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Passing to more personal thanks, a premise is necessary. I have been living in Milan for eleven years and counting. And to me, this accomplishment seems the end of something more than a course of study. For this reason, the number of people to thank is so high that no one should feel offended if they are not mentioned explicitly in these lines. As an applied theorist, I think the best strategy is to proceed in an ordered manner. Therefore, I will mention four different circles of acquaintances: my family, my girlfriend and her family, my fellow friends, and folks at Castelbarco. About the first group, let's face it: words can not tell the feelings and bounds that connect me to my dad Saulo, my mum Renata, my sister Emma, and my grandma Anna. So, I will just say that I owe them what I am... and that's it! A sub-category of 'family' is constituted by my home friends, Pietro, Lapo, and Niccolò, whom I have been knowing for a while. I regard them as crystal-clear evidence that the depth of a connection is not necessarily determined by the time you can spend together. I will use this last sentence to pass to the second category, and publicly express my gratitude for the patience and spirit of sacrifice that Sofía has put in this relationship. I hereby solemnly swear that the time I have stolen from us in this last year will be entirely returned and augmented. I would also like to say thanks to her family for the hospitality they have had for me over the last couple of years. To this aim, I will list them all here, in alphabetical order as not to create any discrimination: Reina,[1] Ben, Jesús, Nuria and Pilar. As it comes to my colleagues, a special thank goes to Angelo, Francesca, and Melika. I feel that I would have never made it without their

[1] I am well aware that Reina (the dog, for those who do not have the privilege of knowing her) should be last. I am actually trying to be ironic.

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Introduction

This dissertation consists of three essays on public finance. The first and second essay analyze worker profiling policies in the context of unemployment assistance programs. In particular, the first essay establishes optimal criteria to allocate reemployment services to jobless welfare recipients according to their profile. The second essay studies the complementarity between worker profiling and the provision of job-search incentives, and derives an analytical tool to conduct a welfare analysis on existing profiling policies. The third essay explores the implications of international cooperation in the handling of a global pandemic, and outlines the drivers that determine the direction of international aids according to an efficiency-based perspective.

In the first chapter, a risk-neutral government, who provides risk-averse unemployed workers with welfare support, finds it optimal to match workers with active or passive labor-market policies, based on workers' human capital. However, when human capital is subject to two-sided uncertainty, the government can decide either to detect it via profiling, or to form expectations about it and match workers and policies accordingly. The paper delivers two findings. First, the government's return from worker's search is increasing and concave in expectations, due to hyperbolic decreasing incentive costs and linear increasing labor taxes upon reemployment. Second, the concavity of returns causes the value of information to be negative for high-end expectations, whenever the loss from putting low-skilled workers at rest outweighs the gain from lowering search incentives to high-skilled workers. If so, profiling should not fully detect human capital, but rather boost the expectations of a share of low-skilled workers and persuade them to search for re-employment at a lower incentive cost for the government.

The second chapter deals with unemployment assistance, that is tailored to workers' human capital in optimum. Since human capital is difficult to infer, assistance is provided on the basis of its expected level. Alternatively, workers can be profiled and their actual level of human capital be detected. A profiling program establishes (i) whom to profile,

(ii) at what stage of the program and (iii) what unemployment benefits to pay, according to the new information obtained. The paper identifies the determinants of optimal profiling along these three dimensions in a dynamic principal-agent framework with non-contractible effort and two-sided uncertainty about workers' human capital. There are two main findings. First, workers with higher expectations on human capital are incentivized to search for a job, thanks to larger returns on search effort. They are profiled only at a successive stage of the unemployment spell, once the savings from fine-tuning of benefits outweigh the cost of profiling. Second, since the cost of incentive provision is increasing in the generosity of benefits, profiling is used also to lower promised benefits to those workers who are requested to search after it.

The third chapter deals with pandemics, considered as global phenomena that are confronted by domestic containment measures. Domestic measures trade off economic and human losses suffered by the country that adopts them. However, domestic policymakers overlook the impact their decisions have on the risk of cross-border contagion and end up adopting too mild restrictions compared to the social optimum. Bilateral transfers, contingent on the evolution of the pandemic, constitute a mutual insurance scheme among countries and a channel for internalizing such spillovers. More infected countries receive larger transfers on the condition that they prove able to limit the spread of the contagion, and this creates incentives for them to adopt more stringent measures. In addition, the productivity of factors being positively correlated with the diffusion of the pandemic produces a concentration of investments toward less infected countries, which may reverse the direction of transfers.

Worker Profiling and Reemployment Services

ABSTRACT

A risk-neutral government, who provides risk-averse unemployed workers with welfare support, finds it optimal to match workers with active or passive labor-market policies, based on workers' human capital. However, when human capital is subject to two-sided uncertainty, the government can decide either to detect it via profiling, or to form expectations about it and match workers and policies accordingly. The paper delivers two findings. First, the government's return from worker's search is increasing and concave in expectations, due to hyperbolic decreasing incentive costs and linear increasing labor taxes upon reemployment. Second, the concavity of returns causes the value of information to be negative for high-end expectations, whenever the loss from putting low-skilled workers at rest outweighs the gain from lowering search incentives to high-skilled workers. If so, profiling should not fully detect human capital, but rather boost the expectations of a share of low-skilled workers and persuade them to search for re-employment at a lower incentive cost for the government.

JEL classification: D82, D83, H21, J64, J65

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Unemployment Insurance, Worker Profiling

1.1 Introduction

Worker Profiling and Reemployment Services (hereafter, WPRS) is a US Federal program created in 1993 "[...] to identify and rank or score unemployment insurance (UI) claimants by their potential for exhausting their benefits for referral to appropriate reemployment services"[2]. The program aims at helping the jobless to exit unemployment and encompasses an assistance part *stricto sensu*, represented by the joint offer of UI benefits and job-search assistance services, and a profiling part, that screens the characteristics of welfare claimants, so as to better allocate them to the proper welfare policy. Similar programs are present in many OECD countries (see Desiere et al., 2019).

Profiling programs play an ancillary role in welfare provision, in so far as they make it possible to achieve allocative efficiency of policy instruments among recipients, by means of detection of their unobservable characteristics. However, the effectiveness of this type of program has been questioned, for they display a lower return than apprenticeship-based programs, like training and public sector employment, and 'work-first' programs, like job-search assistance and monitoring, in terms of both future earnings and reemployment probabilities. Sullivan et al. (2007) conduct an overall review of WPRS implementation strategies in all US States. Rather than focusing on worker-related dimensions, like job-finding rates or productivity upon re-employment, the authors evaluate the performance of the various strategies based on budget savings for the government.[3]

The criterion adopted by Sullivan et al. to assess the various profiling methods is inspired by the objective of cost minimization and consists of selecting the lowest frequency of 'false positives', that is, recipients who are deemed likely to exhaust welfare benefits before being re-employed (and therefore referred to Reemployment Services), but are actually able to find a job in complete autonomy. Moreover, the authors report that State Workforce Agencies, which are in charge of WPRS implementation, usually decide the criteria for workers' referral to Reemployment Services based on available resources. This fact causes referral criteria to be more stringent in those States where the WPRS program is less generous.

However, a rigorous study of the trade-offs existing in any such program where both 'traditional' policy instruments and profiling are jointly implemented has been missing so far. The main objective of this paper is to study the complementarity between unemploy-

[2]See Sullivan et al. (2007).

[3]Even though this is admittedly an extreme view, one must consider that realized budget savings can be reinvested in other welfare programs or used to raise the generosity of the existing ones.

ment assistance and information detection and provide a rationale for assessing different profiling strategies in the context of welfare programs. To do so, the paper develops an amenable framework that embeds both assistance provision and information detection. In particular, any profiling strategy varies according to two dimensions. First, it determines who should be subject to profiling and to what level of accuracy, based on the expected individual characteristics of each worker. Second, it outlines the optimal transfer scheme for each worker, conditional on the profiling outcome.

The starting point of the analysis is the work by Pavoni and Violante (2007), which reads policy instruments as a combination of technologies implemented by the government, and search recommendations and consumption contracts offered to the welfare recipients. The novelty is the introduction of an information detection (or profiling) technology, that is capable of detecting the level of human capital of each worker, to any desired level of accuracy. Ahead of profiling, the welfare recipient is offered a consumption contract depending on the outcome. The combination of technology and contract gives rise to a new policy instrument, which is Worker Profiling (WP, henceforth). Reemployment Services within the WPRS program offer support to any worker who is deemed unlikely to find a job vacancy before exhausting welfare benefits. For this reason, the paper interprets such Services as a form of Job-search Assistance (JA, henceforth), which requires the government to implement a suitable technology for internalizing the job-search.

Worker's job search is an active labor-market policy alternative to JA. Unemployment Insurance (UI, henceforth) arises whenever the government delegates the job search to the worker. By doing so, the government has no direct control over the job search, and this allows the worker to behave opportunistically, by avoiding search and originating a moral hazard motive, thereof. The only way for the government to make the worker search is to promise her incentives through an *ad-hoc* transfer scheme. In particular, the government can promise lower labor taxes/higher wage subsidies to the worker who manages to get re-employed. Incentives cause the contract to display a consumption dispersion in the alternative scenarios of successful and failed job search. Therefore, when the government decides the policy instrument to assign to the worker, it trades off consumption insurance and incentive provision (Shavell and Weiss, 1979).

In alternative to assisted (JA) and private (UI) search, workers may only be provided with income support, with no search incentives, while no technology is adopted by the government. Such a passive policy is referred to as Social Assistance (SA).

Efficiency gains from profiling in the form of budget savings originate from allocative efficiency, by means of incentive alignment between the government-principal and the worker-agent. Given that workers' human capital is crucial for optimal allocation of policies, yet unobservable both by the government and workers themselves, the government can not help pooling *ex-ante* identical workers into the same policy and contract. This source of inefficiency is avoided once workers' human capital is detected and workers are referred to policies according to it. However, two forces contrast profiling. First, the detection technology is costly and makes WP suboptimal when its cost overcomes its expected gain, which always occurs when expectations about human capital are particularly low or high, that is, whenever the fraction of all workers holding that expectation being allocated to the wrong policy is quite low. Second, since the return from job search is increasing in expected human capital, unveiling it produces a loss on those workers who are found to be low skilled, and nonetheless were quite confident about their skills (and therefore easy to persuade to search) before being profiled. This second force advocates for the criterion of referral to Job-search or Social Assistance (JA or SA) that ensues profiling to be stringent rather than informative. Indeed, the denial of JA or SA to any profiled worker is less indicative of high human capital, the more stringent are criteria to access such policies.

The paper provides sufficient conditions to establish when the trade-off between informativeness and stringency falls in favor of the latter, that is, when it is optimal to supply JA or SA to fewer workers than the needy ones. In particular, it shows that this is the case for low generosity levels of the program, whenever the cost of incentive provision under UI, and consequently the comparative advantage of JA/SA over UI, increase in generosity. Thus, if incentives provision is relatively cheap for low generosity levels, it is efficient to resort to it to a larger extent.

The paper first models the profiling policy in a context where no technology is implemented other than the information detecting one (i.e., the only policies implemented are UI, SA and WP). Then, in order to mimic more closely the existing WPRS program, the four policies are studied jointly. The rest of the paper is organized as follows. Section 3.2 contains the literature review. Section 1.3 presents the economic environment. Section 2.4 describes the policies. Section 1.5 describes the optimal WPRS program. Section 1.6 extends the analysis to the case in which workers' expectations are unobservable to the government. Section 3.5 concludes.

1.2 Literature Review

The contribution of this paper is the design of an optimal welfare program under the assumption that recipient's heterogeneity in reemployment chances is *ex-ante* unobservable by both the provider and recipients themselves, and with profiling as a policy instrument to reveal these chances to both parties (with a certain accuracy) and refer recipients to the best job-search policy.

The presence of a Federal program like WPRS that profiles UI claimants according to their chances of reemployment is indicative of the fact that such chances are not easy to infer *prima facie*, at the time jobless workers apply to those services. Differences in human capital among the jobless have been widely documented by the empirical literature, as well as the negative duration dependence employability displays as the unemployment spell progresses (Machin and Manning, 1999). And even if a positive correlation has been documented between the hazard rate out of unemployment and some characteristics which are easily observable prior to displacement, like wage (Meyer, 1990), such characteristics may be themselves difficult to observe with sufficient precision after displacement. For instance, a vast set of empirical studies has documented that wages suffer depreciation during unemployment (Addison and Portugal, 1989), and that workers experience wage losses upon separation (Fallick, 1996). These two facts make on-the-job wages an unlikely proxy for reemployment probabilities. Another strand of the literature assumes heterogeneity not to be (entirely) detectable by agents themselves. Böhein et al. (2011) study how misperception of the human-capital component of wages may lead jobless workers to set too a high reservation wage and refuse 'good' job offers, thus lengthening their unemployment spell for longer than it is optimal. Such a wrong perception originates from multiple components, other than productivity, that add up in wages (e.g., seniority wages) and create a wedge between labor remuneration and real productivity. Similarly to these works, I introduce the novelty that human capital is subject to two-sided uncertainty. This assumption leaves room for acquisition of information, and for a suitable manipulation of expectations accordingly. The government can decide: (i) whether to profile any given worker, (ii) the precision of the signal she should receive.

The moral hazard problem that originates from claimant's job search being private can be solved in three possible ways. First, the provider can monitor jobseekers in their search activity and only compensate them for search effort (Pavoni and Violante, 2007). Second, it can directly conduct the job search on behalf of welfare claimants, who are in turn guaranteed income support in the meantime (Pavoni et al., 2016). Or, in alternative, the provider can fully delegate the job search to claimants and commit to a suitable contract that incentivizes workers to search. In the third case, incentives can be provided by leveraging on both wage taxes upon reemployment and continuation benefits in unemployment (Hopenhayn and Nicolini, 1997). Pavoni and Violante (2007) make the first attempt to embed the monitored and the incentivized job-search within the same welfare-to-work program, later followed by Pavoni et al. (2013) and Setty (2019). Pavoni and Violante (2007) and Pavoni et al. (2013) conclude that the gains from claimants' search are increasing in the level of worker's human capital, due to decreasing incentive costs. This result holds in my framework as well, once human capital is replaced by expectations about it. Differently from these works, my paper introduces worker profiling as a new policy to detect human capital of welfare recipients. Setty (2019) has close connections to my paper. In his work, the provider chooses to monitor worker's effort with some precision and cost. A trade-off arises as precision is positively correlated to monitoring costs and negatively correlated to search-incentive costs. Moreover, the inverse of worker's utility having convex derivative is a necessary and sufficient condition for precision to be increasing in the generosity of the program. Likewise, my paper finds the same condition to be sufficient for the generosity of the program and the precision of profiling to be positively correlated. However, the cost of precision is endogenous in my case, and coincides with the lower return from reallocating low-skilled workers to a passive policy instrument.

The paper is related to the vast and growing literature on information design initiated by Kamenica and Gentzkow (2011), that deal with the design of an optimal signaling strategy from a principal/sender to an agent/receiver. The peculiarity of the present framework is the 'hybrid' nature of the problem, which mixes the design of information with that of an effort-incentivizing contract. Rodina and Farragut (2020), and Boleslavsky and Kim (2021) study a three-players problem of a principal/sender who wants to induce an agent to exert the desired effort level, and a receiver to take the desired action according to the signal realization. Therefore, unlike this paper, they make the agent's effort level determine the distribution of the underlying state. Moreover, while Rodina and Farragut (2020) assume the sender to be concerned only with maximizing agent's effort, Boleslavsky and Kim (2021) -similar to this paper- assume the sender to be only indirectly interested in agent's effort, since the latter affects her payoff only through the receiver's conjecture about the unknown state. In my framework, instead, agent and receiver are the same person (the claimant), and her level of job-search effort matters to the provider/sender only in so far as occurrence of either future scenario ('Re-employment' and 'Not re-employment') depends on it. Bloedel and Segal (2018), Habibi (2020) and Zapechelnyuk (2020) also study the tension between incentive and information provision in the Bayesian persuasion framework applied to situations of agent's rational inattention, agent's time-inconsistency and quality certification, respectively. However, to the best of my knowledge, Bayesian persuasion has not been studied in the context of unemployment insurance so far.

A second departure from the seminal framework by Kamenica and Gentzkow (2011) is that the paper also considers the case where the initial expectation is private information of the worker, who may lie about it, so to enjoy a larger expected utility. Private worker information may originate from independent, temporary shocks hitting income (Thomas and Worral, 1990), or rather be innate, as the chances of finding a new job (Hagedorn et al., 2010) or the job search effort cost (Fuller, 2014), or show some persistency over time, like correlated income shocks (Fernandes and Phelan, 2000). All these works explore the implications of adverse selection on the optimal contract for different types of workers. Yet, none of them considers any possibility for the planner to circumvent adverse selection other than designing contracts that are robust to strategic lying (i.e., information rents versus contract efficiency). In this paper, instead, profiling publicly reveals new information that lowers the value of private information held by agents, hence constituting a way to relax the adverse selection problem for the principal.

1.3 Economic Environment

Players The economic environment is populated by a risk-neutral government (it, principal) and a risk-averse unemployed worker (she, agent). The government designs a welfare program with three objectives: (i) supporting the worker's income with unemployment benefits up to a given generosity level, (ii) favoring the worker's transition toward reemployment, and (iii) minimizing the program's cost.[4] The government addresses this triple trade-off, by implementing a set of policies, as well as by tailoring each to the individual characteristics of the worker.

Human Capital The worker's human capital can either be high ($h = H$) or low ($h = L$). Human capital impacts the on-the-job productivity of the worker ($\omega_H > \omega_L$) and the probability of receiving a job offer after an interview ($\xi_H > \xi_L$). For this reason, workers with

[4]The government solves the 'dual' problem, that is, it selects the less expensive program among those that meet a given standard of assistance. Such standard can be interpreted as the program's generosity, and turns out to be a relevant dimension for the design of an optimal program.

high (resp., low) human capital will also be labelled as high-(resp., low-)skilled.

Expectations When the worker applies to welfare support, government collects her personal data (social background, past working experiences, education, etc.). According to this individual information, the government can make a first assessment of her job-finding chances on the basis of statistical data. Graduated workers, indeed, are statistically more likely to find a new employment than workers with a lower educational attainment. Such evidence, grounded on the observation of a large amount of cases, allows the government to form unbiased expectations about the individual skills of the worker. In particular, the paper defines *expectation* the probability μ that the worker is high-skilled. By a law of large numbers argument, such a probability is unbiased, meaning that the fraction of high-skilled workers among all workers with same expectation coincides with the expectation itself.

Private Search When the job search is delegated to the worker, she holds private information about her search effort level. The job search is composed of two successive steps: application and interview. In the first step, the worker can either apply to a job posting ($a = 1$), or not ($a = 0$), and gets re-contacted by the employer with probability $p(a)$, which is positive if $a = 1$ ($p(1) > 0$) and null otherwise ($p(0) = 0$). In the second step, the worker is interviewed by the prospective employer and offered a job with probability ξ_h, depending on human capital realization h. Therefore, the probability of finding a job for a worker who exerts search effort a is

$$\pi(h, a) = p(a)\xi_h$$

Given that $p(h, 0) = 0$, notation is simplified by setting $\pi_h := \pi(h, 1)$.

Market-sector production Labor productivity is revealed upon reemployment. The economy displays one market sector, populated by identical atomistic firms competing à la Bertrand over job offers, thus paying to workers a wage ω_h equal to their labor productivity.

Preferences and effort Worker's effort arises from the job search activity. The worker can either search ($a = 1$) or rest ($a = 0$). In the former case, she pays cost $\alpha(1) = e$ and enjoys probability of re-employment π_h. Otherwise, her effort cost is null ($\alpha(0) = 0$). The effort cost is separable from consumption, with worker's utility over consumption c and effort given by $v(c, a) = \log(c) - \alpha(a)$.

Assisted-search technology Conditional on payment of cost κ^{ja}, the government can apply to job postings (first step of the job search) on behalf of the worker. The *per-capita* cost of assisted search includes the administrative expenses of the offices which are in charge of looking for vacancies, create a network with prospective employers and maintain contacts with them, circulate the worker's CV, etc.

Profiling technology Conditional on payment of cost[5] κ^{wp}, profiling detects human capital with a chosen accuracy, and returns a publicly observable binary outcome, 'Pass' ($r = p$) or 'Fail' ($r = f$). The probability of observing outcome r is chosen by the government and depends on underlying human capital h, expectation μ and program's generosity U

$$\left\{\sigma(r|h,\mu,U)\right\}_{r\in\{p,f\},\,h\in\{H,L\}}$$

Contract The worker is offered a contract that specifies a transfer scheme (which equals worker's consumption, as the world only lasts one period), contingent on reemployment (c^w) or not (c^u), and on the profiling outcome r (if profiling is adopted).

$$\left\{c_r^w, c_r^u\right\}_{r\in\{f,p\}}$$

Policies Any policy arises as the composition of (i) consumption contract, and (ii) technology adopted. If no technology is implemented, the government can decide whether to recommend positive search effort and pay incentive costs ('Unemployment Insurance', $i = UI$), or not ('Social Assistance', $i = SA$). If the assisted search technology is implemented, it gives rise to 'Job-Search Assistance' ($i = JA$). Finally, the worker undergoes profiling under 'Worker Profiling' ($i = WP$).

Timing The interaction between workers and provider takes place in one period (see Fig. 1.1). Workers enter the program and are requested to report personal data (age, education, past working experiences, etc.). Based on information collected, expectations are formed about worker's individual reemployment perspectives. The government therefore assigns the worker to a welfare instrument. Uncertainty about employment status ('employed'/'unemployed') and profiling outcome ('Pass'/'Fail') clears, and the government pays (resp., raises) the contingent transfer (resp., tax).

[5] The cost includes administrative expenses, as in the case of assisted search.

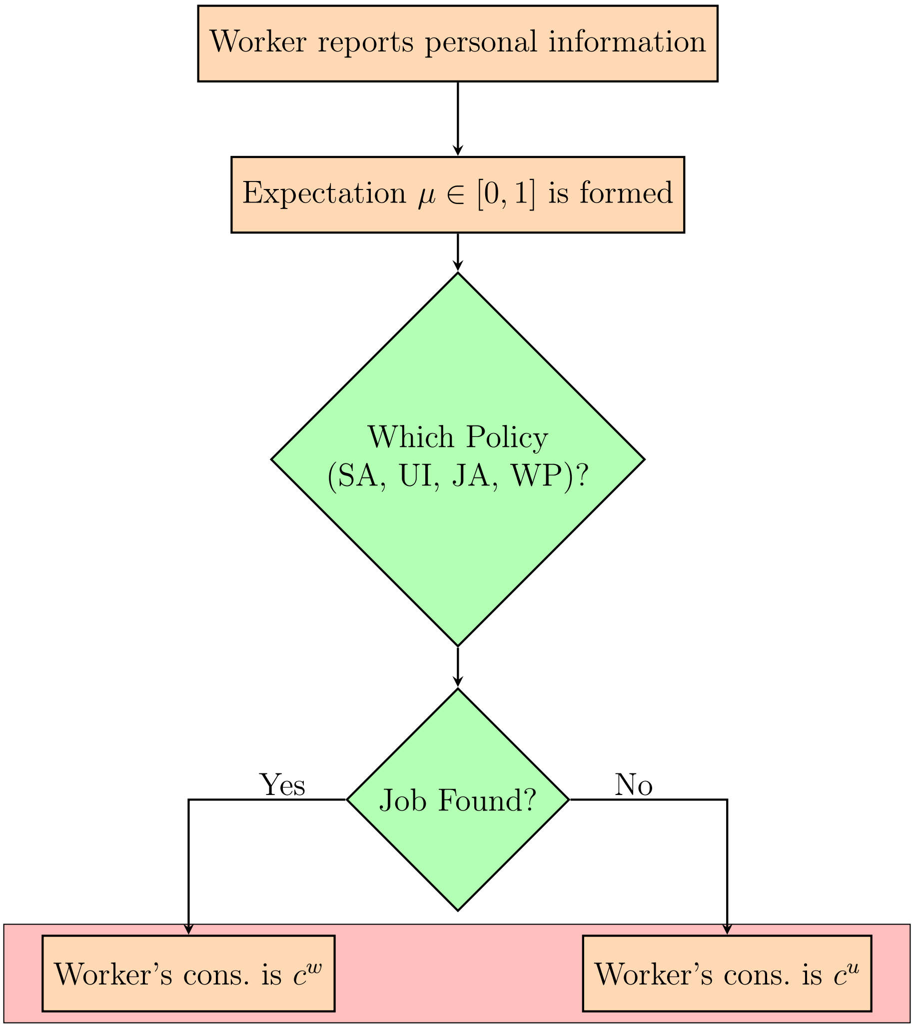


Figure 1.1: Timing of the Model

1.4 Welfare Policies

1.4.1 Unemployment Insurance (UI)

The government has two leverages for incentivizing the worker to actively look for new jobs. On the one hand, it can reward re-employment by opportunely lowering wage taxes (possibly, by pledging on a wage subsidy) so to guarantee a higher net consumption in case of successful job-search. On the other, it can punish workers for failing the job search. Both leverages must meet an Individual Rationality constraint, imposing the expected utility of the worker to be at or above U.

$$\mu\big[\pi_H u(c^w) + \big(1-\pi_H\big)u(c^u) - e\big] + (1-\mu)\big[\pi_L u(c^w) + \big(1-\pi_L\big)u(c^u) - e\big] \geq U \quad \text{(IR-UI)}$$

A second constraint ensures that the contract is incentive compatible for making a worker search. If the worker is successful -a case that occurs with probability $\pi_h,\ h \in \{H, L\}$-, the worker earns a gross wage ω_h and pays a labor tax equal to $\tau_h = \omega_h - c^w$, therefore receiving a net benefit of c^w. If, instead, she fails (with probability $1 - \pi_h$), she receives a benefit of c^u. In either case, she incurs an effort cost e upfront. Alternatively, if she does not engage in the job search, she incurs no effort cost, but enjoys no chance of re-employment either. Thus, the Incentive Compatibility constraint of a worker with expectation μ is

$$\mu\big[\pi_H u(c^w) + \big(1-\pi_H\big)u(c^u) - e\big] + (1-\mu)\big[\pi_L u(c^w) + \big(1-\pi_L\big)u(c^u) - e\big] \geq u(c^u) \quad \text{(IC-UI)}$$

Finally, the provider's expected return of committing to contract (c^w, c^u) is

$$\mu\big[\pi_H\big(\omega_H - c^w\big) - \big(1-\pi_H\big)c^u\big] + (1-\mu)\big[\pi_L\big(\omega_L - c^w\big) - \big(1-\pi_L\big)c^u\big] \qquad \text{(R-UI)}$$

and the value of UI for the government is defined by

$$V^{UI}(\mu, U) = \max_{c^w, c^u} \text{(R-UI)}$$
$$\text{sub:} \quad \text{(IC-UI), (IR-UI)}$$

The following result holds.

Proposition 1 (**Optimal UI Contract**). *In optimum, UI contract paid to a worker with expectation μ satisfies (IR-UI) and (IC-UI) with equality*

$$c^w_{ui} = g\Big(U + \frac{e}{\pi(\mu)}\Big), \; c^u_{ui} = g(U), \quad \textit{with:} \quad g \equiv u^{-1} \tag{1.1}$$

Optimization in the consumption menu under effort imposes both constraints to hold with equality, causing the utility upon failure to equal generosity ($u(c^u) = U$). Moreover, binding (IC-UI) causes the wedge between utilities upon success and upon failure to be

$$u(c^w_{ui}) - u(c^u_{ui}) = \frac{e}{\pi(\mu)} \tag{1.2}$$

with $\pi(\mu) := \mu\pi_H + (1-\mu)\pi_L$ being the expected chance to find a job, conditional on exerting effort and holding expectation $\mu \in [0, 1]$. The policy value $V^{UI}(\mu)$ can be written as the difference between gross returns $a^{UI}(\mu)$, and costs $b^{UI}(\mu, U)$.

$$V^{UI}(\mu, U) = a^{UI}(\mu) - b^{UI}(\mu, U)$$
$$\text{with:} \quad \begin{cases} a^{UI}(\mu) = \mu\pi_H\omega_H + (1-\mu)\pi_L\omega_L \equiv \omega(\mu) \\ b^{UI}(\mu) = \pi(\mu)c^w_{ui} + (1-\pi(\mu))c^u_{ui} \end{cases}$$

It is convenient to draw a distinction between the cost of *effort compensation* and *incentive provision*. If worker's effort were contractible, (IC-UI) would be slack and the principal would insure the risk-averse agent against both sources of risk -i.e., human capital and job-search outcome-, by simply compensating for the effort cost with flat contract $c^w = c^u = g(U + e)$. The effort compensation cost is defined as the difference in cost between this contract and a no-effort contract

$$\text{Effort Compensation Cost} \equiv g(U + e) - g(U)$$

Yet, non-contractible (and unobservable) effort forces the principal to generate a dispersion in transfers upon different job-search outcomes, as in (1.2). The additional risk that the worker faces causes the planner to bear a larger average payment compared to the case of a risk-neutral worker. This additional cost, which compensates the risk-averse agent for her dislike of risky lotteries, is referred to as incentive cost

$$\text{Incentive Cost} \equiv \pi(\mu)c^w_{ui} + (1-\pi(\mu))c^u_{ui} - g(U + e)$$

Looking at the planner's returns under effort, V^{UI} is strictly increasing and concave in μ. Monotonicity originates from two channels, the first of which being the linear increase in the gross return from job search a^{UI}. As the likelihood of high human capital increases, indeed, the probability of reemployment and the expected labor productivity increase as well. The second channel originates from falling incentive costs. The increase of μ causes the utility gap (1.2) to shrink, which boils down to lower risk and hence lower compensation of it. In addition, the utility gap (1.2) being an hyperbolic function of incentives causes a reduction in utility gap to be met by a smaller contraction of the corresponding payment gap at higher μ. In other words, an equal reduction in the wedge (1.2) due to increase of expected hazard rate $\pi(\mu)$ is met by smaller and smaller reduction in the gap of payments as μ approaches 1, which causes incentive cost to be convex decreasing and delivers concave increasing UI returns. Concavity of V^{UI} in μ holds even when the hypothesis of null hazard rate under no search is relaxed (see Appendix A. Features of UI).

1.4.2 Social Assistance (SA)

If the government deems the incentive cost too expensive with respect to the search expected return, it may decide not to pay it and forego the possibility of collecting wage taxes upon worker's reemployment. Therefore, it obtains

$$V^{SA}(\mu, U) = \max_{c^u} -c^u$$
$$\text{sub:} \quad u(c^u) \geq U \quad \text{(IR-SA)}$$

whose the optimal contract $c^w_{sa} = c^u_{sa} = g(U)$ is independent of μ, as well as the policy return. As in UI, V^{SA} can be written as the difference between gross returns ($a^{SA} = 0$) and costs ($b^{SA} = g(U)$).

One may ask whether the government would preferably assign a worker with expectation μ to UI or SA, were these the only two options available. In order to decide whether to forego any chance of reemployment, or delegate the job search to the worker (and pay incentive and effort-compensation costs), the government chooses the higher return between UI and SA for expectation μ and generosity U.

Proposition 2 (**Optimal choice between SA and UI**). *Define U the program's generosity and $\mu_{sa,ui}$ the threshold where the returns of SA and UI are equal*

$$V^{SA}(U) = V^{UI}(\mu_{sa,ui}, U)$$

Then SA is optimal whenever $\mu \leq \mu_{sa,ui}$ and UI is optimal whenever $\mu > \mu_{sa,ui}$.

Proof. It is enough to notice that V^{UI} is increasing, while V^{SA} is constant, in μ. ■

The return of the welfare program for the planner is therefore equal to the value of the upper envelope between V^{UI} and V^{SA}, as reported in Fig. 1.2.

$$\hat{V}(\mu, U) = \max\{V^{UI}(\mu, U), V^{SA}(U)\} = \hat{a} - \hat{b}$$

$$\text{with:} \quad \begin{cases} \hat{a} = 0,\ \hat{b} = g(U), & \forall \mu \leq \mu_{sa,ui} \\ \hat{a} = \omega(\mu),\ \hat{b} = \pi(\mu) g(U + e/\pi(\mu)) + (1 - \pi(\mu)) g(U), & \forall \mu > \mu_{sa,ui} \end{cases}$$

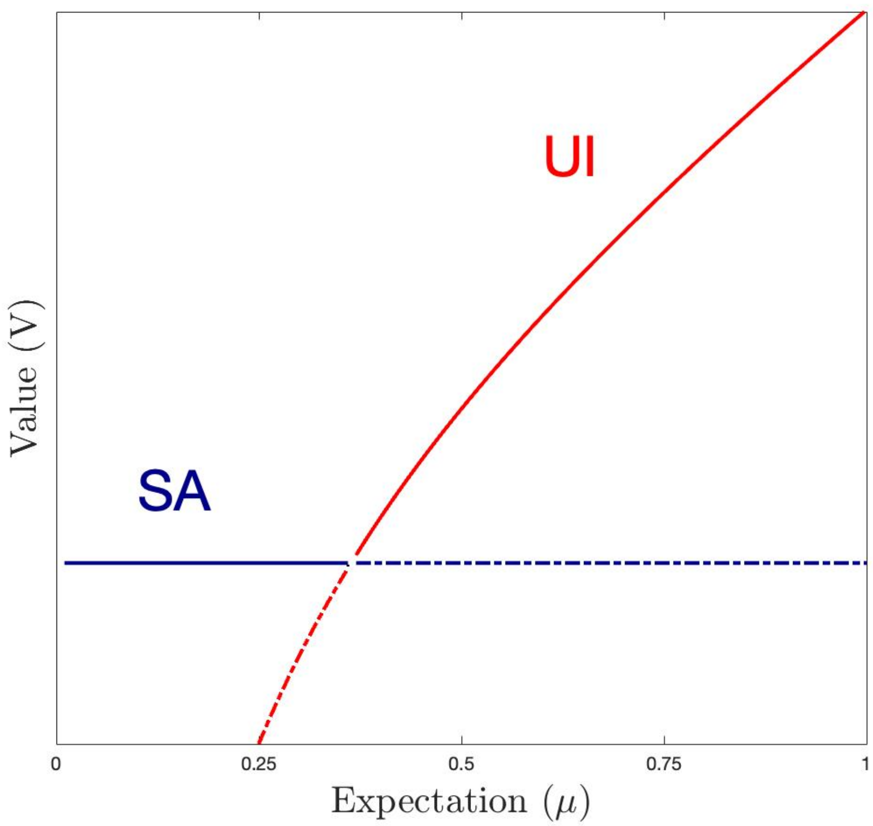


Figure 1.2: Value of welfare program with no profiling or assisted-search technology.

1.4.3 Worker Profiling (WP)

Most SWAs use a statistical approach to profile welfare claimants who pass the initial selection, and rank them according to the probability they exhaust benefits. To this purpose, WPRS systems resort to administrative records, questionnaires and/or personal interviews as data sources to assess the probability that any jobseeker with certain characteristics becomes long-term unemployed. Whatever the method adopted, this is subject to errors of type I (highly re-employable claimants profiled as not so) and II (lowly re-employable claimants profiled as not so). Indeed, either the profiling method assigns a high priority to claimants who are not going to exhaust their benefits (type-I error), or it assigns a low priority to claimants who are later going to be with no job nor welfare support (type-II error). To give a sense of how profiling works and how its type-I and type-II errors can be modelled, one can think of profiling workers on the basis of how they answer to a questionnaire. Choosing the type of questions and/or the minimum standard to be considered eligible to Reemployment Services boils down to choosing the frequency of type-I and type-II errors. Indeed, more stringent requirements to access Reemployment Services lower the risk that high-skilled workers are mistakenly referred to them (type-I error), but this comes at the cost of an increased risk of denying access to such Services to low-skilled workers (type-II error).

The trade-off between the two error types require to choose a preferential target. The 2007 final report on WPRS commissioned by the Department of Labor (Sullivan et al., 2007) focuses on minimization of type-I error only, and ranks the different State-level implementations of WPRS accordingly. Furthermore, the report documents that the criteria for referral to Reemployment Services are often dictated by budgetary reasons, so that the recipients who benefit from assisted search are fewer than the needy ones, resulting in a positive type-II error. This fact seems to constitute *prima facie* a source of inefficiency in the welfare program. In the following, however, type-I error minimization is shown to be a valid preferential target in the design of an optimal profiling strategy, and a positive type-II error to be even desirable under some conditions. In particular, low generosity makes positive type-II error optimal.

Profiling can be modelled as a signal about human capital, with as many outcomes as the number of human capital realizations. And since there are two possible realizations ($h \in \{H, L\}$), restricting the analysis to profiling strategies with a binary outcome is without loss of generality.[6] The design of an optimal profiling strategy, thus, requires choosing the optimal frequency of each outcome ('Pass'/'Fail') as functions of human capital realization, expectation and generosity, and a consumption schedule conditional on the outcome, that implements the desired search effort, e.g.

$$\left\{\sigma(r|H,\mu,U),\ \sigma(r|L,\mu,U),\ c^w(r,U_r),\ c^u(r,U_r)\right\}_{r=\{f,p\}}$$

Any outcome r brings both parties to revise initial expectation μ according to

$$\mu_r = \frac{\mu\sigma(r|H,\mu)}{\mu\sigma(r|H,\mu)+(1-\mu)\sigma(r|L,\mu)}$$

[6]See Bergemann and Morris (2019), Proposition 1: Revelation Principle of Information Design.

where μ_r is the updated probability of the worker being high-skilled.[7] By convention the revised expectation upon 'Pass' μ_p is larger than the one upon 'Fail' μ_f, hence incentives to job search are pledged upon the former outcome, that is, whenever the profiled worker is deemed more likely to get reemployed (by Prop. 2).

Expectations are formed on basis of a large number of past observations of unemployed welfare recipients and their unemployment spell duration, and are consequently unbiased by a law-of-large-numbers argument, meaning that the percentage of high-skilled workers who share the same expectation coincides with the individual probability of being high-skilled represented by that expectation. As such, profiling does not induce a distorting effect in the aggregate of all workers who share the same expectation. Which boils down to require that the distribution of revised expectations is equal in mean to the initial prior (known as Martingale Property).

$$q\mu_p + (1-q)\mu_f = \mu, \quad \mu_f, \mu_p \in [0,1] \qquad \text{(MP)}$$

(MP) can be interpreted as a restriction requiring profiling to be credible. Indeed, if one considers all workers sharing the same expectation μ, inducing any of them to revise it upward to μ_p comes at the cost of inducing a downward revision to μ_f for someone else.

The government may find it convenient to generate also a dispersion in utilities via profiling. Under a mild assumption on the utility function (convex $1/u'$), indeed, incentive costs to search are increasing in generosity (see Appendix B. Design of WP). Therefore, if the government could make continuation utility dependent on the test outcome, it would ease incentive provision under UI by lowering the one upon 'Pass' ($U_p < U$) and increasing the one upon 'Fail' ($U_f > U$). However, this possibility is excluded *a priori*, as there is no actual example, among the existing programs, of a profiling strategy that 'punishes' or 'rewards' profiled claimants depending on their human capital. A possible reason is that such a strategy would be subject to strategic behavior by claimants, who could manipulate the profiling process (for instance, by intentionally underperforming in the questionnaire/interview), so to be assigned to the contract upon 'Fail'. I therefore restrict the analysis to the following type of profiling strategies.

[7] By convention, 'Pass' is assumed to mainly signal high human capital, which boils down to

$$\sigma(p|H,\mu) > \sigma(p|L,\mu) \Longrightarrow \mu_f < \mu < \mu_p$$

A necessary (and sufficient) condition for any profiling outcome to be informative about human capital and induce a revision of expectations is to set a probability differing under both states (e.g., $\sigma(r|H,\mu) \neq \sigma(r|L,\mu)$).

Definition 1 (Soft Profiling). *A profiling strategy is called 'soft' if the worker's expected utility is equal across different outcomes ($U^p = U^f = U$).*

This gives rise to a Soft Constraint (SC, hereafter), that prevents the government from punishing or rewarding the worker depending on the outcome of profiling.

Proposition 3. *Soft profiling generates positive gains only if it allocates workers to different levels of search effort, i.e. $a('Pass') = 1,\ a('Fail') = 0$.*

Incentive costs to search are convex decreasing, while returns are linear increasing, in expectations. On the contrary, the no-search contract provides full consumption insurance and is thus independent of expectations. These two facts are conducive to the search value displaying a positive slope in the space of expectations, and to the worker effort and expectations being complementary ($V_{e\mu} \geq 0$).[8] On the other hand, the marginal incentive cost increases, delivering a tendency toward 'within-policy' concavity in expectations ($V_{\mu\mu} \leq 0$). Therefore, any randomization in the space of expectations inducing the positive effort level under either outcome is suboptimal from the government's perspective, as this would cause the worker to bear the additional risk linked to human capital realization (which she is insured against in UI) and the government to compensate for it (hence, larger incentive costs). Same holds true for any randomization over SA, where no reduction of costs or increase of returns is possible. Thus, the only possible welfare-improving randomization over expectations is across different search effort levels, as graphically suggested by the tendency of V toward 'between-policy' convexity in μ (see Fig. 1.2). These two forces are jointly responsible for the design of the optimal profiling method. The value of WP thus reads

$$
\begin{aligned}
V^{WP}(\mu, U) = &\max_{q,\mu_p,\mu_f,c^w(p),c^u(p),c^u(f)} q\big[\omega(\mu_p) - \pi(\mu_p)c^w(p) - \big(1-\pi(\mu_p)\big)c^u(p)\big] - (1-q)c^u(f) - \kappa^{wp} \\
&\text{sub:} \quad qU_p + (1-q)U_f \geq U \ \ \text{(IR)}, \quad \text{(MP)}, \quad U_p = U_f \ \ \text{(SC)} \\
&\qquad U_p := \pi(\mu_p)u(c^w(p)) + \big(1-\pi(\mu_p)\big)u(c^u(p)) - e \geq u(c^u(p)) \quad (\text{IC}, \mu_p) \\
&\qquad U_f := u(c^u(f))
\end{aligned}
$$

The return for the government is larger upon 'Pass' than upon 'Fail'. For this reason,

[8] Define now V as a function of worker's effort e, expectation μ and continuation utility U, i.e. $V(e,\mu,U)$, such as

$$\hat{V}(\mu, U) = V(e(\mu, U), \mu, U)$$

A function $\phi : \mathbf{R}^n \to \mathbf{R}$ is super- (sub-)modular in arguments x and y, if the cross derivative in those arguments is positive (negative).

one may be tempted to guess that optimal profiling always fully detects high-skilled workers upon 'Pass' (e.g. $\mu_p = 1$) to minimize incentive costs. However, there is a positive correlation between the informativeness of 'Pass' (larger μ_p) and the difficulty to obtain it (lower q). While a more informative 'Pass' increases the government's return on each worker, making it more difficult to receive a 'Pass' increases 'Fail' frequency and lowers returns. Therefore, in optimal WP the marginal gain from increasing informativeness of 'Pass' equals the marginal cost of reallocating the marginal worker to the no-search policy. In case the marginal gain exceeds the marginal cost for every level of μ, the test fully discloses high human capital. The necessary and sufficient condition for this to happen is

Condition 1.

$$V^{UI}(1, U) - V^{SA}(U) \leq V^{UI}_{\mu}(1, U)$$

Prop. 4 features optimal WP.

Proposition 4. *'Fail' contract insures consumption ($c^w_{wp}(f) = c^u_{wp}(f) = g(U)$), while 'Pass' contract creates a consumption dispersion such that (1.2) binds at μ_p*

$$c^w_{wp}(p) = g\Big(\frac{e}{\pi(\mu_p)} + U\Big), \quad c^u_{wp}(p) = g(U)$$

If Condition 1 holds, then WP reveals human capital with full accuracy:

$$\sigma(p|H, \mu) = 1,\ \sigma(p|L, \mu) = 0 \Longrightarrow \mu_p = 1,\ \mu_f = 0$$

Otherwise, the 'Pass' outcome induces expectation $\mu_p < 1$, defined by[9]

$$\frac{V^{UI}(\mu_p, U) - V^{SA}(U)}{\mu_p} = V^{UI}_{\mu}(\mu_p, U) \tag{1.3}$$

Moreover, the optimal program never refers to profiling workers whose initial expectation is higher than μ_p.[10]

Proof. See Appendix B. Design of WP. ■

Any profiled worker who receives a 'Fail' is fully aware of being low-skilled (i.e., $\mu_f = 0$). The return from no search is independent of expectations and lower than the one from

[9]Note that existence of $\mu_p < 1$, whenever Condition 1 does not hold, is guaranteed by the Intermediate Value Theorem, as the function $D(\mu, U) = V^{UI}_{\mu}(\mu, U) - \frac{V^{UI}(\mu,U) - V^{SA}(U)}{\mu}$ is continuous over $[\mu_{sa,ui}, 1]$, $D(\mu_{ui,sa}, U) > 0$ and $D(1) < 0$. Moreover, concavity of V^{UI} guarantees uniqueness of μ_p.

[10]If $\mu \geq \mu_p$, instead, WP is non-informative $\big(\sigma(p|H, \mu) = \sigma(p|L, \mu)\big)$ and dominated by UI.

search. Therefore, fixing the level of informativeness of the 'Pass'-and-search outcome (i.e., holding μ_p constant), it is optimal to increase the chances of referral to UI (i.e., q) for both high- and low-skilled workers, up to the point where only low-skilled ones are referred to SA.[11] This finding provides a theoretical rationale for minimizing type-I error, as in Sullivan et al. (2007).

The non-informativeness (hence, the non-use) of profiling in $\mu \geq \mu_p$ is a direct consequence of Prop. 4. When Condition 1 does not hold, the return of UI in high-end expectations is so large with respect to SA, that the government finds it convenient to make both types of workers search by pooling them within the same contract. If so, no information disclosure to any extent is ever optimal, as the loss on referring the marginal low-skilled worker to SA outweighs the savings realized by lowering incentive costs upon 'Pass' at the margin. A necessary condition for this case to apply is that, if search effort were observable to the government, all workers would optimally be referred to active search. Indeed, in the first-best contract the government only needs to compensate for the effort cost, hence obtaining a return from search delegation larger than any second-best contract. For the value of information acquisition to be negative in high-end expectations, the return from delegating the job search (also) to low-skilled workers (along with incentives) in place of referring them to SA must be positive, so as to outweigh the loss for not revealing information to high-skilled workers. Therefore, search delegation to low-skilled workers under a first-best contract would be *a fortiori* desirable to the government. The following states the result in terms of Condition 1.

Corollary 1. *If the government **never** finds it optimal to delegate search to low-skilled workers, not even in first-best (i.e., absent moral hazard), then Condition 1 holds true.*

Prop. 5 relates Condition 1 to the generosity level of the program, and shows the existence of a monotone relationship between generosity and informativeness of profiling.

Proposition 5. *Assume convexity of $1/u'$ and define U^* as the (unique) generosity level such that Condition 1 holds with equality. Then, Condition 1 holds for $U \geq U^*$, while it does not for $U < U^*$. Moreover, revised expectation upon 'Pass' is monotonically increasing in U.*

Proof. See Appendix B. Design of WP. ■

[11]It is possible to change the frequency of any outcome r, and contemporaneously keep its level of informativeness constant, by changing odds $\sigma(r|L, \mu)$ and $\sigma(r|H, \mu)$ in the same proportion.

If $1/u'$ is convex, the effect of an increase of generosity on the informativeness of 'Pass' outcome is twofold. First, the difference between the value of UI and SA shrinks for any expectation μ, hence causing a reduction of WP gain over referral to UI (i.e., lower left-hand side of (1.3)). Second, it makes incentive costs more reactive to a variation of expectations, hence widening the marginal return of UI in expectations, as well as the WP gain over reducing incentive costs of UI upon 'Pass' (i.e., larger right-hand side of (1.3)).[12] Both effects lead to an increase in informativeness of the 'Pass' outcome, at the expense of lowering its frequency (i.e., higher μ_p and lower q). Prop. 4 and 5 provide a rationale for two facts. First, that minimization of type-I error (i.e., referring high-skilled workers to SA) should be prioritized. And, second, that type-II error (i.e., denying low-skilled workers access to SA) should be minimized only for high generosities. In particular, type-I error should be always null, while a positive type-II error is negatively related to the level of informativeness of profiling and preferable if generosity is low enough (i.e., $U < U^*$). These two facts are consistent with the analysis conducted by Sullivan et al. (2007), where the quality of different State-level implementations of the WPRS program is assessed according to the reduction of type-I error, and type-II error is found to be related to the low level of funding in some US States. In particular, when funds are scarce (i.e., when generosity is low), refusing to supply Reemployment Services to a fraction of needy low-skilled workers, as well as to profile workers with high-end expectations, irrespective of the cost of the policy, respond to efficiency criteria.

Fig. 1.3 features the optimal choice among UI, SA and WP in the space of expectations and for constant generosity.

[12] The first effect is due to the submodularity of $\hat{V}$ in μ and U for different effort levels, while the second is due to supermodularity in μ and U for same effort level. These features of V have been outlined also by Pavoni and Violante (2007).

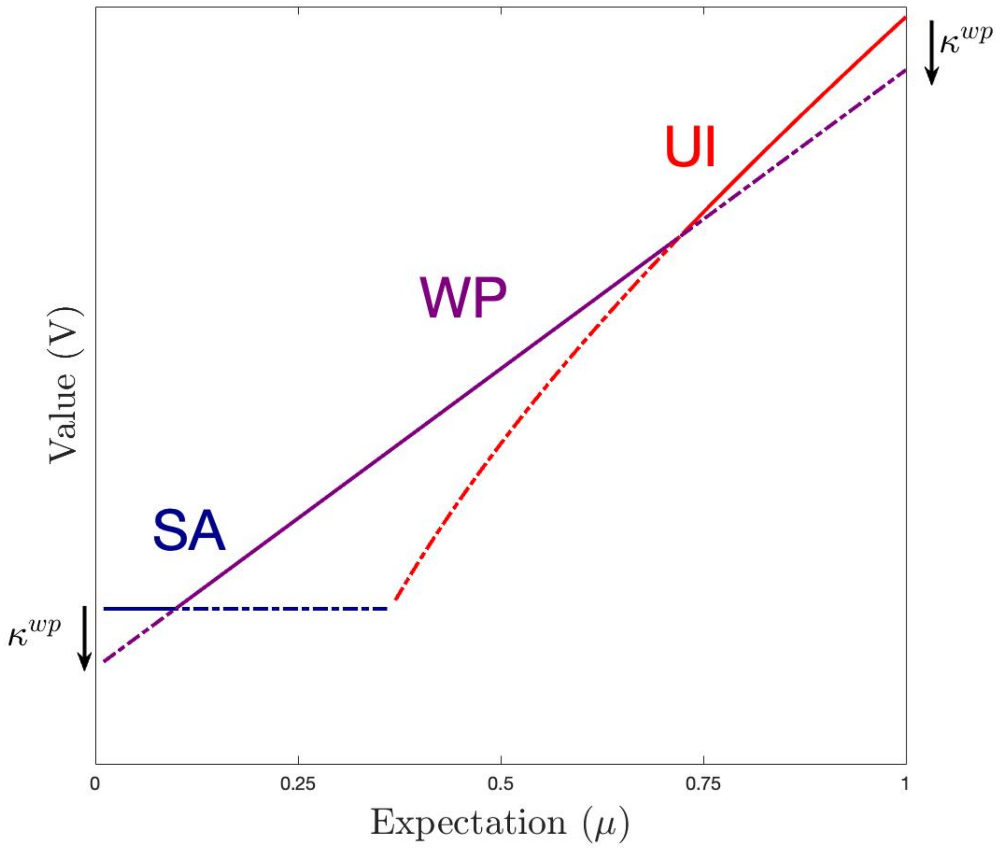


Figure 1.3: Value of welfare program with profiling technology.

The figure shows that the value of WP is largest in $\mu_{sa,ui}$. Indeed, moving right of $\mu_{sa,ui}$, the fraction of workers referred to UI declines together with the fraction of high-skilled workers, leading to a decline in WP value as μ approaches zero. On the other hand, moving left of $\mu_{sa,ui}$ in the space of expectations, the fall in (convex) incentive costs in UI happens to be lower than the one experienced by (linear) costs of WP. This causes the following

Corollary 2. *WP is worth implementing if and only if*

$$\kappa^{wp} \leq \mu_{sa,ui} \frac{V^{UI}(\mu_p, U) - V^{SA}(U)}{\mu_p} \tag{1.4}$$

Moreover, when condition (1.4) is met, only workers with intermediate expectations are profiled.

Workers with very high or low expectations are on average referred to the policy which is more appropriate according to their human capital, and so the cost of the profiling technology outweighs the expected gain from efficient reallocation of workers to the better effort level with a less expensive contract.

1.4.4 Job-Search Assistance (JA)

In WPRS, recipients who are found to be low-skilled after profiling are referred to a Service Provider for reemployment services. Such services encompass a wide array of activities, ranging from core activities, like job-search assistance and job-matching services, to ancillary career-related ones, like financial literacy services, information about supportive services and financial aid, assistance with resume writing and interviewing, etc. In addition, State's income support to welfare recipients within the program is conditional on their active participation to these activities. Therefore, reemployment services can be read as a Job-Search Assistance policy, where the Service Provider searches for new employment on worker's behalf (at cost κ^{ja}), while the worker is requested to attend (part of) the various activities listed above. The value of JA reads

$$\begin{aligned} V^{JA}(\mu, U) = \max_{c^w, c^u} \; & \omega(\mu) - \pi(\mu)c^w - (1-\pi(\mu))c^u - \kappa^{ja} \\ \text{sub:} \quad & \pi(\mu)u(c^w) + (1-\pi(\mu))u(c^u) \geq U \qquad \text{(IR-JA)} \end{aligned}$$

Since the provider is in charge of conducting the job search and no effort is requested to the worker (no IC constraint), the provider finds it optimal to provide full insurance against both state- and outcome-related risks ($c^w_{ja} = c^u_{ja} = g(U)$). Therefore, the value of JA reads

$$V^{JA}(\mu, U) = \omega(\mu) - g(U) - \kappa^{ja}$$

which is linear increasing as a higher μ leads to an increase in expected returns $a^{JA} = \omega(\mu)$, while costs $b^{JA} = g(U) + \kappa^{ja}$ remain constant.

Notice that the gross return in JA is the same as in UI. Therefore, the optimal policy between JA and UI is the one that minimizes costs. And since the cost of JA is constant in the space of expectations, while the cost of UI is decreasing, a result similar to the one contained in Prop. 2 applies.

Proposition 6 (Optimal choice between JA and UI). *Define $\mu_{ja,ui}$ as the expectation threshold where the returns of JA and UI are equal*

$$b^{JA}(U) = b^{UI}(\mu_{ja,ui}, U)$$

Then JA is optimal whenever $\mu \leq \mu_{ja,ui}$ and UI is optimal whenever $\mu > \mu_{ja,ui}$.

The presence of a passive labor-market policy like SA is not contemplated in the existing WPRS program. With the aim of conducting an analysis more adherent to the actual program, this section studies the optimal program when only JA, UI and WP are implemented. This restriction does not bind whenever condition (1.5) is met.

Proposition 7. *SA is dominated by JA for all expectations and generosities if and only if*

$$\kappa^{ja} \leq \pi_L \omega_L \tag{1.5}$$

The parametric restriction imposes a cap on the cost of assisted-search which guarantees that the return of JA when low human capital is certain ($\mu = 0$) is larger than the one of SA. And given that the former is increasing in μ, while the latter is independent of it, such a restriction is necessary and sufficient for SA to be dominated for all expectations and generosity levels.

When reallocating workers between private and assisted search, any gain from profiling comes only from the reduction of costs, and not from an increase in returns. Indeed, no matter how profiling is designed, the expected return will be

$$a^{WP} = q\omega(\mu_p) + (1-q)\omega(\mu_f) = \omega(\mu) = a^{UI} = a^{JA}$$

where the equality is the direct consequence of linearity of returns in μ and the Martingale Property. Hence, the value of WP can be written

$$\begin{aligned} V^{WP}(\mu) =& \omega(\mu) - \kappa^{wp} - \min_{q,\mu_p,\mu_f} \left\{ q b^{UI}(\mu_p, U_p) + (1-q) b^{JA}(U_f) \right\} \\ & \text{sub:} \quad \text{(MP)} + \text{(SC)} \end{aligned}$$

Noticeably, the same results in Prop. 4 and 5 still hold, once Condition 1 is replaced by

$$V^{UI}(1,U) - V^{JA}(0,U) \leq V^{UI}_{\mu}(1,U) \Longrightarrow b^{UI}(1,U) - b^{JA}(U) \geq b^{UI}_{\mu}(1,U) \tag{1.6}$$

and (1.3) by

$$\frac{b^{UI}(\mu_p, U) - b^{JA}(U)}{\mu_p} = b^{UI}_{\mu}(\mu_p, U) \tag{1.7}$$

Therefore, one can conclude that the two main findings of the paper, namely that the optimal profiling strategy is so designed that the job search is always conducted by all high-skilled recipients (i.e., minimum type-I error), as well as by a positive fraction of low-skilled ones (i.e., positive type-II error) in less generous programs, are robust to the adoption of any form of job-search assistance. The explanation is simple. The result about null type-I error always holds if low-skilled workers are referred to a policy whose value is linear in expectations (SA or JA). While positive type-II error is driven by the concavity of UI returns in expectations and the positive relationship between the marginal return of UI to expectations and generosity of the program.

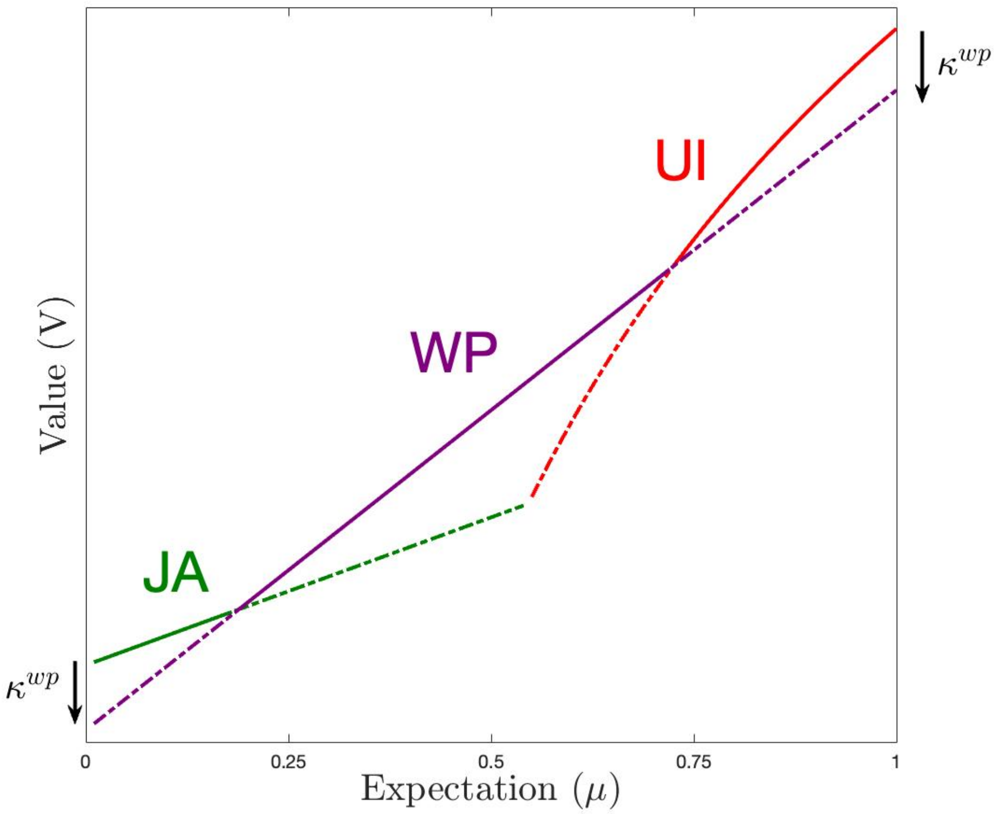


Figure 1.4: Value of welfare program with profiling and assisted-search technology.

1.5 Optimal WPRS Program

Whenever all policy instruments are present in the WPRS program, those low-skilled recipients who are profiled and receive a 'Fail' are either referred to SA or JA. The optimal allocation of search and detection technologies in the space of expectations can be of two types. Moving right from $\mu = 0$ to $\mu = 1$, either JA comes right after SA, followed (in the order) by WP and UI, or WP comes after SA and is dominated by JA for high-end expectations. Prop. 8 shows how generosity impacts the optimal choice between JA and WP.

Proposition 8. *Assume convexity of* $1/u'$ *and also that all policy instruments are present in the optimal program. Define* $\underline{U}$ *the unique generosity level such that*

$$V^{UI}(1, \underline{U}) - V^{SA}(\underline{U}) = V_{\mu}^{JA} \tag{1.8}$$

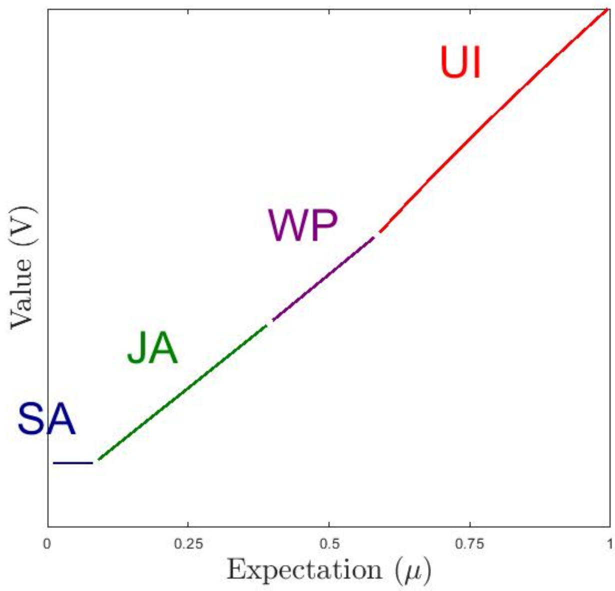
Then, $\underline{U} > U^*$*. Moreover, whenever* $U < \underline{U}$*, JA dominates WP over lower-intermediate expectations. On the contrary, if* $U > \underline{U}$*, JA dominates WP over higher-intermediate expectations.*

Proof. See Appendix B. Design of WP. ■

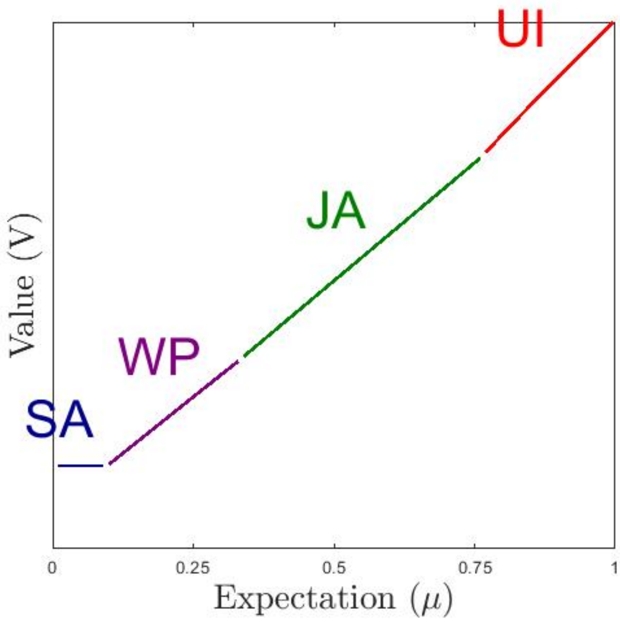
Prop. 8 establishes a relationship between the program's generosity and the design of the optimal WPRS program. When the incentive cost represented by the gap in payments on the left-hand side of (1.8) exceeds a given threshold, profiling is better administered to lower-intermediate expectations, while job-search assistance to upper-intermediate ones, with SA and UI at the low and high end of the interval, respectively. The intuition is that policies with a higher marginal return to expectations are better adopted for higher expectations. Now, the marginal return of JA is constant (a^{JA} is linear in μ). When μ_p is internal, the marginal return of WP is equal to the slope of V^{UI} at μ_p (by (1.3)), and is thus larger than the marginal return of JA, thanks to falling costs b^{UI}. Under Condition 1, generosity inflates the cost of UI contract more than the one of SA contract, hence lowering the distance between returns under either outcome, up until condition (1.8) is met, and the marginal value of JA becomes larger than the one of WP for $U > \underline{U}$.

1.6 Worker's Private Expectations

Even if crucial for program's customization and efficiency-based considerations, worker's personal information may be unobservable to the government and therefore object of strategic misreporting on worker's side, with the aim of obtaining larger expected transfers. Therefore, when the personal data of welfare applicants can not be accessed by the government, the latter faces the trade-off between information elicitation and contract efficiency, that characterizes all situations of adverse selection. The first point to make is



(a) Low Generosity



(b) High Generosity

Figure 1.5: Optimal Policies in the Space of Expectations

that, whenever optimal WP is fully revealing (i.e., under Condition 1), the profiling and consumption contract is equal for every profiled worker, irrespective of their private information. Therefore, personal information of profiled workers is useless under full detection on human capital, and the WP contract does not change.

Proposition 9. *If expectations are private to workers, but Condition 1 is satisfied, then the optimal WP contract is the same as in Prop. 4.*

Proof. See Appendix C. Optimal WP under Private Expectations. ∎

If, instead, Condition 1 fails to hold, the analysis is more complex, as the solution contract where expectations are common knowledge (i.e., absent adverse selection) is no longer implementable. For instance, any profiled worker under common-knowledge expectations (i.e., with $\mu \in [\mu_{sa,wp}, \mu_{wp,ui}]$) has an incentive to lie and induce 'false' expectation $\mu_{sa,wp}$. Indeed, while she enjoys utility U in case of truth-telling, reporting expectation $\mu_{sa,wp}$ allows her to enjoy an expected utility larger than U, given that

$$U'_p = \pi(\mu'_p)U^w(\mu_p) + \big(1 - \pi(\mu'_p)\big)U - e > \pi(\mu_p)U^w(\mu_p) + \big(1 - \pi(\mu_p)\big)U - e = U \quad (1.9)$$

upon 'Pass', as $\mu'_p > \mu_p$[13]. Given this new source of information asymmetry, the principal trades off contract efficiency (i.e., inducing a customized expectation revision and setting the optimal search incentives for each μ) against the cost of information rents. In other words, it chooses between two different types of contracts: pooling and separating. The pooling contract foregoes any fine tuning of incentives to save on information rents, while the separating contract leads workers to reveal their actual expectation (conditional on payment of information rents) and minimizes incentive costs thereof. The optimal WP contract turns out to be the same as in the case of observable expectations. This finding is due to the government's obligation to pledge the same utility level under both profiling outcomes (SC constraint). Therefore, workers enrolled in WP can not receive any compensation for telling the truth via consumption contract, which would otherwise trigger lying by workers enrolled in other policies (SA, for example). Nor the government can slack off incentives by designing a pooling contract, as this would constitute a violation of SC constraint for workers with expectation larger than $\mu_{sa,wp}$, as shown in (1.9). The solution for the government is to offer the same contract as in Prop. 4, so as to formally respect SC, while it is aware that all profiled workers with expectations larger than $\mu_{sa,wp}$ will be lying and that the contracts tailored to them will never be implemented. Therefore, the result holding under Condition 1 can be further extended.

Proposition 10. *Even with private expectations, the optimal contract under WP is the same as in Prop. 4, with the only difference of inducing a larger revision of expectations upon 'Pass' outcome (i.e., μ_p is larger).*

[13]The inequality holds as posteriors are monotone increasing in priors, for any given $\{\sigma(r|h,\mu)\}_{r,h}$:

$$\mu'_p = \frac{\mu'\sigma(p|H,\mu)}{\mu'\sigma(p|H,\mu) + (1-\mu')\sigma(p|L,\mu)} > \frac{\mu\sigma(p|H,\mu)}{\mu\sigma(p|H,\mu) + (1-\mu)\sigma(p|L,\mu)} = \mu_p$$

Proof. See Appendix C. Optimal WP under Private Expectations. ■

To conclude, the main features of the optimal profiling design outlined in Section 2.4 (zero type-I error and positive type-II error for low generosities) are robust to any situation where expectations can be misrepresented by welfare recipients.

1.7 Conclusions

This paper frames profiling of welfare recipients in programs of unemployment assistance as a way to implement an optimal match between recipients and policy instruments and transfers, in a context of two-sided uncertainty about recipients' human capital. Even if these programs may perform poorly in terms of net costs and skill-enhancing effect, compared to apprenticeship-based ones, like on-the-job training or mandatory work,[14] they make possible for any profiled worker to receive ad-hoc information, and to revise her initial expectation on human capital accordingly. This constitutes a twofold gain for the government. First, the detection of hidden human capital improves on allocation of policy instruments and unemployment benefits. Indeed, when no profiling program is adopted, there is always a fraction of workers who are inefficiently matched according to their human capital. Thus, profiling allows for a proper match between workers and policies, by detecting new information about their human capital. Second, when workers are incentivized to search, they can be pledged lower transfers thanks to a higher expected return from job search. However, reallocating highly confident low-skilled workers from private to no search (or to assisted search) also generates a loss for the government, in terms of lower expected labor taxes upon reemployment (if reallocated to no search) or higher search cost (if reallocated to assisted search). If this loss outweighs the gain from incentive reduction on jobseekers, the optimal profiling strategy does not lead to full detection of human capital. In particular, while an optimal strategy always minimizes the number of high-skilled workers who are diverted from search (i.e., null type-I error), it may request a fraction of low-skilled ones to search (i.e., positive type-II error). This occurs whenever the incentive costs to private search are low enough. The paper also shows that convex derivative of the inverse of worker's utility is a sufficient condition that makes informativeness (or precision) of profiling positively correlated to the generosity of the welfare program.

[14] Pavoni et al. (2016) estimate a positive net return of mandatory work programs.

Some questions remain unanswered and are left to further research. First, how does information originating from profiling interact with information that is learnt during the unemployment spell? And second, how does profiling impact on unemployment benefits over time? To address these questions, a dynamic framework is needed, where expectations are revised both upon profiling and upon job search failures. Pavoni and Violante (2007) develop an infinite-horizon model where worker's human capital is known to both parties and depreciates along the spell, and the planner chooses to (re)allocate workers to (other) policy instruments, based on its current level. Capital depreciation could quite naturally be replaced by the expectation revision occurring after every failed attempt to find a job. However, the incentive compatibility constraint in UI differs in the two frameworks, as expectation revision, differently from human capital depreciation, only occurs if the job search actually takes place. Worker's private search can therefore lead to misalignment of expectations between her and the government, to avoid which the latter incurs learning rents, in addition to search incentives and effort compensation. Pavoni and Violante (2007)'s model thus represents an useful starting point for embedding the learning process into the analysis, although its extension to the new framework is not straightforward.

APPENDIX

Appendix A. Features of UI

Proposition 11. *If the agent enjoys some chance $\hat{\pi}$ (with $\hat{\pi} < \pi_L$) to find a job even without looking for it, which does not depend on her hidden state {H,L}, then incentive costs are convex decreasing in μ. Moreover, if $1/u'$ is convex, then the cross derivative of incentive costs by μ and U is negative (i.e., V^{UI} is supermodular in μ and U).*

Proof. While the Individual Rationality constraint remains the same as in Section 2.4, the Incentive Compatibility one becomes

$$\pi(\mu)u(c^w) + \big(1 - \pi(\mu)\big)u(c^u) - e \geq \hat{\pi}u(c^w) + \big(1 - \hat{\pi}\big)u(c^u) \qquad \text{(IC')}$$

And the optimal contract becomes:

$$u(c^w) = U + \frac{1 - \hat{\pi}}{\pi(\mu) - \hat{\pi}}e, \quad u(c^u) = U - \frac{\hat{\pi}}{\pi(\mu) - \hat{\pi}}e$$

Therefore, the incentive cost now looks[15]

$$InC(\mu) = \pi(\mu)g\Big(U + \frac{1 - \hat{\pi}}{\pi(\mu) - \hat{\pi}}e\Big) + \big(1 - \pi(\mu)\big)g\Big(U - \frac{\hat{\pi}}{\pi(\mu) - \hat{\pi}}e\Big) - g(U + e)$$

and its first derivative is

$$\begin{aligned} InC_\mu &= \big(\pi_H - \pi_L\big)(c^w - c^u) + \pi(\mu)c^w_\mu + (1 - \pi(\mu))c^u_\mu \\ \text{with:} \quad c^w_\mu &= -\frac{1}{u'(c^w)}\frac{e(1 - \hat{\pi})(\pi_H - \pi_L)}{(\pi(\mu) - \hat{\pi})^2}, \quad c^u_\mu = \frac{1}{u'(c^u)}\frac{e\hat{\pi}(\pi_H - \pi_L)}{(\pi(\mu) - \hat{\pi})^2} \end{aligned}$$

Thus,

$$InC_\mu < (\pi_H - \pi_L)\Big[c^w - c^u - \frac{1}{u'(c^u)}\frac{e}{(\pi(\mu) - \hat{\pi})}\Big] = (\pi_H - \pi_L)\Big[c^w - c^u - \frac{1}{u'(c^u)}\big(u(c^w) - u(c^u)\big)\Big] < 0$$

where the two inequalities hold by concavity of u, which in particular causes

$$-\frac{1}{u'(c^w)} < -\frac{1}{u'(c^u)}$$

[15] I define $g \equiv u^{-1}$

The second derivative of InC is

$$InC_{\mu\mu} = 2(\pi_H - \pi_L)(c^w_\mu - c^u_\mu) + \pi(\mu)c^w_{\mu\mu} + (1-\pi(\mu))c^u_{\mu\mu}$$
$$\text{with:} \quad c^w_{\mu\mu} = -\Big[\frac{u''(c^w)}{u'(c^w)}c^w_\mu + 2\frac{\pi_H - \pi_L}{\pi(\mu) - \hat{\pi}}\Big]c^w_\mu, \quad c^u_{\mu\mu} = -\Big[\frac{u''(c^u)}{u'(c^u)}c^u_\mu + 2\frac{\pi_H - \pi_L}{\pi(\mu) - \hat{\pi}}\Big]c^u_\mu$$

which can be rewritten as

$$InC_{\mu\mu} = -\Big[2\frac{\pi_H - \pi_L}{\pi(\mu) - \hat{\pi}}(\hat{\pi}c^w_\mu + (1-\hat{\pi})c^u_\mu) + \pi(\mu)\frac{u''(c^w)}{u'(c^w)}(c^w_\mu)^2 + (1-\pi(\mu))\frac{u''(c^u)}{u'(c^u)}(c^u_\mu)^2\Big] > 0$$

where the inequality follows by

$$\hat{\pi}c^w_\mu + (1-\hat{\pi})c^u_\mu = \frac{(\pi_H - \pi_L)\hat{\pi}(1-\hat{\pi})e}{(\pi(\mu) - \hat{\pi})^2}\Big[\frac{1}{u'(c^u)} - \frac{1}{u'(c^w)}\Big] < 0$$

due to concavity of u, which proves concavity of UI return.

Taking the derivative with respect to U yields

$$InC_U = \pi(\mu)\frac{1}{u'(c^w)} + \big(1-\pi(\mu)\big)\frac{1}{u'(c^u)} - \frac{1}{u'\big(g(U+e)\big)} >$$
$$> \frac{1}{u'\big(\pi(\mu)c^w + (1-\pi(\mu))c^u\big)} - \frac{1}{u'\big(g(U+e)\big)} > 0$$

where the first inequality follows from convexity of $1/u'$, while the second from concavity of u as

$$\pi(\mu)g\Big(U + \frac{1-\hat{\pi}}{\pi(\mu) - \hat{\pi}}e\Big) + \big(1-\pi(\mu)\big)g\Big(U - \frac{\hat{\pi}}{\pi(\mu) - \hat{\pi}}e\Big) - g(U+e) > g(U+e) - g(U+e) = 0$$

Finally,

$$InC_{U\mu} = (\pi_H - \pi_L)\Big(\frac{1}{u'(c^w)} - \frac{1}{u'(c^u)}\Big) - \Big(\pi(\mu)\frac{u''(c^w)}{u'(c^w)^2}c^w_\mu + \big(1-\pi(\mu)\big)\frac{u''(c^u)}{u'(c^u)^2}c^u_\mu\Big)$$
$$< (\pi_H - \pi_L)\Big(\frac{1}{u'(c^w)} - \frac{1}{u'(c^u)}\Big) - \frac{u''(c^w)}{u'(c^w)^2}\Big(\pi(\mu)c^w_\mu + \big(1-\pi(\mu)\big)c^u_\mu\Big)$$
$$< (\pi_H - \pi_L)\Big(\frac{1}{u'(c^w)} - \frac{1}{u'(c^u)}\Big) + \frac{u''(c^w)}{u'(c^w)^2}(\pi_H - \pi_L)(c^w - c^u) < 0$$

where the first passage follows from $c^u_\mu > 0$ and the first derivative of $\frac{1}{u'}$, that is $-\frac{u''}{(u')^2}$, being increasing, hence

$$-\frac{u''(c^u)}{u'(c^u)^2} < -\frac{u''(c^w)}{u'(c^w)^2}$$

while the second follows from $InC_\mu < 0$, that is

$$\big(\pi_H - \pi_L\big)(c^w - c^u) + \pi(\mu)c^w_\mu + (1-\pi(\mu))c^u_\mu < 0$$

and the last inequality from convexity of $\frac{1}{u'}$, which implies that

$$\frac{1}{u'(c^w)} - \frac{1}{u'(c^u)} < \frac{\partial}{\partial x}\left\{\frac{1}{u'(x)}\right\}\Big|_{x=c^w}(c^w - c^u) = -\frac{u''(c^w)}{u'(c^w)^2}(c^w - c^u)$$

$V^{UI}_{\mu U} > 0$ and $V^{JA}_{\mu U} = 0$ generates within-policy supermodularity and between-policy submodularity (see Pavoni and Violante, 2007). ■

Appendix B. Design of WP

Appendix B.1. Optimal 'soft' WP

Profiling returns a distribution of two posteriors equal in mean to prior expectation μ, with a consumption contract (c^w, c^u) attached to each posterior. Given the assumption that the expected continuation utility ahead of each profiling outcome is non smaller than U, so that no 'punishment' is possible conditional on the profiling result, the value of downstream policies {UI, SA} is represented by $\hat{V}$ and the test is equivalent to a randomization over it. As a consequence, the value of WP for the provider is

$$V^{WP}(\mu, U) = \max_{q,\mu_p,\mu_f} q\hat{V}(\mu_p, U) + (1-q)\hat{V}(\mu_f, U) - \kappa^{wp} \tag{1.10}$$
$$\text{sub:} \quad q\mu_p + (1-q)\mu_f = \mu, \quad 0 \le \mu_f \le \mu_p \le 1$$

where the (SC) constraint is already accounted for by setting $U_p = U_f = U$, and the (IR, p) constraint is implicit in $\hat{V}$. Furthermore, as shown by Kamenica and Gentzkow (2011), in optimum the test delivers the concave closure of $\hat{V}$ at U.
Therefore, $\mu_f = 0$ and μ_p solves

$$\frac{\partial q}{\partial \mu_p}\big[\hat{V}(\mu_p, U) - \hat{V}(0, U)\big] + q\hat{V}_\mu(\mu_p, U) = 0 \Longrightarrow \frac{\hat{V}(\mu_p, U) - \hat{V}(0, U)}{\mu_p} = \hat{V}_\mu(\mu_p, U)$$

where $\mu_p < 1$ whenever the ratio $\frac{\hat{V}(\mu_p,U)-\hat{V}(0,U)}{\mu_p}$ has an internal (to $[0,1]$ interval) point of maximum, that is, whenever there exists an expectation $\mu \in (0,1)$ such that

$$\frac{\hat{V}(\mu,U)-\hat{V}(0,U)}{\mu} > \hat{V}(1,U)-\hat{V}(0,U) \Longrightarrow \hat{V}(\mu,U) > \mu\hat{V}(1,U)+(1-\mu)\hat{V}(0,U)$$

The inequality establishes that full detection of human capital through fully accurate profiling is harmful from the viewpoint of the principal.
Therefore, when such point of maximum is internal, (1.3) identifies it by equating the first-order derivative of the ratio to zero.

Appendix B.2. Optimal 'non-soft' WP

The result is shown under a more general framework where $\hat{\pi} \in [0, \pi_L)$ is the probability that a job is found with zero effort.

Proposition 12. *Assume $1/u'$ is convex, (SC) constraint is dropped and (IR, μ_p) and (IR, μ_f) in problem (1.10) are replaced by*

$$qU_p + (1-q)U_f \geq U \quad \textit{(IR)}$$

with $U_p := \pi(\mu_p)u(c_p^w) + \big(1-\pi(\mu_p)\big)u(c_p^u) - e$ being the continuation utility after 'Pass' and $U_f := \hat{\pi}u(c_f^w) + (1-\hat{\pi})u(^u c_f)$ the continuation utility after 'Fail'. Then,

$$U_p < U < U_f$$

Proof. The problem looks

$$V^{WP}(\mu,U) = \max_{q,(\mu_r,c_r^w,c_r^u)_{r=p,f}} \big\{q\hat{V}(\mu_p,U_p) + (1-q)\big[\hat{\pi}(\mu_f\omega_H + (1-\mu_f)\omega_L) - \hat{\pi}c_f^w - (1-\hat{\pi})c_f^u\big]\big\} - \kappa^{wp}$$

$$q\mu_p + (1-q)\mu_f = \mu, \quad \mu_f \leq \mu \leq \mu_p \quad \text{(MP)}$$

$$U_p \geq \hat{\pi}u(c_p^w) + (1-\hat{\pi})u(c_p^u) \quad (\text{IC}, \mu_p)$$

$$qU_p + (1-q)U_f \geq U \quad \text{(IR)}$$

First, notice that in optimum $c_f^w = c_f^u = c_f = g(U_f)$, $\mu_f = 0$ and (IC, μ_p) and (IR) are binding. Then, the optimal contract upon becomes:

$$u(c_p^w) = U_p + \frac{1-\hat{\pi}}{\pi(\mu_p)-\hat{\pi}}e, \quad u(c_p^u) = U_p - \frac{\hat{\pi}}{\pi(\mu_p)-\hat{\pi}}e, \quad u(c_f) = \frac{U-qU_p}{1-q}$$

Taking the derivative with respect to U_p yields

$$\frac{\pi(\mu_p)}{u'(c_p^w)} + \frac{1-\pi(\mu_p)}{u'(c_p^u)} = \frac{1}{u'(c_f)}$$

Hence from convexity of $1/u'$, it holds:

$$\frac{1}{u'(c_f)} \geq \frac{1}{u'\big(\pi(\mu_p)c_p^w + (1-\pi(\mu_p))c_p^u\big)} \Longrightarrow c_f \geq \pi(\mu_p)c_p^w + (1-\pi(\mu_p))c_p^u > g(U_p+e) > g(U_p)$$

which implies that $U_p < U < U_f$. ■

Appendix B.3. Optimal 'soft' WP as function of program's generosity

Proof. Consider the function[16]

$$D(\mu, U) = V_\mu^{UI}(\mu, U) - \frac{V^{UI}(\mu, U) - V^{SA}(U)}{\mu}$$

Then, it holds:

$$D_\mu(\mu, U) = V_{\mu\mu}^{UI}(\mu, U) - \frac{D(\mu, U)}{\mu}$$
$$D_U(\mu, U) = V_{\mu U}^{UI}(\mu, U) - \frac{V_U^{UI}(\mu, U) - V_U^{SA}(U)}{\mu} > 0$$

where the inequality follows from convexity of $1/u'$ that causes super-modularity of V^{UI} (see Appendix A. Features of UI), and by

$$V_U^{UI}(\mu, U) - V_U^{SA}(U) = -\pi(\mu)\Big(\frac{1}{u'(c^w(\mu))} - \frac{1}{u'(g(U))}\Big) < 0$$

Hence, μ_p is monotonic increasing in generosity as

$$\frac{\partial \mu_p}{\partial U} = -\frac{D_U(\mu_p, U)}{D_\mu(\mu_p, U)} > 0$$

since $D_\mu(\mu_p, U) = V_{\mu\mu}^{UI}(\mu_p, U) < 0$ by concavity of V^{UI} in μ (see Appendix A. Features of UI).

Moreover, in $\mu_p(U^*) = 1$, and so it is for $U > U^*$. So for $U < U^*$, the solution of (1.3) is internal. And given that $D_U > 0$ and $D(1, U^*) = 0$, $D(1, U)$ is positive (i.e., Condition 1

[16] The value of UI and SA is now a function of both expectation μ and generosity U.

is satisfied) for all $U > U^*$. ■

Appendix B.4. Optimal program design as function of generosity

Proof. The difference between V^{WP} and V^{JA} reads

$$\Delta V := V^{WP} - V^{JA}$$

and its derivative by μ is

$$\Delta V_\mu = \frac{V^{UI}(\mu_p, U) - V^{SA}(U)}{\mu_p} - V_\mu^{JA}$$

Notice that $U^* < \underline{U}$ as $V_\mu^{UI} > V_\mu^{JA}$. Assume $U < U^*$, then μ_p is internal to $[0, 1]$ and defined by (1.3). Thus,

$$\Delta V_\mu = V_\mu^{UI}(\mu_p, U) - V_\mu^{JA} > 0 \tag{1.11}$$

If, instead, $U \geq U^*$, then $\mu_p = 1$ and

$$\Delta V_{\mu U} = V_U^{UI}(1, U) - V_U^{SA}(U) < 0$$

Therefore, if $U \in [U^*, \underline{U})$, by definition of $\underline{U}$, it holds that

$$\Delta V_\mu = V^{UI}(1, U) - V^{SA}(U) - V_\mu^{JA} > 0 \tag{1.12}$$

The inequalities (1.11) and (1.12) prove that, if $U < \underline{U}$, V^{WP} is steeper in μ than V^{JA}, and so WP must dominate JA for μ large enough. However, if $U > \underline{U}$, then

$$\Delta V_\mu = V^{UI}(1, U) - V^{SA}(U) - V_\mu^{JA} < 0$$

and WP dominates JA for μ small enough. And the result follows.

■

Appendix C. Optimal WP under Private Expectations

Proof of Prop. 9

Proof. The second-best contract[17] choice is the one shown in Fig. 1.3, with

- $c^w_{sa} = c^u_{sa} = g(U)$, for $\mu \in [0, \mu_{sa,wp})$;
- $c^w_{wp}(p) = g(U + e/\pi_H)$, $c^u_{wp}(p) = c^w_{wp}(f) = c^u_{wp}(f) = g(U)$, $\mu_p = 1$, $\mu_f = 0$, for $\mu \in [\mu_{sa,wp}, \mu_{wp,ui})$;
- $c^w_{ui}(\mu) = g(U + e/\pi(\mu))$, $c^u_{ui}(\mu) = g(U)$, for $\mu \in [\mu_{wp,ui}, 1]$.

Truth-telling does not bind under SA or WP, as agents, who are granted utility U in equilibrium under SA and WP, have no incentive to report a different initial expectation. Instead, those agents who are assigned to UI find convenient to report a lower expectation, as long as they are still assigned to UI, since search incentives are more generous for lower μ's. In particular, under the second-best contract, they would all lie and report expectation $\mu_{wp,ui}$. The following holds.

Lemma 1. *When implementing UI, the government always finds it optimal to offer a pooling contract.*

Proof of Lemma 1. The UI contract $c^w_{ui}(\mu)$, $c^u_{ui}(\mu)$, $\forall \mu \geq \mu_{wp,ui}$ satisfies truthtelling if and only if:

1. $\delta(\mu) := u(c^w_{ui}(\mu)) - u(c^u_{ui}(\mu))$ is (weakly) increasing in $\mu \geq \mu_{wp,ui}$;
2. $U_\mu\big(\mu, c^w_{ui}(\mu), c^u_{ui}(\mu)\big) = (\pi_H - \pi_L)\delta(\mu), \quad \forall \mu \geq \mu_{wp,ui}$

Define[18]

$$U\big(\mu, c^w(\mu), c^u_{ui}(\mu)\big) := \pi(\mu)u(c^w(\mu)) + (1 - \pi(\mu))u(c^u(\mu)) - e$$

'Only If' Part

Truth-telling imposes that

$$U(\mu, c^w(\mu), c^u(\mu)) \geq U(\mu, c^w(\mu'), c^u(\mu')), \ \forall \mu' \geq \mu_{wp,ui} \tag{TR}$$

[17] The first-best contract is implemented in case of observable effort (and no information asymmetry), the second-best contract is implemented in case of private effort but observable expectations (moral hazard) and the third-best contract in case of private effort and expectations (moral hazard and adverse selection).

[18] To ease notation, I drop the 'UI' subscript in the remainder of the proof.

which can be rewritten as

$$U\big(\mu, c^w(\mu), c^u(\mu)\big) \geq U\big(\mu', c^w(\mu'), c^u(\mu')\big) + (\pi(\mu) - \pi(\mu'))\big[u(c^w(\mu')) - u(c^u(\mu'))\big]$$

and

$$U\big(\mu', c^w(\mu'), c^u(\mu')\big) \geq U\big(\mu, c^w(\mu), c^u(\mu)\big) + \big(\pi(\mu') - \pi(\mu)\big)\big[u(c^w(\mu)) - u(c^u(\mu))\big]$$

Then

$$\big(\pi(\mu) - \pi(\mu')\big)\delta(\mu') \leq U\big(\mu, c^w(\mu), c^u(\mu)\big) - U\big(\mu', c^w(\mu'), c^u(\mu')\big) \leq (\pi(\mu) - \pi(\mu'))\delta(\mu)$$

If $\mu > \mu'$, then $\pi(\mu) > \pi(\mu')$, and $\delta(\mu) \geq \delta(\mu')$. So $\delta(\mu)$ is an increasing function of μ. Moreover, diving the inequalities by $\mu - \mu' > 0$:

$$\begin{aligned}
&\frac{\big(\pi(\mu) - \pi(\mu')\big)}{\mu - \mu'}\delta(\mu') \leq \frac{U\big(\mu, c^w(\mu), c^u(\mu)\big) - U\big(\mu', c^w(\mu'), c^u(\mu')\big)}{\mu - \mu'} \leq \frac{\big(\pi(\mu) - \pi(\mu')\big)}{\mu - \mu'}\delta(\mu) \\
\Longrightarrow &\big(\pi_H - \pi_L\big)\delta(\mu') \leq \frac{U\big(\mu, c^w(\mu), c^u(\mu)\big) - U\big(\mu', c^w(\mu'), c^u(\mu')\big)}{\mu - \mu'} \leq \big(\pi_H - \pi_L\big)\delta(\mu)
\end{aligned}$$

And taking the limit for $\mu \to \mu'$, it follows:

$$U_\mu\big(\mu, c^w(\mu), c^u(\mu)\big) = (\pi_H - \pi_L)\delta(\mu)$$

<u>*'If' Part*</u>

$$\begin{aligned}
U\big(\mu, c^w(\mu), c^u(\mu)\big) =& U\big(\mu', c^w(\mu'), c^u(\mu')\big) + \big(\pi_H - \pi_L\big)\int_{\mu'}^{\mu} \delta(\tilde{\mu})d\tilde{\mu} \\
\geq& U\big(\mu', c^w(\mu'), c^u(\mu')\big) + \big(\pi_H - \pi_L\big)(\mu - \mu')\delta(\mu') \\
=& U\big(\mu', c^w(\mu'), c^u(\mu')\big) + \big(\pi(\mu) - \pi(\mu')\big)\Big(u\big(c^w(\mu')\big) - u\big(c^u(\mu')\big)\Big) \\
=& U\big(\mu, c^w(\mu'), c^u(\mu')\big)
\end{aligned}$$

where the inequality follows from $\delta(\mu)$ being increasing, by hypothesis. Hence, (TR) holds. Expliciting the derivative of the LHS of $U_\mu\big(\mu, c^w(\mu), c^u(\mu)\big) = (\pi_H - \pi_L)\delta(\mu)$ and simplifying yields:

$$\delta_\mu(\mu)\pi(\mu) + u'(c^u(\mu))c^u_\mu(\mu) = 0 \Longrightarrow \delta_\mu(\mu) = -\frac{u'(c^u(\mu))c^u_\mu(\mu)}{\pi(\mu)}$$

Therefore, since (TR) implies $\delta_\mu(\mu) \geq 0$ (by the characterization above), then it must be the case that $c^u_\mu(\mu) \leq 0$. And by the definition of $\delta(\mu)$:

$$\delta_\mu(\mu) = u'\big(c^w(\mu)\big)c^w_\mu(\mu) - u'\big(c^u(\mu)\big)c^u_\mu(\mu) \Longrightarrow c^w_\mu(\mu) = \overbrace{\Big(1 - \frac{1}{\pi(\mu)}\Big)}^{<0}\frac{u'\big(c^u(\mu)\big)}{u'\big(c^w(\mu)\big)}\overbrace{c^u_\mu(\mu)}^{\leq 0} \geq 0$$

Now, taking the derivative of $\pi(\mu)c^w(\mu) + (1-\pi(\mu))c^u(\mu)$ with respect to μ yields:

$$\frac{\partial}{\partial\mu}\big[\pi(\mu)c^w(\mu) + (1-\pi(\mu))c^u(\mu)\big] = \big(\pi_H - \pi_L\big)\big(c^w(\mu) - c^u(\mu)\big) + \pi(\mu)c^w_\mu(\mu) + \big(1-\pi(\mu)\big)c^u_\mu(\mu)$$
$$= \big(\pi_H - \pi_L\big)\big(c^w(\mu) - c^u(\mu)\big) + \big(1-\pi(\mu)\big)c^u_\mu(\mu)\Big[1 - \frac{u'\big(c^u(\mu)\big)}{u'\big(c^w(\mu)\big)}\Big]$$
$$\geq \big(\pi_H - \pi_L\big)\big(c^w(\mu_{wp,ui}) - c^u(\mu_{wp,ui})\big) = \frac{\partial}{\partial\mu}\big[\pi(\mu)c^w(\mu_{wp,ui}) + \big(1-\pi(\mu)\big)c^u(\mu_{wp,ui})\big]$$

where the inequality follows as the gap between c^w and c^u is widening in μ, and marginal utility is decreasing, so $u'(c^u(\mu)) > u'(c^w(\mu))$. Hence, for each $\mu > \mu_{wp,ui}$,

$$\pi(\mu)c^w(\mu) + \big(1-\pi(\mu)\big)c^u(\mu)$$
$$= \pi(\mu_{wp,ui})c^w(\mu_{wp,ui}) + \big(1-\pi(\mu_{wp,ui})\big)c^u(\mu_{wp,ui}) + \int_{\mu_{wp,ui}}^{\mu}\frac{\partial}{\partial\mu'}\big(\pi(\mu')c^w(\mu') + \big(1-\pi(\mu')\big)c^u(\mu')\big)d\mu'$$
$$> \pi(\mu_{wp,ui})c^w(\mu_{wp,ui}) + \big(1-\pi(\mu_{wp,ui})\big)c^u(\mu_{wp,ui}) +$$
$$+ \int_{\mu_{wp,ui}}^{\mu}\frac{\partial}{\partial\mu'}\big(\pi(\mu')c^w(\mu_{wp,ui}) + \big(1-\pi(\mu')\big)c^u(\mu_{wp,ui})\big)d\mu' =$$
$$= \pi(\mu)c^w(\mu_{wp,ui}) + \big(1-\pi(\mu)\big)c^u(\mu_{wp,ui})$$

which shows that the expected payment is lower under a pooling contract than a screening one. Hence the result. ■

Therefore, the planner fixes a threshold $\mu'_{wp,ui} \geq \mu_{wp,ui}$ and offers the pooling third-best contract $c^w_{ui}(\mu'_{wp,ui}) = g(U + e/\pi(\mu'_{wp,ui}))$, $c^u_{ui} = g(U)$, for $\mu \in [\mu'_{wp,ui}, 1]$.
The problem of the planner thus looks

$$\max_{\mu_{sa,wp},\mu'_{wp,ui}} V^{SA}(U)\Phi(\mu_{sa,wp}) + \int_{\mu_{sa,wp}}^{\mu'_{wp,ui}}\big[\mu V^{UI}(1) + (1-\mu)V^{SA}(U) - \kappa^{wp}\big]\phi(\mu)d\mu +$$
$$+ \int_{\mu'_{wp,ui}}^{1}\big[\omega(\mu) - \pi(\mu)c^w_{ui}(\mu'_{wp,ui}) - (1-\pi(\mu))c^u_{ui}\big]\phi(\mu)d\mu$$
$$\text{sub: } 0 \leq \mu_{sa,wp} \leq \mu'_{wp,ui} \leq 1$$

where ϕ is the pdf of expectations.

Therefore, while threshold $\mu_{sa,wp}$ is the expectation where V^{SA} equals V^{WP}, threshold $\mu'_{wp,ui}$ is distorted upward with respect to second-best one, as this reduces the incentive cost of UI. To see it, one just needs to notice that the marginal return of WP in μ is larger than the one of UI. Thus, setting $\mu'_{wp,ui}$ at the intersection of V^{WP} and V^{UI} (as in the second-best case) would cause UI to be dominated by WP for $\mu \in [\mu'_{wp,ui}, 1]$. ■

Proof of Prop. 10

Proof. The second-best contract is the one shown in Fig. 1.3, with

- $c^w_{sa} = c^u_{sa} = g(U)$, for $\mu \in [0, \mu_{sa,wp})$;
- $c^w_{wp}(p) = g(U + e/\pi(\mu_p))$, $c^u_{wp}(p) = c^w_{wp}(f) = c^u_{wp}(f) = g(U)$, $\mu_p < 1$, $\mu_f = 0$, for $\mu \in [\mu_{sa,wp}, \mu_{wp,ui})$;
- $c^w_{ui}(\mu) = g(U + e/\pi(\mu))$, $c^u_{ui}(\mu) = g(U)$, for $\mu \in [\mu_{wp,ui}, 1]$.

When Condition 1 is not satisfied, agents lie not only when assigned to UI, but also when assigned to WP. In particular, those assigned to WP and holding expectation μ' report expectation $\mu_{sa,wp}$, as

$$\arg\max_{\mu} \quad \frac{\mu'}{\mu'_p}\Big[\pi(\mu'_p)\Big(\frac{e}{\pi(\mu_p)} + U\Big) + (1 - \pi(\mu'_p)U - e\Big] + \Big(1 - \frac{\mu'}{\mu'_p}\Big)U, \quad \mu'_p = \frac{\mu'}{\mu' + (1-\mu')\sigma(p|L,\mu)}$$
$$= \arg\min_{\mu \in [\mu_{sa,wp}, \mu_{wp,ui})} \sigma(p|L,\mu) = \mu_{sa,wp}$$

The government must select one of two options, a pooling or a separating contract.
The separating contract is not implementable. Indeed, (SC) constraint and truth-telling would require that μ'-agent enjoys the same continuation utility under both outcomes.

$$U_p(\mu') = U_f(\mu') = U + \varepsilon \geq \max_{\mu} \quad \frac{\mu'}{\mu'_p}\Big[\pi(\mu'_p)\Big(\frac{e}{\pi(\mu_p)} + U\Big) + (1 - \pi(\mu'_p)U - e\Big] + \Big(1 - \frac{\mu'}{\mu'_p}\Big)U$$

However, this would push all agents to report μ'. For instance, any agent with $\hat{\mu} < \mu_{sa,wp}$, would find convenient to report μ' and obtain

$$\frac{\hat{\mu}}{\hat{\mu}_p} \max\Big\{\pi(\hat{\mu}_p)\Big(\frac{e}{\pi(\mu_p)} + U + \varepsilon\Big) + (1 - \pi(\hat{\mu}_p))(U+\varepsilon) - e, U + \varepsilon\Big\} + \Big(1 - \frac{\hat{\mu}}{\hat{\mu}_p}\Big)(U+\varepsilon) \geq U + \varepsilon$$

with $\hat{\mu}_p = \frac{\hat{\mu}}{\hat{\mu} + (1-\hat{\mu})\sigma(p|L,\mu')}$. Which would in turn trigger a generosity increase in all other contracts, resulting in an unfeasible strategy.

Therefore, the planner opts for a pooling consumption contract. However, the following result applies.

Lemma 2. *Under truth-telling, a pooling contract causes the profiling strategy to be pooling as well.*

Proof of Lemma 2. Consider any two expectations-type agents who are referred to WP, $\underline{\mu}, \overline{\mu} \in [\mu_{sa,wp}, \mu_{wp,ui})$ with $\underline{\mu} < \overline{\mu}$, and a pooling contract that promises utilities $U_p^w = U + e/\pi(\mu_p)$ and $U_p^u = U_f^w = U_f^u = U$, with $\mu_p \leq \min\{\underline{\mu}_p, \overline{\mu}_p\}$. Then, truth-telling requires that

$$\frac{\overline{\mu}}{\overline{\mu}_p}\Big[\pi(\overline{\mu}_p)U_p^w + (1-\pi(\overline{\mu}_p)U - e\Big] + \Big(1 - \frac{\overline{\mu}}{\overline{\mu}_p}\Big)U \geq \frac{\overline{\mu}}{\overline{\mu}'_p}\Big[\pi(\overline{\mu}'_p)U_p^w + (1-\pi(\overline{\mu}'_p)U - e\Big] + \Big(1 - \frac{\overline{\mu}}{\overline{\mu}'_p}\Big)U$$
$$\frac{\underline{\mu}}{\underline{\mu}_p}\Big[\pi(\underline{\mu}_p)U_p^w + (1-\pi(\underline{\mu}_p)U - e\Big] + \Big(1 - \frac{\underline{\mu}}{\underline{\mu}_p}\Big)U \geq \frac{\underline{\mu}}{\underline{\mu}'_p}\Big[\pi(\underline{\mu}'_p)U_p^w + (1-\pi(\underline{\mu}'_p)U - e\Big] + \Big(1 - \frac{\underline{\mu}}{\underline{\mu}'_p}\Big)U$$

with

$$\underline{\mu}_p = \frac{\underline{\mu}}{\underline{\mu} + (1-\underline{\mu})\sigma(p|L,\underline{\mu})} < \frac{\overline{\mu}}{\overline{\mu} + (1-\overline{\mu})\sigma(p|L,\underline{\mu})} = \overline{\mu}'_p$$

and

$$\underline{\mu}'_p = \frac{\underline{\mu}}{\underline{\mu} + (1-\underline{\mu})\sigma(p|L,\overline{\mu})} < \frac{\overline{\mu}}{\overline{\mu} + (1-\overline{\mu})\sigma(p|L,\overline{\mu})} = \overline{\mu}_p$$

From the first truth-telling inequality, it follows that $\overline{\mu}_p \geq \overline{\mu}'_p$, hence $\sigma(p|L,\overline{\mu}) \leq \sigma(p|L,\underline{\mu})$ and $\underline{\mu}'_p \geq \underline{\mu}_p$. While, from the second truth-telling inequality, it follows that $\underline{\mu}'_p \leq \underline{\mu}_p$. Therefore, in optimum $\underline{\mu}'_p = \underline{\mu}_p$ which boils down to $\sigma(p|L,\overline{\mu}) = \sigma(p|L,\underline{\mu})$. And the result follows. ■

However, notice that under a pooling contract in WP (the only one possible under truth-telling), higher expectations receive a continuation utility upon 'Pass' larger than the one under 'Fail' (i.e. $U_p > U_f = U$), which constitutes a violation of (SC). This turns out not to be a problem. Indeed, the planner can still design an expectation-indexed profiling strategy equal to the one under observable expectations (so that it formally respects (SC)), knowing however that a so-designed strategy is an off-the-equilibrium one and is never going to be implemented, as all agents with expectation in $[\mu_{sa,wp}, \mu_{wp,ui})$ are going to report $\mu_{sa,wp}$. And the contract for all $\mu \in [\mu_{sa,wp}, \mu_{wp,ui})$ reads

$$c_{wp}^w(p) = g\Big(U + \frac{e}{\pi(\mu_p)}\Big),\ c_{wp}^u(p) = c_{wp}^w(f) = c_{wp}^u(f) = g(U)$$

with $\mu_p \equiv \frac{\mu_{sa,wp}}{\mu_{sa,wp} + (1-\mu_{sa,wp})\sigma(p|L,\mu_{sa,wp})}$.

What is left to show is the contract under UI. An agent holding expectation $\mu_{wp,ui}$ will be reporting $\mu_{sa,wp}$ as well. To induce her to report her actual expectation, the principal must respect the following inequality

$$\pi(\mu_{wp,ui})U_{ui}^{w}+(1-\pi(\mu_{wp,ui}))U_{ui}^{u}-e \geq \frac{\mu_{wp,ui}}{\mu_p'}\Big[\pi(\mu_p')\Big(\frac{e}{\pi(\mu_p)}+U\Big)+(1-\pi(\mu_p'))U-e\Big]+\Big(1-\frac{\mu_{wp,ui}}{\mu_p'}\Big)U$$

with $\mu_p' = \frac{\mu_{wp,ui}}{\mu_{wp,ui}+(1-\mu_{wp,ui})\sigma(p|L,\mu_{sa,wp})} > \mu_p$. It would be natural to set a contract

$$\hat{U}_{ui}^{w} = \hat{U}+\frac{e}{\pi(\mu_{wp,ui})},\ \hat{U}_{ui}^{u} = \hat{U}, \quad \text{with } \hat{U} \equiv \frac{\mu_{wp,ui}}{\mu_p'}\Big[\pi(\mu_p')\Big(\frac{e}{\pi(\mu_p)}+U\Big)+(1-\pi(\mu_p'))U-e\Big]+\Big(1-\frac{\mu_{wp,ui}}{\mu_p'}\Big)U$$

Yet, this would constitute a rationale for agents in SA to lie and obtain a higher utility (the same reason that makes impossible to design a separating consumption contract). This fact forces the principal to set

$$U_{ui}^{w} = \Big[\frac{\mu_{wp,ui}}{\mu_p'}\Big(\frac{\pi(\mu_p')}{\pi(\mu_p)}-1\Big)+1\Big]\frac{e}{\pi(\mu_{wp,ui})}+U, \quad U_{ui}^{u} = U$$

which satisfies truth-telling of $\mu_{wp,ui}$-type agent with equality. The contract is robust to misreporting by all agents assigned to WP. To see it, define

$$\begin{aligned} LHS &= \pi(\mu)U_{ui}^{w}+(1-\pi(\mu))U-e \\ RHS &= \frac{\mu}{\mu_p'}\Big[\pi(\mu_p')\Big(\frac{e}{\pi(\mu_p)}+U\Big)+(1-\pi(\mu_p'))U-e\Big]+\Big(1-\frac{\mu}{\mu_p'}\Big)U \end{aligned}$$

RHS is the expression of the on-the-equilibrium utility upon truth-telling, while LHS is the one of off-the-equilibrium utility upon misreporting. The derivative of the LHS with respect to μ is

$$LHS_{\mu} = \frac{(\pi_H-\pi_L)e}{\pi(\mu_{wp,ui})}\Big[\frac{(1-\mu_p)(\pi(\mu_{wp,ui})-\pi(\mu_{sa,wp}))}{(1-\mu_{sa,wp})\pi(\mu_p)}+1\Big]$$

The derivative of the RHS reads

$$RHS_{\mu} = \frac{(\pi_H-\pi_L)e}{\pi(\mu_p)}\frac{1-\mu_p}{1-\mu_{sa,wp}}$$

The LHS derivative is larger than the RHS derivative (and both are positive). Moreover, the truth-telling constraint being binding at the top of the WP interval (i.e., for $\mu_{wp,ui}$-type agent) guarantees that LHS and RHS are equal in $\mu_{wp,ui}$, which boils down to state that RHS is larger than LHS over $[\mu_{sa,wp}, \mu_{wp,ui})$. Hence, upward truth-telling holds for agents assigned to WP. One can conclude that the contract in UI is also robust against misreporting by agents assigned to SA. Indeed, the RHS is equal to U in $\mu_{sa,wp}$. Therefore, the LHS is lower than U in $\mu \in [0, \mu_{sa,wp})$.

All other claimants with expectation $\mu > \mu_{wp,ui}$ are offered a pooling contract, as outlined in Prop. 9.

To conclude, the problem of the principal reads

$$\max_{\mu_{sa,wp},\mu_{wp,ui},\mu_p} V^{SA}(U)\Phi(\mu_{sa,wp}) + \int_{\mu_{wp,ui}}^{1} \big[\omega(\mu) - c^w_{ui} - (1-\pi(\mu))g(U)\big]\phi(\mu)d\mu +$$
$$+ \int_{\mu_{sa,wp}}^{\mu_{wp,ui}} \Big[\frac{\mu}{\mu'_p}\Big(\omega(\mu'_p) - \pi(\mu'_p)g(U+e/\pi(\mu_p)) - (1-\pi(\mu'_p))g(U)\Big) - \Big(1-\frac{\mu}{\mu'_p}\Big)g(U) - \kappa^{wp}\Big]\phi(\mu)d\mu$$
$$\text{sub:}\quad c^w_{ui} = g\Big\{\Big[\frac{\mu_{wp,ui}}{\mu'_p}\Big(\frac{\pi(\mu'_p)}{\pi(\mu_p)} - 1\Big) + 1\Big]\frac{e}{\pi(\mu_{wp,ui})} + U\Big\}$$
$$\mu'_p = \frac{\mu}{\mu + (1-\mu)\sigma(p|L,\mu_{sa,wp})}, \quad \sigma(p|L,\mu_{sa,wp}) = \frac{\mu_{sa,wp}(1-\mu_p)}{\mu_p(1-\mu_{sa,wp})}$$
$$0 \le \mu_{sa,wp} \le \mu_{wp,ui} \le \mu_p \le 1$$

The 'Pass' outcome is more informative of H state with respect to the first-best case. Indeed, a larger μ_p lowers also the incentive cost of UI, as well the one of the WP-'Pass' contract. To quantify it, the first-order condition with respect to μ_p reads

$$\frac{\mu_{sa,wp}}{\mu_p}\Big(V^{UI}_\mu(\mu_p) - \frac{V^{UI}(\mu_p,U) - V^{SA}(U)}{\mu_p}\Big)\int_{\mu_{sa,wp}}^{\mu_{wp,ui}}(1-\mu)\phi(\mu)d\mu +$$
$$+ \frac{\pi_H(\pi_H-\pi_L)e}{\pi(\mu_p)^2}\Big[\int_{\mu_{sa,wp}}^{\mu_{wp,ui}} \frac{\mu - \mu_{sa,wp}}{u'(g(U+e/\pi(\mu_p)))}\phi(\mu)d\mu + \frac{\mu_{wp,ui} - \mu_{sa,wp}}{\pi(\mu_{wp,ui})u'(c^w_{ui})}\int_{\mu_{wp,ui}}^{1}\pi(\mu)\phi(\mu)d\mu\Big] = 0$$

and given that the second term of the sum is positive, in optimum it must be that

$$V^{UI}_\mu(\mu_p, U) < \frac{V^{UI}(\mu_p,U) - V^{SA}(U)}{\mu_p}$$

which boils down to set a strictly higher μ_p than its second-best counterpart. ■

Optimal Unemployment Insurance with Worker Profiling

ABSTRACT

In unemployment assistance programs, the government profiles recipients according to their traits with the twofold goal of facilitating their reemployment and eliminating overpayments. To this purpose, a profiling program establishes (i) which recipients to profile, (ii) when and (iii) how accurately, and (iv) the transfers to be paid after it. This paper provides criteria to rank existing profiling programs, as well as an estimate of the welfare gains from the adoption of the optimal one. Two types of programs are possible at the optimum. The first type are programs that profile employability with full accuracy, and envisage the job search only for highly employable recipients. The second type are programs in which a fraction of poorly employable recipients are profiled as highly employable and persuaded to search at lower incentive costs. The reasons behind the second type of programs are (i) that expected returns on recipients' search are a concave function of the difference in the probabilities of success under search and no search, and (ii) that, absent incentive costs, also poorly employable recipients were better compensated for their search effort than left at rest. Profiling generates welfare gains also through fine-tuning of payments. Indeed, lowering the generosity of payments to those recipients who are mandated to search upon profiling reduces the cost of compensating them for the search effort.

JEL classification: D82, I38, J65, J68

Keywords: Bayesian Persuasion, Job-Search Assistance, Non-Contractible Effort, Social Assistance, Unemployment Insurance, Worker Profiling

2.1 Introduction

A renewed interest in optimal design of active labor-market policies (ALMPs) started in 2007 amid the financial crisis. Nowadays, following the outbreak of the Covid-19 pandemic, welfare support to the poor and the jobless is at the core of the political agenda of many governments worldwide. Nonetheless, the unprecedented increase in unemployment rates and the contemporaneous economic recession have led to a disproportion between public resources and the need for social security, which ultimately results in a push for optimizing public spending.[19] The trade-off between income support, incentive provision to job search and cost minimization for the public provider has led to policies tailored to recipients' characteristics. As a consequence, tracing a profile of any jobseeker who requests public financial support constitutes an aspect of first-order importance for the design of an effective welfare program. Profiling of welfare claimants is present in most OECD countries[20] and is usually employed as a tool to support and improve the design of existing ALMPs.

In the US, Worker Profiling and Reemployment Services (WPRS) and Reemployment and Eligibility Assessment (REA)[21] are two Federal-funded programs that profile welfare claimants. All workers who request access to public welfare support are asked to report their personal traits, such as education, past working experiences, family background, etc. This information allows for an *early assessment* of reemployment expectations, based on the statistical evidence provided by historical data on claimants' unemployment spells. In addition, both WPRS and REA may implement an *in-depth assessment* of the human capital of each claimant, in the form of one-on-one interviews and/or skill tests, to better tailor the assistance program to their needs.

The two programs generate savings for the provider through distinct channels. First, by improving upon the fit between workers and job-search methods. For instance, in WPRS "UI claimants who are identified through profiling methods as likely to exhaust benefits and who are in need of reemployment services to transition to new employment

[19]Public unemployment spending in the US reached $622 billions in 2021, accounting for 6.7% of the annual Federal budget (USASpending.gov, https://www.usaspending.gov/explorer/agency).

[20]Some examples are given by Worker Profiling and Reemployment Services and Reemployment and Eligibility Assessment programs (US), the Suivi Mensuel Personnalisé (France), 4-Phase Model (Germany) and Work Programme (UK).

[21]In 2015, REA has been replaced by the REemployment Services and Eligibility Assessment (RESEA) program, which provides greater access to reemployment services. I will nonetheless refer to the former version of the program, as it provides a clearer distinction between profiling and reemployment services which eases the exposition.

participate in reemployment services, such as job search assistance" (US Dept. of Labor[22]). Second, by designing transfers based on recipients' needs during the unemployment spell. This holds especially for REA[23], that is devoted to "enhance the rapid reemployment of unemployed workers, identify existing and eliminate potential overpayments, and realize cost savings for UI trust funds" (Poe-Yamagata et al., 2011).[24]

Profiling is complementary to welfare policies, which instead deal with income support and provision of search incentives and assistance. US welfare assistance is funded partly by the Federal government and partly by single States, while the organization and design is mainly deferred to the latter. Profiling programs thus greatly differ along many dimensions, namely (i) who should be profiled, (ii) when and (iii) how accurately, (iv) whether profilees should be requested to search or rest in the meantime and/or upon it, based on the new information obtained, and (v) which payments should accompany it. All these dimensions must therefore be taken into account in the analysis of an optimal profiling program.

The first objective of this paper is to develop a framework suitable to study the main complementarities between profiling and welfare policies. Optimal welfare provision solves the problem of a risk-neutral public welfare provider (hereafter, 'the government'), who needs to maximize the welfare of a risk-averse recipient (hereafter, 'the worker'), subject to a budget constraint and to non-contractible job-search effort of the latter. Following Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997), the design of a welfare program can be formalized as a dynamic principal-agent problem, where the state of the problem is composed by the current utility of the worker/agent, implicit in the structure of future payments, and the level of her expected reemployment skills. Keeping track of the state allows for a recursive formulation of the problem. Job search failures are themselves informative about hidden reemployment perspectives, and cause a revision of expectations. Like in Pavoni et al. (2013), policy instruments arise as the combination of (i) job-search recommendation to workers ('Search' or 'Rest'), (ii) a transfer scheme, made of current consumption and continuation utilities, indexed to future employment status and profiling outcome, and (iii) technologies adopted. The technologies available to the planner are job search assistance and profiling, and can be implemented jointly.

[22] https://www.dol.gov/agencies/eta/american-job-centers/worker-profiling-remployment-services

[23] One of the several purposes of RESEA is to "[...] Strengthen UI program integrity" (US Dept. of Labor, https://www.dol.gov/agencies/eta/american-job-centers/RESEA). Hence, the two versions of the program have similar targets.

[24] The report was commissioned by the Employment and Training Administration of the US Dept. of Labor.

The optimal program arises as a sequence of policies over time.

The paper finds that reemployment expectations and the generosity of payments toward workers are crucial determinants of optimal programs. The cost of search incentives being decreasing in the expected human capital, and the one of search-effort compensation being increasing in the generosity of transfers, make the government save on these costs by delegating the job-search to workers when expected human capital (resp., program's generosity) is high (resp., low). For this reason, the government lowers the payments to job seekers who are profiled as highly employable, in the attempt to further ease incentive provision. Therefore, a REA-like program contemplating this form of 'punishments' should optimally be adopted jointly with direct job search. Furthermore, profiling possibly does not fully detect employability at the optimum. For low program's generosity and absent search incentives, the government would find it optimal to require also poorly employable workers to search. If the additional cost of search incentives is not too large, a fraction of poorly employable workers may be persuaded of being more employable than expected and requested to search afterwards. The paper also states sufficient conditions on workers' utility function that guarantee that the number of poorly employable workers requested to search declines in the level of generosity. On the one hand, indeed, search-effort compensation becomes more expensive and the gains from persuading poorly employable workers shrink accordingly. On the other hand, instead, the assumption on workers' utility function causes the decline of search-incentive costs in response to an increase of expectations to be more sizable when the program is more generous. These two forces are jointly conducive to a more accurate profiling when programs are more generous.

The second objective of the paper is to provide a benchmark to evaluate existing programs. To this aim, an upper bound of returns is estimated by solving for the optimal program, conditional on reemployment expectations of each worker and the implicit generosity of transfers envisaged by the government's welfare policies. Performance assessments about WPRS and REA conducted in the past focus on specific margins and targets. The advantage and main contribution of this paper's approach, instead, lies in the absence of any arbitrary assumption, neither about the margins to focus on, nor about the design of the program (sequence of policies and transfers, timing and accuracy of profiling, job-search methods, etc.).

The rest of the paper is organized as follows. Section 3.2 contains the literature review. Section 2.3 presents the economic environment. Section 2.4 describes the welfare policies. Section 2.5 solves for the optimal program when worker profiling is performance-based.

Section 2.6 solves for the optimal program when worker profiling is based on a statistical assessment. Section 2.7 conducts a quantitative analysis on REA program in the US. Section 2.8 extends the analysis to the case of private worker search. Section 3.5 concludes.

2.2 Literature Review

The main contribution of this paper is the development of a framework suitable to study worker profiling within a welfare program toward the jobless. The paper provides an analysis of the gains and losses of profiling, in conjunction with others labor-market policies, when workers' human capital is not *ex-ante* observable.

Attempts have been made in the past to estimate returns of profiling programs. Sullivan et al. (2007) and Poe-Yamagata et al. (2011) are examples of such attempts. The first paper ranks WPRS programs in US States according to the occurrence of type-I error (i.e., the probability that a highly employable worker is profiled as lowly so). This paper finds that such a choice has a rationale. Indeed, optimal information design always signals low employability with full precision, but noisily detects high employability (i.e., positive probability of type-II error) whenever a share of poorly employable workers is persuaded to be highly so. In the second paper, authors conduct a field study on REA initiative in Florida, Idaho, Nevada and Illinois, and evaluate it over multiple dimensions, such as duration and total amount of unemployment benefits received, likelihood of reemployment and quarterly wage amounts received. In particular, the authors measure a positive impact of REA on public spending in three out of four States.[25] Their estimates of cost savings have the same order of magnitude as this paper's estimates (millions of US Dollars, see Section 2.7.3).

The existence of an agency problem in the contractual relationship between the welfare provider and the recipients has long been acknowledged by the literature. The provider has the possibility to tackle it either by providing recipients with incentives (Atkenson and Lucas, 1995; Wang and Williamson, 1996; Hopenhayn and Nicolini, 1997; Chetty, 2008; Shimer and Werning, 2008), by monitoring them (Pavoni and Violante, 2007; Setty, 2019), or else, by conducting the search on their behalf (Pavoni et al., 2013; 2016). In all cases, the job search produces an extra cost, which possibly outweighs the expected gain from re-employment. For this reason both active and passive policies coexist in a welfare

[25]The absence of any positive impact of REA in Illinois is attributed by the authors to the small number of eligible participants (3,122 in 2009).

program and only workers with better job opportunities are referred to the active ones. When human capital (i.e., job opportunities and on-the-job productivity) is allowed to depreciate during the unemployment spell, workers are reassigned to different policies. Likewise, in this paper any transition to a different policy follows the deterioration of expected reemployment perspectives. Yet, such a deterioration stems from an endogenous learning process which brings agents to revise their initial expectations. Gonzalez and Shi (2010) study unemployment-to-job transitions in a context where workers are heterogeneous in (unobservable) skills and get discouraged by long-lasting unemployment spells. Permanence in unemployment makes them more inclined to accept lower wage proposals. Therefore, the reemployment equilibrium wage is increasing in the perceived probability of being high-skilled. Similarly, in my framework the duration of unemployment spells has a discouragement effect on job-seekers. However, the need of larger search incentives for more discouraged workers produces a contrasting effect on net reemployment wages.

Expectation revision can also be caused by profiling. Profiling can thus become the mean used by the government to persuade jobless recipients to search for new jobs by manipulating their expectations. The paper is related to the vast and growing literature on information design initiated by Kamenica and Gentzkow (2011), that deal with the design of an optimal signaling strategy about a payoff-relevant, yet unknown, state from a principal/sender to an agent/receiver. Strategic signalling may lead to partial information disclosure, with the aim of making the agent choose an action which is most favorable to the principal, yet not to the agent itself. The optimal signal has a geometric characterization, as its return coincides with the concave closure of the pre-signal payoff function. Previous works (Bergemann and Morris, 2018; Kolotilin, 2018; and Galperti and Perego, 2018) have highlighted the limitations of this method and proposed a new formulation of the problem that can be solved with linear programming.[26] The first attempt to model profiling in the context of unemployment insurance has been made by Cappellini (2020). The paper outlines the trade-off in profiling between information detection and principal-agent incentive alignment and shows that it may lead to partial detection of human capital, aimed at persuading the worker to search at lower incentive costs. This paper builds on that result, by highlighting a second channel of gains from profiling that originates from reduction of effort-compensation costs on workers referred to direct search upon profiling. The peculiarity of this paper's framework is the 'hybrid' nature

[26] In particular, Galperti and Perego have proved the existence of a dual of the original problem, where the 'shadow price' of the probability of each state is declining in a measure of agent's persuasion.

of the problem, that requires the principal to deal with both information and incentive design. Consequently, the well-known concavification result à la Kamenica and Gentzkow only holds when utility promised to the agent is kept constant over profiling outcomes, and so the problem can be framed as one of pure information design (like in Cappellini, 2020). Boleslavsky and Kim (2021) extend the concavification result to a setting with three players (sender, agent and receiver) and incentive provision. The sender designs a signal about a hidden state, and determines the receiver's prior belief by convincing the agent to exert private effort that affects the state distribution. Rodina (2020) considers a similar setting where the agent effort is not private. Bloedel and Segal (2018), Habibi (2020) and Zapechelnyuk (2020) also study the tension between incentive and information provision in the Bayesian persuasion framework applied to situations of agent's rational inattention, agent's time-inconsistency and quality certification, respectively. However, to the best of my knowledge, no work has studied the relationship between information design and incentive provision in unemployment insurance so far.

2.3 Economic Environment

Players' Interaction. A risk-neutral government (principal, it) and a risk-averse worker (agent, she) populate the economic environment in discrete time. Each player is infinitely-lived and discount future utility at rate $\beta \in (0, 1)$. The worker can be employed or not, and the government observes her employment status. In period 0, (i) the two players are uncertain about the worker's human capital and hold common expectations about it, (ii) the worker is unemployed, and (iii) the government offers her a contract contingent on any possible future employment status and new information about human capital. The contract is so designed to minimize the expected discounted value of net transfers to the worker, conditional on delivering to her a given expected discounted utility. For each history node, the contract specifies the technology/ies adopted by the government (assisted-search and/or profiling), the effort recommendation to the worker ('Search' or 'Rest') and transfers. Uncertainty about worker's employment status clears at the beginning of each period.

Human Capital and Job Search. Worker's human capital can be high ($h = H$), or low ($h = L$). Workers with high (resp., low) human capital are labelled as high- (resp., low-)skilled. If unemployed, the worker can either rest ($a = 0$) or search for a job ($a = 1$). In the first case, her job-finding probability is null. In the second case, the high-skilled worker finds a job with probability π_H, while the low-skilled one with probability $\pi_L \in (0, \pi_H)$. The job search is public, but non-contractible, and makes any worker incur effort cost e, with worker's utility over consumption c and effort a being separable and given by $v(c, a) = u(c) - e \cdot a$. The first-order derivative of u^{-1}, $1/u'$, is convex.

Market-sector production. Labor productivity is increasing in human capital ($\omega_H > \omega_L$). In the economy there is one market sector only, populated by identical atomistic firms competing à la Bertrand over job offers, and paying wages equal to labor productivity. Reemployment is an absorbing status, since the worker faces no risk of any future lay-off.

Expectations. Any worker who applies to welfare support undergoes an early assessment. The assessment attaches to the worker a probability μ of being high skilled ($\mu = Prob(h = H)$), which is henceforth referred to as *expectation.*[27] In actual programs, welfare claimants report personal information (social background, past working experiences, education, etc.), according to which the welfare provider makes an initial evaluation of their human capital. The evaluation of any claimant is based on historical data that measure the reemployment frequency of claimants with same characteristics. Highly-educated and more experienced workers, for instance, are statistically more likely to exit unemployment than workers with less experience and/or lower educational attainment.

Assisted-search technology. The government can search on behalf of the worker at cost κ^{ja}. The cost includes the administrative expenses of the offices which are in charge of looking for vacancies, create a network with prospective employers and maintain contacts with them, circulate the worker's CV, etc.

Profiling technology. Profiling detects human capital with some accuracy, and returns a publicly observable outcome, at cost κ^{wp}[28]. Profiling can be thought of as a lottery that returns a binary outcome -'Pass' ($r = p$) or 'Fail' ($r = f$)-, with predetermined odds. The government can choose to profile with different levels of accuracy workers holding different expectations. This means that the lottery odds are indexed by expectation μ and program's generosity U

$$\left\{\sigma(r|h, \mu, U)\right\}_{r\in\{p,f\},\, h\in\{H,L\}}$$

[27] By the law of large numbers, such a probability is unbiased, meaning that the fraction of high-skilled workers among all workers with same expectation coincides with the expectation itself.

[28] The cost includes administrative expenses, as in the case of assisted search.

2.4 Policies

Any policy arises as the composition of (i) recommended search effort, (ii) consumption contract, and (iii) assisted-search and/or profiling technology (or neither of them). Combinations of search effort levels and technologies give rise to eight ($2 \times 2 \times 2$) possible policy instruments. However, when the assisted search technology is implemented, it would be redundant to prescribe positive search effort to the worker, which reduces to six the number of policies. If no technology is implemented, the government can decide whether to recommend positive search effort and pay incentives ('Unemployment Insurance', $i = UI$), or not ('Social Assistance', $i = SA$). If only the assisted search technology is implemented, it gives rise to 'Job-Search Assistance' ($i = JS$). Profiling without any search gives rise to 'Assistance and Profiling' ($i = AP$), whereas 'Insurance and Profiling' ($i = IP$) arises when the technology is adopted together with worker's search. Finally, 'Search-Assistance and Profiling' ($i = SP$) originates if both technologies are jointly adopted.

Welfare Policies (No Profiling)			
Recommendation	**Assisted Search**	**Delegated Search**	**No Search**
'Search'	x	Unemployment Insurance (UI)	x
'Rest'	Job-Search assistance (JS)	x	Social Assistance (SA)
Profiling Policies			
Recommendation	**Assisted Search**	**Delegated Search**	**No Search**
'Search'	x	Insurance & Profiling (IP)	x
'Rest'	Search-assistance & Profiling (SP)	x	Assistance & Profiling (AP)

Table 2.1: Policy Instruments

At time $t = 0$, the planner offers the unemployed agent an insurance contract that minimizes transfers and guarantees her an expected discounted utility equal to U. The planner's problem can be written recursively by keeping track of worker's expected human capital and promised utility -henceforth, a proxy for program's generosity- along the unemployment spell. The consumption contract of policy i consists of a menu of today's consumption c^i and tomorrow's continuation utilities $U_i^{s,r}$, contingent on reemployment ($s = w$) or not ($s = u$), and 'Pass' ($r = p$) or 'Fail' ($r = f$) outcome, if job search and/or profiling are conducted. Current expectation μ and the program's generosity U jointly determine the choice of the policy instrument. The government chooses the optimal policy

$i(\mu, U)$ by solving

$$V(\mu, U) = \max_{i \in \{SA, JS, UI, AP, SP, IP\}} V^i(\mu, U) \tag{2.13}$$

The planner is allowed to randomize over worker's utility U under the constraint that the promised utility must be delivered in expectation. To this end, the operator $\mathbf{V}$ is defined as

$$\begin{aligned} \mathbf{V}(\mu, U) &= \max_{\{U(x)\}_{x \in [0,1]}} \int_0^1 V(\mu, U(x))dx \\ \text{sub:} \quad U &= \int_0^1 U(x)dx \end{aligned} \tag{2.14}$$

In the following, I introduce the problem of the welfare provider in case of re-employment and for all six instruments during unemployment. First, I define welfare-oriented policies (SA, JS and UI) and later the profiling ones (AP, SP and IP).

2.4.1 Welfare Policies

Wage Tax/Subsidy (W). In case of successful job search, the worker's productivity is revealed. Therefore, the market-sector value when human capital is equal to $h \in \{H, L\}$ reads

$$W(h, U) = \max_{\tau, U^w} \tau + \beta W(h, U^w) = \max_{c^w, U^w} \omega_h - c^w + \beta W(h, U^w)$$

$$\text{sub:} \quad U = u(c^w) + \beta U^w \tag{PK}$$

Since reemployment is assumed to be an absorbing state (the separation rate between employees and firms is assumed null), the planner is sure to raise tax/pay subsidy also in the next period. The labor tax τ is the wedge between gross (ω_h) and net wage (c^w). The Promise-Keeping (hereafter, (PK)) constraint is the recursive expression of worker's utility. It guarantees that utility flow from current period $u(c^w)$ and continuation utility U^w are large enough to match current utility level U. The optimal contract prescribes constant continuation utility ($U^w = U$).[29] Hence from (PK) one can obtain the closed-

[29] At the optimum, U^w solves

$$W_U(h, U) = -\frac{1}{u'(c^w)} = W_U(h, U^w) \Longrightarrow U^w = U$$

form expression for consumption $c^w = u^{-1}\big((1-\beta)U\big)$. The expression for labor tax/subsidy thus is

$$W(h,U) = \frac{\omega_h - u^{-1}\big((1-\beta)U\big)}{1-\beta}$$

Social Assistance (SA). The planner's problem when neither job search, nor profiling is performed reads

$$V^{SA}(\mu,U) = \max_{c^{sa},U^{sa}} -c^{sa} + \beta\mathbf{V}(\mu,U^{sa})$$

$$\text{sub:}\quad U = u(c^{sa}) + \beta U^{sa} \qquad \text{(PK)}$$

The planner transfers c^{sa} and pledges continuation utility U^{sa}, without requiring the worker to exert any effort. SA is a passive measure, fully devoted to income support, and does not envisage any form of job search. Thus, there is no chance of reemployment for the worker, nor any chance for the provider of raising a labor tax in the incoming period. Differently from the definition of wage tax/subsidy, where reemployment is an absorbing state, the planner can freely select the best policy instrument in the next period. However, the following holds.

Proposition 13 (**Absorbing SA**). *Social Assistance is an absorbing policy and its continuation utility equals current utility ($U^{sa} = U$).*

Proof. See Appendix A: Properties of SA, JS and UI. ■

The proof follows the same steps as in Pavoni et al. (2016). The result implies that, once the worker enters SA, she is never reallocated to any other policy, neither she can exit unemployment, as no search is conducted. This result is admittedly quite extreme for policymakers, who may find it hard to politically defend a welfare program granting lifetime financial support to people who will never have the chance of getting reemployed. Yet, the result is remarkable in that it establishes that any passive policy should be regarded as a policy of last resort, to target only to workers with low expected human capital. Current consumption solves (PK) with $U^{sa} = U$.[30] The value of SA is independent of μ

[30] At the optimum, U^{sa} solves

$$V_U^{SA}(\mu,U) = -\frac{1}{u'(c^{sa})} = V^{SA}(\mu,U^{sa}) \Longrightarrow U^u = U$$

and has a closed-form expression

$$V^{SA}(U) = -\frac{u^{-1}\big((1-\beta)U\big)}{1-\beta} \tag{2.15}$$

No revision of expectations occurs during SA, as no job search is conducted. When, instead, the search is unsuccessful, both the government and the worker downward revise their initial expectation μ, according to the formula

$$\mu' := \frac{\mu\big(1-\pi_H\big)}{\mu\big(1-\pi_H\big)+(1-\mu)\big(1-\pi_L\big)} \leq \mu \tag{2.16}$$

where μ' is the revised probability that worker's human capital is $h = H$. μ' is lower than the initial one, with equality holding only if human capital was already known ($\mu \in \{0, 1\}$). The reason lies in the unbiasedness of μ, that is equal to the actual share of high-skilled workers among those who hold that expectation. Thus, a fraction $\pi(\mu) := \mu\pi_H + (1-\mu)\pi_L$ of them manages to find a new employment, which implies that the high-skilled who remained unemployed after one period are a fraction $\mu(1-\pi_H)/(1-\pi(\mu))$ of the initial group. Therefore, in case of failed search, a higher probability is attached to realization $h = L$.[31]

Job-Search assistance (JS). When resorting to assisted search, the government looks for employment on worker's behalf, an activity that costs him κ^{ja}. The value of JS reads

$$V^{JS}(\mu, U) = \max_{c^{js}, U_H^w, U_L^w, U^u} -c^{js} - \kappa^{ja} + \beta\big[\mu\pi_H W(H, U_H^w) + (1-\mu)\pi_L W(L, U_L^w) + (1-\pi(\mu))\mathbf{V}(\mu', U^u)\big]$$

$$\text{sub:} \quad U = u(c^{js}) + \beta\big[\mu\pi_H U_H^w + (1-\mu)\pi_L U_L^w + (1-\pi(\mu))U^u\big] \qquad \text{(PK)}$$

Two are the sources of risk related to the job search. The first risk is related to its outcome (success or failure). The second one, instead, is connected to human capital realization, conditional on finding a new job for the worker. While the government finds it optimal to

[31] If the failed attempts to exit unemployment are t, one for each period, then initial expectation μ is updated t times according to the formula

$$\mu^{(t)} = \mu^{(t-1)\prime} = \frac{\mu\big(1-\pi_H\big)^t}{\mu\big(1-\pi_H\big)^t + (1-\mu)\big(1-\pi_L\big)^t} \tag{2.17}$$

where the convention that $\mu^{(0)} = \mu$ is used. It is easy to see that:

- $\mu = 0$ and $\mu = 1$ are the only two expectations such that $\mu^{(t)} = \mu$. When players know human capital, no update ever occurs;
- $\lim_{t\to\infty} \mu^{(t)} = 0$, if $\mu^{(0)} < 1$.

insure the agent against the latter, due to her risk aversion ($U_H^w = U_L^w$), the same holds for the former only if no search incentive is to be paid ahead, i.e. if the worker will in no case be referred to UI during the spell.[32] In either case, then the optimal contract solves

$$-\frac{1}{u'(c^{js})} = W_U(\mu, U^w) = \mathbf{V}_U(\mu', U^u)$$

with

$$W(\mu, U) := \frac{\mu \pi_H}{\pi(\mu)} W(H, U) + \frac{(1-\mu)\pi_L}{\pi(\mu)} W(L, U)$$

being the expected wage tax/subsidy, conditional on reemployment.

Unemployment Insurance (UI). The planner may delegate the job search to the agent and provide her with incentives to conduct it. Incentive provision originates from the fact that worker's effort is non-contractible, and boils down to adding an Incentive Compatibility constraint (hereafter, (IC)) to the planner's problem.

$$U \geq u(c^{ui}) + \beta U^u \tag{IC}$$

The (IC) constraint guarantees incentive compatibility of the contract against agent's deviation from recommended search effort. Promise Keeping in UI takes into account the effort cost e exerted by the job-seeker agent

$$U = u(c^{ui}) - e + \beta\big[\pi(\mu)U^w + (1-\pi(\mu))U^u\big] \tag{PK}$$

The problem of the planner reads

$$V^{UI}(\mu, U) = \max_{c^{ui}, U^w, U^u} -c^{ui} + \beta\big[\pi(\mu)W(\mu, U^w) + (1-\pi(\mu))\mathbf{V}(\mu', U^u)\big]$$
$$\text{sub:} \quad \text{(PK) - (IC)}$$

(IC) and (PK) constraints imply the following condition on the difference in continuation utilities between successful (U^w) and failed (U^u) search

$$U^w - U^u \geq \frac{e}{\beta\pi(\mu)} \tag{2.18}$$

[32]The proof is reported in Appendix A: Properties of SA, JS and UI.

The condition in (2.18) is binding at the optimum and accounts for the planner's cost of incentive provision. Incentive cost can be defined by the difference in costs between the cases of contractible and non-contractible effort.[33] Incentive costs are increasing in the cost of effort and decreasing in the level of patience (β) and confidence (μ). Intuitively, it is less expensive to convince the agent to search when she expects larger return on search and weighs more the prospective reward ensuing from it. Condition (2.18) shows that search incentives have a convex hyperbolic shape in the space of expectations. Concavity of V^{UI} in μ follows from convexity of incentive costs and the linearity of returns. Lemma 3 proves these two facts.

Lemma 3 (**Slopes of the value functions with respect to μ and U**). *Every policy return V^i is concave increasing in expectations, and concave decreasing in promised utility. V^{UI} is supermodular (i.e, $V^{UI}_{\mu U}(\mu, U) \geq 0$). And so is V^{JS}, whenever $U^w_{JS} \geq U^u_{JS}$.*

Proof. See Appendix A: Properties of SA, JS and UI. ■

Incentive costs depending negatively on expectations through the utility dispersion generates a comparative advantage of UI for high-end expectations. On the contrary, convexity of $1/u'$ is a sufficient condition for the costs of incentive provision and effort compensation to be increasing in U, and so for UI to suffer a comparative disadvantage for high-end generosities. For this second reason, more generous programs are mainly focused on assistance, in the form of income support and assisted search, and less on search incentives. The following proposition establishes how policies are located over the (μ, U) space.

Proposition 14 (**Welfare Policies in the (μ, U) Space**). *Assume* $\mathbf{V}$ *is 'locally' supermodular, that is, for every (μ, U), there exist ϵ_μ, $\epsilon_U > 0$ such that*

$$\mathbf{V}_U(\tilde{\mu}, U) \geq \mathbf{V}_U(\mu, U) \quad \textit{and} \quad \mathbf{V}_\mu(\mu, \tilde{U}) \geq \mathbf{V}_\mu(\mu, U)$$

for every $\tilde{\mu} \in (\mu, \mu+\epsilon_\mu)$ and $\tilde{U} \in (U, U+\epsilon_U)$, and that $\mu' \in (\mu-\epsilon_\mu, \mu)$ and $U^u_i \in (U-\epsilon_U, U)$. Then,

$$V^{UI}_U(\mu, U) \leq V^{JS}_U(\mu, U) \leq V^{SA}_U(U) = W_U(\mu, U) < 0 \tag{2.19}$$

[33] On the same lines of Pavoni and Violante (2007), one can imagine the existence of a policy which delegates job search to workers whenever effort is contractible. If that is the case, the government will only need to compensate for the worker's effort. If so, the incentive cost is defined as the difference in cost of contract between UI and this new policy.

and

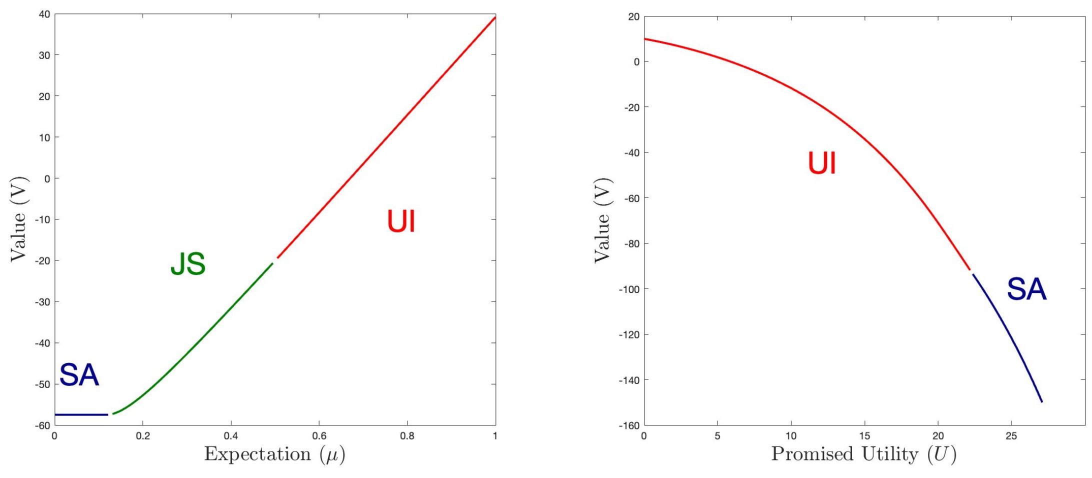
$$0 = V_{\mu}^{SA}(U) \leq V_{\mu}^{JS}(\mu, U) \leq V_{\mu}^{UI}(\mu, U) \tag{2.20}$$

where the last inequality holds (at least) at the crossing point. Lastly, if the program does not allow transition from JS to UI, then $V_{\mu\mu}^{JS}(\mu, U) = 0$ and $V_{U}^{JS}(\mu, U) = V_{U}^{SA}(U)$.

Proof. See Appendix A: Properties of SA, JS and UI. ■

The assumption about local supermodularity of $\mathbf{V}$ descends from supermodularity of each V^i and requires the marginal gain of μ to be increasing in the level of generosity (i.e., $\mathbf{V}_{\mu U}(\mu, U) \geq 0$). A rise in $\pi(\mu)$ increases the return of the job search, no matter who between the government and the worker conducts it. However, the assumption only holds locally. Indeed, the shape of $\mathbf{V}$ is determined by two contrasting forces, within-policy supermodularity and between-policy submodularity. While the former dominates locally, the latter has a global impact on $\mathbf{V}$.

Prop. 14 can be read as follows. Fix μ and move U. Then, UI, JS and SA are optimal for low, intermediate and high U, respectively. Now, fix U and move μ. SA, JS and UI are optimal for low, intermediate and high μ, respectively. The marginal value of μ is increasing in the level of search intensity, duration and effort by the worker.



(a) Constant U ($\exp((1-\beta)U) = \$575$) (b) Constant μ ($\pi(\mu) = 0.15$)

Figure 2.6: Value of welfare policies.[34]

Therefore, fixing generosity and spanning the space of expectations, one observes that policies with higher (resp., lower) marginal returns are optimal for higher (reps., lower)

[34]The parameter values and functional forms used in this Section are: $u(.) = \log(.)$, $\beta = 0.9$, $e = 0.5$, $\kappa^{ja} = 6$, $\kappa^{wp} = 1.5$, $\omega_H = 20$, $\omega_L = 3$, $\pi_H = 0.27$, $\pi_L = 0.14$. All monetary values are divided by 100.

expectations. The upper envelope V thus displays a tendency toward between-policy convexity in the space of μ (see Fig. 2.6a). Such shape of V has deep implications on the choice of optimal profiling. Similarly, Fig. 2.6b plots V over U for constant μ. UI is optimal for low generosity and cost of effort. On the other end of the spectrum, SA is first-best policy for high generosity, as no search is conducted. The slope of JS lies in between the one of UI and SA and depends on the downstream program in case the worker is not reemployed in the next period. The following proposition clarifies the point.

Proposition 15 (**Optimal Dynamics of Promised Utility and Benefits**). *Continuation utility upon failed search is*

- *decreasing when UI is part of the policy sequence ahead;*
- *constant, otherwise.*

Unemployment benefits are constant in SA and JS, and decreasing in UI.

Proof. See Appendix A: Properties of SA, JS and UI. ■

Since incentive costs increase in the level of utility promised to the worker, such utility decreases during UI. Furthermore, whenever the welfare program implements assisted search first and worker's search later, the planner finds it optimal to start decreasing worker's promised utility while the worker is still in JS. This finding sheds light on the possible policy patterns that can arise as a function of worker's initial expectation and program's generosity. Fig. 2.7 shows two instances of optimal policy sequences, for same initial expectation ($\mu_0 = 0.9$) and different generosity. When generosity is higher ($\overline{U}_0 = 28.9$), the worker never enters UI and only moves from JS to SA. Thus, no effort exertion is requested her and so she is granted both consumption and utility insurance along the spell. Hence, she eventually exits unemployment or enters SA with the same utility level and consumption as the entry one. When, instead, generosity is lower ($\underline{U}_0 = 24.7$), the worker is relocated from JS to UI. For this purpose, her utility decreases over time until she finds a job and exits unemployment.

One may ask whether a worker in UI can be referred to JS. Two contrasting forces have an impact on the policy transition over the UI-JS frontier. First, the optimal contracts of UI and JS prescribe a decline in promised utility for values of the (μ, U) space that are close enough to the frontier. This produces a decrease in the difference in contract costs and makes UI more appealing *ceteris paribus*. Second, the downward revision of expectations causes an increase in the incentive cost of UI, which makes JS more appealing. The following proposition establishes a relationship between the concavity of V and the slope of the UI-JS frontier in the μ space that avoid the possibility of any transition from UI to JS.

Proposition 16 (**Policy Transitions**). *Let $\hat{U}(\mu)$ be the promised utility level that makes the government indifferent between administering UI or JS to a worker with expectation μ, i.e.*

$$V^{UI}(\mu, \hat{U}(\mu)) \equiv V^{JS}(\mu, \hat{U}(\mu))$$

In addition, define $\eta^i(\mu, U)$ as the real non-negative number such that

$$V^i_U(\mu, U) \equiv -g'((1-\beta)U + \eta^i(\mu, U)), \quad i = SA, UI, JS$$

where $g \equiv u^{-1}$ is the inverse utility function. Assume that $\eta^i(\mu, U)$ is non-increasing in U for every μ and that

$$\beta\big[\hat{U}(\mu) - \hat{U}(\mu')\big] \leq \eta^{UI}(\mu, \hat{U}(\mu)) \tag{2.21}$$

Then, any worker in UI either remains in UI, enters SA or exits unemployment. In particular, the optimal program never switches from UI to JS.

Proof. See Appendix A: Properties of SA, JS and UI. ∎

The sufficient condition (2.21) establishes an upper bound on the slope of the UI-JS frontier in the μ space. Such slope is determined by the change in the difference of contract costs between UI and JS in response to an increase of μ and U. In particular, (i) incentive costs in UI fall in response to any increase of μ and (ii) both incentive and effort-compensation costs in UI rise in response to any increase in U.[35] An upper bound on the slope of the frontier thus requires that the first determinant does not have a major effect on the difference of contract costs, so that every variation of μ only requires a small variation of U (of equal sign) to reestablish the parity between the value of UI and JS. Condition (2.21) guarantees that the former effect prevails over the latter one.

[35]Assumption on convexity of $1/u'$ guarantees that incentive cost are positively related to promised utility.

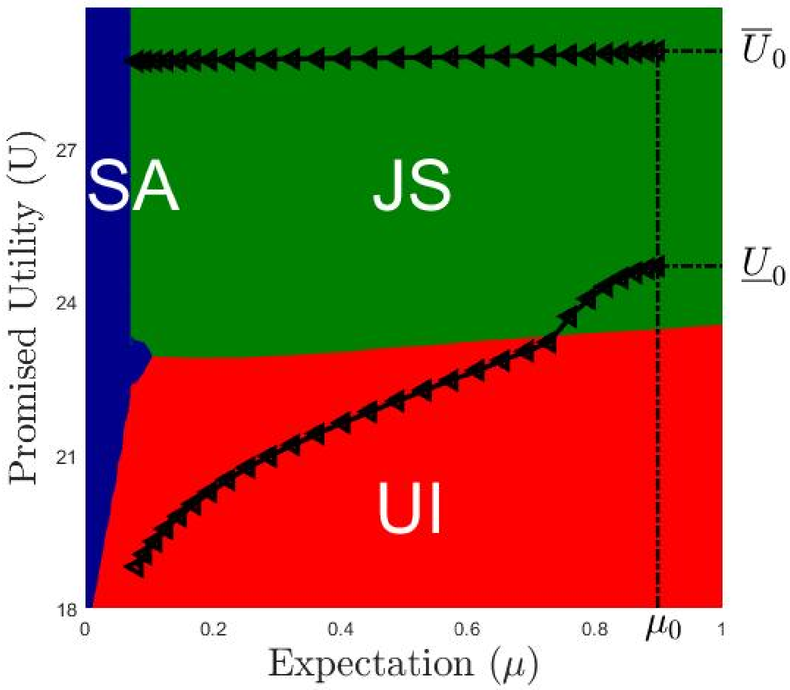


Figure 2.7: Optimal Welfare Policies in the Space of Expectation and Generosity.

2.5 Performance-Based Profiling

Profiling publicly discloses worker's human capital, up to a level of accuracy chosen by the government. For this purpose, profiling programs implement different strategies to infer the level of human capital of the worker. Some assign the profilee a given task and assess human capital based on how she performs. Therefore, profiling is designed as a test with two possible outcomes, 'Pass' ($r = p$) or 'Fail' ($r = f$). Conditional on the outcome of the test, the worker is referred to a different policy. Increasing the difficulty of the task is a way to improve the accuracy of the test, as more low skilled people are failing it and receiving a 'Pass' is more indicative of high human capital. The probability schedule brings both the worker (i.e., the profilee) and the government (i.e., the profiler) to revise expectation μ upon observation of the public outcome r according to the formula

$$\mu_r = \frac{\mu\sigma(r|H,\mu)}{\mu\sigma(r|H,\mu) + (1-\mu)\sigma(r|L,\mu)}$$

A necessary and sufficient condition for profiling to induce a change in expectations is to avoid returning either outcome with the same probability, irrespective of underlying human capital realization (e.g., $\sigma(r|H,\mu) \neq \sigma(r|L,\mu)$). In addition, profiling does not create any type of bias in the aggregate, since expectations are correct on average. Which boils down to require that the revised expectations are equal in mean to the prior (so called Martingale Property, (MP) henceforth).

$$q\mu_p + (1-q)\mu_f = \mu, \quad \mu_f, \mu_p \in [0,1], \quad \mu_f \leq \mu \leq \mu_p \quad \text{(MP)}$$

where q is the probability a 'Pass' is returned to any worker having an initial expectation equal to μ. (MP) can be interpreted as a restriction requiring profiling to be credible. Indeed, considering all workers who share the same expectation μ, inducing any of them to revise their expectation up to μ_p comes at the cost of inducing an expectation revision down to μ_f for someone else.[36] Given the nature of this profiling methodology, any profiled worker could pretend to be low-skilled by intentionally failing the test. It is thus necessary that the continuation utility upon 'Pass' is non smaller than the continuation utility upon 'Fail', in which case the worker retains the same expectation as before being profiled. Indeed, the worker who intentionally fails the test derives no new information about her human capital. Such a requirement, labelled No Discrimination constraint (ND, hereafter) imposes a restriction on the contract offered under either profiling outcome. In principle, the worker's utility of choosing the off-the-equilibrium action may be difficult to compute, as transfers are to be evaluated with the non-revised expectation. For instance, if the worker is requested to search right after, the consumption dispersion present in the UI contract is weighed differently as

$$U_f^u = u(c^{ui}) - e + \beta[\pi(\mu_f)U_{ui}^w + (1-\pi(\mu_f))U_{ui}^u(\mu_f')] < u(c^{ui}) - e + \beta[\pi(\mu)U_{ui}^w + (1-\pi(\mu))U_{ui}^u(\mu')]$$

However, the following result applies.

Proposition 17 (**No Type I Error**). *Only low-skilled workers who undergo profiling receive 'Fail' (i.e., $\mu_f = 0$), and are referred to Social Assistance henceforth.*

Proof. Any disclosure of new information via profiling is equivalent to a randomization over the space of expectations with mean equal to initial μ. Therefore, the government finds it convenient to implement profiling whenever it can exploit the between-policy convexity generated by the different policy slopes as in (2.20). The linearity of V^{SA} in μ causes the concave closure of $\mathbf{V}$ (for constant U) to be obtained by referring failed profilees to SA with $\mu_f = 0$. Intuitively, reducing the likelihood of 'Fail' outcome increases the frequency of 'Pass' (q) more than one to one. This fact, joint with the linearity of

[36] Without loss of generality, the posterior upon 'Fail' (μ_f) is set to be lower than the posterior upon 'Pass' (μ_p).

V^{SA} in μ (see Fig. 2.6a), makes convenient to limit 'Fail' only to low-skilled people (i.e., zero probability of type I error) and induces as many workers as possible to upward revise expectations. ■

Therefore, since SA is an absorbing policy and no job search is conducted ever after, the (ND) constraint has an easy formulation.

The timing of profiling policies is as follows. In the current period, the profiled worker is paid current transfers and possibly asked to search. In the next period, the outcome of the job search (if any) is disclosed first, before the one of test. The timing implies that the government can not index worker's current (c^i) and future re-employment consumption (c_i^w) on the new information derived from profiling.

Assistance-and-Profiling (AP). AP does not envisage any job search. Thus, the planner's problem reads

$$\begin{aligned}
V^{AP}(\mu, U) = & \max_{c^{ap},(U_r^u)_{r=\{p,f\}},\mu_p} -c^{ap} - \kappa^{wp} + \beta\big[q\mathbf{V}(\mu_p, U_p^u) + (1-q)\mathbf{V}(0, U_f^u)\big] \\
& \text{sub:} \quad U = u(c^{ap}) + \beta\big[qU_p^u + (1-q)U_f^u\big] \ \text{(PK)}, \quad \text{(MP)} \\
& \qquad\quad U \geq u(c^{ap}) + \beta U_f^u \ \text{(ND)}
\end{aligned}$$

The (ND) constraint can be rewritten as $U_p^u \geq U_f^u$. Absent (ND), the contract would equate the slopes of $\mathbf{V}$ across the different outcomes (i.e., $\mathbf{V}_U(\mu_p, U_p^u) = \mathbf{V}_U(0, U_f^u)$). By (2.19), this would boil down to set

$$U_f^u \geq U_p^u \tag{2.22}$$

Intuitively, if the government expects to incur an extra cost for incentive provision later during the unemployment spell, then it would save on this cost by decreasing the promised utility of the worker who passes the test and may at some point be referred to UI. Condition (2.22) and (ND) constraint bring the planner to insure the worker against the risk of profiling outcome ($U_f^u = U_p^u = U^u$).

Passing to the choice of μ_p, one may be tempted to adopt a reasoning similar to the one that leads to refer to SA only low-skilled workers ($\mu_f = 0$) and guess that 'Pass' outcome only targets high-skilled workers (e.g. $\mu_p = 1$) at the optimum. However, this is not always the case, due to concavity of $\mathbf{V}$ in μ for high-end expectations. Indeed, while concave returns cause the marginal gain of 'Pass' informativeness about high human capital to decline in the level of informativeness itself, reducing the frequency of 'Pass' and failing more workers cause a loss at the margin. Therefore, the planner trades off informativeness of 'Pass' against its frequency up to the point where the gain of higher informativeness equals the cost of lower frequency. In case the marginal gain exceeds the marginal cost for every μ, the test fully discloses high human capital. Otherwise, the internal solution satisfies

$$\mathbf{V}_\mu(\mu_p, U^u_{AP}) = \frac{\mathbf{V}(\mu_p, U^u_{AP}) - \mathbf{V}(0, U^u_{AP})}{\mu_p} \tag{2.23}$$

Eq. 2.23 shows that the upper posterior does not depend on worker's initial expectations, which means that all profiled workers hold the same revised expectation after receiving a 'Pass'. The downside is that the value of information for the government is negative when workers' initial expectation is larger than μ_p, irrespective of the administrative cost of profiling, as the low-skilled ones among them are mistaken in a direction favorable to the government. Hence, disclosing any information about their actual human capital causes the planner a loss that outweighs the gain of informing high-skilled workers.

Search-Assistance-and-Profiling (SP). Whenever the planner jointly adopts assisted search and profiling technologies, its problem reads

$$\begin{aligned} V^{SP}(\mu, U) = \max_{c^{sp}, U^w, U^u_p, U^u_f, \mu_p} & -c^{sp} - \kappa^{wp} - \kappa^{ja} + \\ & + \beta\big[\pi(\mu)W(\mu, U^w) + q(1-\pi(\mu_p))\mathbf{V}(\mu'_p, U^u_p) + (1-q)(1-\pi_L)\mathbf{V}(0, U^u_f)\big] \\ \text{sub:} \quad & U = u(c^{sp}) + \beta\big[\pi(\mu)U^w + q(1-\pi(\mu_p))U^u_p + (1-q)(1-\pi_L)U^u_f\big] \quad \text{(PK)} \\ & U \geq u(c^{sp}) + \beta\big[\pi(\mu)U^w + (1-\pi(\mu))U^u_f\big] \ \text{(ND)}, \quad \text{(MP)} \end{aligned}$$

As for the case of AP, the planner insures the worker against the risks related to profiling, by committing to a constant continuation utility (i.e., $U^u_p = U^u_f$). About the informativeness of the profiling strategy, the posterior expectation μ_p induced by 'Pass' outcome, is either 1 or solves

$$\mathbf{V}_\mu(\mu'_p, U^u_{SP}) = \frac{\mathbf{V}(\mu'_p, U^u_{SP}) - \mathbf{V}(0, U^u_{SP})}{\mu'_p} \tag{2.24}$$

Indeed, if in case of AP the randomization in the space of expectations occurs over the upper envelope $\mathbf{V}$, now instead the randomization only occurs conditional on job-search failure. Therefore, optimal profiling in SP delivers the concave closure (net of cost κ^{wp}) of

$(1-\pi(\mu))\mathbf{V}(\mu', U^u_{SP})$ in the space of expectations $\mu \in [0,1]$.

Insurance-and-Profiling (IP). When profiling is implemented jointly with delegated search, the planner's problem reads

$$V^{IP}(\mu, U) = \max_{c^{ip}, U^w, U^u_p, U^u_f, \mu_p} -c^{ip} - \kappa^{wp} +$$

$$+ \beta\big[\pi(\mu)W(\mu, U^w) + q(1-\pi(\mu_p))\mathbf{V}(\mu'_p, U^u_p) + (1-q)(1-\pi_L)\mathbf{V}(0, U^u_f)\big]$$

$$\text{sub:} \quad U = u(c^{ip}) - e + \beta\big[\pi(\mu)U^w + q(1-\pi(\mu_p))U^u_p + (1-q)(1-\pi_L)U^u_f\big] \quad \text{(PK)}$$

$$U \geq u(c^{ip}) + \beta\big[qU^u_p + (1-q)U^u_f\big] \quad \text{(IC)}, \quad U^u_p \geq U^u_f \;\; \text{(ND)}, \quad \text{(MP)}$$

Similarly to SP, profiling delivers the concave closure of $(1-\pi(\mu))\mathbf{V}(\mu', U^u_{IP})$ by selecting a posterior upon 'Pass' which equal 1 or solves (2.24).

2.5.1 Optimal Profiling Policies in the (μ, U) Space

I am now ready to locate the optimal profiling policies in each region of the (μ, U) state space. First, no profiling policy is optimal for very high or very low expectations, which can be explained through gains and losses of profiling. Profiling generates savings for the government by delegating search to high-skilled workers with a lower cost of incentives. The losses are of two types: administrative expenses (κ^{wp}), and the loss of revealing low skills to workers who are overconfident about their human capital. Therefore, for very high and very low expectations, workers are on average efficiently matched with policies, and the gains from reallocation and/or transfer reduction are outweighed by the losses. Second, there exists a correspondence between profiling policies and their welfare counterparts. Indeed, each profiling policy dominates the other two in a region of the space of expectations where the welfare policy with the largest return is the one that conducts (or does not conduct) the job search with the same method.

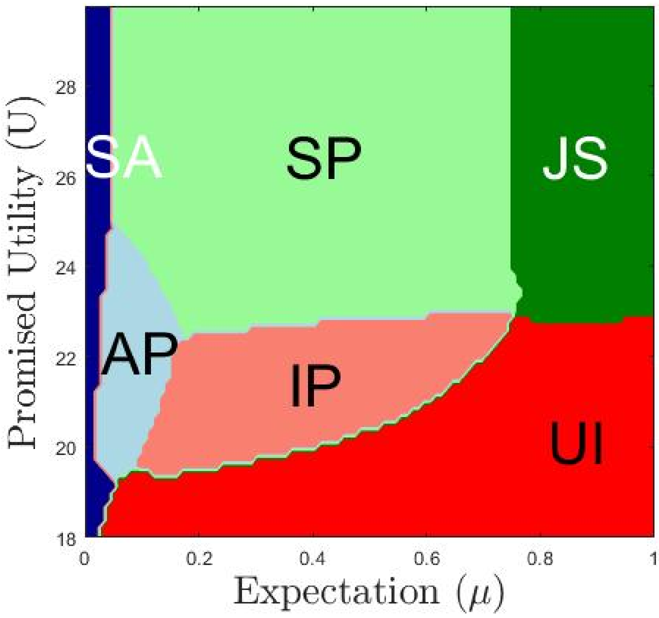


Figure 2.8: Optimal Policies in the Space of Expectation and Generosity.

Fig. 2.8 reports the optimal policies in the (μ, U) space. The complementarity of search effort and expectations is mirrored in the best profiling policy adopted. In particular, AP does not implement search and is thus optimal for lower-intermediate expectations. SP and IP, on the other hand, which contemplate different forms of search, are optimal for upper-intermediate expectations. As generosity rises (moving vertically from bottom to top of Fig. 2.8), the return of job search decreases due to higher costs of search-effort compensation and incentive provision (due to convex $1/u'$[37]), and worker's search (UI and IP) is replaced by assisted search (JS and SP) for high-end expectations, or by no search (SA and AP) for low-end expectations.

An important aspect of a profiling policy is the level of accuracy with which to detect the human capital of each profilee. Indeed, as outlined in Eq. 2.23 and 2.24, the value of information to the planner is possibly negative when the within-policy concavity of $\mathbf{V}$ prevails over its between-policy convexity in μ. If so, the optimal profiling strategy does not completely disclose high human capital upon 'Pass' ($\mu_p < 1$) and refers also a fraction of low-skilled workers to active policies upon it. Thus, any worker who receives a 'Pass' and is referred to an active policy can downward revise her expectation and reenter into IP, SP or AP at any later stage (unless she exits unemployment in the meantime). On the contrary, if profiling is fully accurate, workers do not revise expectation henceforth and never undergo profiling at any successive stage. The following result establishes a

[37]See Lemma 3.

monotone relationship between accuracy and generosity.

Proposition 18 (**Optimal Profiling Accuracy**). *Fix μ. Then, the accuracy of profiling in detecting high human capital is increasing in the level of generosity ($\partial\mu_p/\partial U \geq 0$). In particular, profiling is fully accurate under Search-Assistance-and-Profiling (SP) when no worker's search (UI or IP) is ever implemented in the downstream policy sequence.*

Proof. See Appendix B: Properties of AP, SP and IP. ■

The intuition of the result hinges on the sensitivity of the slope of $\mathbf{V}$ in μ to changes of U. As anticipated in Prop. 14, $\mathbf{V}$ is locally within-policy supermodular and globally between-policy submodular. Looking at the determinants of μ_p in Eq. 2.23 and 2.24, within-policy supermodularity produces an increase in the left-hand side in response to a rise in U, which is accompanied by a decrease in the right-hand side due to between-policy submodularity. To reestablish equality between the two sides of the equation, μ_p must increase. In particular, for rather high generosity, Prop. 14 and 15 have shown that, when the optimal program never resorts to worker's search (for high-end generosities), worker's utility does not fall over time and the return of JS is linear in μ. Therefore, the concave closure of $\mathbf{V}$ delivered by SP is the one realized by the full randomization over expectations ($\mu_f = 0,\ \mu_p = 1$) and constant continuation utility over time ($U^u_{SP} = U$).

Government gains from information acquisition via profiling in two ways. First, by realizing an efficient match between policies and workers. And second, by providing (lower) search incentives only to workers who receive a 'Pass'. For high generosity levels, the gains from profiling exclusively originate from the first channel, as high-skilled workers are referred to Reemployment Services. This type of profiling is reminiscent of the WPRS program, where information on human capital is used as a criterion to allocate services. On the contrary, for low generosity levels, information on human capital is used also to fine-tune transfers, like in REA program. Savings of this second type are even more relevant when no opportunistic behavior is possible for profilees and the program designer can thus index continuation utility to the profiling outcome, as now workers are possibly transferred to 'more generous' or 'less generous' programs based on their profiles (see Section 2.6).

2.6 Statistical Profiling

The presence of (ND) constraint in performance-based profiling prevents 'punishment' on workers who are referred to UI upon it. However, this restriction does not apply whenever human capital is assessed via statistical methods. In particular, some programs profile welfare claimants by conducting a background check and gathering observable data, and estimate a probability of reemployment in accordance with the information collected, on the base of a large number of past observations. Given that the outcome of statistical profiling does not rely on worker's commitment to it, the (ND) constraint does not apply in this new context. Consequently, the planner exploits the additional flexibility originating from the removal of 'no-punishment' restrictions as a leverage for incentive provision, with the target of reducing expected future transfers to recipients.

Profiling and reduction of transfers over time are two complementary instruments that open the way for sizable efficiency gains in the design of the optimal assistance program. Indeed, the planner now finds it optimal to index future transfers to the information detected during worker's profiling. Therefore, the contract of any profiling policy is not only consisting of the lottery odds of each outcome, but also of the schedule of continuation utilities depending on it.

Proposition 19 (**Optimal Statistical Profiling**). *When profiling refers workers to JS (for higher generosities), it is fully accurate. When, instead, profiling refers workers to UI (for lower generosities), the 'Pass' posterior is either 1 (i.e., full accuracy) or solves*

$$\mathbf{V}_{\mu}(\hat{\mu}, U_p^u) = \frac{\mathbf{V}(\hat{\mu}, U_p^u) - \mathbf{V}(0, U_f^u) + \mathbf{V}_U(0, U_f^u)(U_f^u - U_p^u)}{\hat{\mu}} \tag{2.25}$$

with $\hat{\mu} = \mu_p$ *in AP and* $\hat{\mu} = \mu_p'$ *in SP/IP. As in case of performance-based profiling,* $\hat{\mu}$ *is increasing in generosity.*

Proof. See Appendix C: Statistical Profiling. ■

The government sets different continuation utilities according to the profiling outcome. As shown in Prop. 14, the cost of incentive provision and effort compensation is increasing in generosity, which makes the marginal loss of higher generosity larger in UI than in SA. Hence, the government finds it optimal to lower the net discounted value of future payments upon 'Pass'. The result matches a characteristic of the actual REA program, where any worker who is found high-skilled is referred to minimum welfare support in the form of SNAP transfers up until reemployment. The criterion at the base of this rule is that any high-skilled worker does not need more generous transfers as she is likely to find reemployment soon.

The possibility to randomize over continuation utilities modifies the informativeness of the 'Pass' outcome. Eq. 2.25 strikes a new balance between incentive cost reduction of UI contract, the likelihood of being referred to it, and the new channel arising from the relaxation of the Incentive-Compatibility constraint.[38] Increasing informativeness, indeed, also increases the possibility of a 'Fail', conditional on which the planner pledges a larger utility. Hence, expected continuation utility for the agent is larger if the 'Pass' outcome is made more informative (and less likely) *ceteris paribus*, which allows the planner to further lower promised payments in order to restore contract efficiency (i.e., a binding (PK) constraint).

Proposition 20 (**Statistical Profiling Contracts**)**.**

- *If the policy sequence after 'Pass' includes UI, utility upon 'Pass' is lower than current utility. Otherwise, it remains constant.*
- *Unemployment benefits fall over time in IP, and remain constant otherwise. In particular, in IP benefits fall to a larger extent once workers receive a 'Pass'.*
- *In IP (resp., SP), the net wage upon reemployment is larger than (resp., equal to) current unemployment benefits.*

Proof. See Appendix C: Statistical Profiling. ■

Fig. 2.9 plots the patterns of policies, expectation, utility and unemployment benefits for a worker who enters the program with initial expectation of $\mu_0 = 0.95$ and promised utility of $U_0 = 22.4$. The worker is initially assisted in the search and profiled after 5 months. If she is found low-skilled, she is referred to SA with constant transfers. If, instead, she is found high-skilled, she is requested to search autonomously with transfers declining over time. As profiling is fully accurate and human capital entirely detected, any policy under either profiling outcome is absorbing.

[38]The first two forces where already at play in the problem with (ND) constraint (see Section 2.4).

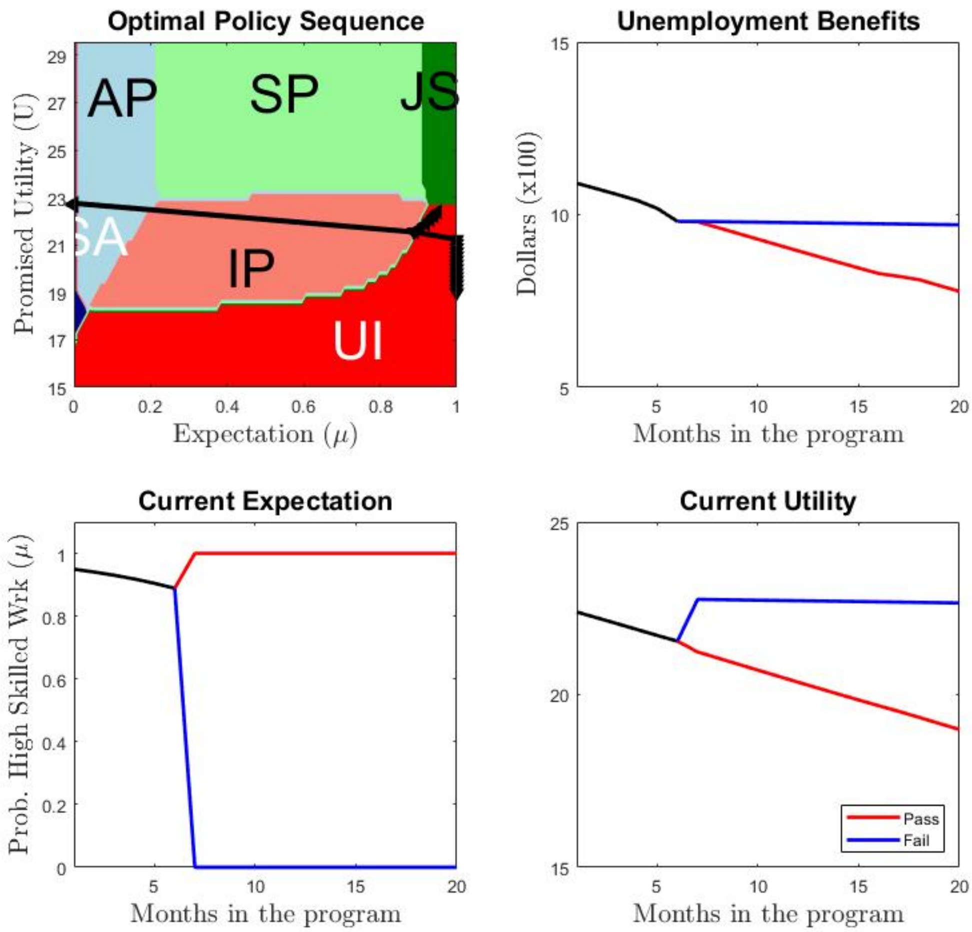


Figure 2.9: Consumption pattern upon profiling in a program with decreasing utility and $\mu_0 = 0.95$ and $c_0 = 100 \times exp((1-\beta)U_0) = \$939$.[39]

2.7 Quantitative Analysis

The main objectives of this paper are to present optimal welfare programs with profiling, and to estimate welfare gains from their implementation. To this aim, this section estimates the return of optimal profiling in the State of Florida, by comparing it to an alternative scenario where only welfare policies SA, JS and UI are implemented.

2.7.1 Parameterization

Many welfare programs worldwide combine UI benefits, profiling and job-search assistance, in the attempt to improve compliance to program requirements and the effectiveness of job search (see Section 2.1). In US, for example, two are the operating programs

[39]Monetary values are scaled down by a factor of 100.

Parameter	Symbol	Value	Source
Preferences			
Discount Factor	β	0.9	
Search Effort Cost	e	0.27	various sources
Labor Market			
Job Search Hazard	$\{\pi_H, \pi_L\}$	{0.27, 0.14}	basic monthly CPS, y. 2019
Net Wage	$\{c_H^w, c_L^w\}$	{$2,498, $1,128}	Poe-Yamagata et al. (2011)
Wage Tax	$\{\tau_H, \tau_L\}$	{$178, -$224}	EIC, FICA
Assisted Search			
Administrative Cost	κ^{ja}	$430	Balducchi and O'Leary (2018)
Worker Profiling			
Administrative Cost	κ^{wp}	$50	Poe-Yamagata et al. (2011)
REA programs (FL,ID,IL,NV)			
Generosity (consumption equivalent)	c	[$1,350,$2,301]	Nicholson and Needels (2011)

Table 2.2: Choice of Parameters Value

that profile workers: the Worker Profiling and Reemployment Services (WPRS) and the Reemployment and Eligibility Assessment (REA). WPRS is a federally-mandated program that supplies job-search assistance to welfare claimants who face a high risk of benefit exhaustion prior to reemployment. REA is, instead, a voluntary program each State can opt in, whose goal is to reduce fraud and fund misallocation by excluding from UI benefits those recipients who either do not conduct any search activity, or do not need any form of welfare support (because they are highly re-employable). In other words, REA and WPRS differ in the use of information they collect with profiling, as with REA efficiency gains realizes via reduction of transfers, whereas with WPRS by implementing the job-search with proper methodology. To meet their target, both programs conduct an in-depth assessment of individual skills, based on which workers receive job-counseling, learn how to develop a resume and/or are directly referred to employers (see Manoli et al., 2018). Moreover, neither program allows workers enrolled in an employment or training program to access any of these services.

Poe-Yamagata et al. (2011) conduct a cost-benefit analysis of the REA programs in Florida, Idaho, Illinois and Nevada, which assisted a total of 134,550 claimants in 2009. Of all claimants, 58% were men, 66% were white and 13% black. The report distinguishes between high and low skilled workers. The weighted mean share of high skilled participants is 48%.[40]

Turning to to the choice of parameters (see Table 2.2), the parameters to be chosen are:

[40] The relative weight assigned to each State depends on the number of participants it assisted. In 2009, Florida, Idaho, Illinois and Nevada supplied UI to 80,531, 18,156, 3,112 and 32,751 jobless workers, respectively (Poe-Yamagata et al., 2011). The report does not make a distinction between high- and low-skilled workers in Illinois. However, this is not a source of major concern, given the small number of welfare recipients in the State.

the functional form of period utility ($u(.)$), the discount factor (β), the effort cost of searching (e), the on-the-job productivity (i.e., the gross wage) and reemployment hazard rates of high- and low-skilled workers ($\{\omega_h, \pi_h\}_{h\in\{H,L\}}$), and the cost of administering profiling (κ^{wp}). The unit of time is set to one month.

I use a logarithmic specification of utility and set the monthly discount factor equal to $\beta = 0.9$. Based on Pavoni et al. (2013), the working effort cost is 49% of the consumption equivalent for men and 62% for women, corresponding respectively to $\overline{e}^m = 0.67$ and $\overline{e}^w = 0.97$ given the logarithmic specification[41]. And given that the percentage of male participants within the four programs is 58%, the working effort cost of the average participant amounts to $\overline{e} = 0.58\overline{e}^m + 0.42\overline{e}^w = 0.8$. Krueger and Muller (2010) conduct an analysis on the cost of search effort based on the American Time Use Survey (ATUS) and find that jobseekers spend on average 160 minutes every day looking for a job. Following Pavoni et al. (2013), I target the search effort to 1/3 (160/480) of the working effort, hence $e = 0.8/3 = 0.27$. The value is consistent with Pavoni et al. (2013), who estimate a cost of effort of $e = 0.22$.

Poe-Yamagata et al. (2011) reports data about net wages earned in the last 10 quarters prior to the start of UI claim. Quarterly wages in all States display a hump-shaped pattern, which increases until it reaches a peak three quarters before displacement and steadily declines later on. The decline is consistent with the *Ashenfelter's dip*, suggesting that wages fall in the pre-layoff period (Ashenfelter, 1978). Preventing this effect from distorting estimates requires to exclude the last three quarters of pre-layoff wage. However, the paper does not consider human capital depreciation along the unemployed spell, which is instead well documented by the empirical literature (Keane and Wolpin, 1997; Neal, 1995) and requires to lower the last wage, in accordance with the duration of unemployment spell. As the two effects tend to offset each other, I simply consider the wage earned in the last quarter. As a consequence, the monthly net wages of Florida, Idaho and Nevada are \$1,833, \$1,367 and \$1,900, respectively.[42] The report, however, does not distinguish between wages of high- and low-skilled workers. Thus, I exploit the cross-sectional variation in wages and the share of high-skilled participants across States.

[41] Logarithm allows for separation of consumption utility from working disutility in a natural way, according to the formula

$$log((1-\xi)c) = \log(c) + \log(1-\xi) = \log(c) - \overline{e}$$

with $\xi \in \{0.49, 0.62\}$ being the consumption equivalent of working disutility.

[42] Poe-Yamagata et al. (2011) does not report the percentage of high-skilled recipients in Illinois, which makes their data on wages useless for the estimation of $\{c_H, c_L\}$.

Given that there are two unknowns and three States, I compute $\{c_H, c_L\}$ as the pair that minimizes the loss function

$$\Lambda(\hat{c}_H, \hat{c}_L) = \sum_{i=1}^{3} \varphi_i(\theta_i \hat{c}_H + (1-\theta_i)\hat{c}_L - c_i)^2, \quad i = \{FL, ID, NV\}$$

with φ_i being the fraction of all welfare recipients in country i. The computation delivers monthly wages equal to $c_H^w = \$2,498$ and $c_L^w = \$1,128$. In order to compute their gross counterpart, I reverse engineer the gross labor income by computing the tax and deductibles that led to net amounts. In US, employees are subject to the Federal Insurance Contribution Act (FICA) tax, which is comprehensive of Social Security and Medicare tax. FICA tax is a net payroll tax which is levied half on employers and half on employees, and amounted to 15.3% of Adjusted Gross Income (AGI) in 2009. Moreover, taxpayers with an AGI lower than a certain amount, that depends on their marital status and number of children, are entitled to an Earned Income Credit (EIC). Since no data on the marital status or the number of children of recipients is available, I assume that the representative recipient is married and has two children. Under 2009 FICA and EIC tax schemes, fiscal neutrality for a married couple with two children is achieved at a gross annual income of $\$26,250$, with the couple paying a tax (resp., receiving a subsidy) for an income above (resp., below) that threshold. Therefore, low-skilled recipients, whose net annual income is $\$13,536$, receive a tax credit under EIC, making their gross income lower than the net one, and precisely equal to $\$10,844$. High-skilled recipients, instead, have a gross income of $\$32,112$ and a net one of $\$29,976$.[43] Therefore, monthly gross wages are equal to $\omega_H = \$2,676$ and $\omega_L = \$904$.

I estimate the hazard rates $\{\pi_H, \pi_L\}$, using data from the basic monthly Current Population Survey (CPS). Following the method-of-moments estimation, the probability of reemployment after t periods is computed as the fraction of workers who exit unemployment at that time. Reemployment probabilities are chosen as the ones that minimize the distance between the probabilities of reemployment so computed and the expected hazard rates, with weights given by the fraction of high- and low-skilled workers in the sample (for a more detailed description, see Appendix E: Estimation of hazard rates). The estimated monthly hazard rates are $\pi_H = 0.27$ and $\pi_L = 0.14$. The assumption that

[43] The net annual income of high- and low-skilled workers is $\$2,498 \times 12 = \$29,976$ and $\$1,128 \times 12 = \$13,535$, respectively. Low-skilled workers pay $\$1,659$ under FICA, i.e. the 15.3% of their gross income, but receive $\$4,350$ under EIC, hence receiving an annual subsidy of $\$2,691$. High-skilled workers, instead, pay a FICA tax of $15.3\% \times \$32,112 = \$4,913$, and are given a tax rebate of $\$2,774$, that account for an annual tax of $\$2,139$.

the worker can exit unemployment only upon search is quite extreme. I therefore assume the rate of reemployment in case no search is conducted either by the worker or by the government to be equal to half the after-search rate of low-skilled workers, i.e. $\hat{\pi} = 0.07$. A positive hazard rate in case of no search has a positive impact on the return of passive labor-market policies, like SA and AP, and a negative impact on the return of effort-incentivizing ones, like UI and IP, due to the increase in incentive costs. The value of off-the-equilibrium zero-effort action, indeed, is higher and the incentive constraint now requires satisfying a tighter condition

$$U^w - U^u \geq \frac{e}{\beta(\pi(\mu) - \hat{\pi})}$$

Passing to the choice of κ^{wp}, the estimates of average per-capita cost of REA in 2009 contained in the report range from \$12 (Idaho) to \$134 (Illinois) and include cost of personnel and operative costs of centers supplying REA services (e.g., State Workforce Agencies and One-Stop Career Centers). I, therefore, set the administrative cost of profiling equal to the weighted average of REA per-capita cost among the four State programs, that is, $\kappa^{wp} = \$50$. Instead, the cost of assisting any worker in the job search is based on Balducchi and O'Leary (2018), who estimate $\kappa^{ja} = \$430$. Such a figure is consistent with other estimates ($\kappa^{ja} = \$500$ in Pavoni et al. (2016)), as well as cost estimates of programs that perform different activities (for instance, search monitoring), but feature a similar set of operations (regular meetings with personnel at One-Stop Career Centers, phone calls to employers, etc.). For instance, Pavoni and Violante (2007) estimate a monthly cost of monitoring of \$478 per claimant.

The generosity of any program depends both on the amount of flow endowments and the duration. Poe-Yamagata et al. (2011) collects data about the average maximum and weekly benefit in each State, as well as the distribution of benefit duration among participants. The weekly benefit amount ranges from \$234 in Florida to \$299 in Nevada, suggesting a substantial variability in generosity of State programs. In the US, unemployment benefits are paid under four distinct schemes, which are activated in succession, depending on the current labor market situation of each US State. Unemployment Insurance (UI) benefits last up to 26 weeks in all States. Workers who are still unemployed at the end of the 26th week, are entitled to additional 53 weeks under the Emergency Unemployment Compensation (EUC) scheme. Moreover, States pay additional benefits up to 20 more weeks under the Extended Benefits (EB) scheme, if their unemployment rate exceeds 8.5%, which was the case for all four States in 2009. Exhaustees of UI, EUC and EB are finally referred to the Supplemental Nutrition Assistance Program (SNAP). This constitutes the typical instance of a purely income-support measure of last resort, consisting of a constant allowance for the purchase of food, with no eligibility assessment or time limit. Transfers decline over time, as claimants move from one program to another. WPRS and REA never profile workers after they have exhausted UI, EUC and EB, as no assisted search or further transfer reduction is possible once the worker enters SNAP. I assume that workers who are entitled to 26 weeks of regular UI benefits are assisted under EUC and EB programs for the whole prospective duration of the programs, i.e. 73 weeks, and that exhaustees who are still unemployed at the end of UI+EUC+EB receive an endowment from the Supplemental Nutrition Assistance Program (SNAP), which replaced the Food Stamps Program in 2008. Average total payment was $7,930 under EUC and $3,844 under EB (Nicholson and Needels, 2011), hence constituting a monthly endowment of $c^{EUC/EB} = \$645$, while a family of four people was receiving a $501 monthly benefit from SNAP.[44] The program's generosity for each of the four States is computed backward from the moment the welfare recipient enters into SNAP or finds reemployment, up until the first month when she receives regular UI benefits. Worker's utility of reemployment in case she is high-(resp., low-)skilled amounts to[45]

$$U_H^w = \frac{u(c_H^w)}{1-\beta} = \frac{\log(24.98)}{1-0.9(1-)} = 32.18 \quad U_L^w = \frac{u(c_L^w)}{1-\beta} = \frac{\log(11.28)}{1-0.9} = 24.23$$

while in SNAP with no search it is equal to

$$U_{e=0}^{SNAP}(\mu) = \frac{u(c^{SNAP}) + \beta\hat{\pi}[\mu U_H^w + (1-\mu)U_L^w]}{1-\beta(1-\hat{\pi})} = 22.32\mu + 19.25(1-\mu)$$

Condition $U_L^w > U_{e=0}^{SNAP}(0) + \frac{e}{\beta(\pi_L - \hat{\pi})}$ implies that the worker always finds it convenient to search also during SNAP. Hence, the value of SNAP can be rewritten as function of entry expectation μ

$$U^{SNAP}(\mu) = \mu\frac{u(c_{SNAP}) - e + \beta\pi_H U_H^w}{1-\beta(1-\pi_H)} + (1-\mu)\frac{u(c_{SNAP}) - e + \beta\pi_L U_L^w}{1-\beta(1-\pi_L)}$$

If the worker is entitled to regular UI, EUC and EB, then her assistance program lasts for 26+53+20=99 weeks, that is, around 25 months. Starting from the last month, the

[44] See SNAP Data Tables at the following link: https://www.fns.usda.gov/pd/supplemental-nutrition-assistance-program-snap.

[45] All monetary amounts are normalized so that 1 consumption unit corresponds to $100.

following recursion is implemented

$$U_{i,j,t} = u(c^j_t) - e + \beta[\mu_i^{t-1}\pi_H U^w_H + (1-\mu_i^{t-1})\pi_L U^w_L + (1-\pi(\mu_i^{t-1}))U_{i,j,t+1}],\ 1 \leq t \leq 25,$$
$$j = \{FL, ID, IL, NV\},\ i = \{< HS, HS, < CD, CD, GD\}$$

with $U_{i,j,26} = U^{SNAP}(\mu_i^{26})$, j indexing States and i indexing education. The initial probability of being high-skilled, μ_i^0, equals the share of high-skilled individuals with same educational attainment, θ_i. The generosity levels of each program and educational attainment, expressed in consumption-equivalent terms,[46] are reported in Table 2.3. Unsurprisingly, the generosity of the program is increasing in the level of educational attainment, due to higher initial expectations and $U^w_H > U^w_L$. Among the four States, Illinois (resp., Idaho) is the most (resp., least) generous one for all levels of education.

States	Less Than HS	HS Diploma	Some College	College	Graduate
Florida	\$1,350	\$1,536	\$1,580	\$1,748	\$1,811
Idaho	\$1,141	\$1,282	\$1,315	\$1,440	\$1,487
Illinois	\$1,666	\$1,920	\$1,981	\$2,212	\$2,301
Nevada	\$1,362	\$1,550	\$1,595	\$1,763	\$1,827

Table 2.3: Program generosity for any State and educational level (consumption equivalent).

2.7.2 Optimal REA Program

Workers are assessed via in-person interviews with REA staff. When issuing the call for the interview, States target those who are predicted to be likely to exhaust their UI benefit. The assessment is based on interviews, that last 45 min on average. After the interview, workers profiled as high skilled suffer a reduction of unemployment benefits. Therefore, the profiling methodology adopted in the REA program of statistical type, as it allows 'punishment' on recipients, based on their skills.

Fig. 2.10 reports the optimal policies in the state-space of programs' generosity and initial expectation, and locates Florida's REA recipients over it according to their ed-

[46] The expression of consumption equivalent of U is

$$c(U) = \exp((1-\beta)U)$$

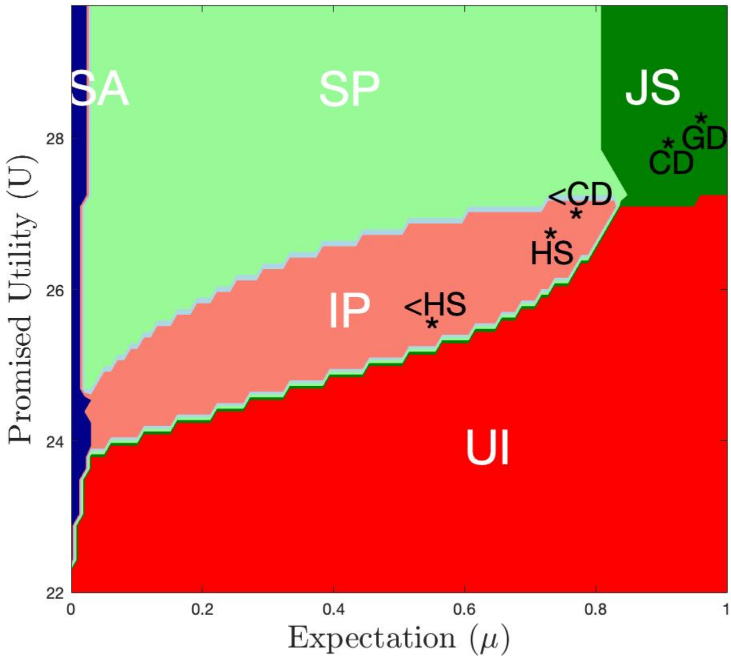


Figure 2.10: Optimal Policies for Florida's REA Recipients.
Note: <HS=Less Than High School, HS=High School Diploma, <CD=Some College, CD=College Degree, GD=Graduate Degree

ucational attainment.[47] Recipients with less than a college degree should better search and at the same time undergo profiling right upon entry into the program, while those with a higher educational attainment should be initially assisted in the search but not profiled. Quite surprisingly, the ones who are assisted in the search are those recipients whose search would have a larger expected return. The reason is that a larger $\pi(\mu_0)$ is correlated with larger consumption upon reemployment and so higher implicit utility U_0. Hence, graduates' effort is too expensive to compensate for the government.

2.7.3 Welfare Gains

A relevant question for policymakers is how large savings they can realize from designing an optimal profiling program. In order to estimate such value, I compare two distinct programs, one with only welfare policies SA, UI and JS ($\mathcal{W}$) and the other encompassing all six policies ($\mathcal{P}$). Fig. 2.11 reports the optimal patterns of promised utility, unemployment benefits and wage taxes/subsidies in the two programs for Florida's jobseekers with a high school degree (i.e., the most numerous group, accounting for 54% of all recipients in Florida in 2009), whose initial expectation and promised utility (in consumption equiv-

[47]The initial generosity of REA programs in the other three States is quite similar.

alent terms) are $\mu_0 = 0.72$ and $c(U_0) = \$1,436$, respectively. As shown in Fig. 2.10, this group of jobseekers are profiled under IP right after entering $\mathcal{P}$ and referred to UI and SA, while in the program with no profiling they never exit UI. The pattern of promised utility and unemployment benefits is thus falling over time only for high-skilled workers in $\mathcal{P}$ and for both high- and low-skilled ones in $\mathcal{W}$. However, the larger incentive costs in the latter case are conducive to a steadier decline in both benefits and utility. The reemployment tax displays a monotone increasing pattern in $\mathcal{P}$ (in the weak sense for low-skilled workers) due to the declining promised utility. In $\mathcal{W}$, instead, this component is contrasted by declining expected productivity (and gross wage thereof) of workers and increasing incentive costs, as expectations are revised downward. For this reason, the pattern of expected wage tax upon reemployment is non monotonic in time.

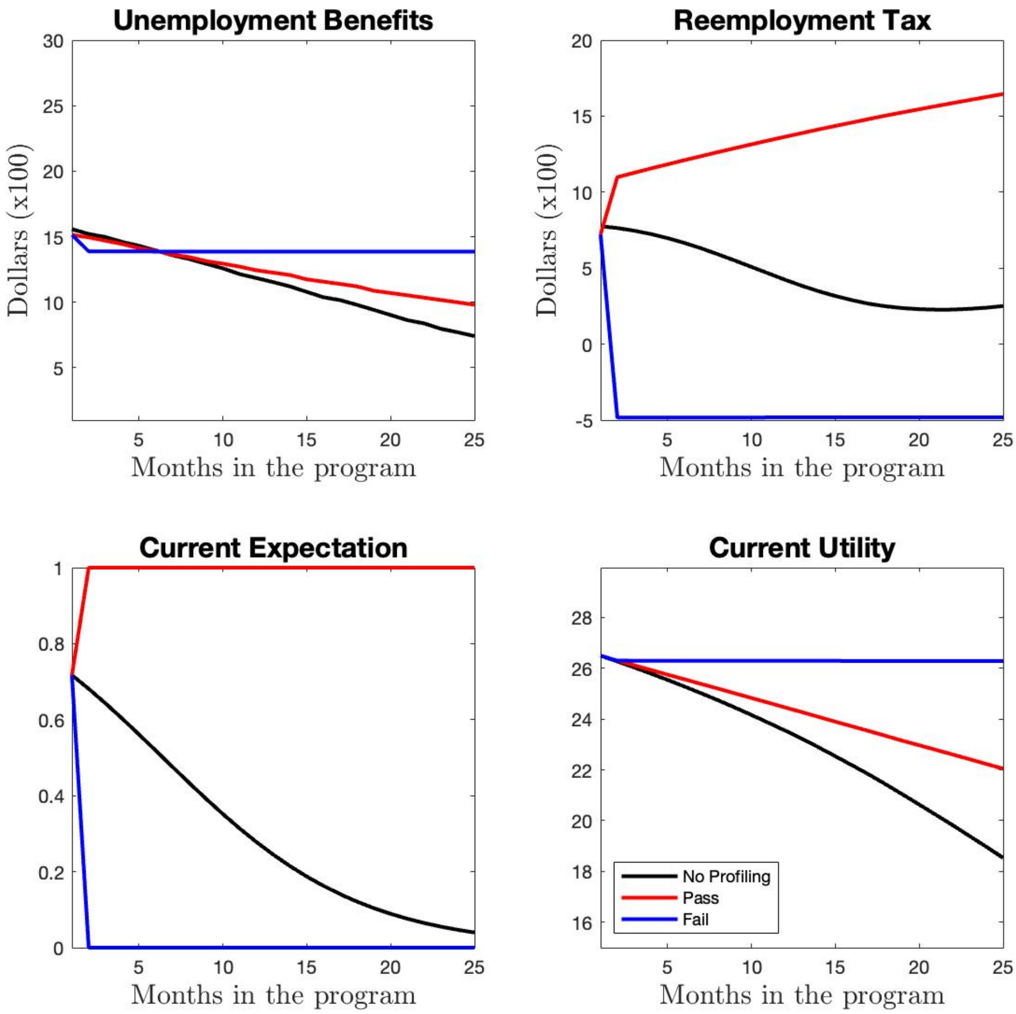


Figure 2.11: Optimal REA program of Florida for recipients with a high-school diploma over a 25-month horizon (UI+EUC+EB). Initial expectation and generosity are $\mu_0 = 0.72$ and $c(U_0) = \$1,436$, respectively.

Table 2.4 reports the per-capita welfare gain of profiling for each educational group. Given the relative size of each group and the annual number (80,531) of REA recipients, the aggregate welfare gain of Florida in 2009 would have been $1,817,200. This figure is consistent with the estimate provided by Poe-Yamagata et al. (2011), who estimate net per-capita savings of $47 for UI recipients and overall savings of $3,784,957.[48]

	Less Than HS	HS Diploma	Some College	College	Graduate
Perc. in Program	0.13	0.54	0.17	0.12	0.04
μ_0	0.54	0.72	0.76	0.9	0.95
$c(U_0)$	$1,275	$1,436	$1,474	$1,617	$1,671
Per-cap. Welfare Gain	$14.5	$29.3	$26.2	$3.2	$0.5

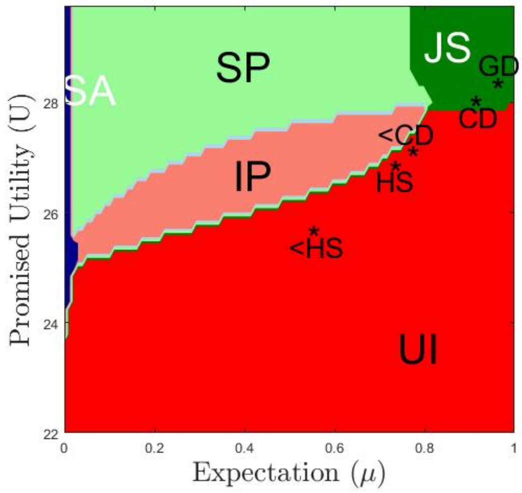
Table 2.4: Welfare Gains of Profiling per Education Group in Florida (y. 2009)

2.7.4 Robustness Checks

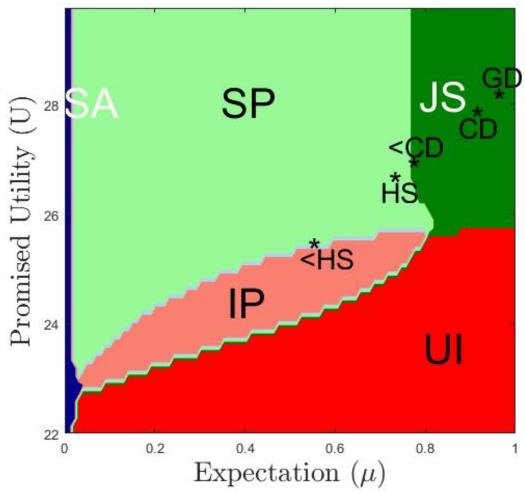
A relevant dimension on which workers display a large heterogeneity is effort cost. Various studies estimate different costs between men and women (Attanasio et al., 2008; Eckstein and Wolpin, 1989). In addition, they document the existence of a work-effort cost, which is not accounted for in the baseline model of this paper. I therefore conduct a robustness check by allowing for the search-effort cost to vary by $\pm 10\%$ with respect to the baseline value. Second, I assume that the reemployed worker incurs a working cost equal to the search-effort one. Optimal policies in the (μ, U) space for $e = 0.24$, $e = 0.3$ and $e^w = e = 0.27$ are reported in Fig. 2.12. When the search-effort cost is lower, then search-delegating policies (UI and IP) expand their areas at the expense of assisted-search ones (JS and SP). The opposite occurs when the search-effort cost is larger and all groups of recipients are offered search assistance upon entry. Positive working-effort cost $e^w = 0.27$ produces a comparative disadvantage for active labor-market policies, as reemployment (and effort compensation upon it) is less likely in SA. Therefore, the gain from reallocating workers across SA and JS is larger the higher is the working-effort cost, via relaxation of the Promise-Keeping constraint. For this reason, SP replaces JS for high-end generosities and expectations (see Fig. 2.12c).

A second parameter displaying great variability is the cost of assisted search and profiling technologies. Poe-Yamagata et al. (2011) estimate that in Nevada, which merges reemployment services with profiling, κ^{ja} equals $148. Such a figure is consistent with

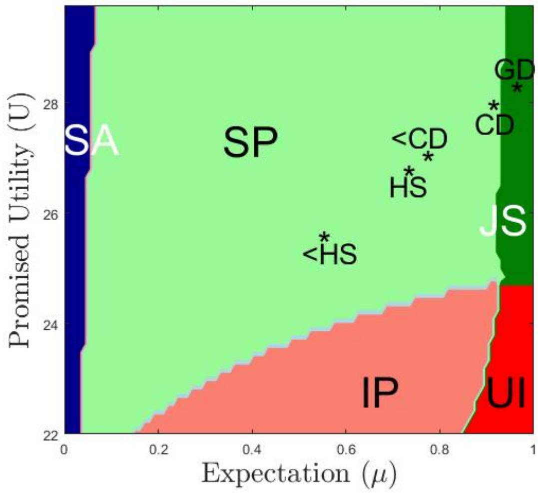
[48]Poe-Yamagata et al. also estimate savings for UI+EUC recipients. However, the authors do not apply any time discount, and this delivers an inflated estimation of net gains. For this reason, I consider only per-capita savings realized on recipients of UI benefits, which come first in chronological order.



(a) $e = 0.24$



(b) $e = 0.3$

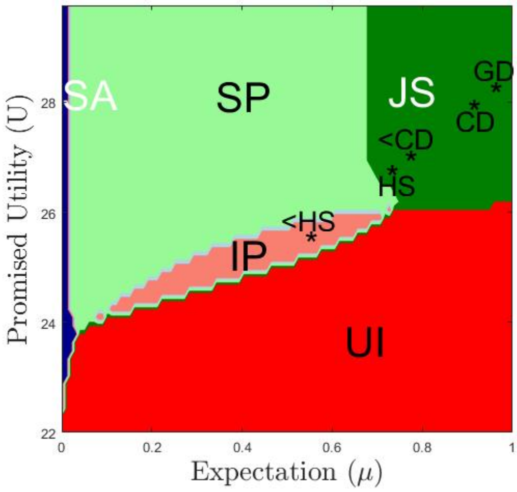


(c) $e^w = e = 0.27$

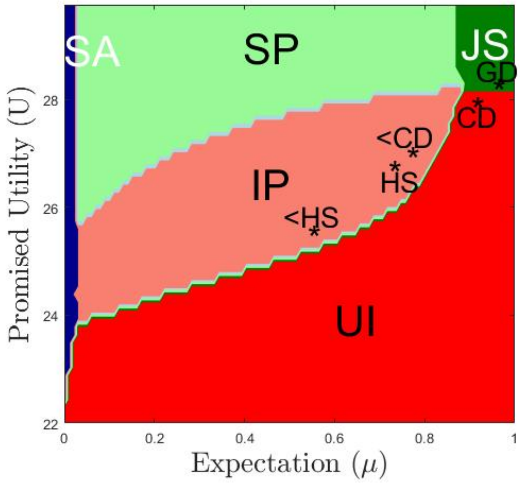
Figure 2.12: Optimal Policies over the (μ, U) Space for Different Effort Costs

past estimates of administrative costs of assisted search. For example, Pavoni et al. (2013) compute an average cost of \$150 per person. Fig. 2.13a and 2.13b report the state space of policies for $\kappa^{ja} = \$387$ (-10%) and $\kappa^{ja} = \$473$ (+10%). When assisting workers is less costly, JS and SP are optimal also for lower generosities, and the opposite occurs when job-search assistance is more costly and incentivizing workers more convenient also for higher effort-cost compensation.

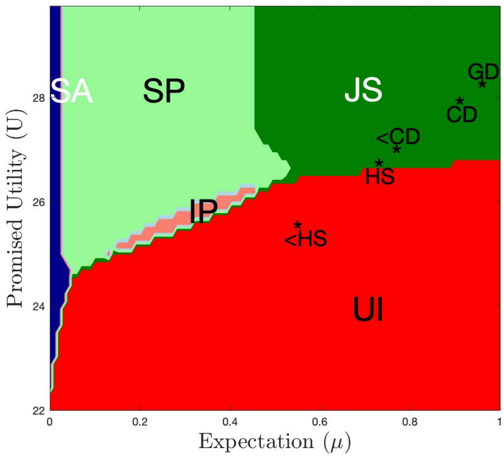
The cost of profiling varies according to the design of the REA program. Given that all possible levels of accuracy are allowed, I select the largest cost among all four States, i.e. $\kappa^{wp} = \$134$ (Illinois). As a consequence, the areas of SP and IP shrink in favor of the respective welfare policies, JS and UI (see Fig. 2.13c).



(a) $\kappa^{ja} = \$387$



(b) $\kappa^{ja} = \$473$



(c) $\kappa^{wp} = \$134$

Figure 2.13: Optimal Policy Space for Different Costs of Assisted Search and Worker Profiling

2.8 Private Search and Moral Hazard

The government may be unable to observe worker's actions. In particular, worker's search may be private and thus unobservable to it. This may create a misalignment of expectations between the two parties, whenever the agent shirks effort and derives no new information about her permanence in unemployment, while the principal assumes that this was the result of a failed search and revises its expectation accordingly. The next period's contract provides larger incentives, consistently with the failed search hypothesis. Therefore, as the worker retains a more optimistic expectation than the on-equilibrium one, she also expects larger transfers. As a consequence, with private search the worker derives an additional advantage from shirking job search, other than saving on effort. To contrast it, the planner promises larger transfers upfront in case of re-employment. These high-powered incentives enlarge in the prospective duration of private search, following the increase in the number of possible deviations from recommended effort by the agent, and are driven to zero if the next-period contract does not request private search. Indeed, the agent can benefit from deviation only if a dispersion in promised transfers exists in the two alternative scenarios of reemployment and unemployment.

In consequence to the possibility of a covert misalignment of expectations, the planner incurs payment of *learning rents* to induce agent's search. The value of UI and IP thus incorporates such rents. Starting with UI, the problem of the planner when private search ends in the next period is

$$
\begin{aligned}
V_1^{UI}(\mu, U) = \max_{c^{ui}, U^w, U^u} & -c^{ui} + \beta\big[\pi(\mu)W(\mu, U^w) + (1-\pi(\mu))\hat{V}(\mu', U^u)\big] \\
\text{sub:} \quad & \hat{V}(\mu, U) := \max_{i \in \{SA, JS, SP, AP\}} V^i(\mu, U) \\
& U = u(c^{ui}) - e + \beta\big[\pi(\mu)U^w + (1-\pi(\mu))U^u\big] \qquad \text{(PK)} \\
& U \geq u(c^{ui}) + \beta U^u \qquad \text{(IC)}
\end{aligned}
$$

The only difference with the non-contractible effort case is a restriction on the basket of policies to choose among in the next period, in order to be consistent with the current provision of learning rents. The dispersion in continuation utilities is the same as in (2.18).

Passing to the case of longer UI duration, define $T(\mu, U)$ as the duration of UI, for any worker with initial expectation μ and utility U

$$
T(\mu, U) := \inf\big\{n : \, i(\mu_n, U_n) \neq UI\big\} \tag{2.26}
$$

where $\mu_n := \mu^{(n)}(\mu)$ (resp., $U_n := U_n(U)$; resp., $i(\mu_n, U_n)$) is defined as the expectation (resp., continuation utility; resp., policy) after failing the job search for n periods. Next step is to design a contract which is robust to any possible deviation from $t = 0$ (today) to $t = T$ periods ahead. The worker could deviate in the first, second, ... T-th period after being assigned to UI. But she may also decide to shirk multiple times, possibly not successive, before reverting to job search, or even shirk forever after. For this reason, the design of a robust contract is in principle a complicated task. The following holds.

Proposition 21. *Any contract incentivizing search effort for T periods is robust against any possible deviation from the sequence of efforts, whenever it is robust against one-shot* *deviations from that sequence.*[49] *Therefore, IC constraint when implementing UI for T periods reads*

$$U \geq u(c^{ui}) + \beta\big[U^u + \Lambda(T,\mu)\big]$$

with $\Lambda(T,\mu)$ defined by the recursion

$$\begin{cases} \Lambda(1,\mu^{T-1}) = 0 \\ \Lambda(T-j,\mu^j) = \Big(\frac{\pi(\mu^{(j)})}{\pi(\mu^{(j+1)})} - 1\Big)e + \beta\pi(\mu^{(j)})\Big(\frac{1}{\pi(\mu^{(j+1)})} - 1\Big)\Lambda(T-j-1,\mu^{(j+1)}), \quad 0 \leq j \leq T-2 \end{cases} \tag{2.27}$$

which thus is (i) independent of U^u, (ii) null in $\mu \in \{0,1\}$ and/or $T = 1$, and (iii) increasing in T.

Proof. See Appendix D: Moral Hazard. ■

The dispersion in utilities between re-employment and unemployment now reads

$$U^w - U^u = \frac{e + \beta\Lambda(T,\mu)}{\beta\pi(\mu)}$$

The gap between U^w and U^u, which proxies the cost of incentives, is increasing in Λ. If UI is implemented for $t < T$ periods, then learning rents are lower and the planner incurs a lower contract cost.

Insurance-and-Profiling (IP). When profiling is adopted jointly with private search,

[49] This result is reminiscent of the way Euler equations are derived. Indeed, Euler equations are conditions imposed on the path of controls (consumption, investment, etc.), which guarantee that the decision maker is never willing to select any different path lying in the feasibility set and differing from the optimal one in one period only. The same property holds for Nash equilibrium strategies in repeated games, which are so if robust to deviations at any single node of the game tree.

the planner's problem reads

$$
\begin{aligned}
V^{IP}(\mu,U) = \max_{c^{ip},U^w,(U^u_r)_{r=\{p,f\}},\mu_p} & -c^{ip} - \kappa^{wp} + \beta\big[\pi(\mu)W(\mu,U^w) + q(1-\pi(\mu_p))V(\mu_p',U^u_p) + \\
& + (1-q)(1-\pi_L)V(0,U^u_f)\big] \\
\text{sub:} \quad & U = u(c^{ip}) - e + \beta\big[\pi(\mu)U^w + q(1-\pi(\mu_p))U^u_p + (1-q)(1-\pi_L)U^u_f\big] \\
& U \geq u(c^{ip}) + \beta\big[q(\hat{\Lambda}^{i(\mu_p,U^u_p)}(\mu_p) + U^u_p) + (1-q)(\hat{\Lambda}^{i(0,U^u_f)}(0) + U^u_f)\big] \\
\text{with:} \quad & \hat{\Lambda}^i(\mu) = \begin{cases} \Lambda(t,\mu), & \text{if } \; i = (UI,t) \\ 0, & \text{otherwise} \end{cases} , \qquad \text{(MP)}
\end{aligned}
$$

Any worker can be profiled at any stage of the unemployment spell, possibly multiple times. If IP is designed to fully reveal the underlying state, the worker is certain to be high-skilled after receiving a 'Pass' and low-skilled otherwise. Hence, she does not revise her expectation henceforth, even if she fails to exit unemployment in the next stages of the welfare program. In other words, the policy she is assigned to under either profiling outcome is absorbing. In case profiling is not designed as a perfect signal, instead, the worker who passes it and is referred to any active policy can downward revise her expectation and reenter into IP at a later stage (unless she escapes unemployment in the meantime).

Proposition 22. *When worker's search is private, accuracy of profiling under IP is determined by the need to reduce learning rents, as the 'Pass' posterior is either 1 or solves*

$$
\begin{aligned}
V_\mu(\mu_p',U^u_p) = & \frac{V(\mu_p',U^u_p) - V^{SA}(U^u_f) + V^{SA}_U(U^u_f)(U^u_f - U^u_p)}{\mu_p'} + \\
& + \big[V^{SA}_U(U^u_f) - W_U(\mu,U^w)\big]\frac{\mu_p\Lambda_\mu(t,\mu_p) - \Lambda(t,\mu_p)}{\mu_p'}
\end{aligned} \tag{2.28}
$$

Proof. See Appendix D: Moral Hazard. ■

The result sheds light on the complementarity between profiling and private search. Indeed, in the case of AP and SP, the government selected the upper posterior more (i.e., 'Pass') by equating the marginal gain of higher informativeness and the marginal cost of lower frequency, which led to the possibility of recommending search also to also a fraction of low-skilled workers. Now, a further component, namely the reduction of learning rents, drives the choice of the upper posterior, in addition to incentive cost reduction.

2.9 Conclusions

This paper provides an estimate of the welfare gains that can be obtained in programs of unemployment assistance via profiling of recipients. The rationale for embedding profiling into a welfare program stems from the difficulty of inferring recipients' job-finding skills and on-the-job productivity. At the optimum, active labor-market policies and workers' expectations about personal skills and productivity are complementary. Workers who are likely to be low-skilled are thus provided income support only, while those who have moderate or high expectations of being high-skilled are supplied with job-search assistance or search incentives, which come in the form of lower wage taxes or higher wage subsidies. Looking at the dimension of program's generosity, instead, search-incentivizing policies are adopted for low-end generosity, while search-assistance ones are adopted for high-end generosity, due to increasing costs of effort compensation. This causes the dynamic of worker's utility which is implicit in the stream of payments to be decreasing along the spell whenever incentives to worker's search are provided.

The effects of implementing worker profiling within the program divide into gains and losses. The gains from workers' profiling stem from incentive alignment between workers and the government. Indeed, rather than pooling into the same policy and contract both high- and low-skilled workers with equal expectations, profiling allows to refer them to the proper job-search method so to minimize the cost of the program. However, worker profiling entails also a loss for the government on those workers whose expectations are positively biased before being profiled as low-skilled. This loss may be conducive to partial detection of hidden skills aimed at strategic persuasion of (a fraction of) low-skilled workers in the sense of Kamenica and Gentzkow (2011). The deep reason at the base of partial detection of skills is that low-skilled workers are sufficiently productive and the generosity of the program sufficiently low that the government would rather compensate them for searching than putting them at rest. Therefore, some low-skilled workers might better receive a boost of expectations and keep searching for a job, rather than staying inactive, even at the cost of higher incentive payments to high-skilled workers.

The profiling outcome is matched by a fine-tuning of payments, whenever followed by search-incentivizing policies. In particular, an optimal program promises lower transfers to recipients who are profiled as high-skilled and required to search for job, due to agency costs increasing in the level of promised utility. This result, which features actual REA programs, should be accompanied with a decreasing-in-time pattern of unemployment benefits, as opposed to the constant subsidy under SNAP.

Some questions remain unanswered. The main shortcoming of the paper is constituted by the assumption on costs and accuracy of profiling. The actual per-capita cost of profiling depends on the accuracy with which skills are detected. A more accurate detection, indeed, leads to a more expensive profiling process (e.g., longer in-person interviews, more elaborate tasks to perform). In addition, any actual profiling program, as well as any sort of tests aimed at detecting a hidden characteristic, contains a given amount of noise that impedes an exact detection of skills. Assuming (i) the cost of profiling to vary in accordance to the change induced on the initial expectation (i.e., known as *entropy cost* in the information design literature),[50] and/or (ii) the accuracy of information detection to be upper bounded, might lead to different estimates of the value of worker profiling. In this sense, the welfare gain computed in Section 2.7 can be read as an upper bound on the return from adopting optimal profiling.

A second aspect on which further inspection is required is the absence of any information asymmetry. Both parties are assumed to share the same initial expectation, as claimants truthfully report their personal data to the provider at the beginning of the program. However, if claimants could conceal their personal information, those with high expectations in reemployment would anticipate being requested to search and choose to misreport it,[51] so as to benefit from larger incentives. If this is the case, the government would need to make search-incentivizing contracts robust to information misreporting and this would further exacerbate the problem of incentive provision. In this sense, as noticed in the case of private worker search (Section 2.8), profiling might constitute a way for the government to curb the information rents that originate from the agency problem.

[50] For a description of costs of information detection that allow the concavification result to survive, see Kamenica and Gentzkow (2014).

[51] In no other case they would find convenient to lie, as all contracts other than incentive-providing ones are independent of expectations.

APPENDIX

Appendix A: Properties of SA, JS and UI

Proof of Prop. 13

Proof. Envelope Theorem and first-order conditions imply

$$V_U^{SA}(\mu, U) = -\frac{1}{u'(c_{sa})} = \mathbf{V}_U(\mu, U^u)$$

Now, given that SA is optimal in (μ, U), then $\mathbf{V}_U(\mu, U) = V_U^{SA}(\mu, U) = \mathbf{V}_U(\mu, U^u)$, and concavity of $\mathbf{V}$ in U implies that $U^u = U$. Therefore, the state space (μ, U) is equal in the next period, proving that SA is optimal forever after.

■

Proof of Lemma 3

Proof. The problem of policy $i \in \{SA, JS, UI\}$ reads

$$V^i(\mu, U) = \max_{(z, U^w, U^u) \in \Gamma(\mu, U)} -g(z) - \kappa^i + \beta\left[p^i(\mu) W(\mu, U^w) + (1 - p^i(\mu))\mathbf{V}(\mu^i, U^u)\right]$$
$$\text{sub:} \quad \Gamma^i(\mu, U) = \left\{(z, U^w, U^u) : \; U = z - e^i + \beta\left[p^i(\mu)U^w + (1 - p^i(\mu))U^u\right], \; U \geq z + \beta U^u\right\}$$

with $p^{SA}(\mu) = 0, \; p^{JS}(\mu) = p^{UI}(\mu) = \pi(\mu)$ and

$$(e^i, \kappa^i) = \begin{cases} (0,0) & \text{if } i = SA \\ (0, \kappa^{ja}) & \text{if } i = JS \\ (e, 0) & \text{if } i = UI \end{cases}$$

The following holds.

Lemma 4. *V^i is decreasing in U and increasing in μ. Moreover, if $\mathbf{V}$ is concave in either argument, then so is V^i.*

Proof. To prove concavity of V^i in U/μ, it suffices to show that:

- the objective function is concave in the choice variables and U/μ;

- the graph of the feasibility set is convex.

Simply notice that $g = u^{-1}$ is convex, and that W and V are concave in U^w and U^u, respectively. Moreover, while $\pi(\mu)W(\mu, U)$ is linear in μ, $(1-\pi(\mu))\mathbf{V}(\mu', U)$ is concave in μ if $\mathbf{V}$ is concave in the first argument, as

$$-2(\pi_H - \pi_L)\frac{\partial \mu'}{\partial \mu}V_\mu(\mu', U) + (1-\pi(\mu))\Big[\frac{\partial^2 \mu'}{\partial \mu^2}V_\mu(\mu', U) + \frac{\partial \mu'}{\partial \mu}V_{\mu\mu}(\mu', U)\Big] < 0$$

as $(1-\pi(\mu))\frac{\partial^2 \mu'}{\partial \mu^2} = 2(\pi_H - \pi_L)\frac{\partial \mu'}{\partial \mu}$.
Furthermore, PK constraint is linear in U, z, U^w and U^u, and so is IC constraint, since U^i is linear in U. This means that the graph of Γ^i_μ (i.e., for constant μ) defined as

$$Gr\Gamma^i_\mu = \big\{(z, U^w, U^u, U) : \; U = z - e^i + \beta\big[p^i(\mu)U^w + (1-p^i(\mu))U^u\big], \; U \geq z + \beta U^u\big\}$$

is convex. Same applies to the graph of Γ^i_U (i.e., for constant U), since PK and IC are linear in μ.

To prove (negative) monotonicity in U, one needs to show:

- (negative) monotonicity of the objective function in U;
- (negative) monotonicity of the feasibility set Γ^i_μ in U, i.e.

$$U < \tilde{U} \Longrightarrow \Gamma^i(\mu, \tilde{U}) \subseteq \Gamma^i(\mu, U)$$

The objective function does not directly depend on U, while monotonicity can be shown by rewriting the IC constraint as

$$U^w - U^u \geq \frac{e^i}{\beta p^i(\mu)}$$

which does not depend on U. Therefore, the PK constraint is tightened by an increase of U, which thus leads to a shrinkage of Γ^i_μ.

Proving (positive) monotonicity of V^i in μ is analogous. Indeed, it follows from:

- (positive) monotonicity of the objective function in μ;
- (positive) monotonicity of the feasibility set Γ^i_U in μ, i.e.

$$\mu < \tilde{\mu} \Longrightarrow \Gamma^i(\mu, U) \subseteq \Gamma^i(\tilde{\mu}, U)$$

The objective function is always monotone in μ, as so are W and V in their first argument, μ' is an increasing function of μ and $W(\mu, U^w) \geq V(\mu^i, U^u)$. Monotonicity of $\Gamma^i(., U)$, instead, holds as an increase of μ leads to a relaxation of (IC)[52]. Indeed, (IC) is more slack since

$$\mu < \tilde{\mu} \Longrightarrow U^w - U^u \geq \frac{e^i}{\beta p^i(\mu)} \geq \frac{e^i}{\beta p^i(\tilde{\mu})}$$

■

V^i and V concave in U

The proof of concavity of V^i in U follows a recursive argument. Guessing concavity of $\mathbf{V}$ in U, it holds that V^i is concave by Lemma 4 and that, from definition (2.14) of $\mathbf{V}$,

$$\mathbf{V}_{UU}(\mu, U) = V^i_{UU}(\mu, U)$$

when no randomization over U is conducted. Otherwise, $\mathbf{V}_{UU}(\mu, U) = 0$.

V^i and V concave in μ

The proof of concavity in μ follows the same steps, as assuming concavity of $\mathbf{V}$ in μ leads to concavity of V^i. However, the second derivative of $\mathbf{V}$ in μ reads

$$\mathbf{V}_{\mu\mu}(\mu, U) = qV^i_{\mu\mu}(\mu, \overline{U}) + (1-q)V^j_{\mu\mu}(\mu, \underline{U}) < 0, \quad \text{with: } q = \frac{U - \underline{U}}{\overline{U} - \underline{U}}$$

V^{UI} and V^{JS} supermodular

The derivative of V^{UI} and V^{JS} wrt U reads

$$\begin{aligned} V^{UI}_U(\mu, U) &= -\frac{1}{u(c_{UI})} = \pi(\mu)W_U(\mu, U^w_{UI}) + (1-\pi(\mu))\mathbf{V}_U(\mu', U^u_{UI}) \\ V^{JS}_U(\mu, U) &= -\frac{1}{u(c_{JS})} = W_U(\mu, U^w_{JS}) = \mathbf{V}_U(\mu', U^u_{JS}) \end{aligned}$$

Thus

$$\begin{aligned} V^{UI}_{\mu U}(\mu, U) = &(\pi_H - \pi_L)(W_U(\mu, U^w_{UI}) - \mathbf{V}_U(\mu', U^u_{UI})) + \pi(\mu)W_{UU}(\mu, U^w_{UI})\frac{\partial U^w_{UI}}{\partial \mu} + \\ &+ (1-\pi(\mu))\mathbf{V}_{UU}(\mu', U^u_{UI})\frac{\partial U^u_{UI}}{\partial \mu} + (1-\pi(\mu))\frac{\partial \mu'}{\partial \mu}\mathbf{V}_{\mu U}(\mu', U^u_{UI}) \\ &= (\pi_H - \pi_L)(W_U(\mu, U^w_{UI}) - \mathbf{V}_U(\mu', U^u_{UI}) + W_{UU}(\mu, U^w_{UI})(U^u_{UI} - U^w_{UI})) + \\ &+ \frac{\partial U^u_{UI}}{\partial \mu}\big(\pi(\mu)W_{UU}(\mu, U^w_{UI}) + (1-\pi(\mu))\mathbf{V}_{UU}(\mu', U^u_{UI})\big) + (1-\pi(\mu))\frac{\partial \mu'}{\partial \mu}\mathbf{V}_{\mu U}(\mu', U^u_{UI}) \end{aligned}$$

[52] (PK) is always relaxed by an increase in μ (recall that $U^w \geq U^u$ at the optimum).

Convexity of $1/u'$ implies concavity of W_U, which boils down to

$$W_U(\mu, U^w_{UI}) + W_{UU}(\mu, U^w_{UI})(U^u_{UI} - U^w_{UI}) > W_U(\mu, U^u_{UI}) = W_U(\mu', U^u_{UI}) \geq \mathbf{V}_U(\mu', U^u_{UI})$$

Assume *per contra* that $V^{UI}_{\mu U}(\mu, U) \leq 0$. Then, it must be that $\partial U^u_{UI}/\partial\mu > 0$, which in turn implies that $\partial c_{UI}/\partial\mu < 0$, as $u(c_{UI}) = U - \beta U^u_{UI}$. But this leads to a contradiction as

$$V^{UI}_{\mu U}(\mu, U) = -\frac{\partial}{\partial c_{UI}}\Big(\frac{1}{u'(c_{UI})}\Big)\frac{\partial c_{UI}}{\partial \mu} > 0$$

Passing to JS,

$$V^{JS}_{\mu U}(\mu, U) = W_{UU}(\mu, U^w_{JS})\frac{\partial U^w_{JS}}{\partial \mu} = \mathbf{V}_{\mu U}(\mu', U^u_{JS})\frac{\partial \mu'}{\partial \mu} + \mathbf{V}_{UU}(\mu', U^u_{JS})\frac{\partial U^u_{JS}}{\partial \mu}$$

Per contra, assume that $V^{JS}_{\mu U}(\mu, U) < 0$. Then, it must be that $\partial U^s_{JS}/\partial\mu > 0,\ s \in \{w, u\}$. However, the PK-JS constraint reads

$$U = (1 - \beta + \pi(\mu))U^w_{JS} + \beta(1 - \pi(\mu))U^u_{JS}$$

And so

$$\frac{\partial U^w_{JS}}{\partial \mu} = -\frac{\beta}{1 - \beta + \beta\pi(\mu)}\Big[(\pi_H - \pi_L)(U^w_{JS} - U^u_{JS}) + (1 - \pi(\mu))\frac{\partial U^u_{JS}}{\partial \mu}\Big] < 0$$

where the inequality follows from assumption $U^w_{JS} \geq U^u_{JS}$. Hence, I have reached a contradiction.

$V^{JS}_{\mu\mu}(\mu, U) = 0$ **and** $V^{JS}_U(\mu, U) = V^{SA}_U(U)$ **with no JS** $\to$ **UI transition**

The period before entering SA, the FOC condition is

$$V^{JS}_U(\mu, U) = -\frac{1}{u'(c_{JS})} = -\frac{1}{u'(c_{SA})} = V^{SA}_U(U^u_{JS}) \Longrightarrow U^u_{JS} = (1 - \beta)u(c_{JS}) = U^w_{JS} = U$$

The value of JS wrt μ after imposing $U^u_{JS} = U^w_{JS} = U$ reads

$$V^{JS}(\mu, U) = -u^{-1}((1 - \beta)U) - \kappa^{ja} + \beta\big[\pi(\mu)W(\mu, U) + (1 - \pi(\mu))\mathbf{V}(\mu', U)\big]$$

Therefore, if $\mathbf{V}$ is linear in the first argument, so is V^{JS}, given linearity of $\pi(\mu)W(\mu, U)$ in μ. But then the proof follows from a recursive argument and linearity of V^{SA} in μ.

■

Proof of Prop. 14

Proof. The derivative of the value of each policy i with respect to U is

$$V^i_U(\mu, U) = \mathbf{V}_U(\mu, U) = -\frac{1}{u'(c_i)} \tag{2.29}$$

which can be obtained by applying the envelope theorem to the problem of each policy. Third, first-order conditions in UI and JS impose

$$\mathbf{V}_U(\mu', U^u_{UI}) - V^{UI}_U(\mu, U) = \mathbf{V}_U(\mu', U^u_{UI}) + \lambda^{UI} - \chi^{UI} = \frac{\pi(\mu)}{1-\pi(\mu)}\chi^{UI} > 0 \tag{2.30}$$

$$V^{UI}_U(\mu, U_{UI}) = -(\lambda^{UI} - \chi^{UI}) = W_U(\mu, U^w_{UI}) + \chi^{UI} > W_U(\mu, U^w_{UI}) \tag{2.31}$$

$$V^{JS}_U(\mu, U) = -\lambda^{JS} = W_U(\mu, U^w_{JS}) = \mathbf{V}_U(\mu', U^u_{JS}) \tag{2.32}$$

where λ_i (resp., χ_i) is the Lagrange multiplier associated to (PK) (resp., (IC)) constraint. Hence,

$$\mathbf{V}_U(\mu', U^u_{UI}) > V^{UI}_U(\mu, U) > \mathbf{V}_U(\mu', U) \Longrightarrow U^u_{UI} < U \tag{2.33}$$

where the first inequality holds by FOC and the second by supermodularity of $\mathbf{V}$. Similarly, in JS consumption is constant over time and employment states by (2.32), which implies that

$$u(c_{JS}) = u(c^w_{JS}) = (1-\beta)U^w_{JS} \Longrightarrow U = \big(1 - \beta + \beta\pi(\mu)\big)U^w_{JS} + \beta(1-\pi(\mu))U^u_{JS}$$

and

$$\mathbf{V}_U(\mu, U) = V^{JS}_U(\mu, U) = \mathbf{V}_U(\mu', U^u_{JS}) \leq \mathbf{V}_U(\mu, U^u_{JS}) \tag{2.34}$$

where the inequality holds since $\mathbf{V}$ is supermodular, the first equality as JS is optimal in (μ, U) and the second equality from FOC (2.32). Thus, by concavity of $\mathbf{V}$ in U, it holds that $U^u_{JS} \leq U \leq U^w_{JS}$. Supermodularity of JS follows from Lemma 3.

Optimal Policies in the U Space

The proof of the first part of the statement consists of showing that at the crossing point

$$V^{UI}_U(\mu, U) \leq V^{JS}_U(\mu, U) \leq V^{SA}_U(U) = W_U(\mu, U) \tag{2.35}$$

First, the closed-form expressions of W and V^{SA} deliver

$$W_U(\mu, U) = -\frac{1}{u'(u^{-1}((1-\beta)U))} = V_U^{SA}(U)$$

Then, by (2.32),

$$V_U^{JS}(\mu, U) = W_U(\mu, U_{JS}^w) \leq W_U(\mu, U) = V_U^{SA}(U)$$

where the inequality follows from $U \leq U_{JS}^w$.

By (2.33), we know that $U_{UI}^u < U$. Hence,

$$u(c_{UI}) = U - \beta U_{UI}^u > (1-\beta)U \Longrightarrow V_U^{UI}(\mu, U) < -\frac{1}{u'(u^{-1}((1-\beta)U))} = V_U^{SA}(U)$$

If JS refers to SA, then

$$u(c_{JS}) = u(c_{JS}^u) = u(c_{SA}) = (1-\beta)U_{JS}^u$$

So $U_{JS}^w = U_{JS}^u = U$ and

$$V_U^{JS}(\mu, U) = -\frac{1}{u'(u^{-1}((1-\beta)U))} = V_U^{SA}(U)$$

So far, I have shown that if JS is followed by SA, then

$$V_U^{UI}(\mu, U) \leq -\frac{1}{u'(u^{-1}((1-\beta)U))} = V_U^{SA}(U) = V_U^{JS}(\mu, U)$$

What is left to show is that $V_U^{UI}(\mu, U) \leq V_U^{JS}(\mu, U)$, even if JS does not refer to SA in (μ, U). *Per contra*, assume that $V_U^{UI}(\mu, U) > V_U^{JS}(\mu, U)$. First, it must be that the inequality

$$-\frac{1}{u'(u^{-1}((1-\beta)U))} \geq V_U^{UI}(\mu, U) > V_U^{JS}(\mu, U)$$

holds and implies that $(1-\beta)U \leq u(c_{UI}) < u(c_{JS})$. And from (2.34), it must be that $U_{JS}^u < U < \frac{u(c_{JS})}{1-\beta}$.
Now, by FOCs (2.30) and (2.32), it holds that

$$\mathbf{V}_U(\mu', U_{UI}^u) > V_U^{UI}(\mu, U) > V_U^{JS}(\mu, U) = \mathbf{V}_U(\mu', U_{JS}^u)$$

By concavity of $\mathbf{V}$ in U, it must hold that $U^u_{UI} < U^u_{JS}$. But this is impossible as

$$u(c_{UI}) + \beta U^u_{UI} = U = u(c_{JS}) + \beta U^u_{JS} + \beta\pi(\mu)\Big[\frac{u(c_{JS})}{1-\beta} - U^u_{JS}\Big] > u(c_{UI}) + \beta U^u_{UI}$$

where the inequality follows from $c_{JS} > c_{UI}$ and $\frac{u(c_{JS})}{1-\beta} > U^u_{JS}$.
Therefore, it has been shown that $V^{UI}_U(\mu, U) \leq V^{JS}_U(\mu, U)$. Hence, V^{UI} dominates V^{JS} for low-end generosity levels and crosses it from above.

Optimal Policies in the μ Space

Passing to the second part of the statement, it is enough to prove that at the crossing point

$$0 = V^{SA}_\mu(U) < V^{JS}_\mu(\mu, U) < V^{UI}_\mu(\mu, U)$$

The derivatives of V^{JS} and V^{UI} wrt to μ

$$\begin{aligned} V^{JS}_\mu(\mu, U) =& \beta(\pi_H - \pi_L)\big[W(\mu, U^w_{JS}) - \mathbf{V}(\mu', U^u_{JS}) - \lambda^{JS}(U^u_{JS} - U^w_{JS})\big] + \\ & + \beta\Big[\pi(\mu)W_\mu(\mu, U^w_{JS}) + (1-\pi(\mu))\frac{\partial\mu'}{\partial\mu}\mathbf{V}_\mu(\mu', U^u_{JS})\Big] \qquad (2.36) \\ V^{UI}_\mu(\mu, U) =& \beta(\pi_H - \pi_L)\big[W(\mu, U^w_{UI}) - \mathbf{V}(\mu', U^u_{UI}) - \lambda^{UI}(U^u_{UI} - U^w_{UI})\big] + \\ & + \beta\Big[\pi(\mu)W_\mu(\mu, U^w_{UI}) + (1-\pi(\mu))\frac{\partial\mu'}{\partial\mu}\mathbf{V}_\mu(\mu', U^u_{UI})\Big] \qquad (2.37) \end{aligned}$$

Consider JS being implemented in the current period. Using FOC $-\lambda^{JS} = W_U(\mu, U^w_{JS})$, it holds that

$$W(\mu, U^w_{JS}) + W_U(\mu, U^w_{JS})(U^u_{JS} - U^w_{JS}) - \mathbf{V}(\mu', U^u_{JS}) > W(\mu, U^u_{JS}) - \mathbf{V}(\mu', U^u_{JS}) > 0 = V^{SA}_\mu(U)$$

where the first inequality follows from concavity of W in U and the second is a necessary condition for optimality of JS.
Consider a program that implements UI with the additional constraint that $U^u_{UI} \geq U^u_{JS}$, and label its value $\hat{V}^{UI}$. Moreover,

$$W_U(\mu, U^w_{JS}) = V^{JS}_U(\mu, U) \geq V^{UI}_U(\mu, U) > W_U(\mu, U^w_{UI}) \Longrightarrow U^w_{JS} < U^w_{UI}$$

where the first inequality follows from the statement proved above and the second one from FOC of UI.[53] Hence, derivatives (2.36) and (2.37) can be rewritten

$$
\begin{aligned}
&V^{JS}_{\mu}(\mu,U) = \\
&\quad = \beta(\pi_H - \pi_L)\big[W(\mu,U^w_{JS}) - \mathbf{V}(\mu',U^u_{JS}) + W_U(\mu,U^w_{JS})(U^u_{UI} - U^w_{JS}) + \mathbf{V}_U(\mu',U^u_{JS})(U^u_{JS} - U^u_{UI})\big] + \\
&\quad + \beta\Big[\pi(\mu)W_\mu(\mu,U^w_{JS}) + (1-\pi(\mu))\frac{\partial\mu'}{\partial\mu}\mathbf{V}_\mu(\mu',U^u_{JS})\Big] \\
&\hat{V}^{UI}_{\mu}(\mu,U) = \\
&\quad = \beta(\pi_H - \pi_L)\big[W(\mu,U^w_{UI}) - \mathbf{V}(\mu',U^u_{UI}) + W_U(\mu,U^w_{UI})(U^w_{JS} - U^w_{UI}) + W_U(\mu,U^w_{UI})(U^u_{UI} - U^w_{JS})\big] + \\
&\quad + \beta\Big[\pi(\mu)W_\mu(\mu,U^w_{UI}) + (1-\pi(\mu))\frac{\partial\mu'}{\partial\mu}\mathbf{V}_\mu(\mu',U^u_{UI})\Big]
\end{aligned}
$$

In order to prove the result, it is enough to show that

$$
\begin{aligned}
&W(\mu,U^w_{JS}) + W_U(\mu,U^w_{JS})(U^u_{UI} - U^w_{JS}) - \mathbf{V}(\mu',U^u_{JS}) + \mathbf{V}_U(\mu',U^u_{JS})(U^u_{JS} - U^u_{UI}) < \\
&< W(\mu,U^w_{UI}) + W_U(\mu,U^w_{UI})(U^w_{JS} - U^w_{UI}) + W_U(\mu,U^w_{UI})(U^u_{UI} - U^w_{JS}) - \mathbf{V}(\mu',U^u_{UI})
\end{aligned}
$$

and

$$
\pi(\mu)W_\mu(\mu,U^w_{JS}) + (1-\pi(\mu))\frac{\partial\mu'}{\partial\mu}\mathbf{V}_\mu(\mu',U^u_{JS}) < \pi(\mu)W_\mu(\mu,U^w_{UI}) + (1-\pi(\mu))\frac{\partial\mu'}{\partial\mu}\mathbf{V}_\mu(\mu',U^u_{UI})
$$

The first inequality holds since:

- $W(\mu,U^w_{JS}) < W(\mu,U^w_{UI}) + W_U(\mu,U^w_{UI})(U^w_{JS} - U^w_{UI})$, by concavity of W in U;
- $W_U(\mu,U^w_{JS})(U^u_{UI} - U^w_{JS}) < W_U(\mu,U^w_{UI})(U^u_{UI} - U^w_{JS})$, as $U^u_{UI} < U \leq U^w_{JS} < U^w_{UI}$;
- $\mathbf{V}(\mu',U^u_{UI}) \leq \mathbf{V}(\mu',U^u_{JS}) + \mathbf{V}_U(\mu',U^u_{JS})(U^u_{UI} - U^u_{JS})$, by concavity of $\mathbf{V}$ in U.

The second inequality holds since $W_{\mu U} = 0$ and $\mathbf{V}_\mu(\mu',U^u_{JS}) \leq \mathbf{V}_\mu(\mu',U^u_{UI})$, by assumption $U^u_{JS} \leq U^u_{UI}$ and supermodularity of $\mathbf{V}$. Therefore, it has been shown that $\hat{V}^{UI}$ crosses V^{JS} from below in the μ space, and so does V^{UI}, which implies that UI dominates JS for high expectations. ■

[53] The additional constraint preserves the FOC $V^{UI}(\mu,U) > W_U(\mu,U^w_{UI})$.

Proof of Prop. 15

Unemployment Benefits

Thus, unemployment benefits fall over time during UI and stay constant in JS, as

$$\mathbf{V}_U(\mu', U^u_{UI}) > V^{UI}_U(\mu, U) \implies c^u_{UI} < c_{UI}$$
$$V^{UI}_U(\mu, U_{UI}) > W_U(\mu, U^w_{UI}) \implies c_{UI} < c^w_{UI}$$
$$V^{JS}_U(\mu, U) = W_U(\mu, U^w_{JS}) = \mathbf{V}_U(\mu', U^u_{JS}) \implies c_{JS} = c^w_{JS} = c^u_{JS}$$

where the implications follow from (2.29).

Continuation Utility

If JS never refers to UI, then one can start computing backward from the point in time where JS refers to SA. Hence, $V^{JS}_U(\mu, U) = V^{SA}_U(U)$. Therefore $U^w_{JS} = U^u_{JS} = U$ and $\mathbf{V}_{\mu U}(\mu, U) = V^{JS}_{\mu U}(\mu, U) = 0$. The last period before the worker enters SA, the contract satisfies

$$V^{JS}_U(\mu, U) = \mathbf{V}_U(\mu', U^u_{JS}) = \mathbf{V}_U(\mu, U^u_{JS}) \implies U^u_{JS} = U$$

The result is shown by induction argument.

Proof of Prop. 16

Assume that UI is the optimal policy in (μ, U). From the first-order condition on UI, it holds that

$$-g'((1-\beta)U + \eta^{UI}(\mu, U)) = V^{UI}_U(\mu, U) = -g'(U - \beta U^u_{UI})$$

From which it follows that

$$U - \frac{\eta^{UI}(\mu, U)}{\beta} = U^u_{UI}$$

Since by assumption $\eta^{UI}_U(\mu, U) \leq 0$, the left-hand side is increasing in U. Thus,

$$U^u_{UI} = U - \frac{\eta^{UI}(\mu, U)}{\beta} \leq \hat{U}(\mu) - \frac{\eta^{UI}(\mu, \hat{U}(\mu))}{\beta} \leq \hat{U}(\mu')$$

The condition guarantees that anytime UI is adopted in (μ, U), then the next-period state in case of job search failure becomes (μ', U^u_{UI}) where it is still optimal to implement UI (or switch to SA) as $U^u_{UI} \leq \hat{U}(\mu')$. To conclude, the program never switches from UI to JS.

Appendix B: Properties of AP, SP and IP

Proof of Prop. 18

Proof. Consider the two first-order conditions of the AP problem

$$\mathbf{V}_\mu(\mu_p, U^u_{AP}) - \frac{\mathbf{V}(\mu_p, U^u_{AP}) - \mathbf{V}(0, U^u_{AP})}{\mu_p} = 0$$
$$q\mathbf{V}_U(\mu_p, U^u_{AP}) + (1-q)\mathbf{V}_U(0, U^u_{AP}) + \lambda^{AP} = 0$$

For the pair (μ_p, U^u_{AP}) to be a point of maximum, it must be that the Hessian matrix H of second-order derivatives has positive determinant.

$$H = \begin{bmatrix} q\mathbf{V}_{UU}(\mu_p, U^u_{AP}) + (1-q)\mathbf{V}_{UU}(0, U^u_{AP}) & q\Big(\mathbf{V}_{\mu U}(\mu_p, U^u_{AP}) - \frac{V_U(\mu_p, U^u_{AP}) - V_U(0, U^u_{AP})}{\mu_p}\Big) \\ \mathbf{V}_{\mu U}(\mu_p, U^u_{AP}) - \frac{V_U(\mu_p, U^u_{AP}) - V_U(0, U^u_{AP})}{\mu_p} & \mathbf{V}_{\mu\mu}(\mu_p, U^u_{AP}) \end{bmatrix}$$

Differentiating the two conditions by U yields

$$\mathbf{V}_{\mu\mu}(\mu_p, U^u_{AP})\frac{\partial \mu_p}{\partial U^u_{AP}} + \mathbf{V}_{\mu U}(\mu_p, U^u_{AP}) - \frac{\mathbf{V}_U(\mu_p, U^u_{AP}) - \mathbf{V}_U(0, U^u_{AP})}{\mu_p} = 0$$
$$\Big\{q\Big[\mathbf{V}_{\mu U}(\mu_p, U^u_{AP}) - \frac{\mathbf{V}_U(\mu_p, U^u_{AP}) - \mathbf{V}_U(0, U^u_{AP})}{\mu_p}\Big]\frac{\partial \mu_p}{\partial U^u_{AP}} +$$
$$+ q\mathbf{V}_{UU}(\mu_p, U^u_{AP}) + (1-q)\mathbf{V}_{UU}(0, U^u_{AP})\Big\}\frac{\partial U^u_{AP}}{\partial U} + \lambda^{AP}_U = 0$$

Plugging the expression of $\frac{\partial \mu_p}{\partial U^u_{AP}}$ from the first equation into the second one, the term in curly brackets becomes

$$\Delta := -\frac{q}{\mathbf{V}_{\mu\mu}(\mu_p, U^u_{AP})}\Big[\mathbf{V}_{\mu U}(\mu_p, U^u_{AP}) - \frac{\mathbf{V}_U(\mu_p, U^u_{AP}) - \mathbf{V}_U(0, U^u_{AP})}{\mu_p}\Big]^2 +$$
$$+ q\mathbf{V}_{UU}(\mu_p, U^u_{AP}) + (1-q)\mathbf{V}_{UU}(0, U^u_{AP})$$

with $\Delta < 0$, as $det(H) > 0$ and $\mathbf{V}_{\mu\mu}(\mu_p, U^{u,p}_{AP}) < 0$. Therefore the second equation becomes

$$\frac{\partial U^u_{AP}}{\partial U}\Delta + \lambda^{AP}_U = 0$$

which shows that $\frac{\partial U^u_{AP}}{\partial U} > 0$, since $V^{AP}_{UU}(\mu, U) = -\lambda^{AP}_U < 0$. From the first equation, local supermodularity of $\mathbf{V}$ and $\mathbf{V}_U(\mu_p, U) < \mathbf{V}_U(0, U)$ (see Prop. 14) yield $\frac{\partial \mu_p}{\partial U^u_{AP}} > 0$, which deliver the result.

The proof of μ_p being monotone increasing in U also under SP and IP follows accordingly.

Finally, full accuracy of profiling whenever 'Pass' refers to JS and no incentive is provided along the spell follows from linearity of V^{JS} in μ (see proof of Lemma 4).

■

Appendix C: Statistical Profiling

Proof of Prop. 19 and 20

Proof. $\mu_f = 0$ descends form the linearity of SA and JS in μ. In addition, any worker who receives a 'Pass' (resp., 'Fail') is referred to UI/JS (resp., SA).

Assistance-and-Profiling (AP)

At the optimum,

$$V_U^{AP}(\mu, U) = \mathbf{V}_U(\mu_p, U_{AP}^{u,p}) = \mathbf{V}_U(0, U_{AP}^{u,f}) \tag{2.38}$$

$$-\frac{1}{u'(c_{AP})} = V_U^{AP}(\mu, U) = \mathbf{V}_U(0, U_{AP}^{u,f}) = -\frac{1}{u'(c_{AP}^{u,f})} \tag{2.39}$$

which implies that $c_{AP} = c_{AP}^{u,p} = c_{AP}^{u,f}$, and $U_p^u \leq U \leq U_f^u$. Indeed, by (2.39), it follows

$$U = u(c_{AP}) + \beta(qU_{AP}^{u,p} + (1-q)U_{AP}^{u,f}) = (1-\beta q)U_{AP}^{u,f} + \beta q U_{AP}^{u,p}$$

where the passage follows from $u(c_{AP}) = u(c_{AP}^{u,f}) = (1-\beta)U_{AP}^{u,f}$, and the expression of consumption in SA (see Prop. 13). If referred to JS -which is optimal only for high-end generosities-, then $\mu_p = 1$ given the linearity of JS in μ. Moreover, for U high enough, JS never refers to UI, and so $U_{JS}^w = U = U_{JS}^u$, which in turn implies that $u(c_{JS}) = (1-\beta)U_{JS}^w = (1-\beta)U = u(c_{SA})$ and

$$V_U^{JS}(\mu, U) = -\frac{1}{u'(c_{JS})} = -\frac{1}{u'(c_{SA})} = V_U^{SA}(U)$$

Therefore, if referred to JS/SA forever after, then $U_{AP}^{u,p} = U_{AP}^{u,f} = U_{AP}^u$. So, nothing changes with respect to the case with ND constraint, whenever AP refers workers to SA and JS forever after, that is, for higher generosities.

Assume, instead, AP refers to UI directly, or to JS which later refers to UI. Then $\mathbf{V}_U(\mu, U) < V_U^{SA}(U)$

$$V_U^{SA}(U_{AP}^{u,f}) = V_U(\mu_p, U_{AP}^{u,p}) \Longrightarrow U_{AP}^{u,p} < U < U_{AP}^{u,f}$$

I now show that the 'Pass' posterior μ_p in AP is increasing in U.

$$\mathbf{V}_U(\mu_p, U_{AP}^{u,p}) + \lambda^{AP} = 0$$
$$-\frac{\mathbf{V}(\mu_p, U_{AP}^{u,p}) - \mathbf{V}(0, U_{AP}^{u,f}) + \lambda^{AP}(U_{AP}^{u,p} - U_{AP}^{u,f})}{\mu_p} + \mathbf{V}_\mu(\mu_p, U_{AP}^{u,p}) = 0$$

The Hessian matrix of second-order derivatives reads

$$H = \begin{bmatrix} \mathbf{V}_{UU}(\mu_p, U_{AP}^{u,p}) & \mathbf{V}_{\mu U}(\mu_p, U_{AP}^{u,p}) \\ \mathbf{V}_{\mu U}(\mu_p, U_{AP}^{u,p}) & \mathbf{V}_{\mu\mu}(\mu_p, U_{AP}^{u,p}) \end{bmatrix}$$

$(\mu_p, U_{AP}^{u,p})$ are a point of maximum of the objective function if and only if

$$\mathbf{V}_{UU}(\mu_p, U_{AP}^{u,p}) < 0, \quad det(H) > 0$$

The first condition holds as $\mathbf{V}$ is concave in each argument (see proof of Lemma 4). Differentiating the two FOCs wrt U yields

$$\mathbf{V}_{\mu U}(\mu_p, U_{AP}^{u,p})\frac{\partial \mu_p}{\partial U} + \mathbf{V}_{UU}(\mu_p, U_{AP}^{u,p})\frac{\partial U_{AP}^{u,p}}{\partial U} = -\frac{\partial \lambda^{AP}}{\partial U}$$
$$\mathbf{V}_{\mu\mu}(\mu_p, U_{AP}^{u,p})\frac{\partial \mu_p}{\partial U} + \mathbf{V}_{\mu U}(\mu_p, U_{AP}^{u,p})\frac{\partial U_{AP}^{u,p}}{\partial U} = \frac{U_{AP}^{u,p} - U_{AP}^{u,f}}{\mu_p}\frac{\partial \lambda^{AP}}{\partial U}$$

and solving the system, one obtains

$$\begin{bmatrix} \frac{\partial \mu_p}{\partial U} \\ \frac{\partial U_{AP}^{u,p}}{\partial U} \end{bmatrix} = det(H)^{-1} \begin{bmatrix} \mathbf{V}_{\mu\mu}(\mu_p, U_{AP}^{u,p}) & -\mathbf{V}_{\mu U}(\mu_p, U_{AP}^{u,p}) \\ -\mathbf{V}_{\mu U}(\mu_p, U_{AP}^{u,p}) & \mathbf{V}_{UU}(\mu_p, U_{AP}^{u,p}) \end{bmatrix} \begin{bmatrix} \frac{U_{AP}^{u,p} - U_{AP}^{u,f}}{\mu_p} \\ -1 \end{bmatrix} \frac{\partial \lambda^{AP}}{\partial U}$$

Both derivatives are positive, since $U_{AP}^{u,p} - U_{AP}^{u,f} < 0$ and an increase in U makes it harder for the planner to satisfy (PK) constraint (i.e., $\partial \lambda^{AP}/\partial U > 0$).

Search-assistance-and-Profiling (SP)

At the optimum

$$V_U^{SP}(\mu, U) = W_U(\mu_p, U_{SP}^{w,p}) = W_U(\mu_f, U_{SP}^{w,f}) = \mathbf{V}_U(\mu'_p, U_{SP}^{u,p}) = \mathbf{V}_U(\mu'_f, U_{SP}^{u,f})$$
$$\Longrightarrow c_{SP} = c_{SP}^{w} = c_{SP}^{u,p} = c_{SP}^{u,f}, \quad U_{SP}^{u,p} \leq U \leq U_{SP}^{u,f} = U_{SP}^{w,p} = U_{SP}^{w,f}$$

since

$$\frac{u(c_{SP})}{1-\beta} = \frac{u(c^{w}_{SP})}{1-\beta} = U^{w}_{SP} = \frac{u(c^{u,f}_{SP})}{1-\beta} = U^{u,f}_{SP}$$

where the last equality follows from referral to SA upon 'Fail'. So

$$U = (1-\beta+\beta\pi(\mu)+\beta(1-q)(1-\pi(\mu_f)))U^{u,f}_{SP} + \beta q(1-\pi(\mu_p))U^{u,p}_{SP}$$

and the same argument in AP applies, meaning that the continuation utility upon 'Pass' falls if and only if the outcome refers workers directly or indirectly to UI.

Insurance-and-Profiling (IP)

The optimal IP contract satisfies

$$\begin{aligned}
&\mathbf{V}_U(\mu_p', U^{u,p}_{IP}) - V^{IP}_U(\mu, U) = \frac{\pi(\mu_p)}{1-\pi(\mu_p)}\chi^{IP} > \frac{\pi_L}{1-\pi_L}\chi^{IP} = \mathbf{V}_U(0, U^{u,f}_{IP}) - V^{IP}_U(\mu, U) \Longrightarrow U^{u,p}_{IP} < U^{u,f}_{IP} \\
&\mathbf{V}_U(\mu_p', U^{u,p}_{IP}) > \mathbf{V}_U(0, U^{u,f}_{IP}) > V^{IP}_U(\mu, U) = -(\lambda^{IP} - \chi^{IP}) > W_U(\mu, U^{w}_{IP}) \Longrightarrow c^{u,p}_{IP} < c^{u,f}_{IP} < c_{IP} < c^{w}_{IP}
\end{aligned} \tag{2.40}$$

Moreover, $U^{u,p}_{IP} < U$, as

$$(1-\beta)U^{u,p}_{IP} \leq u(c^{u,p}_{IP}) < u(c_{IP}) = U - \beta[qU^{u,p}_{IP} + (1-q)U^{u,f}_{IP}] < U - \beta U^{u,p}_{IP}$$

where the first inequality follows from (2.35), the second one from (2.40) and the last one from $U^{u,p}_{IP} < U^{u,f}_{IP}$.

Passing to the equation that determines the 'Pass' posterior in IP, the first-order condition of μ_p reads

$$\begin{aligned}
&\frac{1}{\mu_p}\big[(1-\pi_L)\mathbf{V}(0, U^{u,f}_{IP}) - (1-\pi(\mu_p))\mathbf{V}(\mu_p', U^{u,p}_{IP})\big] - \\
&- (\pi_H - \pi_L)\mathbf{V}(\mu_p', U^{u,p}_{IP}) + \frac{(1-\pi_H)(1-\pi_L)}{1-\pi(\mu_p)}\mathbf{V}_\mu(\mu_p', U^{u,p}_{IP}) + \\
&+ \lambda^{IP}\Big[\frac{1}{\mu_p}\big[(1-\pi_L)U^{u,f}_{IP} - (1-\pi(\mu_p))U^{u,p}_{IP}\big] - (\pi_H - \pi_L)U^{u,p}_{IP}\Big] + \frac{\chi^{IP}}{\mu_p}\big[U^{u,p}_{IP} - U^{u,f}_{IP}\big] = 0
\end{aligned}$$

Rearranging the terms and using the first order condition on $U^{u,f}_{IP}$

$$(1-\pi_L)\big[\mathbf{V}_U(0, U^{u,f}_{IP}) + \lambda^{IP}\big] = \chi^{IP}$$

it yields

$$\mathbf{V}_\mu(\mu'_p, U^{u,p}_{IP}) = \frac{\mathbf{V}(\mu'_p, U^{u,p}_{IP}) - \mathbf{V}(0, U^{u,f}_{IP}) + \mathbf{V}_U(0, U^{u,f}_{IP})(U^{u,f}_{IP} - U^{u,p}_{IP})}{\mu'_p}$$

with $U^{u,f}_{IP} > U^{u,p}_{IP}$. ■

Appendix D: Moral Hazard

Proof of Prop. 21

Proof. The first part of the proof is contained in the Technical Appendix. It shows that multiple deviations can be accounted for by single one-shot deviations, that is, deviations from recommended action lasting only one period. Now consider the recursion (2.27)

$$\begin{aligned}
\Lambda(1, \mu^{T-1}) &= 0 \\
\Lambda(2, \mu^{T-2}) &= u(c_{T-1}) - e + \beta\pi(\mu^{T-2})U^w_T + \beta(1-\pi(\mu^{T-2}))U^u_T - U^u_{T-2} \\
&= U^u_{T-2} + \beta[\pi(\mu^{T-2}) - \pi(\mu^{T-1})](U^w_T - U^u_T) - U^u_{T-2} = \beta[\pi(\mu^{T-2}) - \pi(\mu^{T-1})]\frac{e}{\beta\pi(\mu^{T-1})} \\
\Lambda(T-j, \mu^j) &= u(c_{j+1}) - e + \beta\pi(\mu^j)U^w_{j+2} + \\
&\quad + \beta(1-\pi(\mu^j))\underbrace{[u(c_{j+2}) - e + \beta\pi(\mu^{j+1})U^w_{j+3} + \beta(1-\pi(\mu^{j+1}))[...]]}_{\Lambda(T-j-1,\mu^{j+1})+U^u_{j+2}} - U^u_{j+1} \\
&= U^u_{j+1} + \beta[\pi(\mu^j) - \pi(\mu^{j+1})](U^w_{j+2} - U^u_{j+2}) + \beta(1-\pi(\mu^j))\Lambda(T-j-1, \mu^{j+1}) - U^u_{j+1} \\
&= \beta[\pi(\mu^j) - \pi(\mu^{j+1})]\frac{\beta\Lambda(T-j-1, \mu^{j+1}) + e}{\beta\pi(\mu^{j+1})} + \beta(1-\pi(\mu^j))\Lambda(T-j-1, \mu^{j+1}) \\
&= \Big(\frac{\pi(\mu^j)}{\pi(\mu^{j+1})} - 1\Big)e + \beta\pi(\mu^j)\Big(\frac{1}{\pi(\mu^{j+1})} - 1\Big)\Lambda(T-j-1, \mu^{j+1}), \quad 0 \le j \le T-1
\end{aligned}$$

And notice that the constraint $(\hat{IC}, t)$, defined as

$$U_s(\mathcal{W}, \mu^s, \sigma^s) = u(c_s(\sigma^s)) + \beta\big[U_{s+1}(\mathcal{W}, \mu^{s+1}, (\sigma^s, u)) + \Lambda(T-s, \mu^s)\big]$$

makes the contract robust against any possible deviation after period t, thanks to the recursive definition of Λ. In particular,

$$(\hat{IC}, t) \iff (IC, s),\ \forall s \ge t$$

Hence the whole set of IC constraints can be expressed by

$$U = U_0(\mathcal{W}, \mu, \sigma_0) = u(c) + \beta\big[U^u + \Lambda(T, \mu)\big], \quad (I\hat{C}, 0)$$

Λ is defined by the recursion in (2.27), and is independent of U^u. In addition, $\Lambda(t+1, \mu) \geq \Lambda(t, \mu)$, with inequality being strict for $\mu \in (0, 1)$.[54] Indeed, taking the difference between $\Lambda(t + 1, \mu)$ and $\Lambda(t, \mu)$, it holds:

$$\begin{cases} \Lambda(2, \mu) - \Lambda(1, \mu) = \Big(\frac{\pi(\mu)}{\pi(\mu')} - 1\Big)e > 0 \\ \Lambda(t + 1, \mu) - \Lambda(t, \mu) = \beta\pi(\mu)\Big(\frac{1}{\pi(\mu')} - 1\Big)(\Lambda(t, \mu') - \Lambda(t - 1, \mu')) > 0, \quad \forall t \geq 2 \end{cases}$$

■

Lemma 5. *The value of* $(UI, t)_{t\geq1}$ *is increasing in* μ*.*

Proof. The problem of policy $(UI, t)_{t\geq1}$ reads

$$V_t^{UI}(\mu, U) = \max_{(z,U^w,U^u)\in\Gamma(\mu,U)} -g(z) + \beta\big[\pi(\mu)W(\mu, U^w) + (1 - \pi(\mu))V_{t-1}^{UI}(\mu', U^u)\big]$$
$$\text{sub:} \quad \Gamma(\mu, U) = \Big\{(z, U^w, U^u) : \; U = z - e + \beta\big[\pi(\mu)U^w + (1 - \pi(\mu))U^u\big],$$
$$U \geq z + \beta\big[U^u + \Lambda(t, \mu)\big]\Big\}$$

V_1^{UI} is monotone increasing in μ (see Lemma 3). By induction, assume that V_{t-1}^{UI} is increasing in μ. Positive monotonicity of V_t^{UI} in μ follows from:

- (positive) monotonicity of the objective function in μ;
- (positive) monotonicity of the feasibility set Γ_U^i in μ, i.e.

$$\mu < \tilde{\mu} \Longrightarrow \Gamma_U^i(\mu) \subseteq \Gamma_U^i(\tilde{\mu})$$

The objective function is always monotone in μ, as so are W and V_{t-1}^{UI} in their first argument, μ' is an increasing function of μ and $W(\mu, U) \geq V_{t-1}^{UI}(\mu', U)$ at the optimum. Monotonicity of Γ_U, instead, holds whenever an increase of μ leads to a relaxation of (IC).[55] Now, if $\Lambda(t, .)$ is constant or decreasing, this always holds. Indeed, (IC) is more

[54] In $\mu \in \{0, 1\}$, no learning occurs and learning rents are null.
[55] (PK) is always relaxed by an increase in μ (recall that $U^w \geq U^u$ in optimum).

slack if $\Lambda(t,.)$ is decreasing as

$$\mu < \tilde{\mu} \Longrightarrow U^w - U^u \geq \frac{e/\beta + \Lambda(t,\mu)}{\pi(\mu)} > \frac{e/\beta + \Lambda(t,\tilde{\mu})}{\pi(\tilde{\mu})}$$

To prove that monotonicity holds also when $(\Lambda(t,.))_{t>1}$ is increasing in μ, I prove that the RHS is decreasing in μ.

From the definition of Λ in (2.27), I can rewrite

$$\frac{e/\beta + \Lambda(t,\mu)}{\pi(\mu)} = \frac{e}{\pi(\mu)}\Big(\frac{1}{\beta} - 1\Big) - \beta\Lambda(t-1,\mu') + \beta\Big(\frac{e/\beta + \Lambda(t-1,\mu')}{\pi(\mu')}\Big) \tag{2.41}$$

Define $f(\mu) := \frac{\pi(\mu)}{\pi(\mu')}$, and notice that it is concave in μ. Indeed:

$$\begin{aligned} f_\mu(\mu) &= (\pi_H - \pi_L)^2 \frac{(1-\mu)^2\pi_L(1-\pi_L) - \mu^2\pi_H(1-\pi_H)}{\big[(1-\pi_H)\pi_H\mu + (1-\pi_L)\pi_L(1-\mu)\big]^2} \\ f_{\mu\mu}(\mu) &= -\frac{2(\pi_H-\pi_L)^2\pi_H\pi_L(1-\pi_H)(1-\pi_L)}{\big[(1-\pi_H)\pi_H\mu + (1-\pi_L)\pi_L(1-\mu)\big]^3} < 0 \end{aligned}$$

Thus, the derivative of $\Lambda(t,\mu)$ by μ reads

$$\Lambda_\mu(t,\mu) = f_\mu(\mu)e + \beta\big[f_\mu(\mu) - (\pi_H - \pi_L)\big]\Lambda(t-1,\mu') + \beta\big[f(\mu) - \pi(\mu)\big]\frac{\partial\mu'}{\partial\mu}\Lambda_\mu(t-1,\mu') \tag{2.42}$$

Two cases are possible:

1. $f_\mu(\mu) \geq \pi_H - \pi_L$
2. $f_\mu(\mu) < \pi_H - \pi_L$

If the first case applies, then

$$\Lambda_\mu(t,\mu) > 0 \Longrightarrow \Lambda_\mu(t-1,\mu') > 0$$

Assume *per contra* that $\Lambda_\mu(t-1,\mu') < 0$. But then by (strict) concavity of f, $f_\mu(\mu') > \pi_H - \pi_L$. Which, coupled with the expression of the derivative in (2.42), implies that for the assumption to be true, it must be that $\Lambda_\mu(t-2,\mu'') < 0$, and so on, until

$$f_\mu(\mu^{(t-2)})e = \Lambda_\mu(2,\mu^{(t-2)}) < 0 < \pi_H - \pi_L < f_\mu(\mu^{(t-2)})$$

Therefore, I have reached a contradiction.

Now, I am ready to prove by induction that

$$\Lambda_\mu(t,\mu) > 0 \wedge f_\mu(\mu) \geq \pi_H - \pi_L \Longrightarrow \frac{\partial}{\partial \mu}\Big(\frac{e/\beta + \Lambda(t,\mu)}{\pi(\mu)}\Big) < 0$$

Base Step ($t = 2$)

Notice that the result is always true for $t = 2$, as the expression reads

$$\frac{e/\beta + \Lambda(2,\mu)}{\pi(\mu)} = \frac{e}{\pi(\mu)}\Big(\frac{1}{\beta} - 1\Big) + \frac{e}{\pi(\mu')}$$

Induction Step

Assume *per contra* that

$$\Lambda_\mu(t,\mu) > 0 \ \wedge \ f_\mu(\mu) \geq \pi_H - \pi_L \ \wedge \ \frac{\partial}{\partial \mu}\Big(\frac{e/\beta + \Lambda(t,\mu)}{\pi(\mu)}\Big) > 0$$

Since the first two addends of (2.41) have been shown to be decreasing in μ, for it to be true it must be that $\frac{\partial}{\partial \mu'}\Big(\frac{e/\beta+\Lambda(t-1,\mu')}{\pi(\mu')}\Big) > 0$. However,

$$\Lambda(t,\mu) > 0 \wedge f_\mu(\mu) \geq \pi_H - \pi_L \Longrightarrow \Lambda_\mu(t-1,\mu') > 0 \wedge f_\mu(\mu') > \pi_H - \pi_L \Longrightarrow \frac{\partial}{\partial \mu'}\Big(\frac{e/\beta + \Lambda(t-1,\mu')}{\pi(\mu')}\Big) < 0$$

where the second implication follows by induction hypothesis. Hence the contradiction.

What is left to be shown is the following:

$$\Lambda_\mu(t,\mu) > 0 \wedge f_\mu(\mu) < \pi_H - \pi_L \Longrightarrow \frac{\partial}{\partial \mu}\Big(\frac{e/\beta + \Lambda(t,\mu)}{\pi(\mu)}\Big) < 0$$

Base Step ($t = 2$)

Same as in the case above, as the thesis always applies.

Induction Step

The derivative by μ has the following expression

$$\frac{\partial}{\partial \mu}\Big(\frac{e/\beta + \Lambda(t,\mu)}{\pi(\mu)}\Big) = \frac{1}{\pi(\mu)}\Big[\Lambda_\mu(t,\mu) - (\pi_H - \pi_L)\frac{e/\beta + \Lambda(t,\mu)}{\pi(\mu)}\Big]$$

So assume *per contra* that it is positive. Then this means that $\Lambda_\mu(t,\mu) > (\pi_H - \pi_L)\frac{e/\beta+\Lambda(t,\mu)}{\pi(\mu)}$. Moreover, by (2.41), it either means that $\Lambda_\mu(t-1,\mu') < 0$ or that

$$\frac{\partial}{\partial \mu'}\Big(\frac{e/\beta + \Lambda(t-1,\mu')}{\pi(\mu')}\Big) > 0$$

The first case can not apply, as (2.42) would imply that

$$(\pi_H - \pi_L)\frac{e}{\beta\pi(\mu)} < (\pi_H - \pi_L)\frac{e/\beta + \Lambda(t,\mu)}{\pi(\mu)} < \Lambda_\mu(t,\mu) < f_\mu(\mu)e < (\pi_H - \pi_L)e$$

which is impossible, as $\frac{1}{\beta\pi(\mu)} > 1$. Therefore, it must be the case that

$$\frac{\partial}{\partial\mu'}\Big(\frac{e/\beta + \Lambda(t-1,\mu')}{\pi(\mu')}\Big) > 0 \Longrightarrow \Lambda_\mu(t-1,\mu') > (\pi_H - \pi_L)\frac{e/\beta + \Lambda(t-1,\mu')}{\pi(\mu')} > 0$$

Now, if $f_\mu(\mu') \geq \pi_H - \pi_L$, I have reached a contradiction, since I have shown above that

$$\Lambda_\mu(t-1,\mu') > 0 \wedge f_\mu(\mu') \geq \pi_H - \pi_L \Longrightarrow \frac{\partial}{\partial\mu'}\Big(\frac{e/\beta + \Lambda(t-1,\mu')}{\pi(\mu')}\Big) < 0$$

If, instead, $f_\mu(\mu') < \pi_H - \pi_L$, then

$$\Lambda_\mu(t-1,\mu') > 0 \wedge f_\mu(\mu') < \pi_H - \pi_L \Longrightarrow \frac{\partial}{\partial\mu'}\Big(\frac{e/\beta + \Lambda(t-1,\mu')}{\pi(\mu')}\Big) < 0$$

by induction hypothesis, and a contradiction is reached in this case, too.

■

Proof of Prop. 22

Define $\Lambda(t,\mu)$ as the learning rents necessary to implement UI for t prospective periods ahead, and notice that, if $\mu_f = 0$, $\Lambda(t,\mu_f) = 0$. Then, from the definition of IP, the first-order condition reads

$$\begin{aligned}&\frac{\partial q}{\partial\mu_p}\big[(1-\pi(\mu_p))V(\mu_p',U_p^u) - (1-\pi_L)V(0,U_f^u)\big] - q(\pi_H-\pi_L)V(\mu_p',U_p^u) +\\&+ q\frac{(1-\pi_H)(1-\pi_L)}{1-\pi(\mu_p)}V_\mu(\mu_p',U_p^u) + \lambda\Big[\frac{\partial q}{\partial\mu_p}\big[(1-\pi(\mu_p))U_p^u - (1-\pi_L)U_f^u\big] - q(\pi_H-\pi_L)U_p^u\Big] -\\&-\chi\Big[\frac{\partial q}{\partial\mu_p}\Lambda(t,\mu_p) + q\Lambda_\mu(t,\mu_p)\Big] = 0\end{aligned}$$

Which can be rewritten as

$$V_\mu(\mu_p',U_p^u) = \frac{V(\mu_p',U_p^u) - V(0,U_f^u)}{\mu_p'} + \Big(\lambda - \frac{\chi}{1-\pi_L}\Big)\frac{U_p^u - U_f^u}{\mu_p'} + \frac{\chi}{1-\pi_L}\frac{\mu_p\Lambda_\mu(t,\mu_p) - \Lambda(t,\mu_p)}{\mu_p'}$$

and plugging in $-V_U^{SA}(U_f^u) = -V_U(0,U_f^u) = \lambda - \frac{\chi}{1-\pi_L}$ and $\lambda = -W_U(\mu,U^w)$ delivers the result.

Appendix E: Estimation of hazard rates

In order to infer the hazard rates $\{\pi_H,\, \pi_L\}$, I proceed as follows. First, from the basic monthly Current Population Survey (CPS), I derive the fraction of high- and low-skilled workers for each level of educational attainment θ_i, $i \in$\{*LessHighSc.*, *HighSc.*, *SomeCollege*, *College*, *Graduate*\}.[56] Then, I compute the hazard rate out of unemployment for each time horizon $(\pi_t)_{t\geq 1}$, from the cross-section of jobless workers who report to have been unemployed for t periods of time, using the following formulas

$$\pi_1 = 1 - Prob(t>1) = 1 - \frac{\#\, jobless\, for\, t>1}{\#jobless}$$

$$\pi_1 + (1-\pi_1)\pi_2 = 1 - Prob(t>2) = 1 - \frac{\#\, jobless\ for\ t>2}{\#jobless}$$

....

Third, by looking at the same cross-sections, I compute the share of those with same spell duration (at the time the survey is conducted) who also have attained the same educational level, $\psi_{i,t}$. Lastly, I compute $\{\pi_H,\, \pi_L\}$ that minimize

$$\{\pi_H,\, \pi_L\} = \arg\min_{\hat{\pi}_H, \hat{\pi}_L} \sum_t \big(\sum_i \psi_{i,t}(\theta_i \hat{\pi}_H + (1-\theta_i)\hat{\pi}_L) - \pi_t \big)^2$$

that is,

$$\pi_H = \frac{\sum_t b_t \sum_s \pi_s a_s - \sum_s \pi_s \sum_t a_t b_t}{12 \sum_t a_t^2 - (\sum_t a_t)^2}, \quad \pi_L = \frac{(\sum_t \pi_t)(\sum_s a_s^2) - \sum_s \pi_s a_s \sum_t a_t}{12 \sum_t a_t^2 - (\sum_t a_t)^2}$$

with $a_t = \sum_i \psi_{it}\theta_i,\ b_t = \sum_i \psi_{it}(1-\theta_i) = 1 - a_t$.[57] The results are reported in Table 2.5. The hazard rate π_t is quite stable over time, as well as the share of any education level among all jobless people with same duration of unemployment spell, ψ_{it}. The estimated hazard rates are $\pi_H = 0.27$ and $\pi_L = 0.14$.

[56] High-skilled workers are defined as those who earn a wage higher than the mean of ω_H and ω_L, that is, \$2,527.

[57] First-order conditions for π_H and π_L return the minimizers of the convex objective function.

	Total	< High Sch.	High Sch. D.	< Col.[58]	Col. D.	Grad. D.	
θ_i	39,333	0.54	0.72	0.76	0.9	0.95	
Horizon	Total	ψ_{it}=Pr(Education=i | Horizon $\geq t$)					Haz. Rate (π_t)
t=1	3,481	0.11	0.31	0.29	0.28	0.01	0.22
t=2	2,517	0.11	0.32	0.29	0.27	0.01	0.28
t=3	1,742	0.11	0.32	0.29	0.27	0.01	0.31
t=4	1,316	0.11	0.32	0.29	0.28	0.01	0.24
t=5	1,081	0.11	0.32	0.29	0.28	0	0.18
t=6	815	0.11	0.33	0.28	0.27	0	0.25
t=7	586	0.12	0.33	0.28	0.27	0	0.28
t=8	468	0.12	0.33	0.28	0.27	0	0.2
t=9	356	0.11	0.32	0.29	0.27	0	0.24
t=10	274	0.11	0.31	0.28	0.29	0	0.23
t=11	215	0.11	0.33	0.26	0.29	0	0.22
t=12	167	0.11	0.34	0.25	0.3	0	0.22

Table 2.5: Education-cohort size for any unemployment spell duration.

[58] '< Col.' item includes workers who attended college, but have not earned a degree, and workers with an Associate Degree, which is a post-secondary course of study lasting 2 or 3 years.

Technical Appendix

Setting

- $T < \infty$

- $\sigma_t \in \{u, w\}$ describes the worker status, either unemployed or employed. If $\sigma_t = w$, the worker finds reemployment, which is an absorbing state. Hence $p(\sigma_{t+1} = w|\sigma_t = w) = 1$.

- $\sigma^t = \{\sigma_0, ..., \sigma_t\}$ is a public history describing the employment status of the worker

- $c_t(\sigma^t)$ is the transfer function, with $c_t(\sigma^t) \geq 0$ for every σ^t. Let $\mathbf{c}(\alpha \backslash \sigma^t)$ be the stream of transfers downstream of node σ^t

- $a_t(\sigma^t)$ is the effort level, with

$$a_t(\sigma^t) \in \begin{cases} \{0, e\}, & \text{if} \quad \sigma_t = u \\ e, & \text{if} \quad \sigma_t = w \end{cases}$$

  The effort is unobservable by the government. Denote by $\mathbf{a}(\alpha \backslash \sigma^t)$ the continuation plan of effort costs downstream of node σ^t, and $\mathbf{a}(\sigma^t)$ its upstream counterpart

- $h \in \{L; H\}$ is the hidden state, which is revealed once the worker finds reemployment

- $\mu_t(\sigma^t, \mathbf{a}(\sigma^{t-1}))$ -with $\sigma^t = (\sigma^{t-1}, \sigma_t)$- is the expectation held by the worker during unemployment, expressing the probability about state H. This is clearly a non-contractible variable, as the worker can hide it from the government. $\mu_t(\sigma^t, \mathbf{a}(\sigma^{t-1}))$ impacts the probability of future σ_{t+1}. in particular,

$$p(\sigma_{t+1} = w|\sigma_t = u, \mu_t, a_t(\sigma^t) = e) = \pi(\mu_t), \quad p(\sigma_{t+1} = w|\sigma_t = u, \mu_t, a_t(\sigma^t) = 0) = 0$$

  where I have dropped dependence of μ_t by $(\sigma^t, \mathbf{a}(\sigma^{t-1}))$ to ease notation. Moreover, $\mu_t(\sigma^t, \mathbf{a}(\sigma^{t-1}))$ undergoes an updating process every time the worker exerts effort in t and remains unemployed in $t+1$

$$\mu_{t+1}(\sigma^t, \mathbf{a}(\sigma^{t-1}), \sigma_{t+1} = u, a_t(\sigma^t) = e) = \frac{\mu_t(\sigma^t, \mathbf{a}(\sigma^{t-1}))(1 - \pi_H)}{\mu_t(\sigma^t, \mathbf{a}(\sigma^{t-1})))(1 - \pi_H) + (1 - \mu_t(\sigma^t, \mathbf{a}(\sigma^{t-1})))(1 - \pi_L)} \tag{2.43}$$

Instead, if no effort is exerted, the worker does not revise expectation[59]

$$\mu_{t+1}(\sigma^t, \mathbf{a}(\sigma^{t-1}), \sigma_{t+1} = u, a_t(\sigma^t) = 0) = \mu_t(\sigma^t, \mathbf{a}(\sigma^{t-1})) \tag{2.44}$$

- If $\sigma_s = w$,

$$r_s(\sigma^s, \mathbf{a}(\sigma^{s-1})) = \tilde{\omega}(\mu_t(\sigma^t, \mathbf{a}(\sigma^{t-1}))), \quad \text{if } t = \inf_s\{y_s = w\} - 1$$

Otherwise, if $\sigma_t = u$, $r_s(\sigma^s, \mathbf{a}(\sigma^{s-1})) = 0$.

Worker's Problem in UI

Let $\mathcal{W}(\sigma^t) = (\mathbf{c}, \mathbf{a})(\alpha \backslash \sigma^t) = \big\{c_s(\sigma^s), a_s(\sigma^s)\big\}_{s=t}^T$ denote the contract offered by the government to the worker. Worker's expected utility reads

$$\begin{aligned}
U_t(\mathcal{W}, \mathbf{a}(\sigma^{t-1}), \sigma^t) &= \mathbf{E}\Big\{\sum_{s=t}^T \beta^{s-t}\big(u(c_s(\sigma^s)) - a_s(\sigma^s)\big)\Big|\mathcal{W}(\sigma^t), \mu_t(\sigma^t, \mathbf{a}(\sigma^{t-1}))\Big\} + \\
&+ \beta^{T+1-t}\sum_{\sigma^{T+1}} p(\sigma^{T+1}|\sigma^t, \mu_t, \mathbf{a}(\sigma^T))U_{T+1}(\sigma^{T+1}) \\
&= \sum_{s=t}^T \beta^{s-t}\sum_{\sigma^s} p(\sigma^s|\sigma^t, \mu_t(\sigma^t, \mathbf{a}(\sigma^{t-1})), a_t(\sigma^t))\Big\{u(c_s(\sigma^s)) - a_s(\sigma^s)\Big|\mathcal{W}(\sigma^t)\Big\} \\
&+ \beta^{T+1-t}\sum_{\sigma^{T+1}} p(\sigma^{T+1}|\sigma^t, \mu_t, \mathbf{a}(\sigma^T))U_{T+1}(\sigma^{T+1}) \\
&+ \beta\Big[p(\sigma_{t+1} = w|\sigma_t = u, \mu_t, a_t(\sigma^t))\sum_{s=t+1}^T \beta^{s-(t+1)}\sum_{h\in\{H,L\}} p(h|\sigma_{t+1} = w, \mu_t)\Big\{u(c_s(\sigma^s)) - e\Big|\mathcal{W}'(\sigma^t, w, h)\Big\} + \\
&+ p(\sigma_{t+1} = u|\sigma_t = u, \mu_t, a_t(\sigma^t))\sum_{s=t+1}^T \beta^{s-(t+1)} \times \\
&\qquad \times \sum_{\sigma^s} p(\sigma^s|\sigma_{t+1} = u, \mu_{t+1}^u, a_{t+1}(\sigma^t, u))\Big\{u(c_s(\sigma^s)) - a_s(\sigma^s)\Big|\mathcal{W}'(\sigma^t, u)\Big\}\Big] + \\
&+ \beta^{T+1-t}\sum_{\sigma^{t+1}} p(\sigma^{t+1}|\sigma^t, \mu_t, \mathbf{a}(\sigma^t))\sum_{\sigma^{T+1}} p(\sigma^{T+1}|\sigma^{t+1}, \mu_{t+1}, \mathbf{a}(\sigma^T))U_{T+1}(\sigma^{T+1})
\end{aligned}$$

[59]Notice that $\mu_{t+1}(\sigma^t, \mathbf{a}(\sigma^{t-1}), \sigma_{t+1} = w, a_t(\sigma^t) = 0)$ is not defined as h is disclosed once $\sigma_{t+1} = w$.

$$= u(c_t(\sigma^t)) - e +$$

$$+ \beta \sum_{h\in\{H,L\}} p(h|\mu_t)p(\sigma_{t+1} = w|h)\Big[\sum_{s=t+1}^{T} \beta^{s-(t+1)}\Big\{u(c_s(\sigma^s)) - e\Big|\mathcal{W}'(\sigma^t, w, h)\Big\} + \beta^{T-t}U_{T+1}(\sigma^t, \mathbf{w}, h)\Big] +$$

$$+ \beta(1-\pi(\mu_t))\Big[\sum_{s=t+1}^{T} \beta^{s-(t+1)} \sum_{\sigma^s} p(\sigma^s|\sigma_{t+1} = u, \mu^u_{t+1}, a_{t+1}(\sigma^t, u))\Big\{u(c_s(\sigma^s)) - a_s(\sigma^s)\Big|\mathcal{W}'(\sigma^t, u), \mu^u_{t+1}\Big\} +$$

$$+ \sum_{\sigma^{T+1}} p(\sigma^{T+1}|\sigma^{t+1}, \mu^u_{t+1}, \mathbf{a}(\sigma^T))U_{T+1}(\sigma^{T+1})\Big]$$

$$= u(c_t(\sigma^t)) - e + \beta\Big\{\pi(\mu_t)\Big[\frac{\mu_t\pi_H}{\pi(\mu_t)}U_{t+1}(\mathcal{W}', \mathbf{a}(\sigma^t), (\sigma^t, w, H)) + \frac{(1-\mu_t)\pi_L}{\pi(\mu_t)}U_{t+1}(\mathcal{W}', \mathbf{a}(\sigma^t), (\sigma^t, w, L))\Big] +$$

$$+ (1-\pi(\mu_t))U_{t+1}(\mathcal{W}', \mathbf{a}(\sigma^t), (\sigma^t, u))\Big\}$$

with $\mu^u_{t+1} = \mu_{t+1}(\sigma^t, \sigma_{t+1} = u, \mathbf{a}(\sigma^{t-1}), a_t(\sigma^t))$[60].

The IC constraint starting from time t reads

$$U_t(\mathcal{W}, \mathbf{a}(\sigma^{t-1}), \sigma^t) \geq U_t((\mathbf{c}, \hat{\mathbf{a}})(\alpha\backslash\sigma^t), \mathbf{a}(\sigma^{t-1}), \sigma^t), \quad \forall\, \hat{\mathbf{a}}(\alpha\backslash\sigma^t) \in A_t(\alpha\backslash\sigma^t) \tag{2.45}$$

Government's Problem in UI

The problem for the government reads

$$V^{UI}(U, \mathbf{a}(\sigma^{t-1}), \sigma^t) = \max_{\mathcal{W}} \mathbf{E}\Big\{\sum_{s=t}^{T} \beta^{s-t}\big(r_s(\sigma^s, \mathbf{a}(\sigma^{s-1})) - c_s(\sigma^s)\big)\Big|\mathcal{W}(\sigma^t), \mu_t(\sigma^t, \mathbf{a}(\sigma^{t-1}))\Big\}$$

$$\text{sub:} \quad (2.45), \quad U_s(\mathcal{W}'(\sigma^s), \mathbf{a}(\sigma^{s-1}), \sigma^s) \geq U, \quad \forall s \geq t$$

Given that expectation is revised upon (failure and) effort exerted only in the last period, it follows a Markovian process, meaning that expectation in $t+1$ can be predicted by expectation μ_t and effort in t, and realization of σ_{t+1}. Thus, I define $x_t = (\mu_t(\sigma^t, x_{t-1}), a_t(\sigma^t))$ and write

$$\mu_{t+1}(\sigma^t, \sigma_{t+1} = u, \mathbf{a}(\sigma^t)) = \mu_{t+1}(\sigma^t, \sigma_{t+1} = u, \mu_t(\sigma^t, x_{t-1}), a_t(\sigma^t)) = \mu_{t+1}(\sigma^t, \sigma_{t+1} = u, x_t)$$

And given that reemployment is absorbing and discloses the state, there exists an isomorphism between all unemployment histories $(\sigma^s, \sigma_{s+1} = u)$ and terminal realizations

[60] Note that by assumption on absorbing nature of re-employment, $\sigma^s = (\sigma^t, (y_j = w)^s_{t+1})$, $\forall\, \sigma^s \succsim (\sigma^t, \sigma_{t+1} = w)$.

$\sigma_{s+1} = u$, as no extra information is contained in σ^s which can not be inferred by observing $\sigma_{s+1} = u$. As a result, next expectation μ_{t+1} only depends on current expectation μ_t and effort a_t and future realization of σ_{t+1}:

$$\mu_{t+1}(\sigma^t, \sigma_{t+1} = u, x_t) = \mu_{t+1}(\sigma_{t+1} = u, \mu_t, a_t) = \begin{cases} \mu_t, & \text{if} \quad a_t = 0 \\ \frac{\mu_t(1-\pi_H)}{1-\pi(\mu_t)}, & \text{if} \quad a_t = e \end{cases}$$

Therefore:

$$\begin{aligned} U_t(\mathcal{W}, \mu_t, \sigma^t) &= u(c_t(\sigma^t)) - e + \\ &+ \beta\Big\{\pi(\mu_t)\Big[\frac{\mu_t\pi_H}{\pi(\mu_t)}U_{t+1}(\mathcal{W}', H, \sigma^t, \sigma_{t+1} = w) + \frac{(1-\mu_t)\pi_L}{\pi(\mu_t)}U_{t+1}(\mathcal{W}', L, \sigma^t, \sigma_{t+1} = w)\Big] + \\ &+ (1-\pi(\mu_t))U_{t+1}(\mathcal{W}', \mu^u_{t+1}, \sigma^t, \sigma_{t+1} = u)\Big\} \\ = u(c_t(\sigma^t)) - e + \beta\Big\{&\mu_t\pi_H U_{t+1}(\mathcal{W}', H, \sigma^t, \sigma_{t+1} = w) + (1-\mu_t)\pi_L U_{t+1}(\mathcal{W}', L, \sigma^t, \sigma_{t+1} = w) + \\ &+ (1-\pi(\mu_t))U_{t+1}(\mathcal{W}', \mu^u_{t+1}, \sigma^t, \sigma_{t+1} = u)\Big\} \end{aligned}$$

Government's problem can be rewritten as

$$\begin{aligned} V^{UI}(U, \mu_t, \sigma^t) &= \max_{\mathcal{W}\in\Omega(\mu_t,\sigma^t)} \mathbf{E}\Big\{\sum_{s=t}^{T}\beta^{s-t}\big(r_s(\sigma^s, \mu_s) - c_s(\sigma^s)\big)\Big|\mathcal{W}(\sigma^t), \mu_t\Big\} \\ &= \max_{c_t(\sigma^t), a_t(\sigma^t)\in\Gamma(\mu_t,\sigma^t)} r_t(\sigma^t, \mu_t) - c_t(\sigma^t) + \\ &+ \beta\Big[\pi(\mu_t) \max_{\mathcal{W}'\in\Omega'(\mu_{t+1},\sigma^t,\sigma_{t+1}=w)} \mathbf{E}\Big\{\sum_{s=t+1}^{T}\beta^{s-(t+1)}\big(\overbrace{r_s(\sigma^s, \mu_s)}^{=\tilde{\omega}(\mu_t)} - c_s(\sigma^s)\big)\Big|\mathcal{W}'(\sigma^t, \sigma_{t+1} = w), \mu_{t+1}\Big\} + \\ &+ (1-\pi(\mu_t)) \max_{\mathcal{W}'\in\Omega'(\mu_{t+1},\sigma^t,\sigma_{t+1}=u)} \mathbf{E}\Big\{\sum_{s=t+1}^{T}\beta^{s-(t+1)}\big(r_s(\sigma^s, \mu_s) - c_s(\sigma^s)\big)\Big|\mathcal{W}'(\sigma^t, \sigma_{t+1} = u), \mu_{t+1}\Big\}\Big] \end{aligned}$$

$$\begin{aligned} \text{sub:} \quad & U_t(\mathcal{W}, \mu_t, \sigma^t) \geq U_t((\mathbf{c}, \mathbf{a}')(\alpha\backslash\sigma^t), \mu_t, \sigma^t), \quad \forall\ \mathbf{a}'(\alpha\backslash\sigma^t) \in A_t(\alpha\backslash\sigma^t) \quad (IC) \\ & \mu_t := \mu_t(\sigma^t, \mu_{t-1}, a_{t-1}(\sigma^{t-1})) \end{aligned}$$

Lemma 6. *The government prefers to insure the worker against the risk of* h *realization upon reemployment.*

Lemma 7. *Define (IC, s) the constraint that makes contract* $\mathcal{W}$ *robust to the alternative strategy* $\mathbf{a}'(\alpha\backslash\sigma^s) = (0, e\mathbf{1}_{T-s}) \in A_s(\alpha\backslash\sigma^s)$ *that shirks in* s *and sticks to effort from* $s+1$

to the final period T

$$U_s(\mathcal{W}, \mu_s, \sigma^s) \geq U_s((\mathbf{c}, \mathbf{a}')(\alpha\backslash\sigma^s), \mu_s, \sigma^s) = u(c_s(\sigma^s)) + \beta U_{s+1}(\mathcal{W}, \mu_s, (\sigma^s, u))$$

If (IC, s)$_{s=t}^T$ are all binding under contract $\mathcal{W}$, then contract $\mathcal{W}$ is feasible.

Proof. First, consider that no learning motive or moral hazard problem is present upon reemployment, when state is disclosed, nor there is any chance that any reemployed W falls back into unemployment ($p(y_s = u|y_t = w) = 0, \ \forall s > t$). Hence in order to verify (2.45), one can only focus on continuation histories $\sigma^s \succsim \sigma^t$ where $\sigma^s = \big(\sigma^t, (y_j)_{j=t+1}^s\big) = (\sigma^t, (u)_{j=t+1}^s)$. For this reason, I therefore adopt the convention that $\sigma^{s+1} = (\sigma^s, u)$. Define also μ_s as the expectation in period s if contract $\mathcal{W}$ is followed.
Notice that continuation utility at time $s > t$ upon reemployment ($y_s = w$) follows

$$U_s(\mathcal{W}, h, \sigma^s) = u(c_s(\sigma^s)) - e + \beta U_{s+1}(\mathcal{W}, h, (\sigma^s, w))$$

while upon failure ($y_s = u$), it follows

$$\begin{aligned}U_s(\mathcal{W}, \mu_s, \sigma^s) = u(c_s(\sigma^s)) - e + \beta\Big[&\mu_s\pi_H U_{s+1}(\mathcal{W}, H, (\sigma^s, w)) + (1-\mu_s)\pi_L U_{s+1}(\mathcal{W}, L, (\sigma^s, w)) + \\ &+ (1-\pi(\mu_s))U_{s+1}(\mathcal{W}, \mu_{s+1}^u, (\sigma^s, u))\Big] \\ &\text{with:} \quad \mu_{s+1}^u = \mu_{s+1}(\sigma^s, \sigma_{s+1} = u, \mu_s, e)\end{aligned}$$

By Lemma 6, focusing on contracts $\mathcal{W}$ such that

$$\begin{aligned}&U_{s+1}(\mathcal{W}, H, (\sigma^s, w)) = U_{s+1}(\mathcal{W}, L, (\sigma^s, w)) \\ \Longrightarrow &\mu\pi_H U_{s+1}(\mathcal{W}, H, (\sigma^s, w)) + (1-\mu)\pi_L U_{s+1}(\mathcal{W}, L, (\sigma^s, w)) = \pi(\mu)U_{s+1}(\mathcal{W}, (\sigma^s, w))\end{aligned}$$

is without loss of generality.
Second, the following holds true:

$$\begin{aligned}&U_s(\mathcal{W}, \mu_s, \sigma^s) = u(c_s(\sigma^s)) + \beta U_{s+1}(\mathcal{W}, \mu_s, \sigma^{s+1}) \\ \Longrightarrow &U_s(\mathcal{W}, \mu, \sigma^s) \geq u(c_s(\sigma^s)) + \beta U_{s+1}(\mathcal{W}, \mu, \sigma^{s+1}), \quad \forall \mu : \ \mu > \mu_s\end{aligned} \tag{2.46}$$

The proof of (2.46) will be given by induction, joint with the main statement.
<u>Base Step ($t = T$)</u>

UI contract ends in $t = T$, where the only possible deviation is $\hat{\mathbf{a}}(\alpha\backslash\sigma^T) = \hat{a}_T(\sigma^T) = 0$, Thus, for W to be robust to this deviation, it must be that

$$U_T(\mathcal{W}, \mu_T, \sigma^T) = u(c_T(\sigma^T)) - e + \beta\big[\mu_T \pi_H U_{T+1}(\sigma^T, w, H) + (1-\mu_T)\pi_L U_{T+1}(\sigma^T, w, L) + \\ + (1 - \pi(\mu_T))U_{T+1}(\sigma^T, u)\big]$$

$$U_T(\mathcal{W}, \mu_T, \sigma^T) \geq u(c_T(\sigma^T)) + \beta U_{T+1}(\sigma^T, u)$$

Since (IC, T) is binding by assumption, it holds

$$U_T(\mathcal{W}, \mu_T, \sigma^T) = u(c_T(\sigma^T)) + \beta U_{T+1}(\sigma^T, u), \quad U_{T+1}(\sigma^T, w) - U_{T+1}(\sigma^T, u) = \frac{e}{\beta\pi(\mu_T)} > 0$$

and then, for $\mu > \mu_T$,

$$\begin{aligned} &U_T(\mathcal{W}, \mu, \sigma^T) - \beta U_{T+1}(\mathcal{W}, \mu, (\sigma^T, u)) = \\ =&u(c_T(\sigma^T)) - e + \beta\pi(\mu)\big[U_{T+1}(\mathcal{W}, (\sigma^T, w)) - U_{T+1}(\mathcal{W}, (\sigma^T, u))\big] > \\ >&u(c_T(\sigma^T)) - e + \beta\pi(\mu_T)\big[U_{T+1}(\mathcal{W}, (\sigma^T, w)) - U_{T+1}(\mathcal{W}, (\sigma^T, u))\big] = \\ =&U_T(\mathcal{W}, \mu_T, \sigma^T) - \beta U_{T+1}(\mathcal{W}, \mu_T, (\sigma^T, u)) \end{aligned}$$

which proves (2.46) for $t = T$.

Induction Step ($t \leq T - 1$)

First, notice that

$$\begin{aligned} &U_t(\mathcal{W}, \mu_t, \sigma^t) = u(c_t(\sigma^t)) + \beta U_{t+1}(\mathcal{W}, \mu_t, \sigma^{t+1}) \\ \Longrightarrow& - e + \beta\pi(\mu_t)\big[U_{t+1}(\mathcal{W}, (\sigma^t, w)) - U_t(\mathcal{W}, \mu_{t+1}, \sigma^{t+1})\big] \\ &= \beta\big[U_{t+1}(\mathcal{W}, \mu_t, \sigma^{t+1}) - U_{t+1}(\mathcal{W}, \mu_{t+1}, \sigma^{t+1})\big] \\ &= \beta\big[-e + \beta\pi(\mu_t)\big(U_{t+2}(\mathcal{W}, (\sigma^{t+1}, w)) - U_{t+2}(\mathcal{W}, \mu_{t+1}, \sigma^{t+2})\big] \\ \Longrightarrow& U_{t+1}(\mathcal{W}, (\sigma^t, w)) - U_{t+1}(\mathcal{W}, \mu_{t+1}, \sigma^{t+1}) > \beta\big[U_{t+2}(\mathcal{W}, (\sigma^{t+1}, w)) - U_{t+2}(\mathcal{W}, \mu_{t+1}, \sigma^{t+2})\big] \end{aligned} \tag{2.47}$$

where the equalities hold since (IC, t) and (IC, $t+1$) are binding. Now, by induction hypothesis, $\forall\mu : \mu > \mu_{t+1}$,

$$\begin{aligned} &U_{t+1}(\mathcal{W}, \mu_{t+1}, \sigma^{t+1}) = u(c_{t+1}(\sigma^{t+1})) + \beta U_{t+2}(\mathcal{W}, \mu_{t+1}, \sigma^{t+2}) \\ \Longrightarrow& U_{t+1}(\mathcal{W}, \mu, \sigma^{t+1}) \geq u(c_{t+1}(\sigma^{t+1})) + \beta U_{t+2}(\mathcal{W}, \mu, \sigma^{t+2}) \end{aligned}$$

Therefore, take any $\mu > \mu_t$ and

$$
\begin{aligned}
&U_t(\mathcal{W}, \mu_t, \sigma^t) - \beta U_{t+1}(\mathcal{W}, \mu_t, \sigma^{t+1}) = \\
=&u(c_t(\sigma^t)) - \beta u(c_{t+1}(\sigma^{t+1})) - e(1-\beta) + \beta\pi(\mu_t)\big[U_{t+1}(\mathcal{W}, (\sigma^t, w)) - \beta U_{t+2}(\mathcal{W}, (\sigma^{t+1}, w))\big] + \\
&+ \beta(1-\pi(\mu_t))\big[U_{t+1}(\mathcal{W}, \mu_t^u, \sigma^{t+1}) - \beta U_{t+2}(\mathcal{W}, \mu_t^u, \sigma^{t+2})\big] \\
<&u(c_t(\sigma^t)) - \beta u(c_{t+1}(\sigma^{t+1})) - e(1-\beta) + \beta\pi(\mu)\big[U_{t+1}(\mathcal{W}, (\sigma^t, w)) - \beta U_{t+2}(\mathcal{W}, (\sigma^{t+1}, w))\big] + \\
+&\beta(1-\pi(\mu))\big[U_{t+1}(\mathcal{W}, \mu_t^u, \sigma^{t+1}) - \beta U_{t+2}(\mathcal{W}, \mu_t^u, \sigma^{t+1})\big] \\
<&u(c_t(\sigma^t)) - \beta u(c_{t+1}(\sigma^{t+1})) - e(1-\beta) + \beta\pi(\mu)\big[U_{t+1}(\mathcal{W}, (\sigma^t, w)) - \beta U_{t+2}(\mathcal{W}, (\sigma^{t+1}, w))\big] + \\
&+ \beta(1-\pi(\mu))\big[U_{t+1}(\mathcal{W}, \mu^u, \sigma^{t+1}) - \beta U_{t+2}(\mathcal{W}, \mu^u, \sigma^{t+2})\big] \\
=&U_t(\mathcal{W}, \mu, \sigma^t) - \beta U_{t+1}(\mathcal{W}, \mu, \sigma^{t+1})
\end{aligned}
$$

where the first inequality follows from (2.47) above, as $\pi(\mu) > \pi(\mu_t)$, while the second inequality follows from induction hypothesis. I can thus conclude that (2.46) holds also for t.

I now pass to the proof of the main part of the proposition, that is, that binding IC constraints is a sufficient condition to account for all possible deviations occurring from t onward. By induction hypothesis, $\mathcal{W}$ satisfies all $(IC,\ s)_{s=t}^T$ with equality, and that guarantees robustness to all possible deviations over histories in $\alpha\backslash\sigma^{t+1}$, i.e.

$$U_{t+1}(\mathcal{W}, \mu_{t+1}, \sigma^{t+1}) \geq U_{t+1}((\mathbf{c}, \mathbf{a}')(\alpha\backslash\sigma^{t+1}), \mu_{t+1}, \sigma^{t+1}), \quad \forall\ \mathbf{a}'(\alpha\backslash\sigma^{t+1}) \in A_{t+1}(\alpha\backslash\sigma^{t+1})$$

What it is to show is that $\mathcal{W}$ is robust also to all possible deviations in $\alpha\backslash\sigma^t$, i.e.

$$U_t(\mathcal{W}, \mu_t, \sigma^t) \geq U_t((\mathbf{c}, \mathbf{a}')(\alpha\backslash\sigma^t), \mu_t, \sigma^t), \quad \forall\ \mathbf{a}'(\alpha\backslash\sigma^t) \in A_t(\alpha\backslash\sigma^t)$$

First of all, notice that $A_t(\alpha\backslash\sigma^t) = \{0, e\} \times A_{t+1}(\alpha\backslash\sigma^{t+1})$ can be decomposed into:

- all effort histories with positive effort in t, i.e. $A_e = A_t(\alpha\backslash\sigma^t) \cap \{a_t(\sigma^t) = e\}$;

- all effort histories with zero effort in t, i.e. $A_0 = A_t(\alpha\backslash\sigma^t) \cap \{a_t(\sigma^t) = 0\}$;

Second, assumption on robustness to any $\mathbf{a}'(\alpha\backslash\sigma^{t+1}) \in A_{t+1}(\alpha\backslash\sigma^{t+1})$ guarantees robustness of $\mathcal{W}$ to the first set of deviations A_e, since $\mu_{t+1} = \frac{\mu_t(1-\pi_H)}{1-\pi(\mu_t)} = \mu_t^u$. Indeed, pick any $\mathbf{a}'(\alpha\backslash\sigma^t) \in A_e$. Then, it follows

$$U_t(\mathcal{W}, \mu_t, \sigma^t) = u(c_t(\sigma^t)) - e + \beta\big[\pi(\mu^t)U_{t+1}(\mathcal{W}, \mu_t^w, (\sigma^t, w)) + (1-\pi(\mu^t))U_{t+1}(\mathcal{W}, \mu_t^u, \sigma^{t+1})\big]$$
$$\geq u(c_t(\sigma^t)) - e + \beta\big[\pi(\mu^t)U_{t+1}(\mathcal{W}, \mu_t^w, (\sigma^t, w)) + (1-\pi(\mu^t))U_{t+1}(\mathcal{W}', \mu_t^u, \sigma^{t+1})\big] = U_t(\mathcal{W}', \mu_t, \sigma^t)$$

where the inequality follows from robustness to $\mathbf{a}'(\alpha\backslash\sigma^{t+1}) \in A_{t+1}(\alpha\backslash\sigma^{t+1})$.

What is left to show is robustness of $\mathcal{W}$ to A_0. By assumption, (IC, t) and (IC, $t+1$) are binding, which means that

$$U_t(\mathcal{W}, \mu_t, \sigma^t) = U_t((\mathbf{c}, \tilde{\mathbf{a}})(\alpha\backslash\sigma^t), \mu_t, \sigma^t) = U_t((\mathbf{c}, \hat{\mathbf{a}})(\alpha\backslash\sigma^t), \mu_t, \sigma^t) \tag{2.48}$$

with $\hat{\mathbf{a}}(\alpha\backslash\sigma^t) = (0, e\mathbf{1}_k)$, $\tilde{\mathbf{a}}(\alpha\backslash\sigma^t) = (e, 0, e\mathbf{1}_{k-1})$. Define $\tilde{\mathcal{W}} = (\mathbf{c}, \tilde{\mathbf{a}})(\alpha\backslash\sigma^t)$ and $\hat{\mathcal{W}} = (\mathbf{c}, \hat{\mathbf{a}})(\alpha\backslash\sigma^t)$. Thus, by construction

$$U_{t+2}(\tilde{\mathcal{W}}, \mu_t^u, \sigma^{t+2}) = U_{t+2}(\hat{\mathcal{W}}, \mu_t^u, \sigma^{t+2}) \tag{2.49}$$

Indeed, both alternative strategies prescribe to set effort cost to 0 either at stage t or $t+1$ (but not both), and therefore the expectation at node $\sigma^{t+2} = (\sigma^t, u, u)$ is equal to μ_t^u under both strategies. Moreover, they prescribe positive effort forever after, until the last period T.

Pick any $\mathbf{a}'(\alpha\backslash\sigma^t) \in A_0$. There are two possibilities: $a'_{t+1}(\sigma^{t+1}) = e$ or $a'_{t+1}(\sigma^{t+1}) = 0$. If the first case applies, consider the alternative deviation strategy $\mathbf{a}''(\alpha\backslash\sigma^t) \in A_e$ so constructed:

$$a''_t(\sigma^t) = e, \quad a''_{t+1}(\sigma^{t+1}) = 0, \quad \mathbf{a}''(\alpha\backslash\sigma^{t+2}) = \mathbf{a}'(\alpha\backslash\sigma^{t+2})$$

Likewise, define $\mathcal{W}' = (\mathbf{c}, \mathbf{a}')(\alpha\backslash\sigma^t)$ and $\mathcal{W}'' = (\mathbf{c}, \mathbf{a}'')(\alpha\backslash\sigma^t)$. Hence, by construction,

$$U_{t+2}(\mathcal{W}', \mu_t^u, \sigma^{t+2}) = U_{t+2}(\mathcal{W}'', \mu_t^u, \sigma^{t+2}) \tag{2.50}$$

for the same reason as in (2.49), and

$$U_t(\mathcal{W}', \mu_t, \sigma^t) = U_t(\hat{\mathcal{W}}, \mu_t, \sigma^t) + \beta^2(1-\pi(\mu_t))\big[U_{t+2}(\mathcal{W}', \mu_t^u, \sigma^{t+2}) - U_{t+2}(\hat{\mathcal{W}}, \mu_t^u, \sigma^{t+2})\big]$$
$$U_t(\mathcal{W}'', \mu_t, \sigma^t) = U_t(\tilde{\mathcal{W}}, \mu_t, \sigma^t) + \beta^2(1-\pi(\mu_t))\big[U_{t+2}(\mathcal{W}'', \mu_t^u, \sigma^{t+2}) - U_{t+2}(\tilde{\mathcal{W}}, \mu_t^u, \sigma^{t+2})\big]$$

which follows from the fact that $\mathcal{W}'$ is identical to $\hat{\mathcal{W}}$ in periods t and $t+1$, and the same holds true for $\mathcal{W}''$ and $\tilde{\mathcal{W}}$.

One can easily see that the RHS of the two equations are equal, by (2.48), (2.49) and (2.50), which causes also the LHS to be equal

$$U_t(\mathcal{W}', \mu_t, \sigma^t) = U_t(\mathcal{W}'', \mu_t, \sigma^t)$$

But then, given that $\mathcal{W}$ is robust to any alternative strategy in A_e,

$$\mathbf{a}''(\alpha\backslash\sigma^t) \in A_e \Longrightarrow U_t(\mathcal{W}, \mu_t, \sigma^t) \geq U_t(\mathcal{W}'', \mu_t, \sigma^t) = U_t(\mathcal{W}', \mu_t, \sigma^t)$$

proving that $\mathcal{W}$ is robust to $\mathbf{a}'(\alpha\backslash\sigma^t)$, too.

Now, consider the case where $a'(\sigma^{t+1}) = 0$ and the strategies $\hat{\mathbf{a}}$ and $\tilde{\mathbf{a}}$ defined as above, and also $\ddot{\mathbf{a}}(\alpha\backslash\sigma^t) = (0, 0, e\mathbf{1}_{k-1})$. I first show that

$$U_t(\mathcal{W}, \mu_t, \sigma^t) \geq U_t(\ddot{\mathcal{W}}, \mu_t, \sigma^t)$$

under the assumption of (IC, t) being binding

$$U_t(\mathcal{W}, \mu_t, \sigma^t) = U_t(\hat{\mathcal{W}}, \mu_t, \sigma^t) = u(c_t(\sigma^t)) + \beta U_{t+1}(\hat{\mathcal{W}}, \mu_t, \sigma^{t+1})$$

which boils down to prove that

$$\begin{aligned} & U_{t+1}(\hat{\mathcal{W}}, \mu_t, \sigma^{t+1}) \geq U_{t+1}(\ddot{\mathcal{W}}, \mu_t, \sigma^{t+1}) = u(c_{t+1}(\sigma^{t+1})) + \beta U_{t+2}(\ddot{\mathcal{W}}, \mu_t, \sigma^{t+2}) \\ \Longrightarrow\ & U_{t+1}(\mathcal{W}, \mu_t, \sigma^{t+1}) \geq u(c_{t+1}(\sigma^{t+1})) + \beta U_{t+2}(\mathcal{W}, \mu_t, \sigma^{t+2}) \end{aligned} \tag{2.51}$$

where the first inequality follows from the fact that both strategies prescribe no effort in t, and the second inequality follows from the fact that $\hat{\mathcal{W}} = \mathcal{W}$ (resp., $\ddot{\mathcal{W}} = \mathcal{W}$) over $\alpha\backslash t+1$ (resp., $\alpha\backslash t+2$). By assumption, (IC, $t+1$) is binding

$$U_{t+1}(\mathcal{W}, \mu_t^u, \sigma^{t+1}) = u(c_{t+1}) + \beta U_{t+2}(\mathcal{W}, \mu_t^u, \sigma^{t+2})$$

which, jointly with (2.46) and since $\mu_t^u < \mu_t$, causes (2.51). Now, $\mathbf{a}'$ and $\ddot{\mathbf{a}}$ prescribe the same action in periods t and $t+1$. Therefore, in order to prove that $\mathcal{W}$ is robust against

$\mathbf{a}'(\alpha\backslash\sigma^t)$, it is enough to show that

$$U_{t+2}(\mathcal{W}, \mu_t, \sigma^{t+2}) = U_{t+2}(\ddot{\mathcal{W}}, \mu_t, \sigma^{t+2}) \geq U_{t+2}(\mathcal{W}', \mu_t, \sigma^{t+2}) \tag{2.52}$$

Now, there are two possibilities, $a'_{t+2}(\sigma^{t+2})$ can either be 0 or e. If the first case occurs, in order to prove (2.52) it is enough to show

$$U_{t+3}(\mathcal{W}, \mu_t, \sigma^{t+3}) \geq U_{t+3}(\mathcal{W}', \mu_t, \sigma^{t+3}) \tag{2.53}$$

Indeed, (IC, $t+2$) binding and (2.46) jointly cause

$$U_{t+2}(\mathcal{W}, \mu_t, \sigma^{t+2}) \geq u(c_{t+2}(\sigma^{t+2})) + \beta U_{t+3}(\mathcal{W}, \mu_t, \sigma^{t+3})$$

On the other hand, if $a'_{t+2}(\sigma^{t+2}) = e$, then

$$U_{t+2}(\breve{\mathcal{W}}, \mu_t, \sigma^{t+2}) = u(c_{t+2}(\sigma^{t+2})) - e + \beta\big[\pi(\mu_t)U_{t+3}(\mathcal{W}, (\sigma^{t+2}, w)) + (1-\pi(\mu_t))U_{t+3}(\breve{\mathcal{W}}, \mu_t^u, \sigma^{t+3})\big],$$
$$\breve{\mathcal{W}} = \{\mathcal{W}, \mathcal{W}'\}$$

But then proving (2.52) boils down to show (2.53). I have just established the following implication

$$U_{j+1}(\mathcal{W}, \mu'_{j+1}, \sigma^{j+1}) \geq U_{j+1}(\mathcal{W}', \mu'_{j+1}, \sigma^{j+1}) \Longrightarrow U_j(\mathcal{W}, \mu'_j, \sigma^j) \geq U_j(\mathcal{W}', \mu'_j, \sigma^j), \quad \forall j : t \leq j \leq T$$

where μ'_j is the expectation in period j if strategy $\mathbf{a}'$ is applied. But then the proof is complete, as

$$U_{T+1}(\mathcal{W}, \mu'_{T+1}, \sigma^{T+1}) = U_{T+1}(\mathcal{W}, \sigma^{T+1}) = U_{T+1}(\mathcal{W}', \mu'_{T+1}, \sigma^{T+1}) \Longrightarrow U_t(\mathcal{W}, \mu'_t, \sigma^j) \geq U_t(\mathcal{W}', \mu'_t, \sigma^t)$$

■

Lemma 7 proves to be useful in light of the following result.

Lemma 8. *In optimum, all (IC, $s)_{s=0}^T$ constraints are binding.*

Proof. By contradiction, assume that $\mathcal{W} = (\mathbf{c}, \mathbf{a})(\alpha\backslash\sigma^0)$ is optimum and that (IC, t) is slack

$$U_t(\mathcal{W}, \mu_t, \sigma^t) > u(c_t(\sigma^t)) + \beta U_{t+1}(\mathcal{W}, \mu_t, (\sigma^t, u))$$

Then there exists $\varepsilon > 0$ such that

$$\begin{cases} c'_{t+1}(\sigma^t, w) = c_{t+1}(\sigma^t, w) - \varepsilon \\ U_t(\mathcal{W}', \mu_t, \sigma^t) = u(c_t(\sigma^t)) + \beta U_{t+1}(\mathcal{W}', \mu_t, \sigma^{t+1}) \end{cases}$$

where $\mathcal{W}' = (\mathbf{c}', \mathbf{a})(\alpha \backslash \sigma^0)$ is defined as

$$c'_s(\sigma^s) = c_s(\sigma^s),\ \forall \sigma^s \neq (\sigma^t, w), \quad c'_{t+1}(\sigma^t, w) = c_{t+1}(\sigma^t, w) - \varepsilon$$

Now, government's payoff is larger under $\mathcal{W}'$ than under $\mathcal{W}$, as payment to the worker in history (σ^t, w) is lower in the former case. Moreover, by Lemma 7, $\mathcal{W}'$ is also feasible, since it satisfies all (IC, $s)_{s=0}^T$ constraints with equality. But this contradicts that $\mathcal{W}$ is optimum. ■

Thus, robustness against all one-shot deviations from the prescribed effort sequence constitutes a necessary condition for a contract to be optimum (by Lemma 8) and sufficient one for it to be robust against any multiple deviation (by Lemma 7). Therefore, focusing on the set of contracts with such characteristic is without loss of generality.

Pandemics and Cooperation: an Efficiency-Based Perspective

ABSTRACT

Pandemics are global phenomena confronted by domestic containment measures. Domestic measures trade off economic and human losses suffered by the country that adopts them. However, domestic policymakers overlook the impact their decisions have on the risk of cross-border contagion and end up adopting too mild restrictions compared to the social optimum. Bilateral transfers, contingent on the evolution of the pandemic, constitute a mutual insurance scheme among countries and a channel for internalizing such spillovers. More infected countries receive larger transfers on the condition that they prove able to limit the spread of the contagion, and this creates incentives for them to adopt more stringent measures. In addition, the productivity of factors being positively correlated with the diffusion of the pandemic produces a concentration of investments toward less infected countries, which may reverse the direction of transfers.

JEL classification: F36, H23, H41, H75, I18

Keywords: Arrow-Debreu equilibrium, insurance, negative spillovers, Next-Generation EU, pandemics, policy coordination

3.1 Introduction

Covid-19 has shown how important it is for countries to coordinate policies to contain the spread and limit the human and economic costs of a pandemic. Domestic containment measures have an impact on the risk of cross-border contagion, that national policymakers do not internalize. Indeed, when deciding the severity of measures, national governments face an internal trade-off between public health and the economic losses caused by 'locking down' production, and do not consider the potential damage of their decisions on other countries.[61] If potential external damage were accounted for when deciding domestic restrictions, policymakers would impose more stringent measures *ceteris paribus*. The main obstacle to coordination is that it may give rise to free-riding attitudes by net beneficiaries of the program. In 2020, amid the first pandemic wave in Europe, free-riding was a major concern for a group of EU member States, labelled as the 'frugals', that warned that Southern European countries could spend EU funds in wasteful consensus-driven policies. This caused the Next-Generation EU plan (NGEU, henceforth) to be structured in such a way that transfers to most damaged countries are conditional on the approval of structural reforms according to a precise road map.

How to overcome the limits of domestic policies in the face of a pandemic and avoid any risk of opportunistic behavior at the same time? In other words, how to make policymakers 'think big' and internalize the negative spillovers derived by loose containment measures, and how to do so in an incentive-compatible way? The paper analyzes state-contingent bilateral transfers as a form of mutual insurance. Gains from mutual insurance have two sources. First, countries have the possibility to get insured against the idiosyncratic component of the pandemic shock, as well as to smooth out consumption over different states. Second, transfers lead to the adoption of more severe measures. At the optimum, indeed, 'infected' countries benefit from an inflow of resources from 'non-infected' countries, whose size depends on the future spread of the pandemic. If the pandemic spreads to non-infected countries as well, then the global recession becomes more severe and the size of these transfers shrinks accordingly. As a consequence, governments in the infected countries now have a direct incentive to contain the spread of the virus, given by the gap in payments they are receiving in the different states of the world. This transfer scheme has the advantage that it does not create any risk of free-riding by recipients, as payments to a given country are contingent not only on its epidemiological situation, but also on

[61] The only source of concern would be the boomerang effect if the virus is later re-transmitted from abroad. However, this lagged 'backfire effect' is outside the scope of this paper.

the effects of the policies it implements.

The flow of aids is not heading toward 'infected' countries only. Two elements determine its direction. First, the provision of incentives from non-infected toward infected countries. Second, the need to invest resources in the most efficient way to fight the pandemic. Asking any infected country to produce the medical equipment on its own may result in more contacts among workers and a higher infection rate. In addition, any such country may suffer a fall in the productivity of factors, caused by the disruptive effects of the pandemic. Therefore, an efficient strategy would prescribe to transfer resources from the infected and less productive to the non-infected and more productive countries, hence reversing the flow of international transfers.

The analysis is relevant in the design of actual cooperation programs, like the Next-Generation EU. This paper warns that pandemic damages should not be the only determinant of transfers among EU member States, and that also productivity of factors should guide the allocation of funds aimed at the production of medical supplies. Transfers being (i) contingent on the actual success of measures in limiting the scope of contagion, and (ii) directed also to less affected and most productive countries, would overcome the opposition of 'frugal' countries, which have been most critical of any form of a common European health insurance scheme.

The rest of the paper is organized as follows. Section 3.2 contains the literature review. Section 3.3 presents the optimal insurance scheme in an three-period environment with no investment asset. Section 3.4 extends the analysis to an environment where capital accumulation is possible. Section 3.5 concludes.

3.2 Literature Review

A strand of the literature on Covid-19 studies the economic implications of different policy responses to a pandemic. Guerrieri et al. (2020) model Covid-19 as a negative Keynesian supply shock and study the performance of different fiscal and monetary policy responses to it. The paper provides guidance about the optimal policy response, parametrized to the current state of the pandemic, and estimates its economic gains and losses. However, it does not consider any negative spillover effect of individual behaviors on the overall spread of the pandemic. Jones et al. (2020) study containment measures in a framework where the epidemic dynamics impact the economic activity, and the social-planner is concerned about an infection externality and a healthcare congestion externality. The paper finds that private incentives are too mild to achieve a social optimum, due to a fatalism bias about future infection rates, and that the mitigation policy implemented by the social planner is more drastic and effective than the private one. On the same lines, Eichenbaum et al. (2021) incorporate an extended version of SIR model to study the interaction between economic decisions and epidemics, and argue that the competitive equilibrium is not socially optimal because infected people do not fully internalize the effect of their economic decisions on the spread of the virus.[62] Likewise, this paper studies policy responses in a context of negative externalities caused by individual behavior. However, the paper conducts the analysis at an international level, where national policies bear a negative spillover effect in terms of risk of cross-border contagion. Therefore, no government can adopt policies that implement the social optimum and the only way countries have to mitigate externalities is to coordinate their policies.

A number of papers highlight the importance of policy coordination to earn a global public good (GPG) by decentralized policymakers. The two main obstacles are, first, the lack of commitment power for each agent, who can thus shirk any effort and free ride on others' effort, and second, the presence of spillovers of domestic decisions on other countries. Nordhaus and Yang (1996) have been among the first to study the possible ways of tackling global warming. The authors argue that the non-cooperative solution implies lower effort than the Pareto-efficient solution, where all countries cooperate to achieve the optimum, even though the latter solution delivers much more sizable gains with respect to the former one. Nordhaus (2015) finds that no emission abatement can occur due to free-riding, unless coalitions among countries are created and given the possibility to levy sanctions on non-participants countries. The reason why free-riding is so pervasive in the production of a global public good is that single actions (e.g., carbon emissions or containment measures) impact the world situation (e.g., CO2 concentrations or the spread of the virus) uniformly, irrespective of the place they are taken. In this sense, however, limiting the spread of a pandemic is a different problem. Indeed, it always starts in one country and spreads from there all around the world. Unlike carbon emissions, which are produced by all countries, containment measures only interest those countries who are affected by the pandemic. It is therefore a primal interest of 'healthy' countries to induce the 'sick' ones to adopt containment measures, as this may prevent the virus

[62]Other papers study the interaction between policies and individual behavior. For example, Carnehl et al. (2021) highlight the importance of accounting for the effect of citizens' behavior on policy outcome. In particular, the authors find that policies aimed at decreasing the transmission rate can lead to unintended negative consequences.

from spreading and reduce their human and economic losses. In climate change, instead, the problem can not be circumscribed to any delimited area, as no country is exempted from the abatement of emissions. This fact lowers the prospective gains from cooperation, and makes countries less inclined to undersign any long-term commitment.

3.3 Three-period Model

The world is constituted by two countries -Home (H) and Foreign (F)- which live for three periods ($t \in \{0, 1, 2\}$). Each country is ruled by a government and populated by an unit mass of citizens, who can either be healthy or sick and have no possibility of borrowing or saving (e.g., they are hand-to-mouth). Healthy citizens are endowed with one unit of labor supply ($n_h \in [0, 1]$), whereas sick citizens are labor-constrained ($n_s = 0$) and can not recover. The government acts as a benevolent dictator, by imposing healthy citizens the amount of labor to supply, taxing their income and transferring it to sick people, who have no income otherwise. The government maximizes the present value of social welfare, which comes from citizens' consumption and leisure, and discount future utility by $\beta \in (0, 1)$. In addition to labor supply and consumption,[63] the government also chooses the level of containment measures. Containment measures have a negative impact on labor productivity $A \in [\underline{A}, \overline{A}]$, but lower the possibility of cross-border contagion in the next period. Thus, the probability of cross-border contagion ρ is positively related to the productivity of labor A, e.g. $\rho = \rho(A)$, with $\rho' > 0, \rho'' > 0$. However, containment measures have no impact on the amount of sick citizens in the country where such measures are imposed. Therefore, the share of sick people remains constant over time, once a country is hit by the virus in $t = 1$. Neither government has access to any storage facility or external credit market.[64]

Events unravel according to a specific timeline. In $t = 0$, countries jointly agree on whether to set a mutual insurance scheme in case of any future pandemic shock, by trading a portfolio of state-contingent securities in zero-net supply. In $t = 1$, either country is hit by an idiosyncratic pandemic shock (Home is hit with probability ξ), which infects $(1 - \phi) \times 100\%$ of its population. In $t = 2$, the shock is transmitted with prob $\rho(A)$ to the other country. Each security pays one consumption unit at any given future time and state of the world. In particular, two securities promise payment in $t = 1$, depending

[63] Since the citizens are hand-to-mouth, the government indirectly chooses consumption by selecting labor supply and labor taxation.

[64] This assumption will be relaxed in Section 3.4.

on which of Home and Foreign is infected first. And four securities promise payment in $t = 2$, depending on whether the virus spreads to both countries or not. Countries can fully commit to the insurance scheme and face no risk that their counterpart walks away from the contract in any state of the world.

3.3.1 Autarky

First, the equilibrium is presented under autarky, in which case no transfer is made between countries. Whenever Home is hit by the virus in $t = 1$, it solves the following problem

$$W^s = \max_{c_s, c'_s, A, A', n_h, n'_h} \phi\big[\log(c_h^\gamma(1-n_h)^{1-\gamma}) + \beta\log(c_h'^\gamma(1-n'_h)^{1-\gamma})\big] + (1-\phi)\big[\log(c_s^\gamma) + \beta\log(c_s^\gamma)\big]$$

$$\text{sub:} \quad (1-\phi)c_s + \phi c_h = \phi A n_h \tag{3.54}$$

$$(1-\phi)c'_s + \phi c'_h = \phi A' n'_h \tag{3.55}$$

Given that no storage security is available, the government can not smooth utility over time. It can only tax healthy citizens and transfer wealth to sick ones by balancing the budget constraint at every period. By assumption, the infected country faces no risk of worsening the domestic diffusion of the pandemic. Hence, no internal trade-off exists to determine the level of restrictions.

On the contrary, if Home is not affected by the virus in $t = 1$, it solves the following problem.

$$\begin{aligned} W^h(A^*) = \max_{A, A_+, A_-, c_h, n_h, c_{h,+}, n_{h,+}, c_{s,+}, c_{h,-}, n_{h,-}} & \log(c_h^\gamma(1-n_h)^{1-\gamma}) + \\ & + \beta\rho(A^*)\big[\phi\log(c_{h,+}^\gamma(1-n_{h,+})^{1-\gamma}) + (1-\phi)\log(c_{s,+}^\gamma)\big] + \\ & + \beta(1-\rho(A^*))\log(c_{h,-}^\gamma(1-n_{h,-})^{1-\gamma}) \\ \text{sub:} \quad & c_h = A n_h \\ & c_{h,-} = A_- n_{h,-}, \text{ in case Home is } \textbf{not} \text{ infected in } t = 2 \\ & \phi c_{h,+} + (1-\phi)c_{s,+} = \phi A_+ n_{h,+}, \text{ in case Home is infected in } t = 2 \end{aligned}$$

In this case, Home faces the risk of being infected in the following period, based on the level of restrictions adopted by Foreign. Under such risk, the government would now find optimal to transfer resources from positive to negative state of the world in $t = 2$, as well as over time, i.e. form $t = 1$ to $t = 2$. The following holds.

Lemma (Autarky). *Both countries set the lowest level of containment measures ($A = \overline{A}$). In particular, the optimal policy in Autarky for the country infected in $t = 1$ is:*

$$n_h = n_h' = \frac{\gamma}{\gamma + \phi(1-\gamma)}, \quad c_h = c_h' = c_s = c_s' = \frac{\phi\gamma\overline{A}}{\gamma + \phi(1-\gamma)}$$

The optimal policy in Autarky for the country not infected in $t = 1$ is:

$$c_h = c_{h,-} = \overline{A}\gamma, \quad c_{h,+} = c_{s,+} = \frac{\phi\gamma\overline{A}}{\gamma + \phi(1-\gamma)}, \quad n_h = n_{h,-} = \gamma, \quad n_{h,+} = \frac{\gamma}{\gamma + \phi(1-\gamma)}$$

Proof. See Appendix. ■

Governments of both countries equalize consumption among citizens, by taxing the healthy and subsidizing the sick. However, the absence of any storage security prevents them from achieving any better allocation of resources, i.e. insuring consumption over different periods and states of the world.

The reason why neither country adopts containment measures is that any welfare loss suffered by either country is not the consequence of its own measures. In particular, the country which is infected in $t = 1$ is not concerned about the spread of the virus to its neighbor, as it derives no benefit from limiting the contagion. Such an extreme result originates from two assumptions. First, that no further diffusion of the virus is possible within the country already affected by the virus. And second, that favoring (or not preventing) the spread of the virus to the neighboring country can not backfire in the future, as the world ends in $t = 2$. On the contrary, in an infinite horizon world where contagion can cross frontiers multiple times and in both directions, the country infected in $t = 1$ would also consider that, if the counterpart is infected by the virus at successive stages, the virus could cross back the border and infect an even larger share of the domestic population. However, even in such an alternative (and admittedly more realistic) scenario, the incentive for infected countries to contain the virus is only indirect and too weak to achieve the social optimum.

3.3.2 Cooperation

Countries can cooperate in many ways. One of them is transferring resources directly to the most needy among them. This form of state-contingent risk-sharing is well-known

by the general equilibrium literature to be welfare improving, whenever agents are hit by idiosyncratic shocks, as it allows them to smooth consumption over states. Lemma 3.3.2 shows that bilateral transfers (i.e., state-contingent transfers in zero net supply) are also able to lower the risk that the shock grows in magnitude, by spilling over to other countries. Indeed, the healthy country pledges more generous aids, as far as it remains unaffected by the pandemic in the following period. This creates an incentive for the sick country to lower the risk of cross-border contagion.
Under Cooperation there are 4 possible states of the world in $t = 2$.

$$(s_1, s_2) \in \{+, -\} \times \{+, -\}$$

where $(s_1, s_2) = (+, +)$ means that Home is infected in $t = 1$, and that it does not pass the virus to Foreign in $t = 2$. I label t^+ the transfer in $t = 1$ to Home when infected, and $\{t^{+-}, t^{++}\}$ the transfers in $t = 2$, in case the contagion is passed to Foreign or not, respectively. There are now three budget constraints also in case Home is infected in $t = 1$, as the transfer in $t = 2$ is contingent to the diffusion of the pandemic.

$$t = 1: \quad \phi c_{h,+} + (1 - \phi) c_{s,+} = \phi A_+ n_{h,+} + t^+ \tag{3.56}$$

$$\text{If Foreign not infected in } t = 2: \quad \phi c_{h,++} + (1 - \phi) c_{s,++} = \phi A_{++} n_{h,++} + t^{++} \tag{3.57}$$

$$\text{If Foreign infected in } t = 2: \quad \phi c_{h,+-} + (1 - \phi) c_{s,+-} = \phi A_{+-} n_{h,+-} + t^{+-} \tag{3.58}$$

The problem of the government when Home is hit by the virus in $t = 1$ now reads[65]

$$\begin{aligned} W^s(t^+, t^{+-}, t^{++}) = \max_{\mathbf{c}_+, \mathbf{n}_+, \mathbf{A}_+} & \phi \log(c_+^\gamma (1 - n_+)^{1-\gamma}) + (1 - \phi) \log(c_+^\gamma) + \\ & + \beta \big\{ \rho(A_+) \big[\phi \log(c_{+-}^\gamma (1 - n_{+-})^{1-\gamma}) + (1 - \phi) \log(c_{+-}^\gamma) \big] + \\ & + (1 - \rho(A_+)) \big[\phi \log(c_{++}^\gamma (1 - n_{++})^{1-\gamma}) + (1 - \phi) \log(c_{++}^\gamma) \big] \big\} \\ & \text{subject to (3.56), (3.57) and (3.58)} \end{aligned}$$

[65] To ease notation, I define $\mathbf{x}_i = (x_i, x_{i,+}, x_{i,-})$, $x = \{c, n, A\}$, $i = \{+, -\}$ and already account for the fact that in optimum, consumption is equalized across healthy and sick people in all times and states ($c_{h,.} = c_{s,.}$).

On the contrary, if Home is not infected in $t = 1$, its constraints are

$$t = 1: \quad c_{h,-} = A_- n_{h,-} + t^- \tag{3.59}$$

$$\text{If Home not infected in } t = 2: \quad c_{h,--} = A_{--} n_{h,--} + t^{--} \tag{3.60}$$

$$\text{If Home infected in } t = 2: \quad \phi c_{h,-+} + (1-\phi) c_{s,-+} = \phi A_{-+} n_{h,-+} + t^{-+} \tag{3.61}$$

and government's problem reads

$$\begin{aligned} W^h(t^-, t^{--}, t^{-+}, A_-^*) = \max_{\mathbf{c}_-, \mathbf{n}_-, \mathbf{A}_-} \; & \log(c_-^\gamma (1-n_-)^{1-\gamma}) + \beta(1-\rho(A_-^*)) \log(c_{--}^\gamma (1-n_{--})^{1-\gamma}) \\ & + \beta\rho(A_-^*) \big\{ \phi \log(c_{-+}^\gamma (1-n_{-+})^{1-\gamma}) + (1-\phi) \log(c_{-+}^\gamma) \big\} \\ & \text{subject to (3.59), (3.60) and (3.61)} \end{aligned}$$

The following lemma reports the equilibrium level of restrictions under Cooperation.

Lemma (Cooperation). *Like in Autarky, if Home is* ***not*** *infected in $t = 1$, it chooses the lowest level of restrictions ($A_- = A_{-+} = A_{--} = \overline{A}$). On the contrary, if Home is infected in $t = 1$, it sets $A_{++} = A_{+-} = \overline{A}$ in $t = 2$, while in $t = 1$ it chooses A_+ to be either a boundary value ($A \in \{\underline{A}, \overline{A}\}$) or the solution of*

$$\phi n_+ = \beta\rho'(A_+) \left[\phi \log\left(\frac{c_{++}^\gamma (1-n_{++})^{1-\gamma}}{c_{+-}^\gamma (1-n_{+-})^{1-\gamma}} \right) + (1-\phi) \log\left(\frac{c_{++}}{c_{+-}} \right)^\gamma \right] \tag{3.62}$$

Proof. See Appendix. ■

Eq. 3.62 strikes a balance between gains and costs of loosening restrictions, when bilateral transfers are in place. The left-hand side displays the marginal return of A in terms of higher productivity of factors, while the right-hand side displays the marginal cost of A in terms of higher risk that the unfavorable scenario verifies. In Autarky, the cost component is null as the infected country bears no wealth-related risk in $t = 2$. Therefore, bilateral transfers favor the alignment of incentives between Home and Foreign by generating a wealth dispersion in the future states of the world also for the country that is infected first.

I am now ready to introduce the equilibrium concept that pins down the prices and quantities of state-contingent transfers traded in $t = 0$.

Definition (Arrow-Debreu Equilibrium). *An Arrow-Debreu equilibrium is an alloca-*

tion of consumption, labor supply, total factor productivity, transfers and prices

$$(\mathbf{c}, \mathbf{c}^*, \mathbf{n}, \mathbf{n}^*, \mathbf{A}, \mathbf{A}^*, \mathbf{t}, \mathbf{t}^*, \mathbf{p}), \quad \textit{with} \quad \mathbf{x} = (x_+, x_-, x_{++}, x_{+-}, x_{-+}, x_{--})$$

such that

- $(\mathbf{c}, \mathbf{n}, \mathbf{A}, \mathbf{t})$ *solves the problem of Home*

$$V = \max_{\mathbf{t}} \xi W^s(t^+, t^{+-}, t^{++}) + (1-\xi) W^h(t^-, t^{--}, t^{-+}, A^*_-)$$
$$\textit{sub:} \quad \mathbf{p}'\mathbf{t} = 0$$

- $(\mathbf{c}^*, \mathbf{n}^*, \mathbf{A}^*, \mathbf{t}^*)$ *solves the problem of Foreign*

$$V^* = \max_{\mathbf{t}^*} \xi W^h(t^+_*, t^{++}_*, t^{+-}_*, A_+) + (1-\xi) W^s(t^-_*, t^{-+}_*, t^{--}_*)$$
$$\textit{sub:} \quad \mathbf{p}'\mathbf{t}^* = 0$$

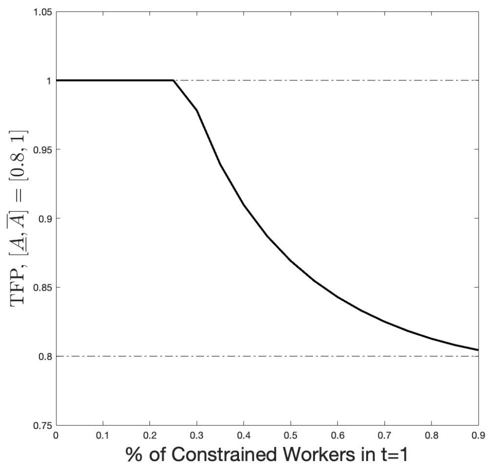
- *markets clear*

$$t^+ + t^+_* = 0, \quad t^{+-} + t^{+-}_* = 0, \quad t^{++} + t^{++}_* = 0,$$
$$t^- + t^-_* = 0, \quad t^{-+} + t^{-+}_* = 0, \quad t^{--} + t^{--}_* = 0$$

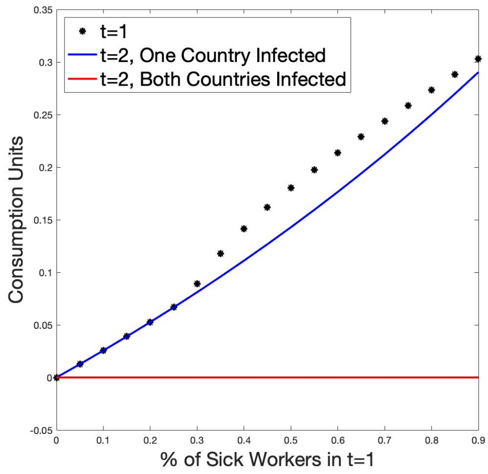
A relevant characteristic of viruses is their contagiousness.[66] Pandemics differ greatly in this respect. Therefore, it is of interest observing how equilibrium containment measures (Fig. 3.14a) and transfers (Fig. 3.14b) vary according to the contagiousness of the virus in terms of share of people infected upon country's infection (i.e., $1-\phi$).[67]

[66] As I write, thirteen variances of Covid-19 have been detected and divided between 'Variants of interest' and 'Variants of concern', according to their contagiousness. Source: https://www.who.int/en/activities/tracking-SARS-CoV-2-variants.

[67] Parameters have been selected as follows: $\beta = 0.99$, $\gamma = 0.5$, $\underline{A} = 0.8$, $\overline{A} = 1$, $\xi = 0.5$ and $\rho(x) = \frac{1}{2}\left(\frac{x-\underline{A}}{\overline{A}-\underline{A}}\right)^2$.



(a) TFP



(b) Transfers

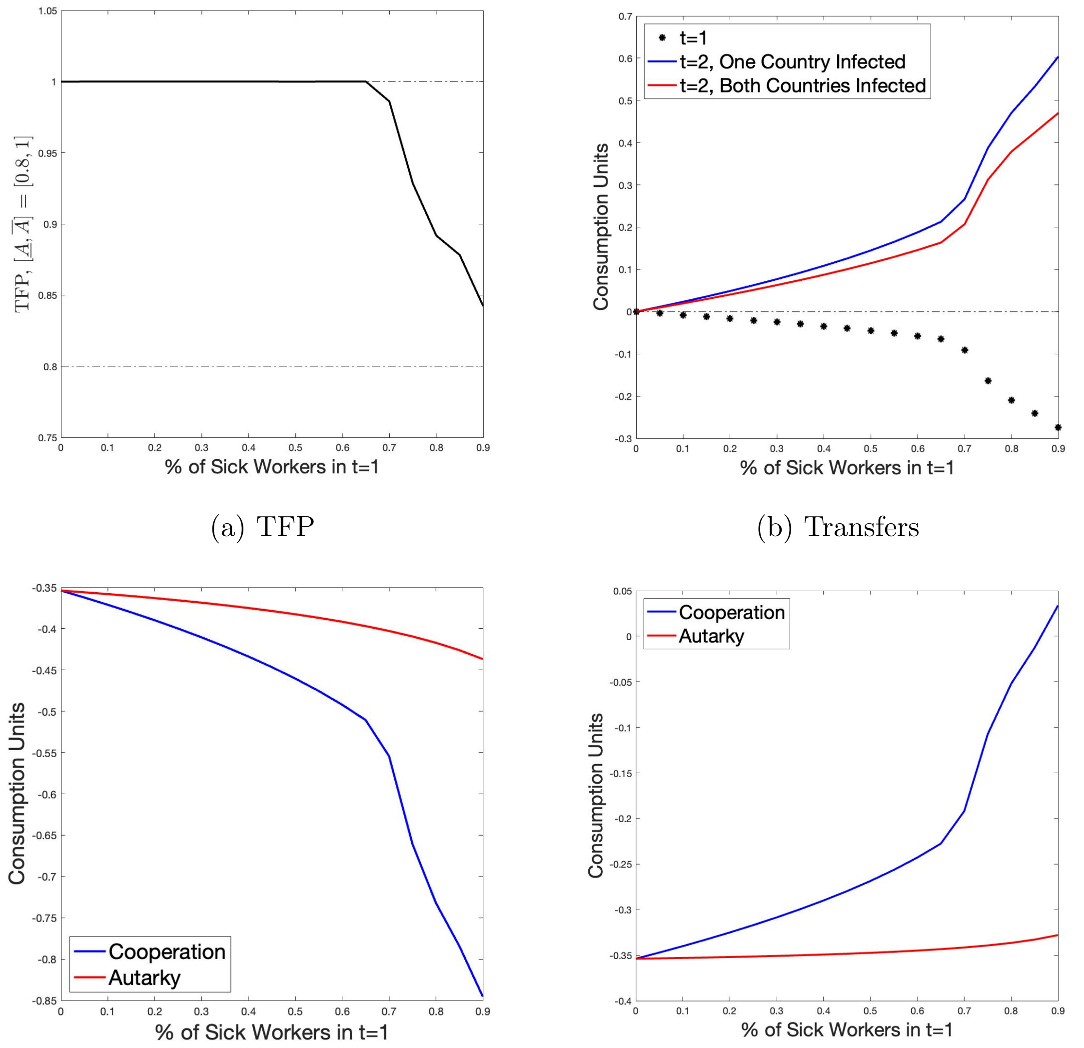
On the x-axis, the figures report the fraction of sick people in the country infected in $t = 1$. Fig. 3.14a shows that the level of containment measures is decreasing in the severity of contagion, and becomes null for $1 - \phi < 0.25$. Indeed, when the fraction of healthy people upon infection is around 1, that is, when almost all people are not affected by the virus, the government does not impose any restriction and leaves labor productivity at its maximum level $A = \overline{A}$, as in autarky. However, when the fraction of healthy people decreases, then government sets increasingly restrictive measures, which in turn cause a fall in labor productivity down to 0.82% of its full potential when 40% of the population is healthy. Fig. 3.14b shows the amount of transfers received by the country. The country benefits from transfers that increase in the contagiousness of the virus only in case the virus is not transmitted. Otherwise, no aid is shipped, as both countries find themselves in the same situation, with only the same fraction of healthy citizens. The difference between the transfers occurring in $t = 2$ according to different state realizations (blue and red lines in Fig. 3.14b) constitutes an incentive for the infected country in $t = 1$ to impose containment measures ($A < \overline{A}$).

3.4 Capital Accumulation

During the first wave of the Covid-19 pandemic (January-June, 2020), many countries were short of medical supplies (face masks, ICU ventilators, PCR tests, etc.) to prevent infections and contain the virus. These items made the difference in terms of reduced infection rate and death toll, and were shipped from non-infected to infected countries. Countries with a larger stock of medical supplies could also achieve the same effects on the prevention of contagion with milder measures, or alternatively adopt more stringent measures and save more lives. In this sense, medical supplies are a way to relax the trade-off between economic and human losses. This Section answer the question of where to locate the production of medical devices. Producing devices in an area with a high infection rate may result in more contacts and infections. Cooperation among countries avoids this problem, as it allows countries to concentrate the production in countries which are initially less affected by the pandemic, and then ship devices to the infected countries. To this aim, countries should use the transfer scheme in a ‘reversed’ way, that is, infected countries should initially invest resources in non-infected ones. The framework now changes as follows. Each country can accumulate capital over time and use it to produce income, according to the function

$$y = Ak^{\alpha}n^{1-\alpha}$$

Capital stock depreciates at 5% rate every period ($\delta = 0.05$) and can be disinvested at no additional cost.



(a) TFP (b) Transfers

(c) Investment - Infected Country (d) Investment - Non-Infected Country

Figure 3.15: Transfers and containment measures with initial capital $k = k^* = 1$

Fig. 3.15 reports the patterns of transfers and containment measures when countries can accumulate capital. In Fig. 3.15a, the pattern of TPF in Home when Home is infected $t = 1$ decreases as a function of the virus' contagiousness, as in Fig. 3.14a. Depending on initial capital, Home can be more or less constrained in $t = 1$ and the TFP curve can shift up or down, accordingly. This finding mirrors the fact that countries with larger capital endowments can afford to adopt more stringent measures and suffer larger economic downturns, in order to reduce the human cost of the pandemic. Fig. 3.15b plots transfers to Home against the percentage of its citizens that are sick. As in Fig. 3.14b, there is a difference in transfers conditional on the contagion spreading also to Foreign or not, which constitutes an incentive for Home's government to adopt more stringent measures. The striking difference with respect to Fig. 3.14b is that transfers in $t = 1$ from the Foreign to Home are now negative, meaning that the infected country is transferring money to the non-infected one. This counterintuitive result is explained by Fig. 3.15c-3.15d, which display capital investment under cooperation (blue line) and autarky (red line), as a function of sick workers in $t = 1$. When Home is affected by the shock (left picture), it disinvests from capital to a much larger extent compared to autarky. The reason is that, by transferring resources to Foreign, Home 'invests' money at higher expected returns, knowing that if the pandemic is circumscribed in $t = 2$ -a scenario whose odds are under its control-, the marginal productivity of capital in Foreign is not scaled down by a factor of $\phi^{1-\alpha}$. Indeed, the marginal productivity of capital in $t = 2$ in Foreign in case of no cross border contagion is

$$MP_k^{++} = \alpha \overline{A} k^{*-(1-\alpha)} {n_{++}^*}^{1-\alpha}, \quad \text{with} \quad n_{++}^* = \gamma$$

Under the alternative scenario, it reads

$$MP_k^{+-} = \alpha \overline{A} k^{*-(1-\alpha)} (\phi n_{+-}^*)^{1-\alpha}, \quad \text{with} \quad n_{+-}^* = \frac{\gamma}{\gamma + \phi(1-\gamma)}$$

and is easily verified to be lower than MP_k^{++}.

In conclusion, when resources can flow between countries, an efficiency argument advocates for polarization of resources toward countries who are spared by the first wave of the pandemic, in search for higher returns. This result is admittedly counterintuitive and reverses the logic of assistance provision that usually lies behind bilateral relationships among countries. Modeling medical supplies as productive capital overlooks a relevant aspect, as it does not allow for supplies to change the transmission rate of the virus. This could be remedied by allowing the probability of cross-border contagion ρ to decrease in both restrictions and capital. In this case, the flow of resources from infected to non-infected countries would be mitigated by a contrasting force advocating for larger investments in medical devices to be realized in infected countries for the active containment of the virus.

3.5 Conclusions

This paper studies bilateral transfers among countries as a form of mutual insurance against pandemic shocks. Two channels determine the direction and size of transfers. First, an incentive-driven channel produces an inflow of resources into those countries that are most affected by the pandemic wave, on the condition that the confinement of the virus is successful. Transfers being dependent on the success of containment policies avoids any risk that recipient countries free-ride on foreign aids and divert resources toward wasteful public spending. Second, a production-driven channel may reverse the flow of aids toward countries that are less affected by the virus and whose productivity of factors is higher, as they bear a lower risk of increasing the number of infections. On the one hand, the incentive-driven channel lowers the risk that the pandemic shock grows in magnitude and worsens the scarcity of resources. On the other hand, the production-driven channel favors efficient investment that leads to a larger aggregate amount of resources in the future. Therefore, bilateral transfers generate aggregate welfare gains by (i) reducing negative externalities that originate from domestic policies, and (ii) allocating investments in the most productive way.

Going forward, an empirical estimation of the size of transfers in the European context constitutes a natural extension of the analysis of this paper. The multi-period interaction between countries that occurs in reality would represent a major difference with respect to the three-period framework analyzed in this paper. In a dynamic framework, indeed, each country would consider also the possibility that transmitting the pandemic to other countries could possibly lead to reinfections at subsequent stages. For this reason, countries would adopt some level of restrictions even in the absence of mutual insurance to minimize the risk of backfire effects.[68] However, bilateral transfers would still constitute an additional incentive for infected countries to adopt more severe measures. A second extension of the paper could analyze the connections between mutual insurance and public debt. If passive interests on debt are directly related to the debt-GDP ratio, bilateral transfers could favor convergence of these ratios across countries by closing the spreads on sovereign debt yields.

[68] In an infinite-horizon framework, countries could also implement a *trigger-strategy equilibrium* where each of them adopts rather stringent containment measures, due to concerns of future retaliation by the other countries.

APPENDIX

Autarky

If Home is hit by the virus in $t = 1$

$$\begin{aligned}\Lambda^s =&\phi\big[\log(c_h^\gamma(1-n_h)^{1-\gamma})+\beta\log({c'_h}^\gamma(1-n'_h)^{1-\gamma})\big]+(1-\phi)\big[\log(c_s^\gamma)+\beta\log({c'_s}^\gamma)\big]+\\ &+\lambda_1\big(\phi A n_h-(1-\phi)c_s-\phi c_h\big)+\lambda_2\big(\phi A' n'_h-(1-\phi)c'_s-\phi c'_h\big)\end{aligned}$$

From which it follows that:

$$c_h = c_s = \frac{\overline{A}\gamma}{1-\gamma}(1-n_h) = \phi\overline{A}n_h \Longrightarrow n_h = \frac{\gamma}{\phi+(1-\phi)\gamma},\ c_h = c_s = \frac{\phi\gamma\overline{A}}{\phi+(1-\phi)\gamma}$$

The same holds in $t = 2$.

If Home is spared by the virus in $t = 1$

$$\begin{aligned}\Lambda^h(A^*) =& \log(c_h^\gamma(1-n_h)^{1-\gamma})+\\ &+\beta\big\{\rho(A^*)\big[\phi\log(c_{h,+}^\gamma(1-n_{h,+})^{1-\gamma})+(1-\phi)\log(c_{s,+}^\gamma)\big]+\\ &+(1-\rho(A^*))\log(c_{h,-}^\gamma(1-n_{h,-})^{1-\gamma})\big\}+\\ &+\lambda_1(An_h-c_h)+\lambda_2(A_-n_{h,-}-c_{h,-})+\lambda_3(\phi A_+n_{h,+}-\phi c_{h,+}-(1-\phi)c_{s,+})\big\}\end{aligned}$$

From which it follows that:

$$A = A_+ = A_- = \overline{A}, \quad (1-\gamma)c = A\gamma(1-n), \quad c = \{c_h,\ c_{h,+},\ c_{h,-}\}, \quad c_{s,+} = c_{h,+}$$

Cooperation

If Home is hit by the virus in $t = 1$

$$\begin{aligned}\Lambda^s(t^+,t^{+-},t^{++}) =&\phi\log(c_{h,+}^\gamma(1-n_{h,+})^{1-\gamma})+(1-\phi)\log(c_{s,+}^\gamma)+\\ &+\beta\big\{\rho(A)\big[\phi\log(c_{h,+-}^\gamma(1-n_{h,+-})^{1-\gamma})+(1-\phi)\log(c_{s,+-}^\gamma)\big]+\\ &+(1-\rho(A))\big[\phi\log(c_{h,++}^\gamma(1-n_{h,++})^{1-\gamma})+(1-\phi)\log(c_{s,++}^\gamma)\big]\big\}+\\ &+\lambda_1\big[\phi An_{h,+}+t^+-\phi c_{h,+}-(1-\phi)c_{s,+}\big]+\\ &+\lambda_2\big[\phi A_{++}n_{h,++}+t^{++}-\phi c_{h,++}-(1-\phi)c_{s,++}\big]+\\ &+\lambda_3\big[\phi A_{+-}n_{h,+-}+t^{+-}-\phi c_{h,+-}-(1-\phi)c_{s,+-}\big]\end{aligned}$$

From which it follows that:

$$c_{s,+} = c_{h,+} = \frac{A\gamma}{1-\gamma}(1-n_{h,+}), \quad c_{s,++} = c_{h,++} = \frac{\overline{A}\gamma}{1-\gamma}(1-n_{h,++}),$$
$$c_{s,+-} = c_{h,+-} = \frac{\overline{A}\gamma}{1-\gamma}(1-n_{h,+-})$$

If Home is spared by the virus in $t = 1$

$$\begin{aligned}\Lambda^h(t^-, t^{--}, t^{-+}, A^*_-) =& \log(c^{\gamma}_{h,-}(1-n_{h,-})^{1-\gamma}) + \\ &+ \beta\big\{\rho(A^*_-)\big[\phi\log(c^{\gamma}_{h,-+}(1-n_{h,-+})^{1-\gamma}) + (1-\phi)\log(c^{\gamma}_{s,-+})\big] + \\ &+ (1-\rho(A^*_-))\log(c^{\gamma}_{h,--}(1-n_{h,--})^{1-\gamma})\big\} + \\ &+ \lambda_1\big[A_- n_{h,-} + t^- - c_{h,-}\big] + \\ &+ \lambda_2\big[A_{--}n_{h,--} + t^{--} - c_{h,--}\big] + \\ &+ \lambda_3\big[\phi A_{-+}n_{h,-+} + t^{-+} - \phi c_{h,-+} - (1-\phi)c_{s,-+}\big]\end{aligned}$$

From which it follows that:

$$(1-\gamma)c = \overline{A}\gamma(1-n), \quad c = \{c_{h,-},\ c_{h,-+},\ c_{h,--}\}, \quad n = \{n_{h,-},\ n_{h,-+},\ n_{h,--}\}, \quad c_{s,-+} = c_{h,-+}$$

Arrow-Debreu Equilibrium

Fix $p_+ = 1$. The equilibrium quantities are determined by a number of equations. From the problem of Home, we have

$$\begin{aligned}p_{+-} &= \frac{W^s_2(t^+, t^{+-}, t^{++})}{W^s_1(t^+, t^{+-}, t^{++})} = \beta\rho(A_+)\frac{c_+}{c_{+-}} \\ p_{++} &= \frac{W^s_3(t^+, t^{+-}, t^{++})}{W^s_1(t^+, t^{+-}, t^{++})} = \beta(1-\rho(A_+))\frac{c_+}{c_{++}} \\ p_{--} &= p_-\frac{W^h_2(t^-, t^{--}, t^{-+}, A^*_-)}{W^h_1(t^-, t^{--}, t^{-+}, A^*_-)} = p_-\beta(1-\rho(A^*_-))\frac{c_-}{c_{--}} \\ p_{-+} &= p_-\frac{W^h_3(t^-, t^{--}, t^{-+}, A^*_-)}{W^h_1(t^-, t^{--}, t^{-+}, A^*_-)} = p_-\beta\rho(A^*_-)\frac{c_-}{c_{-+}} \\ p_- &= \frac{1-\xi}{\xi}\frac{W^h_1(t^-, t^{--}, t^{-+}, A^*_-)}{W^s_1(t^+, t^{+-}, t^{++})} = \frac{1-\xi}{\xi}\frac{c_+}{c_-}\end{aligned}$$

From the problem of Foreign, we have

$$\begin{aligned}
p_{+-} &= \frac{W_3^h(t_*^+, t_*^{++}, t_*^{+-}, A_+)}{W_1^h(t_*^+, t_*^{++}, t_*^{+-}, A_+)} = \beta\rho(A_+)\frac{c_+^*}{c_{+-}^*} \\
p_{++} &= \frac{W_2^h(t_*^+, t_*^{++}, t_*^{+-}, A_+)}{W_1^h(t_*^+, t_*^{++}, t_*^{+-}, A_+)} = \beta(1-\rho(A_+))\frac{c_+^*}{c_{++}^*} \\
p_{--} &= p_-\frac{W_3^s(t_*^-, t_*^{-+}, t_*^{--})}{W_1^s(t_*^-, t_*^{-+}, t_*^{--})} = p_-\beta(1-\rho(A_-^*))\frac{c_-^*}{c_{--}^*} \\
p_{-+} &= p_-\frac{W_2^s(t_*^-, t_*^{-+}, t_*^{--})}{W_1^s(t_*^-, t_*^{-+}, t_*^{--})} = p_-\beta\rho(A_-^*)\frac{c_-^*}{c_{-+}^*} \\
p_- &= \frac{1-\xi}{\xi}\frac{W_1^s(t_*^-, t_*^{-+}, t_*^{--})}{W_1^h(t_*^+, t_*^{++}, t_*^{+-}, A_+)} = \frac{1-\xi}{\xi}\frac{c_+^*}{c_-^*}
\end{aligned}$$

A_+ and A_-^* solve

$$\begin{aligned}
\beta\rho'(A_+)\Big[\phi\log\Big(\frac{c_{+-}^\gamma(1-n_{+-})^{1-\gamma}}{c_{++}^\gamma(1-n_{++})^{1-\gamma}}\Big) + (1-\phi)\gamma\log\Big(\frac{c_{+-}}{c_{++}}\Big)\Big] + \frac{\gamma\phi n_+}{c_+} = 0 \\
\beta\rho'(A_-^*)\Big[\phi\log\Big(\frac{{c_{-+}^*}^\gamma(1-n_{-+}^*)^{1-\gamma}}{{c_{--}^*}^\gamma(1-n_{--}^*)^{1-\gamma}}\Big) + (1-\phi)\gamma\log\Big(\frac{c_{-+}^*}{c_{--}^*}\Big)\Big] + \frac{\gamma\phi n_-^*}{c_-^*} = 0
\end{aligned}$$

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