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Essays on Unemployment Risk, Business Cycles, and Monetary Policy

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Abstract

This thesis consists of three chapters, studying the aggregate business cycle implications of incomplete markets in the form of uninsured unemployment risk.

The first chapter studies the conduct of monetary policy in the presence of heterogeneous exposure to energy price shocks between the employed and the unemployed, as it is documented by data from the euro area Consumer Expectation Survey. I account for this evidence into a tractable Heterogeneous-Agent New Keynesian (HANK) model with Search and Matching (S&M) frictions in the labor market and non-homothetic household preferences: energy price shocks directly impact producers and consumers, weighing more on the jobless due to non-homotheticity and the inability to perfectly insure against unemployment spells. Households' heterogeneous exposure to the shock induces an endogenous trade-off for monetary policy, whose optimal response involves partly accommodating core inflation so as to indirectly sustain employment, and prevent workers from becoming more exposed to the shock through unemployment.

The second chapter studies, both analytically and quantitatively, the occurrence of demand-deficient recessions due to uninsurable unemployment risk when jobs are endogenously destroyed. The ensuing unemployment fears induce a precautionary saving motive that counteracts the desire to borrow during recessions: negative productivity shocks may induce falling natural interest rates and positive unemployment gaps. Analytically, these demand-deficient recessions are shown to already occur with less rigid wages when jobs are endogenously destroyed, compared to the case of exogenous job destruction. Quantitatively, the demand-deficient nature of supply-driven recessions can only be captured when accounting for endogenous job destruction.

The third chapter studies the aggregate implications of uninsurable unemployment risk in an economy where nominal interest rates are constrained at the zero lower bound (ZLB). Precautionary saving desires due to uninsurable unemployment risk can induce sizeable falls in the natural interest rate during productivity-driven recessions. When nominal interest rates are stuck at the ZLB, the only force that can accordingly bring about a fall in the real rate is (expected) inflation. Analytical results suggest that inflationary ZLB recession episodes are indeed a plausible outcome in this setting, as long as the monetary authority targets not only inflation, but also fluctuations in the unemployment rate or output. It is also shown under which restrictions this outcome can occur both in the exogenous and endogenous separation cases.

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Finally, this thesis can can only be dedicated to my parents, Antonella and Girolamo, who have always been my first supporters.

Contents

List of Tables

List of Figures

Chapter 1

Energy Price Shocks, Unemployment, and Monetary Policy

1.1 Introduction

Since the end of 2021, euro area economies have had to cope with exceptionally high energy and overall inflation, and plummeting real wages (Figure 1.1). However, the recent surge in energy prices has impacted workers not only by depressing their real income, but it has also had distributional consequences. Indeed, the shock weighs in a heterogeneous manner on consumers: as energy is a necessity good, poorer households devote a larger share of their income to consuming it, and receive a larger hit when its price increases.[1] Given that workers suffer substantial losses in income and consumption during unemployment spells,[2] these constitute a channel through which households can become more exposed to rising energy prices. Exploiting data from the euro area Consumer Expectation Survey, I document how indeed the unemployed, compared to the employed, devote to energy-intensive consumption (utilities and transport services) a significantly higher share of their overall expenditure in goods and services. Moreover, this share has increased more for the unemployed as the shock hit, despite the fact that most countries have implemented several measures aimed at relieving the pressure on poorer and hence more exposed households (Arregui et al., 2022).

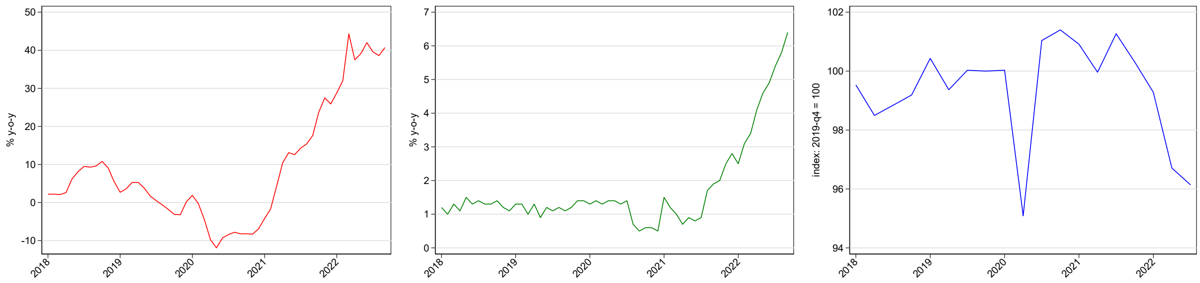
In light of these facts, it then appears crucial to understand how monetary authorities can influence household exposure to energy price shocks via the unemployment rate. This paper studies how monetary policy should optimally react to these shocks through the lens of a tractable Heterogeneous-Agent New Keynesian (HANK) model with Search and Matching (S&M) frictions in the labor market, where employed and unemployed workers are heterogeneously affected by energy price shocks due to their non-homothetic preferences and the inability to perfectly insure against unemployment spells. More specifically, this induces an additional source of consumption risk: at equilibrium, differently from the homothetic case, consumption amounts

[1] Recent studies that have noted the distributional impact of energy price surges due to their effect on heterogeneous consumption baskets include Ari et al. (2022), Bachmann et al. (2022), Battistini et al. (2022), Curci et al. (2022).

[2] Various studies have documented these consumption losses to range from 14% to 26% (Den Haan et al., 2018).

Figure 1.1: Inflation and Real Wages in the Euro Area

(a) Energy Inflation **(b)** Non-energy Inflation **(c)** Real Wages



Notes: The left and middle panels of the figure report monthly year-on-year changes in the the energy and non-energy components of the overall Harmonised Index of Consumer Prices (HICP) in the euro area. The right panel reports, on quarterly basis, the average compensation per employee in the euro area, deflated using the HICP and normalised at 100 in 2019-q4; latest observation: 2022-q3. *Sources:* ECB and Eurostat.

only to what is left after having paid for subsistence energy needs. As a result, higher energy prices end up weighing more on the unemployed, who have lower income and are forced to consume less. Therefore, due to the presence of a frictional labor market, workers can endogenously become unemployed and hence more exposed to the shock. In turn, the monetary authority can then reduce the fraction of more exposed households by trading off higher inflation with lower unemployment.

I show analytically, by relying on a simple positive illustrative case, how the presence of imperfect unemployment insurance amplifies the negative response of employment to a rise in real energy prices through both an unemployment risk channel and the heterogeneous exposure between the employed and the unemployed to the shock. First of all, when energy is a complementary input in production, rising real energy prices induce firms to reduce activity and hence employment. On top of this, in the presence of nominal price rigidity there are two additional sources of reduced activity and employment. First, as unemployment risk increases and households cannot perfectly insure against it, precautionary saving desires increase as well, and aggregate demand is depressed. This, in turn, sets in motion the feedback loop already highlighted by e.g. Ravn and Sterk (2021), whereby production falls even further, feeding back to even greater unemployment risk, and so on. Second, due to the presence of non-homothetic preferences, an increase in the real price of energy rises the consumption losses upon unemployment, thereby strengthening unemployment fears and precautionary saving motives, further depressing aggregate demand and employment.

These two sources of amplification of the negative employment response to rising energy prices —unemployment risk, and the heterogeneous exposure to the shock— pose in turn the normative question of what is the optimal response of monetary policy. First, by relying on the same simplified case from which I can derive analytical characterizations, I show that fluctuations in real energy prices endogenously induce a wedge between efficient and natural employment in this setting. This wedge is due to the presence of subsistence energy consumption and imperfect unemployment insurance, which together imply that employed and unemployed workers are heterogeneously exposed to energy price shocks. As a result, rising energy prices induce an increase in the consumption losses upon unemployment. This generates a trade-off that calls for partly accommodating core inflation in order to indirectly sustain employment, hence preventing households from becoming more exposed to the shock through unemployment spells.

Second, I numerically explore the mechanism for the general case, confirming the analytical results: compared to a policy rule aimed at fully stabilising core inflation, the optimal Ramsey policy is able to achieve a smaller increase in unemployment at the cost of partly accommodating core inflation; this rise in inflation is, however, smaller than if unemployment was instead to be fully stabilised. It is also confirmed that the trade-off arises due to the presence of heterogeneous exposure to the energy price shock between the employed and the unemployed: considering a case of homothetic preferences and hence homogeneous direct exposure to the shock, the optimal policy coincides with one aimed at fully stabilising core inflation.

Related Literature. This paper contributes, first, to the macroeconomic literature on the impact of energy shocks on producers and consumers: earlier studies (Bodenstein et al., 2008, Blanchard and Galí, 2010b, Montoro, 2012, Blanchard and Riggi, 2013) have considered representative-agent settings, providing the normative insight that central banks should be hard on core inflation while accommodating energy price changes (Natal, 2012). My paper moves forward in this context by making a case for not being too hard on core inflation either, as excess unemployment would make heterogeneously exposed households worse off. Gagliardone and Gertler (2023), still using a representative-agent setting with oil as a complementary good in consumption and complementary input in production, show that mainly accounting for the recent inflation surge was a combination of oil price shocks and accommodative monetary policy. In their framework, which also allows for unemployment, a rationale for part of such accommodation is the fact that monetary policy trade-offs arise due to real wage rigidity, while in my heterogeneous-household setting they arise not only due to this but, first and foremost, since energy shocks weigh more on the unemployed. Recent studies have also taken household heterogeneity into account, for instance in Two-Agent New Keynesian (TANK) frameworks à la Debortoli and Galí (2017): Chan et al. (2022) investigate how energy shocks impacting producers can feed back to depressed demand, but without considering how energy shocks *directly* affect consumers, an aspect which I explicitly model with liquidity-constrained households being more exposed to these shocks due to non-homothetic energy consumption. Corsello and Riggi (2023) develop a TANK framework with energy in production and heterogeneously exposed households, as the hand-to-mouth devote a higher (constant) share of their consumption expenditure to energy, hence ending up experiencing a higher inflation rate. Their analysis is positive rather than normative, offering a historical decomposition of inflation inequality into its main drivers including the monetary policy stance. Moreover, while TANK frameworks typically assume a constant fraction of hand-to-mouth households, in my setting the fraction of agents ending up being liquidity-constrained is endogenous and microfounded, as it is captured by the unemployment rate. Other recent studies have accounted instead for household heterogeneity within quantitative HANK models à la Kaplan et al. (2018): Pieroni (2023) introduces non-homothetic preferences in a similar fashion as I do, but in a rich quantitative model with energy consumption by both households and firms, thus relying only on numerical solutions and focusing on positive analysis. In my work, I consider instead a specific dimension of household heterogeneity —stemming from imperfect unemployment insurance— which allows to characterise and investigate analytically the mechanisms at the source of monetary policy trade-offs with their normative implications.

Second, I build on the recent literature studying the macroeconomic implications of incomplete markets in form of imperfectly insurable unemployment risk within tractable HANK-S&M models (Challe et al., 2017, Ravn and Sterk, 2017, 2021, Den Haan et al., 2018, McKay and Reis, 2021). My work relates in particular to Challe (2020), who has investigated the optimal conduct of monetary policy within this class of models in response to productivity shocks and generic cost-push shocks.

I contribute to these strands of literature, in first instance, by modeling energy as an input in both production and consumption within a tractable HANK-S&M framework, and innovating on the existing studies by uncovering a novel precautionary saving motive and source of monetary policy trade-off, both arising endogenously due to non-homothetic preferences and imperfect insurance, focusing on normative implications analytically as well as quantitatively. In this vein, my work relates also to Acharya et al. (2023), who study how (optimal) monetary policy can mitigate consumption risk arising from households' unequal exposure to aggregate shocks in an analytically tractable HANK model. In similar spirit, Smirnov (2022) shows numerically, using a rich quantitative HANK model, how optimal policy aims at mitigating this unequal exposure.

Roadmap. The remainder of this paper is structured as follows. Section 1.2 provides motivating evidence. Section 1.3 describes the model, and Section 1.4 characterises its constrained-efficient allocation. Section 1.5 focuses on the linearised version of the model and deals with positive analysis, while normative analysis is performed in Section 1.6. Section 1.7 concludes.

1.2 Motivating Evidence

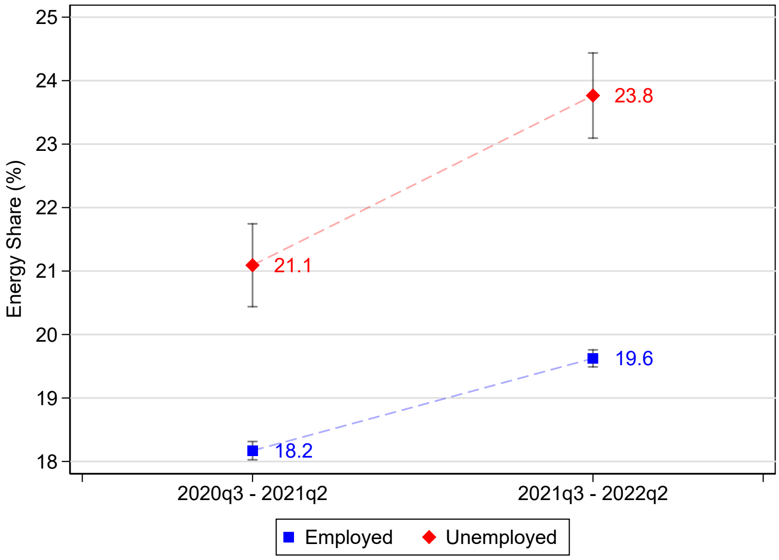
In this section, I exploit the microdata from the European Central Bank's Consumer Expectation Survey (CES) to document some facts about the heterogeneity between the employed and the unemployed in their exposure to energy price shocks. The CES covers the six largest economies in the euro area (Germany, France, Italy, Spain, the Netherlands and Belgium) over the period since April 2020. On quarterly basis, it collects information about individual monthly spending in twelve major categories, including energy-intensive ones (utilities and transport services), as well as their employment situation.[3] This allows to compute an individual-level measure of exposure to energy price shocks as the share of energy-intensive categories in overall expenditure on goods and services.

Figure 1.2 reports the average energy share of the employed and the unemployed over time. Between late 2020 and early 2021, before the surge in energy prices, the measure was around 3 percentage points higher for the unemployed than for the employed, for whom it stood at around 18%. As energy prices surged, this latter share reached nearly 20%, and was around 4 percentage points higher for the unemployed. Therefore, even if many countries have responded to the shock with fiscal measures targeted at alleviating the impact on vulnerable households (Arregui et al., 2022), the energy share of the unemployed has increased by around 1 percentage point more than that of the employed.

To investigate in more depth the presence of different consumption patterns between the employed and the unemployed, with particular focus on energy-intensive consumption, I then focus on the period before the energy price surges (2020-q3 to 2021-q2) and look at average differences in the expenditure shares of the unemployed relative to that of the employed as well as at average percentage differences in overall consumption, utilities, and transport services expenditure. As shown in Table 1.1, while the employed devoted, on average, 11.2% of their overall consumption expenditure to utilities, this share was 3.1 percentage points higher for the unemployed. This is due to the fact that while overall consumption expenditure was $\exp\{-0.247\} - 1 \simeq 22\%$ lower for the unemployed compared to the employed, expenditure on utilities was only around 4% lower. Moreover, despite the fact that expenditure on transport services was around 30% lower for the unemployed, still their expenditure share of energy-intensive consumption (utilities plus transport services) was around 3 percentage points higher than that of the employed, for whom it was slightly above 18%.

[3] Information about the employment situation is available from the October 2020 wave of the survey. Appendix 1.A provides a more detailed description of the data.

Figure 1.2: ENERGY SHARES OF THE EMPLOYED AND THE UNEMPLOYED



Notes: The figure reports the average shares of energy-intensive consumption expenditure over time for employed and unemployed individuals interviewed in the CES, obtained in regressions of the form $e_{it} = \beta_0 + \beta_1\, une_{it} + X_{it}\,\gamma + \varepsilon_{it}$, where e_{it} is the energy share of individual i at time t, $une_{it} = 1$ if the individual is employed and zero if employed, and X_{it} controls for individual-specific characteristics (age, gender, foreign birth, education, presence of a partner). The vertical bars indicate 95% confidence intervals.

Table 1.1: ENERGY-INTENSIVE CONSUMPTION AND UNEMPLOYMENT

(a) Expenditure

	$\ln(consumption)$	$\ln(utilities)$	$\ln(transports)$	$\ln(energy)$
unemployed	-0.247	-0.043	-0.359	-0.124
	[-0.283,-0.212]	[-0.082,-0.005]	[-0.465,-0.252]	[-0.166,-0.082]

(b) Expenditure Shares

	utilities share	transports share	total share
unemployed	0.031	-0.002	0.029
	[0.025,0.037]	[-0.005,0.001]	[0.022,0.036]
employed (baseline)	0.112	0.070	0.182
	[0.111,0.113]	[0.069,0.071]	[0.180,0.183]

Notes: The table reports results of regressions of the form $y_{it} = \beta_0 + \beta_1\, une_{it} + X_{it}\,\gamma + \varepsilon_{it}$, run on 20899 observations from the CES for the period from 2020q3 to 2021q2. y_{it} is, alternatively, the share of energy-intensive items (utilities and transport services) in the overall consumption expenditure of individual i at time t, the log of their overall consumption expenditure, log expenditure on utilities, transport services, and their sum. $une_{it} = 1$ when individual i is unemployed at time t and 0 when employed. X_{it} controls for individual-specific characteristics (age, gender, foreign birth, education, presence of a partner). 95% confidence intervals are reported in brackets.

1.3 The Model

The model tracks a sticky-price economy with uninsurable unemployment risk, along the lines of Challe (2020), enriched by introducing energy as an input both in production and in the consumption bundle. Following Blanchard and Galí (2010b) and most of the subsequent literature on the macroeconomic effects of oil and energy shocks, these are captured by considering energy as a non-produced input, whose real price evolves according to an exogenous stochastic process.

On the household side, there is a unit measure of workers who, due to their inability to borrow, cannot perfectly insure against the risk of becoming unemployed, and who derive utility from a basket of a core consumption good and energy. This latter is consumed above a subsistence level, implying that poorer households (i.e. the unemployed, in this setting) will devote a higher share of their overall consumption expenditure to energy. Additionally, there is a measure $\nu > 0$ of risk-neutral firm owners who simply collect and consume hand-to-mouth the dividends arising in the production sector, net of fiscal transfers.

On the producer side, the core good is assembled by aggregating differentiated varieties from a monopolistically competitive wholesale sector. Each wholesaler uses in turn as inputs energy as well as labor services. These latter are supplied competitively by intermediaries who hire labor from households in a market with standard S&M frictions à la Mortensen and Pissarides (1994).[4]

1.3.1 Households

Workers face idiosyncratic income risk from being either employed or unemployed: due to imperfect insurance, employed and unemployed workers will then make different consumption-saving choices at equilibrium.

The consumption basket of each worker type $i \in \{n, u\}$ (employed, unemployed), c_t^i, is defined as a Stone-Geary aggregator of a core consumption good, g_t^i, and energy, e_t^i, which is consumed above a subsistence level $\xi > 0$,

$$c_t^i = (g_t^i)^{(1-\omega_e)}(e_t^i - \xi)^{\omega_e} \tag{1.1}$$

where $\omega_e \in (0, 1)$ is the quasi-share of energy. Allowing for the presence of the subsistence level ξ will imply, consistently with the empirical evidence, that energy consumption demand is inelastic and that poorer households (i.e. the unemployed, in this setting) will devote a higher share of

[4]Introducing labor market frictions in a separate, upstream sector, is isomorphic to considering these frictions affecting wholesalers who directly hire labor from households.

their income to energy consumption.[5]

The expected lifetime utility of a currently employed (resp. unemployed) worker can be formulated recursively as follows

$$
\begin{aligned}
U_t^n &= \ln\left(c_t^n\right) + \beta\,\mathbb{E}_t\left[\left(1-\lambda_{t+1}\right)U_{t+1}^n + \lambda_{t+1}\,U_{t+1}^u\right] \\
U_t^u &= \ln\left(c_t^u\right) + \beta\,\mathbb{E}_t\left[f_{t+1}\,U_{t+1}^n + \left(1-f_{t+1}\right)U_{t+1}^u\right]
\end{aligned}
$$

where λ is the transition rate from employment to unemployment, while f is the job-finding rate. Each worker type maximises her expected lifetime utility subject to, for all $t \geq 0$,

$$
\begin{cases}
P_{g,t}\,g_t^i + P_{e,t}\,e_t^i + B_t^i = Y_t^i + \left(1+i_{t-1}\right)B_{t-1}^i \\
B_t^i \geq 0
\end{cases}
$$

where $P_{e,t}$ is the price of energy, $P_{g,t}$ the price of the core consumption good, B_t the nominal amount of 1-period bonds held at the end of period t, and i_t the nominal return on these assets. Y_t^i is equal either to W_t (the nominal wage income a worker gets when employed), or Δ_t (nominal home production when unemployed).

Demands for the core consumption good and energy are given by

$$
g_t^i = \left(1-\omega_e\right)\left(\frac{P_t}{P_{g,t}}\right)c_t^i \tag{1.2}
$$

$$
e_t^i = \omega_e\left(\frac{P_t}{P_{e,t}}\right)c_t^i + \xi \tag{1.3}
$$

where

$$
P_t = \left(\frac{P_{g,t}}{1-\omega_e}\right)^{(1-\omega_e)}\left(\frac{P_{e,t}}{\omega_e}\right)^{\omega_e} \tag{1.4}
$$

represents the overall consumer price index (CPI). Therefore, letting $\pi_{t+1} := \frac{P_{t+1}}{P_t} - 1$, to a first order

$$
\pi_t \simeq \left(1-\omega_e\right)\pi_{g,t} + \omega_e\,\pi_{e,t}\,. \tag{1.5}
$$

Euler conditions for employed and unemployed workers are, respectively,

$$
\frac{1}{c_t^n} \geq \beta\,\mathbb{E}_t\left\{\left(\frac{1+i_t}{1+\pi_{t+1}}\right)\left[\left(1-\lambda_{t+1}\right)\frac{1}{c_{t+1}^n} + \lambda_{t+1}\,\frac{1}{c_{t+1}^u}\right]\right\} \tag{1.6}
$$

$$
\frac{1}{c_t^u} \geq \beta\,\mathbb{E}_t\left\{\left(\frac{1+i_t}{1+\pi_{t+1}}\right)\left[f_{t+1}\,\frac{1}{c_{t+1}^n} + \left(1-f_{t+1}\right)\frac{1}{c_{t+1}^u}\right]\right\} \tag{1.7}
$$

[5] See Appendix 1.B.2 for more details about the properties of consumer demand in the presence of the non-homotheticities of Stone-Geary preferences.

each holding with strict inequality if the agent is liquidity constrained (i.e. wishing to borrow), and with equality otherwise. Due to the presence of non-homothetic preferences, captured by $\xi > 0$, consumption does not equal real overall expenditure on goods, and is instead given by

$$c_t^i = \frac{X_t^i}{P_t} - \frac{P_{e,t}}{P_t}\,\xi$$

where $X_t^i = P_{g,t}\, g_t^i + P_{e,t}\, e_t^i$. Intuitively, households derive utility only from what is left after having spent on energy for subsistence purposes. This form of non-homotheticity has the advantage of linking the presence of different consumption patterns directly to the inability of perfectly insuring against job losses, which will force the unemployed to spend less on consumption at equilibrium.

Employment Dynamics. Worker transitions between employment and unemployment give rise to the following laws of motion for the stocks of employed (n_t) and unemployed (u_t) workers

$$n_{t+1} = (1 - \lambda_{t+1})\, n_t + f_{t+1}\, u_t$$
$$u_{t+1} = (1 - f_{t+1})\, u_t + \lambda_{t+1}\, n_t$$

where, since the total measure of workers is normalised to 1, $u_t = 1 - n_t$. Being $s_{t+1} = u_t + \rho\, n_t$ the stock of effective searchers, $\lambda_{t+1} = \rho\,(1 - f_{t+1})$, where ρ is the separation rate.

In other words, a worker employed in the current period faces a risk of being unemployed in the next period given by λ_{t+1}, the joint probability of undergoing separation and not being able to subsequently find another job.

1.3.2 Producers

The core consumption good is produced under perfect competition, by aggregating a continuum of wholesale goods with a constant elasticity of substitution technology

$$Y_t = \left(\int_0^1 y_t(k)^{\frac{\varepsilon-1}{\varepsilon}}\, dk \right)^{\frac{\varepsilon}{\varepsilon-1}} . \tag{1.8}$$

Therefore, demand for variety k is given by

$$y_t(k) = Y_t \left[\frac{P_t(k)}{P_{g,t}} \right]^{-\varepsilon} \tag{1.9}$$

where $P_{g,t} = \left(\int_0^1 P_t(k)^{1-\varepsilon} dk \right)^{1/(1-\varepsilon)}$, and $\varepsilon > 1$.

Wholesalers

Each wholesaler is a monopolistic supplier of the variety k it produces using labor services as well as energy, combined in fixed proportions according to the following production function

$$y_t(k) = \min\left\{\frac{l_t(k)}{1-\gamma_e}, \frac{e_t(k)}{\gamma_e}\right\}$$

where $y_t(k)$ is the amount of variety k produced, $e_t(k)$ and $l_t(k)$ are respectively the amounts of energy and intermediate inputs used in production by wholesaler k, and $\gamma_e \in (0,1)$ captures the factor proportion of energy relative to labor services.

Demand for inputs from wholesalers will then be given by

$$l_t(k) = (1-\gamma_e)\, y_t(k) \tag{1.10}$$

$$e_t(k) = \gamma_e\, y_t(k) \tag{1.11}$$

and the nominal marginal cost by

$$MC_t = (1-\gamma_e)\,\Phi_t + \gamma_e\, P_{e,t} \tag{1.12}$$

where Φ_t is the price of labor services, and $P_{e,t}$ the energy price.

Moreover, wholesale firms are assumed to face Calvo pricing frictions, with θ being the probability that a wholesale firm cannot reset its price. Therefore, the optimal reset price, P_t^*, satisfies

$$p_{g,t}^* = \frac{P_t^*}{P_{g,t}} = \frac{\mathcal{Y}_t}{\mathcal{Z}_t} \tag{1.13}$$

where $\mathcal{Y}_t$ and $\mathcal{Z}_t$ obey the following recursions

$$\mathcal{Y}_t = (1-\tau_y)\left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{MC_t}{P_t}\, Y_t + \theta\,\beta\,\mathbb{E}_t\left[(1+\pi_{g,t+1})^{\varepsilon}\,\mathcal{Y}_{t+1}\right]$$

$$\mathcal{Z}_t = \frac{P_{g,t}}{P_t}\, Y_t + \theta\,\beta\,\mathbb{E}_t\left[(1+\pi_{g,t+1})^{(\varepsilon-1)}\,\mathcal{Z}_{t+1}\right]$$

and τ_y is a production subsidy, financed through a lump-sum tax on firm owners, which is aimed at offsetting the steady-state distortion due to monopolistic competition.

Given Calvo pricing frictions and symmetry in the wholesale sector, gross inflation in the core consumption good price, $1+\pi_{g,t} = P_{g,t}/P_{g,t-1}$, evolves according to

$$1+\pi_{g,t} = \left[\frac{1}{\theta} - \left(\frac{1-\theta}{\theta}\right)\left(p_{g,t}^*\right)^{(1-\varepsilon)}\right]^{1/(\varepsilon-1)}. \tag{1.14}$$

Real dividends collected by firm owners from wholesaler k are then given by

$$d_t^W(k) = \frac{1}{P_t}\left[P_t(k)\, y_t(k) - (1-\tau_y)\,\Phi_t\, l_t(k) - (1-\tau_y)\, P_{e,t}\, e_t(k)\right] \tag{1.15}$$

and aggregate real dividends from the wholesale sector by

$$d_t^W = \int_0^1 d_t^W(k)\, dk = Y_t\,\left\{p_{g,t} - (1-\tau_y)\left[(1-\gamma_e)\,\varphi_t + \gamma_e\, p_{e,t}\right]\mathcal{D}_t\right\} \tag{1.16}$$

where $p_{g,t} = P_{g,t}/P_t$, $\varphi_t = \Phi_t/P_t$, and

$$\mathcal{D}_t := \int_0^1 \left[\frac{P_t(k)}{P_{g,t}}\right]^{-\varepsilon} dk = (1-\theta)\,\left(p_{g,t}^*\right)^{-\varepsilon} + \theta\, \Pi_{g,t}^{\varepsilon}\, \mathcal{D}_{t-1} \tag{1.17}$$

is an index of price dispersion among wholesalers.

Labor Intermediaries

Intermediaries hire labor from households in a frictional market. These frictions are summarised by an aggregate matching function, which is assumed to take a Cobb-Douglas form with constant returns to scale,

$$m_t = s_t^{\alpha}\, v_t^{(1-\alpha)} \tag{1.18}$$

where m_t denotes the total amount of formed matches, v_t is the total amount of vacancies, $\alpha \in (0,1)$, and $s_t = u_{t-1} + \rho\, n_{t-1}$ is the total amount of searching workers, given by those workers who were unemployed plus those workers who were employed but experience separation (at rate ρ). The job-finding and vacancy-filling rates are given, respectively, by

$$f_t = \frac{m_t}{s_t}\,, \tag{1.19}$$

$$q_t = \frac{m_t}{v_t} = f_t^{\frac{\alpha}{\alpha-1}}\,. \tag{1.20}$$

Active matches produce one unit of output at each period. The value of a match is

$$J_t = (1-\tau_z)\,\left(\varphi_t - w_t + S\right) + \beta\,(1-\rho)\,\mathbb{E}_t\left(J_{t+1}\right) \tag{1.21}$$

where $w_t = W_t/P_t$ is the real wage rate, and $\varphi_t = \Phi_t/P_t$ is the price of labor services in terms of the final good. τ_z and S are fiscal instruments that will be aimed at offsetting the steady-state distortions arising from labor market frictions and imperfect unemployment insurance.

When matches break up, inactive intermediaries post vacancies at a cost of κ units of the final good per vacancy per period, and each vacancy is filled with probability q_t. Free entry into vacancy posting gives rise to the following job creation condition

$$f_t^{\frac{\alpha}{1-\alpha}} = \frac{1-\tau_z}{\kappa}\,(\varphi_t - w_t + S) + \beta\,(1-\rho)\,\mathbb{E}_t\left(f_{t+1}^{\frac{\alpha}{1-\alpha}}\right). \tag{1.22}$$

Aggregate real dividends collected by firm owners from labor intermediaries are then given by

$$d_t^I = n_t\,(1-\tau_z)\,(\varphi_t - w_t + S) - \kappa\,v_t\,. \tag{1.23}$$

1.3.3 Equilibrium

Market Clearing

Energy Market. Following Blanchard and Galí (2010b), energy is modeled as a non-produced input, whose real price is assumed to evolve exogenously according to

$$\ln(p_{e,t}) = \rho_e\,\ln(p_{e,t-1}) + \epsilon_t^e \tag{1.24}$$

where $\rho_e \in [0,1)$ and $\epsilon_t^e \sim iid(0,\sigma_e^2)$. In other words, the aggregate market demand for energy from households and firms is assumed to be always cleared at the exogenous real price $p_{e,t}$.

Labor Market. The total supply of labor services is given by the measure of active matches, n_t, while total demand can be obtained by aggregating over wholesalers. Given (1.9) and (1.10), we have

$$\int_0^1 l_t(k) = \int_0^1 (1-\gamma_e)\,y_t(k) = (1-\gamma_e)\,Y_t\,\mathcal{D}_t\,. \tag{1.25}$$

Therefore, labor market clearing requires

$$n_t = (1-\gamma_e)\,Y_t\,\mathcal{D}_t\,. \tag{1.26}$$

Good Market. The supply of the core consumption good is given by Y_t. Its total demand, $n_t\,g_t^n + (1-n_t)\,g_t^u + \nu\,g_t^o$, is obtained by aggregating over employed and unemployed workers and firm owners. Given (1.2), market clearing then requires

$$(1-\omega_e)\,[n_t\,c_t^n + (1-n_t)\,c_t^u + \nu\,c_t^o] = p_{g,t}\,Y_t \tag{1.27}$$

where $c_t^n\,, c_t^u\,, c_t^o$ are, respectively, the final consumption baskets of employed and unemployed workers and firm owners.

Household Consumption and Zero Liquidity Property

Workers. Given the zero-debt limit households face, the supply of assets is always zero at equilibrium, no asset trade actually takes place, and all households turn out to spend all their current income on consumption. Therefore, letting $\delta_t = \Delta_t / P_t$, the consumption bundles of employed workers and unemployed workers are, respectively,

$$c_t^n = w_t - p_{e,t}\,\xi \tag{1.28}$$

$$c_t^u = \delta_t - p_{e,t}\,\xi \tag{1.29}$$

from which it is immediate to see the following,

Proposition 1.1. *When $\delta_t < w_t$, $c_t^u < c_t^n$ and a rise in the real price of energy implies that the consumption of the unemployed falls proportionately more than the consumption of the employed:*

$$\frac{\partial\, c_t^u \,/\, c_t^u}{\partial\, p_{e,t} \,/\, p_{e,t}} < \frac{\partial\, c_t^n \,/\, c_t^n}{\partial\, p_{e,t} \,/\, p_{e,t}} < 0\,.$$

In light of (1.28) and (1.29), the Euler conditions of the employed and the unemployed are, respectively,

$$1 \geq \beta\,\mathbb{E}_t \left\{ \left(\frac{1+i_t}{1+\pi_{t+1}}\right) \left[(1-\lambda_{t+1}) \left(\frac{w_t - p_{e,t}\,\xi}{w_{t+1} - p_{e,t+1}\,\xi}\right) + \lambda_{t+1} \left(\frac{w_t - p_{e,t}\,\xi}{\delta_{t+1} - p_{e,t+1}\,\xi}\right)\right]\right\} \tag{1.30}$$

$$1 \geq \beta\,\mathbb{E}_t \left\{ \left(\frac{1+i_t}{1+\pi_{t+1}}\right) \left[f_{t+1} \left(\frac{\delta_t - p_{e,t}\,\xi}{w_{t+1} - p_{e,t+1}\,\xi}\right) + (1-f_{t+1}) \left(\frac{\delta_t - p_{e,t}\,\xi}{\delta_{t+1} - p_{e,t+1}\,\xi}\right)\right]\right\} \tag{1.31}$$

each holding with strict inequality if the household is liquidity constrained (i.e. wishing to borrow), and with equality otherwise.

It can be shown that at equilibrium, in the steady state neighborhood, the Euler condition of employed workers holds with equality while that of the unemployed holds with inequality.[6] Formally,

Proposition 1.2. *When $\delta < w$, the steady-state Euler conditions of employed and unemployed workers are, respectively,*

$$1 = \beta \left(\frac{1+i}{1+\pi}\right) \left[(1-\lambda) + \lambda \left(\frac{w - p_e\,\xi}{\delta - p_e\,\xi}\right)\right]$$

$$1 > \beta \left(\frac{1+i}{1+\pi}\right) \left[f \left(\frac{\delta - p_e\,\xi}{w - p_e\,\xi}\right) + (1-f)\right]$$

Proof. See Appendix 1.B.3. □

[6] See Ravn and Sterk (2021), Challe (2020) for more details.

In other words, at equilibrium employed workers wish to precautionarily save, but as the unemployed are impeded from borrowing, no one is able to issue the assets that would allow this precautionary saving desire to be actually satisfied. Hence, no asset trade actually takes place at equilibrium. This zero-liquidity property allows to price precautionary saving *desires* without the need of tracking a full wealth distribution over time.

Firm Owners. The final consumption basket of firm owners equals their after-tax real income, net of subsistence energy needs, $c_t^o = (d_t^I + d_t^W + \tau_t) - \xi \, p_{e,t}$, where

$$\tau_t = -\tau_y \, [(1-\gamma_e) \, \varphi_t + \gamma_e \, p_{e,t}] \, \mathcal{D}_t \, Y_t + \tau_z \, n_t \, (\varphi_t - w_t) - n_t \, (1-\tau_z) \, S \, . \tag{1.32}$$

Therefore, at equilibrium firm owner consumption is

$$c_t^o = \left(\frac{p_{g,t} \, \mathcal{D}_t^{-1} - \gamma_e \, p_{e,t}}{1-\gamma_e} - w_t \right) n_t - \kappa \, v_t - \xi \, p_{e,t} \tag{1.33}$$

where, given the equilibrium law of motion of employment, $n_t = (1-\rho) \, n_{t-1} + [1-(1-\rho) \, n_{t-1}]^{\alpha} \; v_t^{1-\alpha}$, v_t can be expressed as

$$v_t = \left\{ \frac{n_t - (1-\rho) \, n_{t-1}}{[1-(1-\rho) \, n_{t-1}]^{\alpha}} \right\}^{\frac{1}{1-\alpha}} \, . \tag{1.34}$$

1.4 Constrained-Efficient Allocation

The constrained-efficient allocation is the solution to the problem of maximizing the aggregate welfare of the economy, taking into account the decentralised equilibrium reactions by consumers and producers.

Considering a utilitarian planner who attaches equal weights to the utility of each household, the flow welfare of the economy is given by

$$U_t = n_t \, \ln(c_t^n) + (1-n_t) \, \ln(c_t^u) + \nu \, c_t^o \tag{1.35}$$

where c_t^n , c_t^u ,and c_t^o are given, respectively, by (1.28), (1.29), and (1.33).

Therefore, the constrained-efficient allocation is the solution to

$$W(\mathcal{D}_{t-1}, n_{t-1}, p_{e,t}) = \max_{\{p_{g,t}^*, w_t, n_t\}} \{U_t + \beta \, \mathbb{E}_t[W(\mathcal{D}_t, n_t, p_{e,t+1})]\} \tag{1.36}$$

subject to (1.14) and (1.17).

Now, $p_{g,t}^* = 1$ at every period is optimal, as it implies $\pi_{g,t} = 0$, thereby ensuring that $\mathcal{D}_t = 1$.

Therefore, the problem reduces to

$$W(n_{t-1}, p_{e,t}) = \max_{\{w_t, n_t\}} \{U_t + \beta\, \mathbb{E}_t[W(n_t, p_{e,t+1})]\} \,. \tag{1.37}$$

The first order condition with respect to w_t gives the following constrained-efficient wage rate

$$w_t^* = \frac{1}{\nu} + \xi\, p_{e,t} \,. \tag{1.38}$$

In other words, the constrained-efficient allocation involves risk-neutral firm owners insuring workers against fluctuations in their equilibrium consumption bundle as given in (1.28). Due to the need of consuming energy for subsistence purposes, a constant wage rate is not sufficient for the purpose in this setting, in contrast with Challe (2020). Insuring workers against consumption fluctuations involves instead the constrained-efficient wage rate adjusting upward (downward) in response to positive (negative) shocks to real energy prices.

The first order condition with respect to n_t gives

$$\ln\left(\frac{w_t^* - \xi\, p_{e,t}}{\delta_t - \xi\, p_{e,t}}\right) + \nu \left[\frac{p_{g,t} - \gamma_e\, p_{e,t}}{1-\gamma_e} - w_t^* - \left(\frac{1}{1-\alpha}\right)\frac{\kappa}{q_t^*}\right] + \beta\, \mathbb{E}_t\left[\frac{\partial\, W(n_t, p_{e,t+1})}{\partial\, n_t}\right] = 0 \tag{1.39}$$

while the envelope condition reads

$$\frac{\partial\, W(n_{t-1}, p_{e,t})}{\partial\, n_{t-1}} = \nu\, \kappa \left(\frac{1}{1-\alpha}\right)(1-\rho)\,\frac{\kappa}{q_t^*}\,(1 - \alpha\, f_t^*) \,. \tag{1.40}$$

Combining (1.39) with (1.40) one period ahead, and exploiting the fact that $q_t = f_t^{\frac{\alpha}{\alpha-1}}$, one can get the following forward recursion for the constrained-efficient job-finding rate

$$f_t^{*\,\frac{\alpha}{1-\alpha}} = \frac{1-\alpha}{\kappa}\left[\frac{p_{g,t} - \gamma_e\, p_{e,t}}{1-\gamma_e} - w_t^* + \frac{1}{\nu}\ln\left(\frac{w_t^* - \xi\, p_{e,t}}{\delta_t - \xi\, p_{e,t}}\right)\right] + \beta\,(1-\rho)\,\mathbb{E}_t\left[f_{t+1}^{*\,\frac{\alpha}{1-\alpha}}\,(1 - \alpha\, f_{t+1}^*)\right] \tag{1.41}$$

from which the constrained-efficient level of employment can be derived by exploiting the law of motion $n_t^* = (1-\rho)\, n_{t-1}^* + f_t^*\,[1 - (1-\rho)\, n_{t-1}^*]$.

1.4.1 Steady State

First of all, the constrained-efficient steady state must entail zero inflation, requiring in turn $p_g^* = 1$, i.e. $\varphi = \frac{1}{1-\tau_y}\left(\frac{\varepsilon-1}{\varepsilon}\right)\left(\frac{1}{1-\gamma_e}\right) p_g - \left(\frac{\gamma_e}{1-\gamma_e}\right) p_e$. Comparing the steady state counterparts of (1.22) and (1.41), which read respectively

$$f^{\frac{\alpha}{1-\alpha}} = \frac{1-\tau_z}{\kappa\,[1-\beta\,(1-\rho)]}\,(\varphi - w + S)$$

$$f^{*\,\frac{\alpha}{1-\alpha}} = \frac{1-\alpha}{\kappa\,[1-\beta\,(1-\rho)\,(1-\alpha\,f^*)]}\,\left[\frac{p_g-\gamma_e\,p_e}{1-\gamma_e} - w^* + \frac{1}{\nu}\,\ln\left(\frac{w^*-\xi\,p_e}{\delta-\xi\,p_e}\right)\right]$$

we can then see that the zero-inflation steady state is constrained-efficient provided that $w = w^*$, and that the tax instruments are such that

$$\tau_y = \frac{1}{\varepsilon}\,, \quad S = \frac{1}{\nu}\,\ln\left(\frac{w^*-\xi\,p_e}{\delta-\xi\,p_e}\right)\,, \quad \tau_z = 1 - \frac{(1-\alpha)\,[1-\beta\,(1-\rho)]}{1-\beta\,(1-\rho)\,(1-\alpha\,f^*)} \tag{1.42}$$

where

$$f^* = \left[\frac{1-\tau_z}{\kappa\,[1-\beta\,(1-\rho)]}\,\left(\frac{p_g-\gamma_e\,p_e}{1-\gamma_e} - w^* + S\right)\right]^{\frac{1-\alpha}{\alpha}}\,. \tag{1.43}$$

Given that the tax instruments are assumed to be constant, the constrained-efficient allocation cannot however be decentralised outside the zero-inflation steady state.[7] Even if this mix of fiscal instruments was allowed to be time-varying, it is evident from equations (1.38)–(1.41) that it would decentralize the efficient allocation also outside of steady state only when coupled with a rising real wage in response to surging energy prices, an implausible occurrence in reality.

1.5 The Linearised Model

In order to investigate how the model responds, to a first order, to fluctuations in energy prices, one needs to specify how the home production of the unemployed, δ_t, behaves in comparison to the income of the employed. Following Challe (2020) as a benchmark, I assume that δ_t fluctuates so that the replacement rate δ_t/w_t is constant. Letting the percentage income loss upon unemployment be $\zeta := 1 - \frac{\delta_t}{w_t} = 1 - \frac{\delta}{w}$, we have

$$\delta_t = (1-\zeta)\,w_t\,. \tag{1.44}$$

Now, letting hatted variables denote level deviations from the zero-inflation steady state, and tilde-variables log (or proportional) deviations, the first-order approximate dynamics of the economy are described by the following set of equations

$$\widehat{n}_t = \frac{\rho}{f+\rho\,(1-f)}\,\widehat{f}_t + (1-\rho)\,(1-f)\,\widehat{n}_{t-1}$$

$$\widehat{y}_t = \widehat{n}_t$$

$$\frac{\kappa}{q\,f}\,\left(\frac{\alpha}{1-\alpha}\right)\,\widehat{f}_t = (1-\tau_z)\,(\widehat{\varphi}_t - \widehat{w}_t) + \beta\,(1-\rho)\,\frac{\kappa}{q\,f}\,\left(\frac{\alpha}{1-\alpha}\right)\,\mathbb{E}_t(\widehat{f}_{t+1})$$

[7] The fact that these tax instruments cannot be adjusted in response to aggregate shocks reflects the idea that fiscal policy is slow to adjust (Acharya et al., 2023).

$$
\begin{aligned}
\widehat{mc}_t &= (1-\gamma_e)\,\widehat{\varphi}_t \;+\; \gamma_e\, p_e\, \widetilde{p}_{e,t} \\
\widehat{mc}_{g,t} &= \frac{1}{p_g}\,\widehat{mc}_t + \frac{\omega_e}{1-\omega_e}\,\widetilde{p}_{e,t} \\
\pi_{g,t} &= \beta\,\mathbb{E}_t(\pi_{g,t+1}) + \Theta\,\widehat{mc}_{g,t} \\
\pi_t &= \pi_{g,t} + \frac{\omega_e}{1-\omega_e}\,(\widetilde{p}_{e,t} - \widetilde{p}_{e,t-1}) \\
\widetilde{I}_t - \mathbb{E}_t(\pi_{t+1}) &= -\frac{\Lambda}{\lambda}\,\mathbb{E}_t(\widehat{\lambda}_{t+1}) + \frac{(1+\Xi_w\,\Lambda\,\Psi)\,\mathbb{E}_t(\widetilde{w}_{t+1}) - \widetilde{w}_t}{1-\Xi_w} - \Xi_w\,\frac{(1+\Lambda\,\Psi)\,\mathbb{E}_t(\widetilde{p}_{e,t+1}) - \widetilde{p}_{e,t}}{1-\Xi_w} \\
\widehat{\lambda}_t &= -\rho\,\widehat{f}_t \\
\widetilde{p}_{e,t} &= \rho_e\,\widetilde{p}_{e,t-1} + \epsilon_t^e
\end{aligned}
$$

where $\Theta = \frac{1-\theta}{\theta}\,(1-\theta\,\beta)$, $\Lambda = \lambda\left[\lambda + \left(\frac{1-\zeta-\Xi_w}{\zeta}\right)\right]^{-1} \in (0,1)$, $\Psi = \frac{1-\Xi_w}{1-\zeta-\Xi_w} > 1$, and $\Xi_w = \frac{p_e\,\xi}{w} \in [0,1)$ is the steady-state share of subsistence energy expenditure in the income of the employed.

Without loss of generality, I let the decentralised wage rate evolve as follows[8]

$$
\widetilde{w}_t = -\chi\,\widetilde{p}_{e,t} \tag{1.45}
$$

where the elasticity with respect to the real price of energy is assumed to be negative, i.e. $\chi > 0$, unless otherwise noted.

The model can be closed, for positive analysis purposes, by specifying an interest rate Taylor-type rule. This is assumed to take the following form,

$$
\widetilde{I}_t = \phi_\pi\,\pi_{g,t} + \phi_f\,\widehat{f}_t \tag{1.46}
$$

targeting core inflation as well as labor market slack, as captured by fluctuations in the job-finding rate, $\widehat{f}_t$. In the model, $\pi_{g,t}$ is the welfare-relevant measure of inflation, as it is the one that is associated with price dispersion, reducing in turn the quantity of output that can be obtained from a given amount of inputs.[9] Blanchard and Galí (2010b) similarly use a Taylor rule targeting core inflation, a variable they argue that many central banks appear to focus on as the basis for their interest rate decisions.[10]

[8] This reduced-form specification can be consistent with wage setting schemes such as nominal wage indexation or Nash bargaining between workers and firm owners.

[9] Similarly, in Bodenstein et al. (2008) it is core inflation that matters for welfare since core prices are sticky while the price of energy is assumed to be completely flexible.

[10] In this respect, in the present setting $\pi_{g,t}$ is what comes closest for instance to the ECB's medium-term orientation of monetary policy: indeed, given a high persistence of energy price shocks, π_t will settle to $\pi_{g,t}$ in the periods subsequent to the shock.

1.5.1 Energy Prices and Aggregate Demand

Focusing on the demand block of the model, as captured by the dynamic IS equation, the percentage deviation of the gross real interest rate from its steady-state level, $\widetilde{R}_t := \widetilde{I}_t - \mathbb{E}_t(\pi_{t+1})$, can be expressed as follows, in first-order approximation,

$$\begin{aligned}\widetilde{R}_t \;\simeq\; & \underbrace{-\,\frac{\Lambda}{\lambda}\,\mathbb{E}_t(\widehat{\lambda}_{t+1})}_{\text{Unemployment Risk}} \;+\; \underbrace{\chi\,(1-\rho_e)\,\widetilde{p}_{e,t}}_{\substack{\text{Standard}\\ \text{Consumption Smoothing}}} \\ & +\; \underbrace{\frac{\Xi_w}{1-\Xi_w}\,(1+\chi)\,(1-\rho_e)\,\widetilde{p}_{e,t}}_{\substack{\text{Non-homothetic}\\ \text{(employed) workers}}} \;\underbrace{-\,\Lambda\,\Psi\,\frac{\Xi_w}{1-\Xi_w}\,(1+\chi)\,\rho_e\,\widetilde{p}_{e,t}}_{\substack{\text{Heterogeneous Exposure}\\ \text{to Energy Price Shocks}}} \qquad (1.47)\end{aligned}$$

First of all, we can notice that in the perfect insurance limit, $\zeta \to 0$, we have $\Lambda = 0$, implying

$$\lim_{\zeta\to 0}\,\widetilde{R}_t \;\simeq\; \chi\,(1-\rho_e)\,\widetilde{p}_{e,t} \;+\; \frac{\Xi_w}{1-\Xi_w}\,(1+\chi)\,(1-\rho_e)\,\widetilde{p}_{e,t}$$

hence, in this case, aggregate demand is always sustained by an increase in real energy prices, by both a standard consumption smoothing channel and the non-homotheticity of consumer preferences. The former channel induces a desire to borrow as future wages are expected to bounce back up. The latter channel induces as well a desire to borrow in order to smooth consumption, as future energy prices —and the consumption losses they directly entail due to non-homothetic preferences— are expected to be lower than in the current period.

In the presence of imperfect insurance, i.e. $\zeta\,,\Lambda \in (0,1)$, we can first notice that when preferences are homothetic, and hence $\Xi_w = 0$, energy prices do not have any direct impact on the demand side of the model as captured by equation (1.47), which therefore reduces to the same considered by Challe (2020),

$$\widetilde{R}_t \;\simeq\; -\,\frac{\Lambda}{\lambda}\,\mathbb{E}_t(\widehat{\lambda}_{t+1}) \;+\; \mathbb{E}_t(\widetilde{w}_{t+1}-\widetilde{w}_t)\,.$$

Still, even if energy prices do not directly depress aggregate demand, they can do it *indirectly* through an increase in unemployment risk, or through real wage fluctuations.

Non-homothetic preferences imply, along with the presence of imperfect unemployment insurance, that employed and unemployed workers are heterogeneously exposed to energy price shocks. This channel, captured by the last term in (1.47), induces a precautionary saving desire that depresses aggregate demand when real energy prices increase. Indeed, the consumption losses upon unemployment are increased as the shock weighs more on the unemployed, causing in turn an incentive to precautionarily save on part of currently employed workers out of the fear of becoming unemployed and hence more exposed to the surge in energy prices.

Therefore, the net effect of non-homotheticty on aggregate demand depends on which channel prevails between the desire to borrow conditional on remaining employed, captured by the third term in (1.47), and the desire to save out of the fear of becoming unemployed and hence more exposed to the shock. One can easily check that the latter channel prevails when the shock is sufficiently persistent, namely when $\rho_e > 1 \big/ (1 + \Lambda\, \Psi)$.

Assuming that a shock to the price of energy is quite persistent does not appear to be unreasonable. For instance Blanchard and Galí (2010b), with reference to oil shocks, argue that its real price would be better characterized as non-stationary. In the present setting, when $\rho_e \to 1$ the *direct* impact of energy price increases on aggregate demand would undoubtedly be negative and amounting to

$$-\Lambda\, \Psi = -\frac{\rho\,(1-f)\,\zeta}{\rho\,(1-f)\,\zeta + (1-\zeta-\Xi_w)} \left(\frac{1-\Xi_w}{1-\zeta-\Xi_w}\right)$$

which is larger, in absolute terms, the larger the proportional loss in income upon unemployment, ζ, as energy price increases end up weighing even more on the unemployed relative to the employed, and hence employed workers have more incentive to precautionarily save in direct response to these shocks.

1.5.2 Positive Analysis: an Illustrative Case

While a quantitative assessment of the behavior of the linearised model will be performed later in Section 1.6 and in comparison to normative analysis, in order to understand the various forces at work it is useful to consider the following illustrative case, which allows to derive a simple and intuitive analytical solution to the employment response to energy price shocks.

I first assume constant real wages, implying in turn that only the two precautionary motives highlighted in equation (1.47) will be at work in determining saving desires. I also assume full worker reallocation ($\rho = 1$) implying $\widehat{n}_t = \widehat{f}_t$, and set the matching function parameter to $\alpha = 0.5$. Lastly, the policy rule parameters are set to $\phi_\pi = 1/\beta\,, \phi_f = 0$.

Under these assumptions, the first-order behavior of the model is characterised by the following equations

$$\begin{aligned}
\widehat{mc}_{g,t} &= \frac{2\,\kappa}{p_g}\,\widehat{n}_t + \left[\frac{p_e}{p_g}\left(\frac{\gamma_e}{1-\gamma_e}\right) + \left(\frac{1}{1-\gamma_e}\right)\frac{\omega_e}{1-\omega_e}\right]\widetilde{p}_{e,t} \\
\pi_{g,t} &= \beta\,\mathbb{E}_t(\pi_{g,t+1}) + \Theta\,\widehat{mc}_{g,t}
\end{aligned}$$

$$\pi_t = \pi_{g,t} + \frac{\omega_e}{1-\omega_e}\left(\widetilde{p}_{e,t} - \widetilde{p}_{e,t-1}\right)$$

$$\frac{1}{\beta}\,\pi_{g,t} - \mathbb{E}_t(\pi_{t+1}) = \frac{\Lambda}{1-f}\,\mathbb{E}_t(\widehat{n}_{t+1}) - \Xi_w\,\frac{(1+\Lambda\,\Psi)\,\mathbb{E}_t(\widetilde{p}_{e,t+1}) - \widetilde{p}_{e,t}}{1-\Xi_w}$$

Combining the last three equations, the dynamic IS curve becomes

$$\frac{\Theta}{\beta}\,\widehat{mc}_{g,t} + (1-\rho_e)\,\frac{\omega_e}{1-\omega_e}\,\widetilde{p}_{e,t} = \frac{\Lambda}{1-f}\,\mathbb{E}_t(\widehat{n}_{t+1}) + (1-\rho_e)\,\frac{\Xi_w}{1-\Xi_w}\,\widetilde{p}_{e,t} - \rho_e\,\frac{\Xi_w}{1-\Xi_w}\,\Lambda\,\Psi\,\widetilde{p}_{e,t}\,.$$

Since the last term on the LHS and the second term on the RHS are approximately equal to zero when $\rho_e \to 1$, we can get the following linear expectational difference equation

$$\widehat{n}_t = \frac{p_g}{2\,\kappa}\,\mathcal{B}_n\,\mathbb{E}_t(\widehat{n}_{t+1}) - \frac{p_g}{2\,\kappa}\,\mathcal{C}_e\,\widetilde{p}_{e,t} \tag{1.48}$$

where $\mathcal{B}_n = \frac{\beta}{\Theta}\left(\frac{\Lambda}{1-f}\right)$, $\mathcal{C}_e = \frac{p_e}{p_g}\left(\frac{\gamma_e}{1-\gamma_e}\right) + \left(\frac{1}{1-\gamma_e}\right)\frac{\omega_e}{1-\omega_e} + \Lambda\,\Psi\,\frac{\beta}{\Theta}\left(\frac{\Xi_w}{1-\Xi_w}\right)$, and local determinacy requires $\frac{p_g}{2\,\kappa}\,\mathcal{B}_n < 1$.

Perfect Insurance Limit

In the perfect insurance limit where the income of the unemployed is equal to that of the employed, the first-order response of employment to a rise in real energy prices is given by

$$\lim_{\zeta\to 0}\,\widehat{n}_t \;=\; -\,\mathcal{F}_e\,\widetilde{p}_{e,t} \tag{1.49}$$

where $\mathcal{F}_e = \frac{p_g}{2\,\kappa}\left[\frac{p_e}{p_g}\left(\frac{\gamma_e}{1-\gamma_e}\right) + \left(\frac{1}{1-\gamma_e}\right)\frac{\omega_e}{1-\omega_e}\right]$.

First of all, it is worth noticing that the perfect insurance limit corresponds, in the particular case considered here, to the natural (flexible-price) response, as the two precautionary motives shaping the response of the natural rate vanish in the perfect insurance limit, implying in turn a steady natural rate when real wages are assumed to remain constant.

This perfect-insurance (and natural) response is, then, entirely driven by what happens on the producer side of the model: in response to rising real energy prices, firms demand less energy; given that this input is a complement with labor in production, and that real wages are assumed to remain fixed, also labor demand decreases, and hence employment falls at equilibrium.

Imperfect Insurance

Coming to the imperfect insurance case, and considering first a homothetic preferences situation, we have

$$\widehat{n}_t \;=\; -\,\frac{\mathcal{F}_e}{1 - \frac{p_g}{2\,\kappa}\,\frac{\beta}{\Theta}\,\left(\frac{\zeta}{1-\zeta\,f}\right)}\,\widetilde{p}_{e,t} \tag{1.50}$$

where, under local determinacy, the denominator shall be positive, and hence the response of employment negative. The presence of imperfect unemployment insurance then amplifies the negative response of employment to a rise in real energy prices: this amplification channel is captured by the presence of the second term at the denominator of equation (1.50). Intuitively, the increase in unemployment risk that follows the reduction in activity, induces a precautionary saving desire on part of households, which depresses aggregate demand. Due to price rigidity, this drag on aggregate demand ends up reducing employment at equilibrium. Moreover, the magnitude of the response is larger the larger the degree of imperfect insurance, as captured by the proportional income loss upon unemployment, ζ.

In the non-homothetic case, we have

$$\widehat{n}_t \;=\; -\,\frac{\mathcal{F}_e + \frac{p_g}{2\,\kappa}\,\frac{\beta}{\Theta}\,\left(\frac{\Xi_w}{1-\zeta-\Xi_w}\right)\,\left[\frac{(1-f)\,\zeta}{1-\zeta\,f-\Xi_w}\right]}{1 - \frac{p_g}{2\,\kappa}\,\frac{\beta}{\Theta}\,\left(\frac{\zeta\,f}{1-\zeta\,f-\Xi_w}\right)}\,\widetilde{p}_{e,t} \tag{1.51}$$

where, again, under local determinacy the denominator shall be positive, and hence the response of employment negative. The presence of heterogeneous exposure to the shock due to non-homotheticity then implies an additional source of amplification of the negative employment response to a rise in real energy prices. First, as before, greater unemployment risk depresses aggregate demand and employment. This channel is again captured by the second term at the denominator of equation (1.51). Second, due to the presence of non-homothetic preferences, as captured by $\Xi_w \in (0,1)$, the rise in real energy prices increases the consumption losses upon employment, thereby strengthening the drag on aggregate demand. This latter amplification channel is captured by the second term at the numerator of equation (1.51). Moreover, as before, when ζ increases the magnitude of the response increases as well, now through both the two channels.

1.6 Normative Analysis

1.6.1 Optimal Policy: an Illustrative Analytical Case

Assuming $\rho = 1$, we have $n_t = f_t = v_t^{1-\alpha}$. Above all, this assumption implies that employment does not act as a state variable in the welfare objective in (1.37), allowing in turn a full analytical treatment of the optimal policy problem.

First of all, in this respect, assuming also $\alpha = 0.5$, the actual, natural, and efficient employment levels read, respectively,

$$n_t = \frac{1}{2\,\kappa}\,(\varphi_t - w_t + S) \tag{1.52}$$

$$n_t^n = \frac{1}{2\,\kappa}\left[\frac{p_{g,t} - \gamma_e\, p_{e,t}}{1-\gamma_e} - w_t + S\right] \tag{1.53}$$

$$n_t^* = \frac{1}{2\,\kappa}\left[\frac{p_{g,t} - \gamma_e\, p_{e,t}}{1-\gamma_e} - w_t^* + \frac{\ln(w_t^* - \xi\, p_{e,t}) - \ln(\delta_t - \xi\, p_{e,t})}{\nu}\right] \tag{1.54}$$

In terms of level deviations from the constrained-efficient steady state we have, in first-order approximations,

$$\widehat{n}_t^n \simeq -\mathcal{F}_e\,\widetilde{p}_{e,t} - \frac{w}{2\,\kappa}\,\widetilde{w}_t \tag{1.55}$$

$$\widehat{n}_t^* \simeq -\mathcal{F}_e\,\widetilde{p}_{e,t} - \frac{w}{2\,\kappa}\,\widetilde{w}_t^* - \frac{w}{2\,\kappa}\,\Psi\,\left[(1-\zeta)\,\widetilde{\delta}_t - \Xi_w\,\widetilde{p}_{e,t}\right] \tag{1.56}$$

How differently efficient and natural employment respond to energy price shocks then depends on how the decentralised wage w_t compares to the efficient wage $w_t^* = \frac{1}{\nu} + \xi\, p_{e,t}$, and on the cyclical behavior of δ_t. In what follows, I assume again as in Challe (2020) and as specified in (1.44) that the decentralised wage and home production fluctuate in such a way that the income loss upon unemployment is constant, implying in turn $\widetilde{\delta}_t = \widetilde{w}_t$, while the decentralised wage is assumed to evolve as in (1.45), i.e. $\widetilde{w}_t = -\chi\,\widetilde{p}_{e,t}$, with $\chi > 0$ unless otherwise noted. Therefore, the wedge between efficient and natural employment is

$$\widehat{n}_t^* - \widehat{n}_t^n = \frac{w}{2\,\kappa}\,\zeta\,\Psi\,\left(\frac{\Xi_w}{1-\Xi_w}\right)\,(1+\chi)\,\widetilde{p}_{e,t}\,. \tag{1.57}$$

Now, given the New Keynesian Phillips Curve

$$\pi_{g,t} = \beta\,\mathbb{E}_t(\pi_{g,t+1}) + \Theta\,\widehat{mc}_{g,t}$$

under the simplifying assumptions made in this section, we have $\widehat{mc}_{g,t} = \frac{2\,\kappa}{p_g}\,\widehat{x}_t^n$, where $\widehat{x}_t^n :=$

$\widehat{n}_t - \widehat{n}_t^n$. Letting $\widehat{x}_t := \widehat{n}_t - \widehat{n}_t^*$ denote the welfare-relevant employment gap,

$$\pi_{g,t} = \beta\,\mathbb{E}_t(\pi_{g,t+1}) + \Theta\,\frac{2\,\kappa}{p_g}\,\widehat{x}_t + \underbrace{\Theta\,\frac{w}{p_g}\,\zeta\,\Psi\left(\frac{\Xi_w}{1-\Xi_w}\right)(1+\chi)\,\widetilde{p}_{e,t}}_{\text{cost-push term}}\,. \tag{1.58}$$

where we can see that energy price shocks act endogenously (also) as a cost-push term. I now turn to discussing its main sources.

Heterogeneous Exposure to Energy Price Shocks. If the efficient wage could be decentralised also outside the constrained-efficient steady state, i.e. $\chi = -\Xi_w$, there would still be a wedge between efficient and natural employment, amounting to

$$\widehat{n}_t^* - \widehat{n}_t^n = \frac{w}{2\,\kappa}\,\zeta\,\Psi\,\Xi_w\,\widetilde{p}_{e,t}\,.$$

In other words, a wedge between efficient and natural employment arises endogenously, first of all, due to the presence of subsistence energy consumption and imperfect unemployment insurance, which together imply that the unemployed are more exposed to energy price shocks compared to the employed. Indeed, as a result of this, rising energy prices induce an increase in the consumption losses upon unemployment. These would be efficiently counteracted by larger employment subsidies, but as these remain constant to their steady-state amount S, a wedge between efficient and natural employment arises.

This, then, constitutes a novel source of a cost-push term in the NK Phillips Curve compared to the benchmark setting of Challe (2020). Indeed, when $\Xi_w = 0$ and hence preferences are homothetic, we can see that the cost-push term vanishes. The term vanishes also in the perfect insurance limit $\zeta \to 0$, as in both these cases there is no heterogeneity between the employed and the unemployed in their direct exposure to energy price shocks.

Real Wage Rigidity. While in Challe (2020) constant real wages can be constrained-efficient (when they correspond to their constrained-efficient steady-state amount) here, as already mentioned, due to the presence of subsistence energy needs a constant real wage rate cannot insure workers against fluctuations in their consumption bundle. As a result, real wage rigidity constitutes another novel source of endogenous cost-push terms, in contrast with Challe (2020). In particular, we can see that when constrained-efficient wage fluctuations cannot be decentralised and real wages remain constant (i.e. $\chi = 0$), the wedge becomes of larger magnitude

$$\widehat{n}_t^* - \widehat{n}_t^n = \frac{w}{2\,\kappa}\,\zeta\,\Psi\left(\frac{\Xi_w}{1-\Xi_w}\right)\widetilde{p}_{e,t}\,.$$

Compared to the constant real wage case, on the one hand downward adjustment (captured by

$\chi > 0$) moderates the fall in natural employment. However, on the other hand, it also causes the home production of the unemployed to fall, calling for a larger employment subsidy on the efficient employment side. This latter effect dominates, causing a wedge between efficient and natural employment which is of larger magnitude compared to the case of constant real wages, as can be seen from (1.57).

Linear-quadratic Problem

In this setting, similarly to Challe (2020), one can derive the following expression for welfare losses from a second-order approximation of the welfare objective[11]

$$L_t = \frac{1}{2}\,\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \,\left(\widehat{x}_{t+j}^2 + \Omega\,\pi_{g,t+j}^2\right) \tag{1.59}$$

where $\Omega = \frac{n\,\varepsilon}{(1-\gamma_e)}\,\frac{p_g}{2\,\kappa}\,/\Theta$. The loss function in (1.59) makes explicit that $\pi_{g,t}$ is the welfare-relevant measure of inflation in this economy, as mentioned before.

Optimal Discretionary Policy. The optimal discretionary policy minimises the period losses

$$\frac{1}{2}\,(\widehat{x}_t^2 + \Omega\,\pi_{g,t}^2)$$

subject to (1.58) taking as given next period expected inflation. The optimality condition of the problem gives

$$\widehat{x}_t = -\left(\frac{n\,\varepsilon}{1-\gamma_e}\right)\,\pi_{g,t} \tag{1.60}$$

which, combined with (1.58), gives

$$\pi_t = \frac{\beta}{1+\Theta\,\frac{2\,\kappa}{p_g}\,\frac{n\,\varepsilon}{(1-\gamma_e)}}\,\mathbb{E}_t(\pi_{t+1}) + \frac{\Theta\,\frac{w}{p_g}\,\zeta\,\Psi\,\left(\frac{\Xi_w}{1-\Xi_w}\right)\,(1+\chi)}{1+\Theta\,\frac{2\,\kappa}{p_g}\,\frac{n\,\varepsilon}{(1-\gamma_e)}}\,\widetilde{p}_{e,t}\,. \tag{1.61}$$

Since $\beta\,\left(1+\Theta\,\frac{2\,\kappa}{p_g}\,\frac{n\,\varepsilon}{(1-\gamma_e)}\right)^{-1} < 1$, the linear expectational difference equation in (1.61) has a unique bounded solution giving, along with the optimality condition in (1.60), the following optimal targets for core inflation and the efficient employment gap under the optimal discretionary policy

$$\pi_{g,t} = \Upsilon\,\zeta\,\Psi\,\left(\frac{\Xi_w}{1-\Xi_w}\right)\,(1+\chi)\,\widetilde{p}_{e,t} \tag{1.62}$$

$$\widehat{x}_t = -\left(\frac{n\,\varepsilon}{1-\gamma_e}\right)\,\Upsilon\,\zeta\,\Psi\,\left(\frac{\Xi_w}{1-\Xi_w}\right)\,(1+\chi)\,\widetilde{p}_{e,t} \tag{1.63}$$

[11]See Appendix 1.B.4 for more details.

where $\Upsilon = \Theta\,\frac{w}{p_g}\left(1-\beta+\Theta\,\frac{2\,\kappa}{p_g}\,\frac{n\,\varepsilon}{(1-\gamma_e)}\right)^{-1}$. We can then see that, in response to an increase in real energy prices, some core inflation is optimally accommodated in order to achieve a smaller (negative) gap between actual and efficient employment. Indeed, one can easily check that if core inflation was fully stabilised, the efficient employment gap would amount to

$$\widehat{x}_t = -\frac{w}{2\,\kappa}\,\zeta\,\Psi\left(\frac{\Xi_w}{1-\Xi_w}\right)(1+\chi)\,\widetilde{p}_{e,t}$$

therefore implying lower employment, as $\frac{w}{2\,\kappa} > \left(\frac{n\,\varepsilon}{1-\gamma_e}\right)\Upsilon$.

Intuitively, the optimal discretionary policy involves partly accommodating core inflation in order to indirectly sustain employment and hence avoid that too many households flow to unemployment thereby becoming more exposed to the shock. We can also notice that when preferences are homothetic, and hence the employed and the unemployed are homogeneously exposed to the shock (corresponding to $\Xi_w = 0$), the trade-off vanishes and the monetary authority can optimally stabilise core inflation and the welfare-relevant employment gap simultaneously.

Lastly, we can see that the targets imply, at equilibrium, that the nominal interest rate consistent with the optimal discretionary policy satisfies

$$\widetilde{I}_t \;\simeq\; \widetilde{R}^n_t \;+\; \frac{\Lambda}{1-f}\,\underbrace{\left(\frac{1-\beta}{\Theta}\right)\frac{p_g}{2\,\kappa}\,\Upsilon\,\zeta\,\Psi\left(\frac{\Xi_w}{1-\Xi_w}\right)(1+\chi)\,\widetilde{p}_{e,t}}_{\mathbb{E}_t(\widehat{x}^n_{t+1})} \;+\; \underbrace{\Upsilon\,\zeta\,\Psi\left(\frac{\Xi_w}{1-\Xi_w}\right)(1+\chi)\,\widetilde{p}_{e,t}}_{\mathbb{E}_t(\pi_{g,t+1})} \tag{1.64}$$

from this, we can first notice that in absence of heterogeneous exposure to energy price shocks, when $\Xi_w = 0$, the optimal policy *outcome* is to align the nominal rate to the real natural rate thereby sterilising core inflation pressures, just as in the textbook Representative-Agent New Keynesian (RANK) model.

By contrast, when unemployed and employed households are heterogeneously exposed to these shocks, this causes a wedge between efficient and natural employment to arise endogenously, and the optimal policy prescription is changed. From (1.64), we can see that the equilibrium real interest rate ends up being higher than the natural real rate, $\widetilde{I}_t - \mathbb{E}_t(\pi_{t+1}) > \widetilde{R}^n_t$: since employment is higher than its natural level, equilibrium demand is kept afloat as the precautionary saving motive due to unemployment risk is dampened.

Optimal Ramsey Policy. The optimal Ramsey policy minimises the lifetime welfare losses in (1.59) subject to the sequence of constraints given by (1.58). The first order conditions of the problem are

$$\Omega\,\pi_{g,t} + \mu_t - \mu_{t-1} = 0 \tag{1.65}$$

$$\widehat{x}_t - \Theta\,\frac{2\,\kappa}{p_g}\,\mu_t = 0 \tag{1.66}$$

for $t \geq 0$, where μ_t is the Lagrange multiplier associated with the period t constraint, and $\mu_{-1} = 0$.

Together with (1.58), these first order conditions imply the following optimal targeted *paths* (formally derived in Appendix 1.B.5) for core inflation and the efficient employment gap under the optimal Ramsey policy

$$\pi_{g,t+j} = \eta^j\,\overline{\Upsilon}\,\zeta\,\Psi\,\left(\frac{\Xi_w}{1-\Xi_w}\right)\,(1+\chi)\,\widetilde{p}_{e,t} \tag{1.67}$$

$$\widehat{x}_{t+j} = -\left(\frac{n\,\varepsilon}{1-\gamma_e}\right)\,\left(\frac{1-\eta^{j+1}}{1-\eta}\right)\,\overline{\Upsilon}\,\zeta\,\Psi\,\left(\frac{\Xi_w}{1-\Xi_w}\right)\,(1+\chi)\,\widetilde{p}_{e,t} \tag{1.68}$$

for $j \geq 0$, where $\eta := \frac{1+\beta+\Theta\,\frac{2\,\kappa}{p_g}\,\frac{n\,\varepsilon}{(1-\gamma_e)}}{2\,\beta}\left[1-\sqrt{1-4\,\beta\,\left(1+\beta+\Theta\,\frac{2\,\kappa}{p_g}\,\frac{n\,\varepsilon}{(1-\gamma_e)}\right)^{-2}}\right] \in (0,1)$ and $\overline{\Upsilon} = \Theta\,\frac{w}{p_g}\,\left(1-\eta\,\beta+\Theta\,\frac{2\,\kappa}{p_g}\,\frac{n\,\varepsilon}{(1-\gamma_e)}\right)^{-1} < \Upsilon$. Therefore, just as in the textbook RANK model, if the monetary authority is able to credibly commit to this policy plan, it is able to achieve a smaller welfare-relevant employment gap on impact and a smaller rise in core inflation relative to the optimal discretionary policy.

The main intuition is, however, the same as in the discretionary case: core inflation is partly accommodated in order to indirectly sustain employment, thereby preventing too many workers ending up being more exposed to the shock by becoming unemployed.

1.6.2 Optimal Policy: the General Case

This section considers the general case of $\rho \in (0,1)$, with the aim of giving a sense of the quantitative properties of the model. However, it shall be stressed that the quantitative predictions of the model, given its stylized nature, should not be taken too literally.

When $\rho < 1$, employment acts as a state variable in the welfare objective since the existing employment stock impacts the current amount of hiring costs, as can be seen from (1.34). This requires resorting to a numerical solution of the Ramsey problem,[12] and hence choosing first of all an appropriate parametrisation for the model.

Parametrisation

The parametrisation, summarised in Table 1.2, is aimed a targeting an average continental European labor market, with each time period taken to correspond to a quarter.

[12]The numerical solution is obtained using Dynare (www.dynare.org).

Preferences. Given a quarterly frequency, the discount factor is set to $\beta = 0.98$, below its usual value of 0.99. Indeed, the presence of uninsured unemployment risk brings the average real rate of return below the value of $1/\beta$ that would prevail under complete markets.[13] Hence, β must be calibrated to a lower value to ensure that real rates of return are not unrealistically small.

The parameters governing the relative weight of energy and the degree of non-homotheticity in consumer preferences (ω_e and ξ, respectively) are calibrated by exploiting the following results (formally derived in Appendix 1.B.2)

$$\omega_e^n := \frac{p_e\, e^n}{w} = \omega_e + (1-\omega_e)\,\Xi_w \tag{1.69}$$

$$\omega_e^u := \frac{p_e\, e^u}{\delta} = \omega_e + (1-\omega_e)\,\frac{\Xi_w}{1-\zeta} \tag{1.70}$$

along with the empirical evidence from Table 1.1 on the energy shares of the employed and the unemployed —appearing on the LHS of equations (1.69) and (1.70)— and on the average percentage difference in overall consumption expenditure between them, which is accordingly set to $\zeta = 22\%$. Reassuringly, this is close to the 20% consumption loss *upon* unemployment documented by Chodorow-Reich and Karabarbounis (2016), and in the range of the estimates surveyed by Den Haan et al. (2018). Therefore, (1.69) and (1.70) jointly imply $\Xi_w = 0.11$ and $\omega_e = 0.08$, with the latter ending up to be in line with the weight of energy in the euro area Harmonized Index of Consumer Prices.

Labor Market. Following Blanchard and Galí (2010a), I target an average job-finding rate $f = 0.25$ and an average unemployment rate $u = 0.10$, implying in turn a separation rate $\rho = 0.037$.

The matching function elasticity is set to $\alpha = 0.6$, which is the midpoint of the range of estimates surveyed by Petrongolo and Pissarides (2001a). As in Challe (2020), flow vacancy costs κ are taken to be 4.5% of the real wage rate. Given the targets, this latter can be recovered from (1.43), and its negative elasticity with respect to the real price of energy is set to $\chi = 0.1$, which would imply a decline in real wages of 4% in light of a 40% increase in the real price of energy, roughly in line with what has been observed in the euro area during the current energy crisis.[14]

Producers. The energy share in production is set to $\gamma_e = 0.04$, as in Bachmann et al. (2022) and Pieroni (2023). The Calvo pricing parameter is set to $\theta = 3/4$, implying an average price duration of one year. The elasticity of substitution among core good varieties is set to $\varepsilon = 4$,

[13] Given the steady state Euler condition $1 = \beta\, R \left[(1-\lambda) + \lambda\left(\frac{w - p_e\,\xi}{\delta - p_e\,\xi}\right)\right]$, one can easily see that since $\left[(1-\lambda) + \lambda\left(\frac{w - p_e\,\xi}{\delta - p_e\,\xi}\right)\right] > 1$, then $R < 1/\beta$.

[14] I check that the real wage remains always in the bargaining set along the simulated path.

Table 1.2: Parametrisation

Targets			
	Description	Value	Source
f	Average job-finding rate	0.25	Blanchard and Galí (2010a)
u	Average unemployment rate	0.10	Blanchard and Galí (2010a)
κ/w	Flow vacancy cost	0.045	Challe (2020)
ζ	Expenditure loss upon unemployment	0.22	ECB Consumer Expectation Survey
ω_e^n	Energy share of the employed	0.182	ECB Consumer Expectation Survey
ω_e^u	Energy share of the unemployed	0.211	ECB Consumer Expectation Survey

Parameters			
	Description	Value	Source/Targets
β	Discount factor	0.98	4% annual interest rate
ω_e	Quasi-share of energy in consumption	0.08	$\zeta, \omega_e^n, \omega_e^u$
ξ	Subsistence consumption of energy	0.109	$\zeta, \omega_e^n, \omega_e^u$
ρ	Separation rate	0.037	f, u
α	Matching function elasticity	0.6	Petrongolo and Pissarides (2001a)
θ	Fraction of unchanged prices	0.75	1 year avg. price duration
ε	Elasticity of Substitution	4	1/3 steady-state markup
γ_e	Share of energy in production	0.05	Bachmann et al. (2022)
ρ_e	Energy price shock persistence	0.97	Blanchard and Galí (2010b)

as in Gagliardone and Gertler (2023), which would imply a steady-state markup of 1/3;[15] even when this is offset by the efficient subsidy $\tau_y = 1/\varepsilon$, the parameter ε might still influence the relative welfare weight: for instance, a smaller ε implies less weight on inflation in the quadratic welfare loss function in (1.59).

Energy Price Shock. I set the shock as a 40% increase in the real price of energy,[16] hitting the economy with high persistence; namely, $\rho_e = 0.97$ as in Blanchard and Galí (2010b), who choose their autoregressive parameter in order to have the real price of oil being very close to a random walk, as it is in the data, while retaining stationarity.

Even if the increase in the energy share of the unemployed relative to that of the employed is not explicitly targeted, the chosen parametrisation is able to match the one percentage point

[15] Kouvavas et al. (2021) document an average markup for the euro area of around 0.45 over the 1995—2018 period.

[16] This is in line with the highest recent rise in the real energy price index above its (stable) longer-run average.

documented in the motivating evidence summarised in Figure 2. Indeed, given the energy shares formally derived in Appendix 1.B.2, $\widehat{\omega}^u_{e,t} - \widehat{\omega}^n_{e,t} \simeq (\omega^u_e - \omega^n_e)(1+\chi)\,\widetilde{p}_{e,t} = 0.01$. This is reassuring, as the increased relative exposure to the shock on part of the unemployed is suggested to be the key driver of the monetary policy trade-off uncovered analytically in Section 1.6.1, and the proposed parametrisation is able to quantitatively match this magnitude with the evidence.

Impulse Responses

Figure 1.3 shows the responses of core inflation and unemployment to a one-off increase in the real price of energy, comparing the behavior of the linearised model under the optimal Ramsey policy with that under two alternative policy rules of the type of equation (1.46): a strict inflation targeting rule (with $\phi_\pi = 15\,, \phi_f = 0$), aimed at suppressing core inflationary pressures, and a rule aimed at stabilising the unemployment rate by targeting also labor market slack and reacting to inflation less aggressively (with $\phi_\pi = 1.5\,, \phi_f = 1.5$). For all these cases, I consider a scenario where the employed and the unemployed are heterogeneously exposed to the shock, as it is suggested by the evidence from Section 1.2, and an alternative scenario where instead preferences are homothetic (i.e. $\xi = \Xi_w = 0$) and hence the employed and the unemployed are homogeneous in their direct exposure to the shock. The top panel of Figure 1.3 reports the impulse responses from the former case, while the bottom panel those from the latter case.

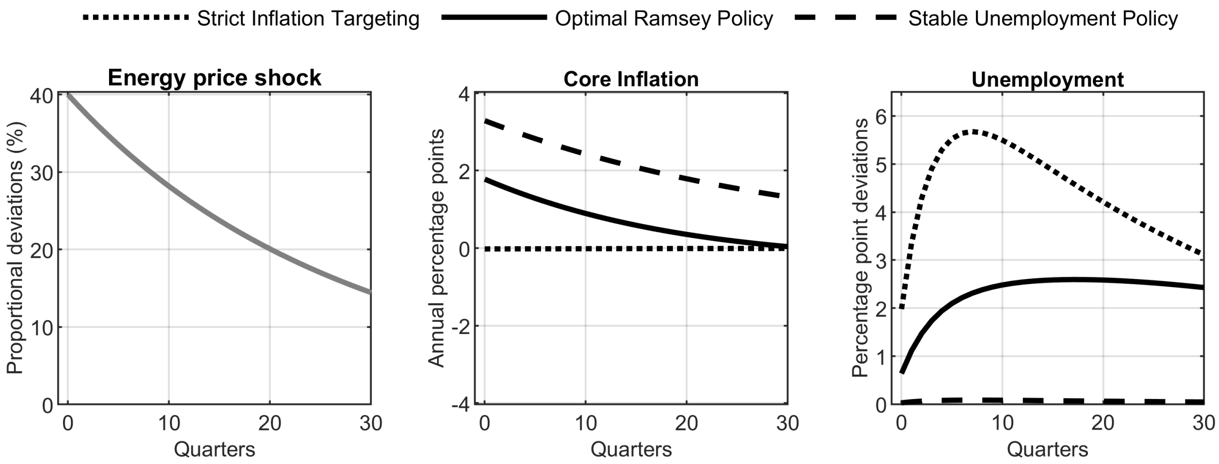
In the heterogeneous exposure case, the surge in the real price of energy causes a sizeable rise in unemployment under strict inflation targeting. By contrast, under the optimal Ramsey policy the monetary authority is able to achieve a notably smaller increase in unemployment, at the cost of partial accommodation of core inflation, as it was suggested by the analytical results. Again, this is intuitively optimal as in such case less workers lose their job and hence avoid becoming more exposed to the surge in energy prices. However, a full stabilization of the unemployment rate neither is optimal, as it implies an overshooting of core inflation compared to the optimal policy: even if unemployment barely increases on impact under this latter, a smaller rise in inflation is achieved by credibly committing to a future recession so as to dampen expectations about future price rises.

In the homogeneous exposure case, as was suggested as well by the analytical results, the trade-off disappears and the optimal Ramsey policy coincides with that achieved under a Taylor rule aimed at aggressively targeting core inflation.

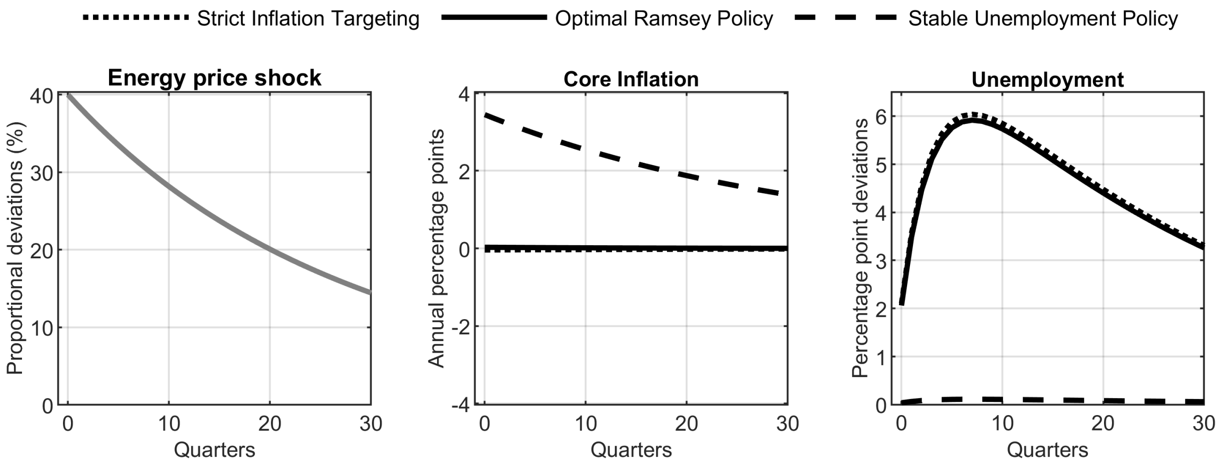
It is also worth mentioning that the results are substantially unchanged, over the medium term, when considering overall inflation instead of core inflation in the target of the Taylor rule: indeed, since $\pi_t = \pi_{g,t} + \frac{\omega_e}{1-\omega_e}\left(\widetilde{p}_{e,t} - \widetilde{p}_{e,t-1}\right)$, and ρ_e is large while ω_e relatively small, we have that with

Figure 1.3: Unemployment and Inflation Effects of an Energy Price Shock

(a) Heterogeneous Exposure to the Shock

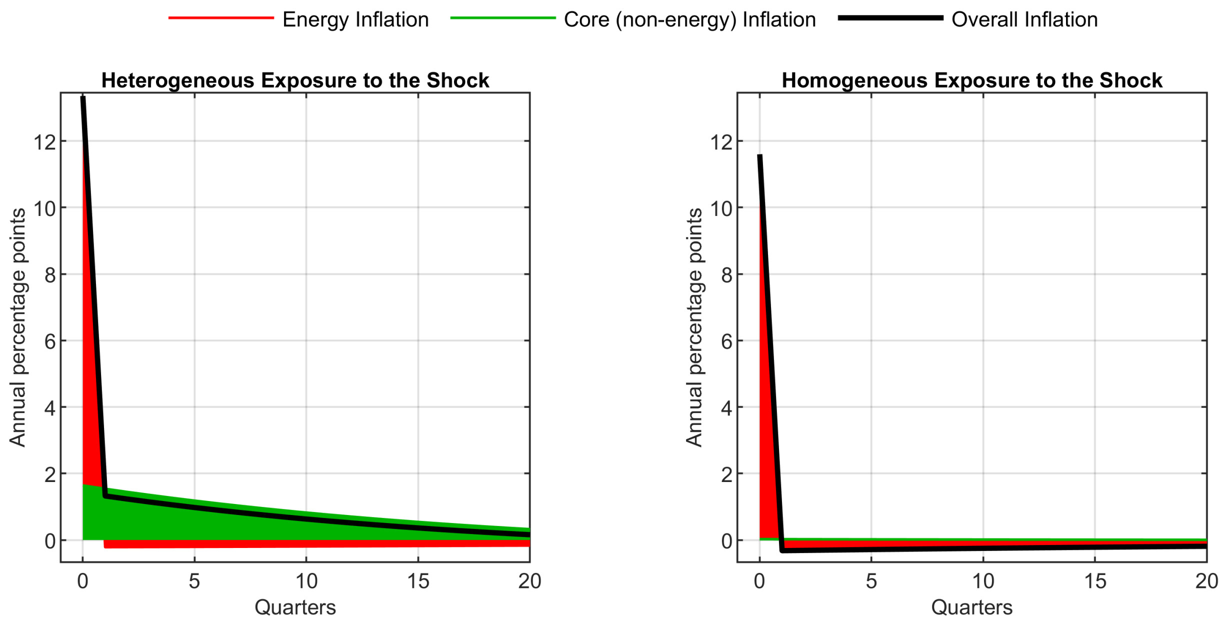


(b) Homogeneous Exposure to the Shock



Notes: The figure reports model-based impulse responses to a 40% real energy price shock of persistence $\rho_e = 0.97$ (left panels), under the parametrisation summarised in Table 1.2. Solid lines correspond to responses under the Ramsey-optimal monetary policy, dotted lines to those under strict inflation targeting ($\phi_\pi = 15\,, \phi_f = 0$), and dashed lines to those under a stable unemployment policy ($\phi_\pi = 1.5\,, \phi_f = 1.5$). Middle and right panels report, respectively, the responses of core inflation and unemployment under the alternative monetary policy regimes. Top panels refer to the heterogeneous exposure case ($\Xi_w = 0.11$) while bottom panels to the homogeneous exposure case ($\xi = \Xi_w = 0$, other parameters unchanged).

Figure 1.4: Overall Inflation and its Components Under the Optimal Ramsey Policy



Notes: The figure reports model-based impulse responses, in the Ramsey-optimal monetary policy regime, to a 40% real energy price shock of persistence $\rho_e = 0.97$, under the parametrisation summarised in Table 1.2. Solid lines correspond to overall inflation, π_t, red areas to its energy component, $\omega_e \, \pi_{e,t}$, and green areas to its core (non-energy) component, $(1 - \omega_e) \, \pi_{g,t}$. The left panel refers to the heterogeneous exposure case ($\Xi_w = 0.11$) while the right panel to the homogeneous exposure case ($\xi = \Xi_w = 0$, other parameters unchanged).

the exception of the period when the shock hits, in subsequent periods $\frac{\omega_e}{1-\omega_e} \left(\widetilde{p}_{e,t} - \widetilde{p}_{e,t-1}\right) \simeq 0$ and $\pi_t \simeq \pi_{g,t}$. However, if overall inflation was to be fully stabilised also in the period of the shock, this would require core deflation on impact, implying a larger surge in unemployment than optimal also in the homogeneous exposure case, where instead stabilising only core inflation at all periods is optimal. In other words, in this case the optimal policy is still accommodative towards energy price changes, but hard on core inflation (similarly to Natal, 2012), as can be seen from Figure 1.4.

Taking stock, even if the model is highly stylized and its exact quantitative predictions should not be taken too literally, its qualitative predictions are likely to matter in practice, and are in line with those suggested by the analytical results: compared to a policy rule aimed at fully stabilising core inflation, the optimal policy is able to achieve a smaller increase in unemployment at the cost of partly accommodating core inflation; the rise in inflation is, however, smaller than if unemployment was instead to be fully stabilised. This trade-off arises due to the heterogeneous exposure to the energy price rise between the employed and the unemployed: in the case of homogeneous direct exposure to the shock, the optimal policy coincides with one aimed at fully stabilising core inflation.

1.7 Conclusions

This paper provides novel evidence on the heterogeneous exposure to energy price shocks between the employed and the unemployed, studying the implications of this fact for the (optimal) conduct of monetary policy in an analytically tractable HANK-S&M model, where non-homothetic preferences and imperfect unemployment insurance endogenously give rise to the heterogeneous exposure documented from the data. Rising energy prices induce a novel precautionary saving motive in this setting, as the consumption losses upon unemployment are increased, posing a drag on aggregate demand and employment. Moreover, the shock acts endogenously as a cost-push term, implying that the monetary authority optimally accommodates some core inflation so as to contain the rise in unemployment and hence avoid households to become more exposed to the shock.

Even if this paper analyses a very specific dimension of heterogeneity, it can lend potentially broader insights: some accommodation of core inflation might be needed to prevent households to become poorer and hence more hit by energy price shocks. Also, these insights can easily extend to similar situations when other subsistence goods than energy receive meaningful relative price shocks.

Lastly, it is worth stressing that the monetary policy trade-off highlighted in this work can likely only be dampened, but not eliminated, by transfers from employed to unemployed workers. Even if for sake of exposure this aspect was not explicitly modeled, these transfers would arguably still provide only partial insurance to the unemployed. Indeed, the fiscal authority would be faced with yet another trade-off, between dampening the increased consumption losses upon unemployment with more generous insurance on the one hand and, on the other hand, the fact that this would depress job search incentives. Investigating this aspect in more depth nevertheless constitutes an interesting avenue for further research.

Chapter 2

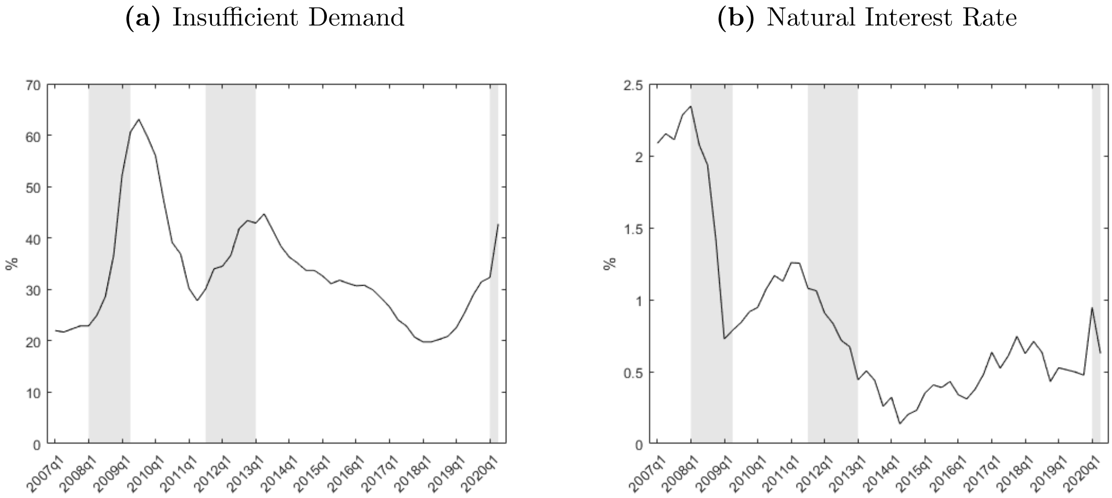
Endogenous Job Destruction Risk and Aggregate Demand Shortages

2.1 Introduction

Recent recessions in the euro area have been characterised by deficient demand. For instance, there has recurrently been a surge in the percentage of businesses reporting insufficient demand as a relevant factor limiting their production (Figure 2.1, panel (a)). Falling natural interest rates, indicating that output fell below potential, suggest as well that these recession episodes have been demand-deficient (Figure 2.1, panel (b)). However, it is difficult to rationalise this demand deficiency as being the result of negative supply-side shocks, at least according to standard macroeconomic theory. Indeed, in the presence of complete markets, a supply-driven recession would imply a desire to borrow out of better future income prospects, thus helping to sustain rather than depressing aggregate demand. Consequently, there has recently been renewed explicit interest on how negative supply shocks can cause demand shortages (Guerrieri et al., 2022). Actually, it has been understood at least since the work of Keynes (1936) that unemployment fears on part of households can make a recession worse, as the desire to hoard a buffer stock of savings in light of a heightened risk of income losses can cause sizable falls in demand. In this respect, the empirical literature has indeed documented that workers suffer substantial losses in both income and consumption during unemployment.[1] Recent studies (Den Haan et al., 2018, Ravn and Sterk, 2021, Challe, 2020, among others) have then explicitly modeled these unemployment fears within macroeconomic models featuring imperfect unemployment insurance, wherein an increased risk of becoming unemployed triggers a precautionary saving behavior on part of households. Yet, these models are mostly silent about how job losses happen in the first place, as they typically assume an exogenous separation rate, implying that all unemployment risk stems only from reduced job creation rates. However, recessions are also characterised by increased job destruction rates, hence an endogenous separation rate cannot be neglected when considering the cyclical properties of worker flows (Fujita and Ramey, 2012). Through the lens of macroeconomic models with uninsurable unemployment risk, it then appears crucial to account

[1] Many studies have reported consumption losses ranging from 14% to 26% (Den Haan et al., 2018).

Figure 2.1: Demand Shortage Indicators for the Euro Area



Notes: The figure reports, on the left panel, the percentage of businesses in the euro area reporting insufficient demand as a limiting factor to production, according to the business climate survey of the European Commission. The right panel reports estimates of the euro area natural rate of interest, conducted by the NY Fed using the Holston et al. (2017) methodology.*Sources:* European Commission's Business Climate Survey and NY Fed.

for endougenously countercyclical job destruction in explaining how unemployment fears, and the precautionary saving behavior they trigger, arise in the first place.

In this paper, I extend the tractable Heterogeneous Agent New Keynesian (HANK) framework with Search and Matching (S&M) frictions of Ravn and Sterk (2021) with endogenous job destruction, showing both analytically and quantitatively how it stands as a key channel in allowing models with uninsurable unemployment risk to be able to be consistent with demand-deficient recessions. Intuitively, this implies that the probability of becoming unemployed increases both due to an increased job destruction rate and a reduced job finding probability, while only this latter channel is at work in the exogenous separation case considered e.g. by Ravn and Sterk (2021). Therefore, endogenous job destruction amplifies unemployment risk and strengthens precautionary saving motives out of fears of becoming unemployed, making a demand deficiency more likely, when realistically paired with the observed presence of rigid real wages.[2]

From an analytical perspective, whether the endogenous job destruction margin becomes relevant in amplifying unemployment risk and precautionary savings during a productivity-driven recession, compared to a reduced job creation margin alone, is formally shown to depend on the degree of real wage rigidity. When wages can flexibly adjust downward in response to negative

[2]Macroeconomic models with S&M frictions typically incorporate rigid real wages, as these stand as a necessary feature for explaining the cyclical volatility of unemployment in this class of models (Christiano et al., 2021). However, endogenous job destruction has largely been ignored, to date, in settings with imperfect unemployment insurance.

productivity shocks, the rise in job destruction as well as the reduction in job creation are contained, and unemployment risk ends up to be contained as well. As a result, the desire to precautionarily save on part of households is not as strong as the desire to borrow in light of the prospects of future income gains from improving economic conditions, conditional on remaining employed. This pre-dominant desire to borrow helps sustaining aggregate demand, as indicated by a rise in the natural interest rate.

By contrast, when real wages respond more sluggishly to the downward pressure exerted by negative productivity shocks, the picture can be overturned. The desire to precautionarily save eventually dominates, causing the natural interest rate to fall, which is a sign that there is a shortage of demand in the economy: with nominal price rigidity, this demand deficiency implies a positive unemployment gap. Formally, it is analytically shown that less rigid wages are already sufficient, in the presence of endogenous job destruction risk, to have such occurrence. Intuitively, with exogenous separation when aggregate productivity falls wage rigidity dampens borrowing desires and reduces job creation incentives, as new matches would be less profitable. When paired with endogenous job destruction, wage rigidity implies also that more existing matches become unprofitable when aggregate productivity falls. As a consequence, the presence of the endogenous job destruction margin amplifies unemployment risk and the precautionary saving motive, making therein demand-deficient recessions a more plausible occurrence.

From a quantitative perspective, accounting for the endogenous job destruction risk channel is shown to be particularly relevant for the so-called "sclerotic" labor markets of continental European countries (Blanchard and Galí, 2010a), characterised by low separation and job-finding rates, and high duration of unemployment spells. Through the lens of models with uninsurable unemployment risk, not considering this channel implies ignoring the potential demand-deficient nature of productivity-driven recessions. Indeed, when calibrating the model to target continental European labor markets, the endogenous job destruction risk channel is quantitatively found to be the predominant driver in causing demand shortages: falling natural rates and positive unemployment gaps are primarily the result of increased job destruction rather than reduced job creation. When the endogenous job destruction margin is shut down, even in the presence of a comparable reduction in job creation the desire to borrow dominates, and the natural interest rate increases, suggesting that aggregate demand is sustained rather than depressed, as indicated as well by a negative unemployment gap.

Related Literature. This paper provides a link between the recent literature on the aggregate impact of uninsured unemployment risk due to incomplete markets (Challe et al., 2017, Ravn and Sterk, 2017, 2021, Den Haan et al., 2018, Challe, 2020), and the earlier literature on search and matching frictions and endogenous job destruction (Mortensen and Pissarides, 1994, Den Haan et al., 2000, Walsh, 2005, Krause and Lubik, 2007, Trigari, 2009).

In Ravn and Sterk (2021) and Challe (2020), due to incomplete markets households precautionarily save as they cannot fully insure against unemployment risk from reduced job creation. Therefore, greater unemployment risk strengthens precautionary savings, potentially causing a fall in aggregate demand, which would in turn feed back to greater unemployment risk due to the presence of nominal rigidities. In my work, greater unemployment risk stems both from reduced job creation and increased job destruction. This strengthens further the precautionary saving motive, causing a more sizeable response in the demand block of the model, i.e. it becomes more plausible that a fall in the natural interest rate and a positive unemployment gap occur during productivity-driven recessions. A closely related paper is Broer et al. (2021), who quantify the unemployment-risk channel in business-cycle fluctuations. On the one hand, they model not only endogenous separations, but also sluggish vacancy creation. On the other hand, they rely only on numerical solutions. By focusing instead on a single specific amplification mechanism, posed by endogenous job destruction, I can characterize and investigate its implications not only numerically but also analytically.

My paper relates, also, to the recent renewed interest on whether negative supply shocks can cause aggregate demand shortages. Guerrieri et al. (2022) present a theory of such so-called *Keynesian supply shocks*, claiming that while in one-sector economies transitory and unanticipated supply shocks are never Keynesian, in economies with multiple sectors Keynesian supply shocks are possible and that incomplete markets, in particular, make the conditions for Keynesian supply shocks more likely to be met. While Guerrieri et al. (2022) study this issue in a two-sector economy hit by transitory and unanticipated supply shocks, such Keynesian supply shocks can occur as well in a one-sector economy à la Ravn and Sterk (2021) due to the presence of uninsurable unemployment risk, when earning risk is sufficiently countercyclical. I build on this by showing that Keynesian supply shocks are indeed most likely when the job destruction margin is endogenised. To my knowledge, this is one on the first attempts to model endogenous job destruction within heterogeneous-household, incomplete-market models.

Roadmap. The remainder of this paper is structured as follows. Section 2.2 provides empirical motivation by reviewing the empirical evidence on the cyclicality of job destruction. Section 2.3 describes the model. Section 2.4 provides an analytical treatment of local fluctuations in response to aggregate productivity shocks. Section 2.5 deals with quantitative analysis. Section 2.6 concludes.

Table 2.1: RELATIVE CONTRIBUTIONS TO UNEMPLOYMENT FLUCTUATIONS

Relative Contribution(%)	Country	Source
25 : 75	US	Shimer (2007, 2012)
43 : 57	Spain	Petrongolo and Pissarides (2008)
29 : 71	UK	Elsby et al. (2011)
15 : 85	Anglo-Saxon	Elsby et al. (2013)
45 : 55	Continental Europe	Elsby et al. (2013)
35 : 65	France	Hairault et al. (2015)

Notes: the table reports the relative contribution (in percentage terms) of the inflow and outflow rates to unemployment fluctuations. Anglo-Saxon countries include Australia, Canada, New Zealand, the United Kingdom, and the United States. Continental European Countries include France, Germany, Italy, Portugal, and Spain.

2.2 Job Destruction Cyclicality: the Empirical Evidence

The conventional wisdom seems to suggest that increased unemployment during recessions stems, primarily, from increased job destruction rather than from diminished job creation. Pioneering empirical research on this respect was conducted by Davis and Haltiwanger (1992): using the US census of manufacturers over the 1972–1986 period, they documented job creation to be less volatile than job destruction, implying, on net, countercyclical job reallocation.

More recently, Shimer (2007, 2012) has cast doubts on this previously established conventional wisdom that recessions are periods characterized primarily by a high exit rate from employment, claiming instead that most of the fluctuations in the US unemployment rate since 1987 were a consequence of movements in the job finding rate alone. However, this conclusion for the US economy has been challenged by Fujita and Ramey (2009) and Elsby et al. (2009), who have concluded instead that a complete understanding of cyclical unemployment requires as well an explanation of countercyclical separation rates, especially for layoffs. This conclusion holds even more strongly for Continental European countries. Some evidence in this regard is summarised in Table 2.1, which reports the relative contributions of inflow (separation) and outflow (job-finding) rates to unemployment fluctuations according to various studies.

All in all, Pissarides (2009) claims that consensus estimates for the relative contribution of the inflow rate lies between one-third and one-half of the total, concluding that one should model cyclicality in both the flows in and the flows out of unemployment.

In other words, a complete understanding of unemployment fluctuations requires modelling fluctuations in both separation and job-finding rates (Elsby et al., 2013). To this respect, Fujita and Ramey (2012) show that standard S&M models with a constant separation rate fail to produce realistic volatility and productivity responsiveness of the separation rate and worker flows, while models with endogenous and countercyclical separation succeed along these dimensions.

2.3 The Model

In the model I construct, households face uninsurable unemployment risk, and job destruction is endogenous. In this vein, my model matches, on the household side, Challe (2020) and Ravn and Sterk (2021) in modelling incomplete markets and imperfect insurance. On the producer side, firms/matches are allowed to differ idiosyncratically in their productivity in a similar way as they do in S&M models with endogenous separation.[3]

2.3.1 Households

On the household side, risk-neutral firm owners simply collect and consume hand-to-mouth the dividends they collect from the firms they manage. Separately from them, there is a unit measure of workers. All these workers cannot borrow. Due to their inability to borrow, workers cannot perfectly insure against the risk of income losses from unemployment.[4]

Formally, at any period a worker $h \in [0,1]$ can be either employed ($n_{ht} = 1$), or unemployed ($n_{ht} = 0$), and solves

$$\max_{\{c_{ht}, a_{ht}\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \ln(c_{ht}) \right]$$

subject to, for all $t \geq 0$,

$$\begin{cases} c_{ht} + b_{ht} = n_{ht}\, w_t + (1 - n_{ht})\, \delta + R_{t-1}\, b_{h,t-1} \\ b_{ht} \geq 0 \end{cases}$$

where c_{ht} is current period consumption, b_{ht} is the amount of assets held at the end of period t, R_t is the gross real return on assets, w_t is wage income deriving from employment, and δ are unemployment benefits obtained when unemployed.

[3]Mortensen and Pissarides (1994) extend Pissarides's (1985) model to allow for idiosyncratic productivity shocks: adverse aggregate shocks raise the threshold for maintaining employment relationships, leading to the termination of less productive matches. Den Haan et al. (2000) model endogenous job destruction within a dynamic general equilibrium setting, while Walsh (2005) builds on this by introducing money and nominal price rigidity.

[4]As will become clear later, when discussing the equilibrium, modelling risk-neutral firm owners separately from workers, and assuming that these cannot borrow, will deliver a high analytical tractability of the model.

2.3.2 Firms

Production occurs using only labor as input, which is hired in a frictional labor market. Active employment relationships (matches) produce a_{it} units of output at each period (provided no separation occurs), and a_{it} is composed by a match-specific and an aggregate component

$$a_{it} = a_t \, \varphi_{it}$$

where the match-specific component, φ_{it}, is *iid* both across firms and over time, and has CDF $G(\varphi)$ and PDF $g(\varphi)$, while a_t represents a common random productivity disturbance which is orthogonal to φ_{it} and assumed to follow an exogenous AR(1) process (in logs)

$$\ln(a_t) = \gamma_a \, \ln(a_{t-1}) + \xi_t$$

with $\xi_t \sim iid(0, \sigma_\xi^2)$, and $\gamma_a \in [0, 1)$.

Matching Technology. Labor market frictions are summarised by an aggregate matching function, which is assumed to take a Cobb-Douglas form with constant returns to scale,

$$m_t = \mathcal{M} \, e_t^{\alpha} \, v_t^{(1-\alpha)}$$

where m_t denotes the total amount of formed matches, $\mathcal{M}$ is a matching efficiency parameter, $\alpha \in (0, 1)$, v_t is the total amount of vacancies, and $e_t = u_{t-1} + \rho_t \, n_{t-1}$ is the total amount of searching workers, including those already unemployed, u_{t-1}, and those (previously) employed workers, n_{t-1}, that experience separation in the current period (at a rate ρ_t).

Letting market tightness be defined as $\vartheta_t = v_t / e_t$, the matching probability for vacancies and the matching probability for searching workers are given, respectively, by

$$q_t = \frac{m_t}{v_t} = \mathcal{M} \, \vartheta_t^{-\alpha}\,, \qquad f_t = \frac{m_t}{e_t} = \mathcal{M} \, \vartheta_t^{(1-\alpha)}$$

Timing. Aggregate and idiosyncratic productivity shocks are realized at the beginning of each period. Once a_t and φ_{it} are realised, job destruction and job creation occur: first, an endogenously determined fraction of old matches breaks up; then, new matches are formed, and active producers and employed workers collectively set the wage rage w_t. After this, households take their consumption/saving decisions.

Value Functions. The value of a match for a firm with idiosyncratic productivity φ is

$$J_t(\varphi) = a_t\,\varphi - w_t + \beta\,\mathbb{E}_t\left[(1-\rho^x)\int_{\varphi^*_{t+1}}^{\infty} J_{t+1}(\varphi)\,g(\varphi)\,d\varphi\right] \tag{2.1}$$

where the wage rate w_t is the same across producers, ρ^x is the exogenous separation rate, and φ^*_{t+1} is the threshold below which a match will become unprofitable, and hence will be endogenously destroyed: since $J_t(\varphi)$ is monotonically increasing in φ, and all firms face the same aggregate shocks, the endogenous job destruction cutoff, φ^*_t, will be determined, at every period, by the condition

$$J_t(\varphi^*_t) = 0$$

and matches with $\varphi_{it} \geq \varphi^*_t$ will be actively producing, while matches with $\varphi_{it} < \varphi^*_t$ will not and break up. Therefore, the total separation rate, associated to φ^*_t, is given by

$$\rho_t := \rho^x + (1-\rho^x)\,G\left(\varphi^*_t\right) \tag{2.2}$$

When matches break up, inactive producers (which are ex-ante symmetric) post vacancies at a cost of κ units of the final good per vacancy per period, and each vacancy is filled with probability q_t. The value of a vacancy is therefore equal across producers, and given by

$$V_t = -\kappa + q_t\,J_t + [1-q_t]\;\beta\,\mathbb{E}_t\left(V_{t+1}\right) \tag{2.3}$$

where

$$J_t := \frac{1}{\left[1-G\left(\varphi^*_t\right)\right]}\int_{\varphi^*_t}^{\infty} J_t(\varphi)\,g(\varphi)\,d\varphi$$

Lastly, given the assumptions on the production structure, aggregate dividends collected by firm owners are given by

$$d_t = n_t\;\left[a_t\,\overline{\varphi}_t - w_t\right] - \kappa\,v_t$$

where n_t denotes total employed workers, $\overline{\varphi}_t := \mathbb{E}(\varphi\,|\,\varphi \geq \varphi^*_t)$, and v_t is the total amount of vacancies.

Employment Dynamics. The laws of motion of employment and unemployment are[5]

$$n_{t+1} = [1-\lambda_{t+1}]\,n_t + f_{t+1}\,u_t \tag{2.4}$$

$$u_{t+1} = 1 - n_{t+1} = \lambda_{t+1}\,n_t + [1-f_{t+1}]\,u_t \tag{2.5}$$

[5]Given that there are $e_{t+1} = u_t + \rho_{t+1}\,n_t$ searching workers, the law of motion for employed workers can equivalently be stated as $n_{t+1} = (1-\rho_{t+1})\,n_t + f_{t+1}\,e_{t+1}$.

where, since a currently employed worker experiences separation with probability given by the total separation rate ρ_{t+1} and, in that case, finds a new job with probability f_{t+1}, the probability that a currently employed worker is without a job next period is given by

$$\lambda_{t+1} = \rho_{t+1} [1 - f_{t+1}] \tag{2.6}$$

2.3.3 Equilibrium

Definition 2.1. *An equilibrium is a set of sequences of optimal household and producer choices such that: (i) markets clear; (ii) labor market variables evolve according to (2.2),(2.4)–(2.6); (iii) wages satisfy the collective wage setting rule; (iv) the Job Destruction and Free Entry conditions hold.*

A relevant feature of the equilibrium lies in its zero-liquidity property, allowing a good tractability of the precautionary motive within the model: given the zero debt limit households face, the supply of assets is zero in equilibrium, no asset trade actually takes place, and all households turn out to consume their current income. This allows the precautionary motive to be operative without the need of tracking a full and time-varying wealth distribution.[6]

Intuitively, in standard complete-market models without capital, the consumption smoothing motive for saving or borrowing is operative also when assets are in zero supply: when the real interest rate goes down, households want to bring consumption to the present; with no assets to liquidate or disinvest from, their income adjusts to deliver equilibrium. Similarly, in this incomplete-market setting, when employed workers wish to save for precautionary reasons, this puts downward pressure on the real interest rate: the natural interest rate must fall to make sure that employed workers are not actually willing to save, so that savings are exactly zero at equilibrium. Given this, unemployed workers and firm owners would wish to borrow to bring consumption to the current period, but cannot do so due to borrowing constraints. Therefore, equilibrium income adjusts accordingly, bringing about the desired reduction in consumption for precautionary motives with no actual saving nor borrowing occurring.

How this property formally arises will now be spelled out in more detail, along with the other equilibrium conditions.

[6] This feature of the model is directly drawn from Challe (2020) and the earlier literature achieving tractability in incomplete-market models by assuming that no agent can borrow: see, among others, Bilbiie (2019), Krusell et al. (2011), McKay et al. (2017), McKay and Reis (2016, 2021), Ravn and Sterk (2017, 2021), Werning (2015).

Market Clearing

Bonds. The market clearing condition for bonds is given by

$$(1 - n_t)\, b_t^u + n_t\, b_t^n = 0 \tag{2.7}$$

where superscripts n and u refer to employed and unemployed workers, respectively.

Given that unemployed workers wish to borrow but are liquidity-constrained, we have $b_t^u = 0$; as a result and as already mentioned, even though employed workers wish to precautionarily save, bonds market clearing implies that at equilibrium no agent is issuing the bonds that would allow them to do so. As a result, also $b_t^n = 0$, and all households indeed consume all their current income.

Goods. The total supply of the final good is $Y_t + (1 - n_t)\,\delta$, where $Y_t = n_t\, a_t\, \overline{\varphi}_t$, and it serves the purposes of providing for household consumption and vacancy posting. Given that all households consume their current income, we have

$$\underbrace{[n_t\, w_t + (1 - n_t)\,\delta]}_{\int_0^1 c_{ht}\, dh} + d_t \,+\, \kappa\, v_t \,=\, Y_t \,+\, (1 - n_t)\,\delta \tag{2.8}$$

Optimal Household Decisions

Taking into account the fact that all workers consume their current income (be it δ or w_t), Euler conditions for employed workers and unemployed workers are, respectively,

$$1 \geq \beta\, \mathbb{E}_t \left[\frac{(1 - \lambda_{t+1})\, u'(w_{t+1}) + \lambda_{t+1}\, u'(\delta)}{u'(w_t)}\, R_t \right] \tag{2.9}$$

$$1 \geq \beta\, \mathbb{E}_t \left[\frac{(1 - f_{t+1})\, u'(\delta) + f_{t+1}\, u'(w_{t+1})}{u'(\delta)}\, R_t \right] \tag{2.10}$$

each holding with strict inequality if the household is liquidity constrained (i.e. wishing to borrow), and with equality otherwise.

At equilibrium, given that employed workers wish to precautionarily save, their Euler condition in (2.9) holds with equality. Conversely, unemployed workers wish to borrow, but face a binding liquidity constraint; their Euler conditions hence hold with inequality. These equilibrium conditions hold at steady state, provided that $\delta < w$, as well as in stochastic equilibrium in the steady state neighbourhood, provided that aggregate shocks are not too large.[7] Formally,

[7] See Ravn and Sterk (2021) and Challe (2020) for more details.

Proposition 2.1. *At steady state, provided that $\delta < w$, Euler conditions for employed workers and unemployed workers are, respectively,*

$$
\begin{aligned}
1 &= \beta \left[\frac{(1-\lambda)\, u'(w) + \lambda\, u'(\delta)}{u'(w)} R \right] \\
1 &> \beta \left[\frac{(1-f)\, u'(\delta) + f\, u'(w)}{u'(\delta)} R \right]
\end{aligned}
$$

Proof. See Appendix 2.B.1. □

It is also worth stressing how a binding liquidity constraint for the unemployed (i.e. their Euler condition holding with inequality) is intimately associated with imperfect insurance against unemployment. If asset trading was not constrained by a borrowing limit, then the employed and the unemployed would trade assets up to the point where their consumption level would be equalised, but this would be at odds with the evidence of consumption dropping upon unemployment. Another, limiting case, where consumption still would be equalised but without any asset trading needed, is when $\delta = w$. In any case, a lower consumption level for the unemployed requires that they face a binding liquidity constraint and that their Euler equation holds with inequality.[8] Formally, if that was not the case,

Proposition 2.2. *Suppose that the Euler conditions of both agents hold with equality at steady state. Then, their consumption is equalised.*

Proof. See Appendix 2.B.2. □

Job Destruction and Free Entry Conditions

The exit cutoff φ_t^* is characterised by the following Job Destruction (JD) condition

$$
J_t(\varphi_t^*) = 0 \iff a_t\, \varphi_t^* - w_t + \beta\, \mathbb{E}_t \left[(1-\rho^x) \int_{\varphi_{t+1}^*}^{\infty} J_{t+1}(\varphi)\, g(\varphi)\, d\varphi \right] = 0 \qquad \text{(JD)}
$$

With free entry into vacancy posting, the value of posting a vacancy ends up to be zero at every period, delivering the following condition

$$
\frac{\kappa}{q_t} = J_t \qquad \text{(FE)}
$$

[8] One can however argue that zero borrowing is an extreme assumption, and that the unemployed could face a binding, but not necessarily zero, borrowing constraint. This possibility is explored numerically in an extension to the quantitative analysis in Section 2.5, as the presence of a non-zero borrowing constraint makes the model analytically intractable.

where, combining (2.1) and (JD), we have $J_t(\varphi) = a_t\,(\varphi - \varphi_t^*)$, for every t, so that

$$\frac{\kappa}{q_t} = \frac{1}{[1 - G(\varphi_t^*)]} \int_{\varphi_t^*}^{\infty} J_t(\varphi)\, g(\varphi)\, d\varphi = a_t\, [\overline{\varphi}_t - \varphi_t^*]$$

Moreover, using (FE) into (JD), we get the following expression for the job destruction cutoff

$$a_t\,\varphi_t^* = w_t - \beta\,\mathbb{E}_t\left[(1 - \rho_{t+1})\,\frac{\kappa}{q_{t+1}}\right] \tag{2.11}$$

from which we can see that, ceteris paribus, a higher wage rate is associated with a higher job destruction cutoff, and an expected tighter market is associated with a lower job destruction cutoff.

A higher wage directly implies a higher productivity cutoff above which the producer can break even. An expected tighter market implies that vacancies will be harder to fill, and hence a producer expects to incur a higher cost if she will decide to break the match and seek to form another, hopefully more productive one; this discourages match destruction, and is hence associated with a lower job destruction threshold.

Also, a higher level of aggregate productivity a_t implies a higher production for all matches; therefore, ceteris paribus, matches of lower idiosyncratic productivity can now break even, and this implies a lower job destruction cutoff. This effect can then amplify the impact of aggregate productivity shocks on output (as emphasized by Den Haan et al., 2000).

However, all of these considerations are of course only valid under partial equilibrium, as aggregate productivity shocks have impact also on the wage rate as well as on the continuation value of the match, κ/q_t.

Wage Setting and Real Wage Rigidity

I assume, in the spirit of Jimeno and Thomas (2013), that the wage is the same across producers, being the result of a collective wage setting rule rather than a match-specific one. This is consistent with the presence of unionised labor markets, and simplifies the treatment of the model both on the producer side and on the household side.[9]

Moreover, it is assumed that wages adjust sluggishly to shocks. Indeed, since the work of Shimer (2005), rigid wages have been shown to be a key feature for S&M models in doing a good job in matching the empirical volatility of unemployment. The fact that wage stickiness vastly increases the sensitivity of these models to driving forces was noticed also by Hall (2005). Firstly,

[9] In most continental European countries, wage setting actually takes place predominantly in form of collective agreements (Du Caju et al., 2008).

he showed that actually any wage rule involving a positive surplus for both parties (workers and producers) can be an equilibrium in the Nash sense, not only the constant surplus splitting scheme commonly known as Nash bargaining. This latter, traditionally adopted by the S&M literature, was however shown by Shimer (2005) to involve excessively flexible wages and hence too much unemployment volatility compared to the data. While focusing in particular on the subset of constant wages in his quantitative assessment, Hall (2005) also emphasized that there are other possible equilibrium wage rules which are consistent with inertial —but not necessarily constant— wages. One of such possibilities is a wage smoothing norm (as adopted e.g. by Challe, 2020, , among others),[10]

$$w_t = \Gamma\, w_t^n + (1 - \Gamma)\, w \tag{2.12}$$

where Γ acts as a smoothing parameter, with $\Gamma = 0$ corresponding to the constant wage case and $\Gamma = 1$ to the flexible wage case, and w_t^n is a notional wage, usually assumed to coincide with the Nash-bargained wage. In Appendix 2.A.1, I show that a collective surplus splitting scheme between workers and producers gives the following notional wage rate, in this setting,

$$w_t^n = \eta\, \{a_t\, \overline{\varphi}_t + \kappa\, \beta\, \mathbb{E}_t\, [(1 - \rho_{t+1})\, \vartheta_{t+1}]\} + (1 - \eta)\, \delta \tag{2.13}$$

where η is the fraction of collective surplus going to workers, with the remaining fraction $(1 - \eta)$ going to producers.

All in all, as already noted by Blanchard and Galí (2010a) how to formalize real wage rigidity still remains an open research question. Hence, to keep the analysis as simple as possible and clearly highlight the role played by wage rigidity, in what follows I assume a wage schedule of the form

$$w_t = w\, a_t^{\chi} \tag{2.14}$$

where $\chi \in [0, \overline{\chi}_w]$ is an index of real wage rigidities, and w is the steady state wage rate. This implies that the mean wage coincides with the mean wage under Nash bargaining. Notice that $\chi = 0$ corresponds to the case of fully rigid wages. $\chi = \overline{\chi}_w$ corresponds instead to the case of flexible wages (under standard Nash bargaining), with the composite parameter $\overline{\chi}_w$ depending on the nature of job separations (exogenous or endogenous) and explicitly outlined in Appendix 2.A.2, where it is also shown that the wage schedule in (2.14) is essentially the same as the

[10] Another possibility emphasised by Hall (2005) —with the analytical shortcoming that the wage becomes a new state variable of the model— is a wage norm depending on past wages, $w_t = \gamma_w\, w_t^n + (1 - \gamma_w)\, w_{t-1}$, where γ_w captures the degree of wage rigidity, and can be interpreted as the fraction of wages renegotiated each period (see Gertler and Trigari, 2009). In the context of unionised labor markets, it actually takes time for renegotiating collective bargaining agreements, hence wages will respond sluggishly to shocks: collective bargaining coverage has been shown to be positively related with downward real wage rigidity both from an empirical perspective (Du Caju et al., 2008, Babecky et al., 2009) and a theoretical perspective (Morin, 2017).

Table 2.2: MODEL SUMMARY

$$1 = \beta\,\mathbb{E}_t\left[\frac{(1-\lambda_{t+1})\,u'(w_{t+1})+\lambda_{t+1}\,u'(\delta)}{u'(w_t)}\,R_t\right]$$

$$w_t = w\,a_t^{\chi}$$

$$q_t = \mathcal{M}\,\vartheta_t^{-\alpha}$$

$$f_t = \mathcal{M}\,\vartheta_t^{(1-\alpha)}$$

$$n_{t+1} = [1-\lambda_{t+1}]\,n_t + f_{t+1}\,(1-n_t)$$

$$u_{t+1} = 1 - n_{t+1}$$

Endogenous Separation	Exogenous Separation
$\rho_t = \rho^x + (1-\rho^x)\,G\,(\varphi_t^*)$	—
$\lambda_{t+1} = \rho_{t+1}\,[1-f_{t+1}]$	$\lambda_{t+1} = \rho\,[1-f_{t+1}]$
$\frac{\kappa}{q_t} = a_t\,[\overline{\varphi}_t - \varphi_t^*]$	$\frac{\kappa}{q_t} = a_t - w_t + \beta\,\mathbb{E}_t\left[(1-\rho)\,\frac{\kappa}{q_{t+1}}\right]$
$a_t\,\varphi_t^* = w_t - \beta\,\mathbb{E}_t\left[(1-\rho_{t+1})\,\frac{\kappa}{q_{t+1}}\right]$	—
$Y_t = n_t\,a_t\,\overline{\varphi}_t$	$Y_t = n_t\,a_t$

Notes: $\rho = \rho^x + (1-\rho^x)\,G\,(\varphi^*)$ is the steady state separation rate of the endogenous job destruction model, as well as the exogenous (and constant) separation rate of the exogenous job destruction model.

wage norm in (2.12), in first order approximations, as there is a 1:1 mapping between Γ and χ. Indeed, the wage schedule in (2.14) can be on its own an equilibrium wage in Hall's (2005) sense once it is ensured that the match surplus remains positive for both workers and producers. As for producers, such circumstance is always satisfied when φ has a positive support, since those matches involving a negative match surplus for the firm will be terminated according to the (JD) condition. As for workers, it instead sufficient that $w_t > \delta$ in the current period and in expectations.

Summary of the Equilibrium Conditions

I end up with 11 equations determining 11 endogenous variables of interest: λ_t, w_t, R_t, Y_t, ϑ_t, φ_t^*, n_t, u_t, ρ_t, q_t, f_t. The full set of equilibrium conditions is summarised in Table 2.2, along with the equilibrium conditions of a comparable model with symmetric matches and exogenous separation.

Steady State. The steady state of the two models is summarised in Table 2.3; the two models are assumed to differ only in their underlying transitional dynamics, while ending up to deliver the same steady state magnitudes.

Proposition 2.3. *At steady state, for given job finding probability f and total separation rate ρ,*

Table 2.3: Steady State

$1 = \beta\, R \left[1 + \lambda \left(\frac{\zeta}{1-\zeta}\right)\right]$	
$\lambda = \rho\,(1 - f)$	
$f = \mathcal{M}\,\vartheta^{(1-\alpha)}$	
$q = \mathcal{M}\,\vartheta^{-\alpha}$	
$u = \frac{\lambda}{\lambda + f}$	
$n = 1 - u$	
$Y = n$	
$w = \eta + \eta\,\kappa\,\beta\,(1-\rho)\,\vartheta + (1-\eta)\,\delta$	
Endogenous Separation	Exogenous Separation
$\rho = \rho^x + (1-\rho^x)\,G\,(\varphi^*)$	—
$\frac{\kappa}{q} = 1 - \varphi^*$	$\frac{\kappa}{q} = 1 - w + \beta\,(1-\rho)\,\frac{\kappa}{q}$
$\varphi^* = w - \beta\,(1-\rho)\,\frac{\kappa}{q}$	—

Notes: Aggregate productivity ,a, and average idiosyncratic productivity at steady state, $\overline{\varphi} = \mathbb{E}\,(\varphi\,|\,\varphi \geq \varphi^*)$, are both normalised to 1 so that the two models deliver the same steady state magnitudes. The parameter $\zeta = [(w-\delta)/w] \in (0,1)$ captures the steady state amount of unemployment insurance.

the wage rate and the job destruction cutoff are given, respectively, by

$$w = \frac{\left[1 + \frac{\beta\, f\,(1-\rho)}{1-\beta\,(1-\rho)}\right]}{\left[\left(\frac{1-\eta}{\eta}\right)\,\zeta + 1 + \frac{\beta\, f\,(1-\rho)}{1-\beta\,(1-\rho)}\right]}$$
$$\varphi^* = \frac{w - \beta\,(1-\rho)}{1 - \beta\,(1-\rho)}$$

where $\zeta := [(w-\delta)/w] \in (0,1)$. *Moreover, the following inequality holds*

$$\varphi^* < w < 1$$

Proof. See Appendix 2.B.3. □

2.4 Local Fluctuations

In this section I study analytically local fluctuations, in the neighbourhood of the steady state, in response to aggregate productivity shocks. As will become clearer later, it shall be stressed that differently to standard, complete market models, focusing on a first-order log approximation

Table 2.4: Summary of the Log-linear Model

$$\widehat{r}_t = -\left[\frac{\rho\,(1-f)\,\zeta}{(1-\zeta)+\rho\,(1-f)\,\zeta}\right]\mathbb{E}_t\left(\widehat{\lambda}_{t+1}\right) + \left\{(1-\zeta)\left[\frac{1-\rho\,(1-f)}{(1-\zeta)+\rho\,(1-f)\,\zeta}\right]\mathbb{E}_t\left(\widehat{w}_{t+1}\right) - \widehat{w}_t\right\}$$

$$\widehat{q}_t = -\alpha\,\widehat{\vartheta}_t$$

$$\widehat{f}_t = (1-\alpha)\,\widehat{\vartheta}_t$$

$$\widehat{n}_{t+1} = -\lambda\,\widehat{\lambda}_{t+1} + (1-\lambda)\,\widehat{n}_t + \lambda\,\widehat{f}_{t+1} + \lambda\,\widehat{u}_t$$

$$\widehat{u}_{t+1} = -\left(\frac{1-u}{u}\right)\widehat{n}_{t+1}$$

$$\widehat{w}_t = \chi\,\widehat{a}_t$$

$$\widehat{a}_t = \gamma_a\,\widehat{a}_{t-1} + \xi_t$$

Endogenous Separation	Exogenous Separation
$\widehat{y}_t = \widehat{n}_t + \widehat{a}_t + [\tau\,(1-\varphi^*)]\,\widehat{\varphi}_t^*$	$\widehat{y}_t = \widehat{n}_t + \widehat{a}_t$
$\widehat{\rho}_t = \left[\tau\left(\frac{1-\rho}{\rho}\right)\right]\widehat{\varphi}_t^*$	—
$\widehat{\lambda}_{t+1} = \widehat{\rho}_{t+1} - \left(\frac{f}{1-f}\right)\widehat{f}_{t+1}$	$\widehat{\lambda}_{t+1} = -\left(\frac{f}{1-f}\right)\widehat{f}_{t+1}$
$\alpha\,\widehat{\vartheta}_t = \widehat{a}_t - \left(\frac{1}{1-\varphi^*}\right)[\varphi^* - \tau\,(1-\varphi^*)]\,\widehat{\varphi}_t^*$	$\alpha\,\widehat{\vartheta}_t = \frac{\widehat{a}_t}{\kappa/q} - \left(\frac{w}{\kappa/q}\right)\widehat{w}_t + \beta\,(1-\rho)\,\mathbb{E}_t\left(\alpha\,\widehat{\vartheta}_{t+1}\right)$
$\widehat{\varphi}_t^* = \left(\frac{w}{\varphi^*}\right)\widehat{w}_t - \widehat{a}_t + \beta\,(1-\rho)\left(\frac{1-\varphi^*}{\varphi^*}\right)\mathbb{E}_t\left(\tau\,\widehat{\varphi}_{t+1}^* - \alpha\,\widehat{\vartheta}_{t+1}\right)$	—

Notes: The parameter $\zeta = [(w-\delta)/w] \in (0,1)$ captures the steady state amount of unemployment insurance. $\rho = \rho^x + (1-\rho^x)\,G(\varphi^*)$ is the total separation rate in the steady state of the endogenous separation model, as well as the constant separation rate of the exogenous job destruction model. $\tau = \left[\frac{\varphi^*\,g(\varphi^*)}{1-G(\varphi^*)}\right]$, where $g(\varphi^*)$ and $G(\varphi^*)$ are, respectively, the PDF and CDF of the idiosyncratic productivity parameter φ, evaluated at the steady state job destruction threshold φ^*.

does not eliminate the precautionary saving motive from this setting. Indeed in standard, complete-market models, a precautionary saving motive can arise due to so-called prudence, i.e. convexity of the first derivative of the utility function, and hence disappears when focusing on first-order approximations. By contrast, in this case the precautionary motive arises due to the presence of liquidity constraints, which imply imperfect insurance against unemployment —as formally shown in propositions 2.1 and 2.2— and whose effect persists regardless of the approximation order.

In what follows, I will first formally show how such precautionary saving motive arises in the presence of imperfect unemployment insurance, posing a drag on aggregate demand and exerting downward pressure on the natural rate of interest. I will then move to analytically comparing the precautionary saving motive in the endogenous job destruction model with the exogenous job destruction case, formally showing how it is amplified in the former case and in the presence of real wage rigidity.

The log-linearized model. Let $\widehat{z}_t$ denote log deviations from their steady state value, z, i.e. $\widehat{z}_t = \ln(z_t) - \ln(z)$. The log-linearised model resulting from first order (log) approximation around the steady state is summarised in Table 2.4.

Focusing on the log-linearised Euler equation we can distinguish —as already highlighted by

Challe (2020)— two main forces shaping the response of the natural interest rate: a standard consumption smoothing motive, and a precautionary motive.

$$\hat{r}_t = \underbrace{-\left[\frac{\rho\,(1-f)\,\zeta}{(1-\zeta)+\rho\,(1-f)\,\zeta}\right]\mathbb{E}_t\left(\widehat{\lambda}_{t+1}\right)}_{\text{precautionary motive}} + \underbrace{\left\{(1-\zeta)\,\left[\frac{1-\rho\,(1-f)}{(1-\zeta)+\rho\,(1-f)\,\zeta}\right]\mathbb{E}_t\left(\widehat{w}_{t+1}\right)-\hat{w}_t\right\}}_{\text{standard consumption smoothing motive}} \tag{2.15}$$

where the parameter $\zeta = [(w-\delta)/w] \in (0,1)$, representing the percentage loss in equilibrium consumption upon unemployment, captures the steady state amount of unemployment insurance ($\zeta = 0$ in the limit of full insurance at steady state).

To understand the two opposite forces shaping the response of the natural interest rate, it is useful to consider, in turn, the extreme cases of perfect insurance and constant wages. In the perfect insurance limit ($\zeta \to 0$), the precautionary motive completely disappears, and the response of the natural rate is entirely driven by the standard consumption smoothing motive; as a consequence, if households expect an increasing wage profile, they wish to borrow, and this puts upward pressure on the natural interest rate. By contrast, in the constant wage case ($\chi = 0$), only the precautionary motive is operative: an increase in the risk of losing the current occupation (and not being swiftly able to find another one), increases the desire to hold a buffer stock of savings, putting downward pressure on the natural interest rate.

Coming back to the general case, for a given real wage elasticity χ, the standard consumption smoothing motive is the same in the exogenous job destruction and endogenous job destruction cases. What differs is the precautionary motive, as driven by unemployment risk, $\mathbb{E}_t\left(\widehat{\lambda}_{t+1}\right)$. In the exogenous job destruction case, its only driver is the job-finding probability, as

$$\widehat{\lambda}_{t+1} = -\left(\frac{f}{1-f}\right)\widehat{f}_{t+1}\,.$$

By contrast, in the endogenous job destruction case, unemployment risk is driven both by changes in the probability of losing a job in the first place, as well as by changes in the job-finding probability after the job loss, as

$$\widehat{\lambda}_{t+1} = \widehat{\rho}_{t+1} - \left(\frac{f}{1-f}\right)\widehat{f}_{t+1}\,.$$

Therefore, in order to have amplification of a contractionary shock in the endogenous case, compared to the exogenous case, one needs countercyclical variations in the job-destruction threshold, as well as procyclical variations in the job-finding probability that are comparable in the two cases. When this occurs, then the presence of an endogenous job destruction margin ends up to indeed amplify unemployment risk and strengthen the precautionary motive, making a fall in the natural interest rate (symptom of demand deficiency in the current period) more likely.

Equilibrium Responses. To formally assess whether and how endogenous job destruction amplifies unemployment risk, strengthening therein the precautionary motive after a contractionary shock, I now solve for local dynamics in the neighbourhood of the steady state. The equilibrium responses to aggregate productivity shocks of the log-linear endogenous job destruction model can be fully characterised starting from those of the job destruction cutoff, $\widehat{\vartheta}_t$, and market tightness, $\widehat{\varphi}^*_t$, from which the dynamics of all the other relevant variables can be determined. In the exogenous separation case, it is instead sufficient to characterise the equilibrium response of market tightness, $\widehat{\vartheta}_t$, to then residually pin down the dynamics of the other variables. These equilibrium responses are summarised in the following proposition.

Proposition 2.4. *In the endogenous job destruction case, the equilibrium response to aggregate productivity shocks of the job destruction cutoff and market tightness are given by*

$$\widehat{\vartheta}_t = \mathcal{A}^a_\vartheta \, \widehat{a}_t \tag{2.16}$$

$$\widehat{\varphi}^*_t = -\mathcal{A}^a_\varphi \, \widehat{a}_t \tag{2.17}$$

with

$$\begin{aligned}
\mathcal{A}^a_\vartheta &= \frac{1}{\alpha} \left(\frac{1}{1-\varphi^*} \right) \left[\frac{1}{1-\beta\,\gamma_a\,(1-\rho)} \right] \left\{ (1-\mathcal{C}^a_\vartheta) - \chi\, w \left[1 - \tau \left(\frac{1-\varphi^*}{\varphi^*} \right) \right] \right\} \\
\mathcal{A}^a_\varphi &= \frac{w}{\varphi^*} \left[\frac{1}{1-\beta\,\gamma_a\,(1-\rho)} \right] (\overline{\chi}_c - \chi)
\end{aligned}$$

where

$$\mathcal{C}^a_\vartheta = \tau \left(\frac{1-\varphi^*}{\varphi^*} \right) w\, \overline{\chi}_c$$

$$\overline{\chi}_c = 1 - \left[\frac{\beta\,(1-\rho)\,(1-\gamma_a)}{1-\beta\,(1-f)\,(1-\rho)} \right] \left(\frac{1-\eta}{\eta} \right) \zeta$$

In the exogenous job destruction case, the equilibrium response to aggregate productivity shocks of market tightness is given by

$$\widehat{\vartheta}_t = \mathcal{B}^a_\vartheta \, \widehat{a}_t \tag{2.18}$$

where

$$\mathcal{B}^a_\vartheta = \frac{1}{\alpha} \left(\frac{1}{\kappa/q} \right) \left[\frac{1}{1-\beta\,\gamma_a\,(1-\rho)} \right] (1-\chi\, w)$$

We can notice, in particular, that more rigid wages (i.e. a lower χ) amplify the response of both market tightness and the job destruction cutoff. Hence, a lower χ implies higher unemployment risk and a stronger precautionary saving motive. This points towards the fact that below a certain value of χ, the precautionary motive will become so strong to offset the standard consumption smoothing motive, causing the natural interest rate to fall during a productivity-driven downturn. This result is summarised in the following corollary for the $\alpha = \eta$ case.

Corollary 2.4.1. *The natural interest rate responds pro-cyclically to aggregate productivity shocks provided that*

$$\chi < \overline{\chi}_r^X = \frac{1}{w} \left\{ \frac{1}{1 + \left(\frac{1}{\rho\, f\, \gamma_a}\right) \left[\frac{1-\beta\,\gamma_a\,(1-\rho)}{1-\beta\,(1-f)\,(1-\rho)}\right] \{[(1-\gamma_a) + \gamma_a\,\rho\,(1-f)]\,(1-\zeta) + \rho\,(1-f)\,\zeta\}} \right\}$$

in the exogenous job destruction case, and provided that

$$\chi < \overline{\chi}_r^E = \frac{\frac{1}{w} + \tau\,\left(\frac{1-\varphi^*}{\varphi^*}\right) \left[\left(\frac{\alpha}{1-\alpha}\right) \left(\frac{1-f}{f}\right) \left(\frac{1-\rho}{\rho}\right) - 1\right] \overline{\chi}_c}{\frac{1}{w\,\overline{\chi}_r^X} + \tau\,\left(\frac{1-\varphi^*}{\varphi^*}\right) \left[\left(\frac{\alpha}{1-\alpha}\right) \left(\frac{1-f}{f}\right) \left(\frac{1-\rho}{\rho}\right) - 1\right]}$$

in the endogenous job destruction case. Moreover, $\overline{\chi}_r^X < \overline{\chi}_r^E$, provided that $\overline{\chi}_r^X < \overline{\chi}_c$.

The requirement $\overline{\chi}_r^X < \overline{\chi}_c$ is in turn equivalent to the following parameter restriction

$$\frac{1 - \left[\frac{\beta\,(1-\rho)\,(1-\gamma_a)}{1-\beta\,(1-f)\,(1-\rho)}\right] \left(\frac{1-\eta}{\eta}\right) \zeta}{1 + \left[\frac{1-\beta\,\gamma_a\,(1-\rho)}{1-\beta\,(1-f)\,(1-\rho)}\right] \left(\frac{1-\eta}{\eta}\right) \zeta} > \frac{1}{1 + \left(\frac{1}{\rho\, f\, \gamma_a}\right) \left[\frac{1-\beta\,\gamma_a\,(1-\rho)}{1-\beta\,(1-f)\,(1-\rho)}\right] \{[(1-\gamma_a) + \gamma_a\,\rho\,(1-f)]\,(1-\zeta) + \rho\,(1-f)\,\zeta\}}$$

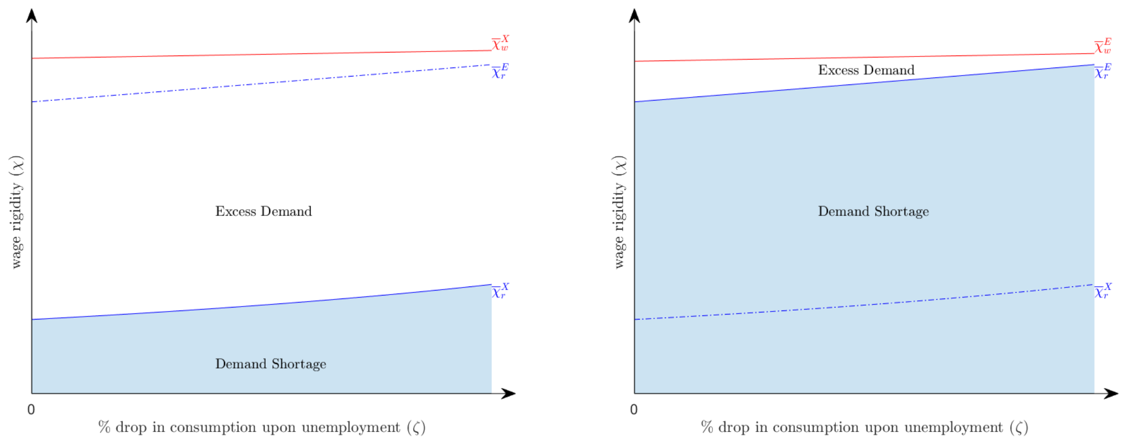
which holds for plausible parameter values and targets.

Figure 2.2 illustrates these results. What we can notice is that, first, the fully-flexible wage case ($\chi = \overline{\chi}_w$) is always characterised by excess rather than deficient demand. Most prominently, we can notice that regardless of the amount of incomplete insurance, as captured by ζ, less rigid wages are sufficient, in the endogenous separation case, to have the precautionary motive dominating on the standard consumption smoothing motive, with a resulting demand shortage.

Intuitively, with endogenous and countercyclical job destruction unemployment risk is amplified, as a contractionary productivity shock implies not only a reduction in the job finding probability (as happens in the exogenous job destruction case), but also a higher probability of losing the job in the first place. As a result, it is much more plausible that a contractionary supply shock causes a demand shortage through the precautionary motive, compared to the exogenous job destruction case. In particular, wages do not need to be as rigid as in the exogenous job destruction case in order to have a fall in the natural interest rate in response to a contractionary aggregate productivity shock.

Figure 2.2: Demand Behavior under a Contractionary Productivity Shock

(a) Exogenous Separation **(b)** Endogenous Separation



Notes: The figure reports upper bounds on the wage elasticity χ. $\overline{\chi}_w^j$, $j \in \{X, E\}$, is the maximum value χ can take, corresponding to the flexible wage case. $\overline{\chi}_r^j$, $j \in \{X, E\}$, are bounds below which the precautionary motive dominates and hence the natural interest rate falls. On the horizontal axis, $\zeta \in (0, 0.5]$, while the other parameters are $\beta = 0.99$, $\rho = 0.02$, $f = 0.2$, $\alpha = \eta = 0.7$. The sorting of the bounds depicted in the figure is robust to plausible alternative parametrizations.

2.4.1 Nominal Price Rigidity

So far, the issue of the presence of demand shortages has been analysed under the assumption of flexible prices. However, this implies that output is supply-determined, and hence there is no feedback from demand shortages to depressed output at equilibrium. Introducing nominal price rigidity allows to uncover how demand-deficiencies, whose symptom is a falling natural interest rate, actually translate in making a recession even worse. Falling natural rates imply indeed a positive unemployment gap under simple inflation targeting, as will be formalised in this section.

With nominal price rigidity (à la Calvo),[11] the log-linear model is first of all characterised by the following Euler condition and Phillips curve[12]

$$i_t - \mathbb{E}_t\left(\pi_{t+1}\right) = -\left[\frac{\rho\left(1-f\right)\zeta}{\left(1-\zeta\right)+\rho\left(1-f\right)\zeta}\right]\mathbb{E}_t\left(\widehat{\lambda}_{t+1}\right) + \left\{\frac{\left(1-\zeta\right)\left[1-\rho\left(1-f\right)\right]}{\left(1-\zeta\right)+\rho\left(1-f\right)\zeta}\right\}\mathbb{E}_t\left(\widehat{w}_{t+1}\right) - \widehat{w}_t$$

$$\pi_t = \beta\,\mathbb{E}_t\left(\pi_{t+1}\right) + \Omega\,\widehat{x}_t$$

where i_t is the nominal interest rate, π_t is the inflation rate, $\Omega = (1-\omega)\,(1-\beta\,\omega)/\omega$, with ω being the probability that price setters cannot reset their price, and x_t is the real marginal cost.

[11]How the structure of the production side of the model is amended to formally introduce price rigidity is outlined in full detail in Appendix 2.C.

[12]Log-linearization is performed around the (zero-inflation) steady state already outlined in Table 2.3.

Additionally, in the exogenous separation case, the job creation condition becomes

$$\alpha\,\widehat{\vartheta}_t = \frac{1}{\kappa/q}\,(\widehat{a}_t + \widehat{x}_t) - \left(\frac{w}{\kappa/q}\right)\widehat{w}_t + \beta\,(1-\rho)\,\mathbb{E}_t\left[\alpha\,\widehat{\vartheta}_{t+1}\right]$$

whereas, in the endogenous separation case the job creation and job destruction conditions become

$$\begin{aligned}\alpha\,\widehat{\vartheta}_t &= \widehat{a}_t + \widehat{x}_t - \left(\frac{1}{1-\varphi^*}\right)[\varphi^* - \tau\,(1-\varphi^*)]\;\widehat{\varphi}^*_t \\ \widehat{\varphi}^*_t &= \left(\frac{w}{\varphi^*}\right)\widehat{w}_t - \widehat{a}_t - \widehat{x}_t + \tau\left(\frac{1-\varphi^*}{\varphi^*}\right)\beta\,(1-\rho)\,\mathbb{E}_t\left(\widehat{\varphi}^*_{t+1}\right) - \left(\frac{1-\varphi^*}{\varphi^*}\right)\beta\,(1-\rho)\,\mathbb{E}_t\left(\alpha\,\widehat{\vartheta}_{t+1}\right)\end{aligned}$$

while the other loglinear model equations remain the same as those in Table 2.4. Lastly, the model is closed by a Taylor rule, assumed to take a simple inflation targeting form

$$i_t = \phi\,\pi_t\,.$$

It is useful to express the model with nominal price rigidity in terms of gaps with respect to the flexible prices benchmark summarised in Table 2.4. Let $\Theta_t = \widehat{\vartheta}_t - \widehat{\vartheta}^l_t$ and $\Phi_t = \widehat{\varphi}_t - \widehat{\varphi}^l_t$, where the l superscript refers to the flexible price benchmark. Then we have

$$\begin{aligned}\alpha\,\Theta^X_t &= \left(\frac{1}{\kappa/q}\right)\widehat{x}_t + \beta\,(1-\rho)\,\mathbb{E}_t\left(\alpha\,\Theta^X_{t+1}\right) \\ \phi\,\pi_t - \mathbb{E}_t\,(\pi_{t+1}) - \hat{r}^l_t &= \left[\frac{\rho\,(1-f)\,\zeta}{(1-\zeta)+\rho\,(1-f)\,\zeta}\right]\left(\frac{f}{1-f}\right)(1-\alpha)\;\mathbb{E}_t\left(\Theta^X_{t+1}\right)\end{aligned}$$

in the exogenous separation case, and

$$\begin{aligned}\alpha\,\Theta^E_t &= \widehat{x}_t - \left(\frac{1}{1-\varphi^*}\right)[\varphi^* - \tau\,(1-\varphi^*)]\;\Phi_t \\ \Phi_t &= -\widehat{x}_t - \left(\frac{1-\varphi^*}{\varphi^*}\right)\beta\,(1-\rho)\,\mathbb{E}_t\,(x_{t+1}) + \beta\,(1-\rho)\,\mathbb{E}_t\,(\Phi_{t+1}) \\ \phi\,\pi_t - \mathbb{E}_t\,(\pi_{t+1}) - \hat{r}^l_t &= \left[\frac{\rho\,(1-f)\,\zeta}{(1-\zeta)+\rho\,(1-f)\,\zeta}\right]\left\{\left(\frac{f}{1-f}\right)(1-\alpha)\;\mathbb{E}_t\left(\Theta^E_{t+1}\right) - \tau\left(\frac{1-\rho}{\rho}\right)\mathbb{E}_t\,(\Phi_{t+1})\right\}\end{aligned}$$

in the endogenous separation case. The equilibrium behavior of the gaps can then be expressed in terms of the response of the natural interest rate, $\hat{r}^l_t$.[13] In the exogenous separation case,

$$\Theta^X_t = \left(\frac{1}{\alpha}\right)\left(\frac{1}{\kappa/q}\right)\left[\frac{1}{1-\beta\,\gamma_a\,(1-\rho)}\right]\left(\frac{1}{\mathcal{D}^X}\right)\hat{r}^l_t$$

[13] The theoretical results are derived under the simplifying assumption that $\phi = 1/\beta$, allowing to tractably investigate the local determinacy properties of the two models, which are discussed in more detail in appendix 2.D. The quantitative analysis in section 2.5 will deal with the more general case of ϕ not necessarily equal to $1/\beta$.

where

$$\mathcal{D}^X = \frac{\Omega}{\beta} - \left[\frac{\gamma_a}{1-\beta\,\gamma_a\,(1-\rho)}\right]\left[\frac{\rho\,f\,\zeta}{(1-\zeta)+\rho\,(1-f)\,\zeta}\right]\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1}{\kappa/q}\right).$$

In the endogenous separation case,

$$\begin{aligned}
\Theta_t^E &= \frac{1}{\alpha}\left[\frac{1}{1-\beta\,\gamma_a\,(1-\rho)}\right]\left(\frac{1}{1-\varphi^*}\right)\left[1-\tau\left(\frac{1-\varphi^*}{\varphi^*}\right)w\,\overline{\chi}_c\right]\left(\frac{1}{\mathcal{D}^E}\right)\hat{r}_t^l\\
\Phi_t &= -\left[\frac{1}{1-\beta\,\gamma_a\,(1-\rho)}\right]\left(\frac{w\,\overline{\chi}_c}{\varphi^*}\right)\left(\frac{1}{\mathcal{D}^E}\right)\hat{r}_t^l
\end{aligned}$$

where

$$\begin{aligned}
\mathcal{D}^E = \frac{\Omega}{\beta} - &\left[\frac{\gamma_a}{1-\beta\,\gamma_a\,(1-\rho)}\right]\left[\frac{\rho\,f\,\zeta}{(1-\zeta)+\rho\,(1-f)\,\zeta}\right]\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1}{1-\varphi^*}\right)\left[1-\tau\left(\frac{1-\varphi^*}{\varphi^*}\right)w\,\overline{\chi}_c\right]\\
&-\left[\frac{\gamma_a}{1-\beta\,\gamma_a\,(1-\rho)}\right]\left[\frac{\rho\,(1-f)\,\zeta}{(1-\zeta)+\rho\,(1-f)\,\zeta}\right]\tau\left(\frac{1-\rho}{\rho}\right)\left(\frac{1}{\varphi^*}\right)w\,\overline{\chi}_c\,.
\end{aligned}$$

Local determinacy ensures that $\mathcal{D}^X > 0$ and $\mathcal{D}^E > 0$, implying in turn that, in the presence of nominal price rigidity, a falling natural interest rate is associated with a larger separation rate and a smaller job-finding rate.

Focusing now on the unemployment gap, $\mathcal{U}_t = \hat{u}_t - \hat{u}_t^l$, we have

$$\begin{aligned}
\mathcal{U}_t^X &= -\left(\frac{f}{1-f}\right)(1-\alpha)\,\Theta_t^X + (1-\rho)\,(1-f)\,\mathcal{U}_{t-1}^X\\
\mathcal{U}_t^E &= \tau\left(\frac{1-\rho}{\rho}\right)f\,\Phi_t - \left(\frac{f}{1-f}\right)(1-\alpha)\,\Theta_t^E + (1-\rho)\,(1-f)\,\mathcal{U}_{t-1}^E
\end{aligned}$$

in the exogenous and endogenous separation cases, respectively. It is then straightforward to see that a falling (rising) natural interest rate is associated with a positive (negative) unemployment gap. In other words, in the presence of nominal price rigidity, a demand-deficient recession (expansion) would be characterised by a falling (rising) natural interest rate as well as with a larger (smaller) surge in unemployment.

2.5 Quantitative Analysis

2.5.1 Calibration

The baseline calibration of the parameters is summarised in Table 2.5. The exogenous and endogenous separation models are calibrated to deliver the same steady state magnitudes, but will differ in their response to aggregate productivity shocks.

Each time period is intended to be a quarter, so I set $\beta = 0.99$. Moreover, I assume log-utility

Table 2.5: Baseline Calibration

Targets			
Variable/Parameter	Notation	Value	Source
Steady state total separation rate	ρ	0.02	Jimeno and Thomas (2013)
Exogenous separation rate	ρ^x	0.014	$= 0.7\,\rho$, Jimeno and Thomas (2013)
Steady state job finding probability	f	0.20	Jimeno and Thomas (2013)
Steady state market tightness	ϑ	0.25	Jimeno and Thomas (2013)

Externally Calibrated Parameters			
Parameter	Notation	Value	Source
Discount rate	β	0.99	Quarterly calibration
Matching function parameter	α	0.7	Fujita and Ramey (2012)
Worker surplus share	η	0.7	Hosios condition
% loss in consumption upon unemployment	ζ	0.2	Chodorow-Reich and Karabarbounis (2016)
Persistence of aggregate productivity shocks	γ_a	0.95	Den Haan et al. (2000)
Std. dev. of aggregate productivity shocks	σ_ξ	0.01	Den Haan et al. (2000)

Internally Calibrated Parameters			
Parameter	Notation	Value	Target
Std. dev. of idiosyncratic productivity	σ_φ	0.1822	φ^* (Proposition ??) & $\overline{\varphi} = 1$
Mean of idiosyncratic productivity	μ_φ	-0.0191	φ^* (Proposition ??) & $\overline{\varphi} = 1$
Matching efficiency	$\mathcal{M}$	0.3031	α, f, and ϑ
Vacancy cost	κ	0.3029	steady state FE

Notes: φ is assumed to follow a log-normal distribution with mean μ_φ and standard deviation σ_φ.

for workers (as in Challe, 2020). The matching function parameter is set to $\alpha = 0.7$, as in Fujita and Ramey (2012) and close to the 0.72 of Shimer (2005), and $\eta = \alpha$. As for aggregate shocks, I set $\sigma_\xi = 0.01$ and $\gamma_a = 0.95$ as in Den Haan et al. (2000). Following Chodorow-Reich and Karabarbounis (2016), consumption is assumed to drop by 20% upon job loss, implying that $\delta = 0.8\,w$.

The main targets of the calibration are the steady state total separation rate and job-finding rate, and are aimed at capturing the characteristics of an average continental European labor market. As in Jimeno and Thomas (2013), the steady state total separation rate is set to be equal to 0.02

$$\rho = \rho^x + (1 - \rho^x)\, G(\varphi^*) = 0.02$$

and the exogenous separation rate is set to be equal to 0.7 of the total separation rate at steady state. The steady state steady job finding rate is set to be $f = 0.20$ and the steady state market tightness to be $\vartheta = 0.25$. Given $\lambda = \rho(1 - f)$, the targeted total separation rate and job finding probability imply a steady state unemployment rate

$$u = \frac{\lambda}{\lambda + f}$$

amounting to approximately 7%. Also, given the targets, I can accordingly calibrate the matching efficiency parameter $\mathcal{M} = f/\vartheta^{(1-\alpha)} = 0.3093$, and compute the vacancy-filling rate as $q = f/\vartheta$.

The steady state wage w, exit cutoff φ^*, and continuation value $\kappa/q = 1 - \varphi^*$ can then be derived from the result in Proposition 2.3, and the vacancy cost parameter κ can be pinned down from the steady state free entry condition, which implies $\kappa = q\,[1 - \varphi^*] = 0.1852$.

Lastly, the idiosyncratic productivity term φ is assumed to follow a log-normal distribution with mean μ_φ and standard deviation σ_φ. Given φ^*, ρ and ρ^x, σ_φ and μ_φ are jointly calibrated so as to normalise $\overline{\varphi} := \mathbb{E}(\varphi \,|\, \varphi \geq \varphi^*) = 1$ at steady state.

Sensitivity

I now perform some sensitivity checks of the results on the interaction between wage rigidity, job destruction, and unemployment risk, and their implications for aggregate demand shortages.

The first dimension along which sensitivity checks are performed regards the persistence of aggregate productivity shocks: I consider, alternatively, $\gamma_a = 0$ (i.e. a purely transitory shock), and shocks of different persistence ($\gamma_a = 0.9$, or 0.95, as in the baseline calibration).

The second parameter involved in the sensitivity checks is the matching function parameter, α.[14] While Shimer (2005) estimates it at 0.72, Mortensen and Nagypal (2007) argue that this value is empirically too high, estimating it at 0.45. Brügemann (2008) proposes, instead, estimates between 0.54 and 0.63. Therefore, I follow Fujita and Ramey (2012) in setting $\alpha = 0.7$ as a benchmark, and in considering $\alpha = 0.5$ as an alternative parametrisation.

Table 2.6 summarises what happens, under alternative parametrisations, to wage responsiveness thresholds for aggregate demand shortages. A more detailed comment on the impulse responses of the model under the baseline calibration is instead the focus of section 2.5.2.

First of all, as far as shock persistence is concerned, purely transitory contractionary shocks cannot possibly cause a demand shortage under the assumed timing: as the shock is unanticipated and purely transitory, it has no impact at all on unemployment risk, hence only the standard consumption smoothing motive is operative. As shock persistence increases, so do the future prospects of income losses due to unemployment, and hence the precautionary saving behavior.

[14] A comprehensive survey on matching functions can be found in Petrongolo and Pissarides (2001b).

Table 2.6: Wage Responsiveness Threshold for Procyclical Natural Interest Rates

Shock persistence (γ_a)	$\overline{\chi}_r^E$ (Endogenous Separation)		$\overline{\chi}_r^X$ (Exogenous Separation)	
	$\alpha = 0.5$	$\alpha = 0.7$	$\alpha = 0.5$	$\alpha = 0.7$
0	0	0	0	0
0.9	0.6307	0.6251	0.0645	0.0636
0.95	0.8300	0.8286	0.1684	0.1659

Notes: The table reports upper bounds on the wage elasticity χ under alternative parametrisations: when $\chi < \overline{\chi}^j$, $j \in \{X, E\}$, the precautionary motive dominates and hence the natural interest rate behaves procyclically, i.e. there is deficient (excess) aggregate demand after a contractionary (expansionary) productivity shock.

As a consequence, demand shortages become a more likely outcome. Given that in the data χ is in the ballpark of 1/3, such outcome occurs in the endogenous job destruction, but not in the exogenous job destruction case, under the chosen parametrization and when shocks are sufficiently persistent.

Lastly, different calibrations of the matching function do not substantially alter the results.

2.5.2 Baseline Results

Figure 2.3 reports impulse responses to a 1% fall in aggregate productivity under the baseline calibration summarised in Table 2.5. In order to better understand the role played by the endogenous job destruction margin, I compare the impulse responses of the endogenous job destruction model with those of the standard S&M model with symmetric matches and exogenous separation summarised in Table 2.2. For both models, I consider the case of flexible wages as well as that of inertial wages. As can be seen from Figure 2.3, both models generate an unrealistically strong response of wages to productivity (almost 1:1) in the flexible wage case, a common feature of S&M models first highlighted by Shimer (2005) and Hall (2005). Therefore, I set the the real wage elasticity to $\chi = 1/3$, as suggested by Challe (2020) with reference to the empirical evidence.

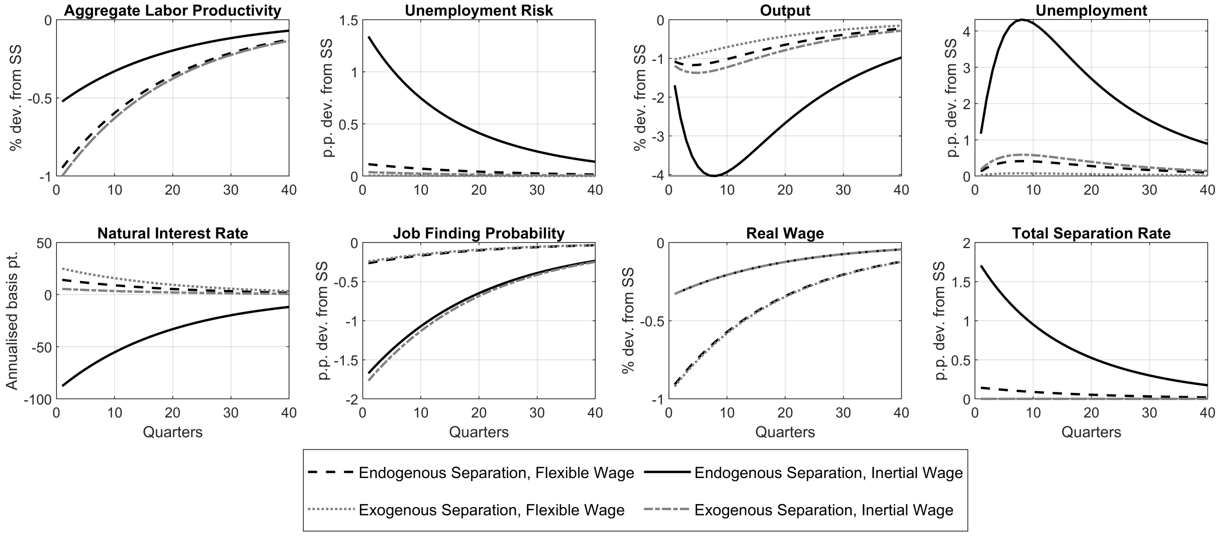
With inertial wages, there is a cleansing effect preventing aggregate labor productivity to fall as much as in the flexible wage case: as more unproductive jobs are endogenously destroyed, the average idiosyncratic productivity of a match is higher, and this partly offsets the aggregate shock (a_t falls while $\overline{\varphi}_t$ rises). However, since more jobs are destroyed, a contained fall in aggregate labor productivity is not enough to prevent a larger fall in output than in the flexible wages case.

Focusing on the inertial wage case, the exogenous and endogenous job destruction models give an almost identical drop in the job finding probability, but of course only the endogenous job destruction model produces a rise in the total separation rate (of about 1.8 percentage points above the steady state level). Given that there is a sizeable increase in unemployment risk only in the endogenous job destruction model, this seems to validate the ability of the model to capture the conventional wisdom that increased unemployment during recessions stems primarily from increased job destruction rates rather than from reduced job creation rates. As a result, unemployment rises, reaching a peak of around 4.5 percentage points above its steady state level, and there is also a sizeable fall in aggregate output (with a through of about -4%), which is proportionally much larger than the size of the initial negative supply shock of 1%.

As unemployment rises, due to incomplete markets workers cannot fully insure against the increased income risk of losing their job; as a consequence, they wish to precautionarily save and cut consumption, potentially inducing an aggregate demand shift larger than the aggregate supply shift, which would manifest as a reduction in the natural interest rate. Indeed, as discussed above, there are two main forces shaping the response of the natural interest rate: standard consumption smoothing, and the precautionary motive. Following a contractionary productivity shock, these two forces push in opposite directions; on the one hand, better future wage prospects induce to smooth consumption by moving it to the present, implying a desire to borrow and hence upward pressure on the natural interest rate; on the other hand, heightened unemployment risk increases the desire to precautionarily save in light of the prospect of becoming unemployed and hence be forced to reduce consumption. This latter force, which puts downward pressure on the natural interest rate, dominates in the endogenous separation and inertial wage case; as a result, the natural interest rate falls by about 100 annualised basis points. Moreover, the precautionary motive is sufficiently strong to cause a fall in the natural interest rate only when the interaction between wage rigidity and endogenous job destruction is at work: when one or both of these two channels are absent, it is actually the standard consumption smoothing motive that dominates, and there is a rise in the natural interest rate, suggesting the presence of excess rather than depressed demand.

Second moments obtained from stochastic simulations, reported in Table 2.7, confirm these results. Unemployment is more volatile with endogenous job destruction, and even more so with rigid real wages. Looking at the correlation between output and the natural interest rate, it is also confirmed that the natural interest rate behaves procyclically only in the endogenous job destruction model with inertial wages, whereas it exhibits a countercyclical behavior in all the other cases.

Figure 2.3: Impulse Responses to a Contractionary Productivity Shock



Notes: The figure reports model-based impulse responses to a 1% decline in productivity under the parametrisation summarised in Table 2.5. Aggregate Labor Productivity refers to $Y_t/n_t = a_t$ in the exogenous separation case, and $Y_t/n_t = a_t\,\overline{\varphi}_t$ in the endogenous separation case. Unemployment Risk refers to $\mathbb{E}_t\left(\widehat{\lambda}_{t+1}\right)$.

Table 2.7: Second Moments

	Endogenous Separation		Exogenous Separation	
	Flexible Wage	Inertial Wage	Flexible Wage	Inertial Wage
$\sigma_{\widehat{y}}$	0.016	0.045	0.014	0.018
$\sigma_{\widehat{w}}\,/\,\sigma_{\widehat{y}}$	0.742	0.096	0.879	0.235
$\sigma_{\widehat{u}}\,/\,\sigma_{\widehat{y}}$	3.681	11.105	0.806	4.538
$\rho_{\widehat{y},\widehat{r}}$	-0.977	0.769	-0.999	-0.965

Notes: The table reports standard deviations of the real wage and unemployment relative to the standard deviation of output, and correlations between output and the natural interest rate. These second moments are obtained from stochastic simulations of the model. The simulated data are HP-filtered with a smoothing parameter of 1600.

2.5.3 Extensions

Nominal Price Rigidity

As showed analytically, nominal price rigidity implies that deficient (excess) demand, whose symptom is a falling (rising) natural interest rate, makes a recession more (less) severe, being associated with a positive (negative) unemployment gap and hence a larger (smaller) surge in unemployment.

In order to quantitatively explore this issue, the fraction of unchanged prices is conventionally set to $\omega = 0.75$ and the Taylor rule parameter to $\phi = 1.5$, while the other parameters and targets are as before, in the baseline calibration summarised in Table 2.5.

Considering the case of inertial wages under both separation regimes, we can see from the impulse responses in Figure 2.4 that the exogenous separation model is indeed characterised by excess demand in terms of a rising natural interest rate as well as a negative unemployment gap. By contrast, in the endogenous separation case, the demand-deficient nature of the recession manifests both as a falling natural interest rate and a positive unemployment gap.

The second moments reported in Table 2.8 confirm these results. While with endogenous job destruction the relative volatility of unemployment is higher in the presence of nominal price rigidity compared to flexible prices (Table 2.7), it is instead lower with exogenous job destruction. Indeed, while in the former case demand-deficient recessions and demand-abundant expansions amplify business cycle fluctuations, the reverse is true in the latter case, where recessions are demand-abundant and expansions are demand-deficient. Looking at the correlation between output and inflation, we can also notice how only the endogenous job destruction model exhibits a procyclical inflation rate.

Non-zero Liquidity

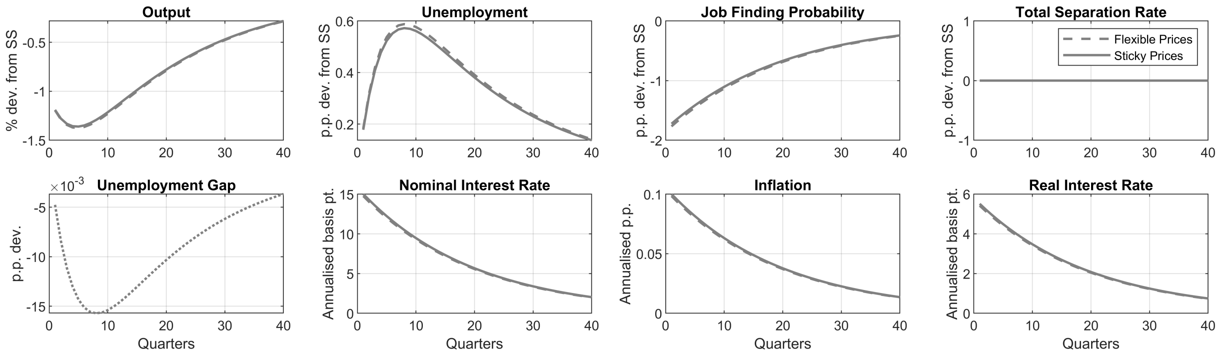
The model can be further extended by allowing workers to actually hold assets for precautionary motives. Following Challe et al. (2017), I do so by making two key assumptions: i) imperfect risk sharing between the unemployed and the employed, due to a debt limit $\underline{b} > 0$ which is non-zero but still tighter than the natural debt limit,[15] and ii) full risk sharing among the employed.

As for the first assumption, a non-zero borrowing limit implies that the unemployed, who have lower income, can borrow from the employed, who in turn precautionarily hold assets for self-insuring themselves against the drop in income that would occur upon unemployment. Furthermore, to be consistent with the fact that the unemployed consume less than the employed,

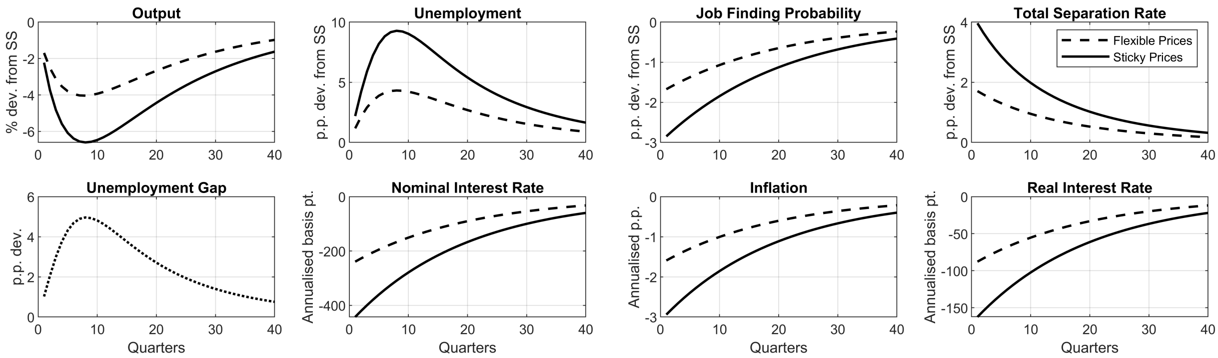
[15] The natural debt limit is the maximum amount a household could borrow, while still being able to repay, in the worst-case scenario income history (corresponding to permanent unemployment, in this setting).

Figure 2.4: Effect of a Contractionary Productivity Shock with Nominal Price Rigidity

(a) Exogenous Separation



(b) Endogenous Separation



Notes: The figure reports model-based impulse responses to a 1% decline in productivity under the parametrisation summarised in Table 2.5; additionally, $\omega = 0.75$ and $\phi = 1.5$.

Table 2.8: Second Moments from the Models with Nominal Rigidity

	Endogenous Separation		Exogenous Separation	
	Flexible Wage	Inertial Wage	Flexible Wage	Inertial Wage
$\sigma_{\widehat{y}}$	0.016	0.072	0.014	0.018
$\sigma_{\widehat{w}} \,/\, \sigma_{\widehat{y}}$	0.716	0.060	0.818	0.237
$\sigma_{\widehat{u}} \,/\, \sigma_{\widehat{y}}$	3.555	12.252	0.750	4.457
$\rho_{\widehat{y},\pi}$	-0.979	0.709	-0.999	-0.967

Notes: The table reports standard deviations of the real wage and unemployment relative to the standard deviation of output, and correlations between output and inflation. These second moments are obtained from stochastic simulations of the model. The simulated data are HP-filtered with a smoothing parameter of 1600.

it is assumed (and later checked) that the borrowing constraint is always binding. As a consequence, the asset market clearing condition in (2.7) now implies $n_t\, b_t^n = (1 - n_t)\, \underline{b}$, i.e. a non-degenerate (but still finite-dimensional) wealth distribution, differently from the zero-liquidity case where $\underline{b} = 0$. In sum, this first assumption implies that there is partial risk sharing between the employed and the unemployed via the asset market. The second assumption of full risk sharing among the employed implies instead, in practice, that those becoming employed pool their debt with the assets of those remaining employed. These model extensions allowing for non-zero liquidity at equilibrium are described in more detail in Appendix 2.E, along with the (re)calibration strategy —aimed at targeting the same steady-state consumption decline for the unemployed of the zero liquidity model ($\zeta = 0.2$).

Figure 2.5 compares the impulse responses of key variables (the natural interest rate, unemployment, and the unemployment gap) in the non-zero and zero liquidity cases and for both separation regimes. Qualitatively, the main result is unchanged when allowing for self-insurance via non-zero asset holdings: only the endogenous separation case is characterised by demand deficiency in response to a contractionary productivity shock, while the exogenous separation case still depicts excess demand. Quantitatively, (partial) self-insurance allows workers to contain the fall in consumption occurring upon unemployment, dampening their unmet precautionary saving desire. As a result, in the endogenous separation model the natural interest rate falls by around 20 annualised basis points less with non-zero liquidity than with zero-liquidity. Since the fall in demand is partially contained, the increase in the unemployment gap is two percentage points smaller, implying a correspondingly smaller surge in unemployment. The quantitative results are, instead, almost unchanged in the exogenous separation case: as demand is partially more sustained, the natural interest rate rises by around one annualised basis point more than in the zero-liquidity case, implying in turn a marginally lower increase in the unemployment rate, due to a slightly larger decline in the unemployment gap.

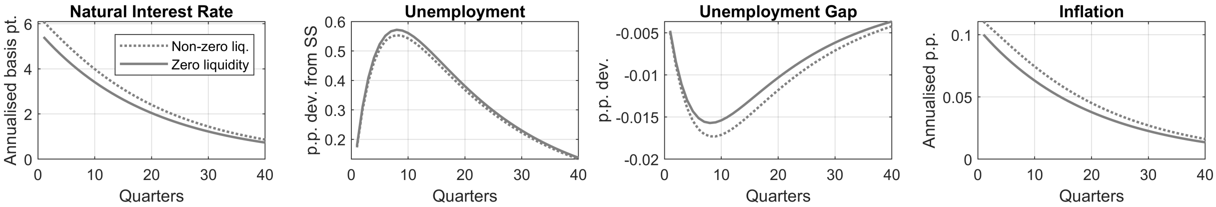
Table 2.9 reports second moments from stochastic simulation of the non-zero liquidity model, confirming how its business cycle properties are essentially in line with those of the zero liquidity model considered in the baseline analysis.

2.6 Conclusions

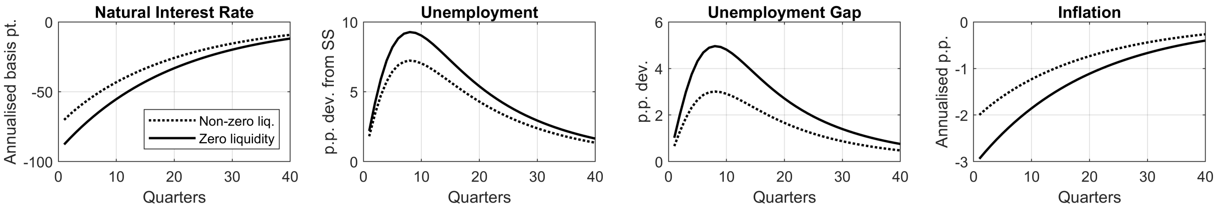
Can negative supply shocks induce sizable falls in demand, and hence make a recession even worse? In models with complete markets the answer is typically negative, as a standard consumption smoothing desire induces households to sustain aggregate demand during a recession, by wishing

Figure 2.5: Effect of a Contractionary Productivity Shock with Non-zero Liquidity

(a) Exogenous Separation



(b) Endogenous Separation



Notes: The figure reports model-based impulse responses to a 1% decline in productivity under the parametrisation summarised in Table 2.5 and Appendix 2.E.

Table 2.9: Second Moments from the Models with Non-zero Liquidity

	Endogenous Separation		Exogenous Separation	
	Non-zero Liquidity	Zero Liquidity	Non-zero Liquidity	Zero Liquidity
$\sigma_{\widehat{y}}$	0.062	0.072	0.018	0.018
$\sigma_{\widehat{w}} \,/\, \sigma_{\widehat{y}}$	0.070	0.060	0.239	0.237
$\sigma_{\widehat{u}} \,/\, \sigma_{\widehat{y}}$	12.002	12.252	4.361	4.457
$\rho_{\widehat{y},\pi}$	0.717	0.709	-0.976	-0.967

Notes: the table reports standard deviations of the real wage and unemployment relative to the standard deviation of output, and correlations between output and inflation. These second moments are obtained from stochastic simulations of the model. The simulated data are HP-filtered with a smoothing parameter of 1600.

to borrow in light of improving income prospects. Introducing incomplete markets in the form of uninsurable unemployment risk allows for a precautionary saving motive that can end up offsetting this desire to borrow, making therein a fall in the natural interest rate —a symptom of demand deficiency— a possible outcome, although not necessarily the likely one as long as unemployment risk stems only from reduced job creation, as it has so far most often been assumed (e.g by Ravn and Sterk, 2021). As shown analytically in this paper, if heightened unemployment risk stems not only from reduced job finding prospects, but also from endogenous job destruction, a fall in the natural interest rate after a negative productivity shock becomes not merely a possible outcome, but a most likely one. Quantitatively, it is then crucial to account for the endogenous job destruction channel in models with uninsurable unemployment risk in order to capture the demand-deficient nature of supply-driven recessions.

Chapter 3

Uninsurable Unemployment Risk and Inflationary Liquidity Traps

3.1 Introduction

Despite their stark differences, the great recession and the Covid-19 recession have had so far at least a feature in common: deflation has been almost completely absent.

Since the onset of the covid-19 pandemic, there has been a wide debate about the supply- or demand-side nature of the shock.[1] In particular, it has gained particular prominence within this debate the fact that supply-side shocks can have so-called *Keynesian* effects (Guerrieri et al., 2022), i.e. they can trigger themselves a demand response that ends up being larger than the initial supply shock. Negative supply shocks of such nature can then exert significant downward pressure on natural and real interest rates. During recessions, a relevant source of income risk is unemployment. If workers cannot completely insure against it, higher unemployment risk spurs precautionary savings, putting itself downward pressure on real interest rates (Ravn and Sterk, 2021).[2] As a result, when nominal rates are stuck at the zero lower bound (ZLB), the real rate can fall below them, with resulting inflationary pressures.

These aforementioned trends of falling natural rates, and nominal interest rates hitting the ZLB during recent recessions, are indeed evident from the data, as can be seen from figure 3.1. The data also confirm that core CPI inflation has always remained in positive territory.

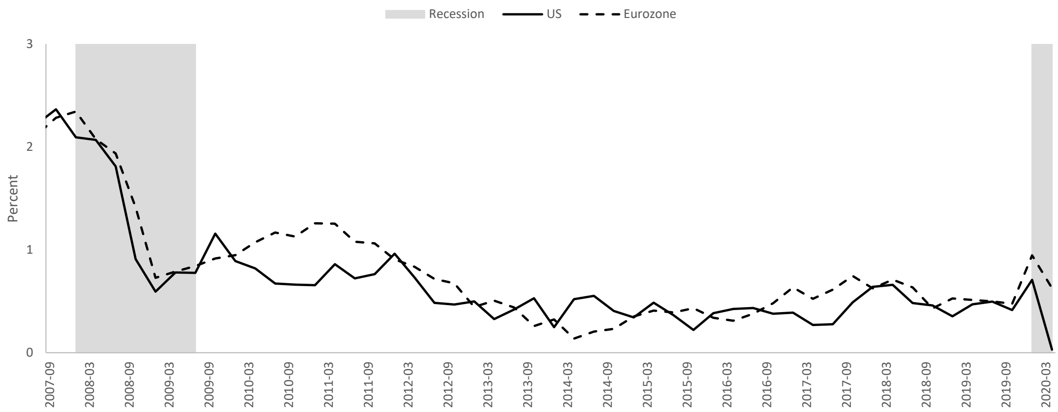
These three facts —falling natural rates, nominal rates stuck at the ZLB, and no deflation— can hardly be reconciled with standard New Keynesian (NK) models with complete markets, for two main reasons. Firstly, in those settings productivity-driven recessions are typically accompanied by rising natural and nominal rates: households wish to borrow during recessions so as to smooth consumption, and this puts upward pressure on real rates. Secondly, even when negative shocks to the natural interest rate are explicitly modelled, these can only be accompanied by deflation when nominal interest rates are stuck at the ZLB. Otherwise, according to standard Taylor rule prescriptions, the presence of inflation would not be consistent with a

[1] See, among others, Eichenbaum et al. (2020a,b), Kaplan et al. (2020).

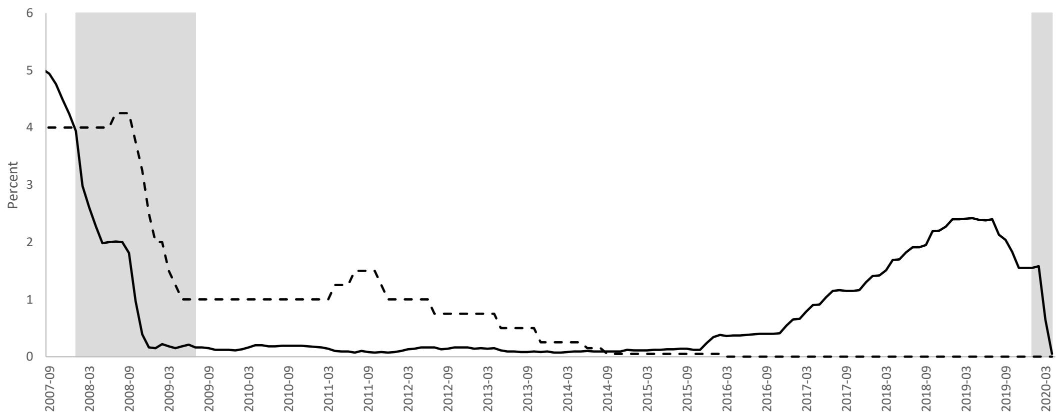
[2] As I show in companion work, the presence of endogenous, as opposed to exogenous separation, makes much more plausible the occurrence of a fall in the natural interest rate in response to contractionary productivity shocks.

Figure 3.1: Interest Rates and Inflation Rates During Recent Recessions

(a) Natural Interest Rate



(b) Nominal Interest Rate



(c) Core CPI Inflation

Notes: Panel (a) reports natural interest rates as estimated with the Holston, Laubach, and Williams (2017) methodology. Reported in panel (b) are the Effective Federal Funds Rate for the US, and the Main Refinancing Operations Rate for the Eurozone. *Sources:* FED, ECB, BLS, OECD.

desirable reduction in the nominal interest rate.

These puzzling outcomes do not necessarily hold true in models with incomplete markets in the form of uninsurable unemployment risk. In these settings, in contrast to standard NK models with complete markets, negative supply shocks (e.g. to productivity) can be themselves the primary source of downward pressure to the natural interest rate, as these shocks might induce a precautionary saving motive that is sufficiently strong to dominate on the desire to borrow implied by standard consumption smoothing during recessions, causing natural and real rates to fall. When the natural interest rate cannot adjust downward, this outcome can only be achieved with (expected) inflation.

Motivated by these facts, I analytically investigate the behavior of tractable macroeconomic models with uninsurable unemployment risk (à la Ravn and Sterk, 2021) when the nominal interest rate cannot fall. Tractability is achieved, first of all, by assuming that no agent can borrow, so that equilibrium properties are studied in a zero-liquidity limit which allows to capture a precautionary saving motive on part of households without the need of tracking a full and time-varying wealth distribution. Then, to tractably model the presence of a zero lower bound on nominal interest rates, it is assumed that the exogenous shock to productivity follows a 2-state Markovian process, whereby the economy is constrained at the ZLB while the shock is in place, and returns to steady state when the shock vanishes.

I show analytically that, firstly, in the presence of real wage rigidity inflationary ZLB recession episodes are indeed an outcome of models with uninsurable unemployment risk, both with exogenous separation and with endogenous separation. Secondly, I show for both of these cases which parameter restrictions on a Taylor rule targeting both inflation as well as labor market conditions (as captured by fluctuations in the unemployment rate) can be consistent with such equilibrium outcome: having the nominal interest rate actually constrained at zero is a possibility, in these settings, as long as the monetary authority cares also about deteriorated labor market conditions and not only about inflation.

Intuitively, the policy prescription of an hypothetically unconstrained Taylor rule should be one in which the monetary authority would want to reduce nominal interest rates even in the presence of inflation and not, instead, rise them. In such occurrence, during the ZLB recession episode the monetary authority is willing to tolerate some current inflation as long as this helps driving up inflation expectations and hence driving down the real interest rate, thereby reducing the desire to save and improving labor market prospects by sustaining aggregate demand.

Related Literature. This paper relates to two main strands of literature. First, it relates to the literature discussing ZLB implications for standard NK models in a tractable way by assuming Markovian processes for the exogenous variables (Eggertsson and Woodford, 2003, Christiano et al., 2011).

Second, the paper relates to tractable macroeconomic models with uninsurable unemployment risk (most notably, Challe, 2020, Ravn and Sterk, 2021). Ravn and Sterk (2021), in their HANK-S&M model with exogenous separation, simply look at a ZLB steady state, briefly arguing that it can possibly be characterised by positive inflation. I depart from them, first of all, by considering not only the case of exogenous separation, but also the endogenous case. Most important, and more in line with the past literature on the ZLB effects on NK models, I provide an analytical characterization of temporary ZLB recession episodes as first-order fluctuations around the intended steady state.

By bridging the HANK-S&M and ZLB episodes literatures, the paper aims at providing an additional possible explanation to the missing deflation puzzle.

Roadmap. The remainder of this paper is structured as follows. Section 3.2 sketches the model. Section 3.3 analyses the behavior of the loglinear stochastic model at the ZLB. Section 3.4 provides an empirical perspective for the analytical results. Section 3.5 concludes.

3.2 The Model

On the producer side, a final (consumption) good is assembled with CES technology, by aggregating differentiated goods from a monopolistically competitive wholesale sector; each wholesale producer uses in turn, as input, a homogeneous intermediate good; this latter is produced by firms differing in their productivity, using only labor, which is hired in a frictional labor market.

On the household side, there is a unit measure of workers who, due to their inability to borrow, cannot perfectly insure against the risk of becoming unemployed. Risk-neutral firm owners simply collect and consume hand-to-mouth the dividends arising in the production sector due to price rigidity and labor market frictions.

3.2.1 Households

Workers face idiosyncratic income risk from being either employed or unemployed: due to imperfect insurance, employed and unemployed workers will then make different consumption-saving choices at equilibrium. Their expected lifetime utilities can be formulated recursively as follows

$$U_t^n = \ln\left(c_t^n\right) + \beta\,\mathbb{E}_t\left[\left(1-\lambda_{t+1}\right)U_{t+1}^n + \lambda_{t+1}\,U_{t+1}^u\right]$$

$$U_t^u = \ln(c_t^u) + \beta\,\mathbb{E}_t\left[f_{t+1}\,U_{t+1}^n + (1-f_{t+1})\,U_{t+1}^u\right]$$

where U_t^n and U_t^u are the expected lifetime utilities of a currently employed or unemployed worker, respectively; λ and f are the transition rates from employment to unemployment, and vice versa. Each worker type $i \in \{n, u\}$ (employed/unemployed) maximises her expected lifetime utility subject to, for all $t \geq 0$,

$$\begin{cases} c_t^i + \frac{1}{1+i_t}\,a_t^i = y_t^i + \frac{1}{1+\pi_t}\,a_{t-1}^i \\ a_t^i \geq 0 \end{cases}$$

where c_t^i is current-period consumption, a_t^i is the amount of assets held at the end of period t, i_t the nominal return on assets, and π_t is the inflation rate in the final good price. y_t^i is equal either to w_t (the wage income a worker gets when employed), or δ (unemployment benefits).

Therefore, Euler conditions for employed and unemployed workers are, respectively,

$$\frac{1}{c_t^n} \geq \beta\,\mathbb{E}_t\left\{\left(\frac{1+i_t}{1+\pi_{t+1}}\right)\left[(1-\lambda_{t+1})\,\frac{1}{c_{t+1}^n} + \lambda_{t+1}\,\frac{1}{c_{t+1}^u}\right]\right\} \tag{3.1}$$

$$\frac{1}{c_t^u} \geq \beta\,\mathbb{E}_t\left\{\left(\frac{1+i_t}{1+\pi_{t+1}}\right)\left[f_{t+1}\,\frac{1}{c_{t+1}^n} + (1-f_{t+1})\,\frac{1}{c_{t+1}^u}\right]\right\} \tag{3.2}$$

each holding with strict inequality if the agent is liquidity constrained (i.e. wishing to borrow), and with equality otherwise.

Employment Dynamics. Worker transitions from employment to unemployment and vice versa give rise to the following laws of motion for the stocks of employed (N_t) and unemployed (U_t) workers

$$N_t = (1-\lambda_t)\,N_{t-1} + f_t\,U_{t-1}$$

$$U_t = (1-f_t)\,U_{t-1} + \lambda_t\,N_{t-1}$$

where, since the total measure of workers is normalised to 1, $U_t = 1 - N_t$. Being $E_t = U_{t-1} + \rho_t\,N_{t-1}$ the stock of effective searchers, $\lambda_t = \rho_t\,(1-f_t)$, where ρ_t is the separation rate and f_t the job-finding rate.

In other words, a worker employed in the current period faces a risk of being unemployed in the next period given by λ_t, the joint probability of undergoing separation and not being able to find another job.

3.2.2 Producers

The final good is produced under perfect competition, by aggregating a continuum of wholesale goods with CES technology

$$Y_t = \left(\int_0^1 y_t(k)^{\frac{\varepsilon-1}{\varepsilon}} dk \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{3.3}$$

Given the CES structure, demand for variety k from the wholesale sector is given by

$$y_t(k) = Y_t \left[\frac{p_t(k)}{P_t} \right]^{-\varepsilon} \tag{3.4}$$

where, given perfect competition in the final good sector, $P_t = \left(\int_0^1 p_t(k)^{1-\varepsilon} dk \right)^{1/(1-\varepsilon)}$.

Wholesalers. Wholesale producers turn intermediate (input) goods into wholesale differentiated products according to a simple linear technology with symmetric productivity

$$y_t(k) = s_t(k)$$

where $y_t(k)$ is the amount of variety k produced, and $s_t(k)$ is the amount of intermediate inputs used in production by wholesaler k. Each wholesaler is a monopolistic supplier of the variety k is produces. Moreover, wholesale firms are assumed to face Calvo pricing frictions, with ω being the probability that a wholesale firm cannot reset its price. Therefore, the optimal reset price, p_t^*, satisfies

$$\tilde{p}_t^* = \left(\frac{\varepsilon}{\varepsilon - 1} \right) \frac{\mathcal{Y}_t}{\mathcal{Z}_t} \tag{3.5}$$

where $\tilde{p}_t^* = p_t^*/P_t$, $\mathcal{Y}_t$ and $\mathcal{Z}_t$ obey the following recursions

$$\mathcal{Y}_t = x_t \, Y_t + \omega \, \beta \, \mathbb{E}_t \left[(1 + \pi_{t+1})^{\varepsilon} \, \mathcal{Y}_{t+1} \right] \tag{3.6}$$

$$\mathcal{Z}_t = Y_t + \omega \, \beta \, \mathbb{E}_t \left[(1 + \pi_{t+1})^{(\varepsilon-1)} \, \mathcal{Z}_{t+1} \right] \tag{3.7}$$

and x_t is the real marginal cost faced by wholesale producers, i.e. the price of intermediate inputs in terms of the final good.

Given Calvo pricing frictions and symmetry in the wholesale sector, gross inflation in the final good price, $1 + \pi_t = P_t/P_{t-1}$, evolves according to

$$1 + \pi_t = \left[\frac{1}{\omega} - \left(\frac{1-\omega}{\omega} \right) (\tilde{p}_t^*)^{(1-\varepsilon)} \right]^{1/(\varepsilon-1)} \tag{3.8}$$

Intermediate good producers. Production in the intermediate good sector occurs using only labor as input, which is hired in a frictional labor market. Labor market frictions are summarised by an aggregate matching function, which is assumed to take a Cobb-Douglas form with constant returns to scale,

$$M_t = \mathcal{M}\, E_t^{\alpha}\, V_t^{(1-\alpha)}$$

where M_t denotes the total amount of formed matches, $\mathcal{M}$ is a matching efficiency parameter, $\alpha \in (0,1)$, V_t is the total amount of vacancies, and $E_t = U_{t-1} + \rho_t\, N_{t-1}$ is the total amount of searching workers, given by those workers who were unemployed plus those workers who were employed but experience separation (at rate ρ_t) in the current period. The matching probability for vacancies and the matching probability for searching workers are given, respectively, by

$$q_t = \frac{M_t}{V_t}\,, \qquad f_t = \frac{M_t}{E_t}$$

Active matches produce z_{it} units of output at each period (provided no separation occurs), and z_{it} is composed by a match-specific and an aggregate component

$$z_{it} = z_t\, \varphi_{it}$$

where the match-specific component, φ_{it}, is *iid* both across firms and over time, and has CDF $G(\varphi)$ and PDF $g(\varphi)$, while z_t represents a common random productivity disturbance which is orthogonal to φ_{it}. The value of a match for a firm with idiosyncratic productivity φ is

$$J_t(\varphi) = x_t\, z_t\, \varphi - w_t + \beta\, \mathbb{E}_t \left[(1-\rho^x) \int_{\varphi^*_{t+1}}^{\infty} J_{t+1}(\varphi)\, g(\varphi)\, d\varphi \right] \tag{3.9}$$

where the wage rate w_t is the same across producers, ρ^x is the exogenous separation rate, and φ^*_{t+1} is the threshold below which a match will become unprofitable, and hence will be endogenously destroyed: since $J_t(\varphi)$ is monotonically increasing in φ, and all firms face the same aggregate shocks, the endogenous job destruction cutoff, φ^*_t, will be determined, at every period, by the condition

$$J_t(\varphi^*_t) = 0$$

and matches with $\varphi_{it} \geq \varphi^*_t$ will be actively producing, while matches with $\varphi_{it} < \varphi^*_t$ will not and break up. Therefore, the total separation rate, associated to φ^*_t, is given by

$$\rho_t := \rho^x + (1-\rho^x)\, G\left(\varphi^*_t\right) \tag{3.10}$$

When matches break up, inactive producers (which are ex-ante symmetric) post vacancies at a cost of κ units of the final good per vacancy per period, and each vacancy is filled with probability q_t. The value of a vacancy is therefore equal across producers. Assuming free entry into vacancy posting, this implies

$$\frac{\kappa}{q_t} = J_t \tag{3.11}$$

where

$$J_t := \frac{1}{[1 - G(\varphi_t^*)]} \int_{\varphi_t^*}^{\infty} J_t(\varphi)\, g(\varphi)\, d\varphi$$

Combining the free entry condition above with the value of the marginal active producer, $J_t(\varphi_t^*) = 0$, gives the following job destruction condition

$$x_t\, z_t\, \varphi_t^* = w_t - \beta\, \mathbb{E}_t \left[(1 - \rho_{t+1})\, \frac{\kappa}{q_{t+1}} \right] \tag{3.12}$$

Wage Setting. To formalise the presence of real wage rigidity in a simple way, I follow Blanchard and Galí (2010a) in assuming a wage schedule of the following form

$$w_t = w\, z_t^{\chi} \tag{3.13}$$

where $\chi \geq 0$ is an index of real wage rigidities. At steady state, $z = 1$ and wages are assumed to be set according to a collective wage setting scheme whereby

$$w = \eta \left[1 + \beta\, \kappa\, (1 - \rho)\, \frac{f}{q} \right] + (1 - \eta)\, \delta$$

3.2.3 Monetary Authority

The monetary authority is assumed to set the nominal interest rate, i_t, by reacting to inflation as well as to labor market conditions. These are assumed to be captured by the unemployment rate, u_t.

$$1 + i_t = \max \left\{ (1 + i) \left(\frac{1 + \pi_t}{1 + \pi} \right)^{\phi_\pi} \left(\frac{U_t}{U} \right)^{-\phi_u}, 1 \right\}. \tag{3.14}$$

3.2.4 Equilibrium

Definition 3.1. *An equilibrium is a set of sequences of optimal household and firm choices, and policy decisions of the monetary authority, such that: (i) markets clear; (ii) labor market variables evolve according to the laws of motion; (iii) the Job Destruction and Free Entry conditions hold.*

Zero Liquidity Property. It can be shown that at equilibrium, in the steady state neighbourhood, employed workers' Euler condition holds with equality while that of unemployed workers holds with inequality[3]

$$\frac{1}{w_t} = \beta\,\mathbb{E}_t \left\{ \left(\frac{1+i_t}{1+\pi_{t+1}} \right) \left[(1-\lambda_{t+1})\,\frac{1}{w_{t+1}} + \lambda_{t+1}\,\frac{1}{\delta} \right] \right\} \tag{3.15}$$

$$\frac{1}{\delta} > \beta\,\mathbb{E}_t \left\{ \left(\frac{1+i_t}{1+\pi_{t+1}} \right) \left[f_{t+1}\,\frac{1}{w_{t+1}} + (1-f_{t+1})\,\frac{1}{\delta} \right] \right\} \tag{3.16}$$

In other words, at equilibrium employed workers wish to precautionarily save, but as unemployed workers are impeded from borrowing, no one is able to issue the assets that would allow the precautionary saving desire to be satisfied. Hence, no asset trade actually takes place at equilibrium. This zero-liquidity property allows to introduce a precautionary saving desire in a tractable fashion, i.e. without the need of tracking a wealth distribution over time.

3.3 The loglinearized stochastic model at the ZLB

Letting hatted variables indicate log-deviations from the zero-inflation steady state, and $\pi_t := \log(1+\pi_t)$, $i_t := \log(1+i_t)$, the loglinearized model is characterised, first of all, by the following Euler condition and Phillips curve

$$i_t - \mathbb{E}_t\,(\pi_{t+1}) = \underbrace{-\left[\frac{\rho\,(1-f)\,\zeta}{(1-\zeta)+\rho\,(1-f)\,\zeta} \right] \mathbb{E}_t \left(\widehat{\lambda}_{t+1} \right)}_{\text{precautionary saving desire}} + \underbrace{\left\{ \frac{(1-\zeta)\,[1-\rho\,(1-f)]}{(1-\zeta)+\rho\,(1-f)\,\zeta} \right\} \mathbb{E}_t\,(\widehat{w}_{t+1}) - \widehat{w}_t}_{\text{standard consumption smoothing}}$$

$$\pi_t = \beta\,\mathbb{E}_t\,(\pi_{t+1}) + \Omega\,\widehat{x}_t$$

where $\widehat{w}_t = \chi\,\widehat{z}_t$, ρ and f are the steady state separation and job finding rates, respectively, $\zeta = [(w-\delta)/w] \in (0,1)$ captures the steady state amount of unemployment insurance, and $\Omega = (1-\omega)\,(1-\beta\,\omega)/\omega$.

Focusing on the Euler condition, the second term on the right hand side captures a standard consumption smoothing desire: an expected increasing wage profile implies a desire to borrow so as to smooth consumption out of better future prospects. The first term on the right hand side captures, instead, a precautionary saving desire: a higher risk of becoming unemployed pushes currently employed workers to wish to save a buffer stock out of fears of future income losses. Since this saving desire cannot actually be satisfied in this zero-liquidity economy, it manifests as downward pressure on the real interest rate.

In the exogenous separation case, unemployment risk λ, hence the precautionary saving

[3] See Ravn and Sterk (2021), Challe (2020) for more details.

desire, are entirely driven by the job finding rate

$$\widehat{\lambda}_t = -\left(\frac{f}{1-f}\right)\widehat{f}_t$$

which evolves according to the following job creation condition

$$\left(\frac{\alpha}{1-\alpha}\right)\widehat{f}_t = \frac{1}{\kappa/q}\left(\widehat{z}_t + \widehat{x}_t\right) - \left(\frac{w}{\kappa/q}\right)\widehat{w}_t + \beta\left(1-\rho\right)\mathbb{E}_t\left[\left(\frac{\alpha}{1-\alpha}\right)\widehat{f}_{t+1}\right].$$

The unemployment rate evolves according to

$$\widehat{u}_t = -\left(\frac{f}{1-f}\right)\widehat{f}_t + \left(1-\rho\right)\left(1-f\right)\widehat{u}_{t-1}$$

In the endogenous separation case, unemployment risk is driven not only by the job finding rate, but also by fluctuations in the separation rate

$$\widehat{\lambda}_t = \widehat{\rho}_t - \left(\frac{f}{1-f}\right)\widehat{f}_t$$

which evolve according to the following job creation and job destruction conditions

$$\left(\frac{\alpha}{1-\alpha}\right)\widehat{f}_t = \widehat{z}_t + \widehat{x}_t - \frac{1}{\tau}\left(\frac{\rho}{1-\rho}\right)\left[\left(\frac{\varphi^*}{1-\varphi^*}\right) - \tau\right]\widehat{\rho}_t$$

$$\widehat{z}_t + \widehat{x}_t + \frac{1}{\tau}\left(\frac{\rho}{1-\rho}\right)\widehat{\rho}_t = \left(\frac{w}{\varphi^*}\right)\widehat{w}_t - \left(\frac{1-\varphi^*}{\varphi^*}\right)\beta\left(1-\rho\right)\mathbb{E}_t\left[\widehat{z}_{t+1} + \widehat{x}_{t+1}\right] + \beta\left(1-\rho\right)\frac{1}{\tau}\left(\frac{\rho}{1-\rho}\right)\mathbb{E}_t\left[\widehat{\rho}_{t+1}\right]$$

where $\tau := \varphi^*\, g(\varphi^*) \,/\, [1 - G(\varphi^*)]$ and φ^* is the steady state job destruction threshold.[4] The unemployment rate, in this case, evolves as follows

$$\widehat{u}_t = f\,\widehat{\rho}_t - \left(\frac{f}{1-f}\right)\widehat{f}_t + \left(1-\rho\right)\left(1-f\right)\widehat{u}_{t-1}$$

Lastly, the Taylor rule in loglinear form reads

$$i_t = \max\left\{0\,,\ \phi_\pi\,\pi_t - \phi_u\,\widehat{u}_t\right\} \tag{3.17}$$

A Markovian shock. Now, to model the ZLB in an analytically tractable form, I follow Eggertsson and Woodford (2003), Christiano, Eichenbaum, and Rebelo (2011) and many subsequent works in assuming a Markovian process for the exogenous shock $\widehat{z}_t$. Differently from them, in this setting one does not need an exogenous shock to the discount factor to increase the desire

[4]Notice that since I normalize $z = 1$ and $\overline{\varphi} := \mathbb{E}\left(\varphi\,|\,\varphi \geq \varphi^*\right) = 1$ at steady state, the steady state job creation condition implies $1 - \varphi^* = \kappa/q$.

to save and hence drive the nominal interest rate at the ZLB. Indeed, a negative productivity shock, by increasing the desire to precautionarily save, can deliver the same result.

I assume, first of all, that the economy is initially at the zero inflation steady state, and that this coincides with a ZLB situation, whereby the steady state Euler condition reads

$$1 = \beta \left[1 + \rho\,(1-f) \left(\frac{\zeta}{1-\zeta} \right) \right] \tag{3.18}$$

and the resulting parameter restriction implies that the log-linear Euler condition becomes

$$i_t - \mathbb{E}_t\,(\pi_{t+1}) = -(1-\beta)\,\mathbb{E}_t \left(\widehat{\lambda}_{t+1} \right) + \{\beta\,[1-\rho\,(1-f)]\;\mathbb{E}_t\,(\widehat{w}_{t+1}) - \widehat{w}_t\}$$

From the initial zero-inflation ZLB steady state situation, an unexpected negative shock to z occurs, which is assumed to be evolving according to the following Markovian process

$$P(\widehat{z}_{t+1} = -\xi \,|\, \widehat{z}_t = -\xi) = \psi$$
$$P(\widehat{z}_{t+1} = 0 \,|\, \widehat{z}_t = -\xi) = (1-\psi)$$
$$P(\widehat{z}_{t+1} = 0 \,|\, \widehat{z}_t = 0) = 1$$

i.e. the shock persists with probability ψ through the next period, and once it vanishes z reverts back to its steady state value forever after.

Regardless of the various cases considered in what follows (fully rigid or inertial wages, endogenous or exogenous separation), one can guess and verify that under the assumptions made above, provided that the ZLB is always binding in the recession regime the main variables driving the model behavior will take on two values: one for the recession state, where $\widehat{z} = -\xi$ and the ZLB binds, and one outside it. In other words, I can proceed by solving for the two regimes, later checking under which conditions (if any) the ZLB on interest rates is always binding in the recession state.

In the exogenous separation case, the main variables to be solved for are the job-finding rate, f, and the inflation rate, π, and are labelled, in log-linear terms, as $(\widehat{f}^X_\xi\,, \pi^X_\xi)$ in the recession state. As the shock vanishes, these revert to steady state.

In the endogenous separation case, among the main variables driving the model behavior we have, in addition to the previous, also the separation rate, ρ_t. In the recession state, these variables are labelled, in log-linear terms, as $(\widehat{\rho}_t\,, \widehat{f}^E_\xi\,, \pi^E_\xi)$, and they are again all reverting to steady state once the shock vanishes.

Unemployment, instead, will not instantaneously revert to its steady state value, but will take time to adjust once the shock vanishes. As the shock hits at $t = 0$, we have

$$\widehat{u}_0 = -\left(\frac{f}{1-f}\right)\widehat{f}_\xi^X \tag{3.19}$$

$$\widehat{u}_0 = f\,\widehat{\rho}_t - \left(\frac{f}{1-f}\right)\widehat{f}_\xi^E \tag{3.20}$$

in the exogenous and endogenous separation cases, respectively.

In subsequent periods, depending on the duration of the shock,

$$\widehat{u}_t = \widehat{u}_0 \sum_{j=0}^{t} \left[(1-\rho)(1-f)\right]^j \tag{3.21}$$

if the shock has persisted through period t (which occurs with probability ψ^t), and

$$\widehat{u}_t = \widehat{u}_0 \, \left[(1-\rho)(1-f)\right]^{\left[(t-K)\,\mathbb{I}_{\{t>K\}}\right]} \sum_{j=0}^{\left[t+(K-t)\,\mathbb{I}_{\{t>K\}}\right]} \left[(1-\rho)(1-f)\right]^j \tag{3.22}$$

if the shock has persisted for K periods and then vanished (which occurs with probability $\psi^K(1-\psi)$).

It is worth stressing that the recession state can be consistent with positive inflation only as long as the Taylor rule targets not only inflation but also labor market conditions. If the central bank targeted only inflation, positive inflation would require interest rates to be risen. However, if the Taylor rule also targets fluctuations in the unemployment rate, as long as $\widehat{u}_t \geq 0$, the policy prescription can be one in which the nominal rate is actually constrained at the ZLB. Formally, in the present setting, an unconstrained Taylor rule should prescribe

$$i_t = \phi_\pi \, \pi_t - \phi_u \, \widehat{u}_t \leq 0$$

As the shock vanishes, $\pi_t = 0$ while $\widehat{u}_t \geq 0$, hence $i_t = -\phi_u \, \widehat{u}_t \leq 0$ for sure. While the shock persists, we always have $\widehat{u}_t \geq \widehat{u}_0$, and π_ξ remains constant. Then, having

$$i_0 = \phi_\pi \, \pi_\xi - \phi_u \, \widehat{u}_0 \leq 0$$

ensures that also

$$i_t = \phi_\pi \, \pi_\xi - \phi_u \, \widehat{u}_t \leq 0$$

in all subsequent periods when the shock persists. Therefore, it is sufficient to have $\phi_\pi \, \pi_\xi - \phi_u \, \widehat{u}_0 \leq 0$ to ensure that that i_t is constrained at 0 along the whole equilibrium path.

3.3.1 Fully rigid wages

In the fully rigid wages case ($\chi = 0$), the precautionary motive is the only force at work in shaping the response of the real interest rate. In this case we therefore surely have downward pressure on the real rate if unemployment risk goes up.

Exogenous Separation

Proceeding by guessing and verifying, I get the following recession state solution for an economy where the separation rate can be assumed to be exogenous

$$\begin{aligned}\widehat{f}_\xi &= -F_0^X\,\xi \\ \pi_\xi &= \left(\frac{f}{1-f}\right)(1-\beta)\,F_0^X\,\xi\end{aligned}$$

where

$$F_0^X = \frac{1}{\frac{\kappa}{q}\left(\frac{\alpha}{1-\alpha}\right)\left[1-\beta\,(1-\rho)\,\psi\right]+\left(\frac{1-\beta\,\psi}{\Omega}\right)\left(\frac{f}{1-f}\right)(1-\beta)} > 0$$

while the unemployment rate is given, at $t = 0$, by

$$\widehat{u}_0 = \left(\frac{f}{1-f}\right)F_0^X\,\xi$$

and evolves in subsequent periods according to the path described in equations (3.21) and (3.22).

Therefore, we indeed have a falling job finding rate rate and a positive inflation rate: the falling job finding rate implies an increased desire to precautionarily save, hence downward pressure on the real interest rate, but since $i_t = 0$, this can only be achieved through some inflation.

However, having this outcome supported at equilibrium requires, as argued above, that $\phi_\pi\,\pi_\xi - \phi_u\,\widehat{u}_0 \leq 0$, i.e. that the nominal interest rate is indeed constrained at 0, or equivalently, that the Taylor rule does *not* prescribe instead a rise in the nominal interest rate. The following proposition summarises under which parameter restriction this occurs.

Proposition 3.1. *In the exogenous job destruction case, a recession at the ZLB is consistent with positive inflation provided that*

$$\phi_u \geq \underline{\phi}_u^{X,0}$$

where

$$\underline{\phi}_u^{X,0} := \phi_\pi\,(1-\beta)$$

Intuitively, the monetary authority should be willing to tolerate some current inflation as long as this helps driving up future inflation expectations and hence driving down the current real interest rate, thereby reducing the desire to save and improving labor market prospects.

Endogenous Separation

Proceeding with the same guess-and-verify procedure as above I get, for the endogenous separation case,

$$\begin{aligned}
\widehat{\rho}_{\xi} &= D_0^E\,\xi \\
\widehat{f}_{\xi} &= -H_0^E\,D_0^E\,\xi \\
\pi_{\xi} &= (1-\beta)\left[1+\left(\frac{f}{1-f}\right)H_0^E\right]D_0^E\,\xi
\end{aligned}$$

and

$$\widehat{u}_0 = \left[f+\left(\frac{f}{1-f}\right)H_0^E\right]D_0^E\,\xi$$

where

$$\begin{aligned}
D_0^E &= \frac{1}{\left\{\frac{\varphi^*\left[1-\beta\,(1-\rho)\,\psi\right]}{\varphi^*+(1-\varphi^*)\,\beta\,(1-\rho)\,\psi}\right\}\frac{1}{\tau}\left(\frac{\rho}{1-\rho}\right)+\left(\frac{1-\beta\,\psi}{\Omega}\right)(1-\beta)\left[1+\left(\frac{f}{1-f}\right)H_0^E\right]} \\
H_0^E &= \frac{1}{\tau}\left(\frac{\rho}{1-\rho}\right)\left\{\frac{\varphi^*\left[1-\beta\,(1-\rho)\,\psi\right]}{\varphi^*+(1-\varphi^*)\,\beta\,(1-\rho)\,\psi}+\left(\frac{\varphi^*}{1-\varphi^*}\right)-\tau\right\}\left(\frac{1-\alpha}{\alpha}\right)
\end{aligned}$$

In this case, the requirement that the Taylor rule prescriptions are such that the nominal interest rate is actually constrained at zero, translates into the following parameter restriction.

Proposition 3.2. *In the endogenous job destruction case, a recession at the ZLB is consistent with positive inflation provided that*

$$\phi_u \geq \underline{\phi}_u^{E,0}$$

where

$$\underline{\phi}_u^{E,0} := \phi_\pi\,(1-\beta)\left[\frac{1+\left(\frac{f}{1-f}\right)H_0^E}{f+\left(\frac{f}{1-f}\right)H_0^E}\right]$$

Compared to the exogenous separation case, there is now a wedge, making the requirement on the magnitude of ϕ_u stronger (as the wedge is larger than 1). Intuitively, countercyclical separation increases unemployment risk and strengthens the desire to precautionary save, thereby exerting more downward pressure on the real interest rate. When the nominal interest rate is stuck at the ZLB, this manifests as more inflationary pressure, compared to the exogenous separation case. Therefore, having the Taylor rule actually prescribing that the nominal rate should remain at the ZLB requires that the monetary authority cares relatively more about labor market conditions, compared to the exogenous separation case.

3.3.2 Inertial wages

In the inertial wage case ($\chi > 0$), the precautionary saving desire is not any more the only force at work in shaping the response of the real interest rate. Even if unemployment risk goes up, the desire to precautionarily save is counteracted by expected higher wages tomorrow. This, on the opposite, increases the desire to borrow. Therefore, while in the fully rigid case a fall in the real rate (achieved through inflation) is the necessary outcome, in this case this requires that the precautionary saving motive is strong enough to offset the desire to borrow.

Exogenous Separation

In the exogenous separation case we have, with inertial wages,

$$\begin{aligned}\widehat{f}_\xi &= -F_\chi^X\,\xi \\ \pi_\xi &= P_\chi^X\,\xi\end{aligned}$$

and

$$\widehat{u}_0 = \left(\frac{f}{1-f}\right) F_\chi^X\,\xi$$

where

$$\begin{aligned}F_\chi^X &= F_0^X \left\{(1-\chi\,w) + \chi\left(\frac{1-\beta\,\psi}{\Omega}\right)\left[\frac{1}{\psi} - \beta\,[1-\rho(1-f)]\right]\right\} \\ P_\chi^X &= \left(\frac{f}{1-f}\right)(1-\beta)\,F_\chi^X - \chi\left[\frac{1}{\psi} - \beta\,[1-\rho(1-f)]\right]\end{aligned}$$

The resulting Taylor rule parameter restrictions are summarised in the following proposition.

Proposition 3.3. *In the exogenous job destruction case with partially flexible wages, a recession at the ZLB is consistent with positive inflation provided that*

$$\phi_u \geq \underline{\phi}_u^X \geq 0$$

where

$$\underline{\phi}_u^X := \phi_\pi\,(1-\beta) - \phi_\pi\,\chi\left(\frac{1}{F_\chi^X}\right)\left(\frac{1-f}{f}\right)\left[\frac{1}{\psi} - \beta\,[1-\rho(1-f)]\right]$$

Compared to the rigid-wage with exogenous separation case, there is now an additional term arising due to (partial) wage flexibility. During the recession episode, wages are higher (in expectation) in the next period. This implies a desire to borrow out of improving income prospects. In this zero liquidity economy this desire to borrow cannot actually be satisfied, and hence manifests as upward pressure on the real interest rate. As the nominal interest rate is stuck at zero, this upward pressure on the real interest rate takes the form of disinflationary pressure. This effect then pushes on the opposite direction of that of the precautionary motive, which has already been highlighted in the fully rigid wages case and is still present, even if mitigated by the desire to borrow, in this case.

Therefore, on the one hand, the desire to borrow dampens the inflationary pressures and hence makes the restrictions on the Taylor rule parameters less demanding. On the other hand, there is an additional condition to be met (captured by the second inequality), i.e. having the precautionary saving motive dominating over the standard consumption smoothing motive, so that the net result is an inflationary one.

Endogenous Separation

In the endogenous separation case we have, with inertial wages,

$$
\begin{aligned}
\widehat{\rho}_\xi &= D_\chi^E\, \xi \\
\widehat{f}_\xi &= -F_\chi^E\, \xi \\
\pi_\xi &= P_\chi^E\, \xi
\end{aligned}
$$

and

$$
\widehat{u}_0 = \left[f\, D_\chi^E + \left(\frac{f}{1-f} \right) F_\chi^E \right] \xi
$$

where

$$
\begin{aligned}
D_\chi^E &= D_0^E - \chi\, D_0^E \left\{ C_\varphi + \left(\frac{1-\beta\,\psi}{\Omega} \right) \left[C_\varphi\,(1-\beta) \left(\frac{f}{1-f} \right) \left(\frac{1-\alpha}{\alpha} \right) + \frac{1}{\psi} - \beta\,[1-\rho(1-f)] \right] \right\} \\
F_\chi^E &= \left[H_0^E\, D_\chi^E + \chi \left(\frac{1-\alpha}{\alpha} \right) C_\varphi \right] \\
C_\varphi &= \left[\frac{w}{\varphi^* + (1-\varphi^*)\,\beta\,(1-\rho)\,\psi} \right] \\
P_\chi^E &= (1-\beta) \left[D_\chi^E + \left(\frac{f}{1-f} \right) F_\chi^E \right] - \chi \left[\frac{1}{\psi} - \beta\,[1-\rho(1-f)] \right]
\end{aligned}
$$

The Taylor rule parameter restrictions are as follows.

Proposition 3.4. *In the endogenous job destruction case with partially flexible wages, a recession at the ZLB is consistent with positive inflation provided that*

$$\phi_u \geq \underline{\phi}_u^E \geq 0$$

where

$$\underline{\phi}_u^E := \phi_\pi (1-\beta) \left[\frac{1 + \left(\frac{f}{1-f}\right) H_0^E + \chi \left(\frac{f}{1-f}\right) \left(\frac{1-\alpha}{\alpha}\right) \frac{C_\varphi}{D_\chi^E}}{f + \left(\frac{f}{1-f}\right) H_0^E + \chi \left(\frac{f}{1-f}\right) \left(\frac{1-\alpha}{\alpha}\right) \frac{C_\varphi}{D_\chi^E}} \right] - \phi_\pi \chi \left[\frac{1}{f\, D_\chi^E + \left(\frac{f}{1-f}\right) F_\chi^E} \right] \left[\frac{1}{\psi} - \beta \left[1 - \rho(1-f)\right] \right]$$

In this case, there are two additional effects compared to the rigid-wage with exogenous separation case. First, as described in the rigid-wage case, there is a wedge due to endogenous separation exerting more downward pressure on the real interest rate, hence more inflationary pressure, compared to the exogenous separation case (as the wedge is larger than one). Also, compared to the endogenous separation case with fully rigid wages, the wedge is now smaller, as wage flexibility mitigates job destruction, and hence the inflationary pressures arising due to the ensuing unemployment risk and precautionary saving behavior.

Second, there is an additional term due to the same borrowing desire described in the partially flexible wage with exogenous separation case, which counteracts the inflationary pressures arising from the precautionary saving motive.

3.4 An Empirical Perspective

How likely are the conditions derived in the previous section to be met in the data? To provide a ballpark figure for the lower bounds on ϕ_u derived in the propositions above, I fix first of all $\phi_\pi = 1.5$ and $\beta = 0.99$, aiming at a quarterly calibration. The probability that the negative shock to productivity persists into the next quarter is set to $\psi = 3/4$, which implies an expected duration of the shock of four quarters.[5]

Following Blanchard and Galí (2010a), I choose two different targets for the steady state job-finding and separation rates, f and ρ. A first calibration aims at targeting the more fluid US labor market, characterised by high values of both ρ and f, and low unemployment duration. For this case, the steady state unemployment rate is set to 5% and the job-finding rate to

[5]Christiano et al. (2011) set their corresponding parameter to 0.8. My choice is, therefore, a conservative one, as a higher ψ would reinforce the precautionary motive and the ensuing inflationary pressures.

$f = 0.7$. Given that at steady state $u = \lambda/(\lambda + f)$ and $\lambda = \rho(1 - f)$, these targets imply in turn $\lambda \approx 0.05$ and $\rho \approx 0.18$. A second calibration aims at targeting the more sclerotic labor market of continental European countries, characterised by low values of both ρ and f, and high unemployment duration. In this case $u = 10\%$ and $f = 0.25$, implying in turn $\lambda \approx 0.03$ and $\rho \approx 0.04$.

The percentage income loss upon unemployment, implied by these targets through the ZLB restriction in (3.18), is $\zeta = 0.25$ in the sclerotic case and $\zeta = 0.16$ in the fluid case, which are close to the 20% consumption drop upon unemployment documented by Chodorow-Reich and Karabarbounis (2016), and both in the range of the empirical estimates reviewed by Den Haan et al. (2018).

As for wage rigidity, I consider both the case of fully rigid wages ($\chi = 0$), as well as that of inertial wages. In this latter case, $\chi = 1/3$ as suggested by Challe (2020).

Table 3.1 reports the resulting lower bounds on ϕ_u derived in the propositions in Section 3.3. Given that these depend on the value chosen for ϕ_π —conventionally set to 1.5— the bottom panel of the table reports also relative lower bounds $\underline{\phi}_{u/\pi} := \underline{\phi}_u/\phi_\pi = \pi_\xi/\widehat{u}_0$ on the required relative magnitude implied by the propositions for the Taylor rule parameters, capturing the increase in inflation relative to unemployment in response to the shock.

First of all, when wages are fully rigid, a more sclerotic labor market, being associated with more separations and hence a stronger precautionary saving motive, implies relatively more inflationary pressure in this setting, making the requirement on the (relative) magnitude of ϕ_u stronger.

In the inertial wage case, while in the fluid labor market calibration reduced job creation alone is sufficient to make the precautionary motive dominating and inducing a fall in the real rate in the first place, this is not the case for the sclerotic labor market calibration, as suggested by the fact that $\underline{\phi}_u^X < 0$ for this latter. Once endogenous separations are added to the picture, however, increased job destruction makes the precautionary motive sufficiently strong to induce a fall in the real rate (and hence inflationary pressure) for both of the target labor markets considered.

3.4.1 Output-targeting Taylor Rule

The results are substantially unchanged when the Taylor rule responds to output instead of unemployment fluctuations,

$$i_t = \max\left\{0\,,\ \phi_\pi\,\pi_t + \phi_y\,\widehat{y}_t\right\}\ . \tag{3.23}$$

Table 3.1: Taylor Rule Parameter Restrictions for Inflationary ZLB Recession Episodes

Lower Bounds: $\underline{\phi}_u := \phi_\pi\, \pi_\xi / \widehat{u}_0$

			Fully Rigid Wages ($\chi = 0$)		Inertial Wages ($\chi = 1/3$)	
Labor Market	ρ	f	$\underline{\phi}_u^{X,0}$	$\underline{\phi}_u^{E,0}$	$\underline{\phi}_u^{X}$	$\underline{\phi}_u^{E}$
Sclerotic (EU)	0.04	0.25	0.0150	0.0510	-0.1227	0.0022
Fluid (US)	0.18	0.7	0.0150	0.0181	0.0031	0.0070

Relative Lower Bounds: $\underline{\phi}_{u/\pi} := \underline{\phi}_u / \phi_\pi = \pi_\xi / \widehat{u}_0$

			Fully Rigid Wages ($\chi = 0$)		Inertial Wages ($\chi = 1/3$)	
Labor Market	ρ	f	$\underline{\phi}_{u/\pi}^{X,0}$	$\underline{\phi}_{u/\pi}^{E,0}$	$\underline{\phi}_{u/\pi}^{X}$	$\underline{\phi}_{u/\pi}^{E}$
Sclerotic (EU)	0.04	0.25	0.0100	0.0340	-0.0818	0.0015
Fluid (US)	0.18	0.7	0.0100	0.0120	0.0020	0.0047

Notes: the table reports lower bounds for the parameters of the Taylor rule in (3.17), consistent with having the nominal interest rate constrained at the zero lower bound, i.e. with $\phi_\pi\, \pi_\xi - \phi_u\, \widehat{u}_0 \leq 0$. The values of the bounds are obtained from the results of the propositions of Section 3.3, under the parametrisation of Section 3.4.

Since (un)employment directly affects the level of output, similar considerations apply to this latter: as long as the recession state persists, $\widehat{y}_t < 0$ and, similarly to the unemployment rate, $\widehat{y}_t \leq \widehat{y}_0$. Therefore, a sufficient condition for having $\phi_\pi\, \pi_t + \phi_y\, \widehat{y}_t \leq 0$ along the equilibrium path is again $\phi_\pi\, \pi_\xi + \phi_y\, \widehat{y}_0 \leq 0$. The resulting analytical lower bounds on ϕ_y are formally derived in the Appendix, while Table 3.2 reports these bounds under the same calibration used in Table 3.1. The message that emerges is broadly the same as that considered for the baseline, unemployment-targeting Taylor rule. In the endogenous separation case, one needs a relatively higher Taylor rule parameter on output fluctuations, compared to the exogenous separation case, but the required lower bound on its magnitude is in any case mild, and below the typically assumed value of 0.5.

3.5 Conclusions

Why has deflation been missing during recent recessions? This question has turned out to be puzzling through the lens of standard complete-market NK models, that would predict substantial deflation during liquidity trap episodes.

This is not necessarily the case in HANK-S&M models with uninsurable unemployment risk. In this work, I have shown that ZLB recession episodes may actually turn out to be

Table 3.2: Parameter Restrictions (Output-targeting Taylor Rule)

Lower Bounds: $\underline{\phi}_y := \phi_\pi \, \pi_\xi / \widehat{y}_0$

			Fully Rigid Wages ($\chi = 0$)		Inertial Wages ($\chi = 1/3$)	
Labor Market	ρ	f	$\underline{\phi}_y^{X,0}$	$\underline{\phi}_y^{E,0}$	$\underline{\phi}_y^{X}$	$\underline{\phi}_y^{E}$
Sclerotic (EU)	0.04	0.25	0.0168	0.1523	-0.1429	0.0066
Fluid (US)	0.18	0.7	0.1060	0.1466	0.0223	0.0584

Relative Lower Bounds: $\underline{\phi}_{y/\pi} := \underline{\phi}_y / \phi_\pi = \pi_\xi / \widehat{y}_0$

			Fully Rigid Wages ($\chi = 0$)		Inertial Wages ($\chi = 1/3$)	
Labor Market	ρ	f	$\underline{\phi}_{y/\pi}^{X,0}$	$\underline{\phi}_{y/\pi}^{E,0}$	$\underline{\phi}_{y/\pi}^{X}$	$\underline{\phi}_{y/\pi}^{E}$
Sclerotic (EU)	0.04	0.25	0.0112	0.1015	-0.0953	0.0044
Fluid (US)	0.18	0.7	0.0100	0.0120	0.0020	0.0047

Notes: the table reports lower bounds for the parameters of the Taylor rule in (3.23), consistent with having the nominal interest rate constrained at the zero lower bound, i.e. with $\phi_\pi \, \pi_\xi + \phi_y \, \widehat{y}_0 \leq 0$. The values of the bounds are obtained from the results of the propositions in the Appendix, under the parametrisation of Section 3.4.

inflationary in these settings: uninsurable unemployment risk induces a precautionary saving motive that counteracts the standard desire to borrow during recessions. If sufficiently strong, the precautionary saving motive can then induce a fall in the real interest rate. When the nominal interest rate is stuck at the ZLB, this can only be achieved through (expected) inflation.

This latter outcome requires, in turn, that the monetary authority cares not only about inflation but also about deteriorated labor market conditions (as captured by fluctuations in the unemployment rate). Otherwise, the Taylor rule would prescribe a rise in the nominal interest rate in the presence of positive inflation and not, instead, that the nominal rate remains constrained at the ZLB.

Analytically, this requires restrictions on the Taylor rule parameters that, on the one hand, are found to be more demanding with endogenous separation than with exogenous separation, as inflationary pressures are higher in the former case: ceteris paribus, in the endogenous separation case one needs a relatively higher Taylor rule parameter on the unemployment rate. On the other hand, a fall in the real rate —which is required to have an inflationary ZLB recession episode in the first place— is more likely with endogenous separation, as increased separations during the recession episode strengthen the precautionary saving motive.

In practice, for a wide range of plausible targets of the steady state separation and job finding rates, the requirements on the magnitude of the Taylor rule parameters are very mild: a Taylor rule parameter on unemployment rate fluctuations of around 0.05 already does the job for both the exogenous and endogenous separation cases. Considering a Taylor rule targeting output, instead of the unemployment rate, on top of inflation, the parameter restrictions also are very mild, with a Taylor rule parameter on output fluctuations of around 0.15 being sufficient in all cases.

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Appendices to Chapter 1

1.A Data Description

The ECB Consumer Expectations Survey (CES) is a panel survey of consumers that has been carried out on a monthly basis starting effectively in April 2020. Its goal is to provide timely high-frequency information on the perceptions and expectations of euro area consumers about the economy, as well as their economic and financial behavior. The microdata are collected in the six main euro area countries: Belgium, Germany, Spain, France, Italy and the Netherlands.[6] The collected sample aims at being representative of the surveyed population by its age, gender, and residence region. Table 1.A.1 summarises the main variables of interest for the analysis of Section 1.2, which are described in more detail in this appendix section.

Expenditure on goods and services. On quarterly basis, interviewed individuals are asked how much did their household spend during the last month on goods and services as listed in twelve major categories. These are: (1) Food, beverages, groceries, tobacco; (2) Restaurants (including take-out food and delivery), cafes/ canteens; (3) Housing (including rent, maintenance/repair costs, home owner/renter insurance, but excluding mortgage payments); (4) Utilities (including water, sewerage, electricity, gas, heating oil, phone, cable, internet); (5) Furnishings, household equipment, small appliances and routine maintenance of the house; (6) Debt repayments (installments in mortgage, consumer loans, car loans, credit cards, student loans, other loans); (7) Clothing, footwear; (8) Health (including personal care products and services), health insurance; (9) Transport (fuel, car maintenance, public transportation fares); (10) Travel, recreation, entertainment and culture; (11) Childcare and education (including tuition fees for child and adult education, costs of after school activities, care of children/ babysitting, but excluding installments on student loans); (12) Other expenditures not mentioned above.

Employment Situation. Starting from the October 2020 wave of the survey, on quarterly basis interviewed individuals are asked what best describes their current employment situation, and can give one of the following answers: (1) Working full-time (self-employed or working for someone else); (2) Working part-time (self-employed or working for someone else); (3) Temporarily laid-off (expecting to return to the previous workplace); (4) On extended leave (disability, sick, parental or other leave); (5) Unemployed and actively looking for a job; (6) Unemployed, interested in

[6] Five additional euro area countries are currently being added: Ireland, Greece, Austria, Portugal and Finland.

Table 1.A.1: Variable Description

Expenditure on goods and services — food
Expenditure on goods and services — restaurants
Expenditure on goods and services — housing
Expenditure on goods and services — utilities
Expenditure on goods and services — furnishing
Expenditure on goods and services — debt repayment
Expenditure on goods and services — clothing
Expenditure on goods and services — health
Expenditure on goods and services — transport services
Expenditure on goods and services — travel and recreation
Expenditure on goods and services — childcare and education
Expenditure on goods and services — other
Employment Situation: employed/unemployed/inactive
Age at start of year: 18-34/35-49/50-70/71+
Gender
Foreign birth
Education: low/middle/high
Presence of a partner

having a job but not actively looking for a job; (7) Unable to work because of disability or other medical reasons; (8) In retirement or early retirement; (9) Studying, at school, or in training; (10) Looking after children or other persons, doing housework; (11) Other.

These 11 categories above are regrouped by the ECB into the following groups: Employed, if working full- or part-time, or if temporarily laid-off or on extended leave. Unemployed, if actively looking for a job, or interested in having a job but not actively looking for it.[7] Inactive, in the remaining cases.

Age at start of year. When recruited for the survey, individuals are asked their birth month and year, and the derived age variable is updated at the beginning of each year based on the reported date of birth. The responses are then regrouped in the age brackets indicated in Table 1.A.1.

Education. During recruitment, individuals are asked what is the highest level of school they

[7] This category includes all respondents that consider themselves unemployed, including those who consider themselves unemployed but are not actively searching for a job. Thus, the definition of unemployment used by the ECB in the CES does not necessarily correspond to that of official EU labor market statistics.

have completed, or the highest degree they have received. Possible responses are: (1) Primary or no education; (2) Lower secondary education; (3) High school diploma (or equivalent professional degree); (4) Some college but no academic degree (for example: no BA, BS); (5) Bachelor's Degree (for example: BA, BS) or equivalent professional degree; (6) Master's Degree (for example: MA, MBA, MS, MSW) or equivalent; (7) Doctoral Degree (for example: PhD) or equivalent. These categories are then regrouped, before data dissemination, into three groups: low (no education, primary, or secondary), middle (high school diploma, or some college but no academic degree), high (bachelor's or higher degree).

1.B Derivations and Proofs

1.B.1 Household Problem

Employed workers

$$V^n(B_{t-1}, p_{e,t}) = \max_{\{c_t^n, g_t^n, e_t^n, B_t\}} \{\ln(c_t^n) + \beta\, \mathbb{E}_t\left[(1-\lambda_{t+1})\, V^n(B_t, p_{e,t+1}) + \lambda_{t+1}\, V^u(B_t, p_{e,t+1})\right]\}$$

$$\text{s.t.} \begin{cases} P_{e,t}\, e_t^n + P_{g,t}\, g_t^n + B_t \leq W_t + (1+i_{t-1})\, B_{t-1} \\ c_t^n \leq (g_t^n)^{(1-\omega_e)}(e_t^n - \xi)^{\omega_e} \\ B_t \geq 0 \end{cases}$$

Unemployed workers

$$V^u(B_{t-1}, p_{e,t}) = \max_{\{c_t^u, g_t^u, e_t^u, B_t\}} \{\ln(c_t^u) + \beta\, \mathbb{E}_t\left[f_{t+1}\, V^n(B_t, p_{e,t+1}) + (1-f_{t+1})\, V^u(B_t, p_{e,t+1})\right]\}$$

$$\text{s.t.} \begin{cases} P_{e,t}\, e_t^u + P_{g,t}\, g_t^u + B_t \leq \Delta_t + (1+i_{t-1})\, B_{t-1} \\ c_t^u \leq (g_t^u)^{(1-\omega_e)}(e_t^u - \xi)^{\omega_e} \\ B_t \geq 0 \end{cases}$$

Letting Γ_t^i be the multiplier associated to the budget constraint of household $i \in \{n, u\}$, and μ_t^i that associated with the consumption aggregator, first order conditions are

$$\left[\frac{\partial}{\partial\, c_t^i}\right]: \quad \frac{1}{c_t^i} = \mu_t^i$$

$$\left[\frac{\partial}{\partial\, g_t^i}\right]: \quad P_{g,t}\, \Gamma_t^i = (1-\omega_e)\, \frac{c_t^i}{g_t^i}\, \mu_t^i$$

$$\left[\frac{\partial}{\partial g_t^i}\right]: \quad P_{e,t}\,\Gamma_t^i = \omega_e\,\frac{c_t^i}{e_t^i-\xi}\,\mu_t^i$$

$$\left[\frac{\partial}{\partial B_t}\right]: \quad \begin{cases}\Gamma_t^n \geq \beta\,\mathbb{E}_t\left[(1-\lambda_{t+1})\,\frac{\partial V^n(B_t,p_{e,t+1})}{\partial B_t} + \lambda_{t+1}\,\frac{\partial V^u(B_t,p_{e,t+1})}{\partial B_t}\right]\\ \Gamma_t^u \geq \beta\,\mathbb{E}_t\left[f_{t+1}\,\frac{\partial V^n(B_t,p_{e,t+1})}{\partial B_t} + (1-f_{t+1})\,\frac{\partial V^u(B_t,p_{e,t+1})}{\partial B_t}\right]\end{cases}$$

Combining the first three FOCs gives the following demand schedules

$$g_t^i = (1-\omega_e)\,\frac{P_t}{P_{g,t}}\,c_t^i \tag{1.B.1}$$

$$e_t^i - \xi = \omega_e\,\frac{P_t}{P_{e,t}}\,c_t^i \tag{1.B.2}$$

where

$$P_t := \frac{\mu_t^i}{\Gamma_t^i} = \left(\frac{P_{g,t}}{1-\omega_e}\right)^{(1-\omega_e)}\left(\frac{P_{e,t}}{\omega_e}\right)^{\omega_e}. \tag{1.B.3}$$

The envelope conditions are

$$\frac{\partial V^i(B_{t-1},p_{e,t})}{\partial B_{t-1}} = (1+i_{t-1})\,\Gamma_t^i$$

hence, since $\Gamma_t^i = 1/(P_t\,c_t^i)$, we get the following Euler conditions

$$\frac{1}{c_t^n} \geq \beta\,\mathbb{E}_t\left\{\left(\frac{1+i_t}{1+\pi_{t+1}}\right)\left[(1-\lambda_{t+1})\,\frac{1}{c_{t+1}^n} + \lambda_{t+1}\,\frac{1}{c_{t+1}^u}\right]\right\} \tag{1.B.4}$$

$$\frac{1}{c_t^u} \geq \beta\,\mathbb{E}_t\left\{\left(\frac{1+i_t}{1+\pi_{t+1}}\right)\left[f_{t+1}\,\frac{1}{c_{t+1}^n} + (1-f_{t+1})\,\frac{1}{c_{t+1}^u}\right]\right\} \tag{1.B.5}$$

Now, given the demand schedules for energy and non-energy goods, and the consumption aggregator and price index, total household expenditure is given by

$$X_t^i := P_{g,t}\,g_t^i + P_{e,t}\,e_t^i = P_t\,c_t^i + P_{e,t}\,\xi \tag{1.B.6}$$

giving in turn

$$c_t^i = \frac{X_t^i}{P_t} - \frac{P_{e,t}}{P_t}\,\xi$$

therefore, since at equilibrium $X_t^u = \Delta_t$ and $X_t^n = W_t$, Euler conditions become

$$\frac{1}{w_t - p_{e,t}\,\xi} \geq \beta\,\mathbb{E}_t\left\{\left(\frac{1+i_t}{1+\pi_{t+1}}\right)\left[(1-\lambda_{t+1})\,\frac{1}{w_{t+1}-p_{e,t+1}\,\xi} + \lambda_{t+1}\,\frac{1}{\delta_t - p_{e,t+1}\,\xi}\right]\right\} \tag{1.B.7}$$

$$\frac{1}{\delta_t - p_{e,t}\,\xi} \geq \beta\,\mathbb{E}_t\left\{\left(\frac{1+i_t}{1+\pi_{t+1}}\right)\left[f_{t+1}\,\frac{1}{w_{t+1}-p_{e,t+1}\,\xi} + (1-f_{t+1})\,\frac{1}{\delta_{t+1} - p_{e,t+1}\,\xi}\right]\right\} \tag{1.B.8}$$

where wage income, w_t, unemployment benefits, δ_t, and the energy price, $p_{e,t}$, are expressed in real terms (i.e. in consumption basket units).

1.B.2 Implications of Non-homotheticity of Household Preferences

Own-price Elasticity of Energy Demand.

$$\varepsilon_{e,e} = \frac{\partial\, e}{\partial\, P_e} \frac{P_e}{e} = -\frac{\omega_e\, X}{\omega_e\, X + (1-\omega_e)\, P_e\, \xi} \tag{1.B.9}$$

demand for energy is inelastic, as $|\varepsilon_{e,e}| < 1$.

Cross-price Elasticity of Demand for Non-energy Goods.

$$\varepsilon_{g,e} = \frac{\partial\, g}{\partial\, P_e} \frac{P_e}{g} = -(1-\omega_e)\, \frac{P_e\, \xi}{P_g\, g} \tag{1.B.10}$$

since $\varepsilon_{g,e} < 0$, g is a gross complement of e.

Income Elasticity of Relative Demand.

$$\varepsilon_{X,e/g} = \frac{\partial\, e/g}{\partial\, X} \frac{X}{e/g} = -\frac{P_e\, \xi}{X} \left(\frac{P_e\, \xi}{X}\right) \left(\frac{X}{X - P_e\, \xi}\right)^2 \tag{1.B.11}$$

therefore, as X falls, households consume more e relative to g.

Income Share of Energy. At equilibrium, since households spend all their income, we have

$$P_{g,t}\, g_t^u + P_{e,t}\, e_t^u = \Delta_t \tag{1.B.12}$$

$$P_{g,t}\, g_t^n + P_{e,t}\, e_t^n = W_t \tag{1.B.13}$$

therefore, given (1.B.6),

$$P_t\, c_t^u = \Delta_t - P_{e,t}\, \xi \tag{1.B.14}$$

$$P_t\, c_t^n = W_t - P_{e,t}\, \xi \tag{1.B.15}$$

hence,

$$P_{e,t}\, e_t^n = \omega_e\, W_t + (1-\omega_e)\, P_{e,t}\, \xi \tag{1.B.16}$$

$$P_{e,t}\, e_t^u = \omega_e\, \Delta_t + (1-\omega_e)\, P_{e,t}\, \xi \tag{1.B.17}$$

and

$$\frac{P_{e,t}\, e_t^n}{W_t} = \omega_e + (1-\omega_e)\, \frac{P_{e,t}\, \xi}{W_t} \tag{1.B.18}$$

$$\frac{P_{e,t}\,e^u_t}{\Delta_t} = \omega_e + (1-\omega_e)\,\frac{P_{e,t}\,\xi}{\Delta_t} \tag{1.B.19}$$

from which we see that since $\Delta_t < W_t$, the unemployed spend a higher share of their income on energy consumption.

1.B.3 Proof to Proposition 1.2

Since $\delta < w$,

$$\left(\frac{w - p_e\,\xi}{\delta - p_e\,\xi}\right) > 1 > \left(\frac{\delta - p_e\,\xi}{w - p_e\,\xi}\right)$$

therefore, since both $f < 1$ and $\lambda = \rho\,(1-f) < 1$,

$$(1-\lambda) + \lambda\left(\frac{w - p_e\,\xi}{\delta - p_e\,\xi}\right) > 1 > (1-f) + f\left(\frac{\delta - p_e\,\xi}{w - p_e\,\xi}\right)$$

implying, in turn, that

$$\begin{aligned} 1 &= \beta\left(\frac{1+i}{1+\pi}\right)\left[(1-\lambda) + \lambda\left(\frac{w - p_e\,\xi}{\delta - p_e\,\xi}\right)\right] \\ 1 &> \beta\left(\frac{1+i}{1+\pi}\right)\left[(1-f) + f\left(\frac{\delta - p_e\,\xi}{w - p_e\,\xi}\right)\right] \end{aligned}$$

1.B.4 Second-order Approximation of the Welfare Objective

When $\rho = 1$ and $\alpha = 0.5$,

$$\begin{aligned} U_t = {} & \ln(\delta_t - \xi\,p_{e,t}) + n_t\left[\ln(w_t - \xi\,p_{e,t}) - \ln(\delta_t - \xi\,p_{e,t})\right] \\ & + \nu\left\{\left[\left(\frac{1}{1-\gamma_e}\right)\frac{p_{g,t}}{\mathcal{D}_t} - \left(\frac{\gamma_e}{1-\gamma_e}\right)p_{e,t} - w_t\right]n_t - \kappa\,n_t^2 - \xi\,p_{e,t}\right\} \end{aligned}$$

and, at the constrained-efficient steady state,

$$\frac{\partial U}{\partial n} = \ln(w^* - \xi\,p_e) - \ln(\delta - \xi\,p_e) + \nu\left\{\left[\left(\frac{1}{1-\gamma_e}\right)p_g - \left(\frac{\gamma_e}{1-\gamma_e}\right)p_e - w^*\right] - 2\kappa\,n^*\right\} = 0\,.$$

In second-order approximation,

$$U_t \simeq -\nu\,\kappa\,\widehat{n}_t^2 + \frac{\partial^2 U}{\partial n\,\partial p_e}\,\widehat{p}_{e,t}\,\widehat{n}_t - \frac{\nu\,n\,p_g}{1-\gamma_e}\,(\mathcal{D}_t - 1) + \text{t.i.p.}$$

Noticing that, since derivatives are evaluated at the constrained-efficient steady state,

$$\frac{\partial^2 U}{\partial n\, \partial p_e}\, \widehat{p}_{e,t} = 2\, \kappa\, \nu\, \widehat{n}_t^*$$

and $\widehat{n}_t = \widehat{x}_t + \widehat{n}_t^*$, we have

$$U_t \simeq -\nu\, \kappa\, \widehat{x}_t^2 - \frac{\nu\, n\, p_g}{1-\gamma_e}\, (\mathcal{D}_t - 1) + \text{t.i.p.}$$

where

$$\mathcal{D}_t - 1 \simeq \ln(\mathcal{D}_t) = \ln \int_0^1 \left[\frac{P_t(k)}{P_{g,t}}\right]^{-\varepsilon} dk \simeq \frac{\varepsilon}{2}\, \text{var}_k \left\{\ln(P_t(k))\right\}$$

Lemma 1.1.

$$\sum_{t=0}^{\infty} \beta^t\, \text{var}_k \left\{\ln(P_t(k))\right\} = \frac{1}{\Theta} \sum_{t=0}^{\infty} \beta^t\, \pi_{g,t}^2 + t.i.p.$$

Proof. See chapter 6 in Woodford (2003). □

Therefore,

$$\mathbb{E}_t \sum_{j=0}^{\infty} U_{t+j} \simeq -\kappa\, \nu\, \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j\, \left(\widehat{x}_{t+j}^2 + \Omega\, \pi_{g,t+j}^2\right) + \text{t.i.p.} \tag{1.B.20}$$

with $\Omega := \frac{n\,\varepsilon}{(1-\gamma_e)}\, \frac{p_g}{2\,\kappa}\, /\Theta$.

1.B.5 Optimal Ramsey Policy

Approaching the problem under a *timeless perspective* (see Woodford, 2003), the first order conditions give

$$\left(\frac{n\,\varepsilon}{1-\gamma_e}\right) \pi_{g,t} + \widehat{x}_t - \widehat{x}_{t-1} = 0\,. \tag{1.B.21}$$

Combined with the NKPC in (1.58), this gives

$$\begin{aligned}\widehat{x}_t = \frac{\beta}{1+\beta+\Theta\, \frac{2\,\kappa}{p_g}\, \frac{n\,\varepsilon}{(1-\gamma_e)}}\, \mathbb{E}_t(\widehat{x}_{t+1}) + \frac{1}{1+\beta+\Theta\, \frac{2\,\kappa}{p_g}\, \frac{n\,\varepsilon}{(1-\gamma_e)}}\, \widehat{x}_{t-1} \\ - \frac{\Theta\, \frac{w}{p_g}\, \frac{n\,\varepsilon}{(1-\gamma_e)}}{1+\beta+\Omega\, \frac{2\,\kappa}{p_g}\, \frac{n\,\varepsilon}{(1-\gamma_e)}}\, \zeta\, \Psi \left(\frac{\Xi_w}{1-\Xi_w}\right) (1+\chi)\, \widetilde{p}_{e,t}\end{aligned}$$

whose unique stationary solution is

$$\widehat{x}_t = \eta\, \widehat{x}_{t-1} - \left(\frac{n\,\varepsilon}{1-\gamma_e}\right) \overline{\Upsilon}\, \zeta\, \Psi \left(\frac{\Xi_w}{1-\Xi_w}\right) (1+\chi)\, \widetilde{p}_{e,t} \tag{1.B.22}$$

where $\eta := \frac{1+\beta+\Theta\,\frac{2\,\kappa}{p_g}\,\frac{n\,\varepsilon}{(1-\gamma_e)}}{2\,\beta}\left[1-\sqrt{1-4\,\beta\,\left(1+\beta+\Theta\,\frac{2\,\kappa}{p_g}\,\frac{n\,\varepsilon}{(1-\gamma_e)}\right)^{-2}}\right] \in (0,1)$
and $\overline{\Upsilon} = \Theta\,\frac{w}{p_g}\,\left(1-\eta\,\beta+\Theta\,\frac{2\,\kappa}{p_g}\,\frac{n\,\varepsilon}{(1-\gamma_e)}\right)^{-1}$.

The paths for $\widehat{x}_{t+j}$ and $\pi_{g,t+j}$ can then be recovered by exploiting (1.B.21) and (1.B.22), imposing a steady-state initial condition (i.e. $\widehat{x}_{t-1} = 0$) and considering a one-off shock at t with persistence $\rho_e \to 1$.

Appendices to Chapter 2

2.A Derivations

2.A.1 Real Wage Rate Under Nash Bargaining

The notional wage, w_t^n, is assumed to be determined by standard Nash Bargaining, involving a constant surplus splitting scheme between workers and producers.

The asset value of a match for a firm with idiosyncratic productivity φ is given by $J_t(\varphi)$ as already expressed in (2.1). The asset value to active producers is then given by

$$J_t := \mathbb{E}_t \left[J_t(\varphi) \,|\, \varphi \geq \varphi_t^* \right] = a_t \, \overline{\varphi}_t - w_t + \beta \, \mathbb{E}_t \left[(1 - \rho_{t+1}) \, \frac{\kappa}{q_{t+1}} \right]$$

given free entry into vacancy posting, implying $V_t = 0$, this also coincides with the average surplus to active producers.

The asset value of employment is

$$W_t = w_t + \beta \, \mathbb{E}_t \left\{ (1 - \lambda_{t+1}) \, W_{t+1} + \lambda_{t+1} \, U_{t+1} \right\}$$

and the asset value of unemployment is

$$U_t = \delta + \beta \, \mathbb{E}_t \left\{ f_{t+1} \, W_{t+1} + (1 - f_{t+1}) \, U_{t+1} \right\}$$

therefore, the surplus enjoyed by an employed worker is

$$W_t - U_t = w_t - \delta + \beta \, \mathbb{E}_t \left[(1 - f_{t+1}) \, (1 - \rho_{t+1}) \, (W_{t+1} - U_{t+1}) \right]$$

and the aggregate surplus arising from all active matches is given by

$$n_t \, \left[(W_t - U_t) + J_t \right]$$

The aggregate surplus is collectively split by firm owners and workers, with a fraction η going to workers, and the remaining fraction $(1 - \eta)$ going to firm owners. This implies the following collective surplus splitting rule

$$W_t - U_t = \left(\frac{\eta}{1 - \eta} \right) \, J_t$$

which, combined with free entry, gives the following wage rate

From the FE condition we have

$$\frac{\kappa}{q_t} = J_t$$

where

$$J_t := \frac{1}{[1 - G(\varphi_t^*)]} \int_{\varphi_t^*}^{\infty} J_t(\varphi)\, g(\varphi)\, d\varphi$$

Combining the FE condition with the surplus splitting rule and led one period ahead, gives

$$(W_{t+1} - U_{t+1}) = \left(\frac{\eta}{1-\eta}\right) \frac{\kappa}{q_{t+1}} \tag{2.A.1}$$

Also, from the surplus splitting rule,

$$W_t - U_t = \left(\frac{\eta}{1-\eta}\right) J_t = \left(\frac{\eta}{1-\eta}\right) \left\{a_t\, \overline{\varphi}_t - w_t^n + \beta\, \mathbb{E}_t \left[(1 - \rho_{t+1})\, \frac{\kappa}{q_{t+1}}\right]\right\} \tag{2.A.2}$$

Using (2.A.1) and (2.A.2) into the recursion for worker surplus,

$$W_t - U_t = w_t^n - \delta + \beta\, \mathbb{E}_t \left[(1 - f_{t+1})\, (1 - \rho_{t+1})\, (W_{t+1} - U_{t+1})\right]$$

gives

$$\left(\frac{\eta}{1-\eta}\right) \left\{a_t\, \overline{\varphi}_t - w_t^n + \beta\, \mathbb{E}_t \left[(1 - \rho_{t+1})\, \frac{\kappa}{q_{t+1}}\right]\right\}$$
$$= w_t^n - \delta + \beta\, \mathbb{E}_t \left[(1 - f_{t+1})\, (1 - \rho_{t+1}) \left(\frac{\eta}{1-\eta}\right) \frac{\kappa}{q_{t+1}}\right]$$

which gives, upon rearranging and since $f_{t+1} = \vartheta_{t+1}\, q_{t+1}$, the wage rate as expressed in (2.13).

2.A.2 Real Wage Rigidity

In the log-linear model, there is a strictly increasing mapping between the wage norm parameter Γ and the wage elasticity χ.

$$\chi^E = \frac{1}{w} \left\{ \frac{\alpha + \left[\frac{\beta\, \gamma_a\, f\,(1-\rho)}{1-\beta\, \gamma_a\,(1-\rho)}\right] - \tau \left(\frac{1-\varphi^*}{\varphi^*}\right) \left[\frac{\alpha + (1-\alpha)\, \beta\, f\, \gamma_a\,(1-\rho)}{1-\beta\, \gamma_a\,(1-\rho)}\right] \overline{X}_c^*}{\frac{\alpha/\eta}{\Gamma} + \left[\frac{\beta\, \gamma_a\, f\,(1-\rho)}{1-\beta\, \gamma_a\,(1-\rho)}\right] - \tau \left(\frac{1-\varphi^*}{\varphi^*}\right) \left[\frac{\alpha + (1-\alpha)\, \beta\, f\, \gamma_a\,(1-\rho)}{1-\beta\, \gamma_a\,(1-\rho)}\right]} \right\}$$

and

$$\chi^X = \frac{1}{w} \left\{ \frac{\alpha + \left[\frac{\beta\, \gamma_a\, f\,(1-\rho)}{1-\beta\, \gamma_a\,(1-\rho)}\right]}{\frac{\alpha/\eta}{\Gamma} + \left[\frac{\beta\, \gamma_a\, f\,(1-\rho)}{1-\beta\, \gamma_a\,(1-\rho)}\right]} \right\}$$

in the endogeonous and exogenous separation cases, respectively, where $\tau := \varphi^* \, g(\varphi^*)/[1-G(\varphi^*)]$ and $\overline{\chi}_c^* = \varphi^* + (1-\varphi^*)\,\beta\,\gamma_a\,(1-\rho)$.

When $\Gamma = 0$, $\chi = 0$. When $\Gamma = 1$, χ is equal to (hence bounded above by)

$$\overline{\chi}_w^E = \frac{1}{w}\left\{\frac{\alpha + \left[\frac{\beta\,\gamma_a\,f\,(1-\rho)}{1-\beta\,\gamma_a\,(1-\rho)}\right] - \tau\,\left(\frac{1-\varphi^*}{\varphi^*}\right)\,\left[\frac{\alpha+(1-\alpha)\,\beta\,f\,\gamma_a\,(1-\rho)}{1-\beta\,\gamma_a\,(1-\rho)}\right]\,\overline{\chi}_c^*}{\frac{\alpha}{\eta} + \left[\frac{\beta\,\gamma_a\,f\,(1-\rho)}{1-\beta\,\gamma_a\,(1-\rho)}\right] - \tau\,\left(\frac{1-\varphi^*}{\varphi^*}\right)\,\left[\frac{\alpha+(1-\alpha)\,\beta\,f\,\gamma_a\,(1-\rho)}{1-\beta\,\gamma_a\,(1-\rho)}\right]}\right\}$$

in the endogenous separation case, and

$$\overline{\chi}_w^X = \frac{1}{w}\left\{\frac{\alpha + \left[\frac{\beta\,\gamma_a\,f\,(1-\rho)}{1-\beta\,\gamma_a\,(1-\rho)}\right]}{\frac{\alpha}{\eta} + \left[\frac{\beta\,\gamma_a\,f\,(1-\rho)}{1-\beta\,\gamma_a\,(1-\rho)}\right]}\right\}$$

in the exogenous separation case.

2.B Proofs

2.B.1 Proof to Proposition 2.1

Starting from the Euler condition of employed workers

$$\frac{1}{\beta} = \left[\frac{(1-\lambda)\,u'\,(w) + \lambda\,u'\,(\delta)}{u'\,(w)}\right]\,R$$

since $\lambda \in (0,1)$, and $\delta < w$ implies in turn $u'(\delta) > u'(w)$, the term in square brackets is

$$\frac{(1-\lambda)\,u'\,(w) + \lambda\,u'\,(\delta)}{u'\,(w)} = (1-\lambda) + \lambda\,\frac{u'\,(\delta)}{u'\,(w)} > 1$$

giving in turn that

$$\frac{1}{\beta} > R$$

Since $f \in (0,1)$,

$$\frac{(1-f)\,u'\,(\delta) + f\,u'\,(w)}{u'\,(\delta)} = (1-f) + f\,\frac{u'\,(w)}{u'\,(\delta)} < 1 < \frac{(1-\lambda)\,u'\,(w) + \lambda\,u'\,(\delta)}{u'\,(w)}$$

implying, in turn,

$$\left[\frac{(1-f)\,u'\,(\delta) + f\,u'\,(w)}{u'\,(\delta)}\right]\,R < \left[\frac{(1-\lambda)\,u'\,(w) + \lambda\,u'\,(\delta)}{u'\,(w)}\right]\,R = \frac{1}{\beta}$$

which establishes that the Euler condition for unemployed workers holds indeed with inequality.

2.B.2 Proof to Proposition 2.2

Let $\Omega := \frac{u'(c^n)}{u'(c^u)}$ be a measure of consumption inequality at steady state. If the Euler conditions of the employed and the unemployed both hold with equality, these would read, respectively

$$1 = \beta \left[(1-\lambda) + \lambda \frac{1}{\Omega}\right]$$
$$1 = \beta \left[f\,\Omega + (1-f)\right]$$

Therefore,

$$(1-\lambda) + \lambda \frac{1}{\Omega} = f\,\Omega + (1-f)$$

But this necessarily implies that $\Omega = 1$, i.e. consumption is equalised. Indeed, suppose that $\Omega > 1$, then since $\lambda, f \in (0,1)$,

$$f\,\Omega + (1-f) > (1-\lambda) + \lambda \frac{1}{\Omega}\,.$$

Similarly, if instead $\Omega < 1$,

$$f\,\Omega + (1-f) < (1-\lambda) + \lambda \frac{1}{\Omega}\,.$$

2.B.3 Proof to Proposition 2.3

Starting from the Free Entry Condition,

$$\frac{\kappa}{q} = \left[1 - w + \beta\,(1-\rho)\,\frac{\kappa}{q}\right]$$

we have

$$\frac{\kappa}{q} = \left[\frac{1}{1-\beta\,(1-\rho)}\right]\,[1-w]$$

From the wage equation,

$$w = \eta + \eta\,\kappa\,\beta\,\vartheta\,(1-\rho) + (1-\eta)\,\delta$$

where $\kappa\,\vartheta = \frac{\kappa}{q}\,f$. Therefore,

$$w = \eta + \eta\,\frac{\kappa}{q}\,f\,(1-\rho) + (1-\eta)\,\delta$$

Letting $\zeta := 1 - \delta/w$, we have $\delta = (1-\zeta)\, w$, hence

$$w = \eta + \eta\,\frac{\kappa}{q}\, f\,(1-\rho) + (1-\eta)\,(1-\zeta)\, w$$

Exploiting the result from the FE condition,

$$w = \eta + \eta\,\left[\frac{\beta\, f\,(1-\rho)}{1-\beta\,(1-\rho)}\right]\,[1-w] + (1-\eta)\,(1-\zeta)\, w$$

Solving for w,

$$w = \frac{\left[1 + \frac{\beta\, f\,(1-\rho)}{1-\beta\,(1-\rho)}\right]}{\left[\left(\frac{1-\eta}{\eta}\right)\,\zeta + 1 + \frac{\beta\, f\,(1-\rho)}{1-\beta\,(1-\rho)}\right]} < 1$$

Turning now to φ^*, combing the Job Destruction condition with the previous result on κ/q from FE, we have

$$\varphi^* = w - \beta\,(1-\rho)\,\frac{\kappa}{q} = w - \left[\frac{\beta\,(1-\rho)}{1-\beta\,(1-\rho)}\right]\,[1-w] = w\,\left[\frac{1}{1-\beta\,(1-\rho)}\right] - \left[\frac{\beta\,(1-\rho)}{1-\beta\,(1-\rho)}\right] < w$$

where the inequality holds in virtue of the fact that $w < 1$.

2.C The Producer Side with Nominal Price Rigidity

Production consists of three layers: a final (consumption) good is assembled with CES technology, by aggregating differentiated goods from a monopolistically competitive wholesale sector; each wholesale producer uses in turn, as input, a homogeneous intermediate good; this latter is produced by firms differing in their productivity, using only labor, which is hired in a frictional labor market.[8]

Final Good

The final good is produced under perfect competition, by aggregating a continuum of wholesale goods with CES technology

$$Y_t = \left(\int_0^1 y_t(k)^{\frac{\varepsilon-1}{\varepsilon}}\, dk\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

[8]Krause and Lubik (2007) embed the price setting and employment adjustment decisions within a single, representative monopolistically competitive producer. By contrast, Trigari (2009) separates price setting decisions and employment adjustment decisions into two distinct sectors.

Given the CES structure, demand for variety k from the wholesale sector is given by

$$y_t(k) = Y_t \left[\frac{p_t(k)}{P_t}\right]^{-\varepsilon}$$

where, given perfect competition in the final good sector, $P_t = \left(\int_0^1 p_t(k)^{1-\varepsilon} dk\right)^{1/(1-\varepsilon)}$.

Wholesale Firms

Wholesale producers turn intermediate goods into wholesale differentiated products according to a simple linear technology with symmetric productivity

$$y_t(k) = s_t(k)$$

where $y_t(k)$ is the amount of variety k produced, and $s_t(k)$ is the amount of intermediate inputs used in production by wholesaler k.

Each wholesaler is a monopolistic supplier of the variety k is produces. The profit of wholesale firm k is, therefore,

$$D_t^W(k) = y_t(k)\ [p_t(k) - X_t] = Y_t \left[\frac{p_t(k)}{P_t}\right]^{-\varepsilon} [p_t(k) - X_t]$$

where X_t is the nominal marginal cost faced by wholesale firms (i.e. the price of the intermediate goods used in production).

Moreover, wholesale firms are assumed to face Calvo pricing frictions, with ω being the probability that a wholesale firm cannot reset its price.

Therefore, the pricing problem solved by wholesale firm k at time t, determining its optimal reset price, is

$$\max_{p_t(k)} \mathbb{E}_t \left\{\sum_{j=0}^{\infty} \omega^j\, \beta^j \left(\frac{P_t}{P_{t+j}}\right) Y_{t+j} \left[\frac{p_t(k)}{P_{t+j}}\right]^{-\varepsilon} [p_t(k) - X_{t+j}]\right\}$$

Also, I assume that an optimal subsidy to wholesalers is in place, so that the steady state distortion of monopolistic competition is offset, and the rigid prices model delivers the same (zero inflation) steady state already outlined in table 2.3.

Intermediate Good Producers

Production in the intermediate goods sector occurs using only labor as input, which is hired in a frictional labor market. The value of a match for a producer with idiosyncratic productivity φ is

$$J_t(\varphi) = x_t\, a_t\, \varphi - w_t + \beta\, \mathbb{E}_t \left[(1-\rho^x) \int_{\varphi^*_{t+1}}^{\infty} J_{t+1}(\varphi)\, g(\varphi)\, d\varphi \right]$$

2.D Local Determinacy Properties

Exogenous Job Destruction

Under the simplifying assumption that $\phi_\pi = 1/\beta$, we get the following forward looking equation for the tightness gap, Θ^X_t,

$$\Theta^X_t = H^X\, \mathbb{E}_t \left(\Theta^X_{t+1}\right) + \left(\frac{1}{\kappa/q}\right) \left(\frac{\beta}{\alpha}\right) \frac{1}{\Omega}\, \hat{r}^f_t$$

hence determinacy requires

$$H^X = \beta\,(1-\rho) + \left(\frac{1}{\kappa/q}\right) \left[\frac{\rho\,(1-f)\,\zeta}{(1-\zeta) + \rho\,(1-f)\,\zeta}\right] \left(\frac{f}{1-f}\right) \left(\frac{1-\alpha}{\alpha}\right) \frac{1}{\Omega} < 1$$

which implies in turn $\mathcal{D}^X > 0$ as $\gamma_a < 1$.

Endogenous Job Destruction

In the endogenous job destruction case, the model can be expressed in the following matrix form

$$\mathbf{A} \begin{bmatrix} \Theta^E_t \\ \\ \Phi_t \end{bmatrix} = \mathbf{B} \begin{bmatrix} \mathbb{E}_t \left(\Theta^E_{t+1}\right) \\ \\ \mathbb{E}_t \left(\Phi_{t+1}\right) \end{bmatrix} + \begin{bmatrix} 1 \\ \\ 0 \end{bmatrix} \hat{r}^f_t$$

where

$$\mathbf{A} = \begin{bmatrix} \frac{\alpha}{\beta}\,\Omega & \frac{1}{\beta}\,\Omega \left(\frac{1}{1-\varphi^*}\right) [1-\tau\,(1-\varphi^*)] \\ & \\ & \\ \alpha & \left(\frac{1}{1-\varphi^*}\right) [1-\tau\,(1-\varphi^*)] \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \left[\frac{\rho\, f\, \zeta}{(1-\zeta)+\rho\,(1-f)\,\zeta}\right] \left(\frac{1-\alpha}{\alpha}\right) & -\left[\frac{\rho\,(1-f)\,\zeta}{(1-\zeta)+\rho\,(1-f)\,\zeta}\right] \tau \left(\frac{1-\rho}{\rho}\right) \\ \\ -\left(\frac{1-\varphi^*}{\varphi^*}\right) \beta\,(1-\rho)\,\alpha & \tau \left(\frac{1-\varphi^*}{\varphi^*}\right) \beta\,(1-\rho) \end{bmatrix}$$

Determinacy then requires that the matrix $\mathbf{A}^{-1}\mathbf{B}$ has both eigenvalues within the unit circle. This gives the following parameter restriction

$$\frac{\Omega}{\beta} - \left[\frac{1}{1-\beta\,(1-\rho)}\right] \left[\frac{\rho\, f\, \zeta}{(1-\zeta)+\rho\,(1-f)\,\zeta}\right] \left(\frac{1-\alpha}{\alpha}\right) \left(\frac{1}{1-\varphi^*}\right) \left[1-\tau\left(\frac{1-\varphi^*}{\varphi^*}\right) \widetilde{\chi}_c^*\right] \\ - \left[\frac{1}{1-\beta\,(1-\rho)}\right] \left[\frac{\rho\,(1-f)\,\zeta}{(1-\zeta)+\rho\,(1-f)\,\zeta}\right] \tau \left(\frac{1-\rho}{\rho}\right) \left(\frac{1}{\varphi^*}\right) \widetilde{\chi}_c^* > 0$$

where $\widetilde{\chi}_c^* = \varphi^* + (1-\varphi^*)\,\beta\,(1-\rho)$. Then, as $\gamma_a < 1$, the parameter restriction above implies in turn that $\mathcal{D}^E > 0$.

2.E The Non-zero Liquidity Model

The non-zero liquidity model entails two main extensions, compared to the baseline, zero liquidity model. First, it is assumed that households face a non-zero borrowing limit, i.e. the borrowing constraint becomes $b_{ht} \geq -\underline{b}$, with the zero borrowing (and zero liquidity equilibrium) case nested by $\underline{b} = 0$. Furthermore, it is assumed that there is perfect risk sharing among the employed, but not between the employed and the unemployed, as $\underline{b}$ is tighter than the natural borrowing limit.

As argued in detail by Challe et al. (2017), such non-zero liquidity economy still retains tractability as workers, once becoming unemployed, start liquidating the assets they have set aside for self-insurance motives when employed, eventually facing a binding borrowing constraint in a finite number of subsequent periods of unemployment. Challe et al. (2017) favor a specification, supported by the data, where the borrowing constraint is hit immediately in the first period of unemployment, hence unemployed workers remain liquidity-constrained thereafter, as long as they do not find a job. In such equilibrium, there are three types of workers, differing in their consumption levels: i) the employed, whose consumption level is denoted by c^n in equation (2.E.2), precautionarily hold the amount of assets b_t, and earn interest payments on those assets that are left after the fraction $\lambda_t\, n_{t-1}$ of previously employed workers becomes jobless —bringing with them the assets they set aside for self-insurance— and the fraction $f_t\,(1-n_{t-1})$ of previously unemployed workers joins with their outstanding debt the pool of mutually insured employed workers; ii) the short-term unemployed, who were employed in the previous period and still retain the assets that were precautionarily set aside, and whose consumption level is denoted by c^{nu} in equation (2.E.3); iii) the long-term unemployed, whose consumption level is denoted by c^{uu} in equation (2.E.4), have already liquidated all their assets and remain liquidity-constrained as long as they do not find a job, hence having to pay interests on their outstanding debt from the previous period.

$$1 = \beta\, \mathbb{E}_t \left[\frac{(1-\lambda_{t+1})\, u'\left(c^n_{t+1}\right) + \lambda_{t+1}\, u'\left(c^{nu}_{t+1}\right)}{u'\left(c^n_t\right)}\, R_t \right] \tag{2.E.1}$$

$$c^n_t = w_t - b_t + R_{t-1}\, b^n_{t-1} \tag{2.E.2}$$

$$c^{nu}_t = \delta + \underline{b} + R_{t-1}\, b_{t-1} \tag{2.E.3}$$

$$c^{uu}_t = \delta + \underline{b} - R_{t-1}\, \underline{b} \tag{2.E.4}$$

$$n_t\, b_t = (1-n_t)\, \underline{b} \tag{2.E.5}$$

$$n_t\, b^n_{t-1} = (1-\lambda_t)\; n_{t-1}\, b_{t-1} - f_t\, (1-n_{t-1})\, \underline{b} \tag{2.E.6}$$

In an equilibrium with liquidity-constrained (short- and long-term) unemployed workers, it must also be the case that

$$b_t > 0 \tag{2.E.7}$$

$$1 > \beta\, \mathbb{E}_t \left[\frac{f_{t+1}\, u'\left(c^n_{t+1}\right) + (1-f_{t+1})\, u'\left(c^{uu}_{t+1}\right)}{u'\left(c^{nu}_t\right)}\, R_t \right] \tag{2.E.8}$$

$$1 > \beta\, \mathbb{E}_t \left[\frac{f_{t+1}\, u'\left(c^n_{t+1}\right) + (1-f_{t+1})\, u'\left(c^{uu}_{t+1}\right)}{u'\left(c^{uu}_t\right)}\, R_t \right] \tag{2.E.9}$$

2.E.1 Calibration

At steady state,

$$c^n = w - b + R\, b^n \tag{2.E.10}$$

$$c^{nu} = \delta + \underline{b} + R\, b \tag{2.E.11}$$

$$c^{uu} = \delta + \underline{b} - R\, \underline{b} \tag{2.E.12}$$

$$n\, b = (1-n)\, \underline{b} \tag{2.E.13}$$

$$b^n = (1-\rho)\, (1-f)\, b \tag{2.E.14}$$

Therefore, there are two additional parameters that need to be (re)calibrated in the non-zero liquidity model: the borrowing limit $\underline{b}$ and the steady state replacement ratio $\delta_w := \delta/w$. In order to nest the zero liquidity model as a special case of the non-zero liquidity model, it is assumed that the average consumption loss upon unemployment, ζ, is the same in the two models. In the zero liquidity model, this maps exactly to the replacement ratio:

$$\frac{c^{u,0}}{c^{n,0}} = \frac{\delta}{w} = 1 - \zeta\,.$$

In the non-zero liquidity model, we have instead

$$\frac{c^u}{c^n} = \frac{\delta_w\, w + [1 - R\,(1-\rho)\,(1-f)]\,\underline{b}}{w - [1 - R\,(1-\rho)\,(1-f)]\,b} = 1 - \zeta$$

where $c^u := \frac{\lambda\, n}{1-n}\, c^{nu} + \left(1 - \frac{\lambda\, n}{1-n}\right)\, c^u = f\, c^{nu} + (1-f)\, c^{uu}$ is the average consumption level of the unemployed at steady state. Exploiting the fact that $b = \frac{1-n}{n}\, \underline{b} = \frac{\lambda}{f}\, \underline{b}$, we then have

$$\underline{b} = w\, \frac{(1-\zeta) - \delta_w}{[1 - R\,(1-\rho)\,(1-f)]\, \left[1 + (1-\zeta)\, \frac{\lambda}{f}\right]}\,. \tag{2.E.15}$$

δ_w (and consequently $\underline{b}$) is then chosen to amount to the minimal value consistent with having liquidity-constrained unemployed households, i.e. the euler conditions in (2.E.8) and (2.E.9) holding both with inequality. Given the baseline calibration, this corresponds to $\delta_w = 0.793$ and $\underline{b}/w = 0.03$, i.e. the unemployed can borrow up to $\underline{b}/\delta = 3.8\%$ in excess of their income. The zero liquidity case is then nested by $\delta_w = 1 - \zeta$, which implies in turn $\underline{b} = 0$ in (2.E.15).

Appendices to Chapter 3

3.A Output-targeting Taylor Rule

The Taylor rule responding to output instead of unemployment fluctuations is, in loglinear form,

$$i_t = \max\{0\,,\, \phi_\pi\, \pi_t + \phi_+\, \widehat{y}_t\} \tag{3.A.1}$$

The recession state solutions are the same as those outlined in Section 3.3. What changes is the ZLB condition to be checked in the recession state, now reading

$$\phi_\pi\, \pi_\xi + \phi_y\, \widehat{y}_0 \leq 0$$

where $\widehat{y}_0 < 0$.

3.A.1 Fully rigid wages

Exogenous Separation

With endogenous separation, the log deviation of output at $t = 0$ is

$$\widehat{y}_0 = \widehat{z}_\xi + \widehat{n}_0$$

where $\widehat{z}_\xi = -\xi$ and $\widehat{n}_0 = -\frac{u}{1-u}\, \widehat{u}_0$. Hence, given the solution described in Section 3.3,

$$\widehat{y}_0 = -\xi - \rho\, F_0^X\, \xi$$

Proposition 3.5. *In the exogenous job destruction case, a recession at the ZLB is consistent with positive inflation provided that*

$$\phi_y \geq \underline{\phi}_y^{X,0}$$

where

$$\underline{\phi}_y^{X,0} := \phi_\pi\, (1-\beta)\, \left(\frac{f}{1-f}\right) \left(\frac{F_0^X}{1+\rho\, F_0^X}\right)$$

Endogenous Separation

With endogenous separation, the log deviation of output at $t = 0$ is

$$\widehat{y}_0 = \widehat{z}_\xi + \widehat{n}_0 + \widehat{\overline{\varphi}}_\xi$$

where $\overline{\varphi}_\xi = \mathbb{E}(\varphi|\varphi \geq \varphi^*_\xi)$ is the average idiosyncratic productivity of active matches in the recession state: as less productive matches are destroyed, average idiosyncratic productivity is higher, and its approximate log deviation relative to the steady state is given by $\widehat{\overline{\varphi}}_\xi = (1-\varphi^*)\left(\frac{\rho}{1-\rho}\right)\widehat{\rho}_\xi$. Given the solution described in Section 3.3,

$$\widehat{y}_0 = -\xi - \rho\left[(1-f) + H^E_0\right] D^E_0\,\xi + (1-\varphi^*)\left(\frac{\rho}{1-\rho}\right) D^E_0\,\xi$$

Proposition 3.6. *In the endogenous job destruction case, a recession at the ZLB is consistent with positive inflation provided that*

$$\phi_y \geq \underline{\phi}^{E,0}_y$$

where

$$\underline{\phi}^{E,0}_y := \phi_\pi\,(1-\beta)\left(\frac{f}{1-f}\right)\left\{\frac{\left(\frac{1-f}{f}\right) + H^E_0}{\frac{1}{D^E_0} + \rho\left[(1-f) + H^E_0 - \left(\frac{1-\varphi^*}{1-\rho}\right)\right]}\right\}$$

3.A.2 Inertial wages

Exogenous Separation

With exogenous separation and inertial wages, the log deviation of output at $t = 0$ is

$$\widehat{y}_0 = \widehat{z}_0 + \widehat{n}_0 = -\xi - \rho\,F^X_\chi\,\xi$$

Proposition 3.7. *In the exogenous job destruction case with partially flexible wages, a recession at the ZLB is consistent with positive inflation provided that*

$$\phi_y \geq \underline{\phi}^X_y \geq 0$$

where

$$\underline{\phi}^X_y := \phi_\pi\,(1-\beta)\left(\frac{f}{1-f}\right)\left(\frac{F^X_\chi}{1+\rho\,F^X_\chi}\right) - \phi_\pi\,\chi\left(\frac{1}{1+\rho\,F^X_\chi}\right)\left[\frac{1}{\psi} - \beta\,[1-\rho(1-f)]\right]$$

Endogenous Separation

With endogenous separation and inertial wages, the log deviation of output at $t = 0$ is

$$\begin{aligned}
\widehat{y}_0 &= \widehat{z}_0 + \widehat{n}_0 + \widehat{\widehat{\varphi}}_0 \\
&= -\xi - \rho\left[(1-f) + H_0^E + \chi\left(\frac{1-\alpha}{\alpha}\right) C_\varphi\right] D_\chi^E\, \xi + (1-\varphi^*)\left(\frac{\rho}{1-\rho}\right) D_\chi^E\, \xi
\end{aligned}$$

Proposition 3.8. *In the endogenous job destruction case with partially flexible wages, a recession at the ZLB is consistent with positive inflation provided that*

$$\phi_y \geq \underline{\phi}_y^E \geq 0$$

where

$$\begin{aligned}
\underline{\phi}_y^E := \phi_\pi\,(1-\beta)\left(\frac{f}{1-f}\right) &\left\{\frac{\left(\frac{1-f}{f}\right) + H_0^E + \chi\left(\frac{1-\alpha}{\alpha}\right) C_\varphi}{\frac{1}{D_\chi^E} + \rho\left[(1-f) + H_0^E + \chi\left(\frac{1-\alpha}{\alpha}\right) C_\varphi - \left(\frac{1-\varphi^*}{1-\rho}\right)\right]}\right\} \\
&- \phi_\pi\,\chi\left[\frac{1}{1+\rho\left[(1-f)\,D_\chi^E + F_\chi^E - \left(\frac{1-\varphi^*}{1-\rho}\right)\right]}\right]\left[\frac{1}{\psi} - \beta\,[1-\rho(1-f)]\right]
\end{aligned}$$