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Essays on Fiscal Policy: Calibration, Estimation and Policy Analysis

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Preface

The financial crisis of 2007-2008 has shifted the focus of economic research considerably. On the one hand, researchers are attempting to construct economic models that can explain how the financial crisis occurred. These models typically try to incorporate financial factors into existing modelling frameworks. On the other hand, considerable effort is made to find appropriate policy responses to the crisis. The nominal interest rate, the traditional instrument of monetary policy, is constrained at the zero lower bound which shifted focus both, toward unconventional monetary policy instruments and toward the use of fiscal stimulus. This dissertation relates to the latter one and consists of three essays in the field of Fiscal Policy. The first essay questions the use of closed economy models to assess the effects of tax cuts. The second essay builds a medium-scale DSGE model for Luxembourg to conduct policy experiments. The third essay challenges the use of linear models to analyze the effects of fiscal policy during times of fiscal stress.

The first chapter titled *Tax Cuts on Open economies* and it studies the dynamic response of government revenues to income tax cuts in an environment in which countries engage in commodity trade. A reduction in capital tax rates generates substantial dynamic responses within the framework of the standard closed-economy neoclassical growth model. The short-run revenue loss after a tax cut is partly — or, depending on parameter values, even completely — offset by growth in the long-run, due to the resulting incentives to further accumulate capital. We study how the dynamic response of government revenue to a tax cut changes if we allow a Ramsey economy to engage in international trade: the open economy's ability to reallocate resources between labor-intensive and capital-intensive industries reduces the negative effect of factor accumulation on factor returns, thus encouraging the economy to accumulate more than it would do under autarky.

The chapter's main contribution consists in a quantitative assessment of the relevance of this intuition. For this purpose, we calibrate our dynamic two-country

model with the US and the rest of the world in mind, and compute the short-run and long-run responses of government revenue to tax cuts. Our results demonstrate that international trade enhances the response of government revenue to tax cuts by a relevant amount. In our benchmark calibration, for example, a capital tax cut is able to finance itself in the long run, whereas the dynamic response to the tax cut in the autarky economy only compensates for 50% of the short-run revenue loss.

The second chapter titled *LSM: A DSGE Model for Luxembourg* and it builds a medium-scale New Open Economy Macroeconomics Dynamic Stochastic General Equilibrium (NOEM-DSGE) model for Luxembourg, labeled LSM (Luxembourg Structural Model). Luxembourg is a small open economy with a set of particular features, including rather limited competition in the domestic goods market, strong union power, and a dual labour market for resident and non-resident workers. LSM aims at assessing the effects of policy reforms such as greater product and labour market competition. Hence, we pay particular attention to modelling the real side of the economy, by combining some original theoretical features with modelling choices aimed at capturing specific characteristics of the Luxembourg economy. In particular, we adopt an over-lapping generation approach for households, and combine it with investment decisions and a right-to-manage specification of a segmented labour market, with both residents and non-resident workers.

The equilibrium conditions resulting from the optimization problems at the cohort and firm level can be analytically aggregated. The resulting model is calibrated in order to to mimic the actual behaviour of the key macroeconomic aggregates of the Luxembourg economy, and use it to conduct policy experiments aimed at relaxing some of the existing rigidities in the goods and labour market.

The third chapter titled *The Macroeconomic Effects of Fiscal Policy Shocks in Good Times and Bad* and it analyzes the macroeconomic effects of fiscal policy shocks conditional on fiscal stress by introducing a model whose parameter

evolution explicitly depends on initial conditions. Using US quarterly data from 1960:1-2009:4 we estimate several regime switching models that differ in the measure of fiscal stress used as a conditioning factor and find that the model with the debt-to-GDP ratio as the threshold variable fits the data best. The regime switch in our benchmark specification is estimated to be triggered at 42.6 percent of the debt-to-GDP ratio, which implies that approximately one quarter of all the observations in our sample are in the high debt-to-GDP regime. We find significant differences between the impulse responses in the two regimes. A positive government spending shock has for most of the response horizon a persistent negative effect on output in the high regime, while the opposite is true in the low regime. Also the responses of output to a positive tax shock are qualitatively different across regimes. Both responses are negative on impact but diverge after few quarters. The response in the high debt regime becomes positive, while the one in the low debt regime remains negative.

The paper contributes to the existing literature in three ways. First, we consider several alternative measures of fiscal stress that can be a potential a source of nonlinearity in the responses to fiscal policy shocks and evaluate empirically which variable is the most relevant. We do so by confronting the models in terms of their goodness of fit using the penalized loglikelihood function. Second, to our best knowledge, we are the first to estimate a threshold value of the debt-to-GDP ratio in a nonlinear model for the US economy. Previous papers has used ad hoc values to draw a line between good and bad times in a large panel of countries ranging from 90 percent to 120 percent. Since the debt-to-GDP ratio in the US has been historically low compared to other countries the estimated results were driven by observations from other countries in the sample and are not necessarily applicable to the US economy. Third, in our benchmark specifications the dynamics of the debt-to-GDP ratio is endogenous to the model. We adopt a specification where the government budget constraint explicitly links the dynamics of the debt-to-GDP ratio to the endogeneous variables in the VAR. Consequently, we can simulate the responses of the variables in the model taking into account the

possibility of a regime switch after the initial shock. We can derive impulse responses not only conditional on the economy being always in the low (high) regime, but also being in the low (high) regime when the fiscal shock hits while allowing the dynamics of the model to determine the evolution of the regime endogenously. The impulse responses are thus not representing the dynamics of each of the regimes in isolation but also take into account the effects that fiscal shocks have on the dynamics of the threshold variable itself.

CHAPTER 1: Tax Cuts in Open Economies*

Alejandro Cuñat[†] Szabolcs Deák[‡] Marco Maffezzoli[§]

Abstract

A reduction in capital tax rates generates substantial dynamic responses within the framework of the standard neoclassical growth model. The short-run revenue loss after a tax cut is partly — or, depending on parameter values, even completely — offset by growth in the long-run, due to the resulting incentives to further accumulate capital. We study how the dynamic response of government revenue to a tax cut changes if we allow a Ramsey economy to engage in international trade: the open economy's ability to reallocate resources between labor-intensive and capital-intensive industries reduces the negative effect of factor accumulation on factor returns, thus encouraging the economy to accumulate more than it would do under autarky. We explore the quantitative implications of this intuition for the US in terms of two issues recently treated in the literature: *dynamic scoring* and the *Laffer curve*. Our results demonstrate that international trade enhances the response of government revenue to tax cuts by a relevant amount. In our benchmark calibration, a reduction in the

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capital-income tax rate has virtually no effect on government revenues in steady state.

Keywords: International Trade; Heckscher-Ohlin; Dynamic Macroeconomics; Taxation; Revenue Estimation; Laffer Curve.

JEL codes: E13, E60, F11, H20.

1 Introduction

This paper studies the dynamic response of government revenues to income tax cuts in an environment in which countries engage in commodity trade. In particular, we construct a model in which two Ramsey economies specialize according to their factor abundance. We show that the long-run negative effect of a reduction in a country's capital-income tax rate on government revenues is much smaller than in the standard closed-economy Ramsey model.

The different behavior of the closed and open economies can be understood in terms of the different ways their sectorial factor allocation mechanisms work. A reduction in the capital-income tax rate raises the after-tax return to capital, thus creating an incentive to accumulate capital. Under autarky, an increase in the aggregate capital-labor ratio implies higher sectorial capital intensities; the diminishing marginal productivity of capital therefore reduces the return to capital and thereby the incentive to accumulate. In the open economy, instead, capital-labor intensities do not respond to exogenous¹ increases in the aggregate capital-labor ratio that much, as resources are reallocated from labor-intensive to capital-intensive industries. This enables the open economy to accumulate capital without affecting the gross return to capital as much as under autarky. Obviously, this generates a stronger reaction of capital income to the initial tax cut, and therefore reduces the negative impact of the tax cut on government revenues.

¹By "exogenous" changes in capital-labor endowments we actually refer to changes in factor endowment ratios that are not a response to technical progress.

The paper's main contribution consists in a quantitative assessment of the relevance of this intuition. For this purpose, we calibrate our dynamic two-country model with the US and the rest of the world in mind, and compute the short-run and long-run responses of government revenue to tax cuts. We relate our results to two issues, dynamic scoring and the Laffer curve, that have been treated recently in the literature.

First, Mankiw and Weinzierl (2006) criticize the way the Congressional Budget Office and the Joint Committee on Taxation score the proposed legislation each time the US Congress considers tax policy changes: the way the revenue impact of tax changes is calculated is usually referred to as *static scoring*, because it ignores the feedback effect from tax changes to any macroeconomic variable.² Mankiw and Weinzierl (2006) take a firm stand in favor of *dynamic scoring*: they use a closed-economy Ramsey model to show that the short-run response of government revenues to tax-rate changes is always stronger than the long-run response.³

Second, in a more recent reference, Trabandt and Uhlig (2006) use the neo-classical growth model to characterize the shape of the Laffer curve in the US and Europe. They find that both the US and Europe are on the upward sloping side of the Laffer curve; however, they point out that Europe is quite close to the Laffer curve's 'slippery slope,' that is, its downward sloping side.

Regarding dynamic scoring, we find that our dynamic trade model generates a much larger response on the factor accumulation side to a tax cut than in the autarky model, as we discussed above.⁴ In our benchmark calibration, for example, a capital tax cut is able to finance itself in the long run, whereas the dynamic response to the tax cut in the autarky economy only compensates for 50% of the short-run revenue loss. As for the Laffer curve, we find that the US reaches the Laffer curve's 'slippery slope': the actual average US tax rate on

²It is static from a macroeconomic point of view only, because feedback effects from micro-dynamic behaviour are incorporated into the forecast. For details, see Auerbach (1996).

³Leeper and Yang (2008) point out that the results of Mankiw and Weinzierl (2006) are sensitive to their assumption on how government deficits are financed.

⁴In fairness to Mankiw and Weinzierl (2006), we should point out that they are aware of the open-economy model yielding a stronger dynamic effect. See section 3.6 in their paper.

capital income is 27.3%, while the revenue-maximizing tax rate in our model, the peak of the Laffer curve, equals 26.7%. In contrast, under autarky, the peak of the Laffer curve occurs at a tax rate of 50.7%.

The main intuition of our paper is based on Ventura (1997), who shows, in the context of the neoclassical growth model, that the negative effect of capital accumulation on the return to capital is reduced by free trade. Although insightful and elegant, the Ventura model turns out not to be a very useful workhorse for performing a quantitative exercise of the kind we have in mind, since it yields international factor price equalization. First of all, this is obviously not a very realistic feature when contrasted with the data; secondly, the treatment of steady states in the presence of taxation becomes somewhat tricky, as the equalization of before-tax interest rates implies different after-tax interest rates if capital-income taxation differs across countries. Therefore, for our calibration exercise we produce a model based on Cuñat and Maffezzoli (2007), which is a dynamic generalization of Dornbusch et al. (1980). This set-up enables us to model Heckscher-Ohlin trade with trade frictions and therefore no factor price equalization (both important features in reality) in a rather straightforward way. See Davis and Weinstein (2001) and Romalis (2004) for empirical evidence supporting this model's predictions.

The link between taxation and international trade is obviously not new.⁵ Whalley (1980) is a good example of a "Computable General Equilibrium" model of taxation and international trade, which compares the welfare implications of

⁵The closed-economy literature on taxation in a dynamic set-up is clearly vast. An early reference that studies the dynamic incidence of labor taxes in a neoclassical growth model analytically is Bernheim (1981). Other notable examples of works that study the dynamic consequences of tax policies in a neoclassical framework are Cooley and Hansen (1992), Ireland (1994), Pecorino (1995), and Stokey and Rebelo (1995). More recent contributions are Bruce and Turnovsky (1999), who present a dynamic scoring exercise with the main focus on the sustainability of the fiscal balance of the government; and Novales and Ruiz (2002), who use a numerically simulated endogenous growth model to compare the feasible pairs of tax rates on capital and labour from a welfare point of view. It is also worth mentioning Backus et al. (2008), who study the empirical relationship between different measures of effective tax rates on capital and the cross-country dispersion of capital-labor ratios for a group of OECD countries.

tax policies under autarky and trade.⁶ Baxter (1992) shows that changes in taxation can affect cross-country specialization patterns within a dynamic model of Ricardian comparative advantage. This is also the case in our model: comparative advantage is influenced by taxation through its effects on factor accumulation. More interestingly, we show that the quantitative effects of taxation depend on an economy's ability to reallocate resources according to the evolution of its comparative advantage. Mendoza and Tesar (1998) studies tax reforms in a one-good, two-country dynamic model calibrated to U.S. and European tax policies, focusing on the role of intertemporal trade in the transmission of fiscal policy shocks across countries, while Mendoza and Tesar (2005) discuss the issue of international tax competition in the same framework. Bianconi (1995) studies tax policy in a neoclassical two-country dynamic model with integrated capital markets analytically. In comparison with these references, we ignore capital mobility; but the international dimension we exploit, goods trade and changes in production structures, also yields striking results from a quantitative perspective. More recently, Andersen (2007) studies tax competition in a static Ricardian trade model, and Epifani and Gancia (2009) studies the empirical relationship between international trade and the size of the government. The value added of our work here is the treatment of fiscal policy effects in a dynamic setting.

The rest of the paper is structured as follows: section 2 lays out a rather general dynamic trade model; in section 3 we develop some intuition by working out a very particular case, while in section 4 we simulate a more realistic version of the model; section 5 checks the sensitivity and robustness of our results; finally, section 6 presents our concluding remarks.

⁶See Shoven and Whalley (1984) for a survey of CGE models of taxation and international trade.

2 The Model

This and the next two sections present the dynamic Heckscher-Ohlin model with which we study the dynamic effects of tax cuts. We first sketch out the main ingredients of the model economy; then solve for a particular case analytically; finally, we calibrate a more realistic, albeit less tractable, case.

2.1 The Representative Household's Problem

Countries, indexed by j , are populated each by a *continuum* of identical households that can be aggregated into a single representative household. The representative household owns the capital stock and supplies capital and labor services inelastically; and either consumes or invests a final good. Governments collect taxes on factors of production (with possibly different rates applied to capital K and labour L); government revenues are paid back to households via lump-sum transfers. The representative households' preferences over consumption streams can be summarized by the following intertemporal utility function:

$$U_j = \sum_{t=0}^T \beta^t \frac{C_{jt}^{1-\frac{1}{\mu}} - 1}{1 - \frac{1}{\mu}}, \quad (1)$$

where β is the subjective intertemporal discount factor, and μ the elasticity of intertemporal substitution. T denotes the representative household's time horizon; C denotes consumption of the final good. The representative households maximize equation (1) subject to the following intratemporal budget constraint

$$P_{jt}(C_{jt} + I_{jt} - R_{jt}) = (1 - \tau_j^L) w_{jt} L_{jt} + (1 - \tau_j^K) r_{jt} K_{jt}, \quad (2)$$

where P is the price of the final good; I denotes investment; r and w are factor prices; τ^L and τ^K are the tax rates on labor and capital, respectively; and

$$R_{jt} = \tau_j^L \frac{w_{jt}}{P_{jt}} L_{jt} + \tau_j^K \frac{r_{jt}}{P_{jt}} K_{jt} \quad (3)$$

denotes real government transfers.⁷ Factor prices are taken as given by the representative household. The capital stocks evolve according to the following accumulation equation:

$$K_{jt+1} = (1 - \delta) K_{jt} + I_{jt}, \quad (4)$$

where $\delta \in [0, 1]$ is the depreciation rate.⁸ L_j is assumed constant. The first-order conditions

$$\beta C_{jt+1}^{-\frac{1}{\mu}} \left[(1 - \tau_j^K) \frac{r_{jt+1}}{P_{jt+1}} + 1 - \delta \right] = C_{jt}^{-\frac{1}{\mu}}, \quad (5)$$

$$K_{jt+1} = (1 - \tau_j^L) \frac{w_{jt}}{P_{jt}} L_{jt} + \left[(1 - \tau_j^K) \frac{r_{jt}}{P_{jt}} + 1 - \delta \right] K_{jt} + R_{jt} - C_{jt}, \quad (6)$$

and the corresponding transversality condition are necessary and sufficient for the representative household's problem. A recursive competitive equilibrium for this economy is characterized by equations (5)-(6) together with the equations that determine prices in the "static" equilibrium, to be discussed below.

2.2 Equilibrium Prices

Capital and labor are assumed to be internationally immobile. In each period, prices are determined in a "static" equilibrium, where we consider both autarky and trade.⁹

2.2.1 The Final Good

The final good, which is assumed to be nontradable, is produced under perfect competition with a *continuum* of intermediate goods. The representative firm

⁷For the sake of simplicity, we assume a balanced budget, and rule out any productive and/or welfare-enhancing role for public expenditure.

⁸For the sake of notational simplicity, we ignore exogenous technical progress. This does not affect our results significantly.

⁹For convenience, in this section we avoid time subscripts on variables, which might vary over time.

operating in the final good sector maximizes profits subject to the following Cobb-Douglas production function, taking all prices as given:

$$Y_j = \exp \left[\int_0^1 \ln x_j(z) dz \right], \quad (7)$$

where $x(z)$ denotes the quantity of intermediate good z used. Hence, the demand for intermediate good z is given by

$$x_j(z) = \frac{P_j Y_j}{p_j(z)}, \quad (8)$$

where $p_j(z)$ represents the price of intermediate good z and P_j is the price of the final good:

$$P_j = \exp \left[\int_0^1 \ln p_j(z) dz \right]. \quad (9)$$

2.2.2 Intermediate Goods

Intermediate goods are produced also under perfect competition. The representative producer in industry z maximizes profits subject to the following Cobb-Douglas production function, taking again all prices as given:

$$y_j(z) = \phi_j k_j(z)^{\alpha(z)} l_j(z)^{1-\alpha(z)}, \quad (10)$$

where $\alpha(z) \in [0, 1]$ denotes the capital share in industry z , $k(z)$ and $l(z)$ the capital and labor allocated to the production of intermediate good z , respectively, and ϕ_j is a time-invariant country-specific technology parameter. We label intermediate goods so as to have their capital intensities increase in z , *i.e.* $\alpha'(z) \geq 0$. Technologies are identical across countries, but for the exogenous factor-augmenting coefficients ϕ_j .

Intermediate goods can be traded. We model trade frictions as iceberg-type transport costs: $\nu \geq 1$ units of a good must be shipped from the country of origin

for one unit to arrive to the country of destination. $\nu = 1$ therefore corresponds to free trade. A high enough ν yields autarky instead.

2.2.3 Autarky Equilibrium Prices

Assume that ν is such that intermediate goods are not traded. Choosing the final good as the numeraire, the autarky static equilibrium conditions (discussed in the appendix) yield the following equilibrium before-tax factor prices:

$$r_j = \phi_j A \left(\frac{1 - \tilde{\alpha}}{\tilde{\alpha}} \right)^{\tilde{\alpha}-1} \left(\frac{K_j}{L_j} \right)^{\tilde{\alpha}-1}, \quad (11)$$

$$w_j = \phi_j A \left(\frac{1 - \tilde{\alpha}}{\tilde{\alpha}} \right)^{\tilde{\alpha}} \left(\frac{K_j}{L_j} \right)^{\tilde{\alpha}}, \quad (12)$$

where $A \equiv \exp \left[\int_0^1 \ln a(z) dz \right]$, $a(z) \equiv \alpha(z)^{\alpha(z)} [1 - \alpha(z)]^{1-\alpha(z)}$ is an industry-specific constant, and $\tilde{\alpha} = \int_0^1 \alpha(z) dz$ is the autarky economy's aggregate capital share.¹⁰

It is easy to show that the allocation of labor to each sector is a constant fraction of the economy's total amount of labor:

$$l_j(z) = \frac{1 - \alpha(z)}{1 - \tilde{\alpha}} L_j. \quad (13)$$

Finally, sectorial capital-labor intensities move one-to-one with the economy's aggregate capital-labor ratio:

$$\frac{k_j(z)}{l_j(z)} = \frac{\alpha(z)}{1 - \alpha(z)} \frac{w_j}{r_j} = \frac{\alpha(z)}{1 - \alpha(z)} \frac{1 - \tilde{\alpha}}{\tilde{\alpha}} \frac{K_j}{L_j}. \quad (14)$$

¹⁰The autarky version of the model is equivalent to a one-sector Ramsey model with a Cobb-Douglas production technology of the form $Y_j = \phi_j A K_j^{\tilde{\alpha}} L_j^{1-\tilde{\alpha}}$; our closed-economy framework is thus comparable to similar papers in the literature.

2.2.4 Trade Equilibrium Prices

We consider two cases here: a free-trade scenario, in which factor prices are equalized across countries; and a more realistic scenario, in which trade frictions prevent the law of one price from holding. Below we use the free-trade case to produce an analytically solvable example providing some intuition; the case with trade frictions is used in the quantitative section of the paper.

Free Trade Assume $\nu = 1$. For simplicity, consider a worldwide factor price equalization (FPE) equilibrium, in which the world as a whole (the integrated equilibrium) works as the autarky economy described above. Provided that countries do not have capital-labor ratios that are “too different” from the world’s aggregate capital-labor ratio, then they will not be completely specialized, and will have the integrated equilibrium’s factor prices.¹¹ This implies factor prices are independent of the country’s own factor endowments. A direct implication of this is the independence of sectorial capital-labor intensities from the country’s own capital-labor ratio.

Trade Frictions Assume there are two countries, North and South, indexed by $j = N, S$, respectively. We assume that the North is capital abundant, *i.e.* $K_N/L_N > K_S/L_S$, and therefore has a comparative advantage in the production of capital-intensive goods. Intermediate goods can be traded, but not freely: $\nu > 1$.¹² A trading equilibrium is characterized by two cut-off values $0 \leq z_N < z_S \leq 1$, that divide the range of intermediate goods into three subregions:

¹¹Below we make sure that the small open economy is in the FPE set. For a discussion on factor price equalization and the integrated equilibrium, see Dixit and Norman (1980).

¹²For the autarky equilibrium to be sustainable, *at autarky prices* transport costs must make it pointless to ship goods across countries. In other words, it has to be the case that

$$\begin{aligned} b(0, \phi_N, r_N, w_N) &\leq \nu b(0, \phi_S, r_S, w_S), \\ b(1, \phi_S, r_S, w_S) &\leq \nu b(1, \phi_N, r_N, w_N), \end{aligned}$$

where r_j and w_j are the autarky prices described above and $b(z, \phi_j, r_j, w_j)$ the unit-cost function of industry z evaluated in country j . This implies that, if $(w_N/r_N)/(w_S/r_S) =$

1. The intermediate goods $z \in [0, z_N)$ are exclusively produced in the South and shipped to the North.
2. The intermediate goods $z \in [z_N, z_S]$ are produced in both countries and nontraded. These commodities are not worth shipping from one country to another despite comparative advantage. This is due to the price wedge the trade cost introduces between countries.
3. The intermediate goods $z \in (z_S, 1]$ are exclusively produced in the North and shipped to the South.

The corresponding equilibrium conditions are discussed in the appendix. For further details, see Cuñat and Maffezzoli (2007).

3 A Simple Free-trade Case

Let us first address a two-period, free-trade case, which we can solve analytically. Assume $K_{j0} > 0$, $L_j = 1$, $T = 1$, $\mu = 1$ (log-utility), $\delta = 0$, $\nu = 1$ (free trade), $\tau_j^L = 0$ and $\phi_j = 1$. We will use the final good as the numeraire: $P = 1$. Furthermore, for the sake of notational simplicity, we will drop time and country indexes, since they turn out to be redundant under the assumptions imposed in this Section. The Euler equation (5) can then be rewritten as:

$$\beta (Y_0 - I_0) [1 + (1 - \tau^K) r_1] = Y_1 + K_1, \quad (15)$$

where $Y_0 = r_0 K_0 + w_0 L$ and $I_0 = K_1 - K_0$. The transversality condition associated to the representative household's maximization problem is simply $K_2 = 0$.

$(K_N/L_N)/(K_S/L_S) \leq \nu^{\frac{2}{\alpha(1)-\alpha(0)}}$, autarky will take place. If, on the other hand, $(K_N/L_N)/(K_S/L_S) > \nu^{\frac{2}{\alpha(1)-\alpha(0)}}$, autarky will not be sustainable and countries will trade.

3.1 Autarky Economy

The Euler equation (15) implicitly solves for K_1 . We can now study the dynamic implications of changes in taxation (around this equilibrium). By the Implicit Function Theorem, we can compute the effect of changes in τ^K on capital accumulation:

$$\left. \frac{dK_1}{d\tau^K} \right|_A = \frac{-\beta r_1 (Y_0 - I_0)}{1 + r_1 + \beta \left\{ 1 + (1 - \tau^K) r_1 \left[\tilde{\alpha} + (1 - \tilde{\alpha}) \frac{Y_0 + K_0}{K_1} \right] \right\}} < 0, \quad (16)$$

as $Y_0 - I_0 > 0$. A fall in τ^K raises the after-tax return to capital, thus encouraging further capital accumulation.

The effect of τ^K on period-1 gross interest rate is also easy to compute:

$$\left. \frac{dr_1}{d\tau^K} \right|_A = \frac{dr_1}{dK_1} \left. \frac{dK_1}{d\tau^K} \right|_A = (\tilde{\alpha} - 1) A \left(\frac{1 - \tilde{\alpha}}{\tilde{\alpha}} \right)^{\tilde{\alpha}-1} \left(\frac{K_1}{L} \right)^{\tilde{\alpha}-2} \left. \frac{dK_1}{d\tau^K} \right|_A > 0. \quad (17)$$

For future reference, it will be useful to also compute:

$$\varepsilon_{r_1} = (\tilde{\alpha} - 1) \varepsilon_{K_1}|_A > 0, \quad (18)$$

where

$$\varepsilon_x \equiv \frac{dx}{d\tau^K} \frac{\tau^K}{x} \quad (19)$$

represents the elasticity of a generic variable x with respect to changes in τ^K .

The gross return to capital falls with the aggregate capital-labor ratio due to the diminishing marginal productivity of capital: from (14), it is easy to see that sectorial capital intensities rise with the economy's aggregate capital-labor ratio. Thus, the effect of a reduction in τ^K on government revenue has two opposing components: an increase in the capital stock, and a reduction in its gross return.

3.2 Small Open Economy

For simplicity, consider a factor price equalization (FPE) equilibrium, in which the world (the integrated equilibrium) has got a capital-labor ratio with a time path identical to that of the autarky economy above. Consider a small open economy that has got the same initial condition K_0 and parameter values as in the autarky equilibrium. Since this economy faces the same factor prices of the autarky economy, the Euler equation (15) must yield the same solution for K_1 as under autarky.¹³

Once again, by the Implicit Function Theorem, we can compute the effect of changes in τ^K on capital accumulation (around this equilibrium):

$$\left. \frac{dK_1}{d\tau^K} \right|_O = \frac{-\beta r_1 (Y_0 - I_0)}{1 + r_1 + \beta [1 + (1 - \tau^K) r_1]} < 0. \quad (20)$$

Comparing (16) and (20), it is easy to see that the effect of a tax cut on K_1 is larger under free trade than under autarky, as $Y_0 + K_0 > K_1$.¹⁴

$$\left| \left. \frac{dK_1}{d\tau^K} \right|_O \right| > \left| \left. \frac{dK_1}{d\tau^K} \right|_A \right|. \quad (21)$$

Since commodity prices are given for the small open economy, $\left. \frac{dr_1}{d\tau^K} \right|_O = 0$.

The different behavior of the closed and open economies can be understood as due to the different ways their factor allocation mechanisms work. A reduction in τ raises the after-tax return to capital in both economies, creating an incentive to raise K_1 . Under autarky, an increase in K_1 implies higher sectorial capital-labor intensities; the diminishing marginal productivity of capital thereby reduces the return to capital and, therefore, the incentive to accumulate. In the open economy, instead, capital-labor intensities do not respond to increases in the aggregate capital-labor ratio (as factor-price ratios remain constant), and the marginal productivity of capital therefore does not fall: full employment of

¹³In the FPE jargon, our open economy is on the diagonal of the FPE set in both periods.

¹⁴The rest of variables and parameters in equations (16) and (20) are identical.

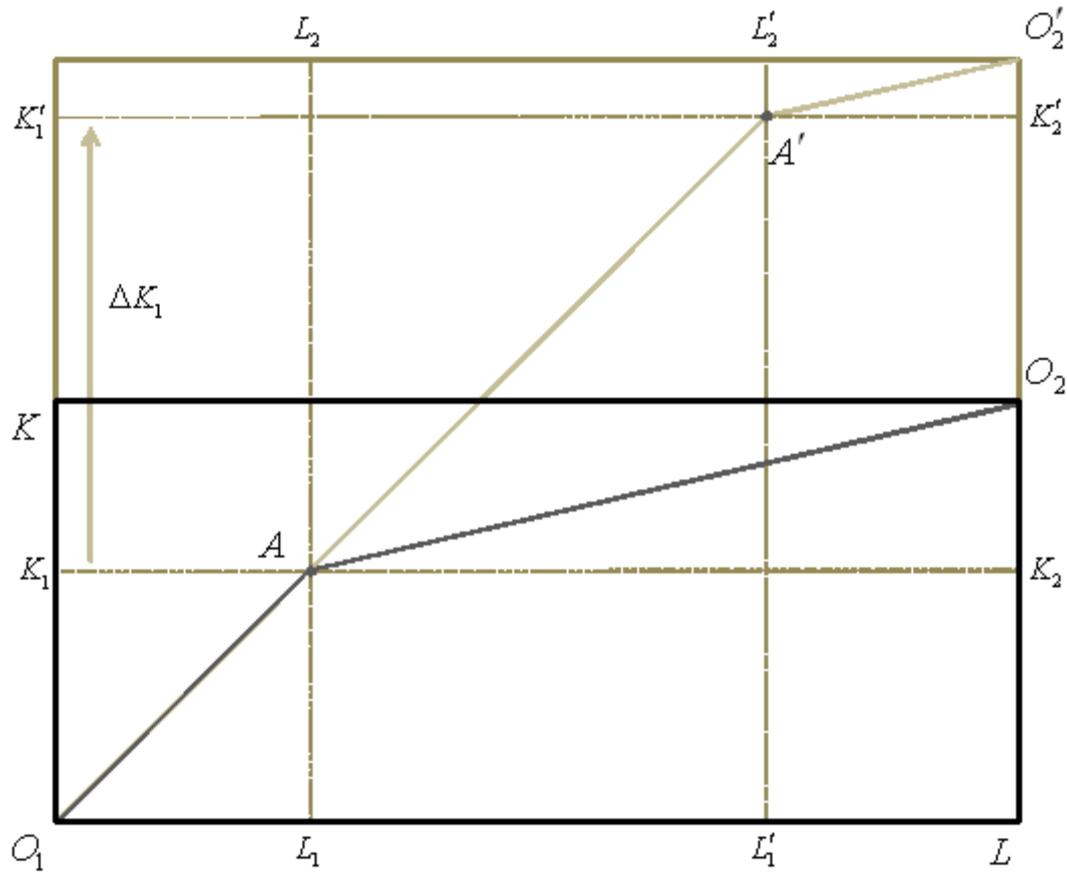


Figure 1: Capital accumulation and allocation under free trade: two goods.

resources is achieved by a reallocation of resources from labor-intensive to capital-intensive industries. In other words, factor price insensitivity to an increase in the aggregate capital-labor ratio is possible if and only if it is accompanied by a shift in production structures. Openness to trade allows this reshuffling and enables an economy to accumulate capital without affecting the gross return to capital.¹⁵

¹⁵See Ventura (1997).

Figures 1 and 2 illustrate this point in the two-good case. The dimensions of the box represent a country's given quantities of the two factors, capital and labor. Factor quantities allocated to sector 1 and 2 are measured, respectively, with respect to O_1 and O_2 . Let us start with the FPE case in Figure 1. Consider an initial endowment (K, L) , and take point A as the corresponding allocation of resources in a small open economy under factor price equalization. The slopes of vectors O_1A and O_2A represent the capital-labor intensities of sectors 1 and 2, respectively. An increase in the capital endowment of the economy implies a vertical enlargement of the box. Given the small-open-economy factor-price-equalization assumption, capital-labor intensities remain constant, and full employment must obtain through the reallocation of factors from the labor-intensive to the capital-intensive sector, i.e., from industry 2 to 1. The new allocation of factors is represented by point A' . Figure 2 illustrates the 2x2 Cobb-Douglas closed-economy case. When the capital endowment increases, the amounts of labor allocated to both sectors do not vary at all. At the same time, the sectoral capital-labor intensities change proportionally as much as the aggregate capital-labor ratio of the economy. Thus, during the growth process the Cobb-Douglas closed economy's allocation shifts vertically from B to B' .¹⁶

3.3 Tax Revenues

Let us now compare the effect of a tax cut on government revenues, $R = \tau^K rK$, in the closed and open economy. Recall that the autarky and open economies

¹⁶The Cobb-Douglas functional forms (and the resulting unitary elasticities of substitution) are not completely harmless in this argument: in the closed economy, higher elasticities in the intermediate-good production function, for example, would deliver a lower increase in sectoral capital-labor intensities and some factor price insensitivity. However, estimated elasticities of substitution between capital and labor are not far from one. See, for example, Lucas (1969). In any case, the open economy would always experience a stronger reallocation process and more factor-price insensitivity than the closed economy unless the elasticities tend to infinity.

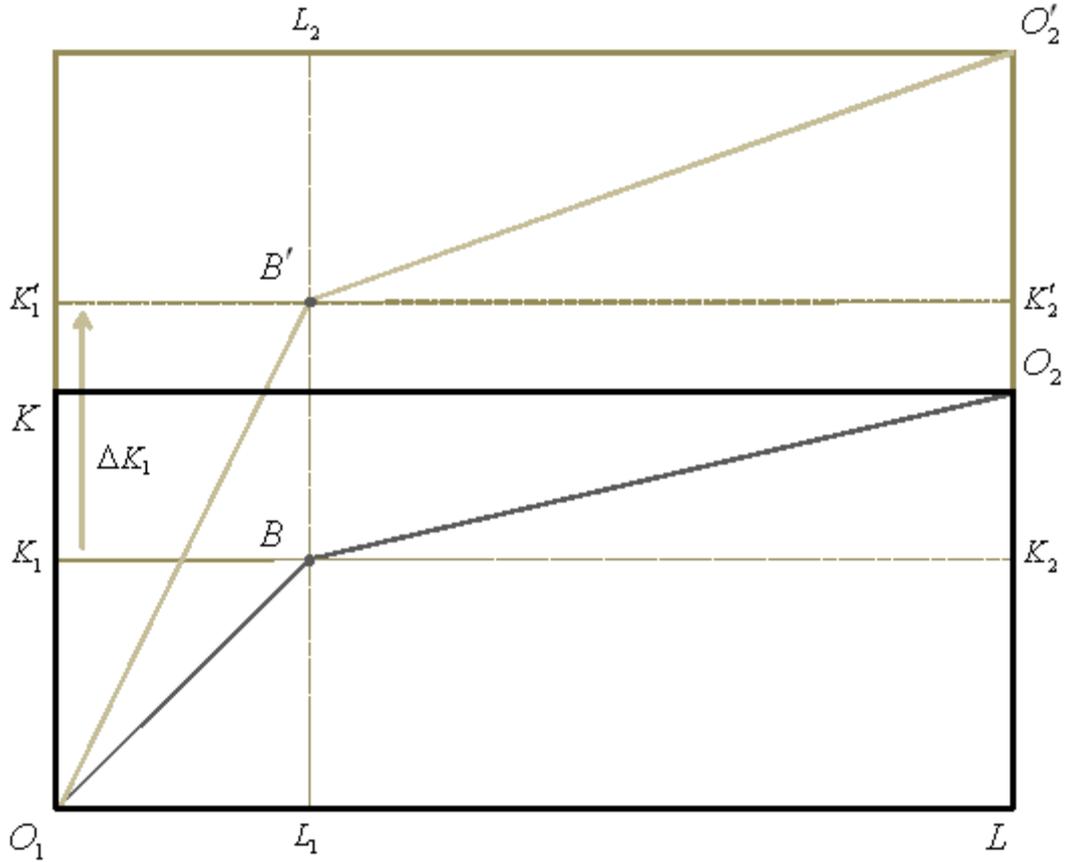


Figure 2: Capital accumulation and allocation in the closed economy: two goods.

have got the same R before the tax cut. Differentiating R_1 with respect to τ^K :

$$\frac{dR_1}{d\tau^K} = r_1 K_1 \left(1 + \tau^K \frac{dr_1/d\tau^K}{r_1} + \tau^K \frac{dK_1/d\tau^K}{K_1} \right), \quad (22)$$

or, in elasticities with respect to τ^K ,

$$\varepsilon_{R_1} = 1 + \varepsilon_{r_1} + \varepsilon_{K_1}. \quad (23)$$

Our results above on the responses of K_1 and r_1 to changes in τ^K imply that under free trade the tax cut is less costly for the government in terms of period-1 revenue than under autarky:

$$\varepsilon_{R_1}|_A - \varepsilon_{R_1}|_O = \tilde{\alpha} \varepsilon_{K_1}|_A - \varepsilon_{K_1}|_O > (\tilde{\alpha} - 1) \varepsilon_{K_1}|_A > 0. \quad (24)$$

This example suggests that openness and autarky display non-trivial quantitative differences in the effects of taxation.

4 Trade with Frictions

Although intuitive, the simple case above is based on a quite unrealistic scenario: free trade, and therefore FPE, are hampered by trade frictions. One other “technical” problem of the dynamic FPE model is that its infinite-horizon case is not that straightforward: the steady-state condition equalizing the after-tax return to capital and the rate of time preference may not hold for all countries if they have got different tax rates, or if their tax rates change. This is due to the before-tax return to capital being equal across countries. This makes steady-state comparisons of the kind Mankiw and Weinzierl (2006) and Trabandt and Uhlig (2006) perform impossible, unless the rate of time preference is assumed endogenous. To study the quantitative aspects of the issue more in detail, therefore, we turn to the trade-frictions scenario we discussed in Section 2. The intuitions of both models, with and without frictions, turn out to be quite similar: in comparison with the closed economy, the possibility to reallocate factor across sectors reduces the negative impact of capital accumulation on the return to capital in the trading economy even when factor price equalization does not apply.

Assume $\nu > 1$ and $T = \infty$. It is convenient to choose a different numeraire: $p_S(0) = 1$. In the appendix we show that in order to remain in a steady state with trade in which $K_N/L_N > K_S/L_S$, we need to impose that $(1 - \tau_N^K) \phi_N >$

$(1 - \tau_S^K) \phi_S$.¹⁷ This assumption, together with the condition that equalizes the steady-state after-tax real rates of return across countries,

$$(1 - \tau_N^K) \frac{r_N}{P_N} = (1 - \tau_S^K) \frac{r_S}{P_S}, \quad (25)$$

enables us to solve the equilibrium conditions for the steady state of the model numerically. We characterize both the autarky and trading equilibrium in order to compare the dynamic feedback from tax cuts for the two different regimes.

4.1 Calibration

To perform our quantitative exercise, we calibrate our trade model in terms of the US (the capital-abundant North) *vs.* the Rest of the World (the labor-abundant South). The basic parametrization is taken from Cuñat and Maffezzoli (2007): we set $\mu = 1$, $\beta = 0.96$, and $\delta = 0.048$. We normalize the size of the world labor endowment by setting $L_W \equiv L_N + L_S = 2$; according to data from Heston et al. (2006), roughly 5% of the global labor force is employed in the US economy: we therefore set $L_N = 0.05L_W$.

Anderson and van Wincoop (2004) show that trade costs represent a 170% *ad-valorem-tax-equivalent* trade barrier for a representative rich country. This number breaks down into a 55% of local trade costs and a 74% of international trade costs. Abstracting away from local distribution costs, we assign the value of ν to represent the *ad valorem* equivalent of international trade costs, *i.e.* we set $\nu = 1.74$.

¹⁷We are simply imposing the condition that, for identical capital-labor ratios in both countries, the after-tax marginal productivity of capital be larger in country N . If, for example, $\beta_N = \beta_S$, $\phi_N = \phi_S$, and $\tau_N^K = \tau_S^K$, both countries would have the same capital-labor ratio in steady state, and there would be no trade. Note that we introduce cross-country differences in TFP levels only to guarantee the existence of international trade in steady state: the actual trade flows are generated by the induced differences in relative factor endowments. Hence, if TFP levels were equal across countries, trade could nonetheless emerge during converge towards the steady state (assuming countries with different initial conditions). A large literature on cross-country comparisons of TFP levels, summarized in Caselli (2005), provides empirical evidence supporting the existence of international differences in TFP levels.

The function $\alpha(z)$ is a key ingredient in our model. Given the Cobb-Douglas production functions for intermediate goods, $\alpha(z)$ should be directly related to the capital shares in value added at the sectorial level. Taking advantage of the *Gross Domestic Product by Industry* published by the US Bureau of Economic Analysis, we collect data on Value Added (VA), Compensation of Employees (COMP), Proprietors' Income (PROINC), Proprietors' Income Inventory Valuation Adjustment (PROIVA), Full-time Equivalent Employees (FTE), and Persons Engaged in Production (PEP), for 56 US sectors, defined according to the SIC87 classification, over the 1987-97 period.¹⁸ These data allow us to compute the labor share in value added at the sectorial level. We follow the two most common approaches in the literature to account for the labor income of self employed workers.¹⁹ The first approach assigns the average wage perceived by employees to self-employed workers, and therefore our first estimate of the labor share is computed as

$$s_N = \frac{\frac{COMP}{FTE} PEP}{VA}. \quad (26)$$

The second approach recognizes that the main problem is the apportionment of proprietors' income, which has components of both labor and capital income, since it mainly represents income of self-employed individuals. We assume that proprietors income, net of inventory valuation adjustment, should be allocated to labor and capital in the same proportions they represent in the remainder of the economy; hence,

$$s_N VA = COMP + s_N (PROINC + PROIVA). \quad (27)$$

¹⁸We drop the government sector and the housing sector, because by construction they include respectively only labor and capital income. See Gomme and Rupert (2004) for a discussion. Furthermore, and for similar reasons, we drop Educational services, Social services, Private households, and Membership organizations. See the appendix for a full list of the sectors included.

¹⁹See Gomme and Rupert (2004) for a recent discussion of the issues at stake, and Cooley and Prescott (1995) for a classical reference.

In other words, our second estimate of the labor share is computed as

$$s_N = \frac{COMP}{VA - PROINC - PROIVA}. \quad (28)$$

These two estimates turn out to be highly positively correlated (with a coefficient around 0.96); however, some relevant differences, in particular for labor-intensive sectors, remain. Since both are rough approximations of the true labor share, and both probably capture some distinct aspects of reality, we take the average of these two alternative estimates as our benchmark distribution. The capital share in value added is simply computed as one minus the labor share. Finally, we order the sectors according to their capital share and get the desired monotonically increasing cross-sector distribution of capital intensity. We approximate the latter with an algebraic polynomial of order 6, fitted using ordinary least squares.

In a closed-economy environment, this would be the end of the story; however, under trade, there is still a further important step. In our numerical experiment, the North, *i.e.* the US economy, is assumed to be the capital-abundant country. Hence, the distribution of capital shares actually observed in the US should correspond to the right-hand tail of the true distribution, *i.e.* the $[z_N, 1]$ interval in our notation. In other words, by focusing on the US sectorial data, we may get an estimate of the highest capital intensity, but not, under trade, an estimate of the lowest one. To bypass this problem, we use the previously fitted polynomial to extrapolate on the left-hand side of the distribution until we hit the horizontal axis, assuming implicitly that the lowest possible capital share is zero. Finally, the domain of this “extended” distribution is mapped into the $[0, 1]$ interval. Figure 3 plots the actual US distribution and the fitted polynomial.²⁰ The fitted polynomial is then used in our simulations.

²⁰In autarky, these values imply an aggregate capital share equal to 0.34, which is close to the 0.33 used in Mankiw and Weinzierl (2006) and the 0.36 used in Trabandt and Uhlig (2006). In the trading equilibrium, these values—together with the calibrated values of the productivity parameters—imply a capital share of 0.37 in the North and 0.33 in the South. Furthermore, the steady-state value of z_N in our model economy reaches 0.037, a value almost identical to its empirical counterpart, as obtained in our calibration procedure (see Figure 3), equal to 0.034.

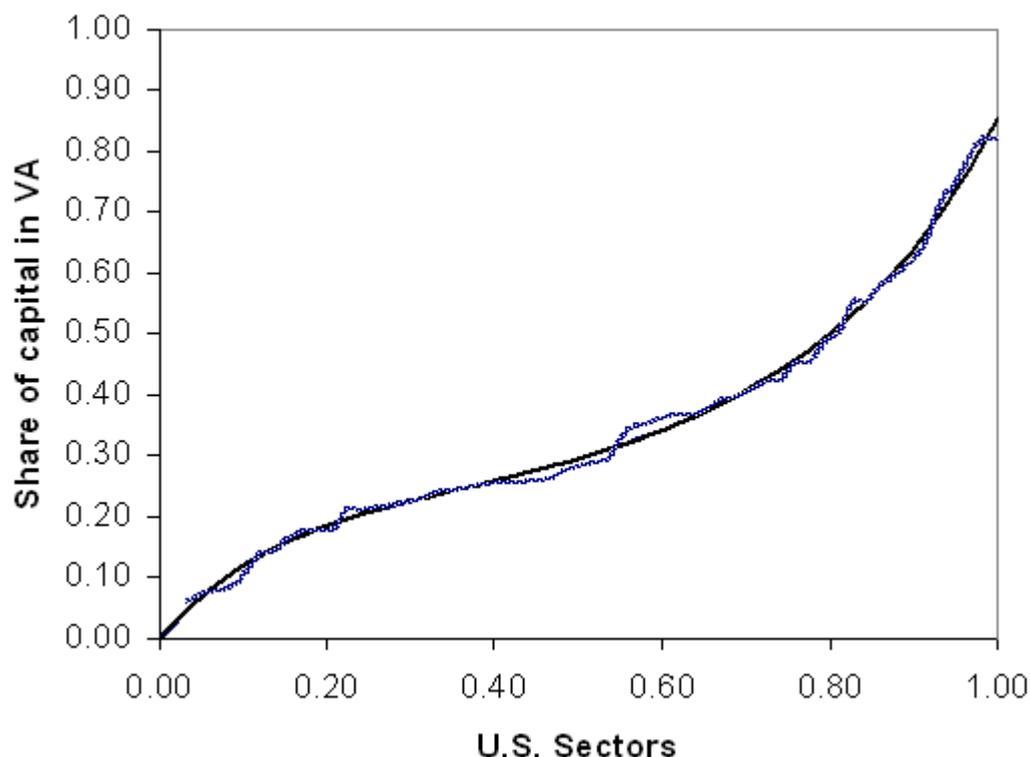


Figure 3: The sectorial distribution of capital shares in VA in the US (1987-1997).

Carey and Rabesona (2002) compute average effective tax rates on factors of production and consumption for 25 OECD countries, extending the Mendoza et al. (1994) methodology: from their Table A2, p. 172, we take the tax rates on capital (based on gross operating surplus) and labor for the 1990-2000 period.²¹ We set the tax rates in the North to reproduce the observed US rates, *i.e.* $\tau_N^K = 27.3\%$ and $\tau_N^L = 23.4\%$; to pin down the tax rates in the South, instead, we

²¹As Carey and Rabesona (2002) point out, the fiscal treatment of depreciation allowances is different across countries, making tax rates based on net operating surplus difficult to compare across countries. Furthermore, our model does not incorporate depreciation allowances explicitly, for the sake of simplicity. Hence, we focus on tax rates based on gross operating surplus.

compute weighted averages of the tax rates on capital and labor for the remaining countries, using the real GDP-PPP levels reported by Heston et al. (2006) for the 1990-2000 period as weights: the resulting values are $\tau_S^K = 28.0\%$ and $\tau_S^L = 30.5\%$.²²

We are left with the country-specific productivity parameters, ϕ_j ; to pin their values down, we (i) normalize the world capital stock setting $K_W \equiv K_N + K_S = 2$, and (ii) calibrate the model to reproduce the observed ratio between the capital-labor ratio in the US and the capital-labor ratio in a Rest-of-the-World aggregate, averaged over the 1990-2000 period, equal to 4.9.²³ The implied values are $\phi_N = 0.778$ and $\phi_S = 0.306$. As already noted before, $(1 - \tau_N^K) \phi_N > (1 - \tau_N^K) \phi_S$ implies $K_N/L_N > K_S/L_S$, so that trade may arise in steady state. The trade share in income (imports plus exports over GDP) generated by our benchmark calibration reaches 7.3%, which is far below the actual overall trade share of the US (21% on average over the 1990-2000 period), but near the US share in income of trade with developing countries (8.7%),²⁴ a group of countries for which US trade is likely to be explained to a great extent by differences in relative factor endowments. The fact that our model generates less trade than the observed is

²²We found no reliable data source for tax rates outside the OECD. Note that our results do not stem from differences in fiscal policy across countries: they do not change qualitatively - and even quantitatively only slightly - if we use the same tax structure in both countries for our model.

²³We collect data from Heston et al. (2006) for 140 countries over the 1950-2003 period on population (pop), real GDP per capita (rgdpl and rgdpch-9, real GDP per worker (rgdpwok), and real investment as a share of GDP (ki). Strictly following Caselli (2005) for consistency with the existing literature, we construct estimates for the net physical capital stock using the Perpetual Inventory Method; we assume infinite service lives and a constant geometric depreciation rate equal to 6% for all countries (please note that this common depreciation rate differs from the one used in our calibration, 4.8%, which is specific to the US and is originally taken from Cooley and Prescott (1995)). The capital-labor ratio is computed as the ratio between our estimate of the capital stock and the labor force. Finally, the RoW aggregate is just computed as the total capital stock in the world, but for the US, over the total labor force, again excluding the US. To check the robustness of these results, we produced alternative estimates assuming fixed expected service lives (20 years), simultaneous exit mortality patterns, and linear depreciation, as in Maffezzoli (2006): the outcomes are almost identical.

²⁴We collect data from the UNCTAD Handbook of Statistics - *International merchandise trade by region* on US trade with developing countries over the 1990-2000 time period. A detailed list of the countries involved can be found on www.unctad.org.

not so surprising, as we ignore Ricardian comparative advantage and “New-Trade Theory” features such as product differentiation and scale economies.

The recursive structure of our problem guarantees that the solution can be represented as a pair of time-invariant policy functions expressing the optimal level of consumption in each country as a function of the two state variables, K_N and K_S . These policy functions have to satisfy the following functional equations:

$$\beta C_j (K'_N, K'_S)^{-\frac{1}{\mu}} \left[(1 - \tau_j^K) \frac{r'_j}{P'_j} + 1 - \delta \right] = C_j (K_N, K_S)^{-\frac{1}{\mu}}, \quad (29)$$

where:

$$K'_j = (1 - \tau_j^L) \frac{w_j}{P_j} L_j + \left[(1 - \tau_j^K) \frac{r_j}{P_j} + 1 - \delta \right] K_j + R_j - C_j (K_N, K_S). \quad (30)$$

Factor prices w_j/P_j and r_j/P_j are obtained by numerically solving the appropriate equilibrium conditions. To solve equations (29) numerically, we apply the Orthogonal Collocation projection method described in Judd (1992).

4.2 Results

4.2.1 Dynamic Scoring

This section studies the dynamic effects of an unexpected and permanent one-percentage-point reduction in the tax rate on capital income in the North, which in our experiment has been calibrated to reproduce the US economy. Figure 4 summarizes the impulse response of the main macroeconomic variables to such a tax cut. We plot income, capital, consumption, together with the trade share in income, under both trade and autarky for comparison purposes. All variables are expressed in terms of the final good and as percent deviations from their initial steady-state values. The left-hand side panels report results for the North, while the right-hand side panels report the corresponding results for the South.

A capital-income tax cut in the North is beneficial in terms of higher income and consumption in steady state, under both autarky and trade. Notice, however,

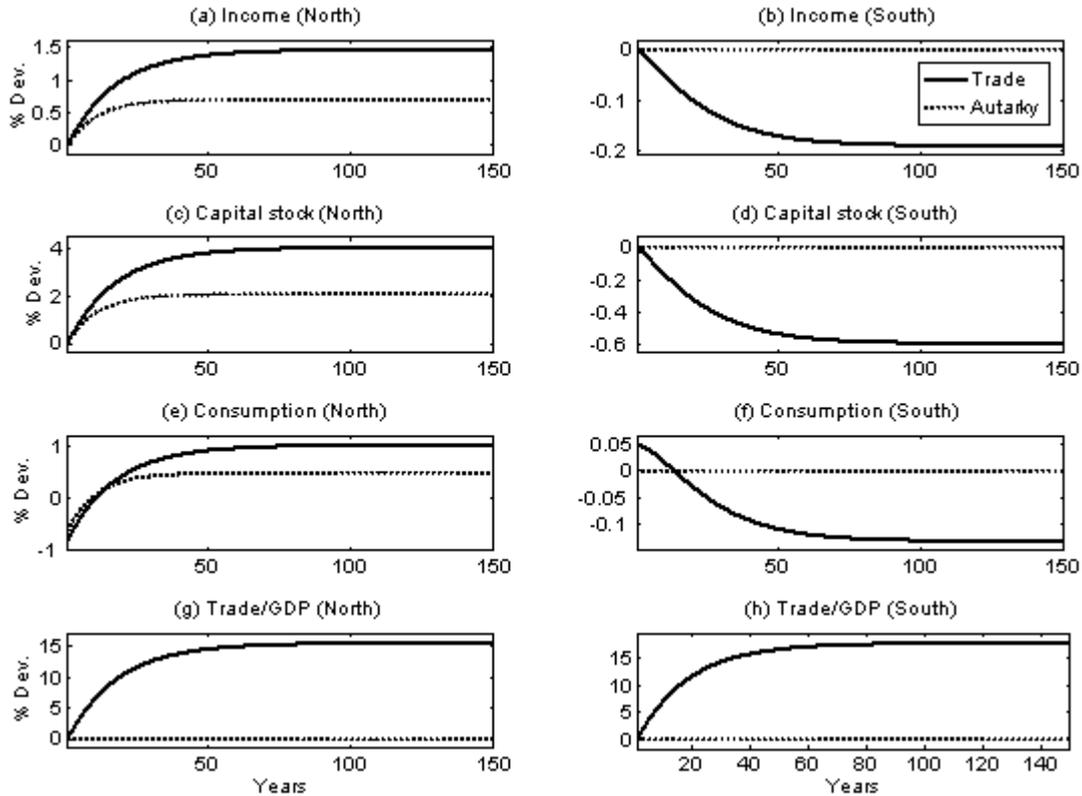


Figure 4: The effects of a capital tax cut in the North: main aggregate variables.

that from a quantitative point of view the long-run effect under trade is more than twice larger than under autarky. As already noted, this is due to the different ways the factor allocation mechanisms work under the two regimes. A reduction in τ_N^K raises the after-tax return to capital in the North, creating an incentive to accumulate capital. Under autarky, capital accumulation implies higher sectorial capital-labor intensities: given diminishing marginal returns, this reduces the return to capital and the incentive to accumulate. In the trade model with

transport costs, instead, an increase in K_N/L_N leads to an increase in z_N .²⁵ This enables the North to accommodate part of the increase in its aggregate capital-labor ratio not through a rise in sectorial capital-labor intensities, $k(z)/l(z)$, but by reshuffling resources from industries with low relative demand for capital (over labor) towards industries with high relative demand for capital. This enables the open economy to accumulate more capital, since the negative effect of capital accumulation on the gross return to capital is much smaller than under autarky.²⁶

In autarky, the South remains completely unaffected by tax cuts in the North. Under trade, however, factor prices in the South are influenced by the North's tax cut: the resulting increase in K_N/L_N not only raises z_N , but also reduces z_S . The South therefore reallocates factors from its most capital intensive industries to more labor intensive industries. This brings about a reduction in the South's return to capital: while the North accumulates capital, the opposite takes place in the South. The latter starts to eat its capital stock, and ends up in a steady state with lower capital, output, and consumption. This process further enhances its comparative advantage in labor intensive goods, and spurs an increase in international trade. These results suggest that fiscal policy decisions may have some spillover effects via international trade.

Figure 5 summarizes the dynamic response of government balances along the transitional path. Panels (a) and (b) (as before, North on the left-hand side and South on the right-hand side) plot the adjustment path for government revenues, while panels (c) and (d) report the share of capital taxes in government revenues;

²⁵The change in z_N is proportional to the change in the North's trade share: the value of North's imports is

$$\int_0^{z_N} p_N(z) x_N(z) dz = z_N P_N Y_N.$$

Thus, the North's trade share is $2z_N$.

²⁶Note that it is not trade *per se* that amplifies the dynamic effects of the tax cut, but the sectorial reallocation of capital induced by international trade. In a numerical experiment not reported in the paper (available upon request), we show that if we keep the specialization patterns constant at the initial steady state, the dynamic effects under trade and autarky are very similar.

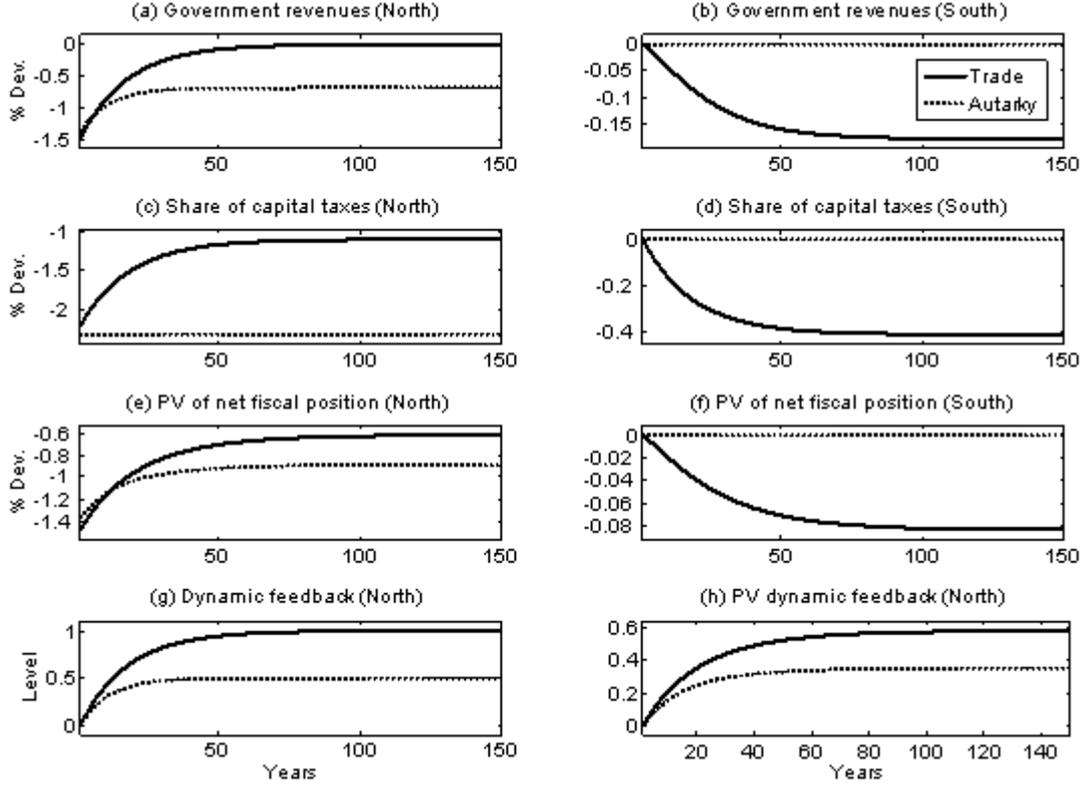


Figure 5: The effects of a capital tax cut in the North: government balances.

both variables are expressed in percentage deviation from their initial steady-state values. Panels (e) and (f) show the present-value *net fiscal position* of the government at different time horizons, defined as

$$\frac{\sum_{s=0}^t \rho_{js} \hat{\Delta} R_{js}}{\sum_{s=0}^t \rho_{j,-1}^s R_{j,-1}}$$

where

$$\rho_{jt} = \prod_{s=0}^t \left[(1 - \tau_j^K) \frac{r_{js}}{P_{js}} + 1 - \delta \right] \quad (31)$$

is the discount factor along the transitional path and $\hat{\Delta}R_{js} \equiv R_{js} - R_{j,-1}$, where $R_{j,-1}$ represents the total tax revenues prevailing before the tax cut. This variable represents the amount of resources the government should borrow or lend, in terms of the present discounted value of its initial revenue plan $R_{j,-1}$, to keep the level of its revenues at the same level as before the tax-cut. If its value is positive, then the tax cut pays for itself as the government could lend some of its revenues and still keep its ‘expenditure’ at their original level.

Panel (g) plots the *dynamic feedback*, which measures the extent to which a tax cut is self-financing in levels over time. Let us define the *static effect* of a tax cut as the revenue loss induced by the tax cut under the assumption that none of the variables adjusts: hence, the static loss always equals the change in the tax rate times the initial tax base. The share of the static effect which is dynamically offset by factor accumulation can be calculated as $(\hat{\Delta}R_{jt} - \Delta\bar{R}_j) / |\Delta\bar{R}_j|$, where $\Delta\bar{R}_j$ denotes the static effect. If the tax cut is more than self-financing, then the change in government revenues is positive and the dynamic feedback is larger than one; if the tax cut is only partially self-financing, then the change in government revenues is negative but larger (smaller in absolute value) than the static effect and the dynamic feedback lies between zero and one. Panel (h) plots the *present-value dynamic feedback*, which represents the extent to which a tax cut is self financing in present discounted values and is computed as

$$\frac{\sum_{s=0}^t \rho_{js} \hat{\Delta}R_{js} - \sum_{s=0}^t \rho_{j,-1}^s \Delta\bar{R}_j}{\sum_{s=0}^t \rho_{j,-1}^s |\Delta\bar{R}_j|}, \quad (32)$$

The value of the present value feedback has the same interpretation as the dynamic feedback: if the tax cut is self-financing, then it is larger than one; if the tax cut is partially self-financing, then it is between zero and one.

Table 1 summarizes the dynamic response of government revenues and the present-value net fiscal position, for the North and the South, and under autarky and trade. The upper part of the table reports the elasticities of both variables

Time	Gov. revenues			PV net fiscal position		
	Autarky	Trade		Autarky	Trade	
		North	South		North	South
	<i>Elasticities</i>					
Impact	-0.37	-0.40	0.00	-0.37	-0.40	0.00
5	-0.31	-0.32	-0.00	-0.34	-0.36	-0.00
10	-0.26	-0.24	-0.01	-0.31	-0.32	-0.01
25	-0.20	-0.10	-0.03	-0.27	-0.24	-0.01
∞	-0.18	0.00	-0.05	-0.24	-0.17	-0.02
	<i>Dyn. feedback</i>			<i>PV dyn. feedback</i>		
Impact	0.00	0.00	-	0.00	0.00	-
5	0.17	0.21	-	0.09	0.11	-
10	0.30	0.42	-	0.16	0.21	-
25	0.46	0.76	-	0.28	0.40	-
∞	0.51	1.01	-	0.36	0.59	-

Table 1: Dynamic feedbacks after a capital tax cut in the North

with respect to changes in τ_N^K :

$$\varepsilon_{R,t} = \frac{\hat{\Delta}R_{jt} \tau_N^K}{\Delta\tau_N^K R_{j,-1}}, \quad (33)$$

$$\varepsilon_{\rho R,t} = \frac{\sum_{s=0}^t \rho_{js} \hat{\Delta}R_{js} \tau_N^K}{\Delta\tau_N^K \sum_{s=0}^t \rho_{j,-1}^s R_{j,-1}}. \quad (34)$$

The lower part reports the dynamic feedbacks. The two first columns report the North's dynamic feedback $(\hat{\Delta}R_{jt} - \Delta\bar{R}_j) / |\Delta\bar{R}_j|$ under autarky and trade, and the third and fourth columns report the North's present-value dynamic feedback, again under autarky and trade.

On impact, the tax cut affects the North's government revenues negatively under both autarky and trade. Actually, this negative impact turns out to be slightly larger under trade, since the elasticity on impact equals -0.37 under autarky and -0.40 under trade: this is a direct consequence of the North's higher steady-state capital-labor ratio in the trading equilibrium. In the South, the tax

cut that took place in the other country has no effect on impact, but under trade it has a significantly negative and permanent effect on government revenues in the long run. In both countries, these effects stem from the different adjustment paths for capital: enhanced capital accumulation in the North, the reverse process in the South. Note that this mechanism explains why in the North the actual decrease in government revenues in the long run is definitely smaller under trade than under autarky: in the former case, government revenues almost converge back to the initial steady-state value.

The role of capital accumulation is clearly reflected in Figure 5, panels (c) and (d): the share of capital taxes in total tax revenues increases steadily in the North and decreases in the South. Panels (e) and (f) show that, if we focus on the net financial position, the cost of the tax cut in terms of revenue to the North's government is much smaller under trade, while exactly the opposite happens in the South. Finally, panels (g) and (h), and the lower part of Table 1, summarize these results in terms of dynamic feedbacks: under autarky, the dynamic feedback in the North converges in the long run to 51%, a value in line with the findings of Mankiw and Weinzierl (2006); under trade, the long-run value of the corresponding dynamic feedback converges to 101%. This implies that in the long run a capital tax cut does not decrease government revenues in the North; on the contrary, it actually improves them slightly. Of course, given that this effect relies on capital accumulation and therefore needs time to build up, the results are less dramatic, but still relevant, if we turn our attention to the present-value dynamic feedback.

4.2.2 Dynamic Feedbacks in the Long Run

The results above imply that the long-run dynamic feedback under trade is larger than its counterpart under autarky. This seems in line with the analytical predictions outlined in Section 3. However, our analytical example was based on two simplifying assumptions: we ignored labor taxation, setting $\tau^L = 0$, and we assumed the same initial capital stock in both the closed and open economies.

These two assumptions are blatantly violated in the simulation exercise presented above: labor taxes are set to a positive value and, in general, steady-state capital stocks are different across trade regimes (even under the same parameterization). In order to evaluate the generality of our conclusions, let us focus on the steady state and consider a generalized version of equation (22) to discuss how long-run tax revenues react to changes in capital taxation in detail:²⁷

$$\frac{dR}{d\tau^K} = \underbrace{\left(\tilde{r} + \tau^K \frac{d\tilde{r}}{d\tau^K} \right) K}_{>0} + \underbrace{\tau^K \tilde{r} \frac{dK}{d\tau^K}}_{<0} + \underbrace{\tau^L \frac{d\tilde{w}}{d\tau^K} L}_{<0}, \quad (35)$$

where $\tilde{r} \equiv r/P$ and $\tilde{w} \equiv w/P$ represent real factor prices. In steady state, the real rates of return are pinned down by the Euler equations (see also equation 25):

$$\tilde{r} = \frac{1 - \beta(1 - \delta)}{\beta(1 - \tau^K)}. \quad (36)$$

Therefore, in steady state both the real rate of return \tilde{r} and its derivative $d\tilde{r}/d\tau^K$ are positive, and remain invariant across trade regimes. The signs of the three terms on the right-hand side of equation (35) are unambiguous. The real rate of return, the tax rate on capital, the derivative of the rate of return, and the capital stock are all strictly positive. Therefore the first term is positive. The second term is negative because the capital stock reacts inversely to changes in the capital tax rate. Finally, the third term is negative too, since the decrease in the capital stock caused by an increase in the tax rate on capital will depress the real wage rate.²⁸

The dynamic feedback (denoted DF hereafter), if computed for *marginal* decreases in τ^K , can then be written as

$$DF \equiv 1 - \frac{\frac{dR}{d\tau^K}}{\tilde{r}K} = - \left(\varepsilon_{\tilde{r}} + \varepsilon_K + \varepsilon_{\tilde{w}} \frac{\tau^L \tilde{w} L}{\tau^K \tilde{r} K} \right). \quad (37)$$

²⁷Country-specific indexes have been omitted for simplicity.

²⁸Note that if $\tau^L = 0$, and therefore $R = \tau^K \tilde{r}K$, then equation (35) can be easily rewritten in elasticity terms as $\varepsilon_R = 1 + \varepsilon_{\tilde{r}} + \varepsilon_K$, which reminds us of equation (23).

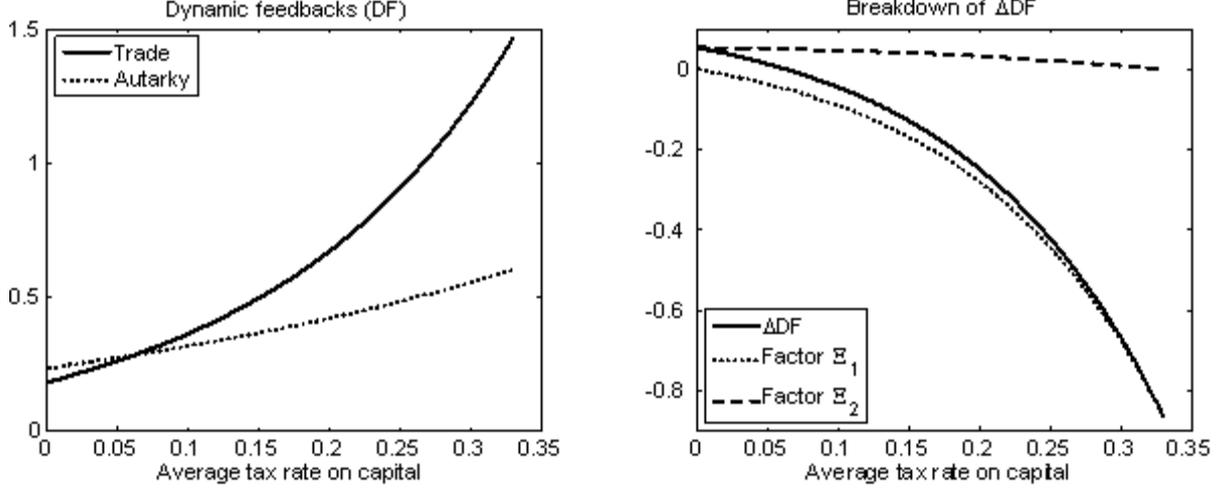


Figure 6: Comparison of long-run dynamic feedbacks across trade regimes.

Since $\varepsilon_{\tilde{r}}$ is invariant across trade regimes, the difference between the dynamic feedbacks (again, for marginal changes in τ^K) under autarky and trade can therefore be expressed as the sum of two factors, denoted Ξ_1 and Ξ_2 :

$$\Delta DF \equiv DF|_A - DF|_O = \underbrace{(\varepsilon_K|_O - \varepsilon_K|_A)}_{\Xi_1 < 0} + \underbrace{\frac{\tau^L L}{\tau^K \tilde{r}} \left[\left(\varepsilon_{\tilde{w}} \frac{\tilde{w}}{K} \right) \Big|_O - \left(\varepsilon_{\tilde{w}} \frac{\tilde{w}}{K} \right) \Big|_A \right]}_{\Xi_2 > 0} \quad (38)$$

The left-hand side panel of Figure 6 compares the North's dynamic feedback for marginal changes in τ_N^K under autarky and trade, as expressed in equation (37) and computed for different initial values of τ_N^K .²⁹ The right-hand side panel reports the breakdown of the difference between dynamics feedbacks into the components described in equation (38). Note that the dynamic feedback under trade is larger than its counterpart under autarky for tax rates on capital above 7%, and becomes larger than one for tax rates above 27%. However, the dynamic feedback under autarky dominates for tax rates below 7%. The breakdown re-

²⁹ We allow for changes in τ_N^K , leaving the rest of parameters unchanged. Notice that different values of τ_N^K lead to different steady-state outcomes within and across trade regimes.

port in the right-hand-side panel shows that the two factors described in equation (38), Ξ_1 and Ξ_2 , have different signs. The first factor, Ξ_1 , is always negative, increases with τ_N^K , and converges to zero when the tax rate does so: this implies that the elasticity of steady-state capital with respect to changes in τ_N^K is always greater under trade than under autarky. The second factor, Ξ_2 , is always strictly positive, and decreases with τ_N^K . For sufficiently low values of τ_N^K the second factor dominates, and the dynamic feedback under autarky exceeds its counterpart under trade.³⁰

The fact that the dynamic feedback under autarky is larger than under trade for some values of τ_N^K does not contradict the intuitions we discussed above. Recall that in our discussion of the free-trade example we fixed parameter values so that the economy had the same steady-state outcomes (capital stocks, factor prices, etc.) under both autarky and trade, and no taxes on labor. This is not the case here, as the autarky and trade regimes yield different steady-state outcomes for the same value of τ_N^K . In this sense, the comparison here is not “perfect:” we are not comparing the effects of a tax cut in two economies that are identical but for the trade regimes they are subject to.³¹

4.2.3 The Laffer Curve

Dynamic scoring studies how tax cuts affect government revenues in the long run and along the transitional path to the new steady state. A closely related approach, typically represented by the Laffer curve, studies the relationship be-

³⁰The second factor, Ξ_2 , turns out to be strictly positive because the tax rate on labor τ^L is strictly positive in our benchmark calibration, and because the steady-state capital stocks differ across trade regimes as long as countries trade in equilibrium.

³¹Actually, a counterfactual experiment in which countries *are* identical but for the trade regime can be devised. The experiment (results are available from the authors upon request) runs as follows: for each value of τ_N^K , compute the minimum value of the trade cost v that makes the open economy converge to the autarky one, *i.e.* the trade cost that makes international trade not feasible. This forces the two economies to be identical, in terms of allocations, at the initial steady state, and consequently yields $\varepsilon_{\bar{w}}$ practically equal under both trade regimes. Then perform the simulations described in the text: the results confirm that $\Delta DF = \varepsilon_K|_O - \varepsilon_K|_A < 0$ for all values of τ_N^K .

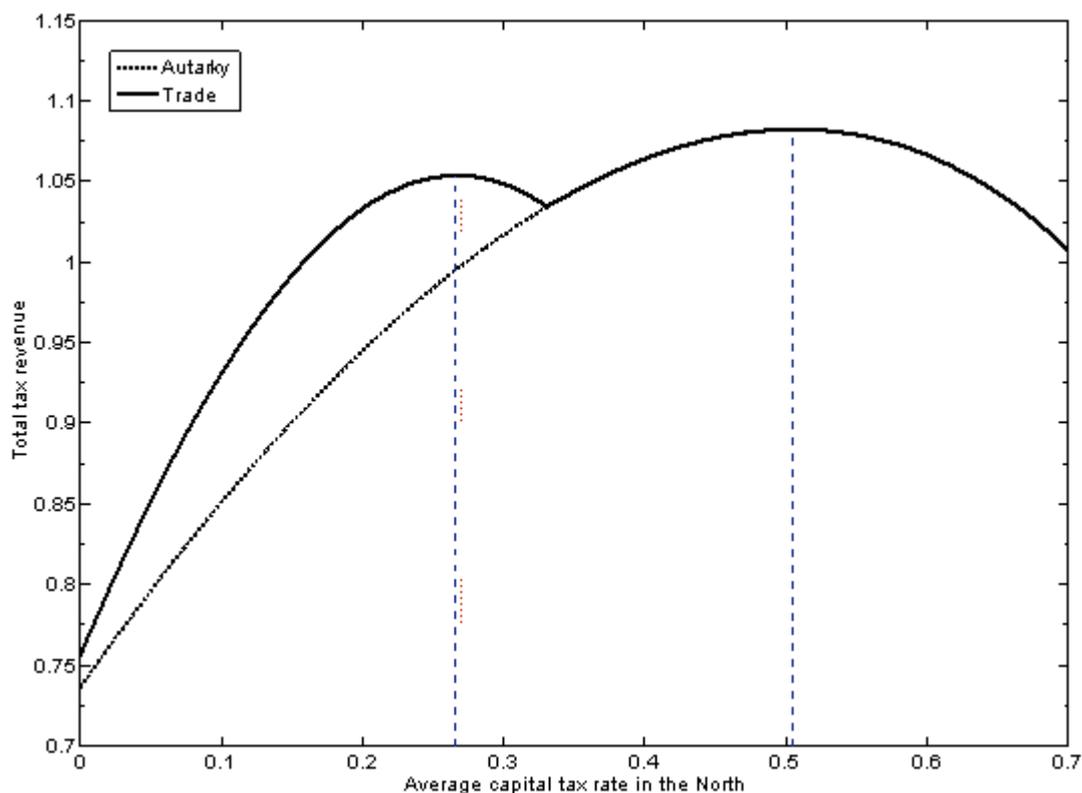


Figure 7: The Laffer curve in the North.

tween steady-state total tax revenues and different tax rates on capital and labor. Figure 7 plots the Laffer curve in the North resulting from our model: it plots steady-state total tax revenues as a function of the average tax rate on capital, *ceteris paribus*.³²

The Laffer curve under trade lies always above, or at least corresponds to, the Laffer curve under autarky. This is of course a direct consequence of specialization: the North will specialize in the production of capital-intensive goods, and this will induce further capital accumulation and therefore generate a higher capital stock in steady state. The steady-state return to capital has to be the same

³²As in the previous Section, we allow for changes in τ^K , leaving the rest of parameters unchanged.

under both trade and autarky, and therefore the overall revenues from capital taxes will be higher with trade. Furthermore, even tax returns from labor taxes will be higher under the trade regime: the labor supply is fixed, but wages will be higher due to the higher capital stock and openness. Hence, total tax revenues have to be higher under trade.

If the tax rate on capital is higher than 33.1%, then - *ceteris paribus* - the trade equilibrium “collapses” into autarky. As the North’s tax rate on capital income rises, the North’s steady-state capital-labor ratio decreases relative to the that of the South. Thus, for a high enough τ_N^K transport costs make trade not profitable. Hence, for higher tax rates the Laffer curves under autarky and trade coincide. As a result, the Laffer curve becomes twin-peaked: the slope is positive initially, becomes negative, turns suddenly positive again and then finally turns negative.

The three dotted vertical lines denote - from left to right - the revenue maximizing tax rate under trade (26.7%), the benchmark tax rate used for calibration (27.3%), and the revenue maximizing tax rate under autarky (50.7%). Note that taking the consequences of trade into account has strong implications as far as fiscal policy is concerned: under trade, the actual marginal tax rate in the US, as measured by Carey and Rabesona (2002), turns out to be slightly larger than the revenue-maximizing rate, and therefore reaches the “slippery slope” of the curve. Under autarky, instead, the actual tax rate remains quite far from the peak.

4.2.4 The Slope of the Laffer Curve

A close look at Figure 7 reveals that the Laffer curve under trade is steeper than its counterpart under autarky for sufficiently low levels of capital taxation. This may appear to contradict our theoretical argument. Once again, this is due to the fact that we are not comparing the effects of a tax cut in two economies that are identical but for the trade regimes they are subject to.

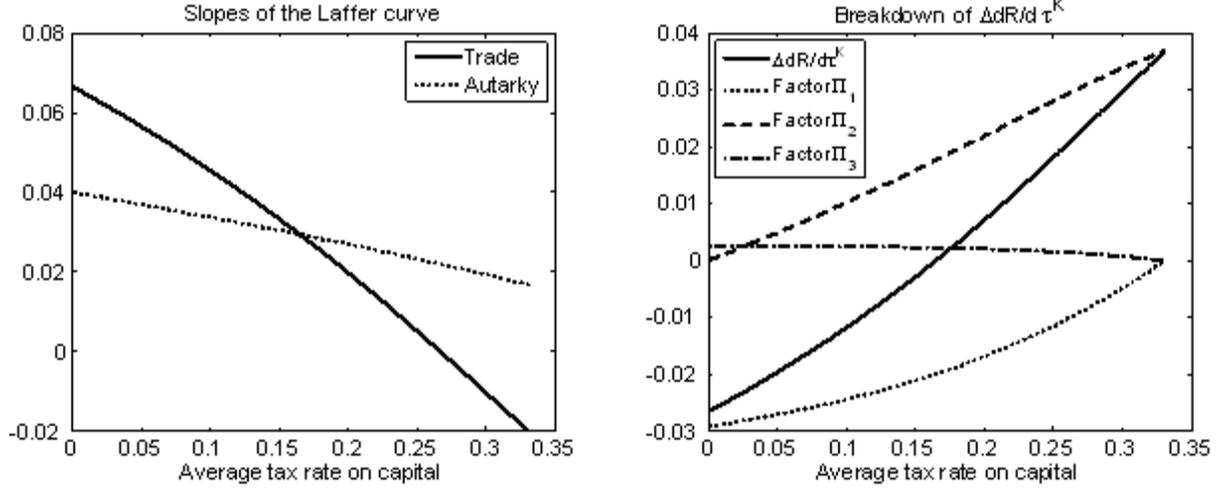


Figure 8: Comparison of the slope of the Laffer curve across trade regimes.

The difference between the slope of the Laffer curve under autarky and its slope under trade can be decomposed in the following way:

$$\begin{aligned} \Delta \frac{dR_N}{d\tau_N^K} &\equiv \frac{dR_N}{d\tau_N^K} \Big|_A - \frac{dR_N}{d\tau_N^K} \Big|_O = \\ &\underbrace{\left(\tilde{r} + \tau_N^K \frac{d\tilde{r}}{d\tau_N^K} \right) (K_N|_A - K_N|_O)}_{\Pi_1 < 0} + \underbrace{\tau_N^K \tilde{r} \left(\frac{dK_N}{d\tau_N^K} \Big|_A - \frac{dK_N}{d\tau_N^K} \Big|_O \right)}_{\Pi_2 > 0} + \\ &\underbrace{\tau_N^L L_N \left(\frac{d\tilde{w}_N}{d\tau_N^K} \Big|_A - \frac{d\tilde{w}_N}{d\tau_N^K} \Big|_O \right)}_{\Pi_3 > 0}. \quad (39) \end{aligned}$$

The first term on the right-hand side of equation (39), denoted Π_1 , takes the role of the initial capital stocks into account, and has a negative sign, since in steady state $K_N|_A < K_N|_O$ for any tax rate on capital (the North will always have an incentive to accumulate more capital under the trade regime than under autarky). The second term, Π_2 , focuses instead on the role of the adjustment dynamics: the theoretical argument described in the previous Sections suggests

that the derivative of the capital stock with respect to the tax rate on capital should always be greater (in absolute value) under trade than under autarky, since capital will be reallocated from labor-intensive to capital-intensive sectors; this implies that the second term should have a positive sign. Finally, the same argument also explains why the derivative of the real wage rate should always be larger (again, in absolute value) under trade, and therefore suggests that the last term, Π_3 , should have a positive sign. The difference in the slope of the Laffer curve across trade regimes will depend on the relative sizes of these three components.

The left panel of Figure 8 plots the slopes of the North's Laffer curves under trade and autarky for different levels of τ_N^K . The right panel plots the difference between the slopes of the North's Laffer curve under autarky and trade, *i.e.* the left hand side of equation (39), and each of the three terms in the right hand side of the same equation. These results confirm that for sufficiently low capital tax rates (approximately below 17%) the Laffer curve under trade is steeper than its autarky counterpart: this is because for this range of values the Π_1 component, linked to the difference between the initial capital stocks, dominates. At low levels of capital taxation, the North's steady-state capital stock is much larger under trade than under autarky. Thus, for low τ_N^K , reductions in τ_N^K yield larger reductions in government revenue under trade than autarky because the subsequent increases in revenue through increases in the tax base do not compensate for the revenue losses arising from taxing the original tax base at a lower rate.

Note that Π_1 increases monotonically with τ_N^K and converges to zero (from below) as soon as the capital tax rate is high enough to make the open economy collapse into autarky. Π_2 is monotonically increasing in τ_N^K , too, but remains always positive. Finally, Π_3 , decreases monotonically with τ_N^K and converges to zero. For values of τ_N^K above the 17% threshold, the positive components start to dominate, and the Laffer curve under autarky becomes steeper than its counterpart under trade.

Parameter value	Trade share	Dynamic feedbacks in steady state		Revenue max. tax rate
		Gov. rev.	Net. fiscal pos. (PV)	
<i>Distribution of capital shares, $\alpha(z)$</i>				
No extrapolation	5.5%	1.02	0.59	26.4%
Benchmark	7.3%	1.01	0.59	26.7%
Two-sided extrap.	11.6%	1.10	0.60	25.5%
<i>Trade cost, ν</i>				
1.70	10.7%	0.98	0.57	27.2%
1.74	7.3%	1.01	0.59	26.7%
1.79	3.3%	1.05	0.61	26.1%

Table 2: Sensitivity analysis: capital intensities and trade costs.

5 Sensitivity Analysis

5.1 The Distribution of Capital Shares

Our calibration procedure parameterizes the $\alpha(z)$ distribution using empirical evidence on the US sectorial structure and taking the implications of our theoretical trade model literally. Two alternative procedures, based on the same data, may seem natural. The first one consists in fitting our polynomial on the actual US distribution of capital intensities without extrapolating the left-hand tail. This is more conservative from an empirical perspective, but not so faithful to our theory, as we attribute the sectorial shares of the interval $(z_N, 1]$ to the whole interval $[0, 1]$. Our second alternative takes our argument to the extreme, and extrapolates the capital intensity distribution not only on the left-hand side, but also on the right-hand one, in order to span the full range of possible capital intensities $[0, 1]$. This second approach could be justified by noting that the level of disaggregation of the available sectorial data is quite coarse, and some of the highest (and lowest) capital shares could have simply been hidden by the aggregation process.

The first panel of Table 2 summarizes the main results for the three parameterizations discussed above. Notice that the cross-sector dispersion of capital intensities has an obvious effect on trade: the more disperse the distribution, the more room for taking advantage of comparative advantage and therefore more trade. More importantly, both alternative procedures generate results that are in line with the outcome of the benchmark parametrization. We do not graph the corresponding Laffer curves, as they are all very similar.

5.2 Trade Costs

The degree of openness, summarized by the trade cost ν , is another key feature in our framework. Figure 9 plots the Laffer curve in the North for four different values of the trade cost (*ceteris paribus*). These generate trade shares in GDP ranging from 50% to 150% of our benchmark's trade share. A lower trade cost boosts trade and specialization and therefore capital accumulation in the North. This makes the Laffer curve expand upwards and to the right, shifting the revenue-maximizing tax rate to the right.

The implications for our results, as summarized in the second panel of Table 2, are straightforward: for the given actual US tax rate, equal to 27.3%, an increase in the degree of openness will reduce the steady-state feedback effect under trade, for both government revenues and the net fiscal position. Furthermore, it will reduce the distance between the actual tax rate and the revenue maximizing tax rate, bringing the North possibly back to the upward sloping side of the Laffer curve.³³

³³Note that, for $\nu = 1.7$, the actual and the revenue-maximizing tax rates reported in Table 2 almost coincide, and therefore the one-percentage-point tax cut we are examining takes the economy to the upward sloping side of the Laffer curve, since the dynamic feedback is less than unity. If the tax rate is reduced by a marginal amount, and therefore the economy remains on the "slippery" side, the dynamic feedback would remain (marginally) larger than one.

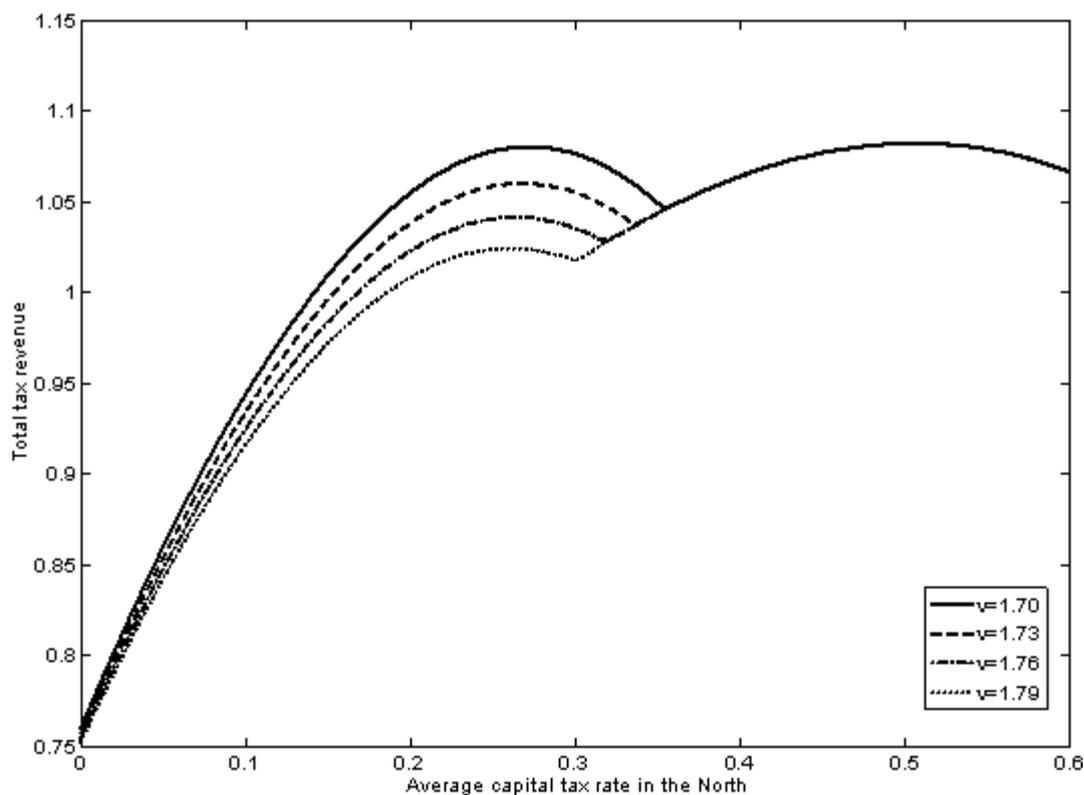


Figure 9: The Northern Laffer curve for different degrees of trade integration.

5.3 The Elasticity of Intertemporal Substitution

The elasticity of intertemporal substitution is key in determining the speed of capital accumulation, and therefore the speed of convergence towards the steady state. Table 3 summarizes our main results for some alternative values of μ that lie symmetrically around our benchmark value. Almost all variables of interest remain unaltered, except for the present-value dynamic feedback for the net fiscal position. The speed of convergence as measured by the half life - the number of periods needed to cover half their way to the new steady state - increases slightly with μ , as expected.

Elast. inter. substitution	PV dyn. feedback in steady state	Dynamic feedback	
		Half life	
μ	<i>PV net fiscal pos.</i>	<i>Gov. rev.</i>	<i>PV net fiscal pos.</i>
0.50	0.47	20	18
0.75	0.54	15	16
1.00	0.59	12	15
1.50	0.65	10	13
2.00	0.69	8	12

Table 3: Alternative elasticities of intertemporal substitution.

6 Concluding Remarks

Opening the neoclassical growth model to trade changes its quantitative implications regarding the effects of taxation in a rather stark way. Given the shrinking size of the US economy relative to the world, and its relatively high degree of openness, our set-up seems to be a better workhorse for understanding the effects of tax policies than the standard closed-economy Ramsey model. At the same time, ours is still an incomplete model, as we ignore sources of comparative advantage other than capital abundance and, perhaps more importantly, “new-trade” theory features explaining intraindustry trade.³⁴ This is left for future work.

For simplicity, we have assumed away international capital mobility, which is an important issue in the area of taxation. Notice, however, that allowing for capital mobility would, if anything, make the model’s results stronger over the transition to the steady state, as the country reducing the capital-income tax rate would attract capital inflows from the rest of the world: this would yield even stronger dynamic feedback measures.³⁵ Conversely, the effects of capital taxation usually discussed in neoclassical models with capital mobility would become much

³⁴See Burstein and Vogel (2009) for a recent paper that extends the Heckscher-Ohlin framework by incorporating productivity differences across producers within a sector.

³⁵The distinction between source-based and residence-based taxation matters here. Our argument assumes implicitly that a tax cut on capital income in the US would mainly affect capital accumulation in the US. The empirical evidence that highlights a “home bias” in portfolio choices, see Obstfeld and Rogoff (2000), may partially support our assumption.

stronger if they were considered within our framework (as opposed to the usual setup with two countries with identical invariant aggregate production functions).

An issue that usually arises in connection with international capital mobility is that of international tax competition. The effects on the South of a tax cut in the North are not dramatic in our benchmark calibration, but suggest that the South might have an incentive to “retaliate.” We also leave this topic for future work, as our model is not well equipped to address the behavior of welfare-optimizing governments: one would need a less simplistic government side, in which government expenditure raises the representative consumer’s utility (or enhances productivity in the production function) and there is thus a trade-off to an income tax cut between the utility loss from lower government expenditure and the efficiency gain from lower taxation.³⁶

Another issue regarding government behavior that we have ignored is the treatment of government deficits. Since we were mainly interested in a comparison between autarky and trade, we do not think our balanced budget assumption is that misleading, and leave this issue as well as part of our research agenda.

³⁶See, for example, Barro (1990).

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A Appendix: Equilibrium Conditions

A.1 Autarky Equilibrium Conditions

1. Unit-cost function:

$$b(z, \phi_j, r_j, w_j) = \frac{r_j^{\alpha(z)} w_j^{1-\alpha(z)}}{\phi_j a(z)}. \quad (40)$$

2. Commodity prices:

$$P_j = \exp \left[\int_0^1 \ln p_j(z) dz \right] = 1, \quad (41)$$

$$p_j(z) = b(z, \phi_j, r_j, w_j). \quad (42)$$

3. Goods market clearing:

$$y_j(z) = x_j(z) = \frac{P_j Y_j}{p_j(z)}, \quad (43)$$

where $P_j Y_j = r_j K_j + w_j L_j$.

4. Factor market clearing:

$$\int_0^1 \frac{\partial b(z, \phi_j, r_j, w_j)}{\partial r} y_j(z) dz = K_j, \quad (44)$$

$$\int_0^1 \frac{\partial b(z, \phi_j, r_j, w_j)}{\partial w} y_j(z) dz = L_j. \quad (45)$$

A.2 Trade Equilibrium in the Presence of Frictions

A.2.1 Equilibrium Conditions

1. Commodity prices: For $z \in [0, z_N)$,

$$p_N(z) = \nu p_S(z) = \nu b(z, \phi_S, r_S, w_S). \quad (46)$$

For $z \in [z_N, z_S]$,

$$p_j(z) = b(z, \phi_j, r_j, w_j). \quad (47)$$

For $z \in (z_S, 1]$,

$$p_S(z) = \nu p_N(z) = \nu b(z, \phi_N, r_N, w_N). \quad (48)$$

2. Goods market clearing: For $z \in [0, z_N]$,

$$y_N(z) = 0 \quad y_S(z) = x_S(z) + \nu x_N(z). \quad (49)$$

For $z \in [z_N, z_S]$,

$$y_j(z) = x_j(z). \quad (50)$$

For $z \in (z_S, 1]$,

$$y_S(z) = 0 \quad y_N(z) = \nu x_S(z) + x_N(z). \quad (51)$$

3. Factor market clearing:

$$\int_{z_N}^1 \frac{\partial b(z, \phi_N, r_N, w_N)}{\partial r} y_N(z) dz = K_N, \quad (52)$$

$$\int_{z_N}^1 \frac{\partial b(z, \phi_N, r_N, w_N)}{\partial w} y_N(z) dz = L_N, \quad (53)$$

$$\int_0^{z_S} \frac{\partial b(z, \phi_S, r_S, w_S)}{\partial r} y_S(z) dz = K_S, \quad (54)$$

$$\int_0^{z_S} \frac{\partial b(z, \phi_S, r_S, w_S)}{\partial w} y_S(z) dz = L_S. \quad (55)$$

4. Marginal commodity conditions:

$$b(z_j, \phi_j, r_j, w_j) = \nu b(z_j, \phi_{-j}, r_{-j}, w_{-j}). \quad (56)$$

5. The numeraire:

$$p_S(0) = 1. \quad (57)$$

B Appendix: Steady State with Trade

This appendix establishes the condition under which we can have a steady state with trade in which countries N and S produce ranges $[z_N, 1]$ and $[0, z_S]$, respectively. Given that the two countries have got the same discount factor and depreciation rate, in steady state,

$$(1 - \tau_N^K) \frac{r_N}{P_N} = (1 - \tau_N^K) \frac{r_S}{P_S}. \quad (58)$$

We need to make this equation compatible with the equilibrium conditions discussed above.

The price indices P_N and P_S can be expressed as

$$\begin{aligned} P_N &= \exp \left[\int_0^{z_N} \ln [\nu b_S(z)] dz + \int_{z_N}^1 \ln [b_N(z)] dz \right] = \\ &= \nu^{z_N} \exp \left[- \int_0^1 \ln [a(z)] dz - z_N \ln \phi_S - (1 - z_N) \ln \phi_N + \right. \\ &+ \int_0^{z_N} \alpha(z) dz \ln r_S + \int_0^{z_N} [1 - \alpha(z)] dz \ln w_S + \\ &\left. + \int_{z_N}^1 \alpha(z) dz \ln r_N + \int_{z_N}^1 [1 - \alpha(z)] dz \ln w_N \right], \end{aligned} \quad (59)$$

$$\begin{aligned} P_S &= \exp \left[\int_0^{z_S} \ln [b_S(z)] dz + \int_{z_S}^1 \ln [\nu b_N(z)] dz \right] = \\ &= \nu^{1-z_S} \exp \left[- \int_0^1 \ln [a(z)] dz - z_S \ln \phi_S - (1 - z_S) \ln \phi_N + \right. \\ &\left. + \int_0^{z_S} \alpha(z) dz \ln r_S + \int_0^{z_S} [1 - \alpha(z)] dz \ln w_S + \right. \end{aligned} \quad (60)$$

$$+ \int_{z_S}^1 \alpha(z) dz \ln r_N + \int_{z_S}^1 [1 - \alpha(z)] dz \ln w_N \Big],$$

where $b_j(z)$ is short hand for $b(z, \phi_j, r_j, w_j)$. From (56), the marginal commodities z_S and z_N must satisfy

$$b_S(z_S) = \nu b_N(z_S), \quad (61)$$

$$\nu b_S(z_N) = b_N(z_N), \quad (62)$$

respectively. From (61) and (62),

$$\frac{w_N}{w_S} = \nu^{\frac{2}{\alpha(z_S) - \alpha(z_N)}} \left(\frac{r_N}{r_S} \right). \quad (63)$$

From (61),

$$\frac{r_N}{r_S} = \left[\nu^{-1} \frac{\phi_N}{\phi_S} \left(\frac{w_N}{w_S} \right)^{\alpha(z_S) - 1} \right]^{\frac{1}{\alpha(z_S)}}. \quad (64)$$

From (63) and (64),

$$\frac{r_N}{r_S} = \nu^{\frac{2[\alpha(z_S) - 1]}{\alpha(z_S) - \alpha(z_N)} - 1} \frac{\phi_N}{\phi_S}. \quad (65)$$

From (58), (59), (60), (63), and (65),

$$\frac{\phi_N (1 - \tau_N^K)}{\phi_S (1 - \tau_S^K)} = \nu^\chi > 1, \quad (66)$$

where

$$\chi = 2 \left[z_N + \frac{(z_S - z_N) \alpha(z_S) - \int_{z_N}^{z_S} \alpha(z) dz + 1 - \alpha(z_S)}{\alpha(z_S) - \alpha(z_N)} \right] > 0. \quad (67)$$

Thus, we need $\phi_N (1 - \tau_N^K) > \phi_S (1 - \tau_S^K)$.

C Appendix: List of Sectors

Farms; Agricultural services, forestry, and fishing; Metal mining; Coal mining; Oil and gas extraction; Nonmetallic minerals, except fuels; Construction; Lumber and wood products; Furniture and fixtures; Stone, clay, and glass products; Primary metal industries; Fabricated metal products; Machinery, except electrical; Electric and electronic equipment; Motor vehicles and equipment; Other transportation equipment; Instruments and related products; Miscellaneous manufacturing industries; Food and kindred products; Tobacco products; Textile mill products; Apparel and other textile products; Paper and allied products; Printing and publishing; Chemicals and allied products; Petroleum and coal products; Rubber and miscellaneous plastics products; Leather and leather products; Railroad transportation; Local and interurban passenger transit; Trucking and warehousing; Water transportation; Transportation by air; Pipelines, except natural gas; Transportation services; Telephone and telegraph; Radio and television; Electric, gas, and sanitary services; Wholesale trade; Retail trade; Banking; Credit agencies other than banks; Security and commodity brokers; Insurance carriers; Insurance agents, brokers, and service; Other real estate; Holding and other investment offices; Hotels and other lodging places; Personal services; Business services; Auto repair, services, and parking; Miscellaneous repair services; Motion pictures; Amusement and recreation services; Health services; Legal services; Miscellaneous professional services.

CHAPTER 2: LSM: A DSGE Model for Luxembourg*

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Abstract

Luxembourg is a small open economy with a set of particular features, including rather limited competition in the domestic goods market, strong union power, and a dual labour market for resident and non-resident workers. In this paper we develop a medium scale DSGE model that captures these features, calibrate it to mimic the actual behaviour of the key macro-economic aggregates, and use it to conduct policy experiments aimed at relaxing some of the existing rigidities in the goods and labour market.

JEL Codes: E13; E32;

Keywords: DSGE, Luxembourg, Small open economy; Dual labor market; Trade union

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1 Introduction

Dynamic Stochastic General Equilibrium (DSGE) models became a popular tool in macroeconomics in the late '90s, in particular for economic policy analysis. These models are based on sound microeconomic foundations, which makes them more robust to the Lucas (1976) critique when evaluating the effects of changes in policy.

In comparison with the Real Business Cycle (RBC) models of the early '90s, DSGE models incorporate rigidities that make them better suited to capture the key stylized macroeconomic facts. They are also often characterized by a more careful and complex specification of the different sectors of the economy, allowing for example for different production sectors, possibly with different market structure and production functions. The latest generation of DSGE models also explicitly accounts for open economy considerations. At the beginning of the 1990s, Paul Krugman stressed three problems in open-economy macroeconomics. He pointed out the need to use open economy macroeconomic models with nominal rigidities (for example, sticky prices or wages); to explicitly consider the role of expectations (for example, to explain the behavior of asset markets); and to better understand the microeconomic foundations of an open economy macro model. Obstfeld and Rogoff (1995) tried to tackle these issues and initiated the development of a new class of open economy macroeconomic models, the so-called New Open Economy Macroeconomics (NOEM).

The key features of a NOEM-DSGE model are an optimization-based dynamic general-equilibrium approach; the presence of sticky prices and/or wages in at least some sectors of the economy; the incorporation of stochastic shocks; and the evaluation of economic (typically monetary) policy based on household's welfare. As in the case of closed-economy DSGE models, the early NOEM-DSGE models were highly theoretical and provided only a very stylized representation of the economy, see e.g. Obstfeld and Rogoff (1995). Later developments, such as Ghironi (1999), Bergin (2003), Lubik and Schorfheide (2005), and Justiniano and Preston (2004), estimated small-scale NOEM-DSGE models, usually by Bayesian

techniques. Current research, often conducted in institutions, aims at further extending NOEM-DSGE models to provide a tool for policy analysis.

We follow this approach and build a medium-scale NOEM-DSGE model for Luxembourg, labeled LSM (Luxembourg Structural Model). LSM aims at assessing the effects of policy reforms such as greater product and labour market competition (as advocated, e.g., by the OECD (2006) and the IMF (2006)). Hence, we pay particular attention to modelling the real side of the economy, by combining some original theoretical features with modelling choices aimed at capturing specific characteristics of the Luxembourg economy. In particular, we adopt an over-lapping generation approach for households, and combine it with Heijdra and Ligthart (2007) style investment decisions and a right-to-manage specification of a segmented labour market, with both residents and non-resident workers.

The equilibrium conditions resulting from the optimization problems at the cohort and firm level can be analytically aggregated. The resulting model is calibrated in order to match specific features of the Luxembourg economy and solved using a nonlinear local solution method.

There already exist three macroeconometric models for Luxembourg (STATEC, Adam (2004, 2007)), the model of the Luxembourg Central Bank (BcL, Guarda (2005)), and LuxMod (STATEC, 2006)), each with useful specific features and suitable for specific purposes but none belonging to the NOEM-DSGE class. This is the distinctive feature of our model, LSM, as will clearly emerge from its description in the following sections. With respect to LuxMod and the BcL models, LSM is substantially more theory-based, but less detailed in terms of the dynamics. Hence, it is more suitable than these models for policy simulations, but perhaps less so for short and medium-term forecasting. With respect to LuxMod, the underlying economic theory is also more developed and coherent, but there is no sectorial disaggregation. Hence, LSM can be more useful than any of the other existing models to evaluate the aggregate effects of changes in economic policy.

To conclude, while some features of LSM are tailored to the specificities of the Luxembourg economy, its overall structure could be easily adapted to assess economic policy in other small open economies.

The paper is structured as follows. In Section 2 we describe the different sectors of LSM. In Section 3 we briefly discuss the equilibrium conditions, with full details provided in Appendix A. In Section 4 we discuss the calibration of LSM, with full details in Appendix B. In Section 5 we use LSM to analyze the effects of increasing competition in the Luxembourg product and labour markets. Finally, in Section 6 we summarize the main results and propose directions for further developments.

2 The structure of LSM

Typically, a DSGE model includes at least three types of agents: Households, the Government, and Firms. Households may be modelled with an infinite life (and horizon in their optimization choices) or with a finite life (and horizon), with a set of overlapping generations with different features in each time period. For LSM, we prefer the overlapping generation (OLG) approach because of four main advantages. First, it allows to evaluate the consequences of changes to some demographic factors such as the birth rate and the mortality rate. Second, it links consumption decisions more closely to current disposable income than to life-time resources, which is relevant for example to study the impact of increases in government consumption. Third, it avoids the need for ad hoc assumptions to ensure consumption is stationary, such as internal habit persistence (e.g., past consumption enters the utility function, see e.g. Pytlarczyk (2005)) or financial constraints, see e.g. Schmitt-Grohe and Uribe (2004).¹

¹Without these assumptions, an infinitely-lived consumer could borrow an infinite amount from the rest of the world in a given time period and repay it in the infinite future, or lend an infinite amount over a long time period, which would violate the assumption that the economy is small with respect to international capital markets, and the existence of a sustainable long-run equilibrium not just for consumption but also for net foreign assets and trade. Finally, the

OLG specifications for households, along the lines of Blanchard (1985) and Yaari (1965), have been used often in macroeconometric models developed in policy institutions, such as the IMF model MULTIMOD (Laxton et al. (1998)), the European Commission model QUEST (Roeger and in't Veld (1997)) or, more recently, the Bank of England model BEQM (Harrison et al. (2005)), the Bank of Belgium model NONAME (Jeanfils and Burggraeve (2005)), and the Bank of Finland model AINO (Kilponen and Ripatti (2006)). The three latter models are the key references that we follow in the specification of LSM. However, we also introduce a set of technical refinements, mostly needed to tackle the additional complications introduced by the OLG structure when deriving the aggregation equations in closed form, to introduce sufficient flexibility in the dynamics of the model, and to model the specificities of the Luxembourg economy.

In particular, we choose a set of assumptions and modelling choices which are not restrictive and permit the derivation of a congruent dynamic model in closed form at the aggregate level. First, perfect unemployment insurance, namely, the labour income of (each member of) each cohort is a weighted average of current wages and unemployment benefits, where the weights coincide with the probabilities of being employed and unemployed. The assumption that wages are all equal is in line with the homogeneity of the home labour input factor that is used for production, see Section 3.2. The assumption that unemployment benefits are equal is related to the working of the labour market, see Section 3.2. The assumption of perfect unemployment insurance is not too unrealistic for Luxembourg given that unemployment benefits are generous for an extended period.

The second assumption is that (each member of) each cohort owns capital and has an equal and exogenously determined share of total firms profits. The assumption of homogenous distribution of the profits could be relaxed by assuming, e.g., that only a certain fraction of households owns capital. However, again, there would be no major changes at the aggregate level. Notice also that the steady state is stable (following a temporary perturbation, the model will converge back to the initial steady-state position), and steady-state consumption is strictly positive and finite.

assumption that households own capital implies that they are also in charge of investment decisions. We will return to this point briefly.

The third assumption is that (each member of) each cohort has an equal and exogenously determined share of net government transfers. Again, some heterogeneity could be allowed without any major consequences at the aggregate level.

The fourth assumption is that financial wealth can be held as government bonds, foreign bonds, and claims to physical capital. A more varied choice could be possible (BEQM, Harrison et al. (2005)), but the focus of LSM is on the real side of the economy, and for this the classification into three assets is sufficient. The three assets are perfect substitutes in the household portfolio, and in equilibrium they earn the same (exogenous) real rate of return (which basically follows from the small open economy assumption).

Fifth, claims to physical capital are in the form of shares in a firm (uniformly distributed across and within cohorts), which operates at the aggregate level and is in charge of investment decisions. This firm, (the households) determines investment by maximizing the firm cash flow subject to the law of motion of physical capital. Finally, and related to the previous point, there are adjustment costs for investment. This assumption is necessary as the OLG structure alone is not sufficient to prevent major (non realistic) changes in the capital stock for small changes in the (exogenous) interest rate.²

In the following subsections we will describe in detail the behavior of the different types of agents in LSM, namely: Households, Government, Firms and Unions.

²This approach follows Heijdra and Ligthart (2007), and it is more convenient for analytical tractability than directly modelling the investment decisions of individuals or firms in the presence of OLG and adjustment costs to capital, while leading to similar conclusions at the aggregate level.

2.1 Households

We provide a detailed description of the household problem at the cohort level in the first subsection. In the second subsection we focus on aggregation. In the third subsection we consider investment and capital accumulation. In the final subsection we discuss the determination of the net foreign asset position.

2.1.1 The consumer's problem at the cohort level

Following the discrete time version of Blanchard (1985), in period t , the representative consumer of generation z maximizes her expected lifetime utility:

$$u_{z,t} = E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} u(x_{z,s}) \right] = \sum_{s=t}^{\infty} (\varphi\beta)^{s-t} u(x_{z,s}),$$

where $\varphi \in (0, 1)$ represents the constant *survival rate*, i.e. the share of individuals that survive in each period, β the subjective discount factor, $x_{z,t} \equiv \{c_{z,t}, d_{z,t}\}$ with c_t denoting non-durable consumption (from now on, consumption *tout court*) and d_t the end-of-period desired stock of durable consumption goods (from now on, durables).

The utility function, $u(x_{z,t})$, is of the constant relative risk aversion (CRRA) type, with CES preferences over consumption and durables:

$$u(x_{z,t}) \equiv \frac{\left\{ \left[\phi c_{z,t}^v + (1 - \phi) d_{z,t}^v \right]^{\frac{1}{v}} \right\}^{1-\sigma} - 1}{1 - \sigma}. \quad (1)$$

In (1), ϕ is related to the expenditure shares of consumption and durables, while if we define by σ^c the (constant) intertemporal elasticity of substitution and by σ^m the elasticity of substitution between consumption and durables, then:

$$\sigma = \frac{1}{\sigma^c}, \quad v = \frac{\sigma^m - 1}{\sigma^m}.$$

The period-by-period budget constraint for the representative agent in generation z can be written as

$$a_{z,t} = \frac{R_t}{\varphi} a_{z,t-1} + \omega_t - (1 + \tau_C) p_t \left[c_{z,t} + \underbrace{\varkappa_t^d \left(d_{z,t} - \frac{1 - \delta_D}{\varphi} d_{z,t-1} \right)}_{\text{Investment in durables}} \right], \quad (2)$$

where

$$R_t \equiv 1 + (1 - \tau_K) i_t.$$

The variables are defined as follows: a_t is the end-of-period asset stock, R_t is gross rate of return common across assets, τ_K is the tax rate on financial asset returns, i_t the *exogenous* (small open economy assumption) gross-of-tax interest rate, ω_t is current non-financial income, p_t is the price of the final good, τ_C is the tax rate on consumption, δ^D is the depreciation rate of durables, and \varkappa_t^d is an exogenous shock to the relative price for durables. Note that we are assuming that the final consumption good can be transformed into durables at a rate \varkappa_t^d . Furthermore, note that $a_{t,t-1} = 0$, for $t \geq z$, meaning that new generations have no endowments.

Following Schmitt-Grohe and Uribe (2004), we assume the existence of a debt-elastic interest-rate premium, i.e. an interest rate that is increasing in the country's net foreign debt:

$$i_t = \bar{i} + \xi_i \left[\exp \left(\bar{f} - \frac{F_t}{GDP_t} \right) - 1 \right] + \varepsilon_{it},$$

where F_t represents the country's net foreign asset position, \bar{i} the long-run, constant, and exogenous interest rate if the country runs its steady-state net foreign asset position (\bar{f}), and ε_{it} an interest-rate shock.

Current non-financial income is defined as

$$\omega_t \equiv \underbrace{(1 - \tau_L) [w_{1,t} n_{1,t} + \bar{w}_{1,t} (1 - n_{1,t})]}_{\text{Labor income}} + (1 - \tau_K) \pi_t + tr_t, \quad (3)$$

where $n_{1,t}$ is the employment rate of resident workers (at the individual level, the unemployment rate can be interpreted as the probability of being unemployed), $w_{1,t}$ their wage rate, τ_L the tax rate on labour related income, $\bar{w}_{1,t}$ the unemployment benefits for resident former workers (to be precisely defined in the following), π_t the exogenous, individual share of total firm profits, and tr_t the net government transfer. Note that the expression for labour income reflects the assumption of perfect unemployment insurance, and the existence of two types of labour, resident and non-resident.

In each period the consumer can use available resources (current income, assets and durables), or borrow in the financial market to finance consumption, or to increase her asset stock (which includes claims on the physical capital stock).³ The intertemporal budget constraint is the following:

$$(1 + \tau_C) \sum_{s=t}^{\infty} R_{t,s} p_s \left[c_{z,s} + \mathcal{Z}_t^d \left(d_{z,s} - \frac{1 - \delta_D}{\varphi} d_{z,s-1} \right) \right] = \frac{R_t}{\varphi} a_{z,t-1} + \sum_{s=t}^{\infty} R_{t,s} \omega_s, \quad (4)$$

where $R_{t,t} \equiv 1$ and, for $s \geq t + 1$, $R_{t,s} \equiv \prod_{j=t+1}^s \frac{\varphi}{R_j}$.

As usual, the representative consumer maximizes intertemporal utility subject to the budget constraint, taking the sequences of prices as given. The resulting first-order conditions can be combined to yield the two Euler equations:

$$u_c(x_{z,t+1}) \beta R_{t+1} \frac{p_t}{p_{t+1}} = u_c(x_{z,t}) \quad (5)$$

$$u_d(x_{z,t}) + \beta (1 - \delta_D) \mathcal{Z}_{t+1}^d u_c(x_{z,t+1}) = \mathcal{Z}_t^d u_c(x_{z,t}) \quad (6)$$

³Notice that, even if the life expectancy of the consumer decreases exponentially, she could still live for an infinite number of periods. Therefore, it is important to impose as an additional constraint the no-Ponzi game condition (NPG): $\lim_{T \rightarrow \infty} \prod_{s=0}^T \varphi \frac{a_{z,t+s}}{R_{t+s}} = 0$, which prevents over-borrowing. This constraint simply ensures that the market will never allow an individual to finance consumption indefinitely via new debt: sooner or later, financial liabilities of any kind have to be honored.

where:

$$u_c(x_{z,t}) = [\phi c_{z,t}^v + (1 - \phi) d_{z,t}^v]^{\frac{1-v-\sigma}{v}} \phi c_{z,t}^{v-1} \quad (7)$$

Combining (5)-(6) and (7), we can express optimal durables in terms of optimal consumption as:

$$d_{z,t} = \xi_t c_{z,t} \quad (8)$$

where:⁴

$$\xi_t \equiv \left\{ \frac{\phi}{1 - \phi} \left[\chi_t^d - \frac{\chi_{t+1}^d (1 - \delta_D)}{R_{t+1} \frac{p_t}{p_{t+1}}} \right] \right\}^{\frac{1}{v-1}}.$$

Therefore, according to (8), the desired stock of durables increases when their user cost decreases, when ϕ decreases (the "consumption share" in the utility function), and when the elasticity of substitution between consumption and durables decreases.

For optimal consumption, from (5) we obtain:

$$c_{z,t+1} = \mathcal{E}_{t+1} c_{z,t}, \quad (9)$$

where:

$$\mathcal{E}_{t+1} \equiv \left\{ \left[\frac{\phi + (1 - \phi) \xi_{t+1}^v}{\phi + (1 - \phi) \xi_t^v} \right]^{\frac{1-v-\sigma}{v}} \beta R_{t+1} \frac{p_t}{p_{t+1}} \right\}^{\frac{1}{\sigma}}.$$

As usual, consumption is postponed when current prices are high relative to future prices and/or interest rates are high. An interesting original element is that the intertemporal path of consumption also depends on the user cost of durables through the ξ terms.

After some manipulations, equations (8) and (9) imply that

$$\sum_{s=t}^{\infty} R_{t,s} (1 + \tau_C) p_s \left[c_{z,s} + \chi_s^d \left(d_{z,s} - \frac{1 - \delta_D}{\varphi} d_{z,s-1} \right) \right] = \zeta_t c_{z,t}$$

⁴The expression $\chi_t^d - \frac{\chi_{t+1}^d (1 - \delta_D)}{R_{t+1} \frac{p_t}{p_{t+1}}}$ can be considered as the user cost of durables, while $\frac{1}{v-1} = -\sigma^m$.

where:

$$\zeta_t \equiv \sum_{j=0}^{\infty} \mathcal{Z}_{t+j} \varphi^j \prod_{s=1}^j \frac{\mathcal{E}_{t+s}}{R_{t+s}}$$

and:

$$\mathcal{Z}_t \equiv (1 + \tau_C) p_t \left[1 + \mathcal{X}_t^d \left(\xi_t - \frac{1 - \delta_D}{\varphi} \frac{\xi_{t-1}}{\mathcal{E}_t} \right) \right]$$

Note that $\mathcal{Z}_t c_{z,t}$ represents the total value of current consumption *and* net investment in durables for generation z in period t , being the demand for durables related to the demand for consumption goods via (8). The term $\zeta_t c_{z,t}$, instead, represents the total *discounted* flow of future consumption levels and net investment in durables. Note also that ζ_t can be defined recursively as:

$$\zeta_t = \mathcal{Z}_t + \mathcal{E}_{t+1} \frac{\varphi}{R_{t+1}} \zeta_{t+1}$$

Multiplying both sides by $c_{z,t}$, we can easily provide a simple interpretation:

$$\zeta_t c_{z,t} = \mathcal{Z}_t c_{z,t} + \frac{\varphi}{R_{t+1}} \zeta_{t+1} \underbrace{(\mathcal{E}_{t+1} c_{z,t})}_{c_{z,t+1}}$$

The discounted flow of future “consumption” $\zeta_t c_{z,t}$ (i.e. consumption plus net investment in durables) equals the current value of “consumption,” $\mathcal{Z}_t c_{z,t}$, plus the discounted value of the one-period-ahead flow, $\zeta_{t+1} c_{z,t+1}$.

Using the intertemporal budget constraint in (4), we can therefore write optimal current consumption as:

$$c_{z,t} = \zeta_t^{-1} \left(\frac{R_t}{\varphi} a_{z,t-1} + m_t \right), \quad (10)$$

where:

$$m_t \equiv \sum_{s=t}^{\infty} R_{t,s} \omega_s \quad (11)$$

represents human wealth.

Notice that both ζ_t in (10) and ξ_t in (8) are independent of z , which simplifies aggregation.

Finally, it is worth mentioning that, in general, changing the arguments in the utility function does not change the structure of the optimal solution for consumption, in the sense that it will remain given by an equation such as (10), even though the expression for ζ_t^{-1} will be properly modified. For example, Harrison et al. (2005) include external habit formation in the model, while Jeanfils and Burggraeve (2005) exclude durables to make utility dependent on consumption only. Similarly, adding other assets to the model, such as money or foreign bonds, only changes the budget constraint and the expression for wealth.

2.1.2 Aggregation

Let us assume that the size of each new-born generation is z_t , where $z_t = \eta^t z_{-\infty}$ and $z_{-\infty}$ is normalized to one. Then, the total population at any date t , Z_t , is equal to:

$$Z_t = \eta^t \sum_{j=0}^{\infty} \left(\frac{\varphi}{\eta} \right)^j = \frac{z_t}{1 - \frac{\varphi}{\eta}},$$

and hence $Z_{t+1} = \eta Z_t$.

The expressions for the aggregate variables can be obtained by linear aggregation of those at the cohort level. Let us start with aggregate assets. We have

$$A_t \equiv \sum_{j=0}^{\infty} \varphi^j z_{t-j} a_{z_{t-j}, t}. \quad (12)$$

Aggregating the budget constraint in (2) over cohorts, we obtain an equation describing the evolution of aggregate assets:

$$A_t = R_t A_{t-1} + W_t - Z_t C_t, \quad (13)$$

where

$$W_t \equiv \omega_t Z_t,$$

since ω_t is not cohort-dependent, and $Z_t C_t$ represents the total aggregate value of current consumption and net investment in durables. Equation (13) can be considered as the budget constraint at the aggregate level.

Next, let us consider aggregate net human wealth, where cohort-level human wealth, m_t , is defined in equation (11). We have:

$$M_t \equiv \sum_{j=0}^{\infty} \varphi^j z_{t-j} m_t = m_t Z_t. \quad (14)$$

The evolution of aggregate net human wealth is given by:

$$M_{t+1} = \frac{\eta}{\varphi} R_{t+1} (M_t - W_t). \quad (15)$$

For aggregate consumption, aggregating equation (10) over cohorts yields:

$$C_t \equiv \sum_{j=0}^{\infty} \varphi^j z_{t-j} c_{z_{t-j},t} = \zeta_t^{-1} [R_t A_{t-1} + M_t], \quad (16)$$

where aggregate assets, A_t , are defined in (12) and aggregate human wealth, M_t , in (14). The evolution of aggregate consumption is governed by the aggregate Euler equation

$$C_{t+1} = \eta \mathcal{E}_{t+1} \left(C_t - \frac{\eta - \varphi}{\eta} \frac{A_t}{\zeta_t - Z_t} \right). \quad (17)$$

For aggregate durables we have

$$D_t \equiv \sum_{j=0}^{\infty} \varphi^j z_{t-j} d_{z_{t-j},t} = \xi_t C_t, \quad (18)$$

and the dynamics of D_t can be determined from that of C_t .

Finally, aggregate financial wealth can be decomposed into government bonds, foreign bonds, and claims to physical capital. Hence,

$$A_t = B_t + F_t + V_t, \quad (19)$$

where B_t represents the value of the end-of-period stock of government bonds, F_t the value of the end-of-period stock of foreign assets, and V_t the value of the end-of-period stock of claims to physical capital, all measured in consumption units. By assuming assets to be perfect substitutes in the household's portfolio, they earn the same (exogenous) real rate of return in equilibrium. We will now analyze in detail the different types of assets.

2.1.3 Physical capital accumulation

Following Heijdra and Ligthart (2007), we assume that households as a whole, which can be considered as a representative "investment firm," are in charge of investment. More specifically, investment is determined by maximizing the cash flow from investing in physical capital, conditional on the law of motion of physical capital.⁵

The cash flow from investing in physical capital is given by:

$$\sum_{s=t}^{\infty} \tilde{R}_{t,s} \left\{ \left[(1 - \tau_K) \frac{r_s}{p_s} + \tau_K \delta_K \right] K_{s-1} - I_s \right\}, \quad (20)$$

where $\tilde{R}_{t,s} \equiv \prod_{j=t+1}^s [R_j (p_{j-1}/p_j)]^{-1}$ is the aggregate discount factor,⁶ r_t is the rental rate on capital, and I_t denotes investment. Note that the investment firm can deduct all depreciation from its taxable income. Physical capital evolves according to:

$$K_t = (1 - \delta_K) K_{t-1} + \Xi \left(\frac{I_t}{K_{t-1}} \right) K_{t-1}, \quad (21)$$

⁵In our Blanchard-Yaari framework, and introducing adjustment costs to physical capital accumulation, the investment decisions taken at the individual level can not be easily aggregated, being highly non linear. Given that Heijdra and Ligthart (2007) faced the same problem, we borrow their elegant solution.

⁶We can show that $\tilde{R}_{t,s} \equiv \prod_{j=1}^s \left[R_{t+j} \frac{p_{t+j-1}}{p_{t+j}} \right]^{-1} = \beta^s \frac{\lambda_{t+s}}{\lambda_t} \frac{p_t}{p_{t+s}}$, where λ_t is the aggregate shadow value of firms' profits in the household budget constraint.

where δ_K is the depreciation rate of capital and the term $\Xi \left(\frac{I_t}{K_{t-1}} \right) K_{t-1}$ indicates that there are adjustment costs. In particular, following Jermann (1998), we assume that those are

$$\Xi \left(\frac{I_t}{K_{t-1}} \right) = \frac{\Xi_1}{\varsigma} \left(\frac{I_t}{K_{t-1}} \right)^\varsigma + \Xi_2.$$

The two parameters Ξ_1 and Ξ_2 are designed to make the adjustment cost vanish in the steady state.

The investment firm maximizes the cash flow subject to the accumulation equation for physical capital; the first order conditions are:

$$\nu_t = \Xi' \left(\frac{I_t}{K_{t-1}} \right)^{-1}, \quad (22)$$

$$R_{t+1} = \frac{(1 - \tau_K) r_{t+1} + p_{t+1} \left(\tau_K \delta_K - \frac{I_{t+1}}{K_t} \right) + p_{t+1} \nu_{t+1} \left[1 - \delta_k + \Xi \left(\frac{I_{t+1}}{K_t} \right) \right]}{p_t \nu_t} \quad (23)$$

Equation (23) corresponds to the standard no-arbitrage condition, where the last term on the right-hand side represents the future marginal contribution of capital to lower installation costs. In other words, the future net-of-tax gross return on claims to physical capital has to be equal to the future return of holding a unit of capital for one period (i.e. the future rental rate plus the future shadow price corrected for depreciation plus the future decrease in installation costs) divided by the current shadow price of the same unit of capital. Thus ν_t corresponds to the well-known Tobin q .

It can be easily shown that:

$$\nu_t K_t = \frac{\left[(1 - \tau_K) \frac{r_{t+1}}{p_{t+1}} + \tau_K \delta_K \right] K_t - I_{t+1} + \nu_{t+1} K_{t+1}}{R_{t+1} \frac{p_t}{p_{t+1}}} \quad (24)$$

Hence, iterating on the previous expression and imposing the *TVC* yields:

$$\nu_t K_t = \sum_{s=t+1}^{\infty} \tilde{R}_{t,s} \{[(1 - \tau_K) r_s + \tau_K \delta_K p_s] K_{s-1} - p_s I_s\} \quad (25)$$

The right-hand side in (25) represents the discounted flow of future cash flows in real terms, i.e. the stock market value of claims to physical capital. This implies that:

$$V_t = p_t \nu_t K_t. \quad (26)$$

2.1.4 Net foreign asset position

Combining (19), (13), (26), (24), and (32), we get the following law of motion for net foreign assets:

$$F_t = R_t F_{t-1} + W_t + [(1 - \tau_K) r_t + \tau_K \delta_K p_t] K_{t-1} - Z_t C_t - p_t I_t - (G_t - T_t).$$

2.2 Firms and Unions

In LSM firms produce intermediate and final goods.⁷ We assume that there is a single representative firm producing the final good Y under perfect competition. This firm combines \mathcal{N} intermediate goods using a CES production function, possibly with increasing returns in the variety of intermediate inputs.

Local firms in the intermediate goods sector produce N varieties of differentiated goods, operating under monopolistic competition. A share Θ of these N locally produced varieties cannot be traded (exported). The remaining $(1 - \Theta)$ can be exported.⁸

Furthermore, other $(1 - \Theta^*) N^*$ varieties can be imported from abroad, where N^* indicates the total number of foreign produced varieties, and Θ^* the share of

⁷The split between final and intermediate goods is common in the literature. See for instance the seminal works by Christiano et al. (2005) and Smets and Wouters (2003).

⁸Also the split between tradables and non-tradables is relatively common in the literature: Justiniano and Preston (2004) discuss the issue in some detail.

them that can be imported in Luxembourg. Hence, the total number of varieties of differentiated intermediate goods in Luxembourg is given by $\mathcal{N} = N + (1 - \Theta^*) N^*$.

Each firm in the local intermediate sector adopts a nested CES production function with capital and two different types of labour as inputs. The different types of labour are introduced to capture the dual labour market in Luxembourg, and represent resident and non-resident workers. The firm chooses the optimal demand of capital and each type of labour by maximizing profits subject to the production function constraint, taking wages and the cost of capital as given. The cost of capital is determined endogenously in order to match demand and supply of capital. For the sake of exposition, we present all the derivations for a generic production function, and then specialize the results to the nested CES case in Appendix A, which requires a more cumbersome notation.

Wages are determined by the interaction between the intermediate sector firms and the unions, which represent the workers (the so-called "right to manage" model). In particular, we assume that there is a union for each type of workers, and that bargaining with the firm takes place in a Nash setting. We assume that there is a separate union for each firm, but this is not a restrictive hypothesis since in symmetric equilibrium firms will make the same choices in terms of demand for labour and capital.⁹

Technically, the interaction between the production and labour markets is represented as a game in two stages, where wage bargaining takes place in the first stage and production in the second. As in Lockwood (1990), the second stage is solved first, and the solution is used in the first stage. Therefore, after discussing the final good sector, we will first describe the problem of the intermediate sector firms (second stage), and then the firm-union bargain (first stage). We will deal, in turn, with producers of non-tradable goods, tradable goods, and importers of foreign intermediate goods.

⁹We assume that for the current wage the supply of non-resident workers adjusts to meet demand. This is of course a limitation of the model, but helps to simplify the framework.

2.2.1 Final good sector

The cost function for the final good producing firm is:¹⁰

$$\begin{aligned} \mathcal{C}_F(\{p_j\}, Y) &\equiv \min_{\{y_j\}} \sum_{j=1}^{\mathcal{N}} p_j y_j \\ \text{s.t. } &\mathcal{N}^{\rho-\mu} \left(\sum_{j=1}^{\mathcal{N}} y_j^{\frac{1}{\mu}} \right)^{\mu} \geq Y \end{aligned} \quad (27)$$

where y_j is the amount of the j^{th} intermediate good used for production of the final good Y , $j = 1, \dots, \mathcal{N}$; $\mu > 1$ is indirectly related to the elasticity of substitution between goods and directly related to the mark-up in the intermediate goods sector; and $\rho \geq 1$ is a parameter that captures increasing returns to variety; see Kim (2004) for details. Cost minimization leads to the usual conditional demand for intermediate good j :

$$y_j = \left(\frac{p_j}{p} \right)^{\frac{\mu}{1-\mu}} Y \mathcal{N}^{\frac{\rho-\mu}{\mu-1}}.$$

2.2.2 Intermediate goods sector - Non-tradable goods: $j \in [1, \Theta\mathcal{N}]$

Second stage: profit maximization The problem of a generic firm in the intermediate goods sector producing a non-tradable good can be formulated as

$$\begin{aligned} \max_{\{h_{zj}^{NT}, k_j^{NT}\}} \pi_j^{NT} &\equiv p_j^{NT} (y_j^{NT}) y_j^{NT} - r k_j^{NT} + \\ &- (1 + \tilde{\tau}_L) \sum_{z=1}^2 w_{zj}^{NT} h_{zj}^{NT} - \psi_j \end{aligned}$$

where $p(y_j^{NT})$ indicates the price of the j^{th} non-tradable intermediate good; h_{zj}^{NT} , $z = 1, 2$, the amount of the two types of labour (resident and non-resident) and k_j^{NT} the capital services; ψ_j is a fixed financial cost to enter the market (the fixed cost generates economies of scale and therefore justifies monopolistic

¹⁰From now on, for the sake of notational simplicity, we are dropping the time index.

competition; see Kim, 2004); and $\tilde{\tau}_L$ represents taxes on labour paid by firms; labour income taxes paid by workers will be taken into account later. In addition:

$$\begin{aligned} p_j^{NT}(y_j^{NT}) &= \mathcal{N}^{\frac{1-\mu}{\mu}} \left(\frac{y_j^{NT}}{Y} \right)^{\frac{1-\mu}{\mu}} p \\ y_j^{NT} &= f(k_j^{NT}, h_{1j}^{NT}, h_{2j}^{NT}) \end{aligned}$$

where the specific functional form for the production function will be discussed later on.

The first order conditions are:

$$\left(\frac{\partial p_j^{NT}}{\partial y_j^{NT}} y_j^{NT} + p_j^{NT} \right) \frac{\partial y_j^{NT}}{\partial h_{zj}^{NT}} = (1 + \tilde{\tau}_L) w_{zj}^{NT} \quad (28)$$

$$\left(\frac{\partial p_j^{NT}}{\partial y_j^{NT}} y_j^{NT} + p_j^{NT} \right) \frac{\partial y_j^{NT}}{\partial k_j^{NT}} = r \quad (29)$$

where $z \in \{1, 2\}$.

Note that, thanks to the Envelope Theorem, (28) implies:

$$\frac{\partial p_j^{NT}}{\partial y_j^{NT}} \left(\frac{\partial y_j^{NT}}{\partial h_{zj}^{NT}} \right)^2 \frac{\partial h_{zj}^{NT}}{\partial w_{zj}^{NT}} + p_j^{NT} \frac{\partial^2 y_j^{NT}}{(\partial h_{zj}^{NT})^2} \frac{\partial h_{zj}^{NT}}{\partial w_{zj}^{NT}} = \mu (1 + \tilde{\tau}_L)$$

Hence:

$$\frac{\partial h_{zj}^{NT}}{\partial w_{zj}^{NT}} = \frac{1}{w_{zj}^{NT}} \left[(1 - \mu) \frac{(1 + \tilde{\tau}_L) w_{zj}^{NT}}{p_j^{NT} y_j^{NT}} + \frac{\partial^2 y_j^{NT}}{(\partial h_{zj}^{NT})^2} \left(\frac{\partial y_j^{NT}}{\partial h_{zj}^{NT}} \right)^{-1} \right]^{-1}$$

since:

$$\frac{\partial p_j^{NT}}{\partial y_j^{NT}} = \frac{1 - \mu}{\mu} \frac{p_j^{NT}}{y_j^{NT}}$$

First stage: firm-union bargaining (Labour market) The loss function of the union representing type z workers in the j^{th} non-tradable sector is

$$\tilde{V}_{U,zj}^{NT} = (1 - \tau_L) \times \left[\frac{w_{zj}^{NT}}{P} h_{zj}^{NT}(w_{zj}^{NT}) + \frac{w_{zj}^T}{P} h_{zj}^T(w_{zj}^T) + \frac{\bar{w}_z}{P} (M_{zj} - h_{zj}^{NT}(w_{zj}^{NT}) - h_{zj}^T(w_{zj}^T)) \right], \quad (30)$$

where $\sum_j M_{1j}$ represents the total working age population of Luxembourg (Z_1), while $\sum_j M_{2j}$ represents total union membership among non resident workers, which is equal to number of employed non-resident workers, and unemployment benefits paid abroad are \bar{w}_2 . Therefore, the union cares about the total resident population (workers and unemployed) since the resident population coincides with the home labour force, and about the non-resident union members (workers and unemployed), but takes the level of unemployment benefits as given.

Each firm-union pair bargains over type- z wage, maximizing the following Nash objective function, taking the firms' labor demand curve into account:

$$\max_{\{w_{zj}^{NT}\}} \Omega_{zj}^{NT} \equiv \left(\tilde{V}_{U,zj}^{NT} - V_{U,zj}^{NT} \right)^{\theta_z} \left[\tilde{\pi}^{NT}(w_{zj}^{NT}) - \pi^{NT} \right]^{1-\theta_z}, \quad (31)$$

where θ_z is a parameter describing the relative bargaining power of the union for type z workers (constant across sectors); and $V_{U,zj}$ and π represent the outside options if the negotiation fails:

$$\begin{aligned} V_{U,zj}^{NT} &= (1 - \tau_L) \frac{\bar{w}_z}{p} \left[M_{zj} - h_{zj}^T(w_{zj}^T) \right] + (1 - \tau_L) \frac{w_{zj}^T}{p} h_{zj}^T(w_{zj}^T), \\ \pi^{NT} &= - (rk_j^{NT} + \phi_j). \end{aligned}$$

Combining (30) and (31), the problem of the union can be rewritten as

$$\max_{\{w_{zj}^{NT}\}} \Omega^{NT} \equiv \left[(1 - \tau_L) \left(\frac{w_{zj}^{NT}}{p} - \frac{\bar{w}_z}{p} \right) h_{zj}^{NT} \right]^{\theta_z} \left[\frac{\tilde{\pi}(w_{zj}^{NT})}{p} \right]^{1-\theta_z}$$

where:

$$\begin{aligned} \tilde{\pi}^{NT}(w_{zj}^{NT}) = & p^{NT} [f(k_j^{NT}, h_{1j}^{NT}, h_{2j}^{NT})] f(k_j^{NT}, h_{1j}^{NT}, h_{2j}^{NT}) + \\ & - (1 + \tilde{\tau}_L) \sum_{z=1}^2 w_{zj}^{NT} h_{zj}^{NT}. \end{aligned}$$

For $j = 1, 2$, the first order conditions can be written as:

$$\theta_z \left(1 + \frac{w_{zj}^{NT} - \bar{w}_z}{w_{zj}^{NT}} \epsilon_{zj}^{NT} \right) \frac{\tilde{\pi}_j^{NT}}{h_{zj}^{NT}} = (1 + \tilde{\tau}_L) (1 - \theta_z) (w_{zj}^{NT} - \bar{w}_z),$$

where:

$$\epsilon_{zj} \equiv \frac{\partial h_{zj}^{NT}}{\partial w_{zj}^{NT}} \frac{w_{zj}^{NT}}{h_{zj}^{NT}}.$$

Several factors affect real wages in LSM. First, as usual, labour productivity. Second, the characteristics of the labour market, such as the union power θ_2 and the replacement ratios \bar{w}_j/w_j . Third, the profit rate, since unions extract some of the producer surplus. Fourth, the relative productivity of the two types of labour, the relative size of the labour forces, and the unemployment rates. Finally, the relative productivity with respect to capital and the amount of capital per worker.

2.2.3 Intermediate goods sector - Tradable goods: $j \in [\Theta N, N]$

Let us now consider the problem of a generic firm in the intermediate goods sector producing tradable goods, y_j^T , such that $y_j^H = s_j^H y_j^T$ is sold at home and $y_j^F = s_j^F y_j^T$ is exported ($s_j^F = 1 - s_j^H$, and $0 \leq s_j^H \leq 1$), with corresponding prices given by p_j^H and p_j^F . The firm should choose the amount of labour and capital to be used for the production of y_j^T (h_{zj}^T and k_j^T , respectively, $z = 1, 2$), and the share of y_j^T sold at home, s_j^H , to optimize the following problem:

$$\max_{\{h_{zj}^T, k_j^T, s_j^H\}} \pi_j^T \equiv p_j^T (y_j^T) y_j^T - r k_j^T - (1 + \tilde{\tau}_L) \sum_{z=1}^2 w_{zj}^T h_{zj}^T - \psi_j,$$

where:

$$\begin{aligned}
p_j^T &= s_j^H p_j^H + s_j^F p_j^F, \\
s_j^F &= 1 - s_j^H, \\
y_j^T &= f(k_j^T, h_{1j}^T, h_{2j}^T), \\
p_j^H &= \mathcal{N}^{\frac{\rho-\mu}{\mu}} \left(\frac{s_j^H y_j^T}{Y} \right)^{\frac{1-\mu}{\mu}} p, \\
p_j^F &= (1 - t^F) (\mathcal{N}^*)^{\frac{\rho-\mu}{\mu}} \left(\frac{s_j^F y_j^T}{Y^*} \right)^{\frac{1-\mu}{\mu}} p^*.
\end{aligned}$$

Note that Y^* and p^* represent foreign output and the foreign aggregate price. Furthermore, note that the elasticity of substitution between intermediate goods is the same at home and abroad, i.e. $\mu^* = \mu$: this assumption is maintained for notational simplicity, but the model can be easily generalized.¹¹ As in the non-tradable sector, ψ_j is a fixed financial cost to enter the market that generates economies of scale and therefore provides a basis for monopolistic competition; see Kim, 2004.

Since the technical aspects of the problem of the firms and of the unions in the tradable sector are similar to those analyzed in detail above for the non-tradable sector, we do not present these derivations (but they are available upon request).

2.2.4 Intermediate goods sector - Imported goods

The importing firms buy goods abroad at the price p_M^* and resell them in the internal market at the price $p_j^M (y_j^M)$. Their problem is

$$\max_{\{y_j^M\}} \pi_j^M \equiv [p_j^M (y_j^M) - (1 + t^M) p_M^*] y_j^M - \psi_j,$$

¹¹The distinction between local and foreign elasticities is important to study shocks to local markups that do not transmit to markups in foreign markets. In this case, we obviously use the generalized version of the model.

where:

$$p_j^M = \mathcal{N}^{\frac{\rho-\mu}{\mu}} \left(\frac{y_j^M}{Y} \right)^{\frac{1-\mu}{\mu}} p.$$

The first order condition is given by

$$p_j^M = \mu (1 + t^M) p_M^*,$$

and the resulting profits are

$$\pi_j^M \equiv (\mu - 1) (1 + t^M) p_M^* y_j^M - \psi_j.$$

2.3 Government

The Government budget constraint is:

$$B_t = R_t B_{t-1} + G_t - T_t \quad (32)$$

where G and T indicate, respectively, total expenses and revenues, while B is government debt.

The Government collects revenues from taxes on the returns on financial assets, on profits, and on labour income (H_1 and H_2 are, respectively, resident and non-resident workers, whose wages are w_1 and w_2 , unemployment benefits are \bar{w} ; workers pay taxes at the rate τ_L and firms pay social contributions at the rate $\tilde{\tau}_L$). Furthermore, the government collects taxes on consumption and on imports. Therefore, total revenues in period t amount to:

$$\begin{aligned} T_t = & \tau_K [i_t F_{t-1} + (r_t - \delta_K p_t) K_{t-1} + \Pi_t] + \\ & + (\tau_L + \tilde{\tau}_L) (w_{1,t} H_{1,t} + w_{2,t} H_{2,t}) + \tau_L \bar{w}_{1,t} (1 - H_{1,t}) + \\ & + \tau_C p_t \left[1 + \varkappa_t^d \left(\xi_t - \frac{1 - \delta_D}{\varphi} \frac{\xi_{t-1}}{\mathcal{E}_t} \right) \right] C_t + \end{aligned}$$

$$+t_M(1 - \Theta^*)N^*p_M^*y^M.$$

where t_M , Θ^* , N^* , p_M^* , and y^M represent respectively the import tariff, the share of foreign varieties that can be traded, the total number of foreign varieties, the price of these foreign varieties, and the quantity imported (this is discussed in more detail in the following sections).

Government expenditure is composed of unemployment benefits for residents ($SUBS$), transfers to non-resident workers (TRF), and core expenditure (\bar{G}), where the latter can be further split into other transfers to resident households (TR), public investment in infrastructures ($INFR_INV$), and general government consumption ($GCON$). Overall, we have:

$$\begin{aligned} G_t &= SUBS_t + TRF_t + \bar{G}_t, \\ SUBS_t &= \bar{w}_{1,t}(1 - H_{1,t}), \\ TRF_t &= TR_t^F(\tau_L + \tilde{\tau}_L)w_{2,t}H_{2,t}, \\ TR_t &= \varrho_1\bar{G}_t, \\ GCON_t &= \varrho_2\bar{G}_t, \\ INFR_INV_t &= (1 - \varrho_1 - \varrho_2)\bar{G}_t. \end{aligned}$$

where $\varrho \in (0, 1)$ represents the share of transfers to resident households from core government expenditure. Note that TRF is modelled as a percentage (TR_t^F) of total labour taxes on non-resident workers. Unemployment benefits for type- j workers are equal to a replacement rate rep_j times the net factor income of the resident workers: $\bar{w}_{j,t} = rep_j \cdot NETINC_t$, where:¹²

$$\begin{aligned} NETINC_t &= (1 - \tau_L)[w_{1,t}H_{1,t} + \bar{w}_{1,t}(1 - H_{1,t})] + \\ &\quad [(1 - \tau_K)r_t + \tau_K\delta_K p_t]K_{t-1} + (1 - \tau_K)\Pi_t \end{aligned}$$

¹²Alternatively, unemployment benefits can be defined in terms of a share of gross wages. This unfortunately leads to indeterminacy in our model. Our formulation guarantees determinacy.

The stock of public infrastructures evolves according to the following accumulation equation:

$$INFR_t = (1 - \delta_{INFR}) INFR_{t-1} + INFR_INV_t, \quad (33)$$

and affects Total Factor Productivity via a purely external effect (see Section 4.1 for further details). Note that δ_{INFR} represents the depreciation rate for public infrastructures.

We further assume that core government expenditure is persistent and depends on the part of the (primary) deficit which excludes core government expenditure, $T_t - (G_t - \bar{G}_t)$:

$$\bar{G}_t = \vartheta \bar{G}_{t-1} + (1 - \vartheta) d^{LR} [T_t - \bar{w}_{1,t} (1 - H_{1,t}) - TR_t^F (\tau_L + \tilde{\tau}_L) w_{2,t} H_{2,t}]. \quad (34)$$

This specification of the Government sector implies a zero public debt and deficit in steady state when $d^{LR} = 1$. Otherwise, a value of $d^{LR} > 1$, combined with that of the other variables and parameters in (34), determines the equilibrium level of debt and deficit. Note that the parameter ϑ measures the persistence of core government expenditure.

2.4 Other variables of interest

Finally, we report the equations for GDP, GNP, net trade, terms of trade, imports, and exports (of intermediate goods):

$$\begin{aligned} GDP_t &= (1 + \tilde{\tau}_L) w_{1t} H_{1t} + (1 + \tilde{\tau}_L) w_{2t} H_{2t} + \\ &\quad r_t K_t + [\Pi_t + (1 - \Theta^*) N^* t_{MP_t, M}^* y_t^M], \\ GNP_t &= GDP_t + i_t F_{t-1} - \underbrace{[TR_t^F (\tau_L + \tilde{\tau}_L) + 1 - \tau_L]}_{\text{Remittances}} w_{2,t} H_{2,t}. \end{aligned}$$

We can easily recover the national accounting identity:

$$\begin{aligned}
 GDP_t = & \underbrace{p_t C_t}_{\text{Priv. cons.}} + \underbrace{p_t C_t \left(\xi_t - \frac{1 - \delta_D \xi_{t-1}}{\varphi} \frac{\mathcal{E}_t}{\mathcal{E}_t} \right)}_{\text{Priv. inv. in durables}} + \\
 & \underbrace{p_t I_t}_{\text{Priv. inv. in capital}} + \underbrace{GCON_t + INFR_INV_t}_{\text{Gov. cons.+gov. inv.}} + \underbrace{NX_t}_{\text{Net trade}}
 \end{aligned}$$

where net trade, NX_t , equals the change in the country's net foreign position plus the remittances of non-resident workers, as implied by the definition of the balance of payments:

$$NX_t = \underbrace{F_t - (1 + i_t) F_{t-1}}_{\text{Change in net foreign position}} + [TR_t^F (\tau_L + \tilde{\tau}_L) + 1 - \tau_L] w_{2,t} H_{2,t}$$

Focusing on intratemporal trade in goods (produced in the intermediate-good sector, but considered final because either exported or imported):

$$\begin{aligned}
 IMP_t^{IG} &= (1 - \Theta^*) N^* p_{t,M}^* y_t^M, \\
 EXP_t^{IG} &= (1 - \Theta) N p_t^F y_t^F, \\
 ToT_t &= \frac{p_t^F}{p_t^M}.
 \end{aligned}$$

3 Symmetric equilibrium

In a symmetric equilibrium, for all firms in a given sector the prices charged for the differentiated goods and the quantities produced are the same, i.e., $p_j^i = p^i$ and $y_j^i = y^i$, where $i = NT, H, F, M$. Furthermore, the equilibrium is characterized by the optimality conditions for households and government.¹³ In Appendix A we provide a detailed derivation of the symmetric equilibrium of LSM. In particular, we specialize the analysis of the production sector and labour market to the case of a CES production function. For the sake of clarity, we do not distinguish

¹³We set the numeraire as the price of the non-traded goods: $p^{NT} = 1$.

between tradable and non-tradable goods, but the same production function is assumed in both sectors:

$$\begin{aligned} y &= A \left[\alpha k^\lambda + (1 - \alpha) (\Lambda h)^\lambda \right]^{\frac{1}{\lambda}} \\ h &= [\varkappa_1 (a_1 h_1)^\kappa + \varkappa_2 (a_2 h_2)^\kappa]^{\frac{1}{\kappa}} \end{aligned}$$

with $\varkappa_2 = 1 - \varkappa_1$. Note that Λ represents a labour-augmenting productivity parameter.¹⁴ We allow for a (purely external) effect of the stock of public infrastructure ($INFR_t$) on the Total Factor Productivity, A . In particular, we model A as:

$$A = (INFR_t)^\varpi \cdot EXOG,$$

where $0 < \varpi < 1$, $EXOG$ represents exogenous technical progress growing at a constant rate γ .

The equilibrium conditions are normalized by the exogenous technological progress and by the cohort size, so that we express variables in efficiency terms. For the sake of simplicity, we maintain the previous notation, but variables are measured in efficiency units.

4 Calibration

Due to the complexity of LSM and the availability of only 15 years of quarterly observations for Luxembourg, the model cannot be estimated and we have to fully calibrate it. In this section we summarize the calibration procedure for the model parameters. Appendix B lists all the parameters of LSM, summarizes their meaning, and discusses their calibration in more detail.

We can divide the model parameters into three groups according to the way we set their values. The values for the parameters in the first group are set directly to standard values in the DSGE literature. In particular, we fix the subjective

¹⁴We use this nested CES specification since it clearly distinguishes the elasticity of substitution between aggregate labour and capital, and that between the two types of labours.

discount rate (β) to 0.995, the elasticity of intertemporal substitution to unity (i.e. $\sigma = 1$ which implies that preferences are logarithmic), the weight of capital in the production function (α) to 0.36 (the implied capital share in production is 25%), the persistence of core government expenditure (ϑ) to 0.9, the returns to variety to zero (which implies that $\rho = 1$), the elasticity of substitution among intermediate goods to 6 (so that $\mu = 1.2$), the relative bargaining power of the unions (θ_z) to 0.5 and the elasticity of substitution between the two labour types in the CES labour aggregator to 1.5 (so that $\kappa = 1/3$).

We follow Backus, Henriksen, and Storesletten (2008) in setting the depreciation rate on physical capital (δ_K) to 8.5%, on durables (δ_D) to 1.5%, and on the stock of public infrastructure (δ_{INFR}) to 4.15%. We set the elasticity of the international interest rate with respect to the national debt/GDP ratio (ξ_i) to 0.000742 based on Schmitt-Grohe and Uribe (2004). Following Boldrin, Christiano, and Fisher (2001) we assume that the elasticity of the adjustment cost with respect to the investment-capital ratio is 0.23 (so that $\varsigma = -3.348$).

We set the parameter related to the elasticity of substitution between durables and non-durables in the utility function (ν) in order to reproduce an elasticity of substitution equal to 1.5. The percentage of total labour taxes on non-resident workers that is transferred back to non-resident workers (TR_t^F) is chosen to be 0.6. We choose a small value for the fixed cost to enter the market of intermediate good j (ψ_j) and set it equal to 0.00001. The parameter related to the elasticity of TFP with respect to public infrastructure (ϖ) is chosen to be equal to 0.01.

Next, we normalize the foreign aggregate price level (P^*), the labour-augmenting productivity parameter (Λ) and the parameters augmenting type-1 (a_1) and type-2 (a_2) labour in the labour CES aggregator to unity. We also assume that Luxembourg and the rest of the world are symmetric in terms of the the share of non-traded varieties, both Θ and Θ^* are equal to 0.5. We normalize the number of traded varieties to unity, which implies that we set both N and N^* equal to 2, again for the sake of symmetry.

For the second parameter group, some values are directly observable or can be estimated. Average life expectancy at birth in Luxembourg was 79.18 years in 2008 (CIA factbook) which implies that the individual survival rate in our model (φ) is 0.987. The average value of net foreign position (\bar{f}) was 85% of GDP at the end of 2007 and 2008 (according to the bulletin of the Central Bank of Luxembourg). The population growth rate in Luxembourg is 1.2% (data from CIA factbook, year 2008) which implies that η equals to 1.012.

Guarda (1997) estimates the elasticity of substitution between capital and labour in a CES production functional form to be 1.012 in the tradables sector in Luxembourg (implies that $\lambda = 0.012$). We set the share of type-1 labour in the labour CES aggregator (χ_1) to 0.6 to reflect the fact that approximately 60% of the employed workforce is resident.

We set the tax rates in LSM according to the values reported in *Taxation trends in the EU*, European Commission, 2008. In particular, the tax rate on consumption (τ_C) equals to 25.1%. The total average effective tax rate on labour equals to related income is 29.6%, but only 67.9% of this amount is paid by the employee while the remaining part is paid by the employer. Thus, we set the tax rate on labour related income (τ_L) to 20.1% and the social contribution rate on labour related income ($\tilde{\tau}_L$) to 9.5%. Estimates of the tax rate on capital income (τ_K) are not reported in the mentioned source due to data availability problems, so we take the average effective tax rate on corporate profits as a useful approximation, and set the parameter equal to 29.6%.

The average TFP growth rate (γ) in Luxembourg over the 1995-2009 period, as reported in the Annual Report of the Central Bank of Luxembourg (2006, p. 54) was 0.6%.

We use the Overall Trade Restrictiveness Index for the European Union equals to 6.6% to set the tariffs in the model. The ad-valorem equivalent of all tariff and non-tariff barriers that the European Union imposed against foreign imports was equal to 6.6% in 2006. However, 94.5% of all imported goods were originated from countries within the EEA in 2007 and no tariffs were applied on them. Thus, the

average effective tariff on imported goods was 0.363%, which is a weighted average of zero and 6.6%, where the weights are the respective import shares. Similarly, 88.2% of all exported goods from Luxembourg in 2007 were sold within the EEA and were exempt from tariffs. The remaining share of exported goods were subject to a tariff rate of 9%, which is the MA-OTRI in 2006 for the European Union. Thus, the effective tariff on exported goods is 1.062%, which is a weighted average of zero and 9%, where the weights are the respective export shares.

In the third group there are nine model parameters that we jointly calibrate by requiring the resulting steady state of the model to match values observed in the data.

The relative weight of durables and non-durables in the utility function (ϕ) is calibrated in order to reproduce the share of durables consumption expenditure from the final consumption expenditure of households equal to 0.116 (average annual share between 1995-2008). The long-run, constant, and exogenous interest rate in the model equals \bar{r} if the country settles down to a net foreign position equals to its steady-state value, so the interest rate risk premium is zero. We calibrate its value to match the observed net foreign position 85% of GDP in Luxembourg (represented by \bar{f}). The implied value is 2.035%. The parameter related to the long-run debt/GDP ratio (d^{LR}) is calibrated in order to reproduce the observed debt/GDP ratio of Luxembourg equal to 0.069.

The share of transfers to resident households (ϱ_1) and the share of public investment into infrastructures (ϱ_2) in core (government) expenditure are calibrated in order to make the model replicate the share of government transfers in total government expenditure (data from OECD annual national accounts, years 2003-2007) and the share of government investment in total government expenditure (data from OECD annual national accounts, years 2003-2007). The replacement ratio of unemployment benefit for domestic workers ($REP1$) and the replacement ratio of unemployment benefit for foreign workers ($REP2$), both are expressed as a share of the total gross income of employed domestic workers, are calibrated in order to replicate a 5% unemployment rate of type-1 workers and a ratio of

type-1 to type-2 workers equal to 1.4238. The calibrated parameter values are reported in Appendix B.

Finally, the foreign real output level (Y^*) and the price of imported goods (p_M^*) are calibrated to match an exports/GDP ratio equal to 0.80 and an imports/GDP ratio equal to 0.45.

5 LSM at work

We now discuss the steady state of the model, which reflects the calibration choices introduced in the previous section. Next, to illustrate the capabilities of LSM, we assess the consequences of an increase in the replacement rate, a measure often proposed to improve the conditions of the unemployed, and of a decrease in the mark-up, associated with liberalizations in the product market, another measure often advocated in the policy debate. For each of the mentioned policy measures, we focus on the effects on a set of key variables. They are the changes in the per-capita wages of resident and non-resident workers, in employment of resident and non-resident workers, in the total wage bill for resident and non-resident workers, in overall firms' profits, in the private demand components (Consumption, Investment, Net exports), in the overall gross domestic product (GDP), in government deficit, and in total factor productivity (TFP, as an overall measure of the productivity of the production factors). We focus on the changes in each variable with respect to its starting value, and use +, ++ and +++ to denote an increase in the range of, respectively, 0-0.5%, 0.5-1% or larger than 1%. The symbols -, --, and --- have a similar interpretation for negative changes. More detailed results and findings for other variables are available upon request.

5.1 Steady state

The steady state values for the endogenous variables of LSM are determined by the interaction of model specification and parameter calibration.

In terms of final demand, the consumption, investment and public expenditure to GDP ratios are about 37.5%, 30.3% and 32.831%, respectively. This leaves a share for net exports smaller than the actual value for Luxembourg: in order to improve this ratio it would be necessary to introduce the re-export service and explicitly model the financial sector, which are left for future research.

GDP can be also decomposed into wages, profits and returns on capital. In this case, the respective shares of GDP are about 44%, 28% and 28%.

In terms of production factors, employment of resident workers is about 95% of the labour force, with about 94% of employment in the tradable sector. Similarly, about 94% of capital is in the tradable sector, and the overall capital to GDP ratio is about 2.9. Employment of the non-resident workers can be interpreted as a percentage of the people who would be willing to work in Luxembourg, and the value in this case is about 67%, much smaller than for the resident population but still considerable and in line with the segmented labour market. The wages of the non-resident workers are about 15% lower than those of the resident workers.

Finally, for the public sector, the deficit is very low (due to a comparable level of tax receipts and expenditures) and the public debt is about 7% of GDP, in line with actual values.

5.2 Higher replacement rate

A commonly advocated measure to attenuate the income loss of unemployed is to increase the replacement rate of both resident and non-resident workers, namely, the fraction of the wage they get in the form of unemployment benefits. We consider a permanent increase of 1% in the replacement rate and report the results in Table 1.

Looking at our simulations, it turns out that, in addition to the expected positive income effect for the unemployed, there is also an unexpected positive effect on the wage of workers that are still employed. Due to the working of the labour market, if the outside option for workers improves, their wage has also to increase.

The ultimate impact of such changes is to affect the employment of workers. This effect will indeed partially offset the positive impact of this policy. Therefore, we have higher wages but lower employment for workers, with the latter effect dominating the former so that the total wage bill actually decreases.

Lower total wages for resident workers imply lower available income, rather than higher as hoped, so that consumption ultimately decreases. The lower demand for consumption shrinks the firms' profits, which in turns reduces investment, which further reduces demand and gross domestic product (GDP). The only positive effects is on net trade, since lower consumption decreases imports. All in all, due to the specific patterns of the labour market, a policy aiming at alleviating the cost of firms' adjustments on the labour market has ultimately further worsened the situation.

In addition, the higher replacement rate combined with lower employment makes public expenditures for unemployment benefits increase. Tax receipts decrease due to lower wages, profits and consumption. And the combination of higher expenditures and lower receipts increases the government deficit. Moreover, there is a compression in government investment (infrastructure, but also research and development, education, etc.), which translates into a negative impact on the evolution of total factor productivity.

In summary, while at first sight desirable, an increase in the replacement rate could have a negative rather than a positive impact on the workers as a whole, and on the entire economy. The magnitude of the reaction of the economy is indeed driven by the calibration of the model. Still, the model is very useful to understand the potential problem with this policy, which is an increase in the wages of the employees associated with the higher unemployment benefits. Hence, a potential solution to implement a policy of this type is to break the link between higher benefits and higher wages. If higher benefits for the unemployed are associated with stable wages for the employees, the negative effects on employment could be avoided, as well as those on the total wage bill, income and consumption. Alternatively, a higher replacement rate associated with tighter

conditions or a limited duration would cushion the adverse effects identified here by providing the right incentives to come back to the labour market when the transitory decline in activity is over. However, it would still be necessary to find a compensation for the higher government expenditure. Since higher taxes could depress income (or profits and investment), the ideal solution would be a reduction in non-productive government consumption.

5.3 Lower mark-up

Liberalization of the product markets is commonly considered as a tool to increase competition, decrease the mark-up and therefore improve the welfare of the consumers. However, a lower mark-up can be expected to lower the profits of the firms, and worse conditions for the firms are sooner or later translated into worse conditions for the workers. Hence, it is a-priori unclear whether the effects of a lower mark-up are overall beneficial or not.

In Table 2 we present the results of a 1% permanent decrease in the mark-up. It turns out that the situation is indeed more complex than what the common sense would suggest. One important reason for this is that profits also depend on sales: price reductions have a favorable effect on sales that more than compensates the lower goods prices.

As a consequence of the bargaining in the labour market, higher profits translate into higher real wages for (both resident and non-resident) workers. In turn, higher wages reduce labour demand and therefore decrease employment. However, the total wage bill is increased, as well as income and therefore consumption. As said before, higher demand further boosts profits, and therefore also investment, which brings about an additional increase in private demand, which is only in part compensated by higher imports.

Moreover, higher profits, total wages and consumption imply higher tax receipts and lower expenditures in unemployment benefits, thus improving substantially the public finances, i.e., the government deficit decreases substantially.

Hence, an expansionary fiscal policy is in principle possible, while this effect is generally not taken into account.

In summary, this is an example of a policy measure whose overall effects are uncertain at first sight but turns out to be even more beneficial than expected when more thoroughly evaluated. The issue is that all the consequences of the policy change should be jointly evaluated, and not only those related to one market or one type of social actor, and this can be done in a model like our LSM.

6 Conclusions

In this paper we have developed a structural macroeconometric model for Luxembourg of the NOEM-DSGE type. The model, labeled LSM for Luxembourg Structural Model, is characterized by a careful theory-based specification of the economy, which is represented by households, government, firms and unions, which interact in the product, labour and financial markets.

A properly calibrated version of LSM provides useful qualitative insights on the expected consequences of changes in economic policy, and can also be relevant to assess the effects and propagation of several types of economic shocks.

While LSM includes a set of specific features of the Luxembourg economy, such as a segmented labour market combined with strong union power, its general structure can be of general interest for modelling small open economies.

To conclude, there is of course scope for additional interesting research in this area, ranging from estimation of a simplified version of the model to the specification of an even more complex model with a more complex structure for the financial sector.

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Table 1: Effects on selected variables of a 1% permanent increase in the replacement rate

<i>LSM mnemonic</i>	<i>Variable</i>	Horizon in years after the shock								
		1y	2y	3y	4y	5y	10y	20y	50y	
GDP	GDP	--	--	--	--	--	--	---	---	
c	Consumption	--	--	---	---	---	---	---	---	
x	Investment	-	-	-	-	-	-	-	-	
NX_IG	Export share - intermediate goods	+++	+++	+++	+++	+++	+++	+++	+++	
govdef	Government deficit	+++	+++	+++	+++	+++	+++	+++	---	
n1	Employment, resident	--	--	--	--	--	--	--	--	
n2	Employment, non resident	--	--	--	--	--	--	--	--	
profit	Profits	---	---	---	---	---	---	---	---	
w1	Wages, resident	+	+	+	+	+	+	-	-	
w2	Wages, non resident	+	+	+	+	+	+	-	-	
wage_bill_1	Total wages, resident	--	--	--	--	--	--	--	--	
wage_bill_2	Total wages, non resident	--	--	--	--	--	--	--	--	
tfp	Total Factor Productivity	-	-	-	-	-	-	-	-	

An increase in the replacement rate.

Table 2: Effects on selected variables of a 1% permanent decrease in the markup

<i>LSM mnemonic</i>	<i>Variable</i>	Horizon in years after the shock								
		1y	2y	3y	4y	5y	10y	20y	50y	
GDP	GDP	+	+	+	+	+	+	+	+	
c	Consumption	++	++	++	++	++	++	++	++	
x	Investment	+	+	+	+	+	+	++	++	
NX_IG	Export share - intermediate goods	---	---	---	---	---	---	---	---	
govdef	Government deficit	---	---	---	---	---	---	---	+	
n1	Employment, resident	-	-	-	-	-	-	-	-	
n2	Employment, non resident	-	-	-	-	-	-	-	-	
profit	Profits	++	++	++	++	++	++	++	++	
w1	Wages, resident	+	+	+	+	+	+	+	+	
w2	Wages, non resident	+	+	+	+	+	+	+	+	
wage_bill_1	Total wages, resident	+	+	+	+	+	+	+	+	
wage_bill_2	Total wages, non resident	+	+	+	+	+	+	+	+	
tfp	Total Factor Productivity	+	+	+	+	+	+	+	+	

A decrease in the markup.

Source: LSM simulations.

Note: +, ++, and +++ indicate, respectively, an increase in the range 0-0.5%, 0.5-1% or larger than 1% with respect to the initial value.

-, --, and --- indicate, respectively, a decrease in the range -0.5 - 0%, -1 -0.5% or smaller than -1% with respect to the initial value.

A Appendix: Derivation of the symmetric equilibrium of LSM

In the following subsections first we specialize the analysis of the production sector and labour market to the case of a CES production function, and then we summarize the equilibrium conditions for the various sectors under the case of a CES production function. The equilibrium conditions are normalized by the exogenous technological progress and by the cohort size, so that we express variables in efficiency terms. For the sake of simplicity, we maintain the previous notation, but now variables are measured in efficiency units.

A.1 The nested CES case

We do not distinguish between tradable and non-tradable goods, but the same production function is assumed in both production processes

$$\begin{aligned} y &= A \left[\alpha k^\lambda + (1 - \alpha) (\Lambda h)^\lambda \right]^{\frac{1}{\lambda}} \\ h &= \left[\varkappa_1 (a_1 h_1)^\kappa + \varkappa_2 (a_2 h_2)^\kappa \right]^{\frac{1}{\kappa}} \end{aligned} \quad (35)$$

with $\varkappa_2 = 1 - \varkappa_1$. Note that Λ represents a labour-augmenting productivity parameter. We use this nested CES specification since it clearly distinguishes the elasticity of substitution between aggregate labour and capital, and that between the two types of labours. A few additional comments are in order. First, if $\lambda \rightarrow 0$ and $\kappa \rightarrow 0$, then both CES aggregators collapse to standard Cobb-Douglas forms:

$$\begin{aligned} y &= A k^\alpha (\Lambda h)^{1-\alpha} \\ h &= (a_1 h_1)^{\varkappa_1} (a_2 h_2)^{\varkappa_2} . \end{aligned}$$

In this case, it is evident that \varkappa_j represents the share of labor income that accrues to type- j employment. In general, these parameters remain strictly linked to the distribution of income across different types of workers. Second, in (35) only

relative labor productivity matters, i.e. a_1/a_2 . Finally, we allow for a (purely external) effect of the stock of public infrastructure ($INFR_t$) on the Total Factor Productivity, A . In particular, we model A as:

$$A = (INFR_t)^\varpi \cdot EXOG,$$

where $0 < \varpi < 1$, $EXOG$ represents exogenous technical progress growing at a constant rate γ . Note also that:

$$\begin{aligned} \frac{\partial y}{\partial h} &= (\Lambda A)^\lambda (1 - \alpha) \left(\frac{h}{y}\right)^{\lambda-1}, \\ \frac{\partial h}{\partial h_z} &= \varkappa_z a_z^\kappa \left(\frac{h_z}{h}\right)^{\kappa-1}, \\ \frac{\partial^2 y}{\partial h^2} &= (\lambda - 1) \frac{\partial y}{\partial h} \left[1 - \frac{(1 - \alpha) h^\lambda}{\alpha(\Lambda k)^\lambda + (1 - \alpha) h^\lambda}\right] \frac{1}{h}, \\ \frac{\partial^2 h}{\partial h_z^2} &= (\kappa - 1) \frac{\partial h}{\partial h_z} \left[1 - \varkappa_z \left(\frac{a_z h_z}{h}\right)^\kappa\right] \frac{1}{h_z}. \end{aligned}$$

It follows that the first order conditions of the firm can be written as:

$$\begin{aligned} \frac{p}{\mu} (\Lambda A)^\lambda (1 - \alpha) \left(\frac{h}{y}\right)^{\lambda-1} \varkappa_z a_z^\kappa \left(\frac{h_z}{h}\right)^{\kappa-1} &= (1 + \tilde{\tau}_L) w_z \\ \frac{p}{\mu} A^\lambda \alpha \left(\frac{k}{y}\right)^{\lambda-1} &= r \end{aligned}$$

Then we have:

$$\begin{aligned} \frac{rk}{py} &= \frac{1}{\mu} \alpha \left(A \frac{k}{y}\right)^\lambda, \\ \frac{(1 + \tilde{\tau}_L) \sum_{j=1}^2 w_j h_j}{py} &= \frac{1}{\mu} (1 - \alpha) \left(A \Lambda \frac{h}{y}\right)^\lambda, \\ \frac{(1 + \tilde{\tau}_L) w_z h_z}{py} &= \frac{1}{\mu} (1 - \alpha) \left(A \Lambda \frac{h}{y}\right)^\lambda \varkappa_z \left(\frac{a_z h_z}{h}\right)^\kappa = \end{aligned}$$

$$\frac{(1 + \tilde{\tau}_L) \sum_{j=1}^2 w_j h_j}{py} \varkappa_z \left(\frac{a_z h_z}{h} \right)^\kappa,$$

and for the labour market:

$$\epsilon_z = \left\{ \left[\frac{1 - \lambda \mu}{\mu} (1 - \alpha) \left(A \Lambda \frac{h}{y} \right)^\lambda + \lambda - \kappa \right] \varkappa_z \left(\frac{a_z h_z}{h} \right)^\kappa + \kappa - 1 \right\}^{-1}, \quad z = 1, 2.$$

A.2 Households

The key equations for the Household sector of LSM are:

$$\begin{aligned} C_{t+1} &= \frac{\mathcal{E}_{t+1}}{\gamma} \left(C_t - \frac{\eta - \varphi}{\eta} \frac{A_t}{\zeta_t - \bar{Z}_t} \right) \\ D_t &= \xi_t C_t \\ A_t &= R_t \frac{A_{t-1}}{\gamma \eta} + W_t - Z_t C_t \\ W_t &= (1 - \tau_L) [w_{1,t} H_{1,t} + \bar{w}_{1,t} (1 - H_{1,t})] + (1 - \tau_K) \Pi_t + \varrho_1 \bar{G}_t \\ \zeta_t &= \bar{Z}_t + \mathcal{E}_{t+1} \frac{\varphi}{R_{t+1}} \zeta_{t+1} \\ \xi_t &= \left\{ \frac{\phi}{1 - \phi} \left[\varkappa_t^d - \frac{\varkappa_{t+1}^d (1 - \delta_D)}{R_{t+1} \frac{p_t}{p_{t+1}}} \right] \right\}^{\frac{1}{v-1}} \\ \mathcal{E}_t &= \left\{ \left[\frac{\phi + (1 - \phi) \xi_t^v}{\phi + (1 - \phi) \xi_{t-1}^v} \right]^{\frac{1-v-\sigma}{v}} \beta R_t \frac{p_{t-1}}{p_t} \right\}^{\frac{1}{\sigma}} \\ Z_t &= (1 + \tau_C) p_t \left[1 + \varkappa_t^d \left(\xi_t - \frac{1 - \delta_D}{\varphi} \frac{\xi_{t-1}}{\mathcal{E}_t} \right) \right] \end{aligned}$$

A.3 Asset Stock

The key equations for the Asset Stock sector of LSM are:

$$F_t = A_t - B_t - p_t \nu_t K_t$$

$$\begin{aligned}
K_t &= \left[1 - \delta_K + \frac{\Xi_1}{\varsigma} \left(\gamma \eta \frac{I_t}{K_{t-1}} \right)^\varsigma + \Xi_2 \right] \frac{K_{t-1}}{\gamma \eta} \\
\nu_t &= \frac{1}{\Xi_1} \left(\gamma \eta \frac{I_t}{K_{t-1}} \right)^{1-\varsigma} \\
p_t \nu_t &= \frac{(1 - \tau_K) r_{t+1} + p_{t+1} \left(\tau_K \delta_K - \frac{\gamma \eta I_{t+1}}{K_t} \right)}{R_{t+1}} \\
&\quad + \frac{p_{t+1} \nu_{t+1} \left[1 - \delta_K + \frac{\Xi_1}{\varsigma} \left(\frac{\gamma \eta I_{t+1}}{K_t} \right)^\varsigma + \Xi_2 \right]}{R_{t+1}}
\end{aligned}$$

A.4 Final good sector

$$\begin{aligned}
Y &= \mathcal{N}^{\rho-\mu} \left(\sum_{j=1}^{\mathcal{N}} y_j^{\frac{1}{\mu}} \right)^\mu \\
y_j &= \left(\frac{p_j}{p} \right)^{\frac{\mu}{1-\mu}} Y \mathcal{N}^{\frac{\rho-\mu}{\mu-1}} \\
p &= \mathcal{N}^{-(\rho-\mu)} \left(\sum_{j=1}^{\mathcal{N}} p_j^{\frac{1}{1-\mu}} \right)^{1-\mu}
\end{aligned}$$

A.5 Intermediate goods sector

A.5.1 Non-tradable goods

The key equations for the non-tradable goods sector and associated labour market are:

$$\begin{aligned}
y^{NT} &= A \left[\alpha (k^{NT})^\lambda + (1 - \alpha) (\Lambda h^{NT})^\lambda \right]^{\frac{1}{\lambda}} \\
h^{NT} &= \left[\varkappa_1 (a_1 h_1^{NT})^\kappa + \varkappa_2 (a_2 h_2^{NT})^\kappa \right]^{\frac{1}{\kappa}}
\end{aligned}$$

$$\begin{aligned}
\mu(1 + \tilde{\tau}_L) w_z^{NT} &= p^{NT} (1 - \alpha) (A\Lambda)^\lambda \varkappa_z a_z^\kappa \left(\frac{y^{NT}}{h^{NT}} \right)^{1-\lambda} \left(\frac{h^{NT}}{h_z^{NT}} \right)^{1-\kappa} \\
\mu r &= p^{NT} \alpha A^\lambda \left(\frac{y^{NT}}{k^{NT}} \right)^{1-\lambda} \\
p^{NT} &= \mathcal{N}^{\frac{\rho-\mu}{\mu}} \left(\frac{y^{NT}}{Y} \right)^{\frac{1-\mu}{\mu}} p \\
\epsilon_z &= \left\{ \left[\frac{1-\lambda\mu}{\mu} (1 - \alpha) \left(A\Lambda \frac{h^{NT}}{y^{NT}} \right)^\lambda + \lambda - \kappa \right] \times \right. \\
&\quad \left. \varkappa_z \left(\frac{a_z h_z^{NT}}{h^{NT}} \right)^\kappa + \kappa - 1 \right\}^{-1} \\
\theta_z \left(1 + \frac{w_z^{NT} - \bar{w}_z}{w_z^{NT}} \epsilon_z^{NT} \right) \frac{\tilde{\pi}^{NT}}{h_z^{NT}} &= (1 + \tilde{\tau}_L) (1 - \theta_z) (w_z^{NT} - \bar{w}_z) \\
\pi^{NT} &\equiv \left(1 - \frac{1}{\mu} \right) p^{NT} y^{NT} - \psi \\
\tilde{\pi}^{NT} &= p^{NT} y^{NT} - (1 + \tilde{\tau}_L) \sum_{s=1}^2 w_s^{NT} h_s^{NT}
\end{aligned}$$

A.5.2 Tradable goods

The key equations for the tradable goods sector and associated labour market are:

$$\begin{aligned}
y^T &= A \left[\alpha (k^T)^\lambda + (1 - \alpha) (\Lambda h^T)^\lambda \right]^{\frac{1}{\lambda}} \\
h^T &= \left[\varkappa_1 (a_1 h_1^T)^\kappa + \varkappa_2 (a_2 h_2^T)^\kappa \right]^{\frac{1}{\kappa}} \\
\mu(1 + \tilde{\tau}_L) w_z^T &= p^T (1 - \alpha) (A\Lambda)^\lambda \varkappa_z a_z^\kappa \left(\frac{y^T}{h^T} \right)^{1-\lambda} \left(\frac{h^T}{h_z^T} \right)^{1-\kappa} \\
\mu r &= p^T \alpha A^\lambda \left(\frac{y^T}{k^T} \right)^{1-\lambda} \\
p^T &= p^H = p^F \\
p^H &= \mathcal{N}^{\frac{\rho-\mu}{\mu}} \left(\frac{s^H y^T}{Y} \right)^{\frac{1-\mu}{\mu}} p
\end{aligned}$$

$$\begin{aligned}
p^F &= (1 - t^F) (\mathcal{N}^*)^{\frac{\rho - \mu}{\mu}} \left(\frac{s^F y^T}{Y^*} \right)^{\frac{1 - \mu}{\mu}} p^* \\
s^F &= 1 - s^H \\
\epsilon_z &= \left\{ \left[\frac{1 - \lambda \mu}{\mu} (1 - \alpha) \left(A \Lambda \frac{h^T}{y^T} \right)^\lambda + \lambda - \kappa \right] \times \right. \\
&\quad \left. \chi_z \left(\frac{a_z h_z^T}{h^T} \right)^\kappa + \kappa - 1 \right\}^{-1} \\
\theta_z \left(1 + \frac{w_z^T - \bar{w}_z}{w_z^T} \epsilon_z^T \right) \frac{\tilde{\pi}^T}{h_z^T} &= (1 + \tilde{\tau}_L) (1 - \theta_z) (w_z^T - \bar{w}_z) \\
\tilde{\pi}^T &= p^T y^T - (1 + \tilde{\tau}_L) \sum_{s=1}^2 w_s^T h_s^T \\
\pi^T &\equiv \left(1 - \frac{1}{\mu} \right) p^T y^T - \psi
\end{aligned}$$

A.5.3 Importers

For the imported good sector we have:

$$\begin{aligned}
p^M &= \mu (1 + t^M) p_M^* \\
y^M &= \left(\frac{\mu (1 + t^M) p_M^*}{p \mathcal{N}^{\frac{\rho - \mu}{\mu}}} \right)^{\frac{\mu}{1 - \mu}} Y \\
\pi^M &\equiv (\mu - 1) (1 + t^M) p_M^* y^M - \psi_j
\end{aligned}$$

A.6 Aggregation

The aggregate variables are given by

$$\begin{aligned}
Y &= \mathcal{N}^{\rho - \mu} \left[\Theta \mathcal{N} (y^{NT})^{\frac{1}{\mu}} + (1 - \Theta) \mathcal{N} (y^H)^{\frac{1}{\mu}} + (1 - \Theta^*) \mathcal{N}^* (y^M)^{\frac{1}{\mu}} \right]^\mu \\
P &= \mathcal{N}^{\mu - \rho} \left[\Theta \mathcal{N} (p^{NT})^{\frac{1}{1 - \mu}} + (1 - \Theta) \mathcal{N} (p^H)^{\frac{1}{1 - \mu}} \right. \\
&\quad \left. + (1 - \Theta^*) \mathcal{N}^* (p^M)^{\frac{1}{1 - \mu}} \right]^{1 - \mu}
\end{aligned}$$

$$\begin{aligned}
H_z &= [\Theta h_z^{NT} + (1 - \Theta) h_z^T] N \\
w_z &= \frac{w_z^{NT} \Theta h_z^{NT} + w_z^T (1 - \Theta) h_z^T}{\Theta h_z^{NT} + (1 - \Theta) h_z^T} \\
K &= [\Theta k^{NT} + (1 - \Theta) k^T] N \\
\Pi &= [\Theta \pi^{NT} + (1 - \Theta) \pi^T] N + (1 - \Theta^*) N^* \pi^M
\end{aligned}$$

A.7 Numeraire

We set the numeraire as the price of the non-traded goods:

$$p^{NT} = 1$$

A.8 Government

The key equations for the Government sector are:

$$\begin{aligned}
B_t &= R_t \frac{B_{t-1}}{\gamma \eta} + G_t - T_t \\
T_t &= \tau_K [i_t F_{t-1} + (r_t - \delta_K p_t) K_{t-1} + \Pi_t] + \\
&\quad (\tau_L + \tilde{\tau}_L) (w_{1,t} H_{1,t} + w_{2,t} H_{2,t}) + \tau_L \bar{w}_{1,t} (1 - H_{1,t}) + \\
&\quad \tau_C p_t \left[1 + \varkappa_t^d \left(\xi_t - \frac{1 - \delta_D}{\varphi} \frac{\xi_{t-1}}{\mathcal{E}_t} \right) \right] C_t + \\
&\quad \tau_M (1 - \Theta^*) N^* p_M^* y^M \\
G_t &= \bar{w}_{1,t} (1 - H_{1,t}) + TR_t^F (\tau_L + \tilde{\tau}_L) w_{2,t} H_{2,t} + \bar{G}_t \\
TR_t &= \varrho_1 \bar{G}_t \\
GCON_t &= \varrho_2 \bar{G}_t \\
INFR_INV_t &= (1 - \varrho_1 - \varrho_2) \bar{G}_t \\
INFR_t &= (1 - \delta_{INFR}) INFR_{t-1} + INFR_INV_t \\
\bar{G}_t &= \vartheta \bar{G}_{t-1} + (1 - \vartheta) d^{LR} \begin{bmatrix} T_t - \bar{w}_{1,t} (1 - H_{1,t}) \\ -TR_t^F (\tau_L + \tilde{\tau}_L) w_{2,t} H_{2,t} \end{bmatrix}
\end{aligned}$$

$$\bar{w}_{1,t} = rep_1 NETINC_t$$

$$\bar{w}_{2,t} = rep_2 NETINC_t$$

where:

$$NETINC_t = (1 - \tau_L) [w_{1,t} H_{1,t} + \bar{w}_{1,t} (1 - H_{1,t})] + \\ [(1 - \tau_K) r_t + \tau_K \delta_K p_t] K_{t-1} + (1 - \tau_K) \Pi_t$$

The last two equations determine unemployment benefits as a function of net income.

A.9 Exogenous variables

The following variables are treated as exogenous in LSM:

$$R_t \equiv 1 + (1 - \tau_K) i_t \\ i_t = \bar{i} + \xi_i \left[\exp \left(\bar{f} - \frac{F_t}{GDP_t} \right) - 1 \right] + \varepsilon_{it} \\ A = A_0 (INFR_t)^\varpi$$

B Appendix: LSM parameters and their calibrated value

As discussed in the main text, we can divide the LSM parameters into three groups according to the way we set their values. Here we provide additional details for the parameters of each group.

B.1 Parameter values based on other papers in the literature or theoretical considerations

- β : the subjective discount factor. We set this parameter to 0.995.
- v : the parameter related to the elasticity of substitution between consumption and dwellings in the utility function. We set the parameter in order to reproduce an elasticity of substitution equal to 1.5.
- σ : this parameter equals $1/\sigma^c$, where σ^c is the elasticity of intertemporal substitution. We assume logarithmic preferences, i.e. we set the parameter equal to unity.
- δ_K : the depreciation rate of physical capital. Following Backus, Henriksen, and Storesletten (2008), we choose a value of 8.5%.
- δ_D : the depreciation rate of the stock of dwellings. Again, following Backus, Henriksen, and Storesletten(2008), we set the parameter equal to 1.5%.
- δ_{INFR} : the depreciation rate of the stock of public infrastructure. The same reference as before suggests a value of 4.15%.
- α : the relative weight of physical capital in the CES production function. This parameter is strictly related to the capital share in output (actually, under a Cobb-Douglas specification, the two coincide). We set the parameter equal to 0.36, a standard value. The implied capital share in production under the benchmark parameterization lies around 25%.

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- ξ_i : the elasticity of the international interest rate with respect to the national debt/GDP ratio. Following Schmitt-Grohe and Uribe (2004), we set the parameter equal to 0.000742.
 - TR_t^F : the percentage of total labour taxes on non-resident workers that is transferred back to non-resident workers. We choose a value equal to 0.6.
 - ϑ : the persistence of core government expenditure. We choose a value equal to 0.9.
 - ς : the elasticity of the adjustment cost with respect to the investment-capital ratio. Following Boldrin, Christiano, and Fisher (2001), we set the parameter equal to $1 - 1/0.23$.
 - Θ : the share of non-traded domestic varieties. We set the parameter equal to 0.5.
 - N : the number of available domestic differentiated intermediate goods. We set the value equal to 2.
 - Θ^* : the share of traded foreign varieties (the share of importable varieties into Luxemburg). We choose a value equal to 0.5 for the sake of symmetry.
 - N^* : the number of available foreign differentiated intermediate goods. We choose a value equal to 2, again for the sake of symmetry.
 - ρ : the parameter capturing the increasing returns to variety. We assume no returns to variety in the benchmark parametrization, and set the parameter equal to 1.
 - μ : the parameter related to the elasticity of substitution among intermediate goods. We set the parameter to obtain an elasticity equal to 1.5.
 - ψ_j : the fixed cost to enter the market of intermediate good j . We choose a small value equal to 0.00001.

- θ_z : the relative bargaining power of the union for type z workers. We choose a value equal to 0.5.
- P^* : the foreign aggregate price level. Normalized to unity.
- Λ : labour-augmenting productivity parameter. We normalize it to unity.
- a_1 : the parameter augmenting type-1 labour in the labour CES aggregator. It is normalized to unity.
- a_2 : the parameter augmenting type-2 labour in the labour CES aggregator. It is normalized to unity.
- κ : the parameter related to the elasticity of substitution between the two labour types in the CES labour aggregator. We set the value of the parameter in order to obtain an elasticity equal to 1.5.
- ϖ : the parameter related to the elasticity of TFP with respect to public infrastructure. We choose a value equal to 0.01.

B.2 Parameter values inferred from direct evidence on the value of the parameter:

- φ : the individual survival rate, i.e. at the individual level, one minus the probability of dying at the end of the current period. Average life expectancy at birth in Luxemburg was 79.18 years in 2008 (CIA factbook): the survival rate that reproduces this outcome is 0.987.
- \bar{f} : The steady-state net foreign position relative to GDP. The average value of net foreign position was 95% and 75% of GDP at the end of 2007 and 2008, respectively (according to the bulletin of the Central Bank of Luxembourg). Thus, we set the parameter to 0.85.
- τ_C : the tax rate on consumption (both durables and non-durables). We choose a value of 25,1%, taken from *Taxation trends in the EU*, European

Commission, 2008. Note that the tax base for consumption tax includes non-durables consumption expenditure and the investment into durables.

- τ_L : the tax rate on labour related income, paid by the employee. We follow again *Taxation trends in the EU*, 2008, and set the value to 20.1%. The figure has been obtained this way: the total average effective tax rate on labour equals 29,6%, but only 67,9% of this amount is paid by the employee. Hence, the average effective tax rate on labour paid by the employee becomes 20.1%.
- $\tilde{\tau}_L$: the social contribution rate on labour related income, paid by the employer. Given the previous result, we set the parameter to 9.5%.
- τ_K : the tax rate on profits and capital income. The source *Taxation trends in the EU*, 2008, does not report, because of data availability problems, an estimate of the average effective tax rate on capital. We take the average effective tax rate on corporate profits as a useful approximation, and set the parameter equal to 29.6%.
- η : the population growth rate. We set the parameter equal to 1.012, since the current population growth rate in Luxemburg is 1.2% (data from CIA factbook, year 2008).
- γ : the rate of exogenous long-run technological progress. We set this parameter equal to 0.6%, which is the average TFP growth rate in Luxemburg over the 1995-2009 period, as reported in the Annual Report of the Central Bank of Luxembourg (2006, p. 54).
- t^M : the tariff on imported goods. The Overall Trade Restrictiveness Index in 2006 for the European Union equals to 6.6%, as computed by the World Bank. This index is the ad-valorem equivalent of all tariff and non-tariff barriers that a country imposes against foreign imports. However, in 2007 94.5% of all imported goods were originated from countries within the EEA

and no tariff was applied on them. Thus, the effective tariff on imported goods is 0.363%, which is a weighted average of zero and 6.6%, where the weights are the respective import shares.

- t^F : the tariff on exported goods. As before, in 2007 88.2% of all exported goods were sold within the EEA and were exempt from tariffs. The remaining share of exported goods were subject to a tariff rate equal to 9%, which is the MA-OTRI in 2006 for the European Union. This is the ad-valorem equivalent of all tariff and non-tariff barriers that a country faces as an exporter. Thus, the effective tariff on exported goods is 1.062%, which is a weighted average of zero and 9%, where the weights are the respective export shares.
- λ : the parameter related to the elasticity of substitution between capital and labour in the CES production function. Guarda (1997) estimates the elasticity to be 1.012 in the tradables sector. We set the value of the parameter in order to obtain the elasticity equal to 1.012.
- χ_1 : the share of type-1 labour in the labour CES aggregator. We choose a value equal to 0.6 to reflect the fact that approximately 60% of the employed workforce is resident.

B.3 Parameter values calibrated so the model matches observed ratios in the data:

- ϕ : the relative weight of durables and non durables consumption in the utility function. We calibrate the parameter in order to reproduce the share of durable goods consumption expenditure from the final consumption expenditure of households equal to 0.116 (average annual share between 1995-2008). The implied value of ϕ is 0.898.
- ϱ_1 : the share of transfers to resident households in core (government) expenditure. We set the parameter equal to 43.154%, in order to make the

model replicate the share of government transfers in **total** government expenditure (data from OECD annual national accounts, years 2003-2007).

- ϱ_2 : the share of public investment in infrastructures in core (government) expenditure. We set the parameter equal to 11.369%, in order to make the model replicate the share of government investment in **total** government expenditure (data from OECD annual national accounts, years 2003-2007).
- \bar{i} the long-run, constant, and exogenous interest rate if the country settles down to a net foreign position equals to its steady-state value, so the interest rate risk premium is zero. We calibrate its value to match the observed net foreign position 85% of GDP in Luxembourg (represented by \bar{f}). The implied value equals to 2.035%.
- d^{LR} : the parameter related to the long-run debt/GDP ratio. We calibrate the parameter in order to reproduce the observed debt/GDP ratio of Luxembourg equal to 0.069. The implied value for the parameter is 1.0009212.
- Y^* : the foreign real output level. We calibrate it in order to reproduce an exports/GDP ratio equal to 0.80. The implied value is 5.016.
- p_M^* : the price of imported goods. We calibrate a value equal to 0.524, in order to make the model reproduce an imports/GDP ratio equal to 0.45.
- $REP1$: replacement ratio of unemployment benefit for domestic workers, expressed as a share of the total gross income of employed domestic workers. We choose a value equal to 21.945% in order to replicate a 5% unemployment rate of type-1 workers.
- $REP2$: replacement ratio of unemployment benefit for foreign workers, expressed as a share of the total gross income of employed domestic workers. We choose a value equal to 15.987% in order to replicate the ratio of type-1 to type-2 workers equal to 1.4238.

The Macroeconomic Effects of Fiscal Policy Shocks in Good Times and Bad*

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Abstract

We analyze the macroeconomic effects of fiscal policy shocks conditional on fiscal stress by introducing a model whose parameter evolution explicitly depends on initial conditions. Using US quarterly data from 1960:1-2009:4 we estimate a multivariate threshold autoregressive model where regime switches are triggered by the dynamics of the government debt-to-GDP ratio. We find that a model with two regimes and with a regime switch at 42.6 percent of the debt-to-GDP ratio fits the data better than the benchmark linear model or specifications of the threshold model using other measures of fiscal stress as a threshold variable. In the regime, where the debt-to-GDP ratio is below the estimated threshold, the predictions of our model are similar to the benchmark linear model: a government spending shock has a positive and a tax receipt shock a negative effect on output

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over time. However, if the debt-to-GDP ratio is above this threshold, then the results are the opposite.

Keywords: fiscal policy, fiscal multiplier, public debt, regime switching threshold autoregression

JEL Classification: E62, C34, H60

1 Introduction

The most striking feature of the current economic crisis from a fiscal point of view is the soaring debt-to-GDP ratio in many advanced economies. The recent stream of government bailouts, the workings of the automatic stabilizers and the increased use of fiscal measures to stimulate aggregate economic activity have all contributed to a series of large fiscal deficits and, consequently, to the rapid accumulation of debt. In the US, the debt-to-GDP ratio has almost doubled in the last 2 fiscal years. It has reached 60 percent, a level that has not been seen since the aftermath of World War II, and is expected to steadily rise further. Can we still expect that fiscal policy has the same effects under these circumstances or does the fiscal multiplier depend on the state of public finances in the US?

We aim to answer this question by analyzing the dynamic effects of fiscal policy shocks in an empirical model, where the conditional mean and variance of key macroeconomic variables depend on initial conditions, in particular on the extent of fiscal stress. More specifically, using US quarterly data, we estimate a multivariate threshold autoregression model (TVAR) with two regimes. This permits us to derive conditional dynamic responses of the variables of interest where both, the transmission mechanism and the stochastic properties of the shocks, are regime dependent.

Our main results are as follows. First, we estimate several regime switching models that differ in the measure of fiscal stress used as a conditioning factor and find that the model with the debt-to-GDP ratio as the threshold variable fits the data best. Second, the regime switch in our benchmark specification is estimated

to be triggered at 42.6 percent of the debt-to-GDP ratio, which implies that approximately one quarter of all the observations in our sample are in the high debt-to-GDP regime. Third, we find significant differences between the impulse responses in the two regimes. A positive government spending shock has for most of the response horizon a persistent positive effect on output in the low regime, while almost no effect in the high regime. Also the responses of output to a positive tax shock are qualitatively different across regimes. Both responses are negative on impact but diverge after few quarters. The response in the high debt regime becomes positive, while the one in the low debt regime remains negative.

The finding that the effects of a fiscal expansion change with the state of public finances is in line with the “expansionary fiscal contraction” view first advocated by Giavazzi and Pagano (1990) in their paper on large fiscal consolidations in Denmark and Ireland in the 1980s. Models in this strand of the literature generate fiscal policy multipliers that change when public debt (Blanchard, 1990; Sutherland, 1997), expected future liabilities of the government (Perotti, 1999) or the ratio of government spending to output (Bertola and Drazen, 1993) reach high levels that signal a consolidation. Households expect that a consolidation via higher taxes in the future will be necessary to prevent government debt from reaching undesirable levels. The wealth effects stemming from the changes in the expectations about the future path of taxes and related tax distortions then add to the direct effects of the fiscal policy measures.

This paper contributes to a large pool of literature studying the dynamic effects of fiscal policy shocks on macroeconomic variables. Numerous empirical papers have analyzed these effects assuming that the transmission mechanism of fiscal policy shocks is independent of initial conditions or that fiscal shocks are realizations of the same underlying stochastic process in the entire sample. Within this linear framework, the literature has mainly focused on different ways to identify structural fiscal shocks, with the main methods being the narrative approach (Ramey and Shapiro, 1998; Edelberg, Eichenbaum, and Fisher, 1999; Burnside, Eichenbaum, and Fisher, 2004; Romer and Romer, 2008, 2009), the

structural VAR approach (Blanchard and Perotti, 2002; Perotti, 2004; Fatás and Mihov, 2001; Favero and Giavazzi, 2009; Perotti, 2010)¹ and the sign restrictions (Mountford and Uhlig, 2008).

While using a standard SVAR approach to analyze the effects of fiscal policy shocks, a few recent contributions have emphasized the importance of accounting for fiscal policy sustainability. Favero and Giavazzi (2007) include the level of debt as an exogenous regressor in their SVAR and add an auxiliary equation based on the government budget constraint, that explicitly links the dynamics of the debt-to-GDP ratio to the endogenous variables in the VAR.² Chung and Leeper (2007), instead, include the debt in their VAR as one of the endogenous variables and impose the intertemporal government budget constraint as a set of cross-equation restrictions. Similarly, Burriel, de Castro, Garrote, Gordo, and Prez (2009) include the growth rate of the debt-to-GDP ratio in their VAR to control for fiscal stress and for potential nonlinearities. All these studies show that the estimated effects of fiscal shocks are sensitive to the inclusion of debt. They, however, all imply a time invariant transmission mechanism and cannot give nonlinear, regime dependent responses.

As an already large literature demonstrates these linear projections may not be reliable in episodes of fiscal stress. Giavazzi and his coauthors (Giavazzi and Pagano, 1990, 1995; Giavazzi, Jappelli, and Pagano, 2000; Giavazzi, Jappelli, Pagano, and Benedetti, 2005) and Alesina and Perotti (1996a,b) find empirical evidence for non monotonic/nonlinear effects of fiscal shocks related to the size and persistence of the shock. Their results show that large and persistent fiscal shocks (consolidations) have different effects than small or less persistent ones. In particular, based on panel data from the OECD and some European countries, they document several episodes of sharp fiscal consolidations that are, in contrast to the conventional wisdom, associated with an expansion of private demand.

¹Among these Blanchard and Perotti (2002) allow for some form of nonlinearity. They introduce a dummy variable associated to a large discretionary tax cut in the second quarter of 1975 in an otherwise linear model since this large shock is too big to be treated as a realization of the same stochastic process.

²Afonso and Sousa (2009) perform a similar exercise by including debt in a Bayesian VAR.

Perotti (1999) and Tagkakalis (2008), find that the effects of government revenue and expenditure shocks depend on initial conditions. Perotti (1999) focuses on the response of private consumption to a government spending shock and concludes that it reacts positively when the debt-to-GDP or deficit-to-GDP ratios are low, i.e. during “normal times”, and negatively during “bad times”, characterized by either a high debt-to-GDP ratio or consecutive quarters of high deficit-to-GDP ratio. Tagkakalis (2008) follows the same methodology but conditions on the business cycle. He finds that a government spending shock has a stronger positive effect on consumption during recessions than in expansions due to countercyclical movements in the fraction of households that face binding liquidity constraints.

A distinctive feature of the studies above is that they allow for regime dependent responses only in one equation, while the transmission mechanism for the rest of the system is assumed to be time invariant. For instance, Perotti (1999) and Tagkakalis (2008) both use time invariant models to identify the structural fiscal shocks, which are later used in a univariate regression with regime dependent responses. Such specification implicitly assumes that the shocks come from a single stochastic process. Moreover, it implies that consumption can have a regime dependent response to fiscal shocks, while output, one of the variables used in the near VAR, cannot. Contrary to this, our approach aims at generalizing the idea of regime specific responses by assuming that both, the stochastic process of the shocks and the transmission mechanism in our dynamic model fully depend on initial conditions.

The rapid deterioration of the state of public finances in most of the advanced countries has stimulated considerable recent effort both, into the asymmetric effects of fiscal policy shocks using nonlinear models conditioning on different factors (see for example Ilzetki, Mendoza, and A., 2010; Auerbach and Gorodnichenko, 2010; Corsetti, Meier, and Müller, 2010; Bom, 2010, among others) as well as into the effects of public debt on macroeconomic performance (see for example Checherita and Rother, 2010; Kumar and Woo, 2010; Nickel, Rother, and

Zimmermann, 2010; Reinhart and Rogoff, 2010; Afonso and Jalles, 2010, among others).

The closest to our work are the papers by Choi and Devereux (2005) and Auerbach and Gorodnichenko (2010) that both look at regime dependent effects of fiscal policy shocks in the US using regime switching models. Choi and Devereux (2005) estimate a SETAR model using the real interest rate as the threshold variable and find significantly different responses of investment growth, output growth and interest rates to fiscal policy shocks in the three regimes. Auerbach and Gorodnichenko (2010), instead, estimate a STAR model where they allow for asymmetric effects of fiscal policy shocks over the business cycle.

We contribute to the existing literature in three ways. First, we consider several alternative measures of fiscal stress that can be a potential a source of nonlinearity in the responses to fiscal policy shocks and evaluate empirically which variable is the most relevant. We do so by confronting the models in terms of their goodness of fit, as advocated by Gonzalo and Pitarakis (2002). In addition, we compare the model to the linear alternative using the penalized log likelihood function and two statistical tests.³

Second, to our best knowledge, we are the first to estimate a threshold value of the debt-to-GDP ratio in a nonlinear model for the US economy. Previous papers has used ad hoc values to draw a line between good and bad times in a large panel of countries ranging from 90 percent to 120 percent (Reinhart and Rogoff, 2010; Kumar and Woo, 2010; Corsetti, Meier, and Müller, 2010). Since the debt-to-GDP ratio in the US has been historically low compared to other countries the estimated results were driven by observations from other countries in the sample and are not necessarily applicable to the US economy.

Third, in our benchmark specifications the dynamics of the debt-to-GDP ratio is endogenous to the model. We adopt the specification of Favero and Giavazzi (2007) who use the government budget constraint to explicitly link the dynamics

³Most of the literature analyzing the nonlinear effects of fiscal policy shocks do not carry out any nonlinearity tests or confront the nonlinear model with the linear one in any way. The only exception we are aware of is the paper by Choi and Devereux (2005).

of the debt-to-GDP ratio to the endogenous variables in the VAR. Consequently, we can simulate the responses of the variables in the model taking into account the possibility of a regime switch after the initial shock. We can derive impulse responses not only conditional on the economy being always in the low (high) regime, but also being in the low (high) regime when the fiscal shock hits while allowing the dynamics of the model to determine the evolution of the regime endogenously. The impulse responses are thus not representing the dynamics of each of the regimes in isolation but also take into account the effects that fiscal shocks have on the dynamics of the threshold variable itself.

The rest of the paper is structured as follows. Section 2 presents the methodology used. We describe the data, the model specification and selection of the threshold variable in Section 3. Section 4 describes the estimated dynamic effects of the fiscal policy shocks in the two regimes. We discuss the possibility of employing alternative model specifications and confront our specification with a structural break model in Section 5. Section 6 concludes.

2 Empirical methodology

This section briefly outlines the empirical methodology used. We employ a multivariate endogenous threshold autoregressive model (ET-VAR) to model explicitly the dependence of the transmission mechanism on initial conditions that differ across regimes. We describe the methodology in terms of the two regime model, but everything can be readily extended to a model with more regimes.

2.1 Threshold VARs

Multivariate threshold autoregressive models (TVARs) combine two piecewise linear models with different sets of coefficients over two subsamples (regimes) into a nonlinear VAR (Tsay, 1998). The two regimes are determined by an observed threshold variable, a value of that threshold variable (threshold value) that separates the two regimes and a delay parameter. The model can be formulated

as:

$$\mathbf{y}_t = F(z_{t-d})[\mathbf{c}^{(1)} + \mathbf{\Phi}^{(1)}\mathbf{X}_t^{(1)} + \varepsilon_t^{(1)}] + (1 - F(z_{t-d}))[\mathbf{c}^{(2)} + \mathbf{\Phi}^{(2)}\mathbf{X}_t^{(2)} + \varepsilon_t^{(2)}]$$

where the transition function is defined to be

$$F(z_{t-d}) = I(z_{t-d} \leq r) = \begin{cases} 1 & \text{if } z_{t-d} \leq r \\ 0 & \text{if } z_{t-d} > r \end{cases}$$

an indicator function, \mathbf{y}_t is an $n \times 1$ vector of endogenous variables, $\mathbf{c}^{(j)}$ is an $n \times 1$ vector of intercepts for regime j , $\mathbf{X}_t^{(j)} = [\mathbf{y}'_{t-1} \ \cdots \ \mathbf{y}'_{t-p_j}]'$ is an $np_j \times 1$ vector of lagged values where p_j is the lag length in regime j , $\mathbf{\Phi}^{(j)}$ is an $n \times np_j$ matrix of coefficients for regime j , z_t is the threshold variable, d is the delay parameter, and r is the threshold value. Notice that we do not restrict the model to have the same number of lags across regimes. Also notice that we allow for regime specific covariance matrices for the residuals, that is $\varepsilon_t^{(j)} \sim N(0, \mathbf{\Sigma}^{(j)})$.

2.2 Estimation

For a given threshold variable and lag lengths p_j the model can be estimated in three steps. The first step estimates the coefficient and covariance matrices conditional on the threshold value and the delay parameter. The second step estimates the threshold value conditional on the delay parameter. The last step estimates the delay parameter.

First, for given values of the threshold variable, r , and the delay parameter, d , the model reduces to two linear VARs. The (regime specific) coefficient and covariance matrices of the two piecewise linear models, $\hat{\mathbf{\Phi}}^{(j)}(r, d)$ and $\hat{\mathbf{\Sigma}}^{(j)}(r, d)$, can be estimated with least squares formula using observations from regime j .

Second, for a given value of the delay parameter d we estimate the threshold value by maximizing the conditional likelihood over a grid of values for the threshold value. Galvão (2006) shows that conditional maximum likelihood works better

than least squares estimation when the covariance matrices are regime specific, hence $\hat{r}(d)$ is obtained as

$$\hat{r}(d) = \arg \min_{r \in R} \sum_{j=1}^2 \frac{T_j}{2} \log |\hat{\Sigma}^{(j)}(r, d)| = \arg \min_{r \in R} \sum_{j=1}^2 \frac{T_j}{2} \log \left| \frac{1}{T_j} \sum_{i=1}^{T_j} \hat{\varepsilon}_t^{(j)}(r, d) \hat{\varepsilon}_t^{(j)}(r, d)' \right|$$

where $|\hat{\Sigma}^{(j)}(r, d)|$ is the determinant of the estimated covariance matrix, T_j is the number of observations in regime j and R denotes the grid of values for the threshold value. We form this grid using all observations of the threshold variable excluding the lowest and highest 18 percent of the observations. This ensures that we have at least 18 percent of the observations in each regime.⁴

Third, the delay parameter is also estimated by maximizing the conditional likelihood over the values $D = \{1, \dots, 4\}$.⁵

$$\hat{d} = \arg \min_{d \in D} \sum_{j=1}^2 \frac{T_j}{2} \log |\hat{\Sigma}^{(j)}(\hat{r}(d), d)|$$

Recall that this estimation procedure is conditional on the lag lengths p_j and the choice of the threshold variable z_t . We will discuss how to select them in subsection 2.4.

2.3 Computing impulse response functions and their confidence intervals

The impulse response functions of the TVARs depend on initial conditions. We can distinguish two cases based on whether we allow for the possibility of a regime switch following the structural shock (Galvão and Marcellino, 2010):

⁴The typical choice in applications is 10 or 15 percent. Our choice reflects the fact that we have a VAR with five endogenous variables and potentially up to four lags are included in the regressions. We also experimented with higher values, but the results are unaffected.

⁵Note that the last two steps are equivalent to maximizing the conditional likelihood over the two dimensional grid $D \times R$. We chose to estimate them in two steps for convenience.

1. If we exclude the possibility of a regime switch following the structural shock, then the impulse response functions depend only on the regime when the shock hits. In this case we have two sets of impulse response functions, one for each piecewise linear model.
2. If we allow for the possibility of a regime switch following the structural shock, then the impulse response functions depend on the (delayed) value of the threshold variable, the history of the endogenous variables leading up to the time of the shock and the (size of the) shock itself. In this case we have a continuum of impulse responses indexed by this triplet and we need to compute generalized impulse responses (Koop, Pesaran, and Potter, 1996).

The first case is a useful benchmark to characterize the different responses of the model variables across regimes. It has the advantage that the computation of the impulse responses requires only a fraction of second and even confidence intervals can be obtained using little computer time. However, it is based on the assumption that if the structural shock hits the economy in times of fiscal stress, for example, then the economy remains in that regime forever. Changes in the threshold variable are part of the impulse responses of the model and we need to relax this assumption if we are interested in a comprehensive characterization of the dynamic effects of structural shocks or in using the model for policy experiments. The cost of endogenizing regime switches is that the generalized impulse responses cannot be evaluated analytically. Therefore we need to resort to the Monte Carlo simulation method of Galvão and Marcellino (2010).

In order to simulate regime switches we need to model the dynamics of the threshold variable. It is useful to introduce at this point a special case of TVARs. If the variable that triggers the regime switch is a combination of the endogenous variables, then the model is called an endogenous threshold autoregressive model (ET-VAR).⁶ The estimation procedure is the same for both types of models but

⁶The limiting case of ET-VARs is the self-exciting threshold autoregressive model (SETAR) which uses (the lagged value) of only one of the endogenous variables as the threshold variable.

they differ in how generalized impulse response functions and their confidence intervals are computed. In particular, if the threshold variable is a combination of the endogenous variables, then there is no need to model the innovations to its dynamics exogenously. In other words, a series of structural shocks to the model determine the dynamics of both the endogenous variables and the threshold variable; given initial conditions. This reduces both the computational burden and the parameter uncertainty of generalized impulse responses.

Note that the dynamics of the threshold variable in the best fitting model, the debt-to-GDP ratio, is fully determined by the dynamics of the endogenous variables. Thus, we describe the algorithm to compute generalized impulse responses assuming that the threshold variable is endogenous to the model.

Suppose we are interested in computing responses up to horizon $t + s$ to the structural shock ε which hits our system at time t . We will refer to ε as “impulse” to distinguish it from the shocks generated to simulate time series from our model.⁷ The regime at time t is $j = 1, 2$ determined by z_{t-d} while the history of endogenous variables leading up to this period is given by $\mathbf{X}_t^{(j)} = [\mathbf{y}'_{t-1} \ \cdots \ \mathbf{y}'_{t-p_j}]'$. We are interested in computing

$$GI_y^{(j)}(\varepsilon, \mathbf{X}_t^{(j)}) = E\{\mathbf{y}_{t,t+s}|\varepsilon, \mathbf{X}_t^{(j)}\} - E\{\mathbf{y}_{t,t+s}|0, \mathbf{X}_t^{(j)}\}$$

where $\mathbf{y}_{t,t+s}$ denotes the history of the endogenous variables between period t and $t + s$ and we use the superscript j to make it explicit that for $GI_y^{(j)}$ denotes the impulse response conditional on being in regime j at the time when the impulse arrives.

We obtain $GI_y^{(j)}(\varepsilon, \mathbf{X}_t^{(j)})$ by going through the following steps:

1. Draw two samples of the endogenous shocks $\varepsilon_{t,t+s}^{(i)}$ ($i = 1, 2$) from the estimated distributions of the residuals of the estimated model, $N(0, \Sigma^{(i)})$.
2. Generate a baseline simulation $\mathbf{y}_{t,t+s}^{bs,l}$ for the endogenous variables by solving the model forward conditional on $\mathbf{X}_t^{(j)}$ and z_{t-d} . The superscript *bs* stands

⁷Artis, Galvão, and Marcellino (2007) refer to ε as the “extraordinary” shock.

for baseline simulation, while l denotes the number of the current replication (see point 5). Use the shocks from step 1 for the simulation.

3. Generate an alternative simulation $\mathbf{y}_{t,t+s}^{as,l}$ for the endogenous variables by solving the model forward conditional on $\mathbf{X}_t^{(j)}$, z_{t-d} . The superscript as stands for alternative simulation. Use the shocks from step 1 for the simulation with the first realization perturbed by the impulse, i.e. replace $\varepsilon_t^{(i)}$ with $\varepsilon_t^{(i)} + \varepsilon$.
4. Compute the impulse response of replication l as $\mathbf{y}_{t,t+s}^{as,l} - \mathbf{y}_{t,t+s}^{bs,l}$. Notice that this response is conditional on the particular histories $\varepsilon_{t,t+s}^{(i)}$ ($i = 1, 2$).
5. Repeat steps 1 to 4 for $l = 1, \dots, L$ to obtain $GI_y^{(j)}(\varepsilon, \mathbf{X}_t^{(j)})$.

The last step averages out the effects of future shocks, which affect similarly both the baseline and the alternative. Notice that we use the same draws $\varepsilon_{t,t+s}^{(i)}$ to compute the two simulations, which guarantees that the only source of difference between them is the impulse ε .

We use $L = 500$ for our simulations. To obtain the regime specific impulse responses $GI_y^{(j)}(\varepsilon)$ we average $GI_y(\varepsilon, \mathbf{X}_t^{(j)})$ out for all histories within the same regime. Given that we have 148 and 48 observations in the two regimes of the best fitting nonlinear specification that we use as initial conditions, we employ 74,000 and 24,000 replications to compute the regime specific impulse responses for the two regimes, respectively.

What makes the methodology computationally very costly is that we nest this Monte Carlo simulation into a bootstrap approach to compute confidence intervals (for a detailed description and discussion see Artis, Galvão, and Marcellino (2007) and Galvão and Marcellino (2010)). We build time series \mathbf{y}_t for the endogenous variables based on the estimated parameters of the model and bootstraps from the (regime specific) residuals (we use 500 replications in our plots). We repeat the first two steps of the estimation procedure for each generated series: we keep the estimated delay parameter \hat{d} fixed, but reestimate the coefficients $\hat{\Phi}^{(j)}(r, d)$ and $\hat{\Sigma}^{(j)}(r, d)$ and the threshold value $\hat{r}(d)$ each time. Since uncertainty about the

threshold value can be a major weakness to regime switching models this approach attempts to address this issue. We use the empirical mean of the obtained impulse response functions and the $\alpha/2$ and $1 - \alpha/2$ empirical percentiles to obtain the $100(1 - \alpha)$ percent confidence intervals around it.

2.4 Choosing between different specifications

We need to compare competing specifications along three different dimensions. First, given a choice of a threshold variable we need to select the lag lengths in the two regimes. Second, we need to compare models that differ in the threshold variable. Third, we need to confront the best fitting nonlinear model with the linear benchmark.

We base the first two choices on a penalized likelihood function. We follow the approach of Artis, Galvão, and Marcellino (2007) and Galvão and Marcellino (2010) for comparing competing specifications that differ in terms of the threshold variable. They use information criteria based on a penalized likelihood function, where the penalty depends on the number of estimated parameters, in particular the Akaike, Hannan and Quinn, and Schwarz information criteria. The results of Gonzalo and Pitarakis (2002) show that the most reliable information criteria to choose among the models is the one with the heaviest penalty function, i.e. the Schwarz criterion.

Following the standard practice in time series analysis, we also use the information criteria to select the best lag structure of our model. In each regime the lag length can take a value from the set $P = \{1, \dots, 4\}$. We allow for different lag lengths in the two regimes and thus for each threshold variable we estimate a model for all possible lag length combinations over the two dimensional grid $P \times P$.

When comparing the best fitting nonlinear specification with the linear model we rely on two statistical tests. The first test is a variable addition test, that

considers as the nonlinear alternative the specification

$$\mathbf{y}_t = \mathbf{c} + \Phi \mathbf{X}_t + \Psi \mathbf{X}_t z_{t-d} + \varepsilon_t$$

given the value of the delay parameter, d (see for example Artis, Galvão, and Marcellino, 2007; Teräsvirta, 1998). If the true model is linear, then the coefficients in Ψ are jointly insignificant which can be tested using an LR test.

The second test was proposed by Tsay (1998) and it uses predictive residuals from an arranged model to construct a test statistic. Given a value for the delay parameter and the threshold variable, z_{t-d} , the observations in the linear model are arranged according to the increasing ordering of the threshold variable z_{t-d} . Then a series of linear models for each value $m = m_0 \dots T$ are estimated using observations $i = 1 \dots m$ from the arranged sample to construct one step ahead prediction errors.⁸ If the data was generated by a threshold model, then this arrangement transforms the model into a structural break model with observations for regime 1 at the beginning and for regime 2 at the end of this arranged sample with a structural break at an unknown date in between. In this case the predictive errors are correlated with the regressors of the arranged model. If, instead, the data was generated by a linear model, then the predictive residuals are uncorrelated with the arranged regressors and the coefficients of the regression of the predictive residuals on the arranged regressors should be jointly insignificant under the null.

There are several alternatives to these two tests in the literature (see for example Andrews and Ploberger, 1994; Hansen, 1999; Altissimo and Corradi, 2002). We choose these two tests because they rely on the choice of the threshold variable to construct a test statistic, but neither the value nor the distribution of the test statistic depend on the threshold value, r , a nuisance parameter that is present only under the alternative. Thus, both of these tests are simple and have familiar limiting distributions. Furthermore, as pointed out by Galvão and

⁸ m_0 is the sample size of the first model and T is the size of the full sample. On the selection of m_0 and details of the procedure see Tsay (1998).

Marcellino (2010), applying these alternative tests in a multivariate setting may be misleading when the variance of the disturbances is regime specific.

3 Model specification and data

In order to keep our exercise comparable to the existing literature, we use a type of structural VAR estimated by Blanchard and Perotti (2002), Perotti (2004) and Favero and Giavazzi (2007). In particular, we adopt the specification of the last of these papers both in terms of our choice of variables and regarding the identification strategy. The specification includes quarterly US data on (federal) government total expenditures net of interest payments (g_t), (federal) government total receipts net of interest receipts (t_t), and GDP (y_t), all in per capita real terms, the GDP deflator inflation rate (Δp_t) and the average nominal cost of financing the debt (i_t).⁹ All variables are in logs except the interest rate, which enters in levels. The specification includes a constant and one lag of the endogenous variables according to the Schwarz information criteria. The sample goes from 1960:1-2009:4.

While Favero and Giavazzi (2007) include the same set of variables in their VAR as Perotti (2004), there are some differences in the definition of the variables worth noting. Transfer payments are considered as part of government expenditure, rather than being subtracted from government receipts. Also, the nominal cost of servicing the debt is used as the interest rate instead of the yield to maturity on long-term government bonds. It is defined as the ratio of net interest payments and the end of last period stock of government debt. Furthermore, since the definition of debt refers to federal government debt we use only federal government expenditures and receipts to construct the endogenous variables.¹⁰

⁹We use the GDP deflator for all variables to obtain the corresponding real values. For details on the construction of the variables and the data sources, see our Appendix or Favero and Giavazzi (2007).

¹⁰Favero and Giavazzi (2007) carefully check whether these differences in data definition alter the estimated effects of fiscal policy shocks. They conclude that the impulse responses are similar to the results of Perotti (2004) both in their full sample (1960:1-2006:4) and in their

3.1 Identification

We identify the structural shocks for the fiscal variables separately for each of the regimes using the Blanchard and Perotti (2002) identification approach, extended by Perotti (2004) for the five variable VAR. Imposing the relationship

$$\begin{bmatrix} 1 & 0 & -\alpha_{gy} & -\alpha_{g\Delta p} & -\alpha_{gi} \\ 0 & 1 & -\alpha_{ty} & -\alpha_{t\Delta p} & -\alpha_{ti} \\ \hline a_{31}^{(j)} & a_{32}^{(j)} & 1 & 0 & 0 \\ a_{41}^{(j)} & a_{42}^{(j)} & a_{43}^{(j)} & 1 & 0 \\ a_{51}^{(j)} & a_{52}^{(j)} & a_{53}^{(j)} & a_{54}^{(j)} & 1 \end{bmatrix} \begin{bmatrix} e_1^{(j)} \\ e_2^{(j)} \\ e_3^{(j)} \\ e_4^{(j)} \\ e_5^{(j)} \end{bmatrix} = \begin{bmatrix} b_{11}^{(j)} & 0 & 0 & 0 & 0 \\ b_{21}^{(j)} & b_{22}^{(j)} & 0 & 0 & 0 \\ \hline 0 & 0 & b_{33}^{(j)} & 0 & 0 \\ 0 & 0 & 0 & b_{44}^{(j)} & 0 \\ 0 & 0 & 0 & 0 & b_{55}^{(j)} \end{bmatrix} \begin{bmatrix} u_1^{(j)} \\ u_2^{(j)} \\ u_3^{(j)} \\ u_4^{(j)} \\ u_5^{(j)} \end{bmatrix}$$

where $e_i^{(j)}$ and $u_i^{(j)}$ denote the reduced form innovation and the structural shock of the i th equation in regime j , respectively. The elasticities α_{gy} , $\alpha_{g\Delta p}$, α_{gi} , α_{ty} , $\alpha_{t\Delta p}$ and α_{ti} represent the automatic response of fiscal variables to economic activity and are computed using external information (see Perotti, 2004, for details). Their values are:

$$\begin{array}{lll} \alpha_{gy} = 0 & \alpha_{g\Delta p} = -0.5 & \alpha_{gi} = 0 \\ \alpha_{ty} = 1.85 & \alpha_{t\Delta p} = 1.25 & \alpha_{ti} = 0 \end{array}$$

Note that we are using the same elasticity values for the two regimes which has both, advantages and disadvantages. On the one hand, one could argue that separate elasticities should be calculated based on the set of observations two subsamples (1960:1-1979:4 and 1980:1-2006:2). We carry out a similar comparison exercise in order to check the effects of data definition on the impulse responses in our slightly longer sample and arrive to the same conclusion.

pertaining to each of the regimes, in order to capture the potential differences in the workings of the automatic stabilizers. On the other hand, using the same elasticities across the two regimes implies that differences in the impulse responses are driven only by the differences in the estimated parameters across regimes but not by differences in the elasticities used for identification.

We have chosen the Blanchard and Perotti (2002) approach among the alternative identification schemes primarily because it facilitates the comparability of our results with a large pool of literature on the effects of fiscal policy shocks. Moreover, other identification approaches in the literature cannot be applied easily to the threshold VAR model setting. The narrative approach along the lines of Ramey and Shapiro (1998) has potentially too few observations to get reliable results after splitting the sample into two. The narrative record used in Romer and Romer (2009) does not lead to the same problem, but we could only use it to identify the effects of tax shocks, while no narrative data of this kind is gathered for government spending.

The other alternative identification method is the sign restrictions approach of Mountford and Uhlig (2008). This identification method requires very few restrictions and can be easily motivated by the theoretical literature when applied to a linear VAR. However, it is not readily generalizable to our regime switching setting. Several papers provide empirical support in favor of contractionary fiscal expansions (Perotti, 1999; Giavazzi and Pagano, 1990, 1995; Alesina and Perotti, 1996a,b, among others), i.e. macroeconomic variables can respond qualitatively differently to fiscal shocks in times of fiscal stress. We therefore prefer to be agnostic and avoid putting any a priori restrictions on the sign of the impulse responses.

Finally, we have to point out that our chosen identification approach is not free of criticism (e.g. Ramey, 2009; Leeper, Walker, and Yang, 2009). It was criticized, for example, for its inability to accommodate fiscal foresight or for the sensitivity of its results to the elasticities used. Our primary goal in this paper is to compare impulse responses obtained from our best fitting nonlinear

model and from the benchmark linear VAR. We do not have reason to believe that these models are affected asymmetrically by the shortcomings of the identification method used and we can safely compare these two sets of results. Furthermore, in a recent paper Chahrour, Schmitt-Grohé, and Uribe (2010) show that the method of Blanchard and Perotti (2002) does a reasonable job to identify the structural shocks of the model.

3.2 Threshold variables considered

Since our primary aim is to analyze whether the responses of macroeconomic variables to fiscal policy shocks are different in times of fiscal stress as opposed to normal times, we restrict our set of potential threshold variables to various proxies for the extent of fiscal stress. We group the variables under four headings: those related to the stock of accumulated debt, those related to the primary deficit, those related to the financing of the debt and those related to the sustainability of debt.

Note that the first three possible approaches to fiscal stress can be motivated by the government's budget constraint. In particular, rearranging the government budget constraint one can show that the dynamics of the debt-to-GDP ratio is governed by:

$$d_t = \frac{1 + i_t}{1 + \Delta p_t} \frac{d_{t-1}}{1 + \Delta y_t} + \frac{\exp(g_t) - \exp(t_t)}{\exp(y_t)} \quad (1)$$

where the first term is the real cost of servicing the debt, the second term is the lagged debt-to-GDP ratio expressed in terms of current GDP and the last term is the deficit-to-GDP ratio. We consider proxies related to all these three terms as listed in Table 1.

We include three variables related to the stock of accumulated debt. The papers by Perotti (1999), Giavazzi, Jappelli, and Pagano (2000), Giavazzi, Jappelli, Pagano, and Benedetti (2005) and Corsetti, Meier, and Müller (2010) are

TABLE 1. Candidate threshold variables

Group	Variable
Debt stock	debt-to-GDP ratio
	quarterly change in the debt-to-GDP ratio
	year-on-year change in the debt-to-GDP ratio
Primary deficit	quarterly primary deficit-to-GDP ratio
	annual primary deficit-to-GDP ratio
	worse of the last two quarters' deficit-to-GDP ratio
Financing the debt	real short-run rate
	real long-run rate
	real cost of financing the debt
	net interest payments-to-receipts ratio
	net interest payments-to-GDP ratio
Sustainability of debt	real short-run rate over GDP growth rate
	real long-run rate over GDP growth rate
	real cost of financing debt over GDP growth rate
	sensitivity of real short-run rate to debt-to-GDP ratio
	sensitivity of real long-run rate to debt-to-GDP ratio
	sensitivity of real cost of financing debt to debt-to-GDP ratio

Note: The real short term rate is computed using the 3-month T-bill rate. The real long term rate is computed using the 10-year government bond rate. The real cost of financing the debt is computed using the nominal cost of financing the debt. Real rates are computed using the GDP deflator inflation. For the definition of the nominal cost of financing the debt and the GDP deflator see the Appendix.

all using the debt-to-GDP ratio as a measure of fiscal stress.¹¹ Further, we consider the change in the debt-to-GDP ratio which has been used as a measure of the increase or decrease of fiscal stress and a potential source of nonlinearity in

¹¹The time series for debt held by the public is available only from 1970:1 on the FRED website. We follow Favero and Giavazzi (2007) and use the observation for 1970:1 as an initial value and the debt dynamics equation (1) to construct a debt-to-GDP series that covers our entire sample.

a number of papers by Giavazzi and Pagano (1990, 1995); Alesina and Perotti (1996a,b); Giavazzi, Jappelli, and Pagano (2000); Giavazzi, Jappelli, Pagano, and Benedetti (2005) and more recently by Burriel, de Castro, Garrote, Gordo, and Prez (2009). Finally, big and persistent year-on-year declines in the debt-to-GDP ratio were used as a measure of fiscal stress reduction in a recent paper by Nickel, Rother, and Zimmermann (2010).

We include three variables related to the primary deficit-to-GDP ratio. The quarterly primary deficit-to-GDP ratio and the annual primary deficit-to-GDP ratio are used in Giavazzi and Pagano (1995) to define protracted and sizable budget cuts or expansions. The worse of the last two periods' quarterly primary deficit-to-GDP ratio follows Perotti's (1999) definition of the bad times dummy.

We include five variables that are meant to capture the cost or the ability of the government to finance debt. The real 3-month treasury bill rate is used by Choi and Devereux (2005) as a direct measure of the cost of financing the debt. We also consider the real cost of financing the debt and the real long term interest rate among our candidates. The former is directly related in our model to the cost of financing the debt while the latter is a more forward looking variable than the short term rate and can be a more sensitive proxy to expectations about the sustainability of fiscal policy. We also include two variables that are potential indicators of the government's ability to repay its debt. Following Haugh, Ollivaud, and Turner (2009) we use the ratio of debt interest payments-to-government receipts to proxy for investor assessments of sovereign risk. Finally, as a joint proxy for the first two terms in (1), we consider the ratio of debt interest payments-to-GDP as an indicator for government indebtedness.

The variables in the last category are meant to capture the sustainability of the debt-to-GDP ratio. We include three variables measuring the difference between the real interest rate and the growth rate of the economy where the interest rate is defined by the short-run real interest rate (3-month T-bill rate), the long-run interest rate (10-year government bond rate) and the real cost of financing the debt. When real interest rates are low relative to the growth rate

of the economy, the debt stock is falling behind GDP and the debt-to-GDP ratio is decreasing. When interest rates are high relative to the growth rate of the economy, then debt is growing faster than GDP and an expansionary fiscal shock accelerates the growth of the debt-to-GDP ratio even further. The last three variables in this category are related to the fiscal limit of the economy.¹² The model of Bi (2010) predicts that the interest rate rises nonlinearly with the level of government liability. When the economy is below the fiscal limit, the sovereign risk premia is stable and interest rates vary little with government debt. As the economy approaches its fiscal limit the probability of sovereign default increases and financial markets start to demand a premium for the government bond. This mechanism increases the slope of the pricing rule of the interest rate that links government debt to the interest rate as the economy approaches its fiscal limit.¹³

3.3 The best fitting nonlinear model

As we discussed in section 2 we estimate a nonlinear model with two regimes for each of the candidate threshold variables. We are considering all possible combinations of the lag lengths in the two regimes up to 4 lags and all possible values for the delay up to 4. Next, we calculate the AIC, SC and HQC information criteria for all the specifications and choose the best lag specification for each threshold variable. Finally, we use the information criteria to decide among the alternative threshold variables.

We report the values of the three different information criteria for all threshold variables in Table 2. We can see that regardless of the IC used, the data prefer

¹²The fiscal limit is defined as the level of debt that the government is able and willing to service. When the debt level exceeds the fiscal limit, a sovereign default occurs.

¹³For the nonlinear relationship described see Figure 6 in Bi (2010) and the discussion therein. Note that the interest rate rule depicted in the graph looks very similar to a threshold function. It shows a very sharp increase in the interest rate at the fiscal limit while it is linear away from it. Thus, the slope of the interest rate has only two values in her model. This is a direct consequence of the assumption that the default rate, δ , is constant and exogenous. If the default rate was an endogenous function of the macroeconomic fundamentals, then the interest rate rule would take off more gradually.

TABLE 2. Values of the information criteria for different models

Threshold Variable	AIC	SC	HQC
debt-to-GDP ratio	-36.09	-34.87	-35.38
quarterly change in the debt-to-GDP ratio	-35.47	-34.21	-34.77
year-on-year change in the debt-to-GDP ratio	-35.50	-34.14	-34.87
quarterly primary deficit-to-GDP ratio	-35.52	-34.21	-34.84
annual primary deficit-to-GDP ratio	-35.37	-34.15	-34.72
worse of the last two quarters' deficit-to-GDP ratio	-35.47	-34.21	-34.82
real short-run rate	-35.38	-34.31	-34.80
real long-run rate	-35.43	-34.29	-34.84
real cost of financing the debt	-35.51	-34.46	-34.95
net interest payments-to-receipts ratio	-35.73	-34.45	-35.06
net interest payments-to-GDP ratio	-35.61	-34.52	-35.06
real short-run rate over GDP growth rate	-35.23	-34.02	-34.55
real long-run rate over GDP growth rate	-35.31	-34.06	-34.63
real cost of financing debt over GDP growth rate	-35.33	-34.17	-34.72
sensitivity of real short-run rate to debt-to-GDP ratio	-35.35	-34.07	-34.68
sensitivity of real long-run rate to debt-to-GDP ratio	-35.34	-34.04	-34.63
sensitivity of real cost of financing debt to debt-to-GDP ratio	-35.12	-33.94	-34.49

Note: The real short term rate is computed using the 3-month T-bill rate. The real long term rate is computed using the 10-year government bond rate. The real cost of financing the debt is computed using the nominal cost of financing the debt. Real rates are computed using the GDP deflator inflation. For the definition of the nominal cost of financing the debt and the GDP deflator see the Appendix.

the model with debt-to-GDP ratio as a threshold variable.¹⁴ The remaining issue is to choose the lag length for this model. The different information criteria

¹⁴Note that the data prefer the model with the debt-to-GDP ratio as the threshold variable for all combinations of the lag lengths in the two regimes. Thus, this result is not driven by the specific lag structure chosen or the particular way one or the other information criterion penalizes the likelihood of the models. In other words the choice of the threshold variable is based on the log likelihood only and is independent of the choice of the lag structure which is selected by the information criteria.

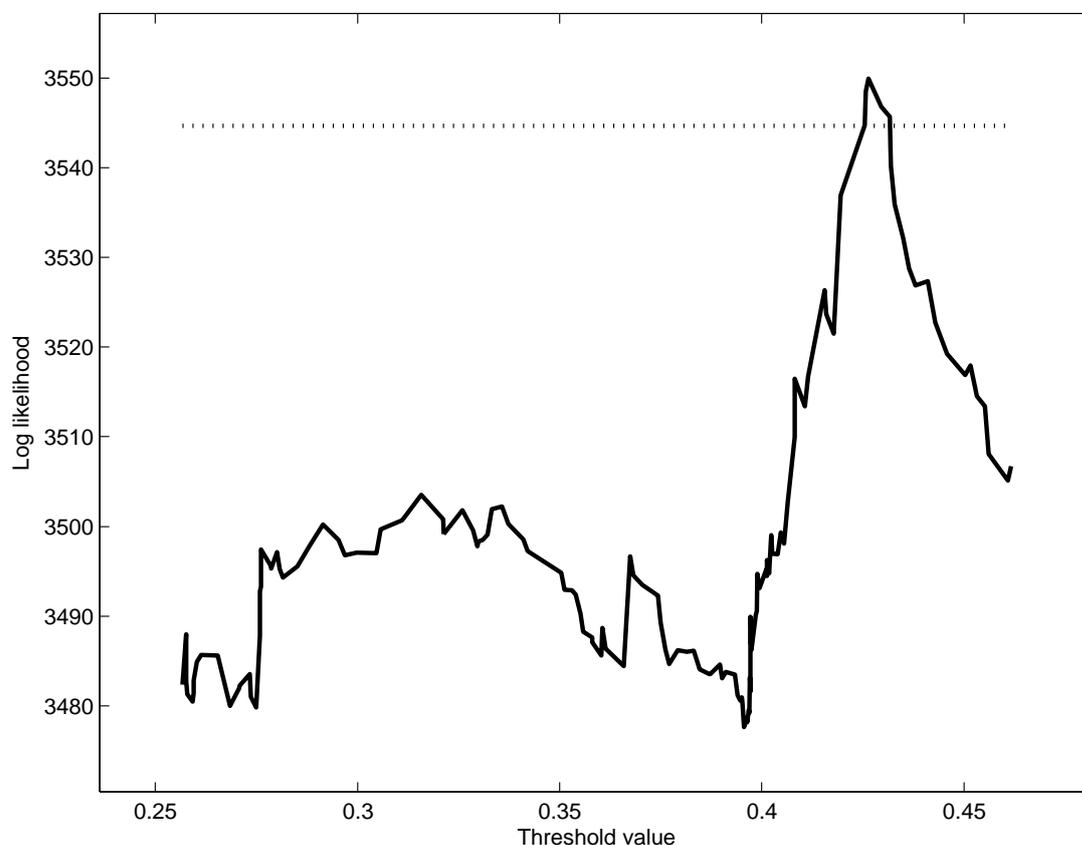
suggest quite different number of lags, ranging from (1,1) by SC to (4,3) by AIC. We decide for the most parsimonious model with one lag in each of the regimes, which was chosen by SC. This leaves us enough degrees of freedom even in the regime with the smaller number of observations to estimate the coefficients of the model.

We also inspect the shape of the log likelihood function plotted against the set of possible values of the threshold variable as an additional way to gauge the fit of the model and, in particular, how strong the threshold effect is in the estimated model (Figure 1). The log likelihood for the model with debt-to-GDP ratio as a threshold variable displays a sharp spike at the estimated threshold value indicating a strong threshold effect. The dotted horizontal line marks the location of the 10 percent confidence interval around the estimated threshold value. The small confidence interval around the estimated threshold value implies a tightly estimated threshold value.

Finally, we perform the two statistical tests, the arranged regression test of Tsay (1998) and the variable addition test of Teräsvirta (1998) to verify whether the data favor the nonlinear model with debt-to-GDP ratio as a threshold variable over the linear VAR. Both tests reject the null hypothesis that the DGP is the linear model at 1 percent significance level. Based on the results presented here we feel confident to use the model with the debt-to-GDP ratio as a threshold variable as our main specification.

4 The Effects of the fiscal policy shocks across regimes

In what follows we present the results from the nonlinear model with the debt-to-GDP ratio as the threshold variable. The estimated threshold (\hat{r}) is at 42.63 percent of debt-to-GDP and the estimated delay of the threshold variable (\hat{d}) is three quarters.

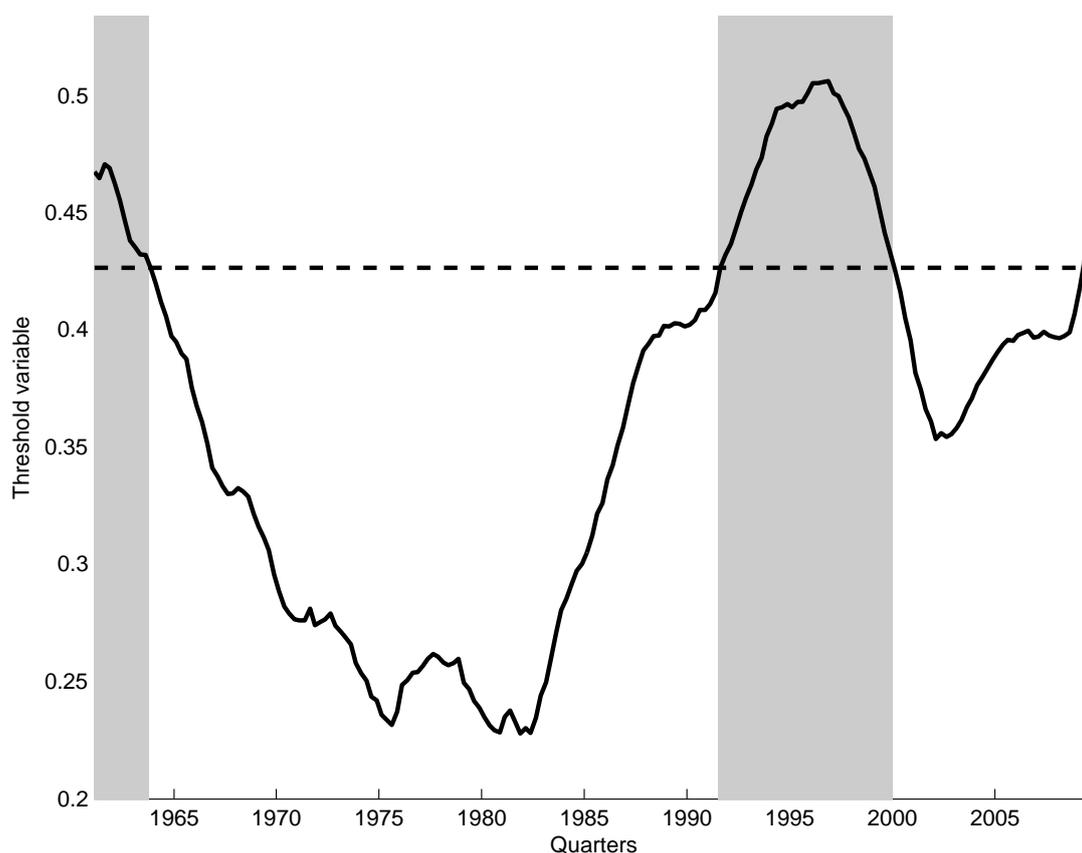


Note: The intersections of the dotted horizontal line with the log likelihood function form the 10 percent confidence interval around the estimated threshold value based on the LR-statistics approach of Hansen (2000).

FIGURE 1. The log likelihood as a function of the threshold value

Figure 2 plots the dynamics of the threshold variable and the threshold value, that splits the chart into two parts. The estimated threshold value implies that approximately three quarters (75.5 percent) of the observations belong to the lower regime (regime 1), as they correspond to the debt-to-GDP ratio (lagged three periods) lower than 42.63 percent. The rest of the observations belong to the high debt-to-GDP ratio regime (regime 2), and are in the plot denoted with the shaded area. In terms of the dates, there are three episodes that belong to

the high regime: the period until 1963:3, the period between 1991:3 and 1999:4 and the period from 2009:2 on.



Note: The dashed horizontal line represents the estimated threshold value. The shaded periods belong to the high regime.

FIGURE 2. The history of regimes

4.1 Identified shocks

In Figure 3 we plot the identified government expenditure and tax receipt shocks from the linear and nonlinear specifications. The timing of the second regime is denoted by shaded areas. For the identified government spending shocks, there is almost no difference between the shocks identified in the two models in the periods

of low regime, while in the high regime the government spending shocks seem to be slightly less volatile in the nonlinear model. For the tax shocks we observe a bit larger discrepancy between the shocks identified from the two models, in particular between 1995 and 2002. However, the shocks identified from the linear model tend to go in the same direction also when they are slightly different in size. This means that the linear benchmark and the regime switching model identify the structural shocks in the similar way. Thus, the main differences between the impulse responses of the two models that we discuss below stem from the differences in the estimated coefficients and not from the differences in the identified shocks.

4.2 Dynamic effects of fiscal shocks in two regimes

The central focus of the study is on the fiscal multipliers in the two regimes that we derive from our estimated nonlinear model. For expositional purposes we present them in two different ways. First, we plot what we refer to as the “basic” responses to structural fiscal shocks where we exclude the possibility of the regime switch following an impulse. The responses in each of the regimes are thus based on the estimated coefficients of one regime only, which shows a clearer picture of how the transmission mechanism differs across the regimes. Second, we plot what we refer to as the “dynamic” responses to structural fiscal shocks, where the system can switch to a different regime after the impulse, based on the endogenous dynamics of the threshold variable. This shows how the inner dynamics of the system leads to switches between the regimes after the initial impulse but in the absence of new structural shocks.¹⁵

4.2.1 Effects in the two regimes without regime switching

Government spending shock. In the left panel of Figure 4 we plot the responses of government spending, tax revenues and GDP to a positive one standard

¹⁵As we explain in section 2 we take into account the shocks that are hitting the system even after the initial impulse but we use Monte Carlo simulations to integrate out their effect.

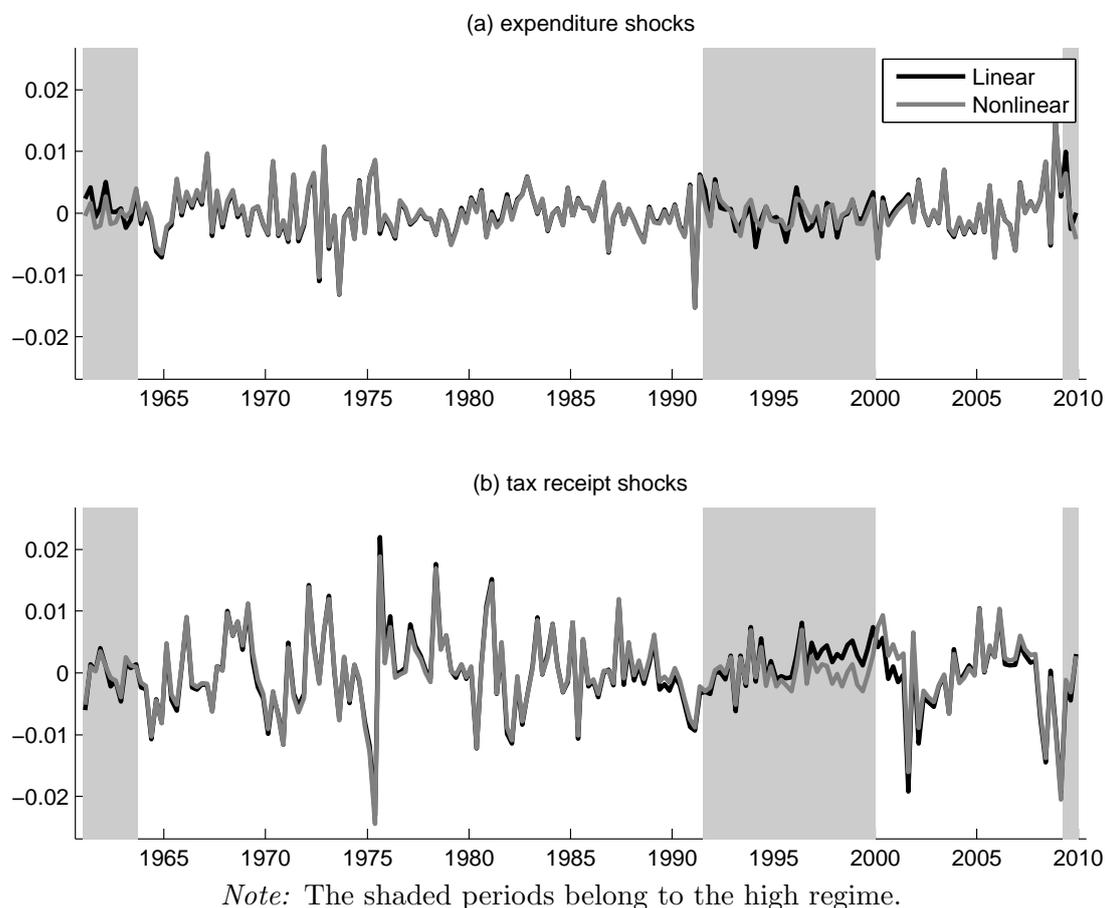
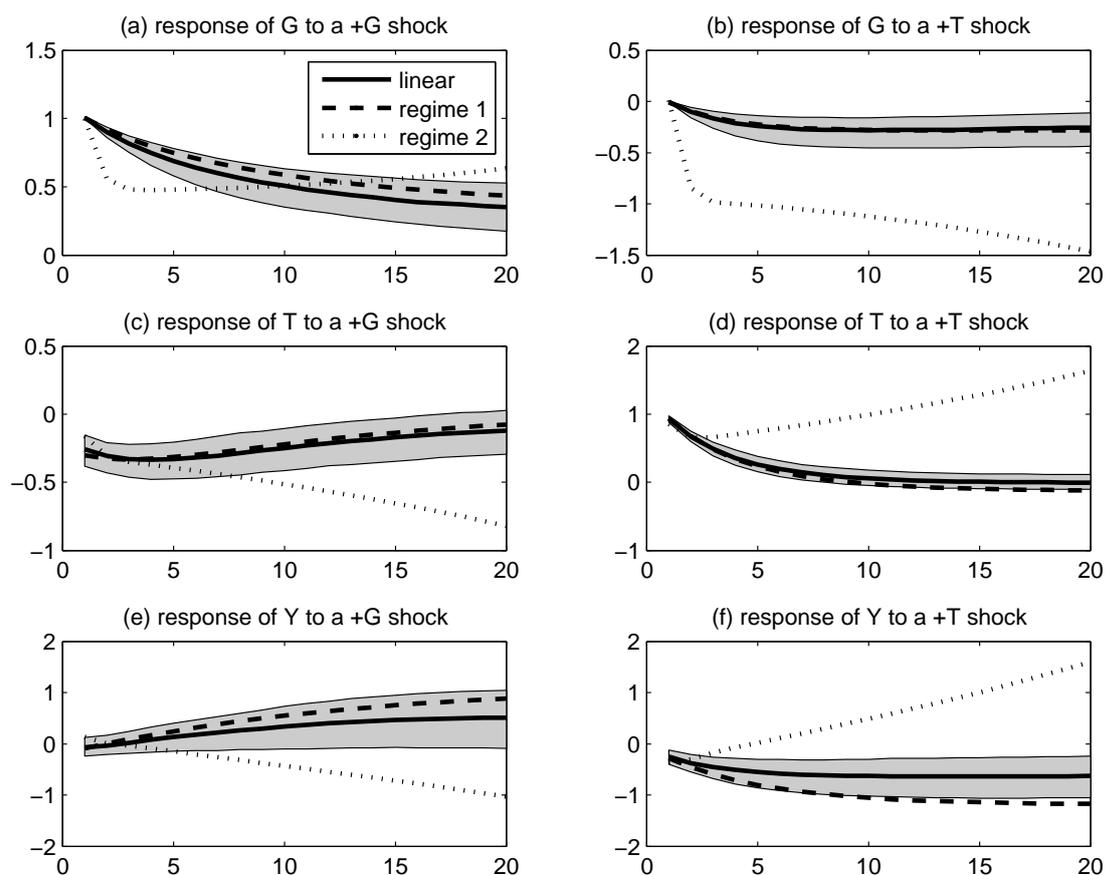


FIGURE 3. The identified structural shocks from the linear and the nonlinear models

deviation government spending shock, rescaled to the size of 1 percent of GDP.¹⁶ We plot the point estimates of the impulse responses in each of the two regimes and the impulse responses from the linear model with $p = 1$, in particular the mean and one standard deviation of the bootstrapped distribution of responses. We find that in the linear model, the government spending increases with the shock, but starts decreasing afterwards while staying positive at least up to 20

¹⁶We present only the responses of government spending, taxes and output, which are of our primary interest. The responses of inflation and interest rate to the shocks are available upon request.

quarters after the shock. Taxes fall on impact after a positive government spending shock and slowly increase thereafter, with the effect never becoming positive. Output does not react much on impact but starts increasing slowly after that. The effect, however, is not significantly different from zero.



Note: All the impulse responses are rescaled following Blanchard and Perotti (2002) and the unit on the vertical axis is 1 percent of GDP for each graph. Regime 1 is the regime with low debt-to-GDP ratio, while regime 2 is the regime with high debt-to-GDP ratio.

FIGURE 4. Confidence interval for the linear model and the basic impulse responses

As noted before, the regime specific impulse responses in this case are simply the linear responses using the estimated parameters in respective regimes. The

impulse responses of the low debt-to-GDP regime turn out not to be significantly different from the linear case. They generally stay within the confidence interval of the linear case, except for the government spending response that following the shock displays more persistence than in the linear model.

Regime 2 impulse responses are, on the other hand, quite different from the linear or regime 1 case. The government spending shock is much less persistent in the second regime, which signals that the policy makers tend to shorten any discretionary government spending measures undertaken in case of a high debt-to-GDP ratio. Response of taxes to the government spending shock is not significantly different from the linear case for the first 6 quarters but starts diverging from it (and from the regime 1 response) later on. The response of output to a positive government spending shock is initially slightly positive and above the confidence interval of the benchmark linear case. A few periods later it becomes negative and starts diverging. This dynamics also explains the response of taxes, as a fall in output inevitably leads to less taxes collected. All in all, it appears that in times of high debt-to-GDP ratio a positive government spending shock has contractionary effects akin to the results of Giavazzi and Pagano (1995); Giavazzi, Jappelli, and Pagano (2000).

Tax receipt shock. We plot the responses of the variables to a positive one standard deviation tax receipt shock, rescaled to the size of 1 percent of GDP, in the right panel of Figure 4. Again we plot the point estimates of the impulse responses in each of the two regimes and the impulse responses from the linear model with $p=1$. In the linear model, the tax receipts increase with the impulse and then slowly decrease back to the previous state. Government spending does not react on impact but slowly decreases and stays stable below the zero line for the rest of the horizon. Output reacts negatively on impact and the response stays below zero and continues to diverge.

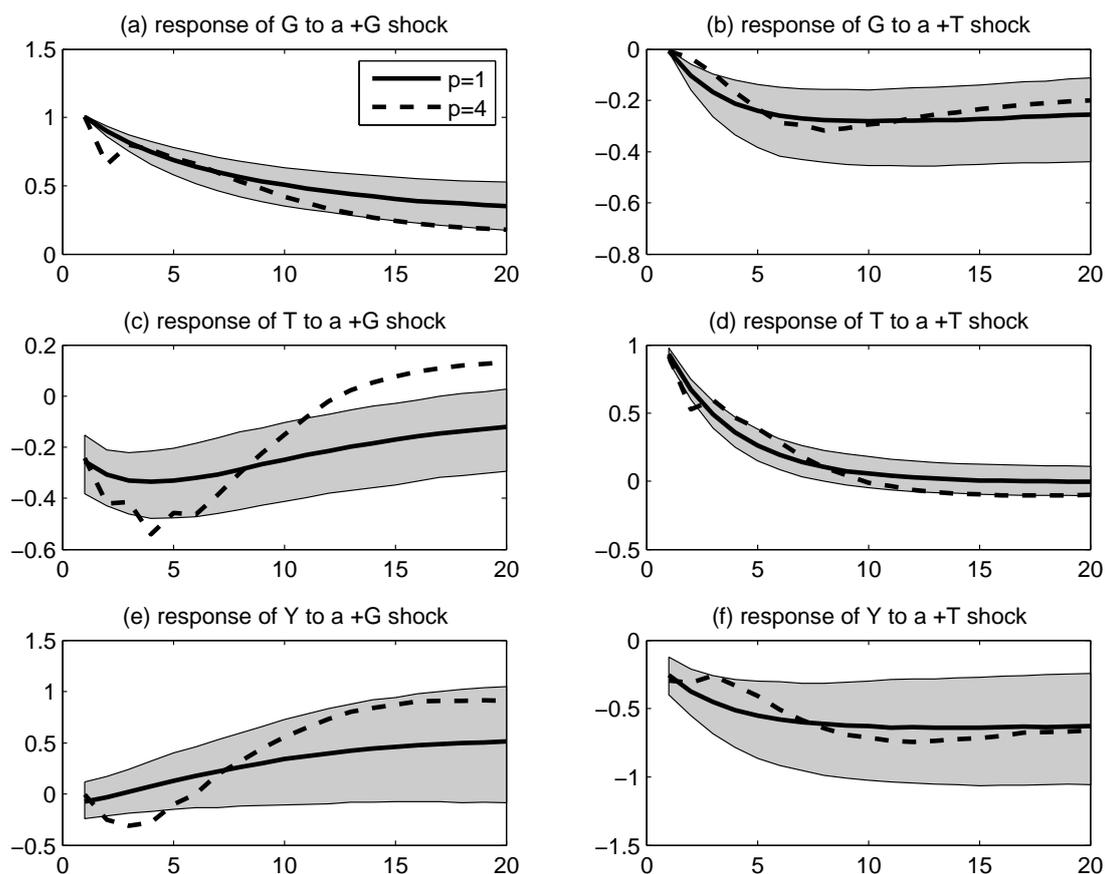
The regime specific impulse responses show the following pattern. The responses of the regime with low debt-to-GDP ratio are not significantly different from the responses of the benchmark linear case, with the exception of the output

response. The latter is slightly more negative than in the linear case, but it is still close to the confidence interval.

By contrast, the impulse responses in the high debt-to-GDP regime diverge considerably from the linear and regime 1 cases. Government spending reacts negatively as in the linear case, but the response is much more pronounced. The difference in responses across regimes are even larger when looking at the response of output. In the high debt-to-GDP regime, the impact effect of the tax shock on output is negative but contrary to the developments in the linear or regime 1 cases it starts increasing soon after. It becomes positive after a couple of quarters and remains increasing rapidly thereafter. The response of tax receipts is driven by the response of output: initially it moves along with the linear case, but when the response of output turns positive also the taxes pick up and start increasing. This scenario means rapid improvement in the debt-to-GDP ratio. The debt stabilization would then push the system quickly into the low debt-to-GDP regime, provided that the regime switching was allowed.

Results of the linear model. When comparing the impulse responses of our linear model to the standard results in the literature, one should take into account that we select the lag length of the linear model based on the SC information criterion to keep comparability with the nonlinear results. By contrast, the standard assumption in the literature is using four lags for quarterly data. In order to assess how much that could affect our conclusions, we compare the impulse responses of the linear specification with two alternative lag lengths (with $p = 1$ and with $p = 4$) in Figure 5. It can be seen, that there are some more substantial differences in the response of taxes and output to a positive government spending shock and in the response of output to a positive tax shock. However, it is evident that only the magnitudes and the shape of responses are affected by the choice of the lag length, while the sign stays the same regardless of the number of lags used. The same characteristic transfers to the basic responses in the case of regime switching model, which we verified by checking the impulse responses for other lag structures than $p_1 = p_2 = 1$. Given that we are primarily focusing

on how the sign of responses changes with the level of fiscal stress and not on the changes in the magnitude of responses, this considerations do not interfere with our analysis.



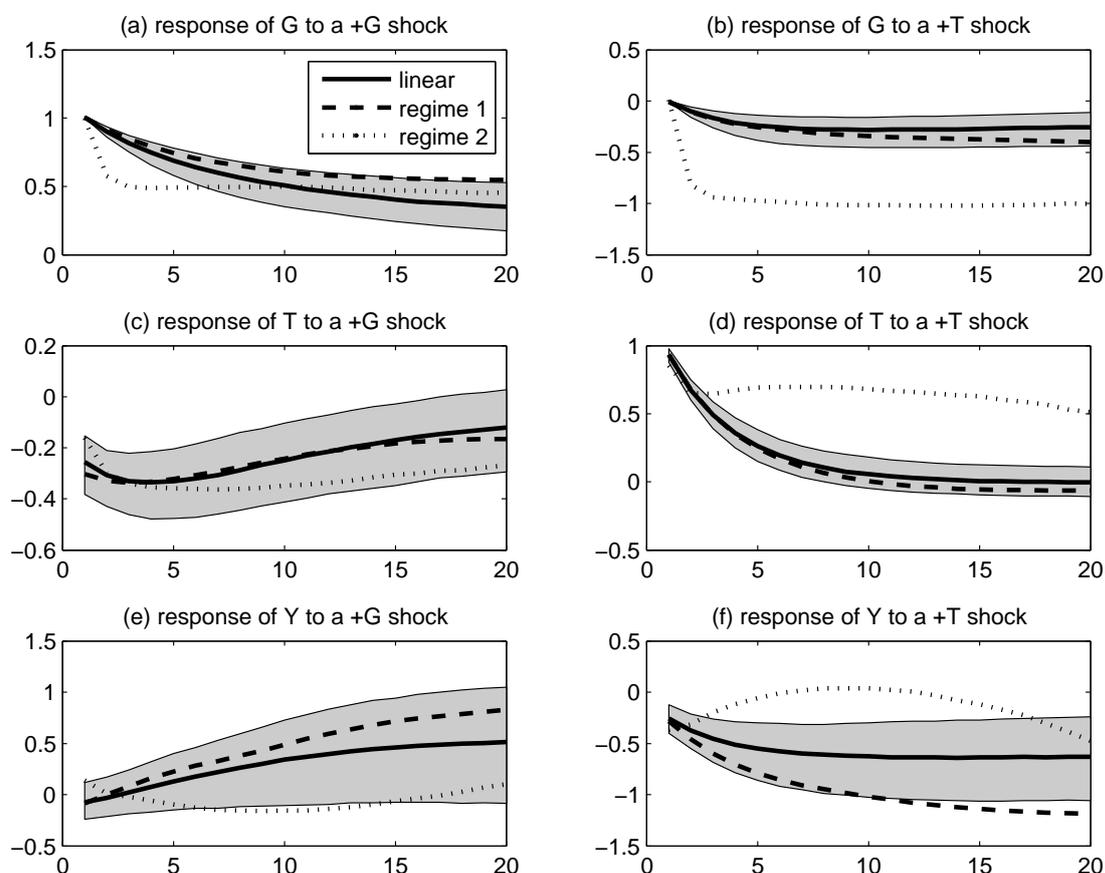
Note: All the impulse responses are rescaled following Blanchard and Perotti (2002) and the unit on the vertical axis is 1 percent of GDP for each graph.

FIGURE 5. Impulse responses for our benchmark linear model and for a linear model with 4 lags

4.2.2 Effects in the two regimes with switching allowed

Finally, we analyze the dynamic impulse responses that account for the possibility of endogeneous regime switch. In Figure 6 we plot the impulse responses for the

benchmark linear model and the mean of the simulated responses, starting from each of the regimes. For instance, the regime 1 responses are an average of simulated responses of the variables conditional on being in regime 1 at the time when the impulse hits, while allowing for endogenous evolution of the regimes based on the dynamics of the threshold variable. If the threshold variable crosses the estimated threshold value, then the system switches to a different regime. The size of the impulse is again adjusted such that it equals 1 percent of GDP.



Note: All the impulse responses are rescaled following Blanchard and Perotti (2002) and the unit on the vertical axis is 1 percent of GDP for each graph.

FIGURE 6. Confidence interval for the linear model and the dynamic impulse responses

The responses starting in regime 1 are very similar to the linear ones and, interestingly, do not differ much from the non-switching case. This can be interpreted in the following way: after a government spending or tax shock, while being in the low debt-to-GDP regime, it is unlikely for internal dynamics of the system to bring the debt-to-GDP ratio high enough to cross the threshold and move the system into a different regime.

A similar argument does not seem to hold for the high debt-to-GDP regime. Comparing the left panels of both Figures 4 and 6, it appears that after a shock the system reaches the threshold and switches back to regime 1 in a couple of periods. This is visible for the response of taxes and output to the government spending shock. Instead of diverging till the end of the horizon like in the case without switches, both responses start to turn back to zero. Also the responses of taxes and output to the tax shock seem to display an endogenous switch. The dynamics in the second regime bring the system back to the first regime approximately 6 quarters after the shock. After the switch, the responses of government spending and taxes stabilize and start moving slowly closer to zero, instead of diverging as we observe in the non-switching case presented in Figure 4. The response of output to the tax shock changes especially dramatically once the switches are possible. Instead of crossing the zero line and turning positive as in the no switch case, the response stays negative and starts diverging away from zero after about 8 quarters. Also here the reason is an endogenous switch to the low debt-to-GDP regime, following an increase in taxes.

5 Robustness

In this section we consider alternative specifications to assess whether our results are due to the particular empirical strategy chosen. As the first robustness check we consider the possibility of a structural break in our sample. Since the observations with high debt-to-GDP ratio are concentrated in the second part of our sample, it could very well be the case that the differences in the two regimes

captured with our ET-VAR are a reflection of an underlying structural change in the economy other than fiscal stress.

While it is reasonable to believe that the data are well described by a threshold model with two regimes, we also check the results of alternative empirical models which can give a better approximation of the underlying nonlinearity of the time series. In particular, we consider an ET-VAR with three regimes and a smooth transition autoregressive (STAR) model as alternatives.

5.1 Structural break

The possible reasons for a structural change can be a change in the way fiscal and/or monetary policies were conducted, or the developments in the variance of shocks known in the literature as the Great Moderation. Several papers studying the effects of fiscal policy shocks split their sample around 1980 (see for example Perotti, 2004; Favero and Giavazzi, 2007; Caldara and Kamps, 2008, among others). Caldara and Kamps (2008) for instance split their sample at 1980 and find that the impulse responses in the second subsample are less pronounced than in the first subsample.

To confront our nonlinear model with the possibility of a structural break we estimate two models. First, we estimate a structural break model (SB-VAR) by splitting our sample into two subsamples (1960:1-1979:4 and 1980:1-2009:4) at roughly the same date as other papers in the literature. Second, we estimate an endogeneous structural break model (ESB-VAR) where we also estimate the most likely date of the structural break in the model based on a penalized likelihood approach. In particular, we estimate a series of SB-VARs with a possible structural break as early as 1978:1 and as late as 1987:4 and compare the penalized likelihoods. The estimated endogeneous structural break that emerges from this exercise is between 1981:4 and 1982:1.

We list the values of information criteria for both models in Table 3, whereby we report the best value of the IC across the possible lag lengths. For convenience, we also repeat the results of the ET-VAR model with the debt-to-GDP ratio as a

threshold variable. We find that none of the two structural break models fits the data better than the model with debt-to-GDP ratio as threshold and that among the two structural break models, the data prefer the one with endogeneously chosen structural break.

TABLE 3. Values of the information criteria for structural break models

Model	AIC	SC	HQC
ET-VAR with debt-to-GDP ratio	-36.09	-34.87	-35.38
SB-VAR	-35.46	-34.35	-34.99
ESB-VAR	-35.51	-34.44	-35.04

5.2 Three regimes

Since there is no *a priori* reason for excluding the possibility that the three regime model gives a better representation of nonlinearities in our framework, we carefully consider the empirical evidence. First, we apply the same nonlinearity tests we previously used on the full sample. Since our nonlinear model consists of two piecewise linear models we can use the same tests on the two regimes separately.¹⁷ We consider only the subsample of observations pertaining to the larger of the two regimes in our best fitting nonlinear model since the smaller subsample is too short for the tests to give any sensible results. We find remaining nonlinearity in the low debt-to-GDP ratio regime which signals that the three regime model would be preferred.

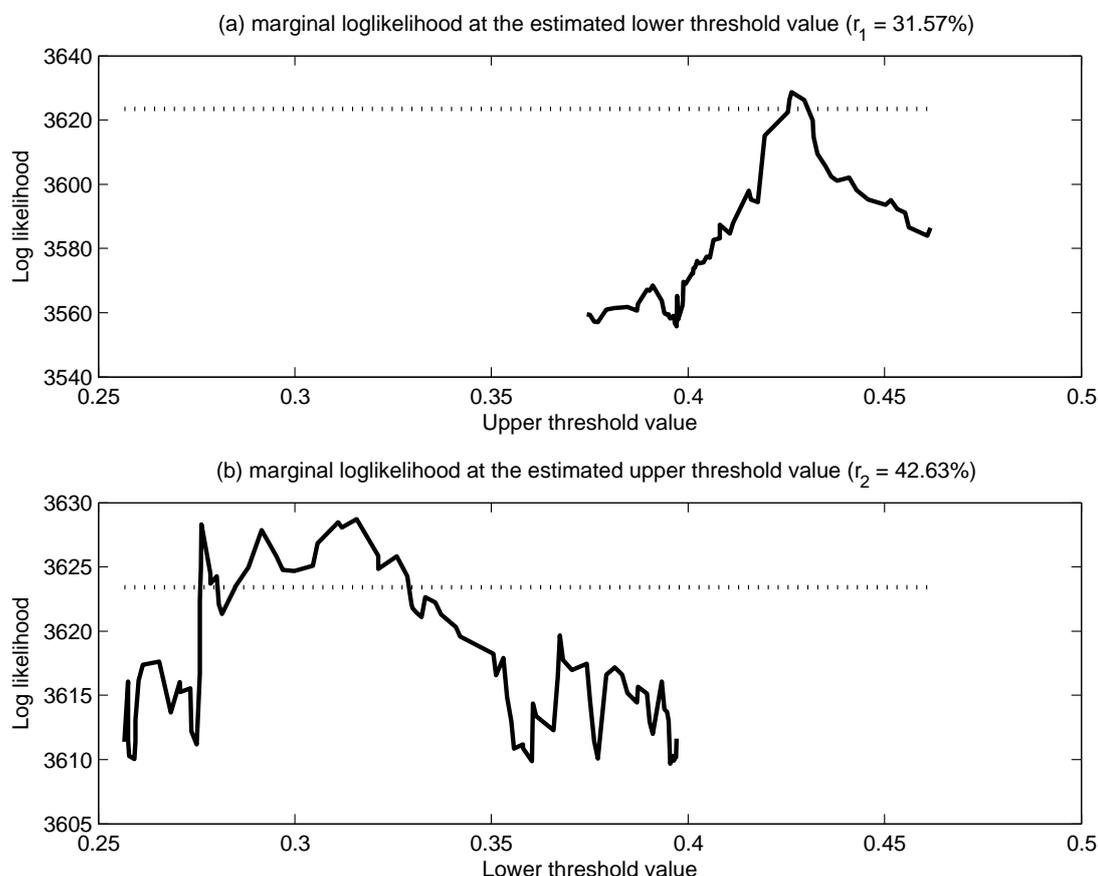
Second, we estimate a nonlinear model with three regimes for each of the candidate threshold variables and for all the possible lag length combinations up to 4 lags. Also in the case of three regimes the debt-to-GDP ratio emerges as the

¹⁷The estimated threshold value in a TVAR with two regimes is a consistent estimate for one of the two estimated threshold values in a model with three regimes (see Hansen, 1999). Consequently, the model with three regimes can be considered as a special case of the model with two regimes where one of the two regimes is modelled using a TVAR with two regimes. Thus, we can use the same tests all over again to test if any of the two regimes can be better characterized by a TVAR with two regimes.

preferred threshold variable. Note that we get this result irrespective of which information criteria we use. Thus, we continue with the analysis of the three regime model with the debt-to-GDP as a threshold variable. The optimal lag length for this particular specification varies from (1,1,1) chosen by SC, (1,1,4) by HQC to (4,3,4) preferred by AIC. We focus again on the most parsimonious version of the model with one lag in each subsample. Doing so, we keep comparability with the two regime case and also reduce the problem of insufficient degrees of freedom, that we could encounter when estimating a model with 4 lags in all three regimes.

In the three regime case we need to estimate two threshold values, $r = (r_1, r_2)$, that split the observations into three regimes. We estimate the model over a two dimensional grid $r \in R \times R$ of possible combinations of values of debt-to-GDP ratio and calculate the log likelihood for each of them.¹⁸ Figure 7 shows the marginal log likelihood functions for each of the two threshold values, obtained as cross-sections of the three dimensional log likelihood surface. In panel (a), we fix the lower threshold value at its point estimate and plot the log likelihood as a function of the higher threshold value only. The dotted horizontal line marks the location of the 10 percent confidence interval around the estimated threshold value. Notice that the marginal likelihood is essentially identical to the log likelihood of the model with two regimes only (Figure 1). The estimated upper threshold value is identical to the estimated threshold value in the two regime model and the confidence bands are also very narrow. This result is expected since the estimated threshold value in a TVAR with two regimes is a consistent estimate for one of the two estimated threshold values in a model with three regimes (Hansen, 1999). In panel (b), we fix the higher threshold value at its point estimate and plot the log likelihood as a function of the lower threshold value only. The estimated lower threshold is 31.57 percent, but we can see that this estimate is very unprecise. The 10 percent confidence band is more than 5 percentage points wide ranging from 27.61 to 32.86 percent.

¹⁸Note that we do not constrain the model to have the same lag lengths in the two regimes and thus the “one-step-at-a-time” approach described by Hansen (1999) cannot be used here to reduce the computational burden.



Note: The intersections of the dotted horizontal lines with the log likelihood functions form the 10 percent confidence intervals around the estimated threshold values based on the LR-statistics approach of Hansen (2000).

FIGURE 7. The marginal log likelihood as a function of the threshold value

Despite the evidence of the nonlinearity tests, we choose to use the model with two regimes for our analysis. There are two reasons for that. First, the large uncertainty around the estimated lower threshold value would translate into wide confidence intervals around the impulse response functions and we would not be able to reliably distinguish between the responses coming from the lower and middle regimes. Second, the upper threshold value in the three regime model corresponds exactly to the estimated threshold value in the two regime case. This

means that the upper regime or bad times regime is the same in both specifications, which is the relevant regime considering the current economic situation.

5.3 Smooth transition autoregressive model

The main disadvantage of the ET-VAR specification is that the sample is split and the estimated coefficients in both regimes are functions of the observations only in their respective subsamples. The econometrician faces a trade-off between the number of regimes and the efficiency of the estimates. The underlying non-linearity of the time series might be approximated better with more regimes, but the smaller subsamples give less precise estimates of the parameters.

An alternative specification used in the literature is the smooth transition autoregressive (STAR) model where the transition function is the logistic function

$$F(z_{t-d}) = \frac{1}{1 + \exp(-\gamma(z_{t-d} - r))}.$$

It allows a smooth transition even with two regimes and can be a better approximation to the underlying nonlinearity than an ET-VAR model with several regimes. An additional advantage of the STAR model is that it utilizes the entire sample to estimate the lag coefficients and the covariance matrices of the model.

In short samples, however, that are typically available in macroeconomics the curvature parameter, γ , and/or the threshold value, r , can be estimated only very imprecisely. This parameter uncertainty translates into wider confidence intervals concerning the regime specific impulse responses. Auerbach and Gorodnichenko (2010) overcome this problem by calibrating rather than estimating the curvature parameter using the NBER classification for recessions as an exogenous source of information. They condition the transmission mechanism of fiscal policy on the business cycle conditions which allows them to calibrate the curvature parameter

to match the average share of recessions in their sample to the implied share of recessions by the transition function.¹⁹

Unfortunately we do not have this type of exogenous information to calibrate the parameters in a STAR model and this approach cannot be applied here. The only possibility is to estimate all the model parameters. Since our model has 5 endogenous variables, as opposed to the three variables of Auerbach and Gorodnichenko (2010), we have 32 parameters to estimate using maximum likelihood even after conditioning the log likelihood function.²⁰ The problem is highly nonlinear and a preliminary grid search suggests that the log likelihood function is ill behaved. Given these we concluded that this econometric specification is computationally not feasible and led us to favour the current ET-VAR model.

6 Conclusion

The aim of this paper is to determine whether the responses of key macroeconomic variables, in particular of GDP, depend on the extent of fiscal stress. To this end, we estimate a multivariate threshold vector autoregression model with two regimes, using US quarterly data, where both the coefficient matrices and the variance of the disturbances can differ across regimes.

The regimes and the transition between them is determined by the debt-to-GDP ratio, which is found to be the threshold variable in the best fitting nonlinear model. The estimated threshold value that separates the two regimes is 42,63 percent, with tight confidence bands. Comparing the output responses in each of the regimes, we conclude the following:

¹⁹Auerbach and Gorodnichenko (2010) set $r = 0$ and $d = 1$ and chooses the value of γ such that $F(z_{t-1}) > 0.8$ for only 20 percent of their sample. They define the threshold variable, z_t , to be the eight-quarter moving average of the output growth rate.

²⁰We have the two parameters of the transition function and the Cholesky factor of each regime specific covariance matrix has 15 entries to be estimated.

- The responses in the periods of low debt-to-GDP ratio are in line with those given by the standard linear specification and are in most points not significantly different from them.
- The responses in the high debt-to-GDP regime differ significantly from the linear specification and are also very different from the low regime estimates. In particular, positive government spending shock has no effect on output for most of the response horizon in the high regime, while the response is positive in the low regime. A positive net tax shock has a negative effect on GDP on impact in both regimes. After the first few quarters, however, the response of the high debt regime becomes positive, while the low regime responses remaining negative.
- Allowing for endogenous regime switches after initial impulse changes the responses for the high debt regime, as the internal dynamics leads the system to the threshold and to a switch to the low regime. The results for the switching impulse responses starting in the low debt regime are not considerably different from the no-switching case.

Finally, we consider also the specification with three regimes. Plotting the likelihood function, however, reveals considerable uncertainty about the lower threshold value. The upper threshold value corresponds exactly to the estimated threshold in the two regime case, so we conclude that the two regime model is less restrictive and more reliable.

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A Data

We use quarterly data for the US economy from 1960:1-2009:4. The source of all the variables are the NIPA accounts (available on the Bureau of Economic Analysis website), except for the time series for the stock of US public debt which is obtained from the FRED database (available on the Federal Reserve of St.Louis website). The definition of the variables is as follows:

- g_t : log of real per capita federal government total nominal expenditures minus net interest payments. Quarterly observations computed as

$$g_t = \log \frac{G - INT_PAY}{POP * GDPDEF}$$

where

- G: federal government total nominal expenditure from line 40 in Table 3.2. It is seasonally adjusted at annual rates.
 - INT_PAY: federal government nominal interest payments from line 29 in Table 3.2. It is seasonally adjusted at annual rates.
 - GDPDEF: The price index for GDP from line 1 in Table 1.1.4. It is seasonally adjusted and the base year is 2005.
 - POP: midperiod population from line 39 of Table 2.1.
- t_t : log of real per capita federal government total receipts . Quarterly observations computed as

$$t_t = \log \frac{T - INT_REC}{POP * GDPDEF}$$

where

- T: federal government total nominal receipts from line 37 in Table 3.2. It is seasonally adjusted at annual rates.

- INT_REC: federal government nominal interest receipts from line 13 in Table 3.2. It is seasonally adjusted at annual rates.
- y_t : log of per capita GDP. Quarterly observations computed as

$$y_t = \log \frac{GDP}{POP * GDPDEF}$$

where

- GDP: nominal GDP from line 1 in Table 1.1.5. It is seasonally adjusted at annual rates.
- Δp_t : GDP deflator inflation rate. Quarterly observations computed as

$$\pi_t = \log GDPDEF_t - \log GDPDEF_{t-1}$$

- i_t : nominal cost of financing the debt. Quarterly observations computed as

$$i_t = \frac{INT_PAY_t}{DEBT_{t-1}}$$

where

- DEBT: Federal Debt Held by the Public recomputed by the authors.