Essays on Monetary Policy and Credit Market Imperfections

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2009

To my family

Acknowledgments

I am in special debt to Tommaso Monacelli for his precious guidance and support during the completion of my graduate studies. Many times he patiently reviewed, criticized, corrected and inspired my research, during instructive and guiding discussions. My toolkit as an economist and a researcher is largely based on those discussions.

I am particularly grateful to Carlo Favero for introducing me to the fascinating world of quantitative macroeconomics and, most importantly, for providing generous encouragement and trust when it was most needed.

Special thanks to Matthieu Darracq Pariès, with whom I co-authored a paper that underlies the contents of chapter 2.

Many people have provided insights, offered suggestions and inspired my work over these years. Among them, I wish to thank Ester Faia, who acted as a member of my thesis committe, and Zvi Hercowitz, who carefully read a preliminary draft of chapter 1 and provided detailed and inspiring comments. I also benefited from discussions with faculty members at Bocconi, in particular Francesco Giavazzi, Luca Sala and Antonella Trigari. Seminar and conference participants at various universities and institutions also provided useful hints.

Finally, I wish to thank, without implicating, the Monetary Policy Strategy Division at the European Central Bank and the Economic Modelling and Forecasting Division of the Research Department at Banca d'Italia for providing an ideal environment where part of this thesis was written.

And most of all, my immense gratitude goes to my family, for supporting me all the way through.

Abstract

The role of credit market imperfections in the business cycle has been the object of study of a large part of modern macroeconomics. The understanding of the interactions of limited access to financial markets with other sources of cyclical fluctuations is of particular relevance in the conduct of macroeconomic policy in general and monetary policy in particular. By altering the cost of financing, monetary authorities intervene directly in the economic structure and can indirectly influence the decisions of the agents.

This thesis analyzes the role of financial frictions in the specific form of borrowing constraints, mainly on the consumers side.

Chapter 1 investigates the role of financial liberalization and monetary policy in explaining the reduced volatility of nondurable consumption, residential investment and household debt in the US business cycle over the last three decades. The question is addressed by using a dynamic stochastic general equilibrium (DSGE) model with household heterogeneity and collateral constraints. The model is estimated, via Bayesian methods, over two distinct time periods (before and after financial liberalization). The estimation exercise shows that housing preference shocks have become more relevant for the business cycle, relative to monetary policy shocks; in addition, the inclusion of household credit frictions helps the model fitting the data better than standard DSGE models.

Chapter 2 considers the relevance of credit market imperfections and household debt in an open-economy model calibrated on US and euro area data. In this framework, the housing sector can be analyzed as a special case of non-traded-goods-producing sector which also provides collateral to the borrowing households. Such a model can explain the observed

simultaneity of a sustained US dollar depreciation against the Euro and an unprecedented increase in housing prices. Moreover, the model can account for some existing open-economy puzzles such as the consumption-real exchange rate anomaly.

Chapter 3 addresses the contribution of household credit market imperfections in an international real business cycle (RBC) model. The existing models typically fail at reproducing the size and sign of the observed cross-country correlations in output, consumption, investment and hours worked. The inclusion of housing collateral helps solving the consumption-correlation anomaly and, under some assumption on individual preferences, generates realistic comovements in hours worked.

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Chapter 1

Credit Frictions and Household Debt in the US business cycle: A Bayesian Approach

1.1 Introduction

The significant reduction in the volatility of most U.S. macroeconomic variables is by now a well-documented phenomenon, that goes under the name of Great Moderation. The large reduction in the standard deviation of GDP growth over the last 30 years is perhaps the most immediate source of such evidence. Consumption of nondurable and durable goods, investment and particularly residential investment also experienced a significant reduction in time variation. A large and growing strand of the literature has provided quantitative analysis on the sources and the cyclical effects of such reduced volatility[1].

During approximately the same period of time, the U.S. financial markets underwent a process of deregulation and liberalization, which deeply modified the access to funds for households and businesses. Developments in the loan markets have substantially improved households' financing conditions. In particular, the amount of collateralized household debt has significantly increased: as documented by Dynan and Sichel (2006), the ratio of household debt to disposable personal income doubled during the period 1960-2004, and the private debt/GDP ratio has grown larger than 100% since 2005. Interestingly, such an increase in levels (and

[1]See, among others, Justiniano and Primiceri (2008) and references therein for a list of contributions.

growth rates) has not been accompanied by a higher volatility. The data show in fact a reduction in the volatility of household debt since 1982, with a new increase after 2000. Figures 1 and 2 report the time series of both the level and the standard deviation of real per capita household debt, while Figures 3 and 4 plot the evolution in the standard deviation of consumption and residential investment. Table 1 and Table 2 document the evolution in the standard deviation of the same variables.

The contemporaneous occurrence of a significant macroeconomic stabilization and a radical evolution of financial market structures has been recently analyzed both empirically and theoretically[2]

This paper proposes the estimation of a Dynamic Stochastic General Equilibrium (DSGE) model with household credit market imperfections. The main objective is an evaluation of the relative importance of financial markets liberalization and changes in the conduct of monetary policy in explaining the reduced volatility of nondurable consumption, residential investment and household debt. Workhorse DSGE models with nominal and real rigidities are usually mute about credit market frictions faced by households. Household debt cannot be treated in the standard New Keynesian model, since the *representative agent* assumption prevents any form of private lending. Therefore, following the seminal contribution of Kiyotaki and Moore (1997), this paper assumes a dual structure on the household side: agents belong to two different groups according to their intertemporal discount factor. The different profile of intertemporal preferences originates a shift of resources across consumers both intratemporally and intertemporally. Household debt thus results in equilibrium from the accumulation of borrowing over time. A second modification relates households' consumption and saving decisions to their balance sheets and the availability of collateralizable durable goods. The relatively impatient agents are in fact assumed to face a collateral constraint, which puts an endogenous limit to the amount of funds they can borrow.

The introduction of these two assumptions has important consequences for the monetary transmission mechanism, which is enriched in three ways. First, the issuing of nominal debt by households generates non-neutrality of monetary policy through the effect of interest rate

[2] See Dynan and Sichel (2006) and Campbell and Hercowitz (2006) as two paradigmatic examples.

movements on both the cost and the real value of debt. Any change in the policy rate produces both a direct change in the cost of servicing debt, and an indirect change in the ex-post real value of outstanding debt, via the effect of monetary policy on the inflation rate. Noticeably, this first channel is independent of nominal rigidities. Second, the existence of a collateral constraint generates a substitution effect between durable and nondurable consumption goods. Intuitively, whenever the collateral constraint tightens (because of changes in the financial markets), agents reduce their borrowing and substitute durable with nondurable consumption. Such an effect is independent of monetary policy decisions, but is likely to interact with them if monetary and financial changes happen to be contemporaneous. The third channel builds on the valuation effect of durable prices on the available amount of collateralizable goods and hence on consumption. Any change in the relative price of durable consumption is reflected into a change in the value of the collateral and implies - ceteris paribus - a change in the demand for durables. The relative degree of price rigidity in the two sectors influences the strength of the valuation channel.

On an independent line of research, Iacoviello and Neri (2007) develop and estimate a similar model, with the aim of explaining the upward trend in US real housing prices over the last forty years. The main difference between their analysis and the one presented here is in the question addressed. The focus of this paper is on the role of monetary policy, housing-related disturbances and financial markets liberalization over the business cycle, regardless of any long-run phenomena such as the run-up in housing prices and collateralized household debt. Using Bayesian methods, the model is estimated on U.S. quarterly data over the period $1965Q1 : 2006Q4$, by considering two distinct sub-sample: the first one goes from $1965Q1$ through $1982Q4$ and the second one goes from $1983Q1$ through $2006Q4$. The choice of such a split is motivated by at least two reasons. First, the last quarter of 1982 is identified in the literature as the end of a high-equity requirement era in financial markets. As documented by Campbell and Hercowitz (2006) and Dynan and Sichel (2006), the Garn-St.Germain Act - passed in late 1982 - started a dramatic increase in loan-to-value ratios and originated an unprecedented development of secondary markets[3]. The time series behavior of household debt also changes completely after that date. Moreover, the second sample leads by only four quarters the conventional beginning of the Great Moderation, usually placed at 1984 Q1[4]. It is therefore natural and somewhat necessary to split the sample in correspondence of such a break point.

The main results of the estimation exercise can be summarized as follows:

1. Housing preference shocks explain almost 46% of the variation in consumption after 1982, as opposed to only 14% in the previous sample. We observe a contemporaneous substantial reduction in the corresponding contribution of monetary policy shocks.

2. The volatility of residential investment and household debt is also largely explained by housing preference shocks, with a quite constant contribution over time. The role of monetary policy and productivity shocks is minor.

3. The estimated standard deviation of all the structural shocks has significantly declined over time. The estimated median values are on average 1.6 times larger in the first period than in the second one.

4. There is a substantial degree of asymmetry in the estimated price stickiness across sectors. The median duration of prices in the nondurable sector is 8.3 quarters - in line with the existing macroeconometric evidence - while prices are almost perfectly flexible in the housing sector. The asymmetry is robust across samples.

The rest of the paper is organized as follows: Section 2 describes the model; Section 3 illustrates the estimation exercise and comments on the results. Section 4 concludes.

1.2 The model

The model describes a two-agent, two-sector economy, with agents belonging to two different groups according to their own intertemporal discount factor, along the lines of Campbell and Hercowitz (2006). Households consume two goods, durables and nondurables, produced in

[3]The Garn-St.Germain Act allowed private savings associations to provide commercial loans and is therefore considered as the beginning of a first significant liberalization process in US financial markets

[4]See McConnell and G.Perez-Quiros (2000).

two different sectors of the economy. Durable goods are interpreted as housing, and serve two purposes: they can be either directly consumed or used as collateral when applying for a loan. Household debt is introduced along with the existence of a collateral constraint on the total amount of borrowing. The next two subsections describe the households' structure in detail.

1.2.1 The impatient agents

The representative *impatient* agent receives utility from the following instantaneous utility function:

$$U(X_t, N_t) = \log(X_t) - \frac{\upsilon}{1+\varphi} N_t^{1+\varphi} \tag{1.1}$$

where X_t is a constant elasticity of substitution (CES) consumption aggregator defined as follows:

$$X_t = \left[(1-\alpha)^{\frac{1}{\eta}} \left(C_t - \theta C_{t-1} \right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} D_t^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{1.2}$$

About notation, C_t denotes consumption of the nondurable good, the parameter θ captures the degree of habit formation in nondurable consumption, and D_t indicates the choice of durable consumption. The term υ is a scale parameter which pins down the amount of hours in steady state, while φ denotes the inverse elasticity of labor supply. The law of motion for durables is defined as follows:

$$D_t = (1-\delta) D_{t-1} + I_t^D$$

with I_t^D denoting durable investment and D_{t-1} being the stock of durables carried over by the previous period.

The agent solves the following intertemporal maximization problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(X_t, N_t) \tag{1.3}$$

subject to the infinite sequence of budget constraints:

$$P_{c,t} C_t + P_{d,t} (D_t - (1-\delta) D_{t-1}) - B_t = -R_{t-1} B_{t-1} + W_t N_t \tag{1.4}$$

where B_t is end-of-period t *nominal* private debt, issued by the impatient agent. The nominal interest rate paid on the existing amount of debt, B_{t-1}, is denoted R_{t-1}, while W_t is the nominal wage received by the agent. The budget constraint can be conveniently rewritten in *real* terms as follows:

$$C_t + q_t(D_t - (1-\delta)D_{t-1}) - b_t = -R_{t-1}\frac{b_{t-1}}{\pi_{c,t}} + \frac{W_t}{P_{c,t}}N_t \tag{1.5}$$

where $q_t \equiv \frac{P_{d,t}}{P_{c,t}}$ is the relative price of durables in terms of nondurable goods, $b_t \equiv \frac{B_t}{P_{c,t}}$ denotes *real* debt (in terms of nondurables), and $\pi_{c,t} \equiv \frac{P_{c,t}}{P_{c,t-1}}$ is nondurable-goods inflation.

Each impatient agent is subject to the following *collateral constraint*:

$$B_t \leq (1-\chi)P_{d,t}D_t \tag{1.6}$$

Following Campbell and Hercowitz (2006), Iacoviello (2005) and Monacelli (2009), we assume that the whole amount of debt is secured by collateral. The parameter $\chi \in [0,1]$ indicates the share of durable goods that cannot be used as a collateral: $(1-\chi)$ thus provides a proxy for the loan-to-value ratio. It is convenient to express (1.6) in terms of nondurable goods as follows:

$$b_t \leq (1-\chi)q_t D_t \tag{1.7}$$

It is possible to show that, whenever $\beta < \gamma$, the collateral constraints always binds in the deterministic steady state[5]. We will therefore assume throughout that the constraint is satisfied with equality in a sufficiently small neighborhood of the steady state, so that the model can be appropriately solved by taking a log-linear approximation around the equilibrium[6].

The impatient agent thus maximizes (1.3) subject to (1.5) and (1.7) satisfied with equality. The corresponding set of first order conditions can be written as follows:

$$q_t U_{c,t} = U_{d,t} + \beta(1-\delta)E_t\left\{U_{c,t+1}q_{t+1}\right\} + (1-\chi)\psi_t U_{c,t} q_t \tag{1.8}$$

$$\psi_t = 1 - \beta E_t\left\{\frac{U_{c,t+1}}{U_{c,t}}\frac{R_t}{\pi_{c,t+1}}\right\} \tag{1.9}$$

Where $U_{c,t}$ and $U_{d,t}$ indicate the marginal utility of nondurable and durable consumption, respectively. Denoting $\lambda_t\psi_t$ the Lagrange multiplier attached to the collateral constraint, it

[5]See Appendix.

[6]The size of the neighborhood directly influences the accuracy of the approximation and is related to the magnitude of the exogenous shocks considered.

is natural to interpret ψ_t in (1.9) as the marginal value of borrowing. More precisely, any rise in ψ_t is equivalent to a tightening of the collateral constraint.

The set of optimality conditions is completed by the intratemporal trade-off between consumption and leisure:

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_{c,t}}$$

with $U_{n,t}$ indicating the marginal utility of working one additional unit of time. The form of such a condition is crucially affected by the labor market structure, and is the object of the next subsection.

1.2.2 Labor Market Structure and Wage Setting

The labor force is made up of *impatient* agents only. This assumption simplifies the model setup and allows for a closed-form computation of the steady state. More precisely, the assumption follows from the observation that the labor supply choice of the patient agents would become irrelevant once their wealth is large enough.

The general setup of the labor market structure follows Erceg, Henderson, and Levin (2000)[7]. There exists a continuum of impatient households (indexed with j) on the unit interval, each supplying a differentiated labor service to the production sector. Each final-good producing firm (in both sectors) uses all of the services in production and perceives each household's labor supply $N_t(j)$ as an imperfect substitute for the labor service provided by another household . We assume the existence of perfectly competitive labor aggregators (or employment agencies) that combine households' specialized labor into labor services available to the intermediate firms. The labor market index N_t^i denotes the amount of labor input used by intermediate firm i:

$$N_t^i = \left(\int_0^1 \left(N_t^i(j) \right)^{\frac{1}{1+\lambda_W}} dj \right)^{1+\lambda_W}$$

where the term $\frac{1+\lambda_W}{\lambda_W}$ represents the elasticity of substitution across differentiated labor services. Labor aggregators minimize the cost of producing a given amount of N_t, taking each

[7]The labor market structure implicitly assumes that the impatient agents are fully insured against wage income shocks, so that their actions can be summarized by the behavior of a representative (impatient) agent. Although the borrowers have limited access to the loan and mortgage markets, we assume in fact that they can trade state-contingent assets among themselves. As a consequence, their consumption and wage profile is unique.

household's wage $W_t(j)$ as given, and sell units of N_t to the production sector at their unit cost W_t, which can be expressed as[8]:

$$W_t \equiv \left(\int_0^1 (W_t(j))^{-\frac{1}{\lambda_W}} \, dj \right)^{-\lambda_W}$$

Total demand for each j-household's labor service is given by:

$$N_t^i(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\frac{1+\lambda_W}{\lambda_W}} N_t^i$$

As already analyzed in Section 2.1, each impatient household maximizes the utility function (1.3) under (1.5) and (1.7). Regarding the choice over the nominal wage, we assume that, each period, only a fraction of households receives a signal that allows for wage changing. The probability that a specific household receives a signal in a given period t is constant and equal to $(1 - \xi_W)$. After receiving the signal, the household sets a new - optimal - nominal wage W_t^*, taking into account the probability of not being allowed to change the wage in the future. For those households that cannot re-optimize, we assume a partial-indexation scheme of the following type:

$$W_t^i = \left(\frac{P_{c,t-1}}{P_{c,t-2}} \right)^{\gamma_w} W_{t-1}^i$$

where $\gamma_w \in [0, 1]$ denotes the degree of indexation to past nondurable inflation[9]. The two extreme cases of no indexation and full indexation correspond to $\gamma_w = 0$ and $\gamma_w = 1$, respectively. The optimality condition for the wage setters thus results in the following dynamic wage mark-up equation:

$$E_t \left\{ \sum_{s=0}^{\infty} \beta^s \xi_w^s N_{t+s}(i) \left[-(1+\lambda_W) U_{n,t+s} + U_{c,t+s} \frac{W_t^*}{P_{c,s}} \left(\frac{P_{c,s-1}}{P_{c,t-1}} \right)^{\gamma_w} \right] \right\} = 0 \tag{1.10}$$

The law of motion of the aggregate nominal wage thus reads:

$$W_t = \left((1-\xi_W)(W_t^*)^{-\frac{1}{\lambda_{W,t}}} + \xi_W \left(\frac{P_{c,t-1}}{P_{c,t-2}} \right)^{-\frac{\gamma_w}{\lambda_{W,t}}} (W_{t-1})^{-\frac{1}{\lambda_{W,t}}} \right)^{-\lambda_{W,t}} \tag{1.11}$$

[8] W_t can be interpreted as the aggregate wage index.

[9] Notice that in the impatient's budget constraint (1.5) the real wage is defined as the ratio between nominal wage (W_t) and nondurable price ($P_{c,t}$). According to this convention, the relevant price index for wage setting is $P_{c,t}$.

Log-linearizing equation (1.10) around the deterministic steady state gives the standard formula:

$$\begin{aligned}\widehat{w}_t = \left(\frac{\beta}{1+\beta}\right) E_t\left\{\widehat{w}_{t+1}\right\} + \left(\frac{\beta}{1+\beta}\right) E_t\left\{\widehat{\pi}_{c,t+1}\right\} + \left(\frac{1}{1+\beta}\right)\widehat{w}_{t-1} \\ - \left(\frac{1+\beta\gamma_w}{1+\beta}\right)\widehat{\pi}_{c,t} + \left(\frac{\gamma_w}{1+\beta}\right)\widehat{\pi}_{c,t-1} - \left(\frac{(1-\xi_W)(1-\beta\xi_W)}{(1+\beta)\,\xi_W\left(1+\frac{1+\lambda_W}{\lambda_W}\varphi\right)}\right)\widehat{\mu}_t^w\end{aligned} \tag{1.12}$$

where variables with a $\widehat{}$ are expressed in log-deviations from their steady-state value. In particular, μ_t^w is the (variable) wage markup, defined as the wedge between the real wage and the marginal rate of substitution between consumption and leisure:

$$\mu_t^w = \frac{\left(-\frac{U_{n,t}}{U_{c,t}}\right)}{\frac{W_t}{P_{c,t}}}$$

The last expression corresponds to equation (1.10) in the case of fully flexible wages.

1.2.3 The patient agents

The representative patient agent receives utility form the following instantaneous utility function:

$$U(\widetilde{X}_t) = \log(\widetilde{X}_t)$$

where $\widetilde{X}_t$ is the CES aggregator:

$$\widetilde{X}_t = \left[(1-\alpha)^{\frac{1}{\eta}}\left(\widetilde{C}_t - \theta\widetilde{C}_{t-1}\right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}\widetilde{D}_t^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{1.13}$$

Notation is analogous to the one adopted for the impatient agent, except that a $\widetilde{}$ is used to denote consumption of each good by the patient agent. In particular, $\widetilde{I}_t^D$ denotes investment in durable consumption:

$$\widetilde{D}_t = (1-\delta)\widetilde{D}_{t-1} + \widetilde{I}_t^D$$

The representative patient agent is characterized by a higher intertemporal discount factor than the impatient agent, denoted $\gamma > \beta$. The patient agents are the owners of firms and capital in the economy, and hence choose consumption plans (over nondurable and durable goods) and investment plans. Patient households choose the utilization rate of capital before renting it to firms at the (nominal) rental market rate R_t^k. Following a large strand of literature, we assume the existence of costs both in changing capital utilization and in physical investment. Denoting $\widetilde{K}_{\iota t-1}$ the stock of existing capital in sector ι at time t, the amount of effective capital that the patient agents can rent to firm j is given by:

$$K_{\iota t}(j) = u_{\iota,t}(j)\widetilde{K}_{\iota,t-1}(j)$$

where $u_{\iota t}$ indicates the degree of capital utilization chosen. The associated cost function is denoted $a(\cdot)$, so that the cost of changing capital utilization is expressed in terms of nondurable goods as follows:

$$P_{ct}a(u_{\iota,t}(j))\widetilde{K}_{\iota,t-1}(j)$$

We assume that, in steady state, $u_{\iota t} = 1$, and $a(1) = 0$. The physical capital accumulation equation reads:

$$\widetilde{K}_{\iota t} = (1-\delta_k)\widetilde{K}_{\iota t-1} + \left[1 - S_\iota\left(\frac{I_{\iota t}}{I_{\iota t-1}}\right)\right] I_{\iota t} \quad ; \quad \iota = c, d \tag{1.14}$$

where δ_k is the depreciation rate of capital, $I_{\iota t}$ is investment in capital, and the function $S(\cdot)$ captures adjustment costs in investment. In particular, we assume that $S'(\cdot) = 0$ and $S''(\cdot) > 0$[10].

The representative patient agent thus solves the following intertemporal maximization problem:

$$\max_{\{\widetilde{C}_t, \widetilde{D}, \widetilde{I}_t, K_t, u_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \gamma^t U(\widetilde{X}_t)$$

subject to:

(i) the infinite sequence of (real) budget constraints:

$$\widetilde{C_t} + q_t\widetilde{I}_t^d + I_{ct} + I_{dt} + b_t - R_{t-1}b_{t-1} = r_t^k\left[u_{c,t}(j)\widetilde{K}_{c,t-1}(j) + u_{d,t}(j)\widetilde{K}_{d,t-1}(j)\right] \tag{1.15}$$
$$- \left[a(u_{c,t}(j))K_{c,t-1}(j) + a(u_{d,t}(j))K_{d,t-1}(j)\right]$$

(where $r_t^k \equiv \frac{R_t^k}{P_{c,t}}$ is the real rental rate of capital in terms of nondurable consumption goods), and

[10] See Appendix for details on the functional form of $a(\cdot)$ and $S(\cdot)$.

(ii) the capital accumulation equation (1.14).

The first order conditions with respect to $\widetilde{C_t}$ and $\widetilde{D}_t$ can be expressed as follows:

$$q_t = \frac{\widetilde{U}_{d,t}}{\widetilde{U}_{c,t}} + \gamma(1-\delta)E_t\left\{\frac{\widetilde{U}_{c,t+1}}{\widetilde{U}_{c,t}}q_{t+1}\right\} \tag{1.16}$$

$$\widetilde{U}_{c,t} = \gamma E_t\left\{\widetilde{U}_{c,t+1}R_t\frac{1}{\pi_{c,t+1}}\right\} \tag{1.17}$$

with $\widetilde{U}_{ct}$ and $\widetilde{U}_{dt}$ denoting the marginal utility of nondurable and durable consumption, respectively, for the patient agent. Equation (1.16) is a standard optimality condition for investment in a durable good: the purchase price of a durable good is equated to the immediate payoff of the purchase (the marginal rate of substitution between durable and nondurable consumption), plus the discounted expected resale value. Equation (1.17) is a standard Euler equation. Turning to the choice of capital, investment and capital utilization, we define $Q_{\iota t}$ to be the ratio between the Lagrange multipliers attached to (1.15) and (1.14) respectively :

$$Q_t^j \equiv \frac{\lambda_t^k}{\lambda_t}$$

The first order conditions can then be rewritten as follows:

$$Q_{\iota t} = E_t\left[\gamma\frac{\widetilde{U}_{c,t+1}}{\widetilde{U}_{c,t}}\left(Q_{\iota t+1}(1-\delta_k) + r_{t+1}^k u_{\iota t+1} - a\left(u_{\iota t+1}\right)\right)\right] \tag{1.18}$$

$$Q_{\iota t}\left[1 - S_\iota\left(\frac{I_{\iota t}}{I_{\iota t-1}}\right) - \frac{I_{\iota t}}{I_{\iota t-1}}S\prime_\iota\left(\frac{I_{\iota t}}{I_{\iota t-1}}\right)\right] + \gamma E_t\left[Q_{\iota t+1}\frac{\widetilde{U}_{c,t+1}}{\widetilde{U}_{c,t}}\left(\frac{I_{\iota t+1}}{I_{\iota t}}\right)^2 S_\iota\prime\left(\frac{I_{\iota t+1}}{I_{\iota t}}\right)\right] = 1 \tag{1.19}$$

$$r_{t+1}^k = a\prime\left(u_{\iota t}\right) \tag{1.20}$$

Following a common practice in the literature[11], $Q_{\iota t}$ can be interpreted as Tobin's Q; it is equal to one in the absence of adjustment costs.

1.2.4 Firms

Production of durable and nondurable goods is modeled in the standard New Keynesian way. In each sector there exists perfectly competitive final-good firms which produce a single

[11] See Smets and Wouters (2007) among others.

good out of a continuum of intermediate goods. The intermediate-good firms operate in a monopolistically competitive market, where each firm produces a single differentiated good and thus exerts some market power. Nominal rigidities are introduced in the form of staggered price setting à la Calvo (1983) in the intermediate-goods sector.

Final-good producers

In each production sector the perfectly competitive final-good firms produce the final consumption good $Y_{\iota t}$ using the intermediate inputs $Y_{\iota t}(j)$:

$$Y_{\iota t} = \left(\int_0^1 Y_{\iota t}^{\frac{1}{1+\lambda_{p\iota}}} (j) dj \right)^{1+\lambda_{p\iota}}$$

where $Y_{\iota t}(j)$ denotes the the quantity of intermediate good of type j demanded by the final good producer in sector ι ($\iota = C, D$) at date t. The term $\frac{1+\lambda_{p\iota}}{\lambda_{p\iota}}$ denotes the sector-specific elasticity of substitution between differentiated varieties. The demand function for each intermediate good j reads:

$$Y_{\iota t}(i) = \left(\frac{P_{\iota t}(j)}{P_{\iota t}} \right)^{-\frac{1+\lambda_{p\iota}}{\lambda_{p\iota}}} Y_{\iota t}$$

where $P_{\iota t}$ denotes the sectoral price index, which is defined using profit maximization and zero-profit conditions as follows:

$$P_{\iota t} = \left(\int_0^1 P_{\iota t}^{-\frac{1}{\lambda_{p\iota}}} (j) dj \right)^{-\lambda_{p\iota}} \tag{1.21}$$

Clearly, λ_{pj} represents the sectoral price markup over marginal costs.

Intermediate-good producers and price setting

Each intermediate-good producing firms j operates the following technology:

$$Y_{\iota t}(j) = \max \left\{ \varepsilon_t^{a\iota} \left((1-\omega) K_{\iota t}(j) \right)^{\alpha} \left(\omega N_{\iota t}(j) \right)^{1-\alpha} - \Phi_{y\iota} \overline{Y_\iota}; 0 \right\} \tag{1.22}$$

where $\varepsilon_t^{a\iota}$ is an exogenous stochastic process describing the evolution of productivity in each sector ι, with $\iota = C, D$:

$$\varepsilon_t^{a\iota} = \rho_{a\iota} \varepsilon_{t-1}^{a\iota} + \eta_t^{a\iota}$$

with $\eta_t^{a\iota}$ IID-Normal. The term ω in (1.22) denotes the fraction of impatient agents in the economy. The total amount of hours supplied to each j firm by the mass of impatient agents is therefore given by $\omega N_{\iota t}(j)$. Analogously, the total amount of capital that the impatient agents rent to firm j is $(1-\omega)K_{\iota t}(j)$. Finally, $\Phi_{y\iota}$ is a fixed cost in production, and $\overline{Y_\iota}$ is the steady state value of $Y_{\iota t}$. Labor and capital are assumed to be fully mobile across sectors, so that the nominal wage and the rental rate of capital are unique. Solving the firms' static profit-maximization problem yields the following definition of marginal cost:

$$MC_{\iota t} = \frac{1}{\varepsilon_t^{a\iota}} \left(\frac{W_t}{1-\alpha} \right)^{1-\alpha} \left(\frac{R_t^k}{\alpha} \right)^{\alpha}$$

Regarding price setting, firms change their prices à la Calvo (1983), i.e. after receiving a random price-change signal, exactly as in the case of wage-setting operated by the impatient agents. The probability that a given firm receives the signal in each period is constant and equal to $(1-\xi_{p\iota})$. Partial indexation to past inflation is assumed for those firms that do not receive the signal. Solving the intertemporal profit-maximization problem for those firms that are allowed to reoptimize, and denoting P_t^* the newly set price, gives the following dynamic mark-up equation:

$$E_t \left\{ \sum_{s=0}^{\infty} \xi_{p\iota}^s \gamma^s \frac{\lambda_{t+s}}{\lambda_t} \frac{P_{\iota t}}{P_{\iota t+s}} Y_{\iota t+s}(j) \left[\frac{P_{\iota t}^*(j)}{P_{\iota s}} \left(\frac{P_{\iota s-1}}{P_{\iota t-1}} \right)^{\gamma_{p\iota}} - (1+\lambda_{p\iota}) MC_{j,s} \right] \right\} = 0 \qquad (1.23)$$

where $\gamma^s \frac{\lambda_{t+s}}{\lambda_t} \frac{P_{\iota t}}{P_{\iota t+s}}$ is the stochastic discount factor for the patient agents, who run the firms. The law of motion of the price index in each sector follows from definition (1.21):

$$P_{\iota t} = \left((1-\xi_{p\iota}) \left(P_{\iota t}^*\right)^{-\frac{1}{\lambda_{p\iota}}} + \xi_{p\iota} \left(\frac{P_{\iota t-1}}{P_{\iota t-2}} \right)^{-\frac{\gamma_{p\iota}}{\lambda_{p\iota}}} \left(P_{\iota t-1}\right)^{-\frac{1}{\lambda_{p\iota}}} \right)^{-\lambda_{p\iota}}$$

Log-linearization of (1.23) around the deterministic steady state yields the following sectoral New Keynesian Phillips Curve:

$$\widehat{\pi}_{\iota t} = \left(\frac{\gamma_{p\iota}}{1+\gamma_{p\iota}} \right) \widehat{\pi}_{\iota t-1} + \left(\frac{\gamma}{1+\gamma\gamma_{p\iota}} \right) E_t \left\{ \widehat{\pi}_{\iota t+1} \right\} + \qquad (1.24)$$
$$+ \left(\frac{(1-\xi_{p\iota})(1-\gamma\xi_{p\iota})}{(1+\gamma)\,\xi_{p\iota}} \right) \widehat{mc}_{\iota t}$$

where, again, a $\widehat{}$ denotes deviations of a variable from its steady state value, and $mc_{\iota t}$ is the *sectoral real* marginal cost in period t:

$$mc_{\iota t} \equiv \frac{MC_t}{P_{\iota t}}$$

1.2.5 Monetary policy

The monetary authority sets the short-term nominal interest rate according to the following log-linearized Taylor-type rule:

$$\widehat{r}_t = \rho_r \widehat{r}_{t-1} + (1-\rho_r)\phi_\pi \widehat{\pi}_{c,t-1} + \phi_{\Delta\pi}\left(\widehat{\pi}_{c,t} - \widehat{\pi}_{c,t-1}\right) + \phi_{\Delta y}\left(\widehat{y}_t - \widehat{y}_{t-1}\right) + \eta_t^r \tag{1.25}$$

where variables in deviations from their steady state are denoted with a $\widehat{}$ and η_t^r is the monetary policy shock, which is assumed to be iid log-normally distributed. We consider nondurable inflation as a target for the central bank although, in principle, the monetary authority could specify rule (1.25) by targeting aggregate inflation π_t, or durable inflation π_{dt}. In particular, it is possible to recover the following relationship between aggregate and sectoral inflation rates:

$$\pi_t = \pi_{\iota t}\frac{g_{\iota t}}{g_{\iota t-1}}, \qquad \iota = c, d$$

where

$$g_{ct} \equiv \frac{P_t}{P_{ct}} = \left[(1-\alpha) + \alpha q_t^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

and

$$g_{dt} \equiv \frac{P_t}{P_{dt}} = \left[\alpha + (1-\alpha)\, q_t^{-(1-\eta)}\right]^{\frac{1}{1-\eta}}$$

1.2.6 Market clearing

The goods market clearing conditions in the two sectors read:

$$Y_{ct} = \omega \widehat{C}_t + (1-\omega)\,\widetilde{C}_t + (1-\omega)\left(I_{ct} + I_{dt}\right) \tag{1.26}$$

and

$$Y_{d,t} = \omega \widehat{I}_{d,t} + (1-\omega)\widetilde{I}_{d,t} \tag{1.27}$$

where clearly:

$$Y_{\iota t} \equiv \int_0^1 Y_{\iota t}(j)dj = (1-\omega)^{\alpha}\omega^{1-\alpha}\varepsilon_t^{a\iota}\int_0^1 K_{\iota t}^{\alpha}(j)N_{\iota t}^{1-\alpha}(j)dj = (1-\omega)^{\alpha}\omega^{1-\alpha}K_{\iota t}^{\alpha}N_{\iota t}^{1-\alpha}$$

Finally, the labor market clearing condition reads:

$$N_{c,t} + N_{d,t} = N_t$$

1.3 The estimation exercise

The overall structure of the artificial economy is enriched with a number of exogenous structural shocks before performing the estimation exercise. In addition to the sectoral technological shifts ε_t^{ac} and ε_t^{ad}, and the monetary policy shock η_t^r, we consider shocks to the intertemporal discount factor, housing preference shocks that perturb the weight assigned to housing in the instantaneous utility function, sectoral investment-specific shocks, labor supply shocks, sectoral cost-push shocks and finally shocks to the loan-to-value ratio[12]. The application of standard log-linearization solution methods permits to solve the model and cast it in state-space form; it is then immediate to compute the likelihood function using the Kalman filter. After specifying independent prior distributions for the structural parameters, the application of Markov Chain Monte Carlo (MCMC) methods delivers estimates of the posterior distributions.

The model is estimated on quarterly U.S. data. The set of observables includes nondurable consumption, residential investment, household debt, nominal interest rate, consumer price inflation, real house prices, nonresidential fixed investment, real output, and hours worked in the consumption-good sector[13]. The estimation is performed on two separate sub-samples, using the last quarter of 1982 as a break point. The choice of such a period is motivated by at least two reasons. First, Campbell and Hercowitz (2006) identify the Monetary Control Act of 1980 and the Garn-St.Germain Act of 1982 as two crucial events that somehow initiated a new era in the U.S. equity requirement legislation. The Garn-St.Germain Act, by allowing savings and loan associations to provide commercial loans, strongly contributed to reduce

[12]Appendix A illustrates the complete model in detail.

[13]See Appendix for a detailed description of the dataset.

equity requirements in the mortgage market. Although other events occurred in the mortgage markets in about the same period that dramatically accelerated the development of a secondary market[14], it seems reasonable to consider the last quarter of 1982 as a break point. Visual inspection of the time series behaviour of household debt confirms the validity of the choice[15]. Second, the chosen break point leads by 4 quarters the conventional starting point of the Great Moderation, usually placed at 1984 Q1[16]. Therefore, the two sub-samples (pre- and post- financial liberalization) approximately coincide with two very different periods of time, characterized not only by different institutional environments in the financial markets, but also by different magnitudes in the volatility of most macroeconomic variables. The main objective of the estimation exercise is to assess the relative importance of changes in exogenous shocks and in the endogenous transmission mechanisms across periods. Noticeably, a similar exercise is reported in Smets and Wouters (2007), who make use of a standard DSGE model for the U.S. economy, providing a natural benchmark to compare the results from an enriched model to.

1.3.1 Calibration and Prior Distributions

Calibrated parameters

Some of the structural parameters have to be calibrated and excluded from the estimation set. In particular, the agents' intertemporal discount factors are chosen as follows: the patient agent's impatience rate γ is calibrated in such a way to obtain a steady-state value of the net nominal interest rate equal to 1% on a quarterly basis. The impatient agent's rate, β, is instead fixed at 0.96: this calibration is in line with the literature on heterogeneous agents models (see Krusell and Smith (1998), Campbell and Hercowitz (2006) and Iacoviello and Neri (2007))[17]. The elasticity of substitution between durable and nondurable goods is set to one, thus implying the limiting case of a Cobb-Douglas function in equations (1.2) and (1.13)[18].

[14] See Gerardi and Willen (2007) for a discussion

[15] See Figure 1.

[16] See McConnell and G.Perez-Quiros (2000).

[17] Notice that β cannot be determined by using steady state ratios, nor does it influence the interest rate. Thus, some degrees of freedom are left in its choice

[18] Ogaki and Reinhart (1998) report estimates for the intratemporal elasticity of substitution between durable and nondurable goods that range between 1.17 and 1.24. Changing the calibrated value of such elasticity does not significantly affect our estimates.

The relative share of durable goods in the aggregator, α, is set to 0.4. Such a value is picked in order to obtain an equilibrium ratio between residential investment and output equal to 0.04, as empirically measured in the sample. About the loan-to-value ratio, the available data on car and mortgage loans show signs of significant changes in equity requirements across period. Looking at the reported values for LTVs may be insufficient, though, given the significant development of the secondary market for mortgages in the second sample. The empirical evidence suggests that after 1982 households typically obtain more than one mortgage on the same house. As an example, consider a first mortgage with a down payment of 25% and a second mortgage, to pay the down payment on the first one, also with a 25% down payment. The two mortgages would appear as separate in the data, and the reported loan-to-value ratio would not change. However, from an economic point of view the correct measure of the down payment is $0.25 * 0.25 = 0.0625$, which corresponds to the amount actually paid by the household[19]. To reflect the observed changes in secondary markets, we calibrate the parameter χ (which in the model is a proxy for the loan-to-value ratio) to 0.75 in the first sub-sample, and to 0.92 in the second one. The depreciation rate for housing, δ, is parameterized to an annual value of 10% corresponding to a quarterly value of 0.0025[20]. The value of δ_k is instead 0.03 on a quarterly basis. The difference reflects the slower depreciation of houses relative to capital. Finally, the elasticities of substitution among differentiated goods and labor types are calibrated to yield a steady-state markup of, respectively, 16% in the nondurable sector, 10% in the durable sector, and 50% in the labor market. The share of capital in the production function, α, is set to 0.3.

Prior distributions

The specification of independent priors is summarized in columns 3, 4 and 5 of Table 1. Priors are quite loose and noninformative in general, especially for the parameters governing nominal rigidities. This implies that no stand is taken a priori on the relative flexibility of prices and wages in the economy, as well as on the magnitude of other behavioural parameters. A Beta distribution is assumed for those parameters that can only assume values in the unit inter-

[19] I am indebted to Zvi Hercowitz for suggesting me this example, as well as how to measure the LTV ratio across samples.
[20] See Campbell and Hercowitz (2006) and Monacelli (2009) for a discussion on how to pick a value for δ.

val. In particular, the mean of the habit persistence parameter, θ, is set to 0.65, consistently with existing estimates (see Christiano, Eichenbaum, and Evans (2005)). The inverse elasticity of substitution is assumed to follow a Gamma distribution, with mean 2 and standard deviation 0.75. Regarding nominal rigidities, we assume a Beta distribution with mean 0.5 and standard deviation 0.28 for all the parameters controlling wage and price stickiness and indexation. Such a distribution is almost flat over the unit interval, with some curvature close to the boundaries, to help the estimation process. About monetary policy, the parameters describing the Taylor rule are centered around standard values. Finally, noninformative priors are used for the standard deviations of the six structural shocks, which are assumed to follow a Uniform distribution over the interval [0,6]. The persistence parameters follow the same Beta distribution used for price and wage stickiness parameters.

1.3.2 Sub-sample Estimation: 1965 I-1982 IV

Column 6 of Table 3 reports the posterior median of the structural parameters over the first sub-sample[21]. The estimated degree of habit persistence is quite lower if compared to the values reported in the existing literature, possibly reflecting some averaging effect between habits of patient and impatient agents. The posterior median of the inverse elasticity of labor supply, φ, is 1.86, quite higher than 1.52, the value reported by Smets and Wouters (2007) for the sub-period 1966 I-1979 II. Regarding nominal rigidities, the median of ξ_{pc} is 0.879, corresponding to an interval of 8 quarters between two consecutive price adjustments in the nondurable-producing sector. The corresponding indexation parameter γ_{pc} is instead very low (0.08). Conversely, price stickiness is very low in the durable sector, where the estimated median for ξ_{pd} is 0.00099, equivalent to only one quarter between two price changes. The model thus captures, at least in the first sub-sample, a clear sectoral asymmetry in terms of price flexibility. Flexible prices in the durable sector are usually assumed - rather than estimated - in the literature. In particular, while Barsky, House, and Kimball (2007) argue that a two-sector New Keynesian model exhibits monetary neutrality under flexible durable prices,

[21] Figure 5 reports prior and posterior distributions for the estimated parameters, obtained after 1,000,000 Metropolis-Hastings simulations. The draws were sufficient to guarantee convergence for all the parameters, according to the criteria illustrated in the Appendix. Convergence diagnostics are available from the author upon request.

Monacelli (2009) shows that non-neutrality arises in the same type of model when a collateral constraint is introduced. Our estimates thus support the existence of asymmetric price rigidities in the presence of credit market frictions. The results can be compared to the existing macro and microeconomic literature. On the one hand, estimated DSGE models generally report quite high estimates for the Calvo parameter, although no distinction is usually made between durable and nondurable goods. Smets and Wouters (2005) report 0.87 as the mode of the posterior distribution of the price stickiness parameter. However, existing microeconometric studies document a much lower degree of observed price stickiness in disaggregated data. Bils and P.Klenow (2004) report a median duration of prices of approximately 1.5 quarters, corresponding to $\xi_p = 0.3$. Recently, Nakamura and Steinsson (2006) have shown that the median duration ranges between 8 and 11 months if sales and price changes due to product substitution are excluded from the sample. The implied value for ξ_p is 0.68. The estimated degree of price stickiness is thus in line with the macroeconomic literature on one side (the nondurable sector), while confirming the intuition that prices are much more flexible in the durable sector, when the latter is identified with the housing sector[22]. The overall degree of nominal rigidities is reduced relative to standard estimated New Keynesian models. This can be at least partially explained by the presence of an additional transmission channel for monetary policy, which operates even in the absence of price and wage stickiness. Such a channel builds on the existence of nominal household debt and works through a collateral constraint which in turn hinges on the availability of durable goods. Differences in the estimated degree of nominal rigidity across sectors must therefore be somewhat related to this new channel.

Turning to wage stickiness, the posterior median of ξ_w is 0.972, with an associated degree of wage indexation equal to 0.038. Overall, these estimates suggest a high degree of nominal rigidities in the labor market. The result is possibly influenced by the assumption that only the borrowers contribute to the labor force, without any sectoral-specific preference for hours worked in their utility function. Exploring different alternative setups for the labor market is the object of future research.

Regarding monetary policy, the estimated response to contemporaneous inflation is 1.567,

[22]See Barsky, House, and Kimball (2007) for an argument.

while the response to output is 0.11756. The coefficient attached to the lagged interest rate is 0.819, which implies a substantial degree of persistence of monetary policy changes.

Finally, about the stochastic structure of the model, all the estimated autocorrelation coefficients of the structural shocks are higher than 0.7, with four exceptions, given by the sectoral investment-specific and cost-push shocks. The medians of ρ_i and ρ_{cp} are very low (0.027 and 0.039, respectively), while the corresponding values for ρ_{i_d} and ρ_{cp_d} are 0.367 and 0.585. Supply side shocks thus seem to have little persistence over the first sample. In terms of volatilities, the highest values are given by investment-specific shocks in both sectors ($\sigma_i = 1.0345$ and $\sigma_{i\_d} = 2.2918$), labor supply shocks ($\sigma_n = 4.042$) and cost-push shocks in the durable sector ($\sigma_{cp\_d} = 3.4427$).

1.3.3 Sub-sample Estimation: 1983 I-2006 IV

The last column of Table 3 reports the estimated posterior medians of the structural parameters over the second sub-sample[23]. Estimated behavioral parameters are lower in the second sub-sample, if compared to the first one. Nominal rigidities instead seem to have increased both in the goods and in the labor market, while indexation has increased, with the only exception of the durable sector. Such evidence confirms the results reported in Smets and Wouters (2007), although the cutoff for the two sub-samples is different. In particular, the sectoral asymmetry in price stickiness is confirmed: the median of the Calvo parameters is equal to 0.88 in the nondurable sector and to 0.0024 in the durable sector. Again, the data seem to point towards price flexibility in the durable sector, which is identified here with the housing sector.

Turning to the monetary policy parameters, the medians of the posterior distributions are quite stable across samples. We do not observe an increase in the response of the nominal interest rate to contemporaneous and lagged inflation, as economic intuition would suggest. The change in the conduct of monetary policy started in 1979 - with the appointment of Paul Volcker as Chairman of the Federal Reserve Board - should in fact be reflected in higher values of ϕ_π in the second sub-sample, in principle. However, the identification of Taylor

[23]Figure 6 reports prior and posterior distributions obtained after 200,000 Metropolis-Hastings simulations.

rule parameters in DSGE models is generally problematic[24]. Posterior estimates are usually highly dependent on prior specification, and the data are not very informative. The prior and posterior plots reported in Smets and Wouters (2003) are a paradigmatic example. It is therefore hard to conclude that higher estimates for ϕ_π over the "Great Moderation" sub-samples are a clear indication of a change in monetary policy. Rather, the use of unchanged priors over the two samples suggests that the data are equally not informative about this parameter. Figure 3 illustrates the problem.

Turning to the structural shocks, we only observe a significant difference in the medians of the posterior distributions of ρ_n and ρ_{cp_d}, which move from 0.73 to 0.89 and from 0.58 to 0.79, respectively. More interestingly, the estimated volatilities of all shocks show signs of a significant reduction. The estimated median volatilities are on average 1.6 times larger in the first sub-sample relative to the second one. In particular, the standard error of monetary policy shocks is more than three times as large in the first period than in the second one. The most pronounced change is in the volatility of intertemporal preference shocks, which declined by 67%. Housing preference shocks display a 25% reduction in volatility, approximately equivalent to the decline in the volatility of loan-to-value ratio shocks. Productivity shocks also show a reduced variability, especially in the nondurable sector. Overall, the empirical evidence seems to suggest that a change occurred in the structural shocks hitting the U.S. economy after 1983, with a substantial reduction in their volatility, and a less pronounced - and less general - increase in their persistence.

Summarizing, the estimation exercise performed over the two samples leads to three main conclusions:

1. There is a substantial degree of asymmetry in the estimated price stickiness across sectors. The median duration of prices in the nondurable sector is 8.3 quarters - in line with the existing macroeconometric evidence - while prices are almost perfectly flexible in the housing sector. The asymmetry is robust across samples.

2. The estimated standard deviation of *all* the structural shocks exhibits a significant de-

[24]See Canova and Sala (2006).

cline over time. The estimated median values are on average 1.6 times larger in the first period (up until 1982) than in the second one. No significant change is observed instead for the persistence of such shocks.

3. No significant change is observed in the estimates of the Taylor rule coefficient across periods. In fact, it is hard to conclude that the coefficients are correctly identified.

1.3.4 Variance decomposition and the role of shocks

Tables 4 and 5 provide the variance decomposition of the forecast error over the two samples. Generally speaking, most of the variability of consumption, residential investment and household debt is explained by housing-specific preference shocks. The relative contribution of technology shocks in both sectors is quite low. Loan-to-value ratio shocks are almost as relevant as technology shocks in the first sample, but their role is larger afterwards. More precisely, 45.69% of the volatility of consumption is explained by housing preference shocks in the second sample, as opposed to only 14.25% in the first one. The contribution of monetary policy shocks has correspondingly declined from 56.51% to 35.77%. Such a change is likely to capture the effects of mortgage markets liberalization, which provides households with more instruments to adjust their consumption profiles, thus making them less dependent on monetary policy decisions. Any increase in housing demand (due to a change in individual preferences, in this case) has a larger impact on consumption, via the availability of more credit (as implied by higher loan-to-value ratios and developed secondary markets).

The contribution of housing preference shocks has remained very large for both residential investment (from 76.92% to 73.36%) and debt (from 90.92% to 85.54%). Looking at residential investment, we observe an increase in the role of loan-to-value ratio shocks, which explain 1.4% of the variance before 1983 and 5.66% afterwards. Intuitively, deregulation and liberalization in financial markets have increased the access to funds for borrowers, thus creating a stronger link between changes in financial conditions (as captured - or proxied - by shocks to the loan-to-value ratio) and investment decisions, exactly as for consumption. Correspondingly, changes in interest rates affect less the borrowers' choice over residential investment. Finally, the role of cost-push shocks in the housing sector has increased in general, the more so in the case of residential investment (from 1.33% to 5.88%).

Overall, the variance decomposition exercise suggests the following conclusions:

1. Housing preference shocks explain most of the volatility of consumption after 1982, compensating for a contemporaneous decrease in the role played by monetary shocks;

2. Residential investment and household debt are also mainly explained by housing preference shocks, with a quite stable contribution over time;

3. The relative contribution of monetary policy and productivity shocks is minor.

1.3.5 Assessing the role of financial deregulation and monetary policy

Having estimated the model on the two samples, it is straightforward to evaluate the relative role played by financial deregulation and changes in the conduct of monetary policy after 1982, in explaining the volatility of consumption, residential investment and household debt. A crucial assumption concerns the magnitude of the exogenous shocks. In order to assess the pure contribution of policy changes, it is necessary to shut down any other possible source of variation in the observed variables. Therefore, we fix the volatility of the shocks to the estimates obtained over the first sample, and perform a counterfactual simulation exercise. More precisely, we calibrate all the structural parameters to the estimated values for the period 1965 IV : 1982 IV, to have a benchmark specification for our model as of 1983 I. Next, we modify the parameters that capture changes in financial market regulation and in the conduct of monetary policy, respectively. In the first case, we change the value of $(1-\chi)$ in the collateral constraint from 0.75 to 0.90. Such a change is meant to reproduce the increased availability of credit to households that characterizes the post-1982 period[25]. The results are reported in Table 7. The effect of increasing the loan-to-value ratio alone implies a 10% decrease in the contribution of monetary policy shocks to the variability of consumption, whereas no significant change is implied in the variations of residential investment and household debt. At the same time, the role of preference shocks - and housing demand shocks in particular - is increased.

[25] See above for an explanation.

1.3.6 Fit of the model

The empirical performance of the model is evaluated by comparing the fit of two alternative specifications. More precisely, we estimate a one-agent version of the model, featuring the same two-sector structure, and the same type of nominal rigidities. Clearly, household debt does not appear in this modified version[26]. The application of Bayesian methods allows for model comparison in a straightforward way: the relative performance of each model is evaluated by measuring the corresponding Marginal Likelihood. Table 6 reports the estimated Marginal Likelihood of each model on the two samples, computed using the Laplace Approximation and the Modified Harmonic Mean method. The results reported in the table suggest two main conclusions. First, the complete model beats the benchmark in each sample, independently of the metric adopted. Second, both models display a higher measure of fit in the second sample. In particular, the relative change in the Marginal Likelihood is higher for the two-agent model, which is rich enough to capture the modified financial market structure of the post-1982 period.

We can thus conclude that the data point towards a model featuring collateral constraints and household debt, as opposed to a simple New Keynesian structure. Such evidence supports the ongoing debate over the role of various types of credit market frictions - in addition to nominal rigidities - in explaining business cycle fluctuations[27].

1.4 Conclusions

The reduction in the volatility of most U.S. macroeconomic variables during the so-called "Great Moderation" has been particularly significant for consumption and residential investment. During approximately the same period, financial markets deregulation and liberalizations gave rise to an increase in the level of household debt, while reducing its volatility. This paper builds and estimates a DSGE model featuring household heterogeneity and collateral constraints, following Kiyotaki and Moore (1997) and Campbell and Hercowitz (2006). The

[26] Basically, the alternative formulation boils down to a standard representative agent model with nondurable consumption goods and residential investment. No private debt arises in such a framework, and no collateral constraint is imposed on the agent.

[27] See Christiano, Motto, and Rostagno (2003) for an alternative way of modeling financial frictions.

presence of collateral constraints faced by the relatively more impatient agents enriches the traditional transmission mechanism of monetary policy shocks along several dimensions. The estimation exercise is performed over two separate samples, corresponding to very different macroeconomic and financial environments. The results lead to four main conclusions: *(i)* housing preference shocks explain almost 46% of the variation in consumption after 1982, as opposed to only 14% in the previous sample; *(ii)* residential investment and household debt are also mainly explained by housing preference shocks, with a quite constant contribution over time; *(iii)* the volatility of all the structural shocks has substantially declined over time; *(iv)* prices are relatively sticky for durable goods, while they are flexible in the housing sector.

This paper thus contributes to the existing literature in two ways. First, it enriches the workhorse DSGE model framework with a detailed description of household credit market imperfections. Second, it provides new evidence about the ongoing debate on the Great Moderation. In particular, our estimates suggest that housing preference shocks are the main determinant of the volatility of consumption, residential investment and household debt. In particular, the role of such shocks in explaining consumption fluctuations has grown remarkably large after 1982, testifying the effects of financial market liberalization and the development of secondary loan and mortgage markets.

Future research will explore how a modified labor market structure - that includes both type of agents - alters the main conclusions of this paper.

1.5 Appendix

1.5.1 Figures and Descriptive Statistics

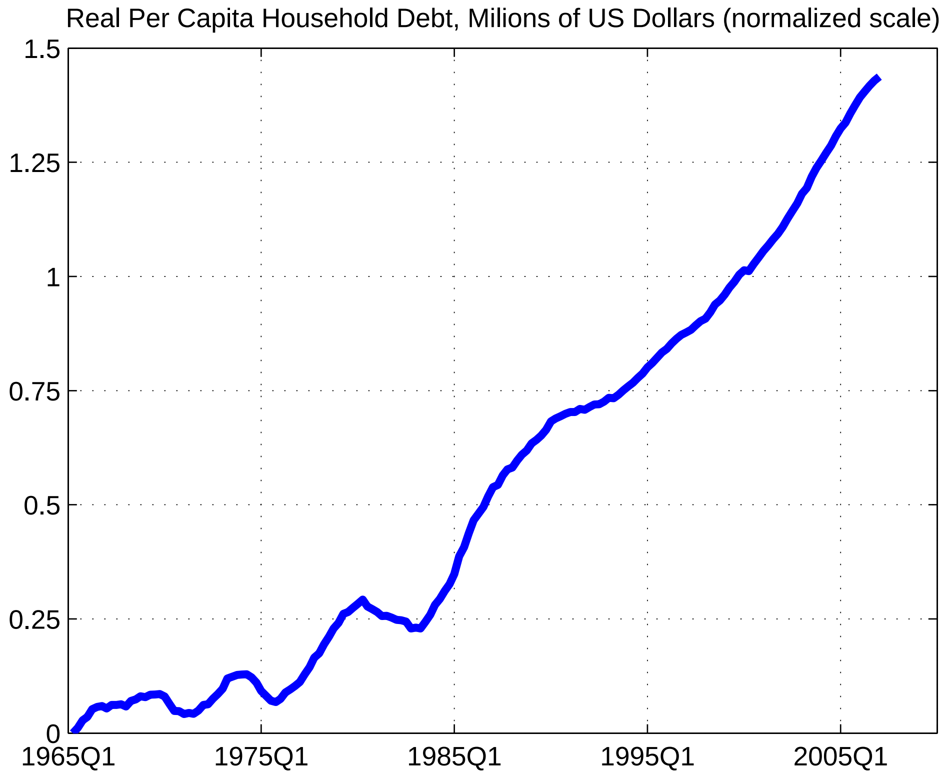

Figure 1.1: Household debt in levels

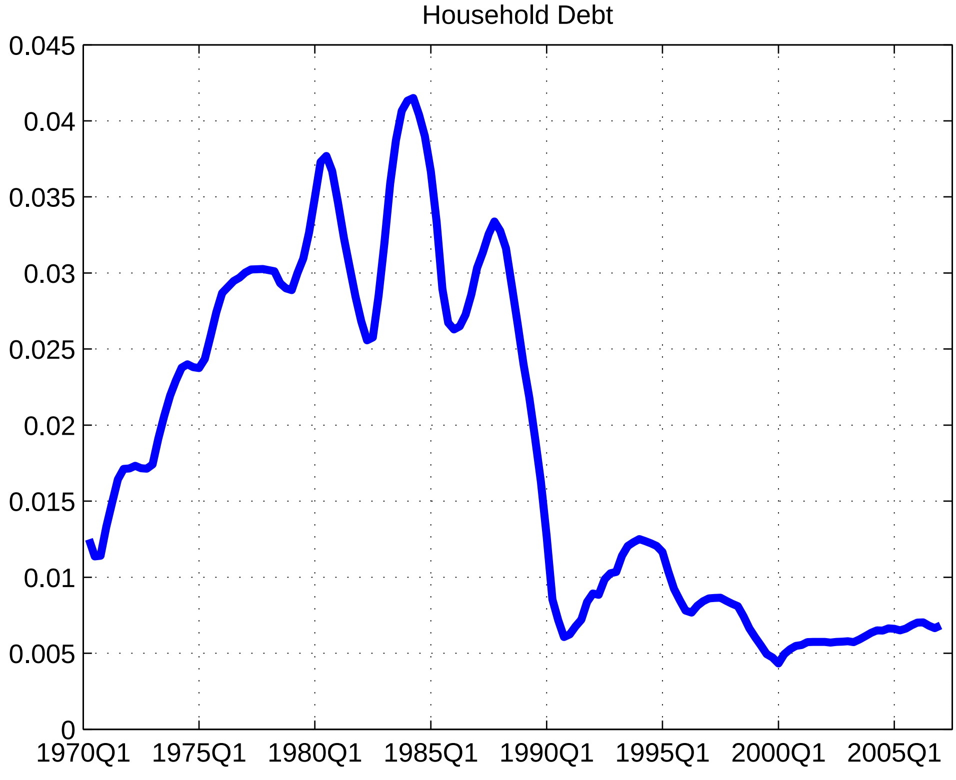

Figure 1.2: Standard deviation of household debt (computed using a 5-year moving window; HP-filtered data)

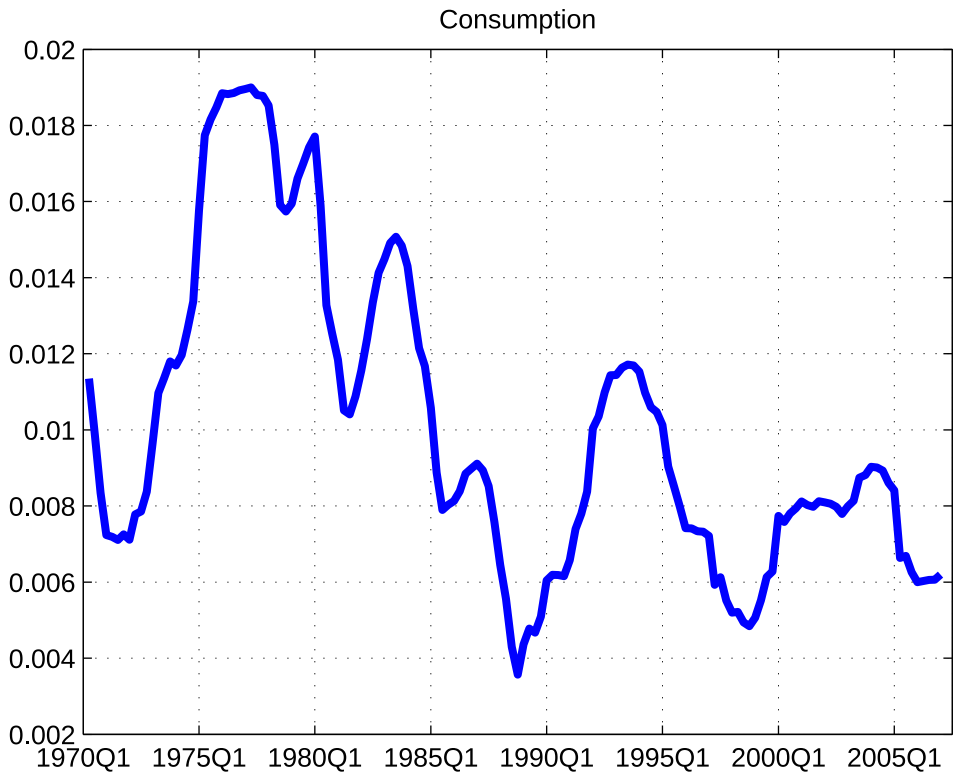

Figure 1.3: Standard deviation of nondurable consumption (computed using a 5-year moving window; HP-filtered data)

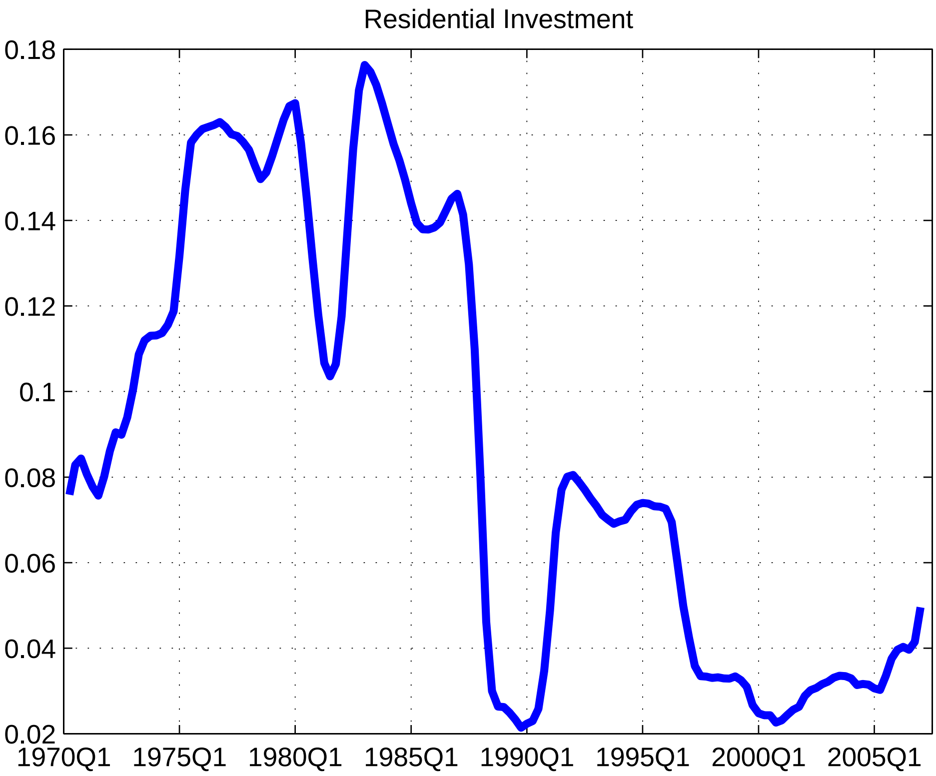

Figure 1.4: Standard deviation of residential investment (computed using a 5-year moving window; HP-filtered data)

Table 1.1: STANDARD DEVIATION OF VARIABLES IN THE TWO SUB-SAMPLES

	$1965Q1 : 1982Q4$	$1983Q1 : 2006Q4$
Std. Dev. of data (%)		
Consumption	4.04	3.71
Residential Investment	19.35	11.34
Household Debt (Total)	11.20	8.56

Table 1.2: DESCRIPTIVE STATISTICS FOR THE STANDARD DEVIATION OF THE VARIABLES (COMPUTED USING A FIVE-YEAR MOVING WINDOW

	$1965Q1 : 1982Q4$		$1983Q1 : 2006Q4$	
	Mean	Std. Dev.	Mean	Std. Dev.
Time Evolution in Std. Dev.				
Consumption	1.63	0.48	0.99	1.30
Residential Investment	15.21	3.57	4.27	1.75
Household Debt (Total)	3.29	1.11	2.32	1.30

1.5.2 The complete model

The model is enriched with a number of structural shocks in order to perform the estimation exercise. In addition to sectoral technological change and monetary policy shocks, we consider intertemporal preference shocks, housing preference shocks, sectoral investment-specific shocks, labor supply shocks, sectoral cost-push shocks, and finally shocks to the loan-to-value ratio. This section illustrates the complete model in detail.

The representative *impatient* agent solves the following intertemporal maximization problem:

$$\max E_0 \sum_{t=0}^{\infty} \varepsilon_t^b \beta^t U(X_t, N_t) \tag{1.28}$$

subject to (1.5) and (1.7) satisfied with equality. The intertemporal preference disturbance ε_t^b evolves exogenously according to an AR(1) process:

$$\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b$$

with $\eta_t^b \sim N(0, \sigma_b)$. We introduce two types of shocks in the specification of $U(X_t, N_t)$. First, a labor supply shock in the form of an exogenous disturbance hitting labor supply:

$$U(X_t, N_t) = \log(X_t) - \frac{\upsilon \varepsilon_t^n}{1+\varphi} N_t^{1+\varphi}$$

with

$$\varepsilon_t^n = \rho_n \varepsilon_{t-1}^n + \eta_t^n,\ \eta_t^n \sim N(0, \sigma_n)$$

Second, a housing-specific preference shock that influences the weight attributed to housing services in the consumption aggregator:

$$X_t = \left[(1 - (\varepsilon_t^d \alpha))^{\frac{1}{\eta}} (C_t - \theta C_{t-1})^{\frac{\eta-1}{\eta}} + (\varepsilon_t^d \alpha)^{\frac{1}{\eta}} D_t^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

with

$$\varepsilon_t^d = \rho_d \varepsilon_{t-1}^d + \eta_t^d,\ \eta_t^d \sim N(0, \sigma_d)$$

Regarding the collateral constraint, the simple formulation for the provided by equation (1.7) can be easily extended to account for variations in equity requirements over time. Time series evidence confirms that, among other things, the average loan-to-value ratio has been increased over time, showing signs of cyclical fluctuations that reflect a more general change in financial constraints faced by households and businesses. A natural way of capturing such evolution is suggested by the interpretation of the parameter χ. As already pointed out, χ indicates the share of durable goods that cannot be used as a collateral, so that $(1-\chi)$ approximately measures the loan-to-value ratio. In a dynamic setting, the loan-to-value ratio is better interpreted as a variable, which moves over time according to some exogenous process. The collateral constraint then modifies to:

$$b_t \leq \varepsilon_t^{ltv}(1-\chi)q_t D_t \tag{1.29}$$

where ε_t^{ltv} denotes an exogenous stochastic term perturbing the loan-to-value ratio in period t. Such term evolves according to the following exogenous process:

$$\varepsilon_t^{ltv} = \rho_{ltv}\varepsilon_{t-1}^{ltv} + \eta_t^{ltv}, \eta_t^{ltv} \sim N(0, \sigma_{ltv})$$

Turning to the patient agent, we assume that the same shock to the intertemporal discount factor of the impatient agent, ε_t^b, is at work, so that the intertemporal utility maximization problem reads:

$$\max E_0 \sum_{t=0}^{\infty} \varepsilon_t^b \gamma^t U(\widetilde{X}_t)$$

We also assume that the housing-specific preference shock ε_t^d is common across agents:

$$\widetilde{X}_t = \left[(1-\varepsilon_t^d\alpha)^{\frac{1}{\eta}}\left(\widetilde{C}_t - \theta\widetilde{C}_{t-1}\right)^{\frac{\eta-1}{\eta}} + \left(\varepsilon_t^d\alpha\right)^{\frac{1}{\eta}}\widetilde{D}_t^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

The patient agent's choice over capital and capacity utilization is also affected by exogenous disturbances. In particular, we model the existence of sector-specific shocks to the price of investment relative to nondurable consumption goods as follows:

$$\widetilde{K}_{\iota t} = (1-\delta_k)\widetilde{K}_{\iota t-1} + \varepsilon_t^{\iota i}\left[1 - S_\iota\left(\frac{I_{\iota t}}{I_{\iota t-1}}\right)\right] I_{\iota t} \quad ; \quad \iota = c, d$$

with

$$\varepsilon_t^{\iota i} = \rho_{\iota i}\varepsilon_{t-1}^{\iota i} + \eta_t^{\iota i},\ \eta_t^{\iota i} \sim N(0, \sigma_{\iota i})$$

We assume the following functional forms:

$$S_\iota(\cdot) = \frac{\phi}{2}\left(\frac{I_{\iota t}}{I_{\iota t-1}} - 1\right)^2; \qquad \iota = c, d$$

and

$$a(\cdot) = \frac{\overline{R}_k}{\psi}([\exp \psi (u_{\iota t} - 1)] - 1)$$

Finally, we include cost-push shocks in the sectoral specifications for the New Keynesian Phillips Curve as in Smets and Wouters (2005). The final version of equation (1.24) thus reads:

$$\begin{aligned}\widehat{\pi}_{\iota t} = &\left(\frac{\gamma_{p\iota}}{1+\gamma_{p\iota}}\right)\widehat{\pi}_{\iota t-1} + \left(\frac{\gamma}{1+\gamma\gamma_{p\iota}}\right)E_t\left\{\widehat{\pi}_{\iota t+1}\right\} + \\ &+ \left(\frac{(1-\xi_{p\iota})(1-\gamma\xi_{p\iota})}{(1+\gamma)\,\xi_{p\iota}}\right)\widehat{mc}_{\iota t} + \varepsilon_t^{\iota p}\end{aligned} \tag{1.30}$$

with

$$\varepsilon_t^{\iota p} = \rho_{\iota p}\varepsilon_{t-1}^{\iota p} + \eta_t^{\iota p},\ \eta_t^{\iota p} \sim N(0, \sigma_{\iota p})$$

1.5.3 The Deterministic Steady State

In this section we derive the steady-state version of the model equations. First, it is immediate to show that the collateral constraint always binds in equilibrium. In fact, by evaluating the Euler equation (1.17) in steady state, one obtains:

$$1 = \gamma R$$

or

$$R = \frac{1}{\gamma}$$

then, evaluating equation (1.9) in steady state gives:

$$\begin{aligned}\psi &= 1 - \beta R \\ &= 1 - \frac{\beta}{\gamma} > 0\end{aligned}$$

where the last inequality follows from the crucial assumption about the two intertemporal discount factors:

$$\beta < \gamma$$

Therefore, the Lagrange multiplier ψ attached to the collateral constraint is strictly positive in steady state, which implies that the constraint holds with equality. Clearly, the result holds true in a sufficiently small neighborhood of the deterministic steady state; this allows to treat the collateral constraint as binding when solving the model up to a log-linear approximation.

Next, we turn to the computation of durable and nondurable consumption. We calibrate the parameter υ in the utility function in such a way to obtain a total amount of hours worked equal to 0.3 in equilibrium[28]. It is immediate to notice that under price and wage (perfect) flexibility, the two blocks of equations for price and wage setting modify substantially. First, when $\xi_W = 0$ and $\eta_t^W = 0$, all agents are allowed to change their wage every period. Therefore, the wage setting condition boils down to the usual, competitive equivalence between the real wage and the marginal rate of substitution between consumption and leisure. However, the presence of a wage markup drives a wedge between the two terms. Thus, the optimality condition for the impatient agent is replaced by:

$$-\frac{U_N}{U_C} = w = \frac{1}{1+\lambda_{pc}}$$

where $w \equiv \frac{W}{P_c}$ is the real wage in terms of nondurable consumption, and

$$-\frac{U_N}{U_C} = wq = \frac{q}{1+\lambda_{pd}}$$

Therefore, the relative price q is pinned down by the following equation:

$$q = \frac{1+\lambda_{pd}}{1+\lambda_{pc}}$$

Next, we turn to the computation of C and D. Evaluating (1.8) in steady state and using (1.1) and (1.2) gives:

$$\widehat{C}/\widehat{D} = \{q\,[1-\beta\,(1-\delta)-(1-\chi)\,\psi]\}^{\eta}\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1}{1-\theta}\right) \tag{1.31}$$

By evaluating the collateral constraint - holding with equality - in steady state we obtain:

$$\frac{b}{\widehat{D}} = (1-\chi)\,q \tag{1.32}$$

[28]We are adopting the usual normalization that the total endowment of hours equals one.

Finally, the level of $\widehat{D}$ is pinned down using the impatient agent's budget constraint (1.5) together with (1.31) and (1.32):

$$\widehat{D} = \frac{wN}{\left(\widehat{C}/\widehat{D}\right) q + \delta - (1-R)(1-\chi)}$$

Then, clearly:

$$\widehat{C} = \left(\frac{\widehat{C}}{\widehat{D}}\right) \widehat{D}$$

The capital-labor ratios in the two sectors can be easily obtained by solving the firm's static cost-minimization problem:

$$\frac{K_j}{N_j} = \frac{w}{r^k} \frac{\alpha_j}{1-\alpha_j} \frac{\omega}{1-\omega} \tag{1.33}$$

Next we focus on the patient agent. Evaluating the market clearing conditions (1.26) and (1.27) in steady state and using the production functions gives:

$$Y_d = \omega \delta \widehat{D} + (1-\omega)\, \delta \widetilde{D} = \frac{((1-\omega)K_d)^{\alpha_d} \, (\omega N_d(i))^{1-\alpha_d}}{\Phi_{yd}} \tag{1.34}$$

and

$$Y_c = \omega \widehat{C} + (1-\omega)\, \widetilde{C} = \frac{((1-\omega)K_c)^{\alpha} \, (\omega N_c(i))^{1-\alpha}}{\Phi_{yc}} \tag{1.35}$$

where, by construction:

$$N_c + N_d = N = 0.3$$

Using (1.34) yields:

$$\widetilde{D} = \left(\frac{1}{(1-\omega)\,\delta}\right) \left\{ \frac{(1-\omega)^{\alpha_d} \omega^{1-\alpha_d} \, (K_d/N_d)^{\alpha_d} \, N_d}{\Phi_{yd}} - \omega \delta \widehat{D} \right\} \tag{1.36}$$

Analogously, using (1.35) yields:

$$\widetilde{D} = \left(\frac{1}{(1-\omega)\,(\widetilde{C}/\widetilde{D})}\right) \left[\begin{array}{c} \frac{(1-\omega)^{\alpha}\omega^{1-\alpha}(K_c/N_c)^{\alpha}(N-N_d)}{\Phi_y} - \omega\widehat{C} + \\ -(1-\omega)\delta_k \left[(K_c/N_c)\,(N-N_d) + (K_d/N_d)\, N_d\right] \end{array} \right] \tag{1.37}$$

where the value of $\widetilde{C}/\widetilde{D}$ is obtained by using the patient agent's Euler equation (1.16):

$$\widetilde{C}/\widetilde{D} = [1-\gamma\,(1-\delta)]^{\eta} \left(\frac{1-\alpha}{\alpha}\right) \left(\frac{1}{1-\theta}\right) \tag{1.38}$$

Then, equating (1.36) and (1.37) and solving for N_d gives:

$$N_d = \frac{\delta\omega\widehat{D}\left(\frac{\widetilde{C}}{\widetilde{D}}\right) - \delta\omega\widehat{C} + N\left[\left(\frac{\delta}{\Phi_y}\right)(1-\omega)^{\alpha}\omega^{1-\alpha}\left(\frac{K_c}{N_c}\right)^{\alpha}\right]}{\left(\frac{\widetilde{C}}{\widetilde{D}}\right)\frac{(1-\omega)^{\alpha_d}\omega^{\alpha_d}(K_d/N_d)^{\alpha_d}}{\Phi_{yd}} + (\delta/\Phi_y)(1-\omega)^{\alpha}\omega^{1-\alpha}\left(\frac{K_c}{N_c}\right)^{\alpha} - \delta(1-\omega)\delta_k\left[\frac{K_c}{N_c} - \frac{K_d}{N_d}\right]} \tag{1.39}$$

and,clearly:

$$N_c = N - N_d$$

Then, using either (1.36) or (1.37) one obtains $\widetilde{D}$; the level of $\widetilde{C}$ is immediately obtained by multiplying expression (1.38) by $\widetilde{D}$. Finally, the level of capital and output in each sector can be easily obtained using (1.33), (1.34) and (1.35).

1.5.4 Data

The dataset includes quarterly data on: nondurable consumption, residential fixed investment, total household debt, short-term nominal interest rate, consumer price inflation, GDP. The sample is 1965 Q1: 2006 Q4. A detailed description of the original data, their source and the transformation applied follows.

- Nondurable consumption: Real Personal Consumption Expenditure: Nondurable Goods (Billions of Chained 2000 Dollars); Source: Bureau of Economic Analysis.
- Residential Fixed Investment: Real Private Residential Fixed Investment; Source: Bureau of Economic Analysis.
- Total Household Debt: Total Outstanding Household Debt-Domestic Nonfinancial Sector. Source: Federal Reserve Bank, Flow of Funds.
- Short-term nominal interest rate: 3-month Treasury bill secondary market rate. Source: Federal Reserve Bank, Board of Governors.
- Consumer Price Inflation: Quarter-on-quarter log-difference, Gross Domestic Product, Implicit Price Deflator. Source: Bureau of Economic Analysis.
- Real House Prices: New One-Family Houses Sold Including Value of Lot, divided by the Implicit Price Deflator for the Nonfarm Business Sector. Source: U. S. Census Bureau.

- Nonresidential Investment: Real Private Nonresidential Fixed Investment, Source: Bureau of Economic Analysis.
- Gross Domestic Product: Real Gross Domestic Product (Billions of Chained 2000 Dollars). Source: Bureau of Economic Analysis.
- Hours worked in the consumption-good sector: Total Nonfarm Payrolls less All Employees in the Construction Sector, multiplied by Average Weekly Hours of Production Workers. Source: Bureau of Labor Statitstics.

All series are seasonally adjusted. Nondurable consumption, residential fixed investment, household debt, nonresidential fixed investment and GDP are expressed in per capita terms by dividing with the population over 16 (Civilian Noninstitutional Population, Source: Bureau of Labor Statistics). The nominal interest rate and the inflation rate are expressed on a quarterly basis, consistently with their definition in the model. The data are expressed in log.

Detrending. The model is a purely business cycle one, and therefore does not display any trend. Once the model is log-linearized around the deterministic steady-state, all variables can be treated as deviations around the mean (the steady state). Therefore, to make the data comparable with the model-generated series, a detrending procedure must be chosen. Following Smets and Wouters (2003), all variables are linearly detrended, while inflation and the nominal interest rate are detrended by the same linear trend in inflation.

1.5.5 Assessing Convergence in the RWMH algorithm

The model is solved up to a log-linear approximation around the deterministic steady state. Once the solution is obtained, the model can be cast in state-space form, and the likelihood function can be computed using the Kalman filter. More precisely, the posterior distributions can be computed once independent prior distributions are specified for each one of the structural parameters to be estimated.

Markov Chain Monte Carlo (MCMC) methods are used to simulate draws from an unknown target distribution, through the generation of a Markov chain, the stationary density of which is assumed to coincide with the target density. A natural question concerns the evaluation of convergence, and the definition of some convergence diagnostics. Following Robert and Casella (1998), one can distinguish between: (i) convergence of the MC to its stationary distribution (which implies exploring the correct distribution of interest and the whole space), (ii) convergence of empirical averages to the appropriate expected values (i.e. the posterior population moments) and (iii) convergence to *iid* sampling. This subsection briefly describes two approaches to the problem of evaluating convergence[29].

Geweke (1992) suggests an empirical evaluation method based on the following intuition. Consider a vector of parameters θ, and a function of interest $g(\theta)$. We are interested in estimating $g(\theta)$ based on the sample draws. For a sufficiently large number of draws, the estimate of $g(\theta)$ based on, say, the first half of the draws, should coincide with the estimate based on the last half. A difference in the two estimates indicates that (i) either too few draws have been taken, or that (ii) the effect of the initial - arbitrary - draw θ^0 is contaminating quite a large part of the draws. Therefore, the total number of draws, S, is divided into a given number of subsets. More precisely, after discarding a fraction S_0 of the initial draws as burn-in replications, the remaining S_1 are divided into, say three subsets: S_A, S_B, S_C. Then, the middle set of replications, S_B, is dropped out, in order to make it more likely for S_A and S_C to be independent of one another. Finally, denoting $\widehat{g_{S_A}}$ and $\widehat{g_{S_C}}$ the estimates of $E\left[g(\theta)|y\right]$ using S_A and S_C respectively, it is possible to construct the numerical standard errors of the two estimates as $\frac{\widehat{\sigma}_A}{\sqrt{S_A}}$ and $\frac{\widehat{\sigma}_C}{\sqrt{S_C}}$. Then a central limit theorem can be invoked to establish that

$$CD \rightarrow N(0,1)$$

where

$$CD = \frac{\widehat{g_{S_A}} - \widehat{g_{S_C}}}{\frac{\widehat{\sigma}_A}{\sqrt{S_A}} + \frac{\widehat{\sigma}_C}{\sqrt{S_C}}}.$$

The method suggested by Brooks and Gelman (1998) is a generalization of the original method of Gelman and Rubin (1992). The method assumes that m parallel chains have been simulated, each starting at a different point, with overdispersion of the starting points over the target distribution. Convergence is assessed by comparing *between* and *within* variances.

[29] See Koop (2003) and Brooks and Gelman (1998).

1.6 Tables and Figures

Table 1.3: Prior and Posterior Distributions

		PRIOR			S1	S2
	Description	Distr.	Mean	Std.Dev.	Median	Median
θ	cons. habit	Beta	0.65	0.1	0.28	0.18
φ	inv. el. labor supply	Gamma	2	0.75	1.86	1.77
ϕ_i	investment adj.	Normal	4	0.5	4.07	4.16
ψ	adj. cost elast.	Gamma	0.2	0.1	0.03	0.01
$\zeta_{p,c}$	Calvo price (nond.)	Beta	0.5	0.28	0.88	0.88
$\zeta_{p,d}$	Calvo price (dur.)	Beta	0.5	0.28	0.00	0.00
ζ_w	Calvo wage	Beta	0.5	0.28	0.97	0.99
$\gamma_{p,c}$	price index. (nond.)	Beta	0.5	0.28	0.80	0.02
$\gamma_{p,d}$	price index. (dur.)	Beta	0.5	0.28	0.47	0.53
γ_w	wage index.	Beta	0.5	0.28	0.04	0.01
ϕ_π	Taylor rule	Normal	1.5	0.1	1.57	1.53
$\phi_{\Delta\pi}$	Taylor rule	Gamma	0.3	0.1	0.26	0.26
$\phi_{\Delta y}$	Taylor rule	Gamma	0.063	0.05	0.55	0.46
ρ_{ζ_r}	Taylor rule	U[0,1]	0.5	0.28	0.82	0.86
ρ_{zetac}	Tech. Shock nond.	Beta	0.5	0.28	0.97	0.99
ρ_{zetad}	Tech. Shock dur.	Beta	0.5	0.28	0.99	0.99
ρ_b	Pref. Shock	Beta	0.5	0.28	0.99	0.96
ρ_{ltv}	LTV shock	Beta	0.5	0.28	0.97	0.99
ρ_i	Inv. Specific (nond.)	Beta	0.5	0.28	0.03	0.03
ρ_{i_d}	Inv. Specific (d.)	Beta	0.5	0.28	0.37	0.33
ρ_{hb}	Housing pref.	Beta	0.5	0.28	1.00	1.00
ρ_n	Labor supply	Beta	0.5	0.28	0.73	0.89
ρ_{cp}	Cost-push (nond.)	Beta	0.5	0.28	0.04	0.01
ρ_{cp_d}	Cost-push (d.)	Beta	0.5	0.28	0.59	0.80

Table 1.4: Prior and Posterior Distributions (continued)

	Description	PRIOR Distr.	S1 Median	S2 Median
σ_{zc}	Tech. Shock nond.	U[0,6]	0.01	0.00
σ_{zd}	Tech. Shock d.	U[0,6]	0.01	0.01
σ_{r}	Monetary policy	U[0,6]	0.01	0.00
σ_{ltv}	LTV shock nond.	U[0,6]	0.01	0.01
σ_{b}	Pref. shock	U[0,6]	0.85	0.03
σ_{i}	Inv. Specific nond.	U[0,6]	1.03	0.65
σ_{id}	Inv. Specific d.	U[0,6]	2.29	1.95
σ_{hb}	Housing pref.	U[0,6]	0.04	0.03
σ_{n}	Labor supply	U[0,6]	4.04	3.95
σ_{cp}	Cost-push nond.	U[0,6]	0.01	0.01
σ_{cp_d}	Cost-push d.	U[0,6]	3.44	2.03

Table 1.5: Variance Decomposition (1965 Q1: 1982 Q4)

	η_{zc}	η_{zd}	η_{r}	η_{ltv}	η_{b}	η_{i}	η_{i_d}	η_{hb}	η_{n}	η_{cp}	η_{cp_d}
C	2.56	1.26	56.51	2.55	13.69	3.27	0.10	14.25	4.38	0.34	0.99
I^D	3.19	6.88	1.73	1.40	2.70	2.15	0.06	76.92	2.81	0.83	1.33
$\widehat{B}$	0.33	1.11	2.67	2.33	1.97	0.13	0.00	90.92	0.41	0.05	0.08
R	0.69	1.17	7.91	3.70	11.10	1.00	0.03	50.81	17.30	0.63	5.65
π	11.49	23.71	5.94	0.09	3.65	0.26	0.01	9.72	14.39	25.36	5.39
q	2.77	90.49	0.06	0.00	0.02	0.00	0.00	0.42	3.37	2.20	0.67
I	0.83	0.58	5.48	1.30	43.10	4.32	0.14	42.98	0.94	0.09	0.26
Y	1.50	0.81	46.35	1.12	22.71	1.45	0.05	22.06	3.02	0.34	0.59
N_c	1.84	0.76	40.83	0.95	25.93	1.69	0.06	24.38	2.84	0.22	0.50

Table 1.6: Variance Decomposition (1983 Q1: 2006 Q4)

	η_{zc}	η_{zd}	η_{r}	η_{ltv}	η_{b}	η_{i}	η_{i_d}	η_{hb}	η_{n}	η_{cp}	η_{cp_d}
C	0.71	0.84	35.77	3.49	3.93	2.41	0.24	45.69	4.22	0.33	2.38
I^D	1.99	9.24	0.97	5.66	0.61	0.85	0.07	73.36	0.87	0.51	5.88
$\widehat{B}$	0.32	1.73	5.13	4.33	0.87	0.35	0.03	85.54	0.67	0.06	0.97
R	1.95	7.76	1.44	12.58	0.71	0.4	0.04	43.63	20.33	0.4	10.77
π	8.12	30.64	0.56	0.02	0.08	0.05	0	1.13	12.17	26.41	20.8
q	1.16	93.51	0	0	0	0	0	0.02	1.05	1.23	3.03
I	0.37	0.79	3.99	3.64	3.17	3.17	0.32	82.15	0.61	0.09	0.8
Y	0.42	0.34	36.95	1.69	2.84	2.84	0.29	46.17	4.21	0.35	1.51
N_c	0.87	0.35	36.18	1.59	2.89	2.89	0.29	46.59	4.16	0.33	1.43

Table 1.7: Fit of the model: marginal likelihood

	1965Q1 : 1982Q4		1983Q1 : 2006Q4	
	Model	Benchmark	Model	Benchmark
Laplace Approximation	1544.4	1407.8	2460.6	2267.6
Modified Harmonic Mean	1540.5	1456.6	2409	2266.8

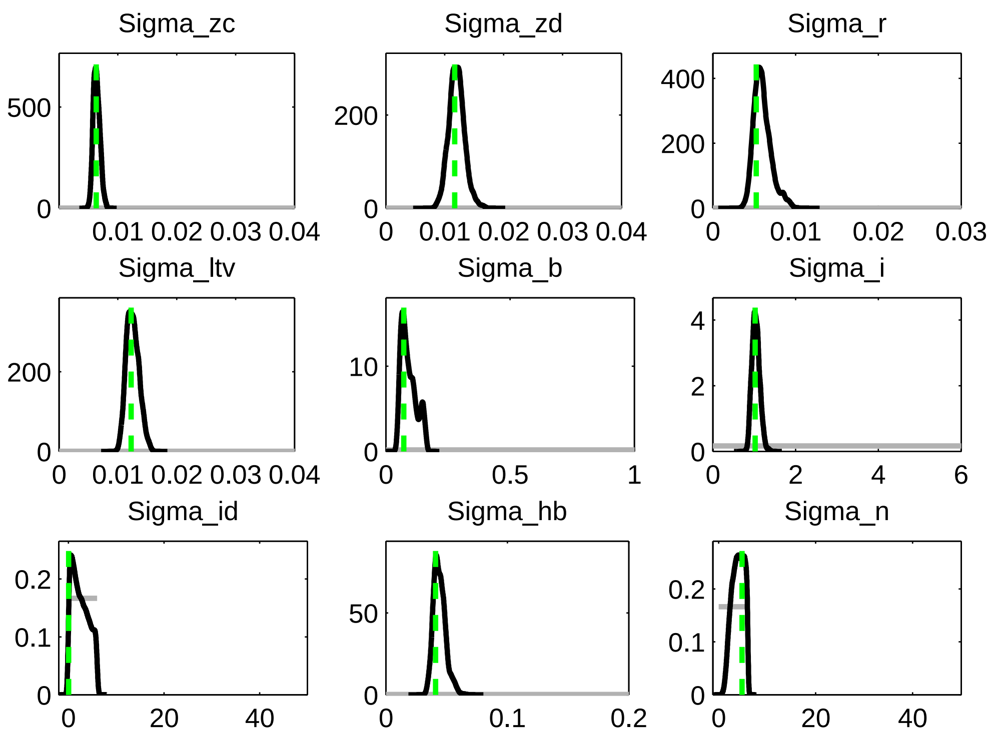

Figure 1.5: Prior and Posterior Distributions (1965 Q1: 1982 Q4). Grey: Prior; Black: Posterior; Dashed: Posterior mode.

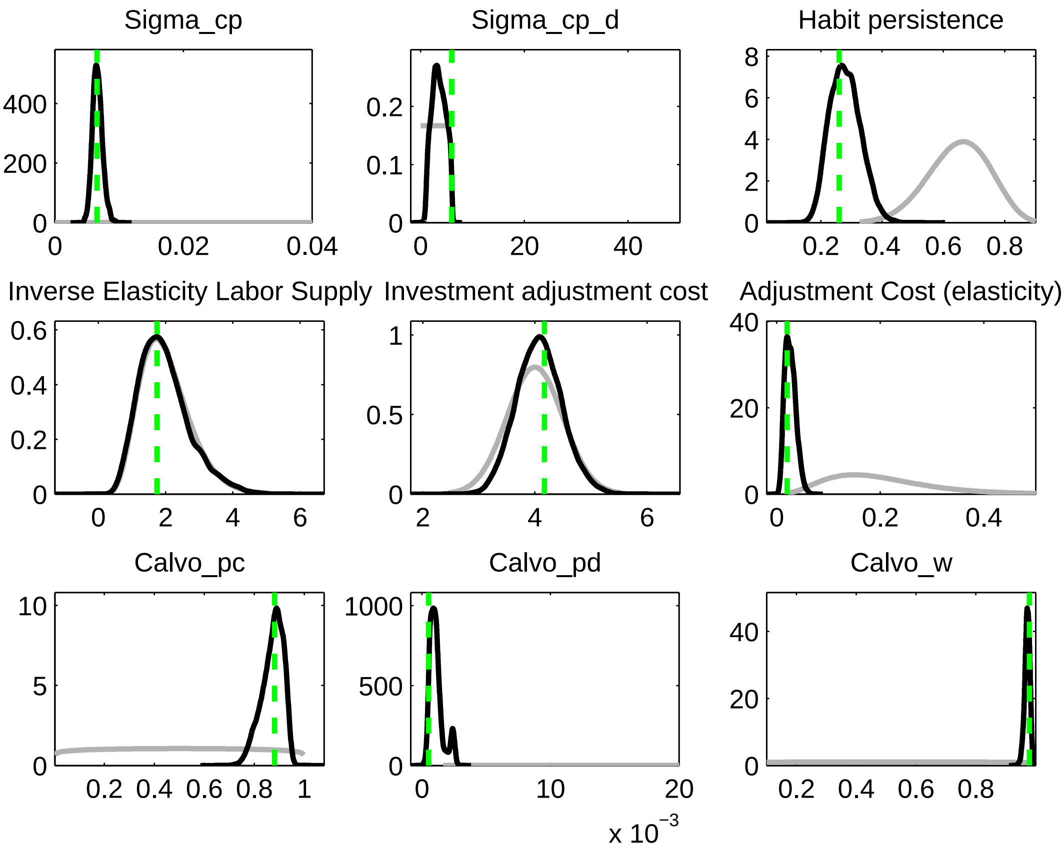

Figure 1.6: Prior and Posterior Distributions (1965 Q1: 1982 Q4). Grey: Prior; Black: Posterior; Dashed: Posterior mode.

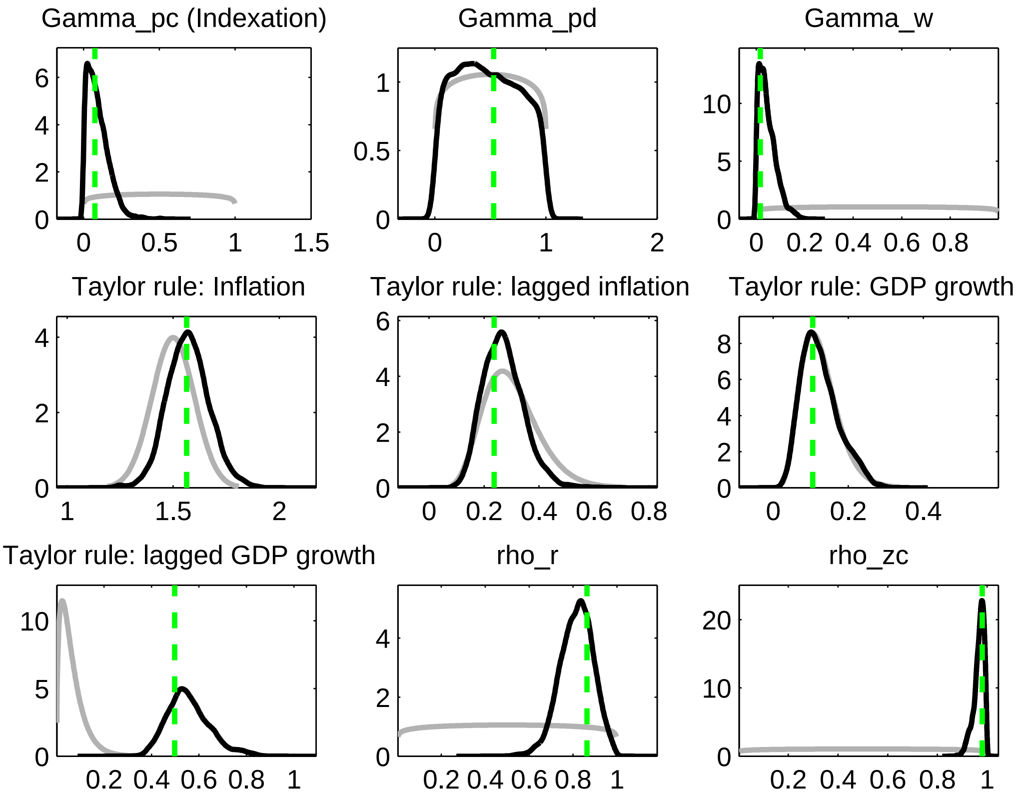

Figure 1.7: Prior and Posterior Distributions (1965 Q1: 1982 Q4). Grey: Prior; Black: Posterior; Dashed: Posterior mode.

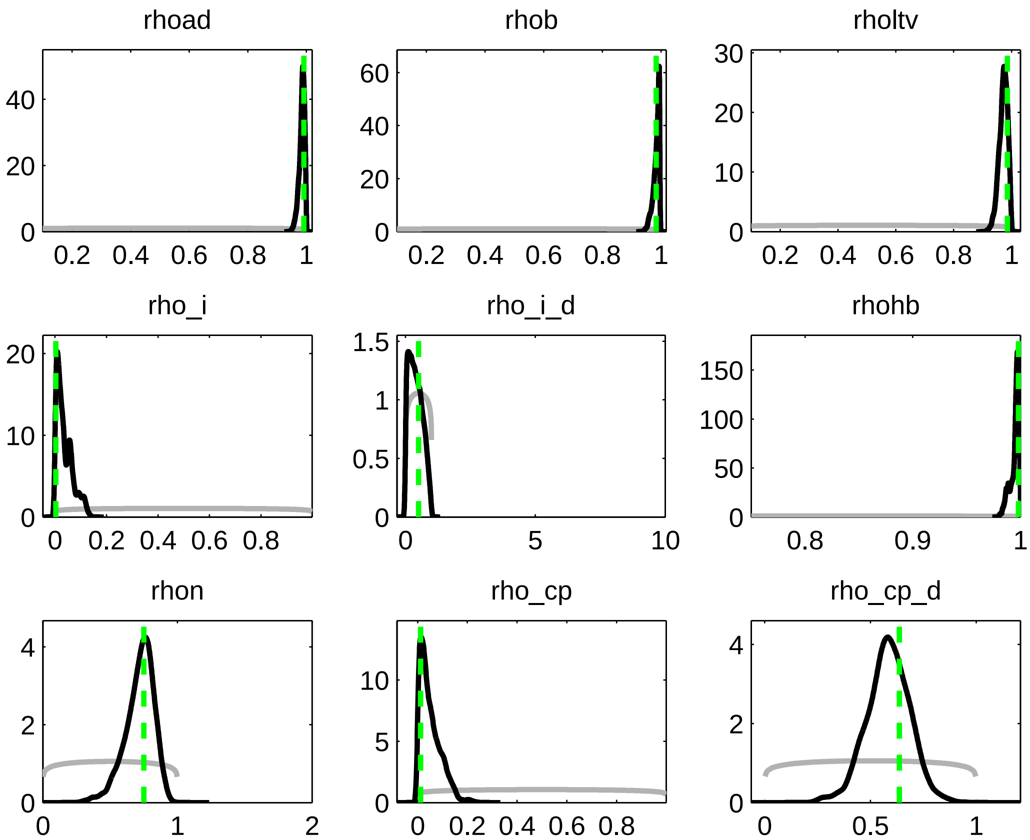

Figure 1.8: Prior and Posterior Distributions (1965 Q1: 1982 Q4). Grey: Prior; Black: Posterior; Dashed: Posterior mode.

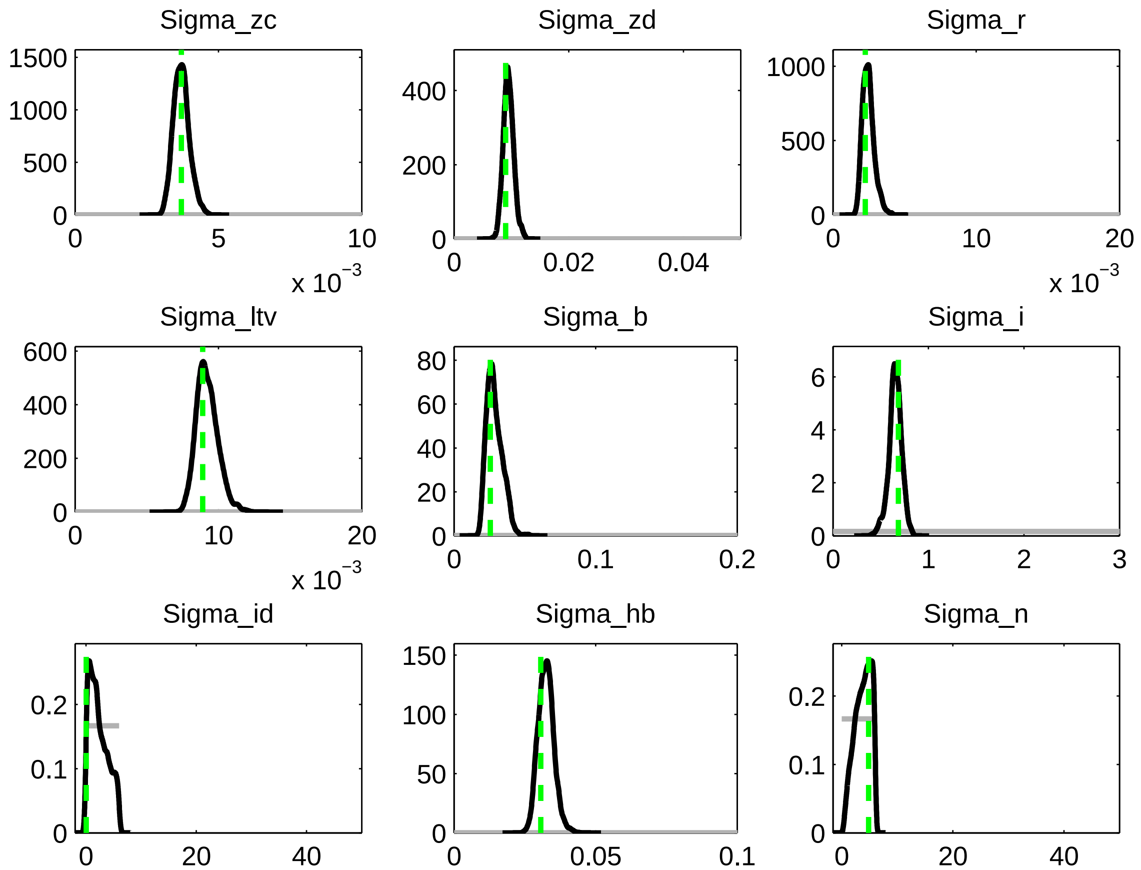

Figure 1.9: Prior and Posterior Distributions (1983 Q1: 2006 Q4). Grey: Prior; Black: Posterior; Dashed: Posterior mode.

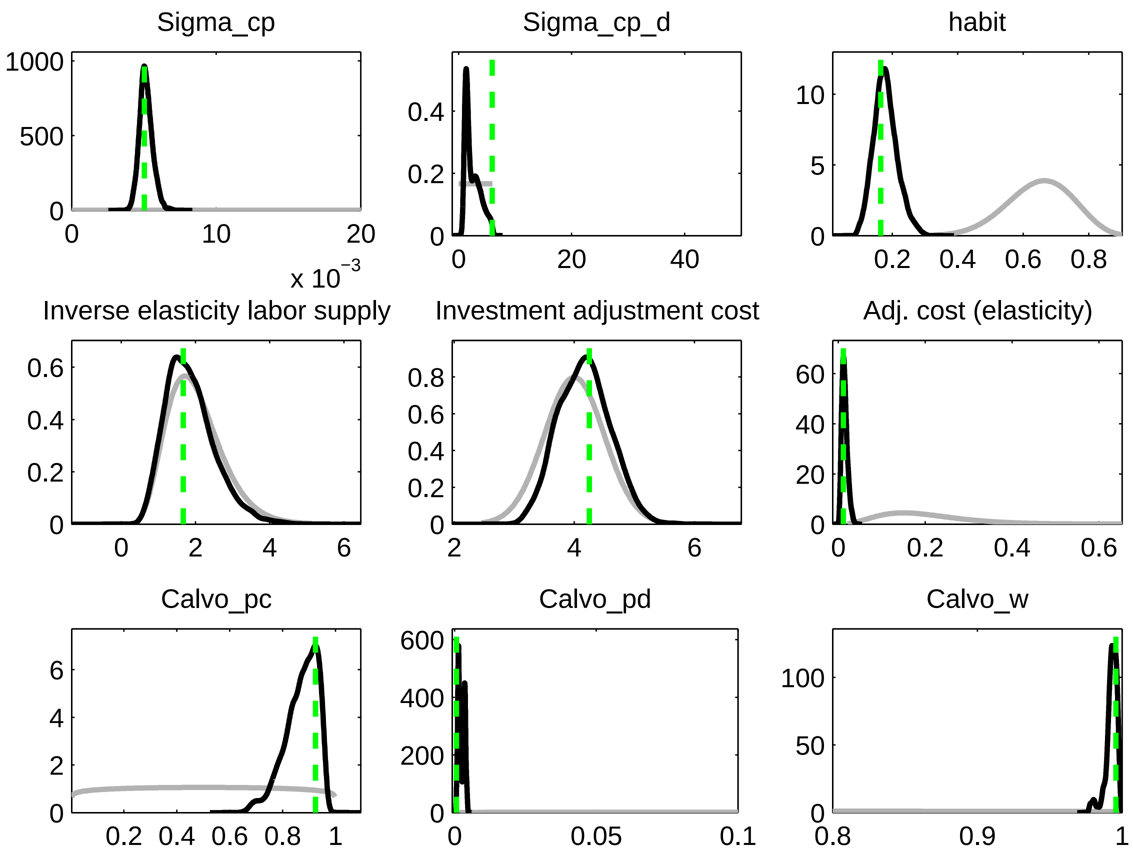

Figure 1.10: Prior and Posterior Distributions (1983 Q1: 2006 Q4). Grey: Prior; Black: Posterior; Dashed: Posterior mode.

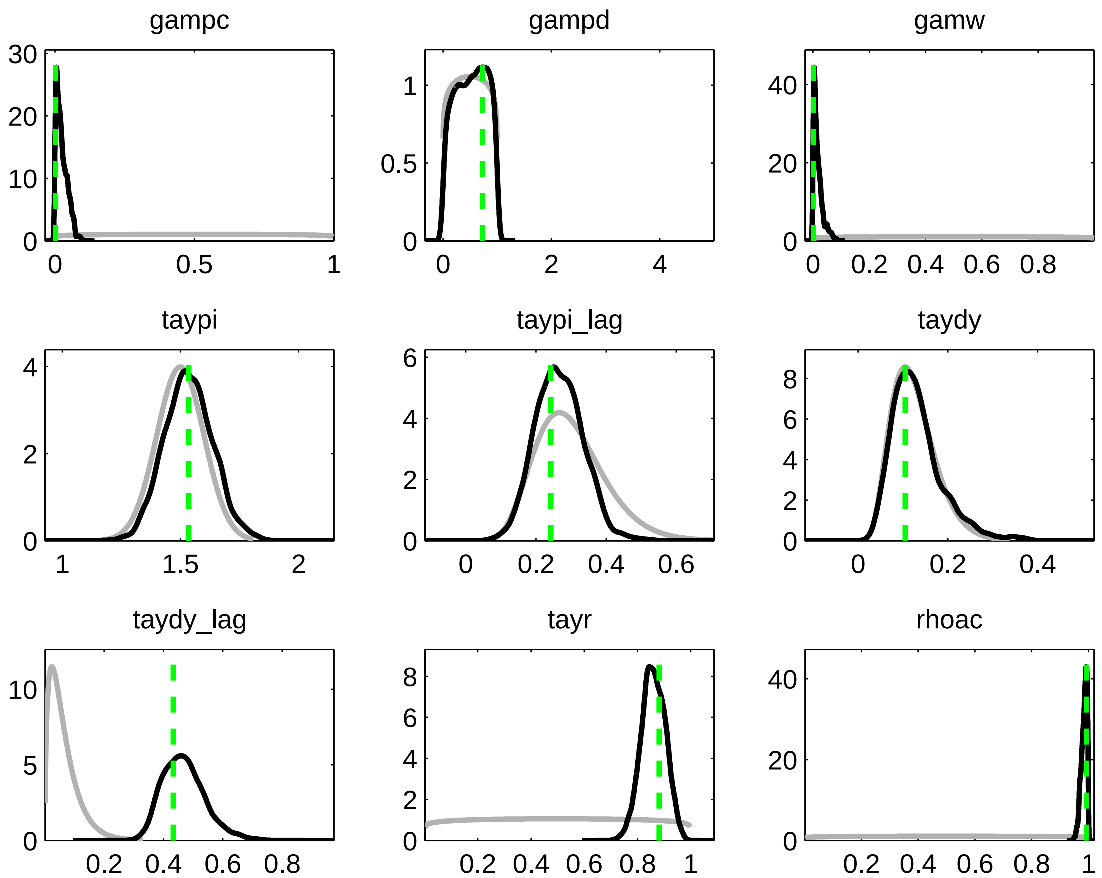

Figure 1.11: Prior and Posterior Distributions (1983 Q1: 2006 Q4). Grey: Prior; Black: Posterior; Dashed: Posterior mode.

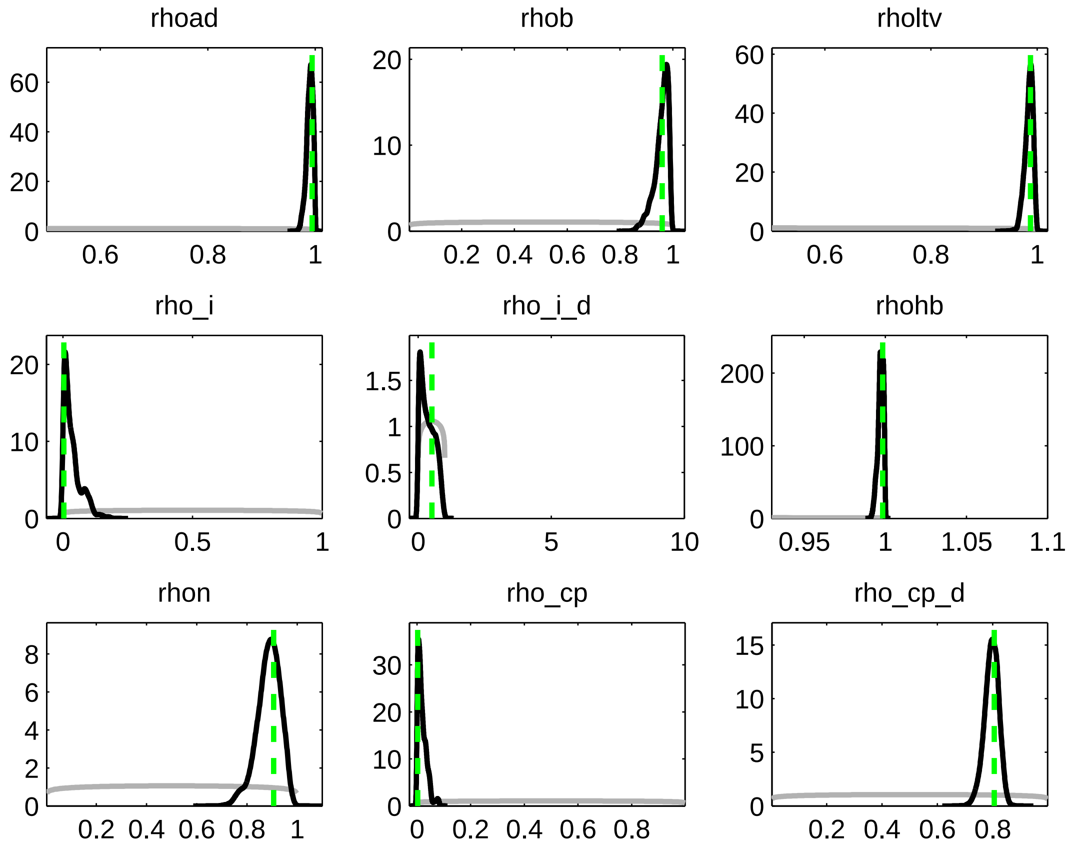

Figure 1.12: Prior and Posterior Distributions (1983 Q1: 2006 Q4). Grey: Prior; Black: Posterior; Dashed: Posterior mode.

Chapter 2

Housing prices, collateral constraints and cross-country dynamics

2.1 Introduction

The behavior of the real exchange rate has captured the attention of many studies in international macroeconomics and finance over the last two decades. Two stylized facts are at the core of the analysis: (i) the *consumption-real exchange rate anomaly* (see Backus, Kehoe, and Kydland (1993) and Chari, Kehoe, and McGrattan (2002)), i.e. the observed negative correlation between the real exchange rate and relative consumption across country and (ii) the *exchange-rate disconnect*, or a high volatility of the real exchange rate which does not reflect a high volatility in real variables. Such facts have motivated a large literature which has engineered different ways of rationalizing a negative comovement between highly volatile exchange rates and cross-country relative consumption [1].

The recent developments in the US housing and financial markets have produced an additional interesting phenomenon, namely a contemporaneous depreciation of the US dollar and increase in house prices. Figure 2.1 shows the cyclical behavior of the two relative prices, using detrended data over the sample 1981Q1: 2005Q4[2]. The evolution of household credit has

[1] To name a few, see Betts and Devereux (2000), Corsetti, Dedola, and Leduc (2008b), Corsetti, Dedola, and Leduc (2008a) among the most recent contributions

[2] The series are detrended using the Hodrick and Prescott filter. See the Appendix for details on the data used

originated large spillovers from the housing market to the non-residential sectors of the economy[3], thus providing an additional source of fluctuations in the terms of trade. The housing sector emerges as a very special case of a non-traded-goods-producing sector, which drives potentially powerful interactions with the consumption and saving decisions of the agents. This paper investigates the role of the housing sector and its interactions with private borrowing in an open-economy environment, devoting special attention to the cross-country dynamics that originate in an otherwise standard New Open Economy Macroeconomics (NOEM) model with a traded and a non-traded sector. In this respect, our contribution is meant to bridge a gap between two growing strands of literature: the one focusing on household credit frictions and housing in closed economy (see Iacoviello (2005), Iacoviello and Neri (2007), Monacelli (2009) and Notarpietro (2007), among others) and the NOEM literature[4].

We provide a systematic explanation of the transmission mechanism of sectoral technology shocks, as well as more standard demand and monetary policy shocks, focusing on the specific role played by household credit constraints in generating domestic and international comovements. Moreover, we relate to the NOEM literature by analyzing the role of traditional open-economy parameters in determining the model predictions. Under the baseline calibration the model accounts for the observed negative comovement of the real exchange rate with relative consumption and broadly captures the domestic cross-sector correlations. The observed positive comovement of the real exchange rate with housing prices is replicated when the model includes a high coefficient of relative risk aversion together with a large degree of credit market frictions. The degree of substitutability between housing and non-housing goods turns out to be of crucial relevance in matching international correlations of real quantities, but has little effect on the house price-real exchange rate correlation. Finally, the large volatility of the real exchange rate can be explained - to some extent - by the pricing-to-market behavior of some firms, as already noted in the literature[5]. The model thus provides an original framework to study the interactions of private borrowing, housing-markets dynamics and cross-country

[3]see Iacoviello and Neri (2007) and Notarpietro (2007) for an empirical analysis of such effects

[4]An exception is the work by Christensen, Corrigan, Mendicino, and Nishyiama (2007), who develop and estimate a small open economy model for the Canadian economy featuring a housing sector.

[5]See Corsetti, Dedola, and Leduc (2008a)

business cycle transmission.

The remainder of the paper is organized as follows. Section 2 describes the theoretical model and the decision problems that the agents face. Section 3 outlines the baseline calibration. The main results are presented and discussed in Section 4 and Section 5. Section 6 concludes.

2.2 Theoretical model

The world economy is composed of two symmetric countries, Home (H) and Foreign (F). Each country is modeled as a two-agent, two-sector economy, producing residential and non residential goods[6]. Non-residential final goods are produced by a continuum of "single-good-firms" indexed on $[0, 1]$, mixing local production with imports. More precisely, in each country final producers for local sales and inputs operate in perfect competition and aggregate a continuum of differentiated products purchased from Home and Foreign intermediate-sector firms. The latter are monopolistic competitors and exert some market power through the setting of prices. The residential-goods sector has a similar structure, but final and intermediate goods are not traded.

We assume that in each country there exists a continuum of infinitely-lived households, the number of which is proportional to the number of firms. Following the seminal contribution of Kiyotaki and Moore (1997), we consider two types of households in each country, differing in their relative intertemporal discount factor. More precisely, a fraction $(1-\omega)$ of households in country H (and, symmetrically, $(1-\omega^*)$ in F) are relatively more patient, and the remaining ω (resp. ω^*) are impatient. Households receive utility from consuming both nonresidential and residential goods, and disutility from labor. Residential goods are treated here as *durable* goods, and serve two purposes: they can be either directly consumed or used as collateral in the mortgage market. Private debt is generated in equilibrium, as the result of intertemporal trade among the patient agents (who act as lenders, or savers), and the impatient agents (who act as net borrowers). The existence of frictions in household credit markets is captured by

[6]We follow closely Iacoviello and Neri (2007) and Notarpietro (2007) in defining the closed-economy setup for each country.

imposing a perpetually binding collateral constraint on the entire group of impatient agents[7]. We present the structure of the model and some derivations for country H only, for the sake of brevity. Analogous derivations hold true for country F.

2.2.1 The borrower's program

Each impatient agent $b \in [0, \omega]$ receives utility from the following instantaneous utility function:

$$W_t^b = E_t \left\{ \sum_{j \geq 0} \beta^j \left[\begin{array}{c} \frac{1}{1-\sigma_X} \left(\widetilde{X}_{t+j}^b \right)^{1-\sigma_X} - \frac{\overline{L}_C}{1+\sigma_{LC}} \left(L_{C,t+j}^b \right)^{1+\sigma_{LC}} \\ - \frac{\overline{L}_D}{1+\sigma_{LD}} \left(L_{D,t+j}^b \right)^{1+\sigma_{LD}} \end{array} \right] \right\} \tag{2.1}$$

where $\widetilde{X}_t^b$ is an index of consumption services derived from non-residential final goods $\left(C^b \right)$ and residential stock $\left(D^b \right)$:

$$\widetilde{X}_t^b \equiv \left[\left(1 - \varepsilon_t^D \omega_D \right)^{\frac{1}{\eta_D}} \left(\widetilde{C}_t^b - h_B \widetilde{C}_{t-1}^b \right)^{\frac{\eta_D - 1}{\eta_D}} + \varepsilon_t^D \omega_D^{\frac{1}{\eta_D}} \left(\widetilde{D}_t^b \right)^{\frac{\eta_D - 1}{\eta_D}} \right]^{\frac{\eta_D}{\eta_D - 1}} \tag{2.2}$$

with the parameter h_B capturing habit formation in consumption of non-residential goods. We introduce a housing preference shock, ε_t^D, which affects the relative share of residential stock, ω_D, and modifies the marginal rate of substitution between non-residential and residential goods consumption.

Households receive disutility from labor in each sector, $L_{C,t}^b$ and $L_{D,t}^b$. The specification of labor supply assumes that households have preferences over providing labor services across different sectors. In particular, the specific functional form adopted implies that hours worked are perfectly substitutable across sectors. $\overline{L}_C$ and $\overline{L}_D$ are level-shift terms needed to ensure that the impatient's labor supply is equal to 1 in steady state.

Impatient agents in each country can trade a nominal risk-less bond denominated in the domestic currency, but they cannot tap the international financial markets to finance their expenditure plans. In addition, they do not save nor accumulate capital. Total savings and investment decisions in each country are implemented by the savers, as we show later.

[7] As a consequence, we will use the terms *impatient (patient)* and *borrower (saver)* as interchangeable throughout, with a slight abuse of terminology.

Under these assumptions, each borrower maximizes her utility function (2.1) subject to an infinite sequence of real budget constraints[8]:

$$\frac{P_t}{\underline{P}_t}\widetilde{C}_t^b + T_{D,t}\left(\widetilde{D}_t^b - (1-\delta)\,\widetilde{D}_{t-1}^b\right) + \frac{R_{t-1}\widetilde{B}_{H,t-1}^b}{\pi_t \underline{P}_{t-1}} = \frac{\widetilde{B}_{H,t}^b}{\underline{P}_t} + \frac{\widetilde{A}_t^b + \widetilde{TT}_t^b}{\underline{P}_t} + (1-\tau_{w,t})\,\frac{W_{C,t}^b L_{C,t}^b + W_{D,t}^b L_{D,t}^b}{\underline{P}_t} \tag{2.3}$$

where $\delta \in (0,1)$ is the depreciation rate and $T_{D,t} \equiv \frac{P_{D,t}}{\underline{P}_t}$ is the relative price of residential goods in terms of non-residential goods, $\widetilde{B}_{H,t}^b$ is the stock of nominal debt issued by the borrower at time t, R_{t-1} is the nominal interest rate paid on the existing amount of debt $\widetilde{B}_{H,t-1}^b$ and π_t is the gross non-residential good inflation rate. $W_{C,t}^b$ and $W_{D,t}^b$ denote the borrower's nominal wages in the two sectors. $\widetilde{TT}_t^b$ are government transfers and $\tau_{w,t}$ is a time-varying labor tax. Finally, $\widetilde{A}_t^b$ is a stream of income coming from state-contingent securities, allowing the borrowers to hedge against wage income risk. Given separability of preferences, trading such assets ensures that all borrowers have identical consumption plans. Therefore, we can drop the superscript $b$and simply use a$\tilde{}$to denote variables related to the borrowers .
We also introduce a consumption tax which affects the price of the distributed goods serving final consumption. The after-tax consumer price index (CPI) is denoted $P_t = (1+\tau_{C,t})\,\underline{P}_t$ where $\underline{P}_t$ is the price of the distribution good gross of consumption tax.
At each period in time, all the borrowers have limited access to credit markets, as summarized by the following (nominal) collateral constraint:

$$\widetilde{B}_{H,t} \leq \varepsilon_t^{LTV}(1-\chi)\,\mathbb{E}_t\left\{P_{D,t+1}\widetilde{D}_t\frac{1}{R_t}\right\}$$

where $\chi \in [0,1]$ is the fraction of the residential good that cannot be used as a collateral. Such a parameter is an indirect measure of the flexibility of the mortgage market. The term $(1-\chi)$ thus provides a proxy for the observed loan-to-value ratio, which is subject to a stationary stochastic shock ε_t^{LTV}. The collateral constraint can be conveniently rewritten in real terms as follows:

$$\widetilde{b}_{H,t} \leq \varepsilon_t^{LTV}(1-\chi)\,\mathbb{E}_t\left\{T_{D,t+1}\widetilde{D}_t\frac{\pi_{t+1}}{R_t\ \varepsilon_{t+1}^{CPI}}\right\} \tag{2.4}$$

[8]We use the non-residential goods price level as a deflator.

where $\widetilde{b}_{H,t} \equiv \frac{B_H}{P_t}$.

Summing up, the impatient agent maximizes (2.1) subject to the infinite sequence of (2.3) and (2.4) holding with equality[9]. We report the first order conditions for this problem in the Appendix.

2.2.2 The saver's program

The patient agents, $s \in [\omega, 1]$, are characterized by a higher intertemporal discount factor than the borrowers, and thus act as net lenders in equilibrium. They own the productive capacities and make decisions on investment plans to build the capital stock which will be rented out to intermediate firms. The savers can trade two nominal risk-less bonds denominated in the domestic and foreign currency. Financial markets are assumed to be incomplete internationally. We introduce a risk premium on the international financing of domestic consumption expenditures. Such risk premium is a function of real holdings of foreign assets in the entire economy. Each patient agent receives instantaneous utility from the same type function (2.1) adopted for the impatient[10]:

$$\mathcal{W}_t^s = \mathbb{E}_t \left\{ \sum_{j \geq 0} \gamma^j \left[\begin{array}{c} \frac{1}{1-\sigma_X} \left(X_{t+j}^s \right)^{1-\sigma_X} - \frac{\varepsilon_{t+j}^{L,s} \widetilde{L}_C}{1+\sigma_{LC}} \left(L_{Ct+j}^s \right)^{1+\sigma_{LC}} \\ - \frac{\varepsilon_{t+j}^{L,s} \widetilde{L}_D}{1+\sigma_{LD}} \left(L_{Dt+j}^s \right)^{1+\sigma_{LD}} \end{array} \right] \varepsilon_{t+j}^{\beta} \right\} \tag{2.5}$$

where X_t^s is given by

$$X_t^s \equiv \left[\left(1 - \varepsilon_t^D \omega_D \right)^{\frac{1}{\eta_D}} \left(C_t^s - h_S C_{t-1}^s \right)^{\frac{\eta_D - 1}{\eta_D}} + \varepsilon_t^D \omega_D^{\frac{1}{\eta_D}} \left(D_t^s \right)^{\frac{\eta_D - 1}{\eta_D}} \right]^{\frac{\eta_D}{\eta_D - 1}} \tag{2.6}$$

The saver maximizes its utility function subject to an infinite sequence of the following budget

[9] It is possible to show that the collateral constraint always binds in the deterministic steady state, under general conditions. We assume here that it continues to bind in a sufficiently small neighborhood of the steady state, so that the model can be solved by taking a first order approximation.

[10] Variables related to the saver are denoted with a superscript s, as opposed to b, used for the borrowers.

constraint:

$$\frac{P_t}{\underline{P}_t}C_t^s+T_{D,t}\left(D_t^s-(1-\delta)\,D_{t-1}^s\right)+I_t^s+\frac{B_{H,t}^s}{\underline{P}_t}+\frac{S_tB_{F,t}^s}{\underline{P}_t\varepsilon_t^{\Delta S}\Psi\left(\frac{\mathbb{E}_tS_{t+1}}{S_{t-1}}-1,\frac{S_t\left(B_{F,t}-\overline{\overline{B}}_F\right)}{\underline{P}_t}\right)}$$
$$=\frac{R_{t-1}B_{H,t-1}^s}{\pi_t\underline{P}_{t-1}}+\frac{S_tR_{t-1}^*B_{F,t-1}^s}{\pi_t\underline{P}_{t-1}}+\sum_{j=C,D}\left[R_t^{k,j}u_t^jK_t^j-\Phi\left(u_t^j\right)K_t^j\right]$$
$$+\frac{(1-\tau_{w,t})\left(W_{C,t}^sL_{C,t}^s+W_{D,t}^sL_{D,t}^s\right)+A_t^s+\Pi_t^s+TT_t^s}{\underline{P}_t}$$

where S_t is the nominal exchange rate, TT_t^s are government transfers and Π_t^s are distributed profits. Capital is sector-specific and the savers have to decide in which sector to invest. The expression

$$R_t^{k,j}u_t^jK_t^j-\Phi\left(u_t^j\right)K_t^j$$

represents the sector-specific nominal return on the real capital stock minus the cost associated with variations in the degree of capital utilization[11]. Savers have access to international bond trading: $B_{H,t}^s$ and $B_{F,t}^s$ are individual holdings of domestic and foreign bonds denominated in local currency The risk premium function $\Psi(\cdot,\cdot)$ is differentiable, decreasing in both arguments and verifies $\Psi(0,0)=1$. The functional form used for the risk premium is $\Psi(X,Y)=\exp\left(-\chi_{\Delta S}X-2\chi Y\right)$. The term $\varepsilon_t^{\Delta S}$ is a unitary-mean disturbance affecting the risk premium. We maintain the assumption that state-contingent assets are traded among the savers, in order to hedge against wage income. The corresponding stream of income is denoted A_t^s. As a result, all savers have identical consumption plans in equilibrium. Therefore, we can drop superscripts s. We also allow for a time-varying labor income tax, given by $1-\tau_{w,t}=\left(1-\overline{\overline{\tau}}_w\right)\varepsilon_t^W$.

The optimality conditions characterizing the solution of the saver's problem are reported in the Appendix.

In the following, we make use of the saver's and borrower's *user costs* of residential investment (the exact definition can also be found in the Appendix). The user cost indicators are driving

[11] Following Smets and Wouters (2007), we assume that the income obtained from renting out capital services depends on the level of capital augmented for its utilization rate. Moreover, the cost of capacity utilization is zero when capacity is fully used ($\Phi(1)=0$). The functional form for the adjustment costs on capacity utilization is $\Phi(X)=\frac{\overline{\overline{R^k}}}{\varphi}\left(\exp\left[\varphi\left(X-1\right)\right]-1\right)$.. We assume here that capacity is always fully used, for simplicity.

the substitution effects between durable and non-durable goods for each household type. The aggregate user cost, denoted $R_D^{aggregate}$, is defined as the weighted average of the saver's and borrower's user costs.

2.2.3 Labor supply and wage setting

In both countries, households provide differentiated labor services. We assume that the fractions of patient and impatient agents are uniformly distributed across the range of labor services. As a consequence, on aggregate the total supply of a given labor service is identical across types. In each sector j ($j = C, D$) a continuum of unions operate as monopolistic suppliers of the differentiated labor services. Every union represents workers of a certain type. For the sake of simplicity, we assume that unions sell their continuum of labor services (over the interval $[0, 1]$) to a perfectly competitive firm, which in turns transforms them into an aggregate labor input using the technology

$$L_t^j = \left[\int_0^1 L_{j,t}(z)^{\frac{1}{\mu_w}} \mathrm{d}z \right]^{\mu_w}$$

where $\mu_w = \frac{\theta_w}{\theta_w - 1}$ and $\theta_w > 1$ is the elasticity of substitution between differentiated labor services. Therefore, in each sector j, union z faces a labor demand curve with constant elasticity of substitution $L_{j,t}(z) = \left(\frac{W_{,t}(z)}{W_t}\right)^{-\frac{\mu_w}{\mu_w - 1}} L_t^j$, where $W_t^j = \left(\int_0^1 W_{j,t}(z)^{\frac{1}{1-\mu_w}} \mathrm{d}z\right)^{1-\mu_{iw}}$ is the aggregate wage rate.

Unions set wages to maximize intertemporal utility under the budget constraint and labor demand. We assume perfect flexibility of nominal wages in both sectors. The first order condition for this program thus reads:

$$\mu_w \widetilde{L} L^{\sigma_l} = \Lambda_t w_t$$

where w_t denotes the real wage: under wage flexibility, the real wage is equal to a markup μ_w over the marginal rate of substitution between consumption and labor.

2.2.4 Investment decisions

The patient agents in each country own capital and rent it out to the intermediate goods-firms at the sector-specific rental rate $R_t^{k,j}$ ($j = C, D$). Investment is constituted by the distributed non-residential good only. The savers choose the investment and capacity utilization in each sector to maximize their intertemporal utility, subject to the intertemporal budget constraint and the capital accumulation equation:

$$K_t^j = (1-\delta_K)K_{t-1}^j + \varepsilon_t^I \left[1 - S\left(\frac{I_t^j}{I_{t-1}^j}\right)\right] I_t^j \tag{2.7}$$

where $\delta_K \in [0,1]$ is the depreciation rate of capital, S is a non-negative adjustment cost function formulated in terms of the gross rate of change in investment, I_t^j / I_{t-1}^j, and ε_t^I is an efficiency shock to the technology of capital accumulation, common to both sectors. The functional forms adopted are $S(x) = \phi/2\ (x-1)^2$ for country H and $S(x) = \phi^*/2\ (x-1)^2$ for country F, with ϕ and ϕ^* constant.

2.2.5 Distribution sector for non-residential goods

Non-residential goods in each country are produced by a continuum of companies that, operating under perfect competition, mix local production with imports. There is a home bias in aggregation, n, which pins down the degree of openness in steady state. The $\iota - th$ distributor technology, $\forall \iota \in [0,1]$, is given by

$$Y_\iota = \left[n_t^{\frac{1}{\xi}} Y_{\iota,H}^{\frac{\xi-1}{\xi}} + (1-n_t)^{\frac{1}{\xi}} Y_{\iota,F}^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}}$$

in the domestic country and

$$Y_\iota^* = \left[(1-n_t^*)^{\frac{1}{\xi}} Y_{\iota,H}^{*\ \frac{\xi-1}{\xi}} + n_t^{*\ \frac{1}{\xi}} Y_{\iota,F}^{*\ \frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}}$$

in the foreign country, with ξ denoting the elasticity of substitution between bundles Y_H and Y_F. The degrees of home bias are subject to shocks. As only the difference of openness rates enters the linearized aggregate equations in the absence of adjustment costs on imports, home bias shocks are given by $n_t = n\sqrt{\varepsilon_t^{\Delta n}}$ and $n_t^* = \frac{n}{\sqrt{\varepsilon_t^{\Delta n}}}$.
Cost minimization determines import demands:

$$Y_{H,t} = n_t \left(T_{H,t}\right)^{-\xi} Y_t,\ Y_{F,t} = (1-n_t)\left(T_t\ T_{H,t}\right)^{-\xi} Y_t$$
$$Y_{F,t}^* = n_t^* \left(T_{F,t}^*\right)^{-\xi} Y_t^*,\ Y_{H,t}^* = (1-n_t^*)\left(\frac{T_{F,t}^*}{T_t^*}\right)^{-\xi} Y_t^* \tag{2.8}$$

Before-tax distribution prices are defined by:

$$\underline{P}_t = \left[n_t P_{H,t}^{1-\xi} + (1-n_t) P_{F,t}^{1-\xi} \right]^{\frac{1}{1-\xi}}$$
$$\underline{P}_t^* = \left[n_t^* P_{F,t}^{*1-\xi} + (1-n_t^*) P_{H,t}^{*1-\xi} \right]^{\frac{1}{1-\xi}}$$

whereas $T_t = \frac{P_{F,t}}{P_{H,t}}$ and $T_t^* = \frac{P_{F,t}^*}{P_{H,t}^*}$ denote the interior terms of trade. We also make use of the relative prices $T_{H,t} = \frac{P_{H,t}}{\underline{P}_t}$ and $T_{F,t}^* = \frac{P_{F,t}^*}{\underline{P}_t^*}$.

2.2.6 Final non-residential goods sector

In country H, final producers for local sales and imports are in perfect competition and aggregate a continuum of differentiated intermediate products from home and foreign intermediate sector. Y_H and Y_F are sub-indexes of the continuum of differentiated goods produced respectively in country H and F. The elementary differentiated goods are imperfect substitutes with an elasticity of substitution denoted $\frac{\mu}{\mu-1}$. Final goods are produced with the following technology $Y_H = \left[\int_0^1 Y(h)^{\frac{1}{\mu}} \mathrm{d}h \right]^{\mu}$ and $Y_F = \left[\int_0^1 Y(f)^{\frac{1}{\mu}} \mathrm{d}f \right]^{\mu}$. In the country F, the corresponding indexes are given by $Y_F^* = \left[\int_0^1 Y^*(f)^{\frac{1}{\mu}} \mathrm{d}f \right]^{\mu}$ and $Y_H^* = \left[\int_0^1 Y^*(h)^{\frac{1}{\mu}} \mathrm{d}h \right]^{\mu}$. For a domestic product h, we denote $p(h)$ its price on local market and $p^*(h)$ its price on the foreign import market. The domestic-demand-based price indexes associated with imports and local markets in both countries are defined as $P_H = \left[\int_0^1 p(h)^{\frac{1}{1-\mu}} \mathrm{d}h \right]^{1-\mu}$, $P_H^* = \left[\int_0^1 p^*(h)^{\frac{1}{1-\mu}} \mathrm{d}h \right]^{1-\mu}$, $P_F^* = \left[\int_0^1 p^*(f)^{\frac{1}{1-\mu}} \mathrm{d}f \right]^{1-\mu}$ and $P_F = \left[\int_0^1 p(f)^{\frac{1}{1-\mu}} \mathrm{d}f \right]^{1-\mu}$. Domestic demand is allocated across the differentiated goods as follows

$$\begin{cases} \forall h \in [0,1] \quad Y(h) = \left(\frac{p(h)}{P_H} \right)^{-\frac{\mu}{\mu-1}} Y_H, \quad Y^*(h) = \left(\frac{p^*(h)}{P_H^*} \right)^{-\frac{\mu}{\mu-1}} Y_H^* \\ \forall f \in [0,1] \quad Y(f) = \left(\frac{p(f)}{P_F} \right)^{-\frac{\mu}{\mu-1}} Y_F, \quad Y^*(f) = \left(\frac{p^*(f)}{P_F^*} \right)^{-\frac{\mu}{\mu-1}} Y_F^* \end{cases}$$

2.2.7 Intermediate non-residential firms

Intermediate goods producers are monopolistic competitors and produce differentiated products using a Cobb-Douglas mixing labour and capital services $\widetilde{K}_t(\bullet) = u_t(\bullet) K_t(\bullet)$:

$$\begin{cases} Y_t(h) = \varepsilon_t^A \left(u_t^C K_{t-1}^C(h) \right)^{\alpha_C} L_t^C(h)^{1-\alpha_C} - \Omega_C & \forall h \in [0,1] \\ Y_t^*(f) = \varepsilon_t^A \left(u_t^{C*} K_{t-1}^{C*}(f) \right)^{\alpha_C} L_t^{C*}(f)^{1-\alpha_C} - \Omega_C^* & \forall f \in [0,1] \end{cases}$$

where ε_t^A and ε_t^{A*} are exogenous technology parameters. Each firm sells its products both in the local and in the foreign market. We denote $Y_H(h)$ and $Y_H^*(h)$ (respectively $Y_F^*(f)$ and $Y_F(f)$) the local and foreign sales of domestic producer h (respectively foreign producer f) and we define $L_H^C(h)$ and $L_H^{C*}(h)$ (respectively $L_F^{C*}(f)$ and $L_F^C(f)$) the corresponding labor demand.

Local firms set prices on a staggered basis *à la* Calvo (1983). In each period, a firm h (resp. f) faces a constant probability $1-\alpha_H$ (resp. $1-\alpha_F^*$) of being able to re-optimize its nominal price. The average duration of a rigidity period is then $\frac{1}{1-\alpha_H}$ (resp. $\frac{1}{1-\alpha_F^*}$). If a firm cannot re-optimize its price, the price evolves according to the following simple rule:

$$p_t(h) = \Pi_{H,t-1}^{\gamma_H} \overline{\Pi}^{1-\gamma_H} p_{t-1}(h)$$

with γ_H denoting price indexation.

Concerning exports, we assume that, in country H, a fraction η (respectively η^* in country F) of exporters exhibit producer-currency-pricing (PCP) while the remaining firms exhibit local-currency-pricing (LCP). Consequently, aggregate export prices denominated in foreign currency are given by

$$P_H^* = \left[\eta \left(\frac{P_{H,t}}{S_t}\right)^{\frac{1}{1-\mu}} + (1-\eta)\, \tilde{P}_H^{*\frac{1}{1-\mu}}\right]^{1-\mu}, \text{ and } P_F = \left[\eta^* \left(S_t P_{F,t}^*\right)^{\frac{1}{1-\mu}} + (1-\eta^*)\, \tilde{P}_F^{\frac{1}{1-\mu}}\right]^{1-\mu}.$$

The aggregate LCP export price indices are accordingly defined as

$$\tilde{P}_H^* = \left[\frac{1}{1-\eta}\int_\eta^1 p^*(h)^{\frac{1}{1-\mu}} dh\right]^{1-\mu}, \text{ and } \tilde{P}_F = \left[\frac{1}{1-\eta^*}\int_{\eta^*}^1 p(f)^{\frac{1}{1-\mu}} df\right]^{1-\mu}.$$

We define the following relative prices $R\tilde{E}R_H = \frac{S\tilde{P}_H^*}{P_H}$, $R\tilde{E}R_F = \frac{\tilde{P}_F}{SP_F^*}$ and $\tilde{T} = \frac{\tilde{P}_F}{P_H}$. Export margins relative to local sales are denoted $RER_H = \frac{SP_H^*}{P_H}$ and $RER_F = \frac{P_F}{SP_F^*}$. In the presence of international price discrimination, these ratios measure the relative profitability of foreign sales compared with the local ones. Finally, $RER_t = \frac{S_t P_t^*}{P_t}$ is the real exchange rate.

In modeling the firms' decision problem we follow closely Adjemian, Darracq Pariès, and Smets (2008), to which the reader is referred for details and derivations.

2.2.8 Residential goods sector

Final producers of residential goods operate in perfect competition and aggregate a continuum of differentiated domestic intermediate products. Final and intermediate residential goods are non-traded. The elementary differentiated goods are imperfect substitutes with elasticity of substitution denoted $\frac{\mu_D}{\mu_D-1}$. Final goods are produced with the following technology $Z_D = \left[\int_0^1 Z_D(h)^{\frac{1}{\mu_D}} \mathrm{d}h\right]^{\mu_D}$. For a domestic product h, we denote $p_D(h)$ its price. The aggregate price index is defined as $P_D = \left[\int_0^1 p_D(h)^{\frac{1}{1-\mu_D}} \mathrm{d}h\right]^{1-\mu_D}$. Domestic demand is allocated across the differentiated goods as follows: $Z_D(h) = \left(\frac{p_D(h)}{P_D}\right)^{-\frac{\mu_D}{\mu_D-1}} Z_D$.

Residential goods are produced by combining capital, labor and land. We assume that in every period of time the savers are endowed with a given amount of land, which they sell to the firms in a fixed quantity. We assume that the supply of land is exogenously fixed and that each residential goods intermediate firm takes the price of land as given in its decision problem. Producers make use of a Cobb-Douglas technology as follows:

$$\begin{cases} Z_{D,t}(h) = \varepsilon_t^{A_D} \left(u_t^D K_{t-1}^D(h)\right)^{\alpha_D} L_t^D(h)^{1-\alpha_D-\alpha_{\mathcal{L}}} \mathcal{L}(h)_t^{\alpha_{\mathcal{L}}} - \Omega_D & \forall h \in [0,1] \\ Z_{D,t}^*(f) = \varepsilon_t^{A_D*} \left(u_t^{D*} K_{t-1}^{D*}(f)\right)^{\alpha_D} L_t^{D*}(h)^{1-\alpha_D*-\alpha_{\mathcal{L}}*} \mathcal{L}^*(f)_t^{\alpha_{\mathcal{L}}*} - \Omega_D^* & \forall f \in [0,1] \end{cases}$$

where $\varepsilon_t^{A_D}$ and $\varepsilon_t^{A_D*}$ are exogenous technology parameters and $\mathcal{L}_t(h)$ denotes the endowment of land used by producer h at time t.

As in the non-residential sector, firms are monopolistic competitors. We assume perfect price flexibility in the residential-goods sector.

The details of the residential goods firms' problem are spelled out in the Appendix.

2.2.9 Government and monetary authority

In each country, public expenditures $\overline{\overline{G}}$ are subject to random shocks ε_t^G. The government finances public spending with labor tax, production and distribution taxes and lump-sum transfers.

Monetary policy is specified in terms of an interest rate rule targetting CPI inflation, detrended log-output and their first difference. In the benchmark specification, we do not include housing prices in the interest rate rules. Written in deviation from the steady state, the interest feedback rule used has the form:

$$r_t = \rho r_{t-1} + (1-\rho)\left(r_\pi \pi_{t-1} + r_y y_{t-1}\right) + r_{\Delta\pi}\left(\pi_t - \pi_{t-1}\right) + r_{\Delta y}\left(y_t - y_{t-1}\right) + \log\left(\varepsilon_t^R\right) \quad (2.9)$$

where lower case letters denote log-deviations of a variable from its deterministic steady-state. We do not consider here the case of a systematic response of the central bank to fluctuations in house prices. A detailed analysis of such assumption is contained in Darracq Pariès and Notarpietro (2008)).

2.2.10 Market clearing conditions

Aggregate investment and capital stock are given by:

$$I_t^j = (1-\omega)\, I_t^{sj} \quad (2.10)$$

$$K_t^j = (1-\omega)\, K_t^{sj} \quad (2.11)$$

for $j = C, D$.

Aggregate domestic demands for non-residential goods are given by:

$$Y_t = \omega \widetilde{C}_t + (1-\omega)C_t + I_t^C + I_t^D + \overline{\overline{G}}\varepsilon_t^G + \Phi\left(u_t^C\right)K_{t-1}^C + \Phi\left(u_t^D\right)K_{t-1}^D \quad (2.12)$$

$$Y_t^* = \omega^* \widetilde{C}_t^* + (1-\omega^*)C_t^* + I_t^{C*} + I_t^{D*} + \overline{\overline{G}}\varepsilon_t^{G*} + \Phi\left(u_t^{C*}\right)K_{t-1}^{C*} + \Phi\left(u_t^{D*}\right)K_{t-1}^{D*} \quad (2.13)$$

Aggregate non-residential productions satisfy:

$$Z_t = \varepsilon_t^A \left(u_t^C K_{t-1}^C\right)^{\alpha_C} \left(L_t^C\right)^{1-\alpha_C} - \Omega_C \quad (2.14)$$

$$Z_t^* = \varepsilon_t^{A*} \left(u_t^{C*} K_{t-1}^{C*}\right)^{\alpha_C} \left(L_t^{C*}\right)^{1-\alpha_C} - \Omega_C^* \quad (2.15)$$

Market clearing conditions in non-residential goods markets lead to the following relations:

$$Z_t = n_t \Delta_{H,t}\left(T_{H,t}\right)^{-\xi} Y_t + (1-n_t^*)\,\Delta_{H,t}^*\left(\frac{T_{F,t}^*}{T_t^*}\right)^{-\xi} Y_t^* \quad (2.16)$$

$$Z_t^* = n_t^* \Delta_{F,t}^*\left(T_{F,t}^*\right)^{-\xi} Y_t^* + (1-n_t)\,\Delta_{F,t}\left(T_t T_{H,t}\right)^{-\xi} \quad (2.17)$$

where $\Delta_{H,t} = \int_0^1 \left(\frac{p_t(h)}{P_{H,t}}\right)^{-\frac{\mu}{\mu-1}} \mathrm{d}h$, $\Delta^*_{H,t} = \int_0^1 \left(\frac{p^*_t(h)}{P^*_{H,t}}\right)^{-\frac{\mu}{\mu-1}} \mathrm{d}h$, $\Delta^*_{F,t} = \int_0^1 \left(\frac{p^*_t(f)}{P^*_{F,t}}\right)^{-\frac{\mu}{\mu-1}} \mathrm{d}f$ and $\Delta_{F,t} = \int_0^1 \left(\frac{p_t(f)}{P_{F,t}}\right)^{-\frac{\mu}{\mu-1}} \mathrm{d}f$ measure price dispersions among products of country H and F, either sold locally or exported.

Similarly, aggregate productions of residential goods read:

$$Z_{D,t} = \varepsilon_t^{A_D} \left(u_t^D K_{t-1}^D\right)^{\alpha_D} \left(L_t^D\right)_t^{1-\alpha_D-\alpha_\mathcal{L}} \mathcal{L}_t^{\alpha_\mathcal{L}} - \Omega_D \tag{2.18}$$

$$Z^*_{D,t} = \varepsilon_t^{A_D*} \left(u_t^{D*} K_{t-1}^{D*}\right)^{\alpha_D} \left(L_t^{D*}\right)_t^{1-\alpha_D-\alpha_\mathcal{L}} \mathcal{L}_t^{*\alpha_\mathcal{L}} - \Omega^*_D \tag{2.19}$$

Market clearing conditions for the residential markets are

$$Z_{D,t} = \Delta_{D,t} \left[\omega \left(\widetilde{D}_t - (1-\delta)\widetilde{D}_{t-1}\right) + (1-\omega)\left(D_t - (1-\delta)D_{t-1}\right)\right] \tag{2.20}$$

$$Z^*_{D,t} = \Delta^*_{D,t} \left[\omega^* \left(\widetilde{D}^*_t - (1-\delta)\widetilde{D}^*_{t-1}\right) + (1-\omega^*)\left(D^*_t - (1-\delta)D^*_{t-1}\right)\right] \tag{2.21}$$

where $\Delta_{D,t} = \int_0^1 \left(\frac{p_{D,t}(h)}{P_{D,t}}\right)^{-\frac{\mu_D}{\mu_D-1}} \mathrm{d}h$ and $\Delta^*_{D,t} = \int_0^1 \left(\frac{p^*_{D,t}(h)}{P^*_{D,t}}\right)^{-\frac{\mu_D}{\mu_D-1}} \mathrm{d}h$ measure price dispersions among non-residential intermediate goods of country H and F.

Equilibrium in the bond markets implies that $B^*_{F,t} + B^*_{H,t} = 0$ and $B_{H,t} + B_{F,t} = 0$. Moreover, demand for bonds denominated in currency F issued by agents in country H is given by

$$\begin{aligned}\frac{S_t B^s_{F,t}}{P_t R^*_t} - \frac{B^*_{H,t}}{P_t R_t} &= \frac{S_t B^s_{F,t-1}}{P_t} - \frac{B^*_{H,t-1}}{P_t} \\ &\quad + T_{H,t} Y_{H,t} + RER_t \frac{T^*_{F,t}}{T^*_t} Y^*_{H,t} - Y_t\end{aligned} \tag{2.22}$$

The aggregate conditional welfare measures for each type of agent in each country are defined by $\mathcal{W}^B_{H,t} = \int_0^\omega \mathcal{W}^b_t \mathrm{d}b$ and $\mathcal{W}^S_{H,t} = \int_{1-\omega}^1 \mathcal{W}^s_t \mathrm{d}s$, and $\mathcal{W}^{B*}_{F,t} = \int_0^{\omega^*} \mathcal{W}^{b*}_t \mathrm{d}b$ and $\mathcal{W}^{S*}_{F,t} = \int_{1-\omega^*}^1 \mathcal{W}^{s*}_t \mathrm{d}s$, respectively.

2.3 Calibration

The baseline calibration is reported in Table 1. The two countries are perfectly symmetric under our parametrization, which draws heavily on the estimates of Darracq Pariès and Notarpietro (2008). More precisely, whenever the estimated values for one parameter differ across countries, we pick the value corresponding to the US economy.

The discount factors are set to 0.99 for the patient agents and 0.96 for the impatient agents, so that the implied equilibrium real interest rate is 4% in annual terms. The depreciation rate for housing is equal to 0.01, corresponding to an annual rate of 4%, whereas the depreciation rate of capital is set to 0.1. Markups are constant across countries and equal to 1.3 in the goods markets (for both nonresidential and residential goods) and 1.5 in the labor market (in each sector).

The relative share of residential goods in the utility function, ω_D, is is set to 0.1, while the intratemporal elasticity of substitution between residential and nonresidential investment, η_D, is equal to 1[12]. Noticeably, η_D and ω_D correspond, in our two-sector model, to the elasticity of substitution between traded and non-traded goods and the relative share of non-traded goods in consumption, respectively. Therefore, the calibration of the two parameters is of particular relevance in matching both housing-market and open-economy stylized facts. We choose the baseline calibration in order to match the steady-state ratios of private debt and residential investment to GDP for the US. However, in our sensitivity analysis we allow the two parameters to assume the values usually reported in the open-economy macroeconomics literature for models that feature traded and non-traded goods.

The relative shares of inputs in production are 0.3 for capital and 0.7 for labor in the nonresidential goods sector, while in the residential sector we assign a weight equal to 0.1 to land and reduce the share of capital to 0.2, in order to maintain the level of labor intensity unchanged. About price setting, we assume price flexibility in the residential goods sector, whereas we set the Calvo parameter for non-residential goods producers to 0.88, corresponding to eight quarters between two consecutive price changes. The price-indexation parameter, γ_p, is equal to 0.55. The share of firms exhibiting producer currency pricing (PCP), η, is equal to 0.98.

About credit market frictions, we calibrate the loan-to-value ratio (determined by the terms $(1-\chi)$ and $(1-\chi^*)$), to 0.8 in both areas. The share of borrowers (ω) is equal to 0.23, following the estimated value for the US reported in Darracq Pariès and Notarpietro (2008).

[12] Ogaki and Reinhart (1998) estimate the elasticity of substitution between durable and nondurable goods to be in between 1.17 and 1.24.

Regarding the utility function, the degrees of habit in consumption for the two agents, h and h_B, are equal to 0.5 and 0.3, respectively. The degree of relative risk aversion, σ_X, is equal to 0.64. Finally, the parameters of the monetary policy rule are taken from Darracq Pariès and Notarpietro (2008).

2.4 The transmission mechanism and the role of credit frictions

This section explores the transmission mechanism of the model by looking at the impulse response functions of prices and quantities to some selected shocks.

The benchmark specification of the model includes three broad categories of structural shocks: *(i)* technology shocks in each sector (ε_t^A, $\varepsilon_t^{A_D}$), *(ii)* monetary policy shocks (ε_t^R) and *(iii)* housing-related shocks, defined as housing preference shocks (ε_t^D) and loan-to-value ratio shocks (ε_t^{LTV}). In addition, we include a shock to the modified UIP condition ($\varepsilon_t^{\Delta S}$) and common factors in tradeable technology and monetary policy across countries (f_t^A and f_t^R, respectively) [13] . The parametrization of the shock processes is reported in Table 1 and is based on Darracq Pariès and Notarpietro (2008).

For each shock, we systematically analyze the contribution of household credit frictions in generating the main results. For the sake of simplicity, we focus on the transmission of US shocks.

2.4.1 Non-residential-goods technology shock

The effects of a positive productivity shock in the non-residential, traded goods sector are reported in Figure 2.5. Under sticky non-residential prices, labor demand decreases on impact, implying a fall in the real marginal cost. Monetary policy accommodates the supply shock by reducing the nominal interest rate. Production increases in both sectors, with house

[13] Such common factors are meant to capture shocks originating from the rest of the world (not included in our two-country model), or other unspecified spillovers (see Darracq Pariès and Notarpietro (2008) for an explanation).

prices featuring a hump-shaped response. The impact on aggregate consumption is quite sensitive to the share of borrowers in the economy. The borrowers, in fact, respond to the shock by reducing their present consumption of both goods, as they do not perceive the consumption-smoothing motive. With a high share of borrowers, the substitution effect from non-residential consumption towards residential investment is amplified; consumption falls significantly, which accentuates the downward pressures on inflation and the extent of policy accommodation. Conversely, with a very low share of borrowers the response of consumption is positive.

In all of the cases the exchange rate depreciates, driven by the fall in the interest rate. The *expenditure-switching effect* dominates and foreign consumption increases.

Interestingly, under the baseline calibration, a positive productivity shock in the traded (non-residential) sector generates a *positive* comovement between the real exchange rate and housing prices and a *negative* comovement between the real exchange rate and relative consumption, as observed in the data.

2.4.2 Housing technology shock

A positive productivity shock in the residential sector generates a sharp and persistent reduction in house prices, which are assumed to be perfectly flexible (see Figure 2.3). Intuitively, firms in the housing sector can fully exploit the technology improvement by adjusting prices and quantities in opposite directions. Residential investment indeed increases significantly on impact, with a persistent effect. The behavior of domestic demand follows from the individual responses to the shock. For the savers, an increase in housing supply, accompanied by a reduction in prices, generates a higher demand for residential investment. Moreover, the decrease in house prices lowers the savers' user cost of housing, generating a substitution effect from nonresidential to residential goods. Wage income also increases for the savers, who are accommodating the increase in labor demand in the residential-goods sector through the provision of more labor supply. The reverse holds true for the borrowers. A sustained decrease in house prices induces a negative *valuation effect* on the existing collateral, making borrowing more costly. As a result, the borrowers demand less of both goods compared with the savers, and debt decreases on impact in the source country. In addition, the borrowers have a stronger incentive to substitute residential investment for consumption goods in order to relax their collateral constraint. The resulting negative aggregate effect on consumption reduces CPI inflation, while aggregate output increases slightly.

Turning to the open-economy dimension, the transmission crucially depends on the response of monetary policy to aggregate demand. In the benchmark case, aggregate demand decreases and inflation falls. The central bank responds by lowering the interest rate. As a result, the exchange rate depreciates. Foreign demand for domestically-produced goods increases, as well as house prices. Hence, in this case the model predicts a *negative* comovement between the real exchange rate and housing prices, as opposed to what is observed in the data. The result is robust to variations in ω.

2.4.3 Housing preference shock

A housing preference shock is defined here as an exogenous stochastic perturbation to the marginal rate of substitution between non-residential and residential consumption in the utility function of each agent. A positive housing preference shock thus generates a persistent surge in housing demand and house prices (see Figure 2.4). The most interesting effect concerns the impact response of consumption, which is positive and increasing in the share of borrowers. The positive response of aggregate consumption is due to the *collateral channel*: by issuing more debt - made possible by the positive valuation effect of higher house prices on the existing collateral - the borrowers can finance extra consumption, thus consuming immediately more. Monetary policy responds by raising the interest rate, which in turn causes an exchange rate appreciation. The current account deteriorates by a small amount on impact and the excess domestic demand in the source country leads to a positive effect on foreign output.

All the above-mentioned effects are amplified when the share of borrowers increases: the amplification reflects the mechanical aggregation effect across the two different groups of consumers. Without borrowing instead, the response of domestic consumption to the housing preference shock is muted. Intuitively, without impatient households, there is no positive effect of higher house prices on consumption, because there is no collateral to be affected. However, the overall increase in domestic aggregate demand is lower than in the presence of borrowers. Most importantly, under the baseline parametrization the responses of housing prices and the real exchange rate always have *opposite* signs.

2.4.4 Loan-to-value (LTV) ratio shock

A positive shock to the LTV ratio corresponds to an exogenous, temporary increase in the availability of funds to the borrowers in the domestic economy (see Figure 2.6). The borrowers thus demand more of both goods, driving house prices up. In particular, the absence of nominal rigidities in the housing sector originates a sharp and sustained increase in house prices, whereas consumption-goods inflation moves slowly, due to tradeable-goods price stickiness. Debt increases, fostered by the positive valuation effect of higher house prices on the existing collateral, and by the exogenous increase in the LTV ratio. Inflation increases, inducing an interest rate rise. As a consequence, the exchange rate appreciates on impact and the current account deteriorates in the short term. However, the initial exchange rate appreciation is rapidly followed by a depreciation of a similar magnitude. A broad substitution effect is observed in the Foreign economy, away from residential goods and domestically produced non-residential goods.

An increase in the share of borrowers reinforces the collateral channel in the Home country[14]. Demand and prices increase more in both sectors, thus requiring a stronger response of monetary policy to inflation. The exchange rate swings are more pronounced, and all of the previously-described spillover effects on the Foreign country are amplified.

As for the case of a housing preference shock, the co-movement of house prices and the real exchange rate is *negative*, as opposed to what is observed in the data.

2.4.5 Monetary policy shock

A positive shock to the short-term interest rate reduces domestic demand in both sectors and generates a contraction of output (see Figure 2.8). The combination of higher interest rates

[14]Clearly, we do not consider the case of no borrowers, which would imply the absence of a debt channel, and prevent the existence of any LTV-ratio shocks.

and lower house prices leads the borrowers to reduce debt, thus amplifying the negative effect of the shock on aggregate consumption and output. Noticeably, the reduction in the inflation rate, brought about by the weakening of economic activity, is detrimental to the borrowers, since it increases the ex-post value of existing debt. As a result, the monetary shock becomes more contractionary when the share of borrowers in the economy increases. The rise in the interest rate induces an exchange rate appreciation; the current account deteriorates. The domestic monetary contraction produces a positive spillover effect on foreign aggregate output. The exchange rate adjustment together with monetary policy tightening in the euro area generates substantial substitution effects away from residential investment and domestic demand for tradable goods.

Without the possibility of borrowing (i.e. when $\omega = \omega^* = 0$), the monetary transmission mechanism only works through nominal rigidities, as in a standard New Open Economy Macroeconomics model. Absent the collateral channel, the impact response of domestic consumption and output is significantly weaker. The interest rate increase is still sufficient to generate an exchange rate appreciation. The resulting adjustment in net imports implies a larger positive spillover on foreign output with respect to the baseline case.

Interestingly, under the baseline calibration both real house prices and the real exchange rate respond negatively to a monetary policy tightening, thus displaying a *positive* comovement after the shock.

2.4.6 UIP shock

The UIP shock - defined as a risk-premium shock in the modified UIP condition - causes a strong exchange rate appreciation and a current account deterioration (see Figure 2.7). The interest rate decreases in the Home country and increases by almost the same magnitude in the Foreign country. Domestic demand increases in both sectors, while output decreases, reflecting a fall in net exports. Inflation is correspondingly lower in the Home country and higher in the Foreign economy. Domestic household debt jumps on impact: the combined effect of higher house prices and a lower interest rate offsets in fact the negative impact of lower CPI inflation on the borrowers' balance sheets. The opposite holds true in the Foreign country, where household debt falls. As the shock directly hits the interest rate, the effects of varying the share of borrowers are mainly concentrated on the response of consumption. Absent the borrowers, the partial offsetting effect of lower inflation on consumption is eliminated, and the demand for nonresidential goods is correspondingly higher. The opposite effect is observed in the Foreign country, where consumption falls more. Again, the observed positive co-movement of real house prices and the real exchange rate is *not* reproduced.

2.4.7 Summing up

This section has provided a detailed description of the internal propagation mechanism of the model, with a special focus on the role played by housing markets and collateral constraints. Overall, the presence of credit frictions has the effect of altering the relative responses of aggregate consumption and non-residential investment to exogenous shocks. More precisely, moving the share of borrowers is implicitly equivalent to attributing more or less importance to credit constraints and thus influences the propagation of standard demand and supply shocks. The introduction of housing-specific shocks generates nontrivial dynamics. In particular, housing preference and loan-to-value ratio shocks are based on the *collateral channel*, and determine an immediate increase in household debt. The impact on aggregate consumption and investment is hence positively influenced by an increase in the share of borrowers. The housing sector plays a special role in the model, due to its dual nature of flexible-price, non-traded goods producing sector. On the one hand, prices and quantities are free to adjust almost instantaneously to external shocks, so that impact responses are usually large. On the other hand, as the residential investment good is non-traded, sectoral shocks typically generate small, indirect spillover effects on foreign country variables. The residential sector is therefore somewhat unaffected by shifts in the share of borrowers: the impact responses of prices and quantities are large enough that varying the size of the borrowers' group only marginally affects the overall adjustment. Nonetheless, structural housing-related shocks generate significant spillovers to non-residential consumption through the collateral channel and therefore the share of borrowers in the economy.

Under the baseline calibration, the model can generate a positive comovement in the real exchange rate and housing prices, in response to either a traded-goods productivity shock, or a monetary policy shock, as summarized in Figure 2.9 .

2.5 Theoretical Results

In this section we analyze the main theoretical implications of the model and perform some sensitivity analysis. Table 2 reports the unconditional second moments and correlations generated by the model under the baseline calibration, along with their empirical counterparts. The statistics are computed by applying the Hodrick and Prescott filter to the data and to the model-generated series[15] . We proceed as follows. First, we analyze the general performance of the model under the benchmark calibration and briefly comment on the role of credit constraints. We then consider the effects of varying a few parameters of interest: the degree of home bias in consumption (n), the degree of risk aversion (σ_X), the elasticity of substitution between Home and Foreign consumption goods (ζ), the elasticity of substitution between residential and non-residential goods (η_D), and the amount of international price discrimination, captured by η (the fraction of exporting firms that exhibit producer currency pricing).

2.5.1 The benchmark model and the role of credit frictions

Under the baseline calibration, the model generates excess relative volatility of consumption with respect to GDP and large fluctuations in house prices. The relative standard deviations of CPI inflation and the nominal interest rates are instead precisely reproduced. On the open-economy side, the model replicates the relative volatility of the nominal exchange rate and the current account, but clearly fails at generating enough volatility in the real exchange rate.

In terms of correlations, the baseline version of the model generates too little international co-movement in output and consumption and predicts no correlation in housing. Nonetheless, the stochastic structure of the model is rich enough to reproduce the observed negative and sizeable correlation between the real exchange rate and relative consumption, thus solving the well-known *consumption-real exchange rate puzzle* (see Backus, Kehoe, and Kydland (1993)).

[15] See the Appendix for a detailed description of the data used.

The observed positive correlation between the real exchange rate and housing prices cannot be reproduced under the baseline parametrization: the theoretical correlation is in fact small and negative.

The importance of credit frictions is summarized by the value of ω, the relative share of borrowers in the economy. An increase from 0.23 (as in the baseline calibration) to 0.5 produces a sizeable increase in the international co-movements of GDP, consumption and residential investment, while it slightly deteriorates the cross-correlation of housing prices. Increasing the share of borrowers amplifies the transmission of housing and credit-related shocks to the non-residential, non-traded sector. Intuitively, a larger fraction of consumers are exploiting the beneficial effect of larger credit supply, which allow them to increase consumption of traded goods. The cross-country transmission of non-traded shocks is therefore amplified, as reflected by higher cross-correlations in output and consumption. We will return to this mechanism later.

2.5.2 Sensitivity analysis

We consider now the effects of variations in some crucial parameters. In each of the cases considered, we also allow for changes in the share of borrowers and in the loan-to-value ratio, thus exploring the interaction of traditional open-economy factors with housing and credit frictions.

Table 3 reports the effects of reducing the home bias in consumption (n) from 0.98 to 0.6. The attribution of more weight to imported goods reduces the domestic correlation of output with consumption. However, reducing n is not sufficient to change the sign of the real exchange rate-relative consumption correlation, which indirectly proves the importance of other sources of frictions in generating such negative comovement. Interestingly, the combination of low home bias with a large share of borrowers (see column 4) significantly increases international comovements of real variables. Intuitively, the effects of housing-related shocks are more heavily transmitted to the foreign country, because of two forces. On the one hand, shocks originating in the residential sector are transmitted to the nonresidential sector via the valuation effect of higher house prices on the existing collateral. The magnitude of such effect is amplified by the increased number of borrowers. In addition, domestic final consumption in each country depends almost equally on local and imported consumption goods, so that the cross-country transmission of the shock is indirectly amplified. However, the model generates excess consumption risk-sharing, so that the consumption-real exchange rate anomaly emerges. The puzzle disappears, though, when combining a low home bias with a high loan-to-value ratio (see column 5). In this case, the model captures the observed correlation between house prices and consumption (a typical by-product of the collateral channel) and generates more realistic patterns for cross-country correlations. In addition, the observed positive correlation between real house prices and the real exchange rate is reproduced.

An increase in the coefficient of relative risk aversion, σ_X, reduces all cross-correlations without affecting the real exchange rate-relative consumption pattern, even for high values of ω (see the last three columns of Table 3). However, when also the LTV ratio increases, the model generates cross-country correlations that are in line with the data. Such result mainly depends on the consumption choices of the borrowers, who are not influenced by a change in σ_X, insofar as they do not smooth consumption intertemporally.

Table 4 reports the effects of reducing the intratemporal elasticity of substitution between nonresidential and residential goods (η_D). As already noted, (η_D) represents the degree of substitutability between housing and non-housing, but also and more generally, between non-traded and traded goods. Therefore, when modifying our baseline calibration to allow for a lower elasticity, we follow Benigno and Thoenissen (2008) and set η_D=0.44 [16] . As reported in column 3, varying the value of η_D in this range does not significantly alter the patterns of cross-correlations and relative volatilities [17] . The result is robust to increases in the loan-to-value ratio. However, when a lower substitutability between residential and nonresidential goods is mixed with a high share of borrowers, the model generates negative cross-country correlations in consumption and output. Intuitively, the increased impact of credit frictions

[16] Any change in η_D must be accompanied by a corresponding change in the relative share of housing in consumption, ω_D, in order to keep the steady-state ratio of private-debt-to-output constant. We account for this in the exercise, although we do not report the values of ω_D, to save space.

[17] Benigno and Thoenissen (2008), using a two-sector model with traded and non-traded goods, show that reducing the intratemporal elasticity of substitution between traded and non-traded goods from 0.74 to 0.44 has no significant effect on the cross-correlation of relative consumption and the real exchange rate, nor on the relative volatility of the real exchange rate.

would call for larger spillovers across sectors, but the low substitutability across goods types counteracts such force and reduces international risk sharing. Consumption, residential investment and output are therefore less correlated among themselves, both domestically and internationally.

The role of trade elasticity is analyzed in Table 5. In the case of a high trade elasticity (ζ=2) combined with either high ω or high LTV ratio, the model generates positive international risk sharing in consumption and matches the correlation of relative consumption with the real exchange rate. With a low trade elasticity (ζ=0.5) instead, the model predicts a very volatile nominal exchange rates and matches international correlations in real quantities. The result reinforces the argument in Corsetti, Dedola, and Leduc (2008b) in favor of either low trade elasticity or highly persistent shocks in generating a low degree of risk sharing. Conversely, the model performs relatively poorly with a very high trade elasticity (ζ=8).

Finally, we consider variations in η, the share of exporting firms that exhibit producer currency pricing (PCP). As the theory suggests, pricing-to-market (PTM) and imperfect exchange-rate pass-through imply small effects on real allocations of variations in the nominal exchange rate, via a reduction in the expenditure-switching effect. Correspondingly, real exchange rate fluctuations are very large [18]. Subsequently, a higher degree of PTM corresponds to a lower cross-correlation in consumption and a higher cross-correlation in output. While the baseline calibration considers the case of almost no PTM (η=0.98, equivalent to only 2% of firms adopting local currency pricing), in our sensitivity exercise we set η=0.5 to allow for a high degree of PTM. As reported in Table 6, the model with high local currency pricing (LCP) generates: *(i)* a high relative volatility in the real exchange rate; *(ii)* almost no cross-correlation in real quantities; *(iii)* a negative correlation between relative consumption and the real exchange rate. In particular, results *(i)* and *(iii)* are consistent with those arising in the theoretical economy of Corsetti, Dedola, and Leduc (2008a), in the case of sticky prices and high LCP. Interestingly, increasing the size of borrowers helps the model reproducing the observed cross-correlations, but implies a very low relative volatility in the real exchange rate. The result hinges on the reduction in the volatility of both residential investment and house prices,

[18] See Betts and Devereux (2000).

brought about by an increase in ω.

2.6 Conclusions

In this paper we have outlined and analyzed on original framework to analyze the interactions of housing-market dynamics, household credit frictions and international business cycle transmission. The model assigns a special role to the housing sector, which provides a consumption and collateral good to the borrowers, but also acts as an otherwise standard non-traded-goods-producing sector. The dual nature determines interesting cross-country dynamics, among which the correlation between housing prices and the real exchange rate is analyzed. The observed simultaneity of the sustained depreciation of the US dollar against the Euro and the increase in housing prices can be rationalized in an economy featuring collateral constraints, high risk aversion and a large fraction of borrowers. Moreover, the model can account for the consumption-real exchange rate anomaly and, in the case of a high degree of pricing-to-market, at least partially capture the large volatility of the real exchange rate.

2.7 Appendix

2.7.1 Supplementary model description

The borrower's program

The impatient agent maximizes (2.1) under (2.3) and (2.4) holding with equality[19]. We report the corresponding first order conditions in the next paragraph.

Let us denote

$$\widetilde{\mathcal{U}\prime}_{X,t} = \varepsilon_t^{\beta} \widetilde{X}_t^{-\sigma_C} \tag{2.23}$$

$$\begin{aligned} \widetilde{\mathcal{U}\prime}_{C,t} &= \left(1-\varepsilon_t^D\omega_D\right)^{\frac{1}{\eta_D}} \left(\widetilde{C}_t - h_B\widetilde{C}_{t-1}\right)^{-\frac{1}{\eta_D}} \widetilde{X}_t^{\frac{1}{\eta_D}} \widetilde{\mathcal{U}\prime}_{X,t} \\ &\quad - \beta h_B \left(1-\varepsilon_t^D\omega_D\right)^{\frac{1}{\eta_D}} \left(\frac{\widetilde{C}_t}{\widetilde{X}_t}\right)^{-\frac{1}{\eta_D}} \mathbb{E}_t \left\{ \begin{array}{c} \left(1-\varepsilon_{t+1}^D\omega_D\right)^{\frac{1}{\eta_D}} \\ \left(\widetilde{C}_{t+1} - h_B\widetilde{C}_t\right)^{-\frac{1}{\eta_D}} \widetilde{X}_{t+1}^{\frac{1}{\eta_D}} \widetilde{\mathcal{U}\prime}_{X,t+1} \end{array} \right\} \end{aligned} \tag{2.24}$$

[19] It is possible to show that the collateral constraint always binds in the deterministic steady state, under general conditions. We assume here that continues to hold in a sufficiently small neighborhood of the steady state, so that the model can be solved by taking a first order approximation.

$$\widetilde{\mathcal{U}\prime}_{D,t} = \varepsilon_t^D \omega_D^{\frac{1}{\eta_D}} \left(\frac{\widetilde{D}_t}{\widetilde{X}_t}\right)^{-\frac{1}{\eta_D}} \widetilde{\mathcal{U}\prime}_{X,t} \tag{2.25}$$

The first order condition related to non-residential consumption and residential stock are respectively,

$$\widetilde{\Lambda}_t = \widetilde{\mathcal{U}\prime}_{C,t} \tag{2.26}$$

and

$$\begin{aligned} \widetilde{\Lambda}_t T_{D,t} \quad = \quad & (1-\chi)\,\varepsilon_t^{LTV} \Psi_t \widetilde{\Lambda}_t \mathbb{E}_t \left\{ T_{D,t+1} \frac{\pi_{t+1}}{R_t \varepsilon_t^{CPI}} \right\} \\ & + \widetilde{\mathcal{U}\prime}_{D,t} + \beta\,(1-\delta)\,\mathbb{E}_t \left\{ \widetilde{\Lambda}_{t+1} T_{D,t+1} \right\} \end{aligned} \tag{2.27}$$

where $\frac{\widetilde{\Lambda}_t}{1+\tau_{C,t}}$ and $\frac{\widetilde{\Lambda}_t}{1+\tau_{C,t}}\Psi_t$ are the multipliers associated to constraint (2.3) and (2.4), respectively.

Finally, the marginal value of additional borrowing is defined by the following "modified" version of the standard Euler equation

$$\Psi_t = 1 - \beta \mathbb{E}_t \left\{ \frac{\widetilde{\Lambda}_{t+1}}{\widetilde{\Lambda}_t} \frac{R_t}{\pi_{t+1}} \right\} \tag{2.28}$$

The set of optimality conditions is completed by the intratemporal trade-off between consumption and leisure, which is analyzed in detail later. By rearranging equation (2.27) it is possible to define the borrower's *user cost* of residential investment as follows:

$$\widetilde{R}_D = T_{D,t}\left(1 - \varepsilon_t^{LTV}\,(1-\chi)\,\Psi_t E_t \left\{ \frac{T_{D,t+1}}{T_{D,t}} \frac{\pi_{t+1}}{R_t \varepsilon_t^{CPI}} \right\} - \beta\,(1-\delta)\,E_t \left\{ \frac{\widetilde{\Lambda}_{t+1}}{\widetilde{\Lambda}_t} \frac{T_{D,t+1}}{T_{D,t}} \right\}\right) \tag{2.29}$$

The saver's program

Let us denote

$$\mathcal{U}\prime_{X,t} = \varepsilon_t^{\beta} X_t^{-\sigma_C} \tag{2.30}$$

$$\begin{aligned} \mathcal{U}\prime_{C,t} = & \left(1-\varepsilon_t^D \omega_D\right)^{\frac{1}{\eta_D}} (C_t - hC_{t-1})^{-\frac{1}{\eta_D}} X_t^{\frac{1}{\eta_D}} \mathcal{U}\prime_{X,t} \\ & - \gamma h \left(1-\varepsilon_t^D \omega_D\right)^{\frac{1}{\eta_D}} \left(\frac{C_t}{X_t}\right)^{-\frac{1}{\eta_D}} \mathbb{E}_t \left\{ \begin{array}{c} \left(1-\varepsilon_{t+1}^D \omega_D\right)^{\frac{1}{\eta_D}} (X_{t+1})^{\frac{1}{\eta_D}} \\ (C_{t+1} - hC_t)^{-\frac{1}{\eta_D}} \mathcal{U}\prime_{X,t+1} \end{array} \right\} \end{aligned} \tag{2.31}$$

$$\mathcal{U}\prime_{D,t} = \varepsilon_t^D \omega_D^{\frac{1}{\eta_D}} \left(\frac{D_t}{X_t}\right)^{-\frac{1}{\eta_D}} \mathcal{U}\prime_{X,t} \tag{2.32}$$

The first order condition related to non-residential consumption and residential stock are respectively,

$$\Lambda_t = \mathcal{U}\prime_{C,t} \tag{2.33}$$

and

$$\Lambda_t T_{D,t} = \mathcal{U}\prime_{D,t} + \gamma (1-\delta) \mathbb{E}_t \left\{\Lambda_{t+1} T_{D,t+1}\right\} \tag{2.34}$$

where $\frac{\Lambda_t}{1+\tau_{C,t}}$ is the multiplier associated with the budget constraint.

Patient households in both countries are allowed to trade in two one-period nominal bonds, a domestic and a foreign one. First order conditions corresponding to the quantity of contingent bonds imply that

$$\Lambda_t = R_t \gamma \mathbb{E}_t \left[\Lambda_{t+1} \frac{P_t}{P_{t+1}}\right] \tag{2.35}$$

$$\Lambda_t = R_t^* \varepsilon_t^{\Delta S} \Psi \left(\frac{\mathbb{E}_t S_{t+1}}{S_{t-1}} - 1, \frac{S_t \left(B_{F,t} - \overline{\overline{B}}_F\right)}{\underline{P}_t}\right) \beta \mathbb{E}_t \left[\Lambda_{t+1} \frac{S_{t+1} P_t}{S_t P_{t+1}}\right]$$

where R_t and R_t^* are one-period-ahead nominal interest rates for country H and F respectively.

The previous equations imply an arbitrage condition on bond prices which corresponds to a modified uncovered interest rate parity (UIP):

$$\frac{R_t}{R_t^* \varepsilon_t^{\Delta S} \Psi \left(\frac{\mathbb{E}_t S_{t+1}}{S_{t-1}} - 1, \frac{S_t \left(B_{F,t} - \overline{\overline{B}}_F\right)}{\underline{P}_t}\right)} = \frac{\mathbb{E}_t \left[\Lambda_{t+1} \frac{S_{t+1} P_t}{S_t P_{t+1}}\right]}{\mathbb{E}_t \left[\Lambda_{t+1} \frac{P_t}{P_{t+1}}\right]} \tag{2.36}$$

Rearranging equation (2.34) yields the definition of the saver's *user cost* of residential investment:

$$R_D = T_{D,t} \left(1 - \gamma (1-\delta) E_t \left\{\frac{\Lambda_{t+1}}{\Lambda_t} \frac{T_{D,t+1}}{T_{D,t}}\right\}\right) \tag{2.37}$$

2.7.2 Data

The data for the US are obtained from the BEA, the BLS, the Census Bureau and the Federal Reserve Board. In particular, real house prices in the US are computed using the Census Bureau index (house price index for new one-family houses sold including value of lot).

Euro area data are taken from Fagan, Henry, and Mestre (2005) and Eurostat. House prices for the euro area are based on national sources and taken from the ECB website. Residential investment is taken from Eurostat national accounts and is backcasted using national sources. The exchange rate is the euro/dollar exchange rate. Due to statistical problems in computing long series of bilateral current account and current account for the euro area, we use the US current account as a share of US GDP. Aggregate real variables are expressed in per capita terms by dividing through with working age population. All the data are detrended by applying the Hodrick and Prescott filter.

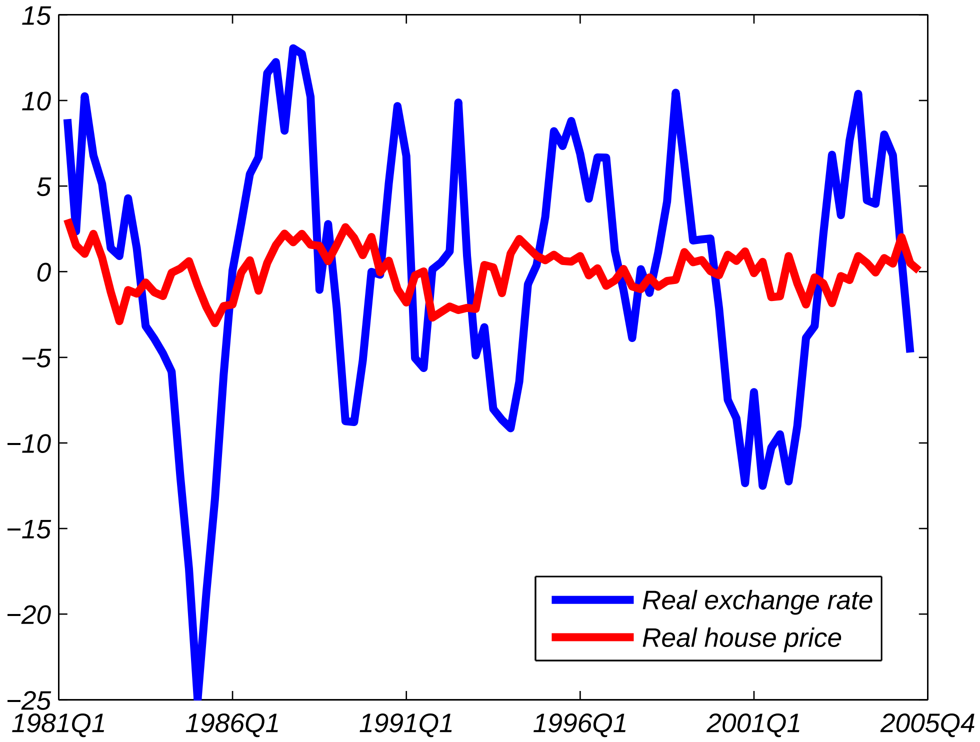

Figure 2.1: Euro/US Dollar Real Exchange Rate and US Housing Prices (HP-filtered series)

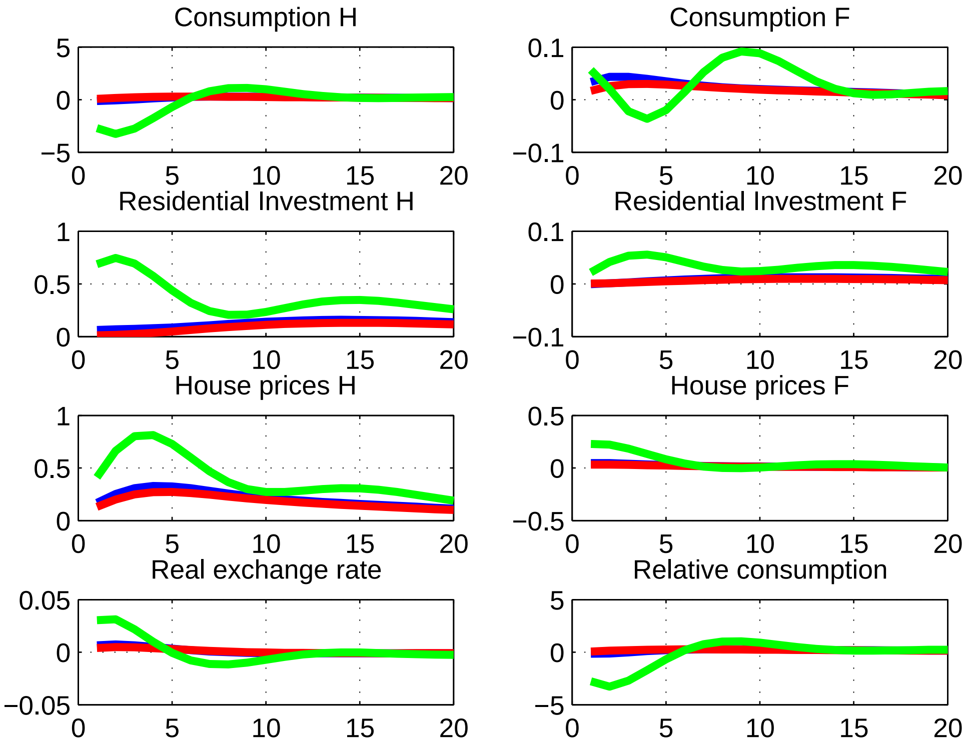

Figure 2.2: Impulse Responses to a Productivity Shock in the Non-residential Goods Sector: Baseline (Blue), No borrowers (Red), High Borrowers (Green)

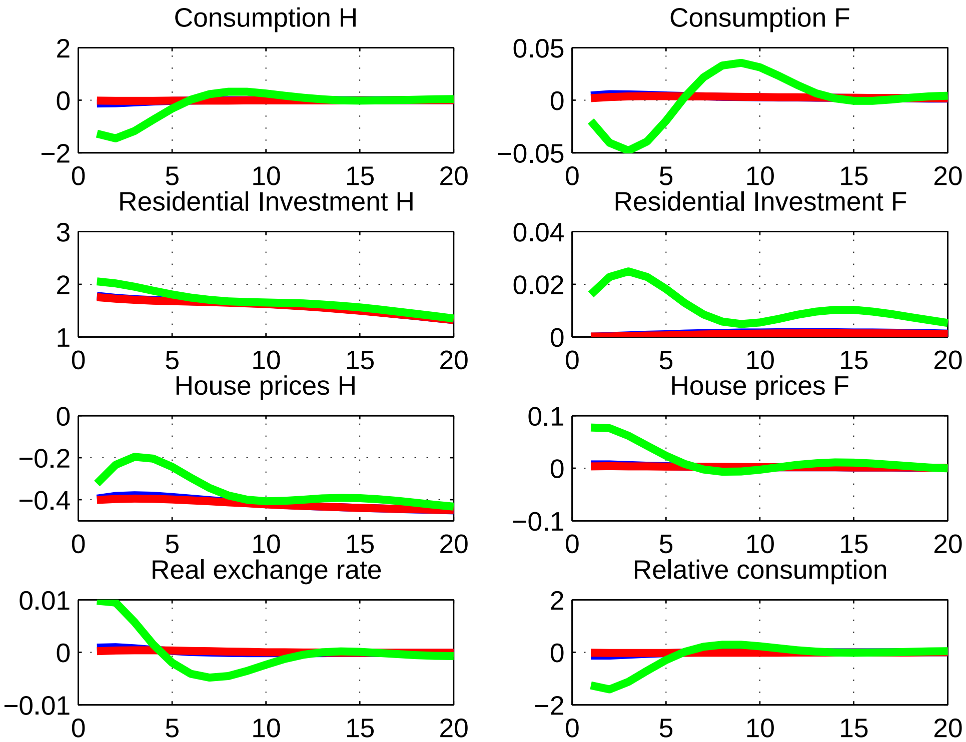

Figure 2.3: Impulse Responses to a Productivity Shock in the Residential Goods Sector: Baseline (Blue), No borrowers (Red), High Borrowers (Green)

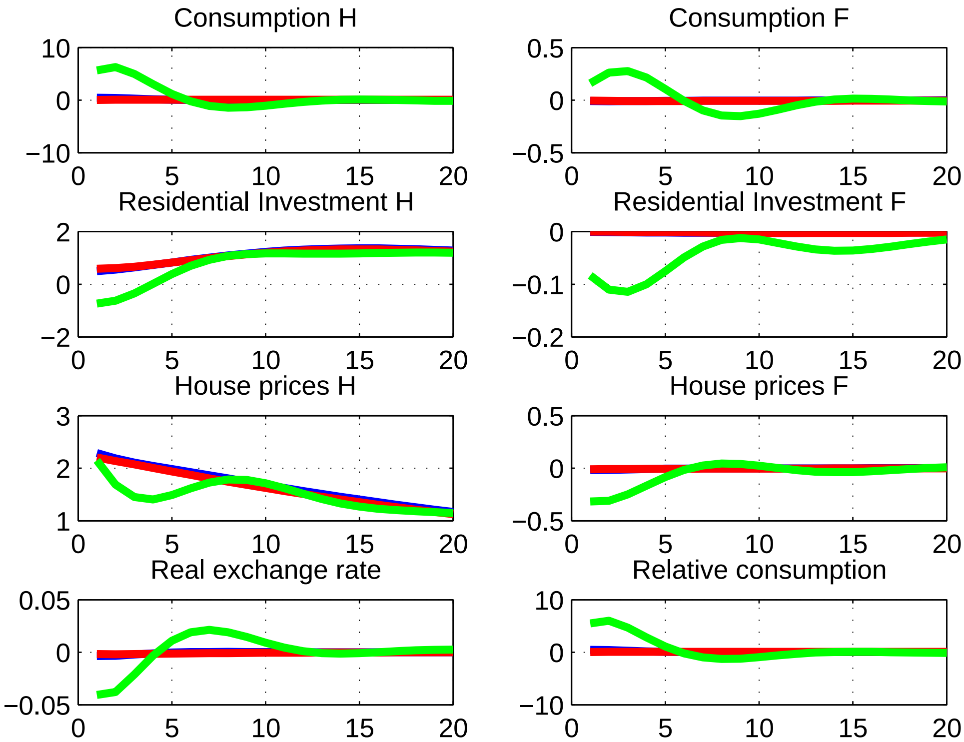

Figure 2.4: Impulse Responses to a Housing Preference Shock: Baseline (Blue), No borrowers (Red), High Borrowers (Green)

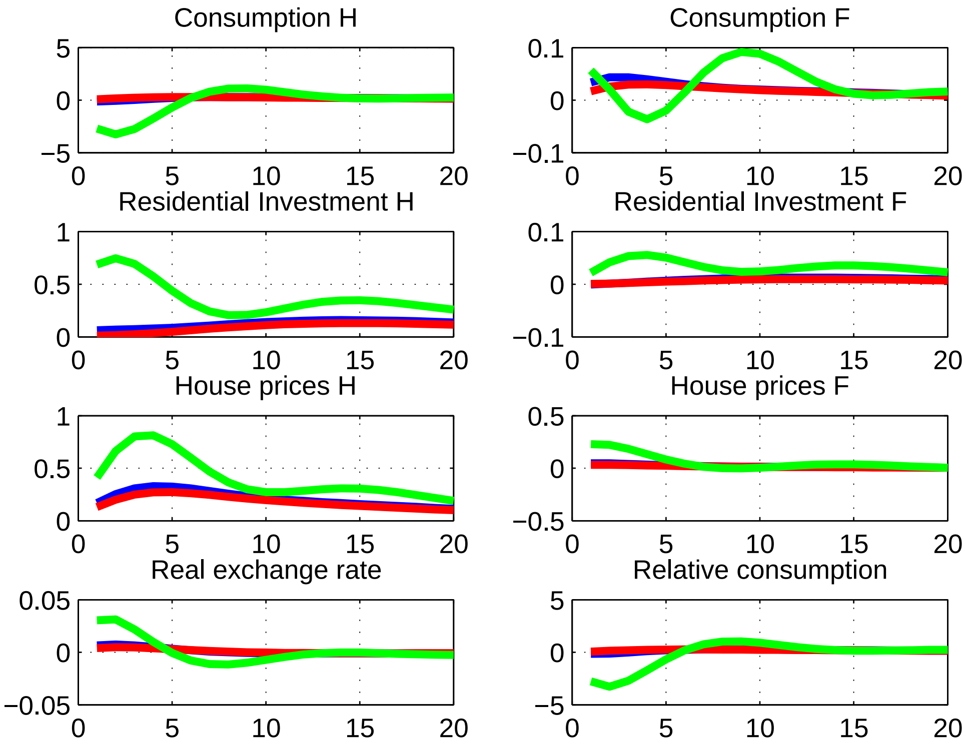

Figure 2.5: Impulse Responses to a Productivity Shock in the Non-residential Goods Sector: Baseline (Blue), No borrowers (Red), High Borrowers (Green)

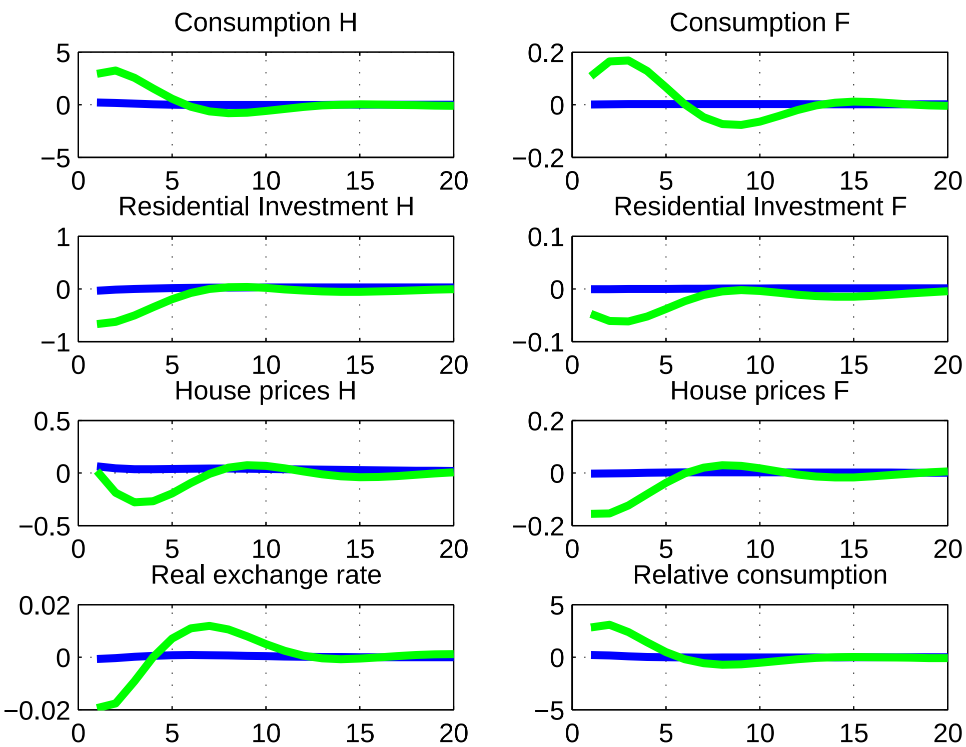

Figure 2.6: Impulse Responses to a Loan-to-Value Ratio Shock: Baseline (Blue), No borrowers (Red), High Borrowers (Green)

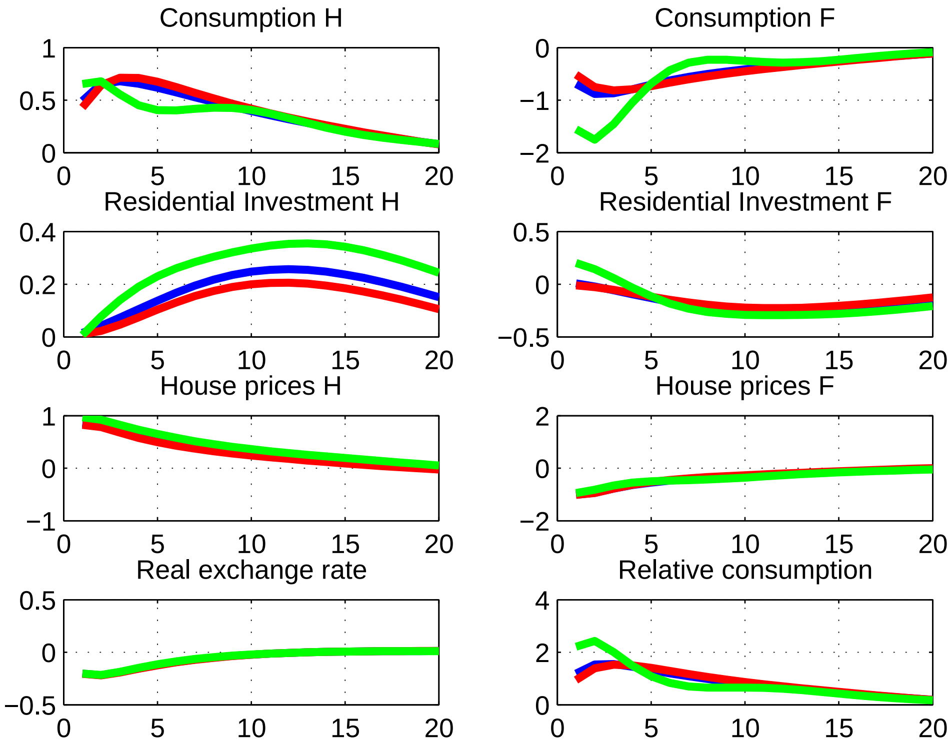

Figure 2.7: Impulse Responses to a UIP-Shock: Baseline (Blue), No borrowers (Red), High Borrowers (Green)

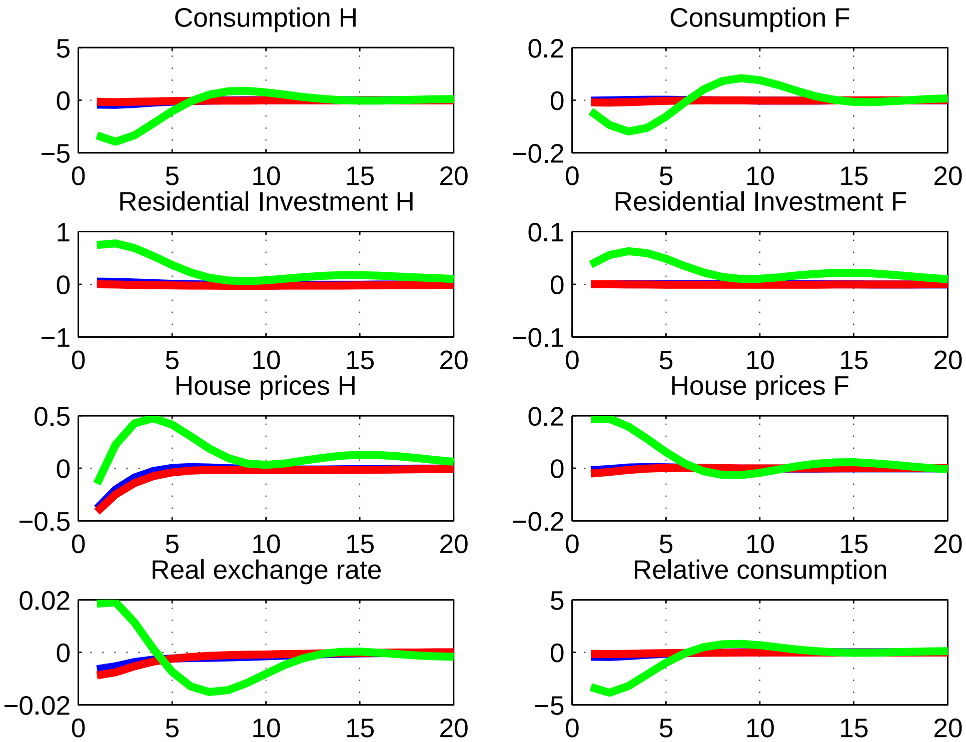

Figure 2.8: Impulse Responses to a Monetary Policy Shock: Baseline (Blue), No borrowers (Red), High Borrowers (Green)

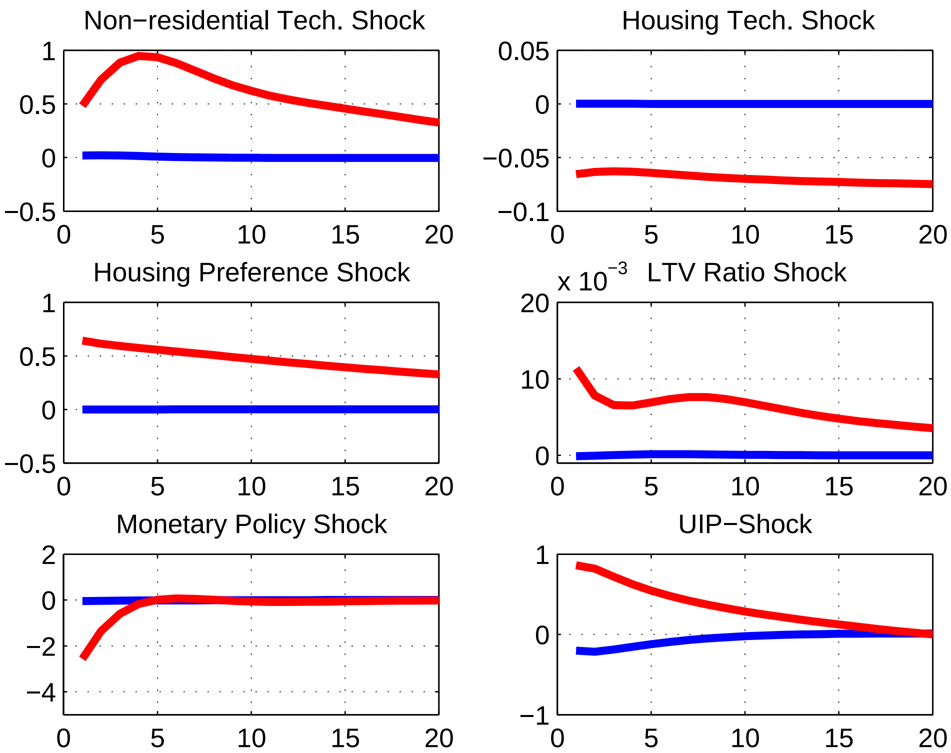

Figure 2.9: Impulse Responses of Real Exchange Rate (Blue) and Real House Prices (Red)

Table 2.1: Baseline Calibration

	Description	Value
γ	Saver's discount factor	0.96
β	Borrower's discount factor	0.99
δ	Housing depreciation rate	0.01
δ_K	Capital depreciation rate	0.10
μ	Goods market markup	1.30
μ_w	Labor market markup	1.50
ω_D	Share of housing in consumption	0.10
η_D	Elasticity of substitution housing/non-housing	1.00
α	Capital share of income (non-residential sector)	0.30
$\alpha_{\mathcal{L}}$	Land share of income (housing sector)	0.10
ζ_p	Calvo price stickiness (non-residential sector)	0.88
$\gamma_{\mathcal{L}}$	Price indexation (non-residential sector)	0.55
η	PCP share of firms	0.98
χ	Down-payment ratio	0.20
ω	Share of borrowers	0.23
h	Consumption habit (saver)	0.50
h_B	Consumption habit (borrower)	0.30
σ_X	Relative risk aversion	0.64
Taylor rule parameters		
ρ		0.79
r_π		1.78
r_y		0.10
$r_{\Delta\pi}$		0.26
$r_{\Delta y}$		0.17
Exogenous shocks: persistence		
ρ^A		0.90
ρ^{A_D}		0.70
ρ^D		0.97
ρ^{LTV}		0.93
$\rho^{\Delta S}$		0.92
f^A		0.80
f^R		0.80
Exogenous shocks: standard deviation		
σ^A		0.35
σ^{A_D}		1.21
σ^R		0.15
σ^D		3.60
σ^{LTV}		1.11
$\sigma^{\Delta S}$		0.26
F^A		0.02
F^R		0.01

Table 2.2: Moments analysis

	Data	Baseline	$\omega = 0.5$	No borrowing
Standard deviations relative to GDP				
Consumption	0.67	1.66	2.09	1.30
Residential Investment	5.21	3.93	0.55	6.22
Investment	3.02	0.87	0.62	1.28
Real house prices	1.07	4.82	0.43	7.53
CPI inflation	0.14	0.36	0.21	0.47
Nominal interest rate	0.24	0.24	0.26	0.35
Nominal exchange rate	3.51	4.24	0.69	6.80
Real exchange rate	7.59	0.13	0.02	0.21
Current account	0.37	0.25	0.69	6.80
Relative consumption	0.01	2.22	2.04	2.27
Domestic correlations				
GDP with				
Consumption	0.84	0.91	0.99	0.76
Residential investment	0.62	0.03	-0.69	0.16
Investment	0.65	0.69	-0.09	0.81
Residential investment with				
Consumption	0.68	-0.09	-0.72	0.02
Real house prices	0.35	0.08	-0.04	0.10
Consumption with				0.10
Real house prices	0.47	0.53	0.12	0.37
Cross correlations between Home and Foreign				
GDP	0.27	0.06	0.41	0.00
Consumption	0.09	-0.02	0.22	-0.05
Residential investment with	0.23	0.00	0.10	0.00
Real house prices	0.06	-0.01	-0.05	0.00
Cross correlations between real exchange rate and				
Relative consumption	-0.33	-0.35	-0.75	-0.38
Real house prices	0.31	-0.11	-0.16	-0.10
Nominal exchange rate	0.99	0.47	0.42	0.47
AR(1) coefficients				
GDP	0.84	0.75	0.75	0.86
Consumption	0.79	0.72	0.76	0.88
Residential investment	0.82	0.75	0.75	0.74
Real exchange rate	0.82	0.77	0.78	0.77

Table 2.3: Sensitivity Analysis 1: The Role of Home Bias and Risk Aversion

	Data	Baseline	n=0.6	n=0.6 $\omega = 0.5$	n=0.6 (1-χ)=0.95	$\sigma_X = 2$	$\sigma_X = 2$ $\omega = 0.5$	$\sigma_X = 2$ (1-χ)=0.95
Stand. dev. relative to GDP								
Consumption	0.67	1.66	1.77	2.18	2.22	1.56	2.37	2.11
Resid. Inv.	5.21	3.93	3.65	0.84	2.15	3.48	1.46	1.50
Investment	3.02	0.87	1.05	0.66	0.60	1.21	1.42	0.52
Real house prices	1.07	4.82	4.67	0.92	2.57	4.44	1.68	0.28
CPI inflation	0.14	0.36	0.48	0.25	0.29	0.46	0.38	0.22
Nominal int. rate	0.24	0.24	0.32	0.27	0.23	0.31	0.41	0.24
Nominal exch. rate	3.51	4.24	0.88	0.21	0.51	3.56	1.75	0.49
Real exch. rate (RER)	7.59	0.13	0.02	0.01	0.01	0.11	0.05	0.01
Current account	0.37	0.25	0.99	0.30	0.69	0.21	0.13	0.05
Relative cons.	0.01	2.22	2.92	1.23	2.50	2.03	3.23	2.08
Domestic correlations								
GDP with								
Consumption	0.84	0.91	0.20	0.95	0.74	0.89	0.93	0.99
Resid. Inv.	0.62	0.03	0.06	-0.45	-0.16	-0.20	-0.63	-0.97
Investment	0.65	0.69	0.14	-0.09	0.03	0.75	-0.06	-0.15
Resid. Inv. with								
Consumption	0.68	-0.09	-0.06	-0.50	-0.26	-0.39	-0.70	-0.97
Real house prices	0.35	0.08	0.08	-0.02	0.01	-0.20	0.53	-0.29
Consumption with								
Real house prices	0.47	0.53	0.56	0.24	0.45	0.15	-0.66	0.16
Cross correlations: H and F								
GDP	0.27	0.06	0.02	0.95	0.66	0.04	0.05	0.57
Consumption	0.09	-0.02	-0.34	0.83	0.25	-0.03	-0.13	0.19
Resid. Inv. with	0.23	0.00	-0.01	0.23	0.02	0.00	-0.02	0.07
Real house prices	0.06	-0.01	-0.13	-0.24	-0.13	0.00	0.16	-0.03
Cross correlations: RER and								
Relative consumption	-0.33	-0.35	-0.55	0.17	-0.37	-0.33	-0.77	-0.76
Real house prices	0.31	-0.11	-0.24	-0.17	-0.24	-0.06	0.30	-0.12
Nominal exch. rate	0.99	0.47	0.59	0.57	0.59	0.47	0.43	0.46
AR(1) coefficients								
GDP	0.84	0.75	0.75	0.75	0.47	0.85	0.79	0.57
Consumption	0.79	0.72	0.77	0.76	0.39	0.83	0.82	0.59
Resid. Inv.	0.82	0.75	0.75	0.75	0.72	0.75	0.75	0.49
Real exch. rate	0.82	0.77	0.73	0.80	0.73	0.77	0.76	0.66

Table 2.4: Sensitivity Analysis 2: The Role of the Elasticity of Substitution between Residential and Nonresidential Goods

	Data	Baseline	$\eta_D = 0.4$	$\eta_D = 0.4$ $\omega = 0.5$	$\eta_D = 0.4$ $(1\text{-}\chi)$=0.95
Stand. dev. relative to GDP					
Consumption	0.67	1.66	1.82	2.15	1.95
Resid. Inv.	5.21	3.93	3.33	0.42	1.21
Investment	3.02	0.87	0.64	0.45	0.23
Real house prices	1.07	4.82	6.90	0.23	2.18
CPI inflation	0.14	0.36	0.27	0.17	0.11
Nominal int. rate	0.24	0.24	0.22	0.26	0.19
Nominal exch. rate	3.51	4.24	3.00	0.96	1.02
Real exch. rate (RER)	7.59	0.13	0.09	0.03	0.03
Current account	0.37	0.25	0.18	0.10	0.07
Relative cons.	0.01	2.22	2.26	4.15	2.26
Domestic correlations					
GDP with					
Consumption	0.84	0.91	0.95	0.99	0.99
Resid. Inv.	0.62	0.03	0.04	-0.96	-0.40
Investment	0.65	0.69	0.41	-0.46	-0.08
Resid. Inv. with					
Consumption	0.68	-0.09	-0.02	-0.97	-0.42
Real house prices	0.35	0.08	0.30	0.00	0.13
Consumption with					
Real house prices	0.47	0.53	0.59	0.11	0.43
Cross correlations: H and F					
GDP	0.27	0.06	0.09	-0.19	0.15
Consumption	0.09	-0.02	0.01	-0.31	0.07
Resid. Inv. with	0.23	0.00	0.00	-0.26	0.01
Real house prices	0.06	-0.01	-0.01	-0.36	0.00
Cross correlations: RER and					
Relative consumption	-0.33	-0.35	-0.30	-0.85	-0.25
Real house prices	0.31	-0.11	-0.09	-0.33	-0.07
Nominal exch. rate	0.99	0.47	0.47	0.40	0.47
AR(1) coefficients					
GDP	0.84	0.75	0.64	0.78	0.22
Consumption	0.79	0.72	0.62	0.78	0.22
Resid. Inv.	0.82	0.75	0.80	0.77	0.72
Real exch. rate	0.82	0.77	0.77	0.71	0.75

Table 2.5: SENSITIVITY ANALYSIS 3: THE ROLE OF TRADE ELASTICITY

	Data	Baseline	$\zeta = 2$	$\zeta = 2$ $\omega = 0.5$	$\zeta = 2$ (1-χ)=0.95	$\zeta = 0.5$	$\zeta = 8$
Stand. dev. relative to GDP							
Consumption	0.67	1.66	1.63	2.11	1.95	1.67	1.36
Resid. Inv.	5.21	3.93	3.83	0.58	1.66	3.98	3.12
Investment	3.02	0.87	0.85	0.62	0.36	0.87	0.76
Real house prices	1.07	4.82	4.72	0.48	1.87	4.83	3.88
CPI inflation	0.14	0.36	0.35	0.22	0.17	0.37	0.30
Nominal int. rate	0.24	0.24	0.24	0.26	0.18	0.24	0.22
Nominal exch. rate	3.51	4.24	3.57	0.61	1.50	6.42	1.59
Real exch. rate (RER)	7.59	0.13	0.11	0.02	0.04	0.20	0.05
Current account	0.37	0.25	0.39	0.09	0.17	0.03	0.77
Relative cons.	0.01	2.22	2.19	1.98	2.33	2.14	1.89
Domestic correlations							
GDP with							
Consumption	0.84	0.91	0.85	0.90	0.98	0.95	0.62
Resid. Inv.	0.62	0.03	0.03	-0.66	-0.27	0.02	0.03
Investment	0.65	0.69	0.65	-0.09	0.17	0.71	0.47
Resid. Inv. with							
Consumption	0.68	-0.09	-0.08	-0.69	-0.32	-0.10	-0.08
Real house prices	0.35	0.08	0.08	-0.06	-0.01	0.08	0.08
Consumption with							
Real house prices	0.47	0.53	0.53	0.18	0.41	0.51	0.53
Cross correlations: H and F							
GDP	0.27	0.06	0.41	0.54	0.13	0.10	-0.29
Consumption	0.09	-0.02	0.22	0.35	0.04	0.03	-0.09
Resid. Inv. with	0.23	0.00	0.10	0.14	0.00	0.00	0.00
Real house prices	0.06	-0.01	-0.05	-0.08	-0.02	0.03	-0.05
Cross correlations: RER and							
Relative consumption	-0.33	-0.35	-0.75	-0.72	-0.26	-0.29	-0.36
Real house prices	0.31	-0.11	-0.16	-0.22	-0.13	-0.01	-0.21
Nominal exch. rate	0.99	0.47	0.42	0.42	0.47	0.49	0.50
AR(1) coefficients							
GDP	0.84	0.75	0.75	0.76	0.40	0.75	0.73
Consumption	0.79	0.72	0.76	0.77	0.38	0.72	0.73
Resid. Inv.	0.82	0.75	0.75	0.75	0.71	0.75	0.75
Real exch. rate	0.82	0.77	0.78	0.75	0.76	0.78	0.75

Table 2.6: Sensitivity Analysis 4: The Role of Pricing-to-Market

	Data	Baseline	no LCP	no LCP $\omega = 0.5$	no LCP (1-χ)=0.95	high LCP	high LCP $\omega = 0.5$
Stand. dev. relative to GDP							
Consumption	0.67	1.66	1.66	2.09	1.95	1.66	2.06
Resid. Inv.	5.21	3.93	3.92	0.55	1.65	3.95	0.53
Investment	3.02	0.87	0.87	0.62	0.36	0.86	0.62
Real house prices	1.07	4.82	4.82	0.43	1.86	4.83	0.39
CPI inflation	0.14	0.36	0.36	0.21	0.17	0.35	0.21
Nominal int. rate	0.24	0.24	0.24	0.26	0.18	0.24	0.26
Nominal exch. rate	3.51	4.24	4.23	0.69	1.72	4.61	0.73
Real exch. rate (RER)	7.59	0.13	0.00	0.00	0.00	3.26	0.49
Current account	0.37	0.25	0.25	0.06	0.11	0.23	0.06
Relative cons.	0.01	2.22	2.22	2.04	2.33	2.16	2.07
Domestic correlations							
GDP with							
Consumption	0.84	0.91	0.90	0.99	0.98	0.94	0.98
Resid. Inv.	0.62	0.03	0.03	-0.69	-0.27	0.03	-0.72
Investment	0.65	0.69	0.69	-0.09	0.17	0.72	-0.1
Resid. Inv. with							
Consumption	0.68	-0.09	-0.09	-0.72	-0.32	-0.09	-0.75
Real house prices	0.35	0.08	0.08	-0.04	-0.01	0.08	-0.01
Consumption with							
Real house prices	0.47	0.53	0.53	0.13	0.41	0.53	0.07
Cross correlations: H and F							
GDP	0.27	0.06	0.06	0.41	0.12	0.07	0.31
Consumption	0.09	-0.02	-0.02	0.23	0.04	0.01	0.15
Resid. Inv. with	0.23	0.00	0.00	0.10	0.00	0.00	0.07
Real house prices	0.06	-0.01	-0.01	-0.05	-0.01	0.00	-0.03
Cross correlations: RER and							
Relative consumption	-0.33	-0.35	-0.35	-0.75	-0.25	-0.25	-0.75
Real house prices	0.31	-0.11	-0.11	-0.16	-0.1	-0.08	-0.09
Nominal exch. rate	0.99	0.47	0.47	0.42	0.47	0.47	0.42
AR(1) coefficients							
GDP	0.84	0.75	0.75	0.75	0.40	0.75	0.76
Consumption	0.79	0.72	0.72	0.77	0.38	0.72	0.78
Resid. Inv.	0.82	0.75	0.75	0.75	0.71	0.75	0.75
Real exch. rate	0.82	0.77	0.77	0.75	0.76	0.77	0.75

Chapter 3

Financial frictions and international real business cycles

3.1 Introduction

Among the most relevant puzzles in international macroeconomics and finance, two have attracted the attention of a large number of studies over the last two decades, namely the *consumption cross-correlations puzzle* and the *quantity puzzle*. The first one (see Obstfeld and Rogoff (2000)) refers to the discrepancy between an observed low correlation of consumption levels across the most developed countries and the theoretical implications of a standard model with complete financial markets. When state-contingent assets are traded, full risk sharing is achieved, implying that consumption is isolated from country-specific shocks in the theoretical economy. Even though the result depends on the complete-markets assumption, Kollmann (1996) and Chari, Kehoe, and McGrattan (2002) have shown that resorting to an incomplete-markets setup (where the risk sharing condition only holds in expectations) is not enough to generate plausible degrees of cross-country consumption correlations. The so-called *quantity puzzle* (see Backus, Kehoe, and Kydland (1992)), concerns the evidence of larger cross-country correlations in output than in consumption, as opposed to what the theory would suggest. Taken together, the two puzzles highlight the main shortcomings of prototypical international real business cycle models: an unrealistically high cross-country correlation in consumption and an unrealistically low cross-correlation in output. The introduction of correlated technology shocks, which creates room for international spillover effects, does not suffice to solve the problem, as shown in Backus, Kehoe, and Kydland (1992), Chari, Kehoe, and McGrattan (2002).

This paper aims at reconciling the empirical evidence with the theory, through an analysis of the role of credit market imperfections. The importance of credit market frictions in the international economy has recently been the object of a large portion of the literature (see Gilchrist, Hairault, and Kempf (2002) and Faia (2007) among others). In particular, some studies have focused on the role of collateral constraints in influencing investment decisions in open economies (see Caballero and Krishnamurty (2001)). Recently, Iacoviello and Minetti (2006) have developed a two-country model where entrepreneurs can borrow - upon the provision of some collateral - from domestic and foreign lenders. Importantly, the latter differ in their relative ability to liquidate the collateral in case of insolvency so that, when the value of the collateral increases, it is relatively more costly for foreign lenders to provide funds, due to a typical agency problem. The entrepreneurs thus have the opportunity to adjust optimally the allocation of their collateral between the two types of lenders after a productivity shock. As a consequence, a model with such frictions can substantially improve over a standard international real business cycle model in tracking the observed output correlations across countries. Admittedly, though, the model cannot generate a realistic degree of correlation in consumption, which remains counterfactually high, mainly reflecting the operation of the permanent income hypothesis (see Backus, Kehoe, and Kydland (1992)).

This paper builds on the theoretical model of Iacoviello and Minetti (2006) by introducing a borrowing problem for a fraction of households, who - symmetrically to firms - can allocate their collateral between domestic and foreign lenders. The same type of agency problem introduced in the original model is considered here, in order to generate an endogenous choice over collateral allocation at the consumption and investment stage. Crucially, the interaction of financial markets frictions at different levels produces more realistic dynamics in real quantities. In particular, the model generates a lower degree of correlation in consumption than is usually obtained.

In addition, we elaborate on the preference structure, by assuming non-separability in the utility function between consumption an labor, in the form introduced by Greenwood, Hercowitz, and Huffman (1988). Such type of preferences has recently been exploited in two-country business cycle models by Raffo (2008), who shows that the absence of an income effect on labor supply allows the model to reproduce the cyclical dynamics of real quantities. We argument here that, in the presence of both household credit frictions and non-separable utility, the model can precisely account for the observed cross-country comovements in consumption, hours worked, investment and output.

3.2 The model

The model builds on Iacoviello and Minetti (2006) and enriches it with an additional credit channel. The world economy is composed of two symmetric countries, which produce a unique and homogeneous final good. The good is used for consumption and can be accumulated over time to build productive capital. We assume that it is tradeable in the international market. Each country is inhabited by three different subjects: entrepreneurs, patient households and impatient households[1]. Entrepreneurs produce the final good by combining labor (provided by the two types of households) and capital stock, plus a fixed amount of a durable, collateralizable good. The collateralizable goods is neither produced nor traded: in each period, agents are endowed with a fixed amount of it, which they can use as an input in production, as a consumption good, or as a collateral, in the case of impatient households[2]. We will identify the durable good with housing throughout, for the sake of exposition. However, such interpretation should by no means be considered as restrictive: a generic durable good would serve the purpose just as well. Entrepreneurs face collateral constraints in the production process. Exactly as in the original model of Iacoviello and Minetti (2006), domestic and foreign lenders differ in the liquidation technology they can operate in case of insolvency. The asymmetry is typically motivated by agency problems faced by financiers operating in a foreign country and generates endogenous fluctuations in the allocation of collateral by the borrowers.

[1]The closed-economy structure is very close to the one in Iacoviello (2005), with the exception that we consider here a real business cycle model without nominal rigidities.

[2]See below.

We assume the existence of two types of households, who differ in their relative impatience rate. Households of both types consume and work. More precisely, the impatient households can borrow from the patient after providing some fraction of the durable asset as collateral. We maintain the assumption that foreign lenders incur in higher liquidation costs than domestic lenders, as in the case of the firms' financing problem. The assumption is rationalized by the fact that there exists a unique group of lenders in this economy (represented by the patient households), so that any diseconomy to scale faced in the financing activity will be equally reflected on both firms and consumers.

The presence of an additional group of borrowers in the economy is essentially motivated by the need to reduce the counterfactually high degree of risk sharing generated by the original model. Intuitively, the existence of collateral constraints for the additional group of consumers serves to dampen the response of consumption to a technology shock. We analyze the model's transmission mechanism in the next section. For the sake of exposition, we concentrate on the domestic country; it is understood that analogous derivations hold for the foreign country.

3.2.1 Entrepreneurs

The representative firm operates a Cobb-Douglas production function mixing labor services, housing and capital, as follows:

$$Y_t = A_t h_{t-1}^{\nu} K_{t-1}^{\mu} L_t^{\alpha(1-\mu-\nu)} L_t^{"(1-\alpha)(1-\mu-\nu)} \tag{3.1}$$

where L_t and L_t'' indicate the labor supply of the patient and impatient households, respectively. The parameter α measures the relative size of the patient households in terms of their corresponding wage income share. Capital, denoted by K_t, depreciates at the rate δ and can be used as collateral by firms. Entrepreneurs also face a cost in adjusting investment, so that their period budget constraint reads:

$$Y_t + b_t^H + b_t^F = C_t + I_t + q_t(h_t - h_{t-1}) + R_{t-1}^H B_{t-1} + R_{t-1}^F B_{t-1} + w_t' L_t' + w_t'' L_t'' + AC_{K,t} \tag{3.2}$$

with

$$I_t = K_t - (1-\delta)K_{t-1} \tag{3.3}$$

and

$$AC_{K,t} \equiv \frac{\psi_K}{2\delta}\left(\frac{I_t}{K_{t-1}} - \delta\right) K_{t-1} \tag{3.4}$$

where q_t denotes the relative price of the collateralizable good (h_t) in terms of consumption. As already mentioned, entrepreneurs face borrowing constraints. Given the asymmetric structure described above, firms can optimally allocate their capital between domestic and foreign lenders, according to the following constraints:

$$R_t^H b_t^H \leq z_H \sigma_t K_t \tag{3.5}$$

$$R_t^F b_t^F \leq K_t \left(1 - \sigma_t\right) - \frac{\left(1 - z_F\right)}{K} \left(1 - \sigma_t\right)^2 K_t^2 \tag{3.6}$$

where b_t^H and b_t^F denote domestic and foreign borrowing, z_H and z_F denote domestic and foreign loan-to-value (LTV) ratios and σ_t and $(1 - \sigma_t)$ are the fractions of collateral allocated to domestic and foreign lenders, respectively [3] . Summarizing, the entrepreneurs choose consumption, demand labor, housing, capital, domestic and foreign borrowing and allocate capital between domestic and foreign lenders to solve the problem of maximizing the intertemporal utility function

$$E_0 \sum_{t=0}^{\infty} \gamma^t \ln C_t$$

subject to (3.2), (3.3), (3.4), (3.5) and (3.6). We report the corresponding first order conditions in the Appendix.

3.2.2 Patient Households

The patient households act as lenders towards firms and impatient households (see below), both domestically and abroad. Every period, they rent labor services to the entrepreneurs (L_t') and lend the quantities b_t^H and b_t^{F*} to domestic and foreign firms, $b_t''^H$ and $b_t''^{F*}$ to domestic and foreign impatient households and b_t to foreign patient households, respectively. Formally, they solve the following problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t (\ln C_t + \mathbf{j} \ln h_t - \frac{\tau}{\eta} L_t^{\eta})$$

[3] Two observations are in order. While we assume full depreciation of the durable good for simplicity, as in Iacoviello and Minetti (2006), we depart from the original model in that we consider capital as the only collateral provided by firms.

s.t.

$$C'_t + q_t(h'_t - h'_{t-1}) + b^H_t + b^{F*}_t + b_t + \psi\frac{\left(b_t - \overline{b}\right)^2}{2} + b''^H_t + b''^{F*}_t =$$

$$R^H_{t-1}B^H_{t-1} + R^{F^*}_{t-1}B^{F^*}_{t-1} + R_{t-1}B_{t-1} + R^H_{t-1}B''^H_{t-1} + R^F_{t-1}B''^F_{t-1} + w_t L'_t$$

where $\mathbf{j}$ is the weight on housing in utility and the elasticity of labor supply is determined as $\frac{1}{1-\eta}$. The constant term τ is chosen to ensure that hours worked amount to 30% of the time endowment in equilibrium. The term $\psi\frac{\left(b_t - \overline{b}\right)^2}{2}$ captures a (small) cost of adjusting the holding of bonds traded with the foreign patient households[4]. Noticeably, we impose $\gamma < \beta$, which is a necessary condition for the collateral constraints to bind. Also, we adopt here the same functional specification of Iacoviello and Minetti (2006) for the agents' momentary utility, to facilitate the comparison of relative results. We will consider the role of preferences in section 4.

The optimality conditions for the impatient agents' problem are reported in the Appendix.

3.2.3 Impatient Households

The impatient households have a lower discount factor than entrepreneurs and patient households ($\beta'' < \gamma$), and act as net borrowers in equilibrium. They consume both types of goods and provide labor services to the entrepreneurs. The financing problem they face is isomorphic to the one faced by the entrepreneurs, so they also issue two types of bonds, according to the origin of the lender (denoted b''^H_t and b''^F_t, respectively). The intertemporal optimization problem they solve thus assumes the following form:

$$\max E_0 \sum_{t=0}^{\infty} \beta''^t (\ln C''_t + \mathbf{j} \ln h''_t - \frac{\widetilde{\tau}}{\eta} L''^\eta_t)$$

s.t.

$$C''_t + q_t(h''_t - h''_{t-1}) - b''^H_t - b''^F_t = -R^H_{t-1}B''^H_{t-1} - R^F_{t-1}B''^F_{t-1} + w''_t L''_t \tag{3.7}$$

$$R^H_t b''^H_t \leq E_t\{m^b_H \rho_t q_{t+1} h''_t\} \tag{3.8}$$

$$R^F_t b''^F_t \leq E_t \left\{ q_{t+1} h''_t (1-\rho_t) - \frac{1 - m^b_F}{qh}(q_{t+1} h''_t (1-\rho_t))^2 \right\} \tag{3.9}$$

[4] Such term is introduced in order to make the behavior of bonds holding stationary. See Schmitt-Grohe and Uribe (2003) for an explanation and a discussion on alternative ways of solving the stationarity problem. In our calibration, we choose small enough values for ψ and $\overline{b}$, so that the dynamics of the model around the steady state are not altered and the equilibrium is well-defined.

where m_H^b and m_F^b indicate domestic and foreign LTV ratios on housing collateral, while ρ and $(1-\rho)$ denote the optimal allocation of housing collateral between domestic and foreign lenders. The rest of the notation follows immediately, with the convention that a $''$ denotes the variables referring to the impatient agents. The corresponding optimality conditions are reported in the Appendix.

3.2.4 Technology shocks

We introduce a stochastic structure for the exogenous productivity processes that follows closely Backus, Kehoe, and Kydland (1992). More precisely, we assume that productivity follows a stationary autoregressive process and allow for contemporaneous correlations between domestic and foreign productivity, as well as for a non-zero contemporaneous covariance between the shocks:

$$\begin{bmatrix} A_{t+1} \\ A^*_{t+1} \end{bmatrix} = \Gamma \begin{bmatrix} A_t \\ A^*_t \end{bmatrix} + \varepsilon_t \tag{3.10}$$

where

$$\Gamma \equiv \begin{bmatrix} 0.8 & 0.09 \\ 0.09 & 0.8 \end{bmatrix}$$

and the variance-covariance matrix of the shocks is denoted $E_t \varepsilon_t \varepsilon'_t = \Omega$, with

$$\Omega \equiv \begin{bmatrix} 0.726 & 0.187 \\ 0.187 & 0.726 \end{bmatrix}$$

3.3 Inspecting the mechanism

This section analyzes the internal propagation mechanism of the model. In particular, we consider the endogenous determination of the optimal allocation of collateral between domestic and foreign lenders for entrepreneurs and impatient households. While the first problem is partially addressed in Iacoviello and Minetti (2006), the second one characterizes our model and hence deserves particular attention. We treat the two choices separately, to emphasize similarities and differences.

3.3.1 The optimal allocation of collateral

We consider the entrepreneurs' problem first. Deriving the optimality conditions for the choice of b_t^H and b_t^F yields:

$$\lambda_t = \beta E_t \{\lambda_{t+1}\} R_t^H + \lambda_t^H R_t^H \tag{3.11}$$

and

$$\lambda_t = \beta E_t \{\lambda_{t+1}\} R_t^F + \lambda_t^F R_t^F \tag{3.12}$$

where λ_t corresponds to the marginal utility of consumption and λ_t^H, λ_t^F are the Lagrange multipliers associated to constraints (3.5) and (3.6). Then, imposing a no-arbitrage condition for the returns on the two types of bonds implies the following equality:

$$\lambda_t^H = \lambda_t^F$$

. Deriving the first order condition for σ_t and making use of the previous equation, it is immediate to obtain an expression for the optimal quantity of collateral capital allocated to domestic lenders:

$$\sigma_t = 1 - \frac{(1 - z_H)K}{2K_t(1 - z_F)} \tag{3.13}$$

Clearly, any decrease in the domestic collateral requirement, which corresponds to an increase in the value of z_H, increases the optimal value of σ_t, ceteris paribus. Analogously, an increase in z_F, the foreign loan-to-value (LTV) ratio, reduces the optimal σ_t. Moreover, for given LTV ratios, a higher stock of capital will be directly assigned to domestic, as opposed to foreign lenders. The latter result follows from the existence of diseconomies to scale in the foreign lenders' liquidation technology. The liquidation cost faced by foreign financiers increases more than linearly with the increase in the collateral value K_t, so that it becomes more convenient for firms to allocate collateral to domestic lenders, as the stock of capital grows.

It is immediate to obtain the optimal allocation of housing collateral between domestic and foreign lenders. In particular, deriving the optimality condition for ρ_t and imposing the no-arbitrage restriction on interest rates yields the following expression:

$$\rho_t = 1 - \frac{(1 - m_H^b) q h''}{2E_t \{q_{t+1} h_t''\} (1 - m_F^b)} \tag{3.14}$$

The optimal allocation of housing collateral depends on domestic and foreign LTV ratios - just as in the case of capital collateral - but also on the expected future value of the collateral itself. Given the form of the borrowing constraints (3.8) and (3.9), in fact, the maximum amount of borrowing is bounded above by the expected future-period value of the collateral[5]. Hence, an expected increase in asset prices increases the optimal amount of domestic, as opposed to foreign borrowing. The rationale for such substitution effect is analogous to the one for the optimal choice of firms' collateral. In brief, foreign lenders face a quadratic cost in lending, which is proportional to the amount of collateral. Whenever the total value of the collateral increases, foreign lenders are penalized vis-à-vis domestic lenders, because of the negative returns to scale displayed by their lending technology. As a result, domestic (impatient) households prefer to allocate their collateral where they can obtain more favorable conditions. This simple mechanism generates an endogenous fluctuation in the optimal allocation of housing between domestic and foreign financiers and allows the impatient households to adjust their borrowing positions in response to exogenous shocks. We analyze below the transmission of a technology shock.

3.3.2 Calibration

The baseline calibration of the model is reported in Table 1. We keep the choice of the structural parameters close to Iacoviello and Minetti (2006), to facilitate the comparison of the main results. We also refer to the parametrization of Iacoviello (2005), who considers a closed economy with a very similar structure to ours.

The discount factors are set to 0.99 for the patient households, 0.98 for the entrepreneurs and 0.95 for the impatient households. In particular, the choice of β implies an equilibrium interest rate of 4% per year. The depreciation rate of capital is equal to 0.03, while the adjustment cost parameter ψ_K is equal to 5. The shares of capital and housing in production (μ and ν)

[5] See Iacoviello (2005) for a discussion.

are equal to 0.25 and 0.03, respectively.

Regarding the utility function, the weight on housing services **j** is equal to 0.1, while the elasticity of labor supply (corresponding to $\frac{1}{1-\eta}$ in our notation) is equal to 0.05. In the next section, we explore the effects of variations in the functional form adopted for the agents' utility, as well as in the parametrization of elasticities, in detail. The labor share of the patient households (α) is set at 0.66, following the estimates of Iacoviello (2005) on US data. Darracq Pariès and Notarpietro (2008), in a different setup, estimate a share of borrowing households in the US economy equal to 0.23, which in our setup corresponds to α=0.7. We analyze the effect of varying such value in the next section. About financial frictions, we fix the LTV ratios on domestic borrowing for capital and housing at 0.9 in both countries, while the corresponding values for the foreign borrowing are equal to 0.8.

3.3.3 The response to a technology shock

We analyze the model's behavior after a positive technology shock. Our main focus is on the international comovement in real quantities and asset prices. In particular, we address here the ability of the model to generate a lower cross-country correlation in consumption than is usually obtained in the international business cycle literature.

Figure 1 reports the impulse responses of the main variables of interest to a technology shock in the foreign country. Output increases in both countries, although the effect is not immediate in the domestic economy, which is only indirectly hit by the shock (both by construction, via the spillovers, and through general-equilibrium effects). The increase in productivity generates an increase in investment (although partially mitigated by the presence of adjustment costs) and labor demand in the source country. By implication, consumption increases and house prices increase as well, due to a generalized raise in local demand[6]. Importantly, entrepreneurs and borrowing households shift collateral towards *local* lenders, both domestically and abroad (not reported). An increase in productivity determines a higher marginal product of capital for foreign firms, who would like to increase investment in the immediate. As

[6]The term consumption indicates *aggregate* consumption, or the sum of individual consumption for the three groups of agents.

already noted, in the presence of asymmetric borrowing constraints *vis-à-vis* domestic and foreign lenders, any increase in K_t generates a relative inefficiency in the foreign lenders lending technology, so that it is optimal for the entrepreneurs to shift collateral towards *local* lenders. This generates a larger increase in foreign versus domestic demand. Noticeably, in our framework - as opposed to Iacoviello and Minetti (2006), house prices play no role in such reallocation, since housing cannot be used as collateral by the entrepreneurs.

However, asset prices do play a role in our model, since housing serves as collateral for households. Again, higher productivity in the foreign country fosters aggregate demand and drives the price of housing up. The corresponding positive valuation effect on housing collateral provides additional funds to the impatient consumers, ceteris paribus. As in the case of firms, the existence of diseconomies to scale in the liquidation technology of foreign lenders implies an optimal reallocation of housing collateral towards *local* financiers. As a result, local consumption increases more where more extra-funds can be obtained through borrowing. The result is an imperfect correlation in consumption levels across countries, which would not arise absent the borrowing channel for households, or with a fixed allocation of collateral between different lenders (in this case $\rho_t = \rho$ in every period t.) The latter case can be understood by looking at the expression for the optimal ρ_t in (3.14): when ρ is constant (at its optimal level), any increase in q_t has no impact on the impatient households' collateral allocation, and it is not possible to generate ample asymmetries in the responses of consumption to a productivity shock across countries. Figure 3.2 clarifies the mechanism by reporting the responses of consumption to a foreign productivity shock under three cases: no collateral constraints on consumption (in which case no impatient agents are considered), fixed allocation of collateral across financiers, and the case of a variable ρ_t. Clearly, while in the first case the consumption paths are almost identical in the two countries, allowing for household borrowing, and in particular for an optimal choice of collateral allocation, generates a more pronounced response on impact in the country where the shock occurs, via the mechanism explained above.

Hence, under the baseline calibration, the model generates the following responses to a foreign productivity shock: (i) output, consumption and investment comove positively; (ii) hours worked are negatively correlated; (iii) asset prices are positively correlated and display a hump-shaped response.

Figure 3.1 also reports the impulse responses of the main variables for the case α=0.2, corresponding to 80% of labor income share going the impatient households. Interestingly, in this case we observe a larger increase in foreign consumption on impact, mainly reflecting the larger share of the impatient households. The latter, who do no smooth consumption intertemporally, optimally choose to immediately consume all the income effect deriving from the increase in productivity. The same occurs in the domestic country, although with some lag, due to the indirect effect of the shock.

Table 2 reports the business cycle statistics generated by the model[7]. We compare our results to two main references: Backus, Kehoe, and Kydland (1992), which we consider as a standard international real-business-cycle model and Iacoviello and Minetti (2006), on which our model builds and which we consider as our primary benchmark. The first column reports the corresponding moments in the data[8].

The model clearly improves upon the benchmarks on two relevant dimensions: output correlation is higher and, most importantly, consumption correlation is substantially lower, although still far from the data. Investment correlations are also much larger than the observed counterparts. Finally, employment correlation is *negative* in the model, as opposed to the data. The latter result directly follows from the preference structure adopted. In the next section, we show that introducing non-separability between consumption and labor services in the utility function can solve the puzzle and help the model generate a positive cross-correlation in hours worked[9]. In terms of volatilities, our model economy performs better than the one of Backus, Kehoe, and Kydland (1992), especially for investment, which is crucially affected by the presence of adjustment costs.

[7] The statistics are computed after simulating the model a large number of times over a 100 periods. Both the data and the model-generated series are filtered using the Hodrick and Prescott filter

[8] See the Appendix of Chapter 2 for a description of the data used.

[9] With a slight abuse of notation, we use the terms "hours worked" and "employment" interchangeably here. However, given the absence of an intensive margin of adjustment on labor supply, the two expressions can be considered as equivalent.

3.4 The role of preferences: non-separable utility

In the previous section we have highlighted the main characteristics of the model and explained how it is possible to generate a positive comovement in investment, consumption and output. We have emphasized in particular the model's ability to reproduce a low consumption cross-correlation, through a violation of the permanent income hypothesis for a fraction of agents. However, as we have pointed out, the baseline version of our model cannot reproduced the observed positive correlation in hours worked across countries.

This section introduces a modification on the type of utility function adopted for the patient and impatient households. We consider here a class of functions that feature non-separability between consumption and hours worked, and adapt two well-known preference specifications in this class to our model. We show that, under the type of utility function introduced by Greenwood, Hercowitz, and Huffman (1988), the model can account for international business cycle facts much better than in the baseline version presented above. In particular, consumption and output cross-correlations are very close to the data and the cross-country correlation in hours worked displays a positive sign. We argument that the interaction of our assumption on non-separability in preferences and the existence of collateral constraints at the consumption and investment stage are crucial to match the empirical stylized facts.

Since the seminal work of Devereux, Gregory, and Smith (1992), a number of studies in the literature have introduced non-separable preferences in two-country business cycle models, mainly with the objective of generating plausible consumption cross-correlations. More recently, Raffo (2008) has adopted the preference specification of Greenwood, Hercowitz, and Huffman (1988) in a two-country model, to show that consumption volatility increases and net exports become countercyclical, as observed in the data. However, to the best of our knowledge, no studies have incorporated non-separable preferences in a two-country model with household credit frictions of the type considered here.

Following Jaimovich and Rebelo (2009), we consider the following generic specification for the utility function of both the patient and the impatient agents:

$$U(Z_t, L_t) = \frac{\left(Z_t - \psi L_t^{\theta} X_t\right)^{1-\sigma_X}}{1-\sigma_X}$$

where Z_t is a Cobb-Douglas aggregator that includes consumption and housing goods:

$$Z_t \equiv c_t^{\zeta} h_t^{1-\zeta}$$

with ζ denoting the elasticity of substitution between the two goods. The index variable X_t evolves as follows:

$$X_t = Z_t^{\gamma} X_{t-1}^{1-\gamma}$$

with $\sigma_X > 0$, $\theta > 1$ and $\psi > 0$. As Jaimovich and Rebelo (2009) note, the presence of X_t makes consumption of the aggregate good Z_t and hours worked non-separable. We adopt this general specification for convenience, since it nests two interesting cases. When $\gamma = 1$ the function above belongs to the general class analyzed by King, Plosser, and Rebelo (1988), which we will refer to as KPR henceforth:

$$U(Z_t, L_t) = \frac{\left(Z_t\left(1 - \psi L_t^{\theta}\right)\right)^{1-\sigma_X}}{1 - \sigma_X} \tag{3.15}$$

When $\gamma = 0$ instead, the utility function assumes the form proposed by Greenwood, Hercowitz, and Huffman (1988), which we will refer to as GHH[10]:

$$U(Z_t, L_t) = \frac{\left(Z_t - \psi L_t^{\theta}\right)^{1-\sigma_X}}{1 - \sigma_X} \tag{3.16}$$

We consider the implications of assuming either (3.15) or (3.16) in the optimization problem of the patient and impatient agents[11].

Figure 2 compares the responses to a technology shock in the foreign country in the baseline model and under the two alternative preference specifications considered here. The main difference regards the response of consumption and hours worked. Under KPR-utility, we observe an increase in investment in both countries after the shock; consumption also increases - driven by the positive discounted effect of future higher productivity - which in turn generates a surge in house prices. Interestingly, the responses of hours worked have opposite signs in the two countries. In the foreign country, where the shock originates, higher productivity implies a positive income effect on consumption. At the same time, firms have an incentive

[10] Since X_t is constant when $\gamma = 0$, we normalize its value to one.
[11] We do not alter the utility function of the entrepreneurs, since by assumptions they only consume, without providing labor services. Therefore, the non-separability issue does not arise in their case.

to increase labor demand over the steady-state value after the increase in productivity. As it is well known, the KPR preference specification implies that the marginal rate of substitution between consumption and hours worked depends linearly on the consumption level. Hence, the optimal determination of labor supply requires (for both types of households):

$$W_t = \frac{Z_t \psi \theta L_t^{\theta-1}}{1 - \psi L_t^{\theta}} \tag{3.17}$$

The overall effect on hours thus depends on the relative intensity of the change in aggregate consumption. We can intuitively understand this mechanism by log-linearizing the previous equation around the steady state, after substituting the expression for W_t obtained from the firms' optimality conditions:

$$\zeta \widehat{c}_t + (1-\zeta)\widehat{h}_t + \left(\theta - 1 + \frac{\psi \theta L^{\theta}}{1 - \psi L^{\theta}}\right)\widehat{l}_t = \alpha_1 \widehat{y}_t - \alpha_2 \widehat{l}_t \tag{3.18}$$

where the coefficients α_1 and α_2 depend on whether we consider patient or impatient workers. In the foreign country, where the shock occurs, the increase in output is large, so that consumption (of both goods) and hours can increase at the same time. In the domestic country instead, where the effects of the shock are smaller, the increase in output is reduced, so that the positive income effect on consumption generates a *fall* in hours worked.

Summarizing, under KPR preferences (and similarly to the general specification of utility adopted so far) a positive technology shock generates opposite effects on hours worked in the two countries. In particular, under the KPR specification the model produces a lower cross-correlation in output compared to the baseline and a general increase in volatilities.

Under GHH preferences, the marginal rate of substitution between consumption and hours worked is independent of consumption, so that the labor supply schedule is modified as follows:

$$W_t = \psi \theta l_t^{\theta-1} \tag{3.19}$$

which can be immediately rewritten in log-linear terms:

$$(\theta - 1)\widehat{l}_t = \alpha_1 \widehat{y}_t - \alpha_2 \widehat{l}_t \tag{3.20}$$

Clearly, the income effect due to the shock disappears. As a result, any increase in productivity (direct or indirect) will result in an *increase* in hours worked. Therefore, the elimination of the income effect is sufficient to generate a positive comovement of hours worked across countries, as reported in column 3 of Table 2.

Interestingly, the GHH-utility model predicts a cross-correlation in output and consumption that is much closer to the data than in the reported benchmarks. Moreover, we account for the observed positive correlation of hours worked, as explained above. However, the model tends to generate excess volatility in all the four variables considered.

3.5 Conclusions

In this paper we have provided a framework to account for the existence of credit market imperfections in open economy. In particular, we have focused on the problem of optimally allocate capital and housing collateral between domestic and foreign lenders, motivating our framework with the existence of agency problems for the financiers. We have shown that the introduction of credit frictions at the household level, in addition to borrowing limits in the production sector, can - at least partially - contribute to solve some of the main existing puzzles in open economy macroeconomics.

In addition, we have shown that the preference specification adopted is crucial for matching the observed pattern of correlation in hours worked. By adapting the utility function introduced by Greenwood, Hercowitz, and Huffman (1988) to out setup, we have shown that the interaction of credit frictions and non-separable preferences is of crucial relevance in explaining international business cycles.

3.6 Appendix

3.6.1 The entrepreneurs' problem

We report here the optimality conditions for the representative firm:

$$w'_t = \alpha(1-\mu-\nu)\frac{Y_t}{L_t}$$

$$w''_t = (1-\alpha)(1-\mu-\nu)\frac{Y_t}{L''_t}$$

$$\frac{1}{C_t} = \gamma E_t\left\{\frac{R_t^H}{C_{t+1}}\right\} + \lambda_t^H R_t^H$$

$$\frac{1}{C_t} = \gamma E_t\left\{\frac{R_t^F}{C_{t+1}}\right\} + \lambda_t^F R_t^F$$

$$\frac{q_t}{C_t} = \gamma\nu E_t\left\{\frac{Y_{t+1}}{C_{t+1}}\frac{1}{h_t}\right\} + \gamma E_t\left\{\frac{q_{t+1}}{C_{t+1}}\right\}$$

$$\frac{1}{C_t} = \gamma E_t\left\{\frac{1}{C_{t+1}}\left(1-\delta+\mu\left(\frac{Y_{t+1}}{K_t}\right)\right)\right\}+\lambda_t^H z_H\sigma_t+\lambda_t^F\left\{(1-\sigma_t)\left[1-2\frac{(1-z_F)(1-\sigma_t)K_t}{K}\right]\right\}$$

$$z_H = \frac{\lambda_t^F}{\lambda_t^H}\left\{1-\frac{2(1-z_F)(1-\sigma_t)K_t}{K}\right\}$$

3.6.2 The patient households' problem (baseline model)

Under the baseline preference specification, the first order conditions for the patient households' problem can be written as follows:

$$R_t^H = R_t^F$$

$$w_t = \tau C'_t L_t^{\eta-1}$$

$$\frac{q_t}{C'_t} = \frac{\mathbf{j}}{h'_t} + \beta E_t\left\{\frac{q_{t+1}}{C'_{t+1}}\right\}$$

$$\frac{1}{C'_t}\left(1+\psi\left(b_t-\overline{b}\right)\right) = \beta E_t\left\{\frac{R_t}{C'_{t+1}}\right\}$$

$$R_t^H = \frac{R_t}{\left(1+\psi\left(b_t-\overline{b}\right)\right)}$$

3.6.3 The impatient households' problem (baseline model)

The optimality conditions for the impatient households read:

$$\frac{1}{C''_t} = \beta'' E_t \left\{ \frac{R^H_t}{C''_{t+1}} \right\} + \widetilde{\lambda}^H_t R^H_t$$

$$\frac{1}{C''_t} = \beta'' E_t \left\{ \frac{R^F_t}{C''_{t+1}} \right\} + \widetilde{\lambda}^F_t R^F_t$$

$$\widetilde{\lambda}^H_t m^b_H = \widetilde{\lambda}^F_t \left\{ 1 - 2q_{t+1} h''_t \frac{(1-m^b_F)}{qh''}(1-\rho_t) \right\}$$

$$w''_t = \widetilde{\tau} C''_t L''^{\eta-1}_t$$

$$\frac{q_t}{C''_t} = \frac{\mathbf{j}}{h''_t} + \beta'' E_t \left\{ \frac{q_{t+1}}{C''_{t+1}} \right\} + E_t \left\{ \widetilde{\lambda}^H_t m^b_H \rho_t q_{t+1} \right\} +$$
$$\widetilde{\lambda}^F_t E_t \left\{ (1-\rho_t) q_{t+1} \left[1 - 2\frac{(1-m^b_F)}{qh''} q_{t+1} h''_t (1-\rho_t) \right] \right\}$$

Table 3.1: Calibration

	Description	Value
β	Patient households' discount factor	0.99
γ	Entrepreneurs' discount factor	0.98
$\beta^{''}$	Impatient households' discount factor	0.95
δ	Capital depreciation rate	0.03
ψ_K	Adjustment cost	5.00
μ	Capital share in production	0.25
ν	Housing share in production	0.03
j	Weight on housing in utility	0.10
$1/(1-\eta)$	Elasticity of labor supply	0.05
α	Labor share of patient households	0.75
m_H	Domestic LTV capital	0.90
m_F	Foreign LTV capital	0.80
m_H^b	Domestic LTV housing	0.90
m_F^b	Foreign LTV housing	0.80

Table 3.2: Business cycle properties of the baseline model economy

	Data	BKK	IM	Baseline
Cross-country correlations				
Output	0.35	-0.18	0.32	0.44
Consumption	0.21	0.88	0.96	0.71
Investment	0.42	na	na	0.78
Hours worked	0.24	na	na	-0.61
Standard deviations				
Output	1.40	1.55	na	1.70
Consumption	1.31	0.62	na	1.62
Investment	3.88	16.91	na	3.01
Hours worked	1.56	0.76	na	1.66

Table 3.3: Business cycle properties of the model economy: alternative preference specifications

	Data	Baseline	KPR	GHH
Cross-country correlations				
Output	0.35	0.42	0.20	0.44
Consumption	0.21	0.76	0.70	0.63
Investment	0.42	0.78	0.86	0.81
Hours worked	0.24	-0.66	-0.81	0.44
Standard deviations				
Output	1.40	1.67	1.93	2.99
Consumption	1.31	1.56	1.63	2.89
Investment	3.88	3.17	3.30	4.74
Hours worked	1.56	0.02	0.42	1.95

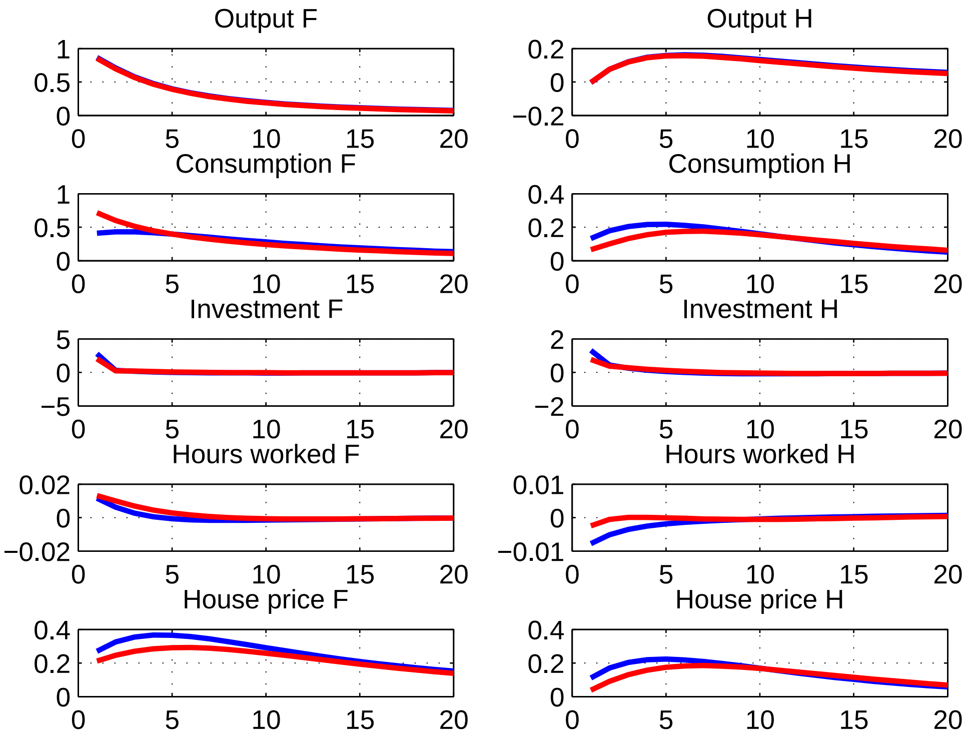

Figure 3.1: Impulse responses to a temporary productivity shock in the foreign country: Baseline (Blue) and α=0.2 (Red)

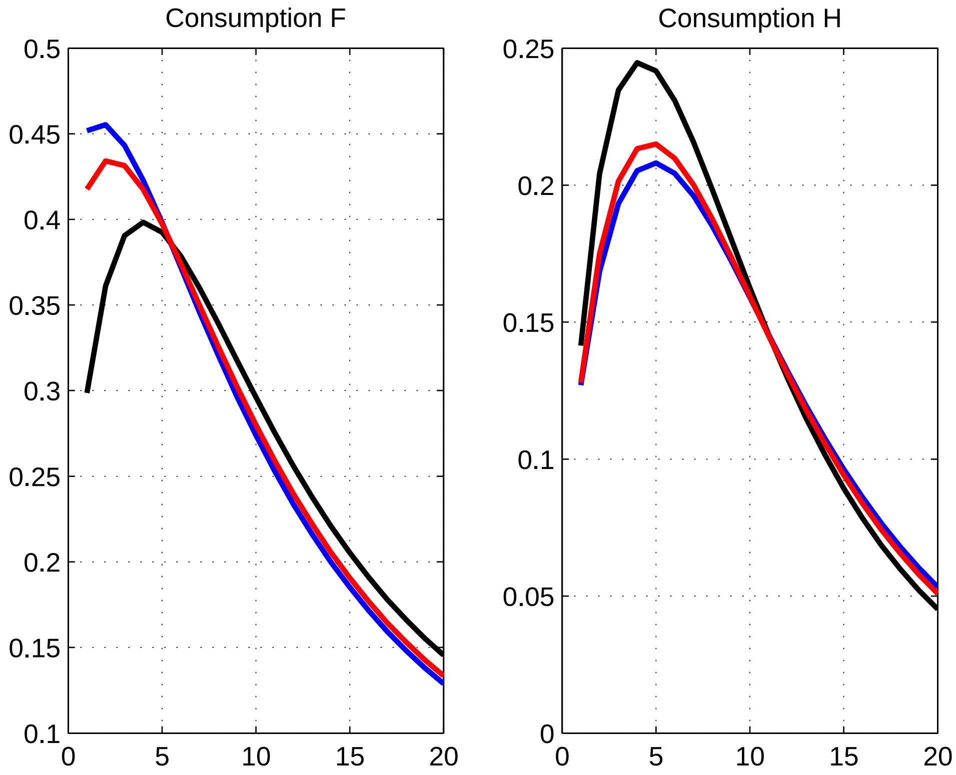

Figure 3.2: Impulse responses of aggregate consumption to a temporary productivity shock in the foreign country: no households borrowing (Black), fixed allocation of housing collateral (Red), variable allocation of housing collateral(Blue).

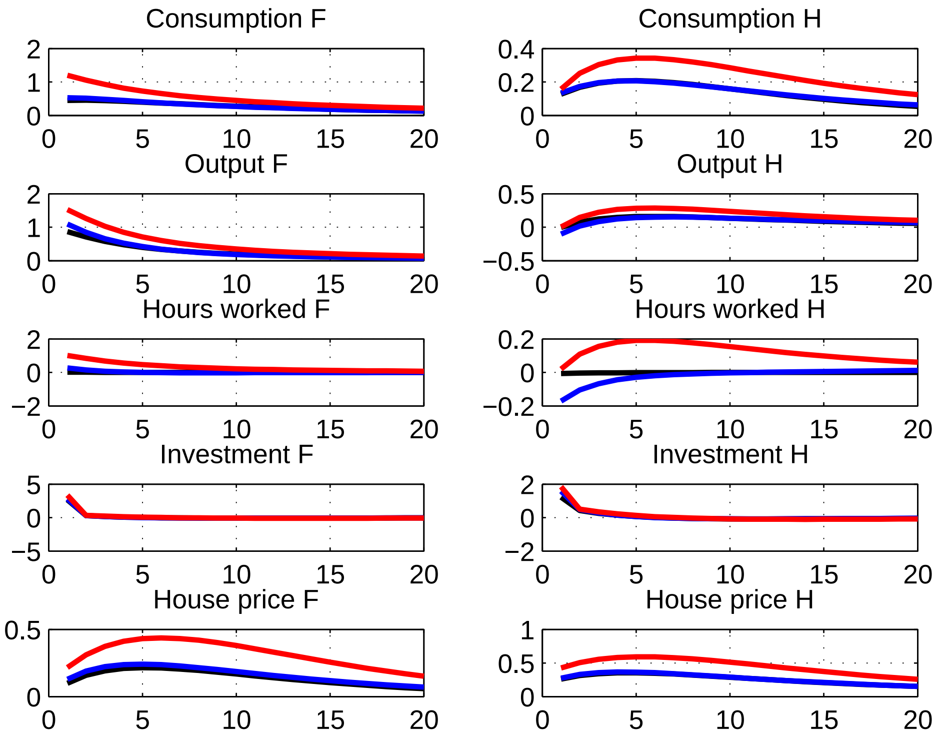

Figure 3.3: Impulse responses to a temporary productivity shock in the foreign country: Baseline Model (Black), KPR (Blue) and GHH (Red)

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