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Information in Networks and Political Economy: Three Models

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Abstract

This thesis spans over topics that share a need for theoretical insights. Taking a game theoretical approach, I study the link between social media and the quality of digital news; the role of a communication network in the provision of information; and the mechanisms behind diversionary conflicts.

The quality of information provided in a network is endogenous to it. The first chapter studies the news producers' role in it. Social media create a new type of incentives for producers. Consumers share content, influence the visibility of articles and determine the advertisement revenues ensuing. I study these incentives and evaluate the potential quality of ad-funded online news. Producers rely on a subset of rational and unbiased consumers to spread news articles. The resulting news has low precision and ambiguous welfare effects. Producers' incentive to invest in quality increases with the private knowledge of the topic; hence, when information is most needed, the generated news tends to be of lesser quality. Competition does not necessarily improve news quality – it does so only if the network is *sufficiently dense*. While ad-funded online news occasionally helps consumers take better decisions, it creates welfare mostly through entertainment. Some interventions, such as flagging wrong articles, substantially improve the outcome; other approaches, such as quality certification, do not.

The second chapter tackles the issue of the provision of information by a designer with misaligned interests. A sender wants to induce connected receivers to take some actions by committing to a signal structure about a payoff-relevant state. I wonder about the role of the network on information provision when signals are shared among neighbors. Receivers differ in their priors; the sender wants to persuade some receivers without dissuading the others. I present and characterize novel strategies through which the network is exploited. Were receivers' priors homogenous, such strategies would underperform with respect to a public signal. However, when priors are heterogenous, these strategies can prove useful to the sender. In particular, if the average degree of the nodes who should not be dissuaded is sufficiently low, strategies exploiting the network convince more receivers than public signals, conditional on the adverse state realizing. Furthermore, connectivity can be beneficial to the sender, in particular in segregated networks; and strategies exploiting the network perform better when one group is especially hard to persuade.

The last chapter offers a different approach: its main objective is to formalize a mechanism and micro-found well-documented behavior. It revisits the diversionary argument of war by proposing a new mechanism: a population that rebels during a conflict weakens the country's military position; this threat discourages the population to attempt a coup.

Being at war thus allows a leader to impose demanding policies without being overthrown. I show how rally-around-the-flag reactions to conflict can be both rational and efficient. Furthermore, purely diversionary incentives exist: international tensions can be initiated with the *only* goal of raising popular support about the conflict. Finally, long-run effects are addressed. When rebellion means are flexible, the population can voluntarily renounce to the freedom to rebel; alternatively, conflicts occur in equilibrium. The strength of the enemy's threat increases the prevalence of barriers to rebellion, while open conflicts are non-monotonically linked to it.

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Introduction

Economics is concerned with understanding human interactions. It studies incentives and their consequences. There are countless approaches to it. I chose a theoretical one.

Why would building models be useful? After all, they tend to be very simplistic. Why should one trust their insights? Like a map does not render the details of a landscape, a model does not integrate all the complexity of the economic world. This is what makes a map readable and a model interesting. Abstracting from confounding forces allows to put into perspective the important mechanisms at play. It also permits to disentangle necessary from sufficient factors: *must* agents be irrational to justify their behavior; or does the environment by itself warrant such behaviors? With this thesis, I answer that, often, one does not need to resort to irrationality in order to explain inefficient outcomes.

My first two chapters relates this observation to social media. Is the irrationality of social media users the one to blame for the poor quality of online news? Not necessarily. My first chapter explores the consequences of the advertisement-based business model on the provision of qualitative news when the producers rely on social media shares to gain visibility. I show how publishers? investment in news quality remains low, even when social media users are Bayesian who only care about sharing true news. Because news consumers can only judge the veracity of an article through the information they already have, it is *too* easy to convince enough of them to share, and producers do not need to invest. The intensive competition between online news outlets is also expected to ultimately worsen the quality of information: because visibility is so hard to gain in very competitive environments, news producers have no interest in ensuring that the articles they publish are true. Eventually, consumers are not necessarily better informed when online news outlets are present on the market. To change this, fact checking matters. However, it needs to help news readers distinguish true from false articles. Therefore, inaccurate articles should be flagged. However, the effectiveness of flagging is limited by the competitiveness of the online news environment.

The emergence of social media has changed the environment of information in many ways; but the role of network is central in all of them. While my first chapters studies the incentives of producers whose only interest lies in visibility, my second chapter wonders about the consequences of social media on the provision of information by a designer trying to induce users to act a certain way. I show how the network can be exploited in order to better tailor the information to each type of agent differently. Here too, even rational agents can be hurt by such environments as social media that allow information to be individually targeted. A sender of information who would only know the receivers' inclinations and their distribution in the network would still be able to adjust the information provider in order to capitalize on the receivers' dispositions, especially if agents with different propensity to believe the receiver are not well integrated together. Actually, exploiting the network becomes more beneficial to the sender when the network is more segregated, or when some receivers are especially hard to convince. It should thus not be surprising that social media are an ideal tool for agents who want to communicate about divisive issues.

With my third chapter, I move my focus from information to political conflicts, and from communication to commitment. Imagine a politician who faces backlash domestically and decides that an international conflict is warranted. Many would argue that said politician, by instigating a conflict, is manipulating the public's opinion to divert attention from the domestic issues. It would be a *diversionnary* conflict: a conflict that is initiated in order to protect the position of the country's leader. Do such conflicts occur because citizens are irrational and let themselves be distracted? Again, not necessarily. Once a leader has initiated a conflict, the country becomes more vulnerable to attacks and to foreign interferences; questioning the leader's legitimacy during such a vulnerable period would put the population at risk on the international front. For instance, if a war had been declared, the country might lose the war, or even be invaded by an hostile neighbor. This would cause more damage than preserving the leader's position. The citizens would thus prefer to support the leader. Actually, they might even prefer to support the conflict, because this would make the country more vulnerable internationally: if the country is relatively vulnerable already, the leader does not need to escalate the conflict in order to further secure his position. This avoids a more severe conflict. Which is optimal. And rational.

With this thesis, I hope to contribute to the comprehension of the causes of inefficient outcomes. By better understanding the mechanisms at play, better frameworks of interactions can be designed. Promoting fact checking is necessary for online news outlets to be informative; on divisive issues, communication should be made public; and institutions should be strengthen to avoid populations held-up between domestic and foreign threats. One can always blame consumers, or Facebook users, or citizens, for being irrational and causing their own misery. But sometimes, it is the environment that must be changed.

Chapter 1

4 Things Nobody Tells you About Online News: a Model for the New News Market

1.1 Introduction

The media landscape has evolved throughout history. From the press to radio, television and the rise of the internet age, many past revolutions gave rise to concerns about news quality. Nowadays, social media are under the spotlight. The idea that the online news market may be worse than traditional media is puzzling as it arises in a highly competitive environment. Yet, in the last decade, the rise in competition was accompanied by a decrease in media trust.[1] Understanding the effect of social media on the provision of information is important as the prevalence of online news is growing; the majority of American and European adults include online outlets to their media diet.[2] Observers increasingly fear market segmentation: this could result in a two-tier market where only those paying for articles would be well-informed. Is there hope for the ad-funded outlets to provide quality news, so that even free articles would be informative? Should competition be encouraged or has social media metamorphosed the news market in a way that makes

[1] See e.g. survey from Gallup, news.gallup.com/poll/321116/americans-remain-distrustful-mass-media.aspx

[2] See Pew Research Center, pewresearch.org/fact-tank/2021/01/12/; pewresearch.org/journalism/2018/10/30/

standard theory inapplicable?

While advertisement revenues and producers' reduced cost of entry date back to more than a century ago, online outlets brought something new: sharing. With social media, consumers play an active role in spreading news article, raising their visibility, thereby producers' advertisement revenues. Hence, news producers behind ad-funded online outlets respond to new incentives. Because of advertisement revenues, articles now need to be shared online. In this sense, the very presence of a news sharing network changes the effects of the previously existing market environment. In this paper, I evaluate the performance of such ad-funded online news outlets, focusing on the incentives linked to sharing behaviors. Three dimensions of the market environment are explored: the amount of private knowledge, the connectivity of the communication network and the presence of competition. After studying the effects of the environment on the provision of information, I question whether such outlets are welfare enhancing and propose possible interventions.

I explore this question by introducing a general setup to represent the online news market. The market is populated by consumers on one side and producers on the other. The agents are concerned with some state of the world, for instance, whether vaccines are effective or not. All consumers observe a private signal, e.g. whether a vaccinated friend has developed the illness. In addition, some consumers, called *seeds*, come across news articles about vaccination directly and can decide to share it on an exogenous network to other consumers, called *followers*. Seeds care about sharing true news; followers read articles that seeds share, they are not part of the strategic interaction.

An article is a signal whose realization is informative about the state of the world. Given seeds' sharing behavior, producers decide on the quality of their outlet, i.e. the precision of the signal they send. Producers choose neither the state of the world on which to report nor the news realization, only the probability for the realization to correspond to the state of the world. In other words, producers only choose how many journalists to hire for their outlets, not what these journalists report; the more journalists, the higher the likelihood of reporting the true efficacy of vaccines. Each producer publishes one article about the same underlying state of the world, vaccine efficacy in this example, and only cares about how many consumers view their article. While the number of seeds reading a producer's outlet is exogenous, the number of followers seeing their article is endogenous. When several producers co-exist in the market, they compete *through seeds* to reach other consumers, as each consumer is restricted to see only one article.

This model brings interesting insights. Even when consumers are not behavioral, the market fails to deliver precise news. Thus, incentives created by social media do not suffice to induce high quality online news, even in a market populated by rational and unbiased agents. The business model based on advertisement revenues is flawed both because of the way it shapes producers' investment and because seeds imperfectly channel all information. The market environment then has counter intuitive effects on news quality: a lack of private knowledge is not substituted by more informative articles; the influence of competition on news quality is tied to the connectivity of the social network on which news are shared. Furthermore, the presence of news outlets has ambiguous welfare consequences, that not all interventions can overcome.

These results rely on two key mechanisms. First, the producers' incentive to invest in news quality is determined by the difference between the value of a true and a false article. Private knowledge, connectivity and competition all affect the value of true articles differently than that of false news, thus inefficiently modulating the producers' response to the market environment. Second, the market is shaped by consumers' sharing decision, which is determined by their private knowledge. Consumers' private knowledge thus bounds news quality. Below, I discuss in more detail how these two mechanisms drive all four main results.

First, ad-funded online outlets tend to fail when informative news would be the most beneficial. News quality is less valuable for a producer in an environment with low private knowledge: either because the consumers are not well-informed by their signals, or because the state of the world is *ex ante* very uncertain. As private signals get noisier, the value of a true article decreases while that of false information increases: consumers struggle distinguishing true and false news, leading them to treat any news article very similarly. As one state of the world becomes more likely, investing in news quality gets more attractive for producers, since the difference between the value of true and false information is greater when the most likely state of the world realizes. These leads producers' incentives to be misaligned with the consumers' need.

Second, competition can be detrimental; its effect depends on the network connectivity. For any market structure, high connectivity negatively affects news quality; but it does so less strongly if the market is competitive. A monopolist's incentive to invest vanishes as the network gets very dense: one single node sharing would reach almost all other consumers then. The monopolist can thus create false content and rely on a few seeds receiving an erroneous private signal to reach many followers. This intuition does not follow through in competitive markets. Producers cannot rely on these few seeds anymore; articles need to be sufficiently shared in order to survive in the network. In this sense, competition decreases the value of a false article. This force thus pushes the producers towards more investment.

Yet, the effect of competition is ambiguous. There is indeed a second opposite effect of competition. Splitting the market might be detrimental to investment, since the cost of news quality does not depend on the size of the market served. By accessing less initial seeds, the producer cannot reach as many followers, even if his articles was shared by all seeds reading it. Competition decreases the value of a true article. The strength of these forces depend on the network degree: as connectivity increases, producers have access to more and more followers while competition inside the network becomes more biting. Therefore, competition is detrimental below a connectivity threshold.

Third, the welfare value of ad-funded online news is ambiguous. Any equilibrium is Pareto inefficient. To go beyond the Pareto criterion, I consider different aspects of consumers' welfare. *Entertainment* – the utility derived from sharing – increases with news quality. To capture the value of information, I introduce an additional action, a bet, in which consumers must match the true state of the world. Agents are brought to better decision by news outlets if their expected utility from betting increases after having observed a news article.

Generally, the market fails to let seeds take better decisions. This does not rely on the presence of competition or the timing of the game; but on the central channeling role of the seeds. Producers have no incentive to publish more precise articles than the consumers' private signals, since this would suffice to being shared all the time. Therefore, the news quality is bounded by the consumers' knowledge and, for symmetric priors, seeds are always as well off by trusting their private signal for the bet. Followers, however, might take better decisions if the market is competitive: as the network tends to filter out false articles, the articles they end up seeing might be more precise than their private signal. Still, their utility from betting is bounded by their private signal.

Studying the decision to enter this bet at a cost allows to analyze whether online news pushes consumers towards action. Unsurprisingly, there exists a range of costs for which online news indeed helps agents enter the bet when it is beneficial. More surprisingly, under mild conditions, there also exists a range of costs for which news outlets are detrimental. By creating noise to consumers' private signals, news outlets too often discourage consumers to enter the bet, as more agents are wrong than right to opt out of the action. The existence a range of entry cost making the mere presence of news outlets detrimental again results from the bound placed by consumers' private knowledge on news quality.

Fourth, I analyze the effects of fact checking. I distinguish between *flagging*, fact checking articles before they are shared; and *quality certification*, fact checking past articles from news outlets in order to assess the outlets quality. The former has substantial effects on welfare by removing the bound placed on news quality; the second might marginally improve the news quality but does not remove the bound from private knowledge, hence it does not significantly affect welfare.

Flagging reduces the value of producing false information by improving the seeds' private knowledge. Interestingly, competition dilutes the effect of flagging. Actually, for any environment, there exists a level of flagging that makes competition detrimental. Indeed, flagging, like competition, reduces the value of false information; however, unlike competition, it does not decrease the value of true information. Flagging can then be seen as a substitute for competition: any outcome from competition is actually reproducible in uncompetitive markets through flagging. Certifying news outlets' quality allows producers to internalize the effects of their investment on the seeds' sharing strategy; however, the best outcome for producers is still to be shared all the time, which happens when they match consumers' private knowledge. Therefore, news quality is still bounded by the consumers' private knowledge.

The paper is organized as follows. Related literature is discussed in the remainder of this section. The general model is presented in Section 1.2. Section 3.4 analyzes the equilibrium resulting from a monopoly and a duopoly respectively; in particular, it assesses the role of the market environment and competitiveness on the outcome. Section 1.4 proposes a framework to assess welfare and analyzes it accordingly. Section 1.5 evaluates the effect of fact checking. Section 1.6 discusses the robustness of results and Section 3.7 concludes. Further extensions are provided in the Appendices 1.A,1.B and 1.C. Appendix 1.D presents the proofs omitted in the main text.

Related literature

I contribute to several strands of the literature. I particularly relate to theoretical works on news markets, media economics and the spread of news in networks.

First, as to **news markets**, the existing theoretical literature accounts for the existence of bad quality news in a competitive but unconnected world. Allcott and Gentzkow [2017] find that uninformative news can survive if news quality is costly and if consumers cannot perfectly infer accuracy or if they enjoy partisan news. My setup is similar in that quality is costly and consumers cannot perfectly distinguish true from false articles. However, my mechanism does not fundamentally rely on outlets' quality being hidden. Furthermore, I introduce to such models an explicit network of information sharing to catalyze the spread of information.

In such unconnected news markets, the ambiguous effects of competition between news providers has been widely explored. Namely, Gentzkow and Shapiro [2008] find that competition is effective at reducing supply-driven biases, while its effects with demand-driven biases are ambiguous. Consistently with this conclusion, other authors find that competition has ambiguous effects when news consumers lack sophistication. For instance, Levy et al. [2017] study how media companies can exploit consumers' correlation neglect. They find that competition reduces the producers' ability to bias readers' beliefs, but that diversity has a cost in terms of optimal consumers' responses. Hu and Li [2018] and Perego and Yuksel [2018] study how rational inattention biases the provision of political information. Both find that competition inflates disagreement. Chen and Suen [2016] also find that competition is detrimental to the accuracy and clarity of news when readers endogenously allocate attention between outlets whose editors are biased. Interestingly, my results on competition is not motivated by biases of either side of the market.

Second, as to **media economics**, this paper relates in particular, to the influence of digitalization on media. Representative of this literature are the following papers. Anderson [2012] combines empirical and theoretical insights to offer an overview of the ad-financed business model in the internet age. Wilbur [2015] documents trends following digitalization for the mass media and how their business models has evolved. Finally, Peitz and Reisinger [2015] review various novel features resulting from new Internet media. I contribute to this literature by explicitly modeling one such new feature of online news market: shared content. I study its effects on producers incentives and equilibrium outcomes.

Note that Peitz and Reisinger [2015] briefly discuss how sharing decision might affect available content and link it to more general media biases. In this perspective, Hu [2021] studies the impact of media regulation in the digital age and finds that government regulation is rendered less effective by media biases inherent to the digital age. Because their model does not take into account any communication network, their analysis does not study interventions targeting the sharing behavior of consumers. My intervention evaluations, in contrast, only accounts for such incentives resulting from consumers' sharing decisions.

Third, as to **news in networks**, a connected world has rarely been the setup for news market models in the literature. To the best of my knowledge, only Kranton and McAdams [2020] study the effect of communication networks on the quality of information provided on the news market. My model is largely inspired by the setup they propose. While Kranton and McAdams [2020] give a compelling argument on how a network of consumer can change a producer's investment incentives, their mechanism abstracts from the role of competition. Furthermore, they do not address welfare effects of market outcomes. A key contribution of this paper is the introduction of competition and welfare considerations to the model.

Following the cascade literature,[3] the recent working paper Hsu et al. [2019] provide optimal conditions on a signal's precision for a cascade to occur when sharing is endoge-

[3] This literature is studies learning in networks when agents learn from actions. See Bikhchandani et al. [1992], Banerjee [1992] for their seminal work.

nous and strategic. This could, in turn, relate to a producer's objective, although no producer is featured in their setup. However, just as Kranton and McAdams [2020], Hsu et al. [2019] is set in an uncompetitive world. Finally, recent works explore the particular setup of learning on social media. Bowen et al. [2021] study learning via shared news and find that polarization emerges when agents hold misconceptions about their friends' sharing behavior. They find that news aggregators help curb polarization. Neither of these papers addresses the effects of competition between news providers in a connected world.

1.2 Model

1.2.1 Environment

The market is populated by news producers and news consumers. Consumers learn about an unknown state of the world $\omega \in \{0,1\}$ through news articles and private signals.[4] There is a common prior across all agents, $\Pr(\omega = 0) = w_0$. All agents are Bayesian.

I denote the set of news consumers I, which can be finite or infinite. All consumers receive an informative binary private signal s about ω. These signals are i.i.d. among consumers with $\Pr(s = \omega|\omega) = \gamma$ for $\omega = 0, 1$. I further impose $\gamma \geq w_0$, so that consumers trust their private signal more than their prior.

In addition to private signals, the consumers can come across news articles in the following ways: they can be exposed to it directly – in such a case they are called initial *seeds* and denoted i; or they can read such news because a seed shared it. If consumers are not seeds, they are called followers and denoted f. All consumers are exposed to at most one article, but some followers might be exposed to none. When I do not want to explicitly distinguish seeds from followers, I denote the news consumers j.

The consumers are arranged on a regular network of degree d.[5] Consumers are randomly drawn to be seeds with probability b. Hence, all consumers, regardless of their role, have the same number of *random* neighbors d. The network allows followers to see articles shared by neighboring seeds. A follower sees no article if none of its neighbors shared content – either because none of the neighbors are seeds or because none of the seeds decided to share. If several neighbors shared content from different sources, the article that a follower f ends up seeing is determined stochastically. Any of f's sharing neighbor

[4] w denotes the outcome of ω. For the remainder of the paper, the distinction between random variables and their outcome is not made at long as it is clear from the context.

[5] In a regular network, all nodes have the same number of connections. Note this assumption is not fundamental to the analysis, but greatly simplifies the notation. The extension to non-regular networks is discussed in Section 1.6.

has the same probability to be seen. Therefore, the probability with which the follower sees a given source is proportional to the number of neighbors sharing this source relative to the number of neighbors having shared any article. In other words, the probability that f sees a given article k is:

$$Pr(f \text{ sees } k | A \text{ neighbors shared } k, B \text{ neighbors shared}) = \frac{A}{B}$$

For instance, say four of f's neighbors shared a piece of information, but only one of them shared k, then, f sees k with probability one fourth, although f does see *some* article with probability one.

On the other side of the market, I consider a finite set of producers K.[6] Each producer, denoted by k, publishes exactly one article.[7] Each producer reaches a seed with the same exogenous probability $\frac{b}{K}$. The producer chooses the overall quality of the news that is published. However, he does not choose the article's content, which is randomly determined. The content of an article is denoted n.

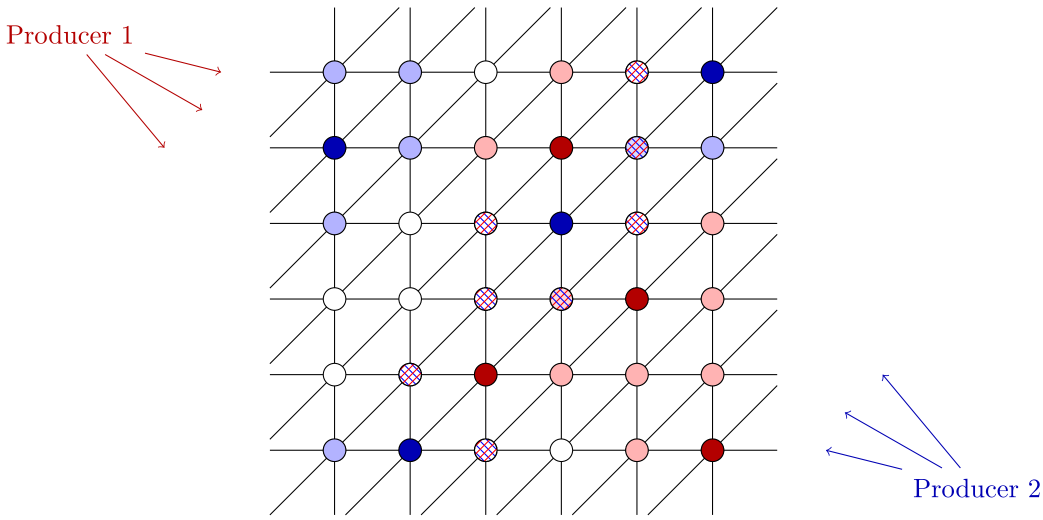


Figure 1.1: Illustration of a possible outcome on the news market

Figure 1.1 depicts the environment described with two producers. Dark colored nodes are seeds; for the illustration, assume all articles are shared. Light colored nodes are followers who see a given producer with probability one. Hatched colored nodes are followers whose source is determined at random – neutrally hatched nodes see each producer with probability 1/2, hatched nodes with a hue see the given producer with probability 2/3.

[6] I abuse notation by denoting K both the set of producers and its cardinality
[7] Therefore, I can abuse notation by also denoting articles by k.

1.2.2 Timing, Objectives and Equilibrium Concept

All strategic interactions are assumed to be simultaneous.[8] The only agents active in the game are initial seeds and producers.

The producers choose the quality of their outlet to maximize their profits. The quality or precision of outlet k is defined as the probability of documenting the true state of the world, $q_k := \Pr(n = \omega|\omega)$ for $\omega = 0, 1$. Producers derive revenue from advertisement, hence from the visibility of their outlet. Their revenue is thus defined as the share of the network that sees their article [9]. Their (total) cost is determined by cost function C. C is common to all producers. I denote c the marginal cost function. I assume C increasing and strictly convex, i.e. $c(q) > 0$ and $c'(q) > 0$. Finally, I assume that without any investment in quality, the outlet produces uninformative news, that is, $q_k = 1/2$. Furthermore, $c(1/2) = 0$.

Seeds like sharing true information and dislike sharing false information. Accordingly, they choose the probability with which they share an article. This can depend on the content that the article they read reports and whether it corresponds to the private signal they received. The article's reported content, i.e. the realization of the news signal, is denoted n; the congruence with the private signal is denoted $S = +, -$ where $S = +$ if the news content is the same as the seed's private signal, and $S = -$ otherwise.[10] The probability with which a seed shares an article from producer k whose content is n is denoted by $z_{S|n,k}$. Therefore, the seeds' strategy is a vector: $(z_{S|n,k})_{(S,n,k)\in\{+,-\}\times\{0,1\}\times K}$. As seeds want to share an article only when its content is truthful, they are assumed to receive a positive payoff from sharing true information and a negative payoff when sharing false information.[11] Seeds have the following payoff from sharing:[12]

$$u(\text{sharing article reporting } n|\omega = w) = \begin{cases} 1 & \text{if } n = w \\ -1 & \text{otherwise} \end{cases}$$

They receive payoff 0 if they do not share.

[8]Equilibria of the sequential game when $w_0 = 1/2$ are provided in the Appendix 1.C

[9]Intuitively, the revenues are scaled for size population because they relate to advertisement revenues. One might expect advertisers to be interested in the *portion* of the population a given news outlet is able to reach. Furthermore, with this representation, the model becomes scale-free. Finally, it allows their profits to be bounded below 1

[10]The outcome of the private signal is $s \in \{0, 1\}$ while the congruence is $S \in \{+, -\}$. For instance $s = 1$ is a *positive* signal towards n being true if $n = 1$, and a negative signal towards the article being true if $n = 0$. If additionally $\omega = 1$, $s = 1$ and $S = +$ are said to be *correct* while they would be *wrong* if $\omega = 0$.

[11]This assumption can represent the interests of truth-seeking consumers. Implicitly, it also accounts for wider concerns such as reputation or attention. In fact, Appendix 1.B assumes that seeds seek attention for themselves. Their best-response is qualitatively similar.

[12]In Appendix 1.A, I consider more general payoffs. Most results follow through, but additional equilibria might appear.

I focus on Nash Equilibria.

1.2.3 Best Responses

For ease of exposition, I derive the best-responses of initial seeds and producers for $w_0 = 1/2$. I then provide the best-response for general w_0 in a dedicated paragraph; details can be found in Appendix 1.D.

Seeds' Problem

Take a seed who received private signal s and read a news article reporting n. Then, the seeds expected utility from sharing is:

$$p(n,s) + \Big(1 - p(n,s)\Big)(-1) = 2p(n,s) - 1$$

where $p(n,s)$ is i's posterior on the probability that the producer published a true article.

A piece of news is true if it matches the state of the world. Hence, the posterior is the probability that the state of the world is the one prescribed by the news, given what was written in the news and what the consumers themselves experienced from the world. That is, $p(n,s) := Pr(\omega = n|n,s)$. Let seeds attribute prior probability q_k to an article from k being true. Recall $w_0 = 1/2$. Using Bayes' rule:

$$\Pr(\omega = n|n,s) = \tfrac{\Pr(n,s|\omega=n)\Pr(\omega=n)}{\Pr(n,s)} = \tfrac{\Pr(\omega=n)\Pr(s|\omega=n)q_k}{\sum_w \Pr(\omega=w)\Pr(s|\omega=w)\Pr(n|\omega=w)}$$

Therefore:

$$p(0,0) = p(1,1) = \tfrac{\gamma q_k}{\gamma q_k + (1-\gamma)(1-q_k)} \qquad \text{and} \qquad p(0,1) = p(1,0) = \tfrac{(1-\gamma)q_k}{(1-\gamma)x_k + \gamma(1-q_k)}$$

As one would expect, all posteriors are increasing in q_k. A seed shares an article when its expected utility from doing so is greater than the outside option 0. Therefore, i shares news n from producer k upon receiving signal s when:

$$p(n,s) \geq 1/2$$

Since no state of the world is ex-ante more likely, any realization of the news is as likely; therefore, the reported content is only relevant in conjunction with private signals, $p(0,s) = p(1,\text{-}s)$. In other words, accounting for the possible (dis)agreement between private signal and news article is sufficient, and the subscript n can be omitted from the seeds' strategy. The identity of the news' producer being irrelevant to seeds beyond q_k, the subscript k is omitted as well. The seeds' best-response is summarized by $z = (z_+, z_-)$

and is characterized as follows:

$$(z_+^*(q_k), z_-^*(q_k)) = \begin{cases} (0,0) & \text{if } q_k < \underline{t} \\ (e,0) & \text{if } q_k = \underline{t} \\ (1,0) & \text{if } q_k \in (\underline{t}, \bar{t}) \\ (1,e) & \text{if } q_k = \bar{t} \\ (1,1) & \text{if } q_k > \bar{t} \end{cases}$$

for any $e \in [0,1]$, where $\underline{t} = (1-\gamma)$ and $\bar{t} = \gamma$.

Since $\underline{t} < \bar{t}$, the seeds' best response are weakly monotonic in q_k: $z_-^* \geq z_+^*$. In other words, one shares an article reporting the opposite of their private signal only if one would be ready to share this article, were it to report the same as their private signal. Hence, when q_k increases, the *ex-ante* probability for a node to share increases. Therefore, although the strategy z is multi-dimensional, the set of undominated z can be represented on a line.[13] Figure 1.2 represents how sharing decisions is affected by different news quality, and the monotone aspect of it; Figure 1.3 displays seeds' best-response to news' quality q_k. The same applies for any k.

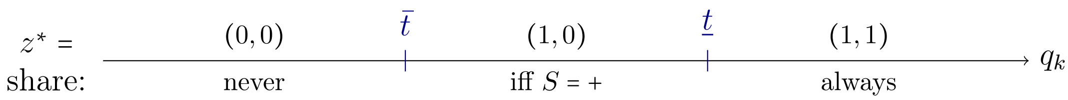


Figure 1.2: Sharing Decisions of Seeds for Different News Quality

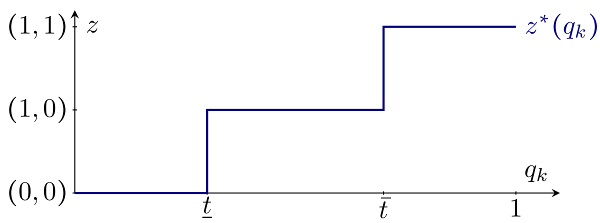


Figure 1.3: Best Response of Seeds as a Function of q_k

Best-response for general w_0

For $w_0 > 1/2$, the news realization n matters in the beliefs that the article is true since a news reporting the most likely state of the world is more probable to be true: $p(0,s) > p(1,\text{-}s)$. However, conditional on reading a given news content n, the seeds' best-response

[13] Formally, $z_k = (z_{+|0,k}, z_{+|1,k}, z_{-|0,k}, z_{-|1,k})$; for $w_0 = 1/2$, it is undominated to treat any news content the same way: $z_{+|0,k} = z_{+|1,k}$, $z_{-|0,k} = z_{-|1,k}$. For $q_k = \underline{t}$, any $z_{+|0,k} \neq z_{+|1,k}$ would also be undominated; however, setting $z_{+|0,k} = z_{+|1,k}$ leads to an equivalent analysis. The same applies to z_- for $q_k = \bar{t}$.

are as before:

$$(z^*_{+|n}(q_k), z^*_{-|n}(q_k)) = \begin{cases} (0,0) & \text{if } q_k < \underline{t}_n \\ (e,0) & \text{if } q_k = \underline{t}_n \\ (1,0) & \text{if } q_k \in (\underline{t}_n, \bar{t}_n) \\ (1,e) & \text{if } q_k = \bar{t}_n \\ (1,1) & \text{if } q_k > \bar{t}_n \end{cases}$$

for any $e \in [0,1]$, where $\underline{t}_n = \frac{(1-\gamma)\Pr(\omega \neq n)}{(1-\gamma)\Pr(\omega \neq n)+\gamma \Pr(\omega = n)}$ and $\bar{t}_n = \frac{\gamma \Pr(\omega \neq n)}{\gamma \Pr(\omega \neq n)+(1-\gamma)\Pr(\omega = n)}$.

Because $\underline{t}_0 < \underline{t}_1 < \bar{t}_0 < \bar{t}_1$, the seeds' best response are again weakly monotonic in q_k; the set of undominated strategies $z^* = (z^*_{+|0}, z^*_{+|1}, z^*_{-|0}, z^*_{-|1})$ can be represented on a line for any $w_0 < \gamma$.

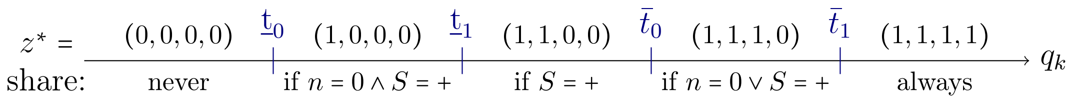


Figure 1.4: Sharing Decisions of Seeds for Different News Quality

Notice that $z^*_{S|0} \geq z^*_{S|1}$: one shares an article reporting the least likely state of the world only if one would be ready to share this article, were it to report the most likely state of the world, given the same (dis)agreement with private signals.

Producers' Problem

Consider a producer k. Let R_k take value 1 if a consumer sees producer k's article. Assume that k is facing seeds who have strategy $\mathbf{z}$, while the other producers ℓ are investing $\mathbf{q}_\ell$. Then, the expected profits for producer k who invests to reach quality q_k is:

$$\mathbb{E}(R_k | q_k; \mathbf{z}, \mathbf{q}_\ell) - C(q_k)$$

The expected share of reader as a function of k's investment in quality is found as follows. For a random node to share the article from producer k, one needs: the consumer to be a seed – with probability b –, to come across k's article – with probability $1/K$ – and to share. The probability to share, z, depends on whether the news article corresponds to the private signal, which depends on whether k produced a true or false article. Recall that $w_0 = 1/2$ so that the news realization n is irrelevant beyond its (dis)agreement with private signals. Seeds receive a private signal corresponding to the state of the world with probability γ; this private signal corresponds to the news' content $S = +$ if the news content is true, and it is not congruent $S = -$ if the news content is false. Hence:

$$p_{T_k} = \frac{b}{K}(\gamma z_+ + (1-\gamma)z_-) \quad \text{and} \quad p_{F_k} = \frac{b}{K}(\gamma z_- + (1-\gamma)z_+)$$

The *ex ante* probability that a consumer reads k's article, which reports a true/false content – denoted $X = T, F$–,[14] represents the value of such article for producer k; it is denoted V_{X_k}. If producer k has no competitor, the probability to be read by publishing a news X is simply:

$$V_{X_k}(z) := \Pr(j \text{ seed}) + \Pr(j \text{ follower} \wedge \geq 1 \; j\text{'s neigh. shared}) = b + (1-b)\big(1-(1-p_{X_k})^d\big)$$

However, when producer k is not alone on the market, it is not enough that a follower's neighbor shared k's article; this follower also needs to see k against all other producers ℓ's articles. Therefore:

$$V_{X_k}(z) := \Pr(j \text{ seed}) + \Pr(j \text{ follower}) \Pr(\geq 1 \; j\text{'s neigh. shared}) \Pr(j \text{ sees } k \text{ against } \ell)$$

$\Pr(j \text{ sees } k \text{ against } \ell)$ depends on the number of f's neighbors having shared k against ℓ. The number of f's neighbors having shared ℓ depends on the content produced by other producers, which we denote $Y_\ell := (Y_l)_{l \neq k}$.

The *ex ante* probability that a consumer reads k's article, which is $X = T, F$, $V_{X_k}(z)$, is then:

$$V_{X_k}(z) = \sum_{Y_\ell} V_{X_k Y_\ell} \Pr(Y_\ell) = \frac{b}{K} + \sum_{Y_\ell} (1-b) \frac{p_{X_k}}{p_{X_k} + p_{Y_\ell}} \big(1-(1-p_{X_k}-p_{Y_\ell})^d\big) \Pr(Y_\ell) \text{ [15]}$$

where $V_{X_k Y_\ell}$ is the value for k of producing an article that is $X = T, F$ when other producers have published articles which are true or false as described in Y_ℓ; and denoting $p_{Y_\ell} = \sum_{l \neq k} p_{Y_l}$, with $Y_l = T, F$.

The probability for a follower to read information k given the other articles Y_ℓ has two factors. The former, $\frac{p_{X_k}}{p_{X_k}+p_{Y_\ell}}$ represents the expected share of followers k would get, conditional on them being reached by any news, whereas the latter factor $1-(1-p_{X_k}-p_{Y_\ell})^d$ represents the probability of any news to reach followers. It means that sharing affects the producer's revenue through two channels: the size of the total readership and the portion of readers viewing a given producer. For instance, if the seeds of a producers' competitor start sharing more often, the total readership increases but the portion of the readership seeing that producer decreases. The relative strength of these two effects depends on the connectivity of the network d. Both factors are however increasing in p_{X_k}. Hence, as long

[14] -X is the alternative, so $X = T$ means -$X = F$ and conversely

[15] $\Pr(Y_\ell|\omega) = \prod_{l:Y_l=T} q_\ell \cdot \prod_{l:Y_l=F}(1-q_\ell)$. For instance, with two other producers ℓ, $\Pr(T, F) = q_1(1-q_2)$.

as true articles are shared more than false articles, i.e. $z_+ \geq z_-$, true information is more visible, no matter the outcome of the competitor.

Finally, the expected portion of the network reached given an investment q_k is:

$$\mathbb{E}(R_k|q_k) = q_k V_{T_k}(z) + (1 - q_k)V_{F_k}(z)$$

Because the profits are $\mathbb{E}(R_k|q_k) - C(q_k)$, the maximization of profits implies:

$$q_k^*(z) = c^{-1}\big(V_{T_k} - V_{F_k}\big) := c^{-1}\big(\Delta V_k(z; q_\ell)\big)$$

Because $c'(q) \geq 0$, the equilibrium investment $q^*(z)$ is (weakly) increasing in $\Delta V_k(z; x_\ell)$. Thus, $\Delta V_k(z)$ denotes producer k's incentive to invest. Intuitively, it corresponds to the additional number of views the producer gets in expectation from producing a true rather than a false article. Section 3.4 analyzes the function shape for one and two producers.

Best-response for general w_0

For $w_0 > 1/2$, in addition to the veracity of a news article, its realization n matters, as $n = 0$ tends to be shared more $p_{X|0,k} \geq p_{X|1.k}$. In other words, the value of producing a $X = T, F$ article also depends on the state of the world. The analysis of the producers' problem is however very similar:

$$\mathbb{E}(R_k|q_k) = w_0[q_k V_{T|0,k}(z) + (1 - q_k)V_{F|1,k}(z)] + (1 - w_0)[q_k V_{T|1,k}(z) + (1 - q_k)V_{F|0,k}(z)]$$

where $V_{X|n,k}$ is the value of a $X = T, F$ article reporting content n.[16]

Finally, the best-response is:

$$q_k^*(z) = c^{-1}\big(w_0[V_{T|0,k} - V_{F|1,k}] + (1 - w_0)[V_{T|1,k} - V_{F|0,k}]\big) := c^{-1}\big(\Delta V_k(z; q_{-k})\big)$$

Intuitively, $V_{T|0,k} - V_{F|1,k}$ corresponds to the additional number of views the producer gets in expectation from producing a true rather than a false article when the most likely state of the world realizes, $\omega = 0$; while $V_{T|1,k} - V_{F|0,k}$ correspond to the same concept for $\omega = 1$.

[16]Similarly as above, $V_{X|n,k}(z) = \frac{b}{K} + \sum_Y (1 - b)\frac{p_{X|n,k}}{p_{X|n,k} + p_{Y|m,\ell}}\big(1 - (1 - p_{X|n,k} - p_{Y|m,\ell})^d\big)\Pr(Y|\omega)$, where $m := (m_l)_{l \neq k}$ is defined implicitly by Y given X and n, e.g. $X = T, n = 0$ means $\omega = 0$, so $Y_l = T \Leftrightarrow m_l = 0$.

1.3 Equilibrium

In this section, I characterize possible equilibria in both a non-competitive and a competitive market. I say that a market is competitive when consumers are see less articles than the amount available on the market. In such case, producers are indeed forced to compete through seeds in order to capture followers' views. Because this setup restricts consumers to receive only one piece of information, I analyze the outcome from a monopoly and a duopoly respectively. I study the equilibrium on each market with symmetric prior $w_0 = 1/2$. For the monopoly, I furthermore characterize the equilibrium and discuss the role of the environment for general w_0.

1.3.1 Equilibrium without Competition

Consider a market with only one producer. For clarity purposes, I omit the k index in this section. The monopolist's incentive to invest is denoted $\Delta V_M(z)$ and can be rewritten explicitly:

$$\Delta V_M(z) = (1-b)\left[(1-p_F)^d - (1-p_T)^d\right]$$

Let us now analyze the shape of such best-response:

Lemma 1. *The monopolist's best-response to sharing $q^*(z)$ is single-peaked in z_+, with maximum $\bar{z} \in (0;1]$; it is strictly decreasing in z_-. $q^*(z)$ is continuous in z.*

Proof. See Appendix 1.D. □

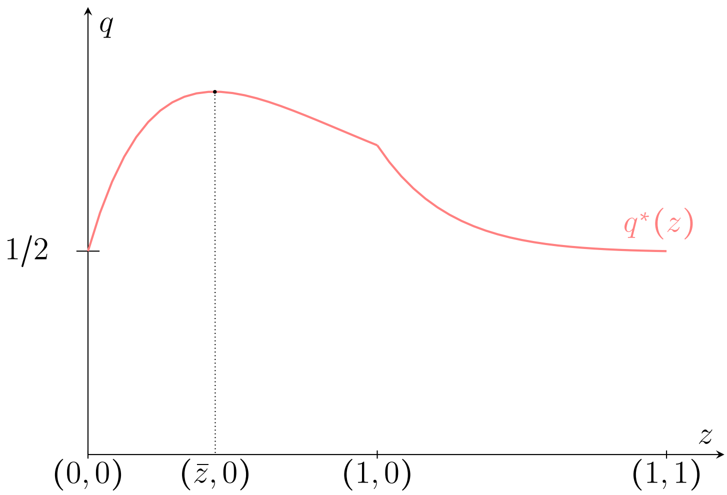


Figure 1.5: Producer's Best Response

Figure 1.5 illustrates the shape of the producer's best response. Because the seeds' strategy is not a unidimensional object, I illustrate the shape of the producer's best-response on the set of seeds' undominated strategy. As before, I represent the seeds' strategy on a line and map the corresponding image as if the argument was unidimensional. The resulting function is non-monotonic. The hump shape is explained by the effect of the network. At first, when the probability for agents to share is low, every additional node sharing reaches an almost constant number of additional followers; because the probability that this share occurs after having issued a true article is higher, true information gains much more followers than false information – the best-response is increasing. But when *enough* shares would occur, any increase in the probability of sharing would lead to shares which are likely to reach followers that would have been reached anyways; the marginal value of the probability of sharing is decreasing, because of redundant path to followers in the network. Therefore, the number of followers reached with a false article, that is rarely shared, is increasing faster with z_+ than the number of followers reached with true news; and the best-response is decreasing. Subsequently, the best response is decreasing. On the decreasing segment, agents start sharing news that does not correspond to their private signals. Therefore, the probability that this concerns a false article is higher than the probability that it applies to true information. It follows that false information accumulates views faster than true news. The difference between the value of true and false information thus decreases, making the best-response decreasing.

Now, recall that $q^* \geq 1/2$, as no investment would lead to $q = 1/2$. Furthermore, $\underline{t} < 1/2$. Therefore, we can characterize the Nash equilibrium of the monopoly.

Proposition 1. *There exists a Nash equilibrium. Any equilibrium displays positive investment and is uniquely characterized by news quality* $q^*_M = \min\{q^*(1,0), \bar{t}\}$

Proof. See Appendix 1.D. □

The sketch of the proof is as follows. An intersection of $q^*(z)$ and $z^*(q)$ exists because both best responses are continuous and that in $z = (0,0)$ the producer's best response is above the value ensuring some sharing, while in $z = (1,1)$ the producer's best response is below the value ensuring full sharing. The intersection in the space $(q, (z_+, z_-))$ is unique because any equilibrium displays $z_+ = 1$; so that any intersection would occur on the decreasing part of the producers' best response, while the seeds' best-response is weakly increasing.[17] The point of intersection depends on the parameters; it can occur on the vertical part of the seeds' best-response, then $q^*(1,0) < \bar{t}$; or on the horizontal part of the seeds' best response, then $q^*(1,0) \geq \bar{t}$.

[17]Technically, for $q^*_M = \bar{t}$, any $z_{-|0} \neq z_{-|1}$ is undominated, so there would exist equilibria with $z_{-|0} \neq z_{-|1}$. I abstract from this technicality as all results follow through.

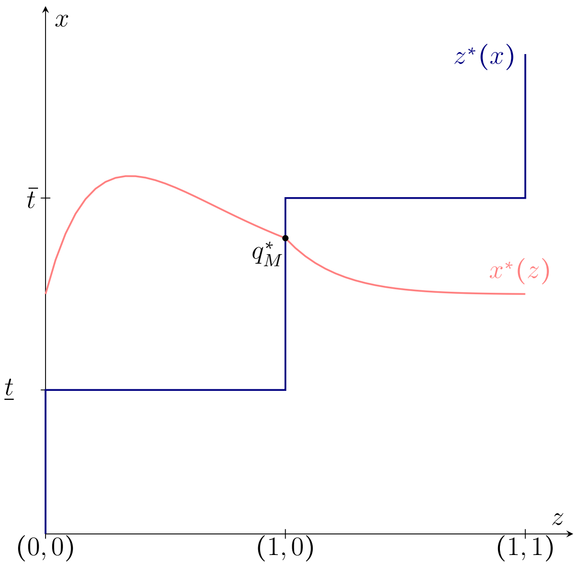


Figure 1.6: Equilibrium with $q^*_M = q^*(1,0)$

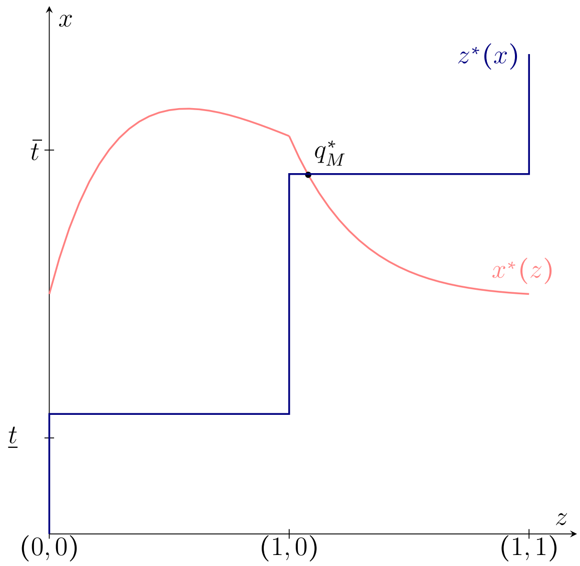


Figure 1.7: Equilibrium with $q^*_M = \bar{t}$

Figure 1.6 shows the equilibrium with $q^*(1,0) < \bar{t}$. Figure 1.7 shows the equilibrium with $q^*(1,0) > \bar{t}$. Notice that Proposition 1 implies that $q^*_M < \max_z\{q^*(z)\}$. As in Kranton and McAdams [2020], the highest investment that a producer would be willing to pay is never achieved.

Below, I derive the corresponding results for general w_0 in order to describe the role of the environment on the equilibrium outcome, in particular, connectivity and private knowledge, through both prior and precision of private signal.

Characterization for general w_0

The producer's best-response for general w_0 is very similar to that for $w_0 = 1/2$. In particular, with $\Delta V_M(z) = w_0\left[(1-p_{F|1})^d - (1-p_{T|0})^d\right] + (1-w_0)\left[(1-p_{F|0})^d - (1-p_{T|1})^d\right]$, the shape of the monopolist's best-response is as before given any realization of content n. The characterization of the equilibrium follows.

Corollary 1.

- *The monopolist's best-response* $q^*(z)$ *is single-peaked in* $z_{+|n}$, *with local maxima* $\bar{z}_n \in (0;1]$; *it is strictly decreasing in* $z_{-|n}$.; $q^*(z)$ *is continuous in* z.
- *There exists a unique Nash equilibrium. It displays positive investment and is characterized by news quality* $q^*_M = \max\{\min\{q^*(1,0),\bar{t}_0\},\min\{q^*(1,1,1,0),\bar{t}_1\}\}$.

Proof. See Appendix 1.D. □

Figures illustrating the producers' best response and the equilibrium are provided in Appendix 1.D. The intuition behind the characterization of the equilibrium is as before.

However, when $w_0 > 1/2$, there are two horizontal and vertical parts of the seeds' best-response for a potential crossing. The inequalities describing the conditions for intersection on the different parts are summarized by the min and max operators as characterized in Corollary 1. Detailed explanations can be found in Appendix 1.D.

Below, I explore the role of the market environment on the equilibrium outcome. The results apply for general w_0, but, when possible, intuitions are kept general for either $w_0 = 1/2$ or $w_0 > 1/2$.

The Role of Connectivity

High connectivity is generally detrimental to investment. In fact:

Lemma 2. *For any sharing behavior z, the monopolist's incentive to invest is single peaked in d; that is, there exists a threshold $\bar{d}$ so that $\Delta V_M(z)$ is increasing for any $d < \bar{d}$ and decreasing for any $d > \bar{d}$.*

Proof. See Appendix 1.D □

In particular notice that as soon as $z_+ > 0$,[18] $\Delta V(z;d) \to 0$ as $d \to \infty$. This means that as the network grows more connected, the producer's incentive to invest vanishes. This insight echoes Kranton and McAdams [2020]'s Proposition 3. To illustrate this point, consider a complete network, that is a network in which every consumer is connected with every other consumer. In such a context, a monopolist would need only a single share in order to reach every single consumers on the market; therefore, the monopolist can reach as many consumers by publishing a false article, than with true information, as long as one seed receives a different private signal than the others.

Role of Private Knowledge

Private knowledge encompasses two parameters: the prior about the state of the world, w_0; and the precision of private signals, γ. These represent respectively how *ex-ante* uncertain the documented topic is, and how well-informed agents privately are. Through different channels, both of these parameters have a similar effect on the producer's incentive to invest.

Proposition 2. *A decrease in private knowledge implies a decrease in the producer's incentive to invest. In particular, for any undominated z, $\Delta V_M(z)$ is increasing in both γ and w_0.*

[18] Technically, $z_{+|n} > 0$ for both $n = 0, 1$ is required.

Proof. See Appendix 1.D □

Intuitively, when γ is low, consumers are not good at distinguishing true from false articles; hence, false information tends to be shared almost as often as true information. The value of a true article is low while that of a false article is high. Therefore, the incentive for the producer to invest is low, as publishing a true article would not raise his visibility by a lot.

The channel through which *ex-ante* uncertainty affects the incentive to invest is different. Because seeds share more often if the article content corresponds to the most likely state of the world, the difference of value between true and false news is greater when the most likely state of the world realizes. Hence, investment is more beneficial to the producer when $\omega = 0$. The expected profits from any given investment thus increases when the most beneficial state becomes more likely.

Proposition 2 does not specify the effect of private knowledge on the equilibrium outcome. Indeed, γ and w_0 also affect the seeds' best-response. However, the equilibrium outcome is generally affected by a change in private knowledge in the same way as the producer's incentive to invest.

Corollary 2. *Generally, a decrease in private knowledge implies a decrease in the equilibrium investment. In particular:*

(i) q_M^* *increases with* γ*.*

(ii) q_M^* *increases with* w_0 *if and only if* $q_M^* \neq \bar{t}_0$*.*

Proof. See Appendix 1.D □

Interestingly, a lack of private knowledge tends not to be compensated for by the market. Indeed, a decrease in private knowledge generally leads to worse information provision. Hence, the online news market fails exactly when it is the most needed: when the state of the world is very uncertain, either because of little prior knowledge, or because of poorly informative private signals. This seems to indicate that the inefficiencies from the market structure, and in particular from the fact that online outlets derive revenues from advertisement, can be very problematic.

Another source of inefficiency linked to private knowledge appears from the strategic interaction, and in particular, from the seeds' imperfect knowledge.

Remark 1. *In equilibrium,* $q_M^* \leq \bar{t}_1$*. Therefore, news quality is bounded by agent's private knowledge* w_0 *and* γ*.*

A proper setup to formally study such inefficiencies is introduced in Section 1.4.

1.3.2 Equilibrium with Competition

I now assume that two producers coexist on the market. Because of tractability concerns, results are provided only for $w_0 = 1/2$. Let the two competitors be denoted by k and ℓ. The producer k's best response given ℓ's investment and sharing strategy z can be rewritten:

$$\Delta V_k(z; q_\ell) = (1-b)\Big[V_{T_k} - V_{F_k}\Big] = (1-b)\Big[q_\ell\Big(V_{T_k T_\ell} - V_{F_k T_\ell}\Big) + (1-q_\ell)\Big(V_{T_k F_\ell} - V_{F_k F_\ell}\Big)\Big]$$

Where:

$$V_{X_k Y_\ell} = \frac{p_{X_k}}{p_{X_k} + p_{Y_\ell}}\Big(1 - (1 - p_{X_k} - p_{Y_\ell})^d\Big)$$

For any article published by ℓ, whether true or false $Y = T, F$, the visibility of k is higher for true articles than for false news, i.e. $V_{T_k Y_\ell} \geq V_{F_k Y_\ell}$. Therefore, $\Delta V_k(z_k; z_\ell, q^*_\ell) \geq 0$. In particular, the incentive to invest is strictly positive as long as true news is shared more often than false news, i.e. for any $z_{T_k} > z_{F_k}$; it is null for $z_{T_k} = z_{F_k}$.

The shape of $\Delta V_k(z_k; z_\ell, q_\ell)$ in z_k is similar to the monopoly case; however, k's best-response also depends on the sharing behavior of seeds reached be ℓ, as well as ℓ's investment.

Lemma 3. *Duopolist k's best-response is as follows:*

(i) $q^*_k(z_k; z_\ell, q_\ell)$ *is single-peaked in* z_{T_k} *with maximum* $\bar{z}_k \in (0;1]$*; it is strictly decreasing in* z_{F_k}.

(ii) $q^*_k(z_k; z_\ell, q_\ell)$ *relation with* z_ℓ *depends on d. For small d, it is decreasing in* z_ℓ. *For large d, it is decreasing in* z_ℓ *for* $p^2_{F_\ell} > p_{T_k} p_{F_k}$.

(iii) $q^*_k(z_k; z_\ell, q_\ell)$ *is decreasing in* x_ℓ *for any* $z_{X_\ell} \geq z_{X_k}$.

(iv) $q^*_k(z_k; z_\ell, q_\ell)$ *is continuous in* z_k, z_ℓ *and* x_ℓ.

Proof. See Appendix 1.D □

As before, the best-response of producer k in non-monotonic to his own seeds' sharing z_k. Interestingly, if the sharing pattern is the same for either producer, their investment are strategic substitutes.

I can now characterize the NE. I call *symmetric equilibria* any equilibrium in which $z_k = z_\ell$ and $q_k = q_\ell$. In this case, $\Delta V_k = \Delta V_\ell$. I denote this common function $\Delta V_D((z_T, z_F), q)$, and omit the producers' indices on the seeds' best response z.

Proposition 3. *The only symmetric equilibrium features positive investment and is characterized by news precision* $q^*_D = \arg\min_{q \in [1/2, \gamma]} \big|\Delta V_D\big((1,0); q\big) - c(q)\big|$.

Proof. See Appendix 1.D □

The proof of existence is similar to that of the monopoly case. There are two cases to distinguish, as illustrated on Figure 1.8 and Figure 1.9.

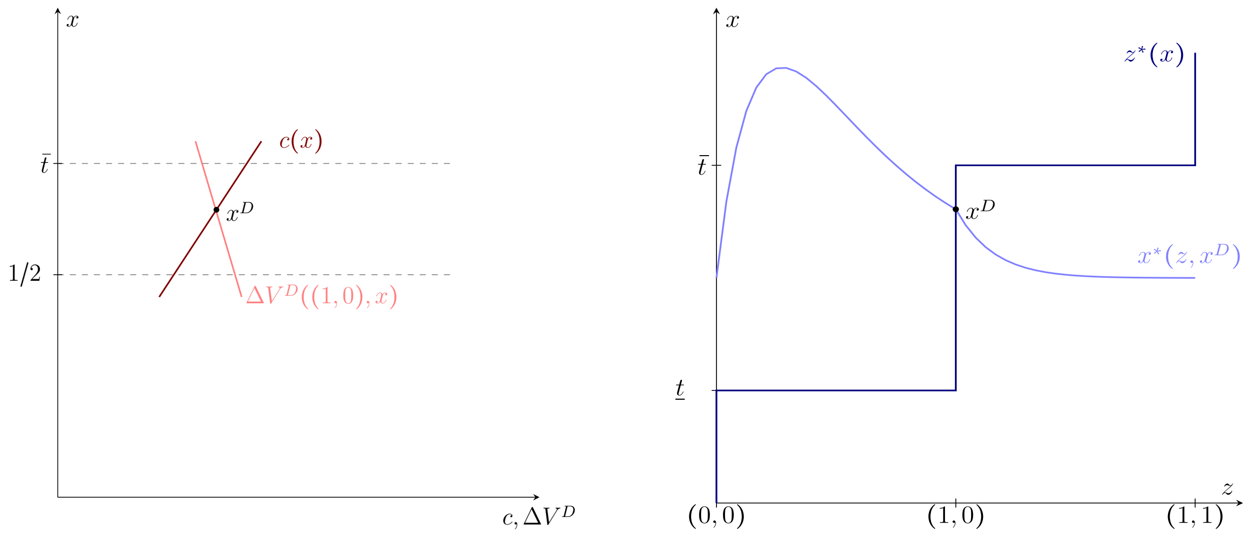


Figure 1.8: Illustration of a case for which $q_D^* \in (1/2, \bar{t})$

When $c(\bar{t}) \geq \Delta V_D((1,0),\bar{t})$, as in Figure 1.8, there exists an intersection between $c(q)$ and $\Delta V_D((1,0),q)$ in the interval $(1/2,\bar{t}]$ (left panel). Given that ℓ invests q_D^*, k's best response crosses the seeds' best response in $((1,0),q_D^*))$. Therefore, this intersection is a NE.

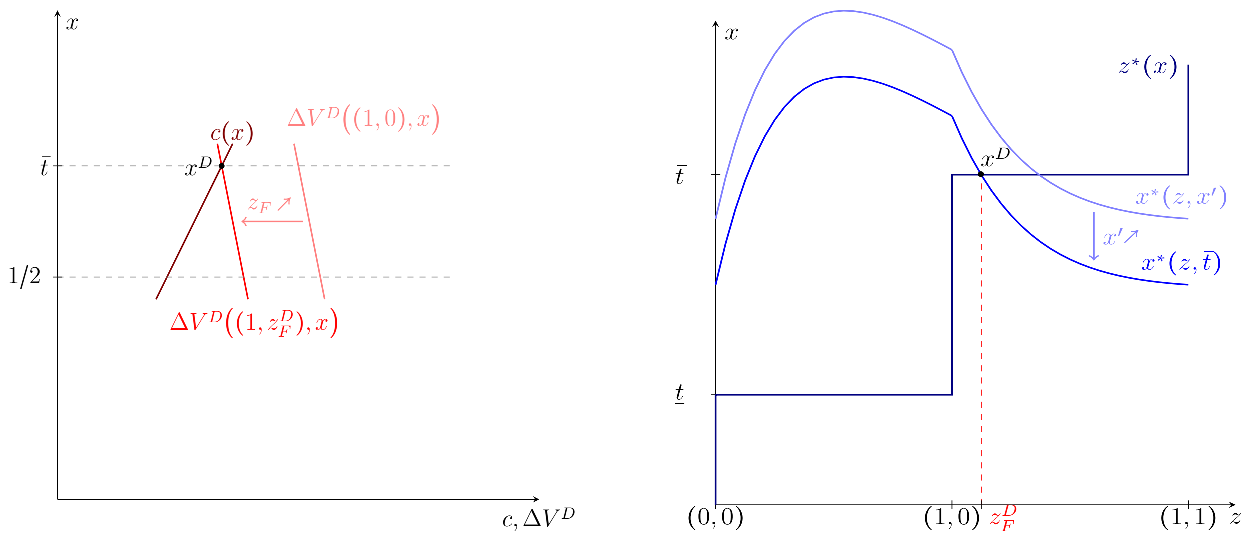


Figure 1.9: Illustration of a case for which $q_D^* = \bar{t}$

When $c(\bar{t}) < \Delta V_D((1,0),\bar{t})$, as in in Figure 1.9, $c(q)$ lies completely on the left of $\Delta V_D((1,0),q)$, without ever intersecting in the interval $q \in (1/2,\bar{t})$ (left panel). Equivalently, k's best response to z given that ℓ invests $q_\ell < \bar{t}$ intersects the seeds' best response

z above $\bar{t}$ (right panel). This means that there does not exist a symmetric NE in which $z^* = (1,0)$. However, if $z_F > 0$, $\Delta V_D(z,q)$ is shifted to the left in the space $(\Delta V_D, q)$, so that it now crosses $c(q)$ (left panel). Furthermore, because q_ℓ increases to $\bar{t}$, the curve $q_k^*(z, q_\ell)$ is shifted downwards in the space (z, q_k) (right panel). For some $z_F > 0$, $\Delta V_D((1, z_F), \bar{t}) = c(\bar{t})$, so that $q_D^* = \bar{t}$.

While the symmetric equilibrium is unique, asymmetric equilibria generally exist and are not unique.

Remark 2. *Let* $S := \frac{1}{2}(1-b\gamma)^d + \frac{1}{2}(1-b(1-\gamma))^d - (1-\frac{1}{2}b)^d$. *If the marginal cost function is linear with slope different than* S*, there are no equilibria with* $q_k \neq q_\ell$ *and* $(q_k, q_\ell) \in (1/2, \gamma)$.

Proof. Assume $q_D^* \in (1/2, \gamma)$. Assume that there exists an $q_k > q_\ell$, with $(q_k, q_\ell) \in (1/2, \gamma)$. Then, $c(q_k) = \Delta V_k((1,0),(1,0),q_\ell)$ and $c(q_\ell) = \Delta V_\ell((1,0),(1,0),q_k)$, so that $c(q_k) - c(q_\ell) = S(q_k - q_\ell)$, which is impossible if c has a slope different from S. □

1.3.3 Effects of Competition

The symmetric equilibrium q_D^* is compared to q_M^*. For cases to be comparable, let $w_0 = 1/2$. I confront the two types of markets in terms of connectivity and signal precision. Furthermore, $q_M^* \geq q_D^*$ only if $\Delta V_M(z) > \Delta V_D(z, q_D^*)$; therefore, I focus on $\Delta V_M(z)$ and $\Delta V_D(z, q_D^*)$ in this section.

The Role of Connectivity

The comparison between monopoly and duopoly depends on the connectivity of the network. Indeed, the presence of a competitor has an ambiguous effect on a producer's incentive to invest: on the one hand, investment might increase because each follower is harder to reach, so that the producer needs to be *sufficiently* shared; on the other hand, news quality might decrease because each producer reaches fewer seeds, so that fewer followers can be reached. In other words, by making the number of shares more important, competition decreases the value of false information; by reducing the producers' potential readership, it decreases the value of true information.

The strength of both of these forces depends on the connectivity. In a very connected network, each seed is connected to most followers, so that a producer *can* reach almost all followers even when a competitor is present. In a sparsely connected network, each follower is unlikely to be connected to several seeds, so that the probability to reach a follower is almost independent from the other competitor's outcome. Therefore, competition should lead to lower investment in sparse network, but would be beneficial to news quality in dense networks. Theorem 1 formalizes this; in particular, there is a unique threshold for a network connectivity that determines which of the two forces dominate.

Theorem 1. *There exists a unique threshold* $\bar{d}$ *such that* $q_M^*(d) \geq q_D^*(d)$ *for all* $d < \bar{d}$ *and* $q_M^*(d) \leq q_D^*(d)$ *for all* $d > \bar{d}$

Proof. Define $DV(d) := \frac{\Delta V_M(z;d) - \Delta V_D(z,q;d)}{1-b}$. First, notice it that for $d = 1$, $DV(d) > 0$; however for $d \to \infty$, $DV(d) < 0$. Therefore, there must exist some d_0 such that $DV(d_0) \geq 0 > DV(d_0 + 1)$. All that is left to do is to show that such d_0 is unique. This is the case because if $DV(d_1) > DV(d_1 + 1)$ for some d_1, then DV is decreasing for all subsequent $d > d_1$. See Appendix 1.D for technical details. □

To further illustrate the two mechanisms at hand, I consider a few specific instances. Take $d = 1$. Figure 1.10 depicts such a network with 12 nodes. On the left panel, the producer, was his investment sufficient to make all seeds share, would reach four additional nodes; on the right panel, the same producer who now shares the market with a competitor can, at best, reach only two followers. Therefore, his incentive to invest when a competitor is present is half that of the case with no competition.

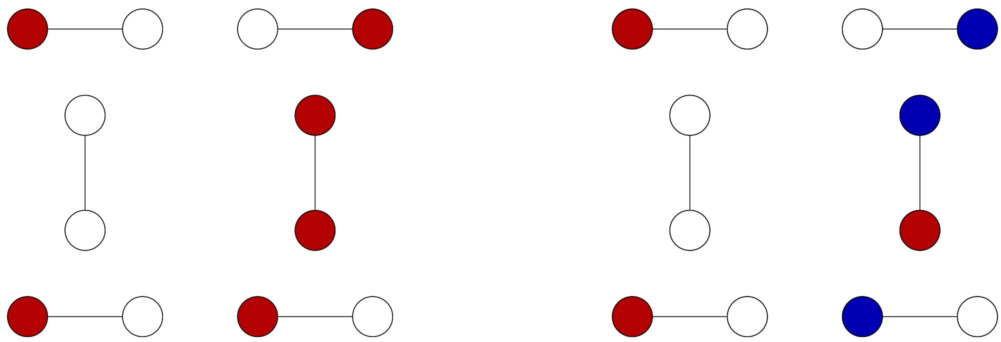


Figure 1.10: Possible followers reached with one vs. two producer(s) on the market

This insight applies in any network with $d = 1$. Indeed, the number of followers a produce can reach is linearly proportional to the number of seeds who share content. Therefore, the producers' incentive to invest is linearly proportional to the additional number of seeds who share when publishing a true article: for each additional share, the producers expect one additional view from a follower. Because a monopolist exogenously reaches twice as many seeds as each duopolist, a duopolist's incentive to invest is half that of a monopolist:

$$\Delta V_M(z;1) = (1-b)(2\gamma - 1)(z_+ - z_-) > (1-b)\tfrac{1}{2}(2\gamma - 1)(z_+ - z_-) = \Delta V_D(z,q;1) \quad \forall z_+ > z_-$$

As the network's connectivity grows, this force vanishes, while the intensity of the competition increases. Figure 1.11 underlines how the strength of competition is made greater by a denser network. In the part of the network depicted, the node in the center happens to be a follower surrounded by seeds reading different articles. Most nodes reached by producer 1 (in red) do not share, as represented by a grey circle. In a network where $d = 4$, producer 1 still has one chance out of two to reach the central node; with $d = 8$, his chances are only 1 out of 4 given the same sharing pattern.

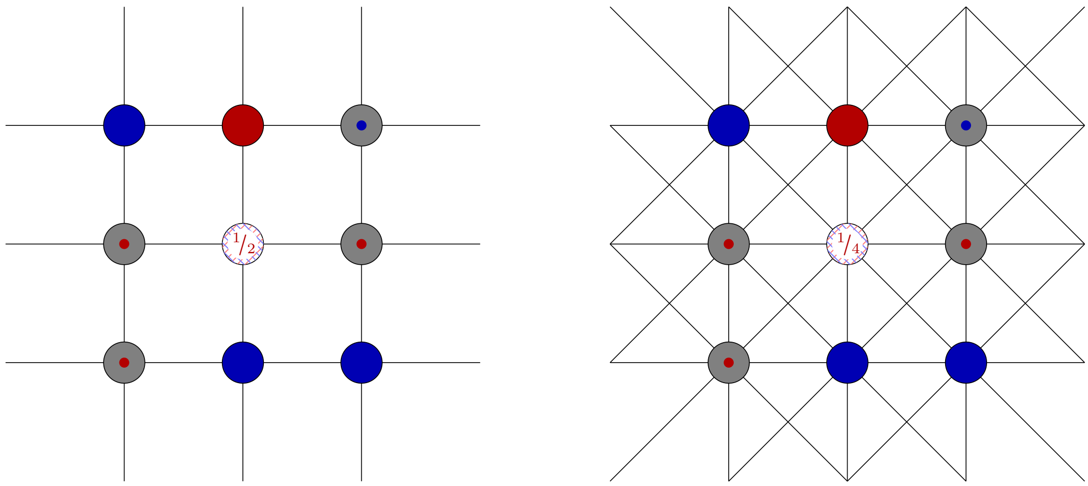


Figure 1.11: Probability of reaching a follower with $d = 4$ vs. $d = 8$

This insight continues to apply as d grows. Take $d \to \infty$ such that that all nodes are connected to each other.[19] All followers will be reached by an article as soon as the probability of sharing is not null, however arbitrarily small. For a monopolist, the incentive to invest vanishes, since only one seed sharing suffices to reach the entirety of the network. It is quite the opposite in a duopoly. The probability to reach a follower depends on the decision of every seed. Hence, the incentive to invest is proportional to the ratio of additional seeds of his sharing, to all sharing seeds:

$$\Delta V_M(z;\infty) = 0 < (1-b)\frac{(2\gamma-1)(z_+-z_-)}{z_++z_-} = \Delta V_D(z,q;\infty) \quad \text{since } (1-\varepsilon)^d \to 0 \ \forall \varepsilon > 0$$

Remark 3. *A further increase in competition, beyond two producers, is either always detrimental, or detrimental for sparser networks.*

Proof. Take any competition with K competitors and a symmetric incentive to invest. Add one producer k'. The difference in the incentive to invest between the K and $K+1$ is positive in $d = 1$ and has a sign which depends on the parameters for $d \to \infty$. See Appendix 1.D for the computation. □

[19] This requires the network to be infinite. Because the finiteness of the network is not essential to the specification, let us assume, for convenience, that $|I| \to \infty$, and that d grows as fast as $|I|$.

The Role of Signal Precision

Remark 4.

(i) When the seeds get perfectly informative private signal, monopoly yields higher investments than duopoly.

(ii) When the seeds get perfectly uninformative private signal, the incentive to invest vanishes for both a monopoly and a duopoly.

Proof. It suffices to derive the value of $\Delta V_M(z) - \Delta V_D(z)$ for $\gamma \to 1$ and $\gamma \to \frac{1}{2}$. Computations can be found in Appendix 1.D. □

When the signal is perfectly informative, seeds only share true information. Then, the monopolist has the highest possible incentive to invest: false information is worthless; with true information, he reaches all the followers the network allows him to reach. For the duopolist, false information is also worthless, but true information is less beneficial. Indeed, if the competitor released true information, they together reach the same portion of followers as the monopolist would have, but they split this audience in two; if the competitor released false information, the duopolist gets the whole share of followers reached, but he reaches fewer followers than the monopolist would have since he is read by fewer seeds.

When the signal is perfectly uninformative, the result is very intuitive: as the private signal is noisy, the agents are not able to tell true from false information, so that they treat both type of news without accounting for their private signal. Because the game is simultaneous, the producer does not internalize the effect of his investment on the consumers' prior, so that no investment is featured in equilibrium.

Numerically, for b and d high enough, there exists a threshold for γ such that duopoly is yielding a higher investment than monopoly for any private signal with lower precision. It indicates that a higher signal precision has stronger effects in a monopoly than in a duopoly. Intuitively, when the signal precision is high, consumers are relatively good at distinguishing true from false articles; therefore, very few seeds are sharing false news, and the monopolist cannot rely on them to reach enough followers. Therefore, the positive effect of competition, that making followers harder to reach, is marginal; while its negative effect, that reducing the number of followers who are reachable, is still significant. Overall, competition is then detrimental.

The example below details some numerical applications.

Example 1. Consider the sign of $\Delta V_M - \Delta V_D$ as a function of γ. In the competitive case, the incentive for one producer to invest is influenced by the competitor's investment. In a symmetric equilibrium, a change in γ indirectly influences ΔV_D through q_D^*. In this example, I only consider the direct effects.[20] In particular, throughout the example, I consider $q^*_\ell = 0.6$ and study k's best response. Furthermore, I assume $z^* = (1,0)$ for both producers. I report the threshold for γ above which $\Delta V_M - \Delta V_D > 0$. For any γ greater than the threshold reported, the incentive to invest in a monopoly is greater. I consider three level of broadcast reach and connectivity. The thresholds rounded to three digits are reported in Table 1.1.

	$b = 0.25$	$b = 0.5$	$b = 0.75$
$d = 5$	–	–	0.612
$d = 10$	–	0.820	0.914
$d = 20$	0.811	0.941	0.964

Table 1.1: Private signal precision thresholds with $q^*_D = 0.6$

Note that the thresholds reported are all above $q^*_D = 0.6$, which is consistent with $z^* = (1,0)$. No threshold reported means that no matter the signal precision, monopoly creates a bigger incentive to invest than competition. Echoing Theorem 1, Table 1.1 illustrates how duopoly leads to a higher incentive to invest for a bigger range of signal precision as d and b increase.

1.4 Welfare

So far, I have only been interested in the market outcomes, as measured by investment. Welfare has not been addressed.

First, I note that the market outcome is inefficient.

Proposition 4. *Any equilibrium outcome on the news market with revenues derived from ads is Pareto inefficient*

Proof. Take the case of a monopoly, with equilibrium $e^* = (q^*, z^*)$. Define $q^c(z; e^*)$ as the level of news quality that makes a consumer whose sharing decision is z indifferent between (q^c, z) and e^*. Likewise, define $q^p(z; e^*)$ as the level of news quality that ensures to the producer faced with sharing decision z the same revenue as e^*. If $\frac{\partial q^c}{\partial z} < \frac{\partial q^p}{\partial z}$ [21], there is room for Pareto improvement since consumers require less investment to marginally increase their sharing than the producer is ready to offer for the same marginal increase

[20] Considering indirect effects would require to specify a marginal cost function.

[21] We abuse notation here in order to keep the intuition as clear as possible. While z is a vector, recall that, when q increases, the consumers would first share the most likely congruent news, then any congruent news, then the most likely news anyways, and then any news. Therefore, with ∂z, I mean to designate a marginal change in the sharing probability **in the relevant dimension**. So for instance if z=(1,0,0,0), ∂z is actually $\partial z_{+|1}$; if $z = (0.5, 0, 0, 0)$, then we mean $\partial z_{+|0}$.

in sharing. Now, the FOC of equilibrium imply $0 \leq \frac{\partial q^c}{\partial z} < \infty$ while $\frac{\partial q^p}{\partial z} \to \infty$. The same reasoning applies to duopolists. □

To analyze the welfare resulting from the production of news, I propose two approaches. The first one relates to the *entertainment* purpose of possible sharing behavior. In this sense, only seeds and producers are part of the analysis. The seeds' decision to share an article depends on the utility of sharing as defined above. However, this does not capture how *informative* the article is. In particular, it does not allow to judge whether agents are making, on average, better choices. To address this question, I introduce an additional action to be taken by all news' consumers after the strategic interactions have unfolded. This permits to analyze whether, on average, agents are able to take better decision; as well as whether the information contained in articles published in online outlets that derive revenues from advertisement can motivate agents to take actions they would have opted out from, were they informed only privately. Furthermore, this measure of welfare accounts for followers' well-being.

1.4.1 Framework of Analysis

Once the game is played out, I assume that a further action takes place: all consumers can chose $a \in \{0, 1\}$ to match the state of the world. I think of this as a financial bet, but it can capture a wider range of utility derived from information. This action can depend on the private signal they receive and on the content of the article they read (if any). I assume that this bet has entry price r, and that consumers might decide to opt out of the bet after having observed their signal and the news article (if any). I represent the case in which the consumers cannot opt out of this action with $r = 0$. The benefits from matching the state of the world is assumed to be the same as their loss from a mismatch:

$$u_j(a|\omega = w) = \begin{cases} 1 & \text{if } a_j = w \\ -1 & \text{otherwise} \end{cases}$$

From the consumer perspective, I study three different utilities: the utility derived by seeds from sharing, the utility derived by any consumer from betting and the utility derived by any consumer from choosing to enter the bet. I refer to the utility from sharing as $u_i(z)$ and to the utility from betting as $u_j(a)$, while the utility from entering the bet also depends on the betting price r.

Lemma 4. *The relevant expected utilities are as follows:*

- *Seeds' expected utility from sharing is:*

$$\mathbb{E}(u_i(z)) = \sum_{S,n,k} z_{S|n,k}\Big[q_k \Pr(X, \omega = n) - (1-q_k)\Pr(\text{-}X, \omega \neq n)\Big]\frac{1}{K}$$

- *Seeds betting $a(n,s) = n$ with probability $z_{S|n,k}$ have expected utility:*

$$\mathbb{E}(u_i(a)) = \sum_{S,n,k} (2z_{S|n,k} - 1)\Big[q_k \Pr(X, \omega = n) - (1-q_k)\Pr(\text{-}X, \omega \neq n)\Big]\frac{1}{K}$$

Upon reading some news, followers betting n with probability $z^f_{S|n}$ have expected utility:

$$\sum_{m,S,n,k}\left(2z^f_{S|n,k} - 1\right)\left[q_k \frac{p_{T|n,k}}{p_{T|n,k}+p_{Y|m,-k}}\Pr(m, S, \omega = n) - (1-q_k)\frac{p_{F|n,k}}{p_{F|n,k}+p_{\text{-}Y|m,-k}}\Pr(m, \text{-}S, \omega \neq n)\right]$$

- *Consumers' expected utility from entering the bet is:* $\mathbb{E}\left(\max\{u_j(a) - r; 0\}\right)$

Proof. See Appendix 1.D. □

Recall that, $S = +$ iff $n = s$ and $S = -$ otherwise. Therefore, $\Pr(+, \omega = n) = \gamma \Pr(\omega = n)$ and $\Pr(-, \omega = n) = (1-\gamma)\Pr(\omega = n)$.[22]

The seeds' expected utility from sharing news reporting n after private signal s is $2p(n,s) - 1$, as shown in Section 1.2.3. To find their expected utility from sharing, it suffices to incorporate their decision to share or not, $z_{S|n,k}$, and the probability for the news of producer k to report n after private signal s. Note that the utility from sharing of a random consumer is $b\,\mathbb{E}(u_i(z))$.

Because of the similarity in payoff structure, the seeds' betting decision follows the same threshold rule as their sharing decision: they bet that the state of the world is the one reported in the article if the probability for the true state of the world to correspond to the news, $p(n,s)$, is greater than 1/2. Therefore $a(n,s) = n$ is played with probability $z_{S|n}$.[23] By a slight abuse of notation, I refer to this betting strategy as $z_{S|n}$ as well.

Remark 5. *For any strategy with $z_{S|n,k} > 0$ for some (S,n,k), the expected utility from sharing is strictly increasing in q_k; the expected utility from betting is not.*

[22]Likewise, Y is implicitly determined by m and ω. So for instance, take $n = \omega = 0$, then $m = 0$ would lead to $Y = T$ and $\text{-}Y = F$, while $m = 1$ would mean $Y = F$ and $\text{-}Y = T$.

[23]Formally, $\mathbb{E}(\mathbb{1}_{a=n|n,s}) = z_{S|n}$. When $p(n,s) = 1/2$, the seeds are indifferent between betting the news content or its opposite. The tie rule was chosen in order to keep consistency. Because when indifferent between several strategies, by definition, their utility is equal among all strategies, this assumption does not influence the welfare analysis.

Despite the similarity in strategies, the expected utility from sharing and betting are different. This occurs because seeds are constraint not to share if they do not believe the news content, while they can still bet their private signal rather than the news in such a case. For instance, an outlet that would systematically report an erroneous content, i.e. $x_k = 0$, would lead to no sharing and so a null utility from sharing; but it would be perfectly informative: betting the opposite than the article reports would always ensure to correctly match the state of the world, thus granting the maximal possible utility from betting.

Notice that followers can bet according to a different strategy than seeds. All consumers have the same preferences and priors; however, in competitive markets, the precision of articles received by followers is higher than that of the outlets which issue them. Indeed, the network filters out false articles: true articles are shared more, so they reach followers with higher probability. Internalizing this effect, followers require less precision from the outlet in order to start betting what the news reports. The decision rule can be defined implicitly as $z^f_{X|n,k} = 1$ when the expected utility from betting the news content is higher than that from betting the private signal.[24]

The expected utility from betting of a consumer taken at random can be defined by accounting for the probability for the consumer to assume the role of either a seed or a follower; and, upon being a follower, for the endogenous probability to read an article.[25]

To understand whether news drives agent to take a bet from which they would have otherwise opted out, I allow consumers to chose whether to enter the bet after observing (n, s). They opt out from it if its expecting value, as described by $u_j(a)$, is lower than the cost of entry $r > 0$. Again, this depends on the content reported in the news they read (if any) and its congruence with their private signal.

1.4.2 Welfare for symmetric priors

Throughout this section, I assume $w_0 = 1/2$. Accordingly, I evaluate whether the presence of news outlets has welfare benefits for consumers and I discuss the effect of competition on total welfare.

Theorem 2. *When no state of the world is ex ante more likely, the existence of news*

[24] I.e. a follower bets article content n by producer k after receiving a private signal $X = T, F$ with probability:

$$z^f_{X|n,k} = 1 \Leftrightarrow \sum_m \left[q_k \Pr(X, m|\omega = n) \frac{p_{T|n,k}}{p_{T|n,k} + p_{Y|m,-k}} \Pr(\omega = n) - (1 - q_k) \Pr(X, \text{-}Y|\omega \neq n) \frac{p_{F|n,k}}{p_{F|n,k} + p_{\text{-}Y|m,-k}} \Pr(\omega \neq n) \right] > 0$$

[25] Formally, conditional on being a follower, the expected utility from betting is:

$$\sum_{m,X,n,k} \left\{ (2\gamma - 1)(1 - p_{X|n,k} - p_{Y|m,-k})^d Pr(m, n, \omega) + \left(2z^f_{X|n,k} - 1\right) \left[q_k V_{T_k Y_{-k}} \Pr(X, m, \omega = n) - (1 - q_k) V_{F_k Y_{-k}} \Pr(X, m, \omega \neq n) \right] \right\}$$

outlets has ambiguous effects on consumers' welfare:

(i) For seeds, the presence of news outlets does not improve their betting decision and is detrimental to their capacity to enter the bet for a non-zero measure set of r. It is beneficial to their expected utility from sharing.

(ii) For followers, the presence of news outlets does not improve the betting decision and is detrimental to their capacity to enter the bet for a non-zero measure set of r if the market is not competitive or if the competitive symmetric investment $q_D^* \leq \frac{\gamma - \sqrt{\gamma(1-\gamma)}}{2\gamma - 1}$

Proof. See Appendix 1.D. □

As seeds cannot derive utility from sharing without any articles to share, the presence of news outlets has a positive effect on seeds' sharing utility. This relates to a concept of entertainment: news consumers might be entertained by ad-funded online news outlets. The other results from Theorem 2 show how consumers might however not be informed by such outlets.

With $w_0 = 1/2$, the news quality in any equilibrium is such that $q^* \leq \gamma$. Seeds are thus either better off betting their private signal, or indifferent between betting their signal or the news content. Therefore, news' outlets are not improving on their choice. In other words, the presence of news outlets does not bring seeds to better decisions.

Followers, however, might benefit from competition. In a competitive market, true news is more visible to followers as the network filters out false articles. This raises the quality of the articles perceived by followers, which might become more precise than the followers' private signal. The quality of news reaching followers is exactly γ for $q_D^* = \frac{\gamma - \sqrt{\gamma(1-\gamma)}}{2\gamma - 1}$.

Note that, although articles reaching are true more often than q^*, the quality of news perceived by followers is still bounded by γ.

Lemma 5. *In a competitive market, the utility from betting for followers upon receive some news is bounded by the precision of their private signal. In particular,* $\mathbb{E}(u_f(a)|\textit{seeing some article})$ $\left[2\gamma - 1; \frac{3}{2}(2\gamma - 1)\right]$.

Proof. See Appendix 1.D. □

The upper bound is found by computing $\mathbb{E}(u_f(a))$ for $q_D^* = \gamma$ and $z^* = (1, 0)$. Indeed, the network is the best at filtering out false information for $z_F = 0$.

Theorem 2 also shows how the presence of news outlets can be detrimental to news consumers, even if they are fully Bayesian. Intuitively, for low to moderate entry costs r, consumers would enter the bet without the presence of any news outlets, as their private signal is informative enough to justify the cost r. However, upon reading a news article whose content disagrees with their private signal, consumers are too uncertain about the state of the world to enter the bet. Now, because the news outlets are more noisy than the private signals, consumers are more often wrongly than rightly dissuaded.

Note that for moderate to high entry costs r, the presence of news outlets can be beneficial. In Lemma 6, I further characterize the cases in which news outlets are beneficial or detrimental to consumers' ability to enter the bet.

Lemma 6.

(i) News' outlets are for seeds' capacity to enter the bet: beneficial for $r \in [r_s, \bar{r}]$, *detrimental for* $r \in [\underline{r}, r_s)$, *and neutral otherwise. These effects are strict for* $q^* < \gamma$.

(ii) For followers: the same applies for uncompetitive market; similar thresholds $\underline{r}'$ *and* $\bar{r}'$ *exist if the market is competitive with symmetric investment* $q_D^* < \frac{\gamma - \sqrt{\gamma(1-\gamma)}}{2\gamma - 1}$; *and news outlets are never detrimental otherwise.*

With $\underline{r} = 2\frac{\gamma(1-q)}{\gamma(1-q)+(1-\gamma)q} - 1; r_s = 2\gamma - 1; \bar{r} = 2\frac{\gamma q}{\gamma q + (1-\gamma)(1-q)} - 1$

Proof. See Appendix 1.D. □

The proof compares the behavior of consumers with and without the presence of news for any r. In particular, for $r \in [\underline{r}, \bar{r}]$, seeds only participate to the bet if $n = s$. If, without an article, they would not have participated to the bet, the information transmitted thanks to news outlets is beneficial, as most seeds being prompted to participate are placing the right bet. However, if without an article, they would have participated to the bet, then the information transmitted thanks to news outlets is detrimental. Indeed, in such a case, the article is wrong more often than the private signal, so most seeds who opted out should have placed a bet. Figure 1.12 illustrates this intuition.

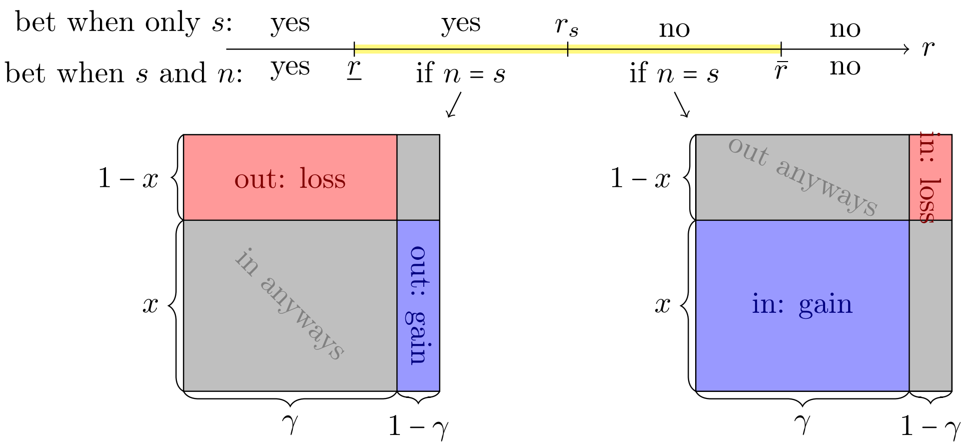


Figure 1.12: Illustration of Lemma 6's proof

The same reasoning applies to followers. However, in a competitive market, the quality of information perceived by followers can be greater than the precision of the individual news outlets. Therefore, $q^* \leq \gamma$ is not sufficient for the articles read by followers to be more noisy than their private signal. Furthermore, the values of r that make followers change their behavior after having received a news article has to account for the quality of the news they perceive.[26]

The welfare consequences of competition can now be assessed more carefully, by comparing consumers' expected gains from betting in a monopoly and a duopoly.

Proposition 5. *Irrespective of the aspect of consumer welfare considered, competition can hinder total welfare even if $q_D^* > q_M^*$.*

Proof. See Appendix 1.D □

When considering the total welfare effect of competition, both sides of the markets have to be considered. Seeds are not made better off and their quantity does not change in expectation. Followers might be better off if q_D^* is close enough to $\bar{t}$; however, the number of followers encountering an article might be affected by competition. As seeds share more types of news, more followers come across possibly informative news, but the least informative becomes the news, as the network fails to filter out wrong articles. Producers split their readership while the total production cost doubles. Therefore, the effect of competition on welfare depends on the level of news quality, the connectivity of the network and the ratio of seeds in the population.

1.4.3 Welfare for asymmetric priors

When the prior about the state of the world w_0 is different from 1/2, the quality of the news is bounded by private *knowledge*. It means that the article published by news outlets might be more precise than private signals. However, generally, the insights from Section 1.4.2 are still applicable: the presence of news outlets have ambiguous effects on consumers' welfare.

Corollary 3. *In uncompetitive markets, for any prior on the state of the world $w_0 < \gamma$:*

(i) Consumers are brought to better decisions iff $q^* > \gamma$.

(ii) The gains from betting are still bounded by the precision of the private signal. In particular, $\mathbb{E}(u_j(a)) \in \left[2\gamma - 1; \frac{2\gamma-1}{1-2\gamma(1-\gamma)}\right]$

Proof. (i) follows from Theorem 2; for (ii), see Appendix 1.D. □

[26] In particular, $\underline{r}' = 2\frac{\sum_Y \gamma(1-q)V_{FY}}{\gamma(1-q)V_{FY}+(1-\gamma)qV_{T\text{-}Y}} - 1$ and $\bar{r}' = 2\frac{\sum_Y \gamma q V_{TY}}{\gamma q V_{TY}+(1-\gamma)qV_{F\text{-}Y}} - 1$

This results echoes Theorem 2: the expected gains from the bet are restricted by the consumers' private knowledge. The bound is derived by considering a prior so strong that agents are indifferent between betting their private signal and their prior.

If the bet has an entry cost r, as in the symmetric prior case, the presence of news outlets has ambiguous effects on the capacity for consumers to enter the bet.

Corollary 4. *In uncompetitive markets, for any prior on the state of the world $w_0 < \gamma$, and any equilibrium outcome $q_M^* < \max\left\{\gamma, \frac{{w_0}^2}{{w_0}^2+(1-w_0)^2}\right\}$:*

(i) There exists a non-zero measure interval for r for which news outlets reduce consumers' capacity to decide to enter the bet.

(ii) The outlets effect on consumers' capacity to enter the bet can be non-monotonic in r.

Proof. The analysis is similar to Lemma 6. See Appendix 1.D for the computations. □

1.5 Fact Checking

In this section, I study the effect of fact checking when applied to articles or to outlets. Applied to articles, I study how flagging false information affects welfare and its differential effect on news quality in non-competitive and competitive markets. Applied to news outlets, I question how much can quality certification improve producers investment.

1.5.1 Flagging

I wonder how flagging false information helps the provision of information on the market. In particular, let us assume that with some probability ρ, an information that does not correspond to the state of the world would be flagged by the platform on which seeds share before they decide whether to share. Because they care about truth only, such flagged information will never be shared. Hence, we can see flagging as perfectly informative signals, substituting the need for private signal. Therefore, one would expect this intervention to improve the outcome by decreasing the value of false information.

Remark 6. *The presence of flagging removes the bound placed on news quality from the precision of private information.*

Another interesting feature of flagging is that the marginal benefit of increasing the probability of being flagged depends on the market structure; in particular, when there is competition, there are strategic consideration to take into accounts. On one hand, an increased ρ makes false information relatively less valuable than true information; on the other hand, an increased ρ might make one's competitor more prone to being flagged, which in term decreases one's incentive to invest, as false information might be enough to survived faced to a flagged competitor.

To see this, let us rewrite the producers' best responses in a monopoly and duopoly when facing a probability ρ that false information is flagged. For the monopolist it is proportional to:

$$V_T - (1-\rho)V_F$$

For duopolists that behave symmetrically, it is proportional to:

$$qV_{TT} + (1-q)\big[(1-\rho)V_{TF} + \rho V_{T\varnothing}\big] - q(1-\rho)V_{FT} - (1-q)(1-\rho)\big[(1-\rho)V_{FF} + \rho V_{F\varnothing}\big]$$

with $V_{X\varnothing}$ denoting the value of publishing a $X = T, F$ article when the competitor has been flagged, that is, $V_{X\varnothing} = 1 - (1 - \frac{p_X}{2})^d$.

To analyze the tradeoff described above, I study how $\Delta V_M(z, q; \rho) - \Delta V_D(z, q; \rho)$ evolves with ρ. I find that flagging is more efficient in a monopoly.

Proposition 6.

(i) Flagging has a stronger effect in monopolies than in duopolies; i.e. $\frac{\partial(\Delta V_M(z;\rho) - \Delta V_D(z,q;\rho))}{\partial \rho} > 0$

(ii) For any environment, there exists a level of flagging that makes competition detrimental; i.e. $\forall(\gamma, b, d), \exists \rho' : \Delta V_M(z; \rho') \geq \Delta V_D(z, q_D^*; \rho')$*, where the inequality is strict for any positive probability of sharing,* $z_T > 0$*.*

Proof. See Appendix 1.D. □

Proposition 6 underlines how intervening is more difficult in competitive news markets. First, the same intervention has a stronger effect on a monopolist than on duopolists. Intuitively, competition dilutes the effect of flagging because of the strategic interaction between producers' investment. If flagging occurs more often, the value for any producer of publishing false information decreases; however, for a duopolist, it is more likely that the competitor has been flagged, and thus not to have to compete in the network in order to reach followers.

To make this intuition more tangible, consider the two forces discussed to put into perspective Theorem 1. Recall that the effect of competition on news quality depends on the connectivity of the network because of the trade-off between two forces: on the one hand, followers are harder to reach; on the other hand, the potential readership is reduced. Now, flagging false articles makes followers harder to reach anyways, with or without competition. Therefore, the benefits of competition are less and less relevant as flagging increases; the negative effect of competition however remains, since the maximum number of readers that can be reached is independent of the flagging probability.

The second result from Proposition 6 shows that competition is always detrimental to the incentive to invest if flagging occurs often enough. Therefore, one can think of this intervention as a substitute for encouraging a change in the market structure towards more competitive markets. In fact, any market outcome from competition is reproducible though flagging.

Corollary 5. *Any outcome $q_D^* > q_M^*$ is reproducible in a monopoly with some level of flagging ρ'; i.e. $\exists \rho' : \Delta V_M(z;\rho') = \Delta V_D(z, q_D^*; 0)$.*

Proof. Follows from Proposition 6; see Appendix 1.D for details. □

Proposition 6 and Corollary 5 both use the following element: if all false articles are flagged, $\rho = 1$, monopoly yields higher incentive to invest than duopoly. This echoes Remark 4 as both $\rho = 1$ and $\gamma = 1$ make false information worthless. As explained above, competition can be positive in that it worsens the value of false information; but if false information is useless anyways, only the reduction of the readership remains. More generally, flagging can be seen as a substitute for consumers' private signal. Interestingly, flagging forces outlets to provide news that goes beyond consumers private knowledge, and thus to create informative content.

Remark 7. *When false articles are flagged, news quality is not bounded by private knowledge anymore.*

While these conclusions rely on a setup that ignores any type of partisanship and distrust of the flagging institutions, they still underline the importance of flagging to counteract the weak incentives created by the business model of ad-funded online news outlets.

1.5.2 Quality Certification

I now wonder how welfare could be improved upon if the consumers were observing the actual quality of information. In terms of policy, this could for instance correspond to the role of a third party institution in charge of certifying the average quality of a news source, or an average *fact checking* score to be displayed on the online outlet.[27]

To understand the implication of such a policy, we need to assume a sequential move game. Appendix 1.C presents the SPE of the monopoly when $w_0 = 1/2$. The outcome depends on the shape of the total cost function $C(q)$.

[27] Such initiatives already exist, such as The Trust Project or Media Bias/Fact Check.

However, in a sequential move game, the seeds' best-responses would not change. Therefore, the threshold on news quality for which they would share any type of information, regardless of their private signal, does not change. This threshold is also the maximum achievable quality in a sequential game, which is set by the consumers' private knowledge.

Remark 8. *Even when observable, news quality is bounded by private knowledge: $q^* < \bar{t}_1$. Therefore, the presence of news outlets still has ambiguous effects on consumers welfare.*

As emphasized in Remark 8, most results from Section 1.4 still apply when the quality of news outlets is observable. In particular, Theorem 2, Lemma 5 and 6, and Corollary 3 and 4 all apply.

Interestingly, both flagging and quality certification rely on the same type of policy: fact checking; yet, they have very different implications. This indicates that a major barrier to high quality online news is consumers' limited private knowledge; improving consumers' trust in news outlets is not sufficient to correct for the inefficiency generated by a business model in which revenues are generated from visibility.

1.6 Discussion

Most of the results exposed in this paper rely on the two following insights: the producers' incentive to invest is determined by the difference between the value of true and false articles and the consumers' private knowledge bounds news quality. These insights are robust to many extensions of the model.

1.6.1 Different setup

The setup studied so far analyzes a simplistic market in which all consumers are identical. All have the same number of neighbors, the same probability of being a seed, the same amount of private knowledge. These assumptions make my analysis more transparent; below, I show that it does not drive any of my results.

Irregular Networks & Seeds' Selection

The setup assumes a regular network, in which every node has exactly d neighbors and has the same probability b to be a seeds. Relaxing these assumptions would be inconsequential to the results as long as seeds' are not targeted, that is, as long as there is no strategic component to the seeds' identity. Denote $V_X(d_j)$ the probability that a node j with degree

d_j is reached by the producer producing a $X = T, F$ article. This would be defined as before. Given any network with degree distribution $\delta(d_j)$, the producer's incentive to invest is simply $\sum_{d_j} \delta(d_j) \left[V_T(d_j) - V_F(d_j)\right]$. This allows for the probability of being a seed to be degree-specific; denoting this probability $b(d_j)$, and taking a monopolist best-response for exposition ease, $V_X(d_j) = \frac{b(d_j)}{K} + \left(1 - b(d_j)\right)\left(1 - (1 - p_X)^{d_j}\right)$. All results would follow through. Details are provided in Appendix 1.D.

Heterogenous signal precision

Consider that consumers receive private signals according to different signal precision γ_j. Most results would directly apply. The seeds' problem would not significantly change: it would be precision-specific – possibly seed-specific. For producers, the probability to be shared after publishing a $X = T, F$ article would incorporate the heterogeneity of signal precision; denoting $\psi(\gamma_i)$ the proportion of seeds with signal precision γ_i, the probability to be shared becomes: $p_X = \frac{b}{K} \sum_i \psi(\gamma_i)\left(\gamma_i z_S + (1 - \gamma_i) z_{\text{-}S}\right)$ where $S = +$ for $Y = T$. The analysis would then be directly applicable.

Note that the news quality in such a context would be bounded by the highest signal precision. Therefore, nodes whose signal precision is noisier would benefit from the presence of news outlets; results from Section 1.4 would then be mitigated to account for the distribution ψ. In particular, a low proportion of nodes with maximal signal precision would make the presence of news outlets more beneficial if the bound is reached; however, the bound would be less likely to be reached as the producers' could rely on the large proportion of nodes with noisier private signals to be shared.

There would not be such a trade-off if signal precision was to differ between seeds and followers. In particular, if the seeds' private information was more precise than that of followers, the presence of news outlets would more often be beneficial to followers. In the opposite case, news outlets would likely be uninformative to both seeds and followers. In any case, the benefits from news outlets are still bounded by agents private knowledge.

1.6.2 Different Objectives

The model I analyze considers agents whose objectives are straightforward and, potentially, simplistic. While such assumptions offer tractability and clarity, one might wonder to which extend the results are robust to further considerations. Below, I explore how the main mechanisms at play would carry through in richer context.

Seeds' Problem

Strategic Considerations

In the appendix, I propose two extensions for the seeds' objectives. Appendix 1.A studies the market outcomes for more general payoff structure; while more equilibria might exist, most insights carry through. In particular, I assume that sharing false news entails a different loss than the benefits of sharing true news. When the loss from sharing a false article is greater than the benefits from sharing true information, seeds are more demanding in terms of news quality in order to share; therefore, the producers has to be more precise than private knowledge. Consumers are more then brought to better decision more easily; news quality is still bounded, and the bound is still a function of private knowledge. Furthermore, all results pertaining to producers' incentive to invest directly follow through since producers' best-response is unaffected by the seeds' problem.

Appendix 1.B characterizes the best-response of seeds seeking *likes* rather than truth. The problem is more complex and less tractable; however, seeds' best-response have a similar shape: it is weakly monotonic in news' quality; true information is still shared more often than false articles. In particular, I assume that seeds share if they expected the number of *likes* from their post to be higher than an exogenous threshold τ; followers are behavioral, in that they like the article they see if its content correspond to their private signal. Then, true articles bring about more likes than false articles, so that seeds might want to share news only if it correspond to their private signal. This depends on how many likes they require to share, and how likely it is that their neighbor will see their post over that of another seed. However, for reasonable τ, there still exists a level of news quality that would induce them to always share, which depends on the precision of their private knowledge. Therefore, private knowledge still bounds news quality. Again, all results about producers' incentive to invest directly apply.

Behavioral Biases & Partisanship

Consumers' behavioral biases are expected to worsen the market outcome. Consider for instance confirmation bias as modeled in **?**: with some probability consumers misinterpret the news content if it disagrees with their private signal; they would then share it as if it was congruent with their private signal. Confirmation bias would not affect the seeds' best-response but would lead both true and false information to be shared more often. The value of false information would then increase faster than that of true information: more agents receive contradicting information when an article is false, increasing the probability for false articles to be shared faster. Eventually, for any environment and sharing pattern, confirmation bias would lower the producers' incentive to invest.

Another cognitive bias that could worsen the outcome is a taste for sensationalism. If consumers' payoff is affected by both the veracity and the sensationalism of the news, seeds are expected to be less demanding in terms of news quality to be willing to share news that is not congruent with their private signal. Sharing a false article in such a case would indeed be perceived less damageable because of their taste for sensationalism. This would reduce the value of the upper bound placed on news quality, while leaving the producers' incentive to invest unchanged. Appendix 1.D proposes a payoff structure to model such a mechanism.

Finally, partisan consumers' could be modeled as seeds sharing a given news content regardless of the realization of their private signal. The expected effects of this type of partisanship would be similar to the effects of confirmation bias. Instead of any node having a probability to misinterpret the news and share when they should not have, there is a probability for any node to be partisan and sharing when others would not have. From the producers' perspective, the value of false information increases more than the value of true information and investment is less attractive.

Producers' Problem

Beyond Visibility

How would the producers' problem be affected if news quality mattered beyond visibility? Producers might indeed get further benefits from being a reputable source of information. Such a setup would not affect the effect that the environment and competitiveness of the market would have on producers' incentives. The bound placed on news quality could be removed, although this would not necessarily occur.

In Appendix 1.D, I propose two frameworks to consider these effects. The underlying intuition is as follows. Assume that these benefits from reputation continuously depend on the level of news quality. The producers' incentive to invest would be shifted upwards; News quality would thus increase in equilibrium. Whether the bound is removed depends on the marginal value of reputation benefits and the marginal cost function: if the marginal value of reputation is lower than the marginal cost of the news quality at the bound, then all results follow through. Otherwise, producers invest until the marginal cost of news quality equated marginal benefits from reputation. Assume that these benefits from reputation discretely depend on the level of news quality: either they occur or they do not. As the producers' reputation benefits would not be affected by an increase in news' quality at the margin, producers' incentive to invest remain unaffected. Whether news quality increases in equilibrium depends on the level of quality required to benefit from reputation, as well as the total profits.

Subscription-Based Revenues

Would a business model that allows the producers to internalize the value of information of consumers decrease inefficiencies? While the game studied in this paper is not set to explore this question in details, it can still underline an interesting tradeoff. Let producers derive their revenues from subscription rather than advertisement. One could see the betting gains from reading news as opposed to relying on private signals as consumers' willingness to pay for information, i.e. willingness to pay for subscription. With subscription, the marginal cost of investment for the producer equates the marginal value of information for the consumer, in equilibrium; therefore, there are no Pareto inefficiencies on this front. However, this comes at the cost of losing advertisement revenues. Assuming that such advertisement creates a surplus for the society, it is not clear which business model should be preferred.

1.7 Conclusion

In this paper, I evaluate the performance of ad-funded online news outlets. I find that, without any intervention, they tend to be highly inefficient. First, news quality is bounded by the amount of private knowledge existing on the topic. The market does not compensate for a lack of private knowledge. High news quality is thus achievable only when the topic documented is already well-known: either because the outcome about this topic is rather certain; or because consumers are privately informed about it. The incentive created by sharing behaviors are a first cause for this result. Producers only care about being shared; as seeds rely on their knowledge to judge whether a content is worth sharing, having them share is not demanding when they are ill-informed. The second cause is the higher value of investment when the more likely state of the world realizes. Indeed, seeds are then more ready to share news documenting an expected state of the world. Thus, uncertain topics generate a lesser incentive to invest than topics for which information is less needed.

I additionally show that competition does not necessarily lead to better news quality. By comparing the outcomes of a monopoly and a duopoly, I conclude that monopoly is preferable in sparser networks populated by well-informed agents. This result puts into light two important forces appearing with competition. On the one hand, followers are harder to reach. This reduces the value of false information, as false articles would barely survive in the network when competing with true news. On the other hand, fewer followers can be reached. This reduces the value of true information, as an article shared by all seeds reading it would still reach few followers. When the network is sparse, the latter force dominates, making competition detrimental to news quality. This shows the limits of competition as a mean towards efficiency.

Furthermore, any online news market based on advertisement revenue is Pareto inefficient. I provide a framework to study welfare and find that online news outlets create value from entertainment but are not necessarily informative. In particular, the existence of online news rarely bring news consumers to take better decision; even when it does, their gain from it are still bounded by the precision of their private information. Furthermore, the presence of online news can be detrimental to Bayesian consumers, as it might discourage them from taking a costly and risky action that would have actually been beneficial to them. A range of entry cost that makes online news detrimental generally exist.

Finally, I discuss how fact checking could improve news informativeness. Flagging false articles reduces their value, thus incentivizing producers to publish true articles more often. Because flagging substitutes private information, news quality is not bounded by private knowledge anymore. However, flagging is less efficient in competitive markets; actually, if false articles are flagged sufficiently often, competition is detrimental to news quality in any market environment. Therefore, one can substitute the positive effects of competition with flagging. To the contrary, allowing consumers to observe the quality of news outlets, for instance through a certification from an external institution, would not remove the bound placed on news quality by private knowledge.

This analysis is attractive because it gives consumers an endogenous control over information flow but not over news content. Furthermore, distortions that are inherent to a social network should be essential in underlining the differences between social media and other historical instances of ad-based business models for news. The central role of competition in this paper is reflected by its predominance in online outlets, as well as online networks. My analysis is robust to many extensions and puts into perspective the limits of the business model of ad-funded online news outlets; under such business models, the information provided online cannot be reliable, even when all news consumers are rational and unbiased.

Appendix

1.A Asymmetric Loss From Sharing

1.A.1 Best Response

In this section of the appendix, I characterize the results derived in Section 3.4 for more general payoffs from sharing. In particular, while I restrict the benefit from sharing true news to 1, I consider a loss λ when false news is shared. The seeds' payoff thus becomes:

$$u(\text{sharing article with content } n|\omega = w) = \begin{cases} 1 & \text{if } n = w \\ -\lambda & \text{otherwise} \end{cases}$$

This changes the seeds' best-response. In particular, it modifies the thresholds according to which they start sharing different news content after their private signal. We can redefine:

$$\underline{t}_0^\lambda = \frac{\lambda(1-\gamma)(1-w_0)}{\lambda(1-\gamma)(1-w_0)+\gamma w_0} \qquad \bar{t}_0^\lambda = \frac{\lambda\gamma(1-w_0)}{\lambda\gamma(1-w_0)+(1-\gamma)w_0}$$
$$\underline{t}_1^\lambda = \frac{\lambda(1-\gamma)w_0}{\lambda(1-\gamma)w_0+\gamma(1-w_0)} \qquad \bar{t}_1^\lambda = \frac{\lambda\gamma w_0}{\lambda\gamma w_0+(1-\gamma)(1-w_0)}$$

The producers' best response does not change.

1.A.2 Equilibrium without Competition

The Nash equilibria might change. In particular, because the thresholds can now be all above or all below the no-investment quality 1/2, the best-responses of seeds and producers might cross in many ways. We define $\bar{q}_0 := q^*(\bar{z}_{+|0}, 0, 0, 0)$; $\bar{q}_F = q^*(1, \bar{z}_{+|1}, 0, 0)\}$ and $\tilde{q}_0 := q^*(1, 0, 0, 0)$; $\tilde{q}_1 := q^*(1, 1, 0, 0)$.

Proposition (1.1.A)**.** *If either* $1/2 \geq \bar{t}_1^\lambda$*; or both* $\bar{q}_0 < \underline{t}_0^\lambda$ *and* $\bar{q}_1 < \underline{t}_1^\lambda$*, then there is a unique equilibrium with zero investment and* $q_M^* = 1/2$*. Otherwise, an equilibrium with positive* *investment exists, which is determined as follows:*

- $q^*_M = \max\{\tilde{q}_0, \underline{t}^\lambda_0\}$ *if* $\bar{q}_1 < \underline{t}^\lambda_1$,
- $q^*_M = \max\{\tilde{q}_1, \underline{t}^\lambda_1\}\}$ *if* $\underline{t}^\lambda_1 \leq \bar{x}_1$ *and* $\tilde{x}_1 \leq \bar{t}^\lambda_0$
- $q^*_M = \max\{\bar{t}^\lambda_0, \min\{q^*(1,1,1,0), \bar{t}^\lambda_1\}\}$ *otherwise.*

In essence, the proof considers all possible crossing given the shape of the respective best-responses. Intuitively, the seeds' best-response is not assumed to be above the producer's best-response in $z = (0,0,0,0)$ anymore, which allows for more possible crossings.

Remark (1.1.A)**.** *In equilibrium,* $q^*_M \leq \bar{t}^\lambda_1$. *Therefore, news quality is still bounded by agent's private knowledge* w_0 *and* γ.

The consequences on the comparative statistics are overall the same. In particular, all results pertaining to the effect of a parameter on the producer's incentive to invest can directly be applied as the producer's best-response is identical in this extension. Note the following change:

Corollary (2.1.A)**.** *Take any increase in* w_0.

- *For a marginal increase, the inequalities detailed in Proposition 1. 1.A do not change, so that the maximal equilibrium investment* q^*_M *increases iff* $q^*_M \neq \underline{t}_0$ *and* $q^*_M \neq \bar{t}_0$
- *For bigger increases, the maximal equilibrium investment* q^*_M *increases iff* $q^*_M \neq \underline{t}_0$, $q^*_M \neq \bar{t}_0$ *and* c^{-1} *is steep enough, i.e.* c^{-1} *is such that, for any* $w'_0 > w_0$, $q^* > \underline{t}_1$ *implies* $q^{*\prime} > \underline{t}'_1$.

Proof. See Appendix 1.D □

1.A.3 Equilibrium with Competition

As in the monopoly, new equilibria might appear when seeds' best-response depends on λ. Let $\bar{q}_k := \max_{z_{T_k}} q^*_k((z_{T_k}, 0); 0)$ and $\tilde{q}_m = q^*((1,0),(0,0);0)$

Remark ((Additional))**.** *There might exist other Nash Equilibria.*

(i) For $\underline{t}^\lambda > 1/2$ *always exist a set of equilibria with zero investment* $x^*_k \in [0, 1/2] \forall k \in K$.

(ii) If $1/2 < \bar{t}^\lambda$ *and* $\min_k \bar{q}_k \geq (\underline{t}^\lambda)$, *there exists a set of equilibria in which exactly one producer invests* $q_m = \max\{\underline{t}^\lambda), \min\{\tilde{q}_m, \bar{t}^\lambda\}$, *and the other does not invest.*

Define $\bar{z}_D := \arg\max_{z_T} \left\{\Delta V\big((z_T, 0), \underline{t}^\lambda\big)\right\}$ and $\bar{q}_D = \Delta V(\bar{z}^D_T, \underline{t}^\lambda)$.

Proposition (3.1.A)**.** *If* $1/2 < \bar{t}^\lambda$ *and* $\underline{t}^\lambda \leq \bar{q}_D$*, there exists a symmetric equilibrium that features positive investment and* $q_D^* = \arg\min_{q \in [\underline{t}^\lambda, \bar{t}^\lambda]} \left|\Delta V_D\big((1,0); q\big) - c(q)\right|$.

Furthermore, we can distinguish equilibria with respect to their stability.

Corollary ((Additional))**.** q_D^* *is the only equilibrium with symmetric positive investment that is stable for the interaction between seeds and producers .*

Finally, I wonder about other asymmetric equilibria and find:

Remark (2.1.A)**.** *(i) If* $\bar{q}_m < \underline{t}^\lambda$*, the unique equilibrium is that featuring no investment.*
(ii) If $\bar{q}_D < \underline{t}^\lambda \leq \bar{q}_m$*, the only equilibria with positive investment have one producer investing* q_m *while the other does not invest.*
(iii) If $\bar{q}_D \geq \underline{t}^\lambda = q_m$*, the only equilibria with positive investment for both producers feature* $q_D^* = \underline{t}^\lambda$*.*
(iv) If the cost function is linear, there are no equilibrium with $q_k \neq q_\ell$ *and* $(x_k, x_\ell) \in (\underline{t}^\lambda, \bar{t}^\lambda)$ *as long as* $c(q)$*'s slope is different from* S*.*

Because the following comparison between monopoly and duopoly focuses on the producers' best-responses, all results follow through.

1.B Attention-Seeking Seeds

In this appendix, I explore an extension of the model with $w_0 = 1/2$ and symmetric behavior for all seeds' $z_k = z_\ell = z$. I assume that seeds do not intrinsically care whether the news they share is true or false; but they do care about receiving good feedback about it, e.g. a lot of *likes*. I characterize the best response of attention-seeking seeds to news quality q.

1.B.1 The Attention-Seeker Problem

I assume that seeds, contrary to producers, cannot observe the actual number of followers they reach; however, they can observe how many followers reacted to their shared post, as, typically, social media feature some sort of feedbacks, be it comments, likes, or re-shares. I focus on positive reactions, that I call *likes*, and assume that followers like a post if they receive a private signal consistent with it. In the context exposed previously, it means that followers receive a binary signal that can agree or disagree with the news, and like only if their private signal is congruent with the news – regardless of the prior probability for news to be true.

As before, seeds simultaneously choose whether to share the piece of news issued by ℓ, given their private signal s. Seeds decide to share if the amount of likes they expect to collect with their post exceeds a threshold $\tau \leq d$. It can be interpreted as the value of an outside option – e.g. posting another type of article would yield τ likes – or, simply, the cost of sharing.

For consistency, I still denote R_{fi} the random variable which is one if f sees the post from i. As before, a follower sees only one post. If more than one neighbor shared a post, the follower sees the post from one random sharing neighbor, with uniform probability, that is:

$$Pr(R_{fi} = 1 | \ s \text{ neighbors of } f \text{ shared}) = \frac{1}{s}$$

where s is the outcome of the random variable S counting the number of f's neighbors who shared.

Define the random variable L_{fi} which is one if f likes the post shared by i. Recall that s is the random private signal that a follower receives. Then:

$$Pr(L_{fi} = 1) = Pr(L_{fi} = 1 | R_{fi} = 1) Pr(R_{fi} = 1) = Pr(S = +) Pr(R_{fi} = 1)$$

An seed expects a different amount of likes for true and false information because, if read, true news gets more likes than false information. The expected number of likes also depends on the visibility of the news, which in turn depends on the sharing decisions of all neighbors of each followers. Define n as the random variable counting the number of shares from f's neighbors, excluding i. The expected number of likes i gets from sharing a piece of information which is $X \in \{T, F\}$ is thus:

$$\mathbb{E}\left(\sum_{f \in \mathcal{N}_i} L_{fi} = 1 \Big| X\right) = dPr(f \text{ is a follower})Pr(S = +|X)\mathbb{E}\left(\frac{1}{P+1}\Big|X\right)$$

Now recall, upon reading a piece of news, seed i, too, gets a private signal about the truthfulness of the news, whose precision is γ. As before, all seeds have a common prior x_k about the probability for producer k to release true information. Let $p(n, s)$ denote i's posterior upon receiving signal s and reading news n. Then, a seed decides to share a piece of information if and only if:

$$p(n,s)d(1-b)\gamma\mathbb{E}\left(\frac{1}{P+1}\Big|T\right) + (1-p(n,s))d(1-b)(1-\gamma)\mathbb{E}\left(\frac{1}{P+1}\Big|F\right) \geq \tau$$

Notice that the seeds' utility now depends on more than the producers' investment; it also depends on the behavior of other seeds. In particular, because seeds compete for likes, which occur only upon being seen, they would prefer a situation in which they are the only sharer. If true information is shared more, then this coordination concern would make them less prone to share true news; however, true information also brings more likes. Thus, there is a trade-off between visibility and veracity.

1.B.2 Seeds' Best Response

In this section, I focus on symmetric strategies $z_i = z \ \forall i$ and, by a slight misuse of language, I call best-response the pair of functions $(z_+^*(q), z_-^*(q))$ which maps q into $[0,1]$ such that $z^*(q, \mathbf{z}^*(\mathbf{q})) = z^*(q)$.[28] Hence, given any investment q, I look at the subset of strategies which can be consistent with a symmetric equilibrium on the seeds' side.

As usual, p_X denotes the probability that a $X = T, F$ news gets shared. Then, $n \sim \mathcal{B}(p_X, d-1)$ We can rewrite:

$$\mathbb{E}\left(\sum_{f \in \mathcal{N}_i} L_{fi} = 1|X\right) = d(1-b)Pr(S = +|X)\frac{1}{dp_X}\left(1 - (1-p_X)^d\right)$$

[28] Technically, each seed's best response would be a pair of $(I+1)$-dimensional function, that each maps q and $\mathbf{z_{-i}}$ into $[0,1]$, with I the random variable counting the number of seeds, and whose expectation is bI.

Thus, the expected number of like is:

$$p(n,s)\gamma\frac{1-b}{p_T}\big(1-(1-p_T)^d\big)+(1-p(n,s))(1-\gamma)\frac{1-b}{p_F}\big(1-(1-p_F)^d\big)$$

Lemma 7. *For any q, $z_+^*(q) \geq z_-^*(q)$.*

Corollary 6. *$\mathbb{E}($ # likes $|X)$ is increasing in $p(n,s)$, for any $S \in \{+,-\}, X \in \{T,F\}$.*

Proof. It is enough to notice that, since $p_T > p_F$: $\frac{p_T}{(1-(1-p_T)^d)}\frac{(1-(1-p_F)^d)}{p_F} > 1$ so that the coefficient of $p(n,s)$ is positive. □

I can now characterize the symmetric best-response of attention-seeking seeds

Proposition 7.

(i) For any $\tau \leq \gamma\delta$, $z_+^*(q;\tau) = z_-^*(q;\tau) = 1$ if and only if $q \geq \hat{q}(\tau)$.

(ii) For any $\tau \geq (1-\gamma)d(1-b)$, $z_+^*(q;\tau) = z_-^*(q;\tau) = 0$ if and only if $q \leq \check{q}(\tau)$.

(iii) For any $\tau \in [\tau_1,\tau_2]$, $z_+^*(q;\tau) = 1, z_-^*(q;\tau) = 0$ if only if $q \in [q_1(\tau), q_2(\tau)]$.

Where:

$$\delta(b) = \tfrac{1-b}{b}[1-(1-b)^d], \quad \tau_1(b) = \tfrac{1-b}{b}[1-(1-b(1-\gamma))^d], \quad \tau_2(b) = \tfrac{1-b}{b}[1-(1-b\gamma)^d]$$

And, given $Q = \frac{\frac{b\tau}{1-b}-1+(1-b(1-\gamma))^d}{(1-b(1-\gamma))^d-(1-b\gamma)^d}$,

$$\hat{q}(\tau) = \frac{\gamma}{2\gamma-1}\frac{\tau-(1-\gamma)\delta}{\tau}, \quad \check{q}(\tau) = \frac{1-\gamma}{2\gamma-1}\frac{\tau-(1-\gamma)d(1-b)}{d(1-b)-\tau}, \quad q_1(\tau) = \frac{(1-\gamma)Q}{(1-\gamma)Q+\gamma(1-Q)}, \quad q_2(\tau) = \frac{\gamma Q}{\gamma Q+(1-\gamma)(1-Q)}$$

Corollary 7.

(i) For any $\tau \leq \gamma\delta$, if $q \geq \hat{q}(\tau)$, $z_+(q,\mathbf{z_{-i}};\tau) = z_-(q,\mathbf{z_{-i}};\tau) = 1$ is the only best response for any (non symmetric) vector of seeds $-i \neq i$'s actions.

(ii) For any $\tau \geq (1-\gamma)d$, if $q \leq \check{q}(\tau)$, $z_+(q,\mathbf{z_{-i}};\tau) = z_-(q,\mathbf{z_{-i}};\tau) = 0$ is the only best response for any (non symmetric) vector of seeds $-i \neq i$'s actions.

Proof. Again, it is enough to recall that the number of likes is decreasing in the probability for another seed to share □

Corollary 8. *Define z_{ps} as the restriction of z to pure strategies. For any (q,τ), $z_{ps}^*(q;\tau)$ either does not exist or is unique.*

Proof. Consider the parameter space (τ, q). Theorem 7 describes three subsets of best-responses that do not intersect. No other pure strategies is sustainable, as, by proposition **??**, $(0,1)$ is never a best-response. □

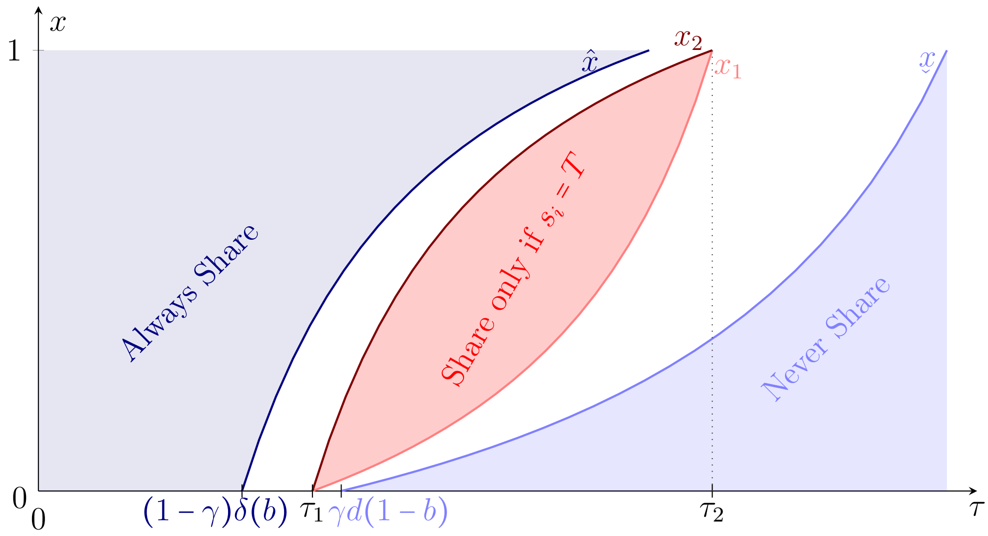


Figure 1.B.1: Illustration of $z^*_{ps}(q;\tau)$ with $b = 0.2, \gamma = 0.75, d = 5$

Figure 1.B.1 illustrates the different region of pure strategy best-responses in space (τ, q). First, one can notice that for some values of τ, the investment of the producer has no effect on the sharing decision of seeds. If τ is *too* low, seeds are not very demanding in terms of likes, so that they are always willing to share. If τ is *too* high, seeds are too demanding in terms of likes, and they never share any information.

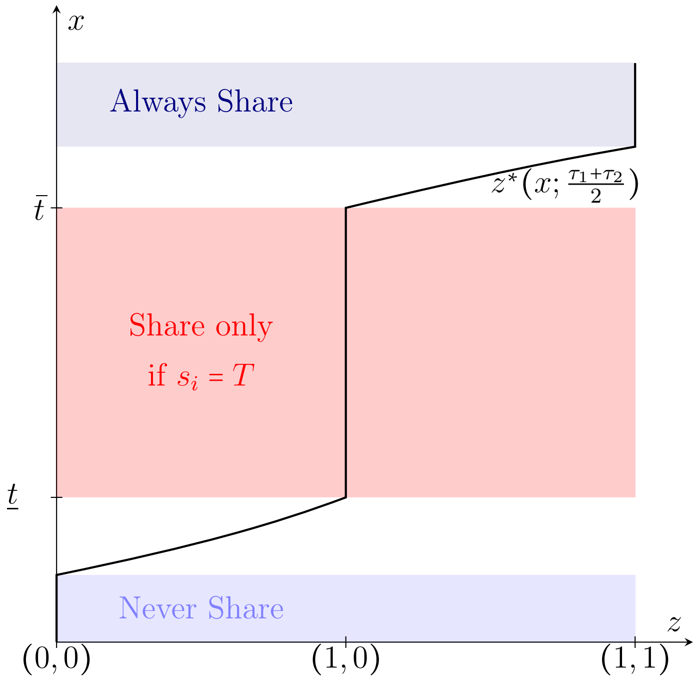


Figure 1.B.2: Illustration of $z^*(q;\tau)$ in $\tau = \frac{\tau_1+\tau_2}{2}$ with $b = 0.2, \gamma = 0.75, d = 5$

For intermediate values of τ, however, the symmetric best-response of seeds is fairly similar to that studied in the benchmark model. To understand so, let us fix a particular value for τ; we want to understand z^* as a function of q. This means fixing one value of τ on Figure 1.B.1 and translating the different areas in term of z. This results in Figure 1.B.2, which illustrates the symmetric best-response $z^*(q)$ for $\tau = \frac{\tau_1+\tau_2}{2}$. Notice that, for this particular τ, $q_1 = 1-\gamma$ and $q_2 = \gamma$. It means that, for q between $1-\gamma$ and γ, the symmetric best-response of attention seeking seeds exactly corresponds to that of naive seeds in the benchmark model.

However, for $q \notin [1-\gamma, \gamma]$, attention-seekers' best response changes. Say the producer invests exactly $1-\gamma$. In the benchmark model, upon receiving a positive private signal, seeds were indifferent between sharing or not, as the probability the news was true in such a case was exactly one half. But now, attention seekers' strategies are substitutes; therefore, upon receiving a positive private signal, they can be indifferent between sharing or not only for one particular sharing strategies of the other seeds. This latter strategy is the unique only symmetric best-response to q. For $\check{q}\left(\frac{\tau_1+\tau_2}{2}\right) < q < 1-\gamma$, z_+^* is strictly increasing in q;[29] for $\hat{q}\left(\frac{\tau_1+\tau_2}{2}\right) > q > \gamma$, z_-^* is strictly increasing in q [30]

The best-response of attention-seeking seed is thus fairly similar to that of naive seeds for the right value of τ. The problem of seeds as studied in the main text can thus be thought of as a simplification of more complex preferences.

[29] z_+^* is implicitly determined by:

$$\frac{\gamma q}{(1-\gamma)(1-q)} = -\frac{\frac{\tau_1+\tau_2}{2}\frac{b}{1-b} - \frac{1-(1-b(1-\gamma)z_+)^d}{z_+}}{\frac{\tau_1+\tau_2}{2}\frac{b}{1-b} - \frac{1-(1-b\gamma z_+)^d}{z_+}}$$

[30] z_-^* is implicitly determined by:

$$\frac{(1-\gamma)q}{\gamma(1-q)} = -\frac{\frac{\tau_1+\tau_2}{2}\frac{b}{1-b} - \frac{1-(1-b(1-\gamma)-b\gamma z_-)^d}{1+\frac{\gamma}{1-\gamma}z_-}}{\frac{\tau_1+\tau_2}{2}\frac{b}{1-b} - \frac{1-(1-b\gamma-b(1-\gamma)z_-)^d}{1+\frac{1-\gamma}{\gamma}z_-}}$$

1.C Equilibria with Sequential Moves

In this appendix, I solve the model presented in Section 1.2 with $w_0 = 1/2$, $K = 1$ and $z_{+|0} = z_{+|1} := z_T$, $z_{-|0} = z_{-|1} := z_F$, as a sequential game. In particular, I assume the following timing:

t=1 Producers k simultaneously choose their precision level $Pr(\text{news } k \text{ is T}) = x_k$.

* Network is formed. One piece of news per producer is issued. Consumers receive a private signal.

t=2 Seeds i simultaneously choose whether to share the article they read.

Note that, as the seeds play last, their problem does not change. q is now the actual investment and not their prior about it; and the best-response is now their contingent strategy. Nothing else changes. Thus, I will only analyze the choice of the producer in the first period.

Now, the producer is internalizing his effect on seeds' action. Because their strategy is not smooth, the producer's consider different cases. Recall that the producer wants to maximize:

$$q\Delta V(z(q)) + V_F(z(q)) - C(q)$$

Because the existence of some SPE might rely on the particular tie rule chosen when the seeds' are indifferent between sharing or not, I always take the tie rule the most advantageous to investment.

Any other level of news quality than $1/2, x'_M, \gamma$ where $x'_M = c^{-1}(\Delta V(1,0))$ is suboptimal. Now for $x'_M \neq 1/2$, $1/2$ cannot be part of a SPE. Indeed, because c is increasing, we know that:

$$q'_M \Delta V(1,0) - C(q'_M) = \int_{1/2}^{q'_M} \Delta V(1,0) - c(q)dq \geq 0$$

Furthermore, if $q'_M > \gamma$, then q'_M cannot be part of any SPE. Otherwise, the total profits have to be compared in q'_M and γ.

For clarity concerns, I only characterize the producer's investment prescribed in the SPE:

- If $q'_M < \gamma$:
  - If $V(1,1) - V(1,0) > C(\gamma) - C(q'_M)$, then the SPE prescribe such that the producer invests γ.
  - If $V(1,1) - V(1,0) < C(\gamma) - C(q'_M)$, then the SPE prescribe such that the producer invests q'_M.

- If $V(1,1) - V(1,0) = C(\gamma) - C(q'_M)$ then both investments described above are part of an SPE.

- If $q'_M \geq \gamma$, then the SPE prescribe that the producer invests γ.

1.D Proofs and computations

1.D 1.2.3 The Producers' Problem

Multinomial: the Distribution of an Outcome conditional on a Sum of Outcomes

Consider a random vector $X \sim Multi(n,p)$ of dimension k. By definition, we have:

$$Pr(X_1 = a, X_2 = b) = {p_1}^a {p_2}^b (1 - p_1 - p_2)^{n-a-b} \frac{n!}{a!b!(n-a-b)!}$$

Now, because each trial is independent, we have that $X_1 + X_2 \sim \mathcal{B}(p_1 + p_2, n)$. Hence:

$$Pr(X_1 + X_2 = s) = (p_1 + p_2)^s (1 - p_1 - p_2)^{n-s} \frac{n!}{s!(n-s)!}$$

Therefore, we find the following conditional distribution:

$$Pr(X_1 = a | X_1 + X_2 = s) = \frac{Pr(X_1 = a, X_2 = s - a)}{Pr(X_1 + X_2 = s)}$$

$$= \frac{{p_1}^a {p_2}^{s-a} (1 - p_1 - p_2)^{n-s} \frac{n!}{a!(s-a)!(n-s)!}}{(p_1 + p_2)^s (1 - p_1 - p_2)^{n-s} \frac{n!}{s!(n-s)!}} = \frac{{p_1}^a {p_2}^{s-a}}{(p_1 + p_2)^s} \frac{s!}{a!(s-a)!}$$

Note that it can be rewritten as:

$$Pr(X_1 = a | X_1 + X_2 = s) = \frac{{p_1}^a {p_2}^{s-a}}{(p_1 + p_2)^{s-a+a}} \frac{s!}{a!(s-a)!} = \left(\frac{p_1}{p_1 + p_2}\right)^a \left(\frac{p_2}{p_1 + p_2}\right)^{s-a} \frac{s!}{a!(s-a)!}$$

Hence, the conditional random variable $X_1|_{X_1+X_2} \sim \mathcal{B}\left(n, \frac{p_1}{p_1+p_2}\right)$.

The Probability of Being Read by a Follower (as a Producer)

First, note that:

$$\begin{aligned} Pr(\text{follower sees } k) &= \sum_{s=0}^{d} Pr(\text{follower sees } k \text{ and } s \text{ neighbors shared}) \\ &= \sum_{s=0}^{d} Pr(\text{follower sees } k \mid s \text{ shares}) Pr(s \text{ shares}) \end{aligned}$$

Now, we also have:

$$\begin{aligned} Pr(\text{follower sees } k \mid s \text{ shares}) &= \sum_{\nu=s}^{d} Pr(\text{follower sees } k \text{ and } \nu \text{ neighbors shared } k \mid s \text{ shares}) \\ &= \sum_{e=s}^{d} Pr(\text{follower sees } k \mid \nu \text{ and } s) Pr(\nu \text{ shares of } k \mid s \text{ shares}) \end{aligned}$$

Finally, using the conditional probability derived above, we rewrite the probability for a follower to see a piece of news n from producer k which is X, given ℓ produced m which is Y, as:

$$\sum_{s=1}^{d} \sum_{\nu=0}^{s} \frac{\nu}{s} \left(\frac{p_{X|n,k}}{p_{X|n,k} + p_{Y|m,\ell}} \right)^{\nu} \left(\frac{p_{Y|m,\ell}}{p_{X|n,k} + p_{Y|m,\ell}} \right)^{s-\nu} \frac{s!}{\nu!(s-\nu)!} (p_{X|n,k}+p_{Y|m,\ell})^{s} (1-p_{X|n,k}-p_{Y|m,\ell})^{d-s} \frac{d!}{s!(d-s)!}$$

where $p_{X|n,k}$ and $p_{Y|m,\ell}$ are the probability that a neighbor shares a piece of news from k, ℓ, given it is true/false. Defining $f(\nu)$ as the pmf of a $\mathcal{B}(s, \frac{p_{X|n,k}}{p_{X|n,k}+p_{Y|m,\ell}})$, we simplify the latter expression by:

$$\begin{aligned} &\sum_{s=1}^{d} \frac{1}{s} (p_{X|n,k} + p_{Y|m,\ell})^{s} (1 - p_{X|n,k} - p_{Y|m,\ell})^{d-s} \frac{d!}{s!(d-s)!} \sum_{\nu=0}^{s} \nu f(\nu) \\ =&\sum_{s=1}^{d} \frac{1}{s} (p_{X|n,k} + p_{Y|m,\ell})^{s} (1 - p_{X|n,k} - p_{Y|m,\ell})^{d-s} \frac{d!}{s!(d-s)!} s \frac{p_{X|n,k}}{p_{X|n,k} + p_{Y|m,\ell}} \\ =&\frac{p_{X|n,k}}{p_{X|n,k} + p_{Y|m,\ell}} \sum_{s=1}^{d} (p_{X|n,k} + p_{Y|m,\ell})^{s} (1 - p_{X|n,k} - p_{Y|m,\ell})^{d-s} \frac{d!}{s!(d-s)!} \\ =&\frac{p_{X|n,k}}{p_{X|n,k} + p_{Y|m,\ell}} \left(\sum_{s=0}^{d} (p_{X|n,k} + p_{Y|m,\ell})^{s} (1 - p_{X|n,k} - p_{Y|m,\ell})^{d-s} \frac{d!}{s!(d-s)!} \right. \\ &\left. - (p_{X|n,k} + p_{Y|m,\ell})^{0} (1 - p_{X|n,k} - p_{Y|m,\ell})^{d-0} \frac{d!}{0!(d-0)!} \right) \\ =&\frac{p_{X|n,k}}{p_{X|n,k} + p_{Y|m,\ell}} \left(1 - (1 - p_{X|n,k} - p_{Y|m,\ell})^{d} \right) \end{aligned}$$

We conclude by writing:

$$Pr(R_k = 1|\omega, n, m) = \frac{b}{K} + \frac{p_{X|n,k}}{p_{X|n,k} + p_{Y|m,\ell}}\left(1 - (1 - p_{X|n,k} - p_{Y|m,\ell})^d\right)$$

1.D 3.4 Equilibrium

1.D 1.3.1 Equilibrium without Competition

Lemma 1: shape of monopolist best-response & interpretation

- Because $c^{-1}(\cdot)$ is, by assumption, increasing in its argument, $q^*(z)$ is increasing (resp. decreasing) in $z_{S|n}$ iff $\Delta V(z)$ is increasing (resp. decreasing) in $z_{S|n}$. Now, we have:
  - I call single peaked a function which admits a single maximum point; therefore, any non-constant concave function $f(x)$ defined on a closed interval is single-peaked in x. Hence, it suffices to show that $\Delta V(z)$'s first derivative is decreasing in $z_{+|n}$.

    For $z_{+|0}$, we have:

    $$\frac{\partial \Delta V(z)}{\partial z_{+|0}}\frac{1}{1-b} = -d(1-\gamma)(1-w_0)\big(1-b(1-\gamma)z_{0,0})\big)^{d-1} + dw_0\gamma\big(1-b\gamma z_{0,0}\big)^{d-1}$$

    Whose sign is ambiguous. It is positive for $z_{+|0} = 0$ and decreasing in $z_{+|0}$ since $\frac{1-b\gamma z_{0,0}}{1-b(1-\gamma)z_{0,0}} \leq 1$ and decreases with $z_{+|0}$
    The derivation is similar for $z_{+|1}$.
  - $\Delta V(z)$'s first derivative w.r.t. to $z_{-|0}$, is negative. Indeed:

    $$\frac{\partial \Delta V(z)}{\partial z_{0,1}}\frac{1}{1-b} = -d\gamma(1-w_0)\big(1-b(\gamma z_{0,1} + (1-\gamma))\big)^{d-1} + dw_0\gamma\big(1-b(\gamma + (1-\gamma)z_{0,1})\big)^{d-1} < 0$$

    Where the last inequality comes from $\frac{\gamma}{1-\gamma} > \frac{w_0}{1-w_0} \geq 1$. The derivation is similar for $z_{-|1}$.
- For any given $z_{S|n}$, $\Delta V(z)$ is a polynomial function of $z_{S|n}$, so it is continuous within each segment $z_{S|n} \in (0,1)$. The function is also continuous between segments. Indeed, $\lim_{z_{+|0}\to 1} \Delta V(z) = \lim_{z_{+|1}\to 0} \Delta V(z)$ and $\lim_{z_{+|1}\to 1} \Delta V(z) = \lim_{z_{-|0}\to 0} \Delta V(z)$.

In addition, note that the global maximum is in $z = (1, \bar{z}_1, 0, 0)$ for some priors, and in $z = \{(\bar{z}_0, 0, 0, 0)\}$ for all other priors. Indeed, for $w_0 = \gamma$, $\bar{q}_0 > \bar{q}_1$, while for $w_0 = 1/2$, $\bar{q}_0 < \bar{q}_1$. Figure 1.D.1 and 1.D.2 illustrate the shape of the producer's best response.

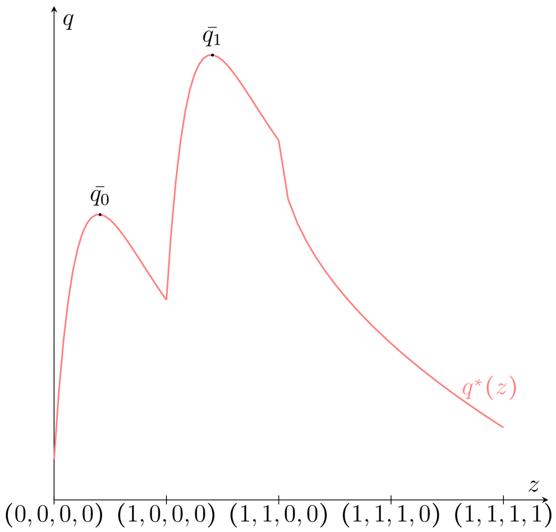


Figure 1.D.1: Producer's Best Response, $\bar{q}_0 < \bar{q}_1$

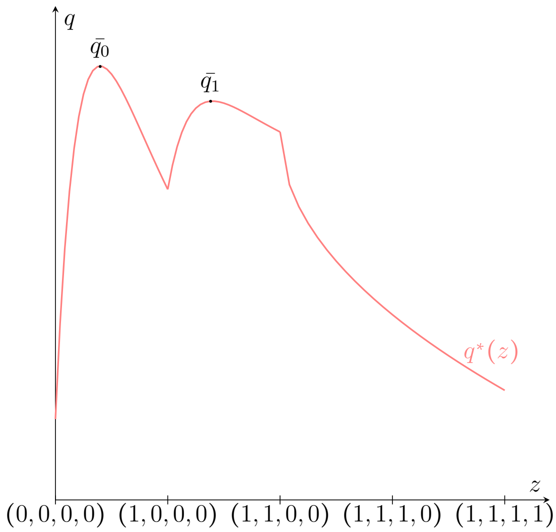


Figure 1.D.2: Producer's Best Response, $\bar{q}_0 > \bar{q}_1$

Again, I represent the seeds' strategy on a line and map the corresponding image as if the argument was unidimensional. The resulting function is non-monotonic. Each hump shaped segment is explained by the effect of the network as in the main text. The two humps follow from the same mechanism applying in two different cases: when news 0 is produced first, and then when news 1 is published.

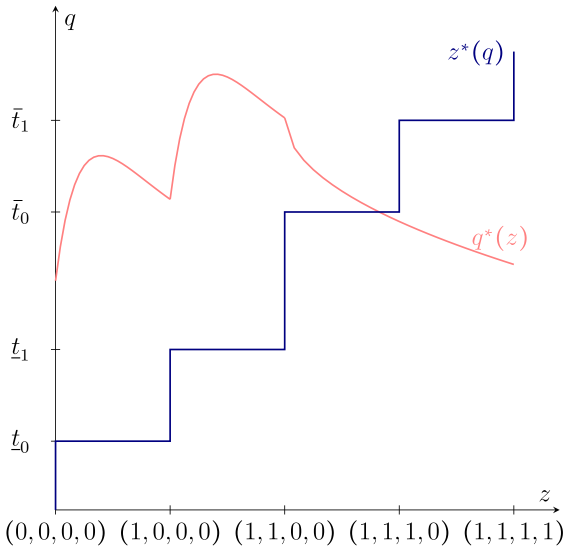


Figure 1.D.3: Equilibrium with $q_M^* = q^*(1,0)$

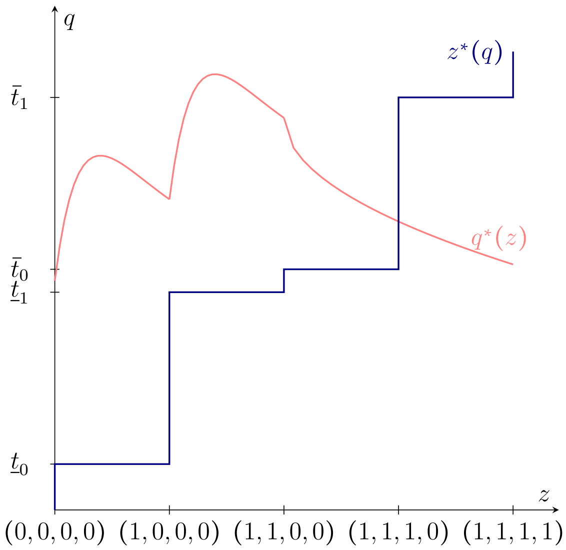


Figure 1.D.4: Equilibrium with $q_M^* = \bar{t}$

Furthermore, Figure 1.D.3 and 1.D.4 illustrate two cases. Figure 1.6 shows the equilibrium with $q^*(1,1,0,0) > \bar{t}_0 > q^*(1,1,1,0)$. Figure 1.7 shows the equilibrium with $\bar{t}_1 > q^*(1,1,1,0) > \bar{t}_0$.

Proposition 1: characterization of the monopoly equilibrium

First, notice that any positive equilibrium investment has to achieve $q \leq \bar{t}_1$. Indeed, $q = \bar{t}_1$ is enough to insure that the producer's news is always share, so that any additional investment would increase costs without increasing benefits. Furthermore, note that even if no investment occurs, sharing can occur. Indeed, faced to completely uninformative news' outlet, agents will still share an article whose content matches their private signal, because the private signal is informative. Therefore, any equilibrium displays $z_{0,0} = z_{1,1} = 1$; and the equlibrium will occur on the decreasing part of the producers' best response. Furthermore, note that $q^*(1,1,1,1) = 1/2$. Indeed, if news gets systematically shared, the producer has no incentive to invest since true news is treated as false news. Because the relevant portion of $q^*(z)$ is strictly decreasing, while $z^*(q)$ is weakly increasing, any intersection has to be unique. Because both best responses are continuous and that in $z = (0,0,0,0)$ the producer's best response is above the value ensuring some sharing, while in $z = (1,1,1,1)$, the producer's best response is below the value ensuring full sharing, the intersection must exist. Therefore, a NE must exist and is unique.
Because the cost function will determine different levels for $q^*(1,1,0,0)$ and $q^*(1,1,1,0)$, we need to understand how these values compare to $\bar{t}_0 < \bar{t}_1$. If $q^*(1,1,0,0) < \bar{t}_0$, from the shapes of the best responses, we have $q^*(1,1,1,0) < q^*(1,1,0,0) < \bar{t}_0 < \bar{t}_1$ so that $q^*_M = q^*(1,1,0,0)$. Indeed, in such a case, because $q^*_M < \bar{t}_0$, the seeds will share an article only if its content matches their private signal: $z^*(q^*_M) = (1,1,0,0)$. This is also optimal for the producer, as, by definition $c(q^*_M) = c(\Delta V(1,1,0,0))$. Furthermore, no other investment is optimal as c is strictly increasing. The same reasoning applies for $\bar{t}_0 < q^*(1,1,1,0) < \bar{t}_1$. Now, consider $\bar{t}_0 < q^*(1,1,0,0)$ but $q^*(1,1,1,0) < \bar{t}_0$. Then, $q^*_M = \bar{t}_0$. Indeed, as $q^*(1,1,1,0) < \bar{t}_0 < q^*(1,1,0,0)$, and because $q^*(z)$ continuous, there must exist some $z^*_{0,1}$ such that $c^{-1}(\Delta V(1,1,z^*_{0,1},0)) = \bar{t}_0$. It is easy to verify that this constitutes a NE. The same reasoning applies for $\bar{t}_1 < q^*(1,1,1,0)$.

When $w_0 = 1/2$, $\bar{t}_0 = \bar{t}_1$, so that the characterization simplifies to $q^*_M = \min\{q^*(1,0), \bar{t}\}$.

Lemma 2: the role of connectivity

$\Delta V(z)$ is single-peaked in d because it is the weighted sum of two hump-shaped single-peaked function of d. Indeed, $f(d) := (1 - p_{F|1})^d - (1 - p_{T|0})^d$ is single peaked as $f(d+1) - f(d) = -p_{F|1}(1 - p_{F|1})^d + p_{T|1}(1 - p_{T|1})^d$ whose sign depends on d. It is positive for $d = 0$ and negative for d big enough. Furthermore, $f(d+1) - f(d) < 0 \Rightarrow f(d+2) - f(d+1) < 0$. The same applies to $(1 - p_{F|0})^d - (1 - p_{T|1})^d$.

Proposition 2 and Corollary 2: the effects of private knowledge

Recall that $\Delta V(z) = w_0\left((1-p_{F|1})^d - (1-p_{T|0})^d\right) - (1-w_0)\left((1-p_{F|0})^d - (1-p_{T|1})^d\right)$, with $p_{X|n} = b\left(\gamma z_{S|n} + (1-\gamma)z_{\text{-}X|n}\right)$. Therefore:

- $\frac{\partial \Delta V(z)}{\partial \gamma} \geq 0$ as $z_{+|0} - z_{-|1} \geq 0$ and $z_{+|1} - z_{-|0} \geq 0$.
- $\frac{\partial \Delta V(z)}{\partial w_0} \geq 0$ as $(1-p_{F|1})^d - (1-p_{F|0})^d \geq 0$ and $(1-p_{T|1})^d - (1-p_{T|0})^d \geq 0$.

Where the derivative is null only for $z = (0,0,0,0)$ and $z = (1,1,1,1)$.

Furthermore, γ and w_0 have the following effects on the seeds' best-response:

- $\frac{\partial \underline{t}_0}{\partial \gamma} < 0$, $\frac{\partial \underline{t}_1}{\partial \gamma} < 0, \frac{\partial \bar{t}_0}{\partial \gamma} > 0, \frac{\partial \bar{t}_1}{\partial \gamma} > 0$
- $\frac{\partial \underline{t}_0}{\partial w_0} < 0$, $\frac{\partial \bar{t}_0}{\partial w_0} < 0, \frac{\partial \bar{t}_1}{\partial w_0} > 0, \frac{\partial \bar{t}_1}{\partial w_0} > 0$.

Therefore, q_M^* unambiguously increases with γ. For w_0, as $q^*(1,1,0,0)$ and $q^*(1,1,1,0)$ are weakly increasing in w_0; no increase in w_0 would change the inequalities detailed in Proposition 1. Therefore, q_M^* increases iff $q_M^* \neq \bar{t}_0$.

1.D 1.3.2 Equilibrium with Competition

Lemma 3: shape of duopolist best-response

(i) Because $c^{-1}(\cdot)$ is, by assumption, increasing in its argument, $q^*(z)$ is increasing (resp. decreasing) in z_{X_k} iff $\Delta V(z_k; z_\ell, q_\ell)$ is increasing (resp. decreasing) in z_{X_k}. Now, we have:

  – I show that $V_{T_k Y_\ell} - V_{F_k Y_\ell}$ is concave in z_{T_k} for $z_{T_k} \in [0,1]$ and any $Y = T, F$. We have:

$$\begin{aligned} V_{T_k Y_\ell} - V_{F_k Y_\ell} &= \frac{p_{T_k}}{p_{T_k} + p_{Y_\ell}}\left(1 - (1 - p_{T_k} - p_{Y_\ell})^d\right) - \frac{p_{F_k}}{p_{F_k} + p_{Y_\ell}}\left(1 - (1 - p_{F_k} - p_{Y_\ell})^d\right) \\ &= \frac{p_{T_k}}{p_{T_k} + p_{Y_\ell}}\left((1 - p_{F_k} - p_{Y_\ell})^d - (1 - p_{T_k} - p_{Y_\ell})^d\right) \\ &\qquad + \left(\frac{p_{T_k}}{p_{T_k} + p_{Y_\ell}} - \frac{p_{F_k}}{p_{F_k} + p_{Y_\ell}}\right)\left(1 - (1 - p_{F_k} - p_{Y_\ell})^d\right) \end{aligned}$$

  We know that $\frac{p_{T_k}}{p_{T_k}+p_{Y_\ell}}$ and $\left(1-(1-p_{F_k}-p_{Y_\ell})^d\right)$ are both strictly increasing and weakly concave in z_{T_k}. From the analysis of the monopolist's best response, we also know that $\left((1-p_{F_k}-p_{Y_\ell})^d - (1-p_{T_k}-p_{Y_\ell})^d\right)$ is single-peaked. As the product of weakly concave functions is weakly concave, all that is left to do is to show that $\frac{p_{T_k}}{p_{T_k}+p_{Y_\ell}} - \frac{p_{F_k}}{p_{F_k}+p_{Y_\ell}}$ is single peaked. We have:

$$\frac{\partial \frac{p_{T_k}}{p_{T_k}+p_{Y_\ell}} - \frac{p_{F_k}}{p_{F_k}+p_{Y_\ell}}}{\partial z_{T_k}} = \frac{\partial \frac{p_{Y_\ell}(p_{T_k}-p_{F_k})}{(p_{T_k}+p_{Y_\ell})(p_{F_k}+p_{Y_\ell})}}{\partial z_{T_k}} = \frac{\partial \frac{p_{Y_\ell}(\frac{1}{2}b(2\gamma-1)z_{T_k})}{(\frac{1}{2}b\gamma z_{T_k}+p_{Y_\ell})(\frac{1}{2}b(1-\gamma)z_{T_k}+p_{Y_\ell})}}{\partial z_{T_k}}$$

$$= \frac{p_{Y_\ell}\Big[\frac{1}{2}b(2\gamma-1)(\frac{1}{4}b^2\gamma(1-\gamma)z_{T_k}{}^2 + \frac{1}{2}bp_{Y_\ell}z_{T_k} + p_{Y_\ell}{}^2) - \big(\frac{1}{2}b^2\gamma(1-\gamma)z_{T_k} + \frac{1}{2}bp_{Y_\ell}\big)\frac{1}{2}b(2\gamma-1)z_{T_k}}{(p_{T_k}+p_{T_\ell})^2(p_{F_k}+p_{Y_\ell})^2}$$

$$= \frac{p_{Y_\ell}\frac{1}{2}b(2\gamma-1)\Big[p_{Y_\ell} - \frac{1}{4}b^2\gamma(1-\gamma)z_{T_k}{}^2\Big]}{2(p_{T_k}+p_{Y_\ell})^2(p_{F_k}+p_{T_\ell})^2}$$

Which is positive in $z_{T_k} = 0$ and decreases with z_{T_k}.

– For z_{F_k}, we have:

$$\frac{\partial \Delta V(z_k; z_\ell, q_\ell)}{\partial z_{F_k}} = (1-b)\left(\Pr(Y_\ell)\frac{\partial V_{T_kY_\ell} - V_{F_kY_\ell}}{\partial z_{F_k}}\right) < 0$$

Indeed,

$$\frac{\partial V_{T_kY_\ell} - V_{F_kY_\ell}}{\partial z_{F_k}} = \frac{1}{2}b\Bigg[p_{Y_\ell}\Bigg(\frac{(1-\gamma)}{(p_{T_k}+p_{Y_\ell})^2}\big(1-(1-p_{T_k}-p_{Y_\ell})\big)^d - \frac{\gamma}{(p_{F_k}+p_{Y_\ell})^2}\big(1-(1-p_{F_k}-p_{Y_\ell})^d$$

$$+d\frac{(1-\gamma)p_{T_k}}{p_{T_k}+p_{Y_\ell}}(1-p_{T_k}-p_{Y_\ell})^{d-1} - d\frac{\gamma p_{F_k}}{p_{F_k}+p_{Y_\ell}}(1-p_{F_k}-p_{Y_\ell})^{d-1}$$

Which is a sum of negative terms. Indeed, the first term is negative because $\frac{1-(1-x)^d}{x^2}$ is decreasing in x and $p_{T_k} \geq p_{F_k}$. We know that $\frac{1-(1-x)^d}{x^2}$ is decreasing in x because:

$$\frac{\partial \frac{1-(1-x)^d}{x^2}}{\partial x}x^4 = d(1-x)^{d-1}x^2 - 2x(1-(1-x)^d) = (1-x)^{d-1}x\big(dx+2(1-x)\big) - 2x < x(-x+2) - 2x < 0$$

where the first inequality follows from $(1-x)^{d-1}\big((d-2)x+2\big)$ being decreasing in d so that among all d, $d = 1$ maximizes the expression.

The second term is negative as $(1-p_{T_k}-p_{Y_\ell})^{d-1} < (1-p_{F_k}-p_{Y_\ell})^{d-1}$; and $\frac{(1-\gamma)p_{T_k}}{p_{T_k}+p_{Y_\ell}} < \frac{\gamma p_{F_k}}{p_{F_k}+p_{Y_\ell}}$. The last inequality holds because:

$$(1-\gamma)p_{T_k}(p_{F_k}+p_{Y_\ell}) - \gamma p_{F_k}(p_{T_k}+p_{Y_\ell}) = p_{F_k}p_{T_k}(1-2\gamma) + p_{Y_\ell}\big((1-\gamma)p_{T_k} - \gamma p_{F_k}\big)$$

is the sum of two negative terms; indeed: $1-2\gamma<0$ and

$$(1-\gamma)p_{T_k}-\gamma p_{F_k}=\frac{1}{2}b\big[(1-\gamma)(\gamma+(1-\gamma)z_{F_k})-\gamma(1-\gamma+\gamma z_{F_k})\big]=\frac{1}{2}b\big[(1-\gamma)^2-\gamma^2\big]z_{F_k}<0$$

(ii) For $d=2$, $\frac{\partial V_{T_kY_\ell}-V_{F_kY_\ell}}{\partial z_{Y_\ell}}$ is decreasing in z_{Y_ℓ} for $Y=T,F$. We have:

$$\begin{aligned}\frac{\partial V_{T_kY_\ell}-V_{F_kY_\ell}}{\partial p_{Y_\ell}} &= -\frac{p_{T_k}}{(p_{T_k}+p_{Y_\ell})^2}\Big[1-(1-p_{T_k}-p_{Y_\ell})^{d-1}\big(1+(d-1)(p_{T_k}+p_{Y_\ell})\big)\Big]\\ &\quad+\frac{p_{F_k}}{(p_{F_k}+p_{Y_\ell})^2}\Big[1-(1-p_{F_k}-p_{Y_\ell})^{d-1}\big(1+(d-1)(p_{F_k}+p_{Y_\ell})\big)\Big]<0\end{aligned}$$

If $-\frac{p_{T_k}}{(p_{T_k}+p_{Y_\ell})^2}\big(-(p_{T_k}+p_{Y_\ell})^2\big)>\frac{p_{F_k}}{(p_{F_k}+p_{Y_\ell})^2}\big(-(p_{F_k}+p_{Y_\ell})^2\big)$ which is ensured by $p_{T_k}\geq p_{F_k}$.[31]

When $d\to\infty$, the expression is determined by the sign of $-\frac{p_{T_k}}{(p_{T_k}+p_{Y_\ell})^2}+\frac{p_{F_k}}{(p_{F_k}+p_{Y_\ell})^2}$ which is negative for $p_{Y_\ell}^2>p_{F_k}p_{T_k}$.

(iii) It is enough to prove that $\Big(V_{T_kT_\ell}-V_{F_kT_\ell}\Big)-\Big(V_{T_kF_\ell}-V_{F_kF_\ell}\Big)\leq 0$. This expression can be rewritten:

$$\begin{aligned}&\tfrac{p_{T_k}}{p_{T_k}+p_{T_\ell}}\left(1-(1-p_{T_k}-p_{T_\ell})^d\right)-\tfrac{p_{F_k}}{p_{F_k}+p_{T_\ell}}\left(1-(1-p_{F_k}-p_{T_\ell})^d\right)\\ &\quad-\tfrac{p_{T_k}}{p_{T_k}+p_{F_\ell}}\left(1-(1-p_{T_k}-p_{F_\ell})^d\right)+\tfrac{p_{F_k}}{p_{F_k}+p_F}\left(1-(1-p_{F_k}-p_{F_\ell})^d\right)\\ =&\tfrac{p_{T_k}}{(p_{T_k}+p_{T_\ell})(p_{T_k}+p_{F_\ell})}\left(p_{F_\ell}-p_{T_\ell}-(1-p_{T_k}-p_{T_\ell})^d(p_{T_k}+p_{F_\ell})+(1-p_{T_k}-p_{F_\ell})^d(p_{T_k}+p_{T_\ell})\right)\\ &\quad-\tfrac{p_{F_k}}{(p_{F_k}+p_{T_\ell})(p_{F_k}+p_{F_\ell})}\left(p_{F_\ell}-p_{T_\ell}-(1-p_{F_k}-p_{T_\ell})^d(p_{F_k}+p_{F_\ell})+(1-p_{F_k}-p_{F_\ell})^d(p_{F_k}+p_{T_\ell})\right)\end{aligned}$$

Let us define α such that $p_{T_k}+p_{F_\ell}=\alpha(p_{T_k}+p_{T_\ell})+(1-\alpha)(p_{F_k}+p_{F_\ell})$; therefore $p_{F_k}+p_{T_\ell}=(1-\alpha)(p_{T_k}+p_{T_\ell})+\alpha(p_{F_k}+p_{F_\ell})$. Because $(1-x)^d$ is convex, we have:

$$\begin{aligned}&(1-p_{T_k}-p_{F_\ell})^d(p_{T_k}+p_{T_\ell})+(1-p_{F_k}-p_{T_\ell})^d(p_{F_k}+p_{F_\ell})\\ &\quad-(1-p_{T_k}-p_{T_\ell})^d(p_{T_k}+p_{F_\ell})-(1-p_{F_k}-p_{F_\ell})^d(p_{F_k}+p_{T_\ell})\\ <\;&\alpha(1-p_{T_k}-p_{T_\ell})^d(p_{T_k}+p_{T_\ell})+(1-\alpha)(1-p_{F_k}-p_{F_\ell})^d(p_{T_k}+p_{T_\ell})\\ &\quad+(1-\alpha)(1-p_{T_k}-p_{T_\ell})^d(p_{F_k}+p_{F_\ell})+\alpha(1-p_{F_k}-p_{F_\ell})^d(p_{F_k}+p_{F_\ell})\\ &\quad-(1-p_{T_k}-p_{T_\ell})^d(p_{T_k}+p_{F_\ell})-(1-p_{F_k}-p_{F_\ell})^d(p_{F_k}+p_{T_\ell}) \qquad =0\end{aligned}$$

Therefore the second factor of the first term of the sum is lower than the second factor of the second term of the sum, which is itself negative. If the first factor of the first term is greater than the first factor of the second term, we are done. And

[31] Notice that a similar inequality holds for $d=3$. From numerical insights, the difference is expected to be increasing then decreasing in d.

indeed, if $p_{uX} < p_{vX}$, we have:

$$\frac{p_{T_k}}{p_{T_k}^2 + p_{T_k}(p_{T_\ell} + p_{F_\ell}) + p_{T_\ell}p_{F_\ell}} > \frac{p_{F_k}}{p_{F_k}^2 + p_{F_k}(p_{T_\ell} + p_{F_\ell}) + p_{T_\ell}p_{F_\ell}}$$
$$p_{T_\ell}p_{F_\ell}(p_{T_k} - p_{F_k}) > p_{T_k}p_{F_k}(p_{T_k} - p_{F_k}) = p_{T_k}^2 p_{F_k} - p_{T_k}p_{F_k}^2$$

(iv) As in the monopoly case, for any given z_{X_k}, $\Delta V(z_k; z_\ell, q_\ell)$ is a polynomial function of z_{X_n} and is also continuous between segments, as $\lim_{z_{T_k}\to 1} \Delta V(z) = \lim_{z_{F_k}\to 0} \Delta V(z_k; z_\ell, q_\ell$. Furthermore, $\Delta V(z_k; z_\ell, q_\ell)$ is also polynomial function of z_ℓ and x_ℓ, so it is continuous.

Proposition 3: characterization of the duopoly symmetric equilibrium

First note that any equilibrium news quality lies in $[1/2, \bar{t}]$. Indeed, recall that $\Delta V_D((0,0), q) = \Delta V_D((1,1), q) = 0$. Clearly, for any $q > \bar{t}$, $c(q) > 0 = \Delta V_D(z^*(q), q)$, which would be suboptimal for the producer.

First I prove that a symmetric equilibrium exists. Then, I show it is unique.

Consider two cases:

1. If $c(\bar{t}) \geq \Delta V_D((1,0), \bar{t})$, then $\exists \tilde{q} \in [1/2, \bar{t}]$: $c(\tilde{q}) = \Delta V((1,0), \tilde{q})$. Indeed, recall that c is weakly increasing in q and $\Delta V_D((1,0), q)$ strictly decreasing in q. We also notice that $c(\tilde{q}) = \Delta V((1,0), \tilde{q})$ while $c(\tilde{q}) = \Delta V((1,0), \tilde{q})$. Because both c and ΔV are continuous in q, they must intersect on $[1/2, \bar{t}]$. Clearly, $(\tilde{q}, (1,0))$ is a NE.
   This equilibrium is unique. First notice that for $z = (1,0)$, the intersection must be unique given the shape of the respective best responses. Let us show that no other undominated sharing rule can be consistent with an equilibrium in this case. A sharing rule $(z,0)$ with $z < 1$ would require $q < 1/2$, which is impossible. A sharing rule $(1,z)$ with $z > 0$ would require $q \geq \bar{t}$. This cannot occur in equilibrium since, for any $z \in [0,1]$, $\Delta V_D((1,z), \bar{t}) < \Delta V_D((1,z), q_D^*) = c(q_D^*) < c(\bar{t})$. Hence, $c(\bar{t}) > \Delta V_D((1,z), \bar{t})$, so that $q^*((1,z), \bar{t}) < \bar{t}$ for any z.

2. If $c(\bar{t}) < \Delta V_D((1,0), \bar{t})$, then $\exists \tilde{z}_F \in [0,1]$: $c(\bar{t}) = \Delta V((1, \tilde{z}_F), \bar{t})$. Indeed, by assumption $c(\bar{t}) < \Delta V_D((1,0); \bar{t})$ and we know that $c(\bar{t}) > 0 = \Delta V_D((1,1); \bar{t})$. Because $\Delta V_D(z;q)$ is continuous in z_F, there must exist such $\tilde{z}_F$. Because $V_D(z;q)$ is strictly decreasing in z_F, this equilibrium is unique.

1.D 1.3.3 Effects of Competition

Theorem 1: shape of $\Delta V_M(z) - \Delta V_D(z,q)$ ***in*** d

Given $DV(d) := \frac{\Delta V_M(z;d) - \Delta V_D(z,q;d)}{1-b}$, we want to show that $DV(d) > DV(d+1) \Rightarrow DV(d+1) > DV(d+2)$. For readability, let us define for this proof:

$$c_1 = 1 - \frac{q}{2} \qquad c_2 = \frac{1+q}{2} \qquad c_3 = \frac{p_T}{p_T + p_F} - q$$

Note that $c_1 > 0$, $c_2 > 0$ and c_3's sign depends on z and q.

We begin by rewriting the assumption $DV(d) - DV(d+1) > 0$ as:

$$-c_1\left((1-p_T)^d - (1-p_T)^{d+1}\right) + c_2\left((1-p_F)^d - (1-p_F)^{d+1}\right) + c_3\left(\left(1 - \tfrac{p_T+p_F}{2}\right)^d - \left(1 - \tfrac{p_T+p_F}{2}\right)^{d+1}\right) > 0$$

$$-c_1\left((1-p_T)^d p_T\right) + c_2\left((1-p_F)^d p_F\right) + c_3\left(\left(1 - \frac{p_T+p_F}{2}\right)^d \frac{p_T+p_F}{2}\right) > 0$$

Therefore, defining for readability again:

$$A := c_1\left((1-p_T)^d p_T\right) - \tfrac{1}{2}c_3\left(\left(1 - \tfrac{p_T+p_F}{2}\right)^d \tfrac{p_T+p_F}{2}\right)$$

$$B := c_2\left((1-p_F)^d p_F\right) + \tfrac{1}{2}c_3\left(\left(1 - \tfrac{p_T+p_F}{2}\right)^d \tfrac{p_T+p_F}{2}\right)$$

$DV(d+1) - DV(d) < 0$ is equivalent to $B > A$. Notice that $B > 0$ because when $c_3 > 0$ makes B a sum of positive term, and when $c_3 < 0$ A is a sum of positive term so that $B > A > 0$.

Likewise we develop $DV(d+2) - DV(d+1)$ as:

$$-c_1\left((1-p_T)^d p_T(1-p_T)\right) + c_2\left((1-p_F)^d p_F(1-p_F)\right) + c_3\left(\left(1 - \frac{p_T+p_F}{2}\right)^d \frac{p_T+p_F}{2}\frac{1}{2}(1-p_T+1-p_F)\right)$$

Therefore:

$$DV(d+1) - DV(d+2) = -(1-p_T)A + (1-p_F)B > 0$$

where the last inequality follows from $p_T > p_F$

Remark 3: beyond two competitors

The difference in the incentive to invest between competition with K and $K+1$ producers is proportional to:

$$\begin{aligned}\textstyle\sum_Y \Pr(Y)\Bigg\{&\Big[\tfrac{p_{T_k}}{p_{T_k}+p_{Y_{-k}}}\big(1-(1-\tfrac{p_{T_k}+p_{Y_{-k}}}{K})^d\big)-\tfrac{p_{F_k}}{p_{F_k}+p_{Y_{-k}}}\big(1-(1-\tfrac{p_{T_k}+p_{Y_{-k}}}{K})^d\big)\Big]\\ &-q\Big[q\tfrac{p_{T_k}}{p_{T_k}+p_{k'T}+p_{Y_{-k}}}\big(1-(1-\tfrac{p_{T_k}+p_{T_{k'}}+p_{Y_{-k}}}{K+1})^d\big)-\tfrac{p_{F_k}}{p_{F_k}+p_{T_{k'}}+p_{Y_{-k}}}\big(1-(1-\tfrac{p_{T_k}+p_{u'T}+p_{Y_{-k}}}{K+1})^d\big)\Big]\\ &-(1-q)\Big[\tfrac{p_{T_k}}{p_{T_k}+p_{F_{k'}}+p_{Y_{-k}}}\big(1-(1-\tfrac{p_{T_k}+p_{F_{k'}}+p_{Y_{-k}}}{K+1})^d\big)-\tfrac{p_{F_k}}{p_{F_k}+p_{F_{k'}}+p_{Y_{-k}}}\big(1-(1-\tfrac{p_{T_k}+p_{F_{k'}}+p_{Y_{-k}}}{K+1})^d\big)\Big]\Bigg\}\end{aligned}$$

Whose sign is positive in $d=1$, meaning that the incentive to invest with $K+1$ producer is smaller in the symmetric case than the incentive to invest with K producers. The expression's sign depends on the parameters for $d\to\infty$.

Remark 4: the role of signal precision

(i) When $\gamma\to 1$, note that the set of the seeds' best response reduces to $\{(1,0)\}$. Then: $\Delta V_M(z)=(1-b)(1-(1-b)^d)>(1-b)\frac{1}{2}q(1-(1-b)^d)+(1-q)(1-(1-\frac{1}{2}b)^d)=\Delta V_D(z,q)$ Because $\frac{1-(1-b)^d}{1-(1-\frac{1}{b})^d}>\frac{1-q}{1-\frac{1}{2}q}$ $\forall q\in[0,1]$.

(ii) When $\gamma\to\frac{1}{2}$, $p_T=p_F$ for any z, so that the incentive to invest vanishes on both types of market: $\Delta V_M(z)=0=\Delta V_D(z;q)$

1.D 1.4 Welfare

1.D 1.4.2 Framework of Analysis

Lemma 4: Consumers' expected utility

- Conditional on receiving news n after private signal s, accounting for the optimal decision whether to share or not, the utility from sharing is $\max\{2p(n,s)-1;0\}$; now $2p(n,s)-1>0\Rightarrow z_{S|n,k}>0$ where $X=T$ iff $n=s$. Therefore, $\max\{2p(n,s)-1;0\}=z_{S|n,k}\big(2p(n,s)-1\big)$. The expected utility from sharing is thus: $\sum_k\frac{1}{K}\sum_{s,n}z_{S|n,k}\big(2p(n,s)-1\big)\Pr(n,s)$. Note that suming over possible s is equivalent to suming over possible X as X,n and s,n are isomorphic. The final expression is fund by plugging the expression for $p(n,s)$ and $\Pr(n,s)$ in the sum.

- Conditional on receiving news n after private signal s, accounting for the optimal decision to bet, the utility from sharing is $\max\{2p(n,s)-1;1-2p(n,s)\}$ where the former argument expresses the expected gain from betting $a = n$ and the latter, the expected gain from betting $a \neq n$. As before, $2p(n,s)-1 > 0 \Rightarrow z_{S|n,k} = 1$; therefore, it is optimal for seeds to bet $a = n$ after (n,s) with the same probability as they share n after (n,s). If $z_{S|n,k} = 0$, they bet $a \neq n$ and get $1 - 2p(n,s)$; when $1 > z_{S|n,k} > 0$ they are indifferent as $p(n,s) = 1/2$. Therefore, $\max\{2p(n,s) - 1; 1 - 2p(n,s)\} = \left(2z_{S|n,k} - 1\right)\left(2p(n,s) - 1\right)$. As before, the final expression for the expected utility of seeds from betting is found by plugging he expression for $p(n,s)$ and $\Pr(n,s)$ into: $\sum_k \frac{1}{K} \sum_{s,n} \left(2z_{S|n,k} - 1\right)\left(2p(n,s) - 1\right)\Pr(n,s)$.

Followers, conditional on receiving some news, account for the higher probability to be reached by a true news because of the filtering effect of the network. If there is no competition, the expression for their expected utility from betting is the same as seeds. Otherwise,

$$Pr(\omega = n|n,s,k) = \frac{\Pr(n,s|\omega=n)\Pr(\text{see k over -k}|\omega=n)Pr(\omega=n)}{Pr(n,s,\text{sees } k)} = \frac{q_k \Pr(T)\sum_Y \Pr(\text{see } T \text{ over } Y\ )\Pr(\omega=n)}{\sum_w \Pr(n|\omega=w)\Pr(X)\sum_Y \Pr(\text{sees } X \text{ over } Y\ )\Pr(\omega=w)}$$

and their utility is found as $\max\{2Pr(\omega = n|n,s,k) - 1; 1 - 2Pr(\omega = n|n,s,k)\}$.

Note that upon receiving no news, followers simply bet their private signal and get $2\gamma - 1$ in expectation.

- Upon each possible outcome (n,s), consumers do not enter if $u_j(a|n,s) < r$.

1.D 1.4.2 Welfare for symmetric priors

Theorem 2, Lemma 5 and Corollary 3: outlets' influence on betting decisions

Consider $w_0 = 1/2$.

(i) The expected utility of a seed who would always follow the news article is: $\sum_s q\Pr(s = n) - (1-q)\Pr(s \neq n)$, which is smaller or equal to $(2\gamma - 1)$ for any $q \leq \gamma$. If the influence follows the news article only when $n = s$, then it is equivalent to always following the private signal. $q^* \leq \gamma$ so seeds are always as well off following their private signal.

(ii) Conditional on receiving a news article n, without competition, the expected utility from a follower is the same as the expected utility from the seed; hence follower are always. as well of betting their private signal in a market without competition.
In a competitive market, consider a follower receiving a signal different than the article they read. If the follower is better off following the news in this case, then the presence of news outlets allows him to take better decision. A follower's expected

utility from betting the content of a news article conditional on receiving one with $n \neq s$ is:

$$\sum_m \left[q(1-\gamma)\Pr(m|\omega = n)\frac{p_{T_k}}{p_{T_k}+p_{Y_\ell}} - (1-q)\gamma \Pr(m|\omega \neq n)\frac{p_{F_k}}{p_{F_k}+p_{Y_\ell}} \right]$$

where $Y = T$ when $m = \omega$ and $Y = F$ for $m \neq \omega$ Because the equilibrium is symmetric, $p_{T_k} = p_{T_\ell} = \frac{p_T}{2}$ and $p_{F_k} = p_{F_\ell} = \frac{p_F}{2}$. Therefore, the value for the expected utility of a follower receiving $n \neq s$ when the news quality is q is:

$$\mathbb{E}(u_j(a)|n \neq s) = \left[q^2(1-\gamma)\frac{1}{2} - (1-q)q\gamma\frac{p_F}{p_T+p_F} \right] + \left[q(1-q)(1-\gamma)\frac{p_T}{p_T+p_F} - (1-q)^2\gamma\frac{1}{2} \right]$$

Which is maximized, given any q, at z^* is such that $p_T = b\gamma$; $p_F = b(1-\gamma)$. Then:

$$\mathbb{E}(u_j(a)|n \neq s) = \frac{(q^2(1-\gamma)-(1-q)^2\gamma)}{2} + q(1-q)\frac{(1-\gamma)\left(\gamma b\right)-\gamma\left((1-\gamma)b\right)}{\gamma b+(1-\gamma)b} = \frac{(q^2(1-\gamma)-(1-q)^2\gamma)}{2}$$

The follower is better off following the article rather than his private signal when this expected utility is greater than 0, which requires $q\frac{\gamma-\sqrt{\gamma(1-\gamma)}}{2\gamma-1}$.
The follower's maximal utility is still bounded by γ because $q \leq \gamma$. In particular, for $q = \gamma$ and $z^* = (1,0)$, $z^f = (1,1)$ so that the expected utility of a follower conditional on receiving some news is:

$$q\left(\frac{1}{2} + \frac{p_T}{p_T+p_F}\right) - (1-q)\left(\frac{p_F}{p_F+p_T} + \frac{1}{2}\right) = \frac{3}{2}(2\gamma - 1)$$

Consider any w_0 in an uncompetitive environment.

(i) As in equilibrium, q^* can be greater than γ, consumers can be made better off by betting the news article content: $\sum_w \left(\sum_s \left(q\Pr(s = n) - (1-q)\Pr(s \neq n)\right)\Pr(\omega = w)\right) > 2\gamma - 1$ iff $q > \gamma$.

(ii) Consider $q^* \in (\gamma, \bar{t}_1]$. Then, $z_{+|0} = z_{+|1} = z_{-|0} = 1$. The expected utility of a seed – or of a follower conditional on seeing a news article – is:

$$\mathbb{E}(u_i) \sum_{X,n}(2z_{S|n} - 1)\left[qPr(X)\Pr(\omega = n) - (1-q)Pr(-X)\Pr(\omega \neq n)\right]$$
$$= \left[q\gamma w_0 - (1-q)(1-\gamma)(1-w_0)\right] + \left[q\gamma(1-w_0) - (1-q)(1-\gamma)w_0\right] + \left[q(1-\gamma)w_0 - (1-q)(1-\right.$$

Note that this equality is also valid for $z_{+|1} > 0$; Indeed $z_{+|1} > 0 \Rightarrow q^* = \bar{t}_1$ but $\bar{t}_1 Pr(F)\Pr(\omega =$

$n) - (1-\bar{t}_1)Pr(T)\Pr(\omega \neq n) = 0$. Therefore,

$$\begin{aligned}
\mathbb{E}(u_i) &= 2\gamma - 1 + 2(q(1-\gamma)w_0 - (1-q)\gamma(1-w_0)) \\
&\leq 2\gamma - 1 + 2(\bar{t}_1(1-\gamma)w_0 - (1-\bar{t}_1)\gamma(1-w_0)) \\
&= 2\gamma - 1 + \frac{2\gamma(1-\gamma)}{\gamma w_0 + (1-\gamma)(1-w_0)}(2w_0 - 1) \\
&\leq 2\gamma - 1 + \frac{2\gamma(1-\gamma)}{\gamma^2 + (1-\gamma)^2}(2\gamma - 1) \\
&= 2\gamma - 1\left(1 + \frac{2\gamma(1-\gamma)}{1-2\gamma(1-\gamma)}\right)
\end{aligned}$$

Where the last inequality follows from

$$\frac{\partial \frac{2w_0-1}{\gamma w_0+(1-\gamma)(1-w_0)}}{\partial w_0} = \frac{2\Big(\gamma w_0+(1-\gamma)(1-w_0)\Big)-(2w_0-1)(2\gamma-1)}{\Big(\gamma w_0+(1-\gamma)(1-w_0)\Big)^2} = \frac{1}{\Big(\gamma w_0+(1-\gamma)(1-w_0)\Big)^2} > 0$$

Theorem 2 and Corollary 4: outlets' influence on entering the bet

Consider again $w_0 = 1/2$. Let us compare the decision to enter the bet with and without news. Without news, all consumers take the same action: they opt out of the bet if $r > r_s$ and enter the bet for $r \leq r_s$. With news, seeds would opt out for $r > \bar{r}$, enter following any news with $r \leq \underline{r}$ and enter only for $n = s$ with $\underline{r} < r \leq \bar{r}$. Their behavior changes only in the interval $[\underline{r}, \bar{r}]$.

- For $r \in (r_s, \bar{r}]$, news articles push agents to enter the bet. All agents with $n = s$ place a bet. Given any state of the world, there are γq agents receiving $n = s$ corresponding to the right state of the world, i.e. who win the bet; and $(1-\gamma)(1-q)$ seeds who lose the bet. As $\gamma q > (1-\gamma)(1-q)$, there are more winners than losers.
- For $r \in [\underline{r}, r_s]$, news articles discourage agents to enter the bet. All agents with $n \neq s$ opt out. Given any state of the world, there are $(1-\gamma)q$ agents receiving $n \neq s$ who had the wrong private signal so who are better off opting out; and $\gamma(1-q)$ seeds who are worse off. As $\gamma(1-q) > (1-\gamma)q$, there are more losers than winners.

Consider any w_0 in an uncompetitive environment. Consumers might decide to enter the bet conditional on the private signal content's they receive. Let r_s be the bet price that makes consumers indifferent between betting or not, were they to only observe their private signal s. Then:

$$r_0 = 2\frac{\gamma w_0}{\gamma w_0 + (1-\gamma)(1-w_0)} - 1 > 2\frac{\gamma(1-w_0)}{\gamma(1-w_0) + (1-\gamma)w_0} - 1 = r_1$$

Furthermore, let $\underline{r}_s$ be the bet price that makes consumers indifferent between betting or not when receiving signal s and news content $n \neq s$. Then:

$$\underline{r}_0 = 2\frac{\gamma(1-q)w_0}{\gamma(1-q)w_0 + (1-\gamma)q(1-w_0)} - 1 > 2\frac{\max\{\gamma(1-q)(1-w_0);(1-\gamma)qw_0\}}{\gamma(1-q)(1-w_0)+(1-\gamma)qw_0} - 1 = \underline{r}_1$$

Finally, let $\bar{r}_s$ be the bet price that makes consumers indifferent between betting or not when receiving signal s and news content $n = s$. Then:

$$\bar{r}_0 = 2\frac{\gamma q w_0}{\gamma q w_0 + (1-\gamma)(1-q)(1-w_0)} - 1 > 2\frac{\gamma q(1-w_0)}{\gamma q(1-w_0)+(1-\gamma)(1-q)w_0} - 1 = \bar{r}_1$$

Now, $\underline{r}_s < r_s < \bar{r}_s$ for both $s = 0, 1$; furthermore, as in the proof of Lemma 6, conditional on receiving a signal s, consumers are expected to be worse at deciding whether to enter the bet for $r \in [\underline{r}_s, r_s]$ for $s = 0$ and for $s = 1$ and $q_M^* \leq \underline{t}_0$; and better for $r \in [r_s, \bar{r}_s]$.

One must thus determine the relative order of the different thresholds, and compare the gains and losses from the presence of outlets for each case.

- If $q < w_0$, then $\underline{r}_1 < r_1 < \bar{r}_1 < \underline{r}_0 < r_0 < \bar{r}_0$. Then, for $r \in [\underline{r}_0; r_0]$, the presence of news outlets does not change the consumers decision to enter the bet if $s = 1$ but dissuades them to enter for $s = 0 \neq 1 = n$. This dissuasion happens with probability $\gamma(1-q)$ for $\omega = 0$ and with probability $(1-\gamma)q$ for $\omega = 1$. The total expected effect is thus $-\gamma(1-q)w_0 + (1-\gamma)q(1-w_0) < 0$ for any equilibrium $q < \underline{t}_1$.
- If $q \in \left[w_0; \frac{w_0^2}{w_0^2+(1-w_0)^2}\right]$, then $\underline{r}_1 < r_1 < \underline{r}_0 < \bar{r}_1 < r_0 < \bar{r}_0$. Then, for $r \in [\bar{r}_1; r_0]$, the presence of news outlets does not change the consumers decision to enter the bet if $s = 1$ but as before dissuades to many of them to enter for $s = 0 \neq 1 = n$.
- If $q > \frac{w_0^2}{w_0^2+(1-w_0)^2}$, then $\underline{r}_1 < \underline{r}_0 < r_1 < r_0 < \bar{r}_1 < \bar{r}_0$. Then, for $r \in [\underline{r}_0; r_1]$, the presence of news outlets dissuades consumers to enter for any $s \neq n$. This creates a loss for $s = w$, which happens with probability $\gamma(1-q)$ for either $\omega = w$; and a gain for $s \neq w$, which happens with probability $(1-\gamma)q$ for either $\omega = w$. The total expected effect is thus negative as long as $q < \gamma$.

Finally notice how, for instance for $q < w_0$, the presence of news outlets is positive for $r \in [r_1, \bar{r}_1]$ and $r \in [r_0, \bar{r}_0]$ but negative for $r \in [\underline{r}_0; r_0]$, while $\bar{r}_1 < \underline{r}_0 < r_0$.

Proposition 5: effect of competition on total welfare

- About the expected utility from sharing: taking $d \to \infty$, the difference in profits is $-2C(q_D)$ while the difference in expected utility from sharing is $q_D\gamma - (1-q_D)\gamma$.

There exists a cost function $C(q)$ such that $2C(q_D) > q_D\gamma - (1-q_D)\gamma$, for instance $C(q) = \frac{q^2}{2}$.

- About the expected utility from betting: Consider a cost function such that $q_M^* < q_D^* < \frac{\gamma-\sqrt{\gamma(1-\gamma)}}{2\gamma-1}$. Then, by Theorem 2, neither seeds nor followers are made better off by the presence of a second news outlet. The difference in total revenues from producers is:

$$q(1-q)\left[(1-p_T)^d + (1-p_F)^d - 2\left(1 - \frac{p_T+p_F}{2}\right)\right]$$

The total cost of production doubles. For $d \to \infty$ the revenues are the same, so that any cost function $C(q)$ would surpass that the total gain in revenues.

The same applies to the utility from entering the bet for $r < \underline{r}$ and $r > \bar{r}$.

1.D 1.5 Evaluation of Intervention

1.D 1.5.1 Flagging

Proposition 6 and Corollary 5: effect of flagging with or without competition

Let the difference of incentive to invest with flagging be $FDV(z,q;\rho) := \Delta V_M(z;\rho) - \Delta V_D(z,q;\rho)$. Let us first rewrite:

$$\frac{\partial FDV(z,q;\rho)}{\partial \rho} = V_F + (1-q)V_{TF} - (1-q)V_{T\varnothing} - qV_{FT} - 2(1-q)(1-q)V_{FF} + (1-q)(1-2\rho)V_{F\varnothing}$$

To prove that this derivative is positive, I show that $\frac{\partial^2 FDV(z,q;\rho)}{\partial\rho\partial q} \geq 0$, so that $\frac{\partial FDV(z,q;\rho)}{\partial \rho} \geq \frac{\partial FDV(z,q;\rho)}{\partial \rho}\Big|_{q=1/2}$ I then move to show that $\frac{\partial FDV(z,q;\rho)}{\partial \rho}\Big|_{q=1/2} > 0$.

To show that $\frac{\partial^2 FDV(z,q;\rho)}{\partial\rho\partial q} \geq 0$, let us rewrite:

$$\begin{aligned}
\frac{\partial^2 FDV(z,q;\rho)}{\partial\rho\partial q} &= -V_{TF} + V_{T\varnothing} - V_{FT} + 2(1-\rho)V_{FF} - (1-2\rho)V_{F\varnothing} \\
&= -\tfrac{p_T}{p_T+p_F}\Big(1 - (1 - \tfrac{p_T+p_F}{2})^d\Big) + \Big(1 - (1 - \tfrac{p_T}{2})^d\Big) - \tfrac{p_F}{p_T+p_F}\Big(1 - (1 - \tfrac{p_T+p_F}{2})^d\Big) \\
&\quad + (1-\rho)\Big(1 - (1-p_F)^d\Big) - (1-2\rho)\Big(1 - (1 - \frac{p_F}{2})^d\Big) \\
&= \rho + (1 - \tfrac{p_T+p_F}{2})^d - (1 - \tfrac{p_T}{2})^d - (1-\rho)(1-p_F)^d + (1-2\rho)(1 - \tfrac{p_F}{2})^d \\
&= \rho\Big[1 + (1-p_F)^d - 2(1 - \tfrac{p_F}{2})^d\Big] + \Big[(1 - \tfrac{p_T+p_F}{2})^d - (1 - \tfrac{p_T}{2})^d - (1-p_F)^d + (1 - \tfrac{p_F}{2})^d\Big]
\end{aligned}$$

Now, this expression is the sum of two positive terms. Indeed:

- the first term is increasing in p_F so that $1+(1-p_F)^d - 2(1-\frac{p_F}{2})^d \geq 1 + (1-p_F)^d - 2(1-\frac{p_F}{2})^d\Big|_{p_F=0} = 0$

- the second term is increasing in p_T so that $(1-\frac{p_T+p_F}{2})^d-(1-\frac{p_T}{2})^d-(1-p_F)^d+(1-\frac{p_F}{2})^d \geq (1-\frac{p_T+p_F}{2})^d-(1-\frac{p_T}{2})^d-(1-p_F)^d+(1-\frac{p_F}{2})^d\Big|_{p_T=p_F}=0$

We can thus conclude that $\frac{\partial^2\left(\Delta V_M(z;\rho)-\Delta V_D(z,q;\rho)\right)}{\partial\rho\partial q}\geq 0$

Let us now show that $\frac{\partial FDV(z,q;\rho)}{\partial\rho}\Big|_{q=1/2}>0$. We can rewrite:

$$\frac{\partial FDV(z,q;\rho)}{\partial\rho}\Big|_{q=1/2}=V_F+\frac{1}{2}V_{TF}-\frac{1}{2}V_{T\varnothing}-\frac{1}{2}V_{FT}-(1-\rho)V_{FF}+\frac{1}{2}(1-2\rho)V_{F\varnothing}$$

Noting that $V_{FF}=\frac{1}{2}V_F$, we get:

$$\begin{aligned}\frac{\partial FDV(z,q;\rho)}{\partial\rho}\Big|_{q=1/2}&=\left[\frac{(1+\rho)}{2}V_F-\rho V_{F\varnothing}\right]+\frac{1}{2}\left[V_{TF}-V_{FT}-V_{T\varnothing}+V_{F\varnothing}\right]\\&=\left[\tfrac{1+\rho}{2}V_F-\rho V_{F\varnothing}\right]+\tfrac{1}{2}\left[\tfrac{p_T-p_F}{p_T+p_F}\left(1-(1-\tfrac{p_T+p_F}{2})^d\right)+(1-\tfrac{p_T}{2})^d-(1-\tfrac{p_F}{2})^d\right]\end{aligned}$$

Again, this is the sum of two positive terms.

- The first term is positive as $\frac{1+\rho}{2}>\rho$ and $V_F\geq V_{F\varnothing}$. Note that the term is strictly positive for $p_F>0$.
- It is more cumbersome to show that the second term is positive. We show that it is non-decreasing in d and then show it is weakly positive for $d=1$.To show that it is non-decreasing in d, we proceed by induction. For ease of notation, let us define for this proof:

  $$E(d):=\tfrac{p_T-p_F}{p_T+p_F}\left(1-(1-\tfrac{p_T+p_F}{2})^d\right)+(1-\tfrac{p_T}{2})^d-(1-\tfrac{p_F}{2})^d$$

  Then,

  $$\begin{aligned}E(d)-E(d+1)&=-\tfrac{p_T-p_F}{p_T+p_F}\tfrac{p_T+p_F}{2}(1-\tfrac{p_T+p_F}{2})^d+\tfrac{p_T}{2}(1-\tfrac{p_T}{2})^d-\tfrac{p_F}{2}(1-\tfrac{p_F}{2})^d\\&=\underbrace{\tfrac{p_T}{2}\left[(1-\tfrac{p_T}{2})^d-(1-\tfrac{p_T+p_F}{2})^d\right]}_{:=A}-\underbrace{\tfrac{p_F}{2}\left[(1-\tfrac{p_F}{2})^d-(1-\tfrac{p_T+p_F}{2})^d\right]}_{:=B}\end{aligned}$$

  Therefore $E(d)-E(d+1)<0$ for $A<B$. We want to show that if $E(d)$ it is non-decreasing at some d', then it is non-decreasing for all subsequent $d>d'$. The inductive step requires us to show that for $A\leq B$, $E(d+1)-E(d+2)\leq 0$. This is indeed the case as:

$$\begin{aligned}E(d+1)-E(d+2)=&\tfrac{p_T}{2}\left[(1-\tfrac{p_T}{2})^d(1-\tfrac{p_T}{2})-(1-\tfrac{p_T+p_F}{2})^d(1-\tfrac{p_T}{2})+(1-\tfrac{p_T+p_F}{2})^d(-\tfrac{p_F}{2})\right]\\&-\tfrac{p_F}{2}\left[(1-\tfrac{p_F}{2})^d(1-\tfrac{p_F}{2})-(1-\tfrac{p_T+p_F}{2})^d(1-\tfrac{p_F}{2})+(1-\tfrac{p_T+p_F}{2})^d(-\tfrac{p_T}{2})\right]\\=&(1-\tfrac{p_T}{2})A-(1-\tfrac{p_T}{2})B\end{aligned}$$

Because $(1-\frac{p_T}{2}) \leq (1-\frac{p_F}{2})$ and $A \geq 0$, we do have: $A \leq B \Rightarrow (1-\frac{p_T}{2})A \leq (1-\frac{p_T}{2})B$. Finally, it is easy to verify that for $d=1$, $A=B$, so that $E(1)-E(2)=0$. [32]

We can thus conclude $\frac{\partial FDV(z,q;\rho)}{\partial \rho}\Big|_{q=1/2} \geq 0$ for any $p_F \geq 0$ and $\frac{\partial FDV(z,q;\rho)}{\partial \rho}\Big|_{q=1/2} > 0$ for any $p_F > 0$, which concludes the proof of the stronger effect of flagging in a monopoly.

To show that there exists a level of flagging that makes competition detrimental to the producers' incentive to invest, it is enough to note that $\Delta V_M(z;\rho)$ is continuous in ρ and that with $\rho=1$, $\Delta V_M(z;1) > \Delta V_D(z,q;1)$ since $\Delta V_M(z;1) - \Delta V_D(z,q;1) = V_T - qV_{TT} - (1-q)V_{T\varnothing} > 0$. To show that any outcome $q_D^* > q_M^*$ is reproducible in a monopoly, notice $\Delta V_D(z,q;1) > \Delta V_D(z,q;0)$.

1.D 1.6 Discussion

Irregular Networks and Seeds' Selection

Denote $\Delta V(d_j)$ the producer's incentive to invest in a regular network of degree d_j as derived in the main text. Let $\Delta V(\delta)$ be the producer's incentive to invest in a network with degree distribution δ. $\Delta V(d_j)$ is continuous in d_j; hence, there exists a representative degree $\tilde{d}$ such that $\Delta V(\tilde{d}) = \sum_{d_j} \delta(d_j)\Delta V(d_j)$. The equilibria can be characterized applying Proposition 1 and 3 with $d=\tilde{d}$. The role of private knowledge is qualitatively the same for every $\Delta V(d_j)$, hence for $\Delta V(\delta)$: Proposition 2 and Corollary 2 apply. The role of connectivity on the producer's incentive to invest can be assessed in terms of $\tilde{d}$. The effects of competition through connectivity also carry through as $\Delta V_M(d_j) - \Delta V_D(d_j)$, is continuous in d_j; hence, there exists a representative degree $\check{d}$ such that $\Delta V_M(\check{d}) - \Delta V_M(\check{d}) = \sum_{d_j} \delta(d_j)\Big(\Delta V_M(d_j) - \Delta V_D V(d_j)\Big)$. All other results directly apply.

Behavioral Biases and Partisanship

Consider confirmation bias. When $S=-$, with probability ϵ, seeds misinterpret the news content and believe it corresponds to their private signal. Then, the probability for an article to be shared becomes: $p_T = \frac{b}{K}\Big[\Big(\gamma + (1-\gamma)\epsilon\Big)z_+ + (1-\gamma)(1-\epsilon)z_-\Big]$ and $p_F = \frac{b}{K}\Big[\Big(\gamma\epsilon + (1-\gamma)\Big)z_+ + \gamma(1-\epsilon)z_-\Big]$. The analysis would then be directly applicable. For instance, take a monopoly. $\frac{\partial \Delta V(z)}{\partial \epsilon} = -d(z_+ - z_-)\left(\gamma(1-p_F)^{d-1} - 1 - \gamma(1-p_T)^{d-1}\right) \leq 0$ as $z_+ - z_- \geq 0$, $\gamma < 1-\gamma$ and $(1-p_F)^{d-1} \leq (1-p_T)^{d-1}$. The same applies to the duopoly case.

Consider sensationalism. Seeds are assumed to enjoy sharing an article that is not

[32]Note that if $A > 0$ and $p_T > p_F$, $A \leq B \Rightarrow (1-\frac{p_T}{2})A > (1-\frac{p_T}{2})B$. Therefore, the term is strictly increasing for any $d \geq 2$, $p_T > p_F$.

congruent with their private signal because of their taste for sensationalism. In particular, assume that they get a utility premium from such a share of ϵ. Their payoff from sharing is then:

$$u(\text{sharing article } n|\omega = w, S) = \begin{cases} 1 + \epsilon \mathbb{1}_{S=-} & \text{if } n = w \\ -1 + \epsilon \mathbb{1}_{S=-} & \text{otherwise} \end{cases}$$

It follows that their expected utility from sharing when $S = -$ is $2p(n,s) - 1 + \epsilon$, so that $z_{-|n} > 0$ if $q > \tilde{t}_n = \bar{t}_n = \frac{(1-\epsilon)\gamma \Pr(\omega \neq n)}{(1-\epsilon)\gamma \Pr(\omega \neq n) + \epsilon(1-\gamma)\Pr(\omega = n)}$. Therefore, the news quality is now bounded by $\tilde{t}_1$ and $\tilde{t}_1 < \bar{t}_1$.

Beyond Visibility

Consider that news quality affects reputation benefits continuously. In particular, assume that the producers revenues can be written $\mathbb{E}(R_k|q) + \nu q$. Then the best-response of the producer would be: $\tilde{q}^*(z) = c^{-1}(\Delta V(z) + \nu) > c^{-1}(\Delta V(z)) = q^*(z)$. However, to understand whether news quality could surpass $\bar{t}_1$, one needs to understand whether the producers' best-response might lie completely above the seeds' best-response. This would occur if $c^{-1}(\nu) = c^{-1}(\Delta V((1,1)) + \nu) > \bar{t}_1$. When this is the case, the equilibrium news quality is $c^{-1}(\nu)$; otherwise, one can apply Proposition 1 and 3 with $\tilde{q}^*(z) = c^{-1}(\Delta V(z) + \nu)$.

Consider that news quality affects reputation benefits discretely. In particular, assume that the producers revenues can be written $\mathbb{E}(R_k|q) + \nu \mathbb{1}_{q > \bar{q}}$. Then the best-response of neither side of the market would be affected. However, if $q^* < \bar{q}$, the producer would invest $\bar{q}$ iff $\mathbb{E}(R_k|\bar{q}) + \nu - C(\bar{q}) > \mathbb{E}(R_k|q^*) + \nu - C(q^*)$.

1.D 1.A Asymmetric Loss From Sharing

Proposition 1.1.A: characterization of the equilibrium without competition

First notice that any positive equilibrium investment has to lie within $[\underline{t}_0^\lambda, \bar{t}_1^\lambda]$. Indeed, it is easy to see that for any $q < \underline{t}_0^\lambda$, no news is ever shared so that the producer has no incentive to invest; likewise, $q = \bar{t}_1^\lambda$ is enough to insure that the producer's news is always share, so that investing more than this does not increase the producer's benefit.
It follows that, if $1/2 \geq \bar{t}_1^\lambda$, the producer will never want to invest more than $1/2$ – intuitively, the producer's best response lies above the seeds' best response. If $\bar{q}_0 < \underline{t}_0^\lambda$ and $\bar{q}_1 < \underline{t}_1^\lambda$, it is too costly for the producer to invest more than $1/2$, as for any sharing strategy z, the marginal benefit from investing $q > 1/2$ is lower than its marginal cost – intuitively, the producer's best response lies below the seeds' best response. Indeed, we know that $q < \underline{t}_0^\lambda$ cannot be an equilibrium. For any $q \in [\underline{t}_0^\lambda, \underline{t}_1^\lambda)$, by definition $\bar{q}_0 = c^{-1}(\Delta V(z_{\bar{0},0}, 0, 0, 0)) \geq c(\Delta V(z^*(q))$ so that $c(q) \geq c(\underline{t}_0^\lambda) > c(\bar{q}_0) \geq c(\Delta V(z^*(q))$. Likewise,

for $q \in [\underline{t}_1^\lambda, \bar{t}_0^\lambda)$, as $\bar{q}_1 < \underline{t}_1^\lambda$, we have $c(q) \geq c(\underline{t}_1^\lambda) > c(\bar{q}_1) \geq c(\Delta V(z^*(q))$. For any $q \geq \bar{t}_0^\lambda$, $c(q) \geq c(\bar{t}_0^\lambda) > c(\max\{\bar{q}_0, \bar{q}_1\}) \geq c(\Delta V(z^*(q))$
Now, let us understand what happens if positive investment is possible. If $\bar{q}_1 < \underline{t}_1^\lambda$, as argued above, the investment has to be such that $q \in [\underline{t}_0^\lambda, \underline{t}_1^\lambda)$. Because $\bar{q}_0 > \underline{t}_0^\lambda$, and $q^*(z)$ continuous, there there must exist some $z^*_{T|0}$ such that $c^{-1}(\Delta V(z^*_{T|0}, 0, 0, 0)) = \underline{t}_0^\lambda$. If $\tilde{q}_0 < \underline{t}_0^\lambda$, the maximal investment equilibrium is thus $\underline{t}_0^\lambda$; otherwise, $\tilde{q}_0$ is an equilibrium as $\tilde{x_{00}} \in [\underline{t}_0^\lambda, \underline{t}_1^\lambda)$ and by definition, $c(\tilde{q}_0) = \Delta V(1, 0, 0, 0)$, and leads to more investment. A similar reasoning applies to $\bar{q}_1 \geq \underline{t}_1^\lambda$ and $\tilde{q}_1 \leq \bar{t}_0^\lambda$.
Finally, if $\bar{q}_1 \geq \underline{t}_1^\lambda$ and $\tilde{q}_1 > \bar{t}_0^\lambda$, because $q^*(z)$ is decreasing in $z_{-|0}$ and $z_{-|1}$, and continuous, there must exist a $q' \geq \bar{t}_0^\lambda$ and a $z' = (1, 1, z_{-|0}, z_{-|1})$ such that $c(q') = \Delta V(z')$. It is easy to verify that $\max\{\bar{t}_0^\lambda, \min\{q^*(1,1,1,0), \bar{t}_1^\lambda\}\}$ yield the highest q on $[\underline{t}_0^\lambda, \underline{t}_1^\lambda]$ such that $c(q') = \Delta V(z')$.

Additional Remark: existence of other equilibria

(i) $x_k^*((0,0), z_{-k}, q_{-k}) \in [0, \min\{1/2, \underline{t}^\lambda\}]$ and $z_{T_k}(q_k) = z_{F_k}(q_k) = 0$ for $q_k \in [0, \min\{1/2, \underline{t}^\lambda\}]$.

(ii) Notice that for $p_{X_{-k}} = q_{-k} = 0$, $\Delta V_k(z, q_{-k}) = (1-b)\left[(1 - 1/2b\underline{t}^\lambda)^d - (1 - 1/2b\bar{t}^\lambda)^d\right]$, which corresponds to the monopoly case up to 1/2, which is accounted for when defining $\bar{q}_k$. Furthermore, it is a best response for $-k$ to not invest if $z_{-k} = (0,0)$, which is a best response if $q_k \in [0, \min\{1/2, \underline{t}^\lambda\}]$.

Proposition 3.1.A: characterization of the equilibrium with competition

First note that any equilibrium investment bigger than 1/2 has to lie in $[\underline{t}^\lambda, \bar{t}^\lambda]$. Indeed, recall that $\Delta V_D((0,0), q) = \Delta V_D((1,1), q) = 0$. Hence, clearly, for any $q < \min\{\underline{t}^\lambda, \bar{q}\}$ or $q > \max\{\bar{t}^\lambda, 1/2\}$, $c(q) > 0 = \Delta V_D(z^*(q), q)$, which would be suboptimal for the producer.

Given $1/2 < \bar{t}^\lambda$ and $\underline{t}^\lambda \leq \bar{q}_D$, different parameters allow for two cases:

1. If $c(\underline{t}^\lambda) \leq \Delta V_D((1,0), \underline{t}^\lambda)$ and $\Delta V_D((1,0), \bar{t}^\lambda) < c(\bar{t}^\lambda)$, then $\exists \tilde{q} \in [\underline{t}^\lambda, \bar{t}^\lambda]$: $c(\tilde{q}) = \Delta V((1,0), \tilde{q})$. Indeed, recall that c is weakly increasing in q and $\Delta V_D((1,0), q)$ strictly decreasing in q. Clearly, $(\tilde{q}, (1,0))$ is a NE.
   It is the symmetric NE which leads to the highest investment. Indeed, assume there exists another symmetric equilibrium with investment $q' > q_D^*$. As argued above, $q' \in \{\underline{t}^\lambda, \bar{t}^\lambda\}$.

   For $q' = \bar{t}^\lambda > q_D^*$ to be part of an equilibrium, there must exist a $z' = (1, z'_F)$ with $z'_F > 0$ such that $V_D(z', q') = c(\bar{t}^\lambda)$. It is impossible, because $c(\bar{t}^\lambda) > c(q_D^*) = \Delta V_D((1,0), q_D^*) > \Delta V_D((1,0), \bar{t}^\lambda) > \Delta V_D((1, z_F), \bar{t}^\lambda)$ $\forall z_F > 0$, where the last inequality uses that $\Delta V_D(z; q)$ is decreasing in z_F.

2. If $c(\underline{t}^\lambda) > \Delta V_D((1,0),\underline{t}^\lambda)$, then $\exists \tilde{z}_T \in [\bar{z}_T^D, 1]$: $c(\underline{t}^\lambda) = \Delta V(\tilde{z}_T, \underline{t}^\lambda)$. Indeed, by assumption $\Delta V_D((\bar{z}_T^D, 0); \underline{t}^\lambda) > c(\underline{t}^\lambda) > \Delta V_D((1,0); \underline{t}^\lambda)$, and $\Delta V_D(z;q)$ is continuous and decreasing on $[\bar{z}_T^D, 1]$. Clearly, $(\underline{t}^\lambda, (\tilde{z}_T, 0))$ is a NE.

3. If $c(\bar{t}^\lambda) < \Delta V_D((1,0),\bar{t}^\lambda)$, then $\exists \tilde{z}_F \in [0,1]$: $c(\bar{t}^\lambda) = \Delta V_D((1,\tilde{z}_F),\underline{t}^\lambda)$. Indeed, by assumption $\Delta V_D((0,0);\bar{t}^\lambda) = 0 < c(\bar{t}^\lambda) < \Delta V_D((1,0);\underline{t}^\lambda)$, and $\Delta V_D(z;q)$ is continuous and decreasing in z_F. Clearly, $(\bar{t}^\lambda, (1, \tilde{z}_F))$ is a NE.

Additional Corollary and Remark 2.1.A: discussion of other equilibria

About the equilibria's stability:
First if $1/2 > \underline{t}^\ell$, $\Delta V_D(z, \underline{t}^\ell) > \underline{t}^\ell$ for any z. Because $\Delta V_D(\bar{z}_T, q)$ is continuous and increasing on $[0, \bar{z}_T]$, we know that, given any q, $c^{-1}(\Delta V_D(z, \underline{t}^\ell))$ crosses $z_v^*(q)$ only once. So for any q_0, there is a unique $q', z^*(q')$. We pick the q_0 that leads to equilibrium $q_0, z^*(q_0)$, which must be unique.
If $\bar{q}_D \geq \underline{t}^\lambda > 1/2$, then given any $q = \underline{t}^\lambda$, $c^{-1}(\Delta V_D(z,q))$ crosses $z_v^*(q)$ twice: once for some $z'_T < \bar{z}_T$ with $\Delta V_D(z'_T, q)) = c(\underline{t}^\lambda)$; and once afterwards. The slope of $\Delta V_D(z,q))$ in $z'_T < \bar{z}_T < 1$ is strictly increasing. The investment required for seeds to share upon receiving congruent private signal with probability z'_T is equal to $\underline{t}^\lambda$ with slope 0. Therefore, the equilibrium $(\underline{t}^\lambda, (z'_T, 0))$ cannot be stable. In particular, any stable equilibrium must have $z_k, z_\ell > z'_T$.
Finally, we prove that q_D^* is the only stable equilibrium with symmetric investment by noting that $q_k = q_\ell = q^*$ implies $z_k = z_\ell > z'_T$. Indeed, any equilibrium investment q^* requires $\Delta V_u(z_k, z_\ell; q^*) = \Delta V_v(z_k, z_\ell; q^*)$. Now, because $\frac{\partial \Delta V_k}{\partial z_k} \neq -\frac{\partial \Delta V_k}{\partial z_v}$ for every $z_k, z_\ell > z'_T$, the unique z_k, z_ℓ supporting q_D^* must be defined by $\Delta V(z,q)$; therefore, $z_u = z_v$.

About other asymmetric equilibria (i) $\bar{q}_m < \underline{t}^\lambda$ means that, even if the network is free of competition, there is no sharing rule that could convince a producer to invest. Therefore, no investment can occur.
(ii) $\bar{q}_D < \underline{t}^\lambda$ means that there does not exist a symmetric equilibrium with positive investment, i.e. $\forall z_k, \underline{t}^\lambda > \Delta V_k(z_k, z_k; \underline{t}^\lambda)$. Furthermore, no other equilibrium with positive investment can exist. By using the proof of the Additional Corollary above, $z_k = z_\ell$ if $q_k = q_\ell$, so $z_k \neq z_\ell$ is inconsistent with $q_k < q_\ell$. Finally, $z_k < z_\ell$ and $q_k = \underline{t}^\lambda < q_\ell$ cannot be an equilibrium as $\underline{t}^\lambda > \Delta V_k(z_k, z_k; \underline{t}^\lambda) > \Delta V_k(z_k, z_\lambda; q_\ell)$ for $z_\lambda > z_k$ $q_\lambda > \bar{t}^\lambda$.
(iii) $\underline{t}^\lambda = q_m$ implies that $\Delta V_k(z_k, z_\lambda; q_\ell) < \Delta V_k(z_k, (0,0); 0) < \underline{t}^\lambda$ for any z_k. Hence x_k cannot exceed $\underline{t}^\lambda$. As the same applies to q_ℓ, both producers must be investing the minimum $\underline{t}^\lambda$ if they do invest. Furthermore, $x_k = q_\ell$ implies $z_k = z_\ell$. (iv) Assume $q_D^* \in (\underline{t}^\lambda, \bar{t}^\lambda)$. Assume that there exists an $q_k > q_\ell$, with $(q_k, 1_\ell) \in (\underline{t}^\lambda, \bar{t}^\lambda)$. Then, $c(q_k) = \Delta V_k((1,0),(1,0),q_\ell)$ and $c(q_\ell) = \Delta V_\ell((1,0),(1,0),q_k)$, so that $c(q_k) - c(q_\ell) = S(q_k - q_\ell)$, which is impossible if c has a slope different from S.

If $q_D^* \in \{\underline{t}^\lambda, \bar{t}^\lambda\}$, then for any q_ℓ, $c(q_k) \neq \Delta V_k((1,0),(1,0),q_\ell)$ so there cannot be any equilibrium where both producer invest away from the minimum.

1.D 1.B Attention-Seeking Seeds

1.B.2 Seeds' Best Response

The Probability of Being Read by a Follower (as an Seed)

Because we take the perspective of a given seed i, we now define the random variable $S \sim \mathcal{B}(d-1, p_X)$ as the number of times i's followers' neighbors' have shared, **in addition to** i.

$$\begin{aligned}
\mathbb{E}\left(\frac{1}{S+1}\right) &= \sum_{s=0}^{d-1} \frac{1}{s+1}(p_X)^s(1-p_X)^{d-1-s}\frac{(d-1)!}{s!(d-1-s)!} \\
&= \frac{1}{dp_X}\sum_{s=0}^{d-1}(p_X)^{s+1}(1-p_X)^{d-s-1}\frac{d!}{(s+1)!(d-s-1)!} \\
&= \frac{1}{dp_X}\sum_{\tilde{s}=1}^{d}(p_X)^{\tilde{s}}(1-p_X)^{d-\tilde{s}}\frac{d!}{\tilde{s}!(d-\tilde{s})!} \\
&= \frac{1}{dp_X}\left[\sum_{\tilde{s}=0}^{d}(p_X)^{\tilde{s}}(1-p_X)^{d-\tilde{s}}\frac{d!}{\tilde{s}!(d-\tilde{s})!} - (p_X)^0(1-p_X)^{d-0}\frac{d!}{0!d!}\right] \\
&= \frac{1}{dp_X}\left[1-(1-p_X)^d\right]
\end{aligned}$$

where $\tilde{s} = s+1$.

Lemma 7: true news are shared more

By contradiction, suppose that $z_F^*(q) > z_T^*(q)$, so that $p_F > p_T$. For this to be sustainable, we need $\mathbb{E}(\#\text{likes}|s_i = T) \leq \tau \leq \mathbb{E}(\#\text{likes}|s_i = F)$. However, this happens only when:

$$\frac{\gamma}{1-\gamma} < \frac{p_T}{(1-(1-p_T)^d)}\frac{(1-(1-p_F)^d)}{p_F}$$

Indeed, we have:

$$p_\ell(T;x_\ell)\gamma\frac{1-b}{p_T}\big(1-(1-p_T)^d\big)+(1-p_\ell(T;x_\ell))(1-\gamma)\frac{1-b}{p_F}\big(1-(1-p_F)^d\big) < p_\ell(F;x_\ell)\gamma\frac{1}{p_T}\big(1-(1-p_T)^d\big)+(1-p_\ell(F$$

$$[p_\ell(T;x_\ell)-p_\ell(F;x_\ell)]\gamma\frac{1}{p_T}\big(1-(1-p_T)^d\big) < [p_\ell(T;x_\ell)-p_\ell(F;x_\ell)](1-\gamma)\frac{1}{p_F}\big(1-(1-p_F)^d\big)$$

$$\gamma \frac{1}{p_T}\left(1-(1-p_T)^d\right) < (1-\gamma)\frac{1}{p_F}\left(1-(1-p_F)^d\right)$$

Because $\frac{\gamma}{1-\gamma} > 1$, we need $\frac{p_T}{(1-(1-p_T)^d)}\frac{(1-(1-p_F)^d)}{p_F} > 1$. Now $f(x) = \frac{x}{(1-(1-x)^d)}$ is an increasing function; indeed, we have:

$$\text{sign}\left(\frac{\partial f}{\partial x}\Big|_{x\in(0,1)}\right) = \text{sign}\left(\frac{1-(1-x)^d - xd(1-x)^{d-1}}{\left(1-(1-x)^d\right)^2}\right) = \text{sign}\left(1-(1-x)^d - xd(1-x)^{d-1}\right)$$

Now, $g(x) := (1-x)^d - xd(1-x)^{d-1} < 1$ over $x \in [0,1]$. Indeed, $g(0) = 1, g(1) = 0$, and g strictly decreasing in-between, since:

$$\frac{\partial g}{\partial x}\Big|_{x\in[0,1]} = (d-1)(1-x)^{d-2}\Big[(1-x) - \big(1 + x(d-1)\big)\Big] = (d-1)(1-x)^{d-2}[-xd] \leq 0$$

As f is continuous on $[0,1]$ with $f(0) = \frac{1}{d}$ and $f(1) = 1$, f is indeed increasing.

Therefore, we conclude that $p_T > p_F$, a contradiction.

Proposition 7: characterization of the seeds' best-response

(i) Given $\tau \leq \gamma\delta$, if $q \geq \hat{q}(\tau)$, it is easy to verify that always sharing is a best response, i.e. $\mathbb{E}(\#\text{ likes } | T, \mathbf{z_{-i}} = (\mathbf{1},\mathbf{1})) > \mathbb{E}(\#\text{ likes } | F, \mathbf{z_{-i}} = (\mathbf{1},\mathbf{1})) \geq \tau$. Indeed, if every other seeds always share, $p_T = p_F = b$, then the expected number of likes upon receiving a false signal is:

$$[p(F;q)\gamma + (1-p(F,q))(1-\gamma)]\frac{1-b}{b}(1-(1-b)^d)$$

Which is higher than τ iff: $p(F;q) \geq \frac{\frac{\tau}{\delta(b)} - (1-\gamma)}{2\gamma - 1}$. Given that $p(F;q) = \frac{(1-\gamma)q}{(1-\gamma)q+\gamma q}$, this happens iff $q \geq \frac{\gamma}{2\gamma-1}\frac{\tau-(1-\gamma)\delta}{\tau} = \hat{q}(\tau)$. Because $\tau \leq \gamma\delta$, $\hat{q}(\tau) \leq 1$; for $\tau < (1-\gamma)\delta$, $\hat{q}(\tau) < 0$, the condition is always fulfilled.

For proving the converse, recall that $\frac{1-(1-p)^d}{p}$ is decreasing in p. Suppose there exists another $p' < b$ that is sustained in equilibrium. Then, $\mathbb{E}(\#\text{ likes } | F, p' < b) > \mathbb{E}(\#\text{ likes } | F, p = b) \geq \tau$ so that i would have an incentive to deviate towards $p_i = 1$.

(ii) Likewise, given $\tau \geq (1-\gamma)d(1-b)$, if $q \leq \check{q}(\tau)$, then even $d(1-b)[p(T,q)\gamma + (1-p(T,q))(1-\gamma)]$ likes are not enough for anyone to share, so that $(0,0)$ is a the best response to any p given q and τ. Indeed, if every other seeds never share, $p_T = p_F = 0$. Then, the expected number of likes upon receiving a true signal is:

$$[p(T;q)\gamma + (1-p(T,q))(1-\gamma)]d(1-b)$$

Which is lower than τ iff: $p(T;q) \leq \frac{\frac{\tau}{d(1-b)}-(1-\rho)}{2\rho-1}$. Given that $p(T;q) = \frac{\gamma q}{\gamma q+(1-\gamma)q}$, this happens iff $q \leq \frac{1-\gamma}{2\gamma-1}\frac{\tau-(1-\gamma)d(1-b)}{d(1-b)-\tau} = \check{q}(\tau)$. Because $\tau \geq (1-\gamma)d(1-b)$, $\check{q}(\tau) \geq 0$; for $\tau > \gamma d(1-b)$, $\hat{q}(\tau) > 1$, the condition is always fulfilled.

(iii) Again, we can simply verify that, given $\tau \in [\tau_1, \tau_2]$, if $q \in [q_1(\tau), q_2(\tau)]$ and every $-i$ seed is sharing only when they receive a positive signal, $\mathbb{E}(\#\text{ likes }|T) \geq \tau \geq \mathbb{E}(\#\text{ likes }|F)$. Any $z_{-i,F} > 0$ would lower the $\mathbb{E}(\#\text{ likes }|F)$ further away from τ, making i set $z_{i,F} = 0$; any $z_{-i,T} < 1$ would increase the $\mathbb{E}(\#\text{ likes }|T)$ further away from τ, making i set $z_{i,T} = 1$.

Indeed, if every other seeds share only upon receiving a positive signal, $p_T = 1, p_F = 0$. Then, i also only shares upon receiving a positive signal iff:

$$p(T;q)\frac{1-(1-b\gamma)^d}{b}+(1-p(T,q))\frac{1-(1-b(1-\gamma))^d}{b}) > \tau > p(F;q)\frac{1-(1-b\gamma)^d}{b}+(1-p(F,q))\frac{1-(1-b(1-\gamma))^d}{b}$$

Which is possible only if $\tau \in [\tau_1, \tau_2]$. Note that if $\tau \in \{\tau_1, \tau_2\}$, $q_1 = q_2 \in \{0, 1\}$.

Replace $p(T;q)$ and $p(F;q)$ by the adequate expression to find the range q_1, q_2.

Chapter 2

Persuasion in Networks: a Model with Heterogenous Agents

2.1 Introduction

Information is transmitted through social networks constantly. Social media, for instance, are an ever more prevalent source of information for many. The structure of the network is essential to understand how information fares. But, not only does it constrain the *spread* of information; it also shapes the *content* available. How different is communication on a network?

A sender who wants to communicate a payoff-relevant state to multiple receivers might want to communicate privately to different receivers. This would allow him to tailor information differently to different receivers. This would be particularly relevant if receivers differ substantially, for instance in their priors. Yet, receivers can communicate among themselves. Hence, the information delivered in the network can spread to unintended receivers and lead to suboptimal outcomes. Can the sender exploit a communication network in any way? When should she prefer it to public communication? What role do the differences between receivers and the network structure play?

To answer these questions, I propose a model of persuasion in networks with multiple heterogenous receivers. As is standard with persuasion, the sender is assumed to commit to a signal structure in order to induce as many receivers as possible to take some given action. Receivers are Bayesian and want to take the right action for the right state.

Differently from the classical persuasion problem, receivers can hold different priors, which is common knowledge. Priors are such that they would induce different actions. The sender thus needs to persuade one group of receivers without dissuading the other one. Furthermore, receivers are arranged on a communication network. Each receiver does not only observe their own signal realization, but also that of their neighbors.

The sender knows the distribution of connectivity within each group and between groups. She can design the signal structure to allow for different probability distribution for each group; she can correlate signals between groups too. As she does not know the exact network structure, she cannot target nodes individually; she can only design the information conditional on receivers' priors. She wants to maximize the persuasion value, that is, the probability for a receiver taken at random to take her preferred action.

This framework corresponds to many contexts. For instance, a firm might use social media to credibly advertise a new technology. Some users are enthusiastic enough to adopt the technology without further information; others might require proof in order to switch to the firm. I consider the following motivating example: a pharmaceutical company claims that one of their previously approved products is an effective preventive treatment against COVID-19. The company encourages individuals to skip the vaccine and to purchase their treatment instead. A spokesperson can decide to go on television to announce the launch of a clinical trial; and later on present the evidence. Alternatively, the company can reach the same people through Facebook. One or several community managers can announce clinical trials to different users, and follow up with results. The community managers cannot withhold or falsify evidence. However, the company can decide how to hold these clinical trials. In particular, they can decide whether to advertise the same trials for all Facebook users; whether to hire the same scientific team to hold the trials; and how precisely to design each individual trials. Users frequent different groups: some are active on anti-vax groups, and would buy the treatment against COVID-19 without further evidence; the other users frequent pro-vax groups, and need to be convinced about the effectiveness of the treatment.

What would the company do? First, the community managers can do as good as the spokesperson by advertising the same trial to every Facebook user: the persuasion value of a public signal is reproducible in any network. Furthermore, if both types of users have more connection within themselves than between them, the company has no reason to advertise clinical trials to anti-vaxers, as these users would buy the treatment in any case. However, anti-vaxers might have pro-vaxers friends that will display trials result on their wall. How can the company utilize the difference in pattern connections in order to persuade pro-vaxers without dissuading anti-vaxers?

I focus on strategies that ensure for the state favoring the sender to be signaled when it realizes. I propose two novel strategies that exploit the network and show that there does not exist any other. Both strategies rely on a similar mechanism. The sender splits information among different receivers; in particular, she designs the signal structure so that each bit of information is not very informative. Therefore, among agents who need to be persuaded, only those who observe enough bits of information can be persuaded; but as each bit of information is less informative, nodes who should not be dissuaded are less likely to actually detect the adverse state. This relies on the assumption that the nodes who should not be dissuaded observe less signals from the other group than the other group does within itself.

For instance, say that the pharmaceutical company focuses on ensuring that users buy the treatment when the treatment is actually effective. Then, the proposed strategies would be equivalent to the company designing a lot of small scale, not well-controlled trials. The result of any of these trials taken individually would not be sufficient to convince pro-vaxers; but the accumulation of congruent signals would eventually persuade them. As each trial is less likely to give adverse results, anti-vaxers are less likely to come across a result disproving the effectiveness of the treatment, as long as pro-vaxers are friends with more pro-vaxers than anti-vaxers.

The two strategies rely on the same mechanism but play on different parameters of the distribution. While, with what I call a *multiple-message* (MM) strategy, each bit of information is delivered by the actual signal realization; the *network-specific* (NS) strategy exploits correlation of signals. It is then the similarity of signals that is informative about the state. With a MM strategy, nodes must only observe successes in order to take the sender's preferred action; with a NS strategy, receivers must observe all identical signals, i.e. that all or none are successful.

How well do these strategies do? With a public signal, as everybody observes the result from the same trial, they all take the same action; in particular, they buy the treatment if the treatment is effective; if it is not, the results are still sometimes encouraging and they buy it. With a strategy exploiting the network, some pro-vaxers might not be connected enough to ever be receive enough information to buy the treatment. The other pro-vaxers, those who are connected enough, buy the treatment less often than with a public signal; however most anti-vaxers, those who are poorly-connected to pro-vaxers, buy the treatment more often than with a public signal.

Now, if anti-vaxers were as skeptical on the treatment as pro-vaxers, introducing this heterogeneity of informativeness would be suboptimal: the company should go on television! However, when the company does need to convince these two groups differently, strategies exploiting the network can be very useful. When the treatment happens not to be effective, and if the average degree of users susceptible to buy the treatment is small enough compared to the minimum connectivity required to persuade pro-vaxers, the strategies exploiting the network are doing better than the public signal. Furthermore, the NS strategy is better at exploiting degree differences than the MM strategy, so that under the same condition on average degree, the company is better off by relying on correlation rather than on frequency of signal realizations.

I further study these strategies in simplified networks. Take a regular network, that is, pro-vaxers all have the same number of pro-vaxers friends; and anti-vaxers all have the same number of pro-vaxers friends. Then, all pro-vaxers are connected well-enough to be persuaded as often as with a public signal; anti-vaxers, however, are less likely to be dissuaded as they observe the outcome of only a few, poorly informative trials. Therefore, the strategies exploiting a regular network are always better than public communication.

This simplified case underlines how connectivity is not harmful *per se*. The company is happy to divide informativeness into bits that is spread within a well connected group of pro-vaxers; it allows the true effectiveness of the treatment to be hidden to anti-vaxers more often, while still being well disclosed to pro-vaxers. The type of connectivity detrimental to the company is the between group connectivity: the company is better off if anti-vaxers do not talk to too many pro-vaxers. This result has an intuitive link with homophily and segregation on social networks. It underlines how those social habits can be detrimental to the provision of information.

Furthermore, it also emphasize the importance of pro-vaxers' skepticism. The harder are pro-vaxers to convince, the more the strategies exploiting the network outperform public signals. Indeed, when pro-vaxers are so skeptical, a public signal would need to be extremely informative; hence, it is very likely for the anti-vaxers to be dissuaded from purchasing the treatment. It is thus all the more useful to design a strategy that exploits the network and allows to dissuade anti-vaxers less often.

These two intuitions are expected to carry on to general networks. Future versions of this work should contain more precise results in this respect. Furthermore, further research will explore another category of strategies: instead of ensuring for the treatment to be bought when it is effective, the sender could focus on never letting some receivers detect that it does not work. Strategies exploiting the networks can accommodate this objective. Therefore, the same comparison to public communication should be carried out with this new category of strategy. This analysis should eventually allow to study better the role of polarization and its interaction with the network.

2.1.1 Related literature

I mainly contribute to the information design literature, in particular the strands interested in Bayesian persuasion and the endogenous provision of information in networks. To the best of my knowledge, this paper is the first to combine multiple receivers with heterogenous priors and a communication network between receivers.

The seminal work Kamenica and Gentzkow [2011] exposes a persuasion problem with one sender and one receiver. The particularity of this approach is that the sender can commit to a signal distribution about a payoff-relevant state. The authors characterize the sender-optimal signal structures: the sender optimally designs a distribution which is minimally informative to still induce receivers to take the sender's preferred action upon receiving the relevant signal realization. Taneva [2019] generalizes this result by proposing a systematic methodological approach to finding the optimal information structure for the sender in static finite environments. While, in Kamenica and Gentzkow [2011], the emphasis is put on the case of a unique receiver, the authors underline how their analysis can directly be applied to multiple receivers, but only if the latter have homogenous priors and the influence of their actions on the sender's payoff is separable.

The relaxation of either of these two conditions has been explored in the literature. About the latter, a few papers have considered the importance of persuasion in a voting context. For instance, Wang [2013] asks whether private or public communication is preferable for a sender facing strategic voters with heterogenous preferences. Alonso and Câmara [2016] also consider receivers with homogenous priors and heterogenous preferences in a voting context. Kerman et al. [2020] wonder about correlation of private signals between homogenous strategic voters.

The role of heterogenous priors among receivers has seldom been explored. Some authors have studied cases in which priors are unknown to the sender, which could be read as an interpretation of heterogenous priors. Kosterina [2018] studies the problem of a sender who wants to optimize the signal distribution when the worst-case-scenario priors realize. Castiglioni et al. [2021] study an online persuasion problem, in which the sender repeatedly faces receivers with homogenous but unknown priors. Finally, Guo and Shmaya [2019] consider a unique receiver with unknown priors and proposes a nested-interval structure as optimal.

This is reminiscent of Innocenti [2021] who explicitly assumes multiple receivers with heterogenous but known priors. Receivers can hold two different priors, each inducing different actions. He shows how, with a public signal, the tradeoff between persuading some receivers and avoiding to dissuade others lead to two type of strategies, which he coins hard and soft news. These echo, respectively, fully pooling and separating mechanisms in Guo and Shmaya [2019]. I adopt a similar environment with receivers who hold either of two priors inducing different actions. I also borrow Innocenti's terminology. However, rather than assuming a public signal, I allow for partially private signals, by introducing a network structure and the possibility for the sender to target different groups with different signal structures, possibly correlated.

A persuasion problem involving receivers communicating over signals in a network is the object of Kerman and Tenev [2021]'s analysis. However, the authors assume homogenous receivers. Furthermore, in their model, the network structure is known to the sender who can thus target individual nodes. Finally, their analysis takes place in a voting context; by contrast I consider a payoff for the sender that is linear in the number of receivers taking the preferred action. Reminiscent of some of the insights from my model, Kerman and Tenev [2021] find that a higher network density does not necessarily translate into a decrease in the sender's gain from persuasion.

More generally, I contribute to the information design literature. Bergemann and Morris [2019] provides an overview of the literature by unifying the strand of the literature interested in Bayesian persuasion with the rest of the literature. The authors underline how the presence of multiple receivers naturally raises the question of the sender's preference for public or private information. While they consider the possibility for the sender to have preferences or strategical interests in correlated actions, they ignore the possibility for agents to communicate among themselves. My paper contributes to endogenize how privately signals are communicated, subject to the constraints of the communication network.

Finally, a few recent papers have introduced networks in information design. Egorov and Sonin [2020] study the receivers choice to subscribe to a signal structure designed by a sender. Candogan [2019] considers the problem of a sender who wants to persuade receivers arranged on a network by communicating publicly. The receivers payoffs are subject to strategic complementarities. Finally, Galperti and Perego [2019] provide a general framework to study the maximal impact that information revealed to some seeds in a network can have. Evocative of the strategies I propose to exploit the network in the specific context considered, Galperti and Perego underline how messages can be coded to be understood only be nodes who receive all individual signals composing the message.

The remainder of the paper is organized as follows. The model is presented in Section 2. In Section 3, the private and public information cases are explained and their relation to networks are underlined. Section 4 proposes novel strategies exploiting the network, studies their performance and explores future developments. Section 5 concludes.

2.2 Model

2.2.1 Environment

Consider a persuasion problem with one sender (she) and N receivers (they/he). They communicate over a payoff-relevant binary state $\omega \in \{0,1\}$. I allow for agents to hold different priors over the state. The sender's prior is denoted $\mu := \Pr(\omega = 1)$. The receivers are partitioned in two groups A and B. Let $a := A/N$ and $b := B/N$ so that $a + b = 1$.[1] The groups are characterized by their members' prior beliefs, with $\mu_A > \mu_B$. The group to which a node belongs is common knowledge among all players.

Receivers are arranged on an undirected network.[2] Any node i observes his own signal realization $s_i \in \{0,1\}$. In addition, he observes the signal realization of all of his neighbors. Let $\mathcal{N}_i$ be i's neighborhood including himself; and $\mathcal{N}_i^I := \mathcal{N}_i \cap I$ for $I \in \{A, B\}$ be i's I-neighborhood. Finally, i's I-degree, denoted d_i^I is defined as $|\mathcal{N}_i^I|$.

Both the network structure and the communication patterns are exogenous. Furthermore, the network's topology is unknown to the sender; only the degree distribution is accessible to her.[3] Denote $\delta_I(d_i^I)$ the portion of node in group $I \in \{A, B\}$ who have d_i neighbors belonging to the same group; and $\delta_I(d_i^{-I})$ the portion of node in I who have d_i neighbors belonging to the other group.

Receivers update their beliefs about the state according to Bayes' rule. Let $\beta_I(\mathcal{s}_i)$ be the belief of agent i from group I after observing $\mathcal{s}_i$, where $\mathcal{s}_i$ is the sum of relevant signal realizations, possibly accounting for group belonging.[4] Given the binary nature of the state, $\mathcal{s}_i$ contains all information accessible to i. The index i is kept in $\mathcal{s}_i$ to account for the i's neighborhood's size, that is, the number of signals observed.

Finally, as the sender can positively correlate signals, the probability for the joint realization of signals must be defined. I assume that the number of success for dependent and simultaneous Bernoulli trials is characterized by the following distribution:

$$\tilde{p}_k^n = \Pr(X = k | r, n, \alpha) = (1-\alpha)\binom{n}{k} r^k (1-r)^{n-k} + \alpha\big[(1-r)\mathbb{1}_{k=0} + r\mathbb{1}_{k=n}\big]$$

where X is the number of successes in a sequence of n dependent trials, r is the success probability of any trial and α is the pair-wise correlation of any two trials. Appendix A details how this distribution is consistent with the parameters r and α. For the remainder

[1] To ease notation, A and B refer to both the sets of nodes and its cardinality.

[2] The analysis in a directed network would be identical. Instead of degrees, the sender would focus on in-degrees. The rest of the analysis would not be affected.

[3] This can be interpreted as a random network, or as limited information on the side of the sender.

[4] Relevant signals are those which are informative. For now, we can define $\mathcal{s}_i := (\mathcal{s}_i^A, \mathcal{s}_i^B)$ where $\mathcal{s}_i^I := \sum_{j \in \mathcal{N}_i^I} s_j$. The definition is provided in Section 2.3.2.

of this work, let p_k^n be the probability of k successes among n independent trials, that is, the probability prescribed by the standard Binomial variable.

2.2.2 Objectives, Timing and Equilibrium Concept

All receivers have the same preferences. Their payoff is such that there is a unique preferred action for any realized state. The optimal action given a state is denoted by the state. For instance, if receivers were to know that the state is 0, they would take action 0. However, because they cannot observe the state, their action is conditional on their belief $\beta(s_i)$. Let t be the belief that makes them indifferent between action 0 and 1. They are assumed to take action 1 if and only if they believe with probability greater or equal to t that the state is 1, i.e. iff $\beta(s_i) \geq t$. Furthermore, t is such that the receivers' different priors induce different actions, i.e. $\mu_A > t > \mu_B$. I denote $\alpha_A := \frac{\mu_A(1-t)}{(1-\mu_A)t}$ and $\alpha_B := \frac{\mu_B(1-t)}{(1-\mu_B)t}$, so that $\alpha_A > 1 > \alpha_B$. α_I indicates how much information is needed to make them change action.

The sender's objective is to induce receivers to take her preferred action, regardless of the state. Without loss of generality, I assume her preferred action is 1. I refer to 1 as the *favorable* action or state. The sender only cares about the expected number of receivers taking the favorable action.[5] I denote V the *value of persuasion*, i.e. the sender's expected payoff from the persuasion problem; it is defined as the probability that a receiver takes action 1. In order to persuade receivers, she commits to a signal structure π. Conditional on the state, the distribution specifies a probability of success for each group I, $\Pr(s_I = 1|\omega)$, as well as the signal correlation within groups $\mathrm{Corr}(s_I, s_I|\omega) \geq 0$ and between groups $\mathrm{Corr}(s_A, s_B|\omega) \geq 0$. The notation is as follows: $p_I := \Pr(s_I = 1|\omega = 0)$, $q_I := \Pr(s_I = 1|\omega = 1)$, $\rho_{IJ} := \mathrm{Corr}(s_I, s_J|\omega = 0)$ and $\varphi_{IJ} := \mathrm{Corr}(s_I, s_J|\omega = 1)$.

The game is played sequentially. The timing is the following:

1. The sender commits to a signal structure π.
2. All uncertainty realizes, in particular ω and $s_i \forall i \in N$.
   Receivers observe π and s. They update their beliefs about ω using Bayes rule.
   Given their updated beliefs $\beta(s_i)$, they take action $a_i \in \{0, 1\}$.

Note that receivers observe π fully, including correlations between signals. I look at Subgame Perfect Equilibria (SPE), using backward induction.

[5]Implicitly, this means that she is risk neutral, as is standard with Bayesian persuasion. However, in the context of multiple receivers, this assumption is stronger. Each receivers' action needs to enter the sender's payoff linearly. Therefore, I abstract from global social effects. For instance, I ignore cases in which the sender might benefit increasingly or decreasingly from each marginal adopter; or in which she needs a quota of receivers to take the action, such as voting context.

2.3 Benchmarks: Private and Public Information

2.3.1 Unconnected World

For now, let us abstract from the existence of a network. A sender facing multiple receivers might be able to communicate with each of them in isolation or, alternatively, might need to commit to signals that are publicly observed. Those are referred to as the private information and public information cases respectively.

In the private information case, the sender can optimize over each receiver separately. Indeed, because each signal realization is only observed by the given receiver, the sender does not need to worry about information spreading to unintended receivers. Furthermore, because her payoff is separable in each receiver's decision, she can optimize the signal structure for each receiver separately. Her strategy therefore corresponds to the one prescribed in a standard persuasion problem, i.e. in Kamenica and Gentzkow [2011].

The sender optimally designs a signal structure which is minimally informative but informative enough to persuade receivers upon receiving some given signal realization. It translates into $q_I = 1$ and $p_I = \max\{\alpha_I, 1\}$. I refer to it as the *standard* strategy. Any receiver belonging to group A would take action 1 without any information; so $p_A = 1 = q_A$.[6] For any receiver belonging to B, the sender sets $q_B = 1$ and $p_B = \alpha_B$.

As for correlations, since receivers only observe their own signal, any correlation structure would be optimal as long as it is consistent with the signal described above, i.e. such that $\rho_{AB} = 0$. In particular, $\rho_{II} = \varphi_{II} = 1$ would correspond to one common signal realization for all members of each group I, and would lead to the same outcome, in expectation, as letting all agents get a different signal realizations $\rho_{II} = \varphi_{II} = 0$. The persuasion value is:

$$V_{PI} = \mu + (1 - \mu)[a + \alpha_B b]$$

When signal is public, there exists a tradeoff between persuading agents in B and avoiding members of A to be dissuaded. Innocenti [2021] studies the case of a unique public signal and shows that there are two potential optimal strategies: persuade both groups as often as possible with the favorable signal, but sometimes dissuading group A – a *hard news* strategy –; or never dissuade group A at the price of persuading group B less often – a *soft news* strategy. The former implies $q = 1, p = \alpha_B$;[7] the latter prescribes

[6] Any signal π with $p_A = q_A$ would be uninformative, so the strategy specified in the main text is not unique.

[7] Because the signal is public, $q_A = q_B$ and $p_A = p_B$. I omit the index in this case.

$q = \frac{\alpha_A - 1}{\alpha_A - \alpha_B}, p = \alpha_B q$. The persuasion value are respectively:

$$V_{HN} = \mu + (1-\mu)\alpha_B \quad \text{and} \quad V_{SN} = a + b\Big[\mu q + (1-\mu)\alpha_B q\Big]$$

The determination of the optimal strategy between these two candidates depends on: the polarization of prior beliefs, defined as $\alpha_A - \alpha_B$; the probability to be in an unfavorable state according to the sender, μ; and the groups' relative sizes, a, b.

Because all signal realizations are observable, the sender can replicate the public information case in an unconnected world by sending the same signal realization to every receiver – $\rho_{IJ} = \varphi_{IJ} = 1$; or by sending any amount of informative signals, as long as the set of signals inducing agents to take the favorable action realizes with the probabilities specified above.

2.3.2 Implementation in a Network

These two benchmarks can relate to a connected world. An empty network – i.e. $\delta_A(d^A = 1) = \delta_B(d^B = 1) = 1$ – corresponds to the private information case: the sender can persuade each group in isolation. Actually, any network in which there are no connections between group A and group B – i.e. $\delta_A(d^B = 0) = 1$ – can deliver the same value of persuasion. However, the correlation structure now matters. Setting $\rho_{II} = \varphi_{II} = 1$ is optimal. [8]

On the other hand, a complete network – i.e. $\delta_I(d^A = A) = \delta_I(d^B = B) = 1$ – corresponds to the public information case: every agent can observe the signal realization of every other agent. However, the public information case is reproducible in any network.

Lemma 8. *The persuasion value of the public information is reproducible in any network by setting* $\rho_{IJ} = \varphi_{IJ} = 1$.

Proof. If $\rho_{IJ} = \varphi_{IJ} = 1$, then $\forall i, j \in N, s_i = s_j$. Because receivers are fully Bayesian and observe correlations in π, their effective information set is composed of a unique signal realization, common to every receiver. This is equivalent to the sender designing a unique signal observed by everyone. □

In other words, the presence of a network can never hurt the sender. If anything, she might prefer sending a common message rather than exploiting the network. Therefore, if the network is exploited, it is because it improves upon the persuasion value with respect

[8]If the degree distribution within B was homogenous, the correlation structure would be irrelevant as long as the set of signal realizations inducing nodes in B to take the favorable action occurs with the optimal probability. If the degree distribution within B is not homogenous, however, not perfectly correlated signals introduces a suboptimal heterogeneity in the effective informativeness of the signal structure. This intuition will be made more formal in Lemma 11

to a public signal. On the other hand, a private message is the best the sender can hope for. Therefore, the value persuasion in a network will never exceed that of private communication.

Remark 9. *The value of persuasion in a network with heterogenous priors is bounded between the value of persuasion of the public information case and that of the private information case.*

Intuitively, the sender would like to provide information on the state to B without it spreading to A. Therefore, it can never be optimal to provide information to A, even if the sender relies on this to inform B. Indeed, in the latter case, providing that information to B would be (at least weakly) preferable.

Lemma 9. *The sender designs π so that $(s_i)_{i \in I}$ is uninformative for some group $I \in \{A, B\}$, that is $p_I = q_I$, $\rho_{II} = \varphi_{II}$ and $\rho_{IJ} = \varphi_{IJ} = 0$.*

Proof. Let us assume that π is designed to deliver informative signals to B. As members of group A do not need to be persuaded, any informativeness spent on them would be wasted. The sender does not need to exploit signals to group A in order to inform members of group B, as this can be achieved through targeting nodes in B directly. Therefore, provide group A with informativeness would reduce the probability for agents belonging to A to take the favorable action without allowing to increase the number of nodes in B taking the actions beyond what is achievable through the informativeness to B. The same argument applies if informative signals are delivered to A, in order to convince B. □

Remark 10. *Because the sender designs uninformative signals for some group I, the distribution parameters related to s_I are irrelevant as long as they are consistent with Lemma 9. The group to which to provide informativeness is the group with higher connectivity with B members than with A members.*[9] *For the remainder of the analysis, s_A will be assumed to be uninformative. To ease notation, denote $p_B = p$, $q_B = q$, $d_i^B = d_i$ $\rho_{BB} = \rho$ and $\varphi_{BB} = \varphi$. Note that all results would carry on if only A was delivered informative news, reinterpreting in particular d_i as nodes' A-degree. Furthermore, without loss of generality I focus on cases with $p \geq 1/2$.*[10]

This section closes with a few formal concepts that should allow for more efficient analysis.

Definition 1. The set of *messages* a node $i \in I$ can receive is $\mathcal{M}_i := \{s_i \in \mathbb{N} : s_i = \sum_{j \in \mathcal{N}_i^B} s_j\}$.

[9] See Theorem 4.

[10] This relates to the general irrelevance of the content of the signal; only the distribution of signals matter to interpret them. To simplify exposition, I do not expose strategies that mirror the ones discussed with inverted signal realizations.

Definition 1a. The set of *favorable messages* for node $i \in I$ is $\mathcal{FM}_i := \{s_i \in \mathcal{M}_i : \beta_I(s_i) \geq t\}$.

Definition 1b. The set of *persuading messages* for node $i \in I$ is $\mathcal{PM}_i := \{s_i \in \mathcal{M}_i : \beta_I(s_i) = t\}$.

A message is simply a realization of signals in i's B-neighborhood, that is aggregated but contains all relevant information. A favorable message corresponds to a realization of signals in i's neighborhood that would induce i to take the favorable action. The set of *unfavorable* messages is similarly defined as the set of signal realization that would induce i to take the unfavorable action, i.e. $\mathcal{M}_i - \mathcal{FM}_i$. The set of persuading messages corresponds to the messages that would make node i indifferent between action 0 and 1. This set can be empty.

Definition 2a. The *targeted nodes* is the subset of nodes for whom the set of persuading messages is non-empty, i.e. $\{i \in N : \mathcal{PM}_i \neq \varnothing\}$.

Definition 2b. The *susceptible nodes* is the subset of nodes for whom the set of favorable messages is non-empty, i.e. $\{i \in N : \mathcal{FM}_i \neq \varnothing\}$.

The targeted nodes are the nodes that the sender constraints herself to make indifferent between actions. In standard Bayesian persuasion, the sender uses minimal informativeness to persuade nodes as often as possible. In that case, such persuaded nodes are targeted, because they are the ones made *just* indifferent between actions. If nodes differ in terms of priors, persuading some nodes as often as possible might result in dissuading the others too often. Therefore, the sender might want to avoid ever dissuading some other nodes. Then, such non-dissuaded nodes are also targeted: sender still makes them indifferent; she wants to send a favorable message to the former nodes as often as possible given that it does not cause the latter nodes to ever be dissuaded. In the networks considered, in addition to their priors, nodes might differ in their B-degree; a same realization of signals in the network could then lead to different posteriors. Therefore, the sender might be able to only target a subset of nodes, defined not only by their priors but also by their degree.

The susceptible nodes are simply those susceptible to receive favorable message. In other words, for each of them, there exists a signal realization that would induce them to take the favorable action. In standard Bayesian persuasion, whether or not agents hold heterogenous priors, all nodes are susceptible under an optimal information structure. However, in a network, posteriors might depend on the degree in addition to the group belonging; this means that B nodes with some certain degrees might never reach a posterior high enough to be persuaded, irrespective of the realization of signals in the network, such nodes would not be susceptible. They would be referred to as *non-susceptible*.

2.4 The Role of the Network

2.4.1 Strategies Exploiting the Network

Definition 3. A hard news strategy is a signal structure π such that for any targeted node i, $\Pr(s_i \in \mathcal{PM}_i | \omega = 1) = 1$

Definition 3a. The *unconnected* (U) hard news strategy is a signal structure π that is independent of any of the targeted nodes' degree and in which all nodes $i \in B$ are targeted.

Definition 3b. A *multiple-message* (MM) hard news strategy is a signal structure π that is dependent of the targeted nodes' degree and in which for any targeted node $i \in B$, $\mathcal{PM}_i = \{d_i\}$.

Definition 3c. A *network-specific* (NS) hard news strategy is a signal structure π that is dependent of the targeted nodes' degree and in which for any targeted node $i \in B$, $\mathcal{PM}_i = \{0, d_i\}$.

As introduced by Innocenti [2021], a hard news strategy convinces targeted nodes in B as often as possible. The unconnected hard news strategy generally replicates the persuasion value of the public information case.[11]

The multiple-message strategy relies on a message composed of multiple successful signals in order to convince targeted nodes. Hence, such strategy is irrelevant if agents do not observe multiple signals. The sender designs π such that each individual signal realization is less informative than under an unconnected hard news strategy. Therefore, only the accumulation of successful signals can persuade agents in B; however, it is also less likely for nodes in A to observe an unfavorable message as the probability for any signal realization to be a success is greater under this strategy.

The network-specific strategy relies on the information contained in the similarity of signal realizations rather than on individual realizations. Hence, it cannot exist outside a network. The sender informs receivers about the state through correlation rather than a probability of success. Therefore, individual signal realizations are not informative *per se*; it is a consensus among messages that should inform receivers on the state. In particular, the sender renders the realization of identical signals more likely in the favorable state than in the unfavorable one, so that agents are persuaded by observing identical signals in their neighborhood. As the MM strategy, the NS strategy capitalizes on the B-degree difference among nodes of different groups: it is more likely for nodes in A than in B

[11]It does in networks such that $\delta_A(d_i = 0) = 0$. The persuasion value of this strategy is provided in Proposition 8.

to observe identical but uncorrelated signal realizations in their B-neighborhood because their B-neighborhood is smaller.

I consider multiple-messages and network-specific strategies that target members of B with a given B-degree. The B-degree of the targeted nodes is denoted $\hat{d}$. Note that these strategies could be declined to never dissuade targeted agents in A; this would correspond to a soft-news strategy. These strategies are omitted in the current version of this work.

However, a natural question arises: are there other hard news strategy that should be considered? I show that there does not exist any such strategies.

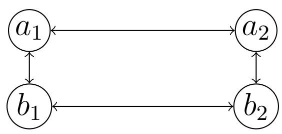
Theorem 3. *If a signal structure is a hard news strategy, then it is either an unconnected strategy, or a multiple-message strategy, or a network-specific strategy.*

Proof. See Appendix A. □

The proof relies on a very intuitive argument: there are only limited tools to insure that all targeted nodes receive a persuading message in the favorable state. In particular, with a hard news strategy, nodes who observe an unfavorable message perfectly infer that the unfavorable state of the world realized. Therefore, any a persuading message different from 0 or d_i would require for the set of all messages to be persuading; but this is impossible, as the signal structure must be informative in order to persuade nodes.

Example 2 below illustrates these strategies in a simple network and shows how they can outperform public information.

Example 2. Assume $\mu = t = 0.5, \mu_A = 2/3, \mu_B = 1/3$ so that $\alpha_A = 2, \alpha_B = 1/2$. Furthermore, consider the network depicted below.



The hard news strategy with $\rho = \varphi = 1$ would lead to a value of persuasion $V_{HN} = \mu + (1-\mu)\alpha_B = 3/4$; the soft news strategy with $\rho = \varphi = 1$ would mean $q = 2/3$ and $p = 1/3$ so that $V_{SN} = 1/2 + 1/2[\mu \cdot q + (1-\mu) \cdot p] = 3/4$. If the sender was able to communicate privately within each group, the value of persuasion would become $V_{PI} = 1/2 + 1/2[\mu + (1-\mu) \cdot \alpha_B] = 7/8$.

Now, the two alternative strategies MM and NS would entail the following specification:

1. For MM, the sender does not need to use correlations. Hence she sets $\rho = \varphi = 0$. One can study the sender's strategy using the following table:

	$\omega = 0$	$\omega = 1$
$s_B = 0$	$(1-p)^2$	$(1-q)^2$
$s_B = 1$	$2p(1-p)$	$2q(1-q)$
$s_B = 2$	p^2	q^2

Because the sender is designing a hard news strategy, $q = 1$. Agents in B are persuaded iff they observe $s_{b_1} = s_{b_2} = 1$. Therefore, the sender would like to maximize the probability of this realization in the unfavorable state. She would thus set: $p^2 = \alpha_B$. It means that $s_B < 2$ is an unfavorable message for all members of B. However, if $s_B = 1$ was to realize, only half of A's member would be dissuaded because only half would observe $s = 0$. This is an advantage compared to the unconnected hard news strategy, that dissuades all nodes in A upon the realization of an unfavorable message. In other words, because $p_{MM} = \sqrt{2}^{-1} > 2^{-1} = p_{HN}$, it is less likely for nodes in A to observe an unfavorable message signal realizations of their neighbors in B. Hence the persuasion value under MM is: $V_{MM} = \mu + (1-\mu)p_B^2 + (1-\mu)\frac{a}{2}2p(1-p) = 3/4 + 1/2 \cdot 1/4 \cdot 2\frac{\sqrt{2}-1}{2} \approx 13/16$

2. In this case, the sender mainly uses correlations. Let her set $\rho = 0$ and $\varphi = 1$. The sender's strategy is now represented by :

	$\omega = 0$	$\omega = 1$
s_B=0	$(1-p)^2$	$1-q$
s_B=1	$2p(1-p)$	0
s_B=2	p^2	q

The set of persuading messages contains both $s_B = 0$ and $s_B = 2$. For this, she needs $(1-q)\alpha \geq (1-p)^2$ and $q\alpha \geq p^2$ so $p^2 + (1-p)^2 \leq \alpha$. Because V is increasing in the probability that $s_B = 0$ or $s_B = 2$ realizes, the constraint is binding. The sender sets $p_B = 1/2 = q_B$. This replicates the persuasion value of private information. Indeed, a_1 and a_2 would never be dissuaded, as observing one signal realization of B is fully uninformative. Hence, members of group A are always persuaded. On the other hand, b_1 and b_2 are persuaded upon seeing $s_{b_1} = s_{b_2}$. In the unfavorable state, this persuasion occurs with probability $Pr(s_1 = s_2|\omega = 0) = p^2 + (1-p)^2 = 1/2$. Therefore, the persuasion value under NS is: $V_{NS} = \mu + (1-\mu)(a + b\alpha_B) = 7/8$.

Let us generalize the insights from Example 2 to any network. Recall that $\hat{d}$ is the B-degree of the targeted nodes. $\hat{d}$ corresponds to the minimum number of signal realizations agents in B need to observe in order to potentially be persuaded, with either MM or NS. For MM, these realizations all need to be a success. For NS, these realizations all need to be the same.

Lemma 10. *Consider any MM or NS strategy.*

(i) $\forall i \in A, \mathcal{FM}_i \neq \varnothing$ *and* $\forall i \in B, d_i \geq \hat{d} \Leftrightarrow \mathcal{FM}_i \neq \varnothing$

(ii) $\forall i \in N : \mathcal{FM}_i \neq \varnothing,\ \Pr(s_i \in \mathcal{FM}_i | \omega = 1) = 1$

(iii) $\forall i \in N : \mathcal{FM}_i \neq \varnothing,\ \Pr(s_i \in \mathcal{FM}_i)$ *is decreasing in* d_i*.*

Proof. See Appendix A. □

The first result characterizes the susceptible nodes in strategies exploiting the network. Because the multiple-message and network-specific strategies exploit the differences of connectivity within B and between A and B, they are meant to convey different level of informativeness to nodes with different B-degree. However, this comes at the price of introducing heterogeneity within groups of otherwise identical nodes. In particular, if members of B are too scarcely-connected in B, they do not observe enough signals and thus perceive messages to be too little informative for them to ever be induced to take the favorable action.

Furthermore, the definition of hard news strategy insures that all susceptible nodes are induced to take the favorable action in the favorable state of the world. The only realization of signals permitted by a hard news strategy are easily characterized: because targeted nodes must observe a persuading message with probability 1, either all signals to B are successes, or, all signals to B are identical. In either case, this insures that all susceptible nodes receive a favorable message.

Finally, the probability that a susceptible node receives a favorable signal is decreasing in the node's B-degree. Indeed, the more signals a node observes, the more informative his message. Therefore, if nodes are *too* connected to members of B, they are induced to take the favorable action less often than with a public signal. In such a case, they are hit by messages that are too informative. This applies to both groups.

Proposition 8 below characterizes the persuasion value of such strategies in a general network.

Proposition 8.

(i) The unconnected strategy targeting all agents in B *is such that* $\varphi = \rho = q = 1$ *and* $p = \alpha_B$*. The persuasion value associated with such strategy* $V_U = \mu + (1-\mu)\Big[\alpha_B + a\ \delta_A(0)(1-\alpha_B)\Big]$*.*

(ii) The multiple-message strategy targeting agents in B *whose* B*-degree is* $\hat{d}$ *is such that* $q = 1$ *and* $\rho p + (1-\rho)p^{\hat{d}} = \alpha_B$*. The persuasion value associated,* V_{MM}*, can be written*

as:

$$\mu\Bigg[a + b\sum_{d_i=\hat{d}}^{B}\delta_B(d_i)\Bigg] + (1-\mu)\Bigg[a\delta_A(0) + \sum_{d_i=1}^{B}\Big[a\delta_A(d_i) + \mathbb{1}_{d_i\geq\hat{d}}\, b\delta_B(d_i)\Big]\Big[\rho\, p + (1-\rho)p^{d_i}\Big]\Bigg]$$

(iii) The network-specific strategy targeting agents in B whose B-degree is $\hat{d}$ is such that $\varphi = 1$, $\rho + (1-\rho)\big(p^{\hat{d}} + (1-p)^{\hat{d}}\big) = \alpha_B$, and $q = \alpha_B^{-1}\big(\rho p + (1-\rho)p^{\hat{d}}\big)$. The persuasion value associated, V_{NS}, can be written as:

$$\mu\Bigg[a+b\sum_{d_i=\hat{d}}^{B}\delta_B(d_i)\Bigg]+(1-\mu)\Bigg[a\delta_A(0)+\sum_{d_i=1}^{B}\Big[a\delta_A(d_i)+\mathbb{1}_{(d_i\geq\hat{d})}\, b\delta_B(d_i)\Big]\Big[\rho+(1-\rho)\big(p^{d_i}+(1-p)^{d_i}\big)\Big]\Bigg]$$

This strategy is implementable if and only if $\alpha_B > 2^{-(\hat{d}-1)}$.

Proof. See Appendix A □

While the technical details of the proofs are reported in the appendix, the expressions for V_{MM} and V_{NS} are intuitive. When the favorable state realizes, $s_i = 1$ for all $i \in B$ if the MM strategy is adopted; and $s_i = s_j$ for all $i, j \in B$ if the NS strategy is adopted. This cannot dissuade any agent in A. Furthermore, the realization of signals persuades all agents with $d_i \geq \hat{d}$. Indeed, members of group B require at least $\hat{d}$ sucesses or identical signals to be persuaded. When the unfavorable state realizes, nodes of either group are taking the unfavorable action as soon as they observe: an unsuccessful signal realization for MM; or a signal realization for some neighbor in B that is different from the others for NS. For any node with B-degree d_i, the probability to observe a favorable message is $\sum_{s=0}^{B-d_i}\tilde{p}^B_{s+d_i}$. In case of NS, the probability to observe a favorable message is $\sum_{s=0}^{B-d_i}\tilde{p}^B_{s+d_i} + \sum_{s=0}^{B-d_i}\tilde{p}^B_{s}$.

The general characterization of the optimal $\hat{d}$ is left for a future version of this work. However the next sections, I provide sufficient conditions to fully characterize the signal structure of each strategy and the ranking of persuasion value for different strategies.

2.4.2 Homogenous Priors

First, let us compare the different strategies when agents do not differ in their prior. In such a context, the standard strategy and the unconnected hard news strategy are the same. It is however interesting to study how well the hard news MM and NS strategies do.

Lemma 11. *For any random network populated by receivers with homogenous priors, the multiple-message and network-specific strategies weakly underperform with respect to the* *standard strategy.*

Proof. See Appendix A. □

Intuitively, the MM and NS strategies level on the potential differences in connectivity patterns between groups with different priors. These strategies allow group A, which is expected to be connected to B less than B is to itself, to observe a less informative signal exactly because of their lack of connectivity. But this comes at a price: the strategy will perfectly target nodes with some degree, but is too informative for nodes that are more connected, and not informative enough for agents that are less connected.

Now, when the priors are homogenous, it is suboptimal to let group A with less informative signals. In other words, when groups have the same priors, exploiting the network introduces suboptimal heterogeneity. In particular, the sender loses all nodes with $d_i < \hat{d}$ in any state, and she wastes informativeness on nodes with $d_i > \hat{d}$ in the unfavorable state.

2.4.3 Heterogenous Priors

When receivers hold different priors, the strategies exploiting the network allow the sender to design a signal structure that is less informative for some nodes. This is a double edge-sword: on the one hand, the sender can design π to be less informative to some nodes in A; on the other hand, this might cause some nodes to receive too informative messages.

Theorem 4 provides sufficient conditions to characterize the optimal signal structure with such strategies and for such strategies to be beneficial to the sender.

Theorem 4. *Denote* $\tilde{\delta}(d_i) := a\delta_A(d_i) + \mathbb{1}_{d_i \geq \hat{d}}\, b\delta_B(d_i)$.

(i) If $\mathbb{E}_{\tilde{\delta}}(d_i) \leq \hat{d}$, $\rho_{MM} = 0$ *and* $V_{MM} < V_{NS}$. *If furthermore* $\mathbb{E}_{\tilde{\delta}}({d_i}^2) \leq \hat{d} \cdot \mathbb{E}_{\tilde{\delta}}(d_i)$, $\rho_{NS} = 0$.

(ii) If $\mathbb{E}_{\tilde{\delta}}(d_i) \leq \hat{d}$, *when the unfavorable state realizes, the multiple-message strategy outperforms the unconnected strategy.*

The sender designing MM or NS strategies has to tradeoff ρ for p, as both are decreasing the informativeness of favorable messages. Why does the sender prefer increasing p than ρ?

For the MM strategy, if the sender was decreasing p in order to increase ρ, it would increase the probability for a node in A to observe an unfavorable message, and hence to be dissuaded. Intuitively, a MM signal structure is designed to diversify the type of unfavorable messages that realize for a node in B. While all unfavorable messages have the same consequence on B, the ones containing more successes have a lower chance to dissuade members of A that have a lower B-degree. Therefore, because she is constraint, the sender prefers to allow for different signal realizations.

This intuition applies if the gain from exploiting the lower connectivity of A members overcompensates for the nodes who are hit with *too much* informativeness due to their higher connectivity. Indeed, when the sender relies on a MM strategy, it is less likely for a poorly-connected member of A to see an unfavorable message, but it is impossible for a poorly-connected member of B to observe a favorable message. Because of the rate at which the probability varies with connectivity, it is sufficient – but not necessary – that the average degree of susceptible nodes is lower than the degree of targeted nodes.

To sum up, the strategy relies on the difference in connection patterns between groups. Correlating messages renders this difference less salient. Hence, it is suboptimal for the sender to correlate signals when this difference compensates the excess of informativeness for well-connected nodes and the loss of non-susceptible members of B.

A similar reasoning applies to the NS strategy. Only successes in a node's neighborhood can be due to highly correlated signals, or to a very high probability for any of them to be a success. As before, agents with bigger B-neighborhood are better at distinguishing whether signals are identical out of luck – that is, because of a high (or low) probability of success – or because of correlation. For instance, the probability for two uncorrelated signals to be identical, even for outcomes as random as $p = 1/2$, is relatively high, 50% in this case; but it vanishes fast when more signals are considered, to slightly more than 6% in the example considered if five uncorrelated signals are observed. If the signals are correlated, the number of signals observed becomes less and less relevant in determining the informativeness of the message. Because this effect is more extreme with changes in d_i, the sufficient condition for the NS strategy, compared to the MM one, is more demanding, as it does not only consider average degree, but also include a notion of the variance of degrees of susceptible nodes.

The second result in Theorem 4 underlines that the NS strategy is better at capitalizing on degree differences than the MM strategy when the unfavorable state of the world realizes. The effective informativeness of messages issued from a strategy designed for a given B-degree varies more for different d_i with NS than with MM. As long as the average degree of susceptible nodes is lower than the degree of targeted nodes, the average probability for the susceptible nodes to actually receive a favorable message is also higher with NS than with MM. Because both strategies exploit the network, both might equally lose non-susceptible nodes when the favorable state of the world realizes.

A similar intuition applies regarding the comparison between MM and U strategies. When the unfavorable state of the world realizes, MM might, on average, deliver more favorable signals than U, as the higher probability of favorable signals for poorly-connected signals in A more than compensates the lower probability of favorable signals for highly connected nodes. This is indeed the case if the probability of a favorable signal is higher for a node with the average degree of susceptible node than the minimum probability required for agents in B to be persuade, which is the probability of persuasion upon the unfavorable state realizing. However, in the favorable state, the unconnected strategy convinces all nodes, while the multiple-message strategy loses the non-susceptible nodes.

The relative size of these effects depends on the parameters. In particular, one would expect that losing the non-susceptible is compensated by the higher average probability of persuading nodes with the MM strategy when the favorable state of the world is rather unlikely or when few nodes are non-susceptible.

Corollary 9. *For* $\mathbb{E}_{\tilde{\delta}}(d_i) < \hat{d}$*,* $V_{NS} > V_{MM} > V_U$ *for relatively small* μ*,* b *or* $\sum_{d_i=1}^{\hat{d}} \delta_B(d_i)$*.*

Proof. See Appendix A □

To compare the strategies exploiting the network to the unconnected strategy, one needs to compare the benefits of MM against U in the unfavorable state of the world with the loss from MM against U in the favorable state of the world; as well as the relative probability that each state will occur.

When μ is small, the unfavorable state of the world is much more likely than the favorable one, so it is much more likely for benefits to occur than losses, making MM better than U. Likewise, when b or $\sum_{d_i=1}^{\hat{d}} \delta_B(d_i)$ are small, the magnitude of the cost is rather small, so that the benefits from the unfavorable state are more likely to compensate for the small costs from the favorable state.

To better understand the performance of strategies exploiting the network when receivers differ in their prior, I consider specific or simplified network structures below.

Regular Networks

Generalizing example 2, consider a network in which all nodes within one group have the same number of B-neighbors. In such a context, the targeted nodes are straightforward to define. This approach also has the advantage of a reduced set of relevant parameters, which still allows powerful predictions.

Let d_B be the B-degree of B nodes, while d_A is the number of neighbors of A members who belong to B.

Corollary 10. *Take a network in which* $\delta_B(d_i^B = d_B) = 1$ *and* $\delta_A(d_i^B = d_A) = 1$ *with* $d_B > d_A$*.*

(i) A sender designing a multiple-message or a network-specific strategy would set $\rho = 0$*.*

(ii) The network-specific strategy outperforms the multiple-message strategy, which outperforms the unconnected hard news strategy: $V_{NS} > V_{MM} > V_U$

(iii) V_{NS}*,* V_{MM} *and* $V_{NS} - V_{MM}$ *are increasing in* d_B *but decreasing in* d_A*.*

(iv) V_{MM}*,* V_U *and* $V_U - V_{MM}$ *are increasing in* α_B*.*

Proof. See Appendix A □

Following Theorem 4, it is optimal for the sender to design the information structure such that $\rho = 0$. Indeed, in such a regular network there is no variance of degree: all nodes in B have the same degree, so they are all targeted and hence all delivered optimal informativeness. Likewise, all nodes in A have a lower degree than the targeted ones, so they all observe effectively less informative messages than B members. This implies that it is optimal for the designer to exploit degree differences by setting $\rho = 0$; and that such strategy exploiting degree differences are performing better than the ones ignoring it.

As explained above, the NS strategy is better at capitalizing on degree differences than MM. The effective informativeness of messages issued from a strategy designed for a given B-degree varies more for different d_i with NS than with MM. Furthermore, because all nodes are susceptible, there is no loss from exploiting the network. As a result, any strategy relying on the connectivity differences does better than the standard unconnected strategy.

The third result of the proposition reinforces the intuitions previously underlined. The strategies relying on the difference of B-degree between groups perform better, the larger are these differences. Furthermore, as the NS strategy is better than the MM strategy at exploiting such differences, the former gets increasingly better compared to the latter as the difference increases. This echoes the condition $\mathbb{E}_{\tilde{\delta}}(d_i) \leq \hat{d}$ from Theorem 4. The following equivalent for a general network is expected to appear in future version of this work:

Conjecture 1. V_{NS}*,* V_{MM} *and* $V_{NS} - V_{MM}$ *are increasing in* $\mathbb{E}_{\delta_B}(d_i)$ *but decreasing in* $\mathbb{E}_{\delta_A}(d_i)$*.*

This result also shows how connectivity *per se* is not detrimental to the sender. Actually, it can benefit her, if this connectivity allows a group to be more interconnected. The problem is the number of connections between the groups. One can informally link this to homophily. While it could be that group A is simply less connected overall, and therefore not necessarily display homophily, it is interesting to note that, were all nodes having the same overall degree, the pattern $d_A < d_B$ would indicate homophily. In this context, homophily would serve the sender, as it would increase the difference between d_A and d_B.

Finally, the last result underlines how the magnitude of skepticism from the members of B influences the persuasion value, but also which type of strategy to use. Unsurprisingly, as very skeptical receivers are harder to convince, the sender needs to make the signal structure more informative, lower the probability for any node to take the favorable action. This applies given any strategy. However, when members of B are very hard to convince, exploiting the network becomes more beneficial. When α_B is low, the differences in connection patterns between groups matter more, as decreasing the effective informativeness of messages observed by nodes in A is more important. I expect this intuition to carry through in general networks:

Conjecture 2. *For $\mathbb{E}_{\tilde{\delta}}(d_i) < \hat{d}$, $V_{NS} > V_{MM} > V_U$ for relatively small α_B.*

This conjecture echoes Corollary 9. It posits that since the benefits from exploiting the networks in the unfavorable state are higher for more skeptical members of B, these benefits are more likely to compensate the loss from never persuading non-susceptible nodes.

High, medium and low connectivity within B

So far, only the persuasion value of the different strategies exploiting the network have been analyzed for any arbitrary $\hat{d}$. To gain insights into this problem, I consider the following simple network: all nodes in A have a medium number of connection to B, denoted d_M; while nodes in B can have a low, a medium or high number of connections, denoted d_L, d_M and d_H respectively. Let $d_L < d_M < d_H$.

Lemma 12. *Take a network in which $\delta_B(d_i^B = d_L) + \delta_B(d_i^B = d_M) + \delta_B(d_i^B = d_H) = 1$ and $\delta_A(d_i^B = d_M) = 1$ with $d_L < d_M < d_H$. The NS and MM strategies targeting $\hat{d} \in \{d_L, d_M\}$ underperforms the U strategy.*

Proof. See Appendix A □

Targeting nodes with higher degree have two effects: increase the number of non-susceptible nodes in B; and decrease the informativeness of messages to A. The former reduces the persuasion value, the latter raises it. For the persuasion value of MM or NS strategies surpass that of the unconnected strategy, the informativeness of messages to A has to be decreased below its value for targeted node; hence the targeted nodes have to have a higher degree than nodes in A.

Intuitively, targeting d_L would underperform with respect to the unconnected strategy because in the unfavorable state of the world, the effective informativeness of messages would be weakly higher than the one needed to persuade nodes in B *for everyone*. Targeting d_M would underperform with respect to the unconnected strategy because in the favorable state of the world, all nodes in B with degree d_L would be lost. This cannot be compensated by the outcome when the unfavorable state of the world realizes, as the effective informativeness of messages for A would still be the same as the one in the unconnected strategy.

While exploiting the network has non-linear effects on the persuasion value, one would expect that this reasoning still applies when nodes in A have heterogenous degrees. This leads to the following conjecture.

Conjecture 3. *If the targeted nodes have a B-degree $\hat{d} < \mathbb{E}_{\delta_A}(d_i)$, then $V_{NS} < V_U$ and $V_{MM} < V_U$*

Further research is needed to understand the conditions characterizing the optimal targeted nodes.

2.4.4 Further Research

The first element that needs to be addressed is which degree in B to target. Using Proposition 8, one can study the different persuasion value under the same strategy with different targeted nodes. This should allow to derive some necessary conditions for a certain degree to be optimal to target.

Similarly, the conditions provided in Theorem 4 are sufficient but not necessary for the ranking of the considered strategies. It will be important to determine necessary conditions for strategies exploiting the network to be beneficial to the sender.

A second step in this work will be to consider soft news strategies exploiting the network. Such a strategy would be defined as a signal structure such that for any targeted node i in A, $\Pr(s_i \in \mathcal{PM}_i) = 1$; in other words, the sender choses some nodes in A and makes sure that they can never be dissuaded. This requires for the favorable state not to be signaled for sure to members of B. The proposed strategies would then be adapted. In particular, as before, any targeted nodes in B should be made indifferent, i.e. $\Pr(s_i \in \mathcal{PM}_i|\omega = 0, i \in B) = \alpha_B \Pr(s_i \in \mathcal{PM}_i|\omega = 1, i \in B)$. But now, targeted nodes in A should not detect $\omega = 0$, which requires $\Pr(s_i \in \mathcal{PM}_i|\omega = 1, i \in B) \neq 1$. Actually, for the targeted nodes in A to be indifferent upon seeing any message, we need $1 - \Pr(s_i \in \mathcal{PM}_i|\omega = o, i \in B) = \alpha_A\Big(1 - \Pr(s_i \in \mathcal{PM}_i|\omega = 1, i \in B)\Big)$. This forms a system that implicitly determines q, φ, p and ρ for the MM and NS strategies.

Proposition 8 would then have a soft news strategy equivalent, in which the persuasion value of such strategies would be characterized for any degree of targeted nodes in A and in B. The persuasion value would thus be determined by two degree thresholds. These strategies could then be compared between themselves, but also with equivalent hard news strategies. The role of polarization and its relation to the network could thus be underlined.

2.5 Conclusion

This paper explores the role of a network in shaping the provision of information in a persuasion problem. How and when should the sender exploit a network in order to induce communicating agents with heterogenous priors to act a certain way?

I find that there exists strategies which exploit the network in order to send messages with different informativeness to different nodes. In particular, the information necessary to persuade one type of receiver can be split among many of them. This applies particularly well when the network is rather segregated, that is, when nodes with different priors are connected less than nodes with the same prior. It allows receivers who need to be persuaded to observe an informative message by combining each bit of information that has been disseminated among themselves; while preventing the other receivers from accessing too informative messages, as they cannot observe many bits of information, each of which are barely informative.

I present two ways in which the sender can implement such scheme. She can make each bit of information informative by itself. Then, each signal is informative, but poorly so; it thus takes many positive signals to persuade the skeptical receivers. Alternatively, the sender can code information in the *similarity* of signal realizations. In such a case, the signal is not informative by itself; but, in association with other messages, it is. Again, the more signals one observes, the more informative of a message they deliver when combined. However, the informativeness varies relatively more with degree when the sender utilizes the similarity of signals rather than their individual realization.

Which of these strategies is more valuable to the sender? Well, it depends on the context. If the average degree of nodes who need to be persuaded is different enough from those who should not be dissuaded, the sender would prefer to exploit the network through correlated signals than through low informativeness of individual realization. This is due to the greater capacity for the strategy relying on correlation to utilize degree differences. Whether the sender wants to exploit the network in the first place also depends on the relative degree averages. If the receivers who should not be dissuaded are connected poorly enough, then exploiting the network is better when the adverse state realizes, as the sender would, on average, persuade more nodes. However, when the favorable state realizes, the standard strategy that does not exploit the network persuades all receivers; whereas some nodes who need to be persuaded and are too poorly connected are left out if the network is exploited. Therefore, when the favorable state is unlikely to begin with, or when only very few nodes are impossible to persuade, exploiting the network is better.

These results are put in perspective in a regular network. I consider the case in which all nodes who should not be dissuaded have the same number of connections to nodes who should be persuaded, who are themselves equally connected to each other. In this case, the strategies exploiting the network are unambiguously better; and the one who uses correlation even more so.

In such a context, the sender benefits from greater cohesion within the group who she needs to persuade. This shows how connectivity is not necessarily detrimental to the sender. Even though agents can communicate, the sender can easily adapt the informativeness of each signal so that the messages observed are not too informative. This scheme is of course more efficient if each group's connectivity is rather homogenous. However, higher connectivity between groups *is* detrimental to the sender. This puts into perspective the role of homophily in the network. The sender is not necessarily hurt by connectivity; but by cross-group connectivity, she is. The reverse applies to the amount of information provided to the receivers: it does not necessarily decrease with connectivity, but it surely does if the network is more segregated. It underlines how the network structure, and in particular homophily, is detrimental to the provision of information.

Using the same regular network, I show that strategies exploiting the network benefit the sender even more when agents are harder to persuade. Indeed, relying on the network to decrease the effective informativeness of messages observed by agents who should not be dissuaded is all the more valuable when a lot of information is delivered to other agents. Therefore, one would expect that the sender would always prefer to communicate about polarizing topics on the network.

To better asses the role of polarization on the value of strategies exploiting the network, other strategies should be considered. The strategies presented above are the unique strategies exploiting the network and ensuring that the favorable state is signaled when it occurs. However, the sender might prefer to never dissuade some nodes. The previous strategies, adapted for this objective, would allow to explicitly contrast beliefs of each group.

Yet, the results presented already offer many interesting insights. The way existing information spreads in a network depends on its structure; this has been largely documented. However, the current work offers insights into the role of the network structure on the provision of information in the first place. It addresses topical concerns about polarization and segregation in social media, and shows how these weaknesses can be exploited even when agents are fully rational.

Appendix

2.A Computations and Proofs

Distribution of n dependent Bernoulli trials

Recall that the probability of k successes among n dependent trials is denoted $\tilde{p}_k^n$. Furthermore, denote any individual trial x_i.

The probability for any single trial x_i to be a success is p, hence $p = \mathbb{E}(x_i) = \sum_{k=0}^{n-1} \tilde{p}_{k+1}^n$. Likewise, conditioning on $x_i = 0$, we find $1 - p = \sum_{k=0}^{n-1} \tilde{p}_k^n$

By the definition, $\text{Corr}(x_i, x_j)[\mathbb{E}(x_i^2) - \mathbb{E}(x_i)^2] + \mathbb{E}(x_i)\mathbb{E}(x_j) = \mathbb{E}(x_i x_j)$. Therefore $\rho[p - p^2] + p^2 = \sum_{k=0}^{n-2} \tilde{p}_{k+2}^n$ as the expectation of $\mathbb{E}(x_i x_j)$ is the probability for both x_i, x_j to be successes, regardless of what happens with all other trials.

Therefore, $\tilde{p}_k^n$ must satisfy: (i) $p = \sum_{k=0}^{n-1} \tilde{p}_{k+1}^n$; (ii) $1 - p = \sum_{k=0}^{n-1} \tilde{p}_k^n$; (iii) $\rho(1-p)p + p^2 = \sum_{k=0}^{n-2} \tilde{p}_{k+2}^n$.

In particular, for $\rho = 0$, $\tilde{p}_k^n$ is the standard Binomial PMF, while for $\rho = 1$, $\tilde{p}_k^n = \begin{cases} 1-p & \text{if } k = 0 \\ p & \text{if } k = n \\ 0 & \text{otherwise} \end{cases}$

It is easy to verify that the PML proposed fulfills (i), (ii) and (iii). Among the PMLs of the form: $(1-\rho)^x \binom{n}{k} p^k (1-p)^{n-k} + \rho^y \big[(1-p)\mathbb{1}_{k=0} + p\mathbb{1}_{k=n}\big]$, only $x = y = 1$ fulfills all three conditions. If $x \neq 1$ or $y \neq 1$, then $\sum_{k=0}^{n} \tilde{p}_k^n \neq 1$.

Proof of Theorem 3

Let us first derive $\Pr(s_i = x|\omega = 0)$ and $\Pr(s_i = x|\omega = 1)$.

For any given node i with B-degree d_i, all realizations of s_B with at least x successes could allow for s_i to equate x. For any number of successes $s \geq x$ within B-nodes, there are $\binom{B-d_i}{s-x}\binom{d_i}{x}$ out of the $\binom{B}{s}$ possible realizations of signals that allow for s_i to be x. Indeed, there are $\binom{d_i}{x}$ ways to arrange the x successes within i's neighborhood; and $\binom{B-d_i}{s-x}$ ways to arrange the other $(s-x)$ successes outside of i's neighborhood. Therefore, the probability for this given node i to observe x successes in his B-neighborhood when s successes occurred among the B nodes is $\frac{\binom{B-d_i}{s-x}\binom{d_i}{x}}{\binom{B}{s}}\tilde{p}_s^B$. Fixing x successes within i's neighborhood, anything can happen in the rest of the network, i.e. from $s-x=0$ to $B-d_i$ other successes. Hence:

$$\Pr(s_i = x|\omega = 0) = \sum_{s-x=0}^{B-d_i} \frac{\binom{B-d_i}{s-x}\binom{d_i}{x}}{\binom{B}{s}}\tilde{p}_s^B = (1-\rho)\sum_{s-x=0}^{B-d_i}\binom{B-d_i}{s-x}\binom{d_i}{x}p^s(1-p)^{B-s} + \rho\big[(1-p)\mathbb{1}_{x=0} + p\mathbb{1}_{x=n}\big]$$

$$= (1-\rho)\binom{d_i}{x}\underbrace{\sum_{s-x=0}^{B-d_i}\binom{B-d_i}{s-x}p^{s-x}(1-p)^{B-d_i-(s-x)}}_{=1} p^x(1-p)^{d_i-x} + \rho\big[(1-p)\mathbb{1}_{x=0} + p\mathbb{1}_{x=n}\big]$$

Likewise, $\Pr(s_i = x|\omega = 1) = (1-\varphi)\binom{d_i}{x}q^x(1-q)^{d_i-x} + \varphi\big[(1-q)\mathbb{1}_{x=0} + q\mathbb{1}_{x=n}\big]$.

By definition of a hard news strategy, we need $\Pr(s_i \in \mathcal{PM}_i|\omega = 1) = 1$. Therefore, for any hard news strategy, it must be that $\sum_{s_i\in\mathcal{PM}_i}\Pr(s_i = x|\omega = 1) = 1$. We show that the only sets of messages that fulfill this condition are associated to one of the three strategies U, MM or NS.

Assume by contradiction that $0 < s_i < d_i$ is a persuading message for the targeted nodes. Then $q \neq 1 \neq \varphi$, hence:

$$\sum_{x\in X}(1-\varphi)\binom{d_i}{x}q^x(1-q)^{d_i-x} + \varphi\big[q\mathbb{1}_{x=d_i} + (1-q)\mathbb{1}_{x=0}\big] = 1 \Leftrightarrow X = \{0, 1..., d_i\}$$

This would thus require $\mathcal{PM}_i = \mathcal{M}_i$, which is impossible, as messages have to be informative in order to be persuading.

Proof of Lemma 10

Proof. Recall that $\Pr(s_i = x|\omega = 1) = \varphi\binom{d_i}{x}q^x(1-q)^{d_i-x} + (1-\varphi)\big[q\mathbb{1}_{x=d_i} + (1-q)\mathbb{1}_{x=0}\big]$.

(i) By definition, $\mathcal{FM}_i \neq \varnothing$ requires that there exists a s_i such that $\beta(s_i) \geq t$. First, because MM and NS are hard news strategies, upon observing any signal realization that is not a success for MM, or any signal realization different to other signals for NS, nodes perfectly detect that the unfavorable state realizes. Hence, $\mathcal{FM}_i \in \{\varnothing, \{d_i\}\}$ for MM and $\mathcal{FM}_i \in \{\varnothing, \{0, d_i\}\}$ for NS. Now, $\forall i \in A$, $\beta(s_i = d_i) \geq t$. Indeed, because they induce informative signals, both strategies insure that $\Pr(s_i = d_i|\omega = 0) \leq \Pr(s_i = d_i|\omega = 1)$; hence, $\forall i \in A, \Pr(s_i = d_i|\omega = 0) \leq \alpha_A \Pr(s_i = d_i|\omega = 1)$. Furthermore, $\forall i \in B : d_i \geq \hat{d}$, $\Pr(s_i = d_i|\omega = 0) \leq \Pr(s_i = \hat{d}|\omega = 0) = \alpha_B \Pr(s_i =$

$\hat{d}|\omega = 1) = \alpha_B \Pr(s_i = d_i|\omega = 1)$ where the first inequality follows from the expression for $\Pr(s_i = d_i|\omega = 0)$ being decreasing in d_i; and the last equality follows from the definition of hard news strategy. Finally $\forall i \in B : d_i < \hat{d}$, $\beta(s_i = 0) < t$ and $\beta(s_i = d_i) < t$. Indeed, $d_i < \hat{d}$ implies $\Pr(s_i = d_i|\omega = 0) > \Pr(s_i = \hat{d}|\omega = 0)$. Therefore, $\Pr(s_i = d_i|\omega = 0) > \alpha_B \Pr(s_i = d_i|\omega = 1)$. Likewise, $\forall i \in B : d_i < \hat{d}$, $\Pr(s_i = 0|\omega = 0) > \alpha_B \Pr(s_i = 0|\omega = 1)$.

(ii) For U and MM, $q = 1$ insures $\forall i \in B s_i = 1$, so that $\forall i \in N, s_i = d_i$; for NS, $\varphi = 1$ implies $\forall i, j \in B, s_i = s_j$, so that $\forall i \in N, s_i \{0, d_i\}$.

(iii) $\Pr(s_i = x|\omega = 0)$ is decreasing in d_i.

□

Proof of Proposition 8

Proof. By definition, the beliefs of targeted nodes upon observing a persuading signal must equate t. Because $\beta(s) = \frac{\Pr(s|\omega=1)}{\Pr(s|\omega=0)+\Pr(s|\omega=1)}$, the sender needs to set $\Pr(s|\omega = 0) = \alpha_B \Pr(s|\omega = 1)$. Because we consider hard news strategies, $\Pr(s|\omega = 1) = 1$.

(i) For the strategy to be independent of the targeted nodes' degree, the posterior has to be independent of the number of signals a node observes. Therefore, it requires $\varphi = \rho = 1$. Because it is a hard news strategy, $q = 1$. Because nodes in B are targeted, $p = \alpha_B q = \alpha_B$.

Following Lemma 9, all nodes in A whose B-degree is zero, observe no informative signals and thus always take the favorable action. Therefore, if the unfavorable state realizes, the probability of a node taking the favorable action is the probability for the favorable message to realize, plus the probability that the favorable message not to realize times the probability of the node being member of A without connection to B. Hence, the persuasion value of the unconnected strategy is $V_U = \mu + (1 - \mu)\Big[\alpha_B + a\ \delta_A(0)(1 - \alpha_B)\Big]$.

(ii) Under a MM strategy, for the targeted nodes i, $\mathcal{PM}_i = \{d_i\}$. From Lemma 10's proof, $\Pr(s_i = d_i|\omega = 1) = (1 - \rho)p^{d_i} + \rho\ p$ and $\Pr(s_i = d_i|\omega = 0)(1 - \rho)q^{d_i} + \rho\ q$. Because B-nodes with B-degree $\hat{d}$ are targeted, we have $\Pr(s_i = \hat{d}|\omega = 1) = \alpha_B \Pr(s_i = \hat{d}|\omega = 0)$. Furthermore, $\Pr(s = \hat{d}|\omega = 0) = \varphi q + (1 - \varphi)q^{\hat{d}} \stackrel{!}{=} 1$ requires $q = 1$. Therefore, the sender sets $q = 1$ and $\rho p + (1 - \rho)p^{\hat{d}} = \alpha_B$.

Now, by Lemma 10, $s_i = d_i$ induces node i to take the favorable action iff $i \in A$ or ($i \in B$ and $d_i \geq \hat{d}$). Therefore, the probability that any random node takes the favorable action, i.e. the value of persuasion, under this strategy is:

$$\sum_{\omega} \Pr(\omega) \Bigg[\Pr(i \in A) \sum_{d=0}^{B} \Pr(d_i = d) \Pr(s_i = d_i | \omega) + \Pr(i \in B) \sum_{d=\hat{d}}^{B} \Pr(d_i = d) \Pr(s_i = d_i | \omega) \Bigg]$$

Which translates into:

$$\mu \Bigg[a + b \sum_{d_i=\hat{d}}^{B} \delta_B(d_i) \Bigg] + (1-\mu) \Bigg[a\delta_A(0) + \sum_{d_i=1}^{B} \Big[a\delta_A(d_i) + \mathbb{1}_{d_i \geq \hat{d}}\, b\delta_B(d_i) \Big] \Big[\rho\, p + (1-\rho) p^{d_i} \Big] \Bigg]$$

(iii) Under a NS strategy, for the targeted nodes i, $\mathcal{PM}_i = \{0, d_i\}$. Therefore, $\Pr(s_i \in \mathcal{PM}_i | \omega = 0) = \rho + (1-\rho)\Big[p^{d_i} + (1-p)^{d_i}\Big]$ and $\Pr(s_i \in \mathcal{PM}_i | \omega = 1) = \varphi + (1-\varphi)\Big[q_i^d + (1-q)_i^d\Big]$.
Because B-nodes with B-degree are targeted, we have $\Pr(s_i \in \{0, d_i\} | \omega = 1) = \alpha_B \Pr(s_i \in \{0, d_i\} | \omega = 0)$. Furthermore, $\Pr(s \in \{0, \hat{d}\} | \omega = 0) = \varphi + (1-\varphi)\Big[q^{\hat{d}} + (1-q)^{\hat{d}}\Big] \stackrel{!}{=} 1$ requires $\varphi = 1$. Therefore, the sender sets $\varphi = 1$ and $\rho + (1-\rho)\Big[p^{\hat{d}} + (1-p)^{\hat{d}}\Big] = \alpha_B$. Finally, for $\Pr(s_i = \hat{d} | \omega = 1) = \alpha_B \Pr(s_i = \hat{d} | \omega = 0)$, we need $q = \alpha_B^{-1}\big(\rho p + (1-\rho) p^{\hat{d}}\big)$ Similarly to above, $s_i \in \{0, d_i\}$ induces node i to take the favorable action iff $i \in A$ or ($i \in B$ and $d_i \geq \hat{d}$). Therefore, the probability that any random node takes the favorable action, i.e. the value of persuasion, under this strategy is:

$$\sum_{\omega} \Pr(\omega) \Bigg[\Pr(i \in A) \sum_{d=0}^{B} \Pr(d_i = d) \Pr(s_i \in \{0, d_i\} | \omega) + \Pr(i \in B) \sum_{d=\hat{d}}^{B} \Pr(d_i = d) \Pr(s_i \in \{0, d_i\} | \omega) \Bigg]$$

Which translates into:

$$\mu \Bigg[a + b \sum_{d_i=\hat{d}}^{B} \delta_B(d_i) \Bigg] + (1-\mu) \Bigg[a\delta_A(0) + \sum_{d_i=1}^{B} \Big[a\delta_A(d_i) + \mathbb{1}_{(d_i \geq \hat{d})}\, b\delta_B(d_i) \Big] \Big[\rho + (1-\rho)\big(p^{d_i} + (1-p)^{d_i}\big) \Big] \Bigg]$$

□

Proof of Lemma 11

Proof. • Consider the MM strategy. From Proposition **??** the persuasion value of this strategy without difference in group priors is:

$$V_{MM} = \mu \sum_{d_i=\hat{d}}^{N} \delta(d_i) + (1-\mu) \sum_{d_i=\hat{d}}^{N} \delta(d_i) \big[\rho\, p + (1-\rho) p^{d_i} \big]$$

This is to be compared with the value of persuasion with a standard strategy which is:

$$V_{std} = \mu + (1-\mu)\alpha$$

Because $\hat{d} \geq \text{argmin}_i d_i$, $\sum_{d_i=\hat{d}}^{N} \delta(d_i) \leq 1$. Furthermore, $\sum_{d_i=\hat{d}}^{N} \delta(d_i)\big[\rho\, p + (1-\rho)p^{d_i}\big] \leq \alpha$. Indeed, by definition, $\rho\, p + (1-\rho)p^{\hat{d}} = \alpha$. Therefore, $\forall d_i > \hat{d}, \rho\, p + (1-\rho)p^{\hat{d}} < \alpha$. We conclude $V_{MM} \leq V_{std}$. The equation holds with equality iff $\forall i \in N, d_i = \hat{d}$.

- Consider the NS strategy. From Proposition **??** the persuasion value of this strategy without difference in group priors is:

$$\mu \sum_{d_i=\hat{d}}^{N} \delta_B(d_i) + (1-\mu) \sum_{d_i=\hat{d}}^{N} \delta_B(d_i)\Big[\rho + (1-\rho)\big(p^{d_i} + (1-p)^{d_i}\big)\Big]$$

This is to be compared with the value of persuasion with a standard strategy which is:

$$V_{std} = \mu + (1-\mu)\alpha$$

Because $\hat{d} \geq \text{argmin}_i d_i$, $\sum_{d_i=\hat{d}}^{N} \delta(d_i) \leq 1$. Furthermore, $\sum_{d_i=\hat{d}}^{N} \delta(d_i)\big[\rho + (1-\rho)(1-\rho)\big(p^{d_i} + (1-p)^{d_i}\big)\big] \leq \alpha$. Indeed, by definition, $\rho + (1-\rho)(1-\rho)\big(p^{\hat{d}} + (1-p)^{\hat{d}}\big) = \alpha$. Therefore, $\forall d_i > \hat{d}, \big(p^{d_i} + (1-p)^{d_i}\big) < \alpha$. We conclude $V_{NS} \leq V_{std}$. The equation holds with equality iff $\forall i \in N, d_i = \hat{d}$.

□

Proof of Theorem 4

(i) I consider each strategy separately. I first show that the two conditions imply $\rho_{MM} = 0$ and ρ_{NS} respectively. I then proceed to show that the first condition is sufficient for $V_{MM} < V_{NS}$.

- For the multiple-message strategy, the sender sets (p, ρ) subject to the constraint $\rho\, p + (1-\rho)p^{\hat{d}} = \alpha_B$ to maximize:

$$(1-\mu) \sum_{d_i=1}^{B} \tilde{\delta}(d_i)\Big[\rho\, p + (1-\rho)p^{d_i}\Big]$$

One can rewrite this objective as:

$$\begin{aligned}
\sum_{d_i=1}^{B} \tilde{\delta}(d_i)\Big[\rho\, p + (1-\rho)p^{d_i}\Big] &= \sum_{d_i=1}^{B} \tilde{\delta}(d_i)\Big[\alpha_B + (1-\rho)(p^{d_i} - p^{\hat{d}})\Big] \\
&= \sum_{d_i=1}^{B} \tilde{\delta}(d_i)\left[\alpha_B + \left(1 - \frac{\alpha_B - p^{\hat{d}}}{p - p^{\hat{d}}}\right)(p^{d_i} - p^{\hat{d}})\right] \\
&= \sum_{d_i=1}^{B} \tilde{\delta}(d_i)\left[\alpha_B + \left((p-\alpha_B)\frac{p^{d_i} - p^{\hat{d}}}{p - p^{\hat{d}}}\right)\right]
\end{aligned}$$

Which is increasing in p if $\mathbb{E}_{\tilde{\delta}}(d_i) < \hat{d}$. Indeed, both $p - \alpha_B$ and $\sum_{d_i=1}^{B} \tilde{\delta}(d_i)\frac{p^{d_i} - p^{\hat{d}}}{p - p^{\hat{d}}}$

are increasing in p. While it is trivial to show this for $p-\alpha_B$, it is less straightforward for $\sum_{d_i=1}^{B}\tilde{\delta}(d_i)\frac{p^{d_i}-p^{\hat{d}}}{p-p^{\hat{d}}}$. We have:

$$\frac{\partial\sum_{d_i=1}^{B}\tilde{\delta}(d_i)\frac{p^{d_i-1}-p^{\hat{d}-1}}{1-p^{\hat{d}-1}}}{\partial p}$$
$$=\sum_{d_i=1}^{B}\tilde{\delta}(d_i)\left[\frac{\left[(d_i-1)p^{d_i-2}-(\hat{d}-1)p^{\hat{d}-2}\right](1-p^{\hat{d}-1})+(\hat{d}-1)p^{\hat{d}-2}(p^{\hat{d}-1}-p^{\hat{d}-1})}{(1-p^{\hat{d}-1})^2}\right]$$
$$=\sum_{d_i=1}^{B}\tilde{\delta}(d_i)\left[\frac{(d_i-1)p^{d_i-2}-(\hat{d}-1)p^{\hat{d}-2}+(\hat{d}-d_i)p^{\hat{d}+d_i-3}}{(1-p^{\hat{d}-1})^2}\right]$$

Whose sign is proportional to numerator, which itself is proportional to

$$\sum_{d_i=1}^{B}\tilde{\delta}(d_i)\left[(d_i-1)-(\hat{d}-1)p^{\hat{d}-d_i}+(\hat{d}-d_i)p^{\hat{d}-1}\right]$$

This expression is decreasing in p:

$$\frac{\partial\sum_{d_i=1}^{B}\tilde{\delta}(d_i)\left[(d_i-1)-(\hat{d}-1)p^{\hat{d}-d_i}+(\hat{d}-d_i)p^{\hat{d}-1}\right]}{\partial p}$$
$$=\sum_{d_i=1}^{B}\tilde{\delta}(d_i)\left[-(\hat{d}-1)(\hat{d}-d_i)p^{\hat{d}-d_i}+(\hat{d}-d_i)(\hat{d}-1)p^{\hat{d}-1}\right]\geq 0$$

since $0<\hat{d}-\mathbb{E}_{\tilde{\delta}}(d_i)<\sum_{d_i=1}^{B}\tilde{\delta}(d_i)(\hat{d}-d_i)p^{-d_i+1}$. The inequality holds strictly for $p<1$. Therefore,

$$\sum_{d_i=1}^{B}\tilde{\delta}(d_i)\left[(d_i-1)-(\hat{d}-1)p^{\hat{d}-d_i}+(\hat{d}-d_i)p^{\hat{d}-1}\right]$$
$$\geq\sum_{d_i=1}^{B}\tilde{\delta}(d_i)\left[(d_i-1)-(\hat{d}-1)p^{\hat{d}-d_i}+(\hat{d}-d_i)p^{\hat{d}-1}\right]\Bigg|_{p=1}=0$$

Again, the inequality holds strictly for $p<1$. We conclude that to maximize $\sum_{d_i=1}^{B}\tilde{\delta}(d_i)\left[\rho\, p+(1-\rho)p^{d_i}\right]$, the sender sets p as high as possible, which requires $\rho_{MM}=0$.

– For the network-specific strategy, the sender sets (p,ρ) subject to the constraint $\rho+(1-\rho)\left(p^{\hat{d}}+(1-p)^{\hat{d}}\right)=\alpha_B$ to maximize:

$$(1-\mu)\sum_{d_i=1}^{B}\tilde{\delta}(d_i)\left[\rho+(1-\rho)\left(p^{d_i}+(1-p)^{d_i}\right)\right]$$

One can rewrite this objective as:

$$\sum_{d_i=1}^{B} \tilde{\delta}(d_i)\Big[\rho + (1-\rho)\big(p^{d_i} + (1-p)^{d_i}\big)\Big]$$

$$= \sum_{d_i=1}^{B} \tilde{\delta}(d_i)\left[\frac{\alpha - p^{d_B} - (1-p)^{\hat{d}}}{1-p^{\hat{d}}-(1-p)^{\hat{d}}} + \left(1 - \frac{\alpha - p^{\hat{d}} - (1-p)^{\hat{d}}}{1-p^{\hat{d}}-(1-p)^{\hat{d}}}\right)\left(p^{d_i} + (1-p)^{d_i}\right)\right]$$

$$= \sum_{d_i=1}^{B} \tilde{\delta}(d_i)\left[\frac{\alpha_B - p^{\hat{d}} - (1-p)^{\hat{d}} - (1-\alpha_B)\left(p^{d_i}+(1-p)^{d_i}\right)}{1-p^{\hat{d}}-(1-p)^{\hat{d}}}\right]$$

$$= \sum_{d_i=1}^{B} \tilde{\delta}(d_i)\left[1 - \frac{1-p^{d_i}-(1-p)^{d_i}}{1-p^{\hat{d}}-(1-p)^{\hat{d}}}\right]$$

Which is increasing in p for $p \geq 1/2$. Indeed, $\sum_{d_i=2}^{B} \tilde{\delta}(d_i)\frac{1-p^{d_i}-(1-p)^{d_i}}{1-p^{\hat{d}}-(1-p)^{\hat{d}}}$ is decreasing in $p \geq 1/2$.[12]

$$\frac{\partial \sum_{d_i=2}^{B} \tilde{\delta}(d_i)\frac{1-p^{d_i}-(1-p)^{d_i}}{1-p^{\hat{d}}-(1-p)^{\hat{d}}}}{\partial p} = \sum_{d_i=2}^{B} \tilde{\delta}(d_i)\frac{-d_i\left[p^{d_i-1}-(1-p)^{d_i-1}\right](1-p^{\hat{d}}-(1-p)^{\hat{d}}) + \hat{d}\left[p^{\hat{d}-1}-(1-p)^{\hat{d}-1}\right](1-p^{d_i}-(1-p)^{d_i})}{(1-p^{\hat{d}}-(1-p)^{\hat{d}})^2}$$

Is equal to 0 in $p = 1/2$, but it is null for no other $p \in [0,1]$. Hence, the function is monotone between $[0,1/2)$ and $(1/2,1]$, and $p = 1/2$ is a stationary point. Using Hospital rule twice, we have:

$$\left.\frac{\partial \sum_{d_i=2}^{B} \tilde{\delta}(d_i)\frac{1-p^{d_i}-(1-p)^{d_i}}{1-p^{\hat{d}}-(1-p)^{\hat{d}}}}{\partial p}\right|_{p=0} = \frac{-\sum_{d_i=2}^{B} \tilde{\delta}(d_i)d_i(d_i-1)\hat{d} + \hat{d}(\hat{d}-1)\sum_{d_i=1}^{B} \tilde{\delta}(d_i)d_i}{2\hat{d}^2} \geq 0$$

for $\mathbb{E}_{\tilde{\delta}}({d_i}^2) \leq \hat{d} \cdot \mathbb{E}_{\tilde{\delta}}(d_i)$; and likewise:

$$\left.\frac{\partial \sum_{d_i=2}^{B} \tilde{\delta}(d_i)\frac{1-p^{d_i}-(1-p)^{d_i}}{1-p^{\hat{d}}-(1-p)^{\hat{d}}}}{\partial p}\right|_{p=1} = \frac{\sum_{d_i=2}^{B} \tilde{\delta}(d_i)d_i(d_i-1)\hat{d} - \hat{d}(\hat{d}-1)\sum_{d_i=1}^{B} \tilde{\delta}(d_i)d_i}{2\hat{d}^2} \leq 0$$

Therefore, for $\mathbb{E}_{\tilde{\delta}}({d_i}^2) \leq \hat{d} \cdot \mathbb{E}_{\tilde{\delta}}(d_i)$, $p = 1/2$ is a maximum. We conclude that to maximize $\sum_{d_i=1}^{B} \tilde{\delta}(d_i)\frac{1-p^{d_i}-(1-p)^{d_i}}{1-p^{\hat{d}}-(1-p)^{\hat{d}}}$ the sender sets p as high as possible, which requires $\rho_{NS} = 0$.

Furthermore, consider again $\hat{d} \leq \mathbb{E}_{\tilde{\delta}}(d_i)$, so that $\rho_{MM} = 0$. I will show that $V_{MM} < V_{NS}|_{\rho_{NS}=0} \leq V_{NS}$ where the last inequality follows from the sender's optimization. Hence, assume $\rho_{NS} = 0$. The same reasoning applies for picking $\hat{d}$, so I can compare them assuming the targeted nodes within each strategy are the same.
As $p_{MM} = \alpha_B^{\frac{1}{\hat{d}}}$ and $\alpha_B = p_{NS}^{\hat{d}} + (1-p_{NS})^{\hat{d}}$, $p_{MM} = \left(p_{NS}^{\hat{d}} + (1-p)_{NS}^{\hat{d}}\right)^{\frac{1}{\hat{d}}}$. To ease

[12]We can sum from $d_i = 2$ as in d_i, the expression is 0

notation, we denote p_{NS} with p and p_{MM} with $\tilde{p}$ in this proof. Note that $\tilde{p} \geq p$ as $\tilde{p}$ is decreasing in $\hat{d}$ with $\lim_{d \to \infty} \tilde{p} = p$. We have:

$$V_{NS} - V_{MM} = (1-\mu) \sum_{d_i=1}^{B} \tilde{\delta}(d_i) \left[p^{d_i} + (1-p)^{d_i} - \tilde{p}^{d_i} \right]$$

For the remainder of this proof, I denote $f(d) = \left(p^{\hat{d}} + (1-p)^{\hat{d}}\right)^{\frac{1}{\hat{d}}} - p^{d_i} - (1-p)^{d_i}$. I derive the shape of this function. In particular, it is increasing, then decreasing. I show that as long as it is increasing, it is concave; and that if it is decreasing for some d_i, then it is decreasing for all subsequent d_i. I consider the discrete variations in $f(d)$ as $d \in \mathbb{N}$. $\Delta f(d) := f(d+1) - f(d) = (1-p)p^d + p(1-p)^d - (1-\tilde{p})\tilde{p}^d$; and $\Delta\Delta f(d) := \Delta f(d+1) - \Delta f(d) = (1-p)^2 p^d + p^2(1-p)^d - (1-\tilde{p})^2 \tilde{p}^d$. It is easy to verify that for any $d < \hat{d}$, $f(d) < 0$ and $\Delta f(d) > 0$. Indeed a^x is concave for any $x < 1$ so that $(a+b)^x > a^x + b^x$; i.e. $\left(p^{\hat{d}} + (1-p)^{\hat{d}}\right)^{\frac{d}{\hat{d}}} > p^d + (1-p)^d$.
Now, if $f(d)$ is increasing, $(1-p)p^d + p(1-p)^d > (1-\tilde{p})\tilde{p}^d$; which implies $(1-p)^2 p^d + p^2(1-p)^d > (1-\tilde{p})(1-p)p^d + (1-\tilde{p})p(1-p)^d > (1-\tilde{p})^2\tilde{p}^d$ where the first inequality follows from $p > 1-p > 1-\tilde{p}$. Therefore, $\Delta f(d) > 0 \Rightarrow \Delta\Delta f(d) < 0$.
Furthermore, as soon as $f(d)$ is decreasing, it stays decreasing. Indeed, $f(d) < 0$ means $(1-p)p^d + p(1-p)^d < (1-\tilde{p})\tilde{p}^d$ so that $(1-p)p^{d+1} + p(1-p)^{d+1} < (1-p)p^d\tilde{p} + p(1-p)^d\tilde{p} < (1-\tilde{p})\tilde{p}^{d+1}$ i.e. $f(d+1) < 0$, using again $\tilde{p} > p > 1-p$.
We want to show that $\sum_{d_i=1}^{B} \tilde{\delta}(d_i) f(d_i) < 0$. To show this, consider:
$\tilde{f}(d) := \begin{cases} f(d) & \text{if } \Delta f(d) > 0 \\ \max f(d) & \text{if } \Delta f(d) \leq 0 \end{cases}$. $\tilde{f}(d)$ function is concave; and $f(d) \leq \tilde{f}(d)$.
We have:

$$\sum_{d_i=1}^{B} \tilde{\delta}(d_i) f(d_i) \leq \sum_{d_i=1}^{B} \tilde{\delta}(d_i) \tilde{f}(d_i) < \tilde{f}\left(\sum_{d_i=1}^{B} d_i\right) = f\left(\sum_{d_i=1}^{B} d_i\right) < 0$$

where the second inequality follows from $\tilde{f}(d)$ being concave;[13] and the equality because $\mathbb{E}_{\tilde{\delta}}(d_i) < \hat{d}$ such that $f\left(\mathbb{E}_{\tilde{\delta}}(d_i)\right) < 0$.

(ii) For $\rho_{MM} = 0$, $p_{MM} = \alpha_B^{\frac{1}{\hat{d}}}$, so that conditional on $\omega = 0$, the difference of probability for a node to take the favorable action between MM and U is:

$$\sum_{d_i=1}^{B} \tilde{\delta}(d_i) \alpha_B^{\frac{d_i}{\hat{d}}} - \alpha_B > 0$$

as $f(x) = \alpha_B^{\frac{x}{\hat{d}}}$ is a decreasing convex function for $\alpha_B < 1$, so that $\sum_{d_i=1}^{B} \tilde{\delta}(d_i) f(d_i) > f\left(\sum_{d_i=1}^{B} \tilde{\delta}(d_i) d_i\right)$, i.e. $\sum_{d_i=1}^{B} \tilde{\delta}(d_i) \alpha_B^{\frac{d_i}{\hat{d}}} > \alpha_B^{\frac{\mathbb{E}_{\tilde{\delta}}(d_i)}{\hat{d}}} \geq \alpha_B$ for $\mathbb{E}_{\tilde{\delta}}(d_i) \leq \hat{d}$.

[13] It is strict because $\hat{d} \leq \mathbb{E}_{\tilde{\delta}}(d_i)$ implies $\exists i : d_i < \hat{d}$, and that $\tilde{f}(d)$ is strictly concave for $d_i < \hat{d}$.

Proof of Corollary 9

When the favorable state of the world realizes, the difference of probability for a node to take the favorable action between MM and U is: $-b\sum_{d_i=1}^{\hat{d}}$. Therefore, $V_{MM} > V_U$ for $\mu b\sum_{d_i=1}^{\hat{d}} \delta_B(d_i) < (1-\mu)\sum_{d_i=1}^{B} \tilde{\delta}(d_i)\alpha_B^{\frac{d_i}{\hat{d}}} - \alpha_B$. Now $\mu b\sum_{d_i=1}^{\hat{d}}$ is increasing in μ, b and $\sum_{d_i=1}^{\hat{d}}$. Likewise $(1-\mu)\sum_{d_i=1}^{B} \tilde{\delta}(d_i)\alpha_B^{\frac{d_i}{\hat{d}}} - \alpha_B$ is decreasing in μ and b – through $\tilde{\delta}$. Therefore, the inequality is more likely to hold for small μ, b and $\sum_{d_i=1}^{\hat{d}}$.

Proof of Corollary 10

Proof. (i) Using Theorem 4, as $\mathbb{E}_{\tilde{\delta}}(d_i) = ad_a + bd_B < d_B = \hat{d}$ and $\mathbb{E}_{\tilde{\delta}}({d_i}^2) = ad_a^2 + bd_B^2 < d_B(ad_A + bd_B) = \hat{d}\cdot\mathbb{E}_{\tilde{\delta}}(d_i)$, we know $\rho = 0$. Let us consider each strategy separately:

(ii) As $\sum_{d_i=1}^{\hat{d}-1} \delta(d_i) = 0$, the value of persuasion of MM and U are the same in the favorable state. Therefore, using Theorem 4, $V_{MM} > V_U$.

(iii) The persuasion value when using a MM strategy is:

$$V_{MM} = \mu + (1-\mu)\Big[a\ {\alpha_B}^{\frac{d_A}{d_B}} + b\ \alpha_B\Big]$$

Because $\alpha_B < 1$, V_{MM} is decreasing in $\frac{d_A}{d_B}$, i.e. increasing in d_B but increasing in d_A. The persuasion value when using a NS strategy is:

$$V_{NS} = \mu + (1-\mu)\Big[a\ \left({p_{NS}}^{d_A} + (1-p_{NS})^{d_A}\right) + b\ \alpha_B\Big]\ \text{ s.t } {p_{NS}}^{d_B} + (1-p_{NS})^{d_B} = \alpha_B$$

For any given p, ${p_{NS}}^{d_A} + (1-p_{NS})^{d_A}$ is decreasing in d_A, making V_{NS} decreasing in d_A. For any given d_A, ${p_{NS}}^{d_A} + (1-p_{NS})^{d_A}$ is increasing in p_{NS}, which is increasing in d_B due to the constraint.
Finally, $V_{NS} - V_{MM} = \left({p_{NS}}^{d_A} + (1-p_{NS})^{d_A}\right) - \left({p_{NS}}^{d_B} + (1-p_{NS})^{d_B}\right)^{\frac{d_A}{d_B}}$. Because $f(x) = a^x$ gets less concave as $x \in (0,1]$ approaches 1, $\left({p_{NS}}^{d_B} + (1-p_{NS})^{d_B}\right)^{\frac{d_A}{d_B}}$ approaches ${p_{NS}}^{d_A} + (1-p_{NS})^{d_A}$ as $\frac{d_A}{d_B}$ approaches 1, hence when d_A increases or d_B decreases.

(iv) The derivative of the MM persuasion value is:

$$\frac{\partial V_{MM}}{\partial \alpha_B} = (1-\mu)\left[a\frac{d_A}{d_B}\alpha_B^{-1+\frac{d_a}{d_B}} + b\right] > 0$$

Furthermore,

$$\frac{\partial V_U - V_{MM}}{\partial \alpha_B} = (1-\mu)\left[1 - b - a\frac{d_A}{d_B}\alpha_B^{-1+\frac{d_a}{d_B}}\right] > 0$$

since $\frac{d_A}{d_B}\alpha_B^{-1+\frac{d_a}{d_B}} < 1$. Indeed, $\ln\left(\frac{d_A}{d_B}\right) + \ln(\alpha_B)\left(-1+\frac{d_a}{d_B}\right) < \left(\frac{d_A}{d_B}-1\right) + \ln(\alpha_B)\left(-1+\frac{d_A}{d_B}\right) = \left(\frac{d_A}{d_B}-1\right)(1-\ln(\alpha_B)) < 0$

□

Proof Lemma 12

Proof. With $\hat{d} = d_L$, $\frac{1-p^{d_i}-(1-p)^{d_i}}{1-p^{d_L}-(1-p)^{d_L}} \leq 1$ for any d_i, so the probability for a node to take the favorable action with NS is smaller than α_B. Likewise, $\frac{p^{d_i}-p^{\hat{d}}}{p-p^{\hat{d}}} \leq 0$ for any d_i, so the probability for a node to take the favorable action with NS is smaller than α_B.
With $\hat{d} = d_M$, there is a probability $b\delta_B(d_L)$ for a random node to be non-susceptible.Furthermore $\frac{1-p^{d_i}-(1-p)^{d_i}}{1-p^{d_L}-(1-p)^{d_L}} \leq 1$ for d_i of any susceptible node; hence, $V_{NS} < V_U$. Likewise, $\frac{p^{d_i}-p^{\hat{d}}}{p-p^{\hat{d}}} \leq 0$ for d_i of any susceptible node; hence $V_{MM} < V_U$. □

Chapter 3

When Conflict is a Political Strategy: a Model of Diversionary Incentives

3.1 Introduction

Do diversionary conflicts exist? The idea is surely attractive: diverting the public's attention abroad in order to hide domestic issues, how easy! But wars are costly. And is the public really that distractible? To believe that diversionary wars exists, one has to believe that all agents are naive : the public, for being distracted so easily, and the instigators of such wars, for not understanding that distraction could be less expensive.

This paper reinterprets diversionary conflicts under a rational light. When no one is naive, can diversionary conflicts exist? I argue that they can. Rather than a distraction, they create a pressure for domestic obedience that no other type of spending could generate. The public does not forget about domestic issues; it cannot act upon it in times of war. This allows leaders to implement demanding domestic policies without risking insurgency. Actually, it can even be rational for a population to support such a conflict.

I begin by exposing three historical examples whose patterns are particularly relevant to the mechanism underlined in this paper. Specifically, I consider Wawro [2005]'s rendition of the 1870 Franco-Prussian war to show how conflict has been designed as a necessary tool to bring independent states to relinquish their sovereignty and rally Germany. Furthermore, I also exploit Zubok [2009]'s vision of the Cold War, to underline the relationship between the official sentiment towards the West and internal popular opposition. Finally, I use Gagnon Jr [2006]'s reinterpretation of the recent Yugoslavian war that emphasizes how conflict has been used as political strategy for demobilizing domestic opposition. These anecdotal evidence seems to point towards patterns that would, in parts, be inconsistent with the classical diversionary argument: patriotic rhetoric and victory on the front are not all that is at stake. Also important seems to be the fact of being at war; and the actual threat posed by the enemy.

In order to formalize these arguments, I introduce a game between two players: a leader, who decides on domestic and foreign policies, and her population, who choses whether to support both domestic and foreign policies. The threat posed by the enemy is captured by a penalty on the population's payoff in case they decide to rebel when a conflict is raging. The leader uses taxation rates to extract wealth from the population. A conflict would then have two consequences: decrease the tax base, as the conflict burns resources; and increase the tax rate, as the threat posed by the conflict would allow the leader to implement harsher policies without risking insurgency. If the latter effect is stronger than the former, the leader benefits from using war as an extractive policy.

The setup distinguishes between low- and high- level conflict. To enter an open war, the leader first has to initiate tensions. However, between these two decisions, the population can express support if tensions were initiated. Because such support would advertise the population's hostility towards the enemy nation, this action is assumed to increase the penalty on the population's payoff in case they rebel during a conflict. Because rebellion decision occurs last, popular support of a belligerent policy can thus be seen as a commitment mean: by advertising their enmity to the other nation, they implicitly tie their hands, as a rebellion would now be more expensive; this allows the leader to implement demanding policies without having to resort to open war. In this sense, rallies around the flag are both rational and efficient.

However, the very existence of such an opportunity for the population to show support might be detrimental. Indeed, some low-level conflicts are initiated with the only goal of raising popular support. That is, there are contexts in which, if the population were not able to advertise endorsement of the foreign policy, the leader would stay at peace; but the very fact they can show such support is sufficient for the leader to initiate conflict? Using this very restrictive definition of diversionary incentive, I prove that the set of parameters for which such incentive is the driver of conflict is non zero-measure.

Finally, I wonder about long-run effects. Can a population have the incentive to set up barriers to rebellion in order to preserve external peace? When the leader can effectively use the threat of war with an hostile nation as an extraction tool, the population would either preserve peace, at the price of their freedom of rebellion, or preserve their freedom of rebellion, at the price of war. I find that they choose the latter in a non-monotonically function of the strength of the threat. A more threatening enemy makes preserving peace more important; but it also makes it more expensive in terms of commitment to the leader. These two counteracting forces create the hump shaped relationship between the prevalence of conflict and the strength of external threat. Therefore, for important threat, the population would decide to preserve peace by voluntarily renouncing to their freedom of rebellion. This result can account for the evolution of institutions in the long run, in particular in dependence with the institutions of neighboring countries: countries with weaker institutions pose a greater threat, forcing the population to weaken the country's institutions in order to avoid a diversionary conflict, thus spreading bad institutions internationally.

3.1.1 Related literature

This paper contributes to several strand of the literature. On a doctrinal level, it is built on parts of the classical theory on diversionary wars, as seen in Bodin and Tooley [1955] or Mayer [1969], by underlying the importance of fear in the success of belligerent strategies; and to Levy [1989], I answer that scapegoating does not create nationalism, but does make it salient. Much of the literature on diversionary conflict pertains to foreign policy analysis [see for instance Hagan, 2017, for a survey]. In this strand, the existence of diversionary use of force is either tested or assumed. The empirical evidence on its existence is mixed [Levy, 1989; Chiozza et al., 2004]. Some [Powell, 2014; Jung, 2014; Murray, 2017] assume diversionary conflict in order to empirically study the drivers, implementations or performances of such policies. Others [Oakes, 2012; Davies, 2016] simply assumes them as part of a leader's set of possible strategies to secure her political position. Mostly, this literature recognizes two motivations for diversionary use of force: obtain rally effects; or carry information.

The later justification emerged from economic theory works [Richards et al., 1993; Downs and Rocke, 1994; Hess and Orphanides, 1995]. These papers rely on principal-agent models, in which the outcome of the war is a signal that can determine reelection of the agent. Chiou et al. [2014] twists the argument by arguing that a leader's strategy affects the information environment and thus changes the outcome of a coordination game. Further theoretical contributions [Tarar, 2006; Gent, 2009] include this incentive in international relation models. The former justification, that of rally effects, has been largely highlighted in many related literatures. To the best of my knowledge, they have never been micro founded. It has been justified, as in Theiler [2018], through the social identity theory from sociology; or assumed in theoretical works such as Arena and Bak [2013]. My contribution on this front is thus double: I formalize a new mechanism to justify diversionary conflicts, and I provide a game theoretical insight to diversionary wars justified by rally effects.

In a larger perspective, about conflict theory, I propose a mechanism that contributes to solve the paradox of war, as underlined by Fearon [1995], and which would fall in the agency problem category of Jackson and Morelli [2011]. In particular, the diversionary benefit of war would relate to a political bias as coined by Jackson and Morelli [2007]. Accordingly, Tarar [2006] proves how assuming diversionary motives can lead to the disappearance of the bargaining range. In the political economy literature, I relate to some works on extraction. My mechanism closely relates to Padró i Miquel [2007], that shows how ethnic discrimination within a country permits extraction. Finally, while Acemoglu and Robinson [2005] and Ticchi and Vindigni [2008] see political regimes as a commitment mean for the elite, here a lack of rebellion opportunities is seen, in the long run, as a commitment device for the population.

The remainder of this work is organized as follows. Some anecdotal historical evidence is briefly exposed in Section 3.2. Section 3.3 introduces the setup, while Section 3.4 presents and discusses the equilibrium. Section 3.5 proves the existence of a diversionnary incentive. Long run effects are explored in Section 3.6. Section 3.7 concludes.

3.2 Motivating examples

In this section, I briefly review three historical examples of the use of war as a policy tool. This anecdotal evidence serves as motivation for model considered. Each example is exposed through the lens of the setup, and mainly relies on one referential work. These expositions are not meant to be exhaustive; they do not – and could not – expose the complete picture of each of the political and historical context. However, these examples should give interesting insights on some recurrent patterns, that are explicited at the end of the section.

3.2.1 Franco-Prussian War

In the middle of the 19^{th}, what is currently a federated Germany was still heavily divided. Otto von Bismarck, who had just won a war, was now actively working towards a German unification. However, the South States that Bismarck wanted to encompass in the Empire were reluctant to relinquish their independence. As reported by [Wawro, 2005, p.30], these states actually had strong animosities towards Prussia, to which they would prefer France. Bismarck, however, exploited France's territorial ambitions on Belgium, Luxembourg and Rhineland in order to change their opinion. Quickly, the States promised men; but no annexation was effective yet. Three years of tensions and diplomatic crisis followed.

Instead of simply declaring war on France, it was important for Bismarck "*to make the French declare war on Prussia, so as to trigger the south German alliances*" [Wawro, 2005, p.37]. First, he instigated a dispute over the Spanish crown succession; but when this did not resolve in war, he wrote a dispatch to the press, falsifying the content of the discussion between a French ambassador and the Prussian king to make it look like the ambassador had been insulted. French opinion ignited and on the 19 June 1870, France had to declare war. Six month later, after a victory on the field, Bismarck also won the four intractable South States. The Constitution of the German Empire was signed; Germany was unified.

First, notice that Bismarck's behavior is difficult to reconcile to the only other economic argument behind diversionary war. Indeed, Bismarck would not have had to signal his competence, as he had just won the Austro-Prussian wars. Another argument often cited is that of war as a mean to revive a patriotic sentiment. But again, this explanation falls short: the South States had no strong nationalist feeling. Furthermore, a mere war was not enough here. Bismarck indeed multiplied tactic in order to not be the instigator of the war. I thus take two lessons from this example. First, it is widely accepted that Bismarck used these tensions with France as a political tool in order to extract concession from the South States. Second, for this to be an efficient tool, France had to look threatening. Here, Bismarck could benefit from this costly war; but, this worked only because the concessions he was extracting from the South States were less costly to them than the threat posed by a French victory.

3.2.2 Cold War

According to Zubok [2009], in the first month of peace after WWII, the Soviet people were yearning for peace. The war had shaped a strong sense of identity in them, but also deeply scarred their economy. Despite these preferences, both of the people and the elite, Stalin multiplied the action in order to deteriorate the relationship with the West. [Zubok, 2009, p. 29] After 1946 Churchill's iron curtain speech, Stalin replied by accusing the British politician to seek world domination. This changed the public sentiment, as "*the common public wish from now on would not be cooperation with the Western powers but the prevention of war with them. This fear was exactly what Stalin needed to promote his mobilization campaign.*" [Zubok, 2009, p. 53]. Furthermore, "*the winds of a new war also helped Stalin to stamp out any potential discontent and dissent among the elites. The majority of state officials and military officers in the Soviet Union were convinced that the West was on the offensive and had to be contained.*" [Zubok, 2009, p. 60]. After the death of Stalin in 1953, his successor, Khrushchev, launched a de-Stalinization of the country, which involved more liberal policies both internally and externally: from there on, the West was not a threat anymore [Zubok, 2009, p. 104]. This period interestingly corresponds to domestic uprisings and internal turmoil. While the Hungarian Revolution of 1956 is arguably the archetypical mass uprising, Kozlov and MacKinnon [2002] actually documents numerous other popular insurrections, from 1953 to the late 80s. The Cold War ended in 1991 with the collapse of the Soviet Union. This downfall is generally imputed to Gorbachev, who famously advocated liberal policies, and the integration of capitalist principles into the economical system. However, the inadequacy between the ideological systems lead to deceiving results; and popular discontentment rose. Eventually, various revolutions in the Soviet block – coupled with an attempted coup – caused the disintegration of the Soviet Union.

Of course, given the complexity and the length of this conflict, the above exposition barely introduces the issue. However, these few elements all underline a very interesting relationship between internal compliance and external threat. First notice, the ideological confrontation with the West helped Stalin implement policies that were impoverishing his population and discontenting the *nomenklatura*. Of course, other forceful implementations and preventive repressions also accompanied the Stalinian regime, as for example attested by the Trial of the Generals in 1951 or the Slánský Trial in 1952. Yet, it is still interesting to note how the threat of Western victory complemented the other means of repression. One should also note that, despite the absence of open violence in the core of the two blocks, the Soviet Union still had to bear huge due to the conflict, in particular for arming. The second striking element is the timing of popular uprising after Stalin's death, that seem to follow a shift in the state's assessment of the external threat. In February 1956, Khrushchev advertises his *peaceful intentions*; and in October 1956, the major Hungarian Revolution breaks out. Then, Gorbachev comes and establishes a new strategy, with the introduction of capitalist elements to the Soviet system; the Union ends up disintegrating.

3.2.3 The Yougoslavian War

In the late 20$^{\text{th}}$, the Western world was looking in horror at the heart of Europe: a war was raging. Interpreted by most commentators as an ethnic conflict belonging to a premodern society, Gagnon Jr [2006] shows how the ethnic rhetoric was used and abused by the conservative ruling party, and its president Milošević, in order to instigate a war. It all began in the early 1990, when the Serbian ruling party's position was threatened from all parts: from Slovenia and Croatia, whose population seemed to impose political change; from rival political parties, whose policies were widely endorsed by the public; and from their population, who was mobilizing to ask for competitive elections [Gagnon Jr, 2006, pp. 90-91]. But this conservative party had a plan: induce external violence to demobilize internal opposition. In May 1990, they began instigating violence with Croatia, all the while presenting Croatia as the perpetrator of unjust violence against innocent Serbs. [Gagnon Jr, 2006, pp. 94-95]. They continued provoking as much counteroffensive as possible. "*In Serbia, these conflicts were portrayed as evidence of the Croatian regime's intentions to rid itself of its Serb population*" [Gagnon Jr, 2006, pp. 100]. For years, this worked. Back and forth, popular opposition was rising, and the conflict was worsening [Gagnon Jr, 2006, ch. 4]. Finally, in 1996, the conflict came to an end, and Milošević had to agree to peace. "*Once the Bosnian conflict ended, a popular mass mobilization movement once again surfaced in Serbia. (...) In response, the grievances that had existed since at least 1990 but which had been demobilized by the wars and the images of threat now burst into the open.*" [Gagnon Jr, 2006, p. 121]. In February 1998, the conflict in Kosovo began, once again ignited by Serbian forces. "*Meanwhile, faced with growing dissatisfaction and anti-regime mobilization at home, Milošević pointed to the growing unrest in Kosovo as evidence of a continued threat.*"[Gagnon Jr, 2006, p. 125]. However, this time, the international community entered the stage with an ultimatum for Milošević to sign a peace agreement. Upon his refusal, the United States launched several bombing on Serbia, which united all of the Serbian opposition on Milošević's side [Gagnon Jr, 2006, p. 124]. The opposition was again successfully demobilized. Finally in 2000, once the hostilities came to an end, the opposition united, and, endorsed by large popular rallies, successfully demanded elections. Milošević lost.

Gagnon Jr [2006] multiplies evidence to show that the *ethnic* Yougoslavian War was actually a political strategy to weaken internal opposition. Notice how Milošević did not merely declare war, but orchestrated an escalation of violence in order for Croatia and the rest of Yugoslavia to look threatening. In particular, he used state television and newspaper propaganda to show how Serbs' life would be endangered if they were in contact with Croats. This strategy goes beyond nationalist rhethoric. Gagnon Jr [2006, (ch. 2)] indeed provides poll data to show how mild was the ethnic divide before the war. It seems that Milošević's strategy could not have only rested on a tribal instinct of violence against *the others*. The key to the success of his strategy was for the enemy to look threatening.

3.2.4 From History to Theory

These three examples put into perspective two interesting patterns. First, *being at war* seems to give a leader particular privileges that are revoked once peace is achieved again. Therefore, it seems that that *state of war* is the peculiarity that leaders are after during internal turmoil. Second, more than an enemy, it seems that leaders need a *threatening* enemy to successfully use war for their end. It follows that leaders do not only rely on patriotic sentiments, as often claimed by political commentators.

The only other micro-founded contribution to the diversionary nature of wars does not account for such behaviors; in this setup, in contrast, they are the focal point. In particular, I argue that the state of war is particular because it adds a cost on rebelling. This cost depends on how threatening is the other country. Internal uprising should indeed weaken a country's position on the front; the more threatening the enemy, the more dangerous it is to weaken one's position in the war.

3.3 Setup

Under study is the incentive for a leader to instigate a conflict with another nation, rather than the bilateral occurrence of war. Therefore, I limit the analysis to one country, which is populated by two players: a leader (she) and citizens whose interests are aligned (they). The leader decides on both internal policies and external policies, while the population's role is to express its agreement with either type of policies. In particular, the population can decide to support belligerent policies; but also to oppose internal policies. In the latter case, the population rebels and ousts the leader in a costly manner.

The policy decided within the country is captured by a continuous scalar τ, which is interpreted as a tax rate.[1] The foreign policy is however discrete, with three level of conflicts: peace, tensions and open war. Open war can occur only if tensions were previously initiated. [2] The decision to initiate tensions is denoted $\theta \in \{0,1\}$, and that of declaring war $\omega \in \{0,1\}$. The population can express support on foreign policy, but only concerning tensions; this choice is captured by $\eta \in \{0,1\}$. Furthermore, the population decides whether to rebel: $\psi \in \{0,1\}$.

The total production from the country is normalized to 1. All costs are expressed as ratio of the country's production. The cost of conflict is κ_t for instigating tensions and κ_w for fighting war. Both of these include tangible and intangible costs. For instance, κ_t can be interpreted as a diplomatic loss of power but also as trade impairments or arming costs. I abstract from the issue of winning or losing: κ_w represents the net cost of war, net of potential advantages derived from winning the war and including further disadvantages from losing it.

Rebelling also comes at a cost. In case of rebellion, the population pays a price ρ in order to oust the leader. However, the population raises this cost to ρ' when they

[1] The interests of the leaders might not be pecuniary. Likewise, the concerns of the population might go beyond economical disagreements. This however capture misaligned interests between the leader and her population in a very conservative way.

[2] There is no reason to assume that the outcome from a war instigated after tensions would be different from that of a war that was not preceded by tensions. Therefore, allowing for open war without tension would not change the conditions for war, but could create multiple equilibria. For clarity concerns, I impose this restriction.

decide to support the leader's decision to initiate tension. It is indeed assumed that a public support has an intrinsic value: coordination and organization would be harder for inconsistent public opinion. This additionally captures how a population that advertised animosities towards a foreign nation would be worse off forfeiting against them. [3]. Indeed, if a rebellion is attempted in time of conflict, the country has to forfeit on the international front. The additional cost of forfeiting, irrespective of the support decision is defined as ϕ_t in case of tensions and as ϕ_w in case of war. In case of rebellion, the leader's payoff is normalized to 0. The population grabs what is left $(1-\phi(h))(1-\rho(h))$, where h represents the history that led to the rebellion.

Finally, taxes are modeled as a transfer of utility. Furthermore, differences in risk attitudes are ignored. Any cost is perceived similarly by both player. For instance, if a war is declared and no revolution occurs, the available interior product is reduced from 1 to $1-\kappa_w$ for both players, who then share this amount according to the taxation rate proposed by the leader.

The timing is as follows:

1. The leader decides whether to initiate tension, $\theta \in \{0,1\}$.
2. If $\theta = 1$, the population decides on support, $\eta \in \{0,1\}$; no decision otherwise.
3. If $\theta = 1$, the leader decides whether to intensify the conflict into an open war, $\omega \in \{0,1\}$; no decision otherwise.
   The leader sets the taxation, $\tau \in [0,1]$.
4. The population decides whether to rebel, $\psi \in \{0,1\}$.

The extensive form of this game is depicted in Figure 3.1.

Finally, consider the following tie rule: when a player is indifferent, they choose the status quo – i.e. the more peaceful decision for the leader; no revolution and no support for the population.

3.4 Equilibrium

I characterize the Subgame Perfect Equlibrium (SPE) of this game by using backward induction. Note that the only tie rule necessary for the existence of an equilibrium is the one pertaining to revolution; however, I do not specify the equilibria under other tie rules, as no results is driven from it.

[3]Appendix 3.B considers alternative specifications in which the population's decision to support tension, rather than impacting ρ: (i) impacts the entire payoff by a factor $(1-x)$; (ii) directly impacts ϕ_t and ϕ_w. I show how all results carry out.

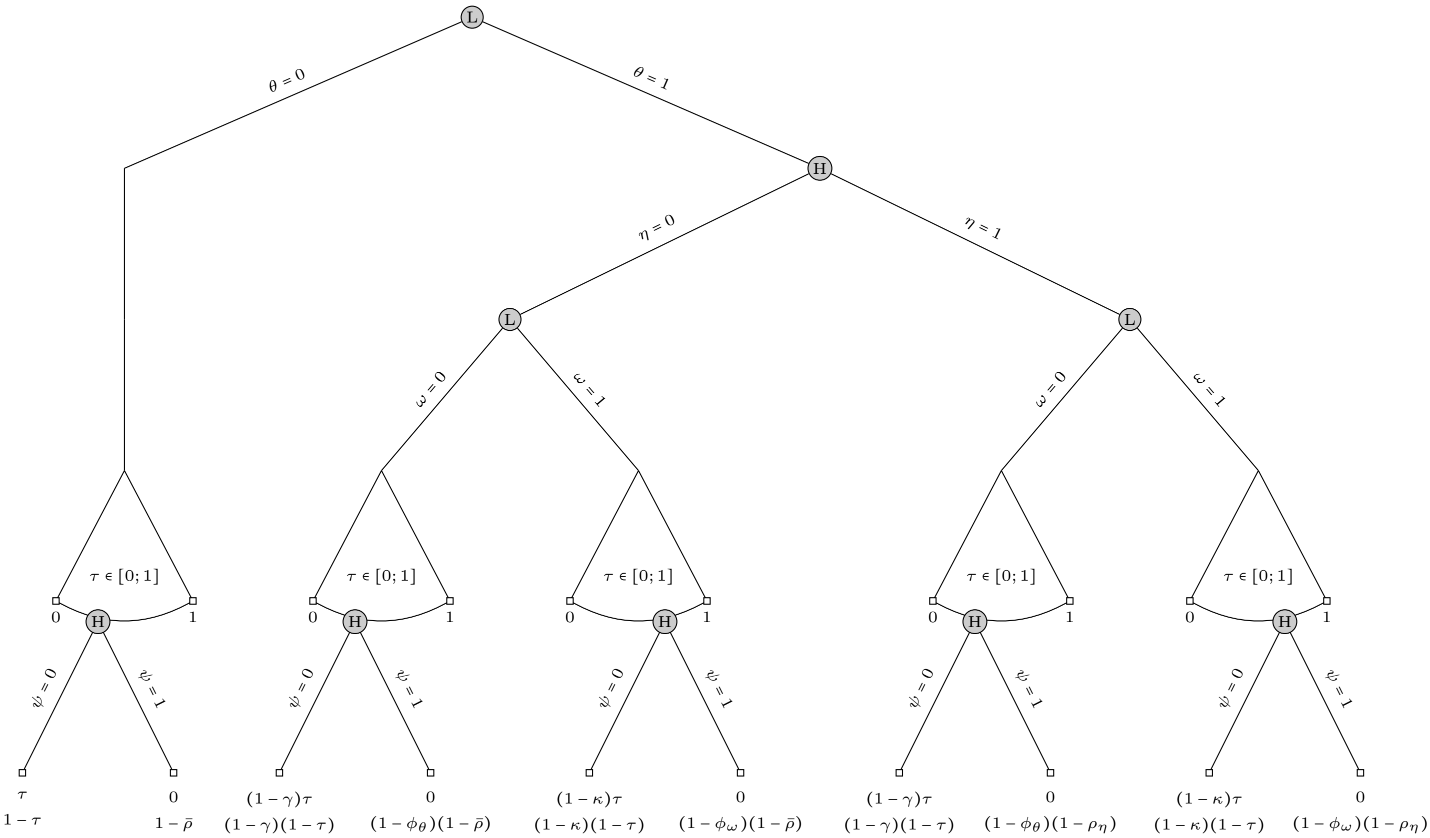


Figure 3.1: Extensive Form Game

In the last stage, the population rebels if the tax rate is high enough for a rebellion to lead to a higher payoff than the status quo. Hence, in the penultimate stage, the leader sets the tax rate to make the population indifferent between rebelling or not, i.e.

$$\tau(h) = 1 - \frac{(1-\phi(h))(1-\rho(h))}{1-\kappa(h)}$$

at every partial histories $h \in H_3$ [4] [5] .

When deciding whether to escalate, the leader internalizes all subsequent moves and compares $1 - \kappa_w - (1-\phi_w)(1-\rho(\eta))$ to $1 - \kappa_t - (1-\phi_t)(1-\rho(\eta))$ [6]. Hence, he choses:

---

[4] Let H_3 be the set of histories until the tax decision node:
$H_3 = \{(\theta=0), (\theta=1, \eta=0, \omega=0), (\theta=1, \eta=0, \omega=1), (\theta=1, \eta=1, \omega=0), (\theta=1, \eta=1, \omega=1)\}$

[5] Let $\rho(h) = \begin{cases} \rho' & \text{if } h \in \{(\theta=1, \eta=1, \omega=0), (\theta=1, \eta=1, \omega=1)\} \\ \rho & \text{otherwise} \end{cases}$. Likewise,

$\phi(h) = \begin{cases} \phi_w & \text{if } h \in \{(\theta=1, \eta=0, \omega=1), (\theta=1, \eta=1, \omega=1)\} \\ \phi_t & \text{if } h \in \{(\theta=1, \eta=0, \omega=0), (\theta=1, \eta=1, \omega=0)\} \\ 0 & \text{if } h = (\theta=0) \end{cases}$ and $\kappa(h) = \begin{cases} \kappa_w & \text{if } h \in \{(\theta=1, \eta=0, \omega=1), (\theta=1, \eta=1, \omega=1)\} \\ \kappa_t & \text{if } h \in \{(\theta=1, \eta=0, \omega=0), (\theta=1, \eta=1, \omega=0)\} \\ 0 & \text{if } h = (\theta=0) \end{cases}$

[6] By a slight abuse notation, I refer to history $(\theta = 1, \eta)$ as (η) for the remainder of this paper.

- to always escalate to war if $\kappa_w - \kappa_t < (1-\rho')(\phi_w - \phi_t)$
- to never escalate to war if $\kappa_w - \kappa_t \geq (1-\rho)(\phi_w - \phi_t)$
- to escalate only if he is not supported if $(1-\rho')(\phi_w - \phi_t) \leq \kappa_w - \kappa_t < (1-\rho)(\phi_w - \phi_t)$

The population will decide to support the tension only if this decision avoids open war. Indeed, if the parameters are such that the leader takes the same action, irrespectively of the population's decision, then support always worsen the payoff for the population. Hence, for the population to support the tensions, it must be that $(1-\rho)(\phi_w - \phi_t) \leq \kappa_w - \kappa_t < (1-\rho')(\phi_w - \phi_t)$. For such parameters, the population must actually be willing to pay the price of support. In particular, the price of support must be lower than the price of escalating to war. Hence, $\eta = 1$ additionally requires $(1-\phi_t)(1-\rho') > (1-\phi_w)(1-\rho)$.

In the first stage, the leader decision depends on the parameters:

- If $\kappa_w - \kappa_t < (1-\rho)(\phi_w - \phi_t)$, the leader compares peace to war (with no support); he chooses peace if $\rho \geq 1 - \kappa_w - (1-\phi_w)(1-\rho)$.
- If $\kappa_w - \kappa_t \geq (1-\rho')(\phi_w - \phi_t)$, the leader compares peace to tensions (with no support); he chooses peace if $\rho \geq 1 - \kappa_t - (1-\phi_t)(1-\rho)$.
- If $(1-\rho)(\phi_w - \phi_t) \leq \kappa_w - \kappa_t < (1-\rho')(\phi_w - \phi_t)$ and $(1-\phi_t)(1-\rho') > (1-\phi_w)(1-\rho)$, the leader compares peace to supported tensions; he chooses peace if $\rho \geq 1 - \kappa_t - (1-\phi_t)(1-\rho')$.
- If $(1-\rho)(\phi_w - \phi_t) \leq \kappa_w - \kappa_t < (1-\rho')(\phi_w - \phi_t)$ but $(1-\phi_t)(1-\rho') \leq (1-\phi_w)(1-\rho)$, the leader compares peace to war (with no support); hence the condition for peace is again $\rho \geq 1 - \kappa_w - (1-\phi_w)(1-\rho)$.

In order to lighten notation, I define the following values:

- $\underline{\kappa}_t = \phi_t(1-\rho')$
- $\bar{\kappa}_t = \phi_t(1-\rho)$
- $r_\rho = \frac{1-\rho'}{1-\rho}$
- $\underline{\kappa}_w = \phi_w(1-\rho')$
- $\bar{\kappa}_w = \phi_w(1-\rho)$
- $r_\phi = \frac{1-\phi_w}{1-\phi_t}$

$\bar{\kappa}_t$ (resp. $\bar{\kappa}_w$) can be interpreted as the cost from which tensions (resp. war) are beneficial *per se* for the leader; and $\underline{\kappa}_t$ (resp. $\underline{\kappa}_w$) as the cost from which tensions (resp. war) are beneficial for the leader only under support. Finally, r_ρ is a ratio representing how affordable support is for the population while r_ϕ indicates how cheap war is to them relatively to tensions.

Given the previous discussion, the SPE is defined in Lemma 13

Lemma 13. *The SPE is as follows:*

- *The leader's equilibrium strategy is:*
  - $\theta = 1$ *if* $\kappa_w < \bar{\kappa}_w$*; or if* $\kappa_t < \bar{\kappa}_t$*; or if* $\underline{\kappa}_w - \underline{\kappa}_t \leq \kappa_w - \kappa_t < \bar{\kappa}_w - \bar{\kappa}_t$*, and* $r_\phi < r_\rho$*, and* $\kappa_t < \underline{\kappa}_t + \rho' - \rho$*;*
    $\theta = 0$ *otherwise.*
  - $\omega(\eta = 0) = 1$ *if* $\kappa_w - \kappa_t < \bar{\kappa}_w - \bar{\kappa}_t$*;* $\omega(\eta = 0) = 0$ *otherwise.*
    $\omega(\eta = 1) = 1$ *if* $\kappa_w - \kappa_t < \underline{\kappa}_w - \underline{\kappa}_t$*;* $\omega(\eta = 1) = 0$ *otherwise.*
  - $\forall h \in H_3, \tau(h) = 1 - \frac{(1-\phi(h))(1-\rho(h))}{1-\kappa(h)}$
- *The population's equilibrium strategy is:*
  - $\eta = 1$ *if* $\underline{\kappa}_w - \underline{\kappa}_t \leq \kappa_w - \kappa_t < \bar{\kappa}_w - \bar{\kappa}_t$ *and* $r_\phi < r_\rho$*;* $\eta = 0$ *otherwise.*
  - $\forall h \in H_3 \times [0,1]$ *and corresponding* $h' \in H_3$*:*
    $\psi(h) = 1$ *if* $\tau > 1 - \frac{(1-\phi(h'))(1-\rho(h'))}{1-\kappa(h')}$*;* $\psi(h) = 0$ *otherwise.*

Notice that rebellion never occurs in equilibrium,. This directly follows from perfect information. The leader will always set the taxation rate just low enough to avoid popular uprising. Likewise, a war following support never happens in equilibrium. Again, this follows directly from the assumptions that are meant to reflect this paper's interpretation of such behaviors: a way to avoid more severe conflict.

Theorem 5. *There are four possible outcomes: peace, (unsupported) war, unsupported tensions and supported tensions.*

- *War occurs iff* $\kappa_w < \bar{\kappa}_w$*; and* $\kappa_w - \kappa_t < \bar{\kappa}_w - \bar{\kappa}_t$*; and either* $\kappa_w - \kappa_t < \underline{\kappa}_w - \underline{\kappa}_t$*, or* $r_\phi \geq r_\rho$*.*
- *Supported tensions occur iff* $\underline{\kappa}_w - \underline{\kappa}_t \leq \kappa_w - \kappa_t < \bar{\kappa}_w - \bar{\kappa}_t$*; and* $r_\phi < r_\rho$*; and* $\kappa_t < \underline{\kappa}_t + \rho' - \rho$*.*
- *Unsupported tensions occur iff* $\kappa_t < \bar{\kappa}_t$*; and* $\kappa_w - \kappa_t \geq \bar{\kappa}_w - \bar{\kappa}_t$*.*
- *Peace occurs iff* $\kappa_w \geq \bar{\kappa}_w$*; and* $\kappa_t \geq \bar{\kappa}_t$*; and either* $\kappa_t \geq \underline{\kappa}_t + \rho' - \rho$*, or* $r_\phi \geq r_\rho$*, or* $\kappa_w - \kappa_t < \underline{\kappa}_w - \underline{\kappa}_t$*, or* $\kappa_w - \kappa_t \geq \bar{\kappa}_w - \bar{\kappa}_t$ *.*

Remark 11. *In equilibrium, the population can indeed rally around the flag.*

Proof. Lemma 13 is used to find the set of conditions determining the action at each node. They can then be groupped by possible outcomes and redundant conditions are simplified. Details can be found in Appendix 3.A. □

Hence, for peace to exist, it is necessary that neither type of conflict is beneficial *per se*, and that escalation is not either; but it is not sufficient. It must also be that the leader does not have an incentive to initiate conflict in order to initiate a rally around the flag –

so either, support is simply not worth the cost of tension, or support is too expensive for the population, or it would be inefficient in preventing escalation, or aggravating conflict is not credible for the leader. This is actually what supported tensions require: for support to be necessary and efficient at preventing escalation, but also for it to be preferred by the population, and by the leader. Unsupported tensions occur when low-level conflict is beneficial to the leader *per se*, but escalation is not. Finally, for a war to happen, it is necessary that war is beneficial *per se* but also that escalation is. Again, these two conditions are not sufficient, as it must also be that the population cannot or is not willing to avoid escalation through support.

3.4.1 Discussion

Let us now focus one some particular sets of parameters, in order to have a better grasp on the importance of each parameter. The parameters' importance will then be discussed in general.

First, what if raising international tensions was not costly, i.e. $\kappa_t = 0$? If additionally, such tensions can be freely forfeited, that is $\phi_t = 0$, then no unsupported tensions can occur. Indeed, this would mean that tensions are without consequences for either player, so that the leader would never have a strict incentive to initiate tension for the sake of it. However, it is interesting to notice that peace is not ensured either. If $\kappa_w < \bar{\kappa}_t$, war is beneficial *per se*, that is, it decreases the tax base less than it allows to increase the tax rate. There are two possible outcomes then: either the population can and is willing to avoid escalation through support, resulting in supported tensions; or they are not, resulting in war. Now, if $\kappa_t = 0$ but $\phi_t > 0$, peace can never occur. Indeed, this means that the tax base does not decrease for the leader, but the population is threatened by such tensions. Therefore, the leader can freely increase her payoff by initiating tensions. Whether this leads to war, supported or unsupported tensions depends on the other parameters.

On the other end, huge costs of tensions are not enough to insure peace, as it is not even sufficient to avoid tensions. In particular, if $\kappa_t > \phi_t$, that is, if tensions are reducing the countries wealth more when affirmed than when forfeited, it is still possible for tensions to occur in equilibrium, but only if they are supported. Indeed, the population can credibly support such a conflict in order to avoid war, and not rebel in equilibrium. It is surprising because, staying in a low-level conflict creates an additional incentive to rebel, that must be balanced off by a smaller taxation rate. However, while the leader has to pay people in order to avoid rebellion under tensions, he is still paid by the support the population gives her in order to avoid war. Therefore, it can be that, in equilibrium, tensions occur despite their huge cost.

Another interesting case regarding costs occurs when escalation to war has proportional effects for both players, that is $\frac{\kappa_t}{\kappa_w} = \frac{\phi_t}{\phi_w}$. In such a context, unsupported tensions never occur, because if tensions were beneficial *per se*, war *a fortiori* would be. The occurrence of peace depends on the level of fighting costs κ, while the occurrence of war depends on their ratio. Indeed, the highest the above ratio, the more similar the effects of war relative to tensions are on the population, hence the more likely war becomes. Support gets indeed relatively too costly for high value of this ratio.

However, it is worth noting that, in general, higher costs of fighting tend to decrease the occurrence of either type of conflicts. Hence, it tends to have beneficial effects for the society – any type of conflict is indeed Pareto inefficient. Yet, while increasing costs *sufficiently* to ensure peace is unambiguously desirable, some increases in fighting costs might be counter-productive. Indeed, if the increase is not sufficient to insure peace, it ends up creating a higher deadweight loss from fighting a conflict that is now destroying more wealth.

Likewise, if low rebellion costs should, at first sight, be desirable, they are not unambiguously so. First, notice that $\rho = 0$ is rarely desirable, as peace would then rarely be possible. Indeed, in time of peace, the cost of revolution is the only pressure the leader can use in order to extract wealth from his population without being ousted. Decreasing revolution costs below the level that insures the leader the same revenue as in time of conflict would thus worsen the situation for both players, as this would create the inefficiency of actually having to create a conflict in order for the leader to extract wealth from the population without risking to be ousted. Therefore, a very high ρ can actually be optimal from a social perspective. I further analyze this tradeoff in Section 3.6.

The role of the commitment price ρ' is even more ambiguous. Indeed, if it is too small, it could be inefficient in convincing the leader to renounce to war, and thus would lead to a high-level conflict rather than reaching Pareto preferred supported tensions. But a ρ' that is too high could dissuade the population from preventing war through support. If the leader initiates tensions only for diversionary reasons[7], then a commitment price that is high enough to make popular support impossible would actually insure peace. In such a case, a very high ρ' is preferable. However, if war is beneficial *per se*, a commitment price that is too high could make war avoidable but not avoided. Then, lower ρ' would be socially preferred.

Finally, it can unambiguously be concluded that the higher the costs of forfeiting, the more menacing the threat of conflict. This means a bigger extraction power for the leader, who is hence more prone to enter conflicts, which creates inefficiencies. In Section 3.6, I further analyze how the mere existence of this threat leads the population to both

[7]That is, only in anticipation of popular support. I formalize the concept of *diversionary incentives* considered in Section 3.5.

rationally and efficiently renounce to some of their freedom.

3.5 Diversionary Incentive

In this section, I prove the existence of a *diversionary incentive* for conflicts. Loosely speaking, one could call a war *diversionary* when it is used to prevent a leader's position to be questioned. In this setup however, the incentive to use conflict is, by assumption, always that of preventing a rebellion while implementing demanding policies. While I do find that conflict can occur in equilibrium, this does not fully address the question.Tthe population always accepts the internal policy in equilibrium, but popular support of an external conflict is not given. Theorem 5 shows that rally around the flag do occur. The next logical question is thus: does the anticipation of such a support suffice for a leader to initiate war?

The definition of a diversionary incentive thus follows:

Definition 4. A leader initiates conflict because of a *diversionary incentive* when:

- the leader anticipates the public to support it; and
- the leader would not have initiated it, were it not supported by the public.

Using Lemma 13 and Theorem 5, this definition can be translated in terms of the setup's parameters.

Corollary 11. *A diversionary incentive emerges when:*

1. *The leader initiates conflict:* $\kappa_t < \underline{\kappa}_t - \rho' + \rho$.
2. *Support can credibly be anticipated:* $\underline{\kappa}_w - \underline{\kappa}_t \leq \kappa_w - \kappa_t < \bar{\kappa}_w - \bar{\kappa}_t$ *and* $r_\phi < r_\rho$.
3. *Neither conflict would be initiated otherwise:* $\kappa_t \geq \bar{\kappa}_t$ *and* $\kappa_w \geq \bar{\kappa}_w$.

Proof. Conditions in 1. and 2. correspond to the conditions for supported tensions derived in Theorem 5. Conditions in 3. are derived using the equilibrium strategies specified in Lemma 13; they correspond to the conditions for peace for the alternative game in which the population is not active until the last period (i.e. $\eta \in \{0\}$). □

If war was not beneficial *ex ante*, how could it be a credible threat to escalate the conflict? If the threat is not credible, popular support is impossible. Yet, the threat *can* be credible: once tensions have been instigated, war might become more attractive, making escalation credible. Likewise, while tensions would not be beneficial *per se*, the additional benefit the leader would derive from popular support would make them attractive *ex ante*. Finally, one could argue that, if the support is valuable enough to make the leader willing to instigate tensions that would otherwise be too expensive, then the population cannot be willing to pay the price of support. Again, this does not stand when confronted to the respective formal conditions. Actually, all conditions of Corollary 11 can be met at the same time.

Theorem 6. *There exists a non-zero measure parameter space $\mathcal{D}$ for which the leader has a diversionary incentive to initiate conflict.*

Proof. To prove that $\mathcal{D}$ is non-zero measure, let us find a subset $\mathcal{D}' \subseteq \mathcal{D}$ that is non zero measure. $\mathcal{D}'$ is arbitrarily chosen as follows:

$$\mathcal{D}' = \left\{ (\kappa_t, \kappa_w, \phi_t, \phi_w, \rho, \rho') : \forall (x, \epsilon) \in \mathcal{E}, \begin{array}{lll} \rho = x; & \phi_t = x + \epsilon_2; & \kappa_t = x + \epsilon_4; \\ \rho' = x + \epsilon_1; & \phi_w = x + \epsilon_3; & \kappa_w = x + \epsilon_5; \end{array} \right\}$$

$$\text{where } \mathcal{E} = \left\{ (x, \epsilon) \in (0,1)^6 : \begin{array}{c} 1 - x > \epsilon_4 + \epsilon_1 > \epsilon_5 > \epsilon_4 + (1 - \epsilon_1)(\epsilon_3 - \epsilon_2) \\ \epsilon_5 > \epsilon_3 > \frac{3}{2}\epsilon_1 + \epsilon_2 \end{array} \text{ and } \min\{t, \tfrac{1}{3}\} > x \right\}$$

$$\text{with } t = \frac{\sqrt{(\epsilon_1+\epsilon_2)^2 + 4(\epsilon_1+\epsilon_3-\epsilon_5-\epsilon_1\epsilon_3)} - (\epsilon_1+\epsilon_2)}{2}$$

The conditions in $\mathcal{E}$ in particular imply $\epsilon_1 > \max\left\{\frac{\epsilon_5-\epsilon_3}{1-\epsilon_3}; \frac{\epsilon_3-\epsilon_2}{1+\epsilon_3-\epsilon_2}; \frac{1}{3}\right\}$. Indeed, for the interval for ϵ_5 to exist, it must be that $\epsilon_1 > (1-\epsilon_1)(\epsilon_3 - \epsilon_2)$, meaning $\epsilon_1 > \frac{\epsilon_3-\epsilon_2}{1+\epsilon_3-\epsilon_2}$. On the other hand for t to be well-defined and positive, it must be that $4(\epsilon_1 + \epsilon_3 - \epsilon_5 - \epsilon_1\epsilon_3) > 0$, hence $\epsilon_1 > \frac{\epsilon_5-\epsilon_3}{1-\epsilon_3}$. Finally, $\epsilon_1 > \frac{\epsilon_3-\epsilon_2}{1+\epsilon_3-\epsilon_2}$ and $\epsilon_3 - \epsilon_2 > \frac{3}{2}\epsilon_1$ implies $\epsilon_1 > \frac{1}{3}$. Because none of these implications contradict any other defined in $\mathcal{E}$, $\mathcal{E}$ is non-zero measure.

It is easy to verify that any vector from $\mathcal{D}'$ belongs to the space of feasible parameters. Indeed $1 - x > \max_{i=1,\ldots,6}\{\epsilon_i\}$ so all parameters are lesser than 1. Because $\min_{i=1,\ldots,6}\{\epsilon_i\} > 0$ and $x > 0$, all parameters are strictly positive.

All that is left to do is to prove that $\mathcal{D}' \subseteq \mathcal{D}$. I show that for any $v \in \mathcal{D}'$, the conditions from Corollary 11 are all fulfilled:

1. Leader prefers supported tensions to peace:
   Indeed: $\kappa_t - \phi_t(1-\rho') - \rho' + \rho = \epsilon_4 - \epsilon_1 - \epsilon_2 + (x + \epsilon_1)(x + \epsilon_2) < 0$
   It follows from $\epsilon_4 < \epsilon_5 - (1 - \epsilon_1)(\epsilon_3 - \epsilon_2) < \epsilon_1 + \epsilon_2 - (x + \epsilon_1)(1 + \epsilon_2)$ where the last inequality holds because $x < t$.

2. Support can credibly be anticipated:

   (a) Escalation without support is credible:
   Indeed: $\kappa_w - \phi_w(1-\rho) - \kappa_t + \phi_t(1-\rho) = \epsilon_5 - \epsilon_4 - (1-x)(\epsilon_3 - \epsilon_2) < 0$.

It follows from the definition of $\mathcal{E}$, in particular: $\epsilon_3 - \epsilon_2 > \frac{3}{2}\epsilon_1$, $\epsilon_5 - \epsilon_4 < \epsilon_1$, and $x < \frac{1}{3}$.

(b) Escalation with support is avoided:
Indeed: $\kappa_w - \phi_w(1-\rho') - \kappa_t + \phi_t(1-\rho') = \epsilon_5 - \epsilon_4 - (1-x-\epsilon_1)(\epsilon_3-\epsilon_2) > 0$.
It follows from $\epsilon_5 - \epsilon_4 - (1-\epsilon_1)(\epsilon_3-\epsilon_2) + x(\epsilon_3-\epsilon_2) > \epsilon_5 - \epsilon_4 - (1-\epsilon_1)(\epsilon_3-\epsilon_2) > 0$

(c) Preferred to war by the population:
$(1-\phi_w)(1-\rho) - (1-\phi_t)(1-\rho') = -(1-x)(\epsilon_3-\epsilon_2-\epsilon_1) - \epsilon_1\epsilon_2 < 0$
If follows from $\epsilon_3 > \frac{3}{2}\epsilon_1 + \epsilon_2$.

3. Neither conflict would be initiated otherwise:

   (a) Tensions are not beneficial *per se*:
   Indeed: $\kappa_t - \phi_t(1-\rho) = \epsilon_4 - \epsilon_2(1-x) + x^2 > 0$
   It follows from $\epsilon_4 - \epsilon_2(1-x) > \epsilon_4 - \epsilon_2 > 0$, since $\epsilon_4 + \epsilon_1 > \frac{3}{2}\epsilon_1 + \epsilon_2$.

   (b) War is not beneficial *per se*:
   Indeed: $\kappa_w - \phi_w(1-\rho) = \epsilon_5 - \epsilon_3(1-x) + x^2 > 0$
   It follows from $\epsilon_5 - \epsilon_3(1-x) > \epsilon_5 - \epsilon_3 > 0$.

□

Therefore, in a case for which the set of parameters belongs to $\mathcal{D}$, the very possibility for the population to commit through support decreases their payoff. The population would be better off without the freedom of rallying around the flag. The outcome would also be more efficient. However, this is not a general result. The population is better off with this choice when popular support prevents an otherwise unavoidable war. I further wonder about efficiency and rationality of flexible commitment means in section 3.6.

3.6 Long Run Effects

In this section, I wonder about the optimality and efficiency of flexible rebellion costs. In the long-run, the cost of popular uprising can indeed be considered endogenous: the institutional, military and legal impairments put in place in order to restrain rebellions are changeable, and the population can contribute to shape them.

Rather than discretely changing ρ to ρ' through the support decision, the population can freely chose the level of ρ. This decision is meant to represent the incentive for a population to put into place the institutional framework that would allow them to easily oust a leader. Obviously, this is a long run consideration. It particularly depends on the diplomatic and military condition of a country; but could not *follow* a short term decision to initiate international tensions. Therefore, from now on, I do not differentiate between tensions and open war anymore: a leader can either chose to initiate conflict, at cost κ, or not. If war is initiated, a population who decides to rebel would still suffer a cost of forfeit, uniquely defined as ϕ. These two parameters are now the only exogenous elements.

The new timing is as follows:

1. The population sets the cost of rebellion, $\rho \in [0, 1]$.

2. The leader decides her international position, $\omega \in \{0, 1\}$;
   she sets the taxation rate accordingly, $\tau \in [0, 1]$.

3. The population decides their domestic position, $\psi \in \{0, 1\}$.

The extensive form of the game is depicted in Figure 3.1. The same tie rules as before are kept. Furthermore, if the population is indifferent between $\rho = 0$ and $\rho > 0$, they set $\rho = 0$.

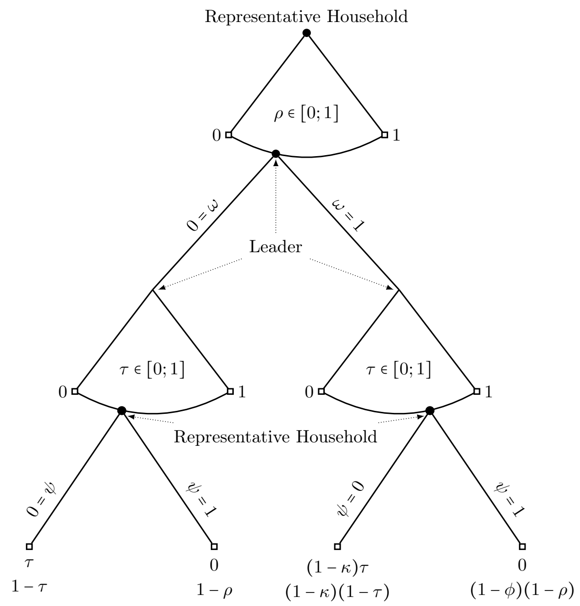


Figure 3.1: Long-Run Game: Extensive Form

3.6.1 Equilibrium

I characterize the SPE of this game using backward induction.

Because in the last stage, the population choses to rebel only if their payoff doing so is strictly greater than through the acceptance of the taxation rate, in the penultimate period, the leader sets the taxation rate to make them indifferent between $\phi = 0$ and $\phi = 1$. That is: $\tau : (1-\kappa)(1-\tau) = (1-\phi)(1-\rho)$. It follows that:

$$\tau(h) = 1 - \frac{(1-\phi(h))(1-\rho(h))}{1-\kappa(h)}$$

at every partial histories $h = (\rho, \omega) \in [0,1] \times \{0,1\}$. [8] .

When deciding whether to enter in a conflict, the leader trades off the loss in tax base induced by war with the increased tax rate he could be imposing during conflict. Hence, at history $h = (\rho) \in [0,1]$, he choses peace if $\rho(h) \geq 1 - \frac{\kappa}{\phi}$; and war otherwise.

In the first period, the population can preventing conflict by increasing rebellion cost. They know that any ρ lower than $1-\frac{\kappa}{\phi}$ would result in a conflict. Therefore, they compare their payoff with $\rho = 1-\frac{\kappa}{\phi}$ and $\omega = 0$ to $\rho = 0$ and $\omega = \mathbb{1}_{(\kappa<\phi)}$. Obviously, if $\kappa \geq \phi$, a conflict raises more costs than benefits, so that the leader would never start a conflict, and the population has no reason to set anything else than $\rho = 0$. Now, if $\kappa < \phi$, the population compares committing – $\frac{\kappa}{\phi}$ – to being at war – $1-\phi$. If $\kappa \geq \phi(1-\phi)$, they decide to increase rebellion costs above 0; otherwise, they do not and let war be used as an extraction tool.

The SPE is characterized in Lemma 14

Lemma 14. *The SPE of the LR game is as follows:*

- *The population's equilibrium strategy is:*
  - $\rho = 1 - \frac{\kappa}{\phi}$ *if* $\phi < \kappa < \phi(1-\phi)$; $\rho = 0$ *otherwise.*
  - $\forall h = (\rho, \omega, \tau) \in [0,1]\times\{0,1\}\times[0,1]$ *and corresponding* $h' = (\rho, \omega) \in [0,1]\times\{0,1\}$*:* $\psi(h) = 1$ *if* $\tau > 1 - \frac{(1-\phi(h'))(1-\rho(h'))}{1-\kappa(h')}$ *;* $\psi(h) = 0$ *otherwise.*
- *The leader's equilibrium strategy is:*
  - $\forall \rho \in [0,1]$*,* $\omega(\rho) = 1$ *if* $\rho < 1 - \kappa\phi$*;* $\omega(\rho) = 0$ *otherwise.*
  - $\forall h \in [0,1] \times \{0,1\}$*,* $\tau(h) = 1 - \frac{(1-\phi(h))(1-\rho(h))}{1-\kappa(h)}$

Again, because of perfect information, no rebellion ever occurs.

Theorem 7. *There are three possible equilibrium outcomes: conflict, committed peace and uncommitted peace.*

- *Conflict occurs iff* $\kappa \leq \phi(1-\phi)$*.*
- *Committed peace occurs iff* $\phi < \kappa < \phi(1-\phi)$*.*

[8] $\rho(h)$ is trivially defined in h. Let $\phi(h) = \begin{cases} \phi & \text{if } h \in [0,1] \times \{1\} \\ 0 & \text{if } h \in [0,1] \times \{0\} \end{cases}$ and $\kappa(h) = \begin{cases} \kappa & \text{if } h \in [0,1] \times \{1\} \\ 0 & \text{if } h \in [0,1] \times \{0\} \end{cases}$

- *Uncommitted peace occurs iff $\kappa \geq \phi$.*

Remark 12. *(i) In equilibrium, conflicts can occur despite perfectly flexible commitment means;*
(ii) The strength of the foreign threat has a non-monotonic effect on the prevalence of conflict;
(iii) The prevalence of commitment is positively linked with the strength of the foreign threat.

Proof. This directly follows from Lemma 14. □

Figure 3.2 depicts the different outcomes as a function of the parameters.

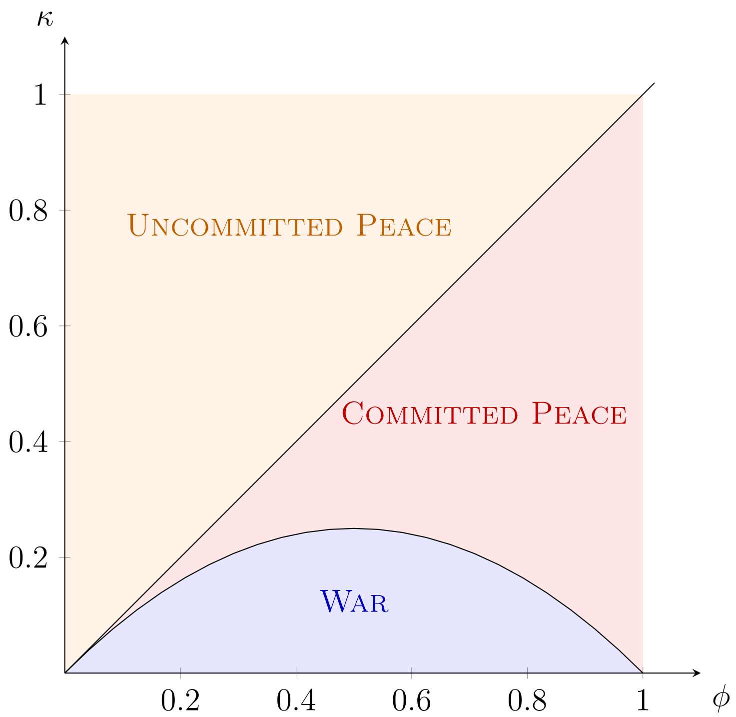


Figure 3.2: Possible Outcomes in Parameters Space

3.6.2 Discussion

As seen in Theorem 7, for the leader to use conflict as a tool of fiscal extraction, it has to be that conflict is relatively cheap. Interestingly, there are two effects at play. The cheapest conflict, the more the leader can benefit from war, as tax base decreases relatively less than the possible increase in tax rate. But the cheapest the conflict, the less prone is the population to commit in order to avoid it. These two forces create the non-monotonic effect of ϕ on the prevalence of conflict.

Because conflict is Pareto inferior, ϕ has the same non-monotonic effect in social efficiency. However, it has an unambiguous negative effect on the populations' equilibrium payoff. Indeed, an increase in ϕ has two negative effects for the population: on the one hand, in time of conflict, the leader can extract more from them, as the threat is bigger; on the other hand, the population has to pay a higher price in terms of ρ in order to preserve peace. The overall effect is negative for the population but mainly positive for the leader. Indeed, her payoffs tend to increase as a function of ϕ; however, because the deadweight loss created by conflict is born by the leader, her payoff is discontinuous at $\phi(1-\phi) = \kappa$; there, the payoff shifts down. Therefore, there are values of ϕ whose decrease could benefit both players. Figures 3.3 and 3.4 illustrate this conclusion. They depict the equilibrium payoff of each agents as a function of ϕ, given various κ.

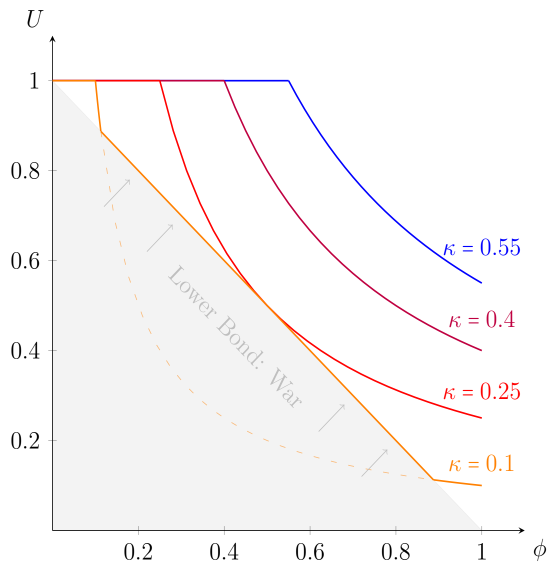


Figure 3.3: Role of ϕ on Population's Payoff

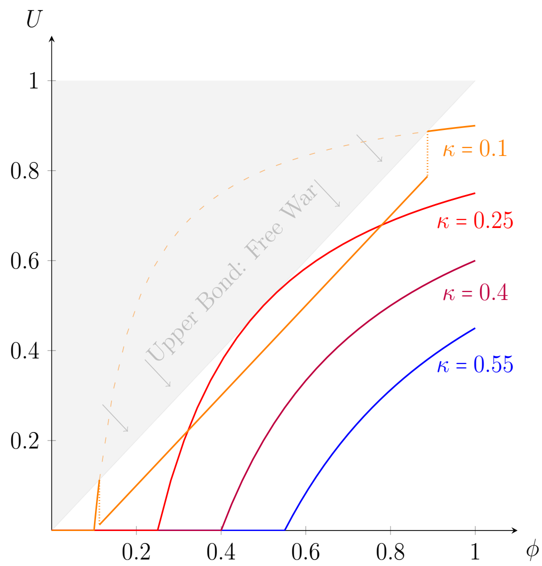


Figure 3.4: Role of ϕ on Leader's Payoff

Because the deadweight loss created by conflict is born by the leader, the population's payoff is constant in terms of κ as long as $\kappa \leq \phi(1-\phi)$. It is increasing afterwards, as the leader is easier to satisfy through a lower ρ when conflict gets more costly. Again, the leader's payoff is discontinuous in $\kappa = \phi(1-\phi)$. Therefore, although her payoff is decreasing piecewise, there are values of κ whose increase could benefit both players. Furthermore, small κ is also inefficient from a social point of view, as it allows for conflicts to take place. Therefore, for $\kappa < \phi(1-\phi)$, it is unambiguously beneficial to increase κ to at least $\phi(1-\phi)$. Again, Figures 3.5 and 3.6 depict the equilibrium payoff of each agent as a function of κ, given various ϕ.

Finally, it is worth emphasizing again how $\rho = 0$ is rarely optimal. Indeed, it is optimal only when the leader cannot use conflict as a fiscal extraction tool, i.e. $\kappa \geq \phi$. Otherwise, the leader can threaten the population to use conflict as a way to implement demanding policies while avoiding rebellion. Then, the population might find optimal to voluntary

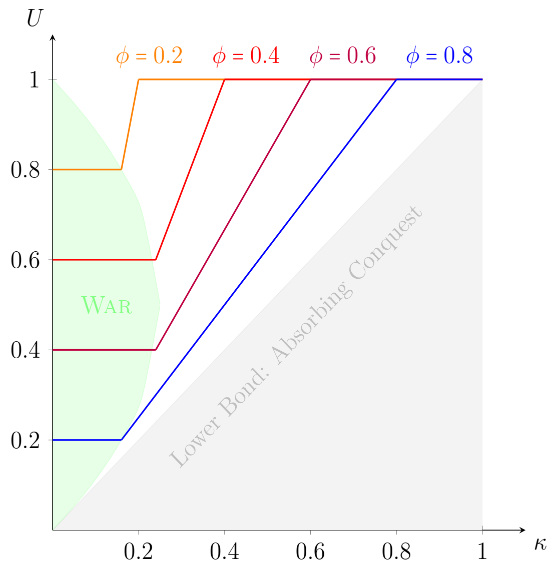


Figure 3.5: Role of κ on Population's Payoff

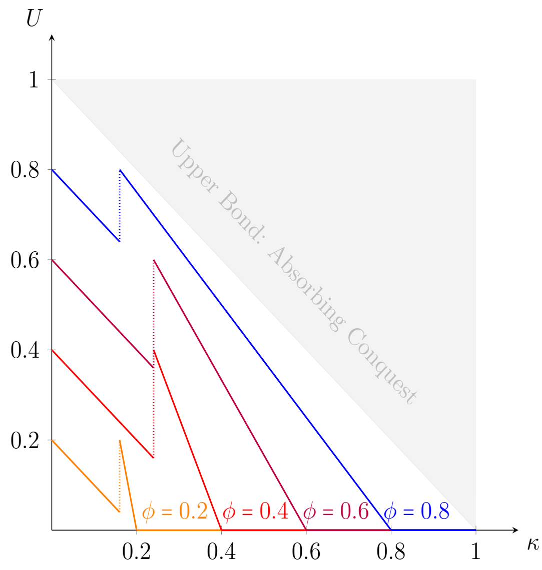


Figure 3.6: Role of κ on Leader's Payoff

enchain: prevent the threat posed by the instigation of an external conflict, and set up relatively inefficient institutions. More than rational, this choice is optimal as it prevents an inefficient occurrence of conflict.

In such a context, the population is threatened by their environment as much as by their leader. Indeed, the leader can use conflict as an extraction tool because of the threat of losing the conflict if the population decides to rebel. Therefore, it is the very existence of potential enemy that becomes the threat. The more hostile this enemy, the bigger ϕ, the bigger the threat. Indeed, a very hostile enemy would make any forfeit terrible, thus granted a huge extraction power to the leader. This in terms might mean that the population is better off with very inefficient institutions, i.e. a very high ρ. Notice that neither forfeit, nor conflict actually needs to occur; the mere threat of the environment is sufficient.

These insights might shed new light on the contagion of bad institutions. A country with weak institutions would be a rather menacing enemy, leading neighboring countries to fall prey to their own leaders, and to either weaken their institutions willingfully, or to be impoverished by wars. Because the prevalence of commitment is positively linked to ϕ, one would expect threatening environments to unambiguously weaken institutions. In a similar reasoning, one could justify democratic peace through an opposite virtuous circle: a democratic country would not be much of a threat to another democratic country, were one to forfeit a conflict after its leader initiated it. This would not give a leader any extraction power; thus, neither violence nor depletion of institutions should be feared.

3.7 Conclusion

This paper wonders whether diversionary conflicts can exist in a rational world. They can. It questions the rationality of rallies around the flag. They are. It further inquires about the possibility for a leader to initiate war with the only goal of generating such rallies around the flag. She can. It finally examines the optimality of voluntary enchainment through restriction to rebellion means in the long run. It is.

I first describe how a conflict can be successfully used to avoid rebellion even when no agent is naive. To do so, I formalize a mechanism that has been alluded to in much of the diversionary literature in political sciences. I argue that beyond patriotism and diversion, insurgency is avoided through fear; in particular, the fear of weakening one's international position, whose intensity is intuitively represented by the *threat* posed by the enemy. In this context, a leader does not need to *win* a war, as argued by the signalling interpretation of Richards et al. [1993], but to *stay* at war. Likewise, she does not need a patriotic or distractible population, as alluded by Levy [1989], but a fierce enemy. A belligerent strategy can then be beneficial to the leader if the decrease in tax base due to the cost of conflict is more than compensated by the increase in tax rate allowed by the state of war.

I then show how rallies around the flag can be interpreted as a commitment to internal peace. Indeed, supporting an aggressive foreign policy tends to make a one's enemies more hostile towards oneself. Thus, rallying around the flag allows a population's leader to implement harsher internal policies without having to resort to open war. Such rally effects in fact occur in equilibrium. Rallying around the flag is thus rational: it is the lesser of two evils; and it is efficient: it prevents further destruction of resources through war.

Because in the setup, a leader's rationale to use force is always that of preventing a rebellion while implementing demanding policies, such conflicts occur in equilibrium. In this sense, diversionary conflicts do occur. However, I further wonder whether conflict can be initiated in the only goal of gathering the popular support about foreign policy, rather than just domestic obedience. This is the considered definition of *diversionary incentive*. I find that such incentive exists. Actually, the set of parameters for which they exist is non-zero measure.

Finally, I explore the long-run effects of a threatful environment on a country's rebellion opportunities. A population might want to purposely set up barriers to rebellion in order to prevent the leader from using inefficient extraction tool. I prove that, indeed, this occurs in equilibrium. By a slight shift in the interpretation of freedom of rebellion, seen as general strong institutions, this results sheds light on the spread of weak institutions: a country whose institutions are weak is a more serious threat to its neighbors, who might, in turn, want to set up weak institutions to preserve peace. I also show how, for other parameters, the population does not commit and conflicts occur. The prevalence of such conflicts is non-monotonically related to the intensity of the enemy's threat is hump shaped. The function is hump-shaped. It reflects two opposite forces: a more hostile country renders peace more valuable, but also more expensive, as the leader is harder to dissuade from war.

This work is relevant in that it reclaims an argument that has been extensively discussed in political sciences, refines it and formalizes the mechanism that underlies it. This allows to rationalize altogether diversionary conflicts and rallies around the flag; it also permits to theoretically prove the existence of pure diversionnary incentives; and to question long terms effect on institutions. Because the mechanism is simple and the setup pared-down, the predictions about the role of each parameters should be clear enough to be brought to the data. Such rigorous empirical analysis could test the empirical credibility of the conclusions presented in this paper while potentially restoring some consensus to the discordant empirical literature on diversionary wars.

Appendix

Appendix

3.A Logical Conditions and Equilibrium Outcomes

3.A.1 List of Conditions for SPE and their Logical Relationships

First, let us list the conditions on parameters discussed in the backward induction of Section ??.

Condition	Name
$\kappa_t \geq \phi_t(1-\rho)$	A
$\kappa_w \geq \phi_w(1-\rho)$	B
$\kappa_t \geq \phi_t(1-\rho') + (\rho' - \rho)$	C
$\kappa_w - \kappa_t \geq (1-\rho)(\phi_w - \phi_t)$	D
$\kappa_w - \kappa_t \geq (1-\rho')(\phi_w - \phi_t)$	E
$\frac{1-\phi_w}{1-\phi_t} \geq \frac{1-\rho'}{1-\rho}$	F

Table 3.A.1: List of the Conditions on Parameters

From them, and using Lemma 13, the four outcomes mentioned in Theorem 5 can be derived, as seen in Table 3.A.2.

Some of the conditions along a given equilibrium path are redundant. First, let us recall that $\kappa_w > \kappa_t, \phi_w > \phi_t$ and $\rho' > \rho$. From there, we find that:

- $C \Rightarrow A$, as $\kappa_t \geq \phi_t(1-\rho') + (\rho' - \rho) > \phi_t(1-\rho') + \phi_t(\rho' - \rho) = \phi_t(1-\rho)$.
- $D \Rightarrow E$, as $\kappa_w - \kappa_t \geq (1-\rho)(\phi_w - \phi_t) > (1-\rho')(\phi_w - \phi_t)$.
- $\bar{A}$ *and* $B \Rightarrow D$, as $-\kappa_t > -\phi_t(1-\rho)$, and $\kappa_w \geq \phi_w(1-\rho)$ implies $\kappa_w - \kappa_t > (1-\rho)(\phi_w - \phi_t)$.

D		$\bar{D}$ *and* E				$\bar{E}$	
		F		$\bar{F}$			
A	$\bar{A}$	B	$\bar{B}$	C	$\bar{C}$	B	$\bar{B}$
$\theta = 0$	$\theta = 1$	$\theta = 0$	$\theta = 1$	$\theta = 0$	$\theta = 1$	$\theta = 0$	$\theta = 1$
–	$\eta = 0$	–	$\eta = 0$	–	$\eta = 1$	–	$\eta = 0$
–	$\omega = 0$	–	$\omega = 1$	–	$\omega = 0$	–	$\omega = 1$
PEACE	TENSIONS	PEACE	WAR	PEACE	TENSIONS	PEACE	WAR
-	~~SUPPORT~~	-	~~SUPPORT~~	-	SUPPORT	-	~~SUPPORT~~

Table 3.A.2: Equilibrium Path by Parameters' Conditions

- A *and* $\bar{B} \Rightarrow \bar{D}$, as $-\kappa_t \leq -\phi_t(1-\rho)$ and $\kappa_w < \phi_w(1-\rho)$ implies $\kappa_w - \kappa_t < (1-\rho)(\phi_w - \phi_t)$.
- C *and* $E \Rightarrow B$, as $\kappa_t \geq (1-\rho')\phi_t + \rho' - \rho$ and $\kappa_w - \kappa_t \geq -(1-\rho')\phi_t + (1-\rho')\phi_w$ implies $\kappa_w \geq (1-\rho')\phi_w + \rho' - \rho > (1-\rho')\phi_w + \phi_w(\rho' - \rho) = (1-\rho)\phi_w$.
- C *and* $F \Rightarrow B$, as F can be rewritten as $\rho'(1-\phi_t) + \phi_t - \rho \geq (1-\rho)\phi_w$; and with $\kappa_t \geq \rho'(1-\phi_t) + \phi_t - \rho$, it implies $\kappa_w > \kappa_t \geq \phi_w(1-\rho)$.

3.A.2 Derivation of the Logical Condition by Outcome

In this subsection, I derive conditions grouped by outcome.

To simplify exposition, I use set notation. Referring to the conditions in Table 3.A.1, I denote:

$$X = \left\{(\kappa_w, \kappa_t, \phi_w, \phi_t, \rho, \rho') \in (0;1)^6 \ : \ \textit{condition } X \textit{ fulfilled}\right\} \quad \forall X \in \{A, B, C, D, E, F\}$$

Table 3.A.3 summarizes the logical relations between the conditions derived above, along with their set notation.

It is useful to acknowledge the following implications of Table 3.A.3:

- $C \cap A = C$
- $D \cap E = D$
- $\bar{D} \cup \bar{E} = \bar{D}$
- $\bar{D} \cap \bar{E} = \bar{E}$
- $(A \cup \bar{B}) \cap \bar{D} = \bar{D}$
- $(\bar{A} \cup B) \cap D = D$
- $(C \cap E) \cup B = B$
- $C \cap E \cap B = C \cap E$

Logical Condition:	Equivalent for Sets:	Logical Condition:	Equivalent for Sets:
$C \Rightarrow A$	$C \subset A$	$\bar{A}$ *and* $B \Rightarrow D$	$\bar{A} \cap B \subset D$
$D \Rightarrow E$	$D \subset E$	A *and* $\bar{B} \Rightarrow \bar{D}$	$A \cap \bar{B} \subset \bar{D}$
C *and* $F \Rightarrow B$	$C \cap F \subset B$	C *and* $E \Rightarrow B$	$C \cap E \subset B$

Table 3.A.3: Summary of the Conditions' Interrelation

Condition for War

A high-level conflict can occur in either of the cases described in Table 3.A.2. We can write:

$$(\bar{D} \cap E \cap F \cap \bar{B}) \cup (\bar{E} \cap \bar{B}) = \bar{B} \cap [\bar{E} \cup (\bar{D} \cap E \cap F)] = \bar{B} \cap (\bar{E} \cup \bar{D}) \cap (\bar{E} \cup F)$$

Therefore, war is defined by:

$$\boxed{\bar{B} \cap \bar{D} \cap (\bar{E} \cup F)}$$

Condition for Tensions

A low-level conflict that meet no popular enthusiasm is straightforward to define as:

$$\boxed{\bar{A} \cap D}$$

On the other hand, international tensions that are supported exist when:

$$\boxed{\bar{C} \cap \bar{D} \cap E \cap \bar{F}}$$

Condition for Peace

Let us re-write the conditions for each state of peace. We have:

- $A \cap D = A \cap (\bar{A} \cup B) \cap D = A \cap B \cap D$
- $B \cap \bar{D} \cap E \cap F = B \cap (A \cup \bar{B}) \cap \bar{D} \cap E \cap F = A \cap B \cap \bar{D} \cap E \cap F$
- $B \cap \bar{E} = B \cap \bar{D} \cap \bar{E} = B \cap (A \cup \bar{B}) \cap \bar{D} \cap \bar{E} = B \cap A \cap \bar{D} \cap \bar{E} = A \cap B \cap \bar{E}$
- $C \cap E \cap \bar{D} \cap \bar{F} = C \cap E \cap B \cap \bar{D} \cap \bar{F} = A \cap C \cap E \cap B \cap \bar{D} \cap \bar{F}$

Therefore, we can define peace as:

$$
\begin{aligned}
&(A \cap D) \cup (B \cap \bar{D} \cap E \cap F) \cup (C \cap E \cap \bar{D} \cap \bar{F}) \cup (B \cap \bar{E}) \\
&= (A \cap B \cap D) \cup (A \cap B \cap \bar{D} \cap E \cap F) \cup (A \cap B \cap C \cap \bar{D} \cap E \cap \bar{F}) \cup (A \cap B \cap \bar{E}) \\
&= A \cap B \cap [D \cup (\bar{D} \cap E \cap F) \cup (C \cap \bar{D} \cap E \cap \bar{F}) \cup \bar{E}] \\
&= A \cap B \cap \{D \cup \bar{E} \cup [\bar{D} \cap E \cap (F \cup (C \cap \bar{F}))]\} \\
&= A \cap B \cap \{D \cup \bar{E} \cup [\bar{D} \cap E \cap (F \cup C)]\}
\end{aligned}
$$

It follows that peace exists when:

$$
\boxed{A \cap B \cap (D \cup \bar{E} \cup F \cup C)}
$$

While not minimal, I keep this formulation in order to underline its intuitive meaning. Note that the expression also brings:

$$
\begin{aligned}
&A \cap B \cap (C \cup F \cup D \cup \bar{E}) \\
&\quad = [A \cap B \cap (C \cup \bar{E})] \cup (A \cap B \cap D) \cup (A \cap B \cap F) \\
&\quad = \{A \cap [(B \cup (C \cap E)) \cap (C \cup \bar{E})]\} \cup (A \cap B \cap D) \cup (A \cap B \cap F) \\
&\quad = \{A \cap [(B \cup C \cup \bar{E}) \cap ((C \cap E) \cup C \cup \bar{E})]\} \cup (A \cap D) \cup (A \cap B \cap F) \\
&\quad = \{A \cap [(B \cup C \cup \bar{E}) \cap (C \cup \bar{E})]\} \cup (A \cap D) \cup (A \cap B \cap F) \\
&\quad = [A \cap (C \cup \bar{E})] \cup (A \cap D) \cup (A \cap B \cap F) \\
&\quad = C \cup [A \cap (\bar{E} \cup D)] \cup (A \cap B \cap F)
\end{aligned}
$$

Therefore, condition C is sufficient but not necessary to the preservation of peace.

3.B Alternative Payoff Specification

3.B.1 $\eta = 1$ affects the overall payoff

Instead of modifying ρ, let us assume that support $\eta = 1$ implies a change in the total payoff that becomes $(1-p)(1-\rho)(1-\theta(h))$. Similar to the notation in the main text, for every partial history $h \in H_3$, I define $p(h) = \begin{cases} p & \text{if } h \in \{(\theta = 1, \eta = 1, \omega = 0), (\theta = 1, \eta = 1, \omega = 1)\} \\ 0 & \text{otherwise} \end{cases}$.
Furthermore, I consider the following alternative definitions: $\underline{\kappa}_w = (1-p)(1-\rho)\phi_t$ and $\underline{\kappa}_w = (1-p)(1-\rho)\phi_w$. I can thus reformulate Lemma 13, Theorem 5.

. **Lemma 13.3.B.1** *The SPE is as follows:*

- *The leader's equilibrium strategy is:*

- $\theta = 1$ *if* $\kappa_w < \bar{\kappa}_w$; *or if* $\kappa_t < \bar{\kappa}_t$; *or if* $\underline{\kappa}_w - \underline{\kappa}_t \leq \kappa_w - \kappa_t < \bar{\kappa}_w - \bar{\kappa}_t$, *and* $r_\phi < (1-p)$, *and* $\kappa_t < \underline{\kappa}_t + p(1-\rho)$;
  $\theta = 0$ *otherwise.*
- $\omega(\eta = 0) = 1$ *if* $\kappa_w - \kappa_t < \bar{\kappa}_w - \bar{\kappa}_t$; $\omega(\eta = 0) = 0$ *otherwise.*
  $\omega(\eta = 1) = 1$ *if* $\kappa_w - \kappa_t < \underline{\kappa}_w - \underline{\kappa}_t$; $\omega(\eta = 1) = 0$ *otherwise.*
- $\forall h \in H_3, \tau(h) = 1 - \frac{(1-p(h))(1-\rho)(1-\phi(h))}{1-\kappa(h)}$

• *The population's equilibrium strategy is:*

- $\eta = 1$ *if* $\underline{\kappa}_w - \underline{\kappa}_t \leq \kappa_w - \kappa_t < \bar{\kappa}_w - \bar{\kappa}_t$ *and* $r_\phi < (1-x)$; $\eta = 0$ *otherwise.*
- $\forall h \in H_3 \times [0,1]$ *and corresponding* $h' \in H_3$:
  $\psi(h) = 1$ *if* $\tau > 1 - \frac{(1-p(h'))(1-\rho)(1-\phi(h'))}{1-\kappa(h')}$; $\psi(h) = 0$ *otherwise.*

Proof. This follows from backward induction as carried out in the main text, using the new specification of payoff. □

. **Theorem 5.3.B.1** *There are four possible outcomes: peace, (unsupported) war, unsupported tensions and supported tensions.*

- *War occurs iff* $\kappa_w < \bar{\kappa}_w$; *and* $\kappa_w - \kappa_t < \bar{\kappa}_w - \bar{\kappa}_t$; *and either* $\kappa_w - \kappa_t < \underline{\kappa}_w - \underline{\kappa}_t$, *or* $r_\phi \geq 1 - p$.
- *Supported tensions occur iff* $\underline{\kappa}_w - \underline{\kappa}_t \leq \kappa_w - \kappa_t < \bar{\kappa}_w - \bar{\kappa}_t$; *and* $r_\phi < 1 - p$; *and* $\kappa_t < \underline{\kappa}_t + p(1-\rho)$.
- *Unsupported tensions occur iff* $\kappa_t < \bar{\kappa}_t$; *and* $\kappa_w - \kappa_t \geq \bar{\kappa}_w - \bar{\kappa}_t$.
- *Peace occurs iff* $\kappa_w \geq \bar{\kappa}_w$; *and* $\kappa_t \geq \bar{\kappa}_t$; *and either* $\kappa_t \geq \underline{\kappa}_t + p(1-\rho)$, *or* $r_\phi \geq 1 - p$, *or* $\kappa_w - \kappa_t < \underline{\kappa}_w - \underline{\kappa}_t$, *or* $\kappa_w - \kappa_t \geq \bar{\kappa}_w - \bar{\kappa}_t$.

Proof. I show that all implications shown in Appendix 3.A.1 still hold. From there, Appendix 3.A.2 applies directly.
Conditions C, E and F have changed. Condition C now reads $\kappa_t \geq (1-p)(1-\rho)\phi_t + p(1-\rho)$, condition E is $\kappa_w - \kappa_t \geq (1-p)(1-\rho)(\phi_w - \phi_t)$ and F becomes $r_\phi \geq 1 - p$. All implications concerned by this change still hold. Indeed:

- $C \Rightarrow A$, as $\kappa_t \geq (1-p)(1-\rho)\phi_t + p(1-\rho) = \phi_t(1-\rho) + p(1-\rho)(1-\phi_t) > \phi_t(1-\rho)$.
- $D \Rightarrow E$, as $\kappa_w - \kappa_t \geq (1-\rho)(\phi_w - \phi_t) > (1-p)(1-\rho)(\phi_w - \phi_t)$.
- C *and* $E \Rightarrow B$, as $\kappa_t \geq (1-p)(1-\rho)\phi_t + p(1-\rho)$ and $\kappa_w - \kappa_t \geq -(1-p)(1-\rho)\phi_t + (1-p)(1-\rho)\phi_w$ implies $\kappa_w \geq (1-p)(1-\rho)\phi_w + p(1-\rho) = (1-\rho)\phi_w + p(1-\rho)(1-\phi_w) > (1-\rho)\phi_w$.

- C *and* $F \Rightarrow B$, as C is rewritten $\kappa_t \geq \phi_t(1-\rho) + p(1-\rho)(1-\phi_t)$; and F means $p \geq 1 - \frac{1-\phi_w}{1-\phi_t}$; so $\kappa_w > \kappa_t \geq \phi_t(1-\rho) + p(1-\rho)(1-\phi_t) \geq \phi_t(1-\rho) + (1-\rho)(1-\phi_t) - (1-\rho)(1-\phi_w) = \phi_w(1-\rho)$.

□

Furthermore, let us adapt the proof of Theorem 6 in this context. In particular, I provide an alternative characterization for $\mathcal{D}' \subseteq \mathcal{D}$. I let the reader verify that all parameters are acceptable and that all conditions of Corollary 11 hold for:

$$\mathcal{D}' = \left\{ (\kappa_t, \kappa_w, \phi_t, \phi_w, \rho, \rho') : \forall (x, \epsilon) \in \mathcal{E}, \begin{array}{lll} \rho = x; & \phi_t = x + \epsilon_2; & \kappa_t = x + \epsilon_4; \\ p = \frac{\epsilon_1}{1-x}; & \phi_w = x + \epsilon_3; & \kappa_w = x + \epsilon_5; \end{array} \right\}$$

where $\mathcal{E}$ is as before

3.B.2 $\eta = 1$ affects the parameters ϕ

Instead of modifying ρ, let us assume that support $\eta = 1$ implies a change in ϕ. In particular, $\phi(\theta = 1, \eta = 1, \omega = 1) = \phi'_w > \phi_w$ and $\phi(\theta = 1, \eta = 1, \omega = 0) = \phi'_t > \phi_t$. Furthermore, I impose $\phi_w - \phi_t > \phi'_w - \phi'_t$. This ensures that the benefit of escalation is higher without than with support. Without this assumption, there is no room for support, as it would only give the leader an additional incentive to escalate the conflict to war.
I consider the following alternative definitions: $\underline{\kappa}_w = \phi'_t(1-\rho)$ and $\underline{\kappa}_w = \phi'_w(1-\rho)$. I can thus reformulate Lemma 13, Theorem 5.

. Lemma 13.3.B.2 *The SPE is as follows:*

- *The leader's equilibrium strategy is:*
  - $\theta = 1$ *if* $\kappa_w < \bar{\kappa}_w$; *or if* $\kappa_t < \bar{\kappa}_t$; *or if* $\underline{\kappa}_w - \underline{\kappa}_t \leq \kappa_w - \kappa_t < \bar{\kappa}_w - \bar{\kappa}_t$, *and* $\phi'_t < \phi_w$, *and* $\kappa_t < \underline{\kappa}_t$;
    $\theta = 0$ *otherwise.*
  - $\omega(\eta = 0) = 1$ *if* $\kappa_w - \kappa_t < \bar{\kappa}_w - \bar{\kappa}_t$; $\omega(\eta = 0) = 0$ *otherwise.*
    $\omega(\eta = 1) = 1$ *if* $\kappa_w - \kappa_t < \underline{\kappa}_w - \underline{\kappa}_t$; $\omega(\eta = 1) = 0$ *otherwise.*
  - $\forall h \in H_3, \tau(h) = 1 - \frac{(1-\phi(h))(1-\rho)}{1-\kappa(h)}$
- *The population's equilibrium strategy is:*
  - $\eta = 1$ *if* $\underline{\kappa}_w - \underline{\kappa}_t \leq \kappa_w - \kappa_t < \bar{\kappa}_w - \bar{\kappa}_t$ *and* $\phi'_t < \phi_w$; $\eta = 0$ *otherwise.*
  - $\forall h \in H_3 \times [0,1]$ *and corresponding* $h' \in H_3$:
    $\psi(h) = 1$ *if* $\tau > 1 - \frac{(1-\phi(h'))(1-\rho)}{1-\kappa(h')}$; $\psi(h) = 0$ *otherwise.*

Proof. This follows from backward induction as carried out in the main text, using the new specification of payoff. □

. **Theorem 5.3.B.2** *There are four possible outcomes: peace, (unsupported) war, unsupported tensions and supported tensions.*

- *War occurs iff* $\kappa_w < \bar{\kappa}_w$*; and* $\kappa_w - \kappa_t < \bar{\kappa}_w - \bar{\kappa}_t$*; and either* $\kappa_w - \kappa_t < \underline{\kappa}_w - \underline{\kappa}_t$*, or* $\phi_t' \geq \phi_w$*.*
- *Supported tensions occur iff* $\underline{\kappa}_w - \underline{\kappa}_t \leq \kappa_w - \kappa_t < \bar{\kappa}_w - \bar{\kappa}_t$*; and* $\phi_t' < \phi_w$*; and* $\kappa_t < \underline{\kappa}_t$*.*
- *Unsupported tensions occur iff* $\kappa_t < \bar{\kappa}_t$*; and* $\kappa_w - \kappa_t \geq \bar{\kappa}_w - \bar{\kappa}_t$*.*
- *Peace occurs iff* $\kappa_w \geq \bar{\kappa}_w$*; and* $\kappa_t \geq \bar{\kappa}_t$*; and either* $\kappa_t \geq \underline{\kappa}_t$*, or* $\phi_t' \geq \phi_w$*, or* $\kappa_w - \kappa_t < \underline{\kappa}_w - \underline{\kappa}_t$*, or* $\kappa_w - \kappa_t \geq \bar{\kappa}_w - \bar{\kappa}_t$ *.*

Proof. I show that all implications shown in Appendix 3.A.1 still hold. From there, Appendix 3.A.2 applies directly.
Conditions C, E and F have changed. Condition C now reads $\kappa_t \geq \phi_t'(1-\rho)$, condition E is $\kappa_w - \kappa_t \geq (1-\rho)(\phi_w' - \phi_t')$ and F becomes $\phi_t' \geq \phi_w$. All implications concerned by this change still hold. Indeed:

- $C \Rightarrow A$, as $\kappa_t \geq \phi_t'(1-\rho) > \phi_t(1-\rho)$.
- $D \Rightarrow E$, as $\kappa_w - \kappa_t \geq (1-\rho)(\phi_w - \phi_t) > (1-\rho)(\phi_w' - \phi_t')$.
- C *and* $E \Rightarrow B$, as $\kappa_t \geq (1-\rho)\phi_t'$ and $\kappa_w - \kappa_t \geq -(1-\rho)\phi_t' + (1-\rho)\phi_w'$ implies $\kappa_w \geq (1-\rho)\phi_w' > (1-\rho)\phi_w$.
- C *and* $F \Rightarrow B$, as $\kappa_t \geq \phi_t'(1-\rho) > \phi_w(1-\rho)$ implies $\kappa_w > \kappa_t \geq \phi_w(1-\rho)$.

□

Furthermore, let us adapt the proof of Theorem 6 in this context. In particular, I provide an alternative characterization for $\mathcal{D}' \subseteq \mathcal{D}$. I let the reader verify that all parameters are acceptable and that all conditions of Corollary 11 hold for:

$$\mathcal{D}' = \left\{ (\kappa_t, \kappa_w, \phi_t, \phi_w, \phi_t', \phi_w', \rho) : \forall (x, \epsilon) \in \mathcal{E}, \rho = x \text{ and } \begin{matrix} \phi_t = x + \epsilon_2; & \phi_t' = x + \epsilon_2 + \epsilon_{01}; & \kappa_t = x + \epsilon_4; \\ \phi_w = x + \epsilon_3; & \phi_w' = x + \epsilon_3 + \epsilon_{02}; & \kappa_w = x + \epsilon_5; \end{matrix} \right\}$$

$$\text{where } \mathcal{E} = \left\{ (x, \epsilon) \in (0,1)^6 : \begin{matrix} (1-x)(\epsilon_3 - \epsilon_2) > \epsilon_5 - \epsilon_4 > \epsilon_{01} - \epsilon_{02} > 0 \\ \epsilon_5 > \epsilon_3 \end{matrix} \right\}$$

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