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PhD in Economics

"R&D networks, market structure and industry evolution"

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ABSTRACT: The goal of this thesis is to investigate at the theoretical level the relationship between R&D networks, from one side, and market structure and industry evolution, from the other. Recent years have seen an upsurge in technological collaborations among firms, which now constitute a structural feature of competition, especially in high tech sectors. In this work, we developed a static and a dynamic model to study the role of R&D partnerships in affecting market structure, industry evolution and firms' performance.

The thesis is constituted by three papers. In the first paper, "Interfirm technological alliances and the evolution of industries: a survey of the empirical literature", we survey the several streams of the empirical literature on interfirm technological alliances. The evidence is rich, coming from several disciplines whose theoretical frameworks are sometimes radically different.

First, this paper proposes a number of stylized facts concerning the relevance of the phenomenon, its evolution over time, the differences across sectors and the most common motivations that lead firms to cooperate. Second, we produce some stylized results concerning the formation of technological alliances, the structural properties of the R&D networks and the effects of firms' cooperative activity on performance and technological capabilities.

The broad picture which emerges is one in which interfirm technological agreements are structural elements of the evolution of high sectors. Cooperation is part of the innovative strategies of large firms, the main actors in the network, which perform R&D also on an individual basis and look for partners with complementary capabilities to introduce new products and processes. The network, which becomes the "locus of innovation", is strongly driven by path dependence mechanisms, in which the central actors tend to increase their prominence, and it significantly affects firms' innovative and economic performance.

In the second paper, "R&D networks with heterogeneous firms", we develop a static model of R&D network formation. The paper models the formation of R&D networks in an industry where firms are technologically heterogeneous, extending previous work by Goyal and Moraga (2001). While remaining competitors in the market side, firms share their R&D efforts on a pairwise base, to an extent that depends on their technological capabilities. In the class of symmetric networks, the complete network is the only stable network, although not necessarily profit or social welfare maximizing. However, when we allow deviations by coalitions, the complete network is not stable because firms have the incentive to alter market structure by coordinating the exclusion of other firms from the network Then, we extend the analysis to asymmetric structures, which turn out to have interesting properties in terms of stability, aggregate profits and social welfare. When technological groups are asymmetric in size, firms in the smaller technological group can exploit their special position in terms of performance, even if this does not result in the role they play in the network.

In the third paper, "The evolution of R&D networks", we model dynamically the formation of R&D networks. In particular, the paper focuses on the coevolutionary process involving firms' technological capabilities, market structure and the network of interfirm technological agreements.
The main result the R&D network can work as a strong selection mechanism in the industry, creating ex post asymmetries among ex ante similar firms. This is due to a self-reinforcing, path-dependent process, in which events in the early stages of the industry affect firms' survival in the long run. In this framework, both market and technological externalities created by the formation of cooperative agreements play a crucial role. Although the R&D network creates profound differences at the beginning, which are reflected by an unequal distribution of links, it tends to eliminate them as it becomes denser and denser. The nature of the technological environment affects the speed of the transition and some of the characteristics of the industry in the long run.

Overall, the two theoretical papers coherently point at the role that R&D networks can play in shaping the nature of competition in industries, creating asymmetric market structures and significantly affecting firms' performance. These results are consistent with the empirical evidence summarized in the first paper.

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Interfirm technological alliances and the evolution of industries: a survey of the empirical literature[1]

1. Introduction

The aim of this paper is to review the empirical literature on interfirm technological agreements. Together with the phenomenon it describes, this literature has increased exponentially in the last years. Several suggestions on the rationale and effects of interfirm technological agreements have been proposed, receiving various degrees of confirmation by the empirical evidence. At the same time, the efforts have been interdisciplinary, with contributions coming from different disciplines, like economics, sociology and management (see Caloghirou *et al.*, 2003; Gulati *et al.*, 2000; Hagedoorn *et al.* 2000; Powell and Grodal, 2004, for previous recent surveys).

After having discussed the empirical evidence, this paper suggests a perspective which is relatively uncommon in the literature: interfirm technological agreements and R&D networks are seen as *structural elements* in the evolution and dynamics of industries. Our main point is that the existing empirical literature can constitute the basis for an appreciative theory of the role of R&D networks in industry evolution, which should be particularly appealing for economists interested in improving their knowledge on the fundamental link between technological progress and market structure. Furthermore, such a theory can be conceived as a step towards further empirical analysis and formal modelling.

The paper is structured as follows. Section 2 is introductory and preliminary: we define interfirm technological agreements, discuss the sources of data, and provide some basic

[1] I thank Franco Malerba, Lorenzo Cassi, Nicoletta Corrocher and Roberto Fontana for very useful and detailed comments on a preliminary version of this paper. The usual disclaimers apply.

evidence on the relevance and the evolution of the phenomenon over time, and on the broad motivations leading firms to collaborate. Section 3 and 4 constitute the core of the paper. Section 3 reviews the studies that consider the formation of technological agreements. First, we consider the characteristics at the firm, industry and dyadic level that affect firms' propensity to enter into cooperative agreements. Second, we discuss the structural properties of the network resulting from the collaborations firms have in place. Section 4 surveys the studies that treat technological agreements as explanatory variables, considering the effects of agreements on firms' innovative and economic performance and on firms' technological profiles. This distinction is mainly adopted for expository reasons, since the two aspects are clearly interrelated. On the basis of the existing empirical evidence, section 5 proposes some themes for an appreciative theory of R&D networks and industry evolution. Finally, section 6 concludes.

2 Definition, data and stylized facts

This section starts with the definition of interfirm technological agreements that we will use in this paper. This will help us in delimiting the object of this survey. Then, we will discuss the main sources of data that have been used in the literature. This is done because the lack or limited availability of data has been a typical concern, weakening the reliability of results. Finally, we will present a number of "stylized facts" concerning the evolution of the phenomenon over time and its motivations.

2.1 Definition

The definition of interfirm technological agreements is adapted from Hagedoorn (2002):

Interfirm technological agreements are defined as common interests between independent industrial partners, which are not connected through majority ownership, and in which R&D is at least part of collaborative effort, through some arrangements for transferring technology or joint research.

This definition immediately excludes from the analysis all the agreements that are only concerned with production (like standard long term buyer-supplier contracts) or marketing joint ventures. Agreements that have *also* production or marketing elements, which are quite common in practice, are included. For instance, an agreement involving the joint development *and* the production of a component to be used by the collaborating firms fits our definition. At the same time, we do not consider informal cooperation among firms, occurring for instance through information exchange among engineers or scientists (Von Hippel, 1987), or cooperation among firms and universities (Mowery and Sampat, 2004).[2]

The definition is broad enough to accomplish several ways in which firm can collaborate. Cooperation can occur through various legal arrangements, implying different degrees of resources commitment, different levels and directions of technological flows, different coordination mechanisms, and different time horizons. Examples of interfirm technological agreements are R&D joint ventures, where two or more firms constitute a new legal entity in order to perform R&D activities; joint R&D agreements, where firms share resources to undertake joint R&D projects; licensing and cross-licensing agreements; research contracts, where one partner, usually a small R&D specialized firm, performs research activity for another firm.

2.2 Data sources

The datasets used in the empirical analyses can be grouped in three classes:

1. Literature-based datasets

Several datasets have been collected by consulting specialized journals, financial newspapers and other publicly available sources of data. The MERIT-CATI dataset (Hagedoorn, 2002), which we discuss more at length in the following sections, is probably the most comprehensive in terms of coverage of industrial sectors and time horizon. At the same time, industry specific datasets have been collected as well, like

[2] Nevertheless, some of the studies we will survey consider datasets including both formal and informal cooperation, and interfirm and firm-university technological agreements.

the ARPA database developed at Politecnico di Milano for ICT sectors (Colombo and Garrone, 1996).

Although the collection of these datasets has greatly improved our knowledge of interfirm technological alliances, these types of data suffer from several limitations: agreements are known only if made public by the firms themselves; a general bias exists in favor of large, well-known firms, more fashionable technologies, Anglo-Saxon countries; information about the dissolution of agreements is less easily available than data on their formation.

2. Surveys

Some works have used data collected through questionnaires, in which firms are asked explicit questions about the extent of their collaborative activities, the motives behind them, and types of collaborators (i.e., competitors, customers, suppliers or universities).

In particular, a number of papers (for instance, Veugelers and Cassiman, 2002: Tether, 2002) used data from the Community Innovation Surveys (CIS), collected by the statistical offices of the Member States according to a common European Standard, for the analysis of innovative inputs and outputs by European firms.

The problems with these kinds of data are those that are usually implied by survey analyses: results may depend on questions formulation; a degree of discretion in respondents' answers cannot be avoided; a careful analysis for non-respondent biases must be performed.

3. Data from public-funded R&D programs and antitrust authorities

A third class of data concerns government-sponsored cooperative agreements and antitrust laws.

In Europe, a cornerstone of technological policy is constituted by the programs (in particular, the Framework programs) promoted by the European Union to foster collaboration among firms (but also universities and research centers). Data on projects resulting from these programs have been recently collected and analyzed (see for instance Breschi and Cusmano, 2004).

For the US, data have been collected using information from the Federal Register at U.S. Department of Justice (Vonortas, 1997). Under the National Cooperative Research

Act, voluntary filings of R&D partnerships give firms benefits in case of anti-trust interventions.
Finally, for Japan Branstetter and Sakakibara (2002) have analyzed R&D consortia with a degree of government subsidization and intervention.
These types of datasets may suffer from selection bias given by the criteria according to which firms ask and obtain funds, or decide to register the partnership.

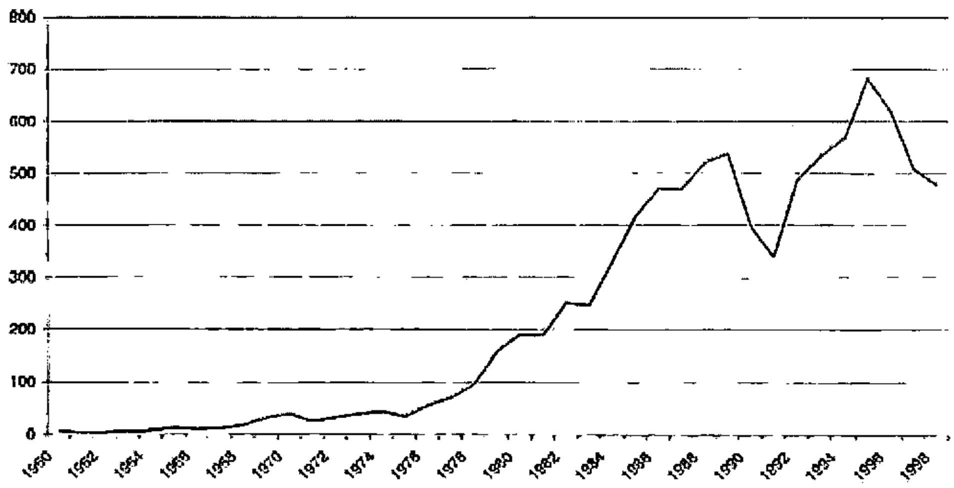
2.3 Stylized facts: relevance and trends

In order to give a flavor of the basic stylized facts concerning interfirm technological agreements, we will refer to the MERIT-CATI database, whose data have been collected through years by John Hagedoorn and colleagues. The references are given by the papers containing a descriptive account of the database (Hagedoorn, 1993; Hagedoorn, 2002). The dataset is constituted by more than 10000 agreements signed among more than 4000 firms worldwide, between 1960 and 1998. Data involve several sectors, at different level of R&D intensity. The dataset excludes from the analysis publicly-funded agreements.

Some basic stylized facts, which find generally confirmation in the analysis of other datasets, can be summarized as follows.

1. In terms of the number of *newly established* agreements, worldwide and for all sectors, we see that, after a limited growth in the 1960s and 1970s, the number of agreements has exhibited highly significant growth rates in the 1980s, and since then it has been showing a cyclical behavior with a positive trend in the 1990s (see Figure 1). We observed no more than ten partnerships established each year during the 1960s; 160 at the end of the 1970s; nearly 700 new partnerships in the peak in 1995.

Figure 1- New established R&D partnership (1960-1998) Source: Hagedoorn (2002)-MERIT-CATI database.



2. *Sectoral differences* exist and are significant. Classifying sectors in high tech, medium tech and low tech, according to their R&D intensities, we can see that the overall increase in the number of agreements has been accompanied by a significant increase of the high tech industries share. Figure 2 shows that, while in 1960 medium tech sectors (instrumentation and medical equipment, automative, consumers electronics and chemicals) accounted for about 70% of the total numbers of newly established agreements, and the remaining share was composed by partnerships in the high tech sectors (Computers, software, microelectronics, telecommunications), in 1998 the situation is reversed, with high tech sectors accounting for more than 80% of the newly established agreements.

At a more disaggregate level, we observe that, within the high tech sectors, the ICT industry (Computers, software, microelectronics, telecommunications) plays a strikingly important role, constituting 50% of the total number of agreements at the end of the sample period. Pharmaceutical (which includes biotechnology) also contributes in a significant way to the agreements in high tech sectors, with approximately 30% of all newly established partnerships.

Figure 2: new R&D partnerships, for low, medium, high tech industries (percentages). Source: Hagedoorn (2002).

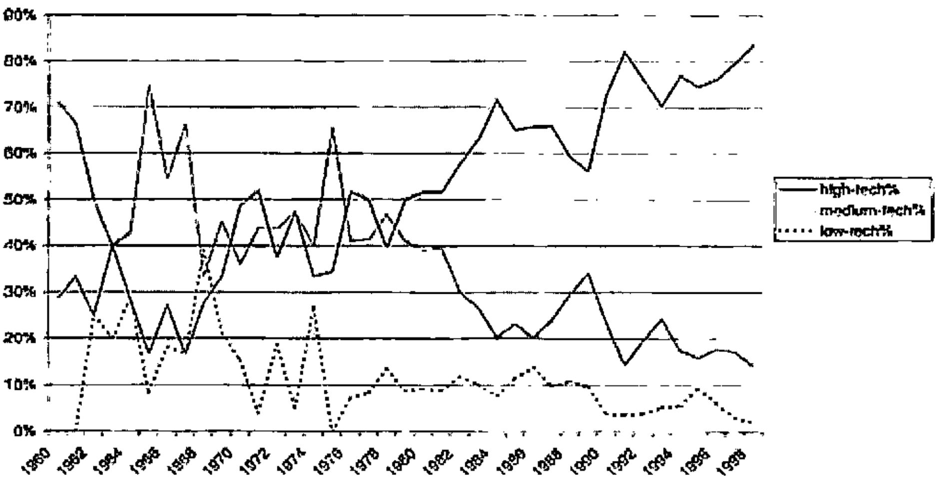
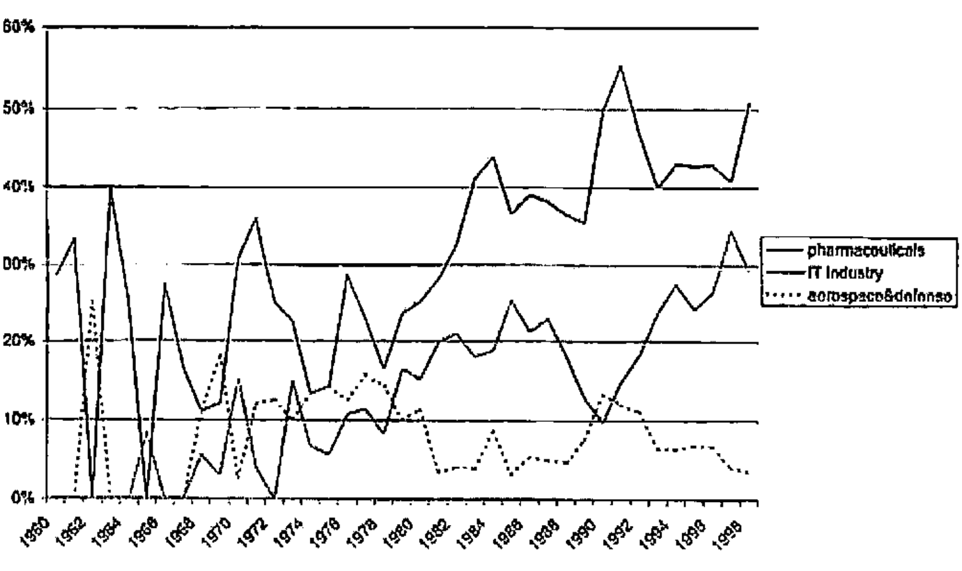


Figure 3: newly established R&D partnerships in high tech industries (percentages). Source: Hagedoorn (2002)



3. Finally, it is possible to investigate the role of different *modes of cooperation*, in different sectors over time. Hagedoorn (2002) divides the modes of cooperation in two broad categories: joint ventures and contractual arrangements (as R&D pacts, customer-supplier relations and licenses). Joint ventures are usually characterized by higher set up costs and a long term orientations, as compared to the flexibility and generally shorter term orientation of contractual forms. Then he defines a relative contractual partnering index for each sector $RCI_i = \frac{CP_i / JV_i}{TCP / TJV}$, where CP_i is the number of sectoral

contractual partnerships, JV_i the number of sectoral joint ventures, and TCP and TJV the total number contractual partnerships and joint ventures, respectively. The value of the index across decades and sectors is reported in Table 1. The range for this index is $[0,\infty)$, where values larger than 1denote a relative importance of contractual forms in that particular sector compared with the average value. We can notice that, especially focusing on the last two decades, contractual forms are prevalent exactly in those industries where partnerships are numerous (ICT and pharmaceuticals).

Table 1. Relative contractual partnering index of selected sectors during 1960-1998 Source: Hagedoorn (2002)

Sectors	*1960-1998*	*1970-1979*	*1980-1989*	*1990-1998*
Pharmaceuticals	2.65	2.48	2.29	1.48
Information Technology	1.06	0.91	1.27	1.64
Aereospace/Defence	7.94	5.34	3.57	0.58
Automative	1.32	3.16	0.46	0.57
Chemicals	0.38	0.26	0.35	0.24
Instruments and medical equipment	0.00	0.18	0.92	1.64
Consumer electronics	0.00	0.99	0.28	1.18

In a nutshell, the stylized facts reported in this section show that what has to be explained is prominently a "recent" phenomenon, concerning flexible, short-term forms of cooperation in high tech industries (ICT and pharmaceuticals).

2.4 Stylized facts: motivations

Several industry case studies have stressed the rationale for cooperation in specific cases. For a broader view, it can be useful again to refer to the CATI database. On the basis of the several and (partially) contrasting motives for cooperation mainly put forth by business scholars in terms of appreciative theoretical or empirically grounded considerations (see Hagedoorn, 1993, for references to relevant theoretical literature), each agreement of the database is assigned to one or more of the following motives:

1.Searching for complementarities, synergies, and cross-fertilization between technological and scientific fields, in sectors characterized by increased technological complexity, in which no firm can master all the relevant knowledge base required to innovate.

2. Reducing costs and risks in R&D and exploiting economies of scale.

3. Performing basic or "pre-competitive" R&D.

4. Monitoring new technological opportunities, possibly followed by the development of new products, and entry in new markets.

5. Facing the shortening of the product life cycle and the reduction of innovation time-span (i.e. the period between discovery and introduction into the market), which lead firms to cooperate to reduce the period of development.

6 Market positioning, i.e. modifying market structure in firm's favor against rivals in domestic and international markets.

7. Mostly "hidden" motives, like capturing rival's tacit knowledge, technology transfer, "technological leaprofrogging".

Several alliances are assigned to more than one category, since some of these categories operate at different levels (i.e. "market" vs. "technology" level) and consequently they are not mutually exclusive. Hagedoorn (1993) provides a ranking of *importance* among the different motives across different sectors.

Table 2: motives for cooperation, 1980-1989. Selected sectors. Source: Hagedoorn, 1993.

	Technological complementarities	Basic R&D	Lack of financial resources	Reduction innovation time span	Market access/ structure	High cost/risks	Monitoring technology/market entry
Pharmaceuticals	35%	10%	13%	31%	13%	1%	15%
Computers	28%	2%	2%	22%	51%	1%	10%
Software	38%	2%	4%	36%	24%	1%	11%
Microelectronics	33%	5%	3%	33%	52%	3%	6%
Aerospace/Defense	34%	0%	1%	26%	13%	36%	8%
Automotive	27%	2%	2%	22%	52%	4%	4%
Chemicals	16%	1%	1%	13%	51%	7%	8%
Instruments and medical equipment	35%	2%	4%	40%	28%	0%	10%
Consumer electronics	19%	0%	4%	19%	53%	2%	11%

Two comments are necessary: first, although sectoral differences exist and are significant, nevertheless some motives exhibit a general prominence. Such motives are searching for technological complementarities, shortening of the innovative time span and influencing market structure.

Second, a general, broad view of the role of R&D partnerships is suggested by these data, which is consistent with a detailed account of specific industries. In high tech industries, innovation is more and more complex, building on several technological fields. This is the case in pharmaceuticals, after the new discoveries in molecular biology in the mid 1970s, and in microelectronics, where innovation hinges on competences in fields as different as solid physics, construction of semiconductor manufacturing and testing equipment, and programming logic. Firms cannot possess all the relevant knowledge required to innovate and therefore they look for partners having complementary capabilities to face an increased rate of introduction of new products and processes, to monitor new opportunities and enter new markets, to sustain long-lasting competitive advantage.

3. The formation of technological agreements

After the broad introduction to the general relevance and underlying motives of technological alliances, in this section we go more in depth reviewing the empirical studies that treat the formation of interfirm technological agreements. Using the terminology by Geroski (1995), in this section and in section 4 we will provide a number of "stylized empirical results". Concerning the formation of technological agreements, we distinguish three levels of analysis.

1. A first series of studies has focused on *the firm level*. Scholars have tried to identify firms' characteristics (for instance, size, age, technological capabilities) and industries' characteristics (for instance, concentration and appropriability of innovation) that affect firms' propensity to enter into collaborative agreements, the total number of agreements and the total number of partners.
2. The second level of analysis is the *dyad* (i.e. the single pair of firms involved in an agreement). In this case, the studies investigate the characteristics of the firms that increase the probability of an agreement between them, and analyze how the choice of mode for cooperation is affected by firms' attributes (including their history of collaboration).
3. The third level is the *network* of R&D alliances. Recent studies have investigated the structural properties of these networks, their evolution over time, and the relation between network measures at the firm level and the propensity to enter into new alliances.

3.1 The formation of technological agreements: the firm's and industry level

Several studies have estimated econometric models that link firms' and industries characteristics to the propensity of setting up cooperative ventures (Logit or Probit models) and to the intensity of collaborative activities, measured by the number of technical agreements firms are involved in or by the number of partners they have (Poisson and negative binomial regressions).

3.1.1 The firm's level

Many empirical studies have recurrently found a *positive* impact of some variables at the firm's level on firm's collaborative activities.

1. *Size.* Firms that are active in interfirm technological agreements are typically large firms. This is one of most robust findings of this literature. A positive relation between size and propensity to form interfirm alliances or between size and the number of technical agreements is found by Link and Bauer (1987), Kleinknecht and Reijen (1992), Hagedoorn and Schakenraad (1994), Colombo (1995), Colombo and Garrone (1996), Siebert (1996),Vonortas (1997), Ahuja (2000a), Fritch and Lukas (2001), Bayona *et al.* (2001), Tether (2002),Veugelers and Cassiman (2002), Hernan *et al.* (2003), Becker and Dietz (2004). This evidence is robust across time, sectors and countries[3]. Large firms are likely to engage in a wide range of economic activities, increasing the opportunities for cooperation. A "cost spreading" argument (Cohen and Klepper, 1996) may apply to technological agreements as to R&D in general: large firms can spread the gain from innovation over a larger base of economic activity, increasing their incentives towards cooperative agreements (as a form of R&D investment). Some forms of cooperative agreements (for instance R&D joint ventures) entail high physical and legal set-up costs for which small firms lack financial resources. Finally, large firms can have significant bargaining power in contracting with their partners.

2. *R&D intensity and technological capabilities.* Using data from the UK CIS 2 survey on 1275 innovating firms, Tether (2002) shows that performing R&D on a continuous basis and intensively has a significantly positive effect on firms' propensity to cooperate. Similar results are obtained by Fritch and Lukas (2001) from a survey on German firms, and by Bayona *et al.* (2001) from a survey on Spanish firms. Link and Bauer (1987) find a positive value for *absolute* R&D in explaining cooperation activity.

[3] For exceptions, see Shan (1990) (who found a negative sign) , Pisano (1989) and Arora and Gambardella (1990) (who found size as non significant) and Burgers Hill and Kim (1993) (who found a non-monotonic relationships). However, Shan focuses on small biotech firms, Pisano and Arora and Gambardella on large pharmaceutical firms, Burgers Hill and Kim on the world largest car producer. All these studies focus on a relatively small number of size classes. This can explain the results.

Ahuja (2000a), in his sample of 97 leading firms in the chemicals industry, shows the stock of patents positively affects the number of agreements. Sakakibara (2002) introduces the variable "R&D capabilities", defined as the difference between firm and industry R&D intensity, and finds that this variable positively affects Japanese firms' participation to government sponsored R&D consortia. Arora and Gambardella (1990) find that increasing the stock of patents that large pharmaceuticals and chemicals firms have in biotechnology increases the number of external linkages these firms have with specialized biotech firms[4]. In a similar vein, Stuart (1998) shows that technologically "prestigious" firms (i.e., firms whose patents are highly cited) are more likely to form technological agreements, in a sample of semiconductor firms.

The possibility that R&D intensity and the number of technical agreements are not strongly exogenous has been successfully tested by Colombo and Garrone (1996). Their sample is composed by agreements by firms in the semiconductor, data processing and telecommunication sectors. Colombo and Garrone (1998) estimate a simultaneous two equations structural model, to find a significantly positive effect of R&D intensity on the number of technical agreements, while the coefficient for the reverse relation is not significant. A similar two equation models is estimated by Becker and Dietz (2004), who find significantly positive effects in both directions.

These results suggest that internal and cooperative R&D should be seen as complementary rather that substitute. The most common explanation for this result is the role of absorptive capacity (Cohen and Levinthal, 1989). In order to evaluate and fully absorb the outcomes from cooperative ventures, firms need to have pre-existing capabilities in those scientific or technological fields. This implies that firms lacking technological capabilities are not in the position to reap the benefits from cooperation. This view is confirmed by Stuart (1998), which shows that firms in more crowded technological areas are more likely to form new agreements. This is explained by claiming that such firms have many potential partners for which they possess the relevant absorptive capacity.

3. *Experience*. Firms that have more experience in managing collaborative ties (usually measured by the cumulated number of past alliances or by the number of partners in previous years) are more likely to enter collaborative agreements. This is the result

[4] However, Pisano (1990) finds that biotech experience decreases the propensity of pharmaceuticals firms to start external projects with specialized biotech firms.

obtained by Gulati (1995a), Powell *et al.* (1996), Ahuja (2000a), Sakakibara (2002), Hernan *et al* (2003), Okamura and Vonortas (2004). In the business literature, this result is usually explained by referring to the notion of "cooperative capability" (Gulati, 1998). With experience, firms learn how to manage their collaborative ties, to develop interfirm knowledge sharing routines and funnel results inside the organization, to govern contractual arrangements where there is room for moral hazard and incompleteness, to initiate necessary changes in the partnership as it evolves over time. This experience increases their returns from technical agreements.

A second, complementary explanation points at the role of previous partners as an important information source about new opportunities for agreements and new potential partners. We will come back to this point in sections 3.2 and 3.3, discussing the dyadic and the network level.

The relevance of these variables (size, R&D intensity, and experience in managing ties) is already enlightening on the "strategic" nature of cooperative agreements. Interfirm technological alliances are an important, persistent part of the innovative strategies by large and technologically leader firms, rather than a defensive tactic by small firms lacking the ability of innovating alone[5]. This perspective is confirmed by the results of surveys (Fritch and Lukas, 2001; Tether, 2002) which show that firms' propensity to enter into collaborative agreements is higher when firms aim at introducing "breakthrough innovation" (e.g., radically new products).

3.1.2 The industry level

Concerning industry level factors, section 2 already showed that the intensity of the phenomenon of interfirm strategic alliances varies across sectors and a positive relationship exists between sectoral R&D intensity and the number of R&D alliances in that sector: technological agreements are particularly common in high tech sectors (Hagedoorn, 1993). More rigorously, Hernan *et al.* (2003) confirm this evidence, finding a significantly positive coefficient for R&D intensity on firms' participation to

[5] See for instance the IBM's webpage dedicated to partners for a practical example (www.pc.ibm.com/ww/alliances).

R&D joint ventures in the Eureka and Framework programs.[6] We focus in this section on two other industry specific variables: concentration and appropriability. It is worth noting, however, that the number of cross industries studies have been restricted by the limited availability of large data sets. For this reason, this evidence seems less robust than the one presented in the previous paragraphs.

1.*Concentration*. Link and Bauer (1987), Sakakibara (2002), Hernan *et al.* (2003) find that R&D cooperation is more likely to occur in concentrated industries. It is argued that in oligopolistic markets is easier to find the appropriate partners or finding the consensus towards cooperation. Furthermore, market power associated with such structures allows firms to appropriate the return from the cooperative investment. This result also emerges in a pioneering study by Pfeiffer and Nowack (1976), who found a positive relation between concentration and the number of joint ventures at the industry level in US manufacturing firms[7].

It is worth mentioning that the opposite result (a negative relation between concentration and the rate of formation of strategic alliances) is found by Eisenhardt and Shooven (1996). These authors consider a sample of 102 US new firms in the semiconductor sector, and find that the number of competitors in the segment in which the firm operate positively affects the rate of alliances formation. The authors relate this to the gains of accessing external resources, when market conditions are difficult. This result can be conciliated with the previous ones if one considers that, while Eisenhardt and Shooven focus on new (and typically small) firms, the papers we previously discussed are concerned with large, established firms. This suggests that the cooperative strategies of new and established firms may differ significantly, as they are differently affected by industry characteristics.

2. *Appropriability*. Different authors inspired by the economic theories of R&D cooperation have tested the link between the degree of appropriability of R&D investments and R&D cooperation. Indeed, models of R&D cooperation in the IO

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[6] However, Becker and Dietz (2004) found that technological intensity has a *negative* impact on the likelihood of cooperation.

[7] On the contrary, Becker and Dietz (2004) find that the concentration is not significant as explanatory variable of cooperation.

tradition (d'Aspremont and Jacquemin, 1988; Kamien, Mueller and Zang, 1992) identify in the internalization of R&D spillovers[8] one of the main rationale for R&D cooperation.

When sectoral measures of R&D appropriability are introduced as explanatory variables for firms' propensity to cooperate, the sign of the corresponding coefficient turns out to be negative, in accordance with the theory: higher spillovers lead to more cooperation (Hernan *et al.*, 2003; Sakakibara, 2002, Okamura and Vonortas, 2004)[9].

3.2 The formation of technological agreements: the dyadic level

The studies considering the characteristics of the dyad and the probability of cooperation have focused on two main dimensions.

1. The first dimension is technological. One concern of the literature has been to assess the probability of two firms forming a collaborative link, as a function of their technological distance, empirically measured on the basis of their patent portfolios.

A first argument claims that firms need to be close in the technological space for being good partners. This is related, again, to an absorptive capacity argument. As long as firms use technological alliances in order to learn, they need to have preexisting knowledge in the partner's field of expertise to better absorb its capabilities. At the same time, cognitive proximity is required for effective communication to occur.

This hypothesis is confirmed by the works by Stuart (1998) and Okanamura and Vonortas (2004). Stuart (1998) defines firms' technological positions using patent citations for a sample of semiconductor firms, and finds that proximity in such a space increases the likelihood of alliance formation. Okanamura and Vonortas (2004) find that an increase in technological proximity (measured by the similarity of patent portfolios) has a positive effect on link formation for US research joint ventures.

[8] R&D spillovers constitute a form of externality, whose relevance is inversely related to the degree of appropriability.

[9] This result partially contrasts with Veugelers and Cassiman (2002). These authors find that the relevance of outgoing spillovers *at the firm level* negatively affects cooperation. These are related, but do not coincide, with the inverse of the degree of appropriability at the industry level, because they are also affected by firms' strategic considerations.

However, if firms are technologically too close, opportunities for learning decrease. Firms need to be sufficiently dissimilar for technological complementarities to be exploited through collaboration. Mowery *et al.* (1998) find evidence of such an effect. In a sample of 151 international joint ventures in several sectors, they find an inverted U relationship between partners' technological overlap (measured by the cross citation rate and common citation rate in patent portfolio) and the probability of alliance formation. In other words, firms need to be "not too distant nor too close" from a technological point of view (Nooteboom, 1999).

2. The second dimension can be defined as "social" or "relational". Technological alliances are usually complex arrangements for which uncertainty and investment appropriability are relevant issues. For the particular nature of the transaction involved, there is significant room for opportunistic behavior, and, conversely, there is a role for trust building among partners.

A quite robust result in this stream of literature is that firms tend to ally with previous partners (Gulati, 1995a; Stuart, 1998; Gulati and Gargiulo, 1999, Okamura and Vonortas, 2004). Firms, with familiarity, can build trust, lowering transaction costs and limiting the risk of opportunistic behaviors; they can also choose organizational forms that are more flexible (Gulati, 1995b). At the same time, they can develop routines and codes in order to increase the effectiveness of communication with the partner and control the flows of knowledge.

Indirect links among firms are important as well. Common previous partners have to play two main roles: first, they constitute sources of information about potential partners for new collaborative opportunities; second, they can reduce the asymmetric information among the potential partners, providing an indirect reputation effect. Gulati and Gargiulo (1999) find that the number of indirect links (common partners) has a positive effect on the probability of link formation at the dyadic level.

3.3 The formation of technological agreements: the network level

In recent years, there has been a substantial shift of attention from the dyadic to the network level, spurred by the massive contributions by sociologists in the field.

The structure of the overall network of alliances resulting from firms' (uncoordinated) choices matters for two reasons. First, theoretical contributions stress that the network structure has an impact on the level of efficiency of the industry (Cowan and Jonard, 2003 and 2004). In other words, the structure of the network of alliances is a factor that may explain cross-sectional variation in the rate of technological progress. Second, firms' position in the network can affect their propensity to enter into new alliances, in general and at the dyadic level, as well as their economic and innovative performance.

The first structural characteristic that has been extensively considered is the existence of *cliques*, or more in general cohesive sub-groups of firms within the network[10].
There are two main reasons for which we should expect cliques to emerge in networks of technological alliances. Both reasons are related to the contributions that cliques give to the building of "social capital", defined as the sum of resources that accrue to a firm by virtue of possessing a durable network of relationships.
The first reason can be labeled as "cognitive". Firms that share many common partners can develop a common language for cooperation, practices and routines, which favors the creation of new knowledge and its transmission among the firms in the clique. The second reason can be labeled as "reputational", and in turn can be divided into two motivations. *Ex post* (once the link is formed), the participation to a clique can favor cooperation in a context of contractual incompleteness, because in presence of opportunistic behavior, the information about a "deviation" by a firm can spread among the partners, increasing its cost. *Ex ante* (before the alliance is formed), common partners can reduce the degree of information asymmetry about firms' competences and trustworthiness, then favoring the formation of links.
The existence of cohesive sub-groups has been shown in a number of sectors. Nohria and Garcia-Pont (1991) consider 35 leading firms in the automobile industry, and the 133 alliances they formed in the 1980s. They detect six "strategic" blocks. It turns out that strategic blocks are composed by firms with complementary capabilities, and are such that firms in each block have access to a similar set of capabilities. The analysis of Gomes-Casseres (1996) shows that competition in the personal digital assistants market has been characterized, since its inception, by alliance groups of firms coming from

[10] See Wasserman and Faust (1994), ch. 7, for a general discussion on the different notions of cohesive sub-groups.

different sectors (computer hardware and software, telecommunications and consumer electronics).

This view of social capital as "closure" (Coleman, 1988), which stresses the benefits of clustering in networks, is often set against the "structural holes" argument (Burt, 1992). Burt considers players (individual or organizations) in a competitive arena (for instance, a market). Such a competitive arena is characterized by a "social" context, defined as a social network among the players.

The theory suggests that the players' position in the network should help explaining their performance in the competition. In particular, a player's performance should be positively correlated with the extent to which the player manages non redundant contacts in its network. Contacts are defined as redundant if they are connected by a strong relationship (cohesion criterion), or when they have, in turn, the same contacts (redundancy by structural equivalence). Whenever two contacts are non redundant, a structural hole is assumed to exist between them.

Players that occupy structural holes can enjoy higher rates of return from their investments. Non redundant contacts are more likely to give them timely access to diverse sources of information (being the players exposed to more rewarding opportunities), as well to give control over such information, in order to secure more favorable terms in the opportunities they choose to pursue.

In the case of technological alliances, the network among firms is mostly seen as a conduit of information about technology (for instance about more or less promising technological directions). In this perspective, firms in a clique have by definition redundant links, and according to this view, a non efficient structure of the ego-network[11].

Burt's argument has clearly a normative flavor. Firms should fill structural holes, because this allows them a higher rate of return. We will mention in the next section studies that test this hypothesis.

A study by Walker *et al.* (1997) considers how the rate of alliance formation depends on the structure of the networks in which firms are embedded, for a sample of biotech firms

[11] Ego (-centered) networks (Wasserman and Faust, 1994) is defined as a network consisting of a focal actor (ego), a set of alters who have ties to ego, and measurements on the ties among these actors. Extensively, some authors have considered as ego network the focal actor, all the actors at a finite distance from ego, and all the ties among them.

in the period 1984-1988. They find that firms endowed with "social capital" (located in dense areas of the network) form more links that firms active in less dense areas (full of structural holes). At the same time, new links tend to increase the level of social capital.

The social capital and structural holes views are not incompatible. If we assume the existence of advantages (at the firm level) of being located in a clique and having (some) non-redundant contacts, we could expect firms in a cliquish network to have some "long-distance" connections. Watts and Strogatz (1998) show that networks with these characteristics exhibit a "small world" property (low average distance, even in a cliquish, sparse network), because some "short-cuts" among otherwise disconnected areas dramatically reduce the average distance among actors. Theoretical models (Cowan and Jonard, 2003 and 2004) have shown that "small world networks" (networks exhibiting both high cliquishness and low average distance) are the most efficient in the process of knowledge creation and diffusion.
From the above considerations, it is natural to ask if firms' innovative networks are "small worlds". The answer from existing studies is generally "yes". Verspagen and Duysters (2004) find a "small world" network for the alliances of the two sectors they analalyze: chemicals and food (639 firms in their sample) and electronics and ICT (837 firms). Cowan and Jonard (2003) find a small world in the network of firms participating in the BRITE/EURAM programme and the network of research institutes from the TSER programme. Breschi and Cusmano (2004) find high clustering and low average distance for the network of firms, universities and research institute participating to the 3rd and 4th Framework programs.

A question that has not been addressed by the empirical literature on the technological networks is the identity of firms that activate "short cuts" between separated cliques. From a methodological point of view, it is worthwhile to mention the work by Baum *et al.* (2003) whose research question is concerned with the formation of "small world" network. Their theory is that a small world structure emerges from a cliquish network, through clique-spanning ties. Baum *et al.* (2003) want to understand the identity of the actors that activate such ties and propose three alternatives explanations: 1) chance: while firms add new links, this increase the probability that some of them will be

outside the cliques; 2): insurgent partnering, activated by peripheral firms in the network that aim at improving their status; 3) control partnering, activated by central firms that attempt to preserve their privileged position. They consider the network of Canadian investment banks, emerging from underwriting syndicates over the period is 1952-1990. They find support for all the three explanations, but especially for the chance and insurgent partnering motives.
This kind of exercise would be worthy to be replicated on interfirm technological alliances networks. It seems interesting to study if the characteristics of the information that circulate in the network (information on technology vs. other kind of information) may affect firms' incentives towards clique-spanning ties.

Finally, the distribution of collaborative links across firms has been studied. Typically, we observe a *hierarchy* within the firms in the network: a few firms have many links and many firms have a few links. The distribution of links typically follows a power law distribution ($P(k) = k^{-\gamma}$, where k is the firms' number of links, and typically $\gamma \approx 2$): these structures are defined as scale-free networks. Barabasi and Albert (1999) show that this structure can emerge in a growing network if a preferential attachment mechanism is at work: the probability of a new connection at time $t+1$ positively depends on the number of connections a firm has at time t. We have seen in section 3 that this property is found at the firm level. Studies that find evidence of scale-free networks are Krebs (2004) for the Internet Industry; Breschi and Cusmano (2004); Riccaboni and Pammolli (2002) for networks in life sciences and ICT. Typically large firms take the role of "hubs" (highly connected firms).

4. Technological agreements and firms' performance and technological capabilities

This section surveys the studies that treat several dimensions of firm R&D cooperative activity as explanatory variables. Sub-section 4.1 considers the fundamental question of the causal relation between technological agreements and economic and innovative performance. Sub-section 4.2 considers the effects of technological alliances on firm's technological specialization.

4.1 Technological agreements and economic and innovative performance

Firms enter technological agreements because they predict to increase in this way their *expected* performance. However, two issues remain relevant: first, the distribution of returns from cooperative ventures; second, a more precise quantitative assessment of such effects in general, and the factors that positively or negatively affect their magnitude.

In general, assessing the success or the failure of a cooperative venture is not an easy task. Often the true goal of cooperation is not known, to the public and to partners, and also when it is the case, side effects can be important. When the termination date of an agreement is not fixed *ex ante*, its dissolution is both consistent with a failure (i.e. the objective of cooperation has not been reached and cannot be reasonably reached in the future), and with a success (i.e. the goal has been reached) (Kogut, 1988).

However, it is less problematic to assess the relationship between the different dimensions of a firm cooperative strategy and its overall economic performance (measured in terms of rate of profits, sales growth, market shares, productivity or survival). In some cases, the object of study has been the link between innovative output and technological agreements (Sampson, 2003; Cusmano, 2005).

Several dimensions of cooperative strategy have been considered.

1. A positive relationship is usually found between firms' participation in cooperative ventures, number of agreements and number of partners, and firms' performance.

A number of studies with a policy orientation have aimed at estimating the effects on firms' performance of their participation to government sponsored agreement. Benfretello and Sembenelli (2002), in their sample of firms from several sectors participating to the Eureka and Third and Fourth Frameworks programs sponsored by the European Union, find a significantly positive effect on the *ex-post* firm performance measured in terms of total factor productivity, labor productivity and price cost margin. On a similar sample, Cusmano (2005) finds a positive effect from participation in research joint ventures on innovative output in the medical and biotechnological sector, but not in the information technology sector. Studying the performance of R&D

Japanese consortia, Branstetter and Sakakibara (2002) show that participation to the consortia increases the productivity of firms in terms of innovative output. Similar results have obtained for non-government sponsored partnerships: Siebert (1996) shows that the elasticity of profit margin to R&D is higher for firms participating in Research Joint ventures filled at the US Federal registered.

The intensity of cooperative activities (measured by the number of technical agreements) has usually a positive effect on company's performance. Hagedoorn and Schakenraad (1994) find a positive effect of the intensity of strategic alliances with an R&D orientation on firm's profitability for a sample of large firms in different sectors and countries. Mitchell and Singh (1996) show a positive effect of the number of technical agreements on firm's survival on a sample of US firms in the hospital software systems industry. In a sample of 85 biotech firms, Shan *et al.* (1994) show that commercial ties have positive effects on innovative output.

Some studies have considered the number of partners of a firm (in the social network analysis terminology, the degree centrality of the firm in the innovative network), and their characteristics, as explanatory variables of its performance. A positive relation between sales growth and the degree centrality in the network is found by Powell *et al.* (1996) in a sample of 225 dedicated biotech firms. For plant biotechnology, Delackere *et al.* (1998) find a positive relation between the number of partners and innovative output, measured by scientific publications. Stuart (2000), in a sample of semiconductor firms, shows that partners' innovativeness has a greater impact on firm's patenting rate and sales growth than the simple count of technical agreements, and finds that partners' sales matter for growth especially if firms are small or young (this is explained with reference to the status enhancing effect of these alliances). Baum *et al.* (2000) consider a sample of 142 start-ups in biotechnology, and show a positive effect on firms' performance (measured by revenues, employment and patents) of the number of alliances with pharmaceutical firms, the variety in the type of partners (pharmaceutical firms, university, biotech firms, etc) and the number of alliances with rivals with a narrower product scope. As indirect evidence for the same effect, Singh and Mitchell (1996) show that a firm's likelihood of survival in the hospital software system industry decreases if a partner shuts down, and the firm does not form a new partnership.

2. A small group of studies has tried to assess the impact of characteristics at the dyadic level (or more, generally, at the project level) on firm's innovative performance. Consistent with the evidence on alliance formation, the results generally show that technological proximity has a positive and significant effect, with evidence of an inverted U relationship. Branstetter and Sakakibara (2002), in their sample of R&D Japanese Consortia, find that technological proximity among consortium members has a positive effect on ex-post firm's patenting activity. Sampson (2003) finds an inverted U relationship between technological distance and *ex post* innovation output (measured by citation weighted patent count) in sample of 463 alliances in the international telecommunication equipment industry.

3. Finally, recent papers have studied how the structure of the ego-networks impacts on firms' performance. The main question concerns the opposition between a notion of a social capital *à la* Coleman and the Burt's structural holes argument.
Ahuja (2000b) considers a sample of 107 chemicals firms, and investigate the roles of direct ties, indirect ties and structural holes in explaining innovation output measured by patents. He finds that direct ties (more concerned with knowledge creation) have a strong positive effect on innovation output; indirect ties (concerned with information diffusion) have a positive effect but smaller than direct ties; filling structural holes has a negative effect on innovative output; the coefficient for the direct tie-indirect tie interaction is negative, indicating a substitution effect between the two. This result supports the "social capital as closure" perspective.
Hagedoorn and Duysters (2002) consider 88 firms in the computer industry, and they use patent intensity (computer patents/size) as measure of technological performance. They find that having non redundant contacts and bridge ties has no significant effect on firm's performance (which contrasts with Burt's view), while multiple, repeated links with the same partner have a positive effect on firms innovative output. They claim that this result is consistent with a learning view of alliances, while it contrasts with the static, efficiency-based view by Burt.

4.2 Technological agreements and firms' technological capabilities

Together with the effects of cooperation on firms' performance, some interest has been raised by the effect of technological alliances on firms' technological profiles. An empirical assessment of this issue is relevant for two main reasons. First, such exercise can be seen as empirical test for the hypothesis of technological alliances as sources of learning. *Ex post* technological convergence among partners would be consistent with such hypothesis. Second, these results have implications for a dynamic theory of partnership formation, as long as the (resulting) technological positions affect the probability of firms to form links in the following periods (Section 3.2).

There is evidence that strategic alliances are significant factors in explaining firms' movement in the technological space. Stuart and Podolny (1996) consider a small sample of 10 Japanese semiconductor firms, and they characterize their technological positions using patent citations. They find that alliances are part of the strategies of firms that want to move from a peripheral to a core position in the technological space. However, there is evidence of an ambiguous effect of alliances on technological positions. Mowery *et al.* (1996) consider a sample of 792 alliances in several sectors. They measure firms' technological overlap by cross citation rates in patent portfolio and test the hypothesis of an increase in the technological overlap after collaboration. They reject this hypothesis, but they find a significant and positive effect of collaboration in the absolute value of variation in cross citation rate. This result leads the authors to distinguish between alliances through which firms acquire new capabilities, causing technological convergence (191 alliances in their sample), and alliances in which firms aim at accessing new capabilities, leading to divergent technological positions (601 alliances in their sample). The authors do not investigate the factors (at the level of industry, technology, or mode of organization of the alliance) that lead to one outcome or the other, and we are not aware of studies that consider this issue. This seems an interesting line of research to pursue.

5 Interfirm technological agreements and industry evolution

As the previous sections have shown, the evidence on interfirm technological agreements is becoming very rich, although some aspects still wait for a satisfactory analysis. In this section we will argue that the existing empirical literature can constitute the basis for an "appreciative theory" (Nelson and Winter, 1982) that links the self-organization of R&D networks to the rate and the direction of technological progress, to the actors involved in the innovative process, and through these, to the evolution of industries. The formation of R&D networks is a self-organizing process because such networks are the result of uncoordinated firms' choices over time, as a function of technological variables (for instance firms' technological positions) and economic variables (for instance firms' size) (section 3). In turn, these variables change over time as a function of the network (section 4), so that the dynamics of the system is characterized by several feedbacks, mostly positive (self-reinforcing) in nature (like, for instance, the "preferential attachment" mechanism). Such an appreciative theory, whose elements have been at least partially already put forth by some authors (Gomes Casseres, 1996), should be of obvious interest to economists. Furthermore, it can be conceived as a step towards further empirical analysis and formal modelling, which are instead missing.

There are at least three, interrelated, themes that emerge as important in the relationship between technological collaborations, R&D networks and industry evolution.

The first is the role of *path dependency*. At the firm level (section 3.1) we saw that experience in managing ties is an important variable in explaining firms' cooperative activities. At the dyadic level (section 3.2), we gave account of several studies that show how firms tend to ally with previous partners. Furthermore, the history of alliances by a firm explains the formation of its technological capabilities (section 4.2), which in turn affect the selection of its partners (section 3.2). If firms that are active in the network in the early stages are more likely to be central actors in the subsequent periods, and this is reflected in firms' performance, events at the beginning of an industry (or network) life cycle can have long lasting effects on firms' competitiveness. Such events can be due to initial, significant differences in capabilities or can consist of "historical accidents", as geographical location or preexisting social contacts among entrepreneurs. Theories of industrial dynamics with an evolutionary flavor (such as the industry life cycle theory, Klepper, 1997) frequently stress the importance of first mover advantages in explaining both the prosperity of firms and some stylized facts of industry evolution, like the shake-outs (i.e. the drastic reduction in the number of firms that often occurs in industry in the early stages). R&D networks seem to work in this direction, and their role of providing first mover advantages deserves further empirical and theoretical analysis.

The second theme, which directly refers to the first one, is related to the role of networks as both mechanism of technological knowledge diffusion for firms within the network and exclusionary mechanism for firms outside the network. This has clear implications for the evolution of industries.

If no firm possesses all the relevant technological capabilities to innovate, it is the network to act as the "locus of innovation" (Powell *et al.*, 1996). This tends to favor competition, since no firm can control the market via distinctive technological capabilities. However, as we just mentioned, path dependency and self-reinforcing mechanisms, both at the firm and at the dyadic level, tend to limit over time the number of actors that actively participate in the network. An oligopolistic market structure emerges, where a core of large firms (section 3.1) controls the rate and the direction of technological progress, erecting barriers to entry and to survival against firms outside the network ("knowledge-based networked oligopoly", in the terminology of Delapierre and Mytelka, 1998). This view is consistent with the so-called Schumpeter Mark II paradigm for the link between market structure and technological progress (Schumpeter, 1942): incumbent, "networked" firms are the main actors of innovation. In a policy perspective (for antitrust authorities and governments' that subsidize technological cooperation) this logic suggests that anti-competitive effects may be more dynamic than static, and these must be traded-off with the dynamic gains from an increased technological progress. Finally, the network can be composed of different cohesive sub-groups, so that competition occurs among groups, rather than at the firm level (section 3.3). In an industrial dynamics perspective, belonging to different groups can explain interfirm differences in exit rates, growth, economic performance and innovativeness (Gulati *et al.*, 2000).

A third theme is related to the role of networks in affecting the "collective" direction of technological change in industries. This is probably the theme for which most of the work is still to be done. The extent to which collaborations lead to technological convergence or technological divergence among firms in the network (section 4.2) is important for two main reasons.

First, from the society point of view, a certain degree of experimentation at the technological level must be preserved. Using an evolutionary terminology, variety generation mechanisms must be present. Firms need to explore different routes in environments characterized by substantive uncertainty, a distinctive feature of Schumpeterian competition. If firms in a network explore collectively the same areas of the technological space, risks of technological "lock-in" are possible. Indeed, some authors have argued that advantages of the network form of organization compared to more integrated forms lie in the capacity of preserving variety at the technological level (Kogut, 2000).

An important role in this respect can be played by the existence of different cliques. Even if lock-in may exist at the level of the single sub-groups of firms, this can be counterbalanced by different groups exploring different technological directions. Similarly, as argued in section 3.3, variety and access to novel information can be guaranteed by short-cuts or clique-spanning ties in a "small world" network.

Second, networks matter when firms face technological discontinuities. A traditional distinction here is between competence-enhancing discontinuities, favoring incumbent firms versus new entrants, and competence-destroying discontinuities, favoring new entrants versus incumbents (Tushman and Anderson, 1986). This distinction has been adapted to networks by Madhavan *et al.* (1998). These authors define structure reinforcing events as those discontinuities which favors incumbent firms in the network, leading to an increase in their centrality, and structure loosing events as those discontinuities which favor more peripheral agents, reducing the degree of centralization in the network. Similarly, Rosenkpof and Tushman (1998) discuss the link between network intensity and the stages of technological life cycles. They show that in the flight simulation industry the rate of founding of technical agreements is high at the discontinuities, and cliques emerge in mature phases. In general there are opportunities, both at the theoretical and empirical level, for studying the role of network structures in mediating between technological discontinuities and their consequences on industry evolution. When we can distinguish between different cliques, their internal structure and the capabilities to which firms have access may influence how they react to environmental shocks.

6. Conclusion

This paper has surveyed the several streams of the empirical literature on interfirm technological alliances. As we tried to show, the evidence is rich, coming from several disciplines whose theoretical frameworks are sometimes radically different.
First, this paper has proposed a number of stylized facts concerning the relevance of the phenomenon, its evolution over time, the differences across sectors and the most common motivations that lead firms to cooperate. Second, we have produced some stylized results concerning the formation of technological alliances, the structural properties of the networks and the effects of firms' cooperative activity on performance and technological capabilities.
The broad picture which emerges is one in which interfirm technological agreements are structural elements of the evolution of high sectors. Cooperation is part of the innovative strategies of large firms, the main actors in the network, which perform R&D also on an individual basis and look for partners with complementary capabilities to introduce new products and processes. The network, which becomes the "locus of innovation", is strongly driven by path dependence mechanisms, in which the central actors tend to increase their prominence, and it significantly affects firms' innovative and economic performance.

These results notwithstanding, there are still promising lines of research, both at the empirical and theoretical level. First, we need theories and empirical studies that identify more precisely the general mechanisms that drive the formation and evolution of alliances and networks. These studies would surely benefit from a more unified framework, where insights from transaction cost economics, game theory, evolutionary economics, sociology and managerial sciences are considered. Second, clear sectoral specificities exist, in the form of intensity of alliances, their content and their mode of organization. This, presumably, may reflect in the structure of the network at the sectoral level. All these characteristics depend on the technological regimes, which are specific to industries (Malerba, 2004). In this respect, we need detailed case studies of network evolution, taxonomies and theories for specific mechanisms of collaboration in specific contexts.

Given the relevance of the phenomenon, its complexity and multidimensionality, these are surely exciting opportunities for future research.

References

Ahuja, G. (2000a), "The duality of collaboration: inducements and opportunities in the formation of Interfirm Linkages", *Strategic Management Journal,* 21, 317-343.

Ahuja, G. (2000b), "Collaboration networks, structural holes, and innovation: a longitudinal study", *Administrative Science Quarterly*, 45, 425-55.

Arora, A. and Gambardella, A.(1990), "Complementarity and external linkages: the strategies of the large firms in biotechnology", *Journal of Industrial Economics*, 38(4), 361-379.

Bayona, C., Garcia-Marco T. and Huerta, E. (2001), "Firms' motivations for cooperative R&D: an empirical analysis of Spanish firms", *Research Policy*, 30, 1289-1307.

Baum, J. A. C., Calabrese, T. and Silverman, B.S. (2000), "Don't go it alone: alliance network composition and start-ups' performance in Canadian biotechnology", *Strategic Management Journal*, 21, 267-294.

Baum, J. A. Shipilov, A. V. and Rowley, T.J. (2003), "Where do small worlds come from?", *Industrial and Corporate Change*, 12(4), 697-725.

Barabasi, A. and Albert, R. (1999), "Emergence of scaling in random networks", *Science* 286, October, 509-512.

Becker, W. and Dietz, J. (2002), "R&D cooperation and innovation activities of firms-evidence for the german manufacturing industry", *Research Policy*, 33, 209-223.

Benfretello, L. and Sembenelli, A. (2002), "Research joint ventures and firm level performance", *Research Policy*, 31, 493-507.

Branstetter, L. G. and Sakakibara, M. (2002), "When do research consortia work well and why? Evidence from japanese panel data", *The American Economic Review*, 92(1), 143-159.

Breschi, S. and Cusmano,L (2004), "Unveiling the Texture of a European Research Area: emergence of oligarchic networks under EU Framework Programmes", *International Journal of Technology Management. Special Issue on Technology Alliances*, 27 (8), 747-772.

Burgers WP, Hill, C.W. and Kim WC (1993), "A theory of global strategic alliances: the case of the global auto industry", *Strategic management journal*, 14(6), 419-432.

Burt R. S. (1992), "Structural Holes: The Social Structure of Competition", Harvard University Press.

Caloghirou, Y. Ioannides, S. and N. Vonortas (2003), "Research joint ventures: A critical survey of theoretical and empirical literature", *Journal of Economic Surveys*, 17(4), 541-570.

Cohen, W. and Levinthal, D. (1989), "Innovation and learning: the two faces of Research and Development", *The Economic Journal*, 99, 569-596.

Cohen, W. and Klepper, S. (1996), "A reprise on size and R&D", *The Economic Journal*, 106, 925-951.

Coleman, J. C. (1988), "Social capital in the creation of human capital", *American Journal of Sociology*, 94, 95-120.

Colombo, M.G. (1995), "Firm size and cooperation: the determinants of cooperative agreements in information technology industries", *International Journal of the Economics of Business*, 2(1), 3-29.

Colombo, M.G. and Garrone, P. (1996), "Technological cooperative agreements and firms' R&D intensity. A note on causality relations", *Research Policy*, 25,923-932.

Colombo, M.G. and Garrone, P. (1998), "A simultaneous equation model of technological agreements and inframural R&D", in M.G. Colombo (ed.), *The changing boundaries of the firm*, Routledge Press, London.

Cowan, R. and Jonard, N. (2003), "The Dynamics of Collective Invention", *Journal of Economic Behavior and Organization*, 52 (4), 513-532.

Cowan, R. and Jonard, N., (2004), "Network Structure and the Diffusion of Knowledge", *Journal of Economic Dynamics and Control*,28(8), 1557-1575.

Cusmano, L. (2005), "Self-selection and learning in European Research Joint Ventures: a microeconometric analysis of participation and patenting" in Y.Caloghirou,

N. Constantellou, N. Vonortas (eds.), *Knowledge Flows in European Industry: Mechanisms and Policy Implications*, Routledge, forthcoming.

d'Aspremont, C. and Jacquemin, A. (1988), "Cooperative and noncóoperative R&D in duopoly with spillovers", *The American Economic Review*, 78,1133-1137.

Debackere, K., Clarysse, B., Rappa, M. (1996), "Dismantling the Ivory Tower: The Influence of Networks on Innovative Output in Emerging Technologies", *Technological Forecasting and Social Change* 53, 139-154.

Delapierre, M. and Mytelka, L. (1998), "Blurring boundaries: new inter-firm relationships and the emergence of networked, knowledge-based oligopolies" in M.G. Colombo (ed.), *The changing boundaries of the firm*, Routledge Press, London.

Eisenhardt, K. M. and Schoonhoven, C. B. (1996), "Resource-Based View of Strategic Alliance Formation: Strategic and Social Effects in Entrepreneurial Firms", *Organization Science* 7(2), 136-150.

Fritsch, M. and Lukas, R. (2001) "Who cooperates on R&D?", *Research Policy*, 30, 297-312.

Geroski, P.A. (1995) "What do we know about entry?", *International Journal of Industrial Organization*, 13, 421-440.

Gomes-Casseres, B. (1996) "The Alliance Revolution", Harvard University Press.

Gulati, R.(1995a), "Social Structure and Alliance Formation Patterns: A Longitudinal Analysis", *Administrative Science Quarterly*, 40, 619-652.

Gulati, R. (1995b), "Does Familiarity Breed Trust? The Implications of Repeated Ties for Contracual Choice in Alliances", *Academy of Management Journal*, 38, 85-112.

Gulati, R. (1998), *"Alliances and Networks"*, Strategic Management Journal, 19, 293-317.

Gulati, R. (1999), "Network Location and Learning: The Influence of Network Resources and Firm Capabilities on Alliance Formation", *Strategic Management Journal*, 20, 397-420.

Gulati, R. and Gargiulo M. (1999), "Where Do Interorganizational Networks Come From?", *American Journal of Sociology*, 104(5), 1439-1493.

Gulati, R. Nohria, N. and Zaheer, A. (2000), "Strategic Networks", *Strategic Management Journal*, 21, 203-215.

Hagedoorn, J. (1993), "Understanding the rationale of strategic technology partnering: interorganizational modes of cooperation and sectoral differences", *Strategic management Journal*, 14, 371-385.

Hagedoorn, J. (2002), "Inter-firm R&D partnerships: an overview of major trends and patterns since 1960", *Research Policy*, 31, 477-492.

Hagedoorn, J. and Schakenraad, J. (1994), "The effect of strategic technology alliances on company performance, *Strategic Management Journal*, 15, 4, 291-311.

Hagedoorn,J. A.N. Link and N.S Vonortas, (2000) "Research partnerships", *Research Policy*, 29, 567-586.

Hagedoorn, J. and Duysters G. (2002), "Learning in dynamic inter-firm networks –the efficacy of multiple contacts", *Organization Studies*, 23, 525-548.

Hernan, R. Marin, P. L. and Siotis, G. (2003), "An empirical evaluation of the determinants of research joint venture formation", *The Journal of Industrial Economics*, LI (1), 75-89

Kamien, M., Mueller, M. and Zang, I. (1992), "Research joint ventures and R&D cartels", *The American Economic Review*, 82, 1293-1306.

Kleinknecht, A. and Reijnen, J.O.N. (1992), "Why do firms collaborate on R&D?", *Research Policy*, 21, 347-360.

Klepper, S (1997), "Industry life cycles", *Industrial and Corporate Change*, 6, 156-181.

Kogut, B. (1988), "Joint ventures: theoretical and empirical perspectives", *Strategic Management Journal*, 9, 319-332.

Kogut, B. (2000), "The Network as Knowledge: Generative Rules and the Emergence of Structure", *Strategic Management Journal* 21, 405-425.

Krebs. V. (2004), www.orgnet.com .

Link, A.N. and Bauer, L.L. (1987), "An Economic Analysis of Cooperative Research", *Technovation*, 6, 247-260.

Madhavan R., Koka B. R., Prescott J. E. (1998), "Networks in Transition: How Industry Events (Re)shape Interfirm Relationships", *Strategic Management Journal*, 19, 439-459.

Malerba, F. (2004), "Sectoral Systems: How and Why Innovation Differs Across Sectors", in Fagerberg, J. and Mowery, D.C. and Nelson, R. (eds), *The Oxford Handbook of Innovation*.

Mitchell, W. and Singh, K (1996), "Survival of Business Using Collaborative Relationships to Commercialise Complex Goods", *Strategic Management Journal* 17(3), 169-195.

Mowery, D. C., Oxley J. E., Silverman B.S., (1996) "Strategic Alliances and Interfirm Knowledge Transfer", *Strategic Management Journal*, 17, Winter Special Issue, 77-91.

Mowery D. C., Oxley J. E., Silverman B.S., (1998), "Technological Overlap and Interfirm Cooperation: Implications for the Resource-Based View of the Firm", *Research Policy*, 27, 507-523.

Mowery, D. C. and Sampat, B. (2004), "Universities in National Innovation Systems" in Fagerberg, J. and Mowery, D.C. and Nelson, R. (eds), *The Oxford Handbook of Innovation*.

Nelson, R. and Winter, S. (1982), "An evolutionary theory of economic change", Harvard University Press, Cambridge (MA).

Nohria, N. and Garcia-Pont, C. (1991), "Global strategic alliances and industry structure", *Strategic management journal*, 12, 105-134.

Nohria, N. and Garcia-Pont, C. (2002), "Local versus global mimetism: the dynamics of alliance formation in the automobile industry", *Strategic management Journal*, 23, 307-321.

Nooteboom, B. (1999), "Inter-Firm Alliances: Analysis and Design", London: Routledge.

Okamura, K. and Vonortas, N. (2004), "Choosing a partner", paper presented at the Schumpeter conference, Milan, June 9-12.

Pfeffer, J. and Nowak, P. (1976), "Joint venture and interorganizational interdependence" *Administrative Science Quarterly*, 21, 398-418.

Pisano, G. P. (1990) "The R&D boundaries of the firm: an empirical analysis", Administrative Science Quarterly, 35(1), 153-176.

Powell, W.W., Koput, K.W., Smith-Doerr L. (1996), "Interorganizational Collaboration and the Locus of Innovation: Networks of Learning in Biotechnology", *Administrative Science Quarterly*, 41, 116-145.

Powell, W.W. and Grodal, S. (2004) "Networks of innovators" in Fagerberg, J. and Mowery, D.C. and Nelson, R. (eds), *The Oxford Handbook of Innovation*.

Riccaboni, M. and Pammolli, F. (2002) "On firm growth in networks", *Research Policy*, 31, 1405-1416.

Rosenkopf, L. and Tushman, M. (1996) "The coevolution of community networks and technology: lessons from the flight simulation industry", *Industrial and Corporate Change*,7, 311-46.

Sampson, R. (2003), "R&D alliances and firm performance: the impact of technological diversity and alliance organization on innovation", mimeo.

Sakakibara, M. (2002), "Formation of R&D Consortia: industry and company effects", *Strategic management Journal*, 23, 1033-1050.

Schumpeter, J. (1942), "Capitalism, socialism and democracy", Harper and Row, New York.

Shan, W. (1990) "An empirical analysis of organizational strategies by entrepreneurial high technology firms", *Strategic Management Journal*, 11,129-139.

Shan, W., Walker, G., Kogut, B. (1994), "Interfirm cooperation and startup innovation in the biotechnology industry", *Strategic Management Journal*, 15(5),387-394.

Siebert, R. (1996), "The impact of research joint venture on firm perfomance: an empirical assessment", Discussion Paper FS IV 96-13 Wissenschaftszentrum Berlin.

Singh, K. and Mitchell, W. (1996), "Precarious collaboration: business survival after partners shut down or form new partnerships", *Strategic Management Journal*, 17, 99-115.

Stuart, T.E. (1998), "Network positions and propensities to collaborate: an investigation of strategic alliance formation in a high-technology industry", *Administrative Science Quarterly*, 43, 668-698.

Stuart T.E. (2000), "Interorganizational alliances and the performance of firms: a study of growth and innovative rates in a High-Technology Industry", *Strategic Management Journal*, 21, 791-811.

Stuart, T. E., Podolny J.M. (1996), "Local search and the evolution of technological capabilities", *Strategic Management Journal*, 17, 21-38.

Tether,B. (2002), "Who co-operates for innovation, and why. An empirical analysis" *Research Policy*, 31,947-967.

Tushman M. and Anderson, D. (1986), "Technological discontinuities and organizational environments", *Administrative Science Quarterly*, 31, 439-465.

Verspagen, B. and Duysters,G. (2004), "The small worlds of strategic technology alliances *Technovation*, 24(7),563-571.

Veugelers, R. and Cassiman, B. (2002), "R&D Cooperation and Spillovers: some empirical evidence", *American Economic Review*, 92(4), 1169-1184.

Von Hippel, E., (1987), "Cooperation between rivals: informal know-how trading"*Research Policy*, 16, 291-302.

Vonortas, N.S. (1997), "Cooperation in Research Development", Kluwer Academic Publishers, Boston.

Walker, G., Kogut, B., Shan W. (1997), "Social Capital, Structural Holes and the Formation of an Industry Network", *Organization Science* 8(2), 109-125.

Wasserman, S. and Faust, K. (1994), "Social network analysis", New York: Cambridge University Press.

Watts, D. J. and Strogatz, S. H. (1998), "Collective Dynamics of 'Small-World' Networks", *Nature* 393, June, 440-442.

R&D networks with heterogeneous firms[12]

1. Introduction

There is significant evidence that technological agreements among firms are becoming increasingly popular (Hagedoorn, 2002). Especially in high tech industries (i.e., ICT and biotechnology), firms more and more collaborate in the technological domain, under different forms, ranging from joint R&D to the exchange of knowledge through cross licensing agreements.

Several scholars in different disciplines have tackled the issue of explaining theoretically the phenomenon. Initially put forth by sociologists, but promptly accepted in the business literature, the *network* perspective has recently gained a prominent role. In a sociological perspective, the overall network emerging from the alliances in an industry matters because typically the position of a firm in the network is associated with variables like power, status and access to information. These variables, in turn, affect firm's performance (Powell *et al.*, 1996).

Recently, economists have shown interest in the formation of economic and social networks, and have developed formal tools to address this issue (Jackson and Wolinski,

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1996). R&D networks represent a natural application of such tools, and they have been studied by Goyal and Moraga (2001), Goyal and Joshi (2003) and Goyal *et al.* (2004).

This paper belongs to this last stream of literature. It extends previous work considering the role of technological heterogeneity. The issue of technological complementarity has been often mentioned by the empirical literature as an important motive for firms to enter into collaborative agreements. In high tech industries, innovation is more and more complex and building on several technological fields. This is the case in pharmaceuticals, after the new discoveries in molecular biology in the mid 1970s, and in microelectronics, where innovation hinges on competences in fields as different as solid physics, construction of semiconductor manufacturing and testing equipment, and programming logic. Firms cannot possess all the relevant knowledge required to innovate and therefore they look for partners having complementary capabilities to face an increased rate in the introduction of new products and processes, to monitor new opportunities and enter new markets, to sustain long-lasting competitive advantage.
Based on the MERIT-CATI database on world wide technological agreements (Hagedoorn, 1993), among the alliances formed in the period 1980-1989 technological complementarity is cited as a key motivation in 35% of alliances in biotechnology, 38% in new materials technology, 41% in the industrial automation sector and 38% in the software industry. In the sample considered by Mariti and Smiley (1983), technological complementarity constitutes the motivation of 41% cooperative agreements.

In the economic literature, there is a consolidated tradition of models of R&D cooperation (D'Aspremont and Jacquemin, 1988, Kamien *et al.*, 1992). These models usually identify R&D spillovers as the factor that can make cooperation among firms welfare improving, and in that respect they have a strong policy orientation. This literature analyzes cooperation occurring at the industry-wide level (Suzumura, 1992), or comparing exogenously given coalitions (Katz, 1986).
The literature on endogenous coalitions (i.e. partition of firms) in oligopolistic industries (Bloch, 1995) can be considered an extension allowing for strategic consideration on the cooperative side. In this paper, we consider networks of R&D collaborations, which is at the same time more restrictive (because we allow exclusively

coalitions of two firms) and less restrictive (because we do not require transitivity in the collaborative relations).

The paper is structured as follows. Section 2 describes the model, focusing on the extensions to the existing literature. Section 3 is concerned with symmetric networks. We first characterize the effect of different degrees of cooperative activity on R&D investments and production costs. Then, we consider the issue of stability of different network structures in a four firms industry, and their properties in terms of aggregate profits and social welfare. In section 4, we extend the analysis to asymmetric networks in a three firms industry. This leads us to consider a situation where the distribution of technological capabilities in the industry is asymmetric. As in section 3, we study the stability of the different network structures, and their properties in terms of aggregate profits and social welfare. In section 5, we introduce a refinement to the notion of stability used in the previous sections, which provides some interesting economic insights. Section 6 concludes.

2. The model

Informally, the model can be described as follows. We consider n firms in an industry, producing a homogenous good. In the product market, firms compete *à la* Cournot, i.e. choosing quantities. Before market competition, firms can engage in an R&D activity in order to reduce their unit cost of production. Firms can share their efforts on a bilateral basis, and this information sharing is what we define as collaboration. Firms are assumed to be heterogeneous from the technological point of view (for sake of simplicity, firms are divided in two groups). Suppose for instance that heterogeneity comes from different firms' specializations in the range of technological or scientific fields that are required for innovation. Technological heterogeneity has an impact on the consequences of collaboration: information sharing is assumed to be more effective for firms with different technological capabilities, due to the existence of technological complementarities between them.

Formally, we deal with a three-stage game Γ, which coincides with the one presented in Goyal and Moraga (2001). In the first stage, firms can form collaborative links, which give raise to a well specified R&D network. Given the network structure, firms choose non-cooperatively their R&D effort. Given the level of R&D efforts, the cost function of each firm is determined. Finally, given costs, firms compete in the market.

Let $N = \{1,...,n\}$ be the set of firms. Firms are identified by an index $r = 1,2$, which corresponds to the technological group a firm belongs to. $N^r \subseteq N$ represents the set of firms of group r. The R&D network resulting from the first stage is denoted by g. When we write $ij \in g$, this implies that there is a collaborative link between i and j. We define $N_i(g) = \{j \in N \setminus \{i\} : ij \in g\}$ as the set of firms having a collaborative link with i. Assume that firm i belongs to the technological group r. We can write $N_i(g) \equiv N_i^r(g) \cup N_i^{3-r}(g)$, that is we can partition the set of firms collaborating with i in the sets of firm belonging to the same technological group, $N_i^r(g) = \{j \in N^r \setminus \{i\} : ij \in g\}$ and to the other technological group, $N_i^{3-r}(g) = \{j \notin N^r : ij \in g\}$. Also, we indicate with $n_i(g) = |N_i(g)|$ the cardinality of the set of partners for firm i in g, and similarly for $n_i^r(g)$ and $n_i^{3-r}(g)$.

If g is the network resulting from the first stage, we denote with $\Gamma(g)$ the corresponding subgame. In such a subgame, firms fix their level of R&D expenditures correctly anticipating the Cournot outcome of the last stage. Firm i's action in this stage is given by $e_i \in [0, \bar{c}]$, where e_i is the effort put by firm i in the R&D activity. The cost associated to e_i is given by $C(e_i) = e_i^2$. Consequently, $e = (e_i)_{i \in N}$ is the action profile of $\Gamma(g)$.

With respect to Goyal and Moraga (2001), we modify the formulation of collaboration effects. Their paper strictly follows the representation of R&D activity that is standard in the literature on R&D collaboration and spillovers. Kamien *et al.* (1992) summarize the approach as follows:

"The R&D process (...) is supposed to involve trial and error. Put another way, it is a multidimensional heuristic rather than a one-dimensional algorithmic process. The individual firm's R&D activity does not involve following a simple path. If this were the case, the only spillover potential would be from the firm that had somehow forged ahead in the execution of the algorithm to the laggards. However, in an R&D process involving many possible paths and trial and error, it is unlikely that individual firms will pursue identical activities. Indeed it is reasonable for each firm to pursue several avenues simultaneously, the differences among the firms being in the greater emphasis each places on one over the others. The spillover effect in this vision of the R&D process takes the form of each firm learning something about the other's experience. This information, which may become available through deliberate disclosure or leak out involuntarily (e.g, at scientific conferences), enables a firm to improve the efficiency of its R&D process by concentrating on the more promising approaches and avoiding the others"

This view of R&D as a trial and errors process implies that the dimension of the space that firms can explore in their efforts is high, and firms are not "constrained" in their exploration. This derives from the hypothesis that, when information sharing is complete, duplication of efforts are completely eliminated. This assumption is justified because the focus is on the effects of different degrees of R&D appropriability on the desiderability of R&D collaboration.

In this paper we propose a different interpretation. We do not consider the question of R&D appropriability and we do not consider the degree of information sharing as a variable of choice. We assume that the capacity of other firms' R&D to be a substitute of a firm's R&D depends on the technological specialization of firms. We assume that the area of the technological space firms can explore that is constrained by their technological specialization. In a sense, firms are characterized by "competences", which implies a process of search which is necessarily local (Nelson and Winter,1982). Whenever firms belong to the same technological group, the probability that firms pursue the same path increases. If firms are heterogeneous in their technological capabilities, this creates possible opportunities for complementarities as the result of information sharing. Since we consider cost reducing R&D, we formalize the argument assuming that the fraction of R&D effort of firm j that is able to reduce firm's i costs when i and j cooperate is $\overline{\beta}$ if firms belong to different technological groups, and $\underline{\beta}$ if firms belong to the same technological group, with $1 \geq \overline{\beta} \geq \underline{\beta}$. The case discussed in Goyal and Moraga implies $\overline{\beta} = \underline{\beta} = 1$.

Two remarks are needed. First, when both $\overline{\beta}$ and $\underline{\beta}$ are high, information sharing is effective, independently of technological groups. In other words, the likelihood of effort duplication is low, or, in terms of our interpretation, firms have "naturally" several possible paths to follow. As long as an economic interpretation is concerned, we can relate this to a situation where the technological space that firms can explore is particularly rich. For that reason, when discussing our results about stability, aggregate profits and social welfare, we will refer to the notions of technological heterogeneity (measured by $\overline{\beta} - \underline{\beta}$) and technological opportunities (measured by the values of $\overline{\beta}$ and $\underline{\beta}$).

Second, the literature on the economics of innovation has argued theoretically and showed empirically the important role played by absorptive capacity (Cohen and Levinthal, 1989): in order to evaluate and absorb fully the outcomes from cooperative ventures, firms need to have pre-existing capabilities in those scientific or technological fields. Then, even if a firm may lack the knowledge possessed by another firm, it can fail in absorbing it. For our model, this implies that β can be more properly seen as the product of two parameters: γ, which captures the extent to which a firm possesses knowledge that is not possessed by the other firm (with $\overline{\gamma} > \underline{\gamma}$); and α, which captures the extent to which a firm can actually learn by the experience of the other firm, due to absorptive capacity (with $\overline{\alpha} < \underline{\alpha}$). According to this interpretation, we are assuming that the first effect prevails, in the sense that $\overline{\gamma\alpha} > \underline{\gamma\alpha}$.

Given the R&D investments e, the unit cost of production for $i \in N$ is determined by:[13]

[13] In line with Goyal and Moraga (2001), we assume that there are no *indirect* effects from link formation. This admittedly strong assumptions implies that a firm can exclude other firms from the returns of its R&D investment if information sharing is not explicitly agreed (say, because knowledge is embodied in machineries or protected by patents).

$$c_i(g,e) = \bar{c} - e_i - \sum_{j \in N_i^{1}(g)} \underline{\beta} e_j - \sum_{j \in N_i^{1-}(g)} \overline{\beta} e_j \tag{1}$$

Finally, given the costs $c_i(g,e)$, firms compete in the market choosing quantities. $q_i(g,e) \in [0, A]$ denotes the action taken by firm i at this stage. The inverse demand function is linear: $p = A - \sum_{i \in N} q_i(g,e)$. In the Cournot-Nash equilibrium, quantities are given by:

$$q_i(g,e) = \frac{A - nc_i(g,e) + \sum_{j \neq i} c_j(g,e)}{n+1} \tag{2}$$

Net profits are given by:

$$\Pi_i(g,e) = (q_i(g,e))^2 - C(e_i) \tag{3}$$

In the next sections, we will analyze the social welfare property of the different networks. In order to do that, we introduce the following social welfare function:[14]

$$W(g,e) = \sum_{i \in N} \Pi_i(g,e) + \tfrac{1}{2} Q(g,e)^2 \tag{4}$$

This is in the spirit of "second best" (Goyal and Moraga, 2001): we assume that for given network structure efforts are still chosen non-cooperatively and quantities are those resulting from the Cournot-Nash equilibrium.

[14] The second term represents consumer surplus, given the hypothesis of linear demand function with a 45° slope.

3. Symmetric networks

This section focuses on symmetric networks. Networks are symmetric when all the firms are equivalent in terms of connections (i.e. they have the same number of links inside and outside their technological group). With technologically homogenous firms, a symmetric network is characterized by a single value k identifying the number of links that any firm has. Goyal and Moraga define k as the degree of collaborative activity. Given the assumption of heterogeneous firms, however, our notion must change accordingly. In our case, a symmetric network is identified by a pair $k \equiv (k^r, k^{3-k})$, corresponding to the number of links that a representative firm has within and outside its technological group respectively, i.e. $n_i^r(g) = k^r$ and $n_i^{3-r}(g) = k^{3-r}$ $\forall i \in N$. We maintain the convention of calling this vector the degree of collaborative activity, and we indicate with g^k the symmetric network with degree of collaborative activity $k \equiv (k^r, k^{3-k})$. We can define a partial ordering over symmetric networks: $k_1 > k_2$ if $k_1^r \geq k_2^r$ and $k_1^{3-r} \geq k_2^{3-r}$, where at least one inequality is strict.

For the notion of symmetric network to be meaningful, we must restrict our attention to cases where N is given by two *equal* size groups of firms in even number. In this section we choose to concentrate and completely characterize the results for the case with n=4. Some results can be extended to generic n, but the complete analysis is quite difficult to obtain (also Goyal and Moraga, in their simpler framework, limit themselves to partial results).[15]

Given the network g and other firms' investments, the representative firm i maximizes $\Pi_i(g, e)$ in e_i subject to $e_i \in [0, \bar{c}]$. We need to consider five types of firms: a) firm i; b) k^r firms linked to firm i and belonging to its technological group (subscript lr); c) k^{3-r} firms linked to i and belonging to a different technological group (subscript $l3$-r);

[15] As explained by Goyal and Moraga (2001), it is difficult to generalize in the study of asymmetric networks. All the set of direct and *indirect* connections determines the maximization problem the firm has to solve. For each asymmetric network, one needs to solve a different system of first order conditions, in which the possibility of invoking symmetry may be limited. As we will see in section 3.1, the study of asymmetric networks is required to apply the definition of pairwise stability.

d) $\frac{n}{2}-k^r-1$ firms that are not linked to firm i and belong to its technological group (subscript mr); e) $\frac{n}{2}-k^{3-r}$ firms that are not linked to i and belong to the other technological group (subscript $m3\text{-}r$). This results in a specific cost structure for each type of firm:

$$c_i(g^k)=\bar{c}-e_i-\overline{\beta}k^{3-r}e_{l3-r}-\underline{\beta}k^r e_{lr} \tag{5a}$$

$$c_{lr}(g^k)=\bar{c}-e_{lr}-\sum_{j\in N_{lr}^{3-r}(g^k)}\overline{\beta}\,k^{3-r}\,e_j-\sum_{j\in N_{lr}^{r}(g^k)}\underline{\beta}k^r e_j \tag{5b}$$

$$c_{l3-r}(g^k)=\bar{c}-e_{l3-r}-\sum_{j\in N_{l3-r}^{3-r}(g^k)}\overline{\beta}\,k^{3-r}\,e_j-\sum_{j\in N_{l3-r}^{r}(g^k)}\underline{\beta}k^r e_j \tag{5c}$$

$$c_{mr}(g^k)=\bar{c}-e_{mr}-\sum_{j\in N_{mr}^{3-r}(g^k)}\overline{\beta}\,k^{3-r}\,e_j-\sum_{j\in N_{mr}^{r}(g^k)}\underline{\beta}k^r e_j \tag{5d}$$

$$c_{m3-r}(g^k)=\bar{c}-e_{m3-r}-\sum_{j\in N_{m3-r}^{3-r}(g^k)}\overline{\beta}\,k^{3-r}\,e_j-\sum_{j\in N_{m3-r}^{r}(g^k)}\underline{\beta}k^r e_j \tag{5e}$$

Plugging (5a-5e) into firm i's profit function and deriving with respect to e_i we obtain the following first order condition:

$$\frac{\partial \Pi_i}{\partial e_i}\equiv 2q_i(g,e)[n-k^r\underline{\beta}-k^{3-r}\overline{\beta}]-2e_i=0 \tag{6}$$

Invoking symmetry across all firms, we impose $e_i=e_{lr}=e_{l3-r}=e_{mr}=e_{m3-r}=e(g^k)$. Rearranging the first order condition, we obtain the equilibrium effort:

$$e(g^k)=\frac{(A-\bar{c})(n-\underline{\beta}k^r-\overline{\beta}k^{3-r})}{(n+1)^2-(n-\underline{\beta}k^r-\overline{\beta}k^{3-r})(1+\underline{\beta}k^r+\overline{\beta}k^{3-r})} \tag{7}$$

Plugging (7) into (5a), one obtains the unit cost of production for the representative firm:

$$c(g^k)=\frac{\bar{c}(n-\underline{\beta}k^r-\bar{\beta}k^{3-r})-A(n-\underline{\beta}k^r-\bar{\beta}k^{3-r})(1+\underline{\beta}k^r+\bar{\beta}k^{3-r})}{(n+1)^2-(n-\underline{\beta}k^r-\bar{\beta}k^{3-r})(1+k^r\underline{\beta}+k^{3-r}\bar{\beta})} \tag{8}$$

It is interesting to study how effort levels and unit costs in equilibrium vary in different symmetric networks. In other words, varying the network g^k, we study the equilibrium values $e(g^k)$ and $c(g^k)$ in the corresponding subgame. The next proposition summarizes the results:

Proposition 1: *there exists a negative relation between the degree of collaborative activity and the equilibrium effort. Furthermore, the effort is decreasing in* $\bar{\beta}$ *and* $\underline{\beta}$. *There exists a non monotonic relation between the unit cost of production and the degree of collaborative activity. In particular, the unit cost is initially declining in the degree of collaborative activity and then possibly increasing. The complete network is cost minimizing for sufficiently low* $\bar{\beta}$ *and* $\underline{\beta}$.

The level of equilibrium effort is declining in the level of collaboration for two reasons. The first one is a "duplication" effect: since firms take advantage of R&D by other firms, they tend to reduce their efforts in order to save on the R&D costs. The second effect is due to the existence of competition among firms. Forming new links, firms share their effort with more firms, making them stronger competitors. This reduces the firms' incentives to invest in R&D.

The negative effect on efforts when β is high is intuitive. In our interpretation, high β means a low "probability" that two firms will pursue the same path in the research activity. For given R&D efforts, the cost-reduction (both for the firm and its collaborators) is increasing in β. This makes both the duplication and the competition effect stronger and results in a more significant reduction in $e(g^k)$.

The *a-priori* ambiguous relation between the degree of collaborative activity and costs comes from two effects that go in opposite direction: the increase in collaborative activity reduces the effort, but a firm can benefit from the research activities of more firms.

Computations show that, for $\overline{\beta}$ and $\underline{\beta}$ sufficiently low (i.e., when the negative effect of an increase of k on $e(g^k)$ is moderate), the positive effect prevails and costs are minimized in a complete network.

3.1 Stability

In this paragraph, we focus on the stability of different symmetric network structures. From now on, we consider the case *n=4*. This allows us to obtain a full characterization of the results. We will verify the stability of six (symmetric) networks, since k^r can take value in the set $\{0,1\}$ and k^{3-r} in the set $\{0,1,2\}$.

Plugging $n = 4$ and equilibrium efforts, costs and quantities in the profit function yields:

$$\Pi(g_k) = \frac{(A-c)^2(25-(4-\overline{\beta}k^{3-r}-\underline{\beta}k^r)^2)}{(25-(4-\overline{\beta}k^{3-r}-\underline{\beta}k^r)(1+\overline{\beta}k^{3-r}+\underline{\beta}k^r))^2} \tag{9}$$

The notion of stability that is used is the notion of pairwise stability introduced by Jackson and Wolinsky (1996). In the definition we denote with $g-ij$ the network obtained by removing *ij* from *g*, and with $g+ij$ the network obtained by adding *ij* to *g*.

Pairwise stability: *A network g is pairwise stable if and only if for all* $i, j \in N$

(i) *If* $ij \in g$, *then* $\Pi_i(g) \geq \Pi_i(g-ij)$ *and* $\Pi_j(g) \geq \Pi_j(g-ij)$

(ii) *If* $ij \notin g$ *and* $\Pi_i(g+ij) > \Pi_i(g)$, *then* $\Pi_j(g) < \Pi_j(g+ij)$

The definition implies that both agents need to agree to form a link, while they can unilaterally sever it. This notion of stability is the weakest one can think of, since it allows a single link to be modified: firms cannot simultaneously form and/or sever more than one link. Consequently, the set of stable networks is the largest, compared with set of stable networks resulting from stricter notions of stability; nevertheless, such a set is relatively small in all the cases we will consider (a singleton in the case of symmetric networks in a four firms industry), so that pairwise stability constitutes a useful solution concept. In section 5, we will consider an alternative, stricter notion of stability, strong stability.

The following proposition summarizes the results. The sketch of the proof is in appendix:

Proposition 2: *for every strictly positive* $\overline{\beta}$ *and* $\underline{\beta}$, *the complete network is the only stable network.*

Proposition 2 strictly follows the result by Goyal and Moraga (2001) They show that for generic n, the empty network is not stable, while the complete network is always stable. It can be shown that this result holds also in our model. They also show that for n=4, the complete network is the only symmetric stable network, as it is the case here.

Then, no matter what are the degrees of technological opportunities and technological heterogeneity, firms have always the incentive to "destabilize" a symmetric network different from the complete network, forming a new link. Starting from a situation in which firms are symmetric, firms which form a new link can create an asymmetric market structure by sharing their R&D effort. In all the cases this leads to some reduction in costs, even if links occur between firms in the same technological group, for which information sharing may be not effective. The complete network is stable because in this case, by definition, it is not possible to form new links, and firms do not find convenient to sever one of their links, weakening their competitive position.

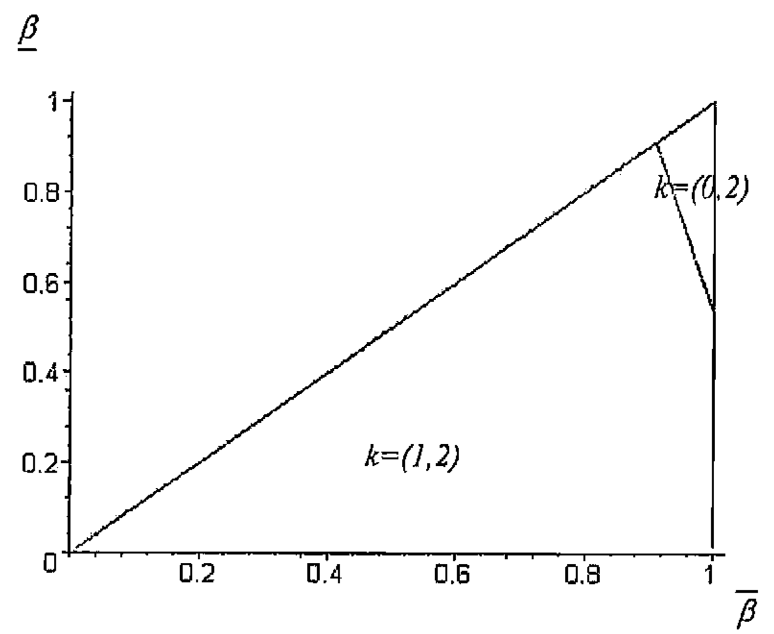
3.2 Aggregate profits

In this section, we consider the behavior of different symmetric networks in terms of aggregate profits. We try to assess the relation between the incentive for individual firms to form collaborative links and what is desirable for them *collectively*. Since in symmetric networks all firms obtain the same level of profits, it is sufficient to compare equilibrium profits for the all possible network structures (denoted with $\Pi(g^k)$, where the subscript is omitted for symmetry), in the range of all conceivable values of $\overline{\beta}$ and $\underline{\beta}$. Proposition 3 summarizes the results.

Proposition 3: *define* $H_1(\overline{\beta},\underline{\beta}) = \Pi(g^{(1,2)}) - \Pi(g^{(0,2)})$. *For all* $\overline{\beta}$ *and* $\underline{\beta}$ *such that* $H_1(\overline{\beta},\underline{\beta}) > 0$, *the complete network maximizes aggregate profits. Otherwise, a network in which all the firms are linked with and only with the firms of the other technological group (*$k^r = 0, k^{3-r} = 2$*) maximizes aggregate profits. In economic terms, the complete network is optimal for firms collectively when technological opportunities are not "too high".*

Figure 1 summarizes graphically proposition 3. This figure represents the set of possible values of parameters, $\{(\underline{\beta},\overline{\beta}) \mid (\underline{\beta},\overline{\beta}) \in [0,1]\times[0,1] \wedge \overline{\beta} \geq \underline{\beta}\}$, and it indicate the areas the parameter space for which a particular network of degree $k \equiv (k^r, k^{3-k})$ is profit maximizing. The following figures must be read in a similar way.

Figure 1: profit maximizing symmetric networks in four firms industry



Firms' private incentive towards link formation can be aligned or excessive with respect to their collective incentive to form links. In fact, for a very significant area in the parameter space, the complete network maximizes aggregate profits.

The increase in the degree of collaborative activity affects net profits in equilibrium through two channels: gross profits and through R&D costs. The effect on gross profit is ambiguous, reflecting the behavior of unit cost (in a symmetric network, $\Pi(g^k) = \left(\frac{(A - c(g^k))}{5}\right)^2 - e(g^k)^2$); while R&D costs are decreasing in *k*. For a large subset of the parameter space, the complete network maximizes aggregate profits: the net effect of increasing network density is always positive. In case of (very) high technological opportunities, the negative effects of an increase in the degree of collaborative activity are more pronounced. The situation, then, resembles a prisoner dilemma's situation. While firms would collectively prefer a lower degree of collaboration, individually they have the incentive to destabilize a symmetric network in order to alter market structure in their favour. This results in a Pareto dominated situation.

3.3 Welfare Analysis

While the previous section has considered the collective incentives for firms to form collaborative links, this section takes into account social welfare, as defined by equation (4).

Proposition 4: *define*

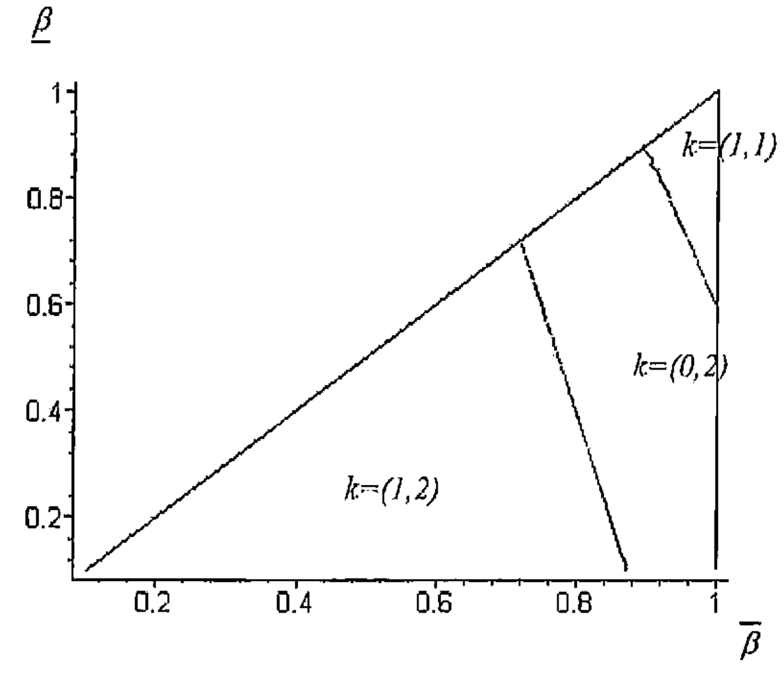
$$H_2(\overline{\beta}, \underline{\beta}) = W(g^{(1,1)}) - W(g^{(0,2)})$$

$$H_3(\overline{\beta}, \underline{\beta}) = W(g^{(0,2)}) - W(g^{(1,2)})$$

It can be shown that $H_2(\overline{\beta}, \underline{\beta}) > 0$ *implies* $H_3(\overline{\beta}, \underline{\beta}) > 0$, and $H_3(\overline{\beta}, \underline{\beta}) < 0$ *implies* $H_2(\overline{\beta}, \underline{\beta}) < 0$.

For all $\overline{\beta}$ *and* $\underline{\beta}$ *such that* $H_2(\overline{\beta},\underline{\beta}) > 0$, *the network where all firms have one link inside and one link outside their technological group* $(k^r = 1, k^{3-r} = 1)$ *is welfare maximizing. For all* $\overline{\beta}$ *and* $\underline{\beta}$ *such that* $H_3(\overline{\beta},\underline{\beta}) > 0 > H_2(\overline{\beta},\underline{\beta})$, *the network where all firms have two links outside and zero link inside their technological group* $(k^r = 0, k^{3-r} = 2)$ *is welfare maximizing. Finally, if* $H_3(\overline{\beta},\underline{\beta}) < 0$, *the complete network is welfare maximizing.*

Figure 2: Welfare maximizing symmetric networks



When technological opportunities are low, the complete network is welfare maximizing, and social interests and firms' private incentives coincide. Social welfare depends on the degree of collaborative activity through its effect on profits and through the total quantity produced, which determines consumer surplus and it is inversely related to the unit cost of production. When technological opportunities are low, the net effect of an increase in the degree of collaborative activity is always positive, and maximal information sharing is optimal. When technological opportunities increase, a less dense network becomes more desirable from a social point of view, because the negative effects from an increased degree of collaboration are higher than in the previous case. Although $k = (0,2)$ and $k = (1,1)$ are equally dense, the latter is socially preferred for very high technological opportunities. This happens because this structure minimizes the negative effects of an increase of k on equilibrium effort: as shown by Proposition 1, such a negative effect is higher when β is higher, and so it can socially optimal to "substitute" a link outside firms' technological group with a link inside firms' technological group.

Finally, it is worth noting that the area of the parameter space for which welfare is maximized by a complete network is included in the area of the parameter space for which aggregate profits are maximized by a complete network. In other words, when the complete network is social welfare maximizing, it is also profit maximizing, but the converse is not true. This is because, when considering social welfare, one needs to add the possibly negative effect that an increase in k generates for consumer surplus, through the reduction of total quantity produced due to higher production costs.

3.4 Discussion

The analysis of symmetric networks has shown that the results of Goyal and Moraga in terms of stability are not significantly modified by introducing a role for technological opportunity and technological heterogeneity the complete network is the only symmetric stable network, independently from $\overline{\beta}$ and $\underline{\beta}$. Firms have always the incentive to alter a symmetric architecture (resulting in an asymmetric market structure) by forming a new link, whenever this is possible.

With respect to networks that maximize aggregate profits and social welfare, we do not find that individual incentives towards link formation are necessarily excessive, as in Goyal and Moraga. Actually, the complete network maximizes aggregate profits for a large set of parameters, while, if technological opportunities are sufficiently low, it is optimal also both from the society point of view to have maximal information sharing.

A more specific role for technological heterogeneity is clearly seen comparing the results about pairwise stability and social welfare. There is an area of the parameter space, where both technological heterogeneity and technological opportunities are high, in which it is socially optimal that information sharing occurs only when it is more effective, that is among firms in different technological groups. However, firms aiming at capturing strategic positions in the network (and consequently a competitive advantage in the industry) have the incentive the share their efforts with firms in their same technological group, which is detrimental in terms of the "collective" incentives to invest in R&D. This leads to a network which is denser that the social optimum.

4. Asymmetric networks

The analysis in section 3 has restricted the attention only to symmetric network. In this section we extend the analysis to the properties of asymmetric networks. We will develop the simplest case of $n=3$. This will lead us to consider a situation where technological groups have different size. We shall assume that firm 1 belongs to group 1, while firms 2 and 3 belong to group 2.

Technological groups that are asymmetric in size represent an interesting case because we can study if and how the firm in the smaller group (which possesses technological capabilities that are rare in the context of the industry) can exploit this situation and obtain an advantageous position in the network and in the market.

We need to compare six typologies of networks:

1. The empty network, denoted with $\varnothing$. In this case all the firms gain in equilibrium the same profit, which we indicate with $\Pi_1^{\varnothing}$.
2. The partially connected network of type 1, where there is one link between firm 1 and one firm in the other technological group (say firm 2). This network is denoted with *p1*, and we indicate with Π_1^{p1}, Π_2^{p1} and Π_3^{p1} profits in equilibrium for firm 1,2 and 3 respectively.
3. The partially connected network of type 2, where there is one link between the two firms in the same technological group. This network is denoted with *p2*, and equilibrium profits are Π_1^{p2} and Π_2^{p2} for firm 1 and firm 2 respectively (the positions of firms 2 and 3 are symmetric).

4. The star network of type 1, where firm 1 is the hub (i.e. it is connected both with firm 2 and firm 3) and firm 2 and firm 3 are the spokes (they are connected only to firm 1). This network is denoted with *s1*, and equilibrium profits are Π_1^{st2} and Π_2^{st2} for firm 1 and firm 2 respectively (again, the positions of firms 2 and 3 are symmetric).
5. The star network of type 2, where say firm 2 is the hub and the remaining firms are the spokes. This network is denoted with *s2*, and we indicate with Π_1^{st2}, Π_2^{st2} and Π_3^{st2} profits in equilibrium for firm 1, 2 and 3 respectively.
6. The complete network, denoted with *c*. We indicate with Π_1^c and Π_2^c equilibrium profits for firm 1 and 2 respectively (the positions of firms and 3 are symmetric).

4.1 Stability

The next proposition summarizes the results about stability. Goyal and Moraga shows that two kinds of structures are possibly stable, when spillovers outside collaboration are absent as in our model: the partially connected network and the complete network.

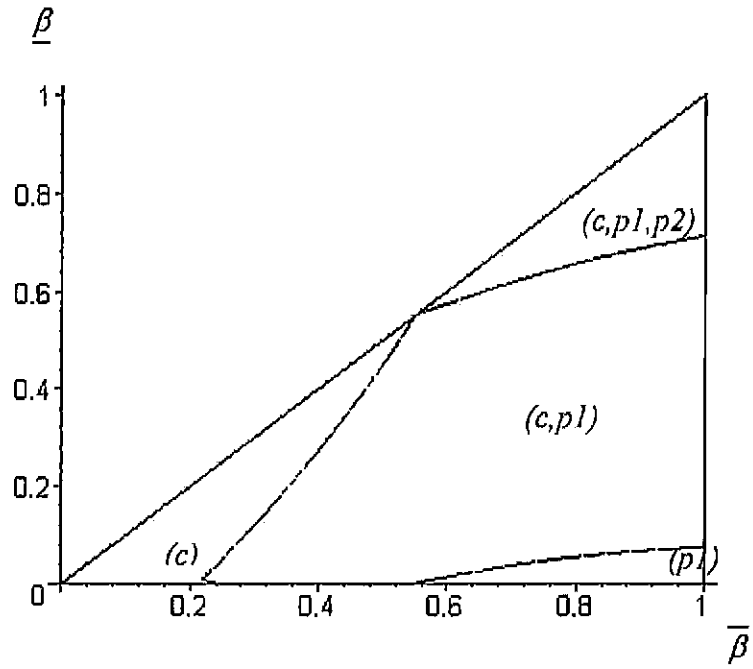
Proposition 5: *the complete network is stable unless technological heterogeneity is very high. There exists a function* $H_4(\overline{\beta},\underline{\beta}) = \Pi_1^c - \Pi_1^{st2}$ *such that, for any value of* $\overline{\beta}$ *and* $\underline{\beta}$ *satisfying* $H_4(\overline{\beta},\underline{\beta}) \geq 0$, *the complete network is stable.*

The partial network of type 1 is stable unless technological opportunities are "low" and heterogeneity is limited. There exists a function $H_5(\overline{\beta},\underline{\beta}) = \Pi_2^{p1} - \Pi_2^{st2}$ *such that for any value of* $\overline{\beta}$ *and* $\underline{\beta}$ *satisfying* $H_5(\overline{\beta},\underline{\beta}) > 0$, *the partial network of type 1 is stable.*

The partial network of type 2 is stable if heterogeneity is limited. There exists a function $H_6(\overline{\beta},\underline{\beta}) = \Pi_2^{p2} - \Pi_2^{st2}$ *such that for any value of* $\overline{\beta}$ *and* $\underline{\beta}$ *satisfying* $H_6(\overline{\beta},\underline{\beta}) > 0$, *the partial network of type 2 is stable.*

Figure 3 summarizes the results about stability in the parameter space.

Figure 3: stability in the three firms industry



Introducing firms' heterogeneity does not impact on the types of networks that are possibly stable, but the stability of different network structures *does* depend on $\overline{\beta}$ and $\underline{\beta}$.

Star networks are never stable. In particular firm 1 will not use its "special" position to become the hub of a star. The star of type 1 is not stable because of two possible deviations.

First, given the existence of a link between firm 1 and firm 2, firm 1 and firm 3 never agree in maintaining a collaborative link. Firm 1 is willing to form a link for low $\overline{\beta}$ ($\overline{\beta}$<0.35). In this case, given that the opportunity of avoiding duplication of efforts is limited, firm 1 does not find the strategy of an exclusive alliance with firm 2 attractive, and it would rather collaborate also with firm 3. At the same time, firm 3 is willing to cooperate with 1 only when $\overline{\beta}$ is sufficiently high ($\overline{\beta}$>0.48). Forming an alliance with 1, firm 3 obtains access to firm 1's R&D effort, but it makes firm 1 even stronger. It turns out that the first effect prevails for $\overline{\beta}$ high.

A second profitable deviation is given by firm 2 and firm 3 forming a link. In this case they can make their position stronger in market competition vis-à-vis firm 1, by sharing their R&D efforts.

In partially connected network of type 1, the position of firm 1 is not "special", in the sense that it obtains the same level of profit as the firm it is connected with. However, whenever heterogeneity is above a minimum threshold (such that we are not in the range in which the partially connected network of type 1 is stable) firm 1 can obtain the maximum industry profit in any stable network.
Firms in the relatively "crowded" technological group, instead, show more variability in the profits associated to stable networks.

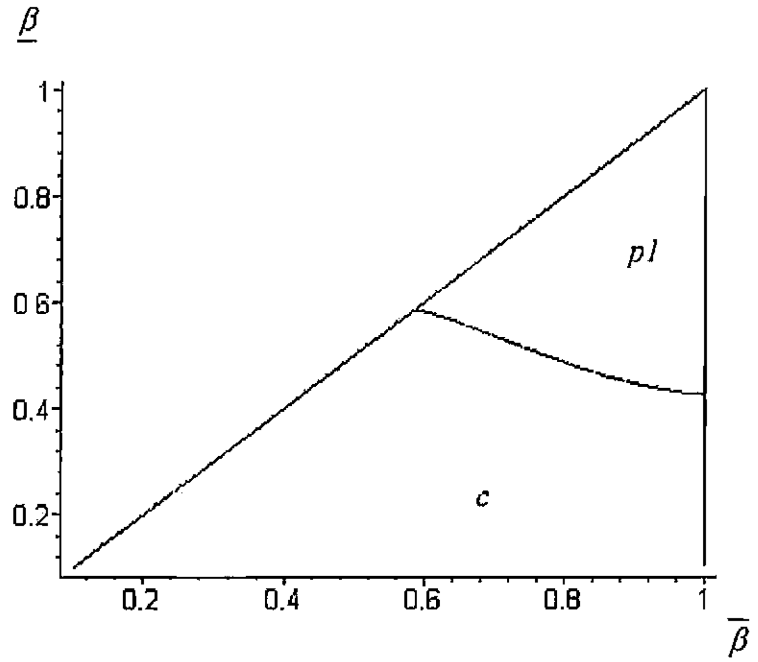
4.2 Aggregate profits

This section considers how aggregate profits vary as a function of the network. In this case it is necessary to sum the profits of the three firms, since it is not possible to talk about a representative firm in the industry (apart from the special case of the empty network)
Define: $\Pi(g) = \Pi_1(g) + \Pi_2(g) + \Pi_3(g)$, with $g \in \{c, p1, p2, st1, st2, \varnothing\}$.

Proposition 6: *when the technological opportunities are sufficiently high, the partially connected network of type 1 maximizes profits; otherwise the complete network does. There exists a function* $H_7(\overline{\beta}, \underline{\beta}) = \Pi(c) - \Pi(p1)$ *such that, for any value of* $\overline{\beta}$ *and* $\underline{\beta}$ *satisfying* $H_7(\overline{\beta}, \underline{\beta}) > 0$, *the complete network maximizes aggregate profits.*

Figure 4 summarizes the results.

Figure 4: profit maximizing networks in three firms industry



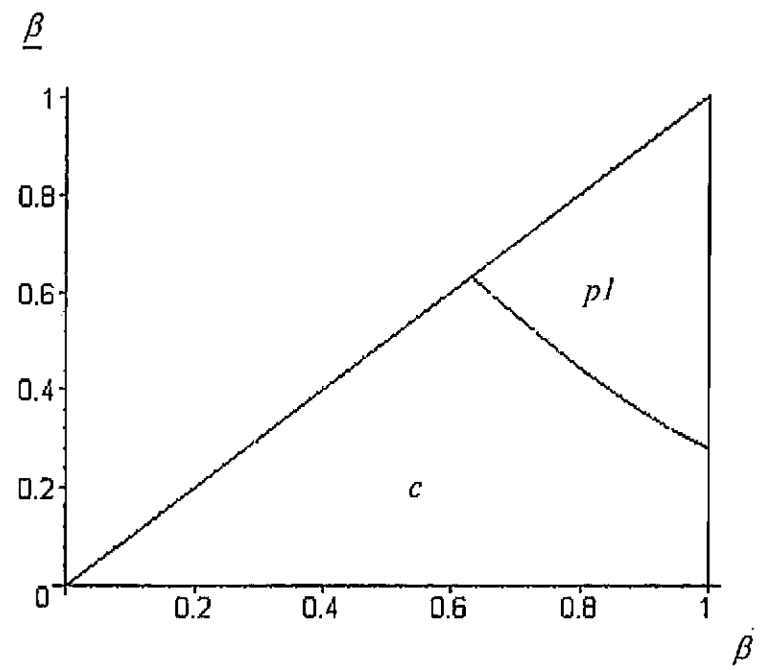
When technological opportunities are high (in particular, when information sharing between technologically heterogeneous firms is effective), allied firms have a strong incentive to invest in R&D and waken the position of the remaining firm in market competition. Then, their costs are low, and their profits high. Although unevenly distributed, aggregate profits in the partially connected network turn out to be higher than in the complete network.

4.3 Social Welfare

Finally, we consider the social welfare properties of asymmetric networks:

Proposition 7: *social welfare is maximized by a partially connected network of type 1 whenever technological opportunities are sufficiently high. Otherwise the complete network maximizes social welfare. There exists a function* $H_8(\overline{\beta},\underline{\beta}) = \Pi(c) - \Pi(p1)$ *such that, for any values of* $\overline{\beta}$ *and* $\underline{\beta}$ *satisfying* $H_8(\overline{\beta},\underline{\beta}) > 0$, *the complete network maximizes social welfare.*

Figure 5: welfare maximizing networks in the three firms industry



It is interesting to notice how firms and social interests substantially coincide (closer inspection reveals that there is a small portion of the parameter space for which these do not coincide). Firms that alter market structure in their favour invest more in R&D, and this reflects in a cost reduction which is beneficial also to consumers.

4.4 Discussion

The properties of asymmetric networks in terms of stability are consistent with previous results in the literature (Goyal and Moraga, 2001; Goyal and Joshi, 2003) and with the emphasis on asymmetric structures that one can find in the firms' coalition literature (Bloch, 1995).

Technological heterogeneity does not impact on the architectures that are possibly stable, but that stability of different network structures *does* depend on $\overline{\beta}$ and $\underline{\beta}$. Very intuitively, the partially connected network of type 1 is the only stable network when heterogeneity is very significant, while if technological opportunities are limited, the partially connected networks (both of type 1 and 2) are not stable.

Firm 1, which is the unique firm belonging to its technological group, does not gain a prominent role in any stable networks. Nevertheless, it obtains the highest profit in the complete network and it can be excluded in pairwise stable networks only in the limited range of parameters where technological opportunities are high and technological heterogeneity is low.

Comparing networks that are pairwise stable and networks maximizing aggregate profits and social welfare, one can observe that in general at least one stable network (if the set of stable networks is not a singleton) is efficient, from firms' and social point of view. The exception is the range in which the partially connected network of type 1 is the only stable network, where profits and social welfare are maximized by a complete network.

5 Strong stability in symmetric and asymmetric networks

In this section, we apply a stronger notion of stability to the two cases studied in the sections 3 and 4. As we said, pairwise stability is a weak notion of stability, because it considers as admissible only a small set of deviations. In particular, it does not allow for coordinated actions of agents that form or sever more than one link. In contexts where the number of agents is small, it seems plausible that agents can arrange more complex deviations, to which a network must resist to be considered as stable.[16]

The notions we will use is the notion of strongly stable networks, discussed in Jackson and van den Nouweland (2003) and Dutta and Mutuswami (1997). In words, a network is strongly stable if there are no coalitions of players that by forming or severing links can strictly increase the payoff of the members of the coalition, where members of the coalition can add links only among them, but they can sever links with all the agents in the network.

[16] In an alternative approach, one could consider network formation as a non cooperative game, in line with Myerson (1991). Firms simultaneously propose the subset of agents they want to be connected with, and links are formed only when the proposals are reciprocated. However, Nash equilibrium is too weak as a solution of concept, due to the coordination problem that arises for the required double coincidence of wants for the formation of a link. The refinement of undominated Nash equilibrium, which is sometimes used in the literature (Goyal and Joshi, 2003), is not of particular help here, because only the empty set as a strategy is weakly dominated for all the parameters values. In the four firms industry, all the symmetric networks can be sustained as Nash equilibrium of the link formation game, and all the symmetric networks but the empty network can be sustained as undominated Nash equilibrium for some range of the parameters. Finally, it is worth noting that the notion of strongly stable networks we discuss in the text coincides with the notion of strong Nash equilibrium in the link formation game.

Formally, strong stability is defined as follows:

***Strong stability**: define $S \subseteq N$ as a coalition in N. A network g' is obtainable from g via S if:*

(i) $ij \in g'$ and $ij \notin g$ implies $\{i, j\} \subset S$.

(ii) $ij \in g$ and $ij \notin g'$ implies $\{i, j\} \cap S \neq \varnothing$.

A network g is strongly stable if there are no coalitions S and network g' obtainable from g via S for which $\Pi_i(g') > \Pi_i(g)$, for all $i \in S$.

This definition of stability is strict, and consequently the existence of strongly stable networks is not guaranteed. When existing, strongly stable networks have nice properties. In particular, strongly stable networks are by definition Pareto efficient. The definition of strong stability that we use here (which is taken from Dutta and Mutuswami, 1997) does not imply pairwise stability as defined in section 3 (which is the original definition by Jackson and Wolinski, 1996): in the former, establishing a new link is an admissible deviation only if both firms are strictly better off; in the latter, one agent can be weakly better off. However, the implication does not hold only for parameters values that constitute the borders between areas of stability of different network structures.[17]

5.1 Strong stability in the four firms' industry

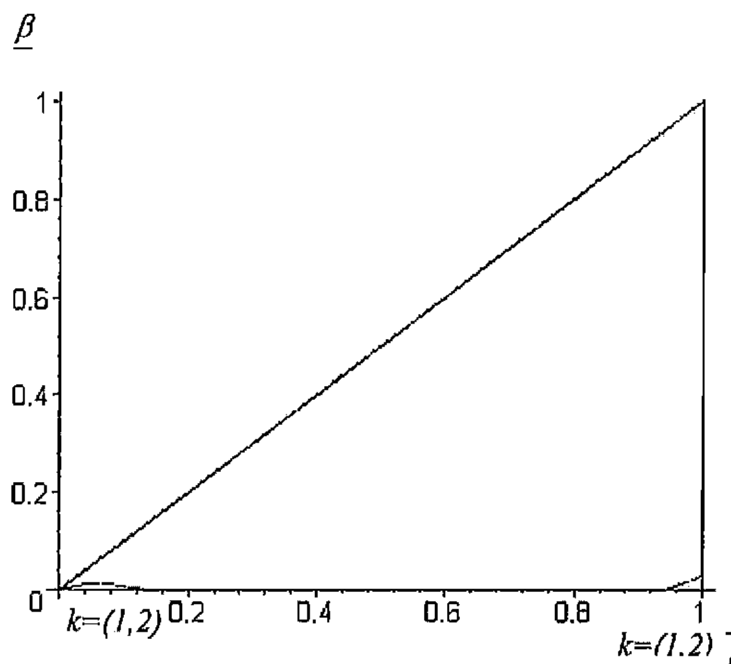
In the case of four firms, only one symmetric network turns out to be pairwise stable. Then, we simply need here to verify if (and when) the complete network, which is always pairwise stable, is also strongly stable.

The results are summarized in proposition 8. In proving Proposition 3, we will refer to a particular asymmetric structure, the *triangle* (denoted with *tr*), where we have a fully connected component of three firms (say 1, 3 and 4, with 3 and 4 belonging to the same technological group) and one firm (firm 2) is isolated. In equilibrium, profits are $\Pi_1(tr), \Pi_2(tr)$ and $\Pi_3(tr)$ (the positions of firm 3 and 4 are symmetric).

[17] We will show in the next subsections why is preferable to adopt this version of strong stability. Another alternative would be to modify the definition of pairwise stability, again with minor differences.

Proposition 8: *the complete network is almost never strongly stable, except that for very low technological opportunities or very high technological opportunities. There exists a function* $H_9(\overline{\beta},\underline{\beta}) = \Pi_3(tr) - \Pi(g^{(1,2)})$ *such that, for all the values of* $\overline{\beta}$ *and* $\underline{\beta}$ *for which* $H_9(\overline{\beta},\underline{\beta}) > 0$ *the complete network is not strongly stable.*

Figure 6: strongly stable networks in the four firms industry



Proposition 8 is very close to a non-existence result: for a largely predominant subset part of the parameter space, the complete network is not strongly stable, so that there are no symmetric networks that are strongly stable. Nevertheless, the result has interesting economic implications for the nature of the coalition and the deviation that turns out to be profitable. Except that for a very limited small area in the parameter space, three firms have the incentive to sever jointly their links towards the fourth firms, creating an asymmetric market structure where three, "networked" firms have a dominant position in the product market. In particular, while firm 1 (which is the only firm in its technological group to have connections) always prefers to be in the triangle network, firm 2 and firm 3 do for the range of parameters shown in the figure. Furthermore, when $\overline{\beta}$ and $\underline{\beta}$ are sufficiently high, the isolated firms is forced out of the market $(q_2 = 0)$.

This result is interesting because it confirms the importance of asymmetric network structures, as shown by the three firms' analysis, and the role played by collaborative ventures in creating *ex post* asymmetries in *ex ante* symmetric situations. A natural question then is when the triangle network turns out to be pairwise stable. It can be shown that for a significant range of parameters (in particular, when technological opportunities are high or technological heterogeneity is high) the triangle network is not pairwise stable because connected firms prefer to form the link with the isolated firm.[18] This leads towards the formation of a complete network, where profits for such firms are generally lower. Although the model is purely static, it suggests a dynamic story in which firms have the *private* incentive to form very dense networks, but then they have the *"collective"* incentive to sever the links towards one firm, to exclude it from the network and create an asymmetric market structures. This has two consequences: it suggests instability of cooperative ventures, and a cycle in alliances formation. Both aspects are consistent with empirical evidence (Kogut, 1988; Hagedoorn, 2002).

5.2 Strong stability in the three firms' industry

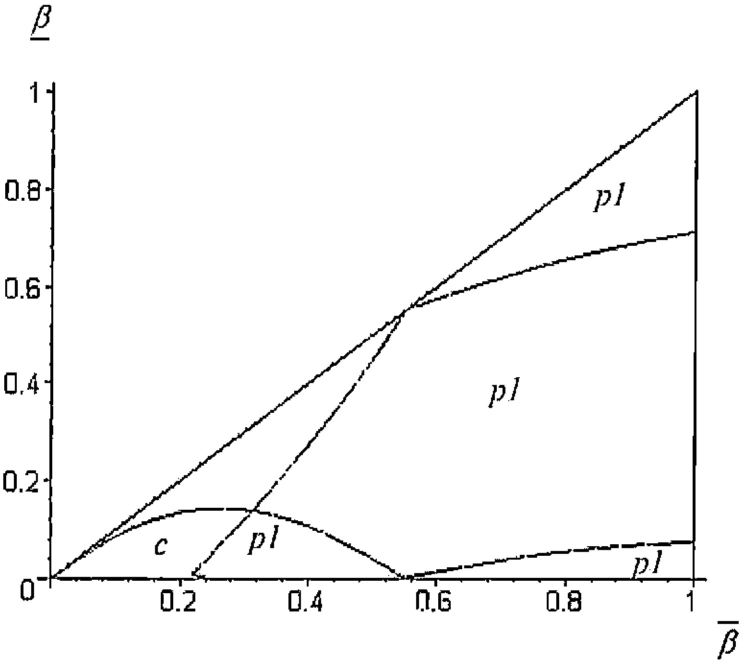
In the case of three firms, three structures turn out to be pairwise stable: the complete network, the partially connected network of type 1 and the partially connected network of type 2.

Proposition 9 summarizes the results.

Proposition 9: *the partially connected network of type 2 is never strongly stable. The partially connected network of type 1 is always strongly stable, when is pairwise stable. The complete network is strongly stable only when technological opportunities are low. There exists a function* $H_{10}(\overline{\beta},\underline{\beta}) = \Pi_1(p1) - \Pi_1(c)$ *such that the complete network is strongly stable for all values of* $\overline{\beta}$ *and* $\underline{\beta}$ *for which* $H_{10}(\overline{\beta},\underline{\beta}) < 0$.

[18] The graphical representation of pairwise stability for the triangle network is reported in the appendix.

Figure 7: strongly stable networks in the three firms industry



In the three firms' case, the complete network, when it is pairwise stable, is very often not robust to a deviation by two firms in different technological group (say firm 1 and 3), which form a coalition and sever jointly the link with firm 2. Apart a small area where technological opportunities are low, profits of connected firms in *pl* are higher than the profits of firm 1 in a complete network.[19]

A partially connected network of type 2 is never strongly stable because firms 2 and 3 have the incentive to substitute their current partner with firm 1.[20]

[19] The firm in the other technological group always gains a higher profit in the partially connected network of type 1.

[20] The same emphasis on the partially connected network is obtained if one refers to a dynamic model of network formation (Watts, 2001; Jackson and Watts, 2002).

Consider the following algorithm for network formation, adapted from Watts (2001). Start from the empty network at $t=0$, and suppose to be in the range of parameters where the partially connected network of type 1 is pairwise stable. From then on, each period a pair of firms is drawn. The two firms can form a link between them, if not existing, or sever the link, if already existing. The agreement is required only to form a new link. Firms form and sever links on the basis of comparison with profits associated with the existing network structure. Firms are myopic: they do not consider the effect of their decision on subsequent choices. The process continues until a stable network is reached. Then firms invest in R&D and market competition occurs.

It is straightforward to see that, under this algorithm, the complete network can emerge only for relatively small class of histories. In particular, apart the consecutive revision of the same link, the complete network emerges only if the sequence is 23-13-12 or 23-12-13. Instead, the partially connected network of type 1 is immediately obtained whenever the first two firms forming a link are firms 1 and 2 or firms 2 and 3.

The partially connected network of type 1 is always strongly stable, because there are no deviations that can make any pair of agents strictly better off. Moreover, firm 2 has no incentive to move to a complete network either.[21]

The refinement of strong stability clearly points out the partially connected of type 1 as a natural solution for the process of network formation. This is interesting for several reasons.

First, on the empirical side, the special role played by this asymmetric network is consistent with the empirical analysis that underlies the motive of altering market structure as an important rationale for interfirm technological agreements (Hagedoorn, 1993). Also the results from the analysis of the four firms' case are in line with this evidence.

Second, the firm in group 2 that "succeeds" in forming the link obtains an advantage in terms of profits, a gain this is increasing in $\bar{\beta}$. This leads naturally to consider the strong competition occurring between the two firms in the larger technological group. There are two ways to tackle this issue. First, one can take the model as it is and solve the problem of multiple equilibria invoking a role for "historical accidents" and path-dependence, in a way that is similar to the one in Zirulia (2005). "Random" events (like social contacts or geographical proximity) leads one firm in group 2 to form a link with 1, with long lasting effects on firms' performance. It is interesting to observe that some business scholars (for instance, Gulati *et al.*, 2000) have underlined the importance for firms to "rush" and form alliances with the "right" partners in the early phases of technological or industrial cycles. Our simple model is consistent with this view. The second solution is to explicitly model such a competition, supposing for instance a role for side payments that allows firm 1 to exploit its strong bargaining power. If side-payments are allowed, we can expect that the firm excluded by the network would "undercut" the other firm, transferring part of the surplus of being connected to firm 1. In this view, firm 1 would exploit the "scarcity" of its technological resources in terms of performance also under this architecture.

[21] If one uses a notion of strong stability where agents in a deviating coalition may be weakly better off, the partially connected network of type 1 is never strongly stable, because the coalition of 1, which is indifferent between the two partners, and the excluded partner from the network is winning.

Third, in terms of policy, we can observe how the partially connected network is welfare maximizing only when technological opportunities are high. There is a significant area in the technological space (with high technological heterogeneity) where welfare is maximized by the complete network. If technological opportunities are not too high, a dense network is not detrimental to R&D efforts, and consequently it has beneficial effects on consumer surplus. However, firms have the incentive to alter market structure in their favor, excluding one firm from the network. In this case, there is possibly room for public intervention to favor industry-wide cooperation.

6. Conclusions and plan for future work

The goal of this paper was to extend the analysis of R&D network formation in a setting when technological heterogeneity among firms is considered. First (Section 3 and 4), the results were derived in terms of pairwise stability, aggregate profits and social welfare associated with different network structures. We wanted to consider the robustness of Goyal and Moraga's results to a modification that seems empirically relevant. We consider two classes of networks. First, we consider symmetric networks in a four firms industry. The complete network is always the only symmetric stable network. Firm have always the incentive of altering the market structure adding a new link, when network is not complete. Aggregate profits and social welfare are also maximized by a complete network, if technological opportunities are not too high, so that private and social incentives are aligned in these cases. Otherwise, less dense networks are optimal from firms' and society point of view. In the class of asymmetric networks, for which the analysis has been performed in the case of three firms, technological heterogeneity matters. Only the complete and the partially connected networks are possibly stable, but which network is stable actually depends on the level of heterogeneity and technological opportunities. Firms belonging to the smaller technological group (having unique technological resources) obtain a special position in the industry, since they can guarantee the maximum profits in the industry in every stable network. The complete and partially connected networks are also the possible welfare and aggregate profit maximizing networks, but social and private incentives do not generally coincide. When technological opportunities are high, the partially

connected network involving two firms of different technological groups is pairwise stable and it maximizes aggregate profits and social welfare.
In section 5, we consider the refinement of strong stability, where all the possible deviations by coalitions of agents are allowed. It turns out that, in the four firms' case, the complete network is very rarely strongly stable, because a coalition of three firms has the incentive to isolate the fourth firm and create an asymmetric market structure. In the three firms' case, the partially connected network where two firms in different technological group are linked is for a large subset of parameter space the only strongly stable network.

In this paper we made a number of restrictive assumptions. In particular, we considered the role of technological heterogeneity independently from the nature and intensity of competition and we kept the assumptions of homogenous good and Cournot competition. Furthermore, we consider a simple representation of technological heterogeneity, allowing only for two types of firms. For the future, we plan to develop a model where firms are located in a technological space that affects both the intensity of competition and the effects of information sharing, and study the stability and efficiency properties of the networks as a function of firms' localization.

References

d'Aspremont, C. and Jacquemin, A. (1988), "Cooperative and noncooperative R&D in duopoly with spillovers", *American Economic Review*, 78,1133-1137.

Bloch, F. (1995), "Endogenous structures of Association in Oligopolies" *RAND Journal of Economics*, 26,537-556.

Cohen, W. and Levinthal, D. (1989), "Innovation and learning: the two faces of Research and development", *The Economic Journal*, 99, 569-596.

Dutta, B. and Mutuswami, S. (1997), "Stable networks", *Journal of Economic Theory*, 76,322-344.

Goyal, S. and Joshi, S. (2003), "Networks of Collaboration in oligopoly, *Games and Economic behavior*,43, *57-85.*

Goyal, S and Moraga, J. (2001), "R&D Networks", *RAND Journal of Economics*, 32, 686-707.

Goyal, S. and Konovalov, A. and Moraga, J. (2004), "Hybrid R&D", mimeo.

Gulati, R Nohria, N. and Zaheer, A. (2000), "Strategic networks", *Strategic management Journal* 21, 203-215.

Hagedoorn, J. (1993), "Understanding the rationale of strategic technology partnering: inter-organizational modes of cooperation and sectoral differences" *Strategic management Journal*, 14, 371-385.

Hagedoorn, J. (2002), "Inter-firm R&D partnerships: an overview of major trends and patterns since 1960", *Research Policy*, 31,477-492.

Jackson, M. and Watts, A. (2002), "The evolution of social and economic networks" *Journal of economic theory*, 106, 265.-295.

Jackson, M. and Wolinski, A. (1996), "A Strategic Model of Social and Economic Networks", *Journal of Economic Theory*, 71, 44-74.

Jackson, M. and van den Nouweland, A. (2003), "Strongly Stable Network", *Games and Economic Behavior*, forthcoming.

Kamien, M. Mueller, M. and Zang, I. (1992), "Research joint ventures and R&D cartels", *American Economic Review*, 82, 1293-1306.

Katz, M. (1986), "An analysis of cooperative research and development", *RAND Journal of Economics*, 17, 527-543.

Kogut, B. (1988) "Joint ventures: theoretical and empirical perspectives", *Strategic Management Journal*, 9, 319-332.

Mariti, P and Smiley, R. (1983), "Cooperative agreements and the organization of industry", The Journal of Industrial Economics, 31, 437-451.

Myerson, R. (1991) "Game Theory: Analysis of Conflict". Harvard University Press.

Nelson, R. and Winter, S. (1982), "An evolutionary theory of economic change", Harvard University Press, Cambridge (MA).

Powell, W.W., Koput, K.W. and Smith-Doerr, L. (1996), "Interorganizational collaboration and the locus of innovation: networks of learning in biotechnology" *Administrative Science Quarterly*, 41, 116-145.

Suzumura, K.(1992), "Cooperative and noncooperative R&D in an oligopoly with spillovers" American Economic Review, 82, 1307-1320.

Watts, A. (2001)"A dynamic model of network formation", *Games and economic behavior*, 34, 331-341.

Zirulia, L. (2005) "The evolution of R&D networks", doctoral dissertation, Università Luigi Bocconi.

Appendix

Proof of Proposition 1:

The proposition immediately derives from the following expression. Symmetric expression holds for k^{3-r}:

$$e(g^{(k^r,k^{3-r})}) - e(g^{(k^r+1,k^{3-r})}) = \frac{(A-\overline{c})\underline{\beta}(n-k^{3-r}\overline{\beta}-k^r\underline{\beta})(n-k^{3-r}\overline{\beta}-k^r\underline{\beta}+\underline{\beta})}{[(n+1)^2-(n-k^{3-r}\overline{\beta}-k^r\underline{\beta})(1+k^{3-r}\overline{\beta}-k^r\underline{\beta})][(n+1)^2-(n-k^{3-r}\overline{\beta}-k^r\underline{\beta}+\underline{\beta})(1+k^{3-r}\overline{\beta}-k^r\underline{\beta})} > 0$$

$$c(g^{(k^r,k^{3-r})}) - c(g^{(k^r+1,k^{3-r})}) = \frac{(A-\overline{c})(n+1)^2\underline{\beta}(n-2k^{3-r}\overline{\beta}-2k^r\underline{\beta}-\underline{\beta}-1)}{[(n+1)^2-(n-k^{3-r}\overline{\beta}-k^r\underline{\beta})(1+k^{3-r}\overline{\beta}-k^r\underline{\beta})][(n+1)^2-(n-k^{3-r}\overline{\beta}-k^r\underline{\beta}+\underline{\beta})(1+k^{3-r}\overline{\beta}-k^r\underline{\beta})}$$

which is positive only for $n-2k^{3-r}\overline{\beta}-2k^r\underline{\beta}+\underline{\beta}+1>0$

and finally,

$$\frac{\partial e_i}{\partial \underline{\beta}} = \frac{-(A-\overline{c})k^r[(n+1)^2-(n-k^{3-r}\overline{\beta}-k^r\underline{\beta})(n+k^{3-r}\overline{\beta}+k^r\underline{\beta}]}{(n+1)^2-(n-k^{3-r}\overline{\beta}-k^r\underline{\beta})(1+k^{3-r}\overline{\beta}-k^r\underline{\beta})} < 0$$

Proof of Proposition 2: Pairwise stability of symmetric networks in the four firms industry

We report here a sketch of the proof of this proposition. All the computations and the relevant plots have been performed with the help of the software Maple, and they are available upon request (to: lorenzo.zirulia@unibocconi.it).

We assume, without loss of generality, that firm 1 and firm 2 belong to the same technological group 1, and firm 3 and 4 belong to the technological group 2. Then, the procedure is as follows:

- For each network (apart from isomorphic networks) one need to consider all the deviations that are considered in the notion of pairwise stability;
- this yields unit cost as a function of efforts for each firm, and consequently profit function;
- the first order conditions for representative firms (i.e. firms playing the same role in the network) are computed;
- the system of first order conditions is solved, invoking symmetry of effort for firms playing the same role in the network;
- equilibrium efforts are computed, and plugged into the profit function of deviating firms;
- equilibrium profits from the deviation and equilibrium profits in the symmetric network under consideration are compared.

The complete network is stable

In this case, the only deviation one needs to take into account is when two firms sever one link. It can be shown that independently from β, such a deviation is not profitable.

The empty network is not stable

In this case, the possible deviations are those where two firms form a link. It can be shown that for any strictly positive value of β, such a deviation is profitable. Furthermore, if $\beta > 3/2 - 1/2\sqrt{5}$, the solution is a corner solution where, for one isolated firm, *e=0* and *q=0*.

The network $k^r = 1, k^{3-r} = 1$ *is not stable*

In this case, the deviation in which two firms belonging to different technological group, say firm 1 and 4, form a link is profitable.

The network $k^r = 0, k^{3-r} = 1$ *is not stable*

In this case, the deviation in which two firms belonging to different technological group, say firm 1 and 4, form a link is profitable.

The network $k^r = 1, k^{3-r} = 0$ *is not stable*

In this case, the deviation in which two firms belonging to different technological group, say firm 1 and 4, form a link is profitable.

The network $k^r = 0, k^{3-r} = 2$ *is not stable*

In this case, it can be shown that the deviation in which two firms belonging to the same technological group, say firm 1and 2, form a link is profitable.

Proof of Proposition 5: Pairwise stability in the three firms industry

In this case, we need to take into account six types of structures, and studying the incentives of firms to move from one structure to other by forming or severing links.

Without loss of generality we assume that firm 1 belongs to technological group 1, while firm 2 and 3 belong to technological group 2. Computations show that:

The empty network is never stable

Any pair of firms has the incentive to form a link and moving to a partially connected network of type 1 or 2, for any strictly positive value of β.

A star network of type 1 is never stable

For any strictly positive value of $\underline{\beta}$, firm 2 and firm 3 find convenient to form a link, and transform the star 1 in a complete network.

A star network of type 2 is never stable

Expect that for high $\overline{\beta}$ and low $\underline{\beta}$, firm 1 would prefer to form a link with firm 3 (which is always willing to form such a link) and make the star network of type 2 a complete network. Furthermore, except that for very low values both of $\overline{\beta}$ and $\underline{\beta}$, firm 2 (supposed to be the hub in the star) wants to sever the link with firm 3 and make the network a partially connected network of type 1. It can be shown that the area in the parameter spaces for which the two deviations are not profitable do not intersect, so that there is always a profitable deviation.

A complete network is stable unless $\overline{\beta}$ is very high and $\underline{\beta}$ is very low.

There is a range of values (as reported in the paper) for which firm 1 would prefer to severe the link say with 3 and make the network a star of type 2. Firm 3 is never willing to sever such a link, while it is never profitable for firm 2 and firm 3 to sever their link.

A partially connected network of type 1 is stable unless technological opportunities are low and technological heterogeneity is limited.

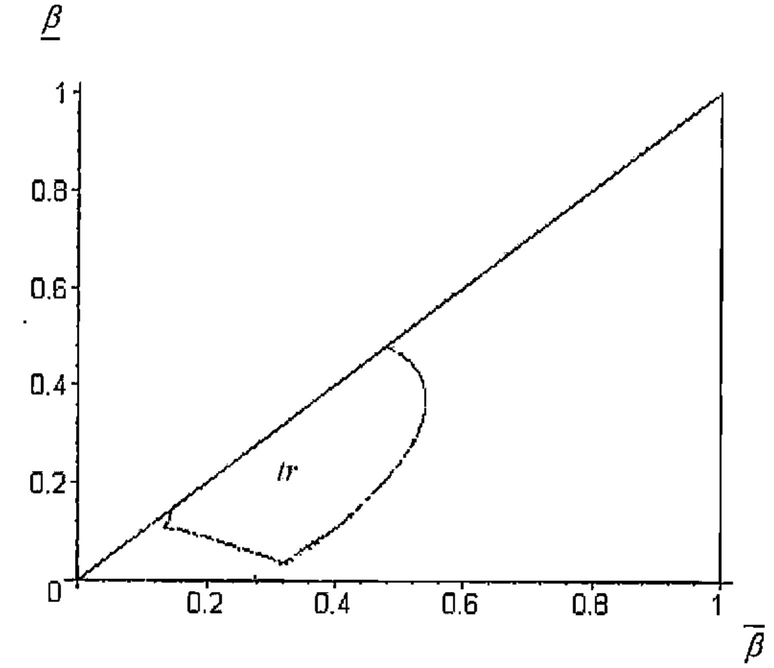
In this case firm 1 and firm 3 never agree on forming the link between them (there are no values of $\underline{\beta}$ for which the double coincidence of wants hold). Firm 2 and firm 3 agree on forming a link between them (making the network a star of type 2) for the range of values of $\overline{\beta}$ and $\underline{\beta}$ specified in the paper. Indeed, firm 3 is always willing to form such a link.

A partially connected network of type 2 is stable if technological opportunities are high and technological heterogeneity is limited.

Firm 2 and 3 are never willing to sever their existing link. While firm 1 always agrees on forming a link with say firm 2, firm 2 gives its consent only for the range shown in the paper.

Proposition 9: Pairwise stability of the triangle network

Figure 8: Pairwise stability of the triangle network



The evolution of R&D networks[22]

1. Introduction

Recently, interfirm technological agreements have played an important role in the innovative activity of high-tech industries (Hagedoorn, 2002). More and more innovation seems the result of joint R&D efforts and information sharing among firms, in a way that has lead some authors to talk about "the network (of collaborating firms) as the locus of innovation" (Powell *et al.*, 1996). The shortening of the product life cycle, the increased competition and the complexity of the knowledge base required for innovation force firms to cooperate even in one of the fundamental source of competitive advantage. At the same time, from a policy point of view, technological cooperation has been considered (and consequently promoted) as a factor which positively affects industries' and countries' competitiveness in the US, Japan and Europe.

An impressive number of empirical studies in the fields of sociology, economics and business have thrown light on this phenomenon, although in a quite unsystematic way (see Zirulia, 2005a, for a review). Similarly, from a theoretical point of view, a rich literature in the game theoretic industrial organization tradition has discussed the effects of R&D cooperation (Katz, 1986; d'Aspremont and Jacquemin 1988; Kamien *et al.*, 1992), while a small but growing number of works have studied the dynamics and the effects of technological networks within an evolutionary framework (Gilbert *et al.*, 2001; Ozman, 2003).

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The present contribution falls within the theoretical literature, proposing a model that focuses on the dynamics of R&D network formation. The model is inspired by the recent papers by Goyal and Moraga (2001) and Goyal and Joshi (2003), which apply the tools of network games (Jackson and Wolisnki, 1996) to study the formation of R&D networks in a static framework. Our model extends their analyses, considering explicitly the dynamic feedbacks between market competition and firms' incentives to engage in collaboration.

The goal of the model is to derive propositions involving the joint dynamics of R&D network and market structure in the context of a model embodying as *assumptions* some of the evidence on interfirm technological agreements. Both empirically and theoretically, the study of this *coevolution* seems a promising direction to pursue, basically missing in the current state of the literature.

The main result of the paper is that the R&D network can work as a strong selection mechanism in the industry, creating ex post asymmetries in ex ante similar firms. This is due to a self-reinforcing, path-dependent process, in which events in the early stages industry affect firms' survival in the long run. In this framework, both market and technological externalities created by the formation of cooperative agreements play a crucial role. Although it creates profound differences at the beginning, which are reflected by an unequal distribution of links across firms, the R&D network tends to eliminate them as it becomes denser and denser. The nature of the technological environment affects the speed of the transition and some of the characteristics of the industry in the long run.

The rest of the paper is organized as follows. Section 2 describes the model, whose analytical properties are the object of section 3. Section 4 presents results from numerical simulations. Finally, section 5 concludes.

2. The model

2.1 An informal description

Informally, the model can be summarized as follows. We consider the evolution of an industry where firms can introduce process innovations *only* through collaborations in an R&D activity, while remaining competitors in the market side[23]. Firms produce a homogenous product, but they are generally different from the technological point of view: they have different levels of efficiency, which result in different levels of production costs, and different technological specializations, which allow complementarities to be exploited when firms collaborate.

We consider a discrete sequence of periods t=0,1,2... Each period can be divided in two sub-periods: the *networking* phase, where firms can modify the network structure according to a procedure described below, and a *market competition* phase, where firms, given the network structure, compete in the product market. Competition is *à la* Cournot, so that firms' different production costs are reflected in firms' different performances. Firms' efficiency level is the result of the history of R&D collaborations for each firm. R&D collaborative projects are modeled as pairwise relationships: for each pair of firms involved in a collaborative agreement, the cost of the project is assumed to be fixed, while its effect (a deterministic reduction in the production cost) depends upon the technological profiles of the two firms.

In the networking phase of each period, two firms are randomly drawn to change the current state of their pairwise relationship, *leaving the state of the remaining R&D network unaltered.* Two firms that are not collaborating can start a collaboration; two firms that are collaborating can decide to interrupt it. Capturing the bounded rationality of agents facing a complex evolution of network and technological capabilities, firms' decisions are based on the short run consequences on their profits. The resulting network for that given period determines firms' level of efficiency, firms' technological

[23] We rule out the possibility of mergers (for instance, for antitrust reasons).

specializations and firms' performance, which will constitute the new initial conditions for the subsequent period.

2.2 Firms and market competition

We consider a market where n firms produce a homogenous product. However, firms are *heterogeneous* from the technological point of view. They are located in a bi-dimensional technological space, and they are identified by the vector $(\gamma_{it},\alpha_{it})$. $\gamma \in [\gamma_0;1)$ is a parameter measuring the productive efficiency of a firm. It determines unit cost of production according to:

$$c_{it} = c(1-\gamma_{it}) \tag{1}$$

$\alpha_{it} \in (0,1)$ characterizes the technological position of a firm, to be intended as its *technological specialization*. We assume that α does not affect *directly* the level of unit cost of production, but it is crucial in determining the value of collaborations.

We will sometime term $(\gamma_{it};\alpha_{it})$ as *firm i's technological capabilities*. Firms move over time in the technological space, and this is the effect of the network structure. Furthermore, we define $\gamma_t \in [\gamma_0;1)^n$ as the n-dimensional vector of variable γ at time t for all the firms; similarly, $\alpha_t \in (0,1)^n$ is the vector of all technological positions at t.

Inverse demand is assumed to be linear:

$$p = A - Q \tag{2}$$

where Q is the total quantity produced by firms.

Firms are characterized by zero fixed costs of production. Given c_{it}, gross profits[24] are given by $\Pi_{it} = (p - c_{it})q_{it}$. Competition is *à la* Cournot, and it is assumed that firms play the (unique) Nash equilibrium in the one-stage game[25]. This means that the quantity produced by each firm at time *t* is:

$$q^*_{it} = \frac{a - n_t c_{it} + \sum_{j \neq i} c_{jt}}{n_t + 1} \tag{3}$$

where $n_t \leq n$ is the number of active firms (i.e. firms producing a strictly positive quantity) at *t*. We define N_t as the subset of such firms.

For sake of simplicity, firms that are inactive at time *t* are supposed to *exit* the market, never to reappear. This in particular implies that at the beginning of period *t*+1 all their existing links with other firms are severed, and since period *t*+1 onward they are no longer considered in the algorithm for network evolution. The discussion below on such an algorithm will make this point clearer. In equilibrium, gross profits are given by $\Pi^*_{it} = (q^*_{it})^2$.

2.3 The effect of the R&D network

At each moment *t*, following the networking phase, the industry is characterized by an R&D network g_t. We define a binary variable $g_{ijt} \in \{0,1\}$: when $g_{ijt} = 1$, a collaborative link exists between firm *i* and *j* at time t. The network $g_t \in \{0,1\}^{\frac{n(n-1)}{2}}$ is then a collection of states for the pair-wise relationships among firms. We indicate with $g + g_{ij}$ the network obtained by replacing $g_{ij} = 0$ in a generic network *g* with $g_{ij} = 1$, and similarly with $g - g_{ij}$ we denote the network obtained by replacing $g_{ij} = 1$

[24] Gross is referred to the cost of R&D. See below.

[25] The assumed functional forms of demand and cost function, together with $A > c(1 - \gamma_0)$, assure the existence and uniqueness of equilibrium in the Cournot game (Wolfstetter, 2000).

with $g_{ij} = 0$. Furthermore we define $N_t(i) \equiv \{j \in N \setminus \{i\} : g_{ijt} = 1\}$, that is the set of firms that have a collaboration with firm i at time t.

Innovation is modeled as a deterministic reduction in the unit cost of production. A network structure corresponds to a list of collaborators for each firm. Suppose to take a generic firm i: for i, collaboration with firm j at time t has a specific value v_{ijt}. The economic interpretation is as follows: whenever $g_{ijt} = 1$, firms i and j start a new R&D project together at time t, which allows them to reduce their unit cost of production to an extent that is function of v_{ijt}.

Therefore, such a value captures the opportunities for firm i to "learn" as a consequence of collaboration with firm j. In this framework, we refer to the process of learning as a process of knowledge "recombination", an idea that dates back to Schumpeter and has been recently rediscovered also in formal models (Weitzman, 1998; Olsson, 2000). According to this interpretation, the creation of new knowledge relies on pre-existing knowledge (of the pair) as major inputs. In the model, firm *i's* knowledge (i.e. its technological capabilities) is completely described by the vector $(\gamma_{it}, \alpha_{it})$. Being exposed to firm *j's* knowledge in the collaboration, firm i recombines its knowledge and improves upon it to an extent that is increasing in firm j level of efficiency (which is taken as a proxy for learning opportunities) decreasing in firm i 's level of efficiency (capturing decreasing returns in learning) and depending on firms' relative technological positions according to a well specified function. Firm's technological positions are modified after collaboration, too[26].

This representation of the learning process has the big advantage of parsimony, since the distribution of technological capabilities in the industry identifies both the outcome of market competition and the effects of technological collaboration.

More specifically, the value from collaboration is given by $v_{ijt} = f(d_t(i,j))\gamma_{jt-1}$. It is increasing in γ_{jt}, since the higher is the level of efficiency of your collaborator (the

[26] A quite similar representation of knowledge, in the context of knowledge creation as knowledge recombination, can be found in Cowan et al. (2003).

more it is "knowledgeable"), the more you can learn from it. It is also increasing in the value assumed by a function f, whose argument is given by the technological distance between firms, as defined by $d_t(i,j) = |\alpha_{it-1} - \alpha_{jt-1}|$. Some authors have argued that firms need to be technologically "not too distant, nor too near" for effective collaboration to take place (Nooteboom, 1999). This is because there are two opposing forces: if firms are distant, their different technological specializations can create opportunities for complementarities and synergies; but if they are too distant, they lack the "absorptive capacity" (Cohen and Levinthal, 1989) to learn from their collaborator and cognitive distance can harm effective communication. This conjecture has found empirical support (Mowery et al., 1998; Sampson, 2003) and it is reflected in the particular functional form chosen for f, which is assumed to be a concave parabola (Cusmano, 2002):

$$f(d_t(i,j)) = a_1 - \frac{a_2^{\,2}}{4a_3} + a_2 d_t(i,j) - a_3 d_t(i,j)^2 \tag{4}$$

$$a_1, a_2, a_3 > 0$$
$$f(d_t(i,j)) \geq 0 \; \forall \; d_t(i,j) \in [0,1]$$

The vector (a_1, a_2, a_3) identifies the technological characteristics of the industry. $\frac{a_2}{2a_3}$ is the optimal technological distance, as the result of the counterbalancing forces of absorptive capacity and search for complementarities[27]. a_1 is a measure of "technological opportunities", being $a_1 = \max_d f(d)$.

Given the total value of collaboration $V_{it}(g_t) = \sum_{j \in N_t(i)} v_{ijt}$, γ_{it} is determined by

$$\gamma_{it} = 1 - e^{-\lambda L_{it}} \tag{5}$$

where

$L_{it} = L_{it-1} + V_{it}(g_t)$ and $\lambda > 0$

[27] Parameters are assumed to be chosen in a way that the maximum point lays in the appropriate interval.

Equation (5) captures the decreasing returns in the innovative process.

Finally, we assume that through collaboration firms modify their technological position. Formally:

$$\alpha_{it} = \rho\alpha_{it-1} + (1-\rho) \sum_{j \in N_t(i)} \frac{\gamma_{jt-1}}{\Gamma_{it-1}} \alpha_{jt-1} \qquad \text{if } N_t(i) \neq \{\varnothing\} \tag{6}$$

$$\alpha_{it} = \alpha_{it-1} \qquad \text{otherwise}$$

where $\Gamma_{it-1} = \sum_{j \in N_t(i)} \gamma_{jt-1}$, $\rho \in (0;1]$.

The final technological position of a firm at time t is a linear combination of its old technological position and a weighted average of technological positions of collaborating firms. A firm is weighted more if it has a high efficiency level (that implies more opportunities of learning). When $\rho < 1$, firms become technologically more "similar" to their collaborators. When $\rho = 1$ (so that technological positions are time-invariant), firms maintain their "identity" in the process of learning (when they recombine their knowledge).

2.4 The evolution of the network

Each period two firms among the ones still in the market are randomly chosen to possibly change their network state. Firms that are not currently collaborating can decide to form a collaborative link, firms that are already collaborating can severe the existing link. Each link has the same probability to be revised.

We assume that maintaining a collaborative link costs each firm a fixed amount $E>0$ in each period. E has to be interpreted as the firm's contribution to the joint R&D project. For a firm involved at time t in $|N_t(i)|$ collaborations, net profits are equal to $\Pi_{it} - |N_t(i)|E$.

The proposed algorithm can be reformulated as follows: each period, two firms are allowed to modify their portfolio of collaborations, starting a new collaboration between each other if it does not exist, or interrupting it if exists. The state of the remaining network is unaltered: all the other collaborations in which these firms are involved, and the collaborations of all the remaining firms are automatically confirmed. In other words, network at time *t-1* and time *t* may differ only for the state of one link.

Suppose that at period *t*, the link *ij* (i.e. the potential or existing link involving firms *i* and *j*) is randomly chosen to be updated. Define $\Pi_{it}(g;\alpha,\gamma)$ as the profit for *i* resulting from market competition when the network is g and the initial technological capabilities are given by (α,γ).

If $g_{ijt-1}=1$, the link is severed if $\Pi_{it}(g_{t-1}-g_{ij};\alpha_{t-1},\gamma_{t-1})>\Pi_{it}(g_{t-1};\alpha_{t-1};\gamma_{t-1})-E$ *or* $\Pi_{jt}(g_{t-1}-g_{ij};\alpha_{t-1},\gamma_{t-1})>\Pi_{jt}(g_{t-1};\alpha_{t-1},\gamma_{t-1})-E$, while in the opposite case it is maintained. This simply means that a firm wants to sever an existing link if profits without the link and the saving on the R&D cost are higher than the profits with the link. If $g_{ijt}=0$, the link is formed if $\Pi_{it}(g_{t-1}+g_{ij};\alpha_{t-1},\gamma_{t-1})-E\geq\Pi_{it}(g_{t-1};\alpha_{t-1};\gamma_{t-1})$ *and* $\Pi_{jt}(g_{t-1}+g_{ij};\alpha_{t-1},\gamma_{t-1})-E\geq\Pi_{jt}(g_{t-1};\alpha_{t-1},\gamma_{t-1})$. If a link does not exist, it is formed when for both players the gain stemming from forming the link is higher than the R&D cost they have to sustain[28].

In terms of behavioral assumptions, the proposed rule implies that agents are myopic, since they decide only on the basis of their current pay-off, but at the same time they have rational expectations *within a given period*, since during the networking phase at time *t* are able to predict *correctly* the marginal cost of their rivals at time *t* and the Nash equilibrium that will be played in the market phase.

[28] In order to avoid that with probability 1 no link is profitable at t=0, we assume that $E\leq E^*$, where

$$E^*=\left(\frac{a-(n-1)c(1-\gamma_1^{\max})+(n-2)c(1-\gamma_0)}{n+1}\right)^2-\left(\frac{a-c(1-\gamma_0))}{n+1}\right)^2 \text{ and } \gamma_1^{\max}=1-e^{-\lambda(I_0+a)}$$

This assumption of myopic behavior aims at the representing the bounded rationality of agents who face a highly complex and uncertain future evolution of the R&D network and of the technological capabilities of firms in the industry.

3. Analytical results

In this section we provide some analytical results. First, we consider the incentives to form collaborative links at the level of the *single pair* of firms. We will also show two numerical examples, for the set of parameters we will consider in the simulations. Then we will turn to the long run properties of the system. Although the stochastic process generated in the model is rather complex, a clear and intuitive result holds for the network state in the long run.

3.1 Firms' cooperative strategies

Let me introduce the following function:

$$F_i(\alpha_j, \gamma_j \mid (\alpha, \gamma)_{-j}) = \left(\frac{A - nc(1-\gamma_i)(e^{-\lambda(\gamma_j f(d(i,j))})+ c(1-\gamma_j)e^{-\lambda f(d(i,j))\gamma_i} + \sum_{k \notin i,j} c_k}{n+1} \right)^2 - \left(\frac{A - nc(1-\gamma_i) + c(1-\gamma_j) + \sum_{k \notin i,j} c_k}{n+1} \right)^2$$

Suppose to take a generic pair of firms i and j. Fix the technological capabilities of the other *(n-2)* firms, and from $\gamma_k, k \in N/\{i,j\}$ derive the unit cost of such firms. Studying $F(\cdot)$ We can answer to the following question: how does the gross gain (i.e. the variation in profits excluding R&D costs) for i of forming a link with firm j vary, as a function of j's and i's technological capabilities?[29]

[29] Notice that implicitly we restrict our attention to the cases where the formation of the link does not lead to the exit of any firms.

In order to make computation easier, we write F as:

$$F_i(\alpha_j;\gamma_j \mid (\alpha;\gamma)_{-j}) = \left(\frac{c(1-\gamma_j)(e^{-\lambda f(d(i,j))\gamma_i} - 1) - nc(1-\gamma_i)(e^{-\lambda\gamma_j f(d(i,j))} - 1)}{n+1} \right)$$
$$\left(q_i(+ij) + q_i(-ij)\right)$$

where $q_i(+ij)$ and $q_i(-ij)$ represent the quantities produced by firm i with and without the link with firm j respectively. The first factor represents a necessary condition for collaboration: the net effect of counterbalancing forces on firm i's profits given by the reduction in its costs and in firm j costs must be positive, i.e. firms must increase the quantity they produce (and consequently their profits). Consistent with the existence of an interior solution, firms i and j are assumed to be close enough so that necessary condition is always satisfied.

We can show that the following propositions hold (the proofs can be found in the appendix):

Proposition 1 *Ceteris paribus, gains from the collaboration increase when firms' technological distance move towards the "optimal technological distance", and decrease otherwise.*

Proposition 2 *Ceteris paribus, the effect of an increase of γ_j on the gains from the collaboration is ambiguous. Possibly, an inverse U relation holds between γ_j and gains from collaboration.*

Proposition 3 *Ceteris paribus, the effect of an increase of γ_i on the gains from the collaboration is ambiguous. Possibly, an inverse U relation holds between γ_i and gains from collaboration.*

Proposition 4 *Ceteris paribus, gains from the collaboration decrease when the remaining firms' average efficiency increases.*

The first proposition is obvious. Proposition 2 is instead more interesting. The rationale for the possibly non-monotonic relationship is straightforward, however. High efficiency of a collaborator is good since your opportunities of learning increase and the extent it can learn from you is limited, but at the same time it is bad since efficiency is correlated with size. If a firm i's potential collaborator is highly efficient, then it is "large". This makes i a "small" firm, in relative terms. Since we deal with process innovation, smaller firms have lower total gains per unit of cost reduction, and their incentive to collaborate and innovate, ceteris paribus, is smaller. This is the so-called "cost spreading" argument, which has been claimed to be one of the advantages in innovation by large firms, and it has found empirical support (Cohen and Klepper, 1996).

The nature of the opposing forces is symmetric in Proposition 3. If firm i is highly efficient, it assures great opportunities of learning to its potential collaborator, and the reduction in its unit cost is smaller in absolute value. At the same time firm i is "large": so that reduction in unit cost of production can be spread over a larger quantity.

Finally, the average efficiency of other firms (Proposition 4) comes into play through the usual channel: its effect on firm's size. Its increase decreases the gains from collaboration, since it makes the firm "smaller" in relative terms.

The results show the complex nature of the interaction between the technological and markets aspects concerning firms' incentives to collaborate. Furthermore they stress the feedbacks between firms' incentives and the evolution of the network. Network evolution affects firms' incentive through market competition and opportunities for learning. In turn, the network changes according to firms' decision. Firms' strategies and the network *coevolve*, a point that has already been raised by business scholars (Koza and Lewin, 1998).

Figure 1 and Figure 2 show the behavior of $F(\cdot)$ under the parameterization of the "standard" simulation discussed in the next session. In the first case (Figure 1), $\gamma_i = 0.35$ and $\sum_{k \neq i,j} \frac{\gamma_k}{n-2} = 0.35$. Firm i is sufficiently small so that the inverse U relationship between gains from collaboration and γ_j emerges. When γ_j is large enough (approximately 0.5), the negative effect on size prevails on the positive effect of technological opportunities. If instead $\gamma_i = 0.5$ (Figure 2), firm i's size guarantees that an increase of γ_j monotonically increases the gains from collaborations.

Figure 1: Gains from collaboration-1

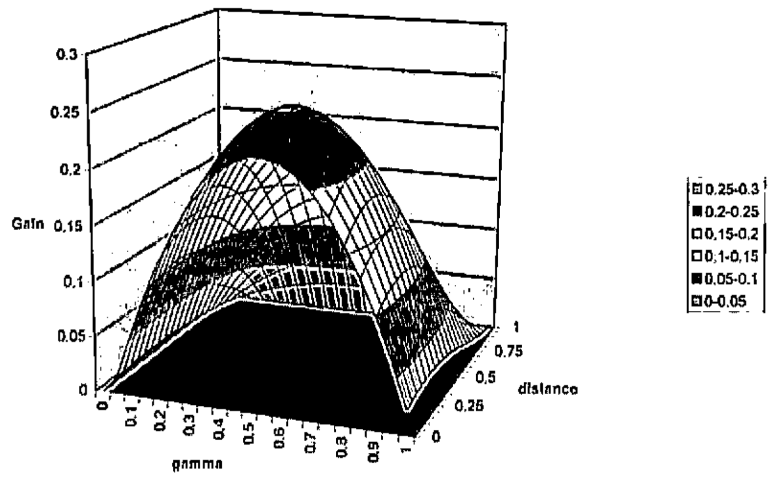
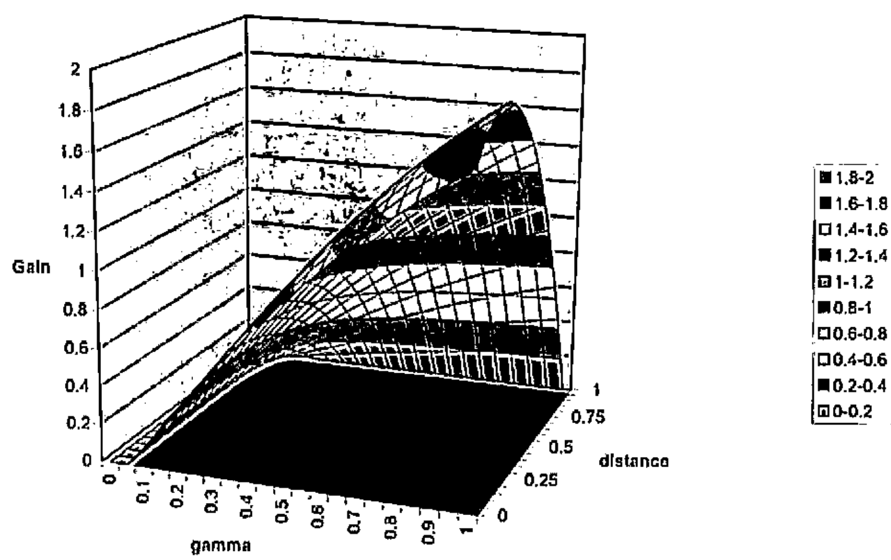


Figure 2- Gains from collaboration-2



3.2 The long run properties of the system

Although the stochastic process describing the evolution of the R&D occurs on a rather complicated state space, it is easy to derive clear results about the limit behavior of the network structure.

The industry at time t is completely characterized by the state $\{g_t, \gamma_t, \alpha_t\}$. Then, it is easy to verify that the underlying stochastic process satisfies the Markov property. Theorem 1, whose proof can be found in the appendix, concerns the long run properties of such a process.

Theorem 1 *As $t \to \infty$, each link is absent with probability 1. The absorbing states of the process are characterized by the empty network, and the set of these states is reached almost surely in the long run.*

The intuition behind this result is very simple and comes directly from the existence of marginal decreasing returns in the outcome of collaboration. Since innovative opportunities become smaller and smaller as firms continuously invest in R&D, while its cost is constant and strictly positive, it will come a time where forming or maintaining collaborative links is not convenient, irrespectively of other firms' technological positions. Loosely speaking, when ("almost"[30]) everything that could be discovered has been discovered, investing in R&D becomes unprofitable. Nevertheless, we are mainly interested in the transition phase of the system, per se and for the way it affects the final equilibrium is reached. This will be the subject of next section, where numerical simulations of the model are reported.

4. Simulation results

In this section we discuss the results emerging from a series of numerical experiments performed on the model. Although several exercises are possible, the ones reported here are illustrative of the basic mechanisms underlying the model.

[30] Obviously, as the simulation will make clear, the precise quantification of "almost" is endogenous to the model.

In the "standard simulation", we consider a situation where competition is rather tough at the beginning. Market size is A=65, 16 firms populate the industry at time 0, and their initial unit cost is about 47.56 (c=50, $L_{i0} = 5\ \forall i \in N$). The initial network is empty. The "optimal" technological distance is 0.25, and technological parameters are chosen in a way that the expected value of $f(d)$ is 0.5, ($a_1 = 0.56, a_2 = 0.5, a_3 = 1$), under the assumption of technological positions that are uniformously distributed along the interval (0,1). The R&D cost is rather "high", E=0.0230, and corresponds to 0.975* E^*, where E^* is the largest R&D cost for which firms at optimal distance will form a link given their initial costs. $\rho = 1$, so that technological positions are time-invariant. We run the experiments for 1000 periods, by which a steady state is reached.
Figure 3 and 4 reports the results for the average of 40 replications.

Figure 3

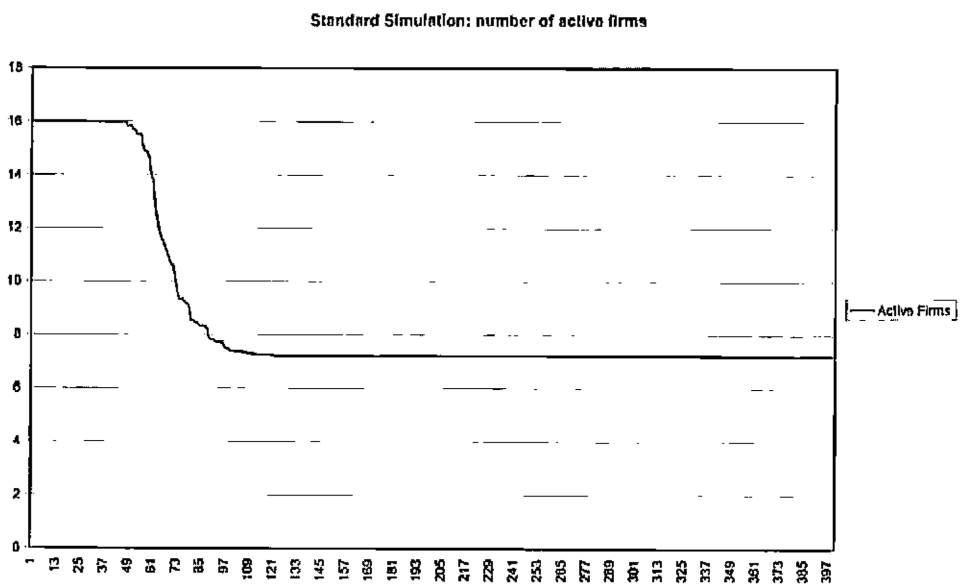


Figure 4

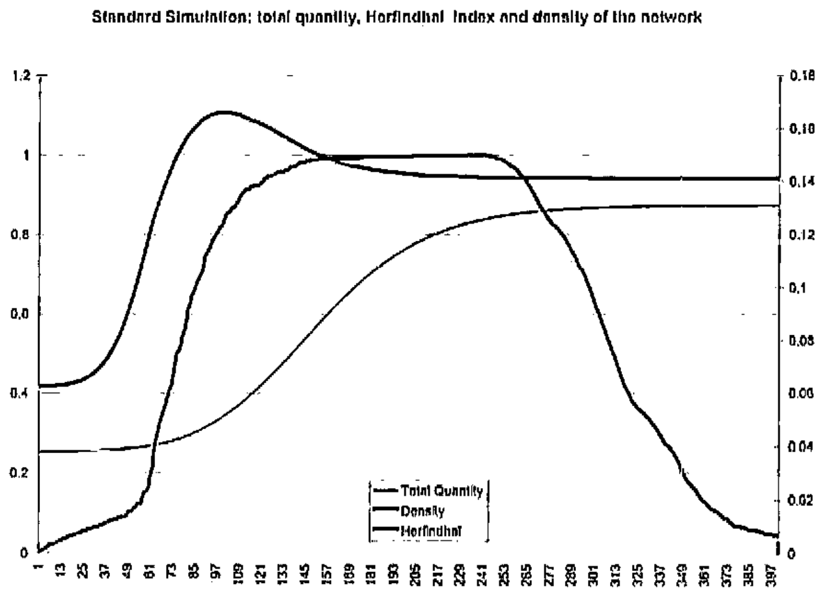


Figure 3 reports the number of active firms over time[31]. The first immediate result is that the number of active firms has a sudden drop around period 45: a shakeout occurs. In the steady state, less than 8 firms on average are in the market, then slightly less than half of the initial number of firms. The shakeout (defined as a significant and rapid reduction in the number of firms active in the market) is indeed a typical feature of the evolution of industries in early stages, as represented by the theory of industry life cycles (Klepper, 1997). In the model, it is the process of network formation that creates the shakeout among firms that are symmetric at the beginning. In other words, the existence of a R&D network (i.e. the possibility for firms to form cost-reducing links) operates as a strong *selection mechanism*.

Figure 4 further elaborates on this point, and shows an interesting dynamics involving market structure and the network of collaborating firms.

[31] Figure 3 reports the average number of firms active in each period across simulations. For this reason, we observe fraction of firms.

The figure reports the dynamics of three variables: total output produced in the market, normalized by market size ($\frac{Q}{A}$); market concentration, measured by the Herfindhal index; and network density, which is the fraction of existing links over the total number of possible links (considering only the firms still in the market). The scales of these variables are different. For preserving readability and comparison of behavior over time, total quantity and density are to be read along the left axis, while the right axis is for the Herfindhal index.

The evolution of the industry can be described in the following terms. At the beginning the density of the network is growing relatively slowly. Since R&D costs are relatively high, market relatively small and the average level of efficiency in the industry low, firms need to find partners located almost at the optimal technological distance, and this process is assumed not be instantaneous. This creates differences in the relative competitiveness of firms, expressed by a sharp in increase in the concentration index. In any case, given the low average level of efficiency in the market, the process of "knowledge recombination" is reflected by a limited growth rate for total output, which, given the assumptions, is only depending on the *average* efficiency of firms.

When the shakeout occurs, the time series for the network density has a break: this is due to the fact that the firms exiting the market have typically no links, and then they were lowering the average number of links. However, the process of links formation continues, until a *complete* network (density 1) emerges for around 100 periods. Concentration continues to grow, but then it starts declining when the density reaches a sufficiently high level: the network operates first as a mechanism creating different efficiency levels and then as a mechanism favoring the "catching-up" of relatively less efficient firms[32].

For around 100 periods, therefore, we can observe a sort of "steady state", where almost equal size firms operate in a complete network.

The behavior of total output, reflecting the behavior of average efficiency, follows an S-shaped curve. The growth rate of total output is the highest during the formation of the

[32] This result is clearly associated to the asymptotic nature of the cost function: knowledge is always created, if a firm is connected, but at a decreasing rate.

network after the shakeout. In this period, the increasing density of the network, together with an increase in the average level of efficiency (creating more opportunities for recombination) and the fact that marginal decreasing returns are not limiting innovative opportunities yet, generate a high level of growth. Interestingly, the inflection point in the output series roughly corresponds to the time in which a complete network is formed. Then, the "steady state" in market structure and network dynamics is accompanied by a low growth of the average efficiency.

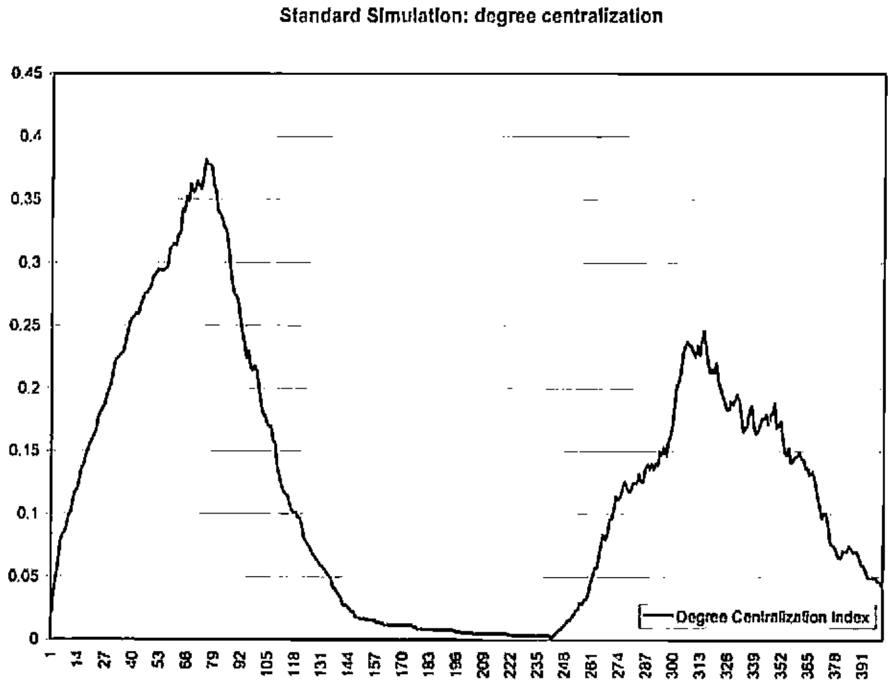
Since we model innovation as a process in which knowledge is both an input *and* an output, we can also interpret the results claiming that while in the early phases network formation mainly drives the creation of knowledge, in the late stage it is existence of a large pool of knowledge which preserves the incentive for firms to form new links (i.e. the cause-effect relation between network formation and knowledge creation is reversed while time elapses).

The final period occurs when technological opportunities have substantially been depleted. Total output and market shares stabilize, and simply some time is required for firms to severe their link. The final long run equilibrium is then reached when the empty network is finally obtained.

It is also interesting to look at the evolution of the network structure over time, especially for the phase immediately preceding and following the shakeout.

Figure 5 reports the behavior of the group degree centralization index over the simulation time.

Figure 5



This index takes a firm's degree (its number of links) as its centrality measure, and it basically summarizes how the links are distributed across firms. It takes value 0 when all the firms have the same number of links (as it happens in a regular network, like the complete network), and value 1 in a star, where there is one firm connected to all the others, and no other links exist (Wasserman and Faust, 1994).

The index shows a marked growth until the shakeout period: this implies, in substance, that in this phase links are more and more unequally distributed across firms. Then, the value of the index falls down in a similar way, to reach the value of zero when the network becomes complete. Then it naturally grows again, when firms start removing their links, and comes back to zero, when the network is empty.

4.1 Discussion

Two main results deserve further explanations. The first major point is that, even firms are symmetric *ex ante*, the opportunity of forming R&D links can generate profound asymmetries *ex post*: in the long run, these are reflected in firms' survival.

Table 1 reports the statistics concerning the number of links for firms at period 40 (just before the shake-out) and their survival in the long run. The variable *link40* takes value 1 if the firm has at least one link at period 40; the variable *surviving* has value 1 if the firm survives the shakeout.

Table 1

	Surviving		
Link40	0	1	Total
0	*303*	*3*	*306*
1	*48*	*286*	*334*
Total	*351*	*289*	

The table clearly shows that firms exiting the market are firms without links. Furthermore, an inspection of the network structure in the initial phase shows that the network structure, at the shake-out, is typically given by a single component of connected firms, while remaining firms are disconnected. A first strong selection occurs between firms that *are* in network, and survive at the first shakeout, and firms that "are not able" to join the network "reasonably" soon[33]. The fact that firms without links eventually exit the market is not obviously surprising, since it is the natural consequence of the assumption that costs are reduced only through collaborations. The interesting point is the mechanism through which some firms are *excluded* by the R&D network.

Second, we need to explain also the evolution of the network structure, in particular the increase in centralization in the initial phase. The firms' polarization in two groups of connected and disconnected firms is a candidate for a first basic explanation, but the evolution *within* the main component can also be an important determinant.

We will show that both the selection process and the evolution of the network structure are driven by a self-reinforcing, path-dependent process, in which events in the early stages of industry affect firms' centrality in the initial network with long term

[33] However, as the following example of a single run will illustrate, belonging to the main component is a necessary but not sufficient condition to survive.

consequences in terms of survival (Arthur, 1990). Forming links at the very beginning (which in the model is due to random factors, and in the real world could correspond to different managerial practices, social contacts or other small "historical accident" affecting firms' networking propensity) propels a positive feedback mechanism that favors the centrality of such firms, and entraps excluded firms in their status. However, *among the surviving firm,* the negative feedbacks end up prevailing, and firms converge in market shares and efficiency levels.

The first mover advantage of firms forming links at the very beginning comes from the net effects of forces described in the previous section. Firms that are "lucky" and form links in the first periods become larger than the other firms. This increases their incentive to form new links, considering also that in this early phase decreasing returns are not substantial yet. At the same time, large firms are more efficient and competent (indeed, they are larger *because* they are more efficient) and they offer their collaborators more opportunities to learn. A complementarity exists between "large" and "small" firms: large firms are willing to cooperate because of the "cost spreading" argument and possibly because of the search for technological complementarities; small firms are willing to collaborate because of the high level of competences they can find. The final effect of this process is the tendency to reinforce the centrality of first movers firms, which results in the sharp increase of the centralization index. This process comes naturally to an end since the number of possible links to be formed is limited. This corresponds to the phase of industry maturity, when the network becomes complete.
At the same time, firms that are not able to form links in the initial phase are excluded by the subsequent process of the network formation: their incentive to start collaborations decreases because such firms are getting smaller and smaller, and they are a limited source of learning opportunities for their potential collaborators.
Overall, this suggests an industrial structure where one can identify three kinds of firms, identified by their position in the network in the initial phase: 1) isolated firms, which are never able to join the network, being trapped in a self-reinforcing mechanism of exclusion, and which end up exiting the market; 2) central actors, whose position is strongly path-dependent and that can gain a (temporary) leadership in the market; 3) (temporarily) peripheral actors, that is firms that are active in the network in relatively

laggard positions, but are destined to catch up with the leader, if able to survive the shakeout.

For a quantitative assessment, we have run two OLS regression on the data generated by the simulations. We considered the variation on the number of links between period 40 and period 10 as dependent variable (*newlink40*), and we regressed it on the number of times a firm has been called to change its network status from period 10 to period 40 (*newcalled40*) and on the number of links the firm have at period 10 (*link10*). In a sparse network, the first variable is clearly supposed to have a positive coefficient. Table 2 shows that, at the beginning of the life cycle, also the sign of the coefficient for the second variable is positive, and significant. We have the confirmation that the "Matthew effect"[34] is at work here: firms that are more central at the very beginning are more likely to attract new collaborators in the following periods. This property is often found in networks of alliances (see, for instance, Powell *et al*, 1996).

Table 2

newlink40	*Coeff.*	*Std. Err*	*t*	*P>|t|*
newcalled	0.2363637	0.0195672	12.08	0.000
link10	0.7089678	0.0574208	12.35	0.000
constant	0.3405201	0.0848473	-4.01	0.000
Number of obs	640			
F(2,637)	149.58			
R-squared	0.3196			

[34] The term refers to the Gospel According to St Matthew: "For unto every one that hath shall be give, and shall have abundance: but from him that hath not shall be taken away even that which he hath".

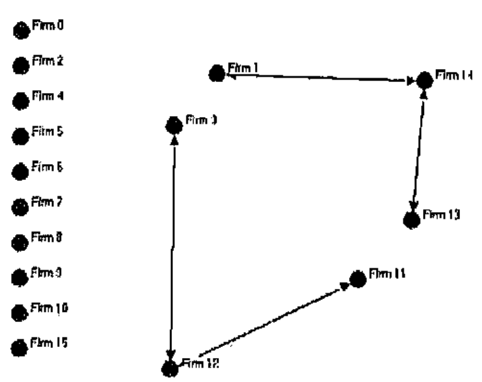
Concerning the selection process, the picture so far must be enriched including the role played by the externalities arising in the process of network formation. When two firms form a link, they always create a negative externality upon the remaining firms ("business stealing" effect). However, when two firms start to collaborate, this also creates a positive "technological externality", but only for firms connected with these two firms. The new projects, indeed, increase the rate of growth of efficiency of the two partners, with a positive effect on the technological opportunities for their collaborator. The increasing network density is strongly penalizing for firms outside the active network at the beginning strongly penalized by the increase in the network density, since they find increasingly difficult to join the network.

In commenting the results, there is an important final remark that has to be done. The results of the model are not purely dependent on the randomness associated to link revision. In particular, the shake-out is not simply driven by the fact that some firms are not drawn to form links. Randomness plays a role because it perturbs an initially symmetric situation, giving some firms an initial advantage. After that, an economic self-reinforcing mechanism operates, which significantly reduces the role of randomness. In other words, the model shows the instability of a symmetric market structure, when firms can form pairwise links. For this reason, it seems reasonable to start with an empty network in a symmetric set-up (in terms' of efficiency levels). If the network at time $t=0$ were a random network, this would simply guarantee some firms (the firms with more collaboration at the starting time) an exogenously given advantage, which would increase the probability of such firms to become central actors in the evolution of the network. The same argument applies if one removes the assumption of equally efficient firms at $t=0$. Furthermore, as section 4.3 will show, the results do not depend on isolated firms being fixed in their level of efficiency.

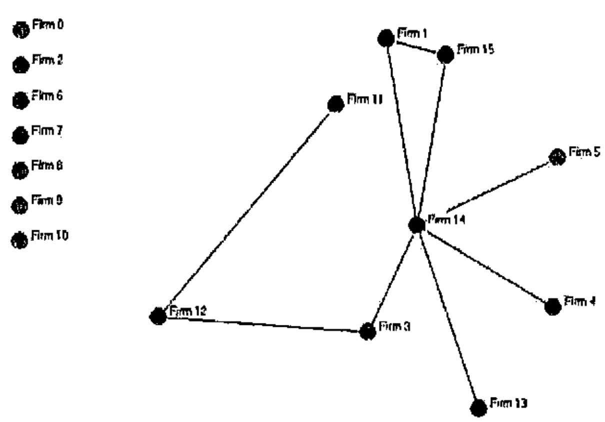
We consider now a single run as an illustrative example. In this history, the final number of surviving firms is 6. The shakeout occurs at period 67, when 7 firms (the ones without links) exit the market. However, the long run number of firms is reached only at period 91.

The following graphs show how network structure at period 20, 40 and 70, respectively. Table 2 summarizes the characteristics of firms at period 70, that is just after the shakeout ("Experience" is the total number of project performed by firms; "Surviving", as before, is 1 if the firm is active in the market in steady state, 0 otherwise).

Single run: R&D network at t=20



Single run: R&D network at t=40



Single run: R&D network at t=70

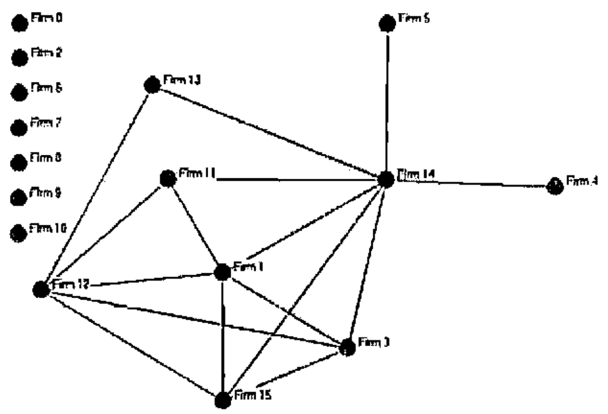


Table 2: Single run, standard simulation, market and network structure at *t=70*

Period 70	Firm 1	Firm 3	Firm 4	Firm 5	Firm 11	Firm 12	Firm 13	Firm14	Firm 15
NumberOfLinks	5	4	1	1	3	5	2	7	4
Profit	6.788736	7.823927	0.269092	0.198898	1.07813	7.401257	0.823664	17.00422	5.964725
Quantity	2.627496	2.813526	0.531123	0.471061	1.071042	2.741579	0.932558	4.143093	2.461041
Alfa	0.284422	0.168737	0.259	0.898658	0.101301	0.013809	0.660202	0.477916	0.228852
Gamma	0.1084	0.112121	0.066473	0.065272	0.077271	0.110682	0.074502	0.138712	0.105071
Market Share	0.147674	0.15813	0.029851	0.026475	0.060196	0.154086	0.052413	0.232856	0.138319
Experience	146	160	34	33	72	181	60	280	134
Surviving	1	1	0	0	1	1	0	1	1

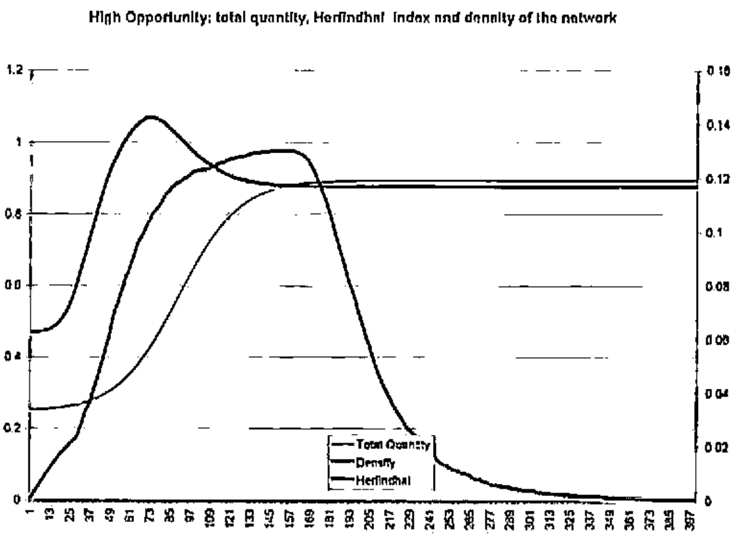
At period 20 the network is very sparse. Only four links are activated, but already two firms show prominence, firm 12 and firm 14, which are the center of the two star components. At period 40, the main component has a structure that resembles a star: firm 14 is very central, while firm 12 has remained stuck with its two initial collaborators. Finally, after the shakeout (period 70), the network is very dense, firm 14 has maintained its prominent position, with other firms catching up in the number of links. Notice how not all firms belonging to the R&D network will survive: not surprisingly, firms in a weak position (firm 4, 5 and 13) will finally exit the market.

4.2 Comparative dynamics

A natural question concerns possible exercises of comparative dynamics. In theory, several different parameterizations can be discussed. Here, we consider briefly two of them.

First, we increase technological opportunities. a_1, a_2, a_3 are chosen in a way that the expected value of *f(d)* becomes 0.75 (instead of 0.5)[35]. The opportunities for "knowledge" recombination within collaborative projects increase, making collaboration more attractive, ceteris paribus. Notice that high opportunity here does not mean that there is "more" to learn in the long run (unit cost is bounded from below, and it always (potentially) converges to 0), but simply that it is easier. The effect on market structure seems ambiguous, a priori. On one hand, more firms can engage in collaboration, especially at the beginning. On the other hand, the average efficiency growth rate is expected to be higher, and this is detrimental for the survival of firms that do not join immediately the network. As figures show, both effects are at work: with "high opportunities", the equilibrium number of firms is higher (the long run level of concentration is lower), but the shakeout occurs typically earlier. Technological progress is faster, as expected. Notice, finally, that the network does not reach density 1. This is easily explained by the fact that the faster depletion of innovative opportunities makes inconvenient the formation of links *before* a complete network is reached.

Figure 6



[35] In particular, $a_1 = 0.84375$, $a_2 = 0.75$ $a_3 = 1.5$

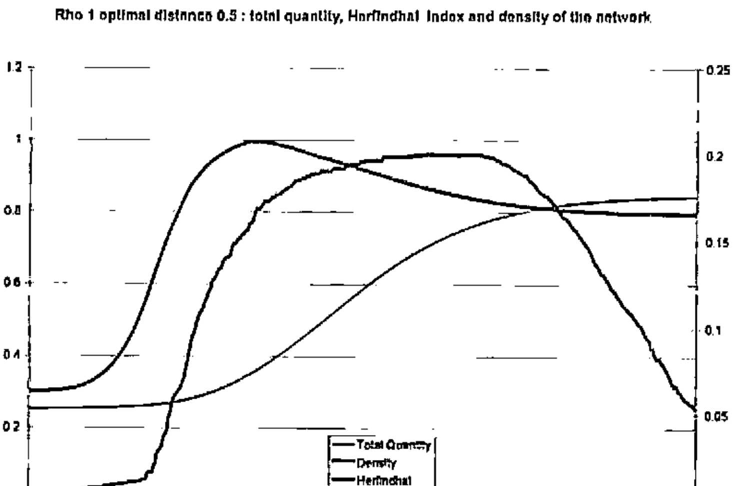
Finally, we have until now considered time invariant technological positions. Empirical evidence suggests indeed that interfirm technological agreements are important in explaining the movement of firms over time, and they can lead firms to become technologically more similar at the dyadic level (Mowery et al, 1998).

For a first study on the impact of variation of ρ on network evolution, we consider the case where technological heterogeneity matters in the outcome of collaboration, fixing the optimal distance at 0.5 (but keeping fixed the expected value of f)[36].

For this case, we run two sets of simulation, one with $\rho = 1$, the other with $\rho < 1$ ($\rho = 0.99$).

The results are reported in Figure 7 and 8.

Figure 7

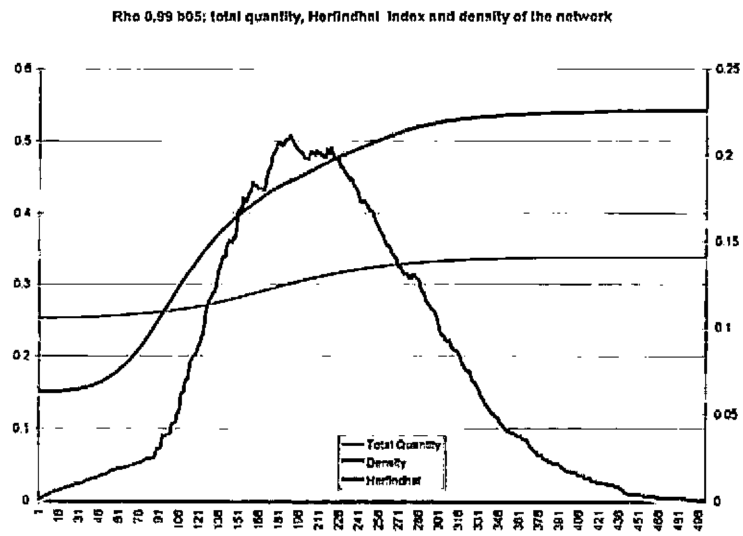


The first remarks concern the comparison between the first case and the standard simulation. Although the qualitative picture is rather similar, one can observe a slightly higher level of concentration in the long run. This is due to the relationship between the optimal distance and the initial distribution of technological positions. It is intuitive to

[36] $a_1 = 0.5517$, $a_2 = a_3 = 2.2069$

see that, once the assumption of uniformously distributed firms is maintained, increasing the optimal technological distance over a certain threshold makes less likely to find a partner around the optimal level, especially for firms in the middle of the technological interval. Since at the beginning this is what really matters, more frictions are introduced in the search of a satisfying partner. Firms lucky enough to find right partners get a stronger advantage. Progress is less rapid, concentration is higher and the network less dense. This is clearly an example which shows that the hypothesis on the initial distribution of firms matters, especially for certain technological environments, because it affects the opportunity for cooperation in the industry. This aspect deserves further analysis in the future.

Figure 8



The changes when $\rho<1$ are indeed quite radical. At the beginning the evolution is pretty much the same. This is not surprising, since in any case the process of technological convergence takes time. The real difference occurs after the shakeout. The process of network formation quite soon comes to an end. The reason for that is simple: the emergence of one single component inevitably lead to the overall convergence to a single technological position, which is detrimental for innovation. In the forty replications, the final value of the average technological distance lies in the interval

[0.002,0.02]. This implies that both technological progress and convergence in market shares stop.

This result shows the important role that entry, a factor not considered in the model, can actually play. In a "relatively" mature industry, in which the technological positions of incumbents have converged, new entrants have an important role to play. They can bring in a precious resource: different capabilities. This also can help the new firms to survive in the market, although less efficient, because of their role in the network. Extending the model to the role of new entrants is an interesting exercise that we plan to realize.

4.3 Extensions

In this section we check the robustness of the results with respect to two main assumptions of the models. First, we implement two other algorithms driving the formation of the R&D network; second, we introduce, although in a very simple way, an alternative way for cost reduction. Overall, the model exhibits robustness with respect to these changes.

Concerning the rules for links revision, it has been maintained the hypothesis of revision of *one* link per period. Given this restriction, two different algorithms have been considered. The first one can be defined as "socially oriented", and it aims at capturing the idea that meetings are more likely between firms that have collaborators in common.

In practice, the algorithm works as follows:

a) One firm is picked up randomly. Each firm has the same the probability to be chosen.

b) With probability $\frac{|N_i(t)|}{n-1}$ the firm revises the state of one of its existing links; otherwise, the firms revise the state of one of its non-existing links.

c) In the case of revision of an existing link, a firm $j \in N_i(t)$ is chosen with uniform probability.
d) In the case of revision of a non-existing link, a given firm j is chosen by i to revise the state of the link with probability:

$$\frac{1+|N_i(t) \cap N_j(t)|}{\sum_{k \notin N_i(t)} 1+|N_i(t) \cap N_k(t)|}$$

i.e. the probability of "meeting" is proportional to the number of collaborators that the two firms have in common.

The second algorithm will be labeled as "economically oriented". It is meant to capture the active, "rational" firm's search for optimal partners.

a) One firm is picked up. Each firm has the same the probability to be chosen.
b) For each $k \neq i$, net profits for i resulting from the meeting with k are computed. In particular, if the link ik does not exist, firm i correctly predicts the willingness of k to cooperate or not. We indicate with $\Pi_{it}(ik)$ such profits.
c) The firm j that is actually chosen is given by:

$$j \equiv \arg \max_{k \in N_i(t)} \Pi_{it}(ik)$$

In case of ties, the one with the highest index is chosen.

Figure 9 and 10 reports the Herfindhal index, the total output and network density for the same parameterization of the "Standard" case, when the algorithms of network formation are respectively the "socially" oriented algorithm and the "economically" oriented-one.

Figure 9

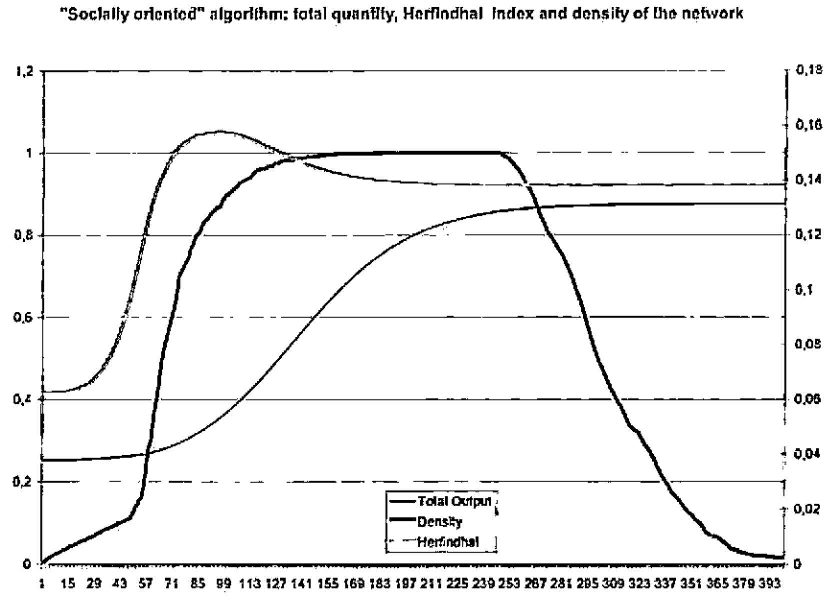
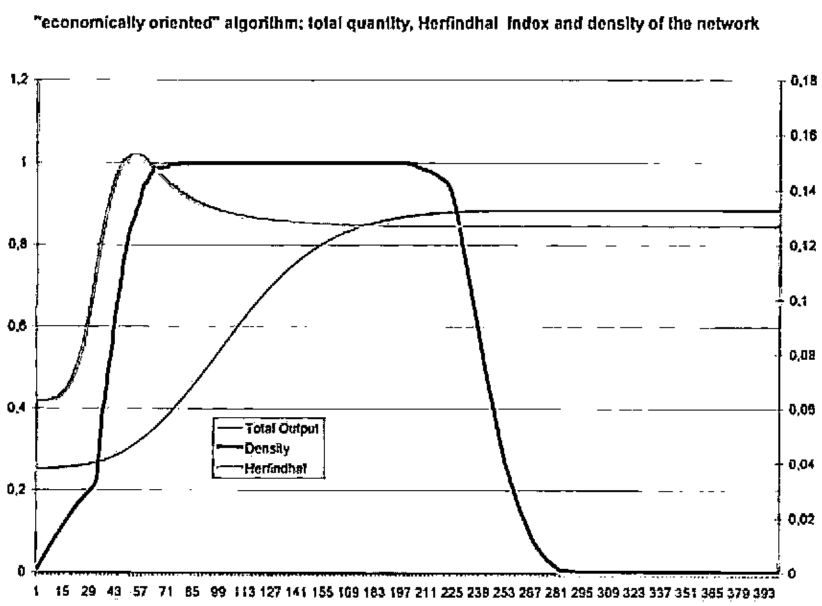


Figure 10



The effects of the "socially" oriented algorithm are negligible. The results are easy to interpret. What is crucial in the model are the first links formed, when the self-reinforcing mechanism is at the work. Since at the beginning the network is sparse, the probability of meeting is basically uniform, and the differences are necessarily of minor importance. When the network has reached a sufficiently high density (i.e. in the periods just preceding the shake out), firms active in the network become significantly more likely to meet. But these firms are also the more likely to be willing to start cooperation, since they are larger and more competent. The effect, then, is simply to make the convergence towards the complete network slightly more rapid, and consequently the shake-out slightly more rapid, without an impact on the qualitative behavior of the series.

The "economically oriented" algorithm has instead a more significant effect. This is similar to an increase in technological opportunities: the shakeout occurs earlier, but involves fewer firms. This algorithm substantially reduces the frictions in the network formation, and then it leads to a stronger role of the first mover advantage. Larger firms at the very beginning have more incentives to form new agreements, so they can look around for complementarities among the "small firms"; small firms can look for the largest firm. In this way, more links are formed: the shakeout is anticipated (because the exclusion process starts in advance) but involves fewer firms. In any case, the selection process is quite strong.

In terms of alternative way of cost reduction, a very simple formulation has been considered. We relax the assumption that costs can be reduced only through collaboration. Each period, each firm is assumed to start an "in-house" R&D project. More generally, other factors (for instance, learning by doing) can lead to this reduction in costs. The strongest assumption is that this process of cost reduction does not require any investment by the firm. Introducing explicitly an R&D cost (say a fixed cost similar to the costs required for cooperative R&D) in the framework of a simultaneous game would create a problem of multiple equilibria, when firms are close enough in efficiency level (i.e. size). Even if one assumed some rule to pick up one equilibrium, this would be too complex to implement.

In economic terms, this assumption can be justified by claiming that collaborative projects are typically started for larger, more costly (and with higher benefits) projects than in-house R&D. The assumption of no cost approximates a situation where each firm can always cover the costs of internal R&D, and the costs can be consequently not modelled. Furthermore, in the present context, we introduce in-house R&D to check the robustness of the results, and not to fully model the choice between in-house and cooperative R&D. Here, one major point is to check the robustness of the selection result due to the network formation. With positive costs (and indivisibility), very small firms would not invest in R&D alone either (for the cost spreading argument). Then the "no cost" situation can be interpreted an upper bound for outcome of the selection process: selection cannot be stronger than the case of "costless" R&D.

Following the notation of the paper, we label v_{iit} the value of such an in-house project. We consider two possible formulations:

$$v_{iit} = \beta f(0)\gamma_{it}$$
$$v_{iit} = \beta f(0)k$$

In the first case, we deal with a cumulative process: more competent firms have more valuable in-house projects; in the second case, instead, the value is independent from firm's level of efficiency. This second case is clearly more favorable to 'laggard' firms, and it is introduced mostly as a benchmark case.

Figure 11

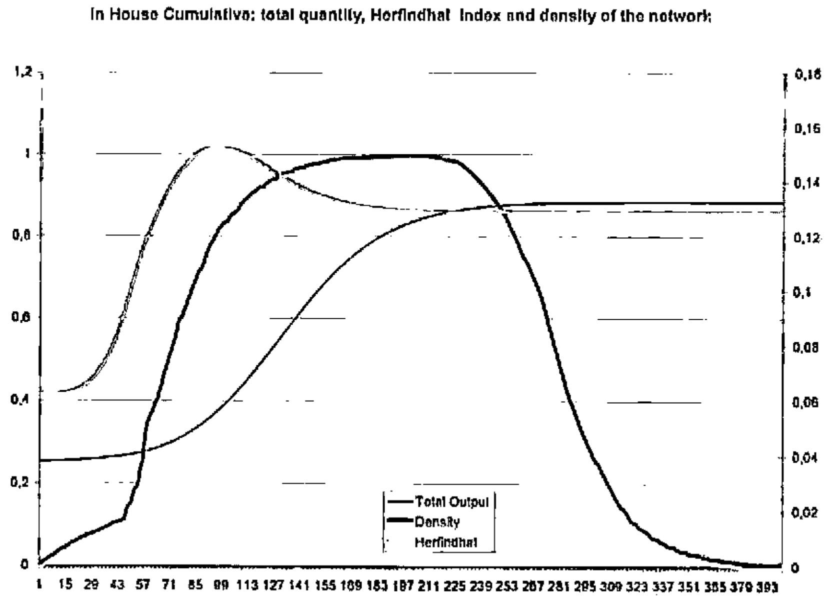
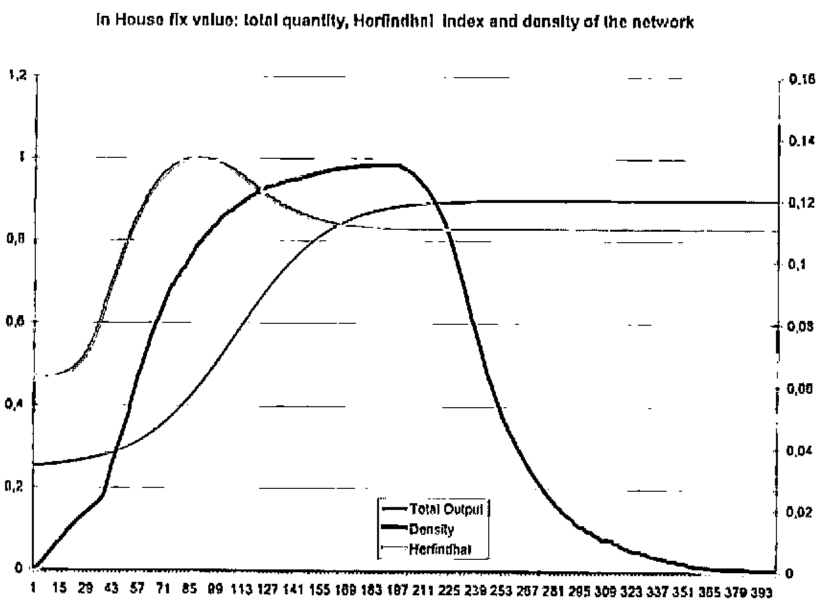


Figure 12



The effect of this modification (Figure 11 and 12) goes indeed in the predicted direction: selection is less strong. In the "cumulative" version (Figure 11, with $\beta = 0.4$), results are very similar to the "standard simulation". Indeed, in this formulation, in house R&D and cooperative R&D are complementary, in the sense that starting cooperative projects increases the value of in-house R&D, and the presence of in-house

R&D increases the incentive of cooperative R&D through its effect on size. Then the two effects, strengthening and weakening the selection process, substantially cancel out.

In the second case, the effect of decreased concentration is stronger. In this case (Figure 12), $\beta = 0.2$ and k=0.5, which means that in-house R&D is equivalent to a collaboration with a firm having the same technological position and efficiency 0.1. However, the main point here is that the results of the model respond "smoothly" to a limited ability of firm to progress autonomously in cost reduction: the logic in the arguments put forth in the previous sub-section is still valid.

5. Conclusion

In this paper we have presented a model of dynamic R&D network formation, in which the focus is explicitly on the joint dynamics of market structure, firms' technological capabilities and network evolution. First, through an analytical experiment, we have spelled out more clearly the effects of market competition and technological opportunities on the firms' incentive to collaborate. Then, we have showed results from numerical simulations. They clearly show the general importance of R&D networks as powerful selection mechanism, leading firms that are not able to join the network or that occupy weak positions to exit the market. At the same time, in the long run the network levels out the differences among the surviving firms, through a process of "densification" of the network that leads to the emergence of a complete or almost complete network in the phase of industry maturity. Also, we have also shown how the rate of technical progress, in the form of "high opportunity" or avaibility of partners, can affect the industry structure in the long run; we have pointed out the detrimental effect on innovation generated by a slow process of technological converge among firms; we have shown that the model is robust to the proposed modifications of the network formation algorithm and to a simple introduction of in-house R&D.

Two final remarks. From a theoretical point of view, it is interesting to compare our results with the analyses by Goyal and Joshi (2003) and Goyal and Moraga (2001). In particular, when they restrict their attention to symmetric networks, Goyal and Moraga show the stability of the complete network. However, in the simplified framework with three firms, Goyal and Moraga show also the stability of some forms of asymmetric networks, in which possibly one of the firms can be forced out of the market when excluded by the network (Zirulia, 2005b, extends their results to heterogeneous firms). In general, they maintain a role for asymmetric networks in having profound effects on market structure, a claim that is consistent with the empirical evidence on the firms' motivation to engage in collaboration (Hagedoorn, 1993). In a sense, our model reconciles these two results, assigning symmetric and asymmetric networks different roles in different phases of the industry.

From the empirical point of view, systematic analyses of the role and effect of R&D network on industry evolution are still missing. However, the model seems consistent with the appreciative argument on the emergence of "knowledge-based networked oligopolies" (Delapierre and Mytelka, 1998). In sectors like pharmaceuticals and ICT, a denser and denser network is emerging, involving the big players at the global level[37]. Furthermore, this web of alliances constitutes a significant barrier to entry (in the model, a barrier to survival), when rapid technical progress and strong competition make impossible a stand-alone strategy.

References

Arthur, W. B.(1990), "Positive feedbacks in the economy", *Scientific American*, 262, 92-99.

Cohen, W. and Levinthal, D. (1989),"Innovation and learning: the two faces of Research and Development", *The Economic Journal*, 99, 569-596.

[37] However, there is some evidence that for pharmaceuticals the rapid formation of alliances has stopped, starting in 1995.

Cohen, W. and Klepper, S. (1996), "A reprise on size and R&D", *The Economic Journal*, 106, 925-951.

Cowan, R. Jonard, N. and Zimmerman, J.B. (2003), "On the creation of networks and knowledge" in M. Marsili and A.P. Kirman, (eds), Proceedings of the 8th WEHIA Conference on Heterogeneous Interacting agents, Springer, forthcoming.

Cusmano L. (2002), "Knowledge creation by R&D interaction: role and determinants of relational research capacity", mimeo.

Delapierre, M. and Mytelka, L. (1998), "Blurring boundaries: new interfirm relationship and the emergence of networked, knowledge-based oligopolies" in M.G. Colombo (ed.) "*The changing boundaries of the firm*", Routledge Press, London.

D'Aspremont and Jacquemin (1988), "Cooperative and noncooperative R&D in duopoly with spillovers", *American Economic Review*, 78, 1133-1137.

Gilbert N., Pyka A. and Ahrweiler, P. (2001), "Innovation Networks-A simulation approach", *Journal of artificial societies and social simulations*, 4(3).

Goyal, S. and Joshi, S. (2003), "Networks of Collaboration in oligopoly, *Games and Economic Behavior*, 43, 57-85.

Goyal,S. and Moraga, J. (2001), "R&D Networks", *RAND Journal of Economics*, 32(4), 686-707.

Hagedoorn, J. (1993), "Understanding the rationale of strategic technology partnering: interorganizational modes of cooperation and sectoral differences", *Strategic management Journal*, 14, 371-385.

Hagedoorn, J. (2002), "Inter-firm R&D partnerships: an overview of major trends and patterns since 1960", *Research Policy*, 31, 477-492.

Hagedoorn J. Link, A. N. and Vonortas, Nicholas S. (2000), "Research partnerships", *Research Policy*, 29, 567-586.

Jackson, M. and Wolinski, A. (1996), "A strategic model of social and economic networks", *Journal of Economic Theory*, 71, 44-74.

Kamien, Mueller and Zang (1992), "Research joint ventures and R&D cartels", *American Economic Review*, 82, 1293-1306.

Katz, M. (1986), "An analysis of cooperative research and development", *RAND Journal of Economics*, 17, 527-543.

Klepper, S (1997), "Industry life cycles" *Industrial and Corporate Change*, 6, 156-181.

Koza, M. and Lewin, A. (1998), "The Co-Evolution of Strategic Alliances", *Organisation Science*, 9(3), 255-264.

Mowery D., Oxley, J. and Silverman, B. (1998), "Technological Overlap and Interfirm Cooperation: Implications for the Resource-Based View of the Firm", *Research Policy*, 27, 507-523.

Nooteboom B. (1999), "Inter-firm Alliances. Analysis and Design", Routledge, London.

Olsson, O. (2000), "Knowledge as a set in idea space: an epistemological view on growth", *Journal of economic growth*, 5, 253-275.

Ozman, M. (2003), "Self organizing inter-firm networks", Research Memoranda, 2003-020, MERIT, Maastricht University.

Powell, W.W., Koput, K.W. and Smith-Doerr, L. (1996), "Interorganizational Collaboration and the Locus of Innovation: Networks of Learning in Biotechnology", *Administrative Science Quarterly*, 41, 116-145.

Sampson, R.(2003), "R&D alliances and firm performance: the impact of technological diversity and alliance organization on innovation", mimeo.

Wasserman, S. and Faust (1994), K. "Social network analysis", New York: Cambridge University Press.

Weitzman, M. (1998) "Recombinant growth", *Quarterly Journal of Economics*, 113: 331-360.

Wolfstetter, E. (2000), "Topics in Microeconomics: Industrial Organization, Auctions and Incentives", Cambridge University Press.

Zirulia, L. (2005a) "Interfirm technological alliances and the evolution of industries: a survey of the empirical literature", doctoral dissertation, Università Luigi Bocconi.

Zirulia, L. (2005b) "R&D networks with heterogeneous firms", doctoral dissertation, Università Luigi Bocconi.

Appendix

Proof of Proposition 1

Simple derivations show that:

$$\frac{\partial F}{\partial d(i,j)} = \lambda(a_2 - a_3 d(i,j))c\left[\frac{n\gamma_j(1-\gamma_i)(e^{-\lambda(\gamma_j f(d(i,j))} - 1) - \gamma_i(1-\gamma_j)(1 - e^{-\lambda f(d(i,j))\gamma_i})}{n+1}\right]$$

As long as firms are close enough, the second factor is positive[38]. The sign of the derivative is then determined by $a_2 - a_3 d(i,j)$, which is positive if firms' distance is lower than the optimal one, and negative otherwise.

Proof of Proposition 2

Deriving one obtains:

$$\frac{\partial F}{\partial \gamma_j} = c\left[\frac{n\lambda f(d(i,j))(1-\gamma_i)e^{-\lambda\gamma_j f(d(i,j))} + 1 - e^{-\lambda\gamma_i f(d(i,j))}}{n+1}\right]\left[q_i(+ij) + q_i(-ij)\right]$$
$$+ c\left[\frac{(1-\gamma_j)(e^{-\lambda f(d(i,j))\gamma_i} - 1) - n(1-\gamma_i)(e^{-\lambda\gamma_j f(d(i,j))} - 1)}{n+1}\right]\left[\frac{dq_i(+ij)}{d\gamma_j} + \frac{dq_i(-ij)}{d\gamma_j}\right]$$

The quantities in the first two square brackets are positive, so it is the first addend. The sign of the second addend depends on

$$\frac{dq_i(+ij)}{d\gamma_j} + \frac{dq_i(-ij)}{d\gamma_j} = \frac{c\left[\lambda f(d(i,j))n(1-\gamma_i)e^{-\lambda\gamma_j f(d(i,j))} - e^{-\lambda\gamma_i f(d(i,j))} - 1\right]}{n+1}$$

which is negative for λ sufficiently small.

From the study of the second derivative, it can be shown that it is negative for λ sufficiently small. Then the point (if any) where the derivative becomes 0 must be a maximum point. If gains from the collaboration are positive, there are consequently three possible cases: the increase in γ_j 1) has always a positive effect; 2) has always a negative effect; 3) has a positive effect initially, and then has a negative effect.

Proof of Proposition 3

Deriving one obtains:

$$\frac{\partial F}{\partial \gamma_i} = c\left[\frac{-\lambda f(d(i,j))(1-\gamma_j)e^{-\lambda\gamma_i f(d(i,j))} + n(e^{-\lambda\gamma_j f(d(i,j))} - 1)}{n+1}\right]\left[q_i(+ij) + q_i(-ij)\right]$$
$$+ c\left[\frac{(1-\gamma_j)(e^{-\lambda f(d(i,j))\gamma_i} - 1) - n(1-\gamma_i)(e^{-\lambda\gamma_j f(d(i,j))} - 1)}{n+1}\right]\left[\frac{dq_i(+ij)}{d\gamma_i} + \frac{dq_i(-ij)}{d\gamma_i}\right]$$

The first addend is negative, while, if the necessary condition for positive gain holds, the sign of the second addend depends on $\left[\frac{dq_i(+ij)}{d\gamma_i} + \frac{dq_i(-ij)}{d\gamma_i}\right]$.

[38] Notice however that the condition of positivity here is stricter than the necessary condition of positive gains from collaboration.

It can be shown that:

$$\left[\frac{dq_i(+ij)}{d\gamma_i}+\frac{dq_i(-ij)}{d\gamma_i}\right]=c\left[\frac{n(1+e^{-\lambda\gamma_j f(d_{ij})})-(1-\gamma_j)\lambda f(d(i,j))e^{-\lambda\gamma_i f(d(i,j))}}{n+1}\right]$$

The first quantity in square brackets is larger than 1, while the second is smaller than 1 for $\lambda f(d(i,j))$ small. Their difference is then positive.

The overall effect is ambiguous. Studying the second derivative, one gets $\dfrac{\partial^2 F}{\partial \gamma_i^{\,2}} < 0$ for λ sufficiently small. Then the point (if any) where the derivative becomes 0 must be a maximum point. There are consequently three possible cases: the increase in γ_i 1) has always a positive effect; 2) has always a negative effect; 3) has a positive effect initially, and then a negative effect.

Proof of Proposition 4

The proposition comes directly from:

$$\frac{\partial F}{\partial \sum_{k\neq i,j} c_k}=\frac{2}{n+1}\left[\frac{(1-\gamma_j)(e^{-\lambda f(d(i,j))\gamma_i}-1)-n(1-\gamma_i)(e^{-\lambda\gamma_j f(d(i,j))}-1)}{n+1}\right]$$

Proof of Theorem 1

We consider the situation where a stable oligopolistic structure has emerged, in the sense that the number of firms will remain constant in the future (the market structure at time t will be maintained in all the periods if $\dfrac{A-n_t c_{it}}{n_t+1}>0\ \forall i\in N_t$). We have to prove that $\lim_{t\to\infty}\Pr(g_{ijt}=1)=0\ \forall ij\in N_t^2$. If $\lim_{t\to\infty}\Pr(g_{ijt}=1)\neq 0$ we would have $\lim_{t\to\infty}\gamma_i=\lim_{t\to\infty}\gamma_j=1$. By continuity of $F(\cdot)$ (which is the gain function defined in section 4.1), this implies $\lim_{t\to\infty}F_i(\alpha_j,\gamma_j)=\lim_{t\to\infty}F_j(\alpha_i,\gamma_i)=0$. But then, since E>0, the link will asymptotically become unprofitable. Given that each link is updated with a positive probability, it will be severed with probability 1 as $t\to\infty$, and then we have the initial claim.