

Optimising flexibility procurement in local flexibility markets under stochastic availability of participants

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ABSTRACT

Local flexibility markets offer a cost-effective solution to overcome the increasing stress on distribution networks by procuring flexibility from distributed energy resources. However, the assets at the distribution level generally have a certain degree of uncertainty and varying costs for operating flexibly. Therefore, distribution system operators will need new mechanisms to structure flexibility procurement, enabling them to manage the risks associated with uncertain resources effectively. This paper proposes an optimisation-based strategy for the optimal selection of flexibility service providers (FSPs) to contract with for mid- to long-term flexibility provision. The strategy alleviates the conservatism of considering individual uncertainties by condensing all FSPs uncertainties into a single uncertainty constraint at the DSO level. The test cases selected electric vehicles and heat, ventilation and air conditioning systems as examples to demonstrate the effectiveness of the strategy. The optimal selection returns the combination of service providers that will ensure the delivery of the needed flexibility at the least cost, given a desired confidence level. The simulation results demonstrate the effectiveness of the strategy in returning the combination of the most beneficial offers that fulfil the trade-off between costs and reliability while ensuring its robustness in uncertain environments; for all test cases, the actual violation level never exceeds the targeted level, and the DSO consistently procures the required flexibility at the lowest available cost.

1. Introduction

The increased penetration rate of renewables within distribution networks [1], combined with the emergence of high energy-consuming appliances on the end-consumer side, challenges system operation at the distribution level. Notably, the energy system becomes more decentralised since more generation is located near end customers and most of it is weather-dependent, which inherently presents a certain degree of uncertainty. Several governments, including China, European Union countries and the UK, aim at the electrification of heat and transport [2]. The electrification of heat and transport brings with it its own challenges [3]. Not only are these two sectors energy-intensive, but their energy consumption also depends on the customer behaviour, for which the pattern is generally highly variable and challenging to accurately predict [4]. In response to these evolving dynamics, distribution network

operators (DNOs)¹ find themselves obligated to undertake costly network upgrades to accommodate the increased injection and withdrawal of high-energy demands within an uncertain operational environment [5]. A cost-effective enhancement to conventional network reinforcement emerges through the exploitation of the inherent flexibility present in distributed assets [6]. The introduction of local flexibility markets (LFMs) is a part of DNOs' transition to the more active role associated with distribution system operators (DSOs) [7]. LFMs are markets that allow distributed energy resources (DERs) to provide their flexibility as a service. They can serve at both the transmission and distribution levels. In [8], four types of flexibility products are identified: sustain, secure, dynamic, and restore. Sustain is a long-term scheduled product, typically contracted months in advance, where flexibility is reserved for recurring use at set times. Secure is a day-ahead pre-fault product, where flexibility is scheduled in advance to manage forecasted constraints. Dynamic

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¹ The operator of the electric power network at the distribution level, which delivers electricity to most end users. It is known under different names, such as local distribution companies (LDC) in Canada and DNO/DSO in the UK.

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is a real-time product, dispatched close to delivery to respond to rapidly emerging network needs, and Restore is a post-fault product, used after a fault has occurred to support recovery. These products differ not only in time horizon but also in function: Sustain supports long-term network planning (e.g., deferring reinforcement), Secure and Dynamic serve operational purposes like congestion management or voltage control [9], while Restore ensures rapid service restoration after outages.

Most of the works in the literature focus on the network operation products (i.e., ‘secure’, ‘dynamic’, and ‘restore’), and propose methodologies and algorithms for the provision and dispatch of flexibility to support operational activities. In [10] for instance, authors propose a local P2P flexibility market that allows the DSO to procure flexibility from downstream assets to avoid the distribution network limits being violated. The market clearing ensures that the resulting outcome is stable and satisfies Pareto efficiency. In [11], a P2P market is proposed to procure flexibility to match PV supply with demand. Time-flexible assets are scheduled to match the forecasted PV power in a day-ahead schedule, and power-flexible assets are called out on a real-time basis to suppress PV forecasting errors. This allows the PV power to be balanced locally without propagating the error upstream. A scheme for a joint TSO-DSO flexibility procurement is proposed in [12]. The flexibility offers frequency regulation services to the TSO and local voltage control and congestion management to the DSO. The first stage matches the offers and the bids, and in the second stage, the DSO checks if the matched flexibility does not trigger network violations. The two stages are repeated until the resulting matching verifies distribution network conditions.

On the other hand, we find relatively fewer papers considering the contracting phase of flexibility provision [13] for the ‘sustain’ product. In [9], a comparison between BAU (business as usual) solutions and flexibility solutions for four different case studies was conducted. Though the authors included the availability cost of flexibility service providers (FSPs) in the operation (Opex) costs, they didn’t detail how these were obtained, and the FSPs were assumed to deliver 100 % of the flexibility they bid. Authors in [14] proposed a cost estimation of various FSPs that could be used as an enhancement to the conventional grid reinforcement. For DR, they gave a detailed description of costs for flexibility provision, but did not include the contract costs in the CAPEX part. Selection of the optimal combination between reinforcement and flexibility was proposed in [15], authors used stochastic mixed-integer linear programming (S-MILP) with Monte Carlo scenarios to derive the optimal combination between network reinforcement and flexibility provision. The impact of uncertainty on the costs of flexibility contracts was assessed in [13]; however, the study only considers uncertainty with respect to renewable generation but not with respect to flexibility availability.

Most of the works investigating flexibility for network planning assume the contracts between the DSO and flexibility service providers (FSPs) have already been decided and the selection procedure is neglected, or they assume a perfect prediction of asset flexibility. However, several key challenges arise when acquiring flexibility from DERs at the distribution level:

- **Uncertainty and volatility at the distribution level:** LFM are characterised by uncertain and volatile resources. While transmission-level resources may also exhibit uncertainty, they are typically supported by more stable generation sources. At the distribution level, managing a larger number of smaller, dispersed assets adds layers of complexity.
- **Localised challenges and requirements:** The distribution level faces unique challenges, such as managing a diverse and heterogeneous array of small-scale DERs with variable generation and consumption patterns.
- **Cost-efficiency in compensation mechanisms:** In flexibility markets, two main types of payments are typically made to participating FSPs: an availability fee paid for reserving capacity, and a utilisation

fee paid only when flexibility is actually activated. Several existing studies assume that all DERs downstream of the network receive availability payments, regardless of whether their flexibility is used. This assumption can place a substantial financial burden on DSOs, as compensating all resources for availability, even those rarely or never activated, is not economically efficient.

Another significant issue associated with long- and mid-term contracts (i.e., contracts for ‘sustain’ product) is that the uncertainty here is considered not on a day-ahead or intra-day basis, but on a months-ahead or years-ahead basis, making the prediction and selection procedures considerably more complex. In fact, as FSPs at the distribution level present a certain degree of uncertainty, there will be a risk of under-delivery. Under-delivery is a term that refers to an FSP’s failure to deliver the total amount of flexibility it contracted for in the market due to the uncertain nature of its flexible assets. Therefore, the selection of participating FSPs is not a straightforward procedure and comes with several challenges:

- Engaging multiple FSPs through contracts may lead to an excess of flexibility compared to the DSO’s actual requirements. Consequently, the DSO would be obligated to compensate all contracted FSPs for their availability, even if their services were not activated.
- On the other hand, contracting with FSPs precisely matching the required flexibility entails the risk of under-delivery, potentially hindering the fulfilment of the DSO’s operational needs.

Hence, one of the DSO objectives is to identify a selection that balances the trade-off between reliability and cost, with a combination of FSPs that ensures the provision of flexibility with a high level of confidence at the least cost. Assigning a reliability score to each FSP, based on historical performance, is a standard practice in transmission-level markets and has been proposed for distribution-level ones. However, strict reliability thresholds (e.g., 90 % as required by some TSOs [16]) risk excluding residential DERs, which are inherently stochastic but potentially cost-effective. Instead of imposing rigid constraints, this work adopts a broader strategy: it incorporates the cost-reliability trade-off directly into the selection process. This inclusive formulation enables the participation of lower-reliability FSPs if their offers are sufficiently competitive, thereby reducing overall procurement costs and widening market access for small-scale, flexible assets. It is also worth noting that although reliability is traditionally not the buyer’s responsibility, in emerging electricity markets, particularly those targeting increased DER participation, experienced stakeholders aim to shield end-users from volatility. In this context, DSOs may share some responsibility for delivery risks, while FSPs are incentivised to avoid under-performance through partial penalties or reduced payments.

In this paper, we propose a novel strategy, formulated as a chance-constrained optimisation problem, which addresses this trade-off and returns the optimal selection of contracts that delivers the required flexibility with the least cost and desired degree of reliability.

The strategy is tailored to long- and mid-term contracts and considers the stochasticity related to flexibility delivery months and years ahead. Additionally, it is technology-agnostic and works for different types of DERs, making it well-suited for DSOs. Our proposed strategy mitigates the conservative approach of addressing individual uncertainties by consolidating all FSPs’ uncertainties into a single constraint at the DSO level. Rather than constraining each FSP to meet a fixed reliability threshold, which is an approach that may be overly conservative and exclude low-reliability but low-cost participants. We model uncertainty collectively at the DSO level. Specifically, the proposed formulation introduces a single chance constraint that ensures the aggregated flexible capacity meets the required target with a given confidence level. This allows the DSO to benefit from the statistical smoothing effect where deviations in some FSPs may offset each other, enabling the selection of more cost-effective combinations that jointly satisfy the reliability requirement, even if some of them are individually less reliable. The strategy then formulates the selection problem as a chance-constrained

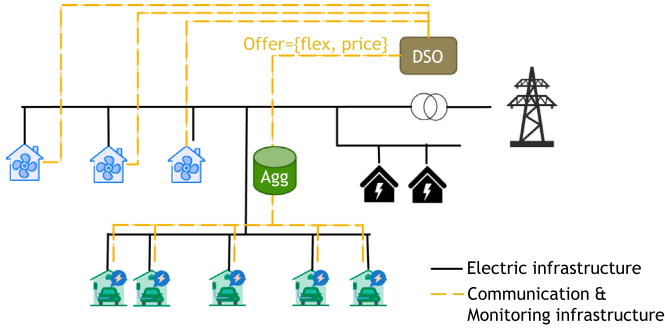


Fig. 1. Network Architecture: The DSO is connected physically to customers with inflexible assets and customers with flexible assets, and communicates with two types of FSPs: aggregators managing the flexible assets portfolio of their customers and customers owning a large capacity flexible asset.

program and returns a combination of FSPs that ensures the delivery of the required flexibility at minimal cost while satisfying a predefined confidence level. Enhancing the procurement process for DERs contributes to the objectives of future electricity markets in managing flexible, dynamic, uncertain and sustainable energy systems and supports Goal 7 of the Sustainable Development Goals. Moreover, it aligns with the emerging regulatory frameworks (e.g., in the UK [17]) that aim at developing new tools and methods to ensure a secure and affordable energy system by leveraging flexibility provision at the local level.

The rest of the paper is organised as follows. Section 2 presents the market design and formulates the problem, and Section 3 explains the adopted methodology for modelling uncertainty and details the solution approach. The results are evaluated and discussed in Section 4, and Section 5 concludes the paper and opens on potential extensions.

2. Problem formulation

This paper focuses on the ‘Sustain’ product, which is a scheduled, forward-looking flexibility service aimed at addressing predictable, longer-term network constraints. It requires providers to commit to availability over defined windows, often aligned with seasonal trends. Sustain contracts are typically arranged months in advance and play a key role in supporting network planning decisions.

2.1. Market structure

The market roles involved in ‘Sustain’ product are the flexible assets, the flexibility service provider (FSP) and the flexibility request party (FRP).

- **Flexible assets:** owned by a customer connected to the distribution grid. Flexible assets are the assets that can adjust their net operational active and/or reactive power injection/withdrawal [18].
- **Flexibility service provider:** The FSP can be either a third entity (e.g., an aggregator) that aggregates flexibility from the prosumers’ flexible assets, or it can be the flexible asset owner if the flexible capacity matches the minimum capacity requirement for the LFM. FSPs offer flexibility in the LFM to an FRP.
- **Flexibility request party:** is the entity that procures flexibility services. In our study, the DSO plays the role of the FRP. It procures flexibility to ensure the safe operation of the distribution network.

Fig. 1 provides a high-level diagram of the considered distribution network. The DSO manages a network that connects two types of customers: customers with inflexible assets and prosumers who, in addition to inflexible loads, own flexible assets. Prosumers can choose to participate in LFMs either directly, if they meet the minimum capacity criteria of the market or through an aggregator. A communication channel is then set up between the DSO and the FSPs. The market process outlined in this paper is inspired by ESO guidelines [8] for procuring distribution



Fig. 2. The timeline of the procedure for contracted flexibility: The DSO launches a call for flexibility (step 3) if a congestion is predicted (step 2). After receiving the offers (step 3), the DSO decides the winning offers (step 4) and signs contracts with the selected providers (step 4). The paper addresses the challenges of step 4, which aims at selecting flexibility providers with the best reliability and lowest costs.

flexibility services. It specifically focuses on the ‘sustain’ product [19] that is defined by Open Networks as the flexibility product that aims to defer network reinforcement. The process is summarized in Fig. 2 and can be summarized as follows:

1. **Pre-qualification:** Interested participants, such as aggregators and DER owners, express their interest in participating in LFMs by registering on an online platform (e.g., [20]) and submitting necessary information, including technical specifications, capacity, operation limits, response time and location of their resources. The DSO conducts a pre-qualification to ensure that the participants meet basic eligibility criteria, such as having appropriate technology and compliance with regulatory requirements. Participants who pass the pre-qualification are then added to a pool of pre-qualified service providers.
2. **Forecasting analysis:** The DSO employs a forecasting model (e.g., [21]) to predict future energy demand and supply over an extended period (such as a season, a year, or five years). For each time step within this period, the DSO runs a power flow analysis to ensure that the energy can be dispatched without violating network constraints. If network violations are recurrently predicted, the DSO identifies the stressed points and, based on its analysis of the trends in energy demand and knowledge of the grid topology, estimates how much flexibility will be needed. For example, if, for a certain node, the forecast predicts periods of congestion with low demand and high generation (e.g., midday of a summer day), the DSO calls for a downward flexibility to decrease generation or increase demand in that node. In the opposite case, i.e., periods of high demand and low generation (e.g., winter nights), the DSO calls for upward flexibility to increase generation or decrease consumption.
3. A call for tenders for flexibility provision is set up per congestion point, and the DSO, calls for flexibility from FSPs located downstream of the points predicted to be stressed through the market platform [20]. Interested FSPs send their offers ($\text{flex}_{\text{offer}}, \text{P}_{\text{offer}}$) to the DSO, where $\text{flex}_{\text{offer}}$ is the maximum flexibility offered and P_{offer} is the offered price. We note that we assume here a perfect competition, given the early stage of LFMs, where participants have limited experience and market power is not yet fully developed, this assumption is reasonable and reflects the current state of the problem.
4. The DSO runs tests to check if the FSPs are able to deliver the flexibility promised in their offers, analyses FSPs’ reliability and then signs contracts ahead of time with the FSPs holding the most beneficial offers.
5. The contracted FSPs should be ready to deliver the flexibility they offered during the contract period at the specified time window.
6. If flexibility is needed, the DSO calls for activation, FSPs respond by delivering the flexibility and the market is cleared.

This paper focuses on the contract selection step, and proposes a strategy that decides the optimal flexibility offers that balance the trade-off between reliability and cost. Fig. 3 roughly classifies offers depending on their reliability and price. Intuitively, the DSO will select the offers that

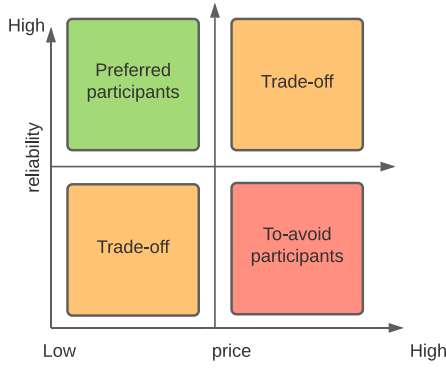


Fig. 3. Rough classification of offers: The offers can fall under one of three categories. ‘Preferred offers’ class refers to the offers with highly reliable service and a low price, the ‘To-avoid offers’ class characterises non-reliable offers with a high price and the ‘trade off’ class can be split into two subclasses: highly reliable offers with high prices and low reliable offers with low prices.

fall under the ‘preferred offers’ class. Nevertheless, the challenge arises when the cumulative capacity of offers within this category falls short of meeting the total capacity required by the DSO. Besides, it is challenging to determine with high precision the thresholds of the classification. The flexibility market in the distribution network is very dynamic: conditions of the network vary, the congestion points are not the same, and the service providers willing to participate will differ upon each call. A generic classification will not be feasible. Secondly, as this is solely a rough classification, there is no precise method to determine the thresholds for the separation, and with the increasing number of offers, the classification will be less obvious. Next, we introduce the mathematical formulation of the strategy that will allow the DSO to select the most beneficial offers while meeting its objectives of a secure, affordable supply.

2.2. Mathematical formulation

As described in step 3) of Section 2.1, we suppose that a node risks being frequently constrained at a specific time slot over a period (e.g., midday slot during the summer season) and requires an amount of flexibility F to alleviate the stress. The challenge the DSO faces is that most FSPs at the distribution level present a degree of uncertainty, as their energy schedules are contingent upon uncontrollable and non-deterministic constraints, costs, and parameters [22] such as EV owner behaviour and weather conditions that impact heating. Therefore, the selection of FSPs under uncertainty of power delivery can be modelled as a chance-constrained problem:

$$\begin{aligned} \min_{\mathbf{b}} \quad & \mathbb{E} \left(\sum_{k \in \mathcal{K}} b_k \times c(\text{flex}_k) \right) \\ \text{s.t.} \quad & \mathbb{P} \left(\sum_{k \in \mathcal{K}} b_k \times \text{flex}_k \geq F \right) \geq 1 - \epsilon \end{aligned} \quad (1)$$

Where $\mathbb{E}$ and $\mathbb{P}$ refer respectively to the expectation and probability functions, $\mathcal{K}$ is the set of FSPs participating in the LFM; FSPs can be the flexible assets’ owners or a third party managing various flexible assets, b_k are the binary decision variables; $b_k = 1$ if the DSO decides to contract with provider k and 0 otherwise, flex_k is the available flexibility that k can provide, and it represents the stochastic input in the problem, F is the total flexibility required by the DSO, ϵ is the violation level that reflects the DSO tolerance for the constraint to be violated, and $c(\text{flex}_k)$ represents the activation remuneration FSP k receives for delivering flex_k . The payment each FSP receives is a function of the delivered flexibility flex_k and is subject to a payment reduction penalty. Generally, to minimise the risk of under-delivery, the DSO adopts a payment reduction penalty for under-delivery that incentivises the participating FSP to deliver the total capacity it contracted for [23,24]. Fig. 4 displays some

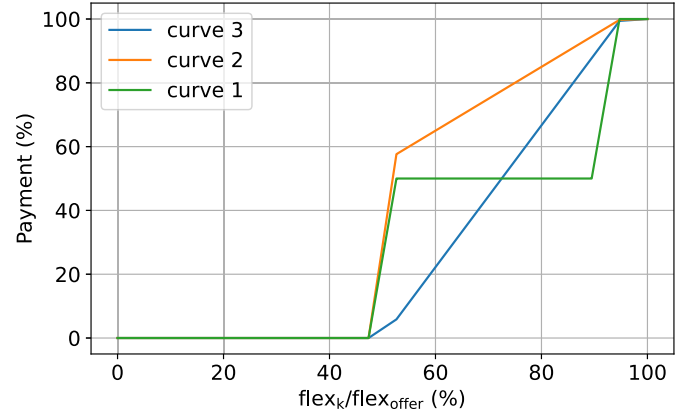


Fig. 4. Examples of penalty curves for under delivery: the FSPs delivering 50 % or less of the contracted flexibility will not receive a payment, while the FSPs delivering more than 95 % of their $\text{flex}_{\text{offer}}$ will receive full payment p_{offer} for their service. The FSPs delivering between 50 % and 95 % will receive a percentage of the price they offer, depending on the curve selected by the DSO. For example, if curve 3 is selected and an asset delivered 60 % of its declared flexibility $\text{flex}_{\text{offer}}$, it will receive 20 % of the full payment p_{offer} .

examples of penalty curves proposed by LEO project [25]. We can see that the FSPs delivering 50 % or less of the contracted flexibility will not receive a payment, while the FSPs delivering more than 95 % of their $\text{flex}_{\text{offer}}$ will receive full payment p_{offer} for their service. Similar penalty functions were adopted in [24,26] to penalise the under-delivery. We note that the curves presented here are illustrative examples, and alternative penalty functions may also be viable. Determining the optimal penalty curve is beyond the scope of this work and will be addressed in future studies.

For a penalty function ρ , the payment a participating FSP receives after each delivery is computed by:

$$c(\text{flex}_k) = \rho \left(\frac{\text{flex}_k}{\text{flex}_{\text{offer}}} \right) \times p_{\text{offer}} \quad (2)$$

where $\text{flex}_{\text{offer}}$ and p_{offer} are the flexibility and price offered by the FSP, and flex_k is the flexibility that is actually delivered. Note that the payment the FSP receives for delivering the flexibility is a cost from the perspective of the DSO. Hence, it is referred to as $c(\text{flex}_k)$ in the formulation of the chance constrained problem 1. Based on 1, the DSO aims to minimise the expected cost of procuring flexibility while ensuring that the procured flexibility $\sum_{k \in \mathcal{K}} b_k \text{flex}_k$ meets the total flexibility F at least $(1 - \epsilon) \times 100$ % of the time.

3. Flexibility estimation and sample generation

To solve the problem 1, we need to estimate the probability density function (PDF) of the random vector flex_k for each asset k . Daily samples are drawn from each month to reflect variability in DER availability. For each asset, this produces a finite set of realisations flex_k^i , where $i \in I$ indexes the days (i.e., samples). These samples form an empirical distribution that is used to evaluate the probabilistic constraint in problem 1.

Flexibility can be defined as an asset’s ability to adjust its scheduled output across time. Therefore, the computation of flex_k^i of an asset k will depend on its baseline, i.e., its scheduled output in the absence of the flexibility market [27]. Thereafter, we will call the baseline output P_{base} . Two of the most common baseline methods are historical baseline [28] and nominal baseline [29]. A historical baseline is computed using an average of historical profiles (e.g., the average of the last 10 days of consumption/generation profiles when flexibility was not provided), while a nominal baseline is computed using a forecast

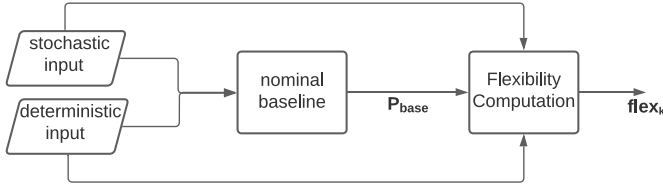


Fig. 5. The process of generating samples for the random variable flex_k . The nominal baseline refers to the day-ahead baseline schedule, deterministic input includes information such as technical parameters of the asset (e.g., EV maximum power rate, HVAC capacity) and stochastic input include dynamic information (e.g., temperature, arrival and departure time of an EV).

looking ahead at what the consumption/generation pattern would be without flexibility. Due to a lack of consumption profiles, we use the second method of baselining. The FSP computes the baseline and sends it to the DSO. While revenue stacking is possible in principle, this work focuses on small-scale DERs for which capacity constraints generally limit concurrent participation across markets within the same time window. Therefore, we assume that the FSPs participate exclusively in the local flexibility market and that revenue stacking is not allowed.

Although the flexibility contracts span mid- to long-term durations (e.g., a season), flexibility delivery and baseline submission operate on a day-ahead basis. The samples flex_k^i used to construct the empirical distribution are generated from daily realisations across the contract horizon (e.g., one or more samples per month over a 4–6 month period). These daily scenarios capture the temporal uncertainty in asset-level flexibility while avoiding the need for long-term forecasts. Once selected, the DER is only required to provide a day-ahead baseline forecast for each contracted delivery window (e.g., 5–7 p.m. on weekdays), making the process operationally feasible.

To prevent gaming (e.g., an FSP inflating its baseline to increase revenue or obscure underperformance), the DSO may conduct data audits in which the submitted baseline is compared against actual metered data in the absence of a flexibility call. Penalties or disqualification can be imposed in the case of manipulation.

The process of generating samples of flex_k is explained in Fig. 5. A day-ahead baseline schedule P_{base} is computed based on the deterministic and stochastic inputs. Then P_{base} is fed as an input along with the other inputs (deterministic and stochastic) to the algorithm of flexibility computation to output flex_k . Every sample i of the stochastic input will result in a sample of the available flexibility flex_k^i .

In a distribution network, different assets owned by customers can provide flexibility ranging from small-scale generation, storage systems such as batteries and electric vehicles (EVs), and flexible controllable loads such as heating, ventilation and air conditioning (HVAC) and electric water heating (EWH). To demonstrate the effectiveness of our model, we focus on two types of flexible assets: building HVACs and EVs, as they are potential sources of flexibility with a high likelihood to be owned by end-users thanks to the governments' efforts in heat and transportation decarbonisation [30–32]. We note that the same method can be extended to other flexible assets without significantly altering the proposed approach. The following subsections explain how the samples of flexibility flex_k were derived, first for HVACs and then for EVs.

3.1. HVAC case

Building heating systems are one of the largest assets connected at the distribution network level and can meet the requirements of the minimum capacity to offer in existing markets. The baseline output of the HVAC is a response to a market incentive, e.g., maximum demand reduction [33] or cost reduction under a time-of-use (TOU) tariff or real-time price (RTP) [34]. The optimisation problem below returns the nominal baseline $P_{\text{base}}(t)$ of an HVAC in response to a maximum demand

reduction incentive.

$$\min_{P_{\text{peak}}, P_{\text{base}}(t)} P_{\text{peak}} \quad (3a)$$

$$\text{s.t. } 0 \leq P_{\text{base}}(t) \leq P_{\text{peak}}, \quad \forall t \in \mathcal{T} \quad (3b)$$

$$T_{\min} \leq T_{\text{in}}(t) \leq T_{\max}, \quad \forall t \in \mathcal{T} \quad (3c)$$

$$T_{\text{in}}(t=0) = T_0, \quad (3d)$$

$$T_{\text{in}}(t) = \alpha T_{\text{in}}(t-1) + \gamma T_a(t-1) + \beta P_{\text{base}}(t-1), \quad \forall t \in \mathcal{T} \quad (3e)$$

$$0 \leq P_{\text{base}}(t) \leq P_{\max}, \quad \forall t \in \mathcal{T} \quad (3f)$$

The problem minimises the maximum power used by the HVAC for a horizon $\mathcal{T}$ while respecting the building constraints (3c), (3d), (3e) and user comfort (3b). The ability of the asset to provide upward flexibility (i.e., to reduce its demand) $\text{flex}_{\text{HVAC}}$ can be then computed as follows:

$$\max_{P_{\text{out}}(t), \text{flex}_{\text{HVAC}}} \text{flex}_{\text{HVAC}} \quad (4a)$$

$$\text{s.t. } 0 \leq \text{flex}_{\text{HVAC}} \leq P_{\text{base}}(t) - P_{\text{out}}(t), \quad \forall t \in \mathcal{T}_{\text{flex}} \quad (4b)$$

$$T_{\text{in}}(t) = \alpha T_{\text{in}}(t-1) + \gamma T_a(t-1) + \beta P_{\text{out}}(t-1), \quad \forall t \in \mathcal{T} \quad (4c)$$

$$0 \leq P_{\text{out}}(t) \leq P_{\max}, \quad \forall t \in \mathcal{T} \quad (4d)$$

$$(3b), (3c) \quad (4e)$$

Where $\mathcal{T}_{\text{flex}} = [T_{\text{flex}}^s, \dots, T_{\text{flex}}^e]$ is the window of the call for flexibility. The problem minimises the consumed power of the HVAC during the flexibility window with respect to the building and user comfort constraints. The output power of an HVAC depends on different deterministic parameters such as building parameters α, β, γ (refer to the building case study in [33] for a detailed explanation of the building model), the upper and lower limits of the indoor temperature T_{in} for the user comfort $T_{\min}$ and $T_{\max}$ and the maximum power of the asset $P_{\max}$. The random parameter that figures in the formulation is the ambient temperature T_a . We note that the same formulation holds for the downward flexibility by replacing the first constraint with $0 \leq \text{flex}_{\text{HVAC}} \leq P_{\text{out}}(t) - P_{\text{base}}(t)$.

3.2. EV case

We focus here on the case where EVs provide flexibility by shifting their charging schedule (i.e., smart charging). Similarly to the HVAC case, the extraction of flexibility for EVs requires a baseline. We assume uncontrolled charging behaviour, where vehicles begin charging immediately upon being plugged in.

The capability of an EV to participate individually in an LFM is context-dependent and can vary based on the product and service type. However, to demonstrate that the methodology is effective for heterogeneous participants, we will assume here that EVs independently do not meet the minimum capacity criteria to offer in an LFM and therefore need to be aggregated to participate.

An EV owner can contract with an aggregator to participate in the LFM and generate extra revenues. Both parties agree on a baseline as part of the contract that manages the relationship between an aggregator and an asset owner, and the owner gives the aggregator control over the asset for a fee. Let $\mathcal{M}$ denote the set of EVs managed by the aggregator, we formulate the problem of computing the upward flexibility as follows:

$$\max_{\text{flex}_{\text{EV}}, P_{\text{ch}}} \text{flex}_{\text{EV}} \quad (5a)$$

$$\text{s.t. } 0 \leq \text{flex}_{\text{EV}} \leq \sum_{m \in \mathcal{M}} (P_{\text{base}}^m(t) - P_{\text{ch}}^m(t)), \quad \forall t \in \mathcal{T}_{\text{flex}} \quad (5b)$$

$$\underline{P}_{\text{ch}}^m \leq P_{\text{ch}}^m(t) \leq \overline{P}_{\text{ch}}^m, \quad \forall t \in \mathcal{T}, \forall m \in \mathcal{M} \quad (5c)$$

$$SOC_{\text{arr}}^m + \frac{1}{C_m} \sum_t (P_{\text{ch}}^m(t) \Delta t) \geq SOC_{\text{des}}^m, \quad \forall m \in \mathcal{M} \quad (5d)$$

For the EV m : SOC_{des}^m and SOC_{arr}^m are the desired state of charge at departure and the state of charge at arrival of its battery, $\underline{P}_{\text{ch}}^m$ and $\overline{P}_{\text{ch}}^m$

are respectively the minimum and maximum charging rate, C_m is the full capacity and P_{base}^m is the EV's schedule charging. Here, P_{ch} is the decision variable that is associated with the new EV charging schedule after considering delivering flexibility. Deriving the available downward flexibility uses the same formulation by replacing maximisation with minimisation. The EV aggregator which is an instance of FSP in this case, as part of its contract with the EV owner, will have permission to collect the data on the availability and charging behaviour of the EV owner.

3.3. Solution approach

The Gaussian model is commonly used in chance-constrained optimisation problems due to its analytical tractability and its capability to approximate the uncertainty of aggregated or poorly characterised random variables in the absence of detailed distributional data. In our case, since total flexibility is modelled as the sum of multiple uncertain asset contributions, the aggregation effect smooths the overall distribution, making the Gaussian approximation both reasonable and practical. Here, HVACs and EVs are used to provide a concrete setting, but other resources with different sources of uncertainty could be included in the proposed framework. The extraction of the PDF of the random vectors $(flex_k)_{k \in \mathcal{K}}$ allows us to relax the chance constraint into a deterministic constraint [35]. Problem 1 becomes equivalent to:

$$\min_{\mathbf{b}} \sum_{k \in \mathcal{K}} b_k \mu_k^c \quad (6)$$

$$\text{s.t. } \mu \mathbf{b} - \Psi(1 - \epsilon) \times \|\Sigma^{\frac{1}{2}} \mathbf{b}\| \geq F$$

Where Ψ refers to the inverse cumulative distribution function (CDF) of the Gaussian distribution and μ_k^c is the mean of the cost function of asset k ($c(flex_k)$). Problem 6 is a mixed-integer second-order cone problem for $(1 - \epsilon) \geq 0.5$ and is solvable by most solvers. Although MISOCP problems scale exponentially with the number of binary variables, state-of-the-art solvers can efficiently handle instances with up to approximately 100 binary variables, depending on the problem structure. In our case, since the application is at the distribution level, e.g., involving flexible assets downstream of a feeder or substation, the number of candidate assets rarely exceeds this limit. Additionally, a pre-selection step can be used to discard assets with low reliability (e.g., below 50 %), further reducing the problem size and ensuring tractability.

4. Results & discussion

4.1. Network settings

To evaluate the effectiveness of our model, we consider flexibility provision during winter, when energy consumption for heating is high and there is the potential for the peak demand to exceed the network's capacity. To prevent this, the DSO launches a call for tenders, months ahead of the winter season, to procure flexibility from assets within its network, and the selected FSPs will sign contracts to deliver their services during that period. A similar market is considered by the Local Energy Oxfordshire (LEO) flexibility platform and is called 'Sustain-Peak Management'.² We simulate the model with a network of 5 medium-sized building HVACs and 284 EVs. The number of EVs is representative of a local region managed by a DSO, and the buildings, though they share the same weather conditions because of their location proximity, have different technical characteristics, which will affect their flexibility potential. The flexibility window was selected to be [7 pm-10 pm], the minimum capacity to offer is 50 kW and we adopted the curve 3 penalty function (Fig. 4). The market settings and the HVACs and EVs parameters are summarised in Table 1. We used temperature data from [36] and the nominal baseline for HVACs was a response to a maximum demand reduction incentive. For EV scenarios, we used data from [37], and charging as soon as plugged in

Table 1
Simulation Settings.

	Type of flexibility	upward
Market settings	Contract settlement	months ahead
	Duration of contract	4 months (winter)
	Capacity _{lim}	50 KW
	Number of HVACs	5
	EVs	284
	Aggregators	1 or 2
HVAC parameters	$\mathcal{T}_{Flex}$	[7pm-10pm]
	P_{max} (kW)	{180, 200, 250}
	CoP	{1.5, 2, 2.5}
	R (°C/kW)	0.0337
	C (kWh/°C)	0.0337
	T_{min} (°C)	16
EV parameters	T_{max} (°C)	18
	P_{max} (kW)	{3.6, 7}
	C_{max} (kWh)	{6.2, 100}

Table 2

Asset fulfilment rate: The rate(%) of successful, partial and unsuccessful deliveries in previous events.

	Unsuccessful	Partial delivery	Successful
HVAC1 180–1.5	26	10	64
HVAC2 180–2.0	16	10	74
HVAC3 200–2.0	10	9	81
HVAC4 200–2.5	13	13	74
HVAC5 250–2.0	10	9	81
Agg1	80	20	0
Agg2	84	16	0
Agg0	73	19	9

as a baseline. It is worth noting that the framework can accommodate different asset/aggregator strategies with different baselines. For the sake of simplicity, we assume that all the FSPs offer the same capacity = 50 kW.

4.2. Asset's fulfilment rate assessment

In this subsection, we study the asset's fulfilment rate, which is a measure that reflects the percentage of successful deliveries based on the historic ability of an FSP to deliver the promised capacity. For privacy issues, the DSO is not allowed to collect information about HVACs and EVs and build an exact model as in (3) and (4) to assess the FSP fulfilment rate. Instead, the DSO assesses it based on historical data. Initially, for newly connected DERs without historical delivery records, the DSO conducts capability testing through trial or simulated flexibility events. During these events, each participant is asked to respond to a controlled flexibility call. The DSO records both the declared and delivered flexibility in a dedicated database. This information is used to compute an initial fulfilment rate, serving as a proxy for the DER's reliability. Once the service becomes operational, this database is continuously updated after each activation event, allowing the reliability estimate to evolve based on actual performance. Table 2 gives the rates of deliveries based on 74 events that were randomly selected. Unsuccessful deliveries refer to deliveries that are less than 50 % of the offered capacity and successful deliveries are events in which the participant succeeds in delivering more than 0.95 % of the offered capacity. The labels for the HVACs refer to the $\{P_{max}, CoP\}$ pair of the corresponding HVAC. For the aggregators, Agg 0 refers to the case where all EVs are managed by a single aggregator, while Agg 1 and Agg 2 refer to the case where the EVs are split randomly between two aggregators.

² <https://project-leo.co.uk/the-energy-challenge/flexibility-services/>

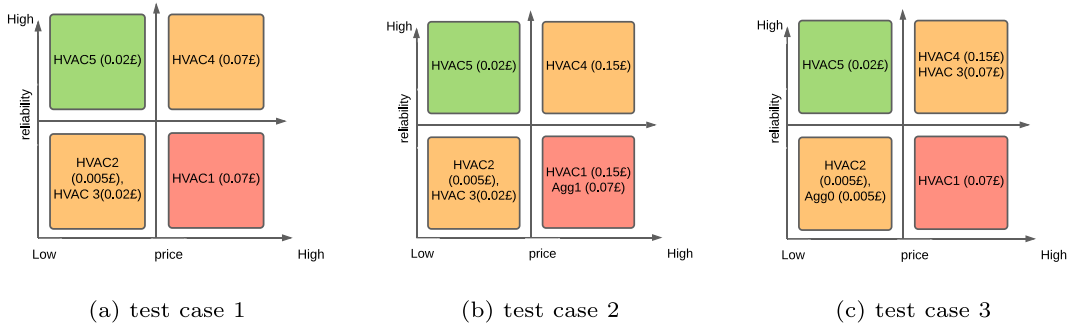


Fig. 6. Reliability-price matrix: Each FSP is assigned a price, the {reliability, price} pair allows us to allocate the FSP to one of the 4 classes of the matrix.

4.3. Asset selection and expected DSO costs

To evaluate the performance of the suggested strategy in selecting FSPs, we split the available flexibility data that we generated into two subsets: the training subset (75 % of the dataset) will be used to extract the parameters of the probability functions of the assets' available flexibility and will be fed into the algorithm to extract the optimal selection of FSPs, and the testing data (25 % of the dataset) will be used to evaluate the performance of the strategy on unseen data. Data was temporally divided to simulate a real-world scenario where the DSO selects participants based on current information, with flexibility delivery expected in the future. Consequently, the training data was treated as "current" data to support DSO decision-making, while the testing data represented "unseen future" data to evaluate the model's credibility. According to [38], the utilisation prices of flexible services range from 60 £/MW to 105 £/MW, we associate prices around this range to the FSPs to assign them to the different classes of the classification chart in Fig. 3. There are four different prices {0.005, 0.02, 0.07, 0.15} £/kW with the first two considered to be low prices and the last two high prices. We select a total flexibility F between 50 kW and 300 kW and a confidence level between 75 % and 90 % corresponding to a violation level ϵ between 0.25 and 0.1 respectively.

We define three test cases:

Test case 1: A network with HVACs

Test case 2: A network with HVACs and Aggregator 1

Test case 3: A network with HVACs and aggregator 0

Fig. 6 demonstrates how the different assets were classified according to their {reliability, price} pair for the different test cases. Tables 3, 5 and 7 show the results of Problem 6 and list the selected FSPs for the three test cases per scenario. Each scenario is defined by a {total flexibility (F), violation level (ϵ)} pair. The cost column displays the expected cost of flexibility procurement from the selected FSPs. The metric $\bar{\epsilon}$ refers to the actual violation level which is computed using the testing (unseen) data.

We compare the results against two baselines:

- Baseline 1 (Robust): Select the most reliable assets sequentially until the required flexibility is met.
- Baseline 2 (Cost-Driven): Select the lowest-cost assets sequentially until the required flexibility is met.

These baselines serve as lower and upper bounds for our proposed strategy: Baseline 1 prioritises reliability at a higher cost, while Baseline 2 emphasises cost-effectiveness but may fail to ensure an acceptable level of reliability.

4.3.1. Test case 1

Intuitively, HVAC5 is the first to be selected by the strategy, since it offers high reliability at a lower price. For the {50 kW, 0.1} scenario, HVAC5 was not sufficient to deliver the flexibility with the required reliability (in Tab. 2 we can see that HVAC5 has a reliability of 81 % while

Table 3

Selected FSPs: Test Case 1.

Flex	ϵ	Cost	Selected FSPs	$\bar{\epsilon}$
50 kW	0.25	0.86	HVAC5	0.08
	0.2	0.86	HVAC5	0.08
	0.15	0.86	HVAC5	0.08
	0.1	1.02	HVAC5 HVAC2	0.04
100 kW	0.25	1.02	HVAC5 HVAC2	0.1
	0.2	1.58	HVAC5 HVAC3	0.1
	0.15	1.74	HVAC5 HVAC2 HVAC3	0.06
	0.1	4.68	HVAC5 HVAC2 HVAC3 HVAC4	0.04
150 kW	0.25	1.74	HVAC5 HVAC2 HVAC3	0.16
	0.2	3.96	HVAC5 HVAC2 HVAC4	0.1
	0.15	4.68	HVAC5 HVAC2 HVAC3 HVAC4	0.075
200 kW	0.25	4.68	HVAC5 HVAC2 HVAC3 HVAC4	0.11
	0.2	7.61	HVAC5 HVAC2 HVAC3 HVAC4 HVAC1	0.09
250 kW	0.25	9.42	HVAC5 HVAC2 HVAC3 HVAC4 HVAC1	0.15

the DSO requires a reliability of 90 %), therefore, the strategy selected HVAC2 to reach the desired confidence level. For the {100 kW, 0.2} scenario, the strategy selected HVAC3 instead of HVAC2, because although it is more expensive than HVAC2, it allows the DSO to reach the desired confidence level. The strategy prefers selecting HVAC2 + HVAC3 over HVAC4, as can be seen in {100 kW, 0.15}, because combined, they provide the same reliability as HVAC4 at a lower total cost. From {200 kW, 0.2} onward, the strategy selects all the available FSPs to fulfil the needed flexibility with the desired confidence level. For all the scenarios, the actual violation level $\bar{\epsilon}$ never surpasses the desired violation level ϵ .

4.3.2. Test case 2

In this test case, we add the aggregator *Agg1* as a low-reliability high-price FSP. We notice similar behaviour as in test case 1. The selection that can seem counter-intuitive is the one associated to {150 kW, 0.2} because the strategy selected *Agg1* (to-avoid class) instead of HVAC4. Nonetheless, *Agg1* has a cheaper price than HVAC4, and even if it has low reliability, combined with the other selected FSPs, they ensure the reliability required by the DSO. Therefore, the introduction of low-reliable high-price FSPs into the market can be beneficial to the DSO, as long as the FSPs in this class offer their service at a relatively lower price than the FSPs in the high-reliability high-price class. This example shows the limitation of the rough classification and the need for a meticulous analysis as provided by the strategy. The introduction of *Agg1* unlocked the procurement of flexibility with a higher confidence level. For instance, procuring 150 kW with 0.1 was not feasible in test case 1 and becomes feasible in this test case. The same applies for the scenarios {200 kW, 0.15} and {250 kW, 0.2}. For this test case, the actual violation level $\bar{\epsilon}$ does not exceed the targeted violation level ϵ .

Table 4Comparison with the baseline scenarios: *Test Case 1*.

Flex	Baseline 1			Baseline 2		
	FSPs	$\bar{\epsilon}$	Cost	FSPs	$\bar{\epsilon}$	Cost
50 kW	HVAC5	0.08	0.86	HVAC2	0.26	0.16
100 kW	HVAC5 HVAC3	0.1	1.58	HVAC2 HVAC3	0.26	0.88
150 kW	HVAC5 HVAC3 HVAC4	0.16	4.52	HVAC2 HVAC3 HVAC5	0.16	1.74
200 kW	HVAC5 HVAC3 HVAC4 HVAC2	0.11	4.68	HVAC2 HVAC3 HVAC5 HVAC1	0.26	4.67
250 kW	HVAC5 HVAC3 HVAC4 HVAC2 HVAC1	0.15	9.42	HVAC2 HVAC3 HVAC5 HVAC1 HVAC4	0.15	9.42

Table 5Selected FSPs: *Test Case 2*.

Flex	ϵ	Cost	Selected FSPs	$\bar{\epsilon}$
50 kW	0.25	0.86	HVAC5	0.08
	0.2	0.86	HVAC5	0.08
	0.15	0.86	HVAC5	0.08
	0.1	1.02	HVAC5 HVAC2	0.04
100 kW	0.25	1.02	HVAC5 HVAC2	0.1
	0.2	1.58	HVAC5 HVAC3	0.1
	0.15	1.74	HVAC5 HVAC2 HVAC3	0.06
	0.1	3.66	HVAC5 HVAC3 Agg1	0.08
150 kW	0.25	1.74	HVAC5 HVAC2 HVAC3	0.16
	0.2	3.82	HVAC5 HVAC2 HVAC3 Agg1	0.1
	0.15	3.82	HVAC5 HVAC2 HVAC3 Agg1	0.1
	0.1	10.13	HVAC5 HVAC2 HVAC3 Agg1	0.06
200 kW	0.25	3.82	HVAC5 HVAC2 HVAC3 Agg1	0.21
	0.2	6.75	HVAC5 HVAC2 HVAC3 Agg1 HVAC1	0.16
	0.15	10.13	HVAC5 HVAC2 HVAC3 Agg1 HVAC4	0.09
250 kW	0.25	10.13	HVAC5 HVAC2 HVAC3 Agg1 HVAC4	0.15
	0.2	13.06	HVAC5 HVAC2 HVAC3 Agg1 HVAC4 HVAC1	0.1

Table 6Comparison with the baseline scenarios: *Test Case 2*.

Flex	Baseline 1			Baseline 2		
	FSPs	$\bar{\epsilon}$	Cost	FSPs	$\bar{\epsilon}$	Cost
50 kW	HVAC5	0.08	0.86	HVAC2	0.26	0.16
100 kW	HVAC5 HVAC3	0.1	1.58	HVAC2 HVAC3	0.26	0.88
150 kW	HVAC5 HVAC3 HVAC4	0.16	4.52	HVAC2 HVAC3 HVAC5	0.16	1.74
200 kW	HVAC5 HVAC3 HVAC4 HVAC2	0.11	4.68	HVAC2 HVAC3 HVAC5 Agg1	0.21	4.27
250 kW	HVAC5 HVAC3 HVAC4 HVAC2 HVAC1	0.15	9.42	HVAC2 HVAC3 HVAC5 Agg1 HVAC1	0.16	7.2

Table 7Selected FSPs: *Test Case 3*:

Flex	ϵ	Cost	Selected FSPs	$\bar{\epsilon}$
50 kW	0.25	0.33	HVAC2 Agg0	0.15
	0.2	0.33	HVAC2 Agg0	0.15
	0.15	0.33	HVAC2 Agg0	0.15
	0.1	1.02	HVAC5 HVAC2	0.04
100 kW	0.25	1.02	HVAC5 HVAC2	0.1
	0.2	1.02	HVAC5 HVAC2	0.1
	0.15	1.02	HVAC5 HVAC2	0.1
	0.1	1.19	HVAC5 HVAC2 Agg0	0.05
150 kW	0.25	1.19	HVAC5 HVAC2 Agg0	0.15
	0.2	1.19	HVAC5 HVAC2 Agg0	0.15
	0.15	3.72	HVAC5 HVAC2 Agg0 HVAC3	0.1
	0.1	6.65	HVAC5 HVAC2 Agg0 HVAC3 HVAC1	0.06
200 kW	0.25	3.72	HVAC5 HVAC2 Agg0 HVAC3	0.18
	0.2	3.72	HVAC5 HVAC2 Agg0 HVAC3	0.18
	0.15	10.03	HVAC5 HVAC2 Agg0 HVAC3 HVAC4	0.08
	0.1	12.96	HVAC5 HVAC2 Agg0 HVAC3 HVAC4 HVAC1	0.07
250 kW	0.25	6.64	HVAC5 HVAC2 Agg0 HVAC3 HVAC1	0.21
	0.2	10.03	HVAC5 HVAC2 Agg0 HVAC3 HVAC4	0.14
	0.15	12.96	HVAC5 HVAC2 Agg0 HVAC3 HVAC4 HVAC1	0.1
300 kW	0.25	12.96	HVAC5 HVAC2 Agg0 HVAC3 HVAC4 HVAC1	0.15

4.3.3. Test case 3

Here, we introduce Agg0 as a low-reliability low-cost FSP. For a small F and a tolerant violation level, the strategy selected the combination {HVAC2, Agg0} instead of HVAC5. This can be explained by the fact that this combination, although composed of low-reliability FSPs, provides the desired flexibility with lower cost compared to HVAC5. Similar behaviour to the previous test cases can be seen across the remaining scenarios. The incorporation of Agg0 as an FSP permits the procurement of $F = 300 \text{ kW}$ with a 75 % confidence level, which was not possible in the previous test cases.

Tables 4, 6 and 8 show the results for the two baselines. In Baseline 1, the most reliable asset is selected first, which in this case is HVAC5, also the least-cost asset. This outcome aligns with our proposed strategy. For a requirement of 100 kW, the next most reliable and least-cost asset is HVAC3, again consistent with the strategy. However, at 150 kW, the selected combination in Baseline 1 achieves a reliability level of 84 % (i.e., $\epsilon = 16$ %) at a cost of 4.68, whereas our strategy achieves the same reliability at a significantly lower cost of 1.74 by selecting a more optimal asset mix.

Cases 1 and 2 exhibit similar patterns when compared with Baseline 2. In most scenarios, Baseline 2 fails to satisfy the minimum required reliability level of 75 % (i.e., $\epsilon = 25$ %). For Case 3, in certain scenarios; such as $F = 150 \text{ kW}$ or 200 kW with $\epsilon = 0.20$ or 0.25 ; the lowest-cost

Table 8
Comparison with the baseline scenarios: *Test Case 3*.

Flex	Baseline 1			Baseline 2		
	FSPs	$\bar{\epsilon}$	Cost	FSPs	$\bar{\epsilon}$	Cost
50 kW	HVAC5	0.08	0.86	HVAC2	0.26	0.16
100 kW	HVAC5 HVAC3	0.1	1.58	HVAC2 Agg0	0.34	0.33
150 kW	HVAC5 HVAC3 HVAC4	0.16	4.52	HVAC2 Agg0 HVAC5	0.15	1.19
200 kW	HVAC5 HVAC3 HVAC4 HVAC2	0.11	4.68	HVAC2 Agg0 HVAC5 HVAC3	0.18	3.72
250 kW	HVAC5 HVAC3 HVAC4 HVAC2 HVAC1	0.15	9.42	HVAC2 Agg0 HVAC5 HVAC3 HVAC1	0.21	6.65

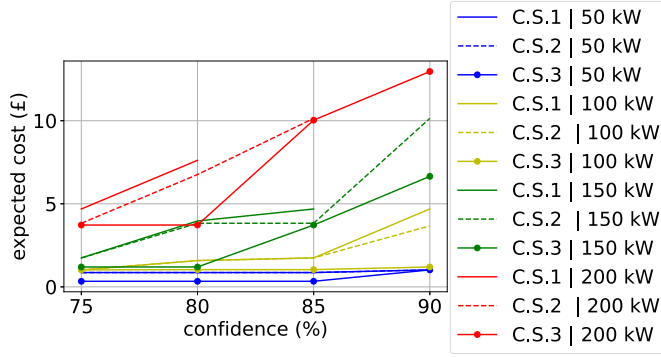


Fig. 7. Expected procurement cost of DSO for different scenarios (F , confidence level) of the three test cases (C.S.).

combination selected by Baseline 2 coincides with the solution from our strategy. However, since Baseline 2 does not incorporate reliability constraints explicitly, it offers only one fixed combination and fails to meet the required flexibility when higher reliability levels (above 80 %) are demanded. Fig. 7 shows the expected cost of procuring flexibility for different scenarios under the three test cases. Obviously, a higher confidence level incurs higher costs. For the same scenario $\{Flex, \epsilon\}$, test case 1 results in the highest costs. Compared to test case 1, the introduction of a low-cost low-reliability FSP in test case 3 (i.e., *Agg0*) and a ‘relatively’ low-cost FSP in test case 3 (i.e., *Agg1*) reduces the costs.

We can see a general difference between the set violation level and the actual one. This difference arises because the available resources do not precisely match the violation level specified by the DSO. Consequently, the algorithm selects the cheapest combination of assets that collectively achieves a violation level as close as possible to the one set by the DSO.

As for the computational time of running the strategy, it ranges between 5 and 35 ms on a laptop with a Processor Intel® Core™ i7 @ 2.80 GHz, with 32 GB RAM using picos python package.

5. Conclusion

In this paper, we proposed an optimal selection strategy for DSO flexibility procurement from uncertain assets. The strategy guarantees to the DSO that the needed flexibility is met with a certain confidence level; for all test cases, the actual violation level never exceeds the targeted level, and the DSO consistently procures the required flexibility at the lowest available cost. The strategy is asset-agnostic and allows for small and medium-scale customers (e.g., residential) to participate in the market individually or via aggregators. The strategy will support DSOs that plan to organise local flexibility markets and settle contracts for a mid-to long-term periods (e.g., months/years ahead) for flexibility provision. It addresses the challenge of the risk related to under-delivery and will support the DSO in the decision-making process to trade off reliability and cost. This is of particular interest to DSOs in countries for which the regulatory framework in settling LFM requires them to allow all technologies to compete without being restricted to a certain asset class

and to secure stable supply while keeping costs down for all customers. Undertaking least-cost procurement will not only lead to lower customer bills but can also mitigate the need for costly network infrastructure upgrades. The proposed scheme will help planners and policymakers in the sector to incentivise the adoption of flexibility and encourage local players and aggregators to participate. Important areas for future work include the development of simplified mechanisms which can improve market transparency without compromising performance, the optimal design of penalty curves which balance the risk of under-delivery with the need for market liquidity and the consideration of simultaneous participation of flexibility providers in multiple flexibility markets.

CRedit authorship contribution statement

C. Essayeh: Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Data curation. **I. Savelli:** Writing – review & editing, Validation, Investigation, Formal analysis, Conceptualization. **T. Morstyn:** Writing – review & editing, Validation, Supervision, Project administration, Investigation, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

This study uses publicly available, open-source data. All datasets and their access links are properly cited and referenced in the manuscript at the first mention of each dataset.

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