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Date : October 31[th] 2012

SURNAME	Suverato	NAME	Davide `

Contents

[1] Results in this section are very preliminary and largely incomplete.

List of Figures

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Motivation and introduction

International trade is a main driver of resource reallocation and recent developments of the trade literature investigate the effect of trade policy shocks on heterogeneous firms. Also in the macroeconomic literature, there is an increasing interest in studying the effect of economic policies on heterogeneous agents. These fields of research benefit from the recent availability of datasets that make micro data suitable for macroeconomic analysis. At the same time, there is a large and valuable effort in the development of theoretical models to understand sources and consequences of labor market frictions, that provide theory behind the matching of individual decisions and aggregate outcomes.

These facts motivate my effort to develop a theoretical framework aiming to study the dynamics of the distributions of resources across firms and income across agents. As a result, this dissertation consists of three theoretical models, at the intersection of international trade, labor economics and macroeconomics.

The first chapter develops a theoretical framework to analyze the effect of a decrease in the sunk cost of export on the dynamics of unemployment and wage inequality. The framework is a dynamic general equilibrium model with on the job search and firm entry and exit. Because of on the job search, wage dispersion arises in equilibrium and firms compete on the labor market to hire workers. Given the equilibrium of the labor market, firms take forward looking decisions of entry, export and exit. The dynamics of all variables that characterize the general equilibrium is solved in exact form. The reallocation of resources from low productive firms to more productive firms, that become exporters, leads to an economy populated by fewer larger firms that are more productive and pay better wages; but it also determines more unemployment and it increases wage inequality. The transitional dynamics delivers rich political implications. Following the fall in the sunk cost of export the unemployment rate overshoots whereas wage inequality smoothly rises to a new higher level. The speed in the rise of inequality diminishes as the share of exporter firms increases.

The second chapter proposes a methodological framework to determine the distribution of equilibrium allocations and income across agents in real business cycle models. The intuition consists of three steps. First, the population of agents is modeled as an exchangeable sequence of agents' types generated according to a Pòlya urn process. Second, the distribution of shares of agent types is theoretically characterized using the de Finetti Theorem. Third, the economy is thought as a collection of groups, each group characterized by a particular share of agents types. The macroeconomic analysis can be performed in each group with the standard representative agent approach. Then, the aggregate macroeconomic variables can be recovered exactly by integration over the distribution determined in the first and second stage. Despite its simplicity, the framework predicts a theoretical density of income distribution that is consistent with the findings of the empirical literature. Perhaps, surprisingly, this result is not due to any calibration, it only depends on the functional form of the theoretical distribution, that is endogenously characterized in the paper.

The third chapter extends the result of the first work to include aggregate uncertainty. Aggregate productivity fluctuations affect wages and profit in an asymmetric way. More productive firms and their employees benefit from a positive productivity shock disproportionately more than low productive firms and their employees. Through this channel, an increase in aggregate productivity induces less productive firms out of the market and it increases the share of incumbent firms that become exporters. The dynamics of the unemployment rate results from the demand of labor of new exporters, the larger firm turnover and the entry of new firms. When the productivity shock hits the economy the selection effect dominates and the unemployment rate increases. The response is more than proportional to the productivity shock. The entry of new firms and the demand of labor by new exporters are responsible for a decrease of the unemployment rate as the economy reaches the steady state.

Chapter 1

Export and Labor Market: a Dynamic Model with on-the-job Search, Firm Entry and Exit

Abstract

This paper develops a theoretical framework to analyze the effect of a decrease in the sunk cost of export on the dynamics of unemployment and wage inequality.

We propose a two-sector, two-factor dynamic general equilibrium model with *on the job search* and *firm entry and exit*. Because of *on the job search*, wage dispersion arises in equilibrium and firms compete on the labor market to hire workers. Employers that become exporters issue more

job offers and attract workers from low productive firms. Through this channel, the export shock induces selection on incumbent firms.

The dynamics of the economy is driven by *firm entry / exit* and the accumulation of a fixed input that is used in entry, production and export activities. Free entry and market clearing determine the mass of firms that enter the market each period. Firms that are not efficient enough exit the market.

The dynamics of all variables that characterize the general equilibrium is solved in exact form. The reallocation of resources from low productive firms to more productive firms, that become exporters, leads to an economy populated by fewer larger firms that are more productive and pay better wages; but it also determines more unemployment and it increases wage inequality. Following the fall in the sunk cost of export the unemployment rate overshoots (because of selection) whereas the Gini index on the wage distribution of the economy smoothly adjusts to a new higher level. The speed in the rise of inequality diminishes as the share of exporter firms increases.

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1.1 Introduction

This paper develops a dynamic general equilibrium model to analyze the effect of a decrease in the sunk cost of export on unemployment and wage distribution. The empirical trade literature finds evidence that exporters are more productive, pay better wages and hire more workers. Nevertheless there is no unique consensus on the effect of trade openness on unemployment and wage inequality.

[Trefler, 2004] documented the effects of trade openness on the labor market, in the case of a US-Canada free trade agreement. The increase in trade openness is associated with a loss of employment by 5% in total manufacturing; 12% for industries that were more affected by the liberalization. At the same time, productivity increases by 6% in total manufacturing; 15% for more impacted industries. [Felbermayr et al., 2011b] provide a complementary analysis of this phenomenon. On a sample of 20 OECD countries from 1983 to 2003, they show that once the business cycle components are taken out of the analysis then there is no evidence of an unemployment-increasing effect of trade openness.

[Goldberg and Pavcnik, 2007] provides large qualitative and quantitative evidence of the increase in inequality after trade liberalization reforms. Using data on Mexico, Colombia, Argentina, Brazil, Chile, India and Hong Kong from the 80s and 90s they show that all countries experienced an increase in wage inequality after trade liberalization policies. Moreover, there is evidence of a stabilization of wage inequality after a decade from the trade liberalization.

This evidence on unemployment and inequality should be understood within a framework that more broadly characterizes the decision to export and the behavior of exporters in the labor market; a comprehensive survey is discussed in [Bernard et al., 2011]. [Roberts and Tybout, 1997] documented the importance of sunk costs of entry in the export market. They also find that prior export experience increases the probability of exporting by 60%. [Bernard et al., 2007] provides extensive evidence of the dimensions through which exporters are "special"[1]. They show that exporters pay a wage that is 6% larger than non exporters, they hire 97% more employees, they ship 108% more output and they are 11% more productive in terms of value added per worker. Controlling for employment

[1] This survey reports data form the 2002 U.S. Census of Manufactures; the statistics we quote are from Table 3.

all these elasticities do not significantly change, but controlling for the volume of output it dramatically falls to 8%. While exporters employ almost twice the number of workers than non exporters, non exporters have more than twice the probability of "death" than exporters have, given the same time span[2].

There is a recent and growing effort in the theoretical literature to propose frameworks that could account for these empirical findings. [Felbermayr et al., 2011a] combine the Diamond Mortensen and Pissarides modeling of the labor market with firm heterogeneity and selection into the export market, as in [Melitz, 2003]. They show that following an increase in the extensive margin of trade unemployment should fall as a consequence of an increase in average productivity. [Helpman and Itskhoki, 2010] develop a two sector framework in which trade liberalization is responsible for the reallocation of workers from a competitive non tradable sector to a tradable sector in which firms are heterogeneous and there are frictions in the labor market. Depending on the strength of this reallocation, overall unemployment may increase. In both these frameworks, as in most of the literature, there is no wage dispersion across firms of the tradable sector. Dealing with wage inequality requires instead a mechanism that is responsible for wage dispersion across firms. [Helpman et al., 2010] propose an argument based on match specific ability and the capability of firms to invest in screening workers after they are matched. Firms hire workers, bargain over wages and separate to pursue the optimal composition of their workforce. Workers have the same ex-ante expected earning across possible employers but they are paid differently depending on the employer with whom they are matched. In this framework, more produc-

[2]These estimates are based on plant data from the Longitudinal Research Database (LRD) of the Bureau of the Census, US manufacturing plants1987 - 1997. A plant is considered exporter if it exports or not, as a dummy variable. A plant is considered"dead" if it is in the LRD in year t but absent from the Census in year t + 5 and beyond.

tive firms pay higher wages and the status of being exporter increases the wage paid by a firm with a given productivity. The increase in trade openness increases wage inequality with respect to the autarky case. Moreover, the theoretical relationship between trade openness and wage inequality exhibits an inverted U shape.

There are at least two dimensions in which this paper contributes to shade light on the relationship between export, unemployment and inequality. First, we solve for the dynamics of unemployment and wage distribution after an export shock; whereas most of the current studies develop static frameworks. Second, the source of worker reallocation and wage dispersion is radically different from what we find in the current frameworks. In this paper wage dispersion and endogenous separation are driven by the fact that workers search while they are employed. This feature builds an additional competition channel across firms. In fact, as a consequence each firm is facing a positively sloped labor supply curve that depends on the distribution of wage offers in the labor market. This channel identifies an interdependence between the labor market and firm entry, exit and export decisions that is new in the trade literature.

We propose to address the research question on two channels: *on the job search* (as driver of worker reallocation) and *firm entry/exit* (to account for the selection effect induced by export shocks). There is one small open economy and two sectors: one producing differentiated consumption goods, the other one producing a service that is used as fixed factor in the consumption sector. Labor is the only variable factor of production. Workers are homogeneous and they search in both sectors, when they are unemployed as well as when they are employed. Firms in the consumption sector are heterogeneous as they face uncertain productivity levels over time. The service sector is perfectly competitive. Each period firms in the consumption sector take forward looking decisions of entry, exit and export.

We show that a fall in the sunk cost of export determines more unemployment and an increase in wage inequality in the economy. The channel that causes this is the allocation of resources from low productive firms to more productive firms, that become exporters. After the export shock, the economy is populated by fewer and larger firms, better paid workers and more unemployment.

The framework is highly tractable and it allows to solve for the exact dynamics of labor market variables. The path of unemployment rate, labor market tightness and the Gini index on the wage distributions are fully characterized. They respond to the reallocation of workers from low productive firms to high productive firms, and among those to the new exporters. The unemployment rate overshoots in response to the an unanticipated fall in the sunk cost of export whereas the Gini index smoothly adjusts to a new higher level. The speed in the rise of inequality diminishes as the share of exporter firms increases.

The structure of the model consists of three building blocks, corresponding to the markets for consumption goods, service and labor. Consumers allocate consumption over varieties of a differentiated good; preferences imply a linear demand for each variety. Two technologies are available: production of varieties of a differentiated consumption good (internationally tradable) and production of physical capital (non internationally tradable). Technologies are linear in labor, that is the only variable factor (immobile across countries). In the differentiated goods sector firms are heterogeneous in terms of productivity and they are monopolists, each one of them produces one variety of the differentiated good. In the capital sector firms are homogeneous and perfectly competitive.

Entry requires investment in a fixed input (sunk cost) and it takes time (one period) to build a plant and be ready for production. Firm investment yields to uncertain efficiency levels, that become known after the irreversible sunk cost of entry has been paid. Once a firm is incumbent it pays for the overhead cost of operations (fixed cost). In addition, exporters pay a sunk cost of export to overcome non-tariff and institutional barriers.

Three (endogenous) productivity cutoffs are relevant in our discussion: i) the minimum productivity above which a firm faces a positive demand, ii) makes a positive profit in the domestic market and iii) makes a positive profit in the export market. At the beginning of each period incumbent firms in the differentiated good sector learn their idiosyncratic productivity and borrow capital from private stockholders to finance production and export (if it is profitable). In addition, potential entrants decide whether to enter or not in the next period. New entrants borrow from private stockholders to finance the sunk cost of entry in the upcoming period. When new productivity shocks occur the period ends and all firms (*old* incumbents and *new* entrants) learn about the unit labor cost they will operate with in the next period. They decide whether to stay in the market or exit; they shut down without costs if the variable profit does not cover the fixed cost of production.

The labor market follows the Diamond Mortensen Pissarides framework, with two main departures: workers search when they are on the job and the mass of employers is endogenous and it evolves over time due to firm entry and exit. Firms and workers bargain over the nominal surplus of a match. Moreover, because of on the job search, firms face a labor supply with positive slope. As a result more productive firms pay better wages and hire more workers.

The evolution of the labor market over time is a sequence of steady state equilibria that takes the mass of employers as endogenous state variable. The state of the dynamic problem updates at the beginning of each period following the entry/exit process. Within a period (between two subsequent waves of incumbent firms) employment inflows and outflows balance at the aggregate level. Labor demand and supply clearing at the firm level determines existence and uniqueness of the distribution of wage offers.

This approach extends the *selection effect* due to [Melitz, 2003] into a dynamic general equilibrium framework in which more productive firms pay better wages, hire more workers and unemployment arises in equilibrium. Firms that become exporters post wage offers at the same wage they would do in a closed economy but they will post more offers and hire more workers, to serve the foreign market in addition to the domestic one (*job creation effect*). Because of on the job search workers reallocate from low productive firms to more productive firms. Less efficient employers posting their optimal wage will not hire enough workers to break even with the fixed costs structure and they will exit the market (*job destruction effect*). Demand and supply clearing in the labor market determine the share of employment in each sector and the unemployment rate. Demand and supply clearing in the service market determines the mass of new entrants.

The dynamic stochastic general equilibrium is characterized by a system of three linear first order difference equations in the mass of employers the stock of service and the number of exporters. Policy analysis and dynamics can be easily worked out analytically in closed form solution. The solution of the dynamic system is exact and it exhibits saddle path stability.

A permanent fall in the sunk cost of export determines: a larger share of exporter firms, a more severe selection of the less efficient incumbent firms out of the market, an increase in average nominal wage and a decrease of prices. There are few employers in the market and the average employment per firm is larger. Following the fall in the sunk cost of export the unemployment rate overshoots (because of selection) whereas the Gini index on the wage distribution of the entire economy smoothly adjusts to a new higher level.

The paper is structured as it follows. The next section provides the discussion of the model. In the third section we define and solve for the dynamic general equilibrium. In section four we discuss the consequences of fall in the fixed cost of export. Section five concludes. In the appendix we provide the characterization of this framework under two different specifications. First we characterize a pure wage posting equilibrium (as in [Burdett and Mortensen, 1998]) instead of the wage bargaining solution. Second, we let firms draw their efficiency level once and forever (as in [Melitz, 2003]) and we perform the steady state analysis of the labor market variable.

1.2 Model

There are two sectors producing a consumption good and a durable good. In the consumption sector a continuum of single product monopolists supply varieties of a differentiated good. In the durable good sector a continuum of perfectly competitive producers supply a composite good that is used as input in the consumption sector. Labor is used in both sectors as variable factor of production. Workers are the owners of production inputs and they rent them to firms.

The consumption goods are treated as internationally tradable manufacturing goods. Instead, the durable good is a composite asset of non tradable inputs (such as land, infrastructure or utilities) and services that support firms in their hiring activity (such as agencies that match workers with firms). These resources have a value and depreciate according to their use in the production process of consumption goods. Each period agents demand consumption goods and hold stocks of the value of the durable input. In this sense, we refer to the durable good as capital.

1.2.1 Consumer preferences

Preferences are non homothetic[3] and the marginal utility of income depends on the number of competitors in the market, first and second moments of their price distribution. Among several utility functions that can rationalize this structure we restrict our choice such that the marginal revenue is linear in price. This is a desirable property when firms bargain with workers over the match surplus.

Consumers allocate consumption over a continuum of varieties indexed by $i \in \Omega(i)$:

$$U\left(\{c^{\omega}(i)\}_{i\in\Omega(i)}\right) = \alpha \int_{i\in\Omega(i)} c^{\omega}(i)\, di - \frac{\gamma}{2}\int_{i\in\Omega(i)} c^{\omega}(i)^2\, di \tag{1.1}$$

where $c^{\omega}(i)$ is the consumption of variety i of agent ω. This utility function is a special case of [Ottaviano et al., 2002] in which agents do not value aggregate consumption and they do not consume an outside good[4]. The demand system for each variety is linear. The parameter $\alpha > 0$ is a demand shifter and parameter $\gamma > 0$ accounts for the sensitivity of demand to price. Condition $c^{\omega}(i) \leq \alpha/\gamma\ \forall i$ guarantees positive and diminishing marginal returns.

Over a population of N consumers, the aggregate demand for each variety is given by:

$$C_t(i) = \frac{N}{\gamma}\left(\alpha - E_{\omega}[\lambda(\omega)]\, p(i)\right) \tag{1.2}$$

where $E_{\omega}[\cdot]$ is the expectation operator over the space of agents $\Omega(\omega)$, $\lambda(\omega)$ is the marginal utility of income for agent ω at time t, $p(i)$ is the price of variety i and it is bounded above $p(i) < \frac{\alpha}{E_{\omega}[\lambda(\omega)]} \equiv p_{\max}$ for every variety that is sold in the market.

[3] As [Fieler, 2011] argues "there is exhaustive evidence that the income elasticity of demand varies across goods and that this variation is economically significant".

[4] Income effect is restored as in [Neary, 2007].

1.2.2 Technology

In both sectors labor is employed with constant returns to scale technology. In the capital sector productivity changes over time. In the consumption sector firms are heterogeneous, therefore each point in time we observe a distribution of productivity across firms.

Let φ be the productivity of labor in the service sector at time t. Because of perfect competition, production in the service sector is equivalent to the output of $E_t^{serv} > 0$ homogeneous firms with productivity φ employing l_t^{serv} workers:

$$A_t = \varphi E_t^{serv} l_t^{serv} \tag{1.3}$$

where A_t are the new units of service produced at time t.

In the consumption sector, let $c \in [\underline{c}, \overline{c}]$ be the random variable that describes the requirement in terms of labor per unit of output per time, with $\underline{c} < \overline{c}$ strictly positive finite values. Define $\Phi(c)$ as the distribution of c over the support $c \in [\underline{c}, \overline{c}]$. The production function in the consumption sector is:

$$q(c) = \frac{l(c)}{c} \tag{1.4}$$

where $l(c)$ is firm employment and $q(c)$ is the output of a firm endowed with unit labor cost c.

As in [Hopenhayn, 1992a], unit labor requirements are independent draws across firms. In addition we assume that all firms draw from the same distribution[5]. This choice increases the tractability of the model, at the cost of loosing information about a single firm over time. Nevertheless, the primary concern of the present work is the effect of a trade shock on

[5] A tractable way to avoid the assumption that all firms draw from the same distribution is to assume that idiosyncratic productivity follows a Markov process with draws that are independent across firms each point in time, that is exactly the specification in [Hopenhayn, 1992b]. However if we believe that there is no reason why workers should know about the firm idiosyncratic productivity process then the tractability and results of this framework will not be affected.

the unemployment rate and the wage distribution. As we discuss later, firms do not commit to binding contracts and workers can reallocate across firms when they are employed, other than move from unemployment to employment. Under this scenario, the assumption of independence will not drive our conclusions on unemployment rate and wage distribution.

1.2.3 Consumption sector: firm level variables

Let $w(c)$ be the wage paid by a firm endowed with unit labor cost c. The profit maximizing price $p(c)$ satisfies the equivalence between marginal cost $w(c)\,c$ and marginal revenue $2p(c)-\frac{\alpha}{E_{\omega}[\lambda(\omega)]}$:

$$p(c)=\frac{1}{2}\left(\frac{\alpha}{\mathbb{E}_{\omega}\left[\lambda(\omega)\right]}+w(c)\,c\right)$$

As in [Melitz and Ottaviano, 2008], define c_D as the maximum unit labor cost below which a firm faces positive demand, $p(c_D)=p_{\max}$, such that $w(c_D)\,c_D=\alpha E_{\omega}\left[\lambda(\omega)\right]$. Then by (3.2) the firm's optimal price is:

$$p(c)=\frac{1}{2}\left(w(c_D)\,c_D+w(c)\,c\right) \tag{1.5}$$

Equilibrium output and labor demand, revenue and variable profit (revenue minus variable cost) associated to the production in the domestic market are:

$$C^{dom}(c) = \frac{N}{2}\frac{\alpha}{\gamma}\left(1-\frac{w(c)}{w(c_D)}\frac{c}{c_D}\right) \tag{1.6}$$

$$l^{dom}(c) = \frac{N}{2}\frac{\alpha}{\gamma}\left(1-\frac{w(c)}{w(c_D)}\frac{c}{c_D}\right)c \tag{1.7}$$

$$r^{dom}(c) = \frac{N}{4}\frac{\alpha}{\gamma}w(c_D)\,c_D\left[1-\left(\frac{w(c)}{w(c_D)}\frac{c}{c_D}\right)^2\right] \tag{1.8}$$

$$\pi^{dom}(c) = \frac{N}{4}\frac{\alpha}{\gamma}w(c_D)\,c_D\left(1-\frac{w(c)}{w(c_D)}\frac{c}{c_D}\right)^2 \tag{1.9}$$

The domestic economy is small and open. Firms of the domestic economy that export face the same preference and technology structure but a competitive environment that is possibly different from the domestic one. All the informations exporters need to know about the foreign market consists of the expected marginal utility of income across foreign consumers $E_\omega [\lambda_t(\omega)]^*$ and the nominal bilateral exchange rate e_t, measured as number of foreign currency units per one unit of domestic currency.

We assume there is no variable cost associated to trade, exporters set the price according to producer currency pricing and the law of one price holds[6]. The optimal price of a good produced by a domestic firm endowed with productivity c is $p(c) = \frac{1}{2}\left(\frac{\alpha}{E_\omega[\lambda(\omega)]} + w(c)c\right)$ in domestic currency and $p^*(c) = \frac{1}{2}\left(\frac{\alpha}{E_\omega[\lambda(\omega)]^*} + ew(c)c\right)$ in foreign currency. The nominal bilateral exchange rate that satisfies the law of one price is equal to the ratio in the average marginal utilities of income in the two countries:

$$E_\omega [\lambda(\omega)] = eE_\omega [\lambda(\omega)]^* \tag{1.10}$$

Levels and fluctuations of the nominal bilateral exchange rate and average marginal utility of income in the foreign economy are exogenous. This paper does not investigate these sources of shocks. Still in a comparative statics exercse, this framework would imply that when the domestic currency loses value in terms of foreign currency, or the average marginal utility of income in the foreign economy decreases, then firms in the domestic economy face positive demand for a larger unit labor requirement:

$$\frac{w(c_D)}{1/c_D} = \frac{\alpha}{E_\omega[\lambda(\omega)]} = \frac{\alpha}{eE_\omega[\lambda(\omega)]^*}$$

The value of c_D is a sufficient statistics to describe the international competitiveness of domestic firms, as $\frac{w(c_D)}{1/c_D}$ is the maximum unit labor cost below which a firm faces positive demand in the domestic market.

[6] [Campa and Goldberg, 2005] documented that the producer currency pricing is a more likely scenario, with respect to the local currency pricing, at least in the long run for many types of imported goods among 23 OECD countries from 1975 to 2003. Still they find evidence of incomplete pass through, ranging in the long run from 40% in US to 80% in Germany.

As a result of this scenario, the marginal revenue associated to sales of the same product in the domestic or foreign markets is the same. Demand, employment and profit associated to the foreign market are the same as in the domestic economy, but for the market size:

$$C^{exp}(c) = \frac{N^*}{N} C^{dom}(c) \ , \ l^{exp}(c) = \frac{N^*}{N} l^{dom}(c) \ , \ \pi^{exp}(c) = \frac{N^*}{N} \pi^{dom}(c) \tag{1.11}$$

where N^* is the population in the foreign economy.

This symmetric structure also implies that the value of a job is the same whether the worker is producing a good that will be sold in the domestic or in the foreign market. How does the value of a job in the domestic labor market depend on the destination goods market is a channel linking trade and labor that deserves investigation. To do so, in this framework we should relax at least one of the three assumptions that restrict the foreign transactions[7].

1.2.3.1 Demand of labor at the firm level

Employment at the firm level is determined by the labor demand (1.7) in the domestic and export market (3.10):

$$l_t^d(w) = \begin{cases} \frac{N}{2}\frac{\alpha}{\gamma}\left(1 - \frac{w(c)}{w(c_D)}\frac{c}{c_D}\right)c & , \textit{if non exporter at time } t \\ \frac{N+N^*}{2}\frac{\alpha}{\gamma}\left(1 - \frac{w(c)}{w(c_D)}\frac{c}{c_D}\right)c & , \textit{if exporter at time } t \end{cases} \tag{1.12}$$

The demand of labor is a continuous decreasing function of the wage. Notice that the unit labor cost for all firms that face a positive demand is lower than the unit labor cost for a firm with zero demand $\frac{w(c)}{1/c} < \frac{w(c_D)}{1/c_D}$.

The main observation that will be crucial for the discussion is that exporters behave in the labor market as *bigger* employers, in the following sense. An exporter demands $\left(1 + \frac{N^*}{N}\right)$ times the units of labor than it

[7] This is the research question for a new project that aims to address the correlation between labor market variables across countries that are trade partners. In this paper we do not address this question.

would do in case it was endowed with the same productivity but it did not have the chance to export. Let E_t be the number of employers that serve the domestic consumption sector and let x_t be the share of exporters out of those. Define an employer unit as an employer that demand labor to serve the domestic market, then the effective number of employer units in the demand side of the labor market is $\left(1 + x_t \frac{N^*}{N}\right) E_t$.

1.2.4 Fixed costs

In the consumption sector firms obey a structure of fixed costs. A firm purchases units of the capital service as fixed input employed in the activities of entry, production and export. The price of each unit of capital is one; hence capital is the numeraire.

Firms demand the services in the capital sector in order to adjust employment over time. Firms pay for the advertising vacancies, hiring and training costs; in addition separations are costly to the extent that employment protection and unemployment insurance are enforced by law. The literature on search in the labor market treats these costs as match specific investments, [Mortensen and Pissarides, 1999]. Under this assumption a two-tiers wage structure arises. Hiring costs are sunk in the case of renegotiations with current employees, but they decrease the value a firm attaches to a match with workers in the market. At the same time, firing costs affect continuation decisions; especially in case a penalty applies to separations that occur without renegotiations.

However in the context of this work, there are limitations to which that discussion can be performed[8]. Firms change their productivity each period exogenously and contracts cannot commit firms and employees to a particular wage or employment level in the future. Therefore, there would not be a clear understanding of what a match specific hiring cost and firing cost mean. Workers are homogeneous and there is complete information,

[8] A more general discussion is developed in [Coşar et al., 2010].

such that interpersonal comparison in a context of costless renegotiations would hardly be reconciled with a two wage structure. Most importantly, each period firms take decisions about enter a market or wait, stay or exit a market. These decisions bring the focus of the discussion on firm turnover, more than job turnover. Under these considerations, we treat fixed costs of employment adjustment as specific to the firm, not to the match[9].

Incumbent firms that adjust employment between two subsequent periods pay a regular price of f_p units of capital service; where regular refers to the fact that the hiring / firing needs are an ordinary consequence of matching demand and production needs in the period. Firms that enter the domestic market for the first time require an exceptional hiring service in order to contact a stock of potential employees to start with in the next period. This is an irreversible investment, as it is specific to a firm in a particular point in time. The sunk cost of entry associated to this investment is f_{ed}. A firm that enters the export market for the first time demand more workers to satisfy the demand of a second market. Therefore also in this case, an exceptional hiring activity is needed. The sunk cost of entry in the export market is f_{ex}. Just for the sake of simplicity, firms that exit the market do not pay any cost[10].

1.2.5 Labor market

Following the approach developed by [Diamond, 1982] and [Mortensen and Pissarides, 1994], the labor market is characterized by search and matching frictions. Workers are homogeneous and they search for better job offers also while they are employed as in

[9] [Ljungqvist, 2002] provides a discussion of the effect of layoff costs in a general equilibrium framework.

[10] In alternative, the value of an exit would be negative instead of being zero. This does not affect the tractability of the model or the conclusions.

[Burdett and Mortensen, 1998][11]. The search and matching process is random and time is discrete. When a worker-firm match occurs the two parties bargain over the match surplus as risk neutral agents[12]. There is complete information and both parties know the distribution of wage offers in the economy.

Contracts are not binding in the sense of [Stole and Zwiebel, 1996], they only specify the current wage and they cannot be made contingent upon outside offers. In every negotiation there is a potentially infinite number of offers and firms move to the production stage only when i) an agreement is reached or ii) the firm and the employee separate[13]. In this context, [Stole and Zwiebel, 1996] show that (i) the firm cares about the profit of the marginal worker in the current period (ii) prior unenforceable agreements cannot affect the outcome at any renegotiation round.

The labor market has a structure similar to [Mortensen, 2009]. In this respect, the contribution of this work is to solve for labor market allocations within a general equilibrium approach. Each period, firms realize their unit labor requirement in the goods market and, given this information, they understand the optimal employment level and the marginal profit they gain from a worker.

Nevertheless, the framework we described simplifies the discussion in several non trivial directions. First, technology exhibits *constant returns on labor*. Therefore, the marginal value of an employee does not depend

[11] [Postel-Vinay and Robin, 2002] develop a structural model with workers and firms heterogeneity based on French panel data.They find that the extent to which worker individual characteristics explain wage differences lies close to 40% for high-skilled white collars, but it falls to be negligeable as the observed skill level decreases.

[12] The assumption upon which workers behave as risk neutral agents in the labor market is common to works that are our benchmark in this topic (see [Helpman and Itskhoki, 2010] for a discussion).

[13] See [Stole and Zwiebel, 1996] footnotes 15 and 17 for a more detailed discussion on the stable bargaining outcome under the infinite number of negotiations, versus alternative specifications.

on firm employment; as in the case with diminishing returns on labor[14]. The optimal number of vacancies issued by a firm is implicitly determined as the number of vacancies that are needed to keep employment at the desired level. The number of vacancies is not chosen at the margin and the cost of employment adjustment is sunk in the decisions of enter and stay in the market.

Second, the reservation wage is fixed in a *competitive outside sector*. Firms in the capital sector pay workers their marginal value. At the same time, they do not have incentives to rise the wage above the reservation wage. As a consequence, the reservation wage in the economy is equal to the value of the marginal productivity of labor in the capital sector.

Third, in each period firms are endowed with *i.i.d. draws of unit labor requirement*. This implies that before the realization of productivity shocks all matches across firms have the same expected value. This assumption reduces uncertainty in the worker problem. In a more general formulation, the value of a match in a given period depends on the endogenous wage offer distributions in the next period, that is an infinite dimensional state variable. [Moscarini and Postel-Vinay, 2012] provide a detailed discussion of the problems that would arise in this context and they give sufficient conditions to solve the model [Burdett and Mortensen, 1998] out of the steady state. [Menzio and Shi, 2010] defines and proves the existence of a *block recursive equilibrium* in the case of directed search. An alternative approach that has been extensively used in macroeconomic models with heterogeneous agents is the *approximate aggregation* argument proposed by [Krusell and Smith, 1998]. The first approach deals with a recursive equilibrium that depends on the aggregate state of the economy only through the aggregate state variables and not through the distribution of workers across employment status and wage. In the latter approach, the distribution of idiosyncratic income across agents is approx-

[14] Notice that under diminishing returns an incentive to over employ arises in exchange for a lower wage paid to all workers.

imated by a finite number of moments, and agents solve their recursive maximization problem taking into account only moments of the actual distribution. Our characterization is consistent with the bargaining design in [Stole and Zwiebel, 1996], (the solution suggested in [Moscarini and Postel-Vinay, 2012] assumes pure wage posting) under random matching ([Menzio and Shi, 2010] assume directed search) and will allow us to determine the wage distribution endogenously.

Notice that despite the independence assumption, the strength of the on the job search argument does not vanishes. When firms hire or have to keep workers they are forced to compete on the current wage, since from the worker's point of view the continuation value of employment is the same across employers. Moreover, since contracts are not binding, workers can freely engage in job to job reallocations. This feature decreases considerably the contribution of the independence assumption in driving our conclusions about the wage distribution; as the following argument shows.

Assume there exists an employer that systematically will draw better unit labor costs in the future with respect to others and consider the difference between workers' choices in that context with respect to our framework. What we do not capture is the possibility that there are workers (*1*) who actually visit the vacancies of this employer and (*2*) discard the offer because their outside offer is better in the current period and (*3*) in the future they do not have the chance to visit the vacancy from this employer when it actually becomes the best choice. By definition, this will not affect unemployed workers, or those that become unemployed because of exogenous separation shocks. More importantly, among employed workers there is a subset that would still choose the same plan they choose in our framework. In fact, workers have the possibility to sample the wage offer from the alternative employer in the future whenever they match. There would be a severe difference in the two scenarios in case the worker had to keep the same wage until he/she becomes unemployed. But, this is not the case because of on the job search. A difference in the optimal choice remains only for those workers who would like to change employer in the future but they do not sample the wage offer from that particular employer. Such a difference therefore is confined to a portion of the entire workforce and its relevance decreases the larger is the probability of finding a match.

Under this framework, the dynamic general equilibrium remains highly tractable. As in [Burdett and Mortensen, 1998] and [Mortensen, 2009] the equilibrium is characterized by a unique wage offer distribution that is a collection of wage offers across firms such that they are the outcome of mutual best response strategies. In addition, we can exploit the general equilibrium framework to characterize the demand side of the labor market and determine the wage offer distribution such that demand and supply of labor clear at the firm level each point in time.

1.2.5.1 Definitions

On the supply side of the labor market a mass of N homogeneous workers randomly look for job offers. On the demand side, a mass of heterogeneous employers opens V_t vacancies at time t. The labor market tightness at time t is:

$$\theta_t = \frac{V_t}{N} \tag{1.13}$$

Workers and firms match randomly. New matches are formed according to a matching function homogeneous of degree one:

$$Matches\left(V_t, N\right) = V_t^{1/2} N^{1/2}$$

The probability that a single worker randomly finds a job offer over the V_t vacancies is an increasing function of the labor market tightness:

$$m\left(\theta_t\right) = \frac{V_t^{1/2} N^{1/2}}{N} = \sqrt{\theta_t} \tag{1.14}$$

as the share of new matches over the entire workforce. The probability that a vacancy is randomly visited by a worker is:

$$\frac{m(\theta_t)}{\theta_t} = \frac{V_t^{1/2} N^{1/2}}{V_t} = \frac{V_t^{1/2} N^{1/2}}{N} \frac{N}{V_t} \tag{1.15}$$

Each period an employed worker separates from the current match with a probability $\delta \in (0, m(\theta_t))$ due to an exogenous job destruction shock. The total number of unemployed workers at time t is u_t. There is perfect information about the distribution of wage offers $F_t(w)$ and the distribution of employed workers across wages $G_t(w)$; the two cdf are defined over a compact wage support $[\underline{w}, \overline{w}]$, for $\underline{w} < \overline{w}$ positive real values. Let R be the reservation wage in the economy, it is equal to the marginal productivity of labor in the capital sector. Since capital is the numeraire, $R = \varphi$.

1.2.5.2 Timing

Time is discrete. Each period is a sequence of four stages:

0. **Shock**. The uncertainty about unit labor costs realizes. Firms understand the optimal wage and employment level they will perform with during the current period.

1 **Adjustment**. Firms that enter the market compare the current employment level with the optimal employment level and they adjust their workforce. Intra-firm costless renegotiations take place based on the new optimal wage and employment levels. At the end of this stage, either incumbent firms are matched with the desired level of employment or they have open vacancies. Moreover there exist a number of workers who entered the period as employed and are waiting for a reallocation.

2 **Reallocation**. All firms face job to job transitions.

- *Exogenous separation.* A share δ of total workers who entered the period as employed at the end of the period will be unemployed.
- *Search and Matching.* All N workers are randomly searching. With probability $m(\theta_t)/\theta_t$ a firm meets a worker. Each period an employed worker has a probability $1-\delta$ to remain employed up to the next period, but he/she may change employer. During the period, an employed worker receives a wage offer from another employer with probability $m(\theta_t)$, that is the same probability at which an unemployed worker visits the same vacancy. With probability $1-\delta-m(\theta_t)>0$ an employed worker receives a wage offer from his current employer, at the new wage.
- *Job to job transitions.* Employed workers accept wage offers that are better with respect to their current employment status. At the end of the period employed workers move to an other employer with probability $m(\theta_t)\ [1-F_t(w)]$, or he/she remains with the current employer with probability $[1-m(\theta_t)+m(\theta_t)F_t(w)]$.
- *Outflows from unemployment.* Unemployed workers accept all wage offers above or equal to the reservation wage. An unemployed worker becomes employed with probability $m(\theta_t)[1-F_t(R)]$, otherwise with probability $1-m(\theta_t)+m(\theta_t)F_t(R)$ stays unemployed.
- *Inflows to unemployment.* Employed workers who separated and unemployed workers that did not find a match end the period as unemployed.

At the end of this stage all firms have the optimal level of employment according to their unit labor cost. Total outflows from unemployment is given by: $m(\theta_t)[1-F_t(R)]u_t$. Total inflows into unemployment is given by: $\delta\ (N-u_t)$.

3. **Production**. Firms start production when they reach the optimal employment level. Over the period they issue a number of vacancies that is sufficient to keep their workforce at the desired level.

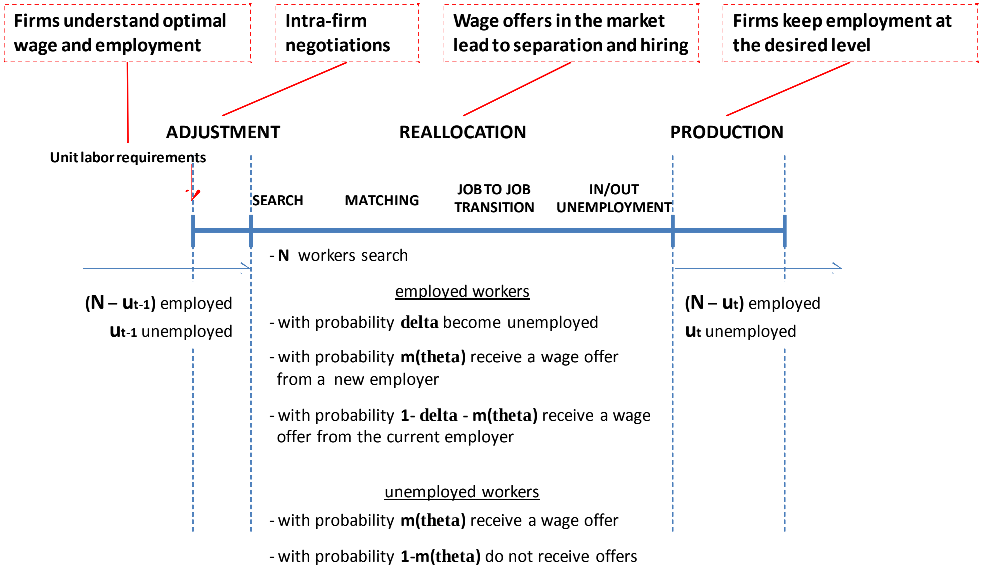

Figure 1.1: Labor market, one period.

Figure 1.1 summarizes the timing of the labor market.

1.2.5.3 Workers

While a worker is employed he/she supplies one unit of labor and receives a wage w_t. After the current period, with probability δ he/she is unemployed. With probability $m(z_{t+1})$ he/she finds a match with an other employer and with probability $1 - \delta - m(z_{t+1}) > 0$ he/she will be matched with the same employer; where we refer to $m(z_{t+1})$ as $m(\theta_{t+1})$ when the state of the economy z_{t+1} is understood. Contracts are not binding therefore the current wage is not an outside option for the worker. He/she compares the value of being employed to the value of unemployment. Since each period unit labor costs are i.i.d. draws across employers, the expected value of employment in the next period given the current information is the same, regardless the productivity in the current match, either the match with the current employer continues or not. The asset value of employment is:

$$W(w_t, z_t) = w_t + \beta E_{z_{t+1}} \left\{ \begin{array}{l} \delta U(z_{t+1}) + \\ +(1-\delta) E_w [\max \{W(w_{t+1}, z_{t+1}), U(z_{t+1})\}] \end{array} \right\} \tag{1.16}$$

where $W(w_t, z_t)$ is the asset value of being employed at a wage w_t when the state of the economy is z_t, $U(\cdot)$ is the asset value of unemployment, $b_t > 0$ is the unemployment benefit at time t, $E_w[\cdot]$ is the expectation operator across wage offers and $E_{z_{t+1}}\{\cdot|z_t\}$ is the expectation operator conditional on the current state of the economy z_t.

Unemployed workers receive a benefit b in the current period. In the next period they have a probability of matching with a firm with open vacancies $m(z_{t+1})$. In this case matches are formed above the reservation wage. Unemployed workers who do not match stay unemployed. The value of unemployment is given by:

$$U(z_t) = b + \beta E_{z_{t+1}} \left\{ \begin{array}{l} (1 - m(z_{t+1})) U(z_{t+1}) + \\ +m(z_{t+1}) E_w [\max \{W(w_{t+1}, z_{t+1}), U(z_{t+1})\}] \end{array} |z_t \right\} \tag{1.17}$$

Define the reservation wage as the wage R such that $W(R, z_t) = U(z_t)$:

$$R = b - \beta E_{z_{t+1}} \{(1 - m(z_{t+1}) - \delta) E_w [\max \{W(w_{t+1}, z_{t+1}) - U(z_{t+1}), 0\}]\} \tag{1.18}$$

The worker's surplus in a match is equal to the difference between the current wage and the reservation wage:

$$W(w_t, z_t) - U(z_t) = w_t - R \tag{1.19}$$

A reservation policy holds. Each point in time, the optimal policy for unemployed workers is:

$$\Upsilon_u(w_t, R) = \begin{cases} \text{"}accept\text{"} & if \quad w_t \geq R \\ \text{"}keep\ searching\text{"} & if \quad w_t < R \end{cases} \tag{1.20}$$

where the equality sign holds as a convention.

In the case of employed workers, notice that the continuation values of employment does not depend on the current wage because of i.i.d. unit labor cost across employers period by period. Therefore a reservation policy holds also for job to job transitions, when the worker who received a wage offer chooses between staying with the same employer or changing employer. The optimal policy for an employed worker at a wage $w_t \geq R_t$ that receives an alternative offer $\widetilde{w}_t$ is simply:

$$\Upsilon_e(w_t, \widetilde{w}_t) = \begin{cases} \text{"}stay\text{"} & if \quad w_t \geq \widetilde{w}_t \\ \text{"}change\text{"} & if \quad w_t < \widetilde{w}_t \end{cases} \tag{1.21}$$

where the equality sign holds as a convention[15].

1.2.5.4 Firms

Firms cannot write binding contracts. The value of filling a vacancy for a firm is equal to the marginal profit per worker in the current period:

$$J_t(w, c) = \frac{p_t(w, c)}{c} - w \tag{1.22}$$

The value of filling a vacancy is the same regardless if the worker is already employed in the firm or not. This is due the assumption that the adjustment cost of employment is fixed. When a firm negotiates with a worker in the market the cost of employment adjustment is sunk, it is not specific to the particular match.

[15]Optimality is due to the fact that any sudden deviation from (1.20) or (1.21) depending on the current employment status, would imply a strictly lower current value and the same continuation value.

1.2.5.5 Employment flows

The measurement of employment flows refers to the value they attain in the production stage and they maintain throughout the rest of the period.

Unemployment. In a given period inflows from unemployment to employment are due to unemployed workers who match with a firm offering at least $w = R_t$, that is $m(\theta_t)[1 - F_t(R_t)]\, u_t$. Outflows from employment to unemployment are $\delta(N - u_t)$. In each point in time we define the equilibrium unemployment level such that flows in and out of employment are equalized:

$$\frac{u_t}{N} = \frac{\delta}{\delta + m(\theta_t)[1 - F_t(R_t)]} \tag{1.23}$$

Wage distribution. Consider the break down of employment flows by wage. In each point in time $m(\theta_t)\, u_t [F_t(w) - F_t(R_t)]$ unemployed workers move into employment accepting an offer at a wage w or lower, $\delta G_t(w)(N - u_t)$ workers who are hired at a wage w or lower separate because of exogenous job destruction, $m(\theta_t)[1 - F_t(w)]\; G_t(w)(N - u_t)$ move to an other job because they receive a better offer. The distribution of effective wages that satisfies the balance in employment flows is:

$$G_t(w) = \frac{\delta}{1 - F_t(R)} \frac{F_t(w) - F_t(R)}{\delta + m(\theta_t)[1 - F_t(w)]} \tag{1.24}$$

where we used the fact that $\frac{u_t}{(N-u_t)} = \frac{\delta}{m(\theta_t)[1-F_t(R_t)]}$ by (3.22).

Separation rate. Firms and workers separate either because of a job destruction shock or because the worker finds a job offer that is better than the worker's current wage. The *separation rate* for a firm that pays a wage w is the sum of the two components:

$$s_t(w) = \delta + m(\theta_t)(1 - F_t(w)) \tag{1.25}$$

Hiring rate. With probability $m(\theta_t)/\theta_t$ a firm meets a worker, the match is formed if the worker accepts the wage offer. With probability $\frac{u_t}{N}$ the worker is unemployed, otherwise he/she is employed at a given wage not lower than the reservation wage. Offering a wage $w \geq R_t$ the firm will hire unemployed workers or employed workers with a current wage that is lower than w. The *hiring rate* per issued vacancy at a given wage w is given by:

$$h_t(w) = \frac{m(\theta_t)}{\theta_t}\left[\frac{u_t}{N} + \left(\frac{N-u_t}{N}\right)G_t(w)\right] \tag{1.26}$$

Employment. When workers search for a job in the consumption sector, on the demand side of the labor market there are $\left(1 + x_t\frac{N^*}{N}\right)E_t$ employer units. For an arbitrarily small $\varepsilon > 0$, the measure of workers employed at a wage lower or equal to w is $[G_t(w) - G_t(w-\varepsilon)](N-u_t)$ and the measure of employer units offering a wage in the same interval is $[F_t(w) - F_t(w-\varepsilon)]\left(1 + x_t\frac{N^*}{N}\right)E_t$. As in [Burdett and Mortensen, 1998], the number of workers who are employed in an employer unit that offers a wage w is given by the following limit:

$$emp_t(w) = \lim_{\varepsilon\to 0}\frac{[G_t(w) - G_t(w-\varepsilon)]}{[F_t(w) - F_t(w-\varepsilon)]}\frac{(N-u_t)}{\left(1 + x_t\frac{N^*}{N}\right)E_t}$$

leading to the employment function:

$$emp_t(w) = \frac{\delta m(\theta_t)}{(\delta + m(\theta_t)[1-F_t(w)])^2}\frac{N}{\left(1 + x_t\frac{N^*}{N}\right)E_t} \tag{1.27}$$

Figure 1.2 shows the employment function at the firm level for a given parameter set and when the wage offer distribution is assumed to be uniform[16].

Vacancies. During the production stage, each employer issues the number of vacancies that is needed in order to keep employment constant. Let v_t be the number of vacancies issued by an employer unit at time t, then the balance in hiring and separations implies $h_t(w)\nu_t = s_t(w)\,emp_t(w)$. Vacancies per employer unit level are given by:

$$v_t = \frac{\theta_t N}{\left(1 + x_t\frac{N^*}{N}\right)E_t} \tag{1.28}$$

[16]The parameter values in the example are $\delta = 0.05$, $m(\theta_t) = 0.30$, $x_t = 0.20$, $N = 100$, $N^* = 20$, $E_t = 100$ and the wage offer distribution is uniform with support $w \in [1, 1.5]$.

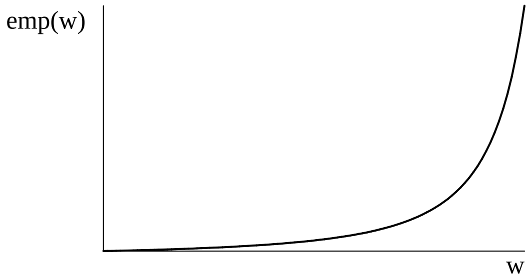

Figure 1.2: Employment function for a uniform distribution of wage offers

The number of vacancies per employer unit during the production stage does not depend on idiosyncratic characteristics. The reason for this result is that a firm paying a relatively high wage has a relatively high employment and hiring success and a relatively low separation rate. The opposite is true for a firm paying a relatively low wage. The two firms have a different optimal level of employment and they open vacancies to restore that employment level given their different hiring and separation rate. The level of open vacancies is a flow variable that acts as a buffer between these firm specific tensions; but the gap the vacancies will cover would be the same, because of random matching.

1.2.5.6 Supply of labor at the firm level

A firm coincides with an employer unit only in the case it serves the domestic market. An exporter is an employer $\left(1+\frac{N^*}{N}\right)$ times bigger the employer unit and it offers the same wage. The labor supply at the firm level is:

$$l_t^s(w) = \begin{cases} emp_t(w) & \text{, if non exporter at time } t \\ \left(1+\frac{N^*}{N}\right) emp_t(w) & \text{, if exporter at time } t \end{cases} \tag{1.29}$$

The labor supply is a positive, continuous, strictly increasing function of the wage, for $F_t'(w) > 0$. The number of vacancies issued by a firm during the production stage is proportional to the employment level:

$$\vartheta_t = \begin{cases} v_t & \text{, if non exporter at time} t \\ \left(1 + \frac{N^*}{N}\right) v_t & \text{, if exporter at time } t \end{cases} \tag{1.30}$$

1.2.5.7 Wage bargaining

Two different bargaining processes occur simultaneously: renegotiation of the firm with current employees and job to job reallocation. Only firms that decided to stay in the market in the current period participate in the labor market. In the bargaining firms choose a wage that is consistent with optimal pricing and ultimately with the entry/exit choice they made.

Workers are homogeneous, they observe each other, then costless renegotiation will rule out the possibility of intra-firm wage dispersion is possible[17]. When a firm matches with a worker the cost of advertising the vacancy has been already paid. Under this scenario the firm will offer one wage in a given period, both to inside workers and outside workers, who can be employed or unemployed[18]. Workers choose according to the policy rules (1.20) and (1.21).

Negotiations between firms and workers follow the extension of [Rubinstein, 1982] discussed in [Stole and Zwiebel, 1996]. Costless alternate bargaining takes place under complete information and non binding contracts. Under this approach the wage determination rule is the (generalized) Nash bargaining solution with threat points the value of a vacant job for the firm (no revenue) and the value of unemployment for the worker

[17] In line with [Mortensen, 2009] "interpersonal comparison of earning of observably identical workers provide a reason for a "non-response" policy. Offer matching induces intra-firm wage dispersion, which can be costly to the extent that equity norms are important. Also dispersion of this form violates anti-discrimination laws when workers are interchangeable within the firm"; as it is our case.

[18] Notice that when a firm match with a worker the cost of advertising the vacancy has been already paid.

(reservation wage). A firm endowed with unit labor cost c at time t offers a wage:

$$w = \arg\max_{w} (w-R)^{\varpi} \left(\frac{p_t(w)}{c} - w \right)^{1-\varpi}$$

where $\varpi \in (0,1)$ is the bargaining power of the workers, and we made explicit that the price is a function of the wage.

It must be noticed that agents bargain over the nominal surplus of the match. In fact, if they were trading over the output of the match (as it is the case in most of the literature) the optimal real wage would be a function of the reservation wage relative to the price of the firm variety and the comparison of real wages would significantly differ from the comparison of nominal wages; that is a primary concern of the present work. This approach is not neutral since the price itself depends on the wage:

$$\varpi \left(\frac{p_t(w)}{c} - w \right) = (1-\varpi) \left(1 - \frac{1}{c} \frac{\partial p}{\partial w} \right) (w-R)$$

Price is linear in the wage by (1.5). The optimal wage is a convex combination of the value of the marginal productivity of labor and the reservation wage:

$$w = \left(\frac{\varpi}{\varpi + \frac{1}{2}(1-\varpi)} \right) \frac{p(w_t)}{c} + \left(\frac{\frac{1}{2}(1-\varpi)}{\varpi + \frac{1}{2}(1-\varpi)} \right) R$$

Imposing the optimal pricing equation (1.5) and solving for $c = c_D$, $w(c_D) = R$ and the general expression for the wage equation simplifies to:

$$w = \varpi \left(R \frac{c_D}{c} \right) + (1-\varpi) R \tag{1.31}$$

Firms with low unit labor costs relative to cutoff c_D pay better wages. Everything else being constant the larger is the reservation wage the higher is the wage. In the reminder of the paper we consider the particular solution of the wage equation that satisfies the Nash axiom of symmetric bargaining:

$$w(c) = \frac{R}{2} \left(1 + \frac{c_D}{c} \right) \tag{1.32}$$

as it is a benchmark of comparison with previous work in our subject. Under this parametrization

Notice that for an hiring firm in the consumption sector it is never optimal to offer a wage equal to the reservation wage or lower. That would be the case if and only if the firm was endowed with a unit labor cost c_D but in that case it would not demand workers since it does not face a positive demand. As a consequence, the minimum wage in the consumption sector is strictly larger than the reservation wage. The service sector employs labor from the subset of unemployed workers only and all workers employed at the reservation wage belong to the service sector.

1.2.6 Entry, exit and export decisions

There is a large (unbounded) number of potential entrants into the domestic market. In order to enter the domestic market firms make an irreversible investment of f_{ed} and they become ready for production in the upcoming period. At that point new entrants and past incumbents learn the unit labor requirement they will be endowed with throughout the period. Given this information, incumbent firms decide to stay in the market or not. In case they stay they pay the fixed cost of production f_p, otherwise they exit and join the population of potential entrants. The first time an incumbent firm exports to a foreign market it faces an irreversible investment of f_{ex} units of capital[19]. The decision to export for the first time is taken with knowledge about the current unit labor cost. Once a firm makes the investment it is able to serve the export market starting

[19] [Das et al., 2007] provide an estimate of the sunk cost of entry in an export market; their dataset consists of plant level data from 32 leather products producers, 40 basic chemicals producers, and 64 knit fabric producers based in Colombia during the period 1981–1991. The average entry costs in the export market range from $430,000 to $412,000 among small producers and from an average of $344,000 to $402,000 for large producers.

from the current period.

Firms decide on the basis of their current profit. Combining (1.9), (3.10) and the wage equation (3.16), profits in the domestic and foreign markets are:

$$\Pi^{dom}(c) = \frac{N}{16}\frac{\alpha}{\gamma}Rc_D\left(\frac{c_D - c}{c_D}\right)^2 - f_p$$

$$\Pi^{exp}(c) = \frac{N^*}{16}\frac{\alpha}{\gamma}Rc_D\left(\frac{c_D - c}{c_D}\right)^2$$

Free entry. Firms make forward looking entry decisions facing uncertainty about their future unit labor requirement. The only source of uncertainty is idiosyncratic to the firms, there is not aggregate uncertainty therefore the average profit across firms is constant over time. Each period new entrants and incumbent firms draw their unit labor requirements from the same distribution over the support $[\underline{c}, \bar{c}]$. Firms that draw a unit labor requirement too high will immediately exit, at a zero value. Let c_{in} be the unit labor requirement below which firms are incumbent and $\Phi(c_{in})$ is therefore the probability that an incumbent continues to be active in the domestic market. The ex ante value of entry is given by:

$$v_e = \sum_{t=s}^{\infty} \Phi(c_{in})^{t-s}\,\overline{\Pi} = \frac{\overline{\Pi}}{1 - \Phi(c_{in})}$$

where $\overline{\Pi}$ is the expected profit across incumbent firms and, for simplicity, firms do not discount future profits. The free entry condition implies:

$$v_e \leq f_{ed} \tag{1.33}$$

it holds with equality when there is positive entry; we restrict the discussion to this scenario.

Exit. Each period an incumbent firm pays a fixed cost f_p. The stream of profit any incumbent expects to gain in an infinite lifetime, not taking into account the current period, is $\frac{\Phi(c_{in})}{1-\Phi(c_{in})}\overline{\Pi}$. An incumbent firm stays open in the domestic market if and only if revenues cover variable costs and the profit is not lower than $-\frac{\Phi(c_{in})}{1-\Phi(c_{in})}\overline{\Pi}$. The cutoff that identifies the marginal incumbent firm satisfies:

$$\frac{1-\Phi\left(c_{in}\right)}{\Phi\left(c_{in}\right)}\left(\pi_{t}\left(c_{in}\right)^{dom}-f_{p}\right)=-\overline{\Pi} \tag{1.34}$$

When a firm chooses the price optimally and offers a wage that satisfies (3.16) then the variable profit is a monotonic decreasing function of the unit labor requirement. The left hand side is monotonically decreasing in c_{in} and it starts above zero at least for more efficient firms. Therefore a value c_{in} that satisfies (1.34) exists and it is unique.

Export. A new exporter pays an initial investment f_{ex} after it knows exactly the unit labor requirement it is endowed with in the current period. The expected lifetime earning from the export market is $v_{e,x} = \sum_{t=s}^{\infty}\Phi\left(c_{in}\right)^{t-s}\overline{\Pi}^{exp} = \frac{\overline{\Pi}^{exp}}{1-\Phi(c_{in})}$. Not taking into account the current period, the future expected profit is $\frac{\Phi(c_{in})}{1-\Phi(c_{in})}\overline{\Pi}^{exp}$. Firms will enter the export market as long as $\frac{\Phi(c_{in})}{1-\Phi(c_{in})}\overline{\Pi}^{exp}+\pi_{t}\left(c\right)^{exp}\geq f_{ex}$. The marginal new exporter makes a profit $\pi_{t}\left(c_{new,x}\right)^{exp} = f_{ex}-\frac{\Phi(c_{in})}{1-\Phi(c_{in})}\overline{\Pi}^{exp}$. If we allow for free entry in the export market then $f_{ex} = \frac{\overline{\Pi}^{exp}}{1-\Phi(c_{in})}$ and the export condition simplifies to: $\pi_{t}\left(c_{new,x}\right)^{exp} = \left[1-\Phi\left(c_{in}\right)\right]f_{ex}$, then:

$$c_{new,x}\left(c_{in}^{(+)},f_{ex}^{(-)}\right)=\left[\pi_{t}^{exp}\right]^{-1}\left(\left[1-\Phi\left(c_{in}\right)\right]f_{ex}\right) \tag{1.35}$$

Entry. The free entry condition implies that the expected profit among incumbent firms is equal to a share of the sunk cost of entry:

$$\overline{\Pi}^{dom}+\frac{\Phi\left(c_{new,x}\right)}{\Phi\left(c_{in}\right)}\overline{\Pi}^{exp}=\left[\frac{1-\Phi\left(c_{in}\right)}{\Phi\left(c_{in}\right)}\right]f_{ed}$$

where $\overline{\Pi}^{dom} = \frac{1}{\Phi(c_{in})}\int_{\underline{c}}^{c_{in}}\left[\pi_{t}\left(x\right)^{dom}-f_{p}\right]d\Phi\left(x\right)$ and $\overline{\Pi}^{exp} = \frac{1}{\Phi(c_{new,x})}\int_{\underline{c}}^{c_{new,x}}\pi_{t}\left(x\right)^{exp}d\Phi\left(x\right)$. Notice that under the assumption that the density function associated to $\Phi\left(\cdot\right)$ is increasing in $[\underline{c},\overline{c}]$, it is possible to rewrite the previous condition as the intersection in the space of profit and unit labor requirement between two conditions. The expected profit in terms of the cutoff of the marginal firm in the market, *marginal profit condition* (MPC), is a monotonic increasing function in c_{in} and the *free entry condition* (FEC) a monotonic decreasing function in c_{in}:

$$\begin{aligned} MPC\overline{\Pi} &= \int_{\underline{c}}^{c_{in}} \pi_t(x)^{dom}\, d\Phi(x) + \int_{\underline{c}}^{c_{new,x}\left(c_{in}^{(+)}, f_{ex}^{(-)}\right)} \pi_t(x)^{exp}\, d\Phi(x) - f_p \\ FEC\overline{\Pi} &= [1-\Phi(c_{in})]\, f_{ed} \end{aligned}$$

Notice that in case of a trade liberalization, modeled as a decrease in the sunk cost of export f_{ex}, then the MPC schedule shifts up, the cutoff c_{in} shifts to the left, the average profit of incumbent firms goes up.

1.2.6.1 Characterization of the free entry condition

The distribution of unit labor requirement is an inverse Pareto truncated in the support $[\underline{c}, \overline{c}]$ with shape parameter $\rho > 2$. The cdf and pdf take the form:

$$\Phi(c) = \frac{c^{\rho} - \underline{c}^{\rho}}{\overline{c}^{\rho} - \underline{c}^{\rho}} \text{ and } \phi(c) = \frac{\rho c^{\rho-1}}{\overline{c}^{\rho} - \underline{c}^{\rho}}$$

where without loss of generality $\overline{c} < c_D$. Under this parametrization the export cutoff is a continuous increasing function of the incumbent cutoff c_{in} and a decreasing function of the sunk cost of export f_{ex}:

$$c_{new,x} = c_D - \left(\frac{\overline{c}^{\rho} - c_{in}^{\rho}}{\overline{c}^{\rho} - \underline{c}^{\rho}} \frac{c_D f_{ex}}{\frac{N^*}{16}\frac{\alpha}{\gamma}R} \right)^{\frac{1}{2}} \tag{1.36}$$

where $f_{ex} < \frac{N^*}{16}\frac{\alpha}{\gamma} c_D R \left(\frac{c_D - \underline{c}}{c_D}\right)^2$ guarantees that $c_{new,x} > \underline{c}$. Autarky is possible, that is there exists a sunk cost of export large enough that $c_{new,x} = \underline{c}$, the unit labor requirement is c_{in}^{aut} such that $\frac{\Theta R}{c_D} NI\left(c_{in}^{aut}\right) - f_p = [1-\Phi(c_{in})]\, f_{ed}$ and $f_{ex}^{aut} = \frac{N^*}{16}\frac{\alpha}{\gamma}\frac{R}{c_D}\left(\frac{\overline{c}^{\rho} - c_{in}^{\rho}}{\overline{c}^{\rho} - \underline{c}^{\rho}}\right)^{-1}(c_D - \underline{c})^2$.

In order to compute the expected profit it is convenient to define the operator $I(\cdot) : [\underline{c}, \overline{c}] \to \mathbb{R}$ that consists of the integral function:

$$\begin{aligned} I(x) &= \int_{\overline{c}}^{x} (c_D - c)^2 c^{\rho-1} dc \\ &= c_D^2 \int_{\underline{c}}^{x} c^{\rho-1} dc - 2c_D \int_{\underline{c}}^{x} c^{\rho} dc + \int_{\underline{c}}^{x} c^{\rho+1} dc \\ &= \frac{c_D^2}{\rho} \left(x^{\rho} - \underline{c}^{\rho}\right) - \frac{2c_D}{\rho+1} \left(x^{\rho+1} - \underline{c}^{\rho+1}\right) + \frac{1}{\rho+2} \left(x^{\rho+2} - \underline{c}^{\rho+2}\right) \end{aligned}$$

The integral function $I(x)$ is continuous and differentiable in the compact support $[\underline{c}, \overline{c}]$, with positive first derivative $I'(x) = x^{\rho-1} (c_D - x)^2 > 0$. Collect the parameters that describe preferences and technology $\Theta = \frac{1}{16} \frac{\alpha}{\gamma} \left(\frac{1}{\overline{c}}\right)^{\rho} \frac{\rho}{1-(\underline{c}/\overline{c})^{\rho}}$. The expected profit among incumbent firms is given by:

$$\overline{\Pi} = \frac{\Theta R}{c_D} \left(N I(c_{in}) + N^* I(c_{new,x})\right) - f_p \tag{1.37}$$

Equation (3.29) characterizes the MPC condition. The free entry condition:

$$\overline{\Pi} = \frac{\overline{c}^{\rho} - c_{in}^{\rho}}{\overline{c}^{\rho} - \underline{c}^{\rho}} f_{ed} \tag{1.38}$$

Provided that the parametrization of the fixed cost structure is not trivial[20], such that $c_{new,x} > \underline{c}$, then there exists an intersection between (3.29) and (3.30) and it is unique.

1.2.7 Service sector: production and accumulation

Firms need knowledge and information flows in order to run the business within a context of regulation compliance, contract enforcement and frictions in the resource markets. In real life the value of this service can be measured as the expenditure due to support functions. In this framework such service is modeled as a durable input that is produced in a service sector, accumulates over time and it is used in the consumption sector as fixed input.

[20] In the figure, the values are $\underline{c} = 0.2$, $\overline{c} = 0.8$, $c_D = 1$, $\rho = 4$, $\frac{\alpha}{\gamma} = 1$, $N = N^* = 100$, $R = 1$, $f_p = 0.1$, $f_{ex} = 0.5$, $f_{ed} = 1$, with a shock that brings the sunk cost of export to $f'_{ex} = 0.25$.

The endowment of service in the economy is subject to a time-to-build lag. As in [Ottaviano, 2011], new units of asset that are produced at time t are available from time $t+1$ on. The supply of service follows an accumulation law given by:

$$S_{t+1} = (1-\delta_s) S_t + A_t \tag{1.39}$$

where $\delta_s \in (0,1)$ is the depreciation rate.

The demand for additional units of service is due to the entry of new firms. Firms that take the entry decision at time t rent f_{ed} units of service. A share $\eta \in (0,1)$ of these units are new and will be available when the firm is ready for production, at the beginning of period $t+1$. Let M_{Et} be the mass of entrants that pay the entry cost at time t. Then the demand of additional units of capital is:

$$A_t = \eta f_{ed} M_{Et} \tag{1.40}$$

The production of units of capital takes place in a competitive sector. Workers are compensated with the value of the marginal productivity of labor. Because of perfect competition, the value of each firm-worker match in the capital sector does not generate profit.

1.3 Equilibrium

For a given mass of employers in the consumption sector E_t a general equilibrium consists of:

(i) *a vector of unit labor requirement cutoffs* $c_{in} > c_{new,x}$

(ii) *a vector of output prices and wages* $\{p_t(c), w_t(c)\}_{c=\underline{c}}^{\overline{c}}$*, consumption good allocations* $\left\{C_t^{dom}(c)\ ,\ C_t^{exp}(c)\right\}_{c=\underline{c}}^{\overline{c}}$ *and employment allocations* $\{l_t(c)\}_{c=\underline{c}}^{\overline{c}}$

(iii) *a wage offer distribution* $F_t(w)$*, a wage distribution* $G_t(w)$ *and a wage support* $[\underline{w}, \overline{w}]$

(iv) *a labor market tightness θ_t, a probability of finding a job $m(\theta_t)$ and an unemployment rate u_t/N*

such that:

a) free entry conditions (3.25)-(**??**) are satisfied

b) the demand and supply of consumption goods and labor clear at the firm level

1.3.1 Wage offer distribution

Entry and exit of firms and labor market adjustment happen simultaneously. Firms draw their unit labor cost, bargain with inside and outside workers. If an equilibrium wage offer distribution does exist it satisfies the clearing in demand and supply of labor at the firm level.

Employment at the firm level is determined by the labor demand (1.7) in the domestic and export market (3.10) and the wage equation (3.16):

$$l_t(w) = \begin{cases} \frac{N}{2}\frac{\alpha}{\gamma}Rc_D\frac{(w-R)}{(2w-R)^2} & \text{, if non exporter} \\ \left(1+\frac{N^*}{N}\right)\left(\frac{N}{2}\frac{\alpha}{\gamma}Rc_D\frac{(w-R)}{(2w-R)^2}\right) & \text{, if exporter} \end{cases} \tag{1.41}$$

Notice that employment goes to zero as $w \to R$. Since a minimum employment level is needed to break even with fixed cost structure, we expect the distribution of wage offers in the consumption sector be defined above a wage $\underline{w} > R$. Employment is increasing in the wage up to a maximum where $w = \frac{3}{2}R$. For larger wages, firms would hire less workers and pay more all their employees, given the same demand and technology. Clearly this outcome cannot be part of an optimal plan by any firm. We expect the wage offer distribution in the consumption sector to be defined in the support $w \in \left[\underline{w}, \frac{3}{2}R\right]$.

A firm that offers a wage w attracts a labor supply given by (3.21). The only candidate correspondence $F_t(w)$ such that demand and supply clear at the firm level is:

$$F_t(w) = 1 - \left(\frac{2\delta}{m(\theta_t) \frac{\alpha}{\gamma} R c_D \left(1 + x_t \frac{N^*}{N}\right) E_t} \frac{(2w - R)^2}{(w - R)} \right)^{\frac{1}{2}} + \frac{\delta}{m(\theta_t)}$$

The candidate function $F_t(w)$ is monotonic increasing in the wage over the entire support $w \in \left[\underline{w}, \frac{3}{2}R\right]$. Imposing the condition $F_t\left(\frac{3}{2}R\right) = 1$, the value of the probability of finding a job, given the number of employers in the consumption sector is given by:

$$m(\theta_t) = \frac{\delta \frac{\alpha}{\gamma} c_D \left(1 + x_t \frac{N^*}{N}\right) E_t}{16} \tag{1.42}$$

The probability of finding a job is monotonically increasing in the number of employers in the market and in the number of exporters. The expression for the cdf of the wage offer distribution in the consumption sector simplifies to:

$$F_t(w) = 1 - \left(\left(\frac{(2w - R)^2}{(w - R)} \frac{2}{R} \right)^{\frac{1}{2}} - 4 \right) \frac{4}{\frac{\alpha}{\gamma} c_D \left(1 + x_t \frac{N^*}{N}\right) E_t}$$

Finally, there exists a wage level $\inf\{w_t\} \in \left(R, \frac{3}{2}R\right]$ s.t. $F_t(\inf\{w_t\}) = 0$. It can be shown that $\inf\{w_t\}$ is increasing in the reservation wage R and it decreases with the number of firms in the consumption sector E_t[21]. Moreover for a given share of offers due to the capital sector, $F_t(R) > 0$ the minimum wage that is actually offered in the consumption sector is larger than $\inf\{w_t\}$, $\underline{w} > \inf\{w_t\}$. The wage offer distribution in the

[21] The wage such that $F_t(\inf\{w_t\}) = 0$ is the lower solution of a quadratic equation:

$$\inf\{w_t\} = \frac{R}{8} \left(\begin{array}{c} \frac{1}{2}\left(4 + \frac{\frac{\alpha}{\gamma} c_D}{4} E_t\right)^2 + 4 \\ -\sqrt{\frac{1}{2}\left(4 + \frac{\frac{\alpha}{\gamma} c_D}{4} E_t\right)^2 \left(\frac{1}{2}\left(4 + \frac{\frac{\alpha}{\gamma} c_D}{4} E_t\right)^2 - 8\right)} \end{array} \right)$$

where the choice of the lower solution delivers is due to the constraint that $\underline{w}_t$ must be bounded for any $E_t > 0$.

entire economy is defined by interval:

$$F_t(w) = \begin{cases} F_t(R) & \text{for } w < \underline{w} \\ 1 - \left(\left(\frac{(2w-R)^2}{(w-R)} \frac{2}{R} \right)^{\frac{1}{2}} - 4 \right) \frac{4}{\frac{\alpha}{\gamma} c_D \left(1 + x_t \frac{N^*}{N}\right) E_t} & \text{for } w \geq \underline{w} \end{cases} \tag{1.43}$$

where $F_t(R) \leq F_t(\underline{w})$.The minimum wage in the consumption sector is a monotonic decreasing function of the cutoff c_{in}:

$$\underline{w} = \frac{R}{2}\left(1 + \frac{c_D}{c_{in}}\right) \tag{1.44}$$

A firm endowed with a given unit labor requirement $c \in [\underline{c}, \overline{c}]$ offers a wage that satisfies the bargaining equilibrium according to the wage equation (3.16). At that levels of wage and unit labor requirement the demand of labor by the firm is determined in the consumption goods market and it is given by (3.11). In the space wage and employment the two conditions are orthogonal straight lines parametric to the value of unit labor requirement $c \in [\underline{c}, \overline{c}]$. The intersection uniquely identifies the point that associates unit labor requirement and employment at the firm level[22].

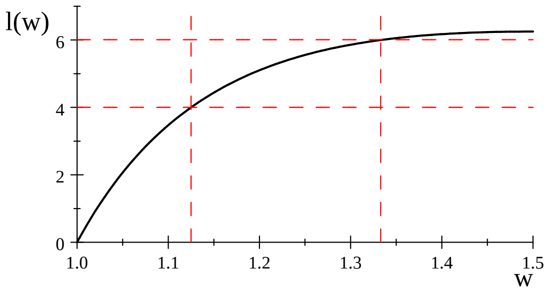

Figure 1.3: Labor demand and supply clearing at the firm level

[22]The parameter values for the supply of labor in the example are $\delta = 0.05$, $m(\theta_t) = 0.30$, $x_t = 0.20$, $N = 100$, $N^* = 20$, $E_t = 100$. The parameter values for the demand of labor in the example are $\frac{\alpha}{\gamma} = 1$, $w(c_D) = 1$, $c_D = 1$. The unit labor requirements are $c = 0.6$, 0.8, the corresponding wages after the bargaining are $w = 1.\overline{3}$, 1.125, the corresponding demands of labor are $l \simeq 6$, 4.

Figure 1.3 illustrates the clearing of labor demand and supply for two firms as an example and the implied equilibrium allocation for the entire population of firms. The wage offer distribution $F_t(w)$ is the unique correspondence $[\underline{w}, \overline{w}] \rightarrow [F_t(R), 1]$ that satisfies all the intersections for $c \in [\underline{c}, \overline{c}] \subset \left[\frac{c_D}{2}, c_D\right]$.

1.3.1.1 Exporter behavior in the labor market

Consider the evidence on exporters that has been documented by [Bernard et al., 2007] and we quoted in the introduction. Two remarks are striking. First, if the increase in employment was due to the wage premium then we should assume an incredibly elastic labor supply. This scenario would be even more not realistic because it should be measured to some extent at the right of the wage support, as exporters pay larger wages. Second, the correlations between wage and employment, and between productivity and employment give a limited information respect to the breakdown between exporters and non exporters. Instead output and employment are strongly linearly correlated, to the extent that when we control for employment we also capture most of the difference between exporters and non exporters in terms of output. This evidence suggests that when a firm becomes exporter hires more workers without a necessary increase of the wage; then, because of this, it produces more output with the new workers that are almost as much as productive than the old ones.

This evidence can hardly be reconciled with frameworks in which the production function shows diminishing returns on labor, the cost of posting a vacancy is match specific and the number of vacancies is optimally chosen to maximize the expected value of hiring. In this context, firm level employment adjusts as a continuous function of the wage. Nevertheless it has to be discontinuous with respect to productivity and the discontinuity occurs at the cutoff productivity level of the marginal exporter; (as a discussion and an exhaustive example of this approach see figure 1 in

[Helpman et al., 2010]).

The framework developed in this work captures this evidence. The demand supply equilibrium in the consumption good market requires that an exporter employs $(1 + N^*/N)$ times workers than in the case it only served the domestic economy, by (3.10). Still the value of the match with a worker does not change with the exporter status, therefore an exporter firm will not offer a higher wage because it is an exporter. The exporter status and the relative high wage are correlated as a result of a relatively high productivity but one does not imply the other. The only channel through which an exporter can match its demand for labor is behaving as an employer $(1 + N^*/N)$ times bigger in terms of employment, with respect to the case he/she was serving the domestic market only.

The segment of the labor supply curve that is relevant for non exporters shifts down when there are exporters in the economy and the opposite is true for the labor supply of exporters.

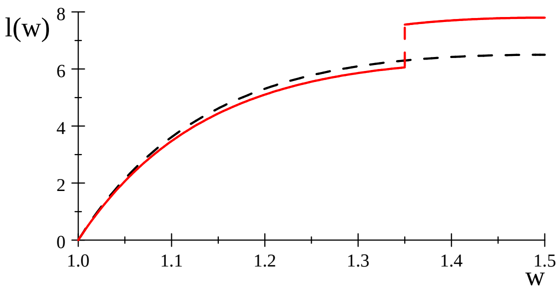

Figure 1.4: Employment at the firm level, in closed (dash) and open economy (solid)

Figure 1.4 illustrates this argument as it describes the firm labor supply function in open economy (bold line) with respect to the closed economy equilibrium, (dash line), for a possible new exporter. The jump occurs at the wage of the marginal new exporter and the labor supply function is right continuous as the cdf $F_t(w)$ does.

1.3.2 Wage distribution

Combining the expression for the wage distribution (3.23) and the probability of matching (**??**). We obtain a closed form equation for the cdf of the wage distribution given the mass of employers:

$$G_t(w) = \left(\frac{N}{N-u_t}\right) 4 \left(\frac{(w-R)}{(2w-R)^2}\frac{R}{2}\right)^{\frac{1}{2}} - \frac{u_t}{N-u_t} \tag{1.45}$$

for $w \in \left[\underline{w}, \frac{3}{2}R\right]$. The density of the wage distribution follows by right differentiation of (3.34) in $\left[\underline{w}, \frac{3}{2}R\right]$:

$$g_t(w) = \frac{\sqrt{2}\sqrt{R}\,(3R-2w)}{(2w-R)^2 (w-R)^{\frac{1}{2}}} \left(\frac{N}{N-u_t}\right) \tag{1.46}$$

Figures (5) show the shape of the cdg and the density of the wage distribution. The two functions are parametric to the value of unemployment rate. An increase in the unemployment rate tilts down the cdf whereas the density function tilts up. Notice that $G_t(\underline{w}) > 0$ because of employment in the service sector.

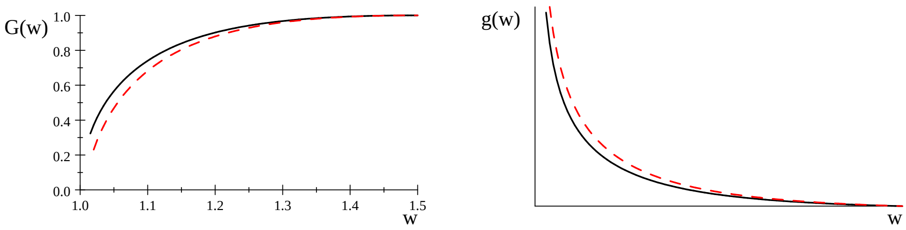

Figure 1.5: Cumulative density function of the wage distribution in the consumption sector (left), and density of the wage distribution (right)

The first moment of the wage distribution over employed workers in the consumption sector is increasing in the minimum wage $\underline{w}$:

$$\mu_t^{cons} = \frac{1}{1 - G_t(\underline{w})} \int_{\underline{w}}^{\frac{3}{2}R} w g_t(w)\, dw \tag{1.47}$$

The Gini coefficient on the wage distribution of employed workers in the consumption sector is given by:

$$gini_t^{cons} = 1 - \frac{1}{\mu_t^{cons}} \int_{\underline{w}}^{\frac{3}{2}R} (1 - G_t(w))^2\, dw \tag{1.48}$$

The Leibniz rule and the monotonicity of the average wage are sufficient to show that (1.48) is a monotonic increasing function of the minimum wage $\underline{w}$. Computing the two measures over the entire population of workers we obtain:

$$\mu_t = RG_t(\underline{w}) + \mu_t^{cons}(1 - G_t(\underline{w})) \tag{1.49}$$

$$gini_t = 1 - \frac{1}{\mu_t}\left[(1 - G_t(\underline{w}))^2 \frac{1}{2}R + \int_{\underline{w}}^{\frac{3}{2}R} (1 - G_t(w))^2\, dw\right] \tag{1.50}$$

The change in average wage and wage inequality over employed workers due to an increase in $\underline{w}$ is necessarily of smaller magnitude than the change in the subset of workers employed in the consumption sector only. In principle, if we allow for a very large reallocation of workers across sectors then the effect of a change in the minimum wage $\underline{w}$ on overall average wage and wage inequality can be offset or even reversed.

1.3.3 Short run equilibrium

A short run equilibrium consists of objects (i)-(iv) of a general equilibrium and in addition it requires:

(v) *a pair of laws of motion for the predetermined state variables* (S_{t+1}, E_{t+1}), *and for the measures of new entrants* M_{Et} *and exporters* X_t

such that conditions (a), (b) are satisfied and:

(c) the service market and the labor market clear at the aggregate level

1.3.3.1 Selection

Let M_{Et} be the mass of firms that decide at time t to pay the sunk cost and enter the market in the next period. Out of the entire population of current incumbents E_t and new entrants M_{Et} only a share $\Phi\left(c_{in}\right)$ decides to be incumbent in the next period:

$$E_{t+1} = \Phi\left(c_{in}\right)\left(E_t + M_{Et}\right) \tag{1.51}$$

Firms that were exporters in the previous period, X_{t-1} and in the current period stay in the market continue to export. We refer to this subgroup as *old* exporters: $X_t^{old} = \Phi\left(c_{in}\right) X_{t-1}$. Firms that were not exporters in the previous period and in the current period are endowed with a unit labor cost $c \leq c_{new,x}$ are *new* exporters: $X_t^{new} = \Phi\left(c_{new,x}\right)$ $\left(M_{Et-1} + E_{t-1} - X_{t-1}\right)$. The total mass of exporters is the sum of the two components:

$$X_t = \Phi\left(c_{new,x}\right) E_t + \left[\Phi\left(c_{in}\right) - \Phi\left(c_{new,x}\right)\right] X_{t-1} \tag{1.52}$$

1.3.3.2 Service market clearing

The total demand of services at time t is given by: $S_t = f_p E_t + f_{ex} X_t^{new} + \left(1-\eta\right) f_{ed} M_{Et}$. The mass of entrants follows by market clearing in the service sector:

$$M_{Et} = \left[\frac{1}{\left(1-\eta\right) f_{ed}}\right] S_t - \left[\frac{f_p + f_{ex}\frac{\Phi\left(c_{new,x}\right)}{\Phi\left(c_{in}\right)}}{\left(1-\eta\right) f_{ed}}\right] E_t + \left[\frac{f_{ex}\Phi\left(c_{new,x}\right)}{\left(1-\eta\right) f_{ed}}\right] X_{t-1} \tag{1.53}$$

The interpretation of equation (3.42) will be crucial for our results. A large stock service allows the entry of new firms. But the allocation of service to new entrants is challenged by the allocation of service to current incumbent firms. A large population of incumbent firms, and out of those a large number of exporters, decrease the stock of service available for new entrants.

Production of new units of capital is driven uniquely by the entry of new firms: $A_t = \eta f_{ed} M_{Et}$. This condition together with the accumulation law of service stock (3.36) characterizes the law of motion for the stock of service in the economy:

$$S_{t+1} = (1 - \delta_s) S_t + \eta f_{ed} M_{Et} \tag{1.54}$$

1.3.3.3 Dynamic system

Combining (3.42) with (3.40) and (1.54), the dynamics of the economy is described by the following system of linear difference equations:

$$\begin{bmatrix} E_{t+1} \\ S_{t+1} \\ X_t \end{bmatrix} = \begin{bmatrix} k_{EE} & k_{ES} & k_{EX} \\ k_{SE} & k_{SS} & k_{SX} \\ \Phi(c_{new,x}) & 0 & \Phi(c_{in}) - \Phi(c_{new,x}) \end{bmatrix} \begin{bmatrix} E_t \\ S_t \\ X_{t-1} \end{bmatrix} \tag{1.55}$$

where the coefficients are:

$$k_{EE} = \Phi(c_{in}) \left(1 - \frac{f_p + \frac{\Phi(c_{new,x})}{\Phi(c_{in})} f_{ex}}{(1-\eta) f_{ed}} \right) < 1 \qquad k_{SE} = \frac{\eta}{1-\eta} \left(\frac{\Phi(c_{new,x})}{\Phi(c_{in})} f \right) > 0$$
$$k_{ES} = \frac{\Phi(c_{in})}{(1-\eta) f_{ed}} > 0 \qquad k_{SS} = \frac{1}{1-\eta} - \delta_s > 0$$
$$k_{EX} = \frac{\Phi(c_{in}) \Phi(c_{new,x})}{(1-\eta)} \frac{f_{ex}}{f_{ed}} > 0 \qquad k_{SX} = \frac{\eta}{1-\eta} f_{ex} \Phi(c_{new,x}) > 0$$

Eigenvalues are real and positive. The system exhibits saddle path stability, where the unstable eigenvalue is associated to the mass of employers in the market.

1.3.3.4 Labor market clearing

Because of perfect competition, the mass of firms in the service sector is undetermined. Nevertheless, employment in the service sector must be equal to the share of total employment that is not allocated to the consumption sector. Labor market clearing at the aggregate level implies: $E_t^{serv} l_t^{serv} = G_t(\underline{w})(N - u_t)$. By imposing (3.22) and (**??**) we obtain the share of total workforce that is employed in the service sector:

$$\frac{G_t(\underline{w})(N-u_t)}{N}=4\left(\frac{(w-R)}{(2w-R)^2}\frac{R}{2}\right)^{\frac{1}{2}}-\frac{u_t}{N} \tag{1.56}$$

The share of unemployed workers is given by (3.22) and (3.13):

$$\frac{u_t}{N}=\frac{16}{16+\frac{\alpha}{\gamma}c_D\left[1-F_t(R)\right]\left(1+x_t\frac{N^*}{N}\right)E_t} \tag{1.57}$$

The share of total workforce that is employed in the consumption sector results from labor market clearing:

$$\frac{(1-G_t(\underline{w}))(N-u_t)}{N}=1-4\left(\frac{(\underline{w}-R)}{(2\underline{w}-R)^2}\frac{R}{2}\right)^{\frac{1}{2}} \tag{1.58}$$

Notice that employment in the consumption sector is decreasing in $\underline{w}$ and it does not depend on the number of employers. Instead the share of unemployment and employees in the capital sector adjusts as the number of employers and exporters change.

The supply and demand of labor in the service sector clear each point in time:

$$\frac{16N}{16+\frac{\alpha}{\gamma}c_D\left[1-F_t(R)\right]\left(1+x_t\frac{N^*}{N}\right)E_t}=4\left(\frac{(2\underline{w}-R)^2}{(\underline{w}-R)}\frac{2}{R}\right)^{-\frac{1}{2}}N-\frac{\eta f_{ed}}{\varphi}M_{Et}$$

Notice that $4\left(\frac{(2\underline{w}-R)^2}{(\underline{w}-R)}\frac{2}{R}\right)^{-\frac{1}{2}}=\frac{16}{16+\frac{\alpha}{\gamma}c_D\left[1-F(\underline{w})\right]\left(1+x_t\frac{N^*}{N}\right)E_t}$. Then $F_t(R)=F(\underline{w})$ if and only if $\eta M_{Et}=0$, otherwise $F_t(R)<F(\underline{w})$. In an equilibrium with positive entry the wage offer distribution is discontinuous to the left of $\underline{w}$. For a given predetermined value E_t and an equilibrium mass of new entrants $M_{Et}>0$ and exporters $X_t=x_tE_t$ there exists one possible value of $F_t(R)$ such that (**??**) is satisfied. The left hand side of (**??**) is increasing in $F_t(R)$ and it shifts down with an increase in X_t. The right hand side is constant in $F_t(R)$, it shifts up with an increase in $F_t(\underline{w})$ and down with an increase in the mass of new entrants M_{Et}.

$$1 - F_t(R) = \frac{16\left(1 - 4\left(\frac{(2\underline{w}-R)^2}{(\underline{w}-R)}\frac{2}{R}\right)^{-\frac{1}{2}} + \frac{\eta f_{ed}}{\varphi}\frac{M_{Et}}{N}\right)}{\frac{\alpha}{\gamma}c_D\left(1 + x_t\frac{N^*}{N}\right)E_t} \tag{1.59}$$

1.3.4 Deterministic steady state

As the mass of employers evolves over time two forces arise. First, on the job search drives the reallocation of workers across firms in the consumption sector. Second, the demand of labor in the capital sector follows entry and exit of firms and it determines flows of workers moving between employment in the capital sector and unemployment.

The labor market clearing condition (**??**) shows that the share of employment in the consumption sector over total workforce does not change with the mass of employers in the sector, E_t. An adjustment in the minimum wage in the consumption sector leads to an immediate adjustment in total employment in the sector, through selection. From the first period following the shock the dynamics is driven by the reallocation of workers from the population of unemployed and employment in the service sector. This process continues as long as workers have an incentive to look for a job in the service sector.

1.3.4.1 Harris-Todaro long run equilibrium

Workers are risk neutral, they have complete information and match randomly with employers. Under this context, the [Harris and Todaro, 1970] condition must hold in the long run. The movement of workers searching for a job leads to a steady state in which the expected earning of being employed in the two sectors is the same:

$$E_T^{serv} l_T^{serv} = \left(1 - 4\left(\frac{(2\underline{w} - R)^2}{(\underline{w} - R)}\frac{2}{R}\right)^{-\frac{1}{2}}\right)\frac{\mu_T^{cons}}{R} \tag{1.60}$$

By substitution of (1.47) and (3.34) is it possible to show that employment in the service sector is a decreasing function of the minimum wage in the consumption sector $\underline{w}$.

Given employment in the service sector, the mass of new entrants in steady state is determined through (1.40),

$$M_{ET} = \frac{\varphi E_T^{serv} l_T^{serv}}{\eta f_{ed}} \tag{1.61}$$

and the steady state stock of service, the mass of employers in the consumption sector, and the mass of exporters are:

$$S_T = \varphi \frac{E_T^{serv} l_T^{serv}}{\delta_s} \tag{1.62}$$

$$E_T = \left(\frac{\Phi\left(c_{in}\right)}{1 - \Phi\left(c_{in}\right)} \right) M_{ET} \tag{1.63}$$

$$X_T = \frac{\Phi\left(c_{new,x}\right)}{1 - \left(\Phi\left(c_{in}\right) - \Phi\left(c_{new,x}\right)\right)} E_T \tag{1.64}$$

The vector $\{E_T, S_T, X_T\}$ is the deterministic steady state of the dynamic system that consists of the three linear difference equations (3.40), (3.41), (1.54) once we substitute for (3.42)[23].

1.4 Consequences of an export shock

We define an export shock as an exogenous fall in the sunk cost of export f_{ex} everything else being constant. This section describes the effect of an unanticipated export shock.

Efficiency. A fall in the fixed cost of export allows more firms to become exporters, the cutoff $c_{new,x}$ shifts to the right everything else being

[23] There exists a necessary structural relationship between the depreciation rate and the rate of use of new units of service in the entry process: $\frac{\eta(1+\delta_s)}{\delta_s} = \left(\left(\frac{f_p + f_{ex}\Phi(c_{new,x})/\Phi(c_{in})}{f_{ed}} \right) - \left(\frac{\Phi(c_{new,x})^2}{1-(\Phi(c_{in})-\Phi(c_{new,x}))} \frac{f_{ex}}{f_{ed}} \right) \right) \left(\frac{\Phi(c_{in})}{1-\Phi(c_{in})} \right)$. This condition is immediately reached once the steady state values of $\Phi\left(c_{in}\right)$ and $\Phi\left(c_{new,x}\right)$ are determined.

constant. In the space defined by the expected profit as a function of the cutoff of incumbent firms, conditions (3.25) and (**??**) identify the new equilibrium value of the cutoff:

Proposition (1). *A fall in the sunk cost of export* f_{ex} *determines a more severe selection, through a lower unit labor requirement* c_{in}.

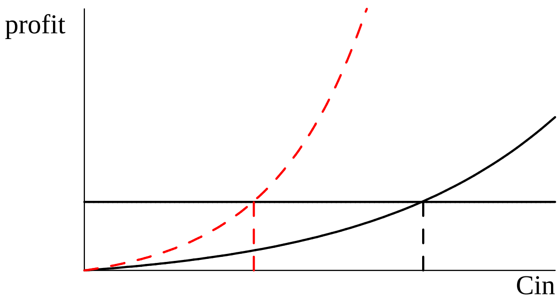

Figure 1.6: A fall in the sunk cost of export increases efficiency

Firm turnover and exporter share. A fall in the unit labor requirement c_{in} implies a lower share of incumbent firms that stay in the market the following period, by (1.34), when (3.25) and (**??**) are satisfied. As a consequence a larger share of incumbent firms exports for the first time.

Proposition (2). *Following the fall in the fixed cost of export we observe a higher turnover of firms and a larger share of firms are new exporters.*

Figure 1.7 shows the change in the population of firms. After the export shock, the cutoff below which firms become exporters shifts to the right, the cutoff above which firms exit shifts to the left.

Wages and prices. The equilibrium condition for the minimum wage in the consumption sector (**??**) clearly implies that a lower cutoff c_{in} leads to an increase of the minimum wage in the consumption sector. The maximum wage does not depend on c_{in}, as well as the wage in the consumption

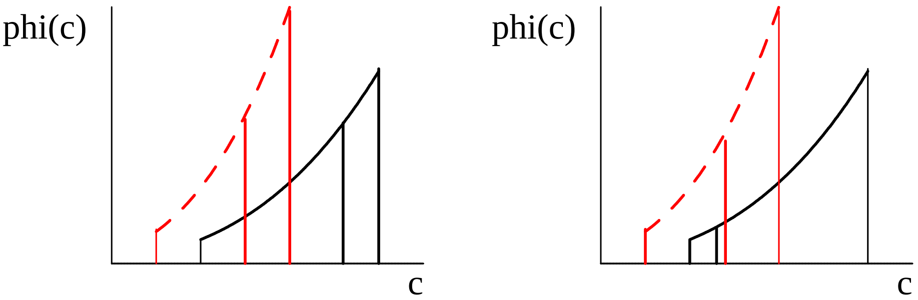

Figure 1.7: A fall in the sunk cost of export increases firm turnover and exporters share

sector. Combining the price equation (1.5) and the wage equation (3.16), the gain in efficiency due to the decrease in c_{in} is responsible for an increase in the real wage, both in the consumption sector and in the capital sector.

Proposition (3). *After the decrease in the fixed cost of export, prices fall, the minimum wage in the consumption sector increases while the maximum wage does not change.*

Proposition (3) establishes that there are gains from trade in real terms both for employed workers in the consumption sector and in the service sector. For employed workers in the consumption sector the gain is also in nominal terms. Nevertheless there are workers who become unemployed because the export shock increased selection of incumbent firms. The next section discusses the three effects that are responsible for a clear understanding of the impact of export shock on labor market variables.

1.4.1 Selection, job destruction, job creation

Three effects are crucial to assess the change unemployment rate and wage inequality following an export shock. Proposition (1) refers to a *selection* *effect*, a decrease in the cutoff c_{in}. From the point of view of the labor market, proposition (2) identifies a *job destruction effect*, as the increase in firm turnover. At the same time, the larger proportion of employers that become exporters and the demand of service to finance new entrants are responsible for a *job creation effect*.

In order to discuss the dynamics of model variables after a fall in the sunk cost of export we use a parametrization of the fixed cost structure that is consistent with the structural estimation in [Coşar et al., 2010]. The depreciation rate δ_s and the relative size of the export market N^*/N are chosen such that during the experiment the unemployment rate varies in the range $(4.5\%, 6\%)$ and the share of exporters varies between $14\%, 20\%$; as these ranges are common to the findings of the empirical literature we quoted in the introduction.

Figure 1.8 shows the dynamics of the employers in the consumption sector, stock of service and exporters, $\{E_t, S_t, X_t\}$, after an unanticipated fall of 5% in the fixed cost of export f_{ex}. The first point in the picture is the initial steady state value, the time length is 24 periods, as we think a period as a quarter.

At the impact, forward looking decisions determines both a decrease in the number of employers (due to selection) and an increase in the number of exporters. In so doing, we observe the effect of propositions (1) and (2). The stock of service decreases for two reasons. First, there is a reallocation of workers to the consumption sector. Second, the increase in firm turnover at the impact decreases the demand for service.

After the shock, the larger turnover induced by the more severe selection is responsible for the decrease in the number of employers. The number of exporters increases because of the persistence that characterizes the exporter status. The probability that a firm becomes a new exporter is the same each period after the shock, but those firms that were exporting in the previous period continue to export in the following periods. The

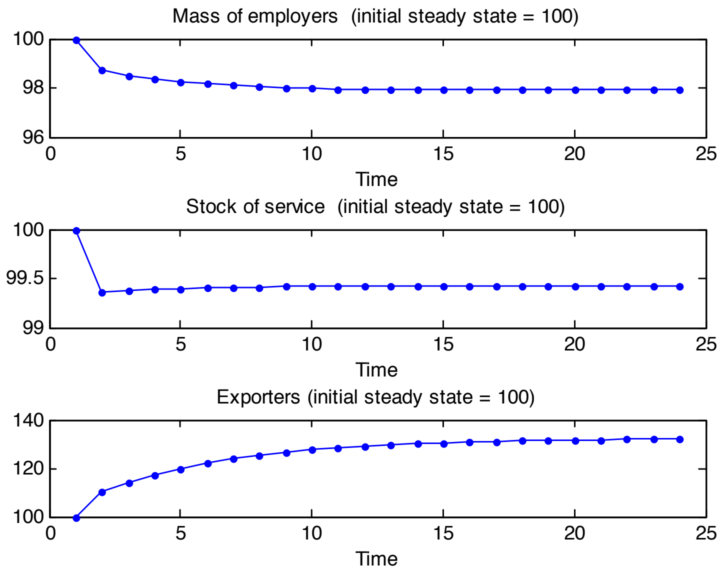

Figure 1.8: Dynamics

additional demand of service due to new entrants drives the production of service in the economy after the shock to the new steady state, given the allocation of workers in the two sector at has been implied by the labor market equilibrium.

Figure 1.9 describes the dynamics of the unemployment rate u_t/N and the Gini index on the wage distribution $G_t(w)$, across all workers employed in both sectors.

At the impact, the selection effect is the only mechanism driving the dynamics. The unanticipated cut in the sunk cost of export forces the exit of firms. The minimum and average wage in the consumption sector reach the new, higher, steady state level. There is worker reallocation within the consumption sector, from low productive firms that exit the market to new exporters. Employment in the consumption sector reaches immediately the new steady state level, lower than in the initial steady state.

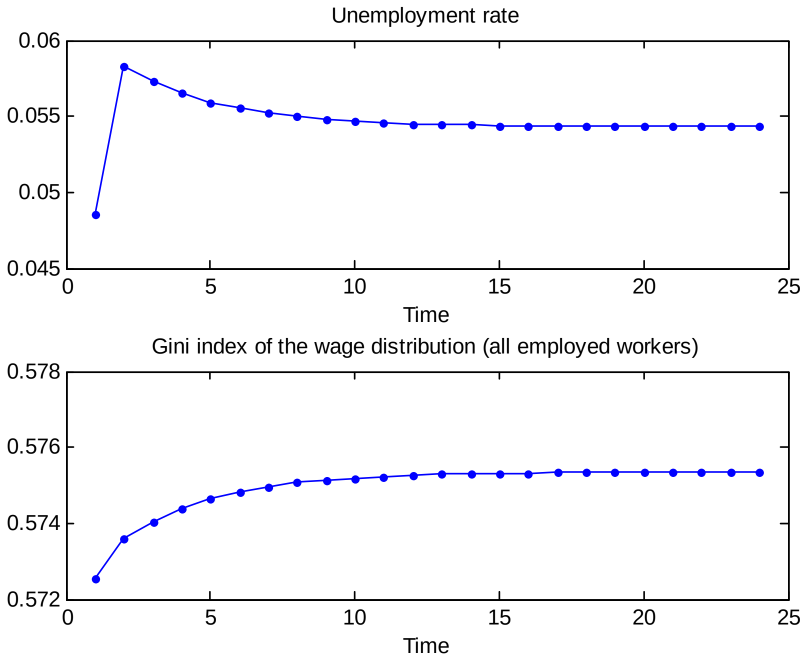

Figure 1.9: Unemployment rate and wage inequality dynamics

Employment in the service sector does not satisfies the long run equilibrium (1.60), at the impact. The dynamics of employment in the service sector is the result of two effects. First, immediately with the export shock and the change in the minimum wage in the consumption sector, there is a disproportionate decrease of the share of wage offers due to the service sector that are accepted and there is more endogenous separation in the service sector. Second, from the first period after the shock, the entry of new firms occurs at a larger turnover. The demand of new units of service increases because the number of new entrants per period increases. Combining the two effects we know that employment in the service sector can only reach the new steady state "from below". That means that the decrease in employment in the service sector is larger than the difference in the two steady state levels. As a consequence the unemployment rate overshoots at the impact and then it converges to the new steady state following the demand of labor in the service sector that adjust with the entry of new firms.

The change of wage inequality results from the reallocation of workers within the consumption sector and from unemployment to employment in the service sector. Wage inequality increases in the consumption sector because of the reallocation of workers from many low productive firms to few and large new exporters. Workers employed in the service sector are all paid the same wage and this homogeneity decreases the overall level of wage inequality, but it does not reverse the rate of change. In order to understand the path of wage inequality over time, consider that the persist ency in the export status is a driver of wage inequality per se. In facts as the share of exporters increases, ceteris paribus the wage inequality in the consumption sector continues to increase and it leads overall wage inequality in the economy.

The reallocation of workers from unemployment to the service sector, after the export shock plays a role in explaining the concavity of the qualitative relationship between wage inequality and share of exporters. In fact as the economy reaches the new steady state the share of employed workers in the service sector increases with respect to immediately after the export shock. In so doing, the reallocation of workers to the service sector smooths the increase in inequality due to the increase in the share of exporters.

1.5 Conclusion

This paper develops a two-sector two-factor dynamic general equilibrium model with *on the job search* and *firm entry/exit*. Under this framework, we analyze the effect of a reduction of the sunk cost of export on the

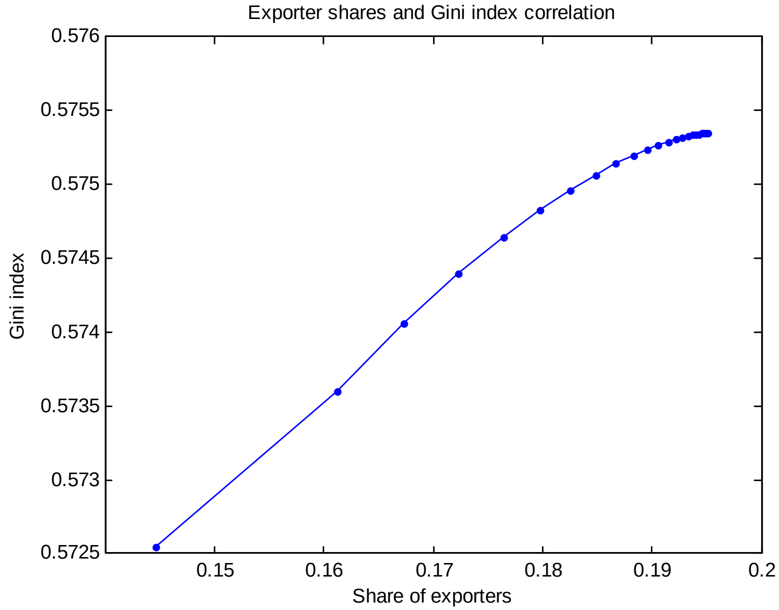

Figure 1.10: Correlation between inequality and exporter share

dynamics of unemployment and wage inequality.

An unanticipated permanent decrease of the sunk cost of export determines an increase of firm turnover and in the share of incumbent firms that become exporters. In the labor market, workers reallocate from low productive firms to more productive firms that became new exporters. As a result, in the (tradable) consumption sector, minimum wage, average wage and employment per firm increase. Nevertheless total employment in the consumption sector decreases, because of the selection of less efficient firms out of the market. The purpose of the service sector is to produce inputs for the firms in the consumption sector. Therefore, a decrease in employment and number of employers in the consumption sector also implies a decrease of employment in the service sector. The comparison of the two steady states before and after a fall in the sunk cost of export leads to the conclusion that unemployment and wage inequality rise after the export shock.

Nevertheless the transitional dynamics to this new steady state contains richer policy implications. Following the fall in the sunk cost of export the unemployment rate overshoots, that is, it reaches a higher level than the long run equilibrium. The level of employment in the consumption sector adjust immediately, but employment in the service sector is too low immediately after the export shock. The demand of service due to the entry of new firms leads the adjustment of unemployment over time to the new steady state level. Wage inequality smoothly adjusts to a new higher level. The driver for the increase in inequality is the reallocation of workers across firms to new exporters. Instead the reallocation of workers from unemployment to employment in the service sector decreases the rate the speed of the rise in inequality. As a result the model predict a positive correlation between the share of exporters and wage inequality but with a concave shape.

Chapter 2

Exchangeability and Pòlya Urn to Determine Distributions in Macroeconomics

Abstract

This paper proposes a simple framework to determine the distribution of equilibrium allocations and income across agents in a real business cycle model. Agents are of two types depending on the human factor of production they own and production takes places in separated markets. We model the population as an exchangeable sequence of agents' types generated according to a Pòlya urn process. This approach allows us to (i) theoretically characterize the matching between agents of different types and (ii) endogenously determine the distribution of agents' types across productive locations. In each market, production is subject to the distri-

bution of types and income dispersion across agents arises in equilibrium.

Despite its simplicity, our framework predicts a theoretical density of income distribution that is consistent with the findings of the empirical literature. Perhaps surprisingly, we obtain this result regardless of ad hoc calibrations, but directly because of the link between the composition of the population and economic activity.

2.1 Introduction

The last three decades of macroeconomic literature have been characterized by an increasing interest in the effect of heterogeneity on macroeconomic models. Although the literature provided many points of view about causes and implications of heterogeneity, only recently we can see a consensus on the need to take heterogeneity into account for macroeconomic modeling.

In his Nobel Lecture, Heckman [Heckman, 2001] points out that: "Problems that appear to be unimportant when examining aggregate averages become central in analyzing micro data". In the excellent survey on macroeconomic literature with heterogeneous agents [Heathcote et al., 2009] we find evidence of this critique: "Macroeconomics is expanding from the study of how average values for the inputs (capital and labor) and outputs (consumption) of production are determined in equilibrium to the study of how the entire distribution of these variables across households is determined".

Recent frameworks have been developed to address this research question. The literature on *social economics*, the *agent based economics* approach and the *combinatorial approach to economics* are examples of recent frameworks in economics that rethink the economic equilibrium as outcome of agent interactions. These solutions can hardly be reconciled with the mainstream macroeconomic literature on heterogeneous households. Moreover this literature, after the contributions of [Huggett, 1993] and [Aiyagari, 1994], developed mostly in the direction of the quantitative performances of existing frameworks than on new theoretical approaches. We contribute to this literature with a theoretical approach that allows us to analytically determine income distributions in standard DSGE frameworks.

The idea of the paper consists on solving an heterogeneous agent model in two stages. First, we split the problem into a collection of representative agent models. Second, we theoretically characterize a law to aggregate the single equilibrium allocations. In our approach we follow an inspiration we found in [Kirman, 2006]: "instead of trying to modify the assumptions on individuals we might want to make assumptions on the distribution of their characteristics". What we do is precisely to give a structure to the macroeconomic problem: (i) we identify a variable that is responsible for heterogeneity across markets, (ii) we organize the economy as a collection of *within* homogeneous *between* heterogeneous markets, (iii) we solve a representative agent equilibrium in each market, (iv) we integrate over the distribution of the variable driving heterogeneity across markets. The outcomes of our model are distributions of macroeconomic variables, in addition to the standard analysis of the mainstream macroeconomic literature.

Our approach is alternative to a number of recent frameworks. The literature on *social interactions approach to economics* (for a review see [Durlauf and Ioannides, 2010] and [Durlauf, 2012]) is based on the assumption that individual choices are influenced by characteristics of other individuals, creating a link among agents and a loop from the past choices to present and future choices. A further alternative approach is the *agent based economics* literature (as an example we refer to [Gaffeo et al., 2008]). This class of models begins from the failure of perfect aggregation, in the sense of Gorman. Under this framework, economics is thought as a complex system, where non linearity of functional relationships and interaction of economic units give rise to the aggregation problem. Functional forms of macro variables arises from laws at the micro level, interaction of agents and adaptive behaviors. A third framework which we have been inspired by is the *combinatorial approach to economics*, extensively discussed in [Aoki, 1998] and [Aoki, 2007]. Following Aoki, the aggregation problem is solved from a stochastic point of view by modeling the economic system as a hierarchically organized population of agents. Under this perspective, agents' interaction becomes tractable as the outcome of the joint distribution of economic characteristics among groups[1].

In the present work agents are of two types according to the human factor of production they own: labor for *workers*, managerial skills for *managers*. Agents are located in separated cities and in each city competitive firms choose the matching technology in the family of constant return to scale technologies and such that they maximize the supply of human service subject to the composition of the local population. To do so, we assume that the population in each city can be modeled as a sequence of exchangeable binary random variables with outcome $\{worker, manager\}$ generated by a Pòlya urn. We finally apply the framework and results that has been developed in [Muliere et al., 2005] to determine the distribution of agent types across markets.

This is the methodological contribution of the paper and it represents a novelty in the macroeconomic analysis, at least to the best of our knowledge. Nevertheless, we argue that the assumption of exchangeability is already common in the matching literature but often it is not explicitly stated. As an example, consider the matching between agents of different

[1] This approach provides a characterization of distribution properties in agents' choice variables (consumption, output, ...) that is sufficient to obtain the patterns of typical economic business cycles and time series with long run memory. We do not support this "sufficiency conjecture", we believe in the economic thought telling us the mechanism behind micro and macro relationships.

types: the outcome of a matching process is (often) a function of the number of agents of different types, whereas the order in which agents match is irrelevant. In this case agents are exchangeable across types, since the probability associated with a given permutation of the sequence of matches does not depend on the order in which types enter the sequence. Exchangeability is a very common property in Bayesian statistics and we choose this approach because it contains a broad class of experiments we are used to deal with in economics[2]. From the methodological point of view, the de Finetti Representation theorem applies to a sequence of exchangeable random variables and it guarantees the existence and uniqueness of a prior distribution for the share of workers over the total population across cities.

A well-known result in Bayesian statistics shows that Pòlya urns generate sequences of exchangeable random variables. Although urns are not the only data generating process with this property, we argue that choosing a Pòlya urn we want to capture the (natural) idea that the composition of the population across productive location and the adoption of a certain technology are two processes that mutually influenced each other over time. In fact, if this is the case, then in our framework: (*i*) the owners of the factor that is most intensively used in production should account for a larger share of the population; (*ii*) the distribution of the share of workers over total population across cities must be unimodal and the mode should be in a neighborhood of that particular factor intensity that maximizes production. Choosing a Pòlya urn as data generating process for the population of agent we obtain a theoretical confirm of this natural idea[3].

[2]Notice that i.i.d. random variables are exchangeable, and any weighted average of i.i.d. sequences of random variables is exchangeable; moreover, exchangeable sequences are stationary. These example shows that most of the current literature on heterogeneous agents that makes use of idiosyncratic shocks is actually working with exchangeable sequences. For a review of exchangeability in economics we suggest McCall (1988).

[3]Using the data from Autor, Katz and Kearney (2008) we also find evidence that supports the shape of the density function we theoretically derive in the paper.

Thanks to the adoption of exchangeability and Pòlya urns we can show that the share of workers over total population across cities is a Beta distribution. Given this result we can micro-found the optimal matching technology and the supply of human service in each productive location. Different productive locations will adopt different human service over physical capital ratio and because of this channel we observe a dispersion in factors prices across cities. The theoretical density functions of factor prices and income distribution are consistent with the humped shape extensively estimated in the empirical literature on income distribution. Moreover, since these distributions are obtained within a standard RBC framework, it is straightforward to assess the evolution of such distributions along the cycle and in response to productivity shocks.

The rest of the paper is structured as follows: in section two we discuss the economic framework, in the third section we use exchangeability and Pòlya urns to determine the distribution of agent types. In the fourth section we define and characterize the dynamic stochastic general equilibrium. In section five we obtain the distributions of factors' prices and we provide a simple example to illustrate our results. Section six concludes and discusses several developments of this project.

2.2 Economic model

Our aim is to characterize an economy segmented in a continuum of markets. Factors' supply in each productive location is determined by the composition of the local population.

2.2.1 Endowments

The economy is populated by a continuum of agents and a continuum of firms. Agents and firms are located in spatially separated markets. We call each of these markets *city* and we index those with the letter j. Agents are of two types, according to the factor of production they own. At a given time t in a given city j workers (W_{jt}) are endowed with one unit of labor, managers (M_{jt}) with one unit of human capital. Labor and human capital are matched to produce a composite human service factor (S_{jt}). A further factor, physical capital (K_{jt}), is employed in the economy and agents of both types may decide to hold stocks of physical capital.

Each city is populated by agents of both types and firms rent factors in competitive factor markets. The same technological solutions are available to all firms in the economy but a firm can hire workers and managers who belongs to the same city the firm is located in. Instead, physical capital and output are freely traded across cities.

In this framework, firms take two decisions sequentially: first, they match workers and managers in order to maximize the availability of human service; second, firms demand human service and physical capital to produce an homogeneous output Y_{jt}. We assume technologies in use in both steps are of the constant return or scale type.

Two sources of uncertainty hit the economy. Across cities, workers shares are independent draws from a non-degenerate distribution $F(\cdot)$. We define the workers share of a given city j:

$$X_j = \frac{W_j}{W_j + M_j} \tag{2.1}$$

In the third section we provide the sufficient assumptions to determine the distribution $F(\cdot)$. Along time, total factor productivity in the economy follows an AR(1) stochastic process[4]:

$$\ln(z_{t+1}) = (1-\rho)\ln(z) + \rho\ln(z_t) + \varepsilon_{t+1} \tag{2.2}$$

where $z > 0$ is the steady state value of total factor productivity and ε_t are i.i.d. random draws from a normal distribution with zero mean and variance $\nu_\varepsilon > 0$.

[4]This choice for the total factor productivity process has the only purpose of comparing this work with the mainstream macroeconomic literature.

2.2.2 Preferences

Agents order their preferences according with an instantaneous utility function that values individual consumption only: $u : \mathbb{R}_+ \to \mathbb{R}_+$. Therefore we do not allow for intensive margin in human factors supply: the total supply of labor L_j and human capital H_j in a city j are: $L_j \equiv W_j$ and $H_j \equiv M_j$.

Let we refer to $u\left(c_j^a\left(t\right)\right)$ as the utility that agent $a \in \{W, M\}$, workers and managers respectively, benefits from due to the consumption of c units of homogeneous good at time t in a city j. The instantaneous utility function is increasing, concave and time invariant. Agents discount future at a rate $\beta \in (0,1)$. The expected lifetime utility satisfies *von Neumann-Morgenstern* form. The expected utility function associated to the lifetime stream of consumption is:

$$U^a\left(c_j^a\left(0\right), c_j^a\left(1\right), ..., c_j^a\left(T\right)\right) = E_0\left\{\sum_{t=0}^{T} \beta^t u\left(c_j^a\left(t\right)\right)\right\}$$

for $T \to \infty$, since agents are infinitely living and E_0 is the expectation operator conditionally on the information set at time 0.

In addition we assume that all agents have a positive consumption each point in time and the indirect utility function associated to individual preferences satisfies the Gorman form. Utility functions of this class prescribe a linear relationship between demand and income, the slope of which does not depend on individual income. As a result, the aggregate demand will not depend on the actual distribution of income across households. Under this assumption a representative agent does exist, both in the *positive* than in the *normative* sense.

Under this framework, in each city j a Pareto optimal allocation of the decentralized problem is a solution to the maximization of a weighted average of individual utilities subject to the resource constraints. In order to understand the rule of consumption allocation across agents we assume an *utilitarian* social welfare function. In each city the decentralized equilibrium maps into the problem of a normative representative agent who maximizes a social welfare function with equal weights:

$$\mathcal{W}_j = x_j U\left(c_j^W(t)\right) + (1-x_j)\, U\left(c_j^M(t)\right) \tag{2.3}$$

The maximization of the utilitarian social welfare function (2.3) implies the following allocation rule:

$$x_j \frac{\partial U\left(c_j^W(t)\right)}{\partial c_j^W(t)} = (1-x_j)\, \frac{\partial U\left(c_j^H(t)\right)}{\partial c_j^H(t)} \tag{2.4}$$

where $C_j(t)$ is consumption of the representative agent in a city j at time t. We normalize the population of each city to be of size 1, then:

$$C_j(t) = x_j c_j^W(t) + (1-x_j)\, c_j^H(t) \tag{2.5}$$

is total consumption in a city j at time t.

2.2.3 Technology

Production requires two technologies. An *intermediate* technology is used to match workers and managers, conditionally on the distribution of workers share across cities. The resulting supply of human service is combined with physical capital to produce output according with a *final* technology.

The human service available at time t in a city j is given by $S_{jt} = T(L_{jt}, H_{jt})$ such that: the function $T : \mathbb{R}_+^2 \to \mathbb{R}_+$ is homogeneous of degree one and it satisfies concavity, Inada conditions[5] and the following normalization holds $T(1,1) = 1$. The choice of the intermediate technology is a solution to the following problem:

$$\begin{aligned} S_{jt} &= \max_{W_{jt}, M_{jt}} T(W_{jt}, M_{jt}) \\ s.t. &: \begin{cases} \Pr(X_{jt} = x_{jt}) = q_{jt} \\ W_j, M_j > 0 \end{cases} \end{aligned} \tag{2.6}$$

[5] Function T satisfies $T_1'(L,H),\ T_2'(L,H) > 0,\ T_1''(L,H),\ T_2''(L,H) < 0$, $\lim_{L\to 0} T_1'(L,H) = \lim_{H\to 0} T_1'(L,H) = \infty,\ \lim_{L\to\infty} T_1'(L,H) = \lim_{H\to\infty} T_1'(L,H) = 0$. The same set of assumptions apply to function G.

where X_{jt} is defined in (2.1) and $q_{jt} \in (0, \infty)$ is the value of the density function associated to the realization x_{jt}.

Firms then employ human service in the city and demand the optimal amount of physical capital to produce the homogeneous consumption good Y_{jt} according with the final technology G. The final technology is a function

homogeneous of degree one, that satisfies concavity, Inada conditions, and such that $G(1,1) = z_t$[6]. In order to make this work comparable with the mainstream macroeconomic literature we assume that G is Cobb-Douglas with human factor's share $\sigma \in (0,1)$:

$$Y_{jt} = z_t S_j^{\sigma} K_{jt}^{1-\sigma} \tag{2.7}$$

2.3 Characterizing the population of agents

We investigate the sequence of agents that describes the population in a city. We proceed building a statistical model, that is a triple $(\mathcal{Z}, \mathcal{F}, \mathcal{P})$ where $\mathcal{Z}$ is the space of possible realizations of the experiment, $\mathcal{F}$ is the $\sigma-$*algebra* associated to $\mathcal{Z}$, and $\mathcal{P}$ is the family of probability measures over the measurable space $(\mathcal{Z}, \mathcal{F})$. Finally, let the probability measure $\mathcal{P}$ be indexed by $\theta \in \Theta$, where Θ is the parameter space.

Define A_i a discrete random variable that takes values $A_i = 1$ if the agent i is a worker and values $A_i = 0$ if the agent i is a manager. Our statistical model is therefore based on a Bernoulli distribution: $\Pr(a_i|\theta) = \theta^{a_i}(1-\theta)^{1-a_i}$, where $a_i = 0, 1$ and $0 \leq \theta \leq 1$. We can refer to the baseline model of our statistical model as $(A, \Pr(a_i|\theta), \theta \in \Theta)$.

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[6] In Jones (2005), the firm's output is given by a technology with technological bias indices, $\kappa > 0$ for physical capital and $\eta > 0$ for human service. Firms choose the couple of technological factors (η, κ) to combine physical capital and human services such that output $Y_{jt} = G(\eta S_{jt}, \kappa K_{jt})$ is maximized, subject to a joint distribution of ideas $\Phi_{\eta,\kappa}$ that is Pareto.

Assume in each city we observe n agents then we can refer to the population in a city as a sequence of random variables $(a_1, ..., a_n)$. The Bernoullian sampling of the population in the economy completes the description of the statistical model we apply to the population of agents[7]: $\left(\mathcal{A}^{(n)}, \prod_{i=1}^{n} \ \Pr(a_i|\theta), \theta \in \Theta\right)$, where $\mathcal{A}^{(n)}$ is the space of events. Subject to θ, the random variables $(A_1, A_2, ..., A_n)$ with realizations in $\mathcal{A}^{(n)}$ are independent and identically distributed.

Our goal is to specify the joint density $\Pr(A_1 = a_1, A_2 = a_2, ..., A_n = a_n)$. Addressing this issue requires to choose the type of dependence that links the elements A_i within the sequence $\{A_i\}_{i=1}^{n}$. There is a number of alternatives, we opt for a very simple form that describes a very large class of experiments in human sciences: *exchangeability*, that we discuss in the next section.

2.3.1 Exchangeability and de Finetti representation theorem

Exchangeability is a powerful and elegant theoretical framework that applies to experiments in which the order of single realizations of events does not matter to determine the probability associated to experiment outcomes and forecasting. In our framework, we apply exchangeability to model the probability associated with a particular composition of the population. In fact, the order in which managers and workers appear as realizations of the variable A_i does not affect the (joint) probability associated with a composition of the population.

DEFINITION (1). *Random variables $(A_1, A_2, ..., A_n)$ are called exchangeable if the cumulative distribution function of each $(A_{\rho_1}, A_{\rho 2}, ..., A_{\rho_n})$,*

[7]The practice to assume a Bernoullian sampling (that is n copies of the baseline model with the joint density equal to n times the marginal densities) is very common in statistics and in this paper is due to technical convenience.

where $(\rho_1, \rho_2, ..., \rho_n)$ *is any permutation of* $(1, 2, ..., n)$*, coincides with the cumulative distribution of* $(A_1, A_2, ..., A_n)$*.*

Definition (2). *The sequence* $\{A_i\}_{i=1}^n$ *is exchangeable if each finite subsequence is exchangeable.*

We assume that the population in each city is an exchangeable sequence of Bernoulli random variables. It follows by definition that all couples, triplets and tuple of variables A_i are identically distributed. It is possible to show that given an exchangeable sequence with n elements, finite and positive variance then let $corr \in [-1, 1]$ be the linear correlation coefficient between any given couple of random variables $corr(A_i, A_k)$ for $i \neq k$. The following result holds: $corr \geq -\frac{1}{n-1}$ for any $n > 1$. If the sequence is infinite, $n \to \infty$, then $corr \geq 0$.

Working with infinite exchangeable sequence we can apply de Finetti representation theorem (1939)[8]:

Theorem (1). *Let* $\{A_i\}_{i=1}^n$ *be an infinite sequence of exchangeable random variables such that each random variable takes on values* 0 *and* 1 *then the probability associated to each of the possible realizations* $(a_1, ..., a_n)$ *takes the form:*

$$\begin{aligned} &\Pr(A_1 = a_1, A_2 = a_2, ..., A_n = a_n) \\ = &\int_0^1 \theta^{\sum_{i=1}^n a_i} (1-\theta)^{n-\sum_{i=1}^n a_i} \, dF(\theta) \end{aligned}$$

where $F(\theta)$ *is the almost sure limit of* $\frac{1}{n}\sum_{i=1}^n A_i$*:*

$$F(\theta) = \lim_{n \to \infty} \Pr\left\{\frac{1}{n}\sum_{i=1}^n A_i \leq \theta\right\} \tag{2.8}$$

[8]We state the theorem in its original version, de Finetti (1937), that applies to the Bernoulli distribution, because this our case. But the theorem has been proved in general and it became a pillar of the subjective probability approach. The interest reader could refer to Hewitt and Savage (1955).

and its distribution is uniquely determined by the sequence $\{A_n\}$.

The probability distribution F is the *de Finetti measure* of the sequence $\{A_n\}$. Let $\{A_{ij}\}_{i=1}^{n}$ be the sequence that describes the population of agents in city j, then notice that the workers share in city j is $X_j \equiv \frac{1}{n}\sum_{i=1}^{n} A_{ij}$. We assume that in each city the population is large enough that the asymptotic case $n \to \infty$ applies then F is the distribution of the workers share across cities. In each point in time

It is common also to term the distribution F as the *prior*, because it adds an à priori information to the data, a subjective believe of the researcher. that does not originate in the data observation but on the personal opinion about the data generating process itself. The selection of a prior is the final step that gives us the posterior distribution of the composition of the population.

2.3.2 Select the prior with a Pòlya urn

One of the simple ways to generate sequences of exchangeable random variables is the *Pòlya urn*, and it is based on the concept of *reinforcement*. A Pòlya urn is a representation of the ideas generating a probabilistic model. The data generating process is described as a sequence of stages that starts with an urn containing an initial number of balls of different colors. At each stage of the experiment a ball is randomly drawn from the urn, the color is observed and the ball is put back in the urn with in addition a given number of balls of the same color; that number of additional balls is called reinforcement. The distribution of the proportion of balls of one color in the urn is thought to be the outcome of repeating draws and reinforcements for a large number of stages.

In our framework, let us assume that we know the number of workers and managers in the economy at the beginning of time (W_0, M_0). The evolution of the population gives rise to the process of formation of cities. The continuum of cities we have in our framework is the outcome of this process and the population in each city is an exchangeable sequence of individuals of two types, workers and managers. Each period cities experience net inflows of workers or managers and that leads to an evolution in the composition of the population. In the long run, the distribution of the workers share across cities is the mixture of the prior distribution and the empirical distributions of the actual realizations.

Call A_k^j the random variable that records the status of the $k-th$ agent in city j, 1 if he is a worker, 0 if he is a manager. The number of workers and managers in the city j before of the $k+1$-th stage are W_k^j and M_k^j The dynamics of processes $\left\{A_k^j\right\}$, $\left\{W_k^j\right\}$ and $\left\{M_k^j\right\}$ in a Pòlya urn are governed by the following:

$$\begin{aligned} A_{k+1} &= \begin{cases} 1 \text{ with probability } \frac{W_k}{W_k+M_k} \\ 0 \text{ with probability } \frac{M_k}{W_k+M_k} \end{cases} \qquad (2.9) \\ \left(W_{k+1}^j, M_{k+1}^j\right) &= \begin{cases} \left(W_k^j + r, M_k^j\right) \text{ with probability } \frac{W_k^j}{W_k^j+M_k^j} \\ \left(W_k^j, M_k^j + r\right) \text{ with probability } \frac{M_k^j}{W_k^j+M_k^j} \end{cases} \end{aligned}$$

with $r > 0$ being the reinforcement.

We assume that the stationary distribution of workers and managers across cities is originated by the data generating process in (9)[9]. Under this restriction, two theorems apply as consequences of the de Finetti's Representation Theorem.

THEOREM (2). *The sequence $\{A_n\}$ generated by a Pòlya urn is exchangeable and its de Finetti measure is a Beta Distribution with parameters* $\left(\frac{W_0}{r}, \frac{M_0}{r}\right)$.

[9]Given the properties of production function, preferences and the absence of cooperation between managers and workers, our economy is a two version of the Lucas and Prescott (1974) search framework. A stationary distribution for the search equilibrium with movements does exist and it is unique, we are not assuming it. Our assumption is that the data generating process in (?) is behind the stationary distribution we will find as a solution of a two factors Lucas and Prescott model.

The next theorem considers the limit behavior of the proportion of workers conditional on the previous information about the city.

THEOREM (3). *Given the Pòlya urn we defined, as n grows to infinity, the share* $X_n = \frac{W_n}{W_n+M_n}$ *converges almost surely to a random limit. Moreover, the distribution of the limit is a Beta with parameters* $\left(\frac{W_0}{r}, \frac{M_0}{r}\right)$.

We conclude that under our framework the prior distribution of the workers share across cities is a Beta Distribution with parameters α_0, $\beta_0 > 0$ that represent $\frac{W_0}{r}$ and $\frac{M_0}{r}$ respectively:

$$F(\theta) = \int_0^\theta \frac{\Gamma(\alpha_0+\beta_0)}{\Gamma(\alpha_0)\Gamma(\beta_0)} \frac{\theta^{\alpha_0-1}(1-\theta)^{\beta_0-1}}{B(\alpha_0,\beta_0)} d\theta \tag{2.10}$$

where $B(\alpha_0,\beta_0) = \int_0^1 v^{\alpha_0-1}(1-v)^{\beta_0-1}\,dv$ is the beta function, a constant for each the couple (α_0, β_0).

2.4 Equilibrium

Firms choose an intermediate technology to optimally match labor and human capital in the composite factor we called human service. Then the firm chooses a final technology that employs human service and physical capital to produce an homogeneous consumption good.

2.4.1 The shape of the intermediate technology

From the previous section we know the distribution of the workers share across cities when the number of agents in each city is infinite. Let each city being populated by a continuum of agents of both types and let $n > 0$ be the arbitrary size of the population. In this scenario, the workers share in a given city reaches its limit value and it stays constant over time,

$X_{jt} = x_j$ for every t. The same holds for the number of workers and managers: $W_{jt} = x_j n$ and $M_{jt} = (1 - x_j)\, n$ for every t [10].

Notice that we can express the density function of the distribution of workers share in terms of human factors. From (2.10) we have:

$$L_{jt}^{(\alpha_0 - 1)} H_{jt}^{(\beta_0 - 1)} = Q_j \tag{2.11}$$

where $L_{jt} = x_j n$, $H_{jt} = (1 - x_j)\, n$ and $Q_{jt} = n^{\alpha_0 + \beta_0 - 2} f(x_j)\, B(\alpha_0, \beta_0)$. Equation (2.11) is the constraint in the choice of the intermediate technology T:

$$\begin{aligned} S_j &= \max_{L_j, H_j} T(W_{jt}, M_{jt}) \\ s.t. &: L_{jt}^{(\alpha_0 - 1)} H_{jt}^{(\beta_0 - 1)} = Q_j \end{aligned}$$

The first order conditions of this problem imply that the intermediate production function S_j belongs to the class of technologies with constant elasticities of substitution between factors:

$$\epsilon = \frac{\frac{\partial S_j}{\partial L_j} \frac{L_j}{S_j}}{\frac{\partial S_j}{\partial H_j} \frac{H_j}{S_j}} = \frac{\alpha_0 - 1}{\beta_0 - 1} \tag{2.12}$$

Notice that from an economic point of view, we want the elasticity of substitution to be positive, and we want to avoid the extreme cases of no substitutability ($\alpha_0 \neq 1$) and perfect substitutability ($\beta_0 \neq 1$). Therefore we come up into two scenarios: $\alpha_0, \beta_0 < 1$ and $\alpha_0, \beta_0 > 1$. In the first scenario the distribution of workers share is bimodal and it shows more concentration on the extreme values of the unit support. The second scenario prescribes a unimodal density with more concentration around one if $\alpha_0 > \beta_0$, and around zero if $\beta_0 > \alpha_0$. In addition we have that S_{jt}

[10] Solving the model for an arbitrary population size constant over time when the workers share attains its limit value in each city implies that the human service S_j is constant over time in each city. This specification allows the comparison with the mainstream macroeconomic models with constant workforce and no intensive margin in the labor supply.

is homogeneous of degree one, by assumption of constant returns to scale. A functional form that satisfies the properties we identified is:

$$S\left(L_{jt}, H_{jt}\right) = n^{\frac{1}{(\alpha_0+\beta_0-2)}} \left[\gamma L_{jt}^{\frac{\epsilon-1}{\epsilon}} + (1-\gamma) H_{jt}^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} \tag{2.13}$$

for a given share $\gamma \in (0,1)$.

Use homogeneity and the normalization $T(1,1)=1$: $\frac{S_j}{H_j} = T\left(\frac{L_j}{H_j}, 1\right)$ to notice that $L_j = H_j \Longrightarrow S_j = H_j = L_j$. Moreover, recall that $L_j = W_j = x_j n$ and $H_j = M_j = (1-x_j)n$; therefore, $L_j = H_j \Longleftrightarrow x_j = \frac{1}{2}$. We conclude that the intermediate technology passes through a point that satisfies $\ln S\left(\frac{n}{2}, \frac{n}{2}\right) = \frac{\ln n}{(\alpha_0+\beta_0-2)} + \ln\frac{1}{2}$. Evaluating the marginal rate of technical substitution at this point we obtain: $\frac{\partial S_j}{\partial L_j} / \frac{\partial S_j}{\partial H_j} = \frac{\alpha_0-1}{\beta_0-1}$. For that to be true we want $\frac{\gamma}{1-\gamma} = \frac{\alpha_0-1}{\beta_0-1}$, that gives us a unique value of factor's share:

$$\gamma = \frac{\alpha_0 - 1}{\alpha_0+\beta_0-2} \tag{2.14}$$

where because from an economic point of view we want positive factors shares, either $\alpha_0 > 1$ and $\beta_0 > 1 > 2-\alpha_0$ or $\alpha_0 < 1$ and $\beta_0 < 1 < 2-\alpha_0$. Remarkably notice that if $\alpha_0, \beta_0 > 1$ then the factor's share is the mode of the Beta distribution describing the composition of the population.

The functional form of the intermediate technology is:

$$S\left(L_{jt}, H_{jt}\right) = n^{\frac{1}{(\alpha_0+\beta_0-2)}} \left[\left(\frac{\alpha_0-1}{\alpha_0+\beta_0-2}\right) L_{jt}^{\frac{\epsilon-1}{\epsilon}} + \left(\frac{\beta_0-1}{\alpha_0+\beta_0-2}\right) H_{jt}^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} \tag{2.15}$$

Notice that none of the results is about the population size, therefore without loss of generality in the proceeding we assume $n = 1$.

In the literature the case in which the elasticity of substitution between factors is one deserved attention. Notice that $\epsilon \to 1$ if and only if $\alpha_0 = \beta_0$, then the distribution of workers share is symmetric about $\frac{1}{2}$. In fact, notice that elasticity of substitution and factors shares in the intermediate technology are linked. Because of the particular form of the distribution of workers share we have: $\epsilon = \frac{\gamma}{1-\gamma}$ or $\gamma = \frac{\epsilon}{1+\epsilon}$. Moreover, the limit case for the technology (2.15) as the elasticity of substitution between factors attains the unit delivers a constant returns to scale Cobb-Douglas production function. When $\alpha_0 = \beta_0$ factors shares are $\frac{1}{2}$, and the intermediate technology in this special case becomes:

$$S(L_{jt}, H_{jt}) = L_{jt}^{1/2} H_{jt}^{1/2} \tag{2.16}$$

Figure 2.1 shows the supply of human service as function of workers share in three cases: the solid line is associated to $\alpha_0 = \beta_0$, that is the Cobb-Douglas case. The dashed line represents the case $\beta_0 > \alpha_0$, the elasticity of substitution between factors is less than one. The dots line represents the case $\alpha_0 > \beta_0$, the elasticity of substitution between factors is larger than one.

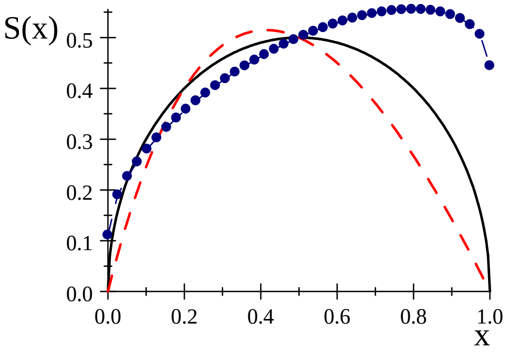

Figure 2.1: The intermediate technology

2.4.2 A representative household in each city

In each city j the aggregate variables are determined as if a representative agent maximizes (2.17) subject to the resource constraint $Y_{jt} = C_{jt} + I_{jt} + NX_{jt}$, where NX_{jt} is the next excess of supply of consumption that is produced in the city and supplied elsewhere in the economy. The law of capital accumulation: $K_{jt+1} = (1-\delta) K_{jt} + I_{jt}$, where I_{jt} is total investment and $\delta \in [0, 1]$ is the depreciation rate of physical capital.

With the purpose of discussing an analytical solution of the model we restrict the general framework to the standard macroeconomic specification, CES preferences, and among those to the log utility specification. Under this framework, we show in the appendix that the representative agent orders consumption bundles according with the following utility function:

$$\mathcal{U}\left(\{C(t)\}_{t=0}^{T}\right) = E_0\left\{\sum_{t=0}^{T}\beta^t \ln\left(C_j(t)\right)\right\} \tag{2.17}$$

The allocation rule specializes into

$$xc_j^H(t) = (1-x)\, c_j^W(t) \tag{2.18}$$

The representative agent' problem is stated by the means of the following

Bellman equation:

$$\begin{aligned} V\left(K_{jt}, z_t\right) &= \sup_{K_{jt+1}} \left\{ \begin{array}{r} \ln\left(z_t S_j^{\sigma} K_{jt}^{1-\sigma} + (1-\delta) K_{jt} - NX_{jt} - K_{jt+1}\right) \\ +\beta E_t\left\{V\left(K_{jt+1}, z_{t+1}\right)|z_t\right\} \end{array} \right\} \\ s.t. &: K_{jt+1} \in \left[0, z_t S_j^{\sigma} K_{jt}^{1-\sigma} + (1-\delta) K_{jt}\right] \text{ and } NX_{jt} \in \end{aligned}$$

Because of the log utility and the Cobb-Douglas production function, a restriction of the space parameter $\delta = 1$ is sufficient to obtain an analytical solution of the Bellman equation[11]. The two policies for the next period of capital and consumption are:

$$K_{jt+1} = (1-\sigma)\,\beta z_t S_j^{\sigma} K_{jt}^{1-\sigma} \tag{2.19}$$

$$C_{jt} = \left[1 - (1-\sigma)\,\beta\right] z_t S_j^{\sigma} K_{jt}^{1-\sigma} \tag{2.20}$$

Because of the properties of aggregation under Gorman form we know that these allocations are Pareto optimal for the decentralized equilibrium.

[11] Since our point is not on the solution method of a DSGE, we restrict the discussion to the log utility, Cobb-Douglas specification in order to obtain an analytical solution. It is fair to say that our approach shares the same pros and cons that any other DSGE solution methods.

In order to solve for the decentralized equilibrium we impose the allocation rule (2.18), the consistency condition (2.5) and the individual budget constraints:

$$c_{jt}^{W}+k_{jt+1}^{W} = (1+i_t)\,k_{jt}^{W}+w_{jt} \tag{2.21}$$

$$c_{jt}^{H}+k_{jt+1}^{H} = (1+i_t)\,k_{jt}^{H}+s_{jt} \tag{2.22}$$

for workers and managers respectively. Taking the consumption good as numeraire and exploit the fact that factors markets are competitive. Real wage is equal to the marginal productivity of labor:

$$w_{jt}=\frac{\sigma}{2}\frac{z_t S_j^{\sigma} K_{jt}^{1-\sigma}}{x_j} \tag{2.23}$$

Real salary is equal to the marginal productivity of human capital:

$$s_{jt}=\frac{\sigma}{2}\frac{z_t S_j^{\sigma} K_{jt}^{1-\sigma}}{1-x_j} \tag{2.24}$$

Real interest rate is equal to the marginal productivity of capital net of the depreciation rate: $i_t=(1-\sigma)\,z_t\,(S_j/K_{jt})^{\sigma}-1$. This must be true in each city and i_t must be the same across cities, since capital is freely traded. For this last condition to be true we must be able to write the physical capital that is demanded in each city as a function of the unique interest rate and the supply of human service that is specific to the city:

$$K_{jt}^{d}=\left[z_t\left(\frac{1-\sigma}{1+i_t}\right)\right]^{\frac{1}{\sigma}} S_j \tag{2.25}$$

where $K_{jt}^{d}\equiv K_{jt}$, the amount pf physical capital employed in production in city j. The total supply of physical capital in a city j at time t is given by (2.26):

$$K_{jt}^{s}=x_j k_{jt}^{W}+(1-x_j)\,k_{jt}^{H} \tag{2.26}$$

The balance of payments across cities implies:

$$K_{jt}^{d}-K_{jt}^{s}=NX_{jt} \tag{2.27}$$

As it common to open economy models, the vector of net export $\underline{D}_{jt}$ is not uniquely determined. Never the less we can state the following consistency condition that applies to the economy: $\underline{NX}_{jt} = \underline{D}_{jt} \times \underline{Y}_{jt} = 0$, that is either $\underline{D}_{jt}$ is the null vector, or the two vectors are linearly independent. In the first case we do not observe trade $\underline{D}_{jt} = 0$ for all j, whereas in the other scenarios we require the less severe assumption that $E_j \left\{ \underline{NX}_{jt} \right\} = E_j \left\{ \underline{D}_{jt} \times \underline{Y}_{jt} \right\} = E_j \left\{ D_{jt} \right\} E_j \left\{ Y_{jt} \right\} = 0$, that implies $E_j \left\{ D_{jt} \right\} = 0$.

For a given pair of values (i_0, z_0), a dynamic stochastic general equilibrium in this economy consists of: (i) the distributions of prices for human factors across cities $\{w(j,t), s(j,t)\}$ that satisfy in each time (2.23) and (2.24) respectively; (ii) a value of the interest rate $i(t)$ that satisfies (2.25); (iii) the distributions of total production, total physical capital stock and total consumption across cities $\{Y(j,t), K(j,t), C(j,t)\}$ that satisfy in each time (2.7), (2.19) and (2.20) respectively; (iv) the distributions of individual physical capital stock holding and consumption across cities and agent's types $\left\{ c^W(j,t), k^W(j,t), c^H(j,t), k^H(j,t) \right\}$ that satisfy in each time the individual budget constraints (2.21), (2.22), the consumption allocation policy implied by (2.4), the consistency conditions (2.5), (2.26), and the balance of payments (2.27) for a given level of current account NX_{jt} such that $E_j \{NX_{jt}\} = 0$ in every period t.

2.4.3 Macroeconomic variables

Define $k_t = \frac{K_{jt}}{S_j}$ and $y_t = \frac{Y_{jt}}{S_j}$ to be respectively physical capital and output per unit of human service. From equation (2.25) it is immediate to obtain $k_t = \left[z_t \left(\frac{1-\sigma}{1+i_t} \right) \right]^{\frac{1}{\sigma}}$ and from the production function we have $\frac{y_t}{k_t} = \frac{1+i_t}{1-\sigma}$. Notice that variables per unit of human service do not vary across cities, that is the reason why the shape of the intermediate technology matters for macroeconomic aggregates. This is the channel through which the distribution of workers share across cities becomes responsible for a distribution of production across cities and factor prices across agents.

Aggregate macroeconomic variables are straightforward: $K_t = k_t \int S(x_j)\, dF(x_j)$, $Y_t = K_t\left(\frac{1+i_t}{1-\sigma}\right)$, $C_t = [1-(1-\sigma)\beta]\, Y_t$ and output per unit of physical capital takes the neoclassical form: $Y_t/K_t = \frac{y_t}{k_t} = \frac{1+i_t}{1-\sigma}$. Notice that the model predicts the very same balanced growth path common to RBC literature: the growth rate of aggregate real macroeconomic variables (per unit of human service) coincides with the interest rate.

2.4.4 Distribution of factor prices

The interest rate is one in the economy, instead wage and salary follow a distribution across agents that is induced by the distribution of workers share across cities.

We can derive the law of motion of the interest rate for a given couple of initial values (i_0, z_0). Notice that $\frac{K_{jt+1}}{K_{jt}} = \left(\frac{(1+i_{t+1})}{(1+i_t)}\frac{z_t}{z_{t+1}}\right)^{-\frac{1}{\sigma}}$, the policy for asset holding implies $\frac{K_{jt+1}}{K_{jt}} = (1-\sigma)\beta z_t S_j^{\sigma} K_{jt}^{-\sigma}$, where $(1-\sigma) z_t (S_j/K_{jt})^{\sigma} = (1+i_t)$ by the marginal productivity of physical capital. We obtain $(1+i_{t+1})^{-\frac{1}{\sigma}} = \beta(1+i_t)^{\frac{\sigma-1}{\sigma}}\left(\frac{z_t}{z_{t+1}}\right)^{\frac{1}{\sigma}}$, taking logs and substituting for the stochastic law of motion of total factor productivity, we have that the law of motion of the interest rate satisfies:

$$\begin{aligned}
\ln(1+i_{t+1}) &= \sigma \ln\left(\frac{1}{\beta}\right) + (1-\sigma)\ln(1+i_t) - (1-\rho)[\ln(z_t)] + \varepsilon_{t+1} \\
&= \sigma \ln\left(\frac{1}{\beta}\right) + (1-\sigma)\ln(1+i_t) + [\ln(z_{t+1}) - \ln(z_t)]
\end{aligned}$$

Consider now output per unit of human service: $y_t = z_t^{\frac{1}{\sigma}}\left(\frac{1-\sigma}{1+i_t}\right)^{\frac{1-\sigma}{\sigma}}$. Factors prices are defined as $w_{jt} = \frac{\sigma}{2} y_t \frac{S(x_j)}{x_j}$ and $s_{jt} = \frac{\sigma}{2} y_t \frac{S(x_j)}{1-x_j}$. Both factors' prices are monotonic functions of the workers share: wage is increasing with x_j the opposite is true for the salary. Following the previous discussion, the human service supply is:

$$S_j(X_j) = \begin{cases} \left[\left(\frac{\alpha_0-1}{\alpha_0+\beta_0-2}\right) X_j^{\frac{\epsilon-1}{\epsilon}} + \left(\frac{\beta_0-1}{\alpha_0+\beta_0-2}\right)(1-X_j)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} & \text{for } \alpha_0 \neq \beta_0 \\ X_j^{1/2}(1-X_j)^{1/2} \ \text{ for } \alpha_0 = \beta_0 \end{cases}$$

where the assumption that the elasticity of substitution between factors is positive and finite rules out the case $\alpha_0 = 1$ or $\beta_0 = 1$. The two factors prices in terms of the workers share random variable are:

$$w_t(X) = \begin{cases} \frac{\sigma}{2} y_t \left[\left(\frac{\alpha_0 - 1}{\alpha_0 + \beta_0 - 2} \right) + \left(\frac{\beta_0 - 1}{\alpha_0 + \beta_0 - 2} \right) \left(\frac{1-X}{X} \right)^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} & \text{for } \alpha_0 \neq \beta_0 \\ \frac{\sigma}{2} y_t \left(\frac{1-X}{X} \right)^{1/2} \text{ for } \alpha_0 = \beta_0 \end{cases}$$

$$s_t(X) = \begin{cases} \frac{\sigma}{2} y_t \left[\left(\frac{\alpha_0 - 1}{\alpha_0 + \beta_0 - 2} \right) X^{\frac{\epsilon-1}{\epsilon}} + \left(\frac{\beta_0 - 1}{\alpha_0 + \beta_0 - 2} \right) \right]^{\frac{\epsilon}{\epsilon-1}} & \text{for } \alpha_0 \neq \beta_0 \\ \frac{\sigma}{2} y_t \left(\frac{X}{1-X} \right)^{1/2} & \text{for } \alpha_0 = \beta_0 \end{cases}$$

We can directly determine the density functions, for the wage:

$$\Pr(w_t = \overline{w}) = \begin{cases} \Pr\left(X = \frac{1}{1 + \mathcal{T}(\overline{w}, y_t)} \right) & \text{for } \alpha_0 \neq \beta_0 \\ \Pr\left(X = \frac{1}{1 + \left(\frac{2}{\sigma} \frac{\overline{w}}{y_t} \right)^2} \right) & \text{for } \alpha_0 = \beta_0 \end{cases} \tag{2.28}$$

$$\Pr(s_t = \overline{s}) = \begin{cases} \Pr\left(X = \mathcal{T}(\overline{s}, y_t) \right) & \text{for } \alpha_0 \neq \beta_0 \\ \Pr\left(X = \frac{\left(\frac{2}{\sigma} \frac{s}{y_t} \right)^2}{1 + \left(\frac{2}{\sigma} \frac{s}{y_t} \right)^2} \right) & \text{for } \alpha_0 = \beta_0 \end{cases} \tag{2.29}$$

where $\gamma = \frac{\alpha_0 - 1}{\alpha_0 + \beta_0 - 2}$ is the labor share in the value of human service and for convenience of exposition we defined $\mathcal{T}(\overline{w}, y_t) = \left[\frac{1}{1-\gamma} \left(\frac{2}{\sigma} \frac{\overline{w}}{y_t} \right)^{\frac{\epsilon-1}{\epsilon}} - \epsilon \right]^{\frac{\epsilon}{\epsilon-1}}$ and $\mathcal{T}(\overline{s}, y_t) = \left[\frac{1}{\gamma} \left(\frac{2}{\sigma} \frac{\overline{s}}{y_t} \right)^{\frac{\epsilon-1}{\epsilon}} - \epsilon \right]^{\frac{\epsilon}{\epsilon-1}}$, that are monotonic increasing functions of $\frac{\overline{w}}{y_t}$ and $\frac{\overline{s}}{y_t}$ respectively. Applying the density f_X, we obtain the two densities f_w and f_s over the support $(0, \infty)$:

$$f_w(\overline{w}) = \begin{cases} \frac{1}{\mathcal{B}(\alpha_0, \beta_0)} \left[\frac{1}{1 + \mathcal{T}(\overline{w}, y_t)} \right]^{\alpha_0 - 1} \left(\frac{\mathcal{T}(\overline{w}, y_t)}{1 + \mathcal{T}(\overline{w}, y_t)} \right)^{\beta_0 - 1} & \text{for } \alpha_0 \neq \beta_0 \\ \frac{1}{\mathcal{B}(\alpha_0, \beta_0)} \left[\frac{\left(\frac{2}{\sigma} \frac{\overline{w}}{y_t} \right)}{1 + \left(\frac{2}{\sigma} \frac{\overline{w}}{y_t} \right)^2} \right]^{2(\alpha_0 - 1)} & \text{for } \alpha_0 = \beta_0 \end{cases}$$

$$f_s(\overline{s}) = \begin{cases} \frac{1}{\mathcal{B}(\alpha_0, \beta_0)} \left[\mathcal{T}(\overline{s}, y_t) \right]^{\alpha_0 - 1} \left(1 - \mathcal{T}(\overline{s}, y_t) \right)^{\beta_0 - 1} & \text{for } \alpha_0 \neq \beta_0 \\ \frac{1}{\mathcal{B}(\alpha_0, \beta_0)} \left[\frac{\left(\frac{2}{\sigma} \frac{\overline{s}}{y_t} \right)}{\left[1 + \left(\frac{2}{\sigma} \frac{\overline{s}}{y_t} \right)^2 \right]} \right]^{2(\alpha_0 - 1)} & \text{for } \alpha_0 = \beta_0 \end{cases}$$

The next figure shows the density of wage and salary in the case $\alpha_0 = \beta_0$, for a benchmark level of y_t (solid line) and a larger level of output per unit of human service (dashed line).

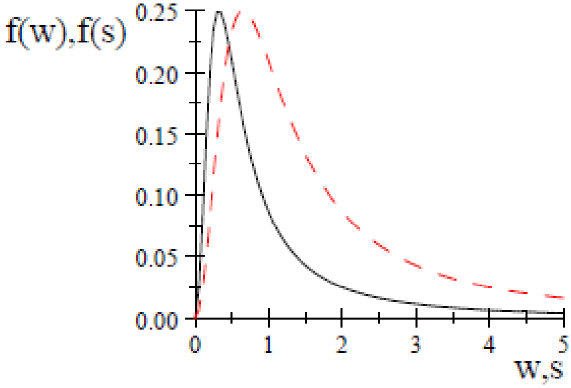

Figure 2.2: Factor prices distributions

The model predicts an hump shaped positive skew density function of income, the moments of which depend on the value of y_t. This theoretical prediction is a desirable result, for three reasons mainly. First, this shape is a prominent feature of the empirical estimation of income distribution, both across agents within countries and across countries. In this respect, notice that an humped shape of income density is a theoretical result of our framework, due to the endogenous distribution of workers share we derived in section two.

Second, the productivity shock becomes crucial in the evolution of the distribution of income over time. It follows that we can add to the standard analysis the evaluation of income dispersion and how does a total factor productivity shock affect income distribution. Without paying any cost in terms of modeling we have a tool to investigate the dynamic of income distribution over the cycle and its correlation with real macroeconomic variables. Finally, from an empirical point of view, notice that in case we buy the assumption that the elasticity between human factors is one then we do not need any additional information or parameter estimation in order to obtain the distribution of income. Nevertheless, the extension to the general framework remains tractable. The following example shows how the general framework can be easily calibrated.

2.4.4.1 A more general specification

In our framework the composition of the population and factors intensity in production are closely related. In order to discuss the more general specification with an elasticity of substitution between human factors different from one we define the measure of *relative abundance*, *intensity* and *excess of demand*, in our two factors framework.

Definition (3) An economy is relative abundant in a human factor if and only if the largest share of the population consists of the type of agent endowed with that human factor. The first moment of the distribution of factor's share across the economy is the measure of factor abundance.

Definition (4) A matching technology is relatively intensive in a human factor if and only if the factor's share is larger than $\frac{1}{2}$.

Definition (5) An economy shows relative excess of demand in a factor if and only the factor share in the matching technology is larger than the share of the population who owns that factor.

For a given mean worker share in the population $\mu \in (0, 1)$ and labor intensity in the matching

$\gamma \in (0, 1)$ we identify the parameters of the distribution of workers share:

$$\alpha_0 = \mu\left(\frac{2\gamma-1}{\gamma-\mu}\right)$$
$$\beta_0 = (1-\mu)\left(\frac{2\gamma-1}{\gamma-\mu}\right)$$

Our representation of the composition of the population requires $\alpha_0, \beta_0 > 0$. This condition imposes a restriction on the link between relative factor abundance and intensity: the matching technology is relatively intensive in one factor if and only if the factor's share in the matching is larger than the relative abundance of that factor in the economy. Two complementary scenarios are possible: (a) the case $\gamma > \frac{1}{2}$ if and only if $\gamma > \mu$ describes a labor intensive economy in excess of labor demand; (b) the case $\gamma < \frac{1}{2}$ if and only if $\gamma < \mu$ describes an economy intensive in human capital and in excess of demand for human capital.

Moreover, we want either $\alpha_0, \beta_0 > 1$ or $\alpha_0, \beta_0 < 1$ to guarantee that both factors add positive value to the matching. Combining the properties of the distribution and the economics requirements we have two possible scenarios that link the shape of the distribution of the worker share and the abundance of labor in the economy: (c) the case $\alpha_0, \beta_0 > 1$ if and only if $\mu > \frac{1}{2}$ describes an economy with unimodal distribution of worker share across productive location and an abundance of workers; (d) the case $\alpha_0, \beta_0 < 1$ if and only if $\mu < \frac{1}{2}$ describes an economy with polarized distribution at the extremum of the support and largely populated by owners of human capital.

Each of the four possible scenarios will have an implication on the shape of the income distribution across agents. As an example, consider an economy with labor share in the intermediate technology $\gamma = \frac{2}{3}$, (that implies human factors are imperfect substitutes with elasticity $\epsilon = 2$) and the distribution of worker share across productive locations is unimodal with mean $\mu = 0.65$. We select the parameters of the worker share distribution such that they represent this idea of the economy: $\alpha_0 = 3.25$,

$\beta_0 = 1.75$. The shape of the distribution is drawn in the following figure:

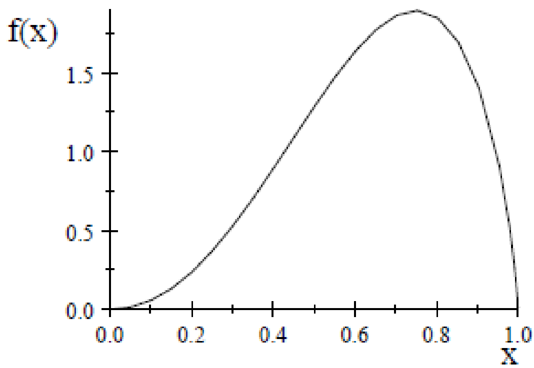

Figure 2.3: Density of the worker share in the example

The next figures show the density of wage and salary respectively for this example; the solid line is drawn for a benchmark level of y_t while the dashed line holds for a larger level of output per unit of human service.

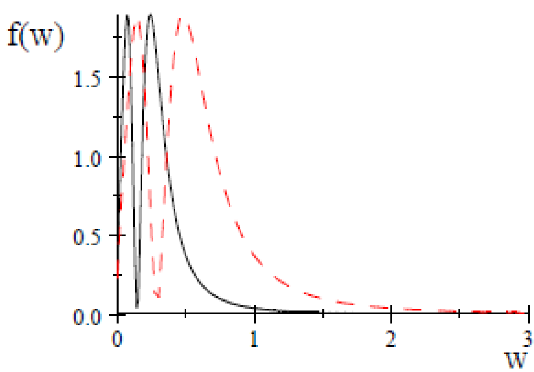

Figure 2.4: Density of wage

In order to obtain distribution of income we weight the two densities, taking into account that a share μ of the population are workers paid with wage, and the complement $(1 - \mu)$ agents are managers paid with salary. The next picture shows that our framework predicts a double humped

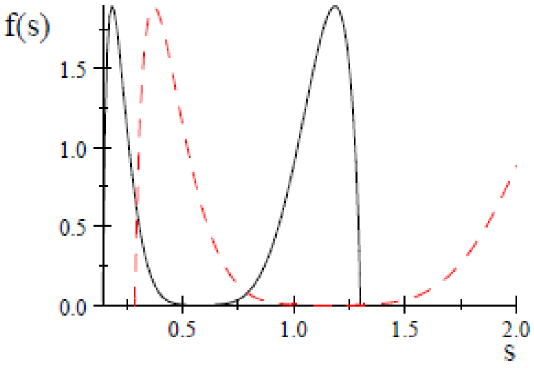

Figure 2.5: Density of salary

shape of the income distribution.

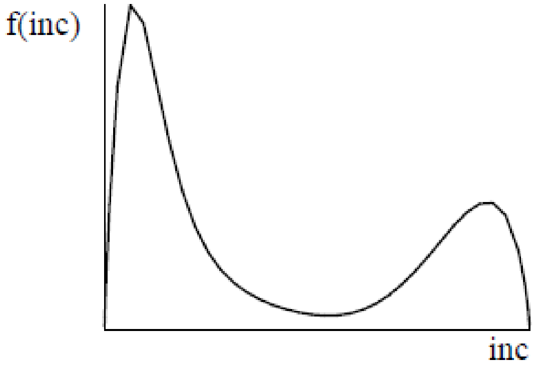

Figure 2.6: Density of income

Remarkably notice that a double humped shape is the result of the interplay between composition of the population and technology. In fact, perhaps surprisingly we obtained a polarization of income distribution despite the distribution of workers share is not polarized and the two types of human factors are likely substitutes.

2.5 Conclusion

This paper provides a simple framework to handle heterogeneity across agents in a real business cycle model. Agents are of two types according with the human factor of production they own. Agents are located in separated cities and in each city competitive firms choose the technology that maximizes output subject to the composition of the local population. Assuming the population of agents is a collection of exchangeable random variables we micro-found the matching function that combine human factors into a human service used in the production of a final good, with physical capital. In each productive location a representative agent exists and physical capital and goods are perfectly traded in the economy across locations. Standard techniques allow us to solve for the dynamic stochastic general equilibrium in each city. The integration over the distribution of types across productive location gives the macroeconomics variables at the economy level.

The contribution of our framework is clearly to allow for the analysis of the distribution of income across agents within the standard RBC framework. We obtain this result assuming that the population can be modeled as a sequence of exchangeable of agents. We recognize from the very same definition of exchangeability that this assumption is already satisfied under the standard macroeconomic. In fact, as long as the matching process is modeled as function of the total number of agents who match the order in which they match is irrelevant and that qualifies the population as a sequence exchangeable agents. In this respect, our theoretical point is to provide a framework that allows to perform a richer analysis of the business cycle properties that extends to the evolution of the distribution of factors prices, income and inequality along the cycle.

The model is highly tractable and identification and calibration of the parameters comes straightforward from the links between the composition of the population and technology. A strong point of this work is to predict shapes of the densities that we find in the empirical literature. In a simple example with standard specification of the parameters, we show that the model predicts humped densities of factor prices and double humped densities of income. Perhaps surprisingly, we obtain this result without imposing a polarization of the composition of the population. A minor but still interesting contribution of the paper is to provide a framework that directly links macroeconomics with economics demography and geography. A line of research that could be worked out in our framework taking both the approach of either exogenous or endogenous growth. Further research in this direction will enhance our understanding about the links among composition of the population, location of productive activities and technology adoption, over the business cycle.

The current framework has at least two major limitations from the empirical point of view. First, the composition of population is independent across productive locations. A natural development of this paper is to introduce dependence across productive locations. In a second work we extend the present framework to allow for the population of agents being a sequence of partial exchangeable random variables in which we can control for the dependence across productive locations. The second limitation is the choice of two types of agents only. This issue can be overcame extending the present framework with a Bernoullian statistical model to a non parametric approach based on the Dirichelet process. We think that this direction of research will provide an extremely tractable framework to fit income distributions and their pattern of change over the cycle.

Chapter 3

Aggregate Productivity Shocks, Exporters and Unemployment Dynamics

Abstract

This paper develops a theoretical framework to analyze the effect of aggregate productivity fluctuations on the unemployment rate dynamics when (i) workers search while they are employed and (ii) firm entry and exit and export decisions are endogenous.

Despite technology is linear in labor and workers are all homogeneous, aggregate fluctuations affect wages and profit in an asymmetric way. More productive firms and their employees benefit from a positive productivity shock disproportionately more than low productive firms and their employees. Furthermore, an increase in aggregate productivity induces less productive firms out of the market. Selection is stronger the larger is the share of exporter firms. The temporary increase in aggregate productivity determines that more firms become exporters. The dynamics of the

unemployment rate results from the demand of labor of new exporters, the larger firm turnover and the entry of new firms. At the time of the shock, the selection effect dominates on the additional demand of labor by new exporters: the unemployment rate increases more than proportionally with the productivity shock. Afterward, the entry of new firms leads the decrease in the unemployment rate as the economy reaches the steady state.

3.1 Introduction

This paper develops a dynamic stochastic general equilibrium model to analyze the effect of productivity cycle fluctuations on labor market dynamics. The contribution of this framework consists in assessing this research question in a framework with (*i*) endogenous entry and exit decisions, (*ii*) endogenous selection into the export market, and (*iii*) labor market is fully developed.

This paper is mainly related to three fields of research. First, it contributes to the literature on macroeconomic fluctuations and endogenous entry in the domestic and export market. As in [Ghironi and Melitz, 2005] and [Bilbiie et al., 2012], a positive aggregate productivity shock induces higher entry rates of perspective new entrants subject to sunk cost of entry and time-to-build lag. Differently from these frameworks, the model I propose allows for endogenous exit decisions.

Second, the model I propose is based on two prominent features of the recent literature in international trade theory. There is a consumption sector with heterogeneous producers of differentiated goods under endogenous markups according to a line of research that has been discussed and developed in [Melitz and Ottaviano, 2008]. In addition, there is a competitive sector producing input that is used by firms in the consumption sector, according to the approach suggested in [Grossman and Helpman, 1991].

Heterogeneity across firms is due to uncertainty about the outcome of investment in research and developments of firm level technology as in [Melitz, 2003]. Nevertheless in this model firms sample their idiosyncratic productivity each period. This feature is responsible for an additional channel of uncertainty that is not present in the Melitz framework and it becomes relevant when aggregate productivity shocks propagate through heterogeneous firms. The supply of fixed input is the result of production and accumulation. With respect to the accumulation process, this paper is close to the framework developed in [Ottaviano, 2011]. The fixed input is a durable asset in which workers save, as a result of an optimal intertemporal consumption allocation. Nevertheless, the production of new assets is the result of the allocation of workers in the two sectors. This process is specific to the framework I propose. It is driven by worker's search and ultimately it depends on the distribution of wage across employers that is endogenously determined.

Third, this framework is related to the recent literature on macroeconomic fluctuations on labor market dynamics [Moscarini and Postel-Vinay, 2011]. In particular the labor market is characterized by on-the-job search, a la [Burdett and Mortensen, 1998] and [Mortensen, 2009]. As a consequence, a large share of worker reallocations are job to job transitions instead of flows from/to unemployment (as in [Diamond, 1982], [Mortensen and Pissarides, 1994]). This feature, jointly with random search and matching, introduces an additional competition channel among firms that takes place in the labor market. Aggregate productivity shocks propagate through this channel in a disproportional way across firms depending on idiosyncratic productivity.

The main result of the paper shows that a temporary increase in aggregate productivity implies that more firms become exporters. The dynamics of the unemployment rate result from the demand of labor of new exporters, the larger firm turnover and the entry of new firms. At the time of the shock, the selection effect dominates on the additional demand of labor by new exporters: the unemployment rate increases, more than proportionally with the productivity shock. Afterward, the entry of new firms leads the decrease in the unemployment rate as the economy reaches the steady state.

The paper is structured as it follows. Section two describes the model with focus on the equilibrium of a given period. Section three characterizes the intertemporal dynamic problem. Section four discusses the dynamic properties of the model and section five concludes.

3.2 Model: one period

There is a small open economy, two sectors producing a consumption tradable good and a durable non-tradable good. In the consumption sector a continuum of single product monopolists supply varieties of a differentiated good. In the durable good sector a continuum of perfectly competitive producers supply a asset of capital that is used as input in the consumption sector. Labor is used in both sectors as variable factor of production. Workers are the owners of production inputs and they rent them to firms.

3.2.1 Consumer preferences

Preferences are non homothetic and the marginal utility of income depends on the number of competitors in the market, first and second moments of their price distribution. Consumers allocate consumption over a continuum of varieties indexed by $i \in \Omega(i)$:

$$U\left(\{c^{\omega}(i)\}_{i\in\Omega(i)}\right) = \alpha \int_{i\in\Omega(i)} c^{\omega}(i)\, di - \frac{\gamma}{2}\int_{i\in\Omega(i)} c^{\omega}(i)^{2}\, di \qquad (3.1)$$

where $c^{\omega}(i)$ is the consumption of variety i of agent ω. This utility function is a special case of [Ottaviano et al., 2002] in which agents do not value aggregate consumption and they do not consume an outside good. The demand system for each variety is linear. The parameter $\alpha > 0$ is a demand shifter and parameter $\gamma > 0$ accounts for the sensitivity of demand to price. Condition $c^{\omega}(i) \leq \alpha/\gamma \; \forall i$ guarantees positive and diminishing marginal returns.

Over a population of N consumers, the aggregate demand for each variety is given by:

$$C_t(i) = \frac{N}{\gamma}\left(\alpha - E_{\omega}\left[\lambda_t(\omega)\right] p_t(i)\right) \tag{3.2}$$

where $E_{\omega}[\cdot]$ is the expectation operator over the space of agents $\Omega(\omega)$, $\lambda(\omega)$ is the marginal utility of income for agent ω at time t, $p(i)$ is the price of variety i and it is bounded above $p_t(i) < \frac{\alpha}{E_{\omega}[\lambda_t(\omega)]} \equiv p_{\max}$ for every variety that is sold in the market.

3.2.2 Technology

Productivity of each firm consists of two components. The aggregate productivity that is common to all firms follows a two states stochastic Markov process:

$$z_t = k_z z_{t-1} + \xi_t \tag{3.3}$$

where $k_z \in (0,1)$ and ξ_t are i.i.d. normally distributed withe noise with zero mean.

In the consumption sector the productivity of labor is the product of aggregate productivity and a firm specific efficiency measure. Define $c \in [\underline{c}, \overline{c}]$ as the inverse of firm idiosyncratic productivity, with $\underline{c} < \overline{c}$ strictly positive finite values and $\Phi(c)$ as the distribution of c across firms over the support $c \in [\underline{c}, \overline{c}]$. Then $\varphi_t = \frac{z_t}{c}$ is the productivity of a firm endowed with unit labor requirement c at time t. The production function in the consumption sector is:

$$q_t(\varphi_t) = \varphi_t l_t \tag{3.4}$$

where l_t is firm employment and q_t is firm output. Unit labor requirements are i.i.d. draws from the same distribution. This modeling assumption is a special case of [Hopenhayn, 1992a] and it significantly increases the tractability of the model. The implications of the model in terms of the evolution of wage distribution and unemployment will not be driven by this assumption. In order to understand why it is necessary to introduce the key features of the labor market equilibrium. Therefore I postpone the discussion.

In the capital sector there is perfect competition and producers are homogeneous. Production in the capital sector is equivalent to the output of $E_t^{cap} > 0$ homogeneous firms with productivity z_t employing l_t^{cap} workers:

$$A_t = z_t E_t^{cap} l_t^{cap} \tag{3.5}$$

where A_t are the new units of capital produced at time t.

3.2.3 Fixed costs

A firm purchases units of the capital as fixed input employed in the activities of entry, production and export. In the trade literature it is common to assume that monopolistic heterogeneous firms pay fixed cost of production to differentiated products in the domestic market and additional fixed costs to customize products for the export market. In this framework I take advantage of the linear demand schedule, firms that are not productive enough do not face a positive demand.

The only source of fixed cost related to technology are sunk costs of entry in the domestic and in the export market. The price of each unit of capital is chosen to be one; hence capital is the numeraire. Firms that enter the domestic market for the first time require f_{ed} units of capital. A firm that enters the export market for the first time need f_{ex} units of capital.

There is an additional fixed cost due to the need to adjust employment over time. Firms pay for advertising vacancies, hiring and training costs, in addition separations are costly to the extent that employment protection and unemployment insurance are enforced by law. The literature on search in the labor market treats these costs as match specific investments, [Mortensen and Pissarides, 1999][1]. However in the context of this work, there are limitations to which that assumption is justified. Firms change their productivity each period exogenously, workers are homogeneous. There is no incentive for either of the firm or the worker to invest in a match specific relationship.

Under these considerations, I assume that the fixed costs of employment adjustment is specific to the firm that decides to stay in the market. Incumbent firms that adjust employment between two subsequent periods pay a price of f_p units of capital.

3.2.4 Firm level variables

Let $w_t(c)$ be the wage paid by a firm endowed with unit labor requirement c at time t. The profit maximizing price $p_t(c)$ satisfies the equivalence between marginal cost $w_t(c)(1/\varphi_t)$ and marginal revenue $2p_t(c) - \frac{\alpha}{E_\omega[\lambda_t(\omega)]}$. As in [Melitz and Ottaviano, 2008], define c_D as the maximum unit labor requirement below which a firm faces positive demand, $p_t(c_D) = p_{\max}$, such that[2]

$$w_t(c_D)\, c_D = \frac{\alpha}{E_\omega[\lambda_t(\omega)]} \tag{3.6}$$

Then equilibrium price, output, variable profit (revenue minus variable

[1]A more general discussion is developed in [Coşar et al., 2010]. [Ljungqvist, 2002] provides a discussion of the effect of layoff costs in a general equilibrium framework.

[2]Notice that I am assuming that the product $w_t(c_D)\, c_D$ varies over time only through the wage whereas the unit labor requirement cutoff c_D is constant over time. This characterization is witot loss of generality since incumbent firms will always be endowed with an endogenous unit labor requirement that is strictly lower than c_D.

cost) associated to the production in the domestic market are:

$$
\begin{aligned}
p(c, z_t) &= \frac{1}{2}\left(w_t(c_D)\, c_D + \frac{w_t(c)}{\varphi_t}\right) && (3.7)\\
C^{dom}(c, z_t) &= \frac{N}{2}\frac{\alpha}{\gamma}\left(1 - \frac{w_t(c)}{w_t(c_D)}\frac{1}{c_D\varphi_t}\right) && (3.8)\\
\pi^{dom}(c, z_t) &= \frac{N}{4}\frac{\alpha}{\gamma} w_t(c_D)\, c_D \left(1 - \frac{w_t(c)}{w_t(c_D)}\frac{1}{c_D\varphi_t}\right)^2 && (3.9)
\end{aligned}
$$

Firms of the domestic economy that export face the same preference and technology structure but a competitive environment that is possibly different from the domestic one. There is no variable cost associated to trade, exporters set the price according to producer currency pricing and the law of one price holds. As a result of this scenario, the marginal revenue associated to sales of the same product in the domestic or foreign markets is the same. Demand, employment and profit associated to the foreign market are the same as in the domestic economy, but for the market size:

$$
C_t^{exp} = \frac{N^*}{N} C_t^{dom}, \; l_t^{exp} = \frac{N^*}{N} l_t^{dom}, \; \pi_t^{exp} = \frac{N^*}{N} \pi_t^{dom} \tag{3.10}
$$

where N^* is the population in the foreign economy.

The demand of labor at the firm level is:

$$
l_t^d = \begin{cases} \frac{N}{2}\frac{\alpha}{\gamma}\left(1 - \frac{w_t(c)}{w_t(c_D)}\frac{1}{c_D\varphi_t}\right)\frac{1}{\varphi_t} & \text{, } \textit{if non exporter at time } t \\ \frac{N+N^*}{2}\frac{\alpha}{\gamma}\left(1 - \frac{w_t(c)}{w_t(c_D)}\frac{1}{c_D\varphi_t}\right)\frac{1}{\varphi_t} & \text{, } \textit{if exporter at time } t \end{cases} \tag{3.11}
$$

The demand of labor of an exporter is $\left(1 + \frac{N^*}{N}\right)$ times larger than if the same firm was not exporting. In order to account for the number of employers in the market I take the following convention. An employer unit is a firm that demands labor to serve the domestic market only. It follows that an exporter accounts for $\left(1 + \frac{N^*}{N}\right)$ employer units.

3.2.5 Labor market

Workers are homogeneous and they search for better job offers also while they are employed as in [Burdett and Mortensen, 1998][3]. The search and matching process is random and time is discrete. When a worker-firm match occurs the two parties bargain over the match surplus as risk neutral agents[4]. In every negotiation there is a potentially infinite number of offers and firms move to the production stage only when an agreement is reached or the firm and the employee separate[5]. There is complete information and both parties know the distribution of wage offers in the economy.

Contracts are not binding in the sense of [Stole and Zwiebel, 1996], they only specify the current wage and they cannot be made contingent upon outside offers or future realizations of aggregate neither idiosyncratic productivity. Technology exhibits constant returns on labor, workers are homogeneous and each of them supplies one unit of labor. In the consumption sector, the source of heterogeneity across firms is exogenous, it only depends on unit labor requirement draws that realize at the beginning of each period. The cost of hiring is not match specific, it has been summarized into the fixed cost of production f_p. Because of perfect competition in the capital sector, the reservation wage is fixed as the value of the marginal productivity of labor in the capital sector $R_t = z_t$.

Under this framework, the dynamic general equilibrium remains highly tractable. The equilibrium is characterized by a wage offer distribution

[3][Postel-Vinay and Robin, 2002] develop a structural model with workers and firms heterogeneity based on French panel data.They find that the extent to which worker individual characteristics explain wage differences lies close to 40% for high-skilled white collars, but it falls to be negligible as the observed skill level decreases.

[4]The assumption upon which workers behave as risk neutral agents in the labor market is common to works that are our benchmark in this topic (see [Helpman and Itskhoki, 2010] for a discussion).

[5]See [Stole and Zwiebel, 1996] footnotes 15 and 17 for a more detailed discussion on the stable bargaining outcome under the infinite number of negotiations, versus alternative specifications.

that is a collection of wage offers across firms such that they are a solution of the wage bargaining. The uniqueness of the wage offer distribution is guaranteed by the additional property that the wage offer distribution must identify a feasible allocation of wage and employment such that demand and supply of labor clear at the firm level for all firms each point in time.

3.2.5.1 Definitions

On the supply side of the labor market a mass of N homogeneous workers randomly look for job offers. On the demand side, a mass of heterogeneous employers opens V_t vacancies at time t. The labor market tightness at time t is:

$$\theta_t = \frac{V_t}{N} \tag{3.12}$$

Workers and firms match randomly. New matches are formed according to a matching function homogeneous of degree one:

$$Matches\,(V_t, N) = V_t^{1/2} N^{1/2}$$

The probability that a single worker randomly finds a job offer over the V_t vacancies is an increasing function of the labor market tightness:

$$m\,(\theta_t) = \frac{V_t^{1/2} N^{1/2}}{N} = \sqrt{\theta_t} \tag{3.13}$$

as the share of new matches over the entire workforce. Each period an employed worker separates from the current match with a probability $\delta \in (0, m\,(\theta_t))$ due to an exogenous job destruction shock. The total number of unemployed workers at time t is u_t. There is perfect information about the distribution of wage offers $F_t\,(w)$ and the distribution of employed workers across wages $G_t\,(w)$; the two cdf are defined over a compact wage support $\left[\underline{w_t}, \overline{w}_t\right]$, for $\underline{w_t} < \overline{w}_t$ positive real values.

3.2.5.2 Timing

Time is discrete. Each period is a sequence of four stages:

0. **Shock**. The uncertainty about unit labor costs realizes. Firms understand the optimal wage and employment level they will perform with during the current period.

1 **Adjustment**. Firms that enter the market compare the current employment level with the optimal employment level and they adjust their workforce. Intra-firm costless renegotiations take place based on the new optimal wage and employment levels. At the end of this stage, either incumbent firms are matched with the desired level of employment or they have open vacancies. Moreover there exist a number of workers who entered the period as employed and are waiting for a reallocation.

2 **Reallocation**. All firms face job to job transitions.

- *Exogenous separation.* A share δ of total workers who entered the period as employed at the end of the period will be unemployed.
- *Search and Matching.* All N workers are randomly searching. With probability $m(\theta_t)/\theta_t$ a firm meets a worker. Each period an employed worker has a probability $1-\delta$ to remain employed up to the next period, but he/she may change employer. During the period, an employed worker receives a wage offer from another employer with probability $m(\theta_t)$, that is the same probability at which an unemployed worker visits the same vacancy. With probability $1-\delta-m(\theta_t)>0$ an employed worker receives a wage offer from his current employer, at the new wage.
- *Job to job transitions.* Employed workers accept wage offers that are better with respect to their current employment sta-

tus. At the end of the period employed workers move to an other employer with probability $m(\theta_t)\ [1-F_t(w)]$, or he/she remains with the current employer with probability $[1-m(\theta_t)+m(\theta_t)\,F_t(w)]$.

– *Outflows from unemployment.* Unemployed workers accept all wage offers above or equal to the reservation wage. An unemployed worker becomes employed with probability $m(\theta_t)\,[1-F_t(R_t)]$, otherwise with probability $1-m(\theta_t)+m(\theta_t)\,F_t(R_t)$ stays unemployed.

– *Inflows to unemployment.* Employed workers who separated and unemployed workers that did not find a match end the period as unemployed.

At the end of this stage all firms have the optimal level of employment according to their unit labor cost. Total outflows from unemployment is given by: $m(\theta_t)\,[1-F_t(R_t)]\,u_t$. Total inflows into unemployment is given by: $\delta\ (N-u_t)$.

3. **Production**. Firms start production when they reach the optimal employment level. Over the period they issue a number of vacancies that is sufficient to keep their workforce at the desired level.

3.2.5.3 Workers and firms, wage and profit

Employed workers have the same probability of unemployed workers to visit a given wage offer in the market. Still there is a difference in the probability of finding a job. Once a worker is matched to an employer he/she has a probability $1-\delta-m(\theta_t)>0$ to receive a wage offer from his/her current employer. In addition to the probability $m(\theta_t)$ of matching with a new employer. Since each period unit labor costs are i.i.d. draws across employers, the continuation value of employment given the current information is the same, regardless the productivity in the current match, either the match with the current employer continues or not.

The asset value of employment is:

$$W(w_t) = w_t + \beta E_{z_{t+1}} \left\{ \begin{array}{l} \delta U(z_{t+1}) + \\ +(1-\delta) E_w \left[\max \left\{ W(w_{t+1}), U(z_{t+1}) \right\} \right] \end{array} |z_t \right\}$$

$$U(b_t, z_t) = b_t + \beta E_{z_{t+1}} \left\{ \begin{array}{l} (1-m(z_{t+1})) U(z_{t+1}) + \\ +m(z_{t+1}) E_w \left[\max \left\{ W(w_{t+1}), U(z_{t+1}) \right\} \right] \end{array} |z_t \right\}$$

where $W(w_t, z_t)$ is the asset value of being employed at a wage w_t when the state of the economy is z_t, $U(b_t, z_t)$ is the asset value of unemployment, $b_t > 0$ is the unemployment benefit at time t, $E_w[\cdot]$ is the expectation operator across wage offers, $E_{z_{t+1}}\{\cdot|z_t\}$ is the expectation operator conditional on the current state of the economy z_t and I refer to $m(z_{t+1})$ as the probability $m(\theta_{t+1})$ when the state of the economy z_{t+1} is understood.

A reservation policy holds for unemployed and employed workers who randomly receive a wage offer. The optimal policy for unemployed workers is to accept any wage offer that is larger or equal the reservation wage, $w_t \geq R_t$. The optimal policy for an employed worker is to accept a wage offer that is strictly larger than the current wage; where the specification of a strict inequality is used as a convention with the aim to capture an implicit switching cost. The optimally of the two policies is clear by noticing that any sudden deviation would imply a strictly lower current value and the same continuation value, because the draws of unit labor requirement are i.i.d. across employers.

The worker's surplus in a match is equal to the difference between the current wage and the reservation wage:

$$W(w_t, z_t) - U(z_t) = w_t - R_t \tag{3.14}$$

The value of filling a vacancy for a firm is equal to the marginal profit per worker in the current period:

$$J(w_t, z_t) = p_t \varphi_t - w_t \tag{3.15}$$

Only firms that decided to stay in the market in the current period participate in the labor market. Two different bargaining processes occur simultaneously: renegotiation of the firm with current employees and job to job reallocation. The value of filling a vacancy is the same regardless if the worker is already employed in the firm or not, as the cost of employment adjustment is not specific to the match. Negotiations between firms and workers follow the extension of [Rubinstein, 1982] discussed in [Stole and Zwiebel, 1996]. Costless alternate bargaining takes place under complete information and non binding contracts. The subgame perfect equilibrium is described by the symmetric Nash bargaining solution with threat points the value of a vacant job for the firm and the value of unemployment for the worker: A firm endowed with unit labor cost c at time t offers a wage:

$$w_t = \arg\max_{w_t} \left(w_t - R_t\right)^{\frac{1}{2}} \left(p_t \varphi_t - w_t\right)^{\frac{1}{2}}$$

Firms choose a wage that is consistent with optimal pricing, as they factor in the price as linear function of the wage by (1.5). The optimal wage is linear in the reservation wage, linear in the idiosyncratic productivity of the firm:

$$w_t = \frac{R_t}{2}\left(1 + c_D \varphi_t\right) \tag{3.16}$$

Notice that the wage can also be expressed as a quadratic function of the aggregate productivity process z_t:

$$w\left(c, z_t\right) = \frac{z_t}{2}\left(1 + \frac{c_D}{c} z_t\right) \tag{3.17}$$

This very simple characterization explains why aggregate productivity shocks affect wage (and consequently employment) disproportionately over the heterogeneous population of firms. Figure 3.1 shows the wage as a function of idiosyncratic unit labor requirement when the aggregate productivity is in steady state (solid line) and after a positive productivity shock (dash line).

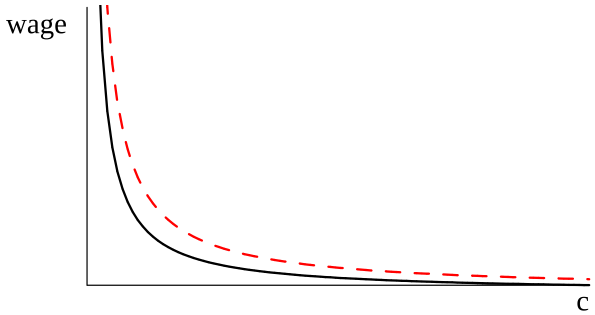

Figure 3.1: A common aggregate productivity shock has an asymmetric impact on wages

The positive effect of aggregate productivity propagates more than proportionally into wages. Moreover the combination of firm heterogeneity and wage dispersion matters. The expansion in wages is asymmetric, it is larger for more efficient employers.

Combining (1.9), (3.10) and the wage equation for c_D, $w\left(c_D, z_t\right) = \frac{z_t}{2}\left(1+z_t\right)$ and c, $w\left(c, z_t\right) = \frac{z_t}{2}\left(1+\frac{c_D}{c}z_t\right)$ then profits in the domestic and foreign markets are:

$$\Pi^{dom}\left(c, z_t\right) = \frac{N}{8}\frac{\alpha}{\gamma}c_D z_t\left(1+z_t\right)\left(\frac{c_D z_t^2 - c}{c_D z_t\left(1+z_t\right)}\right)^2 - f_p \qquad (3.18)$$

$$\Pi^{exp}\left(c, z_t\right) = \frac{N^*}{8}\frac{\alpha}{\gamma}c_D z_t\left(1+z_t\right)\left(\frac{c_D z_t^2 - c}{c_D z_t\left(1+z_t\right)}\right)^2 \qquad (3.19)$$

As in the case of wage, the impact of a common aggregate productivity shock is asymmetric across firms. Figure 3.2 shows the change in profit as a function of unit labor requirement.

The solid line describes profit in steady state, the dash line represents a positive shock to the aggregate productivity z_t. An expansion of aggregate productivity benefits the entire population of firms, but more productive firms gain disproportionately more.

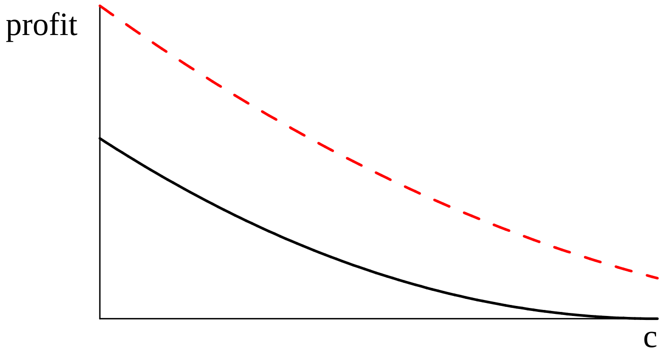

Figure 3.2: A common aggregate productivity shock has an asymmetric impact on profit

3.2.5.4 Employment and wage distribution

The measurement of employment flows refers to the value they attain in the production stage and they maintain throughout the rest of the period. At that stage, a firm coincides with an employer unit if it serves the domestic market only. An exporter is an employer $\left(1+\frac{N^*}{N}\right)$ times bigger, then it accounts for $\left(1+\frac{N^*}{N}\right)$ employer units. There is no incentive for any firm (exporter or not) to increase the wage in order to attract more workers. A firm that paid the fixed cost of production at the beginning of the period can issue as many vacancies it needs to match with workers and reach the optimal employment level given the optimal wage.

When workers search for a job in the consumption sector, on the demand side of the labor market there are $\left(1+x_t\frac{N^*}{N}\right)E_t$ employer units. For an arbitrarily small $\varepsilon>0$, the measure of workers employed at a wage lower or equal to w is $[G_t(w)-G_t(w-\varepsilon)](N-u_t)$ and the measure of employer units offering a wage in the same interval is $[F_t(w)-F_t(w-\varepsilon)]$ $\left(1+x_t\frac{N^*}{N}\right)E_t$. As in [Burdett and Mortensen, 1998], the number of workers who are employed in an employer unit that offers a wage w is given by the following limit:

$$emp_t(w) = \lim_{\varepsilon \to 0} \frac{[G_t(w) - G_t(w-\varepsilon)]}{[F_t(w) - F_t(w-\varepsilon)]} \frac{(N - u_t)}{\left(1 + x_t \frac{N^*}{N}\right) E_t}$$

leading to the employment function:

$$emp_t(w) = \frac{\delta m(\theta_t)}{(\delta + m(\theta_t)[1 - F_t(w)])^2} \frac{N}{\left(1 + x_t \frac{N^*}{N}\right) E_t} \tag{3.20}$$

The labor supply at the firm level is:

$$l_t^s(w) = \begin{cases} emp_t(w) & \text{, if non exporter at time } t \\ \left(1 + \frac{N^*}{N}\right) emp_t(w) & \text{, if exporter at time } t \end{cases} \tag{3.21}$$

In a given period inflows from unemployment to employment are due to unemployed workers who match with a firm offering at least $w = R_t$, that is $m(\theta_t)[1 - F_t(R_t)] u_t$. Outflows from employment to unemployment are $\delta(N - u_t)$. In each point in time we define the equilibrium unemployment level such that flows in and out of employment are equalized:

$$\frac{u_t}{N} = \frac{\delta}{\delta + m(\theta_t)[1 - F_t(R_t)]} \tag{3.22}$$

Each point in time $m(\theta_t) u_t [F_t(w) - F_t(R_t)]$ unemployed workers move into employment accepting an offer at a wage w or lower, $\delta G_t(w)(N - u_t)$ workers who are hired at a wage w or lower separate because of exogenous job destruction, $m(\theta_t)[1 - F_t(w)]\ G_t(w)(N - u_t)$ move to an other job because they receive a better offer. The distribution of effective wages that satisfies the balance in employment flows is:

$$G_t(w) = \frac{\delta}{1 - F_t(R_t)} \frac{F_t(w) - F_t(R_t)}{\delta + m(\theta_t)[1 - F_t(w)]} \tag{3.23}$$

3.2.6 Entry, exit and export decisions

Firms take forward looking decisions of entry, export, stay in the market or exit on the basis of their current profit. Define $\overline{\Pi}(z_t)$ as the average profit across incumbent firms in period t. Let $c_{in,t}$ be the maximum unit labor requirement below which an incumbent firm makes non negative profits in the long run conditionally on staying in the market in period t. Each period new entrants and incumbent firms draw their unit labor requirements from the same distribution over the support $[\underline{c}, \overline{c}]$. Firms that draw a unit labor requirement too high will immediately choose to exit, at a zero value.

Free entry condition (FEC). Then in a given period t an incumbent firm does not exit the market with probability $\Phi(c_{in,t})$. For a given state of the economy in period s the value of entry is:

$$v_e(z_s) = E_s\left[\sum_{t=s}^{\infty} \Phi(c_{in,t})^{t-s}\,\overline{\Pi}(z_t)\right] \tag{3.24}$$

where $E_s[\cdot]$ is the expectation conditional on the information available in period s and for the sake of simplicity I assumed firms do not discount future profits. The free entry condition implies:

$$v_e \leq f_{ed} \tag{3.25}$$

where 3.25 holds with equality when there is positive entry; I restrict the discussion to this scenario.

Marginal firm profit condition (MFPC). The exit decision is taken given the information on the current unit labor requirement. An incumbent firm stays in the market as long as the expected lifetime stream of profit given the current period is larger or equal to the value of exit, that is zero. In a given period s, the marginal incumbent firm realizes negative profit on the domestic market such that:

$$\pi_s(c_{in,s}, z_s)^{dom} - f_p = -E_s\left[\sum_{t=s+1}^{\infty} \Phi(c_{in,s})^{t-s}\,\overline{\Pi} z_t\right]$$

The left hand side of (3.26) is monotonic decreasing in $c_{in,t}$ whereas the right hand side does not depend on $c_{in,t}$ because the distribution of unit labor requirements is time invariant. For a non trivial support $[\underline{c}, \overline{c}]$ such that a cutoff value $c_{in,t}$ does exists given f_p, then $c_{in,t}$ is unique for a given level of the continuation value $E_0[\sum_{t=1}^{\infty} \Phi\left(c_{in,t}\right)^t \overline{\Pi}\left(z_t\right)]$.

A new exporter takes the decision to invest in the export market given the information on the current unit labor requirement. It follows that the profit of the marginal new exporter on the export market satisfies:

$$\pi_t\left(c_{x,s}, z_s\right)^{exp} + E_s\left[\sum_{t=s+1}^{\infty} \Phi\left(c_{in,s}\right)^{t-s} \overline{\Pi}^{exp}\left(z_t\right)\right] = f_{ex} \qquad (3.26)$$

The left hand sinde of (3.26) is monotonic decreasing in $c_{x,s}$ the right hand side is fixed. For a non trivial support $[\underline{c}, \overline{c}]$ such that a cutoff value $c_{x,t}$ does exists given f_{ex}, then $c_{x,t}$ is unique for a given level of the continuation value $E_0[\sum_{t=1}^{\infty} \Phi\left(c_{in,t}\right)^t \overline{\Pi}^{exp}\left(z_t\right)]$.

The expected lifetime stream of profits among incumbent firms $E_s\left[\sum_{t=s}^{\infty} \Phi\left(c_{in,t}\right)^{t-s} \overline{\Pi}\left(z_t\right)\right]$ is given by:

$$\begin{aligned}
&= E_s\left[\sum_{t=s}^{\infty} \Phi\left(c_{in,t}\right)^{t-s}\left(\overline{\Pi}^{dom}\left(z_t\right) + \frac{\Phi\left(c_{x,t}\right)}{\Phi\left(c_{in,t}\right)}\overline{\Pi}^{exp} z_t\right)\right] \\
where &: \\
\overline{\Pi}^{dom}\left(z_t\right) &= \frac{1}{\Phi\left(c_{in,t}\right)}\int_{\underline{c}}^{c_{in,t}}\left[\pi\left(x, z_t\right)^{dom} - f_p\right] d\Phi\left(x\right) \\
\overline{\Pi}^{exp}\left(z_t\right) &= \frac{1}{\Phi\left(c_{x,t}\right)}\int_{\underline{c}}^{c_{x,t}} \pi\left(x, z_t\right)^{exp} d\Phi\left(x\right)
\end{aligned}$$

Together with (3.26) and (3.26), condition (3.27) express the lifetime expected stream of profit as a function of the stream of cutoffs $\{c_{in,t}\}_{t=0}^{\infty}$; for this reason I refer to (3.27) as the marginal firm profit condition. Condition (3.24) and equality in (3.25) characterize the free entry condition. The free entry equilibrium implies:

$$\mathbb{E}_s\left[\sum_{t=s}^{\infty}\left(\int_{\underline{c}}^{c_{in,t}}\left[\pi\left(x, z_t\right)^{dom} - f_p\right] d\Phi\left(x\right) + \int_{\underline{c}}^{c_{x,t}} \pi\left(x, z_t\right)^{exp} d\Phi\left(x\right)\right)\right] = f_{ed} \qquad (3.27)$$

The expected lifetime profit across incumbent firms is constant over time and equal to the sunk cost of entry. The population of firms and the extensive margin of export adjust consistently with $c_{in,t}$ and $c_{x,t}$ such that 3.27 is satisfied each point in time. Condition 3.27 uniquely determines $c_{in,t}$ and $c_{x,t}$ as a function of aggregate productivity z_t for a given choice of the distribution $\Phi(\cdot)$.

3.2.6.1 Characterization of the free entry condition

I assume that the distribution of unit labor requirement is an inverse Pareto truncated in the support $[\underline{c}, \overline{c}]$ with shape parameter $\rho > 2$. The cdf and pdf take the form:

$$\Phi(c) = \frac{c^\rho - \underline{c}^\rho}{\overline{c}^\rho - \underline{c}^\rho} \text{ and } \phi(c) = \frac{\rho c^{\rho-1}}{\overline{c}^\rho - \underline{c}^\rho} \tag{3.28}$$

where without loss of generality $\overline{c} \leq c_D$. The export cutoff is a continuous increasing function of the incumbent cutoff c_{in} and a decreasing function of the sunk cost of export f_{ex}. In order to compute the expected profit it is convenient to define the operator $I(\cdot) : [\underline{c}, \overline{c}] \to \mathbb{R}$ that consists of the integral function: Collect the parameters that describe preferences and technology $\frac{\Theta}{z_t(1+z_t)} = \frac{1}{8}\frac{\alpha}{\gamma}\frac{1}{c_D}$. The expected profit among incumbent firms is a monotonic increasing function of $c_{in,t}$ and z_t:

$$\overline{\Pi}(c_{in,t}, z_t) = \frac{\Theta\left(NI(c_{in,t}) + N^* I(c_{x,t})\right)}{z_t(1+z_t)} - f_p \tag{3.29}$$

where $I(x, z_t) = \int_{\overline{c}}^{x} \left(c_D z_t^2 - x\right)^2 c^{\rho-1} dc$; that is a continuous and differentiable in the compact support $[\underline{c}, \overline{c}]$, with positive first derivative in both arguments. The free entry condition can now be written as:

$$\mathbb{E}_s \left[\sum_{t=s}^{\infty} \left(\frac{c_{in,t}^\rho - \underline{c}^\rho}{\overline{c}^\rho - \underline{c}^\rho}\right) \overline{\Pi}(c_{in,t}, z_t)\right] = f_{ed} \tag{3.30}$$

Each point in time, for a given z_t there exists a unique intersection between (3.29) and (3.30). The left hand side of (3.30) is increasing in the aggregate productivity z_t. This leads to conclude that positive aggregate productivity shocks increase the selection of less productive firms out of the market.

The intuition behind this result is the asymmetric impact of aggregate productivity shock on profit across firms. Following a positive aggregate productivity shock, more productive firms gain market shares and attract disproportionately more workers than less productive firms.

3.2.7 Consumption good market and labor market equilibrium

Entry and exit of firms and labor market adjustment happen simultaneously. Firms draw their unit labor cost, bargain with inside and outside workers. Employment at the firm level is determined by the labor demand in the domestic and export market and the wage equation. The only candidate correspondence $F_t(w, z_t)$ such that demand and supply clear at the firm level is:

$$F_t(w, z_t) = \begin{cases} F_t(R_t) & \text{for } w < \underline{w_t} \\ 1 - \left(\left(\frac{(2w-z_t)^2}{(w-z_t)}\frac{2}{z_t}\right)^{\frac{1}{2}} - 4\right)\frac{4}{\frac{\alpha}{\gamma}c_D\left(1+x_t\frac{N^*}{N}\right)E_t} & \text{for } w \geq \underline{w_t} \end{cases} \tag{3.31}$$

where $F_t(R_t) \leq F_t\left(\underline{w_t}, z_t\right)$.The minimum wage in the consumption sector is a monotonic decreasing function of the cutoff $c_{in,t}$:

$$\underline{w_t} = \frac{z_t}{2}\left(1 + \frac{c_D}{c_{in,t}} z_t\right) \tag{3.32}$$

The probability of finding a job is determined by the boundary condition $F_t\left(\frac{3}{2}z_t, z_t\right) = 1$as $m(\theta_t) = \frac{\delta\frac{\alpha}{\gamma}c_D\left(1+x_t\frac{N^*}{N}\right)E_t}{16}$. In a given point in time the unemployment rate such that consumption good and labor markets are in equilibrium is given by:

$$\frac{u_t}{N} = \frac{16}{16 + \frac{\alpha}{\gamma}c_D\left[1 - F_t(R_t)\right]\left(1 + x_t\frac{N^*}{N}\right)E_t} \tag{3.33}$$

The wage distribution, (3.23) can be written as:

$$G_t(w, z_t) = \left(\frac{N}{N - u_t}\right)4\left(\frac{(w - z_t)}{(2w - z_t)^2}\frac{z_t}{2}\right)^{\frac{1}{2}} - \frac{u_t}{N - u_t} \tag{3.34}$$

3.3 Model: the intertemporal consumption saving problem

Agents in this economy demand unit of consumption goods, save by the means of risk free one period bond and hold stock of physical capital.

3.3.1 Consumer's budget constraints

The nominal period budget constraint of an individual consumer ω is:

$$B^{\omega}_{t+1} + v_t A^{\omega}_t + \int_{i\in\Omega(V_{gt})} p_t(i)\, c^{\omega}_t(i)\, di = (1+i_t)\, B^{\omega}_t + W^{\omega}_t + D^{\omega}_t$$

where B^{ω}_t is individual bond holding, A^{ω}_t is individual asset holding, i_t is the nominal interest rate on one period bond, W^{ω}_t is the labor income and D^{ω}_t is the dividend in nominal terms. Iterating forward we obtain the intertemporal budget constraint for a given horizon T:

$$\left[\prod\nolimits_{s=t+1}^{T}(1+i_s)\right]^{-1} B^{\omega}_{t+T+1} + \sum_{s=t}^{T}\left[\prod\nolimits_{v=t+1}^{s}(1+i_v)\right]^{-1} \left(\int_{i\in\Omega(V_{gt})} p_t(i)\, c^{\omega}_t(i)\, di + v_s A^{\omega}_s\right)$$
$$\leq \left[\sum_{s=t}^{T}\left[\prod\nolimits_{v=t+1}^{s}(1+i_v)\right]^{-1}(W^{\omega}_s + D^{\omega}_s)\right] + (1+i_t)\, B^{\omega}_t$$

where v_t is the value of one unit of asset in time t; we have already anticipated that capital is the numeraire, but for the following discussion it is useful to keep track of the such price.

An optimal plan under the infinite horizon requires the transversality condition to hold:

$$\lim_{T\to\infty}\left[\prod\nolimits_{s=t+1}^{T}(1+i_s)\right]^{-1} B^{\omega}_{t+T+1} = 0$$

The individual intertemporal budget constraint is:

$$\sum_{s=t}^{\infty}\Pi_{t,s}\,(W^{\omega}_s + D^{\omega}_s) + (1+i_t)\, B^{\omega}_t = \sum_{s=t}^{\infty}\Pi_{t,s}\int_{i\in\Omega(V_{gt})} p_t(i)\, c^{\omega}_t(i)\, di + v_s A^{\omega}_s$$

where $\Pi_{t,s} = \left[\prod_{v=t+1}^{s}(1+i_v)\right]^{-1}$ for $s \geq t+1$ and $\Pi_{t,t} = 1$. The total value of new physical capital in the country is investment $I_t = v_t A_t$. Use the law of motion for capital to express investment as $I_t = v_t \left[\frac{K_{t+1}}{1-\delta_k} - K_t\right]$. Define individual investment $I_t^\omega = v_t \left(a_{t+1}^\omega \frac{K_{t+1}}{(1-\delta_k)} - a_t^\omega K_t\right)$, and the demand of asset by agent ω is given by:

$$A_t^\omega = \left[a_{t+1}^\omega \frac{K_{t+1}}{(1-\delta_k)} - a_t^\omega K_t\right] \tag{3.35}$$

Define individual dividend share is $D_t^\omega = a_t^\omega D_t$, where D_t is total profit in the total variable profit of domestic firms.

3.3.2 Capital sector: production and accumulation

The endowment of capital in the economy is subject to a time-to-build lag. As in [Ottaviano, 2011], new units of asset that are produced at time t are available from time $t+1$ on. The supply of service follows an accumulation law given by:

$$K_{t+1} = (1-\delta_k)(K_t + A_t) \tag{3.36}$$

where $\delta_k \in (0,1)$ is the depreciation rate and A_t is the investment in the production of new units of capital.

3.3.3 Consumption and asset holding decisions

Consumer/worker starts each period with a predetermined a_t^ω and he maximizes lifetime utility over consumption, bond and asset holding, subject to the intertemporal budget constraint:

$$\max_{\left\{c_{is}^\omega, a_{s+1}^\omega\right\}_{s=t}^\infty} \mathbb{E}_t \left\{\sum_{s=t}^\infty \beta^{s-t} U\left(\{c_{is}^\omega\}\right)\right\}$$

$$s.t. \sum_{s=t}^\infty \Pi_{t,s}\left(W_s^\omega + a_s^\omega D_s\right) + (1+i_t) B_t^\omega = \sum_{s=t}^\infty \Pi_{t,s}\left[\int_{i\in\Omega(V_{gt})} p_{is} c_{is}^\omega di + v_s\left(\frac{a_{s+1}^\omega K_{s+1}}{1-\delta_k} - a_s^\omega K_s\right)\right]$$

The Lagrangian is:

$$\mathcal{L}\left(c_{is}^{\omega},a_{s+1}^{\omega}\right)=\alpha\int_{i\in\Omega(V_{gt})}c_{it}^{\omega}di-\frac{\gamma}{2}\int_{i\in\Omega(V_{gt})}\left(c_{it}^{\omega}\right)^{2}di+$$
$$+\mathbb{E}_{t}\left\{\sum_{s=t+1}^{\infty}\beta^{s-t}\left[\alpha\int_{i\in\Omega(V_{gt})}c_{is}^{\omega}di-\frac{\gamma}{2}\int_{i\in\Omega(V_{gt})}\left(c_{is}^{\omega}\right)^{2}di\right]\right\}$$
$$+\lambda^{\omega}\Pi_{t,t}\left\{\begin{array}{l}\left(W_{t}^{\omega}+a_{t}^{\omega}D_{t}\right)+\left(1+i_{t}\right)B_{t}^{\omega}\\-\left[\int_{i\in\Omega(V_{gt})}p_{is}c_{is}^{\omega}di+v_{t}\left(\frac{a_{t+1}^{\omega}K_{t+1}}{1-\delta_{k}}-a_{t}^{\omega}K_{t}\right)\right]\end{array}\right\}$$
$$+\lambda^{\omega}\sum_{s=t+1}^{\infty}\Pi_{t,s}\left\{\begin{array}{l}\left(W_{s}^{\omega}+a_{s}^{\omega}D_{s}\right)\\-\left[\int_{i\in\Omega(V_{gt})}p_{is}c_{is}^{\omega}di+v_{s}\left(\frac{a_{s+1}^{\omega}K_{s+1}}{1-\delta_{k}}-a_{s}^{\omega}K_{s}\right)\right]\end{array}\right\}$$

The f.o.c. at two points in time t and $t+1$ are:

$$\begin{array}{l}\frac{\partial\mathcal{L}}{\partial c_{it}^{\omega}}=0:\left(\alpha-\gamma c_{it}^{\omega}\right)-\lambda^{\omega}\Pi_{t,t}p_{it}=0\\\frac{\partial\mathcal{L}}{\partial c_{it+1}^{\omega}}=0:\beta\mathbb{E}_{t}\left\{\left(\alpha-\gamma c_{it+1}^{\omega}\right)\right\}-\lambda^{\omega}\Pi_{t,t+1}p_{it+1}=0\\\frac{\partial\mathcal{L}}{\partial a_{t+1}^{\omega}}=0:-\lambda^{\omega}\Pi_{t,t}\frac{v_{t}K_{t+1}}{1-\delta_{k}}+\lambda^{\omega}\Pi_{t,t+1}\left[D_{t+1}+v_{t+1}K_{t+1}\right]=0\end{array}$$

Compare the f.o.c. of the intertemporal problem for consumption with the intra-temporal equilibrium condition. We conclude that: $\lambda_{s}^{\omega}\equiv\frac{\lambda^{\omega}\Pi_{t,s}}{\beta^{s-t}}$ for any $s\geq t$. Notice that $\Pi_{t,t}/\Pi_{t,t+1}=(1+i_{t+1})$ then explicit for the multiplier λ^{ω} the two f.o.c. with respect to consumption and combine to obtain the Euler equation:

$$\beta\mathbb{E}_{t}\left[\left(\alpha-\gamma c_{it+1}^{\omega}\right)\left(1+i_{t+1}\right)\frac{p_{it}}{p_{it+1}}\right]=\left(\alpha-\gamma c_{it}^{\omega}\right)$$

Rewrite the Euler equation as $\beta\mathbb{E}_{t}\left[\left(1+i_{t+1}\right)\lambda_{t+1}^{\omega}\right]=\lambda_{t}^{\omega}$ and take the expectation over agents,

$$\beta\mathbb{E}_{t}\left[\left(1+i_{t+1}\right)\mathbb{E}_{\omega}\left[\lambda_{t+1}\left(\omega\right)\right]\right]=\mathbb{E}_{\omega}\left[\lambda_{t}\left(\omega\right)\right]\tag{3.37}$$

Write the f.o.c. with respect to asset holding as a no arbitrage condition between physical capital stock holding and bonds:

$$\left(1+i_{t+1}\right)=\left(1-\delta_{k}\right)\frac{v_{t+1}}{v_{t}}\left[1+\frac{D_{t+1}}{v_{t+1}K_{t+1}}\right]\tag{3.38}$$

Consider an agent who can save a nominal value of income v_t by the means of two channels: buy one period bond $(v_t B_{t+1})$, and gain one period later $v_t B_{t+1}(1+i_{t+1})$; or purchase one unit of additional capital (value v_t), and gain at the beginning of the next period the value of the existing capital $v_{t+1}(1-\delta_k)$ plus the dividend per each unit of capital $\frac{D_{t+1}}{v_{t+1}K_{t+1}}$.

3.4 Equations that characterize the dynamics

Optimally of intertemporal allocation. Combining (3.6) and (3.17), the expected marginal utility of income is expressed as a monotonic decreasing function of the aggregate productivity $\mathbb{E}_\omega[\lambda_t(\omega)] = \frac{2\alpha}{z_t(1+z_t)c_D}$. This condition is due to the market structure of the consumption sector and can be combined with the Euler equation (3.37) and the no arbitrage condition (3.38) that characterize the optimality of intertemporal allocations of consumption and asset.

$$\mathbb{E}_t\left[\left(1+\frac{E_{t+1}}{K_{t+1}}\overline{\Pi}(z_{t+1})\right)\frac{z_t}{z_{t+1}}\frac{(1+z_t)}{(1+z_{t+1})}\right] = \frac{1}{(1-\delta_k)\beta} \tag{3.39}$$

where I used the fact that capital is the numeraire, $v_{t+1} = v_t = 1$ and the total dividend is the expected profit across incumbent firms times the number of incumbents $D_{t+1} = \overline{\Pi}(z_{t+1}) E_{t+1}$.

Selection. Let M_{Et} be the mass of firms that decide at time t to pay the sunk cost and enter the market in the next period. The measure of potential entrants that decide to stay in the market in the upcoming period is given by:

$$E_{t+1} = \Phi(c_{in,t+1}, z_{t+1})(E_t + M_{Et}) \tag{3.40}$$

Export decision. The total mass of exporters is the sum of two components:

$$X_t = \Phi(c_{x,t}, z_t) E_t + [\Phi(c_{in,t}, z_t) - \Phi(c_{x,t}, z_t)] X_{t-1} \tag{3.41}$$

A moss of $\Phi\left(c_{in,t}, z_t\right) X_{t-1}$ firms were exporters in the previous period and continue to export, whereas $\Phi\left(c_{x,t}, z_t\right)\left(M_{Et-1}+E_{t-1}-X_{t-1}\right)$ firms become exporters in the current period.

Market clearing in the capital sector. Firms demand capital to enter, run production and export. In a given point in time, the total demand of capital is given by: $K_t = f_p E_t + f_{ex}\Phi\left(c_{x,t}, z_t\right)\left(M_{Et-1}+E_{t-1}-X_{t-1}\right) + f_{ed} M_{Et}$. Market clearing determines the mass of entrants:

$$M_{Et} = \frac{1}{f_{ed}} K_t - \frac{f_p + f_{ex}\frac{\Phi(c_{x,t},z_t)}{\Phi(c_{in,t},z_t)}}{f_{ed}} E_t + \frac{f_{ex}\Phi\left(c_{x,t}, z_t\right)}{f_{ed}} X_{t-1} \tag{3.42}$$

Labor market clearing determines the unemployment rate and the output in the capital sector. Using (3.34), employment in the consumption sector is given by:

$$\left(1-G_t\left(\underline{w}\right)\right)\left(N-u_t\right) = \left(1-4\left(\frac{\left(\underline{w_t}-z_t\right)}{\left(2\underline{w_t}-z_t\right)^2}\frac{z_t}{2}\right)^{\frac{1}{2}}\right) N \tag{3.43}$$

The labor market clearing at the aggregate level implies the unemployment rate:

$$\frac{u_t}{N} = 4\left(\frac{\left(\underline{w_t}-z_t\right)}{\left(2\underline{w_t}-z_t\right)^2}\frac{z_t}{2}\right)^{\frac{1}{2}} - \frac{A_t}{z_t N} \tag{3.44}$$

Given the definition of unemployment in (3.33) the value of output in the capital sector is:

$$A_t = 4\left(\frac{\left(\underline{w_t}-z_t\right)}{\left(2\underline{w_t}-z_t\right)^2}\frac{z_t}{2}\right)^{\frac{1}{2}} z_t - \frac{16 N z_t}{16 + \frac{\alpha}{\gamma} c_D\left[1-F_t\left(R_t\right)\right]\left(E_t + X_t \frac{N^*}{N}\right)} \tag{3.45}$$

The dynamics of the model is characterized by four difference equations, (3.40), (3.41), (3.3), (3.36) in four unknowns $\{E_{t+1}, K_{t+1}, X_t, z_{t+1}\}$. Conditions (3.39), (3.42) and (3.45) are used as auxiliary equations.

3.4.1 Steady state

The steady state values of $\{X_t, M_{Et}, A_t, E_T^{cap} l_T^{cap}\}$ are immediately determined as a function of the steady state values of the model state variables:

$$\begin{aligned} X_T &= \frac{\Phi\left(c_{x,T}, z_T\right)}{1-\left[\Phi\left(c_{in,T}, z_T\right) - \Phi\left(c_{x,T}, z_T\right)\right]} E_T \\ M_{ET} &= \frac{1-\Phi\left(c_{in,T}, z_T\right)}{\Phi\left(c_{in,T}, z_T\right)} E_T \\ A_T &= \frac{\delta_k}{1-\delta_k} K_T \\ E_T^{cap} l_T^{cap} &= \frac{\delta_k}{1-\delta_k} \frac{K_T}{z_T} \end{aligned}$$

The market clearing condition (3.42) in steady state gives:

$$\begin{aligned} K_T = &\left(\frac{1-\Phi\left(c_{in,T}, z_T\right)}{\Phi\left(c_{in,T}, z_T\right)} f_{ed} + f_p + f_{ex} \frac{\Phi\left(c_{x,T}, z_T\right)}{\Phi\left(c_{in,T}, z_T\right)} \right. \\ &\left. - \frac{f_{ex} \Phi\left(c_{x,T}, z_T\right) \Phi\left(c_{x,T}, z_T\right)}{1-\left[\Phi\left(c_{in,T}, z_T\right) - \Phi\left(c_{x,T}, z_T\right)\right]} \right) E_T \end{aligned} \tag{3.46}$$

The optimality condition (3.39) in steady state implies:

$$E_T = \frac{1-\left(1-\delta_k\right)\beta}{f_{ed}\left(1-\Phi\left(c_{in,T}, z_T\right)\right)\left(1-\delta_k\right)\beta} K_T \tag{3.47}$$

where I used the fact that in steady state $\frac{\overline{\Pi}(z_T)}{1-\Phi(c_{in,T}, z_T)} = f_{ed}$ because free entry. There exists a unique couple of values $\left(\Phi\left(c_{in,T}, z_T\right), \Phi\left(c_{x,T}, z_T\right)\right)$ that satisfies (3.46) and (3.47). Moreover, for a given choice of the distribution $\Phi\left(\cdot\right)$, the two values $c_{in,T}$ and $c_{x,T}$ are unique.

The steady state level of employment in the capital sector is determined by the means of Harris-Todaro argument, [Harris and Todaro, 1970]. Agents search for a job under risk neutrality, information is complete and there are no cost of searching. In steady state the ex ante expected earning between sector must be the same. Using (3.43), it is possible to determine the steady state share of employment in the consumption sector. Hence the steady state employment level in the capital sector is:

$$E_T^{cap} l_T^{cap} = \left(1 - 4\left(\frac{\left(\underline{w_T} - z_T\right)}{\left(2\underline{w_T} - z_T\right)^2}\frac{z_T}{2}\right)^{\frac{1}{2}}\right)\frac{\mu_T^{cons}}{R_T} \tag{3.48}$$

where $\mu_T^{cons} = \frac{1}{1-G_T\left(\underline{w_T}\right)}\int_{\underline{w_T}}^{\frac{3}{2}R_T} w dG_T(w)$ by (3.23), $\underline{w_T} = \frac{z_T}{2}\left(1 + \frac{c_D}{c_{in,T}} z_T\right)$ by (3.32) and $R_T = z_T$ due to perfect competition in the capital sector. The steady state level for the stock of capital is:

$$K_T = \frac{1-\delta_k}{\delta_k}\left(1 - 4\left(\frac{\left(\underline{w_T} - z_T\right)}{\left(2\underline{w_T} - z_T\right)^2}\frac{z_T}{2}\right)^{\frac{1}{2}}\right)\mu_T^{cons} \tag{3.49}$$

Finally equation (3.47) determines the steady state mass of employers.

3.4.2 Dynamics[6]

I characterize the dynamics of the model under the restriction that firms decide entry, exit and export taking into account only the long run. That is, the discussion of the dynamics is parametric to the steady state values $\Phi\left(c_{in,T}, z_T\right)$, $\Phi\left(c_{x,T}, z_T\right)$, $\overline{\Pi}\left(z_T\right) = f_{ed}\left(1 - \Phi\left(c_{in,T}, z_T\right)\right)$, $\underline{w_T}$ and $F_t\left(R_t\right) \equiv \digamma$. A polynomial approximation of (3.29), (3.30) and (3.32) would be sufficient to characterize $\Phi\left(c_{in,t}, z_t\right)$, $\Phi\left(c_{x,t}, z_t\right)$, $\overline{\Pi}\left(z_t\right)$, $\underline{w_T}$ as functions of z_t, then solve for $F_t\left(R_t\right)$ in (3.45) and relax this restriction.

Under this framework the dynamics of the model simplifies to:

$$\begin{aligned} E_{t+1} - E_t = &\left[\overline{\Phi_{in}} - \frac{\overline{\Phi_{in}} f_p + f_{ex}\overline{\Phi_x}}{f_{ed}}\right]\left(E_t - E_T\right) + \left[\frac{\overline{\Phi_{in}}}{f_{ed}}\right]\left(K_t - K_T\right) + \\ &\left[\frac{f_{ex}\overline{\Phi_{in}\Phi_x}}{f_{ed}}\right]\left(X_{t-1} - X_T\right) \\ K_{t+1} - K_T = &\left[1 - \delta_k\right]\left(K_t - K_T\right) + \left[\left(1-\delta_k\right)a_{E_t}\right]\left(E_t - E_T\right) + \left[\left(1-\delta_k\right)a_{X_t}\right] \\ &\left(X_t - X_T\right) + \left[\left(1-\delta_k\right)a_{z_t}\right]\left(z_t - z_T\right) \end{aligned}$$

[6] Results in this section are very preliminary and largely incomplete.

$$X_t - X_T = \left[\overline{\Phi_x}\right](E_t - E_T) + \left[\overline{\Phi_{in}} - \overline{\Phi_x}\right] X_{t-1}$$

$$z_{t+1} - z_T = [k_z](z_t - z_T) + \xi_{t+1}$$

where the coefficients $a_{z_t} < 0$, $a_{E_t} > 0$, $a_{X_t} > 0$ belong to the first order Taylor expansion of condition (3.45),

$$A_t - A_T = a_{E_t}(E_t - E_T) + a_{X_t}(E_t - E_T) + a_{z_t}(z_t - z_T) \qquad (3.50)$$

around the steady state and the pair $\left(\overline{\Phi_{in}}, \overline{\Phi_x}\right)$ coincides with the pair of values $(\Phi(c_{in,T}, z_T), \Phi(c_{x,T}, z_T))$ that satisfies (3.46) and (3.47).

Figure 3.3 shows the response of the three endogenous dynamic variables in response to a shock of aggregate productivity of one percentage point of the steady state aggregate productivity level. The first point in the graph is the steady state level, shock occurs at the second point in time.

The shock in aggregate productivity impacts immediately the production of capital. Firms anticipate the future increase in selection and few exporters will enter the market in the next period. Therefore the demand of capital to finance new entry falls in the current period. The response of the mass of employers is lag one period with respect to capital and it is driven by two components. The decrease in the number of new entrants and the increase in firm turnover. The temporary increase in aggregate productivity allows more firms to become exporters. Immediately after the shock, this effect combined with the increase in firm turnover leads to an unambiguous increase in the share of exporters. As the economy adjusts to the steady state, the entry of new firms adds to the previous two effects and the oscillatory dynamics may arise.

Figure 3.4 summarizes the response of unemployment to the aggregate productivity shock. Notice from the previous discussion that at the time of

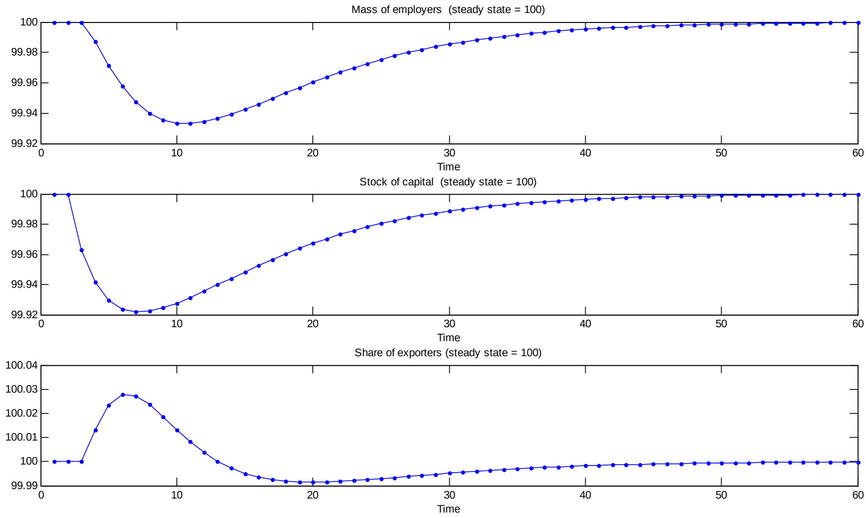

Figure 3.3: Dynamics of endogenous variables

the shock only the stock of capital reacts, and since less capital is needed for the entry of few new entrants the overall impact on employment is negative. After the shock, the demand of labor due to new exporters and the increase in the demand of capital drive the dynamics of unemployment.

3.5 Conclusion

This paper develops a two-sector two-factor dynamic general equilibrium model with uncertainty on aggregate productivity, endogenous entry, export and exit decisions, search and matching frictions in the labor market with on the job search. Under this framework, I show that positive aggregate productivity shocks leads to an increase in the unemployment rate.

The dynamics of the unemployment rate results from the demand of labor of new exporters, the larger firm turnover and the entry of new firms. When the shock hits the economy, firms anticipate the increase

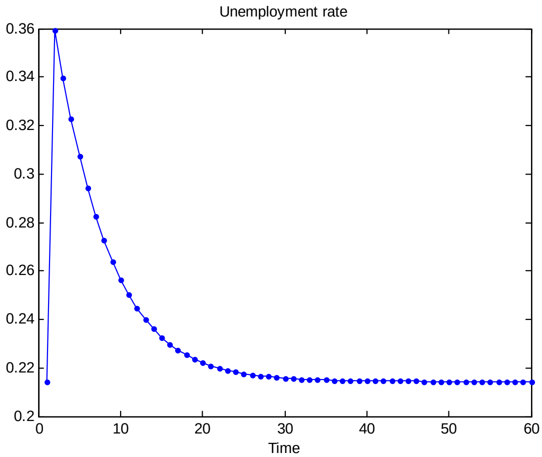

Figure 3.4: Dynamics of the unemployment rate

in turnover and few new entrants demand capital to finance their entry. The overall response of total employment is negative. After the shock, the demand of labor due to new exporters and the increase in the demand of capital drive the dynamics of unemployment. For the entire transition the unemployment rate is larger than in steady state.

Appendix A

Appendix for the Chapter 1

Robustness check: wage posting and unbounded productivity distribution

The purpose of this section is to assess the change in the wage distribution following an export shock under a different wage determination mechanism. We derive the wage offer distribution under pure wage posting. As in [Burdett and Mortensen, 1998] firms post take it or leave offers at a wage that is a solution of the following problem:

$$w = \arg\max_{w>R} \left(\frac{p_t(w)}{c} - w \right) emp_t(w)$$

Firms take as given the demand of consumption goods $\frac{p}{c} = \frac{1}{2}\left(R\frac{c_D}{c} + w\right)$, $\frac{\partial p}{\partial w} = \frac{1}{2}c$ then $\left(\frac{p(w)}{c} - w\right) = \frac{1}{2}\left(R\frac{c_D}{c} - w\right)$ and the employment function that is implied by the search equilibrium: $emp_t(w)$. The necessary first order condition for an interior solution is $-\frac{1}{2}emp_t(w) + \frac{1}{2}\left(R\frac{c_D}{c} - w\right)$ $emp_t'(w) = 0$, and it can be written as:

$$\left(R\frac{c_D}{c} - w\right)\left[\frac{2m(\theta_t)F_t'(w)}{\delta + m(\theta_t)\left[1 - F_t(w)\right]}\right] = 1$$

The sufficient second order condition is: $-\frac{3}{2}emp_t'(w) + \left(R\frac{c_D}{c} - w\right)emp_t''(w) <$ 0, therefore the concavity of the employment function is sufficient to guarantee the optimality of an interior solution. Impose the unique mapping between the distribution of wage offers and the distribution of unit labor requirements $F_t(w(c)) = 1 - \Phi(c)$. For the sake of comparison with the framework in [Burdett and Mortensen, 1998] define $J(a)$ as the distribution of idiosyncratic productivity $a = \frac{Rc_D}{c}$ as it is implied by $\Phi(c)$. Then $\frac{\partial F_t(w(c))}{\partial w} = \left(\frac{\partial w}{\partial a}\right)^{-1} \frac{\partial J(a)}{\partial a}$. The wage is a solution of the differential equation:

$$\frac{\partial w}{\partial a} = \frac{2m(\theta_t) J'(a)}{\delta + m(\theta_t)[1 - J(a)]}(a - w(a))$$

The wage offer distribution satisfies the equality $\left(\frac{p_t(w(c))}{c} - w(c)\right) emp_t(w(c)) = \pi_t(c)$ for all wages, where the variable profit is determined in (1.9) under the assumption that the wage paid by a firm endowed with unit labor requirement c_D is the reservation wage. The cdf and density of the wage offer distribution are:

$$\begin{aligned} F_t(w) &= 1 - \left(\frac{\delta}{m(\theta_t)\left(1 + x_t \frac{N^*}{N}\right) E_t} \frac{2}{\alpha/\gamma R c_D} \frac{a^2}{(a - w)}\right)^{\frac{1}{2}} + \frac{\delta}{m(\theta_t)} \\ F'_t(w) &= -\frac{a^2}{Rc_D}\left(\frac{1}{a - w}\right)^{\frac{3}{2}} \frac{\delta}{\alpha/\gamma m(\theta_t)\left(1 + x_t \frac{N^*}{N}\right) E_t} \end{aligned}$$

Since $\frac{\partial w}{\partial a} F_t(w(c)) = J'(a)$ then the wage productivity profile is given by:

$$w(a) = a - \left(\frac{Rc_D}{a^2} \frac{2\delta}{\delta + m(\theta_t)[1 - J(a)]} \frac{1}{\frac{\alpha}{\gamma}\left(1 + x_t \frac{N^*}{N}\right) E_t}\right)^2$$

Assume that $1 - J(a) = \left(\frac{a}{a}\right)^{\rho}$ then

$$w(a) = a - \left(\frac{2\delta R c_D}{\delta a^2 + m(\theta_t) a^{2-\rho}} \frac{1}{\frac{\alpha}{\gamma}\left(1 + x_t \frac{N^*}{N}\right) E_t}\right)^2$$

Under the restriction $a > \left(\frac{m(\theta_t)(\rho - 2)}{2\delta}\right)^{\frac{1}{\rho}}$, the wage is a monotonic increasing function of idiosyncratic productivity a. The cdf of the wage distribution is:

$$G_t(w(a)) = \frac{\delta}{1 - F_t(R)} \frac{J(a) - F_t(R)}{\delta + m(\theta_t)[1 - J(a)]}$$

For $\rho \to 2$ and $c^4 \to 0$ then $w(a) \simeq a$. The cdf and the density of the wage distribution are:

$$G_t(w) = \frac{\delta}{1 - F_t(R)} \frac{1 - F_t(R) - \left(\frac{\underline{w}}{w}\right)^2}{\delta + m(\theta_t)\left(\frac{\underline{w}}{w}\right)^2}$$
$$g_t(w) = \frac{2\delta \underline{w}^2 w}{\left(m(\theta_t)\underline{w}^2 + \delta w^2\right)^2} \left(\frac{\delta + (1 - F_t(R))\, m(\theta_t)}{1 - F_t(R)}\right)$$

where $G_t(w) \to 1$ as $w \simeq a \to \infty$. A numerical analysis can provide a full comparison of this scenario with the one that has been described in the paper.

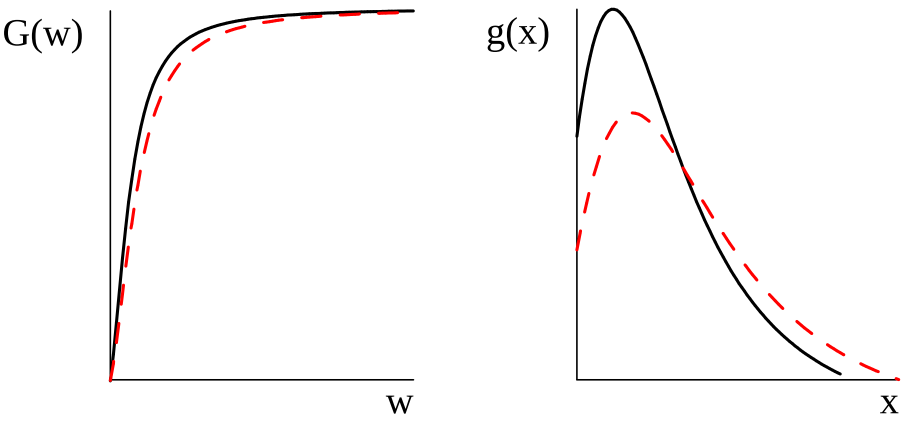

Figure A.1: .Change in the wage distribution due to an increase in the probability of finding a job

Nevertheless for a given $F_t(R)$ if the export shock is associated to an increase in the probability of finding a job then the response of the cdf following to an export shock is the same than under wage bargaining; in the following sense. The average wage increases, the cdf after the export shock is dominated by the cdf before the export shock. The scenario in which the probability of finding a job increases after an export shock arises when the increase in the share of exporters more than compensates the fall in the number of employers in the market, (3.13).

Appendix for the Chapter 2

Proof of the theorems. The following theorems about the processes $\{E_i\}_{i=1,2,...,n}$ and $\{(W_i, M_i)\}_{t=1,2,...,n}$ are consequences of the de Finetti's Representation Theorem (de Finetti, 1937, see Hewitt and Savage 1955). The first theorem establishes that the random sequence of binary random variables $\{E_i\}_{i=1,2,...,n}$ is exchangeable and it is a mixture of Bernoulli random variables with a unique measure.

THEOREM 1**:** *the sequence $\{E_i\}_{i=1,2,...,n}$ generated by a Pòlya urn is exchangeable and its de Finetti measure is a Beta with parameters $\left(\frac{W_0}{r}, \frac{M_0}{r}\right)$.*

Proof**.** Let $e_i \in \{0, 1\}$, and the sequence $(e_1, ..., e_t)$ is such that $1 \leq k \leq t$, where $\sum_{i=1}^{n} e_i = k$, then:

$$
\begin{aligned}
& \Pr\left(E_1 = e_i, ..., E_n = e_n\right) \\
= \quad & \frac{\Pi_{j=0}^{k-1}\left(W_0 + rj\right)\Pi_{j=0}^{n-k-1}\left(M_0 + rj\right)}{\Pi_{j=0}^{n-1}\left(W_0 + M_0 + rj\right)} \\
= \quad & \frac{\Gamma\left(\frac{W_0}{r} + \frac{M_0}{r}\right)}{\Gamma\left(\frac{W_0}{r}\right)\Gamma\left(\frac{M_0}{r}\right)} \frac{\Gamma\left(\frac{W_0+k}{r}\right)\Gamma\left(\frac{M_0+n-k}{r}\right)}{\Gamma\left(\frac{W_0}{r} + \frac{M_0}{r} + n\right)} \\
= \quad & \int_0^1 \phi^k \left(1-\phi\right)^{n-k} \left[\frac{\Gamma\left(\frac{W_0}{r} + \frac{M_0}{r}\right)}{\Gamma\left(\frac{W_0}{r}\right)\Gamma\left(\frac{M_0}{r}\right)} \phi^{\frac{W_0}{r}-1} \left(1-\phi\right)^{\frac{M_0}{r}-1}\right] d\phi
\end{aligned}
\tag{A.1}
$$

where the expression in the square brackets is the de Finetti measure of the sequence $\{E_i\}_{i=1,2,...,n}$ generated by a Pòlya urn, it is a Beta with parameters $\left(\frac{W_0}{r}, \frac{M_0}{r}\right)$. ■

The second theorem pins down the limit distribution of X_∞, that in our economic problem is the workers share across cities in the long run.

THEOREM 2**:** *In a Pòlya urn, as n grows to infinity, the proportion*

$$
X_n = \frac{W_n}{W_n + M_n}
$$

it is a bounded martingale, it converges almost surely to a random limit, and the distribution of the limit is a Beta with parameters $\left(\frac{W_0}{r}, \frac{M_0}{r}\right)$.

Proof. For all $n > 0$, $0 < X_n < 1$

$$\begin{aligned}
& \mathbb{E}\left[X_{n+1}|X_1,...,X_n\right] \\
= & \frac{W_n + r}{W_n + M_n + r}\left(\frac{W_n}{W_n + M_n}\right) + \frac{W_n}{W_n + M_n + r}\left(\frac{M_n}{W_n + M_n}\right) \\
= & \frac{W_n}{W_n + M_n} \\
= & X_n
\end{aligned}$$

Therefore, from Doob's convergence theorem for martingales, the sequence $\{x_n\}$ converges a.s. and L^1 to a random variable x_∞. Apply the law of large numbers, the distribution of the random variable $X_\infty = \lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} E_i$ converges to a Beta with parameters $\left(\frac{W_0}{r}, \frac{M_0}{r}\right)$ since, for all $n \geq 1$ and an arbitrary constant $x \in (0,1)$:

$$\Pr\left(X_n < x\right) = \Pr\left(\frac{1}{n}\sum_{i=1}^{n} A_i \leq x\left(W_0 + M_0 + 1\right) - \frac{W_0}{nr}\right)$$

and

$$\lim_{n\to\infty} \Pr\left(X_n < x\right) = \int_0^x \frac{\Gamma\left(\frac{W_0}{r} + \frac{M_0}{r}\right)}{\Gamma\left(\frac{W_0}{r}\right)\Gamma\left(\frac{M_0}{r}\right)}\phi^{\frac{W_0}{r}-1}\left(1-\phi\right)^{\frac{M_0}{r}-1}d\phi \quad \blacksquare$$

If theorems 1 and 2 apply then we obtain a prior distribution of the workers share in the long run X_∞. Moreover, since the Beta density is a conjugate prior, even the posterior distribution of the workers share belongs to the class of Beta distributions. The choice of the parameters of the final distribution accounts to the researcher. If the researcher could observe the urn for m stages and sample a sequence $\{x_i\}_{i=1,\dots m}$, of which k workers and $n-k$ managers then the parameters of the final distribution were a weighted mean of the initial composition and the information learned during the sampling:

$$a = \sum_{i=1}^{m} x_i + \frac{W_0}{r} \text{ and } b = m - \sum_{i=1}^{m} x_i + \frac{M_0}{r} \qquad (19)$$

The final distribution of the limit workers share across cities in the economy is $X_\infty \sim Beta\left(a,b\right)$ and the density function is given by:

$$f\left(x\right)=\frac{x^{a-1}\left(1-x\right)^{b-1}}{\int_0^1 \phi^{a-1}\left(1-\phi\right)^{b-1}d\phi}$$

in the support $(0,1)$ and the parameter space a, $b>0$.

Beta distribution. For our further purposes it is useful to show how density changes depending on the value of the parameters:

- if $0<a,b<1$ then we have a U-shape density
- If $a=b=1$ then we obtain the uniform distribution on the support $(0,1)$
- If $a,b>1$, a is small compared to b the density is skew toward the left
- If $a,b>1$, a is large compared to b the density is skew toward the right
- If $a=b$ the density is symmetric around $\frac{1}{2}$

The unconditional expectation, mode and variance of the distribution of workers share across cities are:

$$\begin{aligned}
E\left(X_\infty\right) &= \frac{a}{a+b} \\
Mode\left(X_\infty\right) &= \frac{a-1}{a+b-2} \\
V\left(X_\infty\right) &= \frac{ab}{\left(a+b\right)^2\left(1+a+b\right)}
\end{aligned}$$

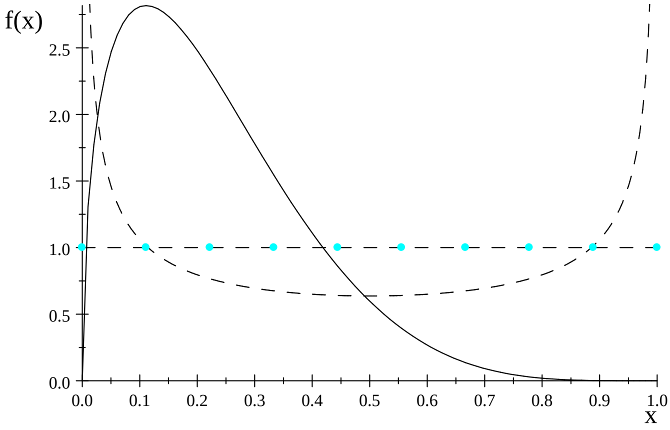

Figure A.2: Beta densities, for $a = b = 0.5$ (dashed line), $a = b = 1$ (dot-dashed line) and $a = 1.5$ $b = 5$ (solid line).

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