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Essays on optimal wealth and income taxation

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Thesis abstract

This thesis examines the design of optimal wealth and income tax policies, focusing on how the equity-efficiency trade-off of taxation is shaped by margins that, although crucial, have been overlooked in the existing literature: heterogeneity in firms' market power, households' portfolio choices, and joint labor supply decisions within couples.

Chapter 1 studies the effects of top wealth taxation on entrepreneurs' choices and macroeconomic aggregates, taking into account that wealthier entrepreneurs operate larger firms and impose larger markups, as observed in the US data. In this chapter I show that entrepreneurs imposing larger markups feature lower production and capital elasticities to the tax. Hence, neglecting the observed markups heterogeneity across entrepreneurs leads to overestimate wage and production losses induced by top wealth taxation. Furthermore, since the burden of top wealth taxation falls onto the entrepreneurs imposing the largest markups, the tax policy reduces the aggregate markup in the economy and increases the labor share of income accruing to workers.

Chapter 2 examines how top wealth taxation affects households' portfolio composition and the allocation of capital in the economy. To this end, I develop a portfolio choice model calibrated to match households' portfolio choices across the US wealth distribution. Simulating a top wealth tax, I find that taxed households shift away from high-productivity, high-growth private equity investments toward safer, lower-return assets. This reallocation of capital reduces GDP and long-run growth, highlighting a new channel through which wealth taxation can affect the economy.

Chapter 3 presents simulations of optimal income taxes for singles and couples in the US economy. The general—although simple—occupational choice framework employed allows to investigate how labor supply assumptions, participation elasticities, and the government's redistributive preferences shape optimal tax schedules.

Overall, these essays provide new insights into the fundamental trade-offs of income and wealth taxation, emphasizing critical margins that policymakers should consider when designing optimal fiscal policies.

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Contents

Chapter 1: wealth tax, entrepreneurship and market power

Abstract

This paper studies how the distortionary and redistributive effects of top wealth taxation change when, beyond productivity, entrepreneurial returns capture their firms' market power. To do so, I develop a model in which entrepreneurs accumulate wealth investing in their firm, face capital income risk and set markups. Consistently with U.S. data, wealthier entrepreneurs operate larger firms and charge higher markups. As a consequence, the burden of a top wealth tax falls onto the entrepreneurs imposing the largest markups, reducing the aggregate markup in the economy and raising the labor share of income accruing to poor workers. Furthermore, I show that entrepreneurs imposing larger markups feature lower production and capital elasticities to the tax. Hence, when observed market power heterogeneity is neglected, a wealth tax raising 1% of GDP imposed on the wealthiest 1% of U.S. households overestimates wage losses induced by the tax by 1.4 pp. and output losses by 1.1 pp.

1 Introduction

Wealth in the United States is highly concentrated and this concentration has steadily increased over recent decades (Saez and Zucman, 2020). Furthermore, empirical studies show that the wealthiest Americans earn the highest returns on their wealth. This is because they hold large portfolio shares in high-yield entrepreneurial activities (Xavier, 2021; Smith et al., 2023). In light of this evidence, scholars and policymakers have extensively debated whether it is desirable to tax the wealth of these rich individuals for redistributive purposes. The answer to this question crucially depends on whether the redistributive gains from a wealth tax outweigh the distortionary effects on the entrepreneurial activities owned by these wealthy Americans. Existing studies typically address this issue under the assumption that the high returns to entrepreneurship - and as a consequence, the high returns to wealth - earned by rich individuals *fully* reflect the productivity of their entrepreneurial investments. If this is the case, a top wealth tax would primarily fall on the individuals investing in the most productive activities in the economy, with potentially severe consequences for output, employment, and wages.

The contribution of this paper is to examine how the distortionary and redistributive effects of top wealth taxation change when this one-to-one relationship between returns and productivity breaks. In particular, I study the effects of a top wealth tax when the heterogeneous returns that entrepreneurs earn from their businesses not only reflect their firm productivity but also the firms' market power. Indeed, American entrepreneurs own firms with sizable and heterogeneous product market power, with some firms imposing exceptionally high markups that they exploit to boost their profits and returns (De Loecker et al., 2020; Autor et al., 2020; Edmond et al., 2023). This paper studies how the equity-efficiency trade-off of top wealth taxation changes when accounting for the observed market power heterogeneity across American entrepreneurs' businesses.

To address this question, I develop a dynamic general equilibrium model that captures wealth inequality, the concentration of entrepreneurial activity at the top of the wealth distribution and the distribution of firm markups observed in U.S. data. To this goal I assume that entrepreneurs face capital income risk, a crucial ingredient for replicating the fat upper tail of the empirical wealth distribution (Benhabib et al., 2015; Benhabib and Bisin, 2018). Consistently with standard models of monopolistic and oligopolistic competition (Atkeson and Burstein, 2008; Edmond et al., 2023), I further assume that firms' market power increases with their market share. Together, these elements yield a novel framework in which to study wealth taxation when entrepreneurs' returns imperfectly reflect their productivity.

Consistently with evidence from the U.S. Survey of Consumer Finances showing that wealthier entrepreneurs manage larger firms, in this framework wealthier (and more skilled) entrepreneurs operate firms producing at a larger scale and charging higher markups. Within this economy, I examine the effects of a progressive top wealth tax whose revenues are lump-sum redistributed across workers and entrepreneurs. I then compare the tax policy outcomes to those arising in a counterfactual scenario in which heterogeneity in market power across entrepreneurs is shut down.

The main result of the analysis is that taking into account that wealthier entrepreneurs own firms imposing larger markups relaxes the equity-efficiency trade-off of top wealth taxation, with respect to the case in which this market power heterogeneity is neglected. In other words, for any given tax-revenue objective, the considered wealth tax induces higher redistribution from rich entrepreneurs to poor workers, at the cost of lower losses in terms of forgone production. This is because when taking into account market power heterogeneity, taxing the wealthiest entrepreneurs means taking away resources from the entrepreneurs imposing the largest markups. These are not only the most productive agents, but also the ones imposing the largest production distortions and featuring the lowest production and capital elasticities with respect to the tax. This limits losses in labor demand and in wage received by workers as an effect of the tax, with respect to the case in which market power heterogeneity across entrepreneurs is neglected. Moreover, when entrepreneurs impose heterogeneous markups, the wealth tax reduces the aggregate markup in the economy, as the tax burden is concentrated onto entrepreneurs who impose the highest markups. This effect diminishes the capital share of income that accrues to wealthy entrepreneurs and increases the labor share of income going to poor workers. This extra redistributive effect of wealth taxation would be neglected when market power heterogeneity across entrepreneurs is not taken into account.

As first step of the analysis, I document the relationship between American entrepreneurs' wealth, their firm size, and the markups they charge. To this end, I employ data from the Survey of Consumer Finances and Compustat. First, I show that American entrepreneurs are concentrated at the top of the wealth distribution and that their entrepreneurial investment is mainly directed toward a single business, from which they earn heterogeneous returns that increase with wealth. I then document that entrepreneurs' firms display substantial heterogeneity in size: firm capital stock, employment and revenues all rise sharply across the entrepreneurs' wealth distribution. Finally, using Compustat data, I show that larger firms in terms of revenues and employees exhibit higher markups.

As a second step, I develop a simple static model to illustrate how entrepreneurs' production elasticities with respect to capital - and hence with respect to wealth taxation - depend on the assumptions on their market power. To study the shape of production elasticities under several market power assumptions, I consider entrepreneurs operating in monopolistic competition and facing Kimball (1995) demand for their products. This modeling choice allows me to obtain demand curves for entrepreneurs' varieties with either constant or variable price elasticity of demand. Within this framework, I analytically show that an entrepreneur's production elasticity with respect to capital positively depends on the price elasticity of demand faced and negatively on the rate at which that elasticity declines. Intuitively, a reduction in available capital raises an entrepreneur's marginal cost and reduces output. However, this "production pass-through" is low when demand elasticity is low, as most of the cost increase is transmitted to prices. This result implies that both the level and the distribution of entrepreneurs' markups - and hence of the demand elasticities they face - are key determinants of the aggregate production elasticity of the economy with respect to the tax. This constitutes a novel insight for the public finance literature.

As a third step I show that, not only the production elasticity with respect to the tax depends on the market power of entrepreneurs, but also the capital elasticity with respect to the tax. To make this point I build a rich dynamic, stochastic, general equilibrium model which I employ to quantify the distortionary and redistributive wealth tax effects under different market power assumptions. In this dynamic setting, entrepreneurs choose not only production and markups but also the amount of capital to allocate to their own business through consumption-saving and portfolio decisions. Each entrepreneur invests a fraction of wealth in his privately owned business - whose return depends on his idiosyncratic entrepreneurial productivity - and allocates the remainder in a mutual fund - conveying capital to a "corporate" sector and yielding a risk-free return. While privately-owned businesses have no access to capital market and entirely rely on entrepreneurs' capital supply as source of financing, firms in the corporate sector have unlimited access to capital markets[1]. In this dynamic economy, I assume that, in addition to entrepreneurs, there are workers who supply labor, receive stochastic labor income, and invest only in the same risk-free asset available to entrepreneurs.

The steady-state of the model is calibrated to the US economy, first assuming entrepreneurs imposing markups increasing in their firms' market shares and then under the assumption of homogeneous and constant markups across all entrepreneurs. In both scenarios the shape of the actual US wealth distribution and the concentration of entrepreneurs at the top of the wealth distribution are matched. In this setting I implement a *permanent* wealth tax policy on the wealth in excess of the 99% percentile of the wealth distribution, which raises 1% of GDP and whose tax-revenues are uniformly lump-sum redistributed across workers and entrepreneurs in the economy. I then compare the steady-states of the model with and without the tax, both when entrepreneurs impose variable markups (increasing in firm's market share) and constant (homogeneous) markups.

First, I show that the wealth tax reduces wealth accumulation of the wealthiest entrepreneurs and the drop in steady-state aggregate capital is 2.6 percentage points smaller in the economy with markups heterogeneity than in the one with homogeneous markups. This is because in the economy with markups heterogeneity the top wealth tax primarily hits high-markup entrepreneurs, that I show feature low capital elasticity to the tax[2]. The finding that entrepreneurs' markups distribution matters for shaping the aggregate capital elasticity to the tax is a novel contribution of this paper.

Combining this result with the lower production elasticity to the tax in the heterogeneous markup economy, I find that GDP losses due to the tax are 1.1 percentage points smaller

[1] Corporations are assumed to have constant return to scale technology and operate in perfect competition, so they make zero profits. Thus, it is irrelevant the ownership of these firms.

[2] The reason is that the elasticity of marginal profits to capital - and hence the sensitivity of capital accumulation choices to taxation - depends on the demand elasticity entrepreneurs face.

in the economy with markup heterogeneity. The smaller GDP losses also reflect a reduction in markup-induced distortions generated by the tax. In particular, the tax burden falls disproportionately on high-markup entrepreneurs, leading to a decline in the economy's aggregate markup. This reduction, in turn, exerts a positive effect on equilibrium output.

I then show that when accounting for market power heterogeneity across entrepreneurs also the redistributive wealth tax effects are larger, with respect to the homogeneous markups case. In the steady-state with wealth taxation implemented the policy generates larger revenues in the setting with markups heterogeneity across entrepreneurs. Hence, in this economy poor workers benefit from a larger transfer. Second, the smaller drop in production in the economy with markups heterogeneity is associated with a smaller drop in labor demand and hence in equilibrium wage. In the economy with markups heterogeneity, hence, the wealth tax reduces equilibrium wage 1.4 percentage points less than in the economy with no markups heterogeneity. Finally, the reduction in the aggregate markup previously highlighted determines an increase in the labor share of income accruing to workers by 0.4%, a redistributive effect that would be neglected if we studied the wealth tax effects under constant markups.

Contribution to the literature: this paper is the first to examine how imperfect competition shapes the effects of top wealth taxation. While Guvenen et al. (2023) also study wealth taxation in a setting with imperfect competition, their framework assumes homogeneous and constant markups across entrepreneurs. Similarly to Boar and Midrigan (2023), their focus lies in comparing wealth and capital income taxation rather than in exploring the role of market power in determining tax outcomes. Existing studies on top wealth taxation emphasize its distortionary effects on capital accumulation (Jakobsen et al., 2020) and on entrepreneurial effort and productivity (Chari et al., 2025). This paper shows that the magnitude of these distortions crucially depends on entrepreneurs' market power. In particular, while the wealthiest entrepreneurs are the most productive, they also exhibit the lowest elasticities of capital and output with respect to taxation.

This paper also contributes to the literature studying cases where tax policies can generate efficiency gains. Piketty et al. (2014), Rothschild and Scheuer (2016), Scheuer and Slemrod (2021), and Gaillard and Wangner (2021) show that taxing the income or wealth of rent-seeking agents can improve overall efficiency. I explore a novel mechanism through which wealth taxation produces such gains: by reducing the aggregate markup in the economy, the tax mitigates the production distortions associated with market power.

Furthermore, this work connects to the recent literature on optimal policy design in the presence of heterogeneous markups across entrepreneurs. Boar and Midrigan (2022) and Edmond et al. (2023) focus on optimal firm subsidies, while Eeckhout et al. (2025) study optimal profit and labor income taxation in a static Mirrleesian framework with entrepreneurs operating in oligopolistic competition. In contrast, I analyze a dynamic economy to highlight and quantify how market power heterogeneity shapes the aggregate elasticities of capital and production in response to taxation.

Layout: the chapter is structured as follows. Section 2 presents the empirical evidence, Section 3 introduces the static framework and discusses the relationship between entrepreneurs' production elasticity with respect to capital and their market power. Section 4 simulates the static model showing how the elasticity of aggregate production with respect to the tax and equilibrium wage depend on entrepreneurs' market power assumptions. Section 5 presents tax simulations in the quantitative dynamic model, highlighting the role of market power assumptions in shaping the aggregate capital elasticity with respect to the tax. Section 6 concludes.

2 Entrepreneurs across the wealth distribution

In this section, I use data from the 2019 *Survey of Consumer Finances* (hereafter, SCF) to study the features of entrepreneurial activity across the U.S. wealth distribution. The evidence reveals large heterogeneity in business size, measured by capital, revenues and employment. In particular, I show that average firm size rises with entrepreneurs' wealth, with substantial heterogeneity in size even within the top percentiles of the wealth distribution. Using Compustat data I then document a positive relationship between firms' size and the markups they impose.

2.1 Data and variables definitions

To study the features of entrepreneurial activity across the wealth distribution, the 2019 wave of the Survey of Consumer Finances is employed. The choice of SCF over other surveys is due to two reasons. First of all, SCF contains detailed information on households' personal wealth and on businesses owned by each household (business income, employees, age, sector...). Furthermore, SCF surveys many more households at the very top of the wealth distribution, with respect to what other surveys do (for details on the sampling procedure see for example Kennickell, 2008). For the scope of this analysis this feature is of particular importance, given that entrepreneurial activity is primarily concentrated at the top of U.S. wealth distribution.

The SCF contains several questions which can be used to classify a household as an entrepreneur:

1. "Do you (and your family living here) own or share ownership in any privately-held businesses, including farms, professional practices, limited partnerships, private equity, or any other business investments that are not publicly traded?"

2. "Do you (or anyone in your family living here) have an active management role in any of these businesses?"

3. "Do you work for someone else, are you self-employed or something else?"

The entrepreneurial status of an household depends on how the term entrepreneur is defined. In this paper I define an entrepreneur as an household who responds affirmatively to questions 1., 2. and 3. The requirement of the household actively managing the business is imposed in order to exclude from the class of entrepreneurs those households who act as "investors" but do not contribute to the management of the business. The requirement of being self-employed is instead imposed in order to exclude from the entrepreneurs' class those households who have a full-time wage-earning job. This definition is consistent with other works in the literature employing SCF data to study entrepreneurship in the US (e.g. Quadrini, 2000; Cagetti and De Nardi, 2006)[3].

[3]Alternative definitions of entrepreneur employed by the literature consider as an entrepreneur an household responding affirmatively to 1., or 1. and 2. (e.g. Boar and Midrigan (2022)). In any case,

2.2 Entrepreneurial activity across the wealth distribution

Table 1 shows that net wealth in the U.S. is extremely unequally distributed, with around 37% of total wealth accruing to the wealthiest 1% of households. Noticeably, entrepreneurial wealth (i.e. the wealth held in actively managed privately owned businesses) is even more unequally distributed, with more than 42% of the overall entrepreneurial wealth owned by the wealthiest 1% of households. The concentration of the entrepreneurial activity at the top of the wealth distribution is further highlighted by Figure 1.

TABLE 1. *Net wealth and entrepreneurial wealth distribution: summary statistics*

Percentile	Net wealth share	Entrepreneurial wealth share
top 10%	76.5%	82.6%
top 5%	64.8%	70.5%
top 1%	37.2%	42.6%
top 0.5%	28.0%	33.4%
top 0.2%	16.4%	23.3%
top 0.1%	12.2%	18.0%

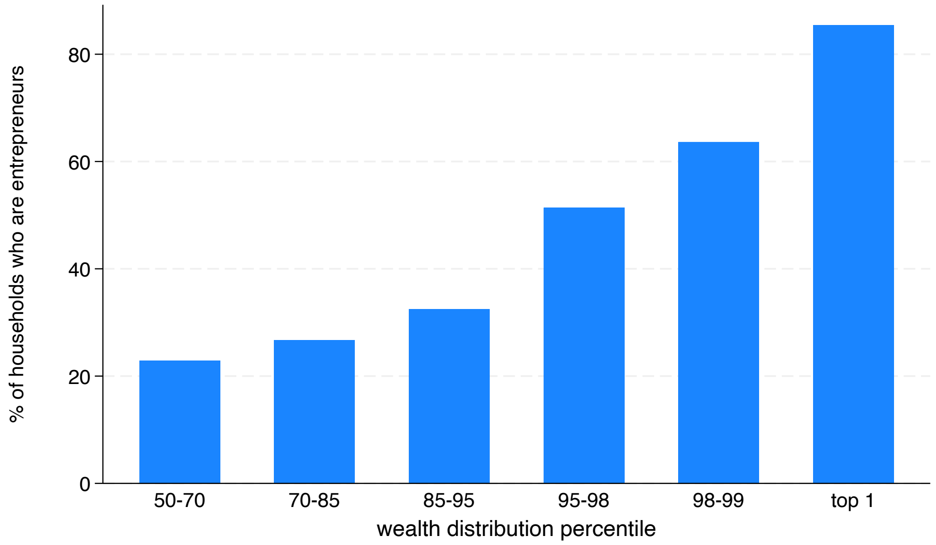
Notes: *column 2 of the table reports the share of net wealth (assets - debts) of U.S. households belonging to different percentiles of the wealth distribution. Column 3, instead, reports the share of wealth held in directly managed private businesses by the wealthiest x% of U.S. entrepreneurs. For details on the definition of entrepreneur see Section 2.1. Data from 2019 Survey of Consumer Finances.*

Figure 1 shows that the fraction of households who are entrepreneurs in a given wealth percentile is increasing across the wealth distribution. In particular, around 40% of the wealthiest 10% of US households are entrepreneurs. This number increases up to 82% for the wealthiest 1% of households. However, this figure does not provide any information on the fraction of overall wealth held in these businesses, compared to other investment opportunities.

Figure 2 fills this gap by reporting the portfolio share (i.e. fraction of net wealth) that U.S. entrepreneurs hold in actively managed private businesses (blue columns). Notice that the fraction of net wealth held in actively managed private businesses is increasing across the wealth distribution and it represents a sizable share of US entrepreneurs' portfolios, especially at the very top of the wealth distribution. Furthermore, the fraction of wealth held in actively managed businesses is significantly larger than the fraction of wealth held

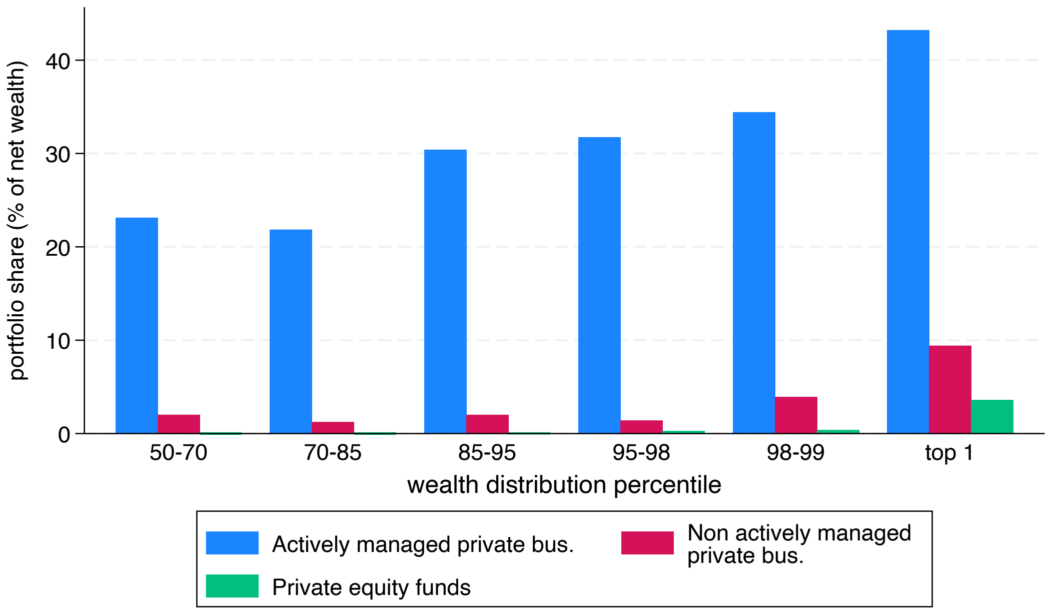
the empirical findings I will present do not significantly change when employing alternative definitions of entrepreneur.

FIGURE 1. *Fraction of US households defined as entrepreneurs across the wealth distribution*



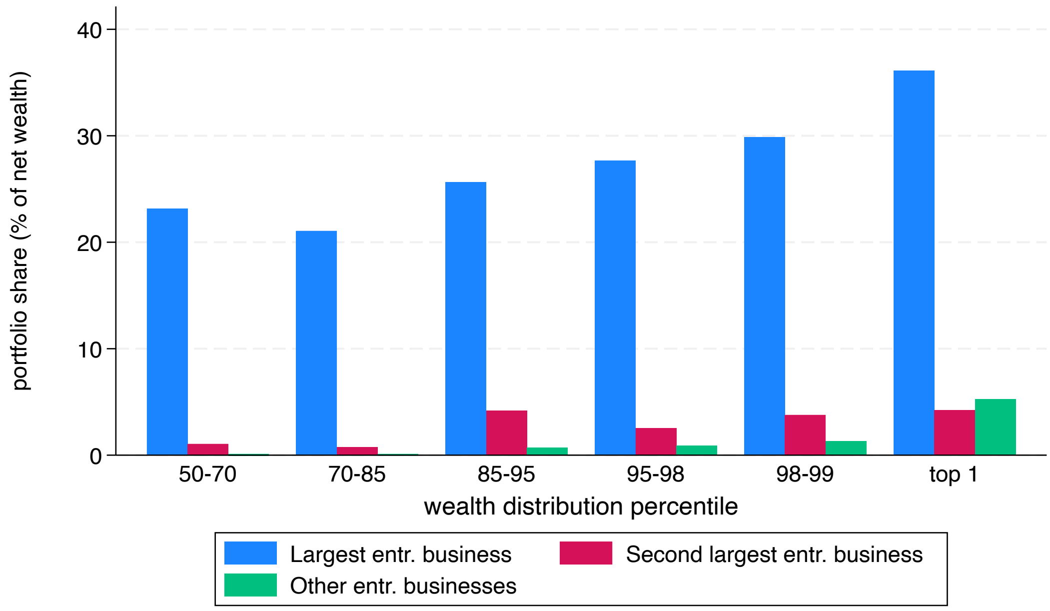
Notes: *the Figure reports the fraction of US households, per given wealth percentiles bin, which satisfy the definition of entrepreneur reported in Section 2.1. Data from 2019 Survey of Consumer Finances.*

FIGURE 2. *Portfolio shares across entrepreneurs: private equity investments*



Notes: *the Figure reports the fraction of net wealth that US entrepreneurs (entrepreneur is defined according to the definition reported in Section 2.1) invest in different private equity investment opportunities. The total amount of private equity investment is disaggregated into: investment in actively managed businesses (blue), investment in non-actively managed businesses (red), other private equity investment (green, mainly private equity funds). The value of each column is computed by averaging the portfolio shares invested in each private equity investment opportunity across the entrepreneurs belonging to a given wealth percentiles bin. Data from 2019 Survey of Consumer Finances* in other private equity investment opportunities such as non-actively managed private equity businesses (red columns) or private equity funds (green columns). This evidence shows that households actively managing businesses at the top of the wealth distribution are really entrepreneurs, more than just investors. Figure 2 also shows that the wealthier the entrepreneur, the more wealth he confers to his own entrepreneurial activities, suggesting that the size of the entrepreneurs' firms, in terms of capital endowment, increases across the wealth distribution. One potential concern on the previous statement is that capital conferred by each entrepreneur is diluted across many entrepreneurial activities. As shown in Figure 3, this is not the case.

FIGURE 3. *Portfolio shares across entrepreneurs: actively managed private businesses*

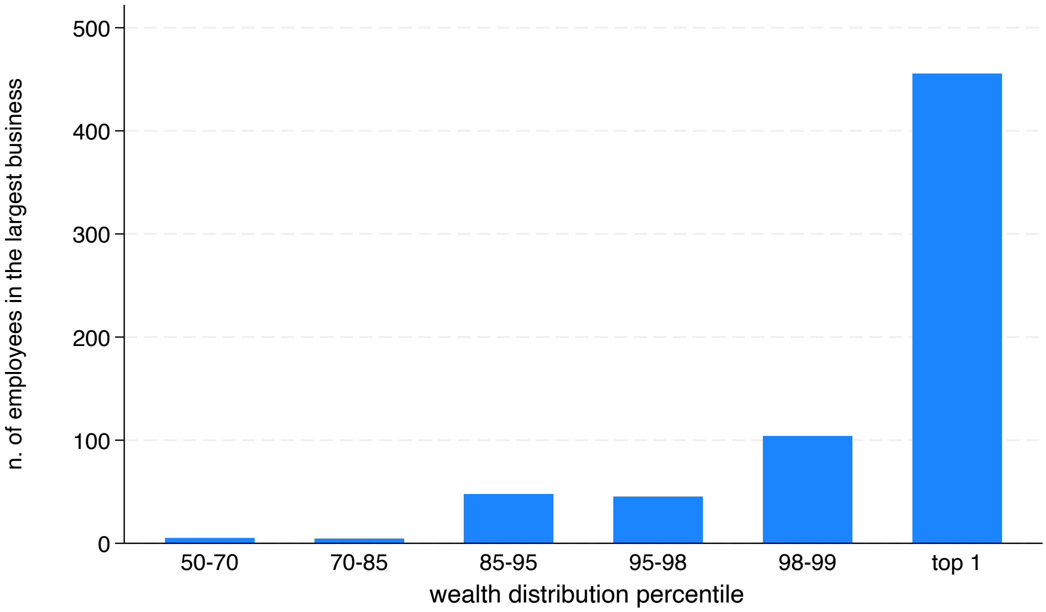


Notes: *the Figure reports the fraction of net wealth that US entrepreneurs (entrepreneur is defined according to the definition reported in Section 2.1) invest in different privately owned actively managed businesses. The total amount of privately owned actively managed business investment is disaggregated into: investment in the largest actively managed businesses (blue), investment in the second actively managed business (red), investment in other privately held businesses (green). The value of each column is computed by averaging the portfolio shares invested in first/second/other actively managed private business across the entrepreneurs belonging to a given wealth percentiles bin. Data from 2019 Survey of Consumer Finances*

Indeed, Figure 3 shows that almost the entire wealth invested in entrepreneurial activities is conveyed towards a single business. Notice that this finding is consistent with the literature arguing that entrepreneurial investment is poorly diversified (Moskowitz and Vissing-Jørgensen, 2002).

However, not only the capital endowment of privately owned businesses is increasing across the wealth distribution, but also their size in terms of number of employees is steeply increasing.

FIGURE 4. *Employees in largest (private) actively managed business*



Notes: *the Figure reports the average number of employees in the largest private actively managed business across the wealth distribution. The value of each column is computed by averaging the number of employees in the largest actively managed business across entrepreneurs belonging to the same wealth percentile bin. The definition of entrepreneur is reported in Section 2.1. Data from 2019 Survey of Consumer Finances*

This pattern is reported in Figure 4, which plots the average number of employees in the largest business of each entrepreneur, for several wealth percentiles bins. A similar pattern could be observed when analyzing the number of employees working in the second largest business owned by each entrepreneur, as well as in further businesses. As Figure C1.1 (Appendix C) shows, a similar result holds for firms' revenues, i.e. the firms owned by entrepreneurs at the top of the wealth distribution have much larger revenues than firms owned by entrepreneurs at lower percentiles.

2.3 Returns to entrepreneurship

In this section I complement the previous evidence showing that beside owning heterogeneous firms (both in terms of capital endowment and employees), entrepreneurs across the wealth distribution receive heterogeneous returns to their entrepreneurial investment. In order to estimate returns to entrepreneurship I use the three latest waves of the SCF, namely the 2013, 2016 and 2019[4]. The following variables are employed:

- GI = *directly managed private business* pre-tax (gross) income reported the year preceding the survey date
- EV = value of the *directly managed private business* equity owned by the household at the date of the survey. It is the answer to the following survey question: "what is the net worth of (your share) of this business?"

The reported pre-tax income has to undergo two major transformations to reflect the perceived capital income obtained through entrepreneurial investment. First of all, taxes paid by each firm are subtracted from gross income. The applied tax adjustment is assumed to be 36% of gross income for C-corporation and 0% for S-corporations[5]. The 36% tax rate is an estimate for the effective corporate tax rate and is chosen consistently with Bhandari and McGrattan (2021). They obtain this figure as a weighted sum of the marginal tax rates on firm earnings.

Furthermore, to identify capital income separately from labor income, a salary is imputed to all entrepreneurs not reporting any. The imputed salary represents the fraction of gross income net of taxes which accrues to labor income. This term is subtracted from gross income net of taxes to obtain net capital income (NI):

$$
\begin{aligned}
NI &= GI \times 0.64 - \text{ imputed salary} \qquad && \text{for C-corp.} \\
NI &= GI - \text{ imputed salary} && \text{for S-corp.}
\end{aligned}
$$

To obtain the imputed salaries I first run a regression (over households reporting a positive salary) of household-level wage over a constant, age, age squared, a dummy for graduating college and a dummy for gender. I then use the estimated coefficients to compute

[4]Returns to entrepreneurship across the wealth distribution are pretty volatile. This motivates my choice of using more than one survey wave for the estimation of returns. On the other hand, using too many waves would induce me to compare returns across the wealth distribution with significantly different underlying wealth distributions. These considerations motivate my choice of using the waves in the period 2013-2019 only, in which wealth inequality in the US has remained pretty stable.

[5]A C-corporation is a legal form for a company in which the owners are taxed separately from the entity. C-corporations are subject to corporate income taxation and the net profits distributed to owner also undergo personal taxation. An S-corporation, instead, is a business legal form that allows to pass its taxable income directly to its shareholders, hence is not subject to corporate income taxation

the fitted wage for those entrepreneurs not reporting any salary. Finally, I obtain the imputed yearly salary by multiplying the wage rate for the total hours worked in a year. This imputation procedure is consistent with other works employing the SCF data in order to obtain estimates of returns to private equity investment (Moskowitz and Vissing-Jørgensen, 2002; Kartashova, 2014; Xavier, 2021).

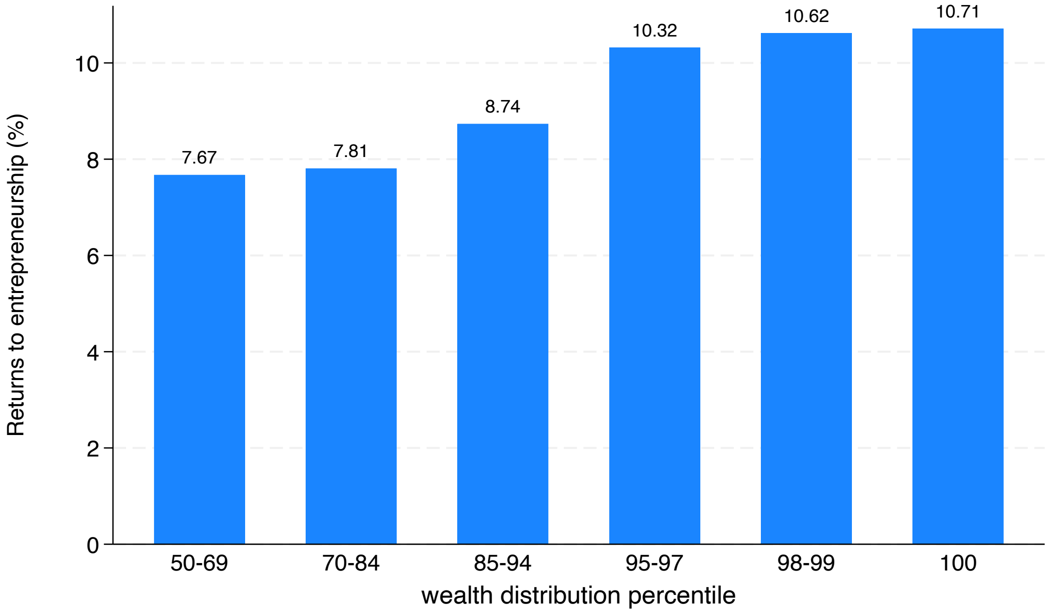
Employing the constructed measure of *net capital income* (NI), I now compute the *annualized returns* to entrepreneurship across the wealth distribution. To do so, for each household i and survey wave $t = \{2013, 2016, 2019\}$ I compute:

$$R_i^t = \left(1 + \frac{3NI_i^t}{EV_i^t}\right)^{\frac{1}{3}} - 1$$

notice that this is the same measure of annualized (SCF is a triennial survey) returns to private equity investment computed by Moskowitz and Vissing-Jørgensen (2002), Kartashova (2014), Xavier (2021) using the SCF data. The returns to entrepreneurship are computed for each household i. Then, by averaging R_i^t across households belonging to the wealth percentile bin $p \in \{50-70, 70-85, 85-95, 95-98, 98-99, \textit{top}\ 1\}$ I obtain the returns to entrepreneurship at wealth percentile bin p and survey wave t: R_p^t. Finally, averaging R_p^t across the three survey waves employed (2013, 2016, 2019) I obtain returns to entrepreneurship at wealth percentile bin p: R_p. The returns estimated through this procedure are reported in Figure 5.

Figure 5 shows that returns to entrepreneurship are increasing across the wealth distribution. In particular, the wealthiest 5% of US households receive returns in the ballpark of 10%, reaching 10.7% at the very top of the wealth distribution. The households below the top 5% receive returns to entrepreneurship around 8.7% while those at lower percentiles around 7.7%. Xavier (2021) analyzes the returns to private equity investment (i.e. returns to investment in all private businesses, not only those actively managed, and private equity funds) across US wealth distribution. She reports increasing returns to private equity investment across almost the entire wealth distribution, although she highlights a drop of returns for the wealthiest 3% of US households. For the top 5% she reports returns to private equity investment in the range 14%-16%, although she highlights that around 20-25% of these returns are due to capital gains (which I have not taken into account

FIGURE 5. *Returns to entrepreneurship across the wealth distribution*



Notes: *the Figure reports the returns to investment in actively managed private businesses across the wealth distribution. For details on the procedure employed see Section 2.3. Data from 2019 Survey of Consumer Finances*

in my procedure) rather than realized income. Fagereng et al. (2020), using Norwegian administrative data still report a positive relationship between private equity returns and net wealth of the entrepreneur, consistently with my findings for the US.

2.4 Firm size and product market power

The previous empirical evidence shows that wealthier entrepreneurs own larger firms in terms of capital, revenues and employees. Do larger firms also possess greater market power? To address this question, I focus on the relationship between firm size and their product market power, measured by firms' markups. In the standard Cournot model, as well as in more recent frameworks of monopolistic and oligopolistic competition with variable markups, a positive relationship between firm size and markups generally holds (Autor et al., 2020; De Loecker and Syverson, 2021; Edmond et al., 2023). Breaking this relationship requires very restrictive assumptions on the demand function faced by entrepreneurs for their products (Biondi, 2022). However, the SCF data do not allow for a direct test of this relationship, as they lack information on firms' production costs

necessary to estimate firm-level markups. To overcome this limitation, I use Compustat data (1980-2019), which provide financial statements for all listed U.S. firms. Following the production approach of De Loecker et al. (2020), I estimate firm-level markups and regress them on firm size, measured by both employment and revenues. The results are reported in Table 2. Columns (1) and (2) show regressions of firm-level markups (as measured by De Loecker et al., 2020) on firms' employment shares (defined as a firm's employees divided by total employment in a given year), including year and sector fixed effects. Columns (3) and (4) instead report regressions of firm-level markups on firms' revenue shares (revenues of a firm over aggregate firms' revenues in a given year), again controlling for year and sector fixed effects. The results indicate a positive and significant within-sector relationship between markups and firm size among listed U.S. firms.
The main limitation of this approach is that Compustat includes only listed firms, which are different - and on average larger - than the privately owned businesses in the SCF data. Nonetheless, Autor et al. (2020) show that a positive relationship between firm size (measured by total assets) and markups also holds in the U.S. Census of Manufactures data, which cover both listed and unlisted firms.

TABLE 2. *Markups - firm size correlation*

	log(markup) (1)	log(markup) (2)	log(markup) (3)	log(markup) (4)
log(employment share)	0.023***	0.040***		
	(0.001)	(0.001)		
log(revenue share)			0.190***	0.197***
			(0.002)	(0.002)
Year FE	Yes	Yes	Yes	Yes
Sector 2-digit FE	Yes	No	Yes	No
Sector 4-digit FE	No	Yes	No	Yes
N	105,175	105,175	105,175	105,175
R^2	0.074	0.149	0.770	0.799
Within-R^2	0.006	0.015	0.114	0.134

Notes: *the first two columns report the results of regressing firm level markups (as measured by De Loecker et al. (2020)) on Compustat (1980-2019) data on firm level employment shares (i.e. employees of a firm over total employment in a given year), including year and sector fixed effects. The third and fourth columns, instead, report the results of regressing firm level markups on the firms' revenue shares.*

Overall, in light of this suggestive evidence and consistently with standard models of oligopolistic and monopolistic competition, it is then reasonable to expect that wealthier

American entrepreneurs manage larger firms imposing larger markups.

The following sections of the paper will investigate how, taking into account this market power heterogeneity across entrepreneurs, shapes the macroeconomic outcomes of top wealth taxation.

3 Static model

This Section introduces a simple static model to illustrate how the distortionary and redistributive effects of top wealth taxation vary with the degree and heterogeneity of market power held by American entrepreneurs. The framework allows me to characterize how entrepreneurs' production elasticities to the tax are shaped by their market power and provides a foundation for interpreting the wealth tax effects in the richer dynamic model of Section 5.

3.1 Households

Let's consider an economy populated by a continuum of households indexed by $i \in [0, 1]$. Each of these households is born either as worker or as an entrepreneur and cannot choose its occupation. For simplicity, assume that households $i \in [0, \omega)$ are workers and households $i \in [\omega, 1]$ are entrepreneurs, where the measure of workers, ω, is exogenously given[6].

Workers: are identical and they all inelastically supply a unit of labor. All workers receive the same wage, w, and use their labor income to consume the amount of final good $c_i = w$. The preferences of each worker i over the final consumption good can be represented by a standard CRRA utility function: $u(c_i) = \frac{c_i^{1-\theta}}{1-\theta}$.

Entrepreneurs: each entrepreneur i is endowed with wealth k_i and entrepreneurial ability z_i. In this static model both k_i and z_i are exogenous and for the moment no assumptions are made on the correlation between the two.

Consistently with the evidence of poor diversification of entrepreneurial investment pre-

[6] In Appendix D I present an extension of this static framework in which I allow household to make an occupational choice between "worker" and "entrepreneur".

sented in Section 2, I assume that each entrepreneur owns and operates one firm only. Furthermore, I assume that each entrepreneur invests all his wealth in his own unique entrepreneurial activity. This choice allows to abstract from portfolio composition effects that wealth taxation may induce[7]. Finally, I also assume that each entrepreneur's firm cannot borrow, so the capital employed for production coincides with the wealth of the entrepreneur k_i. Each entrepreneur's income is the profit, π_i, from his own firm. He uses this income for consumption of a final good, c_i, and shares the same preferences as the model's workers.

Entrepreneurs' technology: each entrepreneur i runs a firm operating in monopolistic competition, producing a differentiated intermediate good over which he has monopoly power. These differentiated products are then purchased by final good producers and used as inputs to produce the final good consumed by both entrepreneurs and workers. To produce these differentiated intermediate goods each entrepreneur i employs the following constant return to scale production function:

$$y_i = z_i k_i^{\nu} n_i^{1-\nu}$$

where $0 < \nu < 1$. Notice that y_i indicates the production of entrepreneur's i firm, which is performed using own capital k_i and workers hired from the labor market, denoted as n_i.

Final good production: final good (to be used for consumption) is produced by identical competitive producers employing a bundle of the entrepreneurs' intermediate varieties as inputs. The employed production technology, thus, combines the intermediate goods $\{y_i\}_{i\in[\omega,1]}$ to produce the amount of final good Y.

The final good production technology is chosen to be flexible enough so to obtain demand curves for entrepreneurs' intermediate varieties with both variable and constant price elasticity of demand. This modeling choice allows me to obtain a framework to study the effects of wealth taxation under a wide range of assumptions on entrepreneurs' market

[7] Although relevant, see Gaillard and Wangner (2021) and Cremonini (2023), analyzing the portfolio composition effects of wealth taxation goes beyond the scope of this analysis.

power. To this aim, I assume that the final good production technology is the Kimball (1995) aggregator:

$$\int_{\omega}^{1} \Upsilon_i \left(\frac{y_i}{Y} \right) di = 1 \tag{1.1}$$

where $\Upsilon_i(\cdot)$ is assumed to be a continuous and twice differentiable function, with $\Upsilon_i'(\cdot) > 0$ and $\Upsilon_i''(\cdot) < 0$ for all i.

Notice that if $\Upsilon_i(\cdot) = \Upsilon(\cdot)$ for all i and $\Upsilon(\cdot)$ is a power function, the technology (1.1) takes the well-known *CES* form.

Demand for intermediate goods: Final good producers, taking input prices $\{p_i\}_{i\in[\omega,1]}$ as given, choose how much to produce of the final good Y and the best input combination $\{y_i\}_{i\in[\omega,1]}$ for doing that. Define the minimal cost of producing Y given prices $\{p_i\}_{i\in[\omega,1]}$ as:

$$\begin{aligned} & C(Y, \{p_i\}_{i\in[\omega,1]}) = YC(1, \{p_i\}_{i\in[\omega,1]}) \\ & \text{where} \quad C(1, \{p_i\}_{i\in[\omega,1]}) := \min_{\{q_i\}_{i\in[\omega,1]}} \int_{\omega}^{1} p_i q_i di \quad \text{s.t.} \quad \int_{\omega}^{1} \Upsilon_i(q_i) di = 1 \end{aligned}$$

where $q_i := y_i/Y$ is the relative demand for input i. Normalize to unit the price of the final good. The profit maximization problem of final good producers writes:

$$\max_{Y} \quad Y - YC(1; \{p_i\}_{i\in[\omega,1]})$$

By solving this problem it is possible to obtain the demand function $p_i(\cdot)$ for the intermediate good produced by each entrepreneur $i \in [\omega, 1]$:

$$p_i(q_i, P) = P\Upsilon_i'(q_i) \tag{1.2}$$

where the price aggregator P is defined as:

$$P := \left(\int_{\omega}^{1} \Upsilon_i'(q_i)\, q_i di \right)^{-1}$$

First, notice that the assumptions $\Upsilon_i'(\cdot) > 0$ and $\Upsilon_i''(\cdot) < 0$ ensure that the demand schedule for each intermediate good i is positive and downward sloped. In particular, the price to be paid for intermediate good produced by entrepreneur i negatively depends on the *relative production* of that good $q_i := y_i/Y$.

Furthermore, notice that the subscript i in $p_i(\cdot)$ highlights that if the function $\Upsilon_i(\cdot)$ is assumed to be heterogeneous across entrepreneurs, then different entrepreneurs will face different demand functions for their own varieties. The price elasticity of demand for the intermediate good produced by entrepreneur i takes the form:

$$\mathcal{E}_i^d(q_i) := \left| \frac{\partial \ln(q_i)}{\partial \ln(p_i)} \right| = -\frac{\Upsilon_i'(q_i)}{q_i \Upsilon_i''(q_i)} \tag{1.3}$$

Notice that in this simple model of monopolistic competition the market power of each entrepreneur is determined by the elasticity of demand he faces for his own intermediate good.

Throughout this paper, I assume entrepreneurs have either market power that increases with their firm's market share, as in standard models of oligopolistic competition, or constant market power, regardless of their production size. This is equivalent to require the following to hold:

Assumption 1. *Assume that the function $\Upsilon_i(q)$ satisfies:*

$$\frac{\partial}{\partial q}\left[-\frac{\Upsilon_i'(q)}{q\Upsilon_i''(q)} \right] \leq 0 \qquad \forall\, q > 0 \qquad \forall\, i \in [\omega, 1]$$

In other words, when the elasticity of demand for an entrepreneur's product decreases as his relative production increases, his firm gains more market power, allowing him to charge higher markups. Instead, when the elasticity of demand function is a constant, the firm's market power and markups are fixed, regardless of the entrepreneur's production scale.

Entrepreneur's problem: each entrepreneur $i \in [\omega, 1]$ maximizes his own utility defined over final good consumption. In order to consume he employs profits received from his own firm, π_i, after hiring n_i workers from the labor market to produce. Formally, each entrepreneur $i \in [\omega, 1]$ solves:

$$\begin{aligned} \max_{c_i, p_i, y_i, n_i} \quad & \frac{c_i^{1-\theta}}{1-\theta} \\ \text{s.t.} \quad & c_i = \pi_i \\ & \pi_i = p_i y_i - w n_i \\ & p_i = P \Upsilon_i' \left(\frac{y_i}{Y} \right) \\ & y_i = z_i k_i^{\nu} n_i^{1-\nu} \\ & z_i, \ k_i \text{ given} \end{aligned} \tag{E}$$

3.2 Optimal entrepreneurs' production choices

Taking the first order conditions of each entrepreneur's $i \in [\omega, 1]$ problem (E) and combining them it is possible to obtain the following equation which characterizes the production choices of each entrepreneur:

$$\underbrace{P \Upsilon_i' (q_i^*)}_{p_i^*} = \underbrace{\frac{\mathcal{E}_i^d(q_i^*)}{\mathcal{E}_i^d(q_i^*) - 1}}_{\text{markup}} \cdot \underbrace{\frac{w Y^{\frac{\nu}{1-\nu}}}{(1-\nu)} \left(\frac{q_i^{*\nu}}{z_i k_i^{\nu}} \right)^{\frac{1}{1-\nu}}}_{\text{marginal cost}} \tag{1.4}$$

Each entrepreneur i sets a price for his own variety p_i^* larger than its marginal cost of production, where the wedge between the two is the markup $\mu_i(q_i^*) = \frac{\mathcal{E}_i^d(q_i^*)}{\mathcal{E}_i^d(q_i^*)-1}$. First of all, notice that the markup chosen by each entrepreneur can be written as a function $\mu_i(q_i)$ of the relative quantity produced q_i. Assumption 1 guarantees that the markup function $\mu_i(q_i)$ is non-decreasing in relative production q_i. In particular, if the elasticity of demand is strictly decreasing in relative production, firms producing at a larger scale face a more rigid demand and choose higher markups. On the other hand, if the elasticity of demand is constant, the markup function is a constant as well and markups imposed by firms do not depend on their production scale.

Equation (1.4) also shows that the optimal relative quantity q_i^* chosen by each entrepreneur depends on his wealth k_i, his skills z_i, as well as on the aggregates w, Y, P.

Let's define the optimal relative quantity function $\mathcal{Q}_i^*(z_i, k_i, w, P, Y)$ which associates to each wealth level k_i, skills z_i and aggregates w, Y, P the optimal relative quantity q_i^* which solves equation (1.4). The properties of this function are summarized by the following Lemma:

Lemma 1. *Assume Assumption 1 holds and let $\mathcal{Q}_i^*(z_i, k_i, w, P, Y)$ be the function which associates to each vector (z_i, k_i, w, P, Y) the optimal relative quantity chosen by entrepreneur i, q_i^*, which solves (1.4). It holds:*

$$\frac{\partial \mathcal{Q}_i^*(\cdot)}{\partial z_i} > 0 \qquad \frac{\partial \mathcal{Q}_i^*(\cdot)}{\partial k_i} > 0 \qquad \frac{\partial \mathcal{Q}_i^*(\cdot)}{\partial w} < 0 \qquad \frac{\partial \mathcal{Q}_i^*(\cdot)}{\partial P} < 0 \qquad \frac{\partial \mathcal{Q}_i^*(\cdot)}{\partial Y} < 0$$

Proof: see Appendix A.

Lemma 1 shows that the higher the entrepreneurial ability z_i or the wealth of the entrepreneur k_i, the larger will be the optimal relative quantity chosen to be produced by the entrepreneur. This holds irrespectively of whether the markup imposed depends on the entrepreneur's production scale. The reason is that wealthier and more skilled entrepreneurs own firms that have lower marginal costs of production, allowing them to produce at a larger scale.
Now, denote with $\mathcal{N}_i^*(z_i, k_i, w, P, Y)$ the function which associates to each vector (z_i, k_i, w, P, Y) the labor demand of entrepreneur i to produce $\mathcal{Q}_i^*(z_i, k_i, w, P, Y)$:

$$\mathcal{N}_i^*(z_i, k_i, w, P, Y) = \left(\frac{\mathcal{Q}_i^*(z_i, k_i, w, P, Y) \cdot Y}{z_i k_i^{\nu}} \right)^{\frac{1}{1-\nu}}$$

Differently from what happens for optimal relative quantity, Assumption 1 is not enough to guarantee a monotonic (increasing) relationship between optimal labor demand and entrepreneur's wealth k_i and skills z_i. The reason is the following. Whenever the entrepreneur gets wealthier or more productive he wants to produce more (Lemma 1) and to do that he could either hire more labor or just exploit his increase in productivity while employing less labor. It is possible to derive a sufficient condition on the function $\Upsilon_i(\cdot)$ which guarantees that optimal labor demand of each entrepreneur i is monotone increasing in his wealth k_i and skills z_i:

Assumption 2. *The function* $\Upsilon_i(q)$ *satisfies:*

$$\frac{(2\Upsilon_i''(q) + q\Upsilon_i'''(q))q}{\Upsilon_i'(q) + q\Upsilon_i''(q)} > -1 \quad \forall q > 0,\ \forall i$$

Assumption 2 requires that the elasticity of marginal revenues with respect to relative quantity produced (left-hand side) is greater than -1. In other terms, it ensures that an entrepreneur's marginal revenues do not decrease too steeply with production. If this assumption were violated, an entrepreneur experiencing a positive productivity shock would have to drastically lower prices to sell the additional output. Consequently, it would become optimal for the entrepreneur to limit the production increase due to the productivity shock by reducing his labor demand. Instead, under Assumption 2 an increase in productivity (or capital endowment) is always complemented by an increase in labor employed for production.

Lemma 2 summarizes the properties of the labor demand function $\mathcal{N}_i^*(\cdot)$ when Assumptions 1 and 2 hold:

Lemma 2. *Let Assumption 1 and 2 hold and let* $\mathcal{N}_i^*(z_i, k_i, w, P, Y)$ *be the function which associates to each vector* (z_i, k_i, w, P, Y) *the labor demand which allows entrepreneur* i *to produce* $\mathcal{Q}_i^*(z_i, k_i, w, P, Y)$ *(i.e. the labor demand which solves entrepreneur's* i *problem (E)). It holds:*

$$\frac{\partial \mathcal{N}_i^*(\cdot)}{\partial z_i} > 0 \qquad \frac{\partial \mathcal{N}_i^*(\cdot)}{\partial k_i} > 0 \qquad \frac{\partial \mathcal{N}_i^*(\cdot)}{\partial w} < 0 \qquad \frac{\partial \mathcal{N}_i^*(\cdot)}{\partial P} < 0 \qquad \frac{\partial \mathcal{N}_i^*(\cdot)}{\partial Y} > 0$$

Proof: see Appendix A

Noticeably, it is possible to show that if the price elasticity of demand that the entrepreneur faces is a constant function then Assumption 2 is met for every value of the elasticity of demand greater than one[8]. Hence, when this is the case, the labor demand of the entrepreneur is monotonic increasing in his skills and wealth. When the elasticity of demand, instead, varies with the relative production of the entrepreneur, whether As-

[8]Notice that if $\mathcal{E}_i^d(q) \leq 1$ then equation (1.4) admits no solution

sumption 2 is satisfied depends on the functional form chosen for $\Upsilon_i(\cdot)$[9].

The profits of each entrepreneur i when making his optimal production choices are:

$$\Pi_i^*(z_i, k_i, w, P, Y) = \left(\frac{\mu_i(\mathcal{Q}_i^*(z_i, k_i, w, P, Y))}{1-\nu} - 1 \right) w\mathcal{N}_i^*(z_i, k_i, w, P, Y) \tag{1.5}$$

where $\mu_i(q) = \mathcal{E}_i^d(q)/(\mathcal{E}_i^d(q)-1)$ is the markup function. The profits that each entrepreneur makes are the product of two terms: the one in parenthesis indicates the marginal profit per dollar of input purchased from the market. The second term $w\mathcal{N}_i^*(z_i, k_i, w, P, Y)$, instead, indicates the total value of inputs purchased from the market by the entrepreneur. As showed in Lemma 3 (Appendix A) under Assumptions 1 and 2 profits are increasing in entrepreneurs' skills z_i and wealth k_i.

3.3 Production elasticities

How do entrepreneurs' production elasticities depend on the assumptions on entrepreneurs' market power? The answer to this question is of crucial importance to study how wealth taxation (or any other policy which redistributes capital across entrepreneurs) affects entrepreneurial choices and as a consequence economic aggregates. For notational convenience denote $q_i^* = \mathcal{Q}_i^*(z_i, k_i, Y, w, P)$ and let $\epsilon^{q_i^*, k_i} = \frac{\partial q_i^*}{\partial k_i} \frac{k_i}{q_i^*}$ indicate the elasticity of entrepreneur's i optimal relative production with respect to capital. Finally, denote as $\epsilon^{\mathcal{E}_i^d} = \left| \frac{\partial \mathcal{E}_i^d}{\partial q_i} \frac{q_i^*}{\mathcal{E}_i^q} \right|$ the super-elasticity of demand, i.e. the rate at which the elasticity of demand varies with relative production. The following Proposition summarizes the properties of the elasticity of relative production with respect to capital:

Proposition 1. *Let Assumption 1 hold. For all entrepreneurs $i \in [\omega, 1]$ the relative production elasticity with respect capital is:*

$$\epsilon^{q_i^*, k_i} = \frac{\nu}{1-\nu} \left(\frac{\nu}{1-\nu} + \frac{1}{\mathcal{E}_i^d(q_i^*)} + \frac{\epsilon^{\mathcal{E}_i^d}(q_i^*)}{(\mathcal{E}_i^d(q_i^*) - 1)} \right)^{-1} \tag{1.6}$$

[9] Assumption 2 will be satisfied by the functional forms for $\Upsilon_i(\cdot)$ that I will employ in the following Sections of the paper. Indeed, if Assumption 2 did not hold, it would not be possible to replicate the strictly increasing relationship between entrepreneurs' wealth and their firm size (in terms of employees) observed in the SCF data, see Section 2.

Hence, the relative production elasticity with respect to capital positively depends on the demand elasticity $\mathcal{E}_i^d(q_i^*)$ and negatively on the super-elasticity of demand $\epsilon^{\mathcal{E}_i^d}(q_i^*)$.

Proof: see Appendix A

To get intuition on this result consider the following two cases:

- First, suppose entrepreneur i faces a *constant elasticity of demand function* for his own variety: $\mathcal{E}_i^d(q) = \sigma_i \quad \forall q$. When this is the case:

$$\epsilon^{q_i^*,k_i} = \frac{\nu}{\nu + (1-\nu)(\sigma_i)^{-1}}$$

  Notice, the smaller the value of σ_i the higher the rate of decline of the entrepreneur's marginal revenues. Consequently, the lower σ_i the smaller the increase in entrepreneur's production in response to a positive capital shock because of the rapid fall of his marginal revenues. In other words, when the entrepreneurs' marginal revenues decline very steeply, he finds optimal to "pass through" his decrease in marginal costs (due to the increase in capital) to prices, rather than on higher quantities produced.

  Since $\sigma_i > 1$[10], it holds $\nu < \epsilon^{q_i^*,k_i} < 1$, and the production elasticity takes its maximum value when the entrepreneur has no market power ($\sigma_i \to \infty$) and the minimum when the entrepreneur has the largest degree of market power ($\sigma_i \to 1$).

- Now suppose entrepreneur i faces a demand curve with *variable elasticity of demand* for his own variety and Assumption 1 holds: $\frac{\partial \mathcal{E}_i^d}{\partial q}(q) < 0 \quad \forall q$. The demand elasticity he faces at q_i^* is $\mathcal{E}_i^d(q_i^*)$. Equation (1.6) shows that his production elasticity is lower than the production elasticity of an entrepreneur facing a *constant* elasticity of demand curve with demand elasticity at the same level $\mathcal{E}_i^d(q_i^*)$. Intuitively, when the elasticity of demand declines with quantity, marginal revenue falls more steeply as output expands. Consequently, the entrepreneur's incentive to raise production in response to a positive capital shock is weaker, leading to a lower elasticity of production.

[10] If $\sigma_1 < 1$ the problem of the entrepreneur would not admit a solution.

This result suggests that the assumptions on entrepreneurs' market power critically shape how entrepreneurs change their production in response to a wealth tax that reduces their capital availability. The simulations of this simple model in Section 4 will illustrate the implications for aggregate production and general equilibrium effects induced by the tax.

3.4 Equilibrium

The equilibrium of this static economy consists of the tuple (w^*, Y^*, P^*), a vector of quantities consumed by each household (workers and entrepreneurs) $(c_i^*)_{i\in[0,1]}$, relative quantity functions $(\mathcal{Q}_i^*(z_i, k_i, w, P, Y))_{i\in[\omega,1]}$, labor demand functions $(\mathcal{N}_i^*(z_i, k_i, w, P, Y))_{i\in[\omega,1]}$, profit functions $(\Pi_i^*(z_i, k_i, w, P, Y))_{i\in[\omega,1]}$ such that:

- Each worker $i \in [0, \omega]$ consumes his labor income $c_i^* = w^*$
- Given (w^*, Y^*, P^*) the functions $\mathcal{Q}_i^*(z_i, k_i, w^*, P^*, Y^*)$, $\mathcal{N}_i^*(z_i, k_i, w^*, P^*, Y^*)$, $\Pi_i^*(z_i, k_i, w^*, P^*, Y^*)$ solve each entrepreneur's $i \in [\omega, 1]$ problem (E)
- Each entrepreneur $i \in [\omega, 1]$ consumes his own profits: $c_i^* = \Pi_i^*(z_i, k_i, w^*, P^*, Y^*)$
- Labor market clears:
$$\omega = \int_\omega^1 \mathcal{N}_i^*(z_i, k_i, w^*, P^*, Y^*)di$$
- Kimball aggregator holds:
$$\int_\omega^1 \Upsilon_i\left(\mathcal{Q}_i^*(z_i, k_i, w^*, P^*, Y^*)\right)di = 1$$
- Aggregate price index P^* is:
$$P^* = \left(\int_\omega^1 \Upsilon_i'\left(\mathcal{Q}_i^*(z_i, k_i, w^*, P^*, Y^*)\right)\mathcal{Q}_i^*(z_i, k_i, w^*, P^*, Y^*)di\right)^{-1}$$

3.5 Aggregation and distortions from markups

I now identify the production distortions induced by markups in this economy, so to study, in the next section, how wealth taxation affects them. The aggregate production Y can be written as:

$$Y = ZK^{\nu}N^{1-\nu}$$

where $K := \int_{\omega}^{1} k_i di$, $N := \int_{\omega}^{1} \mathcal{N}_i^*(z_i, k_i, w, P, Y)di$ and aggregate productivity is defined as:

$$Z := \left(\int_{\omega}^{1} \frac{\mathcal{Q}_i^*(z_i, k_i, w, P, Y)}{z_i} di \right)^{-1} \tag{1.7}$$

In particular, notice that it is possible to interpret aggregate productivity Z as the *harmonic weighted average* of the entrepreneurial productivities z_i, where the individual weight is given by the relative quantity produced by each entrepreneur i.

In this setting aggregate production Y is distorted by markups imposed by entrepreneurs through two channels: the *level* of markups (captured by the aggregate markup in the economy) and markups *dispersion* across entrepreneurs. First, integrating equation (1.4) across all entrepreneurs I get:

$$\frac{wN}{Y} = \frac{1-\nu}{\mathcal{M}} \tag{1.8}$$

where the *aggregate markup* $\mathcal{M}$ is defined as:

$$\mathcal{M} = \int_{\omega}^{1} \mu_i(\mathcal{Q}_i^*(z_i, k_i, w, P, Y)) \frac{\mathcal{N}_i^*(z_i, k_i, w, P, Y)}{N} di$$

that is the aggregate markup is an input-weighted arithmetic average of firm-level markups. Notice that a higher aggregate markup increases the capital share of income accruing to entrepreneurs and reduces the labor share going to workers.

Second, consider the problem of a planner that takes as given the skill and wealth distribution of entrepreneurs and has to decide how to allocate the labor supply L across firms owned by entrepreneurs:

$$\begin{aligned} \max_{\{y_i, n_i\}_{i\in I}, Y} \quad & Y \\ \text{s.t.} \quad & \int_{\omega}^{1} \Upsilon\left(\frac{y_i}{Y}\right) di = 1 \\ & \int_{\omega}^{1} n_i di = L \\ & y_i = z_i k_i^{\nu} n_i^{1-\nu} \qquad \text{for all } i \in I \\ & z_i,\ k_i \qquad \text{given, for all } i \in I \end{aligned}$$

and denote with λ the Lagrange multiplier associated with the labor supply constraint. The first order conditions of the problem imply:

$$P\Upsilon_i'(q_i^*) = \frac{\lambda Y^{\frac{\nu}{1-\nu}}}{(1-\nu)} \left(\frac{q_i^{*\nu}}{z_i k_i^{\nu}}\right)^{\frac{1}{1-\nu}}$$

The planner makes firms produce up to the point in which the marginal value of the production of firm i (left hand side) equalizes its marginal cost of production (right hand side). On the other hand, in the decentralized equilibrium, entrepreneur i produces up to the point in which the marginal value of production (price) equalizes its marginal cost of production times a markup. If markups are increasing in firm size (i.e. when $\partial \mathcal{E}_i^d(q)/\partial q < 0$) the most productive entrepreneurs produce on a larger scale and impose the largest markups. Hence, firms imposing above the average markups (the most productive ones) underproduce with respect to the social optimum. Instead firms imposing below the average markups (the least productive ones) overproduce with respect to the optimum. This misallocation of labor force due to markups dispersion induces a reduction in aggregate productivity Z and hence production Y.

4 Wealth taxation and market power heterogeneity

The static model outlined in Section 3, despite its simplicity, does not have a closed-form solution. Consequently, numerical methods are necessary to solve the model and obtain the effects of top wealth taxation on entrepreneurial decisions and macroeconomic aggregates.

First, I study the wealth tax effects under the assumption that entrepreneurs' market power, and hence markups, increase with their firm market share. In this case, the model parameters and functional forms are chosen so to obtain an empirically plausible equilibrium markup distribution across entrepreneurs. In this setting I investigate how wealth taxation affects the production distortions induced by markups.

These wealth tax effects are then compared to those that would arise in an observationally equivalent economy, in which the markups imposed by entrepreneurs are *constant*, although heterogeneous. Finally, to highlight the role of market power heterogeneity in shaping the wealth tax effects the same wealth tax is implemented in an another observationally equivalent economy in which market power heterogeneity across entrepreneurs has been shut down. To do this, I assume that all entrepreneurs now impose the same constant markup, equal to the average one in the economies with markups heterogeneity.

4.1 Taxing wealth with market power increasing in firms' market shares

Parametrization: the fraction of workers in this economy is $\omega = 0.88$. This value is obtained using the 2019 SCF data, classifying the households not satisfying the definition of an entrepreneur (see Section 2) as workers. Each entrepreneur $i \in [\omega, 1]$ draws his entrepreneurial skills z_i from a Pareto distributed random variable $Pa(x_z, \eta_z)$. The parameters x_z and η_z represent, respectively, the scale and shape parameters and are calibrated to match the observed distribution of returns to entrepreneurship[11] (see Section 2). In a dynamic setting in which entrepreneurs accumulate their own wealth (see Section 5) the correlation between entrepreneurial skills and their wealth arises endogenously. In this static setting it has to be assumed[12]. In particular, I assume that the wealth of each

[11] x_z and η_z are calibrated so to minimize the sum of squared errors between simulated and empirical returns to entrepreneurship across the following wealth groups: $\{50-70p., 70-85p., 85-95p., 95-98p., 98-99p., top\ 1\}$

[12] In this setting there is no distinction between overall wealth of the entrepreneur and wealth held as capital in the business (i.e. "entrepreneurial wealth"). Here the calibration target is the distribution of the latter. The reason of this choice is that the focus of this Section is on how wealth taxation affects entrepreneurial production decisions, which are directly influenced by the availability of capital, not by

entrepreneur is a monotone increasing function of his skills, that is $k_i = k(z_i) = \alpha_0 z_i^{\alpha_1}$ with $\alpha_0, \alpha_1 > 0$. Two remarks are due. First, the positive relationship between skills and wealth is essential to replicate the empirically observed increasing returns to entrepreneurship across the wealth distribution. Second, the function $k(\cdot)$ is specifically chosen to transform the entrepreneurial skill distribution from a Pareto into another Pareto distribution. This is required for replicating the observed entrepreneurial wealth distribution, as its fat upper tail is well-fit by a Pareto distribution. (Benhabib and Bisin, 2018; Vermeulen, 2018).

To model market power (and hence markups) to be increasing in firms' market share I assume the function $\Upsilon_i(\cdot)$ takes the Klenow and Willis (2016) functional form for all i:

$$\Upsilon_i(q;\sigma,\psi) = \Upsilon(q;\sigma,\psi) = 1 + (\sigma-1)e^{1/\psi}\psi^{\frac{\sigma}{\psi}-1}\left[\Gamma\left(\frac{\sigma}{\psi},\frac{1}{\psi}\right) - \Gamma\left(\frac{\sigma}{\psi},\frac{(q)^{\frac{\psi}{\sigma}}}{\psi}\right)\right] \tag{1.9}$$

with $\sigma > 1$ and $\psi \geq 0$, and where $\Gamma(s,x)$ denotes the function:

$$\Gamma(s,x) := \int_x^\infty t^{s-1}e^{-t}dt$$

The reason for this choice is twofold. First, as Edmond et al. (2023) show, once appropriately calibrated it allows to replicate very-well the empirically observed relationship between markups and market shares across US firms. Furthermore, notwithstanding its complicated functional form, it generates easily interpretable elasticity of demand and markups functions (for derivation see Appendix B):

$$\mathcal{E}_i^d(q_i) = \sigma(q_i)^{-\frac{\psi}{\sigma}} \qquad \mu_i(q_i) = \frac{\mathcal{E}_i^d(q_i)}{\mathcal{E}_i^d(q_i)-1} = \frac{\sigma}{\sigma - q_i^{\frac{\psi}{\sigma}}} \tag{1.10}$$

Notice that σ captures the level of the elasticity of demand when $q_i = 1$. Instead, the parameter ψ identifies the sensitivity of the elasticity of demand to changes in q_i (super-elasticity of demand). In this setting it is possible to show that the ratio of parameters $\frac{\psi}{\sigma}$ corresponds to the coefficient in a regression of (a monotone increasing transformation of) firms' markups on firms' market shares[13]. Exploiting this relationship, Edmond et al.

total wealth.

[13] A proof of this statement is provided in Appendix B of Edmond et al. (2023).

Table 3. *Market power increasing in firms' market shares: parametrization summary*

Par.	Description	Value	Target
ω	fraction of workers	0.88	non-entrepreneur household in SCF
ν	capital exponent prod.	0.28	labor share = 0.6
x_z	scale par. entr. ability dist.	0.12	observed returns to entrepreneurship
η_z	shape par. entr. ability dist.	5.0	observed returns to entrepreneurship
σ	demand elasticity when $q = 1$	11.75	$\mathcal{M} = 1.2$
ψ	shape par. demand elasticity	1.90	$\psi/\sigma = 0.16$
α_0	$k_i = k(z_i) = \alpha_0 z_i^{\alpha_1}$	383	min. wealth = 1
α_1	$k_i = k(z_i) = \alpha_0 z_i^{\alpha_1}$	3.97	tail par. ent. wealth 1.25

Notes: *the Table summarizes the chosen parameters values. The first column indicates the symbol used to identify the parameter in the model, the second column the parameter description, the third column the chosen value for the parameter, the fourth column the moment targeted to calibrate each parameter.*

(2023) use 1972-2012 US Census of Manufacturers data to estimate $\frac{\psi}{\sigma}$ across 3-digits NAICS sectors. I choose to target $\frac{\psi}{\sigma} = 0.162$, which is the mid-point of the Edmond et al. (2023) parameter estimates range. The parameter σ is then chosen so to match $\mathcal{M} = 1.2$, a figure consistent with the estimates of Edmond et al. (2023), ranging between $1.05 < \mathcal{M} < 1.35$ for the US economy.

Table 3 summarizes the parameter choices described above and Figure C1.2 (Appendix C) shows that the calibrated model closely replicates the observed returns to entrepreneurial investment.

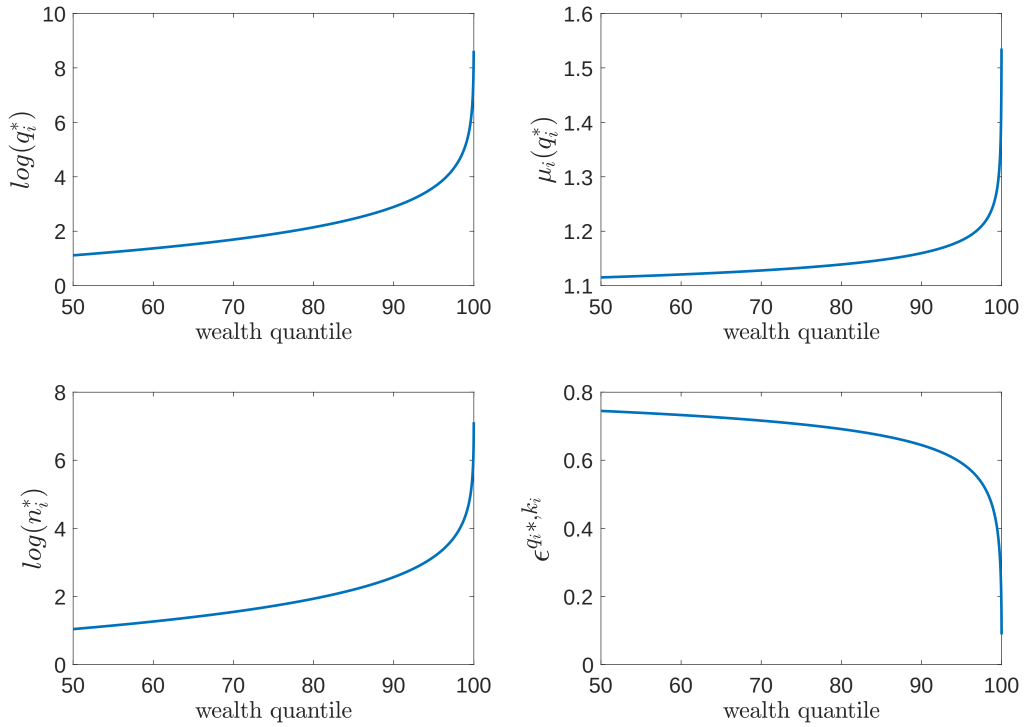
Table 4. *Distribution of markups (cost-weighted)*

	Compustat - Edmond et al. (2023)	Simulated model
aggregate markup $\mathcal{M}$	1.26	1.20
p25	0.97	1.10
p50	1.12	1.16
p75	1.31	1.27
p90	1.69	1.39

Notes: *the Table reports some descriptive statistics of the markups distribution estimated in the data by Edmond et al. (2023) (first column) and simulated by the model (second column). The statistics have been obtained computing the cost-weighted percentiles of the markup distribution, where the weight associated to each observation is given by the share of labor employed by each firm n_i/N.*

Simulation results: Figure 6 shows the simulated relative quantities, markups, labor demand and production chosen by entrepreneurs across the wealth distribution. Notice that, since entrepreneurial ability z_i is assumed to be positively correlated with entrepreneurial wealth k_i, relative quantities produced and hence markups are strictly increasing across

Figure 6. *Entrepreneurs' choices and production elasticities across the wealth distribution*



Notes: *the Figure reports the simulated relative quantities q_i^*, markups $\mu(q_i^*)$, labor demand n_i^*, and production elasticity with respect to capital $\epsilon^{q_i^*,k_i}$ for entrepreneurs at different quantiles of the wealth distribution when the static model presented in Section 3, calibrated as described in Table 3, is simulated.*

the wealth distribution (Lemma 1). Table 4 compares the simulated markup distribution with the empirical one estimated by Edmond et al. (2023) using 2012 Compustat data. The two distributions are quite similar, except for the fact that the simulated one is less right-skewed. The reason behind this discrepancy is that the Compustat dataset is limited to publicly traded firms, including most of the largest American ones. In contrast, the modeled economy has been calibrated using the Census of Manufacturers data. This includes both listed and unlisted firms - and hence it is more representative of the entire American firm population - although it captures less firms at the very top of the size (and hence markups) distribution.

The bottom-right panel of the Figure shows the production elasticities of entrepreneurs with respect to capital. Notably, entrepreneurs at the top of the wealth distribution feature lower production elasticities than poorer entrepreneurs. This is because wealthier entrepreneurs produce at a larger scale and hence face demand curves with lower price elasticities[14] (see discussion of equation (1.6)).

Top wealth tax policy: only three OECD countries currently levy a tax on a comprehensive measure of wealth, that is Norway, Switzerland and Spain. The wealth taxes implemented in these countries share the common feature of being imposed on the wealth in excess of a given threshold. Consistently with this evidence I study a proportional wealth tax, with tax rate $\tau > 0$, on the wealth in excess of an exogenously given threshold $\underline{k} > 0$. The tax revenues collected are uniformly redistributed to all households (workers and entrepreneurs) through a lump-sum transfer T. Each worker $i \in [0, \omega]$, once the tax policy is implemented, consumes $c_i = w + T$. The problem (E) of each entrepreneur $i \in [\omega, 1]$, now becomes:

$$\begin{aligned} \max_{c_i, p_i, y_i, n_i} \quad & \frac{c_i^{1-\theta}}{1-\theta} \\ \text{s.t.} \quad & c_i = \pi_i + T \\ & \pi_i = p_i y_i - w n_i \\ & p_i = P \Upsilon_i' \left(\frac{y_i}{Y} \right) \\ & y_i = z_i \left(k_i - \tau \max\{k_i - \underline{k}, 0\} \right)^{\nu} n_i^{1-\nu} \\ & z_i, \ k_i \text{ given} \end{aligned}$$

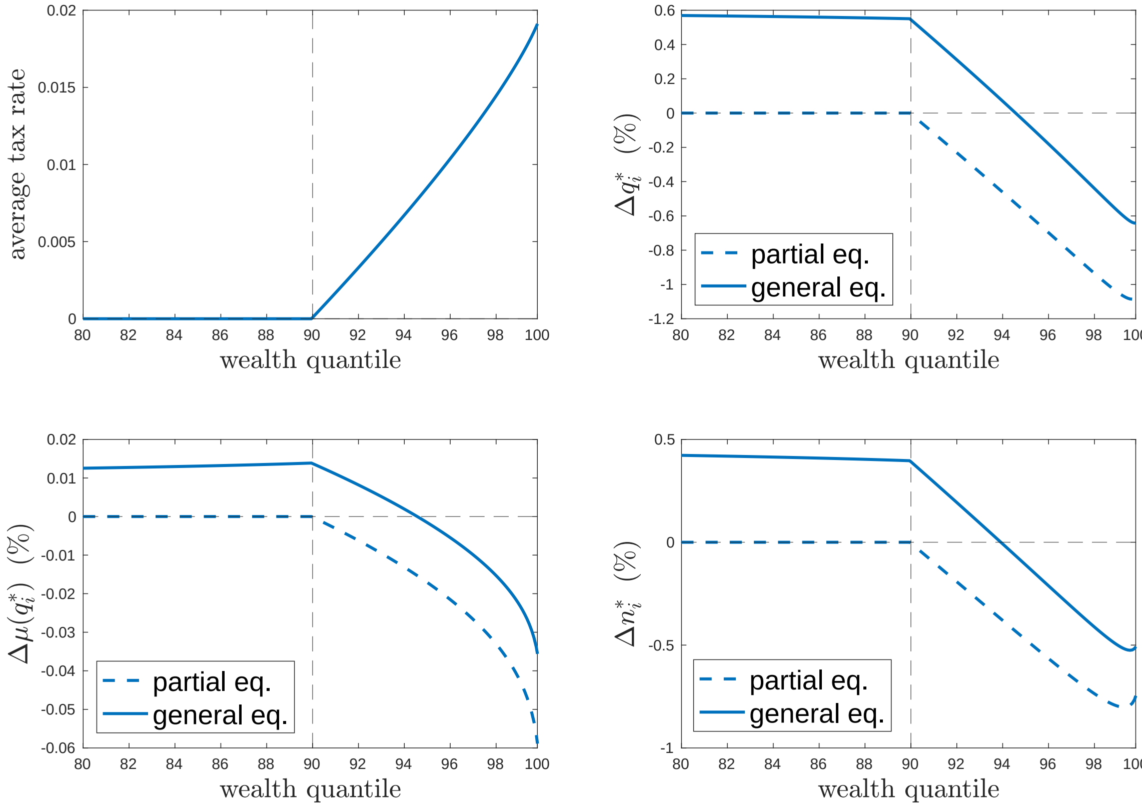
and the lump sum transfer T satisfies:

$$T = \int_{\omega}^{1} \tau \max\{k_i - \underline{k}, 0\} di$$

Just for illustrative purposes I study an extensively discussed policy (Saez and Zucman, 2019), i.e. a wealth tax which falls onto the wealthiest 1% of American households. In my static economy this corresponds to taxing approximately the wealthiest 10% of entrepreneurs. Furthermore, I assume $\tau = 2\%$ so that wealth tax revenues amount approximately to 1% of GDP, a reasonable figure for a top wealth tax absent tax evasion and tax elusion effects (Saez and Zucman, 2022).

[14]Notice that the super-elasticity of demand under this model parametrization is $\epsilon^{\mathcal{E}_i^d}(q) = \frac{\psi}{\sigma}$ for all q and for all entrepreneurs $i \in [\omega, 1]$

FIGURE 7. *Wealth tax simulation: effect on entrepreneurs' choices*



Notes: *the Figure represents the effects of the wealth tax described in Section 4.3 with* $\tau = 0.02$ *on the model calibrated as described in Section 4.2. The first panel indicates the average tax rate (total taxes paid/ total wealth). The other panels represent the differences between entrepreneurs' choices (relative quantities, markups, labor demand) when the tax policy is implemented and the same quantities when the tax policy is not in place. Dotted lines indicate the partial equilibrium effects of the wealth tax (i.e. keeping fixed* w, P, Y*). Solid lines indicate the effect of the wealth tax on entrepreneurs' choices taking into account general equilibrium effects.*

Effects on entrepreneurial choices: Figure 7 illustrates the effects of the aforementioned wealth tax policy on entrepreneurial decisions across the wealth distribution. The first panel of the Figure shows the average tax rate, which is positive and increasing for entrepreneurs beyond the 90^{th} percentile of the wealth distribution.

First of all, consider the partial equilibrium effects of the wealth tax (dotted blue lines). Untaxed entrepreneurs, in partial equilibrium, do not change their production choices, while taxed entrepreneur, experiencing a decrease in their wealth endowment decrease their relative production, the markup they impose and also their labor demand (Lemma 1-2). The wealth tax hence, by taking away resources from productive entrepreneurs and redistributing them as lump-sum transfers used for consumption reduces aggregate production, aggregate labor demand and hence equilibrium wage.

In general equilibrium (solid blue lines) untaxed entrepreneurs exploit the wage and production decrease to expand their relative production and markups they impose. Also the entrepreneurs between the 90^{th} and 95^{th} wealth percentile, although being taxed, experience an increase in their relative production, markups and labor demand due to the general equilibrium effects. Finally, entrepreneurs beyond the 95^{th} wealth percentile still decrease their relative quantity, markups and labor demand, although in a lower extent with respect to the partial equilibrium effects.

The implemented wealth tax unambiguously diminishes aggregate production by both reducing the aggregate stock of capital and lowering overall productivity, as production is reallocated from highly productive to less productive entrepreneurs.

Furthermore, the tax policy decreases the economy's aggregate markup by reducing the markups of the wealthiest entrepreneurs and (in a lower extent) increasing those of the poorest. This effect not only mitigates production distortions arising from the average markup level but also redistributes resources from wealthy entrepreneurs to poor workers by increasing the labor share of income (see equation (1.8)). Now consider the effect of the wealth tax on the misallocation of production factors. Let's denote with $e_i = z_i k_i^{\nu}$ the "effective" productivity of entrepreneur i, which captures the entrepreneur's productivity from fixed factors, i.e. capital and skills. The wealth tax compresses the distribution of effective productivity, by reducing the capital available for the most productive entrepreneurs. This mechanically leads to a decrease in markup dispersion and, consequently, misallocation. However, this reduced misallocation merely reflects a change in the effective productivity distribution rather than an improvement in labor allocation for a *given* effective productivity distribution.

Overall, the distortionary effect of taxing the most productive entrepreneurs' wealth outweighs the benefits from lower markups-induced inefficiencies. This is why the tax policy ultimately reduces aggregate production and wages ($\Delta Y = -0.25\%$ and $\Delta w = -0.21\%$[15]).

[15]These figures will be useful to compare the strength of general equilibrium effects under different market power assumptions in the next section.

4.2 Taxing wealth with constant market power

I now study how the distortionary and redistributive effects of the wealth tax change when entrepreneurs are assumed to impose the same heterogeneous, yet constant, markups. The analysis is conducted within an economy matching the same moments of the one studied in the previous section, but now assuming the entrepreneur's market power is independent of his production scale. This experiment serves to isolate the wealth tax effects arising from *variable* market power from those that simply stem from it being *heterogeneous* across entrepreneurs. Finally, to highlight the role of market power heterogeneity in shaping the wealth tax effects the same wealth tax is implemented in an another economy, differing from the previous ones for having entrepreneurs imposing the same constant markup (equal to the average one in the economies with markups heterogeneity).

Constant markups model parametrization: to have entrepreneurs imposing constant markups (potentially heterogeneous) I assume that each entrepreneur faces a demand function for his own variety featuring constant elasticity of demand. To this aim I assume that for each entrepreneur i:

$$\Upsilon_i(q) = q^{\frac{\sigma_i - 1}{\sigma_i}}$$

with $\sigma_i > 1$ for all i. Under this assumption the demand curve faced by entrepreneur i is:

$$p_i(q) = q^{-\frac{1}{\sigma_i}}$$

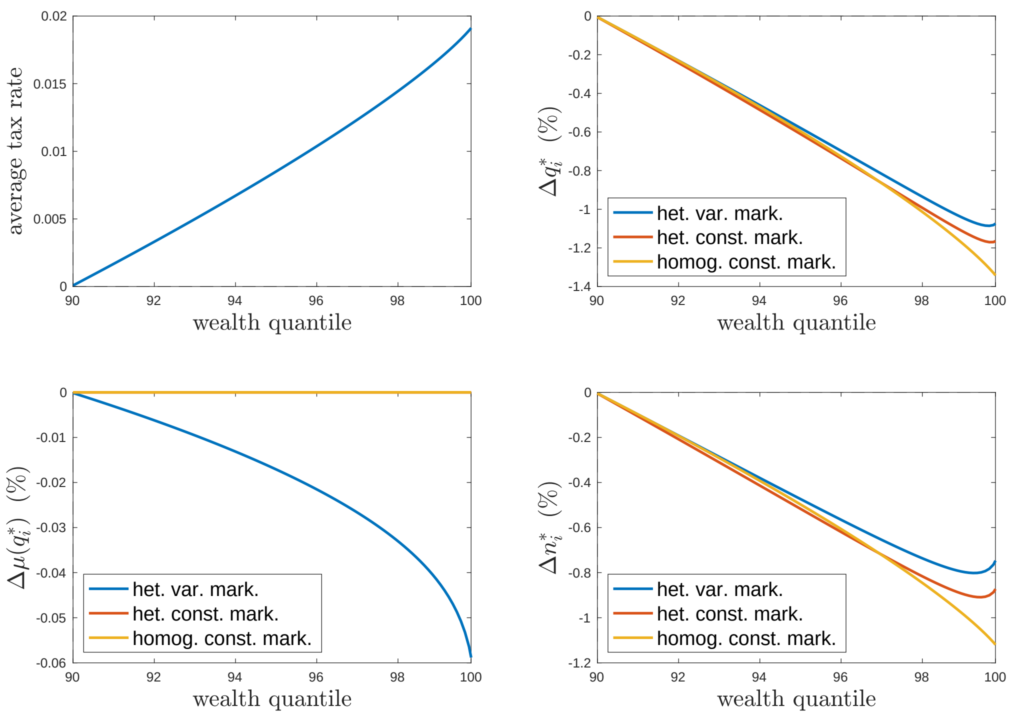
Now each entrepreneur i produces up to the point in which the price of his good equalizes his marginal cost of production times a markup $\frac{\sigma_i}{\sigma_i - 1}$ which is independent of the production size.

First, let's parametrize the economy with constant, although heterogeneous, markups across entrepreneurs. I assume the elasticity of demand σ_i of entrepreneur i to be a monotone decreasing polynomial function of the entrepreneur's skills: $\sigma_i = \sigma(z_i)$. The functional form for $\sigma(\cdot)$ is specifically chosen so that an entrepreneur at a given wealth (and skill) distribution quantile, imposes the exact same markup as the entrepreneur at the same quantile in the variable markups model. In this way I perfectly replicate the markup distribution across entrepreneurs obtained in the variable markups model. I then re-parametrize this model so to match the same targets matched in the economy with variable markups. In particular, to match the observed returns to entrepreneurship distribution I need to suitably change the parameters of the entrepreneurial skills distribution x_z, η_z. The remaining parameters, instead, remain unchanged. Table C1.1 (Appendix C) summarizes the chosen parameter values. Figure C1.3 (Appendix C) shows that the calibrated model closely replicates the observed distribution of returns to entrepreneurship. In the alternative scenario in which all entrepreneurs impose the same constant markup, I assume that the elasticity of demand parameter $\sigma_i = \sigma = 1.2$ for all entrepreneurs i. Again, the parameters of the entrepreneurial skills distribution $Pa(x_z, \eta_z)$ are adjusted so to match the observed return distribution. Table C1.2 (Appendix C), summarizes the parameter choices and Figure C1.4 (Appendix C) shows that the calibrated model is able to closely replicate the observed return distribution.

Wealth tax policy - effects comparison: I now implement the same revenue-equivalent wealth tax studied in the variable markups economy in the two economies described above: one with constant and heterogeneous markups, and the other one with constant and homogeneous markups. The effects of the wealth tax on entrepreneurial choices in all three settings are reported in Figures 8 and 9.

Let's begin by examining the partial equilibrium effects of the wealth tax, as shown in Figure 8. In all three economies, entrepreneurs subject to the tax, that is those at or above the 90^{th} percentile, reduce their production and decrease their labor demand. However, even with the same revenue-equivalent wealth tax, the quantitative effects differ across the three economies. These differences arise from the different shapes of the marginal revenue curves and hence different production elasticities across the considered economies (Proposition 1). First, let's focus on the difference between the economy in which entrepreneurs impose heterogeneous but constant markups (orange curves) and the one in which entrepreneurs impose homogeneous markups (yellow curves). In the economy with markups heterogeneity, entrepreneurs at the very top of the wealth distribution (beyond 97^{th} wealth percentile) impose an above the average markup, larger than the one imposed by entrepreneurs with the same wealth in the economy with homogeneous markups. This is the case because entrepreneurs beyond 97^{th} wealth percentile face a more rigid demand schedule for their own variety in the economy with heterogeneous markups. As equation (1.6) shows, when the price elasticity of demand for the entrepreneur's variety is constant, the lower the elasticity of demand, the lower the production elasticity of the entrepreneur with respect to capital. This explains why the reduction in labor demand and production of taxed entrepreneurs beyond the 97^{th} wealth percentile is smaller in the economy with heterogeneous (constant) markups than in the economy with homogeneous (constant) markups.

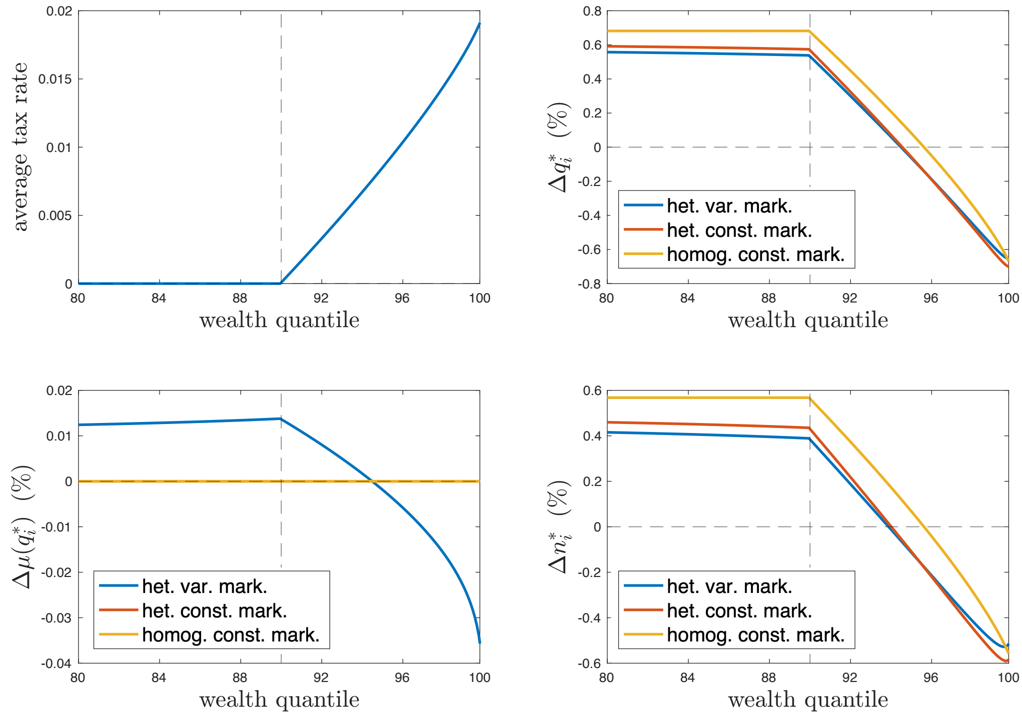
FIGURE 8. *Wealth tax effects comparison: partial equilibrium*



Notes: *Figure represents the partial equilibrium effects (i.e. keeping w, Y, P fixed) of the wealth tax described in Section 4.2 with $\tau = 0.02$ on entrepreneurs' production choices. Blue lines represent the effects of the wealth tax in the economy in which market power positively depends on firm's market share. Orange lines represent the effects of the wealth tax in the economy in which markups are heterogeneous but constant. Yellow lines represent the effects of the tax when entrepreneurs impose homogeneous and constant markups. The first panel indicates the average tax rate. The other panels represent the differences between entrepreneurs' choices (relative quantities, markups, labor demand) when the tax policy is implemented and the same quantities when the tax policy is not in place.*

Now consider the difference between the two economies with heterogeneous markups (constant and variables). In the two economies entrepreneurs across the wealth distribution impose the same markups. However, in the economy in which markups depend on firm's production scale the reduction in quantity produced by the taxed entrepreneurs is asso-

Figure 9. *Wealth tax effects comparison: general equilibrium*



Notes: *Figure represents effects of the wealth tax described in Section 4.3 with* $\tau = 0.02$ *on entrepreneurs' production choices in general equilibrium. Blue lines represent the effects of the wealth tax in the economy in which market power positively depends on firm's market share. Orange lines represent the effects of the wealth tax in the economy in which markups are heterogeneous but constant. Yellow lines represent the effects of the tax when entrepreneurs impose homogeneous and constant markups. The first panel indicates the average tax rate. The other panels represent the differences between entrepreneurs' choices (relative quantities, markups, labor demand) when the tax policy is implemented and the same quantities when the tax policy is not in place.*

ciated with a reduction in their firm's markup. This induces a counterbalancing effect which limits the reduction in production and labor demand of taxed entrepreneurs with respect to the case in which markups are constant.

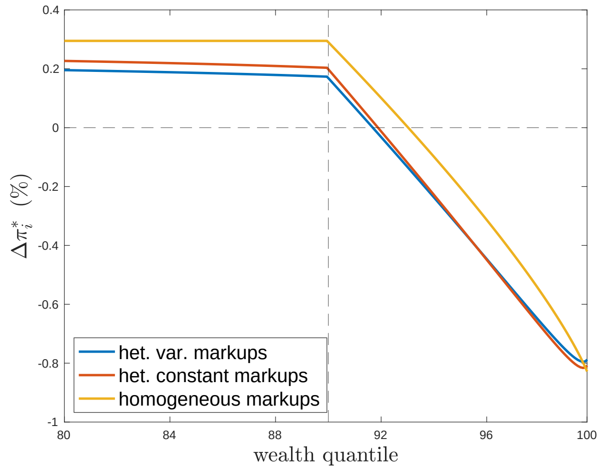
As a result, the largest drop in aggregate labor demand and hence in equilibrium wage is the one in the economy with homogeneous markups, and the smallest in the economy with variable and heterogeneous markups.

Hence workers, although receiving the same transfer in the three economies, experience the lowest reduction in their equilibrium wage in the economy where entrepreneurs impose heterogeneous and variable markups. The redistributive effect of the wealth tax is thus the largest in that case. Figure 9 reports how the wealth tax affects entrepreneurs' choices in the three economies in general equilibrium. The output loss in the economy with heterogeneous and variable markups is the lowest (-0.25% in the economy with heterogeneous variable markups, -0.27% in the economy with heterogeneous and constant markups and -0.28% in the economy with homogeneous markups). The reason is that in spite of the same drop in capital stock, aggregate productivity falls the least in the economy with variable markups. Indeed, in this economy the wealth tax induces the smallest reallocation of production from wealthy and more productive entrepreneurs to poorer and less productive ones.

Overall, in the economies where wealthier (and more productive) entrepreneurs impose larger markups the equity-efficiency trade-off of the wealth tax is relaxed with respect to the case in which all entrepreneurs impose the same (constant) markups. Indeed, for any desired level of tax revenues the wealth tax in the economies with heterogeneous markups induces lower losses in terms of aggregate production and equilibrium wage paid to poor workers.

Figure 10 illustrates the tax's redistributive effects among entrepreneurs. In all three economies, the wealth tax reduces inequality among entrepreneurs by decreasing the profits of the wealthiest entrepreneurs while increasing those of the poorest. However, the most significant redistribution across entrepreneurs occurs in the economy with no market power heterogeneity across them. While the negative effects on the wealthiest entrepreneurs' profits are similar across all three economies, the larger wage drop in the economy with homogeneous market power induces a greater expansion of profits for the poorest entrepreneurs.

FIGURE 10. *Wealth tax effects comparison: profits*



Notes: *Figure represents the effects of the wealth tax described in Section 4 with* $\tau = 0.02$ *on entrepreneurs' profits in general equilibrium. Blue lines represent the effects of the wealth tax in the economy in which market power positively depends on firm's market share. Orange lines represent the effects of the wealth tax in the economy in which markups are heterogeneous but constant. Yellow lines represent the effects of the tax when entrepreneurs impose homogeneous and constant markups.*

5 Dynamic quantitative model

In this section I develop a dynamic, stochastic, general equilibrium model with workers and entrepreneurs. The objective of the Section is twofold. First, this framework allows me to study how the top wealth tax distorts entrepreneurs' capital accumulation under different assumptions about their market power. In particular, I compare the effects of a top wealth tax when entrepreneurs impose markups that increase with their firm's market share against the scenario where all entrepreneurs impose constant and homogeneous markups. Then, appropriately calibrating the model, I quantify how much the steady-state wealth tax effects differ in the two considered scenarios. Through this exercise I show that accounting for the observed market power heterogeneity across American businesses, reduces the drop in production and wages induced by the tax by 40%.

In this model I assume entrepreneurs not only decide how much to produce and which markups to impose, but also how much capital supply to their own business, by making consumption-saving and portfolio choices. Together with privately owned entrepreneurial businesses (credit constrained) in this economy coexist corporations with unlimited access to the capital market and perfectly diversified ownership. This production structure allows me to capture the distortionary effects of wealth taxation that go through lower capital availability for entrepreneurs - and hence for their privately owned businesses - but also through higher cost of financing for corporations[16].

5.1 Setup

The model is infinite horizon and there is a continuum of households indexed by $i \in [0, 1]$. There is no occupational choice: a fraction ω of households are workers and the remaining households are entrepreneurs. There is no aggregate uncertainty.

Entrepreneurs: are heterogeneous in their *stochastic* entrepreneurial skills z_t^i which follow a stationary AR(1) process:

$$\log(z_{t+1}^i) = (1 - \rho_z)\bar{z} + \rho_z \log(z_t^i) + \varepsilon_{t+1}^i \quad \text{where} \quad \varepsilon_t^i \sim N(0, \sigma_z^2) \tag{1.11}$$

Entrepreneurs accumulate wealth by investing either in their own risky business - earning profits - or in a mutual fund that pays the risk-free rate r_t. To capture the heterogeneity in entrepreneurs' portfolio choices observed in the data (see Section 2), I assume that each entrepreneur i with skill level z_t^i invests a fraction $\phi_t^i = \phi(z_t^i)$ of his wealth in his own business. The remaining share, $1-\phi_t^i$, is invested in a mutual fund that channels capital to the economy's "corporate sector" (more details about this sector will follow). The portfolio choice ϕ_t^i is assumed to depend solely on entrepreneurial skill, allowing the model to reproduce in a simple reduced-form way the empirical pattern whereby the wealthiest (and most productive) entrepreneurs hold a larger fraction of their wealth as equity in their own firms. Entrepreneurs' businesses, as in the static model, compete in monopolistic competition and employ the constant return to scale production technology $y_t^i = z_t^i (k_t^i)^\nu (n_t^i)^{1-\nu}$ to produce differentiated intermediate goods. Notice that now, differently from the static model, there is a distinction between overall wealth of the entrepreneur a_t^i and capital used for production in the entrepreneur's firm: $k_t^i = \phi(z_t^i) a_t^i$. Furthermore, I still assume that entrepreneurs' firms are unable to borrow. This modeling assumption allows to capture

[16] I will assume these firms will operate a Cobb-Douglas production function operating in perfect competition. Thus, the ownership structure of these firms is irrelevant since they make zero profits

that these privately owned businesses heavily rely on entrepreneurs' savings and on some collateralized borrowing as the main source of financing (Dyrda et al., 2018).

The timing of each entrepreneur's choices is the following. Let x_t^i denote the cash-on-hand of entrepreneur i, that is his wealth, net of depreciation plus capital income. At the end of every period t the entrepreneur i knows his cash-on-hand level, x_t^i, and his current productivity level z_t^i. Given this information he decides how much to consume out of his cash on hand, c_t^i. Let $a_t^i = x_t^i - c_t^i$ be the level of entrepreneur's i wealth at the end of period t. He then employs his fraction of wealth $\phi(z_t^i)$ as capital for his entrepreneurial activity and the remaining fraction is instead invested in the mutual fund. At the beginning of period $t+1$ his new productivity level z_{t+1}^i realizes, and given this information - and the pre-determined capital - he chooses his optimal production y_{t+1}^i and how much labor n_{t+1}^i to hire from the market at wage w_{t+1}. Production then takes place and each entrepreneur i receives the profits π_{t+1}^i of his own firm. Furthermore, each entrepreneur also receives capital income from investment in the mutual fund $r_{t+1}(1-\phi(z_t^i))a_t^i$. Finally, the wealth used as capital for production depreciates at a rate $0 < \delta < 1$.

Workers: are heterogeneous in their stochastic skills: each worker i in every period t has working skills e_t^i, following a stationary AR(1) process:

$$\log(e_{t+1}^i) = \rho_e \log(e_t^i) + \varepsilon_{t+1}^i \quad \text{where} \quad \varepsilon_t^i \sim N(0, \sigma_e^2) \tag{1.12}$$

Furthermore, each worker is assumed to supply labor inelastically to the two sectors of the economy: ℓ^E units to the firms owned by entrepreneurs and ℓ^C units to the "corporate" sector. Workers do not own firms; hence, apart from supplying labor each period, they choose only how much to consume and how much to invest in the same risk-free asset available to entrepreneurs.

Final good producers: this economy is composed by two sectors: an "entrepreneurial" sector and a "corporate" one. Goods produced in the two sectors are assumed to be perfectly substitutable so that final production at time t, Y_t, writes: $Y_t = Y_t^C + Y_t^E$ where Y_t^C indicates the total production of the corporate sector and Y_t^E indicates the total production of the entrepreneurial sector.

I assume that in the corporate sector operates a continuum of perfectly competitive identical producers employing capital and labor to produce with a standard Cobb-Douglas technology. Capital is rented from the mutual fund and its aggregate is denoted by K_t^C. The problem solved by producers operating in the corporate sector is:

$$\max_{K_t^C, N_t^C} \quad A(K_t^C)^\alpha (N_t^C)^{1-\alpha} - r_t K_t^C - w_t^C N_t^C$$

where A indicates the (time invariant) aggregate productivity of this sector and the associated optimality conditions are:

$$w_t^C = (1-\alpha)\left(\frac{K_t^C}{N_t^C}\right)^\alpha \qquad r_t = (1-\alpha)\left(\frac{N_t^C}{K_t^C}\right)^{1-\alpha} \tag{1.13}$$

The second sector of this economy is the "entrepreneurial sector": in this sector operates a continuum of perfectly competitive producers who combine intermediate goods produced by entrepreneurs to produce the good Y_t^E. To do that they employ the Kimball (1995) aggregator analyzed in Section 3. The problem that each final good producer operating in this sector solves is:

$$\max_{Y_t^E, \{y_t^i\}_{i\in[\omega,1]}} Y_t^E - \int_\omega^1 p_t^i y_t^i di \qquad \text{s.t.} \quad \int_\omega^1 \Upsilon\left(\frac{y_t^i}{Y_t^E}\right) di = 1$$

which, as showed in Section 3, when solved delivers the demand curve for each entrepreneur's variety: $p(q_t^i, P_t) = P_t \Upsilon'(q_t^i)$ where q_t^i now indicates y_t^i / Y_t^E, that is the relative production of entrepreneur i with respect to the aggregate production of the entrepreneurial sector and P_t is the aggregate price index.

5.2 Recursive stationary equilibrium

Assume that all households have the same CRRA preferences for final good consumption. I now write the recursive formulation of the dynamic problems of both workers and entrepreneurs.

To simplify notation let's drop individual indices i. The individual state vector for any household (worker or entrepreneur) is $(x,\ e,\ z)$, i.e. their cash-on-hand (x), their labor market skills (e) and their entrepreneurial productivity (z)[17]. Let $\lambda(x, e, z)$ indicate the density of households at a given state vector (x, e, z).

Furthermore, notice that each household's decision problem not only depends on his idiosyncratic states, but also on some current and future aggregate variables which are determined by the current and future distribution of agents over states. To compute these aggregates, households need to know the current period density function $\lambda(\cdot)$ and its associated law of motion $H(\cdot)$, so to obtain the future density as well: $\lambda' = H(\lambda)$.

Recursive problems: the recursive problem of each worker writes:

$$
\begin{aligned}
V(x, e, z, \lambda) = & \max_{c,a,x'} c^{1-\theta}/(1-\theta) + \beta \mathbb{E}\left(V(x', e', z', \lambda')|(x, e, z, \lambda)\right) \\
\text{s.t.} \quad & x' = (1 + r(\lambda'))a + e'(\ell^C w^C(\lambda') + \ell^E w^E(\lambda')) \\
& a = x - c \\
& \log(e') = \rho_e log(e) + \varepsilon \\
& \lambda' = H(\lambda) \\
& c \geq 0 \quad a \geq 0
\end{aligned}
$$

Workers' optimal intertemporal consumption-saving choices can be characterized by the standard Euler equation:

$$
(c^*)^{-\theta} = \beta \mathbb{E}\left((1 + r(\lambda'))(c^{'*})^{-\theta} \Big| (x, e, z, \lambda)\right)
$$

[17] I assume that every entrepreneur has labor market skills $e = 0$ and entrepreneurial skills $z > 0$, while every worker has entrepreneurial productivity $z = 0$ and labor market skills $e > 0$

The recursive problem of each entrepreneur is instead:

$$
\begin{aligned}
V(x,e,z,\lambda) = \max_{c,a,x',p',y',n'} \; & c^{1-\theta}/(1-\theta) + \beta\mathbb{E}\left(V(x',e',z',\lambda')|(x,e,z,\lambda)\right) \\
\text{s.t.} \quad & x' = (1-\delta)\phi(z)a + (1+r(\lambda'))(1-\phi(z))a + \pi' \\
& a = x - c \\
& \pi' = p'y' - w^E(\lambda')n' \\
& y' = z'(\phi(z)a)^{\nu}(n')^{1-\nu} \\
& p' = P(\lambda')\Upsilon'\left(\frac{y'}{Y^E(\lambda')}\right) \\
& \log(z') = (1-\rho_z)\bar{z} + \rho_z \log(z) + \varepsilon \\
& \lambda' = H(\lambda) \\
& c \geq 0 \quad a \geq 0
\end{aligned}
$$

By combining the FOCs of the entrepreneur's problem it is possible to obtain two equations which characterize the optimal entrepreneurial choices. The first one is a static condition, pinning down the entrepreneur's production decisions given the available capital for production. The second one, instead, is the Euler equation, which captures the intertemporal trade-off of the entrepreneur between consuming today and investing in his own firm and in the mutual fund.

Let's start from the static optimality condition. To save on notation let's denote capital used for production as $k = \phi(z)a$ and, as in the static framework, $q = y/Y^E$:

$$
\underbrace{P(\lambda')\Upsilon'\left(q^{*'}\right)}_{p'} = \underbrace{\frac{\mathcal{E}^d(q^{*'})}{\mathcal{E}^d(q^{*'})-1}}_{\text{markup}} \times \underbrace{\frac{w^E(\lambda')\left(Y^E(\lambda')\right)^{\frac{\nu}{1-\nu}}}{(1-\nu)}\left(\frac{(q'^{*})^{\nu}}{z'(k^*)^{\nu}}\right)^{\frac{1}{1-\nu}}}_{\text{marginal cost}} \tag{1.14}
$$

notice that this condition is identical to (1.4), which characterizes the entrepreneurial production choices in the static model. Hence, given any optimal level of capital employed for production k^*, productivity z' and aggregates $(w^E(\lambda'), P(\lambda'), Y^E(\lambda'))$ the optimal relative quantity produced, labor demand and profits of each entrepreneur can be computed through the functions $\mathcal{Q}^*(z', k^*, w^E(\lambda'), P(\lambda'), Y^E(\lambda'))$,
$\mathcal{N}^*(z', k^*, w^E(\lambda'), P(\lambda'), Y^E(\lambda'))$, $\Pi^*(z', k^*, w^E(\lambda'), P(\lambda'), Y^E(\lambda'))$ whose properties have

been analyzed in Lemma 1-2-3.

The Euler equation of the entrepreneur's problem is:

$$(c^*)^{-\theta} = \beta \mathbb{E}\Bigg[(c'^*)^{-\theta}\Bigg(\phi(z)\Bigg(1-\delta+\frac{\partial \Pi^*(z', \phi(z)a^*, Y^E(\lambda'), w^E(\lambda'), P(\lambda'))}{\partial(\phi(z)a^*)}\Bigg)+$$
$$+(1-\phi(z))(1+r(\lambda'))\Bigg)\Bigg|\,(x,e,z,\lambda)\Bigg]$$

which shows that in equilibrium the intertemporal marginal rate of substitution between consumption across two periods is equated to the marginal return to investment. The latter is equal to a weighted average between marginal return to investment in the entrepreneurial activity (net of depreciation) and in the capital market asset.

Stationary equilibrium definition: let $\mathbf{s} = (x, e, z) \in \mathcal{S}$ denote the vector of individual states and $\{a^j(\mathbf{s}, \lambda),\ c^j(\mathbf{s}, \lambda), x'^j(\mathbf{s}, \lambda)\}_{j=W,E}$ the policy functions for workers and entrepreneurs that solve the previous recursive problems. Denote as $f((e', z')|(e, z))$ the conditional density of an household with skills (e, z) to have in the following period the skills (e', z')[18]. Now define as $H(\cdot)$ the aggregate law of motion, which, given the current states distribution λ, delivers the measure of agents in the state $\mathbf{s}' \in \mathcal{S}$ in the following period:

$$H(\mathbf{s}', \lambda, f, \{x'^j(\mathbf{s}, \lambda)\}_{j=W,E}) = \begin{cases} \int_{\mathbf{s}\in\mathcal{S}:x'=x'^W(\mathbf{s},\lambda)} f((e', z')|(e, z))\lambda(\mathbf{s})d\mathbf{s} & \textit{if } z' = 0 \\ \int_{\mathbf{s}\in\mathcal{S}:x'=x'^E(\mathbf{s},\lambda)} f((e', z')|(e, z))\lambda(\mathbf{s})d\mathbf{s} & \textit{if } e' = 0 \end{cases}$$

From now on, my focus will be on the stationary equilibrium of this economy, i.e. an equilibrium in which the density function $\lambda(\cdot)$ is time-invariant, that is the function $\lambda(\cdot)$ satisfies:

$$\lambda(\mathbf{s}') = H(\mathbf{s}', \lambda, f, \{x'^j(\mathbf{s}, \lambda)\}_{j=W,E}) \quad \forall\ \mathbf{s}' \in \mathcal{S} \tag{1.15}$$

To shorten the notation employed in defining the stationary equilibrium of this economy

[18]Entrepreneurs and workers are assumed never to change occupation in their own life, and an household is *either* a worker *or* an entrepreneur, not both. Because of that we either have $e = e' = 0$ or $z = z' = 0$.

denote:

$$q^*(\mathbf{s},\lambda) = \mathcal{Q}^*(z, \phi(z)a^E(x,e,z), Y^E(\lambda), w^E(\lambda), P(\lambda))$$
$$n^*(\mathbf{s},\lambda) = \mathcal{N}^*(z, \phi(z)a^E(x,e,z), Y^E(\lambda), w^E(\lambda), P(\lambda))$$

Definition 1 (Stationary equilibrium). *The stationary equilibrium of this economy consists of a value function for workers and entrepreneurs:* $\{V^j(\mathbf{s},\lambda),\, V^j(\mathbf{s},\lambda)\}_{j=E,W}$ *and the associated policy functions for workers and entrepreneurs:* $\{a^j(\mathbf{s},\lambda),\, c^j(\mathbf{s},\lambda),\, x'^j(\mathbf{s},\lambda)\}_{j=E,W}$*, a tuple of prices* $(w^E(\lambda), w^C(\lambda), r(\lambda))$ *and aggregates* $(K^C(\lambda), N^C(\lambda), Y^E(\lambda), Y^C(\lambda), Y(\lambda))$ *such that:*

- *The density of individual states* λ *is stationary, that is satisfies equation* (1.15)
- *Given* λ*, workers' policies solve recursive problem (W) and entrepreneurs' policies solve recursive problem (E)*
- *Price functions* $w^C(\lambda)$ *and* $r(\lambda)$ *satisfy profit maximization condition of producers in corporate sector* (1.13)
- *Labor market clearing in "entrepreneurial" sector:*

$$\int_{\mathcal{S}} n^*(\mathbf{s},\lambda)\lambda(\mathbf{s})d\mathbf{s} = \int_{\mathcal{S}} \ell^E e\lambda(\mathbf{s})d\mathbf{s}$$

- *Labor market clearing in "corporate" sector:*

$$N(\lambda) = \int_{\mathcal{S}} \ell^C e\lambda(\mathbf{s})d\mathbf{s}$$

- *Capital market clearing:*

$$K(\lambda) = \int_{\mathbf{s}\in\mathbf{S}:e=0} (1-\phi(z))a^E(\mathbf{s},\lambda)\lambda(\mathbf{s})d\mathbf{s} + \int_{\mathbf{s}\in\mathbf{S}:z=0} a^W(\mathbf{s},\lambda)\lambda(\mathbf{s})d\mathbf{s}$$

- *Kimball aggregator holds:*

$$\int_{\mathcal{S}} \Upsilon\left(q^*(\mathbf{s},\lambda)\right)\lambda(\mathbf{s})d\mathbf{s} = 1$$

- *Price aggregator definition:*

$$P(\lambda) = \int_{\mathcal{S}} \Upsilon' \left(q^*(\mathbf{s}, \lambda)\right) q^*(\mathbf{s}, \lambda)\lambda(\mathbf{s})d\mathbf{s}$$

- *Corporate sector and aggregate production:*

$$Y^C(\lambda) = A(K(\lambda))^\alpha (N(\lambda))^{1-\alpha} \qquad Y(\lambda) = Y^C(\lambda) + Y^E(\lambda)$$

Numerical procedure: the model is solved numerically. I start the procedure by guessing some of the steady-state aggregate state variables. The choice of which variables to guess depends on the features of the demand function for the varieties produced by entrepreneurs. Given the guessed variables, I obtain the policy functions for workers and entrepreneurs using iteration on the Euler equation and the endogenous grid method (Carroll, 2006). I now simulate the stationary distribution of the economy. To do that I draw a sufficiently long history of shocks and using the policy functions obtained I compute the consumption-saving and production choices of workers and entrepreneurs[19]. At this point I recompute the same aggregate variable guessed at the beginning of the procedure and check the distance between the guessed and computed aggregate variables. I iterate this procedure until convergence between actual and guessed aggregate variables.

5.3 Steady-state calibration

I next calibrate the model under two alternative assumptions regarding entrepreneurs' market power. The first one is assuming variable market power, with entrepreneurs setting markups that increase with their firms' market shares. The second is the assumption commonly employed in models used to study wealth taxation effects, namely that market power is constant and homogeneous across entrepreneurs.

[19] There is no result guaranteeing that a stationary distribution of states $\lambda(\cdot)$ exists and is unique. Hence, to check that the obtained states distribution is stationary I repeat the simulation exercise for history of shocks of various (large) length. I then check that the moments of the stationary distribution $\lambda(\cdot)$ do not depend on the chosen length of the shock history

Table 5. *Variable markups steady-state: externally calibrated parameters*

Par.	Description	Value	Target
ω	fraction of workers	0.88	fraction of non-entr.
γ	CRRA par. utility	1	-
ν	capital exponent entr. prod.	0.28	Labor share entr. sect. $= 0.6$
α	capital exponent mkt sector prod.	0.4	Labor share mkt. sector $= 0.6$
ψ	super-elast. demand	3.24	$\sigma/\psi = 0.162$

Notes: *the Table summarizes the parameter choices to calibrate the steady state of the dynamic model presented in Section 5. The first column indicates the symbol used to identify the parameter in the model, the second column the parameter description, the third column the chosen value for the parameter, the fourth column the moment targeted to calibrate each parameter.*

Table 6. *Variable markups steady-state: internally calibrated parameters*

Par.	Description	Value	Target	Data	Model
β	discount factor	0.91	wealth / output	4	4.6
δ	depreciation rate	0.015	entr. wealth fract.	0.44	0.39
σ	elas. demand when $q = 1$	20	av. markups	1.2	1.18
A	TFP market sector	0.35	Y^M/Y	0.43	0.49
$\bar{z}$	av. entrep. skills	0.5	workers in top 1%	0.17	0.2
ρ_e	persistence worker skills	0.95	top 1% wealth	0.36	0.33
σ_e^2	var. innovation worker skill	0.25	top 5% wealth	0.65	0.59
ρ_z	persistence entr. skill	0.95	top 10% wealth	0.77	0.74
σ_z^2	var. innovation entr. skills	0.44	Gini wealth	0.88	0.83
			top 1% capital	0.42	0.46
			top 5% capital	0.71	0.75
			top 10% capital	0.83	0.87

Notes: *the Table summarizes the parameter choices to calibrate the steady state of the dynamic model presented in Section 5. The first column indicates the symbol used to identify the parameter in the model, the second column the parameter description, the third column the chosen value for the parameter, the fourth column the moment targeted to calibrate each parameter, the fifth column the value of the targeted moment in the data, the sixth column the value of the targeted moment in the simulated model,*

Calibration with variable markups: the model is calibrated assuming that the economy is at the steady-state in 2019, so the statistics are targeted for that year. The calibration choices are summarized in Tables 5-6.

Most of the parameters are calibrated similarly to what done for the static model: ω captures the fraction of entrepreneurs in the SCF data, ν and α are chosen to replicate a labor share of income of 60% in both the corporate and entrepreneurial sector. The functional form for $\Upsilon(\cdot)$ is again assumed to be the Klenow and Willis (2016) one, with the parameter σ set so to match the aggregate markup $\mathcal{M} = 1.2$ and the parameter ψ so to capture the empirically estimated relationship between firm level markups and market shares (for details see Section 4). Differently from the static model the steady-state wealth distribution is now an endogenous object. Hence the AR(1) processes for workers' skills (1.12) and entrepreneurial productivity (1.11) are calibrated so to match the top 1%, top 5% and top 10% wealth shares of the overall wealth distribution and same moments for the distribution of wealth that entrepreneurs hold as capital in their own firms. Furthermore, the Gini coefficient of the overall wealth distribution is also targeted. Finally the average of the entrepreneurial productivity process $\bar{z}$ is chosen so to match the observed fraction of entrepreneurs at the top 1% of the wealth distribution. In the following section I will be studying the effects of a wealth tax policy on the wealthiest 1% of American household. Thus, carefully matching the fraction of workers and entrepreneurs at the top of the wealth distribution is particularly important because it determines how much the top wealth tax burden falls onto the wealthiest workers and how much onto the wealthiest entrepreneurs of the economy.

Now consider the function $\phi(\cdot)$, which associates to an entrepreneur with skills z the fraction of his overall wealth he holds as equity in his own business $\phi(z)$. This is assumed to be a power function $\phi(z) = bz^c$, where the parameters $b = 0.23$ and $c = 0.21$ are chosen so that the steady-state relationship between wealth and fraction of wealth held as equity in the entrepreneur's business replicates the one observed in the data (Figure 2). Figure C1.5 in Appendix C shows that the chosen functional form allows to fit well the targeted portfolio choices of entrepreneurs across the wealth distribution.

The discount factor β is calibrated so to match the wealth output ratio of the U.S. economy and the depreciation rate δ to target the fraction of wealth owned by entrepreneurs (44%). Finally, the TFP, A, of the corporate sector is chosen to match the relative production of the corporate sector with respect to that of the privately owned businesses directly managed by the entrepreneurs.

Entrepreneurs' choices and wealth distribution - variable markups: Figure 11 reports the simulated entrepreneurs' choices at the calibrated steady-state. In the first panel notice a monotonic increasing relationship between entrepreneurial productivity and wealth, i.e. the wealthiest entrepreneurs are on average the most productive ones. Since the fraction of net wealth $\phi(z)$ held by the entrepreneur in his own business is increasing in z (and hence in wealth too), at the steady-state there is a monotonic increasing relationship between skills of the entrepreneur and the amount of wealth invested in his own business. Entrepreneurs' production choices across the wealth distribution are similar to the ones analyzed in the static model: the more productive the entrepreneur is, the more wealth he invests in his business, the more produces, the larger the markup he imposes. Table C1.3 in Appendix C compares the simulated steady-state distribution of markups with the corresponding distribution estimated from Compustat data (Edmond et al. (2023)). The two distributions exhibit similar overall patterns, although the simulated one is less right-skewed than its Compustat counterpart. To rationalize this, let's remember that the overall production (and hence size) of firms owned by entrepreneurs is calibrated so to match the production of entrepreneurs' privately owned businesses in the U.S. data. These, on average, tend to be smaller (imposing smaller markups) than the publicly traded ones represented in Compustat.

Column (3) of Table 7 shows the performance of the model in capturing the shape of the wealth distribution observed in the Survey of Consumer Finances data (column (2)). The chosen calibration performs well in capturing the shape of the entire wealth distribution and particularly within the top 1%, where the top wealth tax will be implemented. The shape of the wealth distribution at the top is mainly driven by the wealth accumulation of entrepreneurs, while the wealth distribution at the middle-bottom is instead shaped by workers' choices.

Constant and homogeneous markups calibration: all model parameters are calibrated to match the same targeted moments as in the variable-markups model, except for heterogeneity in markups across entrepreneurs. The calibration choices are summarized in Tables C1.4-C1.5 (Appendix C). To have constant elasticity of demand curves for the entrepreneurs' varieties (and hence constant markups) I assume $\Upsilon(q) = q^{\frac{\sigma-1}{\sigma}}$. The elasticity of demand parameter σ is chosen so to match the same aggregate markup $\mathcal{M} = 1.2$. To obtain the same steady-state wealth and capital distribution I retrieved in the model with heterogeneous markups, the parameters of the entrepreneurs' skill process (1.11), i.e. $\rho_z, \sigma_z^2, \bar{z}$ are suitably recalibrated. In particular, the resulting entrepreneurial skill distribution is less skewed than the one employed in the model with heterogeneous

Table 7. *Steady-state wealth distribution: model vs data*

Percentile	(2) Wealth share (data)	(3) Wealth share (het. markups)	(4) Wealth share (hom. markups)
0–20	−0.001	0.001	0.001
20–40	0.002	0.001	0.001
40–60	0.039	0.065	0.059
60–80	0.105	0.118	0.101
80–90	0.138	0.163	0.154
90–95	0.117	0.148	0.152
95–99	0.246	0.204	0.199
99–99.5	0.092	0.066	0.061
99.5–99.8	0.116	0.095	0.089
99.8–99.9	0.042	0.056	0.055
99.9–100	0.122	0.120	0.118

Notes: *the Table summarizes key moments of the U.S. wealth distribution, comparing empirical estimates with the model's steady-state counterpart. The second column reports wealth shares by percentile bins computed from the 2019 SCF. The third and fourth columns present the corresponding simulated moments under the model with variable markups and with constant, homogeneous markups, respectively*

markups.[20] Furthermore, since the average productivity of entrepreneurs is changed, the TFP of the corporate sector A, has to be adjusted so to keep Y^E/Y unchanged. All remaining parameters remain unaffected.[21] Again, as column (4) of Table 7 shows, the steady-state wealth distribution closely replicates the one observed in the SCF data.

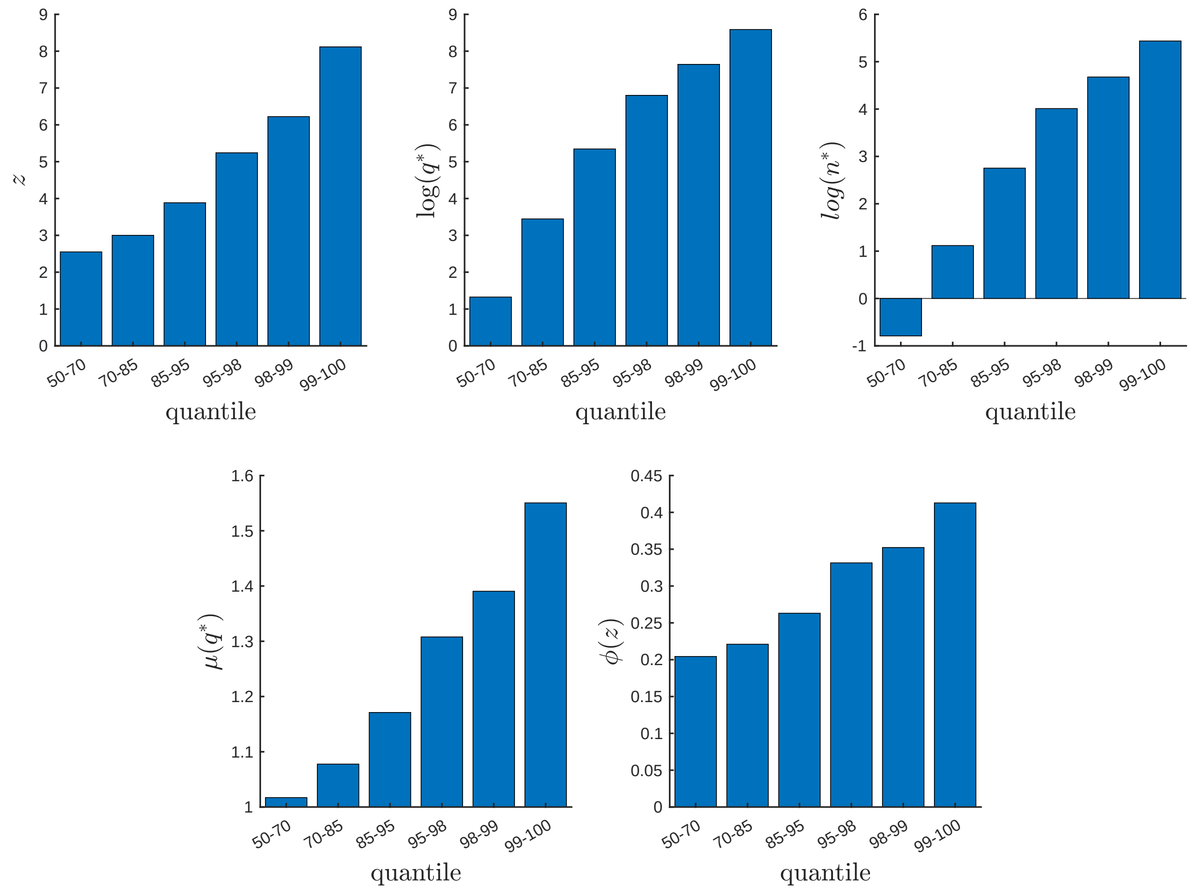
5.4 Wealth tax experiment: steady-state comparison

Suppose that the economy is at the previously calibrated steady-state. Let's implement a *permanent* top wealth tax policy identical to the one analyzed in Section 4 of the paper. In every period t, the tax function which associates to an household i (both workers and

[20]The reason is that in the model with homogeneous markups all entrepreneurs have marginal profits (which determine the steady-state wealth and capital level) decreasing at the same constant rate. Instead, in the model with variable markups marginal profits decrease at an increasing rate. In particular, entrepreneurs producing at a very large scale (imposing above the average markups) face a marginal profits curve decreasing at a higher rate than the one in the homogeneous markups model. This effect dampens wealth accumulation at the top of the wealth distribution. Hence, in the constant and homogeneous markups model a less skewed skill distribution is needed so to match the observed wealth distribution moments.

[21]For completeness notice that the parameters of the polynomial function $\phi(z) = bz^c$ are appropriately re-calibrated so to replicate the same steady-state portfolio choices of the variable markups model.

FIGURE 11. *Simulated entrepreneurs' choices at the steady-state: variable markups*



Notes: *the first panel reports the average productivity of entrepreneurs at different quantiles of the entrepreneurial wealth distribution (i.e. considering entrepreneurs only). The other four panels report simulated relative quantities* $\log(q^*)$*, markups* $\mu(q^*)$*, labor demand* $\log(n^*)$ *and fraction of wealth held as capital in the business* $\phi(z)$ *for entrepreneurs at different quantiles of the steady-state wealth distribution when the dynamic model is calibrated as in Table 5-6*

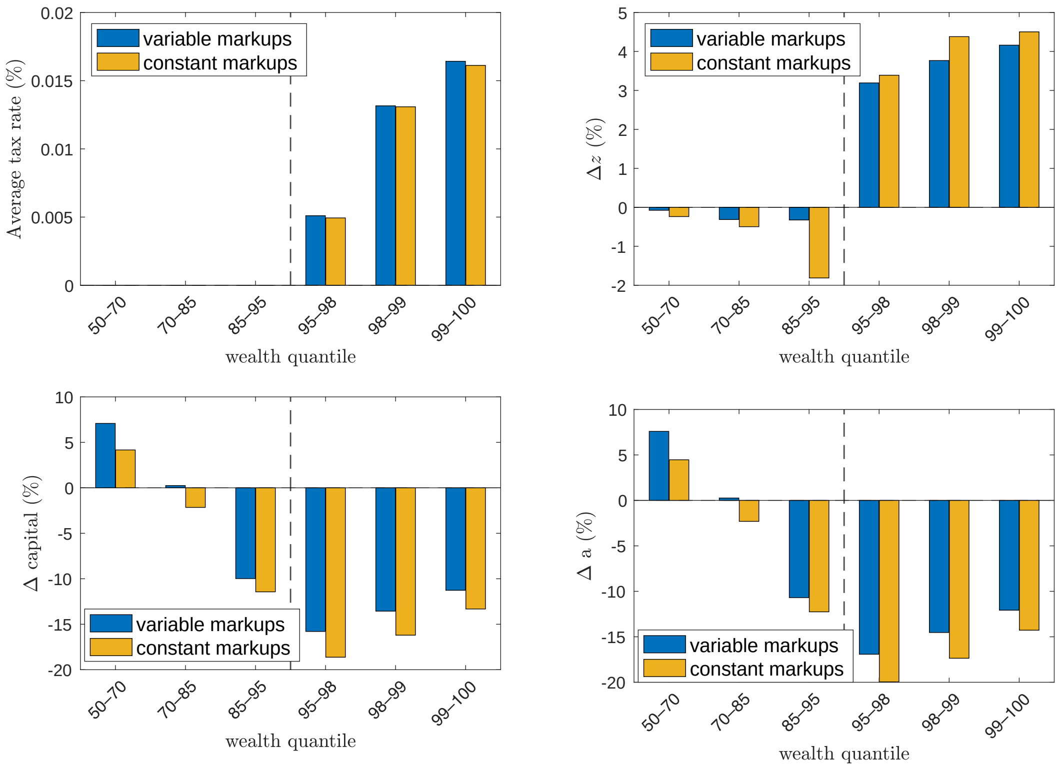
entrepreneurs) with wealth a_t^i the tax to be paid is:

$$\mathcal{T}(a_t^i) = \begin{cases} 0 & \text{if} \quad a_t^i \leq \underline{a} \\ \tau(a_t^i - \underline{a}) & \text{if} \quad a_t^i > \underline{a} \end{cases}$$

where the threshold $\underline{a}$ corresponds to the 99^{th} percentile of the initial steady-state wealth distribution. Furthermore, the tax rate is set to $\tau = 2\%$ so that the tax revenues at the initial steady-state amount to approximately 1% of GDP (a reasonable figure for wealth tax revenues absent tax evasion/elusion effects, Saez and Zucman, 2022). Finally, the tax revenues are lump sum redistributed across workers and entrepreneurs through the lump sum transfer T_t, which amounts to: $T_t = \tau \int_0^1 \max(a_t^i - \underline{a}, 0)di$

How does the new steady state, in which the wealth tax is permanently in place, differ from the initial no-tax steady state? To answer this question, I compare the steady states with and without the tax under the two considered scenarios of entrepreneurs setting variable and constant markups.

FIGURE 12. *Wealth tax effects on entrepreneurs' capital and productivity: steady-states comparison*



Notes: *the first panel represents the average tax rate faced by entrepreneurs in the steady-state where the wealth tax is implemented. The remaining panels represent the difference between entrepreneurs' capital/productivity at the steady-state with no wealth tax and at the steady-state in which the permanent wealth tax is in place.*

Figure 12 compares how entrepreneurs' average productivity, business capital and wealth vary between the no-tax steady state and the steady state with the wealth tax, at different percentiles of the wealth distribution. The first panel reports the average tax rate faced by entrepreneurs in the steady state with the permanent wealth tax. The second panel plots, for each wealth-percentile bin, the change in entrepreneurs' productivity between the wealth-tax and no-tax steady states. The third and fourth panels do the same but for the business capital and wealth of entrepreneurs. Blue bars capture these effects in the economy with variable markups, while yellow bars in the economy with constant markups.

Notice that on the x-axis it's reported the quantile of the wealth distribution including entrepreneurs *only*. Hence, the first panel shows that at the steady state with wealth taxation only the wealthiest 5% of American entrepreneurs pay a positive wealth tax.

Capital accumulation and entrepreneurs' production choices

Selection effect: First of all, notice that the wealth tax changes the average productivity of entrepreneurs across the wealth distribution with respect to the no-wealth tax steady state (second panel of Figure 12). In particular, at the steady state with wealth taxation, top-quantile entrepreneurs display higher average productivity than their counterparts in the no-tax steady state. This is because of a selection effect induced by top wealth taxation. To understand this effect, consider two entrepreneurs with the same wealth level but different productivities. The one with higher productivity has higher returns to wealth than the low productive entrepreneur, hence, once hit by the wealth tax he is relatively less affected and dissaves at a lower rate. This pushes less productive entrepreneurs downward in the wealth distribution rank, keeping only the most productive ones at the very top of the distribution. Hence, in the steady state with wealth taxation, entrepreneurs at the top of the wealth distribution are more productive, receive higher returns to wealth and hold higher fractions of their wealth as capital in their own business with respect to entrepreneurs at the top of the wealth distribution in the steady state with no-wealth tax.

Capital accumulation effects: Now focus on the third panel and notice that both under constant and variable markups, the entrepreneurs at the top of the wealth distribution reduce the capital they supply to their own business (as a result of reduced wealth accumulation induced by the tax). Notice however, that the capital reduction for the entrepreneurs after the 98^{th} wealth percentile is of smaller magnitude than that of entrepreneurs at lower percentiles of the wealth distribution ($95^{th} - 98^{th}$), although the higher average tax rates faced. This is due to the *selection* effect of the tax previously described. Indeed, the average productivity of entrepreneurs beyond the 98^{th} wealth percentile increases more than that of taxed entrepreneurs below the 98^{th} percentile. This dampens the reduction in capital supply at the top of the wealth distribution, with respect to what happens at lower quantiles. Hence, at the very top of the wealth distribution (beyond 98^{th} wealth percentile) the wealth tax reduces capital supplied to entrepreneurs' businesses, but in a lower extent with respect to what happens at lower percentiles of the wealth distribution ($95^{th} - 98^{th}$ percentiles), notwithstanding the higher average tax rates.

Now, let's compare the size of these effects between the two economies with variable and constant markups. The third panel of Figure 12 shows that taxed entrepreneurs reduce their steady-state capital (and wealth) in a larger extent in the economy with homogeneous and constant markups. This is because in the economy with markups heterogeneity the taxed entrepreneurs at the top of the wealth distribution impose larger markups than in the economy with no markups heterogeneity. Entrepreneurs imposing larger markups face more rigid demand curves for their varieties and this implies that they feature *lower capital elasticities with respect to the tax*.

To grasp the intuition behind this result first notice that the intertemporal margin that a wealth tax distorts can be appreciated through the entrepreneurs' Euler equation. A wealth tax reduces perceived marginal profits (to capital), hence the distortionary tax effect on capital accumulation is inversely proportional to the elasticity of marginal profits to capital. It can be shown (Lemma 4 in Appendix A) that the elasticity of marginal profits to capital is inversely proportional to the elasticity of demand the entrepreneur faces. Intuitively, an increase in capital input reduces marginal costs. This increases profits even at constant production, but marginal profits decrease because of decreasing returns to capital. At the same time, an increase in capital input also induces the firm to increase the scale of production, mitigating the tendency of marginal profits to decrease. However, notice that when demand is rigid, the scale of production of the entrepreneur does not change much. This implies that for low-demand-elasticity entrepreneurs, marginal profits decrease faster than for high-demand-elasticity entrepreneurs. As a consequence, when a high-markup entrepreneur is taxed, a small change in capital suffices to generate large changes in the perceived marginal returns to capital, hence, a wealth tax on such an entrepreneur generates limited distortions in capital decisions. In other words, high-markups entrepreneurs feature low capital elasticity with respect to the tax.

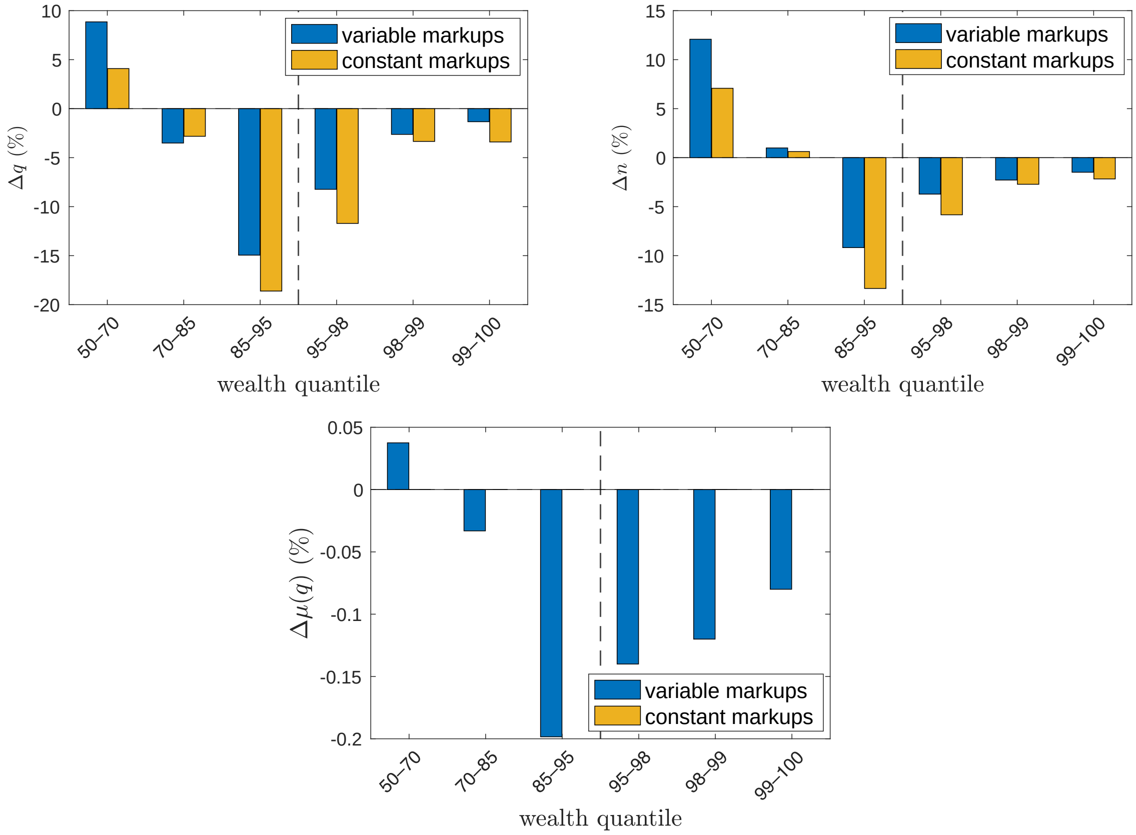
This discussion explains why the considered wealth tax induces a lower reduction in capital accumulation in the economy with markups heterogeneity.

Now notice that in the economy with no markups heterogeneity the selection effect at the very top of the wealth distribution is stronger, namely the average productivity of the wealthiest entrepreneurs raises more than in the economy with variable markups. However, the capital drop at the top of the wealth distribution remains the largest in the economy with no-markups heterogeneity.

Furthermore, notice that at lower quantiles of the wealth distribution (even below the 50^{th}) entrepreneurs experience an increase in the capital they are able to accumulate in both economies with either homogeneous or heterogeneous markups. This is due to several effects. First, the reduction in equilibrium wage in the entrepreneurial sector induced by the tax allows entrepreneurs to expand production, hence profits and capital accumulation. Furthermore, for poor entrepreneurs the lump-sum transfer T they receive is sizable, allowing them to increase investment in their own business. When these effects overcome the negative selection effect at the middle of the wealth distribution (i.e. average productivity of entrepreneurs decreases), then entrepreneurs experience a wealth (and hence capital) increase. For entrepreneurs experiencing a capital increase in a given quantile of the wealth distribution, the increase is larger in the economy with markups heterogeneity. The reason is twofold: in the economy with markups heterogeneity the negative selection effect at the middle-bottom of the wealth distribution is smaller and also the size of the transfer collected in the steady state is larger than the one collected in the economy with no markups heterogeneity.
Overall, aggregating entrepreneurs' positive and negative capital responses to the wealth tax yields a net decline in aggregate steady state capital employed by entrepreneurs. The reason is that wealthiest entrepreneurs operate the largest firms, so their reduction in available capital more than offsets the capital increases among smaller firms owned by poorer entrepreneurs. Furthermore, the distortionary tax effect on aggregate steady-state capital is larger in the economy with markups heterogeneity.
Finally, the fourth panel of Figure 12 shows the effect of the wealth tax on the overall wealth held by entrepreneurs at the steady state. Notice that the displayed patterns

closely follow these of entrepreneurs' capital choices. This is simply because of the chosen exogenous portfolio rule linking capital and wealth of all entrepreneurs $k = \phi(z)a$.

FIGURE 13. *Wealth tax effects on entrepreneurial production choices: variable vs constant markups*



Notes: *Figure represents the difference between entrepreneurs' choices at the steady-state with no wealth tax and at the steady-state in which the permanent wealth tax is in place. The blue columns represent these differences in the model simulated with variable markups and the yellow bars in the case of entrepreneurs imposing homogeneous and constant markups. The first panel indicates the average tax rate. The other panels represent the differences in relative quantities, markups, labor demand in the steady states with and without the tax.*

Entrepreneurs' production choices: Figure 13 shows how the production choices of entrepreneurs change from the steady state with no wealth taxation to the steady steady where wealth taxation is in place. These changes closely track those of capital reported in Figure 12. Similarly to what was happening in the static model, the wealth tax induces a re-shuffling of production from high to low productive entrepreneurs, which is larger in the economy with no-markups heterogeneity. Notice however, that the reallocation is dampened by the selection effect induced by the tax, which increases average productivity of entrepreneurs at the top of the wealth distribution.

As in the static framework (in the variable markups model), the wealthiest entrepreneurs, who set the highest markups, reduce their markups, whereas poorer entrepreneurs raise theirs when production of their firms expands. Overall, the wealth tax lowers the aggregate markup among entrepreneur-managed firms by 0.8%, with a corresponding increase in the sector's labor share of income.

Aggregate effects and redistribution

The wealth tax generates both direct and indirect redistributive effects. The direct effect operates through the lump-sum transfer that redistributes, uniformly across the population, the revenues collected from the wealthiest 1% of households. The indirect effect operates through price adjustments, namely the changes in the equilibrium wages w^E, w^C and the interest rate r. I now compute these price changes between the steady state with and without wealth taxation. I then compare the magnitude of these effects in the economy with and without markups heterogeneity.

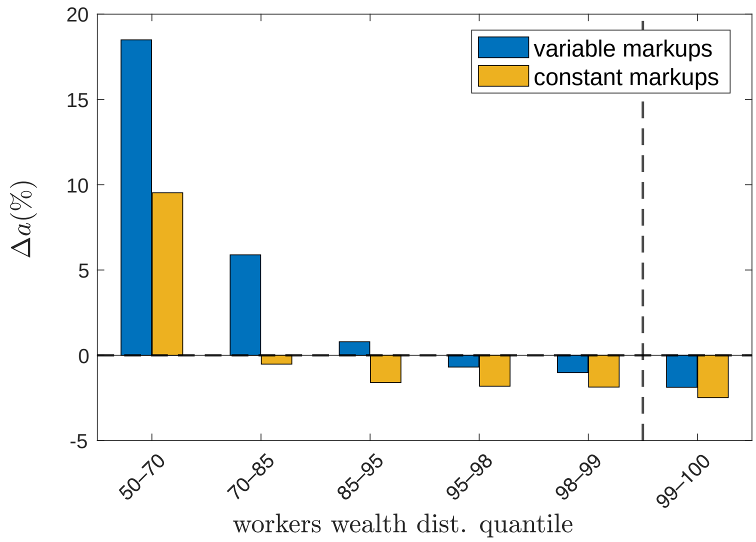
First, the wealth tax raises more revenues, and therefore finances larger transfers, in the economy with heterogeneous markups. Revenues are mainly collected from entrepreneurs, who make up most of the households at the top 1% of the wealth distribution. Consistently with the above discussion on entrepreneurs' choices (see fourth panel of Figure 12) entrepreneurs at the top of the wealth distribution reduce their steady state wealth in a larger extent under the assumption of homogeneous markups. Hence the wealth tax base, and so the transfer, is smaller in this economy, a difference which is estimated to be around 18% of the transfer raised in the economy with homogeneous markups.

Now, let's consider the effects of the tax on wages. Table 8 summarizes the aggregate effects of the wealth tax under the two considered scenarios of constant and variable markups across entrepreneurs. Aggregating the changes in entrepreneurs' capital and production choices described above, we observe a larger drop in equilibrium wage in the "entrepreneurial" sector (w^E) in the economy with homogeneous markups. This result is due to two effects: first of all the larger reduction in capital used for entrepreneurial production in the economy with homogeneous markups (see Table 8). Furthermore, as highlighted in the static framework, even if the changes in steady-state capital used for

production across entrepreneurs had been been the same in the two economies, the reduction in aggregate labor demand would have been larger in the economy with homogeneous markups. This is due to larger production and labor demand elasticities at the top of the wealth distribution in the economy with homogeneous markups (see Section 4 for the detailed discussion). This effect is further amplified in the dynamic framework by the changes in the steady-state capital distribution across entrepreneurs previously described. Now, consider the wealth tax effects on the representative firm of the "corporate" sector. The wealth tax downward distorts the amount of wealth that households accumulate, thereby reducing capital supplied not only to the "entrepreneurial" sector, but also to the "corporate" one. The drop in capital supply in the "corporate" sector is lower than that in the "entrepreneurial" sector. However, the drop in capital supply is still of larger magnitude in the economy with homogeneous markups across entrepreneurs (-12.3% and -13.7% respectively). This induces a decrease in equilibrium wage w^C and an increase in the interest rate r of stronger magnitude in the economy with no markups heterogeneity (see Table 8).
Since in this economy the equilibrium wage received by workers is a weighted average between the salaries in the two sectors in which they work, i.e. entrepreneurial and corporate, the workers in the economy with no-markups heterogeneity experience a reduction in average equilibrium wage which is 1.4 p.p. larger than in the economy with heterogeneous markups across entrepreneurs. On the other hand, the increase in the interest rate received by workers is stronger in the economy with homogeneous markups. Hence, given these opposite effects, in which of the two economies will workers benefit the most from the wealth tax? Figure 14 answers this question showing the change in workers' wealth across the wealth distribution of workers *only*. The larger transfer and the lower wage losses for workers in the economy with heterogeneous markups more than compensate the larger increase in interest rate in the economy with no markups heterogeneity. Hence, workers will increase their wealth accumulation, and ultimately consumption, in a larger extent in the economy in which entrepreneurs impose heterogeneous markups.

Finally, let's compare the effects of the considered wealth tax policy on total production in the two economies we studied. When we analyzed entrepreneurial production choices we

FIGURE 14. *Wealth tax effects on workers' steady state wealth*



Notes: *Figure represents the difference between workers' wealth at the steady state with no wealth tax and at the steady-state in which the permanent wealth tax is in place. The blue columns represent these differences in the model simulated with variable markups and the yellow bars in the model with entrepreneurs imposing homogeneous and constant markups.*

TABLE 8. *Steady-state wealth tax aggregate effects: comparison*

	Heterogeneous variable mark.	**Homogeneous constant markups**
ΔK^E	-14.2%	-16.8%
ΔK^C	-12.3%	-13.7%
ΔY^E	-2.0%	-3.1%
ΔY^C	-1.0%	-2.1%
ΔY	-1.5%	-2.6%
Δw^E	-1.8%	-3.3%
Δw^C	-1.6%	-2.6%
Δr	0.6%	0.9%
$\Delta \mathcal{M}$	-0.8%	0%

Notes: *Table reports the percentage changes of the following aggregates and prices between the steady states with and without wealth taxation. The symbols employed are: capital used for entrepreneurial production K^E and in the corporate sector K^C, entrepreneurial production Y^E, corporate sector production Y^C, aggregate production Y, wages in entrepreneurial and corporate sector w^E, w^C, interest rate r, aggregate markup $\mathcal{M}$*

already highlighted that the decrease in production of the most productive entrepreneurs is not compensated by the production increase of the poorest and least productive ones, thus leading to a drop in production in the entrepreneurial sector. This effect is stronger in the economy with no markups heterogeneity, leading to a production loss of 3.1%, versus a loss of 2% only in the economy featuring markups heterogeneity across entrepreneurs. Furthermore, notice that the drop in corporate sector production is of smaller magnitude than the production drop in entrepreneurial sector (Table 8). The reason is that the lower investment in the corporate sector of wealthy entrepreneurs who are taxed, is partly compensated by an increase of wealth of middle class and poor workers who mainly invest in the corporate sector of the economy. This effect is stronger in the economy with markups heterogeneity across entrepreneurs. This explains why, even in the corporate sector we observe a drop in production which is smaller in the economy with markups heterogeneity. Thus, aggregating the production losses induced by the tax in the two sectors we obtain a GDP loss which is 1.1 p.p. larger in the economy with markups heterogeneity across entrepreneurs.

To sum up, these results suggest that neglecting the role of market power heterogeneity across entrepreneurs in studying the effects of top wealth taxation would have led to overestimate its distortionary effects and underestimate its redistributive power.

In particular, under the considered wealth tax, neglecting market power heterogeneity would have led to overestimate GDP losses by 1.1 percentage points and overestimate the wage losses suffered by workers by 1.4 percentage points.

6 Conclusion

The contribution of this paper is to study the distortionary and redistributive effects of top wealth taxation when heterogeneous returns that entrepreneurs receive from their businesses not only reflect the entrepreneurs' productivity but also their market power.

To do this I build a dynamic stochastic general equilibrium model in which wealthier (and more productive) entrepreneurs manage firms that produce at a larger scale and impose larger markups. This novel setting to study wealth tax effects is consistent not only with the evidence in the Survey of Consumer Finances of wealthier entrepreneurs managing larger firms, but also with empirical evidence from Compustat supporting a positive relationship between firm size and markups in the U.S..

When a progressive top wealth tax is implemented in this setting, the tax burden falls onto the wealthiest entrepreneurs who impose the largest markups. Thus, the tax reduces the aggregate markup in the economy, increasing the labor share of income accruing to poor workers and reducing markups-induced distortions on GDP. However, by taxing the most productive entrepreneurs, the tax still reduces GDP and wages received by workers. I investigate how these effects change when instead market power heterogeneity across entrepreneurs is neglected, and all entrepreneurs impose homogeneous and constant markups, equal to the average one in the heterogeneous markups case.

I show that taking into account that wealthier entrepreneurs own firms with larger market power, relaxes the equity-efficiency trade-off of top wealth taxation with respect to the case in which this market power heterogeneity is neglected. Indeed, top wealth taxation induces smaller losses in capital accumulation, steady-state production and wages in the economy where entrepreneurs impose heterogeneous markups. The reason is that in this economy, taxed entrepreneurs charge higher markups than in the homogeneous-markup case and therefore exhibit lower production and capital elasticities with respect to the tax. This result delivers a broad and novel message in the public finance literature: *the markup distribution of entrepreneurs shapes how the economy reacts to the tax.*

Thus, when observed market power heterogeneity is neglected, a wealth tax raising 1% of GDP imposed on the wealthiest 1% of U.S. households overestimates wage losses induced by the tax by 1.4 p.p. and output losses by 1.1 p.p.

Although the focus of this paper is on the role of *product* market power heterogeneity in shaping the outcomes of top wealth taxation, several contributions have shown that in the U.S. firm heterogeneity is also associated with sizable *labor* market power heterogeneity (Yeh et al., 2022) across them. Exploring whether *labor* market power distortions would dampen or amplify my results is thus a natural extension of this framework. This, would allow me to explore in a more comprehensive way the role of entrepreneurs' market power in shaping top wealth tax outcomes.

Appendix A: Proofs

Proof of Lemma 1

Consider equation (1.4). Using the expression for the elasticity of demand of intermediate good produced by entrepreneur i reported in (1.3):

$$\mathcal{E}_i^d(q_i) = -\frac{\Upsilon_i'(q_i)}{q_i \Upsilon_i''(q_i)}$$

it is possible to re-write equation (1.4) as:

$$P\left(\Upsilon_i'(q_i^*) + q_i^* \Upsilon_i''(q_i^*)\right) q_i^{*-\frac{\nu}{1-\nu}} - \frac{wY^{\frac{\nu}{1-\nu}}}{1-\nu}\left(\frac{1}{z_i k_i^{\nu}}\right)^{\frac{1}{1-\nu}} = 0$$

Define the left hand side of the previous equation as the function $F_i(q_i^*, z_i, k_i, P, Y)$ which allows to re-write it as:

$$F_i(q_i^*, z_i, k_i, P, Y) = 0$$

Now, let's use the Implicit Function Theorem to show that $\frac{\partial q_i^*}{\partial z_i} > 0$ and $\frac{\partial q_i^*}{\partial k_i} > 0$. The proof to obtain the sign of the other partial derivatives reported in Lemma 1 is analogous. It is possible to show that:

$$\frac{\partial F_i(\cdot)}{\partial q_i^*} = P\left(2\Upsilon_i''(q_i^*) + q_i^* \Upsilon_i'''(q_i^*)\right) q_i^{*-\frac{\nu}{1-\nu}} - P\frac{\nu}{1-\nu}\left(\Upsilon_i'(q_i^*) + q_i^* \Upsilon_i''(q_i^*)\right) q_i^{*-\frac{\nu}{1-\nu}-1} < 0 \tag{1.16}$$

The reason why the previous derivative is negative is that both terms are negative. Indeed, using Assumption 1 it is possible to show that $2\Upsilon_i''(q_i^*) + q_i^* \Upsilon_i'''(q_i^*) \leq 0$ for all $q_i^* \geq 0$. Furthermore, $\Upsilon_i'(q_i^*) + q_i^* \Upsilon_i''(q_i^*) > 0$. The way to show it is the following. Equation (1.4) guarantees that a profit maximizing entrepreneur will always choose q_i^* which satisfies $\mathcal{E}_i^d(q_i^*) > 1$. Using the formula for the elasticity of demand (1.3), $\mathcal{E}_i^d(q_i^*) > 1$ rewrites as: $\Upsilon_i'(q_i^*) + q_i^* \Upsilon_i''(q_i^*) > 0$. Now let's compute the following partial derivatives:

$$\frac{\partial F_i(\cdot)}{\partial z_i} = \frac{wY^{\frac{\nu}{1-\nu}}}{(1-\nu)^2}\left(\frac{1}{z_i k_i^{\nu}}\right)^{\frac{1}{1-\nu}} \frac{1}{z_i} > 0 \qquad \frac{\partial F_i(\cdot)}{\partial k_i} = \frac{\nu wY^{\frac{\nu}{1-\nu}}}{(1-\nu)^2}\left(\frac{1}{z_i k_i^{\nu}}\right)^{\frac{1}{1-\nu}} \frac{1}{k_i} > 0 \tag{1.17}$$

Hence, the implicit function theorem guarantees that:

$$\frac{\partial q_i^*}{\partial z_i} = -\frac{\frac{\partial F_i(\cdot)}{\partial z_i}}{\frac{\partial F_i(\cdot)}{\partial q_i^*}} > 0 \qquad\qquad \frac{\partial q_i^*}{\partial k_i} = -\frac{\frac{\partial F_i(\cdot)}{\partial k_i}}{\frac{\partial F_i(\cdot)}{\partial q_i^*}} > 0$$

□

Proof of Lemma 2

First of all, I show that $\frac{\partial \mathcal{N}_i^*(\cdot)}{\partial z_i} > 0$ if Assumption 1 and Assumption 2 hold. The way of showing that $\frac{\partial \mathcal{N}_i^*(\cdot)}{\partial k_i} > 0$ is analogous. Notice that:

$$\frac{\partial \mathcal{N}_i^*(\cdot)}{\partial z_i} > 0 \iff \frac{\partial}{\partial z_i}\left(\left(\frac{\mathcal{Q}_i^*(\cdot)}{z_i k_i^\nu}\right)^{\frac{1}{1-\nu}}\right) Y^{\frac{1}{1-\nu}} > 0 \iff \frac{\partial \mathcal{Q}_i^*(\cdot)}{\partial z_i}\frac{z_i}{q^*(\cdot)} > 1$$

where $\mathcal{Q}_i^*(\cdot)$ is the optimal relative quantity function whose arguments are (z_i, k_i, w, Y, P). To shorten notation let $q_i^* = \mathcal{Q}_i^*(z_i, k_i, w, P, Y)$. Using equations (1.16) and (1.17) is it possible to show that:

$$\frac{\partial q_i^*}{\partial z_i}\frac{z_i}{q_i^*} = -\frac{\Upsilon_i'(q_i^*) + q_i^*\Upsilon''(q_i^*)}{(2\Upsilon_i''(q_i^*) + q_i^*\Upsilon_i'''(q_i^*))q_i^*(1-\nu) - \nu(\Upsilon_i'(q_i^*) + q_i^*\Upsilon_i''(q_i^*))}$$

which is positive if:

$$-\frac{\Upsilon_i'(q_i^*)}{q_i^*\Upsilon_i''(q_i^*)} > 3 + \frac{q_i^*\Upsilon_i(q_i^*)}{\Upsilon_i''(q_i^*)}$$

which holds under Assumption 2. Using the expression for the function $\mathcal{N}_i^*(\cdot)$:

$$\mathcal{N}_i^*(z_i, k_i, w, P, Y) = \left(\frac{\mathcal{Q}_i^*(z_i, k_i, w, P, Y)\cdot Y}{z_i k_i^\nu}\right)^{\frac{1}{1-\nu}}$$

it is immediate to show that $\frac{\partial \mathcal{N}_i^*(\cdot)}{\partial w} < 0$ and $\frac{\partial \mathcal{N}_i^*(\cdot)}{\partial P} < 0$ since Lemma 1 shows that $\frac{\partial \mathcal{Q}_i^*(\cdot)}{\partial w} < 0$ and $\frac{\partial \mathcal{Q}_i^*(\cdot)}{\partial P} < 0$.

Now, let's show that $\frac{\partial \mathcal{N}_i^*(\cdot)}{\partial Y} > 0$. To see that, first of all notice that:

$$\frac{\partial \mathcal{N}_i^*(\cdot)}{\partial Y} = \left(\frac{1}{k_i^\nu z_i}\right)^{\frac{1}{1-\nu}}\left(\frac{\partial q_i^*}{\partial Y}Y + q_i^*\right)^{\frac{1}{1-\nu}}(q_i^* Y)^{\frac{1}{1-\nu}}\frac{1}{1-\nu} > 0 \iff \frac{\partial q_i^*}{\partial Y}\frac{Y}{q_i^*} > -1$$

where, again, to shorten notation, $q_i^* = \mathcal{Q}_i^*(z_i, k_i, w, P, Y)$. Using equation (1.16) it is possible to compute:

$$\frac{\partial q_i^*}{\partial Y}\frac{Y}{q_i^*} = \left(\frac{2\Upsilon_i''(q_i^*) + q_i^*\Upsilon_i''(q_i^*)}{\Upsilon_i'(q_i^*) + q_i^*\Upsilon_i''(q_i^*)} q_i^* \frac{1-\nu}{\nu} - 1\right)^{-1}$$

Hence, to have $\frac{\partial q_i^*}{\partial Y}\frac{Y}{q_i^*} > -1$, rearranging the previous expression, it must hold:

$$-\frac{2\Upsilon_i''(q_i^*) + q_i^*\Upsilon_i''(q_i^*)}{\Upsilon_i'(q_i^*) + q_i^*\Upsilon_i''(q_i^*)} > 0$$

which under Assumption 1 is true since, as the proof of Lemma 1 shows, we both have that $2\Upsilon_i''(q_i^*) + q_i^*\Upsilon_i''(q_i^*) < 0$ and $\Upsilon_i'(q_i^*) + q_i^*\Upsilon_i''(q_i^*) > 0$ for every $q_i^* = \mathcal{Q}_i^*(z_i, k_i, w, P, Y)$ □

Lemma 3. *Let Assumption 1 and 2 hold and let $\Pi_i^*(z_i, k_i, w, P, Y)$ be the function which associates to each tuple (z_i, k_i, w, P, Y) the profits entrepreneur i makes when producing $\mathcal{Q}_i^*(z_i, k_i, w, P, Y)$. It holds:*

$$\frac{\partial \Pi_i^*(\cdot)}{\partial z_i} > 0 \qquad \frac{\partial \Pi_i^*(\cdot)}{\partial k_i} > 0 \qquad \frac{\partial \Pi_i^*(\cdot)}{\partial w} < 0 \qquad \frac{\partial \Pi_i^*(\cdot)}{\partial P} < 0 \qquad \frac{\partial \Pi_i^*(\cdot)}{\partial Y} > 0$$

Proof of Proposition 1: Consider equation (1.4) and re-write it as:

$$F_i(q_i) = P\Upsilon_i'(q_i)q_i^{-\frac{\nu}{1-\nu}}\Lambda(z_i k_i^{\nu})^{\frac{1}{1-\nu}} - \mu_i(q_i) = 0$$

where $\Lambda = \left(\frac{wY^{\frac{1}{1-\nu}}}{1-\nu}\right)^{-1}$ and we employed q_i rather than q_i^* to lighten notation. To use the Implicit function theorem compute:

$$\begin{aligned}\frac{\partial F_i}{\partial q_i} &= (z_i k_i^{\nu})^{\frac{1}{1-\nu}} P\Lambda\left[\Upsilon_i''(q_i)q_i^{-\frac{\nu}{1-\nu}} - \frac{\nu}{1-\nu}\Upsilon_i'(q_i)q_i^{-\frac{\nu}{1-\nu}-1}\right] - \frac{\partial \mu_i(q_i)}{\partial q_i} \\ &= -(z_i k_i^{\nu})^{\frac{1}{1-\nu}} P\Lambda\Upsilon_i'(q_i)q_i^{-\frac{\nu}{1-\nu}-1}\left[\left(\mathcal{E}_i^d(q_i)\right)^{-1} + \frac{\nu}{1-\nu}\right] - \frac{\partial \mu_i(q_i)}{\partial q_i}\end{aligned}$$

Now compute:

$$\frac{\partial F_i}{\partial k_i} = P\Upsilon_i'(q_i)q_i^{-\frac{\nu}{1-\nu}}\Lambda(z_i k_i^\nu)^{\frac{1}{1-\nu}}\frac{\nu}{1-\nu}\frac{1}{k_i}$$

First, notice that $\frac{\partial \mu_i(q_i)}{\partial q_i} = -\frac{1}{(\mathcal{E}_i^d(q_i)-1)^2}\frac{\partial \mathcal{E}_i^d(q_i)}{\partial q_i}$. Using the implicit function theorem and simplifying:

$$\frac{\partial q_i}{\partial k_i}\frac{k_i}{q_i} = \frac{\nu}{1-\nu}\left[\left(\mathcal{E}_i^d(q_i)\right)^{-1} + \frac{\nu}{1-\nu} - \frac{1}{(\mathcal{E}_i^d(q_i)-1)^2}\frac{\partial \mathcal{E}_i^d(q_i)}{\partial q_i}\frac{q_i}{P\Upsilon_i'(q_i)q_i^{-\frac{\nu}{1-\nu}}\Lambda(z_i k_i^\nu)^{\frac{1}{1-\nu}}}\right]^{-1}$$

Using again equation (1.4) notice that: $P\Upsilon_i'(q_i)q_i^{-\frac{\nu}{1-\nu}}\Lambda(z_i k_i^\nu)^{\frac{1}{1-\nu}} = \mu_i(q_i)$. Using this result into the previous equation delivers:

$$\begin{aligned}
\epsilon^{q_i,k_i} &= \frac{\nu}{1-\nu}\left[\left(\mathcal{E}_i^d(q_i)\right)^{-1} + \frac{\nu}{1-\nu} - \frac{\partial \mathcal{E}_i^d(q_i)}{\partial q_i}\frac{q_i}{\mathcal{E}_i^d(q_i)}\frac{\mathcal{E}_i^d(q_i)-1}{(\mathcal{E}_i^d(q_i)-1)^2}\right]^{-1} \\
&= \frac{\nu}{1-\nu}\left[\left(\mathcal{E}_i^d(q_i)\right)^{-1} + \frac{\nu}{1-\nu} + \frac{\epsilon^{\mathcal{E}_i^d}}{\mathcal{E}_i^d(q_i)-1}\right]^{-1}
\end{aligned}$$

which is the required expression for the elasticity of production with respect to capital.

□

Lemma 4. *Let $MP_i^*(k_i) = \frac{\partial \Pi_i^*(k_i,\cdot)}{\partial k_i}$ denote the marginal profits of entrepreneur i when making his optimal production choices (Lemma 3) and define:*

$$\epsilon^{MP_i^*,k_i} = \left|\frac{\partial \log(MP_i^*(k_i))}{\partial \log(k_i)}\right|$$

the elasticity of marginal profits with respect to capital. It holds:

$$\epsilon^{MP_i^*,k_i} = \frac{1}{1-\nu}\left(1-\epsilon^{q_i^*,k_i}\right)$$

where $\epsilon^{q_i^*,k_i}$ is the elasticity of relative production with respect to capital studied in Propo-

sition 1. The $\epsilon^{MP_i^*,k_i}$ decreases with the elasticity of demand faced by entrepreneur i, $\mathcal{E}_i^d(q_i^*)$.

Proof: Consider the static profit maximization problem of an entrepreneur i (E) analyzed in Section 3. To simplify notation, denote the optimal profit function which solves that problem as $\Pi_i^*(k_i)$, neglecting the dependence on the other variables z_i, w, P, Y (see Lemma 3). Using the envelope theorem it is possible to show that:

$$\frac{\partial \Pi_i^*}{\partial k_i} = -\frac{\partial C(q_i^*, k_i)}{\partial k_i}$$

where $C(q_i^*, k_i)$ indicates the total cost function evaluated at the optimal relative production that solves entrepreneur's problem (E), i.e. $q_i^* = \mathcal{Q}_i^*(z_i, k_i, w, P, Y)$:

$$C(q_i^*, k_i) = w\left(\frac{q_i^*}{z_i k_i^{\nu}}\right)^{\frac{1}{1-\nu}} Y^{\frac{1}{1-\nu}}$$

The marginal cost function thus writes:

$$\frac{\partial C(q_i^*, k_i)}{\partial k_i} = -\frac{\nu}{1-\nu} w \left(\frac{q_i^*}{z_i}\right)^{\frac{1}{1-\nu}} Y^{\frac{1}{1-\nu}} k_i^{-\frac{1}{1-\nu}}$$

Thus:

$$\log\left(\frac{\partial \Pi_i^*}{\partial k_i}\right) = \log\left(-\frac{\partial C(q_i^*, k_i)}{\partial k_i}\right) = \log\left(-\frac{\nu}{1-\nu} w \left(\frac{1}{z_i}\right)^{\frac{1}{1-\nu}} Y^{\frac{1}{1-\nu}}\right) + \frac{1}{1-\nu}\log(q_i^*) - \frac{1}{1-\nu}\log(k_i)$$

Taking the derivative of the previous expression with respect to $\log(k_i)$:

$$\frac{\partial \log\left(\frac{\partial \Pi_i^*}{\partial k_i}\right)}{\partial \log(k_i)} = \frac{1}{1-\nu}\left(\frac{\partial \log(q_i^*)}{\partial \log(k_i)} - 1\right) = \frac{1}{1-\nu}\left(\epsilon^{q_i^*,k_i} - 1\right)$$

Hence, using the elasticity definition:

$$\epsilon^{MP_i^*,k_i} = \frac{1}{1-\nu}\left(1 - \epsilon^{q_i^*,k_i}\right)$$

since $\nu < \epsilon^{q_i^*,k_i} < 1$. To show that $\epsilon^{MP_i^*,k_i}$ decreases with $\mathcal{E}_i^d(q_i)$, it is enough to invoke the result of Proposition 1.

□

Appendix B: Klenow and Willis functional form

The Klenow and Willis (2015) functional form for $\Upsilon(\cdot)$ is:

$$\Upsilon(q) = 1 + (\sigma - 1)e^{1/\psi}\psi^{\frac{\sigma}{\psi}-1}\left[\Gamma\left(\frac{\sigma}{\psi}, \frac{1}{\psi}\right) - \Gamma\left(\frac{\sigma}{\psi}, \frac{(q)^{\frac{\psi}{\sigma}}}{\psi}\right)\right]$$

with $\sigma > 1$ and $\psi \geq 0$, and where $\Gamma(s, x)$ denotes the function:

$$\Gamma(s, x) := \int_x^\infty t^{s-1}e^{-t}dt$$

It is possible to show that the first derivative of $\Upsilon(\cdot)$ takes the form:

$$\Upsilon'(q) = \frac{\sigma - 1}{\sigma}\exp\left\{\frac{1 - q^{\psi/\sigma}}{\psi}\right\}$$

starting from $\Upsilon'(q)$, standard algebra also delivers the expression for $\Upsilon''(q)$. Those expressions can be plugged into the formula for the elasticity of demand derived in (1.3):

$$\mathcal{E}^d(q_i) = -\frac{\Upsilon'(q_i)}{q_i\Upsilon''(q_i)}$$

delivering:

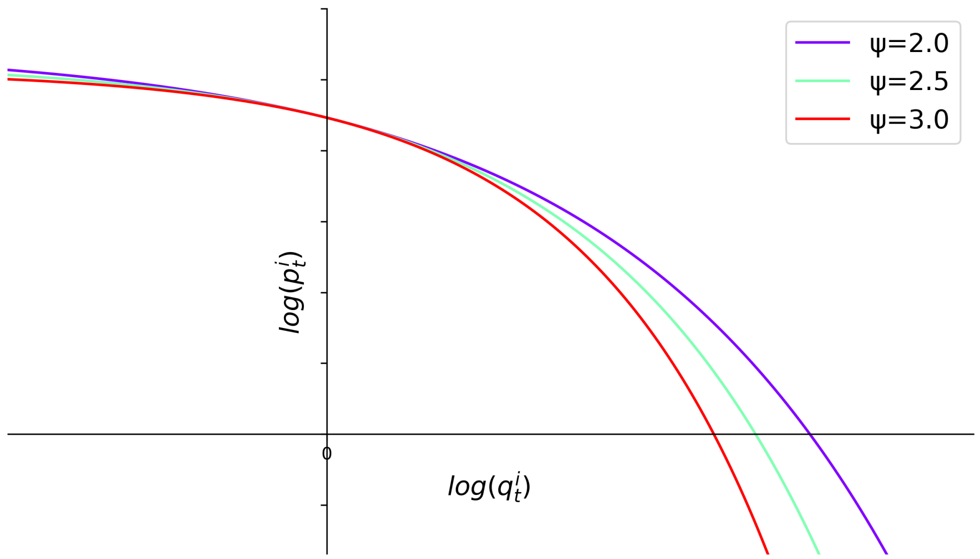
$$\mathcal{E}^d(q_i) = \sigma(q_i)^{-\frac{\psi}{\sigma}}$$

Finally, using the markup definition:

$$\mu(q_i) = \frac{\mathcal{E}^d(q_i)}{\mathcal{E}^d(q_i) - 1} = \frac{\sigma}{\sigma - q_i^{\frac{\psi}{\sigma}}}$$

The following Figure plots an instance of the shape of the demand function for the entrepreneur's $i \in I$ good: $p_i = P\Upsilon'(q_i)$ when $\Upsilon'(\cdot)$ takes the Klenow and Willis (2016)

FIGURE B1.1. *Demand for the intermediate goods with Klenow and Willis functional form for $\Upsilon(\cdot)$, $\sigma = 6$ and varying values for ψ*



functional form. The demand function is plotted for $\sigma = 6$ (employed in the calibration of Section 4.2) and several values of ψ, showing how this parameter regulates the concavity of the demand function.

Appendix C: Additional Tables and Figures

TABLE C1.1. *Model with const. and heterogeneous markups calibration: summary*

Par.	Description	Value	Target
ω	fraction of workers	0.88	non-entrepreneur households in SCF 2019
ν	capital exponent prod.	0.28	labor share = 0.6
x_z	scale par. entr. ability dist.	0.12	observed returns to entrepreneurship
η_z	shape par. entr. ability dist.	4.1	observed returns to entrepreneurship
α_0	$k_i = k(z_i) = \alpha_0 z_i^{\alpha_1}$	868	min. wealth = 1
α_1	$k_i = k(z_i) = \alpha_0 z_i^{\alpha_1}$	3.25	tail parameter entrepreneurial wealth 1.25

Notes: *the Table summarizes the calibrated model's parameters values. The first column indicates the symbol used to identify the parameter in the model, the second column the parameter description, the third column the chosen value for the parameter, the fourth column the moment targeted to calibrate each parameter.*

Table C1.2. *Model with const. and homogeneous markups calibration: summary*

Par.	Description	Value	Target
ω	fraction of workers	0.88	non-entrepreneur households in SCF 2019
ν	capital exponent prod.	0.28	labor share = 0.6
x_z	location par. entr. ability dist.	0.15	observed returns to entrepreneurship
η_z	scale par. entr. ability dist.	5.5	observed returns to entrepreneurship
σ	demand elasticity	6	$\mathcal{M} = 1.2$
α_0	$k_i = k(z_i) = \alpha_0 z_i^{\alpha_1}$	186	min. wealth = 1
α_1	$k_i = k(z_i) = \alpha_0 z_i^{\alpha_1}$	3.96	tail parameter entrepreneurial wealth 1.25

Notes: *the Table summarizes the calibrated model's parameters values. The first column indicates the symbol used to identify the parameter in the model, the second column the parameter description, the third column the chosen value for the parameter, the fourth column the moment targeted to calibrate each parameter.*

Table C1.3. *Steady-state distribution of markups (cost-weighted)*

	Compustat - Edmond et al. (2023)	Simulated model
aggregate markup $\mathcal{M}$	1.26	1.20
p25	0.97	1.11
p50	1.12	1.18
p75	1.31	1.25
p90	1.69	1.45

Notes: *the Table reports some descriptive statistics of the markups distribution estimated in the data by Edmond et al. (2023) (first column) and simulated at the steady-state of the model (second column). The statistics have been obtained computing the cost-weighted percentiles of the steady-state markups distribution, where the weight associated to each observation is given by the share of labor employed by each firm n_i/N.*

Table C1.4. *Constant markups steady-state: externally calibrated parameters*

Par.	Description	Value	Target
ω	fraction of workers	0.88	fraction of non-entr.
γ	CRRA par. utility	1	-
ν	capital exponent entr. prod.	0.28	Labor share entr. sect. = 0.6
α	capital exponent mkt sector prod.	0.4	Labor share mkt. sector = 0.6
σ	elasticity of demand	6	markups = 1.2

Notes: *the Table summarizes the parameter choices to calibrate the steady state of the dynamic model presented in Section 5 with constant and homogeneous markups across entrepreneurs. The first column indicates the symbol used to identify the parameter in the model, the second column the parameter description, the third column the chosen value for the parameter, the fourth column the moment targeted to calibrate each parameter.*

Table C1.5. *Constant markups steady-state: internally calibrated parameters*

Par.	Description	Value	Target	Data	Model
β	discount factor	0.91	wealth / output	4	3.5
δ	depreciation rate	0.015	entr. wealth fract.	0.44	0.49
A	TFP market sector	2.5	Y^M/Y	0.43	0.47
$\bar{z}$	av. entrep. skills	1	workers in top 1%	0.17	0.13
ρ_e	persistence worker skills	0.95	top 1% wealth	0.36	0.37
σ_e^2	var. innovation worker skill	0.25	top 5% wealth	0.65	0.59
ρ_z	persistence entr. skill	0.9	top 10% wealth	0.77	0.73
σ_z^2	var. innovation entr. skills	0.4	Gini wealth	0.88	0.86
			top 1% capital	0.42	0.39
			top 5% capital	0.71	0.75
			top 10% capital	0.83	0.87

Notes: *the Table summarizes the parameter choices to calibrate the steady state of the dynamic model presented in Section 5 with constant and homogeneous markups across entrepreneurs. The first column indicates the symbol used to identify the parameter in the model, the second column the parameter description, the third column the chosen value for the parameter, the fourth column the moment targeted to calibrate each parameter, the fifth column the value of the targeted moment in the data, the sixth column the value of the targeted moment in the simulated model,*

Table C1.6. *Model with occupational choice and heterogeneous markups calibration*

Par.	Description	Value	Target
ν	capital exponent prod.	0.28	labor share = 0.6
x_z	scale par. entrepreneurial ability dist.	0.2	observed returns to entrepreneurship
η_z	shape par. entrepreneurial ability dist.	5	observed returns to entrepreneurship
σ	demand elasticity when $q = 1$	10.6	$\mathcal{M} = 1.2$
ψ	shape par. demand elasticity	1.74	$\psi/\sigma = 0.16$
α_0	$k_i = k(z_i) = \alpha_0 z_i^{\alpha_1}$	157	min wealth = 1
α_1	$k_i = k(z_i) = \alpha_0 z_i^{\alpha_1}$	3.97	tail parameter entrepreneurial wealth 1.25
f	fixed cost	0.011	fraction of entrepreneurs

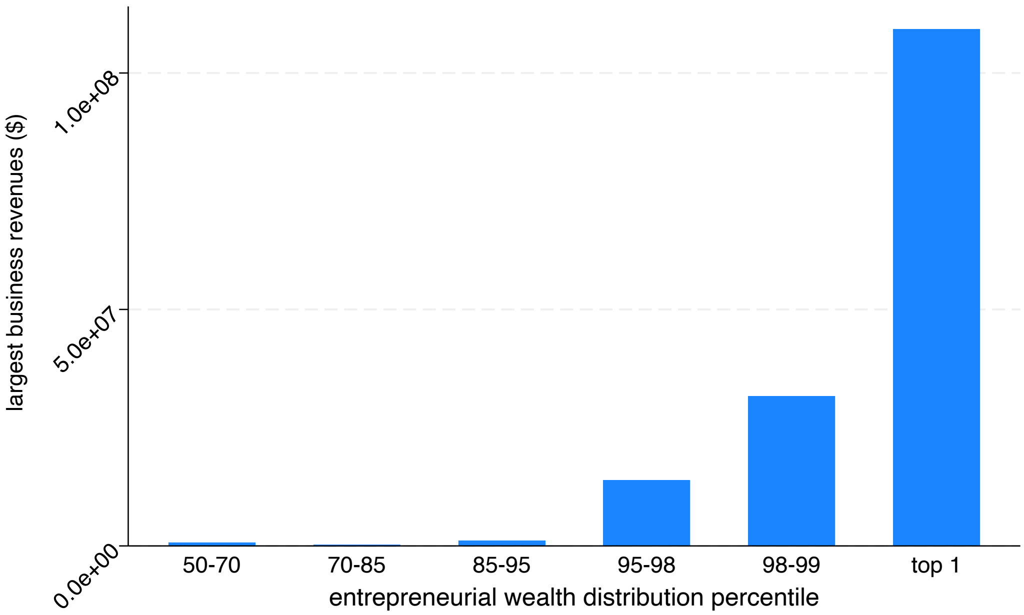
Notes: *the Table summarizes the calibrated parameters values of the model with endogenous occupational choice and entrepreneurs facing demand function for their own variety with variable elasticity of demand. The first column indicates the symbol used to identify the parameter in the model, the second column the parameter description, the third column the chosen value for the parameter, the fourth column the moment targeted to calibrate each parameter.*

Table C1.7. *Model with occupational choice and constant markups calibration*

Par.	Description	Value	Target
ν	capital exponent prod.	0.28	labor share = 0.6
x_z	scale par. entr. ability dist.	0.28	observed returns to entrepreneurship
η_z	shape par. entr. ability dist.	5.4	observed returns to entrepreneurship
σ	demand elasticity	6	$\mathcal{M} = 1.2$
α_0	$k_i = k(z_i) = \alpha_0 z_i^{\alpha_1}$	157	min wealth = 1
α_1	$k_i = k(z_i) = \alpha_0 z_i^{\alpha_1}$	3.97	tail parameter entrepreneurial wealth 1.25
f	fixed cost	0.0515	fraction of entrepreneurs in SCF (2019)

Notes: *the Table summarizes the calibrated parameters values of the model with endogenous occupational choice and entrepreneurs facing demand function for their own variety with constant elasticity of demand. The first column indicates the symbol used to identify the parameter in the model, the second column the parameter description, the third column the chosen value for the parameter, the fourth column the moment targeted to calibrate each parameter.*

Figure C1.1. *Revenues in largest (private) actively managed business*

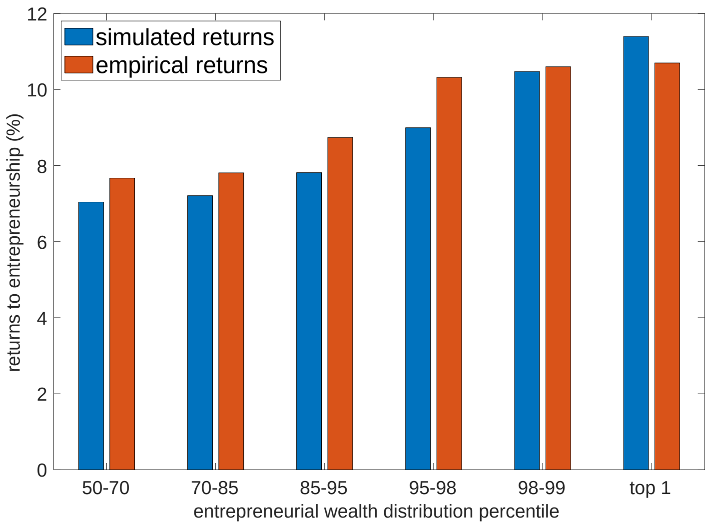


Notes: *the Figure reports the average revenues in the largest private actively managed business across the wealth distribution. The value of each column is computed by averaging the number of employees in the largest actively managed business across entrepreneurs belonging to the same wealth percentile bin. The definition of entrepreneur is reported in Section 2.1. Data from 2019 Survey of Consumer Finances*

Figure C1.2. *Simulated vs empirical returns to entrepreneurship: markups increasing with firm's market share model*

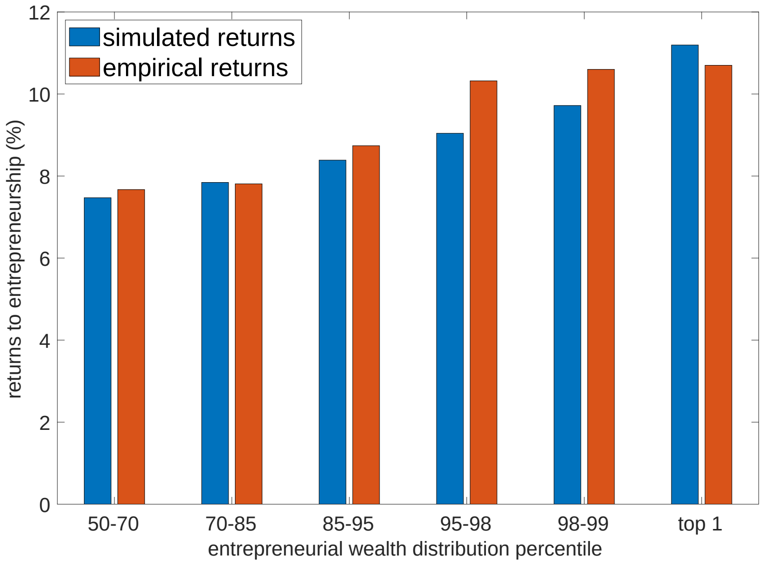
Notes: *the Figure reports the simulated returns to entrepreneurship (blue) and the estimated returns to entrepreneurship (orange). The simulated returns are computed by averaging across wealth percentiles bins the average returns π_i/k_i (for calibration details see Table 3). The estimated returns are those reported in Figure 5.*

FIGURE C1.3. *Simulated vs empirical returns to entrepreneurship: heterogeneous and constant markups model*



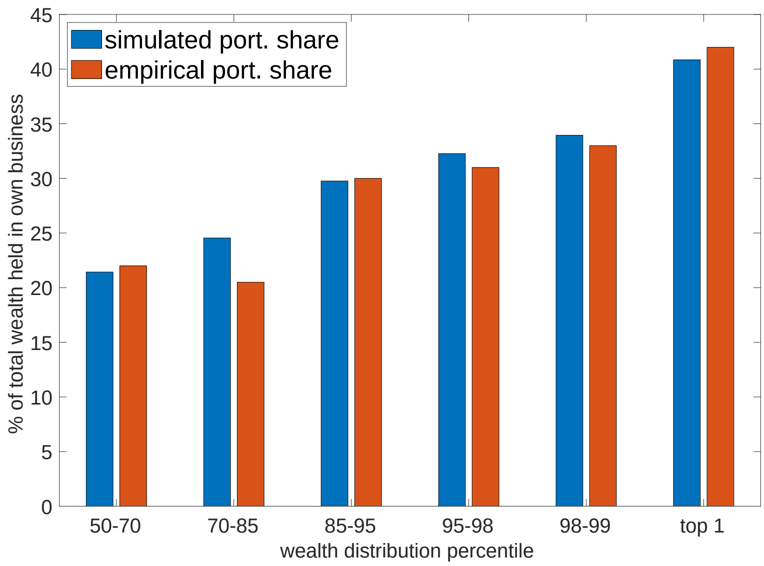
Notes: *the Figure reports the simulated returns to entrepreneurship in the model in which entrepreneurs impose heterogeneous but constant markups (blue) and the estimated returns to entrepreneurship (orange). The simulated returns are computed by averaging across wealth percentiles bins the simulated returns (for calibration details see Table* **??***). The estimated returns are those reported in Figure 5.*

FIGURE C1.4. *Simulated vs empirical returns to entrepreneurship: constant markups model*



Notes: *the Figure reports the simulated returns to entrepreneurship in the model in which entrepreneurs impose constant markups (blue) and the estimated returns to entrepreneurship (orange). The simulated returns are computed by averaging across wealth percentiles bins the simulated returns (for calibration details see Table* **??***). The estimated returns are those reported in Figure 5.*

FIGURE C1.5. *Simulated vs empirical portfolio shares at the steady-state*



Notes: *the Figure reports the simulated fraction of net wealth ($\phi^i = \phi(z^i)$) that entrepreneurs hold in their business at the steady-state (blue) and the estimated portfolio shares (orange, see Figure 2). The simulated portfolio shares are computed by averaging across wealth percentiles bins the simulated portfolio shares.*

Appendix D: Static model with occupational choice

How does the endogenous occupational choice affect wage and output losses in the two economies studied in Sections 4? This Section argues that even when endogenous occupational choice is allowed, the same revenue equivalent wealth tax induces larger output and wage losses in the economy in which entrepreneurs impose constant markups.

D.1 Model and calibration

I now suitably modify the model studied in Section 3 to allow for endogenous occupational choice. Assume that all households $i \in [0, 1]$ are endowed with entrepreneurial skills z_i drawn from a Pareto distribution with cdf $F(z)$ and support $[\underline{z}, \infty)$ (with $\underline{z} > 0$) and wealth $k_i = k(z_i)$. Each household can now choose between becoming a worker or an entrepreneur:

- A worker receives the wage w. For simplicity all households, when workers, are assumed to inelastically supply a unit of labor. Furthermore, when a household is a worker he invests his wealth k_i in a risk-free investment opportunity with zero return. Hence, the consumption of each household $i \in [0, 1]$ who decides to be a worker is: $c_i = w$.

- An entrepreneur receives profits from his entrepreneurial activity. Each entrepreneur solves the profit maximization problem (E)[22]. Furthermore, to become entrepreneur an household has to pay the fixed cost $f > 0$.[23]

Each household $i \in [0, 1]$ makes his occupational choice comparing his consumption when he decides to be a worker with consumption in the entrepreneurial occupation. Formally, each household $i \in [0, 1]$ becomes entrepreneur if:

$$\pi^*(z_i, k(z_i), w, P, Y) - f \geq w$$

[22] When I will study the effects of wealth taxation in the economy in which entrepreneurs impose constant markups the problem to be solved will be (E').

[23] A fixed cost is needed since without it the model would not able to replicate all the calibration targets matched in the previous analysis without occupational choice, *plus* the fraction of workers and entrepreneurs observed in the SCF data (which before was exogenous). More details about calibration will follow.

where $\pi^*(\cdot)$, see equation (1.5), denotes the optimal profits made by entrepreneur i when solving problem (E). If the function $\Upsilon(\cdot)$ takes either the Klenow and Willis (2016) functional form (see (1.9)) or $\Upsilon(q) = q^{\frac{\sigma-1}{\sigma}}$ (which will be the two functional forms used when calibrating the model) it is possible to show that $\pi^*(\cdot)$ is monotonically increasing in z_i while labor income w is independent of z_i. Thus, it is possible to define an occupational choice threshold $\hat{z}$ such that:

$$\pi^*(\hat{z}, k(\hat{z}), w, P, Y) - f = w$$

and all households with skills $z_i \geq \hat{z}$ become entrepreneurs, while all households with skills $z_i < \hat{z}$ become workers.

Equilibrium: The equilibrium of this static economy with occupational choice is a set of aggregates $\{w^*, Y^*, P^*\}$, an occupational choice threshold $\hat{z}$, a vector of quantities consumed by each household (workers and entrepreneurs) $\{c_i^*\}_{i\in[0,1]}$, relative quantity function $q^*(z_i, k(z_i), w^*, P^*, Y^*)$, labor demand function $n^*(z_i, k(z_i), w^*, P^*, Y^*)$, profit function $\pi^*(z_i, k(z_i), w^*, P^*, Y^*)$ such that:

- Each worker i consumes his labor income $c_i^* = w^*$
- Given the aggregates $\{w^*, Y^*, P^*\}$ the functions $q^*(z_i, k_i, w^*, P^*, Y^*)$, $n^*(z_i, k_i, w^*, P^*, Y^*)$, $\pi^*(z_i, k_i, w^*, P^*, Y^*)$ solve the entrepreneur's i problem (E)
- The occupational choice threshold $\hat{z}$ is such that:

$$\pi^*(\hat{z}, k(\hat{z}), w^*, P^*, Y^*) - f = w^*$$

- Labor market clears:

$$\int_{\underline{z}}^{\hat{z}} F(z)dz = \int_{\hat{z}}^{\infty} n^*(z, k(z), w^*, P^*, Y^*)F(z)dz$$

- Kimball aggregator is satisfied:

$$\int_{\hat{z}}^{\infty} \Upsilon\left(q^*(z, k(z), w^*, P^*, Y^*)\right) F(z)dz = 1$$

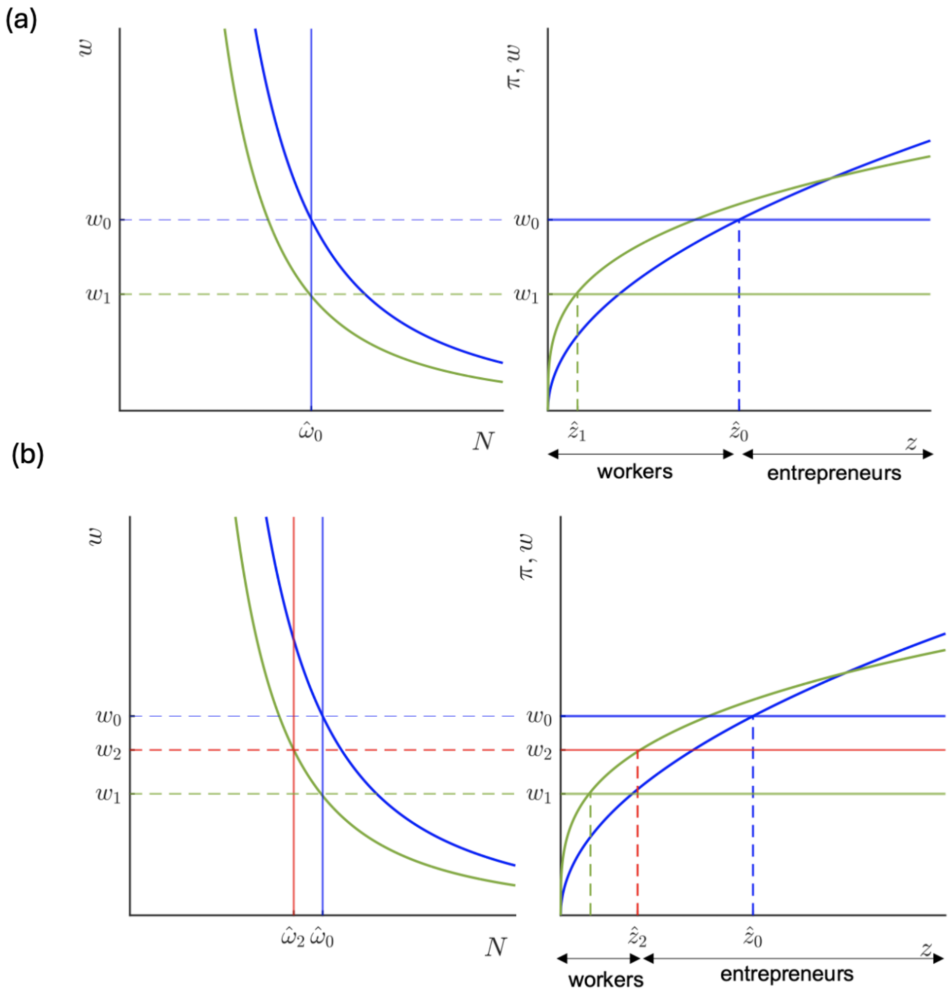
Calibration: the model with occupational choice is calibrated so to match the same targets of the models without occupational choice presented in the previous Sections (observed returns, observed wealth distribution, aggregate markup $\mathcal{M} = 1.2$, labor share). The only difference in the calibration procedure of the model with occupational choice is that the fixed cost f is calibrated so to have the fraction of households who decide to be entrepreneurs equal to the fraction of households defined as entrepreneurs in the SCF 2019 data (0.12). Furthermore, notice that to study the economy in which entrepreneurs impose markups increasing in their market shares, the Klenow and Willis (2016) functional for for $\Upsilon(\cdot)$ will be used (see equation 1.9). Instead, to study the economy in which entrepreneurs impose constant markups the functional form chosen for $\Upsilon(\cdot)$ will be $\Upsilon(q) = q^{\frac{\sigma-1}{\sigma}}$. Details on the calibrated parameters are reported in Appendix C, Tables C1.6 and C1.7.

D.2 Wealth tax experiment with occupational choice

Figure D1.1 shows how allowing for endogenous occupational choice changes the aggregate effects of wealth taxation. First of all consider panel (a). The concave blue line in the right hand plot represents profits as a function of entrepreneurial skills and the horizontal line equilibrium wage. Their intersection at $(w_0, \hat{z}_0)$ identifies the equilibrium wage and occupational choice threshold in the initial equilibrium of the economy, when no tax is implemented. The blue lines in the left plot, instead, represent aggregate labor supply and labor demand functions. Notice that while labor demand is downward sloped, the labor supply curve is vertical. Indeed, when the measure of workers in the economy is $\hat{\omega}_0$, for any wage offered the aggregate labor supply will just be the measure of workers available for production $\hat{\omega}_0$.

As I showed in Figure 10 the introduction the wealth tax reduces profits for wealthiest entrepreneurs and increases profits for poorer entrepreneurs, thus the profits function after the wealth tax is implemented becomes the one in green. Furthermore, the wealth tax reduces aggregate labor demand, which shifts to the left (green curve in the left plot, panel (a)). Suppose just for the moment that the measure of workers is exogenously fixed at $\hat{\omega}_0$ (as if there was no occupational choice). The intersection between labor supply and labor demand at $(\hat{\omega}_0, w_1)$ determines the new equilibrium wage, w_1, once the tax is implemented. Furthermore, notice that all workers between $\hat{z}_1$ and $\hat{z}_0$ now would like to

FIGURE D1.1. *Wealth tax effects on occupational choice threshold and wage*



Notes: *Panel (a): the left plot reports aggregate labor supply and labor demand curves of the analyzed economy. The right plot reports equilibrium wage and profits as a function of productivity. Blue lines represent these curves before the wealth tax is implemented. Green lines represent these curves after the wealth tax is implemented but keeping labor supply fixed at the initial level. Panel (b): the curves in red represent the same curves in panel (a) but once the wealth tax is implemented and labor supply is allowed to vary.*

become entrepreneurs but they cannot since the number of workers has been exogenously fixed.

Now let's look at panel (b) of Figure D1.1 which plots in red the new labor supply and equilibrium wage once I allow households to freely choose their occupation. The workers willing to become entrepreneurs induce a reduction in labor supply (labor supply shifts to the left) and the intersection with labor demand at $(w_2, \hat{\omega}_2)$ determines the new equilibrium wage w_2. Hence, notice that, once I allow occupational choice the reduction in equilibrium wage due to the wealth tax is lower and there are more entrepreneurs producing: $\hat{z}_2 < \hat{z}_0$.

TABLE D1.1. *Wealth tax aggregate effects in the model with occupational choice: simulation results*

	Heterogeneous markups		Constant markups	
(%)	fixed $\hat{z}$	end. $\hat{z}$	fixed $\hat{z}$	end. $\hat{z}$
Δw	-0.16	-0.13	-0.22	-0.148
ΔN	0	-0.032	0	-0.046
ΔK	-0.613	-0.50	-0.613	-0.474
ΔZ	-0.023	-0.025	-0.034	-0.038
$\Delta \mathcal{M}$	-0.025	-0.031	0	0
ΔY	-0.18	-0.18	-0.22	-0.194

Notes: *the Table summarizes the effects of the wealth tax policy described in 4.2 on equilibrium wage, aggregate employment, aggregate capital, aggregate productivity, aggregate markup, aggregate production. These effects are obtained simulating the model with occupational choice calibrated in 6.1. The wealth tax effects are computed first keeping the occupational choice threshold $\hat{z}$ fixed, and then letting $\hat{z}$ vary after the tax implementation*

The model simulations allow to quantify the previously described effects. They are reported in Table D1.1. The first two columns report how the wealth tax affects several aggregates when the model in which entrepreneurs impose heterogeneous markups is simulated, first keeping the occupational threshold $\hat{z}$ fixed and then allowing $\hat{z}$ to change once the wealth tax is implemented. The same exercise is repeated for the economy in which all entrepreneurs impose the same markups and the results are reported in columns 3-4 of Table D1.1.

Notice that in both economies allowing $\hat{z}$ to change once the tax is implemented reduces the drop in equilibrium wage and aggregate capital used for production, with respect to the case in which the measure of workers is fixed. Furthermore, in both economies, the entry of new entrepreneurs (who have low productivity) reduces aggregate productivity and also aggregate markup in the economy in which entrepreneurs impose heterogeneous markups. The reason is that the newly entered entrepreneurs have low productivity, produce at a small scale and hence apply small markups. Finally, notice that the magnitude of all these affects is larger in the economy where entrepreneurs impose constant markups. The reason for this is the larger increase of profits of poorer entrepreneurs and the larger reduction of wage in the economy with constant markups. However, even when allowing households to make an occupational choice the considered wealth tax reduces aggregate production and equilibrium wage more in the economy where entrepreneurs impose constant markups.

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Chapter 2: the portfolio composition effect of wealth taxation

Abstract

In this paper, I study how the introduction of a top wealth tax affects households' portfolio choices and, consequently, the allocation of capital in the economy, GDP, and GDP growth. To do so, I develop a portfolio choice model that replicates U.S. households' investment decisions in private equity, public equity, and safe assets. I use this model as a metering device to quantify the effects of wealth taxation on households' investment choices. I find that taxed households reduce their investment in private equity and, to a lesser extent, in public equity, while increasing their holdings of safe assets. As a result, the wealth tax induces a reallocation of capital from private (and, to a smaller extent, public) equity investments toward safe assets. I further document that private equity investments are primarily directed toward highly productive, high-growth sectors. Therefore, the capital reallocation triggered by the wealth tax leads not only to a reduction in the level of GDP but also to a slowdown in GDP growth.

1 Introduction

The long-standing debate on the desirability of wealth taxation has largely focused its attention on the long-run distortions on saving decisions induced by those taxes. The advocates of wealth taxation, instead, have overlooked those distortionary effects (Saez and Zucman (2019)), claiming that wealth taxation is a powerful tool to restore the progressivity of the tax system at the top of the wealth distribution (Saez and Zucman (2022)). However, the introduction of a wealth tax also generates other effects beside those on household saving choices. When taxed households become poorer: hence they may have access to fewer investment opportunities, face liquidity constraints or change their risk taking behaviour. Those effects induce households to change their portfolio choices, generating effects on the process of capital supply in the economy. The consequence of capital moving across sectors characterized by more or less productive firms, induces changes in GDP. Furthermore, the capital flows across sectors characterized by different productivity growth rates induces changes in GDP growth too.
The contribution of this paper is to analyze the effects of wealth taxation on household portfolio choices and the implied capital allocation effect across sectors. This will be done assuming away the distortionary effect of taxes on the household saving decisions. In this way I will shed light on some mechanisms that have been largely disregarded by the existing literature, such as how wealth taxes affect household portfolio choices and the induced capital allocation across different sectors of the economy.

To analyze the aggregate effects of wealth taxation it is necessary, first of all, to understand how households invest their wealth. In Section 2, indeed, I will explore the portfolio choices of US households across the whole wealth distribution. While those at the bottom and the middle of the wealth distribution invest essentially in real estate and safe assets, those at the top of the wealth distribution are more exposed towards risky assets. In particular, those at the top 1% of the wealth distribution invest significant shares of their wealth into private equity investments. Those are very risky but also very rewarding investment opportunities, since private equity funds have the capabilities of selecting projects with great potential in terms of growth and innovation. Venture capital funds, which invest in extremely promising and innovative start-ups are only an example of that kind of investment. On the contrary, public equity investments, convey funds to less innovative, more traditional firms and sectors.

As a consequence, understanding whether households invest in private, public equity or other investment opportunities becomes crucial to understand to which kind of firms they supply their capital to. As a matter of fact, supplying capital to more or less productive firms has immediate consequences on GDP.

This reasoning explains the choice of introducing in Section 3 a portfolio choice model which captures how households invest their wealth in terms of public equity, private equity and other (safer) assets. In particular, I will consider a two period portfolio choice problem in which households differ for their initial wealth only (and as a consequence risk aversion). Each household at time 0 allocates his initial (after-tax) wealth among public equity, private equity and a safe asset. At time 1 each one consumes the wealth resulting from investment. Notice that in this simple framework households do not make consumption-saving choices. This is consistent with my objective of studying the effects of wealth taxation, assuming away the distortionary effect taxes generate on saving decisions. Once appropriately calibrated the model will be able to reproduce the aggregate investment of households in public equity, private equity and safe assets, as well as some relevant features of the portfolio choices across the wealth distribution.

In Section 4 I will introduce wealth taxation into the framework. In particular, the tax analyzed will be a proportional wealth tax on the wealth in excess of a given threshold (e.g. the 99^{th} percentile). All the tax revenues will be uniformly redistributed across the whole wealth distribution through lump-sum transfers. I will then decompose the effects of the introduction of this tax-transfer schedule into two effects:

1. "Quantity effect": it captures the effect of the wealth tax on investment in the different investment opportunities, under the assumption of unchanged portfolio shares before and after the tax introduction. The larger the amount of an asset detained by the households who experience a reduction in their wealth, the larger the investment drop in that kind of asset.

2. "Portfolio composition effect": it captures how much the wealth tax affects invest-

ment in the different investment opportunities, due to the change in household portfolio shares induced by the tax introduction.

The two combined effects will induce a reduction in private equity supply, a smaller reduction in public equity investment and an increase, instead, in the household investment in safe assets.
In Section 5 I will analyze how the introduction of the wealth tax affects capital allocation across different industries.
In order to do that first of all I will analyze US data on how private equity, public equity and safe assets capital is allocated across different sectors (software, pharma-biotechnology, utilities...). This evidence, together with household endogenous choices of private equity, public equity and safe assets allows me to obtain the capital allocation across the different sectors. Furthermore, I also collect data on TFP and TFP growth across those industries. In this way I show that the firms in the sectors towards which private equity investments are directed have the largest productivities and productivity growth, followed the firms in the sectors which receive more public equity capital. Hence, the reduction of public and private equity investments in favor of safe assets will induce a reallocation of capital towards less productive sectors, determining a GDP reduction, which will be precisely quantified. Not only this, but also capital will be reallocated towards sectors of lower TFP growth, showing a negative impact of the wealth tax on economic growth. The next step of this research project will be the introduction of employment into the framework. An appropriate modelling of the production side of the economy will allow to quantify the employment flows across sectors induced by capital reallocation.
Section 6 will conclude the work.

1.1 Literature review and contribution

One of the main objectives of this work is that of building a household portfolio choice model able to capture some salient features of households investment choices. This is crucial to correctly quantify how the introduction of a wealth tax will affect capital supply in the economy. In this respect, this work relates to the stream of literature which documents heterogeneity of investment behaviour (and hence returns) across the wealth distribution. Two very important contributions on this topic are Bach et al. (2020) and Fagereng et al. (2020). Using administrative Swedish and Norwegian data they both report that individuals at the very top of the wealth distribution tilt their portfolio allocations toward very risky assets (especially private equity). Consistently with that, they also show that households' returns on wealth are increasing across the wealth distributions. Both Bach et al. (2020) and Fagereng et al. (2020) argue that the mechanisms at play in determining this return heterogeneity are: 1- "scale dependence" (larger wealth scale allows households to obtain larger returns) and 2- "type dependence" (individuals at the top of the wealth distribution have personal traits, e.g. investment abilities, which allows them to get higher returns). Since the contribution of Gabaix et al. (2016) several papers have exploited scale and type dependence mechanisms to retrieve heterogeneous returns on wealth across the wealth distribution. For example, Cioffi (2021) builds a dynamic portfolio choice model in which households can choose across three assets (housing, stocks, bonds) and households' relative risk aversion is decreasing in wealth (capturing scale dependence). This allows him to reproduce some relevant features of the empirical household portfolio choices across the wealth distribution. However, he mentions that the choice of not including private equity separately from public equity (as instead I do) does not allow him to precisely capture households behaviour at the very top of the wealth distribution. Gaillard and Wangner (2021), instead, build a static and dynamic version of a portfolio choice model in which households can only choose between a safe and a risky asset. However, they model household preferences so to capture both scale and type dependence mechanisms in their household portfolio choices. Remarkably, they notice that different calibrations of the parameters governing the extent of type and scale dependence can provide observationally equivalent household portfolio choices across the wealth distribution. My portfolio choice problem, apart from being static, differs from the mentioned papers in choice of introducing three assets (one of which is specifically private equity). Beside this, I specify household preferences so to capture scale dependence of returns (and hence increasing shares of private and public equity across the wealth distribution). Furthermore, differently from the previously presented papers, I also introduce an intermediation fee for investing in private equity, which generates heterogeneous net returns in private equity investments (this allows me to replicate some features of households private and public equity choices across the wealth distribution).

Another stream of literature to which this work talks to is that on wealth taxation. Many recent empirical papers on the issue are focused on the estimation of the elasticity of taxable wealth with respect to wealth tax rate, for example Brülhart et al. (2019), Seim (2017), Zoutman (2018) Jakobsen et al. (2020). While the first two focus mainly on the role of tax evasion, the third on the effect of the tax on savings, the latter builds a life-cycle model with utility of bequests. Once the model is appropriately calibrated using evidence from the introduction of a wealth tax reform in Denmark, the authors simulate the long-run elasticity of taxable wealth with respect to the tax rate. The recent work of Akcigit et al. (2018) which shows that corporate and income taxes have reduced the quantity of innovation in the US throughout the 20^{th} century, suggests the need of analyzing deeper the effects of wealth taxation which go beyond the mere effect on capital accumulation. A novel contribution of my work, indeed, is to focus on the distortions that wealth taxation may induce in terms of capital reallocation across production sectors characterized by different productivity and growth. Distortionary effects of wealth taxation (beside those on capital accumulation) have been studied for example by Cagetti and De Nardi (2006), who show that taxing entrepreneurs' wealth, may reduce the number of entrepreneurs. Guvenen et al. (2019), argue that wealth taxation is able to induce capital reallocation from the least to the most productive entrepreneurs. Thus, in his setting wealth taxation (compared to capital income taxation) boosts aggregate productivity and output. Gaillard and Wangner (2021) simulate the effect of the introduction of a wealth tax on GDP. Their contribution is to discuss optimal wealth taxation in presence of scale and type dependence mechanisms together with returns which may not reflect the real productivity of the investment, but the presence of some forms of rent-extraction.

2 Portfolio composition across the wealth distribution

In this Section I will analyze how US households allocate their wealth across different investment opportunities. Understanding the portfolio composition, especially of those at

Table 2. *Summary statistics of the 2019 US wealth distribution*

Wealth percentile	**Wealth share**
0 - 49 percentile	1.49%
50 - 89 percentile	20.28%
Top 10%	76.46%
Top 5%	64.91%
Top 1%	37.20%
Top 0.5%	28.01%

Notes: *The Table represents some features of the US 2019 wealth distribution. The term wealth indicates net wealth = assets - debts of US households. Data are taken from the 2019 wave of the Survey of Consumer Finances.*

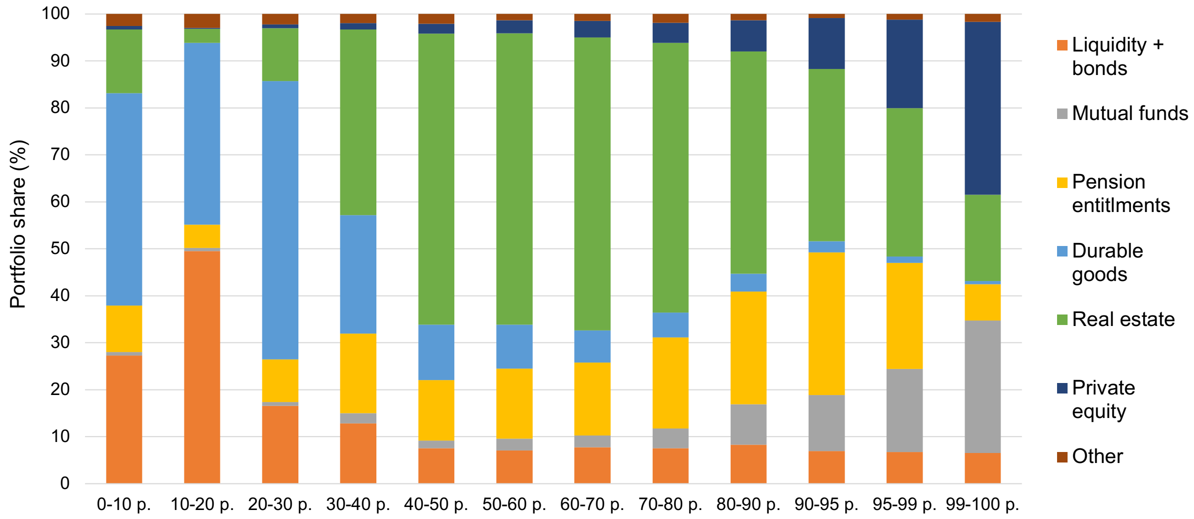
the top of the wealth distribution who will be taxed, is necessary to understand which kind of wealth will be taxed. This is a first step towards understanding how the introduction of the wealth tax will affect aggregate household capital supply to the production side of the economy.

2.1 Data and variables definitions

To obtain the portfolio composition of US households across the wealth distribution, the 2019 wave of the Survey of Consumer Finances (henceforth SCF) is used. The choice of the SCF over other surveys (for example the Panel Survey of Income Dynamics) has two reasons. The first one is the very detailed information on the household wealth composition, the second one is the over-sampling of the very wealthy households (for details on the sampling procedure see for example Kennickell (2008)). The latter motivation is crucial for my analysis: indeed, in order to evaluate how the introduction of a wealth tax on the very wealthy affects aggregate capital supply it is necessary to understand how much and which kind of wealth will be taxed.

Table 2 reports some features of the 2019 US net wealth distribution, showing that wealth is very unequally distributed. Only 1.5% of the overall wealth accrues to the households in the bottom 50% of the wealth distribution, 76% of the overall wealth accrues to the top 10% and 37% to the top 1%. Those figures, hence, show that even introducing a wealth tax on the wealthiest 1% of households only, still means taxing a very sizable share of the aggregate US households' wealth.

FIGURE 2. *US households portfolio composition across the wealth distribution*



Notes: *The Figure represents the portfolio composition of US households in 2019 across the wealth distribution. Each column reports the fraction of aggregate investment in each asset class j by household group i, over aggregate wealth for household group i. The asset classes are $j \in \{$liquidity+bonds, mutual funds, pension entitlments, durable goods, real estate, private equity, other assets$\}$, the description of each asset class is provided in Section 2.1. The household groups i are all the household whose wealth is in between the given wealth percentiles $i \in \{0-10,\ 10-20, ...\}$. Data are taken from 2019 SCF.*

All the possible assets that US households hold can be categorized into 7 different groups: liquidity and bonds (all type of transaction accounts, certificates of deposit, directly held bonds, saving bonds, bonds funds), mutual funds (stock mutual funds or combination mutual funds), pension and insurance entitlements (quasi-liquid retirement accounts and cash value of life insurance), durable goods (e.g. vehicles...), real estate (residences and real estate investment), private equity (privately-held businesses, professional practices, limited partnerships, private equity investments, or any other business investments that are not publicly traded) and other assets (annuities, miscellaneous financial and non-financial assets). Those asset categorization will be used to provide an overview of household portfolio choices across the wealth distribution.

2.2 Portfolio choices across the wealth distribution

Figure 2 shows the portfolio choices of US households among the assets categories described above, across the entire wealth distribution.

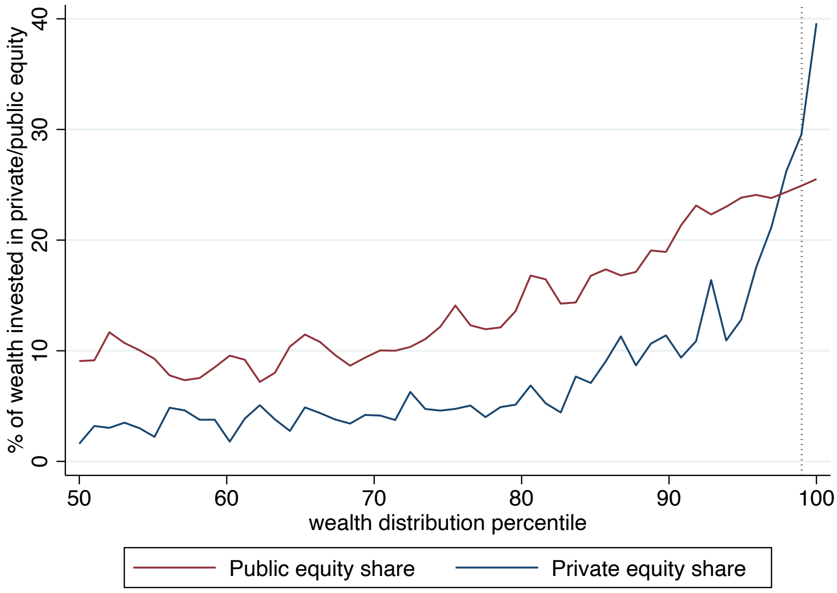
It is worth noticing that the majority of assets held by the households at the bottom of the wealth distribution are essentially durable goods and liquidity. Instead, at the middle of the wealth distribution housing becomes by far the most important asset households hold, together with their insurance and pension entitlements. As long as we move toward the top of the wealth distribution the portfolio shares households invest in private equity and stock mutual funds increase, while the importance of real estate is diminished. In particular, at the top 1% of the wealth distribution mutual funds and private equity make up around 65% of the whole portfolio. Furthermore, notice that even if the portfolio shares invested in private equity are very small across almost the entire wealth distribution, it becomes the most chosen investment opportunity at the very top of the wealth distribution (almost 39% at the top 1%).

As argued in the introduction, private equity is a form of investment which is substantially different from other investment opportunities not only for its high riskiness and potential profitability. Indeed, private equity investments are directed towards extremely promising and innovative firms and start-ups, which do not have the possibilities of obtaining the necessary funds through more "traditional" channels such as bank loans, the emission of corporate bonds or new stocks. This substantial difference in terms of productivity between the firms to which private and public equity investments are directed to, makes necessary to distinguish how much households invest into each of these investment opportunities. This will allow me to study how capital allocation choices made by the US households affect the aggregate US production.

To analyze US households wealth allocation choices in terms of private equity, public equity and safe assets I collapse all the assets classes described in Section 2.1 (and shown in Figure 2) into the three previously mentioned categories:

- **Private equity:** illiquid, very risky and very profitable form of investment. It coincides with the private equity class reported in Figure 2 and described in Section 2.1: privately-held businesses, professional practices, limited partnerships, private equity investments, or any other business investments that are not publicly traded.
- **Public equity:** liquid form of investment, with intermediate level of profitability and riskness. It comprises the total value of financial assets that are invested in stocks (stock mutual funds, combinational mutual funds, directly held stocks, stock

FIGURE 3. *US households portfolio composition: public vs private equity investments*



Notes: *The Figure plots the share of wealth US households invest in private equity and public equity, from the* 50^{th} *wealth percentile onwards. For each wealth percentile, the share invested in asset class* $j \in \{private\ equity,\ public\ equity\}$ *is computed dividing the aggregate investment in asset j of households in a given wealth percentile by the total wealth owned by households in that percentile. The composition of public and private equity asset classes is described in Section 2.2. Data are taken from 2019 SCF.*

investments of pension funds)[1].

- **Safe assets:** all the remaining assets are categorized as "safe" (liquidity, bonds, bond mutual funds, durable goods, life insurance, pension funds shares not invested in stocks, other assets).

Figure 3 reports the portfolio choices of households in terms of private and public equity (the remaining share is invested in safe assets) across the wealth distribution. For the sake of clarity the Figure reports portfolio choices for the households above the 50^{th} percentile only. This choice allows me to better focus on the portfolio decisions of the households who hold most of the aggregate wealth (the wealth held by the top 50% of households in the wealth distribution is 98.5%, see Table 2). Figure 3 shows that across almost the

[1] Combinational mutual funds are included in the "public equity" class for 1/2 of their value only, the rest of value is attributed to the category "safe assets". Pension funds are included in the "public equity" class for 1/2 of their value only if the investment of the fund is split between stocks and other interest earning assets. If that is the case the remaining 1/2 of pension fund value is attributed to the category "safe assets". If the investment of the pension fund is mainly directed toward stock the value of the pension fund is fully included in the category "public equity" Those assumptions are chosen so that the "public equity" variable in my analysis matches the "EQUITY" variable directly provided in the SCF 2019 database, which exactly captures the total value of financial assets that households invest in stocks.

entire wealth distribution people hold a larger portfolio share invested in public equity than in private equity. However, the portfolio share invested in private equity starts increasing very steeply from the 95^{th} percentile onward, overcoming the share invested in public equity around the 97^{th} percentile. This is not a US peculiarity, for example similar empirical evidence has been found for Sweden and Norway by Bach et al. (2020) and Fagereng et al. (2020) using administrative records. This feature of the data can be explained by the own peculiarities of private equity investments. Indeed, investing in private equity funds entails committing very large amounts of money for long periods of time in very risky (but potentially very profitable) projects. Furthermore, the private equity market is not an easily accessible market, as the stock market is. All those features drawn together suggest that only the very wealthy have the resources, capabilities and enough propensity to risk, in order to invest significant shares of their wealth into private equity investments.

On the other hand, stock market investments are very liquid kind of investments which are easily accessible and not too risky. This could be the reason why in the top 50% of the wealth distribution every household has at least 10% of its portfolio invested in public equity. This share is steadily increasing with wealth. Differently to what happens for private equity, however, the share invested in public equity does not grow very steeply at the top of the wealth distribution. Being both public and private equity shares increasing across the wealth distribution the share of wealth invested in safe assets is instead strictly decreasing across the entire wealth distribution, with a big drop at the very top.

In the next Section I will model how agents make their portfolio choices across the wealth distribution. This will allow me to simulate how the introduction of a wealth tax will affect individual portfolio choices and as a consequence the aggregate capital supply to the production side of the economy.

3 A model of portfolio choice

In this Section I introduce a portfolio choice model whose aim is that of capturing how households supply capital to the production side of the economy. Appropriately calibrated the model will be able to reproduce some salient features of the empirical evidence presented in the previous Section. The objective of this model is that of serving as metering device to quantify how much households change their capital supply choices in response to the introduction of a wealth tax. This aim will be accomplished under the modelling assumption that households do not make saving choices. This will allow me to shed light on how the wealth tax affects household portfolio choices and the induced capital allocation, effects often overlooked by the literature aimed at computing the overall elasticity of taxable wealth with respect to the tax rate (e.g.Jakobsen et al. (2020), Zoutman (2018), Brülhart et al. (2019), Seim (2017))

3.1 Setup

I will consider a partial equilibrium model.

The economy is populated by a unit mass of heterogeneous households indexed by $i \in [0, 1]$, who differ only for their initial wealth endowment a_i and absolute risk aversion $\alpha(a_i)$ (which is a deterministic function of the initial wealth level a_i). Each household i solves the following two period problem: in period 0 he has to allocate his initial after-tax disposable wealth $d(a_i)$ (initial wealth endowment a_i minus taxes plus transfers) between a safe asset, a public equity investment and a private equity investment. In period 1 each household i does not make any further choice, he simply uses the wealth resulting from the investment to consume. Notice that the only choice households make in this framework is to choose the composition of their portfolio. No saving decisions are made. This modelling choice is consistent with the aim of this model to be used as a tool for quantifying the effects of wealth taxation on capital supply, assuming away the tax distortions on saving decisions.

Wealth endowment and preferences: Households $i \in [0, 1]$ differ for their value of initial wealth endowment a_i. Their wealth is assumed to be drawn from a Pareto distribution $Pa(1, \eta)$, where $\eta > 0$ is a shape parameter which captures the thickness of the right tail of the distribution. This choice is consistent with the literature analyzing how wealth is distributed. Indeed, since wealth distributions are skewed to the right and display thick upper tails, Pareto distributions, which have exactly those features, are particularly appealing to model wealth (Benhabib and Bisin (2018), Vermeulen (2018)).

Many recent empirical studies show that wealthier households earn larger returns on wealth than poorer households (e.g. Xavier (2021), Bach et al. (2020), Fagereng et al. (2020)). A potential explanation for this phenomenon is that if households hold more wealth, they may have easier access to better investment opportunities or simply dare to take more risky behaviours. This mechanism is named "scale dependence" and it captures why households endowed with larger wealth make riskier portfolio choices (obtaining higher returns) than those holding lower wealth. I will specify household preferences of my portfolio choice problem so to capture exactly this mechanism.
Household preferences over consumption (consumption of household i is denoted as usual with c_i) are specified through the following utility function:

$$u(c_i, a_i) = -\frac{1}{\alpha(a_i)} e^{-c_i \alpha(a_i)}$$

where the coefficient of absolute risk aversion for household i with initial wealth a_i (and disposable wealth $d(a_i)$) is:

$$\alpha(a_i) = \kappa \cdot d(a_i)^{-\gamma}$$

with $\kappa > 0$ and $\gamma > 1$. First of all notice that absolute (and also relative) risk aversion negatively depends on disposable wealth. This is consistent with the "scale dependence" mechanism, capturing that wealthier households have more risk taking behaviours. κ is a scale parameter whose role is that of governing the average level of absolute risk aversion of households. γ, instead, captures the extent in which household choices are affected by "scale dependence". In other words, the parameter γ governs the sensitivity of households absolute risk aversion to the initial value of household disposable wealth. The larger γ, the larger the reduction in absolute (and also relative[2]) risk aversion of a household when his wealth increases.

Assets and returns: Let's remind that each household i can invest in three possible as-

[2] The coefficient of relative risk aversion for household i with initial wealth level a_i is: $r(a_i) = \kappa \cdot d(a_i)^{1-\gamma}$, which is decreasing in disposable wealth $d(a_i)$ as long as $\gamma > 1$.

sets: a safe asset, in public equity or in private equity. Moskowitz and Vissing-Jørgensen (2002) and Kartashova (2014) contributions (using SCF data) have shown that apart from the period 1990-2000, private equity generally earns a premium over public equity. More recent empirical evidence (for US Xavier (2021) and Gaillard and Wangner (2021), for Sweden and Norway Bach et al. (2020) and Fagereng et al. (2020)), has confirmed this result, highlighting the higher level of riskiness of private equity investments.

Coherently with the mentioned evidence I assume that the private equity investment is very risky but its expected return is also very high. The public equity investment, instead, has an intermediate level of profitability and riskiness, while the safe asset is risk-free but it is characterized by a low return.

To be specific, I will denote with δ_i the fraction of disposable wealth of household i invested in private equity, with ω_i the share of disposable wealth invested by household i in public equity, while $1 - \delta_i - \omega_i$ will be the disposable wealth share invested by household i in the safe asset.

The returns of private and public equity are assumed to be normally distributed random variables: $R_v \sim N(\phi_v, \sigma_v^2)$ is the return of the private equity investment, while $R_r \sim N(\phi_r, \sigma_r^2)$ the one of the public equity investment[3]. The return of the safe asset is a non-negative scalar $R_s \in \mathbb{R}_+$. As already argued, I assume that the expected return of private equity investment is the highest and the one of the safe asset is the lowest, that is: $\phi_v > \phi_r > R_s$. Furthermore, being the private equity investment a riskier kind of investment than the public equity one, I assume $\sigma_v^2 > \sigma_r^2$. Furthermore, I also allow private and public equity returns to have a non-zero covariance, namely: $Cov(R_v, R_r) = \theta$.

Private and public equity investments not only differ for their level of riskiness, indeed private equity investments are also less accessible (and less liquid) than public equity ones. To capture this feature of private equity investments I assume that to invest in private equity households have to pay a variable intermediation fee. This induces higher perceived private equity returns at the top of the wealth distribution. In particular, household i

[3] I use the subscript "r" to refer to the private equity investment because it is a "risky" kind of investment, while I use the subscript "v" to refer to the public equity investment because it is a "very risky" kind of investment. This choice is to avoid the use of the letter "p" which may generate confusion between "private" and "public"

which invests a fraction δ_i of his disposable wealth $d(a_i)$ in private equity has to pay:

$$C(d(a_i), \delta_i) = \lambda_1 \delta_i d(a_i)^{1-\lambda_2}$$

with $\lambda_1 > 0$, $0 < \lambda_2 < 1$.

The specific functional form chosen (hence the two parameters λ_1 and λ_2) is needed in order to allow the model to match a peculiarity of the empirical evidence on household portfolio choices previously presented. In particular, it allows to replicate the crossing between the lines representing private and public equity portfolio shares across the wealth distribution (see Figure 3). This is not surprising, indeed this parametrization of the fee households have to pay induces higher private equity net perceived returns (gross return R_v minus unitary investment cost) for households with higher disposable wealth.

Household problem:

To sum up: each household $i \in [0, 1]$, with initial wealth a_i drawn from a Pareto distribution $Pa(1, \eta)$, solves the following problem:

$$\begin{aligned}
\max_{\omega_i, \delta_i} \quad & \mathbb{E}\left[-\frac{1}{\alpha(a_i)} e^{-c_i \alpha(a_i)}\right] \\
\text{s.t.} \quad & c_i = (R_s(1 - \omega_i - \delta_i) + R_r \omega_i + R_v \delta_i) d(a_i) - \lambda_1 \delta_i d(a_i)^{1-\lambda_2} \\
& \omega_i \geq 0, \quad \delta_i \geq 0, \quad 1 - \omega_i - \delta_i \geq 0 \\
& R_v \sim N(\phi_v, \sigma_v^2), \quad R_r \sim N(\phi_r, \sigma_r^2), \quad R_s \in \mathbb{R}_+ \text{ given}, \quad Cov(R_v, R_r) = \theta
\end{aligned} \tag{P}$$

where $\alpha(a_i) = \kappa \cdot d(a_i)^{-\gamma}$, as already discussed.

3.2 Solution

Throughout this Section, for notational convenience, I will denote disposable wealth of household i as d_i, rather than $d(a_i)$.

To understand how the introduction of a wealth tax affects household portfolio choices (and as a consequence capital supply) it is crucial to understand how household portfolio choices depend on their disposable wealth. This is the result of Proposition 1, which provides closed-form expressions for the solution of the household problem (P) previously described.

Proposition 2. *The internal solution to the portfolio choice problem (P) of household $i \in [0,1]$ is:*

$$\omega_i = \frac{1/\kappa}{\sigma_r^2 - \frac{\theta^2}{\sigma_v^2}} \left[\left(\phi_r - R_s - \frac{\theta}{\sigma_v^2}(\phi_v - R_s) \right) d_i^{\gamma-1} + \frac{\theta}{\sigma_v^2} \lambda_1 d_i^{\gamma-\lambda_2-1} \right] \tag{2.1}$$

$$\delta_i = \frac{1/\kappa}{\sigma_v^2 - \frac{\theta^2}{\sigma_r^2}} \left[\left((\phi_v - R_s - \frac{\theta}{\sigma_r^2}(\phi_r - R_s) \right) d_i^{\gamma-1} - \lambda_1 d_i^{\gamma-\lambda_2-1} \right] \tag{2.2}$$

where ω_i is the fraction of wealth that household i, with disposable wealth d_i invests in public equity. Analogously, δ_i represents the fraction of wealth invested in private equity.

Proof: the derivation of the expressions is provided in Appendix A.

To grasp some intuition on the previously derived expressions let's assume for the moment that $Cov(R_v, R_r) = \theta = 0$. Expressions (2.1)-(2.2) simplify to:

$$\omega_i = \frac{\phi_r - R_s}{\kappa \sigma_r^2} d_i^{\gamma-1} \tag{2.3}$$

$$\delta_i = \frac{1}{\kappa \sigma_v^2} (\phi_v - R_s - \lambda_1 d_i^{-\lambda_2}) d_i^{\gamma-1} \tag{2.4}$$

First of all, let's notice that both the wealth shares invested in private and public equity positively depend on their respective Sharpe ratios. In other words, the larger the excess return of an investment opportunity (with respect to the safe asset) and the lower its returns volatility, the larger will be the share invested in that kind of asset. Second of all, assuming for a while that $\lambda_1 = 0$, notice that both portfolio shares positively depend on disposable wealth d_i. This is due to the fact that under the assumption that $\gamma > 1$ households relative risk aversion is decreasing in d_i[4], hence the wealth share households invest in risky assets is larger for wealthier households. Now, let's go back to the case in

[4]Remind that, as remarked in Section 3.1, the coefficient of relative risk aversion for household i with initial wealth level a_i is $r(a_i) = \kappa \cdot d_i^{1-\gamma}$.

which $\lambda_1 > 0$. The third term in parenthesis in (2.4) shows that the share invested in private equity not only depends on the private equity Sharpe ratio and the relative risk aversion of the household, but it is also affected by the costly nature of private equity investments. In particular, wealthier households will experience larger net private equity returns and hence they will invest more into this investment opportunity, with respect to poorer households.

Now, let's go back to the expressions of Proposition 2. When the two risky assets (private equity and public equity) have a positive correlation, $\theta > 0$ agents tend to tilt more their portfolio choices towards the safe asset. The reason is the following. Call "composite risky asset" the asset with expected return $\mathbb{E}(\omega_i R_r + \delta_i R_v)$ and variance $Var(\omega_i R_r + \delta_i R_v)$. When the covariance between private and public equity increases, the variance of the "composite risky asset" goes up, leaving its expected return unaffected. This is the reason why, a household with a given level of risk aversion, if observes a higher covariance between private and public equity investments, decides to shifts its portfolio choices towards the safe asset. Indeed, investing in risky assets has become more dangerous, without extra rewards on the their return.

3.3 Calibration

The model has to be carefully calibrated in order to reproduce the aggregate investment behaviour of households in safe assets, private equity and public equity. This will allow, when introducing a wealth tax, to compute how capital supply will change in response to the wealth tax. Beside this, the calibration aims at capturing, as close as possible, the household portfolio choices across the wealth distribution (especially at the top). This will allow to identify how and how much changes in household portfolio composition will affect the aggregate capital supply (portfolio composition effect).

Table 3 reports the parameters which are externally calibrated.

The first externally calibrated parameter is the shape parameter of the wealth distribution $\eta = 1.35$. The value for η is chosen so to match as close as possible the shape of actual US wealth distribution (SCF 2019 data) at the top 1%. The focus on the top 1% of the wealth distribution is due to the fact that I am interested in simulating the effect of

TABLE 3. *Externally calibrated parameters*

Parameter	Description	Value	Source
η	shape parameter wealth distr.	1.35	SCF data
ϕ_v	expected private equity return	0.156	Gaillard and Wangner (2021)
ϕ_r	expected public equity return	0.058	Gaillard and Wangner (2021)
R_s	safe asset return	0.004	Gaillard and Wangner (2021)
σ_v^2	private equity returns variance	0.377	Gaillard and Wangner (2021)
σ_r^2	public equity return variance	0.174	Gaillard and Wangner (2021)
θ	cov. public/private equity returns	0.064	S&P500 returns, PE buyout index

Notes: *The Table presents the externally calibrated parameters for the household portfolio choice problem presented in Section 3.1-3.2. The first column indicates the symbol used to identify the parameter in the model, the second column the parameter description, the third column the chosen value for the parameter, the fourth column the data sources used to obtain the parameter value.*

wealth taxes imposed at the very top of the wealth distribution. Furthermore, the case in which a tax is imposed on the wealthiest 1% of households will be an exemplifying policy often analyzed in the rest of the paper. The detailed procedure employed to obtain the value $\eta = 1.35$ is described in Appendix B.

The values for the expected returns of private, public equity and safe assets, together with their variances, are taken from Gaillard and Wangner (2021). They split household wealth into private equity, public equity and safe assets in the same way as I do, and compute mean returns and variances using PSID data from 1998 to 2018 (details of their procedure can be found in Appendix B).

Finally, the covariance between public and private equity returns θ is obtained by computing the covariance between the yearly returns of the S&P500 index (as a proxy for public equity returns) and the "Refinitive Private equity buyout index"[5] (as a proxy for private equity returns), from 2003 to 2021.

Table 5 shows in the first two columns the values of the parameters which are internally calibrated. The other columns, instead, present the chosen targets for the calibration of those parameters and the ability of the model to match them. The targets for calibration chosen are: first, the ratio between aggregate households investments in private equity and in safe assets, second the ratio between aggregate households investments in public

[5]The index tracks the gross performance of the U.S. PE buyout industry through a comprehensive aggregation of company values. It is obtained by analyzing over 8,000 U.S. private equity companies. For details see https://www.refinitiv.com/en/financial-data/indices/private-equity-index.

TABLE 4. *Internally calibrated parameters*

Parameter	Value	Target	Model	Data
κ	1.95	ratio aggregate priv.eq./aggregate safe (SCF)	0.419	0.415
γ	1.2	ratio aggregate pub.eq./aggregate safe (SCF)	0.428	0.422
λ_1	0.13	median top 1% priv.eq. portfolio share (SCF)	29.17%	28.61%
λ_2	0.294	median top 1% pub.eq. portfolio share (SCF)	24.29%	24.58%

Notes: *Column 1-2 of the Table report the parameters which are internally calibrated and the chosen values for them. The meaning of the symbols is reported in Section 3.1, where the model is fully described. Columns 3-4-5, instead, present the targets for the calibration and how well the model is able to match the targeted moments.*

equity and in safe assets. Those moments are easily computed using SCF 2019 data. As already argued, the choice of those targets relies in the need of having a model which precisely replicates the aggregate capital supply choices of US households in terms of private equity, public equity and safe assets. Indeed, a crucial use of this model will be that of measuring how much those quantities will change when a wealth tax will be introduced. Beside this, I require the model to match the top 1% median portfolio shares US households invest in public and private equity. The reason why I need the model to replicate those moments is that of having a tool which performs well in replicating the portfolio choices of the individuals at the top of the wealth distribution. This will allow me, when introducing a wealth tax on those agents, to identify which fraction of changes in capital allocation will be due to changes in household portfolio choices.

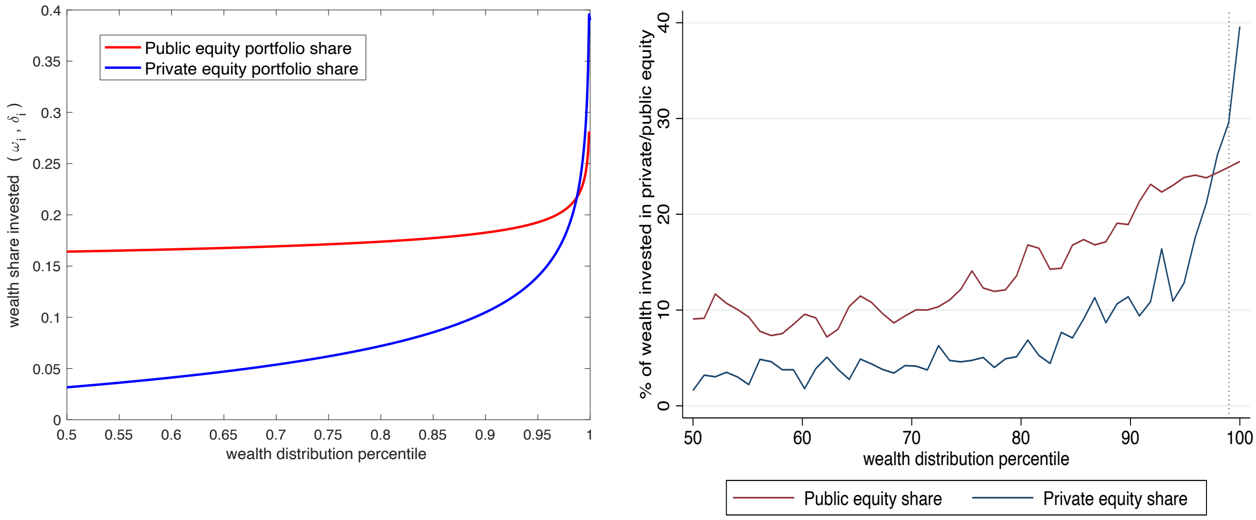
In the next paragraph I will also evaluate whether the calibrated model is able to match other relevant empirical moments, which have not been explicitly targeted.

3.4 Portfolio choices across the wealth distribution

Figure 4 (left panel) shows the household portfolio choices across the wealth distribution obtained through the calibration procedure described in the previous Section. The right panel of Figure 4, instead, is a copy of Figure 3, which is provided to facilitate the comparison between the simulated model and the empirical evidence.

The blue lines of Figure 4 represent the share of disposable wealth that US households invest in private equity, while the red ones the fraction invested in public equity. Both lines are increasing since wealthier households have lower relative risk aversion. Furthermore, the share invested in public equity is larger than the one invested in private equity across

Figure 4. *Simulated portfolio choices vs empirical portfolio choices (SCF 2019 data)*



Notes: *The left panel of the Figure shows the household portfolio choices across the wealth distribution obtained calibrating the model as described in Section 3.3. The right panel reports Figure 2 of this paper, showing the empirical portfolio choices across the wealth distribution in terms of private and public equity.*

almost the entire wealth distribution. The two lines cross at the 98^{th} percentile (in the SCF 2019 data 97^{th}, see Figure 3, right panel) and private equity is steeply increasing at the very top of the wealth distribution. Those features are consistent with the empirical evidence that extremely wealthy people invest large shares of their wealth (up to 39%) in very risky investment opportunities such as private equity. Instead, the majority of US households prefer public equity investments which have an intermediate level of riskiness and return.

The comparison of the left and right panel of Figure 4 shows that the model performs very well in replicating the shares households invest in private equity across the wealth distribution. The performance is slightly worse, instead, for public equity choices. Indeed, while the model predicts a pretty steep increase of the share invested in public equity at the top of the wealth distribution, this does not seem to be the case in the data. I leave to future research the analysis of this discrepancy between the model and the data.

4 The introduction of wealth taxation

In this Section I will introduce into the framework wealth taxation at the top of the wealth distribution. In particular, I will study:

(a) How the introduction of wealth taxation affects the household *portfolio composition*

(b) How the wealth tax affects *aggregate capital supply.*

Tax system: only 5 OECD countries currently have a tax on overall net wealth (or some kind of wealth only) which is still implemented (Colombia, France, Norway, Spain, Switzerland, see OECD (2018)). All those wealth taxes share a common feature: households have to pay a proportional tax on the wealth which exceeds a given threshold. The wealth tax I will consider will have the exact same features.

In particular, I will introduce a wealth tax above a given wealth threshold $\underline{a}$. Households who have wealth larger or equal to $\underline{a}$ have to pay a proportional tax (with tax rate $\tau > 0$) on the wealth exceeding $\underline{a}$. Formally, after the introduction of this wealth tax the expression of disposable wealth of household i, with initial wealth level a_i becomes:

$$d(a_i, \tau) = a_i - \tau \mathbb{I}_{a_i \geq \underline{a}}(a_i - \underline{a}) + T$$

where T denotes a lump-sum transfer (and $\mathbb{I}_{a_i > \underline{a}}$ denotes the indicator function, =1 if $a_i > \underline{a}$). Notice that if the government decides to impose the wealth tax on the wealthiest $x\%$ of the population then the threshold $\underline{a}$ is such that $Prob(A > \underline{a}) = x/100$, where A is the Pareto distributed random variable $A \sim Pareto(1, \eta)$, from which wealth realizations a_i are drawn.

Furthermore, the tax-transfer schedule also included a lump-sum transfer T which is set so that all the tax revenues are uniformly redistributed across the whole wealth distribution. Formally, denoting with $G(\cdot)$ the distribution function of the random variable $A \sim Pareto(1, \eta)$ from which wealth realizations a_i are drawn, the lump-sum transfer T must satisfy:

$$\int_{a_i > 1} T dG(a_i) = \int_{a_i > \underline{a}} \tau(a_i - \underline{a}) dG(a_i)$$

integrating and rearranging the terms it is possible to obtain a closed form expression for the lump-sum transfer T:

$$T = \frac{\tau}{\underline{a}^{\eta-1}(\eta-1)} \tag{2.5}$$

Decomposing the aggregate effect of wealth taxation: in the model considered, the aggregate effect of the introduction of a wealth tax on capital supply can be decomposed into two effects: a "quantity effect" and a "portfolio composition effect".

First of all, suppose that after the introduction of the wealth tax the fraction of wealth each agent invests in each asset remains unaffected. Being taxed, hence poorer, households will have lower wealth to invest in the different investment opportunities. The "quantity effect" captures exactly this phenomenon, i.e. the change in the level of investment in the different investment opportunities, keeping the portfolio choices of the households unchanged.

The second effect, which will be extensively analyzed in the next paragraph is the "portfolio composition effect". The households, when taxed, become poorer and due to the "scale dependance effect" change their risk-taking behaviour, altering their portfolio choices. This phenomenon affects the aggregate investment of households in private equity, public equity and safe assets. This is the "portfolio composition effect".

4.1 Portfolio composition effect

Suppose to introduce the wealth wealth tax previously described on the wealthiest 1% of households. This means that the threshold $\underline{a}$, above which the wealth tax is applied satisfies $Pr(A > \underline{a}) = 0.01$, with $A \sim Pareto(1, \eta)$. Assuming a tax rate $\tau = 1\%$, the disposable wealth of household i with initial wealth level a_i becomes:

$$d(a_i, 0.01) = a_i - 0.01 \times \mathbb{I}_{a_i \geq \underline{a}}(a_i - \underline{a}) + T$$

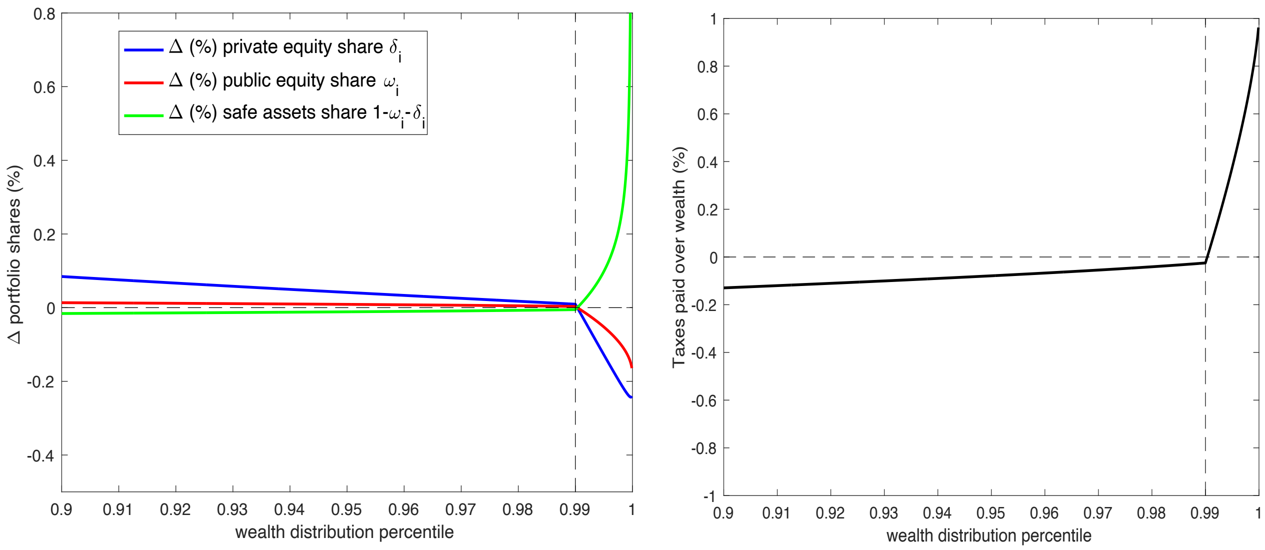
where T is the lump-sum transfer satisfying equation (2.5). How does the introduction of this wealth tax affects the individual portfolio choices across the wealth distribution? Figure 5 answers to this question.

In particular, the left panel of Figure 5 plots the percentage change of public equity, private equity and safe asset portfolio shares, caused by the introduction of the wealth tax. Notice that the focus of the Figure is on the top 10% of the wealth distribution.

First of all, let's notice that the percentile of the wealth distribution in which taxes paid by the household exactly compensate the lump-sum transfer received is the 99.03^{th} wealth percentile. On the right of that percentile it is possible to observe an increase in the share households invest in safe assets and a decrease in the shares invested in public and private equity. Instead, on the left of the 99.03^{th} percentile the behavior is the opposite. The cause of this portfolio composition effect is double. The main reason is the "scale dependence effect": when become poorer households pursue a more risk-averse behaviour. What happens in the model, indeed, is that when the disposable wealth of a household is reduced, its absolute (and relative) risk aversion increases, inducing households to prefer safe assets over risky ones. Furthermore, when poorer, households may face more troubles in accessing private equity investment opportunities, making them less convenient. In the model this effect is captured by the intermediation fees to be paid by households which invest in private equity. In particular, when households become poorer investing in private equity becomes more costly, or in other terms the net return of this investment opportunity goes down. This is another reason behind the private equity investment drop observed in Figure 5.

The previously described changes get larger and larger as long as we move toward the very top of the wealth distribution for two reasons. The first reason is that the wealth tax introduced is progressive, hence the wealthier the household, the larger the taxes to be paid relative to his wealth (this is shown in the right panel of Figure 5). However, even if the tax introduced was proportional, the left panel of Figure 5 would have been qualitatively (but not quantitatively) similar. This is due to the fact that in the model the larger the *absolute* change in disposable wealth, the larger the change in relative (and absolute) risk aversion that households experience. Remind that the magnitude of this effect is captured by the parameter γ. What happens is that those households who lose more wealth due to the introduction of the wealth tax are those who recalibrate more their portfolio choices in order to limit their risk exposure. Indeed, having fewer resources they prefer to be more cautious in their investment behaviour: this is to avoid the risk of consuming too little if their investment results are extremely bad.

FIGURE 5. *The portfolio composition effect induced by a wealth tax imposed on the wealthiest 1%* of households



Notes: *The left panel of the Figure shows the percentage change in private equity, public equity and safe asset portfolio shares induced by the introduction of a wealth tax. The tax considered is a proportional wealth tax with tax rate τ = 1%, applied on the wealth in excess of the 99^{th} percentile of the wealth distribution. All the tax revenues are uniformly redistributed through a lump-sum transfer. The right panel of the Figure, instead, shows the ratio between the amount of taxes paid by each household (according to the previous tax-transfer schedule) and the total amount of household's wealth.*

4.2 Wealth taxation effects on aggregate capital supply

The aggregate effect of wealth taxation on the different household investment opportunities is obtained by combining the "quantity effect" together with the "portfolio composition effect". The aggregate amount of households investments in private equity, public equity and safe assets can be computed as[6]:

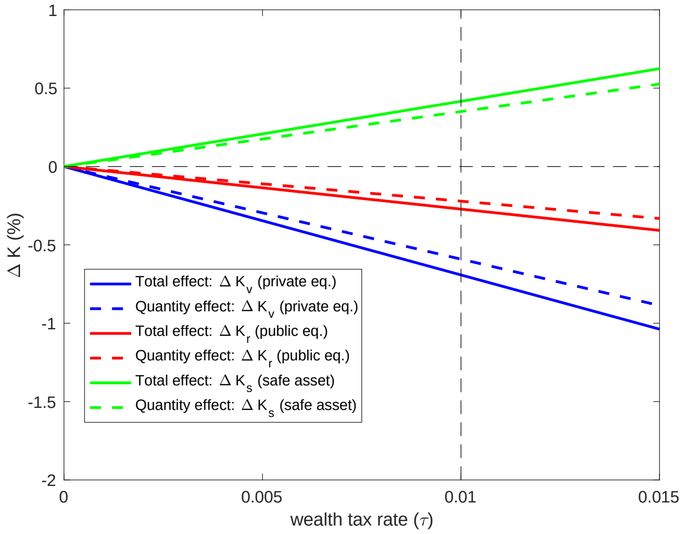
$$K_v(\tau) = \int \delta_i(a_i,\tau)d(a_i,\tau)dG(a_i) \qquad K_r(\tau) = \int \omega_i(a_i,\tau)d(a_i,\tau)dG(a_i)$$
$$K_s(\tau) = \int (1-\omega_i(a_i,\tau)-\delta_i(a_i,\tau))d(a_i,\tau)dG(a_i)$$

where $K_v(\tau)$, $K_r(\tau)$ and $K_s(\tau)$ denote respectively the aggregate investment in private equity, public equity and safe assets and $G(\cdot)$ denotes the distribution function of the random variable (Pareto distributed) from which wealth realizations a_i are drawn.

Figure 6 presents the aggregate effect of wealth taxation on capital supply, for different

[6]To highlight the dependence of the portfolio choices of household i, δ_i and ω_i, on the wealth endowment a_i and the tax rate τ I have chosen the notation $\delta_i(a_i,\tau)$ and $\omega_i(a_i,\tau)$.

FIGURE 6. *The aggregate effects of wealth taxation on capital supply: decomposition*



Notes: *The Figure shows the percentage change in private equity, public equity and safe asset capital supply induced by the introduction of a wealth tax. The tax considered is a proportional wealth tax with tax rate $\tau > 0$, applied on the wealth in excess of the 99^{th} percentile of the wealth distribution. All the tax revenues are uniformly redistributed through a lump-sum transfers. The solid lines represent the aggregate effects of wealth taxation, while the dotted lines represent the "quantity effects". The difference between the two lines is the "portfolio composition effect". The detailed explanation of the "quantity effect" and "portfolio composition effect" is provided in Section 4 of this work.*

tax rates $\tau > 0$. While the solid lines represent the total effects, the dotted lines represent the "quantity effects". Hence the difference between each solid and dotted line represents the "portfolio composition effect" of wealth taxation. In particular, the introduction of the wealth tax on the wealthiest 1% of households described in Section 4.1, with $\tau = 1\%$ induces:

- A reduction of 0.92% of private equity capital supply. The 14% of this change is due to the "portfolio composition effect", the remaining to the "quantity effect".
- A reduction of 0.35% of public equity capital supply. The 19% of this change is due to the "portfolio composition effect", the remaining to the "quantity effect".
- An increase of 0.55% of investment in safe assets. The 19% of this change is due to the "portfolio composition effect", the remaining to the "quantity effect".

TABLE 5. *Empirical US capital allocation across industries*

	Private equity (ψ_v^j)	Public equity (ψ_r^j)	Safe assets (ψ_s^j)
Software	30%	10%	2.6%
Pharma and biotechnology	15%	9%	1.1%
Media and communication	3%	8%	3.9%
Utilities	2%	9%	2.3%
IT hardware	4%	11%	0.3%
Healthcare	6%	4%	1.6%
Commercial prod. and services	18%	12%	1.3%
Consumer prod. and services	12%	18%	7.1%
Finance and insurance	5%	14%	7%
Housing real estate	0%	2%	64.3%
Other (includes government)	5%	1%	8.5%
Total	100 %	100%	100%

Notes: *Column 1 of the Table reports private equity capital allocation across sectors, data are taken from PitchBook-NVCA Venture Monitor 2022. Column 2 of the Table reports public equity allocation across sectors. Data taken from Fidelity Investments Research. Column 3 reports safe assets capital allocation across sectors. Households holdings of safe assets have been disaggregated into housing, government bonds and corporate bonds using SCF data. The investment into corporate bonds has been split among different sectors by using S&P Global data on the US corporate debt market in 2019.*

5 The wealth taxation effect on GDP and growth

The previous Sections have shown that the introduction of wealth taxation affects household capital allocation among private equity, public equity and safe assets. The focus of this Section will be on how this effect does have an impact on GDP and growth.

Private equity, public equity, and safe assets investments not only differ for their level of riskiness and returns, but also they are used for financing very different kind of projects and firms. This is shown in Table 5 which represents how the capital collected through each asset class is allocated across different sectors.

Empirical US capital allocation: in order to obtain the figures of Table 5 the entire US economy has been disaggregated into 12 sectors: software, pharma and biotechnologies, media and communication, utilities, IT hardware, healthcare, commercial products and services, consumer products and services, finance and insurance, housing and real estate and other (which includes government). The capital allocation across those sectors has been obtained by combining several data sources. First of all, private equity capital allocation across sectors has been obtained by using PitchBook data (2022 PitchBook-NVCA Venture Monitor[7]), which provide information on the value of private equity deals by sector in the past 15 years. To compute the allocation across sectors of private equity stock, the investment flows data (i.e. value of yearly private equity deals) have been aggregated assuming an yearly depreciation rate of 5%.

Second of all, to obtain the public equity allocation across sectors, I have retrieved data on US public equity market capitalization by industry. This is the sum of the market value of each listed US company, assigned to the applicable GICS (Global Industry Classification Standard) sector or industry.

Finally, using SCF 2019 data, households holdings of safe assets have been disaggregated into housing, government bonds and corporate bonds[8]. Furthermore, the investment into corporate bonds has been split among different sectors by using S&P Global data on the US corporate debt market in 2019[9].

Table 5 shows that the great majority of investment in private equity is directed toward the software sector (30%), followed by commercial products and services and pharma and biotechnology (15%). Public equity investments, instead, are mainly directed towards the production of consumer products and services (18%), financial and insurance services (14%), commercial products and services (12%) and IT hardware sector (11%). Finally, capital collected through safe assets investment is mainly allocated to the housing sector (64.3%), followed by the government sector (8.5%) and the sector devoted to the production of consumer products and services (7.1%).

Capital allocation, TFP, TFP growth: the portfolio choice model described in the previous Sections allows to endogenously derive the households choices in terms of private equity (K_v), public equity (K_r) and safe assets (K_s). I now assume that those endogenous quantities will be allocated to the different production sectors of the economy according to the proportions shown in Table 5. Formally, let's denote with ψ_v^j the fraction of investment in private equity that is supplied to sector $j \in J = \{\text{software, pharma, ... , housing, other}\}$

[7]https://pitchbook.com/news/reports/q1-2022-pitchbook-nvca-venture-monitor

[8]For the sake of completeness notice that liquidity has been added to the financial sector, durable goods to the housing sector.

[9]https://www.spglobal.com/en/research-insights/articles/u-s-corporate-debt-market-the-state-of-play-in-2019

(first column of Table 5). Analogously let ψ_r^j and ψ_s^j denote, respectively, the fraction of investment in public equity and safe assets that is supplied to sector $j \in J$ (second and third column of Table 5). The total amount of private equity capital supplied to sector $j \in J$ (K_v^j) is computed as $K_v^j = \psi_v^j K_v$. Analogously are computed the total amount of public equity capital and safe assets capital supplied to sector $j \in J$: $K_r^j = \psi_r^j K_r$ and $K_s^j = \psi_s^j K_s$.

The capital allocation obtained in this way is represented in Figures 7-8 (for readability of the Figures the "housing and real estate sector" has not been reported. It accounts for 36% of the aggregate capital in the economy, its TFP is 98 while the TFP growth of the sector 0.5%). Figures 7-8 show that the sectors which receive the largest amounts of private equity capital are those characterized by the highest TFP (Figure 7) and highest TFP growth (Figure 8). The TFP and TFP growth measures reported in the two Figures have been computed by using the 2020 U.S. Bureau of Labor Statistics data, which provide official industry-level total factor productivity statistics. In particular, TFP growth has been computed as an average of the 2010-2020 yearly TFP growth rates.[10] In particular, notice that the three sectors to which the largest quantities of private equity capital is supplied (Software, Pharma-Bio., Commercial products and services) are among those sectors characterized by the highest TFP and TFP growth rates.

5.1 Wealth taxation and capital allocation across industries

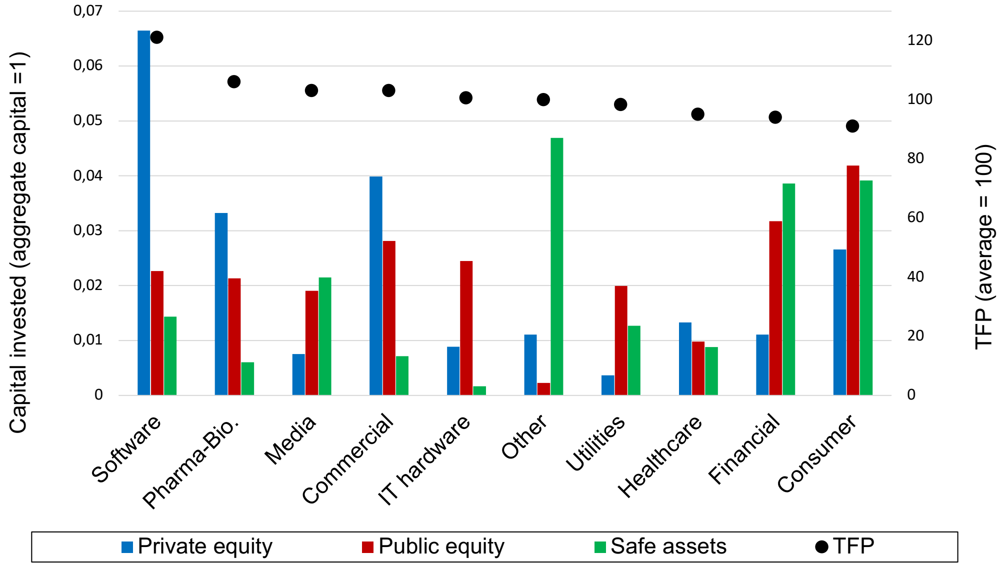
In Section 4.2 I have shown the effects of the introduction of a wealth tax on private equity, public equity and safe assets investment aggregates. Combining those results with the above described evidence on capital allocation across industries it is possible to obtain the effect of the introduction of a wealth tax on capital allocation across US industries.

Let's assume perfect substitutability between private equity, public equity and safe assets capital, the aggregate amount of capital supplied to sector $j \in J = \{$software, pharma, ..., housing, other$\}$ will be:

$$K^j = K_s^j + K_r^j + K_v^j$$

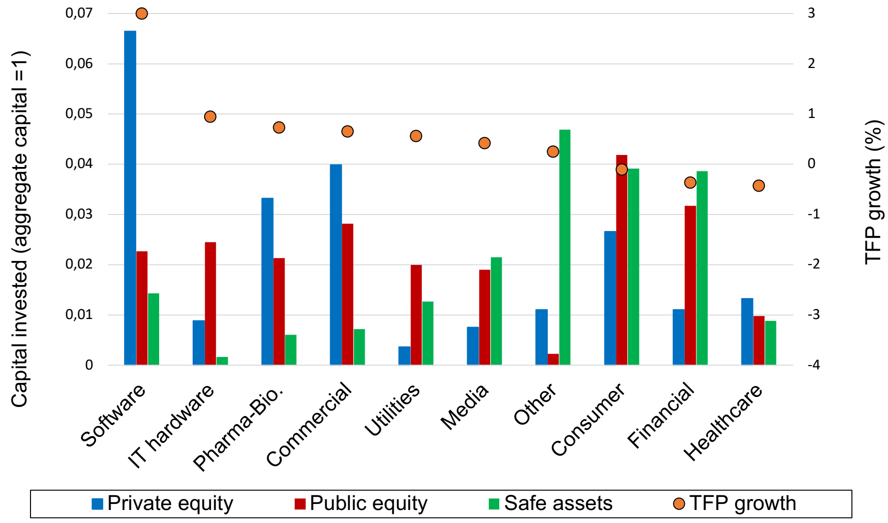
[10]The results would have been qualitatively similar by taking the average of the TFP growth rates from 2015 to 2020. The main difference would have been a significant reduction in the TFP growth rate in the "IT hardware" sector, from 0.94 to 0.5 and an increase of TFP growth in "utilities" sector from 0.56 to 0.9.

Figure 7. *Capital allocation across industries (excluding housing) ordered (decreasingly) by TFP*



Notes: *The Figure shows sector by sector capital allocations (excluding housing, only for readibility reasons. Housing accounts for 36% of the aggregate capital in the economy, its TFP is 98). The capital allocation represented are determined by: 1- the endogenous household choices in terms of private equity, public equity and safe assets; 2- the exogenous capital allocation across sectors presented in Table 4. Sectors have been ordered (decreasingly) by TFP of the sector. Data on TFP have been taken from 2020 Industry level TFP statistics provided by US Bureau of Labor Statistics.*

Figure 8. *Capital allocation across industries (excluding housing) ordered (decreasingly) by TFP growth rate*



Notes: *The Figure shows sector by sector capital allocations (excluding housing, only for readibility reasons. Housing accounts for 36% of the aggregate capital in the economy, its TFP growth is 0.5%). The capital allocation represented is determined by: 1- the endogenous household choices in terms of private equity, public equity and safe assets; 2- the exogenous capital allocation across sectors presented in Table 4. Sectors have been ordered (decreasingly) by average TFP growth between 2010-2020 of the sector. Data on TFP growth have been taken from 2020 Industry level TFP statistics provided by US Bureau of Labor Statistics.*

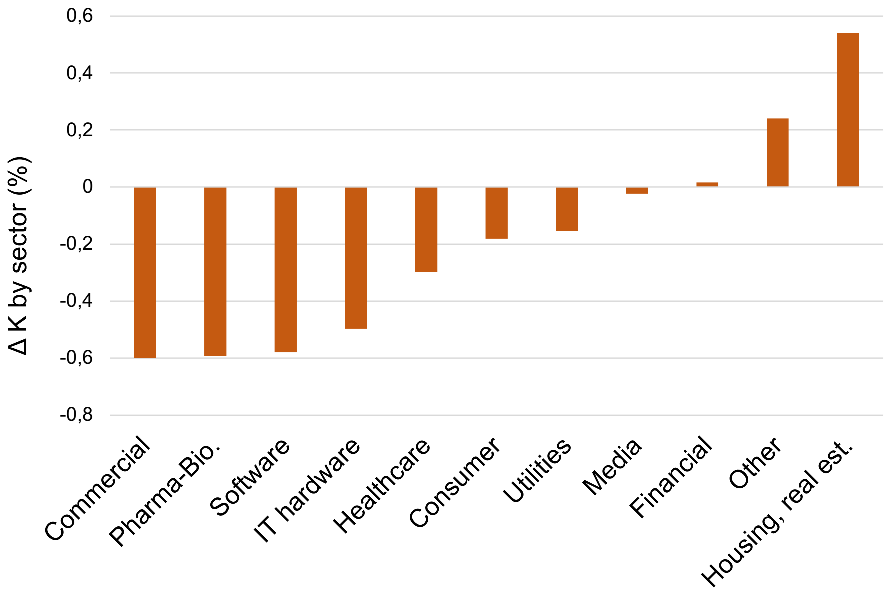
How does the introduction of the wealth tax affects this aggregate? Figure 9 shows the effect on K^j, $j \in J$ of the introduction of a proportional wealth tax (with proportional tax rate $\tau = 1\%$) on the wealth in excess of the 99^{th} percentile of the wealth distribution. The results of Figure 9 have been obtained by assuming that all the revenues collected via the imposition of the tax are uniformly redistributed via a lump-sum transfer. The industries which experience the largest drops are those characterized by the largest amount of private equity investments, namely commercial products and services (-0.6%), pharma-biotechnology (-0.59%) and software (-0.58%). Also the IT hardware sector experiences a significant capital reduction (-0.49%). This is due to the fact that this sector is almost exclusively financed via public equity capital, whose supply, due to the introduction of the wealth tax is reduced (although much less than private equity, see Figure 6). The drops of capital in those sectors are partially compensated by the increase in capital supplied to the housing sector, and the "other" sector which comprises production activities mainly financed via households buying safe assets (e.g. government production of goods and services).

Effects on GDP and GDP growth: the introduction of the previously described wealth tax on the wealthiest 1% of households determines a shift of capital supply from sectors of higher productivity to those of lower productivity. In this way households at the top 1% of the wealth distribution obtain lower returns on their wealth investments, and as a consequence a reduction of their incomes.

In the economy analyzed in this paper the change of GDP can be simply computed as the change in aggregate households consumption (in the considered model there are no savings and no government expenditures). Hence, the GDP reduction due to the introduction of the wealth tax on the wealthiest 1% of households with $\tau = 1\%$ described above, amounts to -0.41%.

However, beside affecting GDP, the introduction of the wealth tax also affects GDP growth. This is caused by capital shifting away from sectors characterized by very high growth (especially software), to sectors of lower growth (e.g. housing and real estate sector). Remember that in the household portfolio choice model introduced in the previous Sections households do not make savings decisions. Hence, the introduction of a wealth

FIGURE 9. *Capital reallocation across industries induced by the introduction of a wealth tax ($\tau = 1\%$) imposed on the wealthiest 1% of households*



Notes: *The Figure shows the percentage change in capital supplied to each sector induced by the introduction of a wealth tax. The tax considered is a proportional wealth tax with tax rate $\tau = 1\%$, applied on the wealth in excess of the 99^{th} percentile of the wealth distribution. All the tax revenues are uniformly redistributed through lump-sum transfers.*

tax, even without inducing a distortionary effect on capital accumulation, is able to generate a GDP growth reduction. The mechanism driving this effect is only the capital misallocation from higher growth to lower growth sectors.

Further research, introducing employment: the next step to be accomplished in this research project will be that of introducing employment into the framework. While the relationship between changes in income inequality (e.g. induced by income taxes) and employment, has been investigated by Lee et al. (2022), analyzing the relationship between wealth taxation and employment would be a new contribution to the literature[11]. Suppose that in each sector a representative firm whose technology is described by a Cobb-Douglas function is responsible for production. Furthermore, also assume that the labor force composition in terms of skilled and unskilled workers employed is different across sectors: then the capital allocation effect induced by the wealth tax will affect labor demand of

[11]Bjorneby et al. (2020) show a positive relationship between household's wealth tax and employment growth in the firms controlled by them. The reason is that higher wealth taxes induce households to invest in non-traded firms (hence hard-do-evaluate) to leave scope for tax elusion behaviours.

skilled and unskilled workers. Consider, for example, the big drop in capital supply in the software sector: the representative firm of this industry will reduce employment in the same proportion as capital. However, if software sector's production mainly requires very skilled workers, in this sector there will be a big drop of skilled workers demand and only a very little reduction of that of unskilled workers. Notice that since in the model considered throughout the paper the aggregate amount of capital does not change after the introduction of the wealth tax (but it's only reallocated), neither employment does. What will happen on the employment side, instead, is going to be a change in the demand of skilled and unskilled workers. If sectors with the largest private equity investments are also those characterized by the largest shares of skilled workers employed, the aggregate labor demand for skilled workers will decrease and that of unskilled workers will increase.

6 Conclusion

In this work I have developed an analytically tractable portfolio choice model which has allowed me to investigate and quantify the effects of a wealth tax, imposed at the top of the wealth distribution, on households' portfolio choices. Differently from the existing literature which aims at capturing the household investment behaviour across the wealth distribution, I introduce in my portfolio choice problem the possibility for households of investing in private equity (besides in public equity and safe assets). This choice is crucial for capturing the behaviour of the individuals at the very top of the wealth distribution (which will be taxed), who invest large fractions of their wealth in private equity.
Handled with this tool I have derived the effects of wealth taxation on aggregate capital allocation across different sectors of the economy.
First of all, the introduction of a wealth tax on the wealthiest households induces the taxed agents to significantly reduce the share of wealth they invest in private equity. Furthermore, they also reduce (in a smaller extent) their portfolio share invested in public equity, while they increase their investment in safe assets. Given the concentration of wealth at the top of the wealth distribution this determines a reduction in aggregate investment in private equity and (in a smaller extent) in public equity, together with an increase in the investment in safe kind of assets. Since private equity investments

are directed towards more productive sectors, this capital misallocation effect induces a reduction of GDP which is quantified in -0.41%. Not only this, but it is shown that the sectors to which private equity investments are directed to (especially the software sector) are characterized by the highest level of TFP growth. Hence, the capital allocation effect induced by the wealth tax results in being detrimental to the economic growth.

The next step of this project will be that of introducing employment into the framework, in order to explore the implications of the wealth tax imposition on labor demand across sectors.

Appendix A: Proof of Proposition 1

The problem to be solved is:

$$\begin{aligned} \max_{\omega_i,\delta_i} \quad & \mathbb{E}\left[-\frac{1}{\alpha(a_i)}e^{-c_i\alpha(a_i)}\right] \\ \text{s.t.} \quad & c_i = (R_s(1-\omega_i-\delta_i) + R_r\omega_i + R_v\delta_i)d(a_i) - \lambda_1\delta_i d(a_i)^{1-\lambda_2} \\ & \omega_i \geq 0, \quad \delta_i \geq 0, \quad 1-\omega_i-\delta_i \geq 0 \\ & R_v \sim N(\phi_v,\sigma_v^2), \quad R_r \sim N(\phi_r,\sigma_r^2), \quad R_s \in \mathbb{R}_+ \text{ given}, \quad Cov(R_v,R_r)=\theta \end{aligned} \tag{P}$$

The normality assumption of returns guarantees that consumption is a normally distributed random variable, with expectation and variance respectively:

$$\mathbb{E}(c_i) = (R_s(1-\omega_i-\delta_i) + \phi_r\omega_i + \phi_v\delta_i)d(a_i) - \lambda_1\delta_i d(a_i)^{1-\lambda_2} \tag{2.6}$$

$$Var(c_i) = d(a_i)^2\sigma_r^2\omega_i^2 + d(a_i)^2\sigma_v^2\delta_i^2 + 2\theta\omega_i\delta_i d(a_i)^2 \tag{2.7}$$

Since consumption c_i is normally distributed then the random variable $e^{-c_i\alpha(a_i)}$ is log-normally distributed. This observation allows to compute[12]:

$$\mathbb{E}\left[-\frac{1}{\alpha(a_i)}e^{-c_i\alpha(a_i)}\right] = -\frac{1}{\alpha(a_i)}e^{-\alpha(a_i)\mathbb{E}(c_i)+\frac{1}{2}\alpha(a_i)^2Var(c_i)} \tag{2.8}$$

At this point it is possible to combine equations (2.6)-(2.7)-(2.8) and compute the derivates of the obtained expression with respect to δ_i and ω_i. Those will be the first order conditions of problem (P):

$$\begin{aligned} \delta_i : \quad & -\alpha(a_i)\left[(\phi_v - R_s)d(a_i) - \lambda_1 d(a_i)^{1-\lambda_2}\right] + \frac{1}{2}\alpha(a_i)^2(2\delta_i\sigma_v^2 d(a_i)^2 + 2\theta\omega_i d(a_i)^2 = 0 \\ \omega_i : \quad & -\alpha(a_i)(\phi_r - R_s)d(a_i) + \frac{1}{2}\alpha(a_i)^2(2\omega_i\sigma_r^2 d(a_i)^2 + 2\theta\delta_i d(a_i)^2) = 0 \end{aligned}$$

Since those expressions are two equations in two unknowns (ω_i and δ_i), it is possible to combine them and solve in order to obtain the closed-form expressions for δ_i and ω_i reported in Proposition 1. □

[12]If a random variable $X \sim N(\mu,\sigma^2)$, then the random variable $Y = e^X$ has expectation $\mathbb{E}(Y) = e^{\mu+\sigma^2/2}$

Appendix B: Model calibration

In this Section I will go through the details of the procedures used to calibrate the portfolio choice model described in Section 3. The chosen values for all the parameters are reported in Section 3.3.

Shape parameter of wealth distribution η: When wealth is Pareto distributed with shape parameter η, then the share of wealth accruing to the top $q\%$ is:

$$\text{wealth share top q\%} = s_{q\%} = \left(\frac{q}{100}\right)^{\frac{\eta-1}{\eta}} \quad \Rightarrow \quad \eta = \frac{\ln(q/100)}{\ln(q/(100 s_{q\%}))} \tag{2.9}$$

The theoretical wealth shares that go to the top $q\%$, where $q \in \{1, 0.9, 0.8, ..., 0.1\}$ are computed for different η using equation (2.9). $\eta = 1.35$ is chosen to minimize the difference between the theoretical wealth shares that go to the top $q\%$ ($q \in \{1, 0.9, 0.8, ..., 0.1\}$) and the empirical ones, computed using the 2019 SCF data.

Assets returns and variances: Gaillard and Wangner (2021) use 1998-2018 PSID data to compute returns and variances of assets categorized as private equity, public equity and safe assets. Those asset classes are defined in the same way as I do in Section 2. In particular, they compute the return for household i, of asset $j \in$ {private equity, public equity, safe asset}, at time t, in the following way:

$$r_{i,j,t} = \frac{R^K_{i,j,t} + R^I_{i,j,t} - R^D_{i,j,t}}{a_{i,j,t-1} + I_{i,j,t}/2}$$

where where $a_{i,j,t-1}$ is the beginning-of-period amount of asset of class j held by household i, and $I_{i,j,t}$ is net investment in the asset. $R^K_{i,j,t}$, $R^I_{i,j,t}$, $R^D_{i,j,t}$ indicate respectively capital gains, income derived from the asset and the cost of debt used to obtain the asset. The mean of $r_{i,j,t}$ across time and households is the expected return of asset class j. Analogously, the variance of $r_{i,j,t}$ across time and households is the variance of asset class j.

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Chapter 3: optimal taxation in occupational choice models: simulations for the US economy

Abstract

Frameworks used to study optimal tax problems are often highly specialized and difficult to simulate in practice. This paper shows that the occupational choice model of Laroque and Pavoni (2017) can be used as a simple and general tool to simulate optimal income tax schedules for individuals and couples in the U.S. economy. The flexibility of the framework allows to compare optimal tax schedules assuming household labor supply on the intensive and extensive margin. Simulated schedules for individual income tax show that extensive margin labor supply leads to negative marginal tax rates at the bottom of the income distribution. Moving to couple taxation, I show that the considered framework can also be employed to recover U.S. government redistributive preferences (inverted optimal tax approach). Given the current governmental preferences I examine how varying redistributive preferences and labor supply elasticities shape optimal couple tax policy.

1 Introduction

The extensive literature on optimal income taxation has developed a wide range of occupational choice models to explore how income tax systems should be optimally structured. Some papers focus on individual income taxation, carefully modeling individual labor supply responses. Others focus on optimal couple taxation, capturing the labor supply decisions of couples and examining the factors influencing these choices.

This paper takes a different approach by using a unified and highly flexible framework - Laroque and Pavoni (2017) - to simulate optimal income taxation for both individuals and couples in the U.S. setting. The strength of this framework lies in its ability to accommodate different assumptions on occupations' features, agents' labor supply and government redistributive preferences. Notwithstanding its generality, this model can be easily solved to obtain optimal tax schedule simulations in a great variety of settings.

The first part of the paper employs Laroque and Pavoni (2017) framework to perform optimal individual income tax simulations. In particular, I compare optimal tax schedules derived under a model where individuals choose how much to work (intensive margin) versus whether to work at all (extensive margin).

When only the extensive margin is active, the optimal tax schedule features negative marginal tax rates at the bottom of the income distribution[1]. This schedule encourages participation inducing small downward distortions of high income agents' labor supply. In contrast, under the intensive margin model, no negative marginal tax rates emerge and the optimal tax schedule features a traditional subsidy to the unemployed households financed through heavier taxation on top earners.

The second part of the paper employs Laroque and Pavoni (2017) to study optimal couple taxation, assuming both spouses make labor supply decisions at the extensive margin.

Using a simplified setting with four occupational types (no earners, 1-earner-female, 1-earner-male, both earners), I apply the so-called "inverted optimal tax" approach to estimate the implicit social welfare weights consistent with the current U.S. tax system. These capture the actual preferences of US government for redistribution across the four considered occupations. These "revealed" Pareto weights, are then used to simulate op-

[1] Cremonini (2020) provides a formal proof of this statement.

timal tax schedules under varying redistributive preferences and participation elasticity scenarios.

When the government has stronger redistributive preferences (with respect to the status-quo) the optimal tax schedule features an increase in subsidies for non-working household and a tax decrease for single-earner-female households. Noticeably, the relative tax burden between one-earner households (men vs female earner) depends on the balance between income levels and labor supply elasticities of these "occupations". When the government strongly values redistribution, poorer one-earner-woman households are taxed less than one-earner-man, due to higher Pareto weights. When the government values little redistribution, the lower labor supply elasticity of 1-earner-women households, leads to optimally set higher tax rates on 1-earner-females than 1-earner-males.

Literature: This paper relates to the long-standing literature on optimal income taxation, which goes back to the seminal contributions of Mirrlees (1971), Diamond (1998), Salanié (1998). In particular this paper contributes to the literature studying optimal taxation in occupational choice models. The works by Rothschild and Scheuer (2013), Ales et al. (2015) study optimal individual income taxation in occupational choice settings where the working decisions of individuals have general equilibrium effects on wages. In this paper I abstract from these general equilibrium effects and allow for occupational choices in occupations characterized by exogenous salaries. To do that I employ the framework of Laroque and Pavoni (2017) which encompasses as particular cases the occupational choice models of Choné and Laroque (2011) (where only the extensive margin of labor supply is active) and Saez (2002) (where both intensive and extensive labor supply margins are active).

This paper also contributes to the literature studying optimal couple income taxation. The frameworks of Kleven et al. (2009) and Immervoll et al. (2011) are closely related to the one in this paper, however they perform optimal tax schedule simulations exogenously assuming the primary earner of each household. In my work I endogenize the decision of becoming primary earner, allowing both female and male workers to become primary earner of the household. The more recent paper by Ales and Sleet (2022) studies optimal couple taxation in a framework which models households' joint occupation choices through a *mixed logit model.* This framework allows them to model rich substitution patterns between different households' work configurations (e.g., male-only, female-only, dual earners, or neither working). Although more simplistic, the framework of Laroque and Pavoni (2017) employed here allows to obtain qualitatively similar result, at much lower computational cost and a more intuitive mapping between model objects and empirical counterparts.

The paper is structured as follows. Section 2 describes the occupational choice framework of Laroque and Pavoni (2017) which is employed for simulating optimal individual and couple tax schedules. Sections 3 employs the model to study optimal individual income taxation, while in Section 4 the model is employed to study optimal couple taxation. Section 5 concludes.

2 Model

In this Section I present the occupational choice framework of Laroque and Pavoni (2017), which I employ as setting for the optimal tax simulations conducted throughout this paper.

Assume a finite number of occupations indexed by $i \in \{0, 1, ..., I\}$ and a continuum of agents, each characterized by a privately known type $\alpha \in \mathcal{A} \subset \mathbb{R}^I$. Let $F(\alpha)$ indicate the cdf according to which these types are distributed. Each occupation i provides a before tax income w^i which is exogenously given and observed by the government. Income received by the worker in occupation i equals the production of the worker in that occupation. Taxes (or transfers) levied on each occupation i are denoted by t^i and the vector of disposable incomes in the various occupations by $c = (w^0 - t^0, w^1 - t^1, ..., w^I - t^I)$. The government is assumed to be able to observe the occupational decision of each agent but not the type of each individual.
Let u^i be the utility function of an individual with skill α when joining occupation i: $u^i(c^i, \alpha)$. I assume that the utility function is increasing and continuously differentiable in both arguments.

Denote with $A^i(c)$ the set of agents who choose occupation i:

$$A^i(c) := \{\alpha \in \mathcal{A} \mid u^i(c^i, \alpha) > u^j(c^j, \alpha) \, \forall \, j \neq i\} \tag{3.1}$$

with $\mu(A^i(c))$ indicating the measure of this set. Since each agent chooses one of the available occupations, it holds:

$$\sum_{i=0}^{I} \mu(A^i(c)) = 1 \tag{3.2}$$

The social planner problem: let's consider the problem of a benevolent planner who chooses the level of disposable income for each occupation:

$$\begin{aligned} \max_{\{c^i\}_{i\in\{0,\dots,I\}}} \quad & \textstyle\sum_{i=0}^{I} \int_{A^i(c)} \beta(\alpha)\psi(u^i(c^i,\alpha))dF(\alpha) \\ \text{s.t.} \quad & \textstyle\sum_{i=0}^{I}(w^i - c^i)\mu(A^i(c)) = G \end{aligned} \tag{3.3}$$

where G denotes the exogenous level of government expenditures. The function $\psi(\cdot)$ is assumed to be weakly increasing and captures the redistributive preferences of the government. Furthermore, $\beta(\alpha)$ is a function through which the government can assign different weights to the various types of agents in the economy[2].
The first order conditions of the problem write:

$$\mu(A^i)\left[P(A^i) - 1\right] = -\sum_{j=0}^{I} \frac{\partial \mu(A^j(c))}{\partial c^i} t^j \qquad \forall \, i \in \{0, ..., I\} \tag{3.4}$$

where $P(A^i)$ denotes the Pareto weight, i.e. the average marginal social welfare weight assigned to agents who choose state i:

$$P(A^i) := \frac{1}{\lambda \mu(A^i)} \int_{A^i} \beta(\alpha)\psi'(u^i(c^i,\alpha))u_1^i(c^i,\alpha)dF(\alpha) \tag{3.5}$$

and λ is the multiplier associated to the government budget constraint. In other terms, $P(A^i)$ represents the value for the government of distributing an extra dollar uniformly to all individuals working in occupation i. It is possible to show that equation (3.4) can

[2] The function $\beta(\alpha)$ satisfies the following requirements: $\beta(\alpha) \geq 0$ and $\int \beta(\alpha)dF(\alpha) = 1$

be re-written for differences rather than levels of taxes: for $i = 0, ..., I$

$$\mu(A^i)\left[P(A^i) - 1\right] = -\sum_{j \neq i}^{I} \frac{\partial \mu(A^j(c))}{\partial c^i}(t^j - t^i) \tag{3.6}$$

The left-hand side captures the social value of increasing consumption for individuals in occupation i, net of the associated resource cost. This net benefit is weighted by the number of individuals in occupation i. The right-hand side, by contrast, reflects the budget effect of increasing consumption for occupation i. An increase in c^i may induce individuals from other occupations to switch to occupation i, potentially altering the total tax revenue collected by the government.

Now, consider equation (3.6) in which the tax differences are written with respect to t^0, this would give the following equation for every $i = 1, ..., I$:

$$\mu(A^i)\left[P(A^i) - 1\right] = -\sum_{j=1}^{I} \frac{\partial \mu(A^j(c))}{\partial c^i}(t^j - t^0)$$

Taking as given the measure of sets and the Pareto weights the first order conditions can be re-written as a linear system of I equations in I unknowns:

$$\begin{bmatrix} -\frac{\partial \mu(A^1)}{\partial c^1} & -\frac{\partial \mu(A^2)}{\partial c^1} & ... & -\frac{\partial \mu(A^I)}{\partial c^1} \\ -\frac{\partial \mu(A^1)}{\partial c^2} & -\frac{\partial \mu(A^2)}{\partial c^2} & ... & -\frac{\partial \mu(A^I)}{\partial c^2} \\ ... & ... & ... & ... \\ -\frac{\partial \mu(A^1)}{\partial c^I} & -\frac{\partial \mu(A^2)}{\partial c^I} & ... & -\frac{\partial \mu(A^I)}{\partial c^I} \end{bmatrix} \begin{bmatrix} t^1 - t^0 \\ t^2 - t^0 \\ ... \\ t^I - t^0 \end{bmatrix} = \begin{bmatrix} \mu(A^1)(P(A^1) - 1) \\ \mu(A^2)(P(A^2) - 1) \\ ... \\ \mu(A^I)(P(A^I) - 1) \end{bmatrix}$$

or, in more compact form:

$$H\Delta t = \mu(P - 1) \tag{3.7}$$

where $\mu(P-1)$ is a $I \times 1$ vector with i^{th} component $\mu(A^i)(P(A^i)-1)$; H is a $I \times I$ matrix with entry $h_{ij} = -\frac{\partial(\mu(A^j))}{\partial c^i}$ for $i = 1, ..., I$ and $j = 1, ..., I$ and Δt a $I \times 1$ vector with i^{th} component $(t^i - t^0)$. Now define the "consumption elasticity" of the set A^j as follows:

$$\eta_i^j = -\frac{\partial \mu(A^j)}{\partial(c^i - c^j)} \frac{c^i - c^j}{\mu(A^j)}$$

which captures the percentage change of agents in occupation j in response to a 1% change of disposable income in occupation i. Any element of matrix H can thus be re-written as:

$$h_{ij} = \eta_i^j \frac{\mu(A^j)}{c^i - c^j} \tag{3.8}$$

Each element of the matrix H, h_{ij}, depends on three components: the elasticity of set A^j with respect to disposable income in occupation i, the measure of set A^j and the difference in disposable income between occupation i and j. Depending on how each occupation is defined these objects will have different empirical counterparts.

If matrix H is invertible the optimal tax schedule (with respect to occupation $i = 0$) can be simply obtained by:

$$\Delta t = H^{-1}\mu(P - 1) \tag{3.9}$$

3 Individual income taxation: intensive vs extensive margin

In this section, I use Laroque and Pavoni (2017) framework to simulate optimal individual income tax schedules in different occupational choice settings. First, I consider a model where individuals choose how much to work along the intensive margin only. I then compare the resulting optimal tax schedule to one derived from an occupational choice model where labor supply decisions are instead made exclusively at the extensive margin. I am going to show that shape of the optimal tax schedule differs substantially in the two settings, especially at the bottom of the income distribution.

To derive the optimal tax schedule, I have to solve the system of equations (3.9). For computational simplicity only, I treat the vector of Pareto weights P as an exogenous object. While this is a simplification, it allows to transform the problem into a linear system that can be simply solved by inverting the matrix H. As outlined in Section 2, calibrating the model requires the following information:

- Data on income and taxes paid by American individuals

- A specification of Pareto weights
- Estimates of labor supply elasticities

The next paragraphs provide an overview of the data sources used to obtain these figures, while more details on the elasticities employed will be reported when performing the optimal tax simulations.

Income and tax data: the unit of reference of these simulations is the individual.
To collect information on labor income and average tax rates faced by American households the 2018 American Community Survey is employed. The labor income distribution is discretized into 20 classes of earnings which correspond to "occupations" (see Appendix A), and a density is associated to each of them. The empirical density in our framework provides a way to estimate the measure of the sets A^i for $i = 0, ..., 20$.
To compute the disposable income of each considered occupation, information on average tax rates is needed. Since these simulations focus on individual income taxation the tax rates for single individuals are employed to compute the average disposable incomes across the 20 occupations. Appendix A summarizes average labor income tax rates faced by American households across the income distribution.

Pareto weights: Pareto weights are computed assuming the following functional (Saez (2002)):

$$P(A^i) = \frac{1}{p(c^i)^{\nu}} \tag{3.10}$$

where $p \geq 0$ is a constant which ensures that $\sum_{i=0}^{I} P(A^i)\mu(A^i) = 1$ and $\nu \geq 0$. Notice that this functional form for Pareto weights can simply be obtained by assuming in equation (3.5) $\beta(\alpha) = 1$ for all $\alpha \in \mathcal{A}$, a standard CRRA utility function for every occupation (solely depending on c^i) and a concave power function $\psi(\cdot)$ for the government redistributive preferences. Notice that, for given occupation specific utility functions, varying ν is equivalent to change the redistributive tastes of the government.

3.1 Extensive margin model: tax simulations

First, I simulate the optimal individual income tax schedule considering the simple case in which only the extensive margin of labor supply is active. In particular, I assume that each individual either works in occupation i corresponding to his type or is unemployed (i.e. he chooses occupation 0). This implies that all entries of the matrix H outside the diagonal are zero, while for the elements on the diagonal the following holds for every $i = 1, ..., I$:

$$h_{ii} = \eta_0^i \frac{\mu(A^i)}{c^i - c^0} \tag{3.11}$$

The linear system to be solved therefore takes the following form:

$$\begin{bmatrix} h_{11} & 0 & ... & 0 \\ 0 & h_{22} & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & h_{II} \end{bmatrix} \begin{bmatrix} t^1 - t^0 \\ t^2 - t^0 \\ ... \\ t^I - t^0 \end{bmatrix} = \begin{bmatrix} \mu(A^1)(P(A^1) - 1) \\ \mu(A^2)(P(A^2) - 1) \\ ... \\ \mu(A^I)(P(A^I) - 1) \end{bmatrix} \tag{3.12}$$

solving this system delivers the tax differences $t^i - t^0$ for all $i = 1, ..., 20$. To obtain the optimal level of taxes in each occupation the government budget constraint is needed:

$$\sum_{i=0}^{I} t^i \mu(A^i) = T \Rightarrow \quad t^0 = T - \sum_{i=0}^{I} (t^i - t^0)\mu(A^i) \tag{3.13}$$

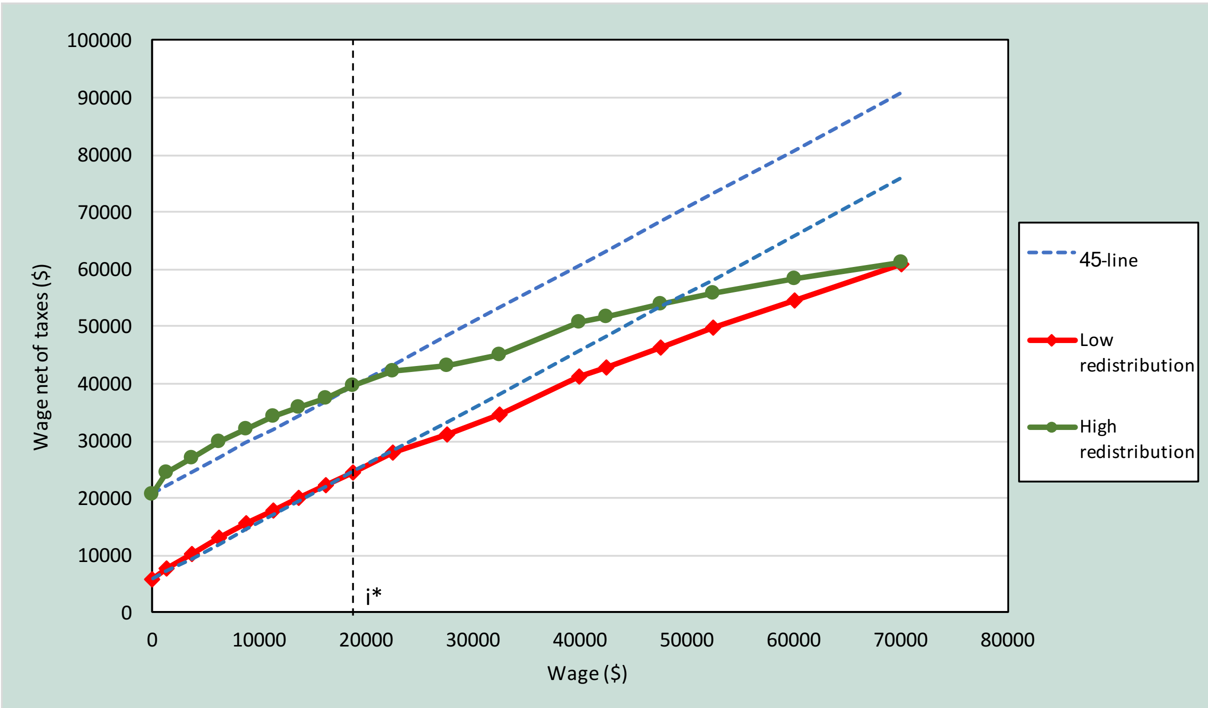
where the tax requirement T is chosen to be $T = 10000\$$, which matches the average per-capita income tax revenues net of transfers (TANF, Food stamps) in 2018 (American Community Survey data).

The solution to the optimal tax problem is highly sensitive to the values of the labor supply elasticities, which determine the elements of the matrix H. The empirical literature on labor supply elasticities is extensive, with a wide range of estimation methods and elasticity concepts employed. For the purpose of this simulation, I rely on the extensive margin elasticities reported by Attanasio et al. (2018). In particular, the elasticity that they call "extensive response" perfectly aligns with the concept of participation elasticity (η_0^i) defined in this framework.

Moreover, Attanasio et al. (2018) provide different estimates of these extensive responses across income quartiles, which is crucial for the calibration in this context. Specifically:

1. for 0p-25p: 1.2
2. for 26p-50p: 0.77
3. for 51p-75p: 0.39
4. for 76p-100p: 0.16

FIGURE 1. *Optimal tax schedule when only extensive margin is active*



Notes: *this Figure representes the optimal tax schedule obtained solving the system of equation (3.12) (i.e. extensive margin model). Details on model calibration are reported in Section 3.1. Red line represents the optimal tax schedule when Pareto weights are computed assuming $\nu = 0.1$, while the green line when $\nu = 1$.*

The assumptions made on the elasticities ensure that the linear system (3.12) admits a unique solution. Specifically, since the matrix H is diagonal with non-zero entries on the diagonal it is possible to show that it is invertible (Cremonini (2020)).

By varying the parameter ν, I also explore how changes in the government's redistributive preferences affect the shape of the optimal tax schedule. The results of these simulations are presented in Figure 1. The red line represents the optimal tax schedule when $\nu = 0.1$ (low redistributive preferences), while the green line represents the optimal tax schedule when the government has higher preferences for redistribution, i.e. when $\nu = 1$.

Figure 1 illustrates that unemployed individuals receive a transfer, which increases as the government's redistributive preferences grow stronger. Notably, the working poor receive a larger transfer than the unemployed. This implies the presence of both a guaranteed minimum income and a negative marginal tax rate at the lower end of the income distribution. Why is it optimal for the working poor to receive more than the unemployed?[3] If they received less (i.e. no negative marginal tax rates), then increasing their transfer would cost one unit of resources while generating a social welfare gain of $P(A^i) > 1$. Additionally, the higher transfer would encourage some unemployed individuals to enter the labor force, increasing total tax revenues. This implies that a scenario with no negative marginal tax rates is suboptimal. Notice that, after the threshold i^*, the negative marginal tax rate turns positive.

The cost of this redistributive scheme is covered by higher taxes on top earners. The structure of the tax schedule is also influenced by the distribution of participation elasticities: high elasticities at the bottom make it relatively inexpensive to incentivize labor market entry, while low elasticities at the top permit heavier taxation on high earners with limited labor supply loss.

Although the simulation uses only four discrete elasticity values (one per earnings quartile), it's plausible that elasticities vary continuously with income. To assess robustness, Appendix B presents a simulation using interpolated elasticity values. The resulting tax schedules are qualitatively unchanged, confirming the consistency of the findings.

3.2 Intensive margin

I now consider the case in which only the intensive margin of labor supply is active. Specifically, I assume that each individual could choose the occupation i which corresponds to his type, occupation $i-1$ or occupation $i+1$. When this is the case it is possible to show that the system (3.9) takes the form:

[3] An intuition for this result is also provided by Christiansen (2015). In a similar framework he argues that a sufficient condition for the negative marginal tax rated faced by the working poor to be optimal is a labor supply elasticity large enough for people in income brackets above that of the working poor.

The linear system (3.9) becomes:

$$\begin{bmatrix} h_{11} & h_{12} & 0 & 0 & ... & 0 \\ h_{21} & h_{22} & h_{23} & 0 & ... & 0 \\ 0 & h_{31} & h_{32} & h_{33} & ... & 0 \\ ... & ... & ... & ... & ... & ... \\ 0 & 0 & ... & 0 & h_{I-1I} & h_{II} \end{bmatrix} \begin{bmatrix} t^1 - t^0 \\ t^2 - t^0 \\ t^3 - t^0 \\ ... \\ t^I - t^0 \end{bmatrix} = \begin{bmatrix} \mu(A^1)(P(A^1) - 1) \\ \mu(A^2)(P(A^2) - 1) \\ \mu(A^3)(P(A^3) - 1) \\ ... \\ \mu(A^I)(P(A^I) - 1) \end{bmatrix} \tag{3.14}$$

Using equation (3.8) it is possible to show that the elements of matrix H take the following form. For each $i = 1, ..., I$:

$$h_{i,i+1} = \eta_i^{i+1} \frac{\mu(A^{i+1})}{c^i - c^{i+1}} \tag{3.15}$$

$$h_{i,i-1} = \eta_i^{i-1} \frac{\mu(A^{i-1})}{c^{i-1} - c^i} \tag{3.16}$$

Moreover, by using the fact that for every $i = 0, ..., I$

$$\sum_{j=0}^{I} \frac{\partial \mu(A^j)}{\partial c^i} = 0$$

$$h_{i,i} = \eta_i^{i+1} \frac{\mu(A^{i+1})}{c^{i+1} - c^i} + \eta_i^{i-1} \frac{\mu(A^{i-1})}{c^i - c^{i-1}} \tag{3.17}$$

The structure of Pareto weights remains the same as in (3.10) and the budget constraint is the one used in the previous simulation.

The values of the intensive elasticities used in the following simulation are again taken from the paper by Attanasio et al. (2018). Specifically:

1. for 0p-25p: 0.44

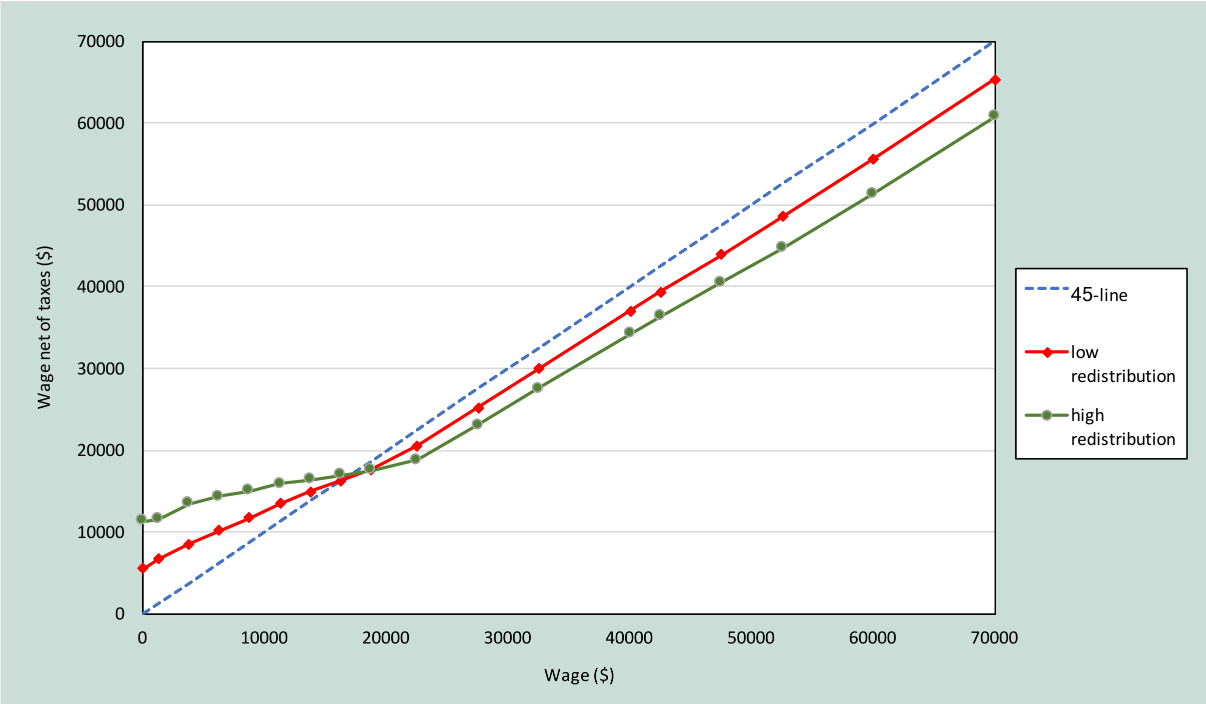
2. for 26p-50p: 0.54

3. for 51p-75p: 0.69

4. for 76p-100p: 1.16

The results of the simulation are presented in the following Figure 2.

FIGURE 2. *Optimal tax schedule when only intensive margin is active*



Notes: *this Figure represents the optimal tax schedule obtained solving the system of equation (3.14) (i.e. intensive margin model). Details on model calibration are reported in Section 3.2. Red line represents the optimal tax schedule when Pareto weights are computed assuming $\nu = 0.1$, while the green line when $\nu = 1$.*

The optimal tax schedule here resembles a standard negative income tax (NIT) system, where redistribution occurs through subsidies targeted at the lowest earners and is financed by taxes on higher income brackets. Unlike the case with only the extensive margin active, there is no negative marginal tax rate. The reasoning is as follows: suppose to slightly increase the negative marginal tax rate of occupation i. If there is any reduction in the labor supply in occupation i this would induce those switching to occupation $i-1$ to pay more taxes. Furthermore, the increase in the marginal tax rate would allow the government to raise more resources for redistribution from all agents in occupation higher than i. Thus, the increase in the marginal tax rate would induce an increase in the social welfare, through larger tax collected employed for redistributive purposes. This shows that the negative marginal tax rate was not an optimal solution of the problem.

The shape of the tax schedule is also highly sensitive to the assumed labor supply elasticities. Even if top earners are associated with low Pareto weights, taxing them too heavily is inefficient due to their relatively high labor supply responsiveness, which would lead to reduce the tax revenues collected by the government. This contrasts with the extensive-margin-only case, where high-income individuals have lower participation elasticities and lower Pareto weights, justifying heavier taxation. Appendix B confirms these findings by interpolating the elasticity values provided by Attanasio et al. (2018). This produces a very similar tax schedule, except for a flatter profile at the bottom of the distribution[4].

4 Couple taxation with extensive margin labor choice

The aim of this section is to show how Laroque and Pavoni (2017) model can be applied to derive optimal tax schedules for couples in the United States. First, I employ the so-called "inverted optimal tax approach" to infer social preferences regarding family taxation in the U.S. I then use the information obtained to conduct optimal tax simulations, analyzing how the optimal tax schedule is shaped by the government's redistributive preferences and by households' labor supply elasticities.

Framework and occupations: suppose that each family consists of one man and one woman, and that each individual decides whether to work or not, that is only the extensive margin of labor supply is active for each family member. In the framework of Laroque and Pavoni (2017), the work status of household members can be represented using the following occupations: $i = 0$ if neither spouse works, $i = 1$ if only the woman works, $i = 2$ if only the man works and $i = 3$ if both spouses work. In cases where only one spouse is employed, that individual is referred to as the primary earner.
To simplify the analysis I rule out the possibility of "double deviations", that is the "primary earner" exiting the labor market and the "secondary earner" entering the labor market or both exiting (or entering) the labor market jointly. Although restrictive, this is a common assumption in the optimal couple taxation literature (e.g. Kleven et al. (2009), Immervoll et al. (2011)). Under these assumptions the system of FOCs (3.7) can

[4]Notice that in the high-redistribution scenario with only an extensive margin (see Figure B3.1), marginal tax rates go above 100% for income larger than, 40,000 USD (thus implying re-ranking of income when moving from pre-tax incomes to post-tax incomes). This seems to indicate that a model with just an extensive margin appears to be too much an oversimplifcation to generate meaningful tax results.

be re-written as:

$$\begin{bmatrix} -\frac{\partial\mu(A^0)}{\partial c^0} & -\frac{\partial\mu(A^1)}{\partial c^0} & -\frac{\partial\mu(A^2)}{\partial c^0} \\ -\frac{\partial\mu(A^0)}{\partial c^1} & -\frac{\partial\mu(A^1)}{\partial c^1} & 0 \\ -\frac{\partial\mu(A^0)}{\partial c^2} & 0 & -\frac{\partial\mu(A^2)}{\partial c^2} \end{bmatrix} \begin{bmatrix} t^0 - t^3 \\ t^1 - t^3 \\ t^2 - t^3 \end{bmatrix} = \begin{bmatrix} \mu(A^0)(P(A^0)-1) \\ \mu(A^1)(P(A^1)-1) \\ \mu(A^2)(P(A^2)-1) \end{bmatrix} \tag{3.18}$$

notice that, in this case, tax differences are written with respect to occupation 3.

Matrix H properties: to compute the optimal couple tax schedule (taking as given the Pareto weights) the key challenge is to link the elements of matrix H to their (estimable) empirical counterparts. Let's proceed step by step.

Consider the first row of matrix H, whose elements capture the changes in the measures of all occupations when disposable income in occupation $i = 0$ varies. It holds:

$$\frac{\partial\mu(A^1)}{\partial c^0} = -\eta_0^1 \frac{\mu(A^1)}{c_1 - c_0} \qquad \frac{\partial\mu(A^2)}{\partial c^0} = -\eta_0^2 \frac{\mu(A^2)}{c_2 - c_0}$$

where η_0^1 corresponds to the participation elasticity of a woman conditional on his spouse not working, and η_0^2 corresponds to the participation elasticity of a man conditional on his spouse not working. Furthermore, given the assumption: $-\frac{\partial\mu(A^3)}{\partial c^0} = 0$ we have $-\frac{\partial\mu(A^0)}{\partial c^0} = \frac{\partial\mu(A^1)}{\partial c^0} + \frac{\partial\mu(A^2)}{\partial c^0}$.

Consider now the second row of H which captures the occupational effects of an increase in c^1, the disposable income of an household where the woman is the only earner. In this case in some families in which both spouses are not working, the woman may find convenient to start working. Alternatively, in households where both family members are working the increase in c^1 may lead male workers to quit their job. These movements are captured by:

$$\frac{\partial\mu(A^0)}{\partial c^1} = -\eta_0^1 \frac{\mu(A^0)}{c_1 - c_0} \qquad \frac{\partial\mu(A^3)}{\partial c^1} = -\eta_1^3 \frac{\mu(A^3)}{c_3 - c_1}$$

where η_0^1 is the participation elasticity of women conditional on her spouse not working, while η_1^3 is the participation elasticity of men conditional on his spouse to be working. Furthermore, given the assumption of no double deviations the following holds:

$-\frac{\partial \mu(A^1)}{\partial c^1} = \frac{\partial \mu(A^0)}{\partial c^1} + \frac{\partial \mu(A^3)}{\partial c^1}$.

An analogous reasoning can be employed to re-write the entries of the third row of the matrix H. Let's assume an increase c^2: men of some couples in which both members are not working may start working. Furthermore, women in couples in which both members are working may decide to quit their job. These movements are captured by:

$$\frac{\partial \mu(A^0)}{\partial c^2} = -\eta_0^2 \frac{\mu(A^2)}{c_2 - c_0} \qquad \frac{\partial \mu(A^3)}{\partial c^2} = -\eta_2^3 \frac{\mu(A^3)}{c_3 - c_2}$$

where η_0^2 is the participation elasticity of men conditional on his spouse not working, while η_2^3 is the participation elasticity of women conditional on his spouse to be working. Furthermore: $-\frac{\partial \mu(A^2)}{\partial c^2} = \frac{\partial \mu(A^0)}{\partial c^2} + \frac{\partial \mu(A^3)}{\partial c^2}$

To sum up, to calibrate this occupational choice model four elasticities are needed: the participation elasticity of women and men conditional on the spouse not working (η_0^1 and η_0^2) or working (η_1^3 and η_2^3). If we assume that all the four mentioned elasticities mentioned are non-zero, it is possible to show that the matrix H in this setting is invertible.

Differently from Section 3, the presented model will be first employed to estimate the vector of Pareto weights P, assuming that the existing tax system is optimal (inverted optimal tax approach).

Then, the observed Pareto weights will be suitably altered so to simulate how the optimal couple tax schedule would change under different governmental redistributive preferences and different elasticity scenarios.

These exercises require data on the disposable incomes and taxes paid by American households, as well as information about the employment status of household members. The following Section summarizes these information and describes the calibration of matrix H.

4.1 Data and matrix H calibration

The reference unit in this analysis is the household as a whole. Since the model focuses on determining optimal tax schedules based solely on the labor supply decisions of couples,

Table 1. *Calibration data*

	$\mu(A^i)$	w^i	c^i
$i = 0$	0.2012	0	22844
$i = 1$	0.0727	48623	42934
$i = 2$	0.2042	57204	50511
$i = 3$	0.5219	91768	75543

Notes: *Measures of column 1 are taken from 2018 Current Population Survey provided by US Bureau of Labor Statistics. Column 2 represents the median income of each "occupation": they are taken from 2018 American Community Survey and they are gross of taxes. In the third column the income tax rate faced by each occupation, according to the income values of column 2, are presented. Finally, the net value of consumption for each occupation is provided in the fourth column. All values are in inflation-adjusted dollars.*

the relevant income considered is exclusively labor income, excluding any income from capital or investments. Accordingly, taxes are defined as labor income taxes, imposed by both central and local governments, minus any benefits received (e.g. guaranteed minimum income, unemployment benefits, family-related government transfers).

Labor income and labor income taxes: the reference year for this analysis is 2018. Data on the number of unemployed households, single-earner households, and dual-earner households in the United States are drawn from the American Current Population Survey. These figures are used to calculate the shares of each defined occupation category and are presented in the first column of Table 1. The second column of Table 1 displays the average gross labor income for each household type, based on data from the 2018 American Community Survey. Given the complexity and variability of tax and benefit systems across U.S. states, estimating average tax liabilities and benefits by occupation is challenging. To address this, I use the OECD tax-benefit calculator. Specifically, I simulate taxes and benefits for U.S. households in which both adults are 50 years old and have two children aged 16 and 12 (figures that reflect the average household composition in the U.S). The resulting disposable incomes are reported in Table 1, column 3.

Participation elasticities: The literature on participation elasticities is extensive, with estimates varying widely depending on the methodological approach and context in which they are derived. Labor force participation elasticities are generally modest and vary across groups. On average, women, low-income individuals, and secondary earners tend to respond more to tax and benefit changes than men or primary earners.

Bartels and Shupe (2023) calculate participation elasticities for primary and secondary earners of both genders in couples for several EU countries between 2008 and 2014 using EUROMOD data. They find that the average elasticity is 0.08 for men and 0.14 for women. However, when distinguishing by their role in the household, the primary earner range is 0.02-0.08 for men and -0.03-0.15 for women whereas the secondary earner range is 0.05-0.2 for men and 0.09-0.22 for women.

Bargain et al. (2014) compute labor supply elasticities not only for EU countries but also for the US., using TAXBEN and EUROMOD programs. Notably, the average participation elasticities reported by Bartels and Shupe (2023) for the EU (0.08 for men and 0.14 for women) fall well within the ranges estimated by Bargain et al. (2014) for the U.S. (which are 0.05-0.15 for men and 0.1-0.2 for women).

Additional studies report similar elasticity estimates. For instance, Lin and Tong (2017) use IRS administrative data from 2000-2009 and the TAXSIM model to estimate slightly lower participation elasticities (0.03 for married men and 0.10 for married women). These will constitute lower bounds in our simulations. Eissa and Hoynes (2004), using 1984-1996 U.S. data and a Probit model, estimate participation elasticities of 0.03 for men and 0.27 for women in response to EITC expansions, highlighting the program's role in encouraging mothers to stay home. While these estimates are broadly consistent with our range, we adopt more conservative elasticity values for women, given that the Eissa and Hoynes (2004) data are dated and gender elasticities have converged over time as women's labor force participation increased Bargain et al. (2014). This discussions leads to deem reasonable the scenarios that are considered in Table 2.

4.2 Inverted optimal tax approach

The optimal taxation literature has traditionally focused on deriving the efficient tax schedule given the government's redistributive preferences. However, a more recent strand of research, beginning with Bourguignon and Spadaro (2012), adopts a reverse approach. Rather than assuming specific social preferences, this approach starts by assuming the existing tax schedule in a country as optimal and then asks which set of Pareto weights

Table 2. *Values for the extensive elasticities employed in simulations*

	Male 1^{st} earner	Male 2^{nd} earner	Fem. 1^{st} earner	Fem. 2^{nd} earner
Scenario 1	0.05	0.1	0.05	0.15
Scenario 2	0.05	0.1	0.05	0.18
Scenario 3	0.05	0.1	0.07	0.18
Scenario 4	0.03	0.09	0.03	0.13

Notes: Participation elasticity values have been chosen consistently with the literature findings summarized in this paragraph. Specifically, scenario (2) considers slightly larger elasticities for female as a secondary earner, scenario (3) consider larger elasticities for female in general, scenario (4) is the lower bound case.

would be consistent with that optimal tax system. In Laroque and Pavoni (2017) this means to solve the system $H\Delta t = \mu(P-1)$ for the vector of Pareto weights P (taking as given Δt) rather than for Δt (taking as given P).

Once retrieved the "revealed Pareto weights" I study whether they are consistent with the properties of Pareto weights derived under standard assumptions for the social welfare function.

Denote with $\hat{P}(A^i)$ the revealed Pareto weight for occupation i. Equation (3.18) allows to estimate the reveled Pareto weights for $i = 1, 2, 3$:

$$\hat{P}(A^i) = 1 + \frac{1}{\mu(A^i)}\Bigg(- \frac{\partial\mu(A^1)}{\partial c^i}(t^1 - t^0) - \frac{\partial\mu(A^2)}{\partial c^i}(t^2 - t^0) - \frac{\partial\mu(A^3)}{\partial c^i}(t^3 - t^0)\Bigg) \quad (3.19)$$

Together with: $\sum_{i=0}^{3} \hat{P}(A^i)\mu(A^i) = 1$ which allows to retrieve $\hat{P}(A^0)$.

Revealed Pareto weights properties: let's recall the definition of Pareto weight (3.5):

$$P(A^i) := \frac{1}{\lambda\mu(A^i)} \int_{A^i} \beta(\alpha)\psi'(u^i(c^i, \alpha))u_1^i(c^i, \alpha)dF(\alpha)$$

they represent the average marginal social welfare weight associated to individuals (or in this case couples) who choose occupation i.

Denote by $G(c) = \int_{A_i} \beta(\alpha)\psi(u^i(c^i, \alpha)))dF(\alpha)$ the social welfare function faced by the planner. If the functions $\psi(\cdot)$ and $u^i(c^i, \cdot)$ are weakly increasing, then the social welfare function $G(c)$ is weakly increasing in c^i, for all $i = 0, ...3$.

Definition 2 (Paretian Social Welfare function)**.** *A Social Welfare function* $G(c^1, ..., c^I)$

is said to be Paretian if $\partial G(c)/\partial c^i \geq 0$ for all $i = 0, ..., I$. Otherwise G is non-Paretian.

Therefore, if the social welfare function of our couple taxation problem is Paretian, it holds: $\int_{A^i} \beta(\alpha)\psi'(u^i(c^i, \alpha))u_1^i(c^i, \alpha)dF(\alpha) \geq 0$ for all $i = 0, ..., 3$, hence $P(A^i) \geq 0$. In other words, only positive Pareto weights could be associated to a Paretian Social Welfare function. I then check whether the estimated "revealed" Pareto weights are consistent with the minimal requirement of a Paretian social welfare function.

FIGURE 3. *Revealed Pareto Weights for US couples*

Notes: *Revealed Pareto Weights for US couples. The scenarios correspond to the values of the participation elasticities of Table 2. The considered couples are: couples in which both members are not working, one-earner couple with woman working, one-earner couple with man working, two earners couple. Computation by the author, data and simulation method presented in the present section.*

Revealed Pareto weights estimates: Figure 3 represents the "revealed" Pareto weights computed using equation (3.19). The different colored lines correspond to the revealed Pareto weights estimated employing the four participation elasticity scenarios described in Table 2.

First, note that the Pareto weights are significantly different from zero across all scenarios, indicating that the estimated revealed Pareto weights are consistent with a Paretian social welfare function.

The next step is to assess whether Pareto weights are decreasing in household disposable income. That would be the case under the standard assumption of concave utility

functions for agents in each occupation and linear or concave function $\psi(\cdot)$ (reflecting utilitarian or inequality averse planner).
Revealed Pareto weights decrease with disposable income for occupations $i = 1, 2, 3$ as expected. However, in some scenarios, the weight assigned to the poorest households (those with no earners) is lower than that of households where only the female spouse works, who have larger disposable income. Why is that the case? Among the possible explanations Saez and Stantcheva (2016) highlight that weights $P(A^i)$ could be not only a function of $c = (c^0, c^1, c^2, c^3)$ but also of net taxes. In fact, assuming that $P(A^i)$ increases with $t = (t^0, t^1, t^2, t^3)$ captures the idea that taxpayers contributing more to the society are more deserving of additional consumption. This perspective helps to explain the observed shape of the revealed Pareto weights: Pareto weights in occupations $i = 1, 2, 3$ mostly reflect government redistributive preferences and concavity of the utility functions. For non-working households, instead, lower weights reflect a political reluctance to allocate significant resources toward those not contributing to the tax system.

An alternative explanation could be the following. Suppose the observed tax system does reflect the government solving the same the planner's optimization problem I have analyzed. As previously noted, estimating participation elasticities is highly contentious and results can vary significantly depending on the methodology and data used. Therefore, the elasticities I used to compute the revealed Pareto weights may differ from those assumed by policymakers when designing the tax schedule. Furthermore, since comprehensive tax reforms are not implemented each year the elasticities used by the government to determine the optimal tax schedule may be based on older data with respect to mine. There is evidence, in fact, that participation elasticities have decreased over time (Lundberg and Norell (2020)), and simply assuming a female participation elasticity (conditional on the spouse not working) of 0.07 (scenario 3) rather than 0.05 or lower delivers a set of Pareto weights strictly decreasing in disposable income. The intuition is very simple: with females more responsive to changes in taxation, given redistributive preferences, female should be taxed less. Therefore, to keep the current tax scheme as optimal with a larger female participation elasticity the government should decrease the weight associated to occupation $i = 1$ and increase the weight associated to $i = 0$.

4.3 Couple tax simulation

In this section I use the previously derived information on governmental preferences to simulate optimal tax schedules under varying assumptions. Specifically, I conduct a series of comparative statics exercises to examine how changes in redistributive preferences or in the assumed participation elasticities affect the resulting optimal tax schedules.
To find the optimal couple tax schedule I need to solve the system of equations (3.18). Notice that, both the Pareto weights $P(A^i(c))$ and the measures of the sets $\mu(A^i(c))$ are endogenous objects depending on the vector of disposable incomes. In Section 3, just for computational simplicity I took these objects as given, now I consider them as *endogenous*. Under this assumption the system is no longer linear and it can only be solved numerically.

Similarly to the approach taken in Section 3, I am going to assume a functional form for the Pareto weights which directly depend on disposable incomes, without specifying the primitives of the social welfare function: $\beta(\alpha)$, $u^i(\cdot,\cdot)$ and $\psi(\cdot)$. I stick to the functional form employed for the individual optimal tax simulations of Section 3:

$$P(A^i) = \frac{1}{p(c^i)^{\nu}} \tag{3.20}$$

This functional form is particularly useful because it allows to model preferences of the government with the use of a single parameter ν: for example, if $\nu = +\infty$ preferences correspond to the Rawlsian case, while if $\nu = 0$ the government displays Utilitarian preferences. More in general, ν can be interpreted in the following way: when disposable income is multiplied by N, the government values N^{ν} times less marginal consumption.
Furthermore, this simple functional form choice makes it possible to solve the non-linear system of equations (3.18), without treating the Pareto weights as given.
Notice, however, that not only the Pareto weights are endogenous objects, but also the measure of each occupation set, $\mu(A^i)$, which so far have been taken as given. To take that into account the endogeneity of occupation measures when computing the optimal tax schedule the following procedure is employed:

1. Start from the existing occupational distribution and disposable income scheme. Given governmental preferences (which amounts to choose ν) the optimal tax schedule could be computed;

2. Compute the new distribution of households across the various occupations under the new tax schedule.

3. The new disposable income and the new $\mu(A^i)$ could be used as the starting point in order to repeat 1. and 2.

4. This procedure has to be iterated up to point of convergence which is the desired solution of the optimal taxation problem

In particular, to compute the new household distribution across occupations in 2., the following relationship is employed:

$$\Delta\mu(A^i) = \sum_{j \neq i} \frac{\partial \mu(A^i)}{\partial c^j} (\Delta c^j - \Delta c^i) \tag{3.21}$$

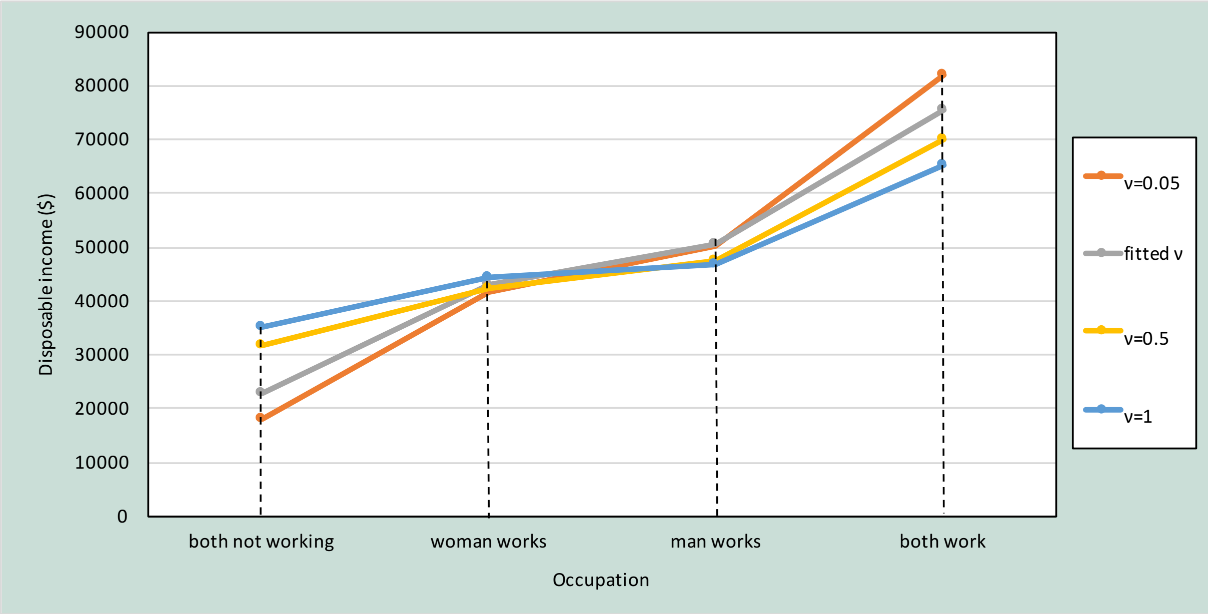
which tell that the change in the measure of occupation i depends on how the disposable income in occupation i changes with respect to that in any other occupations, weighted for a term that positively depends on the elasticity between the two occupations. Given these behavioral responses the new measures of the occupation sets will be $\mu^{'}(A^i) = \mu(A^i) + \Delta\mu(A^i)$. Notice that the structure of this procedure does not guarantee a priori convergence of the algorithm. This issue is discussed in the Appendix C.

To perform simulations with values of Pareto weights that are comparable with actual redistributive preferences of American government, I am going to use the following procedure. By assuming the functional form (3.20) and using the "revealed" Pareto weights computed in the previous paragraph I estimate the ν which best fits their observed shape[5]. In this way, when the ν employed is the fitted one the optimal tax schedule is the existing one. Then, perturbing the chosen value for ν allows me to study how the optimal shape of

[5]The fitted value is $\nu = 0.1412$, which is obtained by averaging the results of the MSE minimization problem across the four elasticity scenarios considered.

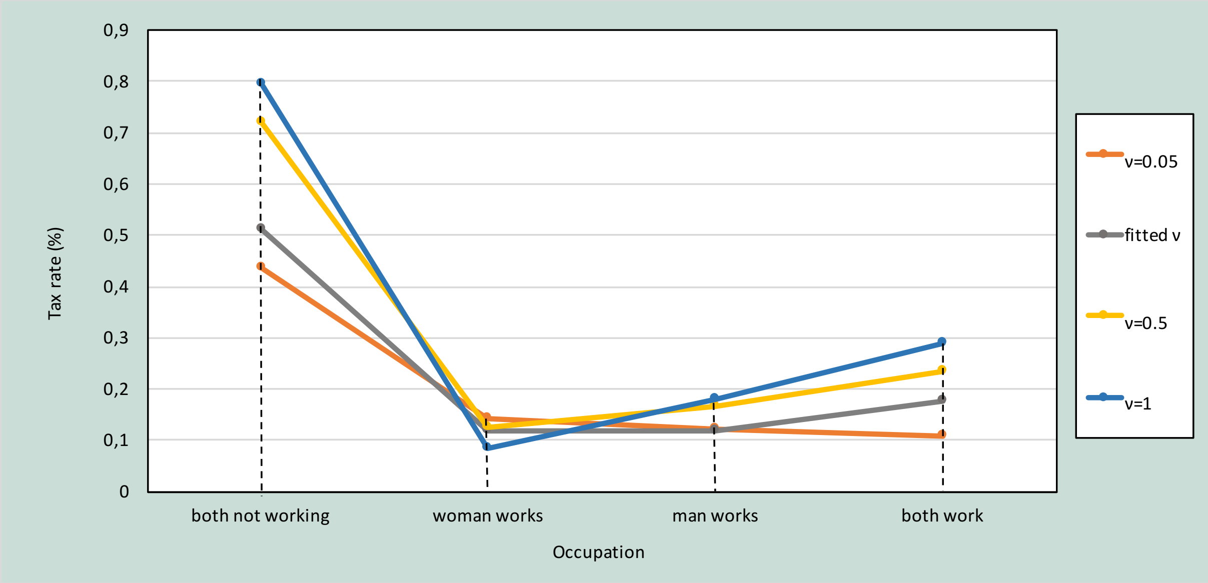
the income tax schedule changes when varying the government redistributive preferences.

FIGURE 4. Simulation of disposable income for US couples



Notes: *Simulation of disposable income for US couples for different governmental redistributive preferences employing the participation elasticities of Table 2, Scenario 1. Fitted* $\nu = 0.1412$*. The considered couples are: couples in which both members are not working, one-earner couple with woman working, one-earner couple with man working, two earners couple. Computation by the author, data and simulation method presented in the present section.*

FIGURE 5. Simulation of optimal tax rates for US couples



Notes: *Simulation of tax rates for US couples for different governmental redistributive preferences employing the participation elasticities of Table 2, Scenario 1. Fitted* $\nu = 0.1412$*. Tax rates are average tax rates, except for the couples in which both are not working, in this case the Participation Tax Rate is presented. The considered couples are: couples in which both members are not working, one-earner couple with woman working, one-earner couple with man working, two earners couple. Computation by the author, data and simulation method presented in the present section.*

Results Figure 4 shows the disposable incomes associated to the optimal tax schedule computed solving the (non-linear) system (3.18). The different colored lines correspond to the optimal disposable incomes under various government redistributive preferences. Figure 5, instead, reports the optimal tax rates associated to the different occupations considered. Notice that in Figure 5, the tax rate associated to occupation $i = 0$ corresponds to the *average participation tax rate* which is computed as follows. First I calculate the participation tax rate for each spouse (conditional on his partner not working), defined as: (benefits of non working + taxes when working) / wage, and then I take the average between men and women participation tax rates. Notice the participation tax rate is inversely proportional to the average financial gain of an household member to enter the labor market (conditional on his/her partner not working). The other tax rates are instead the average tax rates associated to each occupation.

Notice that the gray line closely approximates the existing tax schedule for American households, since it's computed employing the "revealed" Pareto weights estimated in this Section.

The simulation shows that when increasing the redistributive tastes of the government (with respect to the actual preferences, gray line) the greatest variation in tax rates is associated to the non-working and the two-earners families: specifically, more redistribution implies higher subsidies to the non-working families that are taxed away from the two-earners families.

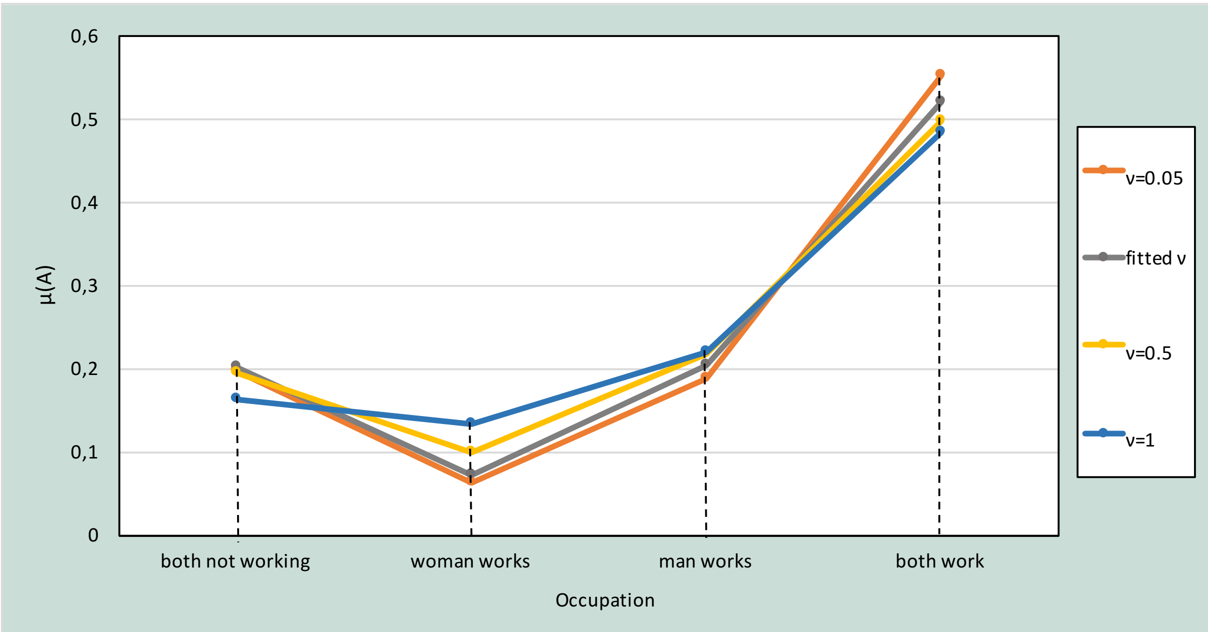
Now, notice that when redistributive preferences of the government are low (orange line) the average tax rate faced by household where only the woman is working is larger than the average tax rate faced by household where only the man is working.

Two contrasting forces determine this effect. One the one hand household with one-earner-woman are poorer, hence associated with a larger Pareto weight. However, the elasticity $\partial\mu(A^i)/\partial c^i$ is larger for the occupation with 1-earner-man with respect to the occupation with 1-earner-women[6]. Thus, when the government highly values redistribution the redistributive effect induced by the Pareto weight dominates, hence the average

[6] Notice that in Scenario 1 (see Table 2) the participation elasticity of male and female as a first earner is the same. Instead, the participation elasticity of female as a second earner is larger than the elasticity of a male as a second earner. This explains why the elasticity of occupation $i = 1$ (only woman working) is lower than the elasticity of occupation 2 (only man working)

tax rate is larger for the occupation with 1-earner-man. Instead, when the redistributive preferences of the government are lower the elasticity effect dominates and the tax rate face by 1-earner-woman household is larger than that faced by 1-earner-man. This is exactly what we can observe in Figure 5: when $\nu = 1$ (that is when government has high preferences for redistribution) we observe a strictly increasing average tax schedule (apart from occupation $i = 0$), instead when governmental preferences for redistribution decrease ($\nu = 0.05$) the tax schedule is strictly decreasing.

FIGURE 6. Equilibrium occupational distribution for US simulation



Notes: *Equilibrium occupational distribution for US couples for different governmental redistributive preferences given the following participation elasticities:* $\eta_1^0 = 0.05$, $\eta_2^0 = 0.05$, $\tilde{\eta}_1^0 = 0.1$, $\tilde{\eta}_2^0 = 0.15$. *Fitted* $\nu = 0.1412$. *The considered couples are: couples in which both members are not working, one-earner couple with woman working, one-earner couple with man working, two earners couple. Computation by the author, data and simulation method presented in the present section.*

Finally, consider the equilibrium occupational distribution. First, notice that the measure of families in which both members work decreases when government preference for redistribution is higher. This is representative of the distortion of labor supply associated with higher taxation for 2-earners couples, which induces in some couples either the man or the woman to stop working. This increases the measure of the sets A^1 and A^2. Lastly, notice that the number of families in which no members are working remains almost constant, only slightly affected by the size of the transfer these households receive. This is due to the low participation elasticity of primary earners. Only in case of $\nu = 1$ the number of families in which nobody works is significantly lower than in the other cases. This is essentially due to a particularly favorable taxation for one-earner families in which the woman is working, inducing many unemployed couples to switch to this occupation.

5 Conclusion

In this paper I used the general occupational choice framework of Laroque and Pavoni (2017) to simulate optimal income tax schedules for both individuals and couples in the U.S. economy. The flexibility of the framework has allowed to show how different assumptions about labor supply behavior (in particular whether agents respond on the intensive or extensive margin) lead to markedly different optimal tax structures.

In the case of individual taxation, simulations reveal that when only the extensive margin is active, the optimal tax system features negative marginal tax rates at the bottom of the income distribution to encourage participation. In contrast, under intensive-margin labor supply, the optimal schedule features a subsidy for unemployed individuals and positive marginal tax rates.

In the case of couple taxation, I specify the model so to account for joint labor supply decisions of couple members at the extensive margin. Using this framework, I recover the Pareto weights implicitly embedded in the current U.S. couple tax system. First, I show that they are consistent with a standard social welfare function (increasing in households' utility) and then use them to simulate optimal tax schedules varying government redistributive preferences. The results reveal how the interaction between Pareto weights and participation elasticities shapes the relative tax burden across household types. Noticeably, 1-earner-female households should receive a more favorable tax treatment then 1-earner-man households when government highly values redistribution, with the opposite result holding when preferences for redistribution are low.

Overall, this work has illustrated the flexibility of Laroque and Pavoni (2017) model to simulate and compare optimal tax schedules across different household structures and labor supply assumptions. Furthermore, the simplicity of the framework makes it especially useful for exploring the key trade-offs in tax design while enabling straightforward simulation of optimal policies across a broad set of economic environments.

Appendix A: Data

TABLE A3.1. Empirical earning distribution calibration US 2018

Earnings level	Density weights	Cumulative distribution
0$	16,38%	16,38%
$1 to $2499	4,55%	20,93%
$2500 to $4999	3,16%	24,09%
$5000 to $7499	3,30%	27,39%
$7500 to $9999	2,27%	29,66%
$10000 to $12499	3,86%	33,52%
$12500 to $14999	1,86%	35,39%
$15000 to $17499	3,16%	38,55%
$17500 to $19999	2,04%	40,59%
$20000 to $24999	6,28%	46,87%
$25000 to $29999	5,23%	52,09%
$30000 to $34999	5,74%	57,84%
$35000 to $39999	4,54%	62,38%
$40000 to $44999	4,65%	67,03%
$45000 to $49999	3,45%	70,47%
$50000 to $54999	4,13%	74,61%
$55000 to $64999	5,45%	80,06%
$65000 to $74999	4,12%	84,18%
$75000 to $99999	6,44%	90,63%
$100000 or more	9,37%	100,00%

Notes: Earnings in year 2018 for working age population. Data taken from U.S. Census Bureau, 2018 American Community Survey 1-Year Estimates. Data are expressed in 2018 inflation adjusted dollars. Column 2-3 represent the density and the cumulative distribution, respectively, of each earning class. The classes have been chosen to be as consistent as possible with Saez (2002).

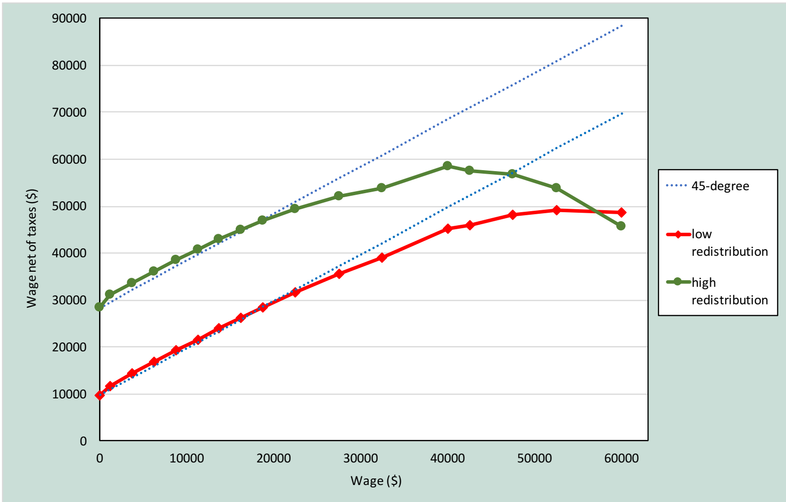
TABLE A3.2. US Marginal tax rates for 2018

Marg. Tax	Single	Married	Married Filing Separately	Head of Household
10%	0 - 9525	0 - 19050	0 - 9525	0 - 13600
12%	9526 - 38700	19051 - 77400	9526 - 38700	13601 - 51800
22%	38701 - 82500	77401 - 165000	38701 - 82500	51801 - 82500
24%	82501 - 157500	165001 - 315000	82501 - 157500	82501 - 157500
32%	157501 - 200000	315001 - 400000	157501 - 200000	157501 - 200000
35%	200001 - 500000	400001 - 600000	200001 - 300000	200001 - 500000
37%	500001+	600001+	300001+	500001+

Notes: 2018 marginal tax rates schedules, source: Department of the Treasury - your federal income tax for individuals. All tax brackets are expressed in dollars.

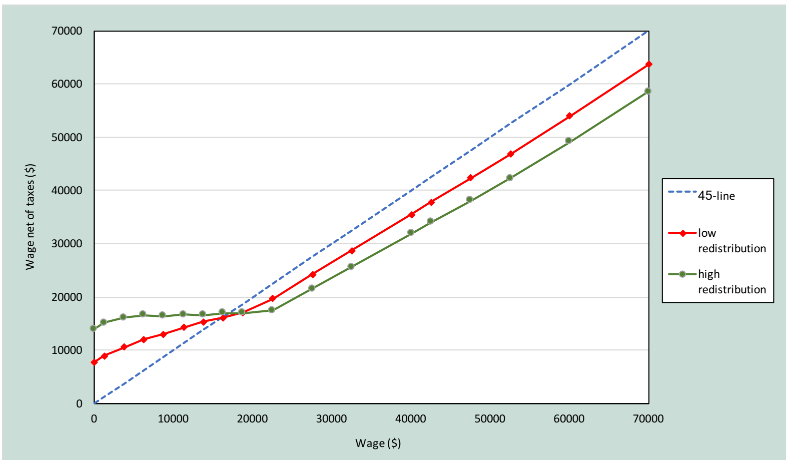
Appendix B: Additional simulations

FIGURE B3.1. Optimal tax schedule with only extensive margin active - with interpolated elasticities



Notes: The simulation is carried on with the assumptions of Section 3.1 with the only difference that the elasticities are interpolated. Specifically, in order to determine the elasticities the following interpolating equation has been computed imposing a third degree polynomial on the participation elasticities provided by Ottaviano (2018): $1.07 \times 10^{-6}x^3 - 8 \times 10^{-5}x^2 - 0.0153x + 1.4025$ where x is the average cumulated density of each earning bracket.

FIGURE B3.2. Optimal tax schedule with only intensive margin active - with interpolated elasticities



Notes: The simulation is carried on with the assumptions of Section 3.2 with the only difference that the elasticities are interpolated. Specifically, in order to determine the elasticities the following interpolating equation has been computed imposing a third degree polynomial on the participation elasticities provided by Ottaviano (2018): $2.88 \times 10^{-6}x^3 - 2.84 \times 10^{-4}x^2 - 0.0123x + 0.3244$ where x is the average cumulated density of each earning bracket.

Appendix C: Convergence of the algorithm for couple taxation simulations

The issue of convergence of the presented algorithm is not trivial and needs to be addressed. Consider the first iteration of the algorithm: the result of Step 1 is the optimal tax schedule given elasticities, redistributive preferences of the government and the initial distribution of couples into occupations. Especially when elasticities are very low (as in our case) for the government it is optimal to impose a lot of taxes on the the occupations associated with the lowest Pareto weights and to give a huge subsidy to those associated with the highest Pareto weights. This may determine as a result of Step 1 negative disposable incomes which do not make sense both from a theoretical point of view (the government cannot tax couples more than their labor gross income) and also from the point of view of the algorithm (it will deliver an error). The solution which is used to solve this problem is the following: the starting point of Step 2 is a weighted average of the solution of Step 1 and the values of the initial tax schedule, where the weights are chosen in order to assigned the maximum weight possible to the solution of Step 1 and to guarantee convergence of the algorithm. Some robustness checks have been performed in order to assess the results when choosing weights capable of ensuring the convergence of the algorithm and close to the optimal ones: this does not alter significantly the results of the procedure.

However, this procedure has a precise economic interpretation: it allows to implicitly include the constraints of non negativity of disposable income and to avoid negative measures of the sets A^i (which arise when the algorithm starts to diverge).

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