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Essays on Confidence and Macroeconomics

PhD in: Economics & Finance

Cycle: 29°

Student's Advisor: Tommaso Monacelli

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**Abstract**

This thesis presents an empirical and theoretical analysis of the role of confidence as a driver of business cycle fluctuations. Confidence is broadly defined as exogenous shifts in economic agents' expectations that are unrelated to current, and possibly future, economic fundamentals. Chapter 1 shows that confidence is an important driver of house prices in the United States. Confidence shocks are identified in a Structural Vector Auto-Regression (SVAR) as exogenous innovations to the University of Michigan Sentiment Index, by exploiting the particular timing and design of the University of Michigan Surveys of Consumers. Empirical results show that confidence can explain on average fifty percent of observed house price volatility and that innovations to confidence on their own can explain up to forty percent of the 2000s' housing boom in the United States. Chapter 1 also introduces the theoretical methodology to model confidence shocks as shocks to higher order beliefs in a small DSGE model with households' heterogeneity and financial frictions. Chapter 2 extends such a model in several dimensions in order to bring the model to the data with the goal of assessing the role of confidence in explaining the boom-bust of credit and house prices in the United States prior and post financial crisis. Results from the Bayesian estimation of the model show that innovations to confidence account for half of the volatility of observed house prices, and can explain up to forty percent of the 2000s' boom in housing prices, consistently with results from Chapter 1. Chapter 3 presents and solve an incomplete market model with heterogeneous households with aggregate innovations to confidence. The goal of the Chapter is to explore the importance of precautionary savings and cross-sectional heterogeneity for the propagation mechanism of belief shocks. The Chapter first shows that, by assuming GHH preferences, the labor supply response after a confidence shock can be derived in closed form, largely simplifying the computational burden of the solution algorithm. Second, a preliminary calibration of the model shows that precautionary savings motives have an important role in the transmission mechanisms of beliefs shocks. The Chapter concludes discussing the planned extensions of the model for future research.

# Introduction

The idea that business cycles can be driven by waves of optimism and pessimism has always been in economists' mind. Keynes' animal spirits and Pigou's expectation driven fluctuations, despite being more than eighty year old, are still highly debated in the macroeconomic literature, especially in the aftermath of the financial crisis.

The reason why the macroeconomic literature started looking more intensively after the financial crisis at the role of sentiments on aggregate fluctuations has to be found mainly in the boom and bust of house prices that characterized the 2000-2008 period. As we are going to see later, expectations played an important role in the building of the boom. It is then important to understand how aggregate economic activity in general can be driven by changes in beliefs and expectations that are not justified by any current or future fundamental.

However this has always been a controversial debate in macroeconomics. The so-called rational expectations revolution started in the seventies relies on the assumption that agents have probability beliefs which coincide with the "real" probabilities. This left almost no space for any alternative assumption about expectations of economic agents. Several attempts to relax the rational expectations assumption have been pursued by macroeconomists, but more in the direction of relaxing the rational expectation assumption than in the direction of making expectations playing per se a role for macroeconomic dynamics.[1].

Nevertheless the suggestion that confidence can affect economic has been explored by the

[1]See Sargent (1993), Evans and Honkapohja (1999) and Woodford (2013) for surveys on the topic.

macroeconomic literature.

Barski and Sims (2012) provide empirical evidence for confidence being able to affect aggregate economic activity: in a three-variable VAR with consumption, income and a measure of confidence they show that impulse responses of consumption and income to innovations in confidence have significant long-lasting effects and, importantly, that confidence does not seem to be Granger-caused by income or consumption, responding mostly to its own innovations.

An important branch of literature recently explored the macroeconomic effects of frictions that agents face in the acquisition of information, relaxing complete information as opposed to rational expectations[2]. The main achievement of macroeconomic models incorporating deviations from perfect information was to provide a framework to understand non-fundamental driven business cycle fluctuations. For instance, in Lorenzoni (2009) the noise component from a public signal for long-run productivity leads to aggregate mistakes in agents' expectations about future productivity. These mistakes result in aggregate fluctuations similar to the ones following an aggregate demand shock. Empirical evidence for the presence of informational frictions and for "news shocks" as the ones considered in Lorenzoni (2009) is provided in Carrol (2003), Coibion and Gorodnichenko (2012), Beaudry and Portier (2006) and Barski and Sims (2011).

This branch of the literature has then identified two ways through which confidence can affect aggregate economic activity; in a "news" approach, confidence can be viewed as containing fundamental information about the future state of the economy. The second way, instead, posits that fluctuations in macroeconomic activity can be actually driven by autonomous fluctuations in agents' beliefs, possibly completely unrelated to fundamentals. This approach is usually labeled the "animal spirits" view. Attempts to rationalize animal spirits have also been done in multiple equilibria settings, where beliefs are self-fulfilling and pin down the equilibrium path of a model; Benhabib and Farmer (1994) and Farmer (2012, 2013) are important contributions in this framework.

[2]See for instance Mankiw and Reis (2002), Sims (2003), Woodford (2001).

Recently, two important contributions developed a framework which is able to model confidence as a force akin to animal spirits without abandoning rational expectations nor equilibrium uniqueness. In Angeletos and La'o (2013) an aggregate shock, which the authors call "sentiment shock", affects the beliefs that an economic agent forms about the choices of other agents; this shock rationalizes shifts in optimistic (or pessimistic) beliefs. For instance, after positive sentiment shock a firm expects the demand for its product to increase and thus raises its demand for labor and capital, stimulating employment and output even without any increase in fundamentals.
Angeletos, Collard and Dellas (2018) propose a similar mechanism that enriches the beliefs structure of economic agents though being very tractable as it bypasses some technical difficulties typical of the noisy information literature. The mechanism is interpreted as confidence, since it affects the uncertainty that economic agents face about one another's choices. They embed such a confidence shock in a textbook RBC model and show that this shock generates realistic business cycle patterns and that it is actually the main source of business cycles in an estimated medium-scale DSGE.

The objective of this thesis is then to contribute to this literature by showing that forces akin to animal spirits can be indeed important for aggregate fluctuations and for house price volatilty. Chapter 1 shows some empirical evidence from a Structural VAR with a focus on house prices; in the data, about fifty percent of house prices' forecast error variance decomposition can be attributed to confidence shocks. Moreover, using a historical decomposition exercise, Chapter I shows that confidence was particularly important both in the building and in the burst of the housing boom that the U.S. experienced before the last financial crisis, confirming evidence in Case and Shiller (2003) and in Piazzesi and Schneider (2009). Based on this evidence, Chapter 2 proposes and estimates a dynamic stochastic general equilibrium model with housing, financial frictions and higher order beliefs (confidence) shocks modeled as in Angeletos et al. (2018). The results from the estimation, which are consistent with the ones in Chapter 1, suggest that "animal spirits" are quantitatively important for house price volatility and, especially, that they were particularly important in the 2000s housing price boom, suggesting that incorporating belief shocks in DSGE models is a step forward in solving the volatility puzzle for house prices. Chapter 3 explores the interactions between beliefs, market incompleteness and heterogeneity through the lenses of an incomplete market model with heterogenous agents and shows that precautionary motives can be of first order importance in the propagation of a shock to beliefs. The Chapter concludes with future research paths.

# Chapter 1

# Confidence and House Prices: Empirical Evidence

## 1.1 Introduction

The 2007-08 financial crisis and the subsequent Great Recession have renewed the interest of economists in the role of credit and housing in the macroeconomy, and in how they interplay over the business cycle. Credit booms usually precede financial crises and, since the size of credit expansions significantly predicts the severity of recessions following a financial crisis, it is then important to understand what drives an excess of credit (Jordà et al. 2013 and 2015b). The last financial crisis makes no exception. Figure 3.1 shows the evolution of a real index for house prices and the mortgage-to-GDP ratio for the United States since the late 1990s: in the seven years before the Great Recession, the mortgage-to-GDP ratio increased from a value of 47% up to about 70% and, contemporaneously, house prices appreciated by approximately 70% in real terms.

In this paper, we provide evidence for the role of confidence, broadly defined as exogenous shifts in expectations unrelated to current economic fundamentals, as a crucial driver of house prices, and quantify its role in explaining the last boom and bust in house prices and credit in the United States. We construct the evidence in two steps. First, through

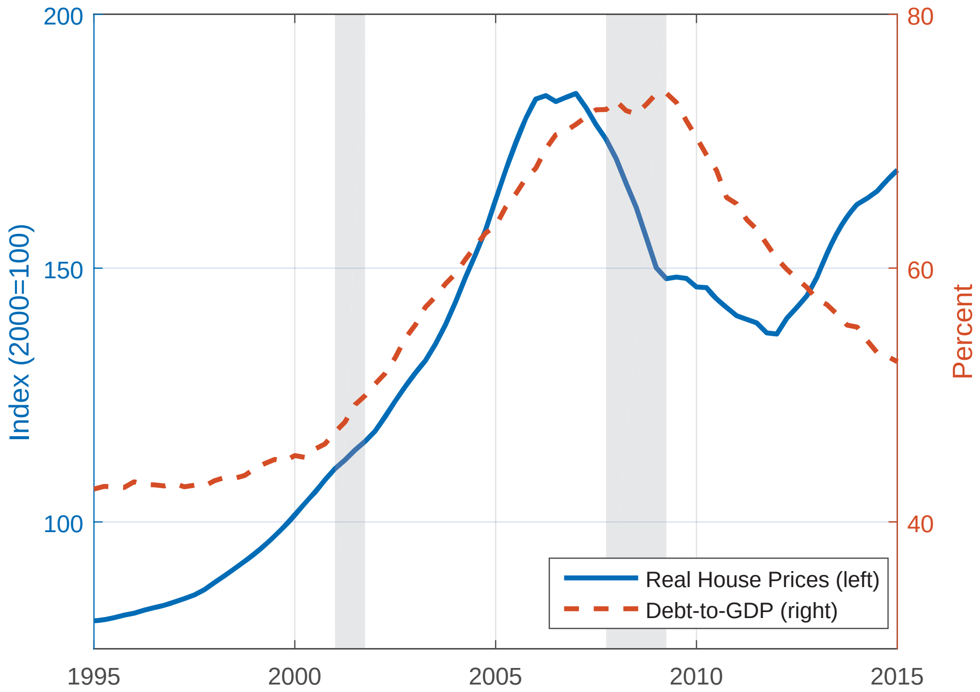


Figure 1.1: Two contemporaneous and unprecedented booms: housing prices and credit. Real house prices are the *S&P/Case-Shiller U.S. National Home Price Index* and Debt-to-GDP is the ratio of *nominal home mortgages from the balance sheet of U.S. households and nonprofit organizations* over *nominal GDP*.

the identification of innovations to an empirical proxy for confidence in a structural vector autoregression (SVAR), and second through the lens of a structural dynamic stochastic general equilibrium model (DSGE) with three main features: households' heterogeneity, financial frictions, and innovations to confidence. In the SVAR, confidence is proxied by the University of Michigan Consumer Sentiment Index, an index based on the University of Michigan Consumer Survey. We exploit the timing and design of the survey to identify structural shocks to confidence. In the DSGE, innovations to confidence are exogenous shifts in economic agents' higher-order-beliefs about current economic conditions.

The macroeconomic literature has identified two other main narratives as possible drivers of the booms mentioned above. The first narrative can be found in Taylor (2007), who argues that a prolonged period of low interest rates such as the one we observed during the Great Moderation made housing finance relatively cheap and attractive. Housing attractiveness translated into an increase in housing demand, which led to an increase in house prices. Also, since houses are typically used as collateral in mortgage contracts, the increase in house prices led to more favorable credit conditions. The second narrative is the one presented in Mian and Sufi (2010), and emphasizes a so-called process of credit liberalization. An overall loosening of credit standards delivered an increase in credit supply, which allowed more borrowing, even against unchanged collateral value, and possibly a collateral appreciation.

The two narratives postulate that some innovation to an economic fundamental is responsible for one or both of the booms. Our paper emphasizes a third narrative based on expectations and beliefs of economic agents that are not necessarily justified by economic fundamentals. Evidence for this narrative is in Case and Shiller (2003) and Piazzesi and Schneider (2009). The former show that, in 2003, home-buyers in the United States were expecting house prices to appreciate, on average, up to 15% per year, while the latter demonstrate that, in 2007, just before the reversal of the boom, the fraction of people believing that house prices would continue to increase doubled with respect to the previous year. From the financial crisis onwards, it became clear that expectations were playing an important role in the 2000s' boom[1]. What is less clear is whether changes in expectations that are not justified by current economic conditions are *quantitatively* important for the observed co-movement of house prices and credit. Also, a proper assessment of how much of the observed boom was belief-driven is still missing in the macroeconomic literature.

This paper provides such an assessment by quantifying the role of belief shocks in the context of an estimated DSGE model with financial frictions and household heterogeneity. Households' heterogeneity is modeled by assuming the existence of two types of representative households: a saver and a borrower. Financial frictions take the form of a collateral constraint, with borrowers being able to borrow up to a fraction of their housing wealth. Following Angeletos, Collard and Dellas (2018), we model confidence shocks as exogenous shifts in beliefs that generate waves of optimism and pessimism orthogonal to current and future economic fundamentals. The main difference between our paper and Angeletos et al. (2018) is that the latter explores the role of confidence shocks in driving business cycles abstracting from financial frictions, households' heterogeneity, and housing.

Our analysis leads to three main results. First, the confidence shock identified through the SVAR explains, on average, half of the forecast error variance of house prices at different horizons, and innovations to confidence are responsible for up to 40% of the housing boom. Second, the Bayesian estimation of the DSGE model delivers similar outcomes: the confidence shock is responsible for half of the unconditional variance of house prices, and the shock decomposition shows that the shock can explain a portion of the housing boom comparable with the 40% estimated in the SVAR. Third, there is a strong positive correlation (0.74) between the confidence series filtered from the estimated DSGE model and the University of Michigan Sentiment Index. The latter result suggests that the belief mechanism through which the shock works in the model is consistent with what economists use to call *confidence*.

The rest of the paper is organized as follows: Section 2 extends on the topics touched in this introduction and reviews the literature more extensively. Section 3 shows the

[1]See Piazzesi and Schneider (2016) for an extensive review of the literature.

empirical evidence, and Section 4 introduces the confidence shock in a baseline DSGE model. Section 5 extends the model in several dimensions typical of the medium scale DSGE literature in order to bring the model to the data. Section 6 presents the results from the estimation and Section 7 concludes.

## 1.2 Literature Review

This paper is related to three strands of literature. In the following we describe them separately and highlight how this work differentiates from existing papers.

### House-price volatility and financial frictions

The first strand of literature is the one investigating the role of housing, financial frictions, and financial shocks in order to understand plausible mechanisms behind large house price and credit swings and their quantitative importance for the recent financial crisis. Theoretical work in Hall (2011) shows how the response of the household sector to a credit tightening is a key ingredient for explaining the Great Recession. In Guerrieri and Lorenzoni (2011), a leverage shock in a Bewley-type model can push the economy into a liquidity trap, like the one the United States experienced in the aftermath of the financial crisis. Jermann and Quadrini (2012) show that a similar shock is important not only for capturing the dynamics of financial variables but also for explaining the volatility of real quantities. However, when it comes to explaining house price volatility this kind of shock performs badly. In Justiniano, Primiceri and Tambolotti (2015) changes in credit conditions, which are modeled as a leverage shock in a DSGE model with financial frictions à la Iacoviello (2005), are not able to reproduce the large boom-bust in house prices we observed before the financial crisis. In their calibrated experiment, what turns out to be important for explaining the housing boom-bust is a housing preference shock that affects the marginal rate of substitution between non-durable consumption and housing. The estimation of a similar model in Guerrieri and Iacoviello (2017) confirms this result:

such a preference shock explains almost the entire observed boom-bust in house prices. Although this shock can be a useful reduced form to obtain large swings in house prices, it is hard to think of such a shock as the main driver of large changes in house prices. A housing preference shock that generates a durable consumption expansion, by affecting the marginal rate of substitution between non-durables and durables, implies a counter-factual contraction in non-durable consumption[2].

Our paper extends the models in the literature by enriching the belief structure of the agents in a way that allows beliefs to play a key role for business cycles and house prices.

### Non-fundamental driven economic activity

The second strand of literature looks at how sentiments can drive aggregate economic activity. This idea, which has been present in economic thinking since at least Pigou (1927) and Keynes (1936), is even more prevalent in the popular press and the business community which often associate recessions to periods of low confidence. However, the so-called rational expectations revolution started in the 1970s, by relying on the assumption that agents have probability beliefs that coincide with the "real" probabilities, left almost no space for beliefs to deviate from rational expectations. Nevertheless, there is evidence that economic activity can be expectation-driven in a way that full information rational expectation models cannot account. Barski and Sims (2012) provide empirical evidence for confidence's ability to affect aggregate economic activity. In a three-variable VAR with consumption, income, and a measure of confidence they show that impulse responses of consumption and income to innovations in confidence have significant long-lasting effects and, importantly, that confidence does not seem to be Granger-caused by income or consumption, responding mostly to its own innovations. Gillitzer and Prasad (2018), using micro-data, get to a similar conclusion: variation in beliefs orthogonal to changes in fundamentals have a causal effect on consumer spending. One way to reconcile this evidence with rational expectations is to relax the assumption of complete informa-

[2] In the building of the housing boom we also observed a boom in non-durable consumption instead.

tion, assuming that economic agents face some frictions in the acquisition of information[3]. The main achievement of macroeconomic models incorporating deviations from perfect information was thus to provide a framework for understanding non-fundamental driven business cycle fluctuations[4]. Within this framework, the literature has identified two ways through which changes in confidence can affect aggregate economic activity. First, in a "news" approach, confidence can be viewed as containing fundamental information about the future state of the economy. Second, fluctuations in macroeconomic activity can be driven by autonomous fluctuations in agents' beliefs, which are completely unrelated to fundamentals at any point in time. This approach is usually named "animal spirits"[5]. Angeletos, Collard and Dellas (2018) provide a convincing mechanism through which confidence affects aggregate activity despite being orthogonal to current and future fundamentals: the mechanism works through an enrichment of the belief structure of economic agents, by relaxing the common-prior assumption, and is interpreted as confidence since it generates waves of optimism and pessimism on economic activity. A textbook RBC model provided with such a confidence shock generates realistic business cycle patterns and, in an estimated medium-scale DSGE model, the confidence shock turns out to be the most important driver of business cycle fluctuations.

Our paper explores the extent to which this kind of belief shock can be a driver not only of business cycles but also of house-price fluctuations through the interaction with financial frictions.

### The role of expectations in housing/credit booms

The third strand of literature includes papers that look at how either reshaping the way economic agents form expectations or enriching their belief structure can deliver mecha-

[3]See for instance Mankiw and Reis (2002), Sims (2003), Woodford (2001).

[4]For instance, in Lorenzoni (2009) the noise component from a public signal for long-run productivity leads to aggregate mistakes in agents' expectations about future productivity. These mistakes result in aggregate fluctuations similar to the ones following an aggregate demand shock.

[5]A similar role for animal spirits is also common in multiple equilibria setting, where beliefs are self-fulfilling and pin down the equilibrium path of a particular model. Benhabib and Farmer (1994) and Farmer (2012, 2013) are important contributions in this sense. However here we are focusing on a single equilibrium model since the final objective is to quantify the importance of confidence in the data.

nisms that generate plausible quantitative boom-bust cycles in housing and credit. Among other things, Berger, Guerrieri, Lorenzoni and Vavra (2018) show how a general equilibrium, incomplete market model with housing can generate a realistic housing boom and bust if households' expectations on future house-price growth can deviate from rational expectations according to a calibrated exogenous process. Their exercise, however, puts some strain on the rationality of economic agents, while the mechanism introduced in this paper does not. In a similar setting Kaplan, Moll and Violante (2018) introduce a belief shock for house prices and show that this shock can be quantitatively important in reproducing the 2000s' housing boom. However, their belief shock is not unrelated to future fundamentals as it affects expected future taste for housing services. Here, the belief shock is, instead, orthogonal to economic fundamentals at any point in time. Models of rational bubbles, summarized in Martin and Ventura (2018), also provide a convincing framework for how asset prices can rationally deviate from their fundamental values due to shifts in market psychology in a multiple equilibria model. It is difficult however to take a multiple equilibria model to the data in order to assess the empirical importance of such a mechanism. The model presented here, generating waves of optimism and pessimism in a single equilibrium linear DSGE setting, can instead be readily estimated and used to test the ability of the confidence shock in explaining the observed variation in house prices. Finally, Bordalo, Gennaioli and Shleifer (2017) propose a dynamic model with diagnostic expectations that can deliver credit market overheating because of a behavioral over-reaction to good or bad news. However, their model also posits some irrational behavior of economic agents, differently from the model presented here.

## 1.3 Empirical Evidence

In this section, we outline the empirical evidence on the ability of confidence in affecting house prices and economic activity in general.

**A proxy for confidence.** One of the most used proxies for confidence in the United States is the University of Michigan Consumer Sentiment Index. The index is based on the following five questions that are part of the broader Michigan Survey of Consumers:

1. *Do you think that today is a good or a bad time to buy a major household item?*

2. *Do you think that your family is worse off financially with respect to one year ago?*

3. *Do you think that a year from now your family will be better off financially, or worse off, or just about the same as now?*

4. *Do you think that during the next twelve months business conditions in the country will experience good times financially, or bad times, or what?*

5. *Which would you say is more likely, that in the country as a whole we'll have continuous good times during the next five years or so, or that we will have periods of widespread unemployment or depression, or what?*

The first two questions are related to present conditions, while the remaining three regard expectations about future economic conditions over the next year and five years. The survey is conducted by interviewing by phone, each month, a minimum of 500 respondents from 1978. Moreover, a mid-month release of the index is published each month, considering only the interviews conducted during the first two weeks of the month. The mid-month release is the series that we are going to use in the empirical model for reasons that are going to be explained later when discussing the identification assumption.
We now want to assess whether innovations in this proxy for confidence have a significant effect on aggregate activity and especially on house price dynamics. In order to do so, we use a seven-variable structural vector autoregression (SVAR) model.

**Data.** Data are from the United States, have monthly frequency and range from January 1985 to June 2015. We include seven variables in the empirical model. This means that each months we observe a column vector $X_t$:

$$X_t \equiv [Conf_t, HP_t, D_t, ResI_t, IP_t, \pi_t, FFR_t]',$$

where $Conf_t$ is a measure for confidence obtained considering the mid-month release of the sentiment index constructed using only the two questions related to current economic conditions, $HP_t$ is a measure of real house prices, $D_t$ is a measure of household debt, and $ResI_t$ is a measure of residential investments. The label $IP_t$ stands for the industrial production index, $\pi_t$ is a measure of inflation, and $FFR_t$ is the Federal Funds Rate. We discuss data sources and construction in Appendix A.

**The model.** We estimate the following vector autoregression with two lags[6]:

$$X_t = \mu_t + B_1 X_{t-1} + B_2 X_{t-2} + \varepsilon_t,$$

where $\mu_t$ is a deterministic component containing a constant and a trend, and $\varepsilon_t$ is a vector of reduced form shocks. Of course, the reduced form shocks obtained as residuals from the regression of confidence on its own lags and lags of all the other variables cannot be considered as structural innovations as they are correlated with the rest other shocks in the VAR.

**Identification.** In order to recover structural innovations, we have to impose some restrictions on the effects of a shock on the dynamic system. Here we set the restrictions by assuming that the confidence measure is not affected contemporaneously by other shocks. The assumption is justified by the timing of the survey, which is mainly conducted during the first two weeks of the month[7], when economic data for that month are not yet released. At a monthly frequency, it is then reasonable to assume that respondents, when they are

[6] Number of lags is selected through information criteria.

[7] Ludvingson (2004) reports that 2/3 of the survey is conducted during the first two weeks on the month.

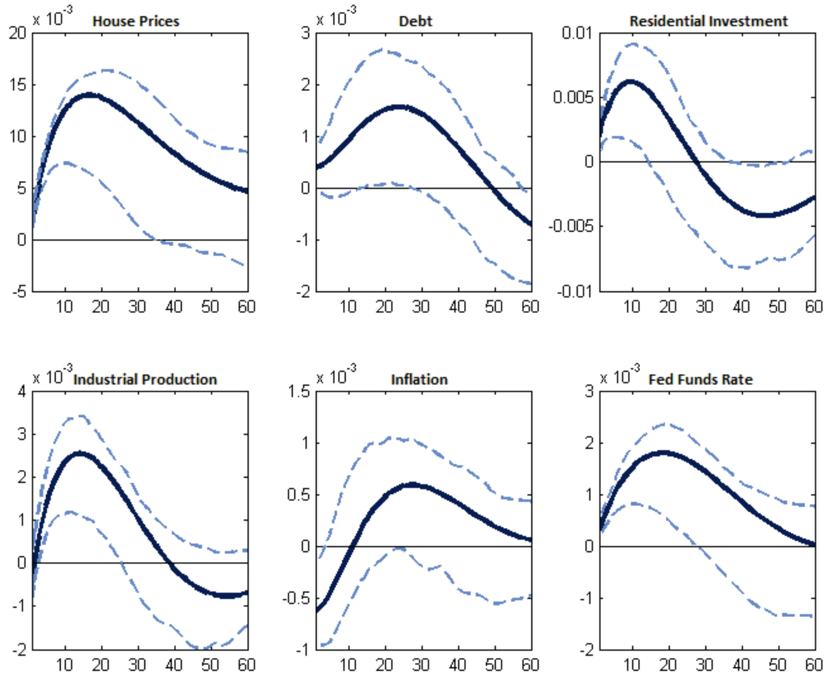


Figure 1.2: Impulse response of economic variables to a one-standard-deviation positive confidence shock. 90% confidence intervals delimited by dashed lines.

called at the phone and asked about their opinions on the state of the economy, have beliefs that are not yet affected by other economic shocks occurred in those very first days of the months. This assumption is even more valid for the mid-month release of the survey, which contains only answers from people interviewed in the first two weeks of the month[8]. The identification assumption results in practice in a Choleski decomposition of the estimated variance-covariance matrix of reduced-form residuals with confidence ordered first[9].

**Impulse responses.** Figure 1.2 plots the dynamic responses of the variables in the system after a one standard deviation confidence innovation, together with 90% confidence

[8]Curtin (2002) shows that the mid-month release and the official release are almost identical, reinforcing the idea that confidence does not move much within a month. The main results showed in this section are robust to the adoption of the official release instead of the mid-month release and to the adoption of the 1-month-lagged confidence.

[9]Confidence is ordered first assuming that the decomposition is in terms of a lower-triangular matrix. The ordering of the other variables is irrelevant as long as the identification of the only confidence shock is concerned.

Table 1.1: Forecast error variance decomposition.

| | Shocks | | | | | | |
|---|---|---|---|---|---|---|---|
| Months | Conf | HP | Borr | ResInv | IP | Infl | FFR |
| 6 | 49.90% | 38.09% | 0.43% | 0.47% | 7.20% | 3.23% | 0.68% |
| 12 | 58.96% | 21.13% | 0.39% | 1.05% | 13.98% | 3.33% | 1.16% |
| 24 | 54.94% | 15.59% | 0.19% | 1.73% | 23.52% | 2.09% | 1.94% |
| 36 | 47.76% | 15.53% | 0.95% | 1.95% | 29.21% | 1.47% | 3.13% |

Note: Forecast error variance decomposition at several horizons (1 to 36 months) for house prices. Each column returns the contribution of a shock to the forecast error variance for the horizon defined by the row (rows sum to one).

bands computed by bootstrapping. A positive shock to confidence increases significantly and for a prolonged period (about 36 months) house prices with a peak of about 1.5%. The increase in house prices is associated with an increase in borrowing, though barely significant, and in residential investments. A positive shock to confidence also results in a mild, though persistent, increase in real activity (proxied by industrial production), which is consistent with previous aggregate and micro evidence[10]. Inflation barely responds.

**Forecast error variance decomposition.** Table 1.1 shows the Forecast Error Variance Decomposition (FEVD) for house prices at different horizons. Each row defines a particular horizon, and a column defines a particular shock. Each entry of the table contains the percentage contribution of a particular shock for the variance of the forecast error on house prices at a given horizon. For instance, the first-row/first-column entry tells us that 49.90% of the variance of the 6-month-ahead forecast errors for house prices made using the estimated VAR is explained by shocks to confidence. The table shows that innovations to confidence explain, on average, about 50% of the variance of house prices at horizons ranging from six months to 3 years. The rest of the variation is mostly explained by shocks to house prices themselves and to industrial production, especially at larger horizons.

[10]Barski and Sims (2012) provide time series evidence for positive innovations to confidence causing an expansion in real activity and Gillitzer and Prasad (2018) find a positive causal effect from non-fundamental driven innovations in beliefs and real activity using disaggregated Australian survey data.

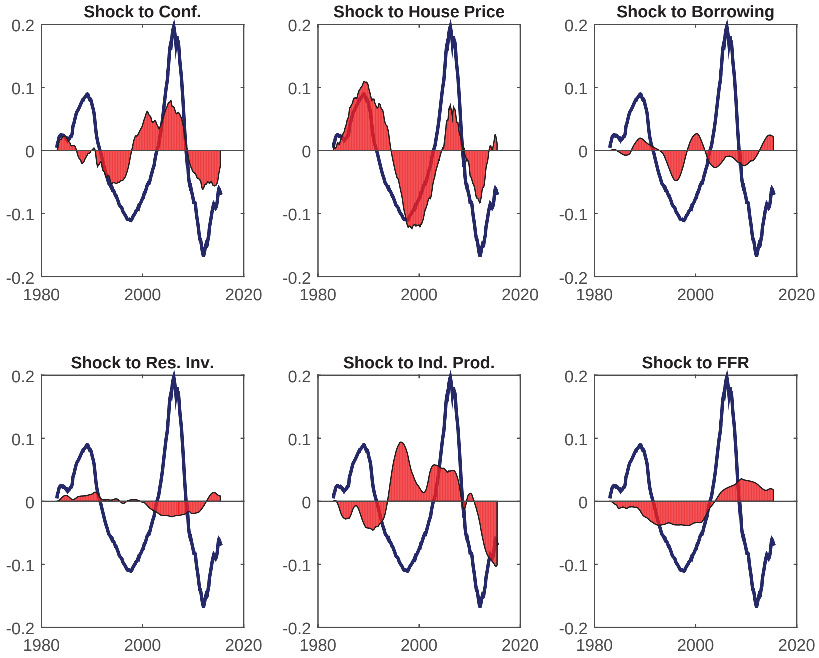


Figure 1.3: Historical decomposition for house prices. In solid blue line the actual, observed value for house prices. The line delimiting the red areas represent the level of house prices when only one particular shock is active.

**Historical decomposition.** From analyzing the impulse response functions and the table summarizing the forecast error variance decompositions we learn that exogenous movements to confidence can be an important source of fluctuations for house prices unconditionally. This does not mean that confidence was indeed playing a significant role during the build-up of the 2000s' housing boom. A historical decomposition exercise helps us in understanding how much of the housing boom is to be attributed to innovations to confidence. The historical decomposition decomposes the observable variables used to estimate the SVAR into different series computed by running several counterfactuals, each one shutting down all the structural shocks but one. Each panel in Figure 1.3 shows with a solid line the observed value of house prices[11], while the colored areas define the

[11] It is actually the observed component of house prices that is not explained by deterministic components in the regression, which are the constant and the deterministic trend.

level of house prices in the counterfactual experiment where only one structural shock is turned on. For example, the top left panel shows the level of house prices for the counterfactual experiment in which only the confidence shock is turned on. The confidence shock by itself is able to explain about 40% of the boom during the early 2000s, driving house prices up especially in the very beginning of the boom in the very late 1990s. Interestingly, the shock had no importance in the previous housing boom during the 1980s.

**Discussion of SVAR results.** From this analysis, we understand that confidence is an important driver for house prices and can explain a significant portion of the boom-bust episode we observed in the 2000s. However, there can be two issues in this exercise.
As mentioned in the introduction, there are two possible alternative drivers for house prices which could not be properly taken into account in this SVAR analysis. For example, if monetary policy was responsible for the increase of house price because of a prolonged period of low-interest rates as suggested by Taylor (2007), exogenous movements in the FFR should explain a significant fraction of house price volatility. Since we do not properly identify monetary policy shocks in the SVAR, it can be that this is not taken into account. We then add the narratively identified monetary policy shocks from Romer and Romer (2004) to the VAR and re-run the analysis estimating a Proxy-SVAR. Table 1.2 shows that confidence does not lose its relative importance in explaining house price movements at different horizons.

The second possible issue is that the Sentiment index, which proxies for consumer beliefs about current economic activity, could contain some information on future economic fundamentals. This is the reason why, as a measure of confidence, we only included the Sentiment Index constructed using only the questions on current economic conditions. We want indeed to reduce the amount of information about the future contained in the Imdex. However, this may be not enough since, in the presence of forward-looking agents, a shock anticipating some change in fundamentals in the future may have an effect on current economic activity and expectations.

Table 1.2: Forecast error variance decomposition adding monetary policy shocks.

| | Shocks | |
|---|---|---|
| Months | Romer & Romer Monetary Policy | Confidence |
| 6 | 0.06% | 49.70% |
| 12 | 0.32% | 54.91% |
| 24 | 1.48% | 46.92% |
| 36 | 3.27% | 35.95% |

Note: Forecast error variance decomposition for house prices when including Romer and Romer (2004) narratively identified monetary policy shocks. Each column returns the contribution of a shock to the forecast error variance for the horizon defined by the row.

## 1.4 A Baseline Model

In this section, we outline a real business cycle model with a durable good, households' heterogeneity, financial frictions and an exogenous shock to beliefs building on Angeletos et al. (2018). We are going to label this shock as *confidence* since it will be generating shifts in beliefs that resemble shifts in optimism or pessimism on current economic conditions. The model resembles the two agents model of Iacoviello (2005) with an enriched belief structure to allow for shifts in beliefs. This simple model is going to be useful as a building block to understand the mechanism through which the confidence shock operates and how it affects durable consumption. In the next section, we will extend the model by adding several exogenous shocks, as well as real and nominal rigidities, for estimation purposes.

**General setting.** Time is discrete. There is a continuum of islands, indexed by $i$, and each island is inhabited by a representative firm and two representative households, a borrower and a saver. Households derive utility from consuming an index of non-durable consumption defined as a CES aggregate of the non-durable local goods produced in each island, and a durable good. The durable good is a local good in fixed supply, equal for all islands. The firm rents capital and hires labor from the households to produce a local differentiated good with some exogenous total factor productivity (TFP) $A_t$, which is common across all islands and follows a stationary first-order autoregressive process.

**Introducing shifts in beliefs.** Each time period contains two stages, and TFP is not perfectly observed in the first stage. During stage 1, in each island, the local household has to decide labor supply, and the local firm chooses optimal labor and capital demand. Decisions are made on the basis of an island-specific noisy signal of TFP: $x_{it} = A_t + \epsilon_{it}$. In stage 2, the true level of $A_t$ is revealed to all islands, and all good markets operate.

Differently from standard incomplete information macroeconomic models, we relax the common-prior assumption, meaning that all agents may not share the same beliefs about one another beliefs. In other words, each island may believe that the signals received by other islands are biased. The bias is given by an exogenous state variable, $\Gamma_t$, which is perfectly observed by each island.

Let $x_{it} = A_t + \epsilon_{it}$ be the signal for $A_t$ received by island $i$, composed by the true level of $A_t$ plus an island-specific error. Imposing the common prior assumption would imply:

$$E_{it}[\epsilon_{it}] = E_{it}[\epsilon_{jt}] = 0, \ \forall i, j \in (0, 1).$$

Hence, each island believes that any other island in the economy is receiving, on average, the same signal.

Here we relax the common-prior assumption by assuming the following:

$$\begin{cases} E_{it}[\epsilon_{it}] = 0, & \forall\ i \in (0,1) \\ E_{it}[\epsilon_{jt}] = \Gamma_t, & \forall\ j \neq i. \end{cases}$$

This condition implies that each island believes, as in the previous case, that its own signal is centered around the true level of $A_t$, but that all the other islands are receiving a biased signal centered around $A_t + \Gamma_t$. We can interpret the variable $\Gamma_t$ as governing confidence: when it is positive, every island thinks that other islands are receiving signals

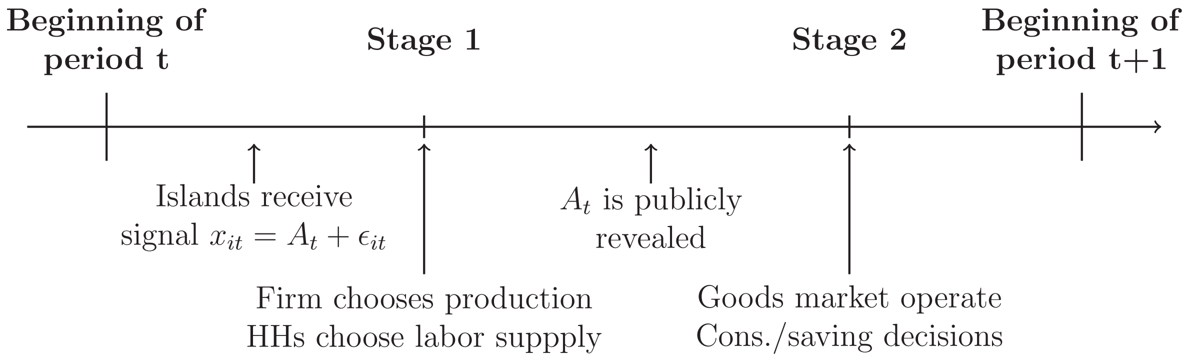


Figure 1.4: Within period timing. Stage 1 decisions are made on the basis of a island-specific signal. Stage 2 decisions are made on the basis of complete information.

about the state of the economy that are more optimistic than its own. If the stochastic process generating $\Gamma_t$ entails some persistence, this optimism is expected to last even in the future. Here, we assume that $\Gamma_t$ follows a stationary AR(1) process with zero mean, meaning that in the steady state all the agents agree on one another's beliefs about the economy. Figure 1.4 summarizes graphically the within-period timing assumptions.

In order to understand the mechanism through which the variable $\Gamma_t$ affects aggregate economic activity by shifting agents' beliefs, let's now analyze separately the optimization problems for the three economic agents living in island $i$: the saver, the borrower, and the representative firm.

**Saver's problem.** The main difference between the two types of households is given by their discount factor[12]. We assume:

$$\beta > \tilde{\beta},$$

where $\beta$ and $\tilde{\beta}$ are, respectively, the saver's and borrower's discount factors.

[12]The labels *borrower* and *savers* derive indeed by this difference: the more patient household in equilibrium will be more willing to postpone consumption and thus to save by lending to the more impatient household. The other difference between the two types of households lies in their budget constraints as we are going to assume that the saver makes the investment decision and owns the representative firm.

Remember that each time period is divided in two stages. Let's start first by the maximization problem in stage 2 when the actual level of $A_t$ is revealed. In this stage, the saver takes as given his own available resources since labor supply is settled in stage 1 and he only has to choose how much to consume and how much to save out of his disposable resources. The representative saver solves the following maximization problem:

$$\max_{c_{it},h_{it},i_{it},d_{it}} E_t\left[\sum_{s=0}^{\infty}\beta^s U(c_{it},h_{it},n_{it})\middle|\mathcal{I}_2\right],$$

$$s.t. \quad P_tc_{it}+P_ti_{it}+Q_{it}h_{it}+P_tr_{i,t-1}d_{i,t-1}=P_td_{it}+Q_{it}h_{i,t-1}+W_{it}n_{it}+R^k_{it}k_{i,t-1}+S_{it} \tag{1.1}$$

$$k_{it}=(1-\delta)k_{i,t-1}+i_{it}. \tag{1.2}$$

It is important to notice that the expectation is taken with respect to the information set in stage 2 ($\mathcal{I}_2$), which is different from the information set in stage 1 ($\mathcal{I}_1$). Equation (1.1) is a nominal budget constraint and equation (1.2) is the low of motion for capital. The variable $c_{it}$ is defined as the final good consumed by the saver living in island $i$ and is a bundle defined as:

$$c_{it}=\left(\int_0^1(c_{i,j,t})^{1-\rho}dj\right)^{\frac{1}{1-\rho}}, \tag{1.3}$$

where $c_{i,j,t}$ is the quantity of the local good produced in island $j$ demanded by island $i$'s saver. $P_t$ is the price of the final good, $Q_{it}$ is the price of the durable good, $W_{it}$ and $R^K_{it}$ are, respectively, the local wage and rental rate of capital, $S_{it}$ are profits from the local firm in the island and $\delta$ is capital depreciation. Finally, $d_{it}$ is end-of-period real debt[13]. Expressing everything in terms of the final non-durable consumption good price and normalizing $P_t=1$, the real budget constraint reads:

$$c_{it}+i_{it}+q_{it}h_{it}+r_{i,t-1}d_{i,t-1}=d_{it}+q_{it}h_{i,t-1}+w_{it}n_{it}+r^k_{it}k_{i,t-1}+s_{it}.$$

[13]Notice that $d_{it}$ can be lower than zero and, in that case, it is credit.

Define the marginal utility from non-durable consumption $\lambda_{it} := U_c(c_{it}, h_{it}, n_{it})$ and the marginal utility from durable consumption $\Delta_{it} := U_h((c_{it}, h_{it}, n_{it}))$. First order conditions can then be written as following:

$$\lambda_{it} = \beta E_t[\lambda_{i,t+1}(r^k_{i,t+1} + 1 - \delta)|\mathcal{I}_2] \tag{1.4}$$

$$\lambda_{it} = \beta E_t[\lambda_{i,t+1} r_{it}|\mathcal{I}_2] \tag{1.5}$$

$$q_{it}\lambda_{it} = \Delta_{it} + \beta E_t[q_{i,t+1}\lambda_{i,t+1}|\mathcal{I}_2] \tag{1.6}$$

Equations (1.4) and (1.5) are standard Euler Equations for non-durable consumption and define a no-arbitrage condition between capital and bond credit accumulation, while Equation (1.6) equates the marginal cost of durable consumption, in terms of non-durable marginal utility, to the marginal benefit of durable. The latter is composed of the sum of current marginal utility from the durable good and expected future gains from reselling the good.

Let's now turn to stage 1. In stage 1 the saver chooses labor supply by maximizing the same expected lifetime utility as before, but with a slightly different information set which affects its budget constraint in the following way:

$$E_t[c_{it} + i_{it} + q_{it}h_{it}|\mathcal{I}_1] = E_t[q_{it}|\mathcal{I}_1]h_{i,t-1} + w_{it}n_{it} + r^k_{it}k_{i,t-1} + E_t[\pi_{it}|\mathcal{I}_1].$$

The first order condition for this problem equates, as usual, the wage to the marginal rate of substitution between expected stage-2 consumption and leisure:

$$w_{it} = \frac{n^{\phi}_{it}}{E_t[\lambda_{it}|\mathcal{I}_1]}. \tag{1.7}$$

**Borrower's problem.** The borrower faces a similar optimization problem, but she does not invest in physical capital and she faces a constraint limiting the amount of credit that she can get. The constraint, following Kiyotaki and Moore (1997), is given by a debt limit that is a fraction $M$ of the value of the durable good owned by the borrower. As shown in Kiyotaki and Moore (1997), this type of collateral constraint can be micro-founded by a problem of costly contract enforcement, where, if the borrower fails to repay the debt, the lender can seize her house. Given some liquidation cost, the effective amount that the lender can seize from the borrower is just a fraction M<1 of her housing wealth[14]. The maximization problem in stage 2 is then the following[15]:

$$\max_{\tilde{c}_{it},\tilde{h}_{it},\tilde{d}_{it}} E_t\left[\sum_{s=0}^{\infty}\tilde{\beta}^s U(\tilde{c}_{it},\tilde{h}_{it},\tilde{n}_{it})\middle|\mathcal{I}_2\right],$$

$$s.t. \quad \tilde{c}_{it} + q_{it}\tilde{h}_{it} + r_{i,t-1}\tilde{d}_{i,t-1} = \tilde{d}_{it} + q_{it}\tilde{h}_{i,t-1} + w_{it}\tilde{n}_{it} \tag{1.8}$$

$$\tilde{d}_{it} \leq M q_{it}\tilde{h}_{it}. \tag{1.9}$$

Defining $\mu_{it}$ the lagrangian multiplier attached to the collateral constraint, the first order conditions can be written as follows:

$$\tilde{\lambda}_{it} - \mu_{it} = \tilde{\beta}E_t[\lambda_{i,t+1}r_{it}|\mathcal{I}_2] \tag{1.10}$$

$$q_{it}\tilde{\lambda}_{it} = \tilde{\Delta}_{it} + \beta E_t[q_{i,t+1}\tilde{\lambda}_{i,t+1}|\mathcal{I}_2] + M\mu_{it}q_{it} \tag{1.11}$$

Compared to the saver's first order conditions, these two equations are slightly different because of the presence of the collateral constraint. Equation (1.10) states that the borrower is not in principle able to equate her current marginal utility to the expected marginal gain of postponing consumption unless the collateral constraint is not bind-

[14] In the following I will refer to the parameter $M$ also using the term Loan-To-Value (LTV) ratio.

[15] Here and also in the fully-fledged model a $\sim$ on top of a variable denotes quantities and parameters related to the borrower.

ing, i.e. $\mu_{it} = 0$. Equation (1.11) still bears the interpretation that, at the margin, the marginal cost of a durable good equates its marginal benefits. The difference with the respect to the saver's optimal condition is that, for the borrower, the marginal benefit contains an additional term that represents a gain in marginal utility coming from the relaxation of the borrowing constraint.

Stage 1 problem is instead exactly the same as the one faced by the saver and delivers an identical optimality condition for labor supply:

$$w_{it} = \frac{\tilde{n}_{it}^{\phi}}{E_t[\tilde{\lambda}_{it}|\mathcal{I}_1]}. \tag{1.12}$$

**Firm's problem.** The only relevant stage for the firm is stage 1, when it has to decide how much labor and capital to demand in order to produce a differentiated local good. The decision is based on a noisy signal on the level of total factor productivity $A_t$ of its production function:

$$y_{it} = A_t k_{it}^{\alpha}(N_{it}^{d})^{1-\alpha}, \tag{1.13}$$

where $N^d$ is labor demand from both households.
The demand that the firm is facing is given by[16]:

$$y_{it} = \left(\frac{p_{it}}{P_t}\right)^{-1/\rho} E_t[Y_t|I_1], \tag{1.14}$$

where $P_t$ is the price of the final good and $Y_t$ is aggregate demand for such a good. Remember that the good market operates in stage 2: in stage 1, the firm has to forecast expected aggregate demand in stage 2. Expected aggregate demand crucially depends on

[16]This demand comes from the fact that the household derives utility from consuming a final good defined as in equation (1.3), which is a CES aggregate of all the local goods produced in the island with an elasticity of substitution between any of the local goods equal to $\frac{1}{\rho}$. Solving the usual expenditure minimization problem yields to an optimal demand as the one in (1.14).

the confidence shock $\Gamma_t$. If $\Gamma_t > 0$ the firm believes that all other islands behave as if they received an average signal $A_t + \Gamma_t > A_t$. This means that the firm is over-optimistic about aggregate demand for the current period. The firm then increases its production, raising its demand for capital and labor. Wages and rental rates for capital thus increase after a positive shock to confidence. Efficiency conditions for the firm are then as follows:

$$w_{it}n_{it} = (1-\rho)(1-\alpha)E_t[Y_t^{\rho}y_{it}^{1-\rho}|\mathcal{I}_1] \tag{1.15}$$

$$r_{it}^{k}k_{it} = (1-\rho)\alpha E_t[Y_t^{\rho}y_{it}^{1-\rho}|\mathcal{I}_1] \tag{1.16}$$

**Market Clearing and Equilibrium.** Market clearing conditions in the four markets (final non-durable consumption good, durable good, labor and debt) have to be satisfied:

$$y_{it}^{1-\rho}Y_t^{\rho} = c_{it} + \tilde{c}_{it} + i_{it} \tag{1.17}$$

$$N_{it}^{d} = n_{it} + \tilde{n}_{it} \tag{1.18}$$

$$d_{it} + \tilde{d}_{it} = 0 \tag{1.19}$$

$$h_{it} + \tilde{h}_{it} = \bar{H}, \tag{1.20}$$

where $\bar{H}$ is the fixed supply of the durable good.

An equilibrium for the model is then, given two exogenous processes for $A_t$ (technology) and $\Gamma_t$ (confidence), a set of processes:

$$\{\lambda_{it}, \tilde{\lambda}_{it}, c_{it}, \tilde{c}_{it}, n_{it}, \tilde{n}_{it}, w_{it}, h_{it}, \tilde{h}_{it}, i_{it}, k_{it}, r_{it}, r_{it}^{k}, q_{it}, \mu_{it}, \tilde{d}_{it}\},$$

satisfying conditions (2), (4)-(13), (15)-(18) and (20).

**Parametrization.** The instantaneous utility function $U(c_{it}, h_{it}, n_{it})$ has the following form:

$$U(c_{it}, h_{it}, n_{it}) = \log(c_{it}) + \psi \log(h_{it}) - \frac{n_{it}^{1+\phi}}{1+\phi}, \tag{1.21}$$

where the parameter $\psi$ governs the marginal rate of substitution between non-durable and durable consumption.

For this section, we keep the calibration close to the RBC literature: the discount factor $\beta$ is set to 0.99, the Frisch elasticity of labor supply to 1, the capital share is $\alpha = 0.3$ and capital depreciation $\delta$ is set to 0.025. The parameter $\psi$ is set to 0.05 to have a steady state housing wealth to consumption ratio equal to 2.3, which is the value reported in Iacoviello (2011).The LTV ratio $M$ is set to 0.85. The persistence of the technology shock is set to 0.95, while the persistence of the confidence shock is set to 0.75.

**Solution algorithm.** The model is log-linearized around its non-stochastic steady state and solved using the algorithm derived in Angeletos et al. (2018). The solution of the model is given by the following dynamic system:

$$\begin{cases} \mathbf{Y}_t = AX^b_{t-1} + BS_t + C\Gamma_t \\ s_t = Rs_{t-1} + \epsilon_t \\ \Gamma_t = Q\Gamma_{t-1} + \nu_t, \end{cases}$$

where $\mathbf{Y}_t$ contains all the endogenous variables of the model, $X^b_t$ contains the endogenous states, $s_t$ are the exogenous states and $\Gamma_t$ is the belief shock, which is also an exogenous state. The intuition behind the solution algorithm is that, after showing that matrices $A$ and $B$ are exactly the ones that one would obtain by solving the model with no fluctuations in higher-order beliefs, the matrix $C$ can be retrieved by a method of undetermined coefficients that takes care of the different information sets in the two stages.

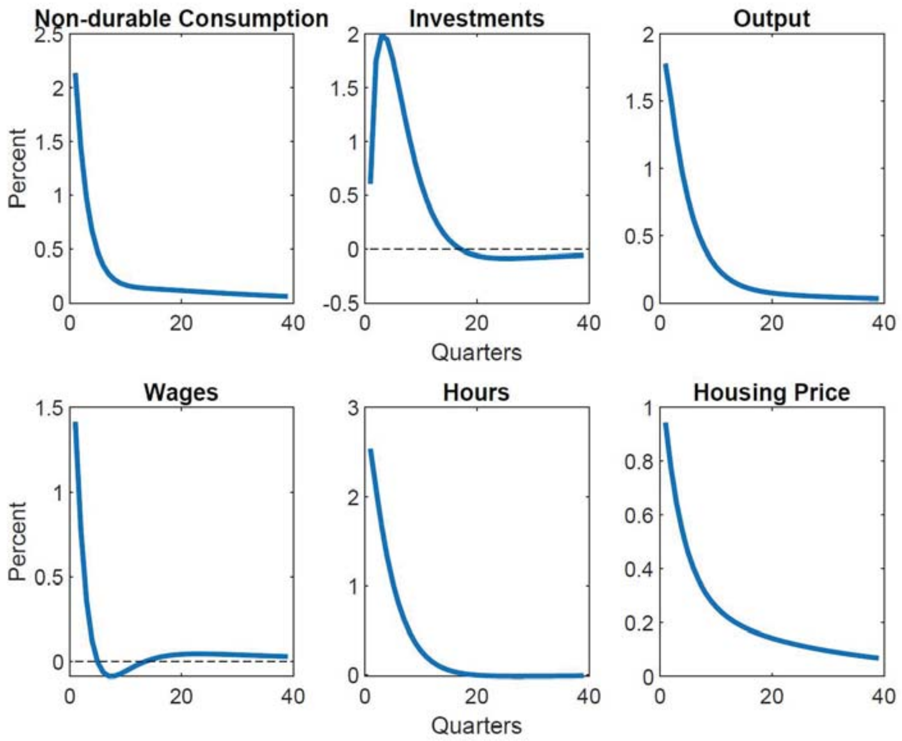


Figure 1.5: Impulse response of aggregate variables to a one standard deviation positive confidence shocks. Units are percent log-deviations from steady state.

**Dynamics.** Figure 1.5 plots the response of aggregate variables to positive confidence shock. As we have seen in the previous section, following a positive shock to confidence the firm in each island expects higher demand from its product and thus raises the demand for capital and labor. The increase in labor demand drives up wages, and households increase labor supply in stage 1. Higher wages and higher hours result in higher income for both the households, which then raise non-durable consumption in stage 2. The saver also raises investment because of the increase in the marginal return to capital. Finally, the price of the durable good also increases. Let's see why by looking at the optimal durable demand conditions for the two representative households, after having substituted the marginal utilities implied by the assumed functional form in (1.21) :

$$\frac{q_{it}}{c_{it}} = \frac{1}{h_{it}} + \beta E_t \left[ \frac{q_{i,t+1}}{c_{i,t+1}} | \mathcal{I}_2 \right] \tag{1.22}$$

$$\frac{q_{it}}{\tilde{c}_{it}} = \frac{1}{\tilde{h}_{it}} + \beta E_t \left[ \frac{q_{i,t+1}}{\tilde{c}_{i,t+1}} | \mathcal{I}_2 \right] + M \mu_{it} q_{it} \tag{1.23}$$

Assume that current and expected housing prices were not moving: the left-hand-sides of equations (1.22) and (1.23), which give the marginal cost of housing in terms of non-durable consumption for the saver and the borrower, would be decreasing given the increase in non-durable consumption. Since the belief shock entails optimism about the *current* state of the economy and is perceived to be short-lived, future non-durable consumption in the right-hand-sides cannot increase enough to compensate the decrease in the marginal cost of durable consumption. If house prices were not moving, marginal utility from durable consumption would have to decrease and durable demand would have to increase for both households. Since housing is in fixed supply, if house prices were not moving there would a competing demand for housing which eventually would have to drive up house prices. Moreover, as soon as house prices increase, the borrower experiences also a relaxation of her borrowing constraint which then amplifies the initial positive income effect of the confidence shock. This amplification mechanism is going to be at work also in the fully-fledged model estimated in the next Chapter: the objective of the estimation is then to evaluate the empirical performance of the mechanism and to understand its importance during the last housing boom.

# Chapter 2

# Confidence and House Price: Model & Estimation

In the previous Chapter we have seen how expectations and beliefs are a potentially crucial driver of house prices, particularly during the boom-bust episode that characterized the last financial crisis in the United States. The Chapter provided some new empirical evidence exploiting the design of the Michigan Survey to identify exogenous movements in Confidence and showed that they indeed played an important role in the financial crisis. In this Chapter we build a medium-scale DSGE model with housing and financial frictions similar to the one in Guerrieri and Iacoviello (2017), extend it with the confidence shock described in the previous chapter, and estimate on aggregate US data in order to quantify the role of beliefs on aggregate fluctuations and especially on the housing boom-bust of the 2000s.

## 2.1 The Fully-Fledged Model

We now describe the full model, which assumes the existence of a continuum of monopolists in each island to introduce nominal rigidities. The model also features real rigidities coming from consumption habits, both in the non-durable and durable goods, and the presence of investment adjustment costs.

**General setting.** The general setting is almost identical to the one in the baseline model, with the only difference that now each island is inhabited by a continuum of monopolistically competitive firms producing differentiated non-durable goods and that the local good produced in each island is a CES aggregate of all the goods produced by monopolists. As in the baseline model, there are two representative households, differing with respect to their discount factor and their budget constraints. Savers are more patient (they have a higher discount factor) and can save either through lending to borrowers or through investing in productive capital. Also, they own the firms in the island and are entitled to their profits. Borrowers are more impatient and cannot invest in capital. In equilibrium, borrowers want to borrow from savers in order to finance their current consumption.

**Real rigidities.** The baseline is enriched with two kinds of real rigidities in order to match the degree of sluggishness and persistence of consumption and investments that we observe in the data. The two kinds of real rigidities are habits in both non-durable and durable consumption and the presence of investment adjustment costs.

Consumption habits consist in assuming that households derive utility from a non-durable consumption basket, $\bar{C}_{it}$, and a durable consumption basket, $\bar{H}_{it}$, depending positively on current consumption and negatively on previous consumption. This assumption makes households concerned with changes in consumption, rather than only with changes in levels: following a shock that affects the consumption decision, consumption will then react gradually instead of reacting strongly on impact, generating an hump-shaped dynamics. In particular, we assume the following functional form for the utility function:

$$U(\bar{C}_{it}, \bar{H}_{it}, n_{it}) = \log(\bar{C}_{it}) + \psi \log(\bar{H}_{it}) - \frac{n_{it}^{1+\nu}}{1+\nu},$$

where

$$\bar{C}_{it} := c_{it} - H_C \bar{C}_{i,t-1} \tag{2.1}$$

$$\bar{H}_{it} := h_{it} - H_H \bar{H}_{i,t-1}. \tag{2.2}$$

The parameters $H_C$ and $H_H$ govern the amount of habit persistence in non-durable and durable consumption: when they are equal to zero, we are back to the case without habit persistence.

The introduction of investment adjustment costs aims at getting the same hump-shaped behavior in investments, by introducing a real cost in changing the level of investments from the one in the previous period. The cost is paid in terms of how much of the investment good becomes new productive capital in the next period. The law of motion of capital changes in the following way:

$$k_{it} = (1-\delta)k_{i,t-1} + i_t \left[1 - \frac{\phi_K}{2}\left(\frac{i_{it}}{i_{i,t-1}} - 1\right)^2\right],$$

where $i_{SS}$ is the level of investments in the steady state.

**Saver's problem.** The saver in one island maximizes expected lifetime utility by choosing non-durable consumption, durable consumption, lending, investments, and labor supply. Lending takes place by the purchase of a one-period bond, paying a nominal risk-free interest rate $r_t$, set by the monetary authority, and common for all islands. As in the baseline model, labor supply is chosen in stage 1 on the basis of an island-specific signal for aggregate productivity and (possibly) distorted beliefs on other islands' signal. The maximization problem is then the following:

$$\max_{c_{it}, h_{it}, i_{it}, d_{it}, n_{it}} E_t \left[\sum_{s=0}^{\infty} \beta^s \log(\bar{C}_{i,t+s}) + \psi \log(\bar{H}_{i,t+s}) - \frac{n_{i,t+s}^{1+\nu}}{1+\nu}\right],$$

$$s.t. \quad c_{it} + i_{it} + q_{it}h_{it} + \frac{r_{t-1}}{\pi_t}d_{i,t-1} = d_{it} + q_{it}h_{i,t-1} + w_{it}n_{it} + r^k_{it}k_{i,t-1} + s_{it}$$
$$k_{it} = (1-\delta)k_{i,t-1} + i_{it}\left[1 - \frac{\phi_K}{2}\left(\frac{i_{it} - i_{i,t-1}}{i_{SS}}\right)\right].$$

The budget constraint is in real terms, in units of the final non-durable consumption good. The variables $\bar{C}_{it}$ and $\bar{H}_{it}$ are defined as in (2.1) and (2.2), $c_{it}$ is defined as the final good consumed by the saver living in island $i$ and is a bundle defined as $c_{it} = \left(\int_0^1 (c_{i,j,t})^{1-\rho}dj\right)^{\frac{1}{1-\rho}}$, where $c_{i,j,t}$ is the quantity of the local good produced in island $j$ demanded by island $i$'s saver. $q_{it}$ is the relative price of housing and $d_{it}$ is real end-of-period debt. The variable $\pi_t := \frac{P_t}{P_{t-1}}$ is gross aggregate inflation of the final non-durable good, $w_{it}$ and $r^k_{it}$ are, respectively, the real local wage and rental rate of capital. Finally, $s_{it}$ are real profits from the local firms in the island owned by the saver.

**Borrower's problem.** The borrower maximizes expected lifetime utility by choosing non-durable consumption, durable consumption, debt, and labor supply. As in the baseline model, borrowing is subject to financial frictions, modeled as a collateral constraint. The maximization problem reads then as follows:

$$\max_{\tilde{c}_{it},\tilde{h}_{it},\tilde{d}_{it},\tilde{n}_{it}} E_t\left[\sum_{s=0}^{\infty}\tilde{\beta}^s \log(\tilde{\bar{C}}_{i,t+s}) + \psi\log(\tilde{\bar{H}}_{i,t+s}) - \frac{\tilde{n}^{1+\nu}_{i,t+s}}{1+\nu}\right],$$

$$s.t. \quad \tilde{c}_{it} + q_{it}\tilde{h}_{it} + \frac{r_{t-1}}{\pi_t}\tilde{d}_{i,t-1} = \tilde{d}_{it} + q_{it}\tilde{h}_{i,t-1} + w_{it}\tilde{n}_{it}.$$
$$\tilde{d}_{it} \leq M q_{it}\tilde{h}_{it}.$$

**Nominal rigidities.** In each island, a competitive sector produces the local goods using a continuum of intermediate inputs according to:

$$y_{it} = \left(\int_0^1 y_{i,j,t}^{(\varepsilon-1)/\varepsilon}dj\right)^{\varepsilon/(\varepsilon-1)},$$

where $\varepsilon$ is the elasticity of substitution between any of the two intermediate inputs. Each intermediate input is produced by a monopolist, indexed by $j$, using labor and capital according to the following technology:

$$y_{i,j,t} = A_t k_{i,j,t}^{\alpha} n_{i,j,t}^{1-\alpha} \tag{2.3}$$

Each monopolist maximizes profits by choosing input quantities in the first stage and setting the price of its good in the second stage. The maximization problem is subject to the following demand function:

$$y_{i,j,t} = \left(\frac{p_{i,j,t}}{p_{i,t}}\right)^{-\varepsilon} y_{it}, \tag{2.4}$$

where $p_{i,j,t}$ is the price set by monopolist $j$ and $p_{it}$ is the price index in island $i$, defined as:

$$p_{it} = \left(\int_0^1 p_{i,j,t}^{1-\varepsilon}\right)^{1/(1-\varepsilon)}.$$

We introduce nominal rigidities in the standard Calvo fashion: each period, only a fraction of monopolists $1 - \theta_P$ is able to adjust the price of its good. When a monopolist gets to update its price in period $t$, it takes instead into account the fact that with some probability it will not be able to reset its price in the future. Hence, he does not maximize current profits, but it sets its price to maximize the present discounted value of profits:

$$\max_{p_{i,j,t}} E_t \left[\sum_{s=0}^{\infty} \theta_P^s Q_{i,t,t+s} \left(\frac{p_{i,j,t}}{P_t} - mc_{i,j,t}\right) y_{i,j,t}\right],$$

where $Q_{i,t,t+s} = \beta^s \frac{\lambda_{i,t+s}}{\lambda_{it}}$ is the relevant stochastic discount factor for real payoffs in island $i$, and $mc_{i,j,t}$ is monopolist-j's real marginal cost. One can show that the optimal ratio of capital to labor $\frac{k_{i,j,t}}{n_{i,j,t}}$ and real marginal cost $mc_{i,j,t}$ are equal across all monopolists in island $i$, meaning that they do not depend on $j$. The optimal prices set by monopolists in each period are then identical. Call this $p_{it}^*$. The price index for island $i$ then becomes:

$$p_{it} = \left(\int_0^1 p_{i,j,t}^{1-\varepsilon}\right)^{1/(1-\varepsilon)} = (\theta_P p_{i,t-1}^{1-\varepsilon} + (1-\theta_P) p_t^{*^{1-\varepsilon}})^{1/(1-\varepsilon)}. \tag{2.5}$$

Equation (2.5) can be rewritten in terms of local inflation:

$$\theta_P \pi_{it}^{\varepsilon-1} + (1-\theta_P)\hat{p}_t^{*1-\varepsilon} = 1,$$

where $\hat{p}_t^* := \frac{p_t^*}{p_{it}}$ is the optimal updated price relative to the price index $p_{it}$.

Aggregating the production function (2.3), equalizing it to the demand schedule in (2.4), and using again the fact that the optimal capital-labor ratio does not depend on $j$ we get:

$$\int_0^1 A_t k_{i,j,t}^{\alpha} n_{i,j,t}^{1-\alpha} dj = A_t \left(\frac{k_{i,t}}{n_{i,t}}\right)^{\alpha} \int_0^1 n_{i,j,t} dj = y_{it} \int_0^1 \left(\frac{p_{i,j,t}}{p_{i,t}}\right)^{-\varepsilon} dj$$

Define aggregate labor demand for island $i$ as:

$$n_{it} = \int_0^1 n_{i,j,t} dj.$$

We then get the following condition for total production in island $i$:

$$y_{it} = \frac{A_t k_{it}^{\alpha} n_{it}^{1-\alpha}}{v_t},$$

where $v_t := \int_0^1 \left(\frac{p_{i,j,t}}{p_{i,t}}\right)^{-\varepsilon} dj$ is the degree of price dispersion. Price dispersion implies then the existence of a wedge between total output and total factor inputs, given that these inputs are inefficiently allocated across monopolists charging different prices.

**Monetary policy.** A Central Bank conducts monetary policy by setting one nominal interest rate for all islands. The interest rate is set by following a Taylor rule responding to deviations of year on year aggregate non-durable inflation, $\pi_t^A$, and of aggregate output from their respective steady state levels. The rule also allows for interest rate smoothing to accommodate for gradual nominal interest rate changes like we observe in modern economies. The rule is reads as follows:

$$R_t = (R_{t-1})^{\rho_R} \left[ (R_{SS})^{1-\rho_R} \left( \frac{\pi_t^A}{\pi_{SS}^A} \right)^{\phi_\pi} \left( \frac{Y_t}{Y_{SS}} \right)^{\phi_Y} \right]^{1-\rho_R}, \tag{2.6}$$

where $R_t$ is the nominal interest rate paid by borrowers to the savers through a 1-period nominal loan and the $SS$ subscript denotes the steady state level of a variable.

**Aggregate Shocks.** We add five additional aggregate shocks, which have been widely used by the medium scale DSGE literature, on top of the technology and confidence shocks.

The first shock is an intertemporal shock to preferences, which acts as a consumption-demand shock by shifting the willingness of households for spending today with respect to tomorrow.

The second shock is a housing preference shock that affects the marginal rate of substitution between durable and non-durable consumption. A positive realization of the shock mechanically increases the demand for housing and thus increases its relative price.

The third shock is an investment shock like the one in Justiniano et al. (2011).

The fourth shock is a monetary policy shock and enters as a deviation of the monetary authority from the Taylor rule in (2.6).

The last shock is a leverage shock and affects the collateral constraint faced by the borrower, by changing the loan-to-value ratio, $M$, in the constraint.

All the shocks follow a first-order autoregressive stochastic process, and all the persistences and volatilities are estimated, among other structural parameters, in the following section. The full set of equilibrium conditions is presented in Appendix B.

## 2.2 Estimation

The model is log-linearized around the non stochastic steady state and is estimated using Bayesian methods: posterior distributions are obtained through a Monte Carlo Markov Chain (MCMC), using a random walk Metropolis-Hastings algorithm. The size of the random walk was calibrated to obtain an acceptance rate of 25% as standard practice in the Bayesian MCMC literature. The size of the generated chain was 300 000 with a burn-in period of 50 000 draws.

**Data.** The estimation is based on six observables for the United States from 1985Q1 to 2016Q4: non-durable consumption, real investments, hours worked, real house prices, CPI inflation and the Federal Funds Rate (FFR). All observables but CPI inflation and hours have been de-trended with a one-sided HP filter with a smoothing parameter of 100 000.

Given that the model is linear and cannot accommodate for the ZLB period the US have experienced from 2008 to 2015, we used, for that period, the shadow rate computed by Wu and Xia (2015) in order to take into account the expansionary monetary policy stance implemented through unconventional policies. We describe exact data sources and construction in Appendix C.

**Calibrated parameters.** A subset of model parameters is calibrated to match some long-run averages over the estimation sample. Table 2.1 reports the calibrated parameters: $\beta$, the saver's discount factor, is set equal to 0.995, implying a to 2% steady state annual real interest rate, $\tilde{\beta} = 0.99$ in order to have a steady state debt-to-output ratio equal to 0.47, which is approximately the US mortgage-to-GDP ratio at the beginning of the housing boom (see Figure 3.1). The steady state housing preference parameter $\psi$ is such that the housing wealth-to-consumption ratio is 2.3 in steady state, which is the value reported in Iacoviello (2011). The capital share $\alpha$ and depreciation rate $\delta$ are set in order to have a steady state capital-to-output and investment-to-output ratios equal

Table 2.1: Calibrated parameter Values.

| Parameter | Description | Value |
|---|---|---|
| $\beta$ | Saver's discount factor | 0.995 |
| $\tilde{\beta}$ | Borrower's discount factor | 0.99 |
| $\psi$ | Housing weight in utility | 0.013 |
| $M$ | Maximum LTV ratio | 0.85 |
| $\delta$ | Capital depreciation | 0.025 |
| $\alpha$ | Capital share | 0.3 |
| $\nu$ | Inverse of Frisch el. | 1 |
| $\varepsilon$ | El. of substitution of goods in one island | 6 |
| $\rho$ | Strategic complementarity between islands | 0.75 |

to, respectively, 2.1 and 0.21, which are standard ratios in the business cycle literature. The steady state maximum LTV ratio is equal to 0.85, and the inverse of Frisch elasticity is 1. Steady state annual inflation is 2%, and the elasticity of substitution between local differentiated goods is 6, implying a gross markup of 1.2. Finally, the degree of strategic complementarity across island $\rho$ is set to 0.75, which is a value estimated by Angeletos et al. (2018) through a reduced-form exercise exploiting the University of Michigan Sentiment Index.

**Estimated parameters.** The remaining parameters are estimated. Prior distributions follow the ones used in the literature[1] and are reported in Tables 2.2 and 2.3 and, as well as the mode and 90% confidence intervals of their posterior distributions. The estimated investment adjustment cost parameter, $\phi_K$, is much lower than the values usually reported in the literature but, as we can see in Figure 1.5 investments in the baseline model already had the hump-shaped dynamic that the introduction of investment adjustment costs aims at replicate[2]. The remaining parameters are instead in line with the literature.

[1]See for instance Smets and Wouters (2007).

[2]Liu and Zha (2013) report a similar low estimation for this parameter.

Table 2.2: Estimated parameter values. Structural parameters.

| Param. | Description | Prior | Posterior | | |
|---|---|---|---|---|---|
| | | (mean, std.) | 5% | Mean | 95% |
| $H_C$ | Non-durable Cons. Habit | Beta (0.7, 0.1) | 0.4898 | 0.5439 | 0.5954 |
| $H_H$ | Durable Cons. Habit | Beta (0.7, 0.1) | 0.3366 | 0.4816 | 0.6150 |
| $\phi_K$ | Inv. adj. cost | Gamma (4,2) | 0.2160 | 0.3390 | 0.4991 |
| $\theta_P$ | Calvo Param. | Beta (0.5, 0.075) | 0.7303 | 0.7643 | 0.7839 |
| $\phi_\pi$ | CB infl. resp. | Normal (1.5,0.25) | 1.5345 | 1.7380 | 1.9231 |
| $\phi_y$ | CB output resp. | Beta (0.125,0.03) | 0.0460 | 0.0652 | 0.0854 |
| $\phi_r$ | IR smooth. | Beta (0.75,0.1) | 0.3831 | 0.5042 | 0.6180 |

Note: The table reports posterior estimates of the parameters. The posterior statistics are based on 250 000 draws from the posterior distribution.

Table 2.3: Estimated parameter values. Shock parameters.

| Param. | Description | Prior | Posterior | | |
|---|---|---|---|---|---|
| | | (mean, std.) | 5% | Mean | 95% |
| $\rho_a$ | Tech. shock pers. | Beta (0.75, 0.1) | 0.9275 | 0.9545 | 0.9805 |
| $\rho_z$ | Demand shock pers. | Beta (0.75, 0.1) | 0.8949 | 0.9104 | 0.9252 |
| $\rho_m$ | Mon. pol. shock pers. | Beta (0.75, 0.1) | 0.5535 | 0.6396 | 0.7216 |
| $\rho_h$ | Housing pref. shock pers. | Beta (0.75, 0.1) | 0.5754 | 0.7635 | 0.9197 |
| $\rho_l$ | Leverage shock pers. | Beta (0.75, 0.1) | 0.4447 | 0.6023 | 0.7540 |
| $\rho_i$ | Investment shock pers. | Beta (0.75, 0.1) | 0.6370 | 0.7035 | 0.7490 |
| $\rho_c$ | Confidence shock pers. | Beta (0.75, 0.1) | 0.8629 | 0.8880 | 0.9122 |
| $\sigma_a$ | Tech. shock vol. | IGamma (0.1, 2) | 0.0083 | 0.0091 | 0.0101 |
| $\sigma_z$ | Demand shock vol. | IGamma (0.1, 2) | 0.0303 | 0.0368 | 0.0439 |
| $\sigma_m$ | Mon. pol. shock vol. | IGamma (0.1, 2) | 0.0204 | 0.0270 | 0.0347 |
| $\sigma_h$ | Housing pref. shock vol. | IGamma (0.1, 2) | 0.0030 | 0.0064 | 0.0122 |
| $\sigma_l$ | Leverage shock vol. | IGamma (0.1, 2) | 0.0029 | 0.0054 | 0.0090 |
| $\sigma_i$ | Investment shock vol. | IGamma (0.1, 2) | 0.0064 | 0.0085 | 0.0110 |
| $\sigma_c$ | Confidence shock vol. | IGamma (0.1, 2) | 0.0188 | 0.0257 | 0.0343 |

Note: The table reports posterior estimates of the parameters. The posterior statistics are based on 250 000 draws from the posterior distribution.

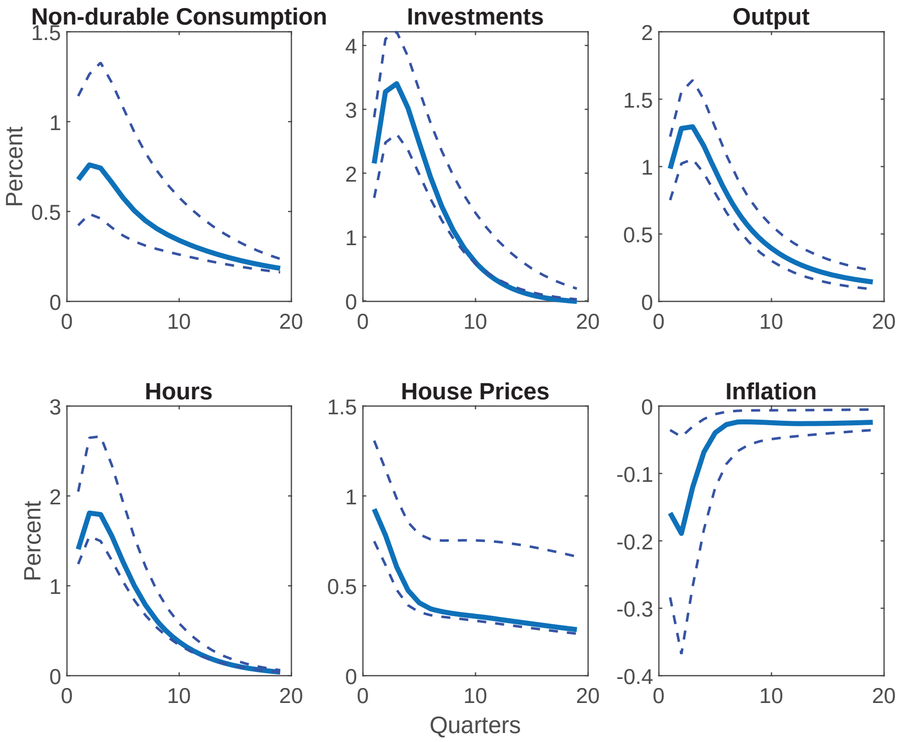


Figure 2.1: Impulse response of aggregate variables to a one standard deviation positive confidence shock. Units are percent log-deviations from steady state.

**Impulse responses.** Figure 2.1 plots impulse response functions (IRFs) to a one standard deviation positive confidence shock. They are quite similar to the ones from the baseline model: non-durable consumption, investments, output and hours reacts positively on impact and come back to the steady state with a hump-shaped dynamics after about 5 years. The shapes and magnitudes of the IRFs for these variables are comparable to the ones estimated in Angeletos et al. (2018). Housing price increases by 1% on impact, which lies on the confidence interval estimated in the SVAR[3]. Interestingly inflation responds negatively though in a barely significant way, which is also consistent with the SVAR.

**Variance decomposition.** Table 2.4 reports the contribution of the different shocks to the volatilities of the observable variables. As in Angeletos et al. (2018), the confidence

[3]When comparing this and following results to the SVAR keep in mind that the University of Michigan Sentiment Index is not an observable in the estimation.

Table 2.4: Variance decomposition

| | Shocks | | | | | | |
|---|---|---|---|---|---|---|---|
| Observable | Conf | Tech | HousPref. | Inv. | Demand | Monetary | Leverage |
| Cons. | 42% | 48% | 1% | 1.5% | 3.5% | 4% | 0% |
| Inv. | 29% | 24% | 0% | 16% | 14% | 16% | 1% |
| Hours | 84% | 1% | 0% | 6% | 1% | 8% | 0% |
| House Prices | 52% | 20% | 0% | 3% | 20% | 4% | 1% |
| Inflation | 11% | 26% | 0% | 3% | 15% | 44% | 1% |
| FFR | 36% | 19% | 0% | 9% | 4% | 30% | 0% |

Note: Contribution of shocks to observables' variances. Rows are observables, columns are shocks (rows sum to 100%).

shock can explain most of the business cycle variation. It explains more than 40% of the variance of consumption, 30% of the variance of investments and most of the variation in hours. Confidence has a relatively small contribution to inflation, still explaining more than one-third of the variation of the Federal Funds Rate. Moreover, confidence is the most important shock in explaining the variance of house prices, explaining half of it, which is remarkably similar to the fraction of variance explained in the SVAR. The investment, technology, demand and monetary policy shocks explain the rest of the volatility of the observables, while the housing preference shock and the financial shock are almost irrelevant.

**Shock decomposition.** Figure 2.2 shows the shock decomposition for house prices in terms of the underlying shocks, by plotting the marginal contribution of the shocks. The shock decomposition consists in running several counter-factual where one computes the level of house prices predicted by the model when feeding into the model just some of the filtered structural shocks. The first counter-factual consider only the preference shock and is the green area in Figure 2.2. Then a second counter-factual is performed by adding the confidence shock to the previous one, thus considering only the preference and the confidence shocks. This counterfactual is the blue area in Figure 2.2: the entire housing boom is then explained by the confidence and preference shocks together. The remaining

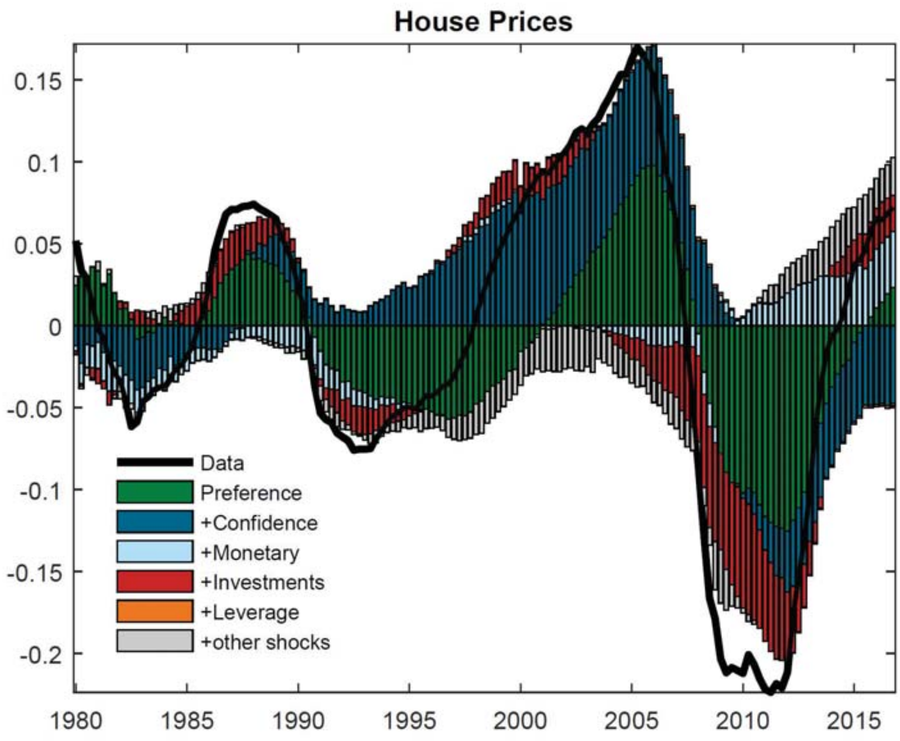


Figure 2.2: Shock decomposition for house prices.

counterfactuals operate in the same way by adding one shock to the previous counterfactual in the order listed in the legend of Figure 2.2[4]. Also, in this case, the result is quite similar to the historical decomposition in the SVAR: the confidence shock is responsible for an important part (approximately a $30 - 40\%$) of the housing boom, especially at the very initial stage of the boom during the late 1990s. The shock has small importance for the previous housing boom during the 1980s instead.

**Theoretical mechanism and sentiment index.** Overall the results from the estimation confirm the results from the empirical model in Section 3. Since the University of Michigan Sentiment index is not used as observable in the estimation of the DSGE model, this result is not trivial. An interesting exercise is then trying to compare the confidence shock series filtered from the estimated model with University of Michigan Sentiment index. Figure 2.3 plots the Sentiment Index in log-deviations from its long-run average

[4]Since the model is linear, the ordering of the marginalization is irrelevant.

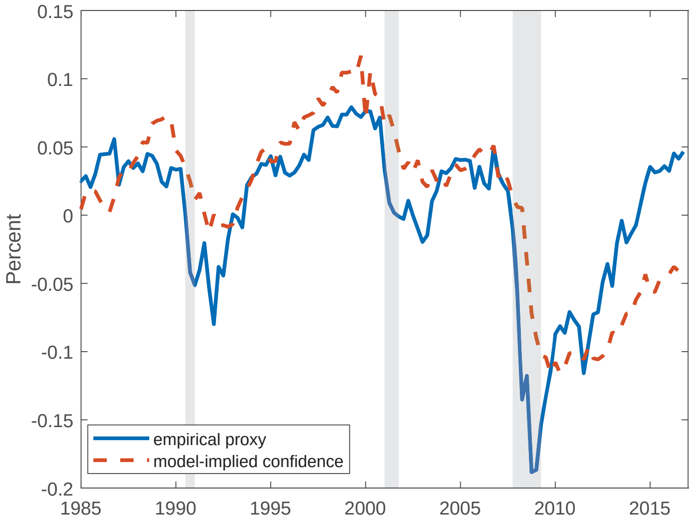


Figure 2.3: The University of Michigan Sentiment Index versus confidence in the model. Shaded areas are recessions.

together with the filtered estimated series of the confidence shock. The two series are remarkably similar, with a 74% correlation. This result suggests two facts:

1. the theoretical mechanism through which the confidence shock operates in the model is consistent with the proxy that the empirical literature used to look at confidence.

2. since the shock series retrieved by the estimation is exogenous and thus orthogonal to current and future economic fundamentals, it is likely that most of the exogenous changes in the University of Michigan Sentiment Index are also unrelated to current and future economic fundamentals.

# Chapter 3

# Confidence and Heterogeneity

The previous Chapter presented a dynamic model with heterogeneous beliefs where exogenous belief fluctuations and their interaction with financial frictions have an important role in explaining house price volatility. The model was a linear DSGE with two representative agents, where financial frictions arise since the impatient representative agent has a borrowing limit defined by a collateral constraint as in Kiyotaki and Moore (1997). The model is solved by linearly approximating aggregate dynamics around the deterministic steady state, implying that the borrowing constraint is always binding as it is indeed binding at the steady state.

Although this can be view as a decent approximation from an aggregate point view, since only a fraction of agents is liquidity constrained while the remaining fraction is not, the literature on financial frictions and aggregate activity has quite stressed the importance of non-linearities and heterogeneity that cannot of course be captured by a linear representative agent model. Guerrieri and Iacoviello (2017) consider a model similar to the one studied in the previous chapter where the collateral constraint can be occasionally binding and show that the constraint was indeed slack during the housing boom of the 2000s and that the collapse in house prices exacerbated the recession through the binding collateral constraint. This kind of asymmetries cannot be captured by a linear model. Moreover, a linear model can have difficulties in featuring big responses of real activity to changes in house prices as, for instance, levered economies are typically more fragile. Mian et al. (2013) find that the marginal propensity to consume out of a loss in housing wealth is typically higher for poor agents, suggesting that an incomplete market model with heterogenous agents may be more appropriate in order to study the mechanisms through which changes in house prices affect consumption and the real activity in general. Berger et al. (2018) show that indeed such a model can deliver sizable consumption elasticity to house prices consistent with the evidence shown in Mian et al. (2013).

Furthermore, belief shock may very well entail re-distributional consequences and precautionary saving motives that a linear representative agent model is not able to properly assess. Bayer et al. (2018) show that these features are indeed quite prominent in the propagation of uncertainty shocks and that this has important implications for monetary and fiscal stabilization. No one, to the best of my knowledge, has done the same analysis for belief shocks.

This Chapter wants to pave the way to provide the analysis above through the lens of a structural equilibrium incomplete market model with heterogeneous agents and an enriched belief structure as the one presented in the previous Chapters, allowing for an aggregate belief shock that generates waves of optimism and pessimism unrelated to economic fundamentals at any point in time. This Chapter shows how the computational challenges that the introduction of the belief shock entails for the characterization of the recursive equilibrium and for aggregate dynamics can be addressed, in a small heterogeneous agent model similar to the one used in Aiyagari (1994), by assuming preferences over labor supply such as the ones used in Greenwood, Hercowitz and Huffman (1988). Thanks to this assumption the stationary equilibrium of the model can be found through standard projection methods and the aggregate dynamics following an aggregate belief shock can be solved by first-order perturbating the dynamic system around the stationary equilibrium as proposed in Reiter (2009).

## 3.1 The Quantitative Model

In the following I outline a dynamic model of heterogeneous households with incomplete markets and two aggregate shocks (productivity $A_t$ and confidence $\xi_t$). The model enriches the belief structure of economic agents, building on Angeletos et al. (2018), in order to have shifts in beliefs, unrelated to current and future fundamentals, that generate optimism/pessimism about the current level of economic activity.

**General setting.** Time is discrete. There is a continuum of islands, indexed by $i \in [0, 1]$ and a mainland. Each island is inhabited by a representative firm, a continuum of ex-ante identical households, indexed by $j \in [0, 1]$ and a government sector. The firm hires labor and rents capital from the households to produce a local good in perfect competition. The government levies a constant labor tax to finance an unemployment benefit for unemployed households. In the mainland a final good firm operates by producing a final good used for consumption an investment.

Each period is divided in two stages. In stage 1 the local firm and households decide how much labor and capital to demand and supply respectively; the decision is made on the basis of incomplete information regarding economic activity in other islands. In stage 2 the level of economic activity is revealed, the market for the final good operates and households decide consumption and investment.

In the following the model is described as if there was no aggregate uncertainty coming from the confidence shock; a separate section is going to explain how the aggregate shock is going to affect the equilibrium.

**Introducing shifts in beliefs.** Figure 3.1 summarizes the within-period timing. In stage 1, the local firm decides production and households decide labor supply on the basis incomplete information about the current level of aggregate productivity. Differently from standard incomplete information macroeconomic models, we relax the common-prior assumption, meaning that all agents may not share the same beliefs about one

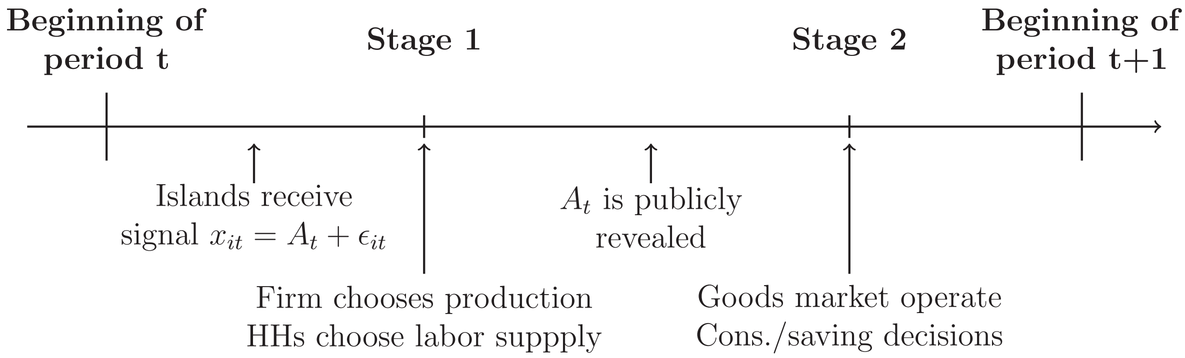


Figure 3.1: Within period timing. Stage 1 decisions are made on the basis of a island-specific signal. Stage 2 decisions are made on the basis of complete information.

another beliefs. In other words, each island may believe that the signals received by other islands are biased. The bias is given by an exogenous state variable, $\Gamma_t$, which is perfectly observed by each island.

Let $x_{it} = A_t + \mu_{it}$ be the signal for $A_t$ received by island $i$, composed by the true level of $A_t$ plus an island-specific error. Imposing the common prior assumption would imply:

$$E_{it}[\mu_{it}] = E_{it}[\mu_{jt}] = 0, \ \forall i, j \in (0, 1).$$

Hence, each island believes that any other island in the economy is receiving, on average, the same signal.

Here we relax the common-prior assumption by assuming the following:

$$\begin{cases} E_{it}[\mu_{it}] = 0, & \forall \ i \in (0,1) \\ E_{it}[\mu_{jt}] = \Gamma_t, & \forall \ j \neq i. \end{cases}$$

This condition implies that each island believes, as in the previous case, that its own signal is centered around the true level of $A_t$, but that all the other islands are receiving a biased signal centered around $A_t + \Gamma_t$. We can interpret the variable $\Gamma_t$ as governing confidence: when it is positive, every island thinks that other islands are receiving signals about the state of the economy that are more optimistic than its own. Here, we assume that $\Gamma_t$ follows a stationary AR(1) process with zero mean, meaning that in the steady state all the agents agree on one another's beliefs about the economy.

**Households.** Each household $j$ in island $i$ earns income from renting out capital $k_{ijt}$ and from supplying labor $n_{ijt}$, being subject to labor-income risk. Indeed, the household's labor efficiency $\epsilon_{jt}$ evolves stochastically and labor income is given by $w_t \epsilon_{jt} n_{ijt}$, with $w_t$ being the aggregate wage rate[1]. Efficiency can be either null or positive. A null efficiency corresponds to unemployment. If $\epsilon_{j,t-1} > 0$, $\epsilon_{jt}$ becomes zero with probability $p_{eu}$, where $p_{eu}$ is then the probability of an household suffering unemployment if employed in the previous. With probability $1 - p_{eu}$ the household remains employed and its efficiency follows an AR(1) process in logs. A unemployed household stays unemployed with probability $1 - p_{ue}$, with $p_{ue}$ being the probability of exiting unemployment. A household exiting unemployment gets an efficiency equal to 1, corresponding to the average households' efficiency.

Unemployed households receive an unemployment insurance $bw_t$ financed by a labor tax $\tau$ on employed households in the island[2]. The stochastic process characterizing the evolution of household's efficiency can then be summarized as follows:

$$\epsilon_{jt} = \begin{cases} (\epsilon_{j,t-1})^{\rho} \exp \eta_{jt} & \textit{with prob. } 1 - p_{eu} \ \textit{if } \epsilon_{j,t-1} > 0 \\ 1 & \textit{with prob. } p_{ue} \ \textit{if } \epsilon_{j,t-1} = 0 \\ 0 & \textit{with prob. } 1 - p_{ue} \ \textit{if } \epsilon_{j,t-1} = 0 \\ 0 & \textit{with prob. } p_{eu} \ \textit{if } \epsilon_{j,t-1} > 0 \end{cases}$$

[1] I dropped the island-specific subscript $i$ for workers' efficiency since the properties of the stochastic process underlying $\epsilon$ are equal across islands. By the law of large number all islands will have the same cross-sectional distribution of workers' efficiency.

[2] The assumption of an unemployment benefit proportional to wage is necessary as the Government needs to balance his budget each period: if tax revenues increase because of an increase in wage also expenditures for unemployment benefits have to increase.

Each household has preferences over consumption $c_{ijt}$ and hours $n_{ijt}$ given by:

$$\sum_{t=0}^{\infty} \beta^t U(c_{ijt}, n_{ijt}; \epsilon_{jt}), \tag{3.1}$$

where $\beta \in (0, 1)$ is the discount factor and $U$ is the instantaneous utility function, taking the so-called Greenwood-Hercowitz-Huffman (GHH) functional form:

$$U(C_{ijt}, n_{ijt}) = \frac{1}{1-\sigma}(C_{ijt} - F(n_{ijt}; \epsilon_{jt}))^{1-\sigma}.$$

The instantaneous utility function $U$ thus exhibits a constant relative risk aversion over a composite demand for the final consumption good and leisure, with risk aversion parameter $\sigma > 0$ and disutility from working given by $F(n_{ijt}; \epsilon_{j,t}) = \nu \epsilon_{jt} \frac{n_{ijt}^{1+\gamma}}{1+\gamma}$. The GHH assumption over preferences serves, as it is going to be outlined later, for two purposes: first, the labor supply decision is not going to depend on idiosyncratic states of the household quite simplifying the computation of the stationary equilibrium, second the labor supply decision is not going to depend on the consumption level of the household, tremendously simplifying the characterization of the recursive equilibrium of the model with an aggregate confidence shock and allowing me to solve the model with incomplete markets, heterogeneous agents and the aggregate confidence shock. As discussed in Bayer et al. (2018) the assumption of disutility from labor depending on the household efficiency is made in such a way that hours worked and efficiency do not appear separately in the budget constraint and in the household's optimality condition. This is not an issue as long as the stochastic process underlying the evolution of household efficiency is calibrated to match the labor income distribution.

Summing up, each household maximizes (3.1) subject to the following budget constraint:

$$P_t C_{ijt} + P_t k_{ij,t+1} = r_{it} k_{ijt} + (1-\tau) w_{it} n_{ijt} \epsilon_{ijt} + (1 - \mathcal{I}_{\epsilon_{ijt}>0}) b w_{it},$$

where $P_t$ is the price of the final good, which can be normalized to 1.

Finally, the maximization is also constrained by the no-borrowing constraint

$$k_{ij,t+1} \geq 0.$$

**Final Good Firm.** The final good firm operating in the mainland produces the final good $Y_t$ using the local goods produced in the islands,$\{y_{it}\}_{i\in[0,1]}$, as inputs with the Cobb-Douglas technology $\log(Y_t) = \int_0^1 \log(y_{it})di$. This implies that the final good firm demands from local firm $i$ the following quantity (remember that $P_t$ has been normalized to 1):

$$y_{it} = \left(\frac{p_{it}}{P_t}\right)^{-1} Y_t = \frac{Y_t}{p_{it}}. \tag{3.2}$$

**Local Firm.** The local firm produces the local good $y_it$ according to the production function:

$$y_it = e^{A_t} K_{it}^{\alpha} N_{it}^{1-\alpha},$$

where $A_t$ is aggregate productivity, $K_{it}$ is the aggregate capital stock in the island and $N_{it}$ is aggregate labour in the island. The production is subject to the demand schedule (3.2). Factor prices are then given by:

$$w_{it} = p_{it}(1-\alpha)e^{A_t}\left(\frac{K_{it}}{N_{it}}\right)^{\alpha}$$

$$r_{it} = p_{it}\alpha e^{A_t}\left(\frac{K_{it}}{N_{it}}\right)^{\alpha-1} + 1 - \delta,$$

where $\delta$ is the depreciation rate of capital.

Aggregate productivity follows an AR(1) process:

$$A_t = \rho_A \log(A_{t-1}) + \sigma_A \mu_t, \; where \; \mu_t \sim \mathcal{N}(0,1).$$

### 3.1.1 Stationary Equilibrium

In this section we characterize the stationary equilibrium with no aggregate uncertainty and thus with no shifts in belief. The model boils down a standard incomplete market models as in Aiyagari (1994). The relevant state variables for the household are[3]:

1. Its own efficiency, $\epsilon$.
2. The joint distribution of $(\epsilon, k)$, $\Lambda$.
3. The aggregate state variables, $S = (A, \xi) = (1, 0)$.

The value function characterizing the household problem is then:

$$V(\epsilon, k, \Lambda, S) = \max_{k', n} U(rk + (1-\tau)wn\epsilon + (1 - \mathcal{I}_{\epsilon>0})bw - k') + \beta E[V(\epsilon', k', \Lambda', S')] \quad (3.3)$$

$$s.t.\ k' \geq 0.$$

A recursive equilibrium with no aggregate uncertainty is a list of a value function $V$, the policy functions $k'(\epsilon, k, \Lambda, S),\ n(\epsilon, k, \Lambda, S)$, the pricing functions $r(\Lambda, S),\ w(\Lambda, S)$, the aggregate capital and labor supply functions $K(\Lambda, S),\ N(\Lambda, S)$, an actual distribution $\Lambda$ over efficiency and capital holdings and a perceived law of motion $\tilde{\lambda}$ such that:

i) Given $V,\ r,\ w,\ \tilde{\lambda}$, the policy functions solve the household's problem and the value function is a solution of the Bellman equation (3.3).

ii) The labor market, the capital market, meaning that the following condition holds:

$$K = \int k d\lambda(\epsilon, k)$$

$$N = \int n d\lambda(\epsilon, k)$$

The final good market then automatically clears because of Walras' law.

[3]In this formulation I drop the indexes $i$,$j$ and $t$ for the sake of notation and a superscript $'$ indicates a $t+1$ variable.

iii) The actual and perceived law of motion coincide, that is:

$$\Lambda' = \tilde{\lambda}(\Lambda).$$

Before going into the details on how to solve for the stationary equilibrium without aggregate uncertainty, let's first have a look to the following Proposition.

**Proposition 1.** *Given the GHH assumption on the utility function, the aggregate labor supply of an island depends only on the aggregate capital of the island.*

*Proof.* Setting up the lagrangian for the generic household with capital holdings $k_{ijt}$ and labor efficiency $\epsilon_{ijt}$ and taking first order conditions with respect to consumption and labor supply we get:

$$\begin{cases} (c_{ijt} - \nu\epsilon_{jt}\frac{n_{ijt}^{1+\gamma}}{1+\gamma})^{-\sigma} = \lambda_{ijt} \\ -(c_{ijt} - \nu\epsilon_{jt}\frac{n_{ijt}^{1+\gamma}}{1+\gamma})^{-\sigma}\nu\epsilon_{ijt}n_{ijt}^{\gamma} = \lambda_{ijt}(1-\tau)w_{it}\epsilon_{jt}, \end{cases}$$

where $\lambda_{ijt}$ is the lagrangian multiplier attached to the budget constraint.
Combining the two and simplifying we notice that the labor supply decision depends only on the local wage:

$$n_{ijt} = \left(\frac{1-\tau}{\nu}w_{it}\right)^{1/\gamma}$$

This means that all households supply the same number of hours $n(w_{it})$ and that aggregate labor supply is:

$$N_{it} = \int_0^1 n(w_{it})\epsilon_{ijt}\mathcal{I}_{\epsilon_{ijt}>0}dj = n(w_{it})\int_0^1 \epsilon_{ijt}\mathcal{I}_{\epsilon_{ijt}>0}dj$$

The last integral is the cross-sectional average of efficiency for households with positive efficiency, which is a time-constant given that the law of large numbers applies[4]. We can

[4]Even though the set-up is with a continuum of agents and a law of large numbers does not hold properly, in the numerical solution everything is discretized and therefore a proper law of large numbers applies.

then define $\int_0^1 \epsilon_{ijt}\mathcal{I}_{\epsilon_{ijt}>0}dj =: \bar{\epsilon}$.

Recall the equation for the wage coming from the firm's foc

$$w_{it} = p_{it}(1-\alpha)e^{A_t}\left(\frac{K_{it}}{L_{it}}\right)^{\alpha},$$

and notice that without aggregate uncertainty $e^{A_t} = 1$ and $p_{it} = 1$.

We then have:

$$N_{it} = \bar{\epsilon}\left(\frac{1-\tau}{\nu}w_{it}\right)^{1/\gamma} = \bar{\epsilon}\left(\frac{1-\tau}{\nu}(1-\alpha)\left(\frac{K_{it}}{N_{it}}\right)^{\alpha}\right)^{1/\gamma} \Leftrightarrow$$

$$N_{it} = \bar{\epsilon}^{\frac{\gamma}{\alpha+\gamma}}\left(\frac{1-\tau}{\nu}(1-\alpha)K_{it}^{\alpha}\right)^{1/(\gamma+\alpha)} =: N^*(K), \tag{3.4}$$

which means that the aggregate labor supply of an island depends only on the aggregate capital of the island. □

An algorithm for solving for the stationary equilibrium is then the following:

0. Guess a level for aggregate capital $K_0$.

1. Use (3.4) to compute the aggregate labor and then use the firm's focs to calculate $w(K)$ and $r(K)$.

2. Given the prices, solve for the household's policy function $k'(\epsilon, k)$.

3. Using the implied policy function, find the invariant distribution of households over individual states $\Lambda$.

4. Use the invariant distribution to compute aggregate capital $K$. If $K \neq K_0$, update the guess $K_0$ and return to step (1.).

In the appendix I am going to elaborate more on the numerical details for implementing steps (2.) and (3.). In a nutshell, I solve for the household's policy function approximating the value function with cubic splines and solve for the value functions using collocation methods over a bi-dimensional grid for capital holdings and efficiency of size $N_k$ $x$ $N_\epsilon$. The invariant distribution is then found, given the policy function using the non-stochastic simulation method outlined in Young (2010).

### 3.1.2 Aggregate Uncertainty

Having found the stationary distribution without aggregate uncertainty, we can solve for aggregate dynamics by first-order perturbation around the stationary equilibrium with respect to the aggregate shock as proposed in Reiter(2009). Again, more details are given in the appendix, but the intuition is that we can perturbate the discretized value functions and distribution $\Lambda$ around the stationary equilibrium. In order to do so, we need to characterize the recursive equilibrium with the aggregate confidence shock $\xi$. Remember that, in stage 1, the households and the firm in island $i$ receive a signal $z$ on aggregate productivity and have beliefs that the signal received by other islands is equal to $z + \xi$. In stage 2, households make their consumption and investment decisions on the basis of complete information.

The value functions characterizing the optimization problems faced by a household in stage 1 and 2 are given by:

$$V^{(1)}(\epsilon, k, \Lambda, \xi, z) = \max_{n} V^{(2)}(\hat{q}, \epsilon, k, \Lambda, \xi, z) \tag{3.5}$$

$$s.t.\ \hat{q} = \hat{r}k + (1-\tau)\hat{w}n\epsilon + (1 - \mathcal{I}_{\epsilon>0})b\hat{w}$$

$$\hat{w} = \hat{p}(1-\alpha)e^{A_t}\left(\frac{K}{N}\right)^{\alpha}$$

$$\hat{r} = \hat{p}\alpha e^{A_t}\left(\frac{K}{N}\right)^{\alpha-1} + 1 - \delta,$$

$$V^{(2)}(q, \epsilon, k, \Lambda, \xi, A) = \max_{c,k'} U(c, n) + \beta E[V^{(1)}(\epsilon', k', \Lambda', \xi', A')] \tag{3.6}$$

$$s.t.\ c + k' = q,$$

where $q$ denotes the available resources in real terms with the final good as a numeraire, and the hat over a variable denotes the stage-1 belief of that variable. Notice that $w$ and $r$ enter with the hat in the value function for stage 1 because they depend on the expected relative price of the local good in terms of the final good, which is a variable realized in the second stage. This expected relative price will be a function depending on:

1. The joint distribution of $(\epsilon, k)$, $\Lambda$

2. the aggregate state, $\xi$

3. the island-specific signal, $z$.

Define this function $\hat{p} := P(\Lambda, z, \xi)$. In a recursive equilibrium $P$ must be consistent with the policy functions implemented by each household and firm in each island. To find a recursive equilibrium we then need to solve for the fixed point of (3.5) and (3.6). The following proposition shows that, for the model considered here, $P(\Lambda, z, \xi)$ can be derived in closed form as a function of only the confidence shock $\xi$.

**Proposition 2.** *Given the GHH assumption on the utility function, the equilibrium consistent $\hat{p}$ is given by:*

$$\hat{p}(\Lambda, z, \xi) = e^{\xi \frac{1-\alpha}{\alpha+\gamma}}.$$

*Proof.* See Appendix. □

Notice that $\hat{p}(z, 0) = 1$, meaning that when there is no belief heterogeneity the expected relative price of the local good is equal to one, which is consistent since, in that case, local output coincides with expected aggregate output.

Proposition 2 is crucial because, combined with Proposition 1, it gives us a closed form solution for $V^{(1)}(\epsilon, k, \Lambda, \xi, z)$. If this was not the case we would have to numerically solve for $V^{(1)}$ and $V^{(2)}$ doubling the size of the system of non-linear difference equations to be perturbated, causing the computation of the solution to be numerically infeasible.

## 3.2 Calibration and Results

The calibration is still preliminary and is based on a model frequency of one quarter. The inverse of intertemporal elasticity of substitution is $\sigma = 2$, the capital share $\alpha = 0.33$, the unemployment benefit $b = 0.15$ and the inverse of Frisch elasticty $\gamma = 1$. The preference parameter $\nu$ is calibrated so that average hours worked is 1/3 of the time endowment. I then calibrate $\beta$ and $\delta$ to have a capital to annual output ratio of 2.1 and an investment to output ratio of 0.21, which are the standard big ratios of the US economy. The calibration yields to $\beta = 0.986$ and $\delta = 0.025$.

The household efficiency process is calibrated to the parameters estimated in Bayer et al. (2018) and is then discretized to a Markov chain using the method described in Tauchen (1986). The probabilities of entering and exiting the unemployment state are calibrated in such a way that unemployment lasts on average 4 quarters and that the fraction of employed households in each period is 0.93.

The persistence and standard deviations of the confidence shock are instead set to the values estimated in the previous chapter.

Figure 3.2 shows the policy functions for consumption and the invariant distribution of households for 4 different levels of household efficiency: 0 (unemployed), 0.6 (low), 1 (medium) and 1.4 (high).

Figure 3.3 shows the impulse response functions of aggregate quantities for a one standard deviation shock to confidence and compare them to the impulse responses of the complete market representative agent counterpart of the model[5]. The main difference arising is that, in the nonlinear model aggregate investment responds more strongly after a positive confidence shock: this means that belief shocks may entail some precautionary saving motives.

[5] The linear model features the same calibration of the heterogeneous agent model in all parameters but the parameter $\nu$, which is re-calibrated in order to have the labor supply of the representative agent in steady state coinciding with the aggregate labor supply of the heterogeneous agent model.

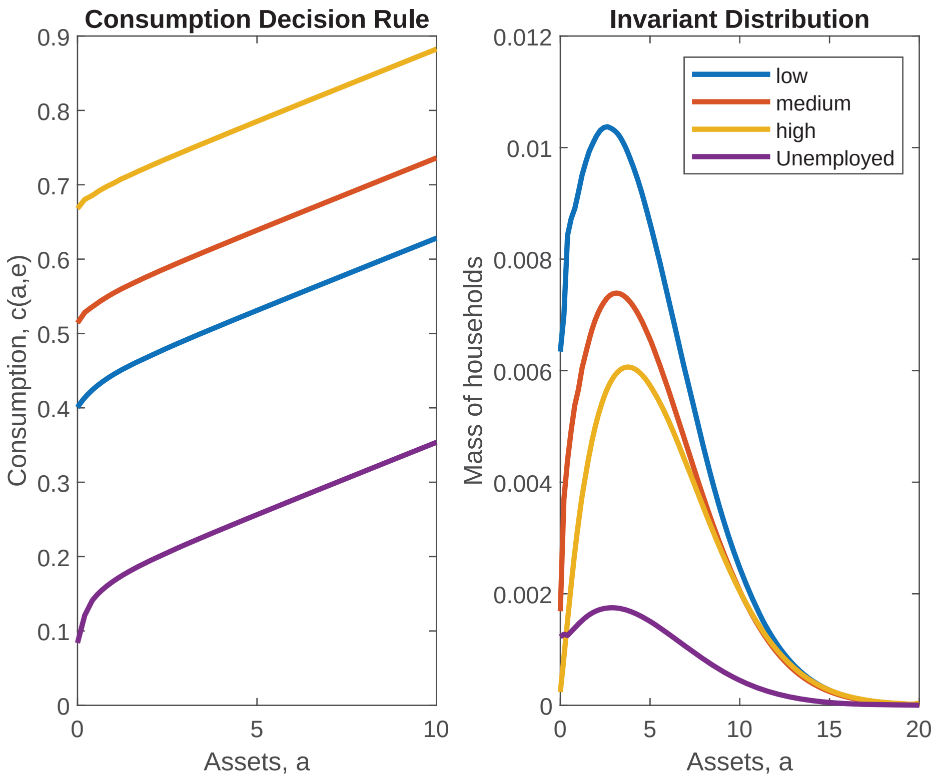


Figure 3.2: Consumption policy functions and invariant distribution of households in the stationary equilibrium for different levels of household efficiency.

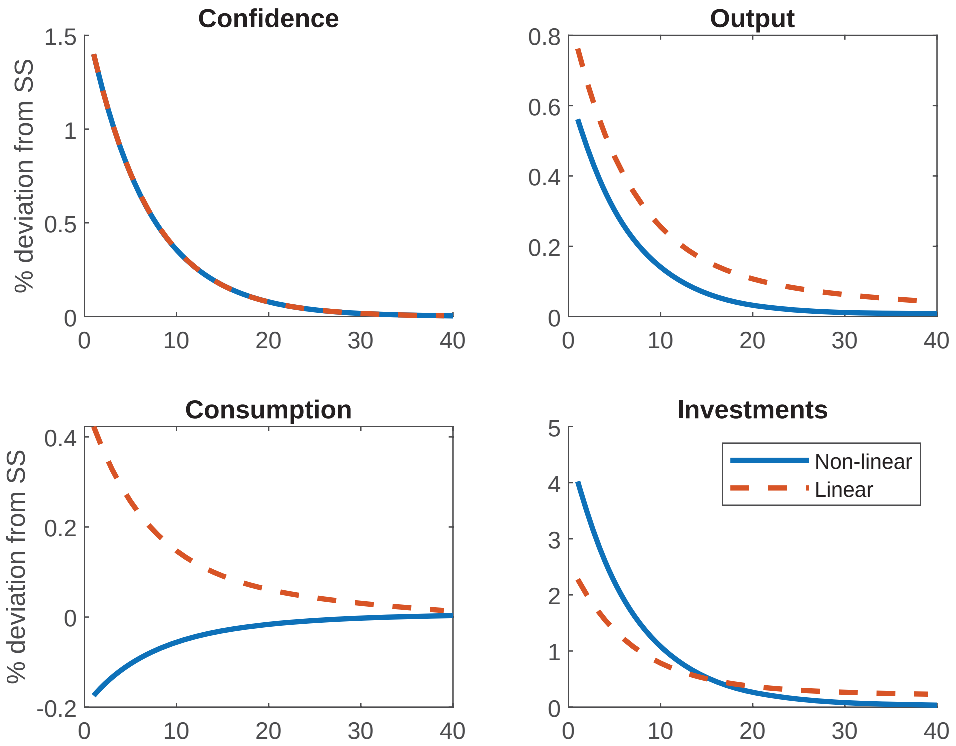


Figure 3.3: Aggregate impulse response functions after a positive, one-standard-deviation confidence shock.

## 3.3 Conclusions and Future Research

This Chapter presented a dynamic model of heterogeneous households with incomplete market and an aggregate confidence shock affecting higher order beliefs of the agents in the economy. The Chapter showed that, by assuming that the utility function takes a convenient form  la Greenwood-Hercowitz-Huffman, the richer structure of higher order beliefs coming from the aggregate confidence shock can be handled from a numerical point view and the model can be solved by perturbation techniques around the stationary equilibrium without aggregate uncertainty. Results from the model suggests then that the interaction between precautionary savings and shocks to beliefs may be quantitative important.

Future research will extend the model on two main dimensions. The extensions are numerically feasible given the numerical tractability of the model inherited by the assumption on the household's instantaneous utility function. The two extensions are:

1. the introduction of an illiquid asset (physical capital) as opposed to a liquid asset (bond). The introduction of an illiquid asset allows, through the appearance of a portfolio choice that disentangles savings from productive investments, to better understand the importance of precautionary savings for the propagation of belief shocks.

2. the introduction of nominal rigidities: nominal rigidities allow to assess the importance of monetary and/or fiscal policies in addressing macroeconomic stability and the possible distributional consequences of belief shocks.

# Bibliography

Aiyagari, S. R. (1994). Uninsured Idiosyncratic Risk and Aggregate Saving. *The Quarterly Journal of Economics*, 109(3):659–684.

Angeletos, G. M., Collard, F., and Dellas, H. (2018). Quantifying Confidence. *Econometrica*, 86(5):1689–1726.

Angeletos, G. M. and La'O, J. (2013). Sentiments. *Econometrica*, 81(2):739–779.

Bachmann, R. and Sims, E. R. (2012). Confidence and the Transmission of Government Spending Shocks. *Journal of Monetary Economics*, 59(3):235–249.

Barsky, R. B. and Sims, E. R. (2011). News shocks and business cycles. *Journal of Monetary Economics*, 58(3):273–289.

Barsky, R. B. and Sims, E. R. (2012). Information, Animal Spirits, and the Meaning of Innovations in Consumer Confidence. *American Economic Review*, 102(4):1343–77.

Bayer, C., Luetticke, R., Pham-Dao, L., and Tjaden, V. (2018). Precautionary Savings, Illiquid Assets, and the Aggregate Cosnequences of Shocks to Household Income Risk. *Econometrica*, forthcoming.

Beaudry, P. and Portier, F. (2006). Stock Prices, News, and Economic Fluctuations. *American Economic Review*, 96(4):1293–1307.

Benhabib, J. and Farmer, R. E. A. (1994). Indeterminacy and Increasing Returns. *Journal of Economic Theory*, 63(1):19–41.

Berger, D., Guerrieri, V., Lorenzoni, G., and Vavra, J. (2018). House Prices and Consumer Spending. *Review of Economic Studies*, 85(3):1502–1542.

Bernanke, B. and Gertler, M. (1989). Agency Costs, Net Worth, and Business Fluctuations. *American Economic Review*, 79(1):14–31.

Bernanke, B. S., Gertler, M., and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. In Taylor, J. B. and Woodford, M., editors, *Handbook of Macroeconomics*, volume 1 of *Handbook of Macroeconomics*, chapter 21, pages 1341–1393. Elsevier.

Blanchard, O. J., L'Huillier, J.-P., and Lorenzoni, G. (2013). News, Noise, and Fluctuations: An Empirical Exploration. *American Economic Review*, 103(7):3045–70.

Bordalo, P., Gennaioli, N., and Shleifer, A. (2017). Diagnostic Expectations and Credit Cycles. *Journal of Finance*, 78(1):199–227.

Carroll, C. D. (2003). Macroeconomic Expectations Of Households And Professional Forecasters. *The Quarterly Journal of Economics*, 118(1):269–298.

Case, K. E. and Shiller, R. J. (2003). Is There a Bubble in the Housing Market? *Brookings Papers on Economic Activity*, 34(2):299–342.

Coibion, O. and Gorodnichenko, Y. (2012). What Can Survey Forecasts Tell Us about Information Rigidities? *Journal of Political Economy*, 120(1):116 – 159.

Eggertsson, G. B. and Krugman, P. (2012). Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo Approach. *The Quarterly Journal of Economics*, 127(3):1469–1513.

Evans, G. W. and Honkapohja, S. (1999). Learning dynamics. In Taylor, J. B. and Woodford, M., editors, *Handbook of Macroeconomics*, volume 1 of *Handbook of Macroeconomics*, chapter 7, pages 449–542. Elsevier.

Farmer, R. E. (2012). Confidence, Crashes and Animal Spirits. *Economic Journal*, 122:155–172.

Farmer, R. E. (2013). Animal Spirits, Financial Crises and Persistent Unemployment. *Economic Journal*, 123:317–340.

Gertler, M. and Kiyotaki, N. (2010). Financial Intermediation and Credit Policy in Business Cycle Analysis. In Friedman, B. M. and Woodford, M., editors, *Handbook of Monetary Economics*, volume 3 of *Handbook of Monetary Economics*, chapter 11, pages 547–599. Elsevier.

Gillitzer, C. and Prasad, N. (2018). The Effect of Consumer Sentiment on Consumption: Cross-Sectional Evidence from Elections. *American Economic Journal: Macroeconomics*, 10(4):234–269.

Guerrieri, L. and Iacoviello, M. (2015). OccBin: A Toolkit for Solving Dynamic Models with Occasionally Binding Constraints Easily. *Journal of Monetary Economics*, 70:22–38.

Guerrieri, L. and Iacoviello, M. (2017). Collateral constraints and macroeconomic asymmetries. *Journal of Monetary Economics*, 90(C):28–49.

Guerrieri, V. and Uhlig, H. (2016). Housing and credit markets: Booms and busts. volume 2 of *Handbook of Macroeconomics*, pages 1427 – 1496. Elsevier.

Hall, R. E. (2011). The Long Slump. *American Economic Review*, 101(2):431–69.

Iacoviello, M. (2005). House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle. *American Economic Review*, 95(3):739–764.

Iacoviello, M. (2011). Housing wealth and consumption. International Finance Discussion Papers 1027, Board of Governors of the Federal Reserve System (U.S.).

Iacoviello, M. and Neri, S. (2010). Housing Market Spillovers: Evidence from an Estimated DSGE Model. *American Economic Journal: Macroeconomics*, 2(2):125–64.

Jensen, H., Ravn, S. H., and Santoro, E. (2015). Changing Credit Limits, Changing Business Cycles. CEPR Discussion Papers 10462, C.E.P.R. Discussion Papers.

Jermann, U. and Quadrini, V. (2012). Macroeconomic Effects of Financial Shocks. *American Economic Review*, 102(1):238–71.

Jordà, O., Schularick, M., and Taylor, A. M. (2013). When Credit Bites Back. *Journal of Money, Credit and Banking*, 45(s2):3–28.

Jordà, O., Schularick, M., and Taylor, A. M. (2015a). Betting the house. *Journal of International Economics*, 96(S1):S2–S18.

Jordà, O., Schularick, M., and Taylor, A. M. (2015b). Leveraged Bubbles. Working Paper 2015-10, Federal Reserve Bank of San Francisco.

Justiniano, A., Primiceri, G., and Tambalotti, A. (2010). Investment Shocks and Business Cycles. *Journal of Monetary Economics*, 57(2):132–145.

Justiniano, A., Primiceri, G., and Tambalotti, A. (2015). Household leveraging and deleveraging. *Review of Economic Dynamics*, 18(1):3–20.

Kaplan, G., Mitman, K., and Violante, G. L. (2017). The Housing Boom and Bust: Model Meets Evidence. NBER Working Papers 23694, National Bureau of Economic Research, Inc.

Keynes, J. M. (1936). *The General Theory of employment, interest and money.*

Kiyotaki, N. and Moore, J. (1997). Credit Cycles. *Journal of Political Economy*, 105(2):211–48.

Krusell, P. and Smith, A. A. (1998). Income and Wealth Heterogeneity in the Macroeconomy. *Journal of Political Economy*, 106(5):867–896.

Kuttner, K. (2014). Low Interest Rates and Housing Bubbles: Still No Smoking Gun. *in Evanoff et al. (eds), The Role of Central Banks in Financial Stability: Has it Changed?*, World Scientific.

Liu, Z., Wang, P., and Zha, T. (2013). Land-Price Dynamics and Macroeconomic Fluctuations. *Econometrica*, 81(3):1147–1184.

Lorenzoni, G. (2009). A Theory of Demand Shocks. *American Economic Review*, 99(5):2050–84.

Ludvigson, S. C. (2004). Consumer Confidence and Consumer Spending. *Journal of Economic Perspectives*, 18(2):29–50.

Maffezzoli, M. and Monacelli, T. (2015). Deleverage and Financial Fragility. Working Papers 546, IGIER (Innocenzo Gasparini Institute for Economic Research), Bocconi University.

Mankiw, N. G. and Reis, R. (2002). Sticky Information Versus Sticky Prices: A Proposal To Replace The New Keynesian Phillips Curve. *The Quarterly Journal of Economics*, 117(4):1295–1328.

Martin, A. and Ventura, J. (2018). The Macroeconomics of Rational Bubbles: A User's Guide. NBER Working Papers 24234, National Bureau of Economic Research, Inc.

Mian, A., Rao, K., and Sufi, A. (2013). Household Balance Sheets, Consumption, and the Economic Slump. *The Quarterly Journal of Economics*, 128(4):1687–1726.

Mian, A. and Sufi, A. (2010). Household Leverage and the Recession of 2007 to 2009. Technical Report 58, IMF Economic Review.

Piazzesi, M. and Schneider, M. (2009). Momentum Traders in the Housing Market: Survey Evidence and a Search Model. *American Economic Review*, 99(2):406–411.

Piazzesi, M. and Schneider, M. (2016). Housing and Macroeconomics. In Taylor, J. B. and Woodford, M., editors, *Handbook of Macroeconomics*, volume 1 of *Handbook of Macroeconomics*, chapter 7, pages 449–542. Elsevier.

Piazzesi, M., Schneider, M., and Tuzel, S. (2007). Housing, Consumption and Asset Pricing. *Journal of Financial Economics*, 83:531–569.

Pigou, A. C. (1927). *Industrial Fluctuations*. Macmillan.

Quadrini, V. (2011). Financial frictions in macroeconomic fluctations. *Economic Quarterly*, 97(3):209–254.

Reiter, M. (2009). Solving Heterogeneous-agent Models by Projection and Perturbation. *Journal of Economic Dynamics and Control*, 33(3):649–665.

Sargent, T. J. (1993). *Bounded Rationality in Macroeconomics*. Number 9780198288695 in OUP Catalogue. Oxford University Press.

Sims, C. A. (2003). Implications of rational inattention. *Journal of Monetary Economics*, 50(3):665–690.

Smets, F. and Wouters, R. (2007). Shocks and frictions in us business cycles: A bayesian dsge approach. *American Economic Review*, 97(3):586–606.

Tauchen, G. (1986). Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions. *Economic Letters*, 20(2):177–181.

Taylor, J. B. (2007). Housing and Monetary Policy. *Proceedings - Economic Policy Symposium - Jackson Hole*, pages 463–476.

Woodford, M. (2003). Imperfect Common Knowledge and the Effects of Monetary Policy. *In Knowledge, Information and Expectations in Modern Macroeconomics, in Honour of Edmund S. Phelps*, Princeton University Press.

Wu, J. C. and Xia, F. D. (2016). Measuring the Macroeconomics Impact of Monetary Policy at the Zero Lower Bound. *Journal of Money, Credit and Banking*.

Young, E. R. (2010). Solving the incomplete markets model with aggregate uncertainty using the Krusell-Smith algorithm and non-stochastic simulations. *Journal of Economic Dynamics and Control*, 34(1):36–41.

# Appendix A

## VAR Data Description

In the following we describe the data used in the SVAR estimated in Section 3. All data have a monthly frequency and range from January 1985 to June 2015.

1. **Confidence.** As a measure of confidence we used the mid-month release of the University of Michigan Consumer Sentiment Index constructed with the two questions related to current economic conditions.
   Source: https://data.sca.isr.umich.edu/

2. **House Prices.** As a measure of house prices we used the S&P/CoreLogic/Case-Shiller Real Home Price
   Source: http://www.econ.yale.edu/∼ shiller/data.htm

3. **Debt.** As a measure of households' debt we used the Total Consumer Credit Owned and Securitized, Outstanding
   Source: https://fred.stlouisfed.org/series/TOTALSL

4. **Residential Investments.** As a measure of residential investments we used The Value of Construction Put in Place Survey (VIP)
   Source: https://www.census.gov/construction/c30/c30index.html

5. **Economic Activity.** As a measure of economic activity we used the Industrial Production Index
   Source: https://fred.stlouisfed.org/series/INDPRO

6. **Inflation** As a measure of inflation we used CPI inflation
   Source: https://fred.stlouisfed.org/series/CPIAUCSL

7. **FFR** As a measure for the Federal Funds Rate we used the United States Shadow Fed Funds Rate computed in Wu and Xia (2016)
   Source: https://sites.google.com/view/jingcynthiawu/shadow-rates
   The United States Shadow Fed Funds Rate computed in Wu and Xia (2016) coincides with the Federal Funds Rate coincide with the official Federal Funds Rate except for the ZLB period (2009-2015).

# Appendix B

## Model Equilibrium Conditions

In the following we present all the equilibrium conditions of the full model. For the sake of notation, we omit the index $i$ that denotes the island, but distinguish between local island variable with lower-case letter and aggregate variable with capital letter. We divide the conditions in different categories:

- Firm's optimal conditions and production function:

$$\alpha mc_t E_t[Y_t^{\rho}] y_t^{1-\rho} = r_t^k k_t \tag{7}$$

$$(1-\alpha) mc_t E_t[Y_t^{\rho}] y_t^{1-\rho} = w_t n_t^d \tag{8}$$

$$v_t y_t = A_t (n_t^d)^{1-\alpha} k_{t-1}^{\alpha} \tag{9}$$

- Borrower's optimal conditions, marginal utilities, budget and collateral constraint:

$$z_t \tilde{n}^{\nu} = \tilde{\lambda}_t w_t \tag{10}$$

$$q_t \tilde{\lambda}_t = \tilde{\Delta}_t + \beta E_t[q_{t+1} \tilde{\lambda}_{t+1}] + f_t M \mu_t q_t \tag{11}$$

$$\tilde{\lambda}_t - \mu_t = \tilde{\beta} E_t[\lambda_{t+1} r_t] \tag{12}$$

$$\tilde{\lambda}_t = \frac{z_t}{\tilde{c}_t - H_C \tilde{c}_{t-1}} - \tilde{\beta} E_t\left[ z_{t+1} \frac{H_C}{\tilde{c}_{t+1} - H_C \tilde{c}_t} \right] \tag{13}$$

$$\tilde{\Delta}_t = \frac{z_t j_t \psi}{\tilde{h}_t - H_H \tilde{h}_{t-1}} - \tilde{\beta} E_t \left[ z_{t+1} j_{t+1} \frac{H_H}{\tilde{h}_{t+1}] - H_H \tilde{h}_t} \right] \tag{14}$$

$$\tilde{c}_t + q_t(\tilde{h}_t - \tilde{h}_{t-1}) + r_{t-1} \frac{\tilde{d}_{t-1}}{\Pi_t} = \tilde{d}_t + w_t \tilde{n}_t \tag{15}$$

$$b_t = f_t M q_t \tilde{h}_t \tag{16}$$

- Saver's optimal conditions and marginal utilities:

$$z_t n^{\nu} = \lambda_t w_t \tag{17}$$

$$q_t \lambda_t = \Delta_t + \beta E_t [q_{t+1} \lambda_{t+1}] \tag{18}$$

$$\lambda_t = \beta E_t [\lambda_{t+1} r_t] \tag{19}$$

$$\lambda_t = \frac{z_t}{c_t - H_C c_{t-1}} - \beta E_t \left[ z_{t+1} \frac{H_C}{c_{t+1} - H_C c_t} \right] \tag{20}$$

$$\Delta_t = \frac{z_t j_t \psi}{h_t - H_H h_{t-1}} - \beta E_t \left[ z_{t+1} j_{t+1} \frac{H_H}{h_{t+1} - H_H h_t} \right] \tag{21}$$

- Market clearing conditions:

$$Y_t^{\rho} y_t^{1-\rho} = c_t + \tilde{c}_t + i_t \tag{22}$$

$$\tilde{h}_t + h_t = \bar{H} \tag{23}$$

$$n_t^d = n_t + \tilde{n}_t \tag{24}$$

- Optimal investment condition, price of capital and capital accumulation:

$$\lambda_t q_t^k = \beta \lambda_{t+1} [r_{t+1}^k + q_{t+1}^k (1 - \delta)] \tag{25}$$

$$\lambda_t \left\{ 1 - \iota_t q_t^k \left[ 1 - \frac{\phi_K}{2} \left( \frac{i_t}{i_{t-1}} - 1 \right)^2 - \phi_K \frac{i_t}{i_{t-1}} \left( \frac{i_t}{i_{t-1}} - 1 \right) \right] \right\} =$$

$$= \beta E_t \left[ \lambda_{t+1} \iota_{t+1} q_{t+1}^k \phi_K \left( \frac{i_{t+1}}{i_t} \right)^2 \left( \frac{i_{t+1}}{i_t} \right) \right] \tag{26}$$

$$k_t = (1-\delta)k_{t-1} + \iota_t i_t \left(1 - \frac{\phi_K}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2\right) \tag{27}$$

- Monetary policy:

$$R_t = R_{t-1}^{\rho_r}\left[R_{SS}\left(\frac{\Pi_t^A}{\Pi_{SS}^A}\right)^{\phi_P}\left(\frac{Y_t}{Y_{SS}}\right)^{\phi_y}\right]^{1-\rho_r}\omega_t \tag{28}$$

- Optimal pricing decision and inflation:

$$\theta_P \pi_t^{\varepsilon-1} + (1-\theta_P)\hat{p}_t^{*\,1-\varepsilon} = 1 \tag{29}$$

$$\hat{p}_t^* F_{1t} = \frac{\varepsilon}{\varepsilon-1}F_{2t} \tag{30}$$

$$F_{1t} = y_t + \theta_P \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t \Pi_{t+1}}\Pi_{t+1}^{\varepsilon}F_{1,t+1}\right] \tag{31}$$

$$F_{2t} = y_t mc_t + \theta_P \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t \Pi_{t+1}}\Pi_{t+1}^{1+\varepsilon}F_{2,t+1}\right] \tag{32}$$

$$v_t = (1-\theta_P)(\hat{p}_t^*)^{-\varepsilon} + \theta_P \Pi_t^{\varepsilon} v_{t-1} \tag{33}$$

- Exogenous variables:

$$\log A_t = \rho_a \log A_{t-1} + \varepsilon_a \tag{34}$$

$$\log j_t = \rho_h \log j_{t-1} + \varepsilon_h \tag{35}$$

$$\log \iota_t = \rho_i \log \iota_{t-1} + \varepsilon_i \tag{36}$$

$$\log z_t = \rho_z \log z_{t-1} + \varepsilon_z \tag{37}$$

$$\log w_t = \rho_m \log w_{t-1} + \varepsilon_m \tag{38}$$

$$\log f_t = \rho_f \log f_{t-1} + \varepsilon_f \tag{39}$$

$$\log \Gamma_t = \rho_c \log \Gamma_{t-1} + \varepsilon_c \tag{40}$$

An equilibrium of the model is, given a set of exogenous processes as in equations (34)-(40), a set of endogenous processes

$$\{n_t, \tilde{n}_t, n_t^d, r_t^k, w_t, i_t, q_t, \pi_t, R_t, \lambda_t, \tilde{\lambda}_t, \Delta_t, \tilde{\Delta}_t, c_t, \tilde{c}_t, h_t, \tilde{h}_t, q_t^k, mc_t, \mu_t, y_t, k_t, d_t, v_t, F_{1t}, F_{2t}, p_t^*\}$$

which satisfy conditions (7)-(33).

# Appendix C

## DSGE Estimation Data

In the following, we describe the construction of the six observables used for the Bayesian estimation. We downloaded the following data from the FRED database:

- PCND:=Personal consumption expenditure: Nondurable goods

  Source: https://fred.stlouisfed.org/series/PCND

- CBI:=Change in Private Inventories

  Source: https://fred.stlouisfed.org/series/CBI

- FPI:=Fixed Private Investment

  Source: https://fred.stlouisfed.org/series/FPI

- HOANBS:=Nonfarm Business Sector: Hours of All Persons

  Source: https://fred.stlouisfed.org/series/HOANBS

- CNP:=Civilian Noninstitutional Population

  Source: https://fred.stlouisfed.org/series/CNP16OV

- GDPDEF:=Gross Domestic Product: Implicit Price Deflator

  Source: https://fred.stlouisfed.org/series/GDPDEF

- HousPr:=S&P/CoreLogic/Case-Shiller Real Home Price

  Source: http://www.econ.yale.edu/∼ shiller/data.htm The data are monthly and present some seasonality: first we seasonally-adjust and then we aggregate them to quarterly frequency by averaging.

- WuXia:= United States Shadow Fed Funds Rate
  Source: https://sites.google.com/view/jingcynthiawu/shadow-rates

The six observables of the model are all particular transformations of the series defined above:

1. **Non-durable consumption.** Non-durable consumption is defined as:

$$\frac{PCND}{GDPDEF \cdot CNP}$$

The series is log transformed and detrended with one-side HP filter with a smoothing parameter equal to 100 000.

2. **Investments.** Investments are defines as:

$$\frac{CBI + FPI}{GDPDEF \cdot CNP}$$

The series is log transformed and detrended with one-side HP filter with a smoothing parameter equal to 100 000.

3. **Hours.** Hours are given by:

$$\frac{HOANBS}{CNP}$$

The series is log transformed.

4. **House Prices.** House prices are defined as:

$$HousPr$$

The series is log transformed and detrended with one-side HP filter with a smoothing parameter equal to 100 000.

5. **Inflation.** Inflation is defined as:

$$\log\left(\frac{GDPDEF}{GDPDEF(-1)}\right),$$

where $GDPDEF(-1)$ is $GDPDEF$ lagged by one quarter.

6. **Federal Funds Rate.** This observable is defined as:

$$WuXia$$

The United States Shadow Fed Funds Rate computed in Wu and Xia (2016) coincides with the Federal Funds Rate coincide with the official Federal Funds Rate except for the ZLB period (2009Q1-2015Q4). The series is log transformed and detrended with one-side HP filter with a smoothing parameter equal to 100 000.

# Appendix D

## Proof of Proposition 2

We want to show that, given the GHH assumption on the utility function, the equilibrium consistent $\hat{p}$ is given by:

$$\hat{p}(\Lambda, z, \xi) = e^{\xi \frac{1-\alpha}{\alpha+\gamma}}.$$

*Proof.* Define with $K$ the aggregate capital implied by $\Lambda$, $K = \int k d\Lambda(\epsilon, k)$. Proposition 1 showed that, given the GHH assumption on the utility function, the optimal labor supply is independent on the consumption level and, especially, on the idionsyncratic state $(\epsilon, k)$. With aggregate uncertainty it will only depends on the distribution $\Lambda$, on the island signal $z$ and on the aggregate state $\xi$. Aggregate labor supply is then:

$$N(\Lambda, z, \xi) = \bar{\epsilon}\left(\frac{1-\tau}{\nu}\hat{w}(\Lambda, z, \xi)\right).$$

From the firm foc we know that:

$$\hat{w}(\Lambda, z, \xi) = e^{z}(1-\alpha)\left(\frac{K}{N(\Lambda, z, \xi)}\right)^{\alpha}\hat{p}(\Lambda, z, \xi)$$

Plugging in and isolating aggregate labor supply we get:

$$N(\Lambda, z, \xi)^{\frac{\alpha+\gamma}{\gamma}} = \bar{\epsilon}\left(\frac{1-\tau}{\nu}(1-\alpha)K^{\alpha}\right)^{\frac{1}{\gamma}} e^{\frac{z}{\gamma}}\left(\hat{p}(\Lambda, z, \xi)\right)^{\frac{1}{\gamma}}$$

Aggregate labor supply is then:

$$N(\Lambda, z, \xi) = \bar{\epsilon}^{\frac{\gamma}{\alpha+\gamma}} \left( \frac{1-\tau}{\nu} (1-\alpha) K^{\alpha} \right)^{\frac{1}{\alpha+\gamma}} e^{\frac{z}{\alpha+\gamma}} (\hat{p}(\Lambda, z, \xi))^{\frac{1}{\alpha+\gamma}}$$

Remember that $\hat{p}$ is related to local output $y$ and expected aggregate $\hat{Y}$ demand through the following demand equation[6]: $y = \frac{\hat{Y}}{\hat{p}}$. $y$ and $\hat{Y}$ are obviously equilibrium object depending on the the aggregate states in the following way:

$$y(\Lambda, z, \xi) = e^{z} K^{\alpha} (N(\Lambda, z, \xi))^{1-\alpha}$$

$$\hat{Y}(\Lambda, z+\xi, \xi) = e^{z} K^{\alpha} (N(\Lambda, z+\xi, \xi))^{1-\alpha}$$

In order for $\hat{p}$ to be consistent with the equilibrium it must hold:

$$\hat{p}(\Lambda, z, \psi) = \frac{\hat{Y}(\Lambda, z+\xi, \xi)}{y(\Lambda, z, \xi)} = \frac{(N(\Lambda, z+\xi, \xi))^{1-\alpha}}{(N(\Lambda, z, \xi))^{1-\alpha}},$$

where:

$$N(z+\xi, \xi) = \bar{\epsilon}_L^{\frac{\gamma}{\alpha+\gamma}} \left( \frac{1-\tau}{\nu} (1-\alpha) K^{\alpha} \right)^{\frac{1}{\alpha+\gamma}} e^{\frac{z+\xi}{\alpha+\gamma}} (\hat{p}(z+\xi, \xi))^{\frac{1}{\alpha+\gamma}}$$

. Simplifying the constant terms we get:

$$\hat{p}(\Lambda, z, \psi) = \frac{e^{(z+\xi)\frac{1-\alpha}{\alpha+\gamma}} (\hat{p}(\Lambda, z+\xi, \xi))^{\frac{1-\alpha}{\alpha+\gamma}}}{e^{z\frac{1-\alpha}{\alpha+\gamma}} (\hat{p}(\Lambda, z, \xi))^{\frac{1-\alpha}{\alpha+\gamma}}}.$$

Isolating $\hat{p}(\Lambda, z, \xi)$ and simplifying:

$$(\hat{p}(\Lambda, z, \xi))^{1+\gamma} = e^{\xi(1-\alpha)} (\hat{p}(\Lambda, z+\xi, \xi))^{1-\alpha}. \tag{41}$$

This condition does not depend either on $\Lambda$ and $z$; a good guess is then that $\hat{p}$ takes the following form:

$$\hat{p}(z, \xi) = cF(\xi).$$

[6]Remember that demand for the local good is given by $y_{it} = \frac{p_{it}}{P_t} Y_t$ and that $P_t$ is normalized to 1.

Plugging this functional form inside the condition inside (41) and solving for $F$:

$$F(\xi) = \frac{1}{c} e^{\xi \frac{1-\alpha}{\alpha+\gamma}}.$$

It then follows that $\hat{p}(\Lambda, z, \xi) = e^{\xi \frac{1-\alpha}{\alpha+\gamma}}$. □