Applied Energy 378 (2025) 124721

Contents lists available at ScienceDirect

Applied Energy

journal homepage: www.elsevier.com/locate/apenergy

Integrating local market operations into transmission investment: A tri-level optimization approach

Yuxin Xia [a,*], Iacopo Savelli [b], Thomas Morstyn [c]

[a] School of Engineering, The University of Edinburgh, Edinburgh, EH9 3FB, UK
[b] GREEN center, Bocconi University, Milano, 20136, Italy
[c] Department of Engineering Science, University of Oxford, Oxford, OX3 7DQ, UK

ARTICLE INFO

Keywords:
Decision analysis
Transmission investment
Local energy markets
Flexibility
Tri-level optimization

ABSTRACT

The rise of Local Energy Markets (LEMs) and increasing local flexibility present a key research question: How do local flexibility and LEM operations impact merchant-regulated transmission investments? This paper introduces a novel tri-level framework to integrate local market dynamics into transmission investment decisions. The framework models the sequential operations of the WSM and LEMs, adhering to their respective network constraints, and includes a regulatory mechanism that incentivizes profit-driven Transmission Companies (Transcos) to make social welfare maximizing investments while accounting for local refinement costs. The tri-level optimization problem is asymptotically approximated by a mixed-integer second-order cone programming problem. Our findings from three case studies reveal that the provision of local flexibility substantially reduces reliance on conventional energy generation supplies. Additionally, transmission investment decisions are influenced by the levels of flexible generation and consumers, while adhering to network constraints. Moreover, the tri-level model enhances Transcos' awareness of the sequential interactions between the WSM and LEMs, enabling them to make investment strategies that are responsive to the changing dynamics of local markets.

1. Introduction

Over the last few decades, the power system paradigm has undergone significant changes as distributed energy resources (DERs) have rapidly been adopted [1], including renewable resources, electric vehicles, and flexible loads. These resources are primarily located in local distribution grids and have led to a significant increase in local flexibility [2]. As a result, passive consumers have transformed into prosumers who can modify their energy usage patterns to provide local flexibility services [2]. To address the intermittent and fluctuating nature of DERs, local energy markets (LEMs) have emerged as a platform for active prosumers and non-active consumers to trade energy in the distribution network [3]. Recently, many studies have focused on designing decentralized energy trading platforms, such as peer-to-peer energy markets [4–6] and transactive energy systems (see the definition in [7]), with the goal of incentivizing local flexibility and coordination between prosumers, primarily for creating operational value. However, there is still a lack of research on the impact of LEMs on transmission system planning, as reviewed and discussed in [8].

The coupling between the Wholesale Market (WSM) and LEMs can be broadly divided into designs based on the WSM Locational Marginal Prices (LMP) signal [9–13], the interface flow between transmission and distribution (T&D) networks [11,13,14], and bidding prices [12]. For instance, Schmitt et al. [9] proposed different LEM configurations, responsive to the WSM LMP signal, and Lezama et al. [14] introduced a LEM framework with a cascading coupling to the WSM. However, despite the detailed modeling of local operations and interactions with the WSM, some approaches such as Schmitt et al. [9], Lezama et al. [14] and Lezama et al. [15] fail to account for distribution network constraints, potentially leading to impractical energy trading outcomes. To address this issue, several studies have incorporated optimal power flow techniques for distribution networks, such as second-order cone programming (SOCP) [13] and *LinDistflow* [16]. Nevertheless, only a few studies have focused on the integration of LEMs into transmission investments, taking into account their coupling to existing markets. For example, recent research by Fuentes González et al. [17] highlights the growing significance of local market operations and community energy initiatives in generation and transmission expansion planning, utilizing biform games and linear production games frameworks. Liu et al. [12] considered the coordinated operation of transmission and distribution networks, with the upper-level transmission system operator (TSO) problem determining dispatch and

* Corresponding author.
E-mail address: yuxin.xia@ed.ac.uk (Y. Xia).

https://doi.org/10.1016/j.apenergy.2024.124721
Received 27 June 2024; Received in revised form 13 September 2024; Accepted 14 October 2024
Available online 12 November 2024

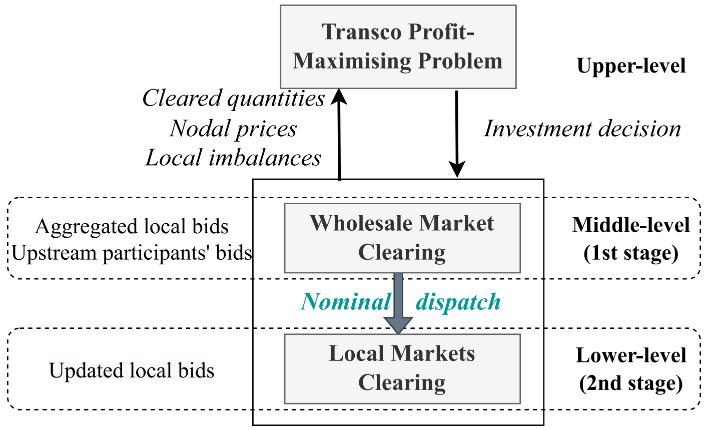

Fig. 1. The proposed tri-level framework.

investment decisions. Alvarez et al. [16] formulated the operation of the Distribution System Operator (DSO) grid and microgrids as a bilevel framework, with the TSO also acting as the centralized planner, operating and planning the transmission network. However, prior studies [12,16,17] adopt a centralized transmission planning paradigm. In contrast, our paper adopts an alternative approach, examining a transmission investment framework that combines merchant and regulatory perspectives.

Merchant transmission investment represents a significant stride towards the deregulation and liberalization of the electricity industry [18], and has been adopted in Australia, Argentina, and Brazil [19]. Nonetheless, ongoing discussions persist regarding the potential of merchant investment to achieve equivalent levels of social welfare compared to centralized planning [20]. To this end, the merchant-regulated investment model is proposed and is a promising approach for transmission companies (Transcos) to invest in transmission infrastructure with profit motives and regulatory compliance [21]. This model allows for the Transco's private initiative, planning, ownership, and execution of investments with regulatory incentives, providing several benefits to Transcos [22]. The H–R–G–V (Hesamzadeh–Rosellón–Gabriel–Vogelsang) mechanism proposed in [22,23] is an incentive merchant-regulated mechanism to promote efficient investment in transmission infrastructure while assuring social welfare maximizing. This mechanism is based on price-cap regulation [24] and surplus subsidy [25], which provides economic incentives to Transcos to invest efficiently by compensating a regulated incentive fee depending on their contribution to economic benefits from their investment. Furthermore, subsequent research [26] extended the H–R–G–V mechanism by incorporating future generation resources such as wind generation and battery storage systems. This extended approach further enhances the efficient jointly optimal investments in large-scale battery-storage, wind-generation, and the transmission network. Despite considering renewable generation, local market operation and the impact of flexible generation located in local grids were not addressed both in [26] and previous research [22,23,27,28]. This transformation necessitates a game-theoretical framework that can effectively manage network expansion planning while accommodating the rising local flexibility.

The key contributions of this paper are threefold. First, this paper contributes to the field of transmission network investment by introducing an innovative merchant-regulatory mechanism based on the H–R–G–V framework, which considers local market operations and acknowledges the inherent lumpy nature of transmission investments. The proposed mechanism provides an efficient solution that accounts for LEMs refinements while maximizing social welfare. Second, the interaction scheme between the WSM and LEM is integrated into the transmission planning problem, enabling both flexible and non-flexible users to coexist in local grids while ensuring transmission and distribution network constraints are complied with. This model is a tri-level optimization and is reformulated as a Mixed-Integer Second-Order Cone Programming (MISOCP) problem. Finally, our tri-level optimization framework effectively captures the sequential interactions between the WSM and LEMs. Unlike traditional transmission planning models that often overlook local market dynamics, our approach integrates local refinements and flexibility into the decision-making process, reducing the risks of over- or under-investment in transmission assets when LEM operations diverge from the initial WSM nominations.

The rest of the paper is organized as follows: Section 2 presents the problem formulation and Section 3 introduces the tri-level optimization and its reformulation as a MISOCP problem. Results for case studies are given in Section 4. Lastly, Section 5 concludes the paper.

2. Problem formulation

In this section, the general tri-level framework and assumptions will be explained in Section 2.1. The mathematical model for the three levels of the problem will be illustrated in Sections 2.2, 2.3, and 2.4. Lastly, a comprehensive discussion of the tri-level model will be presented in Section 2.5.

2.1. The tri-level framework and model assumptions

Fig. 1 depicts a tri-level block diagram that outlines the interaction between the Transco, the WSM, and LEMs. The upper-level problem corresponds to the long-term transmission investment problem, while the middle-level problem corresponds to the WSM clearing. Finally, the lower-level problem pertains to LEMs clearing.

In this paper, we assume that local resources can effectively participate in the upstream market by aggregating through a non-profit entity known as the Local Market Operator (LMO). The LMO represents local users and submits local offer/bid curves to the WSM on their behalf, thereby enabling them to either buy the required energy or sell their excess energy to the upstream market. The WSM is cleared to maximize the social welfare, considering bids and offers from upstream Generation Companies (GENCOs), Transmission Network Loads (TNLs), and the local offer and bid curve. Upon clearing the WSM, several variables are determined, such as the power produced by GENCOs, the accepted demand for

TNLs, transmission line flows, market prices, and the nominal import/export active power between the upstream network and the local network. This WSM clearing and interactions are referred to the first stage. In the second stage, local flexible users will revise their bids and offers based on the new forecasts. The LEMs are cleared by the LMO to determine the assigned quantities and local nodal prices. We assume that imbalance costs are levied on the acquisition of active power that deviates from the initial dispatch determined in the first stage. This initial dispatch is referred to as the *nominal dispatch* in the subsequent sections.

Similar to the work of Lezama et al. [15] and Paredes et al. [29], this study employs a sequential framework where the LEM is modeled as a phase that occurs after the clearing of the WSM. Our approach begins with the aggregation of local bids into the WSM, ensuring more accurate market conditions and determining the desired power injection and consumption across both upstream and local levels. By first allocating resources on a larger scale through the WSM, the model ensures that the most cost-effective energy sources are utilized. This step is crucial as it determines a baseline/*nominal dispatch* for further local refinements. Subsequently, the LEM refines local allocations with respect to the *nominal dispatch*, focusing on matching the updated local supply with local demand while considering the network constraints. We also assume that consumers are price-takers who do not have sufficient market power to strategically influence prices, with the relaxation of this assumption left for future work.

There exists a market structure where LEMs are cleared before the WSM, which can also be integrated into our tri-level framework. This bottom-up approach enables local entities to efficiently manage their resources, with the outcomes of LEM clearances directly influencing subsequent WSM operations. However, we have chosen a WSM-then-LEM sequence because it enhances the co-optimization of resources across the transmission and distribution levels in the WSM stage. Our approach prevents inefficient dispatch scenarios where transmission-connected resources might not compete effectively with those at the distribution network level. However, there is an interesting research question regarding the impact of market clearing sequences on upper-level transmission investment decisions, which we intend to explore in future work. Nevertheless, this paper is the first attempt to model the sequential operations of national and local markets in merchant-regulated transmission network planning, demonstrating the critical role of flexibility in local grids and dynamic market conditions on transmission investment decisions.

This paper assumes that traditional non-flexible consumers and active participants equipped with DER technologies coexist in the local distribution grids [30]. The flexible participants are capable of adjusting their consumption and generation patterns and providing services to the upstream grid. To model power flows in high voltage (HV) transmission networks, the proposed framework employs the DC optimal power flow (DC-OPF) approximation [31], while SOCP relaxation methods are utilized to model power flows in medium voltage (MV) and low voltage (LV) distribution networks [32]. In the absence of sufficient conditions, the second-order cone constraint may remain binding in specific instances, yet this scenario does not universally guarantee exactness. The implementation of an ex-post procedure for feasibility recovery [33] will be subject to future research. In this paper, reactive power is not taken into account in the transmission network, while it is explicitly modeled at the distribution level. We assume that reactive power can be imported/exported from the point of common coupling (PCC).

In this study, we align with the perspective that '*network expansion investments are likely to be lumpy*,' as discussed in [34]. This concept was substantiated through a detailed analysis of merchant investments, with a focus on practical challenges encountered in real-world scenarios. The concept of 'lumpy expansion' indicates that power line capacity expansion are restricted to specific, discrete increments, rather than arbitrary values, as detailed in [30,35]. Our methodology applies this concept to candidate line expansions. We introduce the set $\overline{F}_{l,j}$, representing the capacity ratings for candidate lines l.

2.2. Middle-level problem: WSM clearing

The middle-level problem is a standard WSM clearing problem, taking into account the aggregated local bid curves. The WSM clearing problem is specified in Problem (1).

$$\{d^{agg*}_{t,s,b}, g^{agg*}_{t,s,b}, d^*_{t,s,k,b}, g^*_{t,s,k,b}, f^*_{t,s,m}, \theta^*_{t,s,b}, [\pi^*_{t,s,b}]\} =$$

$$\arg\max \sum_{t\in\mathcal{T}}\sum_{s\in\mathcal{S}}\Bigg(\sum_{b\in\mathcal{B}^{lm}}\left(U^{agg}_{t,s,b}(d^{agg}_{t,s,b}) - C^{agg}_{t,s,b}(g^{agg}_{t,s,b})\right) + \sum_{b\in\mathcal{B}}\Big(\sum_{k\in\Omega^{D\_TN}_{t,s,b}} c^d_{t,s,k,b} d_{t,s,k,b} - \sum_{k\in\Omega^{G\_TN}_{t,s,b}} c^g_{t,s,k,b} g_{t,s,k,b}\Big)\Bigg) \tag{1a}$$

s.t.

$$\Big[\sum_{k\in\Omega^{D\_TN}_{t,s,b}} d_{t,s,k,b} + \sum_{l\in\mathcal{L}} S_{l,b} f_{t,s,l} - \sum_{l\in\mathcal{L}} R_{l,b} f_{t,s,l} = \sum_{k\in\Omega^{G\_TN}_{t,s,b}} g_{t,s,k,b}, \quad \forall b\in\mathcal{B}\setminus\mathcal{B}^{lm}, \quad [\pi_{t,s,b}\in\mathbb{R}], \tag{1b}$$

$$\sum_{k\in\Omega^{D\_TN}_{t,s,b}} d_{t,s,k,b} + \sum_{l\in\mathcal{L}} S_{l,b} f_{t,s,l} - \sum_{l\in\mathcal{L}} R_{l,b} f_{t,s,l} - g^{agg}_{t,s,b} + d^{agg}_{t,s,b} = \sum_{k\in\Omega^{G\_TN}_{t,s,b}} g_{t,s,k,b}, \quad \forall b\in\mathcal{B}^{lm}, \quad [\pi_{t,s,b}\in\mathbb{R}], \tag{1c}$$

$$g^{agg,\min}_{t,s,b} \le g^{agg}_{t,s,b} \le g^{agg,\max}_{t,s,b}, \quad \forall b\in\mathcal{B}^{lm}, \tag{1d}$$

$$d^{agg,\min}_{t,s,b} \le d^{agg}_{t,s,b} \le d^{agg,\max}_{t,s,b}, \quad \forall b\in\mathcal{B}^{lm}, \tag{1e}$$

$$g^{\min}_{t,s,k,b} \le g_{t,s,k,b} \le g^{\max}_{t,s,k,b}, \quad \forall k\in\Omega^{G\_TN}_{t,s,b}, \forall b\in\mathcal{B}, \tag{1f}$$

$$d^{\min}_{t,s,k,b} \le d_{t,s,k,b} \le d^{\max}_{t,s,k,b}, \quad \forall k\in\Omega^{D\_TN}_{t,s,b}, \forall b\in\mathcal{B}, \tag{1g}$$

$$f_{t,s,l} = B_l\Big(\sum_{b\in\mathcal{B}} S_{l,b}\theta_{t,s,b} - \sum_{b\in\mathcal{B}} R_{l,b}\theta_{t,s,b}\Big), \quad \forall l\in\mathcal{L}, \tag{1h}$$

$$f_{t,s,l} \le \mathcal{F}^0_l + \sum_{\hat{t}\in\{2,\ldots,t\}}\sum_{j\in\mathcal{J}} b^F_{\hat{t},l,j}\overline{F}_{l,j}, \quad \forall l\in\mathcal{L}, \tag{1i}$$

$$-f_{t,s,l} \le \mathcal{F}^0_l + \sum_{\hat{t}\in\{2,\ldots,t\}}\sum_{j\in\mathcal{J}} b^F_{\hat{t},l,j}\overline{F}_{l,j}, \quad \forall l\in\mathcal{L}, \tag{1j}$$

$$-\theta^{max}_b \le \theta_{t,s,b} \le \theta^{max}_b, \quad \forall b\in\mathcal{B}, \tag{1k}$$

$$\theta_{t,s,1} = 0,\Big] \quad \forall t\in\mathcal{T}, \forall s\in\mathcal{S}. \tag{1l}$$

The objective function of the WSM clearing problem (1a) is to maximize the social welfare. Nodal power balance in year t and operation period s is modeled by (1b) and (1c). Eq. (1b) defines the active power balance constraints for transmission network nodes which are not directly connected to distribution networks, i.e., $\forall b \in \mathcal{B} \backslash \mathcal{B}^{lm}$. On the other hand, the power balance constraint is modeled in Eq. (1c) for the case where there are local grids connected to transmission node b, $b \in \mathcal{B}^{lm}$. The dual variables $\pi_{t,s,b}$ associated with these two constraints are the WSM clearing prices according to the marginal pricing scheme. Constraints (1d) to (1g) impose maximum and minimum quantity limits for aggregated generation, aggregated demand, GENCOs and TNLs, respectively. Constraints (1h) to (1j) define the active power flow and set the flow limit of the transmission line l. The binary variable $b^F_{\hat{t},l,j}$ is the lumpy expansion decision determined in the upper-level problem. The product $\sum_{\hat{t}\in\{2,\ldots,t\}} \sum_{j\in\mathcal{J}} b^F_{\hat{t},l,j} \overline{F}_{l,j}$ determines the selected amount of lumpy expansion from $\hat{t} \in \{2,\ldots,t\}$ where $t \in \mathcal{T}$ since the investment decision is irreversible. (1k) and (1l) define the range of voltage phase angle of transmission bus b. The reformulated WSM clearing problem is presented in Appendix C, which provides a representation of the aggregated local supply curves $C^{agg}_{t,s,b}(g^{agg}_{t,s,b})$ and demand curves $U^{agg}_{t,s,b}(d^{agg}_{t,s,b})$.

2.3. Lower-level problem: LEMs clearing

The lower-level problem (2) concerns the LEMs clearing process, which involves determining the local dispatch and prices based on the *nominal dispatch* obtained from the WSM clearing results.

$$\{d^{p*}_{t,s,n,k,b}, g^{p*}_{t,s,n,k,b}, d^{q*}_{t,s,n,k,b}, g^{q*}_{t,s,n,k,b}, p^*_{t,s,n,b}, q^*_{t,s,n,b}, W^*_{t,s,ii,b}, W^{p*}_{t,s,ij,b}, W^{q*}_{t,s,ij,b}, [\pi^{p*}_{t,s,n,b}]\}$$

$$= \arg\max \sum_{t\in\mathcal{T}} \sum_{s\in\mathcal{S}} \sum_{b\in\mathcal{B}^{lm}} \left(\sum_{n\in\mathcal{N}^+_b} \sum_{k\in\Omega^{D\_local}_{t,s,n,b}} c^d_{t,s,n,k,b} d^p_{t,s,n,k,b} - \sum_{n\in\mathcal{N}^+_b} \sum_{k\in\Omega^{G\_local}_{t,s,n,b}} c^g_{t,s,n,k,b} g^p_{t,s,n,k,b} - \bar{M}\,|p_{t,s,0,b} - (d^{agg*}_{t,s,b} - g^{agg*}_{t,s,b})| - c^q_{t,s,0,b} q_{t,s,0,b} \right) \tag{2a}$$

s.t.

$$\Big[\, p_{t,s,n,b} = \sum_{k\in\Omega^{G\_local}_{t,s,n,b}} g^p_{t,s,n,k,b} - \sum_{k\in\Omega^{D\_local}_{t,s,n,b}} d^p_{t,s,n,k,b} - D_{t,s,n,b}, \quad \forall n \in \mathcal{N}^+_b \quad [\pi^p_{t,s,n,b} \in \mathbb{R}], \tag{2b}$$

$$q_{t,s,n,b} = \sum_{k\in\Omega^{G\_local}_{t,s,n,b}} g^q_{t,s,n,k,b} - \sum_{k\in\Omega^{D\_local}_{t,s,n,b}} d^q_{t,s,n,k,b}, \quad \forall n \in \mathcal{N}^+_b \quad [\pi^q_{t,s,n,b} \in \mathbb{R}], \tag{2c}$$

$$\begin{aligned} p_{t,s,n,b} = & \sum_{(i,j)\in\mathcal{L}_b: i=n} (W_{t,s,ii,b} a_{i,j,b} - W^p_{t,s,ij,b} a_{i,j,b} + W^q_{t,s,ij,b} e_{i,j,b}) \\ & + \sum_{(i,j)\in\mathcal{L}_b: j=n} (W_{t,s,jj,b} a_{i,j,b} - W^p_{t,s,ji,b} a_{i,j,b} + W^q_{t,s,ji,b} e_{i,j,b}), \quad \forall n \in \mathcal{N}_b, \end{aligned} \tag{2d}$$

$$\begin{aligned} q_{t,s,n,b} = & \sum_{(i,j)\in\mathcal{L}_b: i=n} \left(W_{t,s,ii,b} e_{i,j,b} - W^p_{t,s,ij,b} e_{i,j,b} - W^q_{t,s,ij,b} a_{i,j,b}\right) \\ & + \sum_{(i,j)\in\mathcal{L}_b: j=n} \left(W_{t,s,jj,b} e_{i,j,b} - W^p_{t,s,ji,b} e_{i,j,b} - W^q_{t,s,ji,b} a_{i,j,b}\right), \quad \forall n \in \mathcal{N}_b, \end{aligned} \tag{2e}$$

$$W_{t,s,ii,b} a_{i,j,b} - W^p_{t,s,ij,b} a_{i,j,b} + W^q_{t,s,ij,b} e_{i,j,b} \le \overline{\mathcal{F}}_{i,j,b}, \quad \forall (i,j) \in \mathcal{L}_b, \tag{2f}$$

$$W_{t,s,jj,b} a_{i,j,b} - W^p_{t,s,ji,b} a_{i,j,b} + W^q_{t,s,ji,b} e_{i,j,b} \le \overline{\mathcal{F}}_{j,i,b}, \quad \forall (i,j) \in \mathcal{L}_b, \tag{2g}$$

$$\left(v^{\min}_{t,s,i,b}\right)^2 \le W_{t,s,ii,b} \le \left(v^{\max}_{t,s,i,b}\right)^2, \quad \forall i \in \mathcal{N}^+_b, \tag{2h}$$

$$2 \sum_{(i,j)\in\mathcal{P}_{n,b}} \sum_{k: i\in\mathcal{P}_{k,b}} \left(\frac{a_{i,j,b} p_{t,s,k,b}}{a^2_{i,j,b} + e^2_{i,j,b}} + \frac{e_{i,j,b} q_{t,s,k,b}}{a^2_{i,j,b} + e^2_{i,j,b}}\right) + W_{t,s,00,b} \le \left(v^{\max}_{t,s,n,b}\right)^2, \quad \forall n \in \mathcal{N}^+_b, \tag{2i}$$

$$W^p_{t,s,ij,b} = W^p_{t,s,ji,b}, W^q_{t,s,ij,b} = -W^q_{t,s,ji,b}, \quad \forall (i,j) \in \mathcal{L}_b, \tag{2j}$$

$$-v^2_{\max} \le W^p_{t,s,ij,b} \le v^2_{\max}, -v^2_{\max} \le W^q_{t,s,ij,b} \le v^2_{\max}, \quad \forall (i,j) \in \mathcal{L}_b, \tag{2k}$$

$$-v^2_{\max} \le W^p_{t,s,ji,b} \le v^2_{\max}, -v^2_{\max} \le W^q_{t,s,ji,b} \le v^2_{\max}, \quad \forall (i,j) \in \mathcal{L}_b, \tag{2l}$$

$$\left(W^p_{t,s,ij,b}\right)^2 + \left(W^q_{t,s,ij,b}\right)^2 + \left(\frac{W_{t,s,ii,b} - W_{t,s,jj,b}}{2}\right)^2 \le \left(\frac{W_{t,s,ii,b} + W_{t,s,jj,b}}{2}\right)^2, \quad \forall (i,j) \in \mathcal{L}_b, \tag{2m}$$

$$d^{p,\min}_{t,s,n,k,b} \le d^p_{t,s,n,k,b} \le d^{p,\max}_{t,s,n,k,b}, \quad \forall n \in \mathcal{N}^+_b, \forall k \in \Omega^{D\_local}_{t,s,n,b} \tag{2n}$$

$$g^{p,\min}_{t,s,n,k,b} \le g^p_{t,s,n,k,b} \le g^{p,\max}_{t,s,n,k,b}, \quad \forall n \in \mathcal{N}^+_b, \forall k \in \Omega^{G\_local}_{t,s,n,b}, \tag{2o}$$

$$d^{q,\min}_{t,s,n,k,b} \le d^q_{t,s,n,k,b} \le d^{q,\max}_{t,s,n,k,b}, \quad \forall n \in \mathcal{N}^+_b, \forall k \in \Omega^{D\_local}_{t,s,n,b}, \tag{2p}$$

$$g^{q,\min}_{t,s,n,k,b} \le g^q_{t,s,n,k,b} \le g^{q,\max}_{t,s,n,k,b}, \quad \forall n \in \mathcal{N}^+_b, \forall k \in \Omega^{G\_local}_{t,s,n,b}, \Big] \qquad \forall t \in \mathcal{T}, \forall s \in \mathcal{S}, \forall b \in \mathcal{B}^{lm}. \tag{2q}$$

The objective function (2a) maximizes the local social welfare while accounting for the active and reactive power trades with the transmission grid at the substation level. A penalty term $\bar{M}\,|p_{t,s,0,b} - (d^{agg*}_{t,s,b} - g^{agg*}_{t,s,b})|$ imposes an imbalance cost on the active power injection $p_{t,s,0,b}$ that deviates from the *nominal dispatch* $d^{agg*}_{t,s,b} - g^{agg*}_{t,s,b}$. The revenues (or costs) from selling (or buying) the excess (or deficit) reactive power $q_{t,s,0,b}$ at the slack bus are computed based on the price $c^q_{t,s,0,b}$. Constraints (2b)–(2c) define the active and reactive power injections for $\forall n \in \mathcal{N}^+_b$ of the local market connected to the transmission bus b, $b \in \mathcal{B}^{lm}$. Constraints (2d) and (2e) enforce Kirchhoff's first law, where the sum of power inflows and outflows should be equal to the power injection for active and reactive power. Eqs. (2f) and (2g) impose the limits on active power flows over the distribution line, where parameters $\bar{\mathcal{F}}_{i,j,b}$ and $\bar{\mathcal{F}}_{j,i,b}$ set the maximum power flow leaving node i and j over the line $(i,j) \in \mathcal{L}_b$, respectively. Constraint (2h) enforces the voltage magnitude for distribution node $i \in \mathcal{N}^+_b$. Eq. (2i) is a technical constraint to ensure that the obtained power flows are exact (see Lemma 2 in [32]). Eq. (2j) ensure the Hermitian property of the matrix $W\{t,s,i,j,b\}$. Eqs. (2k) and (2l) state the limits for voltage auxiliary variables, and Eq. (2m) is a second-order cone constraint to ensure the positive semi-definite property of the matrix $W\{t,s,i,j,b\}$ [30]. The dual variables associated with $W^p_{t,s,ij,b}$, $W^q_{t,s,ij,b}$, $(W_{t,s,ii,b} - W_{t,s,jj,b})/2$ and $(W_{t,s,ii,b} + W_{t,s,jj,b})/2$ in (2m) are $\eta^1_{t,s,i,j,b} \in \mathbb{R}$, $\eta^2_{t,s,i,j,b} \in \mathbb{R}$, $\eta^3_{t,s,i,j,b} \in \mathbb{R}$ and $\gamma_{t,s,i,j,b} \geq 0$, respectively. The maximum and minimum quantity limits for flexible consumers and generators are defined by (2n) and (2q) for active and reactive power. The reformulated LEM clearing problem, which addresses the absolute term in the objective function (see Eq. (2a)), is presented in Appendix D.

2.4. Upper-level problem: the Transco investment problem under regulatory incentives

In this section, we introduce an extension to the existing H–R–G–V mechanism [22,23], taking into account the operations of LEMs. We assume that the Transco is making investment decisions based on the anticipated effects these decisions will have on both WSM and LEM operations. The regulated Transco is a profit-maximizing merchant transmission investor [36], who aims to maximize its profits through a two-pronged approach [22]. Firstly, it seeks to expand transmission lines and generate revenue by collecting merchandising surplus. Secondly, it receives compensation in the form of an incentive fee, established by the regulator, for maximizing the social welfare. The incentive fee is determined by the regulator based on the surplus increase due to line investments [22]. In addition, we extend the H–R–G–V mechanism by assuming that any deviation in local injection power leads to a penalty in the form of the LEM imbalance costs, which are factored into the total surplus. As a result, any deviation from the nominal scheduling power determined in the WSM clearing can lead to a reduction in total social welfare. Therefore, the Transco considers the final results of LEMs to make an informed decision regarding the appropriate investment expansion strategy. The upper-level problem is the Transco's long-term planning problem, and is mathematically presented in Problem (3).

$$\max_{u_{t,l}, b^F_{t,l,j}, \Phi_t} \quad \sum_{t\in\mathcal{T}} \frac{1}{(1+r)^{t-1}} \left(MS_t + \Phi_t - C_t \right) - IC_{t=1} \tag{3a}$$

s.t.

$$MS_t = \Psi \sum_{s\in S}\sum_{b\in\mathcal{B}} \pi^*_{t,s,b}\Big(\sum_{k\in\Omega^{D\_TN}_{t,s,b}} d^*_{t,s,k,b} - \sum_{k\in\Omega^{G\_TN}_{t,s,b}} g^*_{t,s,k,b} \Big) + \Psi \sum_{s\in S}\sum_{b\in\mathcal{B}^{lm}} \pi^*_{t,s,b}\big(d^{agg*}_{t,s,b} - g^{agg*}_{t,s,b}\big), \quad \forall t\in\mathcal{T}, \tag{3b}$$

$$C_t = \Psi \sum_{l\in\mathcal{L}} \Big(u_{t,l} K^{fix}_l + K^{var}_l \sum_{j\in\mathcal{J}} b^F_{t,l,j} \bar{F}_{l,j} \Big), \quad \forall t\in\mathcal{T}, \tag{3c}$$

$$IC_t = \Psi \sum_{s\in S}\sum_{b\in\mathcal{B}^{lm}} \bar{M}\left(z^{1*}_{t,s,b} + z^{2*}_{t,s,b} \right), \quad \forall t\in\mathcal{T}, \tag{3d}$$

$$S^L_t = \Psi \sum_{s\in S}\sum_{b\in\mathcal{B}^{lm}} \left(U^{agg}_{t,s,b}(d^{agg}_{t,s,b}) - \pi^*_{t,s,b} d^{agg*}_{t,s,b} \right) + \Psi \sum_{s\in S}\sum_{b\in\mathcal{B}}\sum_{k\in\Omega^{D\_TN}_{t,s,b}} \left(c^d_{t,s,k,b} - \pi^*_{t,s,b} \right) d^*_{t,s,k,b}, \quad \forall t\in\mathcal{T}, \tag{3e}$$

$$S^G_t = \Psi \sum_{s\in S}\sum_{b\in\mathcal{B}^{lm}} \left(\pi^*_{t,s,b} g^{agg*}_{t,s,b} - C^{agg}_{t,s,b}(g^{agg}_{t,s,b}) \right) + \Psi \sum_{s\in S}\sum_{b\in\mathcal{B}}\sum_{k\in\Omega^{G\_TN}_{t,s,b}} \left(\pi^*_{t,s,b} - c^g_{t,s,k,b} \right) g^*_{t,s,k,b}, \quad \forall t\in\mathcal{T}, \tag{3f}$$

$$\Delta\Phi_t = \left(\Delta S^L_t + \Delta S^G_t - IC_t \right), \quad t = 2, \tag{3g}$$

$$\Delta\Phi_t = \left(\Delta S^L_t + \Delta S^G_t - \Delta IC_t \right), \quad \forall t \geq 3, \tag{3h}$$

$$\sum_{t\in\mathcal{T}}\sum_{j\in\mathcal{J}} b^F_{t,l,j} \leq 1, \quad \forall l\in\mathcal{L}, \tag{3i}$$

$$u_{t,l} = \sum_{j\in\mathcal{J}} b^F_{t,l,j}, \quad \forall l\in\mathcal{L}, \forall t\in\mathcal{T}, \tag{3j}$$

$$u_{t,l} \in \{0,1\}, b^F_{t,l,j} \in \{0,1\}, \Phi_{t=1} = 0, u_{t=1,l} = 0. \tag{3k}$$

Eq. (3a) represents the objective of the Transco who aims to maximize its profit over the investment period $\mathcal{T}$. The Discounted Present Values (DPV) of the profit of the Transco is expressed by using the discount rate r. The objective function (3a) consists of the sum of the merchandising surplus MS_t and the incentive fee Φ_t minus the line expansion costs C_t and the first year imbalance costs $IC_{t=1}$. Specifically, the merchandising surplus MS_t, line expansion cost C_t and local imbalance costs IC_t are represented in Eqs. (3b), (3c) and (3d), respectively.

The starred terms represent the optimal solution of the middle-level WSM clearing problem and lower-level LEMs clearing problem (see Fig. 1). The optimal solutions of middle-level WSM clearing (which depends on the upper level variables) such as nodal prices $\pi^*_{t,s,b}$ and cleared quantities $d^{agg*}_{t,s,b}$, $g^{agg*}_{t,s,b}$, $d^*_{t,s,k,b}$, $g^*_{t,s,k,b}$ are used to compute the merchandising surplus MS_t. The expansion cost C_t includes fixed investment cost K^{fix}_l and variable costs K^{var}_l. The term $\bar{F}_{l,j}$ is the lumpy expansion on line l, and the line capacity can be increased (expanded) only by a finite set of discretized quantities $\bar{\mathcal{F}}_l = \bigcup_{j\in J} \bar{F}_{l,j}$ [35]. IC_t represents the cost incurred to offset power imbalances (w.r.t. injections scheduled at the WSM by the first stage) for LEM b's operations in year t and operation period s. The two auxiliary variables $z^{1*}_{t,s,b}$ and $z^{2*}_{t,s,b}$ are primal variables of the lower-level LEM clearing problems used to replace the active power deviation.

The incentive fee Φ_t is calculated based on load surplus S^L_t (Eq. (3e)) and generator surplus S^G_t (Eq. (3f)) and is subject to regulatory constraints (3g) and (3h). Let $\Delta\Phi_t$ represent the change in the value of Φ_t from year $t-1$ to year t. Specifically, this can be mathematically represented as $\Delta\Phi_t = \Phi_t - \Phi_{t-1}$. Similarly, the changes in values for other variables are defined as $\Delta S^L_t = S^L_t - S^L_{t-1}$, $\Delta S^G_t = S^G_t - S^G_{t-1}$, $\Delta IC_t = IC_t - IC_{t-1}$.

The time scaling factor Ψ is to match the yearly investment period with (hourly) operation periods. The investment decisions are made through binary variables $u_{t,l}$ and Eqs. (3i) and (3j) ensure that the decision to expand line l is taken only once for investment period $\mathcal{T}$ and the investment is irreversible. Another binary variable $b^F_{t,l,j}$ is introduced to determine the lumpy expansion of the candidate line l in the investment period $t \in \mathcal{T}$. It is assumed that no expansion is performed in the first year, i.e., $u_{t=1,l} = 0$ and $\Phi_{t=1} = 0$.

Lemma 2.1. *The Transco's profit-maximizing investment strategy under the proposed incentive mechanism leads to social welfare maximization, taking into account the imbalance costs of LEMs. The proof can be found in Appendix A.*

2.5. Discussion on the tri-level framework

The tri-level optimization framework is crucial for capturing the sequential interactions between the WSM and LEMs in merchant-regulated transmission expansion planning (TEP). While these two separate markets operate independently, they are linked through a sequential clearing process. This sequentiality influences market outcomes and has a significant impact on transmission investment decisions. Unlike traditional models that utilize single-level (see Section 2 in [35]) or bi-level [28] approaches, our tri-level model successfully captures this sequence in market structure, which these traditional frameworks cannot address. Given the rapid rate of DER integration and the increasing share of flexible users in LEMs, our model offers a solution for long-term network planning to ensure that the transmission grid is properly reinforced to handle these changes.

Moreover, traditional TEP models fail to account for the sequential operations of the WSM and LEMs, remaining unresponsive to evolving local conditions. In the proposed framework, the WSM is cleared, and local dispatch nominations are determined based on aggregated and approximate data. These nominations do not fully reflect the detailed costs and specifics of local networks. As a result, assuming no deviations from the *nominal dispatch* can be inaccurate. Our tri-level model addresses these limitations by improving Transco's awareness and responsiveness to local market conditions, while accounting for the sequential clearing processes of the WSM and LEMs. This model mitigates the risks of over- or under-investment in transmission assets by integrating local refinements into the decision-making process.

In Section 4.2.1, we examine how various local deviations influence transmission investment decisions within the sequential operational frameworks of the WSM and LEMs. When high penalties for local imbalances compel LEMs to strictly follow WSM nominations, the outcomes of both the traditional central planning model and the tri-level model are identical. However, this case is based on overly restrictive assumptions about local operations, since it presumes there are no deviations from the WSM nominations, which may not be realistic. On the other hand, in a zero-penalty framework where deviations from market nominations are completely ignored, the scenario becomes unrealistic and diminishes the effectiveness of WSM nominations. The traditional central planning model, whether disregarding local conditions, strictly adhering to predefined nominations, or allowing certain deviations, fails to incorporate local refinements and flexibility into investment decisions. In contrast, the tri-level model overcomes these limitations by integrating local refinements into transmission investment decisions, enabling more effective responses to sequential market conditions and adjusting strategic decisions accordingly.

3. Tri-level optimization and reformulation

In this section, we will discuss the approach adopted to reformulate the tri-level problem as a MISOCP problem.

3.1. Compact tri-level optimization model

Let variable subscripts u, w and l denote the upper-level problem, the wholesale and the local energy markets, respectively. The tri-level optimization problem is formulated as (4).

$$\max_{\substack{z\in\{0,1\}^m, x_u,\\ x_w, y_w\\ x_l\in\mathcal{K}, y_l}} \quad h^\top z + c_u^\top x_u + c_{uw}^\top x_w y_w + c_{ul}^\top x_l \tag{4a}$$

s.t.

$$z \in \{0,1\}^m, z \in \mathcal{Z} \tag{4b}$$

$$A x_u + D x_w + E x_w y_w + F x_l \le b_u \tag{4c}$$

$$(x_w, y_w, x_l, y_l) = \arg\max_{\substack{x_w, y_w,\\ x_l\in\mathcal{K}, y_l}} \; c_w^\top x_w \tag{4d}$$

$$\text{s.t.} \quad A_w x_w + B_w z \le b_w \tag{4e}$$

$$(x_l, y_l) = \arg\max_{x_l\in\mathcal{K}, y_l} \; c_l^\top x_l \tag{4f}$$

$$\text{s.t.} \quad A_l x_l + B_l x_w \le b_l \tag{4g}$$

The vectors z and x_u represent the binary and continuous variables, respectively, of the upper-level problem, which is the Transco's planning problem (3). (x_w, y_w) represent the primal and dual variables of the middle-level problem, which is the WSM clearing problem (1), and (x_l, y_l) represent the vectors of primal and dual variables of the lower-level problem (2). The feasible region of the upper-level problem investment decision variables is denoted by $\mathcal{Z}$, while the proper cone denoting the domain of x_l is represented by $\mathcal{K}$. Specifically, $\mathcal{K}$ is the Cartesian product of a collection of second-order cones and non-negative orthants [37]. The mathematical representation with appropriate dimensions can be used to derive the vectors $(h, c_u, c_{uw}, c_{ul}, b_u, c_w, b_w, c_l, b_l)$ and matrices $(A, D, E, F, A_w, B_w, A_l, B_l)$. The nonlinear terms $x_w y_w$ represent the merchandising surplus, which is calculated based on the WSM primal solution x_w (i.e., allocated quantities) and the WSM dual solution y_w (i.e., clearing prices).

The upper-level problem (i.e., Eqs. (4a)–(4c)) formulates the Transco's investment problem under the proposed regulated incentive mechanism (4c). The investment decision variables z from the upper-level problem will be plugged into the middle-level problem (i.e., Eqs. (4d)–(4e)), which describes the WSM clearing problem and determines the dispatch (primal solution), prices (dual solution), and the *nominal dispatch* (calculated from primal solution). The *nominal dispatch* determined in the middle-level will be fed into the lower-level problem (i.e., Eqs. (4f)–(4g)), which formulates the LEMs clearing problem. The lower-level problem determines the nodal prices (dual solution) and local dispatch (primal solution).

3.2. Reformulation as a MISOCP problem

Theorem 3.1. *Problem (4) can be asymptotically approximated by the single-level problem (5) using lexicographic optimization and the weight-sum method for $\gamma \in (0,1)$ [37,38]:*

$$\max \quad \gamma h^\top z + \gamma c_u^\top x_u + \gamma c_{uw}^\top x_w \frac{y_w}{\gamma} + \gamma c_{ul}^\top x_l \tag{5a}$$

s.t.

$$A_u \boldsymbol{x}_u + D\boldsymbol{x}_w + E\boldsymbol{x}_w \frac{\boldsymbol{y}_w}{\gamma} + F\boldsymbol{x}_l \le b_u \tag{5b}$$

$$\boldsymbol{z} \in \mathcal{Z}, \boldsymbol{z} \in \{0,1\}^m \tag{5c}$$

$$A_w \boldsymbol{x}_w + B_w \boldsymbol{z} \le b_w \tag{5d}$$

$$A_l \boldsymbol{x}_l + B_l \boldsymbol{x}_w \le b_l \tag{5e}$$

$$\boldsymbol{y}_w^\top A_w + \boldsymbol{y}_l^\top B_l \ge \gamma c_w^\top \tag{5f}$$

$$\boldsymbol{y}_l^\top A_l \ge_{\mathcal{K}^*} (1-\gamma) c_l^\top \tag{5g}$$

$$\boldsymbol{y}_w^\top (b_w - B_w \boldsymbol{z}) + \boldsymbol{y}_l^\top b_l \le \gamma c_w^\top \boldsymbol{x}_w + (1-\gamma) c_l^\top \boldsymbol{x}_l \tag{5h}$$

$$\boldsymbol{x}_w \ge 0, \boldsymbol{x}_l \in \mathcal{K} \tag{5i}$$

$$\boldsymbol{y}_w \ge 0, \boldsymbol{y}_l \ge 0 \tag{5j}$$

The solution of (5) converge to the solution of (4) when the parameter γ tends to 1. The proof can be found in Appendix B.

Complementary slackness, McCormick Envelopes linearization technique [39] and the big-M method [30,40] are utilized to remove the nonlinear terms, and the final optimization problem results in a single-level MISOCP model. This MISOCP problem can be solved by standard off-the-shelf solvers. To handle large-scale versions of the problem, Benders Decomposition can be used to break down the mixed-integer problem into a relaxed master problem and a Benders subproblem [41].

4. Case study

This section introduces three case studies: one featuring a 2-node transmission network connected to an IEEE 33-bus distribution network (denoted as T2D33), another involving the Garver's 6-node system connected to six IEEE 33-bus distribution networks (denoted as T6D33), and the last one with the modified IEEE 118-bus system connected to fifteen IEEE 33-bus distribution networks (denoted as T118D33). The proposed model is executed on a 3.40 GHz Intel i-7 with 32 GB RAM computer.

For simplicity and clarify of presentation, we assume that each planning period represents one year and includes one operation period $\mathcal{S} = \{1\}$ and the number of operation sub-periods in one year Ψ is 8760. In terms of the distribution network, the voltage is expressed in per-unit with the substation voltage magnitude (at the slack bus) set at 1 p.u., and the maximum and minimum voltage magnitude limits for the remaining nodes set at 1.2 p.u. and 0.8 p.u., respectively [30]. The data for the IEEE 33-bus distribution network and the non-flexible demand located in each distribution network node can be found in [42]. The maximum power flow capacity for distribution network lines is $\overline{\mathcal{F}}_{i,j,b} = 4000$ kW. The base voltage and base apparent power are 12.6 kV and 1 kVA, respectively. In all case studies, at each distribution node $n \in \mathcal{N}_b^+$, where $\mathcal{N}_b^+ = \{1, 2, \dots, 32\}$, $b \in \mathcal{B}^{lm}$, it is assumed there is one flexible consumer, one flexible generator, and one non-flexible demand, i.e., $\Omega_{t,s,n,b}^{G\_local} = \{1\}$ and $\Omega_{t,s,n,b}^{D\_local} = \{1\}$. The price of reactive power traded with the transmission grid $c_{t,s,0,b}^{q}$ is set to £30/MVArh [30]. Bid prices are randomly generated from normal distributions with mean prices of £50/MWh for TNLs $c_{t,s,k,b}^{d}$ and local flexible consumers $c_{t,s,k,b}^{d\_local}$, £20/MWh for GENCOs $c_{t,s,k,b}^{g}$ and £0/MWh for local flexible generators $c_{t,s,k,b}^{g\_local}$, with the same standard deviations of £10/MWh. Negative prices are set to zero.

In what follows, WSM social welfare is calculated by considering the aggregated local dispatch and the dispatch from upstream participants, as determined by the bids and quantities cleared in the WSM, and then subtracting investment and imbalance costs. In the subsequent stage, the LEMs are cleared, accounting for updated bids and distribution network constraints, with the process guided by the nominations from WSM outcomes. The overall social welfare is computed by combining the first-stage WSM clearing results from upstream participants with the actual local dispatch and prices from the second-stage LEM clearing, and then subtracting both investment and imbalance costs.

4.1. Case study 1: T2D33

The system analyzed in this study consists of a two-node transmission network, i.e., $\mathcal{B} = \{1, 2\}$ and $\mathcal{L} = \{1\}$, where node 2 has 50 GENCOs and 50 TNLs and node 1 is connected to an IEEE 33-bus distribution network, as illustrated in Fig. 2. The dashed branch 1 is a new branch that can be built. The set of lumpy capacity expansions is defined as $\mathcal{F}_m = \{1, 2, \dots, 100\}$MW. The reactance of branch 1 is set to 0.2 p.u. and the variable investment cost K_l^{var} is £10/MWh, while the fixed cost K_l^{fix} is £100/h. The limits of GENCOs and TNLs are generated from a uniform distribution ranging from zero to 2 MW. The annual growth rate of load and generation is assumed to be 1%.

4.1.1. Variations in local flexibility ratios: flexible generation

In this section, we aim to demonstrate the impact of local flexible generation on transmission investment decisions, Transco's profit, and social welfare. We assume that the maximum flexible local demand is half of the non-flexible demand. The maximum flexible generation ratios (FGRs) considered ranges from 1× to 2.15× the non-flexible demand. The operational timescale of the planning problem includes ten investment periods, i.e., $\mathcal{T} = \{1, 2, \dots, 10\}$. The optimization problem involved 32,509 variables, 52,751 linear constraints and 640 quadratic constraints, with a computational time of 12.72 s.

For the proposed tri-level model applied to the T2D33 case study with different FGRs, economic and operational outcomes are presented in Figs. 3 and 4. Fig. 3(a) presents the Transco's profit, incentive fee, merchandising surplus, and investment costs for the T2D33 network. In addition, Fig. 3(b) depicts the *nominal dispatch*, indicative of the net demand within the distribution network, alongside line expansion decisions. Fig. 3(c) shows the WSM and overall social welfare with different FGR. Additionally, the clearing volumes for both the WSM and LEMs are illustrated in Fig. 4. Fig. 5 illustrates Transco's profit and line expansion decisions for the T2D33 system over a ten-year investment period, under two different FGR scenarios: FGR = 1, shown in Fig. 5(a), and FGR = 2, shown in Fig. 5(b). In both scenarios, Transco's profit increases steadily over the investment period. Line expansion occurs in year 2 at 2 MW and remains at 0 in subsequent periods.

Based on the results, we characterized the outcomes into three phases:

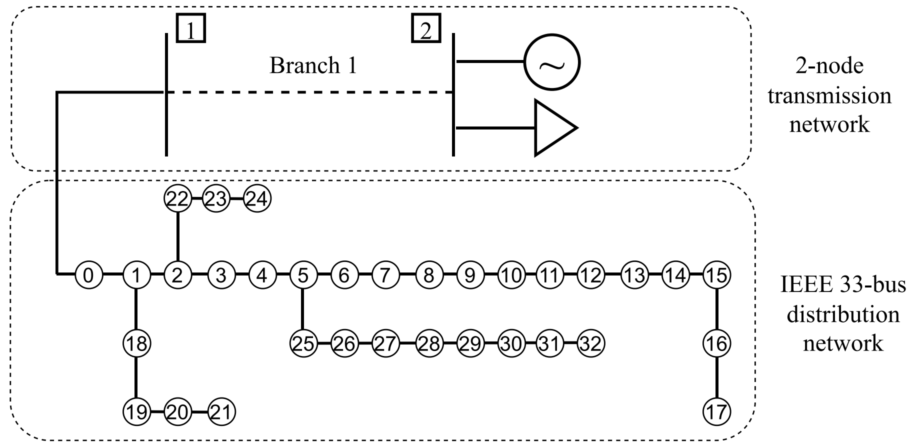

Fig. 2. Topology of the 2-node transmission network with an IEEE 33-bus distribution network connected to node 1.

I: Non-self-sufficient LEM, flow from upstream to local;
II: Self-sufficient, independently operated LEM;
III: Self-sufficient LEM, flow from local to upstream.

In Phase I of Fig. 3, the LEM is not self-sufficient, and additional power is required from GENCOs in node 2, necessitating line expansion. As the FGR increases, there is a reduced reliance on GENCOs, as evidenced by the slight decrease in average imported active power from upstream to the local grid (net distribution network demand) from 1.87 MW to 1.44 MW, as shown in the red line in Fig. 3(b). This reduced reliance can also be evidenced by the clearing volumes of GENCOs Fig. 4(a). In the second year, the Transco invests in Line (1,2) with a capacity of 2 MW due to the lumpiness of investments.

In Phase II of Fig. 3, the LEM becomes self-sufficient as FGR increases. Notably, no expansion is performed during this phase and LEM operates independently. Increasing the proportion from 1.11 to 1.12 results in a decrease in incentive fee, as the local generation is sufficient to meet local demand without requiring additional power from GENCOs in node 2. As presented in Fig. 3(c), social welfare consistently increases to £1,173.39M when the FGR reaches 1.60, driven by an increasing pair of local flexible generation and demand.

In Phase III of Fig. 3, the expansion levels continue to increase from 1 MW (FGR from 1.65 to 1.75) to 2 MW (FGR from 1.80 to 2.00) and eventually to 3 MW (FGR from 2.05 to 2.15), as depicted in Fig. 3(b). It is evident that there is a significant 'drop' in the net distribution network demand (the red line) from 1.87 MW (1.87 MW withdrawn from the transmission network) to −2.54 MW (2.54 MW injected into the transmission network) after the local generation ratio increases from 1.00 to 2.15. This trend indicates that the local generators with lower marginal costs are utilized to meet both upstream and local demand, enabling further line expansion. This trend can be evidenced by the increasing clearing volumes of flexible generation in WSM as shown in Fig. 4(a) and in LEM as shown in Fig. 4(b). In contrast, the red line in Fig. 3(b) displays an inconsistent upward trend in the incentive fee, with a decline observed at ratios 1.75 and 2.00. The yellow line, on the other hand, shows an increase in merchandising surplus at these ratios, indicating the occurrence of congestion in Line (1,2) and the Transco can obtain a greater merchandising surplus.

The results in Fig. 3(c) demonstrate that the WSM social welfare consistently increases with the growing FGR. However, a small decline to £1,173.09M is observed in the overall social welfare at the ratio of 1.65 (the beginning of Phase III) due to the separation between the WSM clearing and LEMs clearing. After increasing the FGR from 1.6 to 1.65, we observe a decrease in the LEM's objective function, which represents the local social welfare, from £2,510.89M to £2,423.56M. Two reasons contribute to this decrease. Firstly, the local aggregation in WSM and the resulting *nominal dispatch* do not account for the distribution network losses, which may lead to imprecise dispatch decisions. Secondly, the Transco receives dispatch information solely from the WSM. This information takes into account the first-stage aggregated local dispatch but does not include the second-stage LEMs individual local dispatch information.

The results presented in Fig. 3(a) demonstrate a U-shaped relationship between the Transco's profit and the FGR, which indicates the level of demand for investment. The rise of FGR has facilitated greater injection of power into the transmission grid, thereby lessening the dependence on GENCOs. However, it is important to consider several factors before concluding whether or not this flexibility leads to decreased or postponed investment in transmission assets. One critical factor is the geographic placement of local grids and transmission loads. If the local grid is not directly connected to upstream demand or the existing line capacity is not sufficient, additional transmission infrastructure investment may be necessary despite the availability of flexible generation, as suggested in Fig. 3 Phase I. Furthermore, the extent of flexible generation also plays a significant role in transmission investment decisions and the Transco's profit. If the amount of flexible generation is not enough to meet local demand and provide excess power to the transmission grid, then additional transmission infrastructure investment and power from GENCOs may still be needed. Conversely, if there is a surplus of available flexible generation and the local market can serve as a power source, then line expansion may still be necessary.

4.1.2. Variations in local flexibility ratios: flexible vs. non-flexible consumers

This test case shows how different proportions of flexible and non-flexible consumers and flexible generation may affect the line expansion decisions. We consider the upper bound of flexible local generation ranges from 1× to 2.1× the predetermined total inflexible demand. The study further investigates varying ratios of flexible consumers to inflexible consumers, i.e., 0, 33%, 50%, 67%, and 100%.

Fig. 6 depicts the transmission line expansion decisions under varying ratios of flexible generation and demand. Additionally, the clearing volumes for both the WSM and LEMs under FGR = 2 are illustrated in Fig. 7. In Fig. 6(a), when the generation ratio lies between 1 and 1.11, elevating

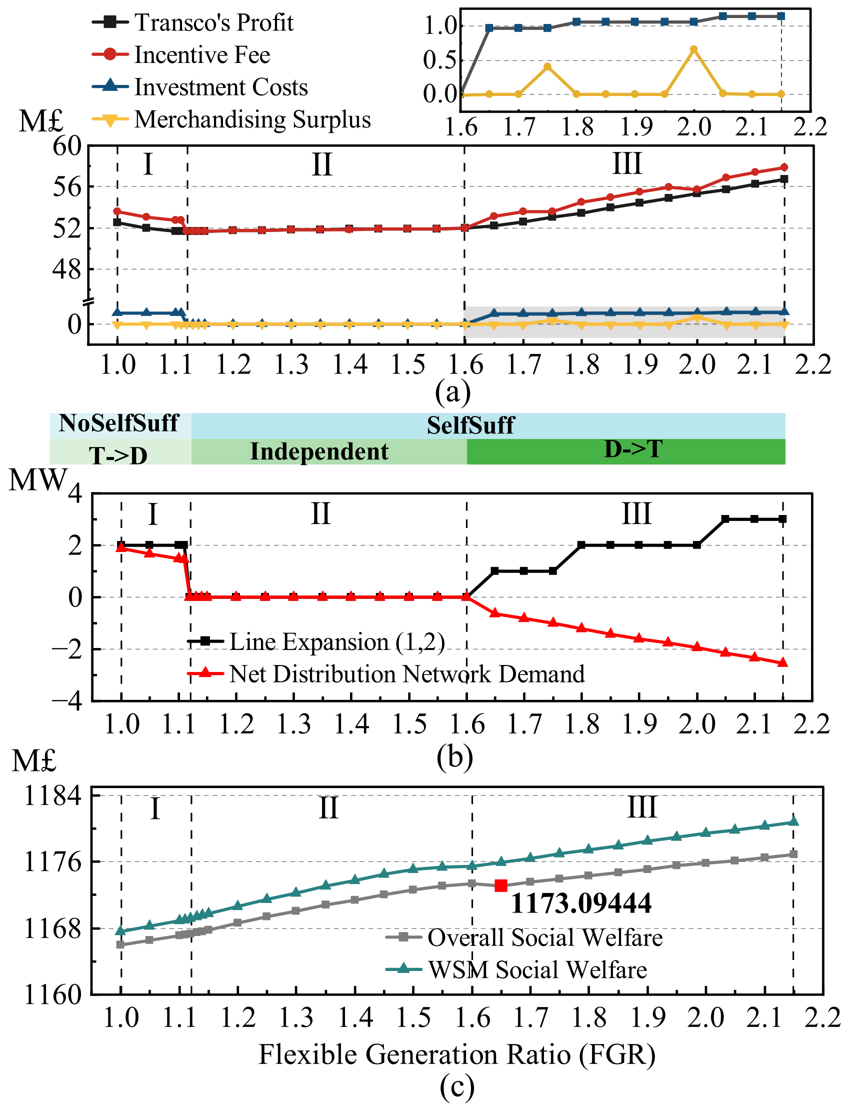

Fig. 3. Results for the T2D33 system with different FGRs. The gray area is shown magnified in the top-right part of the figure. Positive net distribution network demand represents flows withdrawn from the upstream transmission grid, while negative values represent flows injected into the upstream grid.

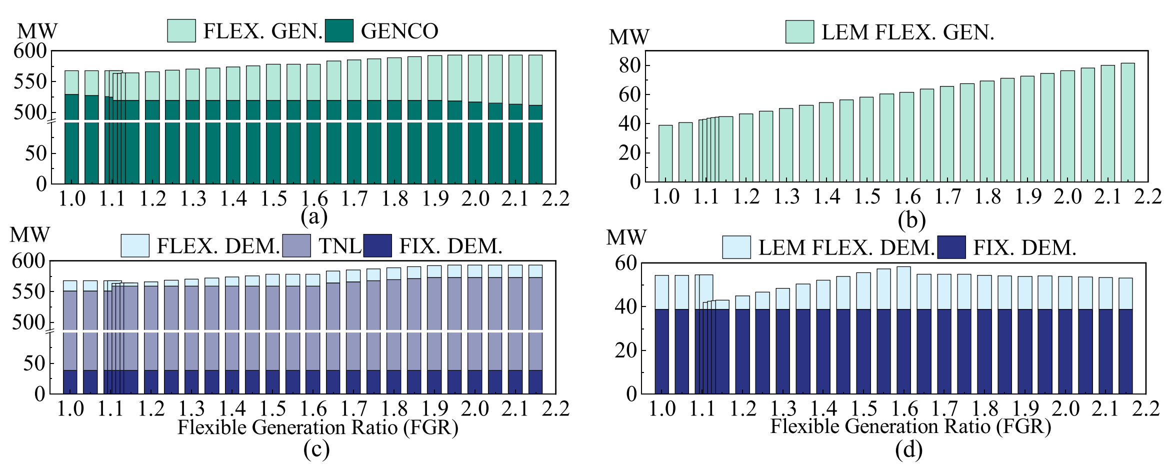

Fig. 4. Clearing volumes for the T2D33 system across different FGRs in both WSM and LEM. (a) details the generation volumes cleared by the WSM, including contributions from GENCOs and flexible local generators (FLEX. GEN.). (b) shows the demand volumes cleared by the WSM, incorporating fixed local demand (FIX. DEM.), transmission network load (TNL), and flexible local generation (FLEX. GEN.). (c) presents the flexible local generation (LEM FLEX. GEN.) clearing amounts within the LEM. (d) displays the demand levels cleared in the LEM, featuring both fixed local demand (FIX. DEM.) and flexible local generation (LEM FLEX. GEN.).

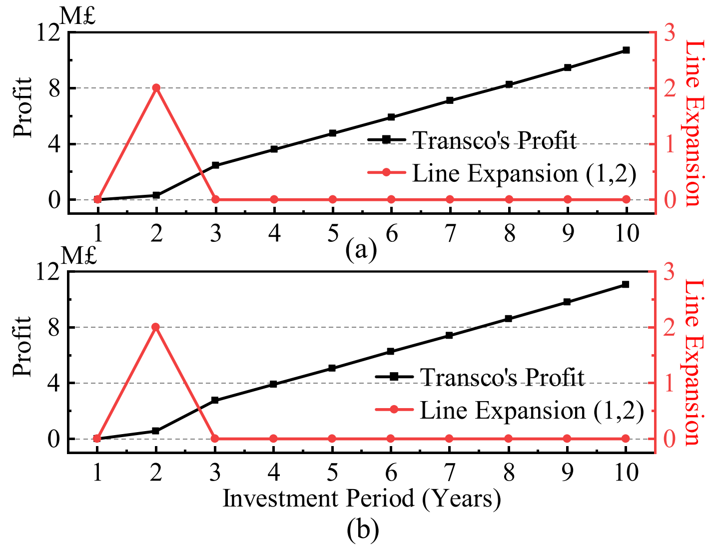

Fig. 5. Transco's profit and investment decisions over the investment periods for the T2D33 system with FGR = 1 (in (a)) and FGR = 2 (in (b)).

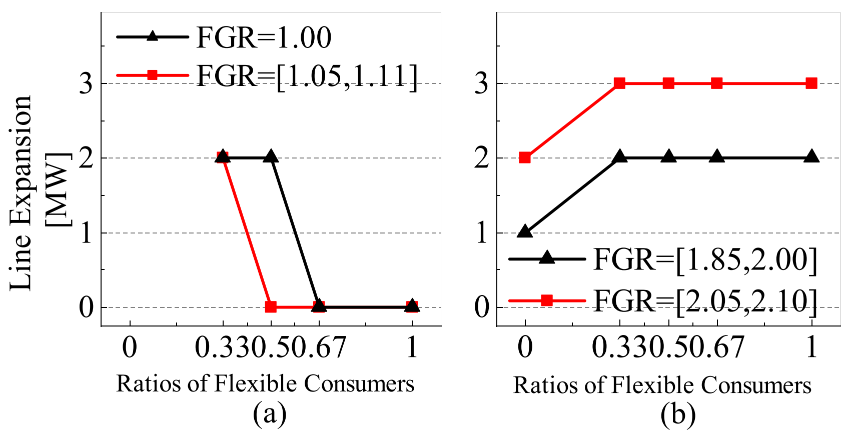

Fig. 6. Results for the T2D33 system with different ratios of local flexible consumers and FGRs. FGR = flexible generation ratio. No solution exists when FGR falls within the range [1,1.11] and the flexible consumer ratio is 0, since local generation is unable to satisfy the fixed demand for the first year.

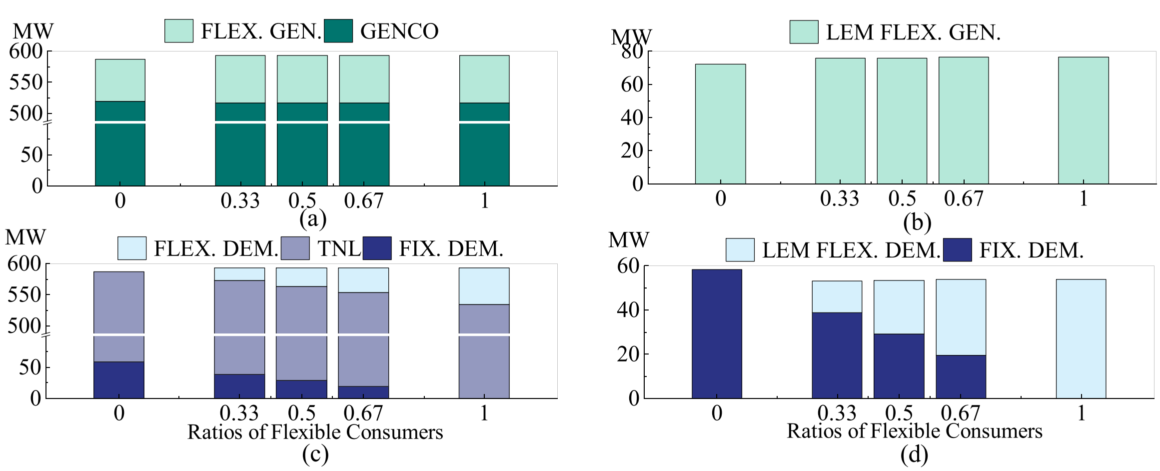

Fig. 7. Clearing volumes for the T2D33 system across different Ratios of Flexible Consumers under FGR = 2 in both WSM and LEM. (a) details the generation volumes cleared by the WSM, including contributions from GENCOs and flexible local generators (FLEX. GEN.). (b) shows the demand volumes cleared by the WSM, incorporating fixed local demand (FIX. DEM.), transmission network load (TNL), and flexible local generation (FLEX. GEN.). (c) presents the flexible local generation (LEM FLEX. GEN.) clearing amounts within the LEM. (d) displays the demand levels cleared in the LEM, featuring both fixed local demand (FIX. DEM.) and flexible local generation (LEM FLEX. GEN.).

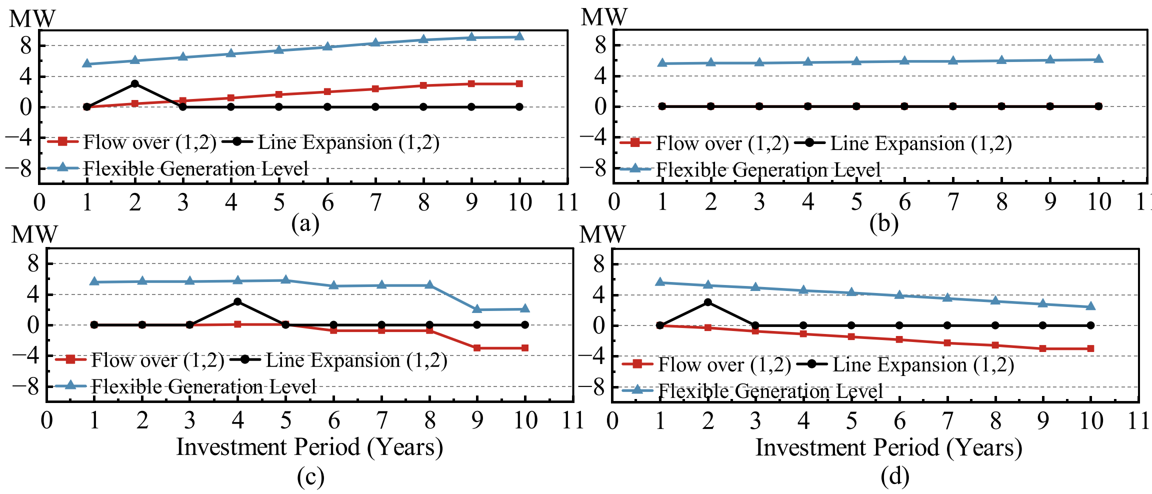

Fig. 8. Effects of temporal changes in generation flexibility on transmission investment. (a) **G↑** for *Steady Increase in Flexibility*, (b) **G-** for *Constant Flexibility*, (c) **G-↓** for *Constant followed by a Sudden Decrease in Flexibility*, (d) **G↓** for *Steady Decrease in Flexibility*.

Table 1
Effects of temporal changes in generation flexibility on transmission investment, social welfare, and market clearing volumes in the T2D33 system. Positive net distribution network demand represents flows withdrawn from the upstream transmission grid, while negative values represent flows injected into the upstream grid.

	G↑	G-	G-↓	G↓
Investment decisions				
Transco's profit (M£)	55.88	51.91	47.27	44.97
Line expansion (MW)	3	0	3	3
Net DN demand (MW)	−1.90	0	0.90	1.81
Social welfare				
Overall social welfare (M£)	1175.48	1172.61	1168.38	1166.40
WSM social welfare (M£)	1179.06	1175.09	1170.45	1168.15
WSM clearing volumes	590.11	577.86	571.63	569.27
GENCO (MW)	514.88	519.56	523.79	528.78
FLEX. GEN. (MW)	75.23	58.30	47.84	40.49
TNL (MW)	532.00	519.56	515.66	512.46
FLEX. DEM. (MW)	19.25	19.43	17.11	17.95
FIX. DEM. (MW)	38.87	38.87	38.87	38.87
LEM clearing volumes[a]				
LEM FLEX. GEN. (MW)	75.70	58.30	47.85	40.49
LEM FLEX. DEM. (MW)	14.89	16.74	14.97	16.25
FIX. DEM. (MW)	38.87	38.87	38.87	38.87

[a] The total LEM clearing volumes are not displayed due to the presence of active and reactive losses in the distribution network and active power injection or withdrawal. Instead, we present the volumes of flexible generation and demand cleared by the LMO, along with the fixed demand.

the ratio of flexible consumers (from 50% to 66.7% at FGR=1.00 and from 33% to 50% for FGR within [1.05,1.11]) reduces the line expansion requirement by 2 MW. This suggests that in scenarios where the LEM is not self-sufficient, augmenting the proportion of flexible consumers could potentially reduce the need for line expansion. Subsequently, as depicted in Fig. 6(b), when the ratio of flexible consumers increases from 0 to 33%, the line expansion rises from 1 MW to 2 MW within the flexible generation ratio of [1.85, 2.00]. This trend persists, with expansions growing from 2 MW to 3 MW as the flexible generation ratio further increases. In situations characterized by a larger share of flexible consumers (i.e., reduced fixed local demand) and an increased presence of flexible generation capabilities, the local generation has the capacity to meet upstream demand, leading to a network expansion.

4.1.3. Temporal dynamics in local flexibility ratios

This section examines the impact of temporal dynamics of local flexibility on transmission planning decisions. It specifically analyzes how changes in the ratio of flexible to non-flexible consumers and local generation flexibility over time influence network planning decisions.

We first analyze how the evolution of the FGR over the investment period impacts transmission investment decisions, assuming that the maximum flexible local demand is half of the non-flexible demand. We identify four patterns of changes in generation flexibility: **G↑** for *Steady Increase in Flexibility*, **G-** for *Constant Flexibility*, **G-↓** for *Constant followed by a Sudden Decrease in Flexibility*, and **G↓** for *Steady Decrease in Flexibility*. Fig. 8 presents the temporal variations in flexible generation, power flow, and line expansion decisions throughout each investment year. Additionally, Table 1 provides a summary of the impact of these changes on transmission investment, social welfare, and market clearing volumes over the entire investment period.

The results reveal a U-shaped relationship between flexibility trends and line expansion decisions, as indicated in Table 1 and the black line in Fig. 8. In the **G↑** scenario, the flow from node 1 to node 2 (red line in Fig. 8(a)) increases as local markets achieve greater self-sufficiency, allowing active power injections into the main grid to meet the TNL at node 2. In contrast, constant flexibility (**G-**), shown in Fig. 8(b), results in no line

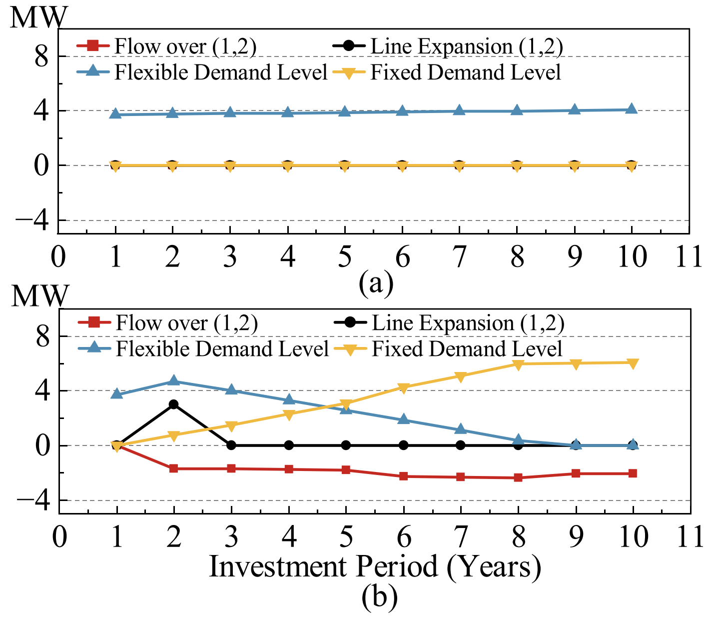

Fig. 9. Effects of temporal changes in demand flexibility on transmission investment. (a) **C-** for *Constant High Flexibility*, (b) **C↓** for *Steady Decrease in Flexibility*.

Table 2
Effects of temporal changes in consumer flexibility on transmission investment, social welfare, and market clearing volumes in the T2D33 system. Positive net distribution network demand represents flows withdrawn from the upstream transmission grid, while negative values represent flows injected into the upstream grid.

	C-	C↓
Investment decisions		
Transco's profit (M£)	7.26	915.32
Line expansion (MW)	0	3
Net DN demand (MW)	0	2.01
Social welfare		
Overall social welfare (M£)	162.41	1070.87
WSM social welfare (M£)	164.39	1072.52
WSM clearing volumes	558.44	568.71
GENCO (MW)	519.57	529.91
FLEX. GEN. (MW)	38.87	38.79
TNL (MW)	519.56	511.84
FLEX. DEM. (MW)	38.87	21.69
FIX. DEM. (MW)	0	35.18
LEM clearing volumes		
LEM FLEX. GEN. (MW)	38.87	38.85
LEM FLEX. DEM. (MW)	37.62	20.47

expansions due to the self-sufficiency of the LEM. Scenarios with constant followed by a sudden decrease in flexibility (**G-↓**), as shown in Fig. 8(c), necessitate a 3 MW line expansion in year 4. This expansion is required as the LEM's flexibility decreases over time, necessitating a delayed line expansion to enable GENCOs at node 2 to compensate for the decline in local generation capacity. In the case of **G↓** (steady decrease in flexibility) as shown in Fig. 8(d), line expansion occurs in year 2 with 3 MW, driven by decreasing flexible generation and increased flow from node 2 to node 1. Furthermore, as FGR decreases over time, both Transco's profit and social welfare decline, driven by reduced availability of lower-cost flexible generation and increased reliance on more expensive generation options, as detailed in Table 1. These findings highlight how temporal variations in local flexibility significantly influence the magnitude and timing of transmission expansion decisions over the investment horizon.

Next, we explore the impact of varying consumer flexibility on transmission investment decisions, with a fixed FGR of 1. Consumer flexibility is categorized into two scenarios: **C-** for *Constant High Flexibility* and **C↓** for *Steady Decrease in Flexibility*. Fig. 9 illustrates the changes in flexible consumers, power flow, and line expansion decisions across each investment year. Additionally, Table 2 details the effects of these changes on transmission investment, social welfare, and market clearing volumes throughout the investment period.

The results from Table 2 and Fig. 9 indicate that a decrease in consumer flexibility results in increased line expansions. This is primarily due to the rising proportion of inelastic local demand that local markets are unable to meet, requiring additional power from upstream sources (GENCOs at node 2). As inelastic demand grows, reliance on external generation increases, thereby driving the need for further line expansions. The increase in inelastic demand, measured by a high Value of Lost Load (VoLL), significantly enhances both social welfare and Transco's profits. However, this shift leads to only a modest rise in WSM clearing volumes, from 558.44 MW to 568.71 MW, as reduced local flexibility necessitates the use of more expensive GENCO resources. This also leads to a decrease in the transmission load that can be met, dropping from 519.59 MW to 511.84 MW.

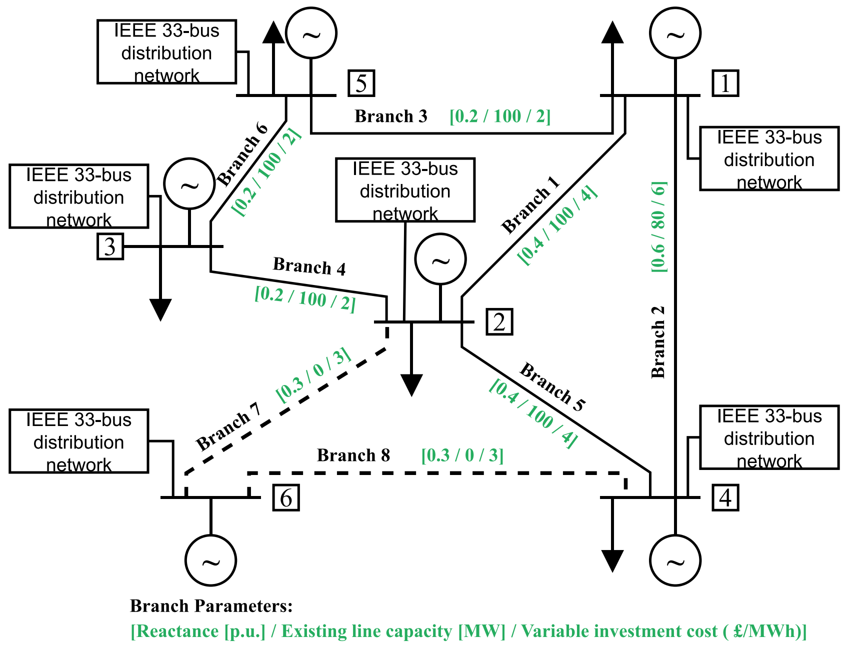

Fig. 10. Topology of Garver's 6-node transmission network with IEEE 33-bus distribution networks.

4.2. Case study 2: T6D33

The case study utilizes the Garver's 6-node transmission network connected to six IEEE 33-bus distribution networks, to investigate investment problem in a more complex setting. This transmission network has 6 nodes and 8 lines, i.e., $\mathcal{B} = \{1, \ldots, 6\}$, and $\mathcal{L} = \{1, \ldots, 8\}$. The network is depicted in Fig. 10, where each transmission node is linked to a local grid, i.e., $\mathcal{B}^{lm} = \{1, \ldots, 6\}$. The reactance of lines and the variable investment cost are shown in Fig. 10. The fixed cost K_l^{fix} is £100/h. The discount rate r is set to 5%. The planning problem involves expanding the existing branches (branch 1 to 6) and constructing new branches (branch 7 and 8). We make the assumption that each transmission network node contains 100 GENCOs and 100 TNLs, with the exception of node 6 which only has GENCOs. The limits of upstream generation and demand are generated from a uniform distribution with values ranging from zero to 250 kW and from zero to 500 kW, respectively. The annual growth rate of load is assumed to be 1%.

4.2.1. Comparative analysis of transmission investment models

This section investigates the impact of excluding LEM operations on transmission expansion decisions by comparing the proposed tri-level model with the central planning (CP) model across two investment periods. The CP problem is modeled as a two-stage, single-level problem where the planner focuses solely on the WSM to maximize WSM social welfare. Nominations obtained from the first stage are then forwarded to LEM operations, with penalties applied for deviations from the WSM outcomes. A detailed formulation of this benchmark optimization model can be found in Appendix E. Key distinctions between the CP and the proposed tri-level model are:

1. Information Assumptions: The CP model operates under the assumption that the central planner has perfect knowledge, which can be relaxed through incentive regulations.
2. Aggregate Resource Consideration: The CP model considers only the aggregation of local resources during the WSM stage and does not account for the subsequent operations in the LEMs.
3. Responsiveness to Local Dynamics: The CP model lacks mechanisms to adapt to changes in local market conditions after initial planning.

In this case study, we set the local FGR value at 1.5 to demonstrate the effectiveness of the proposed model. As outlined in Section 2.5, the CP model does not account for actual LEM operations, leading to constant investment decisions across various penalty levels, that reflect different local deviations. For example, with an FGR of 1.5, investments in lines (6,2) and (6,4) remain fixed at 7 MW and 5 MW, respectively. This lack of responsiveness in the CP model's investment decisions highlights its limitations, particularly its reliance solely on aggregated data from the initial WSM stage, without incorporating the dynamic operations of LEMs. Such deviations can lead to reduced social welfare due to increasing imbalance costs and inefficiencies in investment decisions.

Table 3 presents the results of the tri-level model under varying penalty scenarios for local imbalances in the T6D33 System with an FGR value of 1.5. As penalty levels increase, the absolute power deviation between WSM nominations and actual LEM injections decreases, eventually reaching zero. As expected, the tri-level optimization model aligns with the CP model in scenarios with either no penalty or very high penalties, specifically at penalty levels of 0 and 200 £/MWh. However, in cases where moderate penalties allow for certain local deviations, the tri-level model accounts for these local refinements in making investment decisions. For instance, Table 3 illustrates a U-shaped trend in the expansion decisions for lines (6,2) and (6,4): line (6,2) decreases from 7 MW to 6 MW before increasing back to 7 MW, and line (6,4) follows a similar pattern, decreasing from 5 MW to 4 MW before returning to 5 MW. This reduction in expansion decisions occurs because deviations from the original nominations are permitted, allowing LEMs to diverge from the planned schedule and reduce active power provision from node 6. In contrast to the U-shaped trend in line expansion, local imbalance costs rise from zero to a peak before gradually falling back to zero as penalty levels increase. By incorporating

Table 3
Results of the tri-level model under varying penalty scenarios for local imbalances in the T6D33 System with FGR = 1.5. Investment decisions for lines (6,2) and (6,4) in the CP model remain unchanged at 7 MW and 5 MW, respectively, across all penalty levels.

Penalty level (£/MWh)	0	5	20	30	200
Line expansion (6,2) [MW]	7	7	6	6	7
Line expansion (6,4) [MW]	5	4	5	5	5
Investment costs [£]	1,968,914	1,943,886	1,943,886	1,943,886	1,968,914
Imbalance costs [£]	0	984,944	1,929,839	1,863,550	0
Average nominated power [MW]	−0.66	−0.61	−0.59	−0.58	−0.62
Average actual injection [MW]	2.69	1.27	0.33	0.01	−0.62
Absolute power deviation [MW]	3.35	1.87	0.92	0.59	0

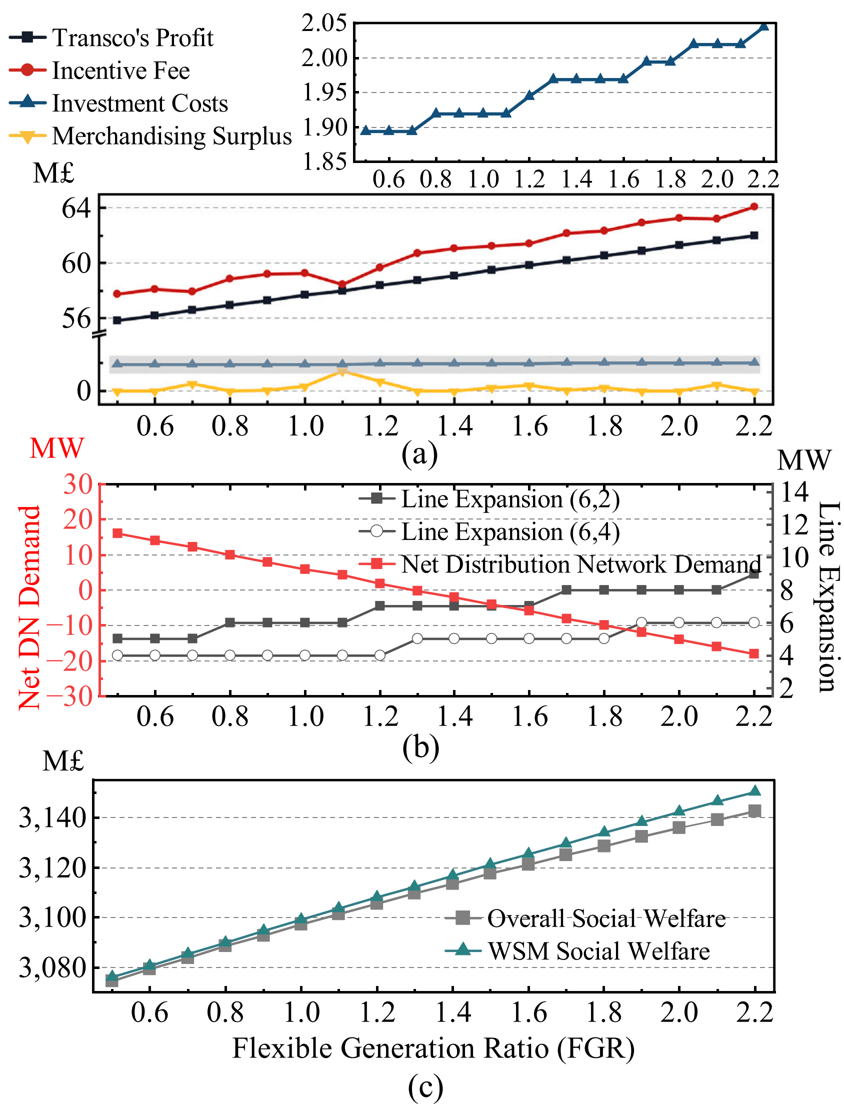

Fig. 11. Results for the T6D33 system with different FGRs. The gray area is shown magnified in the top-right part of the figure.

local refinements into the decision-making process, the model helps prevent over-investment in transmission assets. The tri-level framework enables Transcos to evaluate critical factors such as local deviations, investment costs, and potential surplus increase from investments, while adhering to the sequential operations of the WSM and LEMs.

4.2.2. Variations in local flexibility ratios: flexible generation

In this case study, the operational timescale of the planning problem includes five investment periods, i.e., $\mathcal{T} = \{1, 2, \dots, 5\}$. The maximum flexible local generation considered ranges from $\{0.5\times, \dots, 2.2\times\}$ of the non-flexible demand. The optimization problem has 68,663 variables, 53,326 linear constraints and 1920 quadratic constraints. The computational time was 871.34 s.

The effects of FGR on investment decisions and allocated quantities for the T6D33 system are presented in Fig. 11. In addition, clearing volumes for the T6D33 system across different FGRs in both WSM and LEM is shown in Fig. 12. Fig. 13 displays Transco's profit and line expansion decisions for the T6D33 system across a five-year investment period, considering two FGR scenarios: FGR = 1, as shown in Fig. 13(a), and FGR = 2, as

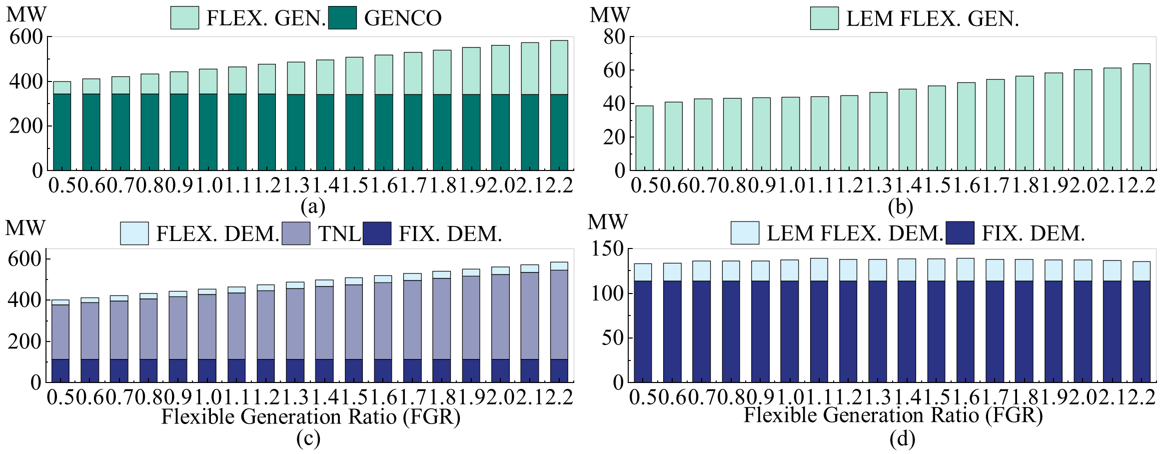

Fig. 12. Clearing volumes for the T6D33 system across different FGRs in both WSM and LEM. (a) details the generation volumes cleared by the WSM, including contributions from GENCOs and flexible local generators (FLEX. GEN.). (b) shows the demand volumes cleared by the WSM, incorporating fixed local demand (FIX. DEM.), transmission network load (TNL), and flexible local generation (FLEX. GEN.). (c) presents the flexible local generation (LEM FLEX. GEN.) clearing amounts within the LEM. (d) displays the demand levels cleared in the LEM, featuring both fixed local demand (FIX. DEM.) and flexible local generation (LEM FLEX. GEN.).

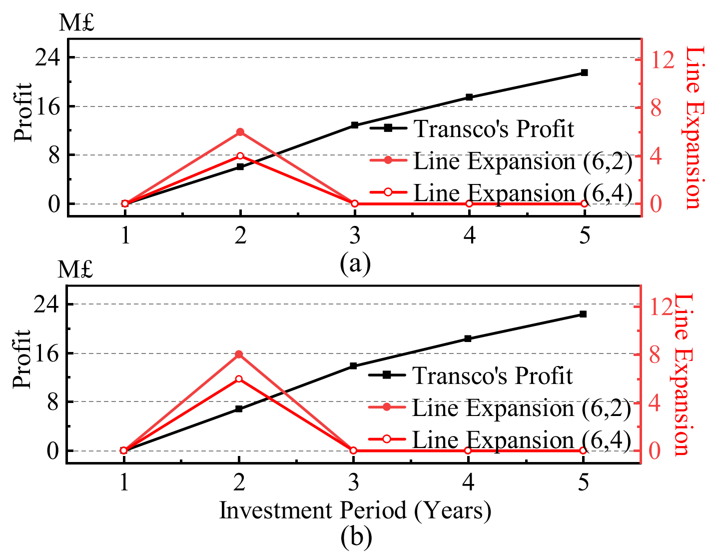

Fig. 13. Transco's profit and investment decisions over the investment periods for the T6D33 system with FGR = 1 (in (a)) and FGR = 2 (in (b)).

shown in Fig. 13(b). Similar to the patterns observed in Fig. 5, Transco's profit steadily increases over the investment period in both scenarios. Line expansions between nodes (6,2) and (6,4) occur in year 2 and remain at 0 in the subsequent periods. Specifically, under the FGR = 1 scenario, the investment decisions lead to expansions of 6 MW on line (6,2) and 4 MW on line (6,4). In contrast, the FGR = 2 scenario results in larger expansions, with 8 MW on line (6,2) and 6 MW on line (6,4), respectively. The results reveal that, unlike the two-node case, increasing the level of FGR leads to a consistent increase in the Transco's profit, overall social welfare and WSM social welfare. The black line in Fig. 11(a) and the teal and gray lines in Fig. 11(c) provide evidence of this trend. Specifically, when the FGR increases from 0.50 to 2.20, the Transco's profit rises by 11%, while the overall social welfare increases by 2.2%. When the FGR reaches 2.2, the injection into the transmission network reaches an average of 18.01 MW as shown in the red line in Fig. 11(b). Additionally, investment decisions made by the Transco for transmission lines (6,2) and (6,4) are represented by the gray lines in Fig. 11(b). All decisions are made in year 2. One of the factors that influences the expansion decision is its dependence on generators (GENCOs and local flexible generators) located in node 6 for supplying power to the upstream market. This creates a need for expanding transmission lines (6,2) and (6,4). However, this situation changes as local markets increase their flexible generation capacity over time and become self-sufficient in meeting their own flexible demand locally and the connected upstream demand. This also enables them to provide extra power to upstream markets through reverse flows on existing transmission lines. As a result, their reliance on GENCOs decreases significantly over time and, in general, more transmission expansion is needed to maximize the utilization of local generation resources available in node 6. Notably, the cleared quantities for GENCOs exhibit a consistent decrease of almost 7% from 47.87 MW to 44.53 MW in this node, while the flexible generation increases significantly from 9.19 MW to 37.80 MW.

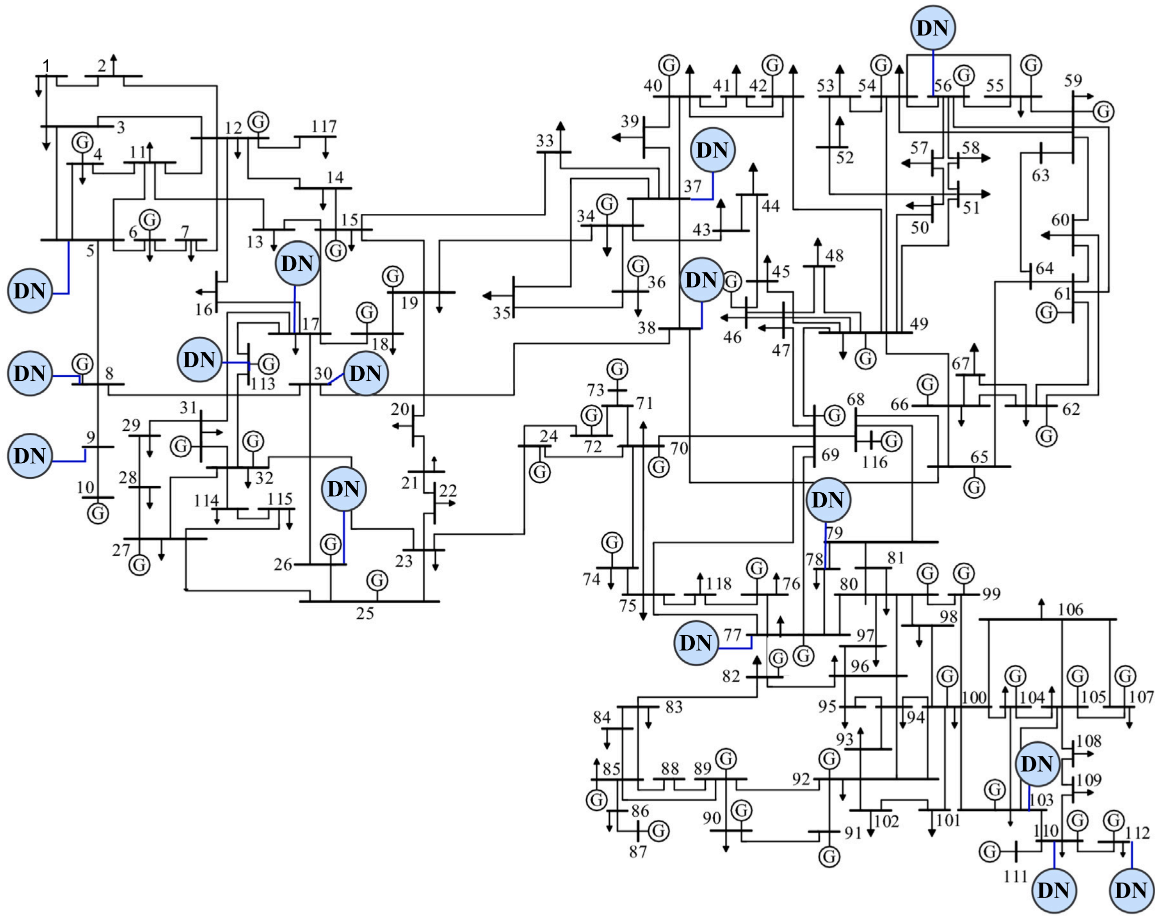

Fig. 14. Topology of the modified IEEE 118-bus network with 15 IEEE 33-bus distribution networks.

4.3. Case study 3: T118D33

The IEEE 118-bus power system as shown in Fig. 14, as detailed in [43], comprises 54 generating units, 99 demand nodes, 186 transmission lines, and a set of 17 candidate lines. Distribution networks are connected to the transmission system at buses 5, 8, 9, 17, 26, 30, 37, 38, 56, 77, 78, 103, 110, 112, and 113. The set of lumpy expansion is defined as $\mathcal{F}_m = \{100\}$MW. The variable investment cost K_l^{var} is £10/MWh, while the fixed cost K_l^{fix} is £100/h. The annual growth rate of load is assumed to be 5%. The operational timescale of the planning problem includes four investment periods, i.e., $\mathcal{T} = \{1, 2, 3, 4\}$ and the discount rate r is set to 5%. The data for branches, candidate lines, generators (including maximum generation and generator bus), and loads (including maximum demand and load bus) can be referenced in [44]. To introduce line congestion in the transmission network, the ratings of these lines have been deliberately decreased, as discussed in [44]. The maximum flexible local generation considered ranges from $\{0.5\times, 1.0\times, 1.5\times, 2.0\times\}$ of the non-flexible demand. The optimization problem involved 106,350 variables, 83,189 linear constraints, and 3840 quadratic constraints, with a computational time of 3972.73 s.

As illustrated in Fig. 15, it is clear that the FGR influences both the investment decisions related to the line (77,78) and the Transco's overall profitability. When the FGR is increased from 0.5 to 1, it might render the expansion of the line (77,78) unnecessary. Given that there are no generators at nodes 78 and 79, the power demand at node 78 is predominantly satisfied by the flows on lines (77,78) and (77,79). Enhancing either the FGR or the ratio of local fixed consumers results in diminished flow demand on the line (77,78) and an increased flow from node 79, mainly sourced from the generators at node 80. Moreover, as highlighted in Fig. 16, the proportion of local flexible consumers significantly shapes the decisions on line expansion. A notable pattern emerges: the expansion need for line (77,78) escalates with the increasing ratio of flexible consumers for different FGR values.

5. Conclusion

This paper proposes a novel approach to integrate the operations of LEMs into the transmission investment problem, taking into account the increasing flexibility of the local grid and the changing local dynamics. To achieve this, the proposed model operates at three levels and uses a sequential clearing process for the WSM and LEMs. Specifically, the Transco uses the clearing results from the WSM at the middle-level, as well as the imbalance information from LEMs at the lower-level, to perform a social welfare maximization investment under a regulated incentive mechanism while considering local deviations. To formulate this problem, the model is first expressed as a tri-level optimization problem, which is then reformulated as a MISOCP problem using various techniques. Future work will also address the uncertainties related to participants' bid prices, load, and generation profiles, focusing on developing a stochastic model that takes into account these uncertainties. Another area for extension would be more detailed bottom-up modeling of local prosumers and local communities to account for complexities such as bounded rationality [45] and community-based cooperation [46].

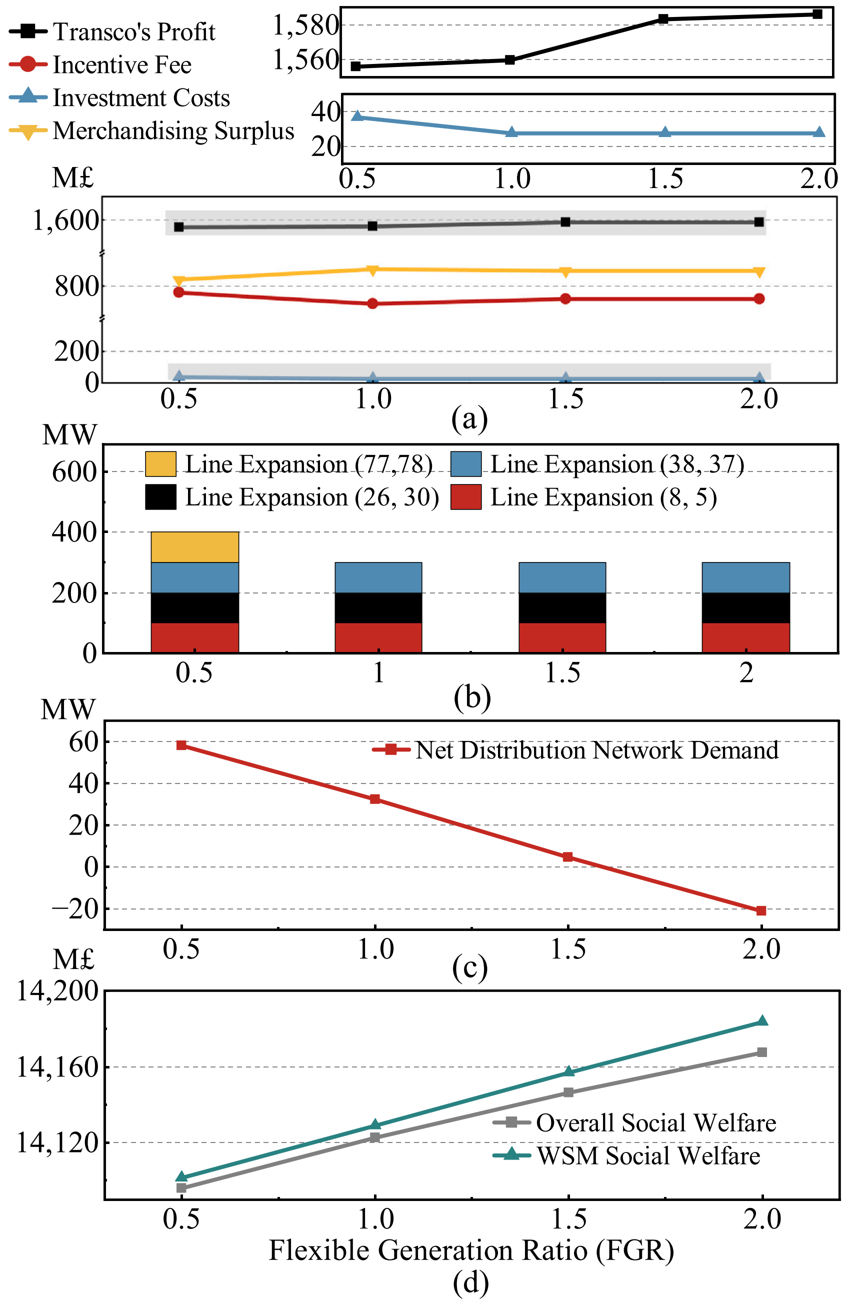

Fig. 15. Results for the T118D33 system with different FGRs. The gray area is shown magnified in the top-right part of the figure.

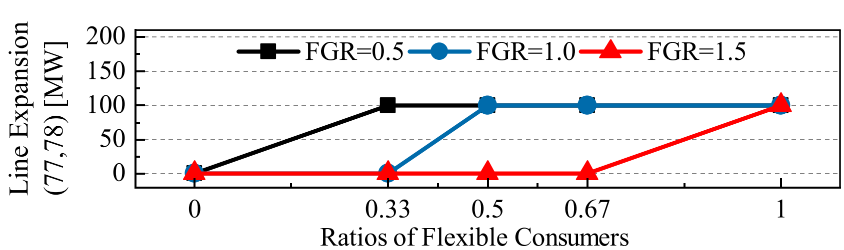

Fig. 16. Results for the T118D33 system with different ratios of local flexible consumers and FGRs. FGR = flexible generation ratio.

Nomenclature

Indices and sets

$b, \mathcal{B}$	Index and Set for all transmission nodes, $b \in \mathcal{B}$.
$b, \mathcal{B}^{lm}$	Index and Set for transmission network nodes connected to local markets, $b \in \mathcal{B}^{lm}, \mathcal{B}^{lm} \subseteq \mathcal{B}$.
$\overline{\mathcal{F}}_{l,j}$	lumpy capacity expansion for line l, with $\overline{\mathcal{F}}_l = \bigcup_{j\in\mathcal{J}} \bar{F}_{l,j}$.
$j, \mathcal{J}$	Index and Set for lumpy capacity indices, $j \in \mathcal{J}$.
$k, \Omega^{G\_TN}_{t,s,b}$	Index and Set for GENCOs at the investment period t and operation period s in node b, $k \in \Omega^{G\_TN}_{t,s,b}, b \in \mathcal{B}$.
$k, \Omega^{D\_TN}_{t,s,b}$	Index and Set for TNLs at the investment period t and operation period s in node b, $k \in \Omega^{D\_TN}_{t,s,b}, b \in \mathcal{B}$.
$k, \Omega^{G\_DN}_{t,s,b}$	Index and Set for aggregated local generators at the investment period t and operation period s in node b, $k \in \Omega^{G\_DN}_{t,s,b}, b \in \mathcal{B}^{lm}$.
$k, \Omega^{D\_DN}_{t,s,b}$	Index and Set for aggregated local consumers at the investment period t and operation period s in node b, $k \in \Omega^{D\_DN}_{t,s,b}, b \in \mathcal{B}^{lm}$.
$k, \Omega^{G\_local}_{t,s,n,b}$	Index and Set for local generators of DNs connected to transmission network node b at the investment period t and operation period s in DN node n, $k \in \Omega^{G\_local}_{t,s,n,b}, b \in \mathcal{B}^{lm}$.
$k, \Omega^{D\_local}_{t,s,n,b}$	Index and Set for local consumers of DNs connected to transmission network node b at the investment period t and operation period s in DN node n, $k \in \Omega^{D\_local}_{t,s,n,b}, b \in \mathcal{B}^{lm}$.
$l, \mathcal{L}$	Index and Set for transmission lines, $l \in \mathcal{L}$.
$l, \mathcal{L}_b$	Index and Set for distribution lines of DNs connected to transmission network node b, $l \in \mathcal{L}_b$.
$n, \mathcal{N}_b$	Index and Set for DN nodes of DNs connected to transmission network node b, $n \in \mathcal{N}_b, b \in \mathcal{B}^{lm}$.
$n, \mathcal{N}^+_b$	Index and Set for distribution nodes of DNs connected to transmission network node b except the slack bus, i.e. $\mathcal{N}^+_b = \mathcal{N}_b \backslash \{0\}$, $n \in \mathcal{N}^+_b, b \in \mathcal{B}^{lm}$.
$s, \mathcal{S}$	Index and Set for operation periods, $s \in \mathcal{S}$.
$t, \mathcal{T}$	Index and Set for investment periods, $t \in \mathcal{T}$.

Parameters

Ψ	Number of operation periods in one investment period.
$VoLL$	The Value of Lost Load, (£/MWh).
B_l	Susceptance of the transmission line l, $l \in \mathcal{L}$ (S).
$\bar{M}$	The price of purchasing/selling deviated active power (£/MWh).
r	The discount rate.
$a_{i,j,b}$	Conductance of the distribution line $(i,j) \in \mathcal{L}_b$ of the local market connected to the transmission node b, $b \in \mathcal{B}^{lm}$ (p.u.).
$e_{i,j,b}$	Negative of the susceptance of the distribution line $(i,j) \in \mathcal{L}_b$ of the local market connected to the transmission node b, $b \in \mathcal{B}^{lm}$ (p.u.).
$S_{l,b}$	Incidence matrix element of sending node b, line l, $b \in \mathcal{B}$, $l \in \mathcal{L}$.
$R_{l,b}$	Incidence matrix element of receiving node b, line l, $b \in \mathcal{B}$, $l \in \mathcal{L}$.
$\mathcal{F}^0_l$	Existing capacity on the transmission line l, $l \in \mathcal{L}$ (MW).
$\bar{\mathcal{F}}_{i,j,b}, \bar{\mathcal{F}}_{j,i,b}$	The maximum power flow leaving distribution network node i and j, respectively, $b \in \mathcal{B}^{lm}$ (MW).
$c^d_{t,s,k,b}$	Demand utility for the TNL k at the investment period t, operation period s in the transmission node b, $k \in \Omega^{D\_TN}_{t,s,b}$, $b \in \mathcal{B}$ (£/MWh).
$c^g_{t,s,k,b}$	Marginal costs for the GENCO k at the investment period t, operation period s in the transmission node b, $k \in \Omega^{G\_TN}_{t,s,b}$, $b \in \mathcal{B}$ (£/MWh).
$c^{d\_local}_{t,s,k,b}$	Demand utility for local flexible consumer k at the investment period t, operation period s in the transmission node b, $k \in \Omega^{D\_DN}_{t,s,b}$, $b \in \mathcal{B}^{lm}$ (£/MWh).
$c^{g\_local}_{t,s,k,b}$	Marginal costs for local flexible generator k at the investment period t, operation period s in the transmission node b, $k \in \Omega^{G\_DN}_{t,s,b}$, $b \in \mathcal{B}^{lm}$(£/MWh).
$c^d_{t,s,n,k,b}$	Demand utility for local flexible consumer k in DN node n at the investment period t, operation period s of local market located in bus b, $k \in \Omega^{D\_local}_{t,s,n,b}$, $b \in \mathcal{B}^{lm}$ (£/MWh).
$c^g_{t,s,n,k,b}$	Marginal costs for local flexible generator k in DN node n at the investment period t, operation period s of the local market connected to the transmission node b, $k \in \Omega^{G\_local}_{t,s,n,b}$, $b \in \mathcal{B}^{lm}$ (£/MWh).
$c^q_{t,s,0,b}$	Price of reactive power traded with the transmission grid at the investment period t, operation period s of the local market connected to the transmission node b (£/MVArh).
$D_{t,s,n,b}$	Active fixed demand required by non-flexible consumers in DN node n at the investment period t and operation period s of the local market connected to the transmission node b, $n \in \mathcal{N}^+_b$ (MW).
$D_{t,s,b}$	Total active fixed demand required by non-flexible consumers at the investment period t and operation period s of the local market connected to the transmission node b, $D_{t,s,b} = \sum_{n\in\mathcal{N}^+_b} D_{t,s,n,b}$ (MW).
$d^{\min}_{t,s,k,b}, d^{\max}_{t,s,k,b}$	Minimum/maximum quantity of active power demanded by TNL k at the investment period t, operation period s in bus b (MW).
$g^{\min}_{t,s,k,b}, g^{\max}_{t,s,k,b}$	Minimum/maximum quantity of active power produced by GENCO k at the investment period t, operation period s in bus b (MW).

$d_{t,s,k,b}^{local,\min}, d_{t,s,k,b}^{local,\max}$	Minimum/maximum quantity of active power demanded by local flexible consumer k at the investment period t, operation period s in bus b (MW).
$g_{t,s,k,b}^{local,\min}, g_{t,s,k,b}^{local,\max}$	Minimum/maximum quantity of active power produced by local flexible generators k at the investment period t, operation period s in bus b (MW).
$d_{t,s,n,k,b}^{p,\min}, d_{t,s,n,k,b}^{p,\max}$	Minimum/maximum quantity of active power demanded by local flexible consumer k in DN node n at the investment period t, operation period s of the local market connected to the transmission node b (MW).
$g_{t,s,n,k,b}^{p,\min}, g_{t,s,n,k,b}^{p,\max}$	Minimum/maximum quantity of active power produced by local flexible generators k in DN node n at the investment period t, operation period s of the local market connected to the transmission node b (MW).
$d_{t,s,n,k,b}^{q,\min}, d_{t,s,n,k,b}^{q,\max}$	Minimum/maximum quantity of reactive power demanded by local flexible consumer k in DN node n at the investment period t, operation period s of the local market connected to the transmission node b (MVAr).
$g_{t,s,n,k,b}^{q,\min}, g_{t,s,n,k,b}^{q,\max}$	Minimum/maximum quantity of reactive power produced by local flexible generators k in DN node n at the investment period t, operation period s of the local market connected to the transmission node b (MVAr).
$v_{t,s,n,b}^{\min}, v_{t,s,n,b}^{\max}$	Minimum/maximum voltage magnitude in node n at the investment period t and operation period s of the local market connected to the transmission node b (p.u.).
K_l^{fix}	Fixed cost of building or expanding line l (£/h).
K_l^{var}	Variable cost of building or expanding line l (£/MWh).
$W_{t,s,00,b}$	A specific value for the voltage magnitude squared at the slack bus in local market connected to the transmission node b at the investment period t, operation period s (p.u.).

Variables

$u_{t,l}$	Binary variable equal to one if line l is expanded at the investment period t, and zero otherwise, $l \in \mathcal{L}_b$.
$b_{t,l,j}^F$	Binary variable equal to one if the lumpy investment in additional capacity $\bar{F}_{l,j}$ for line l is made at the investment period t, and zero otherwise, $l \in \mathcal{L}_b$.
Φ_t	The incentive fee at the investment period t (£).
$g_{t,s,k,b}^{local}$	Allocated active power for local flexible generator k at the investment period t, operation period s in bus b (MW).
$d_{t,s,k,b}^{local}$	Allocated active power for local flexible consumer k at the investment period t, operation period s in bus b (MW).
$g_{t,s,b}^{agg}$	Aggregated allocated active power for local market at the investment period t, operation period s in bus b (MW).
$d_{t,s,b}^{agg}$	Aggregated allocated active power for local market at the investment period t, operation period s in bus b (MW).
$g_{t,s,k,b}$	Allocated active power for GENCO k at the investment period t, operation period s in bus b (MW).
$d_{t,s,k,b}$	Allocated active power for TNL k at the investment period t, operation period s in bus b (MW).
$f_{t,s,l}$	Flow in the line l at the investment period t, operation period s (MW).
$\theta_{t,s,b}$	Voltage phase angle of transmission network bus b at the investment period t, operation period s (rad).
$g_{t,s,n,k,b}^p$	Allocated active power for local flexible generator k in DN node n for the local market connected to the transmission node b at the investment period t, operation period s, $n \in \mathcal{N}_b^+$ (MW).
$g_{t,s,n,k,b}^q$	Allocated reactive power for local flexible generator k in DN node n for the local market connected to the transmission node b at the investment period t, operation period s, $n \in \mathcal{N}_b^+$ (MVAr).
$d_{t,s,n,k,b}^p$	Allocated active power for local flexible consumer k in DN node n for the local market connected to the transmission node b at the investment period t, operation period s, $n \in \mathcal{N}_b^+$ (MW).
$d_{t,s,n,k,b}^q$	Allocated reactive power for local flexible consumer k in DN node n for the local market connected to the transmission node b at the investment period t, operation period s, $n \in \mathcal{N}_b^+$ (MVAr).
$p_{t,s,n,b}$	Active power injection in DN node n for the local market connected to the transmission node b at the investment period t, operation period s (MW).
$q_{t,s,n,b}$	Reactive power injection in DN node n for the local market connected to the transmission node b at the investment period t, operation period s (MVAr).
$W_{t,s,ii,b}$	Voltage magnitude squared in DN node i for the local market connected to the transmission node b at the investment period t, operation period s.
$W_{t,s,ij,b}^p$	Real component of the product $V_{t,s,i,b}\bar{V}_{t,s,j,b}$ for the local market connected to the transmission node b at the investment period t, operation period s.
$W_{t,s,ij,b}^q$	Imaginary component of the product $V_{t,s,i,b}\bar{V}_{t,s,j,b}$ for the local market connected to the transmission node b at the investment period t, operation period s.
$\pi_{t,s,b}$	WSM prices at transmission node b at the investment period t, operation period s (£/MWh).
$\pi_{t,s,n,b}^p$	LEM nodal prices for active power for the local market connected to the transmission node b at the investment period t, operation period s (£/MWh).

Auxiliary variables

$z_{t,s,b}^1, z_{t,s,b}^2$	Replace the absolute value term $\lvert p_{t,s,0,b} - (d_{t,s,b}^{agg*} - g_{t,s,b}^{agg*})\rvert$.
$y_{\hat{t},t,s,l,j}^{\max}$	Replace the product $b_{\hat{t},l,j}^F \mu_{t,s,l}^{\max}$.
$y_{\hat{t},t,s,l,j}^{\min}$	Replace the product $b_{\hat{t},l,j}^F \mu_{t,s,l}^{\min}$.
$w_{t,s,k,b}^{g\_local}$	Replace the product $\varphi_{t,s,b}^{z0} g_{t,s,k,b}^{local}$.

$w^{d\_local}_{t,s,k,b}$ Replace the product $\varphi^{z0}_{t,s,b} d^{local}_{t,s,k,b}$.

CRediT authorship contribution statement

Yuxin Xia: Writing – original draft, Software, Methodology, Conceptualization. **Iacopo Savelli:** Writing – review & editing, Supervision, Conceptualization. **Thomas Morstyn:** Writing – review & editing, Supervision, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgment

This work was supported by the Engineering Scholarship from the University of Edinburgh.

Appendix A. Proof of Lemma 2.1

Proof. As shown in Eq. (3a), the objective function of the Transco is

$$\max_{u_t,\Phi_t} \sum_{t\in\mathcal{T}} \frac{1}{(1+r)^{t-1}} \big(MS_t + \Phi_t - C_t(u_t)\big) - IC_1$$

By substituting the incentive fee Φ_t (Eqs. (3e)) and (3f)) into this objective function, the objective function can be expanded as

$$\begin{aligned}
&\max_{u_t,\Phi_t} \Bigg(MS_1 - IC_1 \\
&+ \frac{1}{(1+r)^1}(MS_2 + S_2^G + S_2^L - IC_2 + \underbrace{\Phi_1 - S_1^G - S_1^L}_{-\left(S_1^G+S_1^L\right)} - C_2(u_2)) \\
&+ \frac{1}{(1+r)^2}(MS_3 + S_3^G + S_3^L - IC_3 + \underbrace{\Phi_2 - S_2^G - S_2^L + IC_2}_{-\left(S_1^G+S_1^L\right)} - C_3(u_3)) \qquad \text{(A.1a)} \\
&+ \frac{1}{(1+r)^3}(MS_4 + S_4^G + S_4^L - IC_4 + \underbrace{\Phi_3 - S_3^G - S_3^L + IC_3}_{-\left(S_1^G+S_1^L\right)} - C_4(u_4)) + \cdots \Bigg) \\
&\Leftrightarrow \max_{u_t,\Phi_t} \sum_{t\in\mathcal{T}} \frac{1}{(1+r)^{t-1}} \Big((MS_t + S_t^G + S_t^L - IC_t) - C_t(u_t) \Big) - T\left(S_1^G + S_1^L\right) \qquad \text{(A.1b)}
\end{aligned}$$

The Transco invests in a way that maximizes social welfare starting from the second investment period given no investment is performed in year 1 (i.e., the term $T\left(S_1^G + S_1^L\right)$ are given) since $\sum_{t\in\mathcal{T}} \frac{1}{(1+r)^{t-1}} \Big((MS_t + S_t^G + S_t^L) - C_t(u_t) \Big)$ represents the overall social welfare. Hence, the objective function of the Transco is equivalent to maximizing social welfare over the planning period. This expression also captures the operations of local markets by incorporating the imbalance costs $\sum_{t\in\mathcal{T}} \frac{1}{(1+r)^{t-1}} IC_t$. □

Appendix B. Proof of Theorem 3.1

Proof. Using strong duality of the lower-level problem, the middle- and lower-level problem (i.e., Eqs. (4d)–(4g)) are equivalent to:

$$\begin{aligned}
(\boldsymbol{x}_w, \boldsymbol{y}_w, \boldsymbol{x}_l, \boldsymbol{y}_l) = \arg &\max_{\substack{x_w\geq 0, y_w \\ x_l\in\mathcal{K}, y_l}} c_w^\top \boldsymbol{x}_w && \text{(B.1a)} \\
\text{s.t.}\quad & A_w \boldsymbol{x}_w + B_w \boldsymbol{z} \leq b_w && \text{(B.1b)} \\
& (\boldsymbol{x}_l, \boldsymbol{y}_l) = \arg\max_{\boldsymbol{x}_l\in\mathcal{K}} c_l^\top \boldsymbol{x}_l && \text{(B.1c)} \\
& \quad \text{s.t.}\quad A_l \boldsymbol{x}_l + B_l \boldsymbol{x}_w \leq b_l && \text{(B.1d)} \\
& \quad \boldsymbol{y}_l^\top A_l \succeq_{\mathcal{K}^*} c_l && \text{(B.1e)} \\
& \quad \boldsymbol{y}_l^\top (b_l - B_l \boldsymbol{x}_w) \leq c_l^\top \boldsymbol{x}_l && \text{(B.1f)}
\end{aligned}$$

where $\mathcal{K}^*$ denotes the dual cone of $\mathcal{K}$. Eq. (B.1d) and (B.1e) represent the primal and dual constraints of the lower-level problem, illustrating primal and dual feasibility. Eq. (B.1f) enforces the strong duality to ensure optimality by posing a reversed weak duality constraint. Notice that the middle-level problem does not anticipate the solutions of the lower-level problem $(\boldsymbol{x}_l, \boldsymbol{y}_l)$, as its constraints and objective function are not dependent on the variables of the lower-level problem. This implies that the solutions of the middle-level problem are not affected by the solutions of the lower-level problem. Therefore, we can solve the middle-level problem and the lower-level problem in two steps:

Step 1: solve the middle-level problem and obtain the optimal primal and dual pairs $(\boldsymbol{x}_w^*, \boldsymbol{y}_w^*)$;

Step 2: solve the lower-level problem with $\boldsymbol{x}_w$ fixed to the values $\boldsymbol{x}_w^*$ and obtain the optimal primal and dual pairs $(\boldsymbol{x}_l^*, \boldsymbol{y}_l^*)$.

Accordingly, Problem (B.1) can be expressed as a Lexicographic optimization problem as follows [37]:

$$\begin{aligned}(\boldsymbol{x}_w, \boldsymbol{y}_w, \boldsymbol{x}_l, \boldsymbol{y}_l) = \mathop{\arg\max}_{\substack{\boldsymbol{x}_w \ge 0, \boldsymbol{y}_w \ge 0,\\ \boldsymbol{x}_l \in \mathcal{K}, \boldsymbol{y}_l \ge 0}} \quad & \langle c_w^\top \boldsymbol{x}_w, c_l^\top \boldsymbol{x}_l \rangle \\ \text{s.t.} \quad & A_w \boldsymbol{x}_w + B_w \boldsymbol{z} \le b_w \\ & A_l \boldsymbol{x}_l + B_l \boldsymbol{x}_w \le b_l \\ & \boldsymbol{y}_l^\top A_l \succeq_{\mathcal{K}^*} c_l \\ & \boldsymbol{y}_l^\top (b_l - B_l \boldsymbol{x}_w) \le c_l^\top \boldsymbol{x}_l \end{aligned} \tag{B.2}$$

Any optimal solution $(\boldsymbol{x}_w^*, \boldsymbol{y}_w^*, \boldsymbol{x}_l^*, \boldsymbol{y}_l^*)$ to the problem (B.2) satisfies the following conditions:

$$\begin{aligned}(\boldsymbol{x}_w^*, \boldsymbol{y}_w^*) = \mathop{\arg\max}_{\boldsymbol{x}_w \ge 0, \boldsymbol{y}_w \ge 0} \quad & c_w^\top \boldsymbol{x}_w \\ \text{s.t.} \quad & A_w \boldsymbol{x}_w \le b_w - B_w \boldsymbol{z} \\ & A_l \boldsymbol{x}_l + B_l \boldsymbol{x}_w \le b_l \end{aligned} \tag{B.3}$$

$$\begin{aligned}(\boldsymbol{x}_l^*, \boldsymbol{y}_l^*) = \mathop{\arg\max}_{\boldsymbol{x}_l \in \mathcal{K}, \boldsymbol{y}_l \ge 0} \quad & c_l^\top \boldsymbol{x}_l \\ \text{s.t.} \quad & A_l \boldsymbol{x}_l \le b_l - B_l \boldsymbol{x}_w^* \\ & \boldsymbol{y}_l^\top A_l \succeq_{\mathcal{K}^*} c_l \\ & \boldsymbol{y}_l^\top (b_l - B_l \boldsymbol{x}_w^*) \le c_l^\top \boldsymbol{x}_l \end{aligned} \tag{B.4}$$

Observe from Eq. (B.3), any feasible solution $(\hat{\boldsymbol{x}}_l, \hat{\boldsymbol{y}}_l)$ to the lower-level problem is optimal. That is because, by strong duality forced in the third constraint of (B.4), the pair $(\hat{\boldsymbol{x}}_l, \hat{\boldsymbol{y}}_l)$ satisfies:

$$\begin{aligned}\hat{\boldsymbol{x}}_l = \mathop{\arg\max}_{\boldsymbol{x}_l \in \mathcal{K}} \quad & c_l^\top \boldsymbol{x}_l \\ \text{s.t.} \quad & A_l \boldsymbol{x}_l \le b_l - B_l \boldsymbol{x}_w^* \end{aligned} \tag{B.5}$$

$$\begin{aligned}\hat{\boldsymbol{y}}_l = \mathop{\arg\max}_{\boldsymbol{y}_l \ge 0} \quad & \boldsymbol{y}_l^\top (b_l - B_l \boldsymbol{x}_w^*) \\ \text{s.t.} \quad & \boldsymbol{y}_l^\top A_l \succeq_{\mathcal{K}^*} c_l \end{aligned} \tag{B.6}$$

Eq. (B.5) is a relaxation of Eq. (B.4) and given (B.3), (B.5) and (B.6), we use the weight-sum method to approximate the lexicographic function Eq. (B.2), with $\gamma \in (0, 1)$ [37,38]:

$$\begin{aligned}\min_{\boldsymbol{x}_w \ge 0, \boldsymbol{x}_l \in \mathcal{K}} \quad & \gamma c_w^\top \boldsymbol{x}_w + (1-\gamma) c_l^\top \boldsymbol{x}_l \\ \text{s.t.} \quad & A_w \boldsymbol{x}_w + B_w \boldsymbol{z} \le b_w \quad [\boldsymbol{y}_w \ge 0] \\ & A_l \boldsymbol{x}_l + B_l \boldsymbol{x}_w \le b_l \quad [\boldsymbol{y}_l \ge 0] \end{aligned} \tag{B.7}$$

where $\boldsymbol{y}_w$ is obtained by the dual solution associated with the first constraint of Eq. (B.7) and $\boldsymbol{y}_l$ is obtained by the dual solution associated with the second constraint of Eq. (B.7). The primal and dual constraints and reversed weak duality associated with Eq. (B.7) are

$$\begin{aligned}\max \quad & \gamma c_w^\top \boldsymbol{x}_w + (1-\gamma) c_l^\top \boldsymbol{x}_l \\ \text{s.t.} \quad & A_w \boldsymbol{x}_w + B_w \boldsymbol{z} \le b_w \\ & A_l \boldsymbol{x}_l + B_l \boldsymbol{x}_w \le b_l \\ & \boldsymbol{y}_w^\top A_w + \boldsymbol{y}_l^\top B_l \ge \gamma c_w^\top \\ & \boldsymbol{y}_l^\top A_l \succeq_{\mathcal{K}^*} (1-\gamma) c_l^\top \\ & \boldsymbol{y}_w^\top (b_w - B_w \boldsymbol{z}) + \boldsymbol{y}_l^\top b_l \le \gamma c_w^\top \boldsymbol{x}_w + (1-\gamma) c_l^\top \boldsymbol{x}_l \\ & \boldsymbol{x}_w \ge 0, \boldsymbol{x}_l \in \mathcal{K} \\ & \boldsymbol{y}_w \ge 0, \boldsymbol{y}_l \ge 0 \end{aligned} \tag{B.8}$$

As a result, the tri-level problem can be approximated by

$$\begin{aligned}\max \quad & \gamma h^\top \boldsymbol{z} + \gamma c_u^\top \boldsymbol{x}_u + \gamma c_{uw}^\top \boldsymbol{x}_w \frac{\boldsymbol{y}_w}{\gamma} + \gamma c_{ul}^\top \boldsymbol{x}_l \\ \text{s.t.} \quad & A_u \boldsymbol{x}_u + D \boldsymbol{x}_w + E \boldsymbol{x}_w \frac{\boldsymbol{y}_w}{\gamma} + F \boldsymbol{x}_l \le b_u \\ & \boldsymbol{z} \in \mathcal{Z}, \boldsymbol{z} \in \{0,1\}^m \\ & A_w \boldsymbol{x}_w + B_w \boldsymbol{z} \le b_w \\ & A_l \boldsymbol{x}_l + B_l \boldsymbol{x}_w \le b_l \\ & \boldsymbol{y}_w^\top A_w + \boldsymbol{y}_l^\top B_l \ge \gamma c_w^\top \\ & \boldsymbol{y}_l^\top A_l \succeq_{\mathcal{K}^*} (1-\gamma) c_l^\top \\ & \boldsymbol{y}_w^\top (b_w - B_w \boldsymbol{z}) + \boldsymbol{y}_l^\top b_l \le \gamma c_w^\top \boldsymbol{x}_w + (1-\gamma) c_l^\top \boldsymbol{x}_l \\ & \boldsymbol{x}_w \ge 0, \boldsymbol{x}_l \in \mathcal{K} \\ & \boldsymbol{y}_w \ge 0, \boldsymbol{y}_l \ge 0 \end{aligned} \tag{B.9}$$

for some $\gamma \in (0,1)$. It remains to show that the single-level problem (B.9) is indeed an asymptotic approximation of the tri-level problem (4). To this end, we introduce two auxiliary variables for two dual variables $\tilde{\boldsymbol{y}}_w, \tilde{\boldsymbol{y}}_l$: $\tilde{\boldsymbol{y}}_w = \frac{\boldsymbol{y}_w}{\gamma}, \tilde{\boldsymbol{y}}_l = \frac{\boldsymbol{y}_l}{1-\gamma}$, then (B.9) becomes

$$\max \quad \gamma h^\top \boldsymbol{z} + \gamma c_u^\top \boldsymbol{x}_u + \gamma c_{uw}^\top \boldsymbol{x}_w \tilde{\boldsymbol{y}}_w + \gamma c_{ul}^\top \boldsymbol{x}_l \tag{B.10a}$$
$$\text{s.t.} \quad A_u \boldsymbol{x}_u + D\boldsymbol{x}_w + E\boldsymbol{x}_w\tilde{\boldsymbol{y}}_w + F\boldsymbol{x}_l \le b_u \tag{B.10b}$$
$$\boldsymbol{z} \in \mathcal{Z}, \boldsymbol{z} \in \{0,1\}^m \tag{B.10c}$$
$$A_w \boldsymbol{x}_w + B_w \boldsymbol{z} \le b_w \tag{B.10d}$$
$$A_l \boldsymbol{x}_l + B_l \boldsymbol{x}_w \le b_l \tag{B.10e}$$
$$\tilde{\boldsymbol{y}}_w^\top A_w + \frac{(1-\gamma)}{\gamma} \tilde{\boldsymbol{y}}_l{}^\top B_l \ge c_w^\top \tag{B.10f}$$
$$\tilde{\boldsymbol{y}}_l^\top A_l \ge_{\mathcal{K}^*} c_l^\top \tag{B.10g}$$
$$\tilde{\boldsymbol{y}}_w^\top (b_w - B_w \boldsymbol{z}) - c_w^\top \boldsymbol{x}_w \le \frac{(1-\gamma)}{\gamma}(c_l^\top \boldsymbol{x}_l - \tilde{\boldsymbol{y}}_l^\top b_l) \tag{B.10h}$$
$$\boldsymbol{x}_w \ge 0, \tilde{\boldsymbol{y}}_w \ge 0, \boldsymbol{x}_l \in \mathcal{K}, \tilde{\boldsymbol{y}}_l \ge 0 \tag{B.10i}$$

Given that the upper level variables $\boldsymbol{z}$ is fixed to $\hat{\boldsymbol{z}}$, we denote $P(\hat{\boldsymbol{z}})$ and $\tilde{P}(\hat{\boldsymbol{z}})$ as the objective value of the Lexicographic optimization problem (B.2) and the modified single-level problem (B.10a)–(B.10i), respectively. Let the pair $\{\hat{\boldsymbol{x}}_w, \hat{\boldsymbol{y}}_w, \hat{\boldsymbol{x}}_l, \hat{\boldsymbol{y}}_l\}$ be the optimal solution of $\tilde{P}(\hat{\boldsymbol{z}})$.

For wholesale market clearing problem, as γ approaches to 1, (B.10f) and (B.10h) becomes:

$$\hat{\boldsymbol{y}}_w^\top A_w \ge c_w^\top \tag{B.11}$$
$$\hat{\boldsymbol{y}}_w^\top (b_w - B_w \hat{\boldsymbol{z}}) \le c_w^\top \boldsymbol{x}_w \tag{B.12}$$

which implies that given fixed upper-level decision $\hat{\boldsymbol{z}}$, $\hat{\boldsymbol{x}}_w, \hat{\boldsymbol{y}}_w$ approximate the optimal and dual solution of problem (B.3). This is because (B.10d) ensures the primal feasibility, and when $\gamma \to 1$, (B.11) and (B.12) ensure the dual feasibility and strong duality condition. Therefore, as $\gamma \to 1$, the pair $(\hat{\boldsymbol{x}}_w, \hat{\boldsymbol{y}}_w)$ is a feasible solution to $P(\hat{\boldsymbol{z}})$.

For the local market clearing problem, we combine Eq. (B.10f) $\times \hat{\boldsymbol{x}}_w$ and Eq. (B.10g) and notice Eq. (B.10d), gives us:

$$\hat{\boldsymbol{y}}_l^\top (b_l - B_l \hat{\boldsymbol{x}}_w) \le c_l^\top \hat{\boldsymbol{x}}_l \tag{B.13}$$

which implies that given fixed middle-level decision $\hat{\boldsymbol{x}}_w$, $\hat{\boldsymbol{x}}_l, \hat{\boldsymbol{y}}_l$ approximate the optimal and dual solution of problem (B.4). This is because (B.10d) ensures the primal feasibility, (B.10f) ensures the dual feasibility and (B.13) enforces the strong duality condition.

In summary, $\hat{\boldsymbol{x}}_w$ is an approximate solution of $P(\hat{\boldsymbol{z}})$ that becomes increasingly close to the optimal solution of Problem $P(\hat{\boldsymbol{z}})$ as $\gamma \to 1$, and the pair $(\hat{\boldsymbol{x}}_l, \hat{\boldsymbol{y}}_l)$ is the exact response of the follower with respect to $\hat{\boldsymbol{x}}_w$ for any $\gamma \in (0,1)$. Therefore, the approximation may sacrifice the leader's optimality when γ is not large enough, but it always gives a feasible solution [37]. □

Appendix C. Reformulated middle-level problem: WSM clearing

This section reports the reformulated middle-level WSM clearing problem (1). In this paper, we assume that the aggregated local curves $C^{agg}_{t,s,b}(g^{agg}_{t,s,b})$ and $U^{agg}_{t,s,b}(d^{agg}_{t,s,b})$ represent the marginal cost and benefit of local participants and information asymmetry and strategic bidding are subjects for future work. Therefore, the cost function for aggregated flexible local generators is rewritten as $C^{agg}_{t,s,b}(g^{agg}_{t,s,b}) = \sum_{k\in\Omega^{G\_DN}_{t,s,b}} c^{g\_local}_{t,s,k,b}\, g^{local}_{t,s,k,b}$ where $g^{local}_{t,s,k,b}$ is the local flexible quantity produced, and the utility function for aggregated flexible local consumers is replaced by $U^{agg}_{t,s,b}(d^{agg}_{t,s,b}) = \sum_{k\in\Omega^{D\_DN}_{t,s,b}} c^{d\_local}_{t,s,k,b}\, d^{local}_{t,s,k,b}$ where $d^{local}_{t,s,k,b}$ is the local flexible quantity demanded.

$$\begin{aligned}&(d^{agg*}_{t,s,b}, g^{agg*}_{t,s,b}, d^*_{t,s,k,b}, g^*_{t,s,k,b}, f^*_{t,s,m}, \theta^*_{t,s,b}, [\pi^*_{t,s,b}]) =\\ &\arg\max \sum_{t\in\mathcal{T}}\sum_{s\in\mathcal{S}}\Bigg(\sum_{b\in\mathcal{B}^{lm}}\sum_{k\in\Omega^{D\_DN}_{t,s,b}} c^{d\_local}_{t,s,k,b}\, d^{local}_{t,s,k,b} - \sum_{b\in\mathcal{B}^{lm}}\sum_{k\in\Omega^{G\_DN}_{t,s,b}} c^{g\_local}_{t,s,k,b}\, g^{local}_{t,s,k,b} + \sum_{b\in\mathcal{B}}\sum_{k\in\Omega^{D\_TN}_{t,s,b}} c^{d}_{t,s,k,b} d_{t,s,k,b} - \sum_{b\in\mathcal{B}}\sum_{k\in\Omega^{G\_TN}_{t,s,b}} c^{g}_{t,s,k,b} g_{t,s,k,b}\Bigg)\end{aligned} \tag{C.1a}$$

s.t.

$$\Bigg[\, d^{agg}_{t,s,b} = \sum_{k\in\Omega^{D\_DN}_{t,s,b}} d^{local}_{t,s,k,b} + D_{t,s,b}, \quad \forall b \in \mathcal{B}^{lm}, \quad [\varrho^{d\_agg}_{t,s,b} \in \mathbb{R}], \tag{C.1b}$$
$$g^{agg}_{t,s,b} = \sum_{k\in\Omega^{G\_DN}_{t,s,b}} g^{local}_{t,s,k,b}, \quad \forall b \in \mathcal{B}^{lm}, \quad [\varrho^{g\_agg}_{t,s,b} \in \mathbb{R}], \tag{C.1c}$$
$$d^{agg}_{t,s,b} - g^{agg}_{t,s,b} \le \mathcal{F}^{\max}_b, \quad \forall b \in \mathcal{B}^{lm}, \quad [\varrho^{\max}_{t,s,b} \ge 0], \tag{C.1d}$$
$$-(d^{agg}_{t,s,b} - g^{agg}_{t,s,b}) \le -\mathcal{F}^{\min}_b, \quad \forall b \in \mathcal{B}^{lm}, \quad [\varrho^{\min}_{t,s,b} \ge 0], \tag{C.1e}$$
$$d^{local,\min}_{t,s,k,b} \le d^{local}_{t,s,k,b} \le d^{local,\max}_{t,s,k,b}, \quad \forall k \in \Omega^{D\_DN}_{t,s,b}, \forall b \in \mathcal{B}^{lm}, \quad [\varphi^{D\_local,\min}_{t,s,k,b} \ge 0, \varphi^{D\_local,\max}_{t,s,k,b} \ge 0], \tag{C.1f}$$
$$g^{local,\min}_{t,s,k,b} \le g^{local}_{t,s,k,b} \le g^{local,\max}_{t,s,k,b}, \quad \forall k \in \Omega^{G\_DN}_{t,s,b}, \forall b \in \mathcal{B}^{lm}, \quad [\varphi^{G\_local,\min}_{t,s,k,b} \ge 0, \varphi^{G\_local,\max}_{t,s,k,b} \ge 0], \tag{C.1g}$$
$$\text{(1b)–(1c)}, \tag{C.1h}$$
$$\text{(1f)–(1l)}, \Bigg] \quad \forall t \in \mathcal{T}, \forall s \in \mathcal{S}. \tag{C.1i}$$

Constraint (C.1b) defines the aggregated demand which consists of the total active power demand for the local flexible consumers $\sum_{k\in\Omega^{D\_DN}_{t,s,b}} d^{local}_{t,s,k,b}$ and the fixed demand required by the local non-flexible consumers $D_{t,s,b}$. The total fixed demand $D_{t,s,b}$ is calculated as the sum of the fixed demands located at all distribution network nodes except for the slack bus, i.e., $D_{t,s,b} = \sum_{n\in\mathcal{N}_b^+} D_{t,s,n,b}$. The aggregated generation is given in (C.1c), which adds up the total active power allocated to the local flexible generators $\sum_{k\in\Omega_{t,s,b}^{G\_DN}} g_{t,s,k,b}^{local}$. The *nominal dispatch* is computed based on $(d_{t,s,b}^{agg} - g_{t,s,b}^{agg})$ and this transmission–distribution (T–D) interface flow limit is enforced by constraints (C.1d) and (C.1e). Eqs. (C.1f) and (C.1g) set the maximum and minimum limits on the local flexible demand and generations. After the clearing of the WSM, the *nominal dispatch* $(d_{t,s,b}^{agg} - g_{t,s,b}^{agg})$ is determined and this decision is then sent to the DSO.

Appendix D. Reformulated lower-level problem: LEMs clearing

This section reports the reformulated lower-level LEMs clearing problem (2). Notice that Problem (2) is nonlinear with absolute values in the objective function (2a). To solve this issue, two auxiliary non-negative variables are introduced, $z_{t,s,b}^1$ and $z_{t,s,b}^2$. The sum $z_{t,s,b}^1 + z_{t,s,b}^2$ is used to replace the deviation between nominal and actual injection $|p_{t,s,0,b} - (d_{t,s,b}^{agg*} - g_{t,s,b}^{agg*})|$, and the difference between $z_{t,s,b}^1$ and $z_{t,s,b}^2$ is constrained by the dispatch deviation.

$$\{d_{t,s,n,k,b}^{p*}, g_{t,s,n,k,b}^{p*}, d_{t,s,n,k,b}^{q*}, g_{t,s,n,k,b}^{q*}, p_{t,s,n,b}^{*}, q_{t,s,n,b}^{*}, W_{t,s,ii,b}^{*}, W_{t,s,ij,b}^{p*}, W_{t,s,ij,b}^{q*}, z_{t,s,b}^1, z_{t,s,b}^2, [\pi_{t,s,n,b}^{p*}]\}$$

$$= \arg\max \sum_{t\in\mathcal{T}}\sum_{s\in\mathcal{S}}\sum_{b\in\mathcal{B}^{lm}}\left(\sum_{n\in\mathcal{N}_b^+}\sum_{k\in\Omega_{t,s,n,b}^{D\_local}} c_{t,s,n,k,b}^{d} d_{t,s,n,k,b}^{p} - \sum_{n\in\mathcal{N}_b^+}\sum_{k\in\Omega_{t,s,n,b}^{G\_local}} c_{t,s,n,k,b}^{g} g_{t,s,n,k,b}^{p} - \bar{M}(z_{t,s,b}^1 + z_{t,s,b}^2) - c_{t,s,0,b}^{q} q_{t,s,0,b}\right) \tag{D.1a}$$

s.t.

$$z_{t,s,b}^1 - z_{t,s,b}^2 = (p_{t,s,0,b} - (d_{t,s,b}^{agg*} - g_{t,s,b}^{agg*})), \quad \forall t\in\mathcal{T}, \forall s\in\mathcal{S}, \forall b\in\mathcal{B}^{lm}, \quad [\varphi_{t,s,b}^{z0} \in \mathbb{R}], \tag{D.1b}$$

$$z_{t,s,b}^1 \geq 0, z_{t,s,b}^2 \geq 0, \quad \forall t\in\mathcal{T}, \forall s\in\mathcal{S}, \forall b\in\mathcal{B}^{lm}, \quad [\varphi_{t,s,b}^{z1} \geq 0, \varphi_{t,s,b}^{z2} \geq 0], \tag{D.1c}$$

(2b)–(2q). (D.1d)

Appendix E. The benchmark model: Central planning (CP) model

This section presents the central planning social welfare maximization model as discussed in Section 4.2.1.

$$\max_{\Xi} \sum_{t\in\mathcal{T}} \frac{1}{(1+r)^{t-1}}\Bigg(\Psi\sum_{s\in\mathcal{S}}\Bigg(\sum_{b\in\mathcal{B}^{lm}}\sum_{k\in\Omega_{t,s,b}^{D\_DN}} c_{t,s,k,b}^{d\_local} d_{t,s,k,b}^{local} - \sum_{b\in\mathcal{B}^{lm}}\sum_{k\in\Omega_{t,s,b}^{G\_DN}} c_{t,s,k,b}^{g\_local} g_{t,s,k,b}^{local}$$

$$+ \sum_{b\in\mathcal{B}}\sum_{k\in\Omega_{t,s,b}^{D\_TN}} c_{t,s,k,b}^{d} d_{t,s,k,b} - \sum_{b\in\mathcal{B}}\sum_{k\in\Omega_{t,s,b}^{G\_TN}} c_{t,s,k,b}^{g} g_{t,s,k,b} - \sum_{l\in\mathcal{L}}\Big(u_{t,l}K_l^{fix} + K_l^{var}\sum_{j\in\mathcal{J}} b_{t,l,j}^{F}\bar{F}_{l,j}\Big)\Bigg)\Bigg) \tag{E.1a}$$

s.t.

(3i)–(3k): binary investment decisions, (E.1b)

(1b)–(1l): WSM primal constraints. (E.1c)

where the variable array of the single-level problem (E.1) is

$$\Xi = \{u_{t,l}, b_{t,l,j}^{F}, d_{t,s,b}^{agg}, g_{t,s,b}^{agg}, d_{t,s,k,b}, g_{t,s,k,b}, f_{t,s,m}, \theta_{t,s,b}, [\pi_{t,s,b}]\}.$$

Based on the first stage results Ξ, we obtain the *nominal dispatch* information and then solve the local market clearing problem (2).

Data availability

Data will be made available on request.

References

[1] Carvallo J-P, Taneja J, Callaway D, Kammen DM. Distributed resources shift paradigms on power system design, planning, and operation: An application of the GAP model. Proc IEEE 2019;107(9):1906–22.
[2] Morstyn T, Savelli I, Hepburn C. Multiscale design for system-wide peer-to-peer energy trading. One Earth 2021;4(5):629–38.
[3] Scot W, Filiberto F, Masaō A, Thomas M, David W, Malcolm M. Local energy markets: From concepts to reality. In: Trading in local energy markets and energy communities: concepts, structures and technologies. Springer Cham; 2023, p. 1–38.
[4] Morstyn T, Niall F, Sarah JD, Malcolm DM. Using peer-to-peer energy-trading platforms to incentivize prosumers to form federated power plants. Nat Energy 2018;3:94–101.
[5] Morstyn T, Teytelboym A, Mcculloch MD. Bilateral contract networks for peer-to-peer energy trading. IEEE Trans Smart Grid 2019;10(2):2026–35.
[6] Le Cadre H, Jacquot P, Wan C, Alasseur C. Peer-to-peer electricity market analysis: From variational to generalized Nash equilibrium. European J Oper Res 2020;282(2):753–71, URL https://www.sciencedirect.com/science/article/pii/S0377221719307921.
[7] Melton RB. GridWise transactive energy framework (DRAFT version). Technical report, United States: U.S. Department of Energy, Office of Scientific and Technical Information (OSTI); 2013, URL https://www.osti.gov/biblio/1123244.
[8] Dudjak V, Neves D, Alskaif T, Khadem S, Pena-Bello A, Saggese P, Bowler B, Andoni M, Bertolini M, Zhou Y, Lormeteau B, Mustafa MA, Wang Y, Francis C, Zobiri F, Parra D, Papaemmanouil A. Impact of local energy markets integration in power systems layer: A comprehensive review. Appl Energy 2021;301:117434, URL https://www.sciencedirect.com/science/article/pii/S0306261921008266.
[9] Schmitt C, Schumann K, Kollenda K, Blank A, Rebenaque O, Dronne T, Martin A, Vassilopoulos P, Roques F, Moser A. How will local energy markets influence the pan-European day-ahead market and transmission systems? A case study for local markets in France and Germany. Appl Energy 2022;325:119913, URL https://www.sciencedirect.com/science/article/pii/S0306261922011710.

[10] Wang J, Xu J, Ke D, Liao S, Sun Y, Wang J, Yao L, Mao B, Wei C. A tri-level framework for distribution-level market clearing considering strategic participation of electrical vehicles and interactions with wholesale market. Appl Energy 2023;329:120230, URL https://www.sciencedirect.com/science/article/pii/S0306261922014878.
[11] Sheikhahmadi P, Bahramara S, Mazza A, Chicco G, Catalão JP. Bi-level optimization model for the coordination between transmission and distribution systems interacting with local energy markets. Int J Electr Power Energy Syst 2021;124:106392, URL https://www.sciencedirect.com/science/article/pii/S0142061520307778.
[12] Liu J, Zeng PP, Xing H, Li Y, Wu Q. Hierarchical duality-based planning of transmission networks coordinating active distribution network operation. Energy 2020;213:118488, URL https://www.sciencedirect.com/science/article/pii/S0360544220315966.
[13] Chen H, Fu L, Zhang R, Lin C, Jiang T, Li X, Li G. Local energy market clearing of integrated ADN and district heating network coordinated with transmission system. Int J Electr Power Energy Syst 2021;125:106522, URL https://www.sciencedirect.com/science/article/pii/S0142061520327630.
[14] Lezama F, Soares J, Faia R, Vale Z, Kilkki O, Repo S, Segerstam J. Bidding in local electricity markets with cascading wholesale market integration. Int J Electr Power Energy Syst 2021;131:107045, URL https://www.sciencedirect.com/science/article/pii/S0142061521002842.
[15] Lezama F, Soares J, Hernandez-Leal P, Kaisers M, Pinto T, Vale Z. Local energy markets: Paving the path toward fully transactive energy systems. IEEE Trans Power Syst 2019;34(5):4081–8.
[16] Alvarez EF, Olmos L, Ramos A, Antoniadou-Plytaria K, Steen D, Tuan LA. Values and impacts of incorporating local flexibility services in transmission expansion planning. Electr Power Syst Res 2022;212:108480, URL https://www.sciencedirect.com/science/article/pii/S0378779622005958.
[17] Fuentes González F, Sauma E, van der Weijde AH. Community energy projects in the context of generation and transmission expansion planning. Energy Econ 2022;108:105859, URL https://www.sciencedirect.com/science/article/pii/S0140988322000433.
[18] De Paola A, Papadaskalopoulos D, Angeli D, Strbac G. A game-theoretic modeling approach for merchant transmission planning. In: 2018 15th international conference on the European energy market. EEM, 2018, p. 1–5.
[19] Fonseka P. Optimal transmission expansion under different market structures. IET Gen Transm Distrib 2007;1:697–706, (9). URL https://digital-library.theiet.org/content/journals/10.1049/iet-gtd_20060391.
[20] De Paola A, Papadaskalopoulos D, Angeli D, Strbac G. Investigating the social efficiency of merchant transmission planning through a non-cooperative game-theoretic framework. IEEE Trans Power Syst 2018;33(5):4831–41.
[21] Majidi M, Baldick R. Definition and theory of transmission network planning. In: Transmission network investment in liberalized power markets. Springer Cham; 2020, p. 17–68.
[22] Vogelsang I. A simple merchant-regulatory incentive mechanism applied to electricity transmission pricing and investment: The case of H-r-g-v. In: Transmission network investment in liberalized power markets. Springer Cham; 2020, p. 353–80.
[23] Hesamzadeh M, Rosellón J, Gabriel S, Vogelsang I. A simple regulatory incentive mechanism applied to electricity transmission pricing and investment. Energy Econ 2018;75:423–39, URL https://www.sciencedirect.com/science/article/pii/S0140988318303670.
[24] Vogelsang I. Price regulation for independent transmission companies. J Regulat Econ 2001;20:141–65.
[25] Sappington D, Sibley DS. Regulating without cost information: The incremental surplus subsidy scheme. Internat Econom Rev 1988;29(2):297–306, URL https://EconPapers.repec.org/RePEc:ier:iecrev:v:29:y:1988:i:2:p:297-306.
[26] Khastieva D, Mohammadi S, Hesamzadeh MR, Bunn D. Optimal transmission investment with regulated incentives based upon forward considerations of firm and intermittent resources with batteries. IEEE Trans Power Syst 2021;36(5):4420–34.
[27] Khastieva D, Hesamzadeh MR, Vogelsang I, Rosellón J, Amelin M. Value of energy storage for transmission investments. Energy Strategy Rev 2019;24:94–110, URL https://www.sciencedirect.com/science/article/pii/S2211467X19300070.
[28] Xia Y, Savelli I, Morstyn T. An incentive regulation approach for balancing stakeholder interests in transmission merchant investment. Electr Power Syst Res 2024;234:110617, URL https://www.sciencedirect.com/science/article/pii/S0378779624005030.
[29] Paredes Á, Aguado JA, Essayeh C, Xia Y, Savelli I, Morstyn T. Stacking revenues from flexible DERs in multi-scale markets using tri-level optimization. IEEE Trans Power Syst 2024;39(2):3949–61.
[30] Savelli I, Morstyn T. Electricity prices and tariffs to keep everyone happy: A framework for fixed and nodal prices coexistence in distribution grids with optimal tariffs for investment cost recovery. Omega (United Kingdom) 2021;103:102450.
[31] Wood AJ, Wollenberg BF. Power generation, operation and control. New York:Wiley; 1996.
[32] Gan L, Li N, Topcu U, Low SH. Exact convex relaxation of optimal power flow in tree networks. 2012, http://dx.doi.org/10.48550/ARXIV.1208.4076, URL https://arxiv.org/abs/1208.4076.
[33] Halilbašić L, Pinson P, Chatzivasileiadis S. Convex relaxations and approximations of chance-constrained AC-OPF problems. IEEE Trans Power Syst 2019;34(2):1459–70.
[34] Joskow P, Tirole J. Merchant transmission investment. J Ind Econ 2005;53(2):233–64, URL http://www.jstor.org/stable/3569878.
[35] Savelli I, De Paola A, Li F. Ex-ante dynamic network tariffs for transmission cost recovery. Appl Energy 2020;258:113979, URL https://www.sciencedirect.com/science/article/pii/S0306261919316666.
[36] Lavrutich M, Hagspiel V, Siddiqui AS. Transmission investment under uncertainty: Reconciling private and public incentives. European J Oper Res 2023;304(3):1167–88, URL https://www.sciencedirect.com/science/article/pii/S0377221722003526.
[37] Byeon G, Van Hentenryck P. Unit commitment with gas network awareness. IEEE Trans Power Syst 2020;35(2):1327–39.
[38] Ehrgott M. Multicriteria optimization. Berlin, Germany: Springer; 2005.
[39] McCormick GP. Computability of global solutions to factorable nonconvex programs: Part I — Convex underestimating problems. Math Program 1976;146–75.
[40] Kraft E, Russo M, Keles D, Bertsch V. Stochastic optimization of trading strategies in sequential electricity markets. European J Oper Res 2023;308(1):400–21, URL https://www.sciencedirect.com/science/article/pii/S0377221722008505.
[41] Li C, Conejo AJ, Liu P, Omell BP, Siirola JD, Grossmann IE. Mixed-integer linear programming models and algorithms for generation and transmission expansion planning of power systems. European J Oper Res 2022;297(3):1071–82, URL https://www.sciencedirect.com/science/article/pii/S0377221721005397.
[42] Baran ME, Wu FF. Network reconfiguration in distribution systems for loss reduction and load balancing. IEEE Trans Power Deliv 1989;4(2):1401–14.
[43] Blumsack SA. Network topologies and transmission investment under electric-industry restructuring (Ph.D. thesis), Carnegie Mellon University; 2006, Available at https://www.cmu.edu/ceic/assets/docs/publications/phd-dissertations/2006/seth-blumsack-phd-thesis-2006.pdf.
[44] Zhang H. Transmission expansion planning for large power systems (Ph.D. thesis), Arizona State University; 2013, Available at https://core.ac.uk/download/pdf/79567383.pdf.
[45] Dinh NT, Karimi-Arpanahi S, Yuan R, Pourmousavi SA, Guo M, Liisberg JAR, Lemos-Vinasco J. Modeling irrational behavior of residential end users using non-stationary Gaussian processes. IEEE Trans Smart Grid 2024;15(5):4636–48.
[46] Safdarian A, Astero P, Baranauskas M, Keski-Koukkari A, Kulmala A. Coalitional game theory based value sharing in energy communities. IEEE Access 2021;9:78266–75.