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Polarization, Regulation and Networks in a Data Market

Advisor: Fernando Vega-Redondo
Co-Advisor: Christoph Wolf

PhD Thesis by
Goonj Mohan
ID Number: 3054738

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Abstract

The thesis focuses on the economy of data; data which is generated by user activity on social networking platforms like Facebook. We know that platforms like Facebook use data or *information* about users to provide services like better product recommendations and also for improving their estimate of a user's willingness to pay for a product. Additionally, scandals like *Cambridge Analytica* have increased concerns about how user data is analysed by platforms. Thus, data collection and analysis done by a platform can be both beneficial and harmful for a user, which brings us to the question -

- What is the optimal privacy policy for users on an online platform? Should users be given more (less) control over their data? How does the optimal privacy policy depend on the purpose for which a platform collects user data?

Another concern is the increasing polarization among users on online platforms. A polarized society can have serious political implications, as we have seen in elections across countries like USA and India. It can also change and perhaps even damage the social fabric of our society. It is therefore important to understand why are online platforms (seemingly) so divisive. This brings us to the second major question of this research -

- Does an online platform have an incentive to increase polarization of its social network?

One can consider another way in which a society may be polarized, namely, when different kinds of users are on different platforms. This results in no interaction between different kinds of users and can increase polarization between users.
For e.g., consider Facebook and Instagram. Facebook is popular among middle aged people and Instagram is popular among the younger generation. This causes

an age based segregation between two groups of people. Again, such segregation maybe harmful for society and brings us to the question -

- Does an online platform have an incentive to segregate different types of users onto different platforms?

I address these three topics in the three chapters of my thesis below.

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Introduction

The thesis has three chapters covering various aspects of the data economy.

Chapter 1

I study whether a social media platform (SMP) has an incentive to increase polarization of its social network. An SMP earns revenue from user data, which is generated on the social network through user interactions. The network comprises of two *groups* of users. The algorithmic influence of an SMP enables it to encourage users to form new links, generate new interactions and earn more revenue. The SMP is said to increase polarization if it *disproportionately* increases links within groups than across groups. I find that an SMP increases polarization if and only if one group is much larger than the other. An SMP can reduce polarization by attracting underrepresented users to join its platform.

Chapter 2

This paper analyzes the effect of different regulations in a data economy, consisting of an online platform and heterogeneous users. The online platform generates revenue from data by steering users. I derive a microfoundation of payoffs in this setup and find that platform payoff is increasing in number of data points and user payoff is non-monotonic in number of data points. I further model users in a network setup which allows analysis of regulations that address the rising concerns about user welfare on online platforms. I find that, first, stricter privacy policy backfires and reduces user welfare. Second, when users own the online platform both total welfare and total user welfare are maximized. Third, when a platform becomes large, user welfare is improved by breaking up the platform.

Chapter 3

This paper analyses whether segregating of users onto different online platforms coincides with platforms' payoff maximisation. When different types of users are on different platforms, there is minimum interaction between them, which can increase polarization between users. This paper analyses whether users are polarized in this manner under different consent rules. I find that, under certain conditions, users are more likely to be segregated by types under a strict privacy policy than under a loose privacy policy. A strict privacy policy may therefore increase polarization between users.

Chapter 1

The Data Economy and Polarization on Social Media

1.1 Introduction

The rise of social media has sparked concerns about its divisive effect on society. While there is some understanding of how human behavior is contributing toward rising online polarization, the role played by social media is unclear. Some evidence suggests that social media increases polarization Allcott et al. (2020), while other evidence suggests that social media either does not affect polarization or may even reduce polarization Asimovic et al. (2021).

This paper contributes toward resolving seemingly contradictory evidence, by giving a unified explanation of the effect of social media on online polarization. I develop a framework which links the payoff incentives of a social media platform (SMP) and polarization of its user network. My model captures how users generate information or *data about themselves and their friends* through interactions on the online social network. Typically, such a network exhibits *weak homophily*, a user has a tendency to *link* with other users who have similar characteristics. The SMP earns revenue from the data which is generated via interactions of these linked users. Therefore, the SMP has an incentive to develop an algorithm which encourages each user to form new (data generating) links. Given that developing such an algorithm is costly, I study if the platform *disproportionately* encourages links between users of the same type rather than between users of different types. In other words, I study whether an SMP has an incentive to increase polarization of its online social network.

Data from social network: Users on an SMP are linked on a network and each link represents an interaction. Therefore, when two users are linked they interact and this produces (interactive) data about *both users*. Specifically, interactive data is informative about the preferences, the willingness to pay (WTP) and the political views of both users. The SMP analyzes this interactive data to generate revenue.

SMP revenue from data: The platform uses data to recommend a product that matches the taste of the user. It also uses data to estimate the WTP of the user and offers the recommended product at that price. As the platform gets more data, the recommended product matches the taste of the user more closely. This increases the probability that a user clicks on the advertisement. Also, as the platform gets more data, it improves its estimate of the WTP of the user and can extract more surplus from the user. Thus, the revenue of the SMP increases as the SMP attains more data. The effect on users' payoff is ambiguous. While users benefit from receiving better product recommendations, they are harmed due to more surplus being extracted. Thus, I develop a microfoundation of SMP's data usage and formally derive these effects. Additionally, since the WTP of a user is finite, the platform can only extract finite surplus from a user. Thus, the marginal value of a new data point is decreasing and platform payoff is concave in the amount of data it attains about a user.

SMP polarization decision: As revenue increases in data, an SMP has an incentive to encourage new links between users. Using a suggestion and ranking algorithm, the SMP influences which new links are formed. Given that developing such an algorithm is costly (e.g. due to hiring coders, investing in servers, etc.), I study whether the platform *disproportionately* increases links between users of the same type rather than between users of different types.

I model the initial network as a 2-islands network. Users are of two types and users of the same type are symmetric. Polarization is defined as the level of homophily, it is a measure of how likely a user is to link with a user of the same type rather than with a user of a different type. The initial network exhibits weak homophily, users are more likely to link with other users of the same type. Therefore, the initial network has some initial level of polarization. Given an initial network, the platform chooses whether to form new links between users of the same type or between users of different types. Equivalently, the platform chooses whether to increase polarization of the initial network or not.

When the marginal cost of developing an algorithm is low, the platform chooses to link all users. This maximizes data collection and hence maximizes platform payoff. As a result, a complete network is formed. Polarization of this complete network is lower than polarization of the initial network. In the complete network, each user is linked to every other user. The initial network has an initial level of polarization as users exhibit weak homophily. Each user is disproportionately linked to other users of the same type. Thus, when the marginal cost of influencing a social network is low, an SMP uses its influence to reduce polarization.

When the marginal cost of developing an algorithm is high, it is not feasible for the platform to link all users. The platform now chooses which new links to encourage for a given initial network. As users are linked on an initial network exhibiting weak homophily, more data is generated between similar users than between dissimilar users. This makes the *marginal value of a new link* between dissimilar users higher than the marginal value of a new link between similar users (due to concavity of platform payoff). Thus, the *marginal effect* incentivizes the platform to form new links between dissimilar users rather than between similar users. Equivalently, the platform is incentivized to reduce polarization.

On the other hand, if the platform has a higher representation of one type of user (for e.g., more democrats than republicans), then more links can be generated by suggesting a user to link with similar users than with dissimilar users. This *size effect* incentivizes the platform to form new links between similar users rather than between dissimilar users. Equivalently, the platform is incentivized to increase polarization. Therefore, the platform increases or reduces polarization based on whether the *marginal effect* or the *size effect* is stronger.

If the polarization of the initial network is low, a user is almost equally likely to link with a similar and a dissimilar user in the initial network. Thus, the marginal effect is weak, making the size effect the only factor relevant to the platform. If the initial network has even a slightly skewed representation of one type of user over the other, the platform will use its influence to disproportionately increase links between similar users rather than between dissimilar users. This will increase the level of homophily in the network, thereby resulting in a more polarized network. Thus, when polarization of the initial network is low, the platform uses its influence to almost always increase polarization.

In contrast, when the polarization of the initial network is high the user is much more likely to connect with a similar user rather than with a dissimilar user. Thus, the initial network already produces a lot of data between similar users and the marginal effect is high. Consequently, the platform is incentivized to increase links between dissimilar users. The size effect incentivizes the platform to increase links between similar users, as before. Thus, the size effect has to be stronger now to counter the marginal effect and for the platform to increase links between similar users. Thus, when polarization of the initial network is high, the platform uses its influence to increase polarization *if* the representation of one type of user is sufficiently high.

There are two effects of the initial network on the platform decision. As polarization of the initial network increases, more data is generated between similar users. This increases the marginal effect and reduces the incentive of the platform to increase polarization. Thus, as initial polarization increases, the platform is less likely to use its influence to increase polarization.

On the contrary, as the size effect increases, representation of one type of user increases, the number of potential links that can be generated between similar users increases and the payoff of the platform increases. Thus, as the size effect increases, the platform is more likely to use its influence to increase polarization.

Related Literature: This paper contributes to two broad streams of literature, the economics of data and polarization on social media. The role of data generated by correlation of user types has been studied in Choi et al. (2019), Acemoglu et al. (2019) and Bergemann et al. (2022). In contrast, this paper looks at the role of data generated by users interacting with each other, so value of data obtained from a user interaction does not depend on the user types per se. The subsequent analysis is complimentary to that of the above papers. In particular, the complimentary setup allows for finding conditions where encouraging new interactions between users of different types is more valuable than new encouraging interactions between users of same type in equilibrium. This paper is able to provide conditions where platform incentives are aligned with reducing polarization, which would not be possible in the other paper since they assume that information between users of same type of users is always more valuable than information between different types of users.

Acemoglu et al. (2021) has looked at the link between misinformation and incentives of a social media platform. In a similar but complementary exercise, this paper looks at the link between polarization and incentives of a platform. While in ACE the platform wants to maximise engagement, in this paper the platform wants to maximize revenue from personalized ads and personalized pricing. Consequently, I provide a microfoundation which derives the value of data under this personalization business model.

Coming to the polarization literature, the seminal theoretical work is by Esteban and Ray (1994) and the measure of polarization in this paper is consistent with the definition provided in their paper. The empirical literature studying polarization on social media is fast growing. Allcott et al. (2020) indicates that SMP reduces polarization and Asimovic et al. (2021) indicates that SMP reduces polarization. These seemingly contradictory results are resolved using the result of this paper - when initial polarization is low, an SMP increases polarization (consistent with Allcott et al. (2020)) and when initial polarization is high, an SMP reduces polarization (consistent with Asimovic et al. (2021)). It is noteworthy that Asimovic et al. (2021) provides evidence that the initial polarization in their setting is in fact exceptionally high since their experiment is carried out in a ethnically divided region with a history of genocide.
Finally, Barbera has extensive work (Barberá (2014),Barberá (2020)) which provides empirical evidence toward the nuanced effect of social media on polarization and the findings of this theoretical paper is consistent with that empirical evidence.

1.2 Model

I consider an online social media platform with users linked by an initial network. Each link between a pair of users represents an interaction, which generates information or *data* about both users. Payoff of the platform is generated by analyzing data.
I provide a microfoundation for the platform payoff and the user payoff generated from data analysis.
Next, I model the initial network of linked users as a random network with 2- types and study if the platform uses its costly algorithm to disproportionately encourage

links between users of the same type, rather than between users of different types. Thus, I study if the platform uses its algorithm to increase polarization.

1.3 Microfoundation

Define the microfounded payoff that a user gets when x amount of data is analysed by the platform as $f(x)$. Define the microfounded payoff that a platform gets when x amount of data is analysed by the platform as $g(x)$.
The effect of data sharing on user payoff can be positive or negative, depending on the purpose for which the platform uses the data. If the platform uses data to improve its estimate of WTP of the user then user payoff decreases with x and if the platform uses data to give better product recommendations to the user then user payoff increases with x.
Suppose the user and the platform know that the user has willingness to pay w, $w \sim Pa(1,3)$ and type / taste for a product is t, where $t \sim N(0,1)$. Note that, the user himself does not know the value of w and t, like the platform, it just knows the distribution.
Each data point that the platform gets is in the form of signals (s_w, s_t), where $s_w|w \sim U(0,w)$ and $s_t|t \sim N(t,1)$.
We first evaluate the effect of improved WTP estimation.

1.3.1 Effect of WTP Estimation

Since $w \sim Pa(1,3)$, $s_w|w \sim U(0,w)$ and Pareto distribution is the conjugate prior of uniform distribution, the Bayesian updated estimate of w also has Pareto distribution, in particular,

$$w|s_1, s_2, ..., s_N \sim Pa(c_N, N+3)$$

where $c_N = max\{s_1, ...s_N, 1\}$.
The distribution of $w|s_1, ...s_N$ is

$$F(w|s_1, ...s_N) = \begin{cases} 1 - \left(\frac{c_N}{w}\right)^{N+3} & w \geq c_N \\ 0 & w < c_N \end{cases}$$

The platform payoff wants to maximise its payoff from selling a product.
If the platform sets the price p for the product then platform payoff is $p[1 - F(p|s_1, ...s_N)]$ as the platform is able to sell the product at price p if and only if user WTP is greater than or equal to p and this probability is denoted by $1 - F(p|s_1, ...s_N)$. The platform thus sets price p which maximises $p[1 - F(p|s_1, ...s_N)]$.

$$p[1 - F(p)] = p\left(\frac{c_N}{p}\right)^{N+3} = \frac{c_N^{N+3}}{p^{N+2}}$$

for $p \geq c_N$ and is maximised at minimum p, $\implies p = c_N$.
$p[1 - F(p)] = p$ for $p < c_N$, which is maximised at maximum p, $\implies p = c_N$.
Therefore, after getting N signals, the platform sets price $c_N = max\{s_1, ..., s_N, 1\}$ and

$$WTPPlatformPayoff = c_N$$

where $WTPPlatformPayoff$ is the *WTP Part of Platform Payoff.*

A user i buys the product if and only if $w > c_N$. Since the user does not know its own w, he will also integrate over all possible w's with support $f(w|s_1, ..., s_N)$.

$$WTPUserPayoff = \int_{c_N}^{\infty} (w - c_N)\, d(F(w|s_1, ..., s_N))$$

where $WTPUserPayoff$ is the *WTP Part of User Payoff.* Substituting $F(w|s_1, ..., s_N)$ from above,

$$\begin{aligned} WTPUserPayoff &= E(w|s_1, ..., s_N) - c_N \\ &= \frac{N+3}{N+2}c_N - c_N = \frac{c_N}{N+2} \end{aligned}$$

Since user payoff depends on the signals generated and the number of signals, I

replace c_N by its expected value so that user payoff is a function of number of signals received by the platform only.
To calculate $E(c_N)$ first define the random variable $x_N := max\{s_1, s_2, ..., s_N\}$.
Since the signals are iid $U(0, w)$,

$$F_{x_N|w}(x) = \begin{cases} 0 & x < 0 \\ \left(\frac{x}{w}\right)^N & 0 \leq x < w \\ 1 & x \geq w \end{cases}$$

Since $c_N = max\{x_N, 1\}$,

$$\begin{aligned} E(c_N|w) = E(x_N|X_N > 1)P(X_N > 1) + E(1|x_N \leq 1)P(x_N \leq 1) \\ = \int_1^\infty x \frac{Nx^{N-1}}{w^N} I_{(0,w)}\, dx + 1.\frac{1}{w^N} \\ = \int_1^w \frac{N}{w^N} x^N\, dx + 1\frac{1}{w^N} \\ = \frac{N}{w_N} \frac{w^{N+1} - 1}{N+1} + \frac{1}{w^N} \end{aligned}$$

Using $w \sim Pa(1, 3)$ we take expectation again to get

$$\begin{aligned} E(c_N) = E(E(c_N|w)) \\ = \frac{N}{N+1} \int_1^\infty \frac{w^{N+1} - 1}{w^N} \frac{3}{w^4}\, dw + \int_1^\infty \frac{1}{w^N} \frac{3}{w^4}\, dw \\ = \frac{3N}{N+1} \left[\int_1^\infty \frac{1}{w^3}\, dw - \int_1^\infty \frac{1}{w^{4+N}}\, dw \right] + \int_1^\infty \frac{3}{w^{N+4}}\, dw \\ = \frac{3N}{N+1} \left[\frac{1}{2} - \frac{1}{N+3} \right] + \frac{3}{N+3} \\ = \frac{3(N+2)}{2(N+3)} = \frac{3}{2} \left[1 - \frac{1}{N+3} \right] \end{aligned}$$

Recall that $WTPPlatformPayoff = c_N$, replacing it by the $E(c_N)$ we get

$$WTPPlatformPayoff = \frac{3(N+2)}{2(N+3)} = \frac{3}{2} \left[1 - \frac{1}{N+3} \right]$$

which clearly increases as the number of signals / data points N increases.

Replacing c_N by its expected value in the user payoff we get

$$WTPUserPayoff = \frac{E(c_N)}{N+2} = \frac{3}{2(N+3)}$$

which clearly decreases as the number of signals / data points N increases. Thus, we have attained the decreasing part of user payoff. Next, we evaluate the effect of production recommendation.

1.3.2 Effect of Product Recommendation

Since $t \sim N(0,1)$, $s_t|t \sim N(t,1)$ and normal distribution is the conjugate prior of itself, the Bayesian updated estimate of t also has normal distribution, in particular,

$$t|s_1, s_2, ..., s_N \sim N\left(\frac{\sum_{i=1}^{N} s_i}{1+N}, \frac{1}{1+N}\right)$$

The quality of recommendation affects the probability that a user buys a book.

Platform payoff after it gets N signals can therefore be formulated as -

$$PlatformPayoff = p[1-F(p)]\left[1 - \frac{Var(t|s_1, ...s_N)}{Var(t)}\right]$$

where p is the price that the platform sets for the recommended book,
$1 - \frac{Var(t|s_1,...s_N)}{Var(t)}$ is the probability that platform recommendation matches user type.

We know that

$$Var(t|s_1, ...s_N) = \frac{1}{1+N}$$
$$Var(t) = 1$$
$$p = c_N, E(p[1-F(p)) = \frac{3(x+2)}{2(x+3)}$$

This gives us

$$PlatformPayoff = \frac{3(N+2)}{2(N+3)}\frac{N}{N+1} = \frac{3N(N+2)}{2(N+1)(N+3)}$$

Coming to user payoff, if the user buys the recommended book then his payoff is

$$\int_{c_N}^{\infty} (w - c_N)\, d(F(w|s_1, ..., s_N))$$

If the user does not buy the book then his payoff is zero. Thus,

$$UserPayoff = \left[1 - \frac{Var(t|s_1, ...s_N)}{Var(t)}\right] \int_{c_N}^{\infty} (w - c_N)\, d(F(w|s_1, ..., s_N))$$

Substituting each value from the previous section,

$$UserPayoff = \frac{N}{(N+1)}\frac{3}{2(N+3)} = \frac{3N}{2(N+1)(N+3)}$$

Extending the function obtained to any $x \geq 0$ one gets,

$$PlatformPayoff = g(x) = \frac{3(x+2)}{2(x+3)}\frac{x}{x+1} = \frac{3x(x+2)}{2(x+1)(x+3)}$$

$$UserPayoff = f(x) = \frac{x}{(x+1)}\frac{3}{2(x+3)} = \frac{3x}{2(x+1)(x+3)}$$

The function $g(x)$ is increasing and the function $f(x)$ is non-monotonic, it increases first, reach the peak at $x = \sqrt{3}$ and then decreases.

1.4 The Network

I now define the initial network which links users on the SMP.
Consider an SMP which has n users. The users are linked to each other by an online network, which is modeled using the islands model, a special case of the multi-type random network.
Given a set of n users or nodes $N = \{1, ..., n\}$, *a network* is represented via its adjacency matrix: a symmetric n-by-n matrix $\mathbf{A}$ with entries in $\{0, 1\}$. The interpretation is that $A_{ij} = A_{ji} = 1$ indicates that nodes i and j are linked, and the

symmetry restricts attention to undirected networks.
Users or nodes have "types", which are the distinguishing features that affect their propensities to connect to each other. Types might be based on any characteristics that influence users' probabilities of linking to each other, including age, race, gender, profession, education level, and even behaviors. The model is quite general in that a type can embody arbitrary lists of characteristics; which characteristics are included depends on the application.
There are m different types in the society. Let $N_k \subset N$ denote the nodes of type k, so the society is partitioned into the m groups, $(N_1, ..., N_m)$. Let $n_k = |N_k|$ denote the size of group k and n denote the total number of users.
A *multi-type random network* is defined by the cardinality vector $\mathbf{n}$ together with a symmetric m-by-m matrix $\mathbf{P}$, whose entries in $[0, 1]$ describe the probabilities of links between various types. The entry P_{kl} captures the probability that a user of type k links to an user of type l. We fill in the remaining entries of $\mathbf{A}(\mathbf{P}, \mathbf{n})$ by symmetry: $A_{ij} = A_{ji}$. We set $A_{ii} = 0$ for each i.
The *islands model* is the special case of the multi-type random networks model, such that, each user only distinguishes between users of one's own group and users of a different group. Moreover, all users are symmetric in how they do this. Formally, in the multi-type random network notation, we say the multi-type random network $(\mathbf{P}, \mathbf{n})$ is a 2-islands network, with parameters (α, p, q) if:

- if there are 2 islands, island k of size n_k and $\frac{n_2}{n_1} = \alpha$
- $P_{kk} = p$ for all k; and
- $P_{kl} = q$ for all $k \neq l$, where $\mathbf{p} \geq \mathbf{q}$ and $p > 0$.

Without loss of generality, assume N_2 is the bigger group, $\alpha \geq 1$.

Therefore, I consider a 2-islands model $G_0 = (\alpha, p_0, q_0)$, where $0 \leq q_0 \leq p_0 \leq 1$. The results of the paper are obtained for a large network, $n \to \infty$.

1.4.1 Data Generated

I now describe the data generated on a network.
Data is generated and attained by the SMP when users interact with each other. Without loss of generality, I normalise the amount of data generated. I assume that

- Each user i interacts with all her neighbors on the network.
- Each interaction between a user i and a user j generates *one unique data point.* This data point reveals information about both i and j.

Therefore, two users being linked on the network is equivalent to them generating a unique data point by interacting on the SMP.
From the assumptions one can conclude that each user i generates A_i data points on the platform, where $A_i = \sum_{j \in N} A_{ij}$ is the number of links of i in the network.
Specifically, for a network $G = (\alpha, p, q)$, a user $i \in N_1$ generates $p(n_1 - 1) + qn_2$ data points and a user $j \in N_2$ generates $p(n_2 - 1) + qn_1$ data points.

Notation: For any $p, q \in [0, 1]$, define

- $k_1 := |A_1| = p(n_1 - 1) + qn_2$
- $k_2 := |A_2| = p(n_2 - 1) + qn_1$

Therefore, for a network $G = (\alpha, p, q)$, a user in N_1 generates k_1 data points and a user in N_2 generates k_2 data points.

1.4.2 The Cost

Consider an initial network $G_0 = (\alpha, p_0, q_0)$ and the microfounded payoffs f, g. The platform uses its suggestion and ranking algorithm to increase interaction between users. Suppose the resulting influenced network is $G = (\alpha, p, q)$. Then the cost incurred by the platform is

$$c(p, q) = c(p - p_0)^2 + c(q - q_0)^2$$

where c is the marginal cost incurred by the platform when encouraging users to link with each other.

1.5 A Large Network

I analyse the platform decision for a large network, that is, as $n \to \infty$. The elements of the model p, q and c also vary with n. For any $p \in [0, 1]$, there exists a $v \in (0, \infty)$

such that

$$p = \frac{v}{n}$$

Similarly, for any $q \in [0, 1]$, there exists a $w \in (0, \infty)$ such that

$$q = \frac{w}{n}$$

The marginal cost of influencing a user, c, increases as the platform becomes larger,

$$c = c_0 n^a$$

for some c_0 and $a \in (0, \infty)$.

This gives us a SMP which satisfies three natural properties,

- The SMP is large since $n \to \infty$.
- Each user has finite number of links. As $n \to \infty$,

$$\mathbf{k_1} \to \frac{v}{n}(n_1 - 1) + \frac{w}{n} n_2 \to \frac{\mathbf{v} + \alpha \mathbf{w}}{\mathbf{1} + \alpha}$$
$$\mathbf{k_2} \to \frac{v}{n}(n_2 - 1) + \frac{w}{n} n_1 \to \frac{\alpha \mathbf{v} + \mathbf{w}}{\mathbf{1} + \alpha}$$

- The cost is increasing in number of users, a natural assumption as investment in hardware and personnel increases as the platform becomes larger.

Having defined the elements of the model for a large network, I can now define the payoffs of the players.

The payoff of the platform is,

$$\begin{aligned} u_P(p, q) &= n_1 g\left(\frac{k_1}{n}\right) + n_2 g\left(\frac{k_2}{n}\right) - c(p, q) \\ &= n_1 g\left(\frac{k_1}{n}\right) + n_2 g\left(\frac{k_2}{n}\right) - c_0 n^a (p - p_0)^2 - c_0 n^a (q - q_0)^2 \end{aligned}$$

For the platform, the value of a data point is *relative*. A SMP values user data relative to the size of the network. Thus, if a SMP attains k_i data points about a user, the corresponding payoff generated for the platform is $g\left(\frac{k_i}{n}\right)$.

With regard to the cost, I allow the rate of increase a to be general. Intuitively, the most relevant case is when the cost and the platform payoff changes at same rate w.r.t n, as one can expect to get an interior solution in this case.

For a user in group N_i, user payoff is

$$u_i(p, q) = f(k_i)$$

For the user, the value of a data point is *absolute*. The user is concerned about the effect of data analysis on himself only. The user values its data in absolute terms, irrespective of the size of the network. Thus, if a platform analyses k_i data points about a user, the corresponding user payoff is $f(k_i)$.

Given an initial network $G_0 = (\alpha, p_0, q_0)$, the platform chooses (p_1, q_1) which maximizes its payoff. Therefore, the platform chooses (p_1, q_1) such that

$$(p_1, q_1) = \arg\max_{(p,q)} \quad n_1 g\left(\frac{k_1}{n}\right) + n_2 g\left(\frac{k_2}{n}\right) - c_0 n^a (p - p_0)^2 - c_0 n^a (q - q_0)^2$$

Lastly, to study the effect of the SMP decision on polarization I formally define polarization for any network $G = (\alpha, p, q)$.

Definition 1.5.1. *For a network $G = (\alpha, p, q)$, polarization is defined as $P(G) = \frac{p}{q}$.*

Thus, polarization of an initial network $G_0 = (\alpha, p_0, q_0)$ is $P(G_0) = \frac{p_0}{q_0}$ and shall henceforth be called *initial polarization.* Polarization of the influenced network $G_1 = (\alpha, p_1, q_1)$ is $P(G_1) = \frac{p_1}{q_1}$ and shall henceforth be called *equilibrium polarization.*

The question of platform increasing polarization can now be answered by comparing initial polarization with equilibrium polarization.

Definition 1.5.2. *A platform is said to increase polarization if and only if equilibrium polarization is weakly higher than initial polarization,*

$$\frac{p_1}{q_1} \geq \frac{p_0}{q_0}$$

1.6 Platform Decision

The platform decision can be analyzed under three cases, depending on the the rate of change of cost w.r.t n, that is, depending on a. I consider the most relevant case below.

Case I Assume $a = 2$

Proposition 1. *For an initial network $G_0 = (\alpha, p_0, q_0)$, the platform increases polarization if and only if $\frac{1+\alpha^2}{2\alpha} \geq \frac{p_0}{q_0}$.*

Proof. Payoff of the platform is

$$\begin{aligned} u_P(p,q) &= n_1 g\left(\frac{k_1}{n}\right) + n_2 g\left(\frac{k_2}{n}\right) - c_0 n^2 (p-p_0)^2 - c_0 n^2 (q-q_0)^2 \\ &= n_1 \left(\frac{\left(\frac{k_1}{n}\right)}{\left(\frac{k_1}{n}\right)+1} \frac{\left(\frac{k_1}{n}\right)+2}{\left(\frac{k_1}{n}\right)+3} \right) + n_2 \left(\frac{\left(\frac{k_2}{n}\right)}{\left(\frac{k_2}{n}\right)+1} \frac{\left(\frac{k_2}{n}\right)+2}{\left(\frac{k_2}{n}\right)+3} \right) - c_0 n^2 (p-p_0)^2 - c_0 n^2 (q-q_0)^2 \end{aligned}$$

As $n \to \infty$,

$$\begin{aligned} u_P(p,q) &\to \frac{2n_1}{3} \frac{\left(\frac{k_1}{n}\right)}{\left(\frac{k_1}{n}\right)+1} + \frac{2n_2}{3} \frac{\left(\frac{k_2}{n}\right)}{\left(\frac{k_2}{n}\right)+1} - c_0 n^2 (p-p_0)^2 - c_0 n^2 (q-q_0)^2 \\ &\to \frac{2k_1}{3(1+\alpha)} + \frac{2\alpha k_2}{3(1+\alpha)} - c_0 n^2 (p-p_0)^2 - c_0 n^2 (q-q_0)^2 \\ &= \frac{2}{3(1+\alpha)}[p(n_1-1)+qn_2] + \frac{2\alpha}{3(1+\alpha)}[p(n_2-1)+qn_1] - c_0 n^2 (p-p_0)^2 - c_0 n^2 (q-q_0)^2 \end{aligned}$$

As $n \to \infty$, we know that there exists $v, v_0,\ w, w_0 \in (0, \infty)$ such that

$$p = \frac{v}{n}, p_0 = \frac{v_0}{n}, q = \frac{w}{n}, q_0 = \frac{w_0}{n}$$

Therefore, as $n \to \infty$,

$$k_1 = v\frac{n_1 - 1}{n} + w\frac{n_2}{n} \to \frac{v + \alpha w}{1 + \alpha}$$
$$k_2 = v\frac{n_2 - 1}{n} + w\frac{n_1}{n} \to \frac{\alpha v + w}{1 + \alpha}$$
$$c_0 n^2 (p - p_0)^2 = c_0 n^2 \left(\frac{v}{n} - \frac{v_0}{n}\right)^2 \to c_0 (v - v_0)^2$$
$$c_0 n^2 (q - q_0)^2 = c_0 n^2 \left(\frac{w}{n} - \frac{w_0}{n}\right)^2 \to c_0 (w - w_0)^2$$

Substituting the above terms into platform payoff we get

$$\lim_{n\to\infty} u_P\left(\frac{v}{n}, \frac{w}{n}\right) = \frac{2(v + \alpha w)}{3(1+\alpha)^2} + \frac{2\alpha(\alpha v + w)}{3(1+\alpha)^2} - c_0(v - v_0)^2 - c_0(w - w_0)^2$$

The platform maximizes its payoff by choosing the optimal p and q, equivalently, by choosing the optimal v and w.

The payoff of the platform is concave and increasing in (p, q), equivalently, in (v, w). Therefore, the optimal v and w are obtained by taking the first order derivative w.r.t. v and w.

$$\frac{\partial}{\partial v}\left[\frac{2(v + \alpha w)}{3(1+\alpha)^2} + \frac{2\alpha(\alpha v + w)}{3(1+\alpha)^2} - c_0(v - v_0)^2 - c_0(w - w_0)^2\right] = 0$$

$$\frac{2}{3(1+\alpha)^2} + \frac{2\alpha^2}{3(1+\alpha)^2} - 2c_0(v - v_0) = 0$$

$$v = \frac{1 + \alpha^2}{3c_0(1+\alpha)^2} + v_0$$

Similarly,

$$\frac{\partial}{\partial w}\left[\frac{2(v + \alpha w)}{3(1+\alpha)^2} + \frac{2\alpha(\alpha v + w)}{3(1+\alpha)^2} - c_0(v - v_0)^2 - c_0(w - w_0)^2\right] = 0$$

$$\frac{2\alpha}{3(1+\alpha)^2} + \frac{2\alpha}{3(1+\alpha)^2} - 2c_0(w - w_0) = 0$$

$$w = \frac{2\alpha}{3c_0(1+\alpha)^2} + w_0$$

The optimal p_1 and q_1 are then

$$p_1 = \frac{v}{n} = \frac{1}{n}\left[\frac{1}{3c_0}\left(\frac{1+\alpha^2}{1+\alpha}\right) + v_0\right]$$

$$q_1 = \frac{w}{n} = \frac{1}{n}\left[\frac{1}{3c_0}\left(\frac{2\alpha}{1+\alpha}\right) + w_0\right]$$

The equilibrium polarization is

$$\frac{p_1}{q_1} = \frac{\frac{1+\alpha^2}{3c_0(1+\alpha)^2} + v_0}{\frac{2\alpha}{3c_0(1+\alpha)^2} + w_0}$$

The platform increases polarization $\iff \frac{p_1}{q_1} \geq \frac{p_0}{q_0}$

$$\iff \frac{\frac{1+\alpha^2}{3c_0(1+\alpha)^2} + v_0}{\frac{2\alpha}{3c_0(1+\alpha)^2} + w_0} \geq \frac{p_0}{q_0} = \frac{v_0}{w_0}$$

$$\iff w_0\left(\frac{1+\alpha^2}{3c_0(1+\alpha)^2}\right) + v_0w_0 \geq v_0\left(\frac{2\alpha}{3c_0(1+\alpha)^2}\right) + v_0w_0$$

$$\iff w_0\left(\frac{1+\alpha^2}{3c_0(1+\alpha)^2}\right) \geq v_0\left(\frac{2\alpha}{3c_0(1+\alpha)^2}\right)$$

$$\iff \frac{1+\alpha^2}{2\alpha} \geq \frac{v_0}{w_0} = \frac{p_0}{q_0}$$

□

1.7 Welfare Analysis

In this section, I carry out the welfare analysis. First, we consider the payoffs of the players as $n \to \infty$.
The revenue generated by the platform as $n \to \infty$ is

$$u_P(p,q) + c(p,q_) \to \frac{2k_1}{3(1+\alpha)} + \frac{2\alpha k_2}{3(1+\alpha)}$$

Thus, the revenue of the platform increases in k_1 and in k_2.
The payoff of a user in group N_i as $n \to \infty$ is

$$f(p,q) = \frac{k_i}{(k_i+1)(k_i+3)}$$

The payoff of a user in N_1 is non - monotonic in k_1 and payoff of a user in N_2 is non - monotonic in k_2.
Thus, the revenue of a SMP increases as it attains more data points. Instead, user payoff is non-monotonic in number of data points. While a platform would always want to attain more data points to increase its revenue, a user would want that the platform attains the amount of data at which user payoff is maximized.
Adding the cost of encouraging new links, which is convex in p and in q, we see that platform payoff is also non - monotonic in number of data points. The platform would also want to attain the amount of data which maximizes its payoff. We have derived this maximum value as (p_1, q_1) in the previous section.
Of course, the optimal p_1, q_1 may not maximize user payoff and more broadly, may not maximize total welfare. Below is the characterization of total welfare attained in equilibrium under p_1, q_1 w.r.t the first best or welfare maximizing (p_{FB}, q_{FB}).

Define the total welfare for $n \to \infty$ as

$$\begin{aligned} TW(p,q) &= u_P(p,q) + n_1 u_1(p,q) + n_2 u_2(p,q) \\ &= \frac{2k_1}{3(1+\alpha)} + \frac{2\alpha k_2}{3(1+\alpha)} + n_1 f(k_1) + n_2 f(k_2) - c_0 n^2 (p-p_0)^2 - c_0 n^2 (q-q_0)^2 \end{aligned}$$

$\implies$

$$TW(v,w) = \frac{2k_1}{3(1+\alpha)} + \frac{2\alpha k_2}{3(1+\alpha)} + n_1 f(k_1) + n_2 f(k_2) - c_0 (v-v_0)^2 - c_0 (w-w_0)^2$$

Taking the first order derivative,

$$\frac{\partial}{\partial v} TW(v,w) = \frac{2 + 3f'(k_1) + 2\alpha^2 + 3\alpha^2 f'(k_2)}{3(1+\alpha)^2} - 2c_0(v-v_0)$$

$$\frac{\partial}{\partial w} TW(v,w) = \frac{4\alpha + 3\alpha\{f'(k_1) + f'(k_2)\}}{3(1+\alpha)^2} - 2c_0(w-w_0)$$

For c_0 large enough, $\frac{\partial^2}{\partial v^2}TW(v,w) < 0$ and $\frac{\partial^2}{\partial w^2}TW(v,w) < 0$.

Therefore, welfare maximizing $v_{FB} = \frac{1}{n}p_{FB}$ and $w_{FB} = \frac{1}{n}q_{FB}$ is attained by solving $\frac{\partial}{\partial v}TW(v,w) = 0$ and $\frac{\partial}{\partial w}TW(v,w) = 0$ respectively.

Let us consider the first order conditions for two cases -

Case 1: $\alpha \to 1$
As $\alpha \to 1$, $k_1 \to \frac{v+w}{2}$ and $k_2 \to \frac{v+w}{2}$. Thus,

$$\frac{\partial}{\partial v}TW(v,w) \to \frac{1}{3} + \frac{1}{2}f'\left(\frac{v+w}{2}\right) - 2c_0(v - v_0)$$
$$\frac{\partial}{\partial w}TW(v,w) \to \frac{1}{3} + \frac{1}{2}f'\left(\frac{v+w}{2}\right) - 2c_0(w - w_0)$$

$\implies v - v_0 = w - w_0$. Substituting $v = w - w_0 + v_0$ in $\frac{\partial}{\partial w}TW(v,w)$ gives

$$\frac{\partial}{\partial w}TW(v,w) \to \frac{1}{3} + \frac{1}{2}f'\left(\frac{2w + v_0 - w_0}{2}\right) - 2c_0(w - w_0)$$

The first order condition under (v_1, w_1) is

$$\frac{\partial}{\partial w}TW(v_1,w_1) \to \frac{1}{3} + \frac{1}{2}f'\left(\frac{2w_1 + v_0 - w_0}{2}\right) - 2c_0(w_1 - w_0)$$
$$\to \frac{1}{2}f'\left(\frac{v_0 + w_0}{2} + \frac{1}{6c_0}\right) = 0$$

$\iff \frac{v_0+w_0}{2} + \frac{1}{6c_0} = \sqrt{3}$. Therefore,

$$\frac{\partial}{\partial w}TW(v_1,w_1) \begin{cases} > 0 & \text{if } \frac{v_0+w_0}{2} + \frac{1}{6c_0} < \sqrt{3} \\ = 0 & \text{if } \frac{v_0+w_0}{2} + \frac{1}{6c_0} = \sqrt{3} \\ < 0 & \text{if } \frac{v_0+w_0}{2} + \frac{1}{6c_0} > \sqrt{3} \end{cases}$$

The first best w_{FB} and equilibrium w_1 are therefore related as

$$\begin{cases} w_{FB} > w_1 & \text{if } \frac{v_0+w_0}{2} + \frac{1}{6c_0} < \sqrt{3} \\ w_{FB} = w_1 & \text{if } \frac{v_0+w_0}{2} + \frac{1}{6c_0} = \sqrt{3} \\ w_{FB} < w_1 & \text{if } \frac{v_0+w_0}{2} + \frac{1}{6c_0} > \sqrt{3} \end{cases}$$

Since $\alpha \to 1$ we get the similar result for v_{FB} and v_1,

$$\frac{\partial}{\partial v} TW(v_1, w_1) \begin{cases} > 0 & \text{if } \frac{v_0+w_0}{2} + \frac{1}{6c_0} < \sqrt{3} \\ = 0 & \text{if } \frac{v_0+w_0}{2} + \frac{1}{6c_0} = \sqrt{3} \\ < 0 & \text{if } \frac{v_0+w_0}{2} + \frac{1}{6c_0} > \sqrt{3} \end{cases}$$

$$\begin{cases} v_{FB} > v_1 & \text{if } \frac{v_0+w_0}{2} + \frac{1}{6c_0} < \sqrt{3} \\ v_{FB} = v_1 & \text{if } \frac{v_0+w_0}{2} + \frac{1}{6c_0} = \sqrt{3} \\ v_{FB} < v_1 & \text{if } \frac{v_0+w_0}{2} + \frac{1}{6c_0} > \sqrt{3} \end{cases}$$

Therefore, when the initial network is *sparse* (v_0, w_0 is small), the network in equilibrium is *not connected enough* to maximize total welfare and when the initial network is *dense*, the network in equilibrium is *too connected* to maximize welfare.

Case 2: $\alpha \to \infty$
As $\alpha \to \infty$, $k_1 \to w$ and $k_2 \to v$.

$$\frac{\partial}{\partial v} TW(v, w) \to \frac{2}{3} + f'(v) - 2c_0(v - v_0)$$
$$\frac{\partial}{\partial w} TW(v, w) \to -2c_0(w - w_0)$$

$\implies w_{FB} = w_0$. Since $w_1 > w_0$,

$$w_{FB} < w_1 \ \forall \ (v_0, w_0)$$

The first order condition $\frac{\partial}{\partial v} TW(v, w) = 0$ under (v_1, w_1) is

$$\frac{\partial}{\partial v} TW(v_1, w_1) = f'\left(v_0 + \frac{1}{3c_0}\right)$$

Therefore,

$$\frac{\partial}{\partial v} TW(v_1, w_1) \begin{cases} > 0 & \text{if } v_0 + \frac{1}{3c_0} < \sqrt{3} \\ = 0 & \text{if } v_0 + \frac{1}{3c_0} = \sqrt{3} \\ < 0 & \text{if } v_0 + \frac{1}{3c_0} > \sqrt{3} \end{cases}$$

$$\begin{cases} v_{FB} > v_1 & \text{if } v_0 + \frac{1}{3c_0} < \sqrt{3} \\ v_{FB} = v_1 & \text{if } v_0 + \frac{1}{3c_0} < \sqrt{3} \\ v_{FB} < v_1 & \text{if } v_0 + \frac{1}{3c_0} < \sqrt{3} \end{cases}$$

Therefore, the network in equilibrium has *too many connections across groups* relative to welfare maximizing network. When the initial network is *sparse*, the network in equilibrium is *not connected enough within groups* to maximize welfare and when the initial network is *dense*, the network in equilibrium is *too connected* to maximize welfare.

1.8 Conclusion

I study whether a social media platform (SMP) has an incentive to increase polarization of its social network. An SMP earns revenue from user data, which is generated on the social network through user interactions. The network comprises of two *groups* of users. The algorithmic influence of an SMP enables it to encourage users to form new links, generate new interactions and earn more revenue. The SMP is said to increase polarization if it *disproportionately* increases links within groups than across groups. I find that an SMP increases polarization if and only if one group is much larger than the other. An SMP can reduce polarization by attracting underrepresented users to join its platform.

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Chapter 2

Regulating Online Platforms in the Data Economy

2.1 Introduction

Online platforms have recently come under heavy scrutiny, in particular regarding the effect they have on users. Online platforms generate revenue from user data and with recent scandals like *Cambridge Analytica* the effect of data collection on users has come into question. Policy makers are contemplating several policies that (hopefully) benefit users of an online platform. This paper analyzes these policies and concludes that while some policies like breaking-up a large platform benefit users, other policies, like stricter privacy rules which give users more control over their data, backfire and harm users.

I develop a framework that the effect of different regulations in a data economy, consisting of an online platform and heterogeneous users. The model takes important aspects of the data economy in consideration. First, users exhibit *information externalities* or *data externalities*. When a user shares his data with a platform, the platform also gathers data about other users. I model this by having a network that connects users with each other. The effect of data externality is then captured as the size of user neighbourhood. Second, information or data has a dual nature. Data can be used to steer different users towards different products, thereby showing them better, but also more expensive products. I tackle this duality by giving a novel microfoundation which inculcates both the positive and negative effect on a user when his data is revealed to a platform.

I find that the microfounded user payoff is an inverted-U w.r.t the number of data points revealed to the platform. When the platform has few data points about a user, the positive effect of product recommendation is stronger than negative effect of personalized pricing. Thus, an additional data point results in higher user payoff. When the platform has many data points about a user, the positive effect of product recommendation is weaker than negative effect of personalized pricing. Thus, an additional data point results in lower user payoff. This gives us the inverted-U shape of user payoff.

Also, since a user will never buy a product priced higher than his willingness to pay (WTP), the microfounded user payoff is always non-negative.

These two properties give us the main findings of the paper. First, a stricter consent policy, which stops all data leakage, results in weakly lower user payoff. This is because denying consent in a stricter privacy setting means that no data about the user is revealed to the platform and the user attains zero payoff. In a loose privacy setting, denying consent may still result in some data leakage, which gives the user a positive payoff. Thus, by non negativity of the microfounded user payoff, a stricter privacy policy backfires on users.

Second, if users own the platform then both total welfare and total user welfare is maximized. Sharing of all data is total welfare maximizing as the platform benefit far exceeds the user loss from data analysis and the platform payoff increases as more data is revealed. When users own the platform then total welfare and total user welfare is simultaneously maximized as the total welfare is distributed amongst the users.

Third, the paper looks at a prevalent regulation that has been proposed recently - breaking up a platform [1]. Since the payoff of a user is an inverted-U, if users have large neighbourhoods, then breaking up a platform increases user welfare. If users have small neighbourhoods, then breaking up a platform backfires and reduces user welfare.

This paper relates to the literature of privacy and its regulatory aspects. The need for better control over one's data has been studied in Posner and Weyl (2018) and Zuboff (2019). Data externalities are a hurdle to any such regulation since they pre-

[1] https://www.theguardian.com/technology/2022/jan/12/lawsuit-aiming-to-break-up-facebook-group-meta-can-go-ahead-us-court-rules

vent users from having control over their data. The importance of externalities has long been recognised, see Fairfield and Engel (2015) and MacCarthy (2010). The working paper by Acemoglu et al. (2019) emphasizes that heterogeneity in privacy concerns is a critical reason for inefficiency in a data market. Since privacy is the main concern, user payoff decreases in number of data points and data externalities lower user payoff. This paper microfounds payoffs when data is used to steer users. Hence, user payoff is non-monotonic in number of data points and data externalities may or may not reduce user payoff. The ambiguous nature of data externalities becomes the main concern when studying regulation. In Choi et al. (2019), excessive loss of privacy due to data externalities is highlighted. The paper further shows that GDPR may fall short in limiting the collection of personal data only up to the socially optimal level. This paper points to the potential backfiring of stricter privacy regulations (e.g. GDPR) and offers user ownership and breaking up of platforms as regulation options that improve total user welfare. The survey of privacy literature by Acquisti et al. (2016) provides a comprehensive review of privacy related papers. The survey concludes that privacy concerns span many different scenarios and it is not feasible to have one blanket policy. This paper reaffirms this conclusion and provides a framework under which data regulation policies can be analyzed.

This paper is also related to the literature of contracting externalities. Seminal work by Segal (1999) and more recent work by Ellingsen and Paltseva (2016), Jackson and Wilkie (2005) have focused on inefficiency in contracting with externalities. They have tested the validity of Coase theorem and find that the theorem does not hold under a wide variety of circumstances. In this model, Coase theorem holds under both, a loose privacy policy and under a strict privacy policy. However, users are worse off under the strict policy than under the loose policy. This paper, with its microfoundation and network structure allows for analysis of regulations that address the rising concerns about user welfare on online platforms.

2.2 Model

Having seen the related literature, we now move on to modelling the key aspects of the economy of data. First, I present a microfoundation of the effect of data analysis. This microfoundation is important as it gives a micro-economic justification to the idea that user payoff can be non-monotonic w.r.t data revealed, that is, user payoff first increases as data is revealed to the platform, reaches a peak and then starts decreasing as more data is revealed.

2.3 Microfoundation

The microfoundation is derived for both the user and the platform. Specifically, $f(x)$ is the microfounded payoff that a user gets when x amount of data is revealed to and analysed by the platform. $g(x)$ is the payoff that a platform gets when x amount of data is revealed to and analysed by the platform.
The effect of data sharing can be both positive or negative, it depends on the purpose for which the platform uses the data. If the platform uses data to improve its estimate of WTP of the user then user payoff decreases and if the platform uses data to give better product recommendations to the user then user payoff increases.
I model this scenario as follows. Suppose a user wants to buy a product. The platform analyses user data to recommend a particular product and to estimate the user WTP for that product.
The user and the platform know that the user has WTP w, $w \sim Pa(1,3)$ and taste for products, $t \sim N(0,1)$. Note that, the user himself does not know the value of w and t, like the platform, it just knows the distribution.
Each data point that the platform gets is in the form of signals (s_w, s_t), where $s_w|w \sim U(0,w)$ and $s_t|t \sim N(t,1)$.
We first evaluate the effect of improved WTP estimation.

2.3.1 Effect of Willingness To Pay Estimation

Since $w \sim Pa(1,3)$, $s_w|w \sim U(0,w)$ and Pareto distribution is the conjugate prior of uniform distribution, the Bayesian updated estimate of w also has Pareto distribution, in particular,

$$w|s_1, s_2, ..., s_N \sim Pa(c_N, N+3)$$

where $c_N = max\{s_1, ...s_N, 1\}$.
The distribution of $w|s_1, ...s_N$ is

$$F(w|s_1, ...s_N) = \begin{cases} 1 - \left(\frac{c_N}{w}\right)^{N+3} & w \geq c_N \\ 0 & w < c_N \end{cases}$$

The platform payoff wants to maximise its payoff from selling a product.
If the platform sets the price p for the product then platform payoff is $p[1 - F(p|s_1, ...s_N)]$ as the platform is able to sell the product at price p if and only if the user WTP is greater than or equal to p and this probability is denoted by $1 - F(p|s_1, ...s_N)$. The platform thus sets price p which maximises $p[1 - F(p|s_1, ...s_N)]$.

$$p[1 - F(p)] = p\left(\frac{c_N}{p}\right)^{N+3} = \frac{c_N^{N+3}}{p^{N+2}}$$

for $p \geq c_N$ and is maximised at minimum p, $\implies p = c_N$.
$p[1 - F(p)] = p$ for $p < c_N$, which is maximised at maximum p, $\implies p = c_N$.
Therefore, after getting N signals, the platform sets price $c_N = max\{s_1, ..., s_N, 1\}$ and

$$WTPPlatformPayoff = c_N$$

where $WTPPlatformPayoff$ is the *Willingness to Pay Part of Platform Payoff.*

A user i buys the product if and only if $w > c_N$. Since the user does not know its own w, he will also integrate over all possible w's with support $f(w|s_1, ..., s_N)$.

$$WTPUserPayoff = \int_{c_N}^{\infty} (w - c_N)\, d(F(w|s_1, ..., s_N))$$

where $WTPUserPayoff$ is the *Willingness to Pay Part of User Payoff.* Substituting $F(w|s_1, ..., s_N)$ from above,

$$\begin{aligned} WTPUserPayoff &= E(w|s_1, ..., s_N) - c_N \\ &= \frac{N+3}{N+2}c_N - c_N = \frac{c_N}{N+2} \end{aligned}$$

Since user payoff depends on the signals generated and the number of signals, I

replace c_N by its expected value so that user payoff is a function of number of signals received by the platform only.
To calculate $E(c_N)$ first define the random variable $x_N := max\{s_1, s_2, ..., s_N\}$.
Since the signals are iid $U(0, w)$,

$$F_{x_N|w}(x) = \begin{cases} 0 & x < 0 \\ \left(\frac{x}{w}\right)^N & 0 \leq x < w \\ 1 & x \geq w \end{cases}$$

Since $c_N = max\{x_N, 1\}$,

$$\begin{aligned} E(c_N|w) = E(x_N|X_N > 1)P(X_N > 1) + E(1|x_N \leq 1)P(x_N \leq 1) \\ = \int_1^\infty x\frac{Nx^{N-1}}{w^N} I_{(0,w)}\, dx + 1.\frac{1}{w^N} \\ = \int_1^w \frac{N}{w^N} x^N\, dx + 1.\frac{1}{w^N} \\ = \frac{N}{w_N}.\frac{w^{N+1}-1}{N+1} + \frac{1}{w^N} \end{aligned}$$

Using $w \sim Pa(1, 3)$ we take expectation again to get

$$\begin{aligned} E(c_N) = E(E(c_N|w)) \\ = \frac{N}{N+1}\int_1^\infty \frac{w^{N+1}-1}{w^N}\frac{3}{w^4}\, dw + \int_1^\infty \frac{1}{w^N}\frac{3}{w^4}\, dw \\ = \frac{3N}{N+1}\left[\int_1^\infty \frac{1}{w^3}\, dw - \int_1^\infty \frac{1}{w^{4+N}}\, dw\right] + \int_1^\infty \frac{3}{w^{N+4}}\, dw \\ = \frac{3N}{N+1}\left[\frac{1}{2} - \frac{1}{N+3}\right] + \frac{3}{N+3} \\ = \frac{3(N+2)}{2(N+3)} = \frac{3}{2}\left[1 - \frac{1}{N+3}\right] \end{aligned}$$

Recall that $WTPPlatformPayoff = c_N$, replacing it by the $E(c_N)$ we get

$$WTPPlatformPayoff = \frac{3(N+2)}{2(N+3)} = \frac{3}{2}\left[1 - \frac{1}{N+3}\right]$$

which clearly increases as the number of signals / data points N increases.

Replacing c_N by its expected value in the user payoff we get

$$WTPUserPayoff = \frac{E(c_N)}{N+2} = \frac{3}{2(N+3)}$$

which clearly decreases as the number of signals / data points N increases. Thus, we have attained the decreasing part of user payoff. Next, we evaluate the effect of production recommendation.

2.3.2 Effect of Product Recommendation

Since $t \sim N(0,1)$, $s_t|t \sim N(t,1)$ and Normal distribution is the conjugate prior of itself, the Bayesian updated estimate of t also has Normal distribution, in particular,

$$t|s_1, s_2, ..., s_N \sim N\left(\frac{\sum_{i=1}^{N} s_i}{1+N}, \frac{1}{1+N}\right)$$

The quality of recommendation affects the probability that a user buys a product. This is feasible in the following scenario - Suppose a user does not enjoy reading autobiographies. Then, it is unlikely that he will buy a book if the recommendation is an autobiography. In this case, the platform is more likely to make a sale if its recommendation is closer to user tatse and platform payoff increases as its recommendation improves. Platform payoff after it gets N signals can therefore be formulated as -

$$PlatformPayoff = p[1 - F(p)]\left[1 - \frac{Var(t|s_1, ...s_N)}{Var(t)}\right]$$

where p is the price that the platform sets for the recommended product, $\left[1 - \frac{Var(t|s_1,...s_N)}{Var(t)}\right]$ is the probability that platform recommendation matches user tatse. We know that

$$Var(t|s_1, ...s_N) = \frac{1}{1+N}$$

$$Var(t) = 1$$

$$p = c_N, E(p[1 - F(p)) = \frac{3(x+2)}{2(x+3)}$$

This gives us -

$$PlatformPayoff = \frac{3(N+2)}{2(N+3)}\frac{N}{N+1} = \frac{3N(N+2)}{2(N+1)(N+3)}$$

Coming to user payoff, if the user buys the recommended product then his payoff is

$$\int_{c_N}^{\infty} (w - c_N)\, d(F(w|s_1, ..., s_N))$$

If the user does not buy the product then his payoff is zero. Thus,

$$UserPayoff = \left[1 - \frac{Var(t|s_1, ...s_N)}{Var(t)}\right] \int_{c_N}^{\infty} (w - c_N)\, d(F(w|s_1, ..., s_N))$$

Substituting each value from the previous section,

$$UserPayoff = \frac{N}{(N+1)}\frac{3}{2(N+3)} = \frac{3N}{2(N+1)(N+3)}$$

Extending the function obtained to any $x \geq 0$ one gets,

$$PlatformPayoff = g(x) = \frac{3(x+2)}{2(x+3)}\frac{x}{x+1} = \frac{3x(x+2)}{2(x+1)(x+3)}$$

$$UserPayoff = f(x) = \frac{x}{(x+1)}\frac{3}{2(x+3)} = \frac{3x}{2(x+1)(x+3)}$$

The function $g(x)$ is increasing and the function $f(x)$ is non-monotonic, it increases first, reach the peak at $x = \sqrt{3}$ and then decreases.

2.4 The Network

Consider an online platform which has n users. The users are connected to each other by an online network, which is modeled using the islands model, a special case of the multi-type random network.

Given a set of n users or nodes $N = \{1, ..., n\}$, *a network* is represented via its adjacency matrix: a symmetric n-by-n matrix $\mathbf{A}$ with entries in $\{0, 1\}$. The inter-

pretation is that $A_{ij} = A_{ji} = 1$ indicates that nodes i and j are linked, and the symmetry restricts attention to undirected networks.
Users or nodes have "types", which are the distinguishing features that affect their propensities to connect to each other. Types might be based on any characteristics that influence users' probabilities of linking to each other, including age, race, gender, profession, education level, and even behaviors. The model is quite general in that a type can embody arbitrary lists of characteristics; which characteristics are included depends on the application.
There are m different types in the society. Let $N_k \subset N$ denote the nodes of type k, so the society is partitioned into the m groups, $(N_1, ..., N_m)$. Let $n_k = |N_k|$ denote the size of group k and n denote the total number of users.
A *multi-type random network* is defined by the cardinality vector $\mathbf{n}$ together with a symmetric m-by-m matrix $\mathbf{P}$, whose entries in $[0, 1]$ describe the probabilities of links between various types. The entry P_{kl} captures the probability that a user of type k links to an user of type l. We fill in the remaining entries of $\mathbf{A}(\mathbf{P}, \mathbf{n})$ by symmetry: $A_{ij} = A_{ji}$. We set $A_{ii} = 0$ for each i.
The *islands model* is the special case of the multi-type random networks model, such that, each user only distinguishes between users of one's own group and users of a different group. Moreover, all users are symmetric in how they do this. Formally, in the multi-type random network notation, we say the multi-type random network $(\mathbf{P}, \mathbf{n})$ is a two islands network with parameters (n, p, q) if:

- there are 2 islands, each island N_k of size n_k, $n_1 + n_2 = n$;
- $P_{kk} = p$ for all k; and
- $P_{kl} = q$ for all $k \neq l$, where $\mathbf{p} \geq \mathbf{q}$ and $p > 0$.

Without loss of generality, assume N_2 is the bigger group, $n_2 \geq n_1$.

2.4.1 Data Generated

I now describe the economy of data based on this network. Each user generates some data on the online platform. I call this *personal data* and it is of two types -

- *individual data*, which is generated by the user and provides the platform information about said user only; and

- *interactive data*, which is is generated by two users who are linked and the platform gets information about *both users*.

Examples of individual data are the ads a user clicks on, login and logout time, the posts that a user pauses on while scrolling. Examples of interactive data are messages between users, photos and videos posted with both users present.
While individual data about a user can be attained only from that particular user, interactive data can be attained from any one of two connected users. This gives rise to *data externality*. For any two users connected on a platform, data about one user is informative about that user and his connections on a platform. This is due to the well documented concept of homophily on online platforms, i.e., users are likely connected with like-minded people. Consider Facebook. It has been shown that highly personal information like sexual orientation can be inferred using the connections. As Jernigan and Mistree (2009) paper shows, a homosexual man is more likely to be connected to other homosexual men than to heterosexual men. This logic can be used in a consumerism context too. An avid reader is likely connected to other book lovers. A football fan is likely connected to other football fans. Thus, a wide range of information related to a user can be obtained from information collected about his friends. A user gives consent to the platform based on how sharing data with the platform affects her, irrespective of the externality it has on any other linked user. We will soon see that the platform is able to access user data due to the presence of data externalities, even when data analysis is harmful for users. In order to do that we expound on how user data is generated on a platform.
Data is generated when users interact with each other or when users use the services of the platform. Without loss of generality, we normalise the amount of data generated,

- A user i interacts with all her neighbors and *one data point* is generated from each interaction. This data point reveals information about both i and j.

- Each user i generates individual data. This data is normalised to one, thus, each user produces one individual data point.

Therefore, two users being linked on the network is equivalent to them generating a unique data point by interacting on the platform.

From the assumptions one can conclude that each user i generates A_i data points on the platform, where $A_i = \sum_{j \in N} A_{ij}$ is the number of links of i in the network. Specifically, for a network $G = (\alpha, p, q)$, a user $i \in N_1$ generates $p(n_1 - 1) + qn_2$ data points and a user $j \in N_2$ generates $p(n_2 - 1) + qn_1$ data points.

Notation: For any $p, q \in [0, 1]$, define

- $k_1 := |A_1| = p(n_1 - 1) + qn_2$
- $k_2 := |A_2| = p(n_2 - 1) + qn_1$

Therefore, for a network $G = (n, p, q)$, a user in N_1 generates k_1 data points and a user in N_2 generates k_2 data points.

2.5 The Game

Consider a two stage game where the platform offers a payment to each user for his data and the user either accepts or rejects the offer. If a user i accepts the offer, he shares all his data with the platform. This in turn reveals the interactive data about his linked nodes to the platform. If a user i does not accept the offer, his personal data is not revealed to the platform. However, if linked nodes of the user accept the platform offer, then his interactive data with these nodes is still revealed to the platform. Summarising,

- **First stage:** The platform moves first and offers $p_i \geq 0$ to each user $i \in N$.
- **Second stage:** User i accepts or rejects the offer, $a_i = 1$ or $a_i = 0$ respectively.

2.6 Privacy Policy: Unilateral Consent and Bilateral Consent

The final component needed to define the payoffs is the *privacy policy* in effect. Regulations like GDPR aim to give users complete control over their data, that is, user data should be shared with the platform if and only if the user has given his consent. Externalities from interactive data make the implementation of this policy difficult. Simply put, if one user does not give consent and a linked user does give consent then should the interactive data be shared with the platform or not? As of now, the status quo is that in such a case the data will be shared with the platform. We compare this status quo with a stricter privacy policy, where interactive data is shared with the platform if and only if both users give consent to share their data. We will see that the stricter privacy policy is not always the optimal policy. But first, let us define these privacy policies precisely.

2.6.1 Unilateral Consent (Loose Policy)

Definition 2.6.1. *Under unilateral consent, the interactive data point generated by any i and j (where $j \in N_i$), is revealed to the platform if $a_i = 1$ or $a_j = 1$.*

Under unilateral consent, an interactive data point is revealed to a platform if any one of the two concerned users gives consent. Thus, each user can *unilaterally* share data that affects his and his neighbors. Thus, if $a_i = 1$, all data about user i is revealed to the platform and i is unaffected by the decision of his neighbors. If $a_i = 0$, then utility of i is affected by whether $a_j = 0$ or $a_j = 1$. Thus, a user i is affected by data externality when $a_i = 0$.
Unilateral consent is representative of the status quo. An online platform is able to access the data of a user once he consents. The fact that this data may affect other people and hence the consent of other people should also be taken is not taken into account in this policy.
The payoff of the platform and the users under unilateral consent then becomes,

$$u_P(\mathbf{p}, \mathbf{a}) = -\sum_{i \in N} p_i a_i + \sum_{i \in N} [g(1 + k_i)a_i + g(M_i)(1 - a_i)]$$

$$u_i(\mathbf{p}, \mathbf{a}) = p_i a_i + f(1 + k_i)a_i + f(M_i)(1 - a_i)$$

where,

- $\mathbf{p} = (p_1, p_2, \ldots, p_n)$ is the payment offer by the platform and $\mathbf{a} = (a_1, a_2, \ldots, a_n)$ is the decision of the users.

- $M_i = |\{j \in N_i : a_j = 1\}|$ **is the number of neighbors of i who share their data**. Clearly, $M_i \in \{0, \ldots, k_i\}$.

In the user utility, i gets the payment (or services) p_i and the payoff $f(1+k_i)$ if he chooses $a_i = 1$ as it reveals all his data to the platform. If he chooses $a_i = 0$, i does not get any payment and the payoff from data revelation is $f(M_i)$.

2.6.2 Bilateral Consent (Strict Policy)

Definition 2.6.2. *Under bilateral consent, the interactive data point generated by any i and j (where $j \in N_i$), is revealed to the platform if and only if, $a_i = 1$ and $a_j = 1$.*

Under bilateral consent, an interactive data point is revealed to the platform if both the users give consent. Thus, *both users* must give consent in order to share data that affects them. Again, users may or may not be affected by the data sharing decision of his neighbours. If $a_i = 0$, his utility is unaffected by the decision of his neighbours. If $a_i = 1$, his utility is affected by whether $a_j = 0$ or $a_j = 1$. The interactive data that i has agreed to share with the platform will be revealed to the platform if and only if $a_j = 1$. Thus, a user i is affected by data externality when $a_i = 1$. The payoff of the platform and the users under bilateral consent then becomes,

$$u_P(\mathbf{p}, \mathbf{a}) = -\sum_{i \in N} p_i a_i + \sum_{i \in N} [g(1+M_i)a_i + g(0)(1-a_i)]$$

$$u_i(\mathbf{p}, \mathbf{a}) = p_i a_i + f(1+M_i)a_i + f(0)(1-a_i)$$

where,

- $\mathbf{p} = (p_1, p_2, \ldots, p_n)$ is the payment offer by the platform and $\mathbf{a} = (a_1, a_2, \ldots, a_n)$ is the decision of the users.
- $M_i = |\{j \in N_i : a_j = 1\}|$ **is the number of neighbors of i who share their data**. Clearly, $M_i \in \{0, \ldots, k_i\}$.

In the user utility, if i chooses $a_i = 1$ he gets payment (or services) p_i and the payoff $f(1+|M_i|)$ as the amount of interactive data revealed is $|M_i|$. If he chooses $a_i = 0$, i gets no payment, 0 data points about his are revealed and the corresponding payoff is $0 + f(0)$.

2.7 Optimal Consent Policy

I present results on the optimal consent rule using the second microfoundation.

$$
\begin{aligned}
f(x) &= \frac{x}{(x+1)}\frac{3}{2(x+3)} = \frac{3x}{2(x+1)(x+3)} \\
g(x) &= \frac{3(x+2)}{2(x+3)}\frac{x}{x+1} = \frac{3x(x+2)}{2(x+1)(x+3)}
\end{aligned}
$$

Note that the function g and f have a neat relationship,

$$g(x) = (x+2)f(x)$$

We will be using this property while solving the model for equilibrium.

Definition 2.7.1. *K_i is the set of neighbours of i. If a user is in group i then he has $|K_i| := k_i = p(n_i - 1) + qn_{-i}$ neighbours in expectation.*

Definition 2.7.2. *The set of users that share data in any subgame is denoted by E. The corresponding platform payoff (post user compensation) is denoted by $u_P(E)$ and corresponding user payoff, of any user in group i, is denoted by $u_i(E)$.*

Before proceeding to the first proposition, I present some lemmas that will be directly used in the proofs of the propositions. Note that, if $f(k_2) \leq f(1+k_2) \implies f$ is increasing in $x \implies$ all users share their data with the platform for free $\implies E = N$ in equilibrium.
Thus, all lemmas and propositions have the underlying assumption $f(k_2) > f(1+k_2)$.

Lemma 1. *For $i = \{1, 2\}$,*

$$
\begin{aligned}
g(1+k_i) - \{f(k_i) - f(1+k_i)\} &= (4+k_i)f(1+k_i) - f(k_i) \\
&= \frac{3}{2}\frac{(k_i+1)^2(k_i+2)+1}{(k_i+1)(k_i+2)(k_i+3)} > 0
\end{aligned}
$$

Lemma 2. *For any $x \geq 0$, $g'(x) \geq -2f'(x)$. Thus, $g(b) - g(a) \geq 2[f(a) - f(b)]$ for any $b \geq a \geq 0$.*

Lemma 3. *For any $a \geq 0$ and any $x \geq 0$*

$$f(x) - f(x+a) = \frac{3}{2}\left[\frac{ax^2 + a^2x - 3a}{(x+1)(x+3)(x+a+1)(x+a+3)}\right]$$

Lemma 4. *For any $p \geq q \geq 0$ and any $n_2 > n_1 > 1$,*

$$(3+k_1)(1+qn_2)(3+qn_2) \leq (1+p(n_2-1))(3+p(n_2-1))(1+k_2)(3+k_2)$$

Lemma 5. *For any $p \geq q \geq 0$ and any $n_2 > n_1 > 1$,*

$$\frac{3}{(3+k_1)(1+qn_2)(3+qn_2)} \geq \frac{2qn_2}{(1+p(n_2-1))(3+p(n_2-1))(1+k_2)(3+k_2)}$$

Proposition 2. *If $(p(n_2-1))(1+p(n_2-1)+qn_1) \geq 3$ then all users share their data under unilateral consent.*

Proof. The individual rationality condition for a user in group i, when all users share their data is

$$IR_i : p_i + f(1+k_i) \geq f(k_i)$$

The compensation offered by the platform is $p_i = \max\{0, f(k_i) - f(1+k_i)\}$. Since $(p(n_2-1))(1+p(n_2-1)+qn_1) \geq 3 \implies f(p(n_2-1)) - f(1+p(n_2-1)+qn_1) \geq 0 \implies f(p(n_2-1)+qn_1) - f(1+p(n_2-1)+qn_1) > 0 \implies p_2 = f(k_2) - f(1+k_2)$.

The platform payoff under the subgame $E = N$ is

$$u_P(N) = n_2g(1+k_2) + n_1g(1+k_1) - n_2[f(k_2) - f(1+k_2)] - n_1[\max\{0, f(k_1) - f(1+k_1)\}]$$

Coming now to the subgame where $E = N_2$ in equilibrium. The individual rationality condition for a user in group N_2 when *only* users of group N_2 share their data is

$$IR_i : p_2 + f(1+k_2) \geq f(p(n_2-1))$$

Thus, the compensation offered by the platform is $p_2 = \max\{0, f(p(n_2-1) - f(1+k_2)\}$ and the corresponding platform payoff is

$$u_P(N_2) = n_2g(1+k_2) + n_1g(qn_2) - n_2[\max\{0, f(p(n_2-1)) - f(1+k_2)\}]$$

Since $f(p(n_2-1)+qn_1) - f(1+p(n_2-1)+qn_1) > 0$, the platform payoff becomes

$$u_P(N_2) = n_2 g(1+k_2) + n_1 g(qn_2) - n_2[f(p(n_2-1)) - f(1+k_2)]$$

The platform wants to maximise its payoff, so, $E = N$ in equilibrium *only if* $u_P(N) \geq u_P(N_2)$, which is equivalent to showing,

$$n_1[g(1+k_1) - g(qn_2)] - n_1[\max\{0, f(k_1) - f(1+k_1)\}] \geq n_2[f(k_2) - f(p(n_2-1))]$$

Substituting functions g and f and using Lemma 1 one can show that,

$$LHS \geq n_1 \left[\frac{2qn_2p(n_1-1) + qn_2 + 3p(n_1-1) + 3}{(k_1+3)(1+qn_2)(3+qn_2)} \right]$$

Substituting f and using Lemma 3 one can show that,

$$RHS = n_2 \left[\frac{3qn_1 - qn_1(p(n_2-1))^2 - (qn_1)^2 p(n_2-1)}{(1+p(n_2-1))(3+p(n_2-1))(1+k_2)(3+k_2)} \right]$$

To prove that $u_P(N) \geq u_P(N_2)$ it is enough to show

$$\begin{aligned} &n_1 \left[\frac{2qn_2p(n_1-1) + qn_2 + 3p(n_1-1) + 3}{(k_1+3)(1+qn_2)(3+qn_2)} \right] \\ &\geq n_2 \left[\frac{3qn_1}{(1+p(n_2-1))(3+p(n_2-1))(1+k_2)(3+k_2)} \right] \end{aligned} \tag{2.1}$$

By Lemma 4 and Lemma 5 we know that,

$$(3+k_1)(1+qn_2)(3+qn_2) \leq (1+p(n_2-1))(3+p(n_2-1))(1+k_2)(3+k_2)$$
$$\frac{3}{(k_1+3)(1+qn_2)(3+qn_2)} \geq \frac{2qn_2}{(1+p(n_2-1))(3+p(n_2-1))(1+k_2)(3+k_2)}$$

These two inequalities prove eq. (2.1), thereby proving $u_P(N) - u_P(N_2) \geq 0$.

Let us now compare $u_P(N)$ and $u_P(N_1)$.

The individual rationality condition for a user in group N_1 when *only* users of group

N_1 share their data is

$$IR_i : p_1 + f(1 + k_1) \geq f(p(n_1 - 1))$$

Thus, the compensation offered by the platform is $p_1 = \max\{0, f(p(n_1 - 1)) - f(1 + k_1)\}$ and the corresponding platform payoff is

$$\begin{aligned} u_P(N_1) = n_1 g(1 + k_1) + n_2 g(qn_1) - n_1[\max\{0, f(p(n_1 - 1)) - f(1 + k_1)\}] \\ \leq n_1 g(1 + k_1) + n_2 g(qn_1) \end{aligned}$$

$$u_P(N) \geq u_P(N_1) \text{ if}$$
$$n_2[g(1 + k_2) - g(qn_1)] \geq n_1[\max\{0, f(k_1) - f(1 + k_1)\}] + n_2[\max\{0, f(k_2) - f(1 + k_2)\}] \tag{2.2}$$

Since g is an increasing function one gets,

$$\frac{1}{2}n_2[g(1 + k_2) - g(qn_1)] \geq$$
$$\max\left\{\frac{1}{2}n_2[g(1 + k_1) - g(k_1)], \frac{1}{2}n_2[g(1 + k_2) - g(k_2)], 0\right\}$$

Using Lemma 2 one gets

$$\max\left\{\frac{1}{2}n_2[g(1 + k_1) - g(k_1)], \frac{1}{2}n_2[g(1 + k_2) - g(k_2)], 0\right\} \geq$$
$$\max\left\{n_1[f(k_1) - f(1 + k_1)], n_2[f(k_2) - f(1 + k_2)], 0\right\}$$

Thus, eq. (2.2) is satisfied, thereby proving $u_P(N) \geq u_P(N_1)$.
Since $u_P(N) \geq u_P(N_1)$ and $u_P(N) \geq u_P(N_2)$ I have shown that all users share their data in equilibrium when $(p(n_2 - 1))(1 + p(n_2 - 1) + qn_1) \geq 3$. □

Proposition 3. *If* $(p(n_2 - 1))(1 + p(n_2 - 1) + qn_1) < 3$ *then all users share their data under unilateral consent.*

Proof. The individual rationality condition for a user in group i, when all users share their data is

$$IR_i : p_i + f(1 + k_i) \geq f(k_i)$$

The compensation offered by the platform is $p_i = \max\{0, f(k_i) - f(1 + k_i)\}$. The platform payoff under the subgame $E = N$ is

$$u_P(N) = n_2 g(1+k_2) + n_1 g(1+k_1) - n_2[\max\{0, f(k_2) - f(1+k_2)\}] - n_1[\max\{0, f(k_1) - f(1+k_1)\}]$$

Coming now to the subgame where $E = N_2$ in equilibrium. The individual rationality condition for a user in group N_2 when *only* users of group N_2 share their data is

$$IR_i : p_2 + f(1 + k_2) \geq f(p(n_2 - 1))$$

Thus, the compensation offered by the platform is $p_2 = \max\{0, f(p(n_2 - 1) - f(1 + k_2)\}$ and the corresponding platform payoff is

$$u_P(N_2) = n_2 g(1 + k_2) + n_1 g(qn_2) - n_2[\max\{0, f(p(n_2 - 1)) - f(1 + k_2)\}]$$

Since $(p(n_2-1))(1+p(n_2-1)+qn_1) < 3$, $f(p(n_2-1)+qn_1) - f(1+p(n_2-1)+qn_1) < 0$ and the platform payoff becomes

$$u_P(N_2) = n_2 g(1 + k_2) + n_1 g(qn_2)$$

The platform wants to maximise its payoff, so, $E = N$ in equilibrium *only if* $u_P(N) \geq u_P(N_2)$, which is equivalent to showing,

$$n_1[g(1+k_1) - g(qn_2)] - n_1[\max\{0, f(k_1) - f(1+k_1)\}] \geq n_2[\max\{0, f(k_2) - f(1+k_2)\}]$$

The above condition is satisfied *if*

$$n_1[g(1 + k_1) - g(qn_2)] - n_1[\max\{0, f(k_1) - f(1 + k_1)\}] \geq n_2[f(k_2) - f(1 + k_2)]$$

Substituting functions g and f and using Lemma 1 one can show that,

$$LHS \geq n_1 \left[\frac{2qn_2p(n_1 - 1) + qn_2 + 3p(n_1 - 1) + 3}{(3 + k_1)(1 + qn_2)(3 + qn_2)} \right]$$

Substituting f and using Lemma 3 one can show that,

$$RHS = n_2 \left[\frac{k_2^2 + k_2 - 3}{(1+k_2)(2+k_2)(3+k_2)(4+k_2)} \right]$$

To prove that $u_P(N) \geq u_P(N_2)$ it is enough to show

$$\left[\frac{2qn_2p(n_1-1) + qn_2 + 3p(n_1-1) + 3}{(3+k_1)(1+qn_2)(3+qn_2)} \right] \geq \frac{n_2}{n_1} \left[\frac{k_2^2 + k_2 - 3}{(1+k_2)(2+k_2)(3+k_2)(4+k_2)} \right]$$

By Lemma 4 we know that,

$$(3+k_1)(1+qn_2)(3+qn_2) \leq (1+p(n_2-1))(3+p(n_2-1))(1+k_2)(3+k_2)$$

Since $k_2 \geq p(n_2-1)$, $(1+p(n_2-1))(3+p(n_2-1)) \leq (2+k_2)(4+k_2)$, which implies $(3+k_1)(1+qn_2)(3+qn_2) \leq (2+k_2)(4+k_2)(1+k_2)(3+k_2)$.
This shows that $denominatorLHS \leq denominatorRHS$. To show that $LHS \geq RHS$ it is enough to show that $numeratorLHS \geq numeratorRHS$.

$$\begin{aligned} numeratorRHS &= \frac{n_2}{n_1}[k_2^2 + k_2 - 3] \\ &= \frac{n_2}{n_1}[(p(n_2-1) + qn_1)(1+k_2) - 3] \\ &= \frac{n_2}{n_1}[p(n_2-1)(1+k_2) - 3 + (qn_1)(1+k_2)] \\ &< \frac{n_2}{n_1}[(qn_1)(1+k_2)] \\ &= qn_2(1+k_2) \end{aligned}$$

Note that we have used the fact that $p(n_2-1)(1+k_2) < 3$ to get the above inequality. Further,

$$\begin{aligned} qn_2(1+k_2) &= q(n_2-1)(1+k_2) + q(1+k_2) \\ &\leq p(n_2-1)(1+k_2) + q(1+k_2) \\ &\leq 3 + q(1+k_2) \text{ as } p(n_2-1)(1+k_2) < 3 \\ &= 3 + q + qp(n_2-1) + q^2 n_1 \\ &\leq 3 + 3p(n_1-1) + 2pqn_2(n_1-1) + qn_2 \\ &= numeratorLHS \end{aligned}$$

Thus, I have shown that $numeratorLHS \geq numeratorRHS$ and $denominatorLHS \leq denominatorRHS \implies LHS \geq RHS \implies u_P(N) \geq u_P(N_2)$.

The proof for $u_P(N) \geq u_P(N_1)$ is same as in proposition 2. Since $u_P(N) \geq u_P(N_1)$ and $u_P(N) \geq u_P(N_2)$, I have shown that all users share their data in equilibrium when $(p(n_2-1))(1+p(n_2-1)+qn_1) < 3$. □

Therefore, under unilateral consent, all users share their data with the platform. Equilibrium price and action are $p^*_{i,Uni} = \max\{0, f(k_i) - f(1+k_i)\}$, $a^*_{i,Uni} = 1 \; \forall \; i \in N$.

Proposition 4. *All users share their data under bilateral consent.*

Proof. The individual rationality condition for a user i is

$$IR_i : p_i + f(1 + |M_i|) \geq 0$$

If i refuses to share data, then no data about i is revealed to the platform and user payoff is zero. Since $f(x) > 0$, i is better off giving his data to the platform for free, irrespective of what other users do. This is true for all users. Thus, all users share their data for free in equilibrium.
Equilibrium price and action are $p^*_{i,Bil} = 0$ and $a^*_{i,Bil} = 1 \; \forall \; i \in N$. □

2.8 Welfare Analysis

2.8.1 Unilateral consent versus Bilateral consent

Define total welfare under unilateral consent rule as $TW_{Uni}(\mathbf{p}, \mathbf{a})$ as the sum of payoff of all players,

$$\begin{aligned} TW_{Uni}(\mathbf{p}, \mathbf{a}) &= \sum_{i \in N} [g(1+k_i)a_i + g(M_i)(1-a_i) + f(1+k_i)a_i + f(M_i)(1-a_i)] \\ &= \sum_{i \in N} \left[\frac{1+k_i}{2+k_i} a_i + \frac{M_i}{1+M_i}(1-a_i) \right] \end{aligned}$$

Since $\frac{x}{x+1}$ is increasing in x, the total welfare is maximized by choosing $a_i = 1 \ \forall \ i \in N$.

Define total welfare under bilateral consent rule as $TW_{Bil}(\mathbf{p}, \mathbf{a})$ as the sum of payoff of all players,

$$\begin{aligned} TW_{Bil}(\mathbf{p}, \mathbf{a}) &= \sum_{i \in N} [g(1+M_i)a_i + g(0)(1-a_i) + f(1+M_i)a_i + f(0)(1-a_i)] \\ &= \sum_{i \in N} \frac{1+M_i}{2+M_i} a_i \end{aligned}$$

Since $\frac{x}{x+1}$ is increasing in x, the total welfare is maximized by choosing $a_i = 1 \ \forall \ i \in N$.

Under unilateral consent, in equilibrium,

- payoff of a user in group N_i is $max\{f(1+k_i), f(k_i)\}$
- platform payoff is $n_2[g(1+k_2) - max\{0, f(k_2) - f(1+k_2)\}] + n_1[g(1+k_1) - max\{0, f(k_1) - f(1+k_1)\}]$
- total welfare is $n_2[g(1+k_2) + f(1+k_2)] + n_1[g(1+k_1) + f(1+k_1)]$.

Since $a_i = 1 \ \forall \ i \in N$ in equilibrium, the total welfare is maximized under unilateral consent, in equilibrium.

Under bilateral consent, in equilibrium,

- payoff of a user in group N_i is $f(1+k_i)$
- platform payoff is $n_2 g(1+k_2) + n_1 g(1+k_1)$
- total welfare is $n_2[g(1+k_2) + f(1+k_2)] + n_1[g(1+k_1) + f(1+k_1)]$.

Since $a_i = 1 \; \forall \; i \in N$ in equilibrium, the total welfare is maximized under bilateral consent, in equilibrium.

Thus, total welfare under both consent rules is maximized in equilibrium but each user is better off under unilateral consent rule than under bilateral consent rule.

2.8.2 Users Owning the Platform

Define total user welfare under unilateral consent rule as $TUW_{Uni}(\mathbf{p}, \mathbf{a})$ as the sum of payoff of all users,

$$TUW_{Uni}(\mathbf{p}, \mathbf{a}) = \sum_{i \in N} [p_i a_i + f(1+k_i)a_i + f(M_i)(1-a_i)]$$

Define total user welfare under bilateral consent rule as $TUW_{Bil}(\mathbf{p}, \mathbf{a})$ as the sum of payoff of all users,

$$TUW_{Bil}(\mathbf{p}, \mathbf{a}) = \sum_{i \in N} [p_i a_i + f(1+M_i)a_i + f(0)(1-a_i)]$$

Form above, we know that maximum total welfare is attained in equilibrium under both consent rules. Since total welfare is the sum of total user welfare and platform payoff (under both consent rules), the total user welfare is maximized by distributing total welfare amongst the users. Thus, if all users are owners of the platform then both the total welfare and the total user welfare is maximized under both consent rules in equilibrium.

$$TW_{Uni}(\mathbf{p^*_{Uni}}, \mathbf{a^*_{Uni}}) = TUW_{Uni}(\mathbf{p^*_{Uni}}, \mathbf{a^*_{Uni}})$$
$$TW_{Bil}(\mathbf{p^*_{Bil}}, \mathbf{a^*_{Bil}}) = TUW_{Bil}(\mathbf{p^*_{Bil}}, \mathbf{a^*_{Bil}})$$

2.8.3 Breaking Up the Platform

The Federal Trade Commission has proposed that Meta should be broken up. I analyze the effect of breaking up a platform under this framework.

Definition 2.8.1. *A platform $G = (n, p, q)$ is broken up into two platforms $G_1 = (m_1, p, q)$ and $G_2 = (m_2, p, q)$ if $m_1 + m_2 = n$.*

Thus, a platform is said to be broken up if the users are split into two new platforms. The group of users in N_i is split into the two platforms as N_{i1} and N_{i2}, where $|N_{ij}| = n_{ij}$ and $n_{i1} + n_{i2} = n_i$.
I assume users on two different platforms do not interact. Consequently, lesser data is generated on each platform and the sum of data points generated on the two platforms is also less than the number of data points generated on the unbroken platform.
The two stage game is played on G_1 and G_2 and as we saw in G, all users share their data in equilibrium. For a user in group N_{ij}, the platform attains $p(n_{ij} - 1) + qn_{-ij}$ interactive data points. Call this k_{ij}. Then payoff of a user in group N_{ij} is $f(1+k_{ij})$. We know that $k_{ij} \leq k_i$ and that the peak of f is attained at $\sqrt{3}$ Thus, if k_i is sufficiently larger than $\sqrt{3}$ $\forall\, i \in \{1, 2\}$ then payoff of each user can be increased by splitting G into G_1 and G_2 such that $k_{ij} > \sqrt{3}$ $\forall\, i, j \in \{1, 2\}$.

Proposition 5. *For k_2 large enough, a platform G can always be broken up into two platforms G_1 and G_2, such that, payoff of each user in G_i is higher than the corresponding user payoff in G.*

Since fewer data points are analysed when the platform is broken up, the sum of the total welfare attained from both platforms is less than the total welfare attained from the unbroken platform. **Thus, breaking up a platform (when it becomes sufficiently large) improves payoff of each use but reduces the total welfare.**

2.9 Conclusion

This paper analyzes the effect of different regulations in a data economy, consisting of an online platform and heterogeneous users. The online platform generates revenue from data by steering users. I derive a microfoundation of payoffs in this setup and find that platform payoff is increasing in number of data points and user payoff is non-monotonic in number of data points. I further model users in a network setup which allows analysis of regulations that address the rising concerns about user welfare on online platforms. I find that, first, stricter privacy policy backfires and reduces user welfare. Second, when users own the online platform both total welfare and total user welfare are maximized. Third, when a platform becomes large, user welfare is improved by breaking up the platform.

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Chapter 3

The Data Economy and Multiple Platforms

3.1 Introduction

A network of users is polarized when there isn't enough interaction between different types of users. One way in which this can happen is studied in Chapter 1.
Another way in which a network is polarized, perhaps even more polarized, is when different type of users are on different platforms. If users of one subgroup are on one platform and users of another subgroup are on another platform then the interaction between these two subgroups is negligible and society is polarized. Arturo Gonzales, Director & Global Head of Policy Advocacy & Research at Facebook[1] has said,

> "they (Facebook) want everyone to be a part of the community and the best way to do that was by *having different subgroups of people join different platforms*".

Thus, segregating different types of users onto different platforms is a goal that Facebook may be pursuing. This paper analyses whether segregating of users coincides with platform payoff maximisation. Further, we will see that under certain conditions, users are more likely to be segregated under a strict privacy policy than under a loose privacy policy.
This work relates to the literature of pricing, network effects and competition on online platforms. Several papers like Fainmesser and Galeotti (2020), Fainmesser and

[1] at the Lancaster Game Theoretic and Behavioural Economics Insights on Social Media Conference, 25-26 February 2021

Galeotti (2021), Aoyagi (2018), Chen et al. (2018) and Sotiropoulos et al. (2019) have studied these interactions. The paper also relates to the polarization literature, Hinich and Ordeshook (1969), Esteban and Ray (1994) and Prummer (2020).

3.2 Model

Ideally, the user payoff attained from data should be the microfounded f and the platform payoff attained from data should be the microfounded g (as seen in the first two chapters). However, this work is at an earlier stage and so the analysis is carried out using general f and g and although they are not microfounded, they do cover many possible user payoff and platform payoff.

3.3 Payoffs Generated from Data

I consider $g(x) = x$ and two versions of f. First, $f(x) = -\alpha x$, $\alpha > 0$. Second, f is non-monotonic,

$$f(x) = \begin{cases} \beta x & x \leq x_0 \\ (\beta + \alpha)x_0 - \alpha x & x > x_0 \end{cases}$$

where $\beta < \alpha$ and $x_0 > 0$. Additionally, I make some simplifying assumptions that are needed at this early stage of the work. Assume that $1 + p(n_1 - 1) + qn_2 \leq x_0$ and $f(p(n_2 - 1) + qn_1) < 0$. These assumptions implies that data from group N_1 is free and getting all the user data has some cost.

3.4 The Network

Consider an online platform which has n users. The users are connected to each other by an online network, which is modeled using the islands model, a special case of the multi-type random network.

Given a set of n users or nodes $N = \{1, ..., n\}$, *a network* is represented via its adjacency matrix: a symmetric n-by-n matrix $\mathbf{A}$ with entries in $\{0, 1\}$. The interpretation is that $A_{ij} = A_{ji} = 1$ indicates that nodes i and j are linked, and the symmetry restricts attention to undirected networks.

Users or nodes have "types", which are the distinguishing features that affect their propensities to connect to each other. Types might be based on any characteristics that influence users' probabilities of linking to each other, including age, race, gender, profession, education level, and even behaviors. The model is quite general in that a type can embody arbitrary lists of characteristics; which characteristics are included depends on the application.

There are m different types in the society. Let $N_k \subset N$ denote the nodes of type k, so the society is partitioned into the m groups, $(N_1, ..., N_m)$. Let $n_k = |N_k|$ denote the size of group k and n denote the total number of users.

A *multi-type random network* is defined by the cardinality vector $\mathbf{n}$ together with a symmetric m-by-m matrix $\mathbf{P}$, whose entries in $[0, 1]$ describe the probabilities of links between various types. The entry P_{kl} captures the probability that a user of type k links to an user of type l. We fill in the remaining entries of $\mathbf{A}(\mathbf{P}, \mathbf{n})$ by symmetry: $A_{ij} = A_{ji}$. We set $A_{ii} = 0$ for each i.

The *islands model* is the special case of the multi-type random networks model, such that, each user only distinguishes between users of one's own group and users of a different group. Moreover, all users are symmetric in how they do this. Formally, in the multi-type random network notation, we say the multi-type random network $(\mathbf{P}, \mathbf{n})$ is a two islands network with parameters (n, p, q) if:

- there are 2 islands, each island N_k of size n_k, $n_1 + n_2 = n$;
- $P_{kk} = p$ for all k; and
- $P_{kl} = q$ for all $k \neq l$, where $\mathbf{p} \geq \mathbf{q}$ and $p > 0$.

Without loss of generality, assume N_2 is the bigger group, $n_2 \geq n_1$.

3.4.1 Data Generated

I now describe the economy of data based on this network. Each user generates some data on the online platform. I call this *personal data* and it is of two types -

- *individual data*, which is generated by the user and provides the platform information about said user only; and
- *interactive data*, which is is generated by two users who are linked and the platform gets information about *both users*.

Examples of individual data are the ads a user clicks on, login and logout time, the posts that a user pauses on while scrolling. Examples of interactive data are messages between users, photos and videos posted with both users present.

While individual data about a user can be attained only from that particular user, interactive data can be attained from any one of two connected users. This gives rise to *data externality.* For any two users connected on a platform, data about one user is informative about that user and his connections on a platform. This is due to the well documented concept of homophily on online platforms, i.e., users are likely connected with like-minded people. Consider Facebook. It has been shown that highly personal information like sexual orientation can be inferred using the connections. As Jernigan and Mistree (2009) paper shows, a homosexual man is more likely to be connected to other homosexual men than to heterosexual men. This logic can be used in a consumerism context too. An avid reader is likely connected to other book lovers. A football fan is likely connected to other football fans. Thus, a wide range of information related to a user can be obtained from information collected about his friends. A user gives consent to the platform based on how sharing data with the platform affects her, irrespective of the externality it has on any other linked user. We will soon see that the platform is able to access user data due to the presence of data externalities, even when data analysis is harmful for users. In order to do that we expound on how user data is generated on a platform.

Data is generated when users interact with each other or when users use the services of the platform. Without loss of generality, we normalise the amount of data generated,

- A user i interacts with all her neighbors and *one data point* is generated from each interaction. This data point reveals information about both i and j.
- Each user i generates individual data. This data is normalised to one, thus, each user produces one individual data point.

Therefore, two users being linked on the network is equivalent to them generating a unique data point by interacting on the platform.

From the assumptions one can conclude that each user i generates A_i data points on the platform, where $A_i = \sum_{j \in N} A_{ij}$ is the number of links of i in the network.

Specifically, for a network $G = (\alpha, p, q)$, a user $i \in N_1$ generates $p(n_1 - 1) + qn_2$ data points and a user $j \in N_2$ generates $p(n_2 - 1) + qn_1$ data points.

Notation: For any $p, q \in [0, 1]$, define

- $k_1 := |A_1| = p(n_1 - 1) + qn_2$
- $k_2 := |A_2| = p(n_2 - 1) + qn_1$

Therefore, for a network $G = (n, p, q)$, a user in N_1 generates k_1 data points and a user in N_2 generates k_2 data points.

3.5 Three Stage Game with Two Platforms

We modify the two stage game seen in Chapter 2 to a three stage game as,

- each user j is offered price p_j^1 to join platform 1 and p_j^2 to join platform 2
- each user j decides to join platform 1, $b_j = 1$ or platform 2, $b_j = 2$
- after joining platform i and observing his neighbors on i, each user j decides whether to share his data, $a_i = 1$ or not, $a_i = 0$

The firm maximises profit by maximising the sum of payoff of platform 1 and platform 2, where payoff of a platform is formulated below The payoff of the users is also formulated below.
I assume that users on two different platforms are not linked. This is plausible when users on different platforms cannot interact with each other and so there is no interactive data generated between them.

3.6 Privacy Policy: Unilateral Consent and Bilateral Consent

The final component needed to define the payoffs is the *privacy policy* in effect. Regulations like GDPR aim to give users complete control over their data, that is, user data should be shared with the platform if and only if the user has given his consent. Externalities from interactive data make the implementation of this policy difficult. Simply put, if one user does not give consent and a linked user does give consent then should the interactive data be shared with the platform or not? As of now, the status quo is that in such a case the data will be shared with the platform. We compare this status quo with a stricter privacy policy, where interactive data is shared with the platform if and only if both users give consent to share their data. Let us define these privacy policies precisely.

3.6.1 Unilateral Consent (Loose Policy)

Definition 3.6.1. *Under unilateral consent, the interactive data point generated by any i and j (where $j \in N_i$), is revealed to the platform if $a_i = 1$ or $a_j = 1$.*

Under unilateral consent, an interactive data point is revealed to a platform if any one of the two concerned users gives consent. Thus, each user can *unilaterally* share data that affects his and his neighbors. Thus, if $a_i = 1$, all data about user i is revealed to the platform and i is unaffected by the decision of his neighbors. If $a_i = 0$, then utility of i is affected by whether $a_j = 0$ or $a_j = 1$. Thus, a user i is affected by data externality when $a_i = 0$.

Unilateral consent is representative of the status quo. An online platform is able to access the data of a user once he consents. The fact that this data may affect other people and hence the consent of other people should also be taken is not taken into account in this policy.

The payoff of the platform and the users under unilateral consent then becomes,

$$u_P(\mathbf{p}, \mathbf{a}) = -\sum_{i \in N} p_i a_i + \sum_{i \in N} [g(1 + k_i) a_i + g(M_i)(1 - a_i)]$$

$$u_i(\mathbf{p}, \mathbf{a}) = p_i a_i + f(1 + k_i) a_i + f(M_i)(1 - a_i)$$

where,

- $\mathbf{p} = (p_1, p_2, \ldots, p_n)$ is the payment offer by the platform and $\mathbf{a} = (a_1, a_2, \ldots, a_n)$ is the decision of the users.
- $M_i = |\{j \in N_i : a_j = 1\}|$ **is the number of neighbors of i who share their data**. Clearly, $M_i \in \{0, \ldots, k_i\}$.

In the user utility, i gets the payment (or services) p_i and the payoff $f(1 + k_i)$ if he chooses $a_i = 1$ as it reveals all his data to the platform. If he chooses $a_i = 0$, i does not get any payment and the payoff from data revelation is $f(M_i)$.

3.6.2 Bilateral Consent (Strict Policy)

Definition 3.6.2. *Under bilateral consent, the interactive data point generated by any i and j (where $j \in N_i$), is revealed to the platform if and only if, $a_i = 1$ and $a_j = 1$.*

Under bilateral consent, an interactive data point is revealed to the platform if both the users give consent. Thus, *both users* must give consent in order to share data that affects them. Again, users may or may not be affected by the data sharing decision of his neighbours. If $a_i = 0$, his utility is unaffected by the decision of his neighbours. If $a_i = 1$, his utility is affected by whether $a_j = 0$ or $a_j = 1$. The interactive data that i has agreed to share with the platform will be revealed to the platform if and only if $a_j = 1$. Thus, a user i is affected by data externality when $a_i = 1$. The payoff of the platform and the users under bilateral consent then becomes,

$$u_P(\mathbf{p}, \mathbf{a}) = -\sum_{i \in N} p_i a_i + \sum_{i \in N} [g(1 + M_i)a_i + g(0)(1 - a_i)]$$

$$u_i(\mathbf{p}, \mathbf{a}) = p_i a_i + f(1 + M_i)a_i + f(0)(1 - a_i)$$

where,

- $\mathbf{p} = (p_1, p_2, \ldots, p_n)$ is the payment offer by the platform and $\mathbf{a} = (a_1, a_2, \ldots, a_n)$ is the decision of the users.

- $M_i = |\{j \in N_i : a_j = 1\}|$ **is the number of neighbors of i who share their data**. Clearly, $M_i \in \{0, \ldots, k_i\}$.

In the user utility, if i chooses $a_i = 1$ he gets payment (or services) p_i and the payoff $f(1+|M_i|)$ as the amount of interactive data revealed is $|M_i|$. If he chooses $a_i = 0$, i gets no payment, 0 data points about his are revealed and the corresponding payoff is $0 + f(0)$.

3.7 Results

Proposition 6. *For* $f(x) = -\alpha x$, $\alpha \geq max\{2, 1 + p(n_1 - 1) + 2qn_2\}$, $p > \frac{1}{n_2-1}$, $\frac{p}{q} > \frac{2n_2}{n_2-n_1}$ *the equilibrium outcome under unilateral consent is* $b_i = 1$ *for all* $i \in N_1$ *and* $b_j = 2$ *for all* $j \in N_2$.

When α is high, the firm values minimising compensation cost over maximising data. It therefore segregates users onto two platforms. The interactive data between groups is lost but neither platform has to compensate users for this between group data that would have existed had all users been on the same platform. The conditions on p and $\frac{p}{q}$ simply ensures that when the users are segregated, the total payoff attained by the platforms is positive.

Proposition 7. *For* $f(x) = -\alpha x$, *the equilibrium outcome under bilateral consent is* $b_i = 1$ *for all* $i \in N$.

Under bilateral consent each user must be compensated by α for each data point. When $\alpha < 1$, the platform maximises profit by getting all the data. If users are segregated, the interactive data between groups is lost and the platform gets a lower payoff.
When $\alpha > 1$, the platform does not compensate any user and the data market shuts down. Thus, users do not segregate under bilateral consent when $f(x) = -\alpha x$.
When $f(x) = -\alpha x$ *users are more likely to be segregated under unilateral consent than under bilateral consent as the data market shuts down under bilateral consent when* $\alpha > 1$.

We carry out the same analysis for f non-monotonic.

Proposition 8. *Suppose* f *is non-monotonic,*

$$f(x) = \begin{cases} \beta x & x \leq x_0 \\ (\beta + \alpha)x_0 - \alpha x & x > x_0 \end{cases}$$

where $\beta < \alpha$. *If* $\frac{p}{q} \geq \frac{2n_1}{n_1-1}$ *(homophily) and* $\alpha \geq 2qn_1$ *the equilibrium outcome under unilateral consent is* $b_i = 1$ *for all* $i \in N_1$ *and* $b_j = 2$ *for all* $j \in N_2$.

For f non-monotonic, we see that users are segregated when there is homophily and when α is high enough. The users are segregated when α is high because the platform prefers minimising cost over maximising data, as seen before.

Proposition 9. *Suppose f is non-monotonic,*

$$f(x) = \begin{cases} \beta x & x \leq x_0 \\ (\beta + \alpha)x_0 - \alpha x & x > x_0 \end{cases}$$

where $\beta < \alpha$. For $\alpha > \frac{2qn_1+\beta x_0}{1+p(n_2-1)+qn_1+x_0}$ the equilibrium outcome under bilateral consent is $b_i = 1$ for all $i \in N_1$ and $b_j = 2$ for all $j \in N_2$.

Note that, $\frac{2qn_1+\beta x_0}{1+p(n_2-1)+qn_1+x_0} > 2qn_1$, thus users are more likely to be segregated under bilateral consent than under unilateral consent.
Users are segregated as the platform prefers minimising cost over maximising data. Since $f(p(n_2-1)+qn_1) - f(1+p(n_2-1)+qn_1) < 0 - f(1+p(n_2-1)+qn_1)$, the cost of compensation is higher under bilateral consent. Due to this, the platform is more likely to be segregated users under bilateral consent than under unilateral consent.
When f is non-monotonic, users are more likely to be segregated under bilateral consent than under unilateral consent as the cost of compensation is higher under bilateral consent.

Additional assumption - We have the standard assumptions $1+p(n_1-1)+qn_2 \leq x_0$ and $f(p(n_2-1)+qn_1) < 0$ for this result. *Additionally*, we also assume that $1+p(n_2-1) \leq x_0$. This assumption implies that when only group N_2 is on platform 2 then the platform gets their consent for free. The result probably holds without this assumption if there are some $k > 2$ number of platforms. The firm would segregate the users into even smaller subgroups than N_1 and N_2 on these k platforms until only some $x < x_0$ amount of data is available on each platform. The platforms will then get all the user data for free and this will maximise firm profit when there is homophily and when α is high enough. This more general and natural result is currently under construction.

3.8 Conclusion

This paper analyses whether segregating of users onto different online platforms coincides with platforms' payoff maximisation. When different types of users are on different platforms, there is minimum interaction between them, which can increase polarization between users. This paper analyses whether users are polarized in this manner under different consent rules. I find that, under certain conditions, users are more likely to be segregated by types under a strict privacy policy than under a loose privacy policy. A strict privacy policy may therefore increase polarization between users.

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