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# Contents



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# Acknowledgements

> There was a Door to which I found no Key:
> There was a Veil through which I could not see:
> Some little Talk awhile of Me and Thee
> There seemed-and then no more of Thee and Me.
> Omar Khayyam[1]

There is not much here in this dissertation, such that it makes sense to *devote* it to someone. This was a just a perliminary attempt to attack some questions based on Karl Popper's advice.[2] I hope that I can answer the questions better in the future.

However, this does not mean that I have not benefitted from others in writing this dissertation. Therefore - and in the language of my chapters- I want to thank two specific *organizations*:[3]

The first organization with hierarchy consists of the following friends. First tier is my supervisor, Nenad, who was very kind to me.[4] He is not in the first tire just because of being my supervisor. He *is* because he did more than what other supervisors typically do; he listened and read what he did not like. Also, he supported me when I needed the most: the time when I changed my research area. Second tier is Pooyan who cares about- sometimes boring- details! He is in the second tier since he helped me with changing my research area, and with the boring introductions of my dissertation chapters. Third tier is Omid who helped me with the first and second chapters of this dissertation. Finally the fourth tier is both Mahdi, Mehdi and Morteza.

The other organization without hierarchy is my family. I think it is unnecessary to mention any reasons about acknowledging my family. Indeed, thank Shima and Sana.

[1]English version by Edward FitzGerald.

[2]Whenever a theory appears to you as the only possible one, take this as a sign that you have neither understood the theory nor the problem which it was intended to solve.

[3]I was not the planner in these organizations, I was and am an owner, so I have been maximizing my benefit.

[4]This is not just a cliche, believe me!

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**Abstract**

This thesis consists of three chapters each addressing different questions related to organizations with private information.

The first chapter considers the bargaining problem in an organization where agents' effort is complementary in view point of mechanism designer. The mechanism designer communicates with the agents. The allocation is characterized and also considers the affect of taxation on the formation of organization and allocations of effort and consumption.

The second chapter considers a different organization in which mechanism designer can not communicate with one agent- the worker- but the other agent- the manager- can communicate with the worker. Mechanism designer aims to maximize social welfare and impose tax on the organization if possible. We address the problem with various cost functions and mechanisms.

The third chapter addresses regulation issues in an industry where there is a vertical relationship between firms. Each firm has private information about its own marginal cost. We characterize allocations of prices, quantitities and subsidies for both upstream firm and downstream firms.

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# Chapter 1

# Organization and Taxation; Micro Approach or Wooden Bridge: Full Communication

## 1.1 Introduction

There are two approaches to study organizations in economic theory. One approach discusses the boundaries of the organizations and markets, the other focuses on the formation of structures and the process of decision making within organizations. The problem of asymmetric information is typically discussed in the latter approach. However, the agency problem also exists between organizations and players outside organization boundaries. A key context is the agency issue between organization and government which would be very relevant for taxation. However, the literature on taxation only considers the agency problem between government and agents, but not *among* the agents within an organization. This paper contributes to the literature by combining these two perspectives, i.e. agency issue within and outside the organization boundaries. We incorporate the within organization agency models into the optimal taxation literature to study how it impacts the organization and taxation from a viewpoint of government.

The model in this study encompasses two ingredients. First, the interdependencies between agents' share of output and second, the impact of such interdependencies on optimal taxation plan of the planner. For this purpose, we consider an organization with two agents, a manager and a worker. Both agents contribute to the final output while their productivity is private information. They can decide to participate in the organization's production activity or not. In case both agents decide to participate, they will decide about the amount of effort, not income, the manager and the worker should agree on the profit sharing scheme after the output is realized. These factors define the existence of organization. To deal with these questions the planner should take in to the account the

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agents' private information.

The above environment is translated to the mechanism in which consists of the agent's effort and the agent's net consumption given tax on the organization or the agents. It raises two questions from the point of view of planner.

The first, how can the government construct the mechanism so that the agents have incentives to engage in the organization? The second, how the taxation affects agents' engagement and the mechanism. Indeed, given the tax rule, can the planner make the optimal mechanism?

To formalize this problem, we develop a model based on Myerson and Satterthwaite [1983]. Their model is concerned with a bargaining problem between a buyer and a seller with independent private valuation functions, in a single good market. In particular there is no interdependence between the agent's utilities. We extend this setup by adding complementarity between the agents' efforts. Therefore, in this paper the decision rule (the agents' effort) has complementary effect as opposed to Myerson and Satterthwaite [1983] where the decision rule has zero-sum game effect. The complementary channel of decision rule makes the taxation meaningful.

We also assume the full communication between the planer and the agents. That is, government can communicate with the both manager and the worker.[1]

The main results of the model in this chapter are as follows.

First, the effort in the first best depends on both agents' type, the structure of the production technology and the output elasticity of effort. In the presence of private information the first best effort is implementable only for lowest productive agents. If the planner wants to implement the first best effort for other types, he can do it only by subsidizing the organization. This result mirrors the "impossibility of trade" in the presence of private information in Myerson and Satterthwaite [1983].

Second, with private information, there is no distortion at the top, organization. If organizational structure is formed by top worker and manager, there is no distortion of the effort of either. Nevertheless, if one of them is not a top agent, distortion would apply on both of them. These results would be in line with Mirrlees [1971], with the interpretation of organization (i.e. manager and worker) as the agent in his model. On the other hand, these differ with the results of team working in the organization economics; Melumad et al. [1995] show no such distortion for the top agent by the non-type in the organization.This difference stems from the objective function in their model that mechanism designer maximizes his profit rather than welfare. Accordingly, it becomes profitable not to make distortion for the top agent in any case because the top agent increases the designer's profit.

[1] In the next chapter this assumption is changed by the limited communication mechanism in which the planner can communicate just with the manager, and the manager can communicate with the worker, see Kos [2012] and Mookherjee and Tsumagari [2014].

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Third, if government levies a tax on the organization, then the formation of the organization depends on the distribution function of agents' productivity and the production function. Indeed there is no determined answer to the existence of organization, when government imposes tax on it. This result shows how taxation affects existence of organization in the presence of private information.

Fourth, it is not always true that high productive agent exerts higher effort than the low productive agent. The level of effort for the low productive agent depends on the distribution of productivity, the output elasticity of effort, and the amount of tax. For instance, when output elasticity of effort is equal for the manager and the worker-and the government does not impose a tax-the agent who has higher virtual productivity, $(\theta_i + \frac{F_i(\theta_i)}{f_i(\theta_i)})^{-1}$, will exert more effort. Taxation impacts this result.

One could think of the following intuition for the above result. Government favors more effort by the higher productive agent. Yet, not being able to observe the productivity, it relies on the agents' virtual productivity levels, given the same output elasticity of effort and without tax.

Fifth, agent $i$'s effort in comparison with the first best effort is increasing if, $\theta_i \frac{f_i(\theta_i)}{F_i(\theta_i)} > \theta_j \frac{f_j(\theta_j)}{F_j(\theta_j)}$. Again in this case, government compares the agents' distribution to make distortion. If manager is more likely to be productive than the worker, then the planner increases manager's effort more than the worker's effort in comparison with the first best. The crucial point is that we assume the output is same in the first and second best. So, if the planner increases the manager's effort then it should decreases the worker's effort.

### 1.1.1 The Iron Bridge

The aim of this study is to make a bridge between public finance and organization economics. However, developing just a micro model, this paper may only succeed to make a *wooden bridge* between these two areas. There are some difficulties involved with a macro approach - *an iron bridge*- that I discuss below.

Given the existence of a matching function between agents, we can describe the environment in the macro perspective as the following. Assume there are two measures corresponding to the manager and worker. These measures match together with - an endogenous or exogenous- matching function.[2][3] Assuming the matching function as given, the problem reduces to measuring one of organizations with different distribution functions for each pair type because the agents will update their belief by the matching function. The advantage of macro approach is the possibility to explain redistribution, failure, and formation within and between organizations, with the disadvantage that the answer depends

[2]To the best of my knowledge, there is no stable matching function in the presence of asymmetric information on the both sides.

[3]The matching function also determines the bargaining process.

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on the matching function.

To develop a robust iron bridge, one should overcome three challenges. The matching function is the first challenge. This function describes the relationship of the managers and the workers inside the population and the organizations. There are many interesting issues there. For example, do they exist or not, and if they do, are they deterministic or stochastic? Are they stable or not? Do agents know them or not?

The second challenge is the bargaining process in the organization. Do all organizations have same bargaining process? How does a specific bargaining process affect matching function and vice versa?

The third challenge is how government preferences are defined over the organization set. Each organization has two dimensional type[4] and government's incentive given to agents is determined by the order on each dimension. Government has many possibilities to order these two dimensions. Which criteria will it adopt?

### 1.1.2 Literature Review

This paper may find relevant studies in different literature. The first one - as mentioned- is the research on incomplete bargaining process, namely the work ofMyerson and Satterthwaite [1983].

Another relevant literature would be the studies on team work that takes information symmetries into the account. Key examples in this line would be Holmstrom [1982] for the moral hazard and Melumad et al. [1995] for the adverse selection. Holmstrom [1982] shows the impossibility of the Pareto optimal, while the first best is implementable. This paper shows the opposite result, where pareto optimal is possible, but not the first best. The difference between this paper and adverse selection models (e.g. the Melumad et al. [1995]), is very similar to the difference between auction and public good in mechanism design literature. This paper has a different objective function and extra constraint for mechanism designer.

There exists the literature in optimal taxation that they use Bayesian Nash equilibrium concept to implement taxation, Piketty [1993]. This concept makes the first best implementable. The difference of this literature and our model is the complementary of effort and bargaining process.

Furthermore, this paper may relate to the part of organizational economics research that considers failure of organization , Garicano and Rayo [2015][5]. This literature focuses on internal sources of failure in organization, not considering the boundary of firm and optimal policy.

[4]Each dimension for one agent.

[5]We mean just part of this literature that considers asymmetric information as a cause of organizational failure.

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And finally, optimal income taxation would be another relevant literature where the most related papers to this are Ales and Sleet [2015] and Scheuer and Werning [2015]. In their models the workers have private information and the managers know it. Thus there is no asymmetric information inside the organization. The only strategic interaction in which exists are between the government and the worker.

The rest of this paper is organized as follows. Section 2 introduces the model with interior solution. Section 3 considers the boundary solution. Section 4 concludes. All proofs are in appendix.

## 1.2 The Model

### 1.2.1 The Environment

The organization consists of two agents, a manager and a worker. For simplicity we use subscript 1 for the manager and 2 for the worker, $O = \{M, O\} = \{1, 2\}$.[6] The agents have a quasilinear utility function which is $u_i = c_i - \theta_i a_i$, where $c_i$ is the consumption or cash that the agent $i$ consumes or recieves, $c_i \epsilon \Re$ and $a_i$ is an effort or an input of production function, not an income, that the agent $i$ exerts, $a_i \in A_i = [\,\underline{a}, \bar{a}\,] \subset \Re_+$.[7] They choose simultaneously their effort, and it is observable by everyone. Organization has a production function $y = f(a_1, a_2)$, $y : \Re_+^2 \rightarrow \Re_+$. $y$ is also observable by everyone and it is split between them such that $y \geq c_1 + c_2$.

The type of agent $i, \theta_i$ and is her private information. $\theta_i$ belongs to the set $\Theta_i = [\,\underline{\theta}, \bar{\theta}\,] \subset \Re_+$ and $\theta_i$ has a probability distribution which is described by a cumulative distribution function $F_i$. The support of $F_i$ is the interval $[\,\underline{\theta}, \bar{\theta}\,]$ which has a density $f_i$ and $f_i(\theta_i) > 0$ for all $\theta_i \in [\,\underline{\theta}, \bar{\theta}\,]$. $\theta_1$ and $\theta_2$ are independent. We denote by $\theta$ the vector $(\theta_1, \theta_2)$, and the support of the $\theta$ is $\Theta = [\,\underline{\theta}, \bar{\theta}\,]^2$. $F$, the distribution of $\theta$, is the product of distributions $F_i$, and the density function is represented by $f$. $f$ and $f_i$ are common knowledge among the agents and the planner.

Therefore, the environment is described as a incomplete information game. There are two players, $O$, and the agent's strategy is a function from $\Theta_i$ to $A_i$, $S_i : \Theta_i \rightarrow A_i$. The outcome of game is a function from $A_1 \times A_2$ to $y \in \Re_+$, $c_1 \in \Re$, and, $c_2 \in \Re$, $C : A_1 \times A_2 \rightarrow \Re_+ \times \Re \times \Re$.

Assumption 1: $\theta_i + \frac{F_i(\theta_i)}{f_i(\theta_i)}$ is increasing in $\theta_i$.

This assumption means that hazard rate is increasing , $\frac{f_i(\theta_i)}{1-F_i(\theta_i)}$, and it can be explained as the conditional probability of dying at time $\theta_i$ of an individual who has survived until

[6] We use she for the manager, he for the worker, and it for the planner, mechanism designer or government.

[7] $c_i < 0$ means that the agent pays cash or consumes a part of his saving.

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time $\theta_i$.[8] In our setup it can be interpreted as the conditional probability of having productivity $\theta_i$ if the agent is less productive than $\theta_i$, hence $\frac{f_i(\theta_i)}{F_i(\theta_i)}$ means the conditional probability of having productivity $\theta_i$ if the agent is more productive than $\theta_i$.

The risk neutrality assumption of the consumption is a crucial for our results. This assumption is needed in the full communication mechanism because it allows to construct an equivalence mechanism for an ex ante budget balance mechanism such that the equivalence mechanism is an ex post budget balance mechanism.[9] Most importantly, with assumption, the incentive compatible condition in Nash equilibrium concept is more tractable.

The production function is represented by a Cobb-Douglas production function $y = a_1^{\lambda} a_2^{1-\lambda}$, $y : \Re_+^2 \rightarrow \Re_+$.

Indeed this function is increasing in each argument, $a_1$ and $a_2$, and it has the constant return to scale property. The production function is bounded since $a_i$ is bounded. The upper bound is $\bar{Y}$, and there is no ex ante uncertainty in the production function if each $a_i$ is observable.

If the production function is assumed to be a general super modular function, then a boundary solution is one scenario. Suppose the manager chooses the marginal product of her effort. Then, the worker would increase his effort as much as possible since it increases the output and his share. Hence, the marginal product of effort or the first order condition would not make a solution and a boundary solution should be considered. Accordingly, I will consider two scenarios for the production function: The first one is the interior solution and the second one is the boundary solution meaning that the first order condition does not apply.

The linearity of the utility function with respect to effort does not imply that the agents are risk neutral regarding their income. Notably, the agents' income depends on the output function $y$ and the bargaining process, $y = y_1 + y_2$. Suppose that the agents get their marginal product of effort, then manager's income becomes: $y_1 = \frac{\partial y}{\partial a_1} a_1 = \lambda a_1^{\lambda} a_2^{1-\lambda}$. And finally $a_1 = (\frac{y_1}{\lambda})^{\frac{1}{\lambda}} a_2^{\frac{\lambda-1}{\lambda}}$. This means that the wage elasticity of labor supply for the manager is not $\epsilon_1 = \frac{\lambda}{1-\lambda}$ because wage elasticity of labor supply depends on $a_2$ as well, and $a_2$is determined in the equilibrium.[10]

[8]In the mechanism design literature, regularity condition is defined as increasing $\frac{f_i(\theta_i)}{1-F_i(\theta_i)}$ and it implies that $\theta_i + \frac{F_i(\theta_i)}{f_i(\theta_i)}$ and $\theta_i - \frac{1-F_i(\theta_i)}{f_i(\theta_i)}$ are increasing, see Borgers et al. [2015]

[9]Full communication mechanism means that both agents can reveal their types to the planner. In other words there are bijective functions between the sets of types and massages which are revealed to the planner for both agents.

[10]Recall that if individual' utility function is $u = c - \theta y^{\alpha}$ where $y$ is income, then wage elasticity of labor supply equals $\epsilon = \frac{1}{\alpha-1}$

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### 1.2.2 The Direct Mechanism

I denote the effort, type and consumption profiles as the following, $a = (a_1, a_2)$, $\theta = (\theta_1, \theta_2)$, and $c = (c_1, c_2)$.

The planner is utilitarian who maximizes the sum of expected agents' utilities with the same weight. The planner cannot observe the agents' types but he takes into account the agents' incentives. I consider the direct mechanism, $\mathcal{M}$, which means that the planner offers outcome of the game to the agents, $(c, a)$, as a function of agents' types. Accordingly, he takes into account the incentive compatibility and the individual's rationality. Hence, the mechanism is defined as functions, $c : \Theta \to \Re^2$ and $a : \Theta \to [\underline{a}, \bar{a}]^2$, $\mathcal{M}(c(\theta), a(\theta))$.

Definition 1: Direct mechanism, $\mathcal{M}(c(\theta), a(\theta))$ are the consumption and the decision rule functions as the following.

The Consumption Rule: $c = (c_1, c_2) : \Theta \to \Re^2$ so $c_i : \Theta \to \Re$

The Decision Rule: $a = (a_1, a_2) : \Theta \to [\underline{a}, \bar{a}]^2$ so $a_i : \Theta \to [\underline{a}, \bar{a}]$

The planner faces a budget balance constraint as well.[11] This indicates that the total consumption should be smaller than or equal to the total production subtracting the tax in each state $\theta$.[12]

In the other words, the planner's problem is

$$\max_{q(\theta), c(\theta)} \int_{\Theta} (c_1(\theta) - \theta_1 q_1(\theta) + c_2(\theta) - \theta_2 q_2(\theta)) dF(\theta)$$

subject to: Incentive Compatibility (IC),
Individual Rationality (IR)
and the Budget Balance Constraint (BC)

### 1.2.3 IR and IC

$\theta_{-i}$ represents the type of player $j \neq i, i, j \in O$. $F_{-i}$ is the cumulative distribution function of $\theta_{-i}$ and $f_{-i}$ is the density function of $F_{-i}$.

Given the direct mechanism and Bayesian Nash equilibrium of the direct mechanism, we define for each agent $i$ functions $\bar{c}_i$ and $\bar{a}_i$ as the following.

$$\bar{c}_i(\theta_i) = \int_{\Theta_{-i}} c_i(\theta_i, \theta_{-i}) dF_{-i}(\theta_{-i}) \ and \ \bar{a}_i(\theta_i) = \int_{\Theta_{-i}} a_i(\theta_i, \theta_{-i}) dF_{-i}(\theta_{-i})$$

$\bar{c}_i(\theta_i)$ is the expected value of the consumption that agent $i$ consumes conditional on reporting her type $\theta_i$ and agent $j$'s truth telling about his type. $\bar{a}_i(\theta_i)$ is the expected

[11] In the macro approach this constraint could be relaxed for the organization if we let the mechanism designer redistribute among the organizations. Consequently, the budget balance constraint exists for the whole economy.

[12] State and type are used interchangeably.

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value of the effort that agent $i$ makes conditional on reporting her type $\theta_i$ and agent $j$'s truth telling about his type.[13] Thus the agent $i$'s expected utility conditional on her type being $\theta_i$ and agent $j$'s truth telling is given by $\bar{u}_i(\theta_i) = \bar{c}_i(\theta_i) - \theta_i \bar{a}_i(\theta_i)$.[14] Now, with the above definitions we can characterize the incentive compatibility and the individual rationality constraints in the form of the following lemmas.

The incentive compatibility constraint is defined as the following.

For every $\theta_i$ *and* $\theta_i^{'}$,

$$\bar{c}_i(\theta_i) - \theta_i \bar{a}_i(\theta_i) \geq \bar{c}_i(\theta_i^{'}) - \theta_i \bar{a}_i(\theta_i^{'}).$$

**Lemma 1.2.1** *If $\mathcal{M}$ is a truth telling mechanism then $\bar{a}_i(\theta_i)$, $\bar{c}_i(\theta_i)$ and $\bar{u}_i(\theta_i)$ are decreasing. $\bar{u}_i(\theta_i)$ is differentiable and $\frac{d\bar{u}_i(\theta_i)}{d\theta_i} = -\bar{a}_i(\theta_i)$ .*

All proofs are in the appendix A.

Lemma describes the properties of a truthful direct mechanism. All restrictions are imposed on the expected interim of allocation and utility.

The individual rationality constraint is defined as the following.

For every $\theta_i$,

$$\bar{u}_i(\theta_i) = \bar{c}_i(\theta_i) - \theta_i \bar{a}_i(\theta_i) \geq 0$$

**Lemma 1.2.2** *The necessary and sufficient conditions for individual rationality constraint is $\bar{u}_i(\bar{\theta}_i) \geq 0$.*

Lemma implies that it suffices to consider the individual rationality constraint for less productive agents.

**Lemma 1.2.3** *The necessary and sufficient conditions for incentive compatibility constraint are as the following:*

*$\bar{a}_i(\theta_i)$ is decreasing and $\frac{d\bar{u}_i(\theta_i)}{d\theta_i} = -\bar{a}_i(\theta_i)$ .*

Lemma simplifies IC constraints, and it is just envelope theorem and necessity condition of Spence-Mirrlees condition. Hence, instead of infinite IC constraints, we will use just requirements of the above lemma. In fact these conditions put the limitation on the conditional expectation of the decision and the consumption rule, so indirectly they put restriction on the decision and the consumption rule.

[13] $\bar{c}_i(\theta_i)$ and $\bar{a}_i(\theta_i)$ are defined on agent $-i$'s truth telling not agent $i$.

[14] $\bar{u}_i(\theta_i)$ is defined on both agents' truth telling.

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### 1.2.4 Budget Balance

The last constraint to be considered by planner is the budget balance constraint. This constraint implies that the production is enough to cover the consumption and agents' tax (or subsidies) which is net consumption. The planner considers the budget balance constraint in two different forms. Based on the first form of the constant, budget is balanced for each realization of state. It is called the ex post budget balance, and for each $\theta \in \Theta$ we have $y(\theta) \geq c_1(\theta) + c_2(\theta) + k(\theta)$, where $k(\theta)$ is tax function. The second form is satisfied in the expectation, and the realization of the agents' types does not matter though. It is called ex ante budget balance when expected output is equal or greater than agents' expected net consumption, and it looks as follows:

$$\int_{\Theta} y(\theta)dF(\theta) \geq \int_{\Theta} (c_1(\theta) + c_2(\theta) + k(\theta))dF(\theta).$$

It is clear that the ex post budget balance is more restrictive than the ex ante budget balance, and implies the ex post budget balance. In this model agents are risk neutral in the consumption, and it can be proved that an equivalent ex post budget balance mechanism exists for the ex ante budget balance mechanism, see Börgers and Norman [2009] and Kos and Messner [2013] . Two direct mechanisms, $\mathcal{M}_1(a(\theta), c(\theta))$ and $\mathcal{M}_2(\hat{a}(\theta), \hat{c}(\theta))$, are equivalent if they have the same decision rule, $a(\theta) = \hat{a}(\theta)$ for every $\theta$, and if for each agent $i \in O$ and for each type $\theta_i, \hat{\theta}_i \in [\underline{\theta}.\bar{\theta}]$, agent $i$' s expected transfers, conditional on agent $i$' s type being $\theta_i$ and reporting to be type $\hat{\theta}_i$, is the same in the two mechanisms, $\bar{c}_i(\hat{\theta}_i) = \bar{\hat{c}}_i(\hat{\theta}_i)$ for every $\hat{\theta}$. In our model it is sufficeint we consider $\bar{c}_i(\theta_i) = \bar{\hat{c}}_i(\theta_i)$ for every $\theta$ and $i$, where $\bar{c}_i(\theta_i) = \int_{\Theta_{-i}} c_i(\theta_i, \theta_{-i})dF_{-i}(\theta_{-i})$.

Definition 2: Two mechanisms, $\mathcal{M}_1(a(\theta), c(\theta))$ and $\mathcal{M}_2(\hat{a}(\theta), \hat{c}(\theta))$, are equivalent if and only if
$I$: $a(\theta) = \hat{a}(\theta)$ for every $\theta$
$II$: $\bar{c}_i(\theta_i) = \bar{\hat{c}}_i(\theta_i)$ for every $\theta$ and $i$.

**Lemma 1.2.4** *If the mechanism is incentive compatible and individually rational then every equivalent mechanism is also incentive compatible and individually rational.*

The definition is silent about the properties that the equivalent mechanism inherits from the main mechanism. Above lemma shows that if we can construct equivalent mechanism for IC and IR mechanism then it is IC and IR too.

**Lemma 1.2.5** *For every ex ante budget balance mechanism there exists equivalent mechanism that is ex post budget balance.*

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The risk neutrality in consumption is the crucial assumption to get the above lemma. With risk neutrality, the plan to construct the equivalence mechanism is that one agent covers the total ex post budget deficit minus expected budget deficit conditional of her type, and the second agent gives expected budget deficit conditional on the agent one's type. If the risk neutrality assumption is relaxed, the planner should give risk premium to one agent. Without risk neutrality the planner can construct equivalent mechanism, but it is costly for the planner and it affects the ex post budget balance.

If we consider the problem in macro perspective then there is no distinction between the ex ante and ex post budget balance. The law of large numbers implies that both constraints are the same.

### 1.2.5 The First Best Allocation and Pivot Mechanism

In this section we characterize the first best allocation rule despite the existence of the private information, and then we consider possibility of implementing the first best allocation rule in the presence of asymmetric information. Henceforth we construct pivot mechanism, and then show that it is impossible to implement it if the planner considers ex ante budget balance.[15]

The first allocation rule means that the planer does not consider the incentive and individual rationality constraints, and it considers just the budget balance constraint. The problem is as follows:

$$\max_{a(\theta),c(\theta)} u_1(\theta) + u_2(\theta)$$

$$s.to : y(\theta) \geq c_1(\theta) + c_2(\theta) + k(\theta)$$

The budget constraint should be binding otherwise we can increase the agents' consumption, and it increases the objective function, $y(\theta) = c_1(\theta) + c_2(\theta) + k(\theta)$. If we put $y$ in the objective function, the problem is changed and planner computes just the decision rule. In that case to compute consumption rule, $c_i(\theta)$, we need another assumption.

Now the structure of the production function and cost function of the agents shows the problem of interior solution. There are two ways to make the interior solution for the objective function. First and in line with growth theory, we assume $a_1$ or $a_2$. Second way is to put some restriction on total production, $B \geq y(\theta)$. Also this constraint should be binding, $B = y(\theta)$, because it increases the social welfare. We will follow the second way because the distortion will be clear in the second way. If we use the first way, always the boundary is the solution, and we can not compare distortion with other models.

The intuition for the boundary of production is as the following.

[15]Pivot mechanism has two properties. First, pivot mechanism implements the first best allocation and second, no mechanism in which implements the first best allocation exists such that it has largest ex ante surplus than pivot mechanism.

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The agents' endowment, $a_i$, is time and they have two options. The first option is that they spend it for leisure and the second one is that they engage in the organization and use it as an input. They need leisure, so they do not use all resource in the organization.[16] This reasoning means that the solution of effort should be interior then we should make the bound for output.

Also $B < \bar{Y}$, which implies the existence of some $a_i \in intA_i$ for both agents such that $B = a_1^{\lambda} a_2^{1-\lambda}$.

The result of above discussion is as follows:

$$\max_{a_1(\theta),a_2(\theta)} y(\theta) - \theta_1 a_1(\theta) - \theta_2 a_2(\theta)$$

$$s.to: \; B = y(\theta)$$

$k(\theta)$ is not choice variable, but it could be the choice variable and it can show the preference of the planer over the organization set.

**Lemma 1.2.6** *The first best allocation is*

$$a(\theta) = (B(\frac{\theta_2 \lambda}{\theta_1(1-\lambda)})^{1-\lambda}, B(\frac{\theta_2 \lambda}{\theta_1(1-\lambda)})^{-\lambda})$$

.

The lemma presents that the first agent will exert more effort than the second agent if $\theta_2 \lambda > \theta_1(1-\lambda)$. Therefore we can not conclude that if first agent is more productive, $\theta_2 > \theta_1$, then she should exert more effort. The effort depends on agent's elasticity.[17]

Similar to Stiglitz [1982], the production function is nonlinear and non separable in effort, but there is a difference with Stiglitz. In Stiglitz model general equilibrium effect determines bargaining procedure between agents. Hence he gets marginal product of the effort as a wage, so the share of each agent is equal to $y_i = \frac{\partial y}{\partial a_i} a_i$, $y = y_1 + y_2$. Sometimes we use Stiglitz procedure for bargaining power, but we do not interpret it as general equilibrium effect, it is just one way of bargaining procedure.

**Corollary 1.2.7** *Stiglitz bargaining procedure is equivalent to ex post individual rationality constraint.* $c_i = \theta_i a_i$, *or* $u_i = 0$, *is equivalent to* $c_1 = B\lambda$, $c_2 = B(1-\lambda)$ *and vice versa.*

[16] It means that the marginal utility of leisure is infinity in zero leisure.

[17] Agent's elasticity means, output elasticity respect to the agent's effort, $\epsilon_i = \frac{\partial y}{\partial a_i} \frac{a}{y}$, so $\epsilon_1 = \lambda$ and$\epsilon_2 = 1-\lambda$.

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It is clear that this mechanism does not satisfy the incentive compatibility constraint[18]. So does an IC, IR, and BC mechanism that implements the first best allocation rule exist? To answer this question we construct a pivot mechanism. We define consumption as:

$$c_1(\theta_1, \theta_2) = a_1(\bar{\theta}_1, \theta_2)\bar{\theta}_1 - (a_2(\theta_1, \theta_2) - a_2(\bar{\theta}_1, \theta_2))\theta_2$$

$$c_2(\theta_1, \theta_2) = a_2(\theta_1, \bar{\theta}_2)\bar{\theta}_2 - (a_1(\theta_1, \theta_2) - a_1(\theta_1, \bar{\theta}_2))\theta_1$$

The first term does not depend on the agent's reported type, $\theta_i$, but the second term does. The second term is used to align the agent's incentives with social welfare. If the manager reports her type $\bar{\theta_1} > \theta_1$ then she decreases the worker's cost, because $a_2$ is increasing in $\theta_1$. Thus he should exert less effort to produce $B$. The pivot mechanism says that the manager should receive incentive based on the objective function that considers just the worker. In this model there are two agents. Hence, consumption of each agent depends on part of the decision rule that depends to other agent. The first term makes IR constraint binding.

**Lemma 1.2.8** *The pivot mechanism is IC and IR.*

The consumption rule of the pivot mechanism serves to make allocation rule IC. The important point is that these properties are satisfied in the ex post and not just interim, so it is too desirable allocation.

**Lemma 1.2.9** *Pivot mechanism has the largest ex ante expected budget surplus among all mechanisms that implement first best allocation rule.*

Now if we show that pivot mechanism is not ex ante budget balanced then we can conclude that the first best allocation rule is not implementable. The following proposition shows this impossibility.

**Proposition 1.2.10** *Pivot mechanism is budget balance if and only if* $\theta_i = \bar{\theta}_i$ *for both agents.*

The main lesson of the above preposition is impossibility of existence any IC, IR, and budget balance mechanism that implements the first best allocation rule.

However, this proposition does not imply that the first allocation rule is not implantable. It says just if planner wants to implement the first best allocation rule then

[18] Suppose the manager has type $\theta_1$ then she has the incentive to tell her type is $\hat{\theta_1}$, $\hat{\theta_1} > \theta_1$. Because the consumption is same for both types, and cost of effort is higher if she tells truth, $B\lambda - \theta_1 B(\frac{\lambda}{\theta_1}) > B\lambda - \theta_1 B(\frac{\lambda}{\hat{\theta_1}}) \Rightarrow \theta_1 > \hat{\theta_1}$, contadiction.

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it is costly, and it should give this cost as subsidies to the agents. Indeed, if government wants to implement the first best allocation rule with subsidies, it can to choose another consumption rule. Of course, the pivot mechanism is the best mechanism that implements the first best, and it generates the largest revenue or the smallest cost for the planner.

In the above proposition it is implicitly assumed that $k(\theta) = 0$, tax levied on agents, and that there is no tax or subsidy on organization. If we want to assume tax or subsidy on organization then budget balance will change as: $k(\theta) < 0$ which means subsidy, and $k(\theta) > 0$ means tax on organization or corporate taxation. The budget constraint with tax is as the following.

$$y(\theta) \geq c_1(\theta) + c_2(\theta) + k(\theta)$$

**Corollary 1.2.11** *If government wants to collect tax from organization, ex ante budget deficit, then it can not implement the first best allocation rule.*

The lemma says that it is impossible to implement the pivot mechanism for the type $\theta = (\bar{\theta}_1, \bar{\theta}_2)$ if the government wants to levy tax on the organization in this state, $k((\bar{\theta}_1, \bar{\theta}_2)) > 0$.

**Corollary 1.2.12** *If government wants to subsidy, ex ante budget surplus, then it is possible to implement the first best allocation rule for some type.*

Subsidies conditional on the states, $k(\theta) > 0$, makes the resource for the planner to implement pivot mechanism.

As proposition **??** shows, it is possible to implement the first best decision rule just for the least productive agents, and the budget constraint is binding for the agents. This result is same as the result obtained by Myerson and Satterthwaite [1983] for trade mechanism.

**Corollary 1.2.13** *The maximum ex post cost to implement the first best decision rule for profile type $\theta_1, \theta_2$ is*

$$a_1(\bar{\theta}_1, \theta_2)\bar{\theta}_1 + a_2(\theta_1, \bar{\theta}_2)\bar{\theta}_2 + a_2(\bar{\theta}_1, \theta_2)\theta_2 + a_1(\theta_1, \bar{\theta}_2)\theta_1 - 2(a_1(\theta_1, \theta_2)\theta_1 + a_2(\theta_1, \theta_2)\theta_2)$$

We use word maximum for the above corollary because we implicitly assume that government uses pivot mechanism as an equivalence mechanism, so it does not change the consumption rule.

In short, if government wants to collect tax, it is impossible to implement the first best decision rule. In contrast, if governments wants that organization exists with the first best allocation rule, organization should receive subsidy.

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### 1.2.6 The Second Best Allocation

Now, given proposition **??**, the planner should pursue its objective with three constraints, IC, IR, BC and boundary of production, BP. Hence it should choose the allocation rule $a(\theta)$, $c(\theta)$, given $k(\theta)$.

The designer's problem is

$$\max_{a(\theta),c(\theta)} \int_{\Theta} (u_1(\theta) + u_2(\theta))dF(\theta)$$

subject to:

$$IC : \begin{cases} \bar{a}_i(.) \; is \; decreasing \\ \dot{\bar{u}}_i(\theta_i) = -\bar{a}_i(\theta_i) \end{cases},$$

$$IR : \bar{u}_i(\bar{\theta}_i) \geq 0,$$

$$BC : \int_{\Theta} y(\theta)dF(\theta) \geq \int_{\Theta} (c_1(\theta) + c_2(\theta) + k(\theta))dF(\theta)$$

$$BP : \; B \geq y(\theta)$$

There is an importance difference between BP and BC, BP is the ex post constraint, while BC is an ex ante. We can not write the BP constraint as the ex ante constraint because there is no technology to transfer the production between the states.

First, we change $c_i(\theta)$ as decision variable with $u_i(\theta)$. To change $c(\theta)$ with $u_i(\theta)$ we integrate from the envelope theorem in IC, so we get

$$\bar{u}_i(\bar{\theta}_i) - \bar{u}_i(\theta_i) = -\int_{\theta_i}^{\bar{\theta}_i} \bar{a}_i(\hat{\theta}_i)d\hat{\theta}_i.$$

Taking integral over $\Theta_i$ on both sides:

$$\int_{\Theta_i} \bar{u}_i(\theta_i)dF_i(\theta_i) = \int_{\Theta_i} \bar{u}_i(\bar{\theta}_i)dF_i(\theta_i) + \int_{\Theta_i}\int_{\theta_i}^{\bar{\theta}_i} \bar{a}_i(\hat{\theta}_i)d\hat{\theta}_i dF_i(\theta_i) = \bar{u}_i(\bar{\theta}_i) + \int_{\Theta_i} \bar{a}_i(\theta_i)F_i(\theta_i)d\theta_i.$$

The last term comes from Fubini's Theorem[19], then we use the definition of $\bar{u}_i(\theta_i)$ and $\bar{a}_i(\theta_i)$. So the final result is as the following.

$$\int_{\Theta} \bar{u}_i(\theta)dF(\theta) = \bar{u}_i(\bar{\theta}_i) + \int_{\Theta} \bar{a}_i(\theta)\frac{F_i(\theta_i)}{f_i(\theta_i)}dF(\theta).$$

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$$\int_{\Theta_{-i}}\int_{\theta_i}^{\bar{\theta}_i} \bar{a}_i(\hat{\theta}_i)d\hat{\theta}_i dF_i(\theta_i) = \int_{\Theta_i}\int_{\underline{\theta}_i}^{\theta_i} \bar{a}_i(\theta_i)dF_i(\hat{\theta}_i)d\theta_i = \int_{\Theta_i} \bar{a}_i(\theta_i)\int_{\underline{\theta}_i}^{\theta_i} dF_i(\hat{\theta}_i)d\theta_i = \int_{\Theta_i} \bar{a}_i(\theta_i)F_i(\theta_i)d\theta_i$$

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Above algebra shows that objective function becomes:

$$\bar{u}_1(\bar{\theta}_1) + \bar{u}_2(\bar{\theta}_2) + \int_{\Theta} (a_1(\theta)\frac{F_1(\theta_i)}{f_1(\theta_i)} + a_2(\theta)\frac{F_2(\theta_i)}{f_2(\theta_i)})dF(\theta)$$

We ignore other IC constraint, increasing $\bar{a}_i(\theta_i)$, and we will check it later.

Also to eliminate $c(\theta)$ from the budget constraint, we use $c_i(\theta) - \theta_i a_i(\theta)$ instead of $u_i(\theta)$ in the extended envelope theorem. The result is:

$$\int_{\Theta} (c_1(\theta)+c_2(\theta))dF(\theta) = \bar{u}_1(\bar{\theta}_1)+\bar{u}_2(\bar{\theta}_2)+\int_{\Theta} (a_1(\theta)(\theta_1+\frac{F_1(\theta_1)}{f_1(\theta_1)})+a_2(\theta)(\theta_2+\frac{F_2(\theta_1)}{f_2(\theta_1)})dF(\theta)$$

Finally the problem is reduced to choose $\bar{u}_i(\bar{\theta}_i)$ and $a(\theta)$, and the IC constraints do not exist in the constraints. Therefore, the final version of problem looks as follows.

$$\max_{a(\theta),\bar{u}_i(\bar{\theta}_i)} \bar{u}_1(\bar{\theta}_1) + \bar{u}_2(\bar{\theta}_2) + \int_{\Theta} (a_1(\theta)\frac{F_1(\theta_1)}{f_1(\theta_1)} + a_2(\theta)\frac{F_2(\theta_1)}{f_2(\theta_1)})dF(\theta)$$

s.to:

$$\bar{u}_i(\bar{\theta}_i) \geq 0$$

$$\int_{\Theta} a_1^{\lambda}(\theta)a_2^{1-\lambda}(\theta)dF(\theta) \geq \bar{u}_1(\bar{\theta}_1)+\bar{u}_2(\bar{\theta}_2)+\int_{\Theta} (a_1(\theta)(\theta_1+\frac{F_1(\theta_1)}{f_1(\theta_1)})+a_2(\theta)(\theta_2+\frac{F_2(\theta_1)}{f_2(\theta_1)}+k(\theta))dF(\theta)$$

$$B \geq a_1^{\lambda}(\theta)a_2^{1-\lambda}(\theta).$$

The objective function shows the similarity between the revenue equivalence property in auction design and social welfare. Two different mechanisms with the same $a(\theta)$ and $\bar{u}_i(\theta_i)$ have same social welfare, like the revenue in auction design.

Indeed, we can not determine $c(\theta)$ uniquely because $\bar{u}_i(\theta)$ and $a_i(\theta)$ determine $\bar{c}_i(\theta)$. With $\bar{c}_i(\theta)$ we can get different $c(\theta)$, and different $c(\theta)$ means that different income tax rules could exist.

**Proposition 1.2.14** *If the planner does not want to collect tax from organization, $k(\theta) = 0$, then optimal decision rule, $a(\theta)$, and expected consumption, $\bar{c}(\theta)$, are as the following.*

$$a(\theta) = (B(\frac{(\theta_2 + \frac{F_2}{f_2})\lambda}{(\theta_1 + \frac{F_1}{f_1})(1-\lambda)})^{1-\lambda}, B(\frac{(\theta_2 + \frac{F_2}{f_2})\lambda}{(\theta_1 + \frac{F_1}{f_1})(1-\lambda)})^{-\lambda})$$

$$\bar{c}_i(\theta_i) = \theta_i\bar{a}_i(\theta_i) + \int_{\theta_i}^{\bar{\theta}_i} \bar{a}_i(\theta_i)d(\theta_i)$$

Like other problems in mechanism design, auction or bargaining, we could not achieve to consumption $c_i(\theta)$, and we get $\bar{c}_i(\theta_i)$. It shows the difficulty or simplicity of distribution in organization. It is difficult because there is no obvious and unique way to distribute

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net consumption between the agents. It is easy because the planner has many options to choose between them. Thus, the model is silent about distribution of income.

The planner who can communicate fully faces two problems. The first one is distribution of gross income, $(y_1, y_2)$, and the second is net income or consumption in this model, $(c_1, c_2)$. The relation between net and gross income is the tax function, $c_i - y_i = T(y_i)$, but given $T(y_i)$ we could not find $c_i$ or $y_i$, and we need another assumption about $y_i$ or $c_i$ to pin down the net income.

Now we need to check whether the expected decision rule, $\bar{a}_i(\theta_i)$, is decreasing in $\theta_i$. It is obvious that if $\theta_i + \frac{F_i(\theta_i)}{f_i(\theta_i)}$ is increasing in $\theta_i$, then $\bar{a}_i(\theta_i)$ is decreasing.

The implicit assumption is that the interior solution exists then we should make it explicitly.[20] We assume that $a_i \in intA_i = [\,\underline{a}, \bar{a}\,]$ then sufficient and necessary conditions are as the following.

$$a_1(\underline{\theta}, \bar{\theta}) = B(\frac{\bar{\theta}_2\lambda}{(\underline{\theta}_1 + \frac{1}{f_1(\underline{\theta}_1)})(1-\lambda)})^{1-\lambda} \in intA_1$$

and

$$a_2(\bar{\theta}, \underline{\theta}) = B(\frac{\underline{\theta}_2 + \frac{1}{f_2(\underline{\theta}_2)}\lambda}{\bar{\theta}_1(1-\lambda)})^{-\lambda} \in intA_2$$

$a_i(\theta_i, \theta_j)$ is increasing in $\theta_j$ and decreasing in $\theta_i$ then it implies $a_i(\underline{\theta}_i, \bar{\theta}_j) \geq a_i(\theta_i, \theta_j)$ for every $\theta$.

The interior solution assumption helps to compare our results with the literature. Mirrlees [1971] and Stiglitz [1982] in taxation and Melumad et al. [1995] in organization economics get the interior solution. In their model when agents can produce as much as they want then there is no restriction on effort and the interior solution is a result.

The optimal effort, $a_i$, depends to the agent's type, $\theta_i$, his or her partner type, $\theta_{-i}$, output elasticity of effort, $\lambda$, and distribution function, $f_i$ and $f_{-i}$.

The necessary and sufficient condition that manager exerts more effort than worker is as follows:

$$(\theta_2 + \frac{F_2}{f_2})\lambda \geq (\theta_1 + \frac{F_1}{f_1})(1-\lambda)$$

The difference of this criteria with the first best criteria is $\frac{F_i}{f_i}$ on the both sides. The term with $\theta_i$ makes well known term in the mechanism design literature, here virtual productivity. The planner compares agents' virtual productivity rather than just agents' productivity.

Thus, given the same elasticity, $\lambda = \frac{1}{2}$, the agent who is more virtual productive, but not neccessarity productive, exerts more effort: $\theta_2 + \frac{F_2}{f_2} \geq \theta_1 + \frac{F_1}{f_1}$ implies $a_1 \geq a_2$. Therefore, it is possible that the manager is more productive than the worker but she exerts less effort because her virtual productivity is smaller.

[20] We use the first order condition to get this result, and the first order condition assumes the interior solution.

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What is the difference between agent's effort in the first and second best? Suppose that manager exerts more effort in the second best then sufficient and necessary condition is:

$$B(\frac{(\theta_2+\frac{F_2}{f_2})\lambda}{(\theta_1+\frac{F_1}{f_1})(1-\lambda)})^{1-\lambda} \geq B(\frac{\theta_2\lambda}{\theta_1(1-\lambda)})^{1-\lambda}$$

Simplification shows that necessary and sufficient condition is $\theta_1\frac{f_1}{F_1} \geq \theta_2\frac{f_2}{F_2}$. The manager, given type of worker, makes more effort if her hazard rate of being productive times her type is greater then his hazard rate of being productive times his type.

The asymmetric information in organization has two effect. The first effect, planner does not know the manager's and the worker's type then it should make incentive for them such that they report their type truthfully. The term $\frac{F_i(\theta_i)}{f_i(\theta_i)}$ shows this effect. The second effect, is the asymmetric information between the manager and the worker. The manager's and the worker's effort are linked together because of the budget balance constraint, so it makes spillover. The planner considers it via the budget constraint and the utilitarian objective function. The term $\frac{F_{-i}(\theta_{-i})}{f_{-i}(\theta_{-i})}$ shows this effect.

The first effect implies that planer should make distortion $\frac{F_i(\theta_i)}{f_i(\theta_i)}$ for less productive agent. The second effect implies that planer should make distortion base on agent's partner in the organization. It causes that planner makes distortion $\frac{F_{-i}(\theta_{-i})}{f_{-i}(\theta_{-i})}$ for agent $i$ regardless of agent's type.

The first effect is in line with team working literature which considers adverse selection in team in view point of owner. Nonetheless, the second effect is out of line with this literature because the owner does not care the agents' welfare, so budget balance does not have any effect on owner. In fact, owner takes all budget that remains after owner gives share of production to the manager and the worker. Therefore, there is no link between the manager and the worker.

Before we compare our result with Mirrlees and Stiglitz, we take logarithm from production function to make it linear. It makes for easier comparison. Now the production function is represented by $ln(y) = \lambda\ ln(a_1) + (1-\lambda)\ ln(a_2)$. We make the assumption that the share of production is $y_1 = \lambda ln(a_1)$ and $y_2 = (1-\lambda)ln(a_2)$, and also the total production is $y$, with a slight abuse of notation.

In Mirrlessian set up the relation between agents' production and total production is $y(\theta_1, \theta_2) = y_1(\theta_1) + y_2(\theta_2)$. In the Mirrlees setup the planner does not know agents' type, but also there is no externality between agents' effort or production, $a_i$ and $y_i$ depends just to $\theta_i$. It allows the planner to just consider agent's information to design the mechanism. The optimal second best agent $i$'s effort just depends to her/his type $\theta_i$. The distortion is represented by $\frac{F_i(\theta_i)}{f_i(\theta_i)}$ for agent $i$, and there is no problem for bargaining procedure for production because by definition the agents' share is defined as, $y_1, y_2$.

In Stiglitz model the relation between agent's production and total production is $y(\theta_1, \theta_2) = \lambda\ y_1(\theta_1, \theta_2)\ +(1-\lambda)\ y_2(\theta_1, \theta_2)$. In this environment there are two implicit

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assumptions. First, the agents' share in production is given. Second, the agents know the partner's type so there is no asymmetric information between agents. For this reason the mechanism is based on the agents' ex post utility, and it is not based on the agents' expected interim utility. [21] However, there is an externality between agents, each agent's income depends on the agent's partner type. So the second best agents' effort depends on both types, but distortion depends just on agent's type, $\frac{F_i(\theta_i)}{f_i(\theta_i)}$.

Another difference between our model and the above models is that those models are macro model and government can redistribute between many agents, but in this model government can redistribute between two agents. We can diminish this problem if we assume that government can collect tax $k(\theta) > 0$ from organization or can subside $k(\theta) < 0$ to organization. As a result this mechanism lets government to redistribute between the agents outside of the organization.

Example: Suppose a firm or an organization consists of one manager and one worker. The distribution of the manager's productivity is $F_1(\theta_1) = \theta_1^2$ and worker is $F_2(\theta_2) = 2\theta_2 - \theta_2^2$ for every $\theta_i \in [0, 1]$. Thus, the worker is more likely to be high productive than the manager, because low $\theta_i$ means that agent is more productive.

The firm has access to three different technologies, different $\lambda$, as $\lambda = \frac{1}{2}$, $\frac{1}{3}$, and $\frac{2}{3}$.

The first case, $\lambda = \frac{1}{2}$: In the first best scenario it is obvious that manager induces more effort if $\theta_2 \geq \theta_1$, and the profile of effort is $(B(\frac{\theta_2}{\theta_1})^{.5}, B(\frac{\theta_2}{\theta_1})^{-.5})$.

In the second best, manager exerts more effort than worker if $(\theta_2 + \frac{F_2}{f_2}) \geq (\theta_1 + \frac{F_1}{f_1})$.[22] Suppose the mean manager, $\theta_1 = \frac{1}{2}$, wants to make an organization if she makes the organization with the worker which his type belongs to the set $(\frac{1}{3}, 1]$ then she will exert higher effort than the worker. It is interesting because in the interval $(\frac{1}{3}, .5)$ we expect and it is optimal that worker exerts more effort, worker is more productive. This result comes from virtual productivity that matters in decision making of the planner.

The second best effort of manager is greater than her first best effort if the type of worker belongs to the set $(\frac{2}{3}, 1]$. So for example the most productive manager does not exert more effort than his first best unless worker's type is greater than $\frac{2}{3}$.[23]

The second case, $\lambda = \frac{1}{3}$: The first base case now depends to $\lambda$, so manager will make more effort than worker if $\theta_2 \geq 2\theta_1$, it means that manager should be two times more productive than worker.

The manager exerts more effort than worker if $(\theta_2 + \frac{F_2}{f_2})\lambda \geq (\theta_1 + \frac{F_1}{f_1})(1 - \lambda)$. It implies that $\theta_2 + \frac{2\theta_2 - \theta_2^2}{2 - 2\theta_2} \geq 2(\theta_1 + \frac{\theta_1^2}{2\theta_1})$. Now mean manager, $\theta_1 = \frac{1}{2}$ exerts more effort than the worker if worker's type $\theta_2$ belongs to $(\frac{7-\sqrt{13}}{6}, 1]$.

[21] It is not precise, but somehow the concept of dominant strategy is used in Stiglitz mechanism.

[22] From condition, $\theta_2 + \frac{2\theta_2 - \theta_2^2}{2 - 2\theta_2} \geq \theta_1 + \frac{\theta_1^2}{2\theta_1}$. Puting $\theta_1 = \frac{1}{2}$ and solving inequality gives the result.

[23] $\theta_1 \frac{2\theta_1}{\theta_1^2} \geq \theta_2 \frac{2 - 2\theta_2}{2\theta_2 - \theta_2^2}$, and it implies that $\theta_2 \in (\frac{2}{3}, 1]$ for every $\theta_1$.

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The manager's second best effort is greater than her first best effort if the worker's type belongs to the set $(\frac{2}{3}, 1]$. The result is the same as the case $\lambda = .5$ because we compare the same agent, same $\lambda$.

The third case, $\lambda = \frac{2}{3}$: Again the first base case depends to $\lambda$. The condition that manager makes more effort than worker is $2\theta_2 \geq \theta_1$. It means that worker should be two times more productive than manager.

The condition that manager exerts more effort than worker is $2(\theta_2 + \frac{2\theta_2 - \theta_2^2}{2-2\theta_2}) \geq \theta_1 + \frac{\theta_1^2}{2\theta_1}$. Now mean manager, $\theta_1 = \frac{1}{2}$ exerts more effort than the worker if the worker's type $\theta_2$ belongs to $(\frac{19-\sqrt{217}}{24}, 1]$. The comparison of manager effort in the first and second best is the same as before because it is independent of $\lambda$.

So far we assume that government does not levy tax on the organization, and it wants just to redistribute output in the organization between agents. Now we change this assumption, and we let the government imposes tax on the organization, $k(\theta) > 0$. We consider tax and not subsidy because then it is possible to use the pivot mechanism for some type, the first best.

We assume that there is no collusion between the agents in the organization. Taxation affects the organization such that they can collude to misreport their type: they can decrease tax if they lie. In addition, when government are informed about agent's types, it can inform agent's partner about agent's type. We assume that these channels do not exist in the mechanism, and the planner asks the agent's type without collusion and reporting it to the agent's partner.

$k(\theta)$ represents the tax in each state, but we use the ex ante budget balance then $K$ matters for the planner. Therefore, it is possible we get the same $K$ with two different tax rule, $k(\theta)$ and $\bar{k}(\theta)$, such that $K = \int\limits_{\Theta} \bar{k}(\theta)dF(\theta) = \int\limits_{\Theta} k(\theta)dF(\theta)$.

$k(\theta)$ cannot be interpreted as an income tax because we can divide $k(\theta)$ between agents, $k(\theta) = T_1(\theta) + T_2(\theta)$, then every $T_i(\theta)$ will be solution, it is enough just we define $T_{-i}(\theta) = k(\theta) - T_i(\theta)$

Given tax $K > 0$ it is not obvious that the optimal mechanism exists or not and it depends to the agents' distribution functions and effort function. The effort function is linear so it makes the problem linear and in a linear problem the interior solution is critical.

The following proposition shows the difficulties that arise in existence of solution in our environment.

**Proposition 1.2.15** *If the planer wants to collect tax from organization, $K > 0$, and $\theta_1 \frac{f_1(\theta_1)}{F_1(\theta_1)} \neq \theta_2 \frac{f_2(\theta_2)}{F_2(\theta_2)}$ , then optimal interior decision rule, $a(\theta)$, and expected consumption, $\bar{c}(\theta)$, are:*

$$a(\theta) = (B(\frac{(\theta_2 + \frac{F_2}{f_2} - \frac{1}{\mu}\frac{F_2}{f_2})\lambda}{(\theta_1 + \frac{F_1}{f_1} - \frac{1}{\mu}\frac{F_1}{f_1})(1-\lambda)})^{1-\lambda}, B(\frac{(\theta_2 + \frac{F_2}{f_2} - \frac{1}{\mu}\frac{F_2}{f_2})\lambda}{(\theta_1 + \frac{F_1}{f_1} - \frac{1}{\mu}\frac{F_1}{f_1})(1-\lambda)})^{-\lambda})$$

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$$\bar{c}_i(\theta_i) = \theta_i \bar{a}_i(\theta_i) + \int_{\theta_i}^{\bar{\theta}_i} \bar{a}_i(\theta_i) d(\theta_i)$$

*if and only if there exists $\mu \geq 0$, and $\mu$ satisfies NBC.*

$$B - K = \int\limits_{\Theta} (a_1(\theta)(\theta_1 + \frac{F_1(\theta_1)}{f_1(\theta_1)}) + a_2(\theta)(\theta_2 + \frac{F_2(\theta_1)}{f_2(\theta_1)}) dF(\theta))$$

*or*

$$\frac{B-K}{B} = \int\limits_{\Theta}$$

$$(\frac{(\theta_2 + \frac{F_2}{f_2} - \frac{1}{\mu}\frac{F_2}{f_2})\lambda}{(\theta_1 + \frac{F_1}{f_1} - \frac{1}{\mu}\frac{F_1}{f_1})(1-\lambda)})^{1-\lambda}(\theta_1 + \frac{F_1(\theta_1)}{f_1(\theta_1)}) + (\frac{(\theta_2 + \frac{F_2}{f_2} - \frac{1}{\mu}\frac{F_2}{f_2})\lambda}{(\theta_1 + \frac{F_1}{f_1} - \frac{1}{\mu}\frac{F_1}{f_1})(1-\lambda)})^{-\lambda}(\theta_2 + \frac{F_2(\theta_1)}{f_2(\theta_1)}) dF(\theta)$$

*and for every $\theta$ there exists $\eta_\theta \geq 0$ such that*

$$\eta_\theta = \mu + \frac{\theta_1 \frac{F_2(\theta_2)}{f_2(\theta_2)} - \theta_2 \frac{F_1(\theta_1)}{f_1(\theta_1)}}{(1-\lambda)(\theta_1 + \frac{F_1(\theta_1)}{f_1(\theta_1)}) a_1^{\lambda}(\theta) a_2^{-\lambda}(\theta) - \lambda(\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)}) a_1^{\lambda-1}(\theta) a_2^{1-\lambda}(\theta)}$$

There are three conditions for the existence of solution.

First, the above solution is not the interior solution of the problem, $\mu$ and $\eta_\theta$ do not exist, if $\theta_1 \frac{f_1(\theta_1)}{F_1(\theta_1)} = \theta_2 \frac{f_2(\theta_2)}{F_2(\theta_2)}$. This problem comes from the linearity of the cost function and it seems we can overcome it if we choose non linear cost function. It makes both constraints, NBC and BP, have the same weight for the planner, therefore the boundary solution will be the solution because of linearity of $a_i$ in the objective function.

The second condition is $\mu \geq 0$ and satisfaction of this condition depends on the agents' distribution function and $K$. Therefore, if there does not exist any $\mu \geq 0$, then there is no interior solution given the parameter of environment.

The last condition, $\eta_\theta \geq 0$ for every $\theta$, depends on $\mu$ and the agents' productivity function. If the difference of agent's virtual productivity is larger then the chance that $\eta_\theta \geq 0$ is smaller.

The second condition is more demanding than the first and last one. If $\mu$ does not exist for some given $k$ then there is no way that the planner levies tax on the organization. On the other hand, if the first and last conditions do not hold then we can construct the new set $A$ such that for every $\theta \in A$ those conditions are satisfied, $\theta_1 \frac{f_1(\theta_1)}{F_1(\theta_1)} \neq \theta_2 \frac{f_2(\theta_2)}{F_2(\theta_2)}$ and $\eta_\theta \geq 0$. Then we should check again the solution on the new set $A$.

**Corollary 1.2.16** *Function from $K$ to $\mu$ is strictly increasing, given existence of solution.*

This corollary does not say that $\mu$ exists for every $K$. It just implies that if $\mu$ exists then it is increasing in $K$ and it means that distortion in the second best with tax is increasing if we compare it with the second best without tax.

Finally, the existence of organization if the governments want to impose tax on it depends on the agent's distribution function and we can conclude with high probability it does not exist if the difference of virtual productivity is large between the agents.

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## 1.3 Conclusion

This chapter considers the environment in which two agents who have private information about their productivity have this opportunity to engage in the organization. If they engage in the organization then they should exert the costly effort. The planner wants to design the mechanism such that it determines the agents' effort and the agents' consumption. In addition, the planner considers tax on the organization. We characterize the effort and the consumption without tax. On the other hand, the existence of the organization in the presence of tax is not obvious and it depends on the production function and distribution function. The view and simple results of this paper, the role of private information in the organization, raise the question about optimal taxation model. What is the cost of ignorance of the private information in organization for taxation and what is optimal tax given existence of organization?

## 1.4 Appendix

Lemma 1.2.1:

Given Bayesian Nash Equilibrium, common knowledge of distribution function, and the agents are expected utility maximizer:

Given Bayesian Nash Equilibrium (truth telling of agent $(-i)$) and known distribution of agent $-i$ for agent $i$, expected agent $i$'s consumption and effort when she or he reports her or his type $\theta_i$ are as the following:

$$\bar{c}_i(\theta_i) = \int\limits_{\Theta_{-i}} c_i(\theta_i, \theta_{-i}) dF_{-i}(\theta_{-i}),$$

$$\bar{a}_i(\theta_i) = \int\limits_{\Theta_{-i}} a_i(\theta_i, \theta_{-i}) dF_{-i}(\theta_{-i}).$$

Given agent $i$'s incentive compatibility constraints for every $\theta_i$ *and* $\theta_i^{'}$,

$$\bar{c}_i(\theta_i) - \theta_i \bar{a}_i(\theta_i) \geq \bar{c}_i(\theta_i^{'}) - \theta_i \bar{a}_i(\theta_i^{'})$$

and

$$\bar{c}_i(\theta_i^{'}) - \theta_i^{'} \bar{a}_i(\theta_i^{'}) \geq \bar{c}_i(\theta_i) - \theta_i^{'} \bar{a}_i(\theta_i)$$

Adding two above inequalities and some basic algebraic calculations imply

$$(\theta_i^{'} - \theta_i)(\bar{a}_i(\theta_i) - \bar{a}_i(\theta_i^{'})) \geq 0.$$

It is obvious that $a(\theta_i)$ is decreasing.

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Multiplication first inequality in $\theta_i^{'}$ and second inequality of in $\theta_i$ and adding two inequality give

$$(\theta_i^{'} - \theta_i)(\bar{c}_i(\theta_i) - \bar{c}_i(\theta_i^{'})) \geq .$$

It is obvious that $c_i(\theta_i)$ is decreasing in $\theta_i$.

From corollary 1 of Milgrom and Segal (2002), $\bar{u}_i(\theta_i)$ is differentiable and differential of $\bar{u}_i(\theta_i)$ is equal $-\bar{a}_i(\theta_i)$,

$$\frac{d\bar{u}_i(\theta_i)}{d\theta_i} = -\bar{a}_i(\theta_i).$$

Because of $a_i(\theta) \geq 0$ and definition of $\bar{a}_i(\theta_i)$ then $\bar{u}_i(\theta_i)$ is decreasing, $\frac{d\bar{u}_i(\theta_i)}{d\theta_i} \leq 0$

Lemma 1.2.2:

Given that the outside option is equal zero, interim individual rationality and decreasing $\bar{u}_i(\theta_i)$,

$$\bar{u}_i(\bar{\theta}_i) \geq 0, \;\; \bar{c}_i(\bar{\theta}_i) - \bar{\theta}_i \bar{a}_i(\bar{\theta}_i) \geq 0$$

Lemma 1.2.3:

Lemma 1.2.1 shows necessity condition so we show sufficient condition. we should show for every $\theta$ and $\hat{\theta}$,

$$\bar{c}_i(\theta_i) - \theta_i \bar{a}_i(\theta_i) \geq \bar{c}_i(\hat{\theta}_i) - \theta_i \bar{a}_i(\hat{\theta}_i).$$

$$\bar{c}_i(\theta_i) - \theta_i \bar{a}_i(\theta_i) \geq \bar{c}_i(\hat{\theta}_i) - \theta_i \bar{a}_i(\hat{\theta}_i) \Longleftrightarrow \bar{u}_i(\theta_i) \geq \bar{u}_i(\hat{\theta}_i) + (\hat{\theta}_i - \theta_i)\bar{a}_i(\hat{\theta}_i) \Longleftrightarrow$$

because $u_i$ is diffrentiable so it is integrable and also by definition $a_i$ is integrable so we can take integral.

$$\int_{\hat{\theta}_i}^{\theta_i} \frac{d\bar{u}_i(\tilde{\theta}_i)}{d\tilde{\theta}_i} d(\tilde{\theta}_i) \geq -\int_{\hat{\theta}_i}^{\theta_i} \bar{a}_i(\hat{\theta}_i) d(\tilde{\theta}_i)$$

suppose $\theta_i > \tilde{\theta}_i$ then $\frac{d\bar{u}_i(\tilde{\theta}_i)}{d\tilde{\theta}_i} = -\bar{a}_i(\tilde{\theta}_i) \geq -\bar{a}_i(\hat{\theta}_i)$ because $\tilde{\theta}_i \geq \hat{\theta}_i$ and $\bar{a}_i(\theta_i)$ is decreasing.

suppose $\tilde{\theta}_i > \theta_i$ then, change upper, lower bound, and sign of integral, $-\dot{\bar{u}}_i(\tilde{\theta}_i) = \bar{a}_i(\tilde{\theta}_i) \geq \bar{a}_i(\hat{\theta}_i)$ because $\hat{\theta}_i \geq \tilde{\theta}_i$ and $\bar{a}_i(\theta_i)$ is decreasing.

Lemma 1.2.4:

Suppose $\mathcal{M}_1(a(\theta), c(\theta))$ and $\mathcal{M}_2(a(\theta), \hat{c}(\theta))$ are equivalent and $\mathcal{M}_1$ is IC and IR.

The definition, $\bar{c}(\theta) = \bar{\hat{c}}(\theta)$ *for every* $\theta$, implies that if $\mathcal{M}_1$ is IC and IR then $\mathcal{M}_2$ is IC and IR.

Because $\mathcal{M}_1$ is IC and IR then $a(\theta) = \hat{a}(\theta)$ for every $\theta$ holds so it is enough to change $\bar{c}(\theta)$ with $\bar{\hat{c}}(\theta)$. IR is obvious.

Lemma 1.2.5:

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The main idea is to use agents' risk neutrality to share ex post budget deficit between them.

Suppose $\mathcal{M}_1(a(\theta), c(\theta))$ is ex ante budget balanced and IC and IR,then, wee construct $\mathcal{M}_2(a(\theta), \hat{c}(\theta))$ such that

$$\hat{c}_1(\theta) = c_1(\theta) + (y(\theta) - c_1(\theta) - c_2(\theta)) - (\bar{y}(\theta_1) - \bar{c}_1(\theta_1) - \bar{c}_2(\theta_1))$$

and

$$\hat{c}_2(\theta) = c_2(\theta) + (\bar{y}(\theta_1) - \bar{c}_1(\theta_1) - \bar{c}_2(\theta_1)).$$

$\bar{c}_2(\theta_1)$ means the expected consumption of agent two when he tells truth and agent one tells her type is $\theta_1$,

$$\bar{c}_2(\theta_1) = \int\limits_{\Theta_2} c_2(\theta_1, \theta_2) dF_2(\theta_2)$$

.

The first case: suppose that we have ex ante budget balance,

$$\int\limits_{\Theta} (c_1(\theta) + c_2(\theta)) dF(\theta) = \int\limits_{\Theta} y(\theta) dF(\theta)$$

.

$$\bar{\hat{c}}_1(\theta_1) = \int\limits_{\Theta_1} (c_1(\theta) + (y(\theta) - c_1(\theta) - c_2(\theta)) - (\bar{y}(\theta_1) - \bar{c}_1(\theta_1) - \bar{c}_2(\theta_1))) dF_1(\theta_1) =$$

$$\bar{c}_1(\theta_1) + \int\limits_{\theta_1} (y(\theta) - c_1(\theta) - c_2(\theta)) dF_1(\theta_1) - (\bar{y}(\theta_1) - \bar{c}_1(\theta_1) - \bar{c}_2(\theta_1))$$

By definition,

$$\int\limits_{\theta_1} (y(\theta) - c_1(\theta) - c_2(\theta)) dF_1(\theta_1) = (\bar{y}(\theta_1) - \bar{c}_1(\theta_1) - \bar{c}_2(\theta_1))$$

so it is not dependent to assumption about budget balanced.

$$\bar{\hat{c}}_2(\theta_2) = \int\limits_{\Theta_2} (c_2(\theta) + (\bar{y}(\theta_1) - \bar{c}_1(\theta_1) - \bar{c}_2(\theta_1))) dF_2(\theta_2) = \bar{c}_2(\theta_2) + \int\limits_{\Theta} (y(\theta) - (c_1(\theta) + c_2(\theta))) dF(\theta)$$

, the last equality comes from definition. Ex ante budget balance implies $\bar{\hat{c}}_2(\theta_2) = \bar{c}_2(\theta_2)$ The second case: suppose that we have ex ante budget surplus.

$$\int\limits_{\Theta} (y(\theta) - (c_1(\theta) + c_2(\theta)) dF(\theta)) = k \geq 0$$

.

It is enough to subtract $k$ from $\hat{c}_2$,

$$\hat{c}_2(\theta) = c_2(\theta) + (\bar{y}(\theta_1) - \bar{c}_1(\theta_1) - \bar{c}_2(\theta_1)) - k$$

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Lemma 1.2.6:

Given existence of interior solution we take partial derivative respect to $a_i$ and then use the constraint $B = y$.

Corollary 1.2.7:

Euler condition says $\frac{\partial y_1}{\partial a_1} a_1 + \frac{\partial y_2}{\partial a_2} = \theta_1 a_1 + \theta_2 a_2$ and it implies.

$$\frac{\partial y_1}{\partial a_1} = \theta_1 \Longleftrightarrow \theta_1 a_1 = \lambda B$$

$$\frac{\partial y_2}{\partial a_2} = \theta_2 \Longleftrightarrow \theta_2 a_2 = (1-\lambda) B$$

Lemma 1.2.8:

We prove strong IR and IC.

For IC just put the definition of $c_i(\bar{\theta}_i, \theta_j)$ in $u_i(\bar{\theta}_i, \theta_j)$

$$u_1(\bar{\theta}_1, \theta_2) = c_1(\bar{\theta}_1, \theta_2) - \bar{\theta}_1 a_1(\bar{\theta}_1, \theta_2) = a_1(\bar{\theta}_1, \theta_2)\bar{\theta}_1 - (a_2(\bar{\theta}_1, \theta_2) - a_2(\bar{\theta}_1, \theta_2))\theta_2 - \bar{\theta}_1 a_1(\bar{\theta}_1, \theta_2) = 0$$

$$u_2(\theta_1, \bar{\theta}_2) = c_2(\theta_1, \bar{\theta}_2) - \bar{\theta}_2 a_2(\theta_1, \bar{\theta}_2) = a_2(\theta_1, \bar{\theta}_2)\bar{\theta}_2 - (a_1(\theta_1, \bar{\theta}_2) - a_1(\theta_1, \bar{\theta}_2))\theta_1 - \bar{\theta}_2 a_2(\theta_1, \bar{\theta}_2) = 0$$

Now strong IC

$$c_1(\theta_1, \theta_2) - \theta_1 a_1(\theta_1, \theta_2) \geq c_1(\hat{\theta}_1, \theta_2) - \theta_1 a_1(\hat{\theta}_1, \theta_2) \Longleftrightarrow$$

$$a_1(\bar{\theta}_1, \theta_2)\bar{\theta}_1 - (a_2(\theta_1, \theta_2) - a_2(\bar{\theta}_1, \theta_2))\theta_2 - \theta_1 a_1(\theta_1, \theta_2) \geq$$

$$a_1(\bar{\theta}_1, \theta_2)\bar{\theta}_1 - (a_2(\hat{\theta}_1, \theta_2) - a_2(\bar{\theta}_1, \theta_2))\theta_2 - \theta_1 a_1(\hat{\theta}_1, \theta_2) \Longleftrightarrow$$

$$-a_2(\theta_1, \theta_2)\theta_2 - \theta_1 a_1(\theta_1, \theta_2) \geq -a_2(\hat{\theta}_1, \theta_2)\theta_2 - \theta_1 a_1(\hat{\theta}_1, \theta_2) \Longleftrightarrow$$

$$B - a_2(\theta_1, \theta_2)\theta_2 - \theta_1 a_1(\theta_1, \theta_2) \geq B - a_2(\hat{\theta}_1, \theta_2)\theta_2 - \theta_1 a_1(\hat{\theta}_1, \theta_2) \Longleftrightarrow$$

$$a_1(\theta_1, \theta_2)^{\lambda} a_2(\theta_1, \theta_2)^{(1-\lambda)} - a_2(\theta_1, \theta_2)\theta_2 - \theta_1 a_1(\theta_1, \theta_2) \geq$$

$$a_1(\hat{\theta}_1, \theta_2)^{\lambda} a_2(\hat{\theta}_1, \theta_2)^{(1-\lambda)} - a_2(\hat{\theta}_1, \theta_2)\theta_2 - \theta_1 a_1(\hat{\theta}_1, \theta_2) \Longleftrightarrow 0 \geq 0$$

Last inequality comes from definition of the first best allocation and Euler condition. With same algebra we can get IC for agent 2

Lemma 1.2.9:

Ex post budget deficit is defined as the following

$$\int_{\Theta} y(\theta) dF(\theta) - \int_{\Theta} (c_1(\theta) + c_2(\theta)) dF(\theta).$$

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Our aim is to maximize above expression and $\int_{\Theta} y(\theta)dF(\theta)$ is same for all the first best allocation rule. We can write remained part of budget deficit as the following and it is enough to maximize the following.

$$-\int_{\Theta_1} \bar{c}_1(\theta_1)dF(\theta_1) - \int_{\Theta_2} \bar{c}_2(\theta_2)dF(\theta_2) =$$

$$-(\int_{\Theta_1} (\bar{u}_1(\theta_1) - \theta_1\bar{a}_1(\theta_1))dF(\theta_1) + \int_{\Theta_2} (\bar{u}_2(\theta_2) - \theta_2\bar{a}_2(\theta_2))dF(\theta_2)).$$

Again the allocation rule is same for all mechanism that we consider in this lemma so it is enough to maximize

$$-(\int_{\Theta_1} \bar{u}_1(\theta_1)dF(\theta_1) + \int_{\Theta_2} \bar{u}_2(\theta_2)dF(\theta_2)) =$$

$$-(\int_{\Theta_1} (\bar{u}_1(\bar{\theta}_1) + \int_{\theta_1}^{\bar{\theta}_1} \bar{a}_1(\hat{\theta}_1)d(\hat{\theta}_1))dF(\theta_1) + \int_{\Theta_2} (\bar{u}_2(\bar{\theta}_2) + \int_{\theta_2}^{\bar{\theta}_2} \bar{a}_2(\hat{\theta}_2)d(\hat{\theta}_2))dF(\theta_2))$$

with same reasoning it is enough to maximize

$$-(\int_{\Theta_1} (\bar{u}_1(\bar{\theta}_1)dF(\theta_1) + \int_{\Theta_2} \bar{u}_2(\bar{\theta}_2)dF(\theta_2)) = -(\bar{u}_1(\bar{\theta}_1) + \bar{u}_2(\bar{\theta}_2))$$

in the pivot mechanism. Given $\bar{u}_i(\bar{\theta}_i) = 0$ it is obvious that pivot mechanism has the largest ex ante budget surplus.

Proposition 1.2.10:

We start with ex post budget deficit and we will show that budget deficit exists for all type except $(\bar{\theta_1}, \bar{\theta_2})$. We use definition of $c_i$ for pivot mechanism and Euler condition for production.

$$c_1(\theta_1, \theta_2) + c_2(\theta_1, \theta_2) - y(\theta_1, \theta_2) =$$

$$a_1(\bar{\theta}_1, \theta_2)\bar{\theta}_1 - (a_2(\theta_1, \theta_2) - a_2(\bar{\theta}_1, \theta_2))\theta_2 + a_2(\theta_1, \bar{\theta}_2)\bar{\theta}_2 - (a_1(\theta_1, \theta_2)$$

$$-a_1(\theta_1, \bar{\theta}_2))\theta_1 - \theta_1 a_1(\theta_1, \theta_2) - \theta_2 a_2(\theta_1, \theta_2)$$

$$a_1(\bar{\theta}_1, \theta_2)\bar{\theta}_1 + a_1(\theta_1, \bar{\theta}_2)\theta_1 - 2a_1(\theta_1, \theta_2)\theta_1 + a_2(\bar{\theta}_1, \theta_2)\bar{\theta}_2 + a_2(\theta_1, \bar{\theta}_2)\bar{\theta}_2 - 2a_2(\theta_1, \theta_2)\theta_2 =$$

, (put the first best allocation instead of $a_i$ and basic algebra computation),

$$\Pi(\theta_1, \theta_2) = B(\frac{\lambda}{1-\lambda})^{-\lambda}\frac{1}{1-\lambda}\theta_2^{1-\lambda}\theta_1^{\lambda}[(\frac{\bar{\theta}_2}{\theta_2})^{1-\lambda} + (\frac{\bar{\theta}_1}{\theta_1})^{\lambda} - 2]$$

It is obvious if $\theta = (\bar{\theta}_1, \bar{\theta}_2)$ then $\Pi(\bar{\theta}_1, \bar{\theta}_2) = 0$ otherwise $\theta \neq (\bar{\theta}_1, \bar{\theta}_2)$ $\Pi(\theta) > 0$, then with probability one we have ex ante budget deficit.

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Corollary 1.2.11:

It is trivial from last preposition that $\Pi + k > 0$ when $k < 0$

Corollary 1.2.12:

Subsidy means that $k > 0$. $\Pi(\theta_1, \bar{\theta}_2)$ is continuous and decreasing in $\theta_1$ then it is possible to find $\theta_1$ such that $\Pi + k > 0$

Corollary 1.2.13:

From definition of consumption rule and Euler condition

$$c_1(\theta_1, \theta_2) + c_2(\theta_1, \theta_2) - y(\theta_1, \theta_2) =$$

$$a_1(\bar{\theta}_1, \theta_2)\bar{\theta}_1 - (a_2(\theta_1, \theta_2) - a_2(\bar{\theta}_1, \theta_2))\theta_2 + a_2(\theta_1, \bar{\theta}_2)\bar{\theta}_2 - (a_1(\theta_1, \theta_2) - a_1(\theta_1, \bar{\theta}_2))\theta_1 -$$

$$(a_1(\theta_1, \theta_2)\theta_1 + a_2(\theta_1, \theta_2)\theta_2) =$$

$$a_1(\bar{\theta}_1, \theta_2)\bar{\theta}_1 + a_2(\theta_1, \bar{\theta}_2)\bar{\theta}_2 + a_2(\bar{\theta}_1, \theta_2)\theta_2 + a_1(\theta_1, \bar{\theta}_2)\theta_1 - 2((a_1(\theta_1, \theta_2)\theta_1 + a_2(\theta_1, \theta_2)\theta_2))$$

Proposition 1.2.14:

This maximization is functional optimization so we should care about the concept we use to solve it.

1- There are two constraint sets, IR and BC, so the solution should satisfy these constraints. We take intersection of these sets, and it means that we should add these constraints. If solution satisfies the new constraint so for sure it satisfies the old ones. The new constraint is as the following. We call it new budget constraint, NBC.

$$\int_{\Theta} a_1^{\lambda}(\theta) a_2^{1-\lambda}(\theta) dF(\theta) \geq (\int_{\Theta} (a_1(\theta)(\theta_1 + \frac{F_1(\theta_1)}{f_1(\theta_1)}) + a_2(\theta)(\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)}) dF(\theta)) \; NBC$$

2- NBC should be binding because the objective function is increasing in $a_i$. Suppose there is a solution, $(a_1, a_2)$, but the NBC is not binding. Then for some set $\mathcal{I} \subset \Theta$ such that $\mathcal{I} = \mathcal{I}_1 \times \mathcal{I}_2$ and $\mathcal{I}_i \subset \Theta_i$ which they do not have measure zero for sure, $\mathcal{I},\ \mathcal{I}_1,\ and\, \mathcal{I}_2$ ,we have the following strict inequality.

$$\int_{\mathcal{I}} a_1^{\lambda}(\theta) a_2^{1-\lambda}(\theta) dF(\theta) > (\int_{\mathcal{I}} (a_1(\theta)(\theta_1 + \frac{F_1(\theta_1)}{f_1(\theta_1)}) + a_2(\theta)(\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)}) dF(\theta))$$

Now suppose we fix some $\theta_2 \in \mathcal{I}_2$ such that $\bar{\mathcal{I}}_1 = \{\theta_1 \in \mathcal{I}_1 : s.t\ (\theta_1, \theta_2) \in \mathcal{I}\}$ does not have measure zero. This set exists otherwise $\mathcal{I}$ has measure zero.

Given $\theta_2$ and fixing function $a_2$, both functions $a_1(\theta)(\theta_1 + \frac{F_1(\theta_1)}{f_1(\theta_1)})$ and $a_1^{\lambda}(\theta) a_2^{1-\lambda}(\theta)$ are increasing and continuous in $a_1$. We can find $\epsilon$ such that if $a_1$ increases in the interval

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$\bar{\mathcal{I}}_1$ the inequality will not be changed or it will be changed to equality. It increases the objective function and so the first solution $a_1, a_2$ is not optimal.

Now there is another issue and it is the boundary of $d\epsilon$. But the growth rate of $a_1(\theta)(\theta_1 + \frac{F_1(\theta_1)}{f_1(\theta_1)})$ is greater than $a_1^{\lambda}(\theta)a_2^{1-\lambda}(\theta)$ because the second derivative of $a_1(\theta)(\theta_1 + \frac{F_1(\theta_1)}{f_1(\theta_1)})$ is greater than $a_1^{\lambda}(\theta)a_2^{1-\lambda}(\theta)$, $(\lambda-1)\lambda a_1^{\lambda-2}(\theta)a_2^{1-\lambda}(\theta)$.

So NBC should be binding and there is no loss in budget constraint and it is difference with moral hazard model.

3: Now we try to find a guess for the NBC constraint. Thanks to Euler condition we can linearize the left hand side in $a_1$ and $a_2$, production function. We get

$$\int\limits_{\Theta} a_1(\theta)(\lambda a_1^{\lambda-1}(\theta)a_2^{1-\lambda}(\theta))dF(\theta) + \int\limits_{\Theta} a_2(\theta)((1-\lambda)a_1^{\lambda}(\theta)a_2^{-\lambda}(\theta))dF(\theta) =$$

$$(\int\limits_{\Theta}(a_1(\theta)(\theta_1 + \frac{F_1(\theta_1)}{f_1(\theta_1)}) + a_2(\theta)(\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)})dF(\theta)).$$

Just some basic algebra

$$\int\limits_{\Theta} a_1(\theta)(\lambda(\frac{a_2(\theta)}{a_1(\theta)})^{1-\lambda} - (\theta_1 + \frac{F_1(\theta_1)}{f_1(\theta_1)}))dF(\theta) +$$

$$\int\limits_{\Theta} a_2(\theta)((1-\lambda)(\frac{a_1(\theta)}{a_2(\theta)})^{\lambda} - (\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)}))dF(\theta) = 0$$

Now it is clear one guess for solution comes if the following conditions are satisfied

$$\lambda(\frac{a_2(\theta)}{a_1(\theta)})^{1-\lambda} - (\theta_1 + \frac{F_1(\theta_1)}{f_1(\theta_1)}) = 0$$

$$(1-\lambda)(\frac{a_1(\theta)}{a_2(\theta)})^{\lambda} - (\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)}) = 0$$

If we divide the first condition on the second condition then we get the ratio between $a_2$ and $a_1$ as the following,

$$\frac{a_2(\theta)}{a_1(\theta)} = \frac{1-\lambda}{\lambda}\frac{\theta_1 + \frac{F_1(\theta_1)}{f_1(\theta_1)}}{\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)}}.$$

Finally given some constraint for boundary of production,$B$, the result is

$$a_1(\theta) = B(\frac{a_1(\theta)}{a_2(\theta)})^{1-\lambda} = B(\frac{\lambda}{1-\lambda}\frac{\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)}}{\theta_1 + \frac{F_1(\theta_1)}{f_1(\theta_1)}})^{1-\lambda}$$

$$a_2(\theta) = B(\frac{a_2(\theta)}{a_1(\theta)})^{\lambda} = B(\frac{1-\lambda}{\lambda}\frac{\theta_1 + \frac{F_1(\theta_1)}{f_1(\theta_1)}}{\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)}})^{\lambda}$$

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$$a(\theta) = (B(\frac{(\theta_2 + \frac{F_2}{f_2})\lambda}{(\theta_1 + \frac{F_1}{f_1})(1-\lambda)})^{1-\lambda}, B(\frac{(\theta_2 + \frac{F_2}{f_2})\lambda}{(\theta_1 + \frac{F_1}{f_1})(1-\lambda)})^{-\lambda}).$$

4: Now we will show that other functions do not satisfy New BC. From Preposition **??** we have the general solution. But we have two explicit assumptions. The first we assumed[24] that the current solution is impossible because $K \neq 0$ and the second assumption is $\mu \neq \eta_\theta$ but if $K = 0$ then $\mu = \eta_\theta$, again thanks Euler condition.

Preposition 1.2.15

1-First we repeat the first and second step argument of preposition **??**. It is obvious that $K = \int_\Theta k(\theta)dF(\theta)$ does not affect the argument because it is constant and given. $K$ matters not $k(\theta)$.

2-Above argument shows that $u_i(\bar{\theta}_i) = 0$ so just the planer should choose $a_i(\theta)$.

3-Second the NBC and objective function are convex so we can write Lagrangian approach to our functional space, theorems 1 and 2 chapter 8 in Luenberger [1997]. According to these prepositions we can write Lagrangian as the following.

$$\mathcal{L} = \max_{a(\theta)} \int_\Theta (a_1(\theta)\frac{F_1(\theta_1)}{f_1(\theta_1)} + a_2(\theta)\frac{F_2(\theta_1)}{f_2(\theta_1)})dF(\theta) + \mu$$

$$\{\int_\Theta a_1^\lambda(\theta)a_2^{1-\lambda}(\theta)dF(\theta) - \int_\Theta (a_1(\theta)(\theta_1 + \frac{F_1(\theta_1)}{f_1(\theta_1)}) + a_2(\theta)(\theta_2 + \frac{F_2(\theta_1)}{f_2(\theta_1)} + k(\theta))dF(\theta))\}+$$

$$\eta_\theta(B \geq a_1^\lambda(\theta)a_2^{1-\lambda}(\theta)).$$

Taking first order derivatives respect to $a_1$ and $a_2$.

$$-\frac{F_1(\theta_1)}{f_1(\theta_1)} = \mu\{\lambda a_1^{\lambda-1}(\theta)a_2^{1-\lambda}(\theta) - (\theta_1 + \frac{F_1(\theta_1)}{f_1(\theta_1)})\} - \eta_\theta\lambda a_1^{\lambda-1}(\theta)a_2^{1-\lambda}(\theta)$$

$$-\frac{F_2(\theta_2)}{f_2(\theta_2)} = \mu\{(1-\lambda)a_1^\lambda(\theta)a_2^{-\lambda}(\theta) - (\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)})\} - \eta_\theta(1-\lambda)a_1^\lambda(\theta)a_2^{-\lambda}(\theta).$$

Determinan of the above linear equations is not zero if and only if

$$\frac{a_2(\theta)}{a_1(\theta)} \neq \frac{1-\lambda}{\lambda}\frac{\theta_1 + \frac{F_1(\theta_1)}{f_1(\theta_1)}}{\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)}}.$$

If above condition does not hold then from Euler condition and NBC we get $K = 0$, so given the assumption $K > 0$ the above condition is true.

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$$\frac{a_2(\theta)}{a_1(\theta)} \neq \frac{1-\lambda}{\lambda}\frac{\theta_1 + \frac{F_1(\theta_1)}{f_1(\theta_1)}}{\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)}}$$

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If the determinan of the above linear condition is not zero then $\mu$ and $\eta$ are determined as the following.

$$\mu = \frac{(1-\lambda)\frac{F_1(\theta_1)}{f_1(\theta_1)}a_1^{\lambda}(\theta)a_2^{-\lambda}(\theta) - \lambda\frac{F_2(\theta_2)}{f_2(\theta_2)}a_1^{\lambda-1}(\theta)a_2^{1-\lambda}(\theta)}{(1-\lambda)(\theta_1+\frac{F_1(\theta_1)}{f_1(\theta_1)})a_1^{\lambda}(\theta)a_2^{-\lambda}(\theta) - \lambda(\theta_2+\frac{F_2(\theta_2)}{f_2(\theta_2)})a_1^{\lambda-1}(\theta)a_2^{1-\lambda}(\theta)}$$

$$\eta_\theta = \frac{(1-\lambda)\frac{F_1(\theta_1)}{f_1(\theta_1)}a_1^{\lambda}(\theta)a_2^{-\lambda}(\theta) - \lambda\frac{F_2(\theta_2)}{f_2(\theta)}a_1^{\lambda-1}(\theta)a_2^{1-\lambda}(\theta) + \theta_1\frac{F_2(\theta_2)}{f_2(\theta_2)} - \theta_2\frac{F_1(\theta_1)}{f_1(\theta_1)}}{(1-\lambda)(\theta_1+\frac{F_1(\theta_1)}{f_1(\theta_1)})a_1^{\lambda}(\theta)a_2^{-\lambda}(\theta) - \lambda(\theta_2+\frac{F_2(\theta_2)}{f_2(\theta_2)})a_1^{\lambda-1}(\theta)a_2^{1-\lambda}(\theta)}$$

If $\theta_1\frac{F_2(\theta_2)}{f_2(\theta_2)} \neq \theta_2\frac{F_1(\theta_1)}{f_1(\theta_1)}$ and it means $\mu \neq \eta_\theta$ then we rewrite the first order conditions as following.

$$\mu(\theta_1+\frac{F_1(\theta_1)}{f_1(\theta_1)}) - \frac{F_1(\theta_1)}{f_1(\theta_1)} = (\mu-\eta_\theta)\lambda a_1^{\lambda-1}(\theta)a_2^{1-\lambda}(\theta)$$

$$\mu(\theta_2+\frac{F_2(\theta_2)}{f_2(\theta_2)}) - \frac{F_2(\theta_2)}{f_2(\theta_2)} = (\mu-\eta_\theta)(1-\lambda)a_1^{\lambda}(\theta)a_2^{-\lambda}(\theta)$$

Finally we will get the result below that determines ratio of efforts.

$$\frac{a_2(\theta)}{a_1(\theta)} = \frac{1-\lambda}{\lambda}\frac{\theta_1+\frac{F_1(\theta_1)}{f_1(\theta_1)} - \frac{1}{\mu}\frac{F_1(\theta_1)}{f_1(\theta_1)}}{\theta_2+\frac{F_2(\theta_2)}{f_2(\theta_2)} - \frac{1}{\mu}\frac{F_2(\theta_2)}{f_2(\theta_2)}}$$

Restriction on upper bound of production $B$ and some simple algebra gives the following results.

$$a_1(\theta) = B(\frac{a_1(\theta)}{a_2(\theta)})^{1-\lambda} = B(\frac{\lambda}{1-\lambda}\frac{\theta_2+\frac{F_2(\theta_2)}{f_2(\theta_2)} - \frac{1}{\mu}\frac{F_2(\theta_2)}{f_2(\theta_2)}}{\theta_1+\frac{F_1(\theta_1)}{f_1(\theta_1)} - \frac{1}{\mu}\frac{F_1(\theta_1)}{f_1(\theta_1)}})^{1-\lambda}$$

$$a_2(\theta) = B(\frac{a_2(\theta)}{a_1(\theta)})^{\lambda} = B(\frac{1-\lambda}{\lambda}\frac{\theta_1+\frac{F_1(\theta_1)}{f_1(\theta_1)} - \frac{1}{\mu}\frac{F_1(\theta_1)}{f_1(\theta_1)}}{\theta_2+\frac{F_2(\theta_2)}{f_2(\theta_2)} - \frac{1}{\mu}\frac{F_2(\theta_2)}{f_2(\theta_2)}})^{\lambda}$$

$$a(\theta) = (B(\frac{(\theta_2+\frac{F_2}{f_2} - \frac{1}{\mu}\frac{F_2(\theta_2)}{f_2(\theta_2)})\lambda}{(\theta_1+\frac{F_1}{f_1} - \frac{1}{\mu}\frac{F_1(\theta_1)}{f_1(\theta_1)})(1-\lambda)})^{1-\lambda}, B(\frac{(\theta_2+\frac{F_2}{f_2} - \frac{1}{\mu}\frac{F_2(\theta_2)}{f_2(\theta_2)})\lambda}{(\theta_1+\frac{F_1}{f_1} - \frac{1}{\mu}\frac{F_1(\theta_1)}{f_1(\theta_1)})(1-\lambda)})^{-\lambda})$$

solves the relaxed maximization problem if and only if there is a Lagrange multiplier

Corollary 1.2.16

Some definition just for simplification

$$X = \frac{(\theta_2+\frac{F_2}{f_2} - \frac{1}{\mu}\frac{F_2}{f_2})\lambda}{(\theta_1+\frac{F_1}{f_1} - \frac{1}{\mu}\frac{F_1}{f_1})(1-\lambda)}$$

$$Y = \theta_1 + \frac{F_1(\theta_1)}{f_1(\theta_1)}$$

$$Z = \theta_2 + \frac{F_2(\theta_1)}{f_2(\theta_1)}$$

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Now the NBC is as the following:

$$\frac{B-K}{B} = \int_{\Theta} (X^{1-\lambda}Y + X^{-\lambda}Z)dF(\theta)$$

$Y$ and $Z$ are not function of $\mu$ so,

$$d(K) = -\int_{\Theta} (\frac{d(X)}{d(\mu)} X^{-\lambda-1}((1-\lambda)XY - \lambda Z)d(\mu)dF(\theta)$$

Taking simple but cumbersome derivatives shows $\frac{d(K)}{d(\mu)} > 0$.

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# Chapter 2

# Organization and Taxation:Micro Approach, Wooden Bridge II: Limited Communication

## 2.1 Introduction

In modelling economic situations, macroeconomists generally consider the impact of private information on macro side,-i.e. between agents and government, see Stiglitz [2002]. What they do not typically consider is the private information on micro side i.e. between agents- and its relation to the macro side. Since such asymmetry exists and can have important implications, we intend to consider this issue by relaxing the assumption in the literature and study its impacts.

This chapter considers an environment in which two agents- a manager and a worker- with private information engage in an organization to produce a good. The organization has a Cobb-Douglas production technology, the agents' effort is costly and complementary. The key information constraint in this organization is that the agents do not know each others productivity level. The planner- which is the tax authority in our case- aims to determine the optimal allocation rule of the effort and consumption to maximize social welfare. However, the planner cannot communicate with the worker, or observe the contract between her and the manager in the labor market.[1]

The above case is built upon two empirically testable assumptions. First that asymmetric information exists between firms and workers about workers' type and agents' productivity (Acemoglu and Pischke [1998] and Gibbons et al. [1991]). Second that firms screen their workers through the labour contracts offered (Autor [2001] and Landers et al. [1996]).

[1]Contract here refers to the function from type to income. The planner can observe the worker's income, but cannot justify the worker's type.

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Since there is no communication between the planner and the worker in this model, one may think of a set of mechanisms from the planner's point of view. In this paper we consider two different mechanisms. In the first one, the planner communicates with the manager and then, the manager communicate with the worker. The planner can observe the agents' effort and output, which yields an adverse selection without moral hazard concern. The second mechanism is similar to the first one with the difference that agents' effort is not observable anymore. Thus both adverse selection and moral hazard are present. We assume two different cost functions for the agents. The first one is a linear cost function, and the other one a strictly convex function.

The result shows no difference between the first and second mechanism since the planner maximizes social welfare and does not care about himself, therefore he gives all the budget minus tax to the manager. This is in contrast with Melumad et al. [1995] where the "owner" of the firm is the planner and the owner cares about his utility. Hence, he does not give all budget to the manager because it decreases the profit due to loss of control over the firm.

As another result of the paper, the manager does not induce any distortion regardless of her type i.e. productivity, because she determines her effort, where the planner cannot control.[2] For that reason, the planner is utilitarian and gives all budget to the manager. The planner can just control the manager's utility with a tax. On the other hand the worker makes distortion because he has private information and the manager determines the worker's effort.[3]

In our model, the planner's ability to impose tax on organization depends on the structure of costs and production functions. In linear case if planner levies any tax on the organization then the organization collapses.[4] The amount of tax that the planner imposes depends on the manager's type i.e. the worker's distribution-, cost function and production function. Assuming reasonable parameters in the model, the tax would be decreasing with respect to the manager's type. This implies that a highly productive manager can pay more tax compared to low productive manager. It is a bit difficult to compare this result with Mirrleesian approach because tax in our model works as a lump sum and not marginal as in that approach. In Mirrlees model the most productive agent has zero marginal tax whereas in this model the most productive manager can pay high amounts of tax as a lump sum.[5]

[2]The manager knows her type and she does not have any incentive to make distortion for herself.

[3]The worker should receive rent from the manager to tell truth.

[4]In the first chapter this depends on the agents' distribution function, production function.

[5]In this model because there is no distortion for the manager then marginal tax is equal zero and government will get lump sum tax.

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### 2.1.1 Literature Review

This paper contributes to various literature. First is the literature on limited communication mechanism design, see Kos [2012] and Mookherjee and Tsumagari [2014]. The key difference with the mentioned papers is that the communication is limited in those, while we allow for full communication between the planner and the manager and no communication between the planner and the worker here. Moreoever, the environment and objective function are different for this model.

The second relevant literature is organization economics, Melumad et al. [1995]. The studies in this literature typically consider internal source. Melumad et al. [1995] consider the organization in view point of the owner who maximizes her profit, so there is a conflict to divide output between the owner, manager, and the worker. In this paper the planner wants to maximize social welfare and there is no conflict. The planner just considers the private information.

And finally the last relevant literature is the optimal taxation research. Very close paper to this study is Stantcheva [2014]. Stantcheva [2014] considers adverse selection in labour market, but assumes that firms are risk neutral and they do not engage in production, they have zero profit, they do not have any private information, and they keep the contract secret.[6] Therefore, Stantcheva [2014] reduces the problem to contract theory and does not study organizational or corporate tax.

The rest of this paper is organized as follows. Section 2 introduces the environment of the model. Section 3 considers the limited mechanism without moral hazard. Section 4 considers the limited mechanism with moral hazard. Section 5 concludes. All proofs are in appendix .

## 2.2 Model: Limited Communication

### 2.2.1 The Environment

The organization consists of two agents, a manager and a worker. For simplicity we use subscript 1 for the manager and 2 for the worker, $O = \{M, W\} = \{1, 2\}$.[7] The agents have quasilinear utility function which is represented by $u_i = c_i - \theta_i v(a_i)$, where $c_i$ is the consumption or cash that the agent $i$ consumes or recieves, $c_i \in \Re$ and $a_i$ is an effort or an input of production function, not the income, that agent $i$ exerts, $a_i \in A_i = [\,\underline{a}, \bar{a}\,] \subset \Re_+$.[8]

[6] Firms can exit or enter and contracts affect the entry and exit.

[7] We use she for the manager, he for the worker, and it for the planner, mechanism designer or government.

[8] $c_i < 0$ means that the agent consumes from saving.

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They choose simultaneously their effort, and it is observable by them.[9] $v$ is a convex and increasing function. We consider two forms of cost function. The first one is $v(a_i) = a_i$ and the second one is $v(a_i) = \frac{a_i^{1+\eta}}{1+\eta}$. Organization has a production function $y = f(a_1, a_2)$, $y : \Re_+^2 \to \Re_+$. $y$ is observable by everyone and it is split between them so that $y \geq c_1 + c_2$.

The type of agent $i$, $\theta_i$, is her private information. $\theta_i$ belongs to the set $\Theta_i = [\underline{\theta}, \bar{\theta}] \subset \Re_+$ and $\theta_i$ is distributed with a cumulative distribution function $F_i$. The support of $F_i$ is the interval $[\underline{\theta}, \bar{\theta}]$ which has a density $f_i$ and $f_i(\theta_i) > 0$ for all $\theta_i \in [\underline{\theta}, \bar{\theta}]$. $\theta_1$ and $\theta_2$ are independent. We denote by $\theta$ the vector $(\theta_1, \theta_2)$, and the support of the $\theta$ is $\Theta = [\underline{\theta}, \bar{\theta}]^2$. $F$, the distribution of $\theta$, is the product of distributions $F_i$, and the density function is represented by $f$. $f$ and $f_i$ are common knowledge among the agents and the planner.

Therefore, the environment is described as a incomplete information game. There are two players, $O = \{M, W\} = \{1, 2\}$, and the agent's strategy is a function from $\Theta_i$ to $A_i$, $S_i : \Theta_i \to A_i$. The outcome of game is a function from $A_1 \times A_2$ to $y \in \Re_+$, $c_1 \in \Re$, and, $c_2 \in \Re$, $C : A_1 \times A_2 \to \Re_+ \times \Re \times \Re$.

We denote the profile of effort, type, and consumption as follows:

$a = (a_1, a_2)$, $\theta = (\theta_1, \theta_2)$, and $c = (c_1, c_2)$.

Assumption 1: $\theta_i + \frac{F_i(\theta_i)}{f_i(\theta_i)}$ is increasing in $\theta_i$.

This assumption means that hazard rate is increasing , $\frac{f_i(\theta_i)}{1-F_i(\theta_i)}$, and it can be explained as the conditional probability of dying at time $\theta_i$ of an individual who has survived until time $\theta_i$.[10] In our setup it can be interpreted as the conditional probability of having productivity $\theta_i$ if the agent is less productive than $\theta_i$, hence $\frac{f_i(\theta_i)}{F_i(\theta_i)}$ means the conditional probability of having productivity $\theta_i$ if the agent is more productive than $\theta_i$.

The production is represented by Cobb-Douglas production function $y = a_1^{\lambda} a_2^{1-\lambda}$, $y : \Re_+^2 \to \Re_+$.[11]

Indeed this function is increasing in each argument, $a_1$ and $a_2$, and it has the constant return to scale property. The production function is bounded since $a_i$ is bounded. The upper bound is $\bar{Y}$, and there is no ex ante uncertainty in the production function if each $a_i$ is observable.

[9]we consider two cases. In the first case the planner observes the agents' effort but in the second case not.

[10]In the mechanism design literature, regularity condition is defined as increasing $\frac{f_i(\theta_i)}{1-F_i(\theta_i)}$ and it implies that $\theta_i + \frac{F_i(\theta_i)}{f_i(\theta_i)}$ and $\theta_i - \frac{1-F_i(\theta_i)}{f_i(\theta_i)}$ are increasing, see Borgers et al. [2015]

[11]We can change it with the general increasing return to scale function $f(a_1, a_2)$ without loss of generality.

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## 2.3 The First case: Limited Communication Mechanism without Moral Hazard

In the first limited communication mechanism the planner observes the agent's effort, but he can not communicate or contract with the worker. Therefore, he can only use $a_2$ for the purpose of contracting with the manager. The planner does not receive any message from the worker. In the literature of limited communication mechanisms, Kos [2012] and Mookherjee and Tsumagari [2014], the message set is restricted such that there is no onto function between the message set and the agent's type. This paper examines the extreme case where the message set is empty.[12]

The mechanism includes two stages where the first and the second stages have respectively two and three moves. In the first stage, the planner offers the manager a contract of the form $x_1(a, \tilde{\theta}_1)$. The manager then accepts or rejects the contract. By accepting the contract, she will report her type $\tilde{\theta}_1$ regarding her own type $\theta_1$. Consequently, she chooses one contract from a menu of alternatives. In the second stage, she offers a subcontract to the worker which might be accepted or rejected. The worker exerts the effort $a_2$ as a part of the subcontract in the case of acceptance. In the third move of the second stage, the manager chooses an action $a_1$ in response to $a_2$.

The second stage interaction between the manager and the worker is not observable by the planner. The planner observes neither the contract offered by the manager nor the subsequent response of the worker. Nevertheless, it does not matter whether the worker knows the contract offered by the planner and the manager's report or not. In the other words, the manager offers the worker a contract $\{a_2(\theta_2), c_2(\theta_2)\}_{\theta_2 \in \Theta_2}$ at the second stage, hence the contract depends only on the worker's type. On the other hand, the manager's preferences over such contract depend on her type $\theta_1$, her report $\tilde{\theta}_1$ to the planner, and the contract $x_1(a, \tilde{\theta}_1)$. Since both the manager's and the worker's utility functions are linear in $c_2$, the informed principal problem does not occur in this mechanism. That is to say, the manager cannot make profit by delaying the revelation of her private information, type and contract, to the worker until the worker responds to the subcontract, Maskin and Tirole [1990].

For a given contract $x_1(a, \tilde{\theta}_1)$, we denote the subcontract, or the mechanism, for the worker by $\{a_2(\theta_2|\theta_1, \tilde{\theta}_1), c_2(\theta_2|\theta_1, \tilde{\theta}_1)\}_{\theta_1 \in \Theta_1}$. This notation implies the dependence of the subcontract to $\theta_1$ and $\tilde{\theta}_1$. After the worker reports his type, $\theta_2$, truthfully, the manager will choose $a_1$ in order to maximize her utility, $x_1(a_1, a_2(\theta_2|\theta_1, \tilde{\theta}_1), \tilde{\theta}_1) - \theta_1 v(a_1) - c_2(\theta_2|\theta_1, \tilde{\theta}_1)$. This sequential optimization problem can be represented as a simultaneous choice of the functions $\{a_1(\cdot|\theta_1, \tilde{\theta}_1), a_2(\cdot|\theta_1, \tilde{\theta}_1), c_2(\cdot|\theta_1, \tilde{\theta}_1)\}$. Finally, the manager with true type $\theta_1$ and

[12]The mechanism between the manager and the worker is not a limited communication mechanism.

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the reported type $\tilde{\theta_1}$ solves the following optimization problem.[13]

$SC(\theta_1|x_1(a,\tilde{\theta_1}))$ : [14]

$$\max_{a(\cdot),c_2(\cdot)} \mathrm{E}_{\Theta_2}[x_1(a(\theta_2|\theta_1,\tilde{\theta_1}),\tilde{\theta_1}) - \theta_1 v(a_1(\theta_2|\theta_1,\tilde{\theta_1})) - c_2(\theta_2|\theta_1,\tilde{\theta_1})]$$

subject to: for all $\theta_2 \in \Theta_2$, and all $\theta_1, \tilde{\theta_1} \in \Theta_1$,

$$\theta_2 \in \arg\max_{\tilde{\theta_2}}[c_2(\tilde{\theta_2}|\theta_1,\tilde{\theta_1}) - \theta_2 v(a_2(\tilde{\theta_2}|\theta_1,\tilde{\theta_1}))] \qquad (i)$$

$$c_2(\theta_2|\theta_1,\tilde{\theta_1}) - \theta_2 v(a_2(\theta_2|\theta_1,\tilde{\theta_1})) \geq 0 \qquad (ii)$$

Constraints $(i)$ and $(ii)$ represent the incentive compatibility and the individual rationality constraints for the worker. Since there exists only one worker in the organization, the relationship between the manager and the worker is the same as the principal-agent relationship. Hence, the manager considers ex post IC and IR.

The implicit assumption in the above mechanism is that the manager considers to make contract with all workers, every $\theta_2 \in \Theta_2$. It means that we do not consider $\Theta_2$ as the manager's choice variable while it is not obvious whether it is profitable or not for her.

**Lemma 2.3.1** *: The necessary and the sufficient conditions for incentive compatibility constraint are as the following.*

$q_2(\theta_2|\theta_1,\tilde{\theta_1})$ *is decreasing in* $\theta_2$ *and* $\frac{du_2(\theta_2|\theta_1,\tilde{\theta_1})}{d\theta_2} = -q_2(\theta_2|\theta_1,\tilde{\theta_1}))$

**Lemma 2.3.2** *: The individual rationality constraint is* $u_2(\bar{\theta}_2|\theta_1,\tilde{\theta_1}) = 0$.

Considering the above lemma, the manager's problem can be rewritten as follows:

**Lemma 2.3.3** *: The planner considers the following reduced form utility function for the manager.*[15]

$$\max_{a(\cdot)} E_{\Theta_2}[x_1(a(\theta_2|\theta_1,\tilde{\theta_1}),\tilde{\theta_1}) - \theta_1 v(a_1(\theta_2|\theta_1,\tilde{\theta_1})) - (\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)})v(a_2(\theta_2|\theta_1,\tilde{\theta_1}))]$$

We denote $\Gamma_1(\theta_1|x_1(a,\tilde{\theta_1})) = \max_{a(\cdot)} \mathrm{E}_{\Theta_2}[x_1(a(\theta_2|\theta_,\tilde{\theta_1}),\tilde{\theta_1}) - \theta_1 v(a_1(\theta_2|\theta_1,\tilde{\theta_1})) - (\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)})v(a_2(\theta_2|\theta_1,\tilde{\theta_1}))]$. $\Gamma_1(\theta_1|x_1(a,\tilde{\theta_1}))$ is the value of the subcontract optimization problem, $SC(\theta_1|x_1(a,\tilde{\theta_1}))$. It represents the manager's reduced form utility in the first stage.

Thus, the planner problem in the first stage reduces to

$$PC :^{16} \quad \max_{x_1(a,\theta_1),a(\theta)} E_{\Theta}(u_1(\theta) + u_2(\theta))$$

[13] $\{a_1(\cdot|\theta_1,\tilde{\theta_1}), a_2(\cdot|\theta_1,\tilde{\theta_1}), c_2(\cdot|\theta_1,\tilde{\theta_1})\}$ means $\{a_1(\theta_2|\theta_1,\tilde{\theta_1}), a_2(\theta_2|\theta_1,\tilde{\theta_1}), c_2(\theta_2|\theta_1,\tilde{\theta_1})\}_{\theta_2\in\Theta_2}$

[14] SC means the subcontract.

[15] In this lemma we ignore that $a_2$ is decreasing. So, we should check it later

[16] PC means the planner contract

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subject to: for all $\theta_1 \in \Theta_1$

$$\theta_1 \in \arg\max_{\hat{\theta}_1} \Gamma_1(\theta_1 | x_1(a, \tilde{\theta_1})) \qquad (i)$$

$$\Gamma_1(\theta_1 | x_1(a, \tilde{\theta_1})) \geq 0 \qquad (ii)$$

$$\text{there exist } a(\cdot|\theta_1, \theta_1), x_2(\cdot|\theta_1, \theta_1) \in \arg\max SP(\theta_1 | x_1(a, \theta_1))$$

$$\text{such that } a(\theta) = a(\theta_2|\theta_1, \theta_1), \text{ for all } \theta_2 \qquad (iii)$$

$$y(a) \geq x_1(a, \theta_1) + k(\theta_1) \qquad (iv)$$

$$B \geq y(a). \qquad (v)$$

Constraint $(i)$ represents the incentive constraint for the manager based on her reduced form utility. Constraint $(ii)$ is the rationality constraint for the manager. Constraint $(iii)$ expresses that the desired effort must coincide with those that manager will choose when she makes subcontract with the worker. The fourth constraint, $(iv)$, is the budget balance constraint and it means that the total production should be greater than the payment to the manager and the tax, $k(\theta_1)$. Now, tax depends only on the manager's type and not on the worker's type. It is different from the previous chapter where the planner can levy the tax on both agents. The last constraint, $(v)$, is the boundary of production which is the same as the previous chapter. We need to impose this constraint only when the cost function is linear.

### 2.3.1 Linear Function: $V(a_i) = a_i$

Following the previous chapter, we consider two cases. In the first case, the government does not impose any tax on the organization, $K = 0$, while in the second case the organization is taxed the amount of $K > 0$.[17,18]

**The first case, $K = 0$:**

The key assumption which yields the results of the following prepositions is that the planner is utilitarian. Thus, the planer has incentive to make the budget constraint binding and it does not matter whether the planner wants to impose tax or not, since $x_1(a, \theta)$ is not optimal if the budget constraint is not binding. Therefore, the fourth constraint of the planner's problem is always binding, for every $\theta$ we have $y(a) = x_1(a, \theta_1) + k(\theta_1)$ in the PC. We also impose the last constraint as the cost function is linear, $V(a_i) = a_i$. When there is not tax, $y(a) = x_1(a, \theta_1)$ and we can easily compute $a_i$.

[17]K=0 means $k(\theta_1) = 0$ for every $\theta_1$

[18]$K > 0$ means $k(\theta_1) > 0$ for some set $I$, $\theta_1 \in I$, such that the measure of this set is not equal to zero.

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With the above discussion, the manager's problem is:

$$\max_{a(\cdot)} \mathrm{E}_{\Theta_2}[a_1^{\lambda}(\theta_2|\theta_1,\tilde{\theta_1})a_2^{1-\lambda}(\theta_2|\theta_1,\tilde{\theta_1}) - \theta_1 a_1(\theta_2|\theta_1,\tilde{\theta_1}) - (\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)})a_2(\theta_2|\theta_1,\tilde{\theta_1})].$$

**Lemma 2.3.4** *The decision rules for the subcontract are as follows.*

$$a(\theta_2|\theta_1,\tilde{\theta_1}) = (B(\frac{(\theta_2+\frac{F_2}{f_2})\lambda}{\theta_1(1-\lambda)})^{1-\lambda}, B(\frac{(\theta_2+\frac{F_2}{f_2})\lambda}{\theta_1(1-\lambda)})^{-\lambda})$$

If the result is compared with the first best and the second best of previous chapter then there is no distortion for the manager. On the other hand, the worker induces distortion if the worker is not top. Hence, the worker with type $\theta_2 \neq \underline{\theta}_2$ makes distortion for the manager and the organization, $\frac{F_2(\theta_2)}{f_2(\theta_2)}$, and the manager with type $\theta_1$ does not make any distortion for the organization. The reason is that the manager makes the contract with the worker and gets the rent of contract. Therefore, she has no incentive to change her effort from the first best.

**Lemma 2.3.5** *The manager's reduced form utility is equal to zero, regardless of his type.* $\Gamma_1(\theta_1|x_1(a,\tilde{\theta_1})) = 0$

The result of the above lemma comes from Euler condition and linear cost function.

The reduced utility is the expected and not the ex post utility. But considering the linearity of the cost function and the constant return to scale production function , the expected and the ex post utilities coincide.

Now we can get the following preposition.

**Proposition 2.3.6** *With the linear cost function and without tax, the organization exists and the decision rule is as follows.*

$$a(\theta) = (B(\frac{(\theta_2+\frac{F_2}{f_2})\lambda}{\theta_1(1-\lambda)})^{1-\lambda}, B(\frac{(\theta_2+\frac{F_2}{f_2})\lambda}{\theta_1(1-\lambda)})^{-\lambda})$$

Under limited communication, the manager will not impose any distortion on the organization but the worker does. Also with a linear cost function the manager's reduce utility is equal zero while it holds for the worker when he is the least productive agent. In this setup, the manager accepts the risk and gets the communication mechanism. On the other hand, the worker gets the rent of the organization.

To solve the above problem, we implicitly assumed that the decision rule satisfies the manager's individual rationality and incentive compatibility constraints and clearly $\Gamma_1(\theta_1|x_1(a,\tilde{\theta_1})) = 0$ for every $\theta_1$ and $\tilde{\theta_1}$.

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**The second case** $K \neq 0$

If the planner decides to collect tax from the organization, then $k(\theta_1) > 0$ for some $\theta_1$. Taxation affects not only the planner's budget constraints, but also the manager's reduced utility. Moreover, the manager's reduced utility affects the planner's individual rationality and incentive compatibility constraints. Taxation does not affect the worker directly but since it might cause the organization to fail, it affects the worker indirectly. However, if the organization continue to exist even with taxation, then taxation does not affect the worker.

In this case, the budget constraint is binding for the planner, $y(a) = x_1(a, \theta_1) + k(\theta_1)$. The planner can increase its utility by increasing $x_1(a, \theta_1)$. Then, the manager's reduced form utility function is as follows.

$$-k(\theta_1) + \max_{a(\cdot)} \mathrm{E}_{\Theta_2}[a_1^{\lambda}(\theta_2|\theta_1, \tilde{\theta_1})a_2^{1-\lambda}(\theta_2|\theta_1, \tilde{\theta_1}) - \theta_1 a_1(\theta_2|\theta_1, \tilde{\theta_1}) - (\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)})a_2(\theta_2|\theta_1, \tilde{\theta_1})].$$

Taxation does not change the manager's decision rule but it affects her reduced utility. The tax, $k(\theta_1)$, does not depend on the worker's type, $\theta_2$, and hence it does not affect his decision rule.

**Lemma 2.3.7** *If the planner decides to collect the tax from the organization, the decision rule is the same as the no taxation case.*

$$a(\theta_2|\theta_1, \tilde{\theta_1}) = (B(\frac{(\theta_2 + \frac{F_2}{f_2})\lambda}{\theta_1(1-\lambda)})^{1-\lambda}, B(\frac{(\theta_2 + \frac{F_2}{f_2})\lambda}{\theta_1(1-\lambda)})^{-\lambda})$$

**Lemma 2.3.8** *The manager's reduced utility is equal to the tax she pays.*
$\Gamma_1(\theta_1|x_1(a, \tilde{\theta_1})) = -k(\theta_1) < 0$

**Proposition 2.3.9** *The organization with the linear cost function and taxation does not form for any type of agents' distribution function.*

If the planner can not communicate with the worker, then he looses the chance to make the organization for any distribution function of agents. In the full communication case, there is a chance to form the organization but depends to the distribution function and the states. This proposition shows the failure of the organization in the presence of taxation with limited communication mechanism for every distribution.

In this environment the Mirrlees tax rule works for the worker but not for the manager. This happens since when taxation exists for the manager, the worker can not find any organization to work. This shows the importance of the organization in the tax design.

Allowing the manager to choose the set of contracts, $I \subset \Theta_2$, does not affect the results considering the linearity of the cost function.

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### 2.3.2 Non Linear Function: $V(a_i) = \frac{a_i^{1+\eta}}{1+\eta}$

Now we use the following non liner cost function, $V(a_i) = \frac{a_i^{1+\eta}}{1+\eta}$. Again we consider two cases as in the previous section. In the first case, government does not impose tax on the organization, $k(\theta_1) = 0$ for every $\theta_1$, and in the second case it imposes tax on the organization, $k(\theta_1) > 0$ for every $\theta_1$.

**First Case:** $K = k(\theta_1) = 0$

Since the budget balance is binding, we can then use the planner's budget constraint to write the manager's reduced utility as follows.

$$\max_{a(\cdot)} \mathrm{E}_{\Theta_2}[a_1^{\lambda}(\theta_2|\theta_1,\tilde{\theta_1})a_2^{1-\lambda}(\theta_2|\theta_1,\tilde{\theta_1}) - \theta_1 \frac{a_1^{1+\eta}(\theta_2|\theta_1,\tilde{\theta_1})}{1+\eta} - (\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)})\frac{a_2^{1+\eta}(\theta_2|\theta_1,\tilde{\theta_1})}{1+\eta}]$$

The only difference with linear case is the form of cost function and constraints are the same.

**Lemma 2.3.10** *The decision rule of the subcontract looks as follows,*

$$a(\theta_2|\theta_1,\tilde{\theta_1}) = ((\frac{\lambda}{\theta_1})^{\frac{1}{\eta}}[\frac{(1-\lambda)\theta_1}{\lambda(\theta_2+\frac{F_2}{f_2})}]^{\frac{1-\lambda}{(1+\eta)\eta}}, (\frac{\lambda}{\theta_1})^{\frac{1}{\eta}}[\frac{(1-\lambda)\theta_1}{\lambda(\theta_2+\frac{F_2}{f_2})}]^{\frac{1+\eta-\lambda}{(1+\eta)\eta}})$$

The result shows that worker makes distortion for the organization and not manager. Now the decision rule does not depend to the boundary of production because the cost function is non linear.

The manager gets the rent of contracting with worker and therefore rent is not zero in this case, moreover the rent depends on the manager's type.

To compute this rent and to make algebra easier we use $\lambda = 0.5$ which means that the elasticity of output is equal for both agents. We are interested in showing the importance of information and communication mechanism in the existence of organization. Therefore, we define the decision rule as follows,

$$a(\theta_2|\theta_1,\tilde{\theta_1}) = (\frac{1}{2\theta_1})^{\frac{1}{\eta}}[\frac{\theta_1}{\theta_2+\frac{F_2}{f_2}}]^{\frac{1}{2(1+\eta)\eta}}, (\frac{1}{2\theta_1})^{\frac{1}{\eta}}[\frac{\theta_1}{\theta_2+\frac{F_2}{f_2}}]^{\frac{1+2\eta}{2(1+\eta)\eta}}).$$

**Lemma 2.3.11** *The manager's reduced utility depends on her type $\eta$ and worker's productivity distribution as follows,*

$$\Gamma_1(\theta_1) = (\theta_1)^{-\frac{1}{2\eta}} \frac{\eta}{1+\eta}(\frac{1}{2})^{\frac{1}{\eta}} E_{\Theta_2}[\frac{1}{\theta_2+\frac{F_2}{f_2}}]^{\frac{1}{2\eta}}$$

We can interpret $\eta$ as inverse wage elasticity of labor supply.[19] Given our interpretation, $\eta$ is greater than zero, see Chetty et al. [2011] for the details. The manager's reduced utility

[19]Recall that if individual' utility function is $u = c - \theta y^{\alpha}$ where $y$ is income, then wage elasticity of labor supply equals $\epsilon = \frac{1}{\alpha-1}$. It is not precise because $a$ is the effort and not income though.

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is decreasing with respect to $\theta_1$ and it means that high productive manager has high reduced utility[20]. The expected profit is increasing with respect to the type. In this setup, reduced utility or expected profit is not translated to the utility or ex post profit. Not surprisingly, it is possible that the manager with high expected profit, $\theta_1$, gets less ex post profit than the manager with low expected profit, $\tilde{\theta}_1 > \theta_1$.

**Proposition 2.3.12** *The organization without tax but with non linear cost function exists and decision rule is,*

$$a(\theta) = ((\frac{1}{2\theta_1})^{\frac{1}{\eta}}[\frac{\theta_1}{\theta_2 + \frac{F_2}{f_2}}]^{\frac{1}{2(1+\eta)\eta}}, (\frac{1}{2\theta_1})^{\frac{1}{\eta}}[\frac{\theta_1}{\theta_2 + \frac{F_2}{f_2}}]^{\frac{1+2\eta}{2(1+\eta)\eta}})$$

The decision rule is decreasing with respect to $\theta_1$ and $\theta_2$, and it means that in every organization productive agent exerts more effort than unproductive agent.

If the manager of type $\theta_1$ reports her type $\tilde{\theta}_1$, then the planner makes contract with her as the following function, and it is obvious that this function is incentive compatible for manager and worker.

$$x_1(a, \tilde{\theta}_1) = a_1^{0.5} a_2^{0.5} \quad if \quad a(\tilde{\theta}_1, \theta_2) = ((\frac{1}{2\tilde{\theta}_1})^{\frac{1}{\eta}}[\frac{\tilde{\theta}_1}{\theta_2 + \frac{F_2}{f_2}}]^{\frac{1}{2(1+\eta)\eta}}, (\frac{1}{2\tilde{\theta}_1})^{\frac{1}{\eta}}[\frac{\tilde{\theta}_1}{\theta_2 + \frac{F_2}{f_2}}]^{\frac{1+2\eta}{2(1+\eta)\eta}})$$

The domain of this function on the first argument $a$ is restricted and it depends on $\tilde{\theta}_1$ and $\theta_2$. We need to consider IC and IR constraints, if we want to extend the domain and therefore we should define appropriate amount for $x_1(a, \tilde{\theta}_1)$.

**Second Case** $K \neq 0$

Now government wants to impose tax on organization, $k(\theta_1) > 0$. The budget constraint is binding, $y(a) = x_1(a, \theta_1) + k(\theta_1)$. Then, the manager's reduced form utility function is:

$$\max_{a(\cdot)} \mathrm{E}_{\Theta_2}[-k(\theta_1, \tilde{\theta}_1) + a_1^{\lambda}(\theta_2|\theta_1, \tilde{\theta}_1) a_2^{1-\lambda}(\theta_2|\theta_1, \tilde{\theta}_1) - \theta_1 \frac{a_1^{1+\eta}(\theta_2|\theta_1, \tilde{\theta}_1)}{1+\eta} - (\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)}) \frac{a_2^{1+\eta}(\theta_2|\theta_1, \tilde{\theta}_1)}{1+\eta}],$$

where $k(\theta_1, \tilde{\theta}_1)$ is the amount of tax that should be paid by the manager with type $\theta_2$ but reports her type $\tilde{\theta}_1$. Since tax does not depend to the worker's type, we can write the manager's problem as follows,

$$-k(\theta_1, \tilde{\theta}_1) + \max_{a(\cdot)} \mathrm{E}_{\Theta_2}[a_1^{\lambda}(\theta_2|\theta_1, \tilde{\theta}_1) a_2^{1-\lambda}(\theta_2|\theta_1, \tilde{\theta}_1) - \theta_1 \frac{a_1^{1+\eta}(\theta_2|\theta_1, \tilde{\theta}_1)}{1+\eta} - (\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)}) \frac{a_2^{1+\eta}(\theta_2|\theta_1, \tilde{\theta}_1)}{1+\eta}].$$

Therefore, the optimal decision rule does not depend on the tax rule given incentive compatibility and individual rationality constraints.

[20]It is enough to take a derivative from $\Gamma_1$ respect to the $\theta_1$.

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**Lemma 2.3.13** *The decision rule of the subcontractor is*

$$a(\theta_2|\theta_1,\tilde{\theta_1}) = ((\frac{\lambda}{\theta_1})^{\frac{1}{\eta}}[\frac{(1-\lambda)\theta_1}{\lambda(\theta_2+\frac{F_2}{f_2})}]^{\frac{1-\lambda}{(1+\eta)\eta}}, (\frac{\lambda}{\theta_1})^{\frac{1}{\eta}}[\frac{(1-\lambda)\theta_1}{\lambda(\theta_2+\frac{F_2}{f_2})}]^{\frac{1+\eta-\lambda}{(1+\eta)\eta}}).$$

This decision rule is the same as the case when government does not levy tax on the organization, given it satisfies the planner's IC and IR constraints.

To make algebra easier we assume that $\lambda$ is equal $\frac{1}{2}$.

**Lemma 2.3.14** *The manager's reduced utility is then,*

$$\Gamma_1(\theta_1) = -k(\theta_1) + (\theta_1)^{-\frac{1}{2\eta}}\frac{\eta}{1+\eta}(\frac{1}{2})^{\frac{1}{\eta}}E_{\Theta_2}[\frac{1}{\theta_2+\frac{F_2}{f_2}}]^{\frac{1}{2\eta}}.$$

Taxation affects the manager's reduced utility and it decreases it as an amount of tax.

**Proposition 2.3.15** *The organization with tax and non linear cost function exists and the decision rule is as the following.*

$$a(\theta) = ((\frac{1}{2\theta_1})^{\frac{1}{\eta}}[\frac{\theta_1}{\theta_2+\frac{F_2}{f_2}}]^{\frac{1}{2(1+\eta)\eta}}, (\frac{1}{2\theta_1})^{\frac{1}{\eta}}[\frac{\theta_1}{\theta_2+\frac{F_2}{f_2}}]^{\frac{1+2\eta}{2(1+\eta)\eta}}).$$

*Maximum amount of tax is equal* $k(\theta_1) = (\theta_1)^{-\frac{1}{2\eta}}\frac{\eta}{1+\eta}(\frac{1}{2})^{\frac{1}{\eta}}E_{\Theta_2}[\frac{1}{\theta_2+\frac{F_2}{f_2}}]^{\frac{1}{2\eta}}$.

The above proposition says that the decision rule is implementable with special tax rule, but not unique. We can choose different tax rules but they should satisfy the IC and IR for the planner's problem.

IR implies, $\Gamma_1(\theta_1) \geq 0$, that the necessary condition for the tax rule is as follows,

$$(\theta_1)^{-\frac{1}{2\eta}}\frac{\eta}{1+\eta}(\frac{1}{2})^{\frac{1}{\eta}}\mathrm{E}_{\Theta_2}[\frac{1}{\theta_2+\frac{F_2}{f_2}}]^{\frac{1}{2\eta}} \geq k(\theta_1) > 0 \;\; for \;\; every \;\; \theta_1 \in \Theta_1.$$

IR makes the lower and upper limits for the tax function.

Also $k(\theta_1)$ should satisfy IC constraint, because planner wants to implement the decision rule. Therefore, it implies that for every $\theta_1$ and $\hat{\theta_1}$ the following constraints should be satisfied.

$$-k(\theta_1)+\mathrm{E}_{\Theta_2}[a_1^{\lambda}(\theta_2|\theta_1,\tilde{\theta_1})a_2^{1-\lambda}(\theta_2|\theta_1,\tilde{\theta_1})-\theta_1\frac{a_1^{1+\eta}(\theta_2|\theta_1,\tilde{\theta_1})}{1+\eta}-(\theta_2+\frac{F_2(\theta_2)}{f_2(\theta_2)})\frac{a_2^{1+\eta}(\theta_2|\theta_1,\tilde{\theta_1})}{1+\eta}] \geq$$

$$-k(\hat{\theta_1})+\mathrm{E}_{\Theta_2}[a_1^{\lambda}(\theta_2|\hat{\theta_1},\tilde{\theta_1})a_2^{1-\lambda}(\theta_2|\hat{\theta_1},\tilde{\theta_1})-\theta_1\frac{a_1^{1+\eta}(\theta_2|\hat{\theta_1},\tilde{\theta_1})}{1+\eta}-(\theta_2+\frac{F_2(\theta_2)}{f_2(\theta_2)})\frac{a_2^{1+\eta}(\theta_2|\hat{\theta_1},\tilde{\theta_1})}{1+\eta}]$$

and

$$-k(\hat{\theta_1})+\mathrm{E}_{\Theta_2}[a_1^{\lambda}(\theta_2|\hat{\theta_1},\tilde{\theta_1})a_2^{1-\lambda}(\theta_2|\hat{\theta_1},\tilde{\theta_1})-\hat{\theta_1}\frac{a_1^{1+\eta}(\theta_2|\hat{\theta_1},\tilde{\theta_1})}{1+\eta}-(\theta_2+\frac{F_2(\theta_2)}{f_2(\theta_2)})\frac{a_2^{1+\eta}(\theta_2|\hat{\theta_1},\tilde{\theta_1})}{1+\eta}] \geq$$

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$$-k(\theta_1) + \mathrm{E}_{\Theta_2}[a_1^{\lambda}(\theta_2|\theta_1,\tilde{\theta_1})a_2^{1-\lambda}(\theta_2|\theta_1,\tilde{\theta_1}) - \hat{\theta}_1 \frac{a_1^{1+\eta}(\theta_2|\theta_1,\tilde{\theta_1})}{1+\eta} - (\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)})\frac{a_2^{1+\eta}(\theta_2|\theta_1,\tilde{\theta_1})}{1+\eta}]$$

It is obvious that $k(\theta_1) = k(\hat{\theta_1})$ is solution because the decision rule maximizes the expected term in the manager's reduced utility.[21]

The decision rule has two parts in the manager's reduced utility. One part depends on the manager's type while the other does not.[22] In this paper we are interested in finding $k(\theta_1)$ such that it makes the decision rule implementable, but we do not want to characterize $k(\theta_1)$.

## 2.4 The Second case: Limited Communication Mechanism with Moral Hazard

Now we consider a case in which the planner can observe the output, but not the effort. The game and sequence of moves are the same as in the first case. Therefore the contract is the function $x_1(y,\theta_1)$. Because linear cost function is special case of non linear function, we just use non linear case.

### 2.4.1 Non Linear Function: $V(a_i) = \frac{a_i^{1+\eta}}{1+\eta}$

The second stage of the game is as before, so we can write the manager's reduced utility and profile of effort as the following.

$$\Gamma_1(\theta_1) = -k(\theta_1) + (\theta_1)^{-\frac{1}{2\eta}} \frac{\eta}{1+\eta} (\frac{1}{2})^{\frac{1}{\eta}} \mathrm{E}_{\Theta_2}[\frac{1}{\theta_2 + \frac{F_2}{f_2}}]^{\frac{1}{2\eta}}.$$

$$a(\theta) = ((\frac{1}{2\theta_1})^{\frac{1}{\eta}}[\frac{\theta_1}{\theta_2 + \frac{F_2}{f_2}}]^{\frac{1}{2(1+\eta)\eta}}, (\frac{1}{2\theta_1})^{\frac{1}{\eta}}[\frac{\theta_1}{\theta_2 + \frac{F_2}{f_2}}]^{\frac{1+2\eta}{2(1+\eta)\eta}}).$$

The planner problem is changed as the following.

$$PC: \quad \max_{x_1(y,\theta_1)} E_{\Theta}(u_1(\theta) + u_2(\theta))$$

subject to: for all $\theta_1 \in \Theta_1$

$$\theta_1 \in \arg\max_{\hat{\theta}_1} \Gamma_1(\theta_1|x_1(a,\tilde{\theta_1})) \qquad (i)$$

$$\Gamma_1(\theta_1|x_1(a,\tilde{\theta_1})) \geq 0 \qquad (ii)$$

[21] $\mathrm{E}_{\Theta_2}[a_1^{\lambda}(\theta_2|\hat{\theta_1},\tilde{\theta_1})a_2^{1-\lambda}(\theta_2|\hat{\theta_1},\tilde{\theta_1}) - \hat{\theta}_1 \frac{a_1^{1+\eta}(\theta_2|\hat{\theta_1},\tilde{\theta_1})}{1+\eta} - (\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)})\frac{a_2^{1+\eta}(\theta_2|\hat{\theta_1},\tilde{\theta_1})}{1+\eta}]$

[22] $\frac{a_1^{1+\eta}(\theta_2|\theta_1,\tilde{\theta_1})}{1+\eta}$ depends on the manager's type. $a_1^{\lambda}(\theta_2|\theta_1,\tilde{\theta_1})a_2^{1-\lambda}(\theta_2|\theta_1,\tilde{\theta_1})$ and $(\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)})\frac{a_2^{1+\eta}(\theta_2|\theta_1,\tilde{\theta_1})}{1+\eta}$ do not depend on the manager's type.

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$$y(a) \geq x_1(a, \theta_1) + k(\theta_1) \qquad (iii)$$

If we compare the above problem with last section then it is clear that two constraints are eliminated. Already we know that the budget balance is binding then the solution that comes from the second stage is solution of problem. It is not amazing because the planer does not incentive to make distortion between agents and he wants to maximize their welfare.

## 2.5 Conclusion

This paper considers the organization in which the planner can not communicate with the worker' but the manager does. The planner wants to maximize the organization welfare and also imposes tax on the organization. We assume two different mechanism, with and with out moral hazard, are same, but taxation depends on the production function and distribution function. With the linear cost the government can not impose tax on the organization.

## 2.6 Appendix

Lemma 2.3.1 The necessary condition is as follows.

For every $\theta_2$ and $\hat{\theta_2}$, IC implies the following inequalities.

$$c_2(\theta_2|\theta_1, \tilde{\theta_1}) - \theta_2 v(a_2(\theta_2|\theta_1, \tilde{\theta_1})) \geq c_2(\hat{\theta}_2|\theta_1, \tilde{\theta_1}) - \theta_2 v(a_2(\hat{\theta}_2|\theta_1, \tilde{\theta_1}))$$

$$c_2(\hat{\theta}_2|\theta_1, \tilde{\theta_1}) - \hat{\theta}_2 v(a_2(\hat{\theta}_2|\theta_1, \tilde{\theta_1})) \geq c_2(\theta_2|\theta_1, \tilde{\theta_1}) - \hat{\theta}_2 v(a_2(\theta_2|\theta_1, \tilde{\theta_1}))$$

Adding two inequalities and some basic algebra give the following.

$$(\hat{\theta}_2 - \theta_2)(v(a_2(\theta_2|\theta_1, \tilde{\theta_1})) - v(a_2(\hat{\theta}_2|\theta_1, \tilde{\theta_1}))) \geq 0$$

If $\hat{\theta}_2 > \theta_2$ then $v(a_2(\theta_2|\theta_1, \tilde{\theta_1})) \geq v(a_2(\hat{\theta}_2|\theta_1, \tilde{\theta_1}))$. Because $v$ is increasing then $a_2$ should be decreasing.

The second condition comes from the envelope theorem, Milgrom and Segal [2002] corollary 1.

The sufficient condition is as follows.

IC condition is equal that for every $\theta_2$ and $\hat{\theta_2}$,

$$c_2(\theta_2|\theta_1, \tilde{\theta_1}) - \theta_2 v(a_2(\theta_2|\theta_1, \tilde{\theta_1})) \geq c_2(\hat{\theta}_2|\theta_1, \tilde{\theta_1}) > \theta_2 v(a_2(\hat{\theta}_2|\theta_1, \tilde{\theta_1}))$$

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By definition of $u_2$ and rearranging of the above inequality

$$u_2(\theta_2|\theta_1,\tilde{\theta_1}) - u_2(\hat{\theta}_2|\theta_1,\tilde{\theta_1}) \geq (\hat{\theta}_2 - \theta_2)v(a_2(\hat{\theta}_2|\theta_1,\tilde{\theta_1}))$$

Integration over the interval $[\hat{\theta_2}, \theta_2]$ on the both sides implies

$$\int_{\theta_2}^{\hat{\theta_2}} \dot{u}_2(\tilde{\theta}_2|\theta_1,\tilde{\theta_1})d(\tilde{\theta}_2) \leq -\int_{\theta_2}^{\hat{\theta_2}} v(a_2(\hat{\theta}_2|\theta_1,\tilde{\theta_1}))d(\tilde{\theta}_2).$$

With the envelope theorem

$$\dot{u}_2(\theta_2|\theta_1,\tilde{\theta_1}) = -v(a_2(\theta_2|\theta_1,\tilde{\theta_1})).$$

We get

$$\int_{\theta_2}^{\hat{\theta_2}} v(a_2(\tilde{\theta}_2|\theta_1,\tilde{\theta_1}))d(\tilde{\theta}_2) \geq \int_{\theta_2}^{\hat{\theta_2}} v(a_2(\hat{\theta}_2|\theta_1,\tilde{\theta_1}))d(\tilde{\theta}_2).$$

Suppose that $\hat{\theta_2} > \theta_2$ because $a_2(\theta_2|\theta_1,\tilde{\theta_1})$ is decreasing and $v$ is decreasing then the above inequality is always true.

If $\theta_2 > \hat{\theta_2}$ then we change the upper and lower limit of integral, and the above reasoning shows that the inequality is true.

Lemma 2.3.2

From the above lemma we know that $u_2$ is decreasing, and the manager can increase her utility if she decreases $c_2$. Therefore, it implies that $u_2(\bar{\theta}_2|\theta_1,\tilde{\theta_1}) = 0$

Lemma 2.3.3

We exploit the envelope theorem, Fubini's theorem and IR condition, and we ignore that $a_2$ is decreasing. After we get the result, we will check it.

Envelope theorem means

$$\int_{\theta_2}^{\bar{\theta}_2} -v(a_2(\hat{\theta}_2|\theta_1,\tilde{\theta_1}))d(\hat{\theta}_2) = \int_{\theta_2}^{\bar{\theta}_2} \dot{u}_2(\hat{\theta}_2|\theta_1,\tilde{\theta_1})d(\hat{\theta}_2) =$$

$$u_2(\bar{\theta}_2|\theta_1,\tilde{\theta_1}) - u_2(\theta_2|\theta_1,\tilde{\theta_1}) = -u_2(\theta_2|\theta_1,\tilde{\theta_1})$$

Fubini's theorem means

$$\int_{\underline{\theta}_2}^{\bar{\theta}_2} -u_2(\theta_2|\theta_1,\tilde{\theta_1})dF_2(\theta_2) = \int_{\underline{\theta}_2}^{\bar{\theta}_2}\int_{\theta_2}^{\bar{\theta}_2} -v(a_2(\hat{\theta}_2|\theta_1,\tilde{\theta_1}))d(\hat{\theta}_2)dF_2(\theta_2) =$$

$$\int_{\underline{\theta}_2}^{\bar{\theta}_2} -v(a_2(\theta_2|\theta_1,\tilde{\theta_1}))\frac{F_2(\theta_2)}{f_2(\theta_2)}dF_2(\theta_2)$$

Lemma 2.3.4

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Taking the first order condition from objective function gives the following conditions.

$$\lambda a_1(\theta_2|\theta_1,\tilde{\theta_1})^{\lambda-1}a_2^{1-\lambda}(\theta_2|\theta_1,\tilde{\theta_1}) = \theta_1$$

and

$$(1-\lambda)a_1(\theta_2|\theta_1,\tilde{\theta_1})^{\lambda}a_2^{-\lambda}(\theta_2|\theta_1,\tilde{\theta_1}) = \theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)}$$

Also we know that

$$B = a_1(\theta_2|\theta_1,\tilde{\theta_1})^{\lambda}a_2^{1-\lambda}(\theta_2|\theta_1,\tilde{\theta_1}).$$

Then with some basic algebra.

$$a(\theta_2|\theta_1,\tilde{\theta_1}) = (B(\frac{(\theta_2+\frac{F_2}{f_2})\lambda}{\theta_1(1-\lambda)})^{1-\lambda}, B(\frac{(\theta_2+\frac{F_2}{f_2})\lambda}{\theta_1(1-\lambda)})^{-\lambda})$$

Lemma 2.3.5

Thanks to Euler equation it is enough to put instead of $y$ the term $a_1\frac{\partial y}{\partial a_1}(a_1,a_2) + a_2\frac{\partial y}{\partial a_2}(a_1,a_2)$.

Also from the above lemma we know that

$$\frac{\partial y}{\partial a_1} = \theta_1$$

and

$$\frac{\partial y}{\partial a_2} = \theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)}$$

Proposition 2.3.6

The decision rule that we get from lemma 2.3.4 satisfies two constraint of the planner, budget constraint and boundary for production function. In addition the manager's reduce form utility satisfies the individual rationality and the incentive compatibility constraint of the planner because her reduce form utility is equal zero then both constraint are satisfied. Therefore, the decision rule is same as the lemma 2.3.4.

Lemma 2.3.7

It is lemma 2.3.4 but the point is that $k(\theta_1)$ does not affect the decision rule because it is independent of $\theta_2$.

Lemma 2.3.8

We use lemma 2.3.7 and lemma 2.3.5 then it gives the result

Proposition 2.3.9

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The individual rationality constraint is not satisfied.

Lemma 2.3.10

$$\max_{a(\cdot)} \mathrm{E}_{\Theta_2}[a_1^{\lambda}(\theta_2|\theta_1,\tilde{\theta_1})a_2^{1-\lambda}(\theta_2|\theta_1,\tilde{\theta_1}) - \theta_1\frac{a_1^{1+\eta}(\theta_2|\theta_1,\tilde{\theta_1})}{1+\eta} - (\theta_2+\frac{F_2(\theta_2)}{f_2(\theta_2)})\frac{a_2^{1+\eta}(\theta_2|\theta_1,\tilde{\theta_1})}{1+\eta}]$$

Taking the first order partial derivative from the manager's reduced utility function gives the following conditions.

$$\lambda a_1^{\lambda-1}(\theta_2|\theta_1,\tilde{\theta_1})a_2^{1-\lambda}(\theta_2|\theta_1,\tilde{\theta_1}) = \theta_1 a_1^{\eta}(\theta_2|\theta_1,\tilde{\theta_1})$$

$$(1-\lambda)a_1^{\lambda}(\theta_2|\theta_1,\tilde{\theta_1})a_2^{-\lambda}(\theta_2|\theta_1,\tilde{\theta_1}) = (\theta_2+\frac{F_2}{f_2})a_2^{\eta}(\theta_2|\theta_1,\tilde{\theta_1})$$

Dividing the first term to the second term and some simple algebra gives the following.

$$a_2(\theta_2|\theta_1,\tilde{\theta_1}) = a_1(\theta_2|\theta_1,\tilde{\theta_1})(\frac{1-\lambda}{\lambda}\frac{\theta_1}{\theta_2+\frac{F_2}{f_2}})^{\frac{1}{1+\eta}}$$

Putting the above term in the one of the first order conditions gives the final results.

Lemma 2.3.11

$$a_1^{\lambda}(\theta_2|\theta_1,\tilde{\theta_1})a_2^{1-\lambda}(\theta_2|\theta_1,\tilde{\theta_1}) = [(\frac{1}{2\theta_1})^{\frac{1}{\eta}}[\frac{\theta_1}{\theta_2+\frac{F_2}{f_2}}]^{\frac{1}{2(1+\eta)\eta}}]^{0.5}[(\frac{1}{2\theta_1})^{\frac{1}{\eta}}[\frac{\theta_1}{\theta_2+\frac{F_2}{f_2}}]^{\frac{1+2\eta}{2(1+\eta)\eta}}]^{0.5} =$$

$$(\frac{1}{2\theta_1})^{\frac{1}{\eta}}[\frac{\theta_1}{\theta_2+\frac{F_2}{f_2}}]^{\frac{1}{2\eta}} = (\theta_1)^{-\frac{1}{2\eta}}(\frac{1}{2})^{\frac{1}{\eta}}[\frac{1}{\theta_2+\frac{F_2}{f_2}}]^{\frac{1}{2\eta}}$$

$$\theta_1\frac{a_1^{1+\eta}(\theta_2|\theta_1,\tilde{\theta_1})}{1+\eta} = \frac{\theta_1}{1+\eta}(\frac{1}{2\theta_1})^{\frac{1+\eta}{\eta}}[\frac{\theta_1}{\theta_2+\frac{F_2}{f_2}}]^{\frac{1}{2\eta}} = (\theta_1)^{-\frac{1}{2\eta}}\frac{1}{2(1+\eta)}(\frac{1}{2})^{\frac{1}{\eta}}[\frac{1}{\theta_2+\frac{F_2}{f_2}}]^{\frac{1}{2\eta}}$$

$$(\theta_2+\frac{F_2(\theta_2)}{f_2(\theta_2)})\frac{a_2^{1+\eta}(\theta_2|\theta_1,\tilde{\theta_1})}{1+\eta} = \frac{(\theta_2+\frac{F_2(\theta_2)}{f_2(\theta_2)})}{1+\eta}(\frac{1}{2\theta_1})^{\frac{1+\eta}{\eta}}[\frac{\theta_1}{\theta_2+\frac{F_2}{f_2}}]^{\frac{1+2\eta}{2\eta}} = (\theta_1)^{-\frac{1}{2\eta}}\frac{1}{2(1+\eta)}(\frac{1}{2})^{\frac{1}{\eta}}[\frac{1}{\theta_2+\frac{F_2}{f_2}}]^{\frac{1}{2\eta}}$$

Putting all terms in the manager's reduced utility gives the result as follows.

$$(\theta_1)^{-\frac{1}{2\eta}}\frac{\eta}{1+\eta}(\frac{1}{2})^{\frac{1}{\eta}}\mathrm{E}_{\Theta_2}[\frac{1}{\theta_2+\frac{F_2}{f_2}}]^{\frac{1}{2\eta}}$$

Proposition 2.3.12

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As it is clear we do not need to consider the budget balance constraint and boundary constrain for production.

The manager's reduce utility satisfies the IR constraint because $\Gamma_1$ is decreasing in $\theta_1$ and $\Gamma_1(\bar{\theta}_1) > 0$.

To check IC constraint we should mention that it is possible only the planner uses the following decision rule then by the manager's reduced utility it is incentive compatible. Because this decision rule comes from the manager's reduced utility then if manager tells lie then she reduces her reduced utility.

$$a(\theta) = ((\frac{1}{2\theta_1})^{\frac{1}{\eta}}[\frac{\theta_1}{\theta_2 + \frac{F_2}{f_2}}]^{\frac{1}{2(1+\eta)\eta}}, (\frac{1}{2\theta_1})^{\frac{1}{\eta}}[\frac{\theta_1}{\theta_2 + \frac{F_2}{f_2}}]^{\frac{1+2\eta}{2(1+\eta)\eta}})$$

Lemma 2.3.13

$k(\theta_1)$ does not affect the decision rule so it is same as lemma 2.3.10.

Lemma 2.3.14

Same algebra of lemma 2.3.11.

Proposition 2.3.15

The planner wants to collect the $K$ then it is enough we define $k(\theta_1) = K$ for every $\theta_1 \in \Theta_1$. Also we have another choice and it is $k(\theta_1) = (\theta_1)^{-\frac{1}{2\eta}}\frac{\eta}{1+\eta}(\frac{1}{2})^{\frac{1}{\eta}}\mathrm{E}_{\Theta_2}[\frac{1}{\theta_2+\frac{F_2}{f_2}}]^{\frac{1}{2\eta}}$. It is the largest amount tax that planer can collect from the organization.

Both above tax rules satisfy IC and IR. The second one makes $\Gamma_1(\theta_1) = 0$ and it is IC and IR.

The second case comes from the following conditions, IC definition.

$$-k(\theta_1)+\mathrm{E}_{\Theta_2}[a_1^{\lambda}(\theta_2|\theta_1,\tilde{\theta_1})a_2^{1-\lambda}(\theta_2|\theta_1,\tilde{\theta_1})-\theta_1\frac{a_1^{1+\eta}(\theta_2|\theta_1,\tilde{\theta_1})}{1+\eta}-(\theta_2+\frac{F_2(\theta_2)}{f_2(\theta_2)})\frac{a_2^{1+\eta}(\theta_2|\theta_1,\tilde{\theta_1})}{1+\eta}] \geq$$

$$-k(\hat{\theta_1})+\mathrm{E}_{\Theta_2}[a_1^{\lambda}(\theta_2|\hat{\theta_1},\tilde{\theta_1})a_2^{1-\lambda}(\theta_2|\hat{\theta_1},\tilde{\theta_1})-\theta_1\frac{a_1^{1+\eta}(\theta_2|\hat{\theta_1},\tilde{\theta_1})}{1+\eta}-(\theta_2+\frac{F_2(\theta_2)}{f_2(\theta_2)})\frac{a_2^{1+\eta}(\theta_2|\hat{\theta_1},\tilde{\theta_1})}{1+\eta}]$$

and

$$-k(\hat{\theta_1})+\mathrm{E}_{\Theta_2}[a_1^{\lambda}(\theta_2|\hat{\theta_1},\tilde{\theta_1})a_2^{1-\lambda}(\theta_2|\hat{\theta_1},\tilde{\theta_1})-\hat{\theta_1}\frac{a_1^{1+\eta}(\theta_2|\hat{\theta_1},\tilde{\theta_1})}{1+\eta}-(\theta_2+\frac{F_2(\theta_2)}{f_2(\theta_2)})\frac{a_2^{1+\eta}(\theta_2|\hat{\theta_1},\tilde{\theta_1})}{1+\eta}] \geq$$

$$-k(\theta_1)+\mathrm{E}_{\Theta_2}[a_1^{\lambda}(\theta_2|\theta_1,\tilde{\theta_1})a_2^{1-\lambda}(\theta_2|\theta_1,\tilde{\theta_1})-\hat{\theta_1}\frac{a_1^{1+\eta}(\theta_2|\theta_1,\tilde{\theta_1})}{1+\eta}-(\theta_2+\frac{F_2(\theta_2)}{f_2(\theta_2)})\frac{a_2^{1+\eta}(\theta_2|\theta_1,\tilde{\theta_1})}{1+\eta}]$$

It is obvious given $k(\theta_1) = k(\hat{\theta_1})$ the above conditions are satisfied. Just we should consider $k(\bar{\theta}_1) = (\bar{\theta}_1)^{-\frac{1}{2\eta}}\frac{\eta}{1+\eta}(\frac{1}{2})^{\frac{1}{\eta}}\mathrm{E}_{\Theta_2}[\frac{1}{\theta_2+\frac{F_2}{f_2}}]^{\frac{1}{2\eta}}$

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# Chapter 3

# Organization and Regulation : Limited Communication

## 3.1 Introduction

Emission standards are among the major policy tools in environmental regulations. Their objective is to reduce greenhouse gas emissions from transportation sources. The welfare effect of these standards largely depends on which policy firms choose in response. Firms typically face three options: sales-mixing (changing prices), down-sizing (releasing smaller cars), and technology adoption. Reynaert [2015] has found that firms embrace the third option as a response to new emission standard in the European car market. Yet, there are new pieces of evidence that car industry has been unsuccessful in adoption of new technologies to achieve standards goal.[1,2] Volkswagen -hereafter VW- emissions scandal is a recent and famous anecdotal evidence. EU as a regulator has proposed new rules to test car emissions after VW scandal.[3] Indeed, the story behind the VW scandal shows another mechanism for miss-adoption of new technology. In 2005, VW had two options to respond to the new standard: to purchase Mercedes' BlueTec system for reducing pollution, or to develop its own system.[4,5] VW rejected the first option and pursued the second one, to develop its own system. This means that VW decided to buy tools from its upstream firm. As Atalay et al. [2014] show for a case of vertical industry, "Roughly

[1] Damian Carrington (Wednesday 30 September 2015) "Wide range of cars emit more pollution in realistic driving tests, data shows". The Guardian.

[2] Damian Carrington (Friday 9 October 2015). "Four more carmakers join diesel emissions row". The Guardian.

[3] From the section Business (27 January 2016) "EU plans new rules for emission tests following VW scandal". BBC.

[4] Jack Ewing (4 October 2015). "Volkswagen Engine-Rigging Scheme Said to Have Begun in 2008". New York Times.

[5] William Boston (5 October 2015). "Volkswagen Emissions Investigation Zeroes In on Two Engineers". The Wall Street Journal.

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one-half of upstream establishments report no shipments to downstream establishments within the same firm." Therefore, the first option would have been the expected choice for VW. Now, the interesting questions are: why VW chose the second option, and how the regulator could have change it?

There are two mechanisms that one can consider to address the above questions. The first is uncertainty and ambiguity in adopting a new technology, and the second one is the different and private marginal cost of upstream firms. We will focus on the second mechanism in this paper. The second mechanism states that private information about marginal cost makes bargaining between the upstream firm and the downstream firm severe. The regulator can mitigate this problem if the regulator considers it in mechanism that is designed to regulate industry. To explain the latter mechanism we consider that the downstream firms- similar to upstream firms- have private information about their marginal cost or productivity.[6,7,8]

We consider an environment where there is a regulator and two firms in which they have private information about their marginal cost. The regulator can communicate just with the downstream firm. The downstream firm can communication with the upstream firm. The upstream firm should get rent from the downstream firm to tell the true information and also the downstream firm should receive rent from the regulator too. Hence, to implement the new technology, the regulator should pay more rents in comparison with standard case of Baron and Myerson [1982].

The key assumption in our model is that the regulator knows the linear production function which is employed in the industry. We also assume that the downstream firm does not add any input to the production function. This means that the industry has just one input and it is produced by the upstream firm.[9] If we assumed that the regulator does not know the production function, it added a new dimension to the private information which would add another channel for rent.

This paper relates to the theory of regulation in industrial economics - see Armstrong and Sappington [2007] for the last survey. The part of this literature studies regulation in the vertical relationships. This literature presumes the regulator is fully informed about industry demand and cost conditions. To the best of our knowledge, there is no study

[6] If we want to model this environment more precisely we should use the bargaining process between the two agents, the downstream firm and upstream firm, with private information.

[7] We do not address the first mechanism but a relevant approach can be found in Kos and Messner [2015]

[8] It is very reasonable explanation because the cost of failure in case the downstream firm could not adopt new technology is very high. Therefore, uncertainty of the downstream firm about which upstream firm has better technology is more reasonable.

[9] If $a_2$ is output of the upstream firm then we assume that the final good is produced by the downstream firm with production function $a_1 = Aa_2$. Another case, the downstream firm produces another input $a_2$ and the final good with the production function $y = a_1^{\lambda} a_2^{1-\lambda}$.

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considering the information constraints in the environment as this paper.

Another related literature is limited communication mechanism design,(Kos [2012] and Mookherjee and Tsumagari [2014]). Our key difference with the mentioned papers is that the communication is limited in those, while we allow for full communication between the regulator and the downstream firm and no communication between the regulator and the upstream firm here. Moreoever, the environment and objective function are different for this model.

The rest of this paper is organized as follows. Section 2 introduces the model. Section 3 concludes. All proofs are in appendix .

## 3.2 The Model

The industry consists of two firms, a downstream firm and an upstream firm. For simplicity we use subscript 1 for the downstream firm and 2 for the upstream firm, $O = \{D, U\} = \{1, 2\}$. The firms have linear cost function. The cost function of upstream firm is represented by $c_2 = \theta_2 q_2$ where $q_2$ is the quantity produced by the upstream firm and $\theta_2$ is the marginal cost of upstream firm that is unknown to the regulator and the downstream firm. The cost function of downstream firm is represented by $c_1 = \theta_1 q_1 + p_2 q_2$ where $q_1$ is the quantity produced by the downstream firm, $\theta_1$ is the marginal cost of downstream firm that is unknown to the regulator and the upstream firm, $p_2$ is the price of input $q_2$ which is paid to the upstream firm by the downstream firm. The industry has a production function $q_1 = \frac{1}{A} q_2$ in which $q_1$ is a final good and it is consumed by a representative consumer. It means that if the regulator wants one final good then the upstream firm should produce at least $A$ amount of $q_2$ because the technology is linear. We assume that $A > 0$. $A$ is known to the regulator and the upstream firm otherwise we mention it explicitly. If $A$ is unknown then it adds another dimension to the private information.

The type of firm $i$, $\theta_i$ and is its private information. $\theta_i$ belongs to the set $\Theta_i = [\underline{\theta}, \bar{\theta}] \subset \Re_+$ and $\theta_i$ has a probability distribution which is described by a cumulative distribution function $F_i$. The support of $F_i$ is the interval $[\underline{\theta}, \bar{\theta}]$ which has a density $f_i$ and $f_i(\theta_i) > 0$ for all $\theta_i \in [\underline{\theta}, \bar{\theta}]$. $\theta_1$ and $\theta_2$ are independent. We denote by $\theta$ the vector $(\theta_1, \theta_2)$, and the support of the $\theta$ is $\Theta = [\underline{\theta}, \bar{\theta}]^2$. $F$, the distribution of $\theta$, is the product of distributions $F_i$, and the density function is represented by $f$. $f$ and $f_i$ are common knowledge among the firms and the regulator.

The representative consumer has the inverse demand function $P(.)$ and it is known by all firms and the regulator. Therefore, $P(.)$ is the price at which the consumer demands the final good $q_1$. The consumer surplus is $\int_0^{q_1} P(\bar{q}_1) d\bar{q}_1 - q_1 P(q_1)$.

Therefore, given the consumer's demand the environment is described as a incomplete

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information game between the firms.[10] There are two players, $O = \{D, U\} = \{1, 2\}$. The strategy of upstream firm is a function from $\Theta_2$ to $p_2$, $S_2 : \Theta_2 \to p_2$. The strategy of the downstream firm is a function from $\Theta_1$ to $p_1$, $S_1 : \Theta_1 \to p_1$, and a function from $\Theta_1$ to $q_2$, $\bar{S}_2 : \Theta_1 \to q_1$ . These strategies imply that both firms have monopoly power in their output market.

The profit of downstream firm is $\pi_1 = p_1 q_1 - \theta_1 q_1 - p_2 q_2 + s_1$ and the profit of upstream firm is $\pi_2 = p_2 q_2 - \theta_2 q_2 + s_2$. $s_1$ and $s_2$ are subsidy that each firm receives and if $s_i < 0$ then it means the regulator levies tax on the firm $i$.

We denote the profile of output, type, price, and subsidy as the following.

$q = (q_1, q_2)$, $\theta = (\theta_1, \theta_2)$, $p = (p_1, p_2)$, and $s = (s_1, s_2)$.

There is another interpretation for this environment. $\theta$ means the technology exists in the industry and $q_2$ is a product that can decrease the emission in the environment. Thus, high $q_1$ means low emission.

Assumption 1: $\theta_i + \frac{F_i(\theta_i)}{f_i(\theta_i)}$ is increasing in $\theta_i$.

This assumption means that hazard rate is increasing , $\frac{f_i(\theta_i)}{1-F_i(\theta_i)}$, and it can be explained as the conditional probability of dying at time $\theta_i$ of an individual who has survived until time $\theta_i$.[11] In our setup it can be interpreted as the conditional probability of having marginal cost $\theta_i$ if the firm has less marginal cost than $\theta_i$, hence $\frac{f_i(\theta_i)}{F_i(\theta_i)}$ means the conditional probability of having marginal cost $\theta_i$ if the firm has more marginal cost than $\theta_i$.

The regulator wants to implement new technology, $\theta$, or to regulate the industry, $(p, q, s)$. The mechanism is the profile of functions $(q_i(\theta), p_i(\theta), s_i(\theta))_{i=1,2}$. $q_i$ means the amount of production that each firm produces, $p_i$ is the price that is set by the firm, and $s_i$ is the subsidy that is received by the firm. We assume that the regulator wants to implement this mechanism so we do not assume the probabilistic mechanism like Baron and Myerson [1982].

The objective of the regulator is:

$$\int_{\underline{\theta}}^{\bar{\theta}} (v(q_1(\theta)) - p_1(\theta)q_1(\theta) - s_1(\theta) - s_2(\theta) + \alpha\pi_1(\theta) + \alpha\pi_2(\theta))dF(\theta)$$

where $v(q_1(\theta))$ is equal to $\int_0^{q_1} P(\bar{q}_1)d\bar{q}_1$ and $1 \geq \alpha \geq 0$. The regulator wants to maximize the above function and the regulator gives less weight to the firms in comparison with the representative consumer. The objective function depends on profile of types.

[10] In this environment the regulator and consumer exist too, but the consumer decision is represented by the inverse demand function and also we want to describe the game that is designed by the planner, so we do not consider them in the game.

[11] In the mechanism design literature, regularity condition is defined as increasing $\frac{f_i(\theta_i)}{1-F_i(\theta_i)}$ and it implies that $\theta_i + \frac{F_i(\theta_i)}{f_i(\theta_i)}$ and $\theta_i - \frac{1-F_i(\theta_i)}{f_i(\theta_i)}$ are increasing, see Borgers et al. [2015]

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### 3.2.1 Limited Communication Mechanism

In the limited communication mechanism the regulator observes the output of downstream firm, but it can not communicate or make contract with the upstream firm. Therefore, it can use just $q_1, p_1, s_1$ for the purpose of contracting with the downstream firm. The regulator does not receive any message from the upstream firm. In the limited communication mechanism literature, Kos [2012] and Mookherjee and Tsumagari [2014], the message set is limited such that there is no onto function between the message set and the type of firm, but in this paper the message set is empty. Therefore, it is the extreme case of this literature.[12]

The mechanism has two stages and stage one has two moves and stage two has three moves. In the first stage the regulator offers the downstream firm a contract of the form $(s_1(\theta_1), p_1(\theta_1), q_1(\theta_1))$. The downstream firm then accepts or rejects the contract. By acceptance of the contract the downstream firm will report her type $\tilde{\theta_1}$ regarding its own type $\theta_1$. Consequently, the downstream firm chooses one contract from a menu of alternatives. In the second stage the downstream firm offers a subcontract to the upstream firm. The upstream firm accepts or reject the contract. The upstream firm produces the amount $q_2$ as part of the subcontract in the case of acceptance. In the third move of the second stage the downstream firm produces the final good $q_1$ in response to the $q_2$.

The second stage interaction between the downstream firm and the upstream firm is not observable by the regulator. The regulator neither observes the contract that the downstream firm offers the upstream firm nor observes the subsequent reply by the upstream firm. Nevertheless, it does not matter whether the upstream firm knows the contract which is offered by the regulator to the downstream firm, and the report made by the latter. In other words, the downstream firm offers the upstream firm a contract $\{p_2(\theta_2), q_2(\theta_2)\}_{\theta_2 \in \Theta_2}$ at the second stage, so the contract just depends to the type of upstream firm. Therefore, this limited mechanism implies that $s_2(\theta) = 0$, but it is not general and it can be changed. In this paper we follow this implication then we assume $s_2(\theta) = 0$ for every $\theta$.

For given contract $(s_1(\tilde{\theta_1}), p_1(\tilde{\theta_1}), q_1(\tilde{\theta_1}))$, we denote the subcontract, or the mechanism, for the upstream firm by $\{p_2(\theta_2|\theta_1, \tilde{\theta_1}), q_2(\theta_2|\theta_1, \tilde{\theta_1})\}_{\theta_1 \in \Theta_1}$. This notation shows that the subcontract depends on $\theta_1$ and $\tilde{\theta_1}$. Given that the upstream firm reports its type, $\theta_2$, truthfully, the downstream firm will choose $q_2$ and $p_2$ in order to maximize its profit as the following.

$$p_1(\tilde{\theta_1})q_1(\tilde{\theta_1}) - \theta_1 q_1(\tilde{\theta_1}) - p_2(\theta_2|\theta_1, \tilde{\theta_1})q_2(\theta_2|\theta_1, \tilde{\theta_1}) + s_1(\tilde{\theta_1})$$

This sequential optimization problem, the second stage of the game, can be represented as

[12] The mechanism between the upstream firm and the downstream firm is not limited communication mechanism.

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a simultaneous choice of the functions $\{p_2(\cdot|\theta_1,\tilde{\theta_1}), q_2(\cdot|\theta_1,\tilde{\theta_1})\}$. Finally, the downstream firm who has type $\theta_1$ and reported her type $\tilde{\theta_1}$ should solve the following optimization problem.[13]

$SC(\theta_1|s_1(\tilde{\theta_1}), p_1(\tilde{\theta_1}), q_1(\tilde{\theta_1}))$ : [14]

$$\max_{p_2(\cdot),q_2(\cdot)} \mathrm{E}_{\Theta_2}[p_1(\tilde{\theta_1})q_1(\tilde{\theta_1}) - \theta_1 q_1(\tilde{\theta_1}) - p_2(\theta_2|\theta_1,\tilde{\theta_1})q_2(\theta_2|\theta_1,\tilde{\theta_1}) + s_1(\tilde{\theta_1})]$$

subject to: for all $\theta_2 \in \Theta_2$, and all $\theta_1, \tilde{\theta_1} \in \Theta_1$,

$$\theta_2 \in \arg\max_{\tilde{\theta_2}}[p_2(\tilde{\theta_2}|\theta_1,\tilde{\theta_1})q_2(\tilde{\theta_2}|\theta_1,\tilde{\theta_1}) - \theta_2 q_2(\tilde{\theta_2}|\theta_1,\tilde{\theta_1})] \qquad (i)$$

$$p_2(\theta_2|\theta_1,\tilde{\theta_1})q_2(\theta_2|\theta_1,\tilde{\theta_1}) - \theta_2 q_2(\theta_2|\theta_1,\tilde{\theta_1}) \geq 0 \qquad (ii)$$

Constraint $(i)$ represents the incentive compatibility constraint for the upstream firm, and Constraint $(ii)$ is the individual rationality constraint for the upstream firm. Because there exists just one upstream firm in the industry or the downstream firm wants to make contract with just one firm then the relationship between the upstream firm and the downstream firm is same as the principal-agent relationship. Hence, the manager considers ex post IC and IR.

The implicit assumption in the above mechanism is that the downstream firm considers to make contract with all upstream firm, all $\theta_2 \in \Theta_2$. It means that we do not consider $\Theta_2$ as the choice variable for downstream firm, but it is not obvious that it is always profitable or not for the downstream firm and it depends on the environment. This implicit assumption works for the regulator too.

**Lemma 3.2.1** *: The necessary and the sufficient conditions for the incentive compatibility constraint are as the following.*

$q_2(\theta_2|\theta_1,\tilde{\theta_1})$ *is decreasing in* $\theta_2$ *and* $\frac{d\pi_2(\theta_2|\theta_1,\tilde{\theta_1})}{d\theta_2} = -q_2(\theta_2|\theta_1,\tilde{\theta_1})$

**Lemma 3.2.2** *: The individual rationality constraint is* $\pi_2(\bar{\theta}_2|\theta_1,\tilde{\theta_1}) = 0$.

With the above lemmata the downstream firm problem is changed as the following.

**Lemma 3.2.3** *: The reduced form profit of downstream firm that the planner considers is as the following.*[15]

$$\max_{q_2(\cdot)} E_{\Theta_2}[p_1(\tilde{\theta_1})q_1(\tilde{\theta_1}) - \theta_1 q_1(\tilde{\theta_1}) + s_1(\tilde{\theta_1}) - (\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)})q_2(\theta_2|\theta_1,\tilde{\theta_1})]$$

[13] $\{p_2(\cdot|\theta_1,\tilde{\theta_1}), q_2(\cdot|\theta_1,\tilde{\theta_1})\}$ and $\{p_2(.), q_2(.)\}$ mean $\{p_2(\theta_2|\theta_1,\tilde{\theta_1}), q_2(\theta_2|\theta_1,\tilde{\theta_1})\}_{\theta_2\in\Theta_2}$

[14] SC means the subcontract.

[15] There is an implicit assumption in this lemma, and it is that We ignore that $q_2$ is decreasing. So, we should check it later.

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We denote $\Gamma_1(\theta_1|(s_1(\tilde{\theta}_1), p_1(\tilde{\theta}_1), q_1(\tilde{\theta}_1))) = \max_{q_2(\cdot)} \mathrm{E}_{\Theta_2}[p_1(\tilde{\theta}_1)q_1(\tilde{\theta}_1) - \theta_1 q_1(\tilde{\theta}_1) + s_1(\tilde{\theta}_1) - (\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)})q_2(\theta_2|\theta_1, \tilde{\theta}_1)]$. $\Gamma_1(\theta_1|(s_1(\tilde{\theta}_1), p_1(\tilde{\theta}_1), q_1(\tilde{\theta}_1)))$ is the value of the sub contract optimization problem, $SC(\theta_1|(s_1(\tilde{\theta}_1), p_1(\tilde{\theta}_1), q_1(\tilde{\theta}_1)))$. It represents the reduced form utility of the downstream type $\theta_1$ when the downstream firm reports to the regulator type $\tilde{\theta}_1$ at the first stage.

**Lemma 3.2.4** *: The reduced form profit of downstream firm, the demand of $q_2$, and the price $p_2$ for downstream firm are as the following.*

$$\Gamma_1(\theta_1|(s_1(\tilde{\theta}_1), p_1(\tilde{\theta}_1), q_1(\tilde{\theta}_1))) = [p_1(\tilde{\theta}_1)q_1(\tilde{\theta}_1) - \theta_1 q_1(\tilde{\theta}_1) + s_1(\tilde{\theta}_1) - Aq_1(\tilde{\theta}_1)E_{\Theta_2}(\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)})]$$

$$q_2(\theta_2|\theta_1, \tilde{\theta}_1) = Aq_1(\tilde{\theta}_1) \ \ for \ \ every \ \ \theta_2$$

$$p_2(\theta_2|\theta_1, \tilde{\theta}_1) = \bar{\theta}_2$$

The downstream firm chooses pool contract because of the production function. Given $q_1$ the downstream firm always chooses same $q_2$ for every $\theta_2$. Suppose $q_1$ is the intermediate good that is produced by the downstream firm, the production function of the upstream firm is $y = q_1^{\lambda} q_2^{1-\lambda}$, and $y$ is the final good then the subcontract is not the pool contract. The cost of reduced profit consists of two parts. The second part of cost depends on the type and distribution of upstream firm. Suppose that both firms have same distribution and they run the industry with corporation and without conflict, usual regulation problem, the above result shows that the cost of the conflict is $Aq_1(\tilde{\theta}_1)\mathrm{E}_{\Theta_2}(\frac{F_2(\theta_2)}{f_2(\theta_2)})$ in comparison with usual regulation problem.

Thus, the regulator problem in the first stage reduces to

$$RC:^{16} \quad \max_{s_1(\theta_1), p_1(\theta_1), q_1(\theta_1)} \int_{\underline{\theta}}^{\bar{\theta}} (v(q_1(\theta)) - p_1(\theta)q_1(\theta) - s_1(\theta) - s_2(\theta) + \alpha\pi_1(\theta) + \alpha\pi_2(\theta))dF(\theta)$$

subject to: for all $\theta_1 \in \Theta_1$

$$\theta_1 \in \arg\max_{\hat{\theta}_1} \Gamma_1(\theta_1|(s_1(\tilde{\theta}_1), p_1(\tilde{\theta}_1), q_1(\tilde{\theta}_1))) \qquad (i)$$

$$\Gamma_1(\theta_1|(s_1(\theta_1), p_1(\theta_1), q_1(\theta_1))) \geq 0 \qquad (ii)$$

Constraint $(i)$ represents the incentive compatibility constraint for the downstream firm based on its reduced form profit. Constraint $(ii)$ is the rationality constraint for the downstream firm.

**Lemma 3.2.5** *: The necessary and the sufficient conditions for the incentive compatibility constraint for the downstream firm are as the following.*

*$q_1(\theta_1)$ is decreasing in $\theta_1$ and* $\frac{d\Gamma_1(\theta_1)}{d(\theta_1)} = -q_1(\theta_1)$

[16] RC means the regulator contract.

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**Lemma 3.2.6** *: The individual rationality constraint is* $\Gamma_1(\bar{\theta}_1) = 0$.

With above lemmata we can write the reduced profit of downstream firm given telling truth as the following.

$$\Gamma_1(\theta_1) = \int_{\theta_1}^{\bar{\theta}_1} q_1(\tilde{\theta}_1)d(\tilde{\theta}_1)$$

Then $s_1(\theta_1)$ is characterized as the following.

$$s_1(\theta_1) = \theta_1 q_1(\theta_1) + Aq_1(\theta_1)\mathrm{E}_{\Theta_2}(\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)}) - p_1(\theta_1)q_1(\theta_1)$$

Given the inverse demand function, $p_1(\theta_1) = P(q_1(\theta_1))$:

$$s_1(\theta_1) = \theta_1 q_1(\theta_1) + Aq_1(\theta_1)\mathrm{E}_{\Theta_2}(\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)}) - P(q_1(\theta_1))q_1(\theta_1)$$

There is no difference if we characterize $p_1$ instead of $q_1$ because from inverse demand function we can characterize $q_1$.

**Proposition 3.2.7** *If* $\theta_1 + (1-\alpha)\frac{F_1(\theta_1)}{f_1(\theta_1)}$ *is increasing in* $\theta_1$ *then*

$$p_1(\theta_1) = \theta_1 + (1-\alpha)\frac{F_1(\theta_1)}{f_1(\theta_1)} + AE_{\Theta_2}(\theta_2 + (1-\alpha)\frac{F_2(\theta_2)}{f_2(\theta_2)})$$

The characterization depends on the distribution and production function. The condition guarantees that $q_1(\theta_1)$ is decreasing. If this condition does not hold for some distribution then we can define $z_\alpha(\theta_1) = \theta_1 + (1-\alpha)\frac{F_1(\theta_1)}{f_1(\theta_1)}$ and use ironing approach of Myerson [1981]. The price has two part. The first part, $\theta_1 + (1-\alpha)\frac{F_1(\theta_1)}{f_1(\theta_1)}$, depends on the type of downstream firm, but the second part, $A\mathrm{E}_{\Theta_2}(\theta_2 + (1-\alpha)\frac{F_2(\theta_2)}{f_2(\theta_2)})$, is fixed and independent of type of downstream firm. The regulator should pay the expectation or mean of rent that the upstream firm receives, to the downstream firm. The fixed term is expectation of rent and is not ex post rent that the upstream firm receives. There are two crucial and implicit assumptions in this model. The first is risk neutrality of the downstream firm and the second is that the downstream firm knows the distribution of the upstream firm. Therefore, it is reasonable to relax these assumptions.

In Baron and Myerson [1982] price depends on the type of firm and there is no fixed term. Suppose we use Baron and Myerson [1982] mechanism then we ignore the second term of the price in our mechanism. Then the price is set lower than the price of our mechanism. It makes that mechanism does not work very well.

In vertical integration literature, see Armstrong and Sappington [2007], the second term is known for both regulator and downstream firm. So there is no uncertainty about the price. On the other hand, the second term in our model comes from uncertainty. It is an expected value but the private information between the upstream firm and downstream firm generates it then it is important the mechanism of uncertainty between the upstream firm and downstream firm.

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## 3.3 Conclusion

This chapter considers regulation or new technology adoption in the environment in which two firm exist. One firm is upstream firm and other one is downstream firm. The regulator can not communicate with upstream firm but downstream firm can do it. There fore, the regulator makes contract with the downstream firm and the regulator takes in to the account this constraint, limited communication. We characterize the price, output, and subsidy for both firms, and we explain the difference of our result with Baron and Myerson [1982] and Armstrong and Sappington [2007]. To interpret this result as an industrial policy it seems we should consider ambiguity of new technology.

## 3.4 Appendix

Lemma 3.2.1

The necessary condition:

If allocation is incentive compatible then for every $\theta_2$ and $\hat{\theta}_2$ the following inequality should hold.

$$p_2(\theta_2|\theta_1,\tilde{\theta_1})q_2(\theta_2|\theta_1,\tilde{\theta_1}) - \theta_2 q_2(\theta_2|\theta_1,\tilde{\theta_1}) \geq p_2(\hat{\theta}_2|\theta_1,\tilde{\theta_1})q_2(\hat{\theta}_2|\theta_1,\tilde{\theta_1}) - \theta_2 q_2(\hat{\theta}_2|\theta_1,\tilde{\theta_1})$$

and

$$p_2(\hat{\theta}_2|\theta_1,\tilde{\theta_1})q_2(\hat{\theta}_2|\theta_1,\tilde{\theta_1}) - \hat{\theta}_2 q_2(\hat{\theta}_2|\theta_1,\tilde{\theta_1}) \geq p_2(\theta_2|\theta_1,\tilde{\theta_1})q_2(\theta_2|\theta_1,\tilde{\theta_1}) - \hat{\theta}_2 q_2(\theta_2|\theta_1,\tilde{\theta_1})$$

Adding both sides of inequalities and basic algebra imply that

$$(\hat{\theta}_2 - \theta_2)(q_2(\theta_2|\theta_1,\tilde{\theta_1}) - q_2(\hat{\theta}_2|\theta_1,\tilde{\theta_1})) \geq 0.$$

then $q_2(\theta_2|\theta_1,\tilde{\theta_1})$ is decreasing.

Another definition of incentive compatibility is

$$\pi_2(\theta_2|\theta_1,\tilde{\theta_1}) = \max_{\hat{\theta}_2} p_2(\hat{\theta}_2|\theta_1,\tilde{\theta_1})q_2(\hat{\theta}_2|\theta_1,\tilde{\theta_1}) - \theta_2 q_2(\hat{\theta}_2|\theta_1,\tilde{\theta_1}).$$

The envelope theorem implies.

$$\frac{d\pi_2(\theta_2|\theta_1,\tilde{\theta_1})}{d\theta_2} = -q_2(\theta_2|\theta_1,\tilde{\theta_1})$$

The sufficient condition:

We should prove the following inequality for every $\theta_2$ and $\hat{\theta}_2$.

$$p_2(\theta_2|\theta_1,\tilde{\theta_1})q_2(\theta_2|\theta_1,\tilde{\theta_1}) - \theta_2 q_2(\theta_2|\theta_1,\tilde{\theta_1}) \geq p_2(\hat{\theta}_2|\theta_1,\tilde{\theta_1})q_2(\hat{\theta}_2|\theta_1,\tilde{\theta_1}) - \theta_2 q_2(\hat{\theta}_2|\theta_1,\tilde{\theta_1})$$

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$$\longleftrightarrow \; definition\; of\; IC$$

$$\pi_2(\theta_2|\theta_1,\tilde{\theta_1}) - \pi_2(\hat{\theta}_2|\theta_1,\tilde{\theta_1}) \geq -(\theta_2 - \hat{\theta}_2)q_2(\hat{\theta}_2|\theta_1,\tilde{\theta_1})$$

$$\longleftrightarrow \; taking\; integral$$

$$\int_{\hat{\theta}_2}^{\theta_2} \frac{d\pi_2(\tilde{\theta}_2|\theta_1,\tilde{\theta_1})}{d\tilde{\theta}_2} d(\tilde{\theta}_2) \geq \int_{\hat{\theta}_2}^{\theta_2} -q_2(\hat{\theta}_2|\theta_1,\tilde{\theta_1})d(\tilde{\theta}_2)$$

$$\longleftrightarrow \; the\; second\; necessary\; condition$$

$$\int_{\hat{\theta}_2}^{\theta_2} -q_2(\tilde{\theta}_2|\theta_1,\tilde{\theta_1})d(\tilde{\theta}_2) \geq \int_{\hat{\theta}_2}^{\theta_2} -q_2(\hat{\theta}_2|\theta_1,\tilde{\theta_1})d(\tilde{\theta}_2)$$

Suppose $\hat{\theta}_2 > \tilde{\theta}_2 > \theta_2$ then $-q_2(\hat{\theta}_2|\theta_1,\tilde{\theta_1}) \geq -q_2(\tilde{\theta}_2|\theta_1,\tilde{\theta_1})$. Taking integral from both sides implies $\int_{\theta_2}^{\hat{\theta}_2} -q_2(\hat{\theta}_2|\theta_1,\tilde{\theta_1}) \geq \int_{\theta_2}^{\hat{\theta}_2} -q_2(\tilde{\theta}_2|\theta_1,\tilde{\theta_1})$, now changing integral limits and sign of inequality give the desired result.

Now suppose $\theta_2 > \tilde{\theta}_2 > \hat{\theta}_2$ then $-q_2(\tilde{\theta}_2|\theta_1,\tilde{\theta_1}) \geq -q_2(\hat{\theta}_2|\theta_1,\tilde{\theta_1})$. Taking integral on both sides implies $\int_{\hat{\theta}_2}^{\theta_2} -q_2(\tilde{\theta}_2|\theta_1,\tilde{\theta_1}) \geq \int_{\hat{\theta}_2}^{\theta_2} -q_2(\hat{\theta}_2|\theta_1,\tilde{\theta_1})$.

Lemma 3.2.2

Suppose $\pi_2(\bar{\theta}_2|\theta_1,\tilde{\theta_1}) = p_2(\bar{\theta}_2|\theta_1,\tilde{\theta_1})q_2(\bar{\theta}_2|\theta_1,\tilde{\theta_1}) - \bar{\theta}_2 q_2(\bar{\theta}_2|\theta_1,\tilde{\theta_1}) > 0$ then the downstream firm can decrease $p_2(\bar{\theta}_2|\theta_1,\tilde{\theta_1})$ and it increases the reduced profit of the downstream firm. This condition is Sufficient because $\pi_2(\theta_2|\theta_1,\tilde{\theta_1})$ is decreasing function.

Lemma 3.2.3

Taking integral from both sides $\frac{d\pi_2(\theta_2|\theta_1,\tilde{\theta_1})}{d\theta_2} = -q_2(\theta_2|\theta_1,\tilde{\theta_1})$ on the interval $[\theta_2, \bar{\theta}_2]$ gives the following result.

$$\pi_2(\theta_2|\theta_1,\tilde{\theta_1}) = \int_{\theta_2}^{\bar{\theta}_2} q_2(\tilde{\theta}_2|\theta_1,\tilde{\theta_1})d(\tilde{\theta}_2)$$

Then $p_2(\theta_2|\theta_1,\tilde{\theta_1})q_2(\theta_2|\theta_1,\tilde{\theta_1}) = \theta_2 q_2(\theta_2|\theta_1,\tilde{\theta_1}) + \int_{\theta_2}^{\bar{\theta}_2} q_2(\tilde{\theta}_2|\theta_1,\tilde{\theta_1})d(\tilde{\theta}_2)$.

Putting above equation in the following equation gives the result.

$$\max_{p_2(\cdot),q_2(\cdot)} \mathrm{E}_{\Theta_2}[p_1(\tilde{\theta}_1)q_1(\tilde{\theta}_1) - \theta_1 q_1(\tilde{\theta}_1) - p_2(\theta_2|\theta_1,\tilde{\theta_1})q_2(\theta_2|\theta_1,\tilde{\theta_1}) + s_1(\tilde{\theta}_1)]$$

$$\max_{q_2(\cdot)} \mathrm{E}_{\Theta_2}[p_1(\tilde{\theta}_1)q_1(\tilde{\theta}_1) - \theta_1 q_1(\tilde{\theta}_1) - \theta_2 q_2(\theta_2|\theta_1,\tilde{\theta_1}) - \int_{\theta_2}^{\bar{\theta}_2} q_2(\tilde{\theta}_2|\theta_1,\tilde{\theta_1})d(\tilde{\theta}_2) + s_1(\tilde{\theta}_1)]$$

Using Fubini's theorem gives the desired result. Recall that $\int_{\underline{\theta}_2}^{\bar{\theta}_2}\int_{\theta_2}^{\bar{\theta}_2} q_2(\tilde{\theta}_2|\theta_1,\tilde{\theta_1})d(\tilde{\theta}_2) = \int_{\underline{\theta}_2}^{\bar{\theta}_2}(\frac{F_2(\theta_2)}{f_2(\theta_2)})q_2(\theta_2|\theta_1,\tilde{\theta_1})d(\theta_2)$.

Lemma 3.2.4

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Given reduced profit of downstream firm.

$$\max_{q_2(\cdot)} \mathrm{E}_{\Theta_2}[p_1(\tilde{\theta}_1)q_1(\tilde{\theta}_1) - \theta_1 q_1(\tilde{\theta}_1) + s_1(\tilde{\theta}_1) - (\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)})q_2(\theta_2|\theta_1, \tilde{\theta}_1)]$$

$p_1(\tilde{\theta}_1),\ q_1(\tilde{\theta}_1)$, and $s_1(\tilde{\theta}_1)$ do not depend on $\Theta_2$ then we can write it as the following.

$$[p_1(\tilde{\theta}_1)q_1(\tilde{\theta}_1) - \theta_1 q_1(\tilde{\theta}_1) + s_1(\tilde{\theta}_1) - \max_{q_2(\cdot)} \mathrm{E}_{\Theta_2}(\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)})q_2(\theta_2|\theta_1, \tilde{\theta}_1)]$$

From demand of input $q_1(\tilde{\theta}_1) = Aq_2(\theta_2|\theta_1, \tilde{\theta}_1)$ we get the results.

$$\Gamma_1(\theta_1|(s_1(\tilde{\theta}_1), p_1(\tilde{\theta}_1), q_1(\tilde{\theta}_1))) = [p_1(\tilde{\theta}_1)q_1(\tilde{\theta}_1) - \theta_1 q_1(\tilde{\theta}_1) + s_1(\tilde{\theta}_1) - Aq_1(\tilde{\theta}_1)\mathrm{E}_{\Theta_2}(\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)})]$$

$$q_2(\theta_2|\theta_1, \tilde{\theta}_1) = Aq_1(\tilde{\theta}_1) \ \ for \ \ every \ \ \theta_2$$

To compute price we use envelope theorem and integrate both sides.

$$\pi_2(\theta_2|\theta_1, \tilde{\theta}_1) = \int_{\theta_2}^{\bar{\theta}_2} q_2(\tilde{\theta}_2|\theta_1, \tilde{\theta}_1)d(\tilde{\theta}_2)$$

Then $p_2(\theta_2|\theta_1, \tilde{\theta}_1)q_2(\theta_2|\theta_1, \tilde{\theta}_1) = \theta_2 q_2(\theta_2|\theta_1, \tilde{\theta}_1) + \int_{\theta_2}^{\bar{\theta}_2} q_2(\tilde{\theta}_2|\theta_1, \tilde{\theta}_1)d(\tilde{\theta}_2)$ and $q_2(\tilde{\theta}_2|\theta_1, \tilde{\theta}_1) = Aq_1(\tilde{\theta}_1) = q_2(\theta_2|\theta_1, \tilde{\theta}_1)$.

$$p_2(\theta_2|\theta_1, \tilde{\theta}_1)Aq_1(\tilde{\theta}_1) = \theta_2 Aq_1(\tilde{\theta}_1) + \int_{\theta_2}^{\bar{\theta}_2} Aq_1(\tilde{\theta}_1)d(\tilde{\theta}_2)$$

After cancellation $Aq_1(\tilde{\theta}_1)$ on both sides we get result.

$$p_2(\theta_2|\theta_1, \tilde{\theta}_1) = \theta_2 + \int_{\theta_2}^{\bar{\theta}_2} d(\tilde{\theta}_2) = \theta_2 + \bar{\theta}_2 - \theta_2 = \bar{\theta}_2$$

Lemma 3.2.5

The Necessary condition:

If allocation is incentive compatible then for every $\theta_1$ and $\tilde{\theta}_1$ the following inequality should hold.

$$p_1(\theta_1)q_1(\theta_1) - \theta_1 q_1(\theta_1) + s_1(\theta_1) - Aq_1(\theta_1)\mathrm{E}_{\Theta_2}(\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)}) \geq$$

$$p_1(\tilde{\theta}_1)q_1(\tilde{\theta}_1) - \theta_1 q_1(\tilde{\theta}_1) + s_1(\tilde{\theta}_1) - Aq_1(\tilde{\theta}_1)\mathrm{E}_{\Theta_2}(\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)})$$

and

$$p_1(\tilde{\theta}_1)q_1(\tilde{\theta}_1) - \tilde{\theta}_1 q_1(\tilde{\theta}_1) + s_1(\tilde{\theta}_1) - Aq_1(\tilde{\theta}_1)\mathrm{E}_{\Theta_2}(\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)}) \geq$$

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$$p_1(\theta_1)q_1(\theta_1) - \tilde{\theta}_1 q_1(\theta_1) + s_1(\theta_1) - Aq_1(\theta_1)\mathrm{E}_{\Theta_2}(\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)})$$

Adding both sides of inequalities and basic algebra imply that

$$(\tilde{\theta}_1 - \theta_1)(q_1(\theta_1) - q_1(\tilde{\theta}_1)) \geq 0.$$

then $q_1(\theta_1)$ is decreasing.

Another definition of incentive compatibility is

$$\Gamma_1(\theta_1) = \max_{\tilde{\theta}_2}[p_1(\tilde{\theta}_1)q_1(\tilde{\theta}_1) - \theta_1 q_1(\tilde{\theta}_1) + s_1(\tilde{\theta}_1) - Aq_1(\tilde{\theta}_1)\mathrm{E}_{\Theta_2}(\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)})].$$

The envelope theorem implies.

$$\frac{d\Gamma_1(\theta_1)}{d\theta_1} = -q_1(\theta_1)$$

The sufficient condition:

We should prove the following inequality for every $\theta_1$ and $\hat{\theta}_1$.

$$p_1(\theta_1)q_1(\theta_1) - \theta_1 q_1(\theta_1) + s_1(\theta_1) - Aq_1(\theta_1)\mathrm{E}_{\Theta_2}(\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)}) \geq$$

$$p_1(\hat{\theta}_1)q_1(\hat{\theta}_1) - \theta_1 q_1(\hat{\theta}_1) + s_1(\hat{\theta}_1) - Aq_1(\hat{\theta}_1)\mathrm{E}_{\Theta_2}(\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)})$$

$$\longleftrightarrow \; definition\ of\ IC$$

$$\Gamma_1(\theta_1) - \Gamma_1(\hat{\theta}_1) \geq -(\theta_1 - \hat{\theta}_1)q_1(\hat{\theta}_1)$$

$$\longleftrightarrow \; taking\ integral$$

$$\int_{\hat{\theta}_1}^{\theta_1} \frac{d\Gamma_1(\tilde{\theta}_1)}{d\tilde{\theta}_1} d(\tilde{\theta}_1) \geq \int_{\hat{\theta}_1}^{\theta_1} -q_1(\hat{\theta}_1)d(\tilde{\theta}_1)$$

$$\longleftrightarrow \; the\ second\ necessary\ condition$$

$$\int_{\hat{\theta}_1}^{\theta_1} -q_1(\tilde{\theta}_1)d(\tilde{\theta}_1) \geq \int_{\hat{\theta}_1}^{\theta_1} -q_1(\hat{\theta}_1)d(\tilde{\theta}_1)$$

Suppose $\hat{\theta}_1 > \tilde{\theta}_1 > \theta_1$ then $-q_1(\hat{\theta}_1) \geq -q_1(\tilde{\theta}_1)$. Taking integral implies $\int_{\theta_1}^{\hat{\theta}_1} -q_1(\hat{\theta}_1) \geq \int_{\theta_1}^{\hat{\theta}_1} -q_1(\tilde{\theta}_1)$, now changing integral limits and sign of inequality give the desired result.

Now suppose $\theta_1 > \tilde{\theta}_1 > \hat{\theta}_1$ then $-q_1(\tilde{\theta}_1) \geq -q_1(\hat{\theta}_1)$. Taking integral implies $\int_{\hat{\theta}_2}^{\theta_1} -q_1(\tilde{\theta}_1) \geq \int_{\hat{\theta}_1}^{\theta_1} -q_1(\hat{\theta}_1)$.

Lemma 3.2.6

Suppose $\Gamma_1(\bar{\theta}_1) = p_1(\bar{\theta}_1)q_1(\bar{\theta}_1) - \bar{\theta}_1 q_1(\bar{\theta}_1) + s_1(\bar{\theta}_1) - Aq_1(\bar{\theta}_1)\mathrm{E}_{\Theta_2}(\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)}) > 0$. The regulator can increase objective function if the regulator decreases $s_1(\bar{\theta}_1)$. It is optimal because $1 \geq \alpha \geq 0$ in the objective function.

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Proposition 3.2.7

The regulator wants to maximize the following objective function.

$$\int_{\underline{\theta}}^{\bar{\theta}} (v(q_1(\theta)) - p_1(\theta)q_1(\theta) - s_1(\theta) - s_2(\theta) + \alpha\pi_1(\theta) + \alpha\pi_2(\theta))dF(\theta)$$

The limited communication mechanism changes the above objective function as the following.

$$\int_{\underline{\theta}}^{\bar{\theta}} [v(q_1(\theta_1)) - p_1(\theta_1)q_1(\theta_1) - s_1(\theta_1) - s_2(\theta) +$$

$$\alpha(p_1(\theta_1)q_1(\theta_1) - \theta_1 q_1(\theta_1) - p_2(\theta)q_2(\theta) + s_1(\theta_1)$$

$$\alpha(p_2(\theta)q_2(\theta) - \theta_2 q_2(\theta) + s_2(\theta)]dF(\theta)$$

Because there is no communication between the regulator and the upstream firm then $s_2(\theta) = 0$ and we know $Aq_1(\theta_1) = q_2(\theta)$ Now the objective function is as the following.

$$\int_{\underline{\theta}_1}^{\bar{\theta}_1} [v(q_1(\theta_1)) + (\alpha - 1)p_1(\theta_1)q_1(\theta_1) + (\alpha - 1)s_1(\theta_1) - \alpha\theta_1 q_1(\theta_1) - \alpha \mathrm{E}_{\Theta_2}\theta_2 A q_1(\theta_1)]dF_1(\theta_1)$$

The integral is changed from $\Theta$ to $\Theta_1$. We use the reduce profit of downstream firm.

$$p_1(\theta_1)q_1(\theta_1) - \theta_1 q_1(\theta_1) + s_1(\theta_1) - Aq_1(\theta_1)\mathrm{E}_{\Theta_2}(\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)}) = \Gamma_1(\theta_1)d(\theta_1) =$$

$$\int_{\theta_1}^{\bar{\theta}_1} q_1(\hat{\theta}_1)d(\hat{\theta}_1)$$

Then $p_1(\theta_1)q_1(\theta_1) = \theta_1 q_1(\theta_1) - s_1(\theta_1) + Aq_1(\theta_1)\mathrm{E}_{\Theta_2}(\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)}) + \int_{\theta_1}^{\bar{\theta}_1} q_1(\hat{\theta}_1)d(\hat{\theta}_1)$ Putting the above equation in the objective function gives the following objective function.

$$\int_{\underline{\theta}_1}^{\bar{\theta}_1} [v(q_1(\theta_1)) + [(\alpha - 1)A\mathrm{E}_{\Theta_2}(\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)}) - \alpha\mathrm{E}_{\Theta_2}\theta_2 A - \theta_1]q_1(\theta_1) +$$

$$(\alpha - 1)\int_{\theta_1}^{\bar{\theta}_1} q_1(\hat{\theta}_1)d(\hat{\theta}_1)]dF_1(\theta_1)$$

Again Fubini's theorem.

$$\int_{\underline{\theta}_1}^{\bar{\theta}_1} [v(q_1(\theta_1)) + [(\alpha - 1)A\mathrm{E}_{\Theta_2}(\theta_2 + \frac{F_2(\theta_2)}{f_2(\theta_2)}) - \alpha\mathrm{E}_{\Theta_2}\theta_2 A - \theta_1]q_1(\theta_1) +$$

$$(\alpha - 1)q_1(\theta_1)\frac{F_1(\theta_1)}{f_1(\theta_1)}]dF_1(\theta_1)$$

Rearranging gives the following result:

$$\int_{\underline{\theta}_1}^{\bar{\theta}_1} [v(q_1(\theta_1)) - q_1(\theta_1)[\theta_1 + (1 - \alpha)\frac{F_1(\theta_1)}{f_1(\theta_1)} + A\mathrm{E}_{\Theta_2}(\theta_2 + (1 - \alpha)\frac{F_2(\theta_2)}{f_2(\theta_2)})]dF_1(\theta_1)$$

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We know that $\frac{d(v(q_1(\theta_1)))}{d(q_1(\theta_1))} = p_1(\theta_1)$, $v(q_1(\theta)) = \int_0^{q_1} P(\bar{q_1})d\bar{q_1}$ and $p_1 = P(q_1)$.

The $v(q_1(\theta_1))$ is decreasing in $\theta_1$ and $-q_1(\theta_1)$ is increasing in $\theta_1$, so there is a solution for every $\theta_1$.

Taking the first order condition for every $\theta_1$ and respect to $q_1$ gives the result.

$$\frac{d(v(q_1(\theta_1)))}{d(q_1(\theta_1))} = p_1(\theta_1) = \theta_1 + (1-\alpha)\frac{F_1(\theta_1)}{f_1(\theta_1)} + A\mathrm{E}_{\Theta_2}(\theta_2 + (1-\alpha)\frac{F_2(\theta_2)}{f_2(\theta_2)})$$

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