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Abstract

Unawareness of outcomes, i.e. the ignorance of some of the possible consequences of an action, is an important form of bounded rationality that is routinely observed in many economic problems. Until recently, however, it has received relatively little attention in the literature. In this PhD thesis, I study from a theoretical point of view how agents' limited awareness affects their behavior in different settings. In "Unawareness of Outcomes in Innovative Activities," I analyze how the awareness of being boundedly aware impacts individuals' decisions to become and remain entrepreneurs. I show that awareness of unawareness implies a reduced value of information, that further discourages pessimistic individuals from entering entrepreneurship and induces optimistic ones to experiment less and continue to advance their projects. This may shed new light on the reasons why many people enter and persist in entrepreneurship despite earning low returns. I also study how agents' attitudes towards the unknown affect the associations among certain characteristics of entrepreneurship, and show how such associations differ from those generated by a model where individuals are fully aware and entrepreneurship is driven solely by non-pecuniary benefits. In "Learning Under Awareness of Unawareness," I develop a model where through a likelihood test individuals can increase their awareness. I study how changes in the probabilities of known outcomes and in agents' optimal actions affect their ability to recognize novel consequences. I show that those individuals who prefer actions involving newly discovered outcomes become less sensitive towards future discoveries and may actually end up learning less. Finally, in "Heterogeneous Awareness in Financial Markets" (joint with Federico Severino, Université Laval), I study a financial market where traders are heterogeneously aware about the possible payoffs of an asset. I show that as unawareness becomes more severe the overall amount of informed agents in the market diminishes and the incentives to gather information are transferred from the partially to the fully aware traders, which keeps the average price high when the asset payoff is low.

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Chapter 1

Introduction

In this PhD thesis, which consists of three main chapters, I study from a theoretical point of view how agents' limited awareness, i.e. their ignorance of some possible outcomes of their actions, affects their behavior in different economic settings.

In Chapter 2, I study the problem of an agent deciding whether to undertake an innovative project, such as founding a startup or entering entrepreneurship more broadly. The agent is not aware of all the possible outcomes of such project, but is aware of his bounded awareness. Conditional on starting the project, the agent has access to a costly experimentation technology providing him with signals about the project's final outcome. I study how the agent's level of awareness and his attitude towards the unknown impact his decisions to enter and remain in entrepreneurship. I show that, under a natural consistency condition on agents' beliefs, awareness of unawareness implies a reduced value of information, that further discourages pessimistic individuals from entering entrepreneurship and induces optimistic ones to experiment less and persist more. I also show that experimentation is least valuable for those agents who are unaware of the most favorable outcomes and/or the most severe issues of their projects. Next, I provide empirical predictions about the association among certain characteristics of entrepreneurship as a function of agents' attitudes towards unawareness and the cost of their projects. Finally, I show that a model where entrepreneurship is driven by non-pecuniary benefits would generate distinctive predictions.

In Chapter 3, I develop a simple choice theoretic model of learning under limited awareness of outcomes. A decision maker conducts a series of clinical trials and in each period she observes only an imprecise measure of the outcome, and uses a likelihood test to decide whether an observation is the result of a known outcome plus a measurement error or of an unknown effect. First, I show that for an agent who does not incorporate new discoveries in

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her treatments, a sort of negative feedback effect in her ability to learn good new outcomes emerges. In particular, the discovery of a beneficial effect increases the perceived likelihood of good outcomes and, in addition, induces the agent to choose treatments involving more extreme effects. Both these reactions inhibit the decision maker's ability to recognize other new favorable outcomes. The changes in the perceived likelihoods and in the optimal treatment instead work in opposite directions when evaluating the impact of new discoveries on the agent's ability to detect adverse side effects. In this case, the ultimate result depends on her sensitivity towards measurement errors, with high sensitivity inducing a positive feedback effect. Finally, I study how an agent's willingness to experiment affects her ability to learn. I show that when new outcomes are sufficiently more extreme than the known ones a decision maker who never experiments learns more, whereas such new outcomes make an agent who loves to experiment more capable of recognizing moderate new effects.

In Chapter 4, which is a joint work with Federico Severino, we consider an order-driven financial market without short sales in which a fraction of investors is only partially aware of the potential payoffs of a risky asset. Such partial awareness induces a change in their investment choices following intermediate signal realizations. We show that, by lessening the shift in the expected asset payoff following any signal realization, more severe degrees of unawareness lead to a reduction in the overall amount of informed traders in the market. This makes large order flows less revealing of a good signal, lowering the price, and increasing the incentives of the fully aware to get informed. As a result, as unawareness rises, incentives to acquire information are transferred to the fully aware investors. The diminished number of partially aware informed agents, who are the only traders buying the asset after intermediate signals, makes low order flows less revealing of a bad signal, keeping the average price high when the asset payoff is low. On the other hand, since the presence of the partially aware informed investors gives rise to new intermediate price levels following a good signal, their reduction due to a higher unawareness level leads to an increased price volatility after such signal.

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Chapter 2

Unawareness of Outcomes in Innovative Activities

2.1 Introduction

When agents engage in substantially innovative activities, such as the development of a new product, a new production technology, or a novel business idea, their stock of knowledge may be insufficient to fully describe the problem at hand, and they may frequently end up facing issues or achieving results that initially were not even imaginable. Such unforeseen issues and achievements may be quite important. The patent lawyers at Bell Labs, for example, were initially unwilling even to apply for a patent on the invention of the laser, since they were skeptical about its relevance to the telephone industry, given that previously optical waves had never been of any importance to communications (Rosenberg, 1996). And yet, in subsequent years the laser technology, used in conjunction with optical fibers, revolutionized telecommunications, and found application in many, apparently unrelated fields, including surgery, textiles and metallurgy. Thalidomide is a powerful sedative that had been sold in many countries in the late 1950s. Early trials did not make its developers aware of the severe side effects it could have on pregnant women. As a result, in the late 1950s and early 1960s more than 10,000 infants around the world were born with severe malformations of the limbs, leading in many cases to death. In the 1980s aspirin was convincingly shown to also work as an anticoagulant and help prevent heart attacks and thrombotic strokes. Sommer *et al.* (2009) provide survey evidence showing that it is quite common for startup companies, even for successful ones, to face unanticipated issues, the so called “unknown unknowns”. These may include unforeseen technical problems, poorly understood customer needs,

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and unanticipated regulatory obstacles.

Agents engaging in innovative activities may have a sense that such activities may lead to unknown outcomes, i.e., outcomes different from those they are currently able to describe, and they may react to this with more or less fear/excitement. They may also try to assess the probability of incurring unknown outcomes based on the familiarity of the problem at hand (e.g., how much the new drug differs from existing ones). Based on this assessment, as well as on which of the known outcomes are the most likely, the agent can then decide to exert some extra effort to gather additional information before deciding whether to advance the innovative project. Such an information gathering activity, commonly referred to as *experimentation*, may involve building prototypes, running clinical trials or conducting market surveys, and it plays a crucial role in the development of new products.[1] In particular, Kerr *et al.* (2014) document a recent trend towards lower costs of experimentation, especially in certain industries, and argue that this has greatly benefited entrepreneurs by allowing them to test their innovative ideas more efficiently.

A natural question that arises in this kind of settings, where an individual engages in an innovative activity and recognizes the possibility of encountering unknown outcomes, is how the innovation decisions of the agent are affected by his awareness level and his "attitude towards unawareness". Indeed, as mentioned above, the sense that the innovation may lead to unknown outcomes whose desirability is difficult to predict may inspire fear or excitement.[2] How do this attitude and the initial level of awareness affect the innovative behavior of the agent? How does the availability of an experimentation technology affect the results? How do the conclusions compare to those one would obtain in a scenario in which the individual is fully aware of the possible outcomes of the innovation, and is only uncertain about which of these known outcomes will eventually take place? These questions have been rarely addressed in the economics literature so far, despite the fact that unforeseen outcomes are an important feature of many innovative activities, and that agents engaging in such activities may recognize the possibility of running into this type of outcomes. This paper offers a first model of how limited awareness of outcomes affects agents' incentives to undertake and persist in innovative activities, so as to provide an answer to the above ques-

[1] For a review on experimentation in new product development, see Thomke (2008).

[2] Karni and Vierø (2017) provide a subjective expected utility representation of an agent's preferences when he is aware of his bounded awareness of outcomes. In their representation, which is the one adopted in the present paper, the agent's attitude towards unawareness is captured by a single parameter. The higher is the parameter value in the utility, the more positive is the agent's reaction to the unknown.

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tions. In particular, I show how different levels of awareness and different attitudes towards unawareness impact agents' decisions of whether and when to innovate, as well as their persistence in pursuing innovative activities.

Specifically, in a discrete-time setting, I consider a decision maker who is born endowed with an innovative idea/project with a stochastic outcome, and must decide whether and when to undertake such project, which can be viewed as the decision of whether and when to found a startup or enter entrepreneurship more broadly. The project requires a fixed per period cost to be kept alive and be completed in a certain number of periods. I assume that, due for example to informational spillovers, the innovative project cannot be restarted once it has been abandoned. In each period in which the agent is not an entrepreneur, he works as a wage worker, receives a fixed per period wage, and at the end of the period his wealth is subject to a random shock. If the individual wishes to enter entrepreneurship, he first needs to accumulate wealth through wage work in order to be able to complete the project. In each period in which the project is in place, the agent can use an experimentation technology and delay the completion of the project by one period in order to receive a signal about the project's final outcome.

The agent is assumed to be only partially aware of the project's final outcomes. He is, however, aware of his unawareness, in the sense that, following Karni and Vierø (2017), he assigns a positive probability and a utility to the event of encountering outcomes different from those he is aware of. The agent is aware of all the signals he can receive via the experimentation technology. He updates his beliefs using Bayes' rule and a simple consistency condition requiring him to have correct posterior beliefs about the outcomes he is aware of.

I study how the agent's attitude towards unawareness (i.e., the utility he assigns to the unknown) and his level of awareness (i.e., the subset of outcomes he is aware of) shape (i) the decision of whether and when to undertake the project/enter entrepreneurship, and (ii) the decisions to experiment and hence the persistence in pursuing the innovative project. First, I focus on agents' experimentation decisions and show that their behavior can be characterized in terms of a threshold period until which, conditional on undertaking the project/entering entrepreneurship, it is optimal to experiment. I show how an agent's attitude towards the unknown can be decomposed into a component driven exclusively by the level of unawareness, in the sense that the individual holds a correct prior evaluation of the project, and a second component that reflects his optimism/pessimism towards the unknown, which biases his initial evaluation.

As for the first component, I show that under the consistency condition on agents' beliefs limited awareness of outcomes implies a reduced signal

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strength, in the sense that good signals are perceived as being less good and bad signals as being less bad. If the confusion generated by unawareness is not large enough to reverse the direction of the signals, this implies a reduced value of information compared to the full awareness case. Intuitively, the fact that the agent is neglecting some outcomes implies that the shift in his expected utility induced by any signal is lessened, and therefore his posterior expected utilities are less dispersed around the prior one. This implies that a partially aware individual will never become an entrepreneur if a fully aware agent with the same initial wealth and the same prior expected utility from the project prefers to work as a wage worker in all periods. Furthermore, if the partially aware agent does become an entrepreneur, then, compared to a fully aware agent with the same initial expected utility from the project and the same initial wealth, he will enter entrepreneurship later and will experiment for a smaller number of periods.

Next, I study the effects of increasing levels of unawareness and provide a sufficient condition under which greater unawareness implies a lower value of information. The condition requires the more unaware agent to be unaware of additional outcomes that are more extreme compared to those the less unaware agent is unaware of. Intuitively, the agents who deem experimentation least valuable are those who are unaware of the most favorable outcomes and/or the most severe issues of their projects. Such agents will be less willing to become entrepreneurs and, if they do wish to become entrepreneurs, they will undertake the project later and experiment for a smaller number of periods.

I then show how agents' initial optimism/pessimism can reinforce or counteract the reduction in the value of information due to bounded awareness. The results may shed new light on the empirical finding that many entrepreneurs seem to enter and persist in entrepreneurship despite earning low risk-adjusted returns (see, for example, Astebro *et al.*, 2014). Indeed, such a finding is consistent with the entrepreneur being only partially aware of the possible outcomes of his innovative idea and having a positive attitude towards the unknown.

I then study the interaction between agents' incentives to experiment and their wealth accumulation decisions, and provide some empirical predictions about how attitudes towards unawareness and the cost of the project affect the association among certain characteristics of entrepreneurship. In particular, if agents are fearful towards the unknown or the cost of the project is high, individuals entering entrepreneurship will experiment in the hope of getting a good signal about the project's outcome. The value of such signal is larger for more confident agents, who will therefore experiment more. Such individuals will also store more wealth in each period, since the project is

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more valuable to them for any number of experiments. In this case we therefore expect an early time of entry into entrepreneurship to be associated with a lower probability of abandoning the project and a higher average duration of completed projects. By contrast, if agents are confident in the unknown or the cost of the project is low, entrepreneurs will experiment out of their concern for bad signals. In this case, the value of such signals is smaller for more confident agents, who will therefore experiment less. Here we therefore expect an early time of entry into entrepreneurship to be again associated with a lower probability of abandoning the project and, differently from before, a lower average duration of completed projects. As an empirical consequence of these results, one may try to infer agents' attitudes towards the unknown from the observed association among the aforementioned characteristics of entrepreneurship, and knowing individuals' awareness levels, one may also estimate their initial bias in terms of overoptimism/overpessimism.

Finally, I compare my results to those one would get in a full awareness scenario where agents' decisions to engage and persist in entrepreneurship are motivated by non-pecuniary benefits. I show that in this case a distinctive empirical pattern can emerge. Individuals with larger non-pecuniary benefits enter entrepreneurship earlier, and if such benefits are sufficiently high they are less willing to experiment so as not to postpone their benefits. If agents are interested in good signals, we therefore expect early entry into entrepreneurship to be associated with a higher probability of abandonment, and a lower average duration of completed projects. This association is not reconcilable with different attitudes towards unawareness, and would therefore be evidence in favor of entrepreneurship being driven by non-pecuniary benefits.

The main contribution of this paper is to provide a first model of how limited awareness of outcomes affects occupational choices and the decisions to experiment and pursue innovative projects. Given that, as argued above, partial awareness seems to be a common feature of entrepreneurship, my findings may advance our knowledge of why many people enter and persist in entrepreneurship despite earning low returns. Moreover, I show how one may empirically distinguish unawareness from other sources of entrepreneurship like non-pecuniary benefits.

In this paper, I take the choice theoretic approach of Karni and Vierø (2017). In Karni and Vierø (2017) the state space is constructed from a set of basic actions and a set of consequences. The agent is assumed to be unaware of some consequences, but aware that his knowledge may be incomplete. The authors provide a tractable subjective expected utility representation of preferences over distributions on acts, where the agent's attitude towards the unknown is captured by a parameter. Another choice

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theoretic paper featuring awareness of unawareness is Grant and Quiggin (2015). Here the authors augment a standard Savage state space with a set of "surprise" states. In addition, they augment a set of "standard" consequences with two unanticipated consequences, one ranked below the worst possible standard consequence and the other ranked above the best standard consequence. These unanticipated consequences can occur only in surprise states. The agent knows that his understanding of the world is incomplete and evaluates acts according to an expected uncertain utility representation. On the applied side, the notion of unawareness has been employed in various settings such as, among others, principal-agent problems (Filiz-Ozbay, 2012; Auster, 2013; Von Thadden and Zhao, 2012), contractual disputes (Grant *et al.*, 2012), and electoral campaigns (Schipper and Woo, 2016). Galanis (2015) shows how, in a multiple state space model, an agent aware of all outcomes but unaware of some contingencies (and not aware of his unawareness) may have a negative value of information. In his model, the agent's awareness level is not constant across states, creating a signal that the agent can only partially understand. This may in turn lead him to commit information processing errors and behave suboptimally in response to additional signals. By contrast, in my model with constant unawareness of outcomes and awareness of unawareness, more information is *ex ante* always valuable, though the lack of knowledge of some outcomes may reduce its value compared to a scenario with full awareness.

The present paper also contributes to a growing literature that views entrepreneurship as experimentation. Kerr *et al.* (2014) highlight how, especially in certain industries, entrepreneurs can engage in cheap experimentations that reveal information about their projects' prospects, creating a real option. Dillon and Stanton (2017) build and estimate a semi-structural model of lifecycle choices and show that the option value of returning to paid work increases substantially the monetary value of entrepreneurship. Similar conclusions are reached also by Manso (2016), who shows, for example, that failed entrepreneurs are not punished when they return to the salaried workforce. Using a case study from Canada, Gottlieb *et al.* (2016) show how extended job-protected leaves can promote entrepreneurship by giving entrepreneurs the ability to test an idea's viability without the risk of long-term negative career consequences. Despite the fact that unforeseen outcomes are common in entrepreneurship, none of the above papers analyzes the impact of bounded awareness on experimentation decisions.

The remainder of the paper is organized as follows. Section 2.2 presents the model. Section 2.3 analyzes the impact of different attitudes towards unawareness and levels of awareness on agents' occupational choices and experimentation decisions. Section 2.4 provides the empirical predictions of

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the model. Section 2.5 concludes. The proofs are collected in Appendix A.

2.2 The model

An agent lives for $T+1$ periods, $t \in \{0, 1, ..., T\}$. He starts period 0 with wealth $W_0 \geq 0$ and an innovative idea/project, and in each period he is endowed with one unit of effort. The innovative project has to progress through $K \leq T$ intermediate stages before delivering a stochastic outcome $y \in Y$ one period after the K-th stage has been reached. In order for the project to progress by one stage the agent needs to exert one unit of effort and pay a fixed cost $c > 0$. In each period t the agent has to decide whether to initiate/advance his project or work as a wage worker, exerting one unit of effort and receiving a fixed wage w.

The agent cannot borrow, but in each period in which he works as a wage worker he has access to a storage technology that allows him to increase his next period wealth by at most w.[3] At the end of each period in which he works, after storage/consumption decisions have been made, the wealth is subject to a random iid shock ϵ with distribution $F(\cdot)$.[4]

As for the innovative project, after reaching each of its intermediate stages, the agent can use an "experimentation technology" that requires one unit of effort, and therefore delays the completion of the project by one period, and that at the end of the period delivers a signal $s \in S$ about the project's final outcome y. Such an experimentation technology may include, for example, analyzing the performance of a prototype, conducting a market survey, or running clinical trials. I assume that the cost of using such an experimentation technology is in terms of effort, but my results could be obtained assuming that this technology entails also a sufficiently small monetary cost.[5] Finally, I assume that if the agent switches to wage work before the completion of the project, he will not be able to restart it. This may reflect, for instance, the fact that once an individual undertakes the project, some information about it is revealed to other agents, who may then be able to complete it first and reap its benefits if the former agent decides to temporarily abandon it.

The set of the project's outcomes Y is finite. Specifically, I take $Y :=$

[3]Evans and Jovanovic (1989) document the importance of liquidity constraints for entrepreneurs.

[4]I assume $W_0 + T \min \epsilon \geq 0$, so that wealth never becomes negative during the life of the agent.

[5]Kerr *et al.* (2014) document a recent trend towards lower costs of experimentation, especially in certain industries, allowing entrepreneurs to test their ideas more efficiently.

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$\{y_1, y_2, ..., y_N\} \subset \mathbb{R}$, with $y_i < y_{i+1}$, $i \in \{1, ..., N-1\}$, and for simplicity I assume that in period 0 all outcomes are *ex ante* equally likely. Being the project innovative, the agent is assumed to be boundedly aware of its possible outcomes. Specifically, he is aware of the subset of outcomes $\hat{Y} \subset Y$. He is, however, aware of his unawareness. This means that he assigns a positive time-0 prior probability, and a utility, to outcome $x := \neg\hat{Y}$ (i.e., to any outcome different from those he is aware of). Formally, each choice to undertake/advance the project with or without using the experimentation technology can be represented as a choice between acts in the state space used in Karni and Vierø (2017) (see the Appendix for details). Their subjective expected utility representation consists of the decision maker's beliefs $\pi(\cdot)$, a Bernoulli utility function $u(\cdot)$ over the known outcomes, and a parameter $\bar{u}(x)$, so that the agent's expected utility from the project's final outcome takes the form

$$\mathbb{E}(u(y)) = \sum_{\hat{y} \in \hat{Y}} \pi(\hat{y})u(\hat{y}) + \pi(x)\bar{u}(x).$$

The parameter $\bar{u}(x)$ captures the individual's attitude towards unawareness, with agents exhibiting more excitement/less fear towards the unknown having higher values of $\bar{u}(x)$. To abstract from risk aversion considerations and highlight the impact of bounded awareness in the neatest possible way, throughout the paper I consider the case $u(\hat{y}) = \hat{y}$ for all $\hat{y} \in \hat{Y}$. Note, however, that while such an assumption simplifies the analysis of wealth accumulation decisions, my main findings do not rely on it. In particular, the results on the relationship between the value of information and the agent's awareness level and attitude towards unawareness hold for any increasing $u(\cdot)$.

Turning to the experimentation technology, I assume that it can yield three possible signals: an uninformative signal, s_N, whose objective conditional probabilities are $Pr(s_N \mid y_i) = Pr(s_N \mid y_j) = \pi_N$ for all $y_i, y_j \in Y$, a good signal, s_G, or a bad signal, s_B. The good and bad signals satisfy the monotone likelihood ratio property, which in this case is equivalent to $Pr(s_G \mid y_i) \leq Pr(s_G \mid y_{i+1})$, $i \in \{1, ..., N-1\}$, and $Pr(s_B \mid y_i) \geq Pr(s_B \mid y_{i+1})$. I further assume that $\pi_N \in (0, 1)$ until an informative signal (i.e., s_B or s_G) is received for the first time, while $\pi_N = 1$ in all subsequent periods. The idea underlying the latter assumption is that the agent must decide how many times to use the costly experimentation technology in the hope of getting an informative signal (e.g., how much time he is willing to spend on a prototype before it reveals whether it is worth completing or abandoning the project). Once such an informative signal is received, no further information can be gathered before the end of the project (e.g., because the prototype

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focused only on certain aspects of the project, while the remaining ones can be understood only once the project is completed).

As for the agent's beliefs, I assume that he knows the true (objective) time-0 prior probability $\pi_0(\hat{y})$ of each known outcome $\hat{y} \in \hat{Y}$, and hence assigns a correct time-0 prior probability $\pi_0(x) = 1 - \sum_{\hat{y} \in \hat{Y}} \pi_0(\hat{y}) > 0$ to outcome x. This means that he does not *ex ante* overestimate nor underestimate the probability of encountering unknown outcomes. The agent is aware of all possible signals and knows the true probabilities of each signal $s \in S$ conditional on any known outcome $\hat{y} \in \hat{Y}$. He updates his beliefs using Bayes' rule, where I assume the following consistency condition:

$$Pr_t(s \mid x) = Pr_t(s \mid Y \setminus \hat{Y}), \quad \forall s \in S, \tag{2.1}$$

Condition (2.1) ensures that, when using Bayes' rule, the agent holds correct posterior beliefs about the known outcomes (and hence correctly updates the overall probability of encountering unknown outcomes). This allows me to focus on a useful benchmark, where I can isolate and highlight the effects of the distortions in the portion of the posterior expected utility involving the outcomes the agent is unaware of. Note that, extending the framework of Karni and Vierø (2017), I am considering a dynamic setting in which, for a given awareness level, the agent updates his beliefs over known and unknown outcomes in response to a signal. In this case, there are a priori no restrictions on how the posterior belief about x is formed. Through condition (2.1) I impose a certain degree of rationality, by requiring the agent to form correct posterior beliefs about the outcomes he is aware of.

Given the above assumptions on beliefs and the experimentation technology, the agent maximizes the sum of discounted expected consumption levels net of effort, using a discount factor $\beta \in (0, 1)$.

2.3 Entry into entrepreneurship and experimentation

In this section, I study the agent's optimal occupational choices and experimentation decisions as a function of his attitude towards unawareness, i.e., his utility from the unknown, $\bar{u}(x)$, and his awareness level, $\hat{Y}$.

First, note that an agent who as of the beginning of period t has not yet entered entrepreneurship solves the following problem, written in recursive form:

$$V(W,t) = \max\left\{V^{entr}(W,t), \max_{cons \leq W+w} cons + \beta \mathbb{E}(V(W+w-cons+\epsilon, t+1))\right\},$$

Tesi di dottorato "Essays on Unawareness in Economics"
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where $V^{entr}(W,t)$ is the value of entering entrepreneurship in period t with wealth W, and *cons* denotes consumption. That is, at the beginning of period t the agent decides whether to enter entrepreneurship or continue to work as a wage worker. In the latter case, he chooses how much of the sum of his beginning-of-period wealth and wage to carry on to the next period.

It is immediate to see that when $t > T - K$ entering entrepreneurship is never optimal, since there are not enough periods for the project to be completed. In this case,

$$V(W,t) = W + \sum_{s=t}^{T} \beta^{s-t} w,$$

i.e., the agent consumes all the acumulated wealth in period t and never enters entrepreneurship. It is then clear that if the agent knew for sure that he would have never been able to afford the project, he would not have accumulated any wealth in the first place. A similar argument can be made if the agent considers the project to be valuable only if he can use the experimentation technology for a sufficiently large number of periods. These observations hint to the fact that when deciding how much to save while working as a wage worker, the agent weighs the benefit of being able to start a valuable project earlier against the cost of risking to wastefully accumulate wealth (i.e., accumulate wealth but subsequently be unable to start the project). As we will see, a key role in these decisions is played by the agent's attitude towards unawareness, $\bar{u}(x)$, and his awareness level, $\hat{Y}$, that affect the *ex ante* value of the project.

The agent will then enter entrepreneurship if and only if he has accumulated enough wealth to complete all the stages of the project and has enough time to experiment for a minimum number of periods that depends on his attitude towards unawareness and his level of awareness. Once the project is completed/abandoned, the agent returns to wage work and, being risk neutral and impatient, consumes all available resources in each period. We can summarize these observations in the following remark.

Remark 1 *The agent enters entrepreneurship in period t if and only if $W_t \geq Kc$ and t is below a threshold $\tilde{t}(\bar{u}(x), \hat{Y})$. Once the agent has exited entrepreneurship, he does not accumulate wealth in any period.*

2.3.1 Attitudes towards unawareness and experimentation decisions

I first study the agent's optimal experimentation decisions, i.e., whether and when he is willing to extend the duration of the project by an additional pe-

Tesi di dottorato "Essays on Unawareness in Economics"
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riod to receive a signal about its final outcome, and show how these decisions are linked to his attitude towards unawareness.

First, I consider the case of an agent whose utility from the unknown $\bar{u}(x)$ is sufficiently high, so that if he had an initial wealth W_0 of at least Kc he would be willing to undertake the project in period 0 and complete it even without the availability of the experimentation technology.[6] I call such an individual an *unawareness loving* agent. Once the project is undertaken, such an agent will be willing to exert one extra unit of effort in order to get a signal about y only if receiving the bad signal s_B induces him, at least in one stage of the project, to change his mind and abandon entrepreneurship. But receiving such a signal is more valuable at earlier stages of the project, since it allows him to save costs c and earn wage w for more periods. Furthermore, recall that if the agent continues to use the experimentation technology after having received the first informative signal, he will get s_N for sure, making such an action unprofitable. We therefore have the following result.

Proposition 1 *An unawareness loving agent that undertakes the project in period $\hat{t}$ either never uses the experimentation technology or uses it after each intermediate stage until an informative signal is received at time $t \leq \bar{t}(\bar{u}(x), \hat{t})$. In the latter case, if the agent receives signal s_B he abandons the project, while if he receives s_G he completes it without further using the experimentation technology. If no informative signal is received up to period $\bar{t}(\bar{u}(x), \hat{t})$, the agent completes the project without further using the experimentation technology.*

Next, I consider an agent whose utility from the unknown $\bar{u}(x)$ is low enough, so that in the absence of the experimentation technology he would never be willing to become an entrepreneur even if he had an initial wealth W_0 greater than or equal to Kc. I call such an individual an *unawareness averse* agent. Such an agent will be willing to undertake the project only if receiving the good signal s_G makes entrepreneurship more attractive than wage work. Once the project is undertaken, the relative value of abandoning it without having received an informative signal, compared to that of using the experimentation technology for an additional period, decreases as the project advances (because fewer costs c remain to be paid and, if the project is abandoned, wages w will be received for a smaller number of periods). Therefore, once the project is started, we may have either one of two cases, depending on the size of $\bar{u}(x)$. On the one hand, if $\bar{u}(x)$ is sufficiently low,

[6] Clearly, if no experimentation technology is available, once the project is undertaken it cannot be optimal to abandon it before completion (otherwise the agent would have been better off by not undertaking the project in the first place).

Tesi di dottorato "Essays on Unawareness in Economics"
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the agent will be interested in the good signal in every intermediate stage and will be willing to use the experimentation technology as many times as possible, given the time of entry into entrepreneurship and the resulting number of available periods. In this case, he will abandon the project in the last intermediate stage if he does not receive any informative signal. On the other hand, if $\bar{u}(x)$ is high enough, the agent will be willing to use the experimentation technology up to a stage where he is interested in the bad signal, and if he does not receive any informative signal up to that stage, he will complete the project without further using the experimentation technology (just like an unawareness loving agent). We therefore have the following result.

Proposition 2 *An unawareness averse agent that undertakes the project in period $\hat{t}$ uses the experimentation technology after each intermediate stage until an informative signal is received at time $t \leq \bar{t}(\bar{u}(x), \hat{t})$. If the agent receives signal s_B he abandons the project, while if he receives s_G he completes it without further using the experimentation technology. If no informative signal is received up to period $\bar{t}(\bar{u}(x), \hat{t})$ with $\bar{u}(x)$ low enough, the agent abandons the project. If no informative signal is received up to $\bar{t}(\bar{u}(x), \hat{t})$ with $\bar{u}(x)$ sufficiently high, the agent completes the project without further using the experimentation technology.*

To better understand the impact of the agent's awareness level and his attitude towards unawareness on his incentives to experiment, I decompose his utility from the unknown, $\bar{u}(x)$, into two components, which will be analyzed in the next subsections. The first component is the one driven exclusively by the level of unawareness, in the sense that the agent holds a correct prior evaluation of the project, while the second one reflects his optimism/pessimism towards the unknown, which biases his initial evaluation. Specifically, for a given level of awareness, $\hat{Y}$, the agent's utility from the unknown, $\bar{u}(x)$, can be decomposed as follows:

$$\bar{u}(x) = \bar{u}_C(x) + \Delta, \tag{2.2}$$

where $\bar{u}_C(x)$ denotes the value of $\bar{u}(x)$ that gives the agent the same time-0 prior expected utility from the project as that of a fully aware individual, i.e.,

$$\bar{u}_C(x) := \frac{1}{\pi_0(Y \setminus \hat{Y})} \sum_{y \in Y \setminus \hat{Y}} \pi_0(y) u(y). \tag{2.3}$$

I call an agent optimistic (resp. pessimistic) if $\Delta > 0$ (resp. $\Delta < 0$).

Tesi di dottorato "Essays on Unawareness in Economics"
di MADOTTO MATTEO
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2.3.2 Ex ante correct agents

In this subsection I study the important benchmark case of an *ex ante* correct individual, i.e., an agent whose utility from the unknown gives him the same time-0 prior expected utility from the risky project as that of a fully aware agent. Formally, I consider the case $\Delta = 0$. In this way, I am able to isolate and highlight the impact of different levels of awareness on agents' incentives to experiment, separating it from the effects of their initial bias, which will be analyzed in the next subsection.

If the agent reaches the first stage of the project at the beginning of period t, uses the experimentation technology and gets a signal $s \in S$ at the end of period t, then, by consistency condition (2.1) and the assumption on the agent's time-0 beliefs, we have that[7]

$$Pr_t(x \mid s)\bar{u}(x) = \frac{\sum_{y \in Y \setminus \hat{Y}} Pr_t(s \mid y)\pi_t(y)}{Pr_t(s)} \cdot \frac{\sum_{y \in Y \setminus \hat{Y}} \pi_t(y)u(y)}{\sum_{y \in Y \setminus \hat{Y}} \pi_t(y)}. \tag{2.4}$$

Note that, again by the consistency condition, $Pr_t(s)$ equals the true (objective) time-t unconditional probability of receiving signal s (which, as already mentioned above, implies that the agent has correct posterior beliefs about the known outcomes). Rearranging the right-hand side of (2.4), we get

$$Pr_t(x \mid s)\bar{u}(x) = \frac{\sum_{y \in Y \setminus \hat{Y}} \left(\frac{\sum_{y \in Y \setminus \hat{Y}} Pr_t(s|y)\pi_t(y)}{\sum_{y \in Y \setminus \hat{Y}} \pi_t(y)} \right) \pi_t(y)u(y)}{Pr_t(s)}. \tag{2.5}$$

From the right-hand side of (2.5) we see that when computing his posterior expected utility, the boundedly aware agent behaves as if he were weighing the utilities from each outcome in $Y \setminus \hat{Y}$ using a unique conditional probability of s, computed as an average of the true conditional probabilities. Such an averaging, combined with the monotone likelihood ratio property, implies:

$$Pr_t(x \mid s_G)\bar{u}(x) \leq \sum_{y \in Y \setminus \hat{Y}} Pr_t(y \mid s_G)u(y), \tag{2.6}$$

$$Pr_t(x \mid s_B)\bar{u}(x) \geq \sum_{y \in Y \setminus \hat{Y}} Pr_t(y \mid s_B)u(y), \tag{2.7}$$

and

$$Pr_t(x \mid s_N)\bar{u}(x) = \sum_{y \in Y \setminus \hat{Y}} Pr_t(y \mid s_N)u(y) = \pi_t(x)\bar{u}(x). \tag{2.8}$$

[7] Clearly, the results below do not depend on the time at which the agent enters entrepreneurship, since he does not get any signal before the project is undertaken.

Tesi di dottorato "Essays on Unawareness in Economics"
di MADOTTO MATTEO
discussa presso Università Commerciale Luigi Bocconi-Milano nell'anno 2020
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After the first informative signal is received, it is easily checked, using again condition (2.1), that, since $Pr(s_N \mid y) = 1$ for all $y \in Y$ in all remaining periods in which the project is in place, the expected utility of the project does not change until its completion.

From (2.6) and (2.7) we see that the presence of awareness of unawareness lessens the shifts in the expected utility induced by signals. As a result, the good signal is perceived as being less good and the bad signal as being less bad. Therefore, as long as the posterior expected utility from the good signal remains above the unconditional one, it is apparent that at any stage of the project the value of information is (weakly) lower for a boundedly aware agent. Indeed, since for such agent posterior expected utilities are less dispersed around the prior one, the benefit he gets from any signal, in terms of allowing him to choose a better action, is reduced.

Note that the averaging of conditional probabilities described above may potentially reverse the direction of the signals: the "confusion" generated by the aggregation of different outcomes within x may be so large that the good signal is mistakenly interpreted as a bad one and vice versa. This is never the case if, for example, x comprises outcomes whose utilities are all above or all below the average. More generally, the condition guaranteeing that the confusion generated is not too large (i.e., that the directions of the posterior expected utilities are preserved) can be written in the following way:

$$\frac{1}{|Y \setminus \hat{Y}|}(Pr_0(Y \setminus \hat{Y} \mid s_B) - Pr_0(Y \setminus \hat{Y} \mid s_G)) \sum_{y \in Y \setminus \hat{Y}} u(y) \leq \sum_{\hat{y} \in \hat{Y}} (Pr_0(\hat{y} \mid s_G) - Pr_0(\hat{y} \mid s_B))u(\hat{y}), \quad (2.9)$$

where $|Y \setminus \hat{Y}|$ denotes the cardinality of $Y \setminus \hat{Y}$. This condition basically states that bad (resp. good) outcomes do not gain too much weight during the averaging of the conditional probabilities of the good (resp. bad) signal due to awareness of unawareness.

The above results have been obtained comparing the case of full awareness to that of a generic level of unawareness. One may wonder whether similar results could be obtained also comparing increasing (in the sense of set inclusion) levels of unawareness. In general, this is not the case. However, there is a specific case in which greater unawareness does imply more restricted posterior expected utilities, and hence a reduced value of information. Consider two agents, agent 1 and agent 2, having the same initial wealth and both having the same time-0 prior expected utility from the project as that of a fully aware agent, but with different levels of unawareness. Specifically,

Tesi di dottorato "Essays on Unawareness in Economics"
di MADOTTO MATTEO
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they are aware of the subsets of outcomes $\hat{Y}_1 \subset Y$ and $\hat{Y}_2 \subset \hat{Y}_1$, respectively, where

$$Y \setminus \hat{Y}_2 = Y \setminus \hat{Y}_1 \cup \{y'\}, \quad \text{with} \quad y' < \min Y \setminus \hat{Y}_1 \text{ or } y' > \max Y \setminus \hat{Y}_1. \quad (2.10)$$

That is, agent 2 is unaware of some extra outcome that is more extreme compared to those agent 1 is unaware of. Letting subscripts denote the agent, it can then be shown that, for any common history of signals up to a generic time $t-1$, we have $\mathbb{E}_{2t}(u(y) \mid s_G) \leq \mathbb{E}_{1t}(u(y) \mid s_G)$, $\mathbb{E}_{2t}(u(y) \mid s_B) \geq \mathbb{E}_{1t}(u(y) \mid s_B)$, and $\mathbb{E}_{it}(u(y) \mid s_N) = \mathbb{E}_{it}(u(y))$, $i \in \{1,2\}$. The following result therefore holds.

Proposition 3 *Under condition (2.9), at any stage of the project the value of information is decreasing in the agent's unawareness level, in the sense of (2.10).*

We thus see that the agents who deem experimentation least valuable are those who are unaware of the most favorable outcomes and/or the most severe issues of their projects. The reason underlying agent 2's lower value of information is the following. From the right-hand side of (2.5) we see that when computing posterior expectations, the more unaware agent behaves as if he were weighing the utilities from a larger set of outcomes using a unique conditional probability of observing the signal, equal to an average of the true conditional probabilities. This induces less dispersed posterior expected utilities if the additional outcome agent 2 is unaware of is more extreme (i.e., larger or smaller than all the outcomes agent 1 is unaware of). Indeed, in this case agent 2 uses the unique (average) conditional probability also to weigh an outcome that, if known, would have greatly affected his posterior expected utility, given that both the conditional probabilities and the utility of such outcome are more extreme than those of the outcomes in $Y \setminus \hat{Y}_1$.

The above result does not hold if y' is any outcome in $\hat{Y}_1$. Indeed, we may have situations in which, for example, y' has a utility that is lower than the average utility over the outcomes in $Y \setminus \hat{Y}_1$ but a conditional probability of signal s_G that is greater than the average over the same set. In this case, it can be checked that after observing s_G agent 2 has a higher posterior expected utility than that of agent 1.

The reduced value of information implied by a greater level of unawareness decreases the benefit the agent can get by using the experimentation technology. As a consequence, a more unaware agent (in the sense of (2.10)) will be willing to store less wealth, and will enter entrepreneurship later or will prefer not to enter it at all. Moreover, once the project is started, he will use the experimentation technology for a smaller number of periods, both

Tesi di dottorato "Essays on Unawareness in Economics"
di MADOTTO MATTEO
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because of his reduced value of information and because of the lower number of available periods implied by his later entry. Formally, the impact of awareness of unawareness on career paths and experimentation decisions can be summarized in the following corollary of Proposition 3.

Corollary 1 *Under condition (2.9), for any common history of wealth shocks a more unaware agent (in the sense of (2.10)) never becomes an entrepreneur if a less unaware agent with the same initial wealth and the same time-0 prior expected utility from the project prefers to work as a wage worker in all periods. If the more unaware agent becomes an entrepreneur, then, compared to the less unaware agent, he enters entrepreneurship later and is willing to experiment for a smaller number of periods.*

Unawareness itself may therefore discourage an agent from ever entering entrepreneurship or, on the other hand, it may induce him to enter later, and experiment less before deciding whether to complete the project or abandon it.

2.3.3 Optimistic and pessimistic agents

In the previous subsection we have seen that if the agent has an *ex ante* correct evaluation of the project, awareness of unawareness implies that signals are less able to affect the perceived project's quality. Such a reduction in the signals' strength may be compensated or reinforced by the agent's initial optimism/pessimism about the unknown. To see this, recall the decomposition in (2.2). In contrast to the case of a correct prior evaluation of the project and differences in the awareness level, differences in the agents' initial bias Δ make both posterior expected utilities (and hence also the prior one) move in the same direction. This may have different implications in terms of the value of information depending on the sign and size of Δ. For example, it is quite straightforward to note that for an agent who would be interested in the bad signal when $\Delta = 0$, being optimistic implies a reduced value of information (because the bad signal is perceived as being less bad). By contrast, for such agent a moderate degree of pessimism (not strong enough to reduce the prior expected utility to the point where the decision maker becomes interested in the good signal) increases his value of information. If instead the pessimism is so strong that the agent is interested in the good signal, the relationship between Δ and the value of information is reversed: since now the agent is interested in s_G, a greater value of Δ will make him more willing to experiment. We can formalize these observations in the following proposition, relating the agent's *ex ante* bias to his value of information.

Tesi di dottorato "Essays on Unawareness in Economics"
di MADOTTO MATTEO
discussa presso Università Commerciale Luigi Bocconi-Milano nell'anno 2020
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Sono comunque fatti salvi i diritti dell'università Commerciale Luigi Bocconi di riproduzione per scopi di ricerca e didattici, con citazione della fonte.

Proposition 4 *For a given value of $\bar{u}_C(x)$ and a given stage of the project, there exists a value $\bar{\Delta}$ of the bias Δ such that the value of information is increasing in Δ over the interval $(-\infty, \bar{\Delta}]$ and decreasing over the interval $[\bar{\Delta}, +\infty)$. The threshold $\bar{\Delta}$ is decreasing in $\bar{u}_C(x)$.*

There is a debate in the economics literature as to why many entrepreneurs seem to enter and persist in entrepreneurship despite earning low risk-adjusted returns. Possible explanations that have been proposed include differences in risk attitudes, biased beliefs under full awareness of outcomes, and non-pecuniary benefits, but the existing empirical evidence has not yet provided conclusive answers (for a review, see Astebro *et al.*, 2014). The results in the last two subsections seem to suggest that individuals who enter and persist in entrepreneurship are boundedly aware individuals with a positive attitude towards the unknown. Specifically, their attitude leads them to engage in endeavors whose possible final outcomes are known only imperfectly, while the reduced value of information implied by the awareness of their ignorance induces them to persist in pursuing their ideas despite the availability of signals that would discourage a fully aware agent.

2.4 Empirical implications

In the previous section I mainly focused on the impact of bounded awareness on agents' incentives to experiment. In this section I instead study how agents' attitudes towards unawareness affect the interaction between wealth accumulation decisions and incentives to experiment, and how this generates some typical associations among certain characteristics of entrepreneurship. I then compare such associations to those we would obtain in a scenario where agents are fully aware and their decisions to enter entrepreneurship are driven by non-pecuniary benefits.

An agent with a higher value of $\bar{u}(x)$ stores more wealth in each period in which he works as a wage worker, since the project is more valuable to him for any number of experiments. As a consequence, for an unawareness averse agent that at any intermediate stage of the project is interested in the good signal, the higher is $\bar{u}(x)$ the earlier he undertakes the project and the larger is the number of stages in which he is able to experiment. As a result, an agent of such type with a higher value of $\bar{u}(x)$ is less likely to abandon the project and, conditional on completing it, spends more time in entrepreneurship. These observations can be summarized in the following proposition describing the qualitative relationships between the agent's attitude towards unawareness and three empirically observable characteristics of

Tesi di dottorato "Essays on Unawareness in Economics"
di MADOTTO MATTEO
discussa presso Università Commerciale Luigi Bocconi-Milano nell'anno 2020
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entrepreneurship: the time of entry into entrepreneurship, the probability of abandoning the project, and the average duration of the project conditional on completing it.

Proposition 5 *There exists an interval* $(-\infty, \bar{u}_G(x)]$ *of values of* $\bar{u}(x)$ *over which for any given history of wealth shocks (i) the time of entry into entrepreneurship is decreasing in* $\bar{u}(x)$*, (ii) the probability of abandoning the project is decreasing in* $\bar{u}(x)$*, and (iii) conditional on completing the project, the average time spent on the project is increasing in* $\bar{u}(x)$*. The threshold* $\bar{u}_G(x)$ *is increasing in the cost per stage* c*.*

The threshold of the interval over which the above relationships hold is increasing in c because a higher cost per stage implies that agents with higher values of $\bar{u}(x)$ are interested in the good signal at all intermediate stages of the project.

Next, consider unawareness loving agents and unawareness averse agents who are willing to experiment up to a stage in which they are interested in the bad signal. Conditional on undertaking the project, for such agents a lower value of $\bar{u}(x)$ implies a positive net value of information for a larger number of stages, where the time-t net value of information is defined as the difference between the time-t prior expected continuation value if the agent exerts an extra unit of effort in order to receive a signal at time t (net of the effort cost) and the time-t prior expected continuation value if he does not exert extra effort. This is because, once the project is undertaken, signal s_B allows agents to avoid advancing bad projects, and the value of such change of plans is higher for an agent with a lower value of $\bar{u}(x)$. It follows that an individual that is more fearful towards the unknown will be willing to use the experimentation technology for a larger number of consecutive stages. Thus, such individual will be more likely to receive a bad signal leading him to abandon the project, and he will complete the project in a larger number of periods. However, as noted earlier, an agent with a lower value of $\bar{u}(x)$ will store less wealth in each period in which he works as a wage worker. Therefore, if we compare two agents of this group of individuals, for any common history of wealth shocks the one with the lower value of $\bar{u}(x)$ will start the project later and, as noted above, will be willing to experiment for a larger number of stages. Whether or not such agent is actually able to experiment for a larger number of stages, however, depends on whether or not there are enough periods left. This is the case when W_0 is high enough or the cost of advancing the project c is sufficiently low, so that the agent with the lower value of $\bar{u}(x)$ is able to undertake the project early enough.

The above observations lead to the following counterpart to Proposition 5.

Tesi di dottorato "Essays on Unawareness in Economics"
di MADOTTO MATTEO
discussa presso Università Commerciale Luigi Bocconi-Milano nell'anno 2020
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Proposition 6 *There exists an interval* $[\bar{u}_B(x), +\infty)$ *of values of* $\bar{u}(x)$*, with* $\bar{u}_B(x) \geq \bar{u}_G(x)$*, over which for any history of wealth shocks (i) the time of entry into entrepreneurship is decreasing in* $\bar{u}(x)$*, (ii) the probability of abandoning the project is decreasing in* $\bar{u}(x)$*, and (iii) conditional on completing the project, the average time spent on the project is decreasing in* $\bar{u}(x)$*. The threshold* $\bar{u}_B(x)$ *is decreasing in the initial wealth* W_0 *and increasing in the cost* c*.*

Comparing Propositions 5 and 6 we can see the main difference between individuals with a positive and a negative attitude towards the unknown. Specifically, the agents in the former group who enter entrepreneurship earlier and are less likely to abandon their project complete it faster, since they experiment out of their concern for bad signals. By contrast, the agents in the latter group experiment in the hope of getting a good signal, and hence those who enter entrepreneurship earlier and are less likely to abandon their project are also more willing to experiment.

Propositions 5 and 6 show how for sufficiently negative/positive attitudes towards unawareness the model predicts a clear-cut relationship between the three characteristics of entrepreneurship, which does not depend on the specific history of wealth shocks considered. In the interval $[\bar{u}_G(x), \bar{u}_B(x)]$, which may well be empty (e.g., if the initial wealth is high enough), the relationship between the three empirical variables is instead history-dependent. As mentioned above, the reason is that in this interval an agent who enters entrepreneurship later is willing to experiment more, but may be able to do so only along certain histories of wealth shocks.

As an empirical consequence of the above results, one may try to infer agents' attitudes towards the unknown from the observed association among the aforementioned characteristics of entrepreneurship. Moreover, knowing individuals' awareness levels, one may also evaluate agents' initial bias, i.e., whether they are overoptimistic/overpessimistic.

2.4.1 An alternative explanation

In this subsection, I consider an alternative explanation given in the literature for why individuals enter and persist in entrepreneurship (e.g., Astebro *et al.*, 2014), namely non-pecuniary benefits.

Non-pecuniary benefits refer to the immaterial gains the agent obtains from working as an entrepreneur, such as being his own boss or achieving personal goals. I model such benefits as a utility gain $\lambda > 0$ the agent gets whenever he completes an intermediate stage of the project, and consider the case where individuals are fully aware, i.e. where $\hat{Y} = Y$.

Tesi di dottorato "Essays on Unawareness in Economics"
di MADOTTO MATTEO
discussa presso Università Commerciale Luigi Bocconi-Milano nell'anno 2020
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If λ is low enough the agent behaves in a way analogous to that under different attitudes towards unawareness: greater values of λ imply higher continuation values conditional on any signal, experimentation takes place from early periods, and the agent never persists in entrepreneurship after receiving the bad signal. In contrast, if λ is sufficiently high the individual is willing to reach the last intermediate stage of the project irrespective of the signal received, prefers to use the experimentation technology in later stages of the project so as to postpone as few non-pecuniary benefits as possible, and agents with higher values of λ are willing to experiment less. This creates a new possibility that was not present under different attitudes towards unawareness. Indeed, if the unconditional expected utility of the project (net of the non-pecuniary benefits) is low enough, so that the individual who experiments is interested in the good signal, then an agent with a higher value of λ will enter entrepreneurship earlier, experiment less, and be more likely to abandon the project. More specifically, we have the following result.

Proposition 7 *For low enough $\mathbb{E}_0(u(y))$, there exists an interval of non-pecuniary benefits $[\underline{\lambda}, +\infty)$ over which for any given history of wealth shocks (i) the time of entry into entrepreneurship is decreasing in λ, (ii) the probability of abandoning the project is increasing in λ, and (iii) conditional on completing the project, the average time spent on the project is decreasing in λ.*

Comparing Proposition 7 to Propositions 5 and 6 we see that, in contrast to the case of different attitudes towards the unknown, those who enter entrepreneurship earlier are more likely to eventually abandon their project. This occurs because such individuals do not expect a higher final outcome, but rather enjoy entrepreneurship more.

The type of relationship among the three empirical variables in Proposition 7 could not occur for any history of wealth shocks under different attitudes towards unawareness. Indeed, in such model if we compare two agents, the one with the greater value of $\bar{u}(x)$ will enter entrepreneurship earlier, and the only case where he is also more likely to abandon the project is when both individuals are interested in the bad signal but the more fearful agent can experiment less because of his later entry into entrepreneurship. But in such case the project completed by the more fearful agent will have a lower average duration. Since the association described in the last proposition is peculiar to non-pecuniary benefits, observing it would provide unambiguous evidence that entrepreneurship is driven by such benefits.

Tesi di dottorato "Essays on Unawareness in Economics"
di MADOTTO MATTEO
discussa presso Università Commerciale Luigi Bocconi-Milano nell'anno 2020
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2.5 Conclusion

In this paper, I analyze the problem of an individual deciding whether to undertake an innovative project, such as founding a startup or entering entrepreneurship more broadly. The agent is not aware of all the possible outcomes of the project, but is aware of his bounded awareness. Conditional on undertaking the project, the agent can use a costly experimentation technology providing him with signals about the project's final outcome. I show that, under a natural consistency condition on agents' beliefs, awareness of unawareness implies a reduced value of information, that further discourages pessimistic agents from entering entrepreneurship and induces optimistic ones to experiment less and persist more. I also provide a sufficient condition under which the value of information is monotonically decreasing in the agent's unawareness. This may help shed some light on the debate in the economics literature as to why many entrepreneurs seem to enter and persist in entrepreneurship despite earning low risk-adjusted returns. Indeed, my results seem to suggest that individuals who enter and persist in entrepreneurship are boundedly aware individuals with a positive attitude towards the unknown. Specifically, their attitude leads them to engage in endeavors whose possible final outcomes are known only imperfectly, while the reduced value of information implied by the awareness of their ignorance induces them to persist in pursuing their ideas despite the availability of signals that would discourage a fully aware individual.

I also provide empirical predictions about the association among some characteristics of entrepreneurship (time of entry, probability of abandoning the project, and average duration of completed projects) as a function of agents' attitudes towards unawareness and the cost of their projects. As an empirical consequence of such results, one may try to infer agents' attitudes towards the unknown from the observed association among the aforementioned characteristics of entrepreneurship. In addition, knowing individuals' awareness levels, one may also evaluate agents' initial bias. Finally, I show that a model where entrepreneurship is driven by non-pecuniary benefits would generate distinctive predictions, which could therefore be used to test the causes of entrepreneurship.

Tesi di dottorato "Essays on Unawareness in Economics"
di MADOTTO MATTEO
discussa presso Università Commerciale Luigi Bocconi-Milano nell'anno 2020
La tesi è tutelata dalla normativa sul diritto d'autore (Legge 22 aprile 1941, n.633 e successive integrazioni e modifiche).
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Appendix A

Awareness of unawareness in the state space of Karni and Vierø (2017)

The model in this paper exploits Karni and Vierø's (2017) subjective expected utility representation of the preferences of a decision maker who is aware of his bounded awareness of outcomes. Formally, Karni and Vierø consider a finite set of "basic actions" A and a finite set of "consequences" (outcomes in the terminology of the present paper) $\hat{Y}$ the agent is aware of. They define $x := \neg\hat{Y}$ as the "none of the above" consequence (i.e., all outcomes the decision maker is unaware of) and let $Y := \hat{Y} \cup \{x\}$. They then construct the state space Ω from the set of basic actions and the set of consequences Y. Namely, they let $\Omega := Y^A = \{\omega : A \to Y\}$, and consider the set of acts F defined as

$$F := \{f : \Omega \to Y \mid f^{-1}(x) \subseteq \Omega \setminus \hat{Y}^A\}.$$

The authors provide a subjective expected utility representation of preferences over lotteries on such acts. As mentioned in the main body of the present paper, such representation consists of the decision maker's beliefs over Ω, a Bernoulli utility function $u(\cdot)$ over the known outcomes in $\hat{Y}$ (which in the present paper has been assumed to be the identity function), and the utility of x (i.e., a parameter), $\bar{u}(x)$, representing the agent's attitude towards unawareness.

Deterministic outcomes, such as earning w when working as a wage worker, can then be viewed as constant acts, i.e., acts assigning the same outcome to all states of the world. As for the experimentation technology, it simply allows the agent to update his beliefs about the states, keeping the level of awareness fixed.

Proof of Proposition 1

Recall that the agent can receive at most one informative signal. It is then clear that if the agent receives the good signal he will complete the project without further using the experimentation technology, while he will respond to the bad signal by abandoning the project and starting to work as a wage worker (otherwise he would have been better off by not using the experimentation technology). Note also that for an unawareness loving agent it cannot be optimal to abandon the project after having received an uninformative signal, since the costs that remain to be paid and the number of periods in which the alternative wage can be received decrease after each intermediate

Tesi di dottorato "Essays on Unawareness in Economics"
di MADOTTO MATTEO
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stage of the project. I now show that it is never optimal to use the experimentation technology after a period in which it has not been used. To see this, assume that the agent has accumulated wealth fast enough so that he can use the experimentation technology $n > 1$ times. Suppose the agent has used the technology without receiving any informative signal after each of the first $n-1$ intermediate stages, and that he decides not to use the technology after stage n, before using it again after stage $n+1$ (the same result could be established if the agent decides not to use the technology after having received any $m < n$ uninformative signals). The expected continuation value from the period after the experimentation technology has been used for the $n-1$-th time to the last period of the project for which the technology has been used n times is given by

$$\begin{aligned} V =& Pr_0(s_G)\beta^{K-n+2}\mathbb{E}(u(y) \mid s_G)+ \\ &+ Pr_0(s_B)(\beta^3(K-n-1)c + \sum_{t=1}^{K-n}\beta^{t+2}w) + Pr_0(s_N)\beta^{K-n+2}\mathbb{E}(u(y)). \end{aligned}$$

But the agent would have been better off if he had used the experimentation technology after stage n, since in the case of a bad signal he would have earned wage w for an additional period and would have saved an additional cost c. Indeed, in this case the expected continuation value between the same periods as before is

$$\begin{aligned} & Pr_0(s_G)\beta^{K-n+2}\mathbb{E}(u(y) \mid s_G))+ \\ &+ Pr_0(s_B)(\beta^2(K-n)c + \sum_{t=0}^{K-n}\beta^{t+2}w) + Pr_0(s_N)\beta^{K-n+2}\mathbb{E}(u(y)) > V, \end{aligned}$$

while the continuation value in the remaining periods is the same.

Proof of Proposition 2

As we have seen in the proof of Proposition 1, an agent is willing to use the experimentation technology only if receiving the good signal induces him to complete the project (without further using the experimentation technology) and receiving the bad signal induces him to abandon it and start working as a wage worker. Then, using the same arguments as in the proof of Proposition 1, it is easily shown that if the unawareness averse agent undertakes the project, he will be willing to use the experimentation technology after each of a certain number of consecutive stages until an informative signal is received.

Recall that the costs that remain to be paid and the number of periods in which the alternative wage can be received decrease after each intermediate

Tesi di dottorato "Essays on Unawareness in Economics"
di MADOTTO MATTEO
discussa presso Università Commerciale Luigi Bocconi-Milano nell'anno 2020
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stage of the project. Therefore, depending on the value of $\bar{u}(x)$, after each stage an unawareness averse agent may be interested in the bad or in the good signal. Therefore an unawareness averse agent completes the project without further experimenting (resp. abandons it after a certain number of uninformative signals) if $\bar{u}(x)$ is sufficiently high (resp. low).

Proof of Proposition 3

Suppose that no informative signal has been received up to period $t-1$. We need to show that

$$\sum_{y \in Y \setminus \hat{Y}_2} Pr_t(y \mid s_G)u(y) - Pr_{2t}(x_2 \mid s_G)\bar{u}_2(x_2) \geq \sum_{y \in Y \setminus \hat{Y}_1} Pr_t(y \mid s_G)u(y) - Pr_{1t}(x_1 \mid s_G)\bar{u}_1(x_1), \quad (2.11)$$

$$\sum_{y \in Y \setminus \hat{Y}_2} Pr_t(y \mid s_B)u(y) - Pr_{2t}(x_2 \mid s_B)\bar{u}_2(x_2) \leq \sum_{y \in Y \setminus \hat{Y}_1} Pr_t(y \mid s_B)u(y) - Pr_{1t}(x_1 \mid s_B)\bar{u}_1(x_1), \quad (2.12)$$

and

$$\sum_{y \in Y \setminus \hat{Y}_2} Pr_t(y \mid s_N)u(y) - Pr_{2t}(x_2 \mid s_N)\bar{u}_2(x_2) = \sum_{y \in Y \setminus \hat{Y}_1} Pr_t(y \mid s_N)u(y) - Pr_{1t}(x_1 \mid s_N)\bar{u}_1(x_1) = 0. \quad (2.13)$$

Here, I will prove only (2.11), but the proofs of (2.12) and (2.13) are analogous.

Recall that at time 0 all outcomes are *ex ante* equally likely. Denote such a common probability by p, and let N_i be the cardinality of the subset $Y \setminus \hat{Y}_i$, $i \in \{1, 2\}$. Moreover, let $\bar{\pi}_{it} := \frac{\sum_{y \in Y \setminus \hat{Y}_i} Pr_t(s|y)p}{N_i p}$ denote the average of the conditional probabilities of signal s over the outcomes agent i is unaware of. We can then rewrite (2.5) as

$$Pr_{it}(x_i \mid s)\bar{u}_i(x_i) = \frac{\sum_{y \in Y \setminus \hat{Y}_i} \bar{\pi}_{it} p u(y)}{Pr_t(s)}.$$

Tesi di dottorato "Essays on Unawareness in Economics"
di MADOTTO MATTEO
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Multiplying the left-hand side of (2.11) by $Pr_t(s_G)$, we can rewrite it as

$$\sum_{y\in Y\setminus\hat{Y}_2}(Pr_t(s_G \mid y)-\bar{\pi}_{2t})pu(y)=\sum_{y\in Y\setminus\hat{Y}_1}(Pr_t(s_G \mid y)-\bar{\pi}_{1t})pu(y)+$$
$$+(Pr_t(s_G \mid y')-\bar{\pi}_{2t})pu(y')+\sum_{y\in Y\setminus\hat{Y}_1}(\bar{\pi}_{1t}-\bar{\pi}_{2t})pu(y).$$

Noting that

$$\bar{\pi}_{2t}=\frac{N_1}{N_1+1}\bar{\pi}_{1t}+\frac{1}{N_1+1}Pr_t(s_G \mid y'),$$

we obtain

$$\sum_{y\in Y\setminus\hat{Y}_2}(Pr_t(s_G \mid y)-\bar{\pi}_{2t})pu(y)-\sum_{y\in Y\setminus\hat{Y}_1}(Pr_t(s_G \mid y)-\bar{\pi}_{1t})pu(y)=$$
$$(Pr_t(s_G \mid y')-\bar{\pi}_{1t})(\frac{N_1}{N_1+1}pu(y')-\frac{1}{N_1+1}\sum_{y\in Y\setminus\hat{Y}_1}pu(y)). \quad (2.14)$$

From the assumptions on the conditional probabilities of s_G we see that the right-hand side of (2.14) is greater than or equal to zero, thus proving inequality (2.11).

Condition (2.9) ensures that the posterior expected utilities of the two agents after a good (resp. bad) signal are greater (resp. lower) than the prior expected utility. Therefore, the result proved above guarantees that both posterior expected utilities of agent 2 are closer to the prior one than those of agent 1. Noting that

$$\mathbb{E}(u_i(y))=\sum_{j\in\{G,B,N\}}Pr(s_j)\mathbb{E}(u_i(y) \mid s_j), \quad (2.15)$$

it immediately follows that the value of information is lower for agent 2.

Proof of Corollary 1

Consider two agents, agent 1 and agent 2, with agent 2 more unaware than agent 1 in the sense of (2.10). Assume that the two agents have the same initial wealth and the same time-0 prior expected utility from the project. From Proposition 3 it follows that under condition (2.9) for any given wealth level W and time t the value of entering entrepreneurship for agent 1, $V_1^{entr}(W,t)$, is greater than or equal to that for agent 2, $V_2^{entr}(W,t)$.

Note that since decision makers are risk neutral and impatient, if agent 2 has not started the project by the end of period $t-1$ and $V_2^{entr}(W,t)$ is such

Tesi di dottorato "Essays on Unawareness in Economics"
di MADOTTO MATTEO
discussa presso Università Commerciale Luigi Bocconi-Milano nell'anno 2020
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that he wishes to enter entrepreneurship in t, then if agent 1 has not started the project by the end of $t-1$ he will also prefer to enter entrepreneurship in t for any $W' \geq W$.

Next, note that for any given common history of wealth shocks and as long as none of the two agents has entered entrepreneurship, at the beginning of every period agent 1 holds a (weakly) greater amount of wealth than agent 2. To see this, simply observe that since $V_1^{entr}(W,t) \geq V_2^{entr}(W,t)\ \forall W, t$, the increase in the overall value function computed in t following any increase in W_t is greater for agent 1.

The observations in the last two paragraphs together imply that agent 2 will never wish to start the project if agent 1 prefers to work as a wage worker in all periods. Moreover, if agent 2 starts the project, he will do it later than agent 1. Finally, given the lower perceived value of the project for any given number of experiments and the smaller number of periods available for experimentation, agent 2 will be willing to experiment for a smaller number of stages.

Proof of Proposition 4

Suppose the agent has reached the beginning of stage j, $j \in \{1, ..., K\}$, without receiving any informative signal. Define $\bar{\Delta}$ as the value of Δ such that the agent is indifferent between completing the project without further using the experimentation technology and switching to wage work, i.e. such that

$$\beta^{K-j+1}\mathbb{E}_0(u(y)) = (K-j+1)c + \sum_{t=0}^{K-j+1} \beta^t w.$$

Note from (2.2) that $\bar{\Delta}$ is decreasing in $\bar{u}_C(x)$. If $\Delta > \bar{\Delta}$ (resp. $\Delta < \bar{\Delta}$) the agent is interested in s_B (resp. s_G) and therefore, recalling (2.15), the value of information is decreasing (resp. increasing) in Δ.

Proof of Proposition 5

Take $\bar{u}_G(x)$ such that $c + w + \beta w = \beta\mathbb{E}_0(u(y))$ (clearly $\bar{u}_G(x)$ is increasing in c). For values of $\bar{u}(x)$ below $\bar{u}_G(x)$ the agent who experiments is interested in s_G at any stage of the project.

To prove (i), note that for any given number of experiments the project is more valuable for an agent with a higher value of $\bar{u}(x)$, and therefore fore any given wealth level W and time t the value of entering entrepreneurship, $V^{entr}(W,t)$, is higher for such agent. Hence, following the same steps as in the proof of Corollary 1, we have that for any common history of wealth

Tesi di dottorato "Essays on Unawareness in Economics"
di MADOTTO MATTEO
discussa presso Università Commerciale Luigi Bocconi-Milano nell'anno 2020
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shocks such agent will hold a larger amount of wealth in any period in which he works and will therefore enter entrepreneurship earlier.

To prove (ii), note that, as seen in (i), an agent with a greater value of $\bar{u}(x)$ will start the project earlier and hence will have more periods available for experimenting. This, together with the fact that a higher value of $\bar{u}(x)$ makes signal s_G more valuable, implies that an individual with a higher $\bar{u}(x)$ will need a larger number of uninformative signals before abandoning the project.

(iii) follows immediately from the facts that agents with $\bar{u}(x) \leq \bar{u}_G(x)$ complete the project if and only if they receive the good signal and that, as seen in (ii), an agent with a higher $\bar{u}(x)$ is willing to experiment for a larger number of periods.

Proof of Proposition 6

Let $\bar{u}_B(x)$ be the minimum value of $\bar{u}(x)$ such that (a) $c+w+\beta w \leq \beta \mathbb{E}_0(u(y))$, and (b) an agent with $\bar{u}(x) = \bar{u}_B(x)$ who enters entrepreneurship has always the possibility to experiment K times. Note that requirement (b) implies that $\bar{u}_B(x)$ is decreasing in the initial wealth W_0, while both (a) and (b) imply that $\bar{u}_B(x)$ is increasing in c.

(i) is proved with the exact same argument as in Proposition 5.

To prove (ii), note that an agent with a lower value of $\bar{u}(x)$ is willing to experiment for a larger number of stages before completing the project, and hence is more likely to receive signal s_B.

(iii) follows immediately from the fact that agents with $\bar{u}(x) \geq \bar{u}_B(x)$ complete the project after a good signal or a number of consecutive uninformative signals that is decreasing in $\bar{u}(x)$.

Proof of Proposition 7

Suppose $\mathbb{E}_0(u(y))$ is low enough so that $c + w + \beta w \geq \lambda + \beta \mathbb{E}_0(u(y))$. Let $\underline{\lambda}$ be such that

$$\sum_{t=0}^{K-n-1} \beta^t \underline{\lambda} \geq \sum_{t=0}^{K-n-1} \beta^t w + (K-n+1)c, \quad \forall n \in \{1, ..., K-1\},$$

so that the agent is always willing to reach the last intermediate stage of the project. For agents with $\lambda \geq \underline{\lambda}$ experimentation takes place in the last stages of the project, so as to postpone as few non-pecuniary benefits as possible.

(i) is proved in the exact same way as in Proposition 5 replacing $\bar{u}(x)$ with λ.

Tesi di dottorato "Essays on Unawareness in Economics"
di MADOTTO MATTEO
discussa presso Università Commerciale Luigi Bocconi-Milano nell'anno 2020
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To prove (ii), simply note that an agent with a higher value of λ is willing to experiment for a smaller number of periods, since postponing nonpecuniary benefits is more costly for him, and hence is less likely to get a good signal.

(iii) follows from the fact that such agents complete the project only after receiving s_G, and an individual with a higher value of λ is willing to experiment for a smaller number of stages.

Tesi di dottorato "Essays on Unawareness in Economics"
di MADOTTO MATTEO
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Bibliography

[1] Astebro, T., H. Herz, R. Nanda, and R. A. Weber (2014) "Seeking the Roots of Entrepreneurship: Insights from Behavioral Economics." *Journal of Economic Perspectives* 28(3):49-70.

[2] Auster, S. (2013) "Asymmetric Awareness and Moral Hazard." *Games and Economic Behavior* 82:503-521.

[3] Dillon, E. W., and C. T. Stanton (2017) "Self-Employment Dynamics and the Returns to Entrepreneurship." NBER Working Paper No. 23168.

[4] Evans, D. S., and B. Jovanovic (1989) "An Estimated Model of Entrepreneurial Choice Under Liquidity Constraints." *Journal of Political Economy* 97(4):808-827.

[5] Filiz-Ozbay, E. (2012) "Incorporating Unawareness into Contract Theory." *Games and Economic Behavior* 76:181-194.

[6] Galanis, S. (2015) "The Value of Information Under Unawareness." *Journal of Economic Theory* 157:384-396.

[7] Gottlieb, J. D., R. R. Townsend, and T. Xu (2016) "Experimenting with Entrepreneurship: The Effect of Job-Protected Leave." NBER Working Paper No. 22446.

[8] Grant, S., J. J. Kline, and J. Quiggin (2012) "Differential Awareness, Ambiguity, and Incomplete Contracts: A Model of Contractual Disputes." *Journal of Economic Behavior & Organization* 82:494-504.

[9] Grant, S., and J. Quiggin (2015) "A Preference Model for Choice Subject to Surprise." *Theory and Decision* 79:167-180.

[10] Karni, E., and M. L. Vierø (2017) "Awareness of Unawareness: A Theory of Decision Making in the Face of Ignorance." *Journal of Economic Theory* 168:301-328.

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[11] Kerr, W. R., R. Nanda, and M. Rhodes-Kropf (2014) "Entrepreneurship as Experimentation." *Journal of Economic Perspectives* 28(3):25-48.

[12] Manso, G. (2016) "Experimentation and the Returns to Entrepreneurship." *Review of Financial Studies* 29(9):2319-2340.

[13] Rosenberg, N. (1996) "Uncertainty and Technology Change". In: Landau, R., Taylor, T., Wright, G. (Eds.), *The Mosaic of Economic Growth.* Stanford University Press, Stanford, CA, pp. 334-356.

[14] Schipper, B. C., and H. Y. Woo (2016) "Political Awareness, Microtargeting of Voters, and Negative Electoral Campaigning." University of California, Davis.

[15] Sommer, S. C., C. H. Loch, and J. Dong (2009) "Managing Complexity and Unforeseeable Uncertainty in Startup Companies: An Empirical Study." *Organization Science* 20(1):118-133.

[16] Thomke, S. H. (2008) "Learning by Experimentation: Prototyping and Testing." In C. H. Loch, and S. Kavadias (Eds.) *Handbook of New Product Development Management*, Elsevier.

[17] Von Thadden, E. L., and X. Zhao (2012) "Incentives for Unaware Agents." *Review of Economic Studies* 79:1151-1174.

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Chapter 3

Learning Under Awareness of Unawareness

3.1 Introduction

A key step in the development of new drugs and medical devices is the use of clinical trials to gather evidence on the efficacy and safety of such products for human subjects. These studies are widely used in several countries. Clinical Trials.gov, the largest online registry for clinical trials available from 2000, in August 2016 contained data on around 180,000 publicly and privately funded trials conducted in 192 countries. The EU Clinical Trials Register in August 2016 displayed information on the protocols and results of around 29,000 clinical trials conducted from 2004 mostly in the European Union. Clinical trials are funded mostly by pharmaceutical companies, academic medical centers, voluntary groups, and national agencies. For example, in the United States the annual support level of the National Institutes of Health (NIH) for clinical trials and supportive activities ranged from 3,136 to 3,221 million dollars in the period 2012-2015.

Given that the treatments tested in clinical trials are new and mostly unexplored, many of their beneficial and adverse effects are unanticipated by the researchers. Such effects may occur during different phases of the trials, from the early stages, where small groups of healthy subjects are usually used, to more advanced phases, where large numbers of heterogeneous subjects are studied, and sometimes they are discovered only after the treatment received marketing approval. They may be more or less frequent, relatively minor or quite serious, and may be due to many different causes (e.g. unforeseen drug-drug interactions, or effects peculiar to a specific group of patients). A crucial question concerning the unanticipated effects of a new treatment is how long

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it takes before such effects are actually detected by the researchers. For example, it was only in the 1980s (more than 80 years after it was patented) that aspirin was convincingly shown to also work as an anticoagulant and help prevent heart attacks and thrombotic strokes. Delayed recognition of a side effect can also have devastating effects. An extreme case is provided by thalidomide, a sedative sold in various countries in the late 1950s. Early trials did not make its developers aware of the severe side effects it could have for pregnant women. As a result, in the late 1950s and early 1960s more than 10,000 infants around the world were born with severe malformations of the limbs, leading in many cases to death.

Intuitively, it is reasonable to expect that an unknown effect producing observations that differ substantially in nature, severity, or frequency from the expected ones (given the treatment and the characteristics of the subjects) will be quickly detected by the researcher, who may then conduct more in-depth analyses.[1] By contrast, new side effects producing observations that are similar to the expected ones may be misunderstood or simply ignored. One may wonder, however, whether a rational investigator would apply such general criteria always with the same rigor, or if instead, for example, past discoveries would affect her sensitivity towards further unexpected effects. And if the latter were true, would the size and the nature of the effects (e.g. beneficial vs. adverse ones) matter? Moreover, may the researcher's propensity to exploit new findings improve or worsen her ability to detect future new effects?

To address these issues, in this paper I develop a simple choice theoretic model of learning under limited awareness of outcomes. Specifically, I start from the state space used in Karni and Vierø (2017), and the results on the subjective expected utility representation of acts and the evolution of beliefs derived therein. I model medical treatments as acts, i.e. functions from the state space to the space of *effects*, also called *outcomes*. Outcomes are unidimensional, and they can be interpreted as overall measures of the efficacy/safety of the tested treatments. Given her prior beliefs, in each period the investigator (hereafter called the agent, or the decision maker) has to choose between alternative treatments. Treatments involving only outcomes known from the beginning of the first trial are assumed to be such that the most efficacious ones have also the most severe adverse side effects (one may think, for example, to different painkillers, such as paracetamol vs. morphine). Each of these treatments has a good known outcome in case

[1] Indeed, the definition of "unexpected adverse events" given by the Office for Human Research Protections in the United States is based on this intuitive idea. See http://www.hhs.gov/ohrp/regulations-and-policy/guidance/reviewing-unanticipated-problems/.

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of success, and a bad known outcome in case of failure. However, all these treatments are assumed to have also some common unknown effects (e.g. drug-drug interactions not yet considered, or peculiar impacts on certain groups of patients not yet treated). The agent does not know such effects, but she is aware of her ignorance in the sense that, as in Karni and Vierø (2017), she assigns a positive probability (here assumed to remain constant over time) and a utility to the event of encountering outcomes different from the known ones.

The agent does not directly observe the true effect of a tested treatment. Instead, as it is often the case in real-world trials, she observes an imprecise measure of it (e.g. because of measurement errors, or simply because the true outcome of interest, like the patient's overall well being, is difficult to measure and surrogate endpoints are needed). After observing a measurement, the agent uses a likelihood test to decide whether such measurement is the result of a known outcome plus an error, or it was generated by an effect the agent is unaware of. Then, she updates the subjective probabilities of the states using reverse Bayesianism, and optimally chooses her next period's treatment. Such an optimization is, however, constrained, since the exploitation of a new beneficial outcome, as well as the removal of a newly discovered adverse effect, entails some costs (e.g. due to the need of further studies or a change in the design of the trial).

First, I investigate how in the absence of experimentation (i.e. inclusion of newly discovered outcomes into future treatments), the awareness of new outcomes changes the state space and hence the relative convenience of the treatments whose effects are known from the beginning of the trials, and how this in turn affects future awareness. I show that the impact of discoveries on the agent's ability to learn new favorable outcomes is unambiguous. In particular, the rules of reverse Bayesianism imply that the relative likelihood of good outcomes increases after the discovery of a new beneficial effect. This in turn makes the agent more daring and willing to choose treatments involving more extreme outcomes. Both the change in the probabilities and that in the optimal treatment decrease the agent's sensitivity, as captured by the likelihood test, towards future good new outcomes. Conversely, becoming aware of a new adverse side effect makes the agent more pessimistic and willing to choose more prudent treatments, increasing her sensitivity towards future beneficial outcomes.

As for the impact of discoveries on the decision maker's ability to become aware of new adverse side effects, I show that it depends on her sensitivity towards the size of measurement errors. Indeed, in this case the changes in the probabilities and in the optimal treatment induced by a discovery have opposite effects on the decision maker's ability to learn. If the probabilities

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of the measurement errors do not vary much with their size, so that a change in the treatment does not have a large impact on the likelihood test, the discovery of a bad new outcome inhibits the agent's ability to detect further side effects. By contrast, if the decision maker is highly sensitive towards measurement errors, the choice of a more prudent treatment following the discovery of a bad new outcome makes her more capable of learning future adverse effects.

Next, I provide some sufficient conditions for a decision maker to want to experiment and for her to never wish to do so. Here the intuition is simple: an agent whose utility is not much affected by favorable discoveries but is quite sensitive to adverse ones may never wish to adopt treatments involving outcomes different from the originally known ones, while a decision maker whose utility behaves in the opposite way will wish to do so.

I then study how an agent's willingness to experiment affects her awareness. I show that after the discovery of a sufficiently beneficial (respectively adverse) new effect, the range of good (respectively bad) new outcomes above (respectively below) a certain threshold that can be learned only by an agent who never experiments expands. This is because for an agent who experiments, the discovery provides her with an additional outcome with which she can confuse future extreme new effects. By contrast, due to the decrease in the likelihood of known outcomes implied by the rules of reverse Bayesianism, the discovery of a highly beneficial effect expands the ranges of moderately favorable new outcomes that can be learned only by an agent who experiments. Conversely, due to the changes in probabilities and treatments explained above, the discovery reduces the ranges of such outcomes that can be detected only by an agent who never experiments. The comparison between the two agents' ability to recognize moderately adverse effects instead depends on their sensitivity towards measurement errors, with high (respectively low) sensitivity favoring the learning ability of the experimenting agent after the discovery of a good (respectively bad) new outcome. As a consequence of such results, in a history of sufficiently favorable and/or adverse new outcomes, the agent who learns more is the one who never experiments.

This paper contributes to the recent and rapidly growing literature on unawareness. In particular, I take the choice theoretic approach of Karni and Vierø (2013, 2017). In Karni and Vierø (2013), the authors construct the state space from a set of basic acts and a set of consequences. They consider a decision maker who is unaware of some consequences, some basic acts, or some links between them, and is unaware of her ignorance. The authors obtain subjective expected utility representation theorems and characterize the evolution of beliefs in the wake of new discoveries. In Karni and Vierø (2017), which is the work most closely related to this paper, the state space

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is constructed from a set of basic actions and a set of consequences. In this case, the agent is unaware of some consequences, but is aware that her knowledge may be incomplete. The authors provide a subjective expected utility representation of preferences over distributions on acts, where the agent's attitude towards the unknown is captured by a parameter. They characterize the evolution of beliefs, showing, in particular, that under their axioms the likelihood ratio between the states associated with known consequences remains unchanged after the discovery of new consequences. Another choice theoretic paper featuring awareness of unawareness is Grant and Quiggin (2015). Here the authors augment a standard Savage state space with a set of "surprise" states. In addition, they augment a set of "standard" consequences with two unanticipated consequences, one ranked below the worst possible standard consequence and the other ranked above the best standard consequence. These unanticipated consequences can occur only in surprise states. The agent knows that her understanding of the world is incomplete and evaluates acts according to an expected uncertain utility representation.

The main contribution of my paper to this literature is to provide a first model of learning under awareness of unawareness. Differently from the other choice theoretic models of unawareness, in my framework new outcomes can be recognized or neglected by an agent, depending on her current and past actions. Though simple, the model allows one to study how the size and the nature of new outcomes affect agents' current and future levels of awareness. Moreover, it sheds some light on how agents' attitudes towards experimentation affect, sometimes in a surprising manner, their awareness. Finally, on the econometrics/statistics side, it may help advance our knowledge of how to employ likelihood tests to select among non-nested models (see, for example, Vuong, 1989) that also take partial awareness into account. Indeed, it seems reasonable to expect that in these models the decision maker may include the possibility of being unaware of some aspects of the world.

My model also contributes to the recent literature aimed at applying the notion of unawareness in various settings. Indeed, though clinical trials are an essential step in the development of almost every drug, and they routinely give rise to unanticipated outcomes with serious consequences, they have not yet being studied within this literature. Filiz-Ozbay (2012) for example considers an insurance problem with asymmetric awareness. She shows that an insurer with superior awareness of the relevant contingencies may strategically offer incomplete contracts, while competition among symmetrically informed insurers enhances the awareness of the insuree. Auster (2013) studies the optimal contract between a fully aware principal and an unaware agent. In deciding whether to make the agent aware of some unforeseen contingencies, the principal faces a tradeoff between participation

Tesi di dottorato "Essays on Unawareness in Economics"
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and incentives, and in equilibrium the decision depends on the probability of such contingencies and on how informative they are about the agent's effort level. Von Thadden and Zhao (2012) consider a moral hazard model with a fully aware principal and an agent who is aware of all relevant contingencies, but has limited awareness of her action space. The principal can increase the agent's awareness and enlarge her action space, relaxing her participation constraint. But this adds more incentive constraints to the principal's problem. The authors show that if the agent's default behavior is sufficiently close to the first best level, the principal will write an incomplete contract where the description of the agent's action is missing, and then derive the optimal menu of contracts when agents have heterogeneous, unobservable levels of awareness. Von Thadden and Zhao (2013) extend these ideas to the case of a multidimensional effort. Grant, Kline, and Quiggin (2012) study a model of contractual disputes where parties have coarse subjective state spaces and may therefore disagree as to which state of the world has taken place, and thus as to what actions are required by the contract. Schipper and Woo (2016) propose a model of electoral campaigning where candidates microtarget voters by making them aware of only certain political issues and providing some information on their preferences over such issues. Galanis (2016) investigates the value of information in a risk-sharing environment where agents may be unaware of some contingencies that, though payoff irrelevant, can be correlated with contingencies that are payoff relevant. He shows that public information can make some agents better off at the expense of others.

The remainder of the paper is organized as follows. Section 3.2 presents the model. Section 3.3 studies how new outcomes affect awareness when agents do not experiment. Section 3.4 investigates how an agent's attitude towards experimentation impacts her awareness of new outcomes. Section 3.5 concludes. A proposal on how one may endogenize the suggested likelihood test, and the proofs of the propositions are in Appendix B.

3.2 The model

In this paper I adopt the framework of Karni and Vierø (2017) for the study of awareness of unawareness, and I implicitly adhere to their axioms. Let $A = \{a\}$ be a singleton set of actions, whose unique element a is interpreted as the action "giving a treatment". Let $Y_t \subset \mathbb{R}$ be the finite set of treatments' outcomes (consequences in the terminology of Karni and Vierø) the decision maker is aware of at time t, $t \in \{1, 2, ..., T\}$. Assume $Y_t = Y_{Bt} \cup Y_{Gt}$, where Y_{Bt} is the subset of the known "bad" outcomes (i.e., the known outcomes

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in case of failure of a treatment) and Y_{Gt} is the subset of the known "good" outcomes (i.e., the known outcomes in case of success of a treatment). I assume $\min Y_{G1} > \max Y_{B1} + \delta$, where $\delta > 0$ is a constant, so that the bad outcomes known at the beginning of the first trial are sufficiently worse than the good ones (this assumption will simplify the derivation of some results). Following Karni and Vierø (2017), I construct the state space from the set of actions and the set of outcomes. Specifically, let $S_t := Y_t^A = \{s_t : A \to Y_t\}$ be the state space at time t. We can partition S_t into the subset of bad states, S_{Bt}, and the subset of good states, S_{Gt}, where $S_{it} := \{s_t \in S_t : s_t(a) \in Y_{it}\}$, $i = B, G$. Let $x_t := \neg Y_t$ denote the "none of the above" outcome (i.e. outcomes the decision maker is currently unaware of), and let $\hat{Y}_t := Y_t \cup \{x_t\}$. Finally, let $\hat{S}_t := \hat{Y}_t^A = \{\hat{s}_t : A \to \hat{Y}_t\}$ be the augmented state space at time t.

At the beginning of each period t in which the agent is aware of the set of outcomes Y_t, the decision maker holds a prior belief $\pi(\cdot; Y_t)$ over the states and has to choose a treatment $f_t : \hat{S}_t \to \hat{Y}_t$. I assume that at the beginning of period 1 the agent has at her disposal a set of treatments which have the following three features:

1. (**Binary known outcomes**) $f_1(S_{i1}) = \{y_{i1}\}$, $y_{i1} \in Y_{i1}$, $i = B, G$. In words, each treatment has one good outcome (in case of success) and one bad outcome (in case of failure) known from the beginning of the first trial;

2. (**Undominated treatments**) For any two treatments f_1 and g_1, we have $f_1(S_{G1}) > g_1(S_{G1})$ if and only if $f_1(S_{B1}) < g_1(S_{B1})$. In words, there is no pair of treatments where one provides a better outcome than the other both in case of success and in case of failure. For instance, one may think of different classes of painkillers, where the most efficacious ones have usually also more severe side effects;

3. (**Not fully controllable treatments**) $f_1(\hat{S}_1 \setminus S_1) = \{x_1\}$. This means that it is never possible for the decision maker to prevent unknown outcomes from taking place. Given that the treatments tested in clinical trials are new and, as argued in the Introduction, there are many potential sources of unexpected effects, such an assumption seems quite reasonable.

At any time t, the agent evaluates treatments according to the subjective expected utility representation in Karni and Vierø (2017), which is based on the prior beliefs, a Bernoulli utility $u(\cdot)$ over the outcomes known from the beginning of period 1, and a parameter $\bar{u}(x_t)$ expressing the decision maker's

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current attitude towards the unknown. So, for example, the agent's expected utility from treatment f_1 in the first trial is given by

$$\pi(S_{B1}; Y_1)u(y_{B1}) + \pi(S_{G1}; Y_1)u(y_{G1}) + \pi(\hat{S}_1 \setminus S_1; Y_1)\bar{u}(x_1).$$

After giving a treatment f_t, the decision maker does not observe directly the outcome of such treatment. Instead, what she observes at time t is the result of a preliminary and imprecise measurement, denoted by $\tilde{y}_t$. One can think of such imprecise measurement as being the investigator's outcome of interest measured with some error, or a variable that proxies for what the researcher is ultimately interested in, which in itself is difficult to measure (e.g. the patient's overall well being). The measurement does not in itself yield any utility to the agent, since this is determined only by what she believes to be the actual effect underlying the measurement. The value $\tilde{y}_t$ can be either the sum of a known outcome $y_t \in f_t(S_t)$ and a measurement error, or the result of an outcome of which the decision maker was not aware at the beginning of period t (e.g. an unsuspected beneficial effect or an unexpected adverse side effect).[2] The decision maker has no idea of whether and how new outcomes are affected by the chosen treatment. The measurement errors are assumed to be i.i.d. and independent of the true outcomes. They are also assumed to have a discrete support with a probability mass function $\phi(\cdot)$ which reaches its maximum in 0, is decreasing on the right of 0 and increasing on the left of it. In what follows, whenever I write $\phi(z)$, I am implicitly assuming that $z \in \text{Supp}\ \phi$.

After observing $\tilde{y}_t$, the agent has to decide whether it is the result of a new outcome or it was generated by a known outcome plus a measurement error. I assume the decision maker simply opts for the scenario she deems the most likely. Specifically, I assume that she adopts the following learning mechanism:

1. She solves
$$\max_{y_t \in f_t(S_t)} \pi(f_t^{-1}(y_t); Y_t)\phi(\tilde{y}_t - y_t). \tag{3.1}$$
That is, the agent maximizes the likelihood that the observation $\tilde{y}_t$ is the result of a known outcome of the current treatment plus an independent error. Denote such maximum likelihood by $L(y_t^*)$, where y_t^* is the maximizer.

[2]This new outcome can potentially be different from $\tilde{y}_t$. In fact, the "mechanism" converting a new outcome into $\tilde{y}_t$ is unknown to the decision maker. For this reason, when the decision maker assigns a probability to the second scenario, she assigns it to $x_t = \neg Y_t$ (see below).

Tesi di dottorato "Essays on Unawareness in Economics"
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2. If $L(y_t^*) \geq \pi(f_t^{-1}(x_t); Y_t)$, then the decision maker concludes that $\tilde{y}_t$ is the result of y_t^* plus a measurement error. Otherwise, she concludes that $\tilde{y}_t$ was generated by a new outcome and uses an "outcome verification technology" (e.g. a more in-depth analysis of the patient) to identify such new effect.

A possible way for endogenizing the use of this likelihood test is proposed in the Appendix. Note that the use of the outcome verification technology precludes cases in which the agent erroneously believes that a new outcome has occurred.

The proposed learning mechanism captures the intuitive idea that new effects are recognized only if they produce observations sufficiently different from the expected ones. Indeed, as noted in footnote 3.1, in clinical trials new effects are considered unanticipated and receive additional attention when they differ significantly in nature, severity, or frequency from the known ones. Clearly, given that the outcome in my framework is unidimensional, and since in each period the agent receives only one observation, severity of the adverse or beneficial effect seems the most natural interpretation. Note that, as it is apparent from (3.1), the two key factors determining whether a new outcome is detected are the known outcomes of the current treatment, which in turn depend on the agent's preferences and on past discoveries, and the probabilities assigned to the good and to the bad states, which, as I will explain shortly, depend on past discoveries.

To make the analysis even more tractable, in what follows I assume $\pi(\hat{S}_t \setminus S_t; Y_t) = \bar{\pi} > 0$ for all Y_t and for all t (constant awareness of unawareness) and $\phi(\delta) \leq \bar{\pi}$, so that a decision maker observing $\tilde{y}_t \leq \max Y_{B1}$ (respectively, $\tilde{y}_t \geq \min Y_{G1}$) can never conclude that such an observation was generated by a good (respectively, bad) known outcome. For simplicity, I also assume that at any time t all new outcomes $y'_t \notin Y_t$ generate precise observations, in the sense that the observation $\tilde{y}_t$ generated by y'_t is equal to y'_t itself.[3]

In this paper I focus on two main types of new outcomes.

Definition 1 *A bad new outcome at time t is an outcome $y'_t \notin Y_t$ such that $y'_t < \min Y_{B1}$. A good new outcome at time t is an outcome $y'_t \notin Y_t$ such that $y'_t > \max Y_{G1}$.*[4]

These two types of new outcomes are perhaps the most important ones in clinical trials. Indeed, once discovered, they can lead to a complete reassessment of the safety or efficacy of the tested treatment.

[3] All my results would still hold if the possible observations generated by a new outcome are sufficiently close to the outcome itself.

[4] Clearly, I assume that, once discovered, bad (respectively, good) new outcomes are included into the next period's subset of known bad (respectively, good) outcomes.

Tesi di dottorato "Essays on Unawareness in Economics"
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To isolate the most fundamental effects of increasing awareness, I assume that when no new outcomes are discovered, prior beliefs remain unchanged. Such beliefs reflect information the researchers gathered on the known outcomes before starting the trial. For example, consider a case in which the severity of a known bad outcome is due to some specific characteristic of the patient. In this case, before starting the trial, the investigator will gather (e.g. from other studies) information on the proportion of individuals in the population who have the characteristic leading to the aforementioned outcome.

When a new outcome $y'_t \notin Y_t$ is discovered at time t, the rules of reverse Bayesianism apply (see Karni and Vierø, 2017). The rules of reverse Bayesianism, together with the definitions of S_{Bt} and S_{Gt}, imply

$$\pi(S_{Bt+1}; Y_{t+1})/\pi(S_{Gt+1}; Y_{t+1}) \geq \pi(S_{Bt}; Y_t)/\pi(S_{Gt}; Y_t)$$

after the discovery at time t of any bad new outcome, and

$$\pi(S_{Bt+1}; Y_{t+1})/\pi(S_{Gt+1}; Y_{t+1}) \leq \pi(S_{Bt}; Y_t)/\pi(S_{Gt}; Y_t)$$

after the discovery at time t of any good new outcome. To see this, it is sufficient to note that by reverse Bayesianism the ratio between the probabilities of the subsets of the states associated with the bad and the good outcomes known at the beginning of period t remains unchanged from t to $t+1$, while, by the definition of S_{Bt} (respectively, S_{Gt}), discovering $y'_t < \min Y_{B1}$ (respectively, $y'_t > \max Y_{G1}$) expands only the subset of bad (respectively, good) states.

Once a good (respectively, bad) new outcome is discovered, the agent can include it in any treatment, where it will occur with a given probability, at a cost k_G^{inc} (respectively, k_B^{inc}). For example, in order to safely implement a newly discovered beneficial effect of a drug, the researcher may need to conduct further studies to make sure that the underlying mechanism, which until then had received little attention, does not also carry harmful side effects. Alternatively, the decision maker can remove the good (respectively, bad) new outcome from any treatment paying a cost k_G^{rem} (respectively, k_B^{rem}). For example, the researchers may need to devote resources in order to modify the drug, or to change the design of the trial so that, for instance, individuals subject to the new side effect are not allowed to participate. To avoid trivial scenarios, I consider the case where side effects are relatively more difficult to get rid of. Specifically, I assume $k_B^{rem} > k_B^{inc}$ and $k_G^{rem} < k_G^{inc}$.

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3.3 Awareness without experimentation

In the above framework, observing a new outcome does not necessarily imply recognizing it as a new outcome. Indeed, the test proposed in the previous section implies that such recognition will occur only if the new outcome (or, to be more precise, the observation it generates, which I have assumed to be equal to the new outcome itself) differs sufficiently from the known effects of the current treatment. A newly discovered outcome in turn affects the choice of the future treatment, and hence what future new outcomes cannot be detected, through two main channels: (i) the impact on the ranking of the treatments with known outcomes, and (ii) the availability of a new outcome itself. Indeed, becoming aware of a new outcome brings the opportunity to include it in future treatments, but it also changes the state space and hence the expected utilities of the treatments whose outcomes were already known.

To better understand the impact of a discovery on future awareness, first in this section I focus on the case of no experimentation (i.e., on channel (i)). That is, I assume that the agent does not incorporate new outcomes in her treatments, and study how discoveries affect the ranking of the treatments with known outcomes, and how this in turn impacts the agent's future awareness. In the next section, I will show that whether or not a decision maker includes new outcomes in future treatments depends on how her utility behaves outside the original range of known effects, and I will study how the inclusion of new outcomes (i.e., channel (ii)) affects learning.

As the following proposition shows, the impact of discoveries on the agent's ability to recognize *good* new outcomes is unambiguous: a sort of negative feedback effect emerges, where the discovery of good (respectively, bad) new outcomes inhibits (respectively, enhances) the agent's ability to learn future good new outcomes.

Proposition 8 *Suppose that at time t an agent who never includes new outcomes in her treatments becomes aware of a good (respectively, bad) new outcome. Then, the range of new outcomes higher than $\max Y_{G1}$ that the agent is unable to recognize as new outcomes expands (respectively, shrinks) from period t to $t+1$.*

The intuition behind this result is the following. After the discovery of a good new outcome the agent becomes more optimistic and daring, and this inhibits her ability to recognize other new favorable outcomes. To see this, first note that when a new good outcome is discovered at time t, the probability of the good states rises. This in turn implies that if the newly discovered outcome is not incorporated in the time-$t+1$ treatment, then the

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agent in $t+1$ will either give the same treatment she gave in t or switch to a more "extreme" treatment (i.e., one involving a lower known outcome in case of failure and a higher known outcome in case of success). The fact that in $t+1$ the decision maker is giving the same or a more extreme treatment and that the prior probability of the good states has risen implies that new favorable outcomes will look closer to the known good effect of the treatment, and thus will be less detectable through the likelihood test. Conversely, suppose that the agent has become aware of a bad new outcome. In this case the discovery makes the decision maker more pessimistic and prudent, leading her to be more sensitive towards new favorable outcomes. Indeed, the discovery of a bad new outcome decreases the probability of the good states. This in turn makes the decision maker willing to choose more prudent treatments (i.e., treatments with a higher known outcome in case of failure and a lower known outcome in case of success). Both these effects make next period's new favorable outcomes easier to detect through the likelihood test.

The effect of discoveries on the decision maker's ability to recognize *bad* new outcomes is instead ambiguous. Consider, for example, what happens after the discovery of a bad new outcome. On the one hand, the increase in the probability of the bad states brought about by such a discovery implies a smaller sensitivity towards new unfavorable outcomes if the agent does not change treatment from t to $t+1$. On the other hand, the choice of a more prudent treatment that may follow the discovery can enhance the agent's ability to detect bad new outcomes. In this case, which of the two effects prevails crucially depends on the agent's sensitivity towards the size of the measurement error. To see this, note that the choice of a more prudent treatment leads the agent to perceive relatively larger measurement errors. If the probabilities of such larger errors are sufficiently smaller, the agent's ability to detect bad new outcomes will be improved.

More precisely, the agent's ability to detect bad new outcomes will deteriorate after the discovery of an unfavorable outcome if, at least for measurement errors below a certain threshold, the probabilities do not decline too fast as the sizes of the errors become larger, i.e., if

$$\phi(\epsilon_i) \geq \bar{k}\phi(\epsilon_j), \quad \forall \epsilon_i, \epsilon_j \leq \bar{\epsilon}, \quad \epsilon_i < \epsilon_j, \tag{3.2}$$

for some high enough $\bar{k} \in (0,1]$ and $\bar{\epsilon} \leq 0$. In this case, where the agent is not very sensitive towards measurement errors, we will observe the sort of negative feedback effect typical of good new outcomes: the discovery of unfavorable outcomes inhibits the decision maker's ability to spot further side effects of her treatment.

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In contrast, the discovery of a bad new outcome will improve the agent's ability to learn further unfavorable outcomes if, at least for measurement errors below a certain threshold, the probabilities decrease fast enough as the sizes of the errors increase, i.e., if

$$\phi(\epsilon_i) \leq \underline{k}\phi(\epsilon_j), \quad \forall \epsilon_i, \epsilon_j \leq \bar{\epsilon}, \quad \epsilon_i < \epsilon_j, \tag{3.3}$$

for some sufficiently low $\underline{k} \in (0, \bar{k}]$ and high enough $\bar{\epsilon} \leq 0$. In this case, where the agent is sufficiently sensitive towards measurement errors, if she is willing to switch to different treatments, we will observe a sort of positive feedback effect, where the discovery of unfavorable outcomes enhances the agent's ability to recognize further side effects.

We can summarize the above discussion in the following proposition.

Proposition 9 *Suppose that at time t an agent who never includes new outcomes in her treatments becomes aware of a bad (respectively, good) new outcome. Then, the range of new outcomes lower than* $\min Y_{B1}$ *that the agent is unable to recognize as new outcomes*

*a. (**Low sensitivity towards errors**) expands (respectively, shrinks) from period t to $t+1$ if condition (3.2) holds for some high enough $\bar{k} \in (0, 1]$ and $\bar{\epsilon} \leq 0$;*

*b. (**High sensitivity towards errors**) shrinks (respectively, expands) from period t to $t+1$ if (i) condition (3.3) holds for some sufficiently low $\underline{k} \in (0, \bar{k}]$ and high enough $\bar{\epsilon} \leq 0$, and, in addition, (ii) there exists a treatment with a higher (respectively, lower) outcome in case of failure than that of the treatment given in period t with a sufficiently high* ex ante *expected utility.*

Requirement (ii) in case b. is needed to ensure that the agent changes treatment after the discovery.

3.4 Experimentation and awareness

In the preceding section I have provided some results on the evolution of awareness under the hypothesis that agents do not experiment (i.e. do not incorporate newly discovered outcomes into their future treatments). In this section I instead study how experimentation affects agents' awareness of new outcomes. To do this, I first provide sufficient conditions for an agent to (a) never wish to experiment, and (b) wish to experiment after the discovery of an outcome more (respectively, less) favorable than the best (respectively,

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worst) outcome she is aware of. Then, I compare two agents, each satisfying one of the two alternative sets of conditions, in terms of their awareness of new outcomes.

Consider two agents, agent 1 and agent 2, who at the beginning of period 1 are aware of the same set of outcomes Y_1 and therefore consider the same state space S_1. Assume the two agents hold common time-1 prior beliefs $\pi(S_{B1}; Y_1) > 0$ and $\pi(S_{G1}; Y_1) > 0$. Use superscripts to denote the agent, $u(\cdot)$ to denote the Bernoulli utility function over the outcomes known from the beginning of time 1,[5] and $u^*(\cdot; Y_t)$ to denote the Bernoulli utility function over outcomes discovered after the beginning of period 1.[6]

Agent 1 has an increasing u^1 function. Her preferences fulfill the following condition, where $\pi_{y'}$ denotes the probability of outcome y':

$$\pi_{y'} u^{*1}(y'; Y_t) + (1 - \bar{\pi} - \pi_{y'}) u^1(\max Y_{G1}) - k_i^{inc} \\ < (1 - \bar{\pi}) u^1(\min Y_{B1}) - k_i^{rem} \quad \forall Y_t, \forall t, i = B, G, \quad (3.4)$$

for all $y' < \min Y_{B1}$ and $y' > \max Y_{G1}$. This means that agent 1 will never be willing to pay k_i^{inc} and give a treatment involving a new outcome y' (whose expected payoff, net of the utility provided by the "none of the above" outcome, can never exceed the left-hand side of (3.4)), and will instead prefer to pay k_i^{rem} to give a treatment involving only outcomes known from the first period (whose expected payoff, again net of the utility of the "none of the above" outcome, will be always larger than or equal to the right-hand side of (3.4)).

Condition (3.4) is fulfilled by good new outcomes if k_G^{inc} is sufficiently larger than k_G^{rem} and agent 1 ranks good new outcomes above the originally known ones, but her evaluation of such new outcomes is never too high. The condition is satisfied by bad new outcomes if agent 1 ranks bad new outcomes below the originally known ones, and such a difference in evaluations is always sufficiently large. One may therefore think to agent 1 as being a decision maker who does not like lotteries involving extreme outcomes (similar to a risk averse agent in a framework with full awareness). In clinical trials, this may be the case of a drug developer whose product is targeted to a wide range of individuals (e.g. paracetamol in the case of painkillers), in which case safety concerns play an essential role.

Agent 2 has an increasing Bernoulli utility function u^2 over the outcomes

[5]Under the axioms of Karni and Vierø (2017), outcomes known at the beginning of time 1 will continue to be evaluated according to the same Bernoulli utility even after the discovery of new outcomes.

[6]Such utility will in general depend on the current set of known outcomes.

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known from the beginning of period 1. Her preferences satisfy

$$\pi_{y'} u^{*2}(y'; Y_t) + (1 - \bar{\pi} - \pi_{y'}) u^2(\min Y_{Bt}) - k_i^{inc} \\ > (1 - \bar{\pi}) u^2(\max Y_{Gt}) - k_i^{rem} \quad \forall Y_t, \forall t, i = B, G, \quad (3.5)$$

for all $y' < \min Y_{Bt}$ and $y' > \max Y_{Gt}$. Thus, at time t agent 2 will always prefer to pay k_i^{inc} and give the optimal treatment involving the outcome discovered in that period (whose expected payoff will be always larger than or equal to the left-hand side of (3.5)) rather than paying k_i^{rem} and giving a treatment involving only outcomes known from the beginning of period t (whose expected payoff never exceeds the right-hand side of (3.5)).

Condition (3.5) is fulfilled if agent 2 ranks more extreme good new outcomes above the previously known ones, and such a difference in evaluations is always sufficiently large. As for unfavorable outcomes, the condition is satisfied if k_B^{rem} is sufficiently larger than k_B^{inc} and agent 2 ranks more extreme bad new outcomes below the previously known ones, but her evaluation of such new outcomes is never too low. One may thus think to agent 2 as a decision maker who likes lotteries involving extreme outcomes (similar to a risk loving agent in an environment with full awareness). This may be the case of a drug developer whose product is targeted to patients suffering from a severe disease (e.g. morphine in the case of the painkillers), in which case efficacy concerns may prevail over concerns about side effects.

Equipped with these results, we can investigate the relationship between experimentation and awareness of new outcomes. First, notice that under our assumption that the agents assign a constant probability to the event of encountering unknown outcomes, experimentation has two main effects on learning. On the one hand, it provides the agent with additional outcomes with which she can confuse future new outcomes. On the other hand, by reverse Bayesianism it lowers the probabilities assigned to all the known outcomes of a treatment, making it more difficult for the agent to confuse them with future new outcomes.

In light of these two effects of experimentation, consider what happens if a sufficiently favorable *good* new outcome $y'_G \notin Y_t^i$, $i = 1, 2$, takes place at time t. In particular, suppose that y'_G is higher than the most favorable known outcome of the treatment given in t by agent 2, and that it can be learned (and hence experimented) by such agent. Moreover, y'_G is high enough so that agent 1 can learn all possible new outcomes that agent 2 would confound with y'_G after its discovery. It is then clear that any good new outcome above the minimum one that can be confounded with y'_G by agent 2 can be learned by either both decision makers or just by agent 1. Furthermore, the range of such good new outcomes that can be learned only by agent 1 expands

Tesi di dottorato "Essays on Unawareness in Economics"
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from period t to $t+1$, exactly because now new outcomes can be confounded by agent 2 also with y'_G. By contrast, new outcomes below the minimum one that can be confounded with y'_G by agent 2 can be learned by both, none, or either one of the two decision makers. However, as we have seen in Proposition 1, after the discovery of y'_G the range of good new outcomes that agent 1 is unable to recognize expands from t to $t+1$. In contrast, as we mentioned above, by virtue of reverse Bayesianism the ranges of (bad and good) new outcomes that agent 2 can confound with outcomes known before the discovery of y'_G shrink. Finally, if agent 1 is highly sensitive towards measurement errors (in the sense of point b. in Proposition 2), then also the range of bad new outcomes that she is incapable of learning expands after the discovery of the good new outcome y'_G.

We can formalize the above observations in the following proposition.

Proposition 10 *Suppose that at time t a sufficiently high good new outcome takes place. Then*

(a) *any good new outcome above a threshold $\bar{y}_G > \max Y_{G1}$ cannot be learned only by agent 2, and the ranges of such outcomes that can be learned only by agent 1 expand from t to $t+1$,*

(b) *the ranges of good new outcomes below $\bar{y}_G$ that can be learned only by agent 1 shrink from t to $t+1$, while those that can be learned only by agent 2 expand, and*

(c) *if agent 1 is highly sensitive towards measurement errors (in the sense of point b. in Proposition 2), the ranges of bad new outcomes that can be learned only by agent 1 shrink from t to $t+1$, while those that can be learned only by agent 2 expand.*

Similar results can be obtained for the case in which a sufficiently unfavorable *bad* new outcome y'_B takes place in period t. Low enough bad new outcomes can be learned by both decision makers or just by agent 1, and after the discovery of y'_B the range of such bad new outcomes that can be learned only by agent 1 expands since y'_B creates additional confusion to agent 2. In contrast, by reverse Bayesianism the confusion generated by the previously known outcomes diminishes after agent 2 experiments, and if agent 1 is not highly sensitive towards measurement errors (in the sense of point a. in Proposition 2) the range of bad new outcomes that she cannot recognize expands after the discovery of y'_B. This leads to the following result.

Proposition 11 *Suppose that at time t a sufficiently low bad new outcome takes place. Then*

Tesi di dottorato "Essays on Unawareness in Economics"
di MADOTTO MATTEO
discussa presso Università Commerciale Luigi Bocconi-Milano nell'anno 2020
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(a) *any bad new outcome below a threshold $\bar{y}_B < \min Y_{B1}$ cannot be learned only by agent 2, and the ranges of such outcomes that can be learned only by agent 1 expand from t to $t+1$, and*

(b) *if the agent is not highly sensitive towards measurement errors (in the sense of point a. in Proposition 2), the ranges of bad new outcomes above $\bar{y}_B$ that can be learned only by agent 1 shrink from t to $t+1$, while those that can be learned only by agent 2 expand*

From the last two propositions it is evident that when new outcomes are sufficiently high and/or low the agent that never experiments learns more.

Corollary 2 *In a history of sufficiently high good and/or sufficiently low bad new outcomes, agent 1 learns more outcomes than agent 2.*

Intuitively, new effects generating observations very different from the originally known ones may be recognized by all agents. However, an agent who is willing to pay to implement good new outcomes and accepts bad new outcomes may be unable to detect some of these extreme effects, since they may look too similar to what she expects from her current treatment. As a result, compared to an investigator who is more concerned about efficacy, chooses extreme treatments and loves to experiment, an investigator who is more concerned about safety, prefers prudent treatments and does not experiment will be always aware of more new outcomes that are sufficiently different from those known from the beginning of the trials.

3.5 Conclusion

Many beneficial and adverse effects of several treatments tested in clinical trials have been unanticipated by the researchers. Crucial questions are then how easy it is for the investigator to detect such effects, and how past discoveries affect the researchers' sensitivity towards further unexpected effects. Moreover, would the size and the nature of the effects (e.g. beneficial vs. adverse) ones matter, and may the researcher's propensity to exploit new findings improve or worsen her ability to recognize future new effects? To explore these issues, I build a simple choice theoretic model of learning under limited awareness of outcomes. A decision maker conducts a series of clinical trials and in each period she observes only an imprecise measure of the outcome, and uses a likelihood test to decide whether an observation is the result of a known outcome plus a measurement error or of an unknown effect.

Tesi di dottorato "Essays on Unawareness in Economics"
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First, I show that for an agent who does not incorporate new discoveries in her treatments, a sort of negative feedback effect in her ability to learn good new outcomes emerges. Specifically, due to the changes in the probabilities of known outcomes implied by the rules of reverse Bayesianism (see Karni and Vierø, 2017) and in the optimal treatment, the discovery of a beneficial effect makes the agent more optimistic and daring, inhibiting her ability to recognize other new favorable outcomes. The impact of new discoveries on the decision maker's ability to detect adverse side effects instead depends on her sensitivity towards measurement errors (i.e., how fast the probabilities of such errors change with their size). In particular, high sensitivity is capable of inducing a positive feedback effect, where the discovery of a bad new outcome makes the agent choose more prudent treatments and increases her ability to recognize further adverse side effects.

I then provide some sufficient conditions for a decision maker to want to experiment (i.e. try newly discovered outcomes) and for her to never wish to do so. Here the intuition is that an agent whose utility is not much affected by favorable discoveries but is quite sensitive to adverse ones may never wish to adopt treatments involving outcomes different from the originally known ones, while a decision maker whose utility behaves in the opposite way will wish to do so. Finally, I study how an agent's willingness to experiment affects her ability to learn. I show that when new outcomes are sufficiently extreme a decision maker who never experiments learns more, since such outcomes are difficult to reconcile with those known from the first trial. In contrast, since by reverse Bayesianism the inclusion of new outcomes in the treatments lowers the probabilities of the known effects, the discovery of extreme outcomes favors the recognition of moderate new effects by those agents who love to experiment.

Tesi di dottorato "Essays on Unawareness in Economics"
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Appendix B

A possible foundation for the suggested learning mechanism

In this subsection, I propose a possible way for endogenizing the learning mechanism proposed in Section 3.2.

Differently from what I did in the main body of the paper, here I assume that the individual who decides what treatment to test in each period (e.g. the sponsor) is different from the individual who has to identify the outcome of each trial (e.g. the researcher). The sponsor ranks treatments according to the same subjective expected utility representation used by the single agent in the main body of the paper. In contrast, the researcher's available actions and payoffs are as follows. If at time t the sponsor decides to give the treatment f_t and the researcher observes $\tilde{y}_t$, then

- The researcher can conclude that $\tilde{y}_t$ was generated by an unknown effect. In this case, she has to use a costly outcome verification technology (e.g. a more in-depth analysis of the patient) to identify in period t the true effect underlying $\tilde{y}_t$. Using this technology entails a fixed cost $c > 0$. However, if a new outcome is discovered, the researcher receives an immediate reward $d > c$. The researcher does not receive any further reward, nor has to pay any further cost, linked to $\tilde{y}_t$ after period t. Therefore, her time-t expected payoff if she concludes that $\tilde{y}_t$ is the result of an unknown outcome is

$$\frac{\pi(f_t^{-1}(x_t); Y_t)}{\eta} d - c,$$

  where

$$\eta := \pi(f_t^{-1}(x_t); Y_t) + \sum_{y_t \in f_t(S_t)} \pi(f_t^{-1}(y_t); Y_t) \phi(\tilde{y}_t - y_t).$$

- Alternatively, the researcher can conclude that $\tilde{y}_t$ is the sum of a known outcome and a measurement error. In this case, she has to wait until period $\bar{T} + t$, with $\bar{T} \geq 1$, in which the effect underlying $\tilde{y}_t$ is revealed (e.g. thanks to new evidence on the efficacy/safety of the drug coming from other comparable studies). In this case, in $\bar{T} + t$ the researcher receives a reward $m > 0$ if in t she correctly identified the precise effect underlying $\tilde{y}_t$, while she has to pay a cost $n > 0$ if she misunderstood the true outcome. If the researcher does not discount the future, her

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time-t expected payoff if she concludes that $\tilde{y}_t$ is the result of the known outcome $\hat{y}_t$ of the treatment f_t plus a measurement error is therefore

$$\frac{\pi(f_t^{-1}(\hat{y}_t);Y_t)\phi(\tilde{y}_t-\hat{y}_t)}{\eta}m \\ -\frac{\sum_{y_t\in f_t(S_t)\setminus\{\hat{y}_t\}}\pi(f_t^{-1}(y_t);Y_t)\phi(\tilde{y}_t-y_t)+\pi(f_t^{-1}(x_t);Y_t)}{\eta}n.$$

If $d-c=m$, and $c=n$, it is straightforward to see that the researcher will conclude that $\tilde{y}_t$ is the result of the scenario she deems the most likely. That is, she will use the likelihood test proposed in Section 3.2.

Proof of Proposition 8

First, note that since at any time t the image of $\hat{S}_t \setminus S_t$ is the singleton set $\{x_t\}$ for every treatment, x_t will play no role in the agent's choice of the time-t treatment. Next, suppose that at the generic time t the agent (who, by assumption, never includes new outcomes in her treatments) gives the treatment f_t with $f_t(S_{Bt})=\{y_{B1}\}$, where $y_{B1}\in Y_{B1}$, and $f_t(S_{Gt})=\{y_{G1}\}$, where $y_{G1}\in Y_{G1}$, and that in t she becomes aware of a good new outcome (i.e., an outcome larger than $\max Y_{G1}$). As argued in Section 3.2 and given the assumption of constant awareness of unawareness, this implies $\pi(S_{Gt+1};Y_{t+1})\geq\pi(S_{Gt};Y_t)$ and $\pi(S_{Bt+1};Y_{t+1})\leq\pi(S_{Bt};Y_t)$. It follows that at time $t+1$ the treatment f_{t+1} with $f_{t+1}(S_{Bt+1})=\{y_{B1}\}$ and $f_{t+1}(S_{Gt+1})=\{y_{G1}\}$ will be ranked above any other treatment g_{t+1} with $g_{t+1}(S_{Bt+1})=\{\hat{y}_{B1}\}$, where $\hat{y}_{B1}\in Y_{B1}$ and $\hat{y}_{B1}>y_{B1}$, and $g_{t+1}(S_{Gt+1})=\{\hat{y}_{G1}\}$, where $\hat{y}_{G1}\in Y_{G1}$ and $\hat{y}_{G1}<y_{G1}$. To see this, simply note that if

$$\pi(S_{Bt};Y_t)u(y_{B1})+\pi(S_{Gt};Y_t)u(y_{G1})\geq\pi(S_{Bt};Y_t)u(\hat{y}_{B1})+\pi(S_{Gt};Y_t)u(\hat{y}_{G1}),$$

and hence

$$\pi(S_{Gt};Y_t)(u(y_{G1})-u(\hat{y}_{G1}))\geq\pi(S_{Bt};Y_t)(u(\hat{y}_{B1})-u(y_{B1})),$$

then it must also be the case that

$$\pi(S_{Gt+1};Y_{t+1})(u(y_{G1})-u(\hat{y}_{G1}))\geq\pi(S_{Bt+1};Y_{t+1})(u(\hat{y}_{B1})-u(y_{B1})),$$

and hence

$$\pi(S_{Bt+1};Y_{t+1})u(y_{B1})+\pi(S_{Gt+1};Y_{t+1})u(y_{G1}) \\ \geq\pi(S_{Bt+1};Y_{t+1})u(\hat{y}_{B1})+\pi(S_{Gt+1};Y_{t+1})u(\hat{y}_{G1}).$$

Tesi di dottorato "Essays on Unawareness in Economics"
di MADOTTO MATTEO
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It follows that the agent's favorite treatment in $t+1$ will have an outcome in case of success greater than or equal to y_{G1} (but, by assumption, lower than or equal to $\max Y_{G1}$).

Suppose that a good new outcome $y'_t > \max Y_{G1}$ cannot be recognized as a new outcome by the agent at time t, since

$$\pi(S_{Gt}; Y_t)\phi(y'_t - y_{G1}) \geq \bar{\pi}. \tag{3.6}$$

As shown above, both terms in the left-hand side of (3.6) (weakly) increase from period t to $t+1$. It follows that y'_t cannot be recognized even in $t+1$.

A symmetric argument shows that if a good new outcome can be detected in t, then it can still be detected in $t+1$ after the discovery of a bad new outcome in t. Formally, suppose that at the generic time t, the agent gives the treatment f_t with $f_t(S_{Bt}) = \{y_{B1}\}$, where $y_{B1} \in Y_{B1}$, and $f_t(S_{Gt}) = \{y_{G1}\}$, where $y_{G1} \in Y_{G1}$, and that in t she becomes aware of a bad new outcome (i.e., an outcome lower than $\min Y_{B1}$). As argued in Section 2 and given the assumption of constant awareness of unawareness, this implies $\pi(S_{Gt+1}; Y_{t+1}) \leq \pi(S_{Gt}; Y_t)$ and $\pi(S_{Bt+1}; Y_{t+1}) \geq \pi(S_{Bt}; Y_t)$. Using arguments analogous to those above, it is easy to see that this in turn implies that at time $t+1$ the treatment f_{t+1} with $f_{t+1}(S_{Bt+1}) = \{y_{B1}\}$ and $f_{t+1}(S_{Gt+1}) = \{y_{G1}\}$ will be ranked above any other treatment g_{t+1} with $g_{t+1}(S_{Bt+1}) = \{\hat{y}_{B1}\}$, where $\hat{y}_{B1} \in Y_{B1}$ and $\hat{y}_{B1} < y_{B1}$, and $g_{t+1}(S_{Gt+1}) = \{\hat{y}_{G1}\}$, where $\hat{y}_{G1} \in Y_{G1}$ and $\hat{y}_{G1} > y_{G1}$. It follows that the agent's favorite treatment in $t+1$ will have an outcome in case of success lower than or equal to y_{G1}.

Suppose that an outcome $y'_t > \max Y_{G1}$ can potentially be recognized as a new outcome by the agent at time t, since

$$\pi(S_{Gt}; Y_t)\phi(y'_t - y_{G1}) < \bar{\pi}. \tag{3.7}$$

As shown above, both terms in the left-hand side of (3.7) (weakly) decrease from period t to $t+1$. It follows that y'_t can be recognized also in $t+1$.

Proof of Proposition 9

Suppose that at the generic time t the agent (who, by assumption, never includes new outcomes in her treatments) gives a treatment f_t with $f_t(S_{Bt}) = \{y_{B1}\}$, where $y_{B1} \in Y_{B1}$, and $f_t(S_{Gt}) = \{y_{G1}\}$, where $y_{G1} \in Y_{G1}$, and that in t she becomes aware of a bad new outcome (symmetric arguments can be made if in t she becomes aware of a good new outcome). As shown in the proof of Proposition 1, this implies $\pi(S_{Bt+1}; Y_{t+1}) \geq \pi(S_{Bt}; Y_t)$ and that the agent's favorite treatment in $t+1$ will have an outcome in case of failure higher than or equal to y_{B1}.

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di MADOTTO MATTEO
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a. Suppose that a bad new outcome $y'_t < \min Y_{B1}$ cannot be recognized as a new outcome by the agent at time t, since

$$\pi(S_{Bt}; Y_t)\phi(y'_t - y_{B1}) \geq \bar{\pi}. \tag{3.8}$$

If in $t+1$ the agent chooses a treatment with the same known outcomes as those of f_t, then it is clear that y'_t cannot be recognized even in $t+1$. Indeed, the first term in the left-hand side of (3.8) increases from period t to $t+1$, while the second one remains unchanged. Suppose instead that in $t+1$ the agent gives a treatment with an outcome in case of failure higher than y_{B1}. Call such outcome y'_{B1}. Assume that condition (3.2) holds with

$$\bar{k} := \max\left\{ \max_{y' < \min Y_{B1}} \frac{\pi(S_{Bt}; Y_t)}{\pi(S_{Bt+1}; Y_t \cup \{y'\})}, \max_{y' > \max Y_{G1}} \frac{\pi(S_{Bt+1}; Y_t \cup \{y'\})}{\pi(S_{Bt}; Y_t)} \right\}$$

and

$$\bar{\epsilon} := \min Y_{B1} - y_{B1}. \tag{3.9}$$

It is then clear that

$$\begin{aligned} \pi(S_{Bt+1}; Y_{t+1})\phi(y'_t - y'_{B1}) &= \pi(S_{Bt+1}; Y_{t+1}) \frac{\phi(y'_t - y'_{B1})}{\phi(y'_t - y_{B1})} \phi(y'_t - y_{B1}) \\ &\geq \pi(S_{Bt+1}; Y_{t+1})\bar{k}\phi(y'_t - y_{B1}) \geq \pi(S_{Bt}; Y_t)\phi(y'_t - y_{B1}) \geq \bar{\pi}. \end{aligned}$$

It follows that y'_t cannot be detected even in $t+1$.

b. Suppose that a bad new outcome $y'_t < \min Y_{B1}$ can potentially be recognized as a new outcome by the agent at time t, since

$$\pi(S_{Bt}; Y_t)\phi(y'_t - y_{B1}) < \bar{\pi}.$$

Suppose that there exists a treatment whose outcome in case of failure is $y'_{B1} > y_{B1}$ and that in case of success is $y'_{G1} < y_{G1}$, and that the time-t *ex ante* expected utility of such treatment differs from that of f_t by an amount Δ, with

$$\Delta \leq (\pi(S_{Bt+1}; Y_{t+1}) - \pi(S_{Bt}; Y_t))(u(y'_{B1}) - u(y_{B1}) + u(y_{G1}) - u(y'_{G1})).$$

It is then clear that in $t+1$ the agent will switch to a treatment with an outcome in case of failure higher than y_{B1}. Call such outcome y''_{B1} (which may or may not coincide with y'_{B1}). Assume that condition (3.3) holds with

$$\underline{k} := \min\left\{ \min_{y' < \min Y_{B1}} \frac{\pi(S_{Bt}; Y_t)}{\pi(S_{Bt+1}; Y_t \cup \{y'\})}, \min_{y' > \max Y_{G1}} \frac{\pi(S_{Bt+1}; Y_t \cup \{y'\})}{\pi(S_{Bt}; Y_t)} \right\}$$

Tesi di dottorato "Essays on Unawareness in Economics"
di MADOTTO MATTEO
discussa presso Università Commerciale Luigi Bocconi-Milano nell'anno 2020
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and $\bar{\epsilon}$ defined as in (3.9). We then have

$$\begin{aligned}\pi(S_{Bt+1}; Y_{t+1})\phi(y'_t - y''_{B1}) &\leq \pi(S_{Bt+1}; Y_{t+1})\underline{k}\phi(y'_t - y_{B1}) \\ &\leq \pi(S_{Bt}; Y_t)\phi(y'_t - y_{B1}) < \bar{\pi}.\end{aligned}$$

It follows that y'_t can be detected also in $t+1$.

Proof of Proposition 10

Suppose that at the generic time t agents 1 and 2 are giving the treatments f^1_t and f^2_t, respectively, and that in t a good new outcome y'_G takes place. Letting superscripts denote the agent, assume $y'_G > \max f^2_t(S^2_t)$ and

$$\max_{y_t \in f^2_t(S^2_t)} \pi((f^2_t)^{-1}(y_t); Y^2_t)\phi(y'_G - y_t) < \bar{\pi}.$$

Moreover, let y'_G be high enough so that

$$\phi(y - \max Y_{G1}) < \bar{\pi} \tag{3.10}$$

for all y such that

$$\pi((f^2_{t+1})^{-1}(y'_G); Y^2_{t+1})\phi(y - y'_G) \geq \bar{\pi}. \tag{3.11}$$

Let

$$\bar{y}_G := \min\{y : y > \max Y_{G1} \wedge \pi((f^2_{t+1})^{-1}(y'_G); Y^2_{t+1})\phi(y - y'_G) \geq \bar{\pi}\}. \tag{3.12}$$

Comparing (3.10), (3.11) and (3.12), it is evident that all good new outcomes above $\bar{y}_G$ can be learned by agent 1 (both in t and $t+1$). By contrast, all good new outcomes in the range $[\bar{y}_G, \hat{y}_G]$ cannot be learned by agent 2 in $t+1$, where $\hat{y}_G \geq y'_G$ is the highest outcome that agent 2 would confound with y'_G in $t+1$. Such range is clearly larger than the range of outcomes above $\bar{y}_G$ that could not be learned by agent 2 in t, since all outcomes greater than or equal to y'_G could be learned by 2 in t.

By reverse Bayesianism (i.e., constant ratios of the states associated with known outcomes) and the assumption of constant awareness of unawareness, we have

$$\pi((f^2_{t+1})^{-1}(y); Y^2_{t+1}) \leq \pi((f^2_t)^{-1}(y); Y^2_t), \quad \forall y \in f^2_t(S^2_t).$$

Next, notice that f^2_{t+1} will assign the same outcomes as f^2_t to the states associated with the outcomes known from the beginning of period t, and that outcomes below $\bar{y}_G$ can never be confounded with y'_G by agent 2 in $t+1$.

Tesi di dottorato "Essays on Unawareness in Economics"
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It follows that the ranges of (good and bad) new outcomes below $\bar{y}_G$ that agent 2 is unable to recognize shrink from period t to $t+1$. In contrast, by Proposition 8 the range of good new outcomes that cannot be learned by agent 1 expands from t to $t+1$. Finally, under the hypotheses of point b. in Proposition 9, also the range of bad new outcomes that agent 1 is unable to detect expands from t to $t+1$.

Proof of Proposition 11

Suppose that at the generic time t agents 1 and 2 are giving the treatments f_t^1 and f_t^2, respectively, and that in t a bad new outcome y'_B takes place. Letting superscripts denote the agent, assume $y'_B < \min f_t^2(S_t^2)$ and

$$\max_{y_t \in f_t^2(S_t^2)} \pi((f_t^2)^{-1}(y_t); Y_t^2)\phi(y'_B - y_t) < \bar{\pi}.$$

Moreover, let y'_B be low enough so that

$$\phi(y - \min Y_{B1}) < \bar{\pi} \tag{3.13}$$

for all y such that

$$\pi((f_{t+1}^2)^{-1}(y'_B); Y_{t+1}^2)\phi(y - y'_B) \geq \bar{\pi}. \tag{3.14}$$

Let

$$\bar{y}_B := \max\{y : y < \min Y_{B1} \wedge \pi((f_{t+1}^2)^{-1}(y'_B); Y_{t+1}^2)\phi(y - y'_B) \geq \bar{\pi}\}. \tag{3.15}$$

Comparing (3.13), (3.14) and (3.15), it is evident that all bad new outcomes below $\bar{y}_B$ can be learned by agent 1 (both in t and $t+1$). By contrast, all bad new outcomes in the range $[\hat{y}_B, \bar{y}_B]$ cannot be learned by agent 2 in $t+1$, where $\hat{y}_B \leq y'_B$ is the lowest outcome that agent 2 would confound with y'_B in $t+1$. Such range is clearly larger than the range of outcomes below $\bar{y}_B$ that could not be learned by agent 2 in t, since all outcomes lower than or equal to y'_B could be learned by 2 in t.

Symmetrically to what I showed in the proof of Proposition 10, the ranges of (good and bad) new outcomes above $\bar{y}_B$ that agent 2 is unable to recognize shrink from period t to $t+1$. By contrast, under the hypotheses of point a. in Proposition 9, the range of bad new outcomes that agent 1 cannot detect expands from t to $t+1$.

Tesi di dottorato "Essays on Unawareness in Economics"
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Bibliography

[1] Auster, S. (2013) “Asymmetric Awareness and Moral Hazard.” *Games and Economic Behavior* 82:503-521.

[2] Filiz-Ozbay, E. (2012) “Incorporating Unawareness into Contract Theory.” *Games and Economic Behavior* 76:181-194.

[3] Galanis, S. (2016) “The Value of Information in Risk-Sharing Environments with Unawareness.” *Games and Economic Behavior* 97:1-18.

[4] Grant, S., J. J. Kline, and J. Quiggin (2012) “Differential Awareness, Ambiguity, and Incomplete Contracts: A Model of Contractual Disputes.” *Journal of Economic Behavior & Organization* 82:494-504.

[5] Grant, S., and J. Quiggin (2015) “A Preference Model for Choice Subject to Surprise.” *Theory and Decision* 79:167-180.

[6] Karni, E., and M. L. Vierø (2013) “Reverse Bayesianism: A Choice-Based Theory of Growing Awareness.” *American Economic Review* 103:2790-2810.

[7] Karni, E., and M. L. Vierø (2017) “Awareness of Unawareness: A Theory of Decision Making in the Face of Ignorance.” *Journal of Economic Theory* 168:301-328.

[8] Schipper, B. C., and H. Y. Woo (2016) “Political Awareness, Microtargeting of Voters, and Negative Electoral Campaigning.” University of California, Davis.

[9] Von Thadden, E. L., and X. Zhao (2012) “Incentives for Unaware Agents.” *Review of Economic Studies* 79:1151-1174.

[10] Von Thadden, E. L., and X. Zhao (2014) “Multi-Task Agency with Unawareness.” *Theory and Decision* 77:197-222.

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[11] Vuong, Q. H. (1989) "Likelihood Ratio Tests for Model Selection and Non-nested Hypotheses." *Econometrica* 2:307-333.

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Chapter 4

Heterogeneous Awareness in Financial Markets

4.1 Introduction

In most of the literature on information acquisition in financial markets, all agents are assumed to have complete knowledge of the risks they are exposed to when trading an asset. Recent evidence, however, shows that this is often not the case. Gennaioli *et al.* (2012), for instance, document several episodes in the recent U.S. history where most investors were not aware of important risks associated with several kinds of financial instruments. The most notorious example is represented by the securitization of mortgages during the 2000s. Indeed, as described by Gennaioli *et al.* (2012), until the summer of 2007 both the holders of mortgage-backed securities and financial intermediaries appeared to be substantially unaware of (i) how fast house prices could decline and mortgage defaults grow, and (ii) the sensitivity of the price of such AAA-rated securities to house prices. In fact, the latter phenomenon was largely overlooked by the models employed by rating agencies (Coval *et al.*, 2009).

Given the past episodes of neglected risks, and in particular the manifest role played by such risks in the global financial crisis, it seems reasonable to consider the idea that, when trading certain assets, investors entertain the possibility of being exposed to unanticipated risks. That is, the investors, being aware of their incomplete knowledge of the risks, may assign a positive probability to the event of facing contingencies different from those they are currently aware of. It is then interesting to investigate how bounded awareness by part of the market participants impacts agents' incentives to acquire information, and how this affects asset prices and market liquidity. Which

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investors would have the greatest incentives to acquire information? How would the equilibrium price, the liquidity of the market, and the equilibrium shares of informed traders vary with the investors' initial levels of awareness? Finally, how would the results differ from those in the full awareness benchmark?

We propose to tackle these issues considering a simple order-driven financial market without short sales *à la* Kyle (1985), where we incorporate the notion of awareness of unawareness as recently axiomatized in Karni and Vierø (2017). More specifically, in our model agents have the opportunity to buy a risky asset or a risk-free security. Competitive market makers aggregate the orders from rational investors and noise traders and, in a later stage, set the price of the asset equal to its expected payoff conditional on the amount of orders received. Investors can buy a costly signal about the payoff of the risky security.

A given positive fraction of the rational traders neglects some risks associated with the risky asset. Specifically, they ignore the most negative payoffs of such security. They are, however, aware of their unawareness, in the sense that they assign a positive probability and a utility to the event of incurring outcomes different from those they are aware of. Fully aware agents know the true proportion of the partially aware in the population, while the latter believe that all market participants are partially aware. We impose a certain degree of rationality on the beliefs of the partially aware agents by requiring that they have a correct prior expected evaluation of the risky asset and that, when updating their beliefs after a signal, they form correct posteriors about the outcomes they are aware of.

We consider an equilibrium in which each rational investor chooses whether to acquire the signal and which asset to buy so as to maximize his expected profit given his level of awareness (and his beliefs about the proportion of partially aware traders). In order to keep the optimal total number of informed investors consistent with both levels of awareness, in our equilibrium the profit maximization of the partially aware traders determines the total number of informed agents, which in turn acts as an upper bound for the solution to the utility maximization of the fully aware. We concentrate on an equilibrium where unawareness changes agents' actions in response to at least one signal. Specifically, we consider an equilibrium in which only the partially aware investors buy the risky asset after a moderately negative signal (the *intermediate signal* in our terminology). Intuitively, the neglect of some outcomes by such investors lead them to perceive this signal as being less negative than it actually is. The different behavior of the partially aware in response to some signal, combined with their erroneous belief that all agents are partially aware, leads them to have a distorted view of the relationships

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among signals, order flows and prices.

We first analyze the equilibrium amounts of informed traders among the fully and the partially aware investors. As mentioned above, the total fraction of the informed is derived from the profit maximization of the partially aware, and is therefore independent of the number of fully aware traders in the market. We show that as unawareness becomes more severe (i.e., the partially aware traders neglect a larger number of possible payoffs), the conditional expected utilities formed by the partially aware investors after the signal realizations become more restricted around the unconditional one. Intuitively, the fact that such agents overlook a larger fraction of possible outcomes weakens their response (in terms of the shift in their expectations) to both positive and negative signals. As a result, the total amount of informed investors in the market is decreasing in the unawareness level of the partially aware. In turn, a smaller amount of informed traders makes large order flows less revealing of a good signal, lowering the price, and increasing the incentives of the fully aware to get informed. As a consequence, as unawareness rises, incentives to acquire information are transferred to the fully aware investors. Indeed, we prove that the equilibrium share of fully aware informed agents is increasing in the unawareness level of the partially aware.

We then analyze the impact of the level of unawareness of the partially aware investors on market makers' ability to recognize, through the order flows they receive, the signal observed by the informed traders, and how this in turn impacts the equilibrium price. First, since a more severe degree of unawareness leads to a lower amount of partially aware informed agents, who are the only traders buying the risky asset after the intermediate signal, as unawareness rises it becomes more difficult for market makers to distinguish between a bad and an intermediate signal. Moreover, the reduction in the total amount of informed investors leads to a greater confusion also between a bad and a good signal. As a result, a higher unawareness level implies that it becomes more difficult for market makers to both correctly detect the presence of a bad signal and correctly recognize its absence. On the other hand, since more unawareness induces a larger fraction of fully aware traders to get informed, and such investors purchase the risky asset only after receiving a good signal, as unawareness rises it becomes easier for market makers to distinguish between an intermediate and a good signal.

The increased inability of market makers to correctly recognize a bad signal due to a more severe degree of unawareness, together with their greater ability to distinguish between intermediate and good signals, has a direct impact on the equilibrium price. In particular, the diminished number of partially aware informed agents, by making low order flows less revealing of a bad signal, keeps average prices high when the asset payoff is low. By

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contrast, since the presence of the partially aware informed, by giving rise to new intermediate price levels, has a moderating effect on price fluctuations after a good signal, their reduction due to a higher unawareness level leads to an increased price volatility following such signal.

Finally, we compare our results to those one would get in an analogous model where instead all traders are fully aware. Given the reduced shift in expected utilities after any signal induced by partial awareness, the highest number of informed traders attains when partially aware agents are absent. As for order flows, we show that if good signals are not too likely expected orders are larger when partially aware investors are present. This occurs because the reduced incentives to gather information of the partially aware induces a large fraction of them to remain uninformed and buy the risky asset. The analysis of aggregate welfare shows that the presence of partial awareness does imply a welfare loss due to the misinterpretation of the intermediate signal. Interestingly, however, such loss diminishes as unawareness becomes more severe, since the incentives to gather the information that leads to the wrong investment decision decrease.

This paper provides a first model of awareness of unawareness for studying information acquisition in financial markets. It shows how some investors, by recognizing that their knowledge of the risks of some assets is incomplete, may lead to an overall reduction in the information acquisition in the market but, at the same time, increase through their impact on asset prices the incentives of the more knowledgeable investors to get informed. Moreover, it clarifies how such changes in traders' incentives induced by the presence of partial awareness, by affecting market makers' beliefs, impact average prices and price volatility.

In our paper we take the choice theoretic approach of Karni and Vierø (2017). In Karni and Vierø (2017) the state space is constructed from a set of basic actions and a set of consequences. The agent is assumed to be unaware of some consequences, but aware that his knowledge may be incomplete. The authors provide a tractable subjective expected utility representation of preferences over distributions on acts, where the agent's attitude towards the unknown is captured by a parameter. Another choice theoretic paper featuring awareness of unawareness is Grant and Quiggin (2015). Here the authors augment a standard Savage state space with a set of "surprise" states. In addition, they augment a set of "standard" consequences with two unanticipated consequences, one ranked below the worst possible standard consequence and the other ranked above the best standard consequence. These unanticipated consequences can occur only in surprise states. The agent knows that her understanding of the world is incomplete and evaluates acts according to an expected uncertain utility representation. Galanis (2015)

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di MADOTTO MATTEO
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shows how, in a multiple state space model, an agent aware of all outcomes but unaware of some contingencies (and not aware of his unawareness) may have a negative value of information. In his model, the agent's awareness level is not constant across states, creating a signal that the agent can only partially understand. This may in turn lead him to commit information processing errors and behave suboptimally in response to additional signals. By contrast, in our model with constant unawareness of outcomes and awareness of unawareness, more information is ex ante always valuable, though the lack of knowledge of some outcomes reduces its value compared to a scenario with full awareness.

As for the financial side, our notion of general equilibrium with unawareness builds upon the one of noisy rational expectation equilibrium mainly developed by Grossman and Stiglitz (1980), Hellwig (1980) and Diamond and Verrecchia (1981). Remarkable contributions can also be found in Gennotte and Leland (1990) and Mele and Sangiorgi (2015), where the interaction between asymmetric information and ambiguity is illustrated. The main differences with our notion of equilibrium are that in our case market participants have heterogeneous knowledge about the possible asset payoffs and that we impose the extra requirement that the total number of informed agents is consistent with all levels of awareness.

The remainder of the paper is organized as follows. In Section 4.2 we present the model. In Section 4.3 we analyze the impact of different unawareness levels on the equilibrium shares of informed traders. In Section 4.4 we study the effects of different degrees of unawareness on market makers' misperception of signals and the equilibrium price. In Section 4.5 we compare our results to those one would get in an analogous model without partially aware investors. Section 4.6 concludes. All proofs are contained in Appendix C.

4.2 The model

Consider a unit mass of risk-neutral investors who have the opportunity to buy a risky asset or a risk-free security with null interest rate. The payoff Y of the risky asset can take a finite number of increasing and *ex ante* equally likely values $y_1, \dots, y_k$. Before deciding whether to buy the risky asset, agents can acquire at a cost $c > 0$ a signal S about Y that can take the realizations s_G, s_M or s_B, that we refer to as the good, intermediate, and bad signal, respectively.[1] Each realization of the signal S has a positive probability and

[1]The use of three signals allows us to study the impact of unawareness on agents' actions in the simplest way without reaching the trivial result that all traders choose the

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we assume that

- $P(S = s_B|Y = y)$ and $P(S = s_M|Y = y)$ are strictly decreasing in y (and hence $P(S = s_G|Y = y)$ is strictly increasing in y);

- $Y|S = s_M$ dominates $Y|S = s_B$ in the sense of first order stochastic dominance;

- $\mathbb{E}[Y \mid S = s_M] > 0$.

From the first two assumptions it follows that the (conditional and unconditional) expected asset payoffs are ordered as follows:

$$\mathbb{E}\left[Y|S = s_B\right] < \mathbb{E}\left[Y|S = s_M\right] < \mathbb{E}\left[Y\right] < \mathbb{E}\left[Y|S = s_G\right].$$

Hence, observing the intermediate signal brings the posterior expected value of the risky asset below the unconditional one. As we will see in the next section, such a negative impact of s_M is lessened by the presence of unawareness. As a result, in a way similar to the episodes of neglected risks mentioned in the Introduction, the partially aware traders overestimate the value of the risky asset and may end up being willing to purchase it. For sake of comparison, in Section 4.3 we will discuss the case in which s_M brings the posterior expectation of the asset payoff above the unconditional one. The third assumption requires that the intermediate signal is not too adverse, so that it does not lead to a negative expected asset payoff, and it is used to simplify the derivation of some results on the moments of the price, but does not affect our findings about the impact of limited awareness on information acquisition.

Investors are heterogeneous in terms of their awareness of the possible payoffs of the risky asset. Specifically, a given fraction η_F of traders is fully aware, in the sense that they know all such possible payoffs. By contrast, the remaining part of agents $\eta_P = 1 - \eta_F$ is partially aware, since they know only the values of Y larger than or equal to some $y_{\hat{k}}$, with $\hat{k} \in \{1, \ldots, k\}$. We denote such subset of values by $\mathcal{E}$. They instead do not know the asset values in the subset $\neg\mathcal{E} \equiv \{y_1, \ldots, y_{\hat{k}-1}\}$. However, they recognize that their knowledge is incomplete and, therefore, assign a positive probability to the event of incurring outcomes different from those they are aware of. They also assign a utility/payoff, that we denote by x, to such an event. Hence, from their perspective the risky asset payoff is captured by a random variable $\hat{Y}$ that can take the values x, $y_{\hat{k}}$,...,y_k (see the axiomatization in

same asset in response to any signal.

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Karni and Vierø, 2017). We refer to the cardinality of $\neg\mathcal{E}$, that is $\hat{k}-1$, as the *unawareness level*.

We make the following assumptions on partial awareness:

$(A1)$ $P(\hat{Y}=y_i)=P(Y=y_i)$ for all $i\geq\hat{k}$;

$(A2)$ $P(S=s|\hat{Y}=y_i)=P(S=s|Y=y_i)$ for all $i\geq\hat{k}$ and all s;

$(A3)$ $P(S=s|\hat{Y}\in\neg\mathcal{E})=P(S=s|Y\in\neg\mathcal{E})$ for all s;

$(A4)$ $\mathbb{E}[\hat{Y}]=\mathbb{E}[Y]$.

The first assumption requires that partially aware traders have correct prior beliefs about the payoffs they are aware of, and hence do not overestimate nor underestimate the probability of incurring unknown outcomes. Indeed, such probability is given by

$$P(\hat{Y}\in\neg\mathcal{E})=1-P(\hat{Y}\in\mathcal{E})=1-P(Y\in\mathcal{E})=P(Y\in\neg\mathcal{E})=\frac{\hat{k}-1}{k}.$$

The second and third assumptions concern the updating of the partially aware agents. They ensure that when using Bayes' rule $P(\hat{Y}=y_i|S=s)=P(Y=y_i|S=s)$ for all $i\geqslant\hat{k}$ and all realizations s of the signal, and that $P(\hat{Y}\in\neg\mathcal{E}|S=s)=P(Y\in\neg\mathcal{E}|S=s)$. In words, we are requiring that agents form correct posterior beliefs on the outcomes they are aware of. In this way we are imposing a certain degree of rationality on the investors' beliefs, meaning that the only mistakes they make when updating come from those outcomes they are unaware of.

Finally, the last assumption ensures the correctness of the unconditional expectation of the partially aware investors, so that they are not *ex ante* overoptimistic nor overpessimistic about the payoff of the risky asset. Note that such assumption is equivalent to requiring that the value of x is the average of the neglected outcomes in $\neg\mathcal{E}$:

$$x=\frac{1}{\hat{k}-1}\sum_{i=1}^{\hat{k}-1}y_i. \tag{4.1}$$

Regularity assumptions $(A1)-(A4)$ imply that the order of the conditional and unconditional expected payoffs are preserved also underpartial awareness. Indeed, it can be shown that

$$\mathbb{E}\left[\hat{Y}|S=s_B\right]<\mathbb{E}\left[\hat{Y}|S=s_M\right]<\mathbb{E}\left[\hat{Y}\right]<\mathbb{E}\left[\hat{Y}|S=s_G\right].$$

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Fully aware traders know the true proportion η_P of the partially aware in the population. By contrast, partially aware agents believe that all traders are partially aware, i.e. they assume $\eta_P = 1$.

In addition to the rational (fully and partially aware) investors, in the market there are noise traders whose orders are collected by the random variable Z. We assume that Z is independent of Y, $\hat{Y}$ and S, and conditionally independent of Y and $\hat{Y}$ given S. In addition, Z is assumed to be uniformly distributed on the interval $[-\ell/2, \ell/2]$, where $0 < \ell \leqslant 1$ is the *noise level.* Note that the size of the support of noise traders does not exceed the total amount of rational investors, so that prices will not be driven solely by the first type of agents. Negative values of Z can be interpreted as sell orders.

The total order flow T is the sum of the noise Z and the orders of the rational traders. Note that the latter investors can be divided into four groups: η_{FI} of fully aware informed, η_{PI} of partially aware informed (with $\eta_{FI} + \eta_{PI} = \eta_I$), η_{FU} of fully aware uninformed, and η_{PU} of partially aware uninformed (with $\eta_{FU} + \eta_{PU} = \eta_U = 1 - \eta_I$). The total order flow is collected by market makers who are fully aware and know the true proportion of partially aware investors in the population. As in Kyle (1985), market makers are assumed to be competitive, they collect the aggregate order flow and, in a *later* stage, set the price $p(T) = \mathbb{E}[Y|T]$.[2] Importantly, however, such price function is known only by the fully aware agents. Indeed, as mentioned above, partially aware traders believe that all investors are partially aware. As a consequence, they consider a *distorted order flow* $\hat{T}$ and a *distorted price* function $\hat{p}(\hat{T}) = \mathbb{E}[\hat{Y}|\hat{T}]$.

Note that perfect competition among market makers together with Assumption $(A4)$ implies that $\mathbb{E}[p] = \mathbb{E}[\hat{p}] = \mathbb{E}[Y]$, so that all uninformed investors are indifferent between buying the risky asset and not doing so. We assume that half of them sets the buy order. Hence, the order flow generated by the uninformed agents (fully plus partially aware) is $\eta_U/2$, where η_U is the total amount of uninformed traders.

We consider the following notion of equilibrium with heterogeneous awareness.

Definition 2 *Given a fraction η_F of fully aware investors, an equilibrium with unawareness is composed of agents' order flows (demands), prices, and proportions of informed traders such that:*

- *market makers set the price $p = \mathbb{E}[Y|T]$;*

[2] Hence, as in Kyle (1985), market makers do not provide investors with additional information through the price.

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- *fully aware agents maximize their (possibly conditional) expected profit, knowing the true η_F and considering the objective $p(T)$;*

- *partially aware agents maximize their (possibly conditional) expected profit believing that $\eta_F = 0$, that the order flow is $\hat{T}$, and that market makers set the distorted price $\hat{p} = \mathbb{E}[\hat{Y}|\hat{T}]$;*

- *given the equilibrium total amount of informed agents, the partially aware informed (resp. uninformed) investors do not have an incentive to become uninformed (resp. informed);*

- *given the equilibrium total amount of informed agents and that of the fully aware informed traders, the fully aware informed (resp. uninformed) investors do not have an incentive to become uninformed (resp. informed).*

In principle, the partially aware could have a distorted perception of the total number of informed agents. However, in our notion of equilibrium, a wrong belief about η_I^* is not admitted: the observation of this number would not contrast the optimization of the partially aware traders. In other words, we require the equilibrium total number of informed traders to be consistent with both levels of awareness. To ensure this, since the partially aware believe that all traders are partially aware, in our equilibrium the maximization of their expected profit determines the total amount of informed agents, η_I^*, which in turn acts as an upper bound on the solution η_{FI}^* of the optimization problem of the fully aware. The equilibrium amount of partially aware informed investors is then found as the difference $\eta_{PI}^* = \eta_I^* - \eta_{FI}^*$.

Throughout the paper we study the case in which the presence of unawareness changes agents' actions for at least one signal. Since the equilibrium price is an average of the conditional expectations of the asset payoff, no informed agent will have an incentive to buy the asset after s_B and all informed traders will purchase it after s_G. We therefore focus on the intermediate signal s_M. In particular, we consider a separating equilibrium in which fully aware informed traders *do not buy* the risky asset after the signal s_M, while the partially aware informed *purchase* it. Intuitively, as we will prove in the next section, unawareness lessens the shift in the expected utility in response to any signal. As a result, the partially aware informed investors perceive a higher conditional expectation of the asset payoff after s_M, and the equilibrium conditions that we provide in the Appendix ensure that

$$\mathbb{E}\left[Y|S = s_M\right] < \mathbb{E}\left[p|S = s_M\right], \qquad \mathbb{E}\left[\hat{Y}|S = s_M\right] > \mathbb{E}\left[\hat{p}|S = s_M\right].$$

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4.3 Information acquisition

In this section, we study how the level of unawareness of the partially aware traders affects investors' incentives to acquire information.

First, we focus on order flows and on the objective and perceived price functions. As we have seen, all uninformed agents are indifferent between buying the risky asset and not doing so, and so half of them sets the order. Noise traders instead set the random order Z. As a result, the true overall order flow in a separating equilibrium as the one described at the end of the previous section is

$$T = Z + \frac{\eta_U}{2} + \begin{cases} 0 & \text{if} \quad S = s_B \\ \eta_{PI} & \text{if} \quad S = s_M \\ \eta_{PI} + \eta_{FI} & \text{if} \quad S = s_G. \end{cases}$$

Since partially aware agents believe that all traders are partially aware, they instead consider the distorted order flow

$$\hat{T} = Z + \frac{\eta_U}{2} + \begin{cases} 0 & \text{if} \quad S = s_B \\ \eta_{PI} + \eta_{FI} & \text{if} \quad S = s_M \\ \eta_{PI} + \eta_{FI} & \text{if} \quad S = s_G. \end{cases}$$

The true price p and the distorted one $\hat{p}$ are piecewise constant increasing functions of T and $\hat{T}$, respectively. Indeed, the partial overlap of the possible realizations of T due to different signals gives rise to a partition of the support of T into five subintervals, whose width depends on the equilibrium amounts of informed traders. The same occurs for $\hat{T}$, whose support is instead divided into three regions. For a graphical representation, see Figure 4.1. Each price level is an average of the (possibly distorted) expected payoffs conditional on the signal realizations written inside the curly brackets.

Focusing on how partially aware agents form their posterior expected evaluation of the risky asset, it can be shown that awareness of unawareness combined with our assumption about the correctness of the posterior beliefs about the known outcomes implies that when updating the partially aware traders behave as if they were weighing every outcome in $\neg\mathcal{E}$ using a unique conditional probability of the signal which is the average of the probabilities within such set (see Chapter 2). This averaging, combined with the monotonicity of the conditional probabilities of the signals, implies that the conditional expected utilities of the partially aware investors get more and more restricted around the unconditional one (while preserving their order) as the unawareness of such agents becomes more severe. Formally, we have the following result (see Figure 4.2 for a graphical representation).

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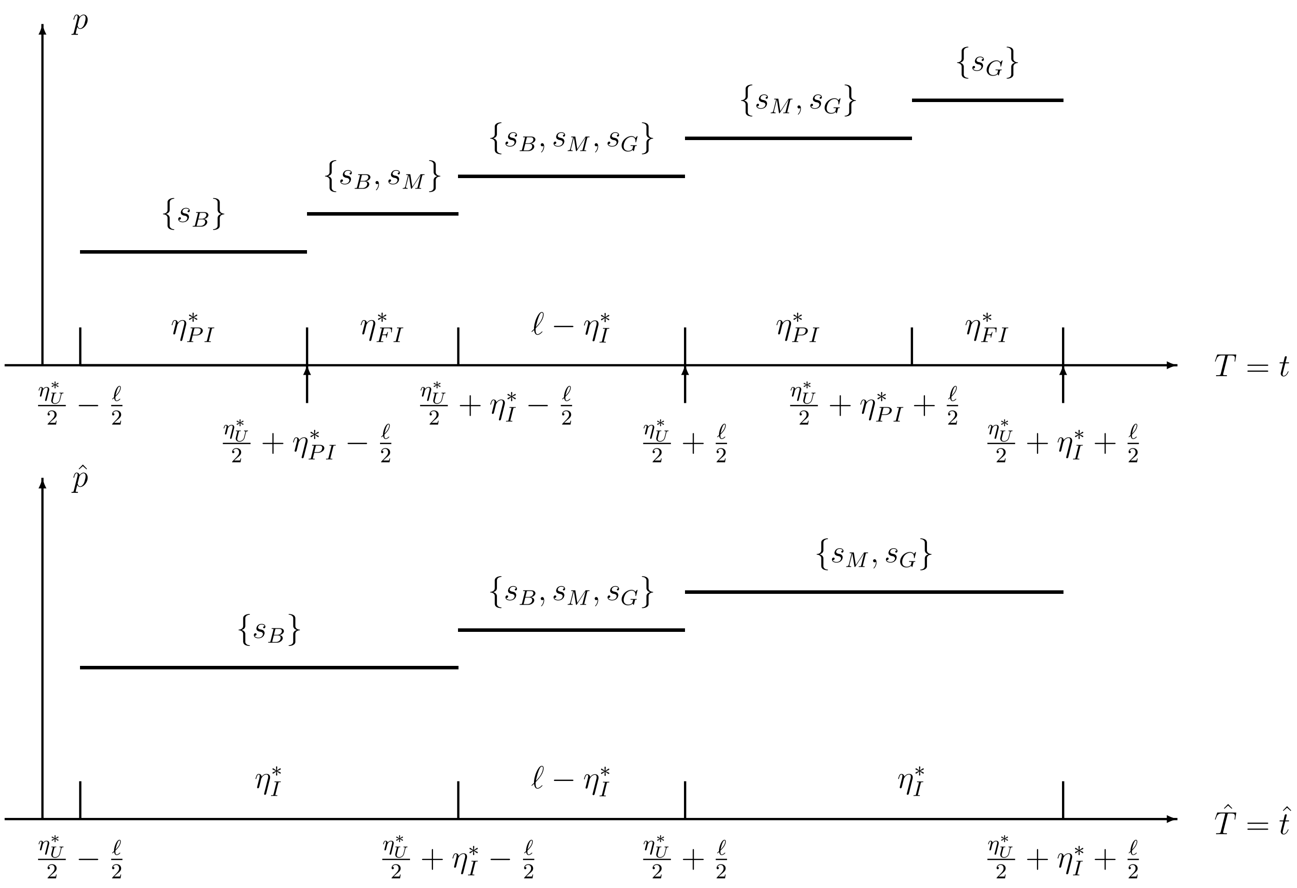

Figure 4.1: Plots of the equilibrium price p and the distorted price $\hat{p}$ as a function of their respective order flows. Each price level is consistent with some of the signal realizations (written inside the curly brackets). The width of each subinterval depends on the equilibrium number of partially and fully aware informed traders.

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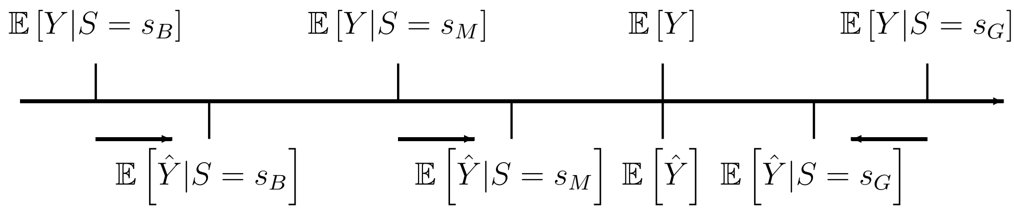

Figure 4.2: Effect of partial awareness on the expectations of the asset payoff, conditional on different signal realizations.

Proposition 12 $\mathbb{E}[\hat{Y}|S=s_B]$ *and* $\mathbb{E}[\hat{Y}|S=s_M]$ *are strictly increasing in the unawareness level, while* $\mathbb{E}[\hat{Y}|S=s_G]$ *is strictly decreasing. In particular,* $\mathbb{E}[\hat{Y}|S=s_B] > \mathbb{E}[Y|S=s_B]$ *and* $\mathbb{E}[\hat{Y}|S=s_M] > \mathbb{E}[Y|S=s_M]$*, while* $\mathbb{E}[\hat{Y}|S=s_G] < \mathbb{E}[Y|S=s_G]$*.*

The more restricted posterior expected utilities induced by higher levels of unawareness have an important effect on the incentives of the partially aware investors to get informed. Recall that the solution of the maximization problem of the partially aware provides the equilibrium total amount of informed investors in the market, η_I^*. Define the threshold c^* of the signal cost above which there is no information gathering:

$$c^* \equiv \left(\mathbb{E}[Y] - \mathbb{E}\left[\hat{Y}|S=s_B\right]\right) P\left(S=s_B\right). \tag{4.2}$$

We have the following result.

Proposition 13 *The equilibrium total amount of informed investors is given by*

$$\eta_I^* = \ell \max\left\{1 - \frac{c}{c^*}; 0\right\}. \tag{4.3}$$

η_I^* *is decreasing in the unawareness level.*

First, note that η_I^* is independent of the amount of fully aware agents η_F. This is because η_I^* comes from the utility maximization of the partially aware agents, who believe that all traders are partially aware. Moreover, the optimal number of informed traders is increasing in the noise parameter ℓ. Indeed, when the noise is larger, the order flows received by the market makers are less informative about the signal, the equilibrium price varies less, and acquiring the signal becomes more valuable.

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As Proposition 13 shows, more severe degrees of unawareness entail less information acquisition in the market. Intuitively, as seen above, the posterior expectations of $\hat{Y}$ become less dispersed as unawareness increases. When the partially aware traders use the price $\hat{p}$, which is an average of the distorted conditional expectations of the asset payoff, this leads to lower incentives to get informed for the more unaware agents and, in equilibrium, to a reduction in the total amount of informed agents.

While a more severe level of unawareness lowers the overall amount of informed traders in the market, it has opposite effects on the amounts of the two types of informed investors. In particular, a reduced η_I makes large order flows less revealing of a good signal, lowering the expected price conditional on s_G, which in turn increases the incentives of the fully aware to get informed. As a result, as unawareness rises, incentives to acquire information are transferred to the fully aware investors. Formally, we have the following result.

Proposition 14 *If η^*_{FI} is an internal solution, η^*_{FI} is increasing in the unawareness level, while η^*_{PI} is decreasing.*

The above result shows the opposite behaviors of fully and partially aware investors with respect to changes in the level of unawareness of the partially aware. In particular, such level of unawareness creates an *externality through the price*. Indeed, the unawareness level affects the width of the intervals over which the objective price is constant. This impacts the conditional expectations of prices p and, therefore, the incentives of the fully aware agents to get informed. More specifically, it can be shown that $\mathbb{E}[p|S = s_G]$ is increasing in η^*_I and in η^*_{FI}, given η^*_I. Intuitively, when the total amount of informed agents diminishes, high order flows become less revealing of a good signal and $\mathbb{E}[p \mid S = s_G]$, which is what fully aware traders look when deciding whether to acquire the signal, decreases. This makes information more attractive and leads to an increase in the equilibrium amount of fully aware informed investors.

For sake of comparison, consider an alternative scenario in which the intermediate signal brings the expected asset payoff above the unconditional one. Specifically, keep all the assumptions of our model but now suppose that $P(S = s_M \mid Y = y)$ and $P(S = s_G \mid Y = y)$ are strictly increasing in y, and that $Y \mid S = s_G$ dominates $Y \mid S = s_M$ in the sense of first order stochastic dominance. In this case, it can be shown that the expectations are ordered as follows:

$$\mathbb{E}\left[Y|S = s_B\right] < \mathbb{E}\left[Y\right] < \mathbb{E}\left[Y|S = s_M\right] < \mathbb{E}\left[Y|S = s_G\right].$$

Tesi di dottorato "Essays on Unawareness in Economics"
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A more severe degree of unawareness again leads to more restricted posterior expectations. Therefore, in this new scenario the counterpart of the separating equilibrium studied so far is one where no informed agent buys the risky asset after s_B, every informed investor purchases it after s_G, and, differently from before, only the fully aware informed buy it in response to s_M (since they have a higher conditional expected evaluation).

Given the reduced shifts in expectations induced by higher unawareness levels, it can be shown that also in this new scenario the overall amount of informed traders in the market is decreasing in the degree of unawareness of the partially aware. In this alternative setup, however, the fully aware agents who acquire information are interested not only in the good, but also in the intermediate signal. In this case, both a higher η_I for a given η_{FI} and a higher η_{FI} for a given η_I have opposite effects on $\mathbb{E}[p \mid S = s_G]$ and $\mathbb{E}[p \mid S = s_M]$. For example, a higher η_{FI} given η_I increases $\mathbb{E}[p \mid S = s_M]$ since low order flows become more revealing of s_B, but it also lowers $\mathbb{E}[p \mid S = s_G]$ because high orders become less revealing of s_G. The ultimate impact of more severe degrees of unawareness on η^*_{FI} and η^*_{PI} depends on the specific values taken by the signal probabilities and the conditional expected payoffs. By contrast, in our main model such contrasting effects are not present, since in that case awareness of unawareness leads agents to set the buy order in response to more signals than those of the fully aware.

4.4 Misperception of signals and equilibrium price

In this section, we analyze the impact of the unawareness level of the partially aware investors on market makers' ability to recognize, through the order flows they receive, the signal observed by the informed traders. We then study how this impacts the equilibrium price.

We start our analysis by studying the changes in the intervals of order flows over which p is constant, that we denote (from left to right) by $I_1, \ldots, I_5$, induced by an increase in the unawareness level. In Figure 4.3 we plot such intervals for low and high unawareness levels. The support of the order flows T sent to the market makers shrinks as the unawareness level rises. In particular, the disappearance of very low order flows is caused by the increasing number of uninformed investors due to the negative impact of unawareness on information gathering. Recall that uninformed agents are indifferent between sending buy orders to the market makers and not doing so, and that half of them buys the risky asset. The growing number of uninformed traders

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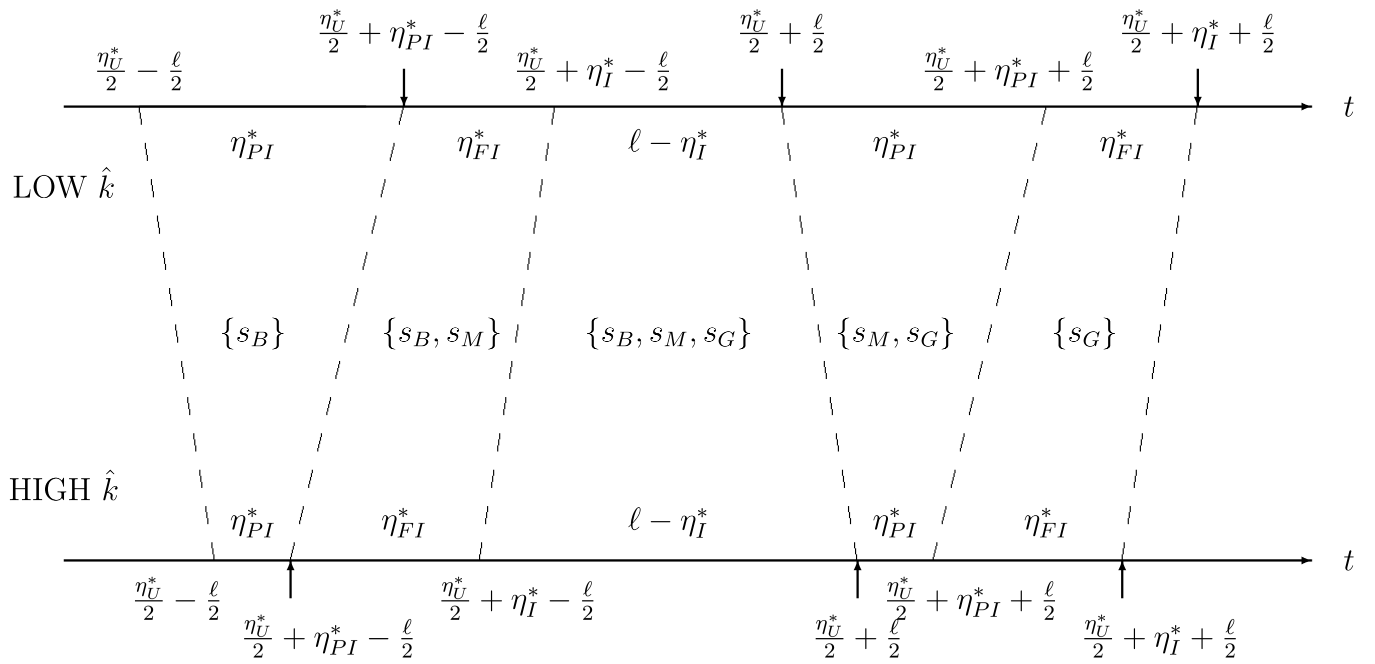

Figure 4.3: The upper line corresponds to a scenario with a low unawareness level, while the lower line corresponds to a scenario with high unawareness. The lines represent order flows t on their support composed of the intervals $I_1, \ldots, I_5$ (from left to right). Each of them is consistent with some of the signal realizations s_B, s_M, s_G and their size depends on the number of partially and fully aware informed traders.

Tesi di dottorato "Essays on Unawareness in Economics"
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induces a shift in the left region of the order flows. Here, information diffusion is limited by unawareness and trading positions reflect an unrealistic optimism due to information shortage. The situation is antisymmetric for high order flows. Here, a more severe degree of unawareness leads to a lower amount of informed investors, who in expectation gain a positive profit after s_G, and hence the highest order flows disappear.

Focusing region by region, we observe an overall change in the internal distribution of orders when the unawareness level increases. The intervals I_2, I_3 and I_5 become larger, while I_1 and I_4 shrink. These effects are due to the fact that, as we have seen, in equilibrium η_I^* and η_{PI}^* are decreasing in the unawareness level, while η_{FI}^* is increasing. The intuition rests, again, on the reduced incentives to gather information of the partially aware investors and on their overoptimistic perception of the intermediate signal. In terms of prices, we observe an increase in the likelihood of high prices (due to order flows in I_5) at the expense of medium-to-high prices, and medium-to-low prices (from order flows in I_2) at the expense of low prices.

To sum up, the reduced incentives to gather information implied by higher levels of unawareness lead to a larger number of uninformed traders, generating a concentration of order flows in the market. In addition, the overoptimistic perception of the intermediate signal by the partially aware agents increases the relative likelihood of medium-to-low and high prices, but per se does not give rise to new price levels.

We can formalize the above observations noting that a sufficiently severe degree of unawareness leads to a negatively skewed price distribution.

Remark 2 *If η_{FI}^* is an internal solution, $P(S = s_G) > 0.5$, and c is not too high, then there exists an unawareness level above which p is negatively skewed.*

The assumption on $P(S = s_G)$ is needed so as not to excessively counteract the shift in the mass of the distribution of p to the right caused by increasing unawareness levels. In Figure 4.4 we depict a plausible distribution of p for increasing unawareness levels and we observe the shift of probabilities. For the empirical literature on skewness of excess returns see, for instance, Harvey and Siddique (2000) and Conrad *et al.* (2013).

The changes in the order flow regions described above have a direct impact on market makers' ability to recognize the signal realization observed by the informed traders, and hence on the price they set. First, since a more severe degree of unawareness leads to a lower amount of partially aware informed agents, who are the only traders buying the risky asset after the intermediate signal, as unawareness increases it becomes more difficult for

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di MADOTTO MATTEO
discussa presso Università Commerciale Luigi Bocconi-Milano nell'anno 2020
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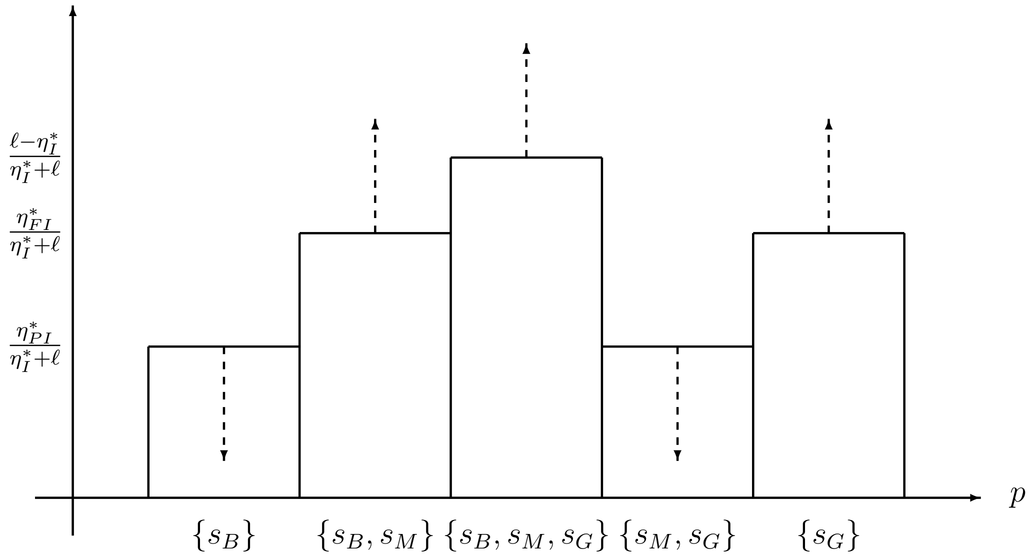

Figure 4.4: A plausible distribution of the equilibrium price p for a given unawareness level. The arrows illustrate the changes in the distribution when $\hat{k}$ rises.

market makers to distinguish s_B from s_M. Indeed, as unawareness rises the regions of order flows consistent with both signals, i.e. I_2 and I_3, expand, while those compatible with only one of such signals, i.e. I_1 and I_4, shrink. In addition, the reduction in the total amount of informed investors leads to a greater confusion also between s_B and s_G. Indeed, as unawareness becomes more severe the only region of order flows consistent with both signal realizations, i.e. I_3, expands. As a result, a greater unawareness level implies that it becomes more difficult for market makers to correctly recognize a bad signal, in the sense that it becomes more likely for them to (i) believe that informed traders received s_B when they actually did not observed it, and (ii) believe that informed traders did not received s_B when they actually observed it.

On the other hand, since more unawareness induces a larger fraction of fully aware traders to get informed, and such investors purchase the risky asset only in response to the good signal, as unawareness rises it becomes easier for market makers to distinguish s_M from s_G. Indeed, the intervals of order flows consistent with only one of such signals, i.e. I_2 and I_5, expand, while the region compatible with both signals, i.e. I_4, shrinks (more than offsetting the expansion of I_3). As a consequence, a more severe degree of unawareness implies that it becomes more unlikely for market maker to believe that informed traders received s_M (resp. s_G) when they actually

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observed s_G (resp. s_M).

The above observations can be summarized in the following proposition.

Proposition 15 *As the unawareness level of the partially aware investors rises, the probability with which market makers*

(i) believe that informed traders received s_B when they actually did not observed it increases;

(ii) believe that informed traders did not received s_B when they actually observed it increases;

(iii) believe that informed traders received s_M (resp. s_G) when they actually observed s_G (resp. s_M) decreases.

The increased confusion of market makers between s_B, on the one hand, and s_M and s_G, on the other, caused by a more severe degree of unawareness, together with their greater ability to distinguish s_M from s_G, pushes the expected prices conditional on the bad and on the intermediate signal closer to each other.[3] Formally, we have the following result.

Proposition 16 *If η^*_{FI} is an internal solution, then $\mathbb{E}[p|S = s_B]$ is increasing in the unawareness level, while $\mathbb{E}[p|S = s_M]$ is decreasing.*

This result can be expressed in terms of the expected price conditional on the asset payoff. In particular, when y is low the prevailing effect is that of $\mathbb{E}[p \mid S = s_B]$ (recall our stochastic dominance assumption). In this case a diminished number of partially aware informed agents, by making low order flows less revealing of a bad signal, keeps average prices high.

Proposition 17 *If η^*_{FI} is an internal solution and $P(S = s_B) \leq P(S = s_M)$, then $E[p|Y = y]$ is increasing in the unawareness level for low enough y.*

The effects of a higher unawareness level on $\mathbb{E}[p \mid S = s_G]$ are instead contrasting. In particular, as we have seen, more unawareness reduces market makers' confusion between s_G and s_M (pointing towards a higher conditional expected price), but it also increases that between s_G and s_B (pointing towards a lower conditional expected price). Indeed, a smaller η^*_{PI} and a larger η^*_{FI} imply that both the region of (relatively high) order flows compatible

[3]The impact of increasing unawareness levels on the volatility of the price conditional on s_B and on s_M involves contrasting effects, and the ultimate result depends on the specific parameter values.

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only with s_G and that of (relatively low) orders consistent with all three signals expand, and it actually turns out that the conditional expected price is constant in $\hat{k}$. By contrast, as the next proposition shows, a more severe degree of unawareness increases the price volatility following a good signal realization.

Proposition 18 *If η^*_{FI} is an internal solution, then $\mathbb{E}[p \mid S = s_G]$ is constant in the unawareness level, while $Var(p \mid S = s_G)$ is increasing.*

The intuition behind the increasing monotonicity of $Var(p \mid S = s_G)$ is as follows. Recall that as unawareness rises the relative amount of fully aware informed investors, who buy the risky asset only after s_G, increases. Consequently, as we have seen, it becomes easier for the market makers to distinguish the good from the intermediate signal, i.e. I_4 shrinks while I_5 expands. At the same time, the region of order flows consistent with all three signals (i.e. I_3) becomes larger. As a result, relatively low and relatively high price levels become prevalent after s_G. That is, the intermediate price level due to the presence of the partially aware informed traders becomes less frequent. Therefore, as unawareness becomes more severe the moderating impact of the partially aware informed investors on price fluctuations after a good signal diminishes, leading to an increased price volatility.

4.4.1 Welfare analysis

In an interior solution the ex ante expected utility of the fully aware informed investors is null, i.e. equal to that of the uninformed, irrespective of the unawareness level of the partially aware. In contrast, the ex ante expected utility of the partially aware informed traders is negative, since they choose the wrong action after s_M. Specifically, since such traders set a buy order after receiving the intermediate or the good signal, this quantity equals

$$\mathbb{E}\left[Y - p|S = s_M\right] P\left(S = s_M\right) + \mathbb{E}\left[Y - p|S = s_G\right] P\left(S = s_G\right) - c$$

and, therefore, it coincides with $\mathbb{E}[Y - p|S = s_M]P\left(S = s_M\right)$, which is negative. Moreover, by Proposition 16, this expression is increasing in the unawareness level. In other words, partially aware informed traders bear an ex ante expected loss due to their distorted perception of the price function, but such loss shrinks when many states of the world are neglected, since fewer partially aware informed traders are present and are therefore less able to affect the price. At the aggregate level, a more severe degree of unawareness lowers the ex ante loss also by diminishing the equilibrium amount of partially aware informed investors, so that fewer agents spend resources buying a signal that in expectation leads to a wrong decision.

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If we compare the ex post expected utilities of the partially and the fully aware agents, that we denote by V_{PI} and V_{FI}, respectively, we have

$$\eta_{PI}^* V_{PI} + \eta_{FI}^* V_{FI} = \begin{cases} -\eta_I^* c & \text{if } S = s_B \\ -\eta_I^* c + \eta_{PI}^* \mathbb{E}\left[Y - p | S = s_M\right] & \text{if } S = s_M \\ \eta_I^* c \left(1/P\left(S = s_G\right) - 1\right) & \text{if } S = s_G. \end{cases}$$

From Propositions 13, 14 and 16 it follows that the aggregate ex post expected utility is decreasing in the unawareness level after s_G, while it is increasing after the other two signals. The exploitation of the good signal requires, indeed, the acquisition of the information. Since a more severe degree of unawareness provides lower incentives to purchase the signal, the aggregate utility after s_G diminishes. On the contrary, neglecting many states of the world is beneficial after a medium or a bad signal, where the information cost constitutes (ex post) a waste of resources, and where therefore a lower number of informed market participants is preferable. In the case of the intermediate signal, this adds up to the beneficial (in terms of welfare) impact of a higher unawareness level on the expected price that we have seen above.

4.5 Comparison with the equilibrium with no partially aware agents

To better assess the impact of partially aware investors on the equilibrium price, we consider a market in which all agents are fully aware, i.e. where $\eta_F = 1$. Recall that uninformed investors are indifferent between buying the risky asset and not doing so, and that we assume that half of them purchases it. By contrast, informed traders set buy orders only after receiving the good signal. Hence, using upper bars to denote the values taken by the variables in this full awareness scenario, the overall order flow is

$$\overline{T} = \overline{Z} + \frac{\overline{\eta}_U}{2} + \begin{cases} 0 & \text{if} \quad S = s_B \\ 0 & \text{if} \quad S = s_M \\ \overline{\eta}_I & \text{if} \quad S = s_G. \end{cases}$$

The support of $\overline{T}$ is divided into the three subintervals illustrated in Figure 4.5.

Observe that the first subinterval corresponds to the union of I_1 and I_2 that we considered when partially aware agents were present. Similarly, the last subinterval corresponds to $I_4 \cup I_5$, while the central interval is the analogue of I_3. When partially aware traders are absent, p takes only three

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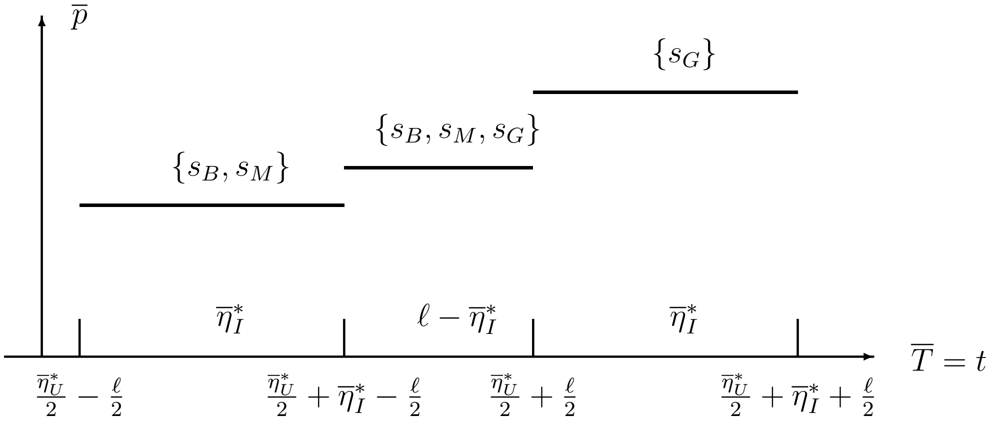

Figure 4.5: Equilibrium price $\overline{p}$ with no partially aware agents as a function of the order flows on their support. Each price level is consistent with some of the signal realizations (written inside the curly brackets). The size of each subinterval depends on the number of informed traders in equilibrium.

of the five values that it takes when partially aware agents are present. This is consistent with setting $\eta_{PI} = 0$ when deriving the equilibrium price in the scenario with partially aware investors: two regions of order flows collapse and the corresponding prices vanish. In particular, the missing price values are the lowest one, given by $\mathbb{E}[Y|S = s_B]$, and the medium-to-high one, given by the average of $\mathbb{E}[Y|S = s_M]$ and $\mathbb{E}[Y|S = s_G]$.

The reduction of order flow regions from five to three is reminiscent of the discussion in Section 4.4 summarized in Figure 4.3. Indeed, increasing the unawareness level induces a shrinkage of the subintervals I_1 and I_4. Such intervals become negligible when unawareness is severe, resulting in an outcome similar to that in the absence of partially aware agents. The intuition behind this common outcome is that most partially aware investors become part of the uninformed population when their unawareness level is high. Indeed, extremely restricted posterior expected utilities can prevent all of the partially aware agents from being incentivized to gather costly information. As a consequence, only fully aware traders get informed and can affect the price through the signal. In a nutshell, the introduction of a moderate degree of unawareness generates novel price values by changing agents' actions in response to some signals while still keeping them incentivized to buy information.

It can be shown that the equilibrium total amount $\overline{\eta}_I^*$ of informed traders

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in the full awareness scenario is given by

$$\overline{\eta}_I^* = \ell \max\left\{1 - \frac{c}{\left(\mathbb{E}\left[Y|S=s_G\right] - \mathbb{E}[Y]\right) P\left(S=s_G\right)}; 0\right\}. \tag{4.4}$$

This expression can be compared with the equilibrium proportion of informed agents η_I^* derived in eq. (4.3) of Proposition 13, where partially aware traders are present. In particular, it can be easily checked that

$$\overline{\eta}_I^* > \eta_I^*.$$

That is, the highest proportion of informed traders attains when partially aware agents are absent. Information acquisition is adversely affected by the presence of partially aware investors. The effect of the presence of partially aware agents on information acquisition is similar to the effect generated by increasing levels of unawareness. Indeed, as shown in Proposition 13, η_I^* is decreasing in the unawareness level because of the restricted posterior expectations of the partially aware. Therefore, both the presence of partially aware investors and their level of unawareness deteriorate information acquisition in the market.

Comparing the order flow $\overline{T}$ in the full awareness scenario with T in the equilibrium with partially aware agents, we have

$$\mathbb{E}[T] = \mathbb{E}\left[\overline{T}\right] + \eta_{PI}^* P\left(S=s_M\right) + \left(0.5 - P\left(S=s_G\right)\right)\left(\overline{\eta}_I^* - \eta_I^*\right).$$

Hence, if $P\left(S=s_G\right) < 0.5$, expected order flows are higher when partially aware investors are present. This occurs because, as we have just seen, partial awareness induces a larger fraction of traders to remain uninformed, which keeps order flows high after the bad and intermediate signals.

As for the aggregate welfare, in an interior solution fully aware informed traders get a zero ex ante expected utility, exactly as in the scenario with a positive fraction of partially aware agents. Focusing signal by signal, and letting $\overline{V}_I$ be the conditional expected utility of the fully aware traders in the full awareness scenario, we have that

$$\overline{\eta}_I^* \overline{V}_I = \begin{cases} -\overline{\eta}_I^* c & \text{if } S = s_B \\ -\overline{\eta}_I^* c & \text{if } S = s_M \\ \overline{\eta}_I^* c\left(1/P\left(S=s_G\right) - 1\right) & \text{if } S = s_G. \end{cases}$$

Since $\overline{\eta}_I^* > \eta_I^*$, it follows that, after s_G, $\overline{\eta}_I^* \overline{V}_I$ is higher than the aggregate ex post expected utility in the market with a positive fraction of partially aware traders, while it is lower after s_B (see Subsection 4.4.1). In other words, the

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larger amount of informed agents implied by the absence of partially aware traders allows an overall better exploitation of the good signal, but it induces more waste of wealth after the bad signal. After s_M the comparison between the aggregate ex post utilities is not unambiguous. Indeed, the presence of partially aware market participants reduces the information gathering, which is beneficial after s_M, but it also entails an incorrect investment by the partially aware informed.

4.6 Conclusion

Recent evidence shows that there have been various episodes in the U.S. history in which many investors were not aware of important risks associated with several kinds of financial instruments. Given these past episodes of neglected risks, and in particular the manifest role of such risks in the recent global financial crisis, it seems reasonable that, when trading certain assets, investors entertain the possibility of being exposed to unknown risks. We study an order-driven financial market in which part of the traders ignores the most negative payoffs of a risky asset, but assigns a positive probability to the event of facing contingencies different from those they are aware of. Such given fraction of partially aware investors believes that all agents in the market are partially aware, and therefore, given the opposite investment choices of partially and fully aware traders after an intermediate signal realization, has a distorted view of the relationships among signals, order flows and asset prices. We study an equilibrium in which each rational agent maximizes his expected profit given his level of awareness, and where the total number of informed traders is consistent with both awareness levels.

We show that when the partially aware agents overlook a larger fraction of possible asset payoffs their response, in terms of the shift in their expectations, to both positive and negative signals gets weaker. As a result, the total amount of informed investors in the market is decreasing in the unawareness level of the partially aware, and the highest number of informed traders is observed when all agents are fully aware. Moreover, a smaller amount of informed traders makes large order flows less revealing of a good signal, lowering the price, and increasing the incentives of the fully aware to get informed. As a consequence, as unawareness rises, incentives to acquire information are transferred to the fully aware investors, and we prove that the equilibrium share of fully aware informed agents is increasing in the unawareness level of the partially aware. We also show that by lowering the incentives of the partially aware to get informed, an increased unawareness level reduces the welfare loss due to their wrong investment decisions

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following an intermediate signal realization.

By lowering the equilibrium amount of partially aware informed traders, who help market makers distinguish between bad and intermediate signal realizations, and the overall fraction of informed investors, who reduce the confusion between bad and good signals, a more severe degree of unawareness makes it harder for market makers to correctly recognize the presence or the absence of bad signals. On the other hand, by increasing the proportion of fully aware traders who get informed and purchase the risky asset only after a good signal, more unawareness implies that it becomes easier for market makers to distinguish between intermediate and good signals. These two main effects on market makers' beliefs have a direct impact on the equilibrium price. On the one hand, the diminished number of partially aware informed agents, by making low order flows less revealing of a bad signal, keeps the average price high when the asset payoff is low. In contrast, since the presence of the partially aware informed, by giving rise to new intermediate price levels, has a moderating effect on price fluctuations after a good signal, their reduction due to a higher unawareness level leads to an increased price volatility following such signal.

Tesi di dottorato "Essays on Unawareness in Economics"
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Appendix C

Conditions for the separating equilibrium

Fully aware traders do not set a buy order after s_M as long as $\mathbb{E}[Y|S = s_M]$ is lower than $\mathbb{E}[p|S = s_M]$. Conversely, the partially aware are willing to buy the risky asset when $\mathbb{E}[\hat{Y}|S = s_M]$ is larger than $\mathbb{E}[\hat{p}|S = s_M]$.

Recalling that $p(t) = \mathbb{E}[Y \mid T = t]$, we have

$$\mathbb{E}\left[p|S = s_M\right] = \int_{\mathrm{supp}T|S=s_M} \sum_{\sigma=s_B,s_M,s_G} \mathbb{E}\left[Y|S = \sigma\right] P\left(S = \sigma|T = t\right) \wp_{T|S=s_M}\left(t\right) dt,$$

where $\wp_{T|S=s_M}(t)$ denotes the conditional probability of $T = t|S = s_M$. An analogous relation holds for $\hat{p}$. Moreover,

$$\begin{aligned}\mathbb{E}\left[p|S = s_M\right] &= \mathbb{E}\left[Y|S = s_G\right]\\ &- \left(\mathbb{E}\left[Y|S = s_G\right] - \mathbb{E}\left[Y|S = s_M\right]\right) \int_{\mathrm{supp}T|S=s_M} P\left(S = s_M|T = t\right) \wp_{T|S=s_M}\left(t\right) dt\\ &- \left(\mathbb{E}\left[Y|S = s_G\right] - \mathbb{E}\left[Y|S = s_B\right]\right) \int_{\mathrm{supp}T|S=s_M} P\left(S = s_B|T = t\right) \wp_{T|S=s_M}\left(t\right) dt\end{aligned}$$

and similarly for the distorted price. As a result, the relations

$$\mathbb{E}\left[Y|S = s_M\right] < \mathbb{E}\left[p|S = s_M\right], \qquad \mathbb{E}\left[\hat{Y}|S = s_M\right] > \mathbb{E}\left[\hat{p}|S = s_M\right]$$

rewrite as

$$\begin{cases}\mathbb{E}\left[Y|S = s_M\right] < \varphi\mathbb{E}\left[Y|S = s_G\right] + (1-\varphi)\mathbb{E}\left[Y|S = s_B\right]\\ \mathbb{E}\left[\hat{Y}|S = s_M\right] > \hat{\varphi}\mathbb{E}\left[\hat{Y}|S = s_G\right] + (1-\hat{\varphi})\mathbb{E}\left[\hat{Y}|S = s_B\right],\end{cases} \tag{4.5}$$

where

$$\varphi \equiv \frac{\int_{\mathrm{supp}T|S=s_M} P\left(S = s_G|T = t\right) \wp_{T|S=s_M}\left(t\right) dt}{\int_{\mathrm{supp}T|S=s_M} \left(P\left(S = s_G|T = t\right) + P\left(S = s_B|T = t\right)\right) \wp_{T|S=s_M}\left(t\right) dt} \tag{4.6}$$

and

$$\hat{\varphi} \equiv \frac{\int_{\mathrm{supp}\hat{T}|S=s_M} P\left(S = s_G|\hat{T} = t\right) \wp_{\hat{T}|S=s_M}\left(t\right) dt}{\int_{\mathrm{supp}\hat{T}|S=s_M} \left(P\left(S = s_G|\hat{T} = t\right) + P\left(S = s_B|\hat{T} = t\right)\right) \wp_{\hat{T}|S=s_M}\left(t\right) dt} \tag{4.7}$$

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Both φ and $\hat{\varphi}$ lie between 0 and 1. In addition, φ is a function of η_I and η_{FI}, while $\hat{\varphi}$ depends solely on η_I. In particular,

$$\hat{\varphi} = \min\left\{\frac{P(S=s_G)\left((1-P(S=s_B))\,\ell + P(S=s_B)\,\eta_I\right)}{\left(P(S=s_G)+P(S=s_B)\,P(S=s_M)\right)\ell - P(S=s_B)\,P(S=s_M)\,\eta_I};\ 1\right\}.$$

The equilibrium value η_I^* is provided in Proposition 13 and $\hat{\varphi}$ becomes

$$\hat{\varphi}(\eta_I^*) = P(S=s_G)\,\frac{c^* - P(S=s_B)\,c}{P(S=s_G)\,c^* + P(S=s_B)\,P(S=s_M)\,c}.$$

Note that this value is independent of the noise parameter.

The expression of φ is more convoluted than the one of $\hat{\varphi}$ (see the proof of the following proposition). However, it is easy to compute the range of φ when η_I^* is fixed and η_{FI} varies in a subinterval of $[0, \eta_I^*]$.

Proposition 19 *The following hold.*

(i) $\hat{\varphi}$ *is increasing in* η_I *and* $\hat{\varphi}(\eta_I^*)$ *is decreasing in the unawareness level.*

(ii) Given η_I^**, if* $P(S=s_B) \geqslant P(S=s_G)$*, then* φ *is decreasing in* η_{FI}*.*

Proof. From eq. (4.7) we derive that $\hat{\varphi}(\eta_I) = 1$ for all $\eta_I > \ell$, while

$$\hat{\varphi}(\eta_I) = \frac{P(S=s_G)\left((1-P(S=s_B))\,\ell + P(S=s_B)\,\eta_I\right)}{\left(P(S=s_G)+P(S=s_B)\,P(S=s_M)\right)\ell - P(S=s_B)\,P(S=s_M)\,\eta_I}$$

for all $\eta_I \leqslant \ell$. Thus, $\hat{\varphi}(\eta_I)$ defines an increasing hyperbola with vertical asymptote at $\eta_I = \ell/(P(S=s_M)+P(S=s_B))$. Restricted on the interval $[0,\ell]$, $\hat{\varphi}$ is therefore increasing in η_I.

By substituting the equilibrium value η_I^* of eq. (4.3) and using c^* defined in eq. (4.2), we obtain

$$\hat{\varphi}(\eta_I^*) = P(S=s_G)\,\frac{c^* - P(S=s_B)\,c}{P(S=s_G)\,c^* + P(S=s_B)\,P(S=s_M)\,c},$$

which defines an increasing hyperbola in c^* with vertical asymptote in $c^* = -P(S=s_B)P(S=s_M)c/P(S=s_G)$. Thus, $\hat{\varphi}(\eta_I^*)$ is increasing in c^* that, in turn, is decreasing in the unawareness level because of its negative relation with $\mathbb{E}[\hat{Y}|S=s_B]$. Hence, $\hat{\varphi}(\eta_I^*)$ is decreasing in the unawareness level.

As for $\varphi(\eta_{FI})$, from eq. (4.6) and considering the equilibrium value η_I^*, we obtain

$$\varphi(\eta_{FI}) = \frac{P(S=s_G)\,(1-P(S=s_G))\,\left(\ell - P(S=s_B)\,(\ell-\eta_I^*) - \eta_{FI}\right)}{D},$$

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where the denominator is

$$D \equiv (1 - P(S = s_G))(P(S = s_G)\ell + P(S = s_M)P(S = s_B)(\ell - \eta_I^*)) \\ + P(S = s_M)(P(S = s_B) - P(S = s_G))\eta_{FI}.$$

If $P(S = s_B) = P(S = s_G)$, then $\varphi(\eta_{FI})$ defines a downward sloping straight line in η_{FI}, and so it is decreasing in η_{FI}.

On the contrary, if $P(S = s_B) \neq P(S = s_G)$, then $\varphi(\eta_{FI})$ is a decreasing hyperbola in η_{FI} with vertical asymptote at

$$\eta_{FI} = \frac{(1 - P(S = s_G))(P(S = s_G)\ell + P(S = s_M)P(S = s_B)(\ell - \eta_I^*))}{P(S = s_M)(P(S = s_G) - P(S = s_B))}.$$

If $P(S = s_B) > P(S = s_G)$, the value of η_{FI} that defines the asymptote is negative and so $\varphi(\eta_{FI})$ is decreasing for all positive values of η_{FI}. ■

From (ii), if $P(S = s_B) \geqslant P(S = s_G)$, it is possible to find a sufficient condition for the first inequality in (4.5), i.e.

$$\mathbb{E}[Y|S = s_M] < \varphi(\min\{\eta_F, \eta_I^*\})\mathbb{E}[Y|S = s_G] + (1 - \varphi(\min\{\eta_F, \eta_I^*\}))\mathbb{E}[Y|S = s_B].$$

Indeed, η_{FI} is constrained between $\max\{\eta_I^* - \eta_P, 0\}$ and $\min\{\eta_F, \eta_I^*\}$.

Proof of Proposition 12

Without loss of generality, consider $\neg\mathcal{E}_1 \equiv \{y_1, y_2, \ldots, y_{\hat{k}-1}\}$ and $\neg\mathcal{E}_2 \equiv \neg\mathcal{E}_1 \cup \{y_{\hat{k}}\}$ and, accordingly, the variables $\hat{Y}_1$ and $\hat{Y}_2$ that take values x_1 and x_2 in the event an unknown outcome occurs.

We only show that $\mathbb{E}[\hat{Y}_2|S = s_G] < \mathbb{E}[\hat{Y}_1|S = s_G]$ because the inequalities with s_M and s_B are analogous. Equivalently, we prove that

$$\mathbb{E}[\hat{Y}_2|S = s_G] - \mathbb{E}[Y|S = s_G] < \mathbb{E}[\hat{Y}_1|S = s_G] - \mathbb{E}[Y|S = s_G].$$

By Assumptions (A2) and (A3), the last inequality can be rewritten as

$$\sum_{i=1}^{\hat{k}} P(Y = y_i|S = s_G)y_i - P(Y \in \neg\mathcal{E}_2|S = s_G)x_2 \\ > \sum_{i=1}^{\hat{k}-1} P(Y = y_i|S = s_G)y_i - P(Y \in \neg\mathcal{E}_1|S = s_G)x_1. \tag{4.8}$$

Tesi di dottorato "Essays on Unawareness in Economics"
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Define, for $j = 1, 2$, the quantity $\bar{\pi}_j \equiv \sum_{i=1}^{|\neg\mathcal{E}_j|} P(S = s_G|Y = y_i)/|\neg\mathcal{E}_j|$, so that

$$P\left(Y \in \neg\mathcal{E}_j|S = s_G\right) x_j = \frac{1}{kP\left(S = s_G\right)} \sum_{i=1}^{|\neg\mathcal{E}_j|} \bar{\pi}_j y_i.$$

Therefore, the inequality in (4.8) becomes

$$\sum_{i=1}^{\hat{k}} \left(P\left(Y = y_i|S = s_G\right) P\left(S = s_G\right) - \frac{\bar{\pi}_2}{k}\right) y_i > \sum_{i=1}^{\hat{k}-1} \left(P\left(Y = y_i|S = s_G\right) P\left(S = s_G\right) - \frac{\bar{\pi}_1}{k}\right) y_i$$

and, since $P(Y = y_i|S = s_G)P(S = s_G) = P(S = s_G|Y = y_i)/k$,

$$\sum_{i=1}^{\hat{k}} \left(P\left(S = s_G|Y = y_i\right) - \bar{\pi}_2\right) y_i > \sum_{i=1}^{\hat{k}-1} \left(P\left(S = s_G|Y = y_i\right) - \bar{\pi}_1\right) y_i. \quad (4.9)$$

The left-hand side of the last inequality equals

$$\sum_{i=1}^{\hat{k}-1} \left(P\left(S = s_G|Y = y_i\right) - \bar{\pi}_1\right) y_i + \sum_{i=1}^{\hat{k}-1} \left(\bar{\pi}_1 - \bar{\pi}_2\right) y_i + \left(P\left(S = s_G|Y = y_{\hat{k}}\right) - \bar{\pi}_2\right) y_{\hat{k}}.$$

As a result, (4.9) becomes

$$\sum_{i=1}^{\hat{k}-1} \left(\bar{\pi}_1 - \bar{\pi}_2\right) y_i + \left(P\left(S = s_G|Y = y_{\hat{k}}\right) - \bar{\pi}_2\right) y_{\hat{k}} > 0.$$

Since $\hat{k}\bar{\pi}_2 = (\hat{k} - 1)\bar{\pi}_1 + P(S = s_G|Y = y_{\hat{k}})$, the last inequality rewrites as

$$\sum_{i=1}^{\hat{k}-1} \left(\bar{\pi}_1 - P\left(S = s_G|Y = y_{\hat{k}}\right)\right) y_i + \left(\hat{k} - 1\right) \left(P\left(S = s_G|Y = y_{\hat{k}}\right) - \bar{\pi}_1\right) y_{\hat{k}} > 0,$$

that is

$$\left(P\left(S = s_G|Y = y_{\hat{k}}\right) - \bar{\pi}_1\right) \left(\left(\hat{k} - 1\right) y_{\hat{k}} - \sum_{i=1}^{\hat{k}-1} y_i\right) > 0.$$

Here, the first factor is positive because of the increasing monotonicity of $P(S = s_G|Y = y)$ in y. The second factor is positive because $y_{\hat{k}} > y_i$ for all $i = 1, \ldots, \hat{k} - 1$. Thus, the proof of the inequalities involving the conditional expected utilities of agents with different unawareness levels is complete.

The last three inequalities in the claim of the proposition follow immediately by setting the unawareness level of one agent equal to zero.

Tesi di dottorato "Essays on Unawareness in Economics"
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Proof of Proposition 13

Recalling that partially aware informed traders buy the risky asset after receiving s_G or s_M, their ex ante expected utility is

$$\begin{aligned}\hat{U}_{PI}(\eta_I) = &\left(\mathbb{E}\left[\hat{Y}|S=s_G\right] - \mathbb{E}\left[\hat{p}|S=s_G\right]\right) P(S=s_G)\\ &+ \left(\mathbb{E}\left[\hat{Y}|S=s_M\right] - \mathbb{E}\left[\hat{p}|S=s_M\right]\right) P(S=s_M) - c.\end{aligned}$$

Note that for any $s = s_B, s_M, s_G$

$$\mathbb{E}\left[\hat{p}|S=s\right] = \int_{\mathrm{supp}\hat{T}|S=s} \sum_{\sigma=s_B,s_M,s_G} \mathbb{E}\left[\hat{Y}|S=\sigma\right] P\left(S=\sigma|\hat{T}=t\right) \wp_{\hat{T}|S=s}(t)\, dt.$$

Since the densities $\wp_{\hat{T}|S=s_G}$ and $\wp_{\hat{T}|S=s_M}$ coincide, $\mathbb{E}[\hat{p}|S=s_G] = \mathbb{E}[\hat{p}|S=s_M]$ and so

$$\begin{aligned}\hat{U}_{PI}(\eta_I) = &\ \mathbb{E}\left[\hat{Y}|S=s_G\right] P(S=s_G) + \mathbb{E}\left[\hat{Y}|S=s_M\right] P(S=s_M)\\ &- \mathbb{E}\left[\hat{p}|S=s_G\right] \left(P(S=s_G) + P(S=s_M)\right) - c. \qquad (4.10)\end{aligned}$$

It holds

$$\begin{aligned}\mathbb{E}\left[\hat{p}|S=s_G\right] = \frac{1}{\ell}\int_{\frac{\eta_U}{2}+\eta_I-\frac{\ell}{2}}^{\frac{\eta_U}{2}+\eta_I+\frac{\ell}{2}} &\left(\frac{\mathbb{E}\left[\hat{Y}|S=s_G\right] P(S=s_G)}{P(S=s_G)+P(S=s_M)+1_{[\eta_U/2]}(t)P(S=s_B)}\right.\\ &+ \frac{\mathbb{E}\left[\hat{Y}|S=s_M\right] P(S=s_M)}{P(S=s_G)+P(S=s_M)+1_{[\eta_U/2]}(t)P(S=s_B)}\\ &\left.+ \mathbb{E}\left[\hat{Y}|S=s_B\right] P(S=s_B)\, 1_{[\eta_U/2]}(t)\right) dt,\end{aligned}$$

and thus

$$\begin{aligned}\mathbb{E}\left[\hat{p}|S=s_G\right] = \frac{1}{\ell}&\left(\mathbb{E}[Y]\max\{\ell-\eta_I;0\}\right.\\ &\left.+ \frac{\mathbb{E}\left[\hat{Y}|S=s_G\right] P(S=s_G) + \mathbb{E}\left[\hat{Y}|S=s_M\right] P(S=s_M)}{P(S=s_G)+P(S=s_M)}\eta_I\right).\end{aligned}$$

We find the equilibrium value η_I^* by comparing the ex ante expected utility of the partially aware informed investors with the one of the partially aware uninformed, which is null. Using the definition of c^* in (4.2), $\hat{U}_{PI}$ can be rewritten as

$$\begin{aligned}\hat{U}_{PI}(\eta_I) = &\left(c^* + \mathbb{E}[Y]\left(1 - P(S=s_B)\right)\right)\left(1 - \frac{\eta_I}{\ell}\right)\\ &- \mathbb{E}[Y]\left(1-P(S=s_B)\right)\max\left\{1-\frac{\eta_I}{\ell};0\right\} - c\end{aligned}$$

Tesi di dottorato "Essays on Unawareness in Economics"
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with $\hat{U}_{PI}(0) = c^* - c$ and $\hat{U}_{PI}(\ell) = -c$. Therefore, if $c \geqslant c^*$, $\hat{U}_{PI}(0) \leqslant 0$ and $\eta_I^* = 0$. On the contrary, if $c < c^*$, $\hat{U}_{PI}$ is null in $\eta_I^* = \ell(1 - c/c^*)$ and eq. (4.3) obtains.

Finally, we analyze the dependence of η_I^* from the unawareness level. If $\hat{k}$ increases, by Proposition 12 $\mathbb{E}[\hat{Y}|S = s_B]$ rises, and so c^* and η_I^* decrease.

Proof of Proposition 14

We build upon the proof of Proposition 13. Fully aware informed agents do not set buy orders after the signal s_M and so their ex ante utility is

$$U_{FI}(\eta_{FI}) = (\mathbb{E}[Y|S = s_G] - \mathbb{E}[p|S = s_G])\, P(S = s_G) - c. \tag{4.11}$$

By fixing η_I^* and η_U^*, we get that $\mathbb{E}[p|S = s_G]$ is equal to

$$\begin{aligned}
&\frac{1}{\ell}\int_{\frac{\eta_U^*}{2}+\eta_I^*-\frac{\ell}{2}}^{\frac{\eta_U^*}{2}+\eta_I^*+\frac{\ell}{2}} \left(\frac{\mathbb{E}[Y|S = s_G]\, P(S = s_G)}{P(S = s_G) + 1_{[\eta_U^*/2+\eta_{PI}]}(t) P(S = s_M) + 1_{[\eta_U^*/2]}(t) P(S = s_B)} \right.\\
&+ \frac{\mathbb{E}[Y|S = s_M]\, P(S = s_M)\, 1_{[\eta_U^*/2+\eta_{PI}]}(t)}{P(S = s_G) + 1_{[\eta_U^*/2+\eta_{PI}]}(t) P(S = s_M) + 1_{[\eta_U^*/2]}(t) P(S = s_B)}\\
&\left. + \frac{\mathbb{E}[Y|S = s_B]\, P(S = s_B)\, 1_{[\eta_U^*/2]}(t)}{P(S = s_G) + 1_{[\eta_U^*/2+\eta_{PI}]}(t) P(S = s_M) + 1_{[\eta_U^*/2]}(t) P(S = s_B)} \right) dt.
\end{aligned}$$

Since $\eta_{PI} = \eta_I^* - \eta_{FI}$, it follows that

$$\begin{aligned}
\mathbb{E}[p|S = s_G] = \frac{1}{\ell}\Bigg(&\mathbb{E}[Y] \max\{\ell - \eta_I^*; 0\}\\
&+ \frac{\mathbb{E}[Y|S = s_G]\, P(S = s_G) + \mathbb{E}[Y|S = s_M]\, P(S = s_M)}{P(S = s_G) + P(S = s_M)} \eta_I^*\\
&+ \frac{\mathbb{E}[Y|S = s_G] - \mathbb{E}[Y|S = s_M]}{P(S = s_G) + P(S = s_M)} P(S = s_M)\, \eta_{FI} \Bigg).
\end{aligned} \tag{4.12}$$

From eq. (4.3) the equilibrium value η_I^* never exceeds ℓ. Therefore, $\mathbb{E}[p|S = s_G]$ becomes

$$\begin{aligned}
\mathbb{E}[p|S = s_G] = \mathbb{E}[Y] + (\mathbb{E}[Y] - \mathbb{E}[Y|\, S = s_B])\, &\frac{P(S = s_B)}{1 - P(S = s_B)} \max\left\{1 - \frac{c}{c^*}; 0\right\}\\
&+ \frac{\bar{c}}{P(S = s_G)} \frac{\eta_{FI}}{\ell},
\end{aligned}$$

Tesi di dottorato "Essays on Unawareness in Economics"
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where $\bar{c} \equiv (\mathbb{E}[Y|S=s_G] - \mathbb{E}[Y|S=s_M]) \frac{P(S=s_M)P(S=s_G)}{1-P(S=s_B)}$. If $c \geqslant c^*$, then $\eta_I^* = 0$ and so $\eta_{FI}^* = 0$. On the contrary, if $c < c^*$,

$$\begin{aligned}\mathbb{E}[p|S=s_G] = &\frac{\mathbb{E}[Y] - \mathbb{E}[Y|\,S=s_B]P(S=s_B)}{1-P(S=s_B)} \\ &- \frac{(\mathbb{E}[Y] - \mathbb{E}[Y|\,S=s_B])\,P(S=s_B)}{1-P(S=s_B)}\frac{c}{c^*} + \frac{\bar{c}}{P(S=s_G)}\frac{\eta_{FI}}{\ell}\end{aligned}$$

and so

$$U_{FI}(\eta_{FI}) = \bar{c} + \frac{(\mathbb{E}[Y] - \mathbb{E}[Y|\,S=s_B])\,P(S=s_B)\,P(S=s_G)}{1-P(S=s_B)}\frac{c}{c^*} - \bar{c}\frac{\eta_{FI}}{\ell} - c$$

for all η_{FI} that satisfy the constraint

$$\max\left\{\ell\left(1-\frac{c}{c^*}\right) - \eta_P; 0\right\} \leqslant \eta_{FI} \leqslant \min\left\{\eta_F; \ell\left(1-\frac{c}{c^*}\right)\right\}.$$

If η_{FI}^* is an internal solution of the equation $U_{FI}(\eta_{FI}) = 0$ obtained from the comparison with the ex ante utility of the fully aware uninformed agents, we have

$$\eta_{FI}^* = \ell\left(1 + \left(\frac{\mathbb{E}[Y] - \mathbb{E}[Y|S=s_B]}{c^*}\frac{P(S=s_B)\,P(S=s_G)}{1-P(S=s_B)} - 1\right)\frac{c}{\bar{c}}\right). \tag{4.13}$$

Hence η_{FI}^*, seen as a function of $\mathbb{E}[\hat{Y}|S=s_B]$, defines an increasing branch of hyperbola. Since $\mathbb{E}[\hat{Y}|S=s_B]$ is increasing in the unawareness level (see Proposition 12), η_{FI}^* is increasing, too.

Finally, consider $\eta_{PI}^* = \eta_I^* - \eta_{FI}^*$. By Proposition 13, η_I^* is decreasing in the unawareness level and so is η_{PI}^*.

Proof of Remark 2

We show that the third centred moment of p is negative for high levels of unawareness. Consider the five intervals of order flows $I_1, \ldots, I_5$ described in Section 4.4. Since p is piecewise constant on them,

$$\mathbb{E}\left[(p-\mathbb{E}[p])^3\right] = \sum_{j=1}^{5}\mathbb{E}\left[(p-\mathbb{E}[Y])^3\,1_{I_j}(T)\right] = \sum_{j=1}^{5}\left(p1_{I_j}(T) - \mathbb{E}[Y]\right)^3\mathbb{E}\left[1_{I_j}(T)\right].$$

Tesi di dottorato "Essays on Unawareness in Economics"
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Hence, $\mathbb{E}\left[(p-\mathbb{E}[p])^3\right]$ is equal to

$$\begin{aligned}
&(\mathbb{E}[Y|S=s_B]-\mathbb{E}[Y])^3\,\mathbb{E}[1_{I_1}(T)]\\
&+\left(\frac{\mathbb{E}[Y|S=s_M]P(S=s_M)+\mathbb{E}[Y|S=s_B]P(S=s_B)}{P(S=s_M)+P(S=s_B)}-\mathbb{E}[Y]\right)^3\mathbb{E}[1_{I_2}(T)]\\
&+\left(\frac{\mathbb{E}[Y|S=s_G]P(S=s_G)+\mathbb{E}[Y|S=s_M]P(S=s_M)}{P(S=s_G)+P(S=s_M)}-\mathbb{E}[Y]\right)^3\mathbb{E}[1_{I_4}(T)]\\
&+(\mathbb{E}[Y|S=s_G]-\mathbb{E}[Y])^3\,\mathbb{E}[1_{I_5}(T)].
\end{aligned}$$

This can be rewritten as

$$\begin{aligned}
&\ell\,\mathbb{E}\left[(p-\mathbb{E}[p])^3\right]=(\mathbb{E}[Y|S=s_B]-\mathbb{E}[Y])^3\,P(S=s_B)\,\eta^*_{PI}\\
&+\left(\frac{\mathbb{E}[Y|S=s_M]P(S=s_M)+\mathbb{E}[Y|S=s_B]P(S=s_B)}{P(S=s_M)+P(S=s_B)}-\mathbb{E}[Y]\right)^3\\
&\cdot(P(S=s_M)+P(S=s_B))\,\eta^*_{FI}\\
&+\left(\frac{\mathbb{E}[Y|S=s_G]P(S=s_G)+\mathbb{E}[Y|S=s_M]P(S=s_M)}{P(S=s_G)+P(S=s_M)}-\mathbb{E}[Y]\right)^3\\
&\cdot(P(S=s_G)+P(S=s_M))\,\eta^*_{PI}\\
&+(\mathbb{E}[Y|S=s_G]-\mathbb{E}[Y])^3\,P(S=s_G)\,\eta^*_{FI}.
\end{aligned}$$

We then have (see the relations in (4.14))

$$\begin{aligned}
\ell\mathbb{E}\left[(p-\mathbb{E}[p])^3\right]=&-(\mathbb{E}[Y]-\mathbb{E}[Y|S=s_B])^3\,P(S=s_B)\,\eta^*_{PI}\\
&-\frac{(P(S=s_G))^3}{(1-P(S=s_G))^3}(\mathbb{E}[Y|S=s_G]-\mathbb{E}[Y])^3(1-P(S=s_G))\,\eta^*_{FI}\\
&+\frac{(P(S=s_B))^3}{(1-P(S=s_B))^3}(\mathbb{E}[Y]-\mathbb{E}[Y|S=s_B])^3(1-P(S=s_B))\,\eta^*_{PI}\\
&+(\mathbb{E}[Y|S=s_G]-\mathbb{E}[Y])^3\,P(S=s_G)\,\eta^*_{FI}
\end{aligned}$$

and so

$$\begin{aligned}
\ell\mathbb{E}\left[(p-\mathbb{E}[p])^3\right]=&(\mathbb{E}[Y]-\mathbb{E}[Y|S=s_B])^3\,\frac{P(S=s_B)(2P(S=s_B)-1)}{(1-P(S=s_B))^2}\eta^*_{PI}\\
&+(\mathbb{E}[Y|S=s_G]-\mathbb{E}[Y])^3\,\frac{P(S=s_G)(1-2P(S=s_G))}{(1-P(S=s_G))^2}\eta^*_{FI}.
\end{aligned}$$

By Proposition 14, η^*_{PI} is decreasing in the unawareness level. Moreover, $\eta^*_{PI}=0$ when the unawareness level ensures that

$$\mathbb{E}\left[\hat{Y}|S=s_B\right]=\mathbb{E}[Y]-\frac{P(S=s_G)}{1-P(S=s_B)}(\mathbb{E}[Y]-\mathbb{E}[Y|S=s_B])-\frac{\bar{c}}{P(S=s_B)}.$$

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Therefore, there exists an unawareness level such that η^*_{PI} is arbitrarily small (and it can be shown that for such unawareness level η^*_{FI} is nonnull). As a result, the prevailing term in the centred third moment of p is the product containing η^*_{FI}. Since $P(S = s_G) > 0.5$, this product is negative. Thus, the skewness of p is ultimately negative.

Proof of Proposition 15

Let $P(s_i \mid s_j)$ denote the probability that market makers assign to signal s_i when informed traders observe signal s_j, with $i, j = B, M, G$. It holds

$$P(s_i \mid s_j) = \int_{\text{supp}T|S=s_i} P(S = s_j | T = t)\, \wp_{T|S=s_i}(t)\, dt.$$

As for (i), we have

$$P(s_B \mid s_M) = \frac{\eta^*_{FI}}{\ell} \frac{P(S = s_B)}{P(S = s_B) + P(S = s_M)} + \frac{\ell - \eta^*_I}{\ell} P(S = s_B)$$

and

$$P(s_B \mid s_G) = \frac{\ell - \eta^*_I}{\ell} P(S = s_B).$$

Since η^*_{FI} is increasing in $\hat{k}$ and η^*_I is decreasing, both $P(s_B \mid s_M)$ and $P(s_B \mid s_G)$ are increasing in the unawareness level.

As for (ii), it holds

$$P(s_M \mid s_B) = \frac{\eta^*_{FI}}{\ell} \frac{P(S = s_M)}{P(S = s_B) + P(S = s_M)} + \frac{\ell - \eta^*_I}{\ell} P(S = s_M)$$

and

$$P(s_G \mid s_B) = \frac{\ell - \eta^*_I}{\ell} P(S = s_G).$$

Since η^*_{FI} is increasing in $\hat{k}$ and η^*_I is decreasing, both $P(s_M \mid s_B)$ and $P(s_G \mid s_B)$ are increasing in the unawareness level.

Finally, as for (iii) we have

$$P(s_M \mid s_G) = \frac{\ell - \eta^*_I}{\ell} P(S = s_M) + \frac{\eta^*_{PI}}{\ell} \frac{P(S = s_M)}{P(S = s_M) + P(S = s_G)}$$

and

$$P(s_G \mid s_M) = \frac{\ell - \eta^*_I}{\ell} P(S = s_G) + \frac{\eta^*_{PI}}{\ell} \frac{P(S = s_G)}{P(S = s_M) + P(S = s_G)}.$$

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Noting that the decrease in η^*_{PI} after a rise in $\hat{k}$ is greater (in absolute value) than that of η^*_I (since $\eta^*_I = \eta^*_{PI} + \eta^*_{FI}$ and η^*_{FI} is increasing in $\hat{k}$), we see that the decrease in the second terms on the right-hand sides of the last two equations following an increase in $\hat{k}$ more than offsets the rise in the corresponding first terms. As a result, both conditional probabilities are decreasing in the unawareness level.

Proof of Proposition 16

First, by setting the ex ante utility U_{FI} of partially aware investors in eq. (4.11) equal to zero, we get

$$\mathbb{E}\left[p|S=s_G\right] = \mathbb{E}\left[Y|S=s_G\right] - \frac{c}{P\left(S=s_G\right)}$$

and so $\mathbb{E}\left[p|S=s_G\right]$ is constant in the unawareness level.

Now we focus on $\mathbb{E}[p|S=s_B]$ and we follow the arguments in Proposition 13:

$$\begin{aligned}
&\mathbb{E}\left[p|S=s_B\right] \\
&= \frac{1}{\ell}\int_{\frac{\eta^*_U}{2}-\frac{\ell}{2}}^{\frac{\eta^*_U}{2}+\frac{\ell}{2}} \left(\frac{\mathbb{E}\left[Y|S=s_G\right]P\left(S=s_G\right)1_{\left[\eta^*_U/2+\eta^*_I\right]}(t)}{1_{\left[\eta^*_U/2+\eta^*_I\right]}(t)P\left(S=s_G\right)+1_{\left[\eta^*_U/2+\eta^*_{PI}\right]}(t)P\left(S=s_M\right)+P\left(S=s_B\right)} \right.\\
&\quad + \frac{\mathbb{E}\left[Y|S=s_M\right]P\left(S=s_M\right)1_{\left[\eta^*_U/2+\eta^*_{PI}\right]}(t)}{1_{\left[\eta^*_U/2+\eta^*_I\right]}(t)P\left(S=s_G\right)+1_{\left[\eta^*_U/2+\eta^*_{PI}\right]}(t)P\left(S=s_M\right)+P\left(S=s_B\right)} \\
&\quad \left. + \frac{\mathbb{E}\left[Y|S=s_B\right]P\left(S=s_B\right)}{1_{\left[\eta^*_U/2+\eta^*_I\right]}(t)P\left(S=s_G\right)+1_{\left[\eta^*_U/2+\eta^*_{PI}\right]}(t)P\left(S=s_M\right)+P\left(S=s_B\right)} \right) dt
\end{aligned}$$

and so

$$\begin{aligned}
\mathbb{E}\left[p|S=s_B\right] &= \frac{1}{\ell}\left(\mathbb{E}\left[Y\right]\max\{\ell-\eta^*_I;0\}+\mathbb{E}\left[Y|S=s_B\right]\eta^*_I\right) \\
&\quad + \frac{1}{\ell}\left(\mathbb{E}\left[Y|S=s_M\right]-\mathbb{E}\left[Y|S=s_B\right]\right)\frac{P\left(S=s_M\right)}{1-P\left(S=s_G\right)}\eta^*_{FI} \\
&= \mathbb{E}\left[Y\right]-\left(\mathbb{E}\left[Y\right]-\mathbb{E}\left[Y|S=s_B\right]\right)\frac{\eta^*_I}{\ell} \\
&\quad + \left(\mathbb{E}\left[Y|S=s_M\right]-\mathbb{E}\left[Y|S=s_B\right]\right)\frac{P\left(S=s_M\right)}{1-P\left(S=s_G\right)}\frac{\eta^*_{FI}}{\ell}.
\end{aligned}$$

Since η^*_I is decreasing in the unawareness level and η^*_{FI} is increasing, we conclude that $\mathbb{E}[p|S=s_B]$ is increasing in the unawareness level.

Tesi di dottorato "Essays on Unawareness in Economics"
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Finally, to establish the decreasing monotonicity of $\mathbb{E}[p|S = s_M]$ with respect to the unawareness level, it is enough to observe that

$$\mathbb{E}[p] = \mathbb{E}[Y] = \mathbb{E}\left[p|S = s_G\right] P\left(S = s_G\right) + \mathbb{E}\left[p|S = s_M\right] P\left(S = s_M\right) + \mathbb{E}\left[p|S = s_B\right] P\left(S = s_B\right),$$

where $\mathbb{E}[p|S = s_G]$ is constant in the unawareness level, while $\mathbb{E}[p|S = s_B]$ is increasing.

Proof of Proposition 17

First, note that

$$\begin{aligned}
&\mathbb{E}\left[p|Y = y\right] \\
&= \mathbb{E}\left[p|S = s_G\right] P\left(S = s_G|Y = y\right) + \mathbb{E}\left[p|S = s_B\right] P\left(S = s_B|Y = y\right) \\
&\quad + \left(\mathbb{E}[p] - \mathbb{E}\left[p|S = s_G\right] P\left(S = s_G\right) - \mathbb{E}\left[p|S = s_B\right] P\left(S = s_B\right)\right) \frac{P\left(S = s_M|Y = y\right)}{P\left(S = s_M\right)} \\
&= \frac{1}{P\left(S = s_M\right)} \big\{ \mathbb{E}[Y] P\left(S = s_M|Y = y\right) \\
&\quad + \mathbb{E}\left[p|S = s_G\right]\left(\left(1 - P\left(S = s_B|Y = y\right)\right) P\left(S = s_M\right) - \left(1 - P\left(S = s_B\right)\right) P\left(S = s_M|Y = y\right)\right) \\
&\quad + \mathbb{E}\left[p|S = s_B\right]\left(P\left(S = s_B|Y = y\right) P\left(S = s_M\right) - P\left(S = s_B\right) P\left(S = s_M|Y = y\right)\right) \big\}.
\end{aligned}$$

As shown in the proof of Proposition 16, $\mathbb{E}\left[p|S = s_G\right]$ is constant in the unawareness level, while $\mathbb{E}\left[p|S = s_B\right]$ is increasing. Assume $P(S = s_B) \leq P(S = s_M)$. Then, recalling that $Y \mid S = s_M$ dominates $Y \mid S = s_B$, there exists $\bar{y} \in Y$ such that for all $y \leq \bar{y}$

$$P\left(S = s_B|Y = y\right) P\left(S = s_M\right) - P\left(S = s_B\right) P\left(S = s_M|Y = y\right) > 0.$$

Consequently, $\mathbb{E}\left[p|Y = y\right]$ is increasing in the unawareness level for all $y \leq \bar{y}$.

Proof of Proposition 18

We already proved the first part of the claim in the proof of Proposition 16. Since $\mathbb{E}[p|S = s_G]$ is constant in the unawareness level, it is enough to study

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$\mathbb{E}[p^2|S=s_G]$. We find

$$\ell\mathbb{E}\left[p^2|S=s_G\right]=\ell\int_{\text{supp}T|S=s_G}p^2(t)\wp_{T|S=s_G}(t)dt=\int_{\frac{\eta_U^*}{2}+\eta_I^*-\frac{\ell}{2}}^{\frac{\eta_U^*}{2}+\frac{\ell}{2}}\left(\mathbb{E}[Y]\right)^2dt$$

$$+\int_{\frac{\eta_U^*}{2}+\frac{\ell}{2}}^{\frac{\eta_U^*}{2}+\eta_{PI}^*+\frac{\ell}{2}}\left(\frac{\mathbb{E}\left[Y|S=s_G\right]P\left(S=s_G\right)+\mathbb{E}\left[Y|S=s_M\right]P\left(S=s_M\right)}{P\left(S=s_G\right)+P\left(S=s_M\right)}\right)^2dt$$

$$+\int_{\frac{\eta_U^*}{2}+\eta_{PI}^*+\frac{\ell}{2}}^{\frac{\eta_U^*}{2}+\eta_I^*+\frac{\ell}{2}}\left(\mathbb{E}\left[Y|S=s_G\right]\right)^2dt$$

$$=\left(\mathbb{E}[Y]\right)^2\left(\ell-\eta_I^*\right)$$

$$+\left(\frac{\mathbb{E}\left[Y|S=s_G\right]P\left(S=s_G\right)+\mathbb{E}\left[Y|S=s_M\right]P\left(S=s_M\right)}{P\left(S=s_G\right)+P\left(S=s_M\right)}\right)^2\eta_{PI}^*$$

$$+\left(\mathbb{E}\left[Y|S=s_G\right]\right)^2\eta_{FI}^*.$$

It is useful to note that

$$\frac{\mathbb{E}\left[Y|S=s_G\right]P\left(S=s_G\right)+\mathbb{E}\left[Y|S=s_M\right]P\left(S=s_M\right)}{P\left(S=s_G\right)+P\left(S=s_M\right)}-\mathbb{E}[Y] \tag{4.14}$$

equals

$$\frac{P\left(S=s_B\right)}{1-P\left(S=s_B\right)}\left(\mathbb{E}[Y]-\mathbb{E}\left[Y|S=s_B\right]\right)$$

and similar formulas hold for all permutations of signal realizations and for $\hat{Y}$. Therefore,

$$\ell\mathbb{E}\left[p^2|S=s_G\right]=\ell\left(\mathbb{E}[Y]\right)^2$$

$$+\left(\left(\mathbb{E}[Y]-\mathbb{E}\left[Y|S=s_B\right]P\left(S=s_B\right)\right)^2-\left(\mathbb{E}[Y]\left(1-P\left(S=s_B\right)\right)\right)^2\right)$$

$$\cdot\frac{\eta_I^*}{\left(1-P\left(S=s_B\right)\right)^2}$$

$$+\Big(\left(\mathbb{E}\left[Y|S=s_G\right]\left(P\left(S=s_G\right)+P\left(S=s_M\right)\right)\right)^2$$

$$-\left(\mathbb{E}\left[Y|S=s_G\right]P\left(S=s_G\right)+\mathbb{E}\left[Y|S=s_M\right]P\left(S=s_M\right)\right)^2\Big)$$

$$\cdot\frac{\eta_{FI}^*}{\left(1-P\left(S=s_B\right)\right)^2}.$$

Tesi di dottorato "Essays on Unawareness in Economics"
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Simple algebraic manipulations lead to

$$\begin{aligned}
&\ell\mathbb{E}\left[p^2|S=s_G\right]=\ell\left(\mathbb{E}[Y]\right)^2+\left(\mathbb{E}[Y]-\mathbb{E}\left[Y|S=s_B\right]\right)\\
&\quad\cdot\left(2\mathbb{E}[Y]-\mathbb{E}\left[Y|S=s_B\right]P\left(S=s_B\right)-\mathbb{E}[Y]P\left(S=s_B\right)\right)\frac{P\left(S=s_B\right)}{\left(1-P\left(S=s_B\right)\right)^2}\eta_I^*\\
&\quad+\left(2\mathbb{E}\left[Y|S=s_G\right]P\left(S=s_G\right)+\mathbb{E}\left[Y|S=s_G\right]P\left(S=s_M\right)+\mathbb{E}\left[Y|S=s_M\right]P\left(S=s_M\right)\right)\\
&\quad\cdot\left(\mathbb{E}\left[Y|S=s_G\right]-\mathbb{E}\left[Y|S=s_M\right]\right)\frac{P\left(S=s_M\right)}{\left(1-P\left(S=s_B\right)\right)^2}\eta_{FI}^*.
\end{aligned}$$

In the last equality we substitute the expressions of η_I^* and η_{FI}^* from eq. (4.3) and eq. (4.13) when they are internal solutions. Then, we write c^* explicitly and we focus only on the terms that depend on the unawareness level, specifically through $\mathbb{E}[\hat{Y}|S=s_B]$. After writing also $\bar{c}$ in explicit form and rearranging the terms, we find that the dependence of $\mathbb{E}\left[p^2|S=s_G\right]$ from the unawareness level is equivalent to the dependence from $\mathbb{E}[\hat{Y}|S=s_B]$ of the following quantity:

$$\begin{aligned}
&\frac{\mathbb{E}[Y]-\mathbb{E}\left[Y|S=s_B\right]}{\mathbb{E}[Y]-\mathbb{E}\left[\hat{Y}|S=s_B\right]}\frac{c}{\left(1-P\left(S=s_B\right)\right)^2}\\
&\quad\cdot\Big(2\mathbb{E}[Y]-\mathbb{E}\left[Y|S=s_B\right]P\left(S=s_B\right)-\mathbb{E}[Y]P\left(S=s_B\right)\\
&\quad+2\mathbb{E}\left[Y|S=s_G\right]P\left(S=s_G\right)+\mathbb{E}\left[Y|S=s_G\right]P\left(S=s_M\right)+\mathbb{E}\left[Y|S=s_M\right]P\left(S=s_M\right)\Big).
\end{aligned}$$

$\mathbb{E}[Y]$ is larger than both $\mathbb{E}[Y|S=s_B]P(S=s_B)$ and $\mathbb{E}[Y]P(S=s_B)$. In addition, both $\mathbb{E}[Y|S=s_M]$ and $\mathbb{E}[Y|S=s_G]$ are positive by assumption. As a result, all factors are positive. Moreover, the whole expression is increasing in the unawareness level because $\mathbb{E}[\hat{Y}|S=s_B]$ is (see Proposition 12).

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Bibliography

[1] Conrad, J., R. F. Dittmar, and E. Ghysels (2013) "Ex Ante Skewness and Expected Stock Returns." *Journal of Finance* 68(1):85-124.

[2] Coval, J., J. Jurek, and E. Stafford (2009) "The Economics of Structured Finance." *Journal of Economic Perspectives* 23(1):3-25.

[3] Diamond, D. W., and R. E. Verrecchia (1981) "Information Aggregation in a Noisy Rational Expectations Economy" *Journal of Financial Economics* 9(3):221-235.

[4] Galanis, S. (2015) "The Value of Information Under Unawareness." *Journal of Economic Theory* 157:384-396.

[5] Gennaioli, N., A. Shleifer, and R. Vishny (2012) "Neglected Risks, Financial Innovation, and Financial Fragility." *Journal of Financial Economics* 104(3):452-468.

[6] Gennotte, G., and H. Leland (1990) "Market Liquidity, Hedging, and Crashes." *American Economic Review* 80(5):999-1021.

[7] Grant, S., and J. Quiggin (2015) "A Preference Model for Choice Subject to Surprise." *Theory and Decision* 79:167-180.

[8] Grossman, S. J., and J. E. Stiglitz (1980) "On the Impossibility of Informationally Efficient Markets." *American Economic Review* 70(3):393-408.

[9] Harvey, C., and A. Siddique (2000) "Conditional Skewness in Asset Pricing Tests." *Journal of Finance* 55(3):1263-1295.

[10] Hellwig, M. F. (1980) "On the Aggregation of Information in Competitive Markets." *Journal of Economic Theory* 22(3):466-498.

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[11] Karni, E., and M. L. Vierø (2017) “Awareness of Unawareness: A Theory of Decision Making in the Face of Ignorance.” *Journal of Economic Theory* 168:301-328.

[12] Kyle, A. S. (1985) “Continuous Auctions and Insider Trading.” *Econometrica* 53(6):1315-1336.

[13] Mele, A., and F. Sangiorgi (2015) “Uncertainty, Information Acquisition, and Price Swings in Asset Markets.” *Review of Economic Studies* 82(4):1533-1567.

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