PhD THESIS DECLARATION

The undersigned

SURNAME *Gorn*
FIRST NAME *Alexey*
PhD Registration Number *1612472*

Thesis title: *Essays on Causes and Consequences of Inequality*

PhD in *Economics & Finance*
Cycle *27th*
Candidate's tutor *Professor Antonella Trigari*
Year of thesis defence *2017*

DECLARES

Under his responsibility:

1) that, according to Italian Republic Presidential Decree no. 445, 28th December 2000, mendacious declarations, falsifying records and the use of false records are punishable under the Italian penal code and related special laws. Should any of the above prove true, all benefits included in this declaration and those of the temporary embargo are automatically forfeited from the beginning;

2) that the University has the obligation, according to art. 6, par. 11, Ministerial Decree no. 224, 30th April 1999, to keep a copy of the thesis on deposit at the "Biblioteche Nazionali Centrali" (Italian National Libraries) in Rome and Florence, where consultation will be permitted, unless there is a temporary embargo pro-

tecting the rights of external bodies and the industrial/commercial exploitation of the thesis;

3) that the Bocconi Library will file the thesis in its "Archivio istituzionale ad accesso aperto" (institutional registry) which permits online consultation of the complete text (except in cases of a temporary embargo);

4) that, in order to file the thesis at the Bocconi Library, the University requires that the thesis be submitted online by the candidate in unalterable format to Società NORMADEC (acting on behalf of the University), and that NORMADEC will indicate in each footnote the following information:

   - thesis *Essays on Causes and Consequences of Inequality*;
   - by *Gorn Alexey*;
   - defended at Università Commerciale "Luigi Bocconi" – Milano in *2017*;
   - the thesis is protected by the regulations governing copyright (Italian law no. 633, 22th April 1941 and subsequent modifications). The exception is the right of Università Commerciale "Luigi Bocconi" to reproduce the same for research and teaching purposes, quoting the source;

5) that the copy of the thesis submitted online to NORMADEC is identical to the copies handed in/sent to the members of the Thesis Board and to any other paper or digital copy deposited at the University offices, and, as a consequence, the University is absolved from any responsibility regarding errors, inaccuracy or omissions in the contents of the thesis;

6) that the contents and organization of the thesis is an original work carried out by the undersigned and does not in any way compromise the rights of third parties (Italian law, no. 633, 22nd April 1941 and subsequent integrations and modifications), including those regarding security of personal details; therefore the University is in any case absolved from any responsibility whatsoever, civil, administrative or penal, and shall be exempt from any requests or claims from third parties;

7) that the PhD thesis is not the result of work included in the regulations governing industrial property, was not produced as part of projects financed by public or private bodies with restrictions on the diffusion of the results, and is not subject to patent or protection registrations, and therefore not subject to an embargo;

Date *31 January 2017*

SURNAME *Gorn*
FIRST NAME *Alexey*

Contents

List of Figures

List of Tables

Acknowledgements

First and foremost I want to thank my adviser Antonella Trigari, an incredible passionate researcher and teacher that I look up to immensely, who I was incredibly lucky to learn from. I am grateful for your support, help, and advice throughout the years!
My first chapter profited from valuable discussions and advice from Alberto Alesina, Nicola Gennaioli, Nicola Pavoni, Nicolas Serrano-Velarde, Tito Boeri, and Yuri Tserlukevich. Furthermore, I am thankful for financial support from the PhD school at Bocconi University as well as the *Research Thesis Grant* of the Cariplo foundation for the research project of my first chapter.
I would like to thank Bocconi University for their support: faculty, stuff, and, of course, my colleagues. The latter group filled my days and I benefited from fruitful discussions with them, especially the ones I could not avoid (or they couldn't avoid me, most likely) - my officemates. Thank you Alexandra and Volker!
I want to thank my friends and family for supporting me throughout the years.
I dedicate this work to the memory of my father.

Abstract

This PhD thesis is composed of three chapters on causes and consequences of inequality.

The first chapter relates the observed increase in levels and dispersion of the U.S. top wages to the increasing prominence of headhunters (professional recruiters). I illustrate the main results with a theoretical model that incorporates headhunters in the labor market framework of random search and two-sided heterogeneity. In the model, headhunters improve assortative matching between firms and their top employees via two channels: passive on-the-job search and screening of candidates. The calibrated model shows that headhunters can account for 40% of the increase in the top 1% wage share and 70% of the increase in the top 10% wage share in the U.S. from 1970 to 2010. I provide supporting empirical evidence on the importance of headhunters for the rise in top wages based on cross-country evidence on headhunter hires/fees and top income growth, as well as on micro evidence for CEO compensation in the U.S.

The second chapter studies the effects of the safety net on the individual well-being of the workers. We use a model with incomplete markets, nominal rigidities, and frictional labor market to study direct and general equilibrium effects of the safety net on consumption and labor market outcomes of individual agents. We study three programs: unemployment insurance, social assistance (SNAP and TANF), and tax credit (EITC). We show that individual consumption given assets is higher with stronger safety net but the distribution over assets shifts to the left. The two effects almost cancel out for aggregate consumption. Distribution of consumption is more concentrated with stronger safety net, incidence of very low consumption disappears and incidence of very high consumption decreases. Similar effects appear for the labor market outcomes (search intensity, job finding rates, unemployment duration). Nash bargained wages for low- and high-skilled workers are also moved apart.

The third chapter studies the effects of safety net on the dynamics of the economy. Two important channels are considered - aggregate demand, and search incentives channel. Safety net reduces the volatility of the economy through the aggregate demand channel due to stabilization of individual consumption, while increases the volatility through the search incentives channel due to higher volatility of hiring and search intensity. We use a simplified version of the model introduced in Chapter 2 without assets distribution as a state. We illustrate the importance of the trade-off between these two channels in the design of optimal safety net.

1. Introduction

This PhD thesis studies the causes and consequences of inequalities across agents. First chapter focuses on one cause of the rise of wage inequality - matching heterogeneous between workers and firms. The chapter shows that improved matching at the top of the wage distribution led to the surge in top wages over the last forty years. The chapter shows that the reason for the improved matching was the rising role of headhunters in the labor market. Headhunters reduce frictions at the top by allowing firms to screen workers and allow for passive search on the workers' side. These two features lead to a disproportionate improvement in matching at the top of the distribution that leads to the surge of top wages. The chapter shows in quantitative experiments that the headhunters contributed to 40% of the increase in the top 1% wage share and 70% of the increase in the top 10% wage share in the U.S. from 1970 to 2010.
Second and third chapters focus on the consequences of income and wealth inequality on the individual outcomes and decisions and how they are affected, in turn, by the social safety net. Both chapters use the model with incomplete markets, nominal rigidities, and frictional labor market. Second chapter focuses on the steady-state of the model, how the safety net programs affect the steady state distribution over assets, consumption, and welfare. It shows that stronger safety net shifts up the policy functions of all agents due to direct effect of transfers and reduced precautionary savings motive. At the same time, the distribution of agents over assets shifts towards zero that has an offsetting effect on consumption. With stronger safety net the distribution of individual consumption is compressed towards higher levels of consumption. Third chapter focuses on the dynamic response of the model to productivity shocks. In order to reach analytical tractability we assume that the agents pull their assets in the end of every period while keeping idiosyncratic uncertainty within each period. We focus on the unemployment insurance and show how the model can be extended to incorporate other safety net programs. We demonstrate the importance of the trade-off between the aggregate demand channel and the labor market channels. When unemployment insurance is too low, the aggregate demand channel dominates and volatility of the economy increases. When, instead, unemployment insurance is too high, the labor market channels dominate increasing the volatility of the economy.

2. The Role of Headhunters in Wage Inequality: It's All about Matching

Abstract

This study relates the observed increase in levels and dispersion of the U.S. top wages to the increasing prominence of headhunters (professional recruiters). I illustrate the main results with a theoretical model that incorporates headhunters in the labor market framework of random search and two-sided heterogeneity. In the model, headhunters improve assortative matching between firms and their top employees via two channels: passive on-the-job search and screening of candidates. The calibrated model shows that headhunters can account for 40% of the increase in the top 1% wage share and 70% of the increase in the top 10% wage share in the U.S. from 1970 to 2010. I provide supporting empirical evidence on the importance of headhunters for the rise in top wages based on cross-country evidence on headhunter hires/fees and top income growth, as well as on micro evidence for CEO compensation in the U.S.

2.1 Introduction

Top wages have been rising sharply in the United States from the early 1970s. One of the reasons for this rise is improved matching between firms and employees at the top positions (especially CEOs and top managers), as documented by Song, Price, Guvenen, Bloom, & von Wachter (2016). The conventional view attributes the improvement in matching to a large extent to skill-biased technological change which raises incentives for firms and workers to be better matched. However, while this explanation is successful in explaining the rise of the upper-middle class, it has difficulties to explain the sharp rise of top wage. The results by Song et al. (2016) reveal a strong non-linearity in the sorting pattern, that is, a disproportional shift of high-skilled workers to high-paying firms in comparison to medium-skilled workers to medium-paying firms, that cannot be generated by standard models of labor markets.

In this study, I develop a model where improved matching at the top is explained by the increasing role of headhunters, or executive search firms, in the labor market. Headhunters, who started to gain market share in the U.S. in the 1970s and now assist to fill more than half of the positions in the top wage segment, enhance matching for two reasons. First, they provide more suitable candidates for the firm because they can screen the candidates better. Second, they induce passive on-the-job search as they contact potential candidates directly, creating opportunities for new matches without active search from workers on the job. To be clear, the headhunters restrict the pool of potential candidates facing firms to only the high-skilled workers, while at the same time, expand the pool of potential candidates to a larger number of those high-skilled workers. These two features guarantee a good match to both the firm and the high-skilled worker, increasing productivity when the match is formed through a headhunter, and therefore allowing wages of such matches to increase. Because executive search firms operate on the top wage segment, these improvements in the matching do not happen (or happen to a lower degree) over the rest of the distribution, leading top wages to increase much more compared to the rest of the distribution. Headhunters generate a strong non-linearity in matching upgrade because they operate exclusively at the very top.

The main indicator of the relative magnitude of top wages used in the literature is the share of total wages that go to the top 1% or top 10% of all employees (from now on, just the top 1% or top 10% wage share). Both shares increased dramatically in the U.S. since the 1970s. The top 1% wage share rose from 5.1% in 1970 to 10.9% in 2010, while the top 10% wage share rose from 25.7% to 34.5%. Figure 2.1 plots both shares for the U.S. and compares them to the case of France. The comparison provides suggestive evidence on the role of headhunters for top wages as in France, a country where headhunters started to gain market share only in the 1990s, top wage shares started to rise only in the mid 1990s.

As we said, among the main reasons behind the sharp increase in top wages considered in the literature is improved matching between firms and employees[1]. Song et al. (2016) show using administrative data that improved sorting between workers and firms was a primary factor for the increase in wage dispersion in the U.S. between 1980s and 2000s.[2] Moreover, a closer investigation of their results in terms of the joint distribution of workers and firms by individual fixed effects (a proxy for unobserved skill and productivity) shows that most of the high-skilled workers moved from low- and medium-productivity firms to high-productivity firms over the years. Improvement in the matching is thus concentrated in the top part of the distribution with high-skilled workers toward the end of the sample almost exclusively employed in high-productive firms[3]. This pattern is consistent with the headhunters creating a separate market for high-skilled workers, thus improving matching at the top.

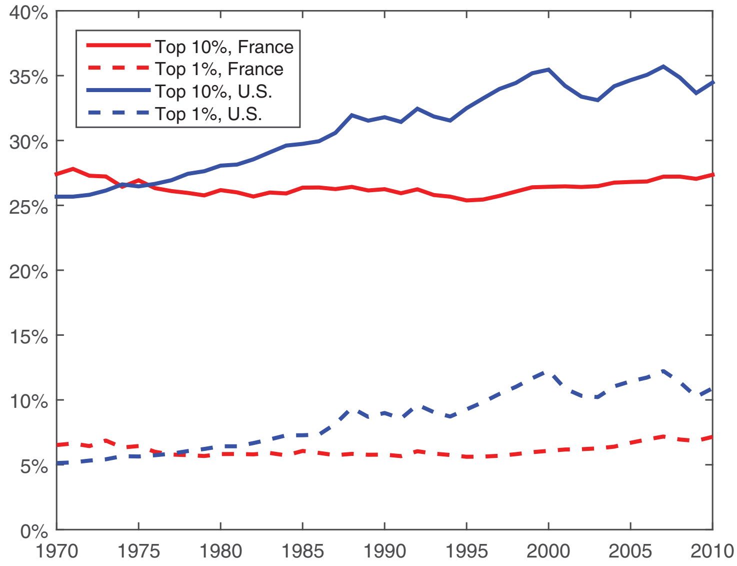
Headhunters were gaining market share in the U.S. at the same time the improvement in sorting at the top was taking place. The fee revenues of top headhunter companies increased by a factor of 10 from the 1970s to 2010s. The International Association of Corporate and Professional Recruiters (IACPR) report (2003) states that 54% of the positions paying above $150,000 a year (around top 5% wages) in 2003 were filled through

[1] Alternative explanations include: decrease in top income taxes - Alvaredo, Atkinson, Piketty, & Saez (2013); direct effects of skill-biased technological change on wages - Acemoglu (2002), Katz, Kearney, et al. (2006), and many others; social norms - Piketty (2014); exogenous changes in random growth theories - Gabaix, Lasry, Lions, & Moll (2016), Jones (2015), Jones & Kim (2015), and Aoki & Nirei (2015); and numerous studies on the increase of CEO pay who constitute a significant part of top 10%, including Gabaix & Landier (2008), Bell & Reenen (2013), and Lemieux, MacLeod, Parent, et al. (2009) among others.

[2] In line with the result of Song et al. (2016), Card, Heining, & Kline (2013) find that the increased covariance between workers' and firms' fixed effects contributed to 30% of the increase in wage dispersion in West Germany between 1985 and 2009. Bagger, Sorensen, & Vejlin (2013) document similar findings using Danish data. They find that the correlation between worker and firm fixed effects increased from -0.07 in 1981 to 0.14 in 2001. Then splitting the sample by the quartiles of worker fixed effects, they find that the aggregate effect was driven by the top quartile of workers where the correlation increased from -0.20 to 0.37, while the correlation stayed almost unchanged at around zero for the rest of the quartiles. Hakanson, Lindqvist, & Vlachos (2015) also document significant increase in sorting in Sweden between 1986 and 2008.

[3] Estimating a fixed effect regression is only one way to evaluate sorting in the labor market. Eeckhout & Kircher (2011) show potential problems with identifying sorting with estimated fixed effects. Other studies propose non-parametric techniques to assess the degree of sorting. In general, such studies find a higher degree of sorting than in the studies using fixed effects regressions. Notable examples are Hagedorn, Law, & Manockii (2012) who find the correlation between worker and firm ranks to be 0.75 in Germany, and Lise, Meghir, & Robin (2016) who find significant sorting for college-graduates in the U.S. These studies mainly focus on the methodology and identifying sorting in a particular period, and not studying the change in sorting over time. An exception is the study by Schulz & Lochner (2016) who show, using non-parametric techniques, that sorting increased in Germany between 1998 and 2008.

Figure 2.1: Top 10% and 1% shares of the total wage bill in the U.S. and France.



This figure plots the top 10% share of the total wage bill in the U.S. (solid blue) and France (solid red), and the top 1% share of the total wage bill in the U.S. (dashed blue) and France (dashed red). Source: Piketty (2014).

headhunters, which constitutes a significant share of the labor market, especially for top-paying positions. Importantly, headhunters are not only focused on specific industries or positions but rather cover a wide range of positions across all industries in the economy. A detailed description of the structure and workings of the headhunter industry, as well as the way it has evolved over time, will be presented in the next section. Their key attribute is that they improve matching between firms and employees as they aim at finding the best candidate for a position. Individual headhunters are typically focused on a specific position or industry and build detailed databases with information on the majority of potential candidates for such position or industry. With this detailed information already in hand, when asked to assist to fill a position they can choose the best fitting candidate and improve matching. This is the first main feature of headhunters - better screening of the candidates. After an headhunter chooses a candidate from its database, it calls the candidate directly and asks whether she wants to consider a job offer (without specifying the offer). The headhunter contacts any candidate who is perceived to be the best fit for the position without the candidate having to signal interest in changing job. A worker who has not to put effort into receiving an offer from an headhunter and who agrees to consider the offer is essentially searching passively on the job. Passive on-the-job search is the second major feature of the headhunter industry. Passive search helps high-skilled workers not getting stuck for long in positions not fitting them, moving them to a better fitting position and improving aggregate matching.

To quantify the contribution of better matching induced by headhunters to the increase in top wages, I develop a labor market model along the lines of Mortensen & Pissarides (1994) augmented with heterogeneous workers and firms[4]. I introduce the headhunter industry in the model by adding a new channel for matching workers and firms. Firms with an open position can either post a vacancy as in the standard model or hire through an headhunter. The difference for the firm is that while it cannot screen workers coming through vacancies, the headhunter guarantees the firm a minimal skill level of the worker with whom the firm is matched. This because only workers with skills above a certain threshold can receive call from headhunters. Consider now the worker's side. Low-skilled workers have access to the standard channel to be matched with a firm with an open position, and to search on the job they have to pay the search costs. I call this active on-the-job search. For high-skilled workers, instead, on top of active on-the-job search, there is also a possibility of passive on-the-job search. A worker is searching passively, if she agrees to consider an offer when a headhunter calls[5]. Screening and passive on-the-job search are exactly the two main features of the headhunter industry that I introduce to the theoretical model.

Having set up the model, I apply the following calibration strategy. First, I calibrate the model without headhunters targeting moments of the wage distribution and aggregate labor market moments in the 1970s in the U.S.. The key calibrated parameters include those characterizing the exogenous distributions of workers over skills and firms over productivity. The idea is that the U.S. labor market in the 1970s is well approximated by one with no, or limited, role for headhunters. Having fixed the parameters not related to the headhunters, I then introduce the headhunter channel to the model and calibrate the related parameters to target the moments of the headhunter industry in the 2010s. On top of this, I introduce skill-biased technological change to match the increase in 90/50 wage ratio from 1970 to 2010 in order to compare the relative contribution of headhunters and skill-biased technological change for the rise of the top wage shares. Skill-biased technological change is modeled as an increase in the degree of complementarity between workers and firms.

To evaluate the role of the headhunter channel, I then exploit the richness of the model and compare several statistics related to the wage distribution with and without the headhunters. Importantly, I perform experiments in line with Song et al. (2016)

[4]Other studies including two-sided heterogeneity in the labor market include Shimer & Smith (2000), Postel-Vinay & Robin (2002), Teulings & Gutier (2014), Gautier & Teulings (2015) and many others.

[5]Capelli & Hamori (2013) show that more than half of executives are willing to consider an offer when a headhunter calls them.

where I compare results from the model generated data to the U.S. data to see if the improvement in matching in the model has similar features to the observed improvement in matching. My calibration strategy answers the question how would the distribution of wages (and therefore also the top wages) have changed between the 1970s and 2010s if skill-biased technological change and headhunters had been the only factors rising top wage inequality. To assess the relative contribution of the two factors, I then shut down one channel at a time: the increase of the degree of complementarity, or the headhunter channel. Finally, I also give a chance to skill-biased technological change to explain all the increase in wage inequality without the headhunters. To do that, I change the increase in the degree of complementarity to match the increase in the top 10% wage share and assess how the model fits other moments. In the baseline calibration, I use a simple sharing rule for wage setting. The reason for this is that I want to isolate the effects of matching. Headhunters may also affect the bargaining power of firms or employees, for example shifting up wages of employees hired through headhunters. I want to abstract from such effects in the main experiment because it would be difficult to disentangle the relative contributions of matching and wage setting in the change of the wage distribution[6].

The main quantitative result of the paper is that the rise of headhunters accounts for 40% of the increase in top 1% and 70% of the increase in top 10% wage shares in the U.S. from the 1970s to 2010s. Skill-biased technological change contributes to another 10% of the top 1% wage share and 23% of the top 10% wage share increase, and interaction between the two factors rises the top shares by further 10% for top 1% and 5% for top 10%. Headhunters and skill-biased technological change also contribute equally to the rise of the upper-middle class measured as 90/50 wage ratio. Thus, the sharp increase of top wages in the model is mainly due to improved assortative matching after introducing headhunters. With headhunters, high-productive firms employ almost exclusively high-skilled workers and high-skilled workers work almost exclusively in high-productive firms. Comparing joint distributions of worker-firm matches in the two steady states reveals a pattern similar to the empirical results of Song et al. (2016), where almost all types of firms, except the highest-paying, lose the highest-skilled workers and where the highest-paying firms gain those workers disproportionally. The headhunter channel generates the strong non-linearity in the change in assortative matching observed in the data, with disproportionate improvement in matching for high-skilled workers. I am not aware of other theoretical models able to generate such non-linearity. If I allow the model to match the increase in the top wages without the headhunter channel, the model overshoots the 90/50 wage ratio by 82%. This happens exactly because of the absence of a strong non-linearity (SBTC affects more

[6] However, I redo the experiments with wages set through explicit bargaining in the appendix.

high-productive workers, but it is not enough), increase in complementarity shifts the whole distribution of wages to the right almost uniformly. The non-linearity generated by the headhunters allows to shift the right tail of the distribution further apart from the rest of the distribution without changing the overall shape of the distribution in the middle.[7]

The model is a significant improvement to other theoretical models showing the importance of assortative matching for wage distribution. Bagger & Lentz (2014) is the closest study. They show that on-the-job search is a crucial mechanism to generate assortative matching in a Diamond-Mortensen-Pissarides model with two-sided heterogeneity. Bagger & Lentz (2014) consider only active on-the-job search, and this paper shows that passive on-the-job search is crucial for generating even more assortative matching, concentrated at the top, consistent with empirical studies. Uren & Virag (2011), instead, show that skill requirements are important to generate an increase in between-group inequality (increased differences between wages of workers with different skill level). Skill requirements play a similar role as the screening by headhunters. While this paper focusing on headhunters and the top part of the distribution, Uren & Virag (2011 study the overall shape of the wage distribution.

Headhunters play a major role at the top paying segment of the labor market but were overlooked by economic research. This paper is first to bring attention to the phenomenon of headhunters in a macroeconomic perspective and to discuss the effects of their activity on the outcomes of the labor market and the economy as a whole. Introducing headhunters in the theoretical models is another contribution of this paper. Particularly as this allows me to introduce the notion of passive on-the-job search within search models. Passive on-the-job search may prove to be a useful notion also applied to the medium wage segment for which contingent headhunters hire medium paying professionals, to job offers received through referrals, or to direct contacts by employers through websites like LinkedIn. Besides labor markets applications, passive on-the-job search might capture phenomena in search models of financial markets where brokers call investors directly offering to buy a particular asset that would fit their portfolio or other theoretical settings.

To provide support for the relevance of headhunters, this paper presents two blocks of independent empirical evidence. First, it uses cross-country differences in the use of headhunters in Europe in 1997 to show that in countries where headhunters made relatively more placements top income shares increased more in the following years. This

[7] The rise of headhunters can be also viewed as a reason for the shift in the mean income growth rates for high-skilled workers in the model of Gabaix et al. (2016). Gabaix et al. (2016) introduce an exogenous increase in mean income growth rate for some workers in 1980s, motivated by globalization and technological change. Headhunters, who allow high-skilled workers to work for the top firms, generate the increase in income growth rate for high-skilled workers due to better matches.

evidence is in line with the prediction of the model that the more the high-productive firms use headhunters, the better is the improvement of matching at the top, and the higher are the top wages. And second, the paper studies micro data on CEO compensation of listed companies in the U.S.. Main results are that firms pay significantly more to new CEOs comparing to the previous ones, and this difference is higher in the periods when headhunters are used more intensively and in the states where there are less legal obstacles to the activity of headhunters[8]. These results suggest that headhunters indeed improve matching between firms and CEOs and therefore increase the wages at the top. Of course this is just a suggestive evidence, ideally, it should be identified which CEO was hired through a headhunter and which CEO was hired through other sources, but such data is not available, partly because of privacy issues.

On the empirical side, the contribution of this paper is to the hires by headhunters in a country (or region) as a proxy for the degree of assortative matching at the top. It is an empirical challenge to identify sorting in the data. Looking through the lens of the model presented in this paper, the hires by headhunters allow to measure the degree of assortative matching. The more hires at the top are done by the headhunters, the better is the assortative matching at the top.

The remainder of the paper is organized as follows. Section 2 discusses the available empirical evidence about headhunters and headhunter industry. Section 3 presents the theoretical model. Section 4 presents the calibration of the model and discusses the implications of the model on wage inequality. Section 5 provides empirical evidence of cross-country differences in the patterns of top income shares over the last thirty years and their relation to the headhunter industry. Section 6 presents empirical results of the effects of changing CEO on the compensation paid by the firm. Section 7 concludes.

2.2 Headhunter industry

This section offers a short description of different types of headhunters, the practices of headhunters, and the size and the development of the industry over time.

First of all, it is important to distinguish between two types of headhunters - retained headhunters and contingency headhunters. Retained headhunters have an exclusive and often permanent contract with the firm to find and hire an employee for a particular position or several employees for a set of positions. Many retained headhunters have long-term contracts with companies and are used when the firm needs to feel a certain position. Retained headhunters operate on the very top positions in the firm structure that pay above $150,000 a year. Retained headhunters are in the focus of

[8]The legal obstacles are instrumented by the enforceability of non-compete agreements as proposed by Garmaise (2009).

this paper. Contingency headhunters, instead, don't have an exclusive contract with the firm, so sometimes the firm contacts several contingency headhunters and only the one that provides the contact with the candidate that is hired is paid by the firm. Contingency headhunters cover mainly the medium skilled positions such as professionals and general managers, the positions paying between $15,000 and $150,000 a year. Even though contingency headhunters also cover a significant share of the labor market they are left aside in this project because their effect on the top wages is thought to be minor.

In order to operate, the headhunters need to establish a database of potential candidates. The databases are often very detailed and include information about contacts, education, prior experience, particular skills, languages spoken, and many others. The headhunters constantly work on expanding and updating the database. When the headhunter needs to fill a position, the candidate is chosen from the database with a goal to have the best fit for the position. The headhunter then contacts the candidate making an offer to consider a new job possibility, without specifying details about the job (employer, wage range e.t.c.). If the candidate agrees to consider the offer the headhunter interviews the candidate to determine the fit for the position. If the interview is successful, the offer is revealed and the candidate is connected with the firm. Now the firm assesses whether it wants to hire the candidate and if it confirms the candidate is hired by the firm.

Because the costs of creating databases of potential candidates are huge, there are different types of headhunter companies. There are many boutiques with few employees that focus on a particular position in a particular industry (or even sub-industry) and a particular region (sometimes even a city). Because of specialization boutiques are able to find the best candidate for the position of specialization. Then, there are local or specialized companies, who target a specific industry or a region. Finally, there are few international companies who can provide service for almost any top position in any industry and region. In general, headhunters cover a wide range of positions: CEOs, board directors, CFOs, senior executives, general management, top professionals in finance and control, information systems, marketing, and sales. They are not focused only on CEOs as sometimes is perceived but cover almost all the top positions in companies. Industry composition of headhunter operations is also dispersed, they operate in all industries. Distribution of fee revenues by industry in the fourth quarter of 2015 is presented in table 2.1.

Geographical distribution of fee revenues instead is not so homogeneous, as presented in table 2.2. Most of the revenues headhunters receive from North America, and mainly the U.S., and Europe is lagging behind with major part of European revenues coming from the U.K. There might be several reasons for the fact that headhunters are more widely used in the U.S. than in Europe. One major reason is the difference in labor market legislation. It is more difficult to be an intermediary in European labor

Table 2.1: Fee revenues of headhunters by industry, 4th quarter 2015.

Industrial	23.5%
Technology	16.2%
Financial	21.0%
Consumer products	18.0%
Life Sciences/Healthcare	15.2%
Non-Profit	4.4%
Other	1.7%

This table presents the distribution of the estimated worldwide fee revenues over industries in the 4th quarter 2015. Source: AESC Insights Q4 2015 State of the Executive Search Industry.

market than in the U.S., especially in some countries. Second possible reason is the cost of creating a database of potential candidates in a new country. The headhunter must know the specifics of the labor market and the companies operating in the country in order to understand the skills demanded by companies and the value of observable signals, such as particular diplomas and experience in particular companies. Because headhunters first appeared in the U.S. they established the databases and acquired the knowledge of the labor market and the signals earlier there and therefore are more efficient.

Table 2.2: Fee revenues of headhunters by region, 4th quarter 2015.

North America	45.5%
EMEA	33.2%
Asia Pacific	16.5%
Latin America	4.8%

This table presents the distribution of the estimated worldwide fee revenues over regions in the 4th quarter 2015. (EMEA - Europe, Middle East, and Africa) Source: AESC Insights Q4 2015 State of the Executive Search Industry.

The history of the rise of headhunters started in the U.S. already in the 1950s with first professional recruiters. However, the first couple of decades were not very successful for them. Only in the late 1970s and early 1980s, the industry started to expand sharply with worldwide fee revenues rising from \$0.75 billion in 1978 to \$3.9 billion in 1990. The fee revenues kept rising up to \$12 billion in 2015 (figure 2.2a). Partly this rise was mechanical because top wages were also rising over the same period (figure 2.2b), but the revenues increase was much larger in proportion to the increase of the wages. Another indicator of the expansion of the industry is the number of hires by headhunters. For example, the number of assignments of one of the historical leaders of the industry Korn/Ferry increased from 42 in 1969 to 8,480 in 2015 according to the financial statements.

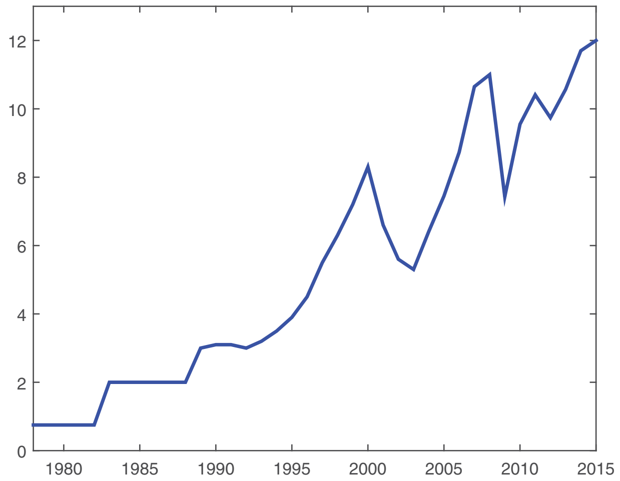
There might be several reasons for the rise of headhunters. Maybe the most important reason is the technological progress in IT and communication that increased the quality of the supply of services by headhunters. Developments and growing availability of computers reduced dramatically the costs of managing and searching through databases of potential candidates. Communication technology (mobile phones and emails), instead, made it easier to contact potential candidates and allowed headhunters to expand the networks of potential candidates. Companies that adopted new technologies earlier were more successful in the market[9]. Another effect of technology goes through the demand side; better technology made it easier to apply for jobs (especially in the late 1990s and 2000s). More applications increased the amount of information that the firms had to evaluate to hire a worker and the higher was the position the more information was there to evaluate. It became more efficient for the firms to delegate the screening of workers for top positions to intermediaries - the headhunters - and the demand for headhunter service increased. One more potential reason for the rise that goes through demand is related to the nature of the skills required from employees in the top positions. Because of technological change, globalization, or change of company structure, it became more important for the firm to hire employees with higher general skill in comparison to 1970s. Firms started to use headhunters more because in the 1990s the skill of the CEO, for example, affected the performance of the company much more than in 1970s, while the nature and efficiency of headhunters stayed the same. Even though there is more evidence of the technological supply story, this paper doesn't exclude other reasons for the rise of headhunters. Because of the nature of the experiments done in this paper, the actual reason for the rise doesn't play a big role for the results of the paper. It will be important, however, to understand the exact reason for the rise of headhunters better in the future studies of dynamics of inequality, and the question of the rise of the industry is interesting by itself.

To determine the exact market share of the very closed and private industry is a very difficult task. One can use IACPR report (2003) that shows that 54% of the positions above $150,000 a year were filled by headhunters in 2003. Another way to determine the market share of the headhunters is to compare the estimates of the fee revenues to the ones implied by the total wage bill. Total fee revenues in the U.S. esti-

[9] Jenn (1995) writes: "The drive towards a more consistent quality of service throughout the world has been greatly assisted by the application of information technology to the search business and the use of global databases. Technological advances have allowed firms to search more widely and communicate more efficiently. Virtually all executive search firms are attempting to modernise their communications and database systems on a global basis. ... This is the area where the search world is changing most dramatically. Firms have a tremendous opportunity to improve their efficiency, achieve better margins and differentiate themselves from their competitors."

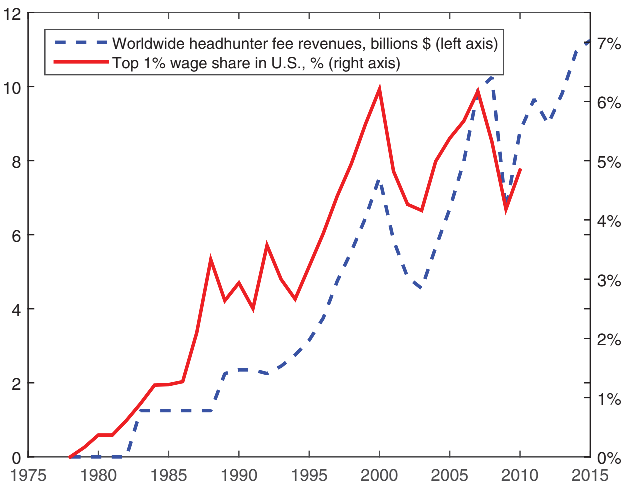
Figure 2.2: Estimated worldwide fee revenues of headhunters and top wages.

(a) Estimated worldwide fee revenues of headhunters.



This figure plots the estimated worldwide fee revenues of the headhunter industry from 1978 to 2015, in $ billions. Source: AESC Insights Q4 2015 State of the Executive Search Industry.

(b) Cumulative change in headhunters fee revenues and top 1% wage share.



This figure plots the cumulative change in the estimated worldwide fee revenues of the headhunter industry from 1978 to 2013 (dashed blue line, left axis), in $ billions, and the cumulative change in the top 1% wage share in the U.S. from 1978 to 2010 (solid red line, right axis), in %. Both series are normalized to 0 in 1978. Sources: AESC Insights Q4 2015 State of the Executive Search Industry, Piketty (2014) , and author's calculations.

mated by AESC are around $4.6 billion. From the other side, it is possible to compute the total wages that go to the top 5% of the U.S. employees from the total wage bill and the top 5% wage share from studies on inequality. Then, using the hiring rate one can determine total wage that goes to the new hires in a given year. From the total wages paid to the new hires, headhunters receive as a fee around 30% of the first year wage. It is possible to determine what would be the aggregate fee revenues if the share of the market of headhunters is known. Using the average for the U.S. hiring rate of 3.5%, to be consistent with the estimates by AESC, the share of the headhunters must be around 15% in the labor market for positions in the top 5%. However, the hiring rate at the top is, in general, much lower than in the lower-paying jobs, with tenures being significantly longer. With more realistic hiring rate, the implied market share of headhunters is more than 30%. This estimate is very imprecise because exact average fee paid to headhunters and exact hiring rate are unknown.

2.3 The model

2.3.1 Environment

The economy is populated by a continuum of heterogeneous workers differing in their skill level, e, who supply inelastically one unit of labor if employed. When a worker is unemployed she benefits from home production activity, unemployment subsidies, leisure and other possible sources she cannot enjoy during employment, collectively represented by b. Also, there is a continuum of heterogeneous firms that differ in their productivity level, p. Each firm can hire one worker. To do this a firm needs to post a vacancy or to contact a headhunter company. Both workers and firms discount their future utility with discounting rate β.

All workers, unemployed and employed, can search for a job. Each period workers decide whether to search for a job checking vacancies (search actively) and/or to be available for a headhunter company (search passively) if her skill is higher than a threshold $\hat{e}$. Workers searching for a job and firms posting a vacancy are matched randomly by a standard CRS matching technology. Firms using headhunters are randomly matched with workers with a skill level above $\hat{e}$ with a possibly different matching technology. In the baseline model the wage in a match is determined period by period as a fraction of resulting production of the match. The production of a match depends on the firm's productivity level and the worker's skill level. Firms can choose only one channel while workers can search both actively and passively (if they are eligible) at the same time. Separation of matches depends on two factors: idiosyncratic exogenous separation shock, s; and worker quitting to another job, $s_Q(.)$. There is no aggregate uncertainty in the model. The paper considers only stationary equilibria.

2.3.2 Timing

The time is discrete. Inside every period, first, existing matches produce and wages and unemployment benefits are paid. Then exogenous separations happen. Workers decide in which markets to participate, new firms decide to enter the market and choose the market to search. After that, workers searching for a job and firms searching for a worker are matched.

2.3.3 Matching

There are two channels in the labor market: the vacancy, V, and the headhunter, H, channels. In channel $i \in \{V, H\}$ workers and firms meet by a standard matching technology: $m_i = m_i(u_i + a_i, j_i)$, where m_i is the number of matches, u_i and a_i are the numbers of unemployed and employed workers participating in the channel, respectively, and $j_i \in \{v, h\}$ is the number of firms participating in the channel. The job finding rate for a worker using channel i is $f_i(u_i, a_i, j_i) = \frac{m_i(u_i+a_i, j_i)}{u_i+a_i}$ and the firm's worker finding rate is $q_i(u_i, a_i, j_i) = \frac{m_i(u_i+a_i, j_i)}{j_i}$.

2.3.4 Wages and production technology

In the baseline model, the wage is proportional to the match productivity[10]: $w(e,p) = \psi \cdot y(e,p)$ with $0 < \psi < 1$. Where $y(e,p)$ is increasing and quasi-concave in both components, that is $y'_p > 0$, $y'_e > 0$, $y''_{pp} \leq 0$ and $y''_{ee} \leq 0$. Moreover, $y(e,p)$ has a property of complementarity between the worker's skill and firm's productivity, having positive cross-derivatives: $y''_{ep} > 0$, $y''_{pe} > 0$. The production function must be supermodular so that it generates the incentives for positive assortative matching.

2.3.5 Worker problem

Consider first the problem of a low-skilled unemployed worker. The *low-skilled unemployed worker* is excluded from the headhunter channel, so the only choice that she does is between searching through vacancies and not searching. Low-skilled worker's value of search can be written as:

$$S_U(e) = \max\{S_{UV}(e), 0\}, \; if \; e < \hat{e}, \tag{2.1}$$

where:

$$S_{UV}(e) \equiv f_V(.)\, E_{p|V}[\max\{W(e,p), U(e)\} - U(e)] - c_{wV}.$$

[10]Other wage setting mechanisms and their implications for the model and main results are considered in the appendix.

If she decides to search through vacancies, with probability $f_V(.)$ she will receive an offer that will be drawn from a distribution of firms posting vacancies. She also has to pay the search cost, c_{wV}, every period of actively searching. When the worker receives an offer she will choose between working in a firm with productivity p and getting the difference between the value of employment, $W(e,p)$, and unemployment, $U(e)$, or staying unemployed and having no gain.

For the *high-skilled unemployed worker*, the problem is the same but she chooses between four options: to search through vacancies, to wait for a headhunter call, to do both, or be inactive. The value of searching through vacancies is the same as for the low-skilled unemployed worker. The value of waiting for a headhunter call, or searching passively, differs in four respects: it has different probability of an offer arrival, $f_H(.)$; it has different search cost, c_{wH}; the search cost is paid only if the offer arrives; and, the offer is drawn from a different distribution - distribution of firms using headhunters. The value of searching through both channels is just a combination of the two, with an implicit assumption that better firms are using the headhunter channel[11]. The value of the search of a high-skilled unemployed worker can be written as:

$$S_U(e) = \max\{S_{UV}(e), S_{UH}(e), S_{UVH}(e), 0\},\ if\ e \geq \hat{e}, \tag{2.2}$$

where

$$S_{UH}(e) \equiv f_H(.)\left(E_{p|H}[\max\{W(e,p), U(e)\} - U(e)] - c_{wH}\right),$$

$$\begin{aligned} S_{UVH}(e) \equiv \quad & f_H(.)\left(E_{p|H}[\max\{W(e,p), U(e)\} - U(e)] - c_{wH}\right) + \\ & + f_V(.)(1 - f_H(.))E_{p|V}[\max\{W(e,p), U(e)\} - U(e)] - c_{wV}. \end{aligned}$$

The unemployed worker who did not receive or reject an offer this period consumes the unemployment benefits and continues the search in the next period. The value of unemployment for a worker can be written as:

$$U(e) = b + \beta(U(e) + S_U(e)). \tag{2.3}$$

Now consider an *employed worker*. She also decides whether to participate in the markets, but with a different outside option. Similarly to the unemployed workers, the employed low-skilled worker may choose between searching through vacancies or not to search at all. Now, the outside option for the worker is her previous employment rather than unemployment. For a *low-skilled employed worker* the value of search can be written as:

[11]This assumption is used already here for the convenience of notation.

$$S_E(e,p) = \max\{S_{EV}(e,p), 0\}, \ if\ e < \hat{e}, \tag{2.4}$$

where

$$S_{EV}(e,p) \equiv f_V(.)\, E_{p'|V}\left[\max\left\{W(e,p'), W(e,p)\right\} - W(e,p)\right] - c_{wV}.$$

The employed high-skilled worker may choose again between four options: searching through vacancy, headhunter, or both channels, or not to search at all. For a *high-skilled employed worker* the value of search can be written as:

$$S_E(e,p) = \max\{S_{EV}(ep), S_{EH}(e,p), S_{EVH}(e,p), 0\}, \ if\ e \geq \hat{e}, \tag{2.5}$$

where the value of search through the vacancies is exactly the same as for a low-skilled worker and

$$S_{EH}(e,p) \equiv f_H(.)\left(E_{p'|H}\left[\max\left\{W(e,p'), W(e,p)\right\} - W(e,p)\right] - c_{wH}\right),$$

$$\begin{aligned} S_{EVH}(e,p) \equiv \ & f_H(.)\left(E_{p|H}\left[\max\left\{W(e,p'), W(e,p)\right\} - W(e,p)\right] - c_{wH}\right) + \\ & + f_V(.)\left(1 - f_H(.)\right) E_{p|V}\left[\max\left\{W(e,p'), W(e,p)\right\} - W(e,p)\right] - c_{wV}. \end{aligned}$$

If a worker decides to stay in a firm or does not receive an offer, she consumes the current wage this period, and next period the match can be exogenously separated with probability s, in which case the worker becomes unemployed, or with probability $(1-s)$ the match survives and the worker can continue to search on-the-job. The value of work for any worker is:

$$W(e,p) = w(e,p) + \beta\left(sU(e) + (1-s)\left(W(e,p) + S_E(e,p)\right)\right). \tag{2.6}$$

2.3.6 Firm problem

Firms also choose channels in the same manner as the workers, but they all solve the same problem (regardless of their productivity level) and they may choose only one channel, having the value of a vacant job defined as:

$$V(p) = \max\{V_V(p); V_H(p); 0\}. \tag{2.7}$$

If the firm decides to post a vacancy, it pays the per-period cost $c_{fV} \cdot p$ and is matched with a worker with probability $q_V(.)$ and hires her if she accepts the offer. It happens when the worker doesn't have a better offer at the same period and if she worked in a worse firm (if searching on-the-job). The value of posting a vacancy for a firm is:

$$V_V(p) = -c_{fV} \cdot p + \beta \left(V(p) + q_V(.) E_{e|V} \left[P(A) \left(J(p,e) - V(p) \right) \right] \right), \quad (2.8)$$

where $P(A)$ is the probability of acceptance of the offer by the worker. In general, this probability might depend on many factors: the skill of the worker, the productivity of this firm, the productivity of the current employer of the worker, another offer to the worker, etc. But in the structure of equilibrium that will be considered here, the exact functional form for this probability is relatively simple.

Similarly, if a firm decides to hire through a headhunter, it pays a per-period cost $c_{fH} \cdot p$ and is matched with a worker with probability $q_H(.)$. The value of hiring through a headhunter is:

$$V_H(p) = -c_{fH} \cdot p + \beta \left(V(p) + q_H(.) E_{e|H} \left[P(A) \left(J(p,e) - V(p) \right) \right] \right). \quad (2.9)$$

If the firm hires a worker or a match stays for this period, the firm receives the product of the match, pays the wage and in the next period the match is separated for exogenous reasons with probability s or because the worker quits to another firm with probability $s_Q(.)$[12], and the firm has to substitute the worker, or survives otherwise. So the value of a firm can be written as:

$$\begin{aligned} J(p,e) = \; & y(e,p) - w(e,p) + \\ & + \beta \left(\left(s + s_Q(.)(1-s) \right) V(p) + \left(1 - s_Q(.) \right)(1-s) J(p,e) \right). \end{aligned} \quad (2.10)$$

The free entry condition for the firms is the following:

$$E_p \left[V(p) \right] = F, \quad (2.11)$$

where F is a fixed cost of creating a firm that is paid once to enter the market. It is assumed that before entering the market, firms do not know their level of productivity. Another interpretation of this assumption can be that a firm is created with not just a single vacancy but with a distribution of positions that it needs to fill.

2.3.7 Steady-state separating equilibrium

In this section, a particular structure of the equilibrium will be considered. The most reasonable scenario, given supermodularity of the production function, is that high-productive firms would hire through the headhunter channel while low-productive firms would use the standard channel.

[12] The arguments of the function are dropped for convenience of notation, but in general it depends on the firm productivity level, worker's skill level, and characteristics of the labor market in both channels.

Distributions

First, we need to specify distributions that will be used in expectations. Let $F(p)$ be an initial distribution of firm productivity levels and $G(p)$ the measure of firms with an open vacancy (both CDFs have support $\left[\underline{p},\overline{p}\right]$). Denote as $\hat{p}$ the cutoff level of firm productivity, such that firms with productivity above $\hat{p}$ hire through a headhunter and firms below $\hat{p}$ post a vacancy, so the fraction of firms posting a vacancy is $\frac{G(\hat{p})}{G(\overline{p})}$.

Let $H(e)$ be the initial distribution of all workers over skill levels, $L_V(e)$ be the measure of employed workers searching for a job through the vacancy channel, $L_H(e)$ the measure of employed workers searching for a job through the headhunter channel, $L_{VH}(e)$ the measure of employed workers searching for a job through both channels, and $U(e)$ the measure of unemployed workers over the skill level (all with support$[\underline{e},\overline{e}]$).

Finally, let $\Phi(e,p)$ be the joint measure of active matches. And $\Lambda_i(e,p)$ be the measure of active matches in which a worker is searching for a new job through channel $i \in \{V, H, VH\}$.

Workers

Given the structure of the equilibrium under consideration and the distributions defined above we can now specify the expectations.

Under our assumptions, low-skilled workers are excluded from the headhunter channel so they can search only through vacancies. Their value of search will be:

$$S_U(e) = S_{UV}(e) \equiv f_V(u_V, a_V, v) \int_{\underline{p}}^{\hat{p}} (W(e,p) - U(e))\, dG(p) - c_{wV}. \tag{2.12}$$

For high-skilled unemployed workers it is optimal to search through both channels, so their value of search is the following:

$$\begin{aligned} S_U(e) = S_{UVH}(e) \equiv\ & f_H(u_H, a_H, h) \left(\int_{\hat{p}}^{\overline{p}} (W(e,p) - U(e))\, dG(p) - c_{wH} \right) + \\ & + f_V(u_V, a_V, v)\,(1 - f_H(u_H, a_H, h)) \int_{\underline{p}}^{\hat{p}} (W(e,p) - U(e))\, dG(p) - c_{wV}. \end{aligned} \tag{2.13}$$

Under our assumption, better firms use the headhunter channel, so if unemployed high-skilled worker receives an offer from a headhunter she will accept it regardless of receiving an offer through vacancy channel or not. Instead, this worker will accept an offer from vacancy channel only if she doesn't receive an offer from the headhunter channel at that period.

Again, for employed workers the value of search is the same as the value of search for unemployed except from the outside option. We can define the value of search of low-skilled employed worker as:

$$S_{EV}(e,p) \equiv f_V(u_V, a_V, v) \int_{\underline{p}}^{\hat{p}} \max\left\{W(e,p') - W(e,p); 0\right\} dG(p') - c_{wV}. \quad (2.14)$$

Or, opening the max inside the integral:

$$S_{EV}(e,p) \equiv f_V(u_V, a_V, v) \int_{p}^{\hat{p}} \left(W(e,p') - W(e,p)\right) dG(p') - c_{wV}.$$

The employed worker accepts a new match only if she is matched with a better firm.

To search from employment, the value of search for a worker with skill e working in a firm with productivity p must be positive:

$$S_{EV}(e,p) \geq 0.$$

This equation (when satisfied with equality) implicitly determines the level of the firm productivity such that a worker with a skill level e does not search for a new job: $\tilde{p}_V(e)$ (for $e < \hat{e}$). So that, if a worker with skill e works in a firm with productivity below $\tilde{p}_V(e)$, she searches for another job and doesn't search otherwise. Her value function of searching will be as before:

$$S_E(e,p) = \max\left\{S_{EV}(e,p); 0\right\}.$$

For a high-skilled employed worker the value of search is now consists of four options, but in this structure of equilibrium, one of them (searching only through vacancies) will never be optimal. So that the value of search can be defined as:

$$S_E(e,p) = \max\left\{S_{EV}(e,p); S_{EH}(e,p); S_{EVH}(e,p); 0\right\} = \max\left\{S_{EH}(e,p); S_{EVH}(e,p); 0\right\}.$$

For a high-skilled worker with skill level e, there are now two cutoff productivity levels $\tilde{p}_{VH}(e)$ and $\tilde{p}_H(e)$, with $\tilde{p}_H(e) \geq \tilde{p}_{VH}(e)$. If the worker is employed in a firm with productivity below $\tilde{p}_{VH}(e)$ she will search for another job through both channels. If she works in a firm with productivity level between $\tilde{p}_{VH}(e)$ and $\tilde{p}_H(e)$, she will search only through headhunter channel. While if she works in a firm with productivity above $\tilde{p}_H(e)$, she will not search for another job at all. Before defining the conditions that determine these cutoffs we need to define the value functions.

Value of search through headhunter channel for a high-skilled worker can be defined as:

$$S_{EH}(e,p) \equiv f_H(u_H, a_H, h) \left(\int_{\max\{\hat{p};p\}}^{\overline{p}} \left(W(e,p') - W(e,p)\right) dG(p') - c_{wH}\right). \quad (2.15)$$

And the value of search through both channel can now be defined as:

$$\begin{aligned} S_{EVH}(e,p) \equiv\ & f_H(u_H, a_H, h) \left(\textstyle\int_{\hat{p}}^{\overline{p}} \left(W(e,p') - W(e,p)\right) dG(p') - c_{wV}\right) + \\ & + f_V(u_V, a_V, v)\left(1 - f_H(u_H, a_H, h)\right) \textstyle\int_{p}^{\hat{p}} \left(W(e,p') - W(e,p)\right) dG(p') - c_{wV}, \end{aligned} \quad (2.16)$$

note that in this case the first integral starts always in $\hat{p}$ because it will never be optimal to search through both channels if a worker is already working in a firm that hires through the headhunter channel. This is the feature of the structure of equilibrium. In other structure of equilibrium, that will be discussed in extensions, not all firms above the cutoff $\hat{p}$ will be using the headhunter channel, so the limits of the integral will be different.

It is easy to see that, given e, $S_{EVH}(e,p)$ is higher than $S_{EH}(e,p)$ for small p, but $S_{EVH}(e,p)$ decreases faster, so they will always have just one intercept. And the equality:

$$S_{EVH}(e,p) = S_{EH}(e,p)$$

defines the cutoff productivity level of searching through both channels for each worker type, $\tilde{p}_{VH}(e)$, while the equality

$$S_{EH}(e,p) = 0$$

defines the cutoff productivity level for searching only through headhunters, $\tilde{p}_H(e)$.

The value functions of working and unemployment are defined as before.

Firms

We can also rewrite the values of posting a vacancy and hiring through headhunter channel given distributions defined above. The value function of firms posting a vacancy in this case is:

$$\begin{aligned} V_V(p) = \; & -c_{fV}\cdot p + \beta\Bigg(V(p) + q_V(u_V,a_V,v)\Bigg(\tfrac{u_V}{u_V+a_V}\int_{\underline{e}}^{\hat{e}}\left(J(p,e)-V(p)\right)dU(e) + \\ & + \tfrac{u_V}{u_V+a_V}\left(1-f_H(u_H,a_H,h)\right)\int_{\hat{e}}^{\overline{e}}\left(J(p,e)-V(p)\right)dU(e) + \\ & + \tfrac{a_V}{u_V+a_V}\int_{\underline{e}}^{\hat{e}}\tfrac{\Lambda_V(e,p)}{\Lambda_V(e,\overline{p})}\left(J(p,e)-V(p)\right)dL_V(e) + \\ & + \tfrac{a_V}{u_V+a_V}\left(1-f_H(u_H,a_H,h)\right)\int_{\hat{e}}^{\overline{e}}\tfrac{\Lambda_{VH}(e,p)}{\Lambda_{VH}(e,\overline{p})}\left(J(p,e)-V(p)\right)dL_{VH}(e)\Bigg)\Bigg), \end{aligned}$$

where the first part in the summation is the expected value of a match after meeting an unemployed low-skilled worker, the second - an unemployed high-skilled worker, the third - an employed low-skilled worker, and the forth - an employed high-skilled worker.

Similarly, the value function of firms using headhunters is:

$$\begin{aligned} V_H(p) = \; & -c_{fH}\cdot p + \beta\Bigg(V(p) + q_H(u_H,a_H,h)\Bigg(\tfrac{u_H}{u_H+a_H}\int_{\hat{e}}^{\overline{e}}\left(J(p,e)-V(p)\right)dU(e) + \\ & + \tfrac{a_H}{u_H+a_H}\int_{\hat{e}}^{\overline{e}}\tfrac{\Lambda_H(e,p)}{\Lambda_H(e,\overline{p})}\left(J(p,e)-V(p)\right)dL_H(e) \\ & + \tfrac{a_H}{u_H+a_H}\int_{\hat{e}}^{\overline{e}}\tfrac{\Lambda_{VH}(e,p)}{\Lambda_{VH}(e,\overline{p})}\left(J(p,e)-V(p)\right)dL_{VH}(e)\Bigg)\Bigg), \end{aligned}$$

where again the first part in the summation is the expected value of a match after meeting an unemployed worker, the second - an employed worker searching only through the headhunter channel, and the third - an employed worker searching through both channels.

One can show that under reasonable conditions on the value of the search costs and production function, the value of hiring through a headhunter, $V_H(p)$, is lower than the value of posting a vacancy, $V_V(p)$, for small p. But, $V_H(p)$ is increasing faster with p. There will be only one intercept between $V_H(p)$ and $V_V(p)$, $\hat{p}$, such that

$$\max\{V_V(p); V_H(p)\} = V_V(p)$$

for $p < \hat{p}$ and

$$\max\{V_V(p); V_H(p)\} = V_H(p)$$

for $p > \hat{p}$. And the cutoff productivity is determined by

$$V_V(\hat{p}) = V_H(\hat{p}).$$

Now, given the distributions, we can also specify the quit rate of a worker with skill e from a firm with productivity p:

$$s_Q(e,p,\omega) = \begin{cases} f_V(u_V,a_V,v)\left(\frac{G(\hat{p})-G(p)}{G(\hat{p})-G(\underline{p})}\right) & if\ p < \tilde{p}_V(e)\ and\ e < \underline{e} \\ f_H(u_H,a_H,h)\left(\frac{G(\overline{p})-G(p)}{G(\overline{p})-G(\hat{p})}\right) & if\ \tilde{p}_{VH}(e) < p < \tilde{p}_H(e)\ and\ e \geq \underline{e} \\ f_H(u_H,a_H,h)\left(\frac{G(\overline{p})-G(p)}{G(\overline{p})-G(\hat{p})}\right) + & if\ p < \tilde{p}_{VH}(e)\ and\ e \geq \underline{e} \\ +(1-f_H(u_H,a_H,h)) \cdot & \\ \cdot f_V(u_V,a_V,v)\left(\frac{G(\hat{p})-G(p)}{G(\hat{p})-G(\underline{p})}\right) & \\ 0 & otherwise, \end{cases}$$

where $\omega = (u_V, a_V, v, u_H, a_H, h)$ is a vector of labor market characteristics.

Finally, the value of an active match for a firm is defined as before, substituting the value functions defined above and the function for quit rates.

Aggregation

The aggregates that enter the matching functions are determined as follows. The number of unemployed workers searching through the vacancy channel:

$$u_V = \int_{\underline{e}}^{\overline{e}} 1 dU(e).$$

The number of unemployed workers searching through the headhunter channel:

$$u_H = \int_{\hat{e}}^{\overline{e}} 1dU(e).$$

The number of employed workers searching through the vacancy channel:

$$a_V = \int_{\underline{e}}^{\hat{e}} \int_{\underline{p}}^{\overline{p}} 1d\Lambda_V(e,p) + \int_{\hat{e}}^{\overline{e}} \int_{\underline{p}}^{\overline{p}} 1d\Lambda_{VH}(e,p).$$

The number of employed workers searching through the headhunter channel:

$$a_H = \int_{\hat{e}}^{\overline{e}} \int_{\underline{p}}^{\overline{p}} 1d\Lambda_{VH}(e,p) + \int_{\hat{e}}^{\overline{e}} \int_{\underline{p}}^{\overline{p}} 1d\Lambda_H(e,p).$$

The number of firms using the vacancy channel:

$$v = \int_{\underline{p}}^{\hat{p}} 1dG(p).$$

And the number of firms using the headhunter channel:

$$h = \int_{\hat{p}}^{\overline{p}} 1dG(p).$$

2.3.8 Equilibrium

The steady state equilibrium, given the initial distributions of workers over skills and firms over productivity, the skill threshold, the matching functions, and the production function, is defined by the value functions, the endogenous distributions, and the decision rules. The decision rules, together with the endogenous distributions, must satisfy the balances such that the endogenous distributions are stationary.

The balances guarantee that the equilibrium distribution is stationary over time. In the equilibrium the inflow of workers to every worker-firm distribution bin must be equal to the outflow of workers from that bin. For a pair of workers with skill e and firms with productivity p, the equilibrium density of active matches, $\phi(e,p)$, must satisfy:

$$\phi(e,p)\left(s + s_Q(e,p)(1-s)\right) = i_E(e,p) + i_U(e,p),$$

where the left-hand side is the total outflow from active matches (exogenous separations plus endogenous quits), and the right-hand side is the total inflow into the matches from employment, $i_E(e,p)$, and unemployment, $i_U(e,p)$, respectively. The inflow from unemployment can be written as:

$$i_U(e,p) = \begin{cases} f_V(u_V, a_V, v)\,\frac{g(p)}{v} u(e) & \text{if } e < \hat{e},\ p < \hat{p} \\ f_H(u_H, a_H, h)\,\frac{g(p)}{h} u(e) & \text{if } e \geq \hat{e},\ p \geq \hat{p} \\ (1 - f_H(u_H, a_H, h))\, f_V(u_V, a_V, v)\,\frac{g(p)}{v} u(e) & \text{if } e \geq \hat{e},\ p < \hat{p} \\ 0 & \text{otherwise,} \end{cases}$$

where the first condition is satisfied when a worker searches only through vacancies and firm posts a vacancy; the second condition is satisfied when a worker searches through headhunters or both and a firm hires through a headhunter; and the third condition is satisfied when a worker searches through both channels and a firm posts a vacancy. Similarly, the inflow from employment can be written as:

$$i_E(e,p) = \begin{cases} f_V(u_V, a_V, v)\,\frac{g(p)}{v}\int_{\underline{p}}^{\min\{p,\tilde{p}_V(e)\}} \phi(e,p')\,dp' & \text{if } e < \hat{e},\ p < \hat{p} \\ f_H(u_H, a_H, h)\,\frac{g(p)}{h}\int_{\underline{p}}^{\min\{p,\tilde{p}_H(e)\}} \phi(e,p')\,dp' & \text{if } e \geq \hat{e},\ p \geq \hat{p} \\ (1 - f_H(u_H, a_H, h))\, f_V(u_V, a_V, v)\,\frac{g(p)}{v}\int_{\underline{p}}^{\min\{p,\tilde{p}_{VH}(e)\}} \phi(e,p')\,dp' & \text{if } e \geq \hat{e},\ p < \hat{p} \\ 0 & \text{otherwise.} \end{cases}$$

2.3.9 Solution method

The steady state equilibrium of the model is solved for using Matlab. To find an equilibrium the following algorithm is used:

1. Guess the search decision rules for workers and firms (the cutoff levels of productivity)
2. State the system of equations corresponding to the balances of endogenous distribution of matched firms and workers
3. Solve the system using non-linear solution methods (trust-region algorithm or Broydn algorithm) for the invariant distribution
4. Compute the value functions given the distributions
5. Compute the decision rules given the value functions
6. Compare the decision rules, if different - update the decision rules and go to 2.

2.3.10 Extensions

The most important extension of the model is introduced to capture the fact that not all firms hire employees for top positions through headhunters. In the baseline model every firm above the threshold $\hat{p}$ hires through the headhunters. To avoid it, I introduce an additional idiosyncratic non-monetary cost for hiring through a headhunter. Every firm with an open position draws a cost c_{fN} every period that is associated with the headhunter channel. This cost might reflect corporate practice, an existence of a preferred candidate inside the firm, or specificity of the position. Firms with high cost will have to post a vacancy even if they would benefit from a match through a headhunter otherwise. Updated value of an open position, $\tilde{V}(p)$, can be written as:

$$\tilde{V}(p) = \max\left\{V_V(p); V_H(p) - c_{fN}\right\}.$$

This extension doesn't alter the model significantly, only making the value functions and the balances more cumbersome[13], but brings the model closer to the data. Because the proportion of firms using headhunters by productivity is unobservable, in the baseline calibration the distribution of the costs will be chosen such that the firm has the same chance to hire through the headhunters regardless of the productivity.

Other important extensions include different wage setting mechanisms (wage bargaining and wage Bertrand competition between firms), and explicit modeling of headhunter's problem (choice of the threshold and the cost for the firm). These extensions are secondary to the main experiments in the paper and therefore will be discussed in the appendix.

2.4 Inequality

2.4.1 Calibration

The calibration strategy is the following. First, the version of the model without the headhunter channel is calibrated to match the labor market in the U.S. in 1970. Then the parameters related to the headhunter channel are calibrated to match the properties of the headhunter industry in the U.S. in the 2010s. To take into account the skill-biased technological change from 1970 to 2010 I also change the degree of complementarity in the production function to match the increase in 90/50 wage ratio.

To calibrate the model we need to specify the exogenous distributions of workers, $H(e)$, and firms, $F(p)$, the productivity function, $y(e,p)$, the matching function, $m(u,v)$, the skill threshold, $\hat{e}$, the search costs, c_{fH}, c_{fV}, c_{wH}, c_{wV}, and the distribution of the non-monetary cost c_{fN}. The functional form for the initial distributions of firms and workers is chosen to be beta with the same parameters, λ_1 and λ_2, for workers and firms, and truncated on $\overline{p} = \overline{e}$. The matching function has standard Cobb-Douglas form:

$$m(u,v) = Mu^{\sigma}v^{1-\sigma},$$

and the production function has a form:

$$y(e,p) = (e \cdot p)^{\gamma},$$

with normalization $\gamma = 1$ in 1970. It is easy to see that this production function is supermodular with $\frac{\partial^2 y(e,p)}{\partial e \partial p} = \frac{\partial^2 y(e,p)}{\partial p \partial e} = 1 > 0$.

All parameters, except for the search costs in the headhunter channel and the skill threshold are calibrated to match the wage distribution and other labor market variables in 1970. There are seven parameters to calibrate for the steady state without the

[13] Equations for the value functions and the balances are presented in the appendix.

headhunters: the parameters of the distribution, λ_1 and λ_2, the maximum type, $\overline{p}$ or $\overline{e}$, the separation rate, s, the matching function efficiency, M, and the search cost through vacancy channel for workers, c_{wV}, and for firms, c_{fV}. To set these seven parameters, seven targets are chosen: the top 1% and 10% wage shares, the 90/50 wage ratio, the unemployment rate, the job finding rate, the quit rate, and the estimate of vacancy cost relative to annual worker's wage. Seven parameters are jointly calibrated to match the targets.

For the headhunter channel, there are just four parameters to calibrate - the search costs for workers, c_{wH}, and firms, c_{fH}, the skill threshold, $\hat{e}$, and the share of firms using headhunters, χ[14]. Four targets are chosen - the estimate of the positive response rate by managers to a call by a headhunter[15], the average fee of headhunters, the range of positions filled by headhunters, and the share of firms that use the headhunters for hiring. This way, the targets do not directly affect the wage distribution, so the calibration strategy does not drive the results. Robustness checks for the choice of parameters for the headhunter channel will be presented in the subsequent sections. On top of the headhunter channel I also increase the degree of complementarity, γ, in order to match the change in the 90/50 wage ratio between 1970 and 2010.

The results of the calibration are presented in Table 2.3. The model matches well the main characteristics of the wage distribution and the labor market in 1970.

2.4.2 Results

The headhunter channel allows separating high-skilled workers reducing frictions for them and providing them with an exclusive opportunity to work in high-productive firms. The presence of this channel changes the distribution of the workers over the wages. Without the headhunter channel, the distribution has a peak close to minimal possible wage and then decreases, having a form close to Pareto (Figure 2.3a). When the headhunter channel is present in the model, the distribution still has similar form but has a fatter right tail (Figure 2.3b), similar to what is observed in the data. The headhunter channel generates the fat tail of the wage distribution in this model. The reason for this is the following, without headhunter channel the probability of matching a high-skilled worker with a high-productive firm is lower than matching a high-skilled worker with a low-productive firm (due to the fact that there are relatively few high-productive firms), so there will be big shares of high-skilled workers working in low-productive firms and low-skilled workers in high-productive firms. Because of skill complementarities, wages of low-skilled workers are lower than wages of high-skilled workers in the same type of firm. And because only some high-

[14] It is equivalent to calibrating a distribution function for the non-monetary cost c_{fN}.

[15] Estimated by Capelli & Hamori (2013)

Table 2.3: Calibrated parameters.

Parameter	Value	Target	Data	Model
Wage distribution, 1970				
Beta parameter, λ_1	1.5	Top 1% wage share	5.1%	4.38%
Beta parameter, λ_2	15	Top 10% wage share	25.7%	25.56%
Maximum types, $\overline{p}$, $\overline{e}$	14	90/50 wage ratio	1.91	2.07
Labor market, 1970				
Separation rate, s	0.026	Unemployment rate	5%	4.9%
Matching function, M	0.9	Job finding rate	50%	49.53%
Search cost - vacancies, c_{wV}	1.05	Quit rate	2%	1.87%
Vacancy cost, c_{fV}	0.17	Vacancy cost estimates	8%	8%
Headhunter industry, 2010				
Headhunter search cost, c_{wH}	1.2	Positive response rate	50%	50.52%
Headhunter firm cost, c_{fH}	2.55	Headhunter average fee	30%	30%
Screening threshold, $\hat{e}$	2.98	Range of positions	top 5%	5.05%
Share of firms using headhunters, χ	0.54	Share of firms	∼54%	54%
Skill-biased technological change, 1970-2010				
Degree of complementarity, γ	1.10	Δ 90/50 wage ratio	0.39	0.39

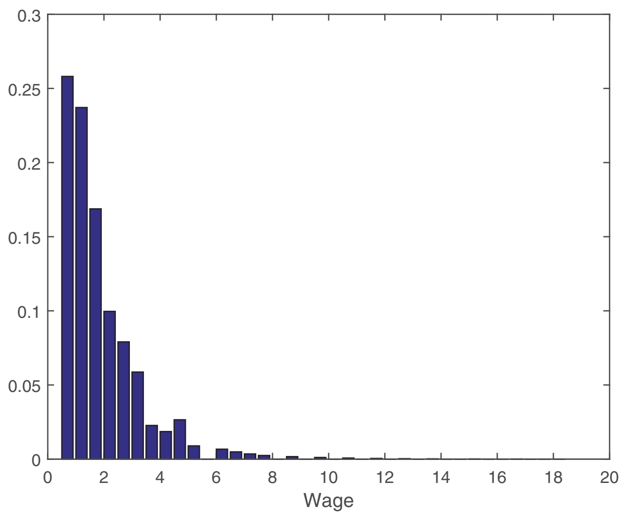
This table presents the result of the baseline calibration.

productive firms will be matched with high-skilled workers there will be a small mass of workers getting very high wages. When, instead, there is a possibility to hire only high-skilled workers through the headhunter channel, high-productive firms will be matched only with high-skilled workers and all of them will receive relatively high wages; this corresponds to the fat tail of the distribution. The difference between the distributions (Figure 2.3c) clear indicates the appearance of fatter right tail with the headhunter channel. However, there are two effects changing the wage distribution in this case - headhunters and the skill-biased technological change. Skill-biased technological change increases wages of all workers and therefore moves the whole distribution to the right.

To see the effect of only the headhunter channel on the wage distribution, Figure 2.4 plots the difference between the distributions without the effects of the skill-biased technological change. An interesting observation about the effect of headhunters on wage distribution can be done - the headhunter channel generates an effect similar to job polarization , namely, decrease in the number of medium-paying jobs and increase in the number of high- and low-paying jobs. This effect also comes from the fact that low-skilled workers move from high-productive to low-productive firms (from the center to the left), and high-skilled workers move from low-productive firms to the

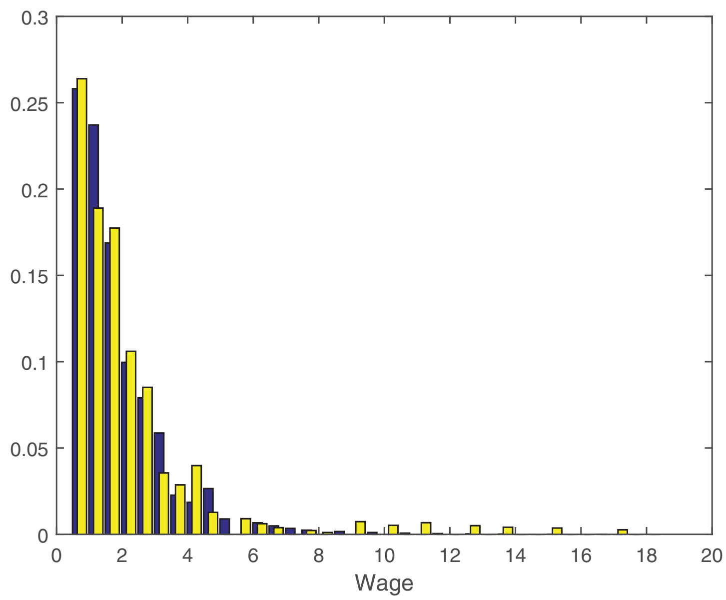
Figure 2.3: Distributions of wages.

(a) Distribution of wages without the headhunter channel.



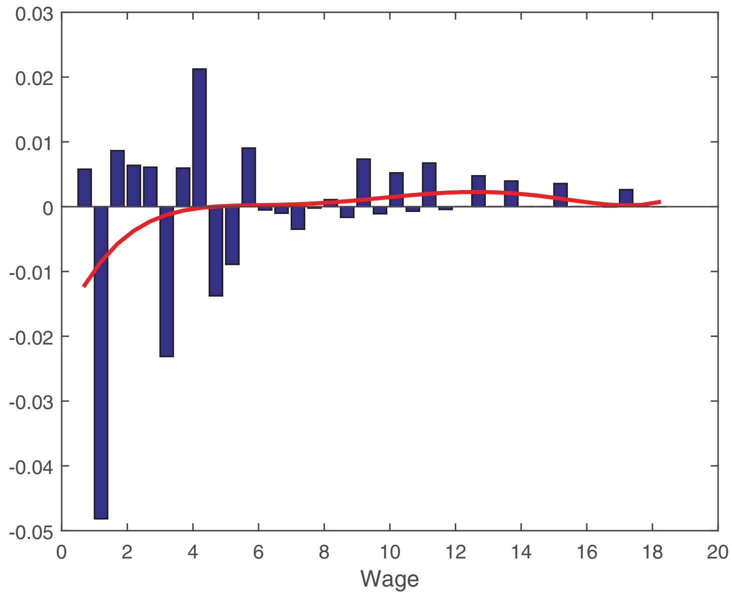
This figure plots the histogram of the distribution of wages in the model without the headhunter channel, baseline calibration.

(b) Distribution of wages with the headhunter channel.



This figure plots the histogram of the distribution of wages in the model without the headhunter channel (blue bars) and the histogram of the distribution of wages in the model with the headhunter channel (yellow bars), baseline calibration.

(c) Difference between the distributions.



This figure plots the difference between the distributions of wages in the model with headhunters and without headhunters (blue bars). The red line plots a fitted polynomial of degree 5. Positive values indicate a larger mass of workers with a given wage in the model with the headhunter channel.

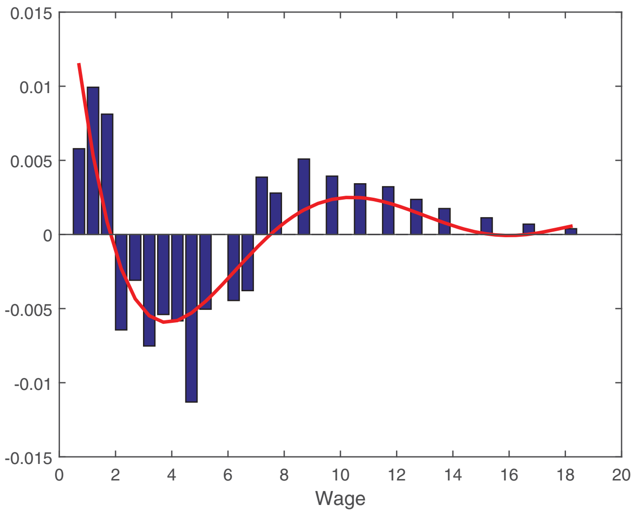
high-productive firms (from the center to the right).

As it was stated before, the increase in wage inequality was mainly driven by the sharp increase of top wages. Figure 2.1 shows the top 10% and the top 1% wage shares in the U.S. from 1970 to 2010. The top 1% share increased from 5.1% in 1970 to 10.9% in 2010, and the top 10% share increased from 25.7% to 34.5%. The shares in 1970 were targeted in the calibration, but the shares in 2010 were not. The results of this experiment show how much of the overall increase in top wages can be explained by the additional channel in the labor market and an increase in the degree of complementarity in production. The results are presented in Table 2.4. In the model, the top 1% share increases by 3.41%, from 4.38% to 7.79%, while in the data it increases by 5.8%. The model is able to explain 59% of the actual increase in the top 1% wage share. For the top 10% wage share, the model predicts 8.58% increase, while the actual increase is 8.8%. The model accounts for 98% of the actual increase in the top 10% wage share.

2.4.3 Skill-Biased Technological Change

The large effect in Table 2.4 comes from skill-biased technological change and the headhunter channel acting together. To assess the relative contributions of the headhunter channel and the skill-biased technological change to the increase of the top wages I

Figure 2.4: Difference between the distributions without SBTC.



This figure plots the difference between the distributions of wages in the model with headhunters and without headhunters (blue bars). The red line plots a fitted polynomial of degree 5. Positive values indicate a larger mass of workers with a given wage in the model with the headhunter channel.

Table 2.4: Top wage shares in the model and data.

Model	Top 1%	Top 10%	Data	Top 1%	Top 10 %
Without HH	4.38%	25.56%	1970	5.1%	25.7%
With HH	7.79%	34.14%	2010	10.9%	34.5%

This table presents the top wage shares in the modal and in the data. Columns 2 and 3 present the results from the model for the baseline calibration without the headhutner channel and the baseline calibration with the headhutner channel. Top 1% and 10% wage shares and 90/50 wage ratio in the U.S. in 1970 and 2010 are presented in columns 5 and 6 (source: Piketty (2014)).

change separately only the matching technology (headhunter channel) or the degree of complementarity (SBTC). I present the results in the Table 2.5. First, I fix the degree of complementarity on the level of 1970 but add the headhunter channel (bottom-left panel). In this case the top 1% wage share is 6.65% ,instead of 7.79% in the baseline calibration (upper-left), and the top 10% wage share is 31.05% (instead of 34.14%). The headhunter channel contributes to explaining of 40% of the increase of top 1% wage share in the data and 70% of the increase in top 10% wage share. The headhunter channel also contributes to explain half of the rise of the upper-middle class, the 90/50 wage ratio rises by 0.20, while in the baseline calibration the rise is 0.39.

If instead, I just increase the degree of complementarity to the level of the baseline calibration without the headhunter channel (upper-right), the top 1% wage share

Table 2.5: Relative contribution of headhunters and SBTC.

		HH	no HH
SBTC	Top 1%	7.79%	4.98%
	Top 10%	34.14%	27.61%
	Δ 90/50	0.39	0.16
no SBTC	Top 1%	6.65%	4.38%
	Top 10%	31.05%	25.56%
	Δ 90/50	0.20	0

This table presents the top wage shares and the change in 90/50 wage ratio for the baseline calibration with headhunters (upper-left), the calibration with SBTC but without headhunters (upper-right), the calibration without SBTC but with headhunters (lower-left), and the calibration without headhunters and no SBTC (lower-right).

increases just to 4.98% and the top 10% wage share increases to 27.61%. The relative contribution of the degree of complementarity is about 10% out of 59% for top 1% wage share, and 23% out of 98% for the top 10% wage share. SBTC also explains almost another half of the increase of the upper-middle class.

Interaction between SBTC and the headhunter channel is also very important. The interaction explains around 9% of the increase in top 1% wage share (59%-40%-10%) and 5% of the increase in top 10% wage share (98%-70%-23%). With higher degree of complementarity, the importance of having a better match increases. Relative productivity of a firm with a high-skilled worker is even higher with respect to a similar firm with a low-skilled worker in case of high degree of complementarity. Better assortative matching on the top reinforces the effects of SBTC.

To give a chance to the SBTC to explain a higher proportion of the rise in top shares I recalibrate the SBTC to match the increase in the top 10% wage share or 90/50 wage ratio without the headhunter channel. I present the results in Table 2.6. We can see that the degree of complementarity must increase up to 1.24 without the headhunter channel to match the increase in 90/50 wage ratio, and up to 1.41 to match the top 10% wage share. When I match the 90/50 wage ratio, the model explains relatively small proportion of the increase in top 1% wage share (26%) and a significant proportion of the top 10% wage share (57%). If the increase in top 10% wage share is comparable to the baseline (or the headhunter channel alone), only SBTC loses a lot for the top 1% wage share. If I match the top 10% wage share, instead, the model explains a larger part of the top 10% wage share (all of it was targeted, while the headhunter channel alone explains 70%) and only slightly smaller increase in the top 1%. However, in this case the model predicts a very large increase in the 90/50 wage ratio (0.72) that is 82% higher than the actual increase. The reason for this is that the rise of the degree of complementarity alone rises all the wages uniformly and the rise must be enormous to match the top 10% wage share. We can see it in Figure 2.5. While with the headhunter channel all the wages rise only slightly due to higher degree of complementarity and the top wages rise more than that due to improvements in the assortative matching. With the headhunter channel, the high-skilled workers both move up with the curve and move along it to the right, and without the headhunter channel they can only move up with the curve.

Table 2.6: Alternative calibration of SBTC.

	Data	Baseline	no HH, SBTC - 90/50	no HH, SBTC top 10
	2010	$\gamma = 1.10$	$\gamma = 1.24$	$\gamma = 1.41$
Top 1%	10.9%	7.79%	5.93%	7.26%
Top 10%	34.5%	34.14%	30.61%	34.43%
Δ 90/50	0.39	0.39	0.40	0.72

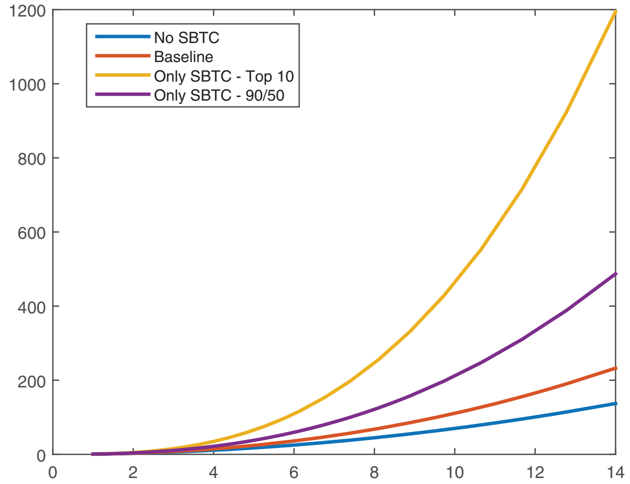
This table presents the top wage shares and the change in 90/50 wage ratio for the U.S. (column 2, source: Piketty (2014)) baseline calibration with headhunters (column 3), the calibration without headhunters and SBTC calibrated to match the change of 90/50 wage ratio (column 4), and the calibration without headhunters and SBTC calibrated to match the top 10

These experiments show that the skill-biased technological change helps to explain the rise in the 90/50 wage ratio that corresponds to the rise of the upper-middle class relative to the bottom, but fails to replicate the sharp increase on the very top. The headhunter channel, instead, has the main effect on the top wages, rather than on the upper-middle class. Skill-biased technological change stretches the whole distribution to the right, while the headhunter channel fixes the left part of the distribution and moves the right tail further apart.

2.4.4 Assortative matching

The main mechanism behind the increase in wage inequality in the model is the increase in sorting between workers and firms, especially at the very top. With headhunters, high-skilled workers have exclusive opportunity to be matched with a high-productive firm, and high-productive firms, instead, have exclusive opportunity to be matched with high-skilled workers. Empirically, there are two most used ways to look at the assortative matching between workers and firms. First, one can directly compare the joint distributions of worker-firm matches over estimated types. And second, one can just look at the correlation between the types. I compute both statistics using the data simulated from the model in the baseline calibration in order to compare them to empirical estimates in the literature. The major drawback of this experiment, however, is that I can observe the real type of workers and firms directly, while in the data it is impossible.

Figure 2.5: Wage of a worker in a best-fit firm.



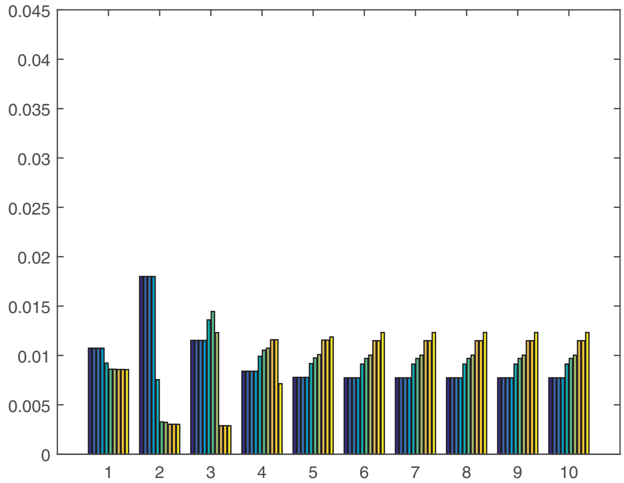
This figure plots for each worker type (horizontal axis) her wage in the best-fit firm (vertical axis). Blue line represents calibration without SBTC, red line represents the baseline calibration, yellow line represents the calibration without the headhunter channel and the degree of complementarity changed in order to match the top 10% wage share in 2010, and purple line represents the calibration without the headhunter channel and the degree of complementarity changed in order to match the change in 90/50 wage ratio between from 1970 to 2010.

First, I study the change in the joint distribution of worker-firm matches. To do it I split workers and firms into ten categories by their skill or productivity level and plot the joint distribution before and after introducing headhunters in the model. Figure 2.6a shows the distribution without headhunters, Figure 2.6b show the distribution with headhunters, and Figure 2.6c shows the change of the distribution. Numbers 1,2,3,..,10 in the figure correspond to the firm type, with 1 being the least productive firms and 10 being the most productive firms, and the color corresponding to the type of workers, with dark blue being the least skilled and yellow being the most skilled workers.

As it can be seen from the figures, with the baseline calibration of the model almost all high-skilled workers (within the top 10%) start working in the best firms (top 10%). All other firms lose significantly in the share of top workers and gain in the share of lower-skilled workers. This pattern is strikingly similar to the findings of Song et al. (2016) who plot similar distributions for estimated workers and firms fixed effects from the U.S. data. Comparing to the data, the model with the baseline calibration exaggerates the increase in the number of top workers working in top firms because most of the firms on top use the headhunter channel and the headhunters don't do mistakes

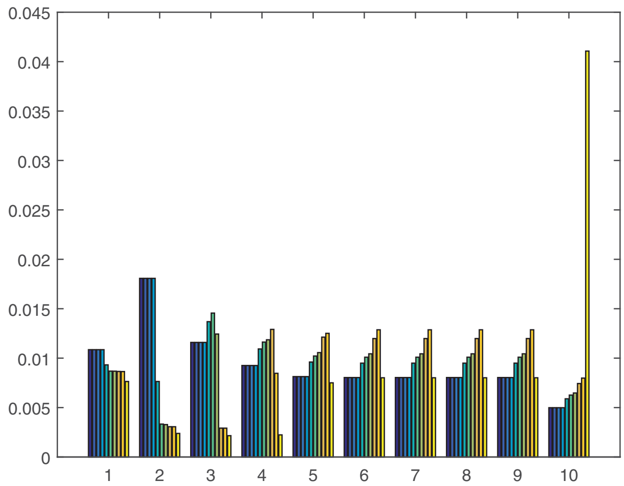
Figure 2.6: Joint distributions of worker-firm matches.

(a) Joint distribution without the headhunter channel.



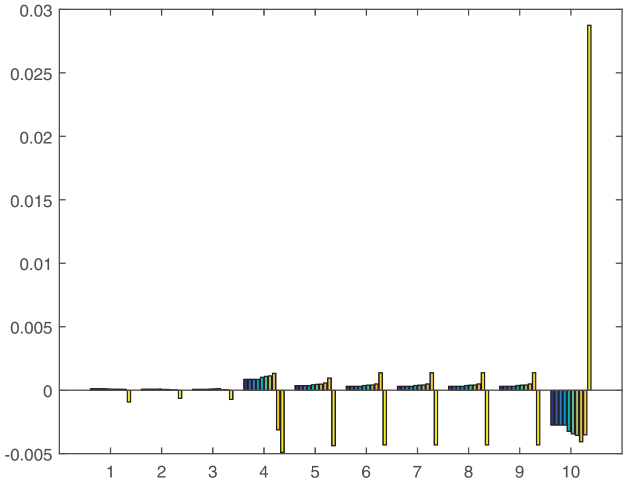
This figure plots the joint distribution of the matches of workers and firms over deciles of workers' and forms' types in the baseline calibration without the headhunter channel. Numbers 1-10 represent the firm type deciles with 1 being the least productive and 10 the most productive firms. Colors represent the worker type deciles with dark blue being the least skilled workers and bright yellow the most skilled workers.

(b) Joint distribution with the headhunter channel.



This figure plots the joint distribution of the matches of workers and firms over deciles of workers' and forms' types in the baseline calibration with the headhunter channel. Numbers 1-10 represent the firm type deciles with 1 being the least productive and 10 the most productive firms. Colors represent the worker type deciles with dark blue being the least skilled workers and bright yellow the most skilled workers.

(c) Change in the joint distribution.



This figure plots the difference between the joint distributions of the matches of workers and firms over deciles of workers' and forms' types in the baseline calibration with and without the headhunter channel. Positive values indicate larger number of matches in the calibration with the headhunter channel. Numbers 1-10 represent the firm type deciles with 1 being the least productive and 10 the most productive firms. Colors represent the worker type deciles with dark blue being the least skilled workers and bright yellow the most skilled workers.

screening workers. In the empirical counterpart, there is an error in estimating the true type of the worker that should smooth the figure, and headhunters might do mistakes by assessing the skill of workers with mistakes, therefore smoothing even more the resulting difference.

The second way to analyze sorting in the labor market is to compute correlations between the types of the workers and firms. In order to do it in the model simulated data, I draw 100000 matches from the joint worker-firm distribution in the model and decompose the variance of log wages into worker type, firm type, and covariance between the two. Table 2.7 presents the results of this experiment for the steady-state without headhunters, with headhunters, and the difference between the two.

Table 2.7: Log wage variance decomposition and correlation of worker and firm types.

	Without HH	With HH	Dif	%
$Var(\log(w))$	0.3020	0.4262	0.1242	100
$Var(\log(e))$	0.1289	0.1549	0.0260	21%
$Var(\log(p))$	0.1302	0.1567	0.0265	21%
$2Cov(\log(e), \log(p))$	0.0428	0.1147	0.0719	58%
$Cor(\log(e), \log(p))$	0.1656	0.3679	0.2023	-

This table presents the results of the wage variance decomposition based on the 100000 draws from the model-simulated data with and without the headhunter channel.

From the results of this experiment ,we can see that covariance and correlation of the worker and firm types increase significantly after the introduction of headhunters to the model. Indeed, the increase is not only in the top part of the distribution but over the whole distribution. This happens because the high-skilled workers that move to the best firms free the positions for low- and medium-skilled workers in the rest of the firms, also improving the matching for them. Again, this result is in line with the empirical results by Song et al. (2016) who show that increased covariance between worker and firm types was one of the major drivers in the increase in the variance of wages in the U.S. between the 1980s and 2000s.

Improvement in assortative matching naturally translates in higher aggregate production in the economy (Table 2.8). Better matches at the top significantly improve average productivity in the economy. In the baseline calibration this improvement is as much as 22% together with the increase because of higher degree of complementarity. To assess the contribution of the matching alone, we can look at the distance of the allocation to the first best. In the calibration without the headhunter channel, the aggregate production is 88% of the first best allocation of workers to firms, while in the calibration with headhunters it is already 91.5%. The improvement in the matching brings the economy by 3.5% closer to the first best allocation, that is a 5-7% increase in production.

Table 2.8: Aggregate production.

	no HH	of FB	HH	of FB	Diff	%	no SBTC	FB
no SBTC	2.82	88.13%	2.97	92.81%	0.15	5.3%	-	3.20
SBTC	3.21	85.6%	3.43	91.47%	0.22	6.85%	0.61/21.63%	3.75

This table presents the aggregate production for the calibration of the model without and with the headhunter channel.

2.4.5 Robustness

To check how much the magnitude of the increase depends on the choice of the skill threshold for the headhunter channel, I do similar experiments for top 1%, 2%, 3%, 7%, or 10% of workers being eligible to the headhunter channel. The results are presented in Table 2.9. Not surprisingly, higher thresholds, except for top 1% threshold, (more efficient screening by headhunters) increase the wage share of top 1% of the workers but decrease the share of top 10%. This happens because, with a higher threshold, the most efficient workers are more concentrated in the top firms, for example, they all work in top 2% of the firms instead of top 5%. Their wages increase even more due to complementarities, so the top 1% wage share increases more. Instead, for the workers in the 10-1% bracket, the probability of working in the best firms decreases with a higher threshold. Workers in 5-2% are excluded from the headhunter channel and many of them end up in bad or average firms, so the top 10% wage share drops relative to baseline calibration despite the top 1% wage share increase. The overall effect of the sorting mechanism is still striking - it explains at least 40% of the actual increase in the top 1% share of wages and 55%% of the top 10% wage share (together with SBTC). We can also see that with the threshold set to top 1% the increase in the top 1% wage share drops relative to the top 2% threshold. This happens because not all firms in top 1% use the headhunter channel and therefore only around a half of the workers in the top 1% are matched through the headhunter channel.

Another target of choice that doesn't have a properly estimated empirical counterpart is the share of firms using the headhunters. In the baseline calibration, it is chosen to fit the estimates by AESC, but I also redo the experiment with different shares to see how sensitive is the result depending on the choice of the target. I set the share of the firms using the headhunter channel to be 20%, 40%, 60%, 80%, or 100%. The results are presented in Table 2.10. Of course, the increase of the top wage shares is decreasing with a lower share of firms using headhunters. But the major part of the effect is still there even if every 5th firm is allowed to use the headhunter channel every period. In this case, the model is still able to explain 53% of the increase in top 1% wage share

Table 2.9: Top wage shares in the model and data for other skill thresholds.

Model	Share	Top 1%	Top 10%	Δ 90/50
Without HH	0	4.38%	25.56%	0
With HH on top 5% (basline)	5.05%	7.79%	34.14%	0.39
With HH on top 10%	10.62%	6.70%	35.81%	0.65
With HH on top 7%	7.47%	7.00%	35.08%	0.39
With HH on top 3%	3.27%	8.01%	32.99%	0.39
With HH on top 2%	2.00%	9.15%	31.51%	0.39
With HH on top 1%	1.16%	8.44%	30.42%	0.16

This table presents the results of the main experiment with alternative calibrations of the skill threshold. The results are presented for the skill thresholds of 5% (baseline calibration), 3%, 2%, and 1%.

and 80% of the increase in the top 10% wage share (again, together with SBTC). Even when the share of firms using headhunters is set to the most conservative estimate the model is still able to predict a large share of the increase in top wages.

Table 2.10: Top wage shares in the model and data for different intensity of the use of headhunter channel.

Model	Top 1%	Top 10%	Δ 90/50
Without HH	4.38%	25.56%	0
With HH, baseline	7.79%	34.14%	0.39
With HH, 100%	9.00%	36.22%	0.39
With HH, 80%	8.12%	34.77%	0.39
With HH, 60%	7.90%	34.28%	0.39
With HH, 40%	7.56%	33.80%	0.39
With HH, 20%	7.46%	32.65%	0.39

This table presents the results of the main experiment with alternative calibrations of the distributions of non-monetary costs of headhunters. The results are presented for the share of firms using headhunters of 54% (baseline), 100%, 80%, 60%, 40%, and 20%.

2.4.6 Discussion

As it was discussed before, the main effect of the headhunter channel on the wage distribution comes from the separation of the labor markets for high-paying and low-paying jobs. With the headhunter channel in place, only high-skilled workers get to high-paying jobs and their wages increase dramatically due to skill and firm productivity complementarities. Moreover, headhunter channel allows high-skilled workers to search on-the-job less costly because they don't have to pay the search cost every period but only when they receive a call from the headhunter. Because of this, high-skilled workers agree to consider an offer even when they work in medium- and high-productive firms, while they would stop searching for a job actively working in such firms without the headhunter channel. Low-skilled workers, in contrast, lose their possibility to work in high-paying jobs, therefore their wages are compressed to lower levels after the introduction of the headhunters.

Simulations presented in this section show that the change of the matching technology can explain a major part of the increase in the wage inequality. The results are quite robust to different calibrations of the headhunter channel and are amplified by skill-biased technological change in the model. Skill-biased technological change, instead, has its main effect on the upper-middle class, contributing significantly to the rise of the 90/50 wage ratio and having a smaller contribution to the rise of top wages. The main reason for this is that the headhunter channel creates the non-linearity in the matching pattern between workers and firms, as observed in the data. SBTC alone gradually improves sorting across all distribution without the sharp increase of top workers in top firms. Only this sharp increase due to strong non-linearity rises the top wages much higher than the middle and low wages.

2.5 Cross-country comparison

This section discusses the cross-country evidence supporting the main mechanism of the paper, that the headhunters by improving matching on the labor market increase the top wages. Headhunters entered labor market of different countries in different periods and therefore are used by firms to a different extent[16]. This variation in headhunters' activity helps to establish causality between the role of headhunters and the growth of top wages. This section uses the data on the major headhunter companies in European labor markets in 1997 and the data on top income shares between 1980 and 2010. Ideally, top wage shares should be used in the analysis, however, such data is not available for all countries over the whole period in question.

Data on major headhunter companies operating in Europe is available in Jenn (1999). The data includes the distribution of fee revenues as well as the number of hires by country[17]. I aggregate the data by country to get total fees and a total number of hires by headhunters in a country in 1997, and I normalize the data by GDP in 1997 (for fee revenues) or total employment in 1997 (for hires). Normalization allows comparing the share of headhunters between countries. The question that this analysis answers is

[16] Different labor market legislation is another source of exogenous differences of headhunter activity across countries.

[17] For some companies the number of hires is estimated and not exactly observed.

what is the relation between headhunter activity and the dynamics of top incomes? To answer this question figures 2.7 and 2.8 plot the relations between normalized hires by headhunters and top income shares, or growth of top income shares. Figures 2.9 and 2.10 plot similar relations for normalized fee revenues. Figures 2.7a and 2.8a show that there is a strong positive correlation between normalized hires by headhunters and the future growth of top incomes. Only Norway falls from the general pattern, but Norway experienced a change in capital income taxation in 2006, so most likely this drop is not related to labor incomes.

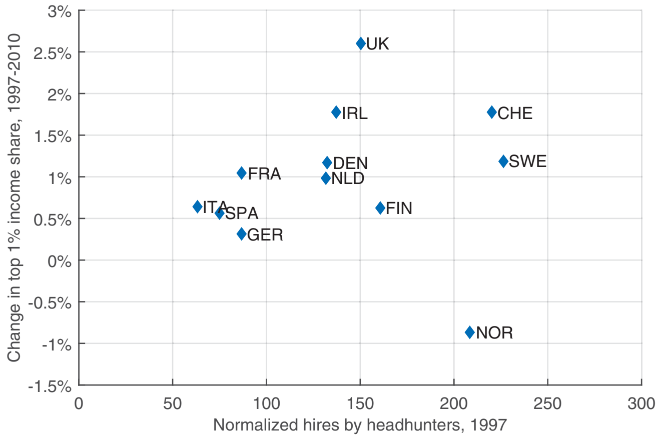
To address the concern that headhunters were more active in countries where top wages were already higher, figures 2.7b and 2.8b plot the relation between top income shares in 1997 and normalized hires by headhunters in 1997. As it is evident from the figures, there is no correlation between headhunter activity and top income shares in 1997. It means that differences of headhunter intensity across countries are driven by other factors, exogenous to top income levels. To further strengthen this claim, figures 2.7c and 2.8c plot the top income shares growth before 1997 against normalized hires in 1997. Lack of positive correlation shows that headhunters intensity is not driven by the previous growth of top incomes. Headhunters didn't choose countries with fast growing top incomes.

Figures 2.9 and 2.10 present similar analysis for normalized fee revenues. All the results for normalized hires hold also for normalized fee revenues. This analysis shows two important facts. First, it shows that headhunters indeed signal future growth of top incomes. In the model the increase of top incomes happens because of improved matching at the top, with headhunters inducing the better matching. This evidence, however, doesn't provide any hints on the mechanism of the top incomes increase, or the degree of quality of the matching. Second, this analysis shows that the distribution of headhunter activity over countries is exogenous to the level of top incomes and the history of the growth of top incomes. There must be other factors limiting headhunter activity in some countries, for example, labor market legislation, or high costs of establishing detailed databases of potential candidates.

The importance of this empirical evidence is in demonstrating the lack of reverse causality. Headhunters might be more active in countries where the income inequality was higher so they came to the market to extract higher fee revenues. In this case, the increasing top wages would drive the rise in the headhunter industry, and the mechanism presented in this paper would not be present. However, the results presented in this section show that only the future change in top incomes is correlated with the headhunter intensity, so reverse causality can be rejected.

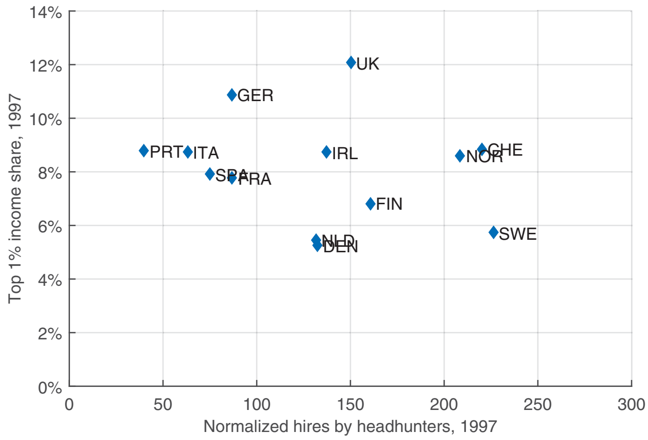
Figure 2.7: Top 1% income share and normalized hires by headhunters.

(a) Growth of top 1% income share, 1997-2010, and normalized hires by headhunters, 1997.



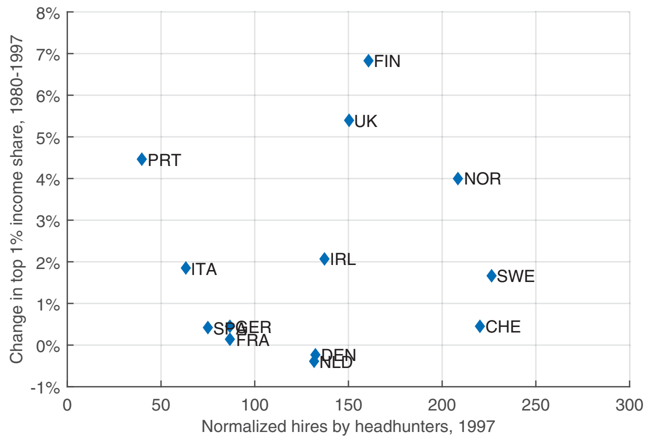
This figure plots the change in the top 1% income share in a country between 1997 and 2010 (vertical axis) against the hires by headhunters in 1997 normalized by total employment in 1997 (horizontal axis). Sources: http://www.wid.world/, Jenn (1999), https://data.oecd.org/, and author's calculations.

(b) Top 1% income share, 1997, and normalized hires by headhunters, 1997.



This figure plots the top 1% income share in a country in 1997 (vertical axis) against the hires by headhunters in 1997 normalized by total employment in 1997 (horizontal axis). Sources: http://www.wid.world/, Jenn (1999), https://data.oecd.org/, and author's calculations.

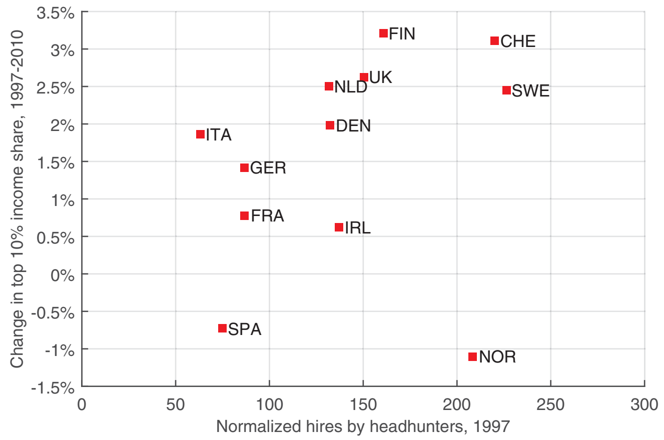
(c) Growth of top 1% income share, 1980-1997, and normalized hires by headhunters, 1997.



This figure plots the change in the top 1% income share in a country between 1980 and 1997 (vertical axis) against the hires by headhunters in 1997 normalized by total employment in 1997 (horizontal axis). Sources: http://www.wid.world/, Jenn (1999), https://data.oecd.org/, and author's calculations.

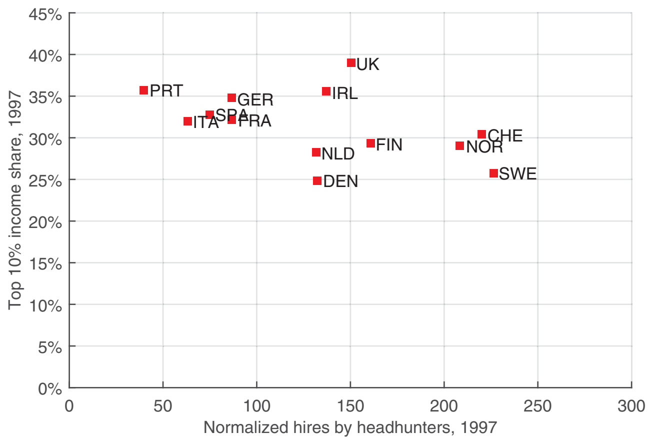
Figure 2.8: Top 10% income share and normalized hires by headhunters.

(a) Growth of top 10% income share, 1997-2010, and normalized hires by headhunters, 1997.



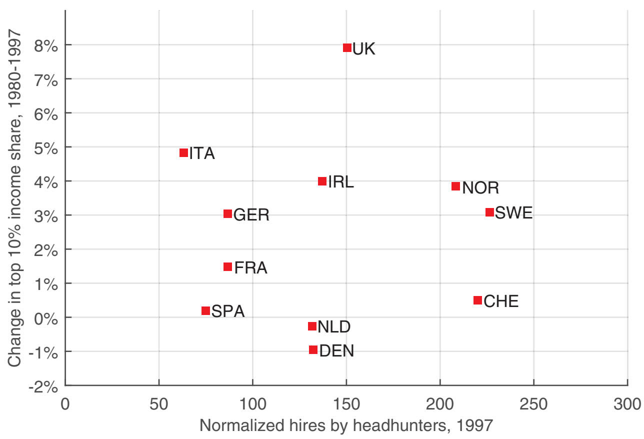
This figure plots the change in the top 10% income share in a country between 1997 and 2010 (vertical axis) against the hires by headhunters in 1997 normalized by total employment in 1997 (horizontal axis). Sources: http://www.wid.world/, Jenn (1999), https://data.oecd.org/, and author's calculations.

(b) Top 10% income share, 1997, and normalized hires by headhunters, 1997.



This figure plots the top 10% income share in a country in 1997 (vertical axis) against the hires by headhunters in 1997 normalized by total employment in 1997(horizontal axis). Sources: http://www.wid.world/, Jenn (1999), https://data.oecd.org/, and author's calculations.

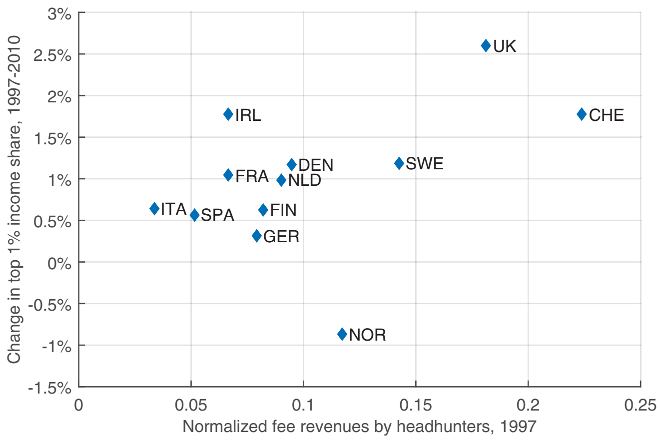
(c) Growth of top 10% income share, 1980-1997, and normalized hires by headhunters, 1997.



This figure plots the change in the top 10% income share in a country between 1980 and 1997 (vertical axis) against the hires by headhunters in 1997 normalized by total employment in 1997 (horizontal axis). Sources: http://www.wid.world/, Jenn (1999), https://data.oecd.org/, and author's calculations.

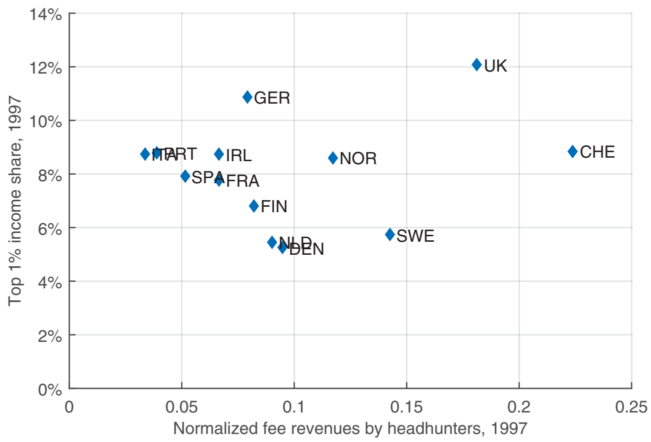
Figure 2.9: Top 1% income share and normalized fee revenues by headhunters.

(a) Growth of top 1% income share, 1997-2010, and normalized fee revenues by headhunters, 1997.



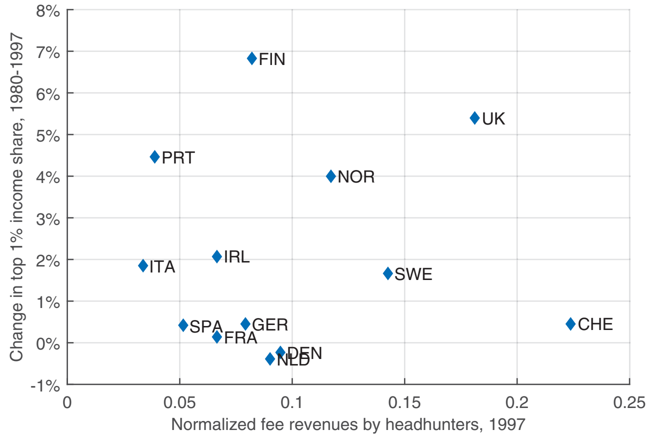
This figure plots the change in the top 1% income share in a country between 1997 and 2010 (vertical axis) against the fee revenues by headhunters in 1997 normalized by GDP in 1997 (horizontal axis). Sources: http://www.wid.world/, Jenn (1999), https://data.oecd.org/, and author's calculations.

(b) Top 1% income share, 1997, and normalized fee revenues by headhunters, 1997.



This figure plots the top 1% income share in a country in 1997 (vertical axis) against the fee revenues by headhunters in 1997 normalized by GDP in 1997 (horizontal axis). Sources: http://www.wid.world/, Jenn (1999), https://data.oecd.org/, and author's calculations.

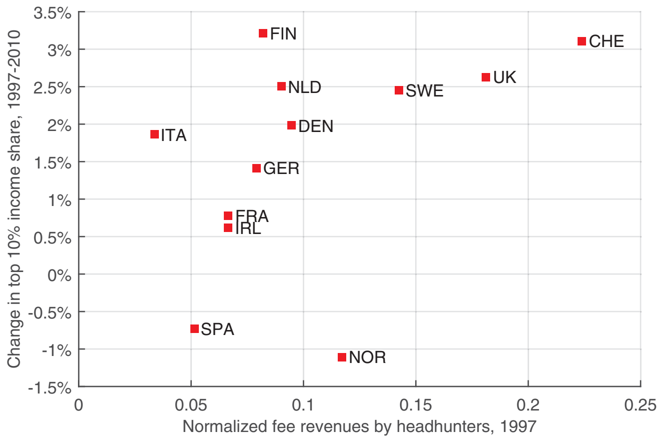
(c) Growth of top 1% income share, 1980-1997, and normalized fee revenues by headhunters, 1997.



This figure plots the change in the top 1% income share in a country between 1980 and 1997 (vertical axis) against the fee revenues by headhunters in 1997 normalized by GDP in 1997 (horizontal axis). Sources: http://www.wid.world/, Jenn (1999), https://data.oecd.org/, and author's calculations.

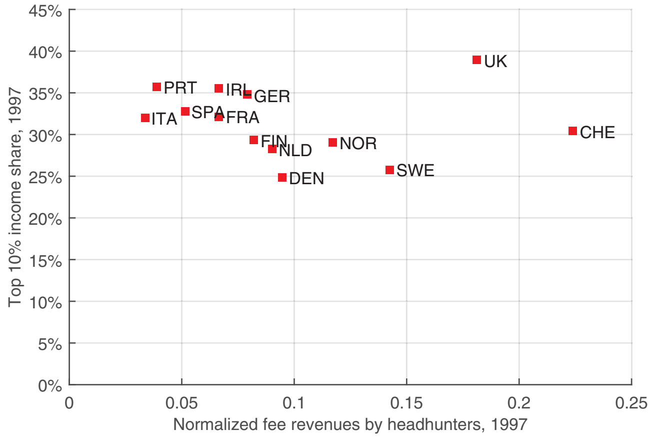
Figure 2.10: Top 10% income share and normalized fee revenues by headhunters.

(a) Growth of top 10% income share, 1997-2010, and normalized fee revenues by headhunters, 1997.



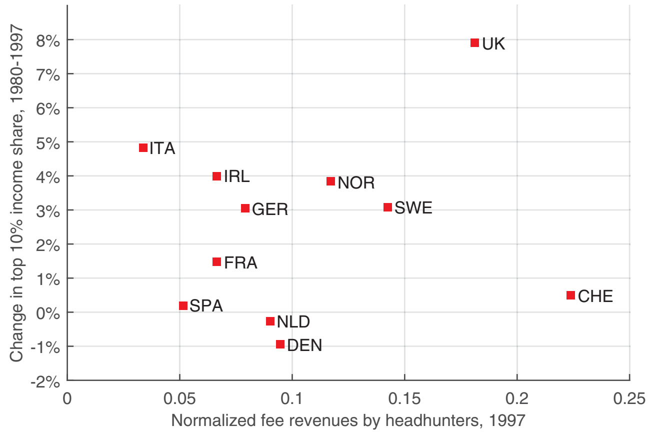
This figure plots the change in the top 10% income share in a country between 1997 and 2010 (vertical axis) against the fee revenues by headhunters in 1997 normalized by GDP in 1997 (horizontal axis). Sources: http://www.wid.world/, Jenn (1999), https://data.oecd.org/, and author's calculations.

(b) Top 10% income share, 1997, and normalized fee revenues by headhunters, 1997.



This figure plots the top 10% income share in a country in 1997 (vertical axis) against the fee revenues by headhunters in 1997 normalized by GDP in 1997 (horizontal axis). Sources: http://www.wid.world/, Jenn (1999), https://data.oecd.org/, and author's calculations.

(c) Growth of top 10% income share, 1980-1997, and normalized fee revenues by headhunters, 1997.



This figure plots the change in the top 10% income share in a country between 1980 and 1997 (vertical axis) against the fee revenues by headhunters in 1997 normalized by GDP in 1997 (horizontal axis). Sources: http://www.wid.world/, Jenn (1999), https://data.oecd.org/, and author's calculations.

2.6 Micro evidence

This section presents the empirical analysis of the potential effect of headhunters on the CEO compensation and firm performance. CEOs constitute a major part of the hires by headhunters, accounting for 20000 hires by headhunters in 2013 in the U.S. alone, and therefore are a good proxy for individual effects of headhunters on matching between workers and firms. There is no information on the identity of the CEOs that were hired through headhunters, so the evidence presented here should not be considered as a direct proof of the work of the mechanism. All conclusions are suggestive.

2.6.1 Data

The data on CEO characteristics and compensation and the firm level data are obtained from COMPUSTAT dataset. Following Gabaix, Landier, & Sauvagnat (2014), four proxies for the firm size will be used, constructed from variables obtained from COMPUSTAT yearly data set. First, firm value is constructed as the sum of the market value of equity, defined as a number of shares outstanding multiplied by the end-of-fiscal-year stock price, and the book value of debt, defined as total assets minus the sum of the book value of equity and deferred taxes. Second, equity value constructed as the number of shares outstanding multiplied by the end-of-fiscal-year stock price. Third, the sales variable from the COMPUSTAT. Fourth, the income is measured as earnings before interest and taxes. For CEO compensation EXECUCOMP panel of the COMPUSTAT will be used that contains information about the top 5 paid executives of the largest firms in the U.S. In particular, the variable TDC1 will be used to measure CEO compensation, that includes salary, bonus, restricted stock granted and the Black-Scholes value of stock options granted. To construct industry dummies the four-digit SIC industry codes are used as in Fama & French (1997). A dummy variable for a change of CEO is constructed such that it is equal to 0 if the CEO is the same as the CEO of the first observation of the company, and 1 otherwise:

$$NewCEO_{i,t} = \begin{cases} 1 & \text{if CEO is different from the first observation of the firm} \\ 0 & \text{otherwise.} \end{cases}$$

Another important variable that will be taking into account is the index of enforceability of non-competition constructed by Garmaise (2009). The index is higher in the states where the non-compete agreements are enforced by courts and low in the states were the non-compete agreements are forbidden. Non-compete agreements restrict job-to-job transitions for workers and therefore limit the activity of headhunters.

The following sample will be used. The maximum available time period in the data set is from 1993 to 2013. The analysis will be restricted only to the CEO of every company. If a firm changes the CEO more than ones in the sample period, all observations starting from the third CEO are dropped. These restrictions leave 3102 firms with 7.95 average years of observation. The reason for leaving only CEOs is motivated by two facts. First, CEO is the highest paid executive in almost all the firms in the sample, therefore they are more likely to be hired through a headhunter. And second, it is perceived that CEOs have the most influence on the firm activities and performance, so matching with a good CEO improves the firm's performance.

2.6.2 Results

I estimate the following equation:

$$\log \left(TDC1_{i,t}\right) = \alpha * NewCEO_{i,t} + \beta * \log \left(Firm\ size_{i,t}\right) + FE_t + FE_i + \varepsilon_{i,t},$$

where $TDC1_{i,t}$ is CEO compensation in firm i and year t, $NewCEO_{i,t}$ is the dummy variable constructed as described above, and $Firm\ size_{i,t}$ is one of the four measures of the firm size described above in firm i and year t. Table 2.11 presents the results on the full sample described before. Columns (1) and (2) present the results of the estimation with firm fixed effects and with or without the year fixed effects. Columns (3) and (4) present the results for similar estimation but without the $NewCEO_{i,t}$ dummy variable. The obtained results show that after a company changes the CEO it pays her from 5% to 16% more than to the previous CEO after conditioning on the firm size.

More interestingly, if we focus on two sub-periods in the sample (Table 2.12) - 1993 to 1998 and 2004 to 2007 - the periods during which the headhunter revenues were increasing particularly fast, as it can be clearly seen from the Figure 2.2a, the coefficient of interest increases from 5% to 9% in the first sub-period and from 5% to 13,6% in the second sub-period. This can be viewed as an indirect evidence of the higher use of headhunters during that periods and therefore improvements in the matching between CEOs and firms that resulted in higher compensation.

Table 2.13 presents the results for individual measures of the firm size. Columns (1) and (2) present the results for the firm value measure and columns (3) and (4) for the equity value measures. The results are consistent with the results for the full sample.

The important question is what is the channel of this effect, why are the firms paying more to the new CEOs. One potential explanation can be that the new CEO has a higher bargaining power than the previous CEO, it can be the case especially if the new CEO was hired with the help of a headhunter while the previous CEO was not. To test for this channel I augment the estimated equation with the interaction term between the *NewCEO* dummy variable and the measures of the firm size. The coefficient of the measure of the firm size in this regression can be viewed as the bargaining power of the CEO, i.e. how much his compensation increases when the firm is growing, and the interaction term can be viewed as the change in the bargaining power of the new

Table 2.11: CEO compensation and change of the CEO, full sample.

	Sample period 1993-2013			
Log of compensation	(1)	(2)	(3)	(4)
New CEO	0.1579***	0.0495**	-	-
	(0.0296)	(0.0180)	-	-
Log of Firm Value	0.1449***	0.0896**	0.1700***	0.0883**
	(0.0395)	(0.0391)	(0.0410)	(0.0393)
Log of Equity Value	0.1936***	0.2243***	0.1763***	0.2234***
	(0.0286)	(0.0310)	(0.0291)	(0.0310)
Log of Income	0.0852***	0.0899***	0.0761***	0.0898***
	(0.0135)	(0.0118)	(0.0138)	(0.0119)
Log of Sales	0.0427	0.0122	0.0809***	0.0105
	(0.0253)	(0.0259)	(0.0809)	(0.0259)
Year FE	No	Yes	No	Yes
Firm FE	Yes	Yes	Yes	Yes
R^2	0.66	0.66	0.66	0.66
Number of observations	24673	24673	24673	24673

* p<10%, ** p<5%, *** p<1%

.

and the previous CEOs. The results are presented in Table 2.14. Columns (1) and (2) present the results for the firm value as a proxy for the firm size, while columns (3) and (4) present the results for the equity value of the firm. The results show that the interaction term is negative or statistically not significantly different from 0. This shows that the increase of CEO compensation does not come from the higher bargaining power.

Now, to test for the channel that works in the model, the increased productivity of a match resulting in a higher wage, the direct effect of a change of CEO on the firm size is tested. Table 2.15 presents the results, with columns (1) and (2) showing the effect on the firm value and columns (3) and (4) showing the effect on the equity value. As it can be seen from the table the effect of the change on the CEO on the firm size is positive. This can be an indicator of the presence of the channel related to the productivity of the match between the new CEO and the firm.

To further explore the matching channel I add the non-competition enforceability index to the analysis. First, Table 2.16 presents the result of the regression including just the enforceability index, but not the new CEO dummy. It confirms the well-known result that CEO compensation is lower in the states that enforce the non-compete agreements.

Table 2.17 presents the result of the interaction between the non-compete enforce-

Table 2.12: CEO compensation and change of the CEO, sub periods of headhunters revenue booms.

	1993-1998		2004-2007	
Log of compensation	(1)	(2)	(3)	(4)
New CEO	0.0906**	-	0.1364**	-
	(0.0315)	-	(0.0382)	-
Log of Firm Value	0.1648	0.1615	0.0497	0.0450
	(0.0943)	(0.0942)	(0.1662)	(0.1653)
Log of Equity Value	0.2359**	0.2370**	0.2487***	0.2521***
	(0.0713)	(0.0714)	(0.0415)	(0.0397)
Log of Income	0.1034***	0.1037***	0.1396**	0.1405**
	(0.0187)	(0.0190)	(0.0263)	(0.0271)
Log of Sales	-0.0542	-0.0665	-0.1549	-0.1697
	(0.0572)	(0.0567)	(0.0989)	(0.1024)
Year FE	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes
R^2	0.724	0.723	0.785	0.785
Number of observations	8304	8304	4505	4505

* p<10%, ** p<5%, *** p<1%

.

ability index and the new CEO dummy. The interaction term increases the magnitude of the effect of the new CEO dummy and has a negative significant coefficient. It means that the effect of the change of the CEO on the compensation that the firm pays is higher in the states with low non-compete enforceability index. And it decreases with the higher index. It is also interesting to notice that in the states where the index would be 1 (the highest index is 0.9 in Florida) the overall effect of the change of CEO on the compensation would be negative.

2.6.3 Discussion

The empirical results presented in this section show that the matches between CEOs and companies improve over time in the U.S. The improvement in the matching results in the higher CEO compensation and larger size of the firms. This is important in the context of this paper because the CEOs constitute a major part of the hires by headhunters, and because CEOs are exactly the people on the top of the wage distribution. The fact that the matching is improving over time supports the mechanism discussed in the paper. Headhunters provide better matches both for the firms and for the CEOs increasing the firm size and the CEO compensation.

Table 2.13: CEO compensation and change of the CEO, individual firm size measures.

	Sample period 1993-2013			
Log of compensation	(1)	(2)	(3)	(4)
New CEO	0.0437**	0.1389***	0.0420**	0.1876***
	(0.0177)	(0.0297)	(0.0171)	(0.0314)
Log of Firm Value	0.4311***	0.4620***	-	-
	(0.0180)	(0.0193)	-	-
Log of Equity Value	-	-	0.3442***	0.3572***
	-	-	(0.0161)	(0.0191)
Year FE	Yes	No	Yes	No
Firm FE	Yes	Yes	Yes	Yes
R^2	0.66	0.653	0.66	0.654
Number of observations	24673	24673	24673	24673

* p<10%, ** p<5%, *** p<1%

.

Table 2.14: CEO compensation and change of the CEO, bargaining power.

	Sample period 1993 - 2013			
Log of compensation	(1)	(2)	(3)	(4)
New CEO	0.3590***	0.3145**	0.4079***	0.3168**
	(0.1293)	(0.1277)	(0.1318)	(0.1302)
Log of Firm Value	0.4746***	0.4466***	-	-
	(0.0173)	(0.0183)	-	-
Log of Equity Value	-	-	0.3720***	0.3629***
	-	-	(0.0159)	(0.0164)
Log of FV*New CEO	-0.0269	-0.0332**	-	-
	(0.0165)	(0.0165)	-	-
Log of EV*New CEO	-	-	-0.0291	-0.0366**
	-	-	(0.0182)	(0.0184)
Year FE	No	Yes	No	Yes
Firm FE	Yes	Yes	Yes	Yes
R^2	0.654	0.659	0.654	0.662
Number of observations	24673	24673	24673	24673

* p<10%, ** p<5%, *** p<1%

.

Table 2.15: CEO compensation and change of the CEO, match efficiency.

	Firm Value		Equity Value	
	(1)	(2)	(4)	(5)
New CEO	0.4784***	0.6138***	0.4023***	0.5208***
	(0.0399)	(0.0450)	(0.0399)	(0.0448)
Year FE	No	Yes	No	Yes
Industry FE	Yes	Yes	Yes	Yes
R^2	0.200	0.231	0.095	0.132
Number of observations	24673	24673	24673	24673

* p<10%, ** p<5%, *** p<1%

.

Table 2.16: CEO compensation and the non-compete enforceability index.

	Sample period 1993 - 2013		
Log of compensation	(1)	(2)	(3)
NCEI	-0.0222***	-0.0286***	-0.0221***
	(0.0028)	(0.0029)	(0.0029)
Log of Firm Value	0.1780***	0.0514***	0.1608***
	(0.0149)	(0.0146)	(0.0149)
Log of Equity Value	0.0982***	0.2114***	0.1025***
	(0.0184)	(0.0149)	(0.0183)
Log of Income	0.0640***	0.04322**	0.0691***
	(0.0122)	(0.0152)	(0.0114)
Log of Sales	0.1268***	0.1571***	0.1397***
	(0.0108)	(0.0082)	(0.0086)
Year FE	No	Yes	Yes
Industry FE	Yes	No	Yes
R^2	0.410	0.384	0.420
Number of observations	24217	24217	24217

* p<10%, ** p<5%, *** p<1%

.

Table 2.17: CEO compensation, new CEOs, and the non-compete enforceability index.

	Sample period 1993 - 2013		
Log of compensation	(1)	(2)	(3)
NCEI*New CEO	-0.1685***	-0.2377***	-0.1805***
	(0.0475)	(0.0477)	(0.0471)
New CEO	0.1245***	0.0778***	0.0663***
	(0.0282)	(0.0190)	(0.0191)
Log of Firm Value	0.1769***	0.0531***	0.1633***
	(0.0161)	(0.0151)	(0.0156)
Log of Equity Value	0.1031***	0.2184***	0.1049***
	(0.0180)	(0.0151)	(0.0183)
Log of Income	0.0626***	0.0390**	0.0673***
	(0.0121)	(0.0152)	(0.0114)
Log of Sales	0.1208***	0.1528***	0.1368***
	(0.0113)	(0.0086)	(0.0089)
Year FE	No	Yes	Yes
Industry FE	Yes	No	Yes
R^2	0.409	0.381	0.419
Number of observations	24217	24217	24217

* p<10%, ** p<5%, *** p<1%

.

Of course, there are shortcomings in this empirical specification because we don't know which CEOs are hired through a headhunter and which CEOs come from internal promotion or other channels. To address this issue directly one needs to collect the data on the origins of the CEO and the way she was hired. Such study would be able to analyze the difference between CEO compensation for a CEO coming through headhunters and not. Most importantly, it would be also able to determine the effect of a CEO hired by a headhunter on the firm performance. However, to collect such data set would require a tremendous amount of work and time. Also, it wouldn't be still guaranteed that such a data set would be full, because not all companies changing CEOs state whether they hired the CEO with a help from a headhunter or in a different way (for example through referrals). Another shortcoming in this analysis is that the firm size may not be a good proxy for the firm performance, other measures should be used to account for the current performance of a firm, such as cash flows or profits.

To try to overcome the lack of data on identities of the CEOs hired by headhunters I use non-compete enforceability index as an instrument for the probability to be hired by a headhunter. In the states with a high NCEI activity of headhunters is limited and therefore very few positions are filled by headhunters. The results show, indeed, that in the states with low NCEI the increase in CEO compensation after a CEO change is larger. This suggests that in the states with low NCEI more CEOs are hired by headhunters, so the improvement in matching is stronger and it leads to higher compensation.

Other studies discussing the increase in CEO compensation over the past decades offer various explanations of this phenomena. Gabaix & Landier (2008) and Gabaix, Landier, & Sauvagnat (2014) argue that the CEO pay increases because the average company size is increasing. Murphy & Sandino (2010) argue that the CEOs may better extract the rent from the company by hiring external compensation consultants that follow their interest. Murphy & Zabojnik (2004) show that the nature of CEO skills required to successfully run a company is changing over time and therefore more firms hire the CEOs from outside of the firm and have to pay her more.

In another paper Murphy & Zabojnik (2007) provide empirical evidence on the CEO origins at the moment of her appointment, i.e. whether she is coming from within the company or from outside, and the effect of the origin on the compensation. They study the S&P 500 companies (the largest 500 firms) during the period from 1970 to 2005. They show that during the 1970s and 1980s only 15% and 17% of CEO appointments account for the outside hires, while it increased to 26% in 1990s and almost 32.7% in 2000s. Even more importantly, they show that the external CEO receives 14.2% higher compensation on average over the full sample, with the difference being just 6% in the1970s, 15,9% in 1980s and 19.6% in 1990s. Not only the companies rely more and more on the outside CEOs but also the pay difference between the internal and external

CEOs is increasing. After reconciling these results with the data that almost all of the outside CEOs are hired by headhunters, this is a strong evidence for the mechanism proposed in this paper.

Among other studies more closely related to the mechanism studied in this paper, Garmaise (2009) shows that tougher non-compete agreements regulation reduces CEO turnover and compensation. Again, this suggests that in the states with higher NCEI the activity of headhunters is limited[18] and therefore it reduces opportunities of CEOs to transit between firms and improve the efficiency of matching, therefore limiting compensation. Pan (2015) shows the importance of assortative matching between CEOs and firms for determination of the CEO compensation and the firm's performance. However, Pan (2015) doesn't consider the change in the degree of assortative matching over time or geographical differences.

2.7 Conclusion

This paper introduces the headhunter channel to the standard model of random matching. The fact that headhunters have better information about a worker's skill level and that they can approach workers who are not actively searching for a (new) job at this moment allows for better screening of workers and reduces labor market frictions at the top part of the wage distribution. Moreover, headhunters separate the labor market for high and low-productive firms allowing the high-productive firms to access only the high-skilled workers. Because of worker skill and firm productivity complementarities, the wages of workers hired through headhunters increase more than proportionally to the rest of the workers. Thus, the presence of headhunters generates a fat tail of the wage distribution with a larger wage share of the top 1% and 10% workers.

Quantitative analysis of the model uses a calibrated version of the model to show that introduction of the headhunter channel in otherwise standard random matching model accounts for 70% of the increase in top 10% share of wages and 40% of the increase of top 1% share of wages in the U.S. between 1970 and 2010. The results are robust to the choice of targets related to the headhunter channel. The main effect comes from the improvement in the assortative matching between workers and firms, especially at the top. The pattern and the amplitude of the improvement are comparable to the empirical estimates of the change in assortative matching in the U.S. over the same period. The headhunter channel helps to generate the strong non-linearity in the pattern of matching observed in the data.

The paper also provides the empirical evidence of the joint increase of the use of headhunters by firms and the top income shares. The paper uses cross-country data

[18]Garmaise (2009) does not talk about headhunters in his study.

on headhunter revenues and number of hires through headhunters together with the top income shares to show that normalized hires by headhunters are a good predictor of the future growth of the top income shares in European countries. Then, it also shows that the new CEOs in the U.S. get higher compensations comparing to the previous CEOs in the same companies and this effect is weaker in the states with high non-compete enforceability index, i.e., in the states that potentially limit the activity of headhunters.

2.8 References

1. Acemoglu, D. (2002). Technical change, inequality, and the labor market. Journal of economic 57 literature, 40(1), 7–72.

2. AESC. (2015). Insights q4 2015 state of the executive search industry (Tech. Rep.). Association of Executive Search and Leadership Consultants.

3. Alvaredo, F., Atkinson, A. B., Piketty, T., & Saez, E. (2013). The top 1 percent in international and historical perspective. The Journal of Economic Perspectives, 27(3), 3–20.

4. Aoki, S., & Nirei, M. (2015). Zipf's law, pareto's law, and the evolution of top incomes in the us. Pareto's Law, and the Evolution of Top Incomes in the US (February 12, 2015).

5. Bagger, J., & Lentz, R. (2014). An empirical model of wage dispersion with sorting (Tech. Rep.). National Bureau of Economic Research.

6. Bagger, J., Sørensen, K. L., & Vejlin, R. (2013). Wage sorting trends. Economics Letters, 118(1), 63–67.

7. Bell, B. D., & Reenen, J. V. (2013). Extreme wage inequality: pay at the very top. The American Economic Review, 103(3), 153–157.

8. Cappelli, P., & Hamori, M. (2013). Who says yes when the headhunter calls? understanding executive job search behavior (Tech. Rep.). National Bureau of Economic Research.

9. Card, D., Heining, J., & Kline, P. (2013). Workplace heterogeneity and the rise of west german wage inequality*. Quarterly Journal of Economics, 128(3).

10. Eeckhout, J., & Kircher, P. (2011). Identifying sorting - in theory. The Review of Economic Studies, rdq034.

11. Fama, E. F., & French, K. R. (1997). Industry costs of equity. Journal of financial economics, 43(2), 153–193.

12. Gabaix, X., & Landier, A. (2008). Why has ceo pay increased so much?*. The Quarterly journal of economics, 123(1), 49–100.

13. Gabaix, X., Landier, A., & Sauvagnat, J. (2014). Ceo pay and firm size: an update after the crisis. The Economic Journal, 124(574), F40–F59.

14. Gabaix, X., Lasry, J.-M., Lions, P.-L., & Moll, B. (in press). The dynamics of inequality. Econometrica.

15. Garmaise, M. J. (2009). Ties that truly bind: Noncompetition agreements, executive compensation, and firm investment. Journal of Law, Economics, and Organization.

16. Gautier, P. A., & Teulings, C. N. (2015). Sorting and the output loss due to search frictions. Journal of the European Economic Association, 13(6), 1136–1166.

17. Hagedorn, M., Law, T. H., & Manovskii, I. (2012). Identifying equilibrium models of labor market sorting (Tech. Rep.). National Bureau of Economic Research. 58

18. Håkanson, C., Lindqvist, E., & Vlachos, J. (2015). Firms and skills: the evolution of worker sorting (Tech. Rep.). Stockholm University, Department of Economics.

19. IACPR. (2003). (Tech. Rep.). International Association of Corporate and Professional Recruiters.

20. Jenn, N. G. (1995). Executive search in the americas: choosing and using a headhunter.

21. Jenn, N. G. (1999). Executive search in europe: choosing and using a headhunter.

22. Jones, C. I. (2015). Pareto and piketty: The macroeconomics of top income and wealth inequality. The Journal of Economic Perspectives, 29(1), 29–46.

23. Jones, C. I., & Kim, J. (2014). A schumpeterian model of top income inequality (Tech. Rep.). National Bureau of Economic Research.

24. Katz, L. F., Kearney, M. S., et al. (2006). The polarization of the us labor market. American Economic Review, 96(2), 189–194.

25. Lemieux, T., MacLeod, W. B., Parent, D., et al. (2009). Performance pay and wage inequality. The Quarterly Journal of Economics, 124(1), 1–49.

26. Lise, J., Meghir, C., & Robin, J.-M. (2016). Matching, sorting and wages. Review of Economic Dynamics, 19, 63–87.

27. Mortensen, D. T., & Pissarides, C. A. (1994). Job creation and job destruction in the theory of unemployment. The review of economic studies, 61(3), 397–415.

28. Murphy, K. J., & Sandino, T. (2010). Executive pay and "independent" compensation consultants. Journal of Accounting and Economics, 49(3), 247–262.

29. Murphy, K. J., & Zabojnik, J. (2004). Ceo pay and appointments: A market-based explanation for recent trends. The American Economic Review, 94(2), 192–196.

30. Murphy, K. J., & Zabojnik, J. (2007). Managerial capital and the market for ceos. Available at SSRN 984376.

31. Pan, Y. (2015). The determinants and impact of executive-firm matches. Management Science.

32. Piketty, T. (2014). Capital in the twenty-first century. Harvard University Press.

33. Postel-Vinay, F., & Robin, J.-M. (2002). Equilibrium wage dispersion with worker and employer heterogeneity. Econometrica, 70(6), 2295–2350.

34. Schulz, B., & Lochner, B. (2016). Labor market sorting in germany.

35. Shimer, R., & Smith, L. (2000). Assortative matching and search. Econometrica, 68(2), 343–369.

36. Song, J., Price, D. J., Guvenen, F., Bloom, N., & von Wachter, T. (2016). Firming up inequality.

37. Teulings, C. N., & Gautier, P. A. (2004). The right man for the job. The Review of Economic Studies, 71(2), 553–580. 59

38. Uren, L., & Virag, G. (2011). Skill requirements, search frictions, and wage inequality*. International Economic Review, 52(2), 379–406.

2.9 Appendix

2.9.1 Non-monetary costs of headhunters

In this case, it is more convenient to define two measures for firms with an open position - $G_V(p)$ for the firms using vacancy channel, and $G_H(p)$ for the firms using the headhunter channel.

Workers

Now the value functions are the following. For low-skilled unemployed workers:

$$S_U(e) = S_{UV}(e) \equiv f_V(u_V, a_V, v) \int_{\underline{p}}^{\overline{p}} (W(e,p) - U(e))\, dG_V(p) - c_{wV}.$$

For high-skilled unemployed workers:

$$\begin{aligned} S_U(e) = S_{UVH}(e) \equiv \quad & f_H(u_H, a_H, h)(1 - f_V(u_V, a_V, v)) \left(\textstyle\int_{\hat{p}}^{\overline{p}} (W(e,p) - U(e))\, dG_H(p) - c_{wH} \right) + \\ & + f_V(u_V, a_V, v)(1 - f_H(u_H, a_H, h)) \textstyle\int_{\underline{p}}^{\overline{p}} (W(e,p) - U(e))\, dG_V(p) + \\ & + f_H(u_H, a_H, h) f_V(u_V, a_V, v) \cdot \\ & \cdot \left(\textstyle\int_{\hat{p}}^{\overline{p}} \int_{\underline{p}}^{\overline{p}} (\max\{W(e,p), W(e,p')\} - U(e))\, dG_H(p)\, dG_V(p') - c_{wH} \right) - c_{wV}. \end{aligned}$$

For low skilled employed workers:

$$S_{EV}(e,p) \equiv f_V(u_V, a_V, v) \int_p^{\overline{p}} \left(W(e,p') - W(e,p)\right) dG_V(p') - c_{wV}.$$

For high-skilled employed workers:

$$S_{EH}(e,p) \equiv f_H(u_H, a_H, h) \left(\int_{\max\{\hat{p};p\}}^{\overline{p}} \left(W(e,p') - W(e,p)\right) dG_H(p') - c_{wH} \right),$$

and:

$$\begin{aligned} S_{EVH}(e,p) \equiv \quad & f_H(u_H, a_H, h)(1 - f_V(u_V, a_V, v)) \left(\textstyle\int_{\max\{\hat{p};p\}}^{\overline{p}} (W(e,p') - W(e,p))\, dG_H(p') - c_{wV} \right) + \\ & + f_V(u_V, a_V, v)(1 - f_H(u_H, a_H, h)) \textstyle\int_p^{\overline{p}} (W(e,p') - W(e,p))\, dG_V(p') + \\ & + f_H(u_H, a_H, h) f_V(u_V, a_V, v) \cdot \\ & \cdot \left(\textstyle\int_{\hat{p}}^{\overline{p}} \int_{\underline{p}}^{\overline{p}} (\max\{\max\{W(e,p''), W(e,p')\} - W(e,p); 0\})\, dG_H(p')\, dG_V(p'') - c_{wH} \right) - c_{wV}. \end{aligned}$$

Firms

The value function of firms posting a vacancy in this case is:

$$\begin{aligned} V_V(p) = \quad & -c_{fV} \cdot p + \beta \Bigg(V(p) + q_V(u_V, a_V, v) \Bigg(\tfrac{u_V}{u_V + a_V} \textstyle\int_{\underline{e}}^{\hat{e}} \left(J(p,e) - V\left(p, c'_{fN}\right) \right) dU(e) + \\ & + \tfrac{u_V}{u_V + a_V} (1 - f_H(u_H, a_H, h)) \textstyle\int_{\hat{e}}^{\overline{e}} \left(J(p,e) - V\left(p, c'_{fN}\right) \right) dU(e) + \\ & + \tfrac{u_V}{u_V + a_V} f_H(u_H, a_H, h) \tfrac{G_H(p)}{G_H(\overline{p})} \textstyle\int_{\hat{e}}^{\overline{e}} \left(J(p,e) - V\left(p, c'_{fN}\right) \right) dU(e) + \\ & + \tfrac{a_V}{u_V + a_V} \textstyle\int_{\underline{e}}^{\hat{e}} \tfrac{\Lambda_V(e,p)}{\Lambda_V(e,\overline{p})} \left(J(p,e) - V\left(p, c'_{fN}\right) \right) dL_V(e) + \\ & + \tfrac{a_V}{u_V + a_V} (1 - f_H(u_H, a_H, h)) \textstyle\int_{\hat{e}}^{\overline{e}} \tfrac{\Lambda_{VH}(e,p)}{\Lambda_{VH}(e,\overline{p})} \left(J(p,e) - V\left(p, c'_{fN}\right) \right) dL_{VH}(e) + \\ & + \tfrac{a_V}{u_V + a_V} f_H(u_H, a_H, h) \tfrac{G_H(p)}{G_H(\overline{p})} \textstyle\int_{\hat{e}}^{\overline{e}} \tfrac{\Lambda_{VH}(e,p)}{\Lambda_{VH}(e,\overline{p})} \left(J(p,e) - V\left(p, c'_{fN}\right) \right) dL_{VH}(e) \Bigg) \Bigg). \end{aligned}$$

The value function of firms using headhunters is:

$$
\begin{aligned}
V_H(p) = \; & -c_{fH} \cdot p + \\
& +\beta \Bigg(V(p) + q_H(u_H, a_H, h) \Bigg(\tfrac{u_H}{u_H + a_H} (1 - f_V(u_V, a_V, v)) \int_{\hat{e}}^{\overline{e}} \left(J(p,e) - V\left(p, c'_{fN}\right)\right) dU(e) + \\
& + \tfrac{u_H}{u_H + a_H} f_V(u_V, a_V, v) \tfrac{G_V(p)}{G_V(\overline{p})} \int_{\hat{e}}^{\overline{e}} \left(J(p,e) - V\left(p, c'_{fN}\right)\right) dU(e) + \\
& + \tfrac{a_H}{u_H + a_H} \int_{\hat{e}}^{\overline{e}} \tfrac{\Lambda_H(e,p)}{\Lambda_H(e,\overline{p})} \left(J(p,e) - V\left(p, c'_{fN}\right)\right) dL_H(e) \\
& + \tfrac{a_H}{u_H + a_H} (1 - f_V(u_V, a_V, v)) \int_{\hat{e}}^{\overline{e}} \tfrac{\Lambda_{VH}(e,p)}{\Lambda_{VH}(e,\overline{p})} \left(J(p,e) - V\left(p, c'_{fN}\right)\right) dL_{VH}(e) + \\
& + \tfrac{a_H}{u_H + a_H} f_V(u_V, a_V, v) \tfrac{G_V(p)}{G_V(\overline{p})} \int_{\hat{e}}^{\overline{e}} \tfrac{\Lambda_{VH}(e,p)}{\Lambda_{VH}(e,\overline{p})} \left(J(p,e) - V\left(p, c'_{fN}\right)\right) dL_{VH}(e) \Bigg)\Bigg).
\end{aligned}
$$

And the value of an open position is:

$$\tilde{V}\left(p, c_{fN}\right) = \max\left\{V_V(p); V_H(p) - c_{fN}\right\}.$$

The quit rate is:

$$
s_Q(e,p,\omega) = \begin{cases}
f_V(u_V, a_V, v) \left(\frac{G_V(\overline{p}) - G_V(p)}{G_V(\overline{p})}\right) & if\ p < \tilde{p}_V(e)\ and\ e < \underline{e} \\
f_H(u_H, a_H, h) \left(\frac{G_H(\overline{p}) - G_H(p)}{G_H(\overline{p})}\right) & if\ \tilde{p}_{VH}(e) < p < \tilde{p}_H(e)\ and\ e \geq \underline{e} \\
(1 - f_V(u_V, a_V, v)) \cdot & if\ p < \tilde{p}_{VH}(e)\ and\ e \geq \underline{e} \\
\cdot f_H(u_H, a_H, h) \left(\frac{G_H(\overline{p}) - G_H(p)}{G_H(\overline{p})}\right) + & \\
+ (1 - f_H(u_H, a_H, h)) \cdot & \\
\cdot f_V(u_V, a_V, v) \left(\frac{G_V(\overline{p}) - G_V(p)}{G_V(\overline{p})}\right) + & \\
+ f_V(u_V, a_V, v) f_H(u_H, a_H, h) \cdot & \\
\cdot \left(1 - \frac{G_V(p)}{G_V(\overline{p})} \frac{G_H(p)}{G_H(\overline{p})}\right) & \\
0 & otherwise.
\end{cases}
$$

Aggregation

The number of firms using the vacancy channel:

$$v = \int_{\underline{p}}^{\hat{p}} 1 dG_V(p).$$

And the number of firms using the headhunter channel:

$$h = \int_{\hat{p}}^{\overline{p}} 1 dG_H(p).$$

And the number of searching workers is determined as before.

Balance

The aggregate balance equation is as before:

$$\phi(e,p)\left(s + s_Q(e,p)(1-s)\right) = i_E(e,p) + i_U(e,p),$$

while the inflows now are:

$$i_U(e,p) = \begin{cases} f_V(u_V, a_V, v)\, \frac{g_V(p)}{v} u(e) & \text{if } e < \hat{e} \\ f_H(u_H, a_H, h)\,(1 - f_V(u_V, a_V, v))\, \frac{g_H(p)}{h} u(e) + & \text{if } e \geq \hat{e} \\ + (1 - f_H(u_H, a_H, h))\, f_V(u_V, a_V, v)\, \frac{g_V(p)}{v} u(e) + & \\ + f_H(u_H, a_H, h)\, f_V(u_V, a_V, v) \left(\frac{g_V(p)}{v} \frac{G_H(p)}{G_H(\overline{p})} + \frac{g_H(p)}{h} \frac{G_V(p)}{G_V(\overline{p})} \right) u(e) & \end{cases}$$

and:

$$i_E(e,p) = \begin{cases} f_V(u_V, a_V, v)\, \frac{g_V(p)}{v} \int_{\underline{p}}^{\min\{p, \tilde{p}_V(e)\}} \phi(e,p')\, dp' & \text{if } e < \hat{e} \\ f_H(u_H, a_H, h)\, \frac{g_H(p)}{h} \int_{\min\{p, \tilde{p}_{VH}(e)\}}^{\min\{p, \tilde{p}_H(e)\}} \phi(e,p')\, dp' + & \text{if } e \geq \hat{e} \\ + f_H(u_H, a_H, h)\,(1 - f_V(u_V, a_V, v))\, \frac{g_H(p)}{h} \int_{\underline{p}}^{\min\{p, \tilde{p}_{VH}(e)\}} \phi(e,p')\, dp' + & \\ + (1 - f_H(u_H, a_H, h))\, f_V(u_V, a_V, v)\, \frac{g_V(p)}{v} \int_{\underline{p}}^{\min\{p, \tilde{p}_{VH}(e)\}} \phi(e,p')\, dp' + & \\ + f_H(u_H, a_H, h)\, f_V(u_V, a_V, v)\, \frac{g_V(p)}{v} \frac{G_H(p)}{G_H(\overline{p})} \int_{\underline{p}}^{\min\{p, \tilde{p}_{VH}(e)\}} \phi(e,p')\, dp' + & \\ + f_H(u_H, a_H, h)\, f_V(u_V, a_V, v)\, \frac{g_H(p)}{h} \frac{G_V(p)}{G_V(\overline{p})} \int_{\underline{p}}^{\min\{p, \tilde{p}_{VH}(e)\}} \phi(e,p')\, dp' & \end{cases}$$

2.9.2 Wage bargaining

In the baseline model wages are set as a constant share of the production, in this extension the wages are determined in period by period wage bargaining between the worker and the firm. This might change the implication of the model because headhunters will affect the outside options of both parties. They improve the value of the vacancy for the firm, so improving firm's bargaining position and driving the wages of top earners down, potentially dampening the effect from better matching. But at the same time, they facilitate job search for high-skilled workers improving also their bargaining position and increasing their wages even more. Moreover, the bargaining position of medium-skilled workers worsens because they lose the possibility to move to better matches, therefore decreasing their wages.

As in standard Nash bargaining, wage in a match between a worker with skill e and a firm with productivity p is a solution of the Nash bargaining problem:

$$w(e,p) = \max_w \left(\hat{W}(e,p,w) - U(e) \right)^{\gamma} \left(\hat{J}(e,p,w) - V(p) \right)^{1-\gamma},$$

where γ is the bargaining power of the worker.

The FOC:

$$\gamma \left(\hat{W}(e,p,w) - U(e) \right)^{\gamma - 1} \left(\hat{J}(e,p,w) - V(p) \right)^{1-\gamma} \frac{\partial \hat{W}(e,p,w)}{\partial w} = \\ - (1-\gamma) \left(\hat{W}(e,p,w) - U(e) \right)^{\gamma} \left(\hat{J}(e,p,w) - V(p) \right)^{-\gamma} \frac{\partial \hat{J}(e,p,w)}{\partial w},$$

or simply

$$\gamma \left(\hat{J}(e,p,w) - V(p) \right) \frac{\partial \hat{W}(e,p,w)}{\partial w} = -(1-\gamma) \left(\hat{W}(e,p,w) - U(e) \right) \frac{\partial \hat{J}(e,p,w)}{\partial w}.$$

From the value functions we can find that:

$$\frac{\partial \hat{W}(e,p,w)}{\partial w} = -\frac{\partial \hat{J}(e,p,w)}{\partial w} = 1,$$

so the equilibrium wage for every match must satisfy the standard sharing rule:

$$\gamma\left(\hat{J}(e,p,w) - V(p)\right) = (1-\gamma)\left(\hat{W}(e,p,w) - U(e)\right).$$

Start with the model without headhunters. RHS of the sharing rule can be written as:

$$\begin{aligned}&\gamma\left(\mathbf{y} - \mathbf{w} + \beta\left((s + \mathbf{s_Q}(1-s))V' + (1-\mathbf{s_Q})(1-s)\mathbf{J}'\right) - \right.\\ &\left. - \left(-c_{fV}\cdot p + \beta\left(V' + q_V E_{e'|V}\left[P(A)(J'-V')\right]\right)\right)\right),\end{aligned}$$

and the LHS can be written as:

$$(1-\gamma)\left(\mathbf{w} + \beta\left(sU' + (1-s)\left(\mathbf{W}' + \mathbf{S}'_{\mathbf{E}}\right)\right) - \left(b + \beta\left(U' + S'_U\right)\right)\right).$$

If the worker does not search on-the-job, the expressions simplify. For the RHS:

$$\gamma\left(\mathbf{y} - \mathbf{w} + \beta\left(sV' + (1-s)\mathbf{J}'\right) - \left(-c_{fV}\cdot p + \beta\left((1-q_V)V' + q_V E_{e'|V} J'\right)\right)\right),$$

and for the LHS:

$$(1-\gamma)\left(\mathbf{w} + \beta\left(sU' + (1-s)\mathbf{W}'\right) - \left(b + \beta\left(U' + S'_U\right)\right)\right).$$

We can solve for $\mathbf{w}$ and apply the sharing rule for the next period to get:

$$\mathbf{w} = \gamma\left(\mathbf{y} + c_{fV}\cdot p\right) + (1-\gamma)b + \beta\gamma\left(q_V V' - q_V E_{e'|V} J'\right) + (1-\gamma)\beta S'_U.$$

Without worker/firm heterogeneity this expression collapses to the standard wage equation - equilibrium value of tomorrow search will be equal to equilibrium value of a job, that in turn will be equal to the expected cost of a vacancy posted (κ/q).

Now consider the case when the worker searches on-the-job. We can solve for the wage, $\mathbf{w}$, from the initial sharing rule, applying the sharing rule of the next period when needed to obtain the following expression for the wage:

$$\begin{aligned}\mathbf{w} = \; &\gamma\left(\mathbf{y} + c_{fV}\cdot p\right) + (1-\gamma)b - (1-\gamma)\beta\left((1-s)\mathbf{S}'_{\mathbf{E}} - S'_U\right)\\ &-\beta\gamma\left(\mathbf{s_Q}(1-s)\left(\mathbf{J}' - V'\right) + q_V E_{e'|V}\left[P(A)(J'-V')\right]\right).\end{aligned}$$

This expression doesn't change in the case of the model with the headhunter channel (except the expectation operator). What changes with the headhunters are the values of the search for the worker, both from unemployment and employment, the value of a vacancy for the firm, and the quit rate. Effects of headhunters on wages are heterogeneous across different matches and depend dramatically on the bargaining power.

For example, for the match between the top-ranked worker and the top-ranked firm, where the worker doesn't search on-the-job and the quit rate is equal to 0, the headhunter channel increases both, the outside option of the worker, S_U', and the outside option of the firm. They have opposite effects on the wage, and which one will be stronger depends fully on the bargaining power. For other matches, the effect is even more complicated. On top of the opposing effects of outside options, there is also an effect on the worker's on-the-job search. With headhunters, the worker doesn't lose the possibility to continue search on-the-job and receive offers from better firms. This puts downward pressure on wages because the worker agrees to the match easier. Moreover, there is an interaction between the worker's search and the value of a vacant position for the firm through the quit rate. The value of a vacant position increases with headhunters, putting upward pressure on wages, but because quit rate increases at the same time, this effect is decreased leaving the overall effect ambiguous.

Numerical simulations show that the overall effect on individual wages, and, especially, on the wage distribution is ambiguous and depends crucially on the choice of the bargaining power. With a high bargaining power of the worker, the effect of headhunters on top wages is higher than in the benchmark model, while with a very low bargaining power the effect is even the opposite, with headhunters reducing wage inequality (even though the value of the bargaining power is not realistic in such simulations). When bargaining power is set to the levels used in the literature, the overall effect is close to the benchmark results.

2.9.3 Headhunters as profit maximizers

In this section, I extend the model to add headhunters as additional agents choosing the fee and the screening standards in order to maximize the profits. To choose the screening standard, the headhunters need to compare the expected payoff from firms willing to use headhunters with the given standard to the cost of screening. Headhunters have correct expectations about the number and the productivity of firms that will use headhunters with each screening standard. The headhunters solve the following problem:

$$\max_{\hat{e}, c_{fH}} \left[\int_{\hat{p}(\hat{e}, c_{fV})}^{\bar{p}} c_{fV} \cdot p dG(p) - \int_{\hat{p}(\hat{e}, c_{fV})}^{\bar{p}} c_H(\hat{e}) \, dG(p) \right],$$

where the first part is the fee revenues from firms using headhunters, and the second term is the total cost of screening the workers. Headhunter balance between the fee and the screening standard. When the screening standard is very high, many firms will want to participate and pay a high fee for it, but the cost of screening for headhunters will be also high, reducing the profits. And when the screening standard is low, firms' willingness to use headhunters decreases, so the headhunter has to reduce the fee, and the profits drop. Solution to this problem crucially depends on the form of the screening cost function.

Modeling headhunters explicitly and calibrating the cost function to match the screening standard and the optimal fee would be equivalent to directly calibrating the standard and the fee, as in the benchmark experiments of this paper. This would change, however, if we studied a dynamic version of the model, but this is the question for future research. Another issue with modeling headhunters is the choice of the market structure. Is it a competitive market, monopoly, or monopolistically competitive market? This question is also left for the future research.

3. Safety net and individual well-being, with Antonella Trigari

Abstract

This chapter studies the effects of the safety net on the individual well-being of the workers. We use a model with incomplete markets, nominal rigidities, and frictional labor market to study direct and general equilibrium effects of the safety net on consumption and labor market outcomes of individual agents. We study three programs: unemployment insurance, social assistance (SNAP and TANF), and tax credit (EITC). We show that individual consumption given assets is higher with stronger safety net but the distribution over assets shifts to the left. The two effects almost cancel out for aggregate consumption. Distribution of consumption is more concentrated with stronger safety net, incidence of very low consumption disappears and incidence of very high consumption decreases. Similar effects appear for the labor market outcomes (search intensity, job finding rates, unemployment duration). Nash bargained wages for low- and high-skilled workers are also moved apart.

3.1 Introduction

During the Great Recession, the US unemployment rate rose from less than 5 percent in 2007 to almost 10 percent in 2009, persisted above 7 percent by 2013, and only recently decreased to 5.5 percent. At the same time, the official poverty rate, the share of people living below the poverty line, rose from 12.5 percent in 2007 to 15.1 percent in 2010, and only decreased to 14.5 percent by 2013. This co-movement between the poverty rate and the unemployment rate is also a feature of earlier business cycles.

The upsurge in unemployment and poverty during recessionary episodes brings about a response of the social safety net, in part automatic (built-in response of the rules of the transfer system) and in part discretionary (arising from discretionary actions by policymakers), which is intended to provide support to the most affected households. While there is no universal definition, the FAO defines the safety net as "cash or in-kind transfer programs that seek to reduce poverty by redistributing wealth and/or protect households against income shocks. Social safety nets seek to ensure a minimum level of well-being, a minimum level of nutrition, or help households manage risk." In 2013, for example, 47.6 million US poor households received food assistance transfers from SNAP (the Supplemental Nutritional Assistance Program, previously called Food Stamp program), 14.4 million unemployed workers received unemployment benefits, 3.7 million needy families received cash welfare from TANF (Temporary Assistance for Needy Families), and 28 million working poor benefitted from the EITC program (the Earned Income Tax Credit program). The size and the cyclicality of the safety net is important for two reasons: First because it directly affects the well-being of different groups of individuals, who are vulnerable in heterogeneous ways to raising unemployment and poverty, by preventing them from losing their incomes, shedding their assets, and/or reducing their consumption. Second, because it influences the dynamics of the business cycle, possibly contributing to stabilizing aggregate fluctuations.

This chapter investigates and measures the effects of the safety net. To do that, we develop a model that captures most of the theoretical channels by which the safety net operates and that includes the key features of the safety net observed in actual economies. To account for the relevant channels and for the actual diversity of transfer programs, the framework merges a standard incomplete market model with heterogeneous agents, a new-Keynesian model with nominal rigidities and a search and matching model of the labor market. The model, in particular, implies a role for aggregate demand; heterogeneous agents in terms of their labor market state, assets, incomes and consumption; a motive for precautionary savings to insure against endogenous unemployment risk; as well as endogenous search intensity decisions of unemployed workers and bargaining over wages. Consequently, the safety net works via the full set of channels that the literature has suggested. These are the stabilization of disposable income and aggregate demand (disposable income channel), the redistribution of resource among heterogeneous agents with different marginal propensities to consume (redistribution channel), the provision of insurance against unemployment risk to liquidity-constrained agents (insurance channel), the (dis)incentive to actively search for a job when unemployed (search incentives channel) and, finally, the influence on the relative position in wage bargaining of firms and workers (bargaining channel). Broadly speaking, the first three channels work via effects on consumption/savings decisions, while the last two channels work via labor markets effects. One distinguishing feature of our analysis is the focus on the interaction between the social safety net and the workings of the labor market. One reason why this is important is that there is a considerable overlapping between the pool of the unemployed workers and the poor.

We calibrate the model to the US economy and to the US safety net and investigate the influence of the safety net on the welfare of individuals that are heterogeneous in terms of their labor market state, assets, incomes, consumption, and skills. For example, we examine how the safety net influences the consumption, savings, income, effort to search for a job, or job-finding rate of the most disadvantaged individuals. Importantly, we account for the fact that the impact of the safety net on the recipients of the transfers also works through their optimal response to the safety net, as well as through general equilibrium effects.

A few very recent papers have also developed theoretical frameworks with incomplete markets, nominal rigidities and search frictions. The objective is mainly to explain the persistent raise in unemployment and decline in job-finding rates during the recent downturn. The focus is on the amplification mechanism produced by the interaction of aggregate demand, precautionary savings and unemployment risk. A drop in aggregate demand that results in reduced new job openings by firms causes an endogenous increase in uninsurable unemployment risk, raising in turn precautionary savings, thus further depressing aggregate demand. Ravn and Sterk (2014), for example, show that in presence of such an amplification mechanism, an exogenous raise in the separation into long-term unemployment has large and persistent effects on unemployment and output. Other examples are Challe et al. (2015), Bayer et al. (2014) and Den Haan et al. (2014). Because the question asked is different and the focus is on precautionary savings, these papers make simplifying assumptions that effectively rule out some of the key channels by which the safety net operates.

Krusell et al. (2010) emphasize the trade-off of unemployment benefits between providing insurance and discouraging job creation within a search and matching model with incomplete-markets and precautionary savings, but no role for aggregate demand. Hagedorn et al. (2013) empirically investigate the bargaining channel of unemployment insurance within the context of a baseline search and matching model.

Both works only focus on one safety net program (unemployment insurance) and most importantly on only one channel (insurance in the first and bargaining in the second).

McKay and Reis (2016) study whether a variety of automatic stabilizers, including some safety net transfers, are effective at reducing business cycle volatility. They put forward a model that nests the standard incomplete markets model and the new-Keynesian model. They find that transfers to the unemployed and the needy are the most effective at stabilizing business cycles through the insurance channel. However, they take unemployment risk to be exogenous. Consequently, they rule out the amplification mechanism arising from the interaction of aggregate demand, precautionary savings and unemployment risk and thus the role of the safety net in moderating it. At the same time, they abstract from both the search incentive and bargaining channels, by which the safety would tend to amplify labor market fluctuations, rather than stabilize them.

Bitler and Hoynes (2014) investigate empirically whether the response of alternative measures of the poverty rate has changed during the Great Recession relative to prior cycles, as well as the mediating role of the four core safety net programs in the US. The find that TANF provided less protection in the Great Recession, possibly explaining more cyclical extreme poverty, while the opposite is true for SNAP and unemployment insurance, in line with evidence of less cyclical overall poverty.

We now describe in more details i) the theoretical channels by which the safety net may work; ii) the key elements of the US safety net and the channels by which they operate; iii) the features of the model needed to capture the theoretical channels and the characteristics of the actual safety net programs.

There are five theoretical channels by which the safety net can affect individual and aggregate outcomes. The first is a disposable income channel, typically emphasized in the literature on automatic stabilizers. By providing transfers to a worker experiencing either a loss of income on the job or a loss of the job itself, the safety net stabilizes her disposable income and consumption level. At the aggregate level, the disposable income channel stabilizes aggregate demand and, in presence of nominal rigidities, stabilizes business cycle fluctuations. Second, the safety net works by an insurance channel, as it provides insurance against the risk of future income drops. The safety net reduces the motive for precautionary savings by better allowing workers, who may be limited in their ability to borrow and be facing liquidity constraints, to smooth consumption over time. A reduction in precautionary savings, in turn, sustains aggregate demand during recessionary times, when the extent of idiosyncratic risk raises. Third, there is a redistribution channel. One direct effect of the safety net is to reduce poverty by transferring resources from rich to poor agents. Redistribution also has aggregate consequences: since poorer individuals have higher marginal propensities to consume than richer individuals do, aggregate demand raises after redistribution. The fourth channel is a search incentives channel. The safety net changes the relative value of work and non-work activity, affecting the marginal incentives to engage in costly search when unemployed. Specifically, transfers to the unemployed workers raise the relative value of unemployment and reduce the marginal incentives to search for a job. This is the well-known "moral hazard effect" of UI. Moreover, the search incentive channel can be amplified when unemployed workers are liquidity constrained, as emphasized in Chetty (2008). The disincentive effect of the safety net on search intensity by the unemployed reduces job-finding rates and thus amplifies cyclical fluctuations in unemployment. Transfers to low-income employed workers, such as EITC, instead, raise the relative value of work and therefore raise the marginal incentives to search when unemployed. Finally, the safety net works by a bargaining channel. By changing the relative value of work to non-work activity, the safety net affects the wages bargained by workers and firms, directly affecting the labor component of income. At the same time, by changing workers' effective bargaining power and wages, the safety net affects firms' incentives to post vacancies and hire new workers. Similarly as the search incentive channel, this effect can change the speed of recoveries in labor markets.

Following Bitler and Hoynes (2014), we focus on the four core elements of the US social safety net. Unemployment Insurance (UI) is a standard earning replacement tool for involuntarily unemployed workers. It is not means tested (eligibility does not depend on income and assets tests) and has a fixed duration, typically up to 26 weeks. Supplemental Nutritional Assistance Program (SNAP, previously called Food Stamps) is a means tested food assistance transfer to households below the poverty line. Cash welfare assistance (TANF, Temporary Assistance for Needy Families, previously called AFDC, Aid to Families with Dependent Children) is a cash transfer to single parent families with children, eligible based on low income and assets. Compared to SNAP, it targets extreme poverty levels (typically less than 50 percent of the poverty guideline). Earned Income Tax Credit (EITC) is a transfer in the form of income tax reductions targeting low-income working individuals. While all programs aim at improving the welfare of the most disadvantage individuals, each program may have different effects via the five channels described above by differently affecting the incentives of workers and firms. All four programs stabilize consumption and aggregate demand via the disposable income, insurance, and redistribution channels as they all target the poor (in case of SNAP, TANF, and EITC) and/or low-income individuals (in case of UI). Nonetheless, their relative quantitative importance is an open question. At the same time, the four programs have heterogeneous effects via the labor market channels. The components of SNAP and TANF that belong to the opportunity cost of employment and UI, when eligible, discourage search efforts and put upward pressure on wages. EITC, instead, encourages search effort and push wages of low-income workers down. At the same time, TANF implies some penalties for non-working making the effects less certain. Further differences come from the fact that the worker loses the UI transfer after finding a job while maintaining eligibility for SNAP and TANF if her income is still low, or from the fact that eligibility conditions in terms of assets limit are stricter for TANF than for SNAP. Thus, many of the labor market effects of the safety net are a priori ambiguous.

The model includes the ingredients needed to capture the five theoretical channels. First, agents are heterogeneous in terms of their labor market state, assets, incomes, and skills (in Bewley fashion) to allow for a redistribution channel. Second, workers are borrowing constrained and thus unable to perfectly smooth consumption, giving room for an insurance channel. Third, nominal rigidities in goods markets imply a disposable income channel at the aggregate level. Fourth, frictional labor markets allows for a bargaining channel. Fifth, the workers choose their job search effort during unemployment, so that there is a search incentive channel. The model is also suitable to properly account for the key elements of the different safety net programs, in terms of both the eligibility criteria and the different characteristics of the program (for example, the type transfer and the duration of participation...). In particular, frictional labor markets allows to model eligibility and duration of UI program. Income and assets heterogeneity permits to model the eligibility to SNAP, TANF and EITC programs.

In this chapter we compare to economies that differ in the strength of the safety net but otherwise identical. We show that the safety net has two important effects on individual consumption decisions. First, a stronger safety net shifts consumption policy functions of all types of workers upward, increasing the consumption of every worker given the assets level. And second, it shifts the distribution of agents over assets towards zero. Stronger safety net weakens precautionary savings motive shifting the savings policy functions downward. This effects almost cancel out if we look on the aggregate consumption, the aggregate consumption is only slightly higher with stronger safety net. However, it has important effects on the distribution of consumption across agents. The distribution becomes more concentrated around the average consumption with stronger safety net. The incidence of very low consumption disappears because the safety net creates a floor for minimum consumption. But at the same time the incidence of very high individual consumption also almost disappears due to lower savings of workers. Similar effects happen on the labor market. Search intensity policy functions shift down with stronger safety net while the distribution shifts to the left. These two effects cancel out completely due to our calibration strategy (we fix aggregate search intensity to be one in the steady state). However it has important effects on the distribution of search intensities, job finding rates and unemployment durations. With stronger safety net, long-term unemployed workers stay unemployed longer while more short-term unemployed exit unemployment faster. On the wage side, weaker safety net compresses the wages of low- and high-skilled workers.

The reminder of this chapter is organized as follows. Section 2 describes the theoretical model. Section 3 demonstrates the quantitative results. Section 4 concludes.

3.2 The model

Time is discrete. There is a measure one of infinitely-lived workers/consumers who can be either employed, short term unemployed or long term unemployed. Asset markets are incomplete and individuals face two sources of uninsurable idiosyncratic risk: endogenous risk on their employment status and exogenous risk on their skills. A safety net system partially insures workers against those risks. There is a measure ν of infinitely-lived capital owners who own the firms, have access to financial markets where all idiosyncratic risks can be insured and enjoy significant wealth. Firms are of three types: final goods firms, retailers and wholesale firms. A competitive final good sector combines differentiated varieties of intermediate goods into final goods. A measure one of monopolistic competitive retailers differentiate a wholesale good into varieties and sell them to the final goods firms. A continuum of wholesale firms hire workers in a frictional labor market to produce the wholesale good and sell it to the retailers in competitive markets.

3.2.1 Timing

Agents' decision problem is formulated according to the following intra-period timing protocol: (i) realization of aggregate shocks, (ii) vacancy posting and matching, (iii) realization of match-level separation shocks and workers' skills shocks, (v) consumption and savings, search intensity by the unemployed, production and wage bargaining. This timing implies that posted vacancies meet with workers unemployed at the end of the previous period and that newly formed matches become productive employment relationship within the same period.

3.2.2 Workers/consumers

Workers/consumers are risk-averse and cannot fully insure against idiosyncratic risk originating in the labor market. Risk takes two forms: employment risk and human capital risk.

A worker is either employed, short term unemployed or long term unemployed. Transitions between the three labor-force states are endogenous. Unemployed workers search for a job with variable search intensity, σ, and differ in their cost of exerting search effort, depending on their unemployment state. Short term and long term unemployed workers also differ in the efficiency with which they match with searching firms. Duration-dependent matching efficiencies help accounting for duration-dependent job finding rates and capture phenomena as the discrimination from the part of employers as the length of the jobless spell raises.[1] Transitions between labor force states take place as follows: an employed worker retains her job with exogenous probability ρ and loses it with complementary probability $1 - \rho$; if she loses her job, she becomes short-term unemployed and each period she finds a new job with endogenous probability ρ_s per unit of search intensity and remains unemployed otherwise; if she fails to find a job for $\bar{k}$ periods (6 months in our baseline calibration, consistently with the definition of long term unemployment in the US), she becomes long term unemployed and from then on she finds new jobs with endogenous probability ρ_l per unit of search intensity and remains long term unemployed otherwise. Workers can only exit the long term unemployment state to employment, that is, there are no transitions from long term unemployment to short term unemployment.

Workers also face idiosyncratic human capital or skill risk, h, capturing shocks to the skills of the worker, or to her productivity on the job when employed. Skill risk generates cross-sectional dispersion in labor income. This is key to model the different programs of the safety net, many of which have both their eligibility and their benefits dependent on the individual's income. Moreover, shocks to the labor income represent a second source of cross-sectional dispersion in assets, besides employment shocks. This helps matching the wealth distribution that we observe in the data. Transitions between different skill levels are exogenous and governed by a transition matrix Π, where the (i, j)th element $\pi\left(h_i, h_j\right)$ is the probability of transition from skill level h_i today to skill level h_j tomorrow.

Employed households work full time (with hours worked normalized to unit), earn a real wage w per unit of skill and suffer a constant disutility from supplying labor denoted with ς.

Market incompleteness is modeled as in the tradition of Bewley (1983), Huggett (1993) and Aiyagari (1994) models. Households can only save in a nominal non state-contingent government bond, a, and face a borrowing limit in real terms equal to $\underline{a}$.

Workers/consumers benefit from three different programs of the safety net: unemployment insurance, social assistance and a tax credit program. The programs target different groups of workers and have different benefit characteristics. Unemployment insurance targets short term unemployed workers. While in extensions we also consider the case where unemployment insurance is extended to the long term unemployed, in the baseline case we assume that unemployment benefits expire after $\bar{k}$ periods of unemployment. Social assistance is a means-tested program targeting the poor. Finally, the tax credit program targets the working poor. Individuals pay a personal

[1] See the recent CV audit studies of Ghayad (2013), Kroft et al. (2013) and Eriksson and Rooth (2014).

income tax. We describe the tax and transfer system in details in Section xx.

We now describe the problem faced by the workers/consumers in the model, depending on their employment status. Let $\mathcal{W}(a,h;\Omega)$ denote the value function of an employed consumer, given her asset holdings, a, her skill level, h, and the aggregate state, Ω. Let $\mathcal{U}^{s_k}(a,h;\Omega)$ denote the value function of a short term unemployed consumer with unemployment spell of length $k = 1,2,...,\bar{k}$, and $\mathcal{U}^{l}(a,h;\Omega)$ the value function of a long term unemployed consumer.

Employed consumers

Let i denote the nominal net interest rate on government bonds and w the wage per unit of skill. An employed consumer earns income from interest on asset holdings, $(i_{-1}/p)\,a$, and from wages, wh. The wage w is determined through Nash bargaining between the worker and the firm and the details of the bargaining process are described later. The consumer pays personal taxes, τ, on earned income, x, and may be eligible for a social assistance transfer, τ_{sa}, and a tax credit benefit, τ_{tc}. Personal taxes, social assistance and tax credits depend on individual characteristics as the assets and income. We describe the exact functional forms in Section xx.

For a given wage w, an employed consumer chooses consumption, c, and beginning-of period asset holdings, a', to solve

$$\tilde{\mathcal{W}}(w,a,h;\Omega) = \max_{c,a'} \left[u(c) - \varsigma + \beta \sum_{h'} \pi(h,h')\, E\left\{\rho \mathcal{W}(a',h';\Omega') + (1-\rho)\,\mathcal{U}^{s_1}(a',h';\Omega')\right\}\right] \tag{3.1}$$

subject to the budget constraint and the borrowing limit

$$\begin{aligned} pc + a' &= p\left[x - \tau(x) + \tau_{sa}(x,a) + \tau_{tc}(x,a)\right] + a \\ x &= x_e(a,h;\Omega) \equiv (i_{-1}/p)\,a + wh \\ a' &\geq p\underline{a} \end{aligned}$$

where ς is the fixed disutility cost of supplying labor, p is the price level, and $\tau_{sa}(\cdot)$ and $\tau_{tc}(\cdot)$ are social assistance and tax credit functions to be described below. Note that an employed worker retains her job with exogenous probability ρ and becomes short term unemployed with unemployment spell of length 1 with probability $1-\rho$.

Let the decision rule for a' for employed consumers be $a' = \tilde{\psi}^e(w,a,h;\Omega)$. Let the wage resulting from Nash bargaining be $w = \omega(a,h;\Omega)$. Then $\mathcal{W}(a,h;\Omega)$ is defined as

$$\mathcal{W}(a,h;\Omega) \equiv \tilde{\mathcal{W}}(\omega(a,h;\Omega),a,h;\Omega)$$

and we also define

$$\psi^e(a,h;\Omega) \equiv \tilde{\psi}^e(\omega(a,h;\Omega),a,h;\Omega)$$

Finally, let the decision rule for c for an employed consumer evaluated at the Nash bargained wage be $c = c^{e}(a,h;\Omega)$.[2]

Short term unemployed consumers

Short term unemployed consumers earn interest income, $(i_{-1}/p)\,a$, and receive unemployment insurance, τ_u. They may also be eligible for social assistance transfers, τ_{sa}. Unemployment benefits are taxed while social assistance benefits are not.

A short term unemployed consumer experiencing an unemployment spell of length $k = 1, 2, ..., \bar{k}$ chooses consumption, c, search intensity, σ, and asset holdings, a', to solve

$$\mathcal{U}^{s_k}(a,h;\Omega) = \max_{c,\sigma,a'} \left[u(c) - \varsigma_s(\sigma) + \beta \sum_{h'} \pi(h,h')\, E\left\{ \sigma\rho_s' \mathcal{W}(a',h';\Omega') + (1-\sigma\rho_s')\, \mathcal{U}^{s_k'}(a',h';\Omega') \right\} \right] \tag{3.2}$$

subject to the budget constraint and the borrowing limit

$$\begin{aligned} pc + a' &= p\left[x - \tau(x) + \tau_{sa}(x,a)\right] + a \\ x &= x_{s_k}(a,h;\Omega) \equiv (i_{-1}/p)\,a + \tau_u(h) \\ a' &\geq p\underline{a} \end{aligned}$$

where $\tau_u(\cdot)$ is the unemployment insurance function to be described below, where the function $\varsigma_s(\cdot)$ denotes the cost of exerting search effort for the short term unemployed, and where ρ_s' is the endogenous job finding probability next period per unit of search intensity for the short term unemployed. We assume that the search effort σ exerted today determines the number of matches formed tomorrow, as formalized in Section xx when we describe matching between workers and firms.[3] The upper-script s_k' for the value of unemployment is given by

$$s_k' = \begin{cases} s_{k+1} & if\ k < \bar{k} \\ l & if\ k = \bar{k} \end{cases}$$

indicating that, failing to find a job, the worker transits from a state of short term unemployment with spell of length k to one of short term unemployment with spell of length $k+1$, until reaching the state of long term unemployment after $\bar{k}$ periods.

[2]The value and policy functions defined for any given wage w, and denoted with a tilde symbol, are needed to formalize the wage bargaining in Section xx.

[3]Think of a worker sending out her CV today but firms only screening it tomorrow. This assumption buys us tractability as it allows to model the consumption and the search intensity decision simultaneously, rather than having search intensity chosen at the start of the period and consumption at the end of the period, which in turn would imply defining two separate value functions for the unemployed.

Let the duration dependent decision rule for a' for unemployed consumers be $a' = \psi^{s_k}(a, h; \Omega)$, with $k = 1, 2, .., \bar{k}$ indicating the length of the unemployment spell. Further, let the policy functions for consumption and search intensity be $c = c^{s_k}(a, h; \Omega)$ and $\sigma = \sigma^{s_k}(a, h; \Omega)$ with k indexing unemployment duration.

Long term unemployed consumers

Long term unemployed consumers earn interest income, may be eligible for social assistance, but have exhausted unemployment benefits.

A long term unemployed worker chooses consumption, c, search intensity, σ, and asset holdings, a', to solve

$$\mathcal{U}^l(a, h; \Omega) = \max_{c,\sigma,a'} \left[u(c) - \varsigma_l(\sigma) + \beta \sum_{h'} \pi(h, h') E\left\{ \sigma\rho_l' \mathcal{W}(a', h'; \Omega') + (1 - \sigma\rho_l') \mathcal{U}^l(a', h'; \Omega') \right\} \right] \tag{3.3}$$

subject to the budget and borrowing constraints

$$\begin{aligned} pc + a' &= p[x - \tau(x) + \tau_{sa}(x, a)] + a \\ x &= x_l(a, h; \Omega) \equiv (i_{-1}/p)\, a \\ a' &\geq p\underline{a} \end{aligned}$$

where the function $\varsigma_l(\cdot)$ denotes the cost of exerting search effort for the long term unemployed and where ρ_l' is the endogenous job finding probability next period per unit of search intensity for the long term unemployed. As for the short-term unemployed workers, the search effort exerted today determines tomorrow's likelihood of finding a job.

Let the decision rules for a', c and σ for a long term unemployed consumer be $a' = \psi^l(a, h; \Omega)$, $c = c^l(a, h; \Omega)$ and $\sigma = \sigma^l(a, h; \Omega)$.

3.2.3 Capital owners

A measure ν of risk-averse infinitely-lived capital owners or capitalists own all firms in the economy and collect dividends. They have access to complete financial markets where idiosyncratic risk can be insured. Since these agents enjoy significant wealth, this is not a strong assumption as they would be close to full self-insurance even without access to complete financial markets. For simplicity, we directly assume capital owners face no employment and skill risk. They are always employed and have fixed skill level $\bar{h}$. They borrow and save using risk-less nominal government bonds, face no borrowing constraint, earn income in the form of interest income, labor income, and dividends, and pay personal income taxes. We assume capitalists have the same period utility of the workers, but we normalize their disutility from work to 0. We assume capital owners are less impatient than workers: they discount the future at rate $\bar{\beta} > \beta$. Heterogeneous discount factors help matching the very skewed wealth distribution that is observed in the data. Finally, we assume capitalists work in wholesale firms, which they own, set up a wage for themselves equal to their revenue per unit of skill.[4]

Let $\bar{\mathcal{W}}(a;\Omega)$ denote the value function of the representative capitalist, given her asset holdings, $\bar{a}$, and the aggregate state, Ω. The representative capital owner chooses consumption, $\bar{c}$, and asset holdings, $\bar{a}'$ to solve

$$\bar{\mathcal{W}}(\bar{a};\Omega) = \max_{\bar{c},\bar{a}'} \left[u(\bar{c}) + \bar{\beta} E \left\{ \bar{\mathcal{W}}(\bar{a}';\Omega') \right\} \right] \tag{3.4}$$

subject to the budget constraint

$$\begin{aligned} p\bar{c} + \bar{a}' &= p\left[\bar{x} - \tau(\bar{x}) + \bar{\tau}\right] + \bar{a} \\ \bar{x} &= (i_{-1}/p)\,\bar{a} + \bar{w}\bar{h} + \bar{d} \end{aligned}$$

where $\bar{\tau}$ is a lump sum transfer to the capitalists and $\bar{d}$ are total dividends.

3.2.4 Final good firms, retailers and price setting

A competitive sector for final goods combines differentiated varieties of intermediate goods according to the production function

$$Y = \left(\int_0^1 (y_j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{3.5}$$

where y_j is the input of intermediate good j and ϵ is the elasticity of substitution across varieties. Final goods firms purchase intermediate good j at price p_j and take as given the final goods price p. From cost minimization, it follows that the demand for variety j is given by

$$y_j = \left(\frac{p_j}{p} \right)^{-\epsilon} Y, \tag{3.6}$$

and the price index p is given by

$$p = \left(\int_0^1 (p_j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}. \tag{3.7}$$

A measure one of monopolistic competitive retailers buy a wholesale good from wholesale firms, differentiate it into varieties y_j with a technology that transforms one

[4] Since capitalists collect total dividends, their income $\bar{x}$ is unaffected by the way their wage is set. Assuming the wage equals revenues per unit of skills just simplifies the equation for wholesale firms' dividends.

unit of wholesale good into one unit of intermediate good and sell it to the final goods producers. Retailers set prices infrequently as in Calvo (1983) with probability of revision θ. At each revision date, a retailer producing variety j chooses an optimal price p_j^* to maximize expected future profits, subject to the demand for its own variety. As retailers are owned by the capitalists, they use their discount factor $\Lambda\left(\Omega,\Omega'\right)$. Let q be the relative price of the wholesale good and χ a fixed cost of production. The price setting problem of retailer j at each revision date t can be written as

$$\max_{p_j^*} \Pi\left(p_j^*;\Omega\right) \tag{3.8}$$

with

$$\Pi\left(p_j;\Omega\right) = d_j\left(p_j;\Omega\right) + (1-\theta)\,\Lambda\left(\Omega,\Omega'\right)\Pi\left(p_j;\Omega'\right)$$

and

$$d_j\left(p_j;\Omega\right) = \left(\frac{p_j}{p} - q\right) y_j - \chi$$

and subject to the demand equation (4.1).

3.2.5 Wholesale goods firms and wage bargaining

Firms hire workers in a frictional labor market and produce the wholesale good. To hire a worker, a firm must post a vacancy. There is a flow cost of posting a vacancy, denoted with κ. Each firm has a single job that can be either filled or vacant.

The value of posting a vacancy, $\mathcal{V}\left(\Omega\right)$, is

$$\mathcal{V}\left(\Omega\right) = -\kappa + \rho_f \bar{\mathcal{J}}\left(\Omega\right) + \left(1-\rho_f\right) E\left\{\Lambda\left(\Omega,\Omega'\right)\mathcal{V}\left(\Omega'\right)\right\}, \tag{3.9}$$

where $\bar{\mathcal{J}}\left(\Omega\right)$ is the average value of a filled job under random search. It depends on the distribution of unemployed workers over assets, skills and duration of the unemployment spell, as these determine the savings and search intensity decisions of the unemployed workers (see the Appendix for its expression). Free entry ensures that in equilibrium $\mathcal{V}\left(\Omega\right) = 0$.

For a given wage w, the value of having a job filled with a worker with assets a and skills h, $\tilde{\mathcal{J}}\left(w,a,h;\Omega\right)$, is given by

$$\tilde{\mathcal{J}}\left(w,a,h;\Omega\right) = qzh - wh + E\left\{\Lambda\left(\Omega,\Omega'\right)\left[\rho\sum_{h'}\pi\left(h,h'\right)\mathcal{J}\left(\tilde{\psi}^e\left(w,a,h;\Omega\right),h';\Omega'\right) + (1-\rho)\,\mathcal{V}\left(\Omega'\right)\right]\right\}, \tag{3.10}$$

where q is the relative price of the wholesale good and z is aggregate labor productivity per unit of skill. The term $qzh - wh$ is profit per worker. Recall $\tilde{\psi}^e\left(w,a,h;\Omega\right)$ is next period asset holdings of an employed worker with assets a and skills h at wage w. $\mathcal{J}(a,h;\Omega)$ is defined as

$$\mathcal{J}(a,h;\Omega) \equiv \tilde{\mathcal{J}}(\omega(a,h;\Omega), a, h; \Omega),$$

where $w = \omega(a,h;\Omega)$ is the solution to the bargaining problem to which we turn now.

The wage is determined by Nash bargaining between wholesale firms and workers. The wage maximizes the Nash product:

$$\max_{w} \left(\tilde{\mathcal{W}}(w,a,h;\Omega) - \mathcal{U}^{s,1}(a,h;\Omega)\right)^{\eta} \left(\tilde{\mathcal{J}}(w,a,h;\Omega)\right)^{1-\eta}, \tag{3.11}$$

where $\eta \in (0,1)$ represents the bargaining power of the worker, $\tilde{\mathcal{W}}(w,a,h;\Omega) - \mathcal{U}^{s,1}(a,h;\Omega)$ is the surplus from employment at wage w for a worker with asset a and skills h, and $\tilde{\mathcal{J}}(w,a,h;\Omega)$ is the firm surplus from a filled job at wage w with a worker with assets a and skills h. The outside option in bargaining for employed workers is short term unemployment with unemployment spell of length 1. The bargained wage depends on both the worker assets and skills.

3.2.6 Labor market matching

Firms with vacancies and unemployed workers randomly match each period according to a matching technology

$$M = \chi \Sigma_{-1}^{\alpha} V^{1-\alpha} \tag{3.12}$$

where M denotes the number of matches within a period when firms post V vacancies and unemployed workers at the end of previous period exert Σ_{-1} units of search intensity. We assume that the search effort exerted by workers unemployed at the end of the previous period determines the number of matches formed today. The matching function has standard properties with χ reflecting the efficiency of the matching process and α the elasticity of matches to units of search effort from unemployed workers. We assume that long term unemployed workers are less efficient than short term unemployed in matching with firms by a factor $0 < \phi < 1$. Aggregate search intensity is given by

$$\Sigma = \int_{a,h} \left[\sum_{k} \sigma^{s_k}(a,h;\Omega) f^{s_k}(a,h;\Omega) + \phi \sigma^{l}(a,h;\Omega) f^{l}(a,h;\Omega)\right] dadh \tag{3.13}$$

where $\sigma^{s_k}(a,h;\Omega)$ and $\sigma^{l}(a,h;\Omega)$ are, respectively, the search intensity of a short term unemployed worker with unemployment spell of length k, assets a and skills h and the search intensity of a long term unemployed worker with assets a and skills h; and where $f^{s_k}(a,h;\Omega)$ and $f^{l}(a,h;\Omega)$ are, respectively, the measure of unemployed workers with unemployment spell of length k over bond holdings and skill levels and the measure of long term unemployed workers over bond holdings and skill levels. The search intensity of the long term unemployed is weighted by ϕ.

The probability a firm fills a vacancy, ρ_f, the probability a short term unemployed workers finds a job per unit of search effort, ρ_s, and the probability a long term unemployed workers finds a job per unit of search effort, ρ_l, are given by

$$\rho_f = \frac{M}{V} = \chi\left(\frac{V}{\Sigma_{-1}}\right)^{-\alpha} \tag{3.14}$$

$$\rho_s = \frac{M}{\Sigma_{-1}} = \chi\left(\frac{V}{\Sigma_{-1}}\right)^{1-\alpha} \tag{3.15}$$

$$\rho_l = \phi\rho_l \tag{3.16}$$

The law of motions for employment N can be described by

$$N = \rho N_{-1} + M \tag{3.17}$$

where separation only hits existing matches and newly-formed matches become productive within the same period.

3.2.7 Government and tax and transfer system

We introduce three programs of the safety net: unemployment insurance, social assistance and a tax credit . These programs target different groups of workers and have different benefit characteristics. Let us describe each of them in turn.

Unemployment insurance targets the short term unemployed workers. We will also consider the case where unemployment insurance is extended to the long term unemployed workers. In the baseline case, we assume that unemployment insurance expires after $\bar{s}$ periods of unemployment and is governed by the following function of the worker's skill:

$$\tau_u(h) = \bar{\tau}_u min\left\{h, \bar{h}_u\right\},$$

where $\bar{\tau}_u$ is the benefit level per unit of skill and $\bar{h}_u$ defines a maximum cap to the level of benefits.[5] Given the persistence of the skill process, making unemployment benefits dependent on the worker's current skill level is a way of capturing the dependence of benefits on past labor incomes.

Social assistance is a means-tested program targeting the poor. A household is eligible for social assistance if his income is below a certain percentage of the poverty line and his liquid assets do not exceed a certain limit. We assume that social assistance is governed by the following function

[5] We borrow this specification from McKay and Reis (2016).

$$\tau_{sa}(x,a) = I_{sa}(a)\, max\left\{(\bar{\tau}_{sa} - \beta_{sa}x), 0\right\},$$

where $I_{sa}(\cdot)$ is an indicator function taking the value of 1 if the individual's current assets are below the eligibility threshold, and taking the value of 0 otherwise. Conditional on asset eligibility, the social assistance transfer equals the maximum benefit amount, $\bar{\tau}_{sa}$, minus the individual's contribution, $\beta_{sa}x$. The recipient's contribution raises with her income at rate β_{sa}, with $\beta_{sa} < 1$. The income eligibility threshold is implicitly defined to be $\bar{\tau}_{sa}/\beta_{sa}$.

Finally, we introduce a tax credit for the working poor, governed by the function

$$\tau_{tc}(x,a) = I_{tc}(a)\, max\left\{min\left\{\bar{\tau}_{tc}, \alpha_{tc} - \beta_{tc}x\right\}, 0\right\},$$

where $I_{tc}(\cdot)$ is an indicator function taking the value of 1 if the individual's current assets are below the eligibility threshold, which is specific to the tax credit program, and the value of 0 otherwise. The tax credit payment depends on the worker's income x, with $\alpha_{tc} > \bar{\tau}_{tc}$ and $\beta_{tc} < 1$. The function $\tau_{tc}(x,b)$ is first constant and then decreasing, capturing the property that the tax credit amount first equals a maximum level and then begins to phase out at higher income levels.[6] The maximum tax credit benefit is $\bar{\tau}_{tc}$ and the income eligibility thresholds is α_{tc}/β_{tc}.

A key element of the model is the personal income tax system. The household's income, x, is taxed according to a tax function $\tau(\cdot)$. We assume the tax system is progressive, that is, the function $\tau(\cdot)$ is increasing and convex.

The government satisfies the budget constraint:

$$\begin{aligned} pG + (1+i_{-1})B - B' = p\int_{a,h}\left[\sum_r \tau(x_r(a,h;\Omega),a) f^r(a,h;\Omega)\right] dadh + p\nu\tau(\bar{x}) \\ - p\int_{a,h}\left[\sum_r \tau_{sa}(x_r(a,h;\Omega),a) f^r(a,h;\Omega)\right] dadh \\ - p\int_{a,h}\left[\tau_{tc}(x_e(a,h;\Omega),a) f^e(a,h;\Omega)\right] dadh \\ - p\int_{a,h}\left[\tau_u(h) f^u(a,h;\Omega)\right] dadh - p\nu\bar{\tau} \end{aligned} \tag{3.18}$$

where G is the government spending, B is nominal government debt, and $r \in \left\{e; s_1; s_2; ...; s_{\bar{k}}; l\right\}$ indexes labor force states.

Monetary policy follows a Taylor rule:

$$i = \bar{i} + \phi\log(\pi) + \varepsilon \tag{3.19}$$

[6]In reality there is an initial phase-in. The benefit is increasing at very low levels of income before reaching the maximum level. However, the rationale for this property has to be to incentivize work at the intensive margin. Since for simplicity we focus on the extensive margin we abstract from the initial phase-in.

with $\phi > 1$, $\pi = p/p_{-1}$ and ε denotes a monetary policy shock that follows an AR(1) process.

Government spending and capital owners transfers follow deterministic policy rules:

$$\log(G) = \log(\overline{G}) - \gamma_G \log\left(\frac{B/p}{\overline{B}}\right)$$

$$\overline{\tau} = \overline{\tau}^* - \gamma_\tau \log\left(\frac{B/p}{\overline{B}}\right)$$

where γ_G and γ_τ measure the sensitivity of reaction of government spending and transfers to the level of government debt.

3.2.8 Equilibrium

A recursive equilibrium is a solution for (i) the value functions $\{\mathcal{W}(a,h;\Omega), \mathcal{U}^{s_k}(a,h;\Omega), \mathcal{U}^{l}(a,h;\Omega)$ and the policy functions $\{\psi^e(a,h;\Omega), \psi^{s_k}(a,h;\Omega), \psi^l(a,h;\Omega), c^e(a,h;\Omega), c^{s_k}(a,h;\Omega), c^l(a,h;\Omega), \sigma$ $\sigma^l(a,h;\Omega)\}$ for workers, with $k = 1, 2, .., \bar{k}$; (ii) the value functions $\{\mathcal{J}(a,h;\Omega), \mathcal{V}(\Omega)\}$ for wholesale firms; (iii) the wage $\omega(a,h;\Omega)$ per unit of skills; (iv) consumption $\bar{c}$, asset holdings $\bar{a}'$ and dividends $\bar{d}$ for capital owners; (v) individual retail firms variables $\{y_j, p_j, d_j\}$; (vi) a set of aggregate quantities and transition rates $\{Y, C, N, M, V, \Sigma, \rho_f, \rho_s, \rho_l\}$; (vii) the final goods price p and the relative price of wholesale goods q; (viii) government choices $\{B', G, i\}$ and; (ix) measures of workers over assets and skill levels $\{f^e(a,h;\Omega), f^{s_k}(a,h;\Omega),$
$f^l(a,h;\Omega)\}$, with $k = 1, 2, .., \bar{k}$. The solution is such that: (i) the value and policy functions for workers solve (3.1)-(3.3); (ii) the value of posting a vacancy satisfies the free entry condition, $\mathcal{V}(\Omega) = 0$; (iii) $\omega(a,h;\Omega)$ solves the Nash bargaining condition (3.11); (iv) $\bar{c}$ and $\bar{a}'$ solve the capital owners problem (3.4) and $\bar{d}$ are defined in the appendix; (v) final goods firms behave according to (4.1) and (4.2); (vi) retailers maximize (4.3); (vii) aggregate employment evolves according to (3.17); (viii) aggregate search intensity is determined by (3.13); (ix) the transition probabilities are determined by (3.14)-(3.16); (x) markets for goods and assets clear as described in the appendix; (xi) fiscal policy satisfies (3.18) and monetary policy satisfies (3.19); (xii) the evolution of the measures of workers over assets and skill levels are consistent with a transition function $\mathcal{Q}$; (xiii) $\mathcal{Q}$ is defined in the appendix.

3.2.9 Computation

To solve for the equilibrium we follow the algorithm by Reiter (2009). First, we solve for the steady state. To do that, we approximate the distributions with histograms, the policy and value functions with vectors over a fixed grid. We use the following iterative procedure: first, we guess disposable income of the capital owners, this determines the steady state interest rate; then, we guess the search cost of unemployed workers, that determines the transition matrix for individual labor market states. Given these two guesses we solve a system of equations that consists of the sharing rules for the Nash bargaining problem, consumers' first order conditions and Bellman equations, and firms' Bellman equations. The solution to the system contains the wage schedule as a function of assets and human capital, decision rules for future assets and search effort, and workers' and firms' value functions. Transition matrix for labor market states and the decision rules for future assets allow us to construct the function Q (that is a matrix in this case) and find an ergodic distribution F (that is a vector). Given F and decision rules for the search effort we find Σ and compare it to 1 (normalization in the steady state). If $\Sigma > 1$ we increase the search cost, and if $\Sigma < 1$ we decrease the search cost. We iterate on the search cost until convergence with $\Sigma = 1$ up to some tolerance. After we found the search cost, we compute other aggregate variables, such as output, profits, aggregate taxes and transfers. We then find the aggregate level of bonds from the government budget constraint and the bonds held by the capital owners from bonds market clearing condition. Given, capital owners' bonds and dividends we update the guess for disposable income of the capital owners. We iterate until convergence.

To solve for the dynamics, following Reiter (2009), we create a function containing all the conditions of the recursive equilibrium given all the variables (aggregate variables, histogram, and the approximations of the decision rules) at period $t-1$ and t, exogenous shocks, and the expectation errors that substitute expectations in the definition of the equilibrium. We linearize the system around the steady state numerically using the first difference approach. Then we use the algorithm by Sims (2001) to solve the linearized system for VAR formulation (also to get rid of the expectation errors). VAR formulation allows us to find impulse responses to the shocks and simulate the model.

3.3 Results

To study the effects of the social safety net it is important to look at the individual level rather than aggregate level variables. Aggregate consumption is not informative about the distribution of consumption, for example. The aggregates mask the heterogeneity of consumption and well-being across individual agents. The model studied in the chapter allows us to do it, analyzing the effects of the safety net on the individual consumption, well-being, and labor market outcomes. For example, if due to a change in the safety net the aggregate consumption increases, it is not very informative about the individual well-being. Aggregate consumption may increase together with the variance of the consumption across individuals. The aggregate consumption rises because the consumption of the rich agents increases a lot while the consumption of poor agents (who are also the targets of the safety net) drops. Such a change in the safety net shouldn't be made. In general, there are several channels through which the safety net affects the individual well-being and many factors mater for the overall effect of the safety net. For example, how distribution of agents over assets reacts, how the policy functions adjust, and how the transition probabilities change. All these factors are also affected by the five channels discussed before.

With a stronger safety net all consumption policy functions shift up, however the distribution over assets also shifts to the left. And due to the concavity of the policy functions even the final change in the individual consumption is ambiguous. At the same time the transition probabilities across individual states (and, hence, policy functions) change. With stronger safety net, consumption of unemployed is higher, but the duration of unemployment could be longer due to lower incentives to search for a job. On top of this, the model also allows to look directly at the individual welfare through the value functions. The value functions take into account not only current and future expected consumption but also expected transitions across labor market states.

In this chapter we compare two steady states of the model with strong and weak safety net. We look at wealth distributions, consumption policy functions, consumption distributions, search intensity policy function, job finding rates, unemployment duration, wages. We make a first step to disentangle the channels of the safety net: redistribution, disposable income, insurance, search intensity, and wage bargaining; and study which are the channels affected the most by each program. If for static distributions it is better to look at the individual level, with dynamics it would be too complicated, so we still look also on the aggregate dynamics.

3.3.1 Calibration

We follow closely McKay and Reis (2016) in our calibration of the model except for the labor market parameters, that is a new element of the model, and the safety net, that we calibrate to follow closely the U.S. safety net programs. Calibrated parameters are presented in Table 3.1.

3.3.2 Safety net

The functional forms of the safety net programs is discussed in section 2.7. Figure 3.1 plots social assistance and tax credit as functions of agent's income. Social assistance is available for every agent, provided that income is low enough so that the payment is positive. Tax credit is available only to the employed agents. Social assistance starts

Parameter	Meaning	Value	Parameter	Meaning	Value
β	Discount factor of CO	0.989	$\overline{\tau}_u$	unemployment benefit	0.25
β^h	Discount factor of HH	0.979	$\overline{\tau}_{sa}$	SA max	0.2
ς	Disutility of work	0	β_{sa}	SA slope	0.2
$\underline{a}$	Borrowing constraint	0	$\overline{\tau}_{tc}$	TC max	0.1
μ	Mark up	1.2	α_{tc}	TC flat region	0.2
$\overline{h}$	CO skill	4	β_{tc}	TC slope	0.1
ν	CO number	0.25	h	productivity vector	[0.8 1.5]
ξ	Fixed cost of retailers	0	p	probability to change skill	0.03
C	LTU search cost $(1+C)$	0.5	$\bar{s}$	STU duration	2
$1-\rho$	separation rare	0.05	ϕ^{Taylor}	Taylor rule	1.55
η	worker bargaining power	0.5	ρ^z	autocor z	0.75
χ	matching efficiency	0.6	σ^z	st dev z	0.0034
α	matching elasticity	0.72	ρ^{mu}	autocor μ	0.85
ϕ	LTU search efficiency	0.9	σ^{mu}	st dev μ	0.025
θ	price stickiness	0.286	ρ^{ε}	autocor ε	0.62
τ_{max}	marginal tax cap	0.4	σ^{ε}	st dev ε	0.0035
τ_{mult}	tax level	0.2			
τ_{power}	progressivity	0.5			
τ_c	consumption tax	0.02			

Table 3.1: Calibration

from a maximal value that guarantees to every agent a minimum level of consumption and then slowly phases out with income. Tax credit has a different structure, it stays on the maximum level for very low incomes before starting to phase out. Figure 3.1a corresponds to a strong safety net with maximum social assistance equal to 0.3 and maximum tax credit equal to 0.1 while 3.1b corresponds to a weak safety net with maximum social assistance and tax credit being 0.1 and 0.05 respectively.

Figure 3.2 plots the social assistance as a function of assets for different types of agents. There are eight types of agents: two skill levels - high and low skill; and four labor market states - employed, short-term unemployed for one or two quarters, and long-term unemployed. We can see from the figure that the social assistance is maximum for the long-term unemployed who has no labor income. Low-skilled short-term unemployed also receive social assistance in both cases (with strong and weak safety net), while high-skilled short-term unemployed and low-skilled employed agents receive the social assistance only in the case of strong safety net. Social assistance slowly phases out with individual assets due to growing capital income.

Figure 3.3 plots tax credit as a function of assets and individual state. Tax credit is available only to employed agents and in both cases only the low-skilled employed agents receive the tax credit. Tax credit also slowly phases out with individual assets due to growing capital income.

In Figure 3.4 we plot the unemployment insurance as a function of assets and individual state. Unemployment insurance is available only to the short-term unemployed and is higher for high-skilled workers. Unemployment insurance doesn't depend on the current non-labor income and asset level, and therefore is constant.

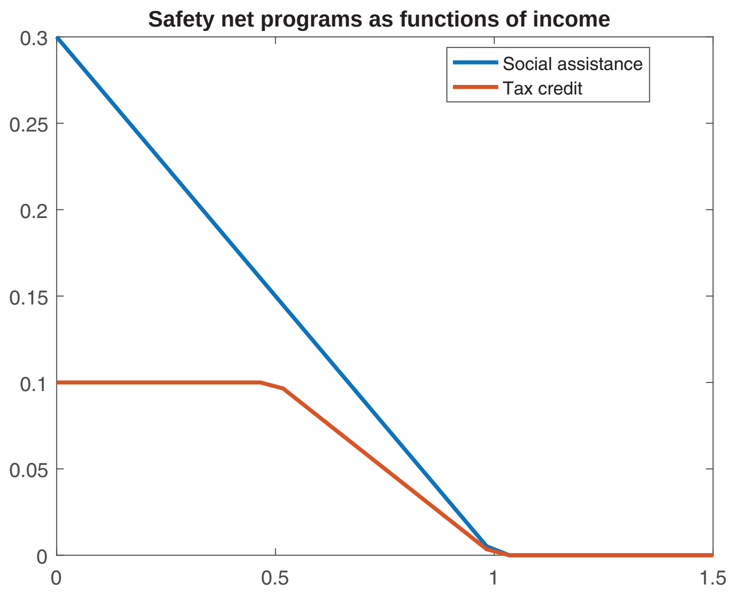
3.3.3 Wealth distribution

First, we study how different strength of the safety net affects the steady state distributions of agents. Figure 3.5 plots conditional and absolute wealth distributions for each employment state with strong safety net. We can see that the distribution is very concentrated on the left with employed and short-term unemployed agents having almost indistinguishable distributions, and long-term unemployed being more concentrated around zero, so that more of long-term unemployed are borrowing constrained. Figure 3.6 plots corresponding distribution for weak safety net. Figure 3.7 further distinguishes between low- and high-skilled employed workers. For both safety net programs it is only the high-skilled employed who save enough to stay far from the borrowing constraint.

The difference between the safety net programs is striking, distributions under weak safety net are much more spread across the asset levels. With weak safety net the maximum asset holdings of a worker is around 5 while with strong safety net it

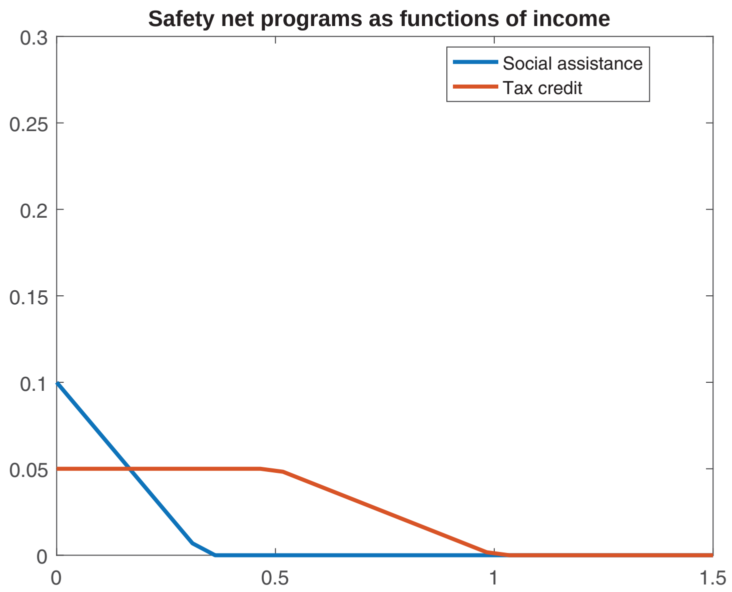
Figure 3.1: Safety net programs as functions of income

(a) Strong safety net



.

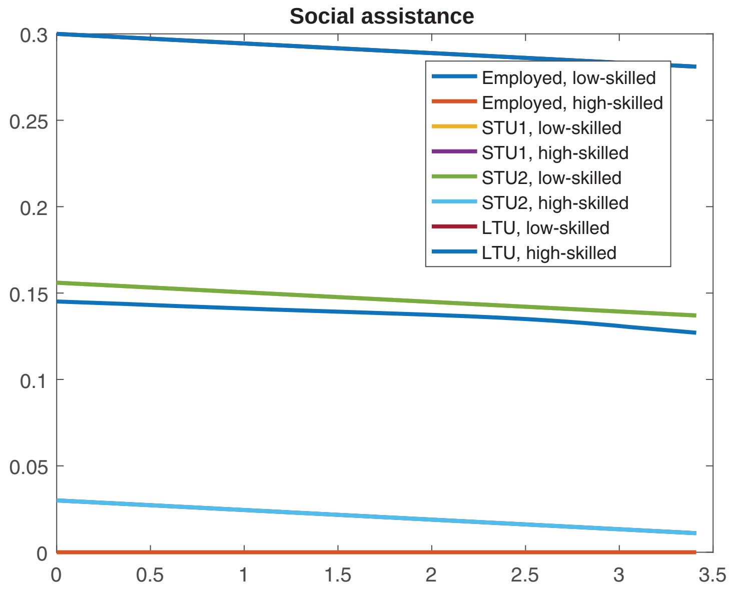
(b) Weak safety net



.

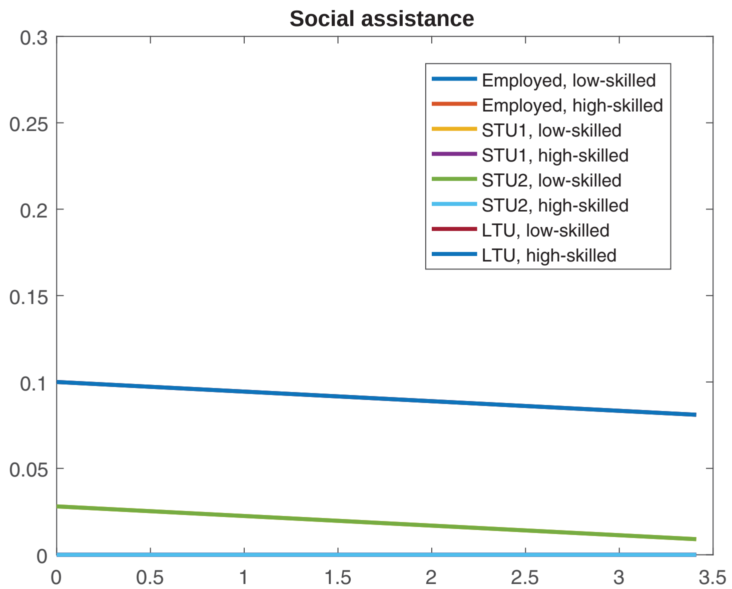
Figure 3.2: Social assistance

(a) Strong safety net



.

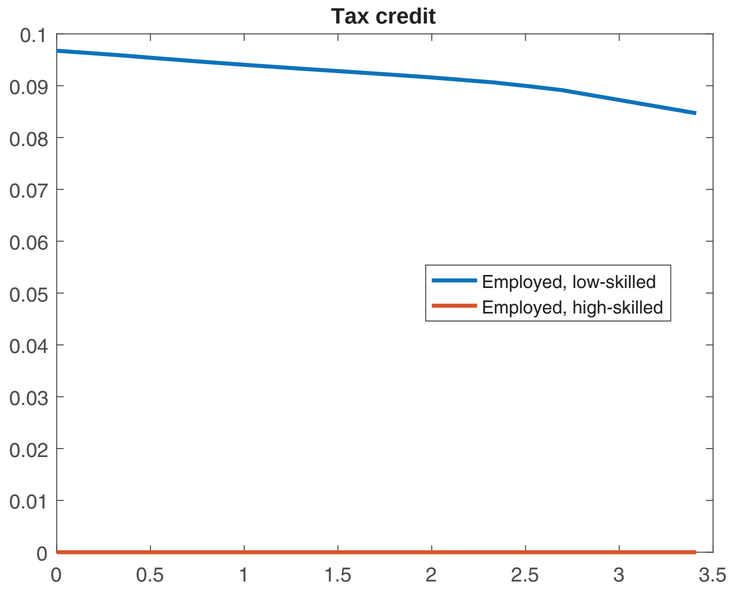
(b) Weak safety net



.

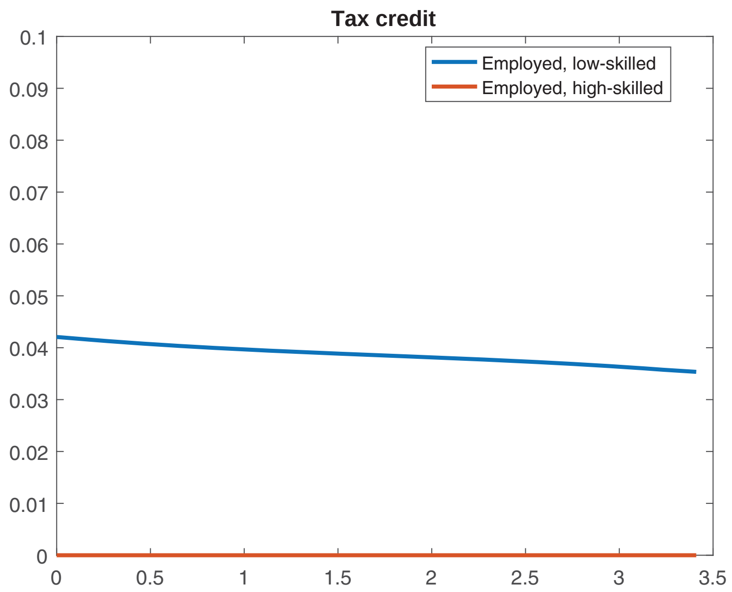
Figure 3.3: Tax credit

(a) Strong safety net



.

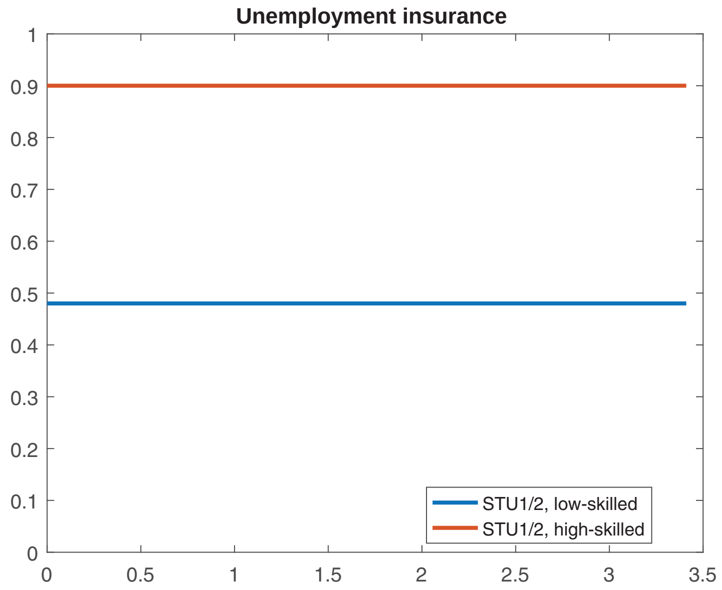
(b) Weak safety net



.

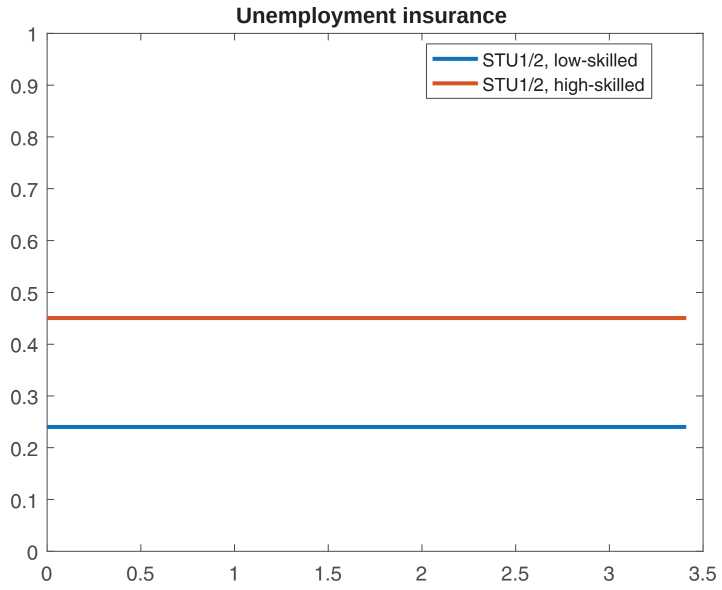
Figure 3.4: Unemployment insurance

(a) Strong safety net



.

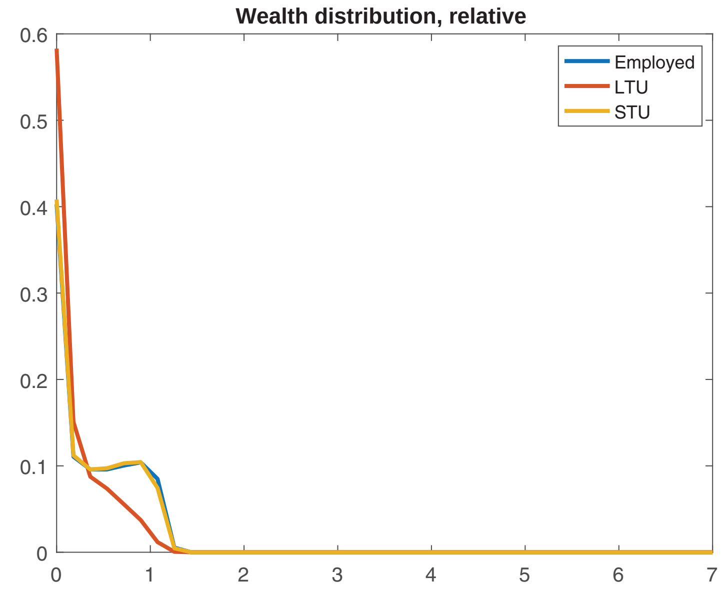
(b) Weak safety net



.

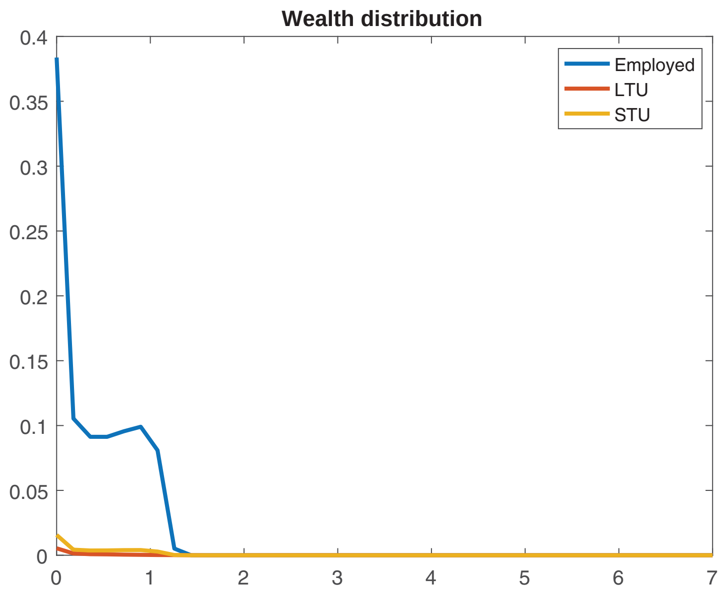
Figure 3.5: Wealth distribtion with strong safety net

(a) Conditional



.

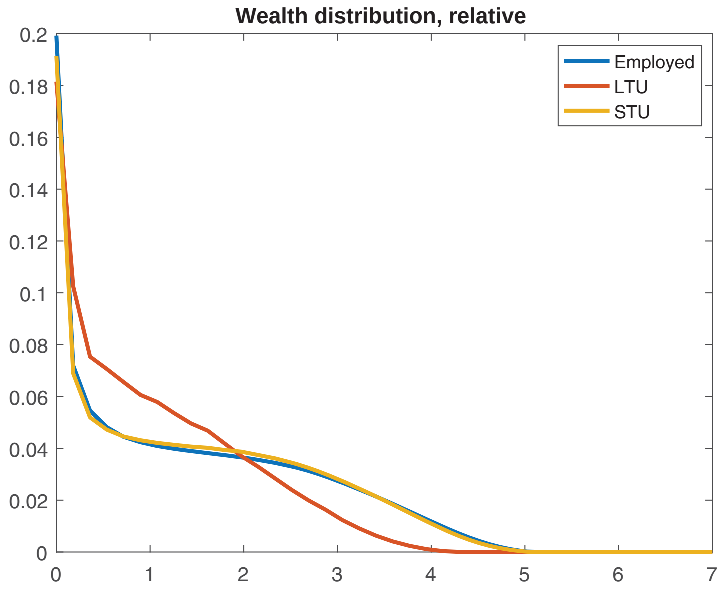
(b) Absolute



.

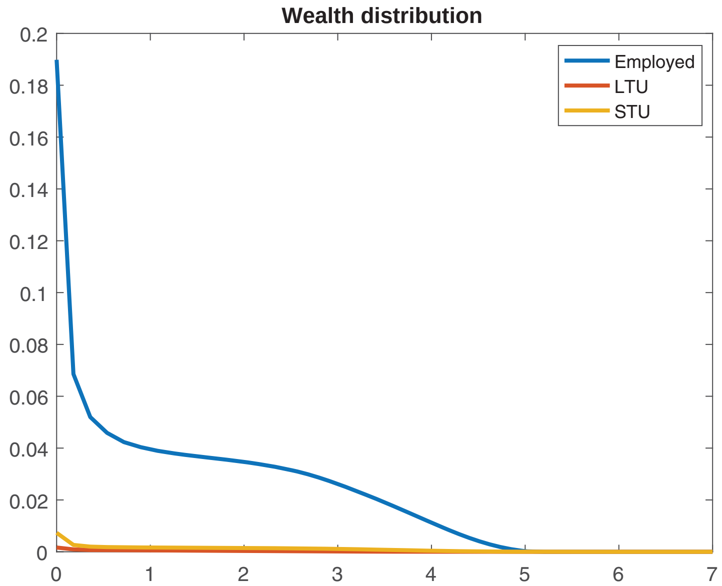
Figure 3.6: Wealth distribtion with weak safety net

(a) Conditional



.

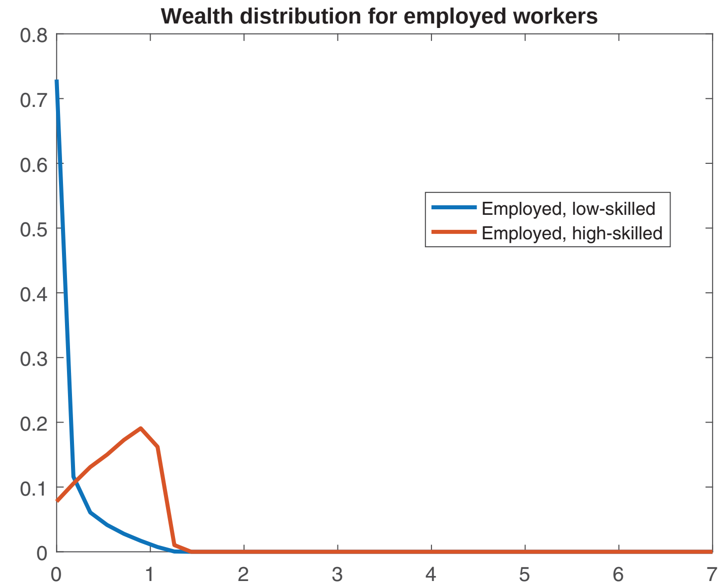
(b) Absolute



.

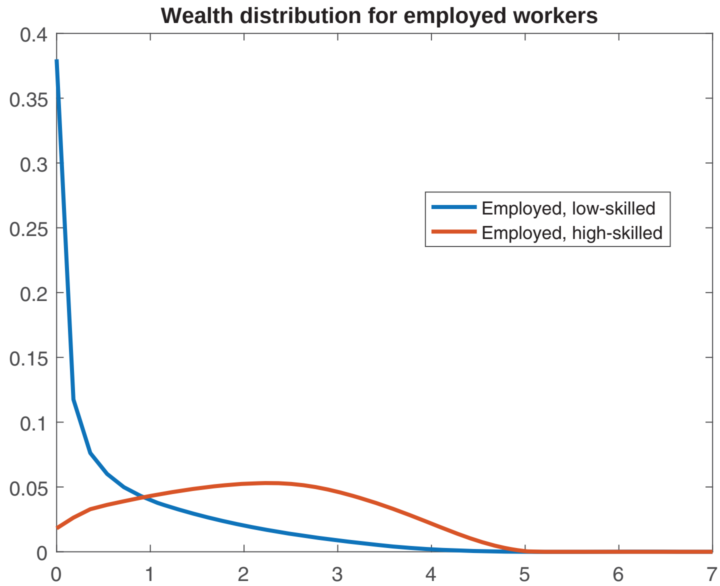
Figure 3.7: Wealth distribution of employed

(a) Strong safety net



.

(b) Weak safety net



.

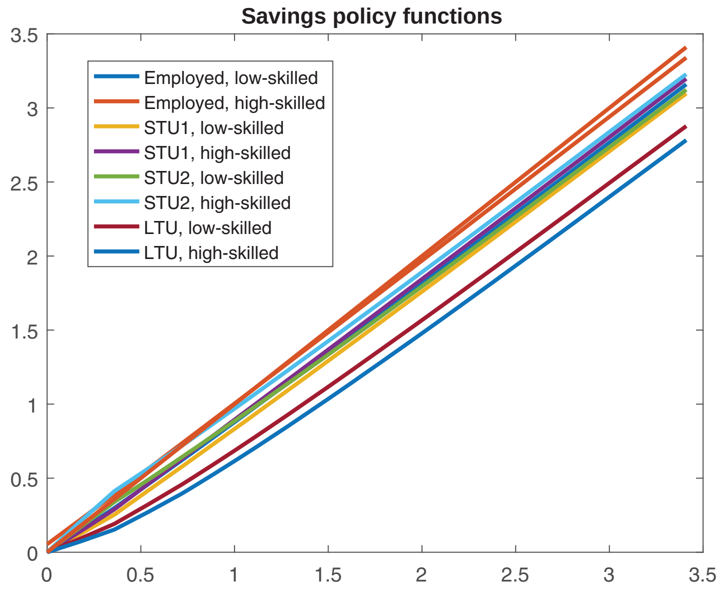
barely above 1. The reason for this difference is the effect of the safety net on precautionary savings. With strong safety net the agents are insured from huge drops in consumptions due to a job loss. The minimum guaranteed consumption with a strong safety net is 0.3, three times higher than with a weak safety net. This has a very large effect on precautionary savings. Agents, knowing that even if they will be very unlucky to lose the job and stay unemployed for long enough to spend all their savings, their consumption won't drop close to zero, have no need to accumulate huge buffers of savings, therefore consuming more in every state. It can be also seen from the savings policy functions (Figure 3.8), policy function of high-skilled employed agents is much higher in the case of weak safety net. The high-skilled employed agents are not directly affected by any of the safety net programs because they do not qualify by income or labor market state eligibility criteria. Despite this, they save more with a weaker safety net. It happens almost purely because of the rising precautionary savings motives (they are also affected by general equilibrium effects) that are very strong even though the probability of transiting to low-skilled long-term unemployment is very low: they have 3% chance to lose their skill, 5% chance to lose the job, and around 10% chance to stay unemployed two quarters in a row, and all these events must happen almost simultaneously.

3.3.4 Consumption

After studying the distribution over assets we can study how individual consumption responds to changes in the safety net. Figure 3.9 plots the consumption policy function depending on the individual assets for all eight individual states. Let us start by making a couple of observation on the general form of the policy functions. First, the value of consumption at zero corresponds to the labor income plus safety net payments for all but high-skilled employed workers. Essentially, the agents at the zero assets are hand-to-mouth, they consume all their income the same period. The only exception are the high-skilled employed workers who have high enough income to save and move away from zero. The hand-to-mouth nature of the consumption at zero assets generates high variance of consumption. Second, the policy functions converge with higher assets. It happens because when agents become richer and richer, their buffer of savings is so high that they can self-insure against the idiosyncratic shocks. Given these two observations, we can observe the differences in curvature of the policy functions across individual states. The policy functions of long-term unemployed have the steepest slope around zero while the policy function of high-skilled employed workers is almost flat, the borrowing constraint is not binding for them. These differences generate a high potential for redistribution channel: taking money from agents with flat policy function and giving them to agents with a very steep policy function gen-

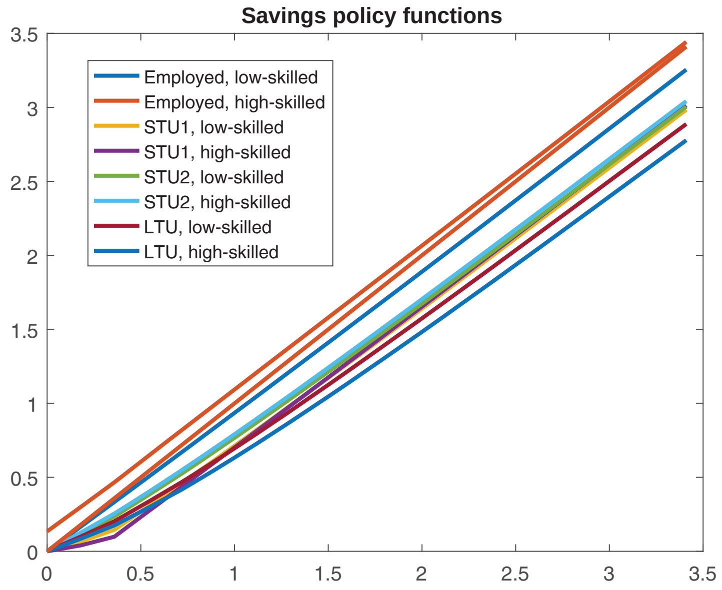
Figure 3.8: Savings policy functions

(a) Strong safety net



.

(b) Weak safety net



.

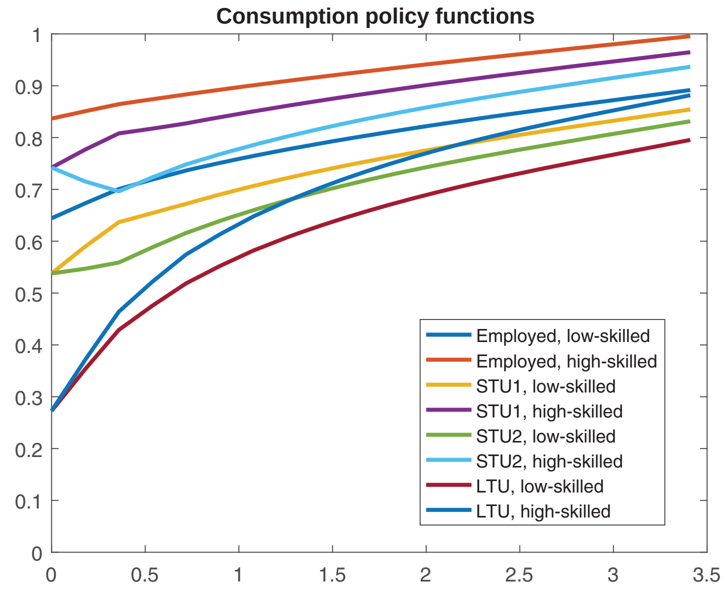
erates an increase in aggregate consumption. Another important consequence of this shape of the policy functions is the importance of the distribution of agents over assets. When policy functions are higher, consumption of each agent given assets level is higher but if there are more agents with low assets at the same time, the overall effect on aggregate consumption is ex ante ambiguous, the curvature effect may dampen or even cancel out the effect of higher policy functions.

Keeping this in mind, we can now study the difference of policy functions between strong and weak safety nets. First of all, the safety net has a direct effect on the hand-to-mouth agents (at zero assets level). With stronger safety net consumption of agent of each individual state is larger at zero (even for high-skilled employed workers who are unaffected directly as discussed above) and the dispersion of the consumption at zero is also lower. So, stronger safety net compresses the policy functions of consumption. If we look at the policy functions away from zero, there are several channels at place. There is still the direct effect of the safety net programs that expand the budget set, and on top of it there is a reduction in the precautionary savings motive that also moves the policy functions up. In general, these effects are difficult to disentangle, but the policy function of high-skilled employed workers helps us to assess the magnitude of the effect of precautionary savings. High-skilled employed workers are unaffected by the transfers directly in both cases, but their policy function still shift up by more than 10% suggesting a strong effect of precautionary savings motive.

The shift of the policy functions per se is not informative about individual consumption because, as we have seen in the previous section, the distribution is more concentrated around zero with strong safety net. Because of concavity of the policy functions, more compressed distribution reduces consumption of each agent (if we fix the policy functions). To see the overall effect, we plot the distribution of individual consumption in Figure 3.10. Comparing the two distributions, we can see that the maximum consumption doesn't change much for the two safety net levels. It means that for the richest agents the curvature and the distribution effects cancel out. For the poorer agents, however, the effects are very strong. Because the social assistance determines a minimum level of consumption, it is much higher with the strong safety net than with weak: there are no agents with consumption below 0.3 in the case of strong safety net while there is a positive mass of such agents in the case of weak safety net. In general, the distribution of consumption is more compressed in the case of strong safety net, and this compression happens on a higher level. For example, almost all mass is to the right of 0.6 in the case of strong safety net, while there is a large mass below 0.6 in case of weak safety net. Given these observations we can conclude that positive effects of a stronger safety net dominate for most of the agents, especially for the poor for whom the safety net is targeted.

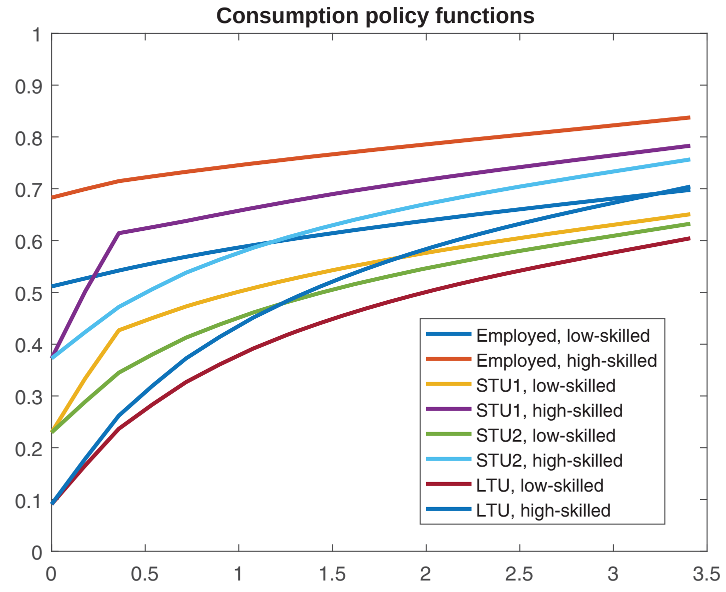
Figure 3.9: Consumption policy functions

(a) Strong safety net



.

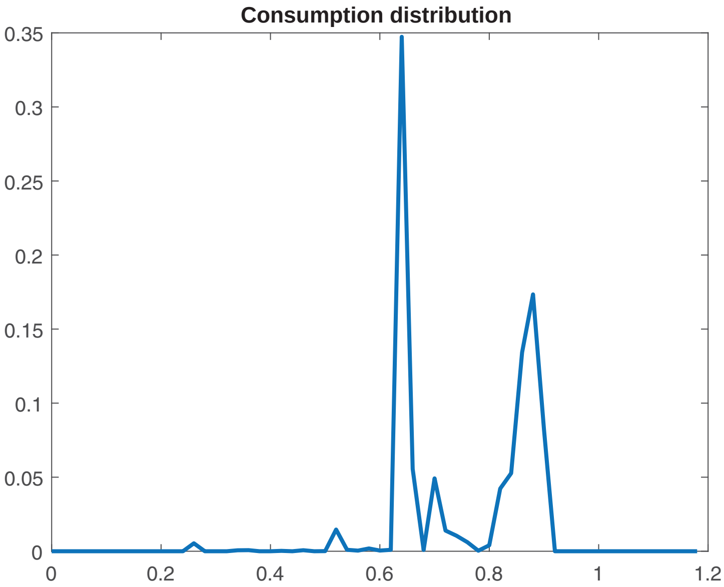
(b) Weak safety net



.

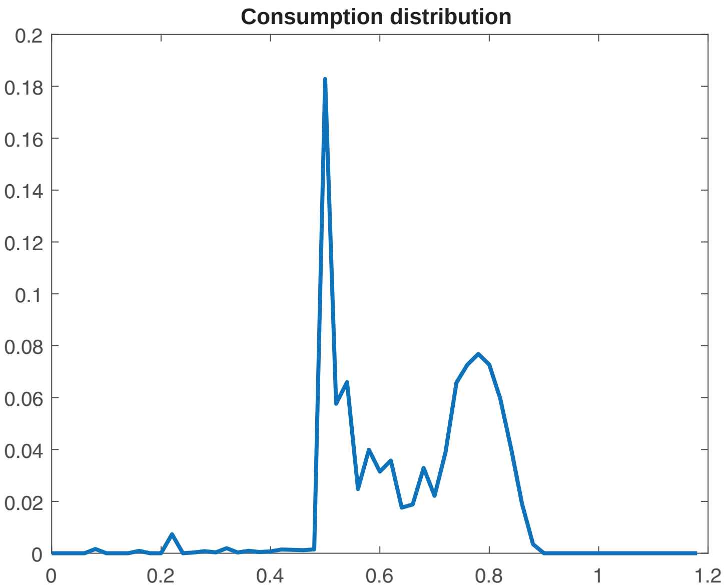
Figure 3.10: Consumption distribution

(a) Strong safety net



.

(b) Weak safety net



.

3.3.5 Search intensity, job finding rates, and unemployment duration

Safety net also has important effects on the search intensity of unemployed workers. Intuitively, with stronger safety net, unemployed have less incentives to look for a job. Figure 3.11 illustrates this fact. The policy functions for search intensity are higher for all unemployed workers under weak safety net. Unemployment insurance and social assistance both have a strong negative effect on search intensity, shifting the policy functions down for all agents. Higher tax credit has a positive effect on the search intensity, but only for low-skilled workers, it results in a lower drop of search intensity for low-skilled unemployed, this effect is rather small however. Again, there is also an effect of the distribution of agents over assets. With strong safety net the distribution is compressed around zero, so search intensity increases due to convexity of the policy function. Because we calibrate the model to fix the unemployment rate across two steady states, these effects cancel out and aggregate search intensity is the same in both steady states. To do it, we recalibrate the search cost. In order to keep the aggregate search intensity constant the search cost must be higher with weak safety net[7]. This will also affect the welfare of unemployed agents.

Naturally, these changes in the search intensity translate almost one to one to the job finding rates and the duration of unemployment, plotted on Figure 3.12 and Figure 3.13, respectively. An important observation from these figures is that expected unemployment duration is higher in case of strong safety net. Thus, even if they enjoy higher consumption while being unemployed, they stay unemployed longer on average and it has a negative effect on expected life-time consumption.

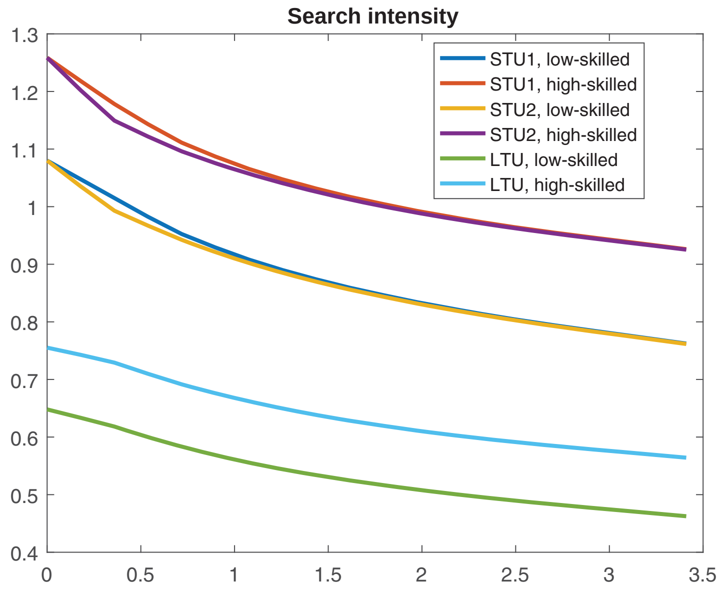
3.3.6 Wages

One of important channels affecting the distribution of consumption is the outcome of the wage bargaining. Figure 3.14 plots the wages of low- and high-skilled workers under strong and weak safety nets. We can see that the wage of high-skilled workers is higher with strong safety net, while the wage of low-skilled workers is lower. The main reason for this heterogeneous effect is the tax credit that affects only the low-skilled employed agents. Despite the fact that unemployment insurance and the social assistance push the outside option of the workers up increasing the wages, tax credit increases the value of employment for a given wage putting negative pressure on wages. We can see that the effect of tax credit is stronger than the effect of unemployment insurance and social assistance.

[7] The drawback of this strategy is the fact that we ignore the effect of safety net on the level of unemployment in the steady state. With weak safety net the unemployment would be lower if we kept the search cost constant, affecting also the distribution of consumption.

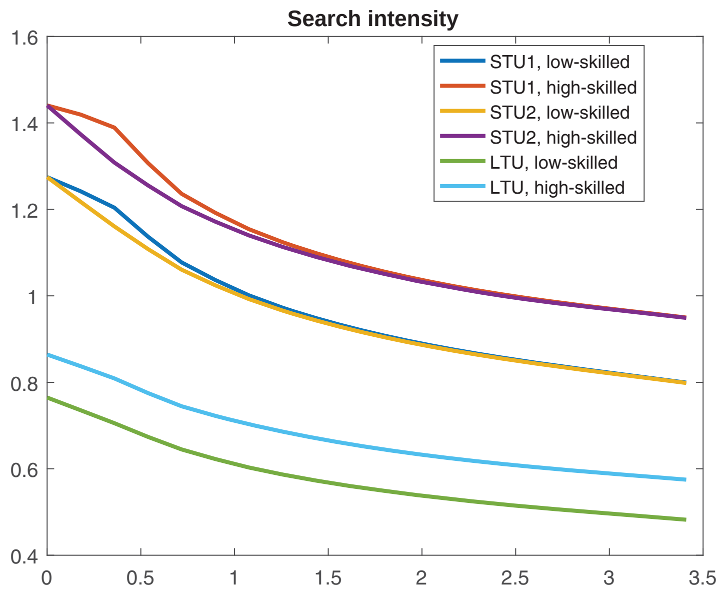
Figure 3.11: Search intensity

(a) Strong safety net



.

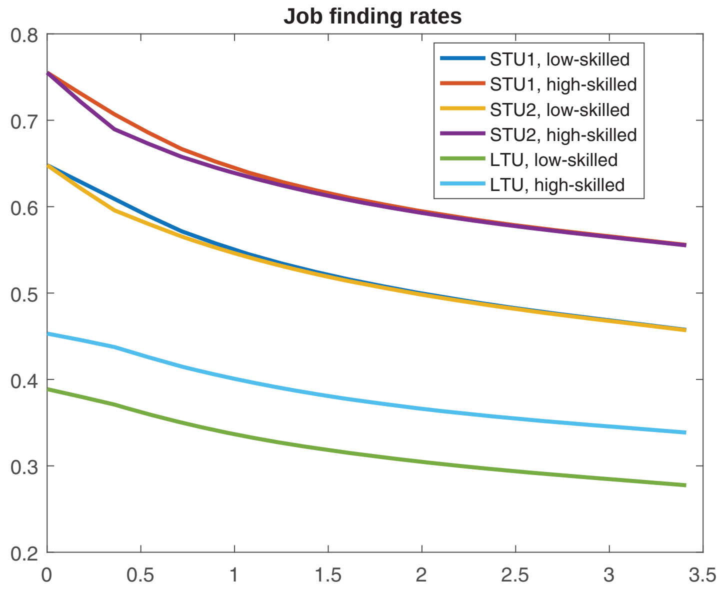
(b) Weak safety net



.

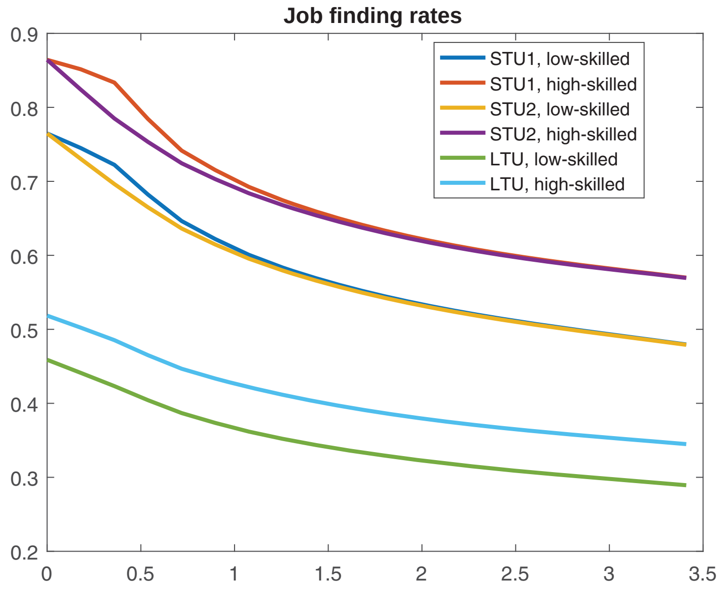
Figure 3.12: Job finding rates

(a) Strong safety net



.

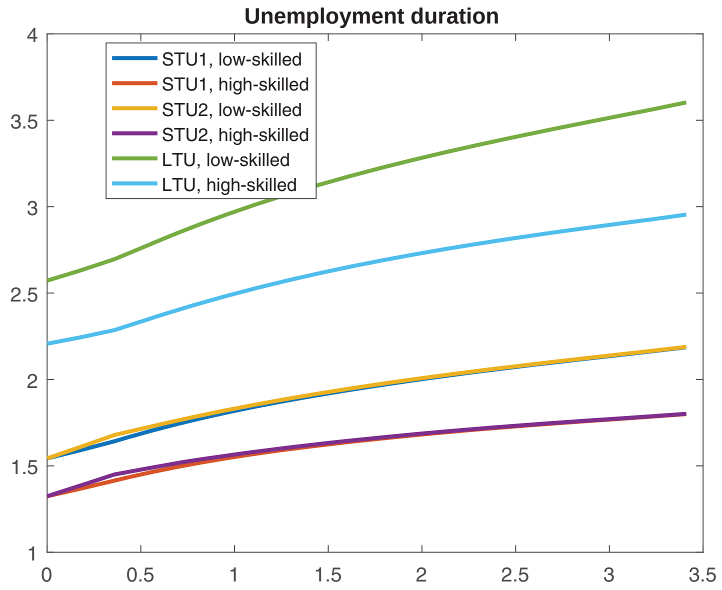
(b) Weak safety net



.

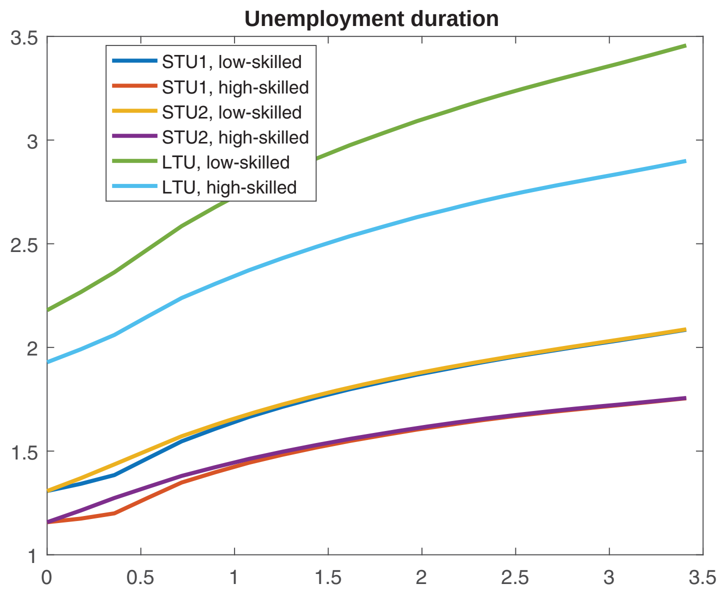
Figure 3.13: Expected unemployment duration

(a) Strong safety net



.

(b) Weak safety net



.

Another important observation is the shape of the wage functions. The wage function is essentially flat with respect to assets. This result differs from the results of Krusell, Mukoyama, and Şahin (2010) who find that the wage function is very steep around zero borrowing constraint and then flattens out fast. An important difference between their and our settings is that they do not have variable search intensity by workers. As we have seen above, search intensity is decreasing with asset holdings and the policy function is convex. Constrained unemployed put the most effort in searching for a job. Because this effort is costly, they must be compensated by a higher wage. This effect makes the wage function flat also for poor workers.

3.3.7 Welfare

Finally, we can study how all the changes in consumption, savings, search intensity, and transition probabilities affect the welfare of the agents. Figure 3.14 plots the value functions of the agents in each individual state for strong and weak safety net. We can see that the welfare is much higher in the case of strong safety net. The shift of the distribution towards zero with strong safety net still has negative effect on welfare (the value functions are increasing with assets) but it is much weaker than the level effect. Overall, the figures show that the individual welfare is undoubtedly higher with strong safety net.

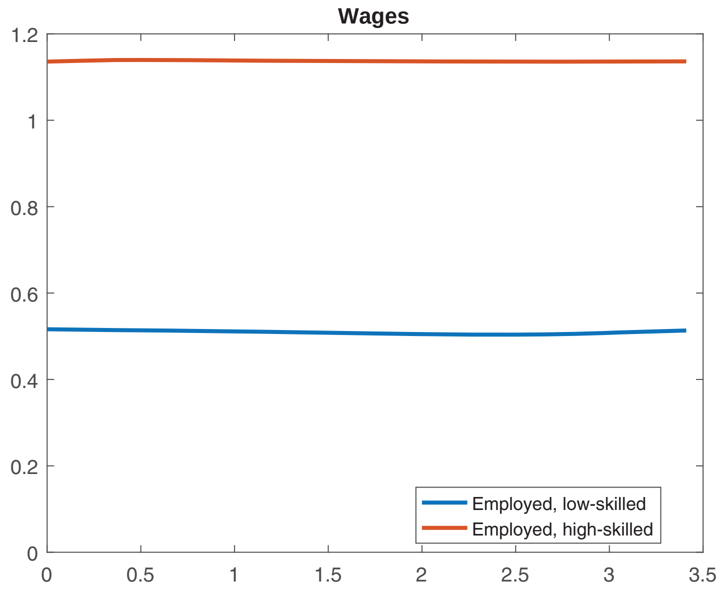
There are several channels that affect the welfare. First, distribution of consumption shifts to the right, increasing welfare. Second, the search intensity decreases reducing the search costs and increasing welfare, but at the same time increasing duration of unemployment decreasing welfare. Positive effects prevail and welfare of all agents increase with stronger safety net.

3.3.8 Aggregate implications

Apart from the steady state implications of the safety net, this model also allows us to study the aggregate dynamics. Figure 3.16 plots the impulse responses of aggregate production, consumption, inflation, and skill-adjusted labor input after a positive productivity shock. The response of the economy is very similar in the case of strong and weak safety net, however, as discussed above it is not very informative about the individual responses. In general, the individual and aggregate dynamics of the model are affected by the same five channels. Some of the channels stabilize the fluctuations of the economy, like redistribution of precautionary savings channel, while some amplify the fluctuations, like search intensity and wage bargaining. Because it is difficult to disentangle these effects in the full model, in Chapter 3 of this thesis we study a simplified version of the model that allows us to find an analytical solution and close the

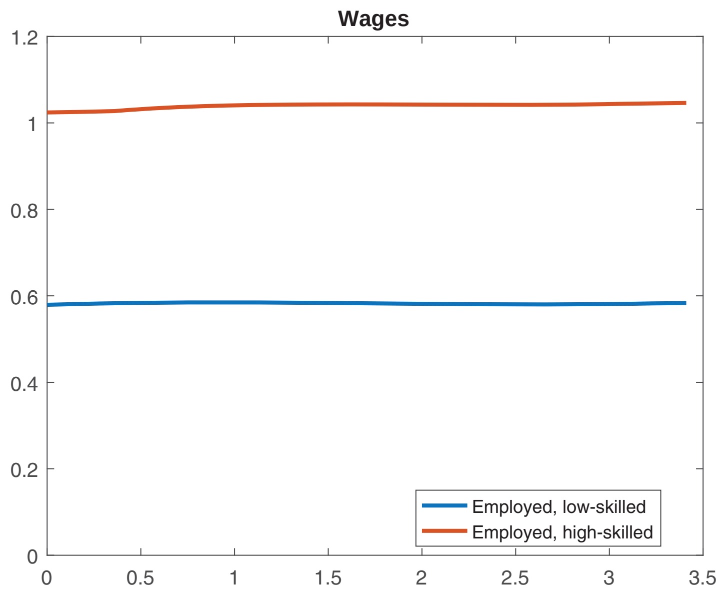
Figure 3.14: Wages

(a) Strong safety net



.

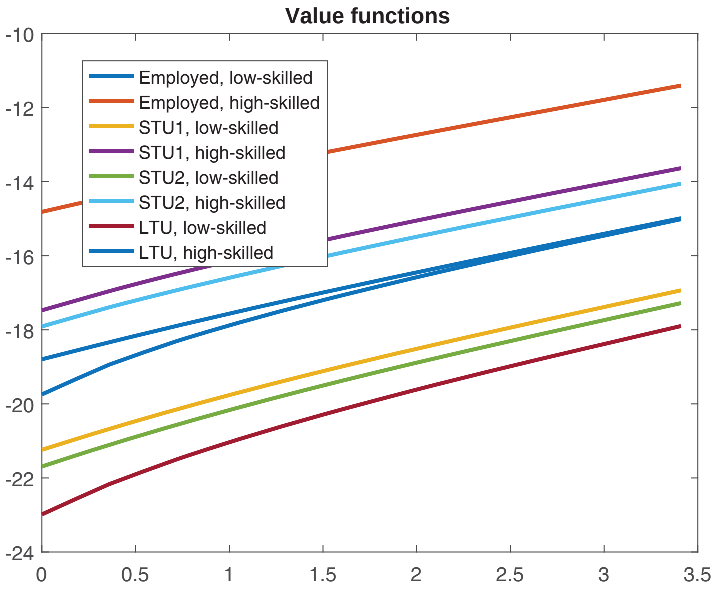
(b) Weak safety net



.

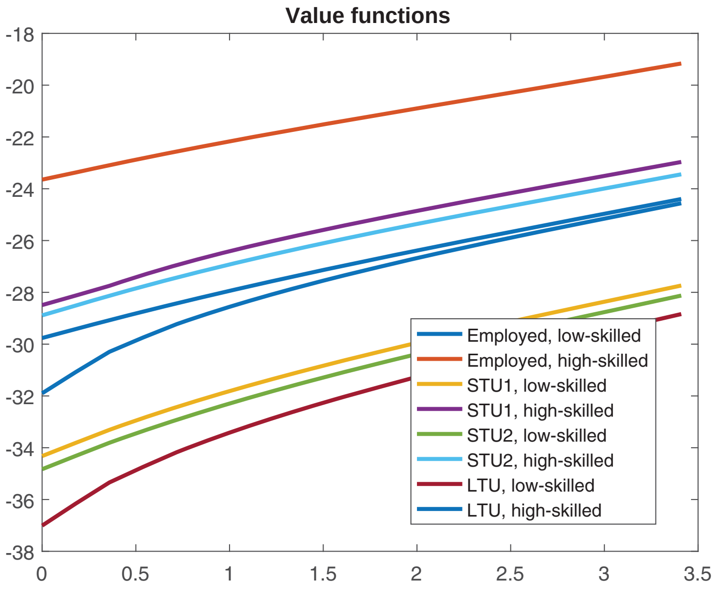
Figure 3.15: Value functions

(a) Strong safety net



.

(b) Weak safety net



.

channels. So that we are able to compare the importance of every channel and get insights for the dynamics of the full model. The study of the importance of the channels in the full model is left for further research.

3.3.9 Main channels

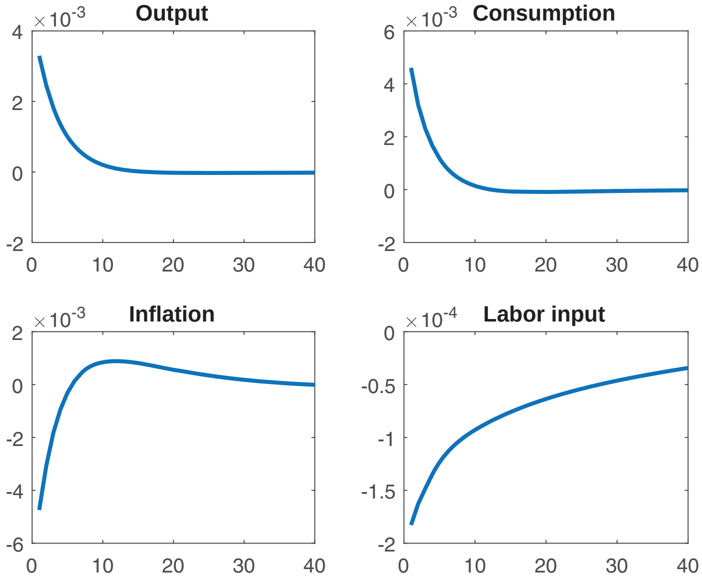
There are five main channels through which the safety net affects the individual well-being and the aggregate economy. Redistribution effect has positive impact on poor agents and negative effect on rich agents, with aggregate consumption rising due to differences in marginal propensity to consume that can be seen via the change in the curvature of the consumption policy function. Precautionary savings has also a positive effect on individual consumption. Consumption policy function rises for all workers, even the high-skilled employed workers who are not affected directly by the safety net. It also has a negative effect that dampens the positive effects through the shift in the distribution of agents over assets towards zero. Disposable income has positive effect as well via compression of the policy functions around zero assets. Even the variance of consumption of hand-to-mouth agents decreases. Search incentive channel dampens the positive effects of first three channels decreasing the incentives of searching for a job. It lowers the search intensity of all agents, decreasing the job finding rate and increasing the expected duration of unemployment. Finally, wage bargaining channel has heterogeneous effects on high- and low-skilled workers. The wage of high-skilled workers rises because of improving value of unemployment, while the wage of low-skilled workers decreases due to higher tax credit that increases non-wage work benefits. Proper isolation of individual channels is the main goal of further research.

3.4 Conclusion

In this chapter we analyze the effect of the safety net on individual well-being of agents in a framework with incomplete markets, nominal frictions, and labor market frictions. We show that stronger safety net increases consumption of all agents given assets level, even the rich high-skilled workers who are not affected directly by the safety net. At the same time the distribution of agents over assets shifts towards zero. Overall, consumption distribution shifts to the right and is more concentrated around higher consumption. Minimum consumption rises together with the social assistance. On the labor market side, stronger safety net decreases the search intensity of all workers, lowering job finding rates and increasing expected duration of unemployment. Wages of high-skilled workers rise due to higher unemployment insurance and social assistance, while wages of low-skilled workers decrease due to higher tax credit. Overall, the welfare of all agents rises with stronger safety net.

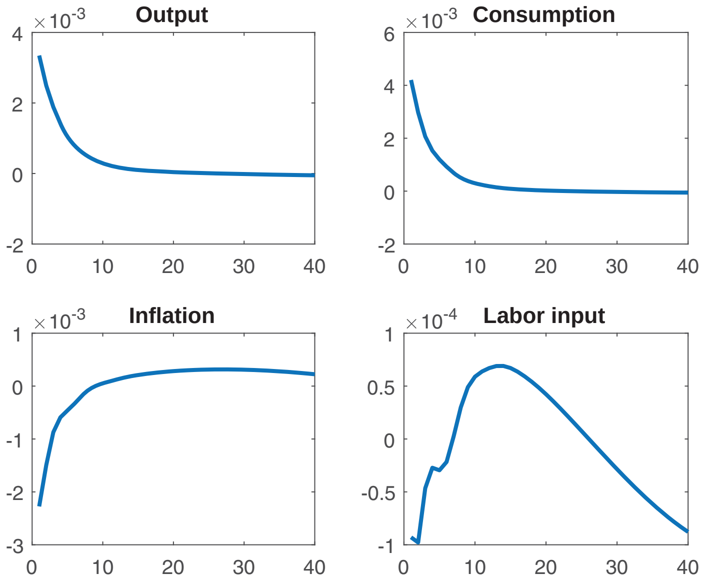
Figure 3.16: Impulse responses to aggregate productivity shock

(a) Strong safety net



.

(b) Weak safety net



.

3.5 References

1. Bayer, Christian, Ralph Lütticke, Lien Pham-Dao, and Volker Tjaden (2014). "Precautionary Savings, Illiquid Assets, and the Aggregate Consequences of Shocks to Household Income Risk".

2. Bitler, Marianne, and Hilary Hoynes (2013). "The More Things Change, the More They Stay the Same? The Safety Net and Poverty in the Great Recession," National Bureau of Economic Research, No. w19449.

3. Challe, Edouard, Julien Matheron, Xavier Ragot, and Juan F. Rubio-Ramirez (2015). "Precautionary Saving and Aggregate Demand".

4. Chetty, Raj (2008). "Moral Hazard versus Liquidity and Optimal Unemployment Insurance," Journal of political economy, 116(2), pp.173-234.

5. Den Haan, Wouter J., Pontus Rendahl, and Markus Riegler (2014). "Unemployment (fears), Precautionary Savings, and Aggregate Demand."

6. Guerrieri, Veronica, and Guido Lorenzoni (2015). "Credit Crises, Precautionary Savings, and the Liquidity Trap."

7. Hagedorn, Marcus, Fatih Karahan, Iourii Manovskii, and Kurt Mitman (2013). "Unemployment benefits and unemployment in the great recession: the role of macro effects", National Bureau of Economic Research, No. w19499.

8. Krusell, Per, Toshihiko Mukoyama, and Ayşegül Şahin (2010). "Labour-market matching with precautionary savings and aggregate fluctuations." The Review of Economic Studies 77.4, pp.1477-1507.

9. Krusell, Per, and Anthony A. Smith, Jr (1998). "Income and wealth heterogeneity in the macroeconomy." Journal of Political Economy 106.5, pp.867-896.

10. McKay, Alisdair, and Ricardo Reis (2016). "The role of automatic stabilizers in the US business cycle.", Econometrica, 84 (1), 141-194, January 2016.

11. Ravn, Morten O., and Vincent Sterk (2014). "Job Uncertainty and Deep Recessions." No. 1501.

12. Reiter, Michael (2009). "Solving heterogeneous-agent models by projection and perturbation." Journal of Economic Dynamics and Control 33.3, pp.649-665.

3.6 Appendix

3.6.1 Average value of a filled job

The average value of a filled job next period for a firm posting a vacancy today conditional on meeting a worker is

$$\bar{\mathcal{J}}(\Omega) = \int_{a_{-1},h_{-1}} \sum_h \pi(h_{-1},h) \left[\sum_k \left(\sigma^{s_k}(a_{-1},h_{-1};\Omega_{-\mathbf{1}}) / \Sigma_{-1}\right) \mathcal{J}\left(\psi^{s_k}(a_{-1},h_{-1};\Omega_{-1}),h;\Omega\right) f^{s_k}(a_{-1},h_{-1};\Omega_{-1}) \right.$$
$$\left. + \phi\left(\sigma^l(a_{-1},h_{-1};\Omega_{-1}) / \Sigma_{-1}\right) \mathcal{J}\left(\psi^l(a_{-1},h_{-1};\Omega_{-1}),h;\Omega\right) f^l(a_{-1},h_{-1};\Omega_{-1}) \right] da_{-1}dh_{-1}$$

where the probability to meet a worker with a given skill and asset level depends on the relative search intensity of this worker chosen last period.

3.6.2 Market clearing for goods, bonds and dividends

Let $r \in \{e; s_1; s_2; ...; s_{\bar{k}}; l\}$ index labor force states. Market clearing conditions for goods:

$$Y = C + G + \kappa V + \chi,$$

with

$$C = \int_{a,h} \left(\sum_r c^r(a,h;\Omega) f^r(a,h;\Omega) \right) dadh + \nu\bar{c},$$

$$sY = z \int_{a,h} h f^e(a,h;\Omega)\, dadh + \nu z \bar{h}$$

and

$$s = \int_0^1 (p_j/p)^{-\epsilon} dj$$

Market clearing conditions for bonds:

$$B' = \int_{a,h} \left(\sum_r \psi^r(a,h;\Omega) f^r(a,h;\Omega) \right) dadh + \nu\bar{a}';$$

Market clearing conditions for dividends:

$$\nu\bar{d} = \int_0^1 d_j dj + \int_{a,h} (qzh - \omega(a,h;\Omega)h) f^e(a,h;\Omega)\, dadh;$$

3.6.3 Law of motion for the distribution of workers

We now define the law of motion for the aggregate distribution of workers. Let **A**, **H** and **R** be the sets of possible assets, skills levels and labor force states. Let $\mathcal{A}$, $\mathcal{H}$ and $\mathcal{R}$ be typical subsets. Let $\mathcal{F}(\mathcal{A},\mathcal{H},\mathcal{R};\Omega)$ be the distribution function of workers over assets, skills and labor force states, given the aggregate state Ω. Let the transition function $Q\left(\left(a,h,r\right),\left(\mathcal{A},\mathcal{H},\mathcal{R}\right);\Omega'|\Omega\right)$ be probability that a worker with current assets, skill level and labor force state $\left(a,h,r\right)$ today transits to the set $\left(\mathcal{A},\mathcal{H},\mathcal{R}\right)$ next period when the aggregate state transits from Ω to Ω'. Then, the law of motion of the aggregate distribution is:

$$\mathcal{F}\left(\mathcal{A},\mathcal{H},\mathcal{R};\Omega'\right)=\int_{(a,h,r)\in(\mathbf{A},\mathbf{H},\mathbf{R})} Q\left(\left(a,h,r\right),\left(\mathcal{A},\mathcal{H},\mathcal{R}\right);\Omega'|\Omega\right)d\mathcal{F}\left(a,h,r;\Omega\right)$$

where $Q\left(\left(a,h,r\right),\left(\mathcal{A},\mathcal{H},\mathcal{R}\right);\Omega'|\Omega\right)$ satisfies

$$Q\left(\left(a,h,r\right),\left(\mathcal{A},\mathcal{H},\mathcal{R}\right);\Omega'|\Omega\right)=\sum_{h'\in\mathcal{H}}\sum_{r'\in\mathcal{R}}\begin{cases}\pi\left(h'|h\right)\varrho\left(r'|r;\Omega\right) & \text{if } \psi^{r}\left(a,h;\Omega\right)\left(p'/p\right)\in\mathcal{A}\\ 0 & \text{otherwise}\end{cases}$$

where $\pi\left(h'|h\right)$ denotes the exogenous transition matrix for skills and $\varrho\left(r'|r;\Omega\right)$ the endogenous transition matrix for labor force states, given by:

	e	s_1	s_2	$\ldots$	$s_{\bar{k}}$	l
e	$1-\rho$	ρ	0	$\ldots$	0	0
s_1	$\sigma^{s_1}\left(a,h;\Omega\right)\rho'_s$	0	$1-\sigma^{s_1}\left(a,h;\Omega\right)\rho'_s$	$\ldots$	0	0
s_2	$\sigma^{s_2}\left(a,h;\Omega\right)\rho'_s$	0	0	$\ldots$	0	0
$\ldots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
$s_{\bar{k}}$	$\sigma^{s_{\bar{k}}}\left(a,h;\Omega\right)\rho'_s$	0	0	$\ldots$	0	$1-\sigma^{s_{\bar{k}}}\left(a,h;\Omega\right)\rho'_s$
l	$\sigma^{l}\left(a,h;\Omega\right)\rho'_l$	0	0	$\ldots$	0	$1-\sigma^{l}\left(a,h;\Omega\right)\rho'_l$

Finally, let $f\left(a,h,r;\Omega\right)$ be the density function, then the measures of workers over different labor force states are defined as follows

$$f^{r}\left(a,h;\Omega\right)\equiv f\left(a,h,r;\Omega\right)$$

with $r\in\left\{e;s_1;s_2;...;s_{\bar{k}};l\right\}$.

4. Safety net and the business cycle, with Antonella Trigari

Abstract

This chapter studies the effects of safety net on the dynamics of the economy. Two important channels are considered - aggregate demand, and search incentives channel. Safety net reduces the volatility of the economy through the aggregate demand channel due to stabilization of individual consumption, while increases the volatility through the search incentives channel due to higher volatility of hiring and search intensity. We use a simplified version of the model introduced in Chapter 2 without assets distribution as a state. We illustrate the importance of the trade-off between these two channels in the design of optimal safety net.

4.1 Introduction

As it was argued in Chapter 2, the safety net not only has important implications for well-being of individuals but also for the dynamics of the aggregate economy. The safety net affects the dynamics through the same five channels: redistribution, disposable income, insurance, search intensity, and wage bargaining. The first three channels, that we call aggregate demand channels, stabilize the economy with stronger safety net. The latter two channels, that we call labor market channels, increase the volatility of the economy with stronger safety net. Aggregate demand channels stabilize the economy because with stronger safety net volatility of individual consumption decreases, it decreases the volatility of aggregate demand, and, through nominal rigidities, stabilizes the economy. Labor market channel works the opposite way. Under stronger safety net unemployed agents search for a job less intensively during a crises, wages are also pushed upwards and firms reduce hiring by more. It translates into higher volatility of the economy.

In this chapter we analyze analytically tractable version of the model developed in Chapter 2. We follow the approach of Midrigan and Philippon (2015) to get rid of the distribution in the state space of the model and reach analytical tractability. We assume that all agents pull the assets in the end of every period. In the beginning of the period the household decides on the amount of cash to give to each member up to a borrowing constraint. Then the members of the households receive idiosyncratic shocks of employment status. On top of the cash, the employed workers receive the wage, unemployed receive the unemployment insurance, and so on. This allows us to keep all the channels and the safety net programs while analyzing the model analytically. Analytical solution allows to close the aggregate demand and labor market channels separately to see their relative contribution. We can still perform similar experiments in the full model, but the simplified model provides more intuition, hence it is a good starting point.

This model allows for explicit modeling of different programs of the safety net while avoiding computational difficulties of endogenous asset distributions. The fact that there are several types of workers every period (employed, unemployed, long-term unemployed) allows to study the effects of the safety net programs through the channels also present in the full model - insurance, search incentives, bargaining, aggregate demand. We lose some of the effects because the individual states are not persistent (i.i.d. across periods), but qualitatively they should be the same. Because the shocks are not persistent and the workers pull assets in the end of the period, we don't need to keep track of the asset distribution that simplifies the solution of the model.

The model allows us to conduct counterfactual experiments and answer quantitatively a number of important questions. For example: What components of the safety net are most effective at stabilizing and minimizing unemployment and poverty rates? What programs can prevent a rising incidence of long-term unemployment or of extreme poverty during recessions? What is the optimal combination of in-work and out-of work transfer programs, and does the optimal mix change over the business cycle? Can a strong safety net prevent the emergence of jobless recoveries? Alternatively, do unemployment insurance and other safety net programs exacerbate fluctuations in labor markets? In this chapter we answer the last question.

To measure the effects of the safety net over the business cycle is essential to consider the full variety of theoretical mechanisms by which the safety net works, as well as to account for general equilibrium effects. The reason is that alternative channels and general equilibrium effects may have either offsetting or reinforcing effects. Consider as an example the case of unemployment insurance. A generous unemployment insurance system stabilizes business cycles through consumption/savings related channels, while it amplifies business cycles through labor market channels. On one hand, unemployment insurance weakens the precautionary savings motive for employed workers, supports aggregate demand and stimulates job creation. On the other, unemployment insurance raises unemployed workers' outside option in bargaining, puts upward pressure on wages and this way discourages job creation. A similar trade-off is generated by the components of SNAP and TANF that belong to the opportunity cost of employment, while EITC has opposite effects on wages. Moreover, the different components of the safety net often target the same group of people (low-income and/ or low-asset individuals) possibly changing individual incentives and welfare in offsetting ways.

Here we start by analyzing the effects of the unemployment insurance on the dynamics of the economy. We confirm our intuition that the volatility of the economy has U-shape depending on the level of the unemployment insurance. When unemployment insurance is too low the aggregate demand channel dominates and volatility is high. And when unemployment insurance is too high the labor market channel dominates and the volatility is high. Medium unemployment insurance is stabilizing the economy. We illustrate it by closing the two channels separately. Then we show how one can extend the model to allow for the role of other safety net programs.

There is a growing stream of literature that studies how the safety net, unemployment insurance in particular, affects the dynamics of the economy. For now aggregate demand and labor market channels were studied separately. On the aggregate demand side, McKay and Reis (2016, a) show that safety net stabilizes the economy. They study a model with heterogeneous agents and nominal rigidities and ad hoc formulation of labor market risk. In a related paper, McKay and Reis (2016, b) study the optimal level of unemployment insurance. They add labor market frictions in the model but use a simplified assumption of zero borrowing limit and zero net supply of assets to reach analytical tractability of the model. Their main result is that the optimal generosity of unemployment insurance increases by 13% after accounting for the stabilization through the aggregate demand channel. In a very similar framework, Kekre (2016) demonstrates analytically the importance of the trade-off between aggregate demand and individual search incentives. Then he also shows in a calibrated model that unemployment benefit extensions prevented an additional increase of unemployment by 2-5% during the Great Recession. On the other side, Mitman and Rabinovich (2015) show that accounting to the labor market channels in a search model optimal unemployment insurance must be pro-cyclical. However, they don't have the aggregate demand channel in their framework. Up to our knowledge, we are first to study the unemployment insurance taking into account both, aggregate demand and labor market channels.

4.2 The model

Time is discrete. There is a family that consists of a measure one of workers. Each worker can be employed or unemployed. The family has assets and can borrow up to a borrowing limit. The family allocates a portion of the assets to each worker in the beginning of the period in the form of cash. After the cash is allocated, the lottery among workers determines which worker is employed and receives the wage, and which worker is unemployed and receives the unemployment insurance. Firms are of three types: final goods, retailers and wholesale firms.

4.2.1 Timing

The intra-period timing is the following: realization of the aggregate shock; vacancy posting by firms, search intensity decision, borrowing and cash by the family, matching and separation shocks; consumption and savings by individual workers, production and wage bargaining.

4.2.2 Household/Family

There is a household with a continuum of members of measure 1. The household decides on the individual consumption and cash on hand of each member, aggregate borrowing and saving, and search intensity of unemployed workers. The household needs to allocate cash to its members in order to consume. In order to provide cash to the agents, the household can use the net assets from the previous period, and borrow today up to a borrowing constraint. On top of cash, employed workers receive the wage, and unemployed workers receive unemployment insurance from the government. Savings for next period are determined after the individual consumption takes place as a sum of all cash that wasn't spent by the agents. Who is employed and unemployed is decided every period by a lottery that every member of the household participates in.

The household maximizes the following utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\int_0^1 \log \left(c_{i,t} \right) di - \varsigma \left(\sigma_t \right) \left(1 - n_{t-1} \right) \right]$$

choosing individual consumption of each member, $c_{i,t}$, search intensity of unemployed, σ_t, cash, x_t, borrowing, b_{t+1}, assets, a_{t+1}, and employment, n_t. The household faces six constraints: 1) budget constraint; 2) borrowing constraint; 3) liquidity constraint for employed workers; 4) liquidity constraint for unemployed workers; 5) law of motion for employment; 6) aggregate asset equation. The budget constraint

$$x_t = \left(1 + i_t \right) \frac{a_t}{p_t} - \left(1 + i_t \right) \frac{b_t}{p_t} + \frac{b_{t+1}}{p_t} + d_t \quad \lambda_t^B$$

equalizes the cash that is paid to the family members to the total available resources of the family - net assets from last period, borrowing this period, and dividends (maybe more logical to put to the assets, for the timing). Borrowing is up to a borrowing constraint with a real value of the borrowing limit $\overline{b}$:

$$b_{t+1} \leq p_t \overline{b} \quad \lambda_t^{BC}$$

Employed workers can consume their cash and the wage net of taxes:

$$c_{i,t} \leq x_t + \left(1 - \tau_t \right) w_t \text{ for employed } \quad \lambda_t^{CN}$$

while unemployed workers consume out of cash and the unemployment insurance:

$$c_{i,t} \leq x_t + \tau_u \text{ for unemployed } \quad \lambda_t^{CU}$$

Employment evolves according to the following law of motion:

$$n_t = \rho n_{t-1} + \rho_{s,t} \sigma_t \left(1 - n_{t-1} \right) \quad \lambda_t^N$$

where ρ is the exogenous survival rate of employment relationships, and $\rho_{s,t}$ is the endogenous job finding rate per unit of search. This is the baseline case, but we also consider a case with σ chosen one period in advance (version 2) in order to be closer to the full model. The reason why we need pre-selected search intensity in the full model is to avoid having two value functions for the workers - one for the moment of choice of search intensity (in the beginning of the period) and one for the moment of bargaining and consumption/savings decision (in the middle of the period when labor market and skill states are realized). When search intensity is pre-selected, both choices take place at the same time and one value function is enough. In the baseline simple model it is not so harmful (in terms of notation) to have separate choice of search intensity and consumption/savings. Implicitely we will have two value functions for the household - one for the moment of choice of search intensity, and one for optimal consumption/savings that will also be used in bargaining. For the firms, however, because in both cases they have instantaneous hiring, there are two value functions - at the moment they post vacancies, and at the moment they bargain. Finally, aggregate assets for the next period are determined by the "left-overs" after consumption of individual members of the family:

$$\frac{a_{t+1}}{p_t} = x_t + (1 - \tau_t) w_t n_t + \tau_u (1 - n_t) + d_t - \int_0^1 c_{i,t} di \ \ \lambda_t^A$$

4.2.3 Final good firms, retailers and price setting

A competitive sector for final goods combines differentiated varieties of intermediate goods according to the production function

$$Y_t = \left(\int_0^1 y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$$

where y_j is the input of intermediate good j and ϵ is the elasticity of substitution across varieties. Final goods firms purchase intermediate good j at price p_j and take as given the final goods price p. From cost minimization, it follows that the demand for variety j is given by

$$y_t(j) = \left(\frac{p_t(j)}{p_t} \right)^{-\epsilon} Y_t, \tag{4.1}$$

and the price index p is given by

$$p_t = \left(\int_0^1 (p_t(j))^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}. \tag{4.2}$$

A measure one of monopolistic competitive retailers buy a wholesale good from wholesale firms, differentiate it into varieties y_j with a technology that transforms one unit of wholesale good into one unit of intermediate good and sell it to the final goods producers. Retailers set prices infrequently as in Calvo (1983) with probability of revision θ. At each revision date, a retailer producing variety j chooses an optimal price $p_t^*(j)$ to maximize expected future profits, subject to the demand for its own variety. As retailers are owned by the whole household, it uses its discount factor $\Lambda_{t,t+1}$. Let q_t

be the relative price of the wholesale good and χ a fixed cost of production. The price setting problem of retailer j at each revision date t can be written as

$$\max_{p_t^*(j)} \Pi_t \left(p_t^*\left(j\right)\right) \tag{4.3}$$

with

$$\Pi_t \left(p_t \left(j\right)\right) = d_t \left(p_t \left(j\right)\right) + \left(1-\theta\right) \Lambda_{t,t+1} \Pi_{t+1} \left(p_{t+1} \left(j\right)\right)$$

and

$$d_t \left(p_t \left(j\right)\right) = \left(\frac{p_t \left(j\right)}{p_t} - q_t\right) y_t \left(j\right) - \chi$$

and subject to the demand equation (4.1).

4.2.4 Wholesale goods firms and wage bargaining

Wholesale goods firms post vacancies to maximize expected profits:

$$E_0 \sum_{t=0}^{\infty} \Lambda_{0,t} d_t^W = E_0 \sum_{t=0}^{\infty} \Lambda_{0,t} \left(q_t z_t n_t - w_t n_t - \kappa v_t\right)$$

subject to the hiring equation:

$$n_t = \rho n_{t-1} + \rho_{f,t} v_t.$$

Each period the firm pays the dividends to the household:

$$d_t^W = q_t z_t n_t - w_t n_t - \kappa v_t.$$

Bargaining

The firm and the household bargain over the surplus of having one additional worker after the matches are already formed. So we need to define the value functions in the middle of the period to be used in the bargaining problem.

The value of the firm after hiring is the following:

$$F\left(w_t, n_t\right) = q_t z_t n_t - w_t n_t + E_t \left\{\Lambda_{t,t+1} \left(F\left(w_{t+1}, n_{t+1}\right) - \kappa v_{t+1}\right)\right\}$$

Marginal value of an additional worker:

$$\frac{\partial F\left(w_t, n_t\right)}{\partial n_t} = q_t z_t - w_t + E_t \left\{\Lambda_{t,t+1} \left(\frac{\partial F\left(w_{t+1}, n_{t+1}\right)}{\partial n_{t+1}} \frac{\partial n_{t+1}}{\partial n_t} - \kappa \frac{\partial v_{t+1}}{\partial n_t}\right)\right\}$$

The value for the household after search:

$$W\left(w_t, n_t\right) = \int_0^1 \log\left(c_{i,t}\right) di + \beta E_t \left\{W\left(w_{t+1}, n_{t+1}\right) - \varsigma\left(\sigma_{t+1}\right)\left(1-n_t\right)\right\}$$

Marginal value of an additional worker:

$$\frac{\partial W\left(w_t, n_t\right)}{\partial n_t}=\log\left(c_t^n\right)-\log\left(c_t^u\right)+\beta E_t\left\{\frac{\partial W\left(w_{t+1}, n_{t+1}\right)}{\partial n_{t+1}}\frac{\partial n_{t+1}}{\partial n_t}-\varsigma'\left(\sigma_{t+1}\right)\left(1-n_t\right)\frac{\partial \sigma_{t+1}}{\partial n_t}+\varsigma\left(\sigma_{t+1}\right)\right\}$$

And the bargaining problem is standard:

$$w_t=\arg\max\left(\frac{\partial W\left(w_t, n_t\right)}{\partial n_t}\right)^{\eta}\left(\frac{\partial F\left(w_t, n_t\right)}{\partial n_t}\right)^{1-\eta}$$

Every period, wage is the solution of this bargaining problem.

4.2.5 Labor market matching

Posted vacancies and unemployed workers are matched every period through a standard matching technology:

$$m_t=\chi\left(\sigma_t\left(1-n_{t-1}\right)\right)^{\alpha} v_t^{1-\alpha}$$

Consequently, probability to fill a vacancy, $\rho_{f,t}$, and find a job, $\rho_{s,t}$:

$$\rho_{f,t}=\chi\left(\frac{v_t}{\sigma_t\left(1-n_{t-1}\right)}\right)^{-\alpha}$$

$$\rho_{s,t}=\chi\left(\frac{v_t}{\sigma_t\left(1-n_{t-1}\right)}\right)^{1-\alpha}$$

The law of motion for employment:

$$n_t=\rho n_{t-1}+m_t$$

4.2.6 Government and tax and transfer system

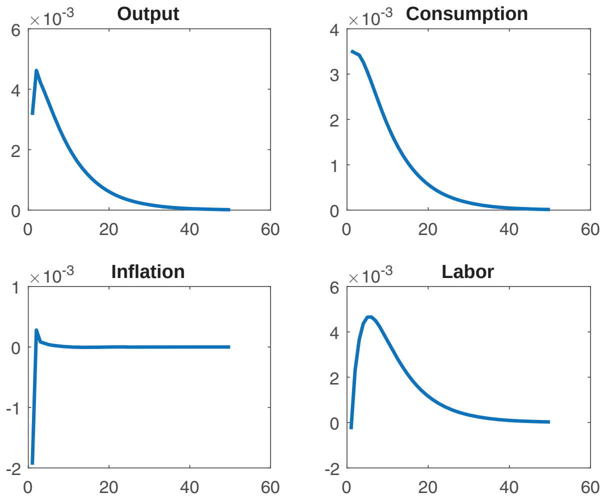
The government provides unemployment insurance τ_u and collects the taxes τ_t to satisfy the government budget constraint:

$$\tau_u\left(1-n_t\right)=\tau_t w_t n_t$$

The government also sets the nominal interest rule through a standard Taylor rule:

$$i_{t+1}=\bar{i}+\phi\log\left(\frac{p_t}{p_{t-1}}\right)+\varepsilon_t$$

Figure 4.1: Impulse responses to a productivity shock



.

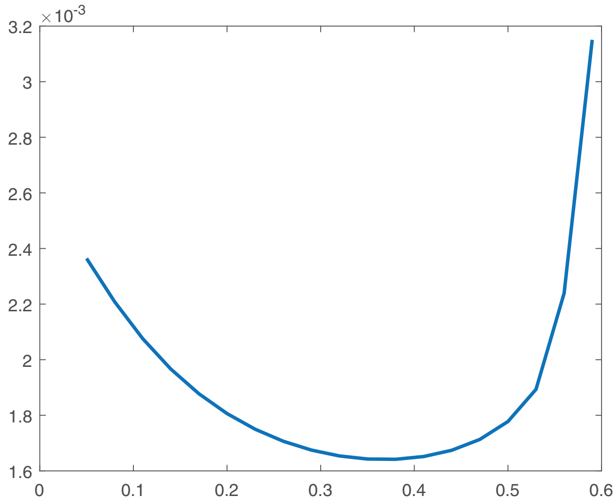
4.3 Results

In this section we analyze the dynamics of the model studying how the economy and, in particular, output responds to a positive productivity shock depending on the level of unemployment insurance. We start with looking at the aggregate dynamics of the model, then we analyze how the dynamic response of output is affected by the unemployment insurance. After that we study how it changes if we close labor market or aggregate demand channels.

4.3.1 Aggregate dynamics

Figure 4.1 plots the impulse responses of output, consumption, inflation, and employment to a positive productivity shock. The impulse responses of output, consumption, and inflation are standard: output and consumption increase, while inflation falls on impact. The impulse response of employment, however, is different from standard models, employment falls slightly on impact but then rises. It might signal that the aggregate demand channel dominates in this model, generating an increase in consumption and employment. Higher consumption results in higher output and employment that then decreases precautionary savings motive increasing consumption more and leading to even higher employment. This effect is absent in the standard models where employment falls following a positive productivity shock.

Figure 4.2: Reaction of output to a productivity shock for different UI



.

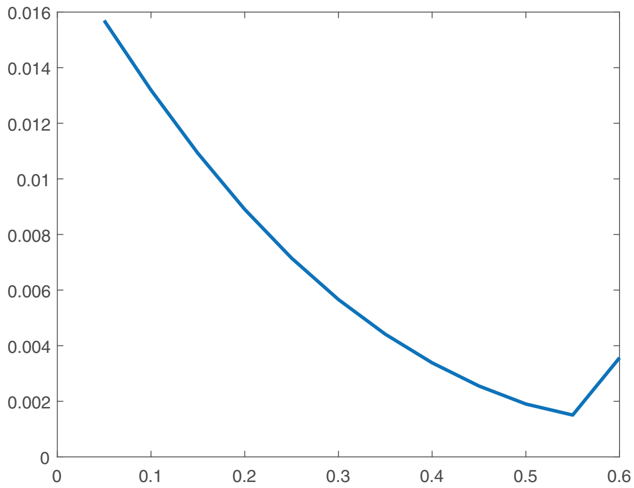
4.3.2 The effect of the UI on the aggregate dynamics

Figure 4.2 plots the reaction of output to a positive productivity shock given different levels of unemployment insurance. We can observe the u-shape of the reaction: when unemployment insurance is very low, the output reaction is very strong, it starts falling with higher unemployment insurance until some point after which it starts rising again. This shape can be explained by the changing dominance of aggregate demand and labor market channels. When unemployment insurance is too low, individual consumption is very volatile and the agents have high incentives for precautionary savings. Following a positive productivity shock the risk of unemployment decreases and so does the precautionary savings motive leading to stronger reaction of consumption and output. When the unemployment insurance is high, instead, the labor market channels dominate. A positive productivity shock leads to stronger reaction of hiring by firms that leads to higher employment and output. This shape of the reaction of output demonstrates the importance of the trade-off between these two channels in design of optimal unemployment insurance and safety net in general. Next we try to isolate the two channels one by one in order to verify our intuition.

4.3.3 Isolating the channels

To confirm our intuition about relative importance of each channel in generating the u-shape of output reaction we close in turn the labor market and the aggregate demand channel. To close the labor market channel, first we substitute the wage equation from

Figure 4.3: Reaction of output to a productivity shock for different UI, wage equation



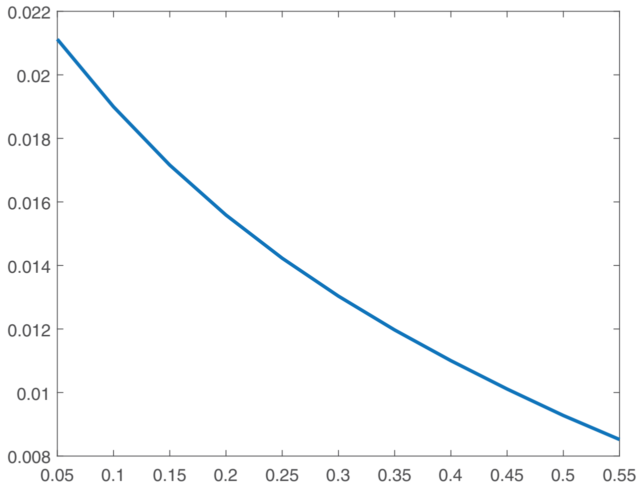
.

wage bargaining with a simple wage equation where wage reacts only to the productivity shock directly. This wage equation is similar to the one used in other studies of the aggregate demand channel. We try to change the reaction of the wage to productivity from fixing the nominal wage to a strong reaction of the wage to productivity and study the reaction of output depending on the unemployment insurance. Then, on top of using the ad hoc wage equation we also fix the search intensity of unemployed to the steady state level. Like this we close the incentives channel of the safety net, that is another part of the labor market channels. Finally, to close the aggregate demand channel we relax the assumption of unemployed workers being liquidity constrained, essentially moving to the representative agents framework. It allows us to close the aggregate demand channel because without liquidity constraints individual consumption does not depend on the individual state and therefore is unaffected by the safety net.

No wage bargaining

Figure 4.3 plots the reaction of output to a productivity shock with an ad hoc wage equation instead of the wage bargaining solution. We can see from the figure that the u-shape almost disappears and the reaction of output is decreasing with unemployment insurance almost for all level of unemployment insurance. It proves our intuition that wage bargaining is an important factor in generating high output volatility with very high unemployment insurance.

Figure 4.4: Reaction of output to a productivity shock for different UI, fixed nominal wage



.

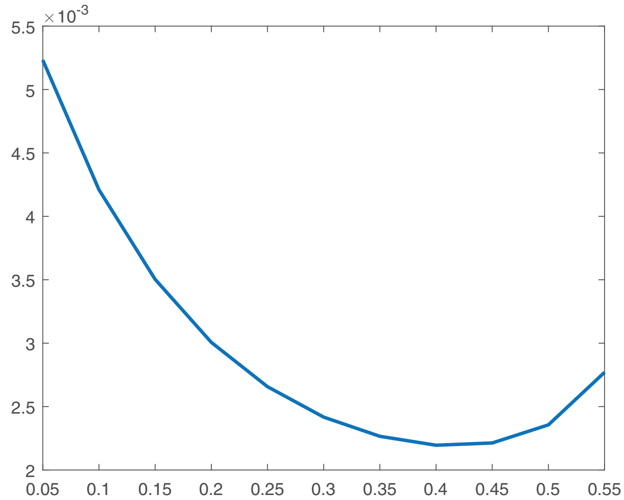
Figure 4.4 plots similar reaction of output with a fixed nominal wage. When wage does not react to productivity the volatility of output decreases with unemployment insurance, again, confirming our intuition.

Finally, Figure 4.5 plots the reaction of output to a productivity shock for different levels of unemployment insurance for an ad hoc wage equation with stronger reaction to productivity. We can see that the u-shape of the reaction returns with higher volatility of wages. This illustrates that under wage bargaining wages react more to the productivity shocks generating a stronger reaction of the labor market to the same productivity shock.

No search intensity

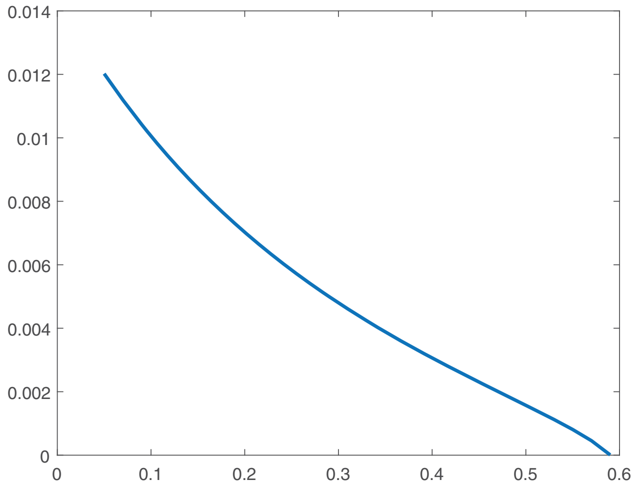
Now on top of having an ad hoc wage equation we also isolate the incentives channel of unemployment insurance. Figure 4.6 plots the reaction of output to a productivity shock for different levels of unemployment insurance with search intensity of the unemployed being fixed on the steady state level. We can see from the figure that the reaction of output is decreasing with unemployment insurance. Moreover, the general magnitude of the reaction without the reaction of search intensity is much lower. This is not surprising because, in general, reaction of search intensity amplifies the labor market fluctuations amplifying the reaction of the output.

Figure 4.5: Reaction of output to a productivity shock for different UI, wage equation with higher sensitivity



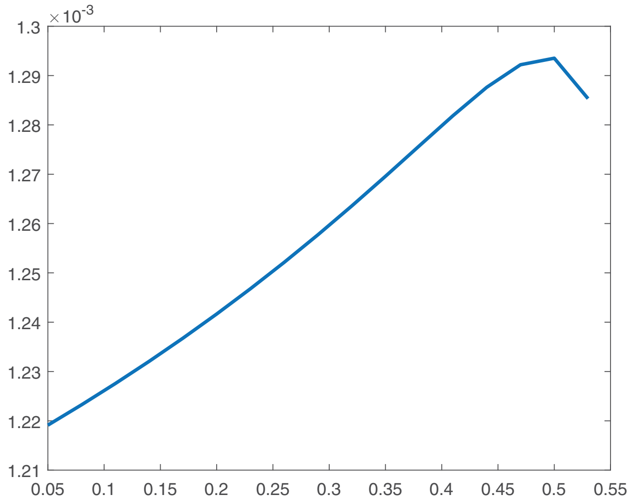
.

Figure 4.6: Reaction of output to a productivity shock for different UI, wage equation, fixed search intensity



.

Figure 4.7: Reaction of output to a productivity shock for different UI, representative agents, wage equation



.

No liquidity constraints

Finally, we isolate the aggregate demand channel by removing the liquidity constraints. Without the liquidity constraints the household can fully insure its members from the idiosyncratic risk of being unemployed providing the same consumption to every member of the household. This stabilizes individual consumption and changes the reaction of the aggregate demand. This model converges to a representative agent model. Without liquidity constraints, if unemployment risk increases it has no effect on individual consumption and therefore there is no need of precautionary savings. Figure 4.7 plots the reaction of output as a function of unemployment insurance in this case. We can see that the reaction of output increases with higher unemployment insurance. The increasing reaction of output is the result of labor market channels, with higher unemployment insurance volatility of labor market increases increasing the volatility of output, even for very small unemployment insurance. This confirms our intuition about relative contribution of the channels to the shape of the output reaction in Figure 4.2. Aggregate demand channels are more important for low levels of unemployment insurance while labor market channels dominate when unemployment insurance is high.

4.4 Other safety net programs

This section discusses an extension of the model that allows to incorporate the other safety net programs discussed in Chapter 2. We need only to change the household side of the model and the government budget constraint.

4.4.1 Household

Different from the section 4.2, every period there are four types of workers - employed with high skill, employed with low skill, short term unemployed, and long term unemployed. There are transfers to different types of the workers after the cash is received. Low-skilled workers receive EITC; short-term unemployed workers still receive unemployment insurance, and long-term unemployed receive social assistance. Let ν^E be the share of high-skilled workers, and ν^U be the share of short-term unemployed in employment and unemployment, respectively.

Utility, budget constraint, and the borrowing constraint of the household are still as before:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\int_0^1 \log\left(c_{i,t}\right) di - \varsigma\left(\sigma_t\right)\left(1 - n_{t-1}\right) \right]$$

$$x_t = \left(1 + i_t\right) \frac{a_t}{p_t} - \left(1 + i_t\right) \frac{b_t}{p_t} + \frac{b_{t+1}}{p_t} \;\; \lambda_t^B$$

$$b_{t+1} \leq p_t \bar{b} \;\; \lambda_t^{BC}$$

Liquidity constraint of high-skilled employed workers is also unchanged:

$$c_{i,t} \leq x_t + \left(1 - \tau_t\right) w_t^H \text{ for employed } \; \lambda_t^{CNH}$$

Now there is also a liquidity constraint of low-skilled employed workers who receive a wage that is lower than the wage of high-skilled workers and therefore are entitled to receive tax credit:

$$c_{i,t} \leq x_t + \left(1 - \tau_t\right) w_t^L + \tau_{tc} \text{ for unemployed } \; \lambda_t^{CNL}$$

Liquidity constraint of short-term unemployed, as before, is:

$$c_{i,t} \leq x_t + \tau_u \text{ for unemployed } \; \lambda_t^{CSU}$$

Finally, liquidity constraint of long-term unemployed differs from the short-term unemployed because they receive the social assistance instead of the unemployment insurance:

$$c_{i,t} \leq x_t + \tau_{sa} \text{ for unemployed } \ \lambda_t^{CLU}$$

Employment law of motion stays as before:

$$n_t = \rho n_{t-1} + \rho_{s,t}\sigma_t \left(1 - n_{t-1}\right) \ \ \lambda_t^N$$

And the assets accumulation equation now incorporates all four types of the agents:

$$\begin{aligned} &\tfrac{a_{t+1}}{p_t} = x_t + \left(1-\tau_t\right) w_t^H \nu^E n_t + \left(\left(1-\tau_t\right) w_t^L + \tau_{tc}\right)\left(1-\nu^E\right) n_t \\ &+\tau_u \nu^U \left(1-n_t\right) + \tau_{sa}\left(1-\nu^U\right)\left(1-n_t\right) + d_t + \textstyle\int_0^1 d_t\left(j\right) dj / p_t - \int_0^1 c_{i,t} di \ \ \lambda_t^A \end{aligned}$$

4.4.2 Government

The government provides the unemployment insurance, τ_u, to the short term unemployed, the social assistance, τ_{sa}, to the long term unemployed, the tax credit, τ_{tc}, to the low-skilled employed, and collects the taxes τ_t to satisfy the government budget constraint:

$$\tau_u \nu^U \left(1-n_t\right) + \tau_{sa}\left(1-\nu^U\right)\left(1-n_t\right) + \tau_{tc}\left(1-\nu^E\right) n_t = \tau_t \left(\nu^E w_t^H + \left(1-\nu^E\right) w_t^L\right) n_t$$

4.4.3 Discussion

With four types of agents there are several possibilities of different agents being liquidity constrained. As before, we can assume that the high-skilled employed workers are not constrained and therefore they determine the interest rate. We can say with certainty that long-term unemployed and, most likely, short-term unemployed are liquidity constrained. Depending on the wages, low-skilled employed may be liquidity constrained or not. These assumptions imply that changes in social assistance and unemployment insurance will translate one-to-one to the increase in aggregate consumption, while tax credit may or may not translate into increase in consumption depending on whether the low-skilled employed workers are constrained or not. This adds a layer of uncertainty into the dynamics of the model and the effects of the safety net. Different types of workers may change their status over the business cycle increasing or decreasing the importance of the safety net. Analysis of these switches is left for future work.

4.5 Conclusion

In this chapter we showed that the reaction of output to a productivity shock as a function of unemployment insurance has a u-shape after one takes into account both, aggregate demand and labor market channels. When unemployment insurance is low, aggregate demand channels dominate and the reaction of output decreases with higher unemployment insurance. When unemployment insurance is high, instead, the labor market channels dominate and the reaction of output increases with higher unemployment insurance. It is important to take into account this trade-off in the design of optimal unemployment insurance.

We also show how the model can be extended to incorporate other safety net programs. A detailed study of the effects of other safety net programs on different channels is left for future research.

4.6 References

1. Chetty, Raj (2008). "Moral Hazard versus Liquidity and Optimal Unemployment Insurance," Journal of political economy, 116(2), pp.173-234.

2. Den Haan, Wouter J., Pontus Rendahl, and Markus Riegler (2014). "Unemployment (fears), Precautionary Savings, and Aggregate Demand."

3. Hagedorn, Marcus, Fatih Karahan, Iourii Manovskii, and Kurt Mitman (2013). "Unemployment benefits and unemployment in the great recession: the role of macro effects", National Bureau of Economic Research, No. w19499.

4. Kekre, Rohan (2016) "Unemployment Insurance In Macroeconomic Stabilization."

5. McKay, Alisdair, and Ricardo Reis (2016). "The role of automatic stabilizers in the US business cycle.", Econometrica, 84 (1), 141-194, January 2016.

6. McKay, Alisdair, and Ricardo Reis (2016). "Optimal Automatic Stabilizers."

7. Midrigan, Virgiliu, and Thomas Philippon (2015). "Household Leverage and the Recession."

8. Mitman, Kurt, and Stan Rabinovich (2015). "Optimal Unemployment Insurance in an Equilibrium Business-Cycle Model.", Journal of Monetary Economics, Vol. 71, 99-118.

9. Ravn, Morten O., and Vincent Sterk (2014). "Job Uncertainty and Deep Recessions." No. 1501.

10. Reiter, Michael (2009). "Solving heterogeneous-agent models by projection and perturbation." Journal of Economic Dynamics and Control 33.3, pp.649-665.

4.7 Appendix

4.7.1 Derivations

Family

FOCs for the second version:

w.r.t. x_t:

$$\lambda_t^B - \lambda_t^A + \int_0^{n_t} \lambda_{i,t}^{CN} di + \int_{n_t}^1 \lambda_{i,t}^{CU} di = 0$$

w.r.t. $c_{i,t}$:

$$\beta^t \frac{1}{c_{i,t}} - \lambda_{i,t}^{CN} + \lambda_t^A = 0 \text{ for employed}$$

with

$$\lambda_{i,t}^{CN} \left(x_t + (1 - \tau_t) w_t - c_{i,t} \right) = 0$$

$$\beta^t \frac{1}{c_{i,t}} - \lambda_{i,t}^{CU} + \lambda_t^A = 0 \text{ for unemployed}$$

with

$$\lambda_{i,t}^{CU} \left(x_t + \tau_u - c_{i,t} \right) = 0$$

w.r.t. b_{t+1}:

$$-\frac{1}{p_t} \lambda_t^B - \lambda_t^{BC} + E_t \left\{ \frac{(1 + i_{t+1})}{p_{t+1}} \lambda_{t+1}^B \right\} = 0$$

with

$$\lambda_t^{BC} \left(p_t \bar{b} - b_{t+1} \right) = 0$$

w.r.t. a_{t+1}:

$$\frac{1}{p_t} \lambda_t^A - E_t \left\{ \frac{(1 + i_{t+1})}{p_{t+1}} \lambda_{t+1}^B \right\} = 0$$

w.r.t. σ_t:

$$-\beta^t \varsigma' (\sigma_t) (1 - n_{t-1}) - \rho_{s,t} (1 - n_{t-1}) \lambda_t^N = 0$$

w.r.t. n_{t+1}:

$$\beta^t \log (c_t^n) - \beta^t \log (c_t^u) + \lambda_t^N -$$
$$-E_t \left\{ (\rho - \rho_{s,t+1} \sigma_{t+1}) \lambda_{t+1}^N - \beta^{t+1} \varsigma (\sigma_{t+1}) \right\} - ((1 - \tau_t) w_t - \tau_u - c_t^n + c_t^u) \lambda_t^A = 0$$

Model with other safety net programs:

FOCs for the second version:

w.r.t. x_t:

$$\lambda_t^B - \lambda_t^A + n_t \nu^E \lambda_t^{CHN} + n_t \left(1-\nu^E\right) \lambda_t^{CLN} + \\ + \left(1-n_t\right) \nu^U \lambda_t^{CSU} + \left(1-n_t\right)\left(1-\nu^U\right) \lambda_t^{CLU} = 0$$

w.r.t. $c_{i,t}$:

$$\beta^t \frac{1}{c_{i,t}} - \lambda_{i,t}^{CHN} + \lambda_t^A = 0 \text{ for high-skilled employed}$$

with

$$\lambda_{i,t}^{CHN} \left(x_t + \left(1-\tau_t\right) w_t^H - c_{i,t}\right) = 0$$

$$\beta^t \frac{1}{c_{i,t}} - \lambda_{i,t}^{CLN} + \lambda_t^A = 0 \text{ for low-skilled employed}$$

with

$$\lambda_{i,t}^{CLN} \left(x_t + \left(1-\tau_t\right) w_t^L + \tau_{tc} - c_{i,t}\right) = 0$$

$$\beta^t \frac{1}{c_{i,t}} - \lambda_{i,t}^{CSU} + \lambda_t^A = 0 \text{ for short-term unemployed}$$

with

$$\lambda_{i,t}^{CSU} \left(x_t + \tau_u - c_{i,t}\right) = 0$$

$$\beta^t \frac{1}{c_{i,t}} - \lambda_{i,t}^{CLU} + \lambda_t^A = 0 \text{ for long-term unemployed}$$

with

$$\lambda_{i,t}^{CLU} \left(x_t + \tau_{sa} - c_{i,t}\right) = 0$$

w.r.t. b_{t+1}:

$$-\frac{1}{p_t} \lambda_t^B - \lambda_t^{BC} + E_t \frac{\left(1+i_{t+1}\right)}{p_{t+1}} \lambda_{t+1}^B = 0$$

with

$$\lambda_t^{BC} \left(p_t \bar{b} - b_{t+1}\right) = 0$$

w.r.t. a_{t+1}:

$$\frac{1}{p_t} \lambda_t^A - E_t \frac{\left(1+i_{t+1}\right)}{p_{t+1}} \lambda_{t+1}^B = 0$$

w.r.t. σ_t:

$$-\beta^t \varsigma' \left(\sigma_t\right) \left(1-n_{t-1}\right) - \rho_{s,t} \left(1-n_{t-1}\right) \lambda_t^N = 0$$

w.r.t. n_t:

$$\int_0^1 \log\left(c_{i,t}\right) di = \int_0^{\nu^E n_t} \log\left(c_{i,t}\right) di + \int_{\nu^E n_t}^{n_t} \log\left(c_{i,t}\right) di + \int_{n_t}^{n_t + (1-n_t)\nu^U} \log\left(c_{i,t}\right) di + \int_{n_t + (1-n_t)\nu^U}^1 \log\left(c_{i,t}\right) di$$

$$\beta^t \left(\nu^E \log\left(c_t^{HN}\right) + \left(1-\nu^E\right) \log\left(c_t^{LN}\right)\right) - \beta^t \left(\nu^U \log\left(c_t^{SU}\right) + \left(1-\nu^U\right) \log\left(c_t^{LU}\right)\right) + \\ + \lambda_t^N - \left(\left(1-\tau_t\right) w_t^H \nu^E + \left(\left(1-\tau_t\right) w_t^L + \tau_{tc}\right)\left(1-\nu^E\right) - \tau_u \nu^U - \tau_{sa}\left(1-\nu^U\right)\right) \lambda_t^A + \\ + \left(\nu^E c_t^{HN} + \left(1-\nu^E\right) c_t^{LN} - \nu^U c_t^{SU} - \left(1-\nu^U\right) c_t^{LU}\right) \lambda_t^A - E_t \left\{\left(\rho - \rho_{s,t+1} \sigma_{t+1}\right) \lambda_{t+1}^N - \beta^{t+1} \varsigma\left(\sigma_{t+1}\right)\right\} = 0$$

Retailers

$$d_t(j) = \left(\frac{p_t(j)}{p_t}\right)^{1-\epsilon} Y_t - q_t \left(\frac{p_t(j)}{p_t}\right)^{-\epsilon} Y_t$$

FOC:

$$(1-\epsilon)\left(\frac{p_t(j)}{p_t}\right)^{-\epsilon} Y_t + \epsilon q_t \left(\frac{p_t(j)}{p_t}\right)^{-\epsilon-1} Y_t = 0$$

Solve for prices:

$$\frac{p_t(j)}{p_t} = \frac{\epsilon}{\epsilon-1} q_t$$

Total dividends:

$$d_t = d_t^W + \int_0^1 d(j)\, dj$$

Price ratio:

$$\frac{p_t^*}{p_t} = \frac{p_t^A}{p_t^B}$$

p^A:

$$p_t^A = \frac{\epsilon}{(\epsilon-1)} q_t Y_t + E\left\{\Lambda_{t,t+1}(1-\theta)(\pi_{t+1})^{\epsilon} p_{t+1}^A\right\}$$

p^B

$$p_t^B = Y_t + E\left\{\Lambda_{t,t+1}(1-\theta)(\pi_{t+1})^{\epsilon-1} p_{t+1}^B\right\}$$

Inflation:

$$\pi_t = \left(\frac{1-\theta}{1-\theta\left(\frac{p_{t+1}^*}{p_{t+1}}\right)^{1-\epsilon}}\right)^{\frac{1}{1-\epsilon}}$$

Production:

$$s_t Y_t = z_t n_t$$

Production loss:

$$s_t = (1-\theta)\, s_{t-1} \pi_t^{-\epsilon} + \theta\left(\frac{p_t^*}{p_t}\right)^{-\epsilon}$$

Total dividends:

$$\int_0^1 d(j)\, dj = Y_t - q_t z_t n_t$$

Wholesale

FOCs:

$$\Lambda_{0,t}(q_t z_t - w_t) - \lambda_t^F + E_t\left\{\rho \lambda_{t+1}^F\right\} = 0$$

$$-\Lambda_{0,t}\kappa + \rho_{f,t}\lambda_t^F = 0$$

optimal job creation:

$$q_t z_t - w_t + E_t\left\{\Lambda_{t,t+1}\rho\frac{\kappa}{\rho_{f,t+1}}\right\} = \frac{\kappa}{\rho_{f,t}}$$

Equations

For households:

$$\beta^t \frac{1}{c_t^n} + \lambda_t^A = 0$$

$$\beta^t \frac{1}{c_t^u} - \lambda_t^{CU} + \lambda_t^A = 0$$

$$\lambda_t^B - \lambda_t^A + (1 - n_t)\,\lambda_t^{CU} = 0$$

$$\lambda_t^A = E_t \frac{(1 + i_{t+1})\,p_t}{p_{t+1}} \lambda_{t+1}^B$$

$$-\frac{1}{p_t}\lambda_t^B - \lambda_t^{BC} + E_t \frac{(1 + i_{t+1})}{p_{t+1}} \lambda_{t+1}^B = 0$$

$$-\beta^t \varsigma'(\sigma_t)(1 - n_{t-1}) - \rho_{s,t}(1 - n_{t-1})\,\lambda_t^N = 0$$

$$\beta^t \log(c_t^n) - \beta^t \log(c_t^u) + \lambda_t^N -$$
$$-E_t \left\{ (\rho - \rho_{s,t+1}\sigma_{t+1})\,\lambda_{t+1}^N - \beta^{t+1}\varsigma(\sigma_{t+1}) \right\} - \left((1 - \tau_t)\,w_t - \tau_u - c_t^n + c_t^u\right)\lambda_t^A = 0$$

$$x_t = (1 + i_t)\frac{a_t}{p_t} - (1 + i_t)\frac{b_t}{p_t} + \frac{b_{t+1}}{p_t}$$

$$x_t + \tau_u - c_t^u = 0$$

$$n_t = \rho n_{t-1} + \rho_{s,t}\sigma_t (1 - n_{t-1})$$

$$\frac{a_{t+1}}{p_t} = x_t + (1 - \tau_t)\,w_t n_t + \tau_u (1 - n_t) + d_t - \left(n_t c_t^n + (1 - n_t)\,c_t^u\right)$$

$$\frac{b_t}{p_{t-1}} = \frac{b_{t+1}}{p_t} = \frac{a_t}{p_{t-1}} = \frac{a_{t+1}}{p_t} = \overline{b}$$

Get rid of the multipliers:

$$\lambda_t^A = -\beta^t \frac{1}{c_t^n}$$

$$\lambda_t^{CU} = \beta^t \frac{1}{c_t^u} + \lambda_t^A = \beta^t \left(\frac{1}{c_t^u} - \frac{1}{c_t^n}\right)$$

$$\lambda_t^B = \lambda_t^A - (1 - n_t)\,\lambda_{i,t}^{CU} = \beta^t \left(-\frac{1}{c_t^n} - (1 - n_t)\left(\frac{1}{c_t^u} - \frac{1}{c_t^n}\right)\right) = -\beta^t \left(n_t \frac{1}{c_t^n} + (1 - n_t)\frac{1}{c_t^u}\right)$$

$$\lambda_t^N = -\frac{\beta^t \varsigma'(\sigma_t)}{\rho_{s,t}}$$

Optimal labor:

$$\log(c_t^n) - \log(c_t^u) - \tfrac{\varsigma'(\sigma_t)}{\rho_{s,t}} +$$
$$+E_t \left\{ (\rho - \rho_{s,t+1}\sigma_{t+1})\,\tfrac{\beta\varsigma'(\sigma_{t+1})}{\rho_{s,t+1}} + \beta\varsigma(\sigma_{t+1}) \right\} + \left((1 - \tau_t)\,w_t - \tau_u - c_t^n + c_t^u\right)\tfrac{1}{c_t^n} = 0$$

Euler:

$$\frac{1}{c_t^n}=E_t\frac{(1+i_{t+1})}{\pi_{t+1}}\beta\left(n_{t+1}\frac{1}{c_{t+1}^n}+(1-n_{t+1})\frac{1}{c_{t+1}^u}\right)$$

Note: In this case the interest rate is lower than in the case of representative agent, so that the household doesn't want to save more in order to insure for the future. Because there is no more bond supply, the household needs to optimally choose to save exactly up to borrowing limit.

$$x_t=(1+i_t)\frac{a_t}{p_t}-(1+i_t)\frac{b_t}{p_t}+\frac{b_{t+1}}{p_{t+1}}\pi_{t+1}$$

$$x_t+\tau_u-c_t^u=0$$

$$n_t=\rho n_{t-1}+\rho_{s,t}\sigma_t u_{t-1}$$

$$\frac{a_{t+1}}{p_{t+1}}\pi_{t+1}=x_t+(1-\tau_t)\,w_t n_t+\tau_u(1-n_t)+d_t-\left(n_t c_t^n+(1-n_t)\,c_t^u\right)$$

$$\frac{b_t}{p_t}=\frac{b_{t+1}}{p_{t+1}}=\frac{a_t}{p_t}=\frac{a_{t+1}}{p_{t+1}}=\bar{b}$$

Firms:

$$q_t z_t-w_t+E_t\left\{\Lambda_{t,t+1}\rho\frac{\kappa}{\rho_{f,t+1}}\right\}=\frac{\kappa}{\rho_{f,t}}$$

$$d_t=q_t z_t n_t-w_t n_t-\kappa v_t$$

$$\frac{p_t(j)}{p_t}=\frac{\epsilon}{\epsilon-1}q_t$$

$$\frac{p_t^*}{p_t}=\frac{p_t^A}{p_t^B}$$

$$p_t^A=\frac{\epsilon}{(\epsilon-1)}q_t Y_t+E\left\{\Lambda_{t,t+1}(1-\theta)(\pi_{t+1})^{\epsilon}p_{t+1}^A\right\}$$

$$p_t^B=Y_t+E\left\{\Lambda_{t,t+1}(1-\theta)(\pi_{t+1})^{\epsilon-1}p_{t+1}^B\right\}$$

$$\pi_t=\left(\frac{1-\theta}{1-\theta\left(\frac{p_t^*}{p_t}\right)^{1-\epsilon}}\right)^{\frac{1}{1-\epsilon}}$$

$$s_t Y_t=z_t n_t$$

$$s_t=(1-\theta)\,s_{t-1}\pi_t^{-\epsilon}+\theta\left(\frac{p_t^*}{p_t}\right)^{-\epsilon}$$

$$\int_0^1 d(j)\,dj = Y_t - q_t z_t n_t$$

Government:

$$\tau_u u_t = \tau w_t n_t$$

Balances:

$$z_t n_t = n_t c_t^n + (1-n_t)\,c_t^u + \kappa v_t$$

$$\rho_{s,t} = \chi\left(\frac{v_t}{\sigma_t\,(1-n_{t-1})}\right)^{1-\alpha}$$

$$\rho_{f,t} = \chi\left(\frac{v_t}{\sigma_t\,(1-n_{t-1})}\right)^{-\alpha}$$

Steady state

$$\frac{1}{c^n} = (1+i)\,\beta\left(n\frac{1}{c^n} + (1-n)\,\frac{1}{c^u}\right)$$

$$\log(c^n) - \log(c^u) - \tfrac{\varsigma'(\sigma)}{\rho_s} +$$
$$+ (\rho - \rho_s\sigma)\,\tfrac{\beta\varsigma'(\sigma)}{\rho_s} + \beta\varsigma(\sigma) + ((1-\tau)\,w - \tau_u - c^n + c^u)\,\tfrac{1}{c^n} = 0$$

$$x = (1+i)\,\frac{a}{p} - (1+i)\,\frac{b}{p} + \frac{b}{p}$$

$$x + \tau_u - c^u = 0$$

$$n = \rho n + \rho_s\sigma\,(1-n) \text{ or } n = \frac{\rho_s\sigma}{1+\rho_s\sigma-\rho}$$

$$\frac{a}{p} = x + (1-\tau)\,wn + \tau_u\,(1-n) + d - (nc^n + (1-n)\,c^u)$$

$$\frac{b}{p} = \frac{a}{p} = \bar{b}$$

$$d^W = qzn - wn - \kappa v$$

$$d = (1-q)\,zn + d^W$$

$$1 = \frac{\epsilon}{\epsilon-1}q$$

$$qz - w + \beta\rho\frac{\kappa}{\rho_f} = \frac{\kappa}{\rho_f}$$

$$\tau_u\,(1-n) = \tau wn$$

$$zn = nc^n + (1-n)\,c^u + \kappa v$$
$$\rho_s = \chi\left(\frac{v}{\sigma(1-n)}\right)^{1-\alpha}$$
$$\rho_f = \chi\left(\frac{v}{\sigma(1-n)}\right)^{-\alpha}$$

4.7.2 Wage bargaining solution

Marginal value of an additional worker:

$$\frac{\partial F(w_t,n_t)}{\partial n_t} = q_t z_t - w_t + E_t\left\{\Lambda_{t,t+1}\left(\frac{\partial F(w_{t+1},n_{t+1})}{\partial n_{t+1}}\frac{\partial n_{t+1}}{\partial n_t} - \kappa\frac{\partial v_{t+1}}{\partial n_t}\right)\right\}$$

because hiring is optimal tomorrow

$$\frac{\partial F(w_t,n_t)}{\partial n_t} = q_t z_t - w_t + E_t\left\{\Lambda_{t,t+1}\left(\frac{\partial F(w_{t+1},n_{t+1})}{\partial n_{t+1}}\rho\right)\right\}$$

Marginal value of an additional worker:

$$\frac{\partial W(w_t,n_t)}{\partial n_t} = \log(c_t^n) - \log(c_t^u) + \beta E_t\left\{\frac{\partial W(w_{t+1},n_{t+1})}{\partial n_{t+1}}\frac{\partial n_{t+1}}{\partial n_t} - \varsigma'(\sigma_{t+1})(1-n_t)\frac{\partial \sigma_{t+1}}{\partial n_t} + \varsigma(\sigma_{t+1})\right\}$$

because search is optimal tomorrow

$$\frac{\partial W(w_t,n_t)}{\partial n_t} = \log(c_t^n) - \log(c_t^u) + \beta E_t\left\{\frac{\partial W(w_{t+1},n_{t+1})}{\partial n_{t+1}}(\rho - \rho_{s,t}\sigma_t) + \varsigma(\sigma_{t+1})\right\}$$

And the bargaining problem is:

$$w_t = \arg\max\left(\frac{\partial W(w_t,n_t)}{\partial n_t}\right)^{\eta}\left(\frac{\partial F(w_t,n_t)}{\partial n_t}\right)^{1-\eta}$$

Sharing rule:

$$\eta\frac{1}{c_t^n}\frac{\partial F(w_t,n_t)}{\partial n_t} = (1-\eta)\frac{\partial W(w_t,n_t)}{\partial n_t}$$

Use firm's FOC

$$E_t\left\{\Lambda_{t,t+1}\left(\frac{\partial F(w_{t+1},n_{t+1})}{\partial n_{t+1}}\right)\right\} = \frac{\kappa}{\rho_{f,t}}$$

Use Sharing rule tomorrow:

$$(1-\eta)\,\beta E_t\left\{\frac{\partial W(w_{t+1},n_{t+1})}{\partial n_{t+1}}\right\} = \eta E_t\left\{\frac{1}{c_{t+1}^n}\Lambda_{t,t+1}\left(\frac{\partial F(w_{t+1},n_{t+1})}{\partial n_{t+1}}\right)\right\} = \eta\frac{\kappa}{\rho_{f,t}}\frac{1}{c_t^n}$$

Now substitute

$$\eta \frac{1}{c_t^n} \left(q_t z_t - w_t + E_t \left\{ \Lambda_{t,t+1} \left(\frac{\partial F(w_{t+1}, n_{t+1})}{\partial n_{t+1}} \rho \right) \right\} \right) =$$
$$= (1-\eta) \left(\log\left(c_t^n\right) - \log\left(c_t^u\right) + \beta E_t \left\{ \frac{\partial W(w_{t+1}, n_{t+1})}{\partial n_{t+1}} \left(\rho - \rho_{s,t}\sigma_t\right) + \varsigma\left(\sigma_{t+1}\right) \right\} \right)$$

$$\eta \frac{1}{c_t^n} \left(q_t z_t - w_t + \rho \frac{\kappa}{\rho_{f,t}} \right) = (1-\eta) \left(\log\left(c_t^n\right) - \log\left(c_t^u\right) + \left(\rho - \rho_{s,t}\sigma_t\right) \frac{\kappa}{\rho_{f,t}} \frac{1}{c_t^n} \frac{\eta}{1-\eta} + \beta E_t \left\{ \varsigma\left(\sigma_{t+1}\right) \right\} \right)$$

Solve for the wage:

$$\left(q_t z_t + \rho \frac{\kappa}{\rho_{f,t}} \right) - \frac{(1-\eta)}{\eta \frac{1}{c_t^n}} \left(\log\left(c_t^n\right) - \log\left(c_t^u\right) + \left(\rho - \rho_{s,t}\sigma_t\right) \frac{\kappa}{\rho_{f,t}} \frac{1}{c_t^n} \frac{\eta}{1-\eta} + \beta E_t \left\{ \varsigma\left(\sigma_{t+1}\right) \right\} \right) = w_t$$

$$w_t = q_t z_t + \rho_{s,t}\sigma_t \frac{\kappa}{\rho_{f,t}} - \frac{(1-\eta)}{\eta \frac{1}{c_t^n}} \left(\log\left(c_t^n\right) - \log\left(c_t^u\right) + \beta E_t \left\{ \varsigma\left(\sigma_{t+1}\right) \right\} \right)$$