Essays in Asset Pricing Anomalies

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Contents

Abstract

Asset pricing anomalies are one of the most lively and debated topic in modern finance.

In chapter one I present and discuss the database that I use for the empirical analyses in the following chapters. I also illustrate the methodology that I used to form two portfolios that capture a couple of asset pricing anomalies, the "size effect" and the "value effect". Finally, I introduce a measure based on the higher-order moments of the return distribution.

In chapter two I develop a theoretical model that explains how the motives behind agent trades make the asset expected returns deviate from those predicted by the one-factor Capital Asset Pricing Model. In addition, I analyze empirically the relation between the two portfolios' excess returns and trading volume.

In chapter three I consider the role that the three measures illustrated in the first chapter play in the risk-return relationship of equity-oriented hedge fund indexes.

Introduction

Since the beginning of the 1980s empirical evidence has mounted on the presence of anomalies in asset pricing. An anomaly is "a documented pattern of price behaviour that is inconsistent with the predictions of traditional efficient markets, rational expectations asset pricing theory"[1] The financial anomalies are mostly departures from the predictions of the Capital Asset Pricing Model (CAPM).

The anomalies were first identified on the U.S. stock markets. Stattman (1980) and Banz (1981) show that variables measuring some firm characteristics, namely the market value and the book-to-market ratio, have reliable power to explain the cross-section of average returns. These deviations from the CAPM are denominated the size and the value effect, respectively.

Later, De Bondt and Thaler (1985) and Jegadeesh and Titman (1993) find evidence of long-term return reversal and short-term return continuation, respectively. The latter pattern is known as momentum effect.

Despite the first studies on these issues date back to much more than a decade ago, financial anomalies, in particular the size effect, the value effect and the momentum effect, are still a very active field of theoretical and empirical research.[2]

[1] See Brav and Heaton (2002), p.1.

[2] Some of the more recent contributions are Jordan (2004), Jostova and Philipov (2004), Zhang (2004) and Zhang (2004).

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Chapter 1

Asset pricing anomalies and multi-factor models*

* I thank Andrea Beltratti for insightful discussions on this issue. All remaining errors are mine.

1.1 Introduction

The empirical analysis carried out in the next chapters deeply rests on a newly assembled database. The database contains data on equities quoted on the main U.S. stock exchanges. Such data are used to create Fama and French portfolios and a factor based on the conditional skewness of the return distribution, constructed by Harvey and Siddique (2000).

1.1.1 The origin of the Fama and French factors

Since the beginning of the 1980s empirical evidence has mounted on the presence of anomalies in asset pricing. An anomaly is "a documented pattern of price behaviour that is inconsistent with the predictions of traditional efficient markets, rational expectations asset pricing theory"[1] The financial anomalies are mostly departures from the predictions of the Capital Asset Pricing Model (CAPM) and are related to the securities characteristics.

The anomalies that were first identified concern the cross-section of U.S. stocks are the size effect and the value effect. Banz (1981) detect the so-called size effect. According to the author, the market value augments the explanation of the cross-section of average returns provided by the market βs. More precisely, average returns on stocks with low market value are too high given their β estimates and average returns on stocks with high market value are too low.

Stattman (1980) discover the so-called value effect, related to the book-to-market ratio. This measure is the ratio of a firm's book value of common equity to its market value. According to the author, firms with high book-to-market ratios earn returns in excess of those predicted by the CAPM.

Fama and French (1993) investigate the relation between the cross-section of returns on stock and bonds and some common risk factors. The authors introduce as possible risk factors two zero-investment portfolios, SMB and

[1] See Brav and Heaton (2002), p.1.

HML, aimed to capture the effect of size and book-to-market ratio.

The SMB (small minus big) factor measures the excess returns on a portfolio long on securities with low market capitalization (small stocks) and short on assets with high market capitalization (big stocks). The HML (high minus low) factor measures the excess returns on a portfolio long on securities with high book-to-market ratio and short on assets with low book-to-market ratio.

The authors conclude that "there are common return factors related to size and book-to-market equity that help capture the cross-section of average stock returns in a way that is consistent with multi-factor asset pricing models".

1.1.2 The origin of the conditional higher-moment factor

The Fama and French three-factor model, based on SMB and HML portfolios in addition to the market portfolio, has been widely adopted in the empirical studies, but also brought into question. Ferson and Harvey (1999) test the empirical performance of the Fama and French model on *conditional* expected returns. The authors concentrate on the ability of the model to explain common dynamic patterns, captured by a set of lagged, economy-wide predictor variables. Their analysis lead to the rejection of the three-factor model as a conditional asset pricing model, even in a sample of equity portfolios similar to the ones used to derive their factors and conclude that "Loadings on lagged instruments reveal information that is not captured by these popular factors for the cross section of expected returns. This should raise a caution flag for researchers who would use the FF factors in an attempt to control for systematic patterns in risk and expected return."[2]

Indeed, several studies highlight the importance of conditional asset pric-

[2] See Ferson and Harvey (1999), p. 33

ing models for the cross-sectional variation in stock returns. Jagannathan and Wang (1996) argue that a conditional version of the CAPM perform well in explaining the cross-section of average returns. Lettau and Ludvingson (1999) find that conditional consumption CAPM perform as well as the Fama and French three-factor model.

Finally, a strand of the literature focus on the role of higher-order moments of the return distribution.[3] In particular, Harvey and Siddique (2000) propose several measures of conditional skewness, thereby linking conditioning information and return distribution higher-order moments.[4]

1.2 A new database of U.S. stocks

As far as I know, the empirical studies on U.S. stock markets employ the SMB and HML portfolios originally formed by Fama and French.[5] Harvey and Siddique (2000) is not an exception. Furthermore, the authors use the same source of data as Fama and French to create the skewness measure. However, some authors have cast doubts on the accuracy of the data use by Fama and French.

1.2.1 Potential weaknesses of the Fama and French data

In order to form the SMB and HML portfolios, Fama and French (1993) use data from the CRSP and the COMPUSTAT databases.[6]

[3]See Ang, Chen and Xing (2001) and Dittmar (2002).

[4]For more details on this issue see chapter 3, section 2.2.

[5]The data are available on Kenneth French's website, http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

[6]The Center for Research in Security Prices is a financial research center at the University of Chicago Graduate School of Business. The COMPUSTAT database is the Standard and Poor's database of U.S. and Canadian public companies.

Kothari, Shanken and Sloan (1995) argue that two related survivorship biases in COMPUSTAT data may partially explain the relation between book-to-market ratios and security returns. They identify two potential sources of selection bias in the COMPUSTAT database. A possible source of bias is data "back-filling". When COMPUSTAT adds a firm to its data file, it might fill in data for the firm back to five years before. Consider high book-to-market firms. These firms were likely to be performing poorly in the subsequent years (low market value relative to book value). After five years, some of them will be no longer on the market, while some will overcome their financial distress. Hence, ex-post the latter are more likely to be added to the COMPUSTAT file.

A second source concerns firms that become financially distressed. They may stop reporting financial results. Then, only firms that ultimately recover from financial distress may report financial data retroactively, for the period they stopped reporting. Again, there is a potential bias toward having data in the database for firms that ultimately recover from distress. Overall, there could be upward bias to the measured relation between returns and book-to-market ratio.[7]

Fama and French (1993) address these issue and construct book-to-market based portfolios in a manner designed to minimize the effects of the biases, i.e. include only firms with 2 years of COMPUSTAT data before the portfolio formation date. They argue that COMPUSTAT rarely includes more than two years of historical data when it adds a firm to the database. However, it is not clear whether this completely eliminates the bias in the COMPUSTAT tapes.

Breen and Korajcyk (1995) and Barber and Lyon (1997) investigate on these potential biases.

Breen and Korajcyk (1995) adopt a new criterion to select securities:

[7] The authors propose to purge the two potential selection bias in the COMPUSTAT files by using an alternative data source, that is they use the book-to-market ratios and share prices for approximately 100 industries reported in the S&P Analyst's Handbook.

firms will only appear in the dataset in the month they were actually added to the file, so that they could avoid the selection biases. As a result, the authors do not detect any value effect in the sample.

In contrast, Barber and Lyon (1997) agree with Chan, Jegadeesh and Lakonishok (1995) that the survivorship bias in COMPUSTAT data is small.

1.2.2 A new source for the data

I collect the data from the Datastream database. This database is maintained by Thomson Financial and provides both market and accounting data on a wide range of securities. My choice is not unique: Annaert, Crombez, Spinel and Van Holle (2002) evaluate the returns for a cross-section of European stocks, using the Datastream data.

The dataset gathers qualitatively and time-series data on stocks quoted on the U.S. major exchanges, Amex, NASDAQ and NYSE.

Qualitatively data include for each stock:

- the name of the issuing company
- the name of the index in which the stock is currently included
- the date on which the stock started to trade on the given exchange
- the date on which the stock stopped to trade on the given exchange

Time-series data include for each stock:

- the market value, that is the product between the stock price and the shares outstanding
- the book-to-market ratio, where the book value is defines as the total asset, excluding the intangible assets, less total liabilities, minority interests and preference stock[8]

[8] Excluding intangible assets (R&D, trademarks and patents), which are tipically high for securities with low book-to-market ratios, makes the comparison between different stocks more transparent.

- the return index, a measure of the theoretical growth in value of a share, assuming that dividends are re-invested: $RI_t = RI_{t-1} * \frac{P_t + D_{t-1}}{P_{t-1}}$.

I consider monthly data.

The sample covers the period from January 1990 to June 2002. However, since the coskewness measure is based on past information, I process also data for the period January 1985 - December 1989.

Some securities have been listed on a given exchange only for a sub-period and then delisted, therefore I also include data on "dead" stocks. This data correct a potential survivorship bias in the sample.

Following Fama and French (1993), I consider only common stocks. Hence, I exclude ADRs, REITs, units of beneficial interest and close-end funds.[9]

However, the Datastream classification is not so detailed as the CRSP, so I focus my attention on the assets classified as "equity". This asset class contains both preferred and common shares, thus I use the Datastream mnemonic code to further restrict the set to common stocks.

In addition, following the authors, stocks with no accounting data at the end of December of year $t-1$. This last restriction, which is rather conservative, ensures that the accounting variables are known before the returns they are used to explain.

1.3 Portfolio formation procedure

In order to form the SMB and HML portfolios, several step are undertaken at the end of June t, $t = 1990, ..., 2002$.

First, stocks listed on the three major U.S. exchanges are ranked according to market capitalization, that is size. The median size of the securities

[9]The ADRs - an acronym for American depositary receipts - are based on foreign company ordinary shares. Their value is determined by several factors beyond the performance of the company, as political risk, exchange rate risk and inflationary risk. The REITs - an acronym for real estate investment trusts - are securities that sell like stocks on the major exchanges and invest in real estate either directly through properties or mortgages.

listed on NYSE is computed and used to split all common stocks into two groups, "small", S and "big", B. Stocks listed on the NYSE are also ranked on the basis of the book-to-market ratio, in order to define the breakpoints for the bottom 30% ("low", L), middle 40% ("medium", M) and top 30% ("high", H) values. In this case, I exclude securities with negative book-to-market ratio. The common stocks listed on the three markets are then assigned to one of the groups defined by the breakpoints. As Fama and French (1993) underline, the splits are arbitrary and essentially follows from the evidence in Fama and French (1992) that the book-to-market value has a stronger role in average stock returns than size.

Second, I construct six portfolios by combining the size and book-to-market group. For example, the SL portfolio contains the stocks that are both in the "small" group and in the "low" group.

Third, I compute monthly value-weighted returns on the six portfolios, from July t to June $t+1$. As Fama and French (1993) underline, using value-weighted returns allows to minimize the variance of firm-specific factors and results in mimicking portfolios that capture the different behaviour of securities with dissimilar characteristics.

Fourth, I compute the simple average of the returns on the portfolios grouped according to stock features. For instance, I compute the simple average of the returns on the three small-stock portfolios, that is SL, SM and SH.

Finally, I form the SMB and HML portfolio. The SMB portfolio returns are the difference between the simple average of the returns on the three small-stock portfolios and the three big-stock portfolios. The HML portfolio returns are the difference between the simple average of the returns on the two high-book-to-market-stock portfolios and the two low-book-to-market-stock portfolios.

Hence, the SMB portfolio contains securities with about the same weighted-average book-to-market value. Thus, the SMB returns should be largely free of the influence of the book-to-market value, focusing instead on the different behaviour of low and high market capitalization stocks. Similarly, the HML portfolio should capture the different behaviour of high and low book-to-market value stocks.

The market portfolio contains all the stocks in the six sorted portfolios plus the negative book-to-market stocks excluded from the portfolios.

All the returns are in excess of the one-month Treasury bill rate.[10]

Table 1 and the following graphs show the resulting portfolios.

Table 1: Summary Statistics

January 1990 - June 2000

Market Portfolio

	Mean	SD	Median	Skewness	Kurtosis	Min.	Max.
Fama-French	0.95	3.98	1.27	-0.77	4.83	-16.2	10.30
New	1.64	3.98	1.47	0.57	5.12	-9.57	17.46

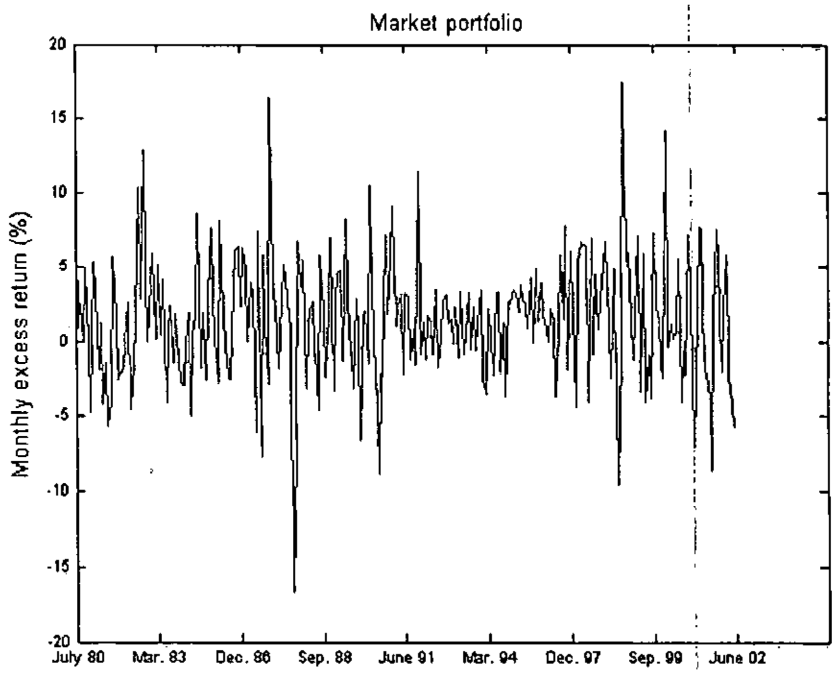
SMB Portfolio

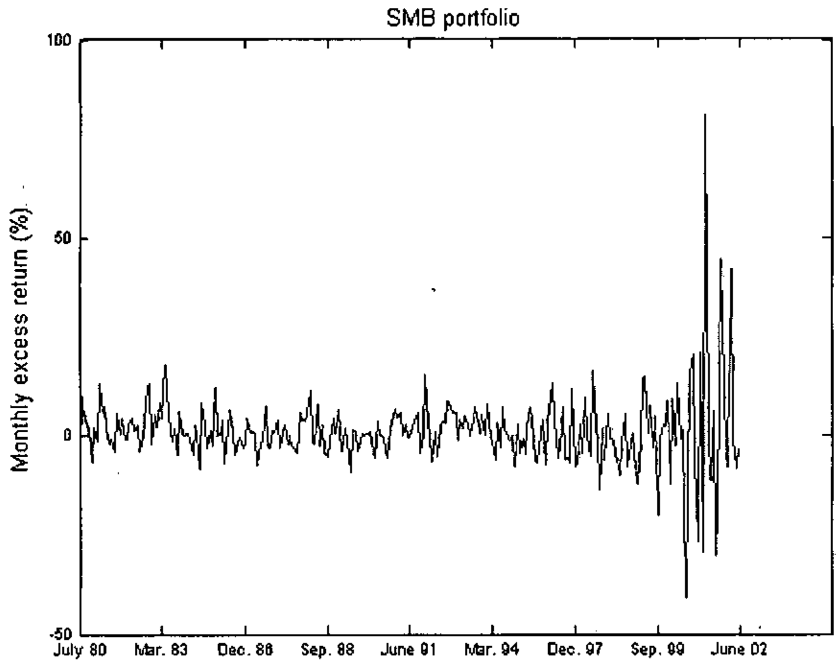
	Mean	SD	Median	Skewness	Kurtosis	Min.	Max.
Fama-French	0.00	4.04	-0.17	1.00	11.12	-16.62	21.83
New	0.39	6.17	0.12	-0.08	3.62	-20.20	16.21

HML Portfolio

	Mean	SD	Median	Skewness	Kurtosis	Min.	Max.
Fama-French	-0.12	3.25	-0.08	-0.40	4.99	-12.65	9.21
New	0.51	3.15	0.61	0.32	4.38	-7.44	12.06

[10] Source: Kenneth French's website.


Market portfolio
Monthly excess return (%)
20
15
10
5
0
-5
-10
-15
-20
July 80
Mar. 83
Dec. 86
Sep. 88
June 91
Mar. 94
Dec. 97
Sep. 99
June 02


SMB portfolio
Monthly excess return (%)
100
50
0
-50
July 80
Mar. 83
Dec. 86
Sep. 88
June 91
Mar. 94
Dec. 97
Sep. 99
June 02

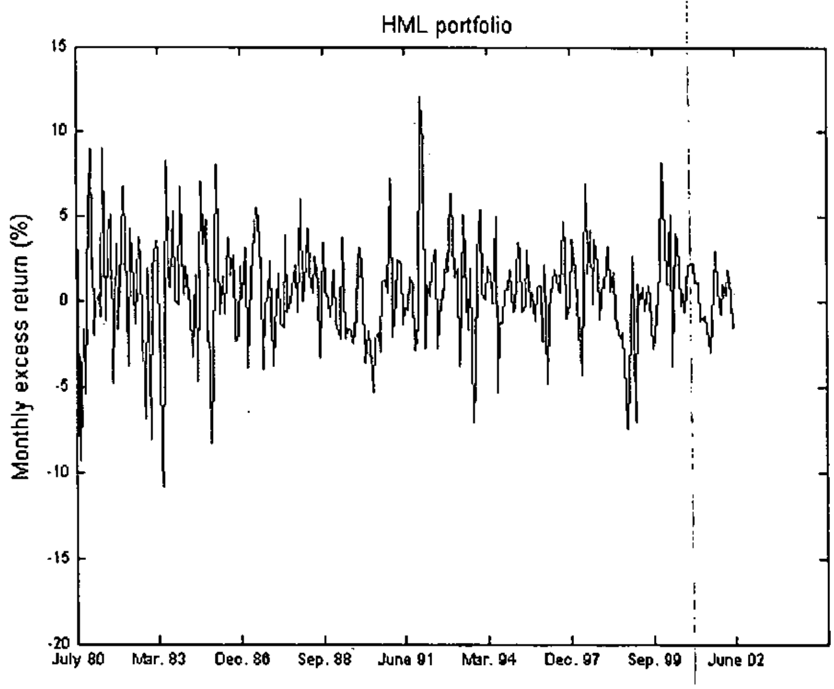


Figure 1 to 3 compare the new portfolios to the Fama and French portfolios.

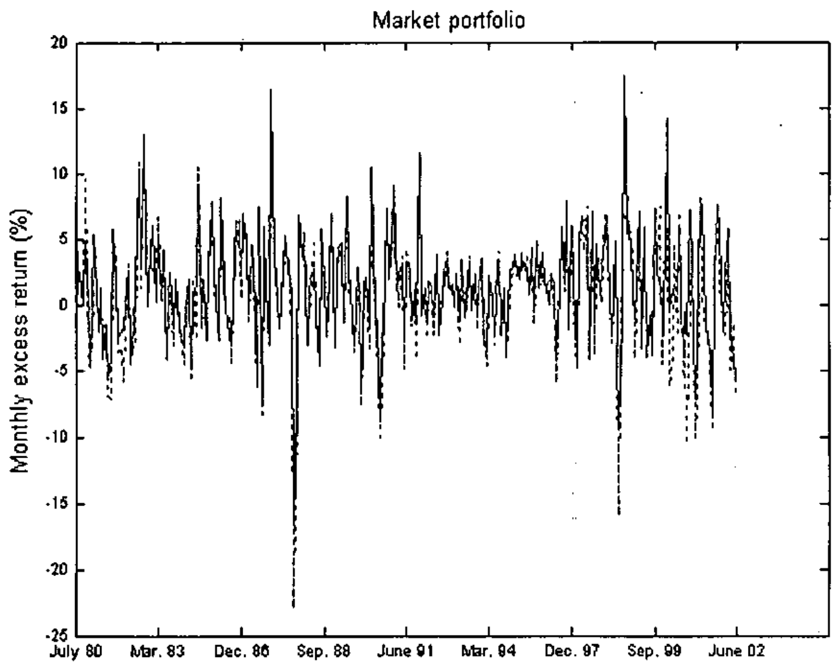


Figure 1 - --- my portfolio ... Fama and French portfolio

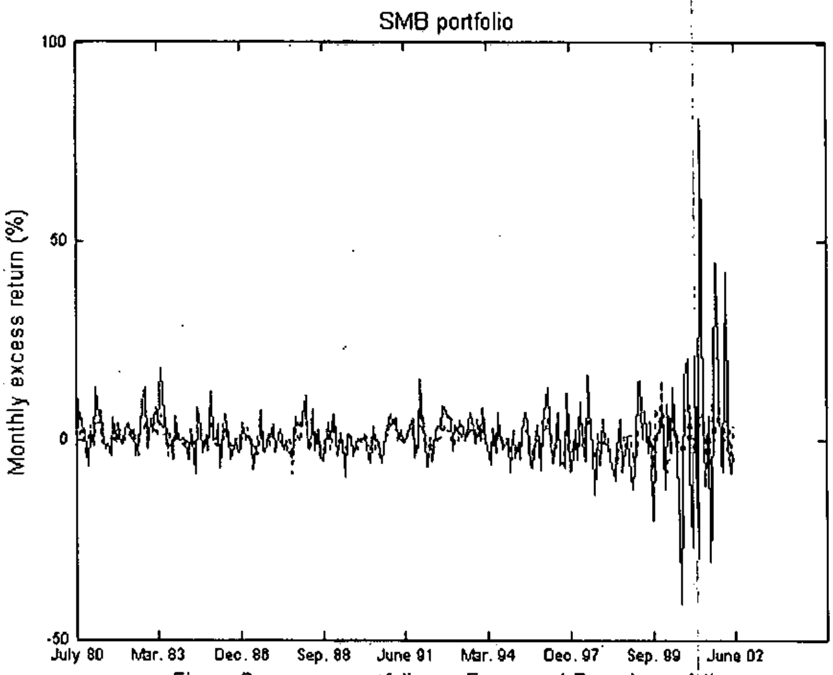


Figure 2 - --- my portfolio ... Fama and French portfolio

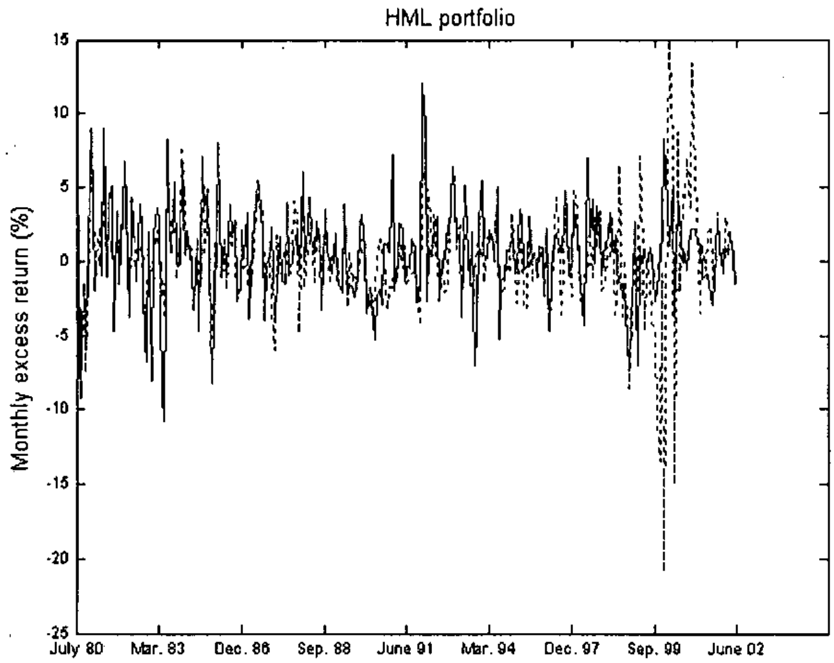


Figure 3 - --- my portfolio ... Fama and French portfolio

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1.5 Appendix - Matlab codes

- **MainNew - tidy time-series data**

(i) MainNew1

Create the file with the time-series data for NYSE, *NyseTS*.

Check that the completeness of time-series data

Add to *NyseQuality* a comment on the completeness of the data

Check whether time-series data are repeated

Clean the file from the series that are not complete and that are double

Check whether the stock was suspended or delisted/dead. If so, put to S or to 0 the following observations

- **SecondPart - compute size and book-to-market breakpoints and form the 6 portfolios**

(i) SecondPartNyseSize

Create a file with the stock code and an index to take into account the quoting year on NYSE, *NyCode*.

Create a file with the median size for each year, *SizeNyse*

Form 2 portfolios, according to the market value of the stock with respect to the median, *SmallNyse* and *BigNyse*.

Note. The median size should be computed only considering NYSE stocks, the portfolios should include all the stocks

(ii) SecondPartNyseBM

Create a file with the 30th and 70th percentile values for book-to-market ratio for each year, *BMNyse*

Form 3 portfolios, according to the book-to-market value of the stock with respect to the percentile values, *LowNyse, MedNyse* and *HighNyse*.

Note. The percentiles should be computed only considering NYSE stocks, the portfolios should include all the stocks

Function: FormPort. Form 6 portfolios by combining size and book-to-market portfolios, *SLPort, SMPort, SHPort, BLPort, BMPort* and *BH-Port.*

- **ThirdPart - compute the 6 portfolios returns and FF factors**

(i) ThirdPart

For each portfolio compute value-weighted returns.

Compute the FF factors, SMB and HML, *SBHL*.

Workspace: AllPort

- **FourthPart - compute the coskewness measure**

(i) FourthPart

Create the market portfolio, taking into account the stocks in *Small-Nyse* and *BigNyse, MKTport.*

Compute excess return on market portfolio, *MKreturn.*

Compute the coskewness factor

(1) compute the residuals of the excess market return

(2) check which stocks have enough (60 months) past data to compute coskewness and create the file *PCosk*

(3) compute the residuals of the stocks in *PCosk*

(4) compute coskewness and cokurtosis of the stocks in *PCosk*

(5) compute the 30th and 40th breakpoints with respect to coskewness and cokurtosis and create the file *Coskew* and *Cokurt.* The first breakpoint should be negative, the second one positive.

(6) form 2 groups, according to the sign of past coskwness and cokurtosis, *NegCos, PosCos* and *NegCok, PosCok*

(7) compute value-weighted returns for S- (most negative past coskewness) and S+ (most positive past coskewness) portfolio returns

Chapter 2

Asset pricing anomalies and trading volume*

* I thank Andrea Beltratti and Eduardo Rossi for insightful discussions on this issue. All remaining errors are mine.

2.1 Introduction

Notwithstanding the attention that academics and practitioners have devoted to asset pricing anomalies, there is not any conclusive explanation about their presence and nature.[1]

One view about anomalies deals with incomplete information.

Merton (1987) show why the size effect might arise in a two-period model with heterogeneous investors. The author assumes that some traders have incomplete information about a subset of the traded assets and so invest only in securities for which they know the parameters describing the return process. An agent invests in a particular stock only if the asset is in his information set. Hence, trade always occurs between equally-informed investors. However, since different traders hold information about different subset of stocks, the distribution of information across agents have price effects. Thus, one can characterize cross-sectional differences in expected returns. The differences depend on the ratio between the relative weight of a particular stock in the market portfolio and the fraction of all investors who know about the stock. However, the author come up with an indeterminate conclusion about the size anomaly since he cannot determine the sign of the relation between the stock's α and its relative market value.

Brav and Heaton (2002) compare different theories about asset pricing anomalies, stressing the importance of the structural uncertainty that arises from incomplete information. The authors consider an economy in which a representative investor has incomplete information on the true distribution of a single stock returns. The investor employ fully Bayesian methods, but he does not know whether the stock expected return is stable or not. Thus, his estimator for the expected return should incorporate this ignorance. The authors also address the issue of the disappearance of the anomalies and consider the role of learning and arbitrage in explaining this point.

[1] Campbell (2000) provide a fairly comprehensive review of the literature.

Following these contributions, I consider a multi-period model in which some agents have incomplete information on the stock return generating processes. Furthermore, I consider an additional source of agents' heterogeneity, related to trading patterns in stocks with different characteristics.

Many empirical studies find that distinct classes of U.S. investors trade in stocks with different features. According to this literature, traders can be classified into three main groups: large institutional investors, that is institutions with more than $100 million of securities under discretionary management, households, that is individuals, small institutions and investment partnerships and insiders, that is investors who are required to report their transactions to SEC according to Section 16(a) of the Securities and Exchange Act of 1934. Large institutional investors mainly trade large and liquid stocks.[2] Households and insiders trade also small and illiquid stocks.[3] The reasons behind diverse trading behaviours could be several: prudence issues, regulatory restrictions, legal liability, transaction-cost motives, costly SEC reporting requirements, agency problems, different access to valuable information.[4]

Given that different trading patterns are due to distinct motives, one can naturally group agents according to the reasons behind their portfolio allocation. In fact, insiders trade essentially to exploit their superior information. Large institutions trade because they are more informed than households, but they also face several constraints that affect their investment choices.[5] Finally, households can use only publicly available information in making portfolio decisions and are not able to distinguish the real motive behind

[2]See, among others, Gompers and Metrick (1998) and Sias (2002). Evidence on value and momentum trading is mixed.

[3]See, among others, Barber and Odean (1998), Cohen (1999) and Lakonishok and Lee (2001).

[4]See Gompers and Metrick (1998) and the literature cited herein. See also Cohen (1999) and Cohen, Gompers and Voulteenaho (2001).

[5]Dennis and Weston (2001) provide evidence about the superior information of insiders and institutions.

insiders' and large institutions' transactions. Thus, heterogeneity in agents' information and some kind of constraints affect the trading pattern of different investors, leading some agents to earn abnormal returns. In other words, the different motives behind distinct investors' trades can justify the presence of stock market anomalies.

This work deeply rests on Wang (1994), which analyze the relation between stock returns and trading volume when agents are heterogeneous in information and investment opportunities. In particular, some agents hold private information about the stock dividend process and invest both in the regulated markets and in assets that are not publicly traded. On the contrary, the remaining agents receive only a noisy signal about future dividends and allocate their portfolio in stocks and bonds. In contrast to Merton (1987), Wang (1994) consider the issues of gaming between informed and uninformed investors since trading occurs between investors holding different information about the securities.

I modify the model in two directions. First, I consider many risky assets in order to take into account different stock characteristics. The generalization of the framework to many stocks allows me to capture the differences in asset allocations from distinct investors.

Second, I re-interpret the private investment opportunity in order to introduce in the model the limits in the trading behaviour of large institutions. More precisely, the private investment opportunity concerns alternative assets that informed agents trade when their investment choice is affected by external constraints. Thus, some investors can either trade because they hold superior information (insider-type behaviour) or because they have access to alternative investment opportunities (large-institution-type behaviour). In contrast, the remaining, uninformed investors behave as households.

As a result, I can highlight the importance of the motives behind the trades of different investors in determining their portfolio allocation and in turn the extra-returns of the portfolios investing in particular stocks. Suppose that risky assets and non-traded assets are substitutes. Then, if informed agents' trading is motivated by a higher expected return from the private investment technology, informed traders reduce their stock holdings and uninformed investors earn abnormal returns. In contrast, if informed investors trade on the basis of their superior information, informed agents increase their stock holdings and earn abnormal returns.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 solves for the equilibrium. Section 4 discuss the model implications on volume and proposes an empirical analysis on the relation between stock volume and prices. Section 5 provides some further comments and concludes.

2.2 The economy

Consider a simple economy with a good that can be either consumed or invested.

The economy is populated by risk-averse agents who are grouped in two classes. The first class, a fraction ϖ of all traders, is given by informed investors and is denoted by I. The second class, a fraction $1-\varpi$ of all traders, is given by uninformed investors and is denoted by U. All agents have a constant absolute risk aversion (CARA) utility function and maximize expected utility of the form:

$$E_t\left\{-\sum_{s=0}^{\infty}\beta^s e^{-\gamma c_{t+s}}\right\},$$

where β is the common time discount factor and γ is the common risk aversion parameter.

Agents trade K risky assets (stocks) and one riskless asset on financial markets.

The riskless asset has an infinitely elastic supply at a positive constant rate of return r. Its gross rate of return is $R = 1 + r$.[6]

The risky assets are classified in two groups, according to some stock peculiarity. Assume that stocks k, $k \epsilon (1, ..., k_j)$ have a common feature and stocks k, $k \epsilon (k_j + 1, ..., K)$ display the opposite feature.[7]

Each share of a stock k, $k = 1, ..., K$, pays a dividend D_t^k at time t. The dividend process is:

$$D_t^k = F_t^k + \varepsilon_{D,t}^k$$

where F_t^k is the persistent component, the "fundamental" of the stock and $\varepsilon_{D,t}^k$ is the idiosyncratic component. The fundamental follows an AR(1) process:

$$F_t^k = a_F F_{t-1}^k + \varepsilon_{F,t}^k \qquad 0 \leq a_F \leq 1$$

and fully determines the expectations of future dividends.

Note that $\varepsilon_{D,t}^k$ and $\varepsilon_{F,t}^k$ are i.i.d. shocks to D_t^k and F_t^k, respectively.

Since dividends have an unconditional mean of zero, the two processes can be thought as deviations in the dividend from its unconditional mean, which can be any positive constant.

The shares of stock k are perfectly divisible and are traded at no cost in a competitive stock market.

Let P_t^k be the ex-dividend share price of stock k. Then, in each period the stock yields a dividend D_t^k and a capital gain $P_t^k - P_{t-1}^k$. Thus, the excess return on one share of stock k, i.e. its excess share return is:

$$Q_t^k = P_t^k + D_t^k - RP_{t-1}^k$$

and the excess rate of return is Q_t^k / P_t^k.

[6] Since the interest rate is given exogenously and the bond supply is elastic at that rate, I do not require the bond market to clear. These assumptions simplify the analysis and allow me to focus on the stock market.

[7] For instance, one can distinguish between small and large stocks or value and growth stocks.

2.2.1 Agents' Heterogeneity

Trading occurs because agents are heterogeneous in both the information that they hold and the effective investment opportunities that they have.

All traders observe realized dividends and market prices of the stocks, but informed agents hold more and better information about the fundamentals that drive stock dividends. They have perfect private information, whereas uninformed traders receive only noisy signals:

$$S_t^k = F_t^k + \varepsilon_{S,t}^k, \qquad k = 1, ..., K$$

where $\varepsilon_{S,t}^k$ is the i.i.d. noise in the signal.

Let the variance of the noise in the uninformed investor's signal about stock k, $\sigma_{S,k}^2$, be a measure of its precision.[8] Then,

$$\sigma_{S,k}^2 > 0 \qquad \forall\, k,$$

i.e. signal precisions determine the information asymmetry in the stock market: the higher $\sigma_{S,k}^2$, the less informative are the private signals to the uninformed agents, the higher the asymmetry between the two classes of traders.

In addition to superior information on the stocks, informed agents have access to an alternative investment opportunity, a risky technology on assets which are not publicly traded.[9]

This opportunity is such that if y_t units are invested at time t, the total payoff at time $t+1$ will be $y_t(1 + r_t + q_{t+1})$, where q_{t+1} is the excess rate of return for period $t+1$. Let

$$q_{t+1} = Z_t + \varepsilon_{q,t+1}$$

[8] I dropped the t subscript because the noise is assumed i.i.d. over time. See next section.

[9] According to Wang (1994) "One can literally interpret them as investing in durables, human capital, non-trade assets, and so forth."

where Z_t is the expected excess rate of return and $\varepsilon_{q,t+1}$ is i.i.d. random shock to the return. Assume that the expected excess return follows an AR(1) process:

$$Z_t = a_Z Z_{t-1} + \varepsilon_{Z,t} \qquad 0 \leq a_Z < 1.$$

The innovation $\varepsilon_{Z,t}$ is assumed to be i.i.d. over time. Indeed Z_t determines the private investment opportunity of the informed agents.

Uninformed traders do not have any information about the returns of the alternative investment opportunity. They only know the return prior distribution.

Given the information structure of the economy, the information set of the agents at time t are:

$I_t^I = \left\{D_s^k, P_s^k, F_s^k, Z_s, k = 1, ..., K | s \leq t\right\}$ for the informed investors and $I_t^U = \left\{D_s^k, P_s^k, S_s^k, k = 1, ..., K | s \leq t\right\}$ for uninformed investors.

Finally, the structure of the economy is common knowledge.

2.2.2 Distributional assumptions

Suppose that the prior of all investors are normal.

Assume also that all shocks, $\varepsilon_{D,t}^k$, $\varepsilon_{F,t}^k$, $\varepsilon_{Z,t}$, $\varepsilon_{q,t}$, $\varepsilon_{S,t}^k$, $\forall$ k, are jointly normal and i.i.d. over time. Normality is assumed purely for mathematical tractability.

Let the matrix of shocks be $\Xi_t = \left[\varepsilon_{D,t}^k \; \varepsilon_{F,t}^k \; \varepsilon_{Z,t} \; \varepsilon_{q,t} \; \varepsilon_{S,t}^k\right]$, $k = 1, ..., K$. Then, $\Xi_t \sim N(0, \Sigma)$, where Σ is the covariance matrix of the shocks.

In order to focus on the second source of heterogeneity, I assume that all shocks at time t are uncorrelated except $\varepsilon_{D,t}^k$, $k = 1, ..., K$ and $\varepsilon_{q,t}$. More precisely,

$$\sigma_{D^k,q} \equiv cov(\varepsilon_{D,t}^k, \varepsilon_{q,t}) > 0 \qquad \text{if } k = 1, ..., k_j$$
$$\sigma_{D^k,q} \equiv cov(\varepsilon_{D,t}^k, \varepsilon_{q,t}) < 0 \qquad \text{if } k = k_j + 1, ..., K.$$

If the covariance between the shock to stock k dividend and the shock to the alternative investment expected excess return is positive, the returns on asset k are positively correlated with the returns on the private technology. Hence, investing in stock k and investing in the private assets are substitutes to the informed traders. Thus, if the expected return on the alternative investment opportunity is high, the informed agents will increase their holdings in non-traded assets to earn higher returns and decrease the holdings in the correlated stock to control the risk of their overall portfolio.

Conversely, if the covariance is negative, the returns on stock k and non-traded assets are negatively correlated. Hence, given a high expected return on the alternative investment, informed agents increase their stock k holdings.

2.3 Equilibrium

Assume that the structure of the economy is common knowledge. Assume that the initial point of the economy is $-\infty$, so that the economy is translationally invariant in time. Then, I shall consider only steady-state equilibria.

To define the equilibrium of the economy I consider agent behaviour. Traders rationally make inferences from the stock prices, so they should conjecture a form for the price functions. Since investors have rational expectations, in equilibrium such conjectures are correct.

Under CARA and time-separable preferences over lifetime consumption and Guassian processes for asset returns and income, one can only consider linear equilibria of the economy, that is equilibria in which the stock price is a function of the state variables of the economy, Z_t and F_t^k, $k = 1, ..., K$.[10] The first variable determines excess returns on non-traded assets, while the

[10] The CARA preferences, under the assumption of normality of the shocks, show some nice properties. The first one is homotheticity, which implies that the results of the model are invariant to proportional scaling of the variances of all shocks and investors' risk aversion. In addition, investors' stock demand are independent of wealth and so there is no income effect. Hence, the equilibrium price of the stocks will be independent of the wealth distribution of the investors as well as the level of aggregate wealth.

remaining variables determine the stock future cash flows.

Informed traders observe the state variables and are able to conjecture price functions on their basis. In contrast, uninformed agents receive only private signals on F_t^k, $k = 1, ..., K$ and observe realized dividends and prices. So, given their information set, they form conditional expectations on the state variable values. Thus, in order to determine conjectured price functions, one solves a Kalman filtering problem to assess the conditional distribution of the state variables.

For simplicity, consider only the case in which $\sigma_{D^k,q} > 0$, $k = 1, ..., k_j$.[11]

It can be shown that, within the class of linear equilibria, only the conditional expectation on stock k fundamental, $\widehat{F}_t^k = E_t(F_t^k | I_t^U)$ is relevant. So, the state of the economy and in turn the steady-state equilibrium are fully determined by Z_t, F_t^k and $\widehat{F}_t^k$. The rational expectation equilibrium is such that stock k price is:

$$P_t^k = (a - p_F^k)\widehat{F}_t^k + p_F^k F_t^k - (p_0^k + p_Z Z_t)$$

where $a = a_F(R - a_F)^{-1}$, p_0^k, $p_Z > 0$ and $0 \leq p_F^k \leq a$.[12]

So, the equilibrium stock k price provides information about future stock k dividends and non-traded asset expected returns to uninformed agents. More precisely, $p_F^k F_t^k - p_Z Z_t$, the linear difference between F_t^k and Z_t, is the information content of the current price. Hence,

$$p_F^k \widehat{F}_t^k - p_Z \widehat{Z}_t = p_F^k F_t^k - p_Z Z_t$$

where $\widehat{Z}_t \equiv E_t(Z_t | I_t^U)$, that is any innovation in dividends and private signals that changes the expectation of F_t^k also changes the expectation of

[11] In the case of negative correlation between the risky assets and the alternative investment opportunity, that is for stocks $k_j + 1, k_j + 2, ..., K$, the following analysis yields similar results.

[12] The proof is a generalization of the proof that Wang (1994) show in Appendix A. Note that in order to determine the set of parameters (p_0^k, p_Z, p_F^k), $\forall\ k$, it is necessary to solve the stock market clearing condition using numerical methods.

Z_t. Moreover, the linear relations between state variables implies that if uninformed investors over- (under-) estimate F_t^k, they also over- (under-) estimate Z_t.

However, market clearing prices are not fully revealing. Since stock k and non-traded assets are substitutes to informed investors, both bad news about future dividends of firm k (low F_t^k) and a high expected return on the alternative investment opportunity (high Z_t) can lead the informed agents to sell stock k and the stock k price to drop. Thus, observing the price is not sufficient for the uninformed investors to identify the truly reason behind informed agent tradings.

Given equilibrium stock prices, one can derive the process of excess stock returns and solve the agents' optimization problems.

The solution to the linear filtering problem shows that $\widehat{F}_t^k$ is determined by two components, the expectation of F_t^k based on previous information and the update in expectations based on new information from surprises in stock k price, dividend and signal. Thus, given P_t^k, $\widehat{F}_t^k$ and $\widehat{Z}_t$ the excess return on stock k is:

$$Q_{t+1}^k = rp_0^k + (R - a_z)p_Z Z_t - (R - a_\Theta)(a - p_F^k)\Theta_t^k + \varepsilon_{Q,t+1}^k$$

where $(R-a_z)p_Z$ and $(R-a_\Theta^k)(a-p_F^k)$ are not negative and $\Theta_t^k \equiv \widehat{F}_t^k - F_t^k$. The latter represents the error of the uninformed agents in estimating stock k fundamental. It can be shown that Θ_t^k follows an AR(1) stationary process without drift:

$$\Theta_t^k = a_\Theta^k \Theta_{t-1}^k + \varepsilon_{\Theta,t}^k, \qquad 0 \leq a_\Theta^k < 1$$

where $\varepsilon_{\Theta,t}^k$ is linear in the shocks Ξ_t. Since the estimation error is mean-reverting to zero, it is expected to be corrected eventually. However, uninformed trader corrections do not eliminate the asymmetry in equilibrium because the state of the economy is changing over time.

The unconditional expectation of stock k excess returns is constant over time:

$$E(Q_{t+1}^k) = rp_0^k \geq 0,$$

as the unconditional expectations of Z_t and Θ_t^k are zero.

For informed traders the conditional expectation of stock k excess returns is:

$$E_t(Q_{t+1}^k|I_t^I) = rp_0^k + (R - a_z)p_Z Z_t - (R - a_\Theta^k)(a - p_F^k)\Theta_t^k.$$

Indeed, Θ_t^k represents a profitable opportunity for the informed investors to forecast and exploit the correction of uninformed traders' expectations and the corresponding price change.

For uninformed agents the conditional expectation of asset k excess returns is:

$$E_t(Q_{t+1}^k|I_t^U) = rp_0^k + (R - a_z)p_Z \widehat{Z}_t$$

where $\widehat{Z}_t$ is the conditional expectation of the alternative investment excess return. Given the linear relation between $\widehat{F}_t^k$ and $\widehat{Z}_t$, innovations in dividends or signals, by affecting $\widehat{F}_t^k$, can change uninformed investors' expected excess return, even though they are uncorrelated with the true value of Z_t.

2.3.1 Optimization problem

Investor i's optimization problem, $i = (I, U)$, is:

$$J^i \equiv \max_{c^i, X^i, y^i} E_t \left[-\sum_{s=0}^{\infty} \beta^s \exp(-\gamma c_{t+s}^i) | I_t^i \right]$$

$$s.t.\ W_{t+1}^i = R(W_t^i - c_t^i) + X_t^i Q_{t+1} + y_t^i q_{t+1}$$

where X_t^i is a vector of stock shares, $X_t^i = \left[X_t^{i,1}\ X_t^{i,2} \ldots X_t^{i,K} \right]$, $Q_{t+1} = [Q_{t+1}^1\ Q_{t+1}^2 \cdots Q_{t+1}^K]$ is the vector of the corresponding excess returns and W_t^i is trader i's wealth at time t.

Since uninformed agents trade only on financial markets, they invest only in stocks and the risk-free asset. Hence, $y_t^U = y^U = 0$.

The solution to informed investors' optimization problem is:

$$J^I(W_t^I; Z_t; \Theta_t; t) \equiv -\beta^t \exp[-\alpha W_t^I - V^I(Z_t, \Theta_t)]$$

$$c_t^I = -\gamma^{-1} \ln(-\gamma^{-1} \partial J^I / \partial W^I)$$

$$\begin{pmatrix} X_t^I \\ y_t^I \end{pmatrix} = \alpha^{-1} \Gamma^I \begin{pmatrix} E_t(Q_{t+1}|I_t^I) \\ E_t(q_{t+1}|I_t^I) \end{pmatrix} - \alpha^{-1} \begin{pmatrix} h_X^I(Z_t, \Theta_t) \\ h_y^I(Z_t, \Theta_t) \end{pmatrix}$$

where Θ_t is the vector of the uninformed traders' estimation errors on stock fundamentals, $\Theta_t = \left[\Theta_t^1 \; \Theta_t^2 \; ... \; \Theta_t^K\right]$.

The solution to uninformed investors' optimization problem is:

$$J^U(W^U; \widehat{Z}_t; t) \equiv -\beta^t \exp[-\alpha W_t^U - V^U(\widehat{Z}_t)]$$

$$c_t^U = -\gamma^{-1} \ln(-\gamma^{-1} \partial J^U / \partial W^U)$$

$$X_t^U = \alpha^{-1} \Gamma^U E_t(Q_{t+1}|I_t^U) - \alpha^{-1} h_X^U(\widehat{Z}_t)$$

where $\alpha = r/\gamma R$ is the intertemporal risk aversion coefficient and Γ^i are positive definite matrices of constants. Finally, $V^i(.)$ and $h_j^i(.)$, $j = X, y$ are functions of the realization of the state variables that agent i either observes or conjectures.

Thus, optimal portfolios have two components. The first component, expressed by the first addend, is a mean-variance efficient portfolio. It obeys the CAPM pricing relation, reflecting the trade-off between expected return and risk. Indeed, the term Γ^i is a function of the inverse of the covariance matrix of asset returns for investor i.

The second component is a portfolio reflecting investor i's hedging needs. In fact, expected returns on stocks and the alternative technology change over time. Since returns on risky assets are correlated with changes in expected future returns, the portfolio provides a vehicle to hedge against changes in future investment opportunities.

2.3.2 Trading strategies

The optimal portfolios of the two classes of agents determine their trading strategies. In order to clarify the trading mechanisms in the economy, restate optimal stockholdings in terms of the variables that characterize investors' opportunities.

Informed investor stockholding are:

$$X_t^I = f_0^I + f_Z^I Z_t + f_\Theta^I \Theta_t, \qquad f_j^I \text{ constant}; j = 0, Z, \Theta.$$

Hence, informed agents' stock allocation is driven by two forces. First, informed investors trade because they have access to the private alternative technology. If the expected excess return on non-traded assets, Z_t changes, informed agents optimally reallocate their portfolios.

Suppose that Z_t increases. Then, since the alternative investment opportunity is a substitute for stock k, $k \in (1, ..., k_j)$, informed agents invest in the non-traded assets. Moreover, they lower stock k holding to control the portfolio risk. The opposite conclusion holds if the excess returns on risky and non-traded assets are negatively correlated.

Second, the informed investors trade because they receive private information on stock cash flows. As Θ_t changes, stock prices deviate from their "fundamental value" due to uninformed traders' estimation errors. Hence, informed agents take speculative positions against expected future corrections. In this case, informed investors trade at favorable prices and learn abnormal returns.

Suppose that Θ_t raises, that is uninformed traders overestimate stock persistent components and in turn expected future dividends. Hence, uninformed investors increase their investment in the risky assets and consequently stock prices rise. In the following period, the fundamentals become publicly known and the realizations of dividends and prices turn out to be lower than expected. As a result, the uninformed traders realize that they have overinvested in the risky assets and sell the stocks. So, stock prices decrease. Since informed traders know the realization of stock fundamentals, they can take advantage of the correction of the uninformed agents' expectation and the corresponding price reduction.

Uninformed investor stockholding are:

$$X_t^U = \alpha^{-1}\Gamma^U E_t(Q_{t+1}|I_t^U) - \alpha^{-1}h_X^U(\widehat{Z}_t)$$
$$X_t^U = f_0^U + f_Z^U\widehat{Z}_t, \qquad f_j^U \text{ constant}, \ j = 0, Z.$$

Thus, $\widehat{Z}_t$, the uninformed agents' expectation on the non-traded asset excess return, is the sole determinant of the uninformed investors' portfolio allocation This finding points out the inability of uninformed agents to infer the real motive behind informed investors' trade, even though uninformed investors know that they face agents with superior information and an alternative investment opportunity.

When informed agents' trade is motivated by the arrival of new information, uninformed investors correct the errors in their previous trading and lose. Indeed, if they overestimate Z_t, they overinvest in stock k and in the following period they will sell the risky asset to correct the bad trades made previously.

In contrast, when informed agents' trade is motivated by changes in Z_t, uninformed investors just take the other side of the trade. If Z_t increases, the informed investors sell stock k shares. The uninformed traders are willing to absorb the shares as the stock k price drops and the expected return rises. Hence, the uninformed agents earn abnormal returns.

Thus the trading motive behind informed and uninformed investors' trade are different and affect the optimal stockholdings in the economy.

2.3.3 Equilibrium trading volume

Since the economy is populated only by two classes of traders, informed and uninformed agents, all the trading takes place between the two groups of investors. Therefore, the equilibrium trading volume, V_t is determined by the changes in the holdings of either the informed or uninformed agents. In order to focus more carefully on the dynamics of the model, consider the trading volume generated by the informed investors:

$$\begin{aligned} V_t = \varpi \left|X_t^I - X_{t-1}^I\right| &= \varpi \left|f_0^I + f_Z^I Z_t + f_\Theta^I \Theta_t - f_0^I + f_Z^I Z_{t-1} + f_\Theta^I \Theta_{t-1}\right| \\ &= \varpi \left|f_Z^I\right| \left|Z_t - Z_{t-1}\right| + \varpi \left|f_\Theta^I\right| \left|\Theta_t - \Theta_{t-1}\right| \end{aligned}$$

where $V_t = \left[V_t^1\ V_t^2\ ...\ V_t^{k_j}\ V_t^{k_j+1}\ ...\ V_t^K\right]$. Thus, volume is determined by the absolute changes in traders' stockholdings. It increases if the expected excess return on the alternative assets, Z_t and/or the error in the uninformed investor conditional expectation of the persistent component in dividends, Θ_t change.

In addition, the trading volume is actually the turnover since the number of shares outstanding is normalized to one.

2.4 An application to asset pricing anomalies

The model outlined so far can be used to shed some light on the presence of asset pricing anomalies related to firm characteristics, like the "size effect" and the "value effect".

Consider the "size effect", that is the superior performance of small-capitalization companies with respect to large-capitalization companies. Fama and French (1993) capture the "size effect" by constructing a portfolio that goes long on "small" stocks and short on "large" stocks, the "small minus big", SMB, portfolio.

Suppose that "small" stocks are the risky assets in the set $(1, ..., k_j)$ whereas "large" stocks are the risky assets in the set $(k_j + 1, ..., K)$.[13] Then, given the empirical evidence on the relation between type of investors and classes of stocks, the model predicts the following trading patterns. The "informational trading" is the trading in the "small" stocks that occurs between

[13] This hypothesis could be easily overturned without significantly affecting the conclusions.

the households (that is the uninformed agents in the model) and the insiders (that is the informed agents in the model). When the are good news about future dividends of a "small" company, the insiders buy the stock and earn abnormal stock returns, since the trade is information-based. In contrast, the "non-informational trading" is the trading in the "large" stocks that occurs between the households (that is the uninformed agents in the model) and the large institutions (that is the informed agents in the model). When there is a high expected return on the alternative investment opportunity, the institutional investor sell the "large" stock and households earn abnormal stock returns, since the trade is due to portfolio rebalancing reasons. Hence, the "size effect" is observable when there are high future dividends on stocks with a positive covariance with the alternative assets, F_t^k, and a high expected excess return on the alternative investment opportunity, Z_t.[14]

2.4.1 Empirical analysis

The empirical literature on volume mainly concentrates on the relationship between trading activity and aggregate returns. Few studies focus on individual stocks.[15] In contrast, the relation between trading volume and specific trading strategies is not expressly investigated.[16] The only exception I am aware of is Lee and Swaminatham (2000), which compare the performance of a volume momentum strategy and a price momentum strategy.

The most used measure of a single stock trading activity is turnover, that

[14]A similar reasoning applies to the "value effect", that is the superior performance of "value", stocks, that is stocks with high book-to-market value, with respect to "growth" stocks, that is stocks with low book-to-market value. The corresponding portfolio is the HML portfolio.

[15]For instance, Campbell, Grossman and Wang (1993) analize the relation between aggregate stock market trading volume and the serial correlation of daily returns for both stock indexes and individual large stocks.

[16]For instance, Daniel and Titman (1997) use a measure of average turnover to investigate if differences in factor loadings for stocks with similar capitalizations are significantly related to trading volume, when portfolios are sorted by size, book-to-market and preformation HML factor loadings.

is the ratio between the number of shares traded to the number of shares outstanding. However, there is no agreement in extending this measure to the portfolio case because of the diversities in the trading activity of portfolios following different strategies.

Lo and Wang (2000) discuss this issue at length and propose the following measure:

$$\tau_t^P = \sum_j \omega_{jt}\tau_{jt}$$

where τ_{jt} is stock j turnover and ω_{jt} is the fraction of portfolio market value invested in stock j, given that there are non-negative holdings in all stocks. However, the authors warn that the measure does not necessarily represent the turnover of a specific trading strategy and that it cannot be applied too broadly. In particular, they consider the case in which short sales are allowed and some portfolios weights are negative. In this case, the measure can be misleading since the volume of short positions offsets the volume of long positions. A possible solution to the problem is to use the absolute values of the portfolio weights. However, this measure correction could be only partially effective, if the turnover of two stocks are identical and the weights sum up to one. In this case, the measure would account for a lower portfolio volume than it really is.

Thus, there is no a completely satisfactory measure of portfolio turnover and, as a result, I follow three criteria. In addition, to gauge the robustness of the volume findings, I consider two versions of the different turnover indexes. The first one has signed weights, so that the weights associated to short-sale positions are negative. The second one has weights in absolute values.

The first index concerns an equally-weighted measure of portfolio volume, namely the turnover weighted by the number of stocks in the portfolio:

$$\tau_t^{EW} = \frac{1}{J}\sum_j \tau_{jt},$$

where J is the total number of stocks. The second volume index is a share-weighted measure:

$$\tau_t^{SW} = \sum_j \frac{N_j}{N} \tau_{jt},$$

where N_j is the total number of shares outstanding for stock j and $N = \sum_j N_j$ is the total number of shares outstanding of all stocks in the portfolio. The third turnover index is a value-weighted measure:

$$\tau_t^{VW} = \left(\sum_j \frac{1}{MV_j} \right) \sum_j MV_j * \tau_{jt},$$

where MV_j is the market value of stock j.[17]

In order to follow the same procedure used to compute the SMB and HML excess returns, the portfolio turnover index is calculated as difference between the simple average of the turnover on six initial portfolios.[18] Hence, the SMB volume is computed from the turnover of three small stocks portfolios and three large stocks portfolios. Similarly, the HML trading activity is computed from the turnover of two high book-to market stocks portfolios and two low book-to market stocks portfolios

Finally, most investigations use CRSP daily or weekly data. In contrast, I collect data from Datastream and I choose the monthly frequency to make my analysis comparable with the Fama and French study. For the same reason, I do not employ any detrending method or take the natural logarithm of the turnover series. Indeed, Lo and Wang (2000) show that, even if turnover data display some kind of nonstationarity, the choice of the detrending adjustment has a substantial impact on the time-series properties of turnover. Therefore, the authors use raw data and consider several sub-periods.[19]

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[17] The formula show the version of the indexes with signed weights.

[18] See the previous chapter on the definition and computation of the Fama and French (1993) factor portfolios.

[19] Lo and Wang (2000) admit that such choice is controversial, but they argue that it is "perhaps the best compromise between letting the data "speak for themselves" and imposing sufficient structure to perform meaningful statistical inference".

It is generally accepted that the "size effect" has been observable till the mid of the 1980s and then, after its disappearance, in the last years.[20] The time-series behaviour of volume measures reflect the performance of the SMB portfolio over time. Figure 1 and 2 show the three turnover measures for the SMB portfolio, with signed weights and weights in absolute value, respectively. The turnover indexes have quite different behaviours, with the equally-weighted measure showing the highest variability in both cases. Nonetheless, irrespective of the weight methodology chosen to construct the indexes, the turnover measures show a common pattern at the beginning of the sample.

From July 1980 to August 1984, on average the portfolio turnover is negative and its volatility is low. In September 1984 the portfolio volume varies noticeably. The shift is very pronounced for all the turnover measures, except the signed value-weighted one. From September 1984 to August 1986, the gain from holding smaller stocks decreases and the portfolio volume changes accordingly. In particular, in August 1986 the equally-weighted signed measure becomes for the first time positive. It remains on average positive during the subsequent months up to June 2000. In this period, on average the gain from the SMB strategy are null or negative. Finally, in the last sample subperiod, from July 2000 to June 2002, all the signed turnover measures are on average negative, with a high volatility. During this period, the "size effect" is again observable. Thus, overall the SMB strategy excess returns seem to be related to a higher trading volume on large-capitalization stocks.

[20] As outlined in the previous chapter, the profitability of holding smaller stocks over time has been put into question by many authors. For instance, Malkiel (2003) note that the gain from holding these stocks have disappeared from the mid 1980s till the end of the 1990s.

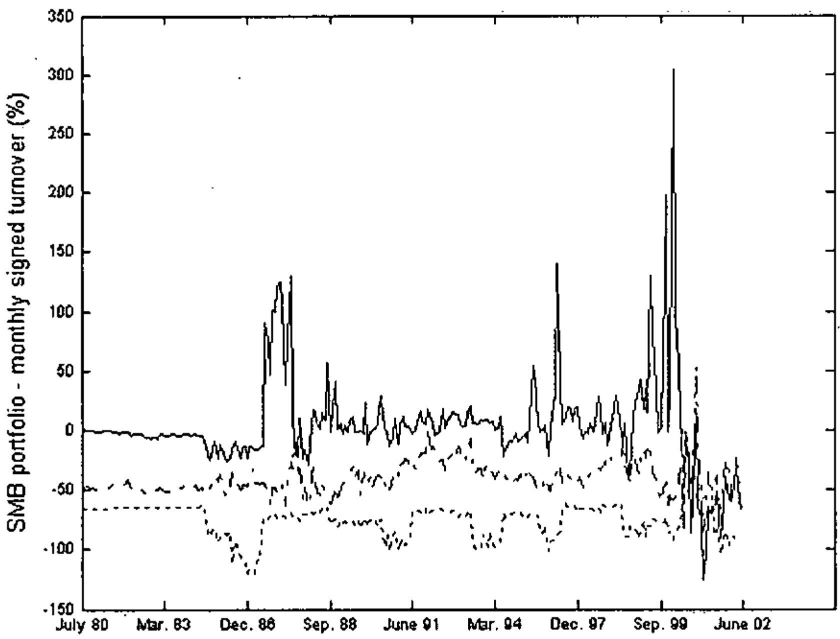

Figure 1 - — Equally-weighted ... Share-weighted -.- Value-weighted

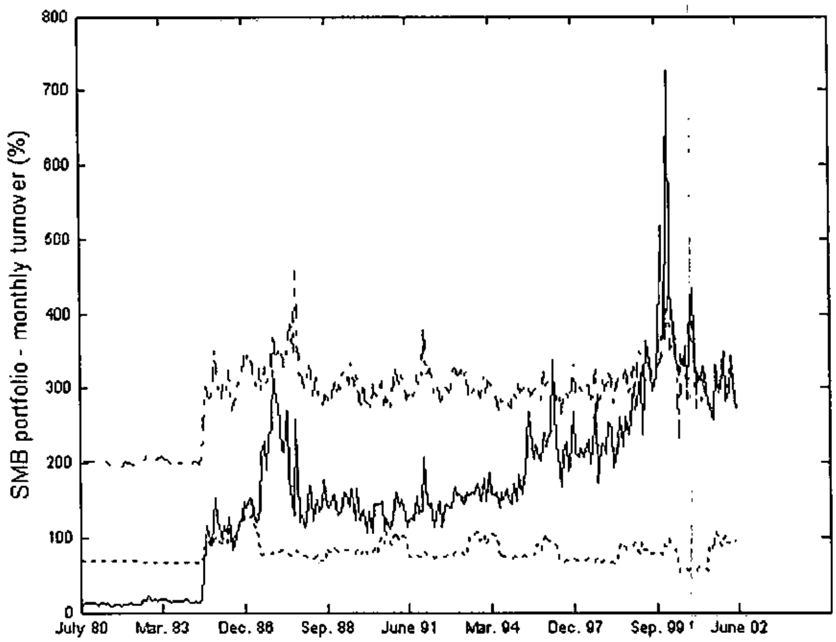

Figure 2 - --- equally-weighted ... share-weighted -.- value-weighted

Figure 3 and 4 show the excess returns and turnover of the SMB portfolio. The turnover weights are signed and the index is value-weighted to make it fully comparable to the excess returns. Since several authors address the issue of stability of stock-specific parameters, I consider two subperiods, namely the period from July 1980 to June 1991 and from July 1991 to June 2002, respectively.[21]

[21] See Llorente, Michaely, Saar and Wang (2001).

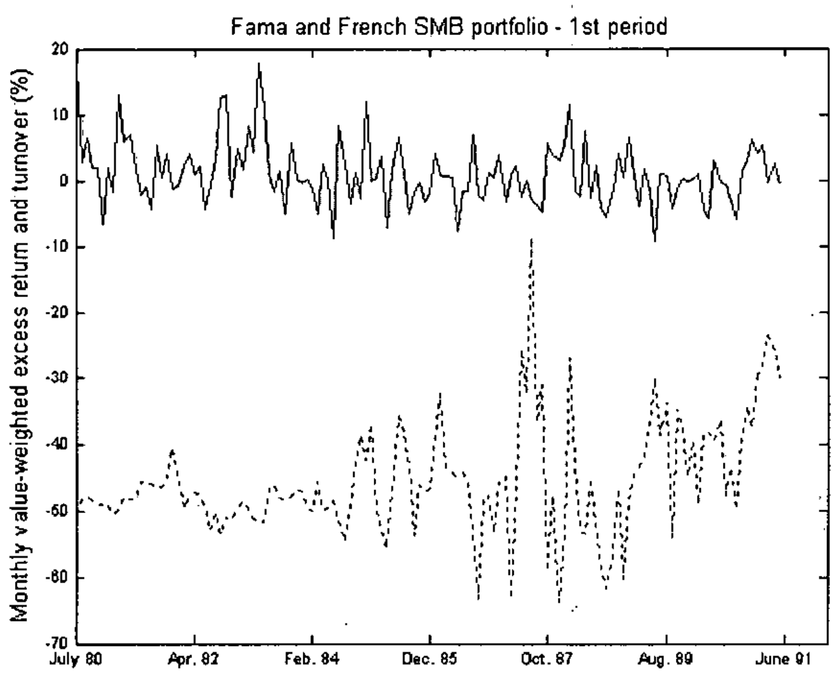

Figure 3 - --- Excess returns ... Turnover

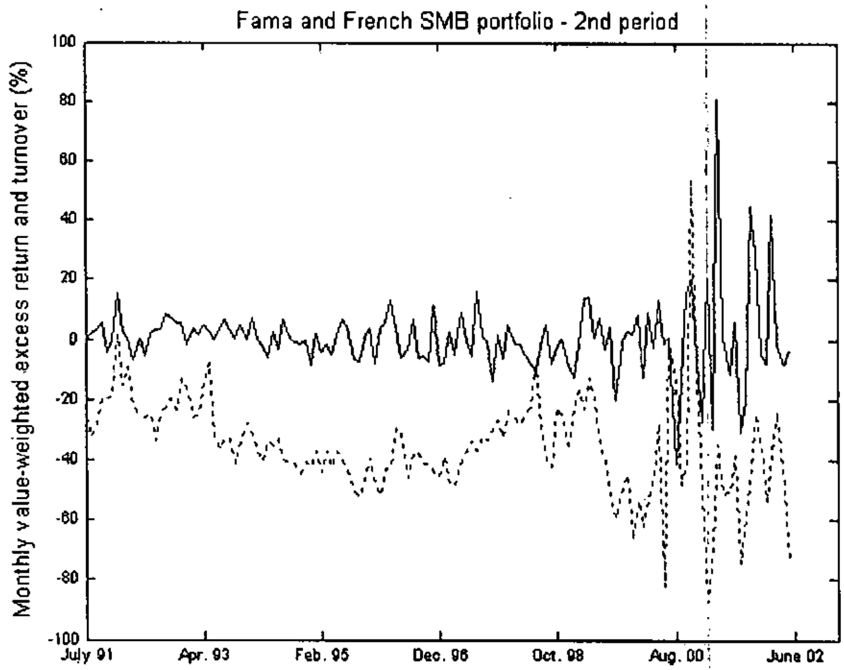

Figure 4 - --- Excess returns ... Turnover

The "value effect" has been not as striking as the "size effect", though it has been more persistent. Indeed, it has not been observable from July 1980 until the end of 1981 and, afterwards, in 1999 and 2000. Figure 5 and 6 show the turnover measures for the HML portfolio, which captures the "value effect", with signed weights and weights in absolute value, respectively. As in the case of the SMB strategy, the turnover measures have a common pattern at the beginning of the sample period, from July 1980 to September 1984. Then, they show different behaviours, although all volume indexes have higher volatilities. As in the previous case, the equally-weighted measures show the highest variability. In addition, the equally-weighted signed measure is almost always negative during the sample period, while the share-weighted and value-weighted turnover show the opposite behaviour. However, all the turnover measures are negative during the second period in which the "value effect" is not observable Then the value stocks produce again higher rates of returns and the turnover indexes change accordingly.

Thus, overall the HML strategy excess returns seem to be related to a higher trading volume on stocks with a high book-to-market value.

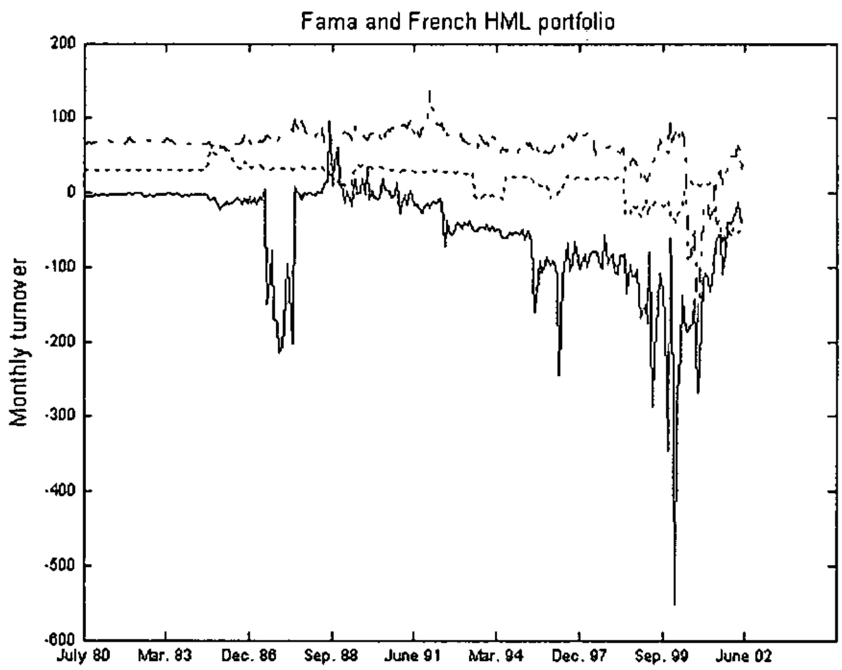

Figure 5 - --- equally-weighted ... share-weighted -.- value-weighted

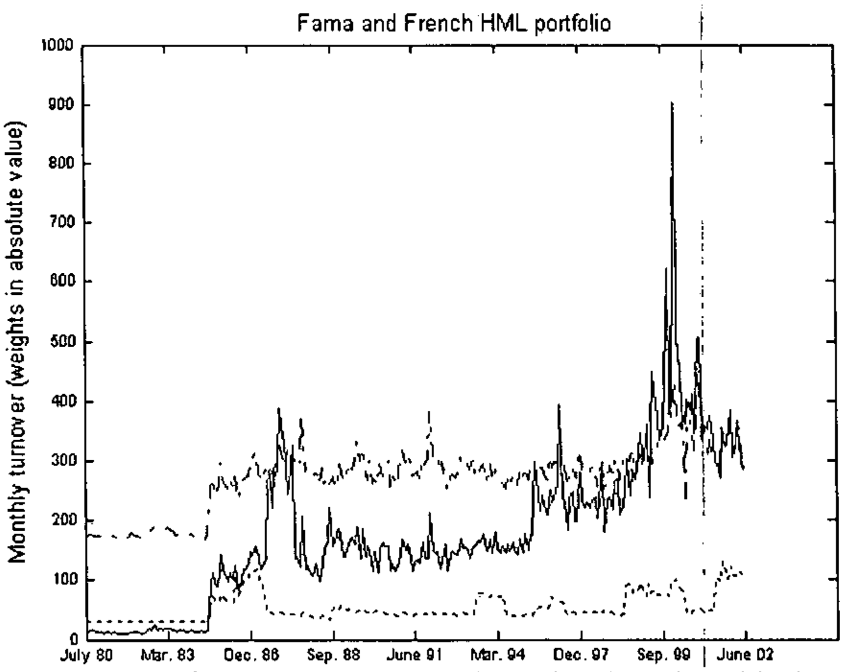

Figure 6 - --- equally-weighted ... share-weighted -.- value-weighted

Figure 7 and 8 show the excess returns and volume on the HML portfolio during the two sub-periods. The measures are value-weighted.

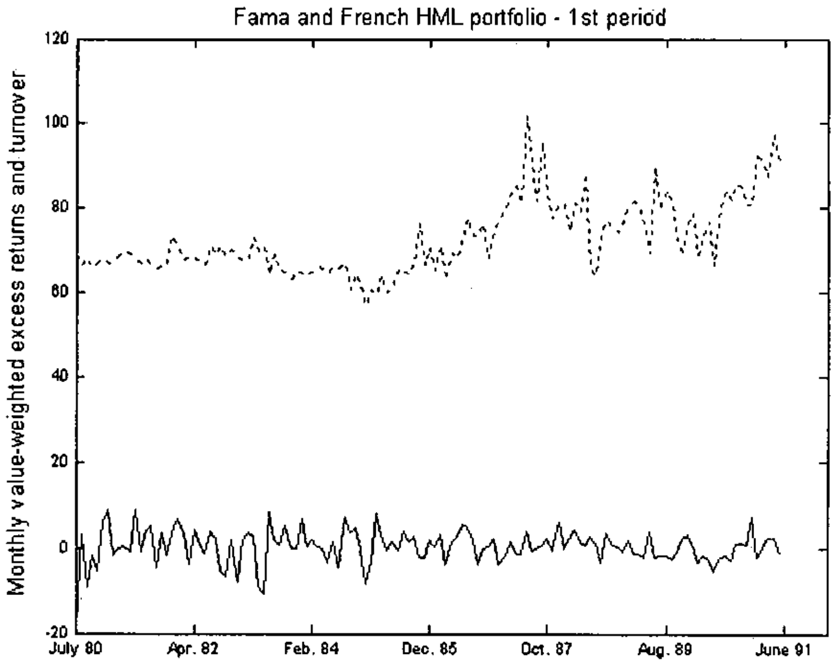

Figure 7 - --- Excess returns ... Turnover

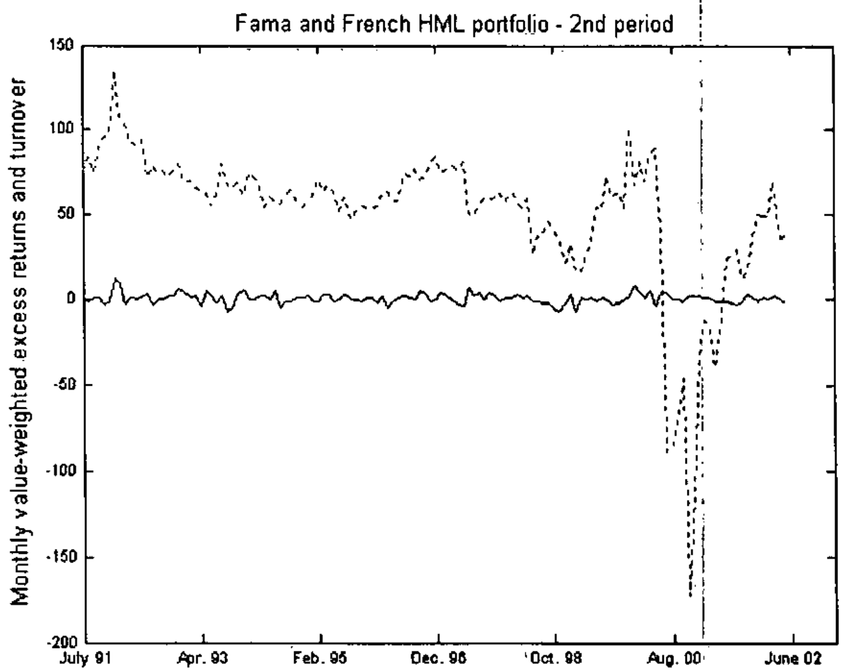

Figure 8 - --- Excess returns ... Turnover

2.5 Conclusion

In an intertemporal economy in which risky assets can be distinguished on the basis of some common features, some agents earn returns in excess of those of a mean-variance portfolio, if the motives behind trade are not publicly known and traders are heterogeneous in information and effective investment opportunities. Thus, these kinds of investor heterogeneity might help in explaining some asset-pricing anomalies.

The empirical evidence on the trading volume of portfolios that capture the "size effect" and the "value effect" confirms the model findings about the relation between specific stock volume and prices.

The theoretical findings in the model are consistent with the main result in Merton (1973). Merton (1973) consider an intertemporal model in which agents have homogeneous expectations. If the investment opportunity set changes over time and one asset is perfectly negative correlated with the change, agents are rewarded, in terms of expected returns, for the risk that an "unfavorable" shift will occur. Hence, in equilibrium the CAPM relationship is not satisfied. In my model, expected stock returns are driven by the change in the non-traded asset expected returns. Indeed, the change occurs exclusively in the investment opportunity set of the informed agents and affects the financial market through the correlation between the stock and the non-traded asset returns. Suppose that agents are identically informed, so that the informational trading motive does not exist. The model predictions are similar to Merton (1973) findings: expected stock returns have an additional component with the respect to the standard CAPM model, since investors are compensated for bearing both the market systematic risk and the idiosyncratic risk associated to specific stock characteristics.

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Chapter 3

Competitive non-linear payoff risk factors in hedge funds*

* I thank Andrea Beltratti, Massimo di Tria and Eduardo Rossi for insightful discussions on this issue. All remaining errors are mine.

3.1 Introduction

In the last years hedge funds have shown a great diffusion as alternative investment vehicles in financial markets.

Unlike more traditional funds, such as mutual funds, hedge funds are not regulated by the United States Investment Company Act. In addition, they are not subject to any SEC regulation.[1] As a result, hedge funds have much broader flexibility in terms of both information disclosure and investment policies.

In fact, in contrast to mutual funds, hedge funds widely employ long/short combinations of assets and leverage and are not evaluated against a passive benchmark. Hedge fund managers have broad investment mandates: they are not subject to any restriction about the types of securities and the degree to which the portfolio may be concentrated in a single security. Hence, the nature of hedge fund strategies is not simply defined by the risk and return characteristics of the underlying securities, but also by the way the assets are managed. Thus, hedge funds can follow strategies that help to capture a wider set of risk-premia than those associated with more traditional investment tools.[2]

The kind of risks which hedge funds are exposed to has drawn growing attention among scholars. However, the complex nature of hedge fund strategies and the limited disclosure requirements make examining this issue a quite challenging task.

A way to deal with hedge fund peculiarities is to focus on hypothetical, specific trading strategies. Fung and Hsieh (2001) and Mitchell and Pulvino

[1] Indeed, as Goetzmann and Ross (2000) remark, there is no official list of hedge funds in the United States and no official clearinghouse for information about them.

[2] For instance, Goetzmann and Ross (2000) identify two major risks in pure hedge fund strategies, the model risk and the convergence risk. The first source of risk deals with the ability of the manager to fully account for the systematic risk of the underpriced security and the position used to hedge it. The second source of risk deals with the convergence process in the expected return for the two securities.

(2001) investigate on the "trend following" and "risk arbitrage" strategy, respectively. The main motivation to their studies is the observation that hedge fund managers typically employ strategies that are not linearly-related to standard asset classes.[3] Indeed, the authors show that both the "trend following" and "risk arbitrage" strategy payoffs are akin to the payoffs of certain option investments.

Their results are in line with previous research on managed portfolio performance showing the importance of option-like features in mutual fund returns.

Merton (1981) analyze a multiple-investor economy in which one agent holds some private information and uses it to make market-timing forecasts. In order to exploit his information edge, the forecaster forms a mutual fund which he manages in return for fees. The other agents invest in the fund without knowing what the forecast will be, although they know it will be correct. The agents also know the fund's investment policy, namely that the manager will either invest all the assets in stocks or in bonds according to his forecast. As a result, the end-of-period value of the fund can be written as the end-of-period value of a portfolio investing in options.[4]

Starting from the insight of Merton (1981), Glosten and Jagannathan (1994) suggest a contingent-claim approach to evaluate the non-linear relation between managed portfolio payoffs and market returns. More precisely, one can approximate the payoff on a managed portfolio using the payoffs on a limited number of options on certain stock index portfolios. If the options are either traded or can be valued using arbitrage methods, one can arrive at an approximate value for the manager's investment by finding the value of the options. Furthermore, Glosten and Jagannathan (1994) suggest that, if prices on traded options are available, excess return on certain selected

[3] See Fung and Hsieh (1997).

[4] One possible strategy consists of holding bonds and one-period call options on shares of the market portfolio. A different strategy consists of holding shares of the market portfolio and one-period put options on shares of the market portfolio.

options could be used as additional factors in a linear model.

Building on this idea, Agarwal and Naik (2004) study hedge fund returns using a modified multifactor model, augmented with simulated option-based risk factors. The authors show that these factors are indeed significantly associated to equity-based hedge fund index returns. This work extends the Agarwal and Naik (2004) analysis in many respects.

First, I introduce a new measure to explain the hedge fund risk-return relationship and evaluate its significance with respect to the option-based factors. In fact, the option-based factors in Agarwal and Naik (2004) are not duly interpretable and are only a small and not representative fraction of all the possible strategies that a hedge fund manager can indeed pursue. The alternative risk factor takes into account conditional higher moments in asset return distributions and stems from research on the cross-section of stock expected returns.[5] Actually, such a measure is potentially suitable to characterize hedge fund returns, which seem to be non-normally distributed.

Dybvig and Ross (1985) argue that managed portfolio returns might follow a non-normal distribution if the information is asymmetric. The authors consider an economy in which the manager may have information that is useful for portfolio selection, but is not possessed by the investor. If market-timing occurs, the information asymmetry may cause managed portfolio returns to be not normal, even if the underlying asset returns and the manager's signal are joint normally distributed. Indeed, the manager's return is the product of normally distributed asset returns and normally distributed asset weights, which are function of his information. If the two normal variables are correlated, the portfolio returns have a chi-squared term and are not normal.

Brooks and Kat (2001) and Kat and Lu (2002) examine two different sets of hedge fund returns. Brooks and Kat (2001) consider the indexes

[5] See Ferson and Harvey (1999), Harvey and Siddique (2000) and Ang, Chen and Xing (2001).

provided by several data vendors and fund advisors over the period January 1995 - April 2001. The collection, classification and aggregation methods differ substantially among the various data vendors, so that there can be considerable heterogeneity between indexes aiming to reflect the same type of strategy.[6] Nonetheless, all the index returns, irrespective of specific style categorizations, show non-normal skewness and kurtosis values. Kat and Lu (2002) study individual hedge funds and equally-weighted portfolios of hedge funds following the same strategy. The data are taken from an unique data vendor and encompass the period June 1994 - May 2001. Both individual hedge fund and portfolio returns exhibit distributions with not null skewness and excessive kurtosis, in contrast to the moments that characterize normal distributions.

Second, I adopt a new method to determine the models explaining the hedge fund strategies. Agarwal and Naik (2004) assess the importance of the competitive risk factors using a stepwise regression. This procedure is one of the most popular selection method because it provides parsimonious models and it is easy to implement. However, it possibly leads to locally rather than globally optimal solutions and invalidates most of the standard statistical inference theory. Given these potential shortcomings, I also follow out a different selection process, based on principal components. As far as I know, this approach has never been used in previous hedge fund studies.

Third, the moment-based measure and the Fama and French (1993) factors are constructed employing a newly assembled database.[7]

The main finding of my study is that the conditional higher moment factor plays a role in determining hedge fund payoffs. In particular, it significantly characterizes the non-linear features of equity-based hedge fund index strategies. The relevance of this result is twofold. First, a coskewness-based factor can and makes option-based risk factors at least less significant.

[6] Fung and Hsieh (2003) also discuss this issue, though from a different perspective.

[7] Chapter 2 of the thesis provides a detailed discussion about the new dataset and the risk factors based on the data that I collected.

The rest of the paper is organized as follows. Section 2 describes the data and the additional risk factors. Section 3 illustrates the variable selection procedures. Section 4 presents the empirical results. Section 5 briefly concludes and provides some further comments.

3.2 Data

3.2.1 Hedge funds and original factors

Agarwal and Naik (2004) focus on equity-oriented hedge fund strategies.[8] More precisely, they study index strategies, instead of individual hedge fund strategies. While this choice is not uncommon in the empirical literature, the analysis of index strategies raises a number of issues, mainly related to data aggregation. Amin and Kat (2002) examine the return distribution features of individual hedge funds and equally-weighted portfolios that contain an increasing number of funds, with the portfolio including the entire population being the index. They show that the aggregation of individual hedge funds in portfolios lead to lower standard deviation and skewness and higher kurtosis. However, while the reduction in the standard deviation is a purely technical matter, the change in skewness and kurtosis is a truly empirical observation and it is not attractive.

To overcome the shortcomings of using index strategies, Agarwal and Naik (2004) collect data from two different distributors, namely the Hedge Fund Research (HFR) and the Credit Suisse First Boston/Tremont (CSFB).

Since there is not a widely accepted classification for hedge funds, Agarwal and Naik (1999) divide the equity-oriented strategies according to the degree of correlation with the market.[9] A first set consists of six HFR indexes

[8]As the authors point out, they analize hedge fund strategies that are mainly related to the stock market because high quality data on exchange-traded options on equity indexes are easily availabile.

[9]Indeed, the term "hedge" can actually be misleading. The traditional hedge fund is actually hedged against market-wide shocks, that is it is market neutral. However, much

which are based on non-directional strategies. These strategies take primarily into account relative mispricing of securities, rather than movements of the market as a whole. Merger Arbitrage, Distressed Securities, Event Driven, Relative Value Arbitrage, Convertible Arbitrage and Equity Hedge form the first group.[10] The second set consists of two hedge funds which are based on directional strategies, that is their payoffs arise primarily from taking directional bets. HFR Equity Non-Hedge and Short Selling form the second group.[11]

The sample consists of net-of-fee monthly returns on the HFR indexes. The HFR provides data from January 1990 to June 2000. The HFR indexes are comprised of domestic and offshore hedge funds, selected among over 1600 listed on the HFR Database. The HFR indexes are equally weighted and so give relatively more weight to the performance of smaller hedge funds.[12]

Table 1, Panel A reports summary statistics for the HFR index returns during the sample period.

In order to characterize hedge fund risks, Agarwal and Naik (2004) estimate the strategy exposures to the risks captured by different security classes. They use a multifactor model in which the risk factors are excess returns on standard assets and options strategies.[13]

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more common are funds that are not hedged. The Appendix provides a brief description of hedge fund strategies.

[10] According to Agarwal and Naik (1999), during the period from January 1994 to September 1998, the Equity Hedge strategy was the least market neutral.

[11] According to Agarwal and Naik (1999), during the period from January 1994 to September 1998, the Short Selling strategy moved in the direction opposite to the market one.

[12] See www.hfr.com for the index construction details. Fung and Hsieh (2003) provide an explanation for why value weighting is naturally unsuitable for the hedge fund industry.

[13] As Fung and Hsieh (1997) show, if one assumes a long-only and buy-and-hold strategy and uses traditional asset categories to explain hedge fund returns, the model has very low explanatory power.

Table 1: Summary Statistics

Panel A: HFR Hedge Fund Indexes

Hedge fund strategy	Mean	SD	Median	Skewness	Kurtosis	Min.	Max.
Non-Directional							
Merger Arbitrage	1.03	1.32	1.33	-3.24	17.18	-6.46	2.90
Distressed Securities	1.29	1.90	1.35	-0.81	8.88	-8.50	7.06
Event Driven	1.33	1.94	1.53	-1.62	9.42	-8.90	5.13
Relative Value Arbitrage	1.14	1.17	1.29	-1.24	12.97	-5.80	5.72
Convertible Arbitrage	0.95	1.01	1.16	-1.48	6.30	-3.19	3.33
Equity Hedge	1.82	2.65	1.82	0.10	4.57	-7.65	10.88
Directional							
Equity Non-Hedge	1.71	4.06	2.28	-0.59	4.17	-13.34	10.74
Short-Selling	0.07	6.40	-0.16	0.13	4.63	-21.21	22.84

Panel B: Risk Factors

Risk factor	Mean	SD	Median	Skewness	Kurtosis	Min.	Max.
Buy-and-Hold Risk Factors							
Equity							
Russell 3000	1.38	3.68	1.65	-0.24	4.11	-11.52	12.70
MSCI World Excluding US	0.66	4.83	0.71	-0.18	3.49	-13.47	14.67
MSCI Emerging Markets	1.01	6.80	1.41	-0.86	5.44	-28.91	16.53
Fama-French SMB Factor	0.39	6.17	0.12	-0.08	3.62	-20.20	16.21
Fama-French HML Factor	0.51	3.15	0.61	0.32	4.38	-7.44	12.06
Harvey-Siddique S-	1.70	3.55	0.64	1.07	6.54	-8.62	17.07
Bond							
SB Government and Corporate Bond	0.63	1.25	0.77	-0.06	3.24	-2.37	4.65
SB World Government Bond	0.63	1.81	0.75	0.16	3.39	-3.63	6.11
Lehaman High Yield	1.27	3.83	1.19	0.63	15.27	-14.82	23.91
Default Spread	1.84	0.31	1.75	0.54	2.38	1.29	2.65
Currency							
FRB Competitiveness-Weighted Dollar	0.43	1.21	0.31	0.39	3.59	-2.79	3.93
Commodity							
Goldam Sachs Commodity	0.65	5.04	0.79	0.54	4.36	-12.28	18.52
Option-based Risk Factors							
S & P 500 At-the-Money-Call	41.99	156.91	4.84	1.16	4.08	-100	593.88
S & P 500 Out-of-the-Money-Call	44.88	203.17	-52.29	1.74	6.05	-100	892.28
S & P 500 At-the-Money-Put	-35.58	123.56	-100	2.47	9.67	-100	555.2
S & P 500 Out-of-the-Money-Put	-36.06	148.08	-100	3.12	13.49	-100	753.99

Buy-and-hold strategies

The first group of factors represents buy-and-hold strategies. It contains:

- indexes representing equities, Russell 3000 index (RUS) and one-period lagged Russell 3000 index (LRUS),[14] MSCI World excluding USA index (MXUS) and MSCI Emerging Markets index (MEM)[15]

- indexes representing bonds, Salomon Brothers Government and Corporate Bond index (SBGC) and Salomon Brothers World Government Bond index (SBWG)[16] and Lehman High Yield index (LHY)[17]

- an index representing currency prices, Federal Reserve Bank Competitiveness-Weighted Dollar index (FRB)[18]

- an index representing commodity prices, Goldman Sachs Commodity index (GSC)[19]

- the change in default spread to capture credit risk (DS). The default spread is defined as the difference between the yield on the BAA-rated corporate bonds and the ten-year constant-maturity-rate Treasury bonds.[20]

The buy-and-hold risk factors include also zero-investment strategies, namely Fama and French (1993) SMB (small minus big), which captures the size effect and HML (high book-to-market minus low book-to-market),

[14] Data source: Datastream. Argawal and Naik (2001) do not use the S&P 500 index because it covers a narrower universe of stocks.

[15] Data source: MSCI website. The MSCI World excluding USA index covers developed markets besides US.

[16] Now the index is denominated Citigroup WGBI.

[17] Data source: Datastream. It takes into account returns from investing in distressed securities.

[18] Data source: Board of Governors of the Federal Reserve System.

[19] Data source: Datastream.

[20] Data source: Board of Governors of the Federal Reserve System.

which captures the value effect. Agarwal and Naik (2001) point out that the Fama and French (1993) portfolios are not truly buy-and-hold strategy factors since they involve periodic trading. However, they are linear, as opposite to non-linear, risk measures and so they are coupled with buy-and-hold factors.

Option-based strategies

The option-based risk factors are based on highly liquid European call and put options on the S&P 500 Composite index (SPX) trading on the Chicago Mercantile Exchange.

The trading strategy involves buying and writing one-month-to-maturity options with different strike prices. Indeed, Glosten and Jagannathan (1994) show that one can approximate fund nonlinear payoffs by a linear combination of options on some benchmark index returns with different degree of moneyness. Hence, Agarwal and Naik (2004) consider at-the-money (ATM) and out-of-the-money (OTM) options.

For example, consider the purchase of an ATM call option. On the first trading day of a month buy a call option at the Black-Scholes price, hold it for one month and on the first trading day of the following month sell it at the Black-Scholes price. Repeating this trading pattern each month provides a time-series of returns on buying an ATM call option.[21]

Due to the lack of data, I compute both strike prices and option prices. Following an Agarwal and Naik (2004) observation about the average behaviour of the selected strike prices, I define the ATM option to be such that the ratio of the index price to the present value of the strike price is 1. Similarly, the ratio of the underlying price to the OTM call (put) strike price is 0.99 (1.01).[22]

[21] If the option expires in the money, one can compute the return on the initial investment. If the option expires out of the money, one assigns a return of -100% for that month.

[22] The choice of the degree of moneyness is arbitrary. A widely-used method is to adopt

In addition, I estimate the volatility in order to compute Black-Scholes prices. I consider daily returns on SPX and assume that are lognormally distributed. Then, I use the exponentially weighted moving average model to compute daily volatilities at the end of each month:

$$\sigma = \sqrt{(1-\lambda)\sum_{t-1}^{T}\lambda^{t-1}(r_t-\bar{r})^2}$$

where r_t is the continuously compounded return on the SPX on day t. The estimated volatilities depend crucially on the decay factor, $\lambda \in (0,1)$, a parameter which determines the relative weight applied to each observation, and the effective amount of data used in the estimation.[23] Indeed, this approach assigns higher weights to later observations than data in the distant past. Therefore, volatility reacts fast to shocks in the market and following a shock (for instance, a large return), the volatility declines exponentially as the weight of the shock observation falls. Following the JP Morgan Risk Metrics, I assume that the mean value of daily returns is zero and the decay factor is 0.97.[24]

Given the daily volatilities for each month I compute the annualized volatility and in turn I calculate the various option prices. Finally, I compute the returns for the different option-based strategies.

The option-based risk factor abbreviations are the following: SPC_a indicates the ATM call option strategy, SPC_o indicates the OTM call option strategy, SPP_a indicates the ATM put option strategy and SPP_o indicates the OTM put option strategy.

an OTM strike price of 2.5 points away from the ATM strike price. The authors, in the robustness check section, consider OTM strike prices ranging from half a standard deviation to two standard deviations away from the ATM strike prices.

[23] I use data from January 1988 onwards.

[24] The Risk Metrics volatility data set is used in many organizations worlwide to estimate value at risk.

3.2.2 Conditional-higher–moment factor

Following recent studies on the importance of conditioning information in explaining CAPM deviations, Harvey and Siddique (2000) introduce a new risk factor to analyze the cross-section of expected stock returns. The factor is based on asset's conditional coskewness with the market portfolio, that is the component of a stock's conditional skewness related to the market's portfolio skewness. The intuition behind the choice of a skewness-based factor is twofold. First, the authors observe that the classes of stocks which give rise to asset pricing anomalies are characterized by most skewed returns. Hence, "skewness may be important in investment decisions because of induced asymmetries in ex post (realized) returns".[25] Second, given the evidence that unconditional return distributions cannot be adequately described by the first two moments, it seems quite natural to better characterize them through the third moment.

To understand how skewness affects the cross-section of asset returns, suppose that investors have non-increasing risk aversion, that is risk aversion do not increase if wealth increases. Then, the authors show that, other things being equal, right-skewed portfolios are preferred to left-skewed portfolios. So, assets that make a portfolio more left-skewed, namely that decrease its total skewness, are less desiderable and should have higher expected returns.

To capture the effect of conditional coskewness, Harvey and Siddique (2000) use 60 months of asset returns to compute the standardized unconditional coskewness for each stock i traded on Amex, NASDAQ and NYSE:

$$\widehat{\beta}_{SKDi} = \frac{E(\epsilon_{i,t+1}\epsilon^2_{M,t+1})}{\sqrt{E(\epsilon^2_{i,t+1})E(\epsilon^2_{M,t+1})}}$$

where $\epsilon_{i,t+1} = r_{i,t+1} - \alpha_i - \beta_i(r_{M,t+1})$ is the residual from the regression of stock i excess return on the contemporaneous market excess return and $\epsilon^2_{M,t+1} = r^2_{M,t+1} - \alpha_M - \beta_M(r_{M,t+1})$ is the residual from the regression of the

[25] Confront Harvey and Siddique (2000), p. 1264.

squared market excess return on the contemporaneous market excess return. This measure represents the contribution of the stock to the coskewness of the market portfolio.

On the basis of the unconditional coskewness, the authors form hedge portfolios whose excess returns from the 61th month onwards are used to proxy for systematic skewness. Following a procedure similar to the Fama and French (1993) approach, they rank stocks according to past coskewness. Then, they form three value-weighted portfolios, one containing the 30 percent of the stocks with the most negative coskewness, S^- one containing the 30 percent of the stocks with the most positive coskewness, S^+ and one containing the remaining 40 percent. Next, Harvey and Siddique (2000) consider both the spread between S^- and S^+ excess returns and S^- excess returns as risk factors. Due to lack of data on the S^+ portfolio, I consider only the second measure. Since the S^- portfolio contains the stocks with the most negative coskewness, a high loading on this hedge portfolio should be associated with high expected returns and so the risk premium associated to the skewness factor should be positive.

Table 1, Panel B reports summary statistics for the buy-and-hold and option-based strategies and the coskewness factor.[26]

3.3 Multifactor model

As Liang (1999) point out, there is a potential collinearity problem among the risk factors. Table 2 shows the correlations among the various strategies and indeed several factors are highly correlated. For instance, the Russell 3000 index exhibits high correlation with the MSCI indexes. The correla-

[26] Excess returns on option-based risk factors have a higher order of magnitude compared to the buy-and-hold strategies. Thus, I use scale them down by dividing the original values by a factor of one hundred.

Note that the table reports the statistics for the default spread, while the risk factor used in the analysis is its change.

tions are even higher between the Russell 3000 index and each option-based risk measure on the one hand, and the coskewness factor on the other hand. Finally, the coskewness measure displays high correlations with the option-based strategies. To mitigate the collinearity problem and avoid the redundancy of predictors, Agarwal and Naik (2004) use a stepwise procedure. This method allows to identify a limited subset of statistically significant factors that ex-post capture variation in hedge fund returns. In order to select such variables, the authors estimate the following regressions:

$$R_t^i = c^i + \sum_{k=1}^{K} \lambda_k^i F_{k,t} + u_t^i$$

where R_t^i is the net-of-fee excess return on hedge fund index i in month t, c^i is the intercept term for hedge fund index i in the sample period, λ_k^i is the average factor loading on factor k for hedge fund index i during the sample period, $F_{k,t}$ is the excess return of factor k in month t and u_t^i is the error term.[27]

3.3.1 Stepwise procedure

Agarwal and Naik (2004) give very few hints on the model selection method that they adopt.[28] Following some comments in their previous work, I apply a forward stepwise procedure.[29] This method identifies the appropriate regression model by gradually enriching a basic, initial model, given a set of potentially useful predictors. The initial model is the regression of hedge fund i excess return on a constant. In order to determine the first variable to add to the model, I compute simple correlations between the hedge fund i

[27]Excess returns are computed using the one-month Treasury bill rate in order to be consistent with the construction of the Fama and French (1993) and option-based factors.

[28]The authors employ a stepwise procedure, that add and delete variables sequentially on the basis of the F-value. In addition, they use robust standard errors. Finally, the authors cite the Bonferroni adjustment in the notes of Table 2.

[29]Confront Agarwal and Naik (1999) and Agarwal and Naik (2001).

Table 2 - Risk factor correlation matrix

	LRUS	RUS	MXUS	MEM	SBWG	SBGC	LHY	FRB	GSC	DS	SMB	HML	SPCa	SPCo	SPPa	SPPo
RUS	-0.01	1														
MXUS	0.06	0.56	1													
MEM	0.11	0.54	0.55	1												
SBWG	-0.25	0.10	0.37	-0.07	1											
SBGC	-0.13	0.33	0.18	0.02	0.56	1										
LHY	0.35	0.44	0.26	0.40	0.00	0.29	1									
FRB	0.09	-0.08	-0.28	-0.18	-0.43	-0.05	0.14	1								
GSC	-0.25	-0.10	-0.04	-0.05	0.04	0.00	-0.14	-0.09	1							
DS	0.08	0.14	0.20	0.27	0.00	-0.05	0.11	-0.02	-0.07	1						
SMB	0.23	-0.14	-0.03	0.16	-0.07	-0.20	0.16	0.03	0.16	0.19	1					
HML	0.01	-0.03	-0.01	-0.01	-0.07	0.12	0.06	0.21	0.02	-0.02	0.01	1				
SPCa	-0.08	0.80	0.45	0.33	0.20	0.40	0.28	-0.10	-0.07	-0.07	-0.29	-0.01	1			
SPCo	-0.08	0.73	0.41	0.28	0.22	0.41	0.25	-0.11	-0.05	-0.09	-0.30	-0.01	0.98	1		
SPPa	0.01	-0.78	-0.44	-0.46	-0.10	-0.39	-0.43	-0.06	-0.02	-0.09	0.10	-0.04	-0.48	-0.39	1	
SPPo	-0.03	-0.71	-0.41	-0.45	-0.11	-0.39	-0.43	-0.05	-0.01	-0.11	0.08	-0.05	-0.40	-0.32	0.98	1
S-	-0.05	0.68	0.40	0.33	0.01	0.16	0.09	-0.04	-0.08	0.10	-0.29	-0.04	0.58	0.51	-0.52	-0.47

excess return and each risk factor excess return. Then, I include in the model the strategy measure that has the highest correlation, in absolute value, with the dependent variable.

Indeed, the selected risk factor is the one that explains the most variation in the hedge fund i excess return. Not surprisingly, three out of eight hedge fund strategies show the highest correlation with option-based factor returns. Also, the two non-directional strategies and the Equity Hedge strategy show the highest correlation with the Russell 3000 index returns.[30]

As expected the augmented model is an improvement with respect to the initial model, therefore I continue the stepwise selection. At each following

[30]See Table 3. The coefficient of the risk factor that has the highest correlation with the hedge fund i strategy is reported in Italics font.

step of the process, there can be one inclusion, followed by at most one exclusion.

The inclusion of the following predictors is based on the partial correlations. At each step, I compute the partial correlations between the dependent variable and the remaining risk factors, controlling for the previously added predictors. The risk measure that exhibits the highest partial correlation, in absolute values, is included in the model.

If the risk factor contributes significantly to explain the hedge fund returns, I keep the augmented model, otherwise I end the selection process. Thus, the procedure continues until a certain stopping rule is encountered. The latter is based on a partial F-statistic that evaluates the new model relatively to the restricted one:

$$(N-k-2)\,\frac{R^2_{full}-R^2_{reduced}}{1-R^2_{full}},$$

where the suffix $full$ refers to the augmented model with $k+1$ explanatory variables, the suffix *reduced* refers to the previous-step model with k regressors and N is the number of observations. The new predictor is entered as long as the statistic p-value remains below a specific maximum value.[31]

If the "full" model is retained, then I check the significance of the regressors that were identified in the previous steps. I perform two-sided t-tests for all the predictors in the model but the last one. If the largest p-value is above an established threshold, then I eliminate the corresponding variable.[32] In this case, I compute the adjusted R^2 value of the resulting regression model.

If the augmented model is not retained, I run again the restricted regression to compute the factor loadings and the adjusted R^2 value at 5% significance level and the process ends.

[31] Note that, for the inclusion step, in fact I test H_0: $\beta_{k+1}=0$ and so the F-statistic value is exactly equivalent to the square of the corresponding t-statistic value.

[32] All the tests are performed using a Newey-West covariance estimator, in order to take into account the presence of heteroskedasticity and autocorrelation in the standard errors.

The stepwise procedure raises two main issues, which are closely related. First, one should define the threshold values for adding and removing the potential predictors. Second, one should consider the effects of multiple comparisons when, as in the case of model selection, one performs several tests on the same data.

To avoid infinite cycling of the stepwise algorithm, the maximum value for the inclusion should be at most equal to the maximum value for the exclusion. However, both values are affected by the number of potential comparisons that the process might perform. Several rules have been proposed in the literature to take into account the multiple comparison issue. The most popular method is the Bonferroni criterion, but, as it is widely recognized, this adjustment is extremely conservative. Thus, I choose to apply more refined rules, that is the Sidak-Holm correction for the entry value and the Finner adjustment for the exit value. Both procedures are step-down rules that apply a different critical value to each test. In both cases, the process works as follows. First, consider a regression model with all possible explanatory variables and get the estimated coefficients. Then, calculate the p-values of the corresponding t-tests at the standard significance level, α and rank them in ascending order. The Sidak-Holm procedure adjusts α to the following inclusion threshold: $\alpha_i^* = 1 - ((1-\alpha)^{1/(K-j+1)})$, where K is the total number of potential predictors and j is the ranking position of the p-value associated to the variable under examination. The Finner method determines the following exclusion threshold: $\alpha_e^* = 1 - ((1-\alpha)^{j/K})$. One can show that $\alpha_i^* \leq \alpha_e^* \ \forall \ j, j = 1, ..., K$.[33]

Table 3, 4 and 5 show the hedge fund models resulting from the forward stepwise approach.

[33]Sidak-Holm and Finner procedures evaluate the regressors according to the p-value ranking. My approach, based on correlations, follow a different ranking and therefore a different ordering in variable selection. Nonetheless, I take into account the p-value ranking when testing the significance of the estimated coefficients.

3.3.2 A principal-components method

Although the forward stepwise procedure is probably the most popular variable selection method, it has some drawbacks.[34] Hence, in order to assess the validity of the previous findings, I use a different approach to determine hedge fund models. The method is an application of Mundlak (1981) technique, which combines the principal components analysis and the theory of multiple comparisons.[35]

First, I compute the principal components from the set of potential predictors. Each principal component is a linear combination of the original risk factors and the new variables are orthogonal to each other. Therefore, there is no redundant information when one considers the principal components and so this method allows to control for the collinearity among the risk factors.

However, the principal components are not interpretable in term of the original explanatory variables. To determine which original predictors are indeed significant given the orthogonalization, Mundlak (1981) propose a decomposition rule on the principal component coefficients vector. The first partition, δ_1 will contain the significant coefficients, while the second partition, δ_2 will contain the non-significant coefficients. The corresponding partitions for the principal components matrix are P_1 and P_2, respectively. Then, the relation between the principal component coefficients vector and the original variable loadings allows to determine the significant risk factors.

The partition is made through the computation of t-tests on the principal component coefficients. The squares of the t-ratios are subsequently ordered from the highest to the lowest to give a rule of numbering for the principal components. Then, an iterative procedure computes an F-statistic, as a weighted sum of t-ratio squares. At each step the algorithm adds the lowest

[34] Miller (2002) discuss at length about the stepwise regression procedure.

[35] Fung and Hsieh (1997) use the principal component analysis to group hedge funds with similar return characteristics together and overcome the potential unreliability of qualitative style categorization.

t-ratio square among the ones that are not yet included in the statistic. The process ends when the statistic value exceeds the critical value at 5% significance level. The principal components which contributes to the F-statistic are those with non-significant coefficients, that is the algorithm establishes P_2 and δ_2. Given the decomposition, the product between P_1 and δ_1 provides the significant factor loadings and consequently the significant predictors are identified.

Table 6, 7 and 8 report the multifactor models determined through the Mundlak (1981) methodology.

3.4 Empirical results

3.4.1 Forward stepwise analysis

Table 3 shows the regression models established by the forward stepwise procedure if the set of potential predictors consists of Agarwal and Naik (2004) risk factors. The table reports, for each hedge fund strategy, the constant term, the significant (at 5% level) loadings on the selected variables and the adjusted R^2 value.

Overall, the models are very parsimonious and display some differences with respect to Agarwal and Naik (2004) findings. For instance, the HML factor is significant only for the Distressed Securities and Relative Value Arbitrage strategies, whereas according to Agarwal and Naik (2004) this measure explains the payoffs of all hedge funds but Convertible Arbitrage and Equity Non-Hedge strategies.[36]

However, my results on option-based risk factors are similar to Agarwal and Naik (2004) outcomes. All directional strategies but the Distressed Securities and the Equity Hedge ones exhibit non-linear risk-return relations. On

[36] The other Fama and French (1993) factor, the SMB variable, is significant for all hedge fund strategies, but the Convertible Arbitrage one. Moreover, the corresponding coefficients have the same sign as in Agarwal and Naik (2004) regressions.

the contrary, all non-directional hedge funds do not show significant betas on option-based factors. Moreover, the loadings on the option-based measures have the same signs as in Agarwal and Naik (2004) models. In particular, three hedge funds show significant exposures to the risk associated to writing a put option on the S&P 500 Composite index. In addition, the Event Driven strategy reveals its non-linear risk-return trade-off through a long position in the OTM call option on the S&P 500 Composite index.

Table 4 reports the multi-factor models originating from the selection process in the case that the initial set of possible regressors includes the coskewness variable. The enlargement of the potential predictor group changes all the resulting regressions and leads to less parsimonious models. In particular, the coskewness risk factor significantly explains four out of eight hedge fund risk-return relationships, even though it does not have the highest correlation with any of the hedge fund strategies.[37]

The most striking result concerns the Merger Arbitrage strategy. In fact the coskewness factor replaces the OTM-put-option strategy as significant regressor. Hence, in the Merger Arbitrage strategy the coskewness measure captures the nonlinear risk-return relation that previously manifested itself through the option-based factor.

Similarly, the coskewness variable replaces the Russell 3000 index factor in the Event Driven model. Furthermore, the loading on the ATM-put-option strategy is no longer significant. A similar outcome occurs in the case of the Equity Non-Hedge strategy. The ATM-put-option factor is, together with the coskewness variable, a newly selected predictor. However, the coefficient on the coskewness factor is significant at the 5% level, while

[37] Indeed, the hedge fund strategies show the highest correlation with the same risk factor as in the case of the original set of potential predictors.

Table 3

Regression models - Agarwal and Naik (2004) risk factors

Forward Stepwise Procedure

	Merger Arbitrage	Distressed Securities	Event Driven	Relative Value Arbitrage	Convertible Arbitrage	Equity Hedge	Equity Non-Hedge	Short-Selling
constant term	0.27	0.61	0.23	0.43	0.19	0.94	0.47	0.88
LRUS			0.08		0.13			
RUS			0.14			*0.48*	*0.79*	*-1.24*
MXUS								
MEM		0.09	0.05				0.11	
SBWG								
SBGC								
LHY	0.08	*0.24*	0.08					
FRB								
GSC								
DS								
SMB	0.04	0.06	0.09	0.06		0.17	0.26	-0.36
HML		0.13		0.09				
SPCa								
SPCo								
SPPa			*-0.51*					
SPPo	*-0.37*			*-0.39*	-0.31			
Adj R squared	33.30%	59.08%	66.82%	36.80%	41.06%	51.83%	77.82%	55.89%

The table shows the regression models resulting from the forward stepwise procedure. It reports the intercept and the significant (at 5% level) coefficients on the risk factors. The last line shows the adjusted R^2. Italics indicates the risk factor which has the highest simple correlation with hedge fund returns.

the coefficient on the option-based measure is not.

The coskewness loading has the expected, positive, sign in all models, but the Equity Non-Hedge one. The Equity Non-Hedge index consists of hedge funds that have relatively high net long exposure and follow a directional strategy. The negative coefficient on the coskewness factor suggests that the hedge fund managers were long on equities which less decrease market skewness.

Table 4

Regression models - Adding Harvey and Siddique (2000) risk factor

Forward Stepwise Procedure

	Merger Arbitrage	Distressed Securities	Event Driven	Relative Value Arbitrage	Convertible Arbitrage	Equity Hedge	Equity Non-Hedge	Short-Selling
constant term	0.32	0.44	0.12	0.49	-0.39	0.92	0.34	0.70
LRUS			*0.07*		*0.13*			
RUS			[redacted]			0.49	*0.76*	*-1.41*
MXUS				*0.05*				
MEM		*0.09*	*0.05*				*0.11*	
SBWG				-0.15				
SBGC								
LHY	*0.13*	*0.20*	*0.12*	*0.05*				
FRB		0.26						
GSC		*0.03*			*0.03*	*0.07*		
DS					0.42			
SMB	*0.05*	*0.07*	*0.09*	*0.05*	*0.03*	0.16	*0.25*	*-0.33*
HML		*0.11*		0.08				
SPCa								
SPCo					0.11			
SPPa		*-0.26*	-0.61				-0.41	
SPPo	[redacted]			*-0.29*	*-0.26*			
S-	***0.15***		***0.11***				***-0.11***	***0.26***
Adj R squared	34.32%	61.53%	59.39%	34.12%	43.64%	53.24%	78.17%	56.62%

The table shows the regression models resulting from the forward stepwise procedure. Italics indicates the factor loadings that are significant at 5% level. The last line shows the adjusted R^2. Grey space indicates a risk factor that is no longer significant when the coskewness measure is considered. Light grey space indicates a new risk factor.

To further evaluate the relative importance of the coskewness factor in explaining hedge fund returns, I add to the regression models identified through the first stepwise selection, the new measure.

Consistent with the findings in Table 4, the coskewness variable replaces the option-based risk factor and has a significant loading in the Merger Arbitrage model. It also enters with a positive coefficient in the Distressed Securities and Relative Value Arbitrage models.

Table 5 reports the augmented-model results.

Table 5

Regression models augmented by the Harvey and Siddique (2000) risk factor

Forward Stepwise Procedure

	Merger Arbitrage	Distressed Securities	Event Driven	Relative Value Arbitrage	Convertible Arbitrage	Equity Hedge	Equity Non-Hedge	Short-Selling
constant term	0.32	0.51	0.15	0.41	0.19	0.94	0.47	0.88
LRUS			0.08		0.13			
RUS						0.48	0.79	-1.24
MXUS								
MEM		0.07	0.06				0.11	
SBWG								
SBGC								
LHY	0.13	0.24	0.10					
FRB								
GSC								
DS								
SMB	0.05	0.08	0.08	0.07		0.17	0.26	-0.36
HML		0.14		0.09				
SPCa								
SPCo								
SPPa			-0.76					
SPPo				-0.33	-0.31			
S-	**0.15**	**0.08**		**0.05**				
Adj R squared	34.32%	60.68%	64.79%	38.18%	41.06%	51.83%	77.82%	55.89%

The table shows the regression models resulting from the forward stepwise procedure and the subsequent introduction of the coskewness factor. It reports the intercept and the significant (at 5% level) coefficients on the risk factors. The last line shows the adjusted R^2.

To sum up, the majority of directional hedge funds, namely Merger Arbitrage, Event Driven, Relative Value Arbitrage and Convertible Arbitrage strategies, shows significant exposures to non-linear risk-return relations. The empirical evidence suggests that such non-linearities can be likewise captured either by option-based strategies or by a coskewness factor. Furthermore, the Merger Arbitrage strategy payoffs seem to be better explained by the coskewness variable. In addition, there is some evidence that Distressed Securities and the two non-directional strategies present non-linear risk-return features, too. In fact, according to my analysis, the coskewness factor - but not the option-based variables - plays a role in explaining the excess returns on these strategies.

3.4.2 Principal components analysis

Table 6 shows the hedge fund models resulting from the Mundlak (1981) approach when the original set of explanatory variables contains only Agarwal and Naik (2004) factors.

The principal component analysis identifies "larger" models than the models selected through the stepwise process. In particular, all the option-based risk factors are significant predictors of hedge funds excess returns. Contrary to the previous findings, also the two non-directional strategies show option-based significant exposures.

Hence, overall the Mundlak (1981) methodology provides very different models from the ones determined by the forward stepwise procedure.

Table 6

Regression models - Agarwal and Naik (2004) risk factors

Principal Components Analysis

	Merger Arbitrage	Distressed Securities	Event Driven	Relative Value Arbitrage	Convertible Arbitrage	Equity Hedge	Equity Non-Hedge	Short-Selling
constant term	0.58	1.51	1.13	0.71	0.17	1.52	1.13	1.09
LRUS		0.15	0.14	0.03	0.11		0.05	0.05
RUS	0.21	0.12	0.33	-0.01	-0.05	0.52	0.76	-1.24
MXUS	0.02		0.02	0.04		0.07	0.08	-0.16
MEM		0.12	0.07		0.01	0.09	0.09	-0.09
SBWG		-0.01	-0.03	-0.20	-0.11	-0.08	-0.06	
SBGC	-0.07	0.06	0.04	0.06	0.12	0.11	0.09	0.22
LHY	0.08			0.04	0.03	-0.01	0.06	-0.08
FRB	0.02	0.38	0.23	-0.10	-0.18	0.08	0.12	-0.02
GSC			0.06	0.02	0.02			0.03
DS	-0.01	-0.59	-0.33	0.12	0.30	-0.34	-0.52	0.32
SMB				0.05	0.02		0.24	-0.33
HML			0.02			-0.04	-0.06	0.04
SPCa	-1.29	-0.53	-0.85	-0.12	-0.13	-1.66	-1.16	2.65
SPCo	0.72	0.19	0.41	0.09	0.22	0.75	0.62	-1.74
SPPa	-0.25	-0.51	0.17	0.14	-0.11	-0.86	-1.41	3.11
SPPo	-0.01	0.16	-0.37	-0.43	-0.21	0.60	0.89	-2.85
Adj R squared	34.34%	61.52%	61.77%	41.54%	38.28%	48.09%	77.65%	54.94%

The table shows the regression models resulting from the principal components analysis. It reports the intercept and the significant (at 5% level) coefficients on the risk factors. The last line shows the adjusted R^2.

Table 7 reports the outcomes of the Mundlak (1981) procedure when the set of original variables contains also the coskewness measure.

The introduction of the coskewness risk factor slightly reduce the dimension of the regression models. However, the principal component analysis still identifies "larger" models than the models determined by the stepwise procedure.

All the option-based risk factors are still significant predictors of hedge funds excess returns. Contrary to the stepwise regression outcomes, also the two non-directional strategies show option-based significant exposures.

The coskewness variable show significant loadings in all hedge fund strategies but the Distressed Securities.

Hence, overall the Mundlak (1981) methodology provides very different models from the ones determined by the forward stepwise procedure.

To sum up, the principal components analysis highlights non-linear risk-return relations in directional and non-directional hedge funds. The empirical evidence suggests that a coskewness risk factor captures such non-linear risk exposures, even though option-based strategies remain significant.

Table 7								
Regression models - Adding Harvey and Siddique (2000) risk factor								
Principal Components Analysis								
	Merger Arbitrage	Distressed Securities	Event Driven	Relative Value Arbitrage	Convertible Arbitrage	Equity Hedge	Equity Non-Hedge	Short-Selling
constant term	0.62	1.51	1.36	0.70	0.17	1.83	1.14	1.06
LRUS	0.06	0.15	0.09	0.02	0.11	0.05	0.06	0.03
RUS	0.17	0.12	0.25	-0.04	-0.07	0.50	0.83	-1.42
MXUS	0.01		0.04	0.04		0.08	0.08	-0.18
MEM		0.12	0.05			0.06	0.09	-0.09
SBWG	-0.03	-0.01	-0.13	-0.20	-0.11	-0.16	-0.07	0.01
SBGC	0.06	0.06	0.12	0.07	0.12	0.27	0.08	0.24
LHY				0.06	0.04	-0.09	0.04	-0.01
FRB	0.02	0.38	0.16	-0.11	-0.19	0.04	0.14	-0.07
GSC	-0.02		0.03	0.02	0.02			0.02
DS	-0.13	-0.59	-0.48	0.10	0.29	-0.55	-0.47	0.19
SMB	0.05		0.10	0.05	0.03	0.15	0.23	-0.30
HML						-0.05	-0.06	0.04
SPCa	-1.36	-0.53	-0.75	-0.16	-0.16	-1.35	-1.08	2.44
SPCo	0.80	0.19	0.45	0.12	0.24	0.71	0.57	-1.60
SPPa	-0.34	-0.51	0.25	0.16	-0.10	-0.82	-1.46	3.21
SPPo	0.02	0.16	-0.53	-0.44	-0.22	0.45	0.92	-2.92
S-	0.09		0.05	0.05	0.04	-0.03	-0.11	0.29
Adj R squared	44.65%	53.27%	71.62%	43.78%	56.21%	58.36%	81.03%	61.68%

The table shows the regression models resulting from the principal components analysis. It reports the intercept and the significant (at 5% level) coefficients on the risk factors.

3.5 Conclusion

Equity-based hedge fund index strategies show non-linear risk-return features. Agarwal and Naik (2004) explain these hedge funds characteristics by means of a multifactor model. In particular, the authors consider some option-based strategies as risk factors and show the importance of such variables in capturing the hedge fund non-linearities.

In this work, I introduce as a possible risk factor a strategy based on the conditional skewness of stock returns. More precisely, I focus on conditional coskewness, that is "the component of an asset's skewness related to the market portfolio's skewness".[38] The empirical evidence shows that the factor based on coskewness captures the nonlinearities in hedge fund excess returns, calling into question the true relevance of option-based strategies. Furthermore, the analysis highlights the importance of conditional higher moment measures in describing the risk-return trade-off of hedge funds.

An interesting extension of this work could be to consider additional factors, such as the momentum strategy or a downside risk measure. Ang, Chen and Xing (2001) shows that a factor based on downside risk explains the variations in the cross-section of stock returns. As the coskewness factor, the downside risk measure is based on conditional information and takes into account the correlations between asset returns and market portfolio returns. Thus, it could play a role in characterizing the risk-return features of equity-based hedge fund index strategies.

Finally, Fung, Hsieh, Naik and Ramadorai (2005) use market events as sample breaks to study the funds of funds performance. An empirical analysis over a longer sample period, split up to take into account the breaks, could allow a better assessment of the importance of competing risk factors in determining hedge fund returns.

3.6 Appendix

Agarwal and Naik (1999) split the hedge fund index strategies in two groups, "non-directional" strategies and "directional" strategies. Non-directional strategies are:

- Merger Arbitrage: it goes long on the acquired company and short the acquiring company
- Distressed Securities: it goes long (and occasionally short) on companies under Chapter 11 and/or under some form of re-organization

[38]See Harvey and Siddique (2000), p. 1265.

- Event Driven: it exploits possible mispricing arising from company event announcements (merger, restructuring, tender offer)
- Relative Value Arbitrage: it goes long on and short via cash or derivative markets in government and corporate bonds and asset-backed securities
- Convertible Arbitrage: it goes long on convertible securities (generally bonds) and short the underlying common stock
- Equity Hedge: it goes long on equities and short stocks and/or stock index options

Directional strategies are:

- Equity Non-Hedge: it goes long on equities. It does not always have a hedge in place
- Short Selling: it sells (over-valued) securities not owned by the seller to take advantage of an anticipated price decline.

Goetzmann and Ross (2000) make a partly different distinction and group strategies into "relative value arbitrage" strategies, "event driven arbitrage" strategies and "intertemporal arbitrage" strategies. Directional strategies are a subclass of the latter.

3.7 Bibliography

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