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Milan, January 30, 2012
Ferjančič Maja

Essays on Uncertainty, Business Cycles and Search Frictions in the Credit Market

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To my family

for always believing in me
and supporting my dreams

Contents

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Maja

Preface

This dissertation consists of three essays in the field of uncertainty and financial frictions.

The first chapter, titled *Uncertainty Shocks and Term Premia Dynamics*, contributes to the existing macro-finance literature by studying the effects of uncertainty shocks on a term structure. By now, is fairly well accepted in the literature that dynamic stochastic general equilibrium (DSGE) models have problems in generating the term premia that are sizeable and volatile enough. Within these models with homoscedastic shocks, the literature has documented that productivity and monetary policy shocks are the most important shocks for the dynamics of the term premium. However, none of the existent literature focuses on the role of uncertainty and its effects on the term premia.

The data, on the other hand, show that in periods of economics downturns, i.e. when uncertainty increases, spreads between yields of securities with different maturities also increase significantly. This, however, can be interpreted as an indication that uncertainty shocks are at work and that they have a significant importance. In this respect, this chapter fills this gap in the literature by quantifying the importance of uncertainty shocks for the term premium.

In order for risk to play a role, the model has to allow for the heteroscedasticity of the shocks. The shocks to volatility of the standard shocks are interpreted as uncertainty shocks. From a computational point of view, the standard approach, i.e. log-linearization around the steady state, removes the risk due to the certainty equivalence at work. Therefore, the choice of an appropriate perturbation method is of the utmost importance.

The results of the simulation show that the uncertainty regarding monetary policy is the main determinant of the term premium. In particular, the uncertainty shock to the volatility of monetary policy shock affects the term premia by 100 times more than the real uncertainty shocks, the effects of which are fairly small. Varying habit persistence significantly affects the relative importance of both real and nominal uncertainty shock. However, even when the habit persistence parameter is as high as 0.9, the uncertainty of monetary policy remains the main determinant of the term premium.

The second chapter, titled *Propagation of Uncertainty Shocks through Risk-Aversion and Investment Costs*, provides a mechanism through which uncertainty shocks have first-order effects on the economy. In this respect, it is shown that allowing the risk-aversion and investment adjustment costs to be related to the uncertainty level in the macroeconomic environment implies a significant role for uncertainty shocks within an otherwise standard dynamic stochastic general equilibrium model with stochastic volatility.

Variations in risk-aversion as a response to the change in the level of uncertainty are shown to significantly amplify the effects of uncertainty shocks on macroeconomic variables. However, the mechanism that works through variation of risk-aversion alone produces a positive response of the investment to higher uncertainty, which is not consistent with what we observe in the data. The model delivers a negative response of the investment to increase in uncertainty if the capital adjustment costs are related to the level of uncertainty. As such, and in contrast with the standard macroeconomic model with heteroscedastic shocks and constant risk-aversion, for which the effects of uncertainty shocks on the economy are fairly small, the model proposed here is able to account for large drops in activity during economic crises.

The third chapter, titled *Search Frictions on the Credit Market*, focuses on credit market frictions. It introduces search and matching frictions to the contractual relationship between productive agents, which are borrowing constrained, and financial intermediaries.

The search frictions could be viewed as a reduced-form representation of a wide array of resource and information frictions characterizing the lending relationship. In the present work, however, the search frictions are interpreted as complementary to credit frictions that might exist simultaneously with the frictions related to asymmetric information.

The financial shocks are modeled in two forms: (i) as a negative credit market efficiency shock, and (ii) as a positive shock to a separation rate. In accordance to the data, both shocks give rise to countercyclical spreads between deposit and lending rates, which in periods of financial distress increase. Further, the model captures the fact that in the downturn firms' financing difficulties increase. The latter, together with the decrease in the bank's willingness to supply loans, has a direct consequence for variability of the credit market liquidity.

The principal results of this chapter are as follows: (i) The economy is, after being hit by the negative credit market efficiency shock, driven into a recession due to a significant increase in the credit spread. At the same time, the liquidity of the credit market increases. (ii) The separation shock increases the credit spread even more, while causing the liquidity of the credit market to decrease. (iii) Because following the separation shock, the increase in the interest rate spread is accompanied by a decrease in the market liquidity, the recession stemming from this shock is relatively stronger when compared to the situation in which the economy is hit by a credit market efficiency shock of an equal size.

Chapter I
Uncertainty Shocks and Term Premia Dynamics

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Abstract

The present paper focuses on the role of uncertainty shocks for the term structure dynamics within the DSGE framework. While previous literature has documented the importance of the first moment productivity and monetary policy shocks for the term premium in the models with homoscedastic shocks, our results indicate that, once allowing the uncertainty shocks to have a role, the volatility of monetary policy is the main determinant of the term premium. Quantitatively, the results show that, for the standard values of parameters, productivity uncertainty shock has small impact on the term premia, while the nominal uncertainty shock seems to affect the term premia by 100 times more than the real uncertainty shock.

Keywords: business cycles, uncertainty shocks, term premium, higher-order perturbation

JEL Classification: E32, E43, E44, G12

1 Introduction

In the light of the recent economic crisis, uncertainty shocks have gained a large attention among economists and policy makers. While recent literature focuses mainly on the effects of uncertainty shocks on the macroeconomy, the present paper focuses on the effects of uncertainty shocks on the term premium.

In modeling the term structure and macroeconomic variables simultaneously, the literature records a number of so called puzzles (for discussions see for example Hördahl et al. (2007), Ravenna and Seppala (2006), Rudebusch and Swanson (2008a), Rudebusch and Swanson (2008b)). In macroeconomic applications it is common to use DSGE models with homoscedastic macroeconomic shocks; this setup by construction does not allow for analysis of the effects of uncertainty shocks on the term structure. The aim of the paper is to fill this gap by merging three strands of literature: the literature analyzing the term structure and macroeconomic implications within the DSGE framework, the literature on stochastic volatility and the literature on uncertainty shocks. To this end, uncertainty shocks are introduced through assuming heteroscedasticity of the macroeconomic shocks and in particular, following Justiniano and Primiceri (2008) and Fernandez-Villaverde and Rubio-Ramirez (2006) macroeconomic shocks are assumed to exhibit stochastic volatility in the form of smooth changes in variances of the structural shocks. In other words, the uncertainty shock is a shock into volatility of each macroeconomic shock.

The starting point for this analysis is by now a fairly accepted result in the literature that DSGE models (with homoscedastic shocks) have problems in generating the term premia that are sizable and volatile enough. Solutions that have been proposed to increase the fit of the term premia are the following: (i) to use recursive preferences instead of power utility preferences (Amisano and Tristani (2008), Rudebusch and Swanson (2008a))[1]; (ii)

[1] In the present paper, the standard power utility preferences are used in order to keep focus on the changes of the basic implications of the standard DSGE model with stochastic volatility and uncertainty shocks for the term structure. The extension with Epstein-Zin preference is in progress.

to increase the size and/or persistence of the structural shocks in the model. For example Hördahl et al. (2007) increase the persistence of consumption shocks, while Ravenna and Seppala (2006), introduce a large taste shock. Such a solution seems appealing at a first sight; however, there are at least two arguments against it: First, as discussed in Rudebusch and Swanson (2008b), this implies a higher volatility of output and consumption and therefore distorts the ability of such a model to fit the main macroeconomic variables. Second, such assumption is in contrast with the literature that analyzes the volatility of the US macroeconomic variables - the main result of which is that time-varying (decreasing) volatility of shocks helps to explain the reduction in standard deviations of output growth and other macro variables (Justiniano and Primiceri (2008)[2], Fernandez-Villaverde and Rubio-Ramirez (2006)).

Term premium depends on the amount of risk and the price of risk. In macroeconomic applications the amount of risk (typically represented by a combination of structural shocks in the model) is related to real risks (uncertainty about future output or consumption) and/or nominal risks (uncertainty about future monetary policy and inflation) and it is allowed to be time-varying. However, these models typically embed the assumption of homoscedastic shocks. This assumption intuitively implies a stationary term structure, i.e. the average term spreads, premia and associated volatilities are assumed to be constant over time. In this respect, the literature on stochastic volatility that documented a decreasing volatility of shocks over the past decades provides another criticism for the DSGE models commonly used to analyze the term structure also on a more general basis. Allowing for heteroscedasticity of shocks in the model seems to be a natural way to further analyze the term structure; first, because it allows to analyze how the decreased volatility in shocks after 1980's contributed to the evolution of the term structure and

[2] Justiniano and Primiceri (2008) (using a DSGE model in which the variances of the structural innovations are allowed to change over time) find that the exogenous structural disturbances hitting the U.S. economy display substantial stochastic volatility. In particular, technology and investment shocks exhibit substantial decline in volatility over the pat decades. Furthermore, they show that the decline in the volatility of output, investment, hours and consumption in the early 1980s is largely driven by a change in the variance of the shock specific to the equilibrium condition of investment.

second, because it allows for analysis of the contribution of uncertainty shocks to the term premia which is particularly relevant in the recent period of financial crisis.

The present paper is the most closely related to Amisano and Tristani (2008) who also build the model with heteroscedastic shocks. However, they model heteroscedasticity in the form of switching regimes in variances of the shocks while in the present paper the heteroscedasticity is modeled in the form of smooth changes in variances over time. This choice is based first, on the results of Justiniano and Primiceri (2008) who show that data prefer smooth changes in variances of the shocks as opposed to one-time switches in variances; second, as discussed in detail below, the data show that the term structure variables seem to change smoothly rather than jumping, third, the set up in the present paper allows to introduce a role for uncertainty shocks.

While macroeconomic literature has documented the importance of the productivity and monetary policy shocks for the term premium, it seems that once taking into account also the uncertainty shocks, the volatility of monetary policy is the main determinant of the term premium. The results show that productivity uncertainty shocks have small impact on the term premia. On the other hand, the uncertainty shock to volatility of monetary policy shock seems to affect the term premia by 100 times more than the real uncertainty shocks.

The rest of the paper is organized as follows: Section 2 presents a short literature review, while section 3 describes data on US securities yields. Section 4 describes the basic New Keynesian model used in the analysis, section 5 discusses the assumptions on macroeconomics shocks and introduces uncertainty shocks, while section 6 provides the term premium definitions. Section 7 discusses calibration, while section 8 shortly describes computational aspects of solving the model presented in section 4. The results are presented and discussed in the section 9. Section 10 is devoted to sensitivity analysis, while section 11 concludes.

2 Helicopter Tour over the Literature

The expectations hypothesis, according to which the long bond yield is the average of the expected short-term rates (or the expected returns from rolling over a series of short bonds) ignores interest rate risk. Unless the bond is held until maturity, the nominal return on a long term bond is uncertain and typical risk-averse investor requires a compensation for this risk, the term premium.[3] In modelling the term premium and macroeconomic variables simultaneously, the literature records a number of so called puzzles. That is, DSGE models have difficulties in generating: Positive serial correlation in consumption growth and a positive slope of the term structure; sufficiently high volatility in long-term bond yields; sizable term premia and large volatility of excess holding period returns (See Hördahl et al. (2007)).

Hördahl et al. (2007) claim that the puzzles with respect to features of bond yields arising within DSGE models can be resolved once the model allows for habit persistence, ARMA evolution of the consumption process and a high degree of interest rate smoothing in the policy rule. In such a set-up, the key features regarding holding returns and bond yields as well as key macroeconomic variables in the data can be matched reasonably well. However, the only macroeconomic variables they consider are consumption growth, inflation and short-term interest rates. Due to second-order approximation their setup does not allow to draw any conclusions about the volatility of the term premium.

Rudebusch and Swanson (2008b) find that in the standard DSGE model the term premia on long-term bonds is far too small and stable relative to data. The fit of the term premia can be improved by introducing the long-memory habits as in Campbell and Cochrane (1999) , however, this result comes only at cost of worsening the fit of macroeconomic variables (they consider a larger specter of macroeconomic variables than Hördahl et al. (2007)). Despite the success of the long-memory habits in endowment economy models,

[3]Kim and Orphanides (2007) define the "term premium" as reflecting such compensation for risk and any other sources of deviation from the expectations hypothesis.

the bond premium puzzle in DSGE models remains even when these models include large and persistent habits and real wage rigidities. The main mechanism behind the result within the model with production is that households face endogenous labour-consumption trade-off. Following a negative shock, household is able to self-insure by increasing the labour supply.

Rudebusch and Swanson (2008a) show that introducing Epstein and Zin (1989) preferences into a DSGE model produces large and variable term premium without compromising the fit of key macroeconomic variables. Long-run productivity risk and inflation risk further improve the model's fit with a lower level of household risk aversion.

Amisano and Tristani (2008) estimate the model approximated up to the second-order, i.e. shocks are assumed to be homoscedastic only able to generate a (non-zero) constant premia. However, they introduce stochastic volatility in the form of regime switching. Heteroscedasticity of the shocks induces a time-variation in risk premia which is entirely due to time variation in the amount of risk. Their nonlinear model is capable to generate variations in risk premia. In particular, the expected excess holding period returns vary over time due to changes in regime and they tend to move countercyclically. Moreover, they tend to be higher and more volatile over the 1980s.

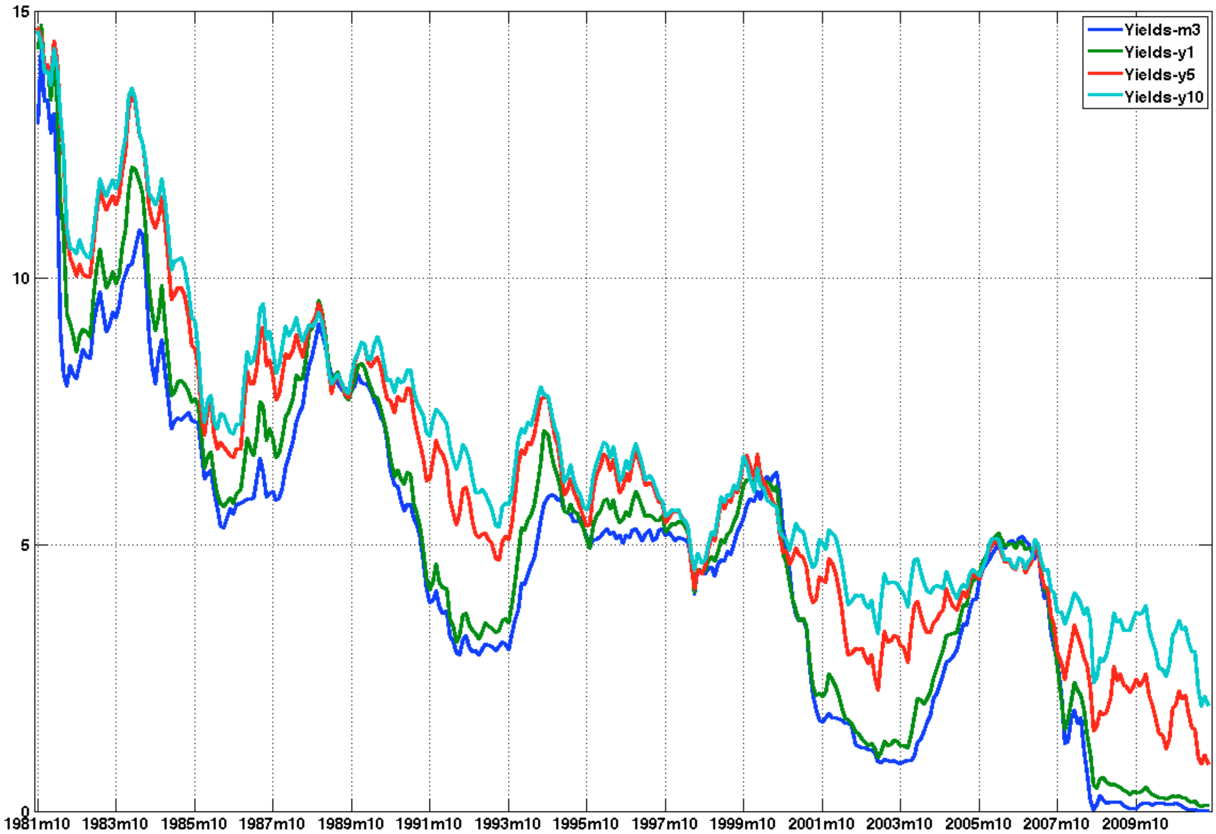
None of the existent literature focuses on uncertainty and its spillovers to the term premium. The data show that in the periods of economics downturns, when uncertainty increases, the spreads between yields of bonds with different maturities increase significantly. This paper fills the gap by quantifying the importance of the uncertainty shocks for the term premium.

3 Data

To emphasize the importance of the heteroscedasticity in the model, we first present the data for securities' yields with different maturities, spreads between them and their associated properties. All data are taken from Federal Reserve Board online database for the period between January 1953 and December 2011.

Figure 1 presents the yields of the securities with maturity 1 quarter, 1 year, 5 years and 10 years in the period between January 1980 and December 2011. Yields are highly persistent, highly correlated and decreasing over time for all types of bonds. On average, yields increase with maturity of the security.

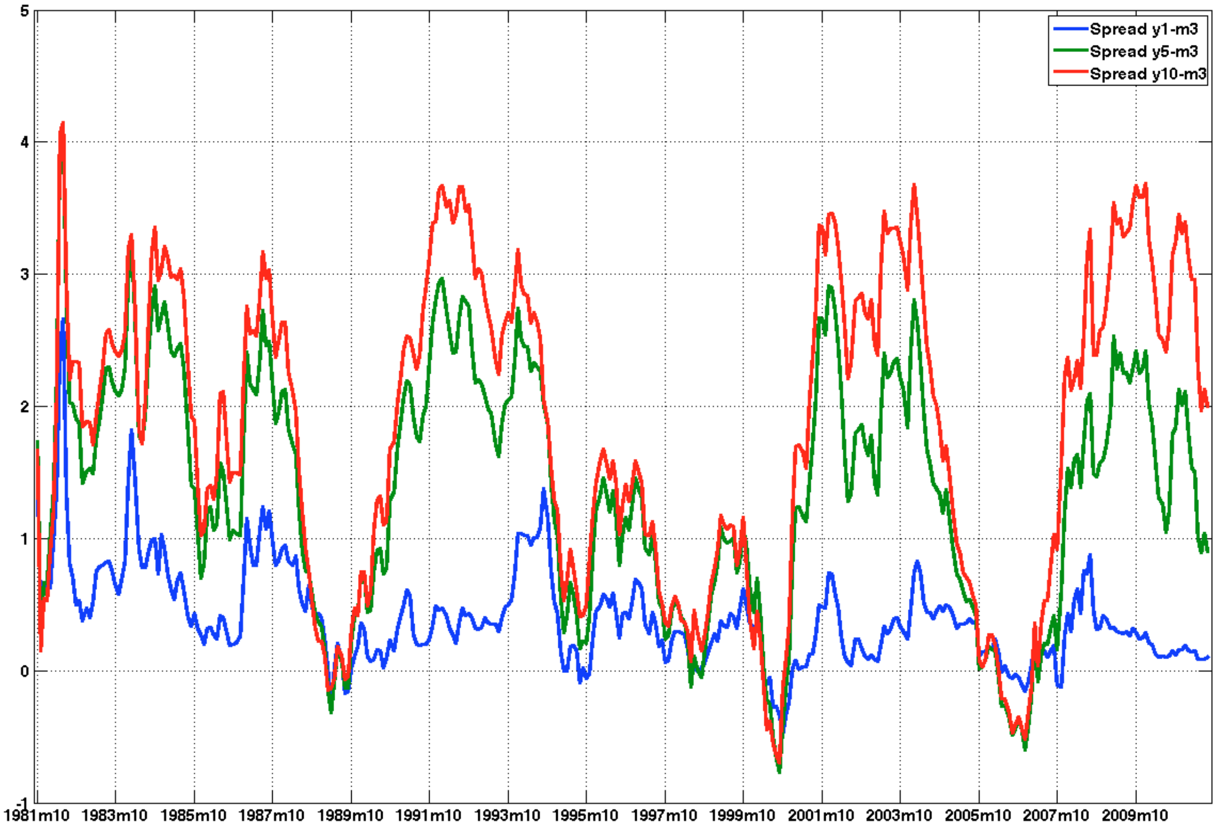
Figure 1: Yields



*Comment: Yields -m3, -y1, -y5 and -y10 stand for the yields of the bonds with maturity 1 quarter, 1 year, 5 years and 10 years, respectively.

Furthermore, in figure 1, we can observe that when economy is in prosperity, the yields are close to each other, however, the difference between them appears to increase in periods of distress. This feature of the data is even more clear in figure 2, which presents the spreads between the yields of securities with maturity 1 year, 5 years, 10 years, respectively, and the yields of securities with quarter 1 year. When the economy is in the downturn, the yields of the securities with longer maturity seem to increase proportionally more than the yields of securities with shorter maturity, thus the spreads exhibit significant increase.[4]

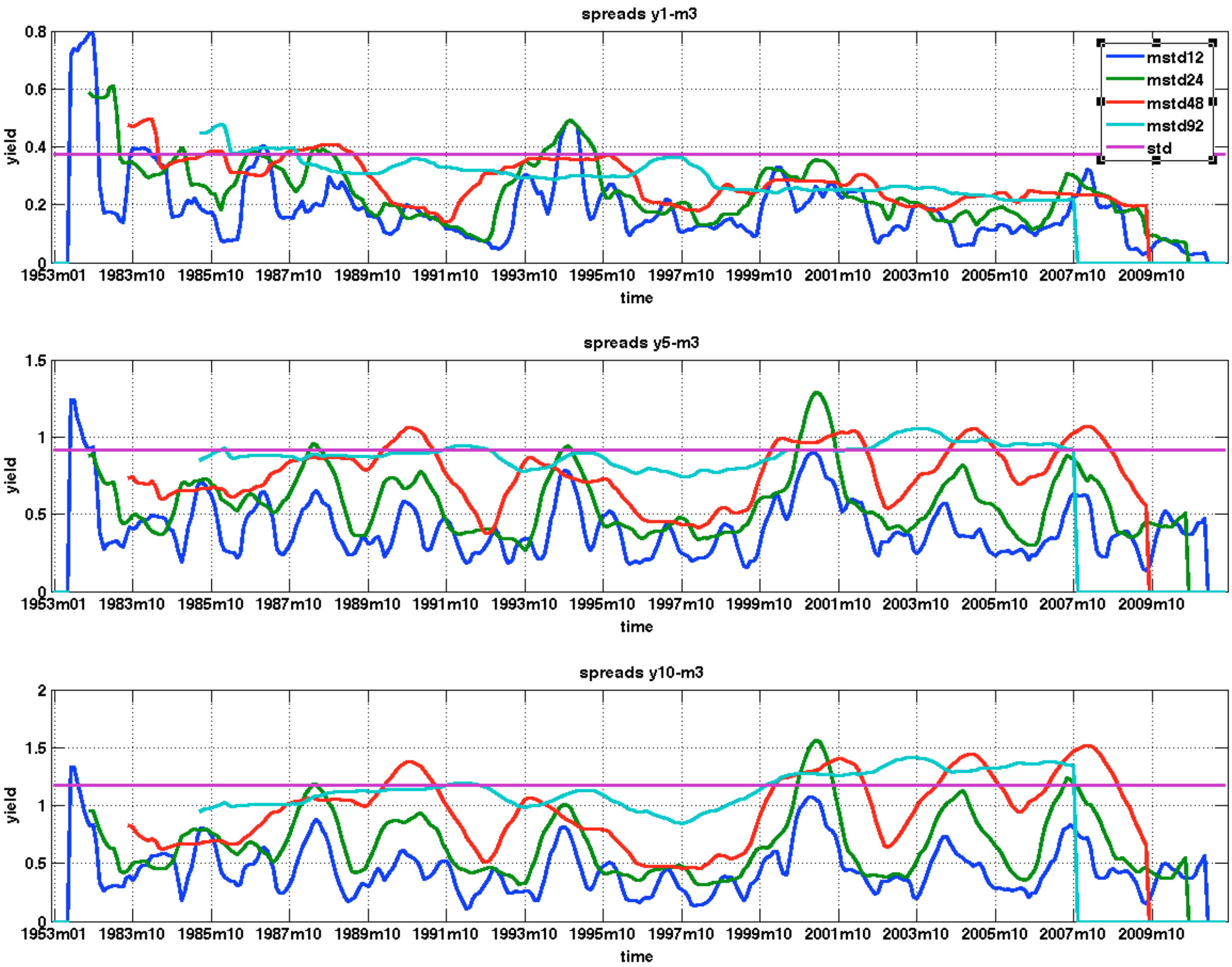
Figure 2: Spreads



*Comment: Spreads y1-m3, y5-m3 and y10-m3 stand for the spreads between the yields of the bonds with maturity 1 year, 5 years and 10 years, respectively, and yield of the bond with maturity 1 quarter.

[4]The conclusions based on the spreads with respect to yield of the securities with maturity 1 quarter are in line with those observed for the spreads with respect to yield of the bonds with maturity 1 year.

Figure 3: Moving Standard Deviations of Spreads (Different Sizes of Rolling Window)

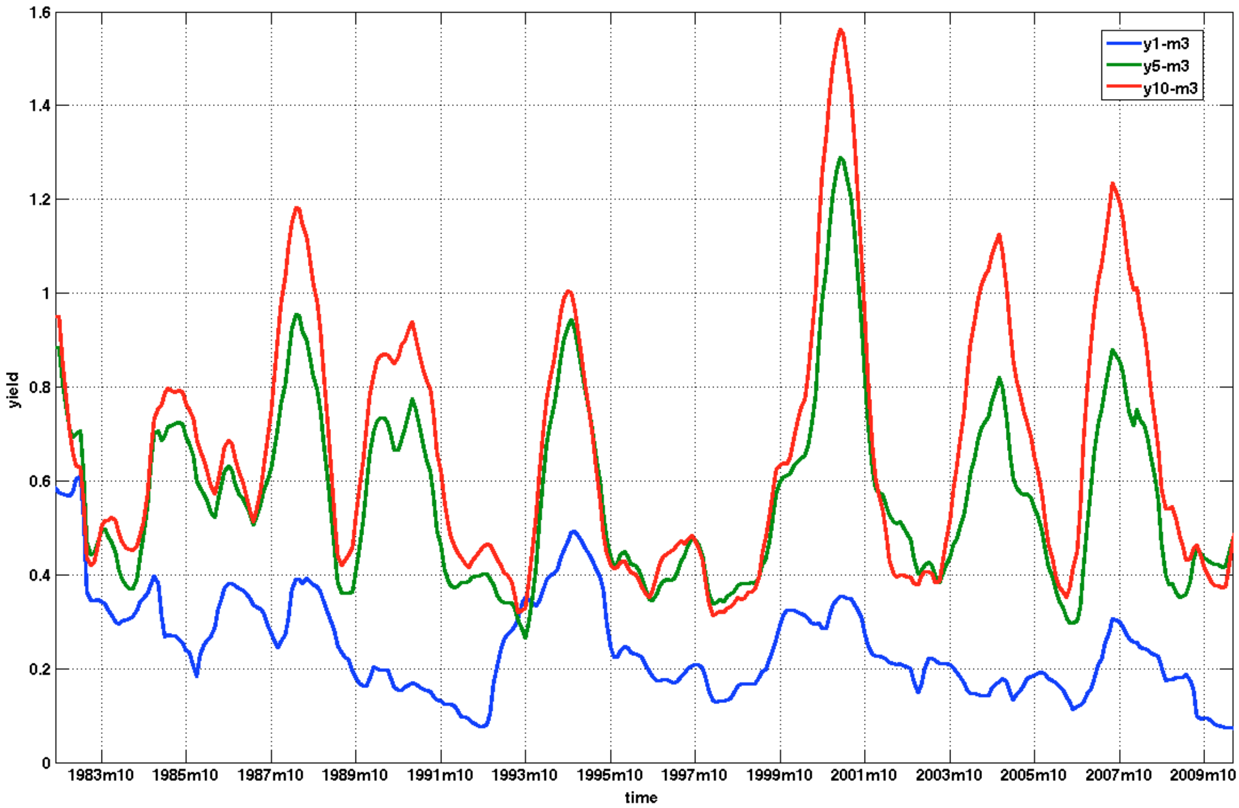


*Comment: mstd12, -24, -48 and -92 stand for the moving standard deviations calculated using the rolling windows of 1 year (12 months), 2 years, 4 years and 7.5 years, respectively.

Figure 3 shows the volatilities of the spreads over the period between January 1983 and December 2011. Each panel presents moving standard deviations computed using different size of the rolling window for spreads between the yields of securities with maturity 1 year, 5 years, 10 years, respectively, and the yields of securities with maturity 1 quarter. All three panels indicate the same conclusions. First, yields are highly volatile over the period and the volatility varies smoothly over time. The longer rolling window to compute the standard deviation of the yields is considered, the higher is associated volatility (which exhibits lower variation over the time). The standard deviation computed based on the whole sample is for the most of the time above the moving standard deviations.

Figure 4 plots the moving standard deviations of the spreads for a shorter time span computed using the rolling window of two years. It enables to see clearly, that the term-structure of the spreads' volatilities is upward-sloping.[5]

Figure 4: Moving Standard Deviations of Spreads (2 years Rolling Window)



*Comment: Spreads y1-m3, y5-m3 and y10-m3 stand for the spreads between the yields of the bonds with maturity 1 year, 5 years and 10 years, respectively, and yield of the bond with maturity 1 quarter.

[5]The choice of the time span and the size of the rolling window in this figure is done for purely expositional purposes and to a great extent arbitrary; however, the property that the term-structure of the spreads' volatilities is upward-sloping is robust to the choice of different periods and/or sizes of the rolling window.

Summing up:

- Levels of yields and spreads are highly correlated over sample period.
- Levels of yields and spreads are highly persistent and they do not exhibit stationarity.
- Yields and their volatilities are in general higher with longer maturity. The yield curve is on average positively sloped and the slope is slightly increasing over time.
- The term structure of yield volatilities and spreads volatilities appears on average upward-sloping.

The implicit assumption in macroeconomics models with homoscedastic shocks is that the yields and associated volatilities are constant over time. However, based on the data presented in this section, it seems that yields and spreads are subject to significant variation in associated volatilities over time. Models with homoscedastic shocks therefore disregard an important feature of the spreads that is clearly observed in the data. Furthermore, assuming constant yields and associated volatilities seems to imply trying to match too high second moments of the yields and the spreads, which is another fact that is common by construction to all of existent analyses of the determinants of the term premium.

4 Model

Households

The baseline model[6] assumes a continuum of agents represented by a representative household. The representative household's preferences are described by the standard power utility specification that is separable in consumption good, C_t, and hours worked, L_t, and exhibits external habit formation, that is, habits are determined by the average consumption in previous period, $\overline{C}_{t-1}$:

$$E_t \sum_{s=0}^{\infty} \beta^s \left[\frac{(C_{t+s} - h\overline{C}_{t+s-1})^{1-\gamma} - 1}{1-\gamma} - \varphi \frac{L_{t+s}^{1+\tau}}{1+\tau} \right] \tag{1}$$

Where E_t denotes mathematical expectations operator conditional on information set available at time t, $\beta \in (0,1)$ denotes household's subjective discount factor, h is habits persistence parameter and γ is the risk-aversion parameter.

Let I_t be investment, K_t physical capital owned by the household, B_t an one-period state-contingent government bond, W_t the nominal wage, R_t^k one-period nominal return on capital, R_t gross nominal interest rate, P_t the aggregate price level, Π_t the nominal lump-sum profit that household receives from the ownership of the firms and T_t the lump-sum taxes. The budget constraint of the household reads as following:

$$P_t C_t + P_t I_t + B_t \leq W_t L_t + R_t^k K_t + R_{t-1} B_{t-1} + \Pi_t + T_t \tag{2}$$

Physical capital, K_t, accumulates according to the following law of motion:

$$K_t = (1-\delta) K_{t-1} + \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) \right] I_t \tag{3}$$

Where δ represents depreciation rate and $S(\cdot)$ is the function that summarizes the technology which transforms current and past investment into installed capital for use in the following period. This function captures a presence of the adjustment costs of investment.

[6]The specification of the model follows very closely Justiniano et al. (2010) and Justiniano and Primiceri (2008).

Capital adjustment costs are quadratic in deviation of investment from its level in the previous period, that is:[7]

$$S_t\left(\frac{I_t}{I_{t-1}}\right)=\frac{\varpi}{2}\left(\frac{I_t}{I_{t-1}}-1\right)^2 \tag{4}$$

The household chooses processes for $\{C_t\}$, $\{L_t\}$, $\{B_t\}$, $\{K_t\}$,and $\{I_t\}$ to maximize lifetime utility subject to the budget constraint and physical capital accumulation process. The first order necessary conditions are:

$$P_t\lambda_t=(C_t-h\overline{C}_{t-1})^{-\gamma_t} \tag{5}$$

$$\beta\frac{\lambda_{t+1}}{\lambda_t}\frac{R_t}{\pi_{t+1}}=1 \tag{6}$$

$$\varphi\, L_t^{\nu}=W_t\lambda_t \tag{7}$$

$$1=q_t\left[1-S_t\left(\frac{I_t}{I_{t-1}}\right)-\frac{I_t}{I_{t-1}}S_t'\left(\frac{I_t}{I_{t-1}}\right)\right]+\beta\frac{\lambda_{t+1}}{\lambda_t}q_{t+1}\left(\frac{I_{t+1}}{I_t}\right)^2S_{t+1}'\left(\frac{I_{t+1}}{I_t}\right) \tag{8}$$

$$q_t=\beta\frac{\lambda_{t+1}}{\lambda_t}\left[rk_t+q_{t+1}\left(1-\delta\right)\right] \tag{9}$$

where λ_t is the marginal utility of consumption, q_t is the marginal value of capital in terms of consumption good, that is Tobin's q, and $\pi_t=\frac{P_t}{P_{t-1}}$ is gross inflation.

Final goods producers

At each point in time t, perfectly competitive firms produce a final consumption good, Y_t, by combining a continuum of intermediate goods, $Y_t(i)$, according to the following technology:

$$Y_t=\left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}}di\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{10}$$

where ε is the elasticity of demand. Profit maximization and the zero profit condition imply that the price of the final good, P_t, is a CES aggregator of the prices of the intermediate goods, $P_t(i)$:

$$P_t=\left[\int_0^1 P_t(i)^{1-\varepsilon}di\right]^{\frac{1}{1-\varepsilon}} \tag{11}$$

[7] In standard macroeconomic applications where models are log-linearized and solved by first order approximation around the steady state, $S(\cdot)$ is restricted to satisfy the following properties: $S(1)=S'(1)=0$, $S''(1)>0$. In the present application the model is approximated up to the third-order and therefore the functional form of $S(\cdot)$ has to be specified. See Schmitt-Grohe and Uribe (2006) for details.

and that the demand function for the intermediate good i is:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\varepsilon} Y_t \tag{12}$$

Intermediate goods producers

Each variety of good is produced by a monopolistically competitive firm $i \in [0,1]$ using as factor inputs capital services, $K(i)_{t-1}$, and labor services, $L(i)_{t-1}$. The production technology is given by:

$$Y\left(i\right)_t = A_t K\left(i\right)_{t-1}^{\alpha} L\left(i\right)_t^{1-\alpha} - A_t F \tag{13}$$

Where A_t is a neutral technological progress and F denotes a fixed cost per unit of output in each period. Cost minimization implies:

$$W_t = (1-\alpha) MC\left(i\right)_t \frac{Y\left(i\right)_t}{L\left(i\right)_t}$$

$$r_t^k = \alpha MC\left(i\right)_t \frac{Y\left(i\right)_t}{K\left(i\right)_{t-1}}$$

Because all the intermediate good producing firms face the same factor prices and they all have access to the same production technology, the marginal costs, $MC(i)_t$, are identical across firms. Indeed, because the above first-order conditions hold for all firms independently of whether they are allowed to reset prices optimally or not, the marginal costs are identical across all firms in the economy. Therefore, we can obtain a common capital-labor ratio and common marginal costs:

$$\frac{K_{t-1}}{L_t} = \frac{W_t}{r_t^k} \frac{\alpha}{1-\alpha} \tag{14}$$

$$MC_t = \frac{1}{\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}} \left(r_t^k\right)^{\alpha} \left(\frac{W_t}{A_t}\right)^{1-\alpha} \tag{15}$$

Staggered prices

The model features staggered prices as in Calvo (1983). In each period only a fraction of randomly picked intermediate firms, $1-\xi_p$, can adjust the prices, with opportunity to adjust in period t following an exogenous Poisson process. Re-optimizing firms set the price that is consistent with maximizing the present value of the future profits subject to demand for good i:

$$E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\lambda_{t+s}}{\lambda_t} \left\{ Y_{t+s}(i) \left[\Pi_{t,t+s} P_t^{new}(i) - MC_{t+s}(i) \right] \right\} \tag{16}$$

$$\text{subject to: } Y_{t+s}(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\varepsilon} Y_{t+s} \text{ ; where } \Pi_{t,t+s} = \prod_{k=1}^{s} \pi_{t+k-1}$$

The first-order condition for the price setting is:

$$E_t \sum_{k=0}^{\infty} (\beta\xi_p)^k \frac{\lambda_{t+k}}{\lambda_t} \left[\frac{P_t^{new}(i)}{P_t} \prod_{k=1}^{s} (\beta\xi_p) \frac{\pi_{t+k}}{\pi_{t+k-1}} \right]^{-\varepsilon} Y_{t+s} \left[P_t^{new}(i) - mc_{t+k}(i) \right] = 0$$

Define $\widetilde{P}_t$ as the relative optimal price, $\widetilde{P}_t = \frac{P_t^{new}}{P_t}$. Then, the first-order condition for the price setting can be rewritten in the following recursive form:[8]

$$ppr_t = (1+\lambda_p)\, pr_t \tag{17}$$

$$pr_t = \widetilde{P}_t^{-\varepsilon-1}\, \overline{Y}_t\, mc_t + \beta\, \xi_p \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\widetilde{P}_t}{\widetilde{P}_{t+1}} \right)^{-\varepsilon-1} \left(\frac{\pi_t}{\pi_{t+1}} \right)^{-\varepsilon} pr_{t+1} \tag{18}$$

$$ppr_t = \overline{Y}_t\, \widetilde{P}_t^{-\varepsilon} + \beta\, \xi_p \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\widetilde{P}_t}{\widetilde{P}_{t+1}} \right)^{-\varepsilon} \left(\frac{\pi_t}{\pi_{t+1}} \right)^{1-\varepsilon} ppr_{t+1} \tag{19}$$

The remaining firms set their prices following the full indexation rule:

$$P_t(i) = P_{t-1}(i)\pi_{t-1} \tag{20}$$

where π_t is the gross inflation. The aggregate price level evolves according to:

$$P_t = \left[\xi P_t(i)^{1-\varepsilon} + (1-\xi)\, P_t^{new}(i)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{21}$$

[8] Since the model is solved up to third-order, the price setting equation is rewritten in recursive form. See Schmitt-Grohe and Uribe (2006) for technical details.

Monetary authority and government

Monetary authority sets the short term nominal interest rate following a Taylor rule that exhibits interest rate smoothing, interest rate responses to deviations of inflation from the steady state inflation, π_{ss}, and deviations of output, Y_t, from its level in the previous period:

$$\frac{R_t}{R_{ss}} = \left(\frac{R_{t-1}}{R_{ss}}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi_{ss}}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_{t-1}}\right)^{\phi_x}\right]^{1-\rho_R} \varepsilon_t^{mp} \tag{22}$$

where R_{ss} is the steady state gross nominal interest rate and ε_t^{mp} is the monetary policy shock.

Aggregation

As shown by Schmitt-Grohe and Uribe (2006) when the model is solved up to the first-order one can neglect resource costs induced by inefficient price dispersion. However, when using higher-order approximation, one has to take care of proper aggregation. In particular, inefficient price dispersion induces resource costs, s_t, which can be written in the recursive form as follows:[9]

$$s_t = \widetilde{P}_t^{(-\varepsilon)} (1 - \xi_p) + \xi_p \left(\frac{\pi_t}{\pi_{t-1}}\right)^{\varepsilon} s_{t-1} \tag{23}$$

The aggregate absorption, $\overline{Y}_t$, is:

$$\overline{Y}_t = C_t + I_t \tag{24}$$

And the relationship between output and aggregate absorption is described through resource costs:

$$Y_t = \overline{Y}_t \, s_t \tag{25}$$

[9] Note that s_t is bounded below by 1.

5 Macroeconomic and Uncertainty Shocks

5.1 Macroeconomic shocks

The processes of the macroeconomic shocks are standard. The productivity shock is assumed to follow an exogenous stochastic $AR(1)$ processes of the form:

$$\log \varepsilon_t^a = (1 - \rho_a)\,\overline{\varepsilon_t^a} + \rho_a \log \varepsilon_{t-1}^a + e_t^a$$

The monetary policy shock evolves as:

$$\log \varepsilon_t^{mp} = e_t^{mp}$$

with $e_t^a \sim N(0, \sigma_{a,t}^2)$ and $e_t^{mp} \sim N(0, \sigma_{mp,t}^2)$.

5.2 Uncertainty shocks

Uncertainty shocks are introduced through assuming that the macroeconomic shocks are heteroscedastic. In particular, following Justiniano and Primiceri (2008) and Fernandez-Villaverde and Rubio-Ramirez (2006) the macroeconomic shocks are assumed to exhibit stochastic volatility, that is, the standard deviations of the productivity and monetary policy shock are allowed to change over time. As it is common in the stochastic volatility literature (see Justiniano and Primiceri (2008) and Kim et al. (1998)) the following stochastic process for the evolution of each standard deviation, $\sigma_{j,t}$, is assumed:[10]

$$\begin{aligned} \log \sigma_{j,t} &= (1 - \lambda_j) \log \overline{\sigma_j} + \lambda_j \log \sigma_{j,t-1} + \tau_{j,t} \\ \tau_{j,t} &\sim N(0, \omega_j^2), \text{ where } j = a, mp \end{aligned}$$

The shock into volatility of each macroeconomic shock, $\tau_{j,t}$, is interpreted as the uncertainty shock. This type of uncertainty shocks are of Fernandez-Villaverde et al. (2009)

[10]Modeling logarithms (as opposed to σ_t) guarantees that the standard deviations of shocks are positive in every point in time.

type as opposed to Bloom (2009) type of uncertainty shocks. The macroeconomic shocks and uncertainty shocks are assumed to be uncorrelated among each other.[11]

6 Term Premium

In modeling the term premium we follow Rudebusch and Swanson (2008b). We consider two measures of the term premium.

The first measure is defined as the difference between the observed yield to maturity on the consol and the risk-neutral yield to maturity.

$$ntp_t = log[\frac{\delta_c \widetilde{pn}_t}{\widetilde{pn}_t - 1}] - log[\frac{\delta_c \widehat{pn}_t}{\widehat{pn}_t - 1}] \tag{26}$$

Where ntp_t is the term premium on the consol, δ_c is the rate of decay of the coupon on the consol, $\widetilde{pn}_t$ is the nominal consols price per one dollar of coupon in period t and $\widehat{pn}_t$ is the risk-neutral price of the consol in period t.

The second measure of the riskiness of long-term bonds is the excess one-period holding return−i.e. the return of holding the bond for one period less the one-period risk-free rate. For the case of an n-period zero-coupon bond, Rudebusch and Swanson show that the excess holding return, ehr_t, is given by

$$ehr_t = \frac{\delta_c \widetilde{pn}_t + e^{i_{t-1}}}{\widetilde{pn}_t - 1} - e^{i_{t-1}} \tag{27}$$

where $\frac{\delta_c \widetilde{pn}_t + e^{i_{t-1}}}{\widetilde{pn}_t - 1}$ is the gross return of holding the consol and includes the one-dollar coupon in period $t-1$ that can be invested in the one-period security. $e^{i_{t-1}}$ is the gross one period risk-free rate.

[11] If correlation was assumed to be non zero, it would be plausible to assume that it is positive, reflecting that when the macroeconomic shocks are larger also the uncertainty is larger. Positive correlation between macroeconomic and uncertainty shocks would result in amplification of the effects of the uncertainty shocks.

7 Calibration

Standard parameters not related to the shocks' processes

The calibrated values of the parameters that are not related to the processes of the shocks are set in line with the values that are standard in the macroeconomic literature that uses the DSGE framework. The time unit is one quarter. The households quarterly subjective discount factor, β, is assumed to be 0.99, which implies an annual steady state interest rate of 4%; the habit persistence parameter, h, is set to 0.66. Risk aversion is assumed to be 2. The latter is on higher end in terms of macroeconomic applications, but standard in analyses of the term premium.

The capital share in the production function, α, is set to 0.33, which corresponds to a steady-state share of capital income roughly equal to 33%, ν is set to imply a Frisch labor supply elasticity equal to 1.5 and φ is set in order to have a steady state equal to 1.

The depreciation rate, δ, is set at 0.025 implying the annual rate of depreciation on capital equal to 10%, while the investment adjustment cost parameter, ω, is set to be 2. The parameter denoting the fixed cost per unit of output in each period, F, is set to guarantee that profits are zero in steady state.

The parameter for interest rate smoothing is set to $\rho_r = 0.8$, the parameter for the response of monetary authority to deviations of inflation from its steady state value is set to $\phi_\pi = 1.8$ and the parameter for the response of monetary authority to deviations of aggregate absorption from its level in the previous period is assumed to be $\phi_x = 0.5$. The price-elasticity of demand for a specific good variety is $\varepsilon = 6$, so that the steady-state markup in the product markets is 20%. The fraction of firms setting the prices optimally each quarter is 0.8. The remaining of the firms set prices following a full indexation rule.

It is worth noting that the values of this block of parameters matter for the effects of the macroeconomic shocks; however, except for the habit persistence parameter, the effects of the uncertainty shocks are not sensitive to changes in the values of these parameters. As it will become clear in the sensitivity analysis, the value of habit persistence parameter is crucial for the relative importance of the effects of real and nominal uncertainty shock on the premium.

Table 1 summarizes the calibrated values of the structural parameters not related to the shocks' processes.

Table 1: Calibration: Parameters not related to the shocks' processes

Parameter	Name	Value
γ	Risk-aversion parameter	2
α	Capital share in production function	0.33
β	Discount factor	0.99
δ	Depreciation rate	0.025
ω	Investment adjustment cost parameter	2
ν	Disutility of labor	0.67
h	Habit persistence parameter	0.66
ρ_r	Monetary authority interest rate smoothing	0.8
ϕ_π	Monetary authority response to deviations of inflation	1.5
ϕ_x	Monetary authority response to deviations output	0.5
ξ_p	Fraction of firms not setting prices optimally	0.8
ε	Elasticity of demand	6

Calibration of the parameters related to the shocks' processes

As previously determined in the literature, the productivity and monetary policy shocks are the most relevant for the term structure. It is therefore reasonable to consider these two shocks and the uncertainty related to their respective processes.

The calibration of the macroeconomics shocks is standard except that the variance is allowed to be time-varying. The technology shock is assumed to be equal to 1 in the steady state and it evolves according to an $AR(1)$ process with persistence parameter set to 0.95. Monetary policy is also equal to 1 in steady state, but not persistent. The steady state standard deviations of the macroeconomic shocks is being set according to Gertler et al. (2008) estimation results for the model that is very close to the model in the present analysis; that is, the standard deviation of the the technology shock is 0.01 and 0.0023 for the monetary policy shock.[12]

The uncertainty shocks are more difficult to calibrate, since there is no direct evidence neither the existent literature has a clear prior on what the relevant size of the uncertainty shock is. The shock to the second moment of the productivity is calibrated following the approach undertaken by Basu and Bundick (2011), while the shock into volatility of monetary policy shock is calibrated based on the data of the spread between the yields of the securities with longer maturities and the yield of security with maturity 1 quarter, whose moving standard deviations have been reported in figure 3. Basu and Bundick (2011) in the first step estimate the autoregressive process of VIX index to obtain the parameters related to the uncertainty. In the second step, they construct VIX index implied by the New Keynesian model and calibrate the shock into volatility of productivity as the value that enables to match the model implied VIX index with the real time VIX data.[13] Turning to the nominal uncertainty shock, inspecting the moving standard

[12]See Table 1 in Gertler et al. (2008) for estimated parameters of SW model.

[13]The second shock they consider is the preference shock. Despite of the analysis of the effects of the uncertainty shocks when the monetary policy faces the zero lower-bound constraint, they do not consider the uncertainty of the monetary policy.

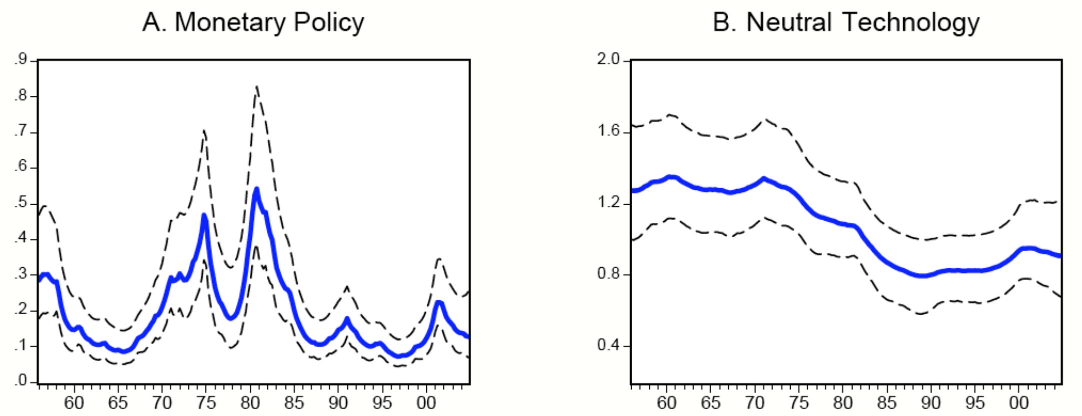
deviations of the spreads in figure 3, one can see that regardless of which type of the spread between yields of securities is considered the maximum values of the standard deviations are about twice as large as the average of the standard deviations of the spreads. Based on these data, it seems reasonable to assume that the uncertainty shock to the second moment of the monetary policy shock is equal to 1 standard deviation. Table 2 gathers the priors on the processes governing the shocks.

Table 2: Calibration: Parameters of the shocks' processes

Parameter	Name	Value
ρ_a	Persistence parameter of the productivity macro.shock	0.95
$\overline{a}$	Productivity macro.shock in steady state	0
$\overline{\sigma_a}$	Std.dev. of productivity macro.shock in steady state	0.01
λ_a	Persistence parameter of the productivity uncertainty shock	0.83
τ_a	Productivity uncertainty shock	$0.4\overline{\sigma}_a$
ρ_{mp}	Persistence parameter of the MP macro.shock	0
$\overline{mp}$	MP macro.shock in steady state	0
$\overline{\sigma}_{mp}$	Std.dev. of productivity macro.shock in steady state	0.0023
λ_{mp}	Persistence parameter of the MP uncertainty shock	0.83
τ_{mp}	MP uncertainty shock	$1\overline{\sigma}_{mp}$

For the sake of transparency, figure 5 reproduces the model implied volatilities reported by Justiniano and Primiceri (2008). Visual inspection of the model implied volatilities they report reveals that the mean values of the volatilities are about 0.01 for productivity shock and 0.002 for monetary policy shock, while the maximum values are 0.0125 and 0.005, respectively. Based on this values, the reasonable value of the uncertainty shock to productivity process would amount to 0.25 standard deviation while the reasonable value of the uncertainty shock to the second moment of the monetary policy shock process would amount to 1.5 standard deviation.

Figure 5: Implied Volatility of Justiniano and Primiceri (2008) model



Source: Figure 1 in Justiniano and Primiceri (2008)

8 Solution Method

The model is solved using perturbation AIM algorithm of Swanson et al. (2006)[14] to compute the third-order approximation solution around the deterministic steady state.[15] The third-order approximation solution is considered for two reasons. First, the model has to be solved up to third-order in order to have a role for uncertainty shocks. In the case of the first-order approximation solution, there is no role for the effects of changing the risk in the model. In the case of the second-order approximation solution, the risk affects the values of the steady state but not the dynamics in the model. In order to properly account for the dynamics induced by uncertainty shocks, the model has to be approximated at least to the third-order. The second reason for using the third-order approximation of the model is that it allows for time-variaton of the term premium.

The impulse responses of the variables induced by the shocks are computed following Fernandez-Villaverde et al. (2009) approach. As they show, when variances of the shocks

[14]Available at Eric Swanson's webpage http://www.ericswanson.us/perturbation.html.
[15]Dynare 4 of Adjemian et al. (2011) as of now does not yield proper effects of the uncertainty shocks.

in the model are allowed to be time-varying, the ergodic distribution moves away from the steady state values of the variables. To take properly into account the effects of the shocks in such a setup, the generalized impulse responses have to be computed with respect to the ergodic distribution.

9 Results

Table 3 compares the ergodic values to the steady state values of term structure variables. One can see, that once the risk is taken into account, the price of bond as well as the neutral price of the bond increase. As the latter is relatively larger, the ergodic term premium is positive even when the economy is in its steady state. Similarly, the yield to maturity increases relatively more than the yield to maturity of the risk-neutral bond, implying positive excess holding return in the equilibrium.

Table 3: Deterministic and ergodic values

Variable	Det.SS	Erg.SS
Bond price	0.40000	0.40396
Risk neutral bond price	0.40000	0.40463
Term premium	0.00000	1.71772
Holding returns	4.02000	3.91852
Risk neutral holding returns	4.02000	3.90134

Figures 6 and 7 show the effects of the first moment shocks while figures 8 and 9 present the effects of the second moment shocks on the term premium.[16] Since the macroeconomic shocks and the uncertainty shocks are assumed to be uncorrelated among each other the

[16]For the sake of consistency the effects of all shocks on the macroeconomic variables are also shown; however, since the focus of the analysis is on the term structure, we discuss only the variables of the latter. For the detailed analysis of the effects of uncertainty shocks on the macroeconomic variables within the New Keynesian model, see Basu and Brent (2011). The responses to productivity first and second moment shocks in the present analysis are consistent with their findings; they, however, do not consider monetary policy shocks.

results presented below should be considered as lower bound on the effects that uncertainty shocks have on the term premium.

Figure 6: Responses to the first moment productivity shock

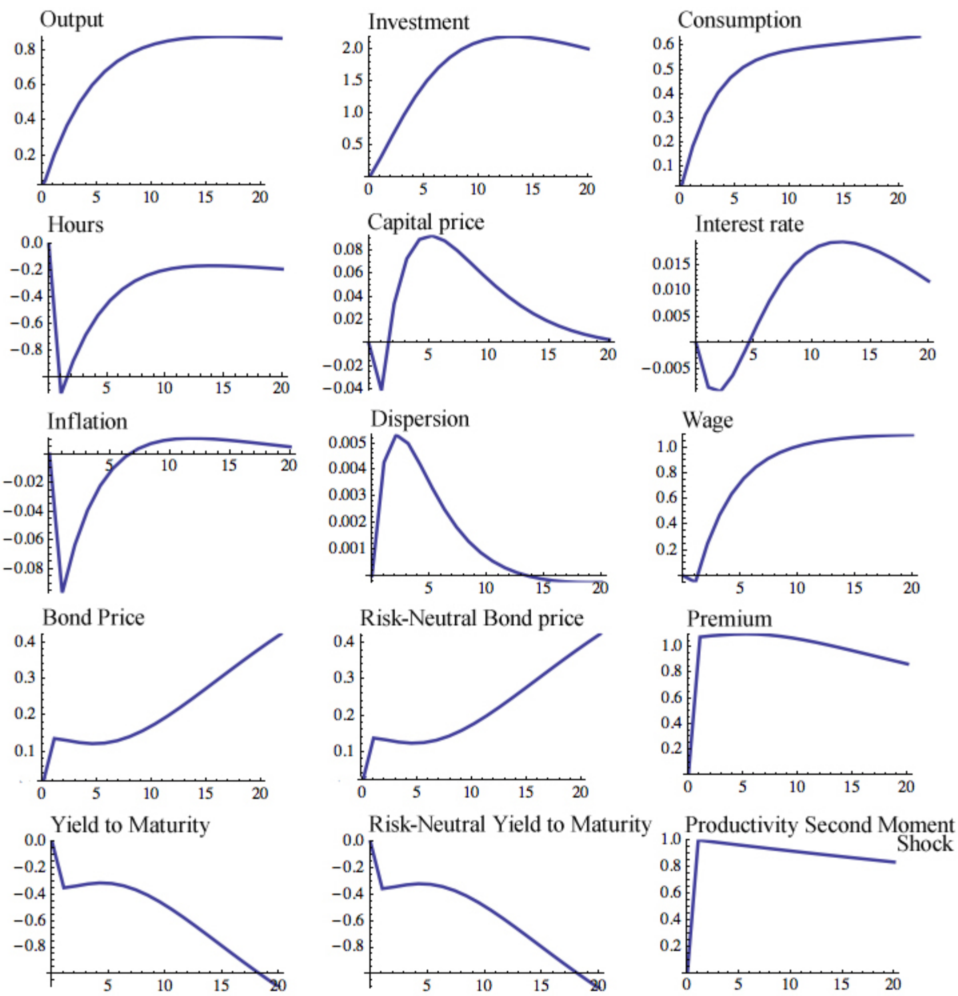
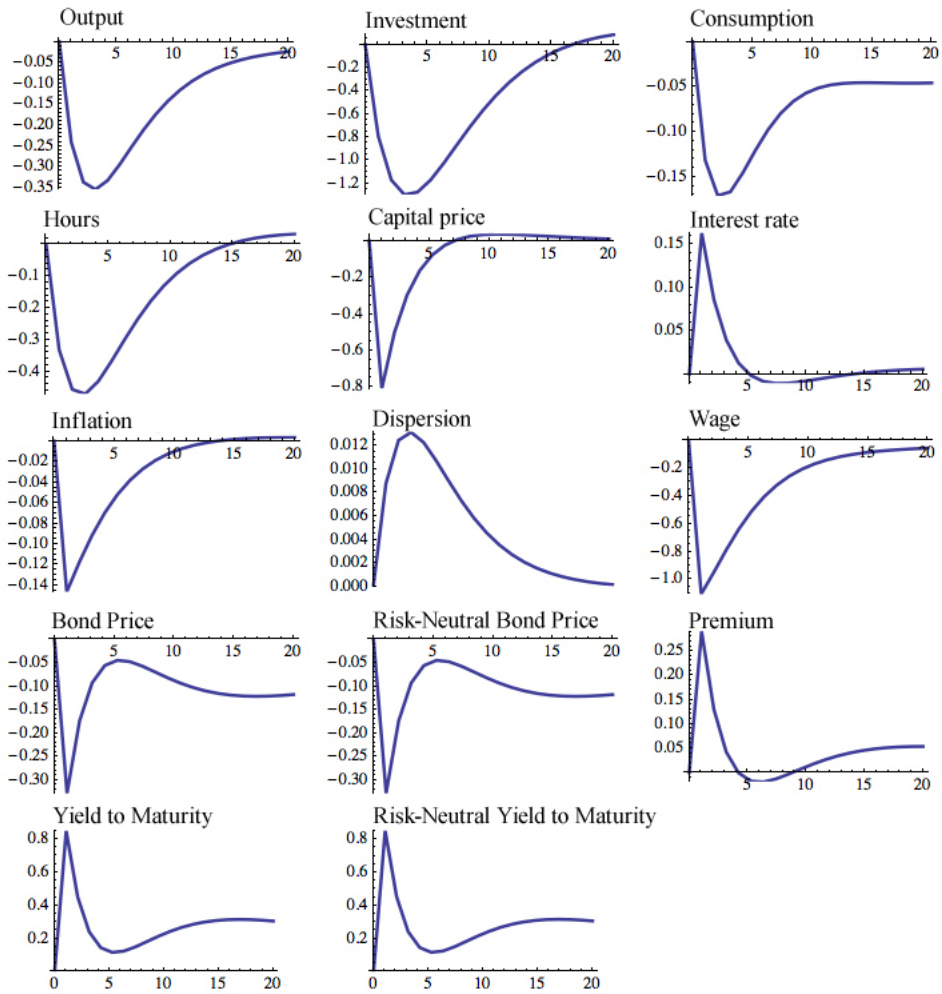


Figure 7: Responses to the first moment monetary policy shock



The term premium increases following the positive first-moment productivity shock as well as following the positive first-moment monetary policy shock. The effects of the productivity shock arefour times as large as the effect of the monetary policy first-moment shock (see figures 6 and 7). The effects of the uncertainty shocks on the term premium are in turn much larger. Figure 8 presents the responses of the term structure variables to productivity second-moment shock while figure 9 presents the responses of the term structure variables to monetary policy second-moment shock. After an increase in the volatility of the productivity shock, the prices of the bonds decrease and term premium increases. Qualitatively similar response is obtained based on the second measure of term premium, the excess holding return. Quantitatively, however, the excess holding return reacts to increased volatility of the productivity shock by less than the theoretical term premium.

Figure 8: Responses to the second moment productivity shock

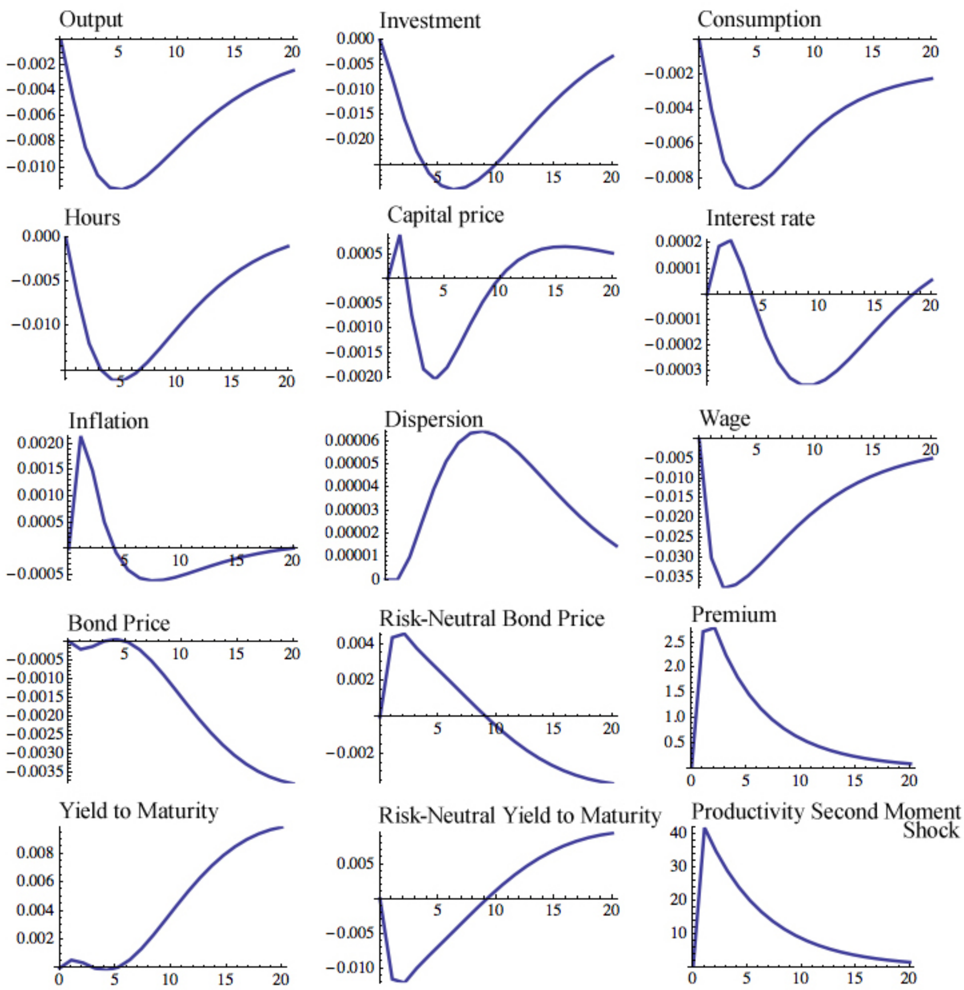
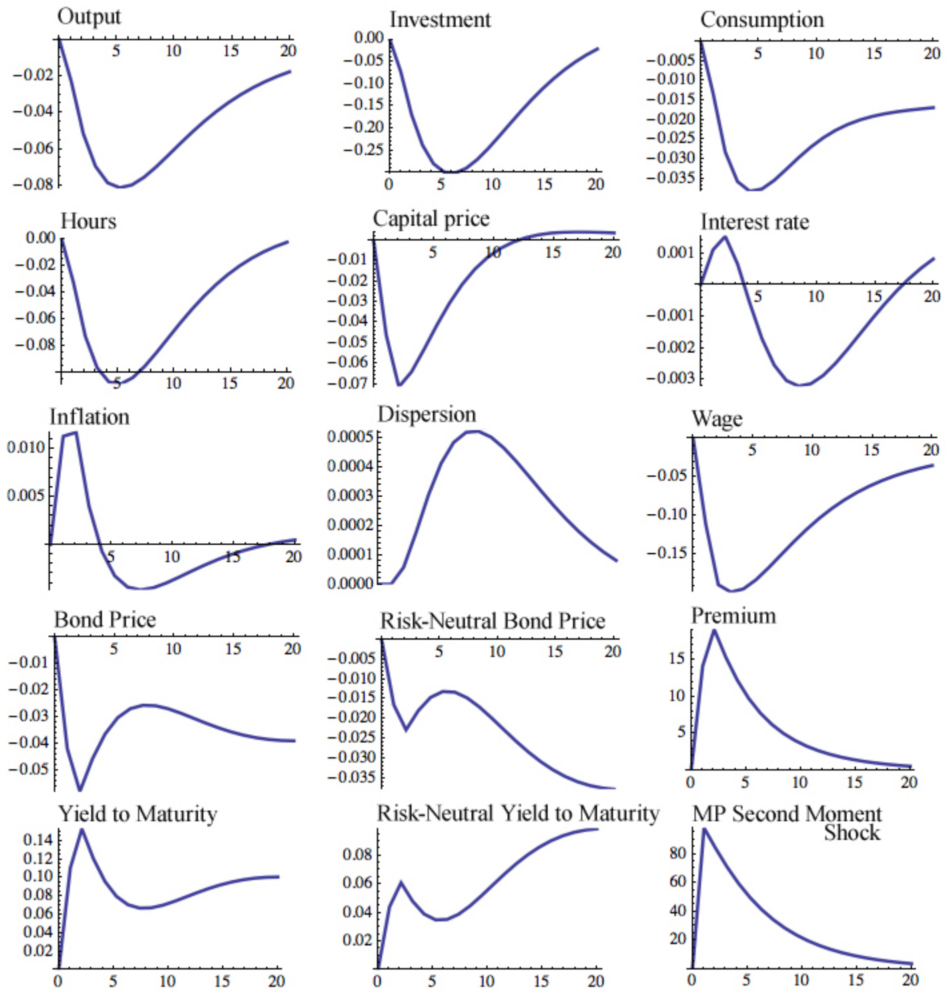


Figure 9 shows the responses of the term structure variables to monetary policy second-moment shock. Similarly, as in the case of the increase in the volatility of the productivity shock, following an increase in the uncertainty about monetary policy, the prices of the bonds decrease and the theoretical term premium as well as the excess holding return increase. Comparing these responses to the responses to productivity second moment shocks, note, that the uncertainty of the monetary policy affects the term premium by roughly 100 times more than uncertainty about productivity.

Figure 9: Responses to the second moment monetary policy shock



While macroeconomic literature has documented the importance of the productivity and monetary policy shocks for the term premium, it seems that once taking into account also the uncertainty shocks, the uncertainty of the monetary policy is the main determinant of additional term premium volatility that standard macroeconomic applications have difficulties to match.

10 Sensitivity analysis

The literature has emphasized a great importance of habit persistence for the term premium. In particular, matching the properties of the term premium (and/or asset prices in general) implied by the model with homoscedastic shocks requires employing high habit persistence. Therefore, in the following we examine the implications for the term premium for different values of the habit persistence parameter.

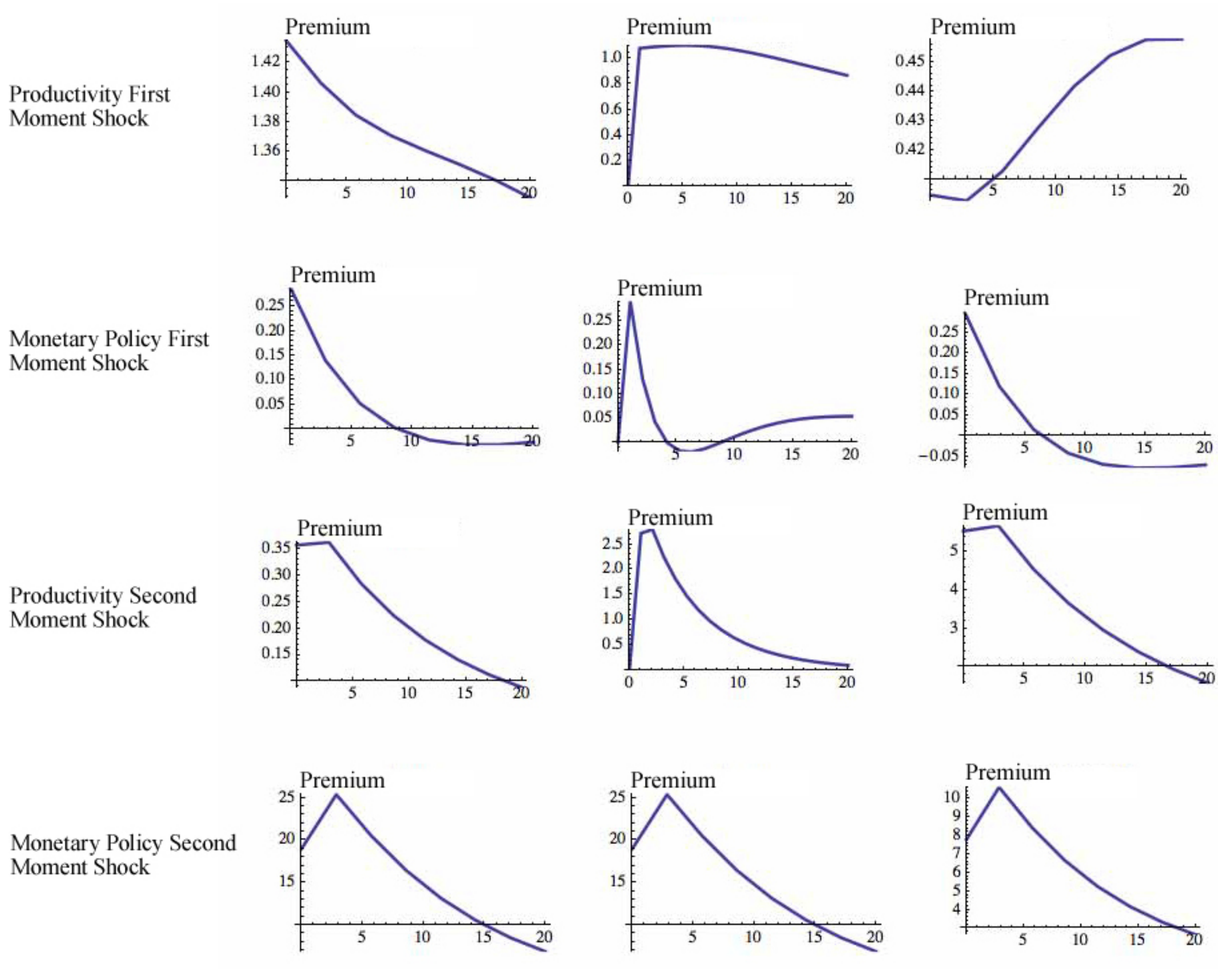
Table 4 gathers the ergodic values of the term structure variables for the different values of the habit persistence parameter. The higher the value of the habit persistence parameter, the higher are the ergodic values of the bond prices and of the term premium. Note, that even when the model employs negligible habit persistence, the ergodic value of the term premium is positive.

Table 4: Deterministic and ergodic values for different values of the habit persistence parameter

Variable	Det.SS	Erg.SS (h=0.01)	Erg.SS (h=0.66)	Erg.SS (h=0.90)
Bond price	0.40000	0.40398	0.40396	0.40397
Risk neutral bond price	0.40000	0.40446	0.40463	0.40539
Term premium	0.00000	1.23811	1.71772	3.63879
Holding returns	4.02000	3.91804	3.91852	3.91828
Risk neutral holding returns	4.02000	3.90566	3.90134	3.88189

Figure 10 compares the impulse responses of the term premium to the first and the second moment shocks for the different values of habit persistence parameter. Panels 1 and 2 show the responses to macroeconomic shocks; that is, responses to the first moment productivity shock and the first moment monetary policy shock. The lower is the habit persistence the higher is the impact of the first moment productivity shock on the term premium. However, the higher habit persistence, the longer lasting is the effect of the productivity shock on the term premium (see panel 1). Habit persistence does not matter significantly for the responses of the term premium to the first moment monetary policy shock (see panel 2).

Figure 10: Comparison of the responses to the first and the second moment shocks for different values of the habit persistence parameter



Panels 3 and 4 show the responses to real and nominal uncertainty shocks; that is, the responses to the second moment productivity shock and the second moment monetary policy shock. While in the case of no habits, shock to the second moment of productivity seems negligible for the term premium, the shock to the second moment of monetary policy shock has a significant effect on the term premium. Increasing the habit persistence seems to lower the importance of the effects of the uncertainty shocks in monetary policy

and increase the effects of the productivity uncertainty shocks on the term premium. Note, that even for the habit persistence parameter as high as 0.9, the uncertainty of the monetary policy seems to be the main determinant of the term premium.
Given the unconventional monetary policy the central banks have conducted to counteract the recent financial crisis that aimed at affecting expectations about inflation in the future and given that inflation is expected to increase in the future, unless monetary policy is conducted very carefully, the present results show that one can expect an increasing volatility in the term premia in the future.

11 Conclusion

The present paper analyzes the effects of the uncertainty shocks on the term structure. To this end, uncertainty shocks are introduced into a standard New Keynesian model through assuming heteroscedasticity of the macroeconomic shocks and in particular, they are assumed to exhibit stochastic volatility in the form of smooth changes in variances of the structural shocks. The uncertainty shock is a shock into volatility of each macroeconomic shock.

While macroeconomic literature has documented the importance of the productivity and monetary policy shocks for the term premium, it seems that once taking into account also the uncertainty shocks, the volatility of monetary policy is the main determinant of the term premium. The results show that productivity uncertainty shocks have small impact on the term premia, while the uncertainty shock to volatility of monetary policy shock seems to affect the term premia by 100 times more than the real uncertainty shocks.

As widely known, habit persistence is of great importance for analysis of the term premia. The results show, that the lower is the habit persistence the higher is the impact of the first moment productivity shock on the term premium, while the habit persistence does not seem to matter significantly for the responses of the term premium to the first moment monetary policy shock. In the case of the uncertainty shocks; i.e. the responses to the second moment productivity shock and to the second moment monetary policy shock the habit persistence affects significantly the relative importance of the real and nominal uncertainty shock. While in the case of no habits, shock to the second moment of productivity seems negligible for the term premium, the shock to the second moment of monetary policy shock has a significant effect on the term premium. Increasing the habit persistence seems to lower the importance of the effects of the uncertainty shocks of monetary policy and increase the effects of the productivity uncertainty shocks on the term premium. Note, that even for the habit persistence parameter as high as 0.9, the uncertainty of the monetary policy seems to be the main determinant of the term premium.

Given the unconventional monetary policy the FED has conducted to counteract the recent financial crisis that aimed at affecting expectations about inflation in the future and given that inflation is expected to increase in the future, and based on the results presented in the paper, one can expect an increasing volatility in the term premia in the future.

References

Adjemian, Stephane, Houtan Bastani, Michel Juillard, Ferhat Mihoubi, George Perendia, Marco Ratto, and Sebastien Villemot, "Dynare: Reference Manual, Version Dynare," Technical Report 1, CEPREMAP 2011.

Amisano, Gianni and Oreste Tristani, "A DSGE model of the term structure with A DSGE model of the term structure with A DSGE model of the term structure with regime shifts," BoF Working Paper, Bank of France 2008.

Basu, Susanto and Brent Bundick, "Uncertainty Shocks in a Model of Effective Demand," Technical Report, Downloadable Boston College Working Papers September 2011.

Bloom, Nicholas, "The Impact of Uncertainty Shocks," *Econometrica*, 2009.

Calvo, Guillermo, "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 1983, *12* (3), 383–398.

Campbell, John Y. and John Cochrane, "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior," *Journal of Political Economy*, April 1999, *107*, 205–251.

Epstein, Larry G. and Stanley.E. Zin, "Substitution, Risk Aversion and the Temporal Behaviour of Consumption and Asset Returns: A Theoretical Framework," *Econometrics*, July 1989, *57* (699), 937–969.

Fernandez-Villaverde, Jesus and Juan F. Rubio-Ramirez, "Estimating Macroeconomic Models: A Likelihood Approach," Technical Report, University of Pennsylvania 2006.

_ , Dario Caldara, Juan F. Rubio-Ramirez, and Yao Wen, "Computing DSGE Models with Recursive Preferences," Technical Report, University of Pennsylvania 2009.

Gertler, Mark, Luca Sala, and Antonella Trigari, "An Estimated Monetary DSGE Model with Unemployment and Staggered Nominal Wage Bargaining," *Journal of Money, Credit and Banking*, 2008, *40* (8), 1713–1764.

Hördahl, Peter, Oreste Tristani, and David Vestin, "The Yield Curve and Macroeconomic Dynamics," ECB Working Paper Series 832, European Central Bank November 2007.

Justiniano, Alejandro and Giorgio Primiceri, "The Time Varying Volatility of Macroeconomic Fluctuations," *The American Economic Review*, June 2008, *98* (3), 604–641.

_ , _ , **and Andrea Tambalotti**, "Investment Shocks and Investment Shocks and Business Cycles," *Journal of Monetary Economics*, March 2010, *57* (2), 132–145.

Kim and Orphanides, "The bond market term premium: what is it, and how can we measure it," BIS Quarterly Review, Bank for International Settlements June 2007.

Kim, S., N. Shephard, and S. Chib, "Stochastic Volatility: Likelihood Inference and Comparison with ARCH Models," *The Review of Economic Studies*, 1998, *65* (3), 361–393.

Ravenna, F. and J. Seppala, "Monetary policy and the term structure of interest rates," Technical Report, University of California, Santa University of California, Santa University of California, Santa Cruz 2006.

Rudebusch, Glen and Eric Swanson, "DSGE Model with Long-Run Real and Nominal Risks," FRBSF Working Paper Series 31, The Federal Reserve Bank of San Francisco November 2008.

_ **and** _ , "Examining the Bond Premium Puzzle Examining the Bond Premium Puzzle with a DSGE Model," *Journal of Monetary Economics*, October 2008, *55.*

Schmitt-Grohe, Stephanie and M. Uribe, "Optimal Inflation Stabilization in a Medium-Scale Macroeconomic Model," NBER Working Papers 11854, The National Bureau of Economic Research 2006.

Chapter II
Propagation of Uncertainty Shocks through Risk-Aversion and Investment Costs

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Abstract

We propose a mechanism through which uncertainty shocks have first-order effects on the macroeconomic environment. In particular, when the agents observe increased uncertainty in the economy, they do not only adjust their actions according to the increased uncertainty but in fact also become more risk-averse. Furthermore, the investment adjustment costs are discussed to be higher in periods when the uncertainty is high. These two simple modifications of the otherwise standard model bring about first-order effects of uncertainty shocks and therefore provide a large amplification mechanism of the effects of uncertainty shocks on the macroeconomic variables.

Keywords: uncertainty shocks, risk aversion, investment adjustment costs, economic downturn

JEL Classification: E32, E43, E44, G12

1 Introduction

In the light of the recent economic crisis, the uncertainty shocks gained a large attention among economists and policy makers. The common wisdom is that larger uncertainty has significant effects on the agents, however, the standard DSGE models cannot catch this fact. The present paper proposes two simple modifications of the standard dynamic stochastic general equilibrium model in order to account for large drops in economic activity due to the uncertainty shocks as observed in the periods of economic distress.

The standard models that have been commonly used by economists and policy makers prior to the recession assumed that macroeconomic shocks are homoscedastic and were solved using the first-order approximation around the steady state. Along this line, the uncertainty shocks by construction have no role. To be able to account for the effects of the uncertainty shocks on the economy, first, the model has to exhibit heteroscedatic shocks (in the present paper the heteroscedasticity is assumed to be in the form of smooth changes in volatility); and second, since the uncertainty shocks do not have the first-order effects on the economy, the model has to be solved using the higher-order approximation around the steady state. However, the second- and the third-order terms in the approximated solution of the model are fairly small for any reasonable calibration of the model and as such induce negligible effects of the uncertainty shocks on the economy.

Crucial assumptions made in this paper that allow for significant role of uncertainty shocks within otherwise standard dynamic stochastic general equilibrium model with stochastic volatility are to allow the risk-aversion to be time-varying and investment adjustment cost to depend on uncertainty. In particular, the risk-aversion is assumed to be correlated with the uncertainty level in the macroeconomic environment. Allowing for small fluctuations in the risk-aversion in response to the change in the level of uncertainty is shown to provide a large amplification of the effects of uncertainty shocks on macroeconomic variables and especially it generates a large drop in consumption due to uncertainty shock. The second assumption that is crucial for the results is that the investment adjustment costs are not only quadratic in the deviation of investment from its steady state, but depend also on uncertainty. The function that allows the adjustment costs to be related to the level of uncertainty also delivers a drop to in macroeconomic variables (though small), but more importantly provides a mechanism through which uncertainty has first-order (and therefore significant) negative effect on investment. As such, the model is able to account for large drops in consumption and investment activity during the economic crisis.

The present paper builds the bridge between macroeconomics, finance, behavioral and experimental economics literature. However, in order to keep the introduction short, the literature review is postponed until the following sections, where the idea and motivation for time-varying risk-aversion and modified adjustment cost function are discussed in details.

The rest of the paper is organized as following: Section 2 relates the paper with the existent literature and motivates the use of the time-variation of the risk-aversion, while section 3 motivates the use of our functional form of the investment adjustment costs. In sections 4 and 5 the theoretical model and the calibration of the model are described, respectively. The results are presented and discussed in the section 6. Section 7 concludes.

2 Uncertainty and (Time-Varying) Risk-Aversion

Macroeconomic models do not distinguish between risk-aversion and risk-appetite. That is, the two concepts are used interchangeably to refer to the sentiment in the asset markets.[1] Both concepts usually rely on the microeconomics interpretation and as such they embed as an inherent assumption that the risk-aversion is a structural parameter and therefore constant. That is, for a given level of risk, a representative agent takes optimal decisions given his attitude towards risk. The relative risk-aversion is, on the other hand, allowed to vary over time. In particular, the common assumption by now is to employ habit persistence in the utility function, in order to fit the consumption data in the macroeconomic models and in order to fit the risk and/or the term premium in the macro-finance models.[2] While until recently these models had been considered to be fairly good in explaining the business cycles fluctuations, it became clear in the recent financial crisis that whether or not the habit persistence (or number of other frictions) is introduced in the model, these models suffer from two shortcomings: first, they are not able to replicate the large drops in the economic activity observed during the economic crisis, and second, they are not able to replicate the facts about the risk and/or the term structure during this economic downturn.

The issue how to explain large fluctuations of the macroeconomic variables during the economic crisis is at the moment a subject to a lively and controversial debate among economists. Part of this discussion is also ongoing research about what are the effects of uncertainty shocks on the macroeconomic variables. See for example, Bloom (2009), Fernandez-Villaverde et al. (2009b), Basu and Bundick (2011). By construction, the canonical dynamic stochastic general equilibrium model of Smets and Wouters (2003, 2007) type that is nowadays widely used theoretically as well as in the policy making

[1] Other terms sometimes used are also risk-perception and risk-sentiment.

[2] To be more specific, the macro-finance literature has, in the attempt to bring the model closer to the data with respect to the term structure properties, resorted to employing habit persistence within complicated preference specifications, for example Epstein and Zin (1989).

institutions, has to be approximated up to the third-order in order for uncertainty shocks to have effects on macroeconomic variables. As the uncertainty shocks do not have first-order effects on the economy and since for a reasonable parameterization of the model, the higher order terms are relatively small (see Basu and Bundick (2011) and Ferjancic (2012)); therefore, these models are not able to replicate significant economic fluctuations during the crises when uncertainty increases significantly.

As stated in the introduction, allowing for the risk-aversion to be time-varying and in particular, to be correlated with the level of macroeconomic uncertainty provides the propagation mechanism through which the uncertainty shocks have the first-order effects on the macroeconomy. This assumption is from macroeconomics view fairly controversial. In particular, the current state of the art macroeconomic models do not take into account that risk-aversion of the agents might vary, i.e. the curvature of the utility function might change over time. To motivate the idea that this should be taken into account, let me put forward that there is plenty of evidence in the finance, behavioral and experimental literature (I return to this point below) about the time-variation of the risk-aversion and that therefore such an assumption is from this point of view a reasonable one. While the evidence shows that these variations are fairly small in normal times, they are important in the periods of distress. Bearing the latter in mind, the constant risk-aversion parameter in the standard macroeconomic models might be a justified assumption for the macroeconomic analyses in normal times, yet such models miss an important channel of propagation mechanism of the shocks when the economy is in recession.

Turning to the macro-financial models focusing on the term premium[3], the common assumption is that the term premium is determined by the risk in the economy (represented as a combination of model's shocks) and price that agents in the model attach to the risk. Further, the price of the risk is commonly assumed to be related to the constant risk-aversion parameter and 'other factors' that affect the term premium. As the price of the

[3]Recall that this group of model does not distinguish between the concept of risk-aversion and risk-appetite and/or risk perception of the economic agents.

risk is usually extracted as a residual between the term premium and risk as represented by shocks in the model, one can say little if anything about micro-foundations of the risk pricing.[4]

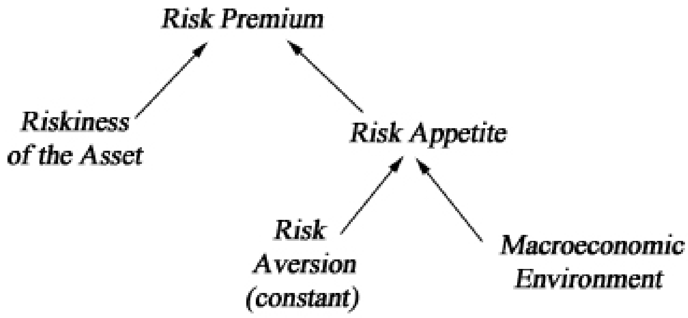
In financial literature, however, the terms risk-appetite and risk-aversion are considered to be different concepts. Gai and Vause (2004) make an attempt to draw a clearer distinction between them: risk-appetite (or the willingness of investors to bear the risk) depends on both the degree to which investors dislike uncertainty and the level of that uncertainty. The level of uncertainty about consumption prospects depends on the macroeconomic environment; that is, in adverse circumstances, an investor will require higher expected excess returns to bear the risk and the risk-appetite will be low; and conversely, high risk-appetite will be associated with low expected excess returns. Risk-appetite is according to their interpretation a broader concept than the microeconomics interpretation of risk-aversion that measures the degree to which investors dislike uncertainty described by the underlying preferences. They further claim, that the risk-appetite is commonly assumed to shift periodically as investors respond to episodes of financial distress and macroeconomic uncertainty, while the risk-aversion is a parameter that should not change frequently over time (note the similarly to the macroeconomics views about risk-aversion).

Further, Gai and Vause (2004) claim that the risk premium (or the expected excess return required to compensate an investor for holding a risky asset) is determined partly by the inherent riskiness of the asset, and partly by the level of risk-appetite. The higher the appetite for risk, the lower the risk premium. Figure 1 illustrates how their concepts are linked.

Focusing on the figure 1, it seems - given that the risk-aversion embedded in preferences is assumed to be constant - as the variation of the risk-appetite is solely due to the changes

[4]This models further are also commonly subject to the criticism that by construction they do not allow for the feedback from the term structure to economy. In this respect, the model presented here provides a step in this direction by allowing for the feedback from uncertainty to economy. The term structure properties are left aside in this model, however, this issue is considered in another paper in progress.

Figure 1: Summary of the Concepts



Source: Figure 1 in Gai and Vause (2004)

in the macroeconomic environment; and in particular, due to changes in the level of uncertainty.[5] It is straightforward to see that given the assumption of constant risk-aversion, the risk-appetite can fluctuate over time solely due to the changes in the macroeconomic environment. Therefore, despite the attempt to distinguish between the concepts on the intuitive level; in terms of the risk-pricing, their model is from the analytical point of view observationally equivalent to macro-finance class of models.

In figure 2 the global risk-aversion indicator constructed by ECB is presented.[6] The indicator exhibits large volatility over time with maximum risk-aversion being reached in the period of the 2008-2009 financial crisis. As such, the indicator in its essence corresponds to common wisdom, that the economic agents might be in certain periods more risk-averse then in other periods. That is, the agents observing higher uncertainty might become more risk-averse. Neither models with constant risk-aversion nor models that allow for time-variation in uncertainty by construction cannot capture the variation

[5]The same conclusion can be drawn from their analytical expression for the term-premium in terms of the risk-appetite and uncertainty in the economy.

[6]I woul like to thank to workers at ECB's statistical office, who kindly provided the data so that figure Chart S.52 (ECB (2011), Financial Stability Review December 2011, Statistical Annex, p.19) could be reproduced. For the description of the methodology used to construct the indicator see ECB (2007), Financial Stability Review December 2011.

in the risk-aversion indicator as presented in figure 2. Therefore, the fact that the models discussed so far are not able to capture the determinants of the risk-pricing might be, at least to some extent, due to the fact that by construction they do not allow for the variation in the risk-aversion.

Figure 2: Global Risk-Aversion Indicator

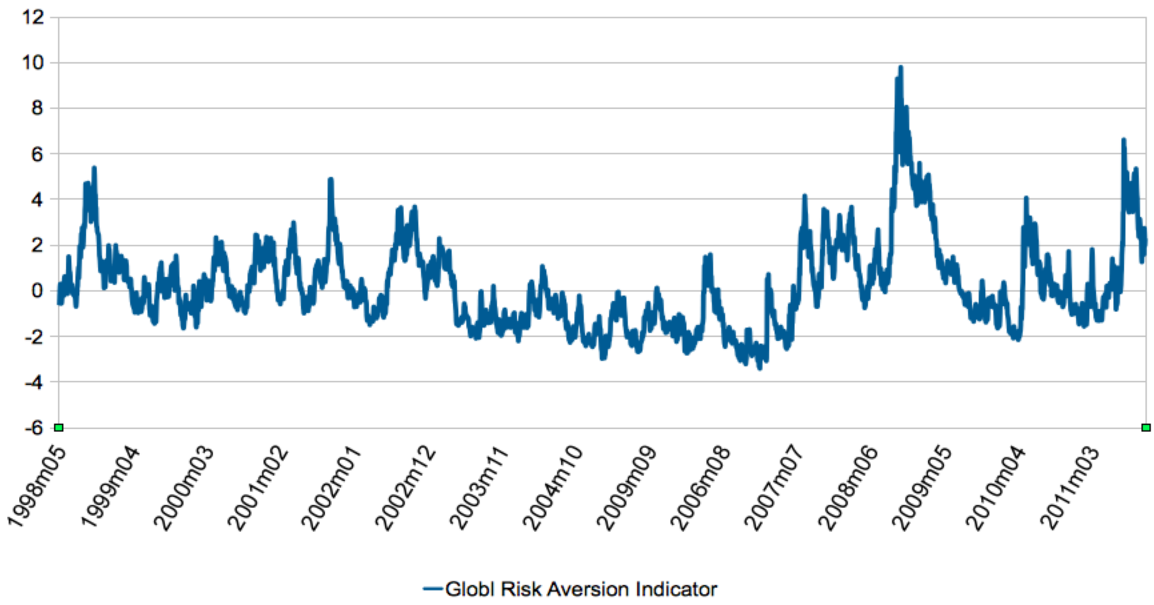
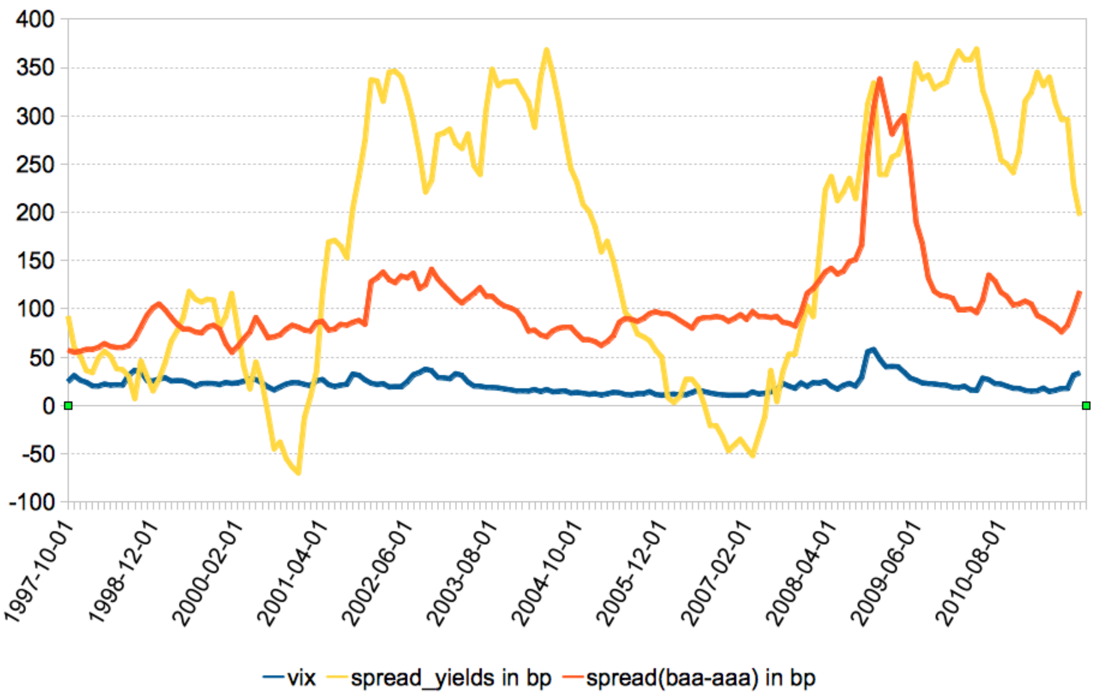


Figure 3 plots the VIX index[7] as a measure of uncertainty, the spread between yields of BAA and AAA corporate bonds and the spread between the yields of the government securities with maturity 10 years and 1 quarter. The data for VIX, US corporate and government bonds yields were obtained at the Federal Reserve Board online database. If pricing of bonds was indeed based on the constant risk-aversion, the spreads should be highly correlated with the VIX index. In the figure we can see that the spread between corporate bonds is indeed highly correlated with the VIX, however, the spread between

[7]VIX is Chicago Board Options Exchange Market Volatility Index that measures implied volatility of SP 500 index options.

yields of government securities does not seem to follow the VIX index closely. It seems that when the VIX index increases significantly, the spread between yields on the government bonds increases but remains higher for a much longer time. Given that, it is reasonable to think that among investors, those who invest in government bonds might be more risk averse and their risk aversion might increase in recession times and remain higher for a certain period.

Figure 3: VIX, Spread(baa-aaa), Spread(10y-1q) - Levels



Therefore, despite the fact that preferences are unlikely to vary over time significantly, they might be subject to minor changes over time. This is a crucial assumption in the present analysis. In particular, the risk-aversion of the representative agent is assumed to be subject to a small variations due to the variations in the macroeconomics risks. When the agents observe large increases in uncertainty, they do not only adjust their optimal decisions according to the new level of uncertainty, but they also become more risk-averse. The idea that agents' preferences change over time is not new. However, the novelty in this paper is to relax the restriction of constant risk-aversion in the standard dynamic stochastic general equilibrium model.

The changes in the risk-aversion are modeled as an AR(1) process that evolves around a constant, but can be affected by uncertainty shock. The implied variations of the risk-aversion are small, definitely smaller than the range of different calibrated values for this parameter that is used in the literature. The way the changes in the risk-aversion are modeled in the present paper is in its nature of the reduced form. More micro-founded approach is certainly needed, however, this reduced form approach provides insights on how the current dynamic stochastic general equilibrium models might be improved to account for the periods of economic distress. While the models with time-varying relative risk-aversion are certainly not able to explain the large drop of macroeconomic activity observed in the recent economic crisis, the model that allows for the risk-aversion being correlated with the economic uncertainty allows for the first-order effects of uncertainty shocks on the economy and as such, can fill this gap.

The nature of modeling the fluctuations of the risk-aversion in the present paper can be easily related to, for example Campbell and Cochrane (1999) and Brandt and Wang (2003). Campbell and Cochrane (1999) model changes in the concept of relative risk-aversion by assuming an exogenous law of motion for external habits. Their approach yields almost constant relative risk-aversion, implication that is at odds with the data. Brandt and Wang (2003) improve the model by assuming that the relative risk-aversion fluctuates around a constant with respect to expected inflation and expected economic growth. Their approach implies more cyclicality of the relative-risk aversion and helps to bring the model closer to the data in terms of the term-structure. Note, however, that for both models, the fluctuations of the relative risk-aversion can be interpreted as coming from the time-varying risk-aversion. In fact, in both models, it is observationally equivalent if the fluctuations in the relative risk-aversion are due to the fluctuations in risk-aversion parameter or due to the other factors affecting relative risk-aversion or both.

As standard macro models that embed endogenous variation in the relative risk-aversion cannot explain neither the term structure properties nor the large drops in the economic variables during the economic downturns, it is reasonable to think that there might be additional cyclicality of the relative risk-aversion induced by the fluctuations in the risk-aversion parameter.

To gain further motivation of the time-varying risk-aversion, we briefly review the empirical finance literature, the behavior economics literature and the experimental economics literature. In the empirical finance literature, the existence of the fluctuations in the aggregate risk-aversion is a quite standard result. The usual interpretation is, that these fluctuations can occur due to the changes in wealth and due to changes in expected consumption path or consumption habits. The modeling of the risk-aversion in this paper is perfectly consistent with these views. Common finance interpretation is that in the recession the wealth decreases as well as the expected consumption decreases and therefore the risk-aversion increases, which is in the reduced form equivalent to the idea of this paper. Scheicher (2003), for example, uses option prices in the German stock market to explain daily movements in the risk-aversion of the investors. His findings are that investors in the German equity market has become increasingly risk-averse since 1998. In addition, their results show that the risk-aversion is strongly linked to the changes in US volatility and to negative news with regard to the exchange rate. Smith and Whitelaw (2009) also find that time-varying risk-aversion is the economic mechanism that appears to match important features of equity return data at the market level. Consistent with theoretical intuition, the price of risk varies over the business cycle, with risk-aversion increasing substantially over the course of economic contractions. Interesting result of their analysis is that it is variation in the price of risk, not in the quantity of risk, that is the dominant component of the equity risk premium. They conclude that this phenomenon may partially account for the puzzling results that often arise in estimating models with the price of risk assumed to be constant.

The standard models, in addition, do not incorporate the concept of perception of risk while in the behavioral economics literature it is widely accepted that perception of risk might vary, might be incomplete and in particular, it can be different from the true value of the risk in the economy. Since the agents cannot fully observe the risk, they estimate it or form their perceptions of risk due to some pre-specified rules. The assumption of the time-varying risk-aversion used in this paper could be interpreted as reduced form of the varying perceptions of risk.

Drawing from the experimental economics literature, Guiso et al. (2011), for example, claim that the fluctuations of the aggregate risk-aversion might be due to the changes in the individual risk-aversion and/or due to the changes in the distribution of wealth across individuals with different risk-aversion. Based on the experiment, they argue that during the recent economic crisis there was a sharp increase in the individual risk-aversion not attributed solely to a worsening of expectations about the distribution of future investments as captured by the relative risk-aversion in macroeconomic models. Guiso and Paiella (2008) use household survey data to construct a measure of risk-aversion, which they relate to consumer's endowments and to measures of the background risk. First, they find that risk-aversion is decreasing function of endowment and thus reject CARA preferences and second, they find that the consumer's environment affects risk-aversion. With respect to the latter finding, they claim that the individuals who are more likely to face income uncertainty or to become liquidity constrained exhibit a higher degree of risk-aversion.

The approach undertaken in the present analysis is from a technical point of view the closest to the work of Gordon and St-Amour (2000), which develops a consumption based asset pricing model in which attitudes towards risk are contingent upon the state of the world. They study asset prices using the two-state Markow preference regimes where the bull and the bear markets reflect alternating periods of low and high risk-aversion. That is, the within-period utility is conditionally iso-elastic. These fluctuations affect the agent's choice over risk-free and risky assets in hedging against IMRS risk. They show that when risk-aversion is state dependent, low (high) level of consumption relative to a subjective metric, counter-cyclical (pro-cyclical) risk-aversion implies that consumption shocks generate larger fluctuations in the marginal utility, against what the agent hedge by readjusting his optimal portfolio. Their estimated parameters point towards infrequent, moderate - but significant - and counter-cyclical shifts in curvature. In particular, their estimation for the two risk-aversion parameters are 2.6513 and 2.2059, implying the difference 0.4454.[8]

3 Uncertainty and Investment Adjustment Costs

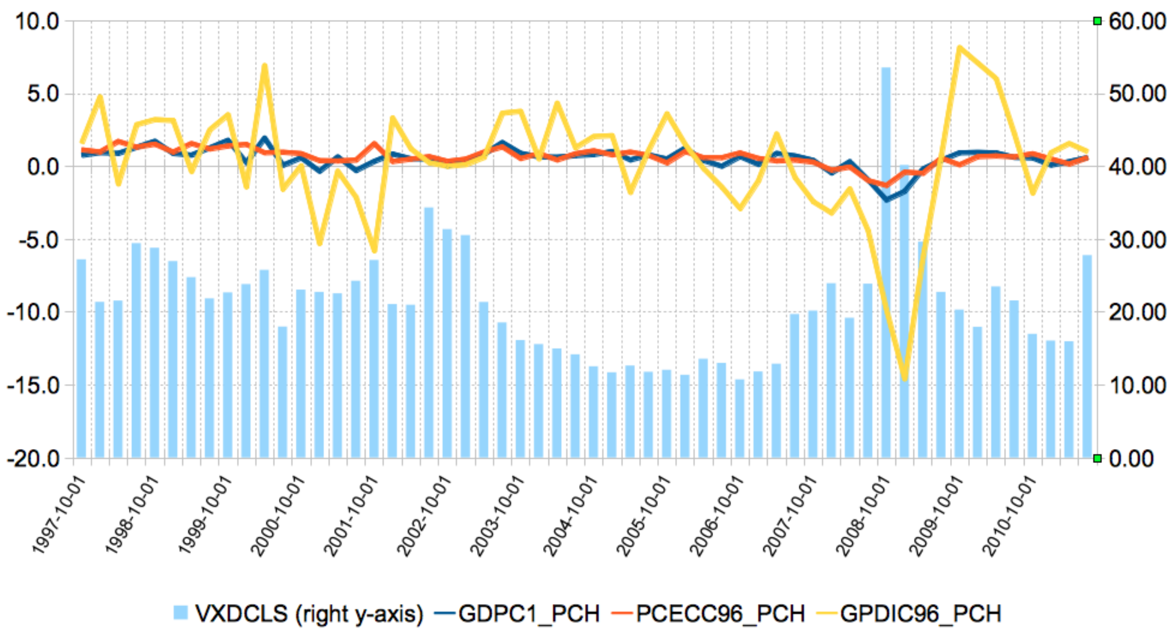
The adjustment costs are introduced into the macroeconomic literature as a reduced form that represents those frictions that: (i) dampen instantaneous response, and (ii) introduce sluggish response of the investment to new information about its profitability. Justiniano and Primiceri (2008) and Justiniano et al. (2010) attach to a function that represents the adjustment costs an investment specific shock. They provide three interpretations of the investment specific shocks. As for the model-based interpretation they interpret the investment shocks as corresponding either to shocks to the relative price of investment in terms of consumption goods or to investment specific technological disturbances. For the former, following Greenwood et al. (1997) they construct a proxy and show that the decline in the volatility of the investment shocks is in line with a decline in the variance of the relative price of investment in terms of consumption goods. The latter interpretation is based on Fisher (2006) as well as on their own estimates that lower variability of investment shocks reflects a decline in the volatility of disturbances to investment specific technology that can be observed in the data. The third interpretation of the investment specific shocks is that, though in reduced form, they reflect financial costs related to the purchase of durable and capital goods. As they argue, this theoretical hypothesis

[8]As will be clear later, this difference in the risk-aversion in the two regimes is much larger that the fluctuations assumed in this paper.

is consistent with the empirical and anecdotal evidence about the expanded access to credit and borrowing for firms and households since the beginning of the 1980s. It seems natural to think that the investment shocks are one of the crucial factors behind the recent economic downturn.

Figure 4 presents the series of the real output growth, real investment growth, real consumption growth and VIX index.[9] There is a clear negative relationship between the growth of real variables and uncertainty as represented by the VIX index and in particular the increase of uncertainty negatively affects the investment growth the most. This is consistent with empirical studies which mostly conclude that the evidence seems to support a negative investment-uncertainty relationship (see for example Bloom (2009), Bloom et al. (2009) for more detailed literature review).

Figure 4: VIX, Output Growth, Consumption Growth and Investment Growth



[9]The data for the real macroeconomic variables are taken from St.Luis Federal Reserve on-line data base.

From a theoretical view, in the investment literature the impact of the effects of uncertainty on investment works through the curvature of the marginal revenue product of capital with respect to uncertainty. The relationship is positive (known as the Hartman-Abel-Caballero effect) if the marginal revenue product of capital is convex in productivity shocks, while the relationship is negative if investment is partially or fully irreversible and the marginal revenue product of capital is concave in productivity shocks. However, in standard macroeconomic applications where the model is approximated up to first-order around the steady state, the uncertainty does not have a role for investment dynamics. Basu and Bundick (2011) analyze in details the effects of the uncertainty on the macroeconomy that work through the third-order effects. While they show that investment responses to increase of uncertainty are negative, the effects of uncertainty on investment (as well as on the other macroeconomic variables) are fairly small. The relatively small impact of uncertainty is clearly at odds with what we observed in the recent period.

In this analysis, the function describing the capital adjustment costs is modified to enable the model to deliver a large negative response of the investment uncertainty increases. In particular, the capital adjustment costs are assumed to have two sources: first, as standard, they are quadratic in deviation of investment from its level in the previous period, and second, they are related to deviation of the macroeconomic uncertainty as represented by the volatility of the productivity shocks from its steady state value. The investment costs are higher when the economy is in recession (for example, because borrowing is more costly when uncertainty is higher). Note that this assumption implies that the investment adjustment costs are not symmetric with respect to the sign of the uncertainty change and as such delivers a mechanism through which the investment responds asymmetrically (and up to the first-order) to changes in the general real uncertainty. In particular, if the uncertainty in the economy increases, the investment adjustment costs increase, while if the uncertainty decreases the investment adjustment costs decrease. Intuition is straight forward, when the uncertainty in the economy increases, it is more costly for the agents to invest.

4 Model

Households

The baseline model assumes a continuum of agents represented by a representative household. The representative household's preferences are described by standard power utility preference that is separable in consumption good, C_t, and hours worked, L_t and exhibits internal habit formation, that is, habits are determined by consumption in previous period

$$E_t \sum_{s=0}^{\infty} \beta^s \left[\frac{C_{t+s}^{1-\gamma_{t+s}} - 1}{1 - \gamma_{t+s}} - \varphi \frac{L_{t+s}^{1+\tau}}{1+\tau} \right] \tag{1}$$

Where E_t denotes mathematical expectations operator conditional on information set available at time t. $\beta \in (0, 1)$ denotes household's subjective discount factor and γ_t is time-varying risk-aversion.

Time-Varying Risk-Aversion

The risk-aversion of the representative agent is assumed to be subject to a small variation due to the variations in the macroeconomics risks. When the agents observe large increases in uncertainty, they do not only adjust their optimal decisions according to the new level of uncertainty, but they also become more risk-averse. That is, the risk-aversion is related to macroeconomic uncertainty or equivalently, the changes in the risk-aversion are related to uncertainty shocks. It is assumed to follow AR(1) process.

$$\log \gamma_t = (1 - \rho_\gamma) \log \overline{\gamma} + \rho_\gamma \log \gamma_{t-1} + \chi \omega_a \tau_t \tag{2}$$

where $\tau_t \sim N(0, 1)$ is the real uncertainty shock, that is, the shock into the second moment of volatility of productivity shock. ω_a is the standard deviation of the uncertainty shock, while χ is the scaling parameter.

Let I_t be investment, K_t physical capital owned by the household, B_t an one-period state-contingent government bond, W_t the nominal wage, R_t^k one-period nominal return on capital, R_t gross nominal interest rate, P_t the aggregate price level, Π_t the nominal lump-sum profit that household receives from ownership of the firms and T_t the lump-sum taxes. The budget constraint of the household then reads as following

$$P_t C_t + P_t I_t + B_t \leq W_t L_t + R_t^k K_t + R_{t-1} B_{t-1} + \Pi_t + T_t \tag{3}$$

Physical capital, K_t, accumulates according to the following law of motion

$$K_t = (1-\delta) K_{t-1} + \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \tag{4}$$

where δ represents depreciation rate and $S(\cdot)$ is the function that describes the investment adjustment costs.

Investment Adjustment Costs

The function that summarizes the technology which transforms current and past investment into installed capital to be used in the following period, $S(\cdot)$, captures a presence of the adjustment costs of investment. In standard macroeconomic applications where models are log-linearized and solved by first order approximation around the steady state, $S(\cdot)$ is restricted to satisfy following properties: $S(1) = S'(1) = 0$, $S''(1) > 0$, while in the applications in which the model is approximated up to the higher-order the functional form of $S(\cdot)$ has to be specified.

In the present analysis, it is assumed that capital adjustment costs have two sources: first, as standard, they are quadratic in deviation of investment from its level in the previous period, and second, they are related to deviation of the macroeconomic uncertainty as represented by the volatility of the productivity shocks from its steady state value.

$$S_t\left(\frac{I_t}{I_{t-1}}\right) = \frac{\varpi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 \left(\frac{\sigma_{a,t}}{\sigma_{a,t-1}} - 1\right) \tag{5}$$

Note that the investment adjustment costs are not assumed to be symmetric with respect to the sign of uncertainty change. This assumption delivers mechanism through which the investment adjust asymmetrically to changes in real uncertainty. In particular, if the uncertainty in the economy increases, the investment adjustment costs increase while if the uncertainty decreases the investment adjustment costs decrease. The intuition behind this assumption is that when the uncertainty in the economy increases, it is more costly for the agents to invest. This further implies that the investment costs are higher when the economy is in recession (for example, because borrowing is more costly when uncertainty is larger).

The household chooses process for $\{C_t\}$, $\{L_t\}$, $\{B_t\}$, $\{K_t\}$, and $\{I_t\}$ to maximize lifetime utility subject to the budget constraint and physical capital accumulation process. The first order necessary conditions are standard:

$$P_t \lambda_t = C_t^{-\gamma_t} \tag{6}$$

$$\beta \frac{\lambda_{t+1}}{\lambda_t} \frac{R_t}{\pi_{t+1}} = 1 \tag{7}$$

$$\varphi L_t^{\nu} = W_t \lambda_t \tag{8}$$

$$1 = q_t \left[1 - S_t \left(\frac{I_t}{I_{t-1}} \right) - \frac{I_t}{I_{t-1}} S_t' \left(\frac{I_t}{I_{t-1}} \right) \right] + \beta \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \left(\frac{I_{t+1}}{I_t} \right)^2 S_{t+1}' \left(\frac{I_{t+1}}{I_t} \right) \tag{9}$$

$$q_t = \beta \frac{\lambda_{t+1}}{\lambda_t} \left[r k_t + q_{t+1} (1-\delta) \right] \tag{10}$$

where λ_t is the marginal utility of consumption, q_t is the marginal value of capital in terms of consumption good, that is Tobin's q and $\pi_t = \frac{P_t}{P_{t-1}}$ denotes gross inflation.

Final goods producers

At each point in time t, perfectly competitive firms produce a final consumption good Y_t by combining a continuum of intermediate goods, $Y_t(i)$, according to the following technology

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{11}$$

where ε is the elasticity of demand. Profit maximization and the zero profit condition imply that the price of the final good, P_t, is a CES aggregate of the prices of the intermediate goods, $P_t(i)$

$$P_t = \left[\int_0^1 P_t(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}} \tag{12}$$

and that the demand function for the intermediate good i is

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\varepsilon} Y_t \tag{13}$$

Intermediate goods producers

Each variety of good is produced by a monopolistically competitive firm $i \in [0,1]$ using as factor inputs capital services, $K(i)_{t-1}$, and labor services, $L(i)_{t-1}$. The production technology is given by

$$Y\left(i\right)_t = A_t K\left(i\right)_{t-1}^{\alpha} L\left(i\right)_t^{1-\alpha} - A_t F \tag{14}$$

Where A_t is a neutral technological progress and F denotes a fixed cost per unit of output in each period. The productivity shock is assumed to follow the exogenous stochastic $AR(1)$ processes of the form

$$\log \varepsilon_t^a = \rho_a \log \varepsilon_{t-1}^a + e_t^a; \text{ with } e_t^a \sim N(0, \sigma_{a,t}^2) \tag{15}$$

Uncertainty shock is introduced through assuming that the productivity shocks are heteroscedastic. As it is common in the stochastic volatility literature (see Justiniano and Primiceri (2008) and/or Kim et al. (1998)) the following stochastic process for the evolution of the standard deviation, σ_t, is assumed[10]

$$\log \sigma_{a,t} = (1 - \lambda_{\sigma_a}) \log \overline{\sigma_a} + \lambda_{\sigma_a} \log \sigma_{a,t-1} + \omega_a \tau_t; \text{ with } \tau_t \sim N(0,1)) \tag{16}$$

The shock into volatility of productivity shock, τ_t, is therefore interpreted as the uncertainty shock. This type of uncertainty shocks are of Fernandez-Villaverde et al. (2009a) type as opposed to Bloom (2009) type of uncertainty shocks. Note, that the macroeconomic shocks and uncertainty shocks are assumed to be uncorrelated among each other while risk-aversion variation, on the other hand, is correlated with uncertainty.

[10]Modeling logarithms (as opposed to σ_t) guarantees that the standard deviations of shocks are positive in every point in time.

Cost minimization implies

$$W_t = (1-\alpha) MC\,(i)_t\, \frac{Y\,(i)_t}{L\,(i)_t}$$

$$r_t^k = \alpha MC\,(i)_t\, \frac{Y\,(i)_t}{K\,(i)_{t-1}}$$

Because in the symmetric equilibrium the marginal costs, $MC(i)_t$, are identical across firms, we can obtain a common capital-labor ratio and common marginal costs

$$\frac{K_{t-1}}{L_t} = \frac{W_t}{r_t^k}\,\frac{\alpha}{1-\alpha} \tag{17}$$

$$MC_t = \frac{1}{\alpha^{\alpha}\,(1-\alpha)^{1-\alpha}}\,\left(r_t^k\right)^{\alpha}\left(\frac{W_t}{A_t}\right)^{1-\alpha} \tag{18}$$

Staggered prices

The model features staggered prices as in Calvo (1983). In each period only a fraction of randomly picked intermediate firms $1-\xi_p$ can adjust the prices, with opportunity to adjust in period t following an exogenous Poisson process. Re-optimizing firms set their price that is consistent with maximizing the present value of the future profits subject to the demand for variety i:

$$E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s\, \frac{\lambda_{t+s}}{\lambda_t}\, \left\{ Y_{t+s}(i)\left[\Pi_{t,t+s} P_t^{new}(i) - MC_{t+s}(i)\right]\right\}$$

$$\text{subject to: } Y_{t+s}(i) \quad = \quad \left[\frac{P_t(i)}{P_t}\right]^{-\varepsilon} Y_{t+s}\ ;\ \text{where}\Pi_{t,t+s} = \prod_{k=1}^{s} \pi_{t+k-1}.$$

The first-order condition for the price setting is

$$E_t \sum_{k=0}^{\infty} (\beta\xi_p)^k\, \frac{\lambda_{t+k}}{\lambda_t}\, \left[\frac{P_t^{new}(i)}{P_t}\prod_{k=1}^{s}(\beta\xi_p)\,\frac{\pi_{t+k}}{\pi_{t+k-1}}\right]^{-\varepsilon} Y_{t+s}\left[P_t^{new}(i) - mc_{t+k}(i)\right] = 0 \tag{19}$$

The remaining firms set their prices following the full indexation rule

$$P_t(i) = P_{t-1}(i)\pi_{t-1} \tag{20}$$

where π_t is the gross inflation.

The aggregate price level evolves according to

$$P_t = \left[\xi P_t(i)^{1-\varepsilon} + (1-\xi)\, P_t^{new}(i)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{21}$$

Monetary authority and government

Monetary authority sets the short term interest rate following the Taylor rule:

$$\frac{R_t}{R_{ss}} = \left(\frac{R_{t-1}}{R_{ss}}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi_{ss}}\right)^{\phi_\pi} \left(\frac{Y}{t} Y_{t-1}\right)^{\phi_x}\right]^{1-\rho} \tag{22}$$

The Taylor rule exhibits interest rate smoothing, interest rate responses to deviations of inflation from the steady state inflation, π_{ss}, and deviations of aggregate absorption, Y_t, from its level in the previous period. R_{ss} is the steady state gross nominal interest rate.

Aggregation

The resource constraint implies that the aggregate absorption is equal to the aggregate production:

$$Y_t = C_t + I_t \tag{23}$$

5 Calibration

The calibrated values of the parameters that are not related to the process of the shocks are set in line with the values that are standard in the macroeconomic analyses within dynamic stochastic general equilibrium framework. The time unit is one quarter. The households quarterly subjective discount factor, β, is assumed to be 0.99, which implies the annual steady state interest rate 4%.

The capital share in the production function, α, is set to 0.33, which corresponds to a steady-state share of capital income roughly equal to 33%, ν is set to imply a Frisch labor supply elasticity equal to 1 and φ set in order to have a steady state labor equal to 1.

The depreciation rate, δ, is set at 0.025 implying the annual rate of depreciation on capital equal to 10%, while the investment adjustment cost parameter $\omega = 2$. The parameter denoting the fixed cost per unit of output in each period, F, is set to guarantee that profits are zero in steady state.

Parameter of the Taylor rule are: $\rho_r = 0.8$ for the interest rate smoothing, $\phi_\pi = 1.5$ for the response of monetary authority to deviations of inflation from the steady state inflation and $\phi_x = 0.5$ for the response of monetary authority to deviations of output from its level in the previous period. The price-elasticity of demand for a specific good variety $\varepsilon = 6$ implying the steady-state markup in the product market of 20%. The fraction of firms not setting prices optimally each quarter $\xi_p = 0.8$. The remaining of the firms set prices following a full indexation rule.

The calibration of the productivity shock is standard except that the variance is allowed to be time-varying. The technology shock is assumed to be equal to 1 in steady state and it evolves according to $AR(1)$ process with persistence parameter set to 0.95. The persistence parameter of the volatility of productivity is set to 0.99, while the real uncertainty shock, is calibrated following the approach undertaken by Basu and Bundick (2011), that is, equal to 0.4 standard deviation of volatility.

The risk-aversion parameter in the steady state is assumed to be 1 and persistence parameter is set to 0.95. The scaling parameter through uncertainty shock affects risk-aversion, κ is set equal to 0.1. κ was chosen is such a way that response of consumption to uncertainty shock is equivalent to response of the consumption to preference shock that is usually used in macroeconomic applications.

Table 1 summarizes the calibrated values of the structural parameters.

Table 1: Calibration of the Parameters

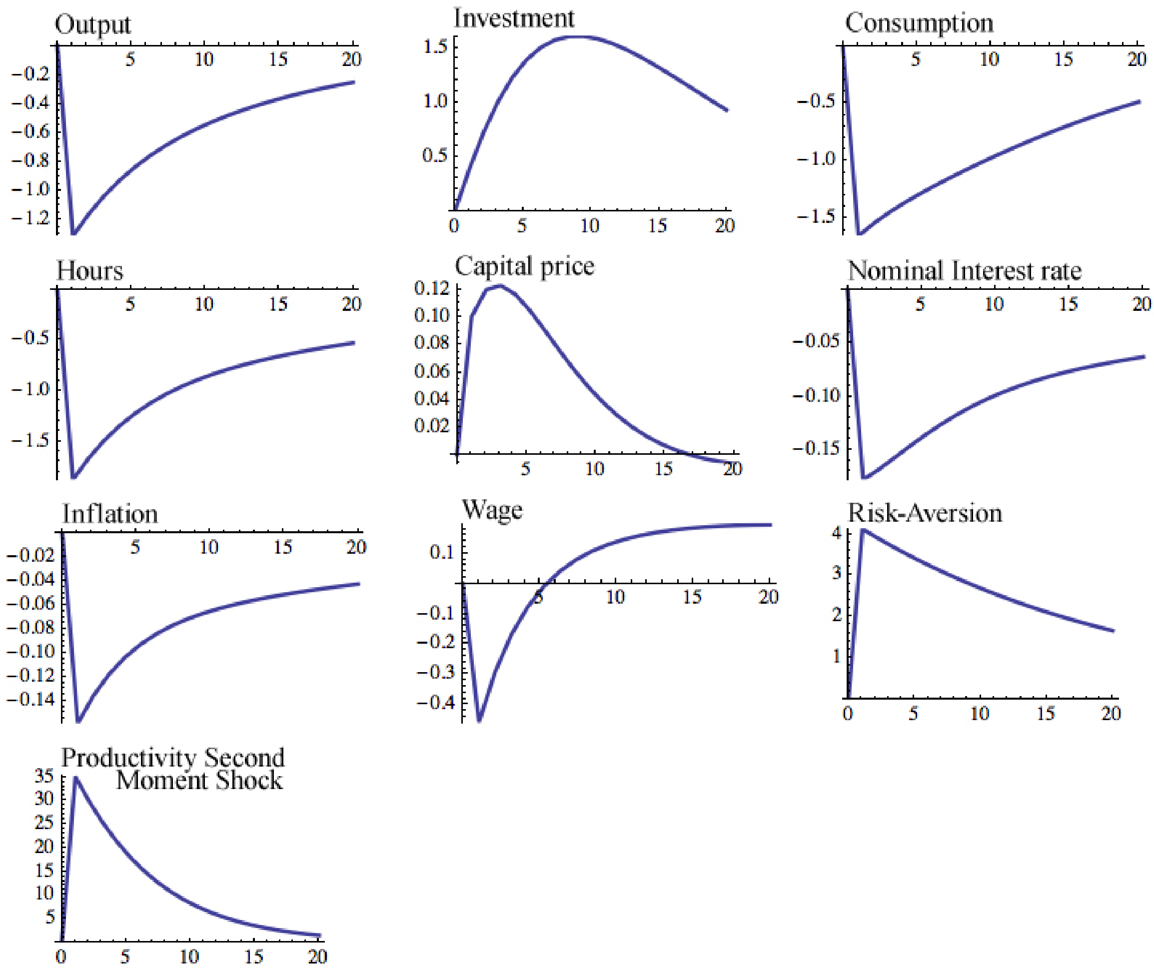
Parameter	Name	Assumed value
α	Capital share in production function	0.33
β	Discount factor	0.99
δ	Depreciation rate	0.025
ω	Investment adjustment cost parameter	2
ν	Disutility of labor	1
ρ_r	Monetary authority interest rate smoothing	0.8
ϕ_π	Monetary authority response to deviations of inflation	1.5
ϕ_x	Monetary authority response to deviations output	0.5
ξ_p	Fraction of firms not setting prices optimally	0.8
ε	Elasticity of demand	6
$\overline{\gamma}$	Risk-aversion in the steady state	1
ρ_γ	Persistence parameter of the risk-aversion	0.95
κ	Scaling parameter	0.1
ρ_a	Persistence parameter of the productivity macro.shock	0.95
$\overline{\sigma_a}$	Std.dev. of productivity macro.shock in steady state	0.01
λ_a	Persistence parameter of the productivity uncertainty shock	0.83
τ_a	Productivity uncertainty shock	$0.4\overline{\sigma_a}$

6 Results

Figure 5 presents the responses of macroeconomic variables to uncertainty shocks that work only through time varying risk-aversion, figure 6 presents the responses that occur through time varying risk-aversion and investment adjustment costs at the same time, while figure 7 present the responses of macroeconomic variables when the mechanism works through investment adjustment costs only. As uncertainty is assumed to affect the risk-aversion and investment adjustment costs, the effects of the uncertainty on the macroeconomic variables are of the first-order and as such enable the model to qualitatively reproduce the economic crunch.

Figure 5 presents the impulse responses of the macroeconomic variables to real uncertainty shock, once the risk-aversion is allowed to be time-varying and in particular to be correlated with the level of economic uncertainty. The mechanism is working through the variation of the risk-aversion. As households observe an increase in the uncertainty level, they become more risk-averse and due to precautionary savings they decrease consumption. As the correlation of the risk-aversion with the uncertainty shocks introduces first-order effects of the uncertainty shocks on the macroeconomy, the effects are large. 0.4 standard deviation productivity uncertainty shock that increases the risk-aversion by 4% induces a significant decrease of output, consumption, employment and real wages with respect to their steady state values.

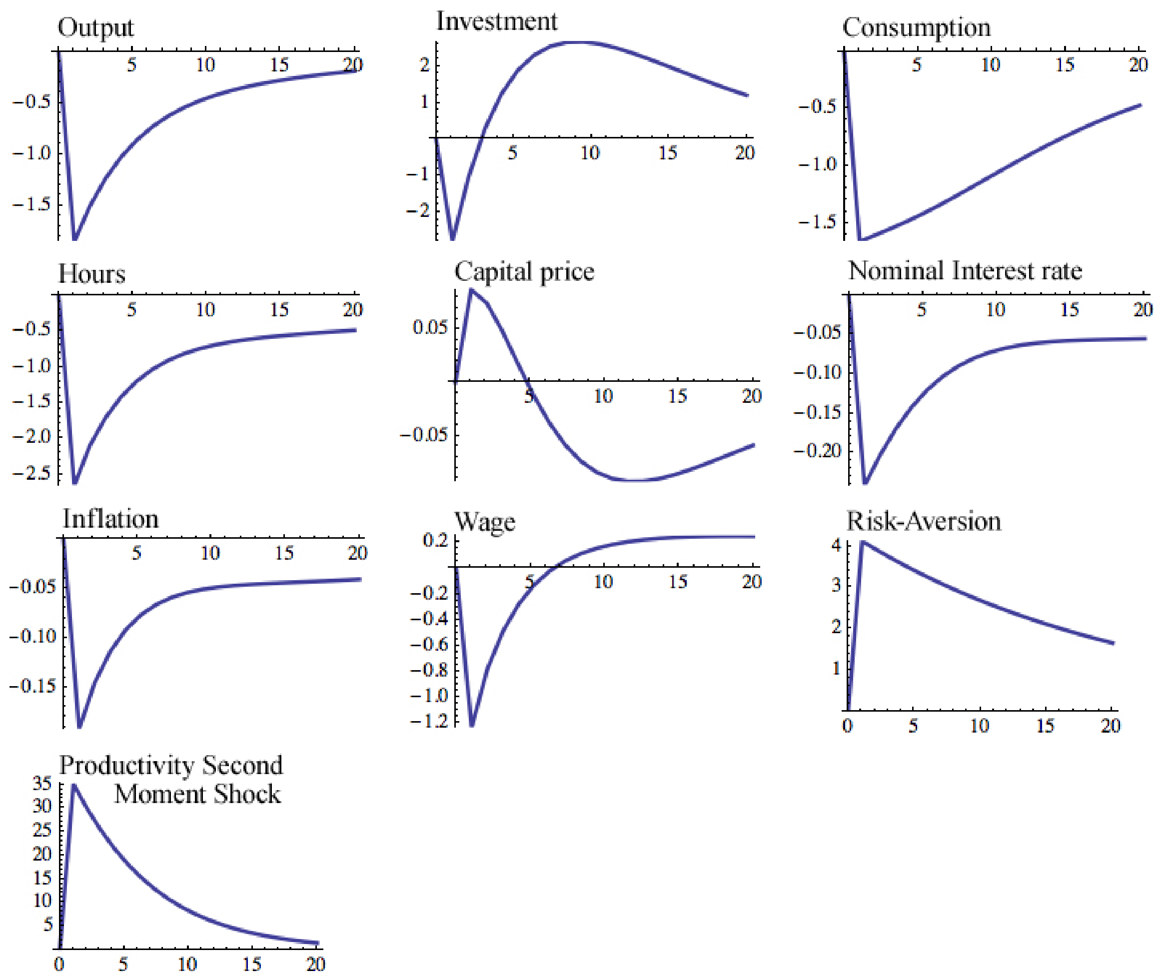
Figure 5: Responses to Uncertainty Shock through the Varying Risk-Aversion



Note, however, that due to uncertainty shock investment and therefore capital stock increase. This is consistent with the model in which the uncertainty does not affect investment adjustment costs, that is, if the adjustment costs are the same regardless of the level of uncertainty in the economy. While households decrease consumption it becomes more profitable to redirect their income into investment that becomes a productive capital and as such allows for higher income in the next periods. However, as argued before, the implication that uncertainty increases investment is contrary to what we observe in the data. This seemingly puzzle is resolved once we allow for investment adjustment costs to depend on uncertainty.

Figure 6 plots the impulse responses of the macroeconomic variables to uncertainty shocks when the risk-aversion responds to the change in uncertainty and the investment adjustment cost depends on the the level of economic uncertainty. Note that the investment adjustment cost is not assumed to be symmetric with respect to the sign of uncertainty change. In particular, if the uncertainty in the economy increases, the investment adjustment costs increase; while if the uncertainty decreases, the investment adjustment costs decrease. The intuition behind is that when the uncertainty in the economy increases it is more costly for the agents to invest due to two reasons: first, the realization of the return on investment can take on the larger set of possible values, with extreme returns being more likely; and second, the investment costs are higher when the economy is in recession simply because borrowing is more costly and agents face more difficulties to access the finance when the uncertainty is higher. The notable difference with respect to results presented in figure 5 is that once the investment adjustment costs depend on the uncertainty also investment decreases due to the uncertainty shock and as such, the model implies the negative relationship between investment and uncertainty as well as the negative relationship between uncertainty and consumption, output and employment. Thus, the model with two simple modifications delivers the effects of the uncertainty shocks that are closer to what we observe in the data.

Figure 6: Responses to Uncertainty Shock through the Varying Risk-Aversion with Uncertainty affecting the Investment Adjustment Costs

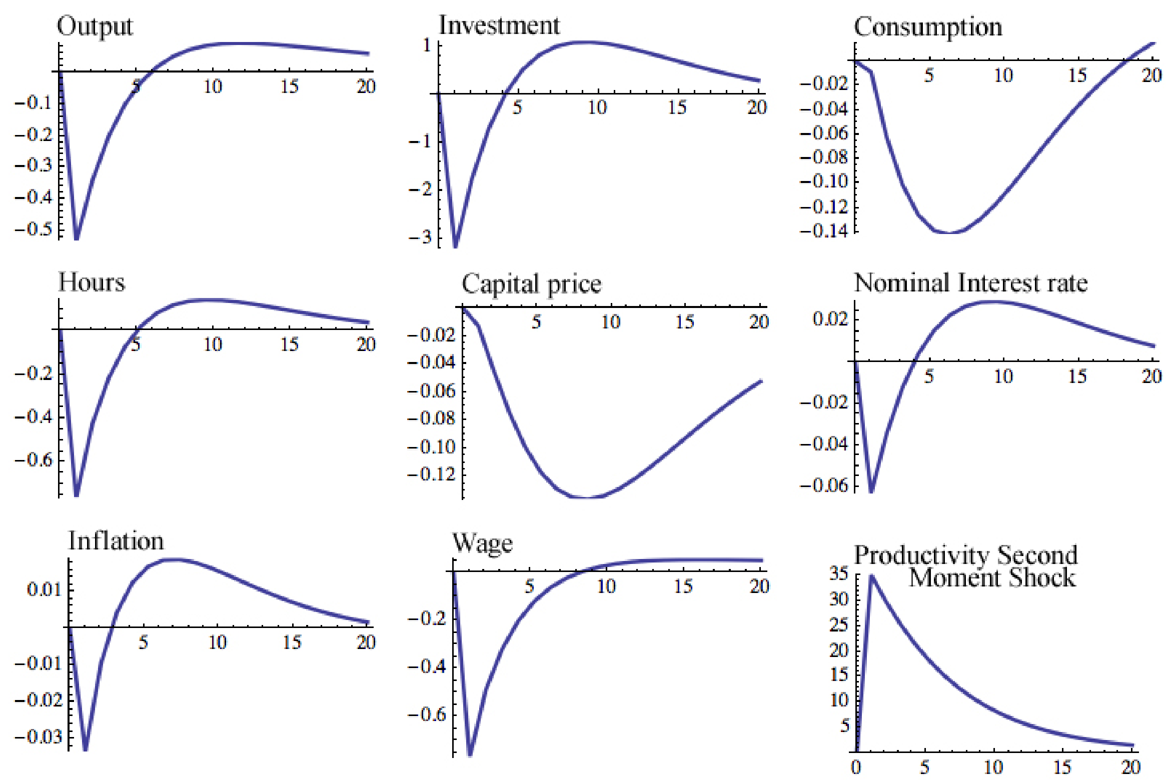


Relating these results with in Basu and Bundick (2011) first note, that they analyze the role of uncertainty in the standard model that is solved up to the third-order approximation around the steady state. As the second- and the third-order terms are small, the uncertainty shocks have a little effects on the macroeconomy. Additional note, they use the sticky price model in which the price setting is assumed to be of Rotenberg type. At this point, it is relevant to emphasize, that the effects of uncertainty shocks in the similar model with Calvo staggered prices are quantitatively smaller than those that they report within the model with Rotenberg pricing (see Ferjancic (2012)). Despite of this observation, however, qualitatively the effects of uncertainty shocks in the present model induce changes in the dynamics of macroeconomic variables that go in the same direction as in the sticky-price model that is solved up to the third-order. The notable difference is that in the present model the uncertainty has the first-order effects. Thus, the responses of the macroeconomic variables to the uncertainty shocks are quantitatively much larger.

To relate the analysis here with Gordon and St-Amour (2000), note that this approach can be seen as a more general case of the model with Markow switching regimes for preferences. Further, if the fluctuations of the risk-aversion are indeed continuous, then Markow switching model overestimates the risk-aversion in high states (that is when economy is in the recession and the risk-aversion is high) and underestimates the risk-aversion in low states (that is when economy is in prosperity and the risk-aversion is low). If this is true then Markow switching model imply larger fluctuations in the risk-aversion, though less frequently and vice versa. As the work of Gordon and St-Amour (2000) also this paper is more descriptive than explanatory. While their study is agnostic about why preferences might shift, in this paper a step forward is done by assuming that the the risk-aversion is related to the level of uncertainty in the economy. Despite of this, further work should be done in determining factors that underlie the movements in attitudes towards risk as embedded in the preferences.

For expositional purposes, figure 7 presents the effects of the uncertainty shock if the uncertainty shocks affects only investment adjustment costs while the risk-aversion is kept constant. As before, higher investment adjustment costs due to increased uncertainty induce a significant decrease of investment, but only a negligible decrease in consumption. Further, an increase of the investment adjustment costs induces also the drop in other macroeconomic variables, but to a much smaller extent. In the light of recent economic developments it is reasonable to assume that the uncertainty shocks worked through both channels, the change in risk-aversion and the change in investment adjustment costs.

Figure 7: Responses to Uncertainty Shock through the Investment Adjustment Costs only



7 Conclusion

The paper provides a mechanism through which uncertainty shocks have first-order effects on the macroeconomic environment. In this respect, is shown that allowing the risk-aversion and investment adjustment costs to be related to the uncertainty level in the macroeconomic environment implies a significant role for uncertainty shocks within otherwise standard dynamic stochastic general equilibrium model with stochastic volatility. Allowing for small fluctuations in the risk-aversion in response to the change in the level of uncertainty is shown to provide a large amplification of the effects of uncertainty shocks to macroeconomic variables. However, the mechanism that works through variation of the risk-aversion alone produces a positive response of investment to higher uncertainty, which is not consistent with what we observe in the data. The model, however, delivers a negative response of the investment to increase in uncertainty if the capital adjustment costs are related to the level of uncertainty. As such, and in contrast with the standard macroeconomic model with heteroscedastic shocks and constant risk-aversion for which the effects of the uncertainty shocks on the economy are fairly small, the model proposed here is able to account for large drops in activity during the economic crisis.

This paper should be considered as an attempt to show in which direction the standard model could be adapted in order to be able to explain the macroeconomic fluctuations during the periods of the recession. Bringing the model to the data is left to a future work.

References

Basu, Susanto and Brent Bundick, "Uncertainty Shocks in a Model of Effective Demand," Technical Report, Downloadable Boston College Working Papers September 2011.

Bloom, Nicholas, "The Impact of Uncertainty Shocks," *Econometrica*, 2009.

_ , Max Floetotto, and Nir Jaimovich, "Really Uncertain Business Cycles," Technical Report, Stanford University 2009.

Brandt, Michael W. and Kevin Q. Wang, "Time-Varying Risk Aversion and Unexpected Inflation," *Journal of Monetary Economics*, 2003, *50.*

Calvo, Guillermo, "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 1983, *12* (3), 383–398.

Campbell, John Y. and John Cochrane, “By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior,” *Journal of Political Economy*, April 1999, *107*, 205–251.

ECB, “Financial Stability Review,” Technical Report, European Central Bank June 2007.

_ , “Financial Stability Review,” Technical Report, European Central Bank December 2011.

Epstein, Larry G. and Stanley.E. Zin, “Substitution, Risk Aversion and the Temporal Behaviour of Consumption and Asset Returns: A Theoretical Framework,” *Econometrics*, July 1989, *57* (699), 937–969.

Ferjancic, Maja, “Uncertainty Shocks and Term Premia Dynamics,” Technical Report, (manuscript) 2012.

Fernandez-Villaverde, Jesus, Dario Caldara, Juan F. Rubio-Ramirez, and Yao Wen, “Computing DSGE Models with Recursive Preferences,” Technical Report, University of Pennsylvania 2009.

_ , Pablo Guerron-Quintana, Juan F. Rubio-Ramirez, and Martin Uribe, “Risk Matters: The Real Effects of Volatility Shocks,” Technical Report, University of Pennsylvania 2009.

Fisher, John M., “The Dynamic Effect of Neutral and Investment-Speci c Technology Shocks,” *Journal of Political Economy*, 2006, *114* (3), 1071–1097.

Gai, Prasanna and Nicholas Vause, “Risk Appetite: Concept and Measurement,” Financial Stability Review, Bank of England December 2004.

Gordon, Stephen and Pascal St-Amour, “A Preference Regime Shift in Bull and Bear Markets,” *The American Economic Review*, September 2000, *90* (4).

Greenwood, Jeremy, Zvi Hercowitz, and Per Krusell, "Long Run Implications of Investment-Specific Technological Change," *American Economic Review*, 1997, *87* (3), 342–362.

Guiso, Luigi and Monica Paiella, "Risk Aversion, Wealth, and Background Risk," *Journal of the European Economic Association*, December 2008, *6* (6), 1109–1150.

_ , Paola Sapienza, and Luigi Zingales, "Time Varying Risk Aversion," Working Paper, Institution 2011.

Justiniano, Alejandro and Giorgio Primiceri, "The Time Varying Volatility of Macroeconomic Fluctuations," *The American Economic Review*, June 2008, *98* (3), 604–641.

_ , _ , and Andrea Tambalotti, "Investment Shocks and Investment Shocks and Business Cycles," *Journal of Monetary Economics*, March 2010, *57* (2), 132–145.

Kim, S., N. Shephard, and S. Chib, "Stochastic Volatility: Likelihood Inference and Comparison with ARCH Models," *The Review of Economic Studies*, 1998, *65* (3), 361–393.

Scheicher, Martin, "What drives investor risk aversion? Daily evidence What drives investor risk aversion? Daily evidence from the German equity market," BIS Quarterly Review, Bank for International Settlements June 2003.

Smith, Danile R. and Robert F. Whitelaw, "Does Risk Aversion Change Over Time?," Working Paper, University 2009.

Search Frictions on the Credit Market

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Abstract

We propose a mechanism through which credit shocks are propagated to the economy in a general equilibrium model. We introduce search and matching frictions in the contractual relationship between productive agents, which are borrowing constrained, and financial intermediaries. We model financial shocks in two forms: (i) as a negative credit market efficiency shock, and (ii) as a positive shock to the separation rate. In accordance to the data, both shocks give rise to countercyclical spreads between deposit and lending rates, which increase in periods of financial distress, and also capture the fact that in the downturn, firms' financing difficulties increase. The latter, together with the decrease in the bank's willingness to supply loans, has a direct consequence on variations in the liquidity of the credit market.

Results show that following both a negative credit market efficiency shock and a positive shock to the separation rate, the credit spread increases. In the case of a credit market efficiency shock, an increase in the credit spread is accompanied by an increase in the liquidity of the credit market, while in the case of the shock to the separation rate, the liquidity of the credit market decreases. Thus, a recession stemming from a credit market efficiency disturbance is relatively mild in comparison to a situation in which the economy is hit by an equally large shock to the separation rate.

Keywords: business cycles, credit market, financial frictions, search and matching frictions, banks, financial crisis

JEL Classification: E32, E43, E44, G01, G21

1 Introduction

The present paper focuses on the banking sector, its role in financial intermediation, the equilibrium bank credit determination and the role of banking sector in propagation mechanism of the shocks on the economy. In modeling the banking sector we borrow from the search and matching frictions literature that has proved to be useful in explaining fluctuations in the labour market. This choice was motivated by the evidence that it is more difficult for firms to find funds for financing their production in the periods of financial distress. This quote from a speech of J.C. Trichet, former governor of the ECB, provides an example of the mechanism we have in mind:

> ”The firms reported that they were having difficulties in finding customers, that their turnover and profits were falling and that: bank loans were harder to come by...”[1]

As the banks play a crucial role in maintaining and extending credit to all other sectors of the economy, and in particular to small and medium size firms, understanding search frictions on the credit market and the propagation mechanism that stems from them is important from a theoretical point of view as well as from a policy makers point of view when implementing the measures to stabilize the credit market.

The fundamental feature of the model is the presence of search and matching frictions on the banking market. These are introduced to capture the idea that productive agents face difficulties in finding the funds. An entrepreneur has to find a bank that provides him a loan to finance her business projects. Thus, the entrepreneur has to spend her resources to find such a loan which will enable her to produce.

[1]Speech by J.C. Trichet, October 15, 2009.

Though in the present paper the asymmetric information is not modeled directly as modeled in Bernanke et al. (1999) nor the frictions arise in the form of credit constraints as in Kiyotaki and Moore (1997), the search frictions could be viewed as reduced-form representation of a wide array of resource and information frictions characterizing the lending relationship. In the present analysis, however, the search frictions are interpreted as complementary credit frictions that might exist simultaneously with the frictions related to the asymmetric information. In order to keep the setup simple and tractable and in order to be able to draw clear implications of the search frictions in the credit market for the rest of economy, the Bernanke-Gertler-Gilrichst and Kiyotaki-Moore type of credit frictions are left aside. As such, the model in a simple and tractable setup gives rise to the positive spread between deposit rates and lending rates, which are in the periods of financial distress typically associated with lower levels of real expenditure and employment [2]. Further, the model captures the fact that in the downturn, firms' financing difficulties increase amplifying the credit spread.

The contributions of the paper are three-fold. First, the paper provides an alternative view on how the equilibrium on the credit market is determined. In particular, the framework allows to construct a loans creation curve, which represents the supply side off the credit market.[3] The demand side for loans is on the other hand described by the lending interest rate schedule. In equilibrium, the two curves determine the equilibrium lending interest rate and liquidity of the market.

[2]See for example Curdia, and Woodford (2009a)

[3]The loan creation curve can be seen as the analogue to the Job Creation curve in the labor search and matching literature.

Second, this framework has a direct implication for how the credit market imperfections matter for market liquidity[4], as well as it is able to capture the fact that in the equilibrium, there is a scope for credit rationing. That is, banks are not willing to provide additional funds to borrowers at the prevailing market interest rate.

Third, the framework in hand allows for alternative dynamic analysis of the mechanism through which the credit shocks propagate to real economy. We are considering two types of credit shocks: first, a shock that affects the efficiency of the matching process one the credit market, and second, a shock that affects the separation rate between the banks and the entrepreneurs.

The main results of our analysis can be illustrated as follows. First, after the economy is hit by any of the two types of shocks that worsen credit market conditions, the credit spread increases significantly. While the shock into credit market efficiency induce an increase in the credit market liquidity, the separation shock decreases the credit market liquidity. Second, following the first type of the shock, the interest rate spread increases relatively by less and since it is accompanied by the increase in the market liquidity, the recession stemming from the credit market disturbance is relatively milder comparing to the situation when the economy is hit by the second type of the shock of equal size.

The rest of the paper is organized as following: In section 2 is a highly selective overview of the credit frictions literature, while section 3 presents data available in Bank lending survey is presented. Section 4 introduces the model in details. Section 5 is split to subsections: After calibration we introduce the shocks that we consider within a model. This is followed by the explanation of the mechanism

[4] The credit market liquidity is defined as the ratio of vacant to unrealized loans. The concept is discussed in details in the subsection 2.2.

that transmits the shocks to economy. The last subsection presents simulation results. Section 6 concludes.

2 Overview of the Credit Frictions Literature

Over the last decades a vast of literature has focused on the interaction between the financial sector and the macroeconomic variables and on the role that financial frictions have on the aggregate economic activity. There has been a number of DSGE models with financial frictions proposed by now, however, they distinguish themselves by the way they model financial frictions, in particular they distinguish themselves by the way in which sector the financial friction is created. The other crucial difference among the existing models with financial frictions is whether they model financial intermediaries explicitly or not.

With respect to how and in which sectors the financial frictions are assumed, the literature can be assigned in one of the two groups. One group of papers develops the theory of financial frictions based on Bernanke et al. (1999) or Carlstrom and Fuerst (87) and Faia and Monacelli (2007) ideas. In this group of models, frictions steam from the agency problems, in particular, they arise because the lenders face the monitoring costs due to imperfect information about the borrowers. Due to this, the external finance premium defined as the difference between the lending rate and the risk free rate evolves endogenously in the model. The second group of papers follows Kiyotaki and Moore (1997), who introduce financial frictions through collateral constraints. These models employ heterogeneous agents, lenders and borrowers, the letter being more impatient and borrowing constrained. These two groups have in common that the financial frictions are introduced on the productive side of economy. Some recent papers based on Kiyotaki and Moore (1997) introduce financial frictions through different assets that can be used as collateral. Such example are Iacoviellio (2005), Monacelli (2007), Monacelli (2009), Calza et al. (2011) who introduced housing and durable goods as collateral.

Literature that incorporates the banking sector in the DSGE framework, model the banks either as operating in the perfectly competitive markets (for example...), as operating on the monopolistically competitive markets (see for example Gerali et al. (2010) or facing the intermediation costs (see Curdia and Woodford (2010a), Curdia and Woodford (2010b)). Among those papers that explicitly model financial intermediaries, not many of them focus on banks, their role in financial intermediation and interdependence with macroeconomy within the search and matching framework.

To my knowledge, there are only some contributions in the literature, which also model the credit market imperfections using the search and matching framework. Dell'Ariccia and Garibaldi (2005) and Dell'Ariccia and Garibaldi (1998) use a somewhat more complex model, which includes also inter-bank sector and possibility that a fraction of borrowers default and therefore a fraction of loans is not repaid. However, since the aim of their analysis is to match the variability of loans and spreads observed on the banking market, they refrain from theoretical analysis of the dynamic effects of the credit shocks and the propagation mechanism through which they are transmitted to economy. Next, Weil and Wasmer (2004) construct a sequential search and matching model. First, the entrepreneur is searching for a bank that to obtain funds to finance the costs of posting vacancies. When she obtains the funds, she searches for a worker. With respect to model characteristics, these credit market imperfections are modeled completely symmetrically to labor market imperfections. Their aim is to analyze how the presence of the credit frictions amplify unemployment and its volatility through a financial accelerator. Given the focus of the analysis, their setup is such, that the economy always adjusts through labor market, i.e. the labor market efficiency is allowed to be vary due to the shocks, while the credit market remains equally efficient in all states of the world. Beaubrun-Diant and Tripier (2009) use the search and matching frictions to explain the countercyclical behavior of interest rate spreads and their response to shocks in production technology and in the cost of banks' resources. In terms of modeling, they focus solely on the banking market. In contrast, the present paper incorporates search frictions in a fully-fledged dynamic general equilibrium model.

3 The Model

3.1 Patient Households

The baseline model assumes continuum of agents represented by a representative household. The representative household's preference is described by standard power utility preference that is separable in consumption good, c_t, and hours worked, n_t:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta_h^t \left[\log c_{h,t} - \frac{n_t^{1+v}}{1+v} \right] \right\}$$

Where E_0 denotes mathematical expectations operator conditional on information set available at time $t = 0$ and $\beta_h \in (0, 1)$ denotes household's subjective discount factor.

Let d_t be one-period deposit, w_t the real wage, R_t^D one-period gross return on deposits and t_t the lump-sum taxes. The budget constraint of the household then reads as following:

$$c_{h,t} + d_t \leq R_{t-1}^D d_{t-1} + w_t n_t + t_t$$

The household chooses process for $\{c_t\}$, $\{n_t\}$ and $\{d_t\}$ to maximize lifetime utility subject to the flow of funds constraint. The first order necessary conditions are standard $\left\{\ c_{h,t}, n_t, d_t\ \right\}$:

$$(c_{h,t}) : \lambda_{h,t} = \frac{1}{c_{h,t}} \tag{1}$$

$$(d_t) : \lambda_{h,t} = \beta_h E_t \left\{ R_t^D \lambda_{h,t+1} \right\} \tag{2}$$

$$(n_t) : n_t^v = w_t \lambda_{h,t} \tag{3}$$

3.2 Unrealized Loans, Vacant Loans and Matching Function

Let the loan demand, $L^D = 1$, be the sum of the unrealized borrowing, L^U, and the realized borrowing, L, from the entrepreneur's point of view and let L^V be the vacant loans or supply of new loans determined by the bank. Define the rate of unrealized loans, $l^U = \frac{L^U}{L^D}$, and rate of vacant loans $l^V = \frac{L^V}{L^D}$.

At time t, the bank posts vacant loans, l_t^V to attract new entrepreneurs to apply for a loan and approves l_t loans. The loan that is approved to entrepreneur is used to finance business decisions in the same period. Rate of unrealized loans, l_t^U, is defined as the difference between 1 (total demand for loans) and the approved loans at the end of period $t-1$: $l_t^U = 1 - l_{t-1}$

The matching function on the credit market is described by a production function that transforms inputs, that are rate of unrealized loans, l^U, and rate of supplied loans, l^V, into matches, that is, $M = m\left(l^V, l^U\right)$. In particular, the matching function is assumed to be of constant returns to scale:

$$m_t = m\left(l_t^V, l_t^U\right) = \sigma_m \left(l_t^V\right)^{\sigma} \left(l_t^U\right)^{1-\sigma} \tag{4}$$

where σ_m represents the efficiency of the matching function.

Let the measure of banking market liquidity be:

$$\theta_t = \frac{l_t^V}{l_t^U} \tag{5}$$

Credit market liquidity in (5) is defined as the ratio of vacant to unrealized loans. To rationalize the concept of liquidity, consider the credit market, in which the supply of loans is relatively high for a given rate of unrealized loans, i.e. θ_t is high. As there is an abundance of funds, it is relatively easy for the entrepreneurs to obtain the funds. The market is liquid. On the other hand, if the supply of loans is relatively low for a given rate of unrealized loans, i.e. θ_t is low, it will be difficult for the entrepreneurs to borrow from the banks. [5]

Then the probability that entrepreneur obtains the loan is:

$$p_t = p(\theta_t) = \frac{m\left(l_t^V, l_t^U\right)}{l_t^U} = m(\theta_t, 1) \tag{6}$$

and the probability that supplied loan finds entrepreneur is:

$$q_t = q(\theta_t) = \frac{m\left(l_t^V, l_t^U\right)}{l_t^V} = m\left(\frac{1}{\theta_t}, 1\right) \tag{7}$$

p_t is increasing in the banking market liquidity, $p'(\theta_t) > 0$, while q_t is decreasing in banking market liquidity $q'(\theta_t) < 0$. Note, that the bank knows the probability of its loan approval rate q_t.

Each period a fraction of the matches, s, exogenously separate. Entrepreneurs loosing their loan as external funds are not allowed to search for a new loan until the beginning of the next period. The total amount of outstanding loans of a bank,

[5] A reader who is familiar with labor search and matching models might notice the similarity of the concept of credit market liquidity and the concept of labor market tightness. The latter is typically defined as a ratio of vacancies to unemployment. The two concepts are indeed related, however, there is a crucial difference in nature of the model. In the labor models, the firms demand for labor. Therefore, posting the vacancies in order to attract workers, is a feature that characterizes the demand side of the market. Unemployment, on the other hand, characterizes the supply side of the labor market. In contrast, in the credit market searching model the opposite is true. Banks supply funds to productive entrepreneurs. Thus the vacant loans are characteristic of the supply side of the credit market, while unrealized loans are related to the entrepreneur's demand for borrowing. In other words, though from notational point of view the two concepts look equivalent, in terms of the content, the market labor liquidity is equivalent to the inverse of the labor market tightness.

l_t, is defined as the sum of the loans being rolled over from the previous period, $(1-s)\, l_{t-1}$ and newly approved loans, $q_t l_t^V$:

$$l_t = (1-s)\, l_{t-1} + q_t l_t^V \tag{8}$$

3.3 Banking Sector

3.3.1 Structure of the banking sector, timing and financial contract

There is one bank that is assumed to consist of two hierarchical type of units. On the lower hierarchical level the bank consist of continuum of units that collect deposits from households, while on the higher hierarchical level there is one large unit that takes all lending decisions and provides loans to a large number of entrepreneurs. That is, on the deposit side of the credit market the bank faces perfect competition, while on the lending side of the credit market, the bank is large enough so that the marginal separation with the entrepreneur does not affect its potential for operating in the following period. Bank's deposit units in the fund raising stage collect deposits from the households, d_t, and transfer them to the lending unit, which immediately after finding the match transforms deposits into loans, l_t, according to the loan production function $l_t = f(d_t) = d_t^{\phi}$ with $\phi = 1$.[6] In producing loans, the bank's lending unit suffers fixed operating cost per unit of newly supplied loan, c.[7]

[6] $\phi = 1$ is assumed for simplicity. $\phi > 1$ would imply that in the bank's balance sheet the loans are larger than deposits, $l_t > d_t$, which could be interpreted as a reduced form representation of intermediation technology that allows for multiplication of the deposits. The direct consequence would be then that the bank has positive net worth. As of now, the capital of the bank does not have role.

[7] Note that the operating cost, c, can be viewed as a special form of the resource costs faced by the banks as modeled in Curdia and Woodford (2009a,b). From technical point of view the operating cost increase the spread between lending and deposit interest rates, but are not necessary for existence of the spread. The crucial feature that gives rise to the positive interest rates spread in the model is that there exist search frictions on the credit market.

The financial contract determines the interest rate at which the bank is willing to lend the entrepreneur. In particular, the lending rate is bargained between the bank's lending unit and entrepreneur. The deposit rate is determined on the financial market by patient household's the supply of funds and demand for deposit by the bank's deposit units. At the end of the period, the entrepreneurial household repays to the bank the amount $R_t^L l_t$,[8] as determined in the the financial contract, while the bank repays deposits to the households, $R_t^D d_t$.

3.3.2 Bank

Given exogenous separation rate, s, the value function of the bank, B_t^M, is the following:

$$B_t(r_t, l_{t-1}) = r_t^L l_t - r_t^D d_t - c l_t^V + \beta_h E_t \left\{ \Lambda_{t,t+1}^h B_{t+1}(r_{t+1}, l_t) \right\}$$

where c is the bank's fixed operating cost per unit of newly supplied loan, β_h is household's subjective stochastic discount factor and $\Lambda_{t,t+1}^h = \frac{\lambda_{h,t+1}}{\lambda_{h,t}}$. Since the bank consists of two independent types of units, the bank's problem can be split in two separate optimization problems. At time t, the deposit units maximize the value of the bank by choosing deposits, d_t, subject to loan production function, taking the lending interest rate, r_t^L as given:

$$\max B_t(r_t, l_{t-1}) = r_t^L l_t - r_t^D d_t - c l_t^V + \beta_h E_t \left\{ \Lambda_{t,t+1}^h B_{t+1}(r_{t+1}, l_t) \right\}$$

subject to

$$l_t = f(d_t)$$

As the deposit side of the credit market bank is perfectly competitive, the first order condition for deposits decision states that for the bank it is optimal to

[8] As will become clear in the next section, because the model assumes perfect risk-sharing within entrepreneurial household, the entrpreneurs' borrowing is always fully repaid.

demand deposits so long that the marginal cost of deposit is equal to the marginal product of deposit:

$$r_t^D = r_t^L f'(d_t)$$

At the same time, t, the bank lending unit maximizes the bank's value by choosing newly supplied loans, l_t^V, given its outstanding amount of loans, l_{t-1}, the probability of a vacant loan being approved to the entrepreneur, q_t, and the current and expected path of lending and deposit rates:

$$\max B_t(r_t, l_{t-1}) = r_t^L l_t - r_t^D d_t - c l_t^V + \beta_h E_t \left\{ \Lambda_{t,t+1}^h B_{t+1}(r_{t+1}, l_t) \right\}$$

subject to

$$l_t = (1 - s)\, l_{t-1} + q_t l_t^V$$

The first order condition with respect to newly supplied loans, l_t^V, reads:

$$\frac{\partial B_t^M(r_t^L, l_{t-1})}{\partial l_t^V} = r_t^L - \frac{c}{q_t} + \beta_h E_t \left\{ \Lambda_{t,t+1}^h \frac{\partial B_{t+1}^M(r_{t+1}, l_t)}{\partial l_t} \right\} = 0 \tag{9}$$

Combining (9) with the envelope condition and forwarding the expression for 1 period yields:

$$\frac{\partial B_{t+1}^M(r_{t+1}, l_t)}{\partial l_t} = (1 - s)\, \frac{c}{q_{t+1}} \tag{10}$$

Using (10), the first order condition (9) can be rewritten as:[9]

$$\frac{c}{q_t} = r_t^L + \beta_h (1 - s)\, E_t \left\{ \Lambda_{t,t+1}^h \frac{c}{q_{t+1}} \right\} \tag{11}$$

Loan creation condition, LCC, in (11) represents the supply side of the loan market. It states, that bank supplies new loans up to the point where the expected operating cost per new loan (lhs) equals the sum of the profit from lending in the current period and the discounted expected savings from creating the new loans in the following period that stem from the fact that the match continues (rhs).

[9]See Appendix $A1$ for detailed derivation of the loan creation condition.

The lending rate is determined by the bargaining between bank's lending unit and entrepreneur about the terms of the financial contract. For the purpose of lending interest rate bargaining, define $J_t(r_t^L)$, the value for the bank of approving another loan at time t after the operating costs are sunk. Differentiating $B_t^M\left(r_t^L, l_{t-1}\right)$ with respect to l_t, taking operating costs as given yields:

$$J_t(r_t^L) = r_t^L + \beta_h(1-s)E_t\left\{\Lambda_{t,t+1}^h J_{t+1}(r_{t+1}^L)\right\} \tag{12}$$

3.4 Entrepreneurial Household

Following Merz (1995), there is a representative entrepreneurial household with a continuum of members of measure equal to unity. The household is assumed to be impatient and productive, which gives rise to borrowing. In particular, the members of the entrepreneurial family can either obtain loans, l_t, and produce or do not obtain loans and therefore do not produce, l_t^U. Realized loans, l_t, are determined through a search and matching process. There is no utility from leisure, which in this framework amounts to not producing. Individuals currently not producing are searching for loans at fixed cost per unrealized loan. Those members who produce, produce homogeneous final good using capital and labor according to the Cobb-Douglas production function $y_t = A_t k_{t-1}^{\alpha} n_t^{1-\alpha}$. The family pools members' incomes before choosing per capita consumption and other uses of funds. In addition to income from production, the entrepreneurs obtain funds by selling the undepreciated capital to capital producers and by borrowing from the banks. The funds are then used to finance consumption, to buy new capital, to pay wages, to repay their loans from the previous period and to find new loans. Therefore, the representative entrepreneurial family provides perfect consumption insurance for its members, implying first, that consumption is the same for each person, regardless of whether the member produces in the current period or not, and second, that previous borrowing is always repaid.

The entrepreneurial household maximizes lifetime utility:

$$\max\ E_0 \left\{ \sum_{t=0}^{\infty} \beta_e^t \log c_{e,t} \right\} \qquad \beta_e < \beta_h$$

subject to the following flow of funds constraint:

$$c_{e,t} + (l_t) qk_t \left[k_t \right] - qk_t \left[\left(1 - \delta \right) k_{t-1} \right] + R_{t-1}^L l_{t-1} + (l_t) w_t n_t + ce(1 - l_t) \leq l_t + (l_t) y_t$$

where qk_t is the price of capital, ce represents the search cost of entrepreneur per unit of unrealized loan, l_t denotes one-period loans obtained from the bank, for which at the end of the period, the entrepreneurial household pays a gross interest rate $R_t^L l_t$. R_t^L is determined in a financial contract by bargaining between entrepreneurs and banks to what I return below.

Note, that the budget constraint consistently with the assumption of the perfect risk sharing within entrepreneurial household implies that all members, those who obtain loans as well as those who do not obtain loans, contribute to the household's income by selling undepreciated capital to capital producers, $qk_t \left(1 - \delta \right) k_{t-1}$, that only some members obtain loans, l_t, and that only those who obtain loans, contribute to the household's income from production, $l_t y_t$. On the side of uses of funds, all members of the household consume, $c_{e,t}$, the entrepreneurial family acquires new capital from capital producers, $l_t qk_t k_t$, and pays wages, $l_t w_t n_t$, for those members who produce. Members currently not producing are searching for loans at costs, $(1 - l_t) ce$; The searching costs are, however, effectively financed by productive members. In addition, the family repays the borrowing from the previous period, $R_{t-1}^L l_{t-1}$. The family always repays all debt from previous period, which implies that the bank never faces losses due to badly performing loans that were not repaid as a consequence of entrepreneurial default.[10]

[10] This is a simplifying assumption that allows to make the model more tractable without loss of generality.

Entrepreneurial family maximizes lifetime utility subject to the flow of funds constraint and production function by choosing optimal path for consumption, $c_{e,t}$, and capital, k_t. The first order conditions are the following:

$$\lambda_{e,t} = \frac{1}{c_{e,t}} \tag{13}$$

$$qk_t = \beta_e \frac{1}{l_t} \frac{\lambda_{e,t+1}}{\lambda_{e,t}} \left[rk_{t+1} + qk_{t+1} (1 - \delta) \right] \tag{14}$$

The first FOC is standard and defines the marginal utility of consumption for the entrepreneurial household. The second FOC is optimality condition for capital. Note, that takes into account also loans, that is, the more loans, the lower demand for loans, the lower price for capital.

Since the inputs markets are assumed to be perfectly competitive, the wages and return on capital, respectively, are determined by their marginal productivity:

$$w_t = (1 - \alpha) \frac{y_t}{n_t} \tag{15}$$

$$rk_t = \alpha \frac{y_t}{k_{t-1}} \tag{16}$$

Note, that the household does not optimize with respect to loans. The treatment of decision for borrowing and derivation of the demand for borrowing is subject of the next subsections.

3.4.1 Entrepreneur's value functions

If p_t is the probability that entrepreneur obtains the loan (match with the bank) and s is exogenous separation rate, the value functions describing the value for the entrepreneur when being matched or non matched, respectively, are the following:

The value for the entrepreneur of obtaining the loan is:

$$E_t^M = y_t - w_t n_t - qk_t \left[k_t - (1 - \delta) k_{t-1} \right] - r_{t-1}^L - ce + \beta_e E_t \left\{ \Lambda_{t,t+1}^e \left[(1 - s) E_{t+1}^M + s E_{t+1}^N \right] \right\}$$

Note, $-ce$ in the value function of the entrepreneur who is matched. This amounts to assuming that despite that the entrepreneurs who are not matched and thus not produce, are on the search for loans, the costs that arise when searching ce are financed by those members of the family who are producing. The motivation for such assumption is, that it is profitable for the producer to finance the costs of searching in order to obtain loan and thus roll over or increase the debt.

The value for the entrepreneur if she is not matched is:

$$E_t^N = qk_t(1-\delta)k_{t-1} - r_{t-1}^L + \beta_e E_t \left\{ \Lambda_{t,t+1}^e \left[p_{t+1} E_{t+1}^M + (1-p_{t+1}) E_{t+1}^N \right] \right\}$$

Define entrepreneur's surplus as $S_t^E = E_t^M - E_t^N$. Taking into account the entrepreneur's value functions, her surplus can be rewritten as:

$$S_t^E = a_t - r_t^L l_t - ce(1-l_t) + \beta_e E_t \left\{ \Lambda_{t,t+1}^e (1-s-p_{t+1}) [S_{t+1}] \right\} \tag{17}$$

where $a_t = y_t - w_t n_t - qk_t k_t$.

3.5 Determination and Dynamics of the Lending Interest Rate

In the bargaining process, the bank and the entrepreneur maximize the wighted total surplus from the realized match by the choosing the value of the lending interest rate, r_t^L. Let η be the bargaining power of the bank and $(1-\eta)$ the bargaining power of the entrepreneur. In the solution, r_t^L must satisfy the following condition:

$$(1-\eta) \left[S_t^E \right] = \eta \left[J_t \right] \tag{18}$$

The equilibrium condition as described by (18) states, that in the equilibrium, the bank and the entrepreneur split the total surplus according to their bargaining powers.

Solving for lending interest rate, we obtain the interest rate schedule:[11]

$$r_t^L = \eta x + \eta \frac{c}{\sigma_m} E_t \left\{ \Phi_{t,t+1} \theta_{t+1}^{1-\sigma} \right\} \tag{19}$$

where $\Phi_{t,t+1} = \beta_e \Lambda_{t,t+1}^e (1 - s - p_{t+1}) - \beta_h (1 - s) E_t \Lambda_{t,t+1}^h$.[12] Treat x as unemployment benefit. In principle, x can be any value equal or larger then ce.

The interest rate schedule, IRS, describes the demand side of the credit market. The entrepreneur is willing to pay higher lending interest rate the lower is liquidity of the credit market.

3.6 Capital Producers

Following Christiano et al, the model assumes a representative perfectly competitive capital producer. The capital producer produces new capital, k_t, using as inputs newly produced equipment at a unit price and the undepreciated physical capital, k_{t-1}, purchased from the entrepreneur at the end of period $t-1$. Production technology of the capital goods is:

$$k_t = (1-\delta)k_{t-1} + (1 - S(i_t/i_{t-1}))i_t$$

Where δ represents depreciation rate and $S(\cdot)$ captures a presence of the adjustment costs of investment. As standard, $S(\cdot)$ is restricted to satisfy following properties: $S(1) = S'(1) = 0$, $S''(1) > 0$.

Capital producer's profit in each period is:

$$\Pi_{k,t} = qk_t(k_t - (1-\delta)k_{t-1} - (1 - S(i_t/i_{t-1}))i_t) - qk_t k_t - i_t$$

And expected lifetime profit is:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta_h^t \Pi_{k,t} \right\}$$

[11] See Appendix $A2$ for details of the derivation.
[12] Note that $\Phi_{t,t+1}$ is strictly negative.

Maximization of the expected profit with respect to investment good, i_t, yields optimality condition linking the price of installed capital, qk, to the price of investment goods.

$$qk_t(1 - S(i_t/i_{t-1}) - S'(i_t/i_{t-1})) + \beta_e \frac{\lambda_{e,t+1}}{\lambda_{e,t}} qk_{t+1} S'(i_{t+1}/i_t) = 1 \tag{20}$$

The aggregate stock of physical capital evolves according to:

$$k_t = (1 - \delta)k_{t-1} + (1 - S(i_t/i_{t-1}))i_t \tag{21}$$

3.7 Aggregation

The resource constraint is:

$$y_t = c_{h,t} + c_{e,t} + i_t + Costs_t \tag{22}$$

where:

$$Costs_t = c * l_t^V + ce * (1 - l_t) \tag{23}$$

The complete set of equilibrium conditions can be found in Appendix $A3$.

4 Dynamic Simulations

4.1 Calibration

Time is in quarters. The quarterly discount factors for patient household and entrepreneurial household are set to $\beta_h = 0.99$ and $\beta_h = 0.98$, respectively. The annual real interest rate is pinned down by the patient households discount rate and is equal to 4%.

The capital share in entrepreneurial production function is set to 0.3 and capital physical depreciation rate is equal to 0.025. The disutility of labor, ν, is set so that Frisch elasticity is equal to 1.5.

Parameter determining the efficiency of the matching function is set to 0.7 and the share of vacant and unrealized loans in the matching function is set to be 0.5. Bank's cost per unit of vacant loan is $c_b = 0.05$, entrepreneur's search cost per unrealized loan is $c_e = 0.15$. Exogenous separation rate, s, is set to 0.2, which implies that 20 percent of the matches are separated at the end of the period. The bank's bargaining power is equal to entrepreneur's bargaining power, that is 0.5.

The choice of parameters relevant for the credit market deserves a few additional comments. The value of the entrepreneur's search cost per unrealized loan, c_e, affects directly credit market probabilities. In particular, the higher c_e, the lower probability that unrealized demand for loan gets realized, p_t, and the higher probability that the vacant loan is approved to entrepreneur, q_t. Further, the value c_e together with the value for exogenous separation rate, s, is crucial for the steady value of the state spread. $c_e = 0.15$ and $s = 0.2$ were chosen simultaneously such that the steady state interest rate spread is equal to 2 percentage points.

Table 1 gathers the priors about structural parameters of the model, while table 2 gathers the steady state values implied by this parameterization.

Table 1: Calibration of parameters

Parameter	Name	Value
β_h	Patient household discount rate	0.99
β_e	Entrepreneurial household discount rate	0.98
α	Share of capital in Production function	0.3
δ	Depreciation of capital	0.025
ν	Disutility of labor	0.67
σ_m	Efficiency of the matching function	0.7
σ	Share of vacant loans in the matching function	0.5
c_b	Bank's cost per unit of vacant loan	0.05
c_e	Entrepreneur's search cost	0.15
s	Exogenous separation rate	0.2
η	Bargaining power of the bank	0.5

4.2 Shocks

There are three shocks in the model: neutral technology shock and two types of the shock that affect the credit market. The first type of credit shock is the shock to efficiency of the credit market and the second type of the credit shock is the shock to the bank-entrepreneur relationship's separation rate. All the processes for the shocks are assumed to be $AR(1)$ with persistence parameters 0.95 and normally distributed exogenous disturbances with zero mean and unit variance, $\sim N(0,1)$.

Table 2: Steady state values

Parameter	Name	Value
y	Output	2.25
c_h	Patient household's consumption	1.6
c_e	Entrepreneurial household's consumption	0.6
k	Capital	14.9
i	Investment	0.4
n	Labor	1
pk	Price of capital	1
w	Wage	1.6
d	Deposits	0.75
R^L	Lending interest rate	1.033
D^L	Deposit interest rate	1.010
$Spread$	Interest rate spread	0.023
l	Realized loans	0.72
l^v	Vacant loans	0.15
l^u	Unrealized loans	0.28
m	Matches	0.14
q	Probability that loan is approved	0.94
p	Probability that unrealized loan is realized	0.52
θ	Liquidity of the credit market	0.56

*Comment: Steady state values are rounded to 2 or 3 decimals.

4.3 Transmission Mechanism

In this section we evaluate the transmission of the shocks. The aim of this exercise is to provide qualitative insights into the mechanism through which the shocks transmit to economy when the credit market is characterized by search frictions, rather than quantitative characterization of the model that would fit the data.

To explain the mechanism that works through the credit market, it is useful to have a closer look into the equilibrium conditions that determine the supply and demand for loans on the credit market amended by the two credit market shocks. As argued above, the supply side of the credit market is represented by the loan creation condition, LCC, which can be rewritten in terms of liquidity as:[13]

$$\theta_t^{1-\sigma} = r_t^L \frac{\sigma_m \epsilon_{m,t}}{c} + \beta_h \left(1 - s\epsilon_{s,t}\right) E_t \left\{ \Lambda_{t,t+1}^h \theta_{t+1}^{1-\sigma} \right\} \tag{24}$$

where $\epsilon_{m,t}$ is the shock to the credit market efficiency and $\epsilon_{s,t}$ is the shock to separation rate.

The demand for loans by entrepreneurs is given by the aggregate lending interest rate schedule:

$$r_t^L = (1-\eta)x + \eta \frac{c}{\sigma_m \epsilon_{m,t}} E_t \left\{ \Phi_{t,t+1} \theta_{t+1}^{1-\sigma} \right\} \tag{25}$$

It is straight forward to show, that the two curves uniquely determine the equilibrium lending interest rate, r^L, and the credit market liquidity, θ:

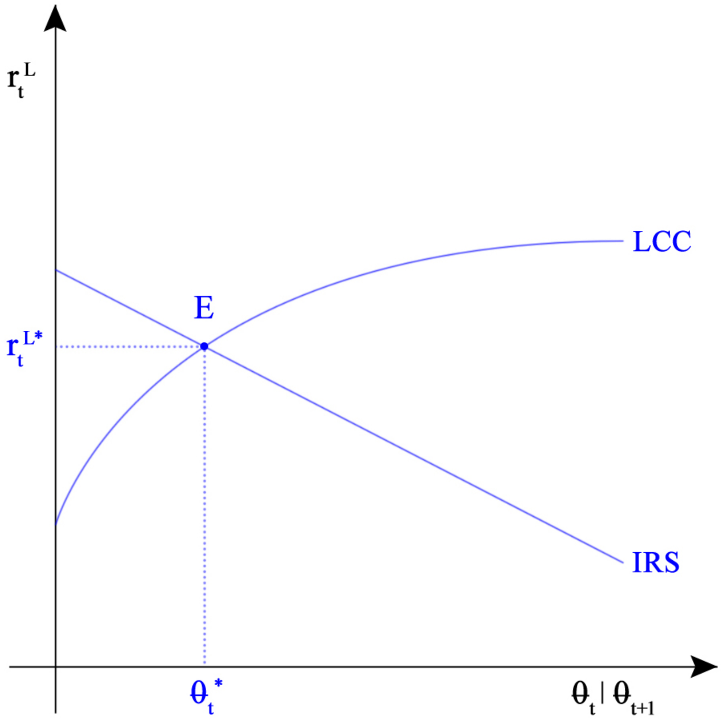
$$\theta^* = \left[\frac{\sigma_m}{c(1-\beta_h\left(1-s\right))} r^L \right]^{\frac{1}{1-\sigma}} \text{ and } r^{L*} = \frac{(1-\eta)x}{1-\eta \frac{\Phi}{(1-\beta_h(1-s))}}$$

where $\Phi = \beta_e \left(1-s-p\right) - \beta_h \left(1-s\right)$.

[13] This expression is equivalent to the one given by equation (11). See Appendix $A2$ for derivation.

Figure 1 graphically represents the supply and demand for loans and equilibrium on the credit market:

Figure 1: Equilibrium on the credit market

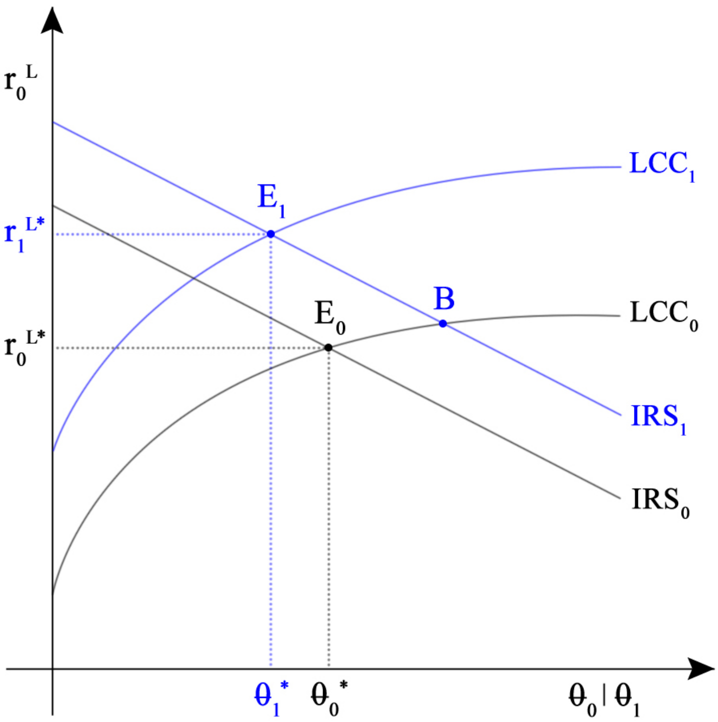


We can see, that the current lending interest rate depends negatively on the expected future credit market liquidity. The effect of higher lending interest rate on the current liquidity is, however, ambiguous. Plugging the interest rate schedule into the loan creation condition reveals, that the current liquidity is determined by the expected future prospects of the credit market. That is, by the expected credit market liquidity. Call the term that causes expected liquidity to affect current liquidity positively, $PT_{t,t+1} = \beta_h \left(1 - s\epsilon_{s,t}\right) E_t \left\{\Lambda_{t,t+1}^h\right\}$, and the term that causes expected liquidity to affect current liquidity negatively, $NT_{t,t+1} = \eta \frac{c}{\sigma_m \epsilon_{m,t}} E_t \left\{\Phi_{t,t+1}\right\}$. It is now easy to see, that if the credit market liquidity is expected to increase in the future, then the current liquidity increases if and only if $PT_{t,t+1}$ is greater than

$NT_{t,t+1}$ in absolute value. If the reverse is true, the current liquidity decreases due to expected increase in future liquidity.

Consider now the two types of the credit market shocks. Figure 2 graphically represents the credit market adjustments within the period, when the credit market is hit by negative credit market efficiency shock.

Figure 2: The effects of negative shock to credit market efficiency



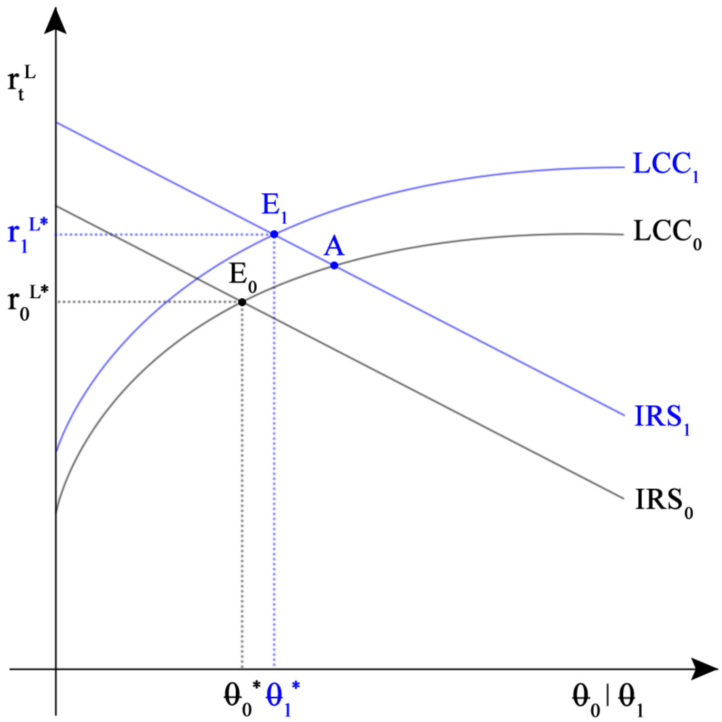
Negative shock to the credit market efficiency, $\epsilon_{m,t}$, means that the credit market becomes less efficient. That is, for any given rate of vacant and unrealized loans, less matches will be created. As a direct consequence of the shock, the current lending interest rate increases. For any given current and expected future liquidity, the entrepreneurs are willing to pay higher interest rate because the credit market is less efficient. Thus, the interest rate schedule shifts upwards. If the supply of vacant loans depended solely on the lending interest rate, the new equilibrium would be in point A, where both the interest rate and current liquidity are higher. However, the bank's optimality condition states that the current liquidity depends on the expected future liquidity as well. For the current set of structural parameters, $PT_{t,t+1}$ is in absolute value smaller than $NT_{t,t+1}$.[14] That is, the current credit market liquidity depends negatively on the expected credit market liquidity implying that following the negative credit market efficiency shock, the current liquidity increases. Thus, the loan creation curve shifts upwards too. The new equilibrium on the credit market is in point E_1, where the lending rate is higher and as the rate of vacant loans increases by relatively more than the rate of unrealized loans the liquidity is higher. This will indeed be the case in the simulations presented below.

The second credit market shock that we consider is a positive shock to the separation rate of the bank-entrepreneur relationship. It is interpreted as exogenous disturbance causing more matches to default at the end of the period. Figure 3 graphically represents the credit market adjustments within the period when the separation shock, $\epsilon_{s,t}$, is at work.

Suppose the shock has hit the economy at the end of the period $t-1$. This implies an increase in both the rate of vacant loans and the rate of unrealized loans. Since the entrepreneurs face search costs, increase in the rate of unrealized loans is relatively greater, therefore the liquidity in period t decrease. Given higher rate of unrealized loans, the entrepreneurs are willing to pay higher interest rate. Thus, the interest rate schedule shifts up-wards. Again, if the expected liquidity remained

[14]Note, that this shock does not affect neither the value of $PT_{t,t+1}$ nor the value of $NT_{t,t+1}$. What happens to the liquidity is ambiguous and as discussed above depends on the relative absolute value of $PT_{t,t+1}$ is greater than $NT_{t,t+1}$. Thus, whether the current liquidity decreases due to expected decrease of future credit market liquidity depends solely on the structural parameters of the model.

Figure 3: The effects of the separation shock



at the same level, the equilibrium point would move along the loan creation curve, as represented by the point B. However, due to lower current liquidity, the future liquidity is expected to rise. Therefore, the bank adjusts its supply of vacant loans, i.e. decreases the rate of vacant loans, so that its inter-temporal optimality condition continues to hold.[15] Loan creation condition shifts upwards. The final

[15]Whether the the expected liquidity affects current liquidity positively or negatively also in this case depends on the relative absolute size of the $PT_{t,t+1}$ and $NT_{t,t+1}$. Note, however, that in the case that $PT_{t,t+1} \leq NT_{t,t+1}$ in absolute value, so that the expected decrease in liquidity reduces current liquidity, and since $\epsilon_{s,t}$, directly decreases $NT_{t,t+1}$, this hock will amplify the decrease in liquidity. If, on the other hand, the expected decrease in liquidity decreases liquidity already in the current period, the shock will reduce this effect. Theoretically, due to the shock $\epsilon_{s,t}$ the effect could be even reversed, but this is highly unlikely and could be achieved only for very limited set of parameters values which would imply that the difference between the two terms is negligible and thus the expected liquidity would be almost irrelevant for the current liquidity.

equilibrium is in point E_1, where the credit market is characterized by higher lending interest rate and lower liquidity.

Comparing the final effects of the two credit market shocks, we can see that following both credit market shocks, the credit spread increases. Note, however, that the increase in the spread is relatively larger in the case of separation shock. The shocks on the other hand drive the credit market liquidity in the opposite directions. That is, the negative credit efficiency shock increases liquidity on impact, while following the positive separation shock liquidity decreases. The absolute size of the effect that the credit shock has on liquidity is relatively larger in the case of credit market efficiency shock.

The productivity shock affects macroeconomy mainly through the real side of the economy. As this is a standard shock, that all models have in common, we omit the detailed analysis of the equilibrium conditions and defer the synthesis of the effects untill the following subsection, where the simulation results are reported.

4.4 Simulation Results

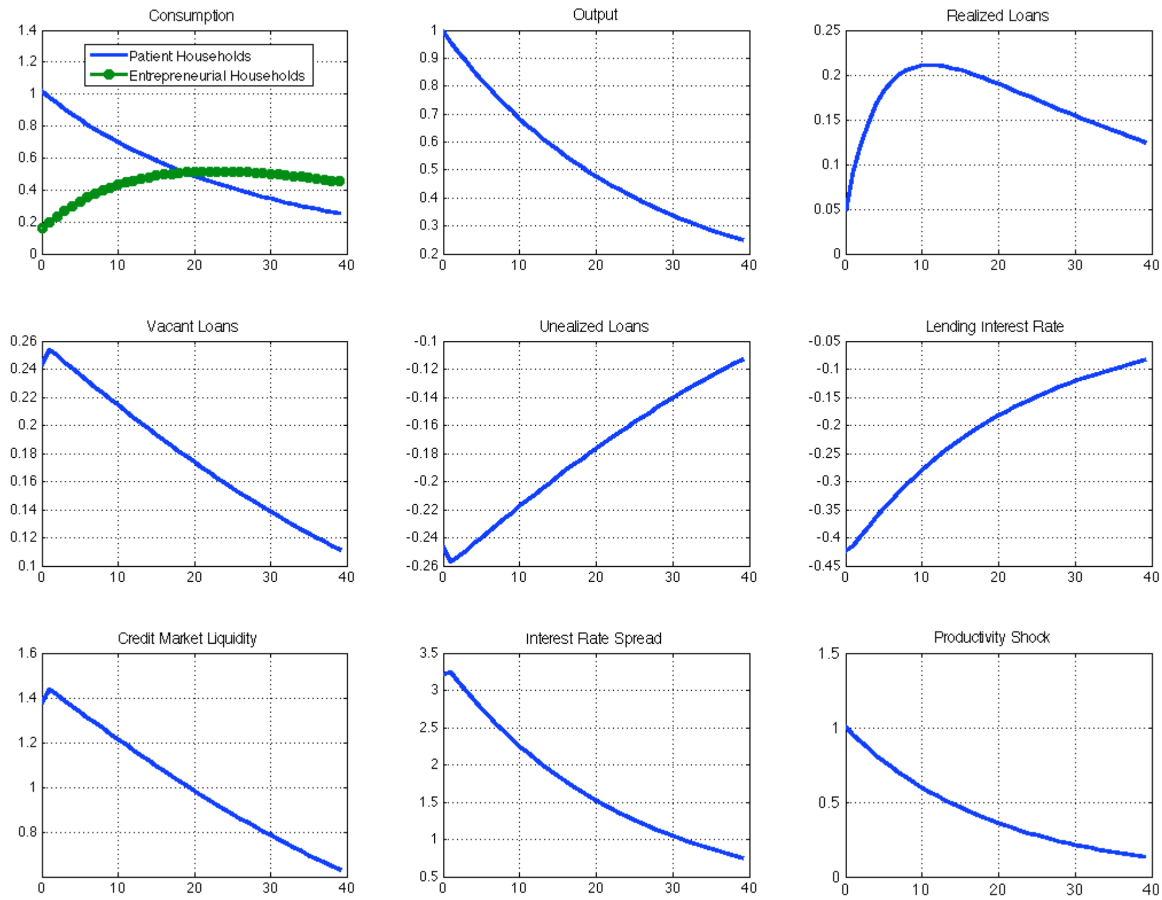
4.4.1 The effects of productivity shock

Having the mechanism above in mind, we continue describing general features of the model by presenting the impulse responses to standard productivity shock. Figure 4 describes the effect of one standard deviation productivity shock on the real sector variables and on the credit market variables.[16] The main channel of the mechanism through which productivity shock works is through the real side of economy. After positive shock output expands and consumption of both types of households increase. The entrepreneurial household's consumption, however, responds relatively less. Since they are productive it is profitable to invest more in production and increase consumption sluggishly. These responses are consistent with a model with credit constraints as in Kiyotaki and More (1997).

Turning to the responses of the credit market variables to productivity shock, we can see that after the economy is hit by positive productivity shock, demand for credit increases and therefore the interest rate spread increases. At a higher lending interest rate, the bank increases supply of new loans. As a result, the entrepreneurs have easier excess to finance and the realized loans increase. The credit market variables, however, do not react significantly after the economy is hit by the productivity shock.

[16]Since there is no prices in the model, the term 'real side' represents all the markets of economy other than the credit market.

Figure 4: Impulse Responses to Productivity Shock

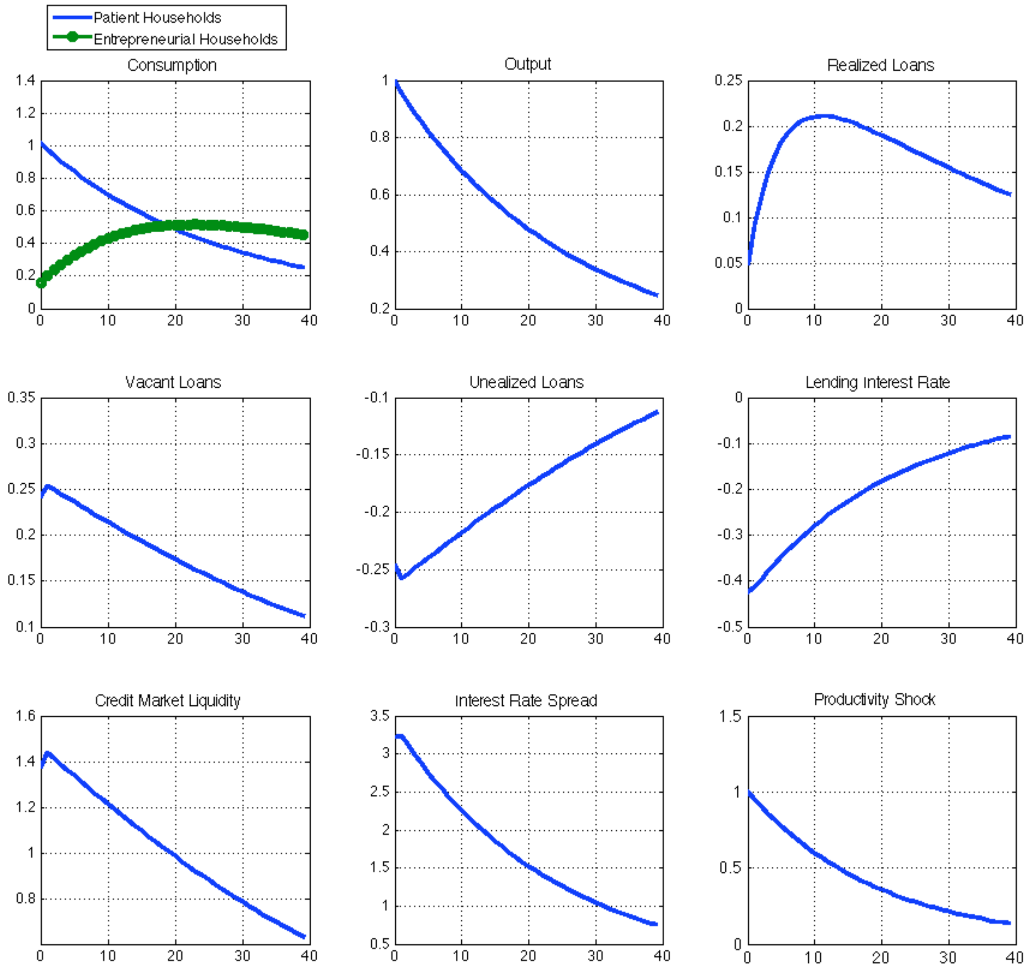


4.4.2 The effects of credit market efficiency shock

The second shock we consider is the shock to the matching function on the credit market. We interpreted it as the credit market efficiency shock. That is, after a negative shock the credit market becomes less efficient and as a result for a given level of vacant and unrealized loans less matches are created. The direct consequence is that less loans are approved.

Figure 5 presents the impulse responses of one standard deviation shock to credit efficiency on the economy. The common wisdom is, that decreased amount of outstanding credit triggers economic recession. This is indeed what happens in the model, but as opposed to the transmission mechanism of productivity shock, the main channel through which the credit efficiency shock is transmitted works through the credit market. As the market becomes less efficient realized loans decrease. From entrepreneurial side, increased rate of unrealized loans implies relatively higher search costs and drop in production, which could induce large drop in consumption of the entrepreneurial household. The entrepreneurs will be in order to counteract potential reduction in production be willing to pay higher interest rate on loans. At a higher lending interest rate for any given expected liquidity, the banks supply relatively more vacant loans. But then, since the supply of vacant loans increases relatively by more than the rate of unrealized loans, the credit market becomes more liquid in the current period and liquidity is expected to decrease in the future periods. Optimal behavior of the banks require weighting the current and expected costs of approving another loan. Therefore, in order to limit expected future fall in liquidity, increase of the supply of vacant loans is lower than it would be if their optimal behavior would have been determined period by period. The mechanism that works through the credit market therefore implies increase of the credit market liquidity, though to the smaller extent, but further increase of the lending rate and thus the credit spread. The negative credit market efficiency shock through the amplification of the credit spread (from 2 to 4 percentage points) induces the recession in the economy.

Figure 5: Impulse Responses to Shock to Credit Market Efficiency

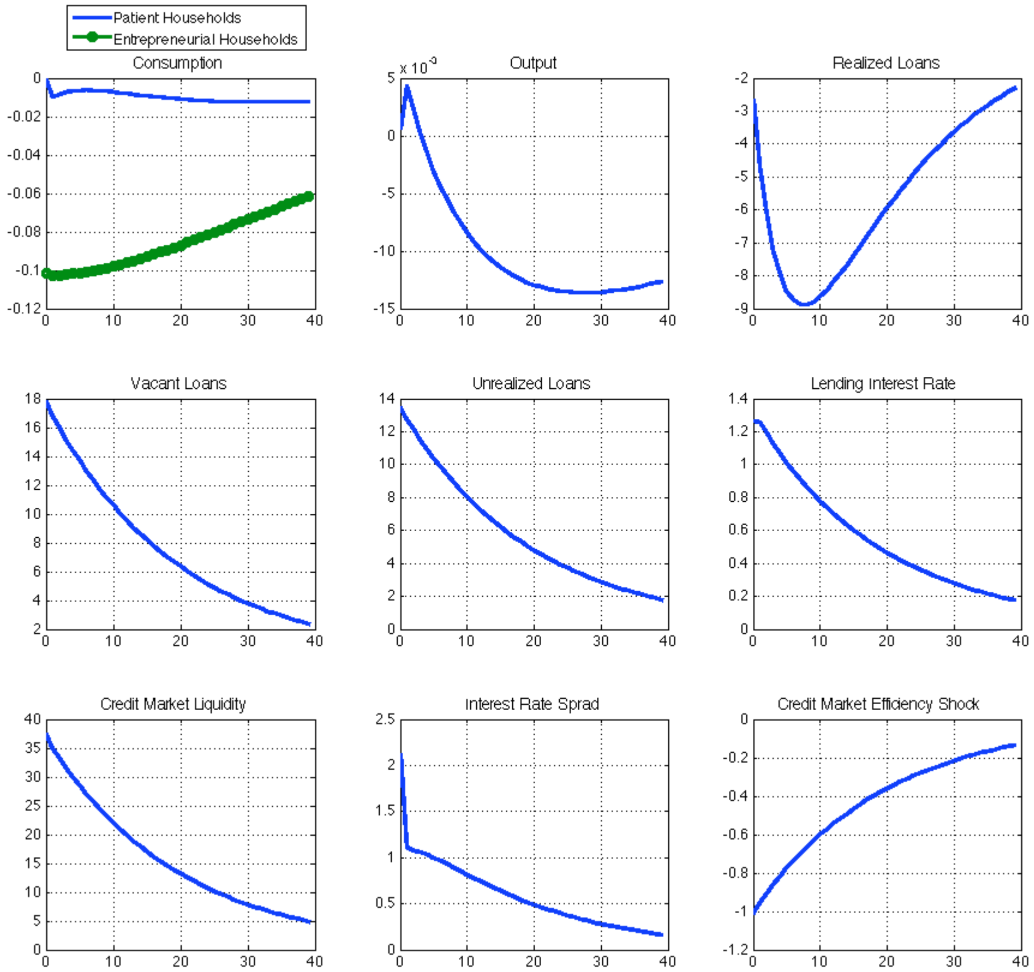


4.4.3 The effects of credit market separation shock

The third shock we consider is the shock to separation rate. Entrepreneurial household at the end of the period repays the loans and applies for new loans. For entrepreneur, it is therefore desirable that the match with the bank continues, since this enables him to continue producing without additional search costs. Thus, rolling over the debt allows him to continue producing and consequently to consume more in its life time. Recall that the entrepreneur whose match with the bank defaults, cannot continue producing but has to search for a new loan.

Figure 6 presents the impulse responses of one standard deviation positive credit market separation shock on the economy. As above, also in this case, decreased amount of outstanding credit will drive economy into the recession. Though the real economy variables, loan rates and interest rates qualitatively respond in the same direction as in the case of market efficiency shock, the mechanism is somewhat different. In particular, when the separation shock hits the economy, there will be relatively more separations at a given efficiency of the credit market. Consequently, more entrepreneurs will be on the search for a new loan and less entrepreneurs will continue producing. Recall, that those members of the family who produce are the ones who effectively finance search costs, which are now relatively higher as a consequence of increased rate of unrealized loans. For a given rate of vacant loans, more entrepreneurs will be on the search. As now the availability of external financing is reduced, they will be willing to pay the higher interest rates to obtain the loans. As before, at higher lending interest rates, the bank is willing to supply more new loans. As both the rate of vacant loans and the rate of realized loans increase, the rate of realized matches will increase. However, due to the fact that the rate of unrealized loans increases relatively more than the supply of new loans, the market liquidity decreases. Since the realization of this shock affects also the extent to which the expected liquidity affects bank's decision about current liquidity, the mechanism induces amplified adjustments of the credit market spread with respect tho those induced by the shock into efficiency market. That is, after the increase in the separation rate, the rate of unrealized loans increases significantly resulting in significant increase in credit spread (form 2 to 5 percentage points). More severe difficulties to access the external finance together with external finances finances becoming more costly results in a credit crunch that leads into greater recession as compared to the one that occurs when the credit efficiency shock is at work.

Figure 6: Impulse Responses to Shock to Credit Market Relationships



5 Conclusion

The present paper proposes a mechanism through which credit shocks are propagated to economy in a general equilibrium model. The fundamental feature of the model, namely, the presence of search and matching frictions on the banking market, is introduced to capture the idea that productive agents face difficulties in finding the funds.

The model in a simple and tractable setup gives rise to the positive spread between deposit rates and lending rates, which in periods of financial distress increases, as well as it captures the fact that in the downturn firms' financing difficulties increase. The latter together with the decrease in the bank's willingness to supply loans, has a direct consequence for variations in the credit market liquidity.

Two types of credit market shocks, the shock into credit market efficiency and separation shock, are considered. The results show that after the economy is hit by the shocks that worsen credit market conditions, the credit spread increases significantly. While the shock into credit market efficiency induces an increase in the credit market liquidity, the separation shock decreases the credit market liquidity. Therefore, following the first type of the credit shock, the interest rate spread increases relatively by less and since it is accompanied by the increase in the market liquidity, the recession stemming from the credit market disturbance is relatively milder comparing to the situation when the economy is hit by the second type of the shock of equal size.

References

Beaubrun-Diant, Kevin E. and Fabien Tripier, "The Credit Spread Cycle with Matching Friction," Technical Report, hal-00430809 2009.

Bernanke, Ben, Mark Gertler, and Simon Gilchrist, *The Financial Accelerator in a Quantitative Business Cycle Framework, in: Handbook of Macroeconomics*, 1 ed., Vol. 1, Elsevier, 1999.

Calza, Alessandro, Tommaso Monacelli, and Livio Stracca, "Housing Finance and Monetary Policy," *Journal of the European Economic Association*, 2011, *Forthcoming.*

Carlstrom, Charles T. and Timothy S. Fuerst, "Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis," *The American Economic Review*, December 87, *5*, 893–910.

Curdia, Vasco and Michael Woodford, "Credit Spreads and Monetary Policy," *Journal of Money, Credit and Banking*, September 2010, *42* (s1), 3–35.

_ **and** _ , "Financial Intermediation and Macroeconomic Analysis," *Journal of Economic Perspectives*, 2010, *24* (4), 21–44.

Dell'Ariccia, Giovanni and Pietro Garibaldi, "Bank Lending and Interest Rate Changes in a Dynamic Matching Model," Technical Report, IMF Working Paper 93 June 1998.

_ **and** _ , "Gross Credit Flows," *Review of Economic Studies*, 2005, *72* (3).

Faia, Ester and Tommaso Monacelli, "Optimal Interest Rate Rules, Asset Prices, and Credit Frictions," *Journal of Economic Dynamics and Control*, October 2007, *31* (10), 3228–3254.

Gerali, Andrea, Stefano Neri, Luca Sessa, and Federico M. Signoretti, "Credit and Banking in a DSGE Model," *Journal of Money, Credit and Banking*, September 2010, *42* (1), 107–141.

Iacoviellio, Matteo, "House Prices, Borrowing Constraints and Monetary Policy in the Business Cycle," *The American Economic Review*, June 2005, *95* (3), 739–764.

Kiyotaki, Nobuhiro and John Moore, "Credit Cycles," *Journal of Political Economy*, April 1997, *105* (2), 211–248.

Monacelli, Tommaso, *Optimal Monetary Policy with Collateralized Household Debt and Borrowing Constraints; in: Asset Prices and Monetary Policy*, John Y. Campbell, University of Chicago Press, 2007.

_ , "New Keynesian Models, Durable Goods, and Collateral Constraints," *Journal of Monetary Economics*, March 2009, *56* (2).

Weil, Philippe and Etienne Wasmer, "The Macroeconomics of Credit and Labor Market Imperfections," *The American Economic Review*, September 2004, *94* (4), 944–963.

Technical Appendix

Equilibrium conditions

The realized loans:

$$l_t = (1-s)\,l_{t-1} + m_t \tag{1}$$

Beveridge-type curve:

$$\Delta l^u = s\,(1-l^u) - pl^u = 0 \tag{2}$$

Matching function:

$$m_t = \sigma_m \left(l_t^V\right)^{\sigma} \left(l_t^U\right)^{1-\sigma} = \sigma_m \theta_t^{\sigma} l_t^U \tag{3}$$

Transition probabilities and panking market tightness:

$$q_t = \frac{m_t}{l_t^V} = \sigma_m \theta_t^{\sigma-1} \tag{4}$$

$$p_t = \frac{m_t}{l_t^U} = \sigma_m \theta_t^{\sigma} \tag{5}$$

$$\theta_t = \frac{l_t^V}{l_t^U} = \frac{p_t}{q_t} \quad \text{(liquidity)} \tag{6}$$

Loan creation condition:

$$\theta_t^{1-\sigma} = \frac{r_t^L - r_t^D}{1-r_t^D}\frac{\sigma_m}{c} + \beta_h\,(1-s)\,E_t\left\{\Lambda_{t,t+1}^h \frac{1-r_{t+1}^D}{1-r_t^D}\theta_{t+1}^{1-\sigma}\right\} \tag{7}$$

Aggregate lending interest rate:

$$r_t^L = \eta r_t^D + \eta \frac{c}{\sigma_m} E_t\left\{\Phi_{t,t+1}\theta_{t+1}^{1-\sigma}\right\} \tag{8}$$

Production function:

$$y_t = A_t k_{t-1}^{\alpha} n_t^{1-\alpha} \tag{9}$$

Marginal products of Labor and capital

$$w_t = (1-\alpha)\,\frac{y_t}{n_t} \tag{10}$$

$$rk_t = \alpha \frac{y_t}{k_{t-1}} \tag{11}$$

Household's and entrepreneurial first order conditions:

$$\lambda_{h,t} = \frac{1}{c_{h,t}} \tag{12}$$

$$\lambda_{h,t} = \beta_h E_t \left\{ R_t^D \lambda_{h,t+1} \right\} \tag{13}$$

$$n_t^v = w_t \lambda_{h,t} \tag{14}$$

$$\lambda_{e,t} = \frac{1}{c_{e,t}} \tag{15}$$

$$qk_t = \beta_e \frac{\lambda_{e,t+1}}{\lambda_{e,t} l_t} \left[rk_{t+1} + qk_{t+1} (1 - \delta) \right] \tag{16}$$

Household's and entrepreneurial budget constraints:

$$c_{h,t} + d_t \leq R_{t-1}^D d_{t-1} + w_t n_t + t_t + \Pi_t \tag{17}$$

$$c_{e,t} + qk_t \left[k_t - (1 - \delta) k_{t-1} \right] + R_{t-1}^L l_{t-1} + w_t n_t + ce * l_t \leq l_t + y_t \tag{18}$$

Capital producers:

$$qk_t (1 - S(i_t/i_{t-1}) - S'(i_t/i_{t-1})) + \beta_e \frac{\lambda_{e,t+1}}{\lambda_{e,t}} qk_{t+1} S'(i_{t+1}/i_t) = 1 \tag{19}$$

The aggregate stock of physical capital evolves as

$$k_t = (1 - \delta) k_{t-1} + (1 - S(i_t/i_{t-1})) i_t \tag{20}$$

Clearing conditions:

$$R_t^L l_t = R_t^D d_t \tag{21}$$

$$y_t = c_{h,t} + c_{e,t} + i_t + Costs_t \tag{22}$$

$$Costs_t = c * l_t^V + ce * (1 - l_t) \tag{23}$$

Derivation of the Loan Creation Condition

$$maxB_t^M(r_t^L) = r_t^L l_t - r_t^D d_t - cl_t^V + \beta_h E_t \left\{ \Lambda_{t,t+1}^h B_{t+1}^M(r_{t+1}) \right\}$$

subject to

$$l_t = (1-s)\, l_{t-1} + q_t l_t^V$$

The first order condition is:

$$\frac{\partial B_t^M(r_t^L)}{\partial l_t^V} = r_t^L \left[\frac{\partial l_t}{\partial l_t^V} \right] - c + \beta_h E_t \left\{ \Lambda_{t,t+1}^h \frac{\partial B_{t+1}^M(r_{t+1})}{\partial l_t} \frac{\partial l_t}{\partial l_t^V} \right\} = 0$$

$$\frac{\partial B_t^M(r_t^L)}{\partial l_t^V} = r_t^L q_t - c + \beta_h E_t \left\{ \Lambda_{t,t+1}^h \frac{\partial B_{t+1}^M(r_{t+1})}{\partial l_t} \frac{\partial l_t}{\partial l_t^V} \right\} = 0$$

$$\frac{\partial B_t^M(r_t^L)}{\partial l_t^V} = r_t^L q_t - c + \beta_h E_t \left\{ \Lambda_{t,t+1}^h \frac{\partial B_{t+1}^M(r_{t+1})}{\partial l_t} \right\} q_t = 0$$

$$\frac{\partial B_t^M(r_t^L)}{\partial l_t^V} = r_t^L - \frac{c}{q_t} + \beta_h E_t \left\{ \Lambda_{t,t+1}^h \frac{\partial B_{t+1}^M(r_{t+1})}{\partial l_t} \right\} = 0$$

$$(1-s)\frac{\partial B_t^M(r_t^L)}{\partial l_t^V} = (1-s)\, r_t^L - (1-s)\frac{c}{q_t} + (1-s)\,\beta_h E_t \left\{ \Lambda_{t,t+1}^h \frac{\partial B_{t+1}^M(r_{t+1})}{\partial l_t} \right\} = 0$$

The envelope condition is:

$$\frac{\partial B_t^M(r_t^L)}{\partial l_{t-1}} = r_t^L \left[\frac{\partial l_t}{\partial l_{t-1}} \right] + \beta_h E_t \left\{ \Lambda_{t,t+1}^h \frac{\partial B_{t+1}^M(r_{t+1})}{\partial l_t} \frac{\partial l_t}{\partial l_{t-1}} \right\}$$

$$\frac{\partial B_t^M(r_t^L)}{\partial l_{t-1}} = r_t^L (1-s) + \beta_h E_t \left\{ \Lambda_{t,t+1}^h \frac{\partial B_{t+1}^M(r_{t+1})}{\partial l_t} \right\} (1-s)$$

$$\frac{\partial B_t^M(r_t^L)}{\partial l_{t-1}} = r_t^L (1-s) + (1-s)\,\beta_h E_t \left\{ \Lambda_{t,t+1}^h \frac{\partial B_{t+1}^M(r_{t+1})}{\partial l_t} \right\} + (1-s)\frac{c}{q_t} - (1-s)\frac{c}{q_t}$$

Using first order condition we obtain:

$$\frac{\partial B_t^M(r_t^L)}{\partial l_{t-1}} = (1-s)\frac{c}{q_t}$$

Forwarding for 1 period:

$$\frac{\partial B^M_{t+1}(r_{t+1})}{\partial l_t} = (1-s)\frac{c}{q_{t+1}}$$

Replacing $\frac{\partial B^M_{t+1}(r_{t+1})}{\partial l_t}$ in the first order condition:

$$0 = r^L_t - \frac{c}{q_t} + \beta_h E_t \left\{ \Lambda^h_{t,t+1} \frac{\partial B^M_{t+1}(r_{t+1})}{\partial l_t} \right\}$$

$$0 = r^L_t - \frac{c}{q_t} + \beta_h E_t \left\{ \Lambda^h_{t,t+1} (1-s) \frac{c}{q_{t+1}} \right\}$$

Finally, the loan creation condition is:

$$\frac{c}{q_t} = r^L_t + \beta_h (1-s) E_t \left\{ \Lambda^h_{t,t+1} \frac{c}{q_{t+1}} \right\} \tag{24}$$

It is useful to rewrite Loan creation condition in terms of liquidity.

Using $\frac{1}{q_t} = \frac{1}{\sigma_m}\theta_t^{1-\sigma}$ we can rewrite it as:

$$\theta_t^{1-\sigma} = r^L_t \frac{\sigma_m}{c} + \beta_h (1-s) E_t \left\{ \Lambda^h_{t,t+1} \theta_{t+1}^{1-\sigma} \right\} \tag{25}$$

Derivation of the Lending Interest Rate Schedule

In the bargaining process, the first order condition is:

$$[S_t] = \frac{\eta}{(1-\eta)}[J_t]$$

Using the expressions for the entrepreneur's and bank's surpluses, J_t and S_t^E, we obtain:

$$\left[\begin{array}{c} -r_t^L + x + \\ \beta_e E_t\left\{\Lambda_{t,t+1}^e\left(1-s-p_{t+1}\right)[S_{t+1}]\right\} \end{array}\right] = \frac{\eta}{(1-\eta)}\left[\begin{array}{c} r_t^L + \\ \beta_h(1-s)E_t\left\{\Lambda_{t,t+1}^h J_{t+1}\right\} \end{array}\right]$$

$$\left[\begin{array}{c} -(1+\frac{\eta}{(1-\eta)})r_t^L \end{array}\right] = \left[\begin{array}{c} \frac{\eta}{(1-\eta)}\beta_h(1-s)E_t\left\{\Lambda_{t,t+1}^h J_{t+1}\right\} \\ -\beta_e E_t\left\{\Lambda_{t,t+1}^e\left(1-s-p_{t+1}\right)[S_{t+1}]\right\} - x \end{array}\right]$$

Forwarding the first order conditions and replacing S_{t+1} yields:

$$\left[\begin{array}{c} (\frac{1}{1-\eta})r_t^L \end{array}\right] = \left[\begin{array}{c} -\frac{\eta}{(1-\eta)}\beta_h(1-s)E_t\left\{\Lambda_{t,t+1}^h J_{t+1}\right\} \\ +\beta_e E_t\left\{\Lambda_{t,t+1}^e\left(1-s-p_{t+1}\right)\frac{\eta}{(1-\eta)}[J_{t+1}]\right\} + x \end{array}\right]$$

$$r_t^L = (1-\eta)x + \eta E_t\left\{\left(\beta_e\Lambda_{t,t+1}^e\left(1-s-p_{t+1}\right) - \beta_h(1-s)E_t\Lambda_{t,t+1}^h\right)J_{t+1}\right\}$$

Using $J_t = \frac{c}{q_t}$ and $\frac{1}{q_t} = \frac{1}{\sigma_m}\theta_t^{1-\sigma}$ we obtain the lending interest rate schedule is:

$$r_t^L = (1-\eta)x + \eta\frac{c}{\sigma_m}E_t\left\{\Phi_{t,t+1}\theta_{t+1}^{1-\sigma}\right\}$$

where $\Phi_{t,t+1} = \beta_e\Lambda_{t,t+1}^e\left(1-s-p_{t+1}\right) - \beta_h(1-s)E_t\Lambda_{t,t+1}^h$. Note, that $\Phi_{t,t+1}$ is negative.