a case in which the central bank might lose the control of inflation and require the supports of the fiscal authorities.

I label my model as Keynesian because the present paper clearly shares the ingredients of Keynesian literature by James Tobin, Hicks (1989), and Minsky (1969); in addition, this paper relates to the concept of speculative liquidity demand, as in Keynes (1936).

The paper is laid out as follows. Section 2 describes the model of incomplete markets with endogenous liquidity. Section 3 presents the results of the model, including the liquidity-augmenting asset prices and asset price fragility. Section 4 extends the baseline model to incorporate nominal rigidities and financial intermediaries in normal and recession times. Section 5 builds the government in the economy as an accountable liquidity planner and analyzes the policy implications. Section 6 provides the conclusion.

10 A Model of Incomplete Markets

This section introduces the incomplete market model, in which a representative household, comprising a consumer and an entrepreneur, makes consumption and investment decisions. Each representative entrepreneur owns a monopolistic firm producing one variety or service. The variety or service should be taken in a broader sense as a physical product, one type of service, or a group or chain of businesses, for example building houses, selling houses, and providing financial services for buyers. The firm has idiosyncratic technology shock. When the entrepreneur makes an investment, he needs collateral in advance. The focus will be on the investment decisions of the entrepreneurs, with consumption largely relegated to the background.

10.1 Environment

I consider a monopolistic competitive economy with a finite number of entrepreneurs, indexed by $i \in \mathcal{I} \equiv \{1, \cdots, \mathbb{N}\}$. There are $\mathbb{N}$ varieties of products, each of which is produced by a monopolistic entrepreneur i. Agents are born at date $t = 0$ and live infinitely and consume a composition of goods in dates $t \in \{0, 1, \cdots, \infty\}$. The entrepreneurs in $\mathcal{I}$ industries are subject to idiosyncratic shocks to their productivity and invest based on their own assessment of the future prospects of the economy; insurance against these shocks is absent and credit may be limited for the producers. The consumer of a generic household performs consumption within the limits of wealth in advance. To focus on the investment decisions of the entrepreneurs, I assume that they always have the cash for personal consumption and then neglect the cash-in-advance constraint for each representative household.

There is a composite final output combined with all the varieties, according to the formula:

$$Y_t \equiv \left(\mathbb{N}^{\frac{-1}{\varepsilon}} \sum_{i \in \mathcal{I}} Y_t\left(j\right)^{1-\frac{1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where $\varepsilon (> 1)$ is the elasticity of substitution across goods. These homogeneous final goods can be used for consumption and investment purposes. The optimal allocation of any given budget across implies that

$$Y_t\left(j\right) = \frac{Y_t}{\mathbb{N}} \cdot \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon}$$

with

$$P_t \equiv \left[\frac{1}{\mathbb{N}} \sum_{j \in \mathcal{I}} P\left(j\right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}.$$

There are two assumptions in my model.

Assumption 1: (Perfect Information) All the idiosyncratic realizations of productivity are known to all the representative households in all industries.

Assumption 2: (Free Entry of the Financial Market) Entrepreneurs have free access to the financial market to purchase assets and to issue assets.

Assumption 1 assures that information friction does not exist and the perfect information is available to all market participants. All the statuses of the portfolios of others are also observable. Assumption 2 simplifies the participation problem for entrepreneurs; that is, the participation costs in the capital and financial market are negligible compared to the income and capital stock of capitalists.

For exposition convenience and clarity, from now on I use i to indicate the entrepreneur running industry i as the asset holder and user to invest in his own industrial firm; I use j to indicate the entrepreneur as asset producer based on his future income, profits, or revenues in general. Both i and j represent generic households, who own one industry to invest physical capital, can invest in another industry's assets, and can also issue assets based their own capital. I use i and j to distinguish two aspects of the generic household as asset user and as asset producer.

10.2 Consumption of Households

Each household $i \in \mathcal{I}$ maximizes the discounted utility function

$$\max_{\left\{C_t(i), I_t(i), [a_t(i,j)]_{j\in\mathcal{I}}\right\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(C\left(i\right)\right) \tag{10.1}$$

where β is the discounting rate, $I_t(i)$ is the investment by i in period t, $a_t(i,j)$ is the amount asset j owned by i, and $C_t(i)$ is a consumption composition given by

$$C_t(i) \equiv \left(\mathbb{N}^{\frac{-1}{\varepsilon}} \sum_{i\in\mathcal{I}} C_t\left(i,j\right)^{1-\frac{1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where $C_t\left(i,j\right)$ is the quantity of good j that was consumed by the household i in period t, and $\varepsilon > 1$ is the elasticity of substitution between any two given varieties j and $j^{'}$.

10.3 The Production Block

There are $\mathbb{N}$ firms, indexed for each representative household $i \in \mathcal{I}$. Each firm produces a different product and chooses a price and production plan at each period. The gross output of entrepreneur i in period t is given by $F\left(z_t^i K_t^i, H_t^i\right)$, where K_t^i is the capital stock of industry i at the beginning of the period, H_t^i is the labor employed in firm i, and z_t^i is the capital-augmenting technology. The function $F(\cdot)$ is common across industries and satisfies Inada conditions:

$$\begin{array}{ccc} F^{'}(\cdot) > 0 & \text{and} & F^{''}(\cdot) < 0 \\ \lim_{K\to 0} F^{'}(\cdot) = +\infty & \text{and} & \lim_{K\to +\infty} F^{'}(\cdot) = 0 \\ \lim_{N\to 0} F^{'}(\cdot) = +\infty & \text{and} & \lim_{K\to +\infty} F^{'}(\cdot) = 0 \end{array} .$$

The capitalist controls K_t^i through invest choice at date $t-1$, but only observes the idiosyncratic productivity z_t^i at date t. We assume that z^i is a Markov chain which is different across industries. The return on investment is thus subject to idiosyncratic uncertainty. Traditional Bewley economies, for example Werning (2015), consider idiosyncratic labor income risk. Instead, this paper focuses on productivity shocks and capital stock evolution.

The labor services are hired in a competitive market, in which both firms and workers take wages as given, denoted as W. The profits of household i in period t are given by

$$\Pi_t^i = P_t^i \cdot F\left(z_t^i K_t^i, H_t^i\right) - W_t H_t^i - P_t I_t^i. \tag{10.2}$$

For simplicity, we assume that the investment goods are formed by a combination of varieties with the same structure of consumption goods

$$I_t(i) \equiv \left(\mathbb{N}^{\frac{-1}{\varepsilon}} \sum_{j\in\mathcal{I}} I_t\left(i,j\right)^{1-\frac{1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

The law of capital accumulation is

$$K_{t+1}^i = (1-\delta)\, K_t^i + I_t(i), \tag{10.3}$$

where $\delta \in [0,1]$ is the capital depreciation rate and is homogeneous across industries.

Let us consider the production and pricing of varieties. According to the assumption that each variety can only be produced by a single firm, which is thus an effective monopolist for this particular variety. All monopolists maximize profits. The static profit maximization gives the price in the form of a constant markup over marginal cost:

$$P_t^i = \frac{\varepsilon}{\varepsilon-1}\frac{W_t}{F_H^{'}\left(z_t^i K_t^i, H_t^i\right)}, \text{ with } \varepsilon > 1.$$

I use the constant elasticity of substitution (CES) production function with Solow-neutral technological progress:

$$Y_t(i) = \left[\eta H_t(i)^{\frac{\sigma-1}{\sigma}} + (1-\eta)\, z_t^i K_t(i)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \text{ with } \sigma > 1 \text{ and } \eta \in (0,1).$$

η is a distribution parameter, which determines the importance of labor and capital service in production, σ is the elasticity of substitution, and H_t^{i*} is the optimal hiring decision by firm i given its productivity z_t^i and the capital stock K_t^i. Hence,

$$H_t^{i*} \equiv \arg\max_{H_t^i} \left\{ P_t^i F\left(z_t^i K_t^i, H_t^i\right) - W_t H_t^i \right\}.$$

I assume that firms can perfectly adjust the employed labor service and use H_t^{i*} to denote the optimal employment in industry i for exposition convenience.

10.4 Market Arrangements and Liquidity

I define the liquid properties of one asset with two intrinsic dimensions of the asset itself. The first dimension is the total marketable amount of this asset, denoted as A, which can be interpreted as the depth of this asset. The second dimension is the distribution of holders and measures the fraction (n) of holders among investors; this is interpreted as the width and recognizability of this asset. The liquid property of one asset is determined by a function of these two dimensions, $L(A, n)$. In the following paragraph, I discuss this definition and measure of asset liquidity in the literature and demonstrate that my construction of liquidity is proper.

Discussion of Liquidity $L(A, n)$ There are two main concepts of liquidity: first, low trading costs, which are the focus of the market microstructure literature; and second, the ability to borrow against an asset (i.e., use it as collateral for a loan), as described by Holmstrom and Tirole (2001). Brunnermeier and Pedersen (2009) calls the first concept "market liquidity" (i.e., the ease which it is traded) and the second "funding liquidity" (i.e., the ease with which the holders can obtain funding) and show that under certain conditions, these two types of liquidity are mutually reinforcing, leading to liquidity spirals. Vayanos and Wang (2012) provide a comprehensive survey on market liquidity. The present construction of liquidity can generalize the two concepts of liquidity. In my construction, the larger amount and wider distribution of one asset naturally implies that it is easier to find a trader and less likely to be restricted by the trading amount; and the larger amount and wider distribution of one asset among investors also implies that the information of the asset value is more commonly known by the market and the potential amount of funding is larger. The present construction of liquidity can be taken as a deeper measure of intrinsic liquidity, determining the market liquidity and funding liquidity. However, the funding liquidity property is used in the scope of the present paper. [10]

[10] Hence, in this paper the interpretation of liquidity is similar to Kiyotaki and Moore (2012) and close the funding liquidity in Brunnermeier and Pedersen (2009) . The early search-based literature, like Lagos and Zhang (2015) and Lagos (2006), focus on acceptability of asset in avoiding information problems.

I define the liquidity of an asset by two fundamental properties of the asset: the marketable stock (A^j) of one asset j and the width of holders (n). The definition of n is $n \equiv \frac{\text{count}\{i \in \mathcal{I}: a(i,j)>0\}}{Size(\mathcal{I})}$, where count$\{\cdot\}$ is the counting function and $Size(\mathcal{I})$ denotes the amount of element $\mathcal{I}$. If one security has sufficiently large stock in transactions, trading demand, buy or sell, can be satisfied in most cases without causing dramatic price volatility; if the width of holder or coverage is bigger, more traders know its prices and value without paying extra information costs.

We explore the properties of the liquidity function $L\left(A_t^j n_t^j\right)$ of one security j at date t. The function of liquidity, $L\left(A_t^j, n_t^j\right)$, satisfies the following conditions:

$$\frac{\partial L\left(A_t^j, n_t^j\right)}{\partial A_t^j} > 0, \quad \frac{\partial L\left(A_t^j, n_t^j\right)}{\partial n_t^j} > 0, \quad \text{and} \quad \frac{\partial^2 L\left(A_t^j, n_t^j\right)}{\partial A_t^j \partial n_t^j} \geq 0\cdot$$

The liquidity of one security j is an increasing function with respect to its marketable amount A and the width n. An increase in n means that more investors hold this security and its value is more publicly known. Intertemporal interpretation of increasing n is that there were more buyers than sellers in the last period, and decreasing n means the opposite. The second-order mixed derivative is non-negative, implying that increases in both dimensions will not decrease the liquidity. Taking securitization as an example, it dramatically increases the liquidity of one asset by dispersing is in the financial market. An IPO in stock markets increases the marketable amount of one stock in the first place; at the same time, it also expands the number of holders. Among all securities, government debts are assumed to have perfect liquidity, i.e., $L(A, n) = 1$.

There are six cases that cause changes in the liquidity of one asset:

Cases	A^j	n^j	$L\left(A_t^j, n_t^j\right)$
I	$A_t^j \uparrow$	$n_t^j \uparrow$	$L\left(A_t^j, n_t^j\right) \uparrow$
II	$A_t^j \uparrow$	$\bar{n}^j$	$L\left(A_t^j, \bar{n}\right) \uparrow$
III	$\bar{A}^j$	$n_t^j \uparrow$	$L\left(\bar{A}, n_t^j\right) \uparrow$
IV	$\bar{A}^j$	$n_t^j \downarrow$	$L\left(\bar{A}, n_t^j\right) \downarrow$
V	$A_t^j \uparrow$	$n_t^j \downarrow$	Not rationally possible
VI	$A_t^j \downarrow$	$\sim$	Not practically possible

The cases, from I to VI, satisfy the properties of $L(A, n)$ defined above. Special attention is given to cases V and VI. I claim that case V is not rationally possible because when n decreases, that is to say, there are more sellers than buyers in the market, it

In empirical works, Covitz and Downing (2007) uses three proxies to measure liquidity: Trade Volume, Dollar Value (face value) and Maturity.

is difficult and not profitable to infuse the same security in the market. Case VI is ruled out as not practically possible, since it is very rare in normal time, as well as in recession times for private securities, that the initial issuers would buy back the security, especially the equity. For numerical exercises, I postulate that one possible specification of the $L(A,n)$ is the Cobb-Dougals aggregator of A and n as follows

$$L(A,n) = \chi A^{\gamma} n^{1-\gamma}, \text{ with } 1 > \gamma > 0 \text{ and } \chi \text{ being constants.}$$

The timing of events is crucial. The household i enters period t with predetermined holdings in treasury bonds, $B_{t-1}(i)$ and a predetermined portfolio of shares $[a_{t-1}(i,j)]_{j\in\mathcal{I}}$of claims to the firm's dividends. He learns the productivity realization (z_t^i) of period t and then has the opportunity to purchase goods at price P_t and make an investment, $I_t(i)$, with his financial securities as collateral. The investment that an entrepreneur of industry $i \in \mathcal{I}$ can make is constrained by this liquidity constraint

$$P_t I_t(i) \le R_{t-1} B_{t-1}(i) + \sum_{j\in J} L_t\left(A_{t-1}^j, n_{t-1}^j\right) \cdot Q_t^j \cdot a_{t-1}(i,j), \tag{10.4}$$

where R_{t-1} is the return of government bonds and Q_t^j is the price or valuation of asset j at the beginning of period t. During each period, entrepreneurs, who plan to make investments, can access credits with their asset portfolios.

After the goods market is closed at the end of period t, he receives his share of dividends in cash. Money and shares are also traded at the end of period t. Household i changes its portfolios according to the following constraint:

$$M_t(i) + \sum_{j\in\mathcal{I}} Q_t^j \cdot a_t(i,j) + B_t(i) + P_t C_t(i) \le M_{t-1}(i) + \Pi_t^i + \sum_{j\in\mathcal{I}} D_t^j a_{t-1}(i,j) +$$

$$\sum_{j\in\mathcal{I}} \left[1 - L\left(A_t^j, n_t^j\right)\right] Q_t^j \cdot a_{t-1}(i,j) + \left[R_{t-1}B_{t-1}(i) + \sum_{j\in\mathcal{I}} L\left(A_{t-1}^j, n_{t-1}^j\right) \cdot Q_t^j \cdot a_{t-1}(i,j) - P_t I_t(i)\right]^{\prime} \tag{10.5}$$

where D_t^j represents the dividends of asset j in period t and $M_t(i)$ is the money holding of capitalist i.

10.5 Equilibrium

I now introduce a natural equilibrium concept for this framework and provide a characterization, reducing the conditions to a few equations.

Definition 3. An equilibrium specifies the asset prices, consumption, and investment decisions that are required to be optimal as well as consistent with aggregate consumption and investment. Formally, an equilibrium is a path for aggregates

$$\left\{C_t, Y_t, \left[A_t^j\right]_{j\in\mathcal{I}}, \left[n_t^j\right]_{j\in\mathcal{I}} \left[Q_t^j\right]_{j\in\mathcal{I}}, R_t\right\},$$

and household decisions, conditional on initial conditions,

$$\left\{C_t\left(i\right), a_t\left(i,j\right), I_t\left(i\right), B_t\left(i\right)|z_t^i\right\}_{i,j\in\mathcal{I}},$$

that satisfies the following conditions:

1. *household (the consumer and producer) optimization:* taking the path for aggregate variables as a given, household choices maximize utility (10.1), subject to flow budget constraints (10.5) and liquidity constraints (10.4);

2. market clearing: for $t = 0, 1, \ldots$ the goods, assets, money, and bond markets clear,
   financial market clearing
   $$\sum_{i\in\mathcal{I}} a_t\left(i,j\right) = A_t^j;$$
   government bond market clearing
   $$\sum_{i\in\mathcal{I}} B_t(i) = B_t;$$
   money market clearing
   $$\sum_{i\in\mathcal{I}} M_t(i) = M_t;$$
   goods market clearing
   $$Y_t = \sum_{i\in\mathcal{I}} C_t\left(i\right) + \sum_{i\in\mathcal{I}} I_t\left(i\right).$$

11 Asset Prices and Liquidity Creation

In this section, I shall examine how asset prices are determined as a function of idiosyncratic production shocks and the liquidity properties of assets. We will also explore asset price stability with the endogenous creation of assets.

In Del-Negro et al. (2017), there are two kinds of risks in the economy: fundamental risk and liquidity risk. The fundamental risk represents uncertainty in the movements of the fundamental value of a security; liquidity risk represents uncertainty in the movements of saleability in financial markets. In the present section, I endogenize the liquidity risk as a strategic complementarity of selling in the financial markets and derive the asset pricing with considerations for liquidity. In the second part of this section, we establish the condition of new issuance of assets and show that asset pricing with liquidity components induces asset issuance and investment in the physical market. The returns of newly issued assets and investments that are intended to be sold in the financial market are low and the asset prices supported by liquidity usage are vulnerable to both the bad news of its yield and good news of other securities. We call this the fragility of asset prices.

11.1 Investment and Asset Prices with Liquidity Constraints

In the each period, a generic household i makes decisions regarding investment and asset holding, with an objective function (11.1); it is repeated here for convenience:

$$\max_{\left\{C_t(i), I_t(i), [a_t(i,j)]_{j\in\mathcal{I}}\right\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t\left(i\right)\right). \tag{11.1}$$

We denote ν_t^i, and λ_t^i as the respective multiplier for the investment constraint and the flow budget constraint:

$$\begin{aligned}
\nu_t^i : & P_t I_t(i) \le R_{t-1} B_{t-1}(i) + \sum_{j\in\mathcal{I}} L\left(A_{t-1}^j, n_{t-1}^j\right) \cdot Q_t^j \cdot a_{t-1}\left(i,j\right); \\
\lambda_t^i : & \begin{array}{c} M_t(i) + \sum_{j\in\mathcal{I}} Q_t^j \cdot a_t\left(i,j\right) + B_t(i) + P_t C_t(i) \le M_{t-1}(i) + \Pi_t^i + \sum_{j\in\mathcal{I}} D_t^j a_{t-1}\left(i,j\right) \\ \sum_{j\in\mathcal{I}} Q_t^j \cdot a_{t-1}\left(i,j\right) + R_{t-1}B_{t-1}(i) - P_t I_t(i) \end{array} \\
& \Pi_t^i = P_t^i \cdot F\left(z_t^i K_t^i, H_t^i\right) - W_t H_t^{i*} - P_t I_t(i), \\
& K_t^i = (1-\delta) K_{t-1}^i + I_{t-1}(i).
\end{aligned}$$

The first order conditions are derived as follows,

$$\beta^t U'\left(C_t(i)\right) = \lambda_t^i P_t \tag{11.2}$$

$$\nu_t^i P_t + \lambda_t^i P_t = \mathbb{E}_t \left\{ \lambda_{t+1}^i \left[P_{t+1}^i \cdot z_{t+1}^i F_K'\left(z_{t+1}^i K_{t+1}^i, H_{t+1}^{i*}\right)\right]\right\}, \left[\nu_t^i \ge 0\right] \tag{11.3}$$

$$\lambda_t^i = \mathbb{E}_t \left\{ \nu_{t+1}^i + \lambda_{t+1}^i \right\} R_t \tag{11.4}$$

$$\lambda_t^i Q_t^j = \mathbb{E}_t \left\{ \lambda_{t+1}^i D_{t+1}^j + \lambda_{t+1}^i Q_{t+1}^j + \nu_{t+1}^i \left[L\left(A_t^j, n_t^j\right) Q_{t+1}^j \right]\right\} \tag{11.5}$$

The derivation of first order conditions and other algebraic derivations of the subsection 11.1 are found Appendix A 15.1.

By (11.2), the multiplier λ_t^i is the marginal utility of wealth and ν_t^i is the marginal utility of financial security hoarding. The expression $[\nu \geq 0]$ denotes the usual complementary slackness condition: if the funding liquidity constraint is not binding, the marginal value of collateral is zero; otherwise, ν is positive. The marginal utility of household i in period t is equal to the sum of the marginal utility of wealth and the marginal utility of liquidity service in making an investment.

When the entrepreneur makes investment, he is facing uncertainty about his production technology in the future as well as a collateral constraint, which entrusts him with credit. When the high technology is realized in his production, he can make profits intertemporally since he used relatively cheaper materials to form relatively high productive capital in his firm. The optimal decision of investment satisfies

$$P_t\left[\nu_t^i + \lambda_t^i\right] = \mathbb{E}_t\left\{\lambda_{t+1}^i P_{t+1}^i z_{t+1}^i F_K\left(z_{t+1}^i K_{t+1}^i, H_{t+1}^{i*}\right)\right\} \tag{11.6}$$

$$\frac{\nu_t^i}{\lambda_t^i} + 1 = \mathbb{E}_t\left\{\frac{\lambda_{t+1}^i}{\lambda_t^i} \cdot \frac{P_{t+1}^i}{P_{t+1}} \cdot \frac{P_{t+1}}{P_t} \cdot z_{t+1}^i F_K'\left(z_{t+1}^i K_{t+1}^i, H_{t+1}^{i*}\right)\right\} \geq 1$$

The circumstance in which the marginal transformation rate of entrepreneur i is larger than one indicates the liquidity constraint binding; that is to say, there is a wedge between the optimal investment and the actual investment because the entrepreneur does not have enough credit.

With a pricing condition for firms, $P_t^i = \frac{\varepsilon}{\varepsilon-1}\frac{W_t}{F_H'\left(z_t^i K_t^i, H_t^{i*}\right)}$, the investment equation can be rewritten as

$$\frac{\nu_t^i}{\lambda_t^i} + 1 = \mathbb{E}_t\left\{\frac{\lambda_{t+1}^i}{\lambda_t^i} \cdot \frac{W_{t+1}}{P_t} \cdot \frac{\varepsilon}{\varepsilon-1}\frac{z_{t+1}^i F_K'\left(z_{t+1}^i K_{t+1}^i, H_{t+1}^{i*}\right)}{F_H'\left(z_t^i K_t^i, H_t^{i*}\right)}\right\}.$$

The central concept is the investment return and the stochastic intertemporal marginal rate of transformation. The investment return is the rate at which the firm can transform date t composite goods to date $t+1$ consumption goods with the operation within his firm. Note that the investment return is not risk-free: the additional sales at $t+1$ depend on events at $t+1$ that are not known at time t, including changes in productivity; furthermore, investment and labor demand decisions made in response to events at $t+1$. When the industry i has higher productivity expectations, it makes a larger investment for profits by transferring a bunch of goods into productive capital exclusively in its factory, namely, there is a cost advantage for the highly productive firm. The intertemporal changes in prices create the profit opportunities for some industries.

The first novel idea of investment decisions lies in the formation of investment. With heterogeneous products and idiosyncratic productivities, the investment that happens in one sector is using a combination of products; the investment is profitable when one sector has the relatively higher technology. The second novel idea is that the entrepreneur may face collateral constraints when he has investment opportunities. In this sense, the holding of financial securities is positive for an entrepreneur waiting for his opportunities. When the financial collateral constraint is binding, $\nu_t^i > 0$, the marginal return from investing one more unit is still larger than the unit; that is to say, the investment has not reached the efficiency level.

The pricing of government bonds, as assets with ideal liquidity in financial markets, also include two components,: the wealth effect of their return and the liquidity service. Hence, they also enjoy a liquidity premium:

$$\frac{1}{R_t} = \mathbb{E}_t \left\{ \frac{\lambda_{t+1}^i}{\lambda_t^i - \nu_{t+1}^i} \right\} > \frac{\mathbb{E}_t \lambda_{t+1}^i}{\lambda_t^i}.$$

This is consistent with results about the convenience premium of government bonds (Krishnamurthy and Vissing-Jorgensen (2012)), because government bonds also serve the function of funding liquidity.

However, government bonds are not the only securities with funding liquidity; privately created securities can also serve the same function, though imperfectly. Combining (11.5) and (11.4), we obtain

$$\mathbb{E}_t \left\{ \frac{U'\left(C_{t+1}(i)\right)}{P_{t+1}} \left[1 + \rho_{t+1} - R_t\right] \right\} = \mathbb{E}_t \left\{ \varphi_{t+1}^i \frac{U'\left(C_{t+1}(i)\right)}{P_{t+1}} \left[\frac{L\left(A_t^j, n_t^j\right) Q_{t+1}^j}{Q_t^j} - R_{t+1} \right] \right\}$$

$$\text{with } 1 + \rho_{t+1} \equiv \frac{Q_{t+1}^j + D_{t+1}^j}{Q_t^j}$$

$$\varphi_{t+1}^i \equiv \frac{\lambda_{t+2}^i}{\lambda_{t+1}^i} \frac{P_{t+2}^i}{P_{t+1}} z_{t+2}^i F_K'\left(z_{t+2}^i K_{t+2}^i, H_{t+2}^{i*}\right) - 1,$$

where $1 + \rho_{t+1}$ indicates the pecuniary return of asset j and φ_{t+1}^i is interpreted as the shadow value of funding liquidity; when $\varphi_t^i > 0$, the firm does not make enough investments. The liquidity properties of privately created securities, $L\left(A, n\right)$, affect the return spread between government bonds and privately created securities. When the liquidity value of private assets worsens, the interest rate of government bonds increases. The second effect of worsening $L(A, n)$ is that the binding of liquidity constraints becomes more likely.

With extended use of financial securities as collateral, not merely for pecuniary returns, the pricing equation of assets is modified by the liquidity value of assets. Expression (11.5) is the capital asset pricing with liquidity concerns, which comprise the value of a claim to dividends of one asset, collateral service, if the investment is positive, and the liquidity holding for the next period.

$$\begin{aligned} Q_t^j =& \mathbb{E}_t \left\{ \frac{\lambda_{t+1}^i}{\lambda_t^i} D_{t+1}^j \right\} + \mathbb{E}_t \left\{ Q_{t+1}^j \left[\frac{\nu_{t+1}^i}{\lambda_t^i} L\left(A_t^j, n_t^j\right) + \frac{\lambda_{t+1}^i}{\lambda_t^i} \right] \right\} \\ \geq& \mathbb{E}_t \left\{ \frac{\lambda_{t+1}^i}{\lambda_t^i} \left[D_{t+1}^j + Q_{t+1}^j \right] \right\}, \left[\nu_{t+1}^i \geq 0 \right] \end{aligned}$$

One clear thing from the above equation is that, given other conditions unchanged, Q_t^j is an increasing function of $L\left(A_t^i, n_t^j\right)$ and indirectly an increasing function of A_t^j and n_t^j. The asset price with liquidity value is higher than the subjectively discounted future dividends and prices. After rearranging, the asset pricing equation can written as:

$$\begin{aligned} Q_t^j =& \mathbb{E}_t \left\{ \frac{\lambda_{t+1}^i}{\lambda_t^i} D_{t+1}^j + \frac{\lambda_{t+1}^i}{\lambda_t^i} Q_{t+1}^j + \frac{\nu_{t+1}^i}{\lambda_t^i} L\left(A_t^j, n_t^j\right) Q_{t+1}^j \right\} \\ =& \mathbb{E}_t \left\{ \frac{\lambda_{t+1}^i}{\lambda_t^i} \left[D_{t+1}^j + Q_{t+1}^j + Q_{t+1}^j L\left(A_t^j, n_t^j\right) \varphi_{t+1}^i \right] \right\} \\ \text{with} \varphi_{t+1}^i \equiv& \frac{\lambda_{t+2}^i}{\lambda_{t+1}^i} \frac{P_{t+2}^i}{P_{t+1}} z_{t+2}^i F_K' \left(z_{t+2}^i K_{t+2}^i, H_{t+2}^{i*} \right) - 1. \end{aligned} \tag{11.7}$$

As we have seen that $L(A, n)$ indicates the liquidity property of one asset, which is ultimately determined by its depth (A) and width (n), the asset price is decided not only by the preference change and expected dividends, but also by the market condition of this asset. Taking A as a constant, the dynamic changes of n describe the aggregate behavior of investors of one generic asset j. When n increases, the price increases, when n decreases, the price decreases. The relationship between asset prices and investor participation (n) demonstrates the pattern of chasing the mass and overpricing in a boom and under-pricing in a bust. The investor's decisions are affected by the market condition by observing the massive behaviors of others; the strategic complementarities may materialize a self-fulfilling trap. The results are summarized by Proposition 3.

Proposition 3. *Asset prices are inflated by their liquidity service; asset prices are determined by the subjective discounting factor, pecuniary dividends, and liquidity property; the liquidity changes amplify asset prices and make a self-fulfilling drop possible.*

Here we only say self-fulfilling drops are possible; in subsection 11.3, I prove that the drop of asset price is necessary if entrepreneurs are given the choice of creating new marketable assets, i.e., through increasing A and n. In subsection 11.1, I examined the behavior of a generic household i as a security buyer and user, as well as the asset pricing with liquidity usage. The next subsection will show how assets are created.

The conclusions regarding asset prices incorporating liquidity services are derived with Lucas tree assets. These results also apply to the bonds. When bonds, private or public, are used as liquidity, it reduces interest claims and causes credit expansion. Appendix 15.5 gives a formal proof and shows the implications of banknotes as liquidity instruments.

11.2 Asset Issuance: Planting Lucas Trees

In this subsection, I analyze the behavior of a generic household j as a supplier of assets. We also demonstrate that the consequence of oversupply of assets caused by liquidity-augmented asset pricing is the induced investment and the fragility of asset prices. The financial operation of supplying assets includes an IPO, new issuance, and securitization. On the timing of the IPO and, analogously, securitization, Ritter and Welch (2002) and Pastor and Veronesi (2005) argue that the number of firms going public changes over time in response to time variation in market conditions. Here, I model the market value changes of an asset based on its liquidity variation and argue that the subsequent outcomes hold the seeds of asset price fragility. To see the difference, I first show market value as a sum of discounted future returns.

In the financial market, the innovation of the market value of asset j can be written as

$$Q^j_{t+1}A^j_{t+1} - Q^j_t A^j_t = Q^j_{t+1}\underbrace{\left[A^j_{t+1} - A^j_t\right]}_{\geq 0} + A_t(j)\left[Q^j_{t+1} - Q^j_t\right].$$

Without the securitization of future incomes or other ways of supplying new assets (that is, the supply of the assets of the firm, j, is the same as the last period, $A^j_{t+1} = A^j_t$) the innovation of market values is reflected in the price. However, the total amount of assets, which has claims on the same or similar future revenue, is variable across time.

Suppose the assets are held by investors who plan to hold them by infinitely long-term. Without other components in its pricing (including liquidity), the price is prescribed by a standard capital asset pricing model; one such model is the Capital Asset Pricing Model (CAPM), which is denoted as Q^{j*}_t. Following Robert E. Lucas (1978), the asset price equals the sum of discounted dividends. Denote ΔA^j_t as the new supply or infusion of asset i in the financial market in the period t. Denote the valuation of the firm with real output by $Q^{j*}_t A^{j*}_t$. The valuation of the firm i satisfies:

$$Q_{t+1}^{j*} = \mathbb{E}_t \sum_{s=t+2}^{\infty} \frac{\lambda_s^i}{\lambda_{t+1}^i} \frac{\Pi_s^j}{\left[A_t^{j*} + \Delta A_t^{j*}\right]}, \tag{11.8}$$

$$Q_t^{j*} = \mathbb{E}_{t-1} \sum_{s=t+1}^{\infty} \frac{\lambda_s^i}{\lambda_t^i} \frac{\Pi_s^j}{A_t^{j*}} \tag{11.9}$$

$$\text{with } \Pi_s^j = F\left(z_s^j K_s^j, H_s^{j*}\right) - P_s I_s(j) - W_s H_s^{j*}$$

$$K_{s+1}^s = (1-\delta) K_s^j + I_s(j) \tag{11.10}$$

The profit from new issuance of assets is written as

$$Q_{t+1}^{j*}\left[A_t^{j*} + \Delta A_t^{j*}\right] - Q_t^{j*} A_t^{j*} = -\Pi_t^j + \underbrace{\mathbb{E}_t \sum_{s=t+2}^{\infty} \frac{\lambda_s^i}{\lambda_t^i}\Pi_s^j - \mathbb{E}_{t-1} \sum_{s=t+2}^{\infty} \frac{\lambda_s^i}{\lambda_t^i}\Pi_s^j}_{\text{Innovation because of the preference shocks}}. \tag{11.11}$$

$$\text{with } \Pi_s^j = F\left(z_s^j K_s^j, H_s^{j*}\right) - P_s I_s(j) - W_s H_s^{j*}$$

The equation above indicates that the market value innovation, without trading in the financial market, equals the net profits paid out to the owner plus the innovation of discounted profits in the future. This valuation changes with the changes in fundamentals. With the purely discounting value of the future, a new issuance of assets would only dilute the dividends and would not change the market value of assets. The gain of a new issuance of assets is zero, given the expected profit path and discounting factor constant. Given the expected profit path and utility path, a new issuance of assets that claim the same path of profits will not bring capital gains for issuers, according to the standard asset pricing model.

However, the entrepreneur can receive capital gains by issuing assets of its firm, this did happen in the forms of a IPO and securitization. The aim of investment and production is not only the profits in the commodity market, but also the firm's market value in the financial market. In the following paragraphs, we show that with the liquidity value in the asset pricing formula, the new issuance of assets does decrease the asset prices and the issuer receives capital gains from the financial market.

We study the asset issuance problem with the results from subsection 11.1. The decision problem for the entrepreneur j with the choice of issuing more assets backed by its capital is written as

$$\max_{\Delta A_t^j \in \mathbb{R}} A_{t-1}^j Q_t^j = \tag{11.12}$$

$$\mathbb{E}_t \left\{ \Pi_t^j + \underbrace{Q_t^j \Delta A_t}_{\text{Capital Gains}} + \frac{\lambda_{t+1}^j}{\lambda_t^j} \left[\underbrace{A_{t-1}^j \frac{\Pi_{t+1}^j}{A_{t-1}^j + \Delta A_t}}_{\text{Dividends after New Issuance}} + \underbrace{A_{t-1} Q_{t+1}^j}_{\text{Market Value Owned}} \right] \right\} \tag{11.13}$$

with

$$Q_s^j = \mathbb{E}_s \left\{ \frac{\lambda_{s+1}^i}{\lambda_s^i} \left[\frac{\Pi_{s+1}^j}{A_s^j} + Q_{s+1}^j + Q_{s+1}^j L\left(A_s^j, n_s^j\right) \varphi_{s+1}^i \right] \right\}, \text{ for } s \geq t,$$

where $A_{t-1}^j Q_t^j$ denotes the market value of asset j in the financial market. The capital gains are distributed to the previous owners, i.e., the owners of the firm or properties before the new issuance.

Proposition 4. *The asset prices will increase as investing participation increases, and the optimal issuance of assets satisfies the rule*

$$\mathbb{E}_s \left\{ -\frac{\lambda_{s+1}}{\lambda_s} \frac{\Pi_{s+1}^j}{A_{s-1}^j + \Delta A_s^j} + \frac{\lambda_{s+1}}{\lambda_s} Q_{s+1}^j \frac{\partial L\left(A_{s-1}^j + \Delta A_s^j, n_s^j\right)}{\partial \Delta A_s^j} \varphi_{s+1}^i \right\} \geq 0 \textit{ for all } s \geq t; \tag{11.14}$$

$$\textit{with } Q_s^j = \mathbb{E}_s \left\{ \frac{\lambda_{s+1}^i}{\lambda_s^i} \left[\frac{\Pi_{s+1}^j}{A_s^j} + Q_{s+1}^j + Q_{s+1}^j L\left(A_s^j, n_s^j\right) \varphi_{s+1}^i \right] \right\} \textit{for all } s \geq t.$$

Namely, if other conditions remain constant, the new issuance of liquidity increases with investment participation in asset j.

Proof. see Appendix 15.2. □

Proposition 4 indicates that with increasing participation (n) in asset j, the previous owners can issue additional assets $\left(\Delta A^j > 0\right)$ without reducing the price of asset j; hence, the capital gains from creating assets are possible in the liquidity-augmenting asset pricing model, even with the backing income path exogenously given and unchanged in the future. In the rising trend of participation in asset j (i.e., the rising of n^j), how many new assets entrepreneur j can issue, without reducing asset pricing, depends on the function of liquidity, $L\left(A^j, n^j\right)$.

Suppose there exists one accountable planner of the market value of asset j. Being accountable means that the planner can issue more assets during an increasing trend of participation of asset j and is responsible to buy back the asset in the decreasing trend. Given a profit process $\{\Pi\}_{s=t}^{\infty}$, exogenous or endogenous, the accountable planner stabilizes the price of asset j by applying the following rule (11.14):

$$\mathbb{E}_s \left\{ -\frac{\lambda_{s+1}}{\lambda_s} \frac{\Pi_{s+1}^j}{A_{s-1}^j + \Delta A_s^j} + \frac{\lambda_{s+1}}{\lambda_s} Q_{s+1}^j \frac{\partial L \left(A_{s-1}^j + \Delta A_s^j, n_s^j \right)}{\partial \Delta A_s^j} \varphi_{s+1}^i \right\} = 0 \text{ for all } s \geq t;$$

this rule indicates that the adjustment of a marketable asset should be proportional to the investment participation in this asset.

Under this rule, the asset prices can be stabilized with the given profit process and the market value changes are contributed by the supply and withdrawal of assets:

$$Q_{t+1}^j A_{t+1}^j - Q_t^j A_t^j = Q_{t+1}^j \underbrace{\left[A_{t+1}^j - A_t^j \right]}_{>=<0} + A_t(j) \underbrace{\left[Q_{t+1}^j - Q_t^j \right]}_{=0}.$$

In this case, the profit for the accountable planner comes from the new issuance in the stable market price; in a similar sense, the central bank adjusts the money stock according to new demand for money. If the participation of one asset is in an increasing channel, the consequence of this change is that the pecuniary return of this asset declines and an increasing component of its asset price comprises its liquidity value since it has larger investor participation (n^j) and greater volume (A^j).

Up to now, we are assuming that the accountable asset planner are creating new assets. But in reality, the accountable planner of assets does not exist. For the accountable asset planner, for example, the modern central banks, when the holders of money shrink, it is reasonable to ask the central bank to buy back the fiat money previously issued to stabilize its value. It also rarely happens that a private agent buys back its securities; one such example occurred when the General Electric Company (GE) bought back its commercial paper. However, it is generally true that private securities are not bought back before the due dates for liabilities and never for equities. In the process of securitization, a Special Purpose Vehicle (SPV) plays a legitimate role that immunizes the initial issuers of securitization immune from buying them back. Hence, privately created liquidity can have a stable price only with increasing participation and with new issuance.

Suppose at the initial time, the asset price of asset j, $Q_t^j = Q_t^{j*}$, which is the price of the sum of the discounted future value. While the participants increase and the liquidity role becomes significant, it is possible that the entrepreneur of this asset issues new assets backed by the same or close revenue in the future with the unchanged price Q_t^{j*}, but the actual returns have been diluted by the larger number of assets claiming the same revenue process in the future. One may argue that the new issuance of the asset is backed by new investment (e.g., new purchase of machines), new properties (e.g., new housing), or new incomes (e.g., labor incomes). In response to this argument, I discuss the relationship between asset issuance and investment below.

Asset Issuance and Investment Discussion Keynes (1936) uses an entire chapter to discuss induced investments; there are also common observations that there is a positive relationship between physical investment and the booming in financial markets. What is the relationship between investment and the asset prices in the financial market? I will answer this question by answering three minor questions.

Does asset issuance require real investment? The theoretical answer is no. Since the investment participation of one generic asset j is increasing, new issuance ($\Delta A^j > 0$) can augment its liquidity property further. The holders of asset j can be compensated with higher liquidity to their diluted dividends, because of the larger amount of asset claiming on the unchanged revenues in the future.

Was the actual investment affected by the asset's financial market condition? The answer, in reality, is yes, for legal, psychological, or market-operation reasons, which are fundamental in the modern economy. When a private entrepreneur issues new securities, rather than trading the existing securities, he must have some new capital, properties, or assets; this is necessary to advertise to new revenues in the future, to make the new issuance legal, to convince the buyers psychologically, or simply to obey market rules. No matter the reason, the private entrepreneur makes an actual investment to accompany the new issuance of assets. In a boom with increasing participation in financial investment, the quality of capital may lessen and even pure deception occurs (Allen and Faulhaber (1988)).

Will the asset issuance reduce asset returns and the capital yields, if the investment is real and has constant quality? The answer is still positive. Even if the investment, including the one purely induced by a new issuance of assets, has constant quality, the marginal decreasing return will necessarily reduce the yield of the investment. We summarize this formally in a remark.

Remark 2. A new issuance of assets causes decreasing pecuniary returns.

The results described in this subsection indicate that the issuer or an entrepreneur in its industry can create more assets and sell them in the financial market without depressing the price; nevertheless, this actually dilutes the pecuniary return. The underlying force is the increased liquidity value of this asset as the number of investors increases; with perfect information, the observable dilution of its dividends is compensated by the increased liquidity value. When the participation of asset investment is booming, the entrepreneur, who accesses the physical investment and asset creation at the same time, can make capital gains through eliminating the wedge between the financial market and the physical capital market. As emphasized above, when there are more sellers and decreasing participation, the initial creator of these assets has no motivation to buy them back and asset prices fall.

11.3 Asset Price Fragility: The Evolution of Participation

The average return in the real economy will inevitably decline as marketable assets and the physical capital backing these assets increase. The asset prices depend more and more on the liquidity properties, i.e., the easiness of trading. Without the motivation to stabilize the asset price which potentially require buying back, the price collapse is inevitable at the end of increasing participation. I show this formally.

Denote the group of investors holding asset j as $\mathbb{I}_t^j \equiv \{i : a_t(i,j) > 0 \text{ for all } i \in \mathcal{I}\}$ and the cumulative probability density function of the productivity i as $Z\left(z_t^i | z_{t-1}^i\right)$. The return of investment, I_t^i, is a function of the technology of firm i, z_t^i. Given the technology z, we write the return rate of investment of entrepreneur i, denoted by $R_{I,t}^i$, as

$$R_{I,t}^i = \mathbb{E}_t \left\{ \frac{P_{t+1}^i \cdot z_{t+1}^i F_K'\left(z_{t+1}^i K_{t+1}^i, H_{t+1}^{i*}\right)}{P_t} | z_t^i \right\} \equiv R_{I,t}^i \left(z_{t+1}^i | z_t^i \right).$$

The expected return of investing in the asset j is $R_{I,t}^j$

$$\begin{aligned} R_{I,t}^j =& \mathbb{E}_t \left\{ \frac{Q_{t+1}^j + D_{t+1}^j}{Q_t^j} \right\} \equiv R_{I,t}^j \left(n_{t+1}^j, z_{t+1}^j | n_t^j, z_t^j \right), \\ \text{with } D_{t+1}^j =& \frac{\Pi_{t+1}^j}{A_t^j}. \end{aligned}$$

We know that $R_{I,t}^j \left(n_{t+1}^j, z_{t+1}^j | n_t^j, z_t^j \right)$ and $R_{I,t}^i \left(z_{t+1}^i | z_t^i \right)$ are increasing functions of technology and the technology of each industry follows an independent stochastic process. However, $R_{I,t}^j \left(n_{t+1}^j, z_{tz+1}^j | n_t^j, z_t^j \right)$ is also affected by investor participation, which decides its liquidity property.

The law of motion of investing participation can be written as

$$\begin{aligned} n_t^j =& n_{t-1}^j + \frac{\text{Acount}\left\{ i \in \mathbb{I}_t^j | i \notin \mathbb{I}_{t-1}^j \right\}}{\text{Size}\left(\mathcal{I}\right)} - \frac{\text{Acount}\left\{ i \notin \mathbb{I}_t^j | i \in \mathbb{I}_{t-1}^j \right\}}{\text{Size}\left(\mathcal{I}\right)}; \\ \mathbb{E}_t \left\{ n_{t+1}^j \right\} =& \\ & \mathbb{E}_t \left\{ n_t^j + \frac{\sum_{i \notin \mathbb{I}_t^j} \text{Prob}\left\{ i \in \mathbb{I}_{t+1}^j | i \notin \mathbb{I}_t^j \right\}}{\text{Size}\left(\mathcal{I}\right)} - \frac{\sum_{i \in \mathbb{I}_t^j} \text{Prob}\left\{ i \notin \mathbb{I}_{t+1}^j | i \in \mathbb{I}_t^j \right\}}{\text{Size}\left(\mathcal{I}\right)} | \mathbb{I}_t^j, z_t^j \right\}. \end{aligned}$$

For any generic representative household i, who is a holder of asset j, the probability that he quit investment in asset j is the probability that his technology has a better return than asset j, and the probability that he join or stay in the investment of asset j is that his own technology is lower than the return of asset j:

$$\text{Prob}\left\{i \notin \mathbb{I}_{t+1}^{j} | i \in \mathbb{I}_{t}^{j}\right\} = \frac{\sum_{i\in\mathbb{I}_t^j} Z\left[R_{I,t}^{i}\left(z_{t+1}^{i}|z_{t}^{i}\right) \geq R_{I,t}^{j}\left(z_{tz+1}^{j}|\mathbb{I}_{t}^{j}, z_{t}^{j}\right)\right]}{\text{Size}\left(\mathcal{I}\right)};$$
$$\text{Prob}\left\{i \in \mathbb{I}_{t+1}^{j} | i \notin \mathbb{I}_{t}^{j}\right\} = \frac{\sum_{i\notin\mathbb{I}_t^j} Z\left[R_{I,t}^{i}\left(z_{t+1}^{i}|z_{t}^{i}\right) < R_{I,t}^{j}\left(z_{t+1}^{j}|\mathbb{I}_{t}^{j}, z_{t}^{j}\right)\right]}{\text{Size}\left(\mathcal{I}\right)}.$$

The probability that the investment participation of asset j decreases in period $t+1$ is written as:

$$\text{Prob}\left\{n_{t+1}^{i} < n_{t}^{i}\right\} = \text{Prob}\left\{i \notin \mathbb{I}_{t+1}^{j} | i \in \mathbb{I}_{t}^{j}, z_{t}^{j}\right\} - \text{Prob}\left\{i \in \mathbb{I}_{t+1}^{j} | i \notin \mathbb{I}_{t}^{j}, z_{t}^{j}\right\}.$$

Based on this, I offer a proposition 5 that relates the probability of participation decreasing to asset issuance and the existing participation level.

Proposition 5. *The asset price drop becomes more likely to occur as the asset issuance and investment participation increase:*

$$\frac{\partial Prob\left\{n_{t+1}^{j} < n_{t}^{j}\right\}}{\partial z_{t}^{j}} < 0 \tag{11.15}$$
$$\frac{\partial Prob\left\{n_{t+1}^{j} < n_{t}^{j}\right\}}{\partial A_{t}^{j}} > 0, \tag{11.16}$$
$$\frac{\partial Prob\left\{n_{t+1}^{j} < n_{t}^{j}\right\}}{\partial n_{t}^{j}} > 0. \tag{11.17}$$

If investing participation turns downward and there are no policy interventions, the asset price will drop to its fundamental value, as CAPM prescribed.

Proof. Appendix 15.3. □

This proposition states that with increasing issuance and participation, the reversing of booming participation in one asset becomes more likely, i.e., the reverse of its liquidity value becomes more likely and a drop in the asset price drop also becomes more likely. As the investor pool and liquidity property of one asset increases, its price also rises; a turning point will necessarily be reached in this prosperous process. There are two forces that make this happen inevitably, corresponding to (11.17) and (11.16), respectively. First, with a larger pool of investors who are producers and can invest in real capital, the probability that some investors receive a better investment opportunity becomes larger; second, the arbitrage between the financial market and physical capital undermines the yields of the security even more due to the new issuance of assets and the over-investment in the real economy. As the investment and issuance increase, the marginal output declines gradually, until the liquidity value cannot compensate for the reduction of the real return.

The asset prices with the decreasing n^j, whose deciding equation is repeated here for convenience,

$$Q_t^j = \mathbb{E}_t \left\{ \frac{\lambda_{t+1}^i}{\lambda_t^i} \left[D_{t+1}^j + Q_{t+1}^j + Q_{t+1}^j L \left(A_t^j, n_t^j \right) \varphi_{t+1}^i \right] \right\}$$

implies decreasing liquidity and lower prices and returns. Given government bonds as the always-available objects for liquidity hoarding, when a privately created asset suffers decreasing participation and worsening liquidity, the optimal choice for each investor is to sell it and buy government bonds, no matter what other investors do. The bottom price will be the fundamental value, as predicted by CAPM; that is, the liquidity property of this asset totally vanishes. The implicit assumption is that the privately created assets, as the claim on future revenues, will not be bought back and nullified through intervention.

For a generic household, the participation of investment in one asset j depends on the relative profitability of its own technology to the expected return of asset j. For simplicity and, more importantly, to focus on the fundamental mechanism, I assume that all the cross-sectional technology and asset trading behaviors are perfectly observable to all agents. To have a flexible solution in the present paper, I assume that all the idiosyncratic technology shocks have the same variance, and investors need only compare the expected returns, according to the mean-variance principle.

12 Extensions

In Section 3, I showed that asset price fluctuations are intrinsic without financial intermediaries. Nevertheless, there are good reasons to study situations with financial intermediaries and nominal rigidities to complete this macroeconomy. The financial intermediaries are in the spotlight after the financial crisis in 2007–2008. The incorporation of financial intermediaries is important and brings my study closer to the recent events, though these will not change the general conclusions in Section 11.

The first subsection presents the price stickiness in my framework. A sticky price modifies markup and investment in normal times and affects the real value of liquidity in a recession. In the second part of this section, we describe a case in which the net worth of financial intermediaries can be damaged by asset price fluctuations, which can cause disruption of the financial sector. I make a comparative analysis between the normal state and the finance-damaged case. From this, we see how the outcomes of the financial disruption cause suddenly binding on liquidity constraints for entrepreneurs and workers. Both investment and consumption drop after one portion (privately created) of liquidity evaporates.

12.1 Nominal Rigidities

Up to this point, we have focused on entrepreneurs' decisions regarding the investment and asset issuance and have left aside household labor supply choices. In the New-Keynesian framework, the emphases have been concentrated on the nature and pattern of price adjustment. This subsection tries to incorporate this non-negligible aspect in my framework; nominal rigidities will play roles in this economy.

Workers decide their consumption and labor supply to maximize the sum of discounted utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[U\left(C_t^l\right) - V\left(H_t(i)\right)\right], \tag{12.1}$$

where C_t^l is the consumption of labor in period t and $V\left(H_t(i)\right)$ is the working disutility for a worker in industry i.

The flow budget constraint of workers is

$$P_t C_t^l + S_t \leq W_t H_t + R_{t-1}^f S_{t-1},$$

where S_t denotes the saving of workers in period t and R_{t-1}^f is the return of worker's savings.

Workers are homogeneous and have the same wage equal to the lowest marginal production across industries,

$$W_t = \min_{i \in \mathcal{I}} \left\{ z_t^i F_H' \left(K_t^i, H_t^{i*}\right)\right\} \equiv \underline{MPL_t},$$

where $\underline{MPL_t}$ denotes the minimal productivity of labor. The wage formation here is simplifying in two ways: homogeneous labor and competitive labor market, neither of which harm my results. The results presented in this paper are robust to monopolistic competitive labor and heterogeneous labor with substitution, with the caveat that the market wage is exogenous to any individual firm. This setup helps us to focus on the motion of capital with a simplified labor market and can be taken as an extreme case in which workers have the least market power. Other setups are possible; for example, we might let wage equal the average of the marginal output of labor.

The labor supply condition is

$$\frac{U'\left(C_t^l\right)}{V'\left(H_t\right)}=\frac{W_t}{P_t}.$$

We use the standard setup to model price stickiness, similar to in the way by Calvo (1983), in that every period there is a fixed probability, $1-\theta$, in which a firm can change the price of its product. The expected prices of its products are written as

$$\mathbb{E}_t P_{t+1}^i=(1-\theta)P_{t+1}^{i*}+\theta P_t^i.$$

The investment equation (11.6)for firm i is modified and repeated as follows:

$$\nu_t^i+\lambda_t^i=\mathbb{E}_t\left\{\frac{P_{t+1}^i}{P_t}\cdot\left[\lambda_{t+1}^i z_{t+1}^i F_K\left(z_{t+1}^i K_{t+1}^i, H_{t+1}^i\right)\right]\right\}.$$

The expected value of an investment also depends on the expected prices in the following period. As stated in subsection 10.3, $P_t^{i*}=\frac{\varepsilon}{\varepsilon-1}\frac{W_t}{F_H'\left(z_t^i K_t^i, H_t^{i*}\right)}$, the higher technology level lowers the price of its products. With the regime of sticky prices, firms and entrepreneurs, who are expecting higher productivity, will set a higher price than the price prescribed by the regime of flexible prices, thereby making a larger investment in the current period.

The demand for products follows the equation$Y_t^j=\frac{Y_t}{\mathbb{N}}\cdot\left(\frac{P_t^j}{P_t}\right)^{-\varepsilon}$. The demand for one specific product increases if its price can decrease relative to the general price level. After a negative technology shock, the firm having a sticky price have a larger demand for its products. After nominal shocks, such the evaporation of private liquidity and the inflated value of money, the sticky price causes reduced demand.

Proposition 6. *In an economy with positive productivity shocks, investment magnitudes are larger with rigid prices than that with perfectly flexible prices; with nominal deflation shock, firms reduce capacity, i.e., reduce hiring.*

With sticky product prices, we have seen that price rigidities can accelerate investment facing high productivity growth. This proposition also predicts unemployment, with the restriction that decapitalization is impossible or highly costly and the lay-off is relatively easier.

12.2 Financial Intermediaries

In this section, we incorporate workers and the net worth of the financial intermediary to make a complete macroeconomy in which workers invest in financial securities through the financial intermediaries. This economy is inhabited by three types of agents: the representative capitalist of industry $i \in \mathcal{I}$, homogeneous workers, and financial intermediaries.

12.2.1 In Normal Times

The setup of financial intermediaries or the financial sector is mechanical; they serve homogeneous asset management services to workers who do not have direct access to the financial market. The economy in normal times is close to the New Keynesian framework.

The newly added financial intermediaries raise funds from workers and purchase assets in the financial market. The financial intermediaries offer asset management services with self-owned capital, K_t^f. The perfectly competitive market forces them to offer the same return to their depositors and the behaviors of all financial intermediaries can be aggregated by a representative financial intermediary. This financial intermediary has full access to the financial market and can trade all securities in the financial market.

The flow budget constraint can be written as

$$\textstyle\sum_{j\in\mathcal{I}} Q_t^j a_t(f,j) + B_t(f) \; = K_t^f + S_t - R_{t-1}^f S_{t-1} + R_{t-1} B_{t-1}(f) + \sum_{j\in\mathcal{I}} D_t^j a_{t-1}(f,j)$$

where $a_t(f,j)$ denotes the holding of asset j in period t with f representing the financial intermediaries, S_t denotes the savings deposits of workers at date t, K_t^f is the equity of the financial intermediary, and R_{t-1}^f the return they offer to depositors.

The net worth of the financial intermediary at the end of the period t is:

$$NW_t^f = \left\{ \sum_{j\in\mathcal{I}} Q_t^j a_t(f,j) + B_t(f) - S_t \right\}.$$

When $NW_t^f \leq 0$, the financial intermediary goes bankrupt and financial disruptions occur, such as bank runs of depositors and fire sales of securities in the financial market.

It has been proven in Proposition (5) that there exist $\bar{n}^i$, such that for $n^j \geq \bar{n}^j$, the asset price of security j is fragile in the sense that small shocks can cause an amplifying spiral of price dropping since everyone in the market observes the decreasing liquidity of this security and chooses to sell. In the fragile financial market after overissuance of financial assets, intrinsic asset price fluctuations inevitably cause damage to the insolvency of the financial intermediaries and lead to financial disruption.

12.2.2 In Financial Disruptions

To examine the outcomes of financial disruptions, I compare two states of the economy: finance working and finance failing. We analyze the outcomes after asset prices drop and the financial intermediaries are in recessions. When comparing the two states, we assume that the nominal prices and wages are fixed on the levels that existed before the financial disruption. These fixed nominal variables have subscript T, which denotes the starting date of the recession; the real variables of agent's choice have subscript $s(> T)$.

The new decision problem in a recession for the entrepreneur producing variety i is:

$$\sum_{s=T}^{\infty} \beta^s U\left(C_s(i)\right)$$

with constraints

$$\begin{aligned}
\nu_s^i :& P_T I_s(i) \leq R_T B_T(i) \\
\lambda_s^i :& \begin{aligned}[t] & M_T(i) + \sum_{j \in \mathcal{J}} Q_s^j a_s\left(i,j\right) + B_s(i) + P_T C_s(i) \\ & \leq M_T(i) + R_T B_T(i) - P_T I_s(i) + \Pi_s^i + \sum_{j \in \mathcal{I}} D_s^j a_s\left(i,j\right) \end{aligned} \\
\text{with } & \Pi_s^i = P_T^i F\left(z_s^i K_s^i, H_s^i\right) - W_T H_s^i \\
& K_s^i = (1-\delta) K_T^i + I_s(i)
\end{aligned}$$

The valuation of government bonds with financial disruption is

$$\begin{aligned}
\left[R_T\right]^{-1} = & \min_{i \in \left\{i \in \mathcal{I} : B_T^i > 0\right\}} \mathbb{E}_T \left\{ \frac{\lambda_{s+1}^i}{\lambda_T^i} \frac{P_T^i}{P_T} z_{s+1}^i F_K'\left(z_{s+1}^i K_{s+1}^i, H_{s+1}^{i*}\right) \right\}, \\
\text{with } & \sum_{i \in \mathcal{I}} R_T B_T(i) = \sum_{i \in \mathcal{I}} P_T I_s(i).
\end{aligned}$$

In a normal time,

$$\begin{aligned}
\left[R_t\right]^{-1} = & \min_{i \in \left\{i \in \mathcal{I} : B_T(i) > 0 \text{ and } L\left(A_t^j, n_t^j\right) a_t(i,j) > 0\right\}} \mathbb{E}_t \left\{ \frac{\lambda_{t+1}^i}{\lambda_t^i} \frac{P_{t+1}^i}{P_t} z_{t+1}^i F_K'\left(z_{t+1}^i K_{t+1}^i, H_{t+1}^{i*}\right) \right\}, \\
\text{with } & \sum_{i \in \mathcal{I}} B_T(i) = B_T \text{ and } \sum_{i \in \mathcal{I}} a_t\left(i,j\right) = A_t^j.
\end{aligned}$$

The investment wedge caused by the lack of funding liquidity is written as

$$\tau_s^i \equiv \mathbb{E}_s \left\{ \frac{\lambda_{s+1}^i}{\lambda_s^i} \frac{P_{s+1}^i}{P_s} z_{s+1}^i F_K' \left(z_{s+1}^i K_{s+1}^i, H_{s+1}^{i*} \right) - \frac{1}{R_t} \right\} = \tau_s^i \left(B_t \right).$$

We see that

$$\underbrace{\left\{ i \in \mathcal{I} : B_T^i > 0 \right\}}_{(1)} \subset \underbrace{\left\{ i \in \mathcal{I} : B_t(i) > 0 \text{ and } L\left(A_t^j, n_t^j \right) a_t(i,j) > 0 \right\}}_{(2)}, \tag{12.2}$$

$$\text{with } \sum_{i \in \mathcal{I}} B_T(i) = \sum_{i \in \mathcal{I}} B_t(i) = B_T$$

$$\sum_{i \in \left\{ i \in \mathcal{I} : B_T^i > 0 \right\}} I_T(i) < \sum_{i \in \left\{ i \in \mathcal{I} : B_T(i) > 0 \text{ or } L\left(A_t^j, n_t^j \right) a_t(i,j) > 0 \right\}} I_t(i).$$

The left side of (12.2) is the set of agents with liquidity access after financial disruption, i.e., private liquidity has evaporated; the right side of (12.2) is the set of agents having liquidity access. After a financial disruption with privately created liquidity has dried up, the pool of entrepreneurs who obtained funding liquidity to invest shrinks. Hence, there is a drop in investment and the value of government bonds rises; that is, lower interest rates are required to hold government bonds.

The decision problem of workers in a recession is

$$\sum_{s=T}^{\infty} \beta^s \left[U\left(C_s^l \right) - V\left(H_s^i \right) \right]$$

with the cash-in-advance constraint and flow budget constraint

$$\begin{aligned} \mu_s^l &: P_T C_s^l \le M_s \\ \lambda_s^l &: P_T C_s^l + M_s \le W_T H_s^i + M_T. \end{aligned}$$

The optimal consumption and labor supply decision is:

$$
\begin{aligned}
\lambda_T^l =& \mathbb{E}_T \left\{ \lambda_s^l + \mu_s^l \right\} \\
\lambda_s^l =& \mathbb{E}_s \left\{ \mu_{s+1}^l + \lambda_{s+1}^l \right\} \\
\lambda_T^l =& \mathbb{E}_T \sum_{s=T}^{\infty} \left\{ \mu_s^l \right\} \text{(after forward iteration)} \\
\beta^T U' \left(C_T^l \right) =& P_T \left(\lambda_T^l + \mu_T^l \right) \\
\beta^T U' \left(C_T^l \right) =& P_T \mathbb{E}_T \sum_{s=T}^{\infty} \left\{ \mu_s^l \right\} \\
\beta^T U' \left(C_T^l \right) =& P_T \lambda_T^l \\
\frac{V' \left(H_s^l \right)}{W_T} =& \frac{U' \left(C_s^l \right)}{P_T} = \beta^{-T} \lambda_T^l = \beta^{-T} \mathbb{E}_T \sum_{s=T}^{\infty} \left\{ \mu_s^l \right\}.
\end{aligned}
\tag{12.3}
$$

Worker's consumption shrinks because of lack of borrowing and liquidity. The wedge caused by the lack of liquidity for self insurance is written as

$$
\tau_s^l \equiv \frac{V' \left(H_s^l \right)}{W_T} - \frac{U' \left(C_s^l \right)}{P_T} = \frac{V' \left(H_s^l \right)}{W_T} - \frac{\mathbb{E}_T \sum_{s=T}^{\infty} \left\{ \mu_s^l \right\}}{\beta^T} = \tau_s^l \left(M_s \right),
$$

where μ^l is the marginal value of cash for workers. An increase in money holding can ease the precautionary motivation of avoiding a binding liquidity constraint in the future.

After liquidity destruction and failure of the financial sector, we see a sharp reduction in both investments of entrepreneurs and the consumption of workers. These occur for the same reason: the lack of liquidity and the dysfunction of the financial sector. In the labor market, workers are willing to work more for wealth. However, given the reduction of aggregate demand, there is a sharp reduction in the demand for labor. We also see that the market values of government assets, fiat money and treasury bonds, increase sharply.

13 The Government as An Accountable Liquidity Planner

Before analyzing the role of government, let us recall what happened to the privately created liquidity. The dropping spiral of asset prices and fire sales can be stopped by a strong buyer, who should be the initial creator of the assets, however, this initial creator obviously has no motivation to buy them back. Private agents can create liquidity, but not be accountable agents to run liquidity. With the new framework of liquidity and assets above, the question of what the government should do arises naturally. This section investigates the role of government as an accountable liquidity planner during or after a financial crisis.

I analyze the associated policies that aim to alleviate economic crises assuming other aspects of government policy, such regulation laws and taxes, are constant. The government has three tools at its disposal: money, treasury bonds, and the central bank's reserves. The treasury decides on government debt, B_t, and pays government expenditures, G_t, with fiat money. The central bank, on the other hand, decides how many government bonds, $B_t(m)$, to hold and how many interest-bearing reserves to offer, RV_t, to offer, with the interest denoted as $R_{\mathrm{RV},t}$. In my analysis, we mainly explore two aspects of policy operations: effects on the real economy and effects on the government itself. The analysis focuses generally on what the government (the fiscal and monetary authorities) can do, rather than what they ought to do. In the analysis, the transversality condition and no-Ponzi scheme are ignored. The policymakers concern only the rollover of its budget and policy operations.

I formalize the cash demand in consumption goods purchasing by adding a cash-in-advance constraint. In period t, the individual, $i \in \{\mathcal{I}, l, f\}$, and the individual and aggregate consumption of households, are subject to

$$P_t C_t(i) \leq M_t(i), \tag{13.1}$$

$$P_t C_t \leq M_t. \tag{13.2}$$

The fiat money also has perfect liquidity as government bonds, since they are legal tender for all debt, public and private.

Two features of the macroeconomic policy after the financial crisis of 2007-2008 are zero-lower bound and interest-bearing reserves. They are related to the financing ability of the government's two branches. We will discuss two policy regimes associated with these two branches of the government and the coordination between these two branches. When analyzing the government's accountable actions, we are aware that government securities, including fiat money, government bonds, and the central bank's reserves, are liquidity instruments used by consumers, entrepreneurs, and financial intermediaries.

13.1 The Fiscal Authority

Suppose that fiscal policy dominates monetary policy. The fiscal authority independently decides its budget, announcing all current and future deficits and surpluses and determining the associated amount of revenue that must be raised through bond sales and seigniorage. In this regime, the monetary authority faces the constraints imposed by the demand for government bonds, for it must finance, with seigniorage and purchasing of the treasury's bonds, any discrepancy between the revenue demand by the fiscal authority and the number of bonds that can be sold to the public. If the fiscal authority's deficits cannot be financed by new bond sales to the public, then the monetary authority is forced to create money and tolerate additional inflation. In this regime, the fiscal authority decides how many government securities, which is also liquidity tools in the financial market, are offered to the public.

Assume that the entire government debt consists of one-period debt. Consider the following identity for the treasury:

$$G_t + R_{t-1}B_{t-1}^T = T_t + \underbrace{B_t(-m) + B_t(m)}_{=B_t^T} + M_t - M_{t-1},$$

where all variables are in nominal terms. The left side comprises government expenditures on goods, services, and transfers G_t, plus interest payments on the outstanding debt $R_{t-1}B_{t-1}^T$; the right side comprises tax revenues T_t, the bond issuance at period t B_t^T, plus any direct receipts from the central bank $M_t - M_{t-1}$. $B_t^T\ [= B_t(-m) + B_t(m)]$ is the total issuance of government bonds, comprising $B_t(m)$ issued to the monetary authority and $B_t(-m)$ issued to the public.

The government deficit is given as

$$D_t = T_t + B_t^T - R_{t-1}B_{t-1}^T + M_t - M_{t-1} - G_t.$$

Except for tax revenues, the government deficit must be financed by bond sales and money creation. After the liquidity crisis happens, the outcome of increased government expenditure at period $s(> T)$, G_s, financed by government bonds and seigniorage money, is written as

$$\begin{aligned} G_s &= M_s - M_{s-1} + B_s - R_{s-1}B_{s-1} \\ P_T C_s^{'}(l) &\leq M_s\ [> M_{s-1}] \\ P_T I_s^{'}(i) &\leq B_s(i)\ [> B_{s-1}(i)] \end{aligned}$$

Remark 3. In the period of liquidity shortage, the government expenditure financed by government securities has an output multiplier larger than one.

The effects of new issuance of government securities can be written as

$$\begin{aligned}
\Delta Y =& \Delta G + \Delta C + \Delta I \\
\text{with } \Delta \mathrm{G} =& \Delta B + \Delta M \\
\frac{\Delta Y}{\Delta G} =& 1 + \frac{\Delta C + \Delta I}{\Delta G} \\
=& 1 + \underbrace{\left[\frac{\Delta C + \Delta I}{\Delta M + \Delta B}\right]}_{+} > 1.
\end{aligned}$$

Using the results in subsection 12.2.2, increasing government securities, i.e., $\Delta M > 0$ and $\Delta B > 0$, can relax the liquidity constraints of workers and entrepreneurs. Hence, we can have $\left[\frac{\Delta C+\Delta I}{\Delta M+\Delta B}\right] > 0$.

The output multiplier of government expenditure is larger than one because of the two-fold effects it generates. First, demand effects arise from the purchase of treasury securities under nominal rigidities; second, the liquidity effects start to facilitate investment and consumption after the transactions between the government and the public. Put another way, the public must first earn the treasury securities and then use them to facilitate investment and consumption. It should be clarified that the strong output effect of government expenditures happens in severe economic depression and financial disruption, i.e., right after a drop in the aggregate economy because of liquidity destruction, and not in normal times. Hence, the fiscal multiplier in a liquidity disruption is also the recovery rate to the normal.

In the fiscal dominant regime, liabilities and assets of the central bank are perfectly matched in payoffs and maturity, so that any returns earned from the government bonds are immediately paid to the reserve holders. This case roughly matches the experience of the U.S. Federal Reserve over the fifty years prior to the 2007-2008 financial crisis (Reis (2016a)).

13.2 The Monetary Authority

The policy of the monetary authority refers to the unconventional monetary policy, comprising changes in the central bank's balance sheet because of asset purchasing in the financial market and the reverse policy for financial intermediaries. In the regime that monetary policy dominates fiscal policy, the monetary authority independently decides the growth rate of money, including reserve offers for the current and future periods, and determines the amount of seigniorage revenue delivered to the fiscal authority.

The conventional central bank has a budget identity that links changes its assets and liabilities. This takes the form

$$B_t(m) + R_{\mathrm{RV},t}\mathrm{RV}_{t-1} = R_{t-1}B_{t-1}(m) + \mathrm{RV}_t,$$

where $B_t(m)$ is the central bank's purchases of government debt, $R_{t-1}B_{t-1}(m)$ is the central bank's receipt of interest payments from the treasury, and $[\mathrm{RV}_t - \mathrm{RV}_{t-1}]$ is the change in the central bank's own liabilities. Interest-bearing reserves are special assets for the financial intermediaries. They are fully repaid and exclusively held by financial intermediaries, mainly thrift banks. Reserves are default free, since the central bank can retire them at will or exchange them one-to-one at any time with fiat money. However, the real value of reserves is affected by market conditions.

The unconventional central bank's identity has government bonds, private assets, and interest-bearing reserves in the form of IOUs issued by the central bank to financial intermediaries. The central bank's reserves function as liquidity among financial intermediaries. The flow budget of the central bank is:

$$B_t(m) + R_{\mathrm{RV},t-1}\mathrm{RV}_{t-1} = \mathrm{RV}_t + R_{t-1}B_{t-1}(m) + \sum_{j\in\mathcal{I}} \left(Q_t^j + D_t^j\right) a_{t-1}\left(m, j\right).$$

The central bank's main liability and the main tool in implementing monetary policy in all modern central banks is the amount of nominal reserves RV_t. Reserves pay an interest rate that is set by the central bank $R_{\mathrm{RV},t}$. These reserves finance the central bank's holdings of assets, including government bonds and private assets, as well as payments to old reserves.

The balance sheet of financial intermediaries in normal time is written as

$$B_t\left(f\right) \ + R_{t-1}^f S_{t-1} = K_t^f + S_t + R_{t-1}B_{t-1}\left(f\right) + \sum_{j\in\mathcal{I}} \left(Q_t^j + D_t^j\right) a_{t-1}\left(f, j\right);$$

After the central bank's purchasing of private liquidity, supposing that the central bank purchases all of them, the financial intermediaries have the balance sheet as follows:

$$B_t\left(f\right) \ + R_{t-1}^f S_{t-1} + \mathrm{RV}_t = K_t^f + S_t + R_{\mathrm{RV},t-1}\mathrm{RV}_{t-1} + R_{t-1}B_{t-1}\left(f\right).$$

After restructuring the balance sheets, financial intermediaries have lower exposure to default risk and stop the fall of net worth. As the main owner of risky assets, the central bank resolves the strategic complementarity by centralizing the risky assets on its own balance sheet. The large-scale asset purchases and reserve policy stop fire sales in the financial market and keep the net worth of financial intermediaries positive. Without paying on interest the reserve, the money in the accounts of financial intermediaries would flow to asset markets and goods markets, breeding more financial speculations and volatility. With the financial sector functioning, the liquidity trap and risk-sharing problem of workers can be eased. The economy is similar to a normal one.

Remark 4. Asset purchases stop asset prices dropping.

With the monetary regime dominant, the government's ability to finance expenditures through bond issuance is constrained by the market conditions. In a monetary regime, regardless of of large government spending or fiscal austerity, the output effects are not significant. The main task for the fiscal authority is maintinaing balance, which happens to be austerity after tentatively increased spending following the crisis.

The output effect of the unconventional monetary policy is small in contrast to the conventional policy in a recession since the financial sector is still functioning and the liquidity trap effect is not severe. The risk of asset collapse and liquidity destruction was stopped by a swapping operation between the central bank and financial intermediaries.

From section 11.2, we know that the risk of private securities, which are created for liquidity use, is fundamental, instead of a sunspot risk. Selling the risky assets back to the financial market may worsen the liquidity. If the central bank intends to avoid losses, it needs to sell the assets it purchased before; however, this makes the financial market fragile again. For an accountable liquidity planner, the first step of quitting Quantitative Easing (QE) is discounting the assets in the central bank's account and admitting the possible losses.

13.3 The Coordinated Fiscal and Monetary Authorities

13.3.1 The Solvency of the Fiscal and Monetary Authorities

The QE policy adopted for the financial crisis of 2007-2008 may cause a solvency risk to the central bank. After purchasing the privately created liquidity and implementing an interest-bearing reserve policy, the real net worth of the central bank is written as

$$
\begin{aligned}
&\mathbb{E}_t\left\{RNW^m_{t+1}\right\} \\
&= \mathbb{E}_t\left\{\left[\underbrace{\sum_{j\in\mathcal{I}}\left[Q^j_t + D^j_t\right]a_t\left(m,j\right)}_{\text{Market Values of Assets}} + \overbrace{R_t B_t(m)}^{\text{Fiscal Support}} - \underbrace{R_{\mathrm{RV},t}\mathrm{RV}_t}_{\substack{\text{Payment}\\ \text{to Reserves}}}\right] \times \underbrace{\frac{1}{P_{t+1}}}_{\substack{\text{Real Value}\\ \text{of Money}}}\right\}
\end{aligned}
$$

$$
\begin{aligned}
\text{with } P_{t+1}&\left[\sum_{i\in\mathcal{I}} C_{t+1}(i) + C_{t+1}(l) + C_{t+1}(f)\right] = \left[\sum_{i\in\mathcal{I}} M_{t+1}(i) + M_{t+1}(l) + M_{t+1}(f)\right] \\
&\frac{1}{P_{t+1}} = \frac{\left[\sum_{i\in\mathcal{I}} C_{t+1}(i) + C_{t+1}(l) + C_{t+1}(f)\right]}{\left[\sum_{i\in\mathcal{I}} M_{t+1}(i) + M_{t+1}(l) + M_{t+1}(f)\right]} = \frac{C_{t+1}}{M_{t+1}} \\
&M_{t+1}(f) \le \left(R_{\mathrm{RV},t} - 1\right)\mathrm{RV}_t
\end{aligned}
$$

where $i \in \mathcal{I}$ represents the entrepreneur producing variety i, l represents labor, and f the financial intermediaries. It is possible that after the central bank issues more interest-bearing reserves, the real value of these reserves and the real net worth of the central bank continue to shrink due to the decreasing real value of money, i.e., higher inflation. In this case, the holding of fiat money shrinks, and investors and savers may seek for more stable store of value, such as commodity currency or simply consumption goods.

According to the results in Section 2, the assets on which investors run are priced higher than their real returns. Hence, it would be difficult to sell the over-priced assets in the financial market, i.e., easily quitting from QE. The central bank is faced with the hazard that revenues are less than payments. The central bank has one free variable, issuing fiat money. If the central bank must issue more fiat money than it receives to pay the reserves, the central bank loses control of inflation.

When the real net worth of the central bank becomes a severe problem, i.e., when runs on the central bank reserves happen, fiscal supports can be used to consolidate the real value of money and the real net worth of the central bank; such supports include issuing interest-bearing government bonds to the central bank.

However, the problems of the central bank's net worth and the want of fiscal supports only exist in theoretical potentials. The real outcome depends on how many private assets he central needs to buy and how many interest-bearing reserves to offer to eliminate financial risk in the financial market and, at the same time, lock the money received by the financial intermediaries in the financial system.

Remark 5. With sufficient coordination, the defaults of the monetary or fiscal authority can be avoided in nominal terms.

The fiscal authority also has monetary support from the central bank, comparsing purchasing of government bonds and the transfer of seigniorage revenues, which are written in the identity as

$$G_t + R_{t-1}B_{t-1}^T = T_t + B_t(-m) + \underbrace{B_t(m) + M_t - M_{t-1}}_{\text{Monetary Supports}}.$$

The central bank is the first buyer of government bonds and the treasury is the receiver of seigniorage revenue. The central bank can generate fiscal revenues by purchasing government bonds and can use the reserve policy to raise funds.

We assume that any such balance-sheet earnings of the central bank are transferred to the treasury; correspondingly, any balance-sheet losses of the central bank are made up by the treasury. By letting $B_t = B_t^T - B_t(m)$ be the stock of government interest-bearing debt held by the public, the budget identities of the treasury and the central bank can be combined to produce the consolidated government budget identity:

$$G_t + R_{t-1}B_{t-1} + R_{\mathrm{RV},t-1}\mathrm{RV}_{t-1} = T_t + \underbrace{B_t + \mathrm{RV}_t + M_t - M_{t-1}}_{\text{Vriables with Freedoms}} + \sum_{j\in\mathcal{I}} \left[Q_t^j + D_t^j\right] a_{t-1}(m,j),$$

which describes all the revenues, expenditures, and possible losses that the government-central bank may suffer from its market interventions.

We see that with sufficient coordination between the fiscal and monetary authorities, neither the treasury nor the central bank would default in nominal terms. It is true, in nominal terms, that *"You (the U.S. government) never have to default because you print the money."* Reis (2016c) argues that QE, which uses interest-paying reserves to purchase government bonds, will not necessarily cause inflation. However, his statement is only true in the short-term; in the long-term, inflation is inevitable. With other conditions unchanged, after smoothly passing a fiscal crisis or a financial crisis, inflation is inevitable.

Can the central bank alleviate fiscal burdens? Reis (2016a) conclude that it can but the effect is small. With monetary support, the fiscal authority can always roll over the government debt; this is enough for policymakers. The inflation effect on alleviating government debt is strong in the long run, given that the fiscal authority can roll over.

13.3.2 Policy Coordination

Following the financial crisis of 2007-2008, there is a striking contrast between the treasury securities, the Zero Lower Bond (ZLB) of government bonds, and the central bank's securities, interest-bearing reserves. In the combined budget of the government, the treasury's bonds and the central bank's reserves are equivalent as the government's securities, but with different nominal returns. The anomaly reflects the lack of coordination between the fiscal authority and the monetary authority.

As I point out in Appendix E (15.5), the price of debts can be written as $[R_t]^{-1}$. The price of the treasury's bonds with zero nominal interest is 1; the price of interest-bearing reserves with positive interest is $[R_{\text{RV},t}(>1)]^{-1} < 1$. With the same one-unit repayment from the government, the treasury's bonds are more expensive than the central bank's reserve for big buyers, most of which are the financial intermediaries with plentiful funds.

The unconventional monetary policy, through offering interest-bearing reserves and purchasing private liquidity, secures the balance sheet of financial intermediaries and locks the funds on the balance sheet of the central bank, as its liabilities. The financial intermediaries have no funds or motivation to purchase government bonds. The ZLB emerges because of the competition of interest-bearing reserves as new government securities. If we imagine in a crisis without an interest-bearing reserve policy, the funds in the financial market would flood to the treasury's bonds. ZLB reflects the constrained security-creation ability of the treasury under the monetary regime.

After the central bank stabilizes the financial market with its reserve policy, a large-scale creation of treasury securities is not necessary. But there is also no harm in doing so, based on our history in dealing with economic crises. However, ZLB constrains the treasury's ability to finance its expenditure. After the government increases expenditure, the government bonds do not enjoy popularity and the government faces the painful task to achieve austerity. Given our existing knowledge and very limit samples of economic crises, increasing government expenditure is still a wise choice from an ex-ante viewpoint.

13.4 The Distributive Effects of Liquidity Creation

The magical economic power of the government comes from the creation of government securities, especially fiat money. To clarify the exposition, I exclude taxes from policy discussion as stated previous and focus on policies that can be accomplished through transactions. The creation of liquidity, both privately created and the government-created, has distributional effects.

The capital gains from issuing private liquidity is given as

$$\sum_{s=t}^{T}\left\{\frac{\lambda_s}{\lambda_t}Q_s^{j*}\Delta A_s^j\frac{\partial L\left(A_{s-1},n_s\right)}{\partial n_s}1_{[n_s>n_{s-1}]}\right\},$$

where T is the date on which the financial crisis occurs, $[n_s > n_{s-1}]$ represents the market condition to issue new assets without decreasing asset prices, and ΔA_s^j is specified by the condition (11.14). The entrepreneur who issued assets during a financial booming has received capital gains.

Any new issuance of money, interest paid to reserves, or purchasing government bonds, will benefit the first receivers of money and the first spender of the money before price-inflating occurs. Suppose that the government increases expenditure or the financial intermediaries spend the new money in the goods market. Before and after the purchase with new money, the aggregate cash-in-advance constraint is

$$PC \leq M \tag{13.3}$$
$$P\Delta C \leq \Delta M$$
$$P^{'}C^{'} \leq M \tag{13.4}$$
$$\text{with } C^{'} + \Delta C = C$$

where ΔM is the new issuance of money and the ΔC is the goods seized by the new money; $P^{'}$ is the price after a new purchase and is higher than before. (13.3) indicates the price level before new money and (13.4) indicates the price level and aggregate consumption after new money purchasing. Since the introduction of new money is surprising or the price is sticky even with anticipation, the immediate purchase by the new money is in the price before this batch of new money emerges in the goods market.

Remark 6. The distribution effect of monetary policies depends on who receives and spends the money first.

The distributional distinctions between the policy regimes lie in who collects the revenues from issuing government securities. In the fiscal regime, the government alleviates the crisis by creating more government securities which bring revenues to the treasury and can be used for government expenditures and payment, G. In monetary regime, the interest-bearing reserves are offered to the financial intermediaries and used to stabilize the asset prices, which are already much higher than their fundamental values. The interest-bearing reserve policy transfers reources to maintain the financial sector's solvency.

14 Concluding Remarks

This paper studied endogenous liquidity, both privately created and government-created, in asset pricing and macroeconomic fragility. It shows one mechanism through which financial markets can create intrinsic instability and fluctuations in the aggregate economy. Compared to the literature on financial shocks and economic crises, the first novelty of this model is that this new mechanism does not rely on exogenous financial shocks.

I first defined the liquidity value of one security with its two properties, the total amount in trading (depth) and the number of holders (width). The liquidity property of securities is an increasing function of these two dimensions. I show that private securities, used as funding collateral, have higher prices than their fundamental returns. For Lucas tree assets, it means higher prices; for debts, it means lower interests. The high price of Lucas tree assets induces more investment and low interest for debts, which means credit expansion.

With the results of liquidity-augmenting asset pricing, I studied the behavior of asset producers (i.e., the planting of Lucas trees) and showed that with increasing participation of investment in one asset, the entrepreneur or capitalist who owns this industry will create more assets, claiming on the future revenues. These newly created assets can be sold in the financial market without decreasing the asset prices, even when the revenue path in the future is unchanged because the holder of this asset is compensated by the increased liquidity value. Hence, there are capital gains for the asset producers through arbitrage between the physical capital market and the financial market.

Through investigating the evolution of investor participation, I showed that the asset prices with liquidity augmenting are fragile and asset price dropping becomes more likely as investment participation increases and the asset issuance increases also. The price collapses cause liquidity destruction and damage the balance sheet of financial intermediaries. With the shortage of sound liquidity and financial dysfunction, the economy falls into recession.

The accountable government can act as an accountable liquidity planner. I investigated and interpreted the implied policies to alleviate economic difficulties from the viewpoint of liquidity. The conventional policy, which is dominated by the fiscal authority, has larger than one output multiplier in economic recessions; the unconventional monetary policy achieved better smoothness through rescuing the financial intermediaries. The distinctions lie in how government securities are created and spent by the two economic branches of government, the treasury and the central bank. The coordination between fiscal and monetary authorities and the distributional effects of government policy were also analyzed.

In the literature, economists often blame often asymmetric information and financial frictions for forging the financial crisis. This paper shows the mechanism of financial crises with the fewest market frictions. The frictions only have roles in shaping the economic process, rather than fundamentally determining the event. The central concept of my model lies in behaviors that create and use liquidity.

My framework follows the thoughts of John Maynard Keynes, John Hicks, and

Ludwig von Mises. In economic history, economic crises are accompanied by asset price collapse; in most cases, objects serving as liquidity are also securities for investment. It is also true that the process of liquidity supply is the process of asset production. These overlapping processes cause the confusion between a liquidity crisis and an investment crisis. When the aggregate economic depression is examined from the viewpoint of liquidity, it can solve the puzzle of why the price collapse of individual asset can cause an aggregate problem in the economy.

Here I make a short discussion and review of the history of economic crises from the perspective of asset/liquidity creation. The liquidity creation without requiring accountability is the gradual and fatal damage to the soundness of liquidity. Historically, running to sound liquidity is the common feature of crises. After the government has printed too much cash, running to metal currency occurs in the crisis; after banks have created too many banknotes, running to cash occurs; after the financial market has created too many securities, running to the government's bonds or reserves occurs. Perhaps next time, after the sophisticated technologies have created too many digital icons, we will run to real securities.

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15 Appendix

15.1 Appendix A

The decision problem of capitalist i is

$$\max_{\left\{C_t(i), I_t(i), [a_t(i,j)]_{j\in\mathcal{I}}\right\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t\left(i\right)\right),$$

subject to constraints

$$\begin{aligned}
\nu_t^i :& P_t I_t(i) \leq R_{t-1} B_{t-1}(i) + \sum_{j\in\mathcal{I}} L\left(A_{t-1}^j, n_{t-1}^j\right) \cdot Q_t^j \cdot a_{t-1}\left(i,j\right); \\
& M_t(i) + \sum_{j\in\mathcal{I}} Q_t^j \cdot a_t\left(i,j\right) + B_t(i) + P_t C_t(i) \leq M_{t-1}(i) + \Pi_t^i + \sum_{j\in\mathcal{I}} D_t^j a_{t-1}\left(i,j\right) \\
\lambda_t^i :& \qquad \sum_{j\in\mathcal{I}} \left[1 - L\left(A_t^j, n_t^j\right)\right] Q_t^j \cdot a_{t-1}\left(i,j\right) \\
& \quad + \left[R_{t-1} B_{t-1}(i) + \sum_{j\in\mathcal{I}} L\left(A_{t-1}^j, n_{t-1}^j\right) \cdot Q_t^j \cdot a_{t-1}\left(i,j\right) - P_t I_t(i)\right] \\
& \Pi_t^i = P_t^i \cdot F\left(z_t^i K_t^i, N_t^i\right) - W_t H_t^i - P_t I_t(i), \\
& K_t^i = (1-\delta) K_{t-1}^i + I_{t-1}(i).
\end{aligned}$$

The Lagrangian problem

$$\begin{aligned}
\mathcal{L}_0 =& \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t(i)\right) + \nu_t(i) \left[R_{t-1} B_{t-1}(i) + \sum_{j\in\mathcal{I}} L\left(A_{t-1}^j, n_{t-1}^j\right) \cdot Q_t^j \cdot a_{t-1}\left(i,j\right) - P_t I_t(i)\right] \\
& \lambda_t(i) \left[\begin{array}{c}
\sum_{j\in\mathcal{I}} Q_t^j \cdot a_{t-1}\left(i,j\right) - \sum_{j\in\mathcal{I}} Q_t^j \cdot a_t\left(i,j\right) \\
+ \left[R_{t-1} B_{t-1}(i) - P_t I_t(i)\right] \\
+ M_{t-1}(i) + \Pi_t^i + \sum_{j\in\mathcal{I}} D_t^j a_{t-1}\left(i,j\right) - M_t(i) - B_t(i) - P_t C_t(i)
\end{array}\right]
\end{aligned}$$

The first order conditions:

$$
\begin{aligned}
\frac{\partial \mathcal{L}_0}{\partial C_t(i)} =& \beta^t U'\left(C_t(i)\right) + \lambda_t(i)\left[-P_t\right] = 0, \\
\frac{\partial \mathcal{L}_0}{\partial I_t(i)} =& \nu_t^i\left[-P_t\right] + \lambda_t^i\left[-P_t\right] + \lambda_{t+1}^i\left[P_{t+1}^i \cdot z_{t+1}^i F_K'\left(z_{t+1}^i K_{t+1}^i, H_{t+1}^i\right)\right] = 0, \left[\nu_t^i \geq 0\right] \\
\frac{\partial \mathcal{L}_0}{\partial B_t(i)} =& \nu_{t+1}^i R_t + \lambda_{t+1}^i R_t + \lambda_t^i(-1) = 0 \\
\frac{\partial \mathcal{L}_0}{\partial a_t\left(i,j\right)} =& \lambda_{t+1}^i D_{t+1}^j + \lambda_{t+1}^i Q_{t+1}^j + \nu_{t+1}^i \cdot L\left(A_t^j, n_t^j\right) Q_{t+1}^j - \lambda_t^i Q_t^j = 0
\end{aligned}
$$

The Euler equation of capitalist

$$
\begin{aligned}
\lambda_t^i \frac{1}{R_t} &= \mathbb{E}_t\left\{\lambda_{t+2}^i \frac{P_{t+2}^i \cdot z_{t+2}^i F_K'\left(z_{t+2}^i K_{t+2}^i, H_{t+2}^i\right)}{P_{t+1}}\right\} \\
\frac{U_c\left(C_t(i)\right)}{P_t R_t} &= \mathbb{E}_t\left\{\frac{\beta^2 U_c\left(C_{t+2}(i)\right)}{P_{t+2}} \frac{P_{t+2}^i \cdot z_{t+2}^i F_K'\left(z_{t+2}^i K_{t+2}^i, H_{t+2}^i\right)}{P_{t+1}}\right\} \\
1 &= \mathbb{E}_t\left\{\frac{R_t}{P_{t+1}/P_t} \frac{\beta U\left(C_{t+1}(i)\right)}{U\left(C_t(i)\right)} \frac{\beta U\left(C_{t+2}(i)\right)}{U\left(C_{t+1}(i)\right)} \frac{P_{t+2}^i \cdot z_{t+2}^i F_K'\left(z_{t+2}^i K_{t+2}^i, H_{t+2}^i\right)}{P_{t+2}}\right\} \\
1 &= \mathbb{E}_t\left\{\frac{R_t}{\pi_{t+1}} SDF_{t+1} SDF_{t+2} \cdot \frac{P_{t+2}^i \cdot z_{t+2}^i F_K'\left(z_{t+2}^i K_{t+2}^i, H_{t+2}^i\right)}{P_{t+2}}\right\} \\
&\text{with } SDF_{t+1} \equiv \frac{\beta U\left(C_{t+1}(i)\right)}{U\left(C_t(i)\right)} \text{ and } \pi_{t+1} \equiv \frac{P_{t+1}}{P_t}.
\end{aligned}
$$

The price of asset j is

$$\begin{aligned}
Q_t^j =&\mathbb{E}_t \left\{ \frac{\lambda_{t+1}^i}{\lambda_t^i} D_{t+1}^j + \frac{\lambda_{t+1}^i}{\lambda_t^i} Q_{t+1}^j + \frac{\nu_{t+1}^i}{\lambda_t^i} L\left(A_t^j, n_t^j\right) Q_{t+1}^j \right\} \\
=&\mathbb{E}_t \left\{ \begin{array}{c} \frac{\lambda_{t+1}^i}{\lambda_t^i} D_{t+1}^j + \frac{\lambda_{t+1}^i}{\lambda_t^i} Q_{t+1}^j + \\ L\left(A_t^j, n_t^j\right) Q_{t+1}^j \cdot \mathbb{E}_{t+1}\left[\frac{\lambda_{t+2}^i}{\lambda_t^i} \frac{P_{t+2}^i}{P_{t+1}} z_{t+2}^i F_K'\left(z_{t+2}^i K_{t+2}^i, N_{t+2}^i\right) - \frac{\lambda_{t+1}^i}{\lambda_t^i} \right] \end{array} \right\} \\
=&\mathbb{E}_t \left\{ \begin{array}{c} \frac{\lambda_{t+1}^i}{\lambda_t^i} D_{t+1}^j + \frac{\lambda_{t+1}^i}{\lambda_t^i} Q_{t+1}^j \left[1 - L\left(A_t^j, n_t^j\right)\right] + \\ \frac{\lambda_{t+2}^i}{\lambda_t^i} L\left(A_{t+1}^j, n_{t+1}^j\right) Q_{t+1}^j \frac{P_{t+2}^i}{P_{t+1}} z_{t+2}^i F_K'\left(z_{t+2}^i K_{t+2}^i, N_{t+2}^i\right) \end{array} \right\} \\
=&\mathbb{E}_t \left\{ \frac{\lambda_{t+1}^i}{\lambda_t^i} \left[D_{t+1}^j + Q_{t+1}^j + Q_{t+1}^j L\left(A_t^j, n_t^j\right) \varphi_{t+1}^i \right] \right\} \\
&\text{with} \varphi_{t+1}^i \equiv \frac{\lambda_{t+2}^i}{\lambda_{t+1}^i} \frac{P_{t+2}^i}{P_{t+1}} z_{t+2}^i F_K'\left(z_{t+2}^i K_{t+2}^i, H_{t+2}^i\right) - 1.
\end{aligned}$$

15.2 Appendix B

The decision problem of new issuance of assets:

$$\max_{\Delta A_t^j \in \mathbb{R}} A_{t-1}^j Q_t^j = \mathbb{E}_t \left\{ \Pi_t^j + \underbrace{Q_t^j \Delta A_t}_{\text{Capital Gains}} + \frac{\lambda_{t+1}^j}{\lambda_t^j} \left[A_{t-1}^j \frac{\Pi_{t+1}^j}{A_{t-1}^j + \Delta A_t} + A_{t-1} Q_{t+1}^j \right] \right\}$$

with

$$Q_s^j = \mathbb{E}_s \left\{ \frac{\lambda_{s+1}^i}{\lambda_s^i} \left[\frac{\Pi_{s+1}^j}{A_s^j} + Q_{s+1}^j + Q_{s+1}^j L\left(A_s^j, n_s^j\right) \varphi_{s+1}^i \right] \right\}, \text{ for } s \geq t,$$

where $A_{t-1}^j Q_t^j$ denotes the market value of asset j in the financial market by manipulating tradable assets claimed on its profits in the future.

F.O.C.

$$\begin{aligned}
\frac{\partial\left[A_{t-1}^{j}Q_{t}^{j}\right]}{\partial\Delta A_{t}^{j}} =&\mathbb{E}_{t}\left\{Q_{t}^{j}+\frac{\partial Q_{t}^{j}}{\partial\Delta A_{t}^{j}}\Delta A_{t}^{j}-\frac{\lambda_{t+1}^{j}}{\lambda_{t}^{j}}A_{t-1}^{j}\frac{\Pi_{t+1}^{j}}{\left(A_{t-1}^{j}+\Delta A_{t}^{j}\right)^{2}}+\frac{\lambda_{t+1}^{j}}{\lambda_{t}^{j}}\frac{\partial A_{t-1}^{j}Q_{t+1}^{j}}{\partial\Delta A_{t}^{j}}\right\} \\
=&\mathbb{E}_{t}\left\{\begin{array}{c}Q_{t}^{j}-\frac{\lambda_{t+1}}{\lambda_{t}}\frac{\Pi_{t+1}^{j}}{A_{t-1}^{j}+\Delta A_{t}^{j}}+\frac{\lambda_{t+1}}{\lambda_{t}}Q_{t+1}^{j}\frac{\partial L\left(A_{t-1}^{j}+\Delta A_{t}^{j},n_{t}^{j}\right)}{\partial\Delta A_{t}^{j}}\varphi_{s+1}^{i} \\ +\frac{\lambda_{t+1}}{\lambda_{t}}\frac{\partial\left[A_{t-1}^{j}Q_{t+1}^{j}\right]}{\partial\Delta A_{t}^{j}}\left[1+L\left(A_{t-1}^{j}+\Delta A_{t}^{j},n_{t}^{j}\right)\varphi_{t+1}^{i}\right]\end{array}\right\}\geq 0.
\end{aligned}$$

The sufficient condition that guarantees the market value of one asset not decreasing after new issuance is that

$$\mathbb{E}_{s}\left\{-\frac{\lambda_{s+1}}{\lambda_{s}}\frac{\Pi_{s+1}^{j}}{A_{s-1}^{j}+\Delta A_{s}^{j}}+\frac{\lambda_{s+1}}{\lambda_{s}}Q_{s+1}^{j}\frac{\partial L\left(A_{s-1}^{j}+\Delta A_{s}^{j},n_{s}^{j}\right)}{\partial\Delta A_{s}^{j}}\varphi_{s+1}^{i}\right\}\geq 0\text{ for all }s\geq t;$$

$$Q_{s}^{j}=\mathbb{E}_{s}\left\{\frac{\lambda_{s+1}^{i}}{\lambda_{s}^{i}}\left[\frac{\Pi_{s+1}^{j}}{A_{s}^{j}}+Q_{s+1}^{j}+Q_{s+1}^{j}L\left(A_{s}^{j},n_{s}^{j}\right)\varphi_{s+1}^{i}\right]\right\},\text{ for all }s\geq t.$$

Q_s^j denotes the price of one asset with the manipulation of the issuer. The condition is analogous to the intuition of managing the value of fiat money.

$$\begin{aligned}
&\frac{\partial Q_{t}^{j}}{\partial\Delta A_{t}^{j}} \\
&=\mathbb{E}_{t}\frac{\lambda_{t+1}^{i}}{\lambda_{t}^{i}}\left\{-\frac{\Pi_{t+1}^{j}}{\left(A_{t-1}^{j}+\Delta A_{t}^{j}\right)^{2}}+\frac{\partial Q_{t+1}^{j}}{\partial\Delta A_{t}^{j}}\left[1+L\left(A_{t-1}^{j}+\Delta A_{t}^{j},n_{t}^{j}\right)\right]+Q_{t+1}^{j}\frac{\partial L\left(A_{t-1}^{j}+\Delta A_{t}^{j},n_{t}^{j}\right)}{\partial\Delta A_{t}^{j}}\right\} \\
&=\mathbb{E}_{t}\left\{\begin{array}{c}\frac{\lambda_{t+1}^{i}}{\lambda_{t}^{i}}\left[-\frac{\Pi_{t+1}^{j}}{\left(A_{t-1}^{j}+\Delta A_{t}^{j}\right)^{2}}+Q_{t+1}^{j}\frac{\partial L\left(A_{t-1}^{j}+\Delta A_{t}^{j},n_{t}^{j}\right)}{\partial\Delta A_{t}^{j}}\right] \\ \frac{\lambda_{t+2}^{i}}{\lambda_{t}^{i}}\left[1+L\left(A_{t-1}^{j}+\Delta A_{t}^{j},n_{t}^{j}\right)\right]\left[-\frac{\Pi_{t+2}^{j}}{\left(A_{t-1}^{j}+\Delta A_{t}^{j}\right)^{2}}+Q_{t+2}^{j}\frac{\partial L\left(A_{t-1}^{j}+\Delta A_{t}^{j},n_{t+1}^{j}\right)}{\partial\Delta A_{t}^{j}}\right] \\ +\frac{\lambda_{t+2}^{i}}{\lambda_{t}^{i}}\frac{\partial Q_{t+2}^{j}}{\partial\Delta A_{t}^{j}}\left[1+L\left(A_{t-1}^{j}+\Delta A_{t}^{j},n_{t}^{j}\right)\right]\left[1+L\left(A_{t-1}^{j}+\Delta A_{t}^{j},n_{t+1}^{j}\right)\right]\end{array}\right\} \\
&=\mathbb{E}_{t}\left\{\sum_{s=t}^{\infty}\frac{\lambda_{s+1}^{i}}{\lambda_{s}^{i}}\left[-\frac{\Pi_{s+1}^{j}}{\left(A_{s-1}^{j}+\Delta A_{s}^{j}\right)^{2}}+Q_{s+1}^{j}\frac{\partial L\left(A_{s-1}^{j}+\Delta A_{s}^{j},n_{s}^{j}\right)}{\partial\Delta A_{s}^{j}}\right]\prod_{l=t}^{s}\left[1+L\left(A_{t-1}^{j}+\Delta A_{t}^{j},n_{l}^{j}\right)\right]\right\}
\end{aligned}$$

$$\frac{\partial Q^j_{t+1}}{\partial \Delta A^j_t} = \mathbb{E}_t \frac{\lambda^i_{t+2}}{\lambda^i_{t+1}} \left\{ \begin{array}{c} -\frac{\Pi^j_{t+2}}{\left(A^j_{t-1} + \Delta A^j_t\right)^2} + \\ \frac{\partial Q^j_{t+2}}{\partial \Delta A^j_t} \left[1 + L\left(A^j_{t-1} + \Delta A^j_t, n^j_{t+1}\right)\right] + Q^j_{t+2} \frac{\partial L\left(A^j_{t-1} + \Delta A^j_t, n^j_{t+1}\right)}{\partial \Delta A^j_t} \end{array} \right\}$$

15.3 Appendix C

The Evolution of participation.

The law of motion of investing participation can be written as

$$n^j_t = n^j_{t-1} + \frac{\text{Count}\left\{i \in \mathbb{I}^j_t | i \notin \mathbb{I}^j_{t-1}\right\}}{\text{Size}\left(\mathcal{I}\right)} - \frac{\text{Count}\left\{i \notin \mathbb{I}^j_t | i \in \mathbb{I}^j_{t-1}\right\}}{\text{Size}\left(\mathcal{I}\right)};$$

$$\mathbb{E}_t\left\{n^j_{t+1}\right\} =$$

$$\mathbb{E}_t \left\{ n^j_t + \frac{\sum_{i \notin \mathbb{I}^j_t} \text{Prob}\left\{i \in \mathbb{I}^j_{t+1} | i \notin \mathbb{I}^j_t\right\}}{\text{Size}\left(\mathcal{I}\right)} - \frac{\sum_{i \in \mathbb{I}^j_t} \text{Prob}\left\{i \notin \mathbb{I}^j_{t+1} | i \in \mathbb{I}^j_t\right\}}{\text{Size}\left(\mathcal{I}\right)} | \mathbb{I}^j_t, z^j_t \right\}.$$

For a generic representative household i, the holder of asset j, the probability that he quit investment in asset i is the probability that his technology has better return than the asset j, and the probability that he join or stay in the investment of asset j is that his own technology is lower than return of asset j,

$$\text{Prob}\left\{i \notin \mathbb{I}^j_{t+1} | i \in \mathbb{I}^j_t\right\} = \frac{\sum_{i \in \mathbb{I}^j_t} Z\left[R^i_{I,t}\left(z^i_{t+1} | z^i_t\right) \geq R^j_{I,t}\left(z^j_{tz+1} | \mathbb{I}^j_t, z^j_t\right)\right]}{\text{Size}\left(\mathcal{I}\right)};$$

$$\text{Prob}\left\{i \in \mathbb{I}^j_{t+1} | i \notin \mathbb{I}^j_t\right\} = \frac{\sum_{i \notin \mathbb{I}^j_t} Z\left[R^i_{I,t}\left(z^i_{t+1} | z^i_t\right) < R^j_{I,t}\left(z^j_{t+1} | \mathbb{I}^j_t, z^j_t\right)\right]}{\text{Size}\left(\mathcal{I}\right)}.$$

The probability that the investment participation of asset j decreases is written as

$$\begin{aligned}
&\text{Prob}\left\{n_{t+1}^{i} < n_{t}^{i}\right\}\\
=&\text{Prob}\left\{i \notin \mathbb{I}_{t+1}^{j} | i \in \mathbb{I}_{t}^{j}, z_{t}^{j}\right\} - \text{Prob}\left\{i \in \mathbb{I}_{t+1}^{j} | i \notin \mathbb{I}_{t}^{j}, z_{t}^{j}\right\}\\
=&\frac{\sum_{i\in\mathbb{I}_{t}^{j}} \int_{z_{t+1}^{i}\in\mathcal{Z}} \int_{z_{t+1}^{j}} 1_{\left[R_{I,t}^{i}\left(z_{t+1}^{i}|z_{t}^{i}\right)\geq R_{I,t}^{j}\left(z_{tz+1}^{j}|\mathbb{I}_{t}^{j},z_{t}^{j}\right)\right]} dH\left(z_{t+1}^{i}, z_{t+1}^{j}|z_{t}^{i}, z_{t}^{j}\right)}{\text{Size}\left(\mathcal{I}\right)}\\
&-\frac{\sum_{i\notin\mathbb{I}_{t}^{j}} \int_{z_{t+1}^{i}\in\mathcal{Z}} \int_{z_{t+1}^{j}} 1_{\left[R_{I,t}^{i}\left(z_{t+1}^{i}|z_{t}^{i}\right)< R_{I,t}^{j}\left(z_{tz+1}^{j}|\mathbb{I}_{t}^{j},z_{t}^{j}\right)\right]} dH(z_{t+1}^{i}, z_{t+1}^{j}|z_{t}^{i}, z_{t}^{j})}{\text{Size}\left(\mathcal{I}\right)}
\end{aligned}$$

1_B is an indicator function defined by a generic condition B.

The condition $R_{I,t}^{i}\left(z_{t+1}^{i}|z_{t}^{i}\right) \geq R_{I,t}^{j}\left(z_{tz+1}^{j}|\mathbb{I}_{t}^{j}, z_{t}^{j}\right)$ can be rewritten as

$$\frac{P_{t+1}^{i} \cdot z_{t+1}^{i} F_{K}^{'}\left(z_{t+1}^{i} K_{t+1}^{i}, H_{t+1}^{i*}\right)}{P_t} > 1 + \frac{\Pi_{t+1}^{j}}{A_{t}^{j} Q_{t}^{j}}$$

$$\left[\frac{P_{t+1}^{i} \cdot z_{t+1}^{i} F_{K}^{'}\left(z_{t+1}^{i} K_{t+1}^{i}, H_{t+1}^{i*}\right)}{P_t} - 1\right] A_{t}^{j} Q_{t}^{j} > P_{t+1}^{j} F\left(z_{t+1}^{j} K_{t+1}^{j}, H_{t+1}^{j*}\right) - W_t H_{t+1}^{j*}.$$

H_t^{j*} is the optimal choice of hiring workers by the entrepreneur j, given the technology z_t^j, capital stock K_t^j, and W_t which is exogenous market wage. Given the process of z_{t+1}^j and associated Π_{t+1}^j, the probability of participation trend reversal, Prob $\left\{n_{t+1}^{i} < n_{t}^{i}\right\}$, is increasing with A and n, two dimension determining the liquidity value of asset j.

$$\frac{\partial\text{Prob}\left\{n_{t+1}^{i} < n_{t}^{i}\right\}}{\partial A_{t}^{j}} > 0,$$
$$\frac{\partial\text{Prob}\left\{n_{t+1}^{i} < n_{t}^{i}\right\}}{\partial n_{t}^{j}} > 0.$$

15.4 Appendix D

1. Nominal Rigidities an The Outcomes of Financial Disruption

The decision problem of the worker to maximize the the sum of discounted utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[U\left(C_t^l\right) - V\left(H_t^i\right)\right]$$

subject the flow budget constraint in normal time

$$P_t C_t^l + S_t \leq W_t H_t^i + R_{t-1} S_{t-1};$$

and subject the flow budget constraint in recession time

$$\begin{aligned}\mu_s^l :& P_T C_s^l \leq M_s \\ \lambda_s^l :& P_T C_s^l + M_s \leq W_T H_s^i + M_T,\end{aligned}$$

where T represents the starting date of crisis and $s(> T)$ the periods after crises.

In normal time, the Lagrangian

$$\mathcal{L}_{normal} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[U\left(C_t^l\right) - V\left(H_t^i\right) \right] + \lambda_t^l \left[W_t H_t^i + R_{t-1} S_{t-1} - P_t C_t^l - S_t \right];$$

F.O.C.

$$\begin{aligned}\frac{\partial \mathcal{L}_{normal}}{\partial C_t^l} &= \beta^t U'\left(C_t^l\right) - \lambda_t^l P_t = 0 \\ \frac{\partial \mathcal{L}_{normal}}{\partial H_t^i} &= -\beta^i V'\left(H_t^i\right) + \lambda_t W_t = 0 \\ \frac{\partial \mathcal{L}_{normal}}{\partial S_t} &= \lambda_t^l(-1) + \lambda_{t+1}^l(R_t) = 0;\end{aligned}$$

After rearranging, we obtain

$$\begin{aligned}\frac{U'\left(C_t^l\right)}{V'\left(N_t^i\right)} &= \frac{P_t}{W_t} \\ U'\left(C_t^l\right) &= \beta R_t U'\left(C_{t+1}^l\right).\end{aligned}$$

In recession time, the Lagrangian

$$\begin{aligned}\mathcal{L}_{recession} =& \mathbb{E}_0 \sum_{s=T}^{\infty} \beta^s \left[U\left(C_s^l\right) - V\left(H_s^i\right) \right] + \mu_s^l \left[M_s - P_T C_s^l \right] \\ &+ \lambda_s^l \left[W_T H_s^i + M_T - P_T C_s^l - M_s \right].\end{aligned}$$

F.O.C.

$$\begin{aligned}\frac{\partial \mathcal{L}_{recession}}{\partial C_s^l} &= \beta^s U'\left(C_s^l\right) - \mu_s^l P_T - \lambda_s^l P_T = 0\\ \frac{\partial \mathcal{L}_{recession}}{\partial H_s^i} &= -\beta^s V\left(H_s^i\right) + \lambda_s^l W_T = 0\\ \frac{\partial \mathcal{L}_{recession}}{\partial M_T^l} &= \mu_T^l - \lambda_s^l = 0.\end{aligned}$$

2. The liquidity auction among entrepreneurs

After privately-created liquidity evaporated, liquidity becomes scarce. I use the bond auction mechanism to allocate bonds to entrepreneurs and show how liquidity shortage limits investment. The entrepreneur, who has better expected return from his own physical investment, will hold liquidity to facilitate his investment. The bonds are sold in auction and in the descending order of bid, the the last bid is the price for all government bonds ask.

In recession time, the only collateral is government bonds; in normal time privately-created securities can also be taken as collateral.

In recession times, with only government bonds as collateral and predetermined amount of bonds, B_t,

$$B_T = \sum B_T(i) = \sum P_t I_t(i), i \in \left\{ i \in \mathcal{I} : \frac{\lambda_{s+1}^i}{\lambda_T^i}\frac{P_T^i}{P_T} z_{s+1}^i F_K'\left(z_{s+1}^i K_{s+1}^i, H_{s+1}^{i*}\right) > R_s \right\}$$

$$[R_s]^{-1} = \min_{i\in\left\{i\in\mathcal{I}:B_T^i>0\right\}} \mathbb{E}_T \left\{ \frac{\lambda_{s+1}^i}{\lambda_T^i}\frac{P_T^i}{P_T} z_{s+1}^i F_K'\left(z_{s+1}^i K_{s+1}^i, H_{s+1}^{i*}\right)\right\}$$

and aggregate investment satisfies

$$\sum I_s(i) = \frac{B_T}{P_T},$$

in the case that there is no policy intervention to change B_T,

In normal times,

$$\underbrace{\sum_i B_t(i)}_{=B_t} + \sum_j L\left(A_t^j, n_t^j\right) Q_t^j A_t(j) = \sum_i P_t I_t(i),$$

$$i \in \left\{ i \in \mathcal{I} : \frac{\lambda_{t+1}^i}{\lambda_t^i}\frac{P_t^i}{P_t} z_{t+1}^i F_K'\left(z_{t+1}^i K_{t+1}^i, H_{t+1}^{i*}\right) > R_t \right\}$$

$$[R_t]^{-1} = \min_{i \in \left\{ i \in \mathcal{I}: B_t^i > 0 \text{ or } a_t(i,j) > 0 \right\}} \mathbb{E}_T \left\{ \frac{\lambda_{t+1}^i}{\lambda_t^i} \frac{P_t^i}{P_t} z_{t+1}^i F_K' \left(z_{t+1}^i K_{t+1}^i, H_{t+1}^{i*} \right) \right\}.$$

with the trade-off between government bonds and privately-created securities,

$$\mathbb{E}_t \left\{ \frac{U'\left(C_{t+1}(i)\right)}{P_{t+1}} \left[1 + \rho_{t+1} - R_t \right] \right\} = \mathbb{E}_t \left\{ \varphi_{t+1}^i \frac{U'\left(C_{t+1}(i)\right)}{P_{t+1}} \left[\frac{L\left(A_t^j, n_t^j\right) Q_{t+1}^j}{Q_t^j} - R_t \right] \right\}.$$

In the normal times, the aggregate investment equals

$$\sum_i I_t(i) = \frac{B_t + \sum_j L\left(A_t^j, n_t^j\right) Q_t^j A_t(j)}{P_t}.$$

After the surprising the crisis, holding the government bonds unchanged B_T, we see investment shrinking,

$$\sum I_s(i) = \frac{B_T}{P_T} < \frac{B_t + \sum_j L\left(A_t^j, n_t^j\right) Q_t^j A_t(j)}{P_t} = \sum_i I_t(i).$$

Recession bursts at date T. Given the recession happen so rapidly and price stickiness, in the periods following T the economy inherits the nominal prices (P_T) and policy variable (B_T).

15.5 Appendix E

1. Credit Markets

We now study the capital investment financed by borrowing to establish a relationship between liability prices and default rates. Following the setup in Bernanke, Gertler, and Gilchrist (1998), we build the value of loans for the financial intermediary (the lender). The ex post gross return on capital for firm j is $z^j F\left(K^j\right)$,when z^j is an idiosyncratic shock to the representative firm j. The random variable z^j is i.i.d. across time with a continuous and once-differentiable c.d.f, $H\left(z^j\right)$, over a non-negative support.

The entrepreneur, j, needs to borrow an mount of B_{t+1}^j to make investment or purchase capital prior to the realization of the idiosyncratic shock. Given K_{t+1}^j and B_{t+1}^j, the optimal contract may be characterized by a gross non-default loan rate, Z_{t+1}^j, and a threshold value of the idiosyncratic shock z^j, called $\bar{z}^j$,such that for values of the idiosyncratic shock greater than or equal to $\bar{z}^j$, the entrepreneur is able to repay the loan at the contractual loan. That is, $\bar{z}^j$ defined by

$$F\left(\bar{z}_t^j K_{t+1}^j\right) = Z_{t+1}^j B_{t+1}^j. \quad (15.1)$$

When $z^j \geq \bar{z}^j$, under the optimal contract the entrepreneur repays the lender the promised amount $Z_{t+1}^j B_{t+1}^j$ and the difference, $F\left(z^j K_{t+1}^j\right) - Z_{t+1}^j B_{t+1}^j$. If $z^j < \bar{z}^j$, the entrepreneur cannot pay the contractual return and thus declares default. In this situation the lending intermediary pays the auditing cost and seize up the remaining capital, $(1-\mu) z^j F\left(K_{t+1}^j\right)$.

The values of $\bar{z}_t{}^j$ and Z_{t+1}^j under the optimal contract are determined by the requirement that the financial intermediary receives an expected return equal to the opportunity cost of its funds. The return that the financial intermediary pays to raise the fund is denoted R_t^f. Accordingly, the loan contract satisfy

$$\left[1 - H\left(\bar{z}_t^j\right)\right] Z_{t+1}^j B_{t+1}^j + (1-\mu) \int_0^{\bar{z}^j} z_t^j F\left(K_{t+1}^j\right) dH(z) = R_{t+1} B_{t+1}^j. \quad (15.2)$$

Combining (15.1) and (15.2) yields the following expression for $\bar{z}^j$:

$$\left\{\left[1 - H\left(\bar{z}_t^j\right)\right] \bar{z}_t^j + (1-\mu) \int_0^{\bar{z}^j} z_t^j dH(z)\right\} F\left(K_{t+1}^j\right) = R_{t+1}^j B_{t+1}^j.$$

$$\left\{\left[1 - H\left(\bar{z}_t^j\right)\right] \bar{z}_t^j + (1-\mu) \int_0^{\bar{z}^j} z_t^j dH(z)\right\} F\left(K_{t+1}^j\right) \cdot Q_{t+1}^j = B_{t+1}^j$$

$$\text{with } Q_{t+1}^j \equiv \left[R_{t+1}^j\right]^{-1}.$$

Now we able to express the lender's expend return as a function of the cutoff value of the firm's idiosyncratic productive shock, $\bar{z}^j$. There are two effects of changing $\bar{z}^j$ on the expected return, and they work in opposite directions. A rise in $\bar{z}^j$ increases the non-default payoff; on the other hand, it also raises the default probability, which lowers the expected payoff. There exists an unique interior value of $\bar{z}^j$: As $\bar{z}^j$ rises above the expected return declines due to the increased likelihood of default. When the value of $\bar{z}^j$ is below the maximum, the function is increasing and concave. Following closely Bernanke, Gertler, and Gilchrist (1998), we consider only equilibria in which the equilibrium value of $\bar{z}^j$ always lies below the maximum feasible value.

2. Banknotes and Bond markets

In case of retail banking, Freeman (1983) presents a model of banks as clearinghouse of private debts where money is used as the means of payment and banknotes are private-created liquidity.

In an economy where banks can issue banknotes, which give the holders the claim on its deposits in the bank $j \in \mathcal{J}$ and the choice of trading banknotes with note holders in the same bank. In this economy, a bank is not only a deposit institution, also a clearing houses within its customers. For an generic bank j, denote the total deposits as A_t^j at date t and the number of customers as n_t^j. Denote the banknotes issued by bank j as b_t^j, paying interest r_t^j. The holders of banknotes b^j can pay and accept payment from other customers of bank j with the banknotes. In this economy the bank serve as clearing house among its customers. As the above, we interpret $L\left(A_t^j, n_t^j\right) \in (0,1)$ the chance that the holder can make transaction, but does not need to redeem the banknotes issued by bank j or the circulation scope of banknotes j. $L\left(A_t^j, n_t^j\right)$of bond B_t^j is interpreted in the same way as resaleability or acceptance in equity markets. $b_t(i,j)$ denotes the amount of bond j held by agent i. We assume there is a cost of money-carrying outside of or across banks.

The collateral and money in advance constraints, which is analogous to (10.4), are

$$\nu_t^i : P_t I_t(i) \leq \sum_{j \in J} L\left(A_{t-1}^j, n_{t-1}^j\right) \cdot R_{t-1}^j b_{t-1}(i,j) + M_{t-1}^i.$$

The flow budget constraint of the entrepreneurs holding banknotes written as

$$\begin{array}{rc} & M_t(i) + \sum_{j \in \mathcal{J}} b_t(i,j) + P_t C_t(i) \\ & \leq M_{t-1}(i) + \left[1 - L\left(A_{t-1}^j, n_{t-1}^j\right)\right] R_{t-1}^j b_{t-1}(i,j) \\ \lambda_t^i : & +\Pi_t^i + \sum_{j \in \mathcal{J}} L\left(A_{t-1}^j, n_{t-1}^j\right) R_{t-1}^j b_{t-1}(i,j) - P_t I_t(i) \\ & \text{with} \quad \Pi_t^i = P_t^i F\left(z_t^i K_t^i, H_t^{i*}\right) - W_t H_t^{i*} \\ & K_t^i = (1-\delta) K_{t-1}^i + I_{t-1}(i) \end{array}$$

After optimize the utility objective (10.1), the valuation of banknote j, R_t^j, is derived as

$$\left[R_t^j\right]^{-1} = \mathbb{E}_t \left\{ \frac{\lambda_{t+1}^i + L\left(A_t^j, n_t^j\right) \nu_{t+1}^i}{\lambda_t^i} \right\} \geq \mathbb{E}_t \left\{ \frac{\lambda_{t+1}^i}{\lambda_t^i} \right\},$$

which indicates that deposits in banks and holding banknotes as liquidity substitutes make depositors satisfactory with lower interest rates. After substituting out ν_{t+1}^i, we obtain

$$\left[R_t^j\right]^{-1} = \mathbb{E}_t \left\{ \frac{\lambda_{t+1}^i}{\lambda_t^i} \left[1 + L\left(A_t^j, n_t^j\right) \varphi_{t+1}^i\right] \right\} \tag{15.3}$$

$$\text{with } \varphi_{t+1}^i \equiv \frac{\lambda_{t+2}^i}{\lambda_{t+1}^i} \frac{P_{t+2}^i}{P_{t+1}} \cdot z_{t+2}^i F_K^{\prime}\left(z_{t+2}^i K_{t+2}^i, H_{t+2}^{i*}\right) - 1.$$

We see straightforwardly that the valuation of banknotes (15.3), issued by a bank, has the analogous form with the valuation of Lucas Trees (11.7), traded in the financial market. The distinction lies in the institutional determination of liquidity. In the financial system consisting of banks, the liquidity property of banknotes issued by a bank is determined by how much deposits and how many customers the bank has; in the financial system with an advanced security market, the liquidity property of one security is determined by its market value and its number of participants. From view of transactions, after taking the banknote as one security issued by the bank and the use of banknotes as transaction with customers of the same bank, the two liquidity problems, in banking system and in advance security market, has the same structure, mechanism, and the same implications as what we derive using Lucas tree assets. The bank with increasing customers and deposits enjoys lower interest rates and make credit expansion. Bank runs happen as depositors have good investment opportunities, preference shocks, or other banks offering higher interest rates. The key point, as we show above with Lucas Trees, is that the probability of runs inflates as more liquidity are created by banks.

Part III

Chapter3: Anti-Corruption and Political Sustainability in China

Abstract: What is happening underneath China's anti-corruption campaigns and how does a political regime secure itself from revolutions? This paper proposes a model studying the dynamic conflicts between two types of politicians, *Good* and *Bad*, in a non-elective regime facing a threat of revolution, a Chinese feature of political regimes. With corruption as the extra revenue extraction from citizens, citizens may overthrow the regime and replace all incumbent politicians (*Good* and *Bad*) through a revolution. Anti-corruption campaigns reduce the total political extraction and secure the regime. Corrupt or *Bad* politicians would attempt to stop the anti-corruption campaign from inside. Another type of politician, the *Good* politician, needs to balance the inside and outside risks when he decides the intensity of the anti-corruption campaign. My results show that an anti-corruption campaign not only obtains support from the *Good* politician, but also partially from the *Bad* politician. In e internal conflicts will not happen in cases with very high or low probabilities of revolution. However, circumstances in the middle range of the percentage of *Bad* politicians has a relatively higher risk of internal conflicts. Regarding one more specific characteristic of Chinese economy, a large and valuable sector of *SOEs* (State-Owned Enterprises) actually makes the regime weaker from inside, which is counter intuitive. I also extend my model to incorporate productivity and myopic shocks in the dynamic environment. Negative productivity shocks cause more intensive anti-corruption and myopic behaviors leads to more toleration and increases of corruption.

Keywords: Anti-Corruption, Conflicts, Political Monopoly and Sustainability in China, Repeated Game.

16 Introduction

Following the Reform and Open in the early 1980s, China has been a successful example of growth and development. China's political institutions, however, remain a puzzle for outsiders. There is a prevailing concern about their sustainability, not only from academics, but also from the political incumbents of China.

How could China keep its party-dominating system relatively stable for so long, when modernization theories, like Acemoglu et al. (2008), predict that economic growth, improved education, and the growth of the middle class would lead to a higher demand for democracy? How can one explain the extremely intensive anti-corruption campaign since the inauguration of Xi Jinping in 2013, when there seems to be no immediate threats to the government and party?

One perspective to understand this is through the political succession, which is at the heart of political decisions (Mesquita and Smith (2015)). Within the ruling elites, there are political conflicts, which undermine the stability of the political regime. These internal conflicts are not only motivated by the desire for power, as studied in Acemoglu, Egorov, and Sonin (2008), but also by another important reason: outside legitimacy.

Corruption has been the focus of extensive literature in political science and economics. Explanations that stress the importance of political accountability explain mostly the variations in the level of "grand" or political corruption (Hollyer and Wantchekon (2011)); others, which emphasize state capacity, focus on the behavior of low-level bureaucrats (Shleifer and Vishny (1993)). I define corruption as extractions from civilians, ignoring the difference between petty corruption and grand corruption, and focus on the distribution between civilians and politicians.

Corruption is typically depicted as the result of two factors: lack of political accountability or insufficient state capacity. The first factor is the most common concern with political monopoly and centralization for several reasons. For example, the inability of the elite to combat corruption arises from one underlying cause: under an authoritarian system, the ruling elite face few constraints in the pursuit of their own self-interest, because they face little or no threat of electoral sanction. In political regimes like China, however, elections are not an effective way to replace politicians. The other two ways are *revolutions* by the rest of society and *the exclusions of some politicians* from the coalition by other politicians, who care more about the sustainability of the coalition. Accountability can be achieved as the result of threats of revolution or internal conflicts [Li and Gilli (2014)].

Corruption in China is endemic and can threaten China's stability directly [Pei (2007)]. The Carnegie Endowment estimates that the cost of corruption in China in

2003 was 3% of the GDP. By 2013, this number had increased to 13% of the GDP[11]. Since Xi Jinping's inauguration, an anti-corruption campaign has started and vowed to crack down on both "Tigers" and "Flies," which metaphorically refers to the corrupt powerful leaders and corrupt low-level bureaucrats respectively. The risk of political conflicts and contests are within the party, and political transitions from inside become significant as a result of the breakdown of the party.

In the political economy, the standard assumption is that the allocation of resources is entrusted to a representative politician (ruler). This assumption captures the notion that society needs to concentrate the monopoly of violence and the power of taxation to a single body. This assumption also simplifies the analysis of interactions between the representative politician and his citizens. In their models, with self-interested politicians, this fundamental dilemma is partly resolved by the control of politicians via elections (Barro (1973)). But in the political monopoly of China, the curbing of myopic and self-interested politicians is through political disciplines within the political system and the threats of revolution from citizens. To sustain this political monopoly, the political decision is a complex mixture of conflict and survival games.

In this study, I abstract from ideological preferences and policy platforms between different politicians and focus on the distribution of incomes between citizens and politicians and the induced conflicts within the political coalition. In this paper, a political system starts with a benevolent political leader in power, called a *Good* politician, who is concerned with the survival of the regime and tries to curb the predatory behaviors of other politicians. Another type of politician is a corrupt or *Bad* politician, who extracts more income from citizens than the *Good* politician, and exists within the same political coalition. The number of *Bad* politicians determines the aggregate extraction of the whole politician system and plays a critical role in the material distribution between citizens and politicians. The distribution of incomes decides the legitimacy and survival rates of the political regime, which are expressed by a probability of revolutions. The conflicts between politicians and citizens are called external conflicts.

At the same time, unaligned interests and motivations within heterogeneous politicians lead to internal conflicts, in which *Good* politicians start an anti-corruption campaign and *Bad* politicians can contest power. The question that I attempt to address is how a good political leader curbs internal conflicts and protects the regime from revolutions. From a theoretical perspective, this is a conflict game with an external threat hanging over both agents.

In my model, the trade-off of the benevolent political leader between two risks, external revolutions and internal conflicts, determines the intensity of anti-corruption efforts. While the trade-off of *Bad* politicians, between benefits from higher regime

[11] See http://carnegieendowment.org/files/pb55_pei_china_corruption_final.pdf

popularity and losses from the anti-corruption campaign, shapes their decisions of whether or not to contest power. The severity of corruption and relative power among politicians, both originating from the fraction of *Bad* politicians, is the ultimate factor in shaping the preferences and actions of agents.

The first result, that *Bad* politicians are willing to support anti-corruption to a mild degree, is counterintuitive. It is straightforward to understand when their decision includes the possibilities of overthrowing the regime, after which all politicians, including the *Bad*, would have zero payoffs. But for *Bad* politicians, the acceptable point of the anti-corruption campaign is lower than *Good* politicians. These unaligned interests forge the main motivations for internal conflicts over power.

The percentage of *Bad* politicians in the regime decides not only the regime popularity, but also the relative power in possible conflicts. If the fraction of *Bad* politicians is very high, close to one, and relatively low, close to zero, internal conflicts do not happen and the *Bad* politicians suffer the political attack silently. However, the same outcomes have different reasons behind them. In the former case, the reason is the low probability of winning a conflict; in the second case, the reason is the willingness to support anti-corruption, aiming to secure the regime and to protect themselves from revolutions. Middle-range corruption is a highly risky case, in which internal conflicts happen because of unaligned interests and an ambiguous chance of winning during conflicts for both types of politicians.

Every internal conflict can likely change the path of the regime transition. If the *Bad* politician wins, anti-corruption efforts will be abolished and the regime popularity will be undermined; authority held by corrupt politicians leads more corruption and will eventually lead to a regime collapse, but if an anti-corruption campaign is successful, political legitimacy and popularity will be regained.

One more distinguishing feature of China's economy is the large and privileged sector of *State-Owned Enterprises* (*SOEs*). Political corruption is also serious in *SOEs*. However, in circumstances with high risks of revolution, that is to say, low regime survival rates, the existence of *SOEs* does not improve the regime's safety. This is because when a revolution is going to happen and the regime has the least legitimacy, two types of politicians can form the coordination equilibrium of unanimously breaking the regime, instead of fighting to protect the regime.

The first contribution of this paper is to provide a benchmark model presenting conflicts in a political monopoly and to evaluate the regime transition. This process is in China because of the lack of political competition. In this paper, I rationalize the political phenomena in China, which are very different from the mature democracies in Europe and the United States.

One more novelty of this paper, from the view of game theory, is the mortality game or survival game. The payoff is not decided by the consumption of goods or wealth, but rather by the chance to survive longer in a repeated game. In my setup, the regime insecurity roots in the inside, the fraction of one type of politician, and his or her corresponding extraction. In this game, one type of player benefits himself and causes negative externality to all the players through increased probability, with which the game may be over and everyone has zero payoffs.

The paper is organized as follows. Section 2 offers background information on China's politics and the recent anti-corruption campaign. Section 3 discusses the relevant literature on the political economy. Section 4 presents the model, the two-period model, and the model with infinite-horizon. In Sections 5 and 6, I solve the models and discuss the results. Section 7 concludes and discusses potential improvements.

17 Chinese Politics and China's Anti-Corruption Campaign

This section presents the Chinese dynastic cycle, the anti-corruption efforts since 1978's Reform and Open, and the anti-corruption efforts since 2013, aiming to familiarize readers with the background and to emphasize that internal conflicts are the substantial forces shaping the political system of China.

17.1 Dynastic Cycles for 2000 years

The dynastic cycle is an alternation between anarchy and despotism, where anarchy is a condition of society without rulers or cooperation and despotism is a society in which rulers tax farmers according to Usher (1989). The replacements of dynasties in Chinese history have generally followed this cyclical pattern since China emerged as a nation and was unified by the first dynasty, Qin Dynasty.

According to this theory, each dynasty rises to a political, cultural, and economic peak and then declines and lose its legitimacy, finally to be replaced by a new dynasty. The cycle then repeats under a surface pattern of repetitive motifs. The cycle appears as follows:

1. A new ruler unites China, founds a new dynasty, and gains emperor power;
2. China, under the new dynasty, achieves prosperity;
3. The population increases;
4. Corruption becomes rampant in the imperial court and the empire begins to enter decline and instability;

5. A natural disaster wipes out farm land (disasters usually cause famines);
6. The famine causes people to rebel and a civil war ensues;
7. The ruler loses legitimacy;
8. The population decreases or stagnates because of violence;
9. China goes through a warring states period;
10. One state emerges victorious;
11. The state starts a new empire.

The cycle repeats itself, as in Figure **??**.

Figure 17.1: Dynastic Cycle

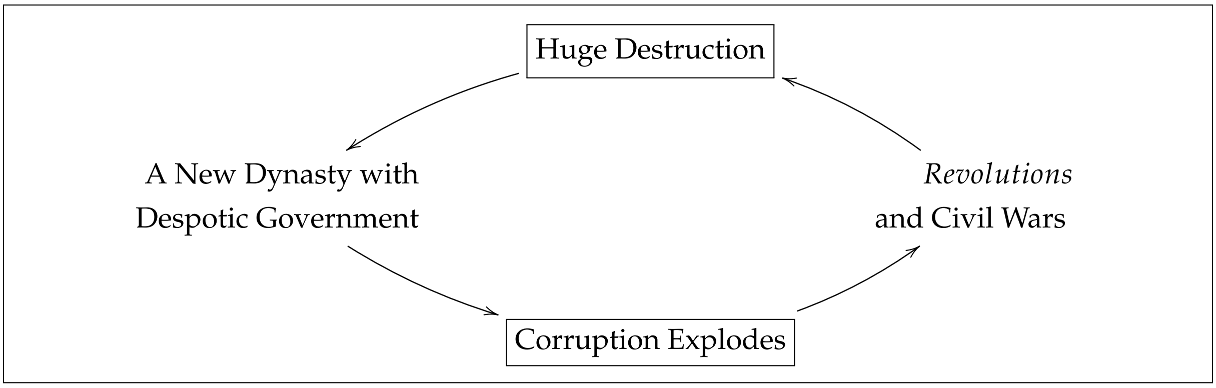
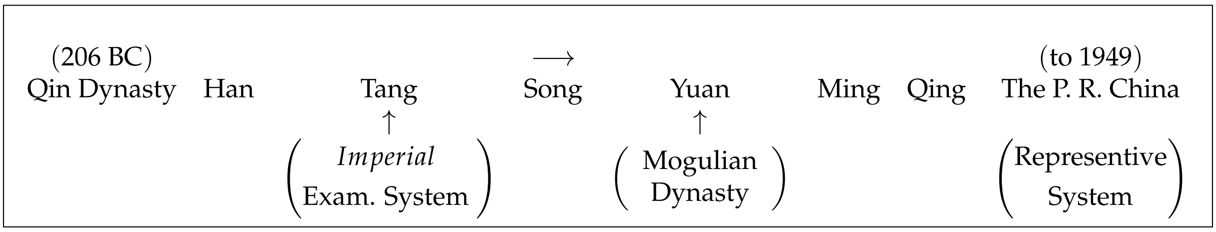


Figure 17.2: The List of Dynasty Replacements for 2000 years



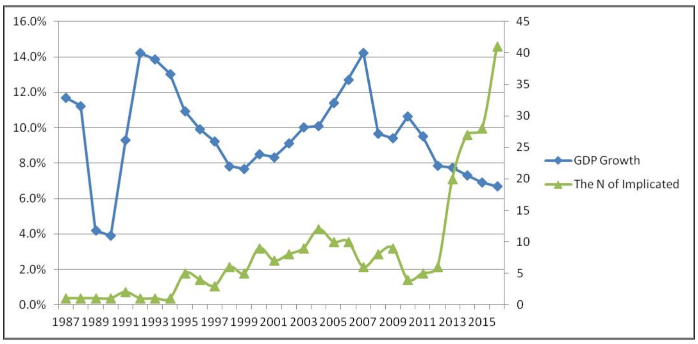
The dynastic cycle is like a business cycle, but with a much longer duration and representing a sequence of fundamental changes in the government of China. Figure **??** shows the dynasty replacements up to 1949. Every dynasty emerges as a winner in civil wars, is fatally damaged by the revolution of farmers/citizens, and collapses in civil wars. Across this cycle, the political institutions have changed very little, except for the Imperial Examination System started from the Tang dynasty[12]. Every dynasty that has unified China is a despotic state. With these historical facts, I emphasize that revolutions were repeated in Chinese history and formed the pressure for each dynasty. Why did a stationary Chinese dynasty/government not survive forever, and why was it finally replaced by a new dynasty? What are the implications of anti-corruption campaigns in Chinese history and the most recent events? These historical questions are the focus of the paper.

Usher (1989) sketches one explanation that population growth leads to a gradual fall in income per head, until eventually the surplus over bare subsistence is insufficient to provide for the ruling class. In my paper, I develop an endogenous explanation that corruption and explosive political extraction are the fundamental causes of dynastic cycles.

17.2 The Anti-Corruption Efforts Since 1980's Reform and Open

Since the Reform and Open in 1978, China has started to build market economy and modern society, and political corruption has grown rampantly. Corruption usually involves trading bribes for political favors, stealing public assets and government expenditures, and illegal benefits in *SOEs* privatization. The changes in the economic environment spur the increasing needs to curb political corruption and keep political legitimacy. I summarize the anti-corruption efforts and derive some of the patterns since 1987.

Figure 17.3: Anti-Corruption and GDP Growth since 1980s



The data is form Chinese government's website (http://www.ccdi.gov.cn/), The Central Commission for Discipline Inspection.

[12] Bai and Jia (2016) finds that after the abolition the regions that used to have higher quotas were associated with a higher probability of revolution, i.e., the Imperial Examination System helped to reduced revolution probabilities.

From Figure 17.3, we can roughly say that the anti-corruption efforts are negatively correlated with economic growth (-0.34); that is to say, when GDP growth rates are low, there are more anti-corruption efforts. We take it as a relationship between business cycles and political cycles.

17.3 Chinese Politics and Xi's Anti-Corruption Campaign

The People's Republic of China has an election system consisting of people's representatives, a standing committee of representatives, and annual meetings of all representatives to audit the government's work. However, China has one-party politics. The incumbency of this one party is protected by the national constitution. All the important positions in any level of government are monopolized by the Party members. The standing committees of the party's political bureau collectively take over the actual presidency.

China's political regime is one type of autocracy, if you classify regimes where the ruling party faces no plausible electoral challenge as autocracies. This definition is in keeping with Przeworski et al. (2000)'s measure of democracy, and is also used in Hollyer and Wantchekon (2011). But the literal definition cannot fit Chinese politicswell. Chinese politics is midway between democracies and autocracies: the political leaders are neither elected through competitive election, nor through violence or inheritance. The leader emerges through selection within the Party and then the election in the national parliament. I define China's political regime as a political monopoly, which has vast participation and ubiquitous organizations, occupies all political powers, and assumes all political responsibilities.

The political activities inside the Party, including internal conflicts or factions, are the key points to understand and analyze the political process in China. Within China's Communist Party (CCP), party politics are largely informal, and the faction plays an important role. A large amount of literature on Chinese politics studies factions and identifies them as the key to understanding political power. Huang (2000) identifies the characteristics of factional links in the CCP, emphasizing one important perspective that conflicts within the Party are indispensable in the theories of China's politics.

After Xi Jinping was inaugurated as General Secretary as the top leader of the Party in November 2012, the anti-corruption campaign started and has been unprecedentedly powerful since China's Reform and Open in the 1980s. Xi's vow to crack down on the "Tiger" (political corruption) and "Flies" (petty corruption) caught large international attention and created tremendous changes in China. Although anti-corruption campaigns have been working in the current political system of China for a long time, the distinguishing feature of this new campaign is its unprecedented intensity and scope, which has even caught one standing member of the Political Bureau. Figure

17.4 is an incomplete list of important political "Tigers."

Figure 17.4: Corrupt "Tigers"

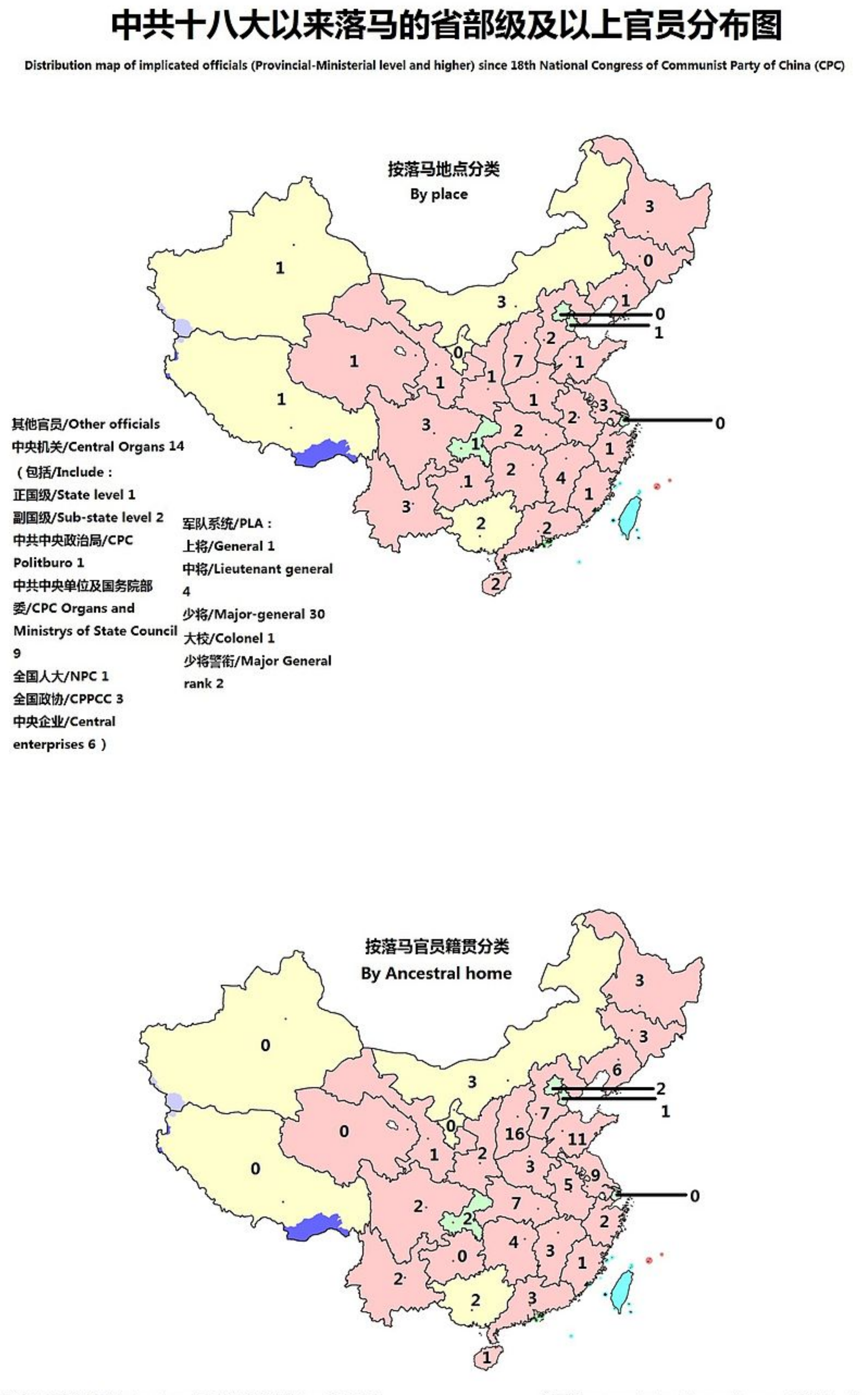
Name	Position	Sector
Zhou Yongkang	Politburo Standing Committee	Government
Bo Xilai	Deputy Party Sectary, Chongqing	Government
Ling Jihua	Head of the General Office of the Party	Government
Jiang Jiemin	Director of the SOEs Commission	Government and SOEs
Li Chuncheng	General Party Sectary, Sichuan	Government
Huang Xingguo	Deputy Party Sectary, Tianjin	Government
Liu Tienan	Vice Minister State Development and Reform Commision	Government
Li Liguo	Director of Ministry of Civil Affairs	Government
Wu Yongwen	Deputy Director Hubei People's Congress Standing Committee	Government
Huang Sheng	Vice Governor Shandong	Government
Ni Fake	Vice Governor Anhui	Government
Tian Xueren	Vice Governor Jilin	Government
Li Chenyun	Vice Governor Sichuan	Government
General Xi Caihou	Vice Chairman PLA Central Military Commision	Army
General Gu Junshan	Deputy Commander PLA General Logistic Department	Army
Ma Jian	Vice Director of Ministry of National Security	Army
General Yan Jinshan	Vice Commander of Chengdu Military District	Army
General Fan Changmi	Vice Commander of Lanzhou Military District	Army
Xu Mingjie	China Ocean Shipping (Group) Company	SOEs
Wang Tianpu	General Manager of Petro China	SOEs
Xu Jian	China First Automobile	SOEs
Sun Zhaoxue	General Manager of Aluminum Corporation of China	SOEs

This is an incomplete list up to 2016, with many corrupt officials on province and city level non included.

Figure 17.5 is a map from Wikipedia summarizing the implicated politicians on the Provincial-Ministerial and higher level. The top map shows the distribution of politicians by working places; the bottom one is the distribution by ancestral home, where he/she grew up.

Someone argues that this anti-corruption campaign is only an excuse for political fighting for the power of its faction and relatives. This argument is not true according to Lu and Lorentzen (2016), which has micro evidence.

Figure 17.5: Distribution Map of Corruption



This figure is from Wikipedia (https://en.wikipedia.org/wiki/Anti-corruption_campaign_under_Xi_Jinping)

18 Related Literature

This paper belongs to the growing literature on nondemocratic politics, which study regime transitions, ruling-coalition formations, and other political phenomena without effective democracy. Gehlback, Sonin, and Svolik (2015) made a systematic review on formal models of nondemocratic politics. Without electoral competitions, the performances of economies and societies are diverse. Without electoral disciplines, as highlighted by the results in Li and Gilli (2014), autocracies are far more likely than democracies to be either the best or the worst performers, in terms of growth and public goods provision. Here, I will discuss only the papers that are closely related to my study.

Many papers by Daron Acemoglu on politics are relevant to my study. Firstly, as a seminal work on conflicts and political transitions, Acemoglu and Robinson (2001) presents a conflict game between the elite and the poor over political regimes, democracy v.s. non-democracy, with political actions of revolutions and coups. In the political economy with self-interested politicians, Acemoglu, Golosov, and Tsyvinski (2007) shows that when politicians are as patient as the citizens, the additional distortions created by the political economy disappear in the long run and the allocation of resources converges to that of a dynamic Mirrlees economy, with an exogenous level of public goods spending. The heterogeneity of politicians in patience and aggregate extraction are key factors for a political regime's sustainability.

My study follows this thought stream of conflicts, but with heterogeneous politicians. The threat to survivorship of the ruling coalition provides incentives for politicians to inspect and exclude corrupt politicians and reduce aggregate extraction, which is the anti-corruption campaign. Facing inspection pressures, corrupt politicians have a motive to fight back and abolish inspection campaigns.

Another possible path to changing the political regime without violence is through political conditions within the elite group. In a comparison between oligarchy and democracy, Acemoglu (2008) characterizes the possible path through which oligarchy may "voluntarily" transform itself into democracy. Under certain conditions, low-skilled entrepreneurs are better off under democracy than remaining in the oligarchic societies. In subsection 20.3, I show that in the political context of China, voluntary transitions are also possible, because the value of *SOEs* offers a motivation for incumbent politicians to break the current political monopoly from inside.

The formation and stability of political coalitions is the key part in a nondemocratic regime. Acemoglu, Egorov, and Sonin (2008) and Acemoglu, Egorov, and Sonin (2012) present an analysis of the political coalition in the absence of strong institutions regulating political decision-making. In such societies, individuals compete for power and are unable to commit to the coalition they form. The coalition in their analysis is not only fragile because of the absence of commitment and competition for power, but also for political reasons. The political monopoly in China can be taken as a coalition, whose cohesion and stability are the research object of this paper.

On dynamic conflicts and repeated games, Polborn (2006) studies a model in which two groups compete with each other for a prize in every time period, assuming a status quo bias or, for example, an incumbent advantage: if there is a fight today, yesterday's winner is in a stronger position than the other group. Implications of his model include lobbying for legislation and political transitions through revolutions. On regime transition, Buchheim and Ulbricht (2016) develops a model that attributes a key role to beliefs held by political outsiders about the vulnerability of regimes, governing the likelihood and outcome of transitions and sharing a similar component with this paper; revolution pressure as the key force affecting regime transmission. Fearon (2011) studies the condition of how democracy can be *self-enforcing*, and points out a similar ingredient of credible threats of protest or rebellion by the citizens if the government performance is problematic. But my model differentiates from theirs in many ways, including the way of power investment and the reasons for regime collapses.

There is also a lot of literature on China's political economy. Regarding the newest anti-corruption campaign in China, there is doubt on the sincerity of the anti-corruption campaign, arguing that it is a cover for intra-elite struggles and a purge of opponents. As a timely paper on China's anti-corruption campaign, Lu and Lorentzen (2016) constructs a dataset of prefecture-level leaders to identify accused officials and also maps their connections and concludes that the crackdown is primarily a sincere effort to cut down on the widespread corruption. Their work find that even personal ties with top leaders have provided little protection. In contrast to the empirical work by Lu and Lorentzen (2016), the present paper builds the theory of political sustainability and discusses the implications of anti-corruption campaigns.

The sector of *SOEs* plays an important economic and political role in China. When examining China's economic growth, Song, Storesletten, and Zilibotti (2011) uses the different credit accesses of *SOEs* and private enterprises to explain China's economic growth. *SOEs* enjoy preferential status in factor and product markets. Song, Storesletten, and Zilibotti (2011)'s results show that the reduction of misallocation between the two sectors explains China's fast growth. The newest politico-economic research on China, Wang (2016), builds the political models of China's hybrid economy under democracy and oligarchy with the *SOEs*. In his model, *SOEs* are the sector to provide political supports to oligarchy. My paper examines the mechanism of political sustainability and the role of anti-corruption campaigns, through which the political system can control political extractions and secure political stability.

19 The Model

19.1 Environment

This political economy consists of citizens and two types of politicians. Citizens produce $y = A \cdot l$, where A denotes the productivity level, and l is the inelastic labor supply. The identical production of citizens is denoted by y. Politicians' income comes only from citizens as extraction or rents. In the benchmark model, I assume away the productivity shock and make the output constant over time to focus on redistribution issues among the three agents: Citizens, *Good* politicians, and *Bad* politicians. In Section 6, I extend the baseline model to include productivity shocks and analyze the effects of business cycles on political officialdom.

In this paper, politicians are distinct from citizens and never engage in goods production. There are two types of identical politicians, simply named *Bad* politicians, indexed by subscript b, and *Good* politicians, indexed by g. They differ in extraction levels or political rents, that is, the *Bad* politician extracts more and the *Good* politician less from the rest of society (Workers and Entrepreneurs), $x_b(> x_g)$. The extraction levels of the two types of politicians are exogenously determined. But this simplification does not miss the essence of corruption that corrupt politicians extract more than non-corrupt politicians.

The discounting factor of politicians is indexed $\delta_i \in (0,1), i \in \{b, g\}$. The rational type choice leads the equalization of the present values of future income for the two politician types:

$$\bar{w} = \frac{x_b}{1-\delta_b} = \frac{x_g}{1-\delta_g}$$

which implies $\delta_b < \delta_g$ and *Bad* politicians are more impatient than the *Good* politicians.

Suppose that the total mass of politicians is constant across time and equal to $N \in R_+$. Denote the number of *Bad* politicians at time t as $N_t^b \in [0, N]$ and the fraction of *Bad* politicians at time t as $n_t \in [0,1]$. The number of good politicians is $N - N_t^b$. The aggregate extraction by politicians at time t is $X_t \equiv N_t^b x_b + \left[(N - N_t^b) x_g\right]$. Define x_t as the average rents of politicians. Then, x_t is written as

$$x_t \equiv \frac{X_t}{N} = \frac{N_t^b}{N} x_b + \frac{N - N_t^b}{N} x_g = n_t x_b + (1 - n_t)\, x_g.$$

The aggregate X_t is always smaller than the aggregate production, Y; the average rents, x_t, is smaller than the average production, y, that is, $X < Y$ and $x < y$. The focus is on the distribution among the agents, so from now on the analysis will use the variables on an average level. For expository convenience, define $\Delta x_{bg} \equiv x_b - x_g$ and hence x_t can be written as $x_g + n_t \Delta x_{bg}$. The average political extraction can be interpreted as the standard rent plus the extra rents of *Bad* politicians.

Discussion of the Setup. For simplicity and tractability, I assume the two types of politicians with different levels of extraction. In reality, the rents or incomes and consumption of politicians disperse and depend on many factors, like powers, positions, talents, and institutions. Each politician can be taken as an identical individual adjusted by power. Therefore, this dichotomy captures the essence of corruption, indicating that some politicians extract an inappropriately higher level of rent from citizens than others. With these two constant levels of extractions across individuals, one being sustainable and the other being non-sustainable, one can immediately see that only the proportion of corrupt politicians matters for the aggregate rents and the long-term sustainability of the political regime. Since the focus of this paper is on how the two types of politicians within one political monopoly coalition interact, rather than why corrupt politicians exist, I assume the existence of corruption and its growth without investigating the micro-foundation of how corruption emerges.

The following assumptions are also made to simplify the analysis and keep the essence:

Assumption 1. *In the absence of any anti-corruption efforts, the growth of Bad politicians is positive.*

After a politician takes more rents than allowed, he has no way to recover his or her clean status, that is, there is no forgiveness mechanism that allows their excess extraction to be accepted or legitimated by the authority of *Good* politicians. Even if a *Bad* politician has not yet been caught, it does not mean that this *Bad* politician is safe from future probes, and the anti-corruption campaign is hanging over their heads. This assumption is generally and intuitively plausible. One immediate implication of this assumption is that the number of *Bad* politicians is non-decreasing without anti-corruption campaigns.

One of the simplest specifications of the law of motion is constant growth rate without the anti-corruption campaign,

$$n_{t+1} = n_t + \underbrace{\gamma \cdot (1 - n_t)}_{\text{Increment of corruption}},$$

wwhere $\gamma \in (0, 1)$ is a constant growth rate. The increment of *Bad* politicians is proportional to the current number of *Good* politicians. The implication is that every period some *Good* politicians convert to *Bad* for higher rents. It is easy to modify this specification and include other behavioral growth rates, like matching and contagion.

One can think of the parameter γ as the speed of contagion: the temptation for a good politician to change his type when he meets a corrupt colleague taking higher political rents, and the willingness to change his type and take more extractions, which can be bribing civilians directly or stealing public assets and expenditures. From the formula above, it is obvious that the proportion of *Bad* politicians will grow until the political regime is revolted, if there are no interventions from the other type of politicians or from a political leader.

Assumption 2: *At the beginning of every period, the fractions of each type of politician are positive, i.e.* $0 < n_t < 1$.

This is to ensure that every period there are two types of politicians and a game between them. The plausibility is guaranteed by the assumption that anti-corruption is costly.

Assumption 3. *Perfect Observability.*

Despite the efforts of *Bad* politicians to hide, their number and fraction in the political coalition can be observed perfectly by others from the inside. This assumption may be subject to the criticism that *Bad* politicians are cunning, which is partially true to outside citizens (entrepreneurs and workers), but the information available to *Good* politicians, who are insiders and work with *Bad* politicians, is better and taken as complete. This assumption is plausible in the sense that inside information is perfectly spread within the coalition.

Assumption 4. *After the ending of a political monopoly, political competition is the automatic outcome.*

In the present paper, political competition for power is the automatic result if the current political monopoly is broken. The first reason for this assumption is that political competition by election or other ways is the common outcome. For example, political competition in democracy is self-enforcing with the conditions listed in Fearon (2011). The after-monopoly politics are not the focus of the present paper.

19.1.1 Conflicts Between Citizens and Politicians

Without effective elections to replace and discipline autocracies, citizens may choose revolution and replace all the incumbent politicians with new ones, who will be paid x_g, the political rents of *Good* politicians. The turmoil in revolutions will cause the destruction of production by a rate, ϕ, which is an exogenous variable and has a time invariant distribution on $[\underline{\phi}, \bar{\phi}]$, with its probability density function denoted as $g(\phi)$. The upper bound, $\bar{\phi}$, satisfies the condition that $\bar{\phi} y > x_b$, ensuring that when all politicians are corrupt, revolutions are desirable for citizens; the lower bound, $\underline{\phi}$, satisfies $x_g \leq \underline{\phi} y$ ensuring that a revolution is undesirable when all politicians are good. The simplest specification of ϕ's distribution is uniform. The production after revolutions is $(1 - \phi) y$.

All citizens have identical preferences and are taken as one representative citizen[13]. When the representative citizen revolts, he or she always succeeds, though he or she has to suffer the random destruction. The probability of revolution, p, depends on the utility loss during revolution and the rents that he or she pays to politicians. Denote $u(\cdot)$ is the utility function of citizens, and $u^{'}(\cdot) > 0$ and $u^{''}(\cdot) \leq 0$. If a revolution happens and the destruction of income is ϕy, the utility loss is $u\left(\phi y\right)$; if they do not revolt, they pay the rents x_t on average to politicians and suffer the utility loss $u\left(x_t\right)$.

Facing two sources of utility losses, the representative citizen solves the problem minimizing the losses of utility, so the probability of revolution at time t, denoted as $p(t)$, is

$$\begin{aligned} p(t) \equiv \mathrm{Pr}_t\left\{\text{Revolution}\right\} &= \mathrm{Pr}_t\left\{u(\phi y) \leq u\left(x_t\right)\right\} \\ &= \mathrm{Pr}_t\left\{\phi y \leq x_t\right\} \equiv \Phi\left(x_t\right). \end{aligned}$$

If revolutions do not happen, the regime survives through this period. Define the survival probability of the regime at time t as $\Gamma_t \in [0,1]$ and $\Gamma_t \equiv 1 - Pr\left(\text{Revolution}\right)$.

$$\begin{aligned} Pr\left\{\text{Survival}\right\} &= 1 - Pr_t\left\{\text{Revolution}\right\} = 1 - \Phi\left(x_t\right), \\ \text{with } x_t &= x_g\left(1-n_t\right) + x_g n_t. \end{aligned} \tag{19.1}$$

This result 19.1 is based on the presumption that the random destruction rate ϕ has a uniform distribution over $[\underline{\phi}, \bar{\phi}]$ with the default setup of $g(\phi)$.

Hence the regime survival depends on average extraction x_t and is determined further by the proportion of *Bad* politicians, n_t, given two fixed rent levels and the constant total number of politicians. Therefore, $\Gamma(n)$ is a decreasing function of n, and $\Gamma(0) = 1$ and $\Gamma(1) = 0$. A larger fraction of *Bad* politicians brings a larger probability of revolution and a smaller regime survival rate. The fraction of *Bad* politicians and the resulting surviving probability are central concepts in this modeling work, because they are the channels through which the two types of politicians can influence each other's strategies and payoffs.

The corner value of p (0 and 1) is not under analysis. Because, first, I assume $n > 0$ and there are always some corrupt politicians, and the political system is never purely good; second, the case of $p = 1$ (equivalently $\Gamma(n) = 0$) amounts to the fact that the revolution happens for certain and the regime will be overthrown, which starts another trend and is not in the scope of this paper. After ruling out these two extreme cases, the analysis will focus on the interior range, where $p \in (0,1)$ and $\Gamma\left(n\right) \in (0,1)$.

[13]There could also be a coordination problem in which all agents expect others not participate a revolution, so do not take part themselves. However, since taking part in a revolution imposes no costs irrespective of whether it succeeds or not, it is a weakly dominant strategy.

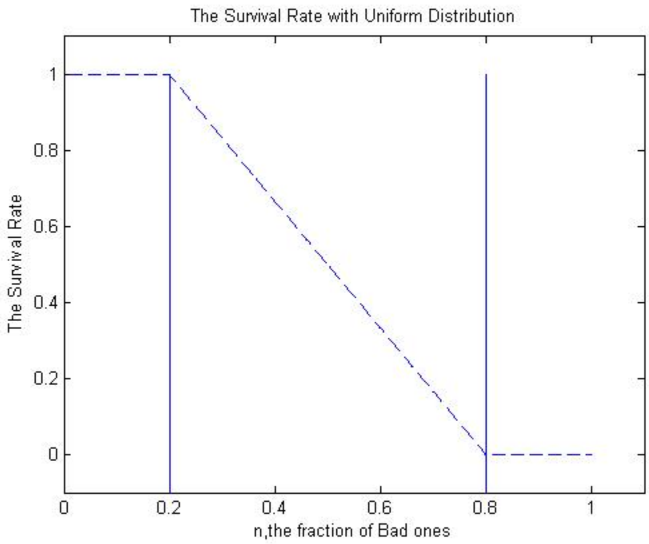
After solving the problem, we have a sustained probability of the political monopoly at time t,

$$\Gamma_1(n_t) = \frac{\bar{\phi}y - [x_g(1-n_t) + x_b n_t]}{\bar{\phi}y - \underline{\phi}y}.$$

This result builds on the condition that citizens have linear utility function and are risk neutral. Figure 19.1 is the graph of survival rates if the destruction parameter follows a uniform distribution. Take derivative of $\Gamma_1(n_t)$

$$\frac{\partial \Gamma_1(n_t)}{\partial n_t} = -\frac{x_b - x_g}{\bar{\phi}y - \underline{\phi}y}.$$

Figure 19.1: Regime Survival PDF with Uniform Distribution of ϕ



with parameterization: $\phi \sim U[0.2, 0.5]$, $x_g = 0.1$, $x_b = 0.6$, and $y = 1$.

In the sequels, if without special announcement on $\Gamma(\cdot)$, it is that $\Gamma(n) = \Gamma_1(n)$. The probability of the regime surviving from time s up to t is

$$\Pr\{\text{No Revolutions up to time} t | n_s\} = \Pi_{i=s}^{t} \Gamma(n_i).$$

19.2 Conflicts Between Politicians

Under revolution pressure for the whole political system, *Good* politicians can start an anti-corruption campaign aiming to exclude corrupt politicians and bureaucrats from the political coalition and monopoly. Nevertheless, another threat which may deter this political leader's action is that *Bad* politicians, as one part of the political coalition, may contest and abolish this campaign. The implicit assumption here is that the benevolent political leader does not have absolute power, as in an autocracy, may not be able to carry out his or her campaign, and even loses his or her power because of internal clashes and obstacles.

The ways to contest for power include resistance, rejecting the implementation of policies, and military coups, because policy implementation relies on other politicians and bureaucrats who have their own policy preferences. These resistances may be even stronger when policies involve anti-corruption and threaten their careers. Military coups can also change power. They propose new leaderships and a shift of power to a subset of their group. In my framework, I do not examine the details of how a subset of politicians contest for power. I classify all possible means by one action, *Contest*, to simplify the conflicts within a political regime. Denote the action of *Bad* politicians by $q \in \{0, 1\}$, with $q = 1$ meaning *Contest* and $q = 0$ not contest.

When *Good* politicians start an anti-corruption campaign and decide on an anti-corruption campaign with the intensity measured by $\alpha \in [0, 1]$, they also face a cost, presented by $c(\alpha)$. The parameter α can be interpreted as the percentage of *Bad* politicians that would be caught.

Assumption 4: *Fighting corruption is costly.*

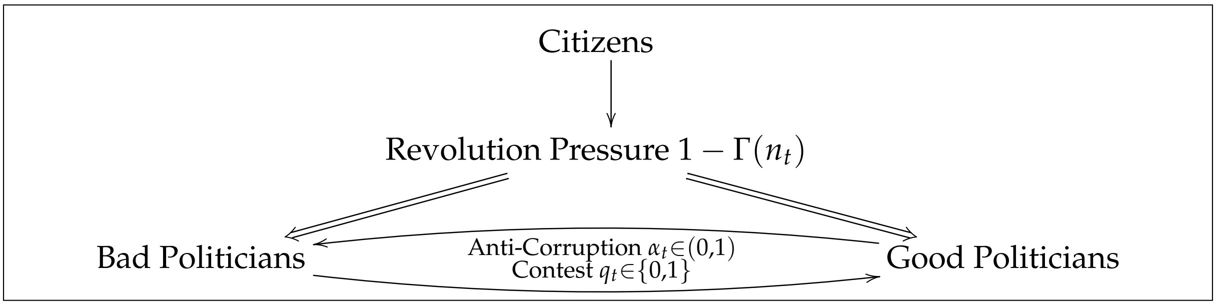
Anti-corruption campaigns are likely to be costly for the political coalition and monopoly. Resources are needed to deter and punish corrupt behaviors conducted by lower-level bureaucrats. The prosecution and replacement of corrupt bureaucrats also causes a loss of skill-specific human capital accumulated over the length of a political career. Finally, the prosecution of some politicians could reveal the inside of the political regime and potentially jeopardize the legitimacy and reputations of the political monopoly. I summarize these costs with one variable, $c(\alpha)$, which depends on the action intensity of the anti-corruption campaign and satisfies $c(0) = 0$ and $c(1) > x_g$.

The two conditions ensure that *Good* politicians will choose the interior value of $\alpha \in (0, 1)$. Because $c(0) = 0$, $\alpha = 0$ is not desirable; with the condition $c'(1) > x_g$, choosing $\alpha = 1$ and catching all corrupt politicians is not desirable. The actual decision of *Good* politicians belongs to $(0, 1)$. The cost function is weakly convex. $c(\alpha)$ is twice continuously differentiable and satisfying $c'(\alpha) > 0$ and $c''(\alpha) \geq 0$.

Figure 19.2 is describing the framework of this game with three agents: *the Representative Citizen*, *Good Politicians*, and *Bad Politician*s.

I simply assume linear cost in static in the two-period model for convenience and strictly convex cost in the infinite-period model. For the conflicts between politicians, I establish first a two-period model and then in the following subsection build a model of infinite-horizon.

Figure 19.2: The Stage Game Summary



Two-Period Model *Good* politicians decide the intensity of the anti-corruption campaign, $\alpha \in [0,1]$. The decision variable is the fraction of corrupt politicians that *Good* politicians want to catch and can also be interpreted as the probability of a *Bad* politician being caught. The decision of *Bad* politicians is whether or not start a contest; $q \in \{0,1\}$: $q = 1$ means that *Bad* politicians initiate a contest for power; $q = 0$, denotes that *Bad* politicians do not contest and accept all investigations silently. Decision variables with an asterisk , $\alpha^* \in (0,1)$ and $q^* \in \{0,1\}$, are the optimal choices of each agent.

The timing of the conflicts is summarized as follows:

1. First Period: There is a fraction of *Bad* politicians, $n_1 > 0$.

   (a) The fraction of corrupt politicians (n_1) is revealed, and also revealed is the probability of revolution, $\Gamma(n_1)$, and the relative power of two types of politicians, $\Omega(n_1)$;

   (b) After observing the proportion of *Bad* politicians, n_1, *Good* politicians decide the intensity of the investigation, α_t, and simultaneously *Bad* politicians choose to *Contest* for power or not, $q_1 \in \{0,1\}$;

   (c) Consumption takes place and the period ends.

2. Second Period: There is a fraction of *Bad* politicians, n_2, which depends on the decision of politicians, (α_1, q_1), and there is a new probability of revolution, $\Gamma(n_2)$.

   (a) If *Bad* politicians have contested power and won, the anti-corruption campaign failed, and α_1 will be decided by the winners; the fraction of *Bad* politicians at the second period, then $n_2 = n_1 + \gamma \cdot (1 - n_1)$.

   (b) If *Bad* politicians have contested and lost, they will be wiped out, then $n_2 = 0$;

   (c) If *Bad* politicians did not contest and accept all investigations, then $n_2 = (1-\alpha_1) n_1 + \gamma (1 - n_1)$.

In conflicts between politicians, the winning probability is defined in two ways: a proportion of the number of corrupt politicians, n_t, or it is determined by their relative material resources, which are given as follows,

$$\begin{aligned}&(1)\ \Omega^1(n_t) = \kappa n_t,\\ &(2)\ \Omega^2(n_t) = \kappa \frac{x_b n_t}{x_b n_t + x_g(1-n_t)},\end{aligned}$$

where $\kappa < 1$ captures the incumbent advantage of non-corrupt politicians, who are holding the leading position at the initial stage. I use $\Omega^1(n_t)$ as the benchmark. Without special announcement on $\Omega(\cdot)$, $\Omega(n_t) = \Omega^1(n_t)$. The probability of the regime being sustained, $\Gamma(n)$, is determined by the conflict between the representative politician and the representative citizen and has been specified in the previous section.

The second period is the last period. No actions happen in the second period. In the first period, *Good* politicians can start an anti-corruption campaign with intensity α_1 and *Bad* politicians can contest for power and fail this campaign with probability $\Omega(n)$. Losers, *Bad* politicians, in conflicts will have zero payoffs, if conflicts happen; if there are no conflicts, *Good* politicians pay the investigation costs generated by their campaign and *Bad* politicians suffer a risk rate, α, of being caught. All payoffs in the second period are discounted by the patience factor and the individual and regime survival rate.

In period 1, *Good* politicians need to pay the investigation cost and the second period income is discounted by the subjective discounting factor (δ) and the regime survival probability, $\Gamma(n_1|\alpha_1)$. If *Bad* politicians contest and win with a probability with $\Omega(n_1)$, the anti-corruption will be zero. *Good* politicians' income in period 1 would be discounted by δ and $\Gamma(n_2|\alpha_1 = 0)$.

For *Good* politicians, the decision objective is to maximize the sum of two-period's consumption with the discount factor δ and the risk of losing in an internal conflict,

$$\sup_{a_1 \in (0,1)} \left\{ \begin{array}{r} (1-q_1)\left[\left(x_g - c\cdot\alpha_1\right) + \delta_g \Gamma(n_2|\alpha_1)\, x_g\right] \\ +q_t^*(1-\Omega(n_1))\left[\left(x_g - c\cdot\alpha_1\right) + \delta_g \Gamma(n_2|\alpha_1 = 1)\, x_g\right] \end{array} \right\} \tag{19.2}$$

with

$$n_2 = (1-\alpha_1)\, n_1 + \gamma\cdot(1-n_1).$$

q_1^* is the optimal decision variable of *Bad* politicians. The first line in the curly bracket is the payoff if there is not an internal conflict; the second line is the payoff if internal conflicts happen and *Bad* politicians win.

For *Bad* politicians, the decision is to contest for power in the first period to maximize their payoff discounted by the patience factor δ and the probability of winning in an internal conflict,

$$\sup_{q_1 \in \{0,1\}} \quad x_b \cdot \left\{ \begin{array}{c} (1-q_1)(1-\alpha_1)\left[1+\delta_b \Gamma\left(n_2\left(\alpha_1^*\right)\right)\right] \\ +q_1 \Omega\left(n_1\right)\left[1+\delta_b \Gamma\left(n_2\left(\alpha_1=0\right)\right)\right] \end{array} \right\}, \tag{19.3}$$

where α_1^* is the optimal decision of *Good* politicians, $q_1 = 1$ amounts to *Bad* politicians' contest for power, and $q_1 = 0$ means not to contest. In the curly bracket, the first line is the payoff if *Bad* politicians choose not to contest and the second line is the payoff if they choose to contest and win.

19.3 Infinite-Horizon Model

Since conflicts and coalitions exist for a long term, it is meaningful to offer a characterization of the repeated game of infinite-horizon. The model of infinite-horizon assumes that conflicts within the political coalition and monopoly continue until the regime is revolted, i.e., an ending period T such that $\Gamma(n_T) = 0$; and it is also true that $\Gamma(n_\infty) = 0$. With a little abuse of notation, I mainly use ∞ as the ending period of the game. The infinitely-repeated conflicts between politicians present the context in which fighting between the two types of politicians is non-stoppable and politicians are endowed with an option of leaving.

Suppose that after observing the investigation intensity, α_t, corrupt politicians can contest for power and, if they win, the anti-corruption efforts would be stopped. According to the relative power at period t, they win with a probability, $\Omega(n_t)$, if they had initiated a contest, while if they lose a contest which reveals themselves as opponents, they lose everything and have zero consumption.

Variables, period payoffs for each agent, and action spaces are the same as in the two-period model. The timing of events within a period is summarized as follows:

1. At the beginning of period t, the fraction of corrupt politicians is revealed, n_t, and the probability of revolution is also revealed, $\Gamma(n_t)$ and the relative power, $\Omega(n_t)$;

2. After observing the proportion of *Bad* politicians, n_t, *Good* politicians decide their action, α_t, being aware that *Bad* politicians can mount *Contest* for power or not, q_t=1 means contest;

3. After observing the size of their side n_t, and the action of their rivals, α_t, *Bad* politicians decide if they should contest for authority; if they win, the inspection intensity α will be fixed on the status quo, $\hat{\alpha}_t$ forever;

4. Consumption takes place;

5. At the end of period t or the beginning of period $t+1$, a new fraction of corrupt politicians is generated according to the process

$$n_{t+1} = (1-\alpha_t)\, n_t + \gamma \cdot (1-n_t)\,,$$

and a new stage game starts.

6. This stage game repeats until the regime collapses.

Table 19.1 summarizes the payoffs for the two types of politicians, with the first item in curly brackets for *Bad* politicians and the second for *Good* politicians.

Table 19.1: Conflict between Two types of Politicians

	Good
Bad	Investigation $[\alpha_t \in (0,1)]$
Contest $[q_t = 1]$	$\left\{ \begin{array}{c} \Omega(n_t)\,[x_b + \delta\Gamma(n_{t+1})\,\mathcal{V}(n_{t+1})]\,, \\ \{x_g - c(\alpha_t) + \beta\Gamma(n_t,\alpha_t)\,[(1-\Omega(n_t))\,\mathcal{W}(\alpha_{t+1},n_{t+1})]\} \end{array} \right\}$
No-Contest $[q_t = 0]$	$\left\{ \begin{array}{c} (1-\alpha_t)\,[x_b + \delta\Gamma(n_t)\,\mathcal{V}(n_{t+1},\alpha_{t+1})]\,, \\ \{x_g - c(\alpha_t) + \beta\Gamma(n_t,\alpha_t)\,(1-\Omega(n_t))\,\mathcal{W}(\alpha_{t+1},n_{t+1})\} \end{array} \right\}$

Payoffs

The recursive problem for *Good* politicians is choosing the probing intensity to maximize the utility discounted by the patience parameter, δ, and by the survival probability, $\Pi_{i=t}^{s}\Gamma(n_i)$,

$$\mathcal{W}(\alpha_t|n_t,q_t) = \max_{\alpha_t\in(0,1)} \left\{ \begin{array}{c} [x_g - c(\alpha_t)] + \\ \delta_g\Gamma(n_t,\alpha_t) \left[\begin{array}{c} (1-q_t)\,\mathcal{W}_g(\alpha_{t+1},n_{t+1}) \\ +q_t\,(1-\Omega_t(n_t))\,\mathcal{W}_g(\hat{\alpha}_t,n_{t+1}) \end{array} \right] \end{array} \right\} \tag{19.4}$$

subject to the constraint

$$(1-\alpha_t)\cdot n_t + \gamma\cdot(1-n_t) \quad = \quad n_{t+1}$$

where $c(\alpha_t)$ denotes the political and economic costs, which is a convex function of α_t. Recall that $\Omega_t(n_t) \in (0,1)$ is the probability that *Bad* politicians win a contest, which is an increasing function of n_t, and that the term $\mathcal{W}(\alpha_{t+1},n_{t+1})$ is a continuation value with the new state variable n_{t+1} and control variable α_{t+1} in the next period.

The recursive problem for *Bad* politicians is deciding whether or not to start a contest for authority, and the stage utility is also discounted by the patience parameter δ

and the probability of being caught, $(1-\alpha_t)$. The recursive problem of *Bad* politicians is written as follows,

$$\mathcal{V}\left(q_t|n_t,\alpha_t\right) = \sup_{q_t\in\{0,1\}} \left\{ \begin{array}{c} \left(1-q_t\right)\left(1-\alpha_t\right)\left[x_b+\delta_b\Gamma\left(n_t\right)\mathcal{V}\left(n_{t+1},\alpha_{t+1}\right)\right] \\ +q_t\cdot\Omega\left(n_t\right)\left[x_b+\delta_b\Gamma\left(n_t\right)\mathcal{V}\left(n_{t+1},\hat{\alpha}_t\right)\right] \end{array} \right\}. \quad (19.5)$$

19.4 *SOEs* and Regime Transmissions

As argued in Acemoglu and Robinson (2000) and Acemoglu and Robinson (2001), political regime changes, for example democratization, are often the direct outcome of such social unrest. Peaceful transmissions, however, are possible under certain circumstances, as in Acemoglu (2008), when the elite group has unanimous agreement on regime transmission. The circumstance in this subsection is more specific to a polity with large number of public assets controlled by politicians, as in China. The large sector of State-Owned-Enterprises (*SOEs*) in China was inherited from its centralized economy before reforms in the 1980s. These *SOEs* are run by the government and are de facto manipulated by political bureaucrats.

Suppose that the value of *SOEs* is equally shared by all politicians. The value of firms is determined by the economic environment, but in the baseline model I assume their value is constant, denoted as $\mathcal{V}(SOEs)$. In this extended circumstance, politicians have one more action: *Break the political coalition*. Assume both types of politician can choose to break the coalition, and they can succeed only when both types of politicians agree to break at the same time. The politicians, who propose to break without positive responses from the rival, will be repressed and lose everything. With the consensus to break, all politicians can seize an equal share of *SOEs*.

Good politicians face the decision problem

$$\mathcal{W}\left(n_t,\alpha_t,q_t\right) = \quad (19.6)$$

$$\sup_{\alpha_t\in(0,1),\mathrm{B}_g\in\{0,1\}} \left\{ \begin{array}{c} \left(1-\mathrm{B}_g\right)\left\{\begin{array}{c} x_g-c\left(\alpha_t\right)+\left(1-q_t\right)\delta_g\Gamma\left(n_t,\alpha_t\right)\mathcal{W}\left(n_t,\alpha_t,q_{t+1}\right) \\ +q_t\left(1-\Omega\left(n_t\right)\right)\delta\mathcal{W}\left(n_t,1,1\right) \end{array}\right\} \\ +\mathrm{B}_g\mathrm{B}_b\cdot\mathcal{U}\left(SOEs\right) \end{array} \right\} \quad (19.7)$$

Bad politicians face the decision problem

$$\mathcal{V}(n_t, q_t, \alpha_t) = \tag{19.8}$$

$$\sup_{q_t \in \{0,1\}, B_b \in \{0,1\}} \left\{ \begin{array}{c} (1 - B_b) \left\{ \begin{array}{c} (1-\alpha_t) x_b + q_t \Omega(n_t) \delta_b \Gamma(n_t, \alpha_t) \delta \mathcal{V}(n_{t+1}, \alpha_t) \\ + (1 - q_t)(1 - \alpha_t) \Gamma(n_t, \alpha_t) \delta \mathcal{V}(n_{t+1}, \alpha_{t+1}, q_{t+1}) \end{array} \right\} \\ + B_g B_b \cdot \mathcal{U}(SOEs) \end{array} \right\} \tag{19.9}$$

where $\mathcal{U}(SOEs)$ is the share of value that each politician can take.

The timing of events in this new context can be summarized as follows:

1. The fraction of corrupt politicians is revealed, n_t, and the probability of revolution is also revealed, $\Gamma(n_t)$ and the relative power, $\Omega(n_t)$.
2. After observing n_t, *Good* politicians decide their action, $\{\alpha_t, Break\}$, being aware that *Bad* politicians may choose *Contest* for power.
3. After observing n_t and the action of his rivals, α_t, *Bad* politicians decide if they should contest for authority or propose *Break*.
4. Repeat this game until both types of politicians agree to break the coalition at the same time, $(\sigma_t^B, \sigma_t^G) = (Break, Break)$.
5. Consumption takes place.
6. At the end of period t and also the beginning of period $t+1$, a new fraction of corrupt politicians is generated according to the process

   $$n_{t+1} = (1 - \alpha_t) n_t + \gamma \cdot (1 - n_t),$$

   and a new stage game starts.
7. The stage game repeats until citizens revolt or the politicians choose to break.

20 Political Equilibrium and Analysis

20.1 Two-Period Model

As per what has been discussed, the fraction of bad politicians is increasing over time without anti-corruption, and this will ultimately cause a significant threat to political sustainability because of the increased chance of revolution. Hence, it is necessary to analyze the dynamic problem of conflict. I start with a two-period model.

For expository convenience, I first define two concepts: *Strictly under control* and *Weakly under control.*

Definition 4. *Strictly under control* and *Weakly under control*:

(I) A political system is called *strictly under control,* if for all $\alpha \in [0,1]$ and all $n \in (0,1)$, internal conflicts will not happen, in other words, *Bad* politicians will not choose to mount a contest;

(II) A political system is called *weakly under control,* if there exists a $0 < \bar{\alpha} < 1$, such that for all $\alpha \in [0, \bar{\alpha}]$, internal conflicts will happen.

In the present paper, the control of corruption is the focus of the analysis. But one associated risk is the internal conflict, which is initiated by *Bad* politicians to contest power and abolish the anti-corruption campaign. The decision of *Bad* politicians depends on the parameters of the incumbent advantage, κ, the relative power, n, and the action intensity of *Good* politicians, α. So I define the two ranges of action space, *Strictly under control* and *Weakly under control*, to distinguish two cases: one is absolutely safe and one is relatively safe. In the range of strictly under control, *Good* politicians do not need to worry about internal conflicts, while in the range of weakly under control, there exists an upper threshold, above which internal conflicts will be triggered.

When corruption is strictly under control, there is only one equilibrium, $(\alpha, q) = [(0,1), 0]$; when weakly under control, there are two possible equilibria, $(\alpha, q) = [(0, \bar{\alpha}), 0]$ or $[1,1]$. The equilibrium $(\alpha, q) = [1,1]$ is the case that internal conflicts happen and the dominant power in the political monopoly may change.

In the two-period model, assume that the political regime inherits a fraction of *Bad* politicians. In the first period, the political leader takes action by choosing an investigation intensity α, which will affect the survival probability of the political regime in the second period, and *Bad* politicians observe α simultaneously and make a decision to conflict or not.

Recall that $\Delta x_{bg} \equiv x_b - x_g$ and the aggregate extraction can written as $x_t = x_g + n_t \Delta x_{bg}$.

Proposition 7. *(i) when $\kappa \leq \bar{\kappa}$, the political system is strictly under control,*

$$\bar{\kappa} = \left\{ \frac{1}{1+\delta} \left[\frac{(\bar{\phi} - \underline{\phi})\, y + \bar{\phi} y - x_g - \gamma \Delta x_{bg}}{(1-\gamma)\, \Delta x_{bg}} \right] \cdot \left(\frac{\delta}{1+\delta} \right) \left[1 + \frac{\bar{\phi} y - x_g - \gamma \Delta x_{bg}}{(\bar{\phi} - \underline{\phi})\, y} \right] \right\}^{-1}$$

with $\Delta x_{bg} \equiv x_b - x_g$.

(ii) when the political system is weakly under control, and the triggering point is

$$\bar{\alpha}(n,\kappa) = \frac{\left(\bar{\phi}y - \left[x_g + \Delta x_{bg}\left(2n_1 + \gamma(1-n_1)\right)\right]\right) + \sqrt{\left\{\begin{array}{c} \left(\bar{\phi}y - \left[x_g + \Delta x_{bg}\left(2n_1 + \gamma(1-n_1)\right)\right]\right)^2 \\ +4n_1\Delta x_{bg}\left(\bar{\phi} - \underline{\phi}\right) \times \\ y\left\{\frac{1}{\delta} + \Gamma(n_1,0) - \frac{\Omega(n_1)\mathcal{V}(n_1,0)}{\delta x_b}\right\} \end{array}\right\}}}{2n_1\Delta x_{bg}}.$$

Proof. See the proof in appendix. □

Since the second period is the last period, citizens have no motivation to revolt and politicians have no reasons for conflict in the second period. All decisions and actions are made in the first period; but analysis starts from the second period following backward induction.

The action of agents crucially depends on the n, which is the fundamental factor defining the relative power in conflicts and the regime survival rate that affects everyone's payoff. Hence, I examine the calculus properties and the actions of *Bad* politicians under conditions that n is sufficiently low ($\underline{n}$) and high $\bar{n}$. Suppose there exists an appropriate n, such that there are two values of α making *Bad* politicians indifferent between conflict or not. Define, among the two, the larger one as $\bar{\alpha}^b$, i.e.,

$$\bar{\alpha}^b \equiv \max\left\{\alpha_1 : \mathcal{D}^b = \begin{bmatrix} \Omega(n_1)\Gamma(n_1)\left[x_b + \delta\Gamma(n_1,0)x_b\right] \\ -\Gamma(n_1)\left[x_b + (1-\alpha_1)\delta\Gamma(n_1,\alpha_1)\right] \end{bmatrix} \geq 0\right\}.$$

Take derivatives with respect to α_1 and n_1 respectively, we have

$$\frac{\partial\mathcal{D}^b}{\partial\alpha_1} = \delta\Gamma\left((1-\alpha_1)n_1 + \gamma\cdot(1-n_1)\right) - (1-\alpha_1)\delta\frac{\partial\Gamma\left((1-\alpha_1)n_1 + \gamma\cdot(1-n_1)\right)}{\partial\alpha_1}$$

$$\frac{\partial\mathcal{D}^b}{\partial n_1} = x_b\left\{\begin{array}{c} \kappa\left[1 + \delta\Gamma\left(n_1 + \gamma\cdot(1-n_1)\right)\right] + \kappa n_1\delta\frac{\Gamma'\left(n_1 + \gamma\cdot(1-n_1)\right)}{\partial n_1} \\ -(1-\alpha_1)\delta\frac{\partial\Gamma\left((1-\alpha_1)n_1 + \gamma\cdot(1-n_1)\right)}{\partial n_1} \end{array}\right\}$$

$$\frac{\partial^2\mathcal{D}^b}{\partial n_1\partial\alpha_1} = x_b\left\{\begin{array}{c} \delta\frac{\partial\Gamma\left((1-\alpha_1)n_1 + \gamma\cdot(1-n_1)\right)}{\partial n_1} \\ -(1-\alpha_1)\delta\frac{\partial^2\Gamma\left((1-\alpha_1)n_1 + \gamma\cdot(1-n_1)\right)}{\partial n_1\partial\alpha_1} \end{array}\right\}.$$

We can see that the increase of α has two effects: one is a higher survival probability, which has globally positive effects, and the other is the negative effect of excluding *Bad* politicians.

20.1.1 A Numerical Example

Table 20.1 gives the parameter values of an illustrative example.

Table 20.1: An Illustrative Example

Parameters	Interpretation	Value
κ	Incumbent Advantage	0.8 or 0.5
$[\underline{\phi}, \bar{\phi}]$	The Distribution Range of ϕ	$[0.2, 0.5]$
γ	The Growth Rate of *Bad* Politicians (for the linear growth)	0.8
γ_2	The Growth Rate of *Bad* Politicians (for the contagion growth)	0.5
δ_b	The Discounting Factor of *Bad* politicians	0.88
δ_g	The Discounting Factor of *Bad* politicians	0.98
x_b	The Extraction of *Bad* politicians	0.6
x_g	The Extraction of *Good* politicians	0.1
x	The Aggregate Extraction of Politicians	$[0.1, 0.6]$
y	The aggregate income of citizens (normalized)	1
c	The linear cost of anti-corruption actions	$c = \frac{x_g}{2}$
	The quadratic cost of anti-corruption, $\frac{\varphi}{2}(\alpha)^2$	$\varphi = x_g$

Figure 20.1 plots the maximum welfare of *Bad* politicians as a function of pairs, (α, n_1). Generally, as α is approaching 1, the welfare of *Bad* politicians is decreasing, with the exception of when α is in the middle range and n_1 is high. In the exceptional case, benefits from mild anti-corruption, and hence the reduced revolution pressures, would outweigh the immediate threats from anti-corruption action in the present parameterization.

When will *Bad* politicians initiate an internal conflict? Figure 20.2 gives the answer. The upper-right area is the area of pairs, (α, n_1), when internal conflicts happen, because the anti-corruption actions are too intensive $(\alpha \to 1)$, or the fraction of *Bad* politicians is too large $(n_1 \to 1)$.

After comparing two optimal choices with different incumbent advantages (measured by $\frac{1}{\kappa}$), $\kappa = 0.8$ or 0.5, we know that the area of internal conflicts are much smaller for a smaller $\kappa = 0.5$. When *Good* politicians, who are incumbent, have a larger incumbent advantage, the possibility of internal conflicts is smaller.

Figure 20.1: The Maximum Welfare of *Bad* Politicians

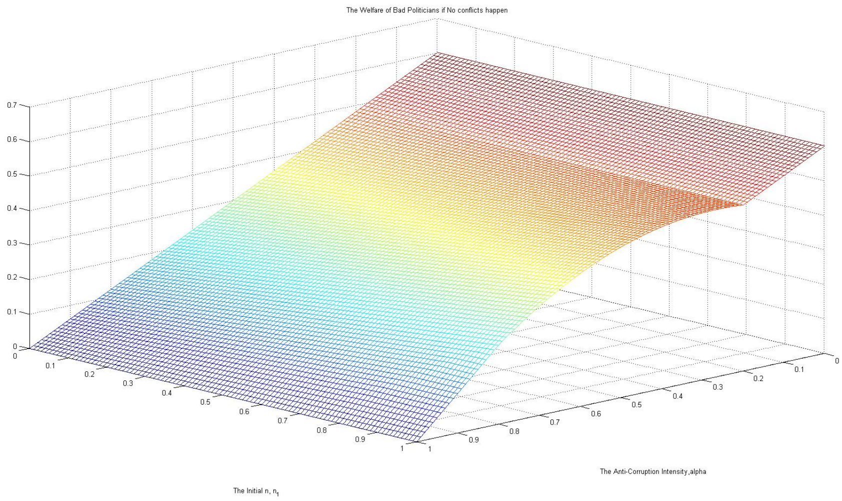
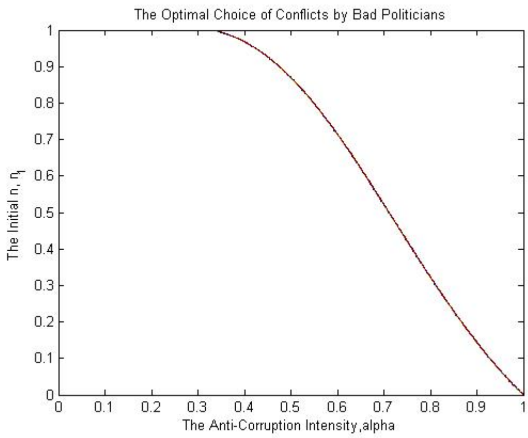
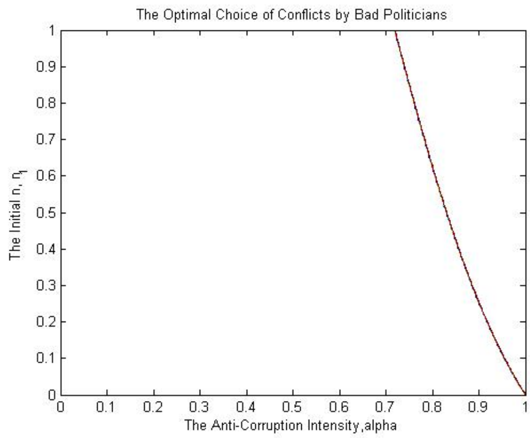


Figure 20.2: The Optimal Choice of *Bad* Politicians

(a) $\kappa = 0.8$



(b) $\kappa = 0.5$



For *Good* Politicians, I plot their maximum welfare in Figure 20.3.

Figure 20.3: The Maximum Welfare of *Good* Politicians

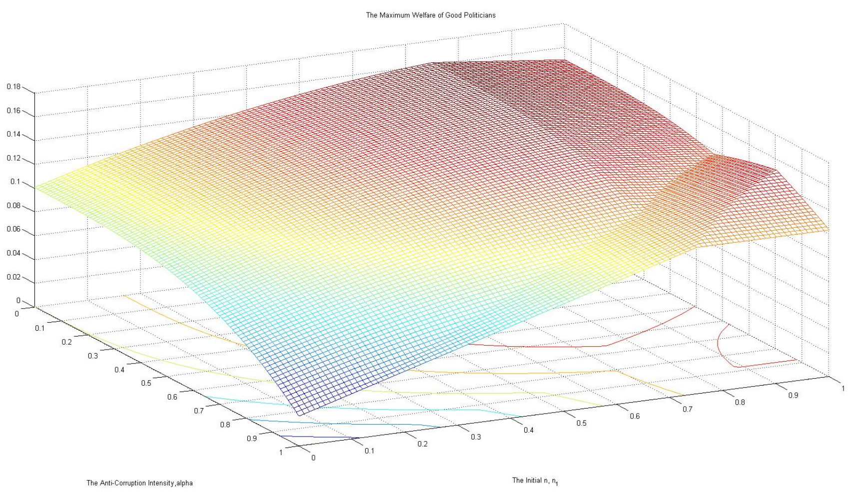
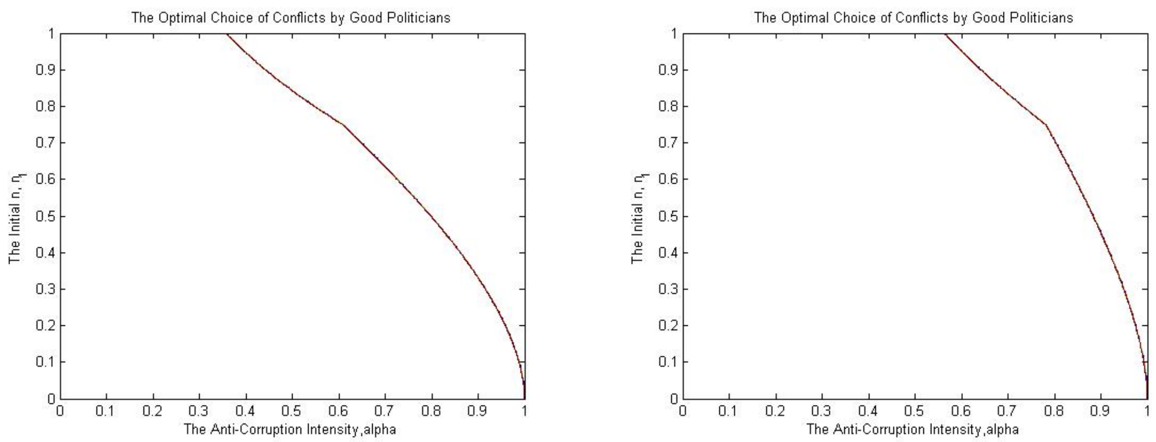


Figure 20.4 plots the frontier dividing the area into two cases: with or without conflicts. The upper-right corner is the area where, with the pair of (n, α), *Good* politicians will prefer an internal conflict and polarize its anti-corruption efforts to the upper bound, 1. We can see that internal conflicts can become desirable for *Good* politicians too, though they are started by *Bad* politicians. During internal conflicts, although *Good* politicians face the hazards of losing incumbency, they can take the chance to clean the political system and reverse it to a perfectly safe state.

Figure 20.4: The Optimal Choice of *Good* Politicians

(a) $\kappa = 0.8$ (b) $\kappa = 0.5$



20.2 Conflicts with Infinite-Horizon

As discussed before, there are two forces shaping the actions of agents and equilibria: the outside threats of revolution and the relative powers of the two types of politicians, which both are determined by the fraction of corrupt politicians at time t, n_t. Figure 20.5 shows the structure of the repeated conflicts with infinite-horizon.

Figure 20.5: The Structure of Conflicts with Infinite-Horizon

$$
\begin{array}{ccccccccc}
n_1 & & \cdots & & n_{t-1} & n_t & & n_{t+1} & \\
(\alpha_1, q_1) & \rightarrow & \cdots & \rightarrow & (\alpha_{t-1}, q_{t-1}) \rightarrow & (\alpha_t, q_t) & \rightarrow & (\alpha_{t+1}, q_{t+1}) & \rightarrow \infty \\
\downarrow & & & & \downarrow & \downarrow & & \downarrow & \\
\begin{pmatrix} \alpha_1 = 1 \\ q_1 = 1 \end{pmatrix} & & \cdots & & \begin{pmatrix} \alpha_{t-1} = 1 \\ q_{t-1} = 1 \end{pmatrix} & \begin{pmatrix} \alpha_t = 1 \\ q_t = 1 \end{pmatrix} & & \begin{pmatrix} \alpha_{t+1} = 1 \\ q_{t+1} = 1 \end{pmatrix} &
\end{array}
$$

Proposition 8. *Define $\underline{\mathcal{V}}(n_t)$ as the present value of a winning conflict,*

$$\underline{\mathcal{V}}(n_t) \equiv x_g \cdot \sum_{s=t}^{\infty} \delta_b^{s-t} \prod_{i=0}^{s-t} \Gamma(n_{t+i}, |\alpha = 0) .$$

(i) The Bad politicians choose conflict at t or later in T by comparing

$$\sup_{q \in \{0,1\}} \left\{ q_t \underbrace{\Omega(n_t)\, \underline{\mathcal{V}}(n_t)}_{\text{Value of Figthing at } t}, \left\{ \begin{array}{c} \underbrace{\sum_{s=t}^{T} \delta_b^{s-t} \prod_{i=t}^{s-1} (1-\alpha_i)\, \Gamma(n_{i+1}, \alpha_{i+1})\, x_b}_{\text{Discounted Payoffs between } t \text{ and } T} \\ \underbrace{+\, \delta_b^{T-t} \prod_{i=t}^{T-1} (1-\alpha_i)\, \Gamma(n_{i+1}, \alpha_{i+1})\, \Omega(n_T)\, \underline{\mathcal{V}}(n_T)}_{\text{Discounted Value of Fighting at } T} \end{array} \right\} \right\} .$$

(ii) For the case of two-period, the Bad politicians choose to wait if

$$(n_t)\, \underline{\mathcal{V}}(n_t) \leq (1-\alpha_t)\, x_b + \delta_b (1-\alpha_t) \Gamma(n_{t+1})\, \Omega(n_{t+1})\, \underline{\mathcal{V}}(n_{t+1})$$

with

$$n_{t+1} = g(n_t, \alpha_t),$$

where $g(n, \alpha)$ is the law of motion of n.

Proof. See the proof in Appendix. □

This proposition gives the condition of when *Bad* politicians would start an internal conflict. The *Bad* politicians compare the present value of fighting at t , $\Omega(n)\underline{\mathcal{V}}(n)$, with fighting at a later time T . When they decide to wait, the expected payoff is the received payoff between t plus a value of fighting in the future. The term,

$$\sum_{s=t}^{T} \delta_b^{s-t} \prod_{i=t}^{s-1} (1-\alpha_i)\, \Gamma(n_{i+1}, \alpha_{i+1})\, x_b$$

, is the present value of payoff received during t and T, whose magnitude is determined by the sequence, $\{(n_i, \alpha_i)\}_{i=t}^{T}$, and $\delta_b^{T-t} \prod_{i=t}^{T-1} (1-\alpha_i)\, \Gamma(n_{i+1}, \alpha_{i+1})\, \Omega(n_T)\, \underline{\mathcal{V}}(n_T)$ is the present value of fighting at a future time T. The two-period model can be taken as a specific case of this result. If *Bad* politicians wait for the next period, they will have a value of power, $n^{'}$, but face the threat of being caught by a chance α. When α is high, internal conflicts will happen. For each n, and in each period, there is an upper threshold of α without triggering internal conflicts.

The case without conflicts Now consider the zone of (n, α) that would not cause internal conflicts; the dynamic decision rule, which is an analogy of the Euler equation, can be derived to present the *Good* politicians' trade-off in deciding the inspection intensity. The equilibrium level of anti-corruption intensity from *Good* politicians and the probability of *Bad* politicians' contest for power can be derived to present the interaction between *Good* politicians and *Bad* ones.

Proposition 9. *In the circumstances without internal conflicts, the optimal decision rule of Good politicians follows the equation as*

$$\Gamma(n_s) \left\{ \underbrace{\delta_g^2 \Gamma_n^{'}(n_{s+1}) \left[x_g - c(\alpha_{s+2}) \right]}_{\text{Benefit in laeger } \Gamma(n_{s+1})} + \underbrace{\delta_g c^{'}(\alpha_{s+1}) \frac{g_n(n_{s+1}, \alpha_{s+1})}{g_\alpha^{'}(n_{s+1}, \alpha_{s+1})}}_{\text{Benefit in reducing effort in } s+1} \right\} = \frac{c^{'}(\alpha_s)}{g_\alpha^{'}(n_s, \alpha_s)}.$$

Proof. If *Good* politicians make smart decisions avoiding the contest, i.e., $(q_s = 0)_{s=t}^{\infty}$, the *Good* politicians' problem reduces to

$$\max_{\alpha} \left\{ \sum_{s=t}^{\infty} \delta_g^{s-t} \prod_{i=t}^{s-1} \Gamma(n_i, \alpha_i) \cdot \left[x_g - c(\alpha_s) \right] \right\}$$

subject to

$$n_{s+1} = g(n_s, \alpha_s).$$

Lagrangian function is

$$\mathcal{L} = \sum_{s=t}^{\infty} \delta_g^{s-t} \prod_{i=t}^{s-1} \Gamma(n_i) \cdot \left[x_g - c(\alpha_s)\right] + \lambda_s \left[g(n_s, \alpha_s) - n_{s+1}\right].$$

The derivatives and Euler equation are as follows:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \alpha_s} =& \delta_g^{s-t} \prod_{i=t}^{s-1} \Gamma(n_i) \cdot \left[-c'(\alpha_s)\right] + \lambda_s \frac{\partial g(n_s, \alpha_s)}{\partial \alpha_s} = 0 \\
\lambda_s =& \delta_g^{s-t} \prod_{i=t}^{s-1} \Gamma(n_i) \cdot c'(\alpha_s) \left[\frac{\partial g(n_s, \alpha_s)}{\partial \alpha_s}\right]^{-1} \\
\frac{\partial \mathcal{L}}{\partial n_{s+1}} =& \delta_g^{s+2-t} \prod_{i=t}^{s} \Gamma(n_i) \cdot \Gamma_n(n_{s+1}) \left[x_g - c(\alpha_{s+2})\right] + (-\lambda_s) + \lambda_{s+1} \frac{\partial g(n_{s+1}, \alpha_{s+1})}{\partial n_{s+1}} \\
=& 0.
\end{aligned}$$

Combine the equations above to have the Euler equation,

$$\begin{gathered}
\delta_g^{s+2-t} \prod_{i=t}^{s} \Gamma(n_i) \Gamma_n'(n_{s+1}) \left[x_g - c(\alpha_{s+2})\right] - \delta_g^{s-t} \prod_{i=t}^{s-1} c'(\alpha_s) \left[\frac{\partial g(n_s, \alpha_s)}{\partial \alpha_s}\right]^{-1} + \\
\delta_g^{s+1} \prod_{i=t}^{s} \Gamma(n_i) c'(\alpha_{s+1}) \left[\frac{\partial g(n_{s+1}, \alpha_{s+1})}{\partial \alpha_{s+1}}\right]^{-1} \frac{\partial g(n_{s+1}, \alpha_{s+1})}{\partial n_{s+1}} = 0 \\
\delta_g^{s-t} \prod_{i=t}^{s-1} \Gamma(n_i) \left\{ \begin{array}{c} \delta_g^2 \Gamma(n_s) \ \Gamma_n'(n_{s+1}) \left[x_g - c(\alpha_{s+2})\right] - c'(\alpha_s) g_\alpha'(\alpha_s)^{-1} \\ + \delta_g \Gamma(n_s) c'(\alpha_{s+1}) \dfrac{g_n(n_{s+1}, \alpha_{s+1})}{g_\alpha(n_{s+1}, \alpha_{s+1})^{-1}} \end{array} \right\} = 0 \\
\delta_g^2 \Gamma(n_s) \Gamma_n'(n_{s+1}) \left[x_g - c(\alpha_{s+2})\right] - \\
c'(\alpha_s) g_\alpha(\alpha_s)^{-1} + \delta_g \Gamma(n_s) c'(\alpha_{s+1}) \frac{g_n(n_{s+1}, \alpha_{s+1})}{g_\alpha(n_{s+1}, \alpha_{s+1})^{-1}} = 0
\end{gathered}$$

Rearrange the equation as follows to interpret conveniently:

$$\Gamma(n_s) \left\{ \underbrace{\delta_g^2 \Gamma_n'(n_{s+1}) \left[x_g - c(\alpha_{s+2})\right]}_{\text{Benefit in laeger } \Gamma(n_{s+1})} + \underbrace{\delta_g c'(\alpha_{s+1}) \frac{g_n(n_{s+1}, \alpha_{s+1})}{g_\alpha(n_{s+1}, \alpha_{s+1})}}_{\text{Benefit in reducing effort in } s+1} \right\} = \frac{c'(\alpha_s)}{g_\alpha'(n_s, \alpha_s)}. \tag{20.1}$$

The first term in the curly bracket is the discounted benefits in period $s+2$ from the increased survival rate by the anti-corruption in period s; and the second term is the cost saving effect from lower n_{s+1}, which is the legacy of the state variable and control variable in period s. The interpretation of this Euler equation is that the effective cost of the action, adjusted by the change of $n_{t+1}\left(=g\left(n_s,\alpha_s\right)\right)$, should be equal to the sum of the benefit of the higher survival rate in period $s+2$ and the cost saving effect in period s. □

With this setup, any action by the political leader has two effects: one working on the stage payoff as a cost of the investigation, and another on the survival rate and reduced costs in the next period; the aggregate effects are their products. In the zone of peace, the sequential decisions of *Good* politicians follow a rule which incorporates the trade-off between the efforts made in this period and the benefits received in future periods. This result gives the intertemporal framework the capacity to incorporate stochastic factors, like productivity shocks, and to examine how the political leader adjusts his anti-corruption efforts sequentially.

This equation also defines the sequence, $\{(n_i,\alpha_i)\}_{i=t}^{T}$, and makes it possible for the *Bad* politicians to calculate the payoffs in peaceful periods. To derive the steady states, appropriate settings of $\Gamma(\cdot)$ and the law of motion n are critical. Here, I make two examples for the law of motion of n, $g(n,\alpha)$.

Example 1: Linear Growth of Corruption Suppose $n_{s+1}=g\left(n_s,\alpha_s\right)=\left(1-\alpha_s\right)n_s+\gamma\cdot\left(1-n_s\right)$, take derivatives of $g(\cdot)$ with respect to n_s and α_s respectively,

$$\frac{\partial g\left(n_s,\alpha_s\right)}{\partial n_s}=1-\alpha_s-\gamma,$$
$$\frac{\partial g\left(n_s,\alpha_s\right)}{\partial \alpha_s}=-n_s.$$

Substitute in the Euler equation and we have the conditions for steady states written as

$$\delta_g\Gamma\left(n_{ss}\right)\left\{x_g-c\left(\alpha_{ss}\right)+\delta_g c'\left(\alpha_{ss}\right)\frac{1-\alpha_{ss}-\gamma}{-n_{ss}}\right\}=\frac{c'\left(\alpha_{ss}\right)}{-n_{ss}}, \tag{20.2}$$
$$\alpha_{ss}=\frac{\gamma\left(1-n_{ss}\right)}{n_{ss}}. \tag{20.3}$$

The equation 20.3 defines the steady state of the political system. Here, nothing can indicate whether or not the action intensity triggers an internal conflict. So, in the steady state, the survival rate of the regime depends on the impatience of the political leader and the initial n. From the non-linearity of this equation, there is more than one local steady state for different n. We can see that the political system is very path-dependent, and the initial status is crucial to the sustainability of the political regime.

Example 2: Contagion Growth of Corruption Following the modeling spirit of Mostagir (2010), the fraction of *Bad* politicians may follow a behavioral process of contagion. In each round of the game, agents randomly meet each other, observe, and compare their behaviors and payoffs. Since the good politicians are only infected by the corrupt politicians if they meet someone who is of a different type and stage consumption to them, the contagion of corruption can only happen if the sharing by the corrupt politicians is initially positive. The contagion process can be written as

$$n_{s+1} = g_2 (n_s, \alpha_s) = (1 - \alpha_s) n_s + \gamma_2 \cdot (1 - n_s) n_s$$

where $\gamma_2 \in (0, 1)$ denotes the rate of contagion and $(1 - n_s) n_s$ is the chance of two politicians with different types meeting.

$$\frac{\partial g_2 (n_s, \alpha_s)}{\partial n_s} = 1 - \alpha_s + \gamma_2 - 2\gamma_2 n_s,$$
$$\frac{\partial g_2 (n_s, \alpha_s)}{\partial \alpha_s} = -n_s,$$

The steady state satisfies the condition

$$\Gamma (n_{ss}) \left\{ \delta_g^2 \Gamma_n' (n_{ss}) \left[x_g - c (\alpha_{ss}) \right] + \delta_g c' (\alpha_{s+1}) \frac{1 - \alpha_{ss} + \gamma_2 - 2\gamma_2 n_{ss}}{-n_{ss}} \right\} = \frac{c' (\alpha_s)}{-n_{ss}},$$
$$\alpha_{ss} = \gamma_2 (1 - n_{ss}).$$

20.3 *SOEs* and Regime Transitions

A large and profitable *SOEs* sector is a distinct feature of the Chinese economy. The extension with *SOEs* would add one more action to all politicians. The new option is breaking the political monopoly and seizing the assets of *SOEs*. After integrating with the capital market, politicians can run a share of *SOEs*, and both types of politicians have the motivation to leave the sinking political regime and convert themselves into businessmen or women by buying shares of *SOEs*, because the enormous size of *SOEs* consists of firms that are run by government bureaucrats.

Denote $\mathcal{U}(SOEs)$ as the value of *SOEs* and the newly added action to politicians is $Break \in \{0, 1\}$. $\mathrm{B}_i = 1, i \in \{Good, Bad\}$ means that politician i chooses to break the current regime, and they can have one share of *SOEs* only if both types of politician agree to break and leave the current regime. The value of *SOEs* will not be discounted by the survival rate of the current regime, and their market values are independent of political regimes.

Good politicians face the decision problem,

$$\mathcal{W}\left(n_t,\alpha_t\right)=$$
$$\sup_{\alpha_t\in(0,1),\mathrm{B}_g\in\{0,1\}}\left\{\begin{array}{c}\left(1-\mathrm{B}_g\right)\left\{\begin{array}{c}x_g-c\left(\alpha_t\right)+\left(1-q_t\right)\delta_g\Gamma\left(n_t,\alpha_t\right)\mathcal{W}\left(n_t,\alpha_t,q_{t+1}\right)\\+q_t\left(1-\Omega\left(n_t\right)\right)\delta\mathcal{W}\left(n_t,1,1\right)\end{array}\right\}\\+\mathrm{B}_g\mathrm{B}_b\cdot\mathcal{U}\left(SOEs\right)\end{array}\right\}.$$

Bad politicians face the decision problem,

$$\mathcal{V}\left(n_t,q_t,\alpha_t\right)=$$
$$\sup_{q_t\in\{0,1\},\mathrm{B}_b\in\{0,1\}}\left\{\begin{array}{c}\left(1-\mathrm{B}_b\right)\left\{\begin{array}{c}\left(1-\alpha_t\right)x_b+q_t\Omega\left(n_t\right)\delta_b\Gamma\left(n_t,\alpha_t\right)\delta\mathcal{V}\left(n_{t+1},\alpha_t\right)\\+\left(1-q_t\right)\left(1-\alpha_t\right)\Gamma\left(n_t,\alpha_t\right)\delta\mathcal{V}\left(n_{t+1},\alpha_{t+1},q_{t+1}\right)\end{array}\right\}\\+\mathrm{B}_g\mathrm{B}_b\cdot\mathcal{U}(SOEs)\end{array}\right\}.$$

The decision problems of the two agents subtly differentiate from the previous model, in that the effects of anti-corruption imrove the regime's survival rate within one period, instead of from the next period. Actually, it does not matter when the discounting calculation starts if the decision is made by comparing the value of staying in the game and the value of quitting the political arena, because the market value of *SOEs* is independent of the political condition and only serves as a threshold.

Proposition 10. *If the game is repeated infinitely, there is a cutoff level of proportion, n_t, such that $(Break, Break)$ is a corporate equilibrium, i.e., the probability of breaking the unique coalition is positive and the event of breaking will eventually happen.*

Proof. For $n\in(0,1)$ the survival rate $\Gamma(n)\in(0,1)$ is a continuous function, with $\Gamma(0)=1$ and $\Gamma(1)=0$. When $\Gamma(1)=0$, the continuation values of all politicians are zero; when$\Gamma(0)=1$, the continuation values for *Good* politicians and existing *Bad* politicians are the discounted present value of future payoff. The continuation value for either type of politician is discounted by this survival rate and it has a continuous range from zero to the discounted present value of stage payoff, i.e., $\mathcal{W}\left(n_t,\alpha_t,q_t\right)\in\left(0,\frac{x_g}{1-\beta}\right)$ and $\mathcal{V}\left(n_t,q_t,\alpha_t\right)\in\left(0,\frac{x_b}{1-\delta}\right)$.

For any reservation value $\mathcal{U}(SOEs)>0$, there exists at least one $\bar{n}^g$, such that $\mathcal{W}\left(\bar{n}^g,\alpha_t,q_t\right)\leq\mathcal{U}(SOEs)$, and there also exists at least one $\bar{n}^b$,such that $\mathcal{V}\left(\bar{n}^b,q_t,\alpha_t\right)\leq\mathcal{U}(SOEs)$. When $n>\max\left\{\bar{n}^g,\bar{n}^b\right\}$, both types of politicians agree to break the regime, since there is a large n and low regime survival rate. □

The intuition of this proposition is reasonably straightforward. As the fraction of *Bad* politicians increases, the political monopoly gets more and more fragile with less and less legitimacy. When the survival rate is low enough, both fighting and staying in the current regime have lower expected payoffs than breaking the regime and buying one share of *SOEs*, whose market value is independent of the regime's legitimacy.

With the presence of *SOEs*, the political monopoly is weaker than the regime without public assets, in which politicians can only live on political rents, since the presence of *SOEs* and their value provides politicians the motivation to break the regime, rather than protecting the regime to their best efforts.

21 Stochastic Environment

The real economy is subject to various shocks, natural disasters, productivity shocks, and preference shocks, and the political system needs to be sustained through these shocks. According to the results in Bruchner and Ciccone (2011), transitory economic shocks can trigger political transitions. This section extends the dynamic model of infinite-horizon to a stochastic environment with economic and preference shocks.

21.1 Productivity and Myopic Shocks

With the assumption that *Good* politicians, or a good political leader, have the complete information, there should be no surprise or changes in the fighting strategy. However, this runs counter to the facts that anti-corruptions are time-varying and countercyclical. As we see in Section 2, anti-corruption campaigns occur periodically and negatively correlate with economic growth. Firstly, there does exist a periodical pattern in anti-corruption efforts; secondly, aggregate shock is a non-negligible force shaping the political environment and agents' action choices.

One more important factor is the patience of politicians. The preference changes of the political leader are an important factor deriving the political action. In this section, I present the results of what would happen to anti-corruption campaigns in the stochastic environment with two shocks: the productivity shocks and the preference shocks.

The final goods is produced from labor and the aggregate output is $y_t = A_t l$, where A_t captures the aggregate productivity and l is the constant labor supply. In particular, I assume that A_t takes two values,

$$A_t = \begin{cases} A^h = 1 & \text{productivity is high} \\ A^l = a & \text{productivity is low} \end{cases}$$

where $A^l = a < 1$ is in the period of recession. The two states, $\{A^h, A^l\}$, transfer following a Markov process,

$$\pi = \begin{bmatrix} \rho & 1-\rho \\ 1-\sigma & \sigma \end{bmatrix}.$$

The diagonal elements are the probabilities that the economy stays in the current status, which are also interpreted as the indicators of persistence.

The cost of revolution is ϕy_t, in which y_t now becomes a random variable in a stochastic environment. The new regime survival rate internalizing this random productivity shock is denoted as $\Gamma(n_t, \alpha_t, A_t)$. In recessions, aggregate outputs drop with proportion a and lower the cost of revolutions. During a recession, the probability of revolution was increased and therefore the regime survival probability was lowered. The sensitivity of the regime safety to n is smaller in a recession, i.e., $\Gamma_n'(n_t, A^l) < \Gamma_n'(n_t, A^h)$. The two specifications of the functional form of $\Gamma(n, A)$ satisfy this property:

$$\frac{\partial \Gamma_1(n_t)}{\partial n_t} = -\frac{x_b - x_g}{\bar{\phi} y - \underline{\phi} y} = -\frac{x_b - x_g}{(\bar{\phi} - \underline{\phi}) A_t l}.$$

The implication of the lower sensitivity of the regime survival rates to n is that, to achieve a same level of regime security, a larger reduction of n is required in recessions. It simply means that in a recession, more intensive anti-corruption is required to secure the political system from outside threats.

With productivity shocks, the new decision problem for the political leader will be written as

$$\max_{\alpha} \mathbb{E}_t \left\{ \sum_{s=t}^{\infty} \delta_g^{s-t} \prod_{i=t}^{s-1} \Gamma(n_i, A_i) \cdot \left[x_g - c(\alpha_s) \right] \right\} \tag{21.1}$$

subject to

$$\begin{aligned} n_{s+1} &= (1-\alpha_s) n_s + \gamma \cdot (1-n_s) \\ \alpha_s &\geq 0 \\ n_s &> 0. \end{aligned}$$

Proposition 11. *(i) The anti-corruption intensity is larger in a recession than in a normal state; a higher persistence of recessions leads to a larger increase of anti-corruption efforts than that in a normal state.*

(ii) After an adverse shock to the politician's patience, the anti-corruption efforts decline; a higher persistence of the preference shock leads to lower anti-corruption efforts.

Proof. See the proof in Appendix. □

After an adverse productivity shock, the range of revolution costs becomes smaller. Given the constant rents of the two types of politicians, if the fraction of *Bad* politicians does not change and the productivity shock is persistent, the political regime will face higher revolution pressure in the next period. To counter the effect of adverse productivity shocks, *Good* politicians will increase anti-corruption efforts and reduce the extraction from citizens by kicking out some *Bad* politicians. Will the adverse productivity shock and the increased anti-corruption efforts trigger internal conflicts? The answer depends on the parameterization and how deep the shock is.

After all the political leaders or *Bad* politicians become more myopic or less patient, they value fewer of the payoffs in the future. The increased safety in the future from present-day anti-corruption becomes less desirable, the cost of anti-corruption efforts in the future generates fewer concerns, and today's comforts become more profitable to the political leader. Hence, an adverse shock to patience leads to fewer anti-corruption efforts. In long run, a gradual decline of patience is the main driver of explosive corruptions.

21.2 How Long Can the Political System Be Sustained?

To answer the question posted at the beginning and relate what I have presented to the dynastic cycles, I offer the general description of how the political system can sustain all the risks studied above.

The probability that the political system survives up to period H can be written as

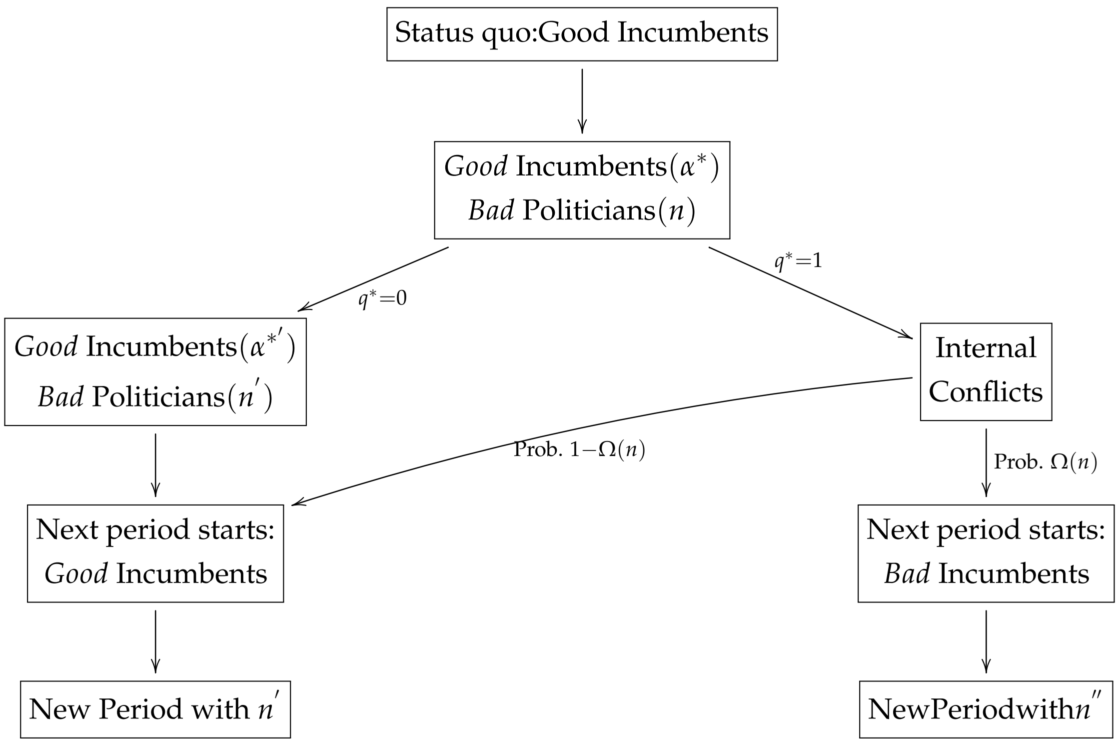
$$\prod_{s=t}^{H}\left[\begin{array}{c}\underbrace{q_s^*\Omega\left(n_s\right)\Gamma\left(n_{s+1}|\alpha_s=0\right)+q_s^*\left[1-\Omega\left(n_t\right)\right]\Gamma\left(n_{s+1}|\alpha_s=1\right)}_{\text{The Path if conflicts happen } (q_s^*=1)}\\ +\quad\underbrace{\left(1-q_s^*\right)\Gamma\left(n_{s+1}|\alpha_s^*\right)}_{\text{The Path if conflict not happen } (q_t^*=0)}\end{array}\right]$$

with

$$\begin{aligned}
q_s^* &= \arg\max_{q_s\in\{0,1\}} \mathcal{V}\left(q_t|n_t,\alpha_t\right),\\
\alpha_s^* &= \arg\max_{\alpha_s\in(0,1)} \mathcal{W}\left(\alpha_t|n_t,q_t\right),\\
n_{s+1} &= g\left(n_s,\alpha_s^*\right),\\
\alpha_s &\geq 0,\\
n_s &> 0.
\end{aligned}$$

The evolution path can be illustrated as in Figure 21.1. We can see that after *Good* politicians and *Bad* politicians decide on anti-corruption and conflict or not, respectively, the possible internal conflicts trigger incumbent changes, with the probability that *Bad* politicians win and anti-corruption efforts are demolished. The change of incumbency causes a larger tolerance of corruption and higher revolution pressure and may eventually lead to the collapse of the status quo system. This is how the dynastic cycle, anti-corruption, and internal conflicts are related.

Figure 21.1: The Evolution Path



Proposition 12. *If the $\Phi(x_t)$ is a strictly decreasing series, then almost certainly, only a finite number of revolutions occur.*

Proof. According to Borel-Cantelli's Lemma, if a sequence $A_1, A_2, \ldots, A_n, \ldots$ is events such that $\sum_{i=1}^{\infty} P(A_i) < \infty$, then almost certainly, only a finite number of the events A_i occur.

As I defined before, $\Phi(x_t) = 1 - \Gamma(x_t)$ is the probability of the event wherein a revolution occurs. If $\Phi(x_t)$ is strictly a decreasing series, then one can say $\sum_{i=1}^{\infty} P(A_i) < \infty$. Then, almost certainly, only a finitenumber of the events, which are revolutions in this context, occur. □

This result says that if the aggregate political extraction grows slower, as a result of the economic development or anti-corruption campaigns, than the growth of citizens' income, the revolution will not infinitely occur and the political system can be sustained in the long run. This result has a similar result to the conclusion by Acemoglu, Golosov, and Syvinsk (2008), which remarks that if the politicians are as patient as the citizens, the best sub-game perfect equilibrium leads to an asymptotic allocation where the aggregate distortions disappear. In my context, if the politicians can make citizens' income grow more than political rents, the political system is generally immune from revolutions. There are two ways to achieve this aim: economic development, or the reduction of aggregate rents through anti-corruption campaigns. This result explains the reasons why Chinese politicians put economic growth as a priority and start more anti-corruption efforts during recessions in China. But meeting this condition cannot be taken for granted. First, in the political economy, politicians are usually taken as more short-sighted than citizens and the degree of short-sightedness can be even worse in the absence of electoral competitions; second, in a stochastic environment, there exists exogenous factors, like deep recessions and famines, which cause a temporarily high threat of revolution. Political sustainability is suffering from both the internal risk and external risk.

22 Conclusions

Although there is no strong electoral control of politicians, political monopoly and centralization are quite stable in China. By which mechanism the incumbent leader secures his or her authority and the current regime is a puzzle in the political economy of non-democracy. From the perspective of dynamic conflicts, this study answers this puzzle by presenting one mechanism through which the political incumbents secure their political regime.

The first contribution of this study is that it establishes a framework including citizens and heterogeneous politicians. When analyzing non-democracies, the existing literature focuses more on the interactions and conflicts between citizens and politicians, while this study has a comprehensive framework of the interactions of citizens and heterogeneous politicians in a social-economic system. The second novelty is the fraction of *Bad* politicians, which is a measure of the heterogeneity of politicians. As a fundamental measure, this fraction defines the relative power in internal conflicts and regime survival rates. For different fractions of *Bad* politicians, the equilibria are very different. When this fraction is very high or low, internal conflicts will not happen, but for different reasons. The reason for the former result is that, when facing a severe regime threat from the outside, politicians in the regime need to cooperate, and the reason for the latter case is that the probability of winning for the *Bad* politicians is low.

With regard to one characteristic of the Chinese economy, I analyze the implication of *SOEs*. A surprising and rational result is that the presence of large *SOEs* can jeopardize the survival capability, since it can reduce the motivation of incumbent politicians to protect the current regime.

One important extension of the benchmark model is to include a stochastic economic environment. Recessions cause more intensive internal conflicts because of higher revolution pressures from citizens. In the stochastic environment, a higher persistence of adverse shocks will make internal fighting more intensive. Anti-corruption campaigns lead to lower political rents and reinforce the political system, or they can change the incumbency, start an era with less control of political corruption, and eventually lead to the collapse of a political system. In the dynamic and stochastic environment, the political decision is to find an optimal path to sustain a political regime. I set up the framework of how to evaluate the political sustainability.

There are possible ways to perfect this project. The first one is to incorporate heterogeneous political leaders. If the political leaders make sophisticated decisions to avoid internal conflicts, this can be classified as the actions of conservative politicians, who are very risk averse. There exists another kind of political leader, named an aggressive politician, who dares to fight. It makes sense to analyze and evaluate the political interactions with conservative and aggressive political leaders. Secondly, although I offered some facts about internal conflicts in China, more evidence of corruption, anti-corruption campaigns, and social unrest across history are absolutely valuable and desirable to validate the model's predictions.

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23 Appendix

23.1 Proof of Proposition 2.

Proof. The conflict decision by *Bad* politicians,

$$\sup_{q_1 \in \{01,\}} \begin{Bmatrix} q_1 \cdot \Omega(n_1)\Gamma(n_1)\left[x_b + \delta\Gamma(n_2|\alpha_1 = 0)x_b\right] \\ (1-q_1)\Gamma(n_1)\left[x_b + (1-\alpha_1)\delta\Gamma(n_2,\alpha_1)\right] \end{Bmatrix}$$

with

$$n_2 = (1-\alpha_1)\, n_1 + \gamma \cdot (1-n_1)\,.$$

The payoff difference between the two actions, conflict or not, denoted as $\mathcal{D}_b$, is

$$\begin{aligned} \mathcal{D}^b =& \Omega\left(n_1\right)\Gamma\left(n_1\right)\left[x_b + \delta\Gamma\left(n_2|\alpha_1 = 0\right)x_b\right] - \Gamma\left(n_1\right)\left[x_b + \left(1-\alpha_1\right)\delta\Gamma\left(n_2,\alpha_1\right)\right] \\ =& \kappa n_1 \Gamma\left(n_1\right)\left[x_b + \delta\Gamma\left(n_1 + \gamma\cdot\left(1-n_1\right)\right)x_b\right] \\ & - \Gamma\left(n_1\right)\left[x_b + \left(1-\alpha_1\right)\delta\Gamma\left(\left(1-\alpha_1\right)n_1 + \gamma\cdot\left(1-n_1\right)\right)\right]. \end{aligned}$$

Give one simplest specification of $\Gamma(\cdot) \equiv \frac{\bar{\phi}y - \left[x_g(1-n) + x_b n\right]}{(\bar{\phi}-\underline{\phi})y}$ to solve the equation $\mathcal{D}_b = 0$,

$$\mathcal{D}^b \le 0$$

$$\kappa n_1 \Gamma\left(n_1\right)\left[x_b + \delta\Gamma((n_1 + \gamma\cdot n_1)x_b\right] - \Gamma\left(n_1\right)\left[x_b + (1-\alpha_1)\delta\Gamma((1-\alpha_1)n_1 + \gamma\cdot n_1)\right] = 0.$$

$$\kappa n_1\left[x_b + \delta\Gamma((n_1 + \gamma\cdot n_1)x_b\right] - \left[x_b + (1-\alpha_1)\delta\Gamma((1-\alpha_1)n_1 + \gamma\cdot n_1)\right] = 0$$

Set $\alpha_1 = 1$ to derive the condition for strictly under control,

$$\kappa n_1\left[x_b + \delta\Gamma\left(n_1 + \gamma\cdot\left(1-n_1\right)\right)\right] - x_b = 0$$

$$\kappa = \frac{1}{n_1\left[1 + \delta\Gamma\left(n_1 + \gamma\cdot\left(1-n_1\right)\right)\right]}.$$

Recall the definition that $\kappa < 1$ is interpreted as the disadvantage of the non-incumbent, smaller value of κ means larger disadvantage for the opposition and also larger incumbent.

Define the

$$
\begin{aligned}
\bar{\kappa} &= \arg\min_{n_1} \frac{1}{n_1\left[1+\delta\Gamma\left(n_1+\gamma\cdot(1-n_1)\right)\right]} \\
&= \arg\max_{n_1} n_1\left[1+\delta\Gamma\left(n_1+\gamma\cdot(1-n_1)\right)\right] \\
&= \left\{\frac{1}{1+\delta}\left[\frac{(\bar{\phi}-\underline{\phi})\,y+\bar{\phi}y-x_g-\gamma\Delta x_{bg}}{(1-\gamma)\,\Delta x_{bg}}\right]\cdot\left(\frac{\delta}{1+\delta}\right)\left[1+\frac{\bar{\phi}y-x_g-\gamma\Delta x_{bg}}{(\bar{\phi}-\underline{\phi})\,y}\right]\right\}^{-1}.
\end{aligned}
$$

Define $\mathbb{E}\mathcal{V}\left(n_1,q_1=1\right)\equiv\Omega(n_1)\mathcal{V}\left(n_1,\alpha_1=1\right)=\kappa n_1\left[x_b+\delta\Gamma\left(n_1+\gamma\cdot(1-n_1)\right)x_b\right]$, which can be interpreted as the expected value of initiating a contest and wining

$$
\begin{gathered}
\mathbb{E}\mathcal{V}\left(n_1,q_1=1\right)-\left[x_b+(1-\alpha_1)\,\delta\Gamma\left((1-\alpha_1)\,n_1+\gamma\cdot(1-n_1)\right)\right]\geq 0 \\
\mathbb{E}\mathcal{V}\left(n_1,q_1=1\right)-\left[x_b+\delta\,(1-\alpha_1)\,x_b\frac{\bar{\phi}y-\left[x_g+\Delta x_{bg}\left((1-\alpha_1)\,n_1+\gamma\cdot(1-n_1)\right)\right]}{(\bar{\phi}-\underline{\phi})\,y}\right]\geq 0 \\
\mathbb{E}\mathcal{V}\left(n_1,q_1=1\right)-\left[x_b+\delta\,(1-\alpha_1)\,x_b\frac{\bar{\phi}y-\left[x_g+\Delta x_{bg}\left(n_1+\gamma\cdot(1-n_1)\right)-\Delta x_{bg}\alpha_1 n_1\right]}{(\bar{\phi}-\underline{\phi})\,y}\right]\geq 0 \\
\mathbb{E}\mathcal{V}\left(n_1,q_1=1\right)-\left[x_b+\delta\,(1-\alpha_1)\,x_b\frac{\bar{\phi}y-\left[x_g+\Delta x_{bg}\left(n_1+\gamma\cdot(1-n_1)\right)\right]+\Delta x_{bg}\alpha_1 n_1}{(\bar{\phi}-\underline{\phi})\,y}\right]\geq 0 \\
\mathbb{E}\mathcal{V}\left(n_1,0\right)-\left[\begin{array}{c} x_b+x_b\cdot\delta\,(1-\alpha_1)\,\dfrac{\bar{\phi}y-\left[x_g+\Delta x_{bg}\left(n_1+\gamma\cdot(1-n_1)\right)\right]}{(\bar{\phi}-\underline{\phi})\,y} \\ +\delta\,(1-\alpha_1)\,x_b\dfrac{\Delta x_{bg}\alpha_1 n_1}{(\bar{\phi}-\underline{\phi})\,y}\end{array}\right]\geq 0 \\
\mathbb{E}\mathcal{V}\left(n_1,0\right)-\left[\mathcal{V}\left(n_1,0\right)-\alpha_1\delta x_b\Gamma\left(n_1,0\right)+\delta x_b\cdot(1-\alpha_1)\,\frac{\Delta x_{bg}\alpha_1 n_1}{(\bar{\phi}-\underline{\phi})\,y}\right]\geq 0 \\
\left(\Omega(n_1)-1\right)\mathcal{V}\left(n_1,0\right)+\alpha_1 x_b\cdot\delta\Gamma\left(n_1,0\right)-\delta x_b\cdot(1-\alpha_1)\left[\Gamma\left(n_1,\alpha_1\right)-\Gamma\left(n_1,0\right)\right]\geq 0 \\
\left(\Omega(n_1)-1\right)\mathcal{V}\left(n_1,0\right)+\alpha_1\delta x_b\left[\Gamma\left(n_1,0\right)-\Gamma\left(n_1,\alpha_1\right)+\Gamma\left(n_1,0\right)\right] \\
-\delta x_b\left[\Gamma\left(n_1,\alpha_1\right)-\Gamma\left(n_1,0\right)\right]\geq 0
\end{gathered}
$$

Rearrange the inequality, we have

$$
\begin{gathered}
\frac{\Omega\left(n_1\right)\mathcal{V}\left(n_1,0\right)}{\delta x_b}-\frac{1}{\delta}-\Gamma(n_1,0)\geq\alpha_1\left(\frac{n_1\Delta x_{bg}}{(\bar{\phi}-\underline{\phi})\,y}-\Gamma\left(n_1,0\right)\right)-\alpha_1^2\frac{n_1\Delta x_{bg}}{(\bar{\phi}-\underline{\phi})\,y} \\
\alpha_1^2\frac{n_1\Delta x_{bg}}{(\bar{\phi}-\underline{\phi})\,y}+\alpha_1\left(\Gamma\left(n_1,0\right)-\frac{n_1\Delta x_{bg}}{(\bar{\phi}-\underline{\phi})\,y}\right)\geq\frac{1}{\delta}+\Gamma(n_1,0)-\frac{\Omega\left(n_1\right)\mathcal{V}\left(n_1,0\right)}{\delta x_b} \\
\alpha_1^2\frac{n_1\Delta x_{bg}}{(\bar{\phi}-\underline{\phi})\,y}+\alpha_1\left(\frac{\bar{\phi}y-\left[x_g+\Delta x_{bg}\left(n_1+\gamma(1-n_1)\right)\right]}{(\bar{\phi}-\underline{\phi})\,y}-\frac{n_1\Delta x_{bg}}{(\bar{\phi}-\underline{\phi})\,y}\right)\geq\frac{1}{\delta}+\Gamma(n_1,0)-\frac{\Omega\left(n_1\right)\mathcal{V}\left(n_1,0\right)}{\delta x_b} \\
\alpha_1^2\frac{n_1\Delta x_{bg}}{(\bar{\phi}-\underline{\phi})\,y}+\alpha_1\left(\frac{\bar{\phi}y-\left[x_g+\Delta x_{bg}\left(2n_1+\gamma(1-n_1)\right)\right]}{(\bar{\phi}-\underline{\phi})\,y}\right)\geq\frac{1}{\delta}+\Gamma(n_1,0)-\frac{\Omega\left(n_1\right)\mathcal{V}\left(n_1,0\right)}{\delta x_b}.
\end{gathered}
$$

Roots of this equation

$$\bar{\alpha}_1 = \frac{\left(\frac{\bar{\phi}y-\left[x_g+\Delta x_{bg}(2n_1+\gamma(1-n_1))\right]}{(\bar{\phi}-\underline{\phi})y}\right) \pm \sqrt{\begin{array}{c}\left(\frac{\bar{\phi}y-\left[x_g+\Delta x_{bg}\left(2n_1+\gamma(1-n_1)\right)\right]}{\left(\bar{\phi}-\underline{\phi}\right)y}\right)^2 \\ +4\frac{n_1\Delta x_{bg}}{\left(\bar{\phi}-\underline{\phi}\right)y}\left\{\frac{1}{\delta}+\Gamma(n_1,0)-\frac{\Omega\left(n_1\right)\mathcal{V}\left(n_1,0\right)}{\delta x_b}\right\}\end{array}}}{2\frac{n_1\Delta x_{bg}}{(\bar{\phi}-\underline{\phi})y}}$$

$$= \frac{\left(\bar{\phi}y-\left[x_g+\Delta x_{bg}\left(2n_1+\gamma(1-n_1)\right)\right]\right) \pm \sqrt{\left\{\begin{array}{r}\left(\bar{\phi}y-\left[x_g+\Delta x_{bg}\left(2n_1+\gamma(1-n_1)\right)\right]\right)^2 \\ +4n_1\Delta x_{bg}\left(\bar{\phi}-\underline{\phi}\right)\times \\ y\left\{\frac{1}{\delta}+\Gamma(n_1,0)-\frac{\Omega\left(n_1\right)\mathcal{V}\left(n_1,0\right)}{\delta x_b}\right\}\end{array}\right\}}}{2n_1\Delta x_{bg}}.$$

Since $\alpha \in (0,1)$, the only reasonable value of $\bar{\alpha}_1$ is

$$\frac{\left(\bar{\phi}y-\left[x_g+\Delta x_{bg}\left(2n_1+\gamma(1-n_1)\right)\right]\right) + \sqrt{\left\{\begin{array}{r}\left(\bar{\phi}y-\left[x_g+\Delta x_{bg}\left(2n_1+\gamma(1-n_1)\right)\right]\right)^2 \\ +4n_1\Delta x_{bg}\left(\bar{\phi}-\underline{\phi}\right)\times \\ y\left\{\frac{1}{\delta}+\Gamma(n_1,0)-\frac{\Omega\left(n_1\right)\mathcal{V}\left(n_1,0\right)}{\delta x_b}\right\}\end{array}\right\}}}{2n_1\Delta x_{bg}}.$$

□

23.2 Proof of Proposition 4

Proof. The problem that need to be solved for political leader is

$$\mathcal{W}\left(n_t,\alpha_t\right) = \sup_{\alpha_t\in(0,1)}\left\{\begin{array}{c} x_g - c\left(\alpha_t\right) + \left(1-q_t\right)\Gamma\left(n_t,\alpha_t\right)\mathcal{W}\left(n_{t+1},\alpha_{t+1}\right) \\ +q_t\left(1-\Omega\left(n_t\right)\right)\Gamma\left(n_t,\alpha_t\right)\mathcal{W}\left(n_{t+1},\hat{\alpha}_t\right)\end{array}\right\}$$

subject to constrain

$$\left(1-\alpha_t\right)\cdot n_t + \gamma\cdot\left(1-n_t\right) \;=\; n_{t+1}$$

where $c\left(\alpha_t\right)$ denotes the inspection cost. Recall that $\Omega\left(n_t\right) \in (0,1)$ is the probability that *Bad* politicians win a contest, which is increasing function of n_t, and the term $\mathcal{W}_g\left(n_{t+1},\alpha_{t+1}\right)$ is continuation value with new state variable n_{t+1} and control variable

α_{t+1} in next period.

Iterate forward the recursive decision problem to infinity and have the expression as follows,

$$\begin{aligned}
\mathcal{W}\left(n_t,\alpha_t,q_t\right) &= \sup_{\alpha_t}\left\{\begin{matrix} x_g - c\left(\alpha_t\right) + \left(1-q_t\right)\beta\mathcal{W}\left(n_{t+1},\alpha_{t+1}\right) \\ +q_t\left(1-\Omega\left(n_t\right)\right)\mathcal{W}\left(n_t,\hat{\alpha}_t\right) \end{matrix}\right\} \\
&= \sup_{\alpha_t\in(0,1)}\left\{\begin{matrix} x_g - c\left(\alpha_t\right) + q_t\left(1-\Omega\left(n_t\right)\right)\mathcal{W}\left(n_t,1\right) \\ +\left(1-q_t\right)\beta\max_{\alpha_{t+1}}\Gamma\left(n_{t+1},\alpha_{t+1}\right)\times \\ \left\{\begin{matrix} x_g - c\left(\alpha_t\right) + \left(1-q_{t+1}\right)\beta\mathcal{W}\left(n_{t+2},\alpha_{t+2}\right) \\ +q_{t+1}\left(1-\Omega\left(n_{t+1}\right)\right)\mathcal{W}\left(n_{t+1},\hat{\alpha}_{t+1}\right) \end{matrix}\right\} \end{matrix}\right\} \\
&= \cdots \\
&= \sup_{\alpha_t\in(0,1)}\left\{\begin{matrix} \sum_{s=t}^{\infty}\beta^{s-t}\prod_{i=t}^{s-1}\Gamma\left(n_i,\alpha_i\right)\left(1-q_i\right)\cdot\left[x_g - c\left(\alpha_t\right)\right] \\ +\beta^{\infty}\prod_{s=t}^{\infty}\Gamma\left(n_{t+s},\alpha_{t+s}\right)\left(1-q_{t+s}\right)\mathcal{W}\left(n_{t+\infty},\alpha_{t+\infty}\right) \end{matrix}\right\} \\
&\quad + \sum_{s=t}^{\infty}\beta^{s-t}\Gamma\left(n_s,\alpha_s\right)\mathcal{W}\left(n_s,1\right)\cdot q_s\left[1-\Omega\left(n_s\right)\right] \\
&= \max_{\alpha_t\in(0,1)}\left\{\begin{matrix} \sum_{s=t}^{\infty}\beta^{s-t}\prod_{i=t}^{s-1}\Gamma\left(n_i,\alpha_i\right)\left(1-q_i\right)\cdot\left[x_g - c\left(\alpha_t\right)\right] \\ +\sum_{s=t}^{\infty}\beta^{s-t}\Gamma\left(n_s,\alpha_s\right)\mathcal{W}\left(n_s,1\right)\cdot q_s\left[1-\Omega\left(n_s\right)\right] \end{matrix}\right\}
\end{aligned}$$

with the condition that

$$\beta^{\infty}\prod_{s=t}^{\infty}\Gamma\left(n_{t+s},\alpha_{t+s}\right)\left(1-q_{t+s}\right)\mathcal{W}\left(n_{t+\infty},\alpha_{t+\infty}\right) = 0,$$

which obviously is true because $\beta < 1$ and $\prod_{i=t}^{s}\Gamma\left(n_{t+i},\alpha_{t+i}\right) < 1$.

Good politicians or the political leader make sophisticated decision avoiding the contest from his rival side and have the summed payoff of discounted value from each period in future,

$$\mathcal{W}\left(n_t,\alpha_t\right) = \sum_{s=t}^{\infty}\beta^{s-t}\prod_{i=t}^{s}\Gamma\left(n_i,\alpha_i\right)\cdot\left[x_g - c\left(\alpha_t\right)\right].$$

And if a conflict between politician happens, *Good* politicians will choose strongest intensity of anti-corruption, $\alpha_t = 1$, and has the value as

$$\mathcal{W}\left(n_t, \hat{\alpha}_t\right) \equiv x_g \cdot \sum_{s=t}^{\infty} \beta^{s-t} \Gamma\left(n_s, 1\right).$$

The recursive problem of *Bad* politicians is that q_t is decision variable, with value 1 amounting to start an attack, and $\Omega\left(n_t\right)$ is the probability of winning a contest if it was started. Define the present value of *Bad* politicians, the they win the conflict and become incumbent, as

$$\underline{\mathcal{V}}\left(n_t\right) = x_g \cdot \sum_{s=t}^{\infty} \delta^{s-t} \prod_{i=0}^{s-t} \Gamma\left(n_{t+i}, |\alpha = 0\right).$$

Obviously, continuation value at t, $\mathcal{V}\left(n_t, \hat{\alpha}_t\right)$, exist and is bounded, because

$$\delta^{T+1-t} \prod_{i=1}^{T+1} \Gamma\left(n_{t+T+1}\right) < 1$$

and then $\lim_{T\to\infty} x_2 \cdot \delta^{T+1-t} \prod_{i=1}^{T+1} \Gamma\left(n_{t+T+1}\right) = 0$. So, $\underline{\mathcal{V}}\left(n_t, 0\right)$ is a well define value for *Bad* politicians.

Rewrite the problem of *Bad* politicians in recursive form,

$$\begin{aligned}
&\mathcal{V}\left(n_t, q_t, \alpha_t\right) \\
= &\sup_{q_t\in\{0,1\}} \left\{ \begin{array}{c} \left(1-q_t\right)\left(1-\alpha_t\right) x_b + q_t \Omega\left(n_t\right) \underline{\mathcal{V}}\left(n_t\right) \\ + \left(1-q_t\right)\left(1-\alpha_t\right) \delta \Gamma\left(n_{t+1}\right) \mathcal{V}\left(n_{t+1}, \alpha_{t+1}, q_{t+1}\right) \end{array} \right\} \\
= &\sup_{q_t\in\{0,1\}} \left\{ \begin{array}{c} \left(1-\alpha_t\right) x_b + q_t \Omega\left(n_t\right) \underline{\mathcal{V}}\left(n_t\right) + \\ \left(1-q_t\right)\left(1-\alpha_t\right) \delta \cdot \\ \left\{ \begin{array}{c} \max_{q_{t+1}\in\{0,1\}} \Gamma\left(n_{t+1}, \alpha_{t+1}\right) \cdot \\ \left\{ \begin{array}{c} \left(1-\alpha_{t+1}\right) x_b + q_{t+1} \Omega\left(n_{t+1}\right) \underline{\mathcal{V}}\left(n_{t+2}\right) \\ + \left(1-q_{t+1}\right)\left(1-\alpha_{t+1}\right) \delta \mathcal{V}\left(n_{t+2}, \alpha_{t+2}, q_{t+2}\right) \end{array} \right\} \end{array} \right\} \end{array} \right\} \\
= &\cdots \text{ iteration} \\
= &\sup_{q\in\{0,1\}} \left\{ \begin{array}{c} \sum_{s=t}^{\infty} \left\{ \begin{array}{c} \delta^{s-t} \prod_{i=t}^{s-1} \Gamma\left(n_i, \alpha_i\right)\left(1-\alpha_i\right)\left(1-q_{t+i}\right) \times \\ \left\{\left(1-\alpha_s\right)\left(1-q_s\right) x_b + q_s \Omega\left(n_s\right) \underline{\mathcal{V}}\left(n_s\right)\right\} \end{array} \right\} \\ + \prod_{i=t}^{\infty} \Gamma\left(n_{t+i}, \alpha_{t+i}\right)\left(1-q_{t+i}\right)\left(1-\alpha_{t+i}\right) \delta^{\infty-t} \mathcal{V}\left(n_{t+\infty}, \alpha_{t+\infty}, q_{t+\infty}\right) \end{array} \right\}
\end{aligned}$$

Because $\Gamma\left(n_{t+i},\alpha_{t+i}\right)<1$,$\left(1-\alpha_{t+i}\right)<1$, and $\delta<1$,

$$\prod_{i=t}^{\infty}\Gamma\left(n_{t+i},\alpha_{t+i}\right)\left(1-q_{t+i}\right)\left(1-\alpha_{t+i}\right)\delta^{\infty-t}\mathcal{V}\left(n_{t+\infty},\alpha_{t+\infty},q_{t+\infty}\right)=0.$$

Hence, recursive problem of *Bad* politicians can be rewritten as with a sequence of q defining the optimal decisions,

$$\mathcal{V}\left(n_t,q_t,\alpha_t\right)=$$
$$\sup_{q\in\{0,1\}}\left\{q_t\Omega\left(n_t\right)\underline{\mathcal{V}}\left(n_t\right),\sum_{s=t}^{T}\left\{\begin{array}{c}\delta^{s-t}\prod_{i=t}^{s-1}\Gamma\left(n_i,\alpha_i\right)\left(1-\alpha_i\right)\left(1-q_{i+1}\right)x_b\times\\ q_T\Omega\left(n_T\right)\underline{\mathcal{V}}\left(n_T\right)\end{array}\right\}\right\}.$$

Let the *Bad* politicians choose conflict now or later, in one period T, they need to compare the stop value now, $\Omega\left(n_T\right)\underline{\mathcal{V}}\left(n_T\right)$, and the stop value at period T plus the payoff between these two periods,

$$\mathcal{V}\left(n_t,q_t,\alpha_t\right)=$$
$$\sup_{q\in\{0,1\}}\left\{q_t\Omega\left(n_t\right)\underline{\mathcal{V}}\left(n_t\right),\sum_{s=t}^{T}\left\{\begin{array}{c}\delta^{s-t}\prod_{i=t}^{s-1}\Gamma\left(n_i,\alpha_i\right)\left(1-\alpha_i\right)\left(1-q_{i+1}\right)x_b\times\\ q_T\Omega\left(n_T\right)\underline{\mathcal{V}}\left(n_T\right)\end{array}\right\}\right\}.$$

Given an initial status of n_t, the optimal decision sequences of politicians define an equilibrium path of $\left(\alpha_s,q_s\right)_{s=t}^{\infty}$ until the regime collapse. □

23.3 Proof of Proposition 6

Proof. (i) Productivity Shocks

Lagrangian

$$\mathcal{L}=\mathbb{E}_t\left\{\sum_{s=t}^{\infty}\delta_g^{s-t}\prod_{i=t}^{s-1}\Gamma\left(n_i,A_i\right)\cdot\left[x_g-c\left(\alpha_s\right)\right]+\lambda_s\left[\left(1-\alpha_s\right)n_s+\gamma\cdot\left(1-n_s\right)-n_{s+1}\right]\right\}$$

F.O.C.

$$\frac{\partial \mathcal{L}}{\partial \alpha_s} = \delta_g^{s-t} \prod_{i=t}^{s-1} \Gamma\left(n_i, A_i\right) \cdot \left[-c^{'}\left(\alpha_s\right)\right] + \lambda_s\left(-n_s\right) = 0$$

$$\lambda_s = \delta_g^{s-t} \prod_{i=t}^{s-1} \Gamma\left(n_i, A_i\right) \cdot \frac{\left[-c^{'}\left(\alpha_s\right)\right]}{n_s}$$

$$\frac{\partial \mathcal{L}}{\partial n_{s+1}} = \mathbb{E}_{A_{s+1}|A_s} \left\{ \begin{array}{c} \delta_g^{s+2-t} \prod_{i=t}^{s} \Gamma\left(n_i, A_i\right) \cdot \Gamma_n^{'}\left(n_{s+1}, A_{t+1}\right) \left[x_g - c\left(\alpha_{s+2}\right)\right] \\ +\lambda_{s+1}\left(1 - \alpha_{s+1} - \gamma\right) \end{array} \right\} - \lambda_s = 0$$

Euler equation:

$$\mathbb{E}_{A_{s+1}|A_s} \left\{ \begin{array}{c} \delta_g^{s+2-t} \prod_{i=t}^{s} \Gamma\left(n_i, A_i\right) \cdot \Gamma_n^{'}\left(n_{s+1}, A_{s+1}\right) \left[x_g - c\left(\alpha_{s+2}\right)\right] \\ -\delta_g^{s+1-t} \prod_{i=t}^{s} \Gamma\left(n_i, A_i\right) \cdot \left[c^{'}\left(\alpha_{s+1}\right)\right] \frac{\left(1 - \alpha_{s+1} - \gamma\right)}{n_{s+1}} \end{array} \right\}$$

$$+\beta \prod_{i=t}^{s-1} \Gamma\left(n_i, A_i\right) \cdot \left[c^{'}\left(\alpha_s\right)\right] \frac{1}{n_s} = 0$$

$$\mathbb{E}_{A_{s+1}|A_s} \left\{ \begin{array}{c} \delta_g^2 \Gamma\left(n_s, A_s\right) \cdot \Gamma_n^{'}(n_{s+1}, A_{t+1}) \left[x_g - c(\alpha_{s+2})\right] \\ -\delta_g \Gamma\left(n_s, A_s\right) \cdot \left[c^{'}\left(\alpha_{s+1}\right)\right] \frac{\left(1 - \alpha_{s+1} - \gamma\right)}{n_{s+1}} \end{array} \right\}$$

$$+\left[c^{'}\left(\alpha_s\right)\right] \frac{1}{n_s} = 0$$

Rearrange the last equation as follows,

$$\Gamma(n_s, A_s) \mathbb{E}_{A_{s+1}|A_s} \left\{ \begin{array}{c} \delta_g^2 \Gamma_n^{'}(n_{s+1}, A_{t+1}) \left[x_g - c(\alpha_{s+1})\right] \\ -\delta_g \left[c^{'}\left(\alpha_{s+1}\right)\right] \frac{\left(1 - \alpha_{s+1} - \gamma\right)}{n_{s+1}} \end{array} \right\} = -\frac{c^{'}\left(\alpha_s\right)}{n_s}.$$

The current productivity state is $A_t = A^l$,

$$-\frac{c^{'}(\alpha_s)}{n_s} = \left\{ \begin{array}{c} \Gamma(n_s, A_s)\mathbb{E}_{A_{s+1}|A_s}\left\{\delta_g^2\Gamma_n^{'}(n_{s+1}, A_{t+1})\left[x_g - c\left(\alpha_{s+1}\right)\right]\right\} \\ -\Gamma(n_s, A_s)\left\{\delta_g\left[c^{'}\left(\alpha_{s+1}\right)\right]\frac{(1-\alpha_{s+1}-\gamma)}{n_{s+1}}\right\} \end{array} \right\}$$

$$\begin{aligned} -\frac{c^{'}(\alpha_s)}{n_s} &= \Gamma\left(n_s, A_s\right)\left\{ \begin{array}{c} \delta_g^2\left[x_g - c\left(\alpha_{s+1}\right)\right]\left\{\underbrace{\rho\Gamma_n^{'}\left(n_{s+1}, A^l\right)}_{\text{Stay in low } A} + \underbrace{(1-\rho)\Gamma_n^{'}\left(n_{s+1}, A^h\right)}_{\text{Switch to high } A}\right\} \\ -\left\{\delta_g\left[c^{'}\left(\alpha_{s+1}\right)\right]\frac{(1-\alpha_{s+1}-\gamma)}{n_{s+1}}\right\} \end{array} \right\} \\ &< \Gamma\left(n_s, A_s\right)\left\{ \begin{array}{c} \delta_g^2\left[x_g - c\left(\alpha_{s+1}\right)\right]\left\{\Gamma_n^{'}\left(n_{s+1}, A^h\right)\right\} \\ -\left\{\delta_g\left[c^{'}\left(\alpha_{s+1}\right)\right]\frac{(1-\alpha_{s+1}-\gamma)}{n_{s+1}}\right\} \end{array} \right\}. \end{aligned}$$

The smaller value of $\Gamma_n^{'}(n, A)$ in recession, i.e.,$\Gamma_n^{'}\left(n_{s+1}, A^l\right) < \Gamma_n^{'}\left(n_{s+1}, A^h\right)$, implies that the term in left side of equation, $-\frac{c^{'}(\alpha_s)}{n_s}$, is smaller than the that in normal time. And $c^{'}(\alpha_s)$ is larger and α_s is larger, since $c^{'}(\alpha)$ is increasing function of α.

The increase of ρ will the α in current period, implying that the persistence of recession will increase the efforts of anti-corruption in current period for safety in the future.

(ii) Preference Shocks

Write the Euler equation here for convenience and rearrange it

$$\Gamma\left(n_s, A_s\right)\mathbb{E}_{A_{s+1}|A_s}\left\{ \begin{array}{c} \delta_g^2\Gamma_n^{'}\left(n_{s+1}, A_{t+1}\right)\left[x_g - c(\alpha_{s+1})\right] \\ -\delta_g\left[c^{'}(\alpha_{s+1})\right]\frac{(1-\alpha_{s+1}-\gamma)}{n_{s+1}} \end{array} \right\} = -\frac{c^{'}(\alpha_s)}{n_s},$$

$$\Gamma\left(n_s, A_s\right)\mathbb{E}_{A_{s+1}|A_s}\left\{ \begin{array}{c} \delta_g^2\left[-\Gamma_n^{'}\left(n_{s+1}, A_{t+1}\right)\right]\left[x_g - c(\alpha_{s+1})\right] \\ \delta_g\left[c^{'}(\alpha_{s+1})\right]\frac{(1-\alpha_{s+1}-\gamma)}{n_{s+1}} \end{array} \right\} = \frac{c^{'}(\alpha_s)}{n_s}.$$

We know that $\Gamma_n^{'}(n, A)$ is negative and $c^{'}(\alpha)$ is positive, so

$$c^{'}(\alpha_s) = n_s\Gamma\left(n_s, A_s\right)\mathbb{E}_{A_{s+1}|A_s}\left\{ \begin{array}{c} \delta_g^2\left[-\Gamma_n^{'}\left(n_{s+1}, A_{t+1}\right)\right]\left[x_g - c\left(\alpha_{s+1}\right)\right] \\ \delta_g\left[c^{'}(\alpha_{s+1})\right]\frac{(1-\alpha_{s+1}-\gamma)}{n_{s+1}} \end{array} \right\}$$

define an increasing function, denoted as$\alpha_s = F\left(\delta_g\right)$. When is δ_g is lower, after a adverse shock to patience, the anti-corruption efforts become lower. □