A Model of Focusing in Political Choice*

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Abstract

This paper develops a model of voters' and politicians' behavior based on the notion that voters focus disproportionately on and, hence, overweigh the policies in which politicians' platforms differ more. We introduce *focusing* in a model of electoral competition between differentiated candidates who invest resources to improve the quality of their policies in multiple common value issues. We show that voters' attention distortion leads to greater investment in policy development, greater platform differentiation (with politicians standing out in the policies they are more competent in), and greater investment in divisive policies. Finally, we show that focusing can contribute to explain puzzling stylized facts such as the entry of single-issue parties with no electoral chances or the inverse correlation between income inequality and redistribution.

Keywords: Electoral Competition; Issue Salience; Issue Ownership; Differentiated Candidates; Bottom-Up Attention; Behavioral Models of Politics; Polarization; Single-Issue Parties; Income Inequality; Redistribution.

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1 Introduction

Evaluating political candidates or parties is a complex, multidimensional task. This is because, in an election, a candidate typically represents a bundle of positions on multiple policy issues. For example, in the 2016 U.S. presidential election, Hillary Clinton was in favor of the Affordable Care Act and the Paris agreement on climate, while Donald Trump opposed both measures. On the other hand, both candidates proposed a plan of public investment in infrastructure and expressed skepticism about the Trans-Pacific Partnership.[1]

This suggests that how citizens weigh a candidate's position on different issues is crucial for the formation of their political preferences. Even when citizens have access to detailed information on candidates' platforms, evaluating them is a complex task, associated with low stakes and no direct feedback from experience, as an individual's political choice is unlikely to be pivotal. This might lead citizens to consistently misperceive the overall value of the available alternatives.

In particular, a large body of experimental research has documented that preferences over alternatives with multiple dimensions are influenced by the environment.[2] This is also true for political preferences outside of the laboratory: Callander and Wilson (2006) show that turnout in U.S. Congressional elections is influenced not only by the attractiveness of each individual candidate but by the entire political offer: voters are more likely to turn out not only as their preferred candidate becomes ideologically closer to them but also as the other candidate becomes ideologically farther away from them.

Building on this evidence, social scientists have recently developed models where the

[1]See 'Policy details lost in the din of debate' in the Financial Times on November 8, 2016.

[2]Manipulating the set of available alternatives affects choice over consumer products which differ in quality and price (Huber, Payne and Puto, 1982; Simonson, 1989; Simonson and Tversky, 1992; Heath and Chatterjee, 1995); choice over lotteries which vary in prizes and probabilities (Allais, 1953; Slovic and Lichtenstein, 1971; Herne, 1999); and choice over monetary allocations which differ in efficiency and fairness (Roth, Murnighan and Schoumaker, 1988; Galeotti, Montero and Poulsen, 2019).

choice set can distort the weights a decision-maker attaches to the features of an alternative (Rubinstein, 1988; Leland, 1994; Bordalo, Gennaioli and Shleifer, 2012, 2013*a*,*b*, 2015; Kőszegi and Szeidl, 2013; Bushong, Rabin and Schwartzstein, 2021). At the same time, the theoretical implications of this *selective focus* for political behavior are largely unexplored and unclear. In fact, most formal models of voting are based on the classic model of choice where the subjective value each alternative gives to a decision-maker is independent of the other available alternatives.

In this paper, we develop a model of voters' and politicians' behavior based on the idea that voters perceive policy issues as more or less *salient* depending on the choice environment. In line with a recent literature in economics and psychology, we assume that a voters' attention is captured by the issues in which the available candidates differ more and that, in turn, these issues are overweighed in the decision-making process. This assumption is based on the notion that our limited cognitive resources are unconsciously attracted by a subset of the available sensory data (Taylor and Thompson, 1982) and, in particular, that "our mind has a useful capability to focus on whatever is odd, different or unusual" (Kahneman, 2011; see also Baumeister and Vohs, 2007).

Importantly, this assumption and its implications for choice under risk and over time have recently found validation in a number of controlled laboratory experiments (Bondi, Csaba and Friedman, 2018; Castillo, 2020; Andersson, Carlson and Wengström, 2021; Dertwinkel-Kalt, Gerhardt, Riener, Schwerter and Strang, 2022). When it comes to political behavior, another reason why voters' limited attention might be attracted by candidates' contrasting features is the fact that the media—which shape voters' perception of what issues are most pressing or important at a given time (McCombs and Shaw, 1972; Weaver, 1996)—neglect the "sphere of consensus" and, instead, give more coverage to non-consensual political topics or the "sphere of legitimate controversy" (Weaver and Elliott, 1985; Hallin, 2005; Shoemaker and Reese, 2013).

In our basic framework, two parties compete for votes in an election offering binding *platforms* composed of $K \geq 2$ *policies*. Parties compete in terms of *valence*: they invest resources to produce policy innovations that increase their proposals' quality on each issue and have different competence in different policy areas (that is, they *own* different issues). The electorate consists of a continuum of voters in different social groups. We assume that voters focus more on policies in which their available options differ more, that is, on policies in which parties' platforms generate a greater range of utility.

We present three sets of results. First, we introduce focusing voters into a classical model of electoral competition and investigate the effect of focusing on the *endogenous* formation of platforms by strategic candidates.[3] We show that focusing increases competition between parties leading them to invest more resources in policy development in all issues. This is due to two effects. First, taking the contrast between platforms as given, focusing increases the effect of a marginal improvement in quality on voters' perceived utility (and, thus, on voters' propensity to vote for the party improving its proposal). Second, focusing gives parties an incentive to manipulate voters' attention, that is, to increase contrast between platforms when offering better policies (thus directing voters' attention towards a strength) and to decrease this contrast when offering worse policies

[3]Political professionals are aware that attention is malleable and try to exploit context-effects. One example is Donald Trump's use of Twitter to increase the prominence of some issues while distracting voters from others. Lewandowsky, Jetter and Ecker (2020) show that increased media coverage of the Mueller investigation of Russian interference in the 2016 election was immediately followed by Trump tweeting about unrelated issues (China, jobs, or immigration). This activity, in turn, was followed by a reduction in media coverage of the Mueller investigation. Another example is from the Persian Gulf crisis of 2019-2020, when U.S. military officials put the option of killing Iran's most powerful commander (Major General Qasem Soleimani) on the menu they offered President Trump when choosing how to respond to recent Iranian-led violence in Iraq. According to media reports of those dramatic days, American officials hoped that including such a drastic measure would push Trump towards a middle course (see 'Why did the Pentagon ever give Trump the option of killing Soleimani?' in the Washington Post on January 10, 2020). Indeed, according to the New York Times, "since the 9/11/2001 attacks, Pentagon officials have often offered improbable options to presidents to make other possibilities appear more palatable" (see 'As Tensions with Iran escalated, Trump opted for most extreme measure' in the New York Times on January 4, 2020).

(thus diverting voters' attention away from a weakness). These two effects change the calculus of the two parties differently, giving the party with a competence advantage in policy k a stronger incentive to increase quality, thereby increasing the contrast between platforms, that is, policy polarization. Both the *focusing-induced increase in quality effect* and the *attention manipulation effect* are stronger in issues where voters' preferences are more dispersed and in issues where the competence differential between parties is larger. As a consequence, the race to offer better policies and policy polarization are more pronounced in more divisive issues and in issues with a stronger ownership. This also means that, as voters' degree of focusing increases, these issues contribute more to their overall evaluation of parties' platforms and, thus, to parties' electoral success.

Second, we show that focusing can encourage the entry of third or "spoiler" candidates. In order to concentrate on the effect these candidates have on voters' attention, we consider a candidate voters do not entertain as a viable option but whose platform contributes to the difference among the available alternatives. This entry has two consequences: first, it can generate larger contrast in policy k (for example, when offering extreme policies), giving both mainstream parties an heightened incentive to invest in this policy; second, it reduces the mainstream parties' ability to manipulate voters' attention since the difference among the available alternatives is no longer determined just by their proposals. We characterize the equilibrium with an exogenous third party and then investigate the endogenous supply of policies by a strategic third party. When the third party's goal is to harm the electoral chances of the party which owns policy k, it finds it optimal to offer a similar policy (and, thus, it acts as the classical spoiler candidate). When, instead, the third party's goal is to help the party which owns policy k, it enters with either a really weak or a really strong policy, depending on how costly the entrant finds it to develop better policies in this issue.

Third, we propose a complementary approach to introducing focusing in formal models of politics and consider how voters' attention is unconsciously attracted to different *consequences* of the same policy (for example, its benefits and its costs). In this framework, we explore the effect of focusing in one important application, fiscal policy. In particular, we assume parties offer a public good funded by a proportional tax rate and show that the model helps explain facts that are puzzling from the perspective of existing political economy theories—the negative correlation between income inequality and redistribution (Ashok, Kuziemko and Washington, 2015; Piketty, Saez and Stantcheva, 2014). Rich voters place more weight on the cost of redistribution and, after an increase in their income, overweigh their higher tax bill. On the other hand, poor voters place more weight on public good consumption and, after a decrease in their income, underweigh the lower cost of increasing redistribution. Our model highlights that policy capture from special interests can be a consequence of the psychology of attention without relying on the coordination and costly collective action necessary for lobbying. When attention and, in turn, preferences are influenced by the choice environment, a small group which neglects one side of the trade-off but is really sensitive on the other can be overly influential in obtaining what it desires.

Our work is primarily related to a recent, yet rapidly growing, research program in formal theory with non-standard preferences or boundedly rational agents (also known as *behavioral political economy*), which studies electoral competition or political agency models when voters employ decision heuristics or are prone to cognitive biases (Bendor, Diermeier, Siegel and Ting, 2011; Minozzi, 2013; Ashworth and De Mesquita, 2014; Bisin, Lizzeri and Yariv, 2015; Levy and Razin, 2015; Ortoleva and Snowberg, 2015; Diermeier and Li, 2017, 2019; Lockwood, 2017; Penn, 2017; Alesina and Passarelli, 2019; Little, 2019; Ogden, 2019; Little, Schnakenberg and Turner, 2022). More closely related to this paper, Callander and Wilson (2006, 2008) and Balart, Casas and Troumpounis (2022) introduce a theory of Downsian competition with *context-dependent voting*. In Callander and Wilson (2006, 2008), the propensity to turn out and vote for the preferred candidate is greater when the other candidate is more extreme. In Balart, Casas and Troumpounis (2022), citizens' votes depend on candidates' (unidimensional) policies when these are sufficiently different and on candidates' advertising expenditure otherwise.[4]

[4]This model can be seen as a special case of focusing in which only the difference in one of two dimensions (candidates' policy position) is relevant to assign the weights and weights take value 0 or 1.

We also contribute to a large literature in economics and political science which has shown that politicians distort their platforms to target each policy to those voters who care the most about it. Incentives to do so have been attributed to the fact that voters are (exogenously or endogenously) more informed about the policies they care more about (Glaeser, Ponzetto and Shapiro, 2005; Gavazza and Lizzeri, 2009; Prato and Wolton, 2016; Matějka and Tabellini, 2021; Devdariani and Hirsch, 2023). Another modeling strategy, however, is to assume that voters ignore information they have but care too little about, up to the point of behaving as *single-issue voters* (List and Sturm, 2006). The latter assumption reflects the long-standing notion of *issue salience* in political science (Converse, 1964; RePass, 1971; Rabinowitz, Prothro and Jacoby, 1982; Niemi and Bartels, 1985). Recent theoretical studies have mostly focused on media coverage (Edwards, Mitchell and Welch, 1995; Epstein and Segal, 2000) or political advertising (Aragonès, Castanheira and Giani, 2015; Dragu and Fan, 2016) as a driver of salience. This paper revisits issue salience and grounds it in a known psychological bias in information processing. In our model, voters have complete information on policies. Our innovation lies in assuming that issue salience for each voter does not depend only on how much the voter cares about different policies, but also on how distant alternative proposals are for different policies.

2 Model

Platforms and Policies. Two office-motivated parties compete for votes in an election. Voters are concerned by $K \geq 2$ issues. Each party drafts a *platform* with *policies* for each issue. As in Aragonès, Castanheira and Giani (2015), a policy is identified by its *quality* and a platform is, thus, a vector of qualities, $p = (p_1, p_2, \ldots, p_K)$, where $p_k \in [0, \infty)$. We think of each element of this vector as the resources the party invests to produce policy innovations that increase its platform's quality on each issue (see also Hirsch and Shotts, 2015, 2018). In other words, we focus on common value issues and, in our model, parties compete in terms of *vertical differentiation* or *valence* rather than in terms of *horizontal* *differentiation* or ideological positioning.

Voters' Preferences. The electorate consists of a continuum of voters who belong to $n \geq 1$ social groups. The fraction of voters in group $i \in N = \{1, \ldots, n\}$ is $m_i > 0$, with $\sum_{i \in N} m_i = 1$. All voters from the same social group have the same policy preferences. At the same time, while greater quality on any issue is unambiguously better for everybody (e.g., a lower crime rate, a higher occupation rate, more effective teaching in public schools), voters in different social groups differ in their relative preference for the K issues, that is, in the rate at which they are willing to trade policy quality across issues. In particular, a voter in group i derives *consumption utility* from platform p equal to:

$$V_i(p) = \sum_{k=1}^{K} u_{ik}(p_k) \tag{1}$$

where $u_{ik} : \mathbb{R}_+ \to \mathbb{R}_+$ is strictly increasing in p_k for all $i \in N$ and all $k \in \{1, \ldots, K\}$. For some results below, we will further assume $u_{ik}(p_k) = \theta_{ik} p_k$, where $\theta_{ik} > 0$ is the marginal benefit voters in group i derive from policies in issue k.

Focusing. Our key assumption and main departure from the classical formal models of voting is that, when evaluating platforms, voters use their *focus-weighted utility* rather than their consumption utility. Consider a non-empty *choice set* composed of a finite number of platforms, $\mathcal{P} = \{p, q, \ldots\}$. Let $\Delta_{ik}(\mathcal{P})$ be the range of consumption utility voters in group $i \in N$ derive from policy k in choice set $\mathcal{P}$:

$$\Delta_{ik}(\mathcal{P}) = \max_{p \in \mathcal{P}} u_{ik}(p_k) - \min_{p \in \mathcal{P}} u_{ik}(p_k). \tag{2}$$

We assume that voters focus more on policies in which their available options differ more, that is, on policies in which candidates' platforms generate a greater range of consumption utility. As discussed above, this assumption is compatible with the psychology of human cognition and has been validated in laboratory experiments.

Formally, for a voter in group $i \in N$, the focus-weighted utility from $p \in \mathcal{P}$ is:

$$\widetilde{V}_i(p, \mathcal{P}) = \sum_{k=1}^{K} g(\Delta_{ik}(\mathcal{P})) \cdot u_{ik}(p_k), \tag{3}$$

where $g(\Delta_{ik}(\mathcal{P}))$ is the focus weight on policy k. As in Kőszegi and Szeidl (2013), we assume that $g(\cdot)$ (i) satisfies $g(0) = 1$ and (ii) is weakly increasing in Δ_{ik}. The first assumption means that voters in group i have *undistorted focus* on policy k if $\Delta_{ik}(\mathcal{P}) = 0$, that is, when the platforms in their choice set do not differ in the consumption utility they offer in this issue. The second assumption means that, when instead $\Delta_{ik}(\mathcal{P}) > 0$, the weight voters in group i place on issue k is larger than the weight used by rational voters and it grows with the contrast the available platforms display in this issue.

When investigating the endogenous formation of choice sets by office-motivated politicians, we will assume $g(\Delta_{ik}(\mathcal{P})) = 1 + \rho\Delta_{ik}(\mathcal{P})$, where $\rho \geq 0$ increases in the severity of focusing. As ρ goes to 0, focusing voters converge to rational voters. When instead, $\rho > 0$, the weight voters place on issue k is larger than the weight used by rational voters and it grows with their degree of focusing, ρ, and with the contrast the available platforms display in this policy, $\Delta_{ik}(\mathcal{P}) > 0$.[5]

Consequences of Focusing on Voters' Preferences. In order to better understand the effect of voters' focusing on the candidates' calculus, we present a preliminary result that assumes an *exogenous choice set* composed of two platforms offering two policies. All proofs are in the Online Appendix.

Lemma 1. *Assume $K = 2$ and $\mathcal{P} = \{a, b\}$. Focusing increases the intensity of preferences between platforms, that is, for all groups $i \in N$, $\widetilde{V}_i(a, \mathcal{P}) - \widetilde{V}_i(b, \mathcal{P}) = c \cdot [V_i(a) - V_i(b)]$, where $c \geq 1$ with strict inequality if $V_i(a) \neq V_i(b)$ and $g(\Delta_{ik})$ strictly increasing.*

Lemma 1 shows that, when elections feature two candidates offering policy solutions

[5] All results presented below continue to hold if we allow focusing to be heterogeneous across social groups, for example, if we assume $g_i(\Delta_{ik}(\mathcal{P})) = 1 + \rho_i\Delta_{ik}(\mathcal{P})$, where $\rho_i \geq 0$ is social group i's severity of focusing. The model with an homogeneous degree of focusing allows us to provide a clearer intuition.

on two issues, focusing voters rank platforms as in the absence of attention distortions. However, it also shows that focusing strengthens voters' intensity of preferences, that is, how much they care about their preferred policy and, thus, the conflict of preferences between members of any two disagreeing groups.

To understand the intuition behind Lemma 1, consider $a \neq b$ with $a_1 > b_1$ and $a_2 < b_2$: a's *relative advantage* lies in its better policy in issue 1, while a's *relative disadvantage* lies in its worse policy in issue 2. Consider a social group i which receives greater consumption utility from a. For these voters, a's relative advantage more than compensates its relative disadvantage. This happens if and only if the range of consumption utility in the first issue, $u_{i1}(a_1) - u_{i1}(b_1)$, is larger than the range of consumption utility in the second issue, $u_{i2}(a_2) - u_{i2}(b_2)$. Given our assumption on the determinants of voters' attention, this leads voters to place larger weight on a's relative advantage than on its relative disadvantage. As a consequence, the difference in focus-weighted utility between the two platforms is larger than the difference in consumption utility, that is, $\widetilde{V}_i(p, \mathcal{P}) - \widetilde{V}_i(q, \mathcal{P}) > V_i(p) - V_i(q)$.

Lemma 1 implies that distorted attention does not affect collective decision making when society chooses between two *exogeneous* platforms composed of two policies. This does not mean that focusing is not important in politics. First, as we show in the following section, focusing shapes the *endogeneous* formation of platforms (even when restricted to two policies) as long as the intensity of preferences affects the likelihood of voting for a particular candidate (for example, with stochastic choice, or whenever other considerations enter voters' decision). Second, in the rest of the paper, we investigate a more general environment where platforms offer policy solutions on more than two issues and, in an extension, we consider elections with three candidates. Indeed, in these cases, focusing can moderate, or even reverse, rational voters' preferences.

Consider a choice set with three platforms composed of two policies, $\mathcal{P} = \{a, b, c\}$. As before, for illustration purposes, consider $a \neq b$ with $a_1 > b_1$ and $a_2 < b_2$ and assume rational voters in group i prefer a to b, that is, $u_{i1}(a_1) - u_{i1}(b_1) > u_{i2}(b_2) - u_{i2}(a_2)$. The range of consumption utility on each policy is no longer determined only by a and b and the addition of c can change the weight focusing voters place on the two policies. For example, if c dominates or is dominated by both original platforms in the first policy (that is, $c_1 > a_1$ or $c_1 < b_1$), the addition of this third platform increases contrast and, thus, attention on the policy where a is advantaged. If the addition of c has a smaller effect on contrast in the policy where a is disadvantaged (for example, when $c_2 \in [a_2, b_2]$ and the consumption utility range in policy 2 is unaffected by the presence of c), voters' preference for a is strengthened by focusing (as it was the case with two platforms). However, if c increases contrast in the second policy more than contrast in the first policy—for example because c is dominated by both policies in the second issue, $c_2 < a_2 < b_2$, but mediocre in the first issue, $b_1 < c_1 < a_1$—the attention of voters in this social group is drawn to the policy where b is advantaged, moderating the lead of a or even reversing the relative preference between the original platforms and leading these voters to prefer b.

We now restrict ourselves again to two candidates but allow their platforms to have more than two policies. Consider a choice set $\mathcal{P} = \{a, b\}$ where, for voters in social group i, platform a has a relative advantage of magnitude p on K_p policies and a relative disadvantage of magnitude m on K_m policies. Moreover, assume that, when voters in this social group have undistorted attention, they prefer platform a, that is, $p \cdot K_p > m \cdot K_m$. How does focusing change these voters' preferences? When voters in this social group focus, they prefer platform a if $g(p) \cdot p \cdot K_p > g(m) \cdot m \cdot K_m$. There are two cases: (i) $p > m$ (that is, a's relative advantages are larger than its relative disadvantages) and (ii) $p < m$ (that is, a's relative advantages are smaller than its relative disadvantages).

Case 1: $p > m$. Note that p (m) is the range of consumption utility on the policies where a enjoys a relative advantage (suffers a relative disadvantage). Thus, when $p > m$, $g(p) > g(m)$ and focusing voters place greater weight on the K_p policies where a has a relative advantage than on the K_m policies where a has a relative disadvantage. This leads to an intensification of preferences with respect to rational voters analogous to what we discussed for bi-dimensional platforms in Lemma 1.

Case 2: $p < m$. In this case, for rational voters to prefer a to b, it must be the case that $K_p > K_m$, that is, a has many small advantages and few large disadvantages or, in other words, a has *diffused advantages and concentrated disadvantages.* In this case, $g(p) < g(m)$ and, thus, focusing voters overweight the concentrated disadvantages of a (where the difference between candidates' platforms is starker) and neglect its diffused advantages (where candidates are more similar to one another). When the magnitude of a's diffused advantages is sufficiently large, focusing voters still prefer platform a more intensely than rational voters. However, voters' preferences can moderate or reverse with respect to the rational benchmark. This is more likely to happen when $g(\Delta_{ik})$ grows faster with the contrast in policies and when the magnitude of a's concentrated disadvantages is larger.[6]

Electoral Competition with Focusing Voters. To investigate the effect of focusing on the *endogenous* supply of policies by political candidates, we introduce focusing voters into a classical model of electoral competition, the probabilistic voting model à la Lindbeck and Weibull (1987). Two parties, $P \in \{A, B\}$, simultaneously announce a binding platform composed of the quality of their policies on each issue k. We denote A's platform with $a = (a_1, a_2, \ldots, a_K)$ and B's platform with $b = (b_1, b_2, \ldots, b_K)$, where $a_k, b_k \in [0, \infty)$.

As mentioned above, parties invest resources to produce policy innovations that increase their proposals' quality on each issue. The cost of producing a policy of quality q_k is suffered by the party, is increasing and convex in the quality, and is decreasing in the

[6]The results presented in this subsection are analogous to the bias towards concentration in unbalanced trade-offs and to the rationality in balanced trade-offs discussed in Kőszegi and Szeidl (2013). Similarly to the informal discussion at the end of this subsection, their Proposition 1 provides conditions for choice from an exogenous choice set to be reversed (with respect to the rational benchmark) when the decision-maker focuses. Similarly to our Lemma 1, their Proposition 3 provides conditions for choice from an exogenous choice set to be unchanged (with respect to the rational benchmark) when the decision-maker focuses. We extend these results by considering the effect of focusing not only on the ranking of alternatives but also on the intensity of preferences over these alternatives.

party's competence in the issue, γ_k^P:[7]

$$C_k^P(q_k^P) = \frac{(q_k^P)^2}{2\gamma_k^P}. \tag{4}$$

Party P's competence advantage in issue k, γ_k^P, reflects, among other things, the expertise of the party staff and members of Congress.[8] This expertise increases the party's ability to develop novel proposals that voters will value. Delivering high quality proposals is costly, but this cost is lower for the party with better expertise on the issue. If $\gamma_k^A > \gamma_k^B$ ($\gamma_k^A < \gamma_k^B$), party A (B) enjoys a competence advantage in issue k or, in the language of Petrocik (1996), party A (B) owns issue k. When, instead, $\gamma_k^A = \gamma_k^B$, both parties are equally good at tackling issue k (or no party owns issue k).

Voters observe parties' platforms, evaluate them with their focus-weighted utility and vote as if they are pivotal (or derive expressive utility from voting). The indirect utility voter v in group i receives when voting for each party is:

$$\begin{aligned} u_{v,i}(A) &= \widetilde{V}_i(a, \mathcal{P}) \\ u_{v,i}(B) &= \widetilde{V}_i(b, \mathcal{P}) + \epsilon_v \end{aligned} \tag{5}$$

where $\mathcal{P} = \{a, b\}$ is voters' choice set and $\epsilon_v \sim U[-\frac{1}{2\phi}, \frac{1}{2\phi}]$ is a parameter that measures voter v's bias towards party B (due to considerations different than the parties' electoral promises, for example, the candidates' charisma). A greater precision in the distribution of this parameter, that is, a greater ϕ, implies that voters' decision about what party

[7] An alternative formulation of the model is that the incumbent party allocates a fixed budget to the production of K public goods and appropriates the unspent resources as rents. In this case, investment in any public good induces an opportunity cost only the incumbent incurs in terms of foregone rents. This alternative formulation generates qualitatively identical results as in our model provided that the budget is large enough. Our formulation has the benefit of greater mathematical tractability.

[8] Formal models of electoral competition between *specialized* or *differentiated candidates* who have different productivites or political capabilities in different policy areas are investigated by Krasa and Polborn (2010, 2012, 2014) or Aragonès, Castanheira and Giani (2015).

to support is affected more strongly by parties' electoral promises and less by other considerations. The shock to voters' preferences is realized after platforms are announced but before the election. Given these assumptions, voter v in group i votes for A if and only if $\widetilde{V}_i(a|\mathcal{P}) > \widetilde{V}_i(b|\mathcal{P}) + \epsilon_v$.

Parties are purely office-motivated and maximize their vote shares minus the total cost of drafting their platform.[9] From the parties' perspective, the expected share of voters in group i who vote for A is:[10]

$$\tfrac{1}{2} + \phi \left[\widetilde{V}_i(a,\mathcal{P}) - \widetilde{V}_i(b,\mathcal{P}) \right]. \tag{6}$$

The two parties' objective functions are:

$$\begin{aligned}
\pi_A(a,b,\mathcal{P}) &= \tfrac{1}{2} + \phi \sum_{i\in N} m_i \left[\widetilde{V}_i(a,\mathcal{P}) - \widetilde{V}_i(b,\mathcal{P}) \right] - \sum\nolimits_{k=1}^{K} C_k^A(a_k) \\
\pi_B(b,a,\mathcal{P}) &= \tfrac{1}{2} + \phi \sum_{i\in N} m_i \left[\widetilde{V}_i(b,\mathcal{P}) - \widetilde{V}_i(a,\mathcal{P}) \right] - \sum\nolimits_{k=1}^{K} C_k^B(b_k).
\end{aligned} \tag{7}$$

3 Equilibrium Platforms with Focusing Voters

In this section, we assume that, for each social group $i \in N$ and policy $k \in \{1,\ldots,K\}$, $u_{ik}(p_k) = \theta_{ik} p_k$ and $g(\Delta_{ik}(\mathcal{P})) = 1 + \rho \Delta_{ik}(\mathcal{P})$. Party A's objective function becomes:

$$\begin{aligned}
\pi_A(a,b,\mathcal{P}) &= \tfrac{1}{2} + \phi \sum_{i\in N} m_i \left[\widetilde{V}_i(a,\mathcal{P}) - \widetilde{V}_i(b,\mathcal{P}) \right] - \sum\nolimits_{k=1}^{K} C_k^A(a_k) \\
&= \tfrac{1}{2} + \phi \sum_{i\in N} m_i \sum\nolimits_{k=1}^{K} g(\Delta_{ik}(\mathcal{P})) \cdot [u_{ik}(a_k) - u_{ik}(b_k)] - \sum\nolimits_{k=1}^{K} C_k^A(a_k) \\
&= \tfrac{1}{2} + \phi \sum_{i\in N} m_i \sum\nolimits_{k=1}^{K} (1 + \rho\theta_{ik}|a_k - b_k|) \cdot \theta_{ik}\,(a_k - b_k) - \sum\nolimits_{k=1}^{K} \frac{(a_k)^2}{2\gamma_k^A}.
\end{aligned} \tag{8}$$

[9] All results we present below are robust to parties maximizing the probability of winning.

[10] As common in probabilistic voting models, we assume that ϕ is small enough to guarantee that vote shares are interior. All results presented below continue to hold if we allow the distribution of the shock to B's relative popularity to be heterogeneous across social groups, that is, if we assume $\epsilon_{v,i} \sim U[-\frac{1}{2\phi_i}, \frac{1}{2\phi_i}]$. The model with homogeneous ϕ's allows us to provide a clearer intuition.

Since the objective function is additively separable in policies and there are no spillovers across policies (that is, the optimal quality in policy k is not affected by the quality offered by either party in policy $j \neq k$), we focus on the problem of choosing the optimal quality in a single policy k. Suppressing all terms independent of a_k and b_k, A's objective function becomes:

$$\phi \sum_{i \in N} m_i (1 + \rho \theta_{ik} |a_k - b_k|) \cdot \theta_{ik} (a_k - b_k) - \frac{(a_k)^2}{2\gamma_k^A}. \tag{9}$$

In an interior equilibrium, the votes gained with a small increase in a_k are perfectly offset by the additional resources the party must devote to policy development to obtain that increase. In other words, the following first-order condition (FOC) must be satisfied:

$$\phi \left[\sum_{i \in N} m_i \theta_{ik} + \rho |a_k - b_k| \sum_{i \in N} m_i \theta_{ik}^2 + \rho |a_k - b_k| \sum_{i \in N} m_i \theta_{ik}^2 \right] = \frac{a_k}{\gamma_k^A}. \tag{10}$$

The left-hand side of equation (10) represents the marginal benefit of increasing the quality of policy k (in terms of greater vote share). The right-hand side, instead, represents its marginal cost (in terms of additional resources required for policy development).

Let $\overline{\theta_k}$ denote the average marginal benefit from policy k in the electorate, that is, $\overline{\theta_k} = \sum_{i \in N} m_i \theta_{ik}$, and let σ_k denote the average squared marginal benefit from policy k in the electorate (a growing function of the heterogeneity of preferences for policy k), that is, $\sigma_k = \sum_{i \in N} m_i \theta_{ik}^2$. Then, A's FOC rewrites as:

$$\phi \left[\underbrace{\overline{\theta_k}}_{\substack{\text{Rational} \\ \text{Quality Effect}}} + \underbrace{\rho |a_k - b_k| \sigma_k}_{\substack{\text{Focusing-Induced Increase} \\ \text{in Quality Effect}}} + \underbrace{\rho |a_k - b_k| \sigma_k}_{\substack{\text{Attention Manipulation} \\ \text{Effect}}} \right] = \frac{a_k}{\gamma_k^A}. \tag{11}$$

The marginal benefit of a_k, that is, the LHS of equation (11), is composed of three terms:

1. The first term, $\mathrm{E}\left[u'_{ik}(a_k)\right] = \overline{\theta_k}$, is the ***rational quality effect*** or the marginal increase in voters' consumption utility (that is, the utility of voters with undistorted attention). This is the only effect present also in the absence of focusing.

2. The second term, $\mathrm{E}\left[\left(g\left(\Delta_{ik}\right)-1\right)u'_{ik}(a_k)\right]=\rho|a_k-b_k|\sigma_k$, is the ***focusing-induced increase in quality effect*** or the boost to the quality effect when voters devote greater attention to policies with greater contrast (for a given contrast $|a_k-b_k|$).

3. The third term, $\mathrm{E}\left[g'\left(\Delta_{ik}\right)\left(u_{ik}(a_k)-u_{ik}(b_k)\right)\right]=\rho|a_k-b_k|\sigma_k$, is the ***attention manipulation effect*** or the impact on the contrast voters observe in policy k and, thus, on the attention they devote to this policy; when party A is advantaged in policy k (that is, $a_k>b_k$), this term represents the benefit from increasing contrast and, thus, increasing voters' attention to a strength (weighted by the size of the advantage); when, instead, party A is disadvantaged in policy k (that is, $a_k<b_k$), this term represents the benefit from reducing contrast and, thus, reducing voters' attention to a weakness (weighted by the size of the disadvantage).

Equation (11) highlights how the three components of the marginal benefit from policy development change with features of the electoral environment. The rational quality effect increases in the average preference for k and does not depend on voters' degree of focusing or the opponent's policy. The focusing-induced increase in the quality effect and the attention manipulation effect are increasing in (a) voters' degree of focusing; (b) the contrast between policies in the choice set; and (c) the heterogeneity of preferences for k.

The relationship between the latter two terms and $|a_k-b_k|$ implies that A's marginal benefit from investing in policy k grows in the initial level a_k when $a_k>b_k$ and, instead, decreases in a_k when $a_k<b_k$. This means that focusing gives the party offering a greater quality in policy k a convex incentive to develop policy innovations and further increase the contrast between policies or its advantage with respect to the competitor. To make sure that this does not make unbounded investment in policy development profitable and that both parties' objective functions are strictly concave in their own action, Assumption 1 requires that voters' degree of focusing is not too pronounced (with respect to the marginal cost parties incur when developing better policies).[11]

Assumption 1. *For each* $k\in\{1,\ldots,K\}$ *and each* $P=\{A,B\}$, $\rho<1/(2\phi\sigma_k\gamma_k^P)$.

[11] See footnote 12 for a discussion of equilibrium platforms when this assumption is violated.

Proposition 1 (Equilibrium of Electoral Competition with Focusing Voters)**.** *Suppose Assumption 1. The electoral competition with focusing voters admits a unique Nash equilibrium in pure strategies in which the parties offer the following qualities in policy k:*

$$\begin{aligned} a_k^* &= \frac{\phi\overline{\theta_k}\gamma_k^A}{1-2\phi\rho\sigma_k|\gamma_k^A-\gamma_k^B|} \\ b_k^* &= \frac{\phi\overline{\theta_k}\gamma_k^B}{1-2\phi\rho\sigma_k|\gamma_k^A-\gamma_k^B|}. \end{aligned} \tag{12}$$

Proposition 1 shows that focusing affects equilibrium qualities in policy k as long as $\gamma_k^A \neq \gamma_k^B$, that is, when parties have a different ability to develop innovations in policy k or, in other words, when one party *owns the issue*. This is the case we consider in the discussion below.[12]

Corollary 1 (Comparative Statics for Equilibrium Policies)**.** *The equilibrium qualities in policy k, (a_k^*, b_k^*), increase in voters' degree of focusing, ρ. Moreover, when $\rho > 0$, they increase in the heterogeneity of voters' marginal benefit from policy k, σ_k.*

Focusing leads to greater investment in policy development thanks to the focusing-induced increase in quality effect and to the attention manipulation effect we discussed above. To understand why the dispersion of voters' preferences in policy k matters with focusing voters, note that the focusing weight on policy k, $g(\Delta_{ik})$, and the consumption utility from policy k, $u_{ik}(a_k)$ are complements in voters' focus-weighted utility as well as in A's objective function. Increasing a_k affects both Δ_{ik} (by $u'_{ik} = \theta_{ik}$) and $u_{ik}(a_k)$ (by $u'_{ik} = \theta_{ik}$) and the two effects reinforce each other. This is why, with focusing voters, the marginal benefit from increasing a_k changes also with $[u'_{ik}]^2 = \theta_{ik}^2$ and not just with $u'_{ik} = \theta_{ik}$ as it is the case with rational voters.

[12] When $\gamma_k^A = \gamma_k^B = \gamma_k$, focusing affects the equilibrium platforms when Assumption 1 is not satisfied for some k. In that case, parties have an incentive to offer a quality as large as possible in policy k. If there is an upper bound on feasible policies, there exists an equilibrium of the electoral competition game with focusing voters where parties offer the largest feasible quality in policy k. This is in contrast with the electoral competition game with rational voters where, regardless of whether Assumption 1 is satisfied, there is a unique equilibrium where both parties offer $\phi\overline{\theta_k}\gamma_k$.

Proposition 1 implies that focusing changes not only the absolute quality of the policies offered by the two parties but also the contrast between the qualities offered by the two parties in the same policy, amplifying (even small) exogenous differences in competence and leading to endogenous issue ownership: in line with the *dominance principle* proposed by Riker (1993), when one party dominates on a particular issue, it brings it to the fore of its campaign.

Corollary 2 (Comparative Statics for Equilibrium Polarization)**.** *The contrast between the equilibrium qualities in policy k (or the equilibrium polarization in policy k) is:*

$$|a_k^* - b_k^*| = \frac{\phi\overline{\theta}_k|\gamma_k^A - \gamma_k^B|}{1 - 2\phi\rho\sigma_k|\gamma_k^A - \gamma_k^B|}. \tag{13}$$

This increases in voters' degree of focusing, ρ. Moreover, when $\rho > 0$, it increases in the heterogeneity of voters' marginal benefit from policy k, σ_k.

Focusing leads to greater platform differentiation because voters' attention distortion has a stronger effect on the equilibrium quality of the party with a competence advantage in policy k. The greater incentive to develop policy innovations when voters focus (due to the focusing-induced increase in quality effect and to the attention manipulation effect) is the same for both parties. However, the marginal cost of the more competent party grows more slowly in the quality offered by the party in policy k. This means that the quality at which the greater marginal benefit is perfectly offset by a greater marginal cost is greater for the more competent party. Consider the optimal qualities of A and B with rational voters. At these levels, each party is perfectly balancing the marginal benefit and the marginal cost of further policy development. Now, increase voters' degree of focusing. This increases the marginal benefit of further investing in the policy for both parties (in the same proportion) but does not affect the marginal cost. This means that the original choice is no longer optimal as the marginal benefit is larger than the marginal cost and there is scope for increasing the objective function with a greater action. However, since the marginal cost of the two parties are growing in the action at different rates, with the less competent party facing a steeper increase in the cost, this party will find a new balance between marginal benefits and marginal costs at a lower level of policy quality than the more competent party. (Note that this remains true even if, in the electoral game with focusing voters, a party's optimal choice depends on the action of the opponent and the convergence to a new equilibrium platform depends also on how the other party changes its policy.)

A larger difference in competence generates a larger difference in equilibrium policies regardless of the degree of focusing (and including when voters have undistorted focus). However, as shown in Corollary 2, the effect of focusing on equilibrium polarization is due to parties' differential ability to develop better policies. Thus, an increase in this gap increases polarization more when voters focus. Corollary 3 shows this formally and also discusses how equilibrium qualities change with parties' competence.

Corollary 3 (Effect of Parties' Competence)**.** *Without loss of generality, assume* $\gamma_k^A < \gamma_k^B$*. (a) The equilibrium policies depend on the opponent's competence in* k *if and only if* $\rho > 0$*. In this case, (b)* b_k^* *decreases in* γ_k^A*; and (c)* a_k^* *increases in* γ_k^B*. Moreover, (d)* $|a_k^* - b_k^*|$ *increases in* $|\gamma_k^A - \gamma_k^B|$ *for any* $\rho \geq 0$ *and this effect is amplified by focusing.*

With rational voters, the opponent's competence does not affect the optimal quality because parties have a dominant strategy and the opponent's policy does not affect their best response. This is different with focusing voters, where both platforms concur to form voters' focusing weights. Suppose that, as in the statement of Corollary 3, party A has a competence disadvantage in policy k, that is, $\gamma_k^A < \gamma_k^B$. In equilibrium, A offers a worse policy than B. If the competence of B increases, its equilibrium policy increases and this increases the contrast between the parties' policies. The resulting increase in voters' attention to this policy generates an exogenous upward shock to the focusing-induced increase to quality effect and strengthens A's incentive to catch up with B (as voters are paying more attention to A's weakness in k). This is the reason why the equilibrium policy of party A *increases* in γ_k^B. Consider now an increase to the competence of A. This leads to an increase in a_k which decreases the contrast between policies (since, in equilibrium, A offers a worse policy than B). The resulting decrease in voters' attention to this policy amounts to an exogenous downward shocks to the focusing-induced increase to quality effect and weakens B's incentive to increase the gap with A (as voters are paying less attention to B's strength in k). This is the reason why the equilibrium policy of party B *decreases* in γ_k^A.

Corollary 4 (Effect of Focusing on Election Outcome)**.** *In equilibrium, the magnitude of the contribution of policy k to voters' overall evaluation of parties' platforms and, thus, to parties' vote shares (a) increases in σ_k if and only if $\rho > 0$; (b) increases in $|\gamma_k^A - \gamma_k^B|$ for any $\rho \geq 0$; and (c) both effects are amplified by focusing.*

As in the discussion for Corollary 3, continue to consider the case where B owns issue k (that is, $\gamma_k^A < \gamma_k^B$). In this case $a_k^* < b_k^*$ and, given the equilibrium policies, the contribution of policy k to A's vote share is given by:

$$\begin{aligned} &\phi\overline{\theta_k}\left(a_k^* - b_k^*\right) + \phi\rho\sigma_k|a_k^* - b_k^*|\left(a_k^* - b_k^*\right) \\ &\qquad = -\phi\overline{\theta_k}\left(\frac{\phi\overline{\theta_k}|\gamma_k^A - \gamma_k^B|}{1 - 2\phi\rho\sigma_k|\gamma_k^A - \gamma_k^B|}\right) - \phi\rho\sigma_k\left(\frac{\phi\overline{\theta_k}|\gamma_k^A - \gamma_k^B|}{1 - 2\phi\rho\sigma_k|\gamma_k^A - \gamma_k^B|}\right)^2. \end{aligned} \tag{14}$$

(Note that, if instead $\gamma_k^A > \gamma_k^B$, both terms in the expression above would have a positive sign). Equation (14) shows that the degree of polarization in policy k is the key element in determining the contribution of this policy to the parties' vote shares. In particular, when policy polarization in k increases, the party with a competence advantage in k gains votes to the expense of the opponent. This is because, from voters' point of view, increased policy polarization means a larger utility differential from the two platforms (in favor of the party with a competence advantage) as well as a larger focus weight on this policy and on this differential.

As shown in Corollary 2 and in Corollary 3, the equilibrium polarization in policy k increases in the heterogeneity of voters' preferences for k and in the competence wedge between parties in this policy. This means that, with focusing voters, more divisive policies and policies with a stronger issue ownership contribute more to parties' vote shares. While the latter is true also with rational voters, $|\gamma_k^A - \gamma_k^B|$ can have a dramatic impact on parties' electoral chances. As the difference in parties' competences approaches the upper bound implied by Assumption 1 and by the $\gamma_k^P > 0$ constraint—that is, as

$|\gamma_k^A - \gamma_k^B|$ approaches $1/(2\phi\rho\sigma_k)$—the equilibrium policy polarization goes to infinity. Therefore, if there exists a policy k that strongly stands out in terms of difference in parties' competence to tackle it, then this policy also stands out in terms of difference in equilibrium policies, possibly to the point where policy k is the main determinant of parties' electoral fortunes. In this case, the party that (almost surely) wins the election is the one with a competence advantage in policy k.

4 Third Candidate Entry

In this section, we study how the entry of a third party changes the electoral competition. We label this party C and its platform—$c = (c_1, c_2, \ldots, c_K)$, where $c_k \in [0, \infty)$—joins A's and B's platforms in voters' choice set, $\mathcal{P} = \{a, b, c\}$. When voters have undistorted focus, C affects the electoral competition only by stealing votes from the other parties. When voters focus, there is an additional channel: C can manipulate voters' attention. In order to understand this latter channel, we assume throughout that voters' *consideration set*—that is, the subset of available alternatives decision-makers consider (Hoyer, 1984; Hauser and Wernerfelt, 1990; Caplin, Dean and Leahy, 2019)—is $\{a, b\}$. This means that, as in the model from the previous section, voters choose between candidates A and B. One reason why voters might not see a third party as a viable option is that it is a "single-issue party" with a narrow platform offering a policy solution in a single domain. Examples of such parties include agrarian parties, environmental parties, anti-immigration parties, eurosceptic parties, and cannabis parties. Another reason is that, in plurality electoral systems, voters might anticipate this party has a negligible chance of winning and concentrate on the two major parties. This is an important and possibly unrealistic assumption, which, however, allows us to cleanly identify the effect third parties have on major parties' platforms and electoral chances *only through* the change in voters' attention to policies.

Initially, we treat C's platform as an exogenous parameter in the competition between A and B. Later in this election, we use the equilibrium to investigate the endogenous supply of policies by a strategic third party (which announces its platform before the major parties). Party A's objective function remains additively separable in policies and, as before, we focus on the problem of choosing the optimal quality in a single policy k. Suppressing all terms independent of a_k and b_k, A's objective function becomes

$$\phi\left(\overline{\theta_k} + \rho\sigma_k\left[\max\{a_k, b_k, c_k\} - \min\{a_k, b_k, c_k\}\right]\right) \cdot (a_k - b_k) - \frac{(a_k)^2}{2\gamma_k^A}. \tag{15}$$

The quality offered by the third party in policy k, c_k, contributes to determine the range of consumption utility and, thus, affects voters' focus weight on this policy. The contrast voters observe in policy k is determined by the largest and the smallest policy in $\mathcal{P}$ and, in general, differs from the contrast observed in the absence of the third party, $|a_k - b_k|$.

To understand how the third party affects A's incentives, consider how it changes its marginal benefit and marginal cost from a small increase in a_k. The marginal cost is unaltered and equals $\frac{a_k}{\gamma_k^A}$. Similarly, the rational quality effect is unchanged and is given by $\overline{\theta_k}$. At the same time, the entry of C changes the other two components of A's marginal benefit, the focusing-induced increase in quality effect and the attention manipulation effect. The nature of the change depends on the value of c_k. For a sufficiently small c_k (that is, $c_k < \min\{a_k, b_k\}$), the two effects are given by:

$$\begin{array}{cccl} \phi\rho|b_k - c_k|\sigma_k & + & 0 & \text{if } a_k < b_k \\ \underbrace{\phi\rho|a_k - c_k|\sigma_k}_{\substack{\text{Focusing-Induced Increase} \\ \text{in Quality Effect}}} & + & \underbrace{\phi\rho|a_k - b_k|\sigma_k}_{\substack{\text{Attention Manipulation} \\ \text{Effect}}} & \text{if } a_k > b_k. \end{array} \tag{16}$$

Since both $|b_k - c_k|$ and $|a_k - c_k|$ are greater than $|a_k - b_k|$, the addition of c_k enhances the focusing-induced increase in quality effect irrespective of whether a_k is greater or smaller than b_k. Keeping everything else constant, this means that voters' place a larger weight on k and that investing in a_k is rewarded with a larger increases in A's vote share. The impact of c_k on the attention manipulation effect depends on whether a_k is greater or smaller than b_k. When $a_k < b_k$, contrast is given by $|b_k - c_k|$ and increasing a_k by a small amount does not change voters' attention to policy k. In this case, the addition of c_k kills the attention manipulation effect, depressing A's incentive to invest in policy development. When $a_k > b_k$, instead, contrast is given by $|a_k - c_k|$ and the attention manipulation effect is unaffected by the addition of c_k.

For a sufficiently large c_k (that is, $c_k > \max\{a_k, b_k\}$), the two effects are given by:

$$\begin{array}{cccl} \phi\rho|c_k - a_k|\sigma_k & + & \phi\rho|a_k - b_k|\sigma_k & \text{if } a_k < b_k \\ \underbrace{\phi\rho|c_k - b_k|\sigma_k}_{\substack{\text{Focusing-Induced Increase} \\ \text{in Quality Effect}}} & + & \underbrace{0}_{\substack{\text{Attention Manipulation} \\ \text{Effect}}} & \text{if } a_k > b_k. \end{array} \tag{17}$$

As in the previous case, the entry of a third party enhances the focusing-induced increase in quality effect (as it increases contrast in policy k) and draws voters' attention to this policy. The impact of a large c_k on the attention manipulation effect is the opposite: it leaves this incentive unchanged for A when $a_k < b_k$ and it shuts it down when $a_k > b_k$.

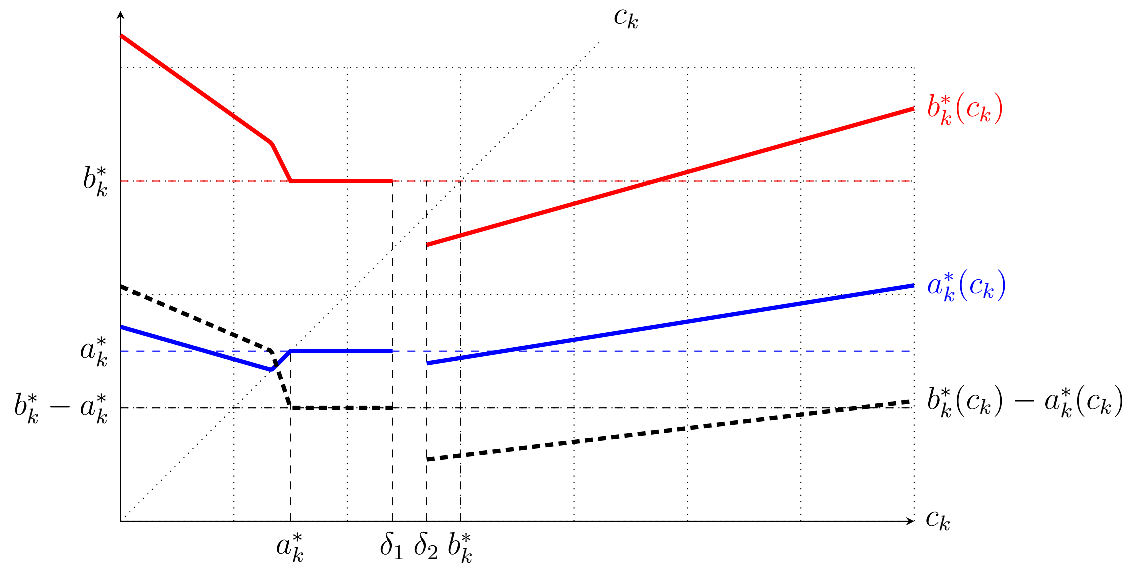
To summarize, the entry of a third party with a policy that either dominates or is dominated by the major parties' policies enhances the focusing-induced increase in quality effect, giving both parties a stronger incentive to invest in policy development. At the same time, the entrant suppresses the attention manipulation effect for the party with a mediocre policy (that is, a policy in between the two opponents' policies), weakening this party's incentive to invest in policy development.

These effects shape the equilibrium policies of parties A and B in elections with an exogenous platform c, as characterized in Proposition A1 in the Online Appendix and shown in Figure 1, where, without loss of generality, we assume $\gamma_k^B > \gamma_k^A$. In the picture, $a_k^*(c_k)$ and $b_k^*(c_k)$ denote equilibrium policies when C competes (as a function of its policy, c_k) while a_k^* and b_k^* denote the equilibrium policies in the absence of C.

For either small or large values of c_k, the equilibrium policies of both A and B are larger than without C—reflecting the enhanced focusing-induced increase in quality effect—and this increase is offset for the party with a mediocre policy, that is, for party A when c_k is low and for party B when c_k is large—reflecting the suppression of the attention manipulation effect.[13] The addition of a mediocre policy, that is, $c_k \in [a_k^*, b_k^*]$, does

[13]For expositional simplicity, our discussion neglects strategic effects: the optimal policy of a party

Figure 1: Equilibrium policies in the presence of third party with policy c_k



Note: Party B owns issue k, $\gamma_k^B > \gamma_k^A$. Third party with policy quality c_k changes the equilibrium policies from (a_k^*, b_k^*) given by Proposition 1 to $(a_k^*(c_k), b_k^*(c_k))$ given by Proposition A1.

not affect the major parties' equilibrium policies when c_k is close to a_k^* but can have a dramatic effect when c_k is close to b_k^*. This is again due to the effect c_k has on the contrast observed by voters in this policy and the suppression of the attention manipulation effect. Consider $c_k = b_k^*$. If B decreases b_k, voters' focus weights remain unchanged, as the presence of c_k keeps contrast in voters' choice set at $|c_k - a_k|$. Thus, B can save in policy development costs, without diverting voters' attention away from a strength. For a similar reason, when $c_k = b_k^* - \epsilon$, deviations to a policy cheaper than b_k^* are tempting for B because voters' attention to this policy will remain high (even if the two major parties become more similar to one another) thanks to C's proposal. The suppression of the attention manipulation effect leads to an equilibrium with B offering a smaller quality or to equilibrium non-existence.[14]

changes when its opponent's policy changes. For example, strategic interactions explains why, in Figure 1, $a_k^*(c_k) < a_k^*$ for $c_k \in [\delta_2, b_k^*]$ even if, in this interval, A's focusing-induced increase in quality effect is heightened and its attention manipulation effect is unchanged with respect to the case with two candidates: offering a lower policy than in elections without C is A's optimal response to $b_k^*(c_k) < b_k^*$.

[14]The addition of c_k introduces discontinuities to parties' best response functions and equilibria fail to exist when $c_k \in (\delta_1, \delta_2)$. As revealed by the expressions for δ_1 and δ_2 in Proposition A1, $\delta_1 \in (a_k^*, b_k^*)$

We can now investigate the endogenous supply of policies by a strategic third party. For simplicity, we assume that C announces its platform before the major parties. We consider two potential objectives by C. First, decreasing the equilibrium policy or the equilibrium vote shares of the party that owns the issue. Second, increasing the equilibrium policy or the equilibrium vote shares of the same party.

Proposition 2. *Suppose Assumption 1, $\rho > 0$, and $\gamma_k^A < \gamma_k^B$. Consider a third party whose platform can affect voters' focusing weight on policy k. If its goal is to either decrease B's equilibrium vote share or B's equilibrium policy, it finds it optimal to either enter with quality $c_k = \delta_2 \in (a_k^*, 2b_k^*)$ or to stay out, depending on γ_k^C. If its goal is to increase these outcomes, then it finds it optimal to enter with either $c_k = 0$ or some $c_k \in (\delta_2, \infty)$, depending on γ_k^C. Moreover, the impact of entry grows with ρ and σ_k: for both sufficiently large and sufficiently small c_k, $a_k^*(c_k)$, $b_k^*(c_k)$ and $b_k^*(c_k) - a_k^*(c_k)$ are linear functions of c_k which become steeper when ρ or σ_k increases.*

The first objective can be achieved only with a set of moderate policies and the smallest (and, thus, cheapest to develop) policy in this set is the most effective. When C's competence in this issue (or the weight on influencing electoral outcomes in C's objective function) is sufficiently large, C prefers to enter and to suffer the cost to develop $c_k = \delta_2 \in (a_k^*, 2b_k^*)$.[15] This policy is in a neighbourhood of the policy offered by the leading candidate in the absence of a third party and, thus, here C acts as the classical "spoiler candidate." Our model highlights that a minor candidate offering policies similar to a major candidate can reduce the major candidate's electoral chances also because of the psychology of voters' attention and not just because of vote splitting. The second objective can be reached with either the smallest possible quality or with a sufficiently large quality. These policies leverage the focusing-induced increase in quality effect that

and $\delta_1 < \delta_2$ for all parameter values. In Figure 1, $\delta_2 < b_k^*$ but this is not generally true.

[15] B's equilibrium vote share is determined by the equilibrium contrast between A and B and by the equilibrium contrast in the whole choice set. When C enters with $c_k = \delta_2$, the former decreases and the latter might increases, but the first effect dominates.

pushes both major parties' qualities upwards. While larger qualities are more effective in shaping electoral outcomes, they are also more expensive than $c_k = 0$. Thus, what policy C enters with depends on γ_k^C.

Proposition 2 also shows that the impact of a third party which enters with an extreme policy grows in voters' degree of focusing and in the heterogeneity of preferences for policy k. Indeed, as these variables increase, the major parties' equilibrium qualities become more sensitive to changes in c_k and, hence, entry is more consequential in terms of shaping these parties' platforms and their electoral chances. The model, thus, predicts that third parties are more likely to enter the arena with proposals on more divisive issues.

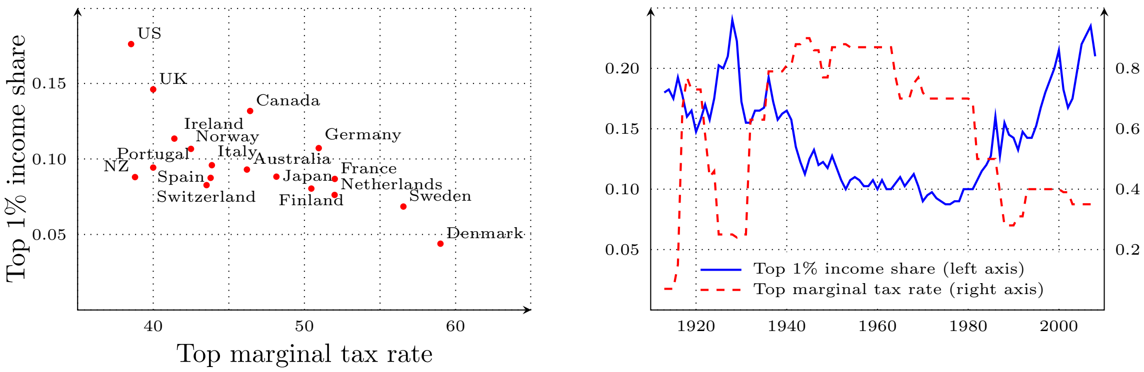
5 Application to Redistribution

Theories of choice-set effects or bottom-up attention have bite when decision-makers choose among options with multiple dimensions. In the model presented above, we considered the problem of voters who evaluate a *platform* composed of multiple *policies*. However, voters can face another multidimensional problem: evaluating a *policy* which generates multiple *consequences*.

For example, increasing redistribution with a higher marginal income tax rate for all income earners means both a reduction in disposable income as well as an increase in the public goods the larger tax revenues will fund; imposing price ceilings can lead to a reduction in inflation but also to supply shortages; increasing visas for foreign workers can make social welfare more sustainable in ageing societies but also increase social turmoil. Indeed, there is ample evidence that voters have a hard time taking into account and properly aggregating all the consequences of a single policy, even in simple environments: Dal Bó, Dal Bó and Eyster (2018) show that participants to a laboratory experiment who are asked to vote in favor or against a reform changing a game's payoff focus on the direct consequences (that is, the change in payoffs for each outcome) and ignore the indirect consequences.(that is, how the change in payoffs will change behavior and, thus, what outcomes are more likely to occur).

Figure 2: Top 1% income share and top marginal tax rate

(a) International (2005-2009 or most recent)   (b) US



Note: Data courtesy of Piketty, Saez and Stantcheva (2014) (see their paper for original sources). Income excludes government transfers and is before individual taxes.

In this section, we propose an alternative approach to introducing bottom-up attention into formal models of politics meant to capture the complexity of this second aggregation problem. In line with the attention distortion we model in the rest of the paper, we continue to assume that what makes a consequence more or less salient is the contrast voters observe in their choice set with respect to this consequence. In particular, we introduce focusing voters in a basic model of fiscal policy (see Persson and Tabellini, 2000, Chapter 3) where candidates choose a level of redistribution and voters can focus on the benefit of redistribution—that is, the greater level of public goods provided by the government—or on its cost—that is, the greater taxation required to finance it.

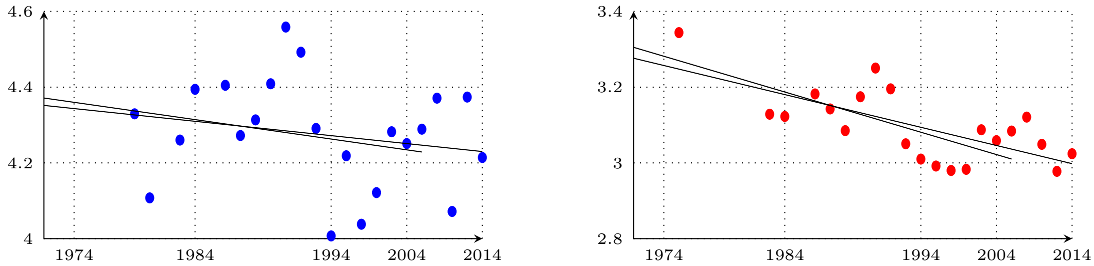
In the last 30 years, the U.S. (as well as other developed economies) have experienced a rapid and sustained increase in the degree of income inequality (see Figure 2b). Contrary to the predictions of standard political economy models, this trend has not been accompanied by an increased demand for redistribution (see Figure 3) or by more redistributive policies (see Figure 2b). To the contrary, the data points to an inverse correlation between these time series. We observe a similar inverse correlation between income inequality and redistribution in a cross-country perspective (see Figure 2a). In this section, we aim at answering the following questions: What is the impact of voters' distorted attention on parties' proposals to redistribute? Can distorted attention help us explain the puzzling empirical patterns from Figures 2 and 3?

Consider a public good, $g \in \mathbb{R}_+$, which is financed by a linear income tax, $\tau \in [0, 1]$.

Figure 3: Preferences for redistribution in General Social Survey (GSS)

(a) Government should reduce income differences (1-7)

(b) Government should improve the standard of living of poor Americans (1-5)



Note: GSS obtained from http://gss.norc.org/. Variables rescaled so that larger values correspond to stronger support for redistribution. Shorter trend ends in 2006. Left panel: Average of *eqwlth* variable. Both trends insignificant. Right panel: average of *helppoor* variable. Both trends significant at 1%. See Ashok, Kuziemko and Washington (2015) for a thorough analysis of the data.

Society is composed of two groups of voters, R for Rich and P for Poor, with different income: $y_R > y_P > 0$. The fraction of voters in group $i \in \{R, P\}$ is $m_i \in (0,1)$. The average income in society is $\overline{y} = m_R y_R + m_P y_P$. Given public good g and tax rate τ, the consumption utility of voters in group i is:

$$V_i(g, \tau) = (1 - \tau) y_i + u(g) \tag{18}$$

where $u : \mathbb{R}_+ \to \mathbb{R}_+$ maps the level of public good provision into its benefits.

Two political parties, A and B, compete in an election by offering binding platforms (g_A, τ_B) and (g_B, τ_B). For each voter, each parties' proposed redistribution plan has a benefit, namely, the level of public good produced, and a cost, namely, the taxed paid to finance it. To aggregate these two consequences and determine the overall worth of the two platforms, voters use voting weights which depend on the range of consumption utility they observe in each consequence in their choice set. In particular, when their choice set is $\mathcal{P} = \{(g_A, \tau_A), (g_B, \tau_B)\}$, the focus-weighted utility voters in group i derive from platform (g, τ) is:

$$\widetilde{V}_i((g, \tau), \mathcal{P}) = g(\Delta_{i\tau}(\mathcal{P})) \cdot (1 - \tau) y_i + g(\Delta_{ig}(\mathcal{P})) \cdot u(g). \tag{19}$$

The weight voters in group i place on post-tax income $(1-\tau)y_i$ is determined by the range of this term in their consumption utility, which equals $\Delta_{i\tau}(\mathcal{P}) = |(1-\tau_A)y_i - (1-\tau_B)y_i|$, whereas the weight they place on the utility from public good provision $u(g)$ is determined by the range of this term in their consumption utility, which equals $\Delta_{ig}(\mathcal{P}) = |u(g_A) - u(g_B)|$. As before, we assume $g(\Delta_{ik}(\mathcal{P})) = 1 + \rho\Delta_{ik}(\mathcal{P})$.

The two parties differ in how efficient they are at providing public goods, as measured by $\gamma^P \geq 0$ for party $P \in \{A, B\}$. A lower γ^P constraints the ability of party P to provide public goods for a given amount of tax revenue because platforms have to satisfy:

$$g_P = \tau_P \overline{y} \gamma^P. \tag{20}$$

This implies that the amount of public goods party P can provide is at most $\overline{y}\gamma^P$.

Parties are office-motivated and maximize their vote shares in a probabilistic voting model as the one we introduced above. The parties' objective functions, given their platforms (g_A, τ_A) and (g_B, τ_B) and choice set $\mathcal{P} = \{(g_A, \tau_A), (g_B, \tau_B)\}$, are:

$$\begin{aligned} \pi_A((g_A, \tau_A), (g_B, \tau_B), \mathcal{P}) &= \tfrac{1}{2} + \phi \sum_{i \in \{P,R\}} m_i \left[\widetilde{V}_i((g_A, \tau_A), \mathcal{P}) - \widetilde{V}_i((g_B, \tau_B), \mathcal{P})\right] \\ \pi_B((g_A, \tau_A), (g_B, \tau_B), \mathcal{P}) &= 1 - \pi_A((g_A, \tau_A), (g_B, \tau_B), \mathcal{P}). \end{aligned} \tag{21}$$

We solve the model assuming that party B's ability to produce public goods is low enough for it not to provide any public good whatsoever ($\gamma^B = 0$). While this is certainly a simplification, it allows us to understand how voters' distorted attention affects parties' incentives to redistribute and how these incentives change with income inequality sidestepping the issue of equilibrium existence and uniqueness (which is exacerbated by the malleable preferences of focusing voters). These incentives arise cleanly in our simple model and would be at work even in a more complex model where both parties offer a positive amount of redistribution. Given this assumption, party B has a dominant strategy to offer ($g_B^* = 0, \tau_B^* = 0$) and we focus on the best response of party A to an opponent who provides no public goods and raises no taxes.

To ensure party A's objective function is strictly concave and its equilibrium policy is interior we make the following assumption on $u(\cdot)$: (i) $u(g)$ is twice continuously differentiable and strictly increasing (ii) $u(0) = 0$, (iii) for some $b < 0$, $u''(g) < b$ for any feasible g, (iv) $u'(0) > 1/\gamma^A > u'(\overline{y}\gamma^A)$.

As a benchmark, we first consider rational voters (that is, $\rho = 0$). The optimal platform in this case does not depend on the opponent's platform. We replace the budget constraint (20) in π_A and express A's objective only in terms of g_A:

$$\phi\left[u(g_A) - g_A/\gamma^A\right]. \tag{22}$$

The equilibrium platform with rational voters balances the average marginal benefit, $\phi u'(g)$, against the average marginal cost, ϕ/γ^A. These two terms are invariant to the income distribution as well as to population shares. Thus, in this case, these two variables have no impact on the equilibrium level of public good provision.[16] When voters focus, instead, the equilibrium redistribution depends on income inequality.

Proposition 3. *There exists $\overline{\rho} > 0$ such that, for any $\rho \in (0, \overline{\rho})$, an equilibrium platform of party A, (g_A^*, τ_A^*), exists, is unique, and is interior, that is, $g_A^* \in (0, \overline{y}\gamma^A)$. A mean preserving spread in income strictly decreases both g_A^* and τ_A^*.*[17]

To understand the intuition behind this result, note that, using the budget constraint

[16]Note that the stylized facts from Figures 2 and 3 are also inconsistent with another workhorse model of electoral competition, the median voter model (Downs, 1957). The median voter model obtains as a special case of the probabilistic voting model when $\epsilon_v = 0$ for all voters. In this case, the equilibrium platform maximizes the consumption utility of the median voters who, with two social groups, belongs to the larger group. If we assume that P voters are the majority and R voters are an elite, that is, $m_R < 1/2$, the equilibrium g_A, which maximizes $u(g_A) - g_A y_P/\overline{y}\gamma^A$, is increasing in income inequality. In short, in the median voter model, larger income inequality leads to larger redistribution.

[17]With a mean preserving spread in income, we refer to an increase in y_R by some $\varepsilon/m_R > 0$ and a simultaneous decrease in y_P by ε/m_P. This is a change that leaves the average income unchanged. The additional advantage of modeling increasing income inequality as a mean preserving spread is that it leaves the tax revenue for given tax rate τ unchanged. An increase in the equilibrium provision of public goods, thus, must be accompanied by an increase in the equilibrium taxation.

to express platforms only in terms of public goods, voter i observes the following range in post-tax income: $y_i\left(1-\frac{g_B^*}{\bar{y}\gamma^B}\right)-y_i\left(1-\frac{g_A^*}{\bar{y}\gamma^A}\right)=y_i\left(\frac{g_A^*}{\bar{y}\gamma^A}-\frac{g_B^*}{\bar{y}\gamma^B}\right)$. For a given pair of platforms, this range is proportional to y_i. On the other hand, the range in public good consumption, $u(g_A^*)-u(g_B^*)$, does not depend on voter i's income. Consider a mean preserving spread in income. This increases the marginal cost of redistribution for R voters and decreases the marginal cost of redistribution for P voters. When voters have undistorted attention, the votes A would gain among P voters with a marginal increase in g_A are perfectly offset by the votes A would lose among R voters. However, the mean preserving spread in income increases the range R voters observe in post-tax income and decreases the range P voters observe in this same dimension. This means that R voters place a larger weight than before the income shock and P voters place a smaller weight than before the income shock to the cost from redistribution. Since the range in public good consumption utility is unaffected by incomes, both social groups place the same weight as before on the benefit from redistribution.

In other words, R voters overweight the increase in their marginal cost of redistribution induced by the upward shock to inequality while P voters underweight the decrease in their marginal cost. In sum, an increase in income inequality amplifies rich voters' marginal sensitivity to taxation, reduces poor voters' marginal sensitivity to taxation and leaves unchanged all voters' marginal sensitivity to public good consumption. This means that, if A proposes a marginal increase in redistribution, the degree to which A gains poor voters does not compensate the degree to which A loses rich voters. As a consequence, rich voters become more influential regardless of their relative size. The psychology of voters' attention can, thus, explain why increased income inequality is associated with constant or decreasing demand for redistribution and, hence, with constant or decreasing implemented levels of redistribution.[18]

[18]The main alternative explanations for the observed correlations (or lack thereof) between income inequality and redistribution are stronger political participation or lobbying by the wealthy (Benabou, 2000), the prospect of upward mobility (Benabou and Ok, 2001), other-regarding preferences (Galasso, 2003), and changes in patterns of social identity (Shayo, 2009). Most of these explanations attenuate

6 Conclusion

How voters allocate their attention is fundamental for understanding political preferences and public policies. Cognitive psychology has pointed to two complementary mechanisms: a goal-driven and ex-ante allocation of attention that is driven by preferences (also called "top-down attention" or "rational inattention") and a stimulus-driven and ex-post allocation of attention that shapes preferences (also called "bottom-up attention"). While the existing literature in political economy has centered on the former, this is the first paper to explore the latter.

We introduce bottom-up attention in a formal model of electoral competition by assuming that, in forming their perception of electoral platforms, voters' attention is attracted by the policies in which candidates' platforms differ more and that, in turn, they weigh disproportionately the policies they focus on. We show that politicians facing focusing voters have greater incentives to develop better policies, especially in issues where preferences are more dispersed (that is, more divisive issues) or where the competence gap between parties is more pronounced (that is, issues with stronger *ownership*); and that voters' distorted attention can contribute to explain puzzling empirical patterns, as the entry of third parties with no electoral chances or the inverse correlation between income inequality and redistribution.

While we explored the consequences of one well-documented attentional distortion on electoral competition, incorporating the psychology of voters' attention in models of campaign rhetoric and agenda setting is likely to generate novel insights. For example, in a monopolistic agenda setting model à la Romer and Rosenthal (1978, 1979), the agenda setter could propose multiple reforms of the status quo in order to affect the focus of its bargaining counterpart (e.g., the median voter or a veto holder) and expand the set of acceptable reforms to his advantage.

Moreover, future research should explore the implications of different models of bottom-

the positive relationship between redistribution and income inequality predicted by political economy models of fiscal policy with rational voters, rather than reversing it. See Borck (2007) for a survey.

up attention for political behavior. For example, Bushong, Rabin and Schwartzstein (2021) propose a model of *relative thinking* where decision makers place a *smaller* weight on a dimension when there is more contrast among alternatives and offer experimental evidence consistent with this hypothesis (see also Somerville, 2022). The authors conjecture that what drives relative thinking in their (as well as in Somerville's) experiment is the *simplicity* of their decision environment, in contrast with the *complexity* of the decision environments in the experiments reporting evidence of focusing (Bondi, Csaba and Friedman, 2018; Castillo, 2020; Andersson, Carlson and Wengström, 2021; Dertwinkel-Kalt et al., 2022). Since we believe that choosing over candidates in an election is more complex than choosing over consumer products, here we assumed focusing rather than relative thinking but it would be interesting to investigate how relative-thinking voters change the electoral calculus of office motivated politicians. More generally, there are many exciting open questions, as what exact features of the political environment trigger voters' attention, how the unconscious allocation of scarce cognitive resources interacts with the conscious search for information by poorly informed voters, and how framing by candidates and the media can shape what dimensions voters evaluate policies on.

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A Online Appendix

Proof of Lemma 1

Assume $K = 2$. Fix $i \in N$, $a, b \in \mathbb{R}^2_+$, and let $\mathcal{P} = \{a, b\}$. Let $d_1 = u_{i1}(a_1) - u_{i1}(b_1)$ and $d_2 = u_{i2}(a_2) - u_{i2}(b_2)$. Then $\delta_{ab} = V_i(a) - V_i(b) = d_1 + d_2$ and $\widetilde{\delta}_{ab} = \widetilde{V}_i(a, \mathcal{P}) - \widetilde{V}_i(b, \mathcal{P}) = g(|d_1|)d_1 + g(|d_2|)d_2$. Note that $g(x) \geq 1$ for any $x \geq 0$ and $g(x) > 1$ for any $x > 0$ if g is strictly increasing.

If $d_1 = d_2 = 0$, then $\delta_{ab} = \widetilde{\delta}_{ab} = 0$, and the lemma follows. If $d_1 \neq 0$ and $d_2 = 0$, then $\delta_{ab} = d_1$ and $\widetilde{\delta}_{ab} = g(|d_1|)d_1$, and the lemma follows. If $d_1 = 0$ and $d_2 \neq 0$, the argument is similar and omitted. If $d_1 > 0$ and $d_2 > 0$, or if $d_1 < 0$ and $d_2 < 0$, the lemma follows because $\widetilde{\delta}_{ab} = \delta_{ab} \frac{g(|d_1|)d_1 + g(|d_2|)d_2}{d_1 + d_2}$, where the fraction is at least unity (strictly above unity if g is strictly increasing).

If $d_1 > 0$ and $d_2 < 0$, or if $d_1 < 0$ and $d_2 > 0$, we need to consider three cases. First, if $|d_1| = |d_2|$, then $\delta_{ab} = \widetilde{\delta}_{ab} = 0$, and the lemma follows. Second, if $|d_1| > |d_2|$, rewrite

$$\widetilde{\delta}_{ab} = g(|d_1|)d_1 + g(|d_2|)d_2 = g(|d_1|)\delta_{ab} + d_2(g(|d_2|) - g(|d_1|)). \tag{A1}$$

From $|d_1| > |d_2|$, both $\delta_{ab} = d_1 + d_2$ and $d_2(g(|d_2|) - g(|d_1|))$ have the same sign as d_1. Thus $\widetilde{\delta}_{ab} \geq g(|d_1|)\delta_{ab} \geq \delta_{ab} > 0$ if $d_1 > 0$, and $\widetilde{\delta}_{ab} \leq g(|d_1|)\delta_{ab} \leq \delta_{ab} < 0$ if $d_1 < 0$. In both cases, the second inequality is strict when g is strictly increasing. The lemma thus follows. Third, if $|d_1| < |d_2|$, the argument is similar and omitted. □

Proof of Proposition 1

By Assumption 1, $2\phi\rho\sigma_k < 1/\gamma_k^P$ for each issue k and both parties P. Fix issue k. The payoff of party with competence $\gamma_k \in \{\gamma_k^A, \gamma_k^B\}$ from contesting the election, on issue k, with policy quality $x \geq 0$ when its opponent runs with policy quality $y \geq 0$, suppressing all constants, is

$$\pi_{\gamma_k}(x, y) = \phi\overline{\theta_k}(x - y) + \phi\rho\sigma_k|x - y|(x - y) - \frac{x^2}{2\gamma_k} \tag{A2}$$

where $\phi\overline{\theta_k} > 0$ and $\phi\rho\sigma_k \geq 0$. The best response of party with competence γ_k to $y \geq 0$ is $br_{\gamma_k}(y) = \arg\max_{x\geq 0} \pi_{\gamma_k}(x, y)$.

The derivative of π_{γ_k} with respect to x for any $x \neq y$ is standard. At $x = y$, the left derivative, $\phi\overline{\theta_k} - \frac{y}{\gamma_k}$, equals the right derivative, and we have

$$\frac{\partial \pi_{\gamma_k}(x,y)}{\partial x} = \begin{cases} \phi\overline{\theta_k} + 2\phi\rho\sigma_k(y-x) - \frac{x}{\gamma_k} & \text{if } x \leq y \\ \phi\overline{\theta_k} + 2\phi\rho\sigma_k(x-y) - \frac{x}{\gamma_k} & \text{if } x > y. \end{cases} \tag{A3}$$

The derivative is strictly positive at $x = 0$, continuous in x, and strictly decreasing in x because $2\phi\rho\sigma_k < 1/\gamma_k$, and thus equals zero for a unique x^*. When $y < \phi\overline{\theta_k}\gamma_k$, then $\left.\frac{\partial \pi_{\gamma_k}(x,y)}{\partial x}\right|_{x=y} < 0$, and thus $\phi\overline{\theta_k} + 2\phi\rho\sigma_k(x^* - y) - \frac{x^*}{\gamma_k} = 0$, or, equivalently, $x^* = \frac{\phi\overline{\theta_k} - 2\phi\rho\sigma_k y}{\frac{1}{\gamma_k} - 2\phi\rho\sigma_k}$. When $y = \phi\overline{\theta_k}\gamma_k$, then $\left.\frac{\partial \pi_{\gamma_k}(x,y)}{\partial x}\right|_{x=y} = 0$, and thus $x^* = \phi\overline{\theta_k}\gamma_k$. When $y > \phi\overline{\theta_k}\gamma_k$, then $\left.\frac{\partial \pi_{\gamma_k}(x,y)}{\partial x}\right|_{x=y} < 0$, and thus $\phi\overline{\theta_k} + 2\phi\rho\sigma_k(y - x^*) - \frac{x^*}{\gamma_k} = 0$, or, equivalently, $x^* = \frac{\phi\overline{\theta_k} + 2\phi\rho\sigma_k y}{\frac{1}{\gamma_k} + 2\phi\rho\sigma_k}$. In summary,

$$br_{\gamma_k}(y) = \begin{cases} \frac{\phi\overline{\theta_k} - 2\phi\rho\sigma_k y}{\frac{1}{\gamma_k} - 2\phi\rho\sigma_k} & \text{if } y \leq \phi\overline{\theta_k}\gamma_k \\ \frac{\phi\overline{\theta_k} + 2\phi\rho\sigma_k y}{\frac{1}{\gamma_k} + 2\phi\rho\sigma_k} & \text{if } y > \phi\overline{\theta_k}\gamma_k. \end{cases} \tag{A4}$$

When $2\phi\rho\sigma_k = 0$, the unique best response of party A on issue k is $\phi\overline{\theta_k}\gamma_k^A$ and the unique best response of party B on issue k is $\phi\overline{\theta_k}\gamma_k^B$. In this case, $(\phi\overline{\theta_k}\gamma_k^A, \phi\overline{\theta_k}\gamma_k^B)$ constitutes a unique Nash equilibrium in pure strategies. Following lemma helps us to identify equilibria in the $2\phi\rho\sigma_k > 0$ case, which we consider next.

Lemma A1. *Consider two best-response correspondences $r_1 : \mathbb{R}_+ \twoheadrightarrow \mathbb{R}_+$ and $r_2 : \mathbb{R}_+ \twoheadrightarrow \mathbb{R}_+$. A Nash equilibrium is $(a, b) \in \mathbb{R}_+^2$ such that $a \in r_1(b)$ and $b \in r_2(a)$. Suppose, for each $i \in \{1, 2\}$, that (i) r_i admits a unique $m_i \geq 0$ such that $m_i \in r_i(m_i)$, (ii) $b_x \in r_i(x)$ implies $b_x \geq m_i$ $\forall x \in [0, m_i)$, and (iii) on $[m_i, \infty)$, r_i is a function such that $r_i(x) \in (m_i, x)$ $\forall x > m_i$. If (a, b) is a Nash equilibrium, then $a \geq m_1$, $b \geq m_2$, $a \leq m_2$ if $m_1 \leq m_2$, and $b \leq m_1$ if $m_2 \leq m_1$.*

Proof. Suppose $(a, b) \in \mathbb{R}_+^2$ constitutes a Nash equilibrium. Note that $m_i \leq b_x \in r_i(x)$ $\forall x \in \mathbb{R}_+$ and $\forall i \in \{1, 2\}$. Thus $a \geq m_1$ and $b \geq m_2$. Consider $m_1 \leq m_2$. The argument for $m_2 \leq m_1$ is analogous and omitted. Suppose, towards a contradiction, that

$a > m_2$. Because $a > m_2$, we have $b = r_2(a) \in (m_2, a)$. Thus $b > m_2 \geq m_1$ and hence $a = r_1(b) \in (m_1, b)$. Therefore, $b < a$ and $a < b$, which is a contradiction. □

Because $2\phi\rho\sigma_k > 0$, Lemma A1 applies to best responses $br_{\gamma_k^A}$ and $br_{\gamma_k^B}$ of the two parties with $m_1 = \phi\overline{\theta_k}\gamma_k^A$ and $m_2 = \phi\overline{\theta_k}\gamma_k^B$. If $\gamma_k^A = \gamma_k^B$, by direct substitution into (A4), $(\phi\overline{\theta_k}\gamma_k^A, \phi\overline{\theta_k}\gamma_k^B)$ constitutes a Nash equilibrium in pure strategies, which is unique by Lemma A1 because $m_1 = m_2$ in this case. If $\gamma_k^A < \gamma_k^B$, by Lemma A1, if (a_k^*, b_k^*) constitutes a Nash equilibrium in pure strategies, then $b_k^* \geq m_2 = \phi\overline{\theta_k}\gamma_k^B > \phi\overline{\theta_k}\gamma_k^A$ and $a_k^* \leq m_2 = \phi\overline{\theta_k}\gamma_k^B$. (a_k^*, b_k^*) thus solves the following system of equations

$$a_k^* = \frac{\phi\overline{\theta_k} + 2\phi\rho\sigma_k b_k^*}{\frac{1}{\gamma_k^A} + 2\phi\rho\sigma_k} \qquad b_k^* = \frac{\phi\overline{\theta_k} - 2\phi\rho\sigma_k a_k^*}{\frac{1}{\gamma_k^B} - 2\phi\rho\sigma_k} \tag{A5}$$

and thus

$$a_k^* = \frac{\phi\overline{\theta_k}\gamma_k^A}{1 - 2\phi\rho\sigma_k(\gamma_k^B - \gamma_k^A)} \qquad b_k^* = \frac{\phi\overline{\theta_k}\gamma_k^B}{1 - 2\phi\rho\sigma_k(\gamma_k^B - \gamma_k^A)}. \tag{A6}$$

The proposition follows because $\gamma_k^B - \gamma_k^A = |\gamma_k^B - \gamma_k^A|$ when $\gamma_k^A < \gamma_k^B$. The argument for $\gamma_k^A > \gamma_k^B$ is analogous and omitted. □

Proof of Corollary 1

From Proposition 1, we have $a_k^* = \frac{\phi\overline{\theta_k}\gamma_k^A}{1-2\phi\rho\sigma_k|\gamma_k^A-\gamma_k^B|}$ and $b_k^* = \frac{\phi\overline{\theta_k}\gamma_k^B}{1-2\phi\rho\sigma_k|\gamma_k^A-\gamma_k^B|}$. Differentiating a_k^* with respect to ρ and σ_k, whenever $\gamma_k^A \neq \gamma_k^B$, we have $\frac{\partial a_k^*}{\partial \rho} = \frac{2\phi^2\overline{\theta_k}\gamma_k^A\sigma_k|\gamma_k^A-\gamma_k^B|}{[1-2\phi\rho\sigma_k|\gamma_k^A-\gamma_k^B|]^2} > 0$ and $\frac{\partial a_k^*}{\partial \sigma_k} = \frac{2\phi^2\overline{\theta_k}\gamma_k^A\rho|\gamma_k^A-\gamma_k^B|}{[1-2\phi\rho\sigma_k|\gamma_k^A-\gamma_k^B|]^2} > 0$ when $\rho > 0$. Similarly, for b_k^*, whenever $\gamma_k^A \neq \gamma_k^B$, we have $\frac{\partial b_k^*}{\partial \rho} = \frac{2\phi^2\overline{\theta_k}\gamma_k^B\sigma_k|\gamma_k^A-\gamma_k^B|}{[1-2\phi\rho\sigma_k|\gamma_k^A-\gamma_k^B|]^2} > 0$ and $\frac{\partial b_k^*}{\partial \sigma_k} = \frac{2\phi^2\overline{\theta_k}\gamma_k^B\rho|\gamma_k^A-\gamma_k^B|}{[1-2\phi\rho\sigma_k|\gamma_k^A-\gamma_k^B|]^2} > 0$ when $\rho > 0$. □

Proof of Corollary 2

From Proposition 1, we have $a_k^* = \frac{\phi\overline{\theta_k}\gamma_k^A}{1-2\phi\rho\sigma_k|\gamma_k^A-\gamma_k^B|}$ and $b_k^* = \frac{\phi\overline{\theta_k}\gamma_k^B}{1-2\phi\rho\sigma_k|\gamma_k^A-\gamma_k^B|}$, and thus $|a_k^* - b_k^*| = \frac{\phi\overline{\theta_k}|\gamma_k^A-\gamma_k^B|}{1-2\phi\rho\sigma_k|\gamma_k^A-\gamma_k^B|}$. Differentiating $|a_k^* - b_k^*|$ with respect to ρ and σ_k, whenever $\gamma_k^A \neq \gamma_k^B$, we have $\frac{\partial|a_k^*-b_k^*|}{\partial \rho} = \frac{2\phi^2\overline{\theta_k}\sigma_k|\gamma_k^A-\gamma_k^B|^2}{[1-2\phi\rho\sigma_k|\gamma_k^A-\gamma_k^B|]^2} > 0$ and $\frac{\partial|a_k^*-b_k^*|}{\partial \sigma_k} = \frac{2\phi^2\overline{\theta_k}\rho|\gamma_k^A-\gamma_k^B|^2}{[1-2\phi\rho\sigma_k|\gamma_k^A-\gamma_k^B|]^2} > 0$ when $\rho > 0$. □

Proof of Corollary 3

From Proposition 1, we have $a_k^* = \frac{\phi\overline{\theta_k}\gamma_k^A}{1-2\phi\rho\sigma_k(\gamma_k^B-\gamma_k^A)}$ and $b_k^* = \frac{\phi\overline{\theta_k}\gamma_k^B}{1-2\phi\rho\sigma_k(\gamma_k^B-\gamma_k^A)}$, where we have substituted $|\gamma_k^A-\gamma_k^B| = \gamma_k^B-\gamma_k^A$ implied by $\gamma_k^A < \gamma_k^B$. Part (a) follows because $a_k^* = \phi\overline{\theta_k}\gamma_k^A$ and $b_k^* = \phi\overline{\theta_k}\gamma_k^B$ when $\rho = 0$. Taking the derivative of b_k^* with respect to γ_k^A, we have $\frac{\partial b_k^*}{\partial\gamma_k^A} = \frac{-2\phi^2\overline{\theta_k}\rho\sigma_k\gamma_B^k}{[1-2\phi\rho\sigma_k(\gamma_k^B-\gamma_k^A)]^2} < 0$ when $\rho > 0$, proving part (b). Taking the derivative of a_k^* with respect to γ_k^B, we have $\frac{\partial a_k^*}{\partial\gamma_k^B} = \frac{2\phi^2\overline{\theta_k}\rho\sigma_k\gamma_A^k}{[1-2\phi\rho\sigma_k(\gamma_k^B-\gamma_k^A)]^2} > 0$ when $\rho > 0$, proving part (c). From Corollary 2, we have $|a_k^* - b_k^*| = \frac{\phi\overline{\theta_k}|\gamma_k^A-\gamma_k^B|}{1-2\phi\rho\sigma_k|\gamma_k^A-\gamma_k^B|}$, and thus $\frac{\partial|a_k^*-b_k^*|}{\partial|\gamma_k^A-\gamma_k^B|} = \frac{\phi\overline{\theta_k}}{[1-2\phi\rho\sigma_k|\gamma_k^A-\gamma_k^B|]^2} > 0$, where the derivative is clearly increasing in ρ, proving part (d). □

Proof of Corollary 4

In equilibrium, the contribution of policy k to the vote share of party A is $\phi\overline{\theta_k}(a_k^* - b_k^*) + \phi\rho\sigma_k|a_k^*-b_k^*|(a_k^*-b_k^*)$ and to the vote share of party B is $\phi\overline{\theta_k}(b_k^*-a_k^*)+\phi\rho\sigma_k|a_k^*-b_k^*|(b_k^*-a_k^*)$. The size of both terms is strictly increasing in $|a_k^* - b_k^*|$. The derivative of $|a_k^* - b_k^*|$ with respect to σ_k is $\frac{\partial|a_k^*-b_k^*|}{\partial\sigma_k} = \frac{2\phi^2\overline{\theta_k}\rho|\gamma_k^A-\gamma_k^B|^2}{[1-2\phi\rho\sigma_k|\gamma_k^A-\gamma_k^B|]^2}$, which, whenever $\gamma_k^A \neq \gamma_k^B$, equals zero when $\rho = 0$ and is strictly positive when $\rho > 0$, proving part (a). The derivative of $|a_k^* - b_k^*|$ with respect to $|\gamma_k^A - \gamma_k^B|$ is $\frac{\partial|a_k^*-b_k^*|}{\partial|\gamma_k^A-\gamma_k^B|} = \frac{\phi\overline{\theta_k}}{[1-2\phi\rho\sigma_k|\gamma_k^A-\gamma_k^B|]^2} > 0$, proving part (b). Both derivatives are clearly increasing in ρ, proving part (c). □

Proof of Proposition 2

Suppose Assumption 1 and $\rho > 0$. Consider a third party with platform $c_k \geq 0$ on policy k, and assume that $\gamma_k^A < \gamma_k^B$. The equilibrium policies $a_k^*(c_k)$ and $b_k^*(c_k)$ are characterized in Proposition A1, and we use the notation $\phi_k = \phi\overline{\theta_k}$ and $\rho_k = 2\phi\rho\sigma_k$ of that proposition. Denote by $C_1 = \left[0, \frac{\phi_k\gamma_k^A(1-\frac{\rho_k}{2}\gamma_k^B)}{1-\rho_k\gamma_k^B+\frac{\rho_k}{2}\gamma_k^A}\right)$, $C_2 = \left[\frac{\phi_k\gamma_k^A(1-\frac{\rho_k}{2}\gamma_k^B)}{1-\rho_k\gamma_k^B+\frac{\rho_k}{2}\gamma_k^A}, a_k^*\right]$, $C_3 = (a_k^*, \delta_1]$, and $C_4 = [\delta_2, \infty)$. These sets are mutually exclusive intervals because $0 < \frac{\phi_k\gamma_k^A(1-\frac{\rho_k}{2}\gamma_k^B)}{1-\rho_k\gamma_k^B+\frac{\rho_k}{2}\gamma_k^A} < a_k^*$, which follows by algebra, and because $a_k^* < \delta_1$, which follows because $a_k^* < \phi_k\gamma_k^B$ and, from the proof of Proposition A1, $\phi_k\gamma_k^B < \delta_1 < \delta_2$. Therefore, $\delta_2 \in (a_k^*, 2b_k^*)$ because $\delta_2 < 2b_k^*$, which follows by straightforward if tedious algebra. Moreover, $C_3 = (a_k^*, \delta_1] \subseteq [a_k^*, b_k^*]$ because $\delta_1 < b_k^*$, which follows by algebra.

Both $a_k^*(c_k)$ and $b_k^*(c_k)$ are continuous functions of c_k on $[0,\delta_1] = C_1 \cup C_2 \cup C_3$ and on $[\delta_2, \infty) = C_4$, and, $\forall c_k \in [0,\delta_1] \cup [\delta_2,\infty)$, $a_k^*(c_k) < b_k^*(c_k)$ when $\gamma_k^A < \gamma_k^B$. Both claims can be readily verified. The equilibrium vote share of party B is $(\phi_k + \frac{\rho_k}{2} r^*(c_k))p^*(c_k)$, where $p^*(c_k) = b_k^*(c_k) - a_k^*(c_k)$ and $r^*(c_k) = \max\{a_k^*(c_k), b_k^*(c_k), c_k\} - \min\{a_k^*(c_k), b_k^*(c_k), c_k\}$.

First, consider $[0,\delta_1]$. We have $a_k^*(c_k) = a_k^*$ and $b_k^*(c_k) = b_k^*$ $\forall c_k \in C_3$. Moreover, because $C_3 \subseteq [a_k^*, b_k^*]$, $a_k^* \leq c_k \leq b_k^*$ and thus $p^*(c_k) = r^*(c_k) = b_k^* - a_k^*$ $\forall c_k \in C_3$. For any $c_k \in C_2$, $b_k^*(c_k)$ is strictly decreasing in c_k, $a_k^*(c_k)$ is strictly increasing in c_k, and $a_k^*(c_k) = c_k$, so that $p^*(c_k)$ and $r^*(c_k)$ are both strictly decreasing in c_k. For any $c_k \in C_1$, $b_k^*(c_k)$ is strictly decreasing in c_k and $c_k < a_k^*(c_k)$, so that $r^*(c_k)$ is strictly decreasing in c_k. $p^*(c_k)$ is strictly decreasing in c_k because $p^*(c_k) = \frac{(\phi_k - \frac{\rho_k}{2}c_k)(\gamma_k^B - \gamma_k^A)}{1-\rho_k\gamma_k^B(1-\frac{\rho_k}{4}\gamma_k^A)}$. In summary, both the equilibrium policy and the vote share of party B are strictly decreasing in c_k on $C_1 \cup C_2$, and on C_3 are constant and are equal to the equilibrium policy, b_k^*, and the vote share of party B, $(\phi_k + \frac{\rho_k}{2}(b_k^* - a_k^*))(b_k^* - a_k^*)$, from Proposition 1. Both are thus uniquely maximized on $[0,\delta_1]$ at $c_k = 0$ and are weakly above the values from Proposition 1 for any $c_k \in [0,\delta_1]$.

Now consider $C_4 = [\delta_2,\infty)$. For any $c_k \in C_4$, $b_k^*(c_k)$ is strictly increasing in c_k. $p^*(c_k)$ is also strictly increasing in c_k because $p^*(c_k) = \frac{(\phi_k + \frac{\rho_k}{2}c_k)(\gamma_k^B - \gamma_k^A)}{1+\rho_k\gamma_k^B(1+\frac{\rho_k}{4}\gamma_k^A)}$. To determine $r^*(c_k)$, we first claim that $b_k^*(c_k) < c_k$ $\forall c_k \in C_4$. This follows because the derivative of $b_k^*(c_k)$ with respect to c_k equals $\frac{\frac{\rho_k}{2}\gamma_k^B + \frac{\rho_k^2}{4}\gamma_k^A\gamma_k^B}{1+\rho_k\gamma_k^A + \frac{\rho_k^2}{4}\gamma_k^A\gamma_k^B} < 1$, and because if $b_k^*(c_k') = c_k'$ for some $c_k' \in C_4$, then $c_k' = \frac{\phi_k\gamma_k^B(1+\frac{\rho_k}{2}\gamma_k^A)}{1-\frac{\rho_k}{2}\gamma_k^B + \rho_k\gamma_k^A}$, where $c_k' < \delta_2$, which can be verified by straightforward if tedious algebra, implies $c_k' \notin C_4$. Thus $r^*(c_k) = c_k - a_k^*(c_k)$ $\forall c_k \in C_4$. That $r^*(c_k)$ is strictly increasing in c_k follows because the derivative of $a_k^*(c_k)$ with respect to c_k equals $\frac{\frac{\rho_k}{2}\gamma_k^A + \frac{\rho_k^2}{4}\gamma_k^A\gamma_k^B}{1+\rho_k\gamma_k^A + \frac{\rho_k^2}{4}\gamma_k^A\gamma_k^B} < 1$. Because $b_k^*(c_k)$, $p^*(c_k)$, and $r^*(c_k)$ are strictly increasing in c_k on C_4, the equilibrium policy and the vote share of party B are uniquely minimized on $[\delta_2,\infty)$ at $c_k = \delta_2$ and grow without bounds with c_k.

We now show that $b_k^*(c_k) < b_k^*$, $p^*(c_k) < b_k^* - a_k^*$, and $p^*(c_k)r^*(c_k) < (b_k^* - a_k^*)^2$ at $c_k = \delta_2$, which implies that the equilibrium policy and the vote share of party B have a global minimum at $c_k = \delta_2$. Because $b_k^*(c_k)$, $p^*(c_k)$, and $r^*(c_k)$ are strictly increasing in c_k on C_4, it suffices to prove the desired inequalities at $c_k = c_k'' = b_k^*\frac{1-\frac{\rho_k}{2}\gamma_k^A + \rho_k\gamma_k^B}{1+\frac{\rho_k}{2}\gamma_k^B} > \delta_2$, where the inequality follows by straightforward if tedious algebra. Direct substitution shows that $(b_k^* - a_k^*) = p^*(c_k'') + b_k^* \frac{\rho_k(\gamma_k^B - \gamma_k^A)}{2+\rho_k\gamma_k^B} = r^*(c_k'') - b_k^* \frac{\rho_k(\gamma_k^B - \gamma_k^A)}{2+\rho_k\gamma_k^B}$, so that $p^*(c_k'') < b_k^* - a_k^*$ and $(b_k^* - a_k^*)^2 = p^*(c_k'')r^*(c_k'') + \left(b_k^* \frac{\rho_k(\gamma_k^B - \gamma_k^A)}{2+\rho_k\gamma_k^B}\right)^2 > p^*(c_k'')r^*(c_k'')$. Moreover, because $a_k^*(c_k'') = a_k^*$ and $p^*(c_k'') = b_k^*(c_k'') - a_k^*(c_k'') < b_k^* - a_k^*$, we have $b_k^*(c_k'') < b_k^*$.

When the objective of the third party is to decrease the equilibrium policy or the vote share of party B, not entering strictly dominates entering with any $c_k \in [0, \delta_1]$, and entering with $c_k = \delta_2$ strictly dominates entering with any $c_k > \delta_2$. Thus entry with $c_k = \delta_2$ or no entry is optimal, depending on γ_k^C. When the objective of the third party is to increase the equilibrium policy or the vote share of party B, entering with $c_k = 0$ strictly dominates not entering as well as entering with any $c_k \in (0, \delta_1] \cup [\delta_2, c_k'']$. Thus entry with $c_k = 0$ or with some $c_k > c_k'' > \delta_2$ is optimal, depending on γ_k^C.

Finally, we show that $a_k^*(c_k)$, $b_k^*(c_k)$, and $b_k^*(c_k) - a_k^*(c_k)$ become steeper functions of c_k when either ρ or σ_k increase. It suffices to take derivatives with respect to ρ_k. For $c_k \in C_1$, we have $\frac{\partial^2 b_k^*(c_k)}{\partial c_k \partial \rho_k} = -\frac{\gamma_k^B}{2} \frac{1-\rho_k\gamma_k^A + \frac{\rho_k^2}{4}\gamma_k^A\gamma_k^B}{[1-\rho_k\gamma_k^B(1-\frac{\rho_k}{4}\gamma_k^A)]^2} < 0$, $\frac{\partial^2 a_k^*(c_k)}{\partial c_k \partial \rho_k} = -\frac{\gamma_k^A}{2} \frac{1-\rho_k\gamma_k^B - \frac{\rho_k^2}{4}\gamma_k^A\gamma_k^B + \frac{\rho_k^2}{2}(\gamma_k^B)^2}{[1-\rho_k\gamma_k^B(1-\frac{\rho_k}{4}\gamma_k^A)]^2} < 0$, as well as $\frac{\partial^2 b_k^*(c_k)}{\partial c_k \partial \rho_k} - \frac{\partial^2 a_k^*(c_k)}{\partial c_k \partial \rho_k} < 0$. For $c_k \in C_4$, we have $\frac{\partial^2 b_k^*(c_k)}{\partial c_k \partial \rho_k} = \frac{\gamma_k^B}{2} \frac{1+\rho_k\gamma_k^A + \frac{\rho_k^2}{2}(\gamma_k^A)^2 - \frac{\rho_k^2}{4}\gamma_k^A\gamma_k^B}{[1+\rho_k\gamma_k^A(1+\frac{\rho_k}{4}\gamma_k^B)]^2} > 0$, $\frac{\partial^2 a_k^*(c_k)}{\partial c_k \partial \rho_k} = \frac{\gamma_k^A}{2} \frac{1+\rho_k\gamma_k^B + \frac{\rho_k^2}{4}\gamma_k^A\gamma_k^B}{[1+\rho_k\gamma_k^A(1+\frac{\rho_k}{4}\gamma_k^B)]^2} > 0$, as well as $\frac{\partial^2 b_k^*(c_k)}{\partial c_k \partial \rho_k} - \frac{\partial^2 a_k^*(c_k)}{\partial c_k \partial \rho_k} > 0$. □

Proposition A1. *Suppose Assumption 1 and $\rho > 0$. Suppose a third party offers quality $c_k \geq 0$ on policy k and, without loss of generality, that $\gamma_k^A \leq \gamma_k^B$. The electoral competition with focusing voters admits a unique Nash equilibrium in pure strategies if $c_k \notin (\delta_1, \delta_2)$ and no such equilibrium otherwise. The parties offer the following qualities in policy k:*

$$(a_k^*(c_k), b_k^*(c_k)) = \begin{cases} \left(\frac{\gamma_k^A(\phi_k - \frac{\rho_k}{2}c_k)(1-\frac{\rho_k}{2}\gamma_k^B)}{1-\rho_k\gamma_k^B(1-\frac{\rho_k}{4}\gamma_k^A)}, \frac{\gamma_k^B(\phi_k - \frac{\rho_k}{2}c_k)(1-\frac{\rho_k}{2}\gamma_k^A)}{1-\rho_k\gamma_k^B(1-\frac{\rho_k}{4}\gamma_k^A)}\right) & \text{if } c_k < \frac{\phi_k\gamma_k^A(1-\frac{\rho_k}{2}\gamma_k^B)}{1-\rho_k\gamma_k^B + \frac{\rho_k}{2}\gamma_k^A} \\ \left(c_k, \frac{\phi_k - \rho_k c_k}{\frac{1}{\gamma_k^B} - \rho_k}\right) & \text{if } c_k \in \left[\frac{\phi_k\gamma_k^A(1-\frac{\rho_k}{2}\gamma_k^B)}{1-\rho_k\gamma_k^B + \frac{\rho_k}{2}\gamma_k^A}, \frac{\phi_k\gamma_k^A}{1-\rho_k(\gamma_k^B - \gamma_k^A)}\right] \\ \left(\frac{\phi_k\gamma_k^A}{1-\rho_k(\gamma_k^B - \gamma_k^A)}, \frac{\phi_k\gamma_k^B}{1-\rho_k(\gamma_k^B - \gamma_k^A)}\right) & \text{if } c_k \in \left(\frac{\phi_k\gamma_k^A}{1-\rho_k(\gamma_k^B - \gamma_k^A)}, \delta_1\right] \\ \left(\frac{\gamma_k^A(\phi_k + \frac{\rho_k}{2}c_k)(1+\frac{\rho_k}{2}\gamma_k^B)}{1+\rho_k\gamma_k^A(1+\frac{\rho_k}{4}\gamma_k^B)}, \frac{\gamma_k^B(\phi_k + \frac{\rho_k}{2}c_k)(1+\frac{\rho_k}{2}\gamma_k^A)}{1+\rho_k\gamma_k^A(1+\frac{\rho_k}{4}\gamma_k^B)}\right) & \text{if } c_k \geq \delta_2 \end{cases} \quad (A7)$$

where $\phi_k = \phi\overline{\theta_k}$, $\rho_k = 2\phi\rho\sigma_k$ *and*

$$\begin{aligned}\delta_1 &= \frac{\phi_k}{\rho_k\gamma_k^B(1-\rho_k(\gamma_k^B-\gamma_k^A))}\left[2(\gamma_k^B-\gamma_k^A)(\sqrt{1-\rho_k\gamma_k^B}-1)+(2\gamma_k^B-\gamma_k^A)\rho_k\gamma_k^B\right]\\ \delta_2 &= \frac{\phi_k\gamma_k^B\left[(4+4\rho_k\gamma_k^A+\rho_k^2\gamma_k^A\gamma_k^B)\sqrt{1-\rho_k\gamma_k^B}+4+4\rho_k\gamma_k^B-2\rho_k\gamma_k^A\right]}{(4+4\rho_k\gamma_k^A+\rho_k^2\gamma_k^A\gamma_k^B)\sqrt{1-\rho_k\gamma_k^B}+4+4\rho_k\gamma_k^A-2\rho_k\gamma_k^B-2\rho_k^2\gamma_k^B(\gamma_k^B-\gamma_k^A)}.\end{aligned} \tag{A8}$$

Proof. Suppose Assumption 1, $\rho > 0$, and $\gamma_k^A \leq \gamma_k^B$. To economize on notation, we use $\phi_k = \phi\overline{\theta_k} > 0$ and $\rho_k = 2\phi\rho\sigma_k > 0$ throughout. The payoff of party with competence $\gamma_k \in \{\gamma_k^A, \gamma_k^B\}$ from contesting the election, on issue k, with policy quality $x \geq 0$ when its opponent runs with policy quality $y \geq 0$, suppressing all constants, is

$$\pi_{\gamma_k}(x, y, c_k) = \phi_k(x-y) + \tfrac{\rho_k}{2}(\max\{x, y, c_k\} - \min\{x, y, c_k\}) \cdot (x-y) - \tfrac{x^2}{2\gamma_k}. \tag{A9}$$

The best response to $y \geq 0$ is $br_{\gamma_k}(y, c_k) = \arg\max_{x\geq 0} \pi_{\gamma_k}(x, y, c_k)$. The proof is lengthy and tedious, if conceptually straightforward: we study the derivative of π_γ to characterize best responses, and we study their fixed points to characterize equilibria.

Best responses. The derivative of π_{γ_k} with respect to x for any x such that $x \neq y$ and $x \neq c_k$ is standard.

Case 1. Suppose that $c_k \leq y$. At $x = c_k$, the left derivative, $\phi_k - \frac{x}{\gamma_k} + \frac{\rho_k}{2}2(y - c_k)$, and the right derivative, $\phi_k - \frac{x}{\gamma_k} + \frac{\rho_k}{2}(y - c_k)$, differ and the derivative does not exists (unless $c_k = y$). At $x = y$, the left derivative, $\phi_k - \frac{x}{\gamma_k} + \frac{\rho_k}{2}(y - c_k)$, equals the right derivative, and we have

$$\frac{\partial\pi_{\gamma_k}(x, y, c_k)}{\partial x} = \begin{cases} \phi_k - \frac{x}{\gamma_k} + \frac{\rho_k}{2}2(y-x) & \text{if } x < c_k \\ \phi_k - \frac{x}{\gamma_k} + \frac{\rho_k}{2}(y-c_k) & \text{if } x \in (c_k, y) \\ \phi_k - \frac{x}{\gamma_k} + \frac{\rho_k}{2}(2x-y-c_k) & \text{if } x \geq y. \end{cases} \tag{A10}$$

The derivative is strictly positive at $x = 0$. It is also continuous and strictly decreasing, because $\rho_k\gamma_k < 1$, in x for any $x \geq 0$ such that $x \neq c_k$. Finally, $\lim_{x\to c_k^-} \frac{\partial\pi_{\gamma_k}(x,y,c_k)}{\partial x} \geq \lim_{x\to c_k^+} \frac{\partial\pi_{\gamma_k}(x,y,c_k)}{\partial x}$. Thus, if x^* such that $\left.\frac{\partial\pi_{\gamma_k}(x,y,c_k)}{\partial x}\right|_{x=x^*} = 0$ exists, then $br_{\gamma_k}(y, c_k) = x^*$.

If x^* does not exist, then $\frac{\partial \pi_{\gamma_k}(x,y,c_k)}{\partial x}$ is strictly positive for any $x < c_k$ and strictly negative for any $x > c_k$, in which case $br_{\gamma_k}(y, c_k) = c_k$. We consider five exhaustive and mutually exclusive cases.

Case 1.1. If $c_k \leq \phi_k \gamma_k$ and $y(\frac{1}{\gamma_k} - \frac{\rho_k}{2}) > \phi_k - \frac{\rho_k}{2} c_k$, then $c_k = y$ in the second inequality implies $c_k > \phi_k \gamma_k$, and hence $c_k < y$. Therefore, $\lim_{x \to c_k^+} \frac{\partial \pi_{\gamma_k}(x,y,c_k)}{\partial x} = \phi_k - \frac{c_k}{\gamma_k} + \frac{\rho_k}{2}(y - c_k) > 0$ and $\lim_{x \to y^-} \frac{\partial \pi_{\gamma_k}(x,y,c_k)}{\partial x} = \phi_k - \frac{y}{\gamma_k} + \frac{\rho_k}{2}(y - c_k) < 0$, so that x^* exists and satisfies $\phi_k - \frac{x^*}{\gamma_k} + \frac{\rho_k}{2}(y - c_k) = 0$, or, equivalently, $x^* = \frac{\phi_k + \frac{\rho_k}{2}(y - c_k)}{\frac{1}{\gamma_k}}$.

Case 1.2. If $c_k \leq \phi_k \gamma_k$ and $y(\frac{1}{\gamma_k} - \frac{\rho_k}{2}) \leq \phi_k - \frac{\rho_k}{2} c_k$, then $\frac{\partial \pi_{\gamma_k}(x,y,c_k)}{\partial x}\Big|_{x=y} = \phi_k - \frac{y}{\gamma_k} + \frac{\rho_k}{2}(y - c_k) \geq 0$, so that x^* exists and satisfies $\phi_k - \frac{x^*}{\gamma_k} + \frac{\rho_k}{2}(2x^* - y - c_k) = 0$, or, equivalently, $x^* = \frac{\phi_k - \frac{\rho_k}{2}(y + c_k)}{\frac{1}{\gamma_k} - \rho_k}$.

Case 1.3. If $c_k > \phi_k \gamma_k$ and $y \rho_k < c_k(\frac{1}{\gamma_k} + \rho_k) - \phi_k$, then $\lim_{x \to c_k^-} \frac{\partial \pi_{\gamma_k}(x,y,c_k)}{\partial x} = \phi_k - \frac{c_k}{\gamma_k} + \frac{\rho_k}{2} 2(y - c_k) < 0$, so that x^* exists and satisfies $\phi_k - \frac{x^*}{\gamma_k} + \rho_k(y - x^*) = 0$, or, equivalently, $x^* = \frac{\phi_k + \rho_k y}{\frac{1}{\gamma_k} + \rho_k}$.

Case 1.4. If $c_k > \phi_k \gamma_k$, $y \rho_k \geq c_k(\frac{1}{\gamma_k} + \rho_k) - \phi_k$, and $y \frac{\rho_k}{2} \leq c_k(\frac{1}{\gamma_k} + \frac{\rho_k}{2}) - \phi_k$, then $\lim_{x \to c_k^-} \frac{\partial \pi_{\gamma_k}(x,y,c_k)}{\partial x} = \phi_k - \frac{c_k}{\gamma_k} + \frac{\rho_k}{2} 2(y - c_k) \geq 0$ and $\lim_{x \to c_k^+} \frac{\partial \pi_{\gamma_k}(x,y,c_k)}{\partial x} = \phi_k - \frac{c_k}{\gamma_k} + \frac{\rho_k}{2}(y - c_k) \leq 0$, so that x^* does not exist and thus $br_{\gamma_k}(y, c_k) = c_k$.

Case 1.5. If $c_k > \phi_k \gamma_k$ and $y \frac{\rho_k}{2} > c_k(\frac{1}{\gamma_k} + \frac{\rho_k}{2}) - \phi_k$, then $\lim_{x \to c_k^+} \frac{\partial \pi_{\gamma_k}(x,y,c_k)}{\partial x} = \phi_k - \frac{c_k}{\gamma_k} + \frac{\rho_k}{2}(y - c_k) > 0$, as well as $\frac{\partial \pi_{\gamma_k}(x,y,c_k)}{\partial x}\Big|_{x=y} = \phi_k - \frac{y}{\gamma_k} + \frac{\rho_k}{2}(y - c_k) < \phi_k - \frac{y}{\gamma_k} + \frac{\rho_k}{2}(y - \phi_k \gamma_k) = (\phi_k \gamma_k - y)(\frac{1}{\gamma_k} - \frac{\rho_k}{2}) < 0$, where the inequalities follow from $y \geq c_k > \phi_k \gamma_k$, so that x^* exists and satisfies $\phi_k - \frac{x^*}{\gamma_k} + \frac{\rho_k}{2}(y - c_k) = 0$, or, equivalently, $x^* = \frac{\phi_k + \frac{\rho_k}{2}(y - c_k)}{\frac{1}{\gamma_k}}$.

Case 2. Suppose that $c_k > y$. At $x = c_k$, the left derivative, $\phi_k - \frac{x}{\gamma_k} + \frac{\rho_k}{2}(c_k - y)$, and the right derivative, $\phi_k - \frac{x}{\gamma_k} + \frac{\rho_k}{2} 2(c_k - y)$, differ and the derivative does not exists. At $x = y$, the left derivative, $\phi_k - \frac{x}{\gamma_k} + \frac{\rho_k}{2}(c_k - y)$, equals the right derivative, and we have

$$\frac{\partial \pi_{\gamma_k}(x, y, c_k)}{\partial x} = \begin{cases} \phi_k - \frac{x}{\gamma_k} + \frac{\rho_k}{2}(y + c_k - 2x) & \text{if } x \leq y \\ \phi_k - \frac{x}{\gamma_k} + \frac{\rho_k}{2}(c_k - y) & \text{if } x \in (y, c_k) \\ \phi_k - \frac{x}{\gamma_k} + \frac{\rho_k}{2} 2(x - y) & \text{if } x > c_k. \end{cases} \tag{A11}$$

The derivative is strictly positive at $x = 0$. It is also continuous and strictly decreasing, because $\rho_k \gamma_k < 1$, in x for any $x \geq 0$ such that $x \neq c_k$. Finally, $\lim_{x \to c_k^-} \frac{\partial \pi_{\gamma_k}(x,y,c_k)}{\partial x} < \lim_{x \to c_k^+} \frac{\partial \pi_{\gamma_k}(x,y,c_k)}{\partial x}$. Thus, x^* such that $\left.\frac{\partial \pi_{\gamma_k}(x,y,c_k)}{\partial x}\right|_{x=x^*} = 0$ exists. If it is unique, then $br_{\gamma_k}(y, c_k) = x^*$. This happens when either $\lim_{x \to c_k^+} \frac{\partial \pi_{\gamma_k}(x,y,c_k)}{\partial x} = \phi_k - \frac{c_k}{\gamma_k} + \rho_k(c_k - y) \leq 0$, or when $\lim_{x \to c_k^-} \frac{\partial \pi_{\gamma_k}(x,y,c_k)}{\partial x} = \phi_k - \frac{c_k}{\gamma_k} + \frac{\rho_k}{2}(c_k - y) \geq 0$. When both of these inequalities fail, then two distinct critical points x^* such that $\left.\frac{\partial \pi_{\gamma_k}(x,y,c_k)}{\partial x}\right|_{x=x^*} = 0$ exist and $br_{\gamma_k}(y, c_k)$ needs to be found by evaluating π_{γ_k} at the critical points. We consider five exhaustive and mutually exclusive cases.

Case 2.1. If $c_k \leq \phi_k \gamma_k$, then $\lim_{x \to c_k^+} \frac{\partial \pi_{\gamma_k}(x,y,c_k)}{\partial x} > \lim_{x \to c_k^-} \frac{\partial \pi_{\gamma_k}(x,y,c_k)}{\partial x} = \phi_k - \frac{c_k}{\gamma_k} + \frac{\rho_k}{2}(c_k - y) \geq 0$, and thus $\phi_k - \frac{x^*}{\gamma_k} + \rho_k(x^* - y) = 0$, or, equivalently, $x^* = \frac{\phi_k - \rho_k y}{\frac{1}{\gamma_k} - \rho_k}$.

Case 2.2. If $c_k > \phi_k \gamma_k$ and $y(\frac{1}{\gamma_k} + \frac{\rho_k}{2}) \geq \phi_k + \frac{\rho_k}{2} c_k$, then $\left.\frac{\partial \pi_{\gamma_k}(x,y,c_k)}{\partial x}\right|_{x=y} = \phi_k - \frac{y}{\gamma_k} + \frac{\rho_k}{2}(c_k - y) \leq 0$. Because $\lim_{x \to c_k^+} \frac{\partial \pi_{\gamma_k}(x,y,c_k)}{\partial x} = \phi_k - \frac{c_k}{\gamma_k} + \rho_k(c_k - y) \leq \phi_k - \frac{y}{\gamma_k} + \frac{\rho_k}{2}(c_k - y) = \left.\frac{\partial \pi_{\gamma_k}(x,y,c_k)}{\partial x}\right|_{x=y}$ rewrites as $(c_k - y)(\frac{1}{\gamma_k} - \frac{\rho_k}{2}) \geq 0$, which holds, we also have $\lim_{x \to c_k^+} \frac{\partial \pi_{\gamma_k}(x,y,c_k)}{\partial x} \leq 0$. Thus $\phi_k - \frac{x^*}{\gamma_k} + \frac{\rho_k}{2}(y + c_k - 2x^*) = 0$, or, equivalently, $x^* = \frac{\phi_k + \frac{\rho_k}{2}(y + c_k)}{\frac{1}{\gamma_k} + \rho_k}$.

Case 2.3. If $c_k > \phi_k \gamma_k$, $y(\frac{1}{\gamma_k} + \frac{\rho_k}{2}) < \phi_k + \frac{\rho_k}{2} c_k$, and $y\rho_k \geq \phi_k - c_k(\frac{1}{\gamma_k} - \rho_k)$, then $\left.\frac{\partial \pi_{\gamma_k}(x,y,c_k)}{\partial x}\right|_{x=y} = \phi_k - \frac{y}{\gamma_k} + \frac{\rho_k}{2}(c_k - y) > 0$ and $\lim_{x \to c_k^+} \frac{\partial \pi_{\gamma_k}(x,y,c_k)}{\partial x} = \phi_k - \frac{c_k}{\gamma_k} + \rho_k(c_k - y) \leq 0$, and thus $\phi_k - \frac{x^*}{\gamma_k} + \frac{\rho_k}{2}(c_k - y) = 0$, or, equivalently, $x^* = \frac{\phi_k + \frac{\rho_k}{2}(c_k - y)}{\frac{1}{\gamma_k}}$.

Case 2.4. If $c_k > \phi_k \gamma_k$, $y\rho_k < \phi_k - c_k(\frac{1}{\gamma_k} - \rho_k)$, and $y\frac{\rho_k}{2} > \phi_k - c_k(\frac{1}{\gamma_k} - \frac{\rho_k}{2})$, then $\lim_{x \to c_k^+} \frac{\partial \pi_{\gamma_k}(x,y,c_k)}{\partial x} = \phi_k - \frac{c_k}{\gamma_k} + \rho_k(c_k - y) > 0$ as well as $\lim_{x \to c_k^-} \frac{\partial \pi_{\gamma_k}(x,y,c_k)}{\partial x} = \phi_k - \frac{c_k}{\gamma_k} + \frac{\rho_k}{2}(c_k - y) < 0$. Moreover, we have $\lim_{x \to c_k^+} \frac{\partial \pi_{\gamma_k}(x,y,c_k)}{\partial x} \leq \left.\frac{\partial \pi_{\gamma_k}(x,y,c_k)}{\partial x}\right|_{x=y}$. Thus, one x^* solves $\phi_k - \frac{x^*}{\gamma_k} + \frac{\rho_k}{2}(c_k - y) = 0$, and hence $x^* = \frac{\phi_k + \frac{\rho_k}{2}(c_k - y)}{\frac{1}{\gamma_k}}$. The other x^* solves $\phi_k - \frac{x^*}{\gamma_k} + \rho_k(x^* - y) = 0$, and hence $x^* = \frac{\phi_k - \rho_k y}{\frac{1}{\gamma_k} - \rho_k}$. Direct substitution into the objective function and tedious but straightforward algebra shows that the latter x^* is the global maximum when $y < \tilde{c}_{\gamma_k}$, the former x^* is the global maximum when $y > \tilde{c}_{\gamma_k}$, and both are global maxima when $y = \tilde{c}_{\gamma_k}$, where $\tilde{c}_{\gamma_k} = c_k + \frac{2(\phi_k \gamma_k - c_k)}{\gamma_k \rho_k (3 + \gamma_k \rho_k)} \left[1 + \gamma_k \rho_k + \sqrt{1 - \gamma_k \rho_k}\right]$. [19]

[19] The steps of the proof that consist solely of algebra manipulation are omitted and are available upon request.

Case 2.5. If $c_k > \phi_k\gamma_k$ and $y\frac{\rho_k}{2} \leq \phi_k - c_k(\frac{1}{\gamma_k} - \frac{\rho_k}{2})$, then $\lim_{x\to c_k^+} \frac{\partial \pi_{\gamma_k}(x,y,c_k)}{\partial x} > \lim_{x\to c_k^-} \frac{\partial \pi_{\gamma_k}(x,y,c_k)}{\partial x} = \phi_k - \frac{c_k}{\gamma_k} + \frac{\rho_k}{2}(c_k - y) \geq 0$, and thus $\phi_k - \frac{x^*}{\gamma_k} + \rho_k(x^* - y) = 0$, or, equivalently, $x^* = \frac{\phi_k - \rho_k y}{\frac{1}{\gamma_k} - \rho_k}$.

Collecting the cases, when $c_k \leq \phi_k\gamma_k$, we have

$$br_{\gamma_k}(y, c_k) = \begin{cases} \frac{\phi_k - \rho_k y}{\frac{1}{\gamma_k} - \rho_k} & \text{if } y < c_k \\ \frac{\phi_k - \frac{\rho_k}{2}(y + c_k)}{\frac{1}{\gamma_k} - \rho_k} & \text{if } y \in \left[c_k, \frac{\phi_k - \frac{\rho_k}{2} c_k}{\frac{1}{\gamma_k} - \frac{\rho_k}{2}}\right] \\ \frac{\phi_k + \frac{\rho_k}{2}(y - c_k)}{\frac{1}{\gamma_k}} & \text{if } y > \frac{\phi_k - \frac{\rho_k}{2} c_k}{\frac{1}{\gamma_k} - \frac{\rho_k}{2}}. \end{cases} \tag{A12}$$

The boundaries of the interval satisfy $c_k \leq \frac{\phi_k - \frac{\rho_k}{2} c_k}{\frac{1}{\gamma_k} - \frac{\rho_k}{2}}$ because $c_k \leq \phi_k\gamma_k$. That $br_{\gamma_k}(y, c_k)$ is continuous in y can be readily verified. Collecting the cases, when $c_k > \phi_k\gamma_k$, we have

$$br_{\gamma_k}(y, c_k) = \begin{cases} \frac{\phi_k - \rho_k y}{\frac{1}{\gamma_k} - \rho_k} & \text{if } y \leq \tilde{c}_{\gamma_k} \\ \frac{\phi_k + \frac{\rho_k}{2}(c_k - y)}{\frac{1}{\gamma_k}} & \text{if } y \in \left[\tilde{c}_{\gamma_k}, \frac{\phi_k + \frac{\rho_k}{2} c_k}{\frac{1}{\gamma_k} + \frac{\rho_k}{2}}\right) \\ \frac{\phi_k + \frac{\rho_k}{2}(y + c_k)}{\frac{1}{\gamma_k} + \rho_k} & \text{if } y \in \left[\frac{\phi_k + \frac{\rho_k}{2} c_k}{\frac{1}{\gamma_k} + \frac{\rho_k}{2}}, c_k\right) \\ \frac{\phi_k + \rho_k y}{\frac{1}{\gamma_k} + \rho_k} & \text{if } y \in \left[c_k, \frac{c_k(\frac{1}{\gamma_k} + \rho_k) - \phi_k}{\rho_k}\right) \\ c_k & \text{if } y \in \left[\frac{c_k(\frac{1}{\gamma_k} + \rho_k) - \phi_k}{\rho_k}, \frac{c_k(\frac{1}{\gamma_k} + \frac{\rho_k}{2}) - \phi_k}{\frac{\rho_k}{2}}\right] \\ \frac{\phi_k + \frac{\rho_k}{2}(y - c_k)}{\frac{1}{\gamma_k}} & \text{if } y > \frac{c_k(\frac{1}{\gamma_k} + \frac{\rho_k}{2}) - \phi_k}{\frac{\rho_k}{2}}. \end{cases} \tag{A13}$$

To see that the boundaries of the intervals are increasing, note that they all equal $\phi_k\gamma_k$ when $c_k = \phi_k\gamma_k$, and increase in c_k with increasing rates, which can be verified by algebra. That $br_{\gamma_k}(y, c_k)$ is continuous in y except at $y = \tilde{c}_{\gamma_k}$ can be readily verified. At $y = \tilde{c}_{\gamma_k}$, $br_{\gamma_k}(y, c_k)$ has two values and straightforward algebra shows that $\frac{\phi_k - \rho_k \tilde{c}_{\gamma_k}}{\frac{1}{\gamma_k} - \rho_k} > \frac{\phi_k + \frac{\rho_k}{2}(c_k - \tilde{c}_{\gamma_k})}{\frac{1}{\gamma_k}}$, and hence for any $y, y' \in \left[0, \frac{\phi_k + \frac{\rho_k}{2} c_k}{\frac{1}{\gamma_k} + \frac{\rho_k}{2}}\right]$ such that $y < y'$, $b_y \in br_{\gamma_k}(y, c_k)$ and $b_{y'} \in br_{\gamma_k}(y', c_k)$ implies $b_y > b_{y'}$.

Equilibria. Because $\rho_k > 0$, $br_{\gamma_k^A}$ and $br_{\gamma_k^B}$ satisfy the conditions of Lemma A1 with $m_1 = \frac{\phi_k - \frac{\rho_k}{2} c_k}{\frac{1}{\gamma_k^A} - \frac{\rho_k}{2}}$ if $c_k \leq \phi_k\gamma_k^A$, $m_1 = \frac{\phi_k + \frac{\rho_k}{2} c_k}{\frac{1}{\gamma_k^A} + \frac{\rho_k}{2}}$ if $c_k > \phi_k\gamma_k^A$, $m_2 = \frac{\phi_k - \frac{\rho_k}{2} c_k}{\frac{1}{\gamma_k^B} - \frac{\rho_k}{2}}$ if $c_k \leq \phi_k\gamma_k^B$, and $m_2 = \frac{\phi_k + \frac{\rho_k}{2} c_k}{\frac{1}{\gamma_k^B} + \frac{\rho_k}{2}}$ if $c_k > \phi_k \gamma_k^B$. Therefore, if $\gamma_k^A = \gamma_k^B$, then $\left(\frac{\phi_k - \frac{\rho_k}{2} c_k}{\frac{1}{\gamma_k^A} - \frac{\rho_k}{2}}, \frac{\phi_k - \frac{\rho_k}{2} c_k}{\frac{1}{\gamma_k^B} - \frac{\rho_k}{2}} \right)$ is the unique Nash equilibrium in pure strategies when $c_k \leq \phi_k \gamma_k^A$, and $\left(\frac{\phi_k + \frac{\rho_k}{2} c_k}{\frac{1}{\gamma_k^A} + \frac{\rho_k}{2}}, \frac{\phi_k + \frac{\rho_k}{2} c_k}{\frac{1}{\gamma_k^B} + \frac{\rho_k}{2}} \right)$ is the unique Nash equilibrium in pure strategies when $c_k > \phi_k \gamma_k^A$. Suppose $\gamma_k^A < \gamma_k^B$ from now on.

Case A. Suppose that $c_k \leq \phi_k \gamma_k^B$. Because $br_{\gamma_k^A}$ and $br_{\gamma_k^B}$ satisfy the conditions of Lemma A1 with m_1 and m_2 such that $m_1 < m_2$, which can be verified by algebra, if (a_k^*, b_k^*) constitutes a Nash equilibrium in pure strategies, then $a_k^* \leq m_2$ and $b_k^* \geq m_2 > m_1$.

Because $br_{\gamma_k^B}$ is a strictly decreasing function on $[0, m_2]$ and because $br_{\gamma_k^A}$ is a non-decreasing function on $[m_1, \infty)$, if (a_k^*, b_k^*) exists, it is unique. To see this, if a distinct (a_k', b_k') also constitutes a Nash equilibrium in pure strategies, then by another application of Lemma A1, $a_k' \leq m_2$ and $b_k' > m_1$, and thus $a_k' > a_k^*$ implies $b_k' < b_k^*$ because $br_{\gamma_k^B}$ is strictly decreasing on $[0, m_2]$, and hence $a_k' \leq a_k^*$ because $br_{\gamma_k^A}$ is non-decreasing on $[m_1, \infty)$, which is a contradiction. By a similar argument, $a_k' < a_k^*$ is not possible, while $a_k' = a_k^*$ implies $b_k' = b_k^*$.

We consider three exhaustive and mutually exclusive cases, and for each find (a_k^*, b_k^*).

Case A.1. Consider $c_k < \frac{\phi_k \gamma_k^A (1 - \frac{\rho_k}{2} \gamma_k^B)}{1 - \rho_k \gamma_k^B + \frac{\rho_k}{2} \gamma_k^A}$. Then the solution to the following system

$$a_k^* = \frac{\phi_k + \frac{\rho_k}{2}(b_k^* - c_k)}{\frac{1}{\gamma_k^A}} \qquad b_k^* = \frac{\phi_k - \frac{\rho_k}{2}(a_k^* + c_k)}{\frac{1}{\gamma_k^B} - \rho_k} \tag{A14}$$

constitutes an Nash equilibrium in pure strategies if $a_k^* \in \left[c_k, \frac{\phi_k - \frac{\rho_k}{2} c_k}{\frac{1}{\gamma_k^B} - \frac{\rho_k}{2}} \right]$, $b_k^* > \frac{\phi_k - \frac{\rho_k}{2} c_k}{\frac{1}{\gamma_k^A} - \frac{\rho_k}{2}}$ when $c \leq \phi_k \gamma_k^A$, and $b_k^* > \frac{c_k(\frac{1}{\gamma_k^A} + \frac{\rho_k}{2}) - \phi_k}{\frac{\rho_k}{2}}$ when $c > \phi_k \gamma_k^A$. The solution is

$$a_k^* = \frac{\gamma_k^A(\phi_k - \frac{\rho_k}{2} c_k)(1 - \frac{\rho_k}{2} \gamma_k^B)}{1 - \rho_k \gamma_k^B (1 - \frac{\rho_k}{4} \gamma_k^A)} \qquad b_k^* = \frac{\gamma_k^B(\phi_k - \frac{\rho_k}{2} c_k)(1 - \frac{\rho_k}{2} \gamma_k^A)}{1 - \rho_k \gamma_k^B (1 - \frac{\rho_k}{4} \gamma_k^A)} \tag{A15}$$

and verifying that it satisfies the required conditions is algebra.

Case A.2. Consider $c_k \in \left[\frac{\phi_k \gamma_k^A (1 - \frac{\rho_k}{2} \gamma_k^B)}{1 - \rho_k \gamma_k^B + \frac{\rho_k}{2} \gamma_k^A}, \frac{\phi_k \gamma_k^A}{1 - \rho_k (\gamma_k^B - \gamma_k^A)} \right]$, and note that $\phi_k \gamma_k^A < \frac{\phi_k \gamma_k^A (1 - \frac{\rho_k}{2} \gamma_k^B)}{1 - \rho_k \gamma_k^B + \frac{\rho_k}{2} \gamma_k^A}$.

Then the solution to the following system

$$a_k^* = c_k \qquad b_k^* = \frac{\phi_k - \frac{\rho_k}{2}(a_k^* + c_k)}{\frac{1}{\gamma_k^B} - \rho_k} \tag{A16}$$

constitutes a Nash equilibrium if $a_k^* \in \left[c_k, \frac{\phi_k - \frac{\rho_k}{2}c_k}{\frac{1}{\gamma_k^B} - \frac{\rho_k}{2}}\right]$ and $b_k^* \in \left[\frac{c_k(\frac{1}{\gamma_k^A} + \rho_k) - \phi_k}{\rho_k}, \frac{c_k(\frac{1}{\gamma_k^A} + \frac{\rho_k}{2}) - \phi_k}{\frac{\rho_k}{2}}\right]$. The solution is

$$a_k^* = c_k \qquad b_k^* = \frac{\phi_k - \rho_k c_k}{\frac{1}{\gamma_k^B} - \rho_k} \tag{A17}$$

and verifying that it satisfies the required conditions is algebra.

Case A.3. Consider $c_k \in \left(\frac{\phi_k \gamma_k^A}{1 - \rho_k(\gamma_k^B - \gamma_k^A)}, \phi_k \gamma_k^B\right]$. Then the solution to the following system

$$a_k^* = \frac{\phi_k + \rho_k b_k^*}{\frac{1}{\gamma_k^A} + \rho_k} \qquad b_k^* = \frac{\phi_k - \rho_k a_k^*}{\frac{1}{\gamma_k^B} - \rho_k} \tag{A18}$$

constitutes a Nash equilibrium if $a_k^* < c_k$ and $b_k^* \in \left[c_k, \frac{c_k(\frac{1}{\gamma_k^A} + \rho_k) - \phi_k}{\rho_k}\right)$. The solution is

$$a_k^* = \frac{\phi_k \gamma_k^A}{1 - \rho_k(\gamma_k^B - \gamma_k^A)} \qquad b_k^* = \frac{\phi_k \gamma_k^B}{1 - \rho_k(\gamma_k^B - \gamma_k^A)} \tag{A19}$$

and verifying that it satisfies the required conditions is algebra.

Case B. Suppose that $c_k > \phi_k \gamma_k^B$. As in the previous case, by Lemma A1, if (a_k^*, b_k^*) constitutes a Nash equilibrium, then $a_k^* \leq m_2$ and $b_k^* \geq m_2 > m_1$. $br_{\gamma_k^A}$ is a non-decreasing function on $[m_1, \infty)$ even in this case. For any $y, y' \in [0, m_2]$ such that $y < y'$, $b_y \in br_{\gamma_k^B}(y, c_k)$ and $b_{y'} \in br_{\gamma_k^B}(y', c_k)$ imply $b_y > b_{y'}$. Thus, by a similar argument as in the previous case, if (a_k^*, b_k^*) and (a_k', b_k') both constitute Nash equilibria in pure strategies, then $a_k^* = a_k'$. If $a_k^* \neq \tilde{c}_{\gamma_k^B}$, then also $b_k^* = b_k'$ because $br_{\gamma_k^B}$ is a function on $[0, m_2]$ except at $\tilde{c}_{\gamma_k^B}$. If $a_k^* = \tilde{c}_{\gamma_k^B}$, then $b_k^* = b_k'$ if $br_{\gamma_k^A}$ is strictly increasing at b_k^*.

We consider three exhaustive and mutually exclusive cases. For the first two, we find (a_k^*, b_k^*), and for the last, we argue that it fails to exist.

Case B.1. Consider $c_k \in (\phi_k \gamma_k^B, \delta_1]$. Confirming that $\phi_k \gamma_k^B < \delta_1$ is algebra. Then the solution to the following system

$$a_k^* = \frac{\phi_k + \rho_k b_k^*}{\frac{1}{\gamma_k^A} + \rho_k} \qquad b_k^* = \frac{\phi_k - \rho_k a_k^*}{\frac{1}{\gamma_k^B} - \rho_k} \tag{A20}$$

constitutes a Nash equilibrium if $a_k^* \leq \tilde{c}_{\gamma_k^B}$ and $b_k^* \in \left[c_k, \frac{c_k(\frac{1}{\gamma_k^A}+\rho_k)-\phi_k}{\rho_k}\right)$. The solution is

$$a_k^* = \frac{\phi_k \gamma_k^A}{1-\rho_k(\gamma_k^B - \gamma_k^A)} \qquad b_k^* = \frac{\phi_k \gamma_k^B}{1-\rho_k(\gamma_k^B - \gamma_k^A)} \tag{A21}$$

and verifying that it satisfies the required conditions is algebra. The equilibrium is unique because $br_{\gamma_k^A}$ is strictly increasing on the entire interval b_k^* belongs to.

Case B.2. Consider $c_k \geq \delta_2$. Then the solution to the following system

$$a_k^* = \frac{\phi_k + \frac{\rho_k}{2}(b_k^* + c_k)}{\frac{1}{\gamma_k^A} + \rho_k} \qquad b_k^* = \frac{\phi_k + \frac{\rho_k}{2}(c_k - a_k^*)}{\frac{1}{\gamma_k^B}} \tag{A22}$$

constitutes a Nash equilibrium if $a_k^* \in \left[\tilde{c}_{\gamma_k^B}, \frac{\phi_k + \frac{\rho_k}{2}c_k}{\frac{1}{\gamma_k^B}+\frac{\rho_k}{2}}\right)$ and $b_k^* \in \left[\frac{\phi_k+\frac{\rho_k}{2}c_k}{\gamma_k^A+\frac{\rho_k}{2}}, c_k\right)$. The solution is

$$a_k^* = \frac{\gamma_k^A(\phi_k + \frac{\rho_k}{2}c_k)(1+\frac{\rho_k}{2}\gamma_k^B)}{1+\rho_k\gamma_k^A(1+\frac{\rho_k}{4}\gamma_k^B)} \qquad b_k^* = \frac{\gamma_k^B(\phi_k + \frac{\rho_k}{2}c_k)(1+\frac{\rho_k}{2}\gamma_k^A)}{1+\rho_k\gamma_k^A(1+\frac{\rho_k}{4}\gamma_k^B)} \tag{A23}$$

and verifying that it satisfies the required conditions is algebra. The equilibrium is unique because $br_{\gamma_k^A}$ is strictly increasing on the entire interval b_k^* belongs to.

Case B.3. Consider $c_k \in (\delta_1, \delta_2)$. Confirming that $\delta_1 < \delta_2$ is algebra. We argue that a Nash equilibrium in pure strategies does not exist when $c_k \in (\delta_1, \delta_2)$. Suppose, towards a contradiction, that (a_k^*, b_k^*) constitutes a Nash equilibrium and $c_k \in (\delta_1, \delta_2)$. Then, by Lemma A1, we have $b_k^* \geq m_2 > m_1$ and $a_k^* \leq m_2$. Because $c_k > \phi_k\gamma_k^B$, $br_{\gamma_k^B}$ is given by one of the first two expressions in (A13) and $br_{\gamma_k^A}$ is given by one of the last four expressions in (A13). This yields eight possible candidate equilibria, and tedious but straightforward algebra shows that none of the candidate equilibria falls into the required intervals.

Finally, direct verification, which is immediate except for δ_1 and δ_2, which both equal $\phi_k\gamma_k^B$ when $\gamma_k^A = \gamma_k^B$, shows that the equilibrium derived under the assumption of $\gamma_k^A < \gamma_k^B$ evaluated at $\gamma_k^A = \gamma_k^B$ collapses to the equilibrium derived for $\gamma_k^A = \gamma_k^B$. □

Proof of Proposition 3

Let $\pi_A(g_A)$ be the payoff of party A from platform (g_A, τ_A) that satisfies the budget constraint (20), evaluated at $(g_B^* = 0, \tau_B^* = 0)$, and expressed in terms of g_A. We have

$$\pi_A(g_A) = \phi\left[u(g_A) - \tfrac{g_A}{\gamma^A} + \rho u(g_A)^2 - \rho\tfrac{m_P y_P^2 + m_R y_R^2}{\overline{y}^2}\left(\tfrac{g_A}{\gamma^A}\right)^2\right]. \tag{A24}$$

The equilibrium platform of party A is (g_A^*, τ_A^*), where $g_A^* \in \arg\max_{g_A \in [0,\overline{y}\gamma^A]} \pi_A(g_A)$ and $\tau_A^* = g_A^*/\overline{y}\gamma^A$. Because $\pi_A(g_A)$ is continuous and $[0, \overline{y}\gamma^A]$ compact, g_A^* exists.

Because u is twice continuously differentiable, we have

$$\begin{aligned}\tfrac{\partial \pi_A(g_A)}{\partial g_A} &= u'(g_A) - \tfrac{1}{\gamma^A} + 2\rho u(g_A)u'(g_A) - 2\rho\tfrac{m_P y_P^2 + m_R y_R^2}{\overline{y}^2}\tfrac{g_A}{(\gamma^A)^2} \\ \tfrac{\partial^2 \pi_A(g_A)}{\partial g_A^2} &= u''(g_A) + 2\rho\left[u(g_A)u''(g_A) + u'(g_A)^2\right] - 2\rho\tfrac{m_P y_P^2 + m_R y_R^2}{\overline{y}^2}\tfrac{1}{(\gamma^A)^2}.\end{aligned} \tag{A25}$$

We first argue that g_A^* is unique because $\frac{\partial^2 \pi_A(g_A)}{\partial g_A^2} < 0\ \forall g_A \in [0, \overline{y}\gamma^A]$. To see this, all terms in $\frac{\partial^2 \pi_A(g_A)}{\partial g_A^2}$ are non-positive with the exception of $2\rho u'(g_A)^2$, and we have $2\rho u'(g_A)^2 < 2\rho u'(0)^2\ \forall g_A \in [0, \overline{y}\gamma^A]$ because u is strictly concave. Moreover, $u''(g_A) < b < 0\ \forall g_A \in [0, \overline{y}\gamma^A]$. Thus $\frac{\partial^2 \pi_A(g_A)}{\partial g_A^2} < 0\ \forall g_A \in [0, \overline{y}\gamma^A]$ if $\rho < \frac{-b}{2u'(0)^2}$, where the bound is strictly positive because $b < 0$ and $u'(0) > 0$.

We now argue that g_A^* is interior. That $g_A^* > 0$ follows because $\left.\frac{\partial \pi_A(g_A)}{\partial g_A}\right|_{g_A=0} = u'(0) - \frac{1}{\gamma^A} > 0$. To see that $g_A^* < \overline{y}\gamma^A$, we have $\left.\frac{\partial \pi_A(g_A)}{\partial g_A}\right|_{g_A=\overline{y}\gamma^A} = u'(\overline{y}\gamma^A) - \frac{1}{\gamma^A} + 2\rho\cdot D$, where $D = u(\overline{y}\gamma^A)u'(\overline{y}\gamma^A) - \frac{m_P y_P^2 + m_R y_R^2}{\overline{y}}\frac{1}{\gamma^A}$. If $D \le 0$, we have $\left.\frac{\partial \pi_A(g_A)}{\partial g_A}\right|_{g_A=\overline{y}\gamma^A} \le u'(\overline{y}\gamma^A) - \frac{1}{\gamma^A} < 0$. If $D > 0$, then $\left.\frac{\partial \pi_A(g_A)}{\partial g_A}\right|_{g_A=\overline{y}\gamma^A} < 0$ if $\rho < \frac{-u'(\overline{y}\gamma^A) + \frac{1}{\gamma^A}}{2D}$, where the bound is strictly positive because $\frac{1}{\gamma^A} > u'(\overline{y}\gamma^A)$.

Because $g_A^* \in (0, \overline{y}\gamma^A)$, it is implicitly defined by

$$u'(g_A^*) - \tfrac{1}{\gamma^A} + 2\rho u(g_A^*)u'(g_A^*) - 2\rho\tfrac{m_P y_P^2 + m_R y_R^2}{\overline{y}^2}\tfrac{g_A^*}{(\gamma^A)^2} = 0. \tag{A26}$$

Only the last term depends on income inequality and, provided $\rho > 0$, g_A^* strictly decreases when $\frac{m_P y_P^2 + m_R y_R^2}{\overline{y}^2}$ strictly increases. A mean preserving spread in income, for some $\varepsilon > 0$, increases y_R by $\frac{\varepsilon}{m_R}$ and decreases y_P by $\frac{\varepsilon}{m_P}$, leaving $\overline{y}$ unchanged. This changes

$m_P y_P^2 + m_R y_R^2$ to

$$m_P \left(y_P^2 - \frac{2 y_P \varepsilon}{m_P} + \frac{\varepsilon^2}{m_P^2} \right) + m_R \left(y_R^2 + \frac{2 y_R \varepsilon}{m_R} + \frac{\varepsilon^2}{m_R^2} \right) \tag{A27}$$

and the change is strictly positive because $-m_P \frac{2y_P\varepsilon}{m_P} + m_R \frac{2y_R\varepsilon}{m_R} > 0$ is equivalent to $2\varepsilon(-y_P + y_R) > 0$. When g_A^* decreases as a result of a mean preserving spread in income, τ_A^* decreases as well because $\tau_A^* = g_A^*/\overline{y}\gamma^A$. □