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The undersigned

SURNAME *Palazzesi*
FIRST NAME *Mario*
PhD Registration Number *1103779*

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Date *31/01/2017*

SURNAME *Palazzesi*
FIRST NAME *Mario*

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Abstract

The present work consists of three chapters, and each of them is the result of the work carried over in the last four years.

In Chapter 1, we investigate the role of uncertainty in a takeover contest. We build a theoretical model that analyzes a tender offer game in which a raider attempts to take over control of a target firm. The raider is assumed to own some positive fraction of shares of the target firm. We study the implications of incomplete information both on the foothold that the raider owns and on prospective improvement that the raider is able to bring to the target firm.

We build a benchmark model with complete information, and then introduce information asymmetries, first purely either on the initial fraction of shares held by the raider or on potential post-takeover valuation of the target. Then we combine both types of uncertainty, building on intuitions and past literature in order to make different sets of assumptions that make the analysis tractable. We fully characterize equilibrium analysis in all different environments and we show that the interaction between the sources of uncertainty plays an important role for the theoretical results. In particular, the predictions that we obtain critically depend on the relationship between the two types of uncertainty that we study.

In Chapter 2 we propose a novel approach to model electoral competition, that includes Knightian uncertainty, i.e. ambiguity. We build a model in which two candidates compete for election and they differ on valence, that is an overall quality of the politician that is unobservable to voters. We argue that introducing ambiguity into the political economics literature can yield interesting novel insights, and we construct a simple model that is able to explain in a very intuitive way a well-documented fact about electoral competition, namely the incumbency effect.

We propose some possible extension of the framework that we introduce, and the directions in which we want to go for future research.

Chapter 3 presents a theoretical model that includes psychological motives behind players' decisions. In particular, we propose a model that explains behavioral data collected in Gneezy (2005), whose experimental setting is briefly introduced, in which agents are guilt and lying averse. In particular, we extend the theoretical framework developed in Battigalli, Charness and Dufwenberg (2013) to include lying averse behavior, and we argue that the model is able to provide sound theoretical justification for the behavior that subjects elicited in the experimental sessions.

Chapter 1

Corporate Control and Incomplete Information

1.1 Introduction

We develop a theoretical model of corporate takeovers, in which the raider, who attempts to gain control of the firm through a takeover operation, already owns a foothold in the target company.

Takeovers are an efficient way in which the free market operates, punishing less capable managements, or rewarding more efficient ones. A novel business idea, a new technology, or simply a better management are simple examples of how a takeover operation can be a means towards a more efficient economic outcome. Extended research efforts have been devoted to understanding corporate takeovers, and we aim at contributing to this literature. In their seminal paper Grossman and Hart (1980) [18] study a theoretical model of takeovers in which ownership is dispersed. Such dispersion implies that the probability of a single shareholder being pivotal is negligible, and this, in turn, implies that shareholders can free ride on the raider's prospective improvement in the firm's market valuation. The immediate consequence of this free-riding inefficiency is that raiders have no opportunity to earn positive profits, because they have to pay out to shareholders the expected improvement in the target's value. Bagnoli and Lipman (1988) [2] and Holmstrom and Nalebuff (1992) [23] study a variation of Grossman and Hart's setting, in which the number of shareholders is finite and, thus, the probability of being pivotal is positive. Also in this case, however, although shareholders decisions have a direct impact on the probability of success of the takeover, in the unique mixed strategy symmetric equilibrium they find the raider's profits are negligible, and go to zero as the number of shares gets large.

Shleifer and Vishny (1986) [31] introduce a large shareholder, who has higher interests in taking over because he owns a foothold α in the target firm before attempting to take over control. They use conditional offers made for exactly $(\frac{1}{2} - \alpha)$ of total shares, and study post-takeover behavior of the large shareholder who gains control, by giving him the opportunity to invest in research activities. They show that the premium paid by the raider is decreasing in the size of the foothold, and that investment in research activities are increasing in α. In the present work we focus on the takeover contest, and we generalize their environment, introducing uncertainty on the size of the toehold. We characterize equilibria in different theoretical frameworks, in which the sources of uncertainty vary. In particular, we model uncertainty both on the size of the initial fraction of shares owned by the raider, and on perpective post-takeover market valuation of the target firm. We want to investigate the optimal behavior of the raider in the takeover attempt and, in particuar, if and how uncertainty over the initial fraction of the shares he owns may affect such behavior. Studying whether or not the raider can effectively conceal to small shareholders his private information regarding the size of his toehold in the target firm obviously limits our attention to unconditional tender offers made by the raider: conditional tender offers would in fact immediately sell out for free the information on the size of the foothold.

The takeover contest's prominent role in the financial literature renders vain any remark regarding its importance here. Despite the vast amount of works that has analyzed corporate takeovers, anyway, the literature lacks, at least to my knowledge, an analysis of how uncertainty on shareholdings can affect agents' behavior, that is the main contribution that this work aims at giving. Our attention is then devoted to the initial share that a potential raider owns in the company he is interested in taking over and, in particular, to the possibility that the size of such toehold is unknown to the rest of the shareholders. We hope that this work can represent a starting point for drawing normative conclusions about the rationale that international financial regulations may have. In many countries transparency legislations require, for example, public disclosure of shares owned in listed companies that exceed given limits: e.g., the Annual Financials regulation in the US (cfr. SEC 10-K) demands shareholders to fully disclose any stocks they own above 5%; in Italy (cfr. art. 120 TUF), individual investors in listed companies must disclose shares above 2%, and listed firms investing in non listed ones must render public information any ownership above 5% of total shares. Needless to say, transparency rules vary greatly across countries, and alongside with these examples there are situations in which such disclosure is not mandatory and a potential raider can deliberately conceal the share he owns in a given company. This kind of transparency rules certainly serve different purposes, among which there might be some kind of protection of the small investors, whose interests may be seen as endagered by opaqueness upon the ownership structure. It is not our scope to discuss the rationale behind these laws in the present work, but using our model we can address the question whether small investors are indeed better off when such legislative requirements are in place and the ownership structure is public information.

The way in which we proceed is to fully characterize equilibrium dynamics in a variety of theroetical frameworks. We study theoretical predictions in a simple model with complete information, that can represents a useful benchmark for later results. Then we introduce uncertainty on the foothold that the raider owns before attempting to take over control. We will see that uncertainty on the fraction of shares owned by the raider can not, by itself, radically change equilibrium dynamics with respect to the full information setting. In this setup, we basically extend Grossman and Hart result: in our framework the raider is able to earn positive profits, because he already owns some fraction of the total shares, but he has to pay out to small shareholders the entire prospective improvement in the company's market value for the shares he need to buy to take over control.

We then introduce uncertainty on the future market valuation of the target firm after a successful takeover, and study the impact of this source of uncertainty on equilibria, initially abstracting from the uncertainty on the toehold. The results of our model suggest that this type of uncertainty gives rise to a plethora of equilibria. Standard separating equilibria arise, in which the raider with the higher prospective improvement on the firm's valuation is able to collect profits on his toehold, while the raider with the lower potential improvement has to give up some of his profits to effectively separate hiself and being able to pay a lower tender price. Moreover, a multiplicity of equilibria in which raiders with different prospects for the target company bid the same tender price arise: in these pooling equilibria we show that the tender offer can be either above or below to the expected post-takeover valuation of the firm.

We then turn to the central part of the analysis, in which we combine uncertainty regarding the foothold and uncertainty regarding post-takeover value. We make some simplifying assumptions that allows us to evaluate the effects of both sources of uncertainty. In particular, we build on existing literature and intuitions in order to develop two distinct frameworks, in which the initial foothold of the raider and propective market valuation of the target are perfectly related. In the first theoretical framework, we make the assumption that the size of the toehold is positively correlated with potential market value after a successful takeover. In other words, we assume that a raider with an high potential improvement in a firm's valuation also owns a larger fraction of the target's shares before the attempt to take over. This stems as a natural prediction of pre-tendering strategic acquisition models, such as in Chowdry and Jegadeesh (1994) [10] for example. In their model, the size of the foothold turns out to be positively related to potential synergistic gains for the target company. Some existing empirical evidence is also consistent with this assumption (see, e.g., Franks (1978) [15] or Mikkelson and Ruback (1985) [28]). The intuition goes as follows: in a financial market that allows pre-tender share acquisition, the raider has incentives to collect as many shares as possible before attempting to take over. In the second theoretical setting, we make an alternative assumption, namely that the size of the foothold and potential market valuation after a successful takeover are negatively related. The basic intuition behind this hypothesis is that prospective improvements in a firm's market valuation may act as an incentive to takeover. In other words, if a shareholder owns almost an half of total shares of a firm, then a modest potential increase in market value may push him to take over control and implement the possible synergies; on the other hand, if the fraction of shares in a firm is small, then relevant prospective improvements are needed in order to incentivate the shareholder to initiate takeover activities. The existence of financial costs needed to be sustained when attempting to take over may then justify the assumption under our second environment. Emprical evidence investigating the relation between toeholds and market value is mixed, and support can be found also of the hypothesis that the size of pre-tender share holdings is negatively related to market valuation after a successful takeover, see e.g. Jarrell and Poulsen (1989) and Eckbo and Langohr (1989).

The theoretical results of the model confirm that a relation between the initial foothold and the prospective improvement in the market value does indeed help in narrowing down equilibria. In particular, in the first setting, in which potential market value is positively related with pre-tender offer shareholdings, no separating equilibrium exists in which the tender offer succeeds for both raiders with positive probability. Conversely, there exist equilibria in which raiders offer the same tender price to small shareholders, and the takeover succeeds with positive probability for both of them. In particular, pooling prices below the ex-ante expected valuation of the target firm can be sustained in equilibrium: these are particularly interesting, because they are equilibrium outcomes in which raiders can effectively conceal the information on the toehold to small shareholders and are indeed able to get control of the target firm even if the tender bid is lower than the ex-ante expected value of the company after a successful takeover operation. In the second environment, in which our basic assumption negatively relates the size of the foothold and post-takeover valuation, the opposite theoretical prediction arises. In this case, in fact, no pooling equilirbia exist, and the only equilibria of the game are those in which the raiders bid different tender prices and, moreover, each of them has a strict incentive to bid truthfully. We fully characterize equilibria and derive conditions on the size of the foothold that support them.

The organization of the paper is as follows: I will review some of the more relevant literature on the topic and then in Section 1.3 we present the theoretical model that we developed, introducing the notation and the main ingredients. In section 1.3.2 we present a benchmark model of complete information, and then we add uncertainty, first on the initial foothold of the raider in section 1.4 and then, in section 1.5 on post-takeover valuation, separately. Thereafter, in section 1.6 we present the complete model in which both types of uncertainty are included. This is the central part of our work, in which we address the question of how the two sources of uncertainty can interact with each other in the equilibrium analysis. In section 1.7 we present additional comments and conclude.

1.2 Related literature

Literature related to corporate takeovers is extensive, and a complete overview would be impossible in this context. I will briefly mention some of the works that are closest to the main ingredients of the current work. A more comprehensive review can be found in Burkart and Panunzi (2006) [9].

A stricking contribution to the literature has been made by Grossman and Hart (1980) who investigated a simple takeover model with complete information and showed a somehow puzzling result, i.e. the free-rider problem, for which the intuition is straightforward: provided that shareholders are only willing to sell for a price that is at least equal to the post-takeover value of the target firm, no potential profits are left for the raider, rendering takeovers a zero-profit generating operation. The way to overcome this lemons problem for Grossman and Hart is to set up some exclusionary device so that benefits can be accrued to the raider not only through the price appreciation of the acquired shares. They use dilution in order to do that, giving the raider the opportunity to dilute the shares of shareholders who did not accept the tender offer. We will not investigate such excluding mechanisms, but limit our analysis to the takeover contest. A similar lemons problem arises in Harrington and Prokop (1993) [19], in which they allow for repeated tender offers after a takeover failure and show that tender bids increase over time. As the time interval between distinct offers goes to zero, so does the raider expected profit. Bagnoli and Lipman (1988) [2] and Holmstrom and Nalebuff (1992) [23] proposed a solution to the lemons problem by abandoning the hypothesis of atomistic shareholders. Assuming a finite number of large shareholders dilute the free-riding incentives because each shareholder does not neglect the possibility of being pivotal for the takeover success. Also in Marquez and Yilmaz (2008) [26] a similar lemons problem arises, although in their model the information structure is different, as they assume that private information is on the side of small shareholders rather than on the raider's. They build a model in which ownership is dispersed only among small shareholders, and the raider is unable to make profits.

Several mechanisms have been proposed to overcome the free-rider problem, such as dilution of outstanding shares, that has first been recognized as a method to extract benefits for the raider in Grossman and Hart (1980) [18]. Alongside, squeeze-outs and debt-financing are other means of achieving a similar objective, namely to reduce post-takeover value of outstanding shares, hereby mitigating the free-rider incentives not to accept the tender offer. Examples of the two instruments can be found respectively in Amihud, Kahan and Sundaram (2004) [1] or Yarrow (1985) [34] and Müller and Panunzi (2004) [29].

The importance that the initial share owned by the raider can have in a takeover contest has first been showed by Shleifer and Vishny (1986) [31], who showed how a large shareholder may play a key role in overcoming the free-rider problem, pointed to by Grossman and Hart. Also in Hirshleifer and Titman (1990) [20], and Chowdry and Jegadeesh (1994) [10], a foothold in target company is useful to overcome the free-rider problem among target shareholders: such a foothold provides in fact to the raider the opportunity to earn profits on the shares he already owns before starting the takeover activities. Whereas both in Grossman and Hart (1980) and in Shleifer and Vishny (1986) the focus is on pooling equilibria, both Hirshleifer and Titman (1990), and Chowdry and Jegadeesh (1994) build separating equilibria in a tender offer game. In both models uncertainty exclusively regards post-takeover value of the target firm. Hirshleifer and Titman (1990) assume, like we do, atomistic shareholders, but they analyze the case in which the raider makes a conditional offer for the shares he needs to gain control and they find a separating equilibrium in which the tender bid perfectly reveals the information on post-takeover value. Chowdry and Jegadeesh (1994) instead depart from our assumptions because they treat the initial toehold as endogenous: in their model pre-tender offer acquisition of shares is a strategic choice that the raider can make. Burkart (1995) [5] studies the bidding behavior of two raiders with a different pre-tender fraction of shares, but in his model, as in the previous ones, the size of the shares is known and the raiders use conditional offers. The presence of a large shareholder is studied also in Ekmecki and Kos (2016) [13], who show that both the presence of a shareholder with an high stake and dispersion of information among small shareholders are crucial for the raider's ability to earn profits. Also Burkart, Gromb, and Panunzi (2006) [6] set up a similar environment, in which they analyze the effects of a minority shareholder on takeovers. They assume complete information and, in their model, the acquirer can extract some private benefits by lowering the share value and the large minority shareholder sells all of his shares in equilibrium. The theoretical predictions are somehow ambiguous: in Ekmecki and Kos (2016), in fact, the larger is the large shareholder the lower is the tender price and the higher are the raider's profits, while in Burkart, Gromb, and Panunzi (2006) equilibrium price increases and the acquirer's profits decrease in the minority share of the large shareholder. In both the previous works, however, the minority shareholder is not the raider. Betton and Eckbo (2000) [3] empirical study covers a large sample of tender offer contests, in which they find that initial foothold is negatively related to bid premium and pre-tender offer stock price jumps, and that it significantly decrease competition, and hence resistance, faced by the raider. Moreover, although in their sample shareholders' expected payoff is decreasing in the raider's toehold, only in half of the cases the bidder does acquire an initial share before attempting to take over. Clearly, pre-tender offer shareholdings crucially depend on the depth of the financial market and on existing legislation regarding financial disclosure requirements. Additional empirical evidence can be found in Franks (1978) [15], in Mikkelson and Ruback (1985) [28], in Jennings and Mazzeo (1993) [22] and in Betton, Eckbo and Thorburn (2009) [4].

The great majority of signalling models focuses on conditional tender offers, or restricted tender offers for an amount of shares that is enough for the raider to get control rights. Burkart and Lee (2010) [7] build up a signalling model in which the information is similar to the one we build up here in section 1.5, namely where post-takeover value is private information of the raider, but they allow only for conditional offers. The distinction between conditional and unconditional offers has been extensively studied. Prokop (2003) [30] shows, by numerical analysis, that profits for the raider are higher under conditional rather than unconditional offers, but the probability of success is lower when conditional tender bids are used. He shows, moreover, that when repetition of tender bids is allowed for after a takeover failure, then profits for the raider go to zero, for both types of tender offer. Karbowski and Prokop (2015) [24] use an experimental setting to show that tendering probabilities are higher than those theoretically predicted for the case of unconditional tender offers, thus mitigating the free-riding problem faced by small shareholders. Marquez and Yilmaz (2007) [25], despite having no large shareholder, reverse the information structure: in their model small shareholders possess private information. Also in Ekmecki and Kos (2012) [12] small shareholders are privately informed about the raider's ability, while the latter is uninformed. They include a large shareholder, who, however, is not the raider attempting to take over. They show that profits for the raider are equivalent using either conditional or unconditional tender offers. Shareholders private information plays a key role in the signalling model studied in Marquez and Yilmaz (2012) [27]. They analyze both the cases in which shareholders have the same information and the case in which such information is private: in the former case they find that increasing the precision of small shareholders' information aggravates the free-riding problem, while in the latter case equilibrium tender offers lie above the expected valuation of the target firm for a shareholder whose information comes from a positive signal. Another interesting recent signalling model can be found in Stepanov (2012) [32] in which the raider can negotiate transactions with an incumbent, and tender offers are only made after failed negotiations. Asymmetric information on the raider's ability to generate profits leads to tender offers that are made by acquirers with higher prospective gains. Finally, Burkart and Lee (2015) [8] analyze a simple tender offer game in which the raider has private information about the post-takeover value improvement and has exogenous private benefits. They obtain an impossibility result that is in line with the one we derive in section 1.4, namely that the bidder can not reveal his type through the tender offer, and then introduce in the model different ways for the raider to relinquish his private benefits to the shareholders.

The relation between initial foothold and target returns represents the main assumption that we make both in section 1.6.1 and in section 1.6.2. Empirical literature on the argument does not identifies a clearcut relation, thus justifying our modelling approach. The relevance of an initial foothold is undisputable. In a sample of 1353 takeover contests between 1971 and 1990, Betton and Eckbo (2000) [3] report that in more than 35% of the cases the raider owns a foothold in the target company that is greater than or equal to 10%. The average toehold in their sample is slightly lower than 15%. As for the relation between the size of the toehold and the target's returns the evidence is more mixed: as a matter of fact, Eckbo and Langohr (1989) [11], Jarrell and Poulsen (1989) [21] and Betton and Eckbo (2000) [3] all provide evidence of a negative effect of toeholds on the target company's returns. On the other hand, Franks and Harris (1989) [16] found that initial footholds increase target returns. Ettinger (2009) [14] studies a takeover game between two raiders, one with a toehold in the target and the other without it. His motivations are close to ours and, alike we do, he takes the size of the initial toehold exogenously. However, his setup is different than ours: the bidders compete in an ascending auction with independent and private valuations and with participating costs. The raider with the initial toehold turns out to be more aggressive, so as to deter participation from the raider with no initial shares, unless the latter's valuation for the target is very high.

The equilibrium concept we use in our analysis is Perfect Bayesian Equilibrium (PBE)[1]. It is worth underlining an important difference with equilibrium analysis that can be found in the preceding literature. In particular, whereas earlier works (e.g. Shleifer and Vishny (1986) [31]) assume that the probability of success of the takeover is 1 when the raider ends up with exactly 50% of the shares after the tender offer, we let this probability to be endogenously derived in equilibrium. This equilibrium concept has been proposed, in the spirit of rational expectations, by Tirole (2010) [33] in a takeover model with complete information, and extended to incomplete and asymmetric information environments by Ekmecki and Kos (2016) [13]. They already adopted such an equilibrium concept in a model with a continuum of atomistic shareholders and show, moreover, that such equilibrium concept captures the behavior of shareholders in a model with a finite number of shares. In other words, as the number of shares goes to infinity, the outcomes of perfect Bayesian Nash equilibrium of the finite-shares model converge to an equilibrium outcome of the model with a continuum of shares.

1.3 The model

We model and study a takeover game with a continuum of atomistic shareholders. We assume that the raider owns an inital foothold in the company he wants to acquire and that he needs to obtain fifty percent of total outstanding shares of the target firm in order to gain control. Obviously, the initial fraction of shares owned by the player that will attempt to takeover is assumed to be below this threshold. In this first section we introduce the notation that we will use throughout the whole work, and present a simple model in which we assume complete information, both on the size of the foothold and on post-takeover valuation. In the following sections we will gradually relax this assumption and introduce asymmetric information. We limit the analysis to uncoditional tender offers p, made by the raider for all outstanding shares of the company. A couple of comments are already worth being made here for the sake of clarity, before going through the notation

[1] For a definition of PBE, see Fudenberg and Tirole (1991) [17].

and hence introducing the model.

First, we assume the toehold as exogenous to our setting, that is we are not considering the decision about the initial share as a strategic decision of the potential acquirer. This is clearly a stand we take: it may be argued that the initial position is a relevant strategic decision of the raider, and there exist works that study the optimal size of the toehold in a takeover game (see, e.g. Chowdry and Jegadeesh (1994) [10]); however, we want to analyze here the strategic behavior of the raider focusing on the tendering game itself. Moreover, financial regulations usually prevent strategic acquisition of shares of a company previous to a takeover attempt, and this explains the motivation behind the perspective we are adopting.

Second, unlike most of the literature on the topic, we study unconditional offers, made for all the shares of the company. The reason for this is pretty straightforward, as we want to analyze the impact of uncertainty on the initial share of the raider, and a conditional tender offer would immediately sell out for free the information on the toehold.

In order to deal with two sources of uncertainy, both on the potential value of the firm after a successful takeover and on the initial fraction of shares owned by the raider, we build two environments, that make two different assumptions, that are meant to render the problem tractable. In the first environment, we assume that the raider who holds an higher fraction of the company is also able to bring the higher enhancement to the firm's value, while the raider with a smaller initial toehold also has lower economic perspectives for the firm. This occurs, for example, as a prediction in signaling models like those described in Burkart and Lee (2014) [7], in which the fraction of shares owned by the raider is inversely related to the fraction of earnings that the raider has to forego in order to remunerate small shareholders: in these models the raider who has the highest potential enhancement on the valuation of the company can retain for himself an higher fraction of total gains, that is an higher initial share of the company in our setting. In the second framework, we assume that the raider who has the smaller initial share of the firm is the one with the bigger potential value-enhancement for the company: this can be the appropriate environment, for example, if we want to model the takeover attempt in a country in which financial regulation forbids, before an attempt to gain control of a firm, to increase the share owned in the target company, in order to have higher gains after a successful takeover. The intuition behind this hypothesis relies on the fact that market gains can serve as an incentive to engage in costly takeover operations: it would then be reasonable to assume that a shareholder with a low toehold would be willing to attempt to takeover only if he expects high-value potential synergies after a successful takeover, while someone with a large initial fraction of shares could be willing to attempt to take over even if the potential enhancement he can bring to the company is relatively small. The two different set of assumptios are thus both logically compelling, and may be relevant in different situations. The existing literature does not investigate empirically on the relationship between the initial fraction held by a raider and the valuation enhancement brought up by a takeover, and hence it does not provide us any hints to which one of the hypothetical environment may be closer to reality. This is why we allow for both the different environments described above, aiming at sheding some light on the theoretical predictions that we can make and hopefully preparing the ground for future empirical research.

1.3.1 Notation

Actions and Payoffs

Total outstanding shares of the company are privately held by a big shareholder, that is the potential raider in our framework, and the rest is publicly dispersed among small shareholders. We label $\alpha \in \left(0, \frac{1}{2}\right)$ the fraction of the shares that is owned at the beginning of the game by the player that attempts to take over corporate control. In particular, the raider's initial share, that is in our setting his private information, can be either high or low, and, from the point of view of small shareholders, it is distributed according to the following prior probabilities,

$$\alpha = \begin{cases} \alpha_l & \Pr\{\alpha = \alpha_l\} = 1 - \lambda \\ \alpha_h & \Pr\{\alpha = \alpha_h\} = \lambda \end{cases} \quad \text{with} \quad \alpha_l < \alpha_h < \frac{1}{2}$$

On the other hand, as it is standard in this literature, we assume that the rest of the shares, i.e. the fraction $(1-\alpha)$, is held by a continuum of atomistic shareholders whose total size has mass $(1-\alpha)$, and as a consequence, they individually possess a single share. They face a simultaneus decision, that is whether to *keep* or to *sell* their share, after any tender offer p made by the raider. Our main interest is on the size of the size of the foothold α.

In order to keep the analysis as simple as possible, we normalize firm's valuation and assume, without losing any generality, that the company's initial value is $v_0 = 0$. In addition, in order for the problem to be economically interesting, we hypothesize that the takeover is always value-enhancing: if the raider manages to acquire full control of the firm, collecting at least one half of total outstanding shares at the end of the game, then total valuation increases to $v_i > v_0$, with $i = l, h$.

In our model, at the beginning of the game the raider makes an unconditional tender offer $p \in [0, \infty)$. We limit our attention to uncoditional offers, in order for the tender offer not to reveal the amount of the toehold owned by the raider, that will be the main source of uncertainty in our theoretical framework. After the raider's bid, a continuation game starts, that is a simple simultaneous game in which small shareholders are the only active players, and they have to choose between keeping or selling their share. We allow them to randomize between these two pure actions. In this setting, randomization can intuitively be thougth of as a percentage of shareholders choosing to sell their share and a remaining fraction choosing to hold on to it. In order to simplify notation we focus on the probability with which small shareholders sell their single share, that is

$$\sigma(p) := \Pr\{sell \mid p\}$$

Final payoffs depend on whether or not the raider manages to obtain more than one-half of total outstanding shares, so that the takeover succeeds. In our analysis a key role will be played by the probability with which the takeover succeeds. If the takeover attempt fails then final valuation of the company is unchanged. We denote q_i the probability of the takeover succeeding, where the i indexes the different types of raider. This probability of success is clearly uniquely determined by small shareholders strategy, that is

$$q_i(\sigma) = \begin{cases} 0 & \text{if } (1-\alpha_i)\sigma < \frac{1}{2} - \alpha_i \\ \in [0,1] & \text{if } (1-\alpha_i)\sigma = \frac{1}{2} - \alpha_i \\ 1 & \text{if } (1-\alpha_i)\sigma > \frac{1}{2} - \alpha_i \end{cases} \quad \text{for } i = h.l \tag{1.1}$$

where we suppressed dependence of σ on the price p. The conditions in (1.1) state that the probability that the takeover succeeds is positive if and only if the probability with which small shareholders sell their share is high enough, where the threshold is the minimal fraction of outstanding shares in the hands of small shareholders that gives exactly one half of total shares to the raider, i.e.

$$\sigma(p) \geq \frac{\frac{1}{2} - \alpha_i}{1 - \alpha_i} \quad \text{for } i = h.l$$

Our equilibrium concept allows the probability of success of the takeover to be determined endogenously in equilibrium, when small shareholders' strategy is such that the raider ends up with exactly one half of total outstanding shares of the target company after the tender offer. As opposed to other models that postulate the latter probabililty to be equal to 1, we use the approach developed in Tirole (2006) [33] and let such probability to be endogenously derived from equilibrium analysis. For another application of this framework, see Ekmekci and Kos (2016) [13].

It is thus not obvious that the potential acquirer ends up with exactly one half of the shares. If the takeover attempt succeeds we call $S_i(p) \in \left[\frac{1}{2}, 1\right]$ the fraction of total shares that the different types of bidders end up with[2]. We can thus write the raider's payoff as a function of his tender offer, that is

$$\pi_i(p) = \begin{cases} S_i(p)v_i - p(S_i(p) - \alpha_i) & \text{if } S_i(p) > \frac{1}{2} \\ S_i(p)q_i(p)v_i - p(S_i(p) - \alpha_i) & \text{if } S_i(p) = \frac{1}{2} \\ 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots & \text{otherwise} \end{cases} \quad \text{for } i = l, h \tag{1.2}$$

where, clearly, S_i is determined by small shareholders behavior. In particular, given the definitions above, it holds

$$S_i(p) = \alpha_i + (1 - \alpha_i)\sigma(p)$$

On the other hand small shareholders are perfectly symmetric, and their payoff is as simple as it could be: it is zero if the takeover does not succeed, while it is either the price p offered by the tenderer, or the value of the share after a succeeding takeover, depending on whether they sold their single share or they kept it.

As a last piece of notation we denote $\mu(p)$ as the posterior probability distribution that small shareholders attach to the raider with the larger foothold, i.e.

$$\mu(p) := \Pr\{\alpha = \alpha_h \mid p\}$$

Equilibrium

The game is dynamic and consists of two stages: at the first stage the raider proposes a public tender offer, while at the second stage small shareholders simultaneously decide whether to accept or reject the raider's offer. The equilibrium concept we use hereafter is perfect Bayesian equilibrium. To formalize it, we define an equilibrium of the continuation game played by the symmetric atomistic shareholders, and an equilibrium of the original game, that includes the first stage at which the bidder attempts to takeover by an unconditional tender offer p.

[2]We are a little abusing notation by expressing the final share S as a function of the price. It should be noticed that S depends on the price through small shareholders strategy σ, as it is the case for the probability of success q introduced above. We will simplify notation with this kind of abuses throughout the chapter.

Definition 1 *$(\widehat{\sigma}, q_l, q_h)$ is a (symmetric) equilibrium of the continuation game if*

$$\widehat{\sigma}(p) = \begin{cases} 0 & \text{if } E_\mu\left[v \mid p\right] > p \\ \in [0,1] & \text{if } E_\mu\left[v \mid p\right] = p \\ 1 & \text{if } E_\mu\left[v \mid p\right] < p \end{cases} \tag{1.3}$$

$$q(p) = \lambda q_h(\widehat{\sigma}(p)) + (1-\lambda) q_l(\widehat{\sigma}(p)) \tag{1.4}$$

Given shareholders equilibrium strategy as defined above, we can now fully describe the first stage optimality problem of the raider in a simple way. The tenderer, in fact, has to choose an optimal price offer p_i^* that maximizes his profits, given the optimal behavior of small shareholders. The optimal bid can thus be obtained as

$$\begin{aligned} \widehat{p}_i &\in \arg\max_p \pi_i(p) \quad \text{for } i = l, h \\ \text{s.t.} \quad S_i &= \alpha_i + (1-\alpha_i)\widehat{\sigma}(\widehat{p}_i) \end{aligned}$$

We can now formally define an equilibrium of our model, which includes the previous definition of the equilibrium in the continuation game, the optimal tender price, and Bayesian computation of posterior probabilities that small shareholders assign to different types of raiders.

Definition 2 *$(\widehat{p}_i, \widehat{\sigma}, q, \mu)$ is an equilibrium of the takeover game if the following conditions hold*

1. *$(\widehat{\sigma}, q)$ is an equilibrium of the continuation game*
2. *$\widehat{p}_i \in \arg\max_p \pi_i(p)$ for $i = l, h$*
3. *$\mu(p) = \frac{\Pr(\alpha_i)\Pr(p|\alpha_i)}{\sum_i \Pr(\alpha_i)\Pr(p|\alpha_i)}$ if $\sum_i \Pr(\alpha_i)\Pr(p \mid \alpha_i) > 0$. Otherwise $\mu(p)$ is any probability distribution on $\{\alpha_l, \alpha_h\}$.*

Notice that at point 3 of the last definition we are not restricting anyway out-of-equilibrium beliefs, but this may potentially give rise to a plethora of equilibria that may be sustained by arguably reasonable beliefs out of the equilibrium path. We will prove existence of equilibria analyzing all possible price offers by the raiders, and assuming convenient out-of equilibrium beliefs. We will comment on this later in this work, and provide the intuition of how the results are robust to different kind of out-of-equilibrium beliefs specification. In particular, we will provide additional comments for the case in which posterior beliefs are assumed to be monotonic, i.e. such that the probability assigned to the type of raider with the highest valuation is not decreasing in the price offer.

1.3.2 Complete information

We start the analysis by characterizing equilibrium dynamics in the simplest case of complete information, that will serve as a useful benchmark for what follows. We assume then here, using the notation introduced in the previous section, that both the size of the toehold, α, owned by the raider at the beginning of the game and the post-takeover valuation of the target firm following a successful takeover are common knowledge, and we further normalize the latter to 1.

Faced with a tender offer p, small shareholders have to decide whether to keep their share of the company, or to sell it at price p. Note that, on one side, profits from selling are certain, because shareholders can collect p by accepting the tender offer; on the other side, valuation after a successful takeover operation is also known, but small shareholders' expected profits from keeping their share depend on the probability that the takeover operation is successful, that we denote q. Clearly such probability of success depends on small shareholders' strategy: in particular, if the raider is able to colletc more than one half of total outstanding shares after the tender offer, the takeover will succeed with probability 1; if the aquired shares are not enough to give him 50% of total shares, the takeover fails with certainty; if the raider ends up with exactly one half of total shares, we let the probability of success of the takeover to be determined endogenously in equilibrium. Formally,

$$q(\sigma) = \begin{cases} 0 & \text{if } \sigma < \frac{\frac{1}{2}-\alpha}{1-\alpha} \\ \in [0,1] & \text{if } \sigma = \frac{\frac{1}{2}-\alpha}{1-\alpha} \\ 1 & \text{if } \sigma > \frac{\frac{1}{2}-\alpha}{1-\alpha} \end{cases} \tag{1.5}$$

Notice that, for shareholders to randomize in equilibrium or, in other words, to choose (optimally) to sell their share with some positive probability $\sigma \in (0,1)$, it has to be the case that expected profits from keeping their share are equal to the tender offer[3]. This will play an important role in the analysis, and helps in understanding the dynamics by which the probability of the takeover success is endogenously determined in equilibrium. We thus propose the argument in the first claim of this benchmark framework.

Claim 3 *In any equilibrium of the game, the probability of the takeover success is equal to the tender price offer, i.e.*

$$p^* = q.$$

[3] Recall that we have normalized post-takeover valuation of the firm to 1.

Proof. As a first step, note that any price offers $p > 1$ are dominated for the raider by $p = 1$, and can thus not be part of an equilibrium.

When the tender offer is 1, the probability of succcess has to be 1. Suppose this were not the case: then each small shareholder obtain a strictly higher payoff by selling their share at the fair value of 1, rather than keeping it, but this leads to a contradiction with the assumption that the probability of success is smaller than 1.

A similar reasoning is valid for the case in which the tender offer is 0, in which case the takeover has to fail with certainty in equilibrium: suppose the probability of success were positive, then each shareholder is strictly better off by holding on to their share rather than selling it, contradicting the assumption of a positive probability of success of the operation.

Suppose then $\widehat{p} \in (0, 1)$: at such a price small shareholders sell, in equilibrium, just enough shares so that the raider ends up with 50% of total outstanding shares, i.e.

$$\widehat{\sigma}(\widehat{p}) = \frac{\frac{1}{2} - \alpha}{1 - \alpha}.$$

If small shareholders sold with an higher probability $\overline{\sigma} > \widehat{\sigma}$, then the probability of success of the takeover implied by (1.5) would be

$$q(\overline{\sigma}) = 1$$

and small shareholders expected profit from keeping their share would be higher than the tender price, i.e.

$$q(\overline{\sigma}) > \widehat{p}$$

so that keeping the share is small shareholders' unique best reply, and this would contradict $q(\overline{\sigma}) = 1$.

Similarly, if they chose $\underline{\sigma} < \widehat{\sigma}$, then the probability of success would be null, i.e.

$$q(\underline{\sigma}) = 0$$

and selling would be the unique best response of small shareholders, as $\widehat{p} > 0$, contradicting $q(\underline{\sigma}) = 0$. Then, the probability of success $q \in (0, 1)$ and small shareholders strategy is $\sigma = \widehat{\sigma}$: in equilibrium, the latter randomization by small shareholder is optimal if and only if the tender price they can earn by selling their share is equal to the expected utility from keeping it. In other words, small shareholders have to be indifferent between selling or keeping the share, i.e.

$$\widehat{p} = q(\widehat{p}).$$

■

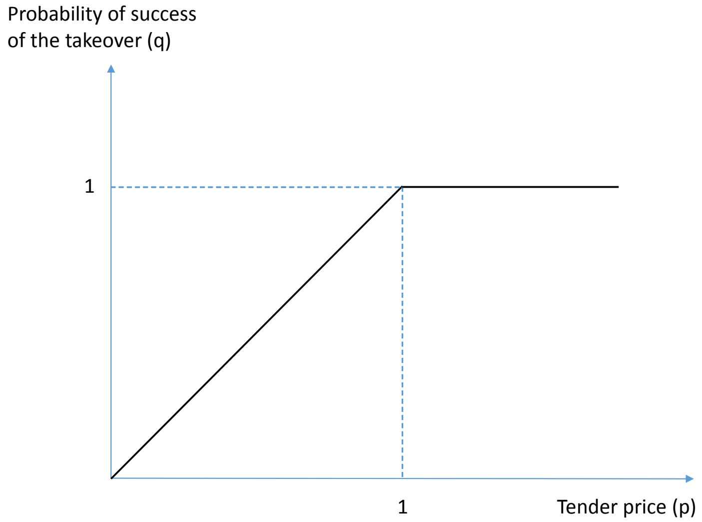

Figure 1.1: Equilibrium probability of success of the takeover, as a function of the tender price offered by the raider.

We now prove a simple claim that states that in every equilibrium the probability of success has to be positive. This is an implication of the fact that the takeover operation is commonly known to be efficient and, for this reason, the result will carry on also in the remainder of the paper.

Lemma 4 *In all equilibria of the game the the takeover succeeds with positive probability.*

Proof. Suppose, as an absurd, that it existed an equilibrium in which the probability of success of the operation was $q(\widehat{p}) = 0$. In particular, denote $\widehat{p} > 0$ the equilibrium price. Small shareholders equilibrium strategy would be

$$\widehat{\sigma}(\widehat{p}) < \frac{\frac{1}{2} - \alpha}{1 - \alpha}.$$

Profits are null for small shareholders, who would have a profitable deviation by selling their share at the positive tender price offer, and hence this can not be our equilibrium. Suppose instead that $\widehat{p} = 0$. This can be an equilibrium if and only if small shareholders strategy prescribes them to reject also all tender offers $p \in (0, 1]$. If this were not the case, in fact, the raider would have a profitable deviation by tendering with a positive price and obtaining positive expected profits, given that he already owns a fraction of shares. But the strategy $\sigma(p) = 0 \; \forall p \in [0, 1]$ can not be part of any PBE.

We have seen above, in the proof of Claim 1, that for a tender offer $p \in (0,1)$ small shareholders' best reply is to sell in equilibrium with a strictly positive probability so that the raider collects exactly half of total outstanding shares of the target company. ■

Claim 5 *In all equilibria of the game* $\widehat{p} = 1$*.*

Proof. When the price is 1 the takeover succeeds with probability 1 and raider's profits are given by (1.2), i.e.

$$\pi(1) = S(1) - (S(1) - \alpha) = \alpha.$$

If p were less than 1 we can use use Claim 3 to simplify the expression of the raider's profits in the following way

$$\pi(p) = S(p)q - p(S(p) - \alpha) = \alpha p.$$

that are always strictly lower than those computed above for $p = 1$. ■

Notice that, when the raider offers $p = 1$ the takeover succeeds with certainty and shareholders are indifferent between accepting the tender offer or keeping their share. The raider is earning positive profits, that come from the foothold he owns before taking over. Hence equilibria are far from being unique, because of potentially infinite permutations on shareholders who are selling or keeping their share, but they are all payoff-equivalent for all the players involved.

1.4 Asymmetric information

We now turn to analyze equilibria of the game introducing incomplete information. In particular, we mantain for now the assumption that the valuation of the target compny after a successful takeover is commonly known, whereas the size of the initial toehold is private information of the raider. To analyze equilibrium dynamics a crucial role is played by small shareholders posterior beliefs about the post-takeover value of the firm, given the observed tender offer p and the takeover succeeding. We will adopt the classical rational expectations approach in order to model equilibrium: posterior beliefs have to be consistent with different types of raiders' behavior in equilibrium. In other words, in equilibria in which different types of raiders (that are, for now, raiders with different initial fraction of total shares) both tender with the same price offer (*pooling*), small shareholders posterior beliefs have to agree with their prior at the beginning of the game, given that the price offer conveys no additional information; conversely, in equilibria in which different types of raider bid different prices (*separating*), posterior beliefs have to assign probability one to the correct type of raider, as the tender price fully reveals the type of acquirer that small shareholders are facing.

Assuming that all the information about the value of the company is publicly known, and normalized to 1 if the takeover succeeds and 0 otherwise, small shareholders payoffs from selling their share are equal to the tender offer p, while their expected payoff from keeping it is q. We can thus simplify the notion of equilibrium in the continuation game in the following way

Definition 6 $(\widehat{\sigma}, q_l, q_h)$ *is a (symmetric) equilibrium of the continuation game if*

$$\widehat{\sigma}(p) = \begin{cases} 0 & \text{if } q > p \\ \in [0,1] & \text{if } q = p \\ 1 & \text{if } q < p \end{cases} \tag{1.6}$$

$$q(p) = \lambda q_h(\widehat{\sigma}(p)) + (1-\lambda) q_l(\widehat{\sigma}(p))$$

We give an initial result that will be of great help in the equilibrium analysis. We omit the proof, that clearly relies on small shareholders strategy $\sigma(p)$.

Lemma 7 *If the probability that the takeover succeeds for the raider with the smaller toehold is strictly positive, then the takeover succeeds with probability* 1 *for the raider with the larger toehold. Moreover, if the probability of success is smaller than 1 for the raider with the larger toehold, then the takeover fails with probability 1 for the raider with the smaller initial fraction of shares. In our notation*[4]

$$\begin{aligned} q_l(p) &> 0 \Rightarrow q_h(p) = 1 \\ q_h(p) &< 1 \Rightarrow q_l(p) = 0 \end{aligned}$$

We then note that Lemma 4, introduced above in the complete information setting, can easily be extended to the current environment. Recall that, with asymmetric information on the size of the initial toehold, the probability of success of the takeover depends crucially on the size of the initial fraction of shares owned by the raider. In particular, given optimal behavior of small shareholders, the probability that the operation succeeds is always higher for the raider with the larger toehold. Then the takeover fails with certainty only if the raider with the larger foothold is not able to

[4] We keep using the indexes l and h to distinguish the two types of raiders: in this environment they denote the size of the toehold owned by the raider.

collect enough shares from small shareholders after the tender bid. We omit the formal proof, but it is easy to show that the probability of success of the takeover has to positive in equilibrium. The reason is clear: given that post-takeover valuation is commonly known, and using Claim 3, each raider has, in this environment, the opportunity to offer as a tender price the fair post-takeover potential value for outstanding shares, given that his profits are, similarly to the complete information case, on the shares he already has in his portfolio at the beginning of the game.

Moreover, it turns out that, in this simple framework, we already obtain an interesting result, that paying the post-takeover valuation to small shareholders turns out to be the only equilibrium of the game. Focusing our attention on the toehold, that is private information of the player that attempts to takeover, the result in this section implies that the initial fraction of shares of the target firm held by the raider before attempting to take over control, does not, by itself, give any potential benefit to the raider. In other words, in no equilibrium of the game the raider can beneficially exploit his private information endangering small shareholders private interests.

As a first step, we provide a result that excludes all equilibria in which raiders with different toeholds tender with different prices. The proof is omitted, but the idea is straightforward: if such an equilibrium existed, due to Bayesian beliefs updating as defined above in Definition 2, small shareholders assign probability 1 to the correct type of raider. This implies that the situation would be analogous to the complete information framework analyzed above.

Claim 8 *No separating equilibria exist.*

This can be viewed as an extension of the classical result in Grossman and Hart (1980) [18]: in such an environment, the optimal choice for each raider is to make a price offer equal to the post-takeover value of the company, thus paynig the fair value to small shareholders and setting to 1 the probability of success of the acquisition. There is no room for different types of raiders to behave in divergent ways, given that the only characteristics in which they differ is the size of the foothold that they own. This is why we will then extend the actual setting to incorporate the possibility that the different types of raiders differ, not only in the size of the toehold, but also in the perspective improvement that they could bring on the valuation of the company. The interaction between asymmetric information on the size of the toehold and on post-takeover valuation of the target firm will be our main concern in the remaining part of the paper. Before turning our attention to the uncertain post-takeover valuation case, we give the result that characterizes all equilibria of this game.

Proposition 9 *In every equilibrium of the game both raiders offer $p = 1$ and the takeover succeeds with probability 1.*

Proof. When the potential raider's tender offer is equal to the post-takeover valuation of the company, it is straightforward, from small shareholders indifference condition and the reasoning behind Claim 3, that the probability of success has to be 1. Both types of raiders would end up with (potentially different) total shares $S_i > \frac{1}{2}$, with $i = l, h$, and we can easily compute profits as

$$\begin{aligned} \pi_l &= S_l - 1(S_l - \alpha_l) = \alpha_l \\ \pi_h &= S_h - 1(S_h - \alpha_h) = \alpha_h \end{aligned}$$

It is immediate to show that no pooling price above the post-takeover valuation can exist. Suppose in fact that there existed a pooling price $\overline{p} > 1$. At such a price it is optimal for small shareholders to sell with the highest probability, given that the price they get is higher than the value of the company, whatever the outcome of the takeover attempt. Set then small shareholders' optimal strategy to $\sigma = 1$, implying that all shares are sold in equilibrium. It is easy to see that profits would be strictly decreasing in $\overline{p}$ and, in particular,

$$\begin{aligned} \pi_l(\overline{p}) &= 1 - \overline{p}(1-\alpha_l) = \alpha_l\overline{p} - (\overline{p} - 1) < \alpha_l = \pi_l(1) \\ \pi_h(p') &= 1 - \overline{p}(1-\alpha_h) = \alpha_h\overline{p} - (\overline{p} - 1) < \alpha_h = \pi_h(1) \end{aligned}$$

Suppose instead that a pooling price $\underline{p} < 1$ existed. In this case the probability that the take-over succeeds will crucially depend on small shareholders' strategy, in the following way

$$q(\sigma) = \begin{cases} 0 \quad \text{if } (1-\alpha_h)\sigma < \frac{1}{2} - \alpha_h \\ \in [0, \lambda] \qquad \text{if } \sigma = \frac{\frac{1}{2}-\alpha_h}{(1-\alpha_h)} \\ \lambda \quad \text{if } \frac{\frac{1}{2}-\alpha_h}{(1-\alpha_h)} < \sigma < \frac{\frac{1}{2}-\alpha_l}{(1-\alpha_l)} \\ \in [\lambda, 1] \qquad \text{if } \sigma = \frac{\frac{1}{2}-\alpha_l}{(1-\alpha_l)} \\ 1 \quad \text{if } (1-\alpha_l)\sigma > \frac{1}{2} - \alpha_l \end{cases}$$

We have already ruled out the case in which the takeover fails with certainty[5]. This implies that $q_l(\underline{p}) > 0$ and hence, by Lemma 7, $q_h(\underline{p}) = 1$. Suppose, on the way to a contradiction, that $q_l(\underline{p}) \in (0, 1)$. Then the expected value of keeping their share for small shareholders would be

$$\mathbb{E}(v \mid \underline{p}) = \lambda + (1-\lambda)q_l(\underline{p})$$

[5] Recall the result that we have proved in Lemma 4, and the discussion above on its straightforward extension to the asymmetric information environment.

and, using shareholders indifference condition we can pin down the value of $q_l(\underline{p})$, that is

$$q_l(\underline{p}) = \frac{\underline{p} - \lambda}{(1-\lambda)} \quad \text{for } \underline{p} \in [\lambda, 1]$$

It is easy to show that profits of the l-raider would be increasing in the equilibrium price $\underline{p}$, and always striclty lower than the ones attainable by tendering $p = 1$. ■

It thus follows that the only pooling equilibria that may arise are those in which both raiders bid their "safe offer", that is to say they offer the actual post-takeover valuation for outstanding shares in order to get control of the company.

Given that uncertainty on the size of the toehold does not, by itself, provide deeper theoretical insights, we extend the current model to include uncertainty on the valuation that the target firm may have after the takeover operation. We will first introduce uncertainty on future valuation by itself, and then proceed by combining it with uncertainty on the size of the toehold.

1.5 Uncertain post-takeover value

In order to obtain a deeper understanding of the theoretical predictions of our model, we now introduce uncertainty on the valuation that the target firm may attain after a successful takeover operation. To isolate the effect of this particular source of uncertainty, as opposed to the one that we have analyze in the previous section, we report a full equilibrium analysis dropping the uncertainty regarding the size of the initial share α that is owned by the raider before the attempt to takeover. We postpone to the following section the analysis of the complete model in which there is uncertainty both on post-takeover valuation and on the size of the raider's toehold. The discussion here is indeed interesting per se, but it is particularly useful in order to understand the kind of reasoning that lies behind our equilibrium analysis, because most of the insights continue to hold in some of the following sections. We will come back more precisely to this later on, hoping that the way of proceeding would then become clearer.

Our aim is to investigate equilibrium dynamics. In particular, in this specific part of the model, we are interested in understanding, first of all, whether any pooling equilibrium can be supported or not. This is extremely interesting because the existence of pooling equilibria signals the opportunity for different types of raiders to successfully conceal their private information to small sharehoders.

We assume, for the time being, that the initial fraction of shares α owned by the raider before attempting to takeover is public information. On the other hand, if the takeover attempt is succcessfull, the target firm would again benefit from the change in corporate control, but post-takeover valuation is uncertain. In particular,

$$v_1 = \begin{cases} v_l & \Pr(v_1 = v_l) = 1 - \lambda \\ v_h & \Pr(v_1 = v_h) = \lambda \end{cases} \quad \text{with} \quad v_h > v_l > 0. \tag{1.7}$$

The takeover operation is still assumed to be value-enhancing, but the size of the improvement is private information of the raider attempting to gain control. From the point of view of small shareholders the size of the jump in market valuation is unknown, and they possess a common prior that attaches probability λ to the highest potential improvement. The rest of the notation, when it is not equivalent, is easily extended from the previous section. For example, the probability that the takeover succeeds is still defined as in (1.5), while the notation for posterior beliefs introduced in part 3 of Definition 2 can be extended to the current framework by denoting

$$\mu(v_h \mid p) = \frac{\Pr(v_h)\Pr(p \mid v_h)}{\sum_j \Pr(v_j)\Pr(p \mid v_j)}.$$

Separating equilibria We first analyze the existence of equilibria in which the different types of raiders separate themselves: that is they behave in different ways, depending on the private post-takeover valuation they have.

For any pair of equilibrium tender offers, it must hold that the probability of success of the take-over is given by

$$q_i = \begin{cases} \frac{p_i}{v_i} & \text{if } p_i \leq v_i \\ 1 & \text{otherwise} \end{cases} \quad \text{for } i = l, h$$

By the same reasoning made above, it can not be the case that the takeover succeeds with probability 0 for both raiders and, in particular it may fail only for the l-type. We now propose a claim that can be seen as a standard result in the signalling literature, that is the following,

Lemma 10 *In any separating equilibrium $p_h \geq p_l$.*

We do not provide a formal proof, but analyze three crucial cases:

(i) when the takeover succeeds with probability 1 for both types of raider;

(ii) when the takeover succeeds with probabilities $q_h, q_l \in (0,1)$ for each raider;

(iii) when the takeover succeeds with probabilities 1 for the l-type and with $q_h \in (0,1)$ for the h-type of raider raiders.

The remaining cases in which $q_h = 1$ and $q_l < 1$ are in fact trivial.

Case (i) Start assuming that the takeover succeeds with probability 1 for both raiders. This implies that

$$\begin{aligned} p_l &\geq v_l \\ p_h &\geq v_h \end{aligned}$$

and that profits of the two raiders are

$$\begin{aligned} \pi_l &= S_l v_l - p_l(S_l - \alpha) \\ \pi_h &= S_h v_h - p_h(S_h - \alpha) \end{aligned}$$

We can write incentive constraints for both types of the raider as

$$\begin{aligned} \pi_l &\geq \pi_l(p_h) \Rightarrow S_l v_l - p_l(S_l - \alpha) \geq S_h v_l - p_h(S_h - \alpha) \\ \pi_h &\geq \pi_h(p_l) \Rightarrow S_h v_h - p_h(S_h - \alpha) \geq S_l v_h - p_l(S_l - \alpha) \end{aligned}$$

and combining the constraints we obtain

$$v_h(S_h - S_l) \geq v_l(S_h - S_l)$$

If we assume that the share acquired is non decreasing in the price offer, the latter condition implies that $p_h \geq p_l$.

Case (ii) Suppose instead that the takeover succeeds with probabilities $q_h, q_l \in (0,1)$. We can write profits as

$$\begin{aligned} \pi_l &= \frac{1}{2} q_l v_l - p_l \left(\frac{1}{2} - \alpha\right) = \alpha p_l \\ \pi_h &= \frac{1}{2} q_h v_h - p_h \left(\frac{1}{2} - \alpha\right) = \alpha p_h \end{aligned}$$

We can then check that the lemma holds imposing the incentive constraint for the h-type of raider, i.e.

$$\alpha p_h \geq \frac{1}{2} q_l v_h - p_l \left(\frac{1}{2} - \alpha\right) = p_l \left[\alpha + \frac{1}{2}\left(\frac{v_h}{v_l} - 1\right)\right]$$

from which it follows that $p_h \geq p_l$.

Case (iii) The last case that we need to consider is the one in which the takeover succeeds with probability 1 for type l and with $q_h \in (0,1)$ for h. Assume then that this is the case and that $p_h < p_l$. It can easily be verified that h will always have an incentive to deviate, that is

$$\pi_h(p_h) = \alpha p_h < \alpha p_l + S_l(v_h - p_l) = \pi_h(p_l)$$

We now show that any price offer above v_h will never be observed. Using the lemma above, it suffices to show that such offers will never be made by type h. To prove this it is enough to notice that, in any separating equilibrium, offering price v_h sets the probablity of the takeover equal to 1 and thus dominates all offers $p > v_h$.

Moreover, it can be showed that it cannot exist a separating equilibrium in which the takeover succeeds with probability 1 for both types of raiders. We already ruled out any price offer above his own valuation for the h-type of raider. Then, if such an equilibrium existed, it has to be the case that

$$\begin{aligned} p_h &= v_h \\ p_l &\leq v_h \end{aligned}$$

The takeover would succeed with probability 1 and raiders would collect shares S_h,$S_l \geq 1/2$ earning profits equal to

$$\begin{aligned} \pi_l &= S_l v_l - p_l(S_l - \alpha) = \alpha p_l \\ \pi_h &= S_h v_h - v_h(S_h - \alpha) = \alpha v_h \end{aligned}$$

but it's straightforward to show that this cannot be an equilibrium, because h would have an incentive to deviate offering p_l and earning higher profits than truthfully offering, i.e.

$$\pi_h(p_l) = S_l v_h - p_l(S_l - \alpha) = \alpha p_l + S_l(v_h - p_l) \geq \alpha v_h.$$

Moreover it can be argued that in no separating equilibrium the h-type would offer less than his valuation. Suppose instead that there were such a separating equilibrium: it has then to be the case that the takeover succeeds with some probability q_h strictly smaller than 1. It is now easy to show that profits of the h raider are indeed increasing in the probability of success, so that no such separating equilibrium can exist

$$\begin{aligned} p_h &< v_h \Rightarrow q_h = \frac{p_h}{v_h} < 1 \\ \pi_h &= \frac{1}{2} q_h v_h - q_h v_h (\frac{1}{2} - \alpha) = \alpha q_h v_h < \alpha v_h \end{aligned}$$

The intuition for this is in the spirit of Grossman and Hart's free riding problem: the raider has to pay out to small shareholders all the expected gain he has on the shares he buys. The h raider will then offer v_h and the takeover succeeds with probability 1.

We have seen above how the l raider offering v_l gives h an opportunity to deviate from offering v_h. It must then be the case that, to sustain a separating equilibrium with h offering his valuation, the takeover has to succeed with some probability strictly lower than 1 after p_l. We can thus conjecture that a separating equilibrium exists, with

$$\begin{aligned} p_l &< v_l \Rightarrow q_l = \frac{p_l}{v_l} < 1 \\ p_h &= v_h \Rightarrow q_h = 1 \end{aligned}$$

We omit the formal proof, but characterize all separating equilibria of this game in the following claim,

Claim 11 *Two types of separating equilibria exist.*

(i) if $\alpha \leq \frac{1}{2}\left(\frac{v_h - v_l}{v_h}\right)$ *then*

$$\begin{aligned} \widehat{p}_l &\in \left[0, \frac{\alpha v_h}{\alpha + \frac{1}{2}\left(\frac{v_h}{v_l} - 1\right)}\right] \\ \widehat{p}_h &= v_h \\ \mu(p) &= \begin{cases} 0 \text{ if } p \leq \widehat{p}_l \\ 1 \text{ if } p > \widehat{p}_l \end{cases} \end{aligned}$$

(ii) if $\alpha \geq \frac{1}{2}\left(\frac{v_h - v_l}{v_h}\right)$ *then*

$$\begin{aligned} \widehat{p}_l &\in \left[v_h - \frac{1}{2\alpha}(v_h - v_l), \frac{\alpha v_h}{\alpha + \frac{1}{2}\left(\frac{v_h}{v_l} - 1\right)}\right] \\ \widehat{p}_h &= v_h \\ \mu(p) &= \begin{cases} 0 \text{ if } p \leq \widehat{p}_l \\ 1 \text{ if } p > \widehat{p}_l \end{cases} \end{aligned}$$

Notice that the among the first type of separating equilibria, there are also those equilibria in which only the high type makes an offer. Suppose, for example, that shareholders beliefs are such that

$$\Pr(V_1 = v_h \mid p) = 1 \; \forall \; p > 0$$

Then if

$$\alpha \leq \frac{1}{2}\left(\frac{v_h - v_l}{v_h}\right)$$

the l raider is unable to make positive profits and will prefer not to offer.

Pooling Equilibria We now turn to analyze the existence of equilibria in which the two types of raiders bid the same tender offer. We keep the assumption that the fraction of total shares that is owned by the raider before he attempts to take over control of the target is commonly known, and all uncertainty is on the perpective valuation that the firm may attain after a successful takeover, that is summarized as in (1.7).

It turns out that any positive price p, that is above a positive lower bound that we will be characterized next, can be sustained in a pooling equilibrium, provided that the initial toehold α satisfies some conditions that depend on the parameters of the model. The full characterization of pooling equilibria will be relegated in an Appendix, while we provide here the main conditions on the parameters that guarantee existence.

Pooling with $\widehat{p} > \mathbb{E}(v)$. We analyze first pooling equilibria in which the price offer $\widehat{p}$ is above the expected valuation of post-takeover value, computed according to the prior λ. Notice that, if such a pooling price existed, then it has to be the case that the probability of success is 1. Moreover, the equilibrium analysis yields an even stronger prediction, that is that both types of raiders get the totality of the target's share on the market after the tender bid $\widehat{p}$. It is easy to see why this have to be the case: in a pooling equilibrium, small shareholders obtain no information on the type of raider from the tender bid observed at the first stage of the game and, hence, the probability that they attach to each type of the raider after the tender offer $\widehat{p}$ needs to coincide with the prior probability, and the expected payoff they obtain from holding their single share is equal to the unconditional expected value of v. Formally

$$\mathbb{E}\left[v \mid \widehat{p}\right] = \lambda v_h + (1-\lambda)v_l = \mathbb{E}(v).$$

If $\widehat{p} > \mathbb{E}(v)$ it then follows that the ecpected value from selling is higher than what small shareholders obtain by keeping their share. We can then derive profits that each type of raider can make in equilibrium, that are

$$\begin{aligned} \pi_h(\widehat{p}) &= v_h - \widehat{p}(1-\alpha) \\ \pi_l(\widehat{p}) &= v_l - \widehat{p}(1-\alpha) \end{aligned}$$

It is worth noticing that raiders always have the opportunity of offering a tender price of 0, and getting 0 profits. We then need to impose that equilibrium profits are non-negative, a condition that has a flavor of what that the mechanism design literature refers to as individual rationality. The most relevant constraint then clearly is the rationality constraint on the low type of raider, that yields a condition on the size of the toehold, that is

$$\alpha \geq 1 - \frac{v_l}{\widehat{p}}. \tag{1.8}$$

The latter condition requires that the size of the toehold is high enough so that the profits that the l-raider can earn on the shares he owns more than compensate the excessive payment that the raider is making in order to buy the remaining shares on the market. This intuition expains why such a lower bound on the toehold is decreasing in the post-takeover valaution v_l and increasing in the equilibrium price $\widehat{p}$.

In order to provide a better understanding of the dynamics, we analyze the limit case in which $\widehat{p} = v_h$. Notice, first, that this is the higher pooling price that can be sustained in equilibrium. Any price $p > v_h$ guarantees, in fact, that the probability of success of the takeover is 1, and that each type of raider collects all outstanding shares of the target firm after the tender offer. When the tender offer is equal to the highest possible post-takeover valuation, profits of each type of raider simplify to

$$\begin{aligned} \pi_h(\widehat{p}) &= \alpha v_h \\ \pi_l(\widehat{p}) &= \alpha v_h - (v_h - v_l) \end{aligned}$$

and rationality of the l-type of raider becomes

$$\alpha \geq 1 - \frac{v_l}{v_h}. \tag{1.9}$$

Assume then that posterior beliefs assign probability 1 to the high type of raider, for any given tender price other than the pooling price, i.e.

$$\mu(p) = 1 \quad \forall p \neq \widehat{p}.$$

We just need to check that there exist no incentive to decrease the tender offer below v_h. Observe that, given the assumption on posterior beliefs, we can use small shareholders indifference condition to pin down the probability of success of the takeover for any value of the tender bid $p < v_h$, that is

$$q(p) = \frac{p}{v_h} \quad \text{for } p \leq v_h$$

and profits of the two types of raider will be

$$\begin{aligned}\pi_h(p) &= \frac{1}{2}q(p)v_h - p\left(\frac{1}{2}-\alpha\right) = \alpha p \\ \pi_l(\widehat{p}) &= \frac{1}{2}q(p)v_l - p\left(\frac{1}{2}-\alpha\right) = p\left[\alpha - \frac{1}{2}\left(1-\frac{v_l}{v_h}\right)\right]\end{aligned}$$

and provided that condition (1.9) on the toehold holds, these are increasing in the tender bid p.

The intuition is that an higher tender price translates into higher expected profits, through an higher probability of success of the takeover: as long as the expected increase in profits justifies the higher bid that has to be paid there will always be an incentive to tender an higher price.

Pooling with $\widehat{p} \leq \mathbb{E}(v)$. We want to describe the characteristics of all equilibria in which both types of raider tender with the same offer that is below the ex-ante expected valuation of the target firm after a successful takeover. Notice that, in this case, the takoever has to fail with positive probability, as long as the equilibrium price is strictly below the ex-ante expected valuation of the firm. Recall that, similarly to what we have argued in the preceding section, Bayesian updating implies that in any pooling equilibrium prior and posterior distributions have to agree, and thus the expected post-takeover valuation has to be the same, accroding to both distributions. If the probability of success were 1, in an equilibrium in which $\widehat{p} < \mathbb{E}(v)$, then small shareholders receive strictly higher expected payoff by not selling at the tender price $\widehat{p}$, and this contradicts the fact that $q(\widehat{p}) = 1$. In equilibrium, indeed, the probability that the takeover succeeds can be derived directly from small shareholders indifference condition, that is

$$q(\widehat{p}) = \frac{\widehat{p}}{\mathbb{E}(v)} \quad \text{for } \widehat{p} \leq \mathbb{E}(v). \tag{1.10}$$

Expected profits that each type of raiders make in such an equilibrium are then

$$\begin{aligned}\pi_h(\widehat{p}) &= \widehat{p}\left[\alpha + S\left(\frac{v_h}{\mathbb{E}(v)} - 1\right)\right] \\ \pi_l(\widehat{p}) &= \widehat{p}\left[\alpha - S\left(1 - \frac{v_l}{\mathbb{E}(v)}\right)\right]\end{aligned}$$

We use S to denote the total fraction of shares that raiders end up with. In this framework, in which the size of the toehold is known, we do not need to distinguish between the two types of raiders. Although we are leaving it implicit here in order to simplify notation, recall that the size of S is uniquely determined by small shareholders equilibrium behavior. In particular,

$$S(p) = \alpha + \sigma(p)(1-\alpha)$$

and the share S and the probability of success of the takeover are univocally related, in the following way

$$\begin{aligned} S(p) &< \frac{1}{2} \Leftrightarrow q(p) = 0 \\ S(p) &= \frac{1}{2} \Leftrightarrow q(p) \in (0,1) \\ S(p) &> \frac{1}{2} \Leftrightarrow q(p) = 1 \end{aligned}$$

The first condition we need, in order to sustain these pooling equilibria, is derived from the rationality constraint of type l, i.e.

$$\pi_l(\widehat{p}) = 0 \Rightarrow \alpha \geq S\frac{\lambda(v_h - v_l)}{\mathbb{E}(v)} \equiv \underline{\alpha}(S) \tag{1.11}$$

This gives us a lower bound on the size of the toehold as a function of total shares that the raiders get in equilibrium. In particular, for prices below the prior expectation of post-takeover value, the probability of success calculated in (1.10) is smaller than 1, and $S = 1/2$. The expression in (1.11) will be recurrent in the analysis and that is why we awarded it with a special notation. Notice, moreover, that the expression in (1.11) can alternatively be seen as a condition on α or as a condition on the share S that the raider ends up with. It is worth noticing here that $\underline{\alpha}(\cdot)$ is strictly increasing in the high valuation and in the probability attached to it, and strictly decreasing in the low valuation.

We then just need to assure that the raider with the highest valuation has no incentive to deviate to higher prices. We will provide the intuition and avoid the burden of heavy computations here. It is rather intuitive that the raider with the higher valuation has the higher incentives to increase the tender bid, in order to increase the probability that the takeover succeeds. Recall, in fact, that he has room to do this, because at any pooling price $\widehat{p} < \mathbb{E}(v)$ he is earning profits not only on the toehold he already owns, but also on the shares he buys from small shareholders. Moreover, the h-type of raider always has the opportunity to tender v_h, that is his "safe offer", and set the probability of success of the takeover to 1 (indipendently of posterior beliefs), earning positive profits, independently from the size of the toehold.

Once again, assume that posterior beliefs assign probability 1 to the raider with the highest valuation, after any price offer that is above the equilibrium $\widehat{p}$, i.e.

$$\mu(p) = 1 \quad \forall p > \widehat{p}.$$

Then profits that the high type of raider can make by deviating from equilibrium behavior are strictly increasing in p. The reason is that our posterior beliefs are such that the loss resulting from an higher tender price are exactly offset by the increase in expected gains due to an higher probability of success. In other words, the h-raider always increases his expected profits by offering an higher tender price and this higher expected profits come exclusively from the initial toehold, because, for the shares he buys at the tender bid he always pays out to small shareholders the expected gains he makes out of them. Therefore, the last condition we need in order to sustain these pooling equilibria is

$$\begin{aligned} \pi_h(\widehat{p}) &= \widehat{p}\left[\alpha + S\left(\frac{v_h}{\mathbb{E}(v)} - 1\right)\right] \geq \alpha v_h = \pi_h(v_h) \\ \widehat{p} &\geq \frac{\alpha v_h}{\left[\alpha + S\left(\frac{(1-\lambda)(v_h - v_l)}{\mathbb{E}(v)}\right)\right]} = \frac{\alpha v_h}{\alpha + \frac{(1-\lambda)}{\lambda}\underline{\alpha}\,(S)} \end{aligned} \tag{1.12}$$

Few observations are worth being made at this point. First of all, the lower bound on the pooling price in (1.12) can be shown to be always below the ex-ante expectation of post-takeover value. As it is intuitive, such lower bound is increasing in post-takeover valuations, both in v_h and in v_l, and is also increasing in the probability attached to the higher valuation, λ. Moreover, prices below the lowest post-takeover valuation can be sustained in a pooling equilibrium, provided that the size of the toehold is not too high. In fact, in order to sustain low pooling prices, and hence low probabilities of success in equilibrium, the relevant constraint will be on the high type of raider, who needs not to have incentives to increase both the tender price and the probability of success. The condition on α reads

$$\alpha \leq \frac{1}{2}(1-\lambda)\frac{v_l}{\mathbb{E}(v)}. \tag{1.13}$$

The last condition, together with the lower bound on the toehold found above in (1.11), imply that pooling prices below v_l can be sustained only if

$$v_l \geq \lambda v_h.$$

Notice, as a last remark, that our attention was on existence of pooling equilibria. Clearly, in order to sustain such equilibria, out-of-equilibrium beliefs can be chosen ad-hoc and, in particular, the easiest way to do this is to assign probability 1 to the raider with the highest valuation whenever the price offer differs from the equilibrium tender bid. We used this in order to prove existence, and it is worth mentioning what changes in the analysis if we introduce some restrictions on out-of-equilibrium beliefs. The most intuitive of these restrictions would be assuming monotonic beliefs. To be clear, this amounts to say that the probability attached to the raider with the highest valuation has to be non-decreasing in the tender offer, namely, given two different possible tender offers p_1 and p_2 (with $p_1 < p_2$) of the raider, the probabiity that the posterior beliefs assign to the high-type of raider can not be higher at the lower tender price p_1, i.e.

$$\Pr\{v = v_h \mid p_1\} \leq \Pr\{v = v_h \mid p_2\}$$

or, in our notation,

$$\mu(p_1) \leq \mu(p_2).$$

It is easy to see how this kind of restrictions breaks down pooling equilibria with prices above the ex-ante expected valation $\mathbb{E}(v)$: in such pooling equilibria, as we have seen above, the posterior probability attached to the high type of raider is equal to the prior λ. This implies that, for a tender bid equal to the expected value of v according to the prior, the probability attached to θ_h can not be higher than λ and thus that the takeover has to succeed with probability 1. However, the results continue to hold for all equilibria in which the price $\widehat{p} \leq \mathbb{E}(v)$.

1.6 Two sources of uncertainty

We assume now that raiders have private information regarding both the size of the fraction of shares of the target firm they initially own and about the potential improvement in the valuation of the company that a takeover could bring on. Small shareholders, on the other hand, are uncertain about which type of raider they are facing, and only observe the tender price the raider offers.

The notation on the actions of the players and on the probability that the takeover attempt succeeds can be extended in a simple way to this part of the paper. We assume, as in section 1.5, that the set to which post-takeover potential valuations belong to remains the same, i.e. $v \in \{v_l, v_h\}$.

Building up a model that incorporates two sources of uncertainty to draw conclusion on how these interact with each other is ambitious and can get extremely cumbersome. In order to simplify the analysis, we will build on intuitions and refer to the existing literature in postulating two different kind of relations among the size of the toehold and post-takeover valuation. We carry out the analysis separately for each framework and then try to tackle the principal results by comparing the analysis in the two environments.

In the first part of this section we will assume that the size of the foothold and post-takeover market value are positively correlated. This assumption is in line with theoretical predictions made in some of the literature we have introduced in the previous sections, such as Chowdry and Jegadeesh (1994) [10]. In their model, differently from the current setting, they use pre-tender offer share acquisition as a signal provided from the raider to small shareholders on the post-takeover improvement in the valuation of the firm. In their model the size of the foothold is thus a strategic choice of the raider and, in the unique separating equilibrium they find, the size of the fraction of target shares bought before taking over fully reveals potential synergistic gains. In particular, the size of the toehold is predicted to be positively correlated to the value of such gains. The intuition behind the result goes as follows. The raider with the lower potential improvement in the firm's valuation prefers to forgo the opportunity to acquire shares at a low pre-tender price in the open market before attempting to takeover, in order to credibly separate himself from the type that has high prospects for the company. For the type of raider who has the higher potential improvement for the firm valuation, the incentives to separate from the low type are given by an higher probability of success of the takeover and the profits obtained on the shares purchased before the tender offer at a lower price. Both factors more than compensate the potential gains from a lower tender offer. Moreover, there exists evidence in the financial empirical literature of a positive relation between the size of the initial fraction of shares held by the raider and valuation of the target firm after a succcessful takeover operation: examples are Franks (1987) [15] or Mikkelson and Ruback (1985) [28]. It is indeed rather intuitive to think that a potential raider, who possesses some private information and knows that he is able to bring an important enhancement in the target company's valuation, has all the interest in keeping for himself the highest possible fraction of the gains, and in our model the only instrument to do so is the toehold, that is by holding a sizeable initial share in the target firm. Recall, in fact, that the idea behind the results in the previous sections is that the raider is willing to pay the fair value on the shares he buys to get control of the company, and in some cases even to overpay them, precisely because he can profit on the shares he already owns before the tender offer.

In the second part of the current section we will instead postulate an alternative assumption and test its theoretical predictions: namely that the size of the share initiallly owned by the player who wants to takeover corporate control is negatively correlated to the improvement he can bring on to the firm. In other words, in this framework, the type of raider who owns a small toehold has the higher perspective improvement for the company's valuation, while the raider with a larger initial share is able to provide a lower post-takeover market value if the takeover attempt succeeds. An intuitive explanation for this assumption has been already hinted at in the previous sections. It is certainly reasonable to think that engaging in a takeover attempt has relevant built-in costs, both financial costs or simple effort costs. While we are not exploring, nor explicitly modeling where these costs come from, it is easy to see that the implications are similar: a raider with a relevant share in a given company may attempt to takeover control even if the potential improvement in the valuation of the target is rather small. On the other hand, the improvement brought on to a target firm after a successful takeover needs to be large in order to induce a shareholder with a low toehold to engage in the attempt to gain corporate control.

Payoffs to the raiders are still defined as in (1.2) and we will adhere to the notions of equilibrium provided above in section (1.3.1). Notice that in this environment, as it was the case in the simpler framework in which uncertainty was only on the toehold α, the direct relationship that we exploited in the previous sections between the tender price p and the probability of success of the takeover q, analyzed in the complete information case in Claim 3, becomes more subtle. We can still assume that small shareholders behavior fully characterizes equilirbium probabilities of success, but in this case the probability that the takeover operation succeeds is different for the different types of raiders. Both these probabilities, together with the prior jointly determine the aggregate probability of success according to (1.4). Note in fact that such probability critically depends on the size of the toehold, that is, as it was defined in (1.1), different for the two types of raiders, i.e.

$$q_i(\sigma) = \begin{cases} 0 & \text{if } (1-\alpha_i)\sigma < \frac{1}{2} - \alpha_i \\ \in [0,1] & \text{if } (1-\alpha_i)\sigma = \frac{1}{2} - \alpha_i \\ 1 & \text{if } (1-\alpha_i)\sigma > \frac{1}{2} - \alpha_i \end{cases} \quad \text{for } i = h, l \tag{1.14}$$

It remains true that, when for small shareholders it is optimal to choose a pure action, the implied probability of success is either null or one. Moreover, also the argument by which a probability of success $q \in (0,1)$ in the interior of the unit interval is possible only if small shareholders use a randomization strategy still holds. In the current environment, anyway, probabilities of success of the takeover are different for the two different types of raider: in particular, it is useful to remind here how aggregate probability of success of the takeover depends on small shareholders equilibrium behavior, i.e.

$$q(\sigma) = \begin{cases} 0 & \text{if } \sigma < \frac{\frac{1}{2}-\alpha_h}{1-\alpha_h} \\ \in (0,\lambda) & \text{if } \sigma = \frac{\frac{1}{2}-\alpha_h}{1-\alpha_h} \\ \lambda \quad \text{if } \frac{\frac{1}{2}-\alpha_h}{1-\alpha_h} < \sigma < \frac{\frac{1}{2}-\alpha_l}{1-\alpha_l} \\ \in (\lambda,1) & \text{if } \sigma = \frac{\frac{1}{2}-\alpha_l}{1-\alpha_l} \\ 1 & \text{if } \sigma > \frac{\frac{1}{2}-\alpha_l}{1-\alpha_l} \end{cases} \tag{1.15}$$

In the following part of this section we analyze both pooling equilibria, in which raiders tender with the same price, and separating equilibria, in which they offer different prices depending on their private information. It is worthwhile to stress here that what we are the most interested in is the existence of pooling equilibria in which different types of raiders behave in the same way: in these equilibria, in fact, raiders with high valuation can effectively strategically conceal to small shareholders the information on the size of their participation in the target firm. In particular, we are after equilibria in which the raiders pool with tender prices below the expected value of the target company after a successful takeover, because those are the equilibria that jeopardize small shareholders' interests.

Notice, moreover, that Lemma 7 that we have introduced in section 1.4 remains valid in this section, and it will be a precious tool in the equilibrium analysis.

It turns out that the theoretical predictions of the model are very interesting: the interaction between uncertainty on the size of the foothold and on post-takeover valuation is indeed helpful in refining attainable equilibria. Equilibrium characterization crucially depends on the relation between the fraction of shares that is in the raider's portfolio before the attempt to takeover and the valuation of the target firm after a successfull takeover, i.e. the potential improvement that the raider is able to generate. This is the main contribution of the paper, that provides different theoretical predictions, based on the different set of assumptions explained above. It is worth reminding at this point that our assumptions mirror different interpretations attached to a raider's profits in a takeover action, but we prefer to abstain, at the moment, from any normative implication, for which further research is still needed.

In the two following sections we provide the analysis for the two frameworks described above: in the first one we make the hypothesis that the raider gains come from the toehold, and thus that the size of the initial share and post-takeover valuation are positively correlated; in the second one we assume that potential synergies serve as an incentive to engage in costly takeover operations, and thus that the size of the toehold is negatively related to the market value of the target following a successful takeover.

1.6.1 Market gains on the toehold

In this subsection we model the theoretical predictions of a model in which a raider's profit originate from the shares he owns in a target company before he attempts to take over. Such a prediction stems, for example, from theoretical models in which pre-tender offer share aquisition is strategically allowed for[6].

In particular, our way to introduce such an assumption into our model is to assume that the higher the potential improvement that a raider can bring to a target firm, the higher his initial toehold in the firm, given that this is the only instrument that our raider has in order to increase his profits. Having in mind a situation in which a potential raider has the opportunity to collect shares of the target company on the open market before engaging in a takeover attempt, this would lead to a strictly positive relation between the fraction of shares held by the raider before initiating the tendering activities and prospective gains in future market value.

We denote type θ_h the raider with the higher initial share, and the higher perspective improvement in the target's market value, i.e.

$$\begin{aligned} \theta_h &: = (\alpha_h, v_h) \\ \theta_l &: = (\alpha_l, v_l) \end{aligned}$$

As before, beliefs held by small shareholders, both at the beginning of the game and in the continuation game that follows the raider's tender offer, play a crucial role in the equilibrium analysis. Given our assumption on the positive relation between post-takeover market value and initial toehold in the raider's portfolio, we adapt the notation on the common prior and on posterior beliefs of small shareholders, that are, respectively

$$\begin{aligned} \lambda &= \Pr\{\theta = \theta_h\} \\ \mu(p) &= \Pr\{\theta = \theta_h \mid p\} \end{aligned}$$

and the expected value of keeping their share for small shareholders is

$$\mathbb{E}_\mu [v \mid p] = \mu(p) q_h(\sigma(p)) v_h + (1 - \mu(p)) q_l(\sigma(p)) v_l \qquad (1.16)$$

The results that we obtain are an interesting extension of the results that we have seen in the section in which uncertainty was only on post-takeover valuation. Equilibrium characterization in the current environment can be

[6] See, e.g. Chowdry and Jegadeesh (1994) [10].

seen as a refinement on the one we have seen in section 1.5. In that case, we obtained both separating equilibria and pooling equilibria. In the former, the raider with the higher valuation offered his private valuation of the target for oustanding share, and the raider with the lower valuation bid below his private valuation, succeeding in getting control of the target with positive probability strictly smaller than 1. In the latter kind of equilibria, we have seen that pooling prices both below and above the expected valuation of the company according to the prior were possible. In the current setting, the first result proves that only the second set of equilibria is possible.

Proposition 12 *There exist no separating equilibrium in which the takeover succeeds with positive probability for both types of raider.*
Proof. *Suppose, on the way to a contradiction, that there exists a separating equilibrium in which the two raiders tender different bids,* $\widehat{p}_l$ *and* $\widehat{p}_h$*. Moreover, suppose, to start with, that the probability of success of the takeover is* 1*, i.e.*

$$\begin{aligned} q_h(\widehat{p}_h) &= 1 \\ q_l(\widehat{p}_l) &= 1. \end{aligned}$$

For this to be a separating equilibrium, posterior beliefs have to assign probability 1 *to the correct type of raider, that is*

$$\begin{aligned} \mu(\widehat{p}_h) &= 1 \\ \mu(\widehat{p}_l) &= 0 \end{aligned}$$

and using (1.16), we can compute the expected value of not selling the share for small shareholder, that is respectively

$$\begin{aligned} \mathbb{E}_\mu \left[v \mid \widehat{p}_h \right] &= v_h \\ \mathbb{E}_\mu \left[v \mid \widehat{p}_l \right] &= v_l. \end{aligned}$$

Now, $q_i(\widehat{p}_i) = 1$ *and shareholders indifference condition together imply that*

$$\begin{aligned} \widehat{p}_h &\geq v_h \\ \widehat{p}_l &\geq v_l \end{aligned}$$

Notice that any price bid $p > v_h$ *can not be made in any separating equilibrium, because tendering* v_h *guarantees a probability of success equal to* 1*, at a lower price. Suppose then that* $\widehat{p}_h = v_h$ *and* $\widehat{p}_l \in [v_l, v_h)$*. It is easy* *to show that type θ_h would always have an incentive to decrease the price. Profits of the raiders are*

$$\begin{aligned} \pi_h(v_h) &= S_h v_h - v_h(S_h - \alpha_h) = \alpha_h v_h \\ \pi_l(\widehat{p}_l) &= S_l v_l - \widehat{p}_l(S_l - \alpha_l) \end{aligned}$$

It the h-raider were to mimic type θ_l his deviation profits would be

$$\pi_h(\widehat{p}_l) = S_h^l v_h - \widehat{p}_l(S_h^l - \alpha_h) > \alpha_h v_h$$

Suppose, as a last step, that $\widehat{p}_l < v_l$, as it was the case in separating equilibria with uncertainty only about post-takeover valuation. In this case the probability of taking over successfully by type θ_l is stricly lower than 1. However, as long as $q_l(\widehat{p}_l) > 0$, Lemma 7 implies that $q_h(\widehat{p}_l) = 1$ and type θ_h would still be able to take over control buying more than half of the shares, thus his incentives will continue breaking down any separating equilibrium. ■

The intuition behind the result is simple: when the size of the fraction of the firm initially owned by the raider is positively correlated with potential future market value the usual mechanism of sustaining a separating equilibrium, that is assigning a probability of success for the l-type in the interior of the unit interval, does not work anymore, because when the type with a low toehold is able to collect exactly half of total shares, the type who owns the larger initial fraction of shares is still able to purchase more than half of the shares. In other words, there is no way of punishing the high-type of raider with a probability of success strictly lower than 1. We can then conclude that the only possible separating equilibria are those in which the takeover fails with certainy for type θ_l.

Proposition 13 *In all separating equilibria*

$$\begin{aligned} \widehat{p}_h &= v_h \\ \widehat{p}_l &= 0 \\ q(\widehat{p}_h) &= 1 \\ q(\widehat{p}_l) &= 0 \\ S_l(\widehat{p}_h) &\geq \alpha_l \frac{v_h}{v_h - v_l} \end{aligned}$$

We omit the proof, and we only underline that the latter condition is needed for the low type of raider not to have an incentive to mimic type θ_h.

It guarantees in fact that the total fraction of shares he ends up with after tendering $\widehat{p}_h$ is such that his profits are non-positive.

We now turn attention to the existence of equilibria in which both types of raiders tender with the same price. We provide the equilibrium analysis for any tender bid, and we identify the most relevant conditions that sustain them. As done before, we split the analysis in subsections, investigating the dynamics for different ranges for the equilibrium price.

First of all, notice that any pooling price can not be higher than the highest valuation. It is easy to see that no pooling price above v_h can be sustained in equilibrium, because bidding v_h guarantees that the takeover succeeds with probability 1, given that, for any posterior $\mu(p)$, equation (1.16) guarantees that the expected value of holding the share for small shareholders is never above v_h.

Pooling equilibria with $\widehat{p} \in (E(v), v_h]$ At such an equilibrium price, the posterior probability has to agree with the prior, and equation (1.16) becomes

$$\mathbb{E}_\mu \left[v \mid \widehat{p} \right] = \lambda q_h(\sigma(\widehat{p}))v_h + (1-\lambda)q_l(\sigma(\widehat{p}))v_l$$

implying that the expected payoff obtained by holding the share is striclty less than the bid price, even if $q_h(\widehat{p}) = q_l(\widehat{p}) = 1$. This means that small shareholder are strictly better off by selling their share, and that raiders end up with the totality of the shares of the target. Equilibrium profits would then be

$$\begin{aligned} \pi_h(\widehat{p}) &= v_h - \widehat{p}(1-\alpha_h) \\ \pi_l(\widehat{p}) &= v_l - \widehat{p}(1-\alpha_l) \end{aligned}$$

For this to be an equilibrium, the only constraint we need to impose is the rationality constraint for the low type. Notice that none of the raiders has any incentive to increase the price, because this can not have a positive effect on the probability of success of the takeover, and thus on profits. We can easily eliminate incentives to decrease the price by imposing out of equilibrium beliefs that assign probability 1 to type θ_h for prices smaller than $\widehat{p}$, i.e.

$$\mu(p) = 1 \quad \text{for } p \neq \widehat{p} \tag{1.17}$$

Notice in fact, that type θ_h will not want to decrease the price, because he will always pay out to small shareholders the expected gains on the shares he buys, implying that his profits are strictly increasing in the price, i.e.

$$\pi_h(p) = \begin{cases} \frac{1}{2}\frac{p}{v_h}v_h - p\left(\frac{1}{2} - \alpha_h\right) = \alpha_h p & \text{for } p < v_h \\ S_h v_h - v_h(S_h - \alpha_h) = \alpha_h v_h & \text{for } p = v_h \end{cases}$$

On the other hand, for type θ_l, if we assign out-of-equilibrium beliefs as in (1.17), small shareholders randomization strategy would be such that type θ_h ends up with exactly half of the shares for any price bid $p \neq v_h$, but this implies that type θ_l would not get enough shares to take over control, and his expected payoff is negative.

Individual rationality for type θ_l boils down to a condition on the pooling price that can be sustained in equilibrium, that is

$$\widehat{p} \leq \frac{v_l}{1-\alpha_l}$$

Notice that the latter upper bound on the pooling price is certainly above v_l, but it may be below the unconditional expected value of v. In particular, for $\widehat{p}$ to fall in the range of prices we are considering here, it has to be the case that

$$\alpha_l > \frac{\lambda(v_h - v_l)}{\mathbb{E}(v)} \equiv \underline{\alpha}(1)$$

As mentioned above, and according to intuition, the lower bound on α found above can be shown to be strictly increasing in the higher valuation v_h and in the prior probability attached to it, λ, and strictly decreasing in v_l.

As a last remark, it is worth noticing that we used ad-hoc out-of equilibrium beliefs to sustain these pooling equiliibria. As noticed in the end of section 1.5, assuming monotonic beliefs immediately breaks down all pooling equilibria with the tender price strictly above the unconditional expected value $\mathbb{E}(v)$. It is easy to see this from equation (1.16): if the probability attached to type θ_l at the tender price $\widehat{p}$ is λ, then monotonic beliefs imply that

$$\mu(p) \leq \lambda \quad \text{for } p < \widehat{p}$$

and this in turn implies that at $p = \mathbb{E}(v)$ the takeover can not fail with positive probability, in particular,

$$q_h(\mathbb{E}(v)) = q_l(\mathbb{E}(v)) = 1.$$

Pooling equilibria with $\widehat{p} \leq \mathbf{E}(\mathbf{v})$ The case in which the equilibrium price is below the unconditional expected post-takeover valuation is slightly more complicated, and this is because the probability of success of the takeover is different for the two types of raider. However, the positive relation between the share and post-takeover valuation helps in sustaining pooling equilibria, and we will now explain why this is the case.

First take the case in which $\widehat{p} = \mathbb{E}(v)$. We can use equation (1.16) and equilibrium posterior $\mu(\widehat{p}) = \lambda$ to see that shareholders' expected value from holding their share is strictly less than the tender price, unless the probability of success $q(\widehat{p}) = 1$. Then, as long as $\pi_l(\mathbb{E}(v)) \geq 0$, it is easy to sustain $\mathbb{E}(v)$ as a pooling equilibrium price, by setting out-of-equilibrium beliefs as

$$\mu(p) = 1 \quad \text{for } p \neq \widehat{p}.$$

The takeover is succeeding with probability 1 at the tender price $\widehat{p}$ for both raiders and it is immediate to see type θ_h has no incentive to deviate from the equilibrium price. As a consequence, neither type θ_l would have any incentive to deviate upward from the equilibrium price. The non-negative profits constraint on the low-type boils down to condition (1.11) we have seen above in section 1.5, relating the foothold α_l and the fraction of shares S_l the l-raider ends up with, i.e.

$$\alpha_l \geq S_l \frac{\lambda(v_h - v_l)}{\mathbb{E}(v)} \equiv \underline{\alpha}(S_l)$$

Deviations by θ_l to lower tender prices are excluded by out-of-equilibrium beliefs: when small shareholders assign probability 1 to type θ_h, for any tender bid below v_h they randomize in such a way that the raider with the higher toehold gets exactly half of the shares, but this implies that the raider with the lower toehold gets strictly less than half of the shares and therefore that the takeover fails with probability 1 for him. This is the key implication that our assumption on the two sources of uncertainty yields in this environment.

When $\widehat{p} < \mathbb{E}(v)$, the aggregate probability of success is strictly lower than 1. This can be shown using equation (1.16) to compute the expected payoff from holding the share for small shareholders, that is

$$\mathbb{E}_\mu [v \mid \widehat{p}] = \lambda q_h(\sigma(\widehat{p}))v_h + (1-\lambda) q_l(\sigma(\widehat{p}))v_l$$

It is easy to see that if $q_h(\sigma(\widehat{p})) = q_l(\sigma(\widehat{p})) = 1$, the latter equality implies that not selling yields a strictly higher expected payoff than selling the share, that earns small shareholders $\widehat{p}$, and this contradicts $q = 1$. Hence,

$$q_l(\sigma(\widehat{p})) < 1$$

We have already argued that the takeover can not fail for the h-type of raider, who is the one with the higher potential improvement for the target. Suppose then

$$q_h(\widehat{p}) > 0,\ q_l(\widehat{p}) = 0.$$

Notice that this can happen in two distinct cases: either the raider with the higher foothold ends up with exactly half of the shares, or he ends up with more than half of the shares, bu the shares he buys at the pooling tender offer $\widehat{p}$ are not enough fot type l to collect half of the shares of the target. In either case, this implies that the takeover fails for the low type of raider, and the aggregate probability of success is bounded above by λ. It is easy to show that this can not happen in any pooling equilibrium. Indeed, when the probability of the l-raider succeding is 0, his expected profits are non positive, and the only equilibrium tender bid can thus be 0. We are then only left to show that a tender offer of 0 can never be observed from the h-raider, and this can immediately be done by recalling the "safe offer" that the raider with the higher improvement has, that is to bid his private valuation v_h. In this case, independently from out-of-equilibrium beliefs, the probability for the takeover to succeed is 1 and raider earns positive profits on the foothold, i.e.

$$\pi_h(v_h) = \alpha v_h$$

We have then showed that the only possibility for a pooling equilibrium is

$$q_l(\widehat{p}) \in (0,1),\ q_h(\widehat{p}) = 1$$

Notice that we can pin down the probability of success of the takeover for type θ_l directly from shareholders indifference condition, i.e.

$$q_l(\widehat{p}) = \frac{\widehat{p} - \lambda v_h}{(1-\lambda)v_l}$$

and this already provides us an interval for equilibrium prices, that is

$$\widehat{p} \in (\lambda v_h, \mathbb{E}(v)) \tag{1.18}$$

Profits of the raiders at the equilibrium tender offer then are

$$\begin{aligned} \pi_h(\widehat{p}) &= S_h v_h - \widehat{p}(S_h - \alpha_h) \\ \pi_l(\widehat{p}) &= \frac{1}{2} q_l(\widehat{p}) v_l - \widehat{p}\left(\frac{1}{2} - \alpha_l\right) \end{aligned}$$

Let's assume, in order to sustain such a pooling equilibrium, that

$$\mu(p) = 1 \quad \text{for } p \neq \widehat{p}.$$

It is straightforward to show that the high type of raider has no incentives to deviate. His tender bid is succeeding with probability 1, and he is paying a tender price that is strictly lower than his private valuation v_h. Moreover, decreasing the price, assumed post-takeover beliefs are such that the expected profits on the shares he buys are exactly equal to the tender bid, so that the expected payoff is linearly increasing in the tender offer.

As for the low type of raider, the first condition we need to impose is the rationality contraint, that is, type θ_l has to get positive profits at the equilibrium bid $\widehat{p}$, i.e.

$$\pi_l(\widehat{p}) = \widehat{p}\left[\alpha_l + \frac{1}{2}\left(\frac{\lambda}{1-\lambda}\right)\right] - \frac{\lambda}{1-\lambda}v_h \geq 0$$

that transaltes into a lower bound on equilibrium pooling prices, that is

$$\widehat{p} \geq \frac{\frac{\lambda}{1-\lambda}v_h}{\alpha_l + \frac{1}{2}\left(\frac{\lambda}{1-\lambda}\right)}$$

it can be showed that such a lower bound falls in the range of prices we have fixed in (1.18) for appropriate values of the toehold, that satisfy

$$\alpha_l > \lambda\underline{\alpha}\left(\frac{1}{2}\right)$$

The assumed out-of-equilibrium beliefs greatly simplify our work in sustaining the equilibrium price. Notice, in fact, that after any deviation small shareholders would assign probability 1 to the high type of raider, implying that expected profits for type θ_l would be negative. The only deviation we then have to check is the one to the highest tender offer v_h, in which the probability of success of the takeover jumps to 1. In such an instance, profits for type θ_l are

$$\begin{aligned}\pi_l(v_h) &= S_l(v_h)v_l - v_h\left(S_l(v_h) - \alpha_l\right) = \\ &= \alpha_l v_h - S_l(v_h)\left[v_h - v_l\right].\end{aligned}$$

A sufficient condition to sustain our pooling equilibria is then that type θ_l can not obtain positive profits by offering the high post-takeover valuation v_h. This, in turn, translates into a condition on the fraction of shares that the raider with the lower toehold would end up with after tendering v_h, that is

$$S_l(v_h) \geq \alpha_l\frac{v_h}{v_h - v_l}.$$

1.6.2 Market value as an incentive to takeover

We now set up an alternative modeling assumption: namely that the size of the toehold is inversely related to the potential market valuation of the target firm following a successfull takeover. In other words, we hypothesize that a raider who owns a small fraction of total shares of a target company is willing to takeover only if the prospective improvement he can bring on to the target firm is relevant. On the other hand, if the initial toehold hold by the raider in his portfolio is relevant, then even a lower post-takeover potential valuation may provide sufficient incentives for the raider to attempt to gain financial control. This intuition translates into the following assumption about the possible types of raider, in which we keep denoting θ_h the raider who has the larger fraction of shares,

$$\begin{aligned} \theta_h &: = (\alpha_h, v_l) \\ \theta_l &: = (\alpha_l, v_h) \end{aligned}$$

Recall that in our setting there is no dispersed information among shareholders, and that the only piece of information that is not public is held by the raider. Moreover, the only signal that small shareholders observe is the tender price offered by the raider. We keep mantaining the simplifying assumptions that a common prior exists, and we denote it by

$$\lambda := \Pr\{\theta = \theta_h\}$$

A crucial role will be played again by the beliefs that small shareholders have after observing the tender offer p made by the raider. We denote small shareholders' posterior beliefs

$$\mu(p) := \Pr(\theta = \theta_h \mid p)$$

The different assumption underlying this section of the paper drastically modify equilibrium characterization. As opposed to what we found in the analysis in the previous section, now it turns out that in all the equilbria of the game the two types of raider separate.

Proposition 14 *There exist no pooling equilibrium.*

Proof. Pooling equilibria with $\widehat{p} \leq v_l$. We will show that for such a pooling equilibrium to exist it would have to be the case that the probability of success for the raider with the lower initial toehold is strictly smaller than 1, and that this always give to type θ_l an incentive to deviate.

Suppose then that there exists a pooling price that is not above the lowest possible valuation, i.e. $\widehat{p} \leq v_l$, and that $q(\widehat{p}) = 1$. It is easy to reach a contradiction from here: in such a pooling equilibrium, and hence at price $\widehat{p}$, posterior beliefs have to agree with the prior and, using equation (1.16) we can note that

$$\begin{aligned}\mathbb{E}_{\mu}\left[v \mid \widehat{p}\right] &= \mu(\widehat{p})q_h(\sigma(\widehat{p}))v_l + (1-\mu(\widehat{p}))q_l(\sigma(\widehat{p}))v_h = \\ &= \lambda v_l + (1-\lambda)v_h = \mathbb{E}_{\lambda}\left[v\right] > v_l \geq \widehat{p}\end{aligned}$$

that imples that selling the share yileds a strictly lower expected payoff than keeping it. Thus, if such a pooling equilibrium existed, the probability of success of the takeover has to be lower than 1. This, in turn, implies that $q_l(\widehat{p}) < 1$.

Notice that $q_l(\widehat{p})$ can not be null in any pooling equilibrium, because this would imply that the raider with the highest valuation can not obtain positive profits in equilibrium. Recall, in fact, that such raider always has the opportunity to tender with the "safe offer", that is his private valuation v_h, and he would earn positive profits on the shares he owns before attempting to take over. Moreover, given that raider θ_l has the highest potential valuation, this is independent of posterior probabilities that small shareholders attach to each type of raider after any given price offer. In other words, for any out-of-equilibrium beliefs, the l-type of raider always has a safe option to ensure himself positive profits. We remark this more formally: equation (1.16) implies that, for any posterior beliefs $\mu(p)$,

$$\mathbb{E}_{\mu}\left[v \mid p\right] \leq v_h$$

Then, at the tender bid $p = v_h$ the probability of success of the takeover $q(v_h)$ has to be 1, and profits for the raider with the highest valuation, θ_l, are strictly positive, i.e.

$$\pi_l(v_h) = S_l v_h - v_h(S_l - \alpha_l) = \alpha_l v_h > 0 \tag{1.19}$$

Given that we ruled out $q_l(\widehat{p}) = 0$, we can use Lemma (7) to rule out also $q_h(\widehat{p}) < 1$. Suppose then that

$$q_h(\widehat{p}) = 1, \text{ and } q_l(\widehat{p}) \in (0,1)$$

We can use equation (1.16) and shareholders indifference condition to pin down the probability with which the takeover succeeds for raider θ_l, that is

$$q_l(\widehat{p}) = \frac{\widehat{p} - \lambda v_l}{(1-\lambda)v_h} \tag{1.20}$$

Now we can set a lower bound on the pooling price, that is $\widehat{p} > \lambda v_l$, and we are left with potential equilibrium prices in the range $(\lambda v_l, v_l]$. We can now calculate profits that are attainable by the l-type of raider: given that, as we have showed already, the takeover has to succeed for him with a positive probability that is strictly smaller than 1, he has to get exactly half of total shares of the firm, and his expected gains are

$$\pi_l(\widehat{p}) = \frac{1}{2}q_l(\widehat{p})v_h - \widehat{p}\left(\frac{1}{2} - \alpha_l\right)$$

It is easy to show that equilibrium profits for the l-type of raider are increasing in $\widehat{p}$, given that

$$\frac{\partial \pi_l(\widehat{p})}{\partial \widehat{p}} = \alpha_l \left[\frac{1}{2}\left(\frac{\lambda}{1-\lambda}\right)\right] > 0$$

and that therefore the highest profits that type θ_l can make are bidding v_l. Now it is enough to show that these profits are always less than those computed above in (1.19), that are obtained through the "safe bid" v_h, that is

$$\pi_l(v_l) = \frac{1}{2}\frac{v_l}{v_h}v_h - v_l\left(\frac{1}{2} - \alpha_l\right) = \alpha_l v_l < \alpha_l v_h = \pi_l(v_h)$$

so that we can rule out any pooling price that is not above the lowest valuation v_l.

Pooling equilibria with $\widehat{p} > v_l$ We will show that these prices can not be sustained in any pooling equilibrium, because we can not get rid of the incentives that type θ_h will have to deviate to lower prices.

We want to show that, for any out-of-equilibrium beliefs $\mu(p)$, type θ_h, that is the raider with the lower post-takeover valuation for the target, would always have an incentive to deviate from equilibrium behavior by decreasing the tender price. We know that, in equilibrium, the probability that the takeover succeeds for the h-type of raider has to be 1. Recall, in fact, that if this were not the case, then Lemma (7) implies that $q_l(\widehat{p}) = 0$ and we have showed at the beginning of this proof that this can not happen in equilibrium. Then, for any pooling price above v_l, profits of the high type of raider would be

$$\pi_h(\widehat{p}) = S_h v_l - \widehat{p}(S_h - \alpha_h)$$

Note that, in such an equilibrium, the raider is overpaying the shares he is buying on the market to get control of the target firm, and he is thus eroding the profits he makes on the shares he owns before the attempt to take over. Given that we have already argued that $q_h(\widehat{p}) = 1$, if we find a potential deviation to a lower tender offer $p' < \widehat{p}$ in which $q_h(p') = 1$ then we are done. If $q_h(p') = 1$, in fact, then it is obvious that type θ_h of raider has an incentive to decrease the price below $\widehat{p}$: if the probability is flat, and the price does not have a direct impact on the probability of success of the takeover, then expected profits are unambiguously strictly decreasing in the tender bid, and the raider would always benefit from offering a lower price.

We then analyze out-of-equilibrium implications of a deviation by the h-type of raider. At any posterior beliefs $\mu(p')$ shareholders can evaluate the expected gains from keeping their single share according to equation (1.16) as

$$\mathbb{E}\left[v \mid p'\right] = \mu(p')q_h(p')v_l + \left[1 - \mu(p')\right] q_l(p')v_h$$

If $q_h(p')$ were null, by Lemma (7) also $q_l(p') = 0$, and therefore aggregate probability of success $q(p') = 0$, that in turn would imply that selling the share is strictly optimal for any positive price offer p', contradicting the fact that $q(p') = 0$.

If $q_h(p') \in (0, 1)$ then Lemma (7) implies that $q_l(p') = 0$ and we can pin down the probability with which the takeover succeeds for type θ_h from shareholders indifference condition, that is

$$q_h(p') = \frac{p'}{\mu(p')v_l} \quad \text{for } p' \in [0, \mu(p')v_l]$$

This means that for $p' = \mu(p')v_l < \widehat{p}$ the probability that the takeover succeeds for type θ_h has to be 1. Moreover, the best choice we can make for out-of-equilibrium posterior beliefs in order to sustain the price $\widehat{p}$ in a pooling equilibrium is to set $\mu(p') = 1$, that is assigning probability 1 to the raider with the higher toehold. Even in this case, the h-type of raider can set the probability of success of the takeover to 1 simply tendering his private valuation of the target company after a successfu takeover.

It is easy at this point to show that profits that type θ_h can achieve in any pooling equilibrium we are assuming here are always strictly lower than those he can obtain by tendering his private valuation v_l, that earns him more than half of the shares with probability 1, i.e.

$$\pi_h(\widehat{p}) = S_h v_l - \widehat{p}(S_h - \alpha_h) < \alpha_h v_l = \pi_h(v_l).$$

■

Let me stress the intuition behind the result. The basic assumption of this part of the model is that the high-valuation raider has the smaller foothold in the firm. This implies that, if small shareholders assigned positive probability to the raider with the higher post-takeover valuation, then they would randomize in such a way to give him exactly half of the shares, and this, in turn, implies that type θ_h would obtain more than half of the shares and the takeover would succeed with probability 1 for him. In other words, the assumption that ties together the two sources of uncertainty does not allow, in this environment, to punish deviations by the raider with the larger toehold with a probability of taking over strictly smaller than 1, because this would imply, by Lemma 7 that the takeover fails for the more efficient raider.

We can then characterize all the equilibria of the game in the following result.

Proposition 15 *In all the equilibria of the game the two types of raider separate, and each one bids his private valuation, i.e.*

$$\begin{aligned} \widehat{p}_h &= v_l \\ \widehat{p}_l &= v_h \\ q_l(\widehat{p}_l) &= 1 \\ q_h(\widehat{p}_h) &= 1 \end{aligned}$$

provided that

$$\widehat{\sigma}(\widehat{p}_h) < \frac{\frac{1}{2} - \alpha_l}{1 - \alpha_l} \tag{1.21}$$

Proof. *We first show that there exists a separating equilibrium with the characteristics described in the claim.*

Suppose then that equilirbium behavior specifies for each raider to offer a tender price equal to his private information on post-takeover valuation of the target firm after a successful takeover, that is

$$\begin{aligned} \widehat{p}_h &= v_l \\ \widehat{p}_l &= v_h \end{aligned}$$

If the proposed prices can be sustained in a separating equilibrium, small shareholders would be able to distinguish between the two types of raiders, and thus, consistency of posterior beliefs with equilibrium behavior yields

$$\begin{aligned} \mu(\widehat{p}_h) &= 1 \\ \mu(\widehat{p}_l) &= 0 \end{aligned}$$

Moreover, equation (1.16) together with the consistency requirement on posterior beliefs yield that

$$\begin{aligned} \mathbb{E}_\mu \left[v \mid \widehat{p}_h \right] &= q_h(\sigma(\widehat{p}_h))v_l \leq v_l = \widehat{p}_h \\ \mathbb{E}_\mu \left[v \mid \widehat{p}_l \right] &= q_l(\sigma(\widehat{p}_l))v_h \leq v_h = \widehat{p}_l. \end{aligned}$$

The two conditions above clearly imply that, in equilibrium, the takeover has to succeed with probability 1 *for both types of raider, i.e.* $q_l(\widehat{p}_l) = q_h(\widehat{p}_h) = 1$*. If this were not the case, in fact, the tender price offered by each type of raider would be strictly higher than the conditional expected value of the share, thus implying that selling would be strictly optimal for small shareholders and that the takeover can not fail with positive probability.*

To summarize, in the separating equilibrium that we have postulated, each type of raider is paying the fair price for the shares he buys on the market, therefore gaining exclusively on the initial toehold he owns at the beginning of the game, before attempting to take over. Formally, equilibrium profits would respectively be

$$\begin{aligned}\pi_l(\widehat{p}_l) &= S_l v_h - v_h(S_l - \alpha_l) = \alpha_l v_h \\ \pi_h(\widehat{p}_h) &= S_h v_l - v_l(S_h - \alpha_h) = \alpha_h v_l\end{aligned}$$

We first need to set conditions on the parameters so that no type of raider has incentives to mimic the other type's equilibrium behavior.

It is easy to show that the h*-type of raider, who has the lower post-takeover valuation for the target firm, never has an incentive to mimic type* θ_l*, in fact*

$$\pi_h(\widehat{p}_l) = S_h^l v_l - v_h(S_h^l - \alpha_h) < \alpha_h v_l$$

where we denote S_i^j *the fraction of shares that the* i*-type of raider gets when he mimics raider* θ_j*, i.e.*

$$S_i^j = \alpha_i + \sigma(\widehat{p}_j)(1 - \alpha_i) \quad \textit{for } i, j = l, h$$

Notice that in this case, it is obvious that $S_h^l > \frac{1}{2}$*. Indeed, if small shareholders behavior when they observe the equilibrium price* $\widehat{p}_l$ *is such that, as we showed above,* $q_l(\widehat{p}_l) = 1$*, it has to be the case that the raider with the higher toehold,* θ_h*, is able to get more than half of the firm's total shares. Then, a fortiori, if type* θ_h *chose* θ_l*s equilibrium behavior, he has to get more than half of the shares*[7]*.*

On the other hand, there may be incentives for type θ_l *to pretend to be the raider with the lower post-takeover valuation. Indeed, we will show that if type* θ_l *is able to reach half of total shares of the target firm and to get control by tendering* $\widehat{p}_h$*, then he will always have incentives to do so. In other words, for the separating equilibrium that we are assuming here to exist, it has to be the case that the takeover attempt fails for type* θ_l *when he tenders* v_l*.*

[7] We can actually conclude more than that, that is $S_h^l > S_l > \frac{1}{2}$, but all we need is that S_h^l exceeds 50% of total shares.

Suppose, in fact, that small shareholders randomization strategy is such that the amount of shares that type θ_h is able to get at the tender price $\widehat{p}_h$ is high enough so that even type θ_l, who initially owns a smaller fraction of total shares, would be able to take over control of the target. If, in other words, $S_l^h > \frac{1}{2}$, or $q_l(\widehat{p}_h) = 1$, then our separating equilibrium breaks down, as

$$\pi_l(\widehat{p}_h) = S_l^h v_h - v_l(S_l^h - \alpha_l) > \alpha_l v_h.$$

Therefore, in order to have a separating equilibrium in which each type of raider offers his post-takeover valuation, we need to impose the condition[8]

$$S_l^h \leq \frac{1}{2}.$$

Recall that the fraction of total shares that raider i ends up with is obtained by shareholders equilibrium strategy. The latter condition thus boils down to a condition on the optimal strategy played by small shareholders, that has to satisfy

$$\widehat{\sigma}(\widehat{p}_h) < \frac{\frac{1}{2} - \alpha_l}{1 - \alpha_l}.$$

In order to fully characterize our separating equilibrium we thus only have to specify out-of-equilibrium beliefs that can sustain it. This can be done in several way; we specify

$$\mu(p) = \begin{cases} 1 & \textit{for } p \leq v_l \\ 0 & \textit{for } p > v_l \end{cases}$$

We analyze potential deviations by type θ_h from $\widehat{p}_h = v_l$, given the assumed posterior beliefs $\mu(p)$.

Decreasing the tender bid to $\underline{p} < \widehat{p}_h$ yileds a probability of success for the takeover that can be computed using equation (1.16) and indifference condition for small shareholders, that yields

$$q_h(\underline{p}) = \frac{\underline{p}}{v_l}$$

and this in turn imply that profits for the h-type of raider are

$$\pi_h(\underline{p}) = \frac{1}{2} q_h(\underline{p}) v_l - \underline{p}\left(\frac{1}{2} - \alpha_h\right) = \alpha_h \underline{p} < \alpha_h v_l$$

[8]When $S_l^h = \frac{1}{2}$ the l- type of raider will be exactly indifferent between following his equilibrium strategy or mimicking the behavior of the raider with the lower post-takeover valuation. As it is usual in the literature we break the indifference in favor of the equilibrium.

On the other hand, it is straightforward to see there can not be any incentive for the raider with type θ_h to offer any tender price $\overline{p} > \widehat{p}_h$, given that this would amount to overpay any share bought on the market at price $\overline{p}$, and thus eroding the profits he makes on his foothold.

As for type θ_l we only need to check that there are no incentives for him to decrease the tender price below $\widehat{p}_l$. Any price $p > v_h$ is in fact dominated by v_h: the probability of success is flat at 1 for $p \geq v_h$ and expected profits are strictly decreasing in the tender price.

We have already discussed the incentives for type θ_l to mimic type θ_h, and condition (1.21), together with out-of equilibrium beliefs, guarantees that no profitable deviation to prices $p \leq v_l$ exists.

For all prices $p' \in (v_l, v_h)$ assumed posterior beliefs are such that small shareholders assign probability 1 to raider θ_h, who has the highest post-takeover valuation, that is $\mu(p') = 0$. This implies that the probability that the takeover succeeds for type θ_l can not be 1, because, if this were the case, equation (1.16) would yield

$$\mathbb{E}_\mu [v \mid p'] = v_h > p'$$

implying that selling at price p' gives a strictly lower expected payoff than keeping the share, and thus that the takeover can not.succeed with certainty. The probability that the takover succeeds for the l-type of raider can be directly pinned down from small shareholders indifference condition, together with equation (1.16), obtaining

$$q_l(p') = \frac{p'}{v_h}$$

which in turn yields expected profits equal to

$$\pi_l(p') = \frac{1}{2} q_l(p') v_h - p' \left(\frac{1}{2} - \alpha_l \right) = \alpha_l p' < \alpha_l v_h.$$

■

1.7 Comments and Conclusions

The paper reports a complete theoretical characterization of agents behavior in a takeover game facing incomplete information both on the value that the target firm may attain after a successful takeover and on the foothold owned by the raider before the attempt to takeover.

We have fully characterized equilibrium behavior in a benchmark case with complete information, extending the classical free-riding argument affecting small shareholders' behavior developed by Grossman and Hart (1980)

[18], also to the setting in which the raider owns a fraction of the shares of the target firm before attempting to take over. moreover, we extended the latter result also to the case of asymmetric information on the size of the raider's toehold.

In section 1.5 we have introdiced uncertainty on the firm's potential market valuation following a successful takeover and we reported a full equilibrium analysis. In the case in which uncertainty purely regards post-takeover value, that is the one that has most extensively been studied in the literature, a multiplicity of equilibria arises. There exist pooling equilibria in which raiders tender with price offers either above and below the ex-ante expected value of the target firm. Aside, there exist separating equilibria in which the takeover succeeds with certainty for the raider with the higher valuation, and with positive probability strictly below 1 for the raider with the lower prospective valuation. This result is standard in the signalling literature: the raider with the higher valuation is able to fully appropriate the profits that he can make on his foothold, while the raider with the lower valuation has to forego some of his profits in order to separate himself and to pay a smaller tender price in equilibrium.

We have then developed a framework in which the two sources of uncertainty could be analyzed together. We have analyzed equilibrium behavior in two alternative environments: in the first one we assumed that the raider's prospective valuation for the target firm is positively related to the size of his foothold, while in the second one we postulated the opposite relation. We interpreted the first environment as one in which pre-tender share acquisition can be a strategic choice for the raider, who can increase his holdings in a target firm before attempting to take over in order to earn higher profits on the shares he does not need to pay the tender offer for. Alternatively, we thought about the second environment as one in which pre-tender share acquisition is not allowed for, and in which the potential improvement that the raider can bring to the target firm serves as an incentive for him to engage in costly takeover activities. The theoretical results show that the relation between the size of the foothold and prospective improvement in firm's market value plays a crucial role for the equilibrium analysis. Theoretical implications of the model are, in fact, radically divergent. Under the first assumption, the only equilibria in which the takeover succeeds with positive probability for both raiders are pooling equilibria. Conversely, the second assumption yields that the only equilibria of the game are those in which the raiders effectively separate, with each of them offering a tender price that is equal to his private information on post-takeover valuation.

The interaction between the two sources of uncertainty can be used to narrow down equilibria, and to make more accurate predictions of actual behavior. We think that the current analysis can be extended to further refine equilibria, and to pin down unique equilibrium outcomes, at least from a payoff equivalence perspective. Moreover, we claim that the present work can be used to draw some normative implications on the rationale that may stand behind international financial regulations pursuing transparency, although we refrain from doing this for the time being, and limit our discussion to the formal theoretical conclusions drawn so far.

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Chapter 2

An Electoral Model with Knightian Uncertainty

2.1 Introduction

In the present paper we present a novel application of ambiguity to the electoral competition, that lacks in the political economics literature.

The notion of ambiguity rose up in the first quarter of the 20th century in the pioneer works of Knight (1921) [34] and Keynes (1921) [31], and has thereafter permeated several fields of economics literature. As opposed to risk, ambiguity captures the level of confidence that a decision maker (DM) may have in his personal evaluation of probabilities. Whereas in many classical situations, e.g. a fair roulette wheel, objective probabilities exist and can reasonably be considered as fully describing the problem at hand, in the vast majority of situations such objective measures do not exist: think as an example to a simple bet on a soccer match (one that has not been set up), where personal assessments of the likelyhood of any outcome are the only instruments driving any decision, and are obviously crucial in determining an agent's behavior. Using Frank Knight's words

> "*the 'degree' of certainty or of confidence felt in the conclusion after it is reached cannot be ignored, for it is of the greatest practical significance. The action which follows upon an opinion depends as much upon the amount of confidence in that opinion as it does upon the favorableness of the opinion itself*"

The clearest and most famous example introducing the concept of ambiguity aversion appeared in Ellsberg (1961) [15], where the author set up thought experiments that brilliantly illustrated the problem we are discussing here. In his classical urn experiment a DM is asked to place a bet on the color of a drawn ball, choosing both the color on which to bet and the urn the ball will have to be drawn from: two potential urns are proposed, with balls of the same two colors, say red (R) and blue (B). One urn has a well known composition, say 50 red and 50 blue, and the other one is characterized by a composition of the same red and blue balls, whose fractions are unknown to the decision maker. It is natural to assign frequency probability of $\frac{1}{2}$ to either color being randomly drawn from the first urn. On grounds of symmetry and an application of the principle of insufficient reason, it may as well be tempting to assume the same probability distribution for the events R (a red ball is randomly drawn) and B (a blue ball is randomly drawn) also considering a random draw from the urn which composition is not known to the decision maker. There should in fact be no compelling reason to assess one of the two events as more or less probable. We would then end up with $p(R) = p(B) = 0.5$ for both the urns, even if it is evident that the "degree of confidence", to which Knight referred in the passage cited above, is obviously different, depending on whether the event R or B describe the outcome of the draw from the urn with the known or the unknown composition. The equivalent probabilities obtained can not be thought to yield equivalent behavioral predictions, neither descriptively nor normatively, and extensive experimental evidence has been acquired that actually confirms the intuition that DMs prefer to bet on the urn whose composition is known, that is on the urn on which the degree of confidence that can be attached to the subjective probability distribution is higher.

The aim of the current work is to apply the concept of ambiguity to a theoretical model of electoral competition. We argue that ambiguity can be an extremely intuitive explanation for a well-documented phenomenon in the political economics literature, that is the incumbency effect. It is well known, in fact, that an incumbent running for reelection faces a disproportionately higher chance of winning against an opponent, even on the grounds of a fair and free democratical election process. Gelman and King (1990) [21] provide statistical estimation of the incumbency advantage from 1900. They find a consistent advantage for incumbents in the House of Representatives and a steady increase of such an advantage through time, from few percentage points in the first decades of the last century, up to around 10% or more in the late decades of the century. Ansolabehere and Snyder (2004) [5] analyzed both state executive and legislative elections from 1942 to 2000, and documented a similar phenomenon, and the same persistently increasing trend over the years. A simple selection argument can be used to provide an intuitive explanation of this phenomenon. Incumbents are, by definition, those politicians who succeeded in winning at the previous election round and hence, as long as some of their characteristics are persistent through time, it may be argued that this yields as a natural consequence that they may be expected to be more successful than opponents[1]. However, several empirical works have advanced such an hypothesis, and proposed methods to deal with the selection issue. Levitt and Wolfram (1997) [36] decomposed the advantage enjoyed by incumbents into different components and found that higher average quality of the incumbents is not a major component behind their political advantage. Lee (2008) [35] compares incumbents who came from a close winning in previous elections to politicians who barely lose at the previous round, in order to evaluate whether there exist benefits for the incumbent when the race can be assumed to be among candidates of similar quality. To sum up, there exist vast consensus that incumbency has *per se* has a direct and positive effect on winning chances of the candidate who is running for reelection.

We argue that the incumbency effect can be a natural consequence of ambiguity aversion, and develop a simple model of electoral competition between two candidates. In order to provide the intuition of this insight without having to increase the complexity of the analysis we build the simplest model we can think of, in which we leave aside political ideology. In other words, we abstract from ideological motives behind voting, although in no way we propose that such motives do not matter. Leaving political patforms and ideological affiliation out of the picture simply allows us to construct a simple framework that is still able to provide the intuition behind our explanation. The only characteristic of the candidates that we take in consideration is what the literature refers to as *valence*, that is a quality indicator of the politician[2]. This feature of the candidates is orthogonal to the policy space, and is positively valued by all the electorate as a whole, independently from political ideologies. Valence is not observable by voters, and they do not have a common prior that describes the distribution of such parameter in the population of politicians. This is where ambiguity kicks in, and we show that ambiguity aversion leads to an incumency effect in an intuitive way. To characterize voters utility we use the α-maxmin expected utility (α-MEU) model, axiomatized by Ghirardato, Maccheroni and Marinacci (2004) [23], that represents an extremely tractable model that is able to introduce ambiguity, allowing for a variety of different attitudes for the individuals' preferences. Obviously, we do not take the stand that the explanation we propose excludes any other

[1] For a theoretical formulation of reasons for systematic difference see Eggers (2015) [14].

[2] For a recent example of a model including both valence and policy platform choices, see Ashworth and Bueno de Mesquita (2009) [8]. In their model, though, valence is costly.

competing one, but we argue that it is sound, albeit simple, and that it can definitely contribute by adding to existing explanations.

On the other hand we advance few observations on recent trends in political outcomes that seem to point in the opposite direction than the one we use to explain the incumbency effect. Namely, several recent political developments, worldwide, seem to go in the direction of higher ambiguity. The 2016 presidential election in the United States represents an example in which the electorate seems to have chosen the most ambiguous candidate, in the sense that we will clarify afterwords. The persistent rising approval among the electorate of 5-Stars Movement in Italy is another striking example that seems to contradict our assumption that voters are ambiguity averse. The latter political movement has in fact built its success by stepping away from classical and established politics, and proposing them as an alternative that is far from the classical political views or even methods. In both the cited examples, although we do not face an incumbent running for reelection, the common feature is the drastic departure from the previous legislatures. We will try to incorporate these recent trends in our framework in order to discuss how our model can capture the dynamics behind them, and try to provide some intuitive explanations of how these developments can be framed and understood in our ambiguous environment.

I will review in section 2.2 some of the literature that is more closely related to our work, and then introduce in section 2.3 the theoretical framework that we setup. In section 2.4 I propose a simple model of ambiguity aversion that is able to explain the incumbency effect. In section 2.5 I will propose some extensions of the theoretical framework in order to discuss political outcomes that go in the direction of higher ambiguity. In section 2.5 I will provide some additional comments and directions in which future research on the topic may go.

2.2 Related Literature

The literature on ambiguity and its applications is extensive, and it is impossible to appropriately cover even also its principal contributions. I will review some of the literature that is most closely related to the present work, for a more comprehensive recent survey on the topic see Machina and Siniscalchi (2013) [37] or Gilboa and Marinacci (2013) [24].

The way in which we introduce ambiguity aversion into our model, in the following section, is using the α-maxmin expected utility (α-MEU) model axiomatized by Ghirardato, Macceheroni and Marinacci (2004) [23]. This model has been proposed by Jaffray (1989) [28], who suggested to combine the famous α-criterion in Hurwicz (1951) [27] with a maxmin approach. We will present our model in the next section, and we just note here that this framework is generalization of the now classical maxmin expected utility (MEU) framework axiomatized by Gilboa and Schmeidler (1989) [25]. The simple and appealing maxmin criterion was proposed by Wald (1950) [50] (c.f. Milnor (1954) [39]), but it was Gilboa and Schmeidler path-breaking contribution that rendered it probably the most widely used tool to apply subjective expected utility theory. Whereas the framework we use allows us to consider diverse attitudes towards ambiguity, the MEU model obtains as a special case, assuming that agents are infinitely averse to ambiguity[3].

Applications are pervasive in all branches of economics and it worth mentioning at least a few of them. Dow and Werlang (1992) [11] apply ambiguity to asset pricing theory showing that when the decision maker is uncertainty averse, there exist a range of prices for which he may strictly prefer to abstain from the financial market rather than holding either a positive or a shorting position on a given asset, something that could obviously never obtain under classical expected utility theory. Epstein and Miao (2003) [16] use ambiguity aversion to explain why investors prefer to invest in their home country assets rather in foreign ones, i.e. the home bias puzzle. This preference may result from the intuition that investors may possess more information on their own country financial markets, and thus have greater confidence in their assessment of risk or, in other words, less ambiguity. In their experimental research Muthukrishnan, Wathieu and Xu (2009) [38] show that ambiguity aversion, characterized through lottery choices, is potively related to the tendency of customers to prefer famous established brands, even when the characteristics of the good are dominated by less established competitors' products. Nishimura and Ozaki (2004) [41] studied the different implications on workers job-searching efforts due respectively to an increase in risk and an increase in ambiguity: they show that theoretical predictions differ substantially, as an increase in risk induces an higher reservation wage, whereas greater uncertainty makes workers more willing to accept arriving job offers. Hansen and Sargent (2001) [26] apply uncertainty aversion to a macroeconomic framework, investigating the robustness of economic policy to variations in the underlying probability distributions, based on the idea that the policy maker does not have a unique prior, but can more realistically be assumed to have a set of possible priors.

[3]For another example of a theoretical model that allows to separate ambiguity (regarding beliefs) and ambiguity attitude see Klibanoff, Marinacci and Mukerji (2005). They show that their model essentially reduces to the MEU model as the degree of ambiguity aversion goes to infinity.

In the political economics literature, the concept of uncertainty has already been introduced, although the point we want to make with the present work is novel and has never been made. In his celebrated work An Economic Theory of Democracy, Downs (1957) [12] makes the point that political candidates have an interest in being ambiguous about the policy they favor, in order to attract more votes. The point made by Downs is further investigated and formalized in Shepsle (1972) [48], although it is worth underlining that, in these works, as in the vast majority of the ones we will mention here, ambiguity is not intended in the way in which we intend it here, but rather as risk. Building on the same idea Alesina and Cukierman (1990) [3] develop a model in which candidates deliberately choose not take a clear stand in their policy declaration, even when voters are risk averse. A similar conclusion is drawn in Aragonès and Postlewaite (2002) [7], in which they introduce ambiguity by the ability that candidates have to restrict voters' beliefs. Rather than on voters' risk preferences, alternative theoretical explanations of Downsian strategic ambiguity rely on the electorate's behavioral characteristics: in Laslier (2006) [33] ambiguity is introduced as a misperception that voters experience in assessing the policy position of each candidate, while in Callander and Wilson (2008) [10] politicians choose uncertain platforms in response to an electorate that displays context-dependent preferences á la Tversky and Simonson (1993) [49]. Aragonès and Neeman (2000) [6] model a two-stage game in which candidates first choose their platform and then their level of ambiguity, and they show that ideological differentiation allows the candidates to choose higher levels of ambiguity. Building on the latter model, Kartik, Van Weelden and Wolton (2016) [30] introduce the possibility of some policy-relevant information that is revealed to the elected candidate after the election: in their model representation causes the winning candidate to be overly ambiguous, and optimal platforms are (possibly degenerate) intervals. Alesina and Holden (2008) [4] construct a model in which voters are uncertain about the candidates' preferences, and both ideology and campaign contributions affect the probability of winning the election. They assume that there exist two types of candidates and allow them them choose intervals for policy platforms and assess the value of strategic ambiguity in the candidates' optimal policy choices. Several papers outside of electoral competition investigate policy choices when voters do not know the type of the decision maker, see Persson and Tabellini (2002) [45] for a survey.

Closer to the point we want to make in the following section, Caselli, Cunningham, Morelli and Moreno Barreda (2014) [9] build a signalling model, in which candidates' type belong to a set of three types and only the incumbent has the ability to signal his quality, in a costly way. In their model, signals sent out by incumbents cluster just above a threshold so that voters perceive more often an high quality incumbent, and this gives rise to an incumency effect, in the spirit of a perceived higher quality of senders as in Kamenica and Gentzkow (2011) [29], in the persuasion literature. In another signalling model, Morelli and Van Weelden (2013) [40] provide an analysis of how willingness to pander to the public opinion by the politicians is affected by asymmetric information between them and the voters. Acemoglu, Egorov and Sonin (2013) [1] use a signalling model to provide an explanation for populist choices, i.e. those that are harmful to the rich elite but do not favor the majority either. In their model such policies are chosen in order to signal an ideology that is in line with the median voter and not with the rich elite. Also Ogden (2016) [42] tries to explain political polarization, but in his model such differentiation stems from polarization in the electorate (imprecise) beliefs formation process. Frenkel (2014) [20] proposes a theoretical framework in which candidates differ in their quality, as it will be the case here, and make costly commitment choices to policy platforms, that impose a costly deviation from their declared policy after receiving new information, upon election. In his model, ambiguity refers to the level of commitment and, in the separating equilibria he finds, the higher the competence the lower will be the ambiguity surrounding the policy platform.

2.3 The Theoretical Framework

We want to develop a theoretical model in which ambiguity is introduced in the electoral competition.

We thus design a framework in which voters have to decide between two candidates, A and B. We will, for the time being, be silent on the politicians' side: we do not model the choices of the candidates running for election, and the only underlying assumption is that they care purely of being elected. Moreover, we don't provide voters the alternative of abstention, that we will add later on, adding a simple cost of taking the time to cast a vote for one of the running candidates. Therefore, for now, voters have to choose between one of the running candidates.

We have in mind a situation in which two candidates run for an office, and they are characterized by their *valence*, i.e. overall quality of the politician. This variable, on which we have already briefly commented in the previous sections, can be thought of as honesty, integrity or appriopriateness, in general, any characteristic that is hortogonal to the policy space and that is unambiguously appreciated by all voters, irrespectively of their political ideologies. Another way to think about θ_j, from a slightly more political point of view, would be to think of the ability of candidate j to take the appropriate policy choice, in any state of the world that may potentially arise, or as the candidate's competence in general. In our framework, θ_j will be unobservable, and the source of ambiguity.

In order to keep the model simple and, at the same time, to focus the attention on the characteristics we are interested in, we keep out of the picture ideological preferences that may direct votes towards one or the other candidate and, in particular, we take no stand in modeling strategic positioning of the candidates on any political issue. In contrast, we include a variabe, ϕ_i that is heterogeneous across voters, and captures the favor that voter i has towards candidate A rather than B. Utility of the voter thus depends on valence θ_j of the elected candidate and on the preference parameter ϕ_i. We assume that voter i's utility if candidate A wins the election is

$$u_i(\theta_A) = \theta_A + \phi_i.$$

We assume that ϕ_i is distributed according to a continuous density $g(\cdot)$, strictly positive on $\left[\underline{\phi}, \overline{\phi}\right]$ with $\underline{\phi}$ and $\overline{\phi}$ potentially being minus and plus infinity, and we denote $G(\cdot)$ the corresponding cumulative distribution function. The distribution characterizing ϕ_i is assumed to be symmetric, with both mean and median equal to 0. Note that these assumptions on the political preferences parameter reflect the fact that voters do not have an a priori bias towards any of the candidates. Recall, in fact, that the preference parameter ϕ_i summarizes the preference for one of the candidates, i.e. $\phi_i = 0$ means that voter i is indifferent between the two candidates, that amounts to say that this voter choice at the election would be the candidate with the higher expected valence. Later on, for intuitive reasons stemming from the hypothesized distribution of ϕ_i, we refer to such a voter as the median voter.

We assume that the valence parameter θ_j is unobservable. Moreover voters, besides not knowing the value of the valence parameter, do not have a prior probability distribution on the support of θ_j either. We assume then that politicians' valence $\theta_j \in \Theta_j$, and we introduce ambiguity by letting the voter have a set of probability distributions over Θ_j, that we denote S_j, that is assumed to be a convex and compact set of probability measures over Θ_j, i.e.

$$S_j \subseteq \Delta(\Theta_j)$$

We let voters attitude towards ambiguity to be flexible. We do not want to impose any given ambiguity aversion nor ambiguity love to voters' preferences, but we want to allow for some kind of ambiguity attitude. To do this, in the most tractable way, we use Hurwicz's α-maxmin expected utility (α-MEU) model, that was axiomatized by Ghirardato, Maccheroni and

Marinacci (2004). According to this, voters evaluate candidates' valence according to

$$\alpha \min_{p \in S_j} \int_{\Theta_j} \theta_j dp(\theta_j) + (1-\alpha) \max_{p \in S_j} \int_{\Theta_j} \theta_j dp(\theta_j)$$

where $S_j \subseteq \Delta(\Theta_j)$ is a convex and compact set of probability measures over Θ_j.

Note that this criterion allows for Gilboa and Schmeidler (1989) maxmin expected utility as a special case, as $\alpha = 1$. This model allows to distinguish in a simple way the source of uncertainty, that is related to the dimension of the set S_j and the attitude of the decision maker towards ambiguity, parametrized by α, i.e. the coefficient of ambiguity aversion. The latter measures, intuitively, the degree of the agent's pessimism and when $\alpha = 1$, as said above, the model collapses to Gilboa and Schmeidler MEU, corresponding to an infinitely pessimistic attitude of an individual who evaluates choices according to the worst possible scenario. On the opposite hand, when $\alpha = 0$, the model reduces to the extremely optimistic behavior of an agent whose decisions are based on the probability distribution that yields the highest expected payoff. When α is in the interior of the unit interval, the model is meant to capture the behavior of a decision maker whose attitude towards ambiguity is intermediate between extreme pessimism and extreme optimism.

2.4 A simple model for the incumbency effect

In this section we will make some additional assumptions on the structure of the theoretical elements that we have introduced in the previous section, in order to simplify the analysis and to describe the behavior that the model is meant to capture.

Suppose then that $\Theta_j = \{0, 1\} = \Theta$ for $j = A, B$. We take the valence parameter space to be binary, because this allows us to simply characterize the set of priors S_j by a 1-parameter representation[4]. We assume that the set of priors on the valence parameter spaces respect the following assumtion,

$$S_A \subset S_B. \tag{2.1}$$

This assumption is meant to characterize an higher amount of information available on candidate A rather than on candidate B. We will further comment on this later on in this work, but, for the time being, let us just say

[4] Notice that our assumption is tantamount to assuming the space Θ_j being an interval on the real line, and the set $S_j \equiv \Delta(\Theta_j)$.

that voters have a better knowledge of A's characteristics rather than B's. Given the simplified assumptions we made, we can think of each politician being either competent or not, with competence being unobservable to voters who, moreover, do not know the probability distribution over Θ, i.e. do not know the actual probability with which either candidate turns out to be competent. Nevertheless, the set of probabilities on Θ is smaller for candidate A rather than for candidate B. This implies, in turn, that the minimal probability that voters can attach to A being competent is always strictly larger than the corresponding probability on candidate B. On the other side, the highest probability attached to a competent politician, is strictly higher for B rather than for A.

We can propose at this point a natural example of a situation that our model is meant to capture. We can think of candidate A as an incumbent, who has been sitting in an office for some time before an election, and of candidate B being the challenger, that runs for election aginst the former. Our assumptions are meant to characterize the fact that voters have greater information on the incumbent, simply because he has already occupied the position and they may have observed actual behavior for years. In particular, we will start with the extreme assumption that uncertainty only regards the challenger, candidate B. In other words, voters know exactly the probability that they attach to the incumbent being competent, while they are uncertain about what the latter probability may be for the challenger. We then assume, in our notation, that the set S_A is a singleton, and, in particular we denote

$$p_A := \Pr\left[\theta_A = 1\right].$$

On the other hand, voters do not know the probability distribution over Θ, and the set of priors S_B is assumed to be

$$S_B = \left\{p \in \Delta(\Theta) : p\left[\theta_B = 1\right] \in \left[\underline{b}, \overline{b}\right]\right\}.$$

It is easy to see how our simplifying assumption on a binary parameter space allows us to simply characterize probability distributions on Θ by using the probability attached to the elected politician being competent. We can as well define the uncertainty on candidate B as the dimension of the set S_B, that, in this setting simply corresponds to the lenght of the interval to which the probability attached to the candidate B being competent belongs to, i.e.

$$d_B = \overline{b} - \underline{b}.$$

The latter measure is useful in our simple setting because it allows us to characterize the dimension of the uncertainty regarding candidate B, with

$d_B \in [0,1]$. A null dimension corresponds to the case in which there is no ambiguity, and the set S_B collapses to a singleton (such as S_A in this simple case); the higest level of ambiguity, for which $d_B = 1$, corresponds instead to the case $S_B \equiv \Delta(\Theta)$.

In this simple case, voter i's utility if candidate A wins the election is simply

$$u_i^A = \mathbb{E}(\theta_A) + \phi_i = p_A + \phi_i$$

while his utility from candidate B is

$$u_i^B = \alpha \underline{b} + (1-\alpha)\,\overline{b}.$$

Note that our assumptions on ϕ_i allow us to characterize the outcome of the electoral process simply looking at the median voter, i.e. the voter with neutral idelogical preferences, that is $\phi_i = 0$. Given that ϕ_i is assumed to be symmetrically distributed, in fact, if the median voter prefers candidate A then this implies that at least 50% of the voters (those with $\phi_i > 0$) prefer the incumbent as well.

In order for the model to yield interesting predictions we assume that the set of priors on the challenger's valence parameter include the prior on the incumbent. Notice, in fact, that if this were not the case, then it is easy to see how the immediate implications would be for the incumbent either to always win (if $a > \overline{b}$) or to always loose (if $a < \underline{b}$). Assume then that

$$p_A \in \left(\underline{b}, \overline{b}\right). \tag{2.2}$$

We argue that an incumbency effect naturally arises, and to see this, it is enough to look at the median voter and calculate that he prefers the incumbent over the challenger as long as

$$p_A > \alpha \underline{b} + (1-\alpha)\,\overline{b}$$

or

$$\alpha > \frac{\overline{b} - p_A}{\overline{b} - \underline{b}}. \tag{2.3}$$

Note that the assumption on p_A made in (2.2) guarantees that the lower bound on α in (2.3) is strictly between 0 and 1. What the latter condition means is that for sufficiently high levels of ambiguity aversion of the electorate, the incumbent will be predicted to obtain more than 50% of the votes. Clearly, the higher the probability attached to the incumbent being competent, the lower the level of ambiguity aversion that is needed in order for the incumbent to be elected. On the other hand, the challenger may benefit from any increase in the perceived probability of him being competent:

both an increase in the lower bound $\underline{b}$ and an increase in the upper bound $\overline{b}$ have in fact a positive effect on the lower bound on α in (2.3), impying that an higher level of ambiguity aversion is needed for the incumbent to have an advantage.

It is easy to generalize the result we have just obtained to a slightly more general environment. We relax the assumption that voters know the probability distribution over the space of the valence parameter for the incumbent and, more realistically, assume that ambiguity affects both candidates. However, we maintain the assumption made in (2.1) reflecting the higher familiarity with the incumbent rather than with the challenger: we assume then that voters are more aware of what the worst case and the best case scenario may be for candidate A, while they may expect either more or less out of candidate B. Using the notation we have developed so far, we assume that our sets of priors are as follows,

$$\begin{aligned} S_A &= \{p \in \Delta(\Theta) : p[\theta_A = 1] \in [\underline{a}, \overline{a}]\} \\ S_B &= \left\{p \in \Delta(\Theta) : p[\theta_A = 1] \in \left[\underline{b}, \overline{b}\right]\right\} \\ 0 &\leq \underline{b} \leq \underline{a} \leq \overline{a} \leq \overline{b} \leq 1. \end{aligned}$$

The dimensions of the two sets S_j, as defined above, would in this case be

$$\begin{aligned} d_A &= \overline{a} - \underline{a} \\ d_B &= \overline{b} - \underline{b}. \end{aligned}$$

We can now obtain a threshold on the level of ambiguity aversion, such that the incumbent runs as a favorite enjoying what the literature refers to as the incumbency advantage.

Proposition 16 *There exist an incumbency advantage, if and only if*

$$\alpha > \frac{\overline{b} - \overline{a}}{d_B - d_A}.$$

Proof. *Notice, first, that, in our setting, proving the existence of an incumbency advantage is equivalent, given the assumed distribution for ϕ_i, to prove that the voter who has neutral political preferences, i.e. $\phi_i = 0$, prefers the incumbent over the challenger.*

For the median voter, the comparison between the candidates reduces to a comparison between expected valence of each one of them, that is he will stricly prefer candidate A if and only if

$$\alpha\underline{a} + (1-\alpha)\overline{a} > \alpha\underline{b} + (1-\alpha)\overline{b}$$

that can be rewritten as

$$\overline{a} - \alpha l_a > \overline{b} - \alpha l_b$$

or

$$\alpha > \frac{\overline{b} - \overline{a}}{l_b - l_a}.$$

■

Note that the threshold on the coefficient of ambiguity aversion is always between 0 and 1, given our assumptions on the sets S_j for $j = A, B$.

The result states that if the electorate has homogeneous attitude towards ambiguity, and, in particular, if voters are sufficiently ambiguity averse, then an incumbency advantage will naturally emerge. Similar considerations to the ones made above for the simplest case in which S_A was a singleton hold in this case as well.

We provide a simple example that may be useful in stressing the dynamics of this simple model: in Figure 1 we depict expected utility of the voter with $\phi_i = 0$ (the dark line in the graph) in a specific example in which the two sets S_j for $j = 1, 2$ are both centered around 0.5.

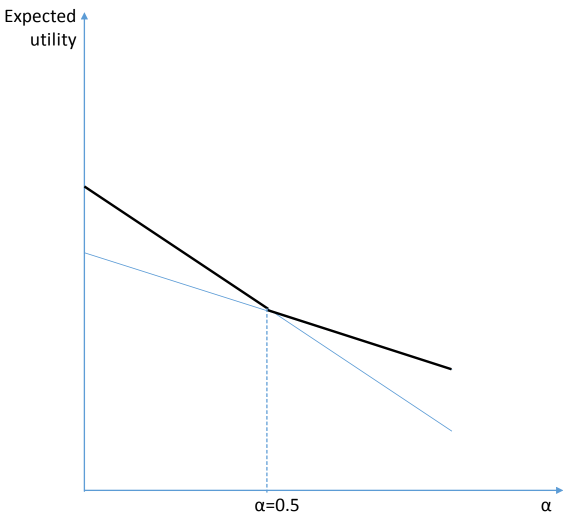

Figure 2.1: The figure reports the α-MEU of the median voter as a function of the parameter of pessimism α. Both $[\underline{a}, \overline{a}]$ and $\left[\underline{b}, \overline{b}\right]$ are centered on $\frac{1}{2}$ and $\widehat{\alpha} = \frac{1}{2}$.

In the simple case in which the intervals to which the probability attached to the competent type of politician belongs are both centered around $\frac{1}{2}$, the threshold on the coefficient of ambiguity aversion α is exactly 0.5. To the left of this point the voter is sufficiently optimistic to prefer the challenger over the incumbent, while to the right ambiguity aversion leads the voter to prefer candidate A, i.e. the incumbent.

Moreover, we can use the model to characterize voters' behavior and obtain a prediction of the electoral outcome that can be summarized in the following proposition, in which we denote W_A the fraction of votes for candidate A.

Proposition 17 *The fraction of votes for the incumbent is given by*

$$W_A = 1 - H^{-1}(\overline{b} - \overline{a} - \alpha\,[d_B - d_A]).$$

Proof. *Define ϕ_s to be the swing voter, i.e. the voter that is exactly indifferent between the two candidates. We can then characterize this swing voter by*

$$\alpha \underline{a} + (1-\alpha)\overline{a} + \phi_s = \alpha \underline{b} + (1-\alpha)\overline{b}$$

from which we can derive

$$\phi_s = \overline{b} - \overline{a} - \alpha(d_B - d_A)$$

From our assumptions on the distribution of the parameter ϕ_i we can thus conclude that all voters for which $\phi_i > \phi_s$ strictly prefer the incumbent over the challenger, i.e.

$$W_A = 1 - H^{-1}(\phi_s).$$

■

Notice that the last result is severly related in an obvious manner to the previous one: the lower bound on the coefficient of ambiguity aversion α found in (2.3) in fact correspond to a negative value for the political preference parameter ϕ_i, that means that the voter who is indifferent between the candidates has a political ideology that is closer to the challenger. This, in turn, implies that more than half of the electorate will vote for the incumbent.

2.5 Extensions and to do list

The simple model developed in the previous section provides an interesting starting point to analyze electoral competition adding a novel component that may provide new interesting insights. As for the simple version presented in section 2.4, it already provides a natural intuition that may help explaining the incumbency effect widely observed in the political economics literature.

The theoretical setting of ambiguity appears to be adapt to describe political dynamics such as the ones considered here, that are relative to the electoral process. The simple model proposed here has the advantage of separating agents attitude towards uncertainty, captured by the pessimism parameter α, and the dimension of uncertainty, that is given here by the dimension of the sets S_j for each of the candidates. This feature is particularly useful in describing a political model, in which both preferences and beliefs can reasonably be assumed to play a crucial role in agents' choices. A very natural extension that could be made is developing a more general theoretical framework applying the smooth ambiguity model developed by Klibanoff, Marinacci and Mukerji (2005) [32]. Similarly to the α-MEU model that we used here, also their model allows to conceptually separate ambiguity characterizing beliefs from ambiguity attitudes characterizing preferences. This may allow to address some initeresting comparative statics exercise in which we may address changes in behavior due to change in voters' preferences or to changes in the available information.

While the key role played by preferences is pervasive in all economic literature, and thus needs no remark here, we can mention some additional consideration regarding voters beliefs. A copious literature has dealt with the high degree of heterogeneity across elections and across voters. For example, media coverage or propaganda is generally higher for national elections rather than for local ones, and voters are known to pay diverse amounts of attention to news or political debates. An interesting model that differentiates voters' interest in politics and their attention is built by Prato and Wolton (2016) [46], who show that, interest and attention are not always positively correlated. In particular, in their model, an higher interest in politics may determine a change in strategic behavior of the candidates that results in uninformative campaigns and, hence, in lower attention paid by voters in equilibrium. A first direction in which we aim to extend the current framework is towards modeling the candidates' side, on which we remained silent here, including political strategic choices on a given set of platforms. We can introduce some instruments for candidates in order to influence the dimension of the information sets S_j. Those measures may be assumed to be costly for the politicians or we may assume that information gathering is costly for the voters.

Another simple extension that may provide new interesting insights may be the introduction of a simple cost of voting. The political literature has extensively debated on voting behavior, and, in particular, on how this may be affected by voting costs[5]. A reviews on both the empirical and the theoretical literature can be found in Aldrich (1993) [2]. Feddersen and Pesendorfer (1996) [18] build an asymmetric information model in which voters have state-dependent preferences and (potentially large) abstention may arise in equilibrium, because less informed voters have an incentive not to vote. In Feddersen and Pesendorfer (1999) [19] they further extend the previous model to heterogeneous preferences and study participation when information and preferences are correlated. Also Ghirardato and Katz (2004) [22] focus on voter participation, and they explain abstention using a decision-theoretic approach, that is closer to the one we have in mind here. They use Gilboa and Schmeidler MEU model and obtain that abstaining may be the best choice when voters perceive both candidates as ambiguous and, moreover, they are perceived to be "ambiguity complements", that is each of them is the best choice under some potential future scenario. When this is the case, abstention may be strictly prefered to voting for each of the candidates, as it allows voters to hedge against ambiguity. With the set up developed so far we can already intuitively show that, for a fixed level of ambiguity aversion α, voters with stronger political preferences would be the ones who decide to vote, while voters whose parameter ϕ_i is closer to 0 may find it optimal to abstain. Perhaps more interestingly, for a given level of ideological preference ϕ, voters participating in an election would be the most ambiguity averse and the most ambiguity loving, i.e. those for which the difference in expected utility from the two candidates is the highest. Abstracting from the probability that voters may attach to the possibility of being pivotal, in our framework voter i strictly prefers to vote for one of the candidates as long as

$$\begin{aligned} \left|\alpha\underline{a} + (1-\alpha)\overline{a} + \phi_s - \alpha\underline{b} - (1-\alpha)\overline{b}\right| &> c \\ \left|\alpha\left(d_B - d_A\right) + \overline{a} + \phi_i - \overline{b}\right| &> c \end{aligned}$$

where c is assumed to be a fixed cost of showing up at the pooling station and casting a vote. Depending on the level of uncertainty that can be attributed to the candidates and on voters preferences, some intriguing theoretical prediction may arise, that would be natural and very interesting to test empirically.

Some last remark that we want to make regard the recent political developments that seem to go in the direction of higher ambiguity. We briefly mentioned two of them in the final part of the introductive section, namely the 2016 US presidential election and the raising consensus that the 5-Stars political movement is obtaining among the public in Italy. From the point

[5]See, for example, Riker and Ordeshook (1968) [47], Palfrey and Rosenthal (1983, 1985) [43], [44] or Feddersen (1992) [17].

of view that we have taken in this theoretical framework, an electoral outcome such as the election of Donald Trump as the President of the United States may be seen from several perspectives. Notice that, even if we did not face an incumbent running in the 2016 presidential campaign, we claim that the assumptions that we have proposed in section 2.4 can reasonably reflect american voters' beliefs. It was evident to all that Trump's presidential campaign was mainly focused on stepping as far as possible from the previous administration. The first days of the new elected President have indeed confirmed such perspective. From this point of view, our theoretical assumptions, reflect the fact that prospective outcomes attainable by Trump are more dispersed than those that would have been attainable by Hilary Clinton, who would have proceeded in some way closer to the Obama administration. In other words, drastic changes yield potentially more dispersed outcomes, and this is the idea behind the assumption we made in section 2.4 in (2.1). Under this viewpoint, it may be interesting to address the empirical question of how ambiguity attitudes may have changed through time or how the active electorate may have changed.

We are aware that the perspective that we propose here is far from being the sole explanation behind voters' behavior, and that it is important to consider the interaction of different theoretical motivations. A risk component may be introduced, so that agents preferences would depend of both ambiguity and risk attitudes. Such a distinction can be relevant, also in the aforementioned case of the US presidency. A different perspective may be to allow for heterogeneous ambiguity attitudes depending on the type of politician: voters may be more pessimistic in evaluating uncertain candidates' valence if they have a political background as opposed to candidates who come from the private sector and are perceived as successful entrepreneurs. Emotions, about which we will discuss in Chapter 3, are likely to be another key component of voters' behavior.

To conclude, we hope to have the opportunity in the future to develop the ideas outined in the present work. This will most likely be done in two main directions: the first one is incorporating novel features in order to develop a richer theoretical framework, the second one is identifying interesting empirical questions that may help clarifying the role of uncertainty on voters behavior.

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Chapter 3

A Model of Lying and Guilt Aversion

3.1 Introduction

This work contributes to the literature on psychological games, introduced by Pearce, Geanakoplos, Pearce and Stacchetti (1989) [19], in which payers' utility does not depend only on what each player does, but also on what he believes other players think, or what he believes other players believe about others, and so on and so forth. In other words, material payoffs, or terminal histories in general, are not the only characteristics of the game that affect players' utilities[1]. This kind of literature allows to consider the impact of emotions such as anger, envy, or reciprocity, and the role that such emotions can play in motivating agents' behavior. It may be argued that payoffs attached to each terminal history for all players in a given game should already incorporate this kind of emotional effect on players utility, and that the issue would then simply be to characterize and define the appropriate utility function. As this explanation may sound reasonable in many simple examples, it does not allow to formally analyze the behavioral motivations by which emotions can affect behavior, besides the fact that incorporating emotional payoffs into players utility could represent a rather challenging and subjective task.

The most famous example of psychological motives for behavior is probably intention-based reciprocity, introduced by Rabin (1993) [26]. In his work

[1]Although in the pioneer framework developed by Geanakoplos, Pearce and Stacchetti (1989) [19], agents payoffs depend on strategies only through terminal histories. We will briefly discuss more recent extensions later in this work, for more detailed comments see Battigalli and Dufwenger (2009).

on fairness he underlines that people like to help who is helping them and to hurt who is hurting them. Tipping behavior in a restaurant provides a classical everyday example of a psychological motive: the act of leaving a tip to the waiter can simply be seen as a rewarding activity for the care with which we have been treated, but more profound psychological reasons can justify this behavior. We are in fact inclined to do so, simply because the waiter expects us to, or, in general, society does. If we travel around and we know that in some country it is customary to tip a given percentage of the bill we get, then we are emotionally pushed to behave accordingly, as not to disappoint what workers expect to receive from us. Interesting results ragarding this example appeared already in Lynn and Latané (1985) [24] who reported empirical results showing that tipping behavior was not related to neither food quality, restaurants' service level, or to the place's general atmosphere. More recently, Parrett (2006) [25] uses both field surveys and laboratory experiments in order to show a similar pattern of behavior, and he proposes, among possible explanations, both reciprocity and "letdown (guilt) aversion". Another classical example of such a psychological motive is represented by pacts: promises represent a form of commitment, even if it may be seen as cheap talk, as the more one believes that others trust a promise the higher is the emotional cost associated with breaking such a promise. Such an interpretation can be found, for example, in Charness and Dufwenberg (2006).

In this work we consider the experimental setting designed by Gneezy (2005) [18], and explain subjects' observed behavior with a theoretical mdoel that includes two emotion-based motivations, namely guilt aversion and lying aversion. We will report a more detailed description of the experiments he designed in the following part of this work, and report only the basic ideas here. In his experimental work, Gneezy set up two games between two players: a deception game and a dictator game. In the former, one player is asked to choose an action that will determine material payoffs for both players. The co-player, by the way, is the only one that is aware of terminal payoffs, but he does not have the opportunity to choose between actions, but only to send a message to the other player, with a suggetion on the action to take. Material payoffs are not aligned, meaning that in each treatment of the experiment, the action that earns more money to the sender of the message is the action that earns less money to the receiver of the message, i.e. the player who has to choose the action to take. Gneezy designed three different treatments of this game, modifying terminal payoffs assigned to each player. Besides the fact of always being misaligned, terminal payoffs vary in order to see how differences in gains and losses that can be produced by lying affect behavior. In particular, in one treatment the difference in payoffs is small, so that a small gain for one player corresponds to a small loss for the other; in the second treatment the sender can gain a slightly higher payoff only at the expense of a large loss for the receiver, while in the third treatment a relevant gain for the sender corresponds to a relevant loss for the receiver. In the dictator game, instead, there is only one active player and the structure of the payoffs is the same as the one designed for the deception game. The active player is now the only informed player, and he takes an action that determines the terminal history of the game and payoffs to both players. The same logic depicted above on gains and losses applies also in this case, so that the active player can increase his final payoffs only at the expense of the other player losing. However, in this variation of the game, there is no deception involved. Clearly, Gneezy's idea was to use such a dictator game as a series of control treatments, in which deception palayed no role, and address the difference in behavior in the two games, that turned out to be significant.

The intuition behind incorporating both guilt aversion and lying aversion in a theoretical model that describes players' behavior in these games should be clear. In both games players suffer from a guilt component, in a way in which they feel they let down the other player's expectations. On the other hand, in the deception game there is an additional component that may motivate behavior, namely attitude towards lying. Battigalli, Charness and Dufwenberg (2013) [3] developed a theroretical model that explained part of Gneezy's results according to the theory of simple guilt that is developed in Battigalli, Dufwenberg (2007) [4]. The basic idea behind simple guilt is that a player suffers from guilt as long as he believes he let down others' expectations. The idea is clearly reminiscent of the simple example provided above about tipping behavior: agents are induced to tip, because they believe that it is what others expect from them. Although the model of simple guilt is well suited to explain the behavior in the deception game, it does not address the divergent behavior that is observed in the dictator game. It may be argued that in the two games the role played by guilt may indeed be different, and we will comment on this later in this work. Anyway, such a difference may not be enough to justify the significantly different behavior of the players in the two games. In order to address this issue, we introduce disutility from lying, that is a cost that is associated with the act of lying *per se*. Clearly, this emotional component will have a role in the deception game in which the sender has the opportunity to lie in order to induce a favorable outcome, while will play no role in the dictator game, in which this chance of lying is not given to the players. To be more precise, in the dictator game, behavior will be determined by material payoffs and by guilt aversion, while in the deception game we add a disutility component that is attached to lying behavior. We thus want to extend the model developed by Battigalli, Charness and Dufwenberg (2013) [3] to include lying aversion, analyze players' behavior under the new theoretical framework, and address the question whether our model can reasonably explain the pattern found in experimental data by Gneezy (2005) [18].

We will show that the model developed here does indeed provide a theoretical foundation of the results reported in Gneezy (2005) [18], and thus that players behavior can be explained adopting the proposed emotional-based motives. In section 3.2 I will review some of the literature that is related to the current work and then, in section 3.3 present a description of the experiments set up by Gneezy. In section 3.4 I present the theoretical framework, introduce the notation and the main assumptions and then develop a theoretical description both of the deception game and the dictator game. In setion 3.5 I will present some additional comments and conclude.

3.2 Related literature

As pointed out in the Introduction, the current works adds on the literature on psychological games pioneered by Geanakoplos, Pearce and Stacchetti (1989) [19]. They inroduce a novel framework to incorporate players' believes as motives behind their behavior, and provide several examples of games in which this may be appropriate. An even more general framework has been developed by Battigalli, Dufwenberg (2009) [5]: they build on Battigalli and Siniscalchi (1999) [6] who presented formally hierarchies of conditional beliefs, and introduce some additional components to the framework developed by Geanakoplos, Pearce and Stacchetti, such as ways of incorporating other players' beliefs, or beliefs that go towards some updating during the game, or the dependence of players' utility on strategies, not only through terminal histories, but also through plans of action. The literature on psychological games, although somehow recent, is relatively extended, both from a theoretical and an experimental point of view. I will mention here some of the works that are more closely related to the ingredients of the present work, for a more general survey see Elster (1998) [14] or Attanasi and Nagel (2008) [2].

One of the best known example of psychological application of Geanakoplos, Pearce and Stacchetti's framework is the work on fairness by Rabin (1993) [26]. He constructs fairness equilibria, assuming that people are willing to give up on their selfish material interests in order to help (punish) other people who have been kind (unkind) in their regards. Dufwenberg and Kirchsteiger (2004) [12] build on Rabin's theoretical model and extend it to a dynamic setting, introducing dependence on beliefs that are updated throughout the game, unlike in Rabin (1993).

In his experimental paper Gneezy (2005) addresses some of the theoretical explanations that may be used to explain the observed behavior. One of this is older than the cited literature on psychological games and is related to models of social preferences, in which the basic assumption is that people are not only concerned about their own interests, but also about others' well-being. An early example of this idea is the pure altruism motive, modeled in Becker (1976) [7]. In his framework agents' utility is assumed to depend both on own consumption and on others' consumption. Such a framework has been later modified by Andreoli (1990) [1], who introduced the notion of "impure altruism", according to which agents not only care about aggregate consumption, but aslo on their personal contribution to it. Other more recent examples can be found in Fehr and Schmidt (1999) [16], who assume that agents care about equity in addition to their individual payoffs, and in Bolton and Ockenfels (2000) [8], who applied a model of equity and reciprocity to various games, such as ultimatum and dictator games, prisoner's dilemmas and gift exchange, and Bertrand market games. Charness and Rabin (2002) [10] test experimentally different theories of social preferences and found that people care about social welfare and, moreover, they found additional evidence of reciprocal behavior. Another example, that goes in the opposite direction, is the model in Kirchsteiger (1994) [21], in which players are assumed to feel envy and this is used to assess different experimental findings.

A different explanation is assuming that people differ in their honesty and they are either honest or they are not. Koford and Penno (1992) [22] categorize agents as "economic", who are always willing to lie to maximize their private interest, or "ethical"[2], who always tell the truth. This pretty severe assumption is tested by Hurkens and Kartik (2009) [20] who designed similar experiments to those in Gneezy (2005) precisely to assess the simple hypothesis that the world may be populated by types *à la* Koford and Penno: they argue that they can not reject this hypothesis, while they confirm evidence of lying aversion behavior among the subjects. Moreover, Gibson, Tanner and Wagner (2012) [17] found experimental evidence of heterogeneity in lying aversion both among subjects and within individuals.

In Battigalli and Dufwenberg (2007) [4] two models of guilt are presented, simple guilt, in which a player's utility loss from guilt derives from letting down the others, and guilt from blame, in which disutility from guilt arises

[2] This is Gneezy's terminology, see Gneezy (2005), pag. 391.

from blame from others, or from how much the others believe that a player intentionally let them down. In Battigalli, Charness and Dufwenberg (2013) [3] the former model of simple guilt is adapted to discuss the experimental data in the deception game in Gneezy (2005) [18], and this is the model we want to extend to incorporate lying aversion and address both games designed by Gneezy in his experimental sessions. As pointed out also in Battigalli, Charness and Dufwenberg (2013), guilt from blame is irrelevant in Gneezy's experimental design in which, as we'll see, player 2 is unaware of the dynamics of the game, and can not reasonably assess player 1's intentions.

Lying aversion has been experimentally studied by Erat and Gneezy (2009) [15]. They analyze both altruistic lies, that, as in the experiment we will describe in the following section, help the other party hurting the liar, and "white lies" that, in their terminology, are beneficial to both the liar and the listener. They find that, even when lying yields a Pareto superior outcome, a significant fraction of the subjects abstain from lying, thus suggesting the existence of an intrinsic aversion to lie. Similar considerations can be found in Lundquist, Ellingsen, Gribbe and Johannesson (2009) [23] who designed experiments with different forms of communication, finding that lying is less attractive the bigger the size of the lie, that is the farther the lie is from the truth, and the larger the strength of the promise. Promises and threats are studied in Ellingsen and Johannesson (2004) [13], in which they show that the former are more credible than the latters. The role of communication was stressed already in Dawes, McTavish and Shaklee (1977) [11], in experiments involving cooperating versus self-interested behavior. Promises are central also in Charness, Dufwenberg (2006) [9], who provide further experimental evidence that communication between players significantly increases the amount of cooperation, and they explain such effect through guilt.

3.3 Gneezy's experiment

Gneezy (2005) designed a simple experiment to investigate empirically the incentives to lie in order to obtain an higher material payoff. In particular he is interested in the impact of different material consequences of lies on behavior, namely the propensity to lie.

The set up of the experiment is extremely simple, and we summarize it here for completeness of exposition. The first type of experiment that was conducted is a "cheap talk sender-receiver" (CTSR) game. He divides the population participating in each experimental session into two equally large subpopulations of senders and receivers: the firsts send a cheap talk message that entails no material consequences, while the seconds take an action that ends the game and assigns material payoffs to both players. Only the sender has the information regarding final payoffs of the game, while the only information that the receiver has throughout the game is the message sent by the sender. The only two actions that the receiver can take are labeled A and B. At the first stage of the game, the sender has only two available messages that he can send to the receiver, that are, again, labeled as A and B, and are the following

Message A : *"Option A will earn you more money than option B"*
Message B : *"Option B will earn you more money than option A"*

In other words each one of the two messages correspond to a suggested action by player 1, the sender, to player 2, the receiver. After having received one of the two messages, the receiver is asked to choose an action, either A or B, without knowing anything about the material payoffs corresponding to each action (such information is known only to the sender). Gneezy conducted three different treatments of the just described experiment, in which he modifies material payoffs assigned to both players in order to study the impact on behavior of such modifications. Material payoffs are described in Table 3.1.

		Payoff	
Treatment	Action	Player 1	Player 2
1	A	5$	6$
	B	6$	5$
2	A	5$	15$
	B	6$	5$
3	A	5$	15$
	B	15$	5$

Table 3.1: Payoffs used in Gneezy's deception game

Common to all treatments is the full misalignement of payoffs: action A consistenly yields an higher payoff for player 2 (the receiver), while action B consistently yields an higher payoff to player 1 (the sender). Therefore, in all treatments, message A is true, while message B is a lie.

The results report evidence that the material consequences of lying do indeed have a substantial impact on lying behavior. In treatment 1, in which both the gain for player 1 from deceiving player 2 and the corresponding loss suffered by player 2 are 1$, 36% of senders lied. In treatment 2, in which the gain from lying for player 1 is 1$ but the loss suffered by player 2 after choosing the "wrong" action is 10$, only 17% of senders lied. Finally, in treatment 3, in which both the gain for player 1 and the loss for player 2 are 10$, 52% of senders lied[3].

In order to evaluate whether such results indicate an aversion to lying or a preference of subjects over the distribution of payoffs, Gneezy used a control experiment in which player 1 is asked directly to choose between action A or action B, and hence between the resulting distribution of payoffs, in the same treatments summarized above. This thus becomes a dictator game, in which player 1 (the dictator) is the only active player, and he still has all the information regarding the payoffs of both players. In order to allow comparability of payoffs between the CTSR game described above and the dictator game, Gneezy sets a probability that the action chosen by the dictator were implemented of 0.8, that corresponds to the actual empirical frequency of the receivers following the suggestion coming from the message of the sender in the deception game. Results of this control experiment show clear evidence that an intrinsic aversion to lying does indeed play an important role in senders' behavior. In the same tratments described for the CTSR game, the percentage of players who chose the selfish option B rather than option A is respectively 66%, 42% and 91%. The pattern of behavior in the dictator game is similar to the one showed in the deception game, but the degree of selfish behavior is significantly higher[4]. Results of the two experiments are compared in 3.2.

Game	**Treatment**		
	1	2	3
CTSR	0.36	0.17	0.52
Dictator	0.66	0.42	0.90

Table 3.2: Percentage of player 1 choosing option B respectively in the deception game and in the dictator game

[3]Differences across treatments are all statistically significant, where p-values come from a one-tailed test of the equality of proportions, using normal approximation to the binomial distribution. Z-scores and p-values reported by Gneezy (2005) for the comparison of the treatments are $Z_{1,2} = 2.58$ and $p_{1,2} = 0.005$; $Z_{1,3} = 1.97$ and $p_{1,3} = 0.024$; $Z_{2,3} = 4.48$ and $p_{2,3} = 0.001$.

[4]For a fixed distribution of payoffs, all differences between the dictator game and the CTSR game are statistically significant at $p < 0.01$. Moreover, differences across treatments in the dictator game are also statistically significant at the 0.01 level.

3.4 A model with guilt and lying aversion

As it is pointed out in Gneezy (2005), the reported evidence gathered in the experiments described in the previous section suggests that subjetcs have an intrinsic aversion to lying. Gneezy used the dictator game as a control experiment to test the strenght of such an aversion to lie, as opposed to distributional preferences, i.e. preferences of subjects over a distribution of payoffs rather than over their own materialistic payoffs. Provided that the results obtained in the dictator game are significantly different than the ones obtained in CTSR game, Gneezy concludes that people are inherently prone not to lie.

In a theoretical model applied to the same experiment, Battigalli, Charness and Dufwenberg (2013) (BCD henceforth) argue that subjects behavior can be explained using the theory of simple guilt, developed in Battigalli and Dufwenberg (2007).

While both these ideas are ideed compelling, a unified theoretical treatment including both aversion to guilt and aversion to lying is still missing. This is the scope of the present work, that aims at developing such a theoretical setting and evaluating its fit to the data collected by Gneezy in his experimental paper. While, on one side, it is evident, as pointed out by Gneezy, that the control experiment suggested clear evidence of an aversion to lying, the diverse behavior of subjects across different treatments can not be explained by such an intrinsic disutility attached to lies. On one side, in fact, significantly more subjects decided to choose the selfish option in the dictator game with respect to the fraction of senders that chose the corresponding message in the CTSR game. It is worthwhile underlining here that this is hardly attributable to strategic considerations of purely economic agents. It could be argued indeed that some senders chose to send the truthful message because they expect receivers not to believe to the cheap talk message. To this matter, Gneezy tries to elicit senders' beliefs by asking them whether they thought that the receiver would have followed their message or not, and paying them for accuracy of the prediction: 82% of senders reported that they expected receivers to follow the message. Moreover, this is in line with actual behavior, as it turned out that 78% of participants who were assigned the role of receiver actually followed the message sent them by the sender and hence chose the option that would have gathered them the more money according to the message. Therefore, it is reasonable to assume the lower fraction of senders choosing the false and selfish message with respect to the fraction of dictators choosing the selfish allocation as evidence of an intrinsic disutility attached to the act of lying. The simplest way to model this characteristic is introducing a fixed cost of lying, that decreases players utility when they deliberately choose to lie. On the other side, such a cost is not enough to explain players behavior across treatments: the different material consequences designed by Gneezy have been reported to significantly modify players behavior. This means that subjects not only are averse to lie, but are also concerned by others well being. In particular, we adopt BCD's theoretical model, in which a player i suffers some utility loss from guilt, to the extent that he believes that player j expected an higher payoff than what he actually gets.

In line with these observations and building on BCD's setting we develop a model that includes both lying and guilt aversion. We define the setting that corresponds to the environment in Gneezy (2005), and evaluate how the theoretical predictions are in line with the gathered experimental data.

3.4.1 Notation

We follow BCD's model that introduces incomplete information into Battigalli and Dufwenberg (2007) in order to adhere to the experimental setting designed by Gneezy (2005). The model will be extended to encompass both the CTSR game and the dictator control game, and we will discuss the specific modifications in the two games in the current section. We will refer to player 1 as the sender in deception game or the dictator in the control game, and with player 2 as the receiver in the former or the inactive player in the latter game. We introduce notation, following BCD, having in mind the setting of the CTSR game.

In each treatment the sender has two strategies, that correspond to the two messages he is allowed to send to the receiver, i.e. $m \in \left\{m^A, m^B\right\}$, where m^j intuitively stands for the two messages specified above, with which the sender suggests action j to the receiver. Given that the sender knows the final payoffs of the game in the different treatments, the message m is a function of the treatment variable $t \in T$. The latter can be thought of as a true state of the world, that is known to the sender and unknown to the receiver. The strategy set for the sender is thus the set of functions from T to the set of available messages, i.e. $S_1 = \left\{m^A, m^B\right\}^T$. Similarly, the receiver's strategies are functions from the set of messages to the two available actions that he has in each treatment, i.e. $S_2 = \{A, B\}^{\left\{m^A, m^B\right\}}$. The receiver, moreover, has no available information on final payoffs of the game, hence the pair of payoff functions for both players is unkwown to him, and we denote it $\pi^t = (\pi_1^t, \pi_2^t) \in \mathbb{R}_+^{\{A,B\}} \times \mathbb{R}_+^{\{A,B\}}$. The size of the set $\{(\pi_1^t, \pi_2^t) : t \in T\}$ reflects the ignorance of the receiver and it is assumed to be large enough, as to comprehend the three particular treatments designed by Gneezy. Moreover, we assume that the latter set is known to the sender: in other words player 1 knows the set of payoff functions that the receiver deems plausible. Terminal histories of the CTSR game are $z \in \left\{m^A, m^B\right\} \times \{A, B\}$. Given that in Gneezy's framework the message yields no consequences on payoffs, we can think of a reduced form game and simplify the notation regarding terminal histories by ignoring the dependence on the set of messages: thus $z \in \{A, B\}$, and, therefore, payoff functions are simply, as already presented above, functions of z (i.e. $\pi_i^t(m^A, A) = \pi_i^t(m^B, A) = \pi_i^t(A)$).

We are interested in the behavior of player 1, given that he is the one who is given the possibility to lie. We define player 1's utility in each of the three treatments t as

$$u_1^t(z, m, \theta_g, \theta_l, \alpha_2) = \pi_1^t(z) - \theta_g \max\left\{0, \left[\mathbb{E}_{\alpha_2}(\pi_2(z)) - \pi_2^t(z)\right]\right\} - \theta_l \mathbb{I}_{m=m^B} \tag{3.1}$$

The sender is characterized by a pair of parameters that are assumed to be non-negative, i.e. $\theta_g \in \Theta_g \subset [0, +\infty)$, $\theta_l \in \Theta_l \subset [0, +\infty)$. We denote F_g and F_l the cumulative distribution functions of the two parameters, and we assume that their distributions are independent from each other. The former characterizes the sender's disutility from guilt, that arises from player 2 receiving a payoff that is lower than what he expected to get; the latter characterizes utility loss from lying, and thus reduces utility only when the sender chooses to send out to the receiver the wrong advice. In order to introduce this latter utility loss, we us the indicator function

$$\mathbb{I}_{m=m^B} = \begin{cases} 1 & \text{if } m = m^B \\ 0 & \text{if } m = m^A \end{cases}$$

Notice, moreover, that this lying cost is not related to the payoffs, but only to actions, and hence characterizes an intrinsic cost of lying, that is independent of consequences.

A crucial role is played by player's beliefs and, in particular, first-order beliefs of the receiver, and hence second-order beliefs of the sender. In particular, in BCD's model, utility loss from guilt arises form letting down the other player's expectations. Thus, utility for player 1 depends on a variable that he does not know, that is player 2's first-order beliefs. This is why player 1's second-order beliefs, which include beliefs about player 2's first-order beliefs, will play a key role in the theoretical analysis.

Player 2 has beliefs over the treatment variable, over the sender characteristics and over his own strategy: we follow BCD in the inclusion of the latter into first-order beliefs, and this simply represents the receiver's plan on how to play the game. We label first-order beliefs of the receiver $\alpha_2 \in \Delta\left(T \times \Theta_g \times \Theta_l \times S_1 \times S_2\right)$, i.e. they are probabilities over the cross product of the space of treatments, the sender's guilt aversion parameter, his cost of lying, the sender's cheap talk strategies and the receiver's own strategy. These can be split in beliefs over himself, $\alpha_{2,2} \in \Delta(S_2)$, and beliefs over the other player and the treatment, $\alpha_{2,1} \in \Delta(T \times \Theta_g \times \Theta_l \times S_1)$. There is no loss of generality in assuming that $\alpha_{2,2}$ assigns probability 1 to a pure strategy and, moreover, given the poor information that is contained in the sender's messages, we focus attention on two particular strategies of the receiver, namely $Y = (A \mid m^A, B \mid m^B)$ and $N = (B \mid m^A, A \mid m^B)$, the two strategies that BCD call respectively the "Yes-man" or "trusting" strategy, and the "contrarian" strategy.

The last thing to notice is that α_2 yields a probability distribution over $T \times \{A, B\}$, and this is the probability distribution used by player 2 to compute the expected value that appears in equation (3.1). Borrowing another piece of notation from BCD we label $\Pi_2^Y = \mathbb{E}_{\alpha_{2,1} \times Y}[\pi_2]$ and $\Pi_2^N = \mathbb{E}_{\alpha_{2,1} \times N}[\pi_2]$, that are, respectively, player 2 epected payoff if he plans to trust the sender's advice or to do the opposite of whatever player 1 suggests.

Given the importance of what player 1 thinks about player 2, as we have underlined above, we introduce here the sender's second-order beliefs as well, and label them $\beta_{1,2} \in \Delta(S_2 \times \Delta(T \times \Theta_g \times \Theta_l \times S_1 \times S_2))$. Lastly, expected disappointment of the receiver can now be defined as either

$$D^S(x) = \mathbb{E}_{\beta_{1,2}}\left[\max\left\{0, \Pi_2^S - x\right\} \mid \Pi_2^S \geq \Pi_2^{S'}\right] \quad \text{for } S, S' = Y, N,\ S' \neq S.$$

3.4.2 Assumptions

We make some hypothesis in order to facilitate the analysis and present them here. We will discuss them and analyze their implications in the following sections. Some of the assumptions are borrowed from BCD's setup, and extended to the present framework in order to incorporate lying costs.

The first assumption regards the sender's paramenters of guilt aversion and lying aversion introduced in the preceding subsection. We assume that these parameters characterizing the sender's utility function described in (3.1) are independent, formally

Conjecture 18 (Assumption 1) *We assume that f_g and f_l are the two strictly positive and continuous densities functions characterizing respectively θ_g and θ_l. Moreover, the distributions of the two parameters are assumed to be independent from each other.*

The second assumption is borrowed as it is from BCD's setup, and regards the receiver's first order beliefs.

Conjecture 19 (Assumption 2) *The first-order beliefs of the receiver about the treatment, the sender's type and the sender's strategy, $\alpha_{2,1}$, are such that expected payoffs by using strategy Y or N conditional on the received message are well defined and independent of m. Therefore, strategy Y (resp. N) is the unique best response of the receiver if and only if $\Pi_2^Y > \Pi_2^N$ (resp. $\Pi_2^N > \Pi_2^Y$)*

As pointed out in BCD, this assumption comes from symmetry considerations, and the application of a principle of insufficient reason.

As we have underlined in the preceding section, the analysis focuses on the behavior of the sender, whose utility crucially depends on first-order beliefs of the receiver. Hence a key role is played by second-order beliefs of the sender, to whom is dedicated the following assumption

Conjecture 20 (Assumption 3) *The second-order beliefs of the sender about the receiver, $\beta_{1,2}$, are independent of the treatment t, and such that*

(i) Assumption 2 holds;

(ii) the receiver is subjectively rational, i.e. he best responds to his first-order beliefs $\alpha_{2,1}$;

(iii) the pair of expected payoffs (Π_2^Y, Π_2^N), that comes from the receiver first-order beliefs $a_{2,1}$ is continuously distributed, with support $\left[0, \overline{\Pi}\right]$, where $\overline{\Pi} \geq 15$;

(iv) the probability, according to the sender's second-order beliefs, that $\Pi_2^Y \geq \Pi_2^N$ is more than 50%: formally, $\mathbb{P}_{\beta_{2,1}}\left[\Pi_2^Y \geq \Pi_2^N\right] > 0.5$

(v) beliefs of the sender on receiver's expected payoff are independent of both distributions characterizing guilt aversion and disutility from lying.

Point (iii) of Assumption 3 is needed to make the space of payoffs large enough, in particular, to comprehend the actual payoffs used by Gneezy in his experiment; point (iv) looks like a strong assumption, but it is indeed consistent with experimental evidence. After asking the senders with which probability they would expect receivers to follow their message, data reported more than 80% of the senders believing receivers to follow the suggestion in the message, and this in turn resulted in line with actual behavior of the receivers in the experiment (Gneezy (2005), p. 386).

Also the last assumption is borrowed from BCD, and will play an important role in simplifying the analysis. It states that expected disappointment of the receivers, in the mind of the senders, only depends on receivers' final payoff.

Conjecture 21 (Assumption 4) *The sender expects that, on average, trusting and contrarian receivers ontain the same disappointment by any payoff in the relevant range, i.e.*

$$D^Y(x) = D^N(x) = D(x) \quad \forall x \in \left[0, \overline{\Pi}\right]$$

It is worth reminding at this point that, in Gneezy's experiments, receivers are given no information on the structures of the payoffs. Given our assumptions about rationality, we have already underlined how receivers play the strategy that yields the higher expected payoffs given their beliefs. We can again use an argument of insufficient reason to justify Assumption 4: given that receivers play what they believe to be their best strategy, there is no reason for senders to expect that receivers' disappointment depends on the strategy they choose to follow. We denote then $D(x)$ the common expectation of the sender of the disappointment of the receivers (both trusters and contrarians), and procede with the analysis of the deception CTSR game.

3.4.3 The deception game

We start the analysis of the sender's behavior, by expliciting his expected utility of sending each one of the two messages available to him. In particular, using equation (3.1) from above, we can write expected utility of sending the true message and expected utility of sending the deceiving message respectively as

$$\begin{aligned} u_A^t(\cdot) &= \left[\pi_1^t(A) - \theta_g D(\pi_2^t(A))\right] P^Y + \left[\pi_1^t(B) - \theta_g D(\pi_2^t(B))\right] \left(1 - P^Y\right) \\ u_B^t(\cdot) &= \left[\pi_1^t(B) - \theta_g D(\pi_2^t(B))\right] P^Y + \left[\pi_1^t(A) - \theta_g D(\pi_2^t(A))\right] \left(1 - P^Y\right) - \theta_l \end{aligned}$$

where $P^Y = \mathbb{P}_{\beta_{2,1}}\left[\Pi_2^Y \geq \Pi_2^N\right]$ is the probability with which the sender expects the receiver to follow the advice contained in the message. We have a little abused of notation by making the preceding expressions only dependent on the two parameters characterizing the sender's type. Notice that first-order beliefs of the receiver are implicit in the preceding expressions of sender's expected utility, through the disappointment terms.

At this point it is easy to derive the utility gain from lying, that is

$$\left\{\pi_1^t(B) - \pi_1^t(A) - \theta_g \left[D(\pi_2^t(B)) - D(\pi_2^t(A))\right]\right\} \left(2P^Y - 1\right) - \theta_l \qquad (3.2)$$

It is easy to see that $D(x)$ is decreasing and convex: recall in fact that it is the expected disappointment of the receiver from the point of view of the sender, and hence according to his second-order beliefs $\beta_{1,2}$. It is therefore the integral of a convex and decreasing function, such as the disappointment of the receiver, that is $\max\{0, \Pi_2 - x\}$, with respect to the unknown expectation of the receiver of his own payoff, Π_2. Lemma 1 and Corollary 1, whose proofs are given in BCD and reported here for completeness, respectively prove that monotonicity and convexity are strict, and a result for the incremental ratio that will be useful later on.

Lemma 22 *Expected disappointment D(x) is strictly decreasing and strictly convex on* $\left[0, \overline{\Pi}\right]$.

Proof. *The assumptions on second-order beliefs of the sender guarantee that there exist a density* $\beta : \left[0, \overline{\Pi}\right] \to \mathbb{R}$ *that is strictly positive on* $\left(0, \overline{\Pi}\right)$ *such that*

$$D(x) = \int_0^{\overline{\Pi}} \max\{0, \Pi_2 - x\} \beta(\Pi_2) \mathrm{d}\Pi_2 = \int_x^{\overline{\Pi}} (\Pi_2 - x)\beta(\Pi_2)\mathrm{d}\Pi_2$$

In order to show that $D(x)$ *is strictly decreasing, fix two payoffs* $x, y \in \left[0, \overline{\Pi}\right]$, *with* $x < y$. *It is immediate to see that, for any expected payoff of the receiver, disappointment is higher at* x, *unless the expected payoff is lower than* x, *in which case disappointment would be null both at* x *and at* y. *Formally, given an expected payoff of the receiver* Π_2,

$$D(x \mid \Pi_2) - D(y \mid \Pi_2) = \begin{cases} 0 & \textit{if } \Pi_2 \leq x \\ \Pi_2 - x > 0 & \textit{if } \Pi_2 \in (x, y) \\ y - x > 0 & \textit{if } \Pi_2 > y \end{cases}$$

Then the result that $D(x)$ *is strictly decreasing follows directly from the fact that* β *is strictly postive on* $\left[0, \overline{\Pi}\right]$ *and, in particular, on* $\left[x, \overline{\Pi}\right]$.

Denote then $\overline{x}(\lambda)$ *the convex combination of* x *and* y, *i.e.* $\overline{x}(\lambda) = \lambda x + (1-\lambda)y$ *for* $\lambda \in (0,1)$. *Then, for a fixed expected payoff of the receiver* Π_2

$$D(\overline{x}(\lambda) \mid \Pi_2) \leq \lambda D(x \mid \Pi_2) + (1-\lambda) D(y \mid \Pi_2)$$

because $D(x \mid \Pi_2)$ *is convex in* x. *Consider then a receiver's expected profit* $\Pi_2' \in (x, \overline{x}(\lambda))$. *It it easy to see that the preceding disequality is strict for* $\Pi_2 = \Pi_2'$, *as* $D(\overline{x}(\lambda) \mid \Pi_2') = D(y \mid \Pi_2') = 0$ *while* $D(x \mid \Pi_2') = \Pi_2' - x > 0$. *Using the fact that* β *is strictly positive on any non-empty open interval and, in particular, on* $(x, \overline{x}(\lambda))$ *we get the result, formally*

$$\begin{aligned} D(\overline{x}(\lambda)) &= \int_{\overline{x}(\lambda)}^{\overline{\Pi}} D(\overline{x}(\lambda) \mid \Pi_2)\beta(\Pi_2)\mathrm{d}\Pi_2 \\ &< \int_x^{\overline{x}(\lambda)} \left[\lambda D(x \mid \Pi_2) + (1-\lambda)D(y \mid \Pi_2)\right]\beta(\Pi_2)\mathrm{d}\Pi_2 + \\ &\quad + \int_{\overline{x}(\lambda)}^{\overline{\Pi}} \left[\lambda D(x \mid \Pi_2) + (1-\lambda)D(y \mid \Pi_2)\right]\beta(\Pi_2)\mathrm{d}\Pi_2 \\ &= \lambda D(x) + (1-\lambda)D(y) \end{aligned}$$

where the last equality follows from the observations that the integral of $D(y \mid \Pi_2)$ *is null for* $\Pi_2 < y$ *and that* $y > \overline{x}(\lambda)$. ■

Corollary 23 *The incremental ratio* $[D(x) - D(x+h)]/h$ *is strictly decreasing in* h *on* $(0, \overline{\Pi} - x)$ *for any* $x \in [0, \overline{\Pi})$
Proof. *Denote* $\Delta_h = D(x) - D(x+h)$. *By definition* $\Delta_0 = 0$. Δ_h *is strictly concave as implied by Lemma 1. Therefore* Δ_h/h *is strictly decreasing.* ■

Now we can precisely analyze equation (3.2): in each one of Gneezy's treatment, in fact, $\pi_1(B) > \pi_1(A)$ and $\pi_2(A) > \pi_2(B)$, and, hence, by Lemma 1, also $D(\pi_2(A)) < D(\pi_2(B))$. Given the assumption on second-order beliefs of the sender $P^Y > 0.5$, the first term in equation (3.2) is strictly decreasing in θ_g. We can then have two cases: either the utility loss from lying is so large that the sender will never try to deceive the receiver, or there exist an upper bound on the guilt aversion parameter below which the sender has an incentive to lie. Such thresholds on the guilt aversion parameter are easily derived from equation (3.2) and it is easy to show that they are ordered in a way that is coherent with experimental data gathered by Gneezy. The threshold in each one of the treatments is (for $t = 1, 2, 3$)

$$\widehat{\theta}_g^t(\theta_l) = \frac{\pi_1^t(B) - \pi_1^t(A)}{D(\pi_2^t(B)) - D(\pi_2^t(A))} - \frac{\theta_l}{\left[D(\pi_2^t(B)) - D(\pi_2^t(A))\right](2P^Y - 1)} \quad (3.3)$$

Then, the sender in treatment t lies if and only if $\theta_g < \widehat{\theta}_g^t(\theta_l)$. As it is intuitive, the thresholds in (3.3) are positively correlated to the gain that the sender can make by lying $(\pi_1^t(B) - \pi_1^t(A))$, and negatively correlated to the cost of lying θ_l: the higher the disutility attached to lying, the lower the threshold on the guilt aversion parameter, and hence the lower guilt aversion needs to be in order to induce the sender to send out to the receiver the truthful message. Moreover, it can be shown that the threshold in (3.3) depends positively on the difference between the probability with which the sender expects the receiver to follow his message and its complement[5]. Intuitively, the incentive to lie is higher when the sender expects the receiver to follow his advice; whereas, for values of P^Y close to 0.5, the message of the sender becomes almost irrelevant for the actual decision of the receiver, so that, if there is a utility cost associated with lying, senders will be pushed to tell the truth.

Before comparing the thresholds in the three different treatments designed by Gneezy, we clarify the point we made earlier. Recall that in Gneezy's

[5]Recall that, following Battigalli, Charness and Dufwenberg (2013), and consistently with experimental data gathered by Gneezy, we assumed $P^Y > 0.5$, and hence $(2P^Y - 1) \in (0, 1]$.

experiments, for $t = 1, 2, 3$, $\pi_1^t(B) > \pi_1^t(A)$ and $\pi_2^t(B) < \pi_2^t(A)$, that $D(x)$ is decreasing and that $\theta_l \geq 0$ by assumption. Given that we assumed that also θ_g is non-negative, it is obvious from (3.3) that for high levels of lying cost, no type of sender will lie. This threshold on the utility loss that results from lying is proportional to the selfish gain from lying.

Proposition 24 *No sender will choose to lie in treatment t if and only if*

$$\theta_l > \left[\pi_1^t(B) - \pi_1^t(A)\right]\left(2P^Y - 1\right) = \underline{\theta_l}^t \tag{3.4}$$

Proof. *Suppose that $\theta_l \geq \underline{\theta_l}^t$. Then equation (3.3) implies that $\widehat{\theta}_g^t < 0$ and our assumptions on θ_g imply that every sender will be better off by sending out to the receiver the true message, A. The only if part comes from the assumption that the density function f_g is strictly positive.* ■

Similar considerations to the ones done before hold for the threshold on θ_l in (3.4): an increase in the potential gains attainable by lying induces an higher threshold on θ_l , and so does an higher probability attached by the sender to the message being believed by the receiver.

A last consideration deserved to be done on the term expressing the difference in the receiver expected disappointment following the two outcomes of the experiment. Recall that expected disappointment, that is $\mathbb{E}_{\beta_{1,2}}[\max\{0, \Pi_2 - x\}]$, is a feature of the sender, because it depends on second-order beliefs of the latter. We can then rewrite the expression for the sender's expected disappointment of the receiver, $D(\pi_2^t(B)) - D(\pi_2^t(A))$, that appears at the denominator of the thresholds in (3.3) as

$$\begin{aligned}
& \int_{\pi_2^t(B)}^{\overline{\Pi}} \left(\Pi_2 - \pi_2^t(B)\right)\beta(\Pi_2)d\Pi_2 - \int_{\pi_2^t(A)}^{\overline{\Pi}} \left(\Pi_2 - \pi_2^t(A)\right)\beta(\Pi_2)d\Pi_2 \\
= & \int_{\pi_2^t(B)}^{\pi_2^t(A)} \left(\Pi_2 - \pi_2^t(B)\right)\beta(\Pi_2)d\Pi_2 + \int_{\pi_2^t(A)}^{\overline{\Pi}} \left(\pi_2^t(A) - \pi_2^t(B)\right)\beta(\Pi_2)d\Pi_2 \\
= & \int_{\pi_2^t(B)}^{\pi_2^t(A)} \Pi_2\beta(\Pi_2)d\Pi_2 - \pi_2^t(B)\left[F_\beta(\pi_2^t(A)) - F_\beta(\pi_2^t(B))\right] \\
& + \left(\pi_2^t(A) - \pi_2^t(B)\right)\left[1 - F_\beta(\pi_2^t(A))\right] \qquad (3.5) \\
= & \left[\Pi_2 F_\beta(\Pi_2)\right]_{\pi_2^t(B)}^{\pi_2^t(A)} - \int_{\pi_2^t(B)}^{\pi_2^t(A)} F_\beta(\Pi_2)d\Pi_2 - \pi_2^t(B)\left[1 - F_\beta(\pi_2^t(B))\right] \\
& + \pi_2^t(A)\left[1 - F_\beta(\pi_2^t(A))\right] \qquad (3.6) \\
= & \pi_2^t(A) - \pi_2^t(B) - \int_{\pi_2^t(B)}^{\pi_2^t(A)} F_\beta(\Pi_2)d\Pi_2 \qquad (3.7)
\end{aligned}$$

The term at the denominator in (3.3) is then increasing in the difference between the payoffs for player 2 in the two outcomes of the game, and decreasing in the integral of cdf that is derived from second-order beliefs of the sender. It is easy to see that the latter integral is larger the more probability mass the sender attaches to low payoffs expected from the receiver. The intuition is as follows: assume that the sender is certain that the receiver expects some payoff that is smaller than $\pi_2^t(B)$, that is lower payoff that player 2 can end up with; then the cdf $F_\beta(\Pi_2)$ is flat at 1 between $\pi_2^t(B)$ and $\pi_2^t(A)$ and expected disappointment of the receiver is 0, because player 2 will be happy, no matter what the actual outocome of the game is. Conversely, if the sender were certain that the receiver expects to get something out of the experiment that is higher than $\pi_2^t(A)$, then integral in (3.7) would be null, and the difference in expeceted disappointment experienced by the receiver following outcomes A and B will always be equal to the difference in payoffs $\pi_2^t(A) - \pi_2^t(B)$.

We can now calculate the thresholds found above for the treatments designed by Gneezy for his experimental work, in order to compare the three different values of $\widehat{\theta}_g^t$. We will use our assumptions 3 and 4, Lemma 22 and Corollary 23 characterizing $D(x)$.

Proposition 25 *Under our assumptions, and provided that $\theta_l < 2P^Y - 1$, the thresholds on θ_g and the fraction of senders that choose to lie in treatment t, $F^t(lies)$, are ordered as follows*

$$\begin{aligned} \widehat{\theta}_g^3 &> \widehat{\theta}_g^1 > \widehat{\theta}_g^2 > 0 \\ 1 &> F^3(lies) > F^1(lies) > F^2(lies) > 0 \end{aligned}$$

Proof. *Notice that the condition imposed on the parameter expressing utility loss from lying, $\theta_l < 2P^Y - 1 \equiv \underline{\theta}^t$ for $t = 1, 2$, implies that each $\widehat{\theta}_g^t$ is strictly positive, for any of the treatments set up by Gneezy. As a consequence, there will be a strictly postive fraction of the population that is better off by sending out to the receiver the deceiving message B.*

The thresholds on guilt aversion in the three treatments designed by Gneezy are

$$\begin{aligned} \widehat{\theta}_g^1(\theta_l) &= \frac{1}{D(5) - D(6)} - \frac{\theta_l}{[D(5) - D(6)]\,(2P^Y - 1)} \\ \widehat{\theta}_g^2(\theta_l) &= \frac{1}{D(5) - D(15)} - \frac{\theta_l}{[D(5) - D(15)]\,(2P^Y - 1)} \\ \widehat{\theta}_g^3(\theta_l) &= \frac{10}{D(5) - D(15)} - \frac{\theta_l}{[D(5) - D(15)]\,(2P^Y - 1)} \end{aligned}$$

Recall that, by the Lemma above $D(5)-D(15) > D(5)-D(6)$ *and by the Corollary* $\frac{10}{D(5)-D(15)} > \frac{1}{D(5)-D(6)}$. *Moreover, it is easy to check that* $\widehat{\theta}_g^1(\theta_l) > \widehat{\theta}_g^2(\theta_l)$ *for* $\theta_l < 2P^Y - 1$.

In order to compute the actual fraction of the population of senders that chooses to lie in treatment t by sending out to the receiver message B, we need to evaluate

$$F^t(lies) = \int\int F_g(\widehat{\theta}_g^t(\beta_{1,2}, \theta_l)) f_\beta(\mathrm{d}\beta_{1,2}) f_l(\mathrm{d}\theta_l)$$

where we explicitly show the dependence of the thresholds both on second-order beliefs and on the parameter characterizing disutility from lying.

Using our simplifying assumptions that the distribution of second-order beliefs is independent both of the distribution of guilt aversion and of the distribution of lying costs, the result on the observed frequencies of lies follows directly from the ordering found for the thresholds on the guilt aversion parameter found in the first part of the proof. ■

The last proposition then successfully generalizes BCD's model to the introduction of a fixed cost of lying. The theoretical predictions made by a model that include utility losses for agents arising both from guilt and from deceiving behavior, are in line with the experimental evidence reported by Gneezy (2005). Increasing the co-player gain (or the loss suffered after a deception), reduces the fraction of senders that choose to lie, while increasing the sender's gain from lying increases the fraction of deceiving messages. Moreover BCD's original model was not developed to capture the different players' behavior in the deception game versus the dictator game, to which we now turn our attention.

3.4.4 The dictator game

In this section we want to analyze the dictator game set up by Gneezy as a control experiment, in the same theoretical framework we set up above for the deception game. In order to do so we have to slightly redefine some of the concepts in the model in order to match the different game. As in the preceding section we will still have two players, a dictator and a receiver. Player 1, who was the sender in the preceding section, is now playing the role of the dictator, while player 2 remains the receiver. Recall that in this game the receiver is inactive: he takes no action, and , therefore, has no strategy, i.e. $S_2 = \varnothing$. The dictator has to choose an option, either A or B, and the distributions of payoff is exactly the same as the one described for the deception game. Recall that, in order to allow comparability between the deception game and the dictator game, Gneezy set up a probability of 0.8 with which the action chosen by the dictator corresponds to the actual distribution of payoffs between the two players. This means, that in our model, the dictator strategies are the set of functions from the space of treatments T to the two lotteries a or b, i.e. $S_1 = \{a, b\}^T$. The fact that the experiment doesn't end with the allocation of payoffs as chosen by the dictator, but with a random allocation, that coincides with the choice of player 1 with probability 0.8 and in the opposite one with probability 0.2, is thus reflected in the dictator choosing a lottery over the allocations rather than an allocation[6]. As it was the case in the deception game, the receiver has no information on the structure of the payoffs in the experiment, and hence the pair of payoff functions for both players is unkwown to him, and we still denote it $\pi^t = (\pi_1^t, \pi_2^t) \in \mathbb{R}_+^{\{A,B\}} \times \mathbb{R}_+^{\{A,B\}}$. The size of the set $\{(\pi_1^t, \pi_2^t) : t \in T\}$ reflects the ignorance of the receiver and it is, again, assumed to be large enough, in particular to comprehend the three particular treatments designed by Gneezy. Similarly to what we have said for the deception game about the message sent out by the sender, the particular lottery chosen by the dictator does not directly affect terminal payoffs of the game. It is the choice made by Nature that sets the allocation of payoffs, and thus we can again reduce terminal histories of the game to two choices available to Nature, i.e. $z \in \{A, B\}$[7].

Our interest is now on the behavior of the player who plays the role of the dictator. We remain aligned to the theoretical framework set up above and its interpretation. In particular, dictators still feel guilt, as expressed by the epected disappointment felt, from their point of view, by the receivers. As opposed to the CTSR game analyzed in the previous section, there is no utility loss resulting from lying now, because players are not given the possibility of deceiving. They are simply asked to choose their prefered lottery, and their expected utility is now

$$u_a^t(\theta_g, \theta_l) = 0.8\left[\pi_1^t(A) - \theta_g \max\left\{0, \mathbb{E}_{\alpha_2}[\pi_2] - \pi_2^t(A)\right\}\right] + \tag{3.8}$$
$$+0.2\left[\pi_1^t(B) - \theta_g \max\left\{0, \mathbb{E}_{\alpha_2}[\pi_2] - \pi_2^t(B)\right\}\right]$$
$$u_b^t(\theta_g, \theta_l) = 0.8\left[\pi_1^t(B) - \theta_g \max\left\{0, \mathbb{E}_{\alpha_2}[\pi_2] - \pi_2^t(B)\right\}\right] + \tag{3.9}$$
$$+0.2\left[\pi_1^t(A) - \theta_g \max\left\{0, \mathbb{E}_{\alpha_2}[\pi_2] - \pi_2^t(B)\right\}\right]$$

[6] The dictator is made aware of this mechanism of choice by the instructions of the experiment designed by Gneezy.

[7] As it was the case in the deception game, terminal histories actually belong to the set $Z = \{a, b\} \times \{A, B\}$. We simplify notation analogously, by writing e.g. $\pi_i^t(a, A) = \pi_i^t(b, A) = \pi_i^t(A)$.

Again, as noticed in the preceding section of the model, expected utility of dictators depend on first-order beliefs of the receiver about expected payoff from the experiment, and, in turn, this implies that second-order beliefs of the dictator will play a crucial role in the analysis. The subjective expected payoff $\mathbb{E}_{\alpha_2}[\pi_2]$ that enters the previous equations, is in fact computed according to the first order beliefs $\alpha_{2,1} \in \Delta(T \times S_1)$ of the receiver on the space of treatments and player 1's strategy.

Notice that Assuption 1 naturally extends to the current environment, in which θ_l plays no role. Moreover, Assumption 2 is irrelevant in the current environment, as player 2 has no strategies available. We introduce a new set of assumptions on second-order beliefs of the dictator, that substitute the ones introduced in Assumption 3 for the sender.

Conjecture 26 (Assumption 3.1) *The second-order beliefs of the dictator about the receiver, $\beta_{1,2}$, are independent of the treatment t, and such that*

(i) the expected payoff of the receiver Π_2, that comes from the receiver first-order beliefs $a_{2,1}$ is continuously distributed, with support $\left[0, \overline{\Pi}\right]$, where $\overline{\Pi} \geq 15$;

(ii) the cumulative distribution function of second-order beliefs F_β is independent of the distribution characterizing guilt aversion F_{θ_g}.

As we did in the previous section, we denote

$$D^d(x) = \mathbb{E}_{\beta_{1,2}}\left[\max\left\{0, \mathbb{E}_{\alpha_2}[\pi_2] - \pi_2^t(z)\right\}\right]$$

the expected disappointment of the receiver after receiving x dollars, from the point of view of the dictator. Notice that both in Assumption 3.1 and in the definition of $D^d(x)$ we are not differentiating player 1's second-order beliefs in this current setting versus the deception game analyzed above. In other words, we will make the relatively strong assumption that second-order beliefs regarding player 2's first-order beliefs held by dictators and by senders are the same. In particular, this implies that the distribution on player 2's expected payoffs implied by player 1's second-order beliefs is the same in the two games. Given the complete ignorance of player 2 on the structure of the experiment, and, in particular, on final payoffs, it may indeed be reasonable to mantain the assumption that we have introduced for the deception game: player 2's expected payoff may depend on what the subject thinks is fair to receive as a remuneration for participating in an experiment, or on what other people referred him they got after participation. On the other hand, it may be argued that in this setting, where the receiver is inactive, he could expect to gain less than in CTSR game described above and, in particular, the dictator, who has more knowledge on the structure on the game, can expect the receiver to expect a lower payoff in this setting rather than in the previous one. We will comment more on this later, while, for the time being, we extend Assumption 4 to the current environment.

Conjecture 27 (Assumption 4.1) *The dictator expects that, on average, receivers ontain the same disappointment as in the deception game by any payoff in the relevant range, i.e.*

$$D^d(x) = D(x) \quad \forall x \in \left[0, \overline{\Pi}\right]$$

We can now simplify equations (3.8) and (3.9) above in the following way

$$u_a^t(\cdot) = 0.8\left[\pi_1^t(A) - \theta_g D(\pi_2^t(A))\right] + 0.2\left[\pi_1^t(B) - \theta_g D(\pi_2^t(B))\right] \quad (3.10)$$
$$u_b^t(\cdot) = 0.8\left[\pi_1^t(B) - \theta_g D(\pi_2^t(B))\right] + 0.2\left[\pi_1^t(A) - \theta_g D(\pi_2^t(A))\right] \quad (3.11)$$

In particular, as long as the random device used to select the implemented allocation of payoffs given the dictator choice gives an higher probability to the option chosen by player 1, it is optimal for the dictator to pick his selfish option as long as his guilt aversion is not high enough. In other words, the probabilities attached by Gneezy to actual outcomes of the game after any choice made by dictators play a similar role to the one described above for P^Y in the deception game.

Similarly to what we have done before, the gain for payer 1 from choosing the option that yields him the higher payoff in treatment t is

$$0.6\left\{\pi_1^t(B) - \pi_1^t(A) - \theta_g\left[D(\pi_2^t(B)) - D(\pi_2^t(A))\right]\right\}$$

Recall that, in each treatment $\pi_1^t(B) > \pi_1^t(A)$ and $\pi_2^t(B) < \pi_2^t(A)$. Moreover expected disappointment is decreasing in x, and the previous equation is strictly decreasing in θ_g. Thus, we can conclude that dictators select the selfish option B, if $\theta_g < \widehat{\theta}_g^d$, where

$$\widehat{\theta}_g^d = \frac{\pi_1^t(B) - \pi_1^t(A)}{D(\pi_2^t(B)) - D(\pi_2^t(A))} \quad \text{for } t = 1, 2, 3. \qquad (3.12)$$

This the same threshold that BCD found for senders in the deception game, as they did not consider any utility loss resulting from lying. Notice that, given our assumptions on the parameters of the model, the threshold for the guilt aversion parameter in equation (3.12) is always higher than the respective threshold computed for the sender in the deception game, for each treatment t. The reason is intuitive: the dictator does not experience any disutility from lying, because he does not have the opportunity to lie. The utility of the sender was negatively affected by both guilt, in the measure in which player 1 expected the receiver to be disappointed by his actual final payoff, and by lying. The dictator suffers only from guilt, as he is still sympathethic with player 2's expected disappointment, but suffers no utility loss from deceiving behavior.

We can now conclude, borrowing some more from BCD, proving that the theoretical predictions that this model generates are consistent with experimental data gathered by Gneezy. We will indeed show that the order of the thresholds on the guilt aversion parameter for dictators in the different treatments is the same as the one predicted in the previous section for the deception game. As a consequence, also the order of the fractions of dictators that choose the selfish allocation B is preserved from the CTSR game analyzed above. Moreover, for each allocation of payoffs of a given treatment t, the fraction of dictators opting for the egoistic choice is higher than the fraction of senders suggesting to the receiver to take the sender-rewarding option through a deceiving message. All these predictions are perfectly in line with experimental data collected by Gneezy in his experimental work.

Proposition 28 *The thresholds on the guilt aversion parameter for the dictator and the fractions of dictators that choose the selfish option B in the different treatments are ordered as follows*

$$\begin{aligned} \widehat{\theta}_g^3 &> \widehat{\theta}_g^1 > \widehat{\theta}_g^2 > 0 \\ 1 &> F^3(B) > F^2(B) > F^1(B) > 0 \end{aligned}$$

Moreover, in each treatment t, the fraction of dictators choosing the option B is higher than the fraction of senders who choose to lie, i.e.

$$F^t(B) > F^t(lies) \quad for\ t = 1, 2, 3.$$

Proof. *The thresholds on guilt aversion for the dictators in the three treatments set up by Gneezy in his experimental sessions are*

$$\begin{aligned} \widehat{\theta}_g^1 &= \frac{1}{D(5) - D(6)} \\ \widehat{\theta}_g^2 &= \frac{1}{D(5) - D(15)} \\ \widehat{\theta}_g^3 &= \frac{10}{D(5) - D(15)} \end{aligned}$$

As it is showed in BCD, by Lemma 22 and Corollary 23, $\frac{1}{D(5)-D(6)} > \frac{1}{D(5)-D(15)}$ *and* $\frac{10}{D(5)-D(15)} > \frac{1}{D(5)-D(6)}$. *Hence,* $\widehat{\theta}_g^3 > \widehat{\theta}_g^1 > \widehat{\theta}_g^2$.

In order to evaluate the fraction of the population of dictators that chooses to pick the selfish option B *in treatment* t*, we need to compute the following integral*

$$F^t(B) = \int F_g(\widehat{\theta}_g^t(\beta_{1,2}))f_\beta(\mathrm{d}\beta_{1,2})$$

where, again, we made explicit the dependence of the thresholds on second-order beliefs of the dictator. Given that we have assumed that second-order beliefs distribute independently of the parameter characterizing guilt aversion, and that both pdfs are strictly positive, the result on the frequencies of dictators choosing the selfish option is an immediate consequence of the order found on $\widehat{\theta}_g^t$*, i.e.*

$$1 > F^3(B) > F^1(B) > F^2(B) > 0.$$

We are left to show that the frequency of selfish dictators is higher than the frequency of senders lying in the respective treatment. First of all, notice that the thresholds for the dictator in equation (3.12) are equivalent to the thresholds in equation (3.3) found above for senders, when utility loss from lying is null, i.e. $\theta_l = 0$. *This means that, if all the mass of probability on the support* Θ_l *were concentrated on the point* $\theta_l = 0$*, that is if all senders suffer no disutility from lying, then, as it is intuitive, the fraction of senders lying in the deception game would be the same as the fraction of dictators choosing the selfish allocation of payoffs in the dictator game. The result then follows directly by noticing that the thresholds* $\widehat{\theta}_g^t$ *found above in equation (3.3) are decreasing in* θ_l*. Hence,*

$$\int F_g(\widehat{\theta}_g^t(\beta_{1,2}))f_\beta(\mathrm{d}\beta_{1,2}) > \int\int F_g(\widehat{\theta}_g^t(\beta_{1,2},\theta_l))f_\beta(\mathrm{d}\beta_{1,2})f_l(\mathrm{d}\theta_l)$$

i.e.

$$F^t(B) > F^t(lies) \quad \textit{for } t = 1,2,3.$$

■

As a last remark, we comment on the point we have made earlier. Notice that in the derivation of the results we greatly simplified the analysis assuming that second-order beliefs of senders and dictators imply the same distribution on expected payoffs for the receiver. As mentioned above, we claim that this assumption is harmless, because it actually works in favor of increasing the thresholds in (3.12) relative to those in (3.3). In order to see this, we make some considerations on player 1's second-order beliefs in the two games. If we relaxed the assumption that player 2's expected disappointment is the same in the two games and we consider the structure of the two games, together with the fact that the sender and the dictator know it, we can safely assume that, if anything changes, player 1 can believe the receiver to expect a lower payoff in the dictator game rather than in the deception game. In other words, take two agents playing respectively the role of the dictator and the role of the sender: it sounds reasonable to assume that the former expects the corresponding receiver to expect less rather than the latter, for the simple fact that one receiver has no choice in the development of the game, while the other one does. If we assumed that the cdf derived from the dictator's second-order beliefs sthochastically dominates the cdf from the sender's second order beliefs (at least in the interval $[\pi_2^t(B), \pi_2^t(A)]$), then the decomposition seen above in (3.7) immediately yields that the difference in expected disappointment is lower for a dictator than for a sender. If this were the case, both lying aversion and guilt aversion would then work in the same direction, that is increasing the proportion of selfish dictators relative to deceiving senders. This is easy to see considering a sender whose disutility from lying is null: comparing the threshold in (3.3) and in (3.12) it is in fact immediate to see that all the difference stems from the difference in expected disappointment. Although the role of guilt may then be different for senders and for dictators, it is arguably far from being enough to justify the significantly different registered behavior of agents in the two roles. This justifies our extension of BCD's framework to incorporate lying aversion.

3.5 Comments and Concluding Remarks

We build a model that includes both guilt aversion and lying aversion, extending the theoretical framework developed in Battigalli, Charness and Dufwenberg (2013). We applied the theoretical model to the experimental findings reported in Gneezy (2005) and we were able to conclude that guilt aversion and lying aversion provide a satisfactory explanation of the patterns found in the data.

The model is able to explain the magnitude of the proportion of liers in the different treatments of the deception game, the magnitude of the proportion of dictators choosing the selfish option in the different treatments of the dictator game, and the difference between the behavior of players across the two games.

Introducing lying aversion has an intuitive direct effect: it decreases the proportion of deceiving senders, for any level of guilt aversion, and implies that agents with low levels of guilt aversion never lie. Such agents would correspond to the "ethical" agents characterized by Koford and Penno (1992) [22]. Moreover, as pointed out by Battigalli, Charness and Dufwenberg (2013) the existence of a cost of lying may induce receivers to have higher expectations in the CTSR game rather than in the dictator game. If this were the case, we showed that the difference in the proportion of decievers and the proportion of egoistic dictators would increase due to the smaller utility loss suffered by dictators as opopsed to senders'. On the other hand, another indirect effect that has not yet been pointed out may be that, under the assumption of a commonly known cost of lying, senders should expect more receivers to follow their message, and this would attenuate the effect of the lying cost, increasing the incentives to deceive.

To sum up, we can draw conclusions that both guilt and lying aversion play an important role in explaining subjects behavior reported by Gneezy (2005). Future research can further investigate the two psychological motives modeled here and, in particular, develop some kind of interaction between them. Differently from what we have done in this paper, several studies report evidence that the utility loss generated by lying can be heterogeneous both across and within agents. Moreover, lying costs are reported to be possibly dependent on potential payoffs or on the type of communication among agents. Richer frameworks would then need to be developed, and we hope to have the opportunity to contribute on this.

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