Information Transmission and Reputation in Financial Markets

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Contents

To my mother

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Introduction

There are many circumstances in which financial institutions can be assumed to have access to privileged information about market fundamentals and hence to be able to provide valuable services to uninformed investors. In this respect, a relevant and largely debated issue is whether these institutions always behave in accordance with the valuable information that they collect, or sometimes have an incentive to ignore it, potentially inflicting non negligible damages to investors who use their services. Conventional wisdom recognizes reputation acquisition as an effective mechanism that may mitigate this problem, as institutions may lose credibility by repeatedly taking the wrong course of action.

In the first chapter, "Investment Banks as Information Providers in IPOs" (joint with F.Pavesi) we analyze the role of Investment Banks (IBs) as information providers in IPOs. An important issue that arises within the context of IPOs is whether investment banks have an incentive to misreport their private information in order to affect the price of the firm they are underwriting in a desired direction. We show that the introduction of reputation may not suffice to eliminate all scope for misreporting: When public information is quite precise, IBs cannot influence fee income much by changing their reports and less talented IBs believe that contrarian signals are probably incorrect. Thus, they distort their actions to agree with the prior. Instead when public information is diffuse, truth-telling can be supported in equilibrium.

The second chapter "Insider Trading under Discreteness" is devoted to the analysis of a model of insider trading that constitutes the building block of the analysis carried out in the third chapter. In particular, I consider the version of Kyle's (1985) model proposed by Rochet and Vila (1994), where both the distribution of noise trade and that of the liquidation value of the risky asset are assumed to be discrete, instead than normal as in the original Kyle's model. The contribution of the chapter is twofold. First, it presents a complete analysis of the out-of-equilibrium beliefs of the market makers. Second, it proposes a more general version of Rochet and Vila's model by allowing for a more general probability distribution of the liquidation value of the asset. This allows me to address the issue of how the trading strategy of the informed trader changes in response to variations in asset riskiness.

In the third chapter, “Mutual Funds, Career Concerns, and Trade Volume”, I analyze the effects of career concerns of portfolio managers on their incentives to trade in a order-driven market that operates as a call auction. Career concerns are shown to lead portfolio managers to trade even without information, and hence even when expecting a negative return from trade. This finding provides a robustness check on Dasgupta and Prat’s (2006) result derived in a quote-driven market. The analysis is then extended to account for portfolio managers’ trading response to changes in asset volatility. The main finding is that uninformed managers with career concerns trade larger quantities as the asset riskiness increases. As a testable empirical implication, the model predicts that increasing levels of institutional ownership in financial markets lead to higher volumes of trade that are positively correlated with assets volatility.

Chapter 1

Investment Banks as Information Providers in IPOs

1.1 Introduction

The role of information providers in reducing the informational asymmetries in financial markets has received considerable attention. In this paper we consider the incentives of Investment Banks (IBs) as information providers in the underwriting process of IPOs. We show that these agents have both the ability and the incentives to manipulate asset prices through strategically distorted announcements.

We address this question in a signalling game with three classes of players: Investment Banks, Firms and Investors. Firms sell shares in an equity market with asymmetric information either directly or through investment banks. Investment banks have better information on firm profitability than the market (although still incomplete) and interact with the equity market, evaluate entrepreneurs' projects and report to investors in return for a fee. We assume that whenever an IB underwrites a firm's equity, the underwriting comes together with a report of the evaluation performed by the IB about the state of the firm.

IBs differ for having a different "evaluation technology". By "evaluation technology" we mean the ability of the IBs in acquiring accurate information about the true state of the firm whose equities are eventually underwritten. We assume that the information technology is exogenously determined by nature and cannot be changed by the IB. The belief held by the market about the ability (i.e. the type of evaluation technology) of an IB represents its reputation and we assume that it affects the IB's payoffs.

In such a framework, we investigate the firm's decision to go public either through an

IB or directly, and the IBs' decisions to underwrite and report their private information to the market.

Since the compensation the IB gets from the firm for the underwriting activity is typically related to the success of the IPO, the IB faces a strong incentive to inflate the price of the firm's stock through distorted reports in order to enhance the short term profits from the underwriting fee. In fact, these incentives are limited by the IB's concerns about its own reputations. Indeed, since the IB interacts with the market repeatedly, biased reports may cause a loss of reputation, which may in turn lead to a loss of future profits.

We show that in some cases reputation is not enough to induce all IBs to truthfully report their private evaluation. We define such equilibria as informationally inefficient because investment banks manage to influence the price of shares without transmitting their private information on firm profitability. In particular, less talented IBs that have a worst evaluation technology endogenously assign less weight to future compensations based on reputation. Therefore, they strategically distort their private information in order to enhance short-run profits. Furthermore, investment banks with the worst technology will have a greater incentive to disregard their signal pretending to be more informed, the greater is the market perception of investment bank ability (initial reputation).

On a theoretical ground, the role of IBs' reputation in IPOs has been explicitly considered by Chemmanur and Fulghieri (1994). However, they focus on the impact that reputation has on the IBs' incentives to improve their evaluation technology, while do not address the issue of information transmission, which is the main object of our analysis.

In this respect, our model is closely related to the burgeoning literature on reputational cheap talk and in particular to the works of Benabou and Laroque (1992) and Ottaviani and Sorensen (2006). In Benabou and Laroque (1992), insiders perform the joint actions of speculating and spreading information at no intrinsic cost, managing to manipulate prices repeatedly, without being fully found out. Insiders do not differ in their forecasting abilities (i.e., they all receive an equally informative signal), but rather in their degree of honesty in reporting their private information. In particular, some types of insiders are constrained to make truthful reports, while other are allowed to act strategically. In our model, IBs are characterized by different forecasting ability and the reporting strategies of all types of IBs are determined endogenously.

In a very general setup, Ottaviani and Sorensen (2006) study information transmission by a privately informed expert concerned about being perceived to have accurate information. They characterize the expert's incentives to deviate from truthtelling in a frame in which the expert is solely concerned with receivers' perception of his forecasting ability. We draw upon their model and frame the analysis of information transmission into the context of underwriting activity for IPOs. The institutional setup we consider allows us to analyze the issue of information reporting in an economic setup in which the expert is not solely concerned about his reputation. Indeed, the expert's concern for being perceived to have accurate information is entwined with the concerned for the impact that his report has on the decision of the firm and eventually on the price of the firm's stock.

Trueman (1994) considers a model where analysts with different forecasting abilities are concerned about building a good reputation for their forecasting accuracy. He finds analysts exhibit herding behavior, whereby analysts disregard their private information and release forecasts similar to those previously announced by other analysts in order to maximize their expected reputation. His finding is in line with Sharfstein and Stein (1990) analysis of managers' herding behavior in a frame in which the expert has to take an investment decision rather than reporting his private information to a third a part. In these papers, experts choose their action sequentially and, as in Ottaviani and Sorensen (2006), are solely concerned about their reputation.

Our work is also related to the recent literature on analysts' interests conflict. On an empirical ground, Michaely and Womak (1999) show that underwriters' analysts tend to release over-optimistic recommendation in the attempt to hype the stock of the firm taken public by their IB. Morgan and Stocken (2003) present a theoretical model that analyze the information content of stock reports when investors are uncertain about an analyst's incentives. Analyst's incentives may be aligned with those of investors or misaligned. They find that any investor uncertainty about incentives makes full revelation of information impossible. Under certain conditions, analysts with aligned incentives can credibly convey unfavorable information, but can never credibly convey favorable information. The first difference with respect to our work is that in their model analysts do not differ for the degree of their signals' informativeness, but for the degree of divergence of their preferences with respect to those of investors. Basically, as in Benabou and Laroque (1992), the analyst is not concerned about being perceived as having accurate information, but about being perceived as honest. Furthermore, though our model suits to study analysts' conflict of interests, it is nevertheless thought to address the issue of information production and transmission in the period preceding the offer date, when the IB's role of information producer and provider affects not only the decisions of the investors, but also those of the firm candidate to go public.

The main departure of our paper from the previous literature comes from recognizing that poorly informed intermediaries may incorrectly discount the reputation "punishment" precisely because of their scarcely informative signals. This may lead them to overvalue the immediate benefit of communicating biased messages to the market, both in the direction of overstating and understating the firm's value. This effect may imply that reputation acquisition is not sufficient to induce all intermediaries to report their private information.

The paper is organized as follows. In section 2, we introduce the general setup of the model by defining what we intend for value of a firm and reputation, where the information structure is such that the bad IBs have less informative private signals. In section 3, we analyze the both condition under which truthtelling by IBs is possible and the incentives that IBs have to deviate from truthtelling. We characterize a family of "partial pooling" equilibria where talented IBs transmit truthful evaluations while untalented IBs transmit untruthful evaluations to the market and manage to influence prices of firms. In section 4 we compare the relative informational efficiency of different market scenarios as defined by the relevant parameters. Section 5 concludes.

1.2 The Model

We consider a financial market populated by a large pool of firms that want to go public, a large pool of investment banks (IBs) that possibly underwrite their shares and a large pool of investors interested in buying the firms' shares. Firms differ in their fundamental values. IBs differ in their ability to recover information about the true value of the firm that they possibly underwrite.

Suppose that there is only one period t. At the beginning of t, a firm and an IB are randomly selected from their respective pools and matched.[1] The IB *privately* evaluates the firm and proposes its underwriting conditions, which consist of: 1) an evaluation of the firm to be *publicly* communicated to the market; 2) a fee that the firm must pay to the IB. The firms observes both the fee and the proposed evaluation and chooses either to be underwritten by the investment bank or to go public directly.

If the IPO occurs through an IB's service, the IB's evaluation reaches the market and, based on this evaluation, investors determine the price of the firm's shares. If the firm

[1] The analysis of how the firm chooses the IB is out of the scope of the present paper.

goes public directly, investors determine the value of its shares based on the observation that the firm has refused to use the IB.

At the end of t, the true value of the firm is revealed and observed by all market participants. Thus, every player in the market can compare the true value of the firm with the actions taken by the IB and the firm and accordingly form his own belief about the IB's ability of recovering information on the true value of a firm that is about to go public. We interpret this belief as the IB's reputation about its ability and assume that all IBs care about their own reputation.

The rest of the section is devoted to explain in more detail the model just described.

1.2.1 Agents

Firms

There are two types of firms. High profits firms, whose true value is 1 and low profits firms, whose value is 0. Let F denote the value of a firm operating at t and assume that $F \in \{0, 1\}$. Let θ be the fraction of high type firms and $1-\theta$ the complementary fraction of low type firms. Notice that θ can be interpreted either as the prior probability at time t that the firm is worth 1, or as the probability at time t that the firm is worth 1 given the past history up to t, that we denote with Ω_{t-1}. Formally, $\theta = \Pr(F = 1 \mid \Omega_{t-1})$. Let us assume that θ is common knowledge and that firms do not know their own type.[2]

A firm can choose either to accept (A) or refuse (R) to be underwritten by an IB. This choice is taken after the IB has assessed the quality of the firm and revealed the firm the evaluation that will be sent to the market in case the firm accept to go public. If the firm were to refuse to be underwritten, it has the outside option of going public directly.

Investment Banks (IBs)

Although IBs do not know firms' types, they receive a private signal about the true type of the firm. This signal is binary and can be either high or low. Let S_h and S_l denote the events that IB receives a high or low signal in period t respectively.

We assume that there are two types of IBs, good (G) and bad (B). Let IB denote a generic Investment bank active at t, so that $IB \in \{G, B\}$. Good IBs receive a more

[2] This seemingly implausible assumption is without loss of generality. Furthermore, notice that in reality, most of the firms that aim to go public do not have accurate information about the way the market is going to react to the IPO. One reason to hire an IB for the IPO is exactly that of getting some help in determining how the market perceive the offer.

informative signal[3] about the true state of the firm than bad IBs, as described by the following probability distributions:

$$\Pr(S_h \mid IB = G, F = 1) = \Pr(S_l \mid IB = G, F_t = 0) = p,\ p \in \left]\frac{1}{2}, 1\right] \tag{1.1}$$

$$\Pr(S_h \mid IB = B, F = 1) = \Pr(S_l \mid IB = B, F_t = 0) = z,\ z \in \left]\frac{1}{2}, p\right] \tag{1.2}$$

This information structure allows each type of IB receiving a signal to update the prior on firm's state and thus form its own belief about the fact that the firm is good. In particular, we have that

$$\begin{aligned} \Pr(F &= 1 \mid IB = G, S_h) = \frac{\theta p}{\theta p + (1-\theta)(1-p)} \\ \Pr(F &= 1 \mid IB = G, S_l) = \frac{\theta(1-p)}{\theta(1-p) + (1-\theta)p} \end{aligned}$$

$$\begin{aligned} \Pr(F &= 1 \mid IB = B, S_h) = \frac{\theta z}{\theta z + (1-\theta)(1-z)} \\ \Pr(F &= 1 \mid IB = B, S_l) = \frac{\theta(1-z)}{\theta(1-z) + (1-\theta)z} \end{aligned}$$

Let α be the fraction of good IBs, while $(1-\alpha)$ the complementary fraction of bad IBs. Also for IBs, α can be interpreted either as the prior probability of being good, or as the probability at time t of being good given all the history up to t, Ω_{t-1}, that is $\alpha = \Pr(IB_t = G \mid \Omega_{t-1})$. We assume that α is common knowledge and that each IB knows its own type.

Once the signal is received, the IB chooses which evaluation to publicly release in the form of a binary message $s \in \{s_h, s_l\}$.[4] The evaluation is observed by the firm, that decides wether to accept or refuse to be underwritten. The IB's message reaches the market if and only if the firm accepts to be underwritten. Let σ_{IB} denote a behavioral strategy for the IB of type IB.

IBs' reputation. We assume that at the end of period t the true value of the firm

[3] We assume that signals are private and non-verifiableAccordingly, a court cannot distinguish whether the analyst received the high or low signal. This prevents a contract from being written with payment contingent on the analyst truthfully reporting the private signal.

[4] Notice that we use S for the private signal recieved by the IB and s for the message sent by the IB. While the first is determined exogenously by the IB's type, the latter is a choice variable for the IB.

F can be observed. Thus, every player in the market can compare it with the observable actions taken by the IB and the firm and accordingly update his own belief about the ability of the IB. We interpret the updated belief about the IB's ability as the new level of reputation acquired by the IB at the end of period t and we denote it with $\widehat{\alpha}$. Formally, in the case in which the firm accepted to be underwritten and evaluation s_j has eventually reached the market, we have that:

$$\widehat{\alpha} = \Pr(IB = G \mid F, s_j)$$

where $F \in \{0, 1\}$ and $s_j \in \{s_l, s_h\}$. On the other hand, in the case in which the firm refused to be underwritten and thus no evaluation has eventually reached the market[5], we have that

$$\widehat{\alpha} = \Pr(IB = G \mid F, R)$$

Clearly, the value of $\widehat{\alpha}$ is endogenous, since it depends on the equilibrium strategies of firms and IBs and on the equilibrium beliefs held by investors, in a way that will be clear soon.

To ease notation, let us define

$$\widehat{\alpha}_{F,s_j} \equiv \Pr(IB = G \mid F, s_j) \text{ in case the firm accepts}$$
$$\widehat{\alpha}_{F,R} \equiv \Pr(IB = G \mid F, R) \text{ in case the firm refuses}$$

Investors

There is a large pool of risk neutral investors interested in buying the shares of the firm that goes public. We assume that investors observe the IB's report and then bid à la Bertrand in order to get the firm's shares. This implies that the stock price of the firm, v, is set equal to its expected value given all publicly available information, Ω_t.[6] Hence, if the firm accepts to be underwritten, the IB's message reaches the market and $\Omega_t = \{\Omega_{t-1}, s\}$ and

$$v = \Pr(F = 1 \mid \Omega_{t-1}, s)$$

On the other hand, if the firm refuses to be underwritten, no IB's message reaches

[5] See appendix 1 for a better insight about IBs' updated reputation and for relative computations
[6] That is, the market for the IPO is semi-strong efficient

the market. The only information available to the market is the refuse of the firm and

$$v = \Pr(F = 1 \mid \Omega_{t-1}, R)$$

To ease notation, let $V(s_j, \alpha) \equiv \Pr(F = 1 \mid s_j, \alpha)$, with $j \in \{h, l\}$ and $d \equiv \Pr(F = 1 \mid R)$. In words, $V(s_j, \alpha)$ is the value of the firm when underwritten by an IB with prior reputation α sending an evaluation s_j. On the other hand, d denotes the value that the market assigns to a firm that chooses to go public directly (refusing to be underwritten by an IB).

It is important to stress that both $V(s_j, \alpha)$ and d will be determined endogenously in equilibrium, that is, they will depend on the equilibrium strategies of IBs and on the equilibrium (and out of equilibrium) beliefs of the investors.[7]

1.2.2 The fee

A firm that goes public has to pay an underwriting fee to the IB. We assume that the firm pays a fee equal to a fraction k of value $V(s_j, \alpha)$ that the IB assures to the firm by underwriting its shares. Formally, the fee is given by $kV(s_j, \alpha)$. We assume that $k \in (0, 1)$ and that it is determined exogenously. It is important to bear in mind that the fee is paid only by firms that go public (i.e., by firms that accepts to be underwritten by an IB).[8]

1.2.3 Payoffs

Firms The payoff of a firm that goes public at time t is assumed to be given by the following function:

$$\pi^F = \begin{cases} (1-k)V(s_j, \alpha) & \text{if the firm accepts} \\ d & \text{if the firm refuses} \end{cases} \tag{1.3}$$

Investment Banks. The underwriting activity is typically characterized by the presence of both explicit and implicit incentives. The explicit incentives are those related to the direct compensation that the IB gets for assisting the firm along the IPO process, that is, the underwriting fee. The implicit incentives are those related to the reputation that the IB acquires about its ability and honesty to provide correct information to

[7] Firm values are derived in appendix 1.

[8] In a very simplified way, this linear fee structure bears some of the essential features present in the contractual arrangements used in practice (see Chemmannur Fulgheri, 1994).

the market. Usually, these incentives works in opposite directions. Indeed, since the compensation the IB gets from the firm is usually proportional to the success of the IPO, the IB has the incentive to inflate the value of the firm. On the other hand, this incentive is mitigated by the fear of building up a bad reputation. Indeed, a bad reputation would translate into a loss of market shares in the market of the underwriting activity (and hence in a loss of future fees), since no firm would use an IB with a bad reputation. Accordingly, wee assume that by sending a message s_j, with $j \in \{h, l\}$, an investment bank $IB \in \{G, B\}$ gets:

$$\pi^{IB} = \begin{cases} kV(s_j, \alpha) + \widehat{\alpha}_{F,s_j} & \text{if the firm accepts} \\ \widehat{\alpha}_{F,R} & \text{if the firm refuses} \end{cases} \tag{1.4}$$

where $kV(s_j, \alpha)$ represents the part of the IB's payoff related to the IB's compensation for the service provided during the IPO, and $\widehat{\alpha}_{F,\cdot}$ represents the reputational component of the IB's payoff.[9],[10] The previous reduced form is meant to represent the trade-off that an IB typically faces while producing and reporting information for the market along the IPO process, where the effects of the evaluation activity persist well beyond the immediate benefits of providing untruthful information to the market.[11],[12]

1.3 Equilibrium Analysis

For the sake of simplicity and w.l.o.g, from now on, let us assume that the good IB has a completely informative signal (i.e., $z < p = 1$). We will focus on equilibria in pure strategies as mixed strategies would complicate computations without significantly

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[9] Notice that we are here assuming for simplicity that the two payoff components (i.e. the fee-related and the reputational component) are equally weighetd. In a more general framework, we should consider a payoff structure given by:

$$\pi^{IB} = \beta k \left(V(s_j, \alpha) + (1 - \beta)\widehat{\alpha}\right.$$

whith $\beta \in [0, 1]$

[10] Notice that from the point of view of an IB, $V(s_j, \alpha)$ is a non-stochastic value. Indeed, $V(s_j, \alpha)$ depends on s_j, which is decided by the IB, and on α, which is a given value at the beginning of the period in which the IB makes its evaluation. On the other hand, $\widehat{\alpha}_{F,\cdot}$ is a stochastic value, since at the moment in which evaluation s_j is proposed (and eventually sent to the market), the IB does not know F.

[11] This reduced form is widely adopted in many papers that deals with experts or managers' reputation or career concerns (see for example Holmstrom 1982, Sharfstein and Stein 1990, Dasgupta and Prat 2004 and Jackson, 2005)

[12] From a technical point of view, since the payoff of the IB (the sender) depends on the belief of investors (the receivers), our game belongs to the class of psycological games. See Battigalli and Dufwenberg (2005) for an analysis of extensive-form psychological games.

altering the intuition behind the results. Within this framework, we first analyze the conditions under which there exist *truthtelling equilibria* where *both* IBs honestly report the signal they have received. We will show that truthtelling by *both* IBs is not always guaranteed and that there exist both *partial pooling equilibria* where the good IB truthfully reports its signal, while the bad IB always pools around the high evaluation, and *partial pooling equilibria* where the good IB truthfully reports its signal, while the bad IB always pools around the low evaluation.

1.3.1 Truthtelling equilibria

We start analyzing whether the presence of a reputational component in the IB's payoffs may induce IBs to truthtell.

In a truthtelling equilibrium, the strategies of the good and bad IBs are such that both types of IBs report the signal received (formally, for every $j \in \{h, l\}$ and $IB = \{G, B\}$ we have that $\sigma_{IB}(s_j \mid S_j) = 1$), conditional on firm accepting to be underwritten. We will focus on (putative) equilibria in which the firm accepts after s_h and refuses after s_l.[13] It follows that in any equilibrium in which IBs truthtell, only evaluation s_h reaches the market. On the other hand, in a truthtelling equilibrium in which the firm refuses after s_l, a refuse by the firm is interpreted as the IB proposing s_l, so that $\widehat{\alpha}_{F,R} \equiv \widehat{\alpha}_{F,s_l}$. Accordingly, $\widehat{\alpha}$ can assume only the following two values *in equilibrium*[14]:

$$\begin{aligned} \widehat{\alpha}_{\min} &= \widehat{\alpha}_{0,s_h} = \widehat{\alpha}_{1,R} \\ \widehat{\alpha}_{\max} &= \widehat{\alpha}_{1,s_h} = \widehat{\alpha}_{0,R} \end{aligned}$$

where $\widehat{\alpha}_{\max} > \alpha > \widehat{\alpha}_{\min}$.

A first consequence of our assumption that the good IB receives completely informative signals [15] is that:

$$\begin{aligned} \widehat{\alpha}_{\min} &= 0 \\ \widehat{\alpha}_{\max} &= \frac{\alpha}{\alpha + z(1-\alpha)} \end{aligned}$$

Let us consider the values the firm can take on in this equilibrium (on and off the

[13] It can be shown that in terms of information transmission by IBs, these equilibria have the same qualitative properties of those equilibria in which the firm accepts to be underwritten both after a low and a high evaluation.

[14] See appendix 1

[15] Again, see appendix 1 and remember that here we are assuming $p = 1$.

equilibrium path). In equilibrium, the firm accepts after s_h and refuses after s_l. Accordingly, let $V^{TT}(s_h, \alpha)$ denote the value of the firm in this truthtelling equilibrium when the firm accepts and evaluation s_h reaches the market. It is easy to show[16] that:

$$V^{TT}(s_h, \alpha) \equiv \Pr(F = 1 \mid s_h, \alpha) = \frac{\theta\left[\alpha + (1-\alpha)z\right]}{\theta\left[\alpha + (1-\alpha)z\right] + (1-\theta)(1-\alpha)(1-z)}$$

Accepting after s_l is an out of equilibrium action by the firm. In this case, s_l would reach the market. Given the equilibrium strategies of the IBs that prescribes truthtelling, the market would evaluate the firm accordingly. In particular, let $V^{TT}(s_l, \alpha)$ denote the (out of equilibrium) value of the firm in this truthtelling equilibrium when the firm has been reported a low signal. In this case, we have that:

$$V^{TT}(s_l, \alpha) \equiv \Pr(F = 1 \mid s_l, \alpha) = \frac{\theta(1-\alpha)(1-z)}{\theta(1-\alpha)(1-z) + (1-\theta)\left[\alpha + (1-\alpha)z\right]}$$

Finally, in equilibrium, the firm refuse after s_l. Since a refuse follows after s_l has been proposed, we have that:

$$d \equiv \Pr(F = 1 \mid R) = \Pr(F = 1 \mid s_l, \alpha) \equiv V^{TT}(s_l, \alpha)$$

We are ready to check under which conditions there exists an equilibrium in which both IB's type truthtell and the firm accepts only after a good evaluation is proposed to be reported.

IBs' problem. An investment bank of type IB receiving private signal S_j will truthtell if and only if the expected profits from reporting the private signal is greater than that obtained when reporting an evaluation that differs from the private signal. Notice that given the assumed equilibrium strategy of the firm (accept if s_h and refuse if s_l), the IB gets no underwriting fee when proposing s_l. Hence, truthtelling is an equilibrium if for every $IB \in \{G, B\}$, the following conditions holds:

$$kV^{TT}(s_h, \alpha) + E\left(\widehat{\alpha}_{F,s_h} \mid IB, S_h\right) \geq E\left(\widehat{\alpha}_{F,R} \mid IB, S_h\right) \qquad (1.5)$$

$$E\left(\widehat{\alpha}_{F,R} \mid IB, S_l\right) \geq kV^{TT}(s_h, \alpha) + E\left(\widehat{\alpha}_{F,s_h} \mid IB, S_l\right) \qquad (1.6)$$

As mentioned previously, $\widehat{\alpha}_{F,\cdot}$ is the investors' belief about IB's ability at the end of period t. This can be interpreted as the new level of reputation acquired by the IB at the end of t, when the true value of the firm has been observed and used to assess the

[16] See appendix 1 for the computation of $V^{TT}(s_h, \alpha)$ and $V^{TT}(s_l, \alpha)$.

ability of the IB. However, by the time the IB proposes its evaluation, the value of the firm is not known. Therefore, an IB computes the *expected* reputation from truthfully reporting (or misreporting) the signal it has received, that is $E\left(\widehat{\alpha}_{F,\cdot} \mid \cdot,\cdot\right)$.

Notice that conditions (1.5) and (1.6) can be written as follows:

$$kV^{TT}(s_h,\alpha) \geq E\left(\widehat{\alpha}_{F,R} - \widehat{\alpha}_{F,s_h} \mid IB, S_h\right) \quad (1.7)$$

$$kV^{TT}(s_h,\alpha) \leq E\left(\widehat{\alpha}_{F,R} - \widehat{\alpha}_{F,s_h} \mid IB, S_l\right) \quad (1.8)$$

In words, $kV^{TT}(s_h,\alpha)$ represents the gain from the underwriting fee (that the IB gets by providing a high evaluation instead of a low evaluation, since the low evaluation would be follow by a refuse of the firm). The RHS of (1.7) is the net expected reputational payoff of misreporting the signal received when the signal is S_h. The RHS of (1.8) represents the net expected reputational payoff of correctly reporting the signal received when the signal is S_l.

Lemma 1 *A sufficient (and necessary) condition for there to be a truthtelling equilibrium is that the truthtelling conditions for the bad IB are satisfied. (proof in appendix 2)*

This result follows directly from the fact that the good IB has a more informative signal and therefore it assigns greater weight to the expected reputation loss of providing an incorrect evaluation. In other words, if a bad IB has the incentive to truthtell, then, *a fortiori,* this must be true for a good IB too. The lemma above allows us to focus on the truthtelling conditions of the bad IB to determine the truthtelling equilibrium. Thus, we just have to prove that the two following conditions are satisfied:

$$kV^{TT}(s_h,\alpha) \geq E\left(\widehat{\alpha}_{F,R} - \widehat{\alpha}_{F,s_h} \mid B, S_h\right) \quad (1.9)$$

$$kV^{TT}(s_h,\alpha) \leq E\left(\widehat{\alpha}_{F,R} - \widehat{\alpha}_{F,s_h} \mid B, S_l\right) \quad (1.10)$$

Lemma 2 *For any values of* $k \in (0,1)$, $\alpha \in (0,1)$ *and* $z \in (\frac{1}{2},1)$, *there always exist a* $\underline{\theta}^{TT} \in (0,1)$ *and a* $\overline{\theta}^{TT} \in (0,1)$ *with* $\overline{\theta}^{TT} > \underline{\theta}^{TT}$ *such that for any* $\underline{\theta}^{TT} \leq \theta \leq \overline{\theta}^{TT}$ *conditions (1.9) and (1.10) are satisfied and IBs truthtell in equilibrium. (proof in appendix 2)*

For an intuition of the previous result, focus on the case of a bad IB. A bad IB receives an informative but imprecise signal. When the values of the prior on firm profitability θ are relatively extreme (so that it is ex ante very likely that the actual value of the firm is 0 or 1), a bad IB gets less confident about a private signal that is contrarian to what indicated by the prior. Consider for example the case in which θ is close to 1 and the bad IB receives S_l. When θ is close to 1, the gain from fees of reporting a high evaluation instead of a low evaluation are close to zero. Indeed, since the prior is very high, the market expects the firm to be valuable, whatever the evaluation sent. Basically, the LHS of (1.8) gets close to zero. On the other hand, when θ is close to 1, the bad IB (which cannot count on a very precise signal), expects that the high state is more likely than the low one (even if the signal received is S_l). Accordingly, the bad IB expects that it will be more likely to be correct and to improve its reputation when sending s_h instead of s_l.

We are left with proving that given the truthtelling strategies of both types of IB, when $\underline{\theta}^{TT} \leq \theta \leq \overline{\theta}^{TT}$, it is in fact optimal for the firm to follow the strategy of accepting after s_h and refusing after s_l. We will show that this in in fact the case whenever the underwriting fee is not to high. Indeed, it is apparent that if the fee were very high, all the benefits from using an IB in order to reduce the information asymmetries in the market would be ripped off by the cost of the IB's service. In particular, we impose that the fee satisfies the following condition:

$$k < \overline{k}^{TT} \equiv \frac{V^{TT}(s_h, \alpha) - V^{TT}(s_l, \alpha)}{V^{TT}(s_h, \alpha)} \tag{1.11}$$

The firm's problem. If the firm is proposed s_l and according to its equilibrium strategy refuses, it gets $d = \Pr(F = 1 \mid s_l) = V^{TT}(s_l, \alpha)$. If the firm deviates by accepting after s_l, then it is valued $V^{TT}(s_l, \alpha)$ and gets $(1-k)\, V^{TT}(s_l, \alpha)$. Thus, for the firm is always optimal to refuse after s_l.

If the firm is proposed s_h and accepts, it gets $(1-k)V^{TT}(s_h, \alpha)$. If it refuses, the market believes that s_l has been proposed and the firm gets $V^{TT}(s_l, \alpha)$. Thus, for the firm is optimal to accept after s_h as long as $(1-k)V^{TT}(s_h, \alpha) \geq V^{TT}(s_l, \alpha)$. Notice that under condition (1.11), this inequality is always satisfied.

We can thus summarize the previous results in the following proposition.

Proposition 1 *Let $\overline{k}^{TT}$ be defined by condition (1.11). Then, for any given value of $\alpha \in (0,1)$, $z \in (\frac{1}{2}, 1)$ and $k \in (0, \overline{k}^{TT})$, there always exist a $\underline{\theta}^{TT} \in (0,1)$ and a $\overline{\theta}^{TT} \in (0,1)$ with $\overline{\theta}^{TT} > \underline{\theta}^{TT}$, such that for any $\theta \in \left[\underline{\theta}^{TT}, \overline{\theta}^{TT}\right]$ there exists an equilibrium in which both good and bad IBs propose to truthfully report their private information, and the firm accepts to be underwritten only when a good evaluation is proposed.*

In such equilibria, only good evaluations reaches the market (because it is never the case that a firm goes public with a low evaluation), and these evaluations are credible. However, we have shown that a truthtelling equilibrium exists only for intermediate values of θ. Whenever the prior on firm's value gets too extreme, truthtelling is destroyed by the incentives of the bad IB to report the signal that is more likely to reveal correct ex-post (that is, once the value of the firm has realized). In words, if for example common sense (represented by the prior) suggests that a firm is very likely to be highly profitable and the IB is not too confident about its low signal, then the IB does not contradict common sense. This result can be interpreted as a sort of conservative and conformist behavior by the bad IB when the prior on firm profitability is too high (or too low) relative to its signal precision. The important conclusion is that this result is driven by exactly those incentives that allow truthtelling to be sustainable in a region of the parameters space, that is the reputational concerns of the IB. When the prior on firm's value is relatively high, the bad IB fails at perceiving the trade off between the temptation to inflate the firm's value and ex-post reputation. In fact, when θ is very high, for the bad IB it is true that ex-ante, the incentives related to the underwriting fee and those related to expected reputation are aligned. On the other hand, when the prior on firm's value is relatively low, the fear to incur in a reputational punishment is much stronger than the temptation to inflate the firm's stocks.

This result allows us to formulate the conjecture that for extreme values of θ, there exist equilibria in which good IBs will truthtell and bad IBs will always provide a negative evaluation when θ is low and always provide a positive evaluation when θ is high. These equilibria are informationally inefficient as the bad IB's private information on firm profitability never reaches the market and is never incorporated in firm values.

1.3.2 Partial pooling

We refer to Partial Pooling (PP) as an equilibrium in which the bad IB always gives a unique evaluation independently from the signal actually received, while the good IB always truthfully reports its signals. Notice that a partial pooling equilibrium is informationallly inefficient, because we are assuming that the signal received (but disregarded) by the bad IB contains some information ($z > \frac{1}{2}$). Again, we will focus on equilibria in which the firm accepts after s_h and refuses after s_l.

We first analyze wether there exists a PP equilibrium in which the bad IB always proposes the high evaluation (independently form the private signal actually received) and the good IB always truthtells. In such equilibrium, both evaluations s_l and s_h are sent in equilibrium. Thus, given the equilibrium strategies of the firm and of the IBs, we have that ex-post reputation $\widehat{\alpha}$ assumes the following values in equilibrium:

$$\begin{aligned}\widehat{\alpha}_{1,s_h} &= \alpha \\ \widehat{\alpha}_{0,R} &= \widehat{\alpha}_{0,s_l} = 1 \\ \widehat{\alpha}_{0,s_h} &= 0\end{aligned}$$

Indeed, in this equilibrium the pair $(F = 1, s_h)$ is non informative about IB's ability, while pairs $(F = 0, R)$ and $(F = 0, s_h)$ reveal completely the IB's type. Notice that the case in which a refused by the firm is followed by a high state of the firm cannot arise on the equilibrium path, that is, the event $(F = 1, R)$ has zero probability to occur in the putative equilibrium that we are considering. Indeed, low evaluations are sent in equilibrium only by good IBs, which in turn have complete information about the true value of the firm and thus do not make mistakes. Accordingly $\widehat{\alpha}_{1,s_l}$ cannot be computed via Bayes rule. However, it is reasonable to assume that when the market observes the outcome $(F = 1, R)$ it believes that the message has been sent by a bad IB. Thus, we assume:

$$\widehat{\alpha}_{1,R} = 0$$

As of the possible values that the firm can take in a partial pooling equilibrium, notice that if the firm refuses, then it is valued $d = 0$. Indeed, when the firm refuses, the market infers that s_l has been proposed. Furthermore, given the equilibrium strategies of good and bad IBs, the market also infers that s_l has been proposed by a good IB that has received S_l (remember that the good IB has complete information about the firms value). On the other hand, if the firm accepts after s_h, then the IB's evaluation reaches the market and the value of the firm is computed accordingly. In particular, let $V_H^{PP}(s_h, \alpha)$ denote the value of the firm in this partial pooling equilibrium when s_h reaches the market. It is possible to show that[17]

$$V_H^{PP}(s_h, \alpha) \equiv \Pr(F = 1 \mid s_h, \alpha) = \frac{\theta}{\theta + (1-\theta)(1-\alpha)}$$

Accepting after s_l is an out of equilibrium action by the firm. Notice that in this case, the low evaluation reaches the market. Let $V_H^{PP}(s_l, \alpha)$ denote the (out of equilibrium) value of the firm in the case in which the low evaluation reaches the market. Given the equilibrium strategies of the IBs, the market sets $V_H^{PP}(s_l, \alpha) = 0$.

[17] See appendix 1

Lemma 3 *In an equilibrium in which the firm accepts only if a high evaluation is proposed and the bad IB always proposes to report a high evaluation, the good IB truthfully reports its private information*

Proof. Given the equilibrium strategies of the firm and of bad IBs, a good IB truthtells if:

$$kV_H^{PP}(s_h, \alpha) + E(\widehat{\alpha}_{F,s_h} \mid G, S_h) \geq E(\widehat{\alpha}_{F,R} \mid G, S_h)$$
$$E(\widehat{\alpha}_{F,R} \mid G, S_l) \geq kV_H^{PP}(s_h, \alpha) + E(\widehat{\alpha}_{F,s_h} \mid G, S_l)$$

Since the good IB has a completely informative signal, it is easy to show that the previous conditions can be written as:

$$kV_H^{PP}(s_h, \alpha) + \alpha \geq 0$$
$$1 \geq kV_H^{PP}(s_h, \alpha)$$

Since $0 < V_H^{PP}(s_h, \alpha) < 1$, the previous conditions are always satisfied for any values of $\theta, \alpha, k \in (0, 1)$. ■

This result is a consequence of two facts. First, given the features of the equilibrium at hand, proposing s_l delivers a very high reputational reward if this turns out to be the correct forecast on firm's state ($\widehat{\alpha}_{0,R} = 1$). Second, since the good IB has a perfectly informative signal, it is sure that $F = 0$ whenever it receives S_l. Thus, a good IB receiving S_l is certain to get the high reputational reward by proposing s_l. Hence, (whatever the value of the prior on firm profitability) the good investment bank will always trust its private signal more than public information. This allows us to focus on the bad IB's conditions to determine the equilibrium,

Lemma 4 *In an equilibrium in which the firm accepts only when a high evaluation is proposed and the good IB truthtells, the bad IB will always propose to report a high evaluation (independently form its private information), provided that the prior on firm's profitability is not too low.*

Proof. Given the equilibrium strategies of the firm and of good IBs, a bad IB always sends s_h if

$$kV_H^{PP}(s_h, \alpha) + E(\widehat{\alpha}_{F,s_h} \mid B, S_h) \geq E(\widehat{\alpha}_{F,R} \mid B, S_h)$$
$$kV_H^{PP}(s_h, \alpha) + E(\widehat{\alpha}_{F,s_h} \mid B, S_l) \geq E(\widehat{\alpha}_{F,R} \mid B, S_l)$$

Since the signal received by the bad IB is informative, the last inequality is stricter than the first one. Thus, it is sufficient to show that the last inequality holds. Notice that the last inequality can be written as:

$$kV_H^{PP}(s_h,\alpha) \geq E(\widehat{\alpha}_{F,R} - \widehat{\alpha}_{F,s_h} \mid B, S_l) \tag{1.12}$$

which reads as follows[18]:

$$k\frac{\theta}{\theta + (1-\theta)(1-\alpha)} \geq \Pr(F=0 \mid S_l, B) - \alpha \Pr(F=1 \mid S_l, B)$$

At $\theta = 0$ the LHS=0 and the RHS=1. At $\theta = 1$, the LHS=$k > 0$ and the RHS=$-\alpha < 0$. Since for every $\theta, \alpha \in (0,1)$ and $z \in \left(\frac{1}{2}, 1\right)$ the LHS is continuous and strictly increasing in θ while the RHS is continuous and strictly decreasing in θ, there always exists a unique value $\theta_H^{pp} \in (0,1)$ such that LHS=RHS and such that for every $\theta > \theta_H^{pp}$ the previous condition holds and it is optimal for the bad IB to pool around s_h. ■

The intuition behind this result lies in the fact that the bad IB's signal is not perfectly precise (though informative). Accordingly, a bad IB receiving S_l is not certain to get the high reputational reward by proposing s_l. At the same time, proposing s_l implies to give in the underwriting fee. The incentives to always propose a high evaluation vanishes when the prior on firm's profitability is low. In this case, the probability that $F = 0$ is high and the bad IB gets more confident about its low signal and about the fact that proposing a low evaluation is the correct choice.

At this point, we are left with proving that given the strategies of the bad and good IBs, it is indeed optimal for the firm to accept after s_h and to refuse after s_l.

The firm's problem. Accepting after s_h gives the firm a payoff equal to $(1-k)V_H^{PP}(s_h,\alpha)$. Deviating to a refuse after s_h gives a payoff of d. Since $d = 0$, for the firm is optimal to accept after s_h as long as $(1-k)V_H^{PP}(s_h,\alpha) \geq 0$, which is always satisfied. What about the case in which s_l is proposed? If the firm follows its equilibrium strategy and refuses after s_l it gets $d = 0$. If the firm deviates and accepts after s_l, the market observes s_l, which is an out of equilibrium outcome. Again, given the equilibrium

[18] Using the results in appendix 1, notice that

$$\begin{aligned} E(\widehat{\alpha}_{F,R} \mid\ & B, S_l) = \widehat{\alpha}_{0,s_l} \Pr(F=0 \mid S_l, B) + \widehat{\alpha}_{1,s_l} \Pr \Pr(F=1 \mid S_l, B) = \\ =\ & \Pr(F=0 \mid S_l, B) \\ E(\widehat{\alpha}_{F,s_h} \mid\ & B, S_l) = \widehat{\alpha}_{0,s_h} \Pr(F=0 \mid S_l, B) + \widehat{\alpha}_{1,s_h} \Pr \Pr(F=1 \mid S_l, B) = \\ =\ & \alpha \Pr \Pr(F=1 \mid S_l, B) \end{aligned}$$

strategies of good and bad IBs, the market believes that s_l comes from a good firm that has received S_l and accordingly sets the firm's value equal to 0. In fact, the firm is indifferent between accepting and refusing after s_l since both actions give a payoff of zero.[19]

We can summarize the previous analysis in the following proposition.

Proposition 2 *for every $k \in (0,1)$, $\alpha \in (0,1)$ and $z \in [1/2, 1)$, there always exists a $\theta_H^{pp} \in (0,1)$ such that for any $\theta > \theta_H^{pp}$ a Partial Pooling equilibrium exists in which the good IB truthfully reports its information, the bad IB always reports a positive evaluation s_h independently from the signal received, and the firm accepts to be underwritten after a good evaluation and refuses after a bad evaluation .*

What happens when the prior on firm's value θ is low? It is possible to show that when θ is below a given treshold, there exist a (partial pooling) equilibrium in which the good IB always truthtells and the bad IB always proposes to report a low evaluation, even though the firm accepts to be underwritten only when a high evaluation is proposed. We refer to this equilibrium as PP_L. We formalize this result in the following proposition.

Proposition 3 *Let $\overline{k}^{PP}$ be defined by condition (1.22). Then, for every value of $z \in (\frac{1}{2}, 1)$ and for every values of $k \in (0, \overline{k}^{PP})$ and $\alpha \in (0,1)$ such that $\alpha > k$, there always exists a $\theta_L^{pp} \in (0,1)$ such that for any $\theta \in (0, \theta_L^{pp})$ there exists an equilibrium in which the good IB truthfully reports its information, the bad IB pools by always reporting a negative evaluation s_l independently from the signal received, and the firm accepts after a good evaluation and refuses after a low evaluation.(proof in appendix 3)*

We will relegate the proof of the previous result in the appendix, since the logic to follow in order to prove it is the same as for the equilibrium PP_H. At an intuitive level, proposition (5) suggests that when the prior on firm profitability is very low, the bad IB is better off disregarding its private information and conforming to public information on θ. In this case the loss in terms of fees that the IB incurs is less than the expected reputation loss that it suffers by not following the prior. In other words, when private information is not complete, the underwriter will tend to attribute less weight to its signal for extreme values of public information.

[19] Notice that assuming a small fixed cost to be paid to the IB for the underwriting service would break the indifference in favour of a refuse by the firm.

1.4 Comparative Statics

We have shown that a unique equilibrium that is invariant with respect to θ does not exist. An equilibrium is thus defined based on the threshold values $\theta_L^{pp}, \theta_H^{pp}, \underline{\theta}^{TT}$, and $\overline{\theta}^{TT}$ that determine the existence of the three different types of pure strategy equilibria PP_L, PP_H and TT.

In the next section we consider how each of the parameters α and k affect the threshold values of θ that respectively guarantee the existence of the inefficient partial pooling equilibria, and the efficient truthtelling equilibrium. The purpose of this section is to identify how variations in the exogenous parameters can lead to more or less efficient equilibria. We thus identify more or less efficient equilibria based on these threshold parameters in the following way. Whenever a variation in a given parameter reduces the parameter space over θ for which the inefficient partial pooling equilibria are satisfied, without reducing the space for which the truthtelling equilibrium is satisfied, we have an improvement in informational efficiency.

1.4.1 Variations in prior reputation (α) and in fees (k)

The threshold values mentioned above are formally defined as the values of θ that make the necessary conditions of each type of equilibrium binding.

Partial pooling equilibria. In the partial pooling equilibria in which bad IBs always report a high evaluation, θ_H^{PP} is defined as the value of θ for which condition (1.12) is binding.

Notice that condition (1.12) can be written as follows:

$$k\frac{\theta}{\theta + (1-\theta)(1-\alpha)} \geq \frac{(1-\theta)z - \alpha\theta(1-z)}{(1-\theta)z + \theta(1-z)} \tag{1.13}$$

Remember that this partial pooling is defined for every value of $\alpha \in (0,1)$, $z \in \left(\frac{1}{2}, 1\right)$ and $k \in (0,1)$ (see proposition 2). In this parameters space, the LHS of (1.13) is increasing in θ while the RHS is decreasing in θ. Thus, as k increases, condition (1.13) is relaxed and θ_H^{PP} decreases. This means that an increase of the underwriting fee makes the partial pooling around the high evaluation more easily sustainable. Notice now that the LHS of (1.13) is increasing in α, while the the RHS is decreasing in α. Hence, as α increases, condition (1.13) is relaxed and θ_H^{PP} has to decrease to keep it binding. Therefore, also an increase in the level of initial reputation makes the the partial pooling around the high evaluation more easily to be sustained. The intuition behind this result is that when α

is high, a positive evaluation is trusted by the market. Thus, sending a positive report increases the firm's value and consequently the underwriting fee (the LHS is increasing in α). On the other hand, the higher is the initial reputation the lower is the net reputational gain that the IB enjoys by correctly reporting its low signal (the RHS is decreasing in α). These findings are represented in in figure 1.

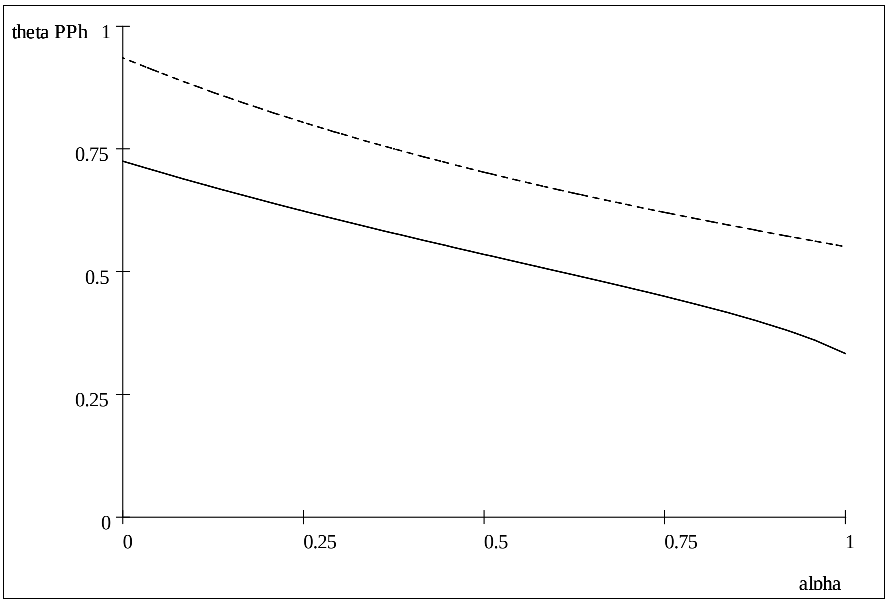

Figure 1: θ_H^{PP} plotted against α; $k = 0.1$ dashed, $k = 0.5$ solid

Consider now the partial pooling equilibrium in which the bad IB always propose a low evaluation. Remember that θ_L^{PP} is the value of θ for which condition (1.21) is binding. Notice that condition (1.21) reads as follows:

$$\frac{\alpha(1-\theta)(1-z)-\theta z}{(1-\theta)(1-z)+\theta z} \geq k \tag{1.14}$$

The partial pooling equilibrium in the low signal was defined for $\alpha \in (0,1)$, $z \in \left(\frac{1}{2},1\right)$ and $k \in \left(0, \overline{k}^{PP}\right)$, with $\alpha > k$ (see proposition 3). Notice that the RHS of (1.14) is a constant while it is easy to show that in the parameters space for which the equilibrium is defined, the LHS is always decreasing in θ. Therefore, as k increases and the RHS increases too, θ_L^{PP} has to decrease to keep (1.14) binding. As expected, an increase in k makes the partial pooling in which the bad IB always proposes a negative report more difficult to be met.

How does θ_L^{PP} varies with α? Notice that the LHS of (1.14) is increasing in α. The reason is that in the equilibrium at hand, the initial level of reputation α is exactly the reward the bad IB gets if it reports a negative evaluation. On the other hand, the reward for a deviation to a positive evaluation is fixed equal to 1. Thus, the higher the initial level of reputation, the more appealing (in expectation, of course) it gets for the IB to follow the equilibrium strategy and report a negative evaluation. As α increases, the LHS increases and θ_L^{PP} has to increase as well to keep condition (1.14) binding. Figure 2 summarizes the previous findings (remember that the equilibrium PP_L holds as long as $\alpha > k$).

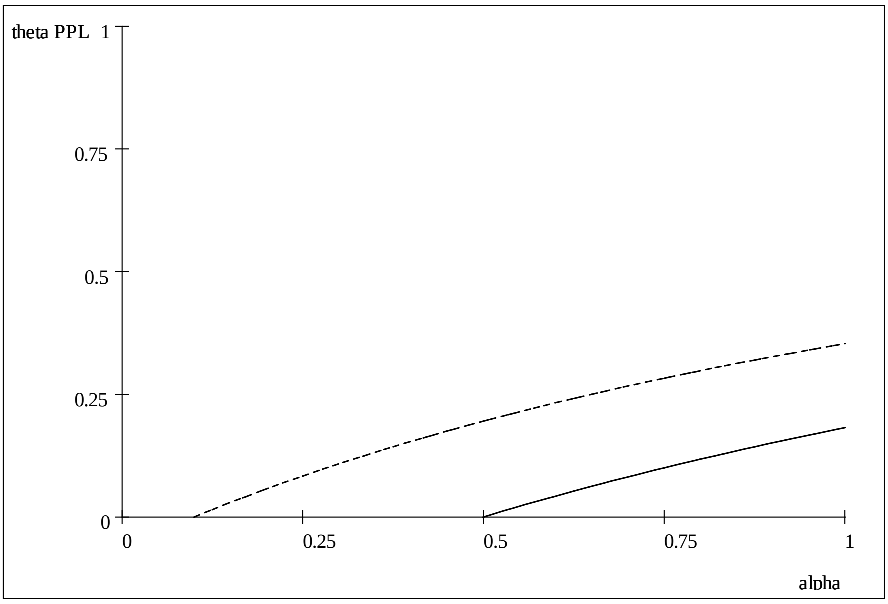

Figure 2: θ_L^{PP} plotted against α; $k = 0.1$ dashed, $k = 0.5$ solid

Remark 1 *An increase in prior reputation α has a perverse effect, since it makes the parameters space for which the inefficient partial pooling equilibria are sustained larger*

Thruthtelling equilibrium In the truthtelling equilibrium outlined in section 4.1, we defined $\overline{\theta}^{TT}$ and $\underline{\theta}^{TT}$ as the values of θ for which respectively conditions (1.9) and (1.10) were satisfied with equality. Let us write again conditions (1.9) and (1.10):

$$kV^{TT}(s_h, \alpha) \geq E(\widehat{\alpha}_{F,R} - \widehat{\alpha}_{F,s_h} \mid B, S_h) \quad (1.15)$$

$$kV^{TT}(s_h, \alpha) \leq E(\widehat{\alpha}_{F,R} - \widehat{\alpha}_{F,s_h} \mid B, S_l) \quad (1.16)$$

Remember that the truthtelling equilibrium at hand was defined for $\alpha \in (0,1)$, $z \in \left(\frac{1}{2}, 1\right)$ and $k \in \left(0, \overline{k}^{TT}\right)$ (see proposition 1). It is immediate to see that in this parameters space, $kV^{TT}(s_h, \alpha)$ is increasing in θ, while both RHSs of (1.15) and (1.16) are decreasing in θ.

Therefore, as k increases, both $\underline{\theta}^{TT}$and $\overline{\theta}^{TT}$ have to decrease to keep (1.15) and (1.16) binding. It is possible to show that $\underline{\theta}^{TT}$ decreases less rapidly than $\overline{\theta}^{TT}$, so that we can conclude that as k increases, the range $\left(\underline{\theta}^{TT}, \overline{\theta}^{TT}\right)$ in which truthtelling is sustainable shifts downwards and shrinks. The intuition for this expected result is that as the underwriting fee increases, the incentives to provide a positive evaluation increase as well. In order for truthtelling to be an equilibrium, the prior on firm profitability must be low so that the reputation punishment of misreporting a low signal is sufficiently high to offset the gain from pooling around a positive evaluation.

How does $\underline{\theta}^{TT}$ and $\overline{\theta}^{TT}$ vary with α? Notice that the signs of the derivatives of $\underline{\theta}^{TT}$ and $\overline{\theta}^{TT}$ with resect to α depend on the values of z. However, as shown in figure 3, it is possible to show that for any given value of $z \in \left(\frac{1}{2}, 1\right)$ and $k \in \left(0, \overline{k}^{TT}\right)$, the range $\left(\underline{\theta}^{TT}, \overline{\theta}^{TT}\right)$ shifts upwards and gets larger as α increases.

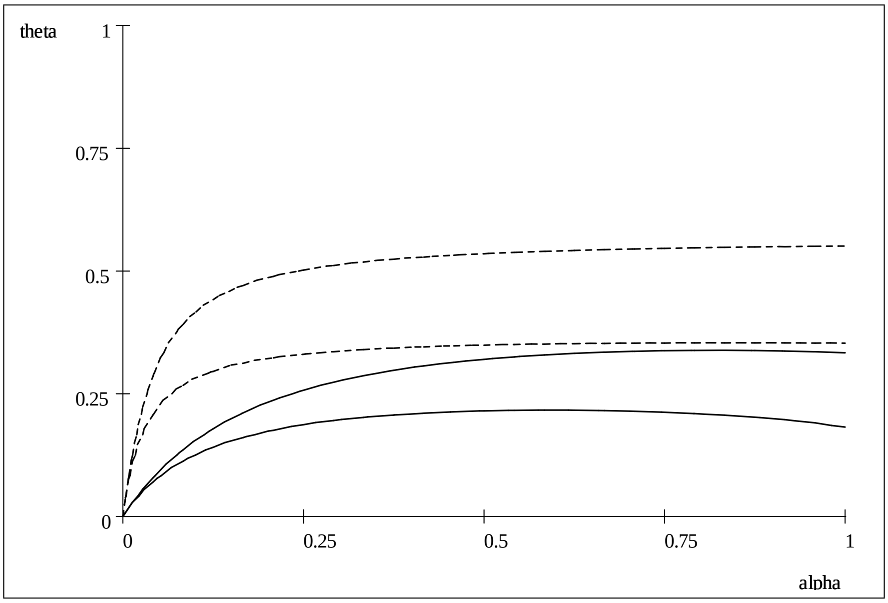

Figure 3: $\underline{\theta}^{TT}$ and $\overline{\theta}^{TT}$ against α; $k = 0.1$ dashed, $k = 0.5$ solid

The previous analysis leads to the following remarks:

Remark 2 *As prior IB reputation α rises, there is a larger parameter space on prior firm profitability θ for which both partial pooling equilibria are sustained. On the other hand, also the range of θ for which truthtelling can be sustained gets larger.*

Thus, in this case nothing can be concluded in terms of informational efficiency.

Remark 3 *An increase in the IB's underwriting fee k shrinks the range of parameters on firm profitability for which truthtelling is sustained and shifts it downward (both $\overline{\theta}^{TT}$ and $\underline{\theta}^{TT}$ decrease, with $\underline{\theta}^{TT}$ decreasing at a lower rate than $\underline{\theta}^{TT}$), increases that for* *which partial pooling PP_H is sustained (θ_H^{PP} decreases) and reduces that for which PP_L is sustained (θ_L^{PP} decreases) .*

As mentioned above, increases in fees have an asymmetric effect on the partial pooling equilibria. Larger fees tend to reduce the chances of having equilibria in which bad IBs pools around low evaluation, but increase the incentive to pool around the good evaluations. Furthermore, there is no doubt that an increase in fees reduces the parameters space where truthtelling is sustainable. Thus, focusing on values of θ relatively high, we can state that increases in fees lead to a worsening in informational efficiency.

1.5 Conclusions

In IPOs, investment banks typically have privileged information on the profitability of firms they are underwriting. They are therefore in a position to reduce the informational asymmetries between firms that are going public and investors, acting as information providers for the market. We introduce reputation to take into account of the fact that providing incorrect evaluations may hinder future profits of the underwriters by reducing their credibility. It turns out however, that in many cases IBs misreport their private information and actually profit from doing so.

Misreporting takes the form of a conformist behavior where IBs tend to disregard their private information once the public signal is extreme. Thus when investors have an ex-ante perception that firm profitability is either very high or very low, underwriters will tend to conform to the prior. In particular when prior reputation is higher underwriters attribute less weight to reputation acquisition increasing the incentives to misreport. Paradoxically, reputation may actually exacerbate the informational inefficiency.

High evaluations tend to inflate the price of firms on which fees are based. We assume that the fee structure is exogenously given and equal to a fraction of the difference between the value of the firm after being underwritten, and the value of the firm that would prevail in case the firm went public by itself. Raising the investment bank fees will lead underwriters to more frequently provide positive evaluations independently from the signal they receive.

The underwriting process is a complex phenomenon. Here we highlight some aspects that have been left out of this paper, and that may be addressed in the future, in a similar framework.

First of all, we assume that the fee structure is exogenously given. An interesting extension would be to endogenously derive underwriting fees as a contract between firms and investment banks.

Furthermore in this model we have concentrated on the strategic information transmission problem faced by investment banks. Another aspect concerns the incentives firms face in disclosing information to the underwriters. Combining these aspects may provide a more complete theory of information disclosure in the IPO underwriting process.

1.6 Appendix 1: Values

1.6.1 Firm's value

The value of a firm that operates at period t and is underwritten with an evaluation s_j by an IB with (prior) reputation α reads:

$$V(s_j,\alpha) = \Pr(F=1 \mid s_j,\alpha) =$$
$$= \frac{\Pr\left(F=1,s_j|\alpha\right)}{\Pr(F=1,s_j \mid \alpha)+\Pr(F=0,s_j \mid \alpha)}, \quad with\ j=h,l$$

where

$$\Pr(F=1,s_j \mid \alpha) = \theta[\sigma_{IB}(s_j \mid S_{h,t},G,\alpha)\Pr(S_h \mid F=1,G)\alpha+$$
$$+\sigma_{IB}(s_j \mid S_{l,t},G,\alpha)\Pr(S_l \mid F=1,G)\alpha+\sigma_{IB}(s_j \mid S_{h,t},B,\alpha)\Pr(S_h \mid F=1,B)(1-\alpha)+$$
$$+\sigma_{IB}(s_j \mid S_{l,t},B,\alpha)\Pr(S_l \mid F=1,B)(1-\alpha)]$$

and

$$\Pr(F=0,u,s_j \mid \alpha) = (1-\theta)[\sigma_{IB}(s_j \mid S_{h,t},G,\alpha)\Pr(S_h \mid F=0,G)\alpha+$$
$$+\sigma_{IB}(s_j \mid S_{l,t},G,\alpha)\Pr(S_l \mid F=0,G)\alpha+\sigma_{IB}(s_j \mid S_{h,t},B,\alpha)\Pr(S_h \mid F=0,B)(1-\alpha)+$$
$$+\sigma_{IB}(s_j \mid S_{l,t},B,\alpha)\Pr(S_l \mid F=0,B)(1-\alpha)]$$

In particular, under our information structure (1.1) and (1.2), we have that

$$\begin{aligned}&\Pr(F=1,s_j\mid\alpha)=\theta[\sigma_{IB}(s_j\mid S_{h,t},G,\alpha)p\alpha+\\&+\sigma_{IB}(s_j\mid S_{l,t},G,\alpha)(1-p)\alpha+\sigma_{IB}(s_j\mid S_{h,t},B,\alpha)z(1-\alpha)+\\&+\sigma_{IB}(s_j\mid S_{l,t},B,\alpha)(1-z)(1-\alpha)]\end{aligned}$$

and

$$\begin{aligned}&\Pr(F=0,u,s_j\mid\alpha)=(1-\theta)[\sigma_{IB}(s_j\mid S_{h,t},G,\alpha)(1-p)\alpha+\\&+\sigma_{IB}(s_j\mid S_{l,t},G,\alpha)p\alpha+\sigma_{IB}(s_j\mid S_{h,t},B,\alpha)z(1-\alpha)+\\&+\sigma_{IB}(s_j\mid S_{l,t},B,\alpha)(1-z)(1-\alpha)]\end{aligned}$$

On the other hand, the value of a firm in period t upon not being underwritten reads:

$$d=\Pr(F=1\mid R)$$

Two things are important to bear in mind.

First, the value of the firm upon underwriting, $V(s_j,\alpha)$, is a function of the Investment Bank's reputation (the higher IB's reputation, the more credible is the evaluation and eventually the more the value of the firm aligns with the evaluation).

Second, as for the case of IB's reputation, the equilibrium value of a firm depends on the equilibrium strategies of both the IB and the firm. Therefore, besides the evaluation itself, the value of a firm depends on the type of equilibrium in which this evaluation occurs.

1.6.2 IB's reputation

Suppose that at the end of period t the true state of the firm is revealed to be $F=1$. Then, the reputation of an IB that in period t has sent message j reads:

$$\begin{aligned}\widehat{\alpha}_{1,s_j}&=\Pr(IB=G\mid F=1,s_j)=\frac{\Pr(IB=G,F=1,s_j)}{\Pr(IB=G,F=1,s_j)+\Pr(IB=B,F=1,s_j)},\\ \textit{with } j&=h,l\end{aligned}$$

where:

$$\begin{aligned}\Pr(IB = G, F = 1, s_j) = [&\sigma_{IB}(s_j \mid S_h, G)p\alpha + \\ &+\sigma_{IB}(s_j \mid S_l, G)(1-p)\alpha \quad]\end{aligned}$$

and

$$\begin{aligned}\Pr(IB = B, F = 1, s_j) = [&\sigma_{IB}(s_j \mid S_h, B)z(1-\alpha) + \\ &+ \sigma_{IB}(s_j \mid S_l, B)(1-z)(1-\alpha)]\end{aligned}$$

Analogously, at the end of period t, if the true state of the firm has revealed to be $F = 0$, the reputation of an IB that in period t has sent message $j = h, l$ reads:

$$\begin{aligned}\widehat{\alpha}_{0,s_j} &= \Pr(IB = G \mid F = 0, s_j) = \frac{\Pr(IB = G, F = 0, s_j)}{\Pr(IB = G, F = 0, s_j) + \Pr(IB = B, F = 0, s_j)}, \\ with\ j &= h, l\end{aligned}$$

where

$$\begin{aligned}\Pr(IB &= G, F = 0, s_j) = [\sigma_{IB}(s_j \mid S_h, G)(1-p)\alpha + \\ &+\sigma_{IB}(s_j \mid S_l, G)p\alpha]\end{aligned}$$

and

$$\begin{aligned}\Pr(IB &= B, F = 0, s_j) = [\sigma_{IB}(s_j \mid S_h, B)(1-z)(1-\alpha) + \\ &+\sigma_{IB}(s_j \mid S_l, B)z(1-\alpha)]\end{aligned}$$

Basically, in each period, an IB's reputation is the Bayesian update on the IB's previous period reputation (starting from the prior $\alpha_0 = \alpha$) given the message sent by the IB in t and the observed true state of the firm revealed at the end of t.

It is important to notice that here we focus on Markov Perfect equilibria where we assume that α is a summary statistic of the $t-1$ period history.

The previous formulas hold whenever the respective denominators are positive. When they are not, α_{1,s_j} and α_{0,s_j} are arbitrary, in the sense that they are not determined via

Bayes rule (although they still enter the perfect Bayesian equilibrium conditions and hence they are co-determined by these conditions together with the behavior strategies σ).

IB'S REPUTATION IN A POOLING EQUILIBRIUM

Consider an equilibrium where both good and bad IB pools around s_h. The previous formulas give us (as it is intuitive):

$$\widehat{\alpha}_{1,s_h} = \widehat{\alpha}_{0,s_h} = \frac{p\alpha + (1-p)\alpha}{p\alpha + (1-p)\alpha + z(1-\alpha) + (1-z)(1-\alpha)} = \alpha$$

In words, when the equilibrium is pooling, the market does not care about the fact that the evaluation is correct, basically assuming that if this occurs, it is just a coincidence.

IB'S REPUTATION IN A TRUTHTELLING EQUILIBRIUM

Consider an equilibrium where both IBs truthtell, when the true state is $F = 1$. If the IB makes a correct evaluation (i.e. has sent s_h) we will have:

$$\widehat{\alpha}_{1,s_h} = \widehat{\alpha}_{0,s_l} = \frac{p\alpha}{p\alpha + z(1-\alpha)} = \widehat{\alpha}_{\max}$$

If instead the IB makes an incorrect evaluation (i.e. has sent s_l) we will have

$$\widehat{\alpha}_{1,s_l} = \widehat{\alpha}_{0,s_h} = \frac{(1-p)\alpha}{(1-p)\alpha + (1-z)(1-\alpha)} = \widehat{\alpha}_{\min}$$

In words, in a truthtelling equilibrium, making a correct evaluation increases reputation above the prior, while making a mistake decreases it below the prior.

IB'S REPUTATION IN A PARTIAL POOLING EQUILIBRIUM

Finally, consider an equilibrium in which the good IB always truthtells while the bad IB always pools around s_h (Partial pooling PP_H). Notice that when the true state is $F = 1$, making a correct evaluation gives:

$$\widehat{\alpha}_{1,s_h} = \frac{p\alpha}{p\alpha + (1-\alpha)} = \widehat{\alpha}_{h,correct}^{PP}$$

while making a mistake (transmitting s_l) gives:

$$\widehat{\alpha}_{1,s_l} = \frac{(1-p)\alpha}{(1-p)\alpha} = 1 = \widehat{\alpha}_{l}^{PP}$$

When the true state is $F = 0$, making a mistake gives:

$$\widehat{\alpha}_{0,s_h} = \frac{(1-p)\alpha}{(1-p)\alpha + (1-\alpha)} = \widehat{\alpha}^{PP}_{h,incorrect}$$

while making a correct evaluation gives:

$$\widehat{\alpha}_{0,s_l} = \frac{p\alpha}{p\alpha} = 1 = \widehat{\alpha}^{PP}_{l}$$

In words, in this kind of equilibrium, it does not matter whether a correct or incorrect evaluation is made if the IB makes a low evaluation, since transmitting s_l immediately identifies the IB as being good. When instead a high evaluation is made, being correct or incorrect makes a difference, since $\widehat{\alpha}^{PP}_{h,incorrect} < \widehat{\alpha}^{PP}_{h,correct}$. Note that when $p = 1$, $\widehat{\alpha}_{1,s_l}$ is an out of equilibrium value since only good IBs send low evaluations and $p = 1$ implies that good IBs have complete information and never make mistakes.

The previous result also tells us that in this partial pooling equilibrium if the IB sends the high message and makes a mistake, it will be highly penalized. Furthermore, sending the high message correctly forecasting the true state is not even enough to increase the IB's reputation above the prior. This is because the market heavily weighs the presence of the bad IBs that pool around the high message.

1.7 Appendix 2

1.7.1 Proof of Lemma 1

Proof. Consider conditions (1.7) and (1.8)for the existence of a truthtelling equilibrium. They can be spelled out as follows:

$$kV^{TT}(s_h, \alpha) \geq E(\widehat{\alpha}_{F,R} - \widehat{\alpha}_{F,s_h} \mid G, S_h) \tag{1.17}$$
$$kV^{TT}(s_h, \alpha) \leq E(\widehat{\alpha}_{F,R} - \widehat{\alpha}_{F,s_h} \mid G, S_l) \tag{1.18}$$

and

$$kV^{TT}(s_h, \alpha) \geq E(\widehat{\alpha}_{F,R} - \widehat{\alpha}_{F,s_h} \mid B, S_h) \tag{1.19}$$
$$kV^{TT}(s_h, \alpha) \leq E(\widehat{\alpha}_{F,R} - \widehat{\alpha}_{F,s_h} \mid B, S_l) \tag{1.20}$$

Lemma 1 implies that (1.20) and (1.19) are necessary and sufficient conditions for the existence of a truthtelling equilibrium. Notice that the good IB has more informative signal than the bad IB. Hence, the following inequalities hold:

$$\begin{aligned} E(\widehat{\alpha}_{F,R}-\widehat{\alpha}_{F,s_h} &\mid B,S_l) < E(\widehat{\alpha}_{F,R}-\widehat{\alpha}_{F,s_h} \mid G,S_l) \\ E(\widehat{\alpha}_{F,R}-\widehat{\alpha}_{F,s_h} &\mid G,S_h) < E(\widehat{\alpha}_{F,R}-\widehat{\alpha}_{F,s_h} \mid B,S_h) \end{aligned}$$

This implies that conditions (1.18) and (1.17) are satisfied whenever (1.20) and (1.19) are satisfied. ■

1.7.2 Proof of Lemma 2

Proof. Consider conditions (1.20) and (1.19). First, consider condition (1.20). To ease notation, let $M^{TT} \equiv kV^{TT}(s_h,\alpha)$ and $R^{TT}(B,s_l,S_l) \equiv E(\widehat{\alpha}_{F,R}-\widehat{\alpha}_{F,s_h} \mid B,S_l)$, so that we can write this condition as follows:

$$M^{TT} \leq R^{TT}(B,s_l,S_l)$$

Using the results appendix 1, it is possible to show that:

$$M^{TT} = k\frac{\theta\left[\alpha+(1-\alpha)z\right]}{\theta\left[\alpha+(1-\alpha)z\right]+(1-\theta)(1-\alpha)(1-z)}$$

and[20]

$$R^{TT}(B,s_l,S_l) = \frac{\alpha}{\alpha+z(1-\alpha)}\left[\frac{z-\theta}{\theta-2\theta z+z}\right]$$

where $\frac{\alpha}{\alpha+z(1-\alpha)} \equiv \alpha_{\max}$.

For every $k,\alpha\in(0,1)$ and $z\in\left(\frac{1}{2},1\right)$, the following properties are satisfied:

[20]Notice that

$$E(\widehat{\alpha}_{F,R}-\widehat{\alpha}_{F,s_h} \mid B,S_l) = E(\widehat{\alpha}_{F,R} \mid B,S_l) - E(\widehat{\alpha}_{F,s_h} \mid B,S_l)$$

with

$$\begin{aligned} & E(\widehat{\alpha}_{F,R} \mid B,S_l) = \widehat{\alpha}_{\max}\Pr(F=0 \mid B,S_l) + \widehat{\alpha}_{\min}\Pr(F=1 \mid B,S_l) = \\ = {} & \frac{\alpha}{\alpha+z(1-\alpha)}\frac{(1-\theta)\,z}{\theta\,(1-z)\,(1-\theta)\,z} \\ & E(\widehat{\alpha}_{F,s_h} \mid B,S_l) = \widehat{\alpha}_{\max}\Pr(F=1 \mid B,S_l) + \widehat{\alpha}_{\min}\Pr(F=0 \mid B,S_l) \\ = {} & \frac{\alpha}{\alpha+z(1-\alpha)}\frac{\theta\,(1-z)}{\theta\,(1-z)\,(1-\theta)\,z} \end{aligned}$$

the results follows from the fact that in a truthtelling equilibrium $\widehat{\alpha}_{\max} = \frac{\alpha}{\alpha+z(1-\alpha)}$ and $\widehat{\alpha}_{\min} = 0$ (see appendix 1)

(i) at $\theta = 0$, $M^{TT} = 0$ and $R^{TT}(B, s_l, S_l) = \alpha_{\max}$. Thus, at $\theta = 0$, $M^{TT} < R^{TT}(B, s_l, S_l)$.

(ii) at $\theta = 1$, $M^{TT} = k$ and $R^{TT}(B, s_l, S_l) = -\alpha_{\max}$. Thus, at $\theta = 1$, $M^{TT} > R^{TT}(B, s_l, S_l)$.

(iii) for $\theta \in (0, 1)$, M^{TT} is a continuous and strictly increasing function of θ, while $R^{TT}(B, s_l, S_l)$ is a continuous and strictly decreasing function of θ.

(i), (ii), guarantee that there exists a value $\theta = \overline{\theta}^{TT} \in (0, 1)$ such that $M^{TT} = R^{TT}(B, s_l, S_l)$. (iii) guarantees that $\overline{\theta}^{TT}$ is unique.

Now consider condition (1.19). Again, let $M^{TT} \equiv kV^{TT}(s_h, \alpha)$ and $R^{TT}(B, s_h, S_l) \equiv E(\widehat{\alpha}_{F,R} - \widehat{\alpha}_{F,s_h} \mid B, S_h)$, so that we can write this condition as follows:

$$M^{TT} \geq R^{TT}(B, s_l, S_h)$$

where

$$M^{TT} = k\frac{\theta\left[\alpha + (1-\alpha)z\right]}{\theta\left[\alpha + (1-\alpha)z\right] + (1-\theta)(1-\alpha)(1-z)}$$

and

$$R^{TT}(B, s_l, S_h) = \alpha_{\max}\left[\frac{1-z-\theta}{1-z-\theta+2\theta z}\right]$$

For every $k, \alpha \in (0, 1)$ and $z \in \left(\frac{1}{2}, 1\right)$, the following properties are satisfied:

(iv) for $\theta = 0$, $M^{TT} = 0$ and $R^{TT}(B, s_l, S_h) = \alpha_{\max}$. Thus, at $\theta = 0$, $M^{TT} < R^{TT}(B, s_l, S_h)$

(v) for $\theta = 1$, $M^{TT} = k$ and $R^{TT}(B, s_l, S_h) = -\alpha_{\max}$. Thus, at $\theta = 1$, $M^{TT} > R^{TT}(B, s_l, S_h)$

(vi) for $\theta \in (0, 1)$, M^{TT} is a continuous and strictly increasing function of θ; $R^{TT}(B, s_l, S_h)$ is a continuous and strictly decreasing function of θ.

(vi), (v), guarantee that there exists a unique value $\theta = \underline{\theta}^{TT} \in (0, 1)$ such that $M^{TT} = 0$ and $R^{TT}(B, s_l, S_h)$. (vi) guarantees that $\underline{\theta}^{TT}$ is unique.

In order to complete the proof we must show that $\underline{\theta}^{TT} < \overline{\theta}^{TT}$

This can easily be seen by observing that for $\theta \in (0, 1)$,for every $k, \alpha \in (0, 1)$ and $z \in \left(\frac{1}{2}, 1\right)$, the following inequality holds:

$$R^{TT}(B, s_l, S_h) < R^{TT}(B, s_l, S_l)$$

Then, at $\theta = \underline{\theta}^{TT}$ we have that

$$M^{TT} = R^{TT}(B, s_l, S_h) < R^{TT}(B, s_l, S_l)$$

Since $R^{TT}(B, s_l, S_l)$ is monotonically decreasing in θ, the equality

$$M^{TT} = R^{TT}(B, s_l, S_l)$$

is satisfied only for $\theta > \underline{\theta}^{TT}$. ■

1.8 Appendix 3

1.8.1 Proof of Proposition 5

Proof. Consider the putative (partial pooling) equilibrium in which the bad IB disregards its private information and always provides a negative evaluation, the good IB truthtells and the firm accepts after s_h and refuses after s_l . In this equilibrium, ex-post reputation $\widehat{\alpha}$ assumes the following values[21]:

$$\begin{aligned} \widehat{\alpha}_{0,R} &= \alpha \\ \widehat{\alpha}_{1,R} &= 0 \\ \widehat{\alpha}_{1,s_h} &= 1 \end{aligned}$$

Notice that the event $(F = 0, s_h)$ has zero probability to occur in the putative equilibrium that we are considering. Indeed, high evaluations are sent in equilibrium only by good IBs, which in turn have complete information about the true value of the firm and thus do not make mistakes. Accordingly $\widehat{\alpha}_{0,s_h}$ cannot be computed via Bayes rule. However, it is reasonable to assume that when the market observes the outcome $(F = 0, s_h)$ it believes that the evaluation has been reported by a bad IB. Thus, we assume:

$$\widehat{\alpha}_{0,s_h} = 0$$

As of the possible values that the firm, let $V_L^{PP}(s_l, \alpha)$ and $V_L^{PP}(s_h, \alpha)$ respectively denote the value of the firm when evaluations s_l and s_h reach the market.

In this equilibrium, the firm refuses after s_l. Accordingly:

$$d = V_L^{PP}(s_l, \alpha) = \frac{\theta(1-\alpha)}{\theta(1-\alpha) + (1-\theta)}$$

Indeed, when the firm refuses, the market infers that s_l has been proposed and set

[21] See appendix 1 (remembering that here we are assuming $p = 1$).

$d \equiv \Pr(F = 1 \mid R) = \Pr(F = 1 \mid s_l)$. Given the equilibrium strategies of IBs, $\Pr(F = 1 \mid s_l) = V_L^{PP}(s_l, \alpha)$.

Analogously, since in equilibrium the firm accepts after s_h, and s_h is sent only by good IB (that have perfect information), then $V_L^{PP}(s_h, \alpha) = 1$.

Finally, accepting after s_l is an out of equilibrium action by the firm. In this case, the IB's low evaluation would reach the market. Notice that given the equilibrium strategies of the IBs, the value of the firm can be computed via Bayes and set equal to $V_L^{PP}(s_l, \alpha)$.

Good IBs' problem. Given the equilibrium strategies of the firm and bad IBs, a good IB truthtells if:

$$kV_L^{PP}(s_h, \alpha) + E(\widehat{\alpha}_{F,s_h} \mid G, S_h) \geq E(\widehat{\alpha}_{F,R} \mid G, S_h)$$
$$E(\widehat{\alpha}_{F,R} \mid G, S_l) \geq kV_L^{PP}(s_h, \alpha) + E(\widehat{\alpha}_{F,s_h} \mid G, S_l)$$

where:

$$\begin{aligned} E(\widehat{\alpha}_{F,s_h} &\mid G, S_h) = \widehat{\alpha}_{1,s_h} \Pr(F = 1 \mid G, S_h) + \widehat{\alpha}_{0,s_h} \Pr(F = 0 \mid G, S_h) = 1 \\ E(\widehat{\alpha}_{F,R} &\mid G, S_h) = \widehat{\alpha}_{1,R} \Pr(F = 1 \mid G, S_h) + \widehat{\alpha}_{0,R} \Pr(F = 0 \mid G, S_h) = \\ &= \alpha \Pr(F = 0 \mid G, S_h) = 0 \end{aligned}$$

$$\begin{aligned} E(\widehat{\alpha}_{F,s_h} &\mid G, S_l) = \widehat{\alpha}_{1,s_h} \Pr(F = 1 \mid G, S_l) + \widehat{\alpha}_{0,s_h} \Pr(F = 0 \mid G, S_l) = 0 \\ E(\widehat{\alpha}_{F,R} &\mid G, S_l) = \widehat{\alpha}_{1,R} \Pr(F = 1 \mid G, S_l) + \widehat{\alpha}_{0,R} \Pr(F = 0 \mid G, S_l) = \\ &= \alpha \Pr(F = 0 \mid G, S_l) = \alpha \end{aligned}$$

Therefore, considering also that $V_L^{PP}(s_h, \alpha) = 1$, our conditions for the good IB can be written as follows:

$$\begin{aligned} &k + 1 \geq 0 \;\rightarrow\; \text{always satisfied} \\ &\alpha \geq k \qquad\;\; \rightarrow \alpha \geq k \end{aligned}$$

Bad IBs' problem. Given the equilibrium strategies of the firm and good IBs, a bad IB pools around s_l if:

$$\begin{aligned} E(\widehat{\alpha}_{F,R} &\mid B, S_h) \geq kV_L^{PP}(s_h, \alpha) + E(\widehat{\alpha}_{F,s_h} \mid B, S_h) \\ E(\widehat{\alpha}_{F,R} &\mid B, S_l) \geq kV_L^{PP}(s_h, \alpha) + E(\widehat{\alpha}_{F,s_h} \mid B, S_l) \end{aligned}$$

Notice that since the bad IB's signal is informative, it is sufficient to show that the first inequality is satisfied. Let us write the first inequality as follows:

$$E(\widehat{\alpha}_{F,R} - \widehat{\alpha}_{F,s_h} \mid B, S_h) \geq kV_L^{PP}(s_h, \alpha) \tag{1.21}$$

After a bit of algebra, it is possible to show that the previous condition can be written as[22]:

$$\frac{\alpha(1-\theta)(1-z) - \theta z}{\theta z + (1-\theta)(1-z)} \geq k$$

At $\theta = 0$, the LHS=α and the RHS=k. At $\theta = 1$, the LHS=-1 and RHS=k. Notice that for $\theta \in (0,1)$, both the LHS is continuous and strictly decreasing in θ. Therefore, provided that $\alpha > k$, there always exists a unique value of $\theta \equiv \theta_L^{pp} \in (0,1)$ such that LHS=RHS and such that for every $\theta \in (0, \theta_L^{pp})$ the previous condition is always satisfied. Thus, we can conclude that given the posited equilibrium strategy of the firm, the good IB truthtells and the bad IB pools around s_l as long as $\alpha > k$ and $\theta \in (0, \theta_L^{pp})$.

The firm's problem. We then have to check wether it is optimal for the firm to follow its equilibrium strategy (accept after s_h and refuse after s_l). Accepting after s_h gives the firm a payoff equal to $(1-k)\,V_L^{PP}(s_h, \alpha)$. If the firm deviates and refuses, it is believed that s_l has been proposed and the firm gets $d = V_L^{PP}(s_l, \alpha)$. Thus accepting after s_h is optimal as long as $1 - k \geq V_L^{PP}(s_l, \alpha)$, or equivalently:

$$k \leq 1 - V_L^{PP}(s_l, \alpha) \equiv \overline{k}^{PP} \tag{1.22}$$

Refusing after s_l gives the firm a payoff equal to $d = V_L^{PP}(s_l, \alpha)$. If the firm deviates and accepts after s_l, then it gets $(1-k)\,V_L^{PP}(s_l, \alpha)$. Thus, for the firm is always optimal to refuse after s_l. ■

[22] Notice that $V_L^{PP}(s_h, \alpha) = 1$, while for expected reputations we have that:

$$\begin{aligned}
E(\widehat{\alpha}_{F,s_h} \quad &\mid \quad B, S_h) = \widehat{\alpha}_{1,s_h} \Pr(F=1 \mid B, S_h) + \widehat{\alpha}_{0,s_h} \Pr(F=0 \mid B, S_h) = \\
&= \quad \Pr(F=1 \mid B, S_h) = \frac{\theta z}{\theta z + (1-\theta)(1-z)} \\
E(\widehat{\alpha}_{F,R} \quad &\mid \quad B, S_h) = \widehat{\alpha}_{1,R} \Pr(F=1 \mid B, S_h) + \widehat{\alpha}_{0,R} \Pr(F=0 \mid B, S_h) = \\
&= \quad \alpha \Pr(F=0 \mid B, S_h) = \alpha \frac{(1-\theta)(1-z)}{\theta z + (1-\theta)(1-z)}
\end{aligned}$$

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Chapter 2

Insider Trading under Discreteness

2.1 Introduction

In "Insider Trading without Normality", Rochet and Vila (1994) present a version of Kyle's (1985) model where both the distribution of noise trade (u) and that of the liquidation value of the risky asset (v) are assumed to be discrete, instead than normal as in the original Kyle's contribution. In particular, Rochet and Vila focus on a specific numerical example to show that under discreteness the property of uniqueness of the linear equilibrium of the original Kyle model does not hold anymore. This result follows from a key feature of the discrete model. Namely, that the set of aggregate orders that are compatible with any specific equilibrium of the discrete game is countable (and not unbounded and continuous as in the original normal model). This property of the equilibrium implies that for any aggregate order that does not belong to this set, the market maker can detect that the insider has made a deviation from his equilibrium strategy. Accordingly, the definition of any perfect Bayesian equilibrium of the discrete game must include a set of out-of-equilibrium beliefs of the market makers about what the insider has observed when deviating from the strategy prescribed by the equilibrium. Basically, out-of-equilibrium beliefs determine the market maker's (price) reaction to an insider's deviation and thus determine whether such deviation is profitable or not.[1] In other words, the existence of a specific equilibrium now depends on its out-of-equilibrium

[1] This is not the case under normality, when the market maker can never detect a deviation by the informed. Indeeed, the unbounded and continuous support of the distribution of v implies that the domain of the linear strategy of the insider is continuous and unbounded too. Since under normality also the support of the distribution of noise trade is unbounded and continuous, any aggregate order observed by a market maker is in principle consistent with the equilibrium strategy of the insider: Any change in the aggregate order can always be attributed to a different realizaation of noise trade. Accordingly, out of equilibrium beliefs of market makers play no role in the normal model.

beliefs.[2] Further, out-of-equilibrium beliefs pay a crucial role in paving the way to equilibria multiplicity.

Despite the critical role played by out-of-equilibrium beliefs, Rochet and Vila propose the existence of a family of equilibria for the specific game that they consider without defining any set of out-of-equilibrium beliefs for market makers. In fact, they leave this task to the reader.

Biais and Rochet (1997) reconsider Rochet and Vila's discrete model and, specifically, focus on the family of perfect Bayesian equilibria that has been proposed in Rochet and Vila (1994). In their analysis, Biais and Rochet do indeed define a set of out-of-equilibrium beliefs for the market makers that (supposedly) support the specific class of equilibria under consideration.

The initial motivation behind the present work comes from recognizing that the out-of-equilbrium beliefs defined in Biais and Rochet (1997) (or if you prefer, the whole price function that stems from these beliefs) are in fact inadequate to support the proposed family of PB equilibria.

Accordingly, section 1 of the present paper is devoted to finding a set of out-of-equilibrium beliefs that supports the equilibrium proposed in Rochet-Vila (1994) and in Biais-Rochet (1997). In doing so, I take a reverse-engeneering approach. First, I will define the whole set of (out-of-equilibrium) price functions that support the equilibria belonging to the class of equilibria proposed by Rochet and Vila. As we will see, it turns out that this class of equilibria is smaller than claimed by Rochet and Vila. Second, I will show that the out of equilibrium beliefs proposed by Bias and Rochet do not support the equilibria under consideration. Finally, I will define a specific price function that support the class of equilibria under consideration and finally retrieve the out-of-equilibrium beliefs that give rise to this price function.

In section 2 of the paper, I will analyze a more general version of the discrete model proposed by Rochet and Vila by allowing for a more general probability distribution of the liquidation value of the asset. This allows me to address the issue of how the trading strategy of the informed trader changes in response to variations in asset riskiness, as measured by the variance of the asset. This analysis provides a starting and benchmark point that might prove useful in building more complex financial and economic models where the descrete Kyle model may have a bite.

[2] Alternatively, the definition of any perfect Bayesian equilibrium of the discrete game could include a price function (that follows from the specified set of out-of-equilibrium beliefs) according to which market makers price aggregate orders that are off-the-equilibrium path and that induce the insider to not deviate.

2.2 The basic model: Rochet-Vila (1994)

Rochet and Vila (1994) propose a version of the Kyle (1985) model where both the distribution of noise trade and that of the liquidation value of the risky asset are assumed to be discrete instead than normal, as in the original Kyle's model. In this respect, their model can be defined as a discrete version of the original Kyle model. Indeed, a part from dropping the assumption of normality, all the other features of the classical Kyle model are maintained unaltered. Specifically, Rochet and Vila consider the market for a risky asset, whose liquidation value v can take on values $-2, -1, 1, 2$ with equal probability $\frac{1}{4}$. The asset is traded in one session of trade during which an informed trader (the insider) and many noise traders submit their trade orders to a fringe of market makers who compete à la Bertrand. All agents are assumed to be risk neutral. In aggregate, orders coming from noise traders u take on values $1, -1$ with equal probability $\frac{1}{2}$. The insider can observe the true liquidation value of the asset (but not the realization of u) and based on this piece of information submits his trade order $x \in R$ to a market maker. The market maker observes the total market order (the sum of the informed order and noise trade), $z = x + u$, and based on this piece of information sets the price p at which trade takes place. Basically, since market makers are competitive and risk neutral, each market maker sets a seming-strong efficient price given by the following function:

$$P(z) = E(v|z) \tag{2.1}$$

At the end of the trade session, the true liquidation value of the asset is realized and payoffs are distributed. The informed trader gets $x(v-p)$, noise traders get $u(v-p)$ and the market maker gets $z(v-p)$.

Given these assumptions, Rochet and Vila propose a family of perfect Bayesian equilibria indexed by $a \in \left[\frac{2}{5}, 1\right)$ where:

(i) the informed trader trades according to the following strategy:

$$X^*(v) = \begin{cases} 1+a & v=2 \\ 1-a & v=1 \end{cases}, \; X^*(v) = -X^*(-v) \tag{2.2}$$

(ii) for the set of aggregate trades that arise in equilibrium, $z = -2-a, -2+$

$a, -a, a, 2-a, 2+a$, the pricing strategy of the market maker reads:

$$P^*(z) = \begin{cases} \frac{1}{2} & z = a \\ 1 & z = 2-a \\ 2 & z = 2+a \end{cases}, \quad P^*(z) = -P^*(-z) \tag{2.3}$$

Basically, in the equilibrium proposed by Rochet and Vila, the informed trader follows a strategy that is an odd function monotonically increasing in the value of the asset, while market makers set prices equal to the expected value of the asset given the information content that they can infer from aggregate orders. Clearly, in contrast to the classical model of Kyle (1985), where the assumption of normality for the distributions of the liquidation value of the asset and of noise trade makes every aggregate order potentially consistent with the equilibrium strategy of the informed trader, discreteness implies the existence of aggregate orders that are not consistent with the equilibrium strategy of the informed trader. Consequently, a price schedule according to which these out-of-equilibrium orders are priced has to be defined as part of any perfect Bayesian equilibrium of the game with discrete distributions. In fact, Rochet-Vila (1994) leaves the task of finding this price schedule to the reader. At page 151, they conclude that in order to complete the proof of their result *"The final step is to define $P(z)$ for every value of z. This can be done for $\frac{2}{5} \leq a < 1$...(the details are left to the reader)"*.

In "Risk Sharing, Adverse Selection and Market Structure" (Financial Mathematics, Lecture Notes in Mathematics, 1997), Biais and Rochet consider the model of Rochet -Vila (1994) discussed above and propose the following out-of-equilibrium beliefs of the market maker (see pag. 15):

$$\begin{aligned} \forall z &> 0,\ z \neq a, 2-a, 2+a,\ \Pr(v = 2|z) = 1 \\ \forall z &< 0,\ z \neq -a, -2+a, -2-a,\ \Pr(v = -2|z) = 1 \end{aligned}$$

Clearly, these beliefs determines the following price schedule:

$$\begin{aligned} \forall z &> 0,\ z \neq a, 2-a, 2+a,\ P(z) = 2 \\ \forall z &< 0,\ z \neq -a, -2+a, -2-a,\ P(z) = -2 \end{aligned}$$

Biais and Rochet conclude that *"given the out-of-equilibrium beliefs we have specified above, the insider never finds it optimal to trade outside the set $\{-2-a, -2+a, -a, a, 2-a, 2+a\}$"*.

Let Ψ denote the set of aggregate orders that are not consistent with the family of equillibria $(X^*(v), P^*(z))$ defined by (2.2) and (2.3), that is:

$$\Psi = \{z \in \mathbb{R},\ z \neq 2-a, -2+a, -a, a, 2-a, 2+a\}$$

In the next three sub-sections, I will provide a general analysis of the out-of-equilibrium price functions that can support the equilibrium proposed by Rochet and Vila, show that the out-of-equilibrium beleifs proposed by Biais and Rochet (and the resulting price function) are not able to support such equilibrium, and finally define a spcific price function (and a corrispondig set of out-of-equilibrium beliefs) that do support the equiibrium under consideration

2.2.1 A note on Rochet-Vila (1994)

In this sub-section, I will define the set of prices for every value of $z \in \Psi$ that do support the class of equilibria $(X^*(v), P^*(z))$. I will show that this class of equilibria is smaller than claimed in Rochet and Vila (1994).

Denote with $B(\cdot)$ the function mapping the pairs (v, x) into informed trader's expected profits:

$$B(v, x) = x(v - \Phi(x))$$

where $\Phi(x) \equiv E\left(P(x+u)\right) = \frac{1}{2}P(x+1) + \frac{1}{2}P(x-1)$ is the price expected by the informed when placing order x.

Focus on the case in which the informed trader observes $v = 1, 2$ (by symmetry, it is easy to show that the same result holds for $v = -1, -2$). In this case, equilibrium expected profits of the informed trader read:

$$\begin{aligned}
B(2, 1+a) &= (1+a)\left(2 - \frac{1}{2}P^*(2+a) - \frac{1}{2}P^*(a)\right) = \frac{3}{4}(1+a) \\
B(1, 1-a) &= (1-a)\left(1 - \frac{1}{2}P^*(2-a) - \frac{1}{2}P^*(-a)\right) = \frac{3}{4}(1-a) \\
B(-1, -1+a) &= (-1+a)\left(-1 - \frac{1}{2}P^*(a) - \frac{1}{2}P^*(-2+a)\right) = \frac{3}{4}(1-a) \\
B(-2, -1-a) &= (-1-a)\left(-2 - \frac{1}{2}P^*(-a) - \frac{1}{2}P^*(-2-a)\right) = \frac{3}{4}(1+a)
\end{aligned}$$

Ultimately, the problem is to define a price function $P(z)$ that for every value of z not compatible with the equilibrium gives rise to prices that support the candidate equilibrium defined by (2.2) and (2.3). Formally, $P(z)$ must be such that the following inequalities are satisfied simultaneously for all x:

$$\begin{aligned}
B(2,1+a) &\geq B(2,x)\\
B(1,1-a) &\geq B(1,x)\\
B(-1,-1+a) &\geq B(-1,x)\\
B(-2,-1-a) &\geq B(-2,x)
\end{aligned}$$

When the previous conditions are satisfied, the pricing strategy of the market maker prevents the informed trader from deviating. It is easy to check that the previous inequalities can be written as follows:

$$\frac{3}{4}(1+a) \geq x\,(2-\Phi(x)) \tag{2.4a}$$
$$\frac{3}{4}(1-a) \geq x\,(1-\Phi(x)) \tag{2.4b}$$
$$-\frac{3}{4}(-1-a) \geq x\,(-2-\Phi(x)) \tag{2.4c}$$
$$-\frac{3}{4}(-1+a) \geq x\,(-1-\Phi(x)) \tag{2.4d}$$

where $\Phi(x) = \frac{1}{2}P(x+1) + \frac{1}{2}P(x-1)$. Notice that perfect Bayesian equilibrium restricts $P(x+1)$ and $P(x-1)$ to be in the support of v, namely in $[-2,2]$. Accordingly, we also have that $\Phi(x) \in [-2,2]$. I will construct the price function $P(z)$ for every z not compatible with the equilibrium in a series of steps.

(a) Consider the case in which $z < -1$ and $z > 1$, with $z \neq -2-a$, $-2+a$, $2-a$, $2+a$. Tentatively, assume that in this case the price function satisfies:

$$P(z) = 2 \quad \forall z \geq 1,\ z \neq 2-a, 2+a \tag{2.5}$$
$$P(z) = -2\ \forall z \leq -1,\ z \neq -2-a, -2+a \tag{2.6}$$

This is sufficient to guarantee that the informed trader will never deviate to any order $x \leq -2$ or $x \geq 2$. Indeed, any deviation to $x \geq 2$ will always generate an aggregate order $z = x + u \geq 1$ and thus will always be met by a price $P(z) = 2$. Accordingly, the expected price will be $\Phi(x) = 2$ and all conditions (2.4a), (2.4b), (2.4c), (2.4d) are satisfied. By symmetry, the same holds true for any deviation to $x \leq -2$.

(b) Consider now the case in which $-1 < z < 1$, with $z \neq -a, a$. In terms of x, this amounts to focus on the case in which $-2 < x < 2$ (with $x \neq -1+a, 1-a$).

(b1) Consider first the case of $x \in (0, 2)$.

Since $\Phi(x) \in [-2, 2]$, conditions (2.4c) and (2.4d) are trivially satisfied when $x \in (0, 2)$. As of conditions (2.4a) and (2.4b), consider the following argument. When $x \in (0, 2)$ and $u = 1$, any realization of the aggregate order z is given by $x + 1 > 1$. Thus, according to (2.5) $P(x+1) = 2$. Accordingly, (2.4a) and (2.4b) can be written as:

$$\begin{aligned} P(x-1) &\geq 2 - \frac{3(1+a)}{2x} \\ P(x-1) &\geq -\frac{3(1-a)}{2x} \end{aligned}$$

On the other hand, when $x \in (0, 2)$ and $u = -1$, any the realization of z is given by $x - 1 \in (-1, 1)$. Expressing the previous two conditions in terms of z, we have that for $z \in (-1, 1)$, $P(z)$ must satisfy:

$$P(z) \geq 2 - \frac{3(1+a)}{2(z+1)} \equiv P_1(z, a) \tag{2.7}$$

$$P(z) \geq -\frac{3(1-a)}{2(z+1)} \equiv P_2(z, a) \tag{2.8}$$

(b2) Consider now the case of $x \in (-2, 0)$.

Again, since $\Phi(x) \in [-2, 2]$, conditions (2.4a), (2.4b) are trivially satisfied when $x \in (-2, 0)$. Following the same line of reasoning above, we know that when $x \in (-2, 0)$ and $u = -1$, (2.6) implies $P(x-1) = -2$ and conditions (2.4c) and (2.4d) can be written as:

$$\begin{aligned} P(x+1) &\leq -2 - \frac{3(1+a)}{2x} \\ P(x+1) &\leq -\frac{3(1-a)}{2x} \end{aligned}$$

On the other hand, $x \in (-2, 0)$ and $u = 1$ imply that any realization of z is $x + 1 \in (-1, 1)$. Thus, the previous conditions can be equivalently written in terms of z so that for every $z \in (-1, 1)$, $P(z)$ must satisfy:

$$P(z) \leq -2 - \frac{3(1+a)}{2(z-1)} \equiv P_3(z, a) \tag{2.9}$$

$$P(z) \leq -\frac{3(1-a)}{2(z-1)} \equiv P_4(z, a) \tag{2.10}$$

b3) Finally, when $x = 0$ it is easy to check that conditions (2.4a), (2.4b), (2.4c), (2.4d) are trivially satisfied.

Summing up, in order to avoid deviations to any $-2 < x < 2$, it must be that for every $z \in (-1, 1)$, $P(z)$ satisfies conditions (3.54), (3.55), (3.58), (3.59) simultaneously. The set of pairs (z, p) that satisfies (3.54) through (3.59) is represented by the shaded area in figure 1. Any continuous function $P(z)$ passing through the shaded area satisfies conditions (3.54) through (3.59) and thus prevents from deviations to any order $-2 < x < 2$. It is apparent that a necessary condition for the existence of a continuous function $P(z)$ satisfying (3.54) through (3.59) is that the shaded area in figure 1 be a connected set.

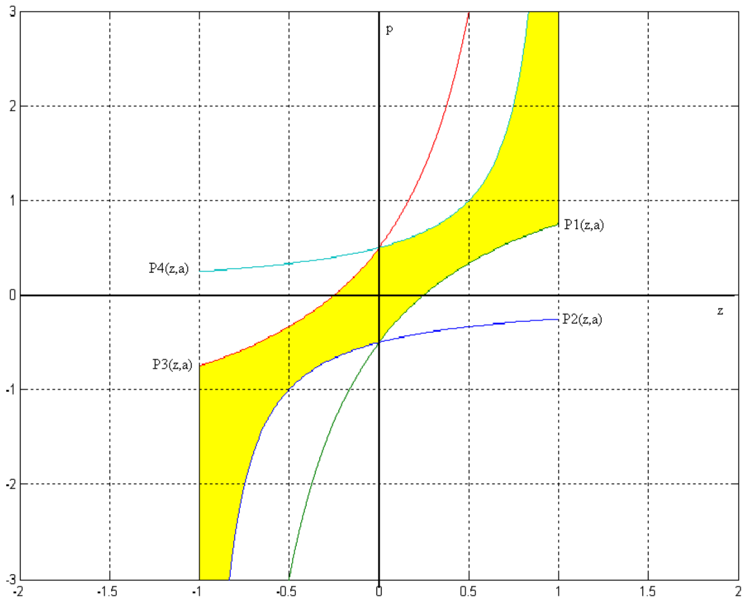

Figure 2.1: conditions (7) through (10)

Notice that the shaded area is a connected set as long as $P_1(z, a)$ and $P_2(z, a)$ lie below $P_3(z, a)$ and $P_4(z, a)$ (remember that we are focusing on $z \in (-1, 1)$). Analytically, this amounts to find the values of a that guaratee that for every $z \in (-1, 1)$, the following inequalities hold simultaneously:

$$P_2(z,a) \leq P_3(z,a) \Leftrightarrow -\frac{3(1-a)}{2(z+1)} \leq -2 - \frac{3(1+a)}{2(z-1)} \quad (2.11)$$
$$P_2(z,a) \leq P_4(z,a) \Leftrightarrow -\frac{3(1-a)}{2(z+1)} \leq -\frac{3(1-a)}{2(z-1)} \quad (2.12)$$
$$P_1(z,a) \leq P_3(z,a) \Leftrightarrow 2 - \frac{3(1+a)}{2(z+1)} \leq -2 - \frac{3(1+a)}{2(z-1)} \quad (2.13)$$
$$P_1(z,a) \leq P_4(z,a) \Leftrightarrow 2 - \frac{3(1+a)}{2(z+1)} \leq -\frac{3(1-a)}{2(z-1)} \quad (2.14)$$

It is easy to check that inequality (2.12) is always satisfied. It can be shown[3] that both the inequalities (2.11) and (2.14) hold for $a \leq \frac{2\sqrt{2}}{3}$ and inequality (2.13) holds for $a \geq \frac{1}{3}$. Therefore, we can conclude that there exists a price function $P(z)$ that prevents the informed trader from deviating to any out-of-equilibrium trade $-2 < x < 2$ if and only if $\frac{1}{3} \leq a \leq \frac{2\sqrt{2}}{3}$. Put in another way, for any $\frac{1}{3} \leq a \leq \frac{2\sqrt{2}}{3}$ it is possible to find a price function $P(z)$ that prevents the informed trader from deviating to out-of-equilibrium trades. This is all as the (out-of-equilibrium) price function is concerned.

It is easy to check that the incentive compatibility constraints for the informed trader:

$$B(2,1+a) \geq B(2,1-a)$$
$$B(1,1-a) \geq B(1,1+a)$$

are satisfied for $a \geq \frac{2}{5}$.

Thus, we can conclude that the equilibrium defined in Rochet and Vila (1994) exists as long as $\frac{2}{5} \leq a \leq \frac{2\sqrt{2}}{3} < 1$ and not for $\frac{2}{5} \leq a \leq 1$ as claimed in the paper.

2.2.2 A Note on Biais-Rochet (1997)

In "Risk Sharing, Adverse Selection and Market Structure" (Financial Mathematics, Lecture Notes in Mathematics, 1997) Biais and Rochet propose the model of Rochet -Vila (1994) discussed above. At pag 15, the authors propose the following out-of-equilibrium beliefs of the market maker (see pag. 15):

$$\forall z > 0,\ z \neq a, 2-a, 2+a,\ \Pr(v = 2|z) = 1 \quad (2.15)$$
$$\forall z < 0,\ z \neq -a, -2+a, -2-a,\ \Pr(v = -2|z) = 1$$

[3] see appendix

At pag 16, they state that *"given the out-of-equilibrium beliefs we have specified above, the insider never finds it optimal to trade outside the set* $\{-2-a, -2+a, -a, a, 2-a, 2+a\}$*.......Therefore to establish the optimality of* $X(.)$ *we only need to prove that:*

$$\begin{aligned} B(2,1+a) &\geq B(2,1-a), \text{ and} \qquad (2.16) \\ B(1,1-a) &\geq B(1,1+a) \qquad (2.17) \end{aligned}$$

An immediate computation shows that these inequalities are satisfied when $a \geq \frac{2}{3}$*."*[4]

It is worth noticing that the market maker's out-of-equilibrium beliefs (2.15) do not support the equilibrium specified above. Indeed, out-of-equilibrium beliefs (2.15) results in the following price function $P(z)$, defined for aggregate trades outside the equilibrium set $\{-2-a, -2+a, -a, a, 2-a, 2+a\}$:

$$\begin{aligned} P(z) &= 2, \ \forall z > 0, \ z \neq a, 2-a, 2+a \qquad (2.18) \\ P(z) &= -2, \ \forall z < 0, \ z \neq -a, -2+a, -2-a \end{aligned}$$

In line with what discussed in the previous section, it must be noticed that such a price function does not support our candidate equilibrium. Indeed, given (2.18), there always exists a profitable deviation for the informed trader.

Focus on the case in which the informed trader observes $v = 1, 2$ (by symmetry, it is easy to show that the same result holds for $v = -1, -2$). We have shown that:

$$\begin{aligned} B(2,1+a) &= \frac{3}{4}(1+a) \\ B(1,1-a) &= \frac{3}{4}(1-a) \end{aligned}$$

Thus, the informed trader will not deviate from his equilibrium strategy as long as the following inequalities hold for every x:

$$\begin{aligned} B(2,1+a) &\geq B(2,x) \\ B(1,1-a) &\geq B(1,x) \end{aligned}$$

[4] Notice that there is a typo, since it is easy to check that conditions (2.16) and (2.17) are satisfied for $a \geq \frac{2}{5}$, as correctly pointed out in the original Rochet-Vila (1994) model.
There is also a typo in proposition 1, were it is stated that the family of equilibria is *"indexed by* $a \in \left]0, \frac{2}{3}\right[$*"*. The correct range of a is the segment $\left[\frac{2}{5}, \frac{2\sqrt{2}}{3}\right]$, as shown in the previous section.

that is, as long as

$$\begin{aligned}\frac{3}{4}(1+a) &\geq x\,(2-\Phi(x)) \\ \frac{3}{4}(1-a) &\geq x\,(1-\Phi(x))\end{aligned}$$

where $\Phi(x) = \frac{1}{2}P(x-1) + \frac{1}{2}P(x+1)$. Consider a candidate deviation $x \in (0,1)$, so that $x+1 > 1$ and $x-1 < 0$. Given the price response (2.18) that follows from the out-of-equilibrium beliefs (2.15) assumed in Biais and Rochet (1997), we have that $\Phi(x) = \frac{1}{2}2 + \frac{1}{2}(-2) = 0$. Therefore, our equilibrium conditions imply that $\forall x \in (0,1)$:

$$\begin{aligned}\frac{3}{4}(1+a) &\geq 2x \\ \frac{3}{4}(1-a) &\geq x\end{aligned}$$

that is:

$$\begin{aligned}\frac{3}{4}(1+a) &\geq 2 \Rightarrow a \geq \frac{5}{3} \\ \frac{3}{4}(1-a) &\geq 1 \Rightarrow a \leq \frac{1}{3}\end{aligned}$$

a contradiction

In words, an order between 0 and 1 will give rise with probability $\frac{1}{2}$ to a negative aggregate order. In this case, the informed trader will pay a negative price (in fact, he will be paid). His expected price is zero and is low enough to compensate the profits that he gains in expectation by following his equilibrium strategy.

2.2.3 A Specific Set of Out-of-equilibrium Beliefs

It is easy to check the the following out-of equilibrium price function:

$$P(z) = \begin{cases} 2 & \forall z > 1 \\ 2z & \forall z \in [-1,1] \\ -2 & \forall z < -1 \end{cases} \quad , \; z \in \mathbb{R} \setminus \Psi$$

do support equilibria caractherized by (2.2) and (2.3).

2.3 Asset Volatility

Consider a more general probability distribution for the liquidation value of the asset v. In particular, let $\Pr(v = 2) = \Pr(v = -2) = w$ and $\Pr(v = 1) = \Pr(v = -1) = \frac{1}{2} - w$, with $0 < w < \frac{1}{2}$. Thus, $E(v) = 0$ and $VAR(v) = 6w + 1$. Let all the other features of the model be unchanged.

The interesting thing is that there still exists a family of equilibria in which the informed trades according to strategy (2.2). However such equilibria exist as long as the variance of the asset is neither too high nor too low. In particular, I will show that the following proposition holds.

Proposition 4 *As long as* $0 < \underline{w} < w < \overline{w} < 1$, *there exists a family of perfect Bayesian equilibria indexed with* $a \in \left[\max\left(\frac{12w-1}{5}, \frac{6w-1}{3-6w}\right), \frac{12w-1}{12w}\right]$ *where:*

(i) the trading strategy of the informed trader, $X(v)$ is such that

$$\begin{aligned} X(2) &= -X(-2) = 1 + a \\ X(1) &= -X(-1) = 1 - a \end{aligned} \tag{2.19}$$

(ii) for the set of aggregate trades that arise in equilibrium, $Z^* = \{-2-a, -2+a, -a, a, 2-a, 2+a\}$ the pricing strategy of the market maker reads:

$$P(z) = -P(-z), \quad P(z) = \begin{cases} 6w - 1 & z = a \\ 1 & z = 2 - a \\ 2 & z = 2 + a \end{cases} \tag{2.20}$$

(iii) for the set of out-of-equilibrium aggregate trades $z \in \mathbb{R} \backslash Z^*$, the pricing strategy of the market maker reads:

$$P(z) = \begin{cases} 2 & z > 1 \\ 2z & -1 \leq z \leq 1 \\ -2 & z < -1 \end{cases} \tag{2.21}$$

Proof. The market maker's problem. Market makers observe the aggregate order z. Given the equilibrium strategy of the informed trader, aggregate demand is a function that satisfies the following properties:

$$\begin{aligned} Z(u,v) &= \begin{cases} 2+a & v=2, u=1 \\ 2-a & v=1, u=1 \\ a & \{v=2, u=-1\} \text{ or } \{v=-1, u=1\} \end{cases} \\ Z(u,v) &= -Z(-u,-v) \end{aligned}$$

Accordingly, when observing $2+a$ and $2-a$, the market maker infers that the value of the asset is 2 and 1 respectively and sets the price accordingly. On the other hand, when the aggregate order is equal to a, the market maker cannot distinguish whether the asset is worth 2 or -1, and updates the prior distribution of the liquidation value of the asset as following:

$$\begin{aligned} \Pr(v &= -2|a) = \Pr(v=1|a) = 0 \\ \Pr(v &= 2|a) = \frac{\Pr(v=2)\Pr(a|v=2)}{\Pr(a)} = 2w \\ \Pr(v &= -1|a) = \frac{\Pr(v=1)\Pr(a|v=1)}{\Pr(a)} = 1-2w \end{aligned}$$

Based on this updated distribution, the market maker computes the expected value of the asset:

$$E(v|a) = 2w \cdot 2 + (1-2w)(-1) = 6w-1$$

Thus, the price response to equilibrium aggregate orders $2+a$, $2-a$ and a reads:

$$P(z) = \begin{cases} 6w-1 & z=a \\ 1 & z=2-a \\ 2 & z=2+a \end{cases}$$

Since the aggregate demand is an odd function, it easy to check that:

$$P(z) = \begin{cases} -6w+1 & z=-a \\ -1 & z=-2+a \\ -2 & z=-2-a \end{cases}$$

and $P(z) = -P(-z)$.

To complete the construction of the price function we are left with defining the way in which the market makers price aggregate orders that are not consistent with the equilibrium. In this respect, market maker's (out of equilibrium) beliefs about the information content of (out of equilibrium) aggregate trade are crucial. It is important to notice that in the present game, PBE does not imply any restriction on market maker's out-of-equilibrium beliefs, a part form the fact that the resulting price belong to the interval $[-2, 2]$. Accordingly, I assume the following out-of-equilibrium beliefs:

$$
\begin{aligned}
&\forall z > 1,\ z \neq 2-a, 2+a,\ \Pr(v = 2|z) = 1 \\
&\forall z \in [0,1],\ z \neq a,\ \Pr(v = 2) = \frac{z+1}{2},\ \Pr(v = -2) = \frac{1-z}{2} \\
&\forall z \in [-1,0],\ z \neq -a, -1,\ \Pr(v = -2) = \frac{z+1}{2},\ \Pr(v = 2) = \frac{1-z}{2} \\
&\forall z < -1,\ z \neq -2+a, -2-a,\ \Pr(v = -2|z) = 1
\end{aligned}
$$

It is then immediate to show that the out-of-equilibrium price function that arises from this beliefs is exactly given by (2.21).

The informed trader. Given the symmetry of the problem, which is given by the fact that the aggregate demand and the pricing strategy of the market maker are odd function, I can focus only on the buy side of the problem. Given the pricing strategy of the market makers, the prices that are expected by the informed trader when following his equilibrium strategy are given by:

$$
\begin{aligned}
\Phi(1+q) &\equiv E\left(P(1+q+u)\right) = \frac{1}{2}P(2+q) + \frac{1}{2}P(q) = \frac{6w+1}{2} \\
\Phi(1-q) &\equiv E\left(P(1-q+u)\right) = \frac{1}{2}P(-q) + \frac{1}{2}P(2-q) = 1-3w
\end{aligned}
$$

Accordingly, the payoffs that the informed trader expects in equilibrium reads:

$$
\begin{aligned}
B(2, 1+q) &= (1+q)\left(\frac{3-6w}{2}\right) \\
B(1, 1-q) &= 3w\,(1-q)
\end{aligned}
$$

Thus, we have to show that the following incentive compatibility constraints for the informed trader are satisfied:

$$
B(2, 1+q) \geq B(2, 1-q) \tag{2.22}
$$

$$
B(1, 1-q) \geq B(1, 1+q) \tag{2.23}
$$

that is

$$\begin{aligned}(1+q)\left(\frac{3-6w}{2}\right) &\geq (1-q)(1+3w)\\ 3w(1-q) &\geq (1+q)\left(\frac{1-6w}{2}\right)\end{aligned}$$

which are both satisfied as long as:

$$q \geq \frac{12w-1}{5} \tag{2.24a}$$

$$q \leq 12w-1 \tag{2.24b}$$

Further, there must be no incentives for the informed trader to deviate to any out-of-equilibrium buy order $x \in \mathbb{R}^{+}\backslash\{1+q, 1-q\}$, that is:

$$(1+q)\left(\frac{3-6w}{2}\right) \geq x\left[2-\Phi(x)\right] \tag{2.25}$$

$$3w(1-q) \geq x\left[1-\Phi(x)\right] \tag{2.26}$$

Given the out-of-equilibrium price strategy of the market maker, it is convenient to consider the case in which $x > 2$ and $0 < x \leq 2$ separately. Notice that when $x > 2$, $\Phi(x) = 2$ and conditions (2.25) and (2.26) are always satisfied for $0 \leq q \leq 1$. When $0 < x \leq 2$, $\Phi(x) = x$ and conditions (2.25) and (2.26) read:

$$\begin{aligned}(1+q)\left(\frac{3-6w}{2}\right) &\geq x(2-x)\\ 3w(1-q) &\geq x(1-x)\end{aligned}$$

A sufficient condition for the previous inequalities to be satisfied is that:

$$\begin{aligned}(1+q)\left(\frac{3-6w}{2}\right) &\geq 1\\ 3w(1-q) &\geq \frac{1}{4}\end{aligned}$$

which amounts to impose that

$$q \geq \frac{6w-1}{3-6w} \tag{2.27}$$

$$q \leq \frac{12w-1}{12w} \tag{2.28}$$

First, notice that condition (3.29) is stricter than condition (3.23). Therefore, conditions (3.20) through (3.29) can be summarized as follows:

$$\max\left(\frac{12w-1}{5},\frac{6w-1}{3-6w}\right)\leq q\leq\frac{12w-1}{12w}$$

Since we are restricting our attention on strategies of the informed trader that are strictly monotonic in the value of the asset, we must impose that

$$\begin{aligned}\frac{12w-1}{12w} &< 1\\ \frac{6w-1}{3-6w} &> 0\\ \frac{12w-1}{5} &> 0\end{aligned}\tag{2.29}$$

The first inequality is always satisfied for $0<w<\frac{1}{2}$. The second and third inequalities are simultaneously satisfied for $w>\frac{1}{12}$.

Furthermore, we must guarantee that

$$\max\left(\frac{12w-1}{5},\frac{6w-1}{3-6w}\right)\leq\frac{12w-1}{12w}\tag{2.30}$$

which is satisfied as long as

$$\begin{aligned}\frac{12w-1}{12w} &\geq \frac{12w-1}{5}\\ \frac{12w-1}{12w} &\geq \frac{6w-1}{3-6w}\end{aligned}$$

The first inequality is satisfied for $\frac{1}{12}<w<\frac{5}{12}$. The second one requires $-\frac{1}{48}\sqrt{33}+\frac{3}{16}<w<\frac{1}{48}\sqrt{33}+\frac{3}{16}$. Since $\frac{1}{48}\sqrt{33}+\frac{3}{16}<\frac{5}{12}$ and $\frac{1}{12}>-\frac{1}{48}\sqrt{33}+\frac{3}{16}$, the final condition on w to have **(2.30)** satisfied is that $\frac{1}{12}<w<\frac{1}{48}\sqrt{33}+\frac{3}{16}$.

Thus, conditions for monotonicity and consistency - **(2.29)** and **(2.30) -** are simultaneously satisfied as long as $0<\frac{1}{12}\equiv\underline{w}<w<\overline{w}\equiv\frac{1}{48}\sqrt{33}+\frac{3}{16}<\frac{1}{2}$. ■

2.4 Appendix

Inequality (2.11)

$$\begin{aligned}
-\frac{3(1-a)}{2(z+1)} &\leq -2-\frac{3(1+a)}{2(z-1)} \\
\frac{3(1-a)}{(z+1)} &\geq 4+\frac{3(1+a)}{(z-1)} \\
3(1-a) &\geq 4(z+1)+\frac{3(1+a)(z+1)}{(z-1)}
\end{aligned}$$

since we are focusing on $-1 < z < 1$, then $z-1 < 0$. Therefore the last inequality reads:

$$3(1-a)(z-1) \leq 4(z^2-1)+3(1+a)(z+1)$$

and can be further simplified as follows:

$$3az \geq -2z^2-1 \tag{2.31}$$

Since we are focusing on the case in which $a \in (0,1)$, it is trivial to check that **(2.31)** is satisfied for every $0 \leq z < 1$. Consider now the case $-1 < z < 0$. Then, **(2.31)** can be written as follows:

$$a \leq \frac{-2z^2-1}{3z}$$

We want this inequality to be satisfied for every value of $-1 < z < 0$, keeping in mind that $a \in (0,1)$. Notice that the minimum value of $\frac{-2z^2-1}{3z}$ in $z \in (-1,0)$ reads $\frac{2\sqrt{2}}{3}$. Thus, the previous condition is satisfied as long as $a \leq \frac{2\sqrt{2}}{3}$.

Inequality (2.13):

$$\begin{aligned}
2-\frac{3(1+a)}{2(z+1)} &\leq -2-\frac{3(1+a)}{2(z-1)} \\
4-\frac{3(1+a)}{2}\left(\frac{1}{z+1}-\frac{1}{z-1}\right) &\leq 0 \\
4-\frac{3(1+a)}{2}\left(-\frac{2}{z^2-1}\right) &\leq 0 \\
4+\frac{3(1+a)}{z^2-1} &\leq 0
\end{aligned}$$

since $-1 < z < 1$, we have that $z^2-1 < 0$. Therefore the last inequality can be written as follows:

$$\begin{aligned} 4\left(z^2-1\right)+3(1+a) &\geq 0 \\ 4z^2-1+3a &\geq 0 \\ a &\geq \frac{1-4z^2}{3} \end{aligned}$$

We want this inequality to be satisfied for every value of $-1 < z < 1$, keeping in mind that $a \in (0,1)$. The maximum value of $\frac{1-4z^2}{3}$ is $\frac{1}{3}$. Thus, the last inequality is satisfied for every value of $z \in (-1,1)$ as long as $a \geq \frac{1}{3}$.

Inequality (2.14)

$$\begin{aligned} 2-\frac{3(1+a)}{2(z+1)} &\leq -\frac{3(1-a)}{2(z-1)} \\ 2-\frac{3}{2}\left(\frac{1+a}{z+1}+\frac{1-a}{z-1}\right) &\leq 0 \\ 2-3\left(\frac{za-1}{z^2-1}\right) &\leq 0 \end{aligned}$$

since $-1 < z < 1$, $z^2-1 < 0$ and therefore:

$$\begin{aligned} 2\left(z^2-1\right)-3za+3 &\geq 0 \\ 2z^2-3za+1 &\geq 0 \\ 3az &\leq 2z^2+1 \end{aligned} \tag{2.32}$$

Since we are focusing on the case in which $a \in (0,1)$, it is trivial to check that **(2.32)** is always satisfied for every $-1 < z \leq 0$. Finally, consider the case in which $0 < z < 1$. In this case, **(2.32)** holds as long as:

$$a \leq \frac{2z^2+1}{3z}$$

Since the minimum value of $\frac{2z^2+1}{3z}$ in $z \in (0,1)$ is $\frac{2\sqrt{2}}{3}$, the last inequality is guaranteed as long as $a \leq \frac{2\sqrt{2}}{3}$.

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Chapter 3

Mutual Funds, Career Concerns and Trade Volume

3.1 Introduction

Most of the standard models in the asset pricing literature assign no role to financial institutions. In these models, it is assumed that individuals invest their savings directly into stock exchanges and financial institutions are regarded as a veil between investors and firms with no real effects on market outcomes. However, casual information suggests both that financial institutions are assuming increasing importance in modern financial markets and that very often these institutions are guided by incentives that are not fully considered in the standard models of asset pricing.[1]

In the present paper, I will focus on one particular financial institution, i.e., mutual funds, and on one particular "non-standard" incentive that might drive mutual fund behavior, i.e., career concerns. I will show that the presence of career concerns is likely to affect the behavior of portfolio managers and eventually the aggregate volume of trade expected to occur on an exchange market.

The mutual fund industry has experienced a remarkable expansion in the past few decades. In the U.S., mutual funds' holdings of corporate equities have increased from \$2.9 billion in 1950 to \$2,005 billion in 2002 (NYSE Figures&Facts, 2005). In the last two decades, mutual funds have increasingly become the main vehicle that channels investors' savings into stock exchanges. From just 1999 to 2005, the number of U.S. households investing in stock mutual funds increased by 9.3 million. By the end of 2005,

[1] For a convincing argument in favour of a natural extension of asset pricing models to take into account the role of financial institutions, see Allen (2001).

90% of all equity-owning households were reported to hold stock mutual funds (ICI and SIA, 2005).[2]

Given the size of the mutual fund industry, it is very likely that the behavior of its key players, fund managers, may affect the features of the financial market in which they are active. Not surprisingly, paralleling the expansion of the mutual fund industry, the financial literature has devoted increasing attention to the behavior of fund managers. In particular, several recent works have pointed out the relevance of career (or reputation) concerns as a strong incentive that may condition the trading strategies of portfolio managers. Chevalier and Ellison (1997) present empirical evidence that career concerns have a perverse impact on fund managers' behavior, leading managers to indulge in excessive risk taking in order to enhance their reputation. This relevance comes from the facts that investors tend to shift their money towards funds that have performed well in the recent past (Ippolito, 1992 and Chevalier and Ellison, 1997) and managers typically receive a fixed fraction of the value of the assets under management. In particular, Chevalier and Ellison show that even when mutual-fund managers do not receive explicit incentive fees, an implicit performance-based compensation structure still arises with periodic proportional fees as a result of the fact that the net investment flow into mutual funds varies in a convex fashion as a function of recent performance. Furtermore, Heinkel and Stoughton (Review of Financial Studies, 1994) find some empirical regularities regarding the form of compensation between clients and fund managers. For example, the overwhelming majority of contracts can be terminated on very short notice and virtually all major institutional clients pay a professional evaluation firm to report ex-post performance that is then used to monitor the manager's performance on a regular basis.

In the present paper, I propose a theoretical model that analyzes the effect of portfolio managers' career concerns on their incentives to trade in an order-driven market in the style of Kyle (1985). The specific analysis is based upon a numerical example presented in Biais and Rochet (1997), who propose a version of Kyle (1985) in which both the distribution of the liquidation value of the risky asset and that of noise trade are discrete. I build on Biais and Rochet's static setup to model a dynamic two-round-of-trade financial market in which at the beginning of each round an uninformed investor chooses whether to delegate trade to a fund manager or stay out of the market. It is assumed that there exists a large population of fund managers composed by two types of

[2] Furthermore, many private individuals who are reported to invest directly in fact rely on professional advisors. In 2005, 77% of investors holding equities outside retirement plans at work usually purchased them through professional financial advisers (ICI and SIA, 2005).

managers: good managers, who are perfectly informed about the true liquidation value of the asset, and bad managers, who are instead completely uninformed. The type of a manager is assumed to be manager's private information. Investors know only the composition of managers' population and thus the probability that a manager randomly drawn from this population is informed.

All the analysis is conducted under the assumption of risk neutrality and of an exogenous linear delegation contract according to which a manager receives a fixed fee plus a fraction of the delivered return. When the delegation contract lasts for all the two rounds of trade (long-term contracting), managers' career concerns are trivially eliminated from the model. Thus, long-term contracting represents the baseline framework against which to compare alternative frameworks where career concerns are at play. In particular, I consider the case in which the delegation contract can be terminated at the end of the first round of trade and the investor can choose whether to renew delegation to the incumbent manager or fire him and possibly hire a new manager (short-term contracting). Clearly, under this contractual arrangement career concerns on the part of managers are likely to arise. Indeed, as long as staying in the market for an additional round of trade is appealing enough to the incumbent manager, he will act as to maximize his chances of being retained. In the particular, since investors are better off when delegating trade to an informed manager, any investor will try to infer the type of the incumbent manager based on the way the manager behaved in the first round of trade: if the manager's performance suggests that the manager is not very likely to be informed, he will be fired and possibly substituted with a new one. Thus, managers will in turn act in ways that suitable to induce the highest probability of being perceived as well-informed. The analysis leads to the following main findings:

1) Career concerns lead portfolio managers to trade even without information, that is, even when the expected return from trade is negative. This result stems form the fact that in equilibrium good managers always trade in order to exploit their information. Accordingly, the absence of trade immediately signals that the manager is bad, implying his instant dismissal. Thus, whenever staying in the market for an additional round of trade turns out to be appealing, an uninformed manager finds himself in the situation of trading off the negative expected return from trade with the expected benefits that this trade might bring about in terms of enhancing the probability of being retained. In particular, bad managers trade according to the strategy that maximizes their chances to be perceived well informed, which amounts to randomly choose the trade order that is more likely to reveal correct among all the orders that are consistent with the strategy

of an informed manager.

2) Career concerns lead to an increase in the expected volume of trade. On the one hand, since trading with no information has an expected negative return, a bad manager never trades in the absence of career concerns. On the other hand, the trading strategy of a good manager is not affected by career concerns: having complete information, a good manager always makes the correct choice and thus is always retained. Hence, an immediate corollary to result 1) is that career concerns lead to an increase in the expected volume of trade, through the contribution of uninformed managers' churning.

3) As the volatility of the asset increases, uninformed managers trade larger quantities. This result comes from the fact that the in equilibrium the strategy of a good manager is monotonically increasing in the value of the asset and a bad manager trades by mimicking the good manager's order with the highest odds to reveal correct ex-post. Thus, when extreme values of the asset become more likely, it is also more likely that a good manager will place a large order and the bad manager will behave accordingly.

4) Since under risk neutrality the trading strategy of informed fund managers is not affected by changes in asset volatility, an empirical implication of the previous result is that in the presence of career concerns the expected volume of trade is positively correlated with asset riskiness.

The present contribution is closely related to the works of Trueman (1988) and Dasgupta and Prat (2006). Both works present theoretical models in which fund managers differ in the degree of their private information about market fundamentals and show that the presence of career concerns induces managers to trade even when uninformed (i.e., "to churn").

While Trueman carries out the analysis within a partial equilibrium framework, Dasgupta and Prat model the financial market as a quote-driven market à la Glosten and Milgrom (1985) in which market makers set bid and ask quotes at which fund managers can buy or sell one unit of a risky asset. The general equilibrium approach taken by Dasgupta and Prat allows them to fully characterize the conditions for delegation and analyze the feedback that churning has on equilibrium prices and volume, showing that trade by uninformed managers leads to non-fully informative prices and high trade volume.

From a formal point of view, the present work provides an extension of Dasgupta and Prat's (2006) result to an alternative trading mechanism, namely an order-driven market that operates as a call auction, with fund managers able to buy or sell any desired amount of the risky asset at the market-clearing price set by the market maker.

The last decade has witnessed a series of upheavals in the ways exchanges are organized, with many markets moving from dealership systems to auction (or hybrid) systems.[3] Most of the on-going debates about dealer systems vs auction systems are in large part attributable to these innovations in trading protocols. In general, theory suggests that multilateral trading systems (such as single-price call auctions) are efficient mechanisms to aggregate diverse information. Consequently, there is interest in how call auctions operate and whether such systems can be used more widely to trade securities.

An advantage of the specific modelling choice adopted in the present paper is that it allows me to investigate the relationship between trade volume and asset volatility, an issue that cannot be addressed in the model developed by Dasgupta and Prat (2006), where managers' are restricted to trade only one unit of the asset.[4]

It is worth noting that the present contribution, as well as the works by Trueman (1988) and Dasgupta and Prat (2006) bear some relation to the financial literature on the "trade volume puzzle." In standard models of trade with rational agents, individuals trade because of information advantages or because they have been hit by an exogenous shock that forces them to trade (or both). This latter type of trade has been named "noise trade" or "uninformed trade" exactly because it is not driven by any information about the underlying asset that is traded. On strictly theoretical terms, in any model with rational traders, the presence of noise trade is a necessary condition for trade based on information, which would otherwise conflict with the no-trade theorem. The classical justifications given by the literature for the presence of noise trade are represented by unexpected liquidity shocks and hedging needs. At an empirical level, there is broad agreement about the fact that only a small fraction of the total volume of trade takes place either for liquidity or hedging purposes. Thus, in keeping with classical explanations, the remaining volume of trade should be due to information. However, it seems highly implausible that a small amount of noise trade can support the remaining large

[3]In October 1997, following demands from the Securities and Investments Board (in turn due to lobbying from the institutional users of the trading systems), the London Stock Exchange changed its trading system in the most liquid securities from a dealership system (SEAQ) to an auction system of limit orders (SETS). London is not alone in changing its trading system. In 1997, the Deutsche Bourse adopted the Xetra electronic order book system. In 2001, the Amsterdam, Brussels, Lisbon, LIFFE and Paris stock exchanges merged to form Euronext, a pan-European exchange, using a single order-driven trading platform based on the French NSC electronic order book system. And NASDAQ, the US technology stock exchange, set up a pan-European technology exchange in 2002 based on the SuperMontage trading platform, a fully integrated central limit order book and quote-driven montage facility and execution system.

[4]Truman (1988) finds a positive relation between the probability that an uninformed manager engages in trade and asset volatility. However, his partial equilibrium framework prevents him to discuss the impact on trade volume.

fraction of informative trade.[5,6]

The distorted effects of reputation on the portfolio choices of fund managers are also considered by Huberman and Kandel (1993). They present an adverse selection model where a manager's objective is the sum of the expected investment outcome plus an (expected) reward entailed by the market's inference regarding the manager's ability. Fund managers have to decide which fraction of the portfolio will be invested in a risky asset and make use of the portfolio weights to signal their ability. Huberman and Kandel show that there exists a unique separating equilibrium in which good managers distort their first best investment decision in order to distinguish themselves from bad managers. The distortion can be both in the direction of an overinvestment or of an under-investment in the risky asset. Thus, the model seems to better explain excessive risk taking rather that excessive trading. Furthermore, they frame the analysis in a partial equilibrium setup.

Huddart (1999) extends Huberman and Kandel (1993) to a set-up in which the reward to reputation is endogenous and both advisors and investors are risk-averse. The general conclusion is that either an increase in the degree of risk aversion by the advisor or in the fee makes separation easier to support. Notice that Huddart retains the partial equilibrium approach and the related limitations of Trueman's and Huberman-Kandel's analysis.

The paper is organized as follows. In section 2, I introduce delegation in the static framework taken from Bias and Rochet (1997) and characterize the equilibrium strategies of investors, fund managers and market makers. In section 3, I extend the analysis to a two-period dynamic setup and compare the equilibrium in which fund managers have career concerns with the equilibrium in which career concerns are absent. In section 4, I analyze how the presence of career concerns affects the trading behavior of fund managers in contexts characterized by different degrees of asset volatility. In section 5, I draw conclusions and discuss the results of the paper.

[5] Hence, the appeal to theories based on the irrationality (or limited rationality) of investors.

[6] Dow and Gorton (1997) present a static (one-round-of-trade) model with complete contracts in which fund managers' churning does not originate from career concerns but as a consequence of the optimal contract that is implemented in order to screen good managers from uninformed ones. The optimal contract is designed such that, in equilibrium, only good managers trade. In fact, however, sometimes good managers engage in churning.

3.2 The basic model

In this section, I will first present the way in which the financial market for the risky asset is modelled. Basically, the structure of the market is taken from Rochet and Vila (1994). This is a *discrete and static* variant of Kyle (1985) in which a perfectly informed trader (the insider) trades along with noise traders and market makers, but both the distributions of the liquidation value of the risky asset and of noise trade are assumed to be discrete. I will take this structure and introduce the delegation process between investors and mutual funds. In the next section, I will consider a dynamic version of the model and reputation issues will be introduced as well.

3.2.1 Structure of the market

Consider the financial market for a risky asset. Denote with v the liquidation value of the asset and assume that v can take on values $-2, -1, 1, 2$ with equal probability $\frac{1}{4}$.

There are four classes of agents: noise traders, fund managers, investors and market makers. For simplicity, they are all assumed to be risk neutral.

Noise traders. I denote with u the total order from noise traders and assume that u can take on values 1 and -1 with equal probability $\frac{1}{2}$.[7] Furthermore, u and v are assume to be independently distributed.

Fund managers. I assume that there is a large population of fund managers. There are two types of fund managers in the population, indexed with $i = \{g, b\}$. Types g are perfectly informed about the true liquidation value of the risky asset, while types b are completely uninformed. I assume that a fund manager knows his own type and that this is private information. It is instead common knowledge that $\Pr(i = g) = \theta$. If a fund manager is hired, he decides the order $x \in \mathbb{R}$ to trade.

Investors. Investors are completely uninformed about the true liquidation value of the asset and cannot invest directly.[8] An investor is given the choice either to refrain from trading or to delegate trade to a fund manager.

Market makers. Finally, there is a large population of market makers who compete à la Bertrand to set the price p at which the asset is traded. Market makers observe only the aggregate market order $z = x + u$ and not the single orders x and u. Based on

[7] A negative order is to be interpreted as a supply of the asset.

[8] This is done for simplicity and is without loss of generality Giving the investor the possibility of trading directly does not alter our results. Indeed, since the investor is assumed to be completely uninformed, the choice of investing directly produces zero payoffs in expectation, that is the same expected payoff obtained by refraining from trading.

the observation of z, the market maker sets the price p at which trade takes place. The assumption of competition à la Bertrand coupled with that of risk neutrality implies that each market maker will set a semi-strong efficient price:

$$p = E(v|z = x + u) \tag{3.1}$$

Basically, (3.1) implies that market makers get zero profits in expectation, reflecting competition à la Bertrand.

All previous agents are assumed to be risk neutral.

The delegation contract. I assume that if an investor chooses to delegate trade, he hires the manager according to the following exogenously specified linear contract. Denote with $R(x) = x(v-p)$ the return that a fund manager delivers by trading quantity x when the liquidation value of the asset is v and the price set by the market maker is p. Then, the fund manager gets the fraction $\alpha \in (0, 1)$ of the return delivered plus a fixed fee $\beta > 0$. Both α and β are exogenous (i.e. they are parameters of the game). Basically, a fund manager trading x gets $\alpha R(x) + \beta$, while the investor who has hired him gets $(1 - \alpha)R(x) - \beta$.[9] There are a couple of considerations that are worth to be done.

First, in the present setup, the linear contract under consideration does not represent a performance-related compensation scheme. Performance-related fees refers to *relative* performance, that is, they are fees that the manager receives (pays) for over-performing (under-performing) a given *benchmark*. They do not refer to absolute performance, which is instead the case of the linear contract $\alpha R(x) + \beta$ at hand. In fact, this linear contract (with $\alpha > 0$) represents pretty well the form of payment used by large in the industry. Indeed, most of the mutual funds charge a fee that is given by a fixed percentage of the total amount of money under management. Now, let this percentage be τ. Then, if the total funds initially received by the manager are equal to F, after management they will be equal to $F(1 + r)$, where r is the percentage return delivered by the manager. Accordingly, the payment from the investor to the manager is equal to $\tau F(1 + r) = \tau F + \tau Fr$, from which it is clear that there is a fixed payment (τF) plus a payment that is related to *absolute* performance (τFr).

Second, in the dynamic extension of the basic model presented here, the fixed component β plays a key role in determining the result that bad managers trade with positive probability when career concerns are at play. Indeed, career concerns are represented

[9] Notice that noise traders get $\pi_n = u(v - p)$, while market makers $\pi_{mm} = -z(v - p)$.

by the concern of a manager of being fired and giving up future profits. As it will be clear later, in the present model β represents the only source of future profits for a bad managers. In other words, while the retaining decision of the investor represents the implicit incentive related to having a good reputation, β is the explicit incentive that makes reputation effects be effective.

Timing. At the beginning of the trading session, the following events take place. First, an investor is randomly selected from the pool of all investors and given the choice of refraining from trading or hiring a fund manager. If the fund manager is hired, he observes his own type and then chooses the quantity x he desires to trade on behalf of the investor. At the same time, noise trade u realizes. The fund manager does not observe the realization of u. Once orders x and u have been formed, they are submitted to the market maker who sets the price according to (3.1) and trade takes place. At the end of the trading session, the true liquidation value of the asset is realized and payoffs are distributed. Of course, payoffs depend on the terms of the delegation contract between the investor and the manager.

3.2.2 Equilibrium analysis

In the absence of career concerns, a set of equilibria is described by the following proposition.

Proposition 5 *Given the previous assumptions and market structure, for any pair* (α, β) *such that* $\alpha \in (0,1)$ *and* $\beta \in \left(0, \frac{3}{4}\theta(1-\alpha)\right]$ *there exists an uncountable family of perfect Bayesian equilibria, indexed by* $q \in \left[\frac{2}{5}, \frac{2}{3}\right)$ *such that:*

i) An investor always hires a fund manager

ii) The trading strategies of good and bad managers are respectively given by:

$$X_{g,q}(v) = \begin{cases} 1+q & \text{when } v = 2 \\ 1-q & \text{when } v = 1 \end{cases}, X^*_{g,q}(-v) = -X^*_{g,q}(v) \tag{3.2}$$

$$X^*_{b,q} = 0 \tag{3.3}$$

iii) The pricing strategy of market makers is such that for the set of equilibrium *trades* $z \in \Psi = \{-2-q,\ -2+q,\ -1,\ -q,\ q,\ 1,\ 2-q,\ 2+q\}$,

$$P_q(z) = \begin{cases} 0 & when\ z = 1 \\ \frac{1}{2} & when\ z = q \\ 1 & when\ z = 2-q \\ 2 & when\ z = 2+q \end{cases},\ P_q^*(-z) = -P_q^*(z) \tag{3.4}$$

while for the set of out-of-equilibrium trades $z \in \mathbb{R} \setminus \Psi$,

$$P_q(z) = \begin{cases} 2 & \forall z > 1 \\ 2z & \forall z \in [0,1] \end{cases},\ P_q(-z) = -P_q(z) \tag{3.5}$$

The formal proof of proposition 1 can be found in the appendix. The intuition of the previous result is straightforward. Due to the presence of noise trade, prices do not fully reflect private information. Hence, good managers with complete information trade whenever they are hired by an investor. Being aware of the fact that with positive probability an informed manager is active in the market, market makers set semi-strong efficient prices to protect themselves from the presence of asymmetric information. Consequently, a manager with no information always expects a negative return on his trade and thus never trades. Since bad managers do not trade while good managers always trade and do it under complete information, on average the return delivered by a manager is positive. Under the linear delegation contract described above, the investor enjoys the fraction $(1-\alpha)$ of this average return, while paying a fixed fee equal to β for delegating trade to a manager. Since an investor has the option of staying out of the market, he will delegate trade if and only if the expected payoff of delegation is positive. In the equilibria identified by proposition 1, the average return of the mutual fund industry is equal to $\frac{3}{4}\theta$. Thus, as long as $\beta < \frac{3}{4}\theta(1-\alpha)$, the cost of hiring a manager is lower that its benefits and delegation always takes place.

The bounds on q guarantee that the optimal trading strategy of the good manager is strictly monotonic with respect to his signal and "non-perverse", that is, the good manager buys when observing positive values of the asset and sells when observing negative values. Notice that these two properties would in general be guaranteed by simply imposing that $0 < q < 1$. In fact, the stricter condition required by proposition 1 comes from the incentive compatible constraints of the good manager and thus also guarantees that the good manager has not any incentive to deviate from the prescribed equilibrium strategy.[10] There is a simple intuition for why q must be not too small. In equilibrium, the "scale of trade" does not affect prices in this model: changing q does not change the prices at which orders $1-q$ and $1+q$ are respectively liquidated. However, choosing $1+q$ will lead on average to higher prices than choosing $1-q$. When informed managers observe that the asset is worth 2, they might be tempted to trade $1-q$ in order to induce a lower price and make a higher profit on each unit of the asset. To avoid this occurrence, q must be high enough to make $1+q$ sufficiently large so as to offset the lower margin that informed mangers make on each unit when trading $1-q$. The same logic applies to explain why q must be not too high. Given the price schedule for out of equilibrium aggregate orders, the good manager could be tempted to place very small orders to induce low prices. Since changing q does not change the prices at which orders $1-q$ are liquidated, the size of any order $1-q$ must be large enough to guarantee that equilibrium profits be sufficiently high to make such deviations not appealing.

It is useful to remind that proposition 1 identifies *a* family of perfect Bayesian equilibria of the static game described above. First of all, the specification of the out of equilibrium beliefs and of the consequent out of equilibrium price function is crucial in determining the bounds of q within which the class of equilibria of proposition 1 is defined.[11] Furthermore, proposition 1 focuses on good managers' "symmetric" strategies, that is, strategies represented by (3.2). If not explicitly specified, in what follows I will focus only on perfect Bayesian equilibria with symmetric strategies for good managers. This keeps the analysis more tractable without losing anything from a qualitative point of view.[12]

3.3 A Dynamic framework

Let me consider the simplest dynamic extension of the basic model analyzed in the previous section. I will assume that the risky asset can now be traded in two sequential rounds of trade $s = 1, 2$. At the end of each round, the asset pays some dividends, which I denote with v_1 and v_2 respectively. For simplicity, I assume that v_1 can take

[10] See conditions (3.26) and (3.32) in the proof of proposition 1

[11] It can be shown that there always exists *a* continuous price function that prevents the good manager from deviating from his equilibrium strategy X_g^* as long as q takes values in subsets of $\left[\frac{1}{3}, \frac{2\sqrt{2}}{3}\right]$. Thus, $\left[\frac{1}{3}, \frac{2\sqrt{2}}{3}\right]$ is also the largest set of values that q can in principle assume in an equilibrium where the good manager trade according to X_g^* (proof available under request).

[12] See appendix A for more details about equilibria where good managers use "non-symmetric" strategies.

on values $-2, -1, 1, 2$ with equal probability $\frac{1}{4}$ and that v_1 and v_2 are *i.i.d.*[13] In each round of trade, noise trade can take on values -1, 1 with equal probability $\frac{1}{2}$. Let $u_1$and u_2 denote respectively noise trade at $s = 1$ and 2 and assume that u_1 and u_2 are *i.i.d.* Furthermore, u_1, u_2, v_1 and v_2 are assumed to be independently distributed.

In each round of trade $s = 1, 2$, the trading protocol is the same as that described for the static model presented in the previous section. A fund manager active at round s observes his own type and accordingly chooses the quantity x_s to trade on behalf of the investor who has hired him. At the same time, noise trade u_s realizes (the active manager does not observe the realization of u_s). Then, orders x_s and u_s are submitted to a market maker who observes only the aggregate order $z_s = x_s + u_s$ and accordingly sets a semi-strong efficient price $p_s = E(v_s|z_s)$.[14] At the end of each round of trade $s = 1, 2$, dividends v_s realize.

I assume that at the beginning of the first round of trade, an investor is randomly drawn from the pool of all investors and given the choice to delegate trade to a fund manager or to stay out of the market. The delegation contract is exogenously specified and given by the linear contract described in the previous section. In order to evaluate the effects produced by the presence of career concerns, I will compare two different contractual setups. In the first setup, the linear delegation contract that is signed at the beginning of $s = 1$ lasts until the end of $s = 2$ (*long-term contracting*). It is apparent that under this contractual arrangement, there is no scope for career concerns since the manager hired at the beginning of the first round of trade is sure to remain in charge in the second round as well.[15] In the second setup, the delegation contract is assumed to last for only one round of trade (*short-term contracting*). Furthermore, it is assumed that at the end of $s = 1$, the investor observes the action taken by the incumbent manager, compares it with the realized value of the dividend and consequently chooses wether to

[13] The idea is that between period 1 and 2 there occurs an event unpredictable at the beginning of period 1 and relevant for the financial situation of the company that issued the asset.

[14] In a dynamic model with more than two periods, say $s = 1, 2, ...T$, it is important to specify what the market maker can observe (on top of v_s) before setting prices at $s + 1$. Indeed, the way the market maker sets prices in a given round of trade, say $s + 1$, depends on his belief about the quality of the manager trading at $s + 1$. And this belief is formed based on all the information the manager has collected up to $s + 1$. This assumption is not necessary in a two-period model. Indeed, as we will see soon, the equilibrium is such that only good managers do trade in the second and last period. Therefore, the market maker knows that the manager active in the last round of trade is good (notice that, in fact, the market does not close because of the presence of noise trade u).

[15] Alternatively, career concerns could have been eliminated by assuming that the decision of an investor to retain or to fire a manager is not affected by manager's performance in the first round, but depends on some other exogenously unspecified reasons. For example, by assuming that the incumbent manager is retained with an exogenous probability equal to $\gamma \in [0, 1]$ (see Dasgupta and Prat 2006).

retain the incumbent manager or fire him.[16] If the incumbent manager is retained, he keeps his own type.[17] If instead the incumbent manager is fired, the investor can further choose either to hire a new manager by drawing him from the existing pool of managers or to stay out of the market.

For simplicity, I assume that there is no discounting.

3.3.1 No career concerns

Assume that the linear delegation contract that is possibly signed between an investor and a fund manager at the beginning of $s = 1$ lasts until the end of $s = 2$ (*long-term contracting*). In particular, *in each round of trade* the investor agrees to pay the manager a fixed fee $\beta > 0$ plus a fraction $\alpha \in (0, 1)$ of the return delivered by the manager. The following proposition characterizes a family of perfect Bayesian equilibria of the dynamic game described above in the case of long-term contracting.

Proposition 6 *Under long-term contracting, for any pair (α, β) such that $\alpha \in (0, 1)$ and $\beta \in \left(0, \frac{3}{4}\theta(1-\alpha)\right]$ there exists an uncountable family of perfect Bayesian equilibria, indexed by $q \in \left[\frac{2}{5}, \frac{2}{3}\right)$ such that delegation takes place and in each round of trade $s = 1, 2$, managers and market makers behave as prescribed by proposition 1*

I will not formally prove the previous result, since it simply amounts to iteratively apply the prove of proposition 1 by backward induction. Notice that delegation takes as long as β is low enough to guarantee that the costs of hiring a manager through the two rounds of trade are less than the its benefits. It can be easily shown that if the investor hires a good manager, the expected return from his trade is equal to $\frac{3}{4}$ in each round. If instead the manager is bad, the return delivered in each round is 0. Thus, the total return expected from an average manager hired at the beginning of $s = 1$ is equal to $\theta\left(\frac{3}{4} + \frac{3}{4}\right) + (1-\theta)\,0 = \frac{3}{2}\theta$ and the expected benefits from delegation $(1-\alpha)\,\frac{3}{2}\theta$. On the

[16] Remember that at the end of $s = 1$, v_1 is realized and observed by all market participants. Investors can then compare it with the performance delivered by their manager and accordingly form a belief about the quality of the manager. As we will see, I will formally model this updating process by letting investors compare v_1 with the order x_1 submitted by the manager, rather than with his performance. In fact, if we assume that investors can also observe the price that has been paid/recieved by the manager, in equilibrium there is a one-to-one relation between performance and trade order.

[17] The rationale behind this assumption is that if a fund manager is well informed about a given company, then he is likely to receive valuable information whenever a new relevant event that affects the company occurs (remember that in the present model there is only *one* asset). Clearly, this is not without loss of generality. One could easily pick "transition probabilities" on the manager's type to reverse or invalidate career concerns altogether.

other hand, the costs of hiring a manager for the two rounds of trade are equal to 2β. Thus, as long as $\beta < \frac{3}{4}\theta(1-\alpha)$ delegation takes place.

Trade volume without career concerns

Let me consider the trade volume that is expected in a financial market where career concerns of portfolio managers are absent. In each round of trade, the volume of trade expected from noise traders amounts to 1:

$$E(V_1^u) = E(V_2^u) = \frac{1}{2}|1| + \frac{1}{2}|-1| = 1$$

Thus, the total volume of trade coming from noise traders is equal to 2. Let me now consider the expected contribution of fund managers in the two rounds of trade. Based on managers' equilibrium strategies outlined in proposition 2, we can easily compute:

$$E(V_1^i + V_2^i) = \theta E(V_1^i + V_2^i|i = G) + (1-\theta)\, E(V_1^i + V_2^i|i = B) = 2\theta$$

where the result comes from the fact that:

$$\begin{aligned} E(V_1^i + V_2^i|i &= G) = 2\left(\frac{1}{4}|1+q| + \frac{1}{4}|1-q| + \frac{1}{4}|-1+q| + \frac{1}{4}|-1-q|\right) = 2 \\ E(V_1^i + V_2^i|i &= B) = 2(1-\theta)0 = 0 \end{aligned}$$

Accordingly, the aggregate volume of trade that is expected to take place in the market reads:

$$E(V_1 + V_2) = 2 + 2\theta \tag{3.6}$$

3.3.2 Career concerns

Consider now the case in which the delegation contract lasts for only one round of trade (short-term contracting). Furthermore, assume that at the end of the first round of trade, the investor can decide whether to retain the incumbent manager or fire him and possibly hire a new one.

Loosely speaking, the investor makes this decision evaluating the behavior of the incumbent manager in the first round of trade: if at the eyes of the investor the manager has done a good job, he will be retained; if his performance is judged to be poorer than that of an average manager, he is fired and possibly substituted with a new manager.

Formally, I assume that at the end of $s = 1$, the investor can observe both the realized value of the dividend, v_1 and the trade choice of the incumbent manager, x_1. Hence, given these observations and the *equilibrium* strategies of good and bad managers, the investor forms a belief that the incumbent manager is good. Let me denote with $\widehat{\theta}$ this belief, with

$$\widehat{\theta} = \Pr(i = g|v_1, x_1)$$

Since at the end of $s = 1$ the investor has the option of replacing the incumbent manager with a new manager (who has probability θ of being good), the incumbent will be retained if and only if $\widehat{\theta} > \theta$, that is, if and only if according to the investor he is more likely to be good than an average manager[18] Thus, as long as staying in the market for an additional round of trade turns out to be appealing to a manager, this decisional process of the investor creates career concerns on the part of managers. Indeed, any manager trading in the first round is aware of the fact that his trade choice will affect his probability of being retained; accordingly, he will be more inclined to make those trade choices that are likely to increase the probability of being retained. As we will see, in the case of bad managers, these choices are not optimal.

In our simplified world, good managers have complete information about the true liquidation value of the asset. This implies that good managers expect a positive return from trading and thus *always* trade. Furthermore, complete information also implies that they always make the correct choice when trading and accordingly are always retained with probability 1 (we could state that in fact good managers do not face career concerns. On the other hand, bad managers are completely uninformed about the true liquidation value of the asset. Thus, the return that they expect on trade is always negative and the optimal choice would be that of not trading. However, since good managers always trade, the absence of trade would immediately signal to the investor the fact that the manager is bad and imply hat the manager is fired with probability 1. Therefore, the only chance that a bad manager has to be retained is that of engaging in uninformative trade in the hope of striking a good performance and being perceived as good by the investor.

Notice that a bad manager will in fact engage in uninformative trade if and only if the option of staying in the market for an additional round of trade turns out to be sufficiently attractive. In the second and *last* round of trade there is no scope for career concerns and therefore bad managers never trade. Accordingly, the profits of a bad manager in the second round of trade amount to the fixed fee β. Hence, a bad manager who is active in the first round has to trade off the negative expected return $\alpha R_b^e(x)$ of

[18]It is important to stress that this decisional rule is *endogenously* determined in equilibrium.

his uninformative trade with the expected benefits that this trade might bring about in terms of enhancing the probability of enjoying future profits β. It is then clear that career concerns lead to (non-optimal) uninformative trade only for a set of (short-term) delegation contracts characterized by values of α relatively small to those of β.

In the following proposition, I formally characterize a family of perfect Bayesian equilibria in which delegation takes place as a rational choice of investors and bad managers trade without information in the first round as a consequence of career concerns.

Proposition 7 *Assume short-term contracting. Assume further that at the end of $s = 1$ the investor can fire the incumbent manager and substitute him with a new one. For any pair of contractual parameters (α, β) such that $\alpha \in \left(0, \frac{5}{9}\right)$ and $\beta \in \left(\frac{1}{3}\frac{\theta^2}{2-\theta}, \frac{1}{3}\theta\right)$, the two-round trade game described above admits an uncountable family of perfect Bayesian equilibria indexed by $q \in \left[\frac{2}{5}, \frac{2}{3}\right)$ such that:*

i) delegation takes place in both rounds of trade. At the end of the first round, the investor retains the incumbent manager if and only if

$$\widehat{\theta} > \theta \tag{3.7}$$

where $\widehat{\theta} = \Pr(i = g|v_1, x_1)$, otherwise he fires him and hires a new manager.

ii) In both rounds, good managers trade according to the following strategy:

$$\begin{aligned} X^*_{s,g,q}(v_s) &= \begin{cases} 1+q & \text{when } v_s = 2 \\ 1-q & \text{when } v_s = 1 \end{cases} , \; s = 1, 2 \\ X^*_{s,g,q}(-v_s) &= -X^*_{s,g,q}(v_2) \end{aligned} \tag{3.8}$$

iii) bad managers do not trade in the second round, while they do trade in the first round according to the following (mixed) strategy:

$$X^*_{1,b,q} = \begin{cases} 1-q & \text{with probability } \frac{1}{2} \\ -1+q & \text{with probability } \frac{1}{2} \end{cases} \tag{3.9}$$

iv) In the first round, the price strategy of market makers is such that for the set of aggregate equilibrium orders z_1 reads:

$$\begin{aligned} P^*_{1,q}(z_1) &= \begin{cases} 2 & \text{when } z_1 = 2+q \\ \frac{\theta}{2-\theta} & \text{when } z_1 = 2-q \\ \frac{\theta}{2} & \text{when } z_1 = q \end{cases} \\ P^*_{1,q}(-z_1) &= -P^*_{1,q}(z_1) \end{aligned} \tag{3.10}$$

In the second round, for equilibrium aggregate orders z_2 market makers set prices according to:

$$P^*_{2,q}(z_2) = \begin{cases} 0 & \textit{when } z_2 = 1 \\ \frac{1}{2} & \textit{when } z_2 = q \\ 1 & \textit{when } z_2 = 2-q \\ 2 & \textit{when } z_2 = 2+q \end{cases} \tag{3.11}$$
$$P^*_{2,q}(-z_2) = -P^*_{2,q}(z_2)$$

In both rounds of trade market makers reacts to out-of-equilibrium aggregate orders z_s by setting prices according to

$$P_{s,q}(z_s) = \begin{cases} 2 & \forall z_s > 1 \\ 2z_s & \forall z_s \in [0,1] \end{cases}, \; s = 1,2 \tag{3.12}$$
$$P_{s,q}(-z_s) = -P^*_{1,q}(z_s)$$

The formal proof of proposition can be found in the appendix. Good managers have complete information. Their buying and selling strategy is always optimal and makes them sure that they will be retained with probability 1. Accordingly, their trading strategy is not affected by career concerns and indeed is invariant across the two rounds and is exactly the same strategy described in propositions 2 and 1 for the case of no career concerns. Thus, the intuition behind the strategy of good managers and the relative bounds on q is exactly the one given in the discussion of proposition 1. For bad managers', things do in fact change with respect to the case of no career concerns. Their trading behavior is strongly conditioned by the concern of being fired. Since bad managers are completely uninformed, the optimal thing to do for them should be that of not trading. In fact, in the second round, where there is no scope for career concerns, they optimally refrain from trading. However, in the first round they know that the absence of trade would immediately signal their bad type, leading to their dismission. Career concerns pushes them to trade even in the presence of a negative expected return due to the absence of information. The upper bound on α and the lower bound on β make sure that the negative expected return $\alpha R^e_b(x)$ of their uninformative trade is low enough, while future profits β are high enough to make it appealing to the bad manager to engage in uninformative trade in the hope of being retained. Bad managers' equilibrium strategy $X^*_{1,b}$ amounts to mimicking the good manager by randomly placing intermediate orders $1-q$ or $-1+q$, in the hope that the chosen order reveals correct ex-post, inducing the investor to perceive the manager as good. Notice that strategy $X^*_{1,b}$ maximizes the probability of being retained, while minimizing the expected losses from trade. Indeed, placing either order $1+q$ or $-1-q$ leads to same probability $\frac{1}{4}$ that the manager is retained as $X_{1,b}^{*}$. However, the (negative) return $\alpha R_b^e(x)$ expected by the bad manager is increasing in the order size. Thus, intermediate orders are preferred to large ones.

Notice that the return that good managers expect in the first round is higher than the one they expect in the second round. These expected returns read respectively:

$$\begin{aligned} E\left(R_{1,g}\right) &= 1-\frac{\theta}{4\left(2-\theta\right)}+q\left(\frac{\theta}{4\left(2-\theta\right)}-\frac{1}{4}\theta\right) \\ E\left(R_{2,g}\right) &= \frac{3}{4} \end{aligned}$$

Since $\theta \in (0,1)$ and q is positive, $E\left(R_{1,g}\right)$ is always greater than $E\left(R_{2,g}\right)$. This result is not surprising if we consider that the uninformative trade of bad managers in the first round increases the level of noise present in the market and consequently allows good managers to strike better prices. In line with his interpreation, $E\left(R_{1,g}\right)$ is decreasing in the probability θ of being a good manager: clearly, the higher the fraction of good managers in the population, the smaller that of traders who lose to market makers, which will in turn react more severely to information asymmetry. In particular, notice that

$$\lim_{\theta\to 1} E\left(R_{1,g}\right) = E\left(R_{2,g}\right)$$

As θ approaches 1, market makers post higher prices and $E\left(R_{1,g}\right)$ decreases to $E\left(R_{2,g}\right)$ which is indeed the return that good managers expects in the second round, when bad managers do not trade. By proposition 2, we can easily compute that in the absence of career concerns $E\left(R_{1,g}\right) = E\left(R_{2,g}\right) = \frac{3}{4}$. Thus, good managers enjoys the inefficient behavior of bad managers' uninformative trade brought in by career concerns. It is important to stress that this result is the consequence of market makers reacting more harshly in the framework with no career concerns. Thus, a possible implication of the analysis is that prices are less volatile when a sizable fraction of traders bear career concerns.

Delegation takes always place because the upper bound on β makes sure that in every round of trade the cost of hiring a manager is less than the expected benefits of delegation. The expected benefits of delegation are strictly positive thanks to the presence of good managers who trade under complete information. It is easy to compute the expected returns from delegation in the two rounds of trade, which read:

$$ER_1 = \left(1 - \frac{1}{4}\theta\right)\theta$$
$$ER_2 = \frac{3}{4}\theta$$

Clearly, these returns are increasing in the probability θ that a good manager is selected to trade. It is interesting to notice that average returns from delegation do not depend on the size of trade q. Furthermore, $ER_2 > ER_1$. This is seemingly surprising if we consider that in the second round bad managers behave optimally (by not trading), while they engage in uninformative trade in the first round. The reason why $ER_2 > ER_1$ has to be found in the higher return that good managers can deliver in the first round thanks to the higher level of noise. The positive effect that better prices have on the return of good managers more than offsets the negative impact that bad managers' trade has on first-round average return. Notice further that the average return on delegation is always positive, no matter how big is the fraction of bad manager (as $\theta \to 0$, it approaches zero). This is seemingly surprising and depends on the fact that prices respond endogenously to the level of information present in the market. Thus, the higher the fraction of bad manager, the lower the prices set on average by market makers and the lower the loss sufferred by bad managers. At the limit, when $\theta \to 0$, market makers know that there is no information in the market and set the price of the asset equal to its expected value, thai is zero and the loss of bad managers is reduced to zero.

A word of caution is needed with respect to the generality of the previous two results. First of all, the model assumes that good managers are the only informed traders present in the market. Thus, they enjoy all the increase in noise.[19] If there were other informed traders, these benefits should be shared among more traders and the increase in good managers' return would be lower as well as the average return on delegation, which could possibly become negative. Second, the result of a higher average return on delegation is achieved also because in the specific model at hand, bad managers' uninformative trade is designed as to minimize the negative return that bad managers expect (randomizing over $1 - q$ and $-1 + q$ rather than on $1 + q$ and $-1 - q$). As we will see in the next section, this is not always the case. Finally, rember that we are assuming investors' risk

[19] This result suggests that in a dynamic model where there is only one insider who does not always have private information, but rather is informed with probability θ in each period, the optimal strategy of the insider could possibly be that of trading also when he does not have information in order to increase the level of noise and enjoy higher returns in periods when he does have information (of course, this presumably occurs when θ is high enough).

neutrality. Thus, in the model under consideration any increase i the average return on delegation is welcome by investors. However, if investors were risk averse, then they would dislike the increase in the variability of managers' returns that occurs with churning.

Staying to our model, it is clear that noise traders are the one who pay for the non optimal behavior of bad managers. It is easy to check that the expected loss of noise traders is higher under career concerns. What if we consider the change in the aggregate return of investors and noise tarders? Focus on the first round of trade. Notice that since market makers xpects zero profits in equilibrium, it must be that

$$E\left(R_{1,u}\right) = -E\left(R_1\right)$$

Investors enjoys $\alpha E\left(R_1\right)$. Thus, the aggregate return of investors and noise traders read

$$-E\left(R_1\right) + \alpha E\left(R_1\right) = -\left(1-\alpha\right) E\left(R_1\right) = -\left(1-\alpha\right)\left(1-\frac{1}{4}\theta\right)\theta$$

What is the situation in the absence of career concerns and churning?

$$-E\left(R_1\right) + \alpha E\left(R_1\right) = -\left(1-\alpha\right) E\left(R_1\right) = -\left(1-\alpha\right)\frac{3}{4}\theta$$

Now, it is apparent that $-\left(1-\alpha\right)\left(1-\frac{1}{4}\theta\right)\theta < -\left(1-\alpha\right)\frac{3}{4}\theta$ and thus, churning makes the class of individuals represented by investors&noiose traders worse.

Trade volume with career concerns

As we did for the case in which there were no career concerns, we can compute the total volume of trade that is expected when career concerns are relevant. As for the model with no career concerns, the volume of trade expected from noise traders is equal to 1 in each round of trade. Let me focus on the expected contribution of the managers across the two rounds of trade. In the first round of trade, both bad and good managers trade and expected volume reads:

$$E(V_1^i) = \theta\left(\frac{1}{4}\left|1+q\right| + \frac{1}{4}\left|1-q\right| + \frac{1}{4}\left|-1+q\right| + \frac{1}{4}\left|-1-q\right|\right) +$$
$$+\left(1-\theta\right)\left(\frac{1}{2}\left|1-q\right| + \frac{1}{2}\left|-1+q\right|\right) = \theta + (1-\theta)(1-q)$$

In the second and last round only good managers trade and the volume of trade expected for in this stage reads:

$$E(V_2^i) = \theta\left(\frac{1}{4}|1+q| + \frac{1}{4}|1-q| + \frac{1}{4}|-1+q| + \frac{1}{4}|-1-q|\right) + \\ + (1-\theta)0 = \theta$$

Hence, the volume of trade that is expected to come from managers is given by:

$$E(V_1^i + V_2^i) = 2\theta + (1-\theta)(1-q)$$

Accordingly, the total expected volume of trade is the sum of noise traders' contribution and managers' contribution over the two rounds of trade, and reads:

$$E(V_1 + V_2) = 2 + 2\theta + (1-\theta)(1-q) \tag{3.13}$$

Remember that in the absence of career concerns, the expected volume of trade was equal to 2θ. It is apparent that for any value of $\theta \in (0,1)$ and $q \in \left[\frac{2}{5}, \frac{2}{3}\right)$, it is true that $2 + 2\theta + (1-\theta)(1-q) > 2 + 2\theta$. Thus, we can conclude that the presence of career concerns leads to an increase in the expected volume of trade. The additional volume $(1-\theta)(1-q)$ stems exclusively from reputation reasons. In fact, this additional amount of trade comes all from bad managers, who do not possess any valuable information about the liquidation value of the asset.

3.4 Trade volume and asset volatility

In the present section, I analyze the effects of changes in asset volatily on the trading strategies of managers concerned with their reputation. In order to investigate this issue, consider the two-period trade game described in the previous section and allow for a more general probability distribution of the the liquidation value of the asset traded in the first round v_1. In particular, let $\Pr(v_1 = 2) = \Pr(v_1 = -2) = w$ and $\Pr(v_1 = 1) = \Pr(v_1 = -1) = \frac{1}{2} - w$, with $0 < w < \frac{1}{2}$. Thus, $E(v) = 0$ and $VAR(v) = 6w + 1$. It is apparent that higher values of w are associated to higher values of the variance for the asset traded at $s = 1$.[20]

I will show that as long as α is small enough, when the variance of the asset increases

[20] Assuming a more general distribution function only for v_1, while keeping the same distribution I used in the previous sections for v_2 is done for simplicity and it without loss of generality

over a given threshold, bad managers increase the volume of their uninformative trade. In fact, the main result is derived under the assumption that $\alpha \to 0$ and $\beta > 0$. Assuming $\alpha \to 0$ allows us to focus on the effects that reputation concerns have on the trading strategies of managers, since in this case the performance of the a manager does not affect his payoff directly, but only through the probability of being retained.

Trueman (1988) shows that the probability that bad managers churn increases with the variance of the asset, predicting that noise trade is more likely to occur for riskier asstes. However, his partial equilibrium framework prevents him to carefully discuss the impact on trade volume. In the present model, the probability that bad managers engage in uninformative trade is fixed at $1-\theta$. However, the size of their orders is found to be sensible to the volatility of the asset. In particular, when volatility gets above a given threshold, bad managers switch their trading strategy of randomizing over $1-q$ and $-1+q$ and start randomizing over orders $1+q$ and $-1-q$.

The intuition is straightforward. In equilibrium, the strategies of good managers are strictly monotonic with respect to the value of the asset. Since good managers are risk neutral and can perfectly observe the value of the asset, their strategies do not depend on the probability distribution of the values of the asset.[21] Bad managers are completely uninformed about the true value of the asset. Since they know what is the optimal equilibrium strategy of good managers, they submit those orders compatible with the strategy of good managers that are more likely to reveal correct ex post. A *low* variance of the values of the asset arises when the intermediate values of the asset (in the present case 1 and -1) are more likely to occur than the extreme values (in our case 2 and -2). Accordingly, from the point of view of our bad managers, it is more likely that good managers submit either order $1-q$ or $-1+q$ than orders $1+q$ or $-1-q$. Therefore, being completely uninformed, bad managers maximize their chances to be correct by randomizing over $1-q$ and $-1+q$. High volatility is associated to extreme values of the asset that are more likely to occur than intermediate ones. In this case, the bad managers maximizes his chances to trade correctly by randomizing over $1+q$ and $-1-q$, since those are the orders that most likely will be placed by a good manager.

[21] Since we are assuming $\alpha \to 0$, one may wonder why good managers should condition their order on the value of the asset they observe. After all, when $\alpha \to 0$, their contract leaves them indifferent to any return they may produce (they get β in any case). In fact, the incentives related to reputation force them to behave in the interest of the investor and follow a trading strategy that maximizes the expected return given the information they possess. Indeed, loosely speaking, the incentives related to reputation are such that if ex post an investor realizes that the manager has adopted an ex-ante non optimal strategy, he fires him. This will formally be translated into equilibrium beliefs by investors about the optimal trading strategy of good managers and a consistent hiring/firing strategy.

Since the optimal strategy of good managers is not affected by the probability distribution of the values of the asset, the aggregate volume of trade is also shown to increase when moving fromstates of low variance to states of high variance. The previous discussion is formalized in the next two propositions. In order to simplify the mathematics, all results have being obtained under the assumption of $\theta = \frac{1}{2}$. It is possible to show that from a qualitative point of view, the result of proposition 4 part a) holds for every $\theta \in (0, 1)$, while the result of part b) holds as long as θ is "not too small".

Proposition 8 *Let $\theta = \frac{1}{2}$ and $\alpha \to 0^+$. Then, the following occurs:*

a) When $0 < w \leq \frac{1}{4}$, as long as $0 < \beta \leq \min\left\{\frac{3}{8}, \frac{5}{2}w - 3w^2\right\}$, there exists an uncountable family of Bayesian Nash equilibria indexed by $q \in \left[\frac{2}{5}, \frac{2}{3}\right)$, in which good managers, bad managers and investors behave as described by proposition 3. In particular, in the first period bad managers randomize with equal probability over $1 - q$ and $-1 + q$.

b) When $\frac{1}{4} < w < \frac{1}{2}$, as long as $0 < \beta \leq \frac{1+4w-12w^2}{4}$, there exists an uncountable family of Bayesian Nash equilibria indexed by $q \in \left[\frac{2}{5}, \frac{2}{3}\right)$ in which good managers and investors still behave as described by proposition 3, while bad managers randomize with equal probability over $1 + q$ and $-1 - q$ in the first period and do not trade in the last one.[22]

The proof of this result is detailed in the appendix. Proposition 4) immediately implies the following result.

Proposition 9 *The maximum value of trade volume expected in the equilibrium with low variance is always lower than the minimum value of trade volume expected in the equilibrium with high variance.*

[22] The pricing strategy of market makers in the family of equilibria in 7a is given by $P_1(z_1) = -P_1(-z_1)$, where

$$P_1(z_1) = \begin{cases} 2 & z_1 = 2 + q \\ \frac{1-2w}{2(1-w)} & z_1 = 2 - q \\ \frac{6w-1}{2} & z_1 = q \end{cases}$$

In the family of equilibria in 7b market makers set prices according to $P_1(z_1) = -P_1(-z_1)$, where

$$P_1(z_1) = \begin{cases} \frac{4w}{1+2w} & z_1 = 2 + q \\ 1 & z_1 = 2 - q \\ \frac{12w-1}{4} & z_1 = q \end{cases}$$

Both in case 7a and 7b, any aggregate order z_1 inconsistent with the equilibrium is met by out of equilibrium beliefs of the market maker that give rise to the following out of equilibrium price function:

$$P_1(z_1) = \begin{cases} 2 & z_1 \geq 1, \ z_1 \neq 2 - q, 2 + q \\ 2z_1 & -1 \leq z_1 \leq 1, \ z_1 \neq -q, q \\ -2 & z_1 \leq -1, \ z_1 \neq -2 - q, -2 + q \end{cases}$$

Proof. Let me denote with I the family of equilibria described in part (a) of proposition 4 and with II the family of equilibria described in part (b) of proposition 4. Since the volume of trade expeted from noise traders is the same in both family of equilibria (it sums up to 1 in each round of trade), let me focus on managers' contribution. It is apparent from proposition 1 that the volume of (noise) trade expected from bad managers in equilibrium of type I is lower than the volume of (noise) trade expected from bad managers in equilibrium of type II.[23] Furthermore, it also follows from proposition 1 that when $0 \leq w \leq \frac{1}{4}$ and thus $1 < VAR(w) \leq \frac{5}{2}$, the expected volume of trade reads:

$$\begin{aligned} V_I^e(w,q) &= \frac{1}{2}\left[w\,|1+q| + \left(\frac{1}{2}-w\right)|1-q| + \left(\frac{1}{2}-w\right)|-1+q| + w\,|-1-q|\right] \\ &+ \frac{1}{2}\left(\frac{1}{2}|1-q| + \frac{1}{2}|-1+q|\right) = \\ &= \frac{1}{2}\left[2w(1+q) + (1-2w)(1-q)\right] + \frac{1}{2}(1-q) = \\ &= 2qw + 1 - q \end{aligned}$$

On the other hand, when $\frac{1}{4} < w < \frac{1}{2}$ and thus $\frac{5}{2} < VAR(w) < 4$, the expected volume of trade reads:

$$\begin{aligned} &V_{II}^e(w,q) = \frac{1}{2}\left(w\,|1+q| + \left(\frac{1}{2}-w\right)|1-q| + \left(\frac{1}{2}-w\right)|-1+q| + w\,|-1-q|\right) + \\ &\frac{1}{2}\left(\frac{1}{2}|1+q| + \frac{1}{2}|-1-q|\right) = \\ &= \frac{1}{2}\left[2w(1+q) + (1-2w)(1-q)\right] + \frac{1}{2}(1+q) = \\ &= 2qw + 1 \end{aligned}$$

Equilibrium multiplicity prevents us from analyzing how the expected volume of trade varies with w within each of the two type of equilibria family I and II (indeed, to any given admissible value of w for which an equilibria family holds it is associated an uncountable number of equilibria indexed by q belonging to that family).

However, notice that the highest expected volume of trade in the first family of equilibria reads:

$$\overline{V_I^e} \equiv V_I^e(\frac{1}{4},q) = 1 - \frac{1}{2}q$$

[23] It may be interesting to compare the proportion of noise trade to that of informed trade in the two types of equilibria.

while the lowest expected volume of trade in the second family of equilibria is given by:

$$\underline{V}^e_{II} = V^e_{II}(w \to \frac{1}{4}, q) = 1 + \frac{1}{2}q$$

Therefore, since $q \in \left[\frac{2}{5}, \frac{2}{3}\right)$, the maximum value of trade volume expected in the equilibrium of type I is always lower than the minimum value of trade volume expected in the equilibrium of type II.[24] ■

3.5 Discussion and Conclusion

In the present paper, I showed that career concerns induce uninformed fund managers to indulge in excessive trading. This result has been derived in a order-driven market for a risky asset in which investors delegate trade to differently informed fund managers concerned about being perceived as well informed. The financial market has been model through a *discrete* variant of the classic Kyle (1985), in which both the distributions of the liquidation value of the risky asset and of noise trade are assumed to be discrete. This represents an extension of Dasgupta and Prat (2006), who showed that the same result holds in a market modelled à la Glosten and Milgrom (1985). The setup I used allowed me to analyze how the trading strategies of fund managers are affected by career concerns in markets characterized by different asset's volatility. I showed that high volatility is likely to induce bad managers to increase the size of their noise trade. This is a generalization of Trueman (1988) to a general equilibrium framework.

Conditions for delegation take place are explicitly derived. Analogously, conditions for career concerns be effective are characterized. In this respect, it is worth noticing that in the present model, career concerns arise if and only if both the two following conditions take place. First, investors have to rationally allocate their funds to managers that are perceived as being better informed than their competitors. Second, the compensation that managers receive for managing the fund has to exhibit a fixed component. In particular, this fixed component should be high enough relatively to the performance component in order to provide the manager with the incentives to managing the fund without worrying about the return he delivers. Notice that these considerations are drawn under the assumption that the contract between a manager and an investor is

[24] It is important to stress that this result does not depend on the assumption that $\theta = \frac{1}{2}$. Indeed, in the more general case, volumes of trade read: $V^e_I(w,q) = 4\theta qw + 1 - q$ and $V^e_{II}(w,q) = 4\theta qw + 1 + q - 2\theta q$. Accordingly, $\overline{V^e_I} = V^e_I(w = \frac{1}{4}, q) = 1 - q\,(1-\theta)$ and $\underline{V}^e_{II} = V^e_{II}(w \to \frac{1}{4}, q) = 1 + q(1-\theta)$, from which it is apparent that $V^e_I(w = \frac{1}{4}, q) < V^e_I(w = \frac{1}{4}, q)$ for every $\theta \in (0,1)$.

exogenously given. It would be interesting to allow for endogenous contracting and check wether under the optimal contract good manager would specify a fixed payment low to prevent bad managers from engaging in *reputation* trade (and eventually kicking them out of the market). On the other hand, notice that all other things being equal, the return delivered by good managers when bad managers trade is higher. This is due to the fact that bad managers' trade is noise trade. The increase of noise in the market allows good managers to strike better prices and thus higher returns. The analysis above has been done under the assumption that the contract between a manager and an investor is exogenously given. It would be interesting to extend the analysis to allow for endogenous contracting and check wether the positive effects that the presence of bad managers have on good managers' profits lead them to optimally choose a contract that does not determine separation.

It is interesting to notice that the (expected) return from delegation is independent from the order size q, and depends only on the exogenous parameters of the model w and θ (see 3.72 and 3.66). While it is apparent that the return from delegation is increasing in θ, we cannot draw a definite conclusions about the way in which it varies with respect to w. Indeed, when w gets higher than $\frac{1}{4}$, there is a switch from an equilibrium of low volume to an equilibrium of high volume. It is worth noticing that in both equilibria, the return from delegation is maximized for $w \to \frac{1}{4}$, when there is the highest degree of information asymmetry between the market maker and the good manager. In particular, the highest return expected in the equilibrium with high volatility (and high volume) is lower than the highest return expected in the equilibrium with low volatility (and low volume). This result is driven by the fact that in the equilibrium with high volatility, bad managers trade more aggressively, generating highest losses.

3.6 Appendix

proof of proposition 1. I will first consider the candidate equilibrium q whereby the trading strategy of the good and the bad managers are given by (3.2) and (3.3) and compute the prices that would arise for these strategies, using the property that prices are set as conditional expectations given by (3.1). I will then show that for these prices, strategies (3.2) and (3.3) do maximize the expected profits of good and bad managers. Finally, given (3.2), (3.3), (3.4) and (3.5), I will work out the conditions under which an investor always finds it convenient to hire a fund manager.

Let $\alpha \in (0, 1)$ and $\beta > 0$. Let $z = Z(u, v, i)$ denote the aggregate order for the asset, with $i \in \{g, b\}$ indicating manager's type. In order to ease notation, from now on I will drop subscript q from the strategies of the players and let $X_{q,i}(\cdot) \equiv X_i(\cdot)$ and $P_q(\cdot) \equiv P(\cdot)$.

The market maker's problem. A market maker has to set the regret-free price at which to liquidate trade, based on the observation of the aggregate demand for the asset. Given fund managers' equilibrium strategies (3.2) and (3.3), and the possible realizations of noise trade and asset value, the equilibrium aggregate demand for the asset is odd and satisfies:

$$\begin{aligned} Z(u, v, i) &= \begin{cases} 2+q & \text{when } \{u=1, v=2, i=g\} \\ 2-q & \text{when } \{u=1, v=1, i=g\} \\ 1 & \text{when } \{u=1, \forall v,\ i=b\} \\ q & \text{when } \{u=-1, v=2, i=g\} \\ & \text{or } \quad \{u=1, v=-1, i=g\} \end{cases} \\ Z(u, v, i) &= -Z(-u, -v, i) \end{aligned}$$

Focus on positive equilibrium orders $z = q, 1, 2-q, 2+q$. Each aggregate order perfectly reveals which type of manager is in the market. When the aggregate order is equal to 1, the market maker infers that a bad manger has been hired and thus that the aggregate order comes all from noise traders. Accordingly, he sets:

$$P(1) \equiv E(v|1) = E(v) = 0$$

In all the other cases ($z = q, 2-q, 2+q$), the market maker realizes that a good manager is trading. Furthermore, when $z = 2-q, 2+q$, he can perfectly recover the information of the good manager. Accordingly, he sets:

$$\begin{aligned} P(2-q) &= 1 \\ P(2+q) &= 2 \end{aligned}$$

On the other hand, when $z = q$, the market maker cannot tell whether the good manager has observed $v = 2$ (and $u = -1$) or $v = -1$ (and $u = 1$). Since the market maker puts equal probability on these two events[25], his optimal price response to aggregate order q

[25] This is a consequence of the specific distributional assumptions we have made for v and u, according to which $v = 2$ and $v = -1$ have equal probability to occur and so do $u = -1$ and $u = 1$, with u and v independent.

reads:

$$P(q) \equiv E(v|q) = \frac{1}{2}2 + \frac{1}{2}(-1) = \frac{1}{2}$$

Since $Z(u,v,i) = -Z(-u,-v,i)$, it is immediate to compute the prices that the market maker sets in response to negative aggregate orders and show that $P(q) = -P(-q)$.

So far, we have shown that the price response of the market maker to equilibrium aggregate orders (i.e., orders that are consistent with strategies (3.2) and (3.3) of good and bad managers) is given by (3.4).

In order to complete the construction of the price function in the candidate equilibrium q, we need to construct the price schedule according to which the market maker sets prices in response to out of equilibrium aggregate orders. In this respect, market maker's out of equilibrium beliefs about information content of trade are crucial. It is important to notice that in the present game, PBE does not imply any restriction on market maker's out of equilibrium beliefs. Thus, the market maker's price response to any out of equilibrium order can be every price belonging to $[-2,2]$. I assume the following out of equilibrium beliefs:

$$\begin{aligned}
&\forall z > 1,\ z \neq 2-a, 2+a,\ \Pr(v=2|z) = 1 \\
&\forall z \in [0,1],\ z \neq a, 1,\ \Pr(v=2) = \frac{z+1}{2},\ \Pr(v=-2) = \frac{1-z}{2} \\
&\forall z \in [-1,0],\ z \neq -a, -1,\ \Pr(v=-2) = \frac{z+1}{2},\ \Pr(v=2) = \frac{1-z}{2} \\
&\forall z < -1,\ z \neq -2+a, -2-a,\ \Pr(v=-2|z) = 1
\end{aligned}$$

It is then immediate to show that the out of equilibrium price function is exactly given by (3.5).[26]

[26]The general idea behind these out of equilibrium beliefs is that whenever the market maker is sure that a manager has posted a positive order, he takes the harshest belief assigning probability 1 to the event that this positive (negative) order is coming from a good manager who has observed $v = 2$ ($v = -2$). Hence, when aggregate order is greater than 1 (less than -1), the market maker is sure that there is a manager posting a positive order (because noise trade is either 1 or -1) and accordingly sets the harshest price $p = 2$ ($p = -2$).

If the aggregate order is between -1 and 1, the situation is more complex because the market maker cannot tell the direction of manager's trade. Consider for example the case $z \in (0,1)$ (being the case $z \in (-1,0)$ symmetric). Any aggregate order between 0 and 1 can be the result of the following two events:

1) $x \in]1,2[$ and $u = -1$

2) $x \in]-1,0[$ and $u = 1$

In the first event, the market maker would set the harshest price $p = -2$. In the second event, he would set the harshest price $p = 2$. Since the market maker cannot distinguish this two events, I assume that when he observes a positive aggregate order $z \in [0,1]$, the probability he assigns to event 1) is

Managers' problem. We have now to show that given (3.4) and (3.5), it is indeed optimal for good and bad managers to follow strategies $X_g^*(\cdot)$ and $X_b^*(\cdot)$ respectively. Given the symmetry of the problem (both $P(\cdot)$ and $Z(\cdot)$ are odd functions), it is sufficient to focus on the analysis of the buy side of the problem, that is, on non negative orders.

Let me define $R(x) = x(v-p)$ as the ex-post return of a manager trading quantity x. Since $p = P(x+u)$, it is useful to write $R(x) = x[v - P(x+u)]$ to highlight that the ex-post return delivered by the manager depends on his order x, on the liquidation value of the asset v and on noise u. It is then clear that ex-ante, the return that manager i expects from placing order x depends on the information he has about v and u. This information crucially depends on the type of manager. Let Ω_i denote the information set of manager i. Formally, the expected return of manager i from order x reads:

$$E(R(x)|\Omega_i) = x[E(v|\Omega_i) - E(P(x+u)|\Omega_i)]$$

Notice that the expected return from order x depends on the expectation that manager i holds about the value of the asset minus the expectation about the price he is going to pay when posting order x. Since $\Omega_g = \{v\}$ and $\Omega_b = \varnothing$, we have that:

$$\begin{aligned} R_g^e(x,v) &\equiv E(R(x)|v) = x[v - E(P(x+u))] \\ R_b^e(x) &\equiv E(R(x)) = x[-E(P(x+u))] \end{aligned}$$

Notice that good managers can make more accurate forecasts of returns thanks to the fact that they are able to perfectly forecast the true value of the asset. However, they are not better than bad manager in predicting trading prices. Since prices depends only on x and u and no manager has some information about u, both types make the same forecast $E(P(x+u))$ about the price they are going to pay by posting order x. To ease notation, let $P^e(x) \equiv E(P(x+u))$ and accordingly write:

$$R_g^e(x,v) = x[v - P^e(x)] \tag{3.14}$$

$$R_b^e(x) = -xP^e(x) \tag{3.15}$$

Notice that since u can take on values 1 and -1 with equal probability, we have that:

$$P^e(x) \equiv E(P(x+u)) = \frac{1}{2}P(x+1) + \frac{1}{2}P(x-1) \tag{3.16}$$

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increasing in the aggregate order z. Basically, the higher the order size, the more likely it is that it is the result of good manager demanding for the asset.

Bad managers. Let me first consider the case in which a bad manager is selected to trade. Using (3.15), it is easy to show that under the linear contract specified in section 2.1, the payoff that a bad manager expects from placing order x can be written as follows:

$$\Pi_b^e(x) = \alpha R_b^e(x) + \beta = -\alpha x P^e(x) + \beta \tag{3.17}$$

Since the equilibrium strategy of the bad manager prescribes to refrain from trading, his expected profits *in equilibrium* read $\Pi_b^e(0) = \beta$.

In order to show that $X_b^* = 0$ is the optimal trading strategy of a bad manager, we have to prove the following two different cases:

1) The bad manager has not any incentives in mimicking a good manager by placing any order $x \in \{1-q, 1+q\}$ consistent with good manager's equilibrium strategy. Formally, this amounts to prove that the following two inequalities hold:

$$\Pi_b^e(0) \geq \Pi_b^e(1+q) \tag{3.18}$$
$$\Pi_b^e(0) \geq \Pi_b^e(1-q) \tag{3.19}$$

Using (3.17), we can write:

$$\Pi_b^e(1+q) = -\alpha (1+q) P^e(1+q) + \beta$$
$$\Pi_b^e(1-q) = -\alpha (1-q) P^e(1-q) + \beta$$

By (3.16) and (3.4), we can easily compute:

$$P^e(1+q) = \frac{1}{2}P(2+q) + \frac{1}{2}P(q) = \frac{5}{4}$$
$$P^e(1-q) = \frac{1}{2}P^*(2-q) + \frac{1}{2}P^*(-q) = \frac{1}{4}$$

Hence, conditions (3.18) and (3.19) can be finally written as follows:

$$\beta \geq -\alpha\frac{5}{4}(1+q) + \beta$$
$$\beta \geq -\alpha\frac{1}{4}(1-q) + \beta$$

It is apparent that the previous inequalities are simultaneously satisfied for

$$-1 \leq q \leq 1 \tag{3.20}$$

2) The bad manager has no incentives to deviate to any other positive order inconsistent with the equilibrium. Let $x \in \mathbb{R}^+ \setminus \{1-q, 1+q\}$ denote an arbitrary positive order inconsistent with the equilibrium. Formally, we have to show that

$$\Pi_b^e(0) = \beta \geq \Pi_b^e(x) = -\alpha x P^e(x) + \beta$$

where

$$P^e(x) = \frac{1}{2}P(x+1) + \frac{1}{2}P(x-1)$$

It is convenient to distinguish the following two cases:

2a) $x \geq 2$. In this case, the aggregate order $z = x + u$ is always greater than 1 for any realization of $u = -1, 1$. Thus, by (3.5), we have that $P(x+1) = P(x-1) = 2$. Accordingly, $P^e(x) = 2$ and our condition boils down to:

$$\beta \geq -2\alpha x + \beta$$

which is clearly always satisfied in our parameters region (remember that x is positive).

2b) $x \in (0, 2) \setminus \{1-q, 1, 1+q\}$. In this case, when $u = -1$, the aggregate order is $z = x - 1 \in (-1, 1)$; when $u = 1$, the aggregate order is $z = x + 1 > 1$. Hence, by (3.5), $P(x-1) = 2(x-1)$ and $P(x+1) = 2$. Therefore, $P^e(x) = x$ and our condition can be written as:

$$\beta \geq -\alpha x^2 + \beta$$

which is clearly always satisfied (again, remember that x is positive).

Good managers. Let me now consider the case in which the good manager is selected to trade. Before trading, the good manager observes the true value of the asset v. By (3.14), the payoff that a good manager expects from placing order x when the asset is worth v can be written as follows:

$$\begin{aligned} \Pi_g^e(x, v) &= \alpha R_g^e(x, v) + \beta = \\ &= \alpha x \left[v - P^e(x)\right] + \beta \end{aligned}$$

Since the symmetry of the problem allows us to focus on the buy side of the market, I will analyze only the cases in which the good manager has observed $v = 1$ and $v = 2$.

Expected profits of the good manager from following his equilibrium strategy $X_g^*(v)$

read:

$$\Pi_g^e(1-q,1) = \alpha(1-q)\left[1 - P^e(1-q)\right] + \beta = \\ = \frac{3}{4}\alpha(1-q) + \beta$$

$$\Pi_g^e(1+q,2) = \alpha(1+q)\left[2 - P^e(1+q)\right] + \beta = \\ = \frac{3}{4}\alpha(1+q) + \beta$$

I will show that $X_g^*(v)$ is the optimal strategy for the good manager by analyzing the following cases:

1) When the good manager observes $v = 1$, he trades $X_g^*(1) = 1 - q$ instead of deviating to any other order consistent with the equilibrium, i.e., $1 + q$ or 0. This amount to show that the following inequalities hold:

$$\Pi_g^e(1-q,1) \geq \Pi_g^e(1+q,1) \tag{3.21}$$

$$\Pi_g^e(1-q,1) \geq \Pi_g^e(0,1) \tag{3.22}$$

It is easy to check that:

$$\Pi_g^e(1+q,1) = \alpha(1+q)\left[1 - P^e(1+q)\right] + \beta = \\ = -\frac{1}{4}\alpha(1+q) + \beta \\ \Pi_g^e(0,1) = \beta$$

so that our conditions (3.21) and (3.22) can be written as:

$$\frac{3}{4}\alpha(1-q) + \beta \geq -\frac{1}{4}\alpha(1+q) + \beta$$

$$\frac{3}{4}\alpha(1-q) + \beta \geq \beta$$

Both these inequalities are always satisfied for

$$q \leq 1 \tag{3.23}$$

2) When the good manager observes $v = 2$, he plays $X_g^*(2) = 1 + q$ instead of

deviating to $1-q$ or 0. Formally, it must be that:

$$\Pi_g^e(1+q,2) \geq \Pi_g^e(1-q,2) \tag{3.24}$$

$$\Pi_g^e(1+q,2) \geq \Pi_g^e(0,2) \tag{3.25}$$

which can be written as:

$$\begin{aligned} \frac{3}{4}\alpha(1+q)+\beta &\geq \alpha(1-q)\left(2-\frac{1}{4}\right)+\beta \\ \frac{3}{4}\alpha(1+q)+\beta &\geq \beta \end{aligned}$$

and simplified to:

$$\begin{aligned} 3(1+q) &\geq 7(1-q) \\ (1+q) &\geq 0 \end{aligned}$$

These inequalities are simultaneously satisfied for

$$q \geq \frac{2}{5} \tag{3.26}$$

3) Following the equilibrium strategy $X_g^*(v)$ gives the good manager expected profits greater than those he would get by deviating to any positive order inconsistent with the equilibrium. Let $x \in \mathbb{R}^+/\{0,1-q,1+q\}$ indicate a positive order inconsistent with the equilibrium. Then, we have to show that the two following conditions hold:

$$\Pi_g^e(1-q,1) \geq \Pi_g^e(x,1) \tag{3.27}$$

$$\Pi_g^e(1+q,2) \geq \Pi_g^e(x,2) \tag{3.28}$$

It is useful to consider the two following cases:

3a) $x \geq 2$. In this case, $z = x+u > 1$ for any realization of u and by (3.5) $P^e(x) = 2$. Accordingly:

$$\begin{aligned} \Pi_g^e(x,1) &= \alpha x\left[1-P^e(x)\right]+\beta = \\ &= \alpha x(1-2)+\beta = -\alpha x+\beta \\ \Pi_g^e(x,2) &= \alpha x\left[2-P^e(x)\right]+\beta = \\ &= \alpha x(2-2)+\beta = \beta \end{aligned}$$

Using these results, it is easy to show that conditions (3.27) and (3.28) can be written as

$$\begin{aligned} \frac{3}{4}(1-q) &\geq -x \\ \frac{3}{4}(1+q) &\geq 0 \end{aligned}$$

which are simultaneously satisfied under condition:

$$-1 \leq q \leq \frac{11}{3} \tag{3.29}$$

3b) $x \in (0,2) \setminus \{1-q, 1, 1+q\}$. We have shown above that in this case $P^e(x) = x$. Thus, we can write:

$$\begin{aligned} &\Pi_g^e(x,1) = \alpha x\left[1 - P^e(x)\right] + \beta = \\ &= \alpha\left(x - x^2\right) + \beta \\ &\Pi_g^e(x,2) = \alpha x\left[2 - P^e(x)\right] + \beta = \\ &= \alpha\left(2x - x^2\right) + \beta \end{aligned}$$

After simple algebra it is easy to show that conditions (3.27) and (3.28) boil down to:

$$\frac{3}{4}(1-q) \geq x - x^2 \tag{3.30}$$

$$\frac{3}{4}(1+q) \geq 2x - x^2 \tag{3.31}$$

A necessary and sufficient condition for the first inequality to be satisfied is that $q \leq \frac{2}{3}$, while a necessary and sufficient condition for the second one is that $q \geq \frac{1}{3}$. Hence, (3.27) and (3.28) are both satisfied as long as:

$$\frac{1}{3} \leq q \leq \frac{2}{3} \tag{3.32}$$

Summing up conditions (3.23), (3.26), (3.29) and (3.32) for the good manager, we have that $X_g^*(v)$ is an equilibrium strategy as long as:

$$q \in \left[\frac{2}{5}, \frac{2}{3}\right) \tag{3.33}$$

Notice that the previous condition is stricter than optimality condition (3.20) for the bad manager. Thus, under (3.33) both good and bad managers will play equilibrium strategies $X_g^*(v)$ and $X_b^* = 0$ as optimal responses to $P(z)$.[27]

The investor's problem. Before trade opens, the investor has to decide whether to delegate trade to a fund manager or stay out of the market. Suppose the investor chooses to hire a manager. With probability θ he hires a good manager. The return that he expects in equilibrium from a good manager reads:

$$\begin{aligned} & E\left[R_g^e\left(X_g^*(v), v\right)\right] = \\ & = \frac{1}{4}(1+q)\left[2 - P^e(1+q)\right] + \frac{1}{4}(1-q)\left[1 - P^e(1-q)\right] + \\ & + \frac{1}{4}(-1+q)\left[-1 - P^e(-1+q)\right] + \frac{1}{4}(-1-q)\left[-2 - P^e(-1-q)\right] = \frac{3}{4} \end{aligned}$$

With probability $1-\theta$ he hires a bad manager. Since a bad manager does not trade in equilibrium, the return that the investor expects for him in equilibrium is zero. Thus, the average return from delegation expected by the investor in equilibrium is equal to $\frac{3}{4}\theta$. Given the linear contract under consideration, the expected equilibrium payoff of delegation reads:

$$\frac{3}{4}\theta(1-\alpha) - \beta$$

If the Investor does not hire any manager, he will get a payoff equal to zero. Therefore, the investor will hire a fund manager if and only if

$$\frac{3}{4}\theta(1-\alpha) - \beta > 0$$

which is satisfied for any $\alpha \in (0,1)$ as long as $\beta < \frac{3}{4}\theta(1-\alpha)$. ■

Equilibria multiplicity. It is useful to remind that proposition 1 identifies *a* family of perfect Bayesian equilibria of the static game described above. In particular, proposition 1 focuses on "symmetric" strategies for the good manager, that is, strategies represented by (3.2). More generally, there exist families of perfect Bayesian equilibria indexed by the pair (q_1, q_2) in which investors delegate trade, bad managers do not trade

[27] In fact, for $q = \frac{2}{5}$, the good type is indifferent between following strategy $X_{q,g}(\cdot)$ and mimicking the bad type by playing $x = 0$

and good managers behave according to the following more general strategy:

$$X^*_{g,q}(v) = \begin{cases} 1 + q_2 & \text{when } v = 2 \\ 1 - q_1 & \text{when } v = 1 \\ -1 + q_2 & \text{when } v = -1 \\ -1 - q_1 & \text{when } v = -2 \end{cases} \tag{3.34}$$

with q_1, q_2 taking values in appropriate subsets of the segment $(0, 1)$. This result can be easily proved by following the reasoning in the proof of proposition 1. At an intuitive level, what we need is just that (at least in some cases) the good manager be able to disguise his information behind noise trade. Notice that strategy (3.34) is such that the market maker cannot distinguish ($v = 2$, $u = -1$) from ($v = -1$, $u = 1$) and ($v = -2$, $u = 1$) from ($v = 1$, $u = -1$).

Basically, (3.34) is derived by considering that the optimal strategy of the good manager must be monotonic with respect to the manager's signal and must satisfy the following *camouflage* conditions:

$$\begin{aligned} X^*_g(2) - 1 &= X^*_g(-1) + 1 \\ X^*_g(-2) + 1 &= X^*_g(1) - 1 \end{aligned}$$

In order to obtain (3.34), let

$$\begin{aligned} X^*_{g,q}(2) - 1 &= X^*_{g,q}(-1) + 1 = q_2 \in \mathbb{R} \\ X^*_{g,q}(-2) + 1 &= X^*_{g,q}(1) - 1 = q_1 \in \mathbb{R} \end{aligned}$$

Optimality requires strictly monotonicity, which restricts q_1 and q_2 to be positive. Optimality also implies that q_1 and q_2 be less than one (values of q_1 and q_2 greater than one implies that the manager is selling when observing positive values of the dividends and buying when observing negative values of the dividends, thus delivering negative returns).[28]

Proof of proposition 3. The prices that the manager expects to pay in the equilibria

[28] Another camouflage case arises when:

$$\begin{aligned} X^*_{g,q}(2) - 1 &= X^*_{g,q}(-2) + 1 \\ X^*_{g,q}(-1) + 1 &= X^*_{g,q}(1) - 1 \end{aligned}$$

however, the strategy satisfying this conditions is not strictly monotonic and thus non optimal.

identified by propositio 3 read:

$$P_2^e(1+q) = -P_1^e(-1-q) = \frac{5}{4} \tag{3.35}$$

$$P_2^e(1-q) = -P_1^e(-1+q) = \frac{1}{4} \tag{3.36}$$

$$P_1^e(1+q) = -P_2^e(-1-q) = 1 + \frac{1}{4}\theta \tag{3.37}$$

$$P_1^e(1-q) = -P_2^e(-1+q) = \frac{\theta}{2(2-\theta)} - \frac{\theta}{4} \tag{3.38}$$

The returns that a good manager expects to deliver in equilibrium read:

$$\begin{aligned} & R_{2,g}^e\left(X_{2,g}^*(v_2), v_2\right) = \\ & = \begin{cases} (1+q)\left(2 - P_2^e(1+q)\right) = \frac{3}{4}(1+q) & \text{if } v_2 = 2 \text{ or } v_2 = -2 \\ (1-q)\left(1 - P_2^e(1-q)\right) = \frac{3}{4}(1-q) & \text{if } v_2 = 1 \text{ or } v_2 = -1 \end{cases} \end{aligned} \tag{3.39}$$

$$\begin{aligned} & R_{1,g}^e\left(X_{1,g}^*(v_1), v_1\right) = \\ & = \begin{cases} (1+q)\left[2 - P_1^e(1+q)\right] = (1+q)\left(1 - \frac{1}{4}\theta\right) & \text{if } v_1 = 2 \text{ or } v_1 = -2 \\ (1-q)\left[1 - P_1^e(1-q)\right] = (1-q)\left(1 - \frac{\theta}{2(2-\theta)} + \frac{1}{4}\theta\right) & \text{if } v_2 = 1 \text{ or } v_2 = -1 \end{cases} \end{aligned} \tag{3.40}$$

The returns that a bad manager expects to deliver in equilibrium read:

$$R_{2,b}^e\left(X_{2,b}^*\right) = 0 \tag{3.41}$$

$$\begin{aligned} & R_{1,b}^e\left(X_{1,b}^*\right) = \\ & = \frac{1}{2}(1-q)\left[-P_1^e(1-q)\right] + \frac{1}{2}(-1+q)\left[-P_1^e(-1+q)\right] = \\ & = -\left(\frac{\theta}{2(2-\theta)} - \frac{\theta}{4}\right)(1-q) \end{aligned} \tag{3.42}$$

Let $\alpha \in (0,1)$, $\beta > 0$ and $q \in (0,1)$. I will prove proposition 3 by backward induction.

Second round of trade. Since the second round of trade is also the last one, managers do not face career concerns. Thus, the second round of trade is equivalent to the basic financial market of section 2. Accordingly, investors, managers and market makers' equilibrium behavior for the second round of trade is the one described in proposition 1: investors always hire a manager, bad managers do not trade, good managers trade according to (3.2) and market makers set prices according to (3.4) and (3.5). Furthermore, we know from proposition 1 that this behavior constitutes an equilibrium as long as:

$$\frac{2}{5} < q < \frac{2}{3} \tag{3.43}$$

$$\beta < (1-\alpha)\frac{3}{4}\theta \tag{3.44}$$

Investor's retaining rule at the end of the first round of trade. The *equilibrium* average return that the investor expects to be delivered by a bad manager in the second round is clearly equal to zero; on the other hand, that expected from a good manager reads:

$$E\left[R^e_{2,g}\left(X_{2,g}\left(v_2\right),v_2\right)\right] = \frac{1}{4}\left[\frac{3}{4}(1+q)\right] + \frac{1}{4}\left[\frac{3}{4}(1-q)\right] + \frac{1}{4}\left[\frac{3}{4}(1+q)\right] + \frac{1}{4}\left[\frac{3}{4}(1+q)\right] = \frac{3}{4}$$

Let $\widehat{\theta} = \Pr(i = g|v_1, x_1)$ denote the investor's believe that the incumbent manager is good. Accordingly, the equilibrium payoff that the investor expects in the second round from retaining the incumbent manager reads:

$$(1-\alpha)\frac{3}{4}\widehat{\theta} - \beta$$

Alternatively, the equilibrium payoff that the investor expects in the second round from hiring a new manager reads:

$$(1-\alpha)\frac{3}{4}\theta - \beta$$

Thus, the investor retains the old manager if and only if:

$$(1-\alpha)\frac{3}{4}\widehat{\theta} - \beta > (1-\alpha)\frac{3}{4}\theta - \beta$$

or equivalently,

$$\widehat{\theta} > \theta$$

Notice that under condition (3.44), this is a necessary and sufficient condition for the old manager to be retained. Indeed, $\widehat{\theta} > \theta$ also implies that $\widehat{\theta}(1-\alpha)\frac{3}{4} - \beta > \theta(1-\alpha)\frac{3}{4} - \beta > 0$, which ensures that the investor will in fact prefer to retain the incumbent manager than refraining from trading (which would give him zero profits).

Investor's belief $\widehat{\theta}$. Based on the observation of x_1 and v_1, the investor updates his belief about the fact that the manager is good. Formally, the investor computes (whenever possible) $\widehat{\theta} \equiv \Pr(i = g|v_1, x_1)$, where

$$\Pr(i = g|v_1, x_1) = \frac{\Pr(i = g)\Pr(x_1|v_1, i = g)}{\Pr(i = g)\Pr(x_1|v_1, i = g) + \Pr(i = b)\Pr(x_1|v_1, i = b)}$$

Let $\Phi = \{-1 - q, -1 + q, 1 - q, 1 + q\}$ denote the set of managers' equilibrium orders. Given the equilibrium strategies $X_{1,g}^*$ and $X_{1,b}^*$, it is easy to show that for any $x_1 \in \Phi$, the following holds true:

$$\Pr(i = g|v_1, x_1) = \begin{cases} 1 & \text{if } (v_1 = 2, x_1 = 1 + q) \text{ or } \\ & \quad (v_1 = -2, x_1 = -1 - q) \\ \frac{2\theta}{1+\theta} & \text{if } (v_1 = 1, x_1 = 1 - q) \text{ or } \\ & \quad (v_1 = -1, x_1 = -1 + q) \\ 0 & \text{if } v_1 \in \{-2, -1, 1, 2\} \text{ and } \\ & \quad x_1 \neq X_{1,g}^*(v_1) \end{cases} \tag{3.45}$$

On the other hand, for orders off the equilibrium path, Bayesian perfection imposes no restrictions on $\Pr(i = g|v_1, x_1)$, which could in principle take any value in $[0, 1]$. I thus assume that:

$$\Pr(i = g|v_1, x_1) = 0, \ \forall x_1 \in \mathbb{R} \setminus \Phi \tag{3.46}$$

Managers' expected probability of being retained. A manager knows that he is retained if $\widehat{\theta} > \theta$. He also knows (3.45) and (3.46), from which he can conclude that

$$\widehat{\theta} > \theta \begin{cases} \text{if } (v_1 = 2, x_1 = 1 + q) \text{ or } (v_1 = -2, x_1 = -1 - q) \\ \text{or } (v_1 = 1, x_1 = 1 - q) \text{ or } (v_1 = -1, x_1 = -1 + q) \end{cases} \tag{3.47}$$

$$\widehat{\theta} < \theta \text{ otherwise} \tag{3.48}$$

Given (3.47) and (3.48), a manager of type i can easily compute the probability of being retained when trading order x_1. In particular, since a good manager observes v_1, he knows that:

$$\Pr(\widehat{\theta} > \theta|x_1, v_1) = \begin{cases} 1 \text{ if } x_1 = X_{1,g}^*(v_1) \\ 0 \text{ if } x_1 \neq X_{1,g}^*(v_1) \end{cases} \tag{3.49}$$

A bad manager does not observe v_1. Therefore, in his case we have that

$$\Pr(\widehat{\theta} > \theta|x_1) = \begin{cases} \frac{1}{4} \text{ if } x_1 = \{-1 - q, -1 + q, 1 - q, 1 + q\} \\ 0 \ \text{ otherwise} \end{cases} \tag{3.50}$$

Good managers' strategy in the first round of trade.[29] Let me define the total profits that a good manager expects from trading order $x_1$at the beginning of $s = 1$ (when he observes v_1) as follows:

$$\Pi^e_{tot,g}(x_1, v_1) = \Pi^e_{1,g}(x_1, v_1) + \Pr(\widehat{\theta} > \theta | x_1, v_1)\Pi^*_{2,g} \tag{3.51}$$

where:

$$\Pi^e_{1,g}(x_1, v_1) = \alpha R^e_{1,g}(x_1, v_1) + \beta = \alpha x_1 \left[v_1 - P^e_1(x_1)\right] + \beta$$

are the profits that the good manager expects to get in the first round of trade from trade x_1. $\Pr(\widehat{\theta} > \theta | x_1, v_1)$ is the probability of being retained if trading x_1. $\Pi^*_{2,g}$ are the *equilibrium* profits that the good manager expects to gain in the second round if he is retained (these are the profits expected at the beginning of $s = 1$ when the manager does not know v_2 yet). We know that the average *equilibrium* return delivered by a good manager in the second round is $\frac{3}{4}$. Thus

$$\Pi^*_{2,g} = \frac{3}{4}\alpha + \beta$$

In order to show that $X^*_{1,g}(\cdot)$ is the equilibrium strategy, I have to show that when the true state is observed to be v_1, order $X^*_{1,q,g}(v_1)$ maximizes (3.51). Again, given the symmetry of the problem, it is sufficient to focus only on the cases in which the good manager observes $v_1 = 1, 2$ and only on the buy side of the problem (i.e. positive orders). Using this fact and expression (3.51), our task amounts to show that:

$$\text{for } v_1 = 1, 2 \text{ and } \forall x_1 \geq 0, \quad \Pi^e_{tot,g}(X^*_{1,g}(v_1), v_1) \geq \Pi^e_{tot,g}(x_1, v_1) \tag{3.52}$$

Using (3.40) and (3.49), the two previous expressions can be written as:

$$\Pi^e_{tot,g}\left(X^*_{1,g}(v_1), v_1\right) = \begin{cases} \alpha(1-q)\left(1 - \frac{\theta}{2(2-\theta)} + \frac{1}{4}\theta\right) + \frac{3}{4}\alpha + 2\beta & \text{if } v_1 = 1 \\ \alpha(1+q)\left(1 - \frac{1}{4}\theta\right) + \frac{3}{4}\alpha + 2\beta & \text{if } v_1 = 2 \end{cases}$$

I will proof (3.52) by considering the following three cases:

a) When the good manager observes $v_1 = 1, 2$, he will follow the equilibrium strategy

[29] Remember that I am assuming that the type of manager does not change over the two rounds of trades.

and play $X^*_{1,g}(v_1)$ instead of refraining from trading. Formally:

$$\text{for } v_1 = 1, 2,\ \Pi^e_{tot,g}\left(X^*_{1,g}(v_1), v_1\right) \geq \Pi^e_{tot,g}\left(0, v_1\right) \tag{3.53}$$

From (3.51) and (3.49) we have that $\Pi^e_{tot,g}\left(0, v_1\right) = \beta$. Therefore, (3.53) can be written as:

$$\alpha(1-q)\left(1 - \frac{\theta}{2\left(2-\theta\right)} + \frac{1}{4}\theta\right) + \frac{3}{4}\alpha + \beta \geq 0 \tag{3.54}$$

$$\alpha(1+q)\left(1 - \frac{1}{4}\theta\right) + \frac{3}{4}\alpha + \beta \geq 0 \tag{3.55}$$

It is apparent that inequalities (3.54) and (3.55) are always satisfied in our parameters region.

b) When the good manager observes $v_1 = 1, 2$, he is better off playing $X^*_{1,g}(v_1)$ instead of either deviating to any other order that could in principle arise in equilibrium, i.e. $x_1 \in \{1+q, 1-q\}$. Formally:

$$\Pi^e_{tot,g}\left(X^*_{1,g}(1), 1\right) \geq \Pi^e_{tot,g}\left(X^*_{1,g}(2), 1\right) \tag{3.56}$$

$$\Pi^e_{tot,g}\left(X^*_{1,g}(2), 2\right) \geq \Pi^e_{tot,g}\left(X^*_{1,g}(1), 2\right) \tag{3.57}$$

which can be written as:

$$\alpha(1-q)\left(1 - \frac{\theta}{2\left(2-\theta\right)} + \frac{1}{4}\theta\right) + \frac{3}{4}\alpha + \beta \geq -\frac{1}{4}\alpha\theta(1+q) \tag{3.58}$$

$$\alpha(1+q)\left(1 - \frac{1}{4}\theta\right) + \frac{3}{4}\alpha + \beta \geq \alpha(1-q)\left(2 - \frac{\theta}{2\left(2-\theta\right)} + \frac{1}{4}\theta\right) \tag{3.59}$$

Inequality (3.58) is clearly always satisfied in our parameters region. Since $\beta > 0$, a sufficient condition for (3.59) is that:

$$(1+q)\left(1 - \frac{1}{4}\theta\right) + \frac{3}{4} \geq (1-q)\left(2 - \frac{\theta}{2\left(2-\theta\right)} + \frac{1}{4}\theta\right)$$

which is satisfied as long as:

$$q \geq \frac{2+\theta-2\theta^2}{2(12-7\theta)}$$

Notice that for every value of $\theta \in (0,1)$, the RHS of the previous inequality is lower than $\frac{2}{5}$ and thus the previous condition is weaker than condition (3.43) and can be ignored.

c) Finally, when the good manager observes $v_1 = 1, 2$, he is better off playing

$X^*_{1,q,g}(v_1)$ instead of deviating to any positive order off the equilibrium path.

Formally, let $x_1 \in \mathbb{R}\backslash\{1+q,\, 1-q\}$. Then, it must be that:

$$\text{for } v_1 = 1, 2,\ \Pi^e_{tot,g}\left(X^*_{1,g}(v_1), v_1\right) \geq \Pi^e_{tot,g}\left(x_1, v_1\right)$$

Focusing on the buy side of the problem, we have to consider the two separated cases in which $x_1 \in (0, 2)$ and $x_1 \geq 2$.

$\mathbf{x}_1 \geq \mathbf{2}$. Given market makers' first-round price strategy for out of equilibrium aggregate orders (3.12), we have that $P^e_1(x_1) = 2$ and thus:

$$E(\Pi_{1,g}\left(x_1\right)|v_1) = \alpha x_1\left[v_1 - P^e_1(x_1)\right] + \beta = \alpha x_1\left[v_1 - 2\right] + \beta$$

Furthermore, given (3.49), it is also true that for $v_1 = 1, 2$, $\Pr(\widehat{\theta} > \theta | x_1, v_1) = 0$. Thus, total expected profits from deviating to any $x_1 \geq 2$ are equal to $\alpha x_1\left[v_1 - 2\right] + \beta$. Notice that $\alpha x_1\left[v_1 - 2\right] \leq 0$ (remember that we are focusing on $v_1 = 1, 2$). Therefore, deviations to any $x_1 \geq 2$ are (weakly) dominated by the choice of not trading, which we have shown to be non optimal.

$\mathbf{x} \in (0, 2)$. Again, given (3.12) we can compute:

$$P^e_1(x_1) = \frac{1}{2}P_1(x_1+1) + \frac{1}{2}P_1(x_1-1) = \frac{1}{2}2 + \frac{1}{2}2(x_1-1) = x_1$$

and accordingly

$$E(\Pi_{1,g}\left(x_1\right)|v_1) = \alpha x\left[v_1 - P^e_1(x_1)\right] + \beta = \alpha x_1\left(v_1 - x_1\right) + \beta$$

Again, given (3.49), we have that for $v_1 = 1, 2$, $\Pr(\widehat{\theta} > \theta | x, v_1) = 0$. Thus, when the true value of the asset is observed to be $v_1 = 1$, the total expected profits from deviating to any $x_1 \in (0, 2)$ are equal to:

$$E(\Pi_{1,g}\left(x_1\right)|v_1 = 1) = \alpha x_1\left[1 - x_1\right] + \beta$$

On the other hand, when the true vale of the asset is observed to be $v_1 = 2$, the total expected profits from deviating to any $x_1 \in (0, 2)$ are equal to

$$E(\Pi_{1,g}\left(x_1\right)|v_1 = 2) = \alpha x_1\left[2 - x_1\right] + \beta$$

Using the previous results, the conditions for the equilibrium can be written as:

$$\begin{aligned}\alpha(1-q)\left(1-\frac{\theta}{2(2-\theta)}+\frac{1}{4}\theta\right)+\frac{3}{4}\alpha+2\beta &\geq \alpha x_1\left[1-x_1\right]+\beta \\ \alpha(1+q)\left(1-\frac{1}{4}\theta\right)+\frac{3}{4}\alpha+2\beta &\geq \alpha x_1\left[2-x_1\right]+\beta\end{aligned}$$

and are always satisfied in our parameters region (remember that here we are focusing on $x_1 \in (0,2)$).

Bad managers' strategy in the first round of trade. A bad manager does not observe v_1. Accordingly, let me define the total profits that he expects from trading order $x_1$at the beginning of $s=1$ as follows:

$$\Pi^e_{tot,b}(x_1) = \Pi^e_{1,g}(x_1) + \Pr(\widehat{\theta} > \theta | x_1)\Pi^*_{2,b}$$

where

$$\Pi^e_{1,b}(x_1) = \alpha R^e_{1,b}(x_1) + \beta = -\alpha x_1 P^e_1(x_1) + \beta$$

are the profits that the good manager expects to get in the first round of trade from trade x_1. $\Pr(\widehat{\theta} > \theta | x_1)$ is the probability of being retained if trading x_1. $\Pi^*_{2,g}$ are the *equilibrium* profits that the good manager expects to gain in the second round if he is retained. Since a bad manager does not trade in the second round, $\Pi^*_{2,b} = \beta$. Thus, we can write:

$$\Pi^e_{tot,b}(x_1) = -\alpha x_1 P^e_1(x_1) + \beta + \frac{3}{4}\Pr(\widehat{\theta} > \theta | x_1) \tag{3.60}$$

In order to show that the mixed strategy $X^*_{1,b}$ is an equilibrium strategy, we have to show that $\forall x_1 \geq 0$ the following condition holds:

$$\Pi^e_{tot,b}(X^*_{1,b}) \geq \Pi^e_{tot,b}(x_1) \tag{3.61}$$

Notice that using (3.42) and (3.50), we have that

$$\begin{aligned}&\Pi^e_{tot,b}(X^*_{1,b}) = \alpha R^e_{1,b}\left(X^*_{1,b}\right) + \beta + \Pr(\widehat{\theta} > \theta | X^*_{1,b})\beta = \\ &-\alpha(1-q)\left(\frac{\theta}{2(2-\theta)} - \frac{\theta}{4}\right) + \frac{5}{4}\beta\end{aligned}$$

I will prove (3.61) by considering the following different cases:

a) Playing $X^*_{1,b}$ is better than deviating to any other order that is consistent with the equilibrium. Notice that

$$\Pi^e_{tot,b}(X^*_{1,b}) = \frac{1}{2}\Pi^e_{tot,b}(1-q) + \frac{1}{2}\Pi^e_{tot,b}(-1+q)$$

It is trivial to show that $\Pi^e_{tot,b}(1-q) = \Pi^e_{tot,b}(-1+q)$ and thus that

$$\Pi^e_{tot,b}(X^*_{1,b}) = \Pi^e_{tot,b}(1-q) = \Pi^e_{tot,b}(-1+q)$$

In fact the bad manager is indifferent between $X^*_{1,b}$ and playing either the pure strategy $1-q$ or $-1+q$. It is also trivial to show that $\Pi^e_{tot,b}(1+q) = \Pi^e_{tot,b}(-1-q)$. Therefore, we just have to check that

$$\Pi^e_{tot,b}(X^*_{1,b}) \geq \Pi^e_{tot,b}(1+q)$$

which can be written as:

$$(1-q)\left(\frac{\theta}{2(2-\theta)} - \frac{\theta}{4}\right) \leq \left(1 + \frac{1}{4}\theta\right)(1+q) \tag{3.63}$$

It is immediate to check that (3.63) is always satisfied in our parameters region. Notice that the bad manager prefers $X^*_{1,b}$ to any other mixed strategy $X_{1,b}$ that randomizes over $\{1+q,\, 1-q,\, -1+q,\, -1-q\}$. Indeed, notice that the expected payoff of any mixed strategy $X_{1,b}$ is a convex combination of $\Pi^e_{tot,b}(1-q)$ and $\Pi^e_{tot,b}(1+q)$. That is,

$$\Pi^e_{tot,b}(X_{1,b}) = h\Pi^e_{tot,b}(1-q) + (1-h)\Pi^e_{tot,b}(1+q), \text{ with } h \in [0,1]$$

But since we have shown that $\Pi^e_{tot,b}(X^*_{1,b}) > \Pi^e_{tot,b}(1+q)$ and that $\Pi^e_{tot,b}(X^*_{1,b}) = \Pi^e_{tot,b}(1-q)$, it immediately follows that $\Pi^e_{tot,b}(X^*_{1,b}) > \Pi^e_{tot,b}(X_{1,b})$.

b) $X^*_{1,b}$ is better than refraining from trading. Formally:

$$\Pi^e_{tot,b}(X^*_{1,b}) \geq \Pi^e_{tot,b}(0)$$

Using (3.60) and (3.50), we find that $\Pi^e_{tot,b}(0) = \beta$. Thus, the previous inequality can be written as:

$$-\alpha(1-q)\left(\frac{\theta}{2(2-\theta)} - \frac{\theta}{4}\right) + \frac{5}{4}\beta \geq \beta$$

and simplified to:

$$\beta \geq \alpha(1-q)\frac{\theta^2}{2-\theta} \tag{3.64}$$

c)$X^*_{1,b}$ is better than deviating to any order off the equilibrium path. Let $x_1 \in \mathbb{R}^+ \setminus \{-1-q, \, -1+q, \, 0, \, 1+q, \, 1-q\}$ and consider the two following cases:

$\mathbf{x}_1 \geq \mathbf{2}$. Given (3.12), $P^e(x_1) = 2$. Given ((3.50), $\Pr(\widehat{\theta} > \theta | x_1) = 0$. Thus:

$$\Pi^e_{tot,b}(x_1) = -2\alpha x_1 + \beta$$

Notice that $-2\alpha x_1 < 0$. Therefore, deviations to $x_1 \geq 2$ are strictly dominated by the choice of not trading, which we have just shown to be non optimal.

$\mathbf{x} \in (0, 2)$. In this case $P^e(x_1) = x_1$ and $\Pr(\widehat{\theta} > \theta | x_1) = 0$. Thus:

$$\Pi^e_{tot,b}(x_1) = -\alpha x_1^2 + \beta$$

Again, $-\alpha x_1^2 < 0$ and consequently also deviations to $x_1 \in (0, 2)$ are strictly dominated by the choice of not trading. Thus, a bad managers will never deviate to out of equilibrium orders.

We can then conclude that under condition (3.64), $X^*_{1,q,b}$ is the equilibrium first-round strategy of the bad managers.

Therefore, we have that good an bad managers follow their equilibrium strategies $X^*_{1,q,g}(\cdot)$, $X^*_{1,q,b}$ and $X^*_{2,q,g}(\cdot)$, $X^*_{2,q,b}$ as long as conditions (3.43) and (3.64) are satisfied, that is:

$$\frac{2}{5} \leq q < \frac{2}{3}$$
$$\beta \geq \alpha(1-q)\frac{\theta^2}{2-\theta}$$

This implies that a sufficient condition on β, expressed in terms of the parameters of the game, can be written as follows:

$$\beta \geq \frac{3\alpha\theta^2}{5\,(2-\theta)} \tag{3.65}$$

Market makers' strategy in the first round of trade. Given bad and good managers' equilibrium strategies (3.9) and (3.8), aggregate order $Z_1(u,v,i)$ satisfies:

$$
\begin{aligned}
Z_1(u,v,i) &= \begin{cases} 2+q & \text{if } \{u=1, v=2, i=g\} \\ 2-q & \text{if } \{u=1, v=1, i=g\} \text{ or } \{u=1, i=b \text{ with } x_{1,b}=1-q\} \\ q & \text{if } \{u=-1, v=2, i=g\} \text{ or } \{u=1, v=-1, i=g\} \\ & \text{or } \{u=1, i=b \text{ with } x_{1,b}=-1+q\} \end{cases} \\
Z_1(u,v,i) &= -Z_1(-u,-v,i)
\end{aligned}
$$

Focus on positive aggregate orders. When a market maker observes $z_1 = 2+q$ he immediately infers that the good type is trading and that the value of the asset is $v_1 = 2$. When he observes $z_1 = 2-q$, he updates the distribution of v_1 as follows:

$$
\begin{aligned}
\Pr(v &= 2|2-q) = \Pr(v=-1|2-q) = \Pr(v=-2|2-q) = \frac{1-\theta}{2(2-\theta)} \\
\Pr(v &= 1|2-q) = \frac{\theta+1}{2(2-\theta)}
\end{aligned}
$$

and accordingly sets $P(2-q) \equiv E(v|2-q) = \frac{\theta}{2-\theta}$. Analogously, when he observes $z_1 = q$, he computes:

$$
\begin{aligned}
\Pr(v &= 2|q) = \frac{1}{4}(1+\theta) \\
\Pr(v &= 1|q) = \frac{1}{4}(1-\theta) \\
\Pr(v &= -1|q) = \frac{1}{4}(1+\theta) \\
\Pr(v &= -2|q) = \frac{1}{4}(1-\theta)
\end{aligned}
$$

and sets $P(q) \equiv E(v|q) = \frac{\theta}{2}$. Therefore, for the set of positive aggregate orders that can arise in equilibrium, we obtain the following market maker's price strategy:

$$
P_1(z_1) = \begin{cases} 2 & \text{if } z_1 = 2+q \\ \frac{\theta}{2-\theta} & \text{if } z_1 = 2-q \\ \frac{\theta}{2} & \text{if } z_1 = q \end{cases}
$$

Following the same logic, it is easy to show that for negative aggregate orders $z_1 = -q, -2+q, -2-q$ we have that

$$
P_1(z_1) = -P_1(-z_1)
$$

This proves that for equilibrium aggregate orders, the price strategy of the market maker is indeed given by (3.10).

In order to complete the construction of the price function in the candidate equilibrium q, we need to construct the price schedule according to which the market maker sets prices in response to out of equilibrium aggregate orders. In this respect, market maker's out of equilibrium beliefs about information content of trade are crucial. It is important to notice that in the present game, PBE does not imply any restriction on market maker's out of equilibrium beliefs. Thus, the market maker's price response to any out of equilibrium order can be every price belonging to $[-2, 2]$. Accordingly, I arbitrarily assume that market makers price out of equilibrium aggregate orders according to (3.12).

Investor's decision at the beginning of the first round. An investor is completely uninformed about the liquidation value of the asset. He has to decide whether to delegate trade to a fund manager or stay out of the market. Suppose the investor chooses to hire a manager. With probability θ he hires a good manager. The first-round return that he expects in equilibrium from a good manager reads:

$$
\begin{aligned}
&E\left[R^e_{1,g}\left(X^*_{1,g}(v_1), v_1\right)\right] = \\
&= \frac{1}{4}(1+q)\left[2 - P^e_1(1+q)\right] + \frac{1}{4}(1-q)\left[1 - P^e_1(1-q)\right] + \\
&+ \frac{1}{4}(-1+q)\left[-1 - P^e_1(-1+q)\right] + \frac{1}{4}(-1-q)\left[-2 - P^e_1(-1-q)\right] = \\
&= \frac{1}{2}(1+q)\left(1 - \frac{1}{4}\theta\right) + \frac{1}{2}(1-q)\left(1 - \frac{\theta}{2(2-\theta)} + \frac{1}{4}\theta\right) = \\
&q\left(\frac{\theta}{4(2-\theta)} - \frac{1}{4}\theta\right) + 1 - \frac{\theta}{4(2-\theta)}
\end{aligned}
$$

On the other hand, with probability $1-\theta$ the investor hires a bad manager. The first-round return that he expects in equilibrium from a bad manager reads:

$$
\begin{aligned}
E\left[R^e_{1,b}\left(X^*_{1,b}\right)\right] &= \frac{1}{2}(1-q)\left(-\frac{\theta}{2(2-\theta)} + \frac{1}{4}\theta\right) + \frac{1}{2}(-1+q)\left(\frac{\theta}{2(2-\theta)} - \frac{1}{4}\theta\right) = \\
&= -\frac{1}{2}(1-q)\left(\frac{\theta}{2-\theta} - \frac{1}{2}\theta\right)
\end{aligned}
$$

Thus, the *equilibrium* average return from delegation expected by the investor in the first round is equal to

$$\begin{aligned} E\left(R_1^e\right) &= \theta E\left[R_{1,g}^e\left(X_{1,g}^*(v_1), v_1\right)\right] + (1-\theta)E\left[R_{1,b}^e\left(X_{1,b}^*\right)\right] = \\ &= \left(1 - \frac{1}{4}\theta\right)\theta \end{aligned} \tag{3.66}$$

and the *equilibrium* payoff that the investor expects from the average manager in the first round is given by:

$$(1-\alpha)\left(1 - \frac{\theta}{4}\right)\theta - \beta$$

Since the investor has the option of staying out of the market, he will hire a manager at the beginning of the first round of trade if and only if the payoff of delegation is positive. This is guaranteed when

$$\beta < (1-\alpha)\left(1 - \frac{\theta}{4}\right)\theta \tag{3.67}$$

Notice that in our parameters region it is always true that

$$\frac{3}{4}\theta < \left(1 - \frac{\theta}{4}\right)\theta$$

Thus, condition (3.67) is weaker than condition (3.44), and can then be ignored.

Summing up, the relevant conditions on β are represented by (3.44) and (3.65), which can be written as:

$$\frac{3\alpha\theta^2}{5\,(2-\theta)} \leq \beta < (1-\alpha)\frac{3}{4}\theta$$

Notice that for the previous inequality to be meaningful, it must be that $\frac{3\alpha\theta^2}{5(2-\theta)} < (1-\alpha)\frac{3}{4}\theta$. It is easy to show that this inequality is guaranteed as long as the following condition on α holds:

$$\alpha < \frac{5\,(2-\theta)}{10-\theta}$$

Notice that the RHS of the previous condition is positive and less than 1 for every value of $\theta \in (0, 1)$, which guarantees that α takes values in its proper range $(0, 1)$. Notice that a sufficient condition on α, would be that $\alpha < \frac{5}{9}$. Accordingly, a sufficient condition on β would be $\frac{1}{3}\frac{\theta^2}{2-\theta} \leq \beta < \frac{1}{3}\theta$. ∎

Proof of proposition 4. I will prove only part a) of the proposition, since the proof of part b) is based on the same logic. Let $\alpha \to 0^+$ and $\theta = \frac{1}{2}$ and proceed by backward induction.

Second round of trade. The last round of the trade game described in section 4 is equivalent to that of the trade game described in section 3.2. Hence, by following the proof of proposition 3, it can be easily shown that as long as the two following conditions are met:

$$\beta \in \left(0, \frac{3}{8}\right) \tag{3.68}$$

and

$$q \in \left[\frac{2}{5}, \frac{2}{3}\right) \tag{3.69}$$

there exists a family of equilibria for the second round of trade where good managers follow

$$X^*_{2,g}(v_2) = \begin{cases} 1+q & \text{when } v_2 = 2 \\ 1-q & \text{when } v_2 = 1 \end{cases}$$
$$X^*_{2,g}(-v_2) = -X^*_{2,g}(v_2)$$

and bad managers do not trade, as prescribed by proposition 3.[30]

Investor's retaining rule at the end of the first round of trade and investors' belief $\widehat{\theta}$. By following the arguments in the proof of proposition 3, it is easy to show that the investor retains the incumbent manager as long as $\widehat{\theta} > \theta$, with $\widehat{\theta}$ given by (3.45) and (3.46).

First round of trade. In the first round of trade, aggregate order for the asset reads:

$$Z(u_1, v_1, i) = \begin{cases} 2+q & \text{when } \{u_1 = 1, v_1 = 2, i = g\} \\ 2-q & \text{when } \{u_1 = 1, v_1 = 1, i = g\} \\ & \text{or } \{u_1 = 1, i = b \text{ with } x_{1,b} = 1-q\} \\ q & \text{when } \{u_1 = -1, v_1 = 2, i = g\} \\ & \text{or } \{u_1 = 1, v_1 = -1, i = g\} \\ & \text{or } \{u_1 = 1, i = b \text{ with } x_{1,b} = -1+q\} \end{cases}$$

[30] Note that for $\alpha = 0$, both the good and the bad manager would in fact be indifferent about any trading strategy in the last round. Indeed, in the last round of trade, good and bad managers' expected profits from placing order x_2 read respectively:

$$\begin{aligned} E(\Pi_{2,g}(x_2)|v_2) &= \alpha R^e_{2,g}(x_2) + \beta = \alpha x_2 \left[v_2 - P^e_2(x_2)\right] + \beta \\ E(\Pi_{2,b}(x_2)) &= \alpha R^e_{2,b}(x_2) + \beta = -\alpha x_2 P^e_2(x_2) + \beta \end{aligned}$$

Therefore, for $\alpha = 0$, any order x_2 delivers the same profit β, both to good and bad managers. However, it also true that for any positive (whatever small) α, the good manager follows $X^*_{2,G}(v)$ and the bad manager does not trade. The case $\alpha \to 0^+$ is meant to represent this situation in which the good manager's indifference is broken in favour of the strategy that maximizes investor's expected return (From a technical point of view, notice that both $E(\Pi_{2,g}(x_2)|v_2)$ and $E(\Pi_{2,g}(x_2))$ are continuous in $\alpha \in (0,1)$).

By following the usual reasoning (see proof of proposition 3), it is easy to show that given bad and good managers' equilibrium strategies in the first round, market maker's pricing strategy in the first round of trade is described by the odd function $P_1(z_1) = -P_1(-z_1)$, with

$$P_1(z_1) = \begin{cases} 2 & z_1 = 2+q \\ \frac{1-2w}{2(1-w)} & z_1 = 2-q \\ \frac{6w-1}{2} & z_1 = q \end{cases} \tag{3.70}$$

As for the out of equilibrium aggregate orders, we assume that the market maker's pricing strategy is the usual one, that is:

$$P_1(z_1) = \begin{cases} 2 & z_1 \geq 1,\ z_1 \neq 2-q, 2+q \\ 2z_1 & -1 \leq z_1 \leq 1,\ z_1 \neq -q, q \\ -2 & z_1 \leq -1,\ z_1 \neq -2-q, -2+q \end{cases} \tag{3.71}$$

Before going on, notice that given this market makers' pricing strategy, we can compute the prices expected by a manager when he places equilibrium orders $1+q$, $1-q$, $-1+q$ and $-1-q$:

$$\begin{aligned} P_1^e(1+q) &= -P_1^e(-1-q) = 1 + \frac{1}{4}(6w-1) \\ P_1^e(1-q) &= -P_1^e(-1+q) = \frac{1}{4}\left(\frac{1-2w}{1-w}\right) - \frac{1}{4}(6w-1) \end{aligned}$$

Good managers' strategy in the first round. Proposition 4 prescribes that good managers trade according to

$$\begin{aligned} X_{1,g}^*(v_1) &= \begin{cases} 1+q & \text{when } v_1 = 2 \\ 1-q & \text{when } v_1 = 1 \end{cases} \\ X_{1,g}^*(-v_1) &= -X_{1,g}^*(v_1) \end{aligned}$$

Let good manager's total expected profits from order x_1 at the beginning of the first round be given by:

$$\Pi_{tot,g}^e(x_1, v_1) = \Pi_{1,g}^e(x_1, v_1) + \Pr\left(\widehat{\theta} > \theta | x_1, v_1\right) \Pi_{2,g}^*$$

For $\alpha \rightarrow 0^+$, order x_1 does not affect good manager's payoff directly, and $\Pi_{1,g}^e(x_1, v_1) =$

$\Pi^*_{2,g} = \beta$. Therefore, we can write:

$$\Pi^e_{tot,g}(x_1, v_1) = \left[1 + \Pr\left(\widehat{\theta} > \theta | x_1, v_1\right)\right] \beta$$

In order to show that $X^*_{1,g}(v_1)$ is optimal for the good manager in the first round, we have to show that for every $v_1 = -2, -1, 1, 2$ and every $x_1 \in \mathbb{R}$, the following holds true:

$$\left[1 + \Pr\left(\widehat{\theta} > \theta | X^*_{1,g}(v_1), v_1\right)\right] \beta \geq \left[1 + \Pr\left(\widehat{\theta} > \theta | x_1, v_1\right)\right] \beta$$

Given investor's beliefs (3.45) and (3.46), for a good manager it is true that:

$$\Pr(\widehat{\theta} > \theta | x_1, v_1) = \begin{cases} 1 \text{ if } x_1 = X^*_{1,g}(v_1) \\ 0 \text{ if } x_1 \neq X^*_{1,g}(v_1) \end{cases}$$

Thus, the optimality condition for $X^*_{1,g}(v_1)$ is always trivially satisfied as long as $\beta > 0$.

In the present context, it important to note that since $\alpha \to 0^+$, the specific order placed by a good manager does not affect his payoff directly, and good manager's and investor's incentives are not obviously aligned. Order x_1 affects the payoff by affecting good manager's probability of being retained. As we know, this probability crucially depends on investor's beliefs about the strategy that a good manager follows in equilibrium. Here, we are (reasonably) focusing on the class of equilibria in which the investor conjectures that in equilibrium good managers do follow the strategy that maximizes the expected return from trade, $X^*_{1,g}(v_1)$.

Remark 4 *For $\alpha \to 0^+$, the good manager follows his equilibrium strategies $X^*_{1,g}(v_1)$ and $X^*_{2,g}(v_2)$ as long as $\beta > 0$ and condition (3.69) is satisfied, that is $q \in \left[\frac{2}{5}, \frac{2}{3}\right)$.*

Bad managers' strategy in the first round. We know that in equilibrium the bad managers does not trade in the second round. Furthermore, the bad manager does not observe v_1. Accordingly, we can write his total expected profits from order x_1 at the beginning of the first round as follows:

$$\Pi^e_{tot,g}(x_1) = \left[1 + \Pr\left(\widehat{\theta} > \theta | x_1\right)\right] \beta$$

In order to show that strategy $X^*_{1,b}$ of randomizing with equal probability over $1 - q$ and $-1 + q$ is the optimal strategy for the bad manager in the first round, we have to show that for every $x_1 \in \mathbb{R}$, the following holds true:

$$\left[1 + \Pr\left(\widehat{\theta} > \theta | X^*_{1,b}\right)\right] \beta \geq \left[1 + \Pr\left(\widehat{\theta} > \theta | x_1\right)\right] \beta$$

Again, given investors' beliefs (3.45) and (3.46), we have that for a bad manager

$$\Pr(\widehat{\theta} > \theta | x_1) = \begin{cases} \frac{1}{2} - w & \text{if } x_1 = 1 - q, -1 + q \\ w & \text{if } x_1 = 1 + q, -1 - q \\ 0 & \text{otherwise} \end{cases}$$

and accordingly

$$\Pr(\widehat{\theta} > \theta | X^*_{1,b}) = \frac{1}{2} - w$$

Thus, it must be true that for every $x_1 \in \mathbb{R}$,

$$\left(1 + \frac{1}{2} - w\right) \beta \geq \left[1 + \Pr\left(\widehat{\theta} > \theta | x_1\right)\right] \beta$$

The previous condition is always trivially satisfied for orders $x_1 \neq 1 + q$, $1 - q$, $-1 + q$, $-1 - q$, for which $\Pr(\widehat{\theta} > \theta | x_1) = 0$.[31] Let $X^+_{1,b}$ denote a mixed strategy that consists in randomizing over the equilibrium orders $1 + q$ and $-1 - q$. In this case

$$\Pr(\widehat{\theta} > \theta | X^+_{1,b}) = w$$

and the equilibrium condition must now satisfy

$$\left(\frac{1}{2} - w\right) \beta \geq w\beta$$

which is always satisfied as long as $\beta > 0$ and $w \in \left(0, \frac{1}{4}\right]$. Note that when w gets greater than $\frac{1}{4}$, the bad manager has an incentive to deviate to from the equilibrium mixed strategy $X^*_{1,b}$ to $X^+_{1,b}$. Hence, the conjecture at the base of part b) of proposition 4 that for $w \in \left(\frac{1}{4}, \frac{1}{2}\right)$ there exist equilibria in which bad managers randomizes over $1 + q$ and $-1 - q$.

Remark 5 *For $\alpha \to 0^+$, the bad managers follows his mixed strategy of randomizing over $1 - q$ and $-1 + q$ in the first round and does not trade in the second round as long*

[31] Clearly, also any mixed strategy $X_{1,b}$ that randomizes over orders $x_1 \in \mathbb{R} \backslash \{1 + q, 1 - q, -1 + q, -1 - q\}$ gives rise to $\Pr(\widehat{\theta} > \theta | X_{1,b}) = 0$ and thus is dominated by $X^*_{1,b}$

as $\beta > 0$ *and* $w \in \left(0, \frac{1}{4}\right]$

The investor's problem. An investor hires a fund manager at the beginning of the first round if the profits he expects to gain from delegating trade are higher than those he would get by staying out of the market. When $\alpha \to 0^+$, the investor finds it convenient to hire a fund manager in the first round of trade as long as:

$$E\left(R_1^e\right) - \beta \geq 0$$

where

$$E\left(R_1^e\right) = \frac{1}{2}E\left(R_{1,g}^e\right) + \frac{1}{2}E\left(R_{1,b}^e\right)$$

and

$$\begin{aligned} E\left(R_{1,g}^e\right) &= 2w(1+q)\left[1 - \frac{1}{4}(6w-1)\right] + (1-2w)(1-q)\left[1 - \frac{1}{4}\frac{1-2w}{1-w} + \frac{1}{4}(6w-1)\right] \\ E\left(R_{1,b}^e\right) &= -(1-q)\left[\frac{1}{4}\frac{1-2w}{1-w} - \frac{1}{4}(6w-1)\right] \end{aligned}$$

After a bit of algebra, one can show that

$$E\left(R_1^e\right) = \frac{5}{2}w - 3w^2 \tag{3.72}$$

Therefore, delegation occurs as long as $E\left(R_1^e\right) - \beta \geq 0$, or equivalently:

$$\beta \leq \frac{5}{2}w - 3w^2$$

Notice that $E\left(R_1^e\right)$ is strictly increasing in $w \in \left(0, \frac{1}{4}\right]$. Furthermore, it is easy to compute that $\lim_{w\to 0^+} E\left(R_1^e\right) = 0^+$, while for $w = \frac{1}{4}$, $E\left(R_1^e\right) = \frac{7}{16}$. Hence, $E\left(R_1^e\right) \in \left(0, \frac{7}{16}\right]$.

Remark 6 *When* $w \in \left(0, \frac{1}{4}\right]$ *and* $\alpha \to 0^+$, *for any* $\beta \leq \frac{5}{2}w - 3w^2$ *an investor finds it convenient to hire a fund manager at the beginning of the first round of trade.*

Note that condition (3.68) implies $\beta \leq \frac{3}{8}$. Since $\frac{5}{2}w - 3w^2 \in \left(0, \frac{7}{16}\right]$, we can safely conclude that a manager is hired in both periods as long as $\beta \leq \min\left(\frac{3}{8}, \frac{5}{2}w - 3w^2\right)$ ■

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