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Information, Markets, and Networks: Essays in Climate Finance

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Introduction

Climate policy requires reliable emissions measurement, carbon prices that reflect environmental and economic conditions, and coordination mechanisms that ensure firms internalize the emissions they generate throughout production networks. Current systems fall short on each dimension. Emissions disclosures rely on heterogeneous assumptions and incomplete reporting; carbon markets operate under informational frictions arising from slow-moving verification, market opacity, and the coexistence of regulated and voluntary mechanisms; and decentralized firms do not account for the externalities transmitted to upstream and downstream partners. These limitations weaken the link between true abatement, reported emissions, investment decisions, and regulatory outcomes. This dissertation examines these issues from three complementary perspectives: the interaction between carbon markets and financial frictions, the measurement of emissions across global supply chains, and the allocation of capital in production networks with unpriced externalities. Each chapter addresses a distinct constraint and together they form a coherent framework for improving climate-policy design.

The first chapter studies how emissions regulation interacts with voluntary offset markets and financial constraints in a dynamic macro-finance environment, following the structure introduced in *Green Coins*. The model incorporates climate damages, an Emissions Trading System (ETS) with fixed annual allowance issuance, and borrowing limits tied to firms' declared net emissions. As emphasized in the paper's introduction, voluntary carbon credit (VCC) markets exhibit substantial heterogeneity in credit quality, limited additionality verification, and long verification lags, which create persistent informational wedges between declared and effective emissions. Firms can use these wedges to present lower net emissions than they actually produce. Because borrowing constraints depend on declared net emissions, financially constrained firms may use VCCs strategically to increase their lending capacity, generating distortions in investment, abatement, and output. This mechanism weakens the informational content of carbon prices and reduces the responsiveness of carbon markets to shocks. The chapter proposes an institutional reform in which each verified sequestration event generates a digital carbon asset recorded on a public ledger, using satellite and IoT evidence for real-time validation as outlined in the paper. A Green Coin Central Bank controls the supply of these assets to stabilize carbon-asset prices and ensure that emissions reductions and financial conditions remain aligned. The analysis shows that such a system reduces the welfare losses associated with the combined ETS–VCC regime and highlights that carbon-market performance depends critically on timely, credible, and standardized emissions information.

The second chapter develops a network-consistent methodology for measuring Scope-3 and embedded emissions across international supply chains, building on the framework introduced in *The Scope of Scope 3*. The introduction of the paper documents large discrepancies across major Scope-3 datasets, driven by differing assumptions about firm-level input–output relationships, limited reporting of upstream and downstream activities, and inconsistent treatment of emissions induced by maritime and land transport. To address these issues, the chapter constructs firm-level emissions estimates using direct emissions, financial information, reconstructed traded quantities, and detailed transport emissions. Quantities are inferred through a general equilibrium network model that incorporates financial constraints using Altman Z-scores, reflecting the empirical observation that distressed firms face distorted prices and trade patterns. The methodology incorporates international shipping routes, port geolocation, and IPCC emission factors, consistent with the workflow described in the paper's introduction, and allocates missing flows using OECD inter-country input–output matrices. This produces firm-level upstream and downstream emissions for a dataset covering more than 50,000 firms in 109 countries. The resulting estimates are transparent, replicable, and consistent with the economic structure of global production. The chapter also provides a basis for counterfactual analysis, including trade disruptions, supply-chain reallocation, carbon border adjustments, and geopolitical shocks, emphasizing that accurate Scope-3 measurement is a precondition for effective carbon accounting and for institutional mechanisms—such as the Green Coin system—that rely on verified emissions information.

The third chapter analyzes production and investment decisions in networks with unpriced emissions externalities and greenness-dependent borrowing constraints, following the framework developed in *Green DAOs for Brown Networks*. Firms produce differentiated goods using an input–output structure and generate direct, upstream, downstream, and embedded emissions. As outlined in the chapter's introduction, firms do not internalize the emissions they induce across the network, and because emissions remain unpriced, the decentralized equilibrium does not incorporate the environmental impact transmitted to suppliers and customers. Borrowing constraints tied to emissions further distort firms' input choices and investment decisions. These combined frictions lead to systematic deviations from the planner's allocation, particularly for firms that are central in the network or rely heavily on emission-intensive suppliers. To address this, the chapter proposes a decentralized governance mechanism in which firms join a Green Decentralized Autonomous Organization that pools borrowing capacity and records production and emissions flows on a shared ledger. Capital is allocated according to rules that incorporate full network-level emissions implications. Under standard conditions, this decentralized mechanism replicates the planner's allocation without requiring centralized enforcement. Preliminary quantitative analyses show substantial efficiency gains, especially in networks characterized by high levels of embedded emissions.

Together, the three chapters provide an integrated assessment of the informational, financial, and network-coordination constraints that limit the effectiveness of current climate policies. Chapter 1 shows that the performance of carbon-asset markets depends on the credibility, granularity, and frequency of emissions information. Chapter 2 provides a measurement framework capable of producing such information at a resolution consistent with global production networks. Chapter 3 shows that, even with accurate measurement and appropriate pricing, decentralized firms do not reach the efficient allocation without governance mechanisms that internalize network-propagated emissions. The dissertation contributes to the literature by integrating macro-finance modelling, supply-chain emissions measurement, and decentralized network governance into a unified framework intended to support more consistent and effective decarbonization policies.

Chapter 1: Green Coins

Green Coins

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Abstract

We propose a novel macrofinance model for the EU area comprising both climate damages and an Emission Trading System that interacts with credit markets with a bias for 'net-zero' brown firms and opaque voluntary carbon credit markets. Our model suggests that the implied allocation is far from the first-best. Relevant welfare gains can be obtained in a setting in which a Green Coin Central Bank (GCCB) runs a transparent blockchain in which coins are mined after a decentralized verification of private offsets, and the GCCB manages green coin prices through open market operations.

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1 Introduction

Emission regulation is a priority that poses many challenges and prompts several key macroeconomic questions. How can we let emissions regulation coexist with decentralized private markets? How can we prevent fraudulent emissions miscounting due to asymmetric information? In addition, how can we design policy tools that promptly react to economic- and climate-related conditions? To address these questions, we need a model of the interactions of public emission trading systems (ETS), private carbon credit markets, and carbon-sensitive credit markets. In addition, we advocate for a novel regulation mechanism that takes advantage of blockchain technologies in order to develop a digital green coin market in which an authority can alter the cost of emissions in real time. More broadly, this study intends to promote a novel *"Green MacroFinTech"* approach to emissions regulation.

Specifically, we take seriously corporate frictions and study an economy in which a brown sector finances its investments with a mix of equity and corporate debt. Corporate debt is preferred to equity because of a tax shield and is limited by a borrowing constraint. This constraint is tied to the net emissions reported by brown firms, enabling firms with lower net emissions to access more credit. Net emissions are computed as gross emissions minus declared offsets. In addition, following of the European ETS, we assume that the brown sector faces an emission constraint, i.e., by regulation, corporate net emissions must be matched by emissions allowances whose supply is decided by a central authority that allocates them once a year.

In this setting, the price of allowances is related to the marginal damage to added value that firms would face if they are unable to match additional production-related emission with allowances. Note that the brown sector has a twofold incentive to buy privately issued carbon credits. First, these credits reduce net emissions and enable firms to get more credit by appearing as 'net-zero' firms. Second, depending on the ETS rules, these credits can lower the need of allowances and relax the regulation constraint.

The economy comprises a second sector that uses green assets, i.e., all assets that con-

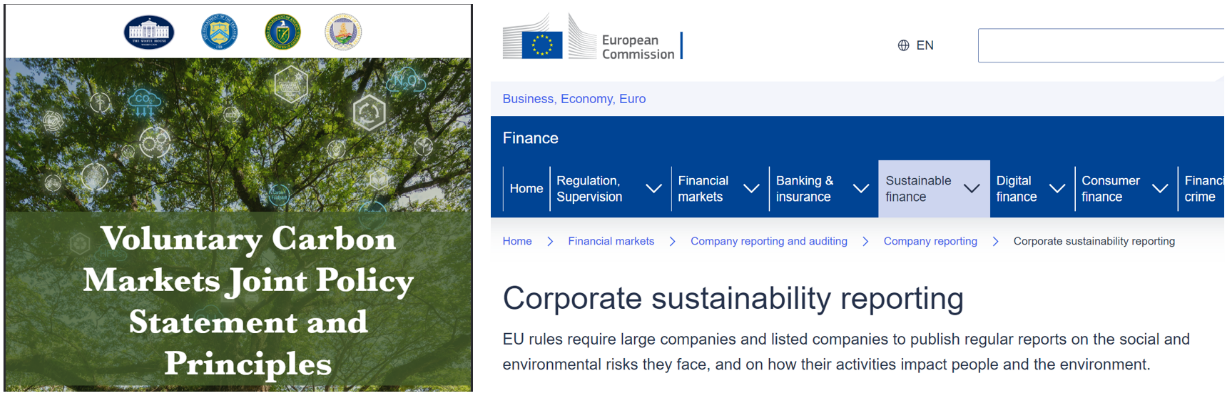


Fig. 1: Policy News in the USA and in the EU

Notes: The left panel refers to the May 2024 White House joint statement about voluntary carbon markets joint policy. The right panel refers to the reporting requirements that the EU has introduced following the Corporate Sustainability Reporting Directive (CSRD, 2022/2464/EU).

tribute to either offsetting or sequestering CO_2 from the atmosphere. For the sake of simplicity, the green sector does not produce a final good. Rather, it provides a valuable service to the brown sector by reducing CO_2 and issuing private carbon credits accordingly. In order to capture the opaqueness of this market, we assume the existence of a non-negative wedge between declared offsets and real offsets. This is a key concern identified by consumer advocates, specialized press, and regulators. Indeed, in the US, the White House has highlighted this issue in the White House joint statement about voluntary carbon markets joint policy. In the EU, the ETS regulation has been modified several times in the last few years in order to avoid an inappropriate use of private carbon credits. Moreover, in the aftermath of the European 2019 ‘Green Deal’, the European Commission has approved a very relevant Corporate Sustainability Reporting Directive (CSRD, 2022/2464/EU).

After compiling a novel dataset that merges macroeconomic aggregate data with prices on emission allowances, futures on private carbon credits, emissions, and green investments for the EU economy, we calibrate this model and show that the model captures important features of macro quantities both in the brown and green sector as well as carbon prices and asset returns in these two sectors. In a second step, we consider a second setting with

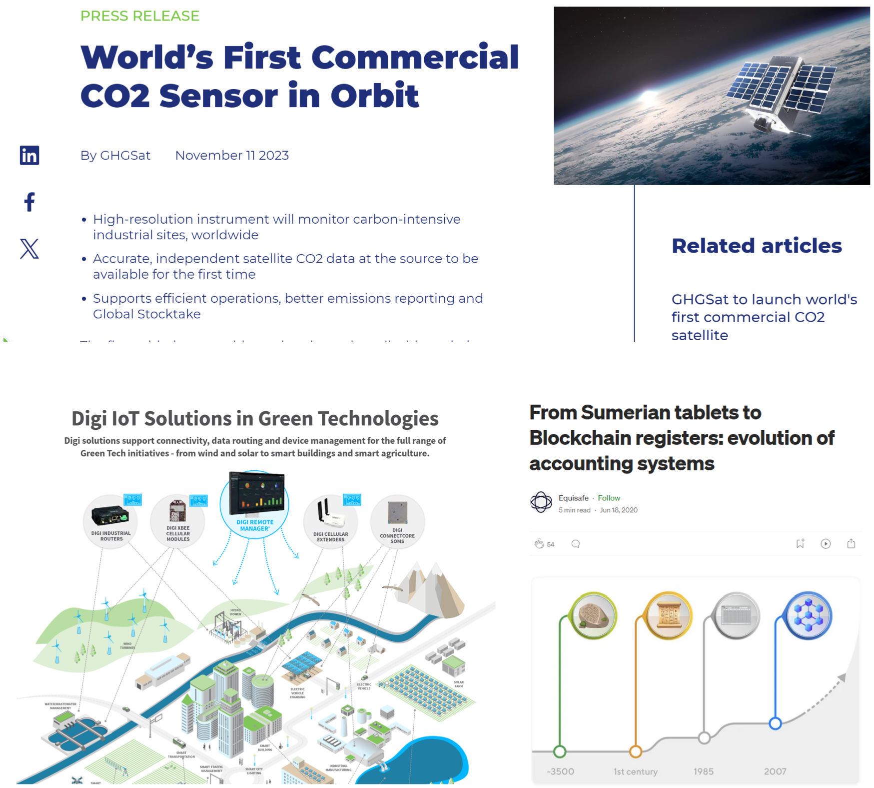


Fig. 2: New Monitoring Technologies

Notes: The top panel refers to new satellite technology that identifies CO_2 emission with great detail offered by GHGSat. The bottom panel refers to the interplay of IoT and Blockchain.

a radically different way to deal with emissions based on the decentralized monetization of emissions. Our results suggest that this setting could produce sizable welfare gains.

Specifically, we propose a simplified structure in which both self-certified carbon credits and auctions of emission allowances are not present. We assume that the following protocol is in place. There is a blockchain that is run and monitored by a Green Coin Central Bank (GCCB, henceforth). A coin is mined by the private sector every time a unit of carbon offset is produced, and it is retired at the end of the period. Thanks to the new technological paradigm called 'Internet of Things' (IoT), these actions can be verified using either location services and barcodes or smart chips. As a result, the supply of green coins tracks in real-time the supply of effective offsets. In addition, the GCCB is assumed to have access to a GHG Satellite technology that enables superior monitoring on ex-post realized offsets. In short, this is a setting with real-time full information on emissions and offsets (see, for example, figure 2).

The GCCB controls the supply of green coins available to the public by running regular open market operations. In this setting, green coins are valuable to the private sector because they are the only assets that provide certified offsets and help firms meeting regulatory emission limits. By tightening or loosing the green coin policy, the GCCB can better steer the economy toward the first best. Although not directly modeled in our study, we believe that a decentralized blockchain can enable a wide range of private actors to be included in the green market. This technology is scalable and can be accessible to both small, medium, and large firms. Most importantly, in a setting in which the GCCB commits to an implementable green policy rule that responds to both economic shocks and state variables, we find that the welfare losses from the first-best are greatly reduced.

Interestingly, the damage originated by emissions acts as a distortional taxation margin since it is proportional to output. We find that the GCCB finds it optimal to implement an 'emission smoothing' policy, in the spirit of what observed in the fiscal policy literature. Exogenous shocks that immediately increase emissions are partially accommodated by the GCCB, meaning that this entity increases the supply of green coins available to the public by buying a smaller share of outstanding coins. As the stock of emissions increases over time, the GCCB slowly tightens the green coin market and increases its holdings of green coins. The welfare benefits can be as large as 6.5% of time-zero first-best consumption.

Related literature. Our policy focus is novel and hence there is a limited number of manuscripts directly related to our research. What follows is not a comprehensive review of the literature, rather it highlights a subset of key contributions. Bustamente and Zucchi (2022) focuses on a firm subject to emission constraint and enabled to buy private Verified Carbon Credits (VCCs). Our research differs from this work in many crucial-and-novel dimensions. In particular, we (i) account for the opaqueness of VCCs by considering a wedge between declared emission reductions and effective ones; (ii) have a quantitative focus; (iii) use new *EU* data; (iii) price carbon emissions in a macrofinance integrated assessment model, and (iv) propose a totally new, policy-relevant, and implementable framework based on green coins. We believe that tokenization can substantially improve emission management and resolve some of the inefficiencies highlighted in Borri, Liu, Tsyvinski, and Wu (2024).

Several modeling assumptions come from the Financial Economics of Climate literature (among others, see Giglio, Kelly, and Stroebel (2021)). We differ for our "Green MacroFinTech" approach that focuses on regulation in models that replicate both quantities and asset prices.

The interplay between macroeconomic aggregates and asset prices has been the main object of interest of the macro-finance literature (see, among many others, Ai (2007); Belo, Bazdresch, and Lin (2014); Corhay, Kung, and Schmid (2015)). The aggregate implications of emissions regulation and its interaction with FinTech in a macro-finance model are still to be assessed. Our research aims to fill this gap in the literature.

FinTech has recently attracted many contributions (see, among others, Biais, Capponi, Cong, Gaur, and Giesecke (2023), Niepelt (2024), Brunnermeier and Payne (2022)). This research proposal distinguishes its position by using FinTech at a macro-level in order to enhance climate change policies. To the best of our knowledge, nobody has focused on the benefits that FinTech can produce in the context of the green transition.

2 The Models

In what follows, we describe the settings that we intend to focus on. We start by describing the planner's problem. We then turn our attention to the setting that should be calibrated using EU data, i.e., a model with an ETS and an opaque market for VCCs. In the last step, we explore a setting with green coins.

2.1 The planner's problem

The household. We assume that the representative agent has Epstein and Zin (1989) recursive preferences so that she prices long-run risks, including those related to climate change:

$$U_t = \left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta(\mathbb{E}_t[U_{t+1}^{1-\gamma}])^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right]^{\frac{1}{1-\frac{1}{\psi}}},$$

where C_t is the amount of output consumed. We do not include leisure in the utility function for the sake of parsimony. As a result, the supply of labor is fixed and normalized to one in both sectors. In this setting, the stochastic discount factor (SDF) is

$$M_{t+1} \quad = \quad \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\left(\frac{U_{t+1}}{\left[\mathbb{E}_t\, U_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma}.$$

The brown sector. The brown sector produces output, Y_t, according to the following production function:

$$Y_t = (Z_t L_{bt})^{1-\alpha} K_{bt-1}^{\alpha}, \tag{1}$$

where Z, L_b, K_b, denote productivity, labor, and capital deployed in this sector, respectively. Capital evolves as follows,

$$K_{bt} = (1-\delta)K_{bt-1} + \Psi(I_{bt}/K_{bt-1})K_{bt-1}, \tag{2}$$

where Ψ denotes Jermann (1998) convex adjustment costs.

Similar to Shapiro and Metcalf (2023), the fraction of output that is not disrupted by climate damage is determined by the following function

$$\chi(s_t) = \exp(-\chi_0 \cdot s_t), \tag{3}$$

where s_t captures carbon dioxide concentration and accumulates depending on net emissions in the economy. In what follows, output net of climate-related damages is denoted by Y_t^n and determined as follows:

$$Y_t^n = \chi\left(s_t\right) \cdot Y_t. \tag{4}$$

Gross emissions produces by the brown sector is assumed to be proportional to gross output:

$$E_t = \lambda_t \cdot Y_t, \tag{5}$$

where the process that determines emissions per unit of added value is subject to the external shocks,

$$\log \lambda_t \quad = (1 - \rho_\lambda)\mu_\lambda + \rho_\lambda \log \lambda_{t-1} + \sigma_\lambda \epsilon_{\lambda,t}. \tag{6}$$

We specify this law of motion in log units so that λ_t is always non negative. As in the data, we model λ_t as a persistent process. Among other channels, external shocks capture unmodeled technology improvements as well as natural disasters (e.g., fires) and changes in energy sources policy (e.g., shifts from renewable to coal-based energy).

In addition, we allow for the negative feedback between CO_2 and output by defining aggregate productivity growth, Δz, as follows:

$$\begin{aligned}\Delta z_{t+1} &= \mu_z + x_t + \sigma_z \epsilon_{z,t+1} \\ x_t &= \rho_x x_{t-1} - \phi_s \cdot (\log(s_{t-1}) - \log(\overline{s})) + \sigma_x \epsilon_{x,t}.\end{aligned}$$

The specification above is common in the long-run risk literature, except for the fact that it accounts for an endogenous disruptive effect of CO_2 on expected long-run growth ($\phi_s > 0$). Thus, green assets are beneficial because not only they help relax emission constraint but also indirectly improves productivity by reducing CO_2 concentration. In what follows, we denote ϵ_z and ϵ_x as short- and long-run productivity shocks, respectively.

The green sector. The green sector does not participate to the goods market. It solely produces CO_2 offsets according to the following production function:

$$G_t = (Z_{gt}L_{gt})^{1-\kappa}K_{g,t-1}^{\kappa}, \tag{7}$$

where capital evolves as follows,

$$K_{gt} = (1-\delta_g)K_{g,t-1} + H(I_{gt}/K_{g,t-1})K_{g,t-1}, \tag{8}$$

and H captures convex adjustment costs as in Jermann (1998). Along the balanced growth path, productivity in this sector is cointegrated with that of the brown sector:

$$\log\frac{Z_{g,t}}{Z_t} - \overline{zc} = (1-\phi_g)\left(\log\frac{Z_{g,t-1}}{Z_{t-1}} - \overline{zc}\right) - (1-\phi_g)(\Delta z_t - \mu). \tag{9}$$

We model the cointegration residual so that the green sector productivity adapts slowly to innovations in the brown sector (small ϕ_g), consistent with the fact that green technology innovations arrive at a slower pace compared to innovations in the traditional brown sector (e.g., slow improvement in solar panels efficiency, slow improvement in the efficiency-emissions ratio for the production of batteries). The parameter $\overline{zc}$ accounts for differences in the levels of productivity which are to be expected given that this sector specializes on offsets, whereas the brown sector focuses on goods and services.

The climate feedback. Let NE_t denote net offsets in the economy:

$$NE_t := E_t - G_t, \tag{10}$$

we model the accumulation of CO_2 as follows:

$$s_t = (1-\delta_s)s_{t-1} + NE_t/Z_t, \tag{11}$$

where net emissions are standardized by productivity and hence must be interpreted as relative to the size of the economy and δ_s captures the fraction of CO_2 that is naturally depleted in the atmosphere.

Key optimality conditions. The full solution of the planner's problem can be found in the Appendix. Here we report the returns of the three endogenous state variables in the model, namely a proxy for the stock of CO_2, brown capital, and green capital, respectively:

$$R^P_{s,t} = \frac{\chi_0 Y^n_t + (1-\delta_s)q^P_{s,t}}{q^P_{s,t-1}} \tag{12}$$

$$R^P_{b,t} = \left(q^P_{bt}\cdot\left[(1-\delta)+\Psi_t-\Psi'_t\cdot\frac{I_{bt}}{K_{bt-1}}\right]+\left(1-q^P_{st}\frac{\lambda_t}{\chi_t}\right)\alpha\frac{Y^n_t}{K_{bt-1}}\right)/q^P_{bt-1} \tag{13}$$

$$R^P_{g,t} = \left(q^P_{gt}\cdot\left[(1-\delta_g)+H_t-H'_t\cdot\frac{I_{gt}}{K_{g,t-1}}\right]+q^P_{st}\kappa\frac{\tilde{G}_t}{K_{g,t-1}}\right)/q^P_{g,t-1}. \tag{14}$$

Variables denoted as q^P_{jt} refer to marginal-q, i.e., the marginal cost of an additional unit of capital of type j. The return associated to the stock of CO_2 reflects variations in the present value of future damages, $q^P_{s,t}$. The return of brown capital, $R^P_{b,t}$, differs from that in a neoclassical model because the marginal product of capital is reduced by the associated loss in value due to the additional emissions, $q^P_{st}\frac{\lambda_t}{\chi_t}$. The return of the green sector is driven by the marginal productivity of green capital, i.e., by the market value of the marginal offsets, $q^P_{st}\kappa\frac{\tilde{G}_t}{K_{g,t-1}}$.

2.2 A model with an ETS and opaque VCCs

In this section, we focus on a decentralized economy with key features that resemble the EU ETS system. We calibrate this setting to EU data and then run a counterfactual analysis of the benefits of a green coin-based system.

Green authority. The green authority observes declared net emission, $\widetilde{NE}$,

$$\widetilde{NE}_t = \lambda_t Y_t - \widetilde{G}_t,$$

where $\widetilde{G}$ refers to reported emission offsets which can differ from the true offsets, G. A penalty is applied when net emissions exceed the emission allowances, EA:

$$C_t^{EA} = \phi_0^{EA} \cdot \exp\left(\phi_1^{EA} \left\{\frac{\lambda_t Y_t - EA_t}{\widetilde{G}_t} - 1\right\}\right) \cdot Z_{t-1}.$$

In practice, this cost function mimics (i.e., it convexifies) the emission constraint

$$\lambda_t Y_t - \widetilde{G}_t - EA_t \leq 0. \tag{15}$$

The authority commits to the following rule for the supply of emission allowances to be auctioned:

$$\begin{aligned} EA_t &= \exp(a_t) \cdot Z_{g,t} > 0 \\ a_t &= (1-\rho_a)(\mu_a - \phi_a \cdot (s_{t-1} - \overline{s})) + \rho_a a_{t-1} + \sigma_a \epsilon_{a,t-j}. \end{aligned}$$

Along the balance growth path, allowances must grow with the economy, and a_t must be interpreted as a cointegration process expressed in log units. We think of this process as an AR(1) with an endogenously time-varying level,

$$\bar{a}_t := \mu_a - \phi_a \cdot (s_{t-1} - \overline{s}),$$

that is reduced when the endogenous stock of emission increases above steady state ($\phi_a > 0$). When ϕ_a is large enough, this policy implies that allowances can be kept constant or even reduced for periods when GHG emissions grows excessively.

We note three additional important points: (i) the stock of accumulated CO_2 depends on true offsets, G, not on declared offsets, $\widetilde{G}$; (ii) ϵ_a captures emission policy shocks; and (iii) we assume that the allowances supply is decided j-period ahead in order to capture the fact in the EU ETS system, the quantity of allowances to be auctioned is chosen ex-ante. In our calibration, j is one year. This exogenous policy rule can be calibrated by jointly studying emission allowances auctioned in the ETS (available at low frequency) as well as the behavior of emission prices (available at high frequency).

As discussed in what follows, we have a well-defined capital structure in the brown sector because the government offers a tax shield, τ, on corporate interest rates, $r_t^B B_{t-1}$. In addition, in the spirit of what observed in Europe, we assume that the cost of new green assets, I_{gt}, is subsidized at a rate τ_g. We assume that these payments are managed by the green authority and, for the sake of simplicity and to keep our analysis transparent, this authority has a balanced budget:

$$p_t \cdot EA_t + T_t = \tau_g I_{gt} + \tau r_t^B B_{t-1},$$

where $p_t \cdot EA_t$ is the amount collected by selling emission allowances, and T_t is a lump sum transfer from or to the household that bridges the gap between cash inflows and outflows.

Brown firm. The brown firm represents all assets and economic activities that produce positive gross emissions and faces the following problem:

$$\begin{aligned}
V_{b,t} &= \max_{B_t, I_{bt}, L_{bt}, \widetilde{G}_t, EA_t} D_{b,t} + \mathbb{E}_t(M_{t+1}V_{t+1}) \\
D_{b,t} &= \underbrace{Y_t^n - W_{bt}L_{bt} - I_{bt}}_{\text{neoclassical}} \underbrace{-(1-\tau)r_t^B B_{t-1} + \Delta B_t - C_t^E - C_t^B}_{\text{deviation from MM}} \\
&\quad - \underbrace{(p_t \cdot EA_t + C_t^{EA})}_{\text{cost of EA}} - \underbrace{(p_{gt}\widetilde{G}_t + C_t^C)}_{\text{cost of VCCs}}
\end{aligned}$$

where Y_t^n, and K_t evolve as described in the previous section. The following cost functions,

$$
\begin{aligned}
C_t^E &= \phi_0 \cdot \exp\left(\phi_1 \left\{1 - \frac{\theta_t K_t}{B_t}\right\}\right) \cdot Z_{t-1} \quad (\text{mimics } B_t \leq \theta_t \cdot K_t), \\
\theta_t &\equiv \theta\left(\frac{\widetilde{G}_t}{\lambda_t Y_t}\right); \quad \theta'(\cdot) > 0 \\
C_t^B &= \eta Z_{t-1}\left(\frac{B_t}{Y_t} - \frac{B_{ss}}{Y_{ss}}\right)^2 \\
C_t^C &= \frac{\zeta}{2}\frac{(\widetilde{G}_t - G_t)^2}{Z_{t-1}},
\end{aligned}
$$

capture the cost of financial distress, debt issuance, and VCC-related reputation, respectively. As in Croce, Kung, Nguyen, and Schmid (2012), the tax shield on corporate interest makes this firm favor debt-financing (deviation from Modigliani-Miller). This aspect is important because it allows us to capture the attention of credit markets for 'net-zero' firms. Specifically, we differ from Croce, Kung, Nguyen, and Schmid (2012) by allowing the borrowing constraint tightness, θ, to be a function of declared net emissions. Hence, this firm has an incentive to buy VCCs even in the absence of an ETS, similar to what we observe in the data. As in Croce, Kung, Nguyen, and Schmid (2012), altering the mix of debt and equity comes at a cost, C^B, and hence the borrowing constraint has significant implications for investment.

In the ETS system, this firm must balance gross emissions by buying either VCCs or emission allowances to minimize the penalty cost C^{EA}. Both in the data and in the model, VCCs are on average cheaper than EU allowances, $\mathbb{E}[p_{gt}] < \mathbb{E}[p_t]$. However, VCCs carry an intrinsic cost C^C related to their opaqueness, i.e., the gap between declared and effective offsets. The firm understands that acquiring overstated offsets could lead to reputational damage and legal consequences (see, for example, the case of Delta Airlines).

The first order conditions imply that investment is distorted by the emission constraint,

$$\begin{aligned} q_{bt} &= \frac{1}{\Psi'_t} = \mathbb{E}_t\left[M_{t+1}\frac{\partial V_{bt+1}}{\partial K_{bt}}\right] - \frac{\partial C_t^E}{\partial K_t} \\ \frac{\partial V_{bt}}{\partial K_{bt-1}} &= \underbrace{\frac{\partial Y_t^n}{\partial K_{bt-1}} + q_{bt}\left(1-\delta-\frac{\Psi'_t I_{bt}}{K_{bt-1}}+\Psi_t\right)}_{\text{neoclassical}} - \underbrace{\left(\frac{\partial C_t^E}{\partial K_{bt-1}} + \frac{\partial C_t^B}{\partial K_{bt-1}}\right)}_{\text{deviation from MM}} - \underbrace{\frac{\partial C_t^{EA}}{\partial K_{bt-1}}}_{\text{EA distress}}. \end{aligned}$$

In addition, the demand of emissions is related to the emission distress cost, or, equivalently, the multiplier assigned to the emission constraint,

$$p_t = \phi_1^{EA} \cdot \frac{C_t^{EA}}{G_t}.$$

Most importantly, the price of VCC is anchored to that of allowances traded on the ETS,

$$p_{gt} = \underbrace{p_t \cdot \frac{(\lambda_t Y_t - EA_t)}{\widetilde{G}_t}}_{\text{link with } p_t} + \underbrace{\phi_1 \cdot \theta'_t \cdot \frac{C_t^E}{B_t} \cdot \frac{K_{bt}}{\lambda_t Y_t}}_{\text{savings on borrowing}} \underbrace{-\zeta \cdot \frac{\widetilde{G}_t - G_t}{Z_{t-1}}}_{<0,\text{ opaque}},$$

and it features two additional margins going in opposite directions. On the one hand, VCCs are more valuable than allowances because they help firms reduce their net emissions and secure better financing terms. On the other hand, because of opaqueness, VCCs are traded at a discount.

Green firm. The green sector is defined as the set of assets that delivers CO_2 offsets and sequestration. This sector determines the endogenous supply of both effective and declared offset. Specifically, the firm faces the following neoclassical problem:

$$\begin{aligned} V_{gt} &= \max_{I_{gt}, L_{gt}} D_{gt} + \mathbb{E}_t(M_{t+1}V_{g,t+1}) \\ D_{gt} &= p_{gt}\widetilde{G}_t^s - (1-\tau_{gt})I_{gt} - W_{gt}L_{gt} \\ \widetilde{G}_t^s &= \xi_t G_t \\ \xi_t &= 1 + b_0 \exp(b_1 \epsilon_{\xi,t}), \end{aligned}$$

where $\epsilon_{\xi,t}$ is independent and identically distributed $\mathcal{N}(0,1)$, and K_{gt} and G_t are defined as in the previous section. This shock is important in order to (i) making this market opaque, and (ii) reduce the correlation between emission price, p_t, and VCC price, p_{gt}, as in the data. In equilibrium, the amount of offsets supplied by the green sector equal to the amount demanded by the brown sector, $\widetilde{G}_t^s = \widetilde{G}_t$.

We note that the positive part of the cash-flow, D_g, originates from (i) selling VCCs to the brown sector and (ii) receiving a subsidy proportional to green investment. In this formulation of the model, we think of the gap between reported and realized offsets as driven by an exogenous process, ξ_t, which is always greater than or equal to one and that captures both changes in moral hazard, ex-post lower efficiency of green assets, and changes in monitoring ability. The financing of this firm is frictionless, in order to both (i) capture the 'benevolence' that capital markets are giving to green projects and (ii) isolate the role of financing frictions only on brown firms.

At the optimum, we have

$$\begin{aligned} q_{gt} &:= \frac{1-\tau_{gt}}{H_t'} = \mathbb{E}_t\left(M_{t+1}\frac{\partial V_{g,t+1}}{\partial K_{gt}}\right) \\ \frac{\partial V_{gt}}{\partial K_{g,t-1}} &= p_{g,t}\frac{\partial \widetilde{G}_t^s}{\partial K_{g,t-1}} + q_{gt}\{1-\delta_g + H_t - H_t' \cdot (I_{gt}/K_{g,t-1})\}, \end{aligned}$$

implying that green investment is driven by the present value of the future offset flows. This flow is crucially driven by the price of VCCs which, in turn, is anchored to the price of allowances on the ETS.

Aggregate resource constraint and consumer budget constraint. Let V_{jt}^{ex} denote the ex-dividend price of asset j at time t, i.e., $V_{jt}^{ex} = V_{jt} - D_{jt}$. The household budget constraint is defined as follows:

$$C_t + V_{bt}^{ex} + V_{gt}^{ex} = V_{bt} + V_{gt} + W_{bt} + W_{g,t} - T_t$$

Since offsets are sold to the brown firm, they represent intermediate goods that do not show up in GDP. Therefore, we have

$$Y_t^n = C_t + I_{bt} + I_{gt} + C_t^{EA} + C_t^B + C_t^E + C_t^C.$$

2.3 A Model with Green Coins

The economy described in the previous section features many sources of inefficiencies, and it is far away from the first-best. In what follows, we consider a setting in which there is scope for a central authority to improve welfare. Specifically, we think of an authority called Green Coins Central Bank (GCCB) that runs and monitors the only authorized system in which offsets are recorded, verified, and monetized or, equivalently, tokenized. The GCCB can alter the price of the green coins with open market operations to bring the economy closer to the first best.

In this setting, offsets are properly measured, but allocations can still be inefficient due to the presence of financial frictions. We will show that the GCCB can improve welfare by responding to exogenous shocks in ways that make investment activities in both the brown and green sector to be more efficient from a social perspective.

Unmodeled frictions. In the context of our model, all we need is to assume that the GCCB has a verification technology and an exchange in which to trade offset units. The model is silent on whether the verification of the transaction should be centralized or decentralized. In what follows, we assume that the system runs on a decentralized blockchain and use blockchain-related terminology. We believe that this technology is superior in at least three dimensions. First, it is very scalable and can be adopted by large, medium, and small businesses. Equivalently, it features smaller adoption fixed costs. Second, in an economy with multiple agents, a decentralized verification of the offsets transactions minimizes the incentives for misreporting. Third, since offsets and emissions may be localized in very different countries, a common blockchain can (i) homogenize offsets accounting and (ii) minimize country-level incentives to overstate offsets and under-report emissions.

The green blockchain. We assume that the GCCB runs a blockchain that mines new coins every period as soon as an offset is verified. Coins are fully retired at the end of the period. As a result, the total supply of green coins in each period equals the total mass of true offsets in every period and every state of the world, G_t. The per-period cost of running the blockchain is $\omega_g G_t$, i.e., it is proportional to all transactions which are verified by the platform. In our model, the green firm is the only miner of green coins, and it has an incentive to sell them to the brown firms who can claim 1 unit of offsets for each coin. The brown firm buys green coins at the spot price p_g. In this setting, all emissions and offsets are verified over the platform, and hence there is no VCC.

The role of the GCCB. In each period, the green authority conducts open market operations in order to buy a share $S_{g,t}$ of the total mass of available green coins, G_t. Thus the total supply of offsets accessible to the brown sector is

$$G_t \cdot (1 - S_{g,t}),$$

and the private sector is now subject to the following constraint:

$$NE_t = \underbrace{\lambda_t Y_t}_{\text{gross emissions}} - \underbrace{G_t(1 - S_{g,t-1})}_{\text{private offsets}} \leq \underbrace{\bar{A} \cdot Z_{g,t-1}}_{\text{emissions allowed}} . \tag{16}$$

By altering the supply of green coins available to the public, the GCCB can manipulate in real time the price of green coins, i.e., the only assets that count for emission regulation. This formulation resembles what central banks do with bonds in order to alter yields. Note that thanks to the monitoring implemented by the blockchain, there is no longer a distinction between true offsets, G, and declared offset ($G_t = \tilde{G}_t \quad \forall t$).

The authority is assumed to follow this Taylor-style green rule,

$$S_{gt} = \frac{\exp(sg_t)}{1+\exp(sg_t)} \in (0,\, 1),$$

where

$$sg_t = A_0 + [A_s\, A_k\, A_{lr}] \cdot \begin{bmatrix} \hat{s}_{t-1} \\ \widehat{\frac{K_{gt-1}}{K_{bt-1}}} \\ x_{t-1} \end{bmatrix} + [A_z\, A_\lambda\, A_x] \cdot \begin{bmatrix} \epsilon_{z,t} \\ \epsilon_{\lambda,t} \\ \epsilon_{x,t} \end{bmatrix}, \tag{17}$$

so that the holdings of green coins is both history dependent (it depends of the demeaned state variables, $\hat{s}_{t-1},\ \widehat{\frac{K_{gt-1}}{K_{t-1}}}$, and x_{t-1}) and sensitive to all contemporaneous shocks in the economy ($\epsilon_{z,t},\ \epsilon_{\lambda,t},$ and $\epsilon_{x,t}$). The coefficients of this rule cannot be estimated in the data since this is a counterfactual setting. In our analysis, we will explore the welfare implications of allowing the GCCB to respond to economic states by changing these parameters.

The authority net payout to the household is:

$$\Pi_t^{GB} = p_{g,t} \cdot G_t \cdot S_{g,t-1} + \omega_g \cdot G_t,$$

which embodies the resources spent to buy new coins and running the blockchain.

Brown Firm. The new problem faced by the brown firm is:

$$\begin{aligned} V_{b,t}^{GC} &= \max_{B_t, I_{bt}, L_{bt}, G_t} D_{b,t}^{GC} + \mathbb{E}_t(M_{t+1}V_{t+1}) \\ D_{b,t}^{GC} &= \underbrace{Y_t^n - W_{bt}L_{bt} - I_{bt}}_{\text{neoclassical}} \underbrace{-(1-\tau)r_t^B B_{t-1} + \Delta B_t - C_t^E - C_t^B}_{\text{deviation from MM}} \\ &\quad - \underbrace{C_t^{EA}}_{\text{cost of net emissions}} - \underbrace{p_{gt}G_t,}_{\text{cost of coins}} \end{aligned}$$

in which there are no longer allowances traded on the ETS and VCCs, only green coins. The variables Y_t^n, K_{bt}, C_t^E, C_t^B, are defined as in the setting with the ETS system. The C_t^{EA} cost function is formulated to reflect the emission constraint in equation (16) as opposed to equation (15):

$$C_t^{EA} \quad = \quad \phi_0^{EA} \cdot \exp\left(\phi_1^{EA}\left\{\frac{(\lambda_t Y_t - \bar{A} Z_{g,t-1})}{\tilde{G}_t} - 1\right\}\right) \cdot Z_{t-1}.$$

At the optimum, the price of green coins is

$$p_{gt} = \underbrace{\phi_1^{EA} \cdot \frac{C_t^{EA}}{G_t^{PD}} \cdot \frac{(\lambda_t Y_t - \bar{A} Z_{g,t-1})}{G_t^{PD}}}_{\text{savings on the cost of NE>0}} + \underbrace{\phi_1 \cdot \theta_t' \cdot \frac{C_t^E}{B_t} \cdot \frac{K_{bt}}{\lambda_t Y_t}}_{\text{savings on borrowing}}.$$

Importantly, this price still reflects both the benefits related to cheaper financing of low net-emission firms and the tightness of the emission constraint. In contrast to the previous setting, now the GCCB can directly control the emission constraint tightness by altering its holding of green coins.

Green firm. The green sector determines the endogenous supply of green coins. Specifically, the firm faces the following neoclassical problem:

$$\begin{aligned} V_{gt}^{GC} \quad &= \quad \max_{I_{gt}, L_{gt}} D_{gt} + \mathbb{E}_t(M_{t+1} V_{g,t+1}) \\ D_{gt} \quad &= \quad p_{gt} G_t - (1-\tau_{gt}) I_{gt} - W_{gt} L_{gt}, \end{aligned}$$

where all variables are defined as before and there is no longer any wedge between declared and actual offsets.

Aggregate resource constraint and consumer budget constraint. In this setting, The household budget constraint is defined as follows:

$$C_t + V_{bt}^{ex,GC} + V_{gt}^{ex,GC} = V_{bt}^{GC} + V_{gt}^{GC} + W_{bt} + W_{g,t} + \Pi_t.$$

Since offsets are sold to the brown firm, they represent intermediate goods that do not show up in GDP. Therefore, we have

$$Y_t^n = C_t + I_{bt} + I_{gt} + C_t^{EA} + C_t^B + C_t^E + \omega_g G_t.$$

3 Data and Calibration

In what follows, we describe the data that we use in order to calibrate our main specification to European data. We then illustrate our calibration and solution methods.

3.1 Data

We focus on the EA19, i.e., the 19 countries in the Euro Area as of 2015, in order to maximize our sample length across time series. All macroeconomic series on national aggregates and price indices are from Eurostat.

Data on green investments and prices for futures related to CO_2 offsets are from Bloomberg. Specifically, with regard to the CME commodity market exchange, we consider daily prices for "XGB1 Comdty: CBL CORE GLOBAL EMISSIONS OFFSET (C-GEO) FUTURES - TR", "LGO1 Comdty: Generic 1st CBL Nature-Based Global Emissions Offset Futures", and "EEXX04EA Index: EEXX European Emission Allowances EUA Spot (Phase 4)" over the sample 2021-2023. In addition, we recover green investments by country in billions of Euros at the yearly frequency over the 2004-2023 sample.

Haver Analytics enables us to obtain daily observations over the sample 2005-2024 for futures and emission allowances "ICE ECX EUA Futures: 1st Position: Settlement Price (Euro/Metric Tonne)", "CME Reg Sess:Carbon Offset Futures: 1st Position: Settlement Price (US/Offset)" and "NYMEX RSes:CBL Nat-Based Glob Emiss Offset".

Emissions as percent of GDP by country at the annual frequency for the 2010-2020 sample are from The United Nations Economic Commission for Europe (UNECE). We complement

Table 1: Calibration

Parameter	Value	Parameter	Value
Preferences		***Climate***	
Discount factor (β)	0.97	GHG depletion rate % (δ_s)	0.55
Risk aversion (γ)	10.00	Mean of log emission rate (μ_λ)	-1.86
IES (ψ)	1.20	Persistent of log emission rate (ρ_λ)	0.50
Damage function parameters % (χ_0)	0.16	Log emission rate volatility (σ_λ)	0.02
GCCB		***VCCs***	
Cost of running blockchain % (ω_g)	0.39	Reporting gap avg. % (b_0)	2.20
Average emissions allowance ($\bar{A} = e^{\mu_a}$)	0.15	Reporting gap volatility (b_1)	0.50
		Reporting costs intensity (ζ)	15.44
Technology (Brown sector)		***Technology (Green sector)***	
Capital share (α)	0.35	Capital share (κ)	0.60
Capital depreciation rate % (δ)	7.60	Capital depreciation rate % (δ_g)	7.60
Elasticity of investment adj. costs (ω)	1.50	Elasticity of investment adj. costs (ν)	15.00
Intensity of debt adjustment costs (η)	0.40	Intensity of emission violation costs (ϕ_1^{EA})	20.00
Debt-to-book avg (θ_0)	0.50	Emission violation costs parameter % (ϕ_0^{EA})	0.05
		Green-brown differences in levels of TFP ($\overline{zc}$)	-0.11
Productivity		***ETS Authority***	
Average productivity growth (μ_z)	0.01	Brown-sector debt interest tax deductible (τ)	0.20
Short-run productivity volatility (σ_z)	0.05	Green-sector investment subsidy (τ_g)	0.30
Long-run productivity persistence (ρ_x)	0.94	Mean emission allowances (μ_a)	-1.92
Long-run productivity volatility % (σ_x)	0.50	Persistent of emission allowances (ρ_a)	0.27
Long-run productivity exposure		Allowance shock volatility (σ_a)	0.12
to GHG emissions (ϕ_s, $\times$ 1000)	0.20	Allowance sensitivity to GHG emissions % (ϕ_a)	8.71
Cointegration parameter (ϕ_g)	0.20	Intensity of distress costs (ϕ_1)	20.00

Notes: This table reports our benchmark annual calibration. See section 3.2 for a detailed discussion.

these data with those from the `Clean Investment Monitor`. This dataset refers mostly to green investments done in the US energy and manufacturing sectors and can be interpreted as providing a lower bound on investment over the sample 2018:Q1-2024:Q1.

Verified carbon credits data are obtained mostly from the Goldstandard Registry of projects that offset emissions. We complement this source with the data on sustainable projects reported by both the International Renewable Energy Agency (IRENA) and the International Energy Agency (IEA).

3.2 Calibration and Solution Method

We report our benchmark calibration for the model with the ETS system in table 1. The main moments that we target are reported in table 2. The AE19 economy is not too dissimilar

Table 2: Simulated Moments

	Data		Model		
	Point Est.	S.Err.	ETS	Green Coins	Planner
Standard Moments					
$\mathbb{E}(\Delta c)$ (%)	0.96	0.40	1.00	1.00	1.00
$\sigma(\Delta c)$ (%)	2.22	0.57	2.70	2.70	3.10
$ACF1(\Delta c)$	0.03	0.23	0.04	0.02	-0.01
$\sigma(\Delta c)/\sigma(\Delta gdp)$	0.73	0.12	0.83	0.83	0.95
$\sigma(\Delta i)/\sigma(\Delta gdp)$	1.98	0.28	1.58	1.49	1.14
$\mathbb{E}(C)/E(GDP)$	0.72	0.43%	0.74	0.74	0.73
$\mathbb{E}(r_f)$ (%)	3.39	2.09	2.52	2.56	2.41
$\mathbb{E}(r^B)$ (%)	6.27	6.52	6.67	6.79	3.73
$\mathbb{E}(r^B - r^G)$ (%)	3.34	1.98	4.29	4.59	1.45
Emissions, ETS, and VCC prices					
$\mathbb{E}(p)/\mathbb{E}(p_g)$	4.15	1.87	1.85		
$\mathbb{E}(\log(\lambda))$	-1.98	0.07	-1.86	-1.86	-1.86
$\sigma(\log(\lambda))$	0.02	0.22%	0.02	0.02	0.02
$ACF1(\log(\lambda))$	0.29	0.17	0.40	0.40	0.40
$\mathbb{E}(a)$	-1.92	0.12	-1.92		
$\sigma(a)$	0.12	0.02	0.12		
$ACF1(a)$	0.27	0.20	0.20		

Notes: Empirical moments are computed using annual data. All data sources are discussed in 3.1. Standard errors are calculated using HAC-adjusted standard errors. The entries for the model are obtained by averaging the results across simulated small samples. Our baseline calibration is detailed in table 1. ACF(1) stands for autocorrelation function with a lag equal to 1.

from the US one in terms of basic macroeconomic statistics. As a result, our calibration is similar to the one in prior work (see, among others, Croce 2014 and Croce, Kung, Nguyen, and Schmid 2012). Damage specification and parameters are in the spirit of those in Shapiro and Metcalf (2023).

The greenium is computed by sorting European public firms according to their direct emission-intensity reported in Trucost. We use the returns of the top (bottom) quintile to measure the return of brown (green) firms in our model. Returns are equally weighted. Looking at equally-weighted returns of emission intensity-sorted firms mitigates bias concerns related to size.

The ratio $\mathbb{E}(p)/\mathbb{E}(p_g)$ considers the average cost of emissions on the European ETS versus the average cost of emissions on the private VCC market. As mentioned in the previous section, data on emissions-per-real-GDP, λ, are from the UNECE. Data on allocated allowances traded on the ETS are from the European ETS system.

We solve our models using the perturbation method. Specifically, we use `Dynare++` to implement a second-order log approximation. We find the performance of our model very satisfactory, given that it reproduces several dimensions of both the macro and the emission data.

4 Main Results

In this section, we share results relevant to show how our model could be used for policy analysis.

4.1 Optimal GCCB Policy Rule.

Given the encouraging results obtained from calibrating the model with the ETS setting, we now proceed with the calibration of our counterfactual economy with green coins. In order to do so, we need to calibrate the parameters in equation (17). Since there is no data counterpart, we collect all of these parameters in a vector θ and choose them in order to solve the following problem:

$$\theta^* := \arg\min_{\theta} \max\{\mathbb{E}\left[(R^P_{bt} - R^{GC}_{bt}(\theta))^2\right], \ \mathbb{E}\left[(R^P_{gt} - R^{GC}_{gt}(\theta))^2\right]\}, \tag{18}$$

where R^P_{bt} and R^P_{gt} are defined in equations (13)–(14) and refer to the return of brown and green capital chosen by planner, respectively. The returns R^{GC}_{bt} and R^{GC}_{gt}, defined in the Appendix, equation (A36) and (A44), respectively, are obtained by solving the setting with green coins given a specific value of θ.

In practice, we simulate both the planner's problem and the green coin economy using the same set of common exogenous shocks and minimize the maximum average discrepancy in the returns of brown and green capital across states and histories. By doing so, we

Table 3: Optimal GCCB Policy Parameters

	A_0	A_s	A_k	A_{lr}	A_λ	A_x	A_z
θ^*	-0.157	1685	-1068	-471	-4.25	-0.58	-2.78
$\frac{\partial u}{\partial(\cdot)}(\times 100)$	78.68	-0.07%	-0.11%	-0.011%	0.08%	-0.005‰	1.38%

Notes: This table reports the values of the parameters determined as in equation (18). In the second row, we report the partial marginal variation of the welfare function with respect to each parameter.

identify a green coin policy that mimics as much as possible the dynamics of the returns and capital stocks that we would observe at the first-best. As shown our figures (C1)–(C2) in the appendix, the biggest average discrepancy is driven by the returns of green capital. As a result, the optimal parameters are mainly driven by the green sector. All parameters belong to the interior, with the exception of A_0. In our model, we require total GHGs concentration to be non-negative, and thus the number of green coins the authority hold cannot be too large. That explains the corner solution with respect to A_0.

Policy parameters and implications. We report our parameter values in first row of table 3. First of all, the value of A_0 implies that it is optimal for the GCCB to hold on average 46% of the total coins, i.e., the institution must commit to a quite 'hawkish' target. The GCCB increases this share with the stock of CO_2 ($A_s > 0$), but it also accommodates long-run growth ($A_{lr} < 0$) in the overall economy. In addition, the GCCB reduces its holdings of coins when the green sector has become particularly large compared to the brown one ($A_k < 0$).

The rightmost three coefficients refer to the contemporaneous response to the three unexpected shocks in the economy. In all cases, the GCCB responds by increasing the supply of green coins available to the private sector, i.e, by reducing its share of coins. We note that this behavior corresponds to implement an 'emission-smoothing policy'.

Specifically, the emission constraint produces costs proportional to output, and hence it has effects similar to those of distortionary taxes. Under the optimal policy, it is optimal to accommodate expansionary productivity shocks by leaving more green coins to the private sector and letting their price decline. Note that accommodating these shocks in the short-run cause an increase in the future stock of CO_2, s. In addition, to the extent that this shock diffuse slowly to the green sector, green capital decreases relative to brown capital. These two channels reinforce each other, and they imply that the short-run accommodation slowly turns into a more stringent emission constraint for the long-run.

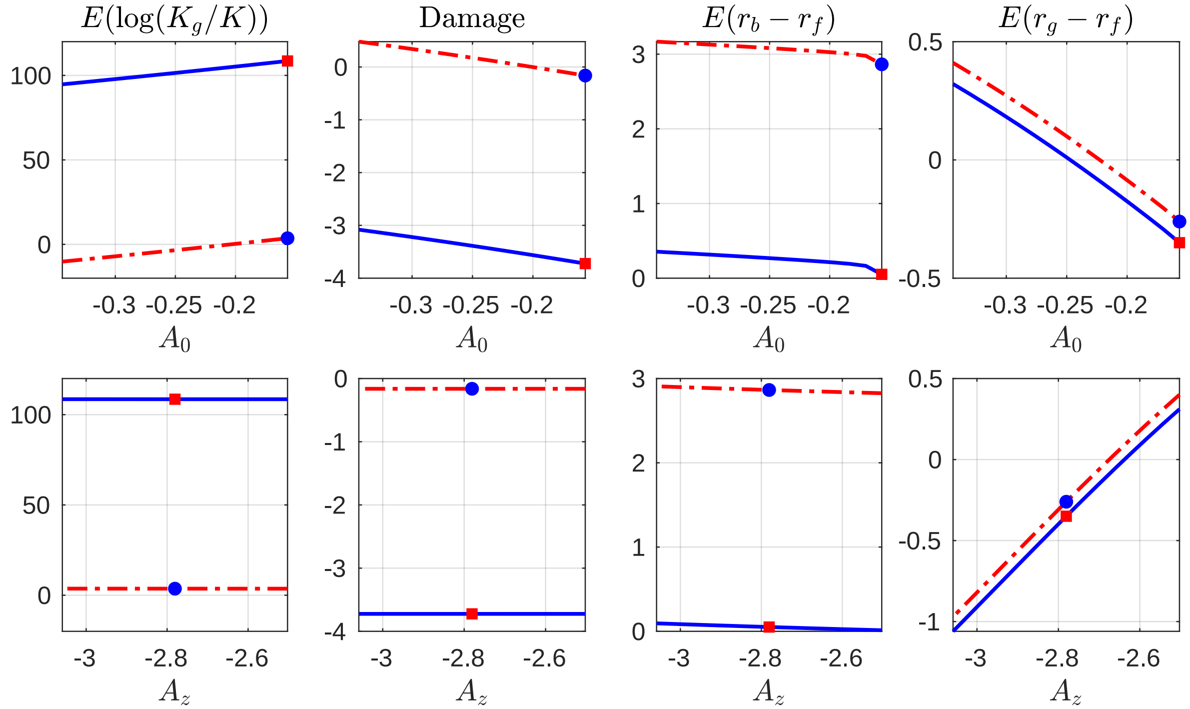
Similarly, if the economy is affected by an adverse shock to the emission-to-GDP intensity, $\epsilon_\lambda > 0$, the GCCB responds by partially offset this shock by leaving more coins to the private sector. As the stock of CO_2 increases, the GCCB increases its purchases of coins in the long-run making them more expensive for the private sector.

Welfare analysis. It is important to assess the convenience of the token-based system compared to the ETS setting. First of all, we must point out that in the setting with the ETS, the welfare costs are equal to 6.93% of time-0 first-best consumption. Our green coin setting overcomes most of this loss as it features a welfare loss of just 0.05% of first-best time-0 consumption. We compare simulated moments across these settings in table 2. In what follows, we 'inspect the mechanism' in order to detail where these welfare gains come from.

The second row of table 3 shows the sensitivity of our welfare function in the settings with green coins with respect to each parameter evaluated at θ^*. Looking at the magnitudes of these derivatives, the most important parameters that we discuss are A_0 and A_z. In figure 3, we plot average moments as we vary these two parameters. Specifically, we focus on how these parameters affect the relative size of the green and brown sector, the average damage, and the risk premia for the green and brown sectors.

In each panel of figure 3 we plot two lines. The dotted (solid) lines refer to difference between the moment obtained in our green coin setting and the corresponding value at the first-best (under the ETS setting). Hence when the red (blue) line gets close to zero, it means that the green coin-based setting mimics the planner (ETS setting) stochastic steady

Fig. 3: Policy Parameters and Stochastic Steady State Values



Notes: In each panel, the blue (red) line shows the difference between the reported moment computed under the green coin setting and the ETS (first-best) setting. All variables are multiplied by 100.

state.

A_0 is a key parameter that allows the green coin setting to mimic the same relative size of the green sector obtained at the first-best. As a result, the implied average damage in this economy is very similar to the first-best level. The ETS setting, instead, features a much smaller green sector and a bigger damage. Interestingly, by committing ex-ante to buy a higher average share of green coins, the GCCB sustains the green sector and reduces its cost of capital while keeping the cost of equity for the brown sector almost constant. At the optimum, the cost of equity in the brown sector is very similar to the level obtained under the ETS system, whereas the cost of capital of the green sector is reduced even below its first-best level. We note that our setting with green coins is still affected by financing frictions, and hence it is not supposed to perfectly mimic the first-best. Given the implicit subsidy to brown capital through the tax shield on debt, the GCCB subsidizes the green sector through a policy that makes the cashflow associated to the offsets safer. By no arbitrage, a lower risk premium stimulates green investment. For a complete comparison of the ETS and the green coin-based economy simulated moments, see table 2.

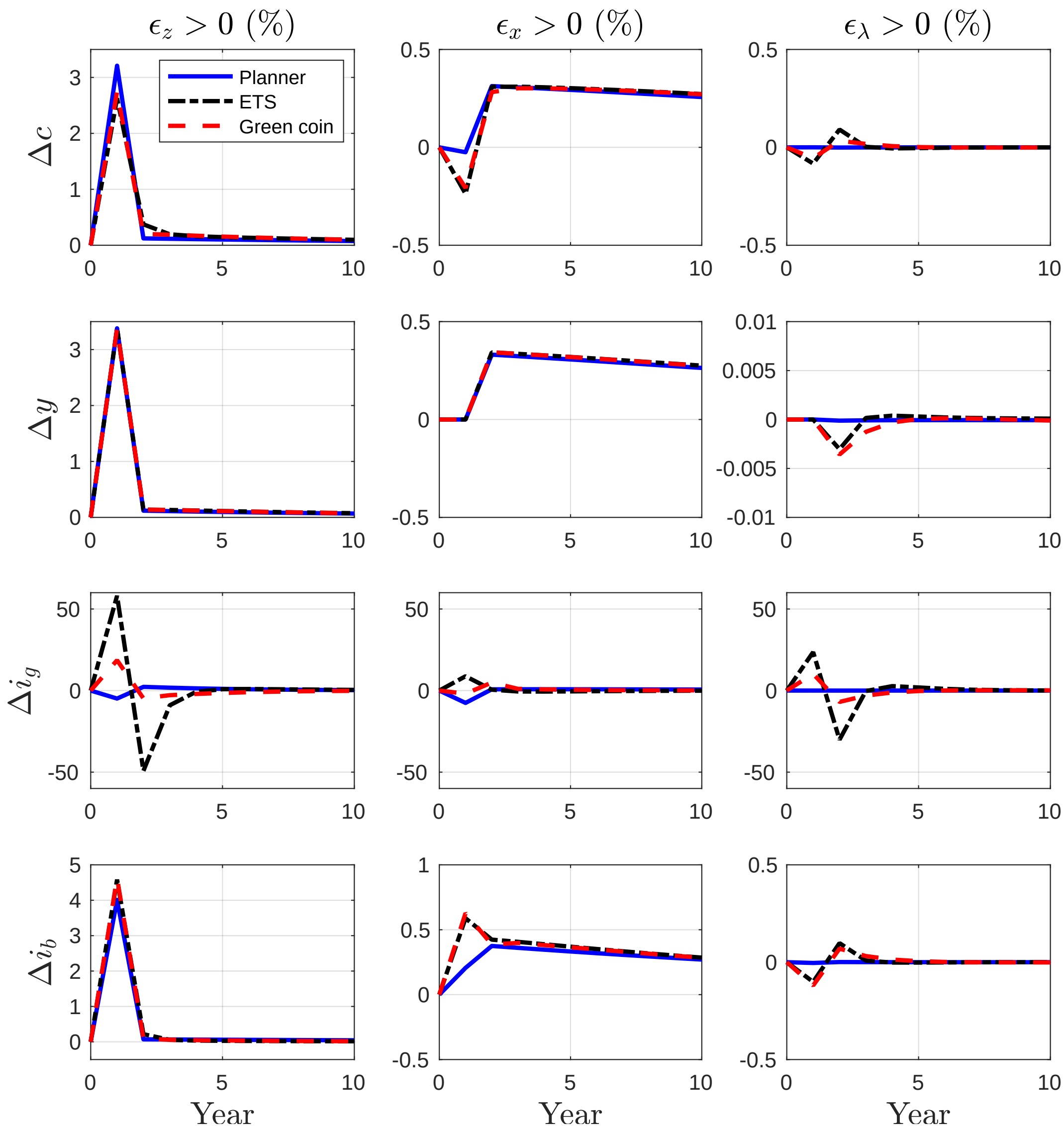
Finally, we note that the riskiness of both the green and the brown sector is sensitive to A_z, i.e., the contemporaneous response of the GCCB to short-run growth shocks. If this parameter is not negative enough, i.e., if the GCCB does not accommodate positive shocks, the risk premium increases. Simultaneously, in the brown sector a less accommodating policy results in cashflows that respond less to long-run news. This is because of hedging effect coming from the cost of offsetting emissions. As a result, as A_z becomes less negative, the risk premium in the brown sector declines. These effects on the cost of capital ultimately affect the relative size of the green and brown sectors. Specifically, a less accommodating policy response to an emission intensity shock reduces the relative share of green capital in the economy. The latter effect is not easily visible in our panel because it is second-order compared to the difference in our steady state moments across the ETS and the first-best settings.

4.2 Dynamic Responses

Analysis the dynamic responses to shocks across the three settings help us further in understanding in which dimensions that green coin setting dominates the ETS. In what follows, we discuss the responses depicted in figure 4.

With respect to a positive short-run productivity shocks, consumption, brown investment, and output behave as in a standard neoclassical model. Under the first best, there is a reallocation of investment away from the green sector in favor of the brown sector. This is because the productivity gain affects immediately the brown sector, but it propagates slowly over time in the green sector. In the ETS economy, in contrast, the opposite happens. The emission allowances do not adjust promptly to this shock and hence there is a strong demand for VCCs which is followed by more investment in green assets. Also the green coin

Fig. 4: Impulse Responses: Quantities



Notes: This figure depicts responses to a one-standard deviation shock. All responses are multiplied by 100. The benchmark calibration is detailed in table 1.

setting features an increase in green investment, but this increase is moderate and hence this response is closer to that measured at the first-best.

When we turn our attention to long-run productivity news, the green investment in the green coin setting mimics very closely the planner's solution. Both the ETS and the green coin settings feature excessive increase in investment and this is due to the additional distortion coming from leverage (Croce, Kung, Nguyen, and Schmid, 2012).

In the ETS system, a positive emission shock, λ, promote a reallocation of investment resources. A higher λ prompts a higher demand of VCCs, and hence it increases green investment. Simultaneously, by resource constraint, brown investment must decline. Because of convex adjustment costs, these adjustments also imply that green assets appreciate, and thus they provide an hedge against emission shocks. Brown stocks, instead, depreciate in a high marginal utility state, and hence they carry an additional risk-premium. These difference in exposure generates a greenium. Interestingly, at the first-best, the response of macroeconomic quantities with respect to this shock go in the same direction, but they are two orders of magnitude smaller. Our green coin setting promotes a reallocation of investment resources that is in between that observed in the ETS setting and at the first-best. Hence, this setting mitigates distortions also with respect to changes in the emission-to-output ratio.

When we turn our attention to the cost of carbon across our three settings. We report our model implied values in table 4. Note that these values refer to marginal prices rescaled by productivity. In order to better interpret these results, we also report the implied equilibrium discount rate for future damages. The marginal cost of CO_2 declines as we move from the Planner's setting toward the ETS system. At first sight, one may find this result surprising as our welfare results go exactly in the opposite direction. In reality, this result is fully driven by the discount rate. The cost of CO_2 is a present value, and it declines with higher discount rate. In the ETS system, the damage is more procyclical, and thus it carries a higher risk premium. The marginal cost of carbon does not reflect welfare because it is silent on the quantity of emissions. In our model, emissions under the ETS system are 20 times higher than at the first-best and hence the implied damage produces significant welfare losses.

Table 4: Emission Price and Discount Rate

Planner		Green Coins		ETS	
q_s	r_s	q_s	r_s	q_s	r_s
0.073	4.28	0.071	4.33	0.067	4.44

Notes: This table reports the implied cost of emissions scaled by productivity and its discount rate across our three settings.

5 Conclusions

We propose a new model to examine emissions regulation in general equilibrium. We study an inefficient decentralized economy with an ETS, opaque VCCs, and credit markets with a bias for 'net-zero' firms. We show that this setting is far away from the first-best. We then analyze a counterfactual setting in which emission offsets are tokenized, and tokens available to the private sectors are managed by a Green Coin Central Bank (GCCB) that controls the cost of emissions with state-contingent open market operations. We show that this setting can produce substantial welfare gains.

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A Planner's problem

The Lagrangian for the planner's problem:

$$
\begin{aligned}
\mathcal{L} = & \left[(1-\beta)\tilde{C}_t^{1-\frac{1}{\psi}} + \beta(\mathbb{E}_t[U_{t+1}^{1-\gamma}])^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right]^{\frac{1}{1-\frac{1}{\psi}}} \\
& + \{(1-\delta)K_{t-1} + \Psi(I_t/K_{t-1})K_{t-1} - K_t\}Q_t \\
& + \{(1-\delta_g)K_{g,t-1} + H(I_{gt}/K_{g,t-1})K_{g,t-1} - K_{gt}\}Q_{g,t} \\
& + \{\chi\left(s_t\right)\left(Z_t L_t\right)^{1-\alpha} K_{t-1}^{\alpha} - (\tilde{C}_t + I_{gt} + I_t)\}\Omega_t \\
& + \left\{s_t - \left((1-\delta_s)s_{t-1} + \{\lambda_t (Z_t L_t)^{1-\alpha} K_{t-1}^{\alpha} - (Z_{gt}L_{gt})^{1-\kappa} K_{g,t-1}^{\kappa}\}/Z_{t-1}\right)\right\} Q_{st},
\end{aligned}
$$

where Q_t, $Q_{g,t}$, Ω_t, and $Q_{s,t}$ are multipliers. The first order conditions are

$$
\begin{aligned}
0 &= U_t^{1/\psi}\beta(\mathbb{E}_t[U_{t+1}^{1-\gamma}])^{\frac{1-\frac{1}{\psi}}{1-\gamma}-1}\,\mathbb{E}_t\left(U_{t+1}^{-\gamma}\frac{\partial U_{t+1}}{\partial K_t}\right) - Q_t && \text{(A1)} \\
q_t := \frac{Q_t}{\Omega_t} &= 1/\Psi'(I_t/K_{t-1}) && \text{(A2)} \\
0 &= U_t^{1/\psi}\beta(\mathbb{E}_t[U_{t+1}^{1-\gamma}])^{\frac{1-\frac{1}{\psi}}{1-\gamma}-1}\,\mathbb{E}_t\left(U_{t+1}^{-\gamma}\frac{\partial U_{t+1}}{\partial K_{gt}}\right) - Q_{gt} && \text{(A3)} \\
q_{gt} := \frac{Q_{g,t}}{\Omega_t} &= 1/H'(I_{gt}/K_{g,t-1}) && \text{(A4)} \\
0 &= (1-\beta)U_t^{1/\psi}\tilde{C}_t^{-\frac{1}{\psi}} - \Omega_t && \text{(A5)} \\
0 &= -\chi_0\cdot\chi_t\cdot Y_t\Omega_t + U_t^{1/\psi}\beta(\mathbb{E}_t[U_{t+1}^{1-\gamma}])^{\frac{1-\frac{1}{\psi}}{1-\gamma}-1}\,\mathbb{E}_t\left(U_{t+1}^{-\gamma}\frac{\partial U_{t+1}}{\partial s_t}\right) + Q_{st}. && \text{(A6)}
\end{aligned}
$$

Plug (A5) into (A1), (A3), and (A6), we obtain

$$
\begin{aligned}
q_t &= \mathbb{E}_t\left(M_{t+1}\frac{1}{\Omega_{t+1}}\frac{\partial U_{t+1}}{\partial K_t}\right) && \text{(A7)} \\
q_{gt} &= \mathbb{E}_t\left(M_{t+1}\frac{1}{\Omega_{t+1}}\frac{\partial U_{t+1}}{\partial K_{gt}}\right) && \text{(A8)} \\
\hat{q}_{st} := \frac{Q_{st}}{\Omega_t} &= \chi_0\cdot\chi_t\cdot Y_t + \mathbb{E}_t\left(M_{t+1}\frac{1}{\Omega_{t+1}}\cdot\left(-\frac{\partial U_{t+1}}{\partial s_t}\right)\right) && \text{(A9)}
\end{aligned}
$$

$$\hat{p}_{st} := \mathbb{E}_t \left(M_{t+1} \hat{q}_{s,t+1} \right) \tag{A10}$$

$$\hat{q}_{st} = \chi_0 \cdot \chi_t \cdot Y_t + (1 - \delta_s) \hat{p}_{st}. \tag{A11}$$

The envelop conditions read

$$\frac{\partial U_t}{\partial K_{t-1}} = Q_t \cdot \left[(1-\delta) + \Psi_t - \Psi'_t \cdot \frac{I_t}{K_{t-1}} \right] + \left(\Omega_t \chi_t - \frac{Q_{st}}{Z_{t-1}} \lambda_t \right) \alpha \frac{Y_t}{K_{t-1}} \tag{A12}$$

$$\frac{\partial U_t}{\partial K_{g,t-1}} = Q_{gt} \cdot \left[(1-\delta_g) + H_t - H'_t \cdot \frac{I_{gt}}{K_{g,t-1}} \right] + \frac{Q_{st}}{Z_{t-1}} \kappa \frac{\tilde{G}_t}{K_{g,t-1}} \tag{A13}$$

$$\frac{\partial U_t}{\partial s_{t-1}} = -(1-\delta_s) Q_{st}. \tag{A14}$$

After defining $q_{st} = \hat{q}_{st}/Z_{t-1}$ and $p_{st} = \hat{p}_{st}/Z_{t-1}$, we obtain the returns as reported in the paper

$$R^P_{s,t} = \frac{Z_{t-1}}{Z_{t-2}} \frac{q_{st}}{p_{s,t-1}} \tag{A15}$$

$$R^P_{I,t} = \left(q_t \cdot \left[(1-\delta) + \Psi_t - \Psi'_t \cdot \frac{I_t}{K_{t-1}} \right] + (\chi_t - q_{st} \lambda_t) \, \alpha \frac{Y_t}{K_{t-1}} \right) / q_{t-1} \tag{A16}$$

$$R^P_{G,t} = \left(q_{gt} \cdot \left[(1-\delta_g) + H_t - H'_t \cdot \frac{I_{gt}}{K_{g,t-1}} \right] + q_{st} \kappa \frac{\tilde{G}_t}{K_{g,t-1}} \right) / q_{g,t-1}, \tag{A17}$$

and the associated Euler equations:

$$1 = \mathbb{E}_t \left[M_{t+1} R^P_{s,t+1} \right] \tag{A18}$$

$$1 = \mathbb{E}_t \left[M_{t+1} R^P_{I,t+1} \right] \tag{A19}$$

$$1 = \mathbb{E}_t \left[M_{t+1} R^P_{G,t+1} \right] \tag{A20}$$

B ETS with opague VCCs

B.1 Brown firm

The brown firm solves

$$V_t = \max_{K_t, B_t, I_t, G_t, EA_t} D_t + \mathbb{E}_t(M_{t+1}V_{t+1}) \tag{A21}$$

subject to

$$D_t = \chi(s_t) \cdot Y_t - W_t L_t - (1-\tau) r_t^B B_{t-1} + \Delta B_t - I_t - C_t^E - C_t^B \tag{A22}$$
$$- p_t \cdot EA_t - p_{gt} G_t - C_t^C - C_t^{EA}$$
$$K_t = (1-\delta) K_{t-1} + \Psi(I_t / K_{t-1}) K_{t-1} \tag{A23}$$

Optimality with respect to debt. The first order condition with respect to corporate debt and the Envelope theorem jointly imply:

$$\begin{aligned} 1 &= \frac{\partial C_t^B}{\partial B_t} + \frac{\partial C_t^E}{\partial B_t} - \mathbb{E}_t\left[M_{t+1}\frac{\partial V_{t+1}}{\partial B_t}\right] \\ \frac{\partial V_t}{\partial B_{t-1}} &= -(1 + (1-\tau) r_{t-1}^B). \end{aligned}$$

Taking into account the fact that at the equilibrium r^B is equal to the risk-free rate:

$$\frac{\partial C_t^B}{\partial B_t} + \frac{\partial C_t^E}{\partial B_t} = \mathbb{E}_t\left[M_{t+1}\tau\right] r_{f,t},$$

which becomes

$$\frac{\partial C_t^B}{\partial B_t} + \frac{\partial C_t^E}{\partial B_t} = \frac{\tau \cdot r_{f,t}}{1 + r_{f,t}},$$

The left-hand side refers to the total marginal cost of issuing an extra unit of debt. The right-hand side measures the value of the corporate interest tax advantage.

Optimality with respect to EA. The demand of emission allowances is related to the marginal environmental distress cost:

$$p_t = -\frac{\partial C_t^{EA}}{\partial EA_t} = \phi_1^{EA} \frac{C_t^{EA}}{G_t}$$

Optimality with respect to declared green credits. The first order condition with respect to G_t imply the following:

$$p_{gt} = \underbrace{p_t \cdot \frac{\lambda_t Y_t - EA_t}{G_t}}_{\text{savings on EA}} \quad \underbrace{-\frac{\partial C_t^E}{\partial G_t}}_{>0,\text{ savings on borrowing}} \quad \underbrace{-\zeta \cdot \frac{G_t - \tilde{G}_t}{Z_{t-1}}}_{<0}$$

Optimality with respect to capital. Envelope and first order conditions with respect to investment and capital imply the following:

$$\begin{aligned}
q_t &= \frac{1}{\Psi_t'} = \mathbb{E}_t\left[M_{t+1}\frac{\partial V_{t+1}}{\partial K_t}\right] - \frac{\partial C_t^E}{\partial K_t} \\
\frac{\partial V_t}{\partial K_{t-1}} &= \chi(s_t)\frac{\partial Y_t}{\partial K_{t-1}} - \frac{\partial C_t^{EA}}{\partial K_{t-1}} - \frac{\partial C_t^E}{\partial K_{t-1}} - \frac{\partial C_t^B}{\partial K_{t-1}} + q_t\left(1 - \delta - \frac{\Psi_t' I_{bt}}{K_{t-1}} + \Psi_t\right).
\end{aligned}$$

B.2 Green firm

$$V_{gt} = \max_{K_{gt}, I_{gt}} D_{gt} + \mathbb{E}_t(M_{t+1}V_{g,t+1}) \tag{A24}$$

$$D_{gt} = p_{gt}G_t^s - (1 - \tau_{gt})I_{gt} - W_{gt}L_{gt} \tag{A25}$$

$$K_{gt} = (1 - \delta_g)K_{g,t-1} + H(I_{gt}/K_{g,t-1})K_{g,t-1}, \tag{A26}$$

where $G_t^s = \xi_t\tilde{G}_t$; $\xi_t = 1 + b_0\exp(b_1\epsilon_{\xi,t})$; and $\tilde{G}_t = (Z_{g,t}L_{gt})^{1-\kappa}K_{g,t-1}^{\kappa}$.

Investment are determined by

$$q_{gt} = (1-\tau_{gt})/H'_t = \mathbb{E}_t \left(M_{t+1} \frac{\partial V_{g,t+1}}{\partial K_{gt}} \right) \tag{A27}$$

$$\frac{\partial V_{gt}}{\partial K_{g,t-1}} = p_{g,t} \frac{\partial G_t^s}{\partial K_{g,t-1}} + q_{gt}\{1-\delta_g + H_t - H'_t \cdot (I_{gt}/K_{g,t-1})\} \tag{A28}$$

C Green Coin Setting

C.1 Brown firm

The brown sector faces the following problem

$$V_t = \max_{K_t, B_t, I_t, G_{gt}^{PD}} D_t + \mathbb{E}_t(M_{t+1}V_{t+1}) \tag{A29}$$

$$D_t = \chi(s_t) \cdot Y_t - W_t L_t - (1-\tau) r_t^B B_{t-1} + \Delta B_t - I_t - C_t^E - C_t^B \tag{A30}$$

$$- p_{gt} G_{gt}^{PD} - C_t^{EA}$$

$$K_t = (1-\delta)K_{t-1} + \Psi(I_t/K_{t-1})K_{t-1} \tag{A31}$$

where

$$C_t^E = \phi_0 \cdot \exp\left(\phi_1 \left\{1 - \frac{\theta_t K_t}{B_t}\right\}\right) \cdot Z_{t-1} \quad (\text{mimics } B_t \leq \theta_t \cdot K_t) \tag{A32}$$

$$C_t^B = \eta Z_{t-1} \left(\frac{B_t}{Y_t} - \frac{B_{ss}}{Y_{ss}} \right)^2 \tag{A33}$$

$$\theta_t \equiv \theta \left(\frac{G_t^{PD}}{\lambda_t Y_t} \right); \quad \theta'(\cdot) > 0 \tag{A34}$$

$$C_t^{EA} = \phi_0^{EA} \cdot \exp\left(\phi_1^{EA} \left\{ \frac{(\lambda_t Y_t - \bar{A} \cdot Z_{g,t-1})}{G_t^{PD}} - 1 \right\} \right) \cdot Z_{t-1} \tag{A35}$$

Optimality with respect to capital. Envelope and first order conditions with respect to investment and capital imply the following:

$$
\begin{aligned}
q_t &= \frac{1}{\Psi_t'} = \mathbb{E}_t \left[M_{t+1} \frac{\partial V_{t+1}}{\partial K_t} \right] - \frac{\partial C_t^E}{\partial K_t} \\
\frac{\partial V_t}{\partial K_{t-1}} &= \chi(s_t) \cdot \frac{\partial Y_t}{\partial K_{t-1}} - \frac{\partial C_t^{EA}}{\partial K_{t-1}} - \frac{\partial C_t^E}{\partial K_{t-1}} - \frac{\partial C_t^B}{\partial K_{t-1}} + q_t \left(1 - \delta - \frac{\Psi_t' I_{bt}}{K_{t-1}} + \Psi_t \right).
\end{aligned}
$$

After defining

$$
R_{bt}^{GC} = \frac{\partial V_t}{\partial K_{t-1}} / \left(q_{t-1} + \frac{\partial C_{t-1}^E}{\partial K_{t-1}} \right), \tag{A36}
$$

the Euler equation associated with brown investment reads

$$
1 = \mathbb{E}_t (M_{t+1} R_{b,t+1}^{GC})
$$

C.2 Green firm

The green sector faces

$$
V_{gt} = \max_{K_{gt}, I_{gt}} D_{gt} + \mathbb{E}_t (M_{t+1} V_{g,t+1}) \tag{A37}
$$

$$
D_{gt} = p_{gt} G_{gt} - (1 - \tau_{gt}) I_{gt} - W_{gt} L_{gt} \tag{A38}
$$

$$
K_{gt} = (1 - \delta_g) K_{g,t-1} + H(I_{gt}/K_{g,t-1}) K_{g,t-1} \tag{A39}
$$

First-order conditions with respect to capital and investment:

$$
q_{gt} = \mathbb{E}_t \left(M_{t+1} \frac{\partial V_{g,t+1}}{\partial K_{gt}} \right) \tag{A40}
$$

$$
q_{gt} = (1 - \tau_{gt}) / H_t' \tag{A41}
$$

$$
\tag{A42}
$$

The envelop condition is

$$\frac{\partial V_{gt}}{\partial K_{g,t-1}} = p_{g,t}\frac{\partial G_t^s}{\partial K_{g,t-1}} + q_{gt}\{1 - \delta_g + H_t - H_t' \cdot (I_{gt}/K_{g,t-1})\} \tag{A43}$$

After defining

$$R_{gt}^{GC} = \frac{\partial V_{gt}}{\partial K_{g,t-1}}/q_{g,t}, \tag{A44}$$

we obtain the Euler equation:

$$1 = \mathbb{E}_t(M_{t+1}R_{g,t+1}^{GC}).$$

C.3 Additional results

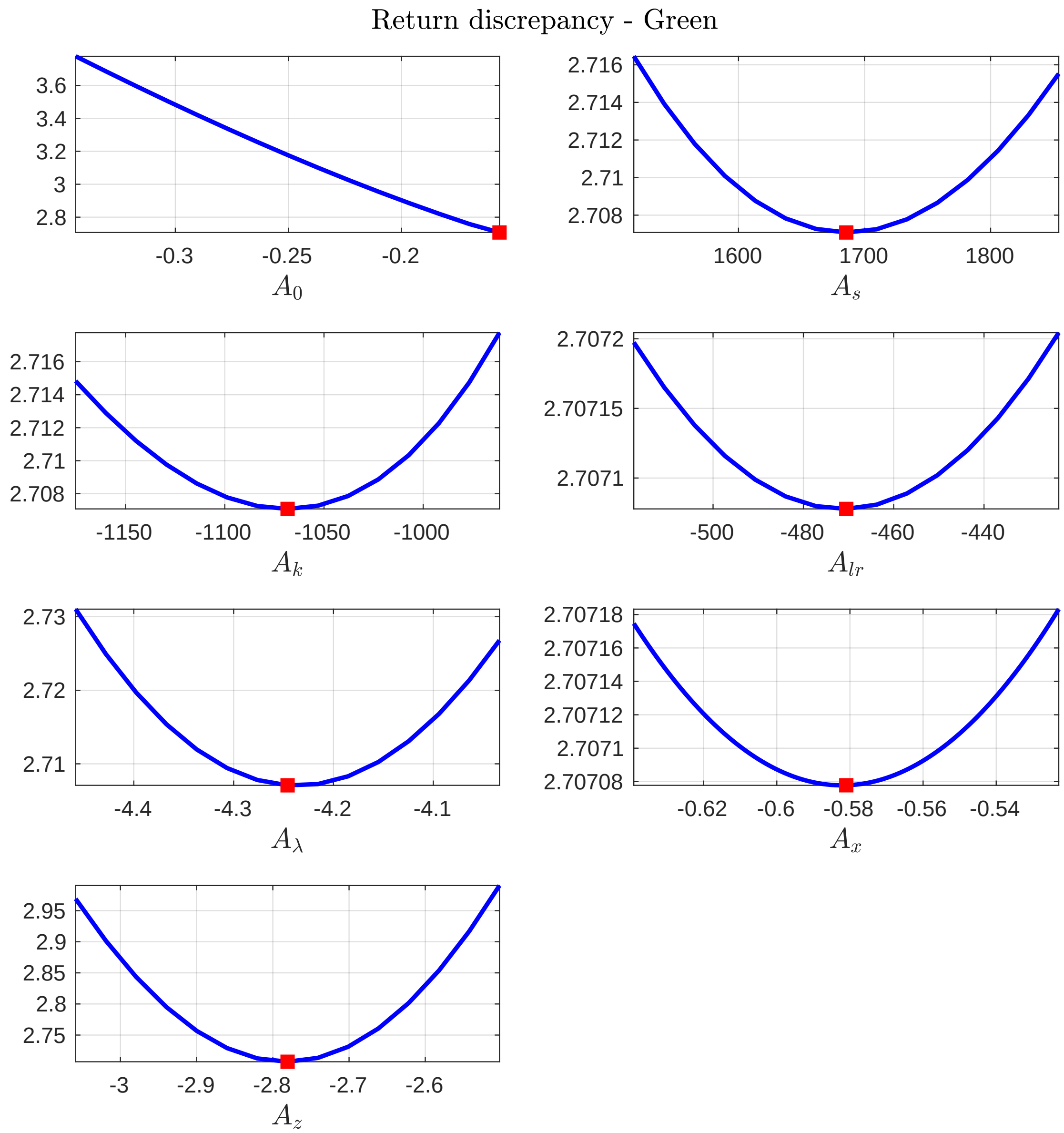

Fig. C1: Green policy (I)

Notes: This figure shows the average squared discrepancy of the returns for the brown sector:

$$\mathbb{E}\left[(R^P_{gt} - R^{GC}_{gt}(\theta))^2\right].$$

The markers denote the value of the parameters that minimize the objective function defined in equation (18) in main text.

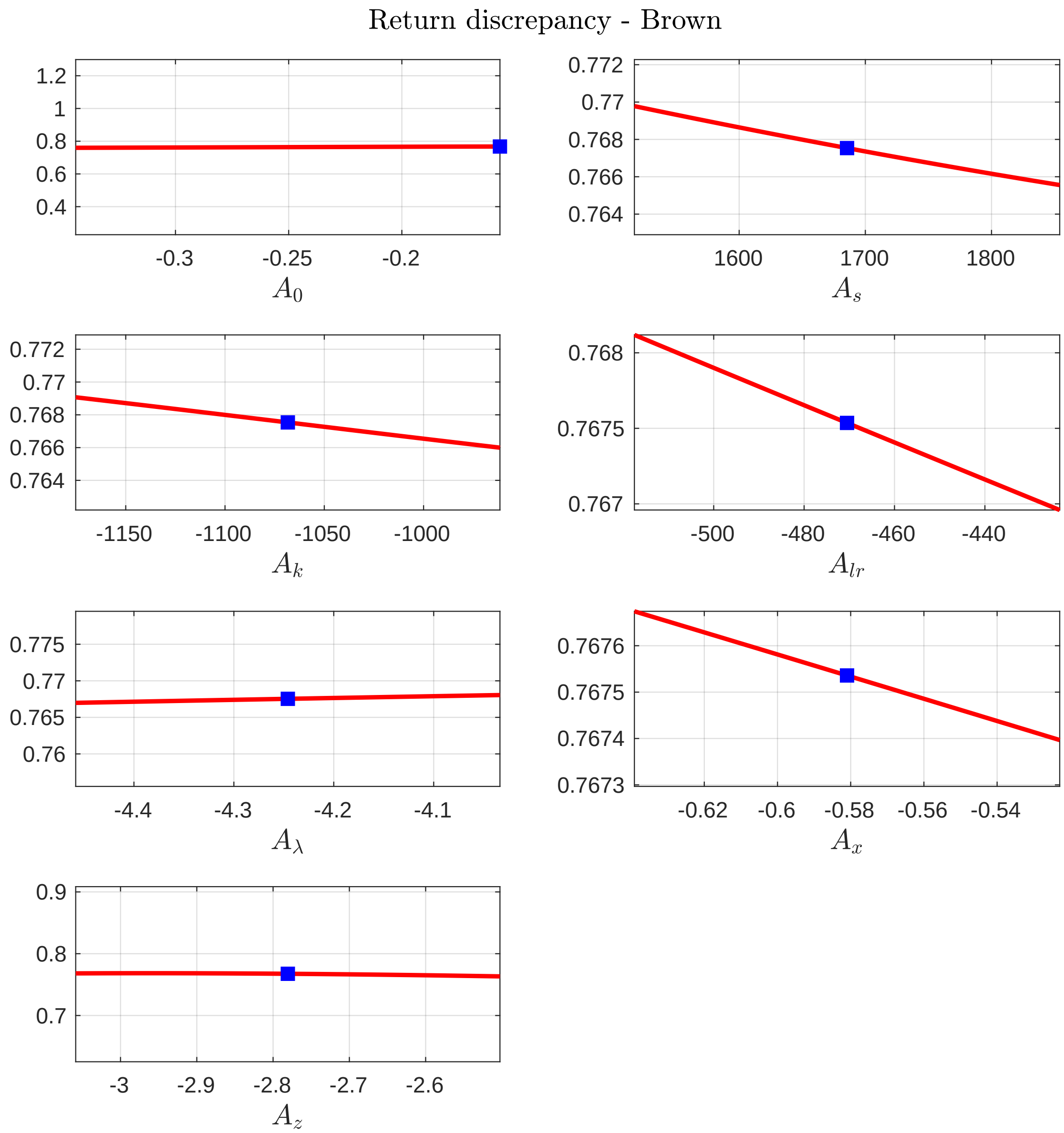

Fig. C2: Green policy (II)

Notes: This figure shows the average squared discrepancy of the returns for the brown sector:

$$\mathbb{E}\left[(R_{bt}^{P} - R_{bt}^{GC}(\theta))^2\right].$$

The markers denote the value of the parameters that minimize the objective function defined in equation (18) in main text.

Chapter 2: The Scope of Scope 3

The Scope of Scope-3

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Abstract

We propose a novel network-based methodology in order to measure emissions along complex international supply chains (SC). Using international data for about 50,000 firms, we characterize SC-level emissions across countries and industries. Our methodology enables us to run counterfactual experiments and formulate granular forecasts about high-scope emissions with respect to many different scenarios such as maritime disruption, military conflicts, trade wars, revised carbon taxes.

JEL classification: E1; E2; G1; O33; O41.
Keywords: Networks, Emissions, Scope-3
This draft: April 30, 2026.

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1 Introduction

The increase in $CO2_e$ concentration in the atmosphere is a global phenomenon that requires thinking about emissions in a global framework. In this spirit, climate experts have defined different scopes of emissions which comprise direct emissions (Scope-1), emissions associated with the acquisition of inputs (Scope-2), and emissions created along the supply chain (Scope-3). While these concepts are well established (see Greenhouse Gas Protocol (2011)), both their measurement and management require additional effort (in this spirit, see, among others, Reichelstein 2025).

In this paper, we propose a transparent, verifiable, and general equilibrium–based methodology to both (i) measure current Scope-3 emissions across different levels of granularity, and (ii) conduct scenario analysis in order to forecast Scope-3 emissions when supply chains adapt to a wide range of shocks.

The first task, that is, measuring current Scope-3 emissions is important for several reasons. First, different data providers offer substantially different measures of high-scope emissions. These discrepancies arise from different assumptions on the input-output matrix across firms, as well as the heterogeneous firm coverage. We propose a methodology which requires only direct emissions and can therefore be independently replicated and verified. Direct emissions tend to be measured with less variability both across data providers, and over time as firms revise their sustainability reports (Cohen et al. 2026).

Second, the accounting of emissions has not fully matured yet. Our methodology is an important new tool to measure high-scope emissions and it can help corporations to better assess their global climate risk exposures, as well better benchmark against other firms operating in their industry and/or country. Importantly, since many companies are not required to disclose their sustainability reports, our methodology increases coverage across thousands of firms. Furthermore, our setting enables us to compute embodied emissions (Hertwich and Wood 2018), that is, a measure that accounts for all emission exchanges along the entire supply chain.

Third, our methodology explicitly accounts for the emissions induced by the international shipping of goods along different roads and maritime routes. By doing so, we increase awareness about high-scope emissions imported from remote, low-cost providers, and enable counterfactual analysis about shocks that alter international trade routes. A key advantage of our methodology is that it highlights firms embedded in local networks with lower transport emissions. In addition, our equations enables us to look at the contribution to the embodied emissions of a focal firm across upstream suppliers, and over different tiers of connections. By doing so, we provide direct guidance on the origins of embodied emissions.

Our second task is to use our general equilibrium approach to run scenario analysis and forecast how emissions across different supply chains respond to relevant shocks. This exercise is relevant for (i) managers who want to make their supply chains more emission-resilient, (ii) investors aiming to build low-carbon portfolios, and (iii) policymakers interested in the link between global emissions and economic and geopolitical shocks.

For the purposes of this manuscript, we focus on the following illustrative questions. First, what would happen to emissions under geopolitical stress causing major maritime route disruptions? Second, what would happen if an entire emerging economy were removed from a supply chain? These experiments illustrate the insights that our measurement of high-scope emissions can deliver. To keep the analysis focused, we select Heidelberg Materials as the focal firm and study its supply chain, a benchmark example in the literature (Reichelstein 2024).

Our methodology is flexible enough to accommodate many additional scenarios. Future research should also address first-order questions such as: what should we expect for high-scope emissions in the aftermath of the EU CBAM? More broadly, how would high-scope emissions change when a large economy imposes highly heterogeneous tariffs across countries?

Methodology and data. We think of a supply chain as a network of firms identified around a focal firm. Using the classic model of Long and Plosser (1983) requires us to identify: (1) N firms; (2) $N \times N$ production function intensity coefficients; (3) N productivity levels; and (4) the total size of a common factor that in our model is capital. After choosing the focal firm, we use FactSet to identify links to customer and supplier firms. This step determines N. Usually, these firms operate in very different countries and sectors. We use the OECD input-output matrix to infer their production function intensity coefficients. From FactSet, we obtain standardized productivity measures for each firm in the network. After matching revenues reported in FactSet, and targeting a cost of capital set to a standard value of 10%, we rescale all productivity measures and recover total capital in the network. In sum, this procedure enables us to uniquely retrieve quantities produced in the network starting from observed data.

Once quantities are obtained, we augment our network setting with an emission matrix that accounts for both direct emissions due to production and additional emissions due to transport. Direct emissions are obtained from Trucost. Granular transport data are not available. As a result, we proceed with a novel AI-powered empirical effort. First of all, we need to identify firms that require physical delivery of products. We apply an LLM to both the FactSet description of each firm and its SIC code to determine which transport the company likely uses, if any. We use (i) Google Maps and SeaRoute (https://eurostat.github.io/searoute/) to identify shipping routes across firms, and (ii) emission data from International Maritime Organization (2024) to convert routes in tons of CO_2e.

When we simulate a disruption in maritime routes, we keep the original network unchanged and hence the equilibrium quantities remain unchanged as well. The only object that needs to be recomputed is the emission matrix. When instead we simulate a ban on an entire country, we use remove all firms located in that country from the network and replace them with comparable firms located nearby. To identify potential replacement we use both FactSet 'known competitors', and an additional list obtained via advanced text analysis. Specifically, we apply cosine similarity to firm description in FactSet and complement this approach with an LLM that determines whether firms are competitors (binary outcome) and the confidence level of such answer. After the network is reformed, the equilibrium model is recalibrated and solved. Equilibrium quantities and emissions are reassessed.

Literature. This paper contributes to a fast-growing literature on the economics of climate and sustainability. For a recent and comprehensive review of the carbon accounting literature, see Stechemesser and Guenther (2012), He et al. (2022), and Stachelscheid and Dutzi (2025). For a seminal review on climate finance and economics, we refer to Giglio et al. (2021), and Hassler et al. (2021) and Heal (2009), respectively.

Our work is related to Hertwich and Peters (2009), who measure carbon footprint across 73 nations and 8 industry aggregates. We differ in our focus on supply-chains, our network equilibrium approach, and the novel AI-powered tools that we employ in order to merge and interpret our large datasets.

Pástor et al. (2021) estimate the present value of future emission-related damages. Greenstone et al. (2023) estimate the monetary value of current damages. Both papers focus on Scope-1 emissions. We do not price damages, instead we promote a new methodology to assess emissions at different scope levels. In addition, we compute firm-level embodied emissions, i.e., a broader measure of emissions (Hertwich and Wood 2018) that can better help in identify 'brown' and 'green' firms.

Pankratz and Schiller (2024) investigates whether in the data companies change their supply chains in response to climate news shocks. We run scenario analysis and find potential replacements via AI-powered algorithms. In contemporaneous and independent work, Kou et al. (2026) use a network general equilibrium model to measure indirect emissions and assess the riskiness of firms according to their supply chain level 'brownness'. Relatedly, Lavia and Luciano (2026) accounts for emissions propagation in a setting with financial frictions. We differ for our attention to carbon accouting, our counterfactual analysis, and our considerations about geolocalization of firms and transport costs. As in Como et al. (2026), we account for financial frictions to better disentangle prices from quantities.

TABLE 1: Comparison of Full FactSet and Trucost Datasets

Panel A: Comparison of source datasets

	FactSet	FactSet Financials	Trucost
Total Number of Unique Companies	275,631	54,912	3,629,434
Total Number of Unique Countries	219	124	203
Percentage of Private Companies	51.5%	22.2%	99.0%
Percentage of Public Companies	14.3%	66.6%	0.5%
Percentage of Other Companies	34.2%	11.2%	0.5%

Panel B: merged dataset

Total Companies	43,117
Perfect Matches (Identifier)	6,259
Average Match Score	96.71
Median Match Score	100.00
Total Countries	109

Panel C: entity type in merged sample

Public Companies (PUB)	26,403	61.2%
Private Companies (PVT)	11,765	27.3%
Other Types	4,949	11.5%

Notes: The top panel of this table compares the full FactSet and Trucost datasets. FactSet provides standardized global financial data and supply chain information. Trucost provides standardized global emissions information, enabling the assessment of corporate environmental impacts across direct operations and value chains. Factset captures 275,631 companies in 219 countries, but only 54,912 companies in 124 countries have the full financial information required by our model. Trucost has 3,629,434 companies in 203 countries over an annual sample starting in 2003 and ending in 2023. In panel B and C, we provide the summary statistics of our matched companies, with full financial information and emission information. In panel C, 'Other types' includes Government entities, Not-for-profit organizations, Subsidiaries, Holding companies, Joint ventures, Limited partnerships, Limited liability companies, Corporations (not specifically public/private), Cooperatives, Trusts, Investment funds, Insurance companies, Utilities, Real Estate Investment Trusts.

2 Data

Merging. We start by merging data from FactSet and Trucost. These two datasets have very different sizes and scope, as shown in the top portion of Table 1. Most importantly, these two datasets do not have a common identifier for their firms, implying that a matching algorithm is required. We describe our merging procedure in detail in Appendix A. Here, we point out that, unlike off-the-shelf entity resolution packages, our solution is specifically designed to handle the hierarchical corporate structures present in financial data, where companies can have complex parent-child relationships across different supply chains.

The system performs multi-pass matching across different organizational levels, combining exact identifier matching, fuzzy string similarity, and AI-based validation to achieve high-quality matches. This approach is particularly valuable for financial analysis, where understanding corporate hierarchies and subsidiary relationships is crucial for accurate emissions accounting and risk assessment.

As detailed in the Appendix, for many public companies the match is perfect because it is based on universal identifiers (CUSIP). Other companies may not have such identifiers, but they may be sufficiently well known that their name and country location coincide across FactSet and Trucost. These matches feature a match score of 100%, reflecting maximum confidence in the entity resolution.

As shown in Panel A of Table 1, our final FactSet sample comprises 54,912 companies with complete financial and geolocation data. To maximize matching coverage while maintaining data quality, we employ a two-pass fuzzy matching procedure augmented with AI validation for ambiguous cases. The first pass applies a 95% similarity threshold, while the second pass uses an 85% threshold with AI review for borderline matches. As reported in Panel B, this approach yields 43,117 successfully matched firms (78.5% of our financial sample) distributed across 109 countries. The matched universe exhibits strong data quality, with an average match score of 96.7 and a median score of 100.0. Match types include 6,259 perfect identifier matches, 30,312 exact name-country matches, and 11,350 high-confidence fuzzy matches. AI validation confirmed an additional 1,868 matches that would have been rejected under purely algorithmic criteria. The resulting merged dataset features a distribution tilted towards public companies, which provides a higher proportion of complete financial and emission records compared to non-public companies.

Geographical distribution. Figure 1 shows both the geographical distribution of our sample and the total number of companies in each country. Our geocoding procedure employs a two-step approach to maximize coverage while maintaining accuracy. First, we geocode companies using structured address data (street address, city, postal code) from FactSet, querying Google's Geocoding API with complete location information. For companies lacking

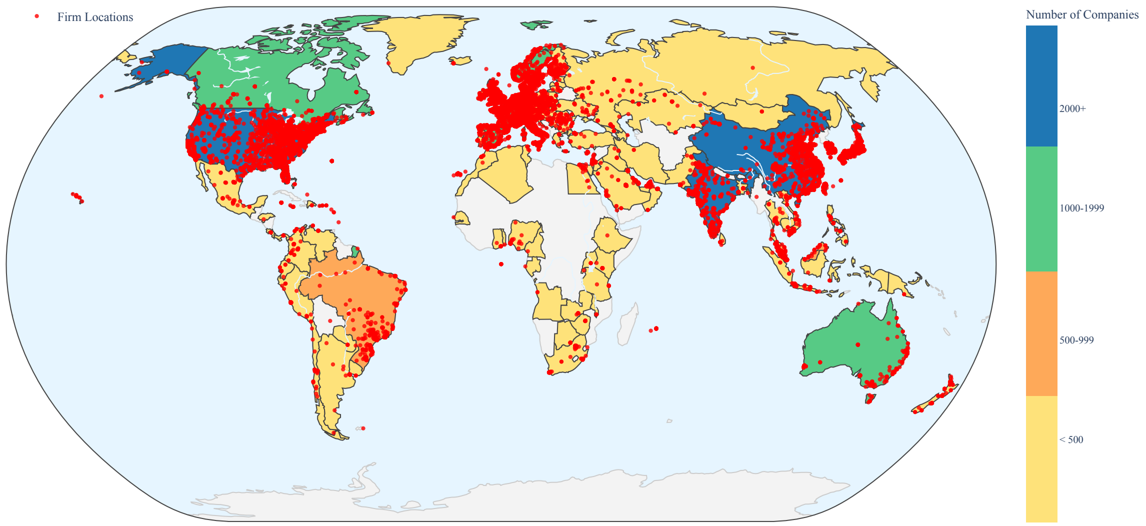


Fig. 1: Global Distribution of Establishments

Notes: This figure shows the geographical distribution of the firms in our dataset denoted as 'Financial FactSet' in table 1. Each dot corresponds to a company location. The color used for each country (right scale) refers to the total mass of firms located in that country.

complete address data, we employ a second geocoding pass using only the company name and country, leveraging Google's ability to locate well-known establishments. This strategy yields geocoding coverage of 100%, providing coordinates for our 54,912 companies with financials. The geocoded sample spans all inhabited continents, with twenty-four countries containing more than 500 companies, ensuring that our results are not driven solely by a small cross-section of major locations. We report summary statistics for the top-15 countries sorted by number of companies in Table B.1 in the Appendix.

Supply chain features. We report basic network statistics in Table 2. In this table, we focus on 2023 data because (i) it is the most recent year in our sample, and (ii) coverage is higher in the second half of our sample. We report the time series of our data in Tables B.2-B.3 in the Appendix. The main takeaway from Table 2 is that, although our merged sample features a smaller cross section of firms, it preserves the main network characteristics reported in the FactSet universe. Notably, our merged sample exhibits higher network connectivity,

Table 2: Supply Chain Network Analysis Comparison

Metric	All Companies	Matched Universe
Analysis Year	2023	2023
Total Firms Analyzed	329,151	54,912
Firms with Suppliers	92,412	29,006
Firms with Customers	33,188	30,806
Firms with Both	17,809	17,338
Firms with Neither	221,360	24,064
Avg Suppliers per Firm	2.99	5.90
Median Suppliers per Firm	1.00	2.00
Avg Customers per Firm	8.33	8.66
Median Customers per Firm	4.00	4.00
Max Suppliers per Firm	1,328	1,083
Max Customers per Firm	370	370
Total Supplier Relationships	276,435	171,073
Total Customer Relationships	276,435	266,989

Notes: This tables compares the network relationships between the entire Factset universe and our matched companies, for a selected year 2023.

i.e., on average our firms feature nearly twice as many suppliers (5.90 vs. 2.99) as in the full sample, while having a similar average of customer relationships. Hence our sample captures established and interconnected firms.

Emissions. In table 3, we report standard statistics for the emissions provided by Trucost. The same statistics for the entire Trucost universe are reported in table B.4, in the Appendix. We note three relevant points. First, both in our merged dataset and in Trucost, emission intensity has been declining in the last twenty years. Second, Trucost has a limited coverage of Scope-3 emissions since it typically does not report downstream emissions. This is one of the dimensions in which our methodology is new and relevant. Third, in figure 2 we plot the distribution of emissions across firms in both Trucost and our our merged dataset. In 2010, the distribution of emissions in our merge dataset is essentially identical to that in the Trucost universe. In recent years, Trucost coverage has changed dramatically, and it has started to include many small private firms with modest reported emissions. As a result, the emission distribution has shifted to the left. In our merged dataset, this effect is mitigated

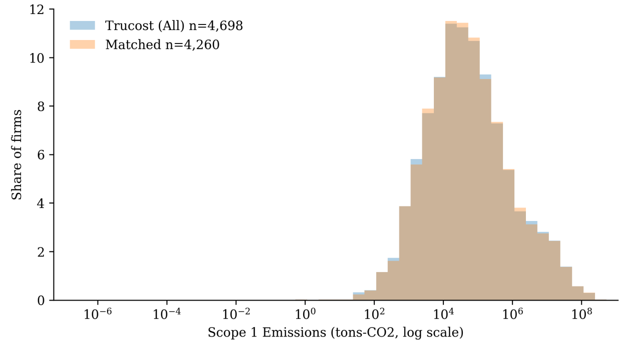
Table 3: Selected GHG Emission Measures in our Dataset

Year	Emission Intensity (Median)			Total Emissions		
	S1	S3 Up	S3 Down	S1	S3 Up	S3 Down
2002	34.96	289.18	0.00	4,186	3,742	0
2003	32.32	252.07	0.00	5,109	3,935	0
2004	30.79	217.27	0.00	6,470	5,074	0
2005	29.47	182.39	0.00	7,384	5,256	0
2006	28.43	169.58	0.00	8,014	5,414	0
2007	27.08	157.65	0.00	8,622	5,872	0
2008	25.78	141.62	0.00	8,960	5,517	0
2009	25.36	153.64	0.00	8,759	5,925	0
2010	26.02	146.20	0.00	9,518	6,207	0
2011	23.98	137.33	0.00	10,489	7,016	0
2012	22.78	129.07	0.00	10,357	7,174	0
2013	22.09	119.31	0.00	11,095	7,380	0
2014	20.67	112.34	0.00	11,089	7,173	0
2015	20.82	112.48	0.00	10,904	6,445	0
2016	18.93	123.01	0.00	12,271	8,113	0
2017	18.29	125.86	0.00	13,633	9,376	0
2018	18.89	116.92	0.00	19,416	11,193	0
2019	18.71	105.53	68.98	19,310	10,267	47,448
2020	18.84	97.77	0.00	19,431	11,595	49,263
2021	17.49	100.13	0.00	22,517	14,583	60,664
2022	16.10	76.99	0.00	20,691	11,704	69,722
2023	15.28	76.99	37.33	18,139	9,267	70,210

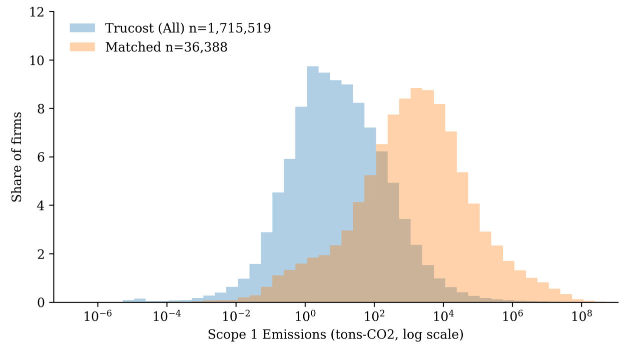
Notes: This table reports medians for emission intensity variables and sums for absolute emission variables from our matched database. Figures are aggregated by fiscal year over an annual sample starting in 2002 and ending in 2023. S1 refers to Scope-1 emissions. S3 Up (Down) refers to upstream (downstream) Scope-3 emissions. 'Emission Intensity' is measured as tons of emissions divided by sales in USD millions. 'Absolute Emissions' is measured in millions of tons.

and hence we have a more stable distribution over time.

International I-O data. In the next section, we present our network-based model to compute high-scope measures. This model can be calibrated only after knowing the shares of sales across all firms belonging to the network. Unfortunately, there is no dataset that provides these data at the firm-pair level. That is, we cannot find the full decomposition of sales (purchases) of a specific firm across customers (suppliers). For the sake of this project, the problem is even more pronounced relative to prior studies because of our international



(a) 2010



(b) 2022

FIG. 2: Emissions Distribution

Notes: This figure shows the distribution of emissions across firms in both the Trucost universe and our merged dataset in 2010 and 2022.

environment.

We address this issue by using the regular Inter-Country Input-Output (ICIO) tables produced by the OECD for their 2023 release (Yamano 2023; https://www.oecd.org/en/data/datasets/inter-country-input-output-tables.html). This matrix is quite granular as it provides data at the sector-country level (45 sectors for 76 individual countries) and hence features a total of 3420×3420 entries. In what follows, we map the ICIO sectors to FactSet sectors, reducing dimensionality to 15 sectors for 76 economies, i.e. 1140×1140 entries.

Shipping. In order to properly compute high-scope emissions, we must account for shipping-related emissions. First, we identify 3,804 main ports across the globe from the World

Port Index (WPI 2019, https://msi.nga.mil/Publications/WPI). We compute maritime routes between these ports using SeaRoute (https://github.com/eurostat/searoute), which determines the length of all possible routes connecting the ports through different straits and passages. From the OECD's maritime transport CO_2 emissions database (Clarke 2023), we obtain the average emission factors (kg CO_2 per nautical mile by vessel type). From the International Maritime Organization (International Maritime Organization 2024), we obtain the average cost intensity of a fleet, measured in USD per tonne-mile. This metric is defined as the annual total cost of a representative fleet, including annualized capital, operational, and fuel expenditures, as well as regulatory revenues and expenses imposed by the policy measures, divided by the total transport work (based on cargo carried). Combining these averages allows us to estimate shipping-related emissions normalized by monetary transport work (USD per nautical-mile). We extend this methodology to road-related shipments by calculating the geodesic distance (conservative approach), and applying terrestrial emission factors (kg CO_2 per-km) from the Intergovernmental Panel on Climate Change guidelines (The Intergovernmental Panel on Climate Change 2023, https://www.ipcc.ch/).

Thanks to our geolocalization effort, we know exactly the routes that can be used to ship goods (we exclude services from these computations) across firm-pairs. A firm-pair comprises a producer (supplier) and user firm (customer). In our benchmark analysis, we select the shortest route to have a conservative estimate. As a result, we account for emissions induced by shipping goods (i) from the supplier firm location to its nearest port; (ii) from the departing to the arrival port using the shortest maritime route; and (iii) from the arrival port to the location of the customer firm. For the final consumers, we use an 'average' destination, namely the center of the country in which they reside. We note that our code is flexible and it enables us to consider many alternative shipping routes. This flexibility allows us to reassess high-scope emissions in counterfactual scenarios.

Figure 3 illustrates an example of alternative shipping routes from Shanghai to Rotterdam, including multi-leg connections through Los Angeles and Cape Town. The map shows

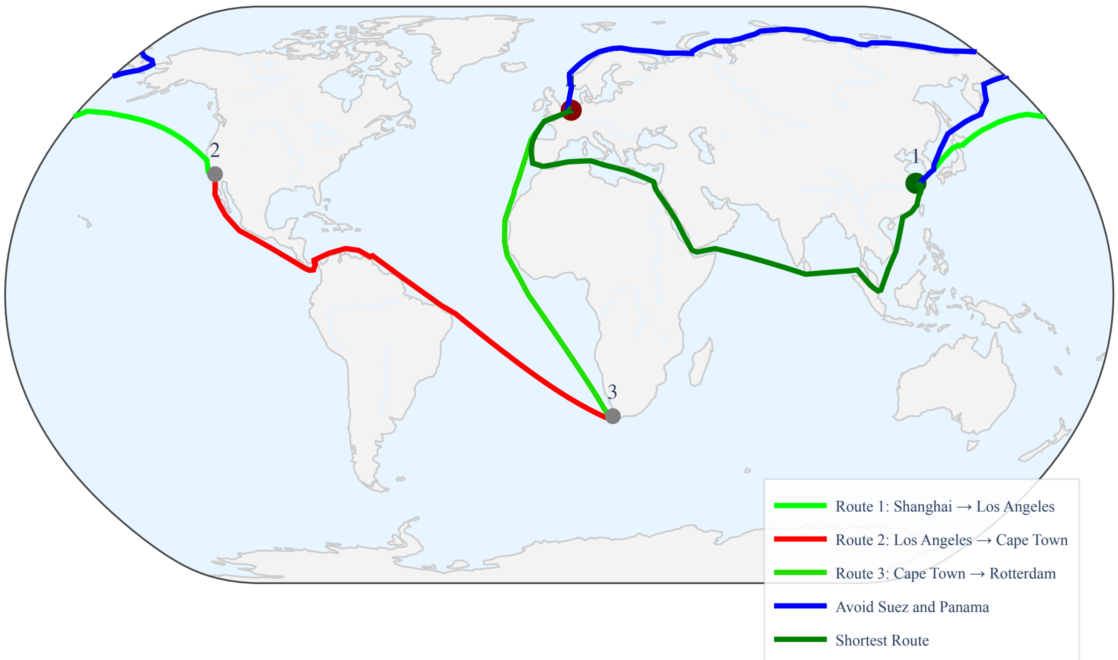


FIG. 3: Multi-leg maritime routes between Shanghai and Rotterdam

Notes: This figure shows the multi-leg maritime route from Shanghai to Rotterdam via Los Angeles and Cape Town. The map illustrates alternative paths between major ports: Route 1 (Shanghai–Los Angeles), Route 2 (Los Angeles–Cape Town), and Route 3 (Cape Town–Rotterdam). Alternative scenarios include the shortest possible route and a diversion avoiding both the Suez and Panama Canals. For a shipment of 50,000 tons of cargo, the benchmark route covers 39,800 km with estimated emissions of 15,920 tCO_2, while the shortest and canal-avoiding alternatives are associated with 6,154.4 tCO_2.

Route 1 (Shanghai–Los Angeles), Route 2 (Los Angeles–Cape Town), and Route 3 (Cape Town–Rotterdam), as well as two alternative scenarios: the shortest possible direct route and the option avoiding both the Suez and Panama Canals. Different route lengths translate into different emissions estimates, which are incorporated in our framework.

Financial constraints. In the data, we have information about sales, but emissions are related to quantities produced, that is, sales divided by prices. In the next section, we show a general equilibrium network model that is used to recover quantities from observed sales. In contrast to the standard Long and Plosser (1983) approach, we account for financial constraints because they can substantially alter prices across firms. To capture firm-level financial constraints in our sample, we calculate the Altman Z-Score (Altman 1983), a widely used bankruptcy prediction model based on accounting ratios. We employ different formulations for public and private firms to account for differences in data availability and capital structure. For public companies, we use the original five-factor model:

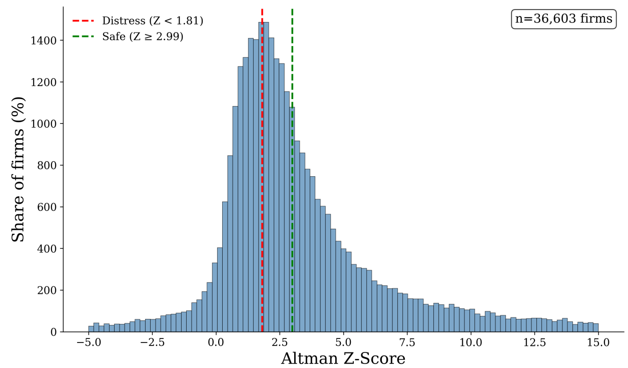
$$Z = 1.2\frac{WC}{TA} + 1.4\frac{RE}{TA} + 3.3\frac{EBIT}{TA} + 0.6\frac{MVE}{TL} + 1.0\frac{S}{TA}, \tag{1}$$

where WC denotes working capital, TA total assets, RE retained earnings, $EBIT$ earnings before interest and taxes, MVE market value of equity, TL total liabilities, and S sales. For public firms, the distress zone is defined as $Z < 1.81$, the gray zone as $1.81 \leq Z < 2.99$, and the safe zone as $Z \geq 2.99$. For private firms, where market value of equity is unavailable, we employ book value of equity (BVE):

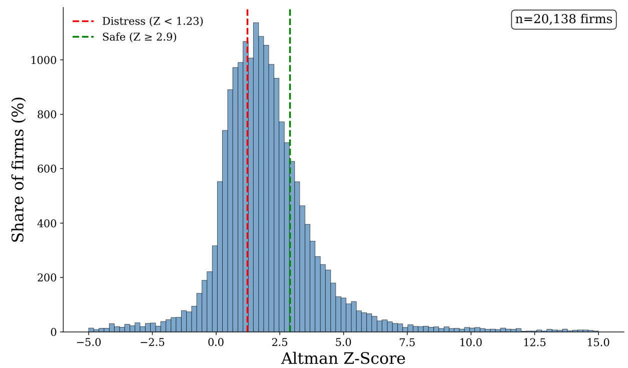
$$Z' = 0.717\frac{WC}{TA} + 0.847\frac{RE}{TA} + 3.107\frac{EBIT}{TA} + 0.420\frac{BVE}{TL} + 0.998\frac{S}{TA}, \tag{2}$$

with corresponding thresholds of $Z' < 1.23$ for distress, $1.23 \leq Z' < 2.90$ for the gray zone, and $Z' \geq 2.90$ for the safe zone. Lower Z-Scores indicate a higher likelihood of financial distress and, by extension, tighter financial constraints. Figure 4 presents the distribution of Z-Scores for both public and private firms in our sample, revealing substantial variation in financial health across entities. This measure allows us to systematically rank firms' financial health and incorporate financial frictions into our analysis of supply chain exposures.

In figure 5, we show the average distress level across country-sector combinations in our sample. The heatmap reveals heterogeneity in financial health across industries and geographies. Some sectors, such as industrial services, transportation and utilities, exhibit higher average distress levels. Gray cells indicate country-sector combinations with insufficient data. This geographic and sectoral variation in financial constraints motivates our granular approach to modeling firm-level production decisions and their implications for emissions allocation.



(a) Public



(b) Private

Fig. 4: Altman Z-Score Distribution

Notes: This figure shows the distribution of Altman Z-Scores for public and private firms. Vertical dashed lines indicate distress and safe zone thresholds for each firm type. Histograms exclude the top and bottom 3% of observations.

3 The model

In what follows, we introduce our general equilibrium model that enables us to conduct counterfactual analyses. In a second step, we introduce equations essential for emissions accounting.

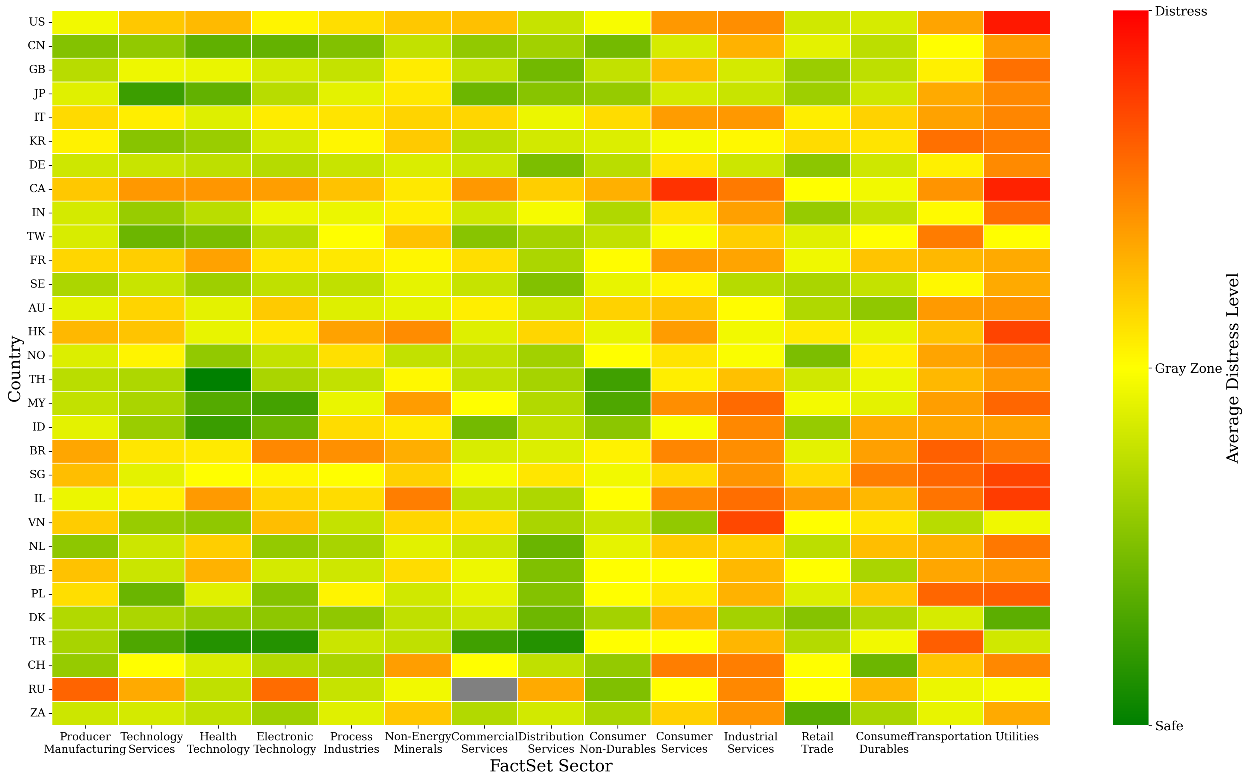


FIG. 5: Average Financial Distress for Selected Country-Sectors

Notes: This figure shows the average distress level calculated from the Altman Z-Score measure among firms in the 30 countries and 15 sectors with the highest observation density in our sample. The color scale ranges from green (safe zone, distress=0) through yellow (gray zone, distress=1) to red (distress zone, distress=2). Gray cells indicate country-sector combinations with no firms in our sample.

3.1 The Decentralized Economy

Consider a network of n companies defined through an input-output matrix, $\mathbf{A} = (\alpha_{m,u})_{u,m}$. Each firm has a Cobb-Douglas production function for good j,

$$y_j = f(k_j, \mathbf{x}_j) = z_j k_j^{\alpha_{0,j}} \prod_{m=1}^{n} x_{m,j}^{\alpha_{m,j}}, \tag{3}$$

where k_j is capital, traded in a common market, and $\mathbf{X} = (x_{m,u})_{m,u=1}^{n}$ comprises all inputs produced by company m (manufacturer) and purchased by company u (user). The household's consumption bundle is standard:

$$\mathrm{C}(\boldsymbol{\chi}) = \prod_{j=1}^{n} \chi_j^{\omega_j}.$$

In order to properly disentangle quantities from prices, we need a model that captures key frictions in the data. We consider a decentralized economy in which (i) emissions are not priced and are taken as given by the household, and (ii) firms access the capital market individually and are subject to individual borrowing constraints. Specifically, firm j solves the following problem:

$$\max_{y_j,\mathbf{x}_j,k_j} \pi_j\left(k_j,\mathbf{x}_j\right) := p_j y_j - \mathbf{p}^{\mathrm{T}}\mathbf{x}_j - r^D g\left(k_j,\mathbf{x}_j\right) - c_B\left(k_j,\mathbf{x}_j\right)$$

$$s.t. \quad g\left(k_j,\mathbf{x}_j\right) = k_j + \delta \underbrace{\mathbf{p}^{\mathrm{T}}\mathbf{x}_j,}_{\text{working capital}} \quad \delta \in [0,1]$$

$$c_B\left(k_j,\mathbf{x}_j\right) = \phi_1 \cdot \exp(\phi_2 \cdot (g_j - \overline{d}_j)), \quad \phi_1 \approx 0^+, \quad \phi_2 >> 0,$$

where g_j represents the total amount that firm j must borrow and it includes working capital if $\delta > 0$ (Bigio and La'O 2016); the convex function c_B convexifies/mimics the hard borrowing constraint $g_j < \overline{d}_j$; and $\overline{d}_j$ is a firm-specific borrowing limit that each firm takes as given. Using the convex distress cost c_B enables us to find the network equilibrium by 'staying in the interior', i.e., by solving a system of first-order conditions without needing to keep track of occasionally binding constraints across firms. As in the model with hard borrowing constraints, the effective cost of capital is firm-specific and depends on the tightness of the constraint:

$$R_j := r^D + \phi_2 c_B\left(k_j,\mathbf{x}_j\right).$$

In practice, when $g_j = \bar{d}_j$, the size of firm j cannot adjust upward, so this financial friction acts as a strong upward adjustment cost. The supply curve of good j becomes nearly rigid, and the entire network is affected as a result.

The household solves the following problem:

$$\max_{\boldsymbol{\chi}} \quad C(\boldsymbol{\chi}),$$
$$s.t. \quad \sum_{j=1}^{n} \left(\underbrace{r^D g_j\left(k_j^*, \mathbf{x}_j^*; \theta\right) + c_B\left(k_j^*, \mathbf{x}_j^*; \theta, \phi\right)}_{\text{tot. capital income}} + \pi_j\left(k_j^*, \mathbf{x}_j^*; \theta\right) \right) = \mathbf{p}^{\mathrm{T}} \boldsymbol{\chi},$$

and all markets clear:

$$y_j = \chi_j + \sum_{u=1}^{n} x_{j,u}^*, \quad j = 1, \ldots, n$$
$$K \geq \sum_j k_j.$$

At the equilibrium, we have the following conditions:

$$k_j = \alpha_{0,j} \frac{p_j y_j}{R_j} \tag{4}$$
$$x_{m,u} = \alpha_{m,u} \frac{p_u y_u}{p_m} \frac{1}{1 + \delta R_u} \tag{5}$$
$$\chi_j = \omega_j \frac{C(\boldsymbol{\chi})}{p_j}. \tag{6}$$

Equation (4) differs from what we would get in a standard Long and Plosser (1983) economy because of the firm-specific cost of capital, R_j. Equation (5) captures the role of working capital borrowing costs through the wedge $(1 + \delta R_u)^{-1}$, which disappears when $\delta = 0$. Equation (6) is standard. Importantly, the capital allocation is distorted when different firms face different effective borrowing costs, and this channel has a general equilibrium effect on quantities and prices.

3.2 Emission networks

Network emissions. Each company m is characterized by a company-specific emission intensity coefficient θ_m. The emissions associated with supplying one unit of good m to user u are $\theta_m \xi_{m,u}$. This term accounts for the fact that the same product can have different emissions when delivered across industries that differ, for example, in their location. When the same good is delivered to the final consumer, direct emissions are $\theta_m \xi_m^c$. Hence total network emissions are:

$$E^{tot} = \sum_{m=1}^{n} \theta_m \xi_m^c \boldsymbol{\chi}_m + \sum_{u=1}^{n} \sum_{m=1}^{n} \theta_m \xi_{m,u} x_{m,u} = \boldsymbol{\theta}^{\mathrm{T}} (\boldsymbol{\xi^c} \circ \boldsymbol{\chi}) + \boldsymbol{\theta}^{\mathrm{T}} (\mathbf{E} \circ \mathbf{X}) \mathbf{1}, \tag{7}$$

in which $\mathbf{E} = (\xi_{m,u})_{m,u=1}^{n}$ is a matrix that comprises all business-to-business (i.e., pair-specific) emissions, $\boldsymbol{\xi^c} = (\xi_m^c)_{m=1}^{n}$ is a vector comprising business-to-consumer (i.e., product-specific) emissions, and the vector $\boldsymbol{\theta} = (\theta_m)_{m=1}^{n}$ reports company-specific emission intensities. Note that this setting can account for offsets, since both $\theta_m \xi_{m,u}$ and $\theta_m \xi_m^c$ are allowed to be negative. Hence, this variable represents the actual *net* level of emissions, i.e., the net amount of molecules of CO_2 (or other greenhouse gas) emitted.

In what follows, we adopt the following normalizations:

$$\xi_m^c \;:=\; 1 + t_m^c, \qquad \xi_{m,u} := 1 + t_{m,u}, \tag{8}$$

that is, we disentangle the emissions from the direct transformation of the inputs from the additional emissions related to shipping, transportation, and potential sequestration. Importantly, in the rest of this study we include in t_m^c only transportation-related emissions. We abstract away from additional emissions stemming from the consumer's use of these goods since we do not have reliable estimates of such emissions. Trucost provides granular data about companies, not about households.

Emissions scopes. Scope-1 is defined as all direct emissions from owned/controlled sources. This definition excludes transportation and shipping. Thus in our model Scope-1 of firm j

is defined as follows:

$$E^1(j) := \theta_j \cdot \left(\underbrace{\chi_j}_{\text{b2c}} + \underbrace{\sum_{u=1}^{N} x_{j,u}}_{\text{b2b}} \right), \tag{9}$$

where 'b2c' ('b2b') refers to the quantities sold to the household (to other businesses).

Scope-3 refers to indirect emissions from the firm's supply chain and it is distinguished between 'upstream' and 'downstream'. Both definitions include several categories, but in our model we can keep track of only a subset of them. Upstream Scope-3 comprises both emissions related to inputs purchased from suppliers and emissions from transportation of inputs. Downstream Scope-3 refers to downstream transport and distribution, as well as processing of sold products. As a result, in our model we can define the following measures:

$$E^3_{up}(j) := \underbrace{\sum_{\substack{m=1,\\ m\neq j}}^{N} \theta_m x_{m,j}}_{\text{Purchased Goods}} + \underbrace{\sum_{\substack{m=1,\\ m\neq j}}^{N} \theta_m t_{m,j} x_{m,j}}_{\text{Inbound Transport}} = \sum_{\substack{m=1,\\ m\neq j}}^{N} \theta_m \xi_{m,j} x_{m,j}, \tag{10}$$

$$E^3_{down}(j) := \underbrace{\sum_{\substack{u=1,\\ u\neq j}}^{N} \theta_u y_u \times \frac{p_j x_{j,u}}{\sum_{m=1}^{N} x_{m,u} p_m}}_{\text{Sold goods}} + \underbrace{\sum_{\substack{u=1,\\ u\neq j}}^{N} \theta_j t_{j,u} x_{j,u}}_{\text{Outbound Transport}}, \tag{11}$$

and

$$E^{tot}_j := E^1(j) + E^3_{up}(j) + E^3_{down}(j). \tag{12}$$

Our definition of downstream Scope-3 deserves further discussion. First, we note that in the data we do not have a decomposition of the emissions produced by firm u while using the inputs acquired from firm j. Perhaps for this reason, downstream Scope-3 is not provided by Trucost. We bridge this gap by assigning to firm j a share of the direct emissions of each customer, i.e, firm $u = 1, \ldots$. The share of direct emissions of firm u assigned to firm j depends on how much firm u spends on the input purchased from firm j ($p_j x_{j,u}$) relative to its total expenditure ($\sum_{m=1}^{N} x_{m,u} p_m$).

Both Scope-3 downstream and Scope-3 upstream use information only from the first layer of suppliers or customers of firm j. Equivalently, these definitions do not account for the full set of emissions along the entire supply chain, and hence $E^{tot}(j)$ is potentially a misleading indicator. In order to overcome this limitation, in what follows we define *rescaled emissions* and use them to recursively compute *embodied emissions* at the firm level (Hertwich and Wood 2018). In contrast to scope-3 measures, embodied emissions account for all possible higher layers of network links.

Rescaled emissions and Scope-3. Let s_j represent the emissions associated with the production and deliver of one unit of good j. Equivalently, s_j must satisfy the following equation:

$$s_j y_j = \xi_j^{\chi} \theta_j \chi_j + \theta_j \sum_{u=1}^{n} \xi_{m,u} x_{m,u}.$$

If we sum across all goods, we get

$$\mathbf{s}^{\mathrm{T}} \mathbf{y} = \sum_{m=1}^{n} \xi_m^{\chi} \theta_m \chi_m + \sum_{m=1}^{n} \theta_m \sum_{u=1}^{n} \xi_{m,u} x_{m,u} = E^{tot}, \tag{13}$$

implying that rescaled emissions enable us to go back to the total emissions $\mathbf{s}^{\mathrm{T}}\mathbf{y}$ produced by the considered supply chain.

In the remainder of this section, we use rescaled emissions, s_j, to obtain embodied emissions, ψ_j. To better understand the meaning of rescaled emissions, it is convenient to define

$$\widehat{\mathbf{X}}^{\mathrm{T}} := \operatorname{diag}(\mathbf{y})^{-1} \mathbf{X}^{\mathrm{T}},$$

which is the matrix of input allocations per unit of output, i.e., whose components are $\hat{x}_{m,u} = \frac{x_{m,u}}{y_u}$.

When $\xi_{m,u} = 1$ for all m, u and $\xi_m^c = 1$ (no heterogeneous transport factors), the accounting identity $y_m = \chi_m + \sum_u x_{m,u}$ implies

$$s_m = \theta_m$$

for all m (with $y_m > 0$). That is, rescaled emissions are equal to the vector comprising emission intensity, θ_m, across firms.

If we additionally assume that there are no self-loops, $x_{j,j} = 0$, we obtain

$$E^3_{up} = \widehat{\mathbf{X}}^{\mathrm{T}}\mathbf{s}. \tag{14}$$

i.e., rescaled emissions and output shares are enough to determine the Scope-3 emissions across our n firms.

In the most general case, one can prove that

$$E^3_{up} = (\widehat{\mathbf{X}}^{\mathrm{T}} - D)\mathbf{s} + \mathbf{tc}, \tag{15}$$

where $D := \mathrm{diag}(\hat{x}_{1,1}, \ldots, \hat{x}_{n,n})$ collects the self-loop coefficients, and $\mathbf{tc}$ is a vector of transport-related correction terms defined as follows:

$$tc_j = \sum_{m \neq j} \theta_m (\xi_{m,j} - \bar{\xi}_m)\, \hat{x}_{m,j}, \tag{16}$$

where

$$\bar{\xi}_m \;:=\; \frac{\xi^c_m\, \chi_m + \sum_u \xi_{m,u}\, x_{m,u}}{y_m}. \tag{17}$$

In short, equation (15) (i) removes emissions comprised in Scope-1 from Scope-3 via the D matrix, and (ii) accounts for the dependence of Scope-3 on direct (i.e., layer-1) transport adjustment, whereas rescaled emissions use average transport coefficients, $\bar{\xi}_m$.

Embodied emissions. Embodied emissions account for emissions "embedded" within a unit of input, thus capturing its entire production process. Let us denote by ψ_j the embodied emissions share associated with good j, implying that $\psi_j y_j$ represents total embodied emissions of good j.

Embodied emissions are defined through the following recursive system of equations:

$$\psi_j y_j = s_j y_j + \sum_{m=1}^{n} \psi_m x_{m,j}, \tag{18}$$

which we can also write as,

$$\operatorname{diag}(\mathbf{y})\,\boldsymbol{\psi} = \operatorname{diag}(\mathbf{y})\,\mathbf{s} + \mathbf{X}^{\mathrm{T}}\boldsymbol{\psi}. \tag{19}$$

Given this definition, we get a closed-form solution for embodied emissions in terms of rescaled emissions $\mathbf{s}$.

$$\boldsymbol{\psi} = \left(\mathbf{I} - \widehat{\mathbf{X}}^{\mathrm{T}}\right)^{-1}\mathbf{s}. \tag{20}$$

Notice that this definition coincides with the authority-based Katz centrality (Sargent and Stachurski, 2022) of the weighted digraph defined by the input-output matrix, $\hat{\mathbf{X}}$. An equivalent interpretation is by means of "downstreamness", i.e., we look at a good j through the lens of all possible emissions that are involved in the production process through the intervention of multiple suppliers. Given these considerations, we are ready to state our main proposition.

Proposition 1. *If the resource constraint is satisfied, total emissions $E^{tot} = \mathbf{s} \circ \mathbf{y}$ equal the embodied emissions of final consumption goods plus an adjustment coming from the individual* ***net*** *flows of inputs' embodied emissions:*

$$\mathbf{s} \circ \mathbf{y} = \boldsymbol{\psi} \circ \boldsymbol{\chi} + \boldsymbol{\psi} \circ \left[\left(\boldsymbol{X} - \boldsymbol{X}^{T}\right)\mathbb{1}\right],$$

or, equivalently,

$$\mathbf{s}^T \boldsymbol{y} = \boldsymbol{\psi}^T \boldsymbol{\chi} + \boldsymbol{\psi}^T \left[\left(\boldsymbol{X} - \boldsymbol{X}^T \right) \mathbb{1} \right]. \tag{21}$$

In order to better understand these results, Figure 6 illustrates a simple supply chain example in which we can measure both direct and embodied emissions. Panel (a) shows the physical production network, where a Factory supplies inputs to two intermediate services: a Brown service that generates emissions and a Green service that does not. Each service then provides goods to a distributor, who delivers them to the final consumer. Panel (b) translates this network into the direct emissions vector and the corresponding input–output structure. Using these inputs, we compute embodied emissions, which account for all possible upstream links in the network and thus reallocate emissions downstream toward the final users. By construction, embodied emissions double count direct emissions, so their total always exceeds direct emissions, similar to what happens with Scope-3 measures. The key point, however, is that embodied emissions allow us to allocate the share of emissions attributable to each member of the supply chain.

From this example, the Scope-3 emissions associated with Distributor 1 equal $0.1E_F + E_B - E_{d_1} = 50$, while those for Distributor 2 equal $0.9E_F + E_G - E_{d_2} = 70$. Hence, although the green service emits nothing directly, the supply chain passing through it is effectively ranked as more polluting because it embodies a larger share of upstream emissions.

Embodied emissions, Scope-3, and spectral decomposition. Equation (20) can be rewritten as follows:

$$\boldsymbol{\psi} = \mathbf{s} + \widehat{\mathbf{X}}^{\mathrm{T}} \mathbf{s} + \widehat{\mathbf{X}}^{\mathrm{T}} \cdot \left(\widehat{\mathbf{X}}^{\mathrm{T}} \mathbf{s} + (\widehat{\mathbf{X}}^{\mathrm{T}})^2 \mathbf{s} + \dots \right).$$

In the special case in which we abstract away from transport costs and self purchases, the expression above collapses to

$$\boldsymbol{\psi} = \underbrace{\mathbf{s}}_{\text{Scope-1}} + \underbrace{E^3_{up}}_{\text{(Scope-3, up)}} + \underbrace{\widehat{\mathbf{X}}^{\mathrm{T}}(I-\widehat{\mathbf{X}}^{\mathrm{T}})^{-1}E^3_{up}}_{\text{All layers}},$$

implying that embodied emissions add to Scope-1 plus Scope-3 emissions all possible additional layers of upstream emissions. In the remainder of this manuscript, however, we will use the following representation, as it is general and accounts also for transport emissions and self-purchases:

$$\boldsymbol{\psi} = \underbrace{\mathbf{s}}_{\text{tier-0}} + \underbrace{\widehat{\mathbf{X}}^{\mathrm{T}}\mathbf{s}}_{\text{tier-1}} + \underbrace{(\widehat{\mathbf{X}}^{\mathrm{T}})^2(I-\widehat{\mathbf{X}}^{\mathrm{T}})^{-1}\mathbf{s}}_{\text{hidden gap (tiers 2+)}}.$$

Let us define $G := (\widehat{\mathbf{X}}^{\mathrm{T}})^2(I-\widehat{\mathbf{X}}^{\mathrm{T}})^{-1}$, and notice that

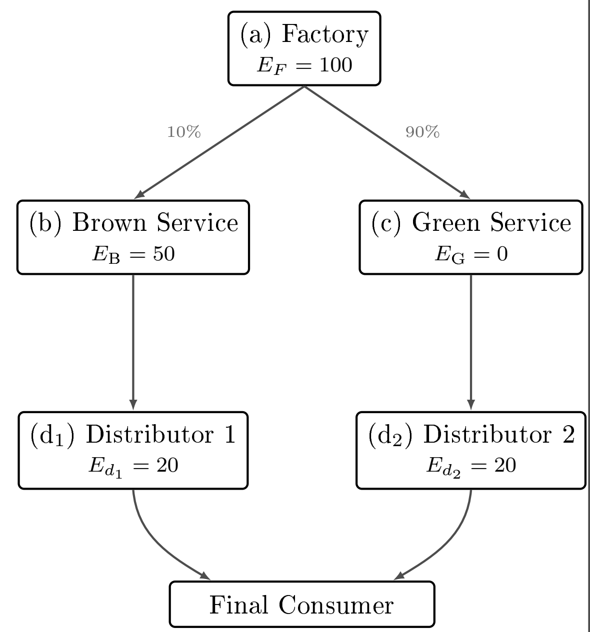
$$\lim_{K\to\infty} \underbrace{\sum_{k=2}^{K}\left(\widehat{\mathbf{X}}^{\mathrm{T}}\right)^k}_{G^K} = G.$$

We can decompose the 'hidden gap' between embodied emissions and tier 0-1 of emissions across firms and tiers. Specifically, the contribution of firm i to firm-j's embodied emissions, ϕ_j, along paths network paths of length K is simply $G^K_{ji}\mathbf{s}_i$. In what follows, we focus on the following two concepts:

$$\rho^i_j := \frac{G_{ji}\mathbf{s}_i}{\sum_{i=1}^n G_{ji}\mathbf{s}_i}, \text{ and } \rho^K_j := \frac{\sum_{i=1}^n G^K_{ji}\mathbf{s}_i}{\sum_{i=1}^n G_{ji}\mathbf{s}_i}. \tag{22}$$

The variable ρ^i_j determines the relative contribution of firm i in the determination of ψ_i when use all possible network paths ($K\to\infty$). ρ^K_j determines contribution of all firms toward ψ_j, when we stop the path length of K.

In Appendix C, we discuss the limitations of a spectral decomposition approach in our setting. While hub and authority scores tend to be very informative in well-integrated



(a) Supply chain network

Direct emissions vector:

$$\boldsymbol{S}^\top = \begin{bmatrix} 100 & 50 & 0 & 20 & 20 \end{bmatrix}$$

Input-Output matrix:

$$X^\top = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0.1 & 0 & 0 & 0 & 0 \\ 0.9 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Embodied emissions:

$$\boldsymbol{\psi} = (I_5 - X^\top)^{-1}\boldsymbol{S}$$
$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.1 & 1 & 0 & 0 & 0 \\ 0.9 & 0 & 1 & 0 & 0 \\ 0.1 & 1 & 0 & 1 & 0 \\ 0.9 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 100 \\ 50 \\ 0 \\ 20 \\ 20 \end{bmatrix} = \begin{bmatrix} 100 \\ 60 \\ 90 \\ 80 \\ 110 \end{bmatrix}$$

(b) Input-output emissions accounting

Fig. 6. Supply chain network emissions mapping to model framework. Panel (a) shows a physical network, where a factory provides inputs to two intermediate manufacturers. The Brown manufacturer creates emissions as a result of its activities, while the Green one does not. The final consumer can purchase the final good from two different distributors that show a similar level of emissions as result of their operation; Panel (b) maps the information on the supply chain network to the key measures in our model.

networks, they add little value in our case. International supply chains have several local clusters that manifest as a multitude of sizeable eigenvalues. Absent a dominant eigenvalue, identifying dominates hubs and authorities becomes challenging. For this reason, instead of reducing dimensionality after the formation of a network, we cluster firms at the port-sector level when creating the network, if necessary.

4 Network Characterization

We use our unique combination of firm-level environmental, supply chain, and financial data, country-sector macroeconomic data, and country port data to calibrate a multi-sector economic model. Our methodology follows a two-stage approach. First, a data pipeline processes and clusters firm-level data into a tractable macroeconomic structure. Second, a calibration process converts this structure into parameters for a competitive equilibrium model. In doing so, our approach addresses two critical challenges: (1) converting transport-based CO2 emissions into economically consistent parameters, and (2) implementing financial constraints to capture the heterogeneous borrowing capacity of firms across the supply chain.

4.1 Network, Transport and Firm Clustering

Network determination. The supply chain of a focal firm is built from FactSet relationship data. Level 1 consists of the firm's direct suppliers and customers, that is, the other firms with which Heidelberg reports a buyer-supplier or customer relationship. Level 2 comprises (i) the direct suppliers of level 1 suppliers, and (ii) the direct customers of level 1 customers. When running counterfactual exercises, we identify alternative firms by using both the "Known competitors" list provided by FactSet and additional AI tools to complement this list with other firms. We provide further details on this step in the next section.

Firm-level transport classification. When assessing transportation-related emissions, we consider only firms that actually require the transport of physical goods. Unfortunately, explicit data on transport logistics for each firm are missing. To overcome this fundamental limitation, we use a two-step text analysis process. First, we determine the company's core business. Second, we identify its most likely mode of transport, that is, either land, maritime, or air. More specifically, FactSet provides a narrative description of each company's business activity, its Standard Industrial Classification (SIC) code, and a list of customers. We employ an LLM that determines whether a company's core business requires the physical transport of goods at all. If so, a second prompt classifies the most likely transport mode based on the product type and the presence of international customers. When entity profiles are vague, we augment the information set by using SIC code descriptions. As a result, our two-step algorithm allows for a more robust and auditable classification of our entire firm universe. This is both a necessary foundation for our emissions calculations, and a major accomplishment in the literature.

Trade flow-based clustering. When a dimensionality reduction is required, we transform a vast network of individual firms into a set of representative clusters. We start by constructing a firm graph around a focal company's supply-chain. Each firm is tagged with its country, sector, and inferred transport mode. To each pair of firms in our network $\{i, j\}$, we assign a weight based on the corresponding dollar entry of the OECD ICIO table, i.e, an input-output matrix created at the country-sector level for a large cross section of economies. The weight computation is described in the next subsection, where we detail our calibration of the $\alpha_{i,j}$ coefficients. We apply the Louvain community detection algorithm (Blondel et al. 2008) to the weighted graph in order to obtain a manageable number of clusters. Each cluster comprises firms that are relatively more densely connected.[1] Finally, firm-level emissions and financial data are aggregated to the cluster level.

4.2 Calibration

Input-Output Matrix (A). The firm-level **A** matrix is constructed from the OECD ICIO tables. The OECD tables feature country-sector level data. Let Z_{ij} denote intermediate inputs from supplying country-sector i to using country–sector j. Letd x_j be the gross output of country-sector j. Thus, OECD country-sector input–output coefficients are $A_{ij}^{\text{OECD}} = Z_{ij}/x_j$.

Using Factset, we map each firm to a specific country-sector using. Unfortunately, this mapping is not trivial since the OECD tables use a slightly different sector nomenclature and, for some countries, a different country coding than Facset. We create FactSet-to-OECD

[1] Louvain is a fast community-detection heuristic that groups nodes into "communities" by maximizing *modularity*—a score that is high when edges are much denser within groups than across them, relative to a random graph with the same degree distribution. It greedily moves nodes to neighboring groups to raise modularity, then contracts each group to a super-node and repeats; the process quickly yields clusters that are tightly connected internally and sparsely connected externally.

sector mapping tables so that each firm inherits the correct row and column of $\mathbf{A}^{\text{OECD}}$. Each OECD country-sector combination is unique, but multiple firms in our dataset may share the same country-sector combination. If the network comprises multiple firms with the same country-sector combination, we allocate the input mass proportionally to their revenues. Stacking these allocations yields a square matrix $\mathbf{s}^{\text{prior}}$ of the same dimension as the desired $\mathbf{A}$, with nonnegative entries and unit column sums wherever the sector is represented in the OECD block.

In a specific supply chain, intermediate flows appear only where we observe supplier-customer links. We define $M_{ij} = 1$ if firm i is a recorded supplier of firm j in the relationship data or if $i = j$ (self-supply). In all other cases, when a link is missing, we set $M_{ij} = 0$. Then, we compute $\tilde{s}_{ij} = s_{ij}^{\text{prior}} M_{ij}$ and renormalise each column of $\tilde{\mathbf{s}}$ to sum to one. We then set $A_{ij} = (1 - \alpha_{0,j}^{\text{OECD}})\, \tilde{s}_{ij}$, where $\alpha_{0,j}^{\text{OECD}}$ is the OECD value-added share for j's country–sector. Column sums therefore reproduce the OECD intermediate intensity for each firm, restricted to the network.

For each firm in the network, we must make sure that intermediate sales are consistent with revenues (R_j), otherwise the resulting $\mathbf{A}$ matrix would be infeasible. To ensure a matrix that is consistent with revenues, we cap self-supply and introduce iterative proportional fitting (two-sided RAS). Specifically, for each column j, a fraction $\tau = 5\%$ of the column total is targeted on the main diagonal. This adjustment is required because self-supply is large at the sector-level, but not at the individual firm level. The remaining $(1 - \tau)$ mass is reallocated to the non-negative off-diagonal terms in proportion to their current values, holding the column sum fixed. Second, we form a matrix of intermediate purchase values $W_{ij} = A_{ij} R_j$, and apply a two-sided RAS on the support where W_{ij} may be positive. Equivalently, we seek nonnegative $\mathbf{W}$ with the same zero pattern such that (1) column sums match desired intermediate purchases, and (2) row sums match a proportional allocation of revenue across suppliers (Deming and Stephan, 1940; Bacharach, 1970). These results are obtained, by defining column targets as

$$c_j \;=\; \left(1 - \alpha_{0,j}^{\mathrm{OECD}}\right) R_j,$$

and row targets as $r_i = \lambda R_i$ with

$$\lambda \;=\; \frac{\sum_j c_j}{\sum_j R_j}\,,$$

so that total intermediate supply equals total intermediate demand.[2]

The coefficient matrix used in the model is then $A_{ij} = W_{ij}/R_j$, for $R_j > 0$. Row-column consistency implies that each firm's intermediate sales $\sum_j A_{ij} R_j$ are proportional to its revenue at the common factor λ, so that revenue net of intermediate sales is nonnegative and aligns with a single economy-wide intermediate-sales-to-revenue ratio. The resulting $\mathbf{A}$ matrix is a sparse firm-by-firm system of value shares, built from OECD ICIO tables and calibrated to match firm revenues and active network linkages.

Productivity (z). Firm-level productivity, z_j, is constructed from FactSet financials as sales-per-employee. For some private firms, number of employees is missing from the financial data. In this case, we use return on assets as an alternative measure of productivity. We note that we use return on assets only for 11.1% of our firms. Because these measures are not directly comparable, we normalize each productivity measure to range from 0 to 100. This scaling convention enables us to retain cross sectional heterogeneity without loss of generality. In our log-linear environment, we can rescale all productivity levels by a common factor in order to match the size of sales. We detail this step in what follows.

Revenue (R), preferences (ω), and the deposit rate (r^D). Revenues are obtained from Factset financials and included in the vector $\mathbf{R}$. According to the model, the implied

[2] Starting from $\mathbf{W}$ after diagonal redistribution, the algorithm alternates column scaling (multiplying column j by $c_j/\sum_i W_{ij}$ where the denominator is positive) and row scaling to match the r_i, reapplying the mask after each sweep; columns with only the diagonal admissible are fixed to meet c_j exactly. Under standard positivity conditions on the restricted support, the iteration converges to the unique biproportional adjustment of the initial $\mathbf{W}$ with those margins (Bacharach, 1970).

income vector can be computed using the $\mathbf{A}$ matrix: $(I - \mathbf{A})\,\mathbf{R}$. Since the consumption bundle weights (ω) represent consumption expenditure shares, we can recover them from the data using the following formula:

$$\omega_j = \frac{(I - \mathbf{A})\,\mathbf{R}_j}{\sum_j (I - \mathbf{A})\,\mathbf{R}_j}.$$

In this case, the composition of final expenditure in the model is aligned with the composition of value added in the data.

In the frictionless Cobb-Douglas benchmark ($\delta = 0$ and no financial constraints), there exists a positive scalar $\kappa > 0$ so that $(I - \mathbf{A})\mathbf{R} = \kappa\,\boldsymbol{\omega}$. As a result,

$$\mathbf{R} \;=\; \kappa\,(I - \mathbf{A})^{-1}\boldsymbol{\omega},$$

and the cross-sectional distribution of revenues is determined by $(\mathbf{A}, \boldsymbol{\omega})$ alone. A uniform rescaling of all productivities changes all prices proportionally and it alters κ, meaning that we can renormalize the productivity vector to match the total scale of revenues in the network.

After normalizing the price of the consumption bundle to one, we set the benchmark deposit rate, r^D, to a target value of 10%. In a frictionless setting, a target deposit rate r^D uniquely determines a common scaling factor applied to the measured productivity vector $\mathbf{z}^0$. In the presence of occasionally non binding constraints, r_j^D becomes endogenous and may differ from the target when some firms are financially constrained ($\zeta_j > 0$ in equation (4)).

Aggregate capital (K), and firm-level capital (k_j). We set the total network capital as follows: $K \approx \frac{\bar{\alpha}_0}{r^D} \sum_j R_j$. Thus K is tied to the data via $\mathbf{A}$ and the targeted r^D.

By contrast, how K is distributed across firms is an equilibrium outcome. Each firm's capital k_j satisfies equation (4). As a result, a firm with higher revenue $p_j y_j$ or larger capital share $\alpha_{0,j}$ demands more capital. A tighter borrowing limit raises its effective borrowing cost, R_j, and it reduces the capital it can employ. An unconstrained firm simply sets $k_j = \alpha_{0,j} p_j y_j / r^D$. In equilibrium, individual capital demands must exhaust the aggregate supply,

$$\sum_j k_j \;=\; \min\left\{ K,\; \sum_j \bar{d}_j \right\}.$$

Financial Constraints ($\mathbf{Z_c}$). Financial distress is tied to firm-level Altman Z-Scores constructed from FactSet balance-sheet and market data. Accounting ratios are winsorized in the tails, after which the appropriate public or private Altman mapping is applied and firms are classified into distress zones using the standard cutoffs. We harmonize firm-level Z-Scores across formulas and standardize within country-sector-year cells.

When working with clusters, each cluster's financial position is the asset-weighted aggregation of firm scores within the cluster:

$$Z_c = \frac{\sum_{f \in c} \text{Assets}_f \, Z_f}{\sum_{f \in c} \text{Assets}_f}.$$

The cluster score is then mapped to a borrowing-capacity multiplier $\lambda_c \in [\lambda_{\min}, \lambda_{\max}]$ using either a piecewise rule tied to the canonical Altman thresholds or a smooth logistic function. The borrowing limits are set as:

$$\bar{d}_c = \lambda_c K_c, \qquad K_c = \sum_{f \in c} K_f,$$

with the understanding that if clustering is not required, this equation is applied at the firm-level.

Transport emissions matrix (E). The **E** matrix augments emissions by accounting for the transport between origin–destination pairs required for the shipping of intermediate goods. For each firm in our dataset, we use Facset to find its address and extract latitude and longitude via Google maps. This step allows us to assign them to a primary seaport, and obtain inland distance from plant to port.

For distinct firms i and j, the route length d_{ij} (kilometres) is the sum of three legs: overland from i to its port, maritime distance between i's port and j's port, and overland from j's port to j. The middle leg is computed with SeaRoute (https://eurostat.github.io/searoute/), which takes port coordinates and returns the shortest feasible sea path length in kilometres, optionally restricting routes that pass through certain canals and straits. When transport is absent, like, for example, for service companies, we have $E_{ij} = 1$ for all $i \neq j$. Additionally, we assume that $E_{ii} = 1$ for each company.

According to equation (8), we must map distances into emission intensities. For each ordered pair with $i \neq j$ and $A_{ij} > 0$, we set

$$t_{ij} \;=\; \kappa\, d_{ij}\,, \tag{23}$$

where $\kappa = 2 \times 10^{-5}$ represents the well-to-wake CO_2 intensity of international maritime shipping (tonnes CO_2 per tonne–kilometre) calibrated from International Maritime Organization (2024). We combine physical emission factors with a single representative valuation of shipped goods per tonne–kilometre so that route length in kilometres maps into a dimensionless augmentation of input-side emissions. All other entries remain at one if $A_{ij} = 0$.

For illustration, suppose firms i and j trade ($A_{ij} > 0$) and their combined routing yields $d_{ij} = 10{,}000\,\text{km}$. Then $\kappa\, d_{ij} = 0.2$ and $E_{ij} = 1.2$: relative to a benchmark link with no associated distance, the emissions intensity attributed to inputs shipped from i to j is multiplied by 1.2 in the direct-emissions formula.

Summary. Summarizing, our calibration approach uses observables to fix the input-output matrix $\mathbf{A}$, the capital shares $\boldsymbol{\alpha}_0$), firm-level productivity $\mathbf{z}$, preference weights $\boldsymbol{\omega}$, aggregate capital K, borrowing ceilings $\bar{d}_j$, emission intensities $\boldsymbol{\theta}$, and the transport matrix $\mathbf{E}$. Given these parameters, the model solves for equilibrium prices $\mathbf{p}$, quantities $\mathbf{y}$, firm-level capital

$\mathbf{k}$, intermediate flows $\mathbf{X}$, final demand $\boldsymbol{\chi}$, and the cost of capital r_j^D.

5 Results

Heidelberg Materials. We illustrate our methodology focusing on a specific company. By doing so, we keep the dimension of the network limited and solve the model without clustering. The advantage of this strategy is twofold. First, we can clearly identify each node in our network as one of the companies in our sample. Second, we can directly compare our results and interpret our counterfactuals. As a case study, we choose Heidelberg Materials. This company is (i) a global leader in building materials (cement, aggregates, and ready-mixed concrete); (ii) the only construction company listed in the German benchmark index DAX; (iii) a regular reporter of emissions, and (iv) a well-known benchmark in the accounting literature (Reichelstein 2024). Heidelberg Materials employs roughly 60,000 people across approximately 3,000 locations in 60 countries, and is widely recognized for its efforts to decarbonize cement production and to advance carbon capture (CCUS) and circular-economy solutions.[3] In what follows, we treat all companies as financially unconstrained.

Figure 7 depicts the level 1 network with a total of 88 firms. We have 68 suppliers, 15 customers, and 5 firms that are both suppliers and customers. When focusing on level 2, we have a total of 534 firms, i.e., too many to visualize in a figure, but still few enough to solve our model without clustering.

In the top portion of Table 4, we compare Trucost emission figures with hand-collected Scope-1, Scope-2, and Scope-3 figures from Heidelberg Materials' annual sustainability reports. Scope-2 and Scope-3 are reported only since 2019. Scope-1 is very similar across the reports and the Trucost dataset. In contrast, Scope-3 exhibits larger percentage gaps and hence it is an interesting object to independently assess with our equilibrium-based approach. In Panel B, we report selected equilibrium objects from the baseline calibration obtained by considering both level 1 and level 2 connections. The rightmost column reports the average

[3] We thank the employees at Entsorga for their insights (https://www.entsorga.it/).

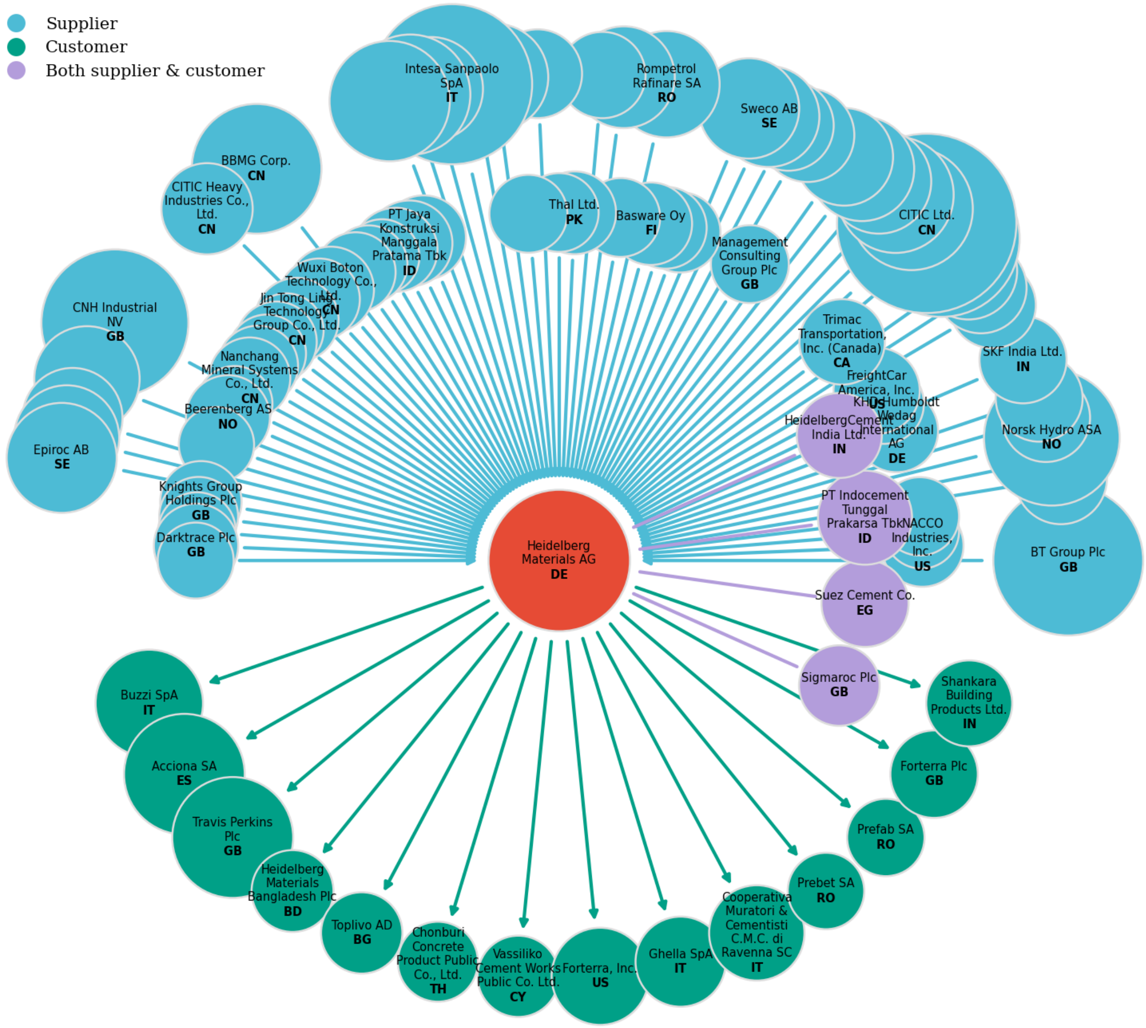


FIG. 7: Heidelberg Materials, Direct suppliers and customers by revenue

Notes: The figure shows Heidelberg Materials AG (center), its first-level suppliers (arrows into the center) and customers (arrows out). A number of entities appear as both supplier and customer. Node size reflects transformed revenue (transformed for visualization purposes), select nodes provide the name and country codes of the counterparties.

values across all firms. These quantities are used to recover our emission values.

To attribute emissions to the transport of goods along the supply chain, we compute shipping and road routes on the extended Heidelberg network using firm locations and port data, to obtain distances. The mapping comprises 44 countries. Figure 8 illustrates the resulting maritime and overland routes between Heidelberg Materials and its counterparties.

Table 5 reports Heidelberg Materials' emissions as well as a summary average across firms. In the appendix, we report also a detailed decomposition that separates purchased-

TABLE 4: Heidelberg Materials

Panel A: Emission Scopes from the data									
	Scope 1			**Scope 2**			**Scope 3**		
Year	Trucost	Report	Δ%	Trucost	Report	Δ%	Trucost	Report	Δ%
2018	77.52	75.40	2.8%	6.12	—	—	14.88	—	—
2019	73.43	72.60	1.1%	5.59	5.40	3.5%	20.14	22.70	−11.3%
2020	68.71	67.90	1.2%	4.96	4.90	1.2%	18.10	21.30	−15.0%
2021	69.38	69.00	0.5%	5.24	4.80	9.1%	21.86	23.90	−8.5%
2022	66.22	65.40	1.3%	4.82	4.50	7.1%	17.87	20.70	−13.7%
2023	62.08	61.20	1.4%	4.75	4.40	8.0%	24.97	15.10	65.3%

Panel B: Endogenous Variables		
Metric	**Heidelberg**	**Average**
Revenue ($p_j y_j$)	21.10	44.89
Output (y_j)	4,592.25	15,745.95
Price (p_j)	0.004595	0.005241
Consumption (χ_j)	3,325.30	11,184.33
Capital (k_j)	157.91	325.57
Capital share ($\alpha_{0,j}$)	0.7484	0.7222
Preference weight (ω_j)	0.000879	0.001873
Value added ($(I - A) \cdot p_j y_j$)	0.02	0.03
Firms (n)	1	534

Notes: In panel A, all emissions are in million tonnes CO_2e (Mt CO_2e). 'Trucost' refers to Trucost estimates. 'Report' refers to hand-collected information from Heidelberg Materials' Annual and Sustainability Reports. Scope 2 and Scope 3 are only self-reported since 2019. Δ% is the percentage difference between Trucost and the Sustainability Report. Scope 1 covers direct emissions, Scope 2 covers indirect emissions from purchased energy, and Scope 3 covers other indirect emissions (both upstream and downstream). Panel B provides equilibrium values for both Heidelberg and the average value for all firms in the extended network. In this panel, y_j is total output; p_j refers to price per unit; $p_j y_j$ is gross revenue (billion USD); χ_j refers to final consumption (household demand for firm j's good); k_j is capital employed; $\alpha_{0,j} = 1 - \sum_m \alpha_{m,j}$ is the capital share (residual from Cobb–Douglas); $\omega_j = C_j / \sum_{j'} C_{j'}$ is weight in the consumption bundle; and $(I - A) \cdot \mathbf{p} \circ \mathbf{y}$ is the value added in billion of USD. Aggregate capital is set to match a borrowing rate of 10%.

goods emissions from transport-related components (see Table B.5). The model calibration is designed to replicate the Trucost Scope-1 measurement, so it should not be surprising that the model matches the figure from Trucost. Importantly, the model delivers a Scope-3 assessment not too far from the number reported by Heidelberg Materials. In the bottom part of the table, we also report summary statistics for the emission intensity coefficients that we estimate. These objects are important for our counterfactual analysis.

Firms in the Heidelberg network can be ranked by their Scope-1 emissions, Scope-3

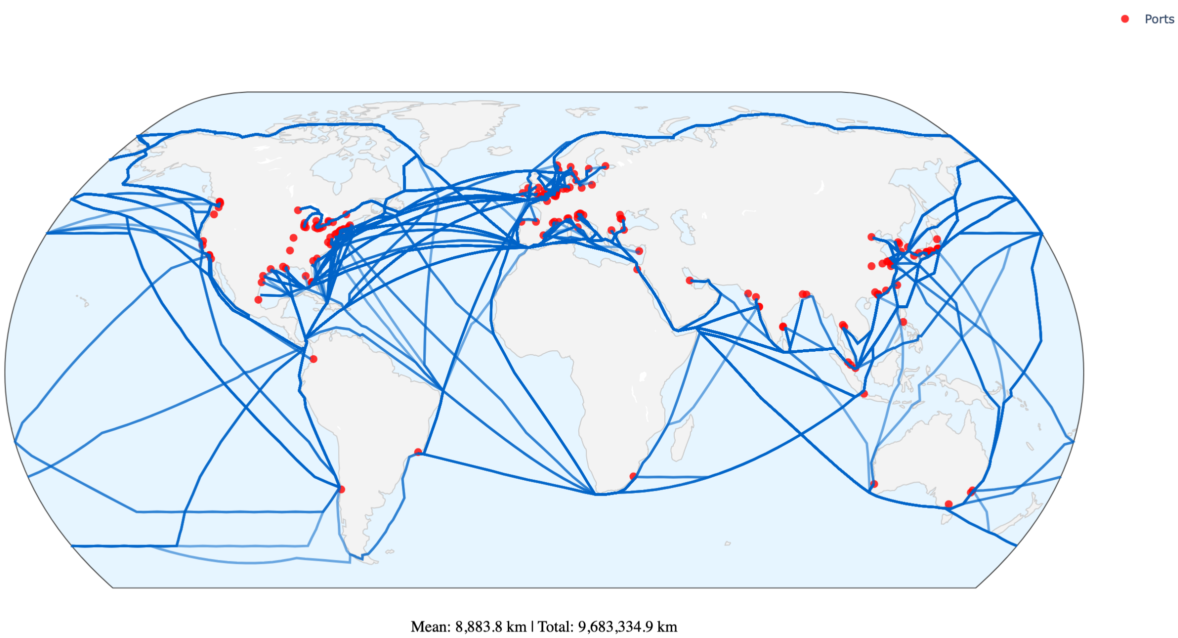


Fig. 8: Heidelberg Materials, Maritime routes on the extended network

Notes: The map shows the routes used to ship or transport goods between Heidelberg Materials (and its direct and second-level counterparties) and their trading partners. Firms are mapped to their nearest port, which is marked as a dot. Segments reflect the shortest maritime route between the ports, using Dijkstra's algorithm as provided by SeaRoute. This maritime network is based on the Oak Ridge National Labs CTA Transportation Network Group, Global Shipping Lane Network, World, 2000, enriched with lines around European coast based on AIS data.

emissions, the sum of both scopes, and their embodied emissions, respectively. Our model imputes emissions along the supply chain and thus yields a complete ranking across all scopes. In contrast, for this network, Trucost provides data on Scope-3 only for 49 firms. In addition, the data are partial, meaning that often they refer to either upstream or downstream Scope-3, not both. As a result, the relative classification of brown and green firms can change dramatically.

We visualize our results in Figure 9 across different emission scopes from the Trucost dataset and our model. We build these figures by focusing on the top-15 firms in the direct Heidelberg network. We note four important results. First, according to our model, Heidelberg Materials ranks in the top three in terms of Scope-3 emissions (panel a). In Trucost, this company scores sixth. Second, focusing on total emissions at the firm level (Scope-1 plus total Scope-3), we see an even more substantial disagreement between our model predictions and Trucost (panel b). Most importantly, ranking firms according to

Table 5: Heidelberg Materials, Emissions and economic indicators

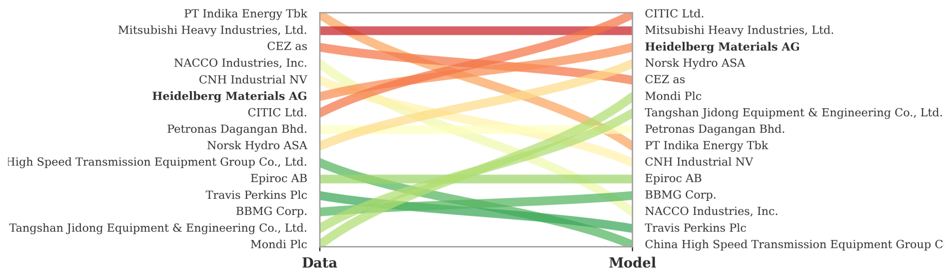
	Trucost	**Report**	**Model**
Scope 1 ($\theta_j y_j$)	69.04	61.20	69.04
Scope 3 Upstream (E^3_{up})	13.24	—	2.19
Scope 3 Downstream (E^3_{down})	5.62	—	7.86
Total Scope 3 ($E^3_{\text{up}} + E^3_{\text{down}}$)	18.87	15.10	10.05
Scope 1 + Total Scope 3	87.91	76.30	79.09
Emission intensity, Heidelberg (θ_j)	—	—	15,035
Emission intensity, network avg. ($\bar{\theta}$)	—	—	2,230
Network Emissions (E^{tot})	—	—	5,264.34

Notes: The table shows emissions for Heidelberg Materials from both Trucost and Heidelberg's sustainability reports. The last column shows the predictions of our model. Scope-3 emissions and firm-level total emissions are defined in equations (**??**)-(12). Supply-chain total emissions are computed as in equation (7) and aggregate emissions across all 534 firms in our network. All emission values are in million tonnes CO_2e (Mt CO_2e). Emission intensity coefficients, $\theta_j := \text{Scope}\,1_j/y_j$, are recovered from our model and are expressed in tonnes of CO_2e per unit of output.

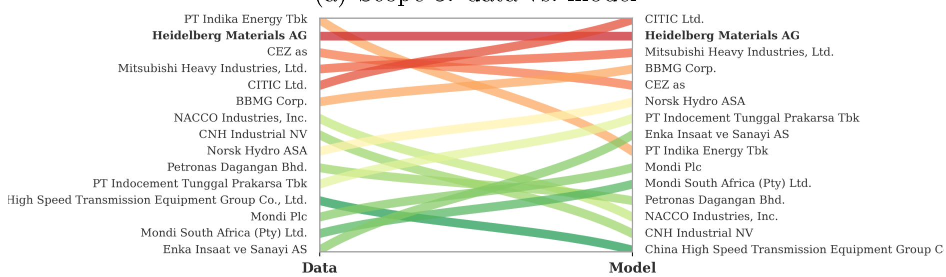
embodied emissions is significantly different from looking at total emissions (panel c and d). Once we account for all possible emission paths in the supply chain, Heidelberg Materials is ranked as a top emitter.

Table 6 reports rank-correlation and shift statistics for the direct network and for the top-15 emitters. In general, the data-based and model-based E_j^{tot} rankings are strongly aligned, with a Spearman $\rho = 0.81$, and a Kendall $\tau = 0.64$. These scores decline when we focus on the top-15 emitters, implying that our model results conform with the Trucost ranking mostly when focusing on mid- and small firms. Most importantly, ranking firms according to their embodied emissions produces a significantly different order relative to using total emissions. This result shows that accounting for the full path of emissions in the network matters when classifying brown and green companies.

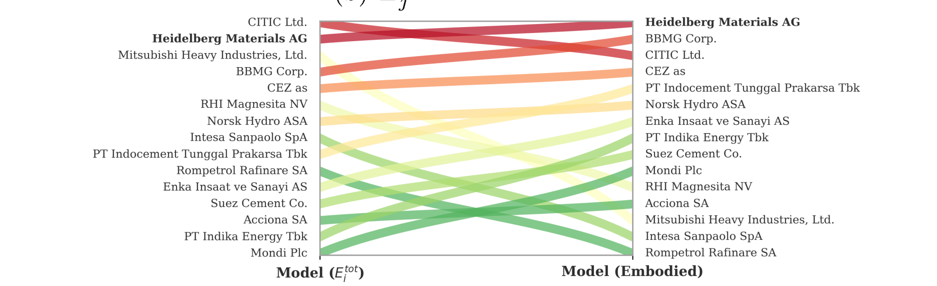
Decomposition. In Figure 10, we depict the coefficients defined in equation (22). We learn three things. First, most of the embodied emissions are captured by the first five tiers in our supply chain. Second, most of the tier 2+ emissions attributed to the focal firm Heidelberg Materials are associated to only 12 firms. The most relevant is located in India



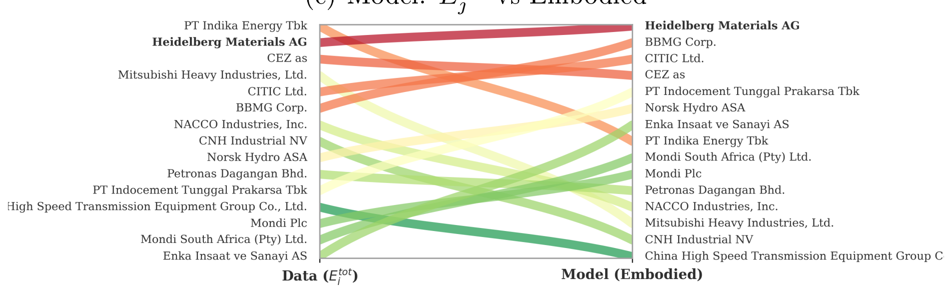
(a) Scope-3: data vs. model



(b) E_j^{tot}: data vs. model



(c) Model: E_j^{tot} vs Embodied



(d) E_j^{tot} data vs Embodied model

Fig. 9: Ranking comparisons for top-15 firms by emissions.

Notes: The figure shows the comparison of different rankings of emissions for the top-15 firms in the Hiedelberg Material's network. Scope-3 refers to the sum of upstream and downstream Scope 3 measures. $E^{tot}(j)$ is the sum of Scope 1 and Scope 3. Embodied emissions are computed as in equation (20). In each panel, the selected top-15 firms are reported at the left-hand side. We then show their alternative ranking on the right-hand side. As a result, the companies at the right-hand side of panel (c) and (d) are different.

and contributes to tier 2+ emissions by nearly 40%.

Table 6: Rank correlations and shift statistics

	Scope 3		E_j^{tot}		**Model**		E_j^{tot} **vs Emb.**	
	Data vs Model		Data vs Model		E_j^{tot} vs Emb.		Data	Model
Metric	L-1	T-15	L-1	T-15	L-1	T-15	L-1	T-15
Spearman ρ	0.543	0.279	0.758	0.525	0.851	0.464	0.869	0.346
Kendall τ	0.399	0.162	0.597	0.352	0.718	0.371	0.722	0.200
Mean shift	18.21	4.13	13.38	3.60	9.90	3.73	9.39	4.27
Max shift	73	9	67	8	67	10	52	9

Notes: The table shows rank correlations and shift statistics for (i) the full level-1 network of firms (L-1), and (ii) the top-15 emitters (T-15). The focal firm is Hiedelberg Materials. 'Scope 3 Data vs Model' refers to the comparison of the ranking obtained by using total Scope 3 data (upstream and downstream) from Trucost and the model. 'E_j^{tot} Data vs Model' compares the sum of Trucost Scope 1 and Scope 3 (upstream and downstream) to the same objected predicted by our model. 'Model E_j^{tot} vs Embodied' the ranking implied by model-based embodied emissions ($\psi_j \times y_j$) and total emissions. 'E_j^{tot} Data vs Embodied Model' compares total emissions from the data and embodied emissions from the model. Both Spearman ρ (correlation of ranks) and Kendall τ (net proportion of pairwise agreements) measure rank-order similarity, where 1 is perfect agreement, and 0 is no association. Mean and max shift report the average and maximum absolute rank difference between the two orderings across firms.

6 Counterfactuals

One key positive aspect of our approach is the ability to run counterfactual analysis, or, equivalently, simulate scenarios. In what follows, we focus on two examples. In the first case, we assume a major disruption to major maritime routes. We think of it as the possible outcome of heightened global geopolitical tension. In this case, we keep the original Heidelberg network unchanged and we revise our $\mathbf{E}$ matrix, i.e., the shipping-related emissions. In the second case, we assume that Heidelberg cannot operate anymore with companies from China, Hong Kong, and Taiwan. We think of this as the result of a major trade war. In this case, we use AI to find likely replacements for these firms in nearby countries. Thus, we change the structure of the network and recompute both equilibrium quantities and routes.

Maritime routes disruption. Assume that the Panama Canal, the Suez Canal, the Strait of Malacca, the Sunda Strait, and the Northwest Passage are blocked. As a result, ships are forced to go over longer alternative routes and thus produce higher transport-

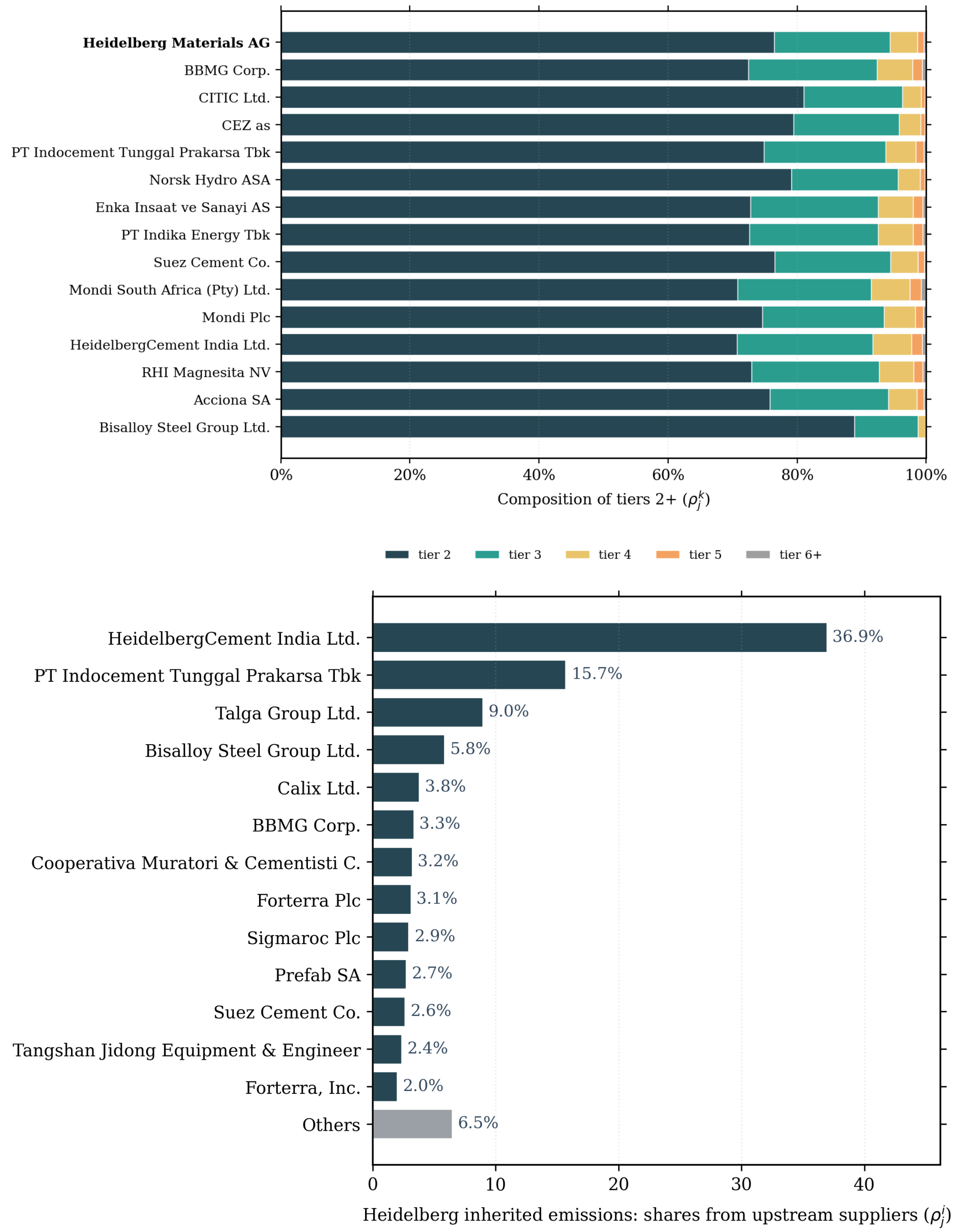


FIG. 10: Decomposing Tier 2+ Emissions

Notes: This figure shows the shares defined in equation (22). The top panel reports the shares of tier 2+ emissions obtained as we increase the length of the network paths. We compute the sum of Scope-1 and Scope-3 emissions and focus on the top-15 firms in the Hiedelberg Material's network. The bottom panel shows the contribution of different suppliers to the tier 2+ emissions of the focal firm, Hiedelberg Material.

related emissions per unit of trade. We assume that Heidelberg keeps the same network, but transport flows take different routes. Specifically, for every firm-pair link that involves sea transport, we recompute the shortest path subject to the above-mentioned route restrictions.

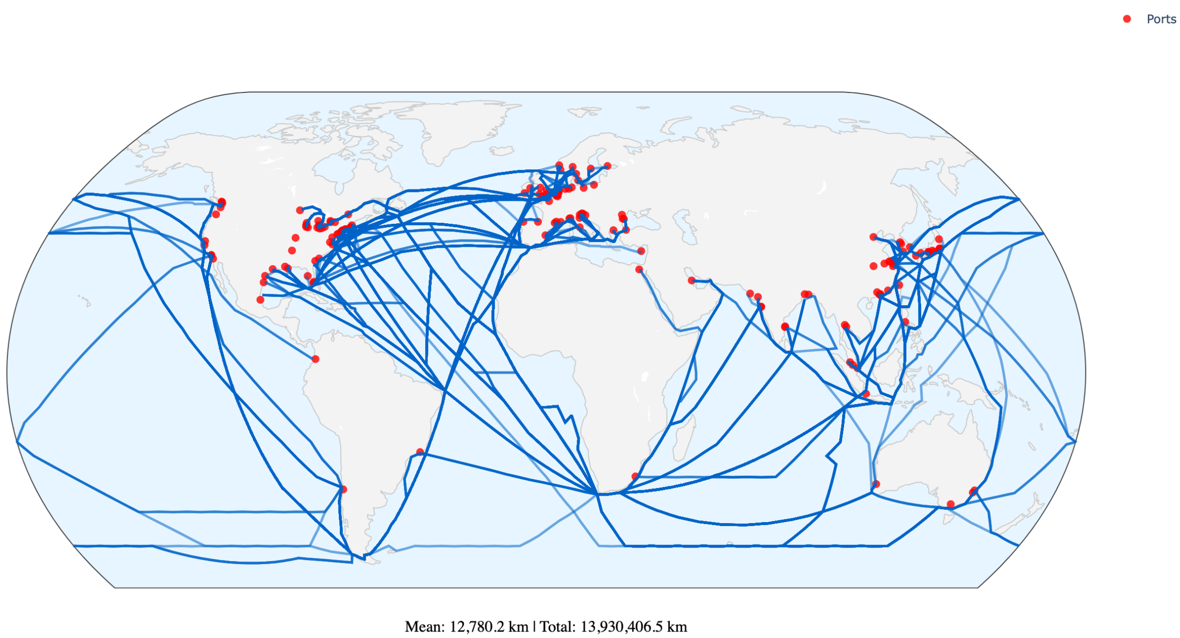


FIG. 11: Heidelberg Materials: Maritime routes disruption.

Notes: The map shows the maritime routes between firms in the Heidelberg supply chain (two-level network, 534 firms) when the Panama Canal, Suez Canal, Strait of Malacca, Sunda Strait, and Northwest Passage are restricted. Firms are mapped to their nearest port; segments are the shortest paths consistent with these restrictions (e.g. via the Cape of Good Hope or northern routes). These rerouted distances feed into the transport augmentation matrix E and thus into the counterfactual Scope-3 and embodied emissions. Route segments are the shortest maritime paths computed using SeaRoute.

Figure 11 shows the new maritime routes for this scenario. Compared to Figure 8, many links between Asia, Europe, and the Americas are forced onto longer itineraries (e.g. via the Cape of Good Hope or the North Atlantic). The mean transport distance between firms increases from 8,884 km to 12,780 km, a 43.9% increase. Hence the transport-augmentation matrix $\mathbf{E}$ plays a more relevant role in the determination of both Scope-3 and embodied emissions. We note that in this case, both Scope-1 and the non-transport components of Scope-3 are unchanged. This is because both the network trade summarized in X and the emission intensity coefficients contained in θ are unchanged.

We report detailed results on the different subcomponents of emissions in Table B.6 in the Appendix. In the main text, we focus on a summary table comprising both economic and emissions data. See Table 7. According to our computations, this disruption could increase the emissions in the Heidelberg network by 7% per year. This is a substantial increase, especially if we take seriously Heidelberg's commitment to reduce emissions over time. In the next experiment, we consider a change in both network structure and economic performance.

Replacements. Assume that firms domiciled in China, Hong Kong, and Taiwan are removed from Heidelberg's supply chain and replaced by comparable non-blocked-country alternatives. The exercise captures a scenario in which geopolitical or regulatory constraints forcefully sever links to suppliers and customers, and the focal firm reconstitutes its network with substitute counterparties. We follow the procedure described below.

Replacement candidates are drawn from the universe of firms in our merged dataset and are restricted to countries with OECD input-output tables and valid sector mappings. Hence, the new firms satisfy the same data requirements applied in the baseline calibration. We proceed in two stages. Breitung and Müller (2025) shows that a cosine similarity approach can successfully recover business networks (i.e., competitor, supplier, and customer links). In the first step, we compute cosine similarities between the business-description embedding of each blocked firm and those of all eligible candidates. The top 1% of candidates by cosine similarity (roughly 530 firms) form a pool of possible replacements.

In the second step, we enhance our algorithm by asking a large language model (ChatGPT-5) to evaluate each firm in this pool. The LLM determines whether it is a competitor of the blocked firm and reports both a binary answer, and a confidence score between 0 and 100. Firms classified as competitors are ranked by their confidence score, and the highest-confidence competitor is selected as the replacement. If no candidate receives a positive classification, the procedure falls back to the firm with the highest cosine similarity in the pool. This algorithm is applied to each supplier or customer to be removed from the network.

Further details on the replacement procedure are provided in Appendix C. When a FactSet-recorded competitor relationship exists and the competitor satisfies the eligibility criteria (present in the data universe, domiciled outside the blocked countries, and passing the data-completeness filters), it is chosen as the replacement.

In the Heidelberg network, 52 firms are domiciled in the blocked countries. Of these, 16 were replaced by a firm that was already in the network, 13 were replaced by a new firm, and 23 were replaced by firms with revenue near zero, making them infeasible replacements. After our replacements, the number of firms in the network declines by 39 units and becomes 495.

For a detailed analysis of the changes in the different scopes of emissions, see Table B.6. We compare emissions and economic outcomes for Heidelberg Materials across the baseline and counterfactual scenarios in Table 7 in the Appendix. For an example in which we reform the network by considering the replacement of a supplier in a single (multiple) supplier network, see Figure C.3 (C.4). According to our model, after such a shock, Heidelberg's total emissions would decline only by 1%. Recall, however, that total emissions are defined so that they comprise only one level of upstream and downstream emissions. If we look at the entire network, a reallocation of activity away from China could reduce network emissions by a staggering 27%. Unfortunately, only the network emissions-to-added value ratio improves only by about 19%, implying that the remaining gap is due to a reduction in network added value. Such a shock, could reduce the network added value by about 10%, i.e., there would be a significant contraction.

On the one hand, this scenario analysis rationalizes why this reallocation is not happening in real life: it would be very costly in terms of added value destruction. On the other hand, it is interesting to see that the impact on emissions abatement would be more substantial, as production would be redirected toward firms with better environmental performance.

7 Conclusion

We propose a novel network-based methodology to measure emissions along complex international supply chains (SC). Using international data for about 50,000 firms, we characterize SC-level emissions across countries and industries.

Table 7: Heidelberg Materials AG, Emissions and Economic Performance

	Baseline	**Blockage** (Δ%)	**Repl.** (Δ%)
Total Emissions, Heildelberg (E_j^{tot})	79.09	0	-1
Network Emissions (E^{tot})	5,264.34	7	-27
Network Revenues ($\sum_j R_j$)	23,972.90	0	-14
Network Added Value ($\sum_j p_j \chi_j$)	17,385.50	0	-10
Network Emissions-to-Added Value (%)	30.2	7	-19

Notes: The table provides emission and economic values under different counterfactual scenarios compared to the baseline. Added value and revenues are in Billions of USD. Emissions are measured in million of tonnes of CO_2e (Mt CO_2e). 'Blockage' assumes that major canals and straits (Panama, Suez, Malacca, Sunda, Northwest Passage) are simultaneously blocked. In this case, the network is unchanged, only transport emissions are recomputed. 'Repl.' refers to the case in which we assume a replacement of firms based in China, Taiwan, and Hong Kong.

Our novel methodology enables us to run counterfactual experiments and formulate granular forecasts about high-scope emissions with respect to many different scenarios such as maritime disruption, military conflicts, trade wars, and changes in carbon taxes. Importantly, our methodology relies only on Scope-1 data to recover high emission scopes and has a broad coverage. In addition, it enables us to compute embodied emissions to better rank 'green' and 'brown' firms within a supply chain.

Future research should use our measurement technology to address ambitious questions related to policy interventions and their unintended consequences.

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Chapter 3: Green DAOs for Brown Networks

Green DAOs for Brown Networks

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Abstract

We propose a novel model to think about networks with emission externalities. We consider a decentralized economy in which (i) emissions are not priced; and (ii) firms are subject to firm-specific borrowing constraints depending on their level of greenness. Greenness is measured by either looking at scope-1, scope-3 and embedded emissions. We show that this economy is highly inefficient. We then look at an economy in which: (i) a DAO borrows on behalf of the entire network allocates capital internally; (ii) all producers belonging to that DAO join a blockchain recording transaction of goods and emissions. We show that this setting replicates the first-best. Preliminary empirical results suggest that the welfare gains produced by a Green-DAO could be significant.

JEL classification: E1; E2; G1; O33; O41.
Keywords: GreenTech, Blockchain, Emissions, Networks
This draft: October 27, 2025.

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1 Model

1.1 The Network

This economy consists of a network of N firms producing a single distinct good each of them, and whose production relations are captured by the input-output matrix (*weighted adjacency matrix*) $\mathbf{A} = (\alpha_{i,j})_{i,j=1}^{n}$, where these numbers are the elasticities of the constant-returns-to-scale Cobb Douglass production function

$$f(k_j, \mathbf{x}_j) = z k_j^{\alpha_{0,j}} \prod_{i=1}^{n} x_{i,j}^{\alpha_{i,j}} \tag{1}$$

where $\boldsymbol{\alpha}_0 = \left(\mathbf{I} - \mathbf{A}^\top\right) \mathbf{1}$.

On the other hand, we take into acount the carbon footprint of each good. For each unit of consumption, each good $j = 1, \ldots, N$ has $\xi_j^c \theta_j$ units of GHG emissions, where $\xi_j^c \in \mathbb{R}$ captures the "sign" of the emitter to account for possible offsets, and $\theta_j \geq 0$ is a firm-specific parameter. For each unit of production of good j by means of the usage of inputs produced by firm i, there are $\xi_{i,j}^x \theta_j$ units of GHG emissions, where $\xi_{i,j}^x \in \mathbb{R}$ captures the "sign" of the emitter to account for possible offsets, and $\theta_j \geq 0$ is the same firm-specific parameter. These definitions allow us to define the *total emissions* as the total aggregate emissions of the economy across goods and links along the supply chain,

$$E\left(\boldsymbol{\chi}, \mathbf{X}\right) = \underbrace{\sum_{j=1}^{N} \theta_j \xi_j^{\chi} \chi_j}_{\text{consumption}} + \underbrace{\sum_{j=1}^{N} \theta_j \sum_{i=1}^{N} \xi_{i,j}^x x_{i,j}}_{\text{production}} \tag{2}$$

where $\boldsymbol{\chi} = (\chi_j)_{j=1}^{N}$ is the vector of consumption of each good, and $\mathbf{X} = (x_{i,j})_{i,j=1}^{N}$ is the matrix of inputs used in the production of each good. We further define the matrix $\mathbf{E} = \left(\xi_{i,j}^x\right)_{i,j=1}^{N}$ as the matrix of emissions of each good along the supply chain and $\boldsymbol{\xi}^{\chi} = \left(\xi_j^{\chi}\right)_{j=1}^{N}$ as the vector of emissions of each good in consumption.

1.2 Emissions

We defined total emissions in (**??**). This number is supposed to represent the actual *net* level of emissions, understood as the *net* amount of mollecules of CO_2 (or other greenhouse gas) emitted. Let's now introduce a new notation and assume that s_j represents the share of direct emissions associated to the production of one unit of good j, which is known. Equivalently, s_j must satisfy the following equation

$$s_m y_m = \xi_m^{\chi} \theta_m \chi_m + \theta_m \sum_{u=1}^{n} \xi_{m,u} x_{m,u}. \tag{3}$$

If we sum across all goods, we get

$$\mathbf{s}^{\top} \mathbf{y} = \sum_{m=1}^{n} \xi_m^{\chi} \theta_m \chi_m + \sum_{m=1}^{n} \theta_m \sum_{u=1}^{n} \xi_{m,u} x_{m,u} = E\left(\mathbf{X}, \boldsymbol{\chi}\right). \tag{4}$$

Throughout this paper, we will use the LHS notation to refer to direct emissions to make clear the role of the direct emissions share s_j. However, the *direct emissions* are not the same than the *embodied emissions*[1]. Essentially, it's the sum of all emissions "embedded" or "embodied" within that unit of input, capturing its entire production lifecycle. Let's denote by ψ_j the "embodied emissions" share associated to good j. Hence, $\psi_j y_j$ will be the embodied emissions of good j. Notice that

$$\psi_j = s_j + \sigma_j, \tag{5}$$

where σ_j can be interpreted as the "indirect emissions", or just the emissions that do not correspond to the manufacturing process.[2] Hence, it must be true that

$$\psi_j y_j = s_j y_j + \sum_{m=1}^{n} \psi_m x_{m,j} \tag{6}$$

for each good, $j = 1, \ldots, n$,where the first term, $s_j y_j$, refers to the direct emissions associated to the production of good $j = 1, \ldots, n$, and the second term. Hence,

$$\operatorname{diag}\left(\mathbf{y}\right) \boldsymbol{\psi} = \operatorname{diag}\left(\mathbf{y}\right) \mathbf{s} + \mathbf{X}^{\top} \boldsymbol{\psi}, \tag{7}$$

[1]For example, if good j is steel, the embodied emissions would include all greenhouse gas emissions from mining the iron ore, transporting it, smelting it into steel, and any other processes up to the point where the steel is ready to be used in another product

[2]Convention: Greek letters indicate embodied or indirect emissions, i.e., emissions that are not directly measured, estimated or (self) reported by firms. Latin letters indicate emissions that are taken as given because they have been estimated or measured.

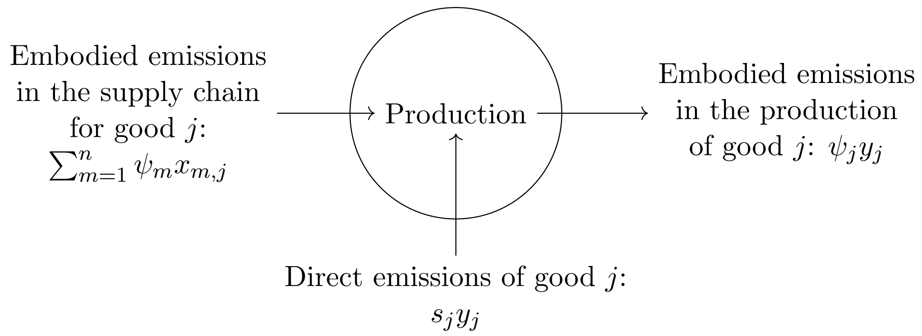

Figure 1: Graphical representation of the embodied and direct emissions of good j. The total embodied emissions of each good j consists of two parts: direct emissions of good j's production and the embodied emissions in the supply chain.

If we define

$$\widehat{\mathbf{X}}^{\top} = \operatorname{diag}(\mathbf{y})^{-1} \mathbf{X}^{\top}, \tag{8}$$

which is the matrix of input allocations per unit of output, i.e., whose components are $\hat{x}_{m,u} = \frac{x_{m,u}}{y_u}$, and left-divide by the inverse of diag $(\mathbf{y})$, then, together with (**??**), we get

$$\boldsymbol{\psi} = \left(\mathbf{I} - \widehat{\mathbf{X}}^{\top}\right)^{-1} \mathbf{s}. \tag{9}$$

Notice that this definition coincides with the authority-based Katz centrality (**?**) of the weighted digraph defined by the input-output matrix, $\hat{\mathbf{X}}$. An equivalent interpretation could be by means of "downstreamness", i.e., we look at a good j through the lens of all possible emissions that are involved in the production process through the intervention of multiple suppliers.

Proposition 1 *If the resource constraint is satisfied, the individual direct emissions (scope 1) equals the embodied direct emissions of consumption plus and adjustment coming from the individual net flows of inputs' embodied emissions, i.e.,*

$$\mathbf{s} \circ \mathbf{y} = \boldsymbol{\psi} \circ \boldsymbol{\chi} + \boldsymbol{\psi} \circ \left[\left(\mathbf{X} - \mathbf{X}^{\top}\right) \mathbb{1}\right]. \tag{10}$$

In this way, the immediate implication of this proposition is that

$$\mathbf{s}^{\top}\mathbf{y} = \boldsymbol{\psi}^{\top}\boldsymbol{\chi} + \boldsymbol{\psi}^{\top} \left[\left(\mathbf{X} - \mathbf{X}^{\top}\right) \mathbb{1}\right]. \tag{11}$$

On the other hand, let's denote by $\mathbf{s}^{SC}$ the *direct* emissions strictly due to supply chain linkages,

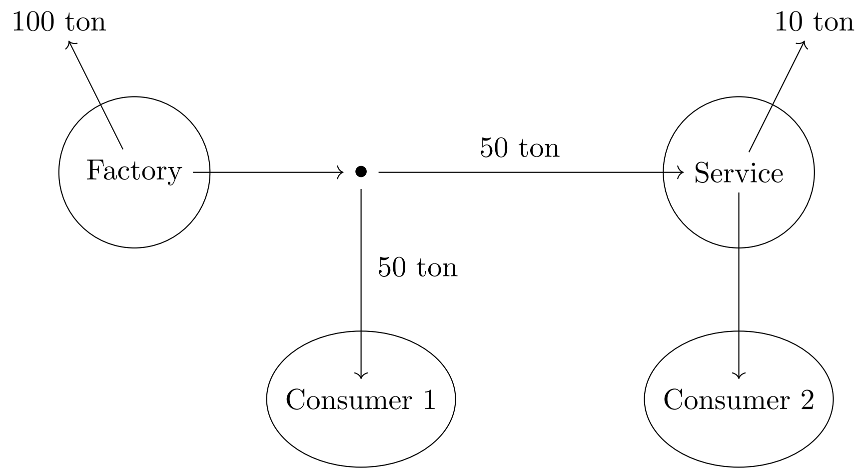

Figure 2: In this figure, there are two firms: Factory and Service; and two consumers: Consumer 1 and Consumer 2. Factory and Service emit 100 and 10 tons of CO_2 (direct emissions). Factory gives half of production to Consumer 1 and to Service. Service provides Consumer 2 with its full output. In this setting, the total embodied emissions are 100 + 60 = 160. However, the total direct emissions are just 100 + 10 = 110.

i.e.,

$$s_j^{SC} y_j = \underbrace{\sum_{u=1}^{n} \theta_j \xi_{j,u} x_{j,u}}_{\text{downstream}} + \underbrace{\sum_{m=1,\ m\neq j}^{n} \theta_m \xi_{m,j} x_{m,j}}_{\text{upstream}} . \tag{12}$$

where the first term accounts for downstream emissions of good j and the second term accounts for upstream emissions of good j. Notice that

$$s_j^{SC} y_j = s_j y_j - \xi_c \theta_j \chi_j + \sum_{m=1,\ m\neq j}^{n} \theta_m \xi_{m,j} x_{m,j}. \tag{13}$$

Hence,

$$s_j^{SC} = s_j + \frac{\sum_{m=1,\ m\neq j}^{n} \theta_m \xi_{m,j} x_{m,j} - \xi_c \theta_j \chi_j}{y_j}, \tag{14}$$

i.e., you add your upstream emissions and substract the emissions related to consumption to your direct emissions. Now imagine that $s_j > s_k$ for any two distinct j, k. The question is: could it be the case that $s_j^{SC} < s_k^{SC}$? For this to happen, we need

$$s_j + \frac{\sum_{m=1,\ m\neq j}^{n} \theta_m \xi_{m,j} x_{m,j} - \xi_c \theta_j \chi_j}{y_j} < s_k + \frac{\sum_{m=1,\ m\neq k}^{n} \theta_m \xi_{m,k} x_{m,k} - \xi_c \theta_k \chi_k}{y_k}, \tag{15}$$

i.e.,

$$s_j - s_k < \frac{\sum_{m=1,\ m\neq k}^{n} \theta_m \xi_{m,k} x_{m,k} - \xi_c \theta_k \chi_k}{y_k} - \frac{\sum_{m=1,\ m\neq j}^{n} \theta_m \xi_{m,j} x_{m,j} - \xi_c \theta_j \chi_j}{y_j}, \tag{16}$$

This implies that goods that switching in the raking is more likely for those firms that are more upstream, i.e., with lower consumption. In the case in which $n = 2$, and, without loss of generality, $j = 1$, we have

$$s_1 - s_2 < \frac{\theta_1 \xi_{1,2} x_{1,2} - \xi_c \theta_2 \chi_2}{y_2} - \frac{\theta_2 \xi_{2,1} x_{2,1} - \xi_c \theta_1 \chi_1}{y_1} \tag{17}$$

For $n = 3$ firms, the switching condition is

$$s_1 - s_2 < \frac{\theta_1 \xi_{1,2} x_{1,2} + \theta_3 \xi_{3,2} x_{3,2} - \xi_c \theta_2 \chi_2}{y_2} - \frac{\theta_2 \xi_{2,1} x_{2,1} + \theta_3 \xi_{3,1} x_{3,1} - \xi_c \theta_1 \chi_1}{y_1}. \tag{18}$$

We wanna make the same question for embodied emissions, i.e., provided that $s_j > s_k$, is it possible to get a switch in embodied emissions ranking, i.e., $\psi_j < \psi_k$. In order to asnwer this question, let's first take a look at the $n = 2$ case and then try to inductively generalise the result. We know that

$$\boldsymbol{\psi} = \left(\mathbf{I} - \hat{\mathbf{X}}^\top\right)^{-1} \mathbf{s}. \tag{19}$$

In the $n = 2$, this is equivalent to

$$\begin{aligned}
\begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix} &= \frac{1}{\det\left(\mathbf{I} - \hat{\mathbf{X}}^\top\right)} \begin{bmatrix} 1 - \hat{x}_{2,2} & \hat{x}_{1,2} \\ \hat{x}_{2,1} & 1 - \hat{x}_{1,1} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} \\
&= \frac{1}{\det\left(\mathbf{I} - \hat{\mathbf{X}}^\top\right)} \begin{bmatrix} \left(1 - \hat{x}_{2,2}\right) s_1 + \hat{x}_{1,2} s_2 \\ \hat{x}_{2,1} s_1 + \left(1 - \hat{x}_{1,1}\right) s_2 \end{bmatrix}
\end{aligned} \tag{20}$$

Notice that

$$\begin{aligned}
\left(1 - \hat{x}_{2,2}\right) s_1 + \hat{x}_{1,2} s_2 &= \left(1 - \frac{\alpha_{2,2}}{\lambda_2 + \theta_2 \xi_{2,2} \mathbf{s}^\top \mathbf{y}} \lambda_2\right) s_1 + \frac{\alpha_{1,2}}{\lambda_2 + \theta_1 \xi_{1,2} \mathbf{s}^\top \mathbf{y}} \lambda_2 s_2 \\
\hat{x}_{2,1} s_1 + \left(1 - \hat{x}_{1,1}\right) s_2 &= \frac{\alpha_{2,1}}{\lambda_2 + \theta_2 \xi_{2,1} \mathbf{s}^\top \mathbf{y}} \lambda_1 s_1 + \left(1 - \frac{\alpha_{1,1}}{\lambda_1 + \theta_1 \xi_{1,1} \mathbf{s}^\top \mathbf{y}} \lambda_1\right) s_2
\end{aligned} \tag{21}$$

Now, $\psi_2 > \psi_1$ if and only if

$$\begin{aligned}
\hat{x}_{2,1} s_1 + \left(1 - \hat{x}_{1,1}\right) s_2 &> \left(1 - \hat{x}_{2,2}\right) s_1 + \hat{x}_{1,2} s_2 \\
\left(1 - \left(\hat{x}_{1,1} + \hat{x}_{1,2}\right)\right) s_2 &> \left(1 - \left(\hat{x}_{2,2} + \hat{x}_{2,1}\right)\right) s_1 \\
\frac{1 - \left(\hat{x}_{1,1} + \hat{x}_{1,2}\right)}{1 - \left(\hat{x}_{2,2} + \hat{x}_{2,1}\right)} &> \frac{s_1}{s_2}
\end{aligned} \tag{22}$$

Recall that the resource constraint is $y_j = \chi_j + \sum_{u=1}^{n} x_{m,u}$. Take $j = 1$ as an example, when $n = 2$. Then, divide by y_1 in both sides and substract $\hat{x}_{1,2}$ in both sides. Hence,

$$1 - \hat{x}_{1,2} = \hat{\chi}_1 + \hat{x}_{1,1} - \hat{x}_{1,2} + \frac{x_{1,2}}{y_1} \tag{23}$$

Hence,

$$1 - \hat{x}_{1,2} - \hat{x}_{1,1} = \hat{\chi}_1 + \left(\frac{x_{1,2}}{y_1} - \frac{x_{1,2}}{y_2}\right) = \hat{\chi}_1 - \left(\frac{x_{1,2}}{y_2} - \frac{x_{1,2}}{y_1}\right). \tag{24}$$

Notice that $\hat{\chi}_1$ is the share of consumption relative to output, $\frac{x_{1,2}}{y_1}$ is the share of inputs given to the other firm relative to own output and $\frac{x_{1,2}}{y_2}$ is the share of inputs given to the other firm relative to the other firm's output. If the difference between the two last shares is positive, it means that firm 2 relies on firm 1's goods significantly, it reflects an economic scenario where firm 1 produces inputs at a relatively low cost in terms of its output share, but these inputs are highly valued by firm 2, consuming a larger proportion of its output. This discrepancy underscores firm 1's efficiency in input production and the critical role its goods play in firm 2's operations, potentially positioning firm 1 as a key supplier in the market with significant influence over firm 2's embodied emissions. Thanks to this, we can rewrite the swtiching condition as

$$\frac{\hat{\chi}_1 - \left(\frac{x_{1,2}}{y_2} - \frac{x_{1,2}}{y_1}\right)}{\hat{\chi}_2 - \left(\frac{x_{2,1}}{y_1} - \frac{x_{2,1}}{y_2}\right)} > \frac{s_1}{s_2} \tag{25}$$

1.3 The Planner's Economy

The social planner maximises welfare making sure that all resources are allocated efficiently, i.e.,

$$\max_{\mathbf{k},\mathbf{X},\boldsymbol{\chi}} \quad w\left(\boldsymbol{\chi},\mathbf{X}\right) \tag{26}$$

$$\text{s.t.} \qquad y_j = \sum_{u=1}^{n} x_{j,u} + \chi_j, \qquad j = 1,\dots,N \tag{27}$$

$$\textstyle\sum_{j=1}^{N} k_j = K. \tag{28}$$

where

$$w\left(\boldsymbol{\chi}\right) = C\left(\boldsymbol{\chi}\right) - c^E\left(\boldsymbol{\chi},\mathbf{X}\right) \tag{29}$$

is welfare,

$$C\left(\boldsymbol{\chi}\right) = \prod_{j=1}^{N} \chi_j^{\omega_j} \tag{30}$$

is consumption

$$c^E\left(\boldsymbol{\chi},\mathbf{X}\right) = \sum_{m=1}^{N} \theta_m \left(\xi_m^{\chi}\chi_m + \sum_{u=1}^{N} \xi_{m,u}^{x} x_{m,u}\right) \tag{31}$$

is the cost of emissions. Notice that (**??**) can be rewritten as

$$\mathbf{y} = \mathbf{X}\mathbf{1} + \boldsymbol{\chi}. \tag{32}$$

Proposition 2 *Given the optimisation problem stated in* (**??**), (**??**) *and* (**??**), *the first-order conditions are given by*

$$k_j = \alpha_{0,j}\frac{\lambda_j y_j}{\mu} \tag{33}$$

$$x_{m,u} = \alpha_{m,u}\frac{\lambda_u y_u}{\lambda_m + \theta_m \xi_{m,u} E\left(\boldsymbol{\chi},\mathbf{X}\right)} \tag{34}$$

$$\chi_j = \omega_j \frac{C(\boldsymbol{\chi})}{\xi_j^{\chi}\theta_j E\left(\boldsymbol{\chi},\mathbf{X}\right)} \tag{35}$$

and the (shadow) prices $(\lambda_j)_{j=1}^N$ *and* μ *are given by*

$$\log \lambda_j = -\log \varepsilon_j^* + \alpha_{0,j} \log \mu + \sum_{m=1}^{n} \alpha_{m,j} \log \left(\lambda_m + \theta_m \xi_{m,j} E\left(\boldsymbol{\chi}, \mathbf{X}\right)\right) \tag{36}$$

$$\mu = \frac{1}{K} \sum_{j=1}^{N} \alpha_{0,j} \lambda_j y_j \tag{37}$$

where $\log \varepsilon_j^* \coloneqq \log z - \mathcal{S}\left(\boldsymbol{\alpha}_j\right)$ *is capturing a pure network effect. Notice that*

$$\mathcal{S}\left(\boldsymbol{\alpha}_j\right) = -\sum_{m=1}^{n} \alpha_{m,j} \log \alpha_{m,j}$$

Furthermore, the resource constraint can be re-written as

$$\boldsymbol{\chi} = \left(\mathbf{I} - \mathbf{B}\right) \mathbf{y} \tag{38}$$

where $\mathbf{B} = (\beta_{i,j})_{i,j=1}^N$ *is the* $N \times N$ *matrix with*

$$\beta_{i,j} = \alpha_{i,j} \frac{\lambda_j}{\lambda_i + \theta_i \xi_{i,j} E\left(\boldsymbol{\chi}, \mathbf{X}\right)}. \tag{39}$$

1.4 The Decentralised Economy

1.4.1 Firms

Each firm $j = 1, \ldots, N$ solves the following problem

$$\max_{k_j, \mathbf{x}_j} \quad \pi\left(k_j, \mathbf{x}_j\right) \tag{40}$$

$$\text{s.t.} \quad g\left(k_j, \mathbf{x}_j\right) \leq \bar{d}_j \tag{41}$$

where

$$\pi\left(k_j, \mathbf{x}_j\right) = p_j y_j - c\left(k_j, \mathbf{x}_j\right) \tag{42}$$

is the profit of firm j, and

$$c^D\left(k_j, \mathbf{x}_j\right) = \mathbf{p}^\top \mathbf{x}_j + r_j^D g\left(k_j, \mathbf{x}_j\right) \tag{43}$$

is the cost of production, and

$$g\left(k_j, \mathbf{x}_j\right) = k_j + \delta\left(\mathbf{p}^\top \mathbf{x}_j\right) \tag{44}$$

is amount borrowed by the firm. This problem can be approximated by the unconstrained problem given by

$$\max_{k_j, \mathbf{x}_j} \quad \pi\left(k_j, \mathbf{x}_j\right) - c^{D,*}\left(k_j, \mathbf{x}_j\right), \tag{45}$$

where

$$c^{D,*}\left(k_j, \mathbf{x}_j\right) = \frac{1}{\phi_1} \exp\left(\phi_2\left(g\left(k_j, \mathbf{x}_j\right) - \bar{d}_j\right)\right), \tag{46}$$

is supposed to mimic the borrowing constraint for the appropriate choice of hyperparameters $\phi_1 > \phi_2 \gg 0$.

1.4.2 Households

The representative household maximises consumption subject to his budget constraint, i.e.,

$$\max_{\boldsymbol{\chi}} \quad \prod_{j=1}^{N} \chi_j^{\omega_j} \tag{47}$$

$$\text{s.t.} \quad \sum_{j=1}^{N}\left(\pi\left(k_j, \mathbf{x}_j\right) + r^D g\left(k_j, \mathbf{x}_j\right)\right) \leq \sum_{j=1}^{N} p_j \chi_j \tag{48}$$

1.4.3 Equilibrium

Proposition 3 *The equilibrium of thi economy is a set of allocations* $\mathbf{k}, \mathbf{X}, \boldsymbol{\chi}$, *and prices,* $\mathbf{p}, r^D$, *such that the first order conditions household*

$$k_j = \alpha_{0,j} \frac{p_j y_j}{r^D + \zeta_j} \tag{49}$$

$$x_{i,j} = \alpha_{i,j} \frac{1}{1 + \delta \left(r^D + \zeta_j\right)} \frac{p_j y_j}{p_i} \tag{50}$$

$$\chi_j = \omega_j \frac{C\left(\boldsymbol{\chi}\right)}{p_j \Lambda} \tag{51}$$

hold for all $i, j = 1, \ldots, N$, *and the prices* $(p_j)_{j=1}^N$ *and* r^D *are given by*

$$\log \mathbf{p} = -\left(\mathbf{I} - \mathbf{A}^\top\right)^{-1} \boldsymbol{\varepsilon} \tag{52}$$

$$\sum_{j=1}^{K} (r^D + \zeta_j) k_j = \sum_{j=1}^{N} \alpha_{0,j} p_j y_j \tag{53}$$

where

$$\log \varepsilon_j = \log \varepsilon_j^* - \alpha_{0,j} \log\left(r^D + \zeta_j\right) - \sum_{i=1}^{N} \alpha_{i,j} \log\left(1 + \delta\left(r^D + \zeta_j\right)\right) \tag{54}$$

and $\log \varepsilon_j^* = \log z - \mathcal{S}\left(\boldsymbol{\alpha}_j\right)$. *Further, the resource constraint can be re-written as*

$$\boldsymbol{\chi} = \left(\mathbf{I} - \mathbf{B}\right) \mathbf{y} \tag{55}$$

where $\mathbf{B} = (\beta_{i,j})_{i,j=1}^N$ *is the* $N \times N$ *matrix with*

$$\beta_{i,j} = \alpha_{i,j} \frac{1}{1 + \delta\left(r^D + \zeta_j\right)} \frac{p_j}{p_i}. \tag{56}$$

1.5 The Decentralised Economy is Always Inefficient

Proposition 4 *The competitive equilibrium in the decentralised economy, if it exists, is always inefficient with respect to the social planner's solution except for the special cases in which either there is no production, i.e.,* $\mathbf{y} = 0$*, or the economy is yet net zero,* $\theta = 0$*.*

Proof. *A necessary condition for the competitive equilibrium to mimic the planner's choice is that the first order conditions of the firms' and the bank's problems match their respective social planner's analogues, i.e.,* (**??**), (**??**) *and* (**??**) *must be equal to* (**??**), (**??**) *and* (**??**)*, respectively.*

For (**??**) *and* (**??**) *to be equal, is must be the case that*

$$\mu = r^D + \zeta_j \tag{57}$$

under the assumption that $\lambda_j = p_j$ *and output levels are the same in both economies.*

Finally, we have another condition that must be satisfied coming from equating (**??**) *and* (**??**)*, i.e.,*

$$\lambda_j + \theta_j \xi_j^{\boldsymbol{\chi}} \left(\mathbf{s}^\top \mathbf{y} \right) = p_j \tag{58}$$

Under the condition that $\lambda_j = p_j$*, this condition is equivalent to*

$$\theta_j \xi_{j,j}^{\mathbf{x}} \left(\mathbf{s}^\top \mathbf{y} \right) = 0, \tag{59}$$

i.e., only if $\theta_j = 0$ *for every* $j = 1, \ldots, n$ *or* $\xi_{j,j}^{\mathbf{x}} \left(\mathbf{s}^\top \mathbf{y} \right) = 0$ *for every* $j = 1, \ldots, n$ *(Net Zero Case), or only if* $\mathbf{s}^\top \mathbf{y} = 0$ *(No Production Case).* □

1.6 A Coin System solves the Inefficiency

1.6.1 Firms

Each firm maximises profits

$$\max_{k_j, \mathbf{x}_j} \pi\left(k_j, \mathbf{x}_j; r^D, \boldsymbol{\tau}^{\mathbf{x}}\right) \tag{60}$$

where

$$\pi\left(k_j, \mathbf{x}_j; r^D, \boldsymbol{\tau}^{\mathbf{x}}\right) = p_j y_j - c^{coins}\left(k_j, \mathbf{x}_j; r^D, \boldsymbol{\tau}^{\mathbf{x}}\right) \tag{61}$$

and

$$c^{coins}\left(k_j, \mathbf{x}_j; r^D, \boldsymbol{\tau}^{\mathbf{x}}\right) = \left(\mathbf{p} \circ \left(1 + \boldsymbol{\tau}_j^{\mathbf{x}}\right)\right)^\top \mathbf{x}_j + r^D k_j \tag{62}$$

and where $\boldsymbol{\tau}^{\mathbf{x}} = \left(\tau_{i,j}^{\mathbf{x}}\right)_{i,j=1}^{N}$ is a set of coints to be used in each transaction.

1.6.2 Household

The represenative household maximises consumption subject to a budget constraint

$$\max_{\boldsymbol{\chi}} \quad C\left(\boldsymbol{\chi}\right) \tag{63}$$

$$\text{s.t.} \quad r^D K = \left(\mathbf{p} \circ \left(1 + \boldsymbol{\tau}^{\boldsymbol{\chi}}\right)\right)^\top \boldsymbol{\chi} \tag{64}$$

and $\boldsymbol{\tau}^{\boldsymbol{\chi}} = \left(\tau_j^{\boldsymbol{\chi}}\right)_{j=1}^{N}$ is a set of coins to be used by the consumer.

1.7 Coins

We have two sets of coins, $\boldsymbol{\tau}^{\mathbf{x}}$ and $\boldsymbol{\tau}^{\boldsymbol{\chi}}$, that are to be automatically determined by means of a smart contract that computes the optimal allocations that the planner would set and re-scales the prices both of consumption and inputs accordingly in order to

1.7.1 Equilibrium

Proposition 5 *The equilibrium will be determined by the optimality conditions from the firms' and the bank's problems plus good and market clearing conditions. The first-order conditions* *are:*

$$k_j = \alpha_{0,j} \frac{p_j y_j}{r^D} \tag{65}$$

$$x_{m,j} = \alpha_{i,j} \frac{p_j y_j}{p_m \left(1 + \tau^{\mathbf{x}}_{m,j}\right)} \tag{66}$$

$$\chi_j = \omega_j \frac{C\left(\boldsymbol{\chi}\right)}{p_j \left(1 + \tau^{\boldsymbol{\chi}}_j\right) \Lambda} \tag{67}$$

for each $j, m = 1, \ldots, N$. *The capital market clearing condition that pins down the cost of capital must be satisfied and can be rewritten as:*

$$r^D K = \sum_{j=1}^{N} \alpha_{0,j} p_j y_j \tag{68}$$

The good market clearing condition must be satisfied and can be rewritten as:

$$\mathbf{y} = \left(\mathbf{I} - \mathbf{B}\right) \boldsymbol{\chi} \tag{69}$$

where

$$\beta_{m,j} = \alpha_{m,j} \frac{p_j}{p_m \left(1 + \tau^{\mathbf{x}}_{m,j}\right)} \tag{70}$$

Finally, coins are determined by

$$p_m \tau^{\mathbf{x}}_{m,j} = \theta_m \xi^{\mathbf{x}}_{m,j} \left(\mathbf{s}^\top \mathbf{y}\right) \tag{71}$$

$$p_m \tau^{\boldsymbol{\chi}}_j = \theta_j \xi^{\boldsymbol{\chi}}_j \left(\mathbf{s}^\top \mathbf{y}\right) \tag{72}$$

In particular, $\lambda_j = p_j$ *and* $\Lambda = 1$.

1.8 A Decentralised Economy with a Bank System

In this case, we are considering an economy in which there is not just one supply chain but M of them that are completely separated, i.e., no firm of one supply chain can produce for another one of another supply chain. That implies the following structure for the input output matrix

$$\mathbf{A} = \begin{pmatrix} \mathbf{A}^1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{A}^{SC} \end{pmatrix} \tag{73}$$

termi where $\mathbf{A}^{sc} = \left(\alpha_{i,j}^{sc}\right)_{i,j=1}^{F_{sc}}$ is the input-output matrix of supply chain sc. Hence

$$\boldsymbol{\alpha}_0^{sc} = \left(\mathbf{I} - (\mathbf{A}^{sc})^\top\right)\mathbf{1} \tag{74}$$

to ensure CRTS in each supply chain, i.e., for each supply chain, $sc = 1, \ldots, SC$,

$$y_f^{sc} = z^{sc} k_f^{\alpha_{0,f}^{sc}} \prod_{m=1}^{F_{sc}} \left(x_{m,f}^{sc}\right)^{\alpha_{m,f}^{sc}} \tag{75}$$

where k_f^{sc} is the capital stock of firm f in supply chain sc, and $x_{m,f}^{sc}$ is the input m used by firm f in supply chain sc.

1.8.1 Firms

There are N firms and M supply chains. Each supply chain, $m = 1, \ldots, M$, consists of $f = 1, \ldots, F^{sc}$ firms, i.e., $N = \sum_{m=1}^{sc} F^{sc}$, and each of them maximises its profits,

$$\max_{k_f^{sc}, \mathbf{x}_f^{sc}} \pi\left(k_f^{sc}, \mathbf{x}_f^{sc}; r^m, \boldsymbol{\tau}^{\mathbf{x},m}\right) := p_f^{sc} y_f^{sc} - c^{coins}\left(k_f^{sc}, \mathbf{x}_f^{sc}; r^m, \boldsymbol{\tau}^{\mathbf{x},m}\right) \tag{76}$$

where the cost function is supply-chain specific as it constains a supply-chain-specific cost of capital, r^m, and coins, $\boldsymbol{\tau}^{\mathbf{x},m}$, i.e.,

$$c^{coins}\left(k_f^{sc}, \mathbf{x}_f^{sc}; r^m, \boldsymbol{\tau}^{\mathbf{x},m}\right) = \left(\mathbf{p}^{sc} \circ (1 + \boldsymbol{\tau}^{\mathbf{x},m})\right)^\top \mathbf{x}_f^{sc} + r^{sc} g\left(k_f^{sc}, \mathbf{x}_f^{sc}; \boldsymbol{\tau}^{\mathbf{x},m}\right) \tag{77}$$

and

$$g\left(k_f^{sc}, \mathbf{x}_f^{sc}; \boldsymbol{\tau}^{\mathbf{x},m}\right) = k_f^{sc} + \delta\left(\left(\mathbf{p}^{sc} \circ \left(1 + \boldsymbol{\tau}^{\mathbf{x},m}\right)\right)^\top \mathbf{x}_f^{sc}\right) \tag{78}$$

with $\delta \in [0,1]$, and where $m = 1, \dots, M$ indicates the supply chain at which the firm, f, belongs to.

1.8.2 Household

The representative household maximises consumption

$$\begin{aligned} \max_{\mathbf{k},\mathbf{X},\boldsymbol{\chi}} \quad & \prod_{m=1}^{sc} \prod_{f=1}^{F^{sc}} \left(\chi_f^{sc}\right)^{\omega_f^{sc}} \\ \text{s.t.} \quad & \sum_{m=1}^{sc} \sum_{f=1}^{F^{sc}} \left(\pi\left(\mathbf{k}_f^{sc}, \mathbf{x}_f^{sc}; r^m, \boldsymbol{\tau}^{\mathbf{x},m}\right) + r^{sc} g\left(\mathbf{k}_f^{sc}, \mathbf{x}_f^{sc}; \boldsymbol{\tau}^{\mathbf{x},m}\right)\right) = \sum_{m=1}^{sc} \sum_{f=1}^{F^{sc}} \chi_f^{sc} p_f^{sc} \left(1 + \tau_f^{\boldsymbol{\chi},m}\right) \end{aligned} \tag{79}$$

where, $\sum_{m=1}^{M} \sum_{f=1}^{F^{sc}} \omega_f^m = 1$, and where $\mathbf{k} = \left(k_f^{sc}\right)_{f=1,m=1}^{F^{sc},M}$, $\boldsymbol{\chi} = \left(\chi_f^{sc}\right)_{f=1,m=1}^{F^{sc},M}$, and $\mathbf{X}$ is defined as

$$\mathbf{X} = \begin{pmatrix} \mathbf{X}^1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{X}^2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{X}^{sc} \end{pmatrix} \tag{80}$$

with $\mathbf{X}^m = \left(x_{i,j}^{sc}\right)_{i,j=1}^{F^{sc}}$ for each $m = 1, \dots, M$.

1.8.3 Bank

The bank decides the capital stock of each supply chain, i.e., $(K^{sc})_{m=1}^{sc}$ by means of a simple rule based on GHG emissions, i.e.,

$$\log \frac{K^{sc}}{K} = 1 - \gamma \, \frac{Emissions_m}{TotalEmissions} \tag{81}$$

where $\gamma \in [0,1]$, $Emissions_m$ may be direct or embodied emissions (computed as in the planner's problem), and K is the total capital stock.

1.9 Equilibrium

Proposition 6 *The equilibrium will be determined by the optimality conditions derived from the firms' and the bank's problems plus good and market clearing conditions.*

$$k_f^{sc} = \alpha_{0,f}^{sc} \frac{p_f^{sc} y_f^{sc}}{r^{sc}} \tag{82}$$

$$x_{i,f}^{sc} = \alpha_{m,f}^{sc} \frac{p_f^{sc} y_f^{sc}}{p_i^{sc}} \tag{83}$$

$$\chi_f^{sc} = \omega_f^{sc} \frac{C(\boldsymbol{\chi})}{\Lambda p_f^{sc}} \tag{84}$$

for all $m = 1, \ldots, M$ and $i, f = 1, \ldots, F^{sc}$. Prices $\left(\mathbf{p}_f^{sc}\right)_{f,m=1}^{F^{sc},M}$ and $(r^{sc})_{m=1}^{sc}$ are determined in equilibrium by

$$\log \mathbf{p}^{sc} = \left(\mathbf{I} - (\mathbf{A}^{sc})^\top\right)^{-1} \log \boldsymbol{\varepsilon}^{sc} \tag{85}$$

where $\boldsymbol{\varepsilon}^{sc} = \left(\varepsilon_{i,f}^{sc}\right)_{i,f=1}^{F^{sc},F^{sc}}$ is given by

$$\log \varepsilon_j = \log z - \mathcal{S}(\boldsymbol{\alpha}^{sc}) - \alpha_{0,j}^{sc} \log r^{sc} \tag{86}$$

, Further, the resource constraint must be satisfied, i.e.,

$$\boldsymbol{\chi}^{sc} = (\mathbf{I} - \mathbf{B}^{sc})\, \mathbf{y}^{sc} \tag{87}$$

where $\mathbf{B}^{sc} = \left(\beta_{i,j}^{sc}\right)_{i,j=1}^{F^{sc}}$ is given by

$$\beta_{i,j}^{sc} = \alpha_{i,j}^{sc} \frac{p_j^{sc}}{p_i^{sc}}; \tag{88}$$

and each supply chain's capital market must clear, i.e.,

$$\sum_{f=1}^{F^{sc}} k_f^{sc} = K^{sc}, \qquad \forall m = 1, \ldots, M \tag{89}$$

Finally,

$$p_m^{sc} \tau_{m,j}^{\mathbf{x},sc} = \theta_m^{sc} \xi_{m,j}^{\mathbf{x},sc} \left(\left(\mathbf{s}^{opt}\right)^\top \mathbf{y}^{opt}\right) \tag{90}$$

$$p_j^{sc} \tau_j^{\chi,sc} = \theta_j^{sc} \xi_j^{\chi} \left(\left(\mathbf{s}^{opt}\right)^\top \mathbf{y}^{opt}\right) \tag{91}$$

and $r^{sc} = \mu$ *for every* $sc = 1, \ldots, SC$*, and where "opt" denotes the optimal values of the planner's problem.*

1.10 Hybrid Network: DAO + non-DAO firms

1.10.1 Firms

Consider en economy of N firms as in Section **??**. $N^{DAO} \leq N$ of these firms belong to a DAO. The remaining $N - N^{DAO}$ firms are non-DAO firms. To ease notation, let's assume that the firms are indexed in such a way that the first N^{DAO} firms are the DAO firms, and the remaining $N - N^{DAO}$ firms are the non-DAO firms. Hence, each $j = 1, \dots, N^{DAO}$ corresponds to a DAO firm, and each $j = N^{DAO} + 1, \dots, N$ corresponds to a non-DAO firm. Each firm solves the following problem

$$\max_{(k_j, \mathbf{x}_j) \in B_j} \quad \pi\left(k_j, \mathbf{x}_j; r^D, \boldsymbol{\tau}^{\mathbf{x}}\right) \tag{92}$$

where $\pi\left(k_j, \mathbf{x}_j; r^D, \boldsymbol{\tau}^{\mathbf{x}}\right)$ is the profit of firm j, and it's defined by

$$\pi\left(k_j, \mathbf{x}_j; r^D, \boldsymbol{\tau}^{\mathbf{x}}\right) = p_j y_j - c\left(k_j, \mathbf{x}_j; r^D, \boldsymbol{\tau}^{\mathbf{x}}\right) \tag{93}$$

and

$$c\left(k_j, \mathbf{x}_j; r^D, \boldsymbol{\tau}^{\mathbf{x}}\right) = \left(\mathbf{p} \circ \left(1 + \boldsymbol{\tau}_j^{\mathbf{x}}\right)\right)^{\top} \mathbf{x}_j + r^D g(k_j, \mathbf{x}_j) \tag{94}$$

and

$$g(k_j, \mathbf{x}_j) = k_j + \delta\left(\mathbf{p}^{\top} \mathbf{x}_j\right) \tag{95}$$

is the amount borrowed by the firm. Here we are mixing DAO and non-DAO firms in the same maximisation problem for convenience. Conceptually, only the DAO firms use coins, and only the non-DAO firms are subject to a borrowing constraint. Therefore,

1. $\boldsymbol{\tau}_j^{\mathbf{x}} = 0 \in \mathbb{R}^N$ for all $j = 1, \dots, N^{DAO}$

2. For each non-DAO firm $j = N^{DAO} + 1, \dots, N$,

$$B_j = \left\{(k_j, \mathbf{x}_j) \in \mathbb{R}^N \times \mathbb{R}^N \ : \ g(k_j, \mathbf{x}_j) \leq \bar{d}_j\right\} \tag{96}$$

   whilst, for each DAO firm $j = 1, \dots, N^{DAO}$, $B_j = \mathbb{R}^N \times \mathbb{R}^N$.

1.10.2 Household

The representative household maximises consumption subject to his budget constraint, i.e.,

$$\max_{\boldsymbol{\chi}} \quad \prod_{j=1}^{N} \chi_j^{\omega_j} \tag{97}$$

$$\text{s.t.} \quad \sum_{j=1}^{N} \left(\pi\left(k_j, \mathbf{x}_j\right) + r^D g\left(k_j, \mathbf{x}_j\right)\right) \leq \sum_{j=1}^{N} \left(1 + \tau_j^{\boldsymbol{\chi}}\right) \chi_j \tag{98}$$

Notice that, mechanically, profits are zero for the DAO firms, and coins are zero for the non-DAO firms.

1.10.3 Equilibrium

Proposition 7 *The equilibrium of this economy is a set of allocations* $\mathbf{k}, \mathbf{X}, \boldsymbol{\chi}$*, and prices,* $\mathbf{p}, r^D$*, such that, for each* $j = 1, \ldots, N^{DAO}$*,*

$$k_j = \alpha_{0,j} \frac{p_j y_j}{r^D} \tag{99}$$

$$x_{m,u} = \alpha_{m,u} \frac{p_u y_u}{p_m \left(1 + \tau_{m,u}^{\mathbf{x}}\right)} \tag{100}$$

$$\chi_j = \omega_j \frac{C\left(\boldsymbol{\chi}\right)}{p_j \left(1 + \tau_j^{\boldsymbol{\chi}}\right) \Lambda} \tag{101}$$

and, for each non-DAO firm $j = N^{DAO} + 1, \ldots, N$*,*

$$k_j = \alpha_{0,j} \frac{p_j y_j}{r^D + \zeta_j} \tag{102}$$

$$x_{m,u} = \alpha_{m,u} \frac{1}{1 + \delta\left(r^D + \zeta_j\right)} \frac{p_u y_u}{p_m} \tag{103}$$

$$\chi_j = \omega_j \frac{C\left(\boldsymbol{\chi}\right)}{p_j \Lambda} \tag{104}$$

and, the equilibrium prices are given by

$$\log \mathbf{p} = \left(\mathbf{I} - \mathbf{A}^{\top}\right)^{-1} \log \varepsilon. \tag{105}$$

where ε_j is given by

$$\varepsilon_j = \begin{cases} \log \varepsilon^* - \alpha_{0,j} \log r^D - \sum_{m=1}^{N} \alpha_{m,j} \log\left(1 + \tau_{m,j}^{\mathbf{x}}\right) & \text{for } j = 1, \ldots, N^{DAO} \\ \log \varepsilon^* - \alpha_{0,j} \log\left(r^D + \zeta_j\right) - \sum_{i=1}^{n} \alpha_{m,j} \log\left(1 + \delta\left(r^D + \zeta_j\right)\right) & \text{for } j = N^{DAO} + 1, \ldots, N \end{cases} \tag{106}$$

with $\log \varepsilon^* = \log z + \mathcal{S}(\mathbf{A})$. FInally, the good market clearing condition is given by

$$(\mathbf{I} - \mathbf{B})\,\mathbf{y} = \boldsymbol{\chi} \tag{107}$$

where $\mathbf{B} = (b_{m,u})_{m,u=1}^{N}$ with

$$b_{m,u} = \begin{cases} \alpha_{m,u} \frac{p_u y_u}{\left(1+\tau_{m,u}^{\mathbf{x}}\right) p_m} & \text{for } m = 1, \ldots, N, \quad u = 1, \ldots, N^{DAO} \\ \alpha_{m,u} \frac{p_u y_u}{(1+\delta(r^D+\zeta_j)) p_m} & \text{for } m = 1, \ldots, N, \quad u = N^{DAO} + 1, \ldots, N \end{cases} \tag{108}$$

and the market clearing condition holds, i.e., $\sum_{j=1}^{N} k_j = K$.

Appendix

Proof of Proposition ??

If the good market clears, then

$$\mathbf{y} = \mathbf{X}\mathbb{1} + \boldsymbol{\chi}.$$

If we premultiply by $\boldsymbol{\psi}$ using the Hadamard product, then

$$\boldsymbol{\psi} \circ \mathbf{y} = \boldsymbol{\psi} \circ (\mathbf{X}\mathbb{1}) + \boldsymbol{\psi} \circ \boldsymbol{\chi}.$$

On the other hand, we know that

$$\boldsymbol{\psi} \circ \mathbf{y} = \mathbf{s} \circ \mathbf{y} + \boldsymbol{\psi} \circ \left(\mathbf{X}^\top \mathbb{1}\right).$$

Hence, equating both things,

$$\mathbf{s} \circ \mathbf{y} + \boldsymbol{\psi} \circ \left(\mathbf{X}^\top \mathbb{1}\right) = \boldsymbol{\psi} \circ (\mathbf{X}\mathbb{1}) + \boldsymbol{\psi} \circ \boldsymbol{\chi},$$

i.e.,

$$\mathbf{s} \circ \mathbf{y} = \boldsymbol{\psi} \circ \boldsymbol{\chi} + \boldsymbol{\psi} \circ \left[\left(\mathbf{X} - \mathbf{X}^\top\right) \mathbb{1}\right]$$

as desired. □

Proof of Proposition ??

The lagrangian of (??) subject to (??), (??) and (??) is

$$a\mathcal{L}\left(\mathbf{k}, \mathbf{X}, \boldsymbol{\chi}; \boldsymbol{\lambda}, \boldsymbol{\mu}\right) = w\left(\boldsymbol{\chi}\right) + \boldsymbol{\lambda}^T\left(\mathbf{y} - \mathbf{X}\mathbf{1} - \boldsymbol{\chi}\right) + \mu\left(K - \sum_{j=1}^{N} k_j\right). \tag{109}$$

It gives us the following first-order conditions:

$$\partial_{k_j}\mathcal{L} = \alpha_{0,j}\frac{\lambda_j y_j}{k_j} - \mu = 0, \quad j = 1, \ldots, N \tag{110}$$

$$\partial_{\chi_j}\mathcal{L} = \omega_j\frac{C(\boldsymbol{\chi})}{\chi_j} - \xi_j^c\theta_j E\left(\boldsymbol{\chi}, \mathbf{X}\right) \tag{111}$$

$$\partial_{x_{m,u}}\mathcal{L} = -\theta_m\xi_{m,u}E\left(\boldsymbol{\chi}, \mathbf{X}\right) - \lambda_m + \alpha_{m,u}\frac{\lambda_u y_u}{\lambda_m} \tag{112}$$

that pin down the following allocations. If we plug (**??**) and (**??**) into the Cobb-Douglass production function (**??**) we get

$$\begin{aligned}\log y_j &= \log z + \alpha_{0,j}\log k_j + \sum_{i=1}^{n}\alpha_{m,j}\log x_{m,j}\\ &= \log z + \alpha_{0,j}\log\alpha_{0,j} + \alpha_{0,j}\log\frac{\lambda_j y_j}{\mu}\\ &+ \sum_{i=1}^{n}\alpha_{m,j}\log\alpha_{m,j} + \sum_{i=1}^{n}\alpha_{m,j}\log\frac{\lambda_j y_j}{\lambda_i + \theta_i\xi_{m,j}E\left(\boldsymbol{\chi},\mathbf{X}\right)}\end{aligned} \tag{113}$$

which implies, if we define $\mathcal{S}\colon \mathbb{R}^N \to \mathbb{R}$ as $\mathcal{S}\left(\boldsymbol{\alpha}\right) = -\sum_{i=0}^{n}\alpha_i\log\alpha_i$

$$0 = \log z - \mathcal{S}\left(\boldsymbol{\alpha}_j\right) + \log\lambda_j - \alpha_{0,j}\log\mu - \sum_{i=1}^{n}\alpha_{m,j}\log\left(\lambda_i + \theta_i\xi_{m,j}E\left(\boldsymbol{\chi},\mathbf{X}\right)\right). \tag{114}$$

□

Proof of Proposition ??

The Lagrangian of the firm's problem is

$$\mathcal{L}\left(k_j,\mathbf{x}_j;\zeta_j\right) = \pi\left(k_j,\mathbf{x}_j\right) + \zeta_j\left(\bar{d}_j - g\left(k_j,\mathbf{x}_j\right)\right) \tag{115}$$

It gives us the following first-order-conditions:

$$\partial_{k_j}\mathcal{L} = \alpha_{0,j}\frac{p_j y_j}{k_j} - r^D - \zeta_j = 0 \tag{116}$$

$$\partial_{x_{m,j}}\mathcal{L} = \alpha_{m,j}\frac{p_j y_j}{x_{m,j}} - p_m - r^D\delta p_m - \zeta_j\delta p_m = 0 \tag{117}$$

which give

$$k_j = \alpha_{0,j}\frac{p_j y_j}{r^D + \zeta_j} \tag{118}$$

$$x_{m,j} = \alpha_{m,j}\frac{1}{1+\delta\left(r^D + \zeta_j\right)}\frac{p_j y_j}{p_m}. \tag{119}$$

The Lagrangian of the household's problem is

$$\mathcal{L}\left(\boldsymbol{\chi};\Lambda\right) = C(\boldsymbol{\chi}) - \Lambda\left(\sum_{j=1}^{N}p_j\chi_j - \sum_{j=1}^{N}\left(\pi\left(k_j,\mathbf{x}_j\right) + r^D g\left(k_j,\mathbf{x}_j\right)\right)\right). \tag{120}$$

However, notice that, for each $j = 1, \ldots, N$,

$$\begin{aligned}
\pi\left(k_j, \mathbf{x}_j\right) + r^D g\left(k_j, \mathbf{x}_j\right) &= p_j y_j - c\left(k_j, \mathbf{x}_j\right) + r^D g\left(k_j, \mathbf{x}_j\right) \\
&= p_j y_j - \mathbf{p}^\top \mathbf{x}_j - r^D g\left(k_j, \mathbf{x}_j\right) + r^D g\left(k_j, \mathbf{x}_j\right) \\
&= p_j y_j - \mathbf{p}^\top \mathbf{x}_j.
\end{aligned}$$

This allows us to rewrite the Lagrangian (**??**) as

$$\mathcal{L}\left(\boldsymbol{\chi}; \Lambda\right) = C(\boldsymbol{\chi}) - \Lambda\left(\sum_{j=1}^{N} p_j \chi_j - p_j y_j - \mathbf{p}^\top \mathbf{x}_j\right).$$

Hence, the first order conditions are

$$\partial_{\chi_j} \mathcal{L} = \omega_j \frac{C(\boldsymbol{\chi})}{\chi_j} - \Lambda p_j = 0. \tag{121}$$

and therefore

$$\chi_j = \omega_j \frac{C\left(\boldsymbol{\chi}\right)}{p_j \Lambda}. \tag{122}$$

On the other hand, we can plug the expressions we have for k_j and $x_{m,j}$ for $m = 1, \ldots, M$ into the (log) production function:

$$\begin{aligned}
\log y_j &= \log z + \alpha_{0,j} \log k_j + \sum_{m=1}^{N} \alpha_{m,j} \log x_{m,j} \\
&= \log z + \alpha_{0,j} \log \alpha_{0,j} \frac{p_j y_j}{r^D + \zeta_j} + \sum_{m=1}^{N} \alpha_{m,j} \log \alpha_{m,j} \frac{1}{1 + \delta\left(r^D + \zeta_j\right)} \frac{p_j y_j}{p_m} \\
0 &= \log z + \mathcal{S}\left(\boldsymbol{\alpha}_j\right) + \log p_j - \sum_{m=1}^{N} \alpha_{m,j} \log p_m \\
&- \alpha_{0,j} \log\left(r^D + \zeta_j\right) - \sum_{m=1}^{N} \alpha_{m,j} \log\left(1 + \delta\left(r^D + \zeta_j\right)\right) \\
&= -\log \varepsilon_j + \log p_j - \sum_{m=1}^{N} \alpha_{m,j} \log p_m
\end{aligned}$$

Hence,

$$\log p_j = \log \varepsilon_j + \sum_{m=1}^{N} \alpha_{m,j} \log p_m. \tag{123}$$

In matrix form,

$$\log \mathbf{p} = \log \boldsymbol{\varepsilon} + \mathbf{A}^\top \log \mathbf{p},$$

i.e.,

$$\log \mathbf{p} = \left(\mathbf{I} - \mathbf{A}^{\top}\right)^{-1} \log \varepsilon. \tag{124}$$

Finally, if we plu the expression for $x_{m,j}$ in the good market clearing condition, we get

$$\begin{aligned} y_j &= \sum_{u=1}^{N} \alpha_{j,u} \frac{p_u y_u}{p_j} + \chi_j, \\ &= \sum_{u=1}^{N} \beta_{j,u} y_u + \chi_j \end{aligned}$$

where $\beta_{j,u} = \alpha_{j,u} \frac{p_u}{p_j}$. Hence, we can finally rewrite it in matrix form amsmath

$$\mathbf{y} - \mathbf{B}\mathbf{y} = \boldsymbol{\chi}. \tag{125}$$

implying (**??**). □

Proof of Proposition ??

Since the firm's problem is now unconstrained, we can just take partial derivatives:

$$\partial_{k_j} \pi\left(k_j, \mathbf{x}_j\right) = \alpha_{0,j} \frac{p_j y_j}{k_j} - r^D = 0 \tag{126}$$

$$\partial_{x_{m,j}} \pi\left(k_j, \mathbf{x}_j\right) = \alpha_{m,j} \frac{p_j y_j}{x_{m,j}} - p_m \left(1 + \tau^{\mathbf{x}}_{m,j}\right) = 0 \tag{127}$$

Hence,

$$k_j = \alpha_{0,j} \frac{p_j y_j}{r^D} \tag{128}$$

$$x_{m,j} = \alpha_{m,j} \frac{p_j y_j}{p_m \left(1 + \tau^{\mathbf{x}}_{m,j}\right)} \tag{129}$$

for each $m, j = 1 \ldots, N$. The lagrangian of the household's problem is

$$\mathcal{L}\left(\boldsymbol{\chi}; \Lambda\right) = C(\boldsymbol{\chi}) - \Lambda \left(\left(\mathbf{p} \circ \left(1 + \boldsymbol{\tau}^{\boldsymbol{\chi}}\right)\right)^{\top} - C\left(\boldsymbol{\chi}\right) \right). \tag{130}$$

Hence,

$$\partial_{\chi_j} \mathcal{L}\left(\boldsymbol{\chi}; \Lambda\right) = \omega_j \frac{C(\boldsymbol{\chi})}{\chi_j} - \Lambda p_j \left(1 + \tau^{\chi}_j\right) = 0 \tag{131}$$

which implies

$$\chi_j = \omega_j \frac{C(\boldsymbol{\chi})}{p_j \left(1 + \tau_j^{\chi}\right) \Lambda}. \tag{132}$$

for each $j = 1, \ldots, N$. Thanks to (**??**), we can rewrite the market clearing condition as

$$K = \sum_{j=1}^{N} k_j = \sum_{j=1}^{N} \alpha_{0,j} \frac{p_j y_j}{r^D} = \frac{1}{r^D} \sum_{j=1}^{N} \alpha_{0,j} p_j y_j \tag{133}$$

and therefore

$$r^D = K \sum_{j=1}^{N} \alpha_{0,j} p_j y_j. \tag{134}$$

On the other hand, if we substitute (**??**) back into the good market clearing condition, we get

$$y_j = \sum_{u=1}^{N} \alpha_{j,u} \frac{p_u y_u}{p_j \left(1 + \tau_{j,u}^{\mathbf{x}}\right)} + \chi_j, \tag{135}$$

which can be rewritten as

$$\mathbf{y} - \mathbf{B}\mathbf{y} = \boldsymbol{\chi}. \tag{136}$$

i.e.,

$$\left(\mathbf{I} - \mathbf{B}\right) \mathbf{y} = \boldsymbol{\chi}. \tag{137}$$

where

$$\beta_{m,j} = \alpha_{m,j} \frac{p_m}{p_j \left(1 + \tau_{m,j}^{\mathbf{x}}\right)}. \tag{138}$$

Finally, in order to determine the value of the coins in such a way that we can recover the first best found by the planner, we need to set the following three equations:

$$r^D = \mu \tag{139}$$

$$\frac{p_j}{p_m \left(1 + \tau_{m,j}^{\mathbf{x}}\right)} = \frac{\lambda_j}{\lambda_m + \theta_m \xi_{m,j}^{\mathbf{x}} \left(s^\top \mathbf{y}\right)} \tag{140}$$

$$\lambda_j + \theta_j \xi_j^{\chi} \left(\mathbf{s}^\top \mathbf{y}\right) = \Lambda p_j \left(1 + \tau_j^{\chi}\right) \tag{141}$$

After the appropriate choice of numèraire, we would have, $\Lambda = 1$ and $p_j = \lambda_j$ for every $j = 1, \dots, N$. Hence,

$$p_m \tau^{\mathbf{x}}_{m,j} = \theta_m \xi^{\mathbf{x}}_{m,j} \left(\mathbf{s}^\top \mathbf{y} \right) \tag{142}$$

$$p_j \tau^{\chi}_{j} = \theta_j \xi^{\chi}_{j} \left(\mathbf{s}^\top \mathbf{y} \right) \tag{143}$$

for every $m, j = 1, \dots, N$. □

Proof of Proposition ??

For each supply chain, we need to solve the Coin Economy Problem, and it's already solved in Proposition **??**. Hence, we just have to write a superindex indicating the supply chain to which the firm belongs to. Hence, for every $sc = 1, \dots, SC$,

$$k^{sc}_j = \alpha_{0,j} \frac{p^{sc}_j y^{sc}_j}{r^{sc}} \tag{144}$$

$$x^{sc}_{m,j} = \alpha_{m,j} \frac{p^{sc}_j y^{sc}_j}{p^{sc}_m \left(1 + \tau^{\mathbf{x},sc}_{m,j}\right)} \tag{145}$$

$$\chi^{sc}_j = \omega^{sc}_j \frac{C(\boldsymbol{\chi})}{p^{sc}_j \left(1 + \tau^{\chi,sc}_{j}\right) \Lambda}. \tag{146}$$

for each $m, j = 1 \dots, N$ and each $sc = 1, \dots, SC$. On the other hand, we are going to have SC different market clearing conditions, i.e,.

$$K^{sc} = \sum_{j=1}^{N} k^{sc}_j = \sum_{j=1}^{N} \alpha^{sc}_{0,j} \frac{p^{sc}_j y^{sc}_j}{r^{sc}} = \frac{1}{r^{sc}} \sum_{j=1}^{N} \alpha^{sc}_{0,j} p^{sc}_j y^{sc}_j \tag{147}$$

and therefore

$$r^{sc} = K^{sc} \sum_{j=1}^{N} \alpha^{sc}_{0,j} p^{sc}_j y^{sc}_j. \tag{148}$$

for every $sc = 1, \dots, SC$. On the other hand, if we substitute (**??**) back into the good market clearing condition, we get

$$y^{sc}_j = \sum_{u=1}^{N} \alpha^{sc}_{j,u} \frac{p^{sc}_u y^{sc}_u}{p^{sc}_j \left(1 + \tau^{\mathbf{x},sc}_{j,u}\right)} + \chi^{sc}_j, \tag{149}$$

which can be rewritten as

$$\mathbf{y} - \mathbf{B}\mathbf{y} = \boldsymbol{\chi}. \tag{150}$$

i.e.,

$$(\mathbf{I} - \mathbf{B})\,\mathbf{y} = \boldsymbol{\chi}. \tag{151}$$

where

$$\beta_{m,j}^{sc} = \alpha_{m,j}^{sc} \frac{p_m^{sc}}{p_j^{sc}\left(1 + \tau_{m,j}^{\mathbf{x},sc}\right)}. \tag{152}$$

Finally, in order to determine the value of the coins in such a way that we can recover the first best found by the planner, we need to set the following three equations:

$$r^{sc} = \mu \tag{153}$$

$$\frac{p_j^{sc}}{p_m^{sc}\left(1 + \tau_{m,j}^{\mathbf{x},sc}\right)} = \frac{\lambda_j^{sc}}{\lambda_m^{sc} + \theta_m^{sc}\xi_{m,j}^{\mathbf{x},sc}\left(\left(\mathbf{s}^{sc}\right)^\top \mathbf{y}^{sc}\right)} \tag{154}$$

$$\lambda_j^{sc} + \theta_j^{sc}\xi_j^{\xi}\left(\left(\mathbf{s}^{sc}\right)^\top \mathbf{y}^{sc}\right) = \Lambda p_j^{sc}\left(1 + \tau_j^{\chi,sc}\right) \tag{155}$$

for every $m, j = 1, \ldots, N$ and $sc = 1, \ldots, SC$. After the appropriate choice of numèraire, we would have, $\Lambda = 1$ and $p_j^{sc} = \lambda_j^{sc}$ for every $j = 1, \ldots, N$. Hence,

$$p_m^{sc}\tau_{m,j}^{\mathbf{x},sc} = \theta_m^{sc}\xi_{m,j}^{\mathbf{x},sc}\left(\left(\mathbf{s}^{sc}\right)^\top \mathbf{y}^{sc}\right) \tag{156}$$

$$p_j^{sc}\tau_j^{\chi,sc} = \theta_j^{sc}\xi_j^{\chi}\left(\left(\mathbf{s}^{sc}\right)^\top \mathbf{y}^{sc}\right) \tag{157}$$

for every $m, j = 1, \ldots, N$. □

Proof of Proposition ??

The first order contidions come directly from Propositions **??** and **??**. If we plug (**??**) and (**??**) back into the (log) production function, we get that, for every $j = 1, \ldots, N^{DAO}$,

$$\begin{aligned}
0 &= \log z + \alpha_{0,j} \log \frac{\alpha_{0,j} p_j y_j}{r^D} + \sum_{m=1}^{N} \alpha_{m,j} \log \alpha_{m,j} \frac{p_j y_j}{\left(1 + \tau^{\mathbf{x}}_{m,j}\right) p_m} \\
&= \log z + \alpha_{0,j} \log \alpha_{0,j} + \sum_{j=1}^{N} \alpha_{m,j} \log \alpha_{m,j} + \alpha_{0,j} \log \frac{p_j}{r^D} + \sum_{m=1}^{M} \alpha_{m,j} \log \frac{p_j}{\left(1 + \tau^{\mathbf{x}}_{m,j}\right) p_m} \\
&= \log z + \mathcal{S}\left(\mathbf{A}\right) + \alpha_{0,j} \log \frac{p_j}{r^D} + \sum_{m=1}^{N} \alpha_{m,j} \log \frac{p_j}{\left(1 + \tau^{\mathbf{x}}_{m,j}\right) p_m} \\
&= \log z + \mathcal{S}\left(\mathbf{A}\right) + \log p_j - \alpha_{0,j} \log r^D - \sum_{m=1}^{N} \alpha_{m,j} \log\left(1 + \tau^{\mathbf{x}}_{m,j}\right) - \sum_{m=1}^{N} \alpha_{m,j} \log p_m \\
&= \log \varepsilon_j + \log p_j - \sum_{m=1}^{N} \alpha_{m,j} \log p_m
\end{aligned} \tag{158}$$

where

$$\varepsilon_j = \log z + \mathcal{S}\left(\mathbf{A}\right) - \alpha_{0,j} \log r^D - \sum_{m=1}^{N} \alpha_{m,j} \log\left(1 + \tau^{\mathbf{x}}_{m,j}\right). \tag{159}$$

Now, for each $j = N^{DAO}+1, \ldots, N$, we need to plug (**??**) and (**??**) back into the (log) production function,

$$\begin{aligned}
0 &= \log z + \alpha_{0,j} \log \frac{\alpha_{0,j} p_j y_j}{r^D + \zeta_j} + \sum_{i=1}^{n} \alpha_{m,j} \log \alpha_{m,j} \frac{p_j y_j}{p_i \left(1 + \delta\left(r^D + \zeta_j\right)\right)} \\
&= \log z + \alpha_{0,j} \log \alpha_{0,j} + \sum_{i=1}^{n} \alpha_{m,j} \log \alpha_{m,j} + \alpha_{0,j} \log \frac{p_j}{r^D + \zeta_j} + \sum_{i=1}^{n} \alpha_{m,j} \log \frac{p_j}{\left(1 + \delta\left(r^D + \zeta_j\right)\right) p_i} \\
&= \log z + \mathcal{S}\left(\mathbf{A}\right) + \log p_j - \alpha_{0,j} \log\left(r^D + \zeta_j\right) - \sum_{i=1}^{n} \alpha_{m,j} \log\left(1 + \delta\left(r^D + \zeta_j\right)\right) p_i \\
&= \log \varepsilon_j + \log p_j - \sum_{i=1}^{n} \alpha_{m,j} \log p_i
\end{aligned} \tag{160}$$

where

$$\varepsilon_j = \log z + \mathcal{S}\left(\mathbf{A}\right) - \alpha_{0,j} \log\left(r^D + \zeta_j\right) - \sum_{i=1}^{n} \alpha_{m,j} \log\left(1 + \delta\left(r^D + \zeta_j\right)\right). \tag{161}$$

That means that we can pin down the equilibrium prices from

$$\log p_j = \log \varepsilon_j + \sum_{m=1}^{N} \alpha_{m,j} \log p_m. \tag{162}$$

In matrix form,

$$\log \mathbf{p} = \left(\mathbf{I} - \mathbf{A}^{\top}\right)^{-1} \log \varepsilon. \tag{163}$$

The relevant thing to bear in mind is that ε_j is different for DAO and non-DAO firms. Finally, markets must clear. The good market clearing condition tells us that, for each $j = 1, \dots, N^{DAO}$,

$$y_m = \sum_{u=1}^{N^{DAO}} \alpha_{m,u} \frac{p_u y_u}{p_m \left(1 + \tau^{\mathbf{x}}_{m,u}\right)} + \sum_{u=N^{DAO}+1}^{N} \alpha_{m,u} \frac{1}{1 + \delta\left(r^D + \zeta_j\right)} \frac{p_u y_u}{p_m} + \chi_m \tag{164}$$

Hence,

$$y_m - \left(\sum_{u=1}^{N^{DAO}} \alpha_{m,u} \frac{p_u y_u}{\left(1 + \tau^{\mathbf{x}}_{m,u}\right) p_m} + \sum_{u=N^{DAO}+1}^{N} \alpha_{m,u} \frac{1}{1 + \delta\left(r^D + \zeta_j\right)} \frac{p_u y_u}{p_m} \right) = \chi_m \tag{165}$$

We can then rewrite it in matrix form as

$$\mathbf{y} - \mathbf{B}\mathbf{y} = \boldsymbol{\chi}. \tag{166}$$

where $\mathbf{B} = \left(b_{m,u}\right)_{m,u=1}^{N}$ with

$$b_{m,u} = \begin{cases} \alpha_{m,u} \frac{p_u y_u}{\left(1+\tau^{\mathbf{x}}_{m,u}\right) p_m} & \text{for } m = 1, \dots, N, \quad u = 1, \dots, N^{DAO} \\ \alpha_{m,u} \frac{p_u y_u}{\left(1+\delta\left(r^D + \zeta_j\right)\right) p_m} & \text{for } m = 1, \dots, N, \quad u = N^{DAO} + 1, \dots, N \end{cases} \tag{167}$$

Finally, the market clearing condition requires that $\sum_{j=1}^{N} k_j = K$.