HEURISTICS AND BIASES IN INVESTMENT DECISION MAKING PROCESSES

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ABSTRACT

"Expectations of market participants have a real effect on the object of these expectations."

John Maynard Keynes

The General Theory of Employment, Interest and Money

In the Rational Beliefs theory developed by Mordecai Kurz, Andrea Beltratti and Maurizio Motolese, the distribution of beliefs in the market is the most important propagation mechanism of economic volatility. They argue that, most of the observed volatility in financial markets is generated by the agents' beliefs and that, several market puzzles, such as the equity premium puzzle, are all driven by the structure of market expectations.

The common economic explanation for these phenomena is the existence of heterogeneous agents with diverse but correlated beliefs. We started our analysis by studying the agents' heterogeneity, drawing inspiration from the Rational Belief Theory by Mordecai Kurz. We observed that, the simulative approach succeeded in reproducing the USA stock markets data but that it was inadequate in describing the psychological determinants of the agents' heterogeneity.

Since we were interested in better understanding the investors' behaviours, we chose to change the research perspective by adopting an experimental approach. We applied psychology and experimental economics to understanding the most representative heuristics and biases driving human behaviours in economic decision making processes.

We focused our analysis on two fundamental aspects for the human decision making processes: the information processing and the recalling activity.

We also analyse the hindsight bias and its effects on learning processes in economics and on several biases, related to the concept of bounded memory.

Our aim is to find psychological insights for the subjects' behaviours in order to better understand the origin of their heterogeneity. The empirical evidence we obtain during the tests allows us to recognise the existence of several biases and to hypothesise their effects in the market dynamics.

We tried to verify the model by Kurz, taking into account the subject's psychological profiles and his behavioural tendencies in updating his beliefs and taking decisions. We were interestingly surprised by the data we gathered and by the behavioural dynamics we observed. We recognised that the agents' psychological characteristics could not be directly applied to Kurz's rational beliefs model, since it was just designed to produce simulations results based only on economic variables.

By using an "ad hoc" model we adopted a different approach: we observed and described the way subjects updated their beliefs whenever they were affected by hindsight bias. Besides, we followed Andrea Wilson's bounded memory model to illustrate the most frequent biases and we hypothesised some calibrations on that model by using our tests results.

Our research showed a definite heterogeneity among individuals, which partly depends on the fact that experiments are artificial tests carried out in a laboratory, so subjects react to this environment in different ways, for example in term of motivations and aims. However, and this is the main important aspect, this heterogeneity also highlights differences in people's beliefs, depending on each person's characteristics, knowledge, ideas, and experience.

> *"If one had to speculate about the future, it would probably be safer to project a continuation of investors psychological reactions than to predict the exact financial performance of companies themselves."*
>
> *Amos Tversky*

Investors and managers, who make an effort in studying the economy fundamentals, should make the same effort in investigating the way people react to information and update their beliefs. Accordingly to this perspective, we projected our research to verify the existence of common paths between the subject's behaviours, and following Kurz's rational belief theory, by paying much more attention to the source of the "endogenous uncertainty", the "endogenous heterogeneity", considered the most responsible element of market volatility.

When Expectations Really Matter

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ABSTRACT

In the Rational Beliefs theory developed by Mordecai Kurz, Andrea Beltratti and Maurizio Motolese, the distribution of beliefs in the market is the most important propagation mechanism of economic volatility; it argues that most of the observed volatility in financial markets is generated by the agents' beliefs and that several market puzzles, such as the equity premium puzzle, are all driven by the structure of market expectations.

The common economic explanation for these phenomena is the existence of heterogeneous agents with diverse but correlated beliefs. Kurz was the first to introduce the concept of "heterogeneity" among economic agents. We drew inspiration from this specific theory, and applied behavioural economics and cognitive sciences to our research. We also followed Andrea Wilson's bounded memory model to illustrate the most frequent biases and we finally hypothesised some calibrations on this model by using our tests results.

INTRODUCTION

The use of rational beliefs and expectations is of paramount importance in economics. Many earlier economists, A. C. Pigou, John Maynard Keynes, John R. Hicks, and Robert E. Lucas attributed a critical role to people's beliefs in the determination of the business cycle. Keynes referred to business cycles as "waves of optimism and pessimism" that helped to determine the level of economic activity. Lucas (1972) developed what he defined as the "rational expectations hypothesis", in which economic agents used available information in the best way.

In this paper we relate Mordecai Kurz's Rational Beliefs Theory with the psychological literature dealing with heuristics and biases.

The Heterogeneous Beliefs Paradigm

The Rational Beliefs Theory was motivated by the empirical observation that intelligent economic agents may hold diverse beliefs even when there is no difference in the information at their disposal. This means that disagreement may simply arise because of the various interpretations of this information. By adopting axioms which allow rational agents to hold different and contrasting beliefs, Kurz's theory does not lead, in general, to a Rational Expectations Equilibrium (REE). However, since the theory of RBE is an extension of the theory of REE, REE can be considered a particular case of RB.

Kurz found ample empirical evidence to support the view that equally informed agents differently interpret the same information and that the heterogeneity of beliefs persists regardless of the amount of past information available. This implies that agents develop different probability beliefs even if they are conditioned by the same available information.

Rational Expectations Model versus Rational Beliefs Theory

One of the crucial question is the following: can heterogeneity be justified by the diversity of information rather than by the diversity of beliefs?

Some significant studies on REE hold that the observed heterogeneity does not arise from the heterogeneity of prior probabilities but, rather, from the diversity of private information.

We retain that this explanation is unsatisfactory both from theoretical and empirical perspectives.

Indeed, theoretical considerations lead to the information revelation of REE, which implies that prices make public all private information and therefore the introduction of asymmetric information, by itself, is not sufficient. It simply transforms the problem into other paradoxes, including the problem of explaining why under REE agents trade at all (e.g. Milgrom and Stokey [1982]); why asset prices fluctuate more than could be explained by "fundamentals" (e.g. Shiller [1981]), indirectly generating an equity premium puzzle (see M&P [1985]); and why any resources are ever used for the production of information (see Grossman and Stiglitz [1980]).

Besides, empirical considerations suggest that the assumption of asymmetric information in financial markets is unsatisfactory. We have already noted the ample empirical evidence in support of the opposite view that, equally informed agents interpret differently the same information.

The arguments presented here highlight the fact that the implications of the common belief assumption in an REE - by itself - are counter factual.

The crucial empirical implications of these models are generated by an added set of assumptions. These include asymmetry of information, rigidity in the transmission of information and outright irrational behaviour of some agents.

These added assumptions introduce "stories" with questionable theoretical and empirical foundations, but these questionable assumptions are the ones which drive the results.

The theory of RB provides the foundation for the use of heterogeneous beliefs as a substitute for the "additional" artificial assumptions which drive most of the REE-based models. The RB theory suggests that the paradigm of diverse beliefs is entirely plausible and generates a powerful propagation mechanism of social risk and market fluctuations.

The paradigm is based on the hypothesis that agents do not know the true structural relationships in the economy. Consequently, rational agents may have contrasting beliefs about what they do not know.

THE RATIONAL BELIEFS ASSUMPTIONS

The central assumption of the RB theory (Kurz, 1994) is that economic agents do not know the exact demand or supply functions, equilibrium maps or true probability laws induced by an equilibrium.

In Kurz's own words, agents do not possess "structural knowledge" (as distinct from "empirical knowledge" or "information"). As they lack structural knowledge, rational agents develop their own theories about the underlying structure and use the available data to test the validity of such theories.

The second assumption refers to the concept of "uncertainty": in Kurz's theory, the economy uncertainty depends on the distribution and on the correlation of the agents' beliefs, the so-called "Endogenous Uncertainty"; the uncertainty about endogenous variables which is not resolved by the knowledge of the exogenous ones. Endogenous uncertainty becomes the most important factor explaining market volatility.

The third assumption which distinguishes the RB theory from the Bayesian perspective is that at each date, an economic agent has at his disposal a vast amount of data about the past performance of the economy, therefore the agent's central point of reference is the empirical distribution derived from the frequency at which events occurred in the past.

The availability of a large amount of past data may lead to speculate that the agents' learning processes may cause heterogeneity of beliefs to disappear; empirical data show that heterogeneity persists over time, therefore, the conclusion of the debate under the heading of "Bayes Consistency" is that the convergence of the posterior to the true distribution is a rare occurrence.

The problem is compounded in learning situations in markets where the data is generated by an unknown process which may be non-stationary and the convergence of the posterior to the true probability is even a less likely event.

The fourth assumption of the RB theory is the observation that the economic life of any agent is short relative to the time at which new data arrives.

What may an economic agent believe ?

REE vs. RB

Information and Knowledge In Rational Expectations model	**Information and Knowledge in Rational Beliefs model**
Agents know: • the structural knowledge; • the equilibrium map between variables; • the true equilibrium probability; distribution of all variables; • the conditional perfect foresight.	Agents know: • the empirical knowledge; • no Structural knowledge; • the long term empirical joint distribution of the observed variables.

The "Uncertainty" concept in the two theories	
"Uncertainty" in Rational Expectations model The uncertainty comes from exogenous shocks. • The expectations do not matter at all: they are simply "Parameters".	**"Uncertainty" in Rational Beliefs model** The uncertainty depends on the distribution and the correlation of beliefs • Endogenous Uncertainty: it is the uncertainty about the endogenous variables not resolved by the knowledge of the exogenous ones. • Endogenous Uncertainty: it is caused by the agents'

	mistakes; wrong beliefs about true probability law generated by equilibrium. • Contrasting beliefs generate a propagation mechanism of market fluctuations.

The role of the "Ad Hoc" assumptions in the two theories	
Rational Expectations model	**Rational Beliefs model**
REE + "ad hoc" irrational assumptions to explain economic fluctuations; • These "ad hoc" irrational assumptions really drive the results • "Ad hoc" assumptions are not empirically justified Examples: Lucasian "islands" with a rigid transmission of public information,	**RB does not need any irrational "ad hoc" assumptions**

<table>
<tr><th colspan="2">The "Heterogeneity" concept in the two theories</th></tr>
<tr><th>"Heterogeneity"
in Rational Expectations model</th><th>"Heterogeneity"
in Rational Beliefs model</th></tr>
<tr><td>Heterogeneity of beliefs as diversity of private information.

Uninformed Noise traders;
They are simply a negation of the theory of rational expectations since the assumption of noise in prices explicitly introduces irrationality of these traders.
Reasons against "noisy traders" irrational assumption:
How does asymmetric information arise in the first place?
Why does a large number of "noisy traders" continue to be present in the market?</td><td>Heterogeneity depends on the different ways people process public information</td></tr>
</table>

EXPERIMENTS AND SIMULATIONS

While there are relatively few papers focused on "cognitive experimental economics", in the last few years there has been a growing diffusion of papers on agent-based simulations. Sometimes the term experiment is used also for simulations with artificial agents, as they shows many features usually related to scientific empirical analysis. Indeed, these models allow us to observe unexpected phenomena, such as a sort of automatic spreading of learning among agents.

Once the modeller has established initial conditions and built the environment, he just observes the events dynamics, as in a "culture-dish laboratory experiment" (Tesfatsion, 2002, p. 2).

The interrelation between experimental economics and agent based computational economics has increased because of the frequent use of common sets and games both with human and artificial agents.

Simulations are today easier to realize than in the past and, in comparison with experiments with human subjects they also show some further advantages: for example there is no risk that players might misunderstand the rules of the game[1] or have unpredictable motivations.

Therefore the following question seems reasonable: why not using simulations instead of experiments? The answer is easy: simulations are not the same as experiments, and artificial agents are not human agents.

Tesfatsion (2002) analyzed some of the possible risks and problems related to simulations, and one of them should be underlined: the results obtained in simulations can be strongly affected by the representation of learning processes and also by the parameters-values setting used.

Therefore experiments and simulations are not alternative ways of gathering data. Tesfatsion (2002) suggested that experiments can help fixing parameters and decision models to be used in simulations. As

[1] This aspect can also be considered as a contros since it may occur in the real life.

we will see in the next chapters, experiments always show unexpected results in the participants' behaviour.
If simulations should start from reality, and not from a hypothetical world (even if it is different from the neoclassical approach), the cooperation with experimental economics is necessary.
Moreover, the analytical power of simulations can be fully expressed if it is used to show the variety and complexity of the real world, as indicated by experiments, which is impossible to manage through usual models.

On the other hand, simulations comparing hypothetical and real agents (Tesfatsion, 2002) can also help to understand and read experimental data.

Rational Beliefs Theory Empirical Testing / How to empirical test RB Theory

Kurz claimed that most of the observed volatility of asset prices and returns is propagated by the agents' beliefs. He suggested that exogenous shocks, central to REE-based explanation of market fluctuations, are insufficient to explain the structure of market volatility. The RB theory shows that market fluctuations arise from the joint effect of exogenous shocks and the states of belief on the markets.
The theory claims that many REE "anomalies" in financial markets, such as the equity premium puzzle, are all propagated by the dynamics of beliefs in the markets.
Kurz concluded his book "Endogenous economic fluctuations" asking whether other ways of testing the validity of the RB theory were possible.

Rational Expectations versus Rational Beliefs Hypotheses

The experimental approach offers an alternative way of testing the implications of the theory. To be more specific, let t be the current period and consider the occurrence of any random variable

$Z^{(t+k)}$ at k dates after t. In the paper "The equity premium is no puzzle" by Kurz and Beltratti 1996, $Z^{(t+k)}$ is the annualized rate of return on the S&P 500 stock index k periods after date t.
In a sample of H sophisticated investors each h is asked to provide forecasts $E_{Q^h}\left[Z^{(t+k)}\middle|I_t\right]$ for $k=1,2,3,\ldots,K$, where I_t represents the conditional information set.

The question is then how such data could allow us to distinguish among the following two hypotheses:

Hypothesis I: The agents hold Rational Expectations;
Hypothesis II: The agents hold Rational Beliefs which are not Rational Expectations;

It can be observed that using all past data agents can estimate the regression model which best fits the data and make, with this empirical model, two forecasts:

- $E_m\left[Z^{(t+1)}\right]$
- $E_m\left[Z^{(t+1)}\middle|I_t\right]$

Since the agents/subjects do not know the true data generating process, we need to define the term "holding Rational Expectations". But, for the REE theory to make sense, the agent should assume that the economy is stationary and agents know that it is stationary.
Therefore, the agents will accept the empirically based forecast as the truth.
This leads us to the following test:

Test A: k = 1

- Under REE all agents <u>**will make and remember**</u> the same conditional forecast of $E_m\left[Z^{(t+1)}\middle|I_t\right]$;

- Under RB the agents **will make and/or remember** diverse conditional forecasts of $E_{Q^h}\left[Z^{(t+1)}\middle|I_t\right]$, $h=1,2,3,\ldots,H$

Interpretation of Test Results.

For Test A the crucial difference between REE and RB is simple:

- Under REE agents agree that **the correct forecast and memory** is $E_m\left[Z^{(t+1)}\middle|I_t\right]$;
- Under RB **we have a distribution of forecasts and/or recalls** even if agents have the same information.

What did empirical evidence show us?

In our experiments we focused on memory dynamics; in particular we estimated the hindsight bias: $Z^{(t+k)}$ described any random variables which the subjects were asked to estimate and to recall later.

In this case the Rational Expectation model would assume $E_m\left[Z^{(t+1)}\middle|I_t\right]$ to be a common value among agents, implying that our subjects should have shown identical estimates and recalls.

Instead, we observed that, when the subjects were asked to recall the original values of their estimates after knowing the real outcome, they showed a wide distribution of forecasts/memories, violating the REE prediction.

In particular 45% of control group subjects showed hindsight bias.

With our tests we ascertained the existence of subjects' heterogeneity in estimating/recalling economic variables and we obtained empirical evidence in favour of Kurz's assumptions.

Beliefs/Biases correlation and the market dynamics

The Rational Beliefs model allows agents to show an endogenous form of heterogeneity; given such diversity, some agents are optimistic and some pessimistic about future capital gains. Kurz developed a model allowing agents to be in these two states of belief, but the identity of the optimists and the pessimists fluctuates over time since any agent may be in these two states of belief at any date. In the RB model there is a unique parameterization under which the model makes all its predictions *simultaneously*. Any parameter choice in this small neighbourhood requires the optimists to be the majority but the rational beliefs condition requires the pessimists to have a higher strength level. This higher strength has a relevant effect on the market: it increases the demand for riskless assets, decreases the equilibrium riskless rate and increases the equity premium.

In simple terms, the large equity premium and the lower equilibrium riskless rate are the result of the fact that, at any moment, there are agents who hold extreme pessimistic beliefs and they often have a relatively stronger impact on the market. The relative impact of these two groups of agents who are, at any date, in the two states of belief is a direct consequence of the rational beliefs condition and in that sense it is unique to an RB model. Kurz also studied the effect of the correlation of beliefs among investors, showing that the main effect of such correlation is on the dynamic patterns of asset prices and returns and is hence important for studying phenomena such as stochastic volatility.

A rather unique characteristic of the RB theory studied earlier is the fact that it describes the agents' heterogeneity by using what Kurz called "assessment variables". This important innovation allows agents to be optimist or pessimist affecting the market dynamics.

Kurz, Beltratti and Motolese introduced specific parameters which regulate the agents' beliefs correlation and argued that this correlation plays a role in the dynamics of prices.

The authors analysed the relationship between the fluctuations of the market state of belief and the correlation among agents, highlighting a fascinating cognitive aspect: beliefs correlations among agents characterise different levels of stock market volatility.

We tried to replace Kurz's "beliefs correlation" with a wider concept about the agents' heuristics and biases; we imagine that the common evolutionary path have defined a human common matrix in solving problems like an evolutionary software.

In particular, we hypothesised that, depending on biases distribution among the investors and on their strength, a sort of "endogenous correlation" among agents is possible, which is the main source of endogenous uncertainty.

BEHAVIOURAL FINANCE

Behavioural Finance is the integration of classical economics and finance with psychology and the decision-making sciences; it is an attempt to explain what causes some of the anomalies that have been observed and reported in finance literature.

Behavioral finance is the study of how investors systematically make errors in judgement and how they can cause stocks and bonds to be overvalued or undervalued. This fact has led to the creation of investments strategies that try to take into account this irrational behaviour.

While investment strategies that exploit emotions have existed for centuries, the newest and most promising area of behavioural finance focuses on identifying mental mistakes regularly made by investors. These strategies do more than just examine fundamentals of companies or the feelings of investors, they also analyse how the brain solves problems and, in certain instances, might be most prone to making mistakes.

From the Rational Expectations model to Behavioural Finance

All economic models make simplifying assumptions about both market conditions and the behaviour of market participants. Sometimes the simplifying assumptions underlying the model are explicitly stated and sometimes the assumptions are implicit - the latter is often the case regarding the behavioural assumptions underlying several models. To be more specific, let us consider the efficient market hypothesis (EMH), an economic model of considerable importance to investors. The simplifying assumptions regarding market conditions that underlie the EMH frequently include, among others, the following assumptions:

- Transaction costs are zero.
- Markets are not segmented.
- Easy (even unlimited) entry into the security markets exists.

The behavioral assumptions that underlie the EMH can be expressed as follows:

- Investors act in an unbiased way to maximize the value of their portfolios.
- Investors always act in their own self-interest.
- Investors have "perfect knowledge"

The first behavioural assumption states that investors are "rational expectations wealth maximizers" - i.e. investors form unbiased expectations of the future and, given these expectations, they buy and sell in the securities markets at prices which they believe will maximize the future value of their portfolios.
Behavioral finance questions whether the behavioral assumptions underlying the EMH are true.

Another aspect of behavioural finance concerns how investors form expectations regarding the future and how these expectations are transferred to security prices. Researchers in cognitive psychology and decision making sciences have shown that, under certain conditions, people systematically make errors in judgement. These mental mistakes may cause investors to have biased expectations about the future, which, in turn, can cause, for example, securities to be mispriced.
Behavioural finance may be able to explain some of the anomalies of the EMH that have been reported in finance literature as it considers that investors may not always act in a wealth maximizing manner and that investors may have biased expectations. Anomalous returns such as those associated with "value" stocks, earnings surprises, short-term momentum and long-term price reversals are fertile ground for researchers in behavioural finance. These anomalies have been documented by many studies. For example, with respect to the "value" anomaly, see Lakonishok, Shleifer and Vishny [1994]; with respect to earnings surprise, see Bernard and Thomas [1990]; with respect to short-term momentum, see Jegadeesh and Titman [1993]; with respect to long-term price reversals, see DeBondt and Thaler [1985, 1987, and 1990].

Expectations as a result of heuristics and biases

In general, making predictions investors use:

- a set of information
- procedures (models) for processing the information

Active investors expect to obtain extra returns by exploiting the following:

1. Superior (Private) Information

Most traditional investment managers try to use "the best" available information. These managers are frequently referred to as "traditional" managers or "fundamental" managers.

2. Better Process in managing Information

Some investment managers assume that most information is commonly available to all investors and focus their energy on trying to develop better procedures for processing this information. Managers that try to do this in a formal way are frequently called "quantitative" managers.

3. Behavioral Biases

Scholars in psychology and decision making sciences have shown that in some circumstances investors do not try to maximize wealth and, in other circumstances, investors make systematic mental mistakes. Both these cases can result in mispriced securities and both are the result of behavioural biases.

However, in trying to investigate all these issues, the key question is: "What are the probabilities that any individual investor or investment manager consistently gathers superior, private information when so many other investors are trying to do the same?".
A similar point can be made about processing information.

Again, the key question is: "What are the probabilities that any single investor or investment manager discovers the true factor model when so many other investors are trying to do the same?".

We discussed the conditions under which behavioural biases occur and how they might affect security pricing in the next sections.

"Fast and Frugal" Heuristics: powerful but imperfect

Psychological research has shown that the human brain often uses shortcuts to solve complex problems. Rather than fully digesting all information before producing an exact answer, the brain sometimes uses "tools" to quickly make predictions. These estimates, however, are not always accurate.
Optical illusions are good examples of how the use of shortcuts can lead to mental mistakes. A set of tools enables the brain to form rapid estimates, which are usually accurate, but on some occasions, these tools cause the brain to incorrectly process information and produce optical illusions.
We tried to summarise the most important behavioural biases that were generally accepted by using functional taxonomy.

The types of behavioural biases observed in securities markets fall into two broad categories:

1. Non Wealth-Maximizing Behaviour

The economist's view of rational behaviour assumes that investors act only to maximize the expected value of their portfolios. Indeed, investors may maximize other elements they feel to be more important than their wealth.

2. Heuristic Biases and Systematic Mental Mistakes

Heuristic biases cause investors to make systematic mental mistakes and, as a result, incorrectly process available information. Investors believe they are correctly processing information and act in a manner which maximizes their expected wealth. Later they may discover their wrong beliefs, but they are frequently not even aware of their error.

Biases and errors in the investment decision making process

Vision is not the only complex problem that the brain solves by using shortcuts. Studies in behavioural finance have shown that the brain uses similar tools for processing financial data and producing estimates when making investments decisions. And, similar to optical illusions, there are certain instances when these tools can cause mistakes.

An interesting and important aspect of heuristic biases is that they are very difficult to detect and contrast.

Heuristics, which have evolved in time, can be thought of as being part of the brain's hardware. Unlike software, these mental shortcuts are such useful and powerful problem solving tools that they simply cannot be easily reprogrammed. However, when a heuristic is used in the wrong situation, when the wrong rule-of-thumb is used to solve a problem, a heuristic bias causes the person to make a mental mistake.

Hindsight Bias

Hindsight bias is a person's tendency to distort a previous judgement in the direction of the new information after learning about the real outcome of a situation or the correct answer to a question. It alters the subject's learning pace over time and it may induce overreaction in the short time but also overreaction in the long run once the subject recognises all his mistakes.

Beliefs Updating Dynamics

Edwards (1968) identified the phenomenon of "conservatism", in which, under appropriate circumstances individuals do not change their beliefs as much as a Bayesian would in the face of new evidence.

Drawing inspiration from A.Wilson's research program we have investigated the connection between bounded memory and biases in information processing.

In our tests we involved the subjects in economic decision making processes (see chapter 3).

The subjects received various sequences of signals providing partial information about the true "state of the economy"; they were later asked to take a decision.

After that we compared observed data with the simulation results obtained by using a Bayesian agent.

In our tests we identified the following several biases:

Polarization

The subjects, after receiving a series of signals, were expected to show a quite stable belief (anchor) without reacting to further signals. Our results did not confirm the existence of "absorbing states". The following observations can effectively summarise the data we obtained.

- The subject was increasingly less influenced by a series of identical signals.
- The longer was the sequence of identical signals we provided, the stronger was the subject's reaction to an opposite signal.

These phenomena took place symmetrically in both directions and are compatible with Shannon's Information Theory.

Confirmatory bias

We analysed the relationships between the a-priori belief and the subjects' reactions to a sequence of signals: we analysed how much the subjects changed their estimate value over time according to their priors and to the signals they received.

For example, depending on the signal reliability, we found that the subjects who expressed an initial belief in favour of an optimistic state of the economy revealed greater reactions to positive signals.

Overconfidence / underconfidence biases

The decision-maker with bounded memory showed two types of overconfidence/underconfidence:

- one based on sample size,
- one based on the quality of the information he received

The decision maker was typically expected to be overconfident after short sequences of information, and underconfident after long sequences.

The underconfidence was expected to result from the fact that bounded memory is implied bounded probability assessments; vice versa, the overconfidence after short sequences, it was expected to result from the fact that beliefs adjusted too rapidly after each signal.

Different experimental settings can lead to under- or over reaction to new signals; people seem to make judgments differently in different situations (Grether 1992; Payne, Battman and Johnson 1992).

First impression matters bias

Exchangeable information is processed in a way that puts too much weight on early signals. This bias is strictly linked to the overconfidence bias.

Last impressions matter bias

Subjects were expected to show a decreasing sensitivity to long lasting series of signals; our results do not confirm this hypothesis.

Representativeness bias and market overreaction

"Representativeness bias" can cause investors to make errors in financial markets. People tend to show an "availability bias", overweighing evidence that comes easily to mind, thereby allowing their decisions to be over-influenced by evidence that is more relevant and attention-grabbing. Therefore, we can ascertain that processing new information and updating beliefs is costly. Information that is presented in a cognitively costly form is weighed less; furthermore, investors may overreact to information that is easily processed as in real case studies.

For example, if a company has repeatedly shown poor results, investors will sometimes loose confidence and think that the company will continue to have poor results in the future. We can recognise some inertia in beliefs updating dynamics. In these instances, investors overreact to the past negative information and ignore valid signs of improvement. Although it may in the future have good results, its stocks may be undervalued.

This is not to say that investors will never change their views; if the company continues to perform well over time, investors will eventually overcome their error, and the company's stocks will become a potentially attractive investment.

The observed dynamics revealed a different updating path in comparison to the one obtained by using a Bayesian's agent.

Anchoring bias and market underreaction

Mental mistakes can also cause investors to underreact to new, positive information about a company. One shortcut that causes this is "anchoring", a tool the brain uses to solve complex problems by selecting an initial reference point and slowly adjusting to the correct answer as it receives additional information.

Anchoring can also cause securities to be mispriced. For example, should a company suddenly report substantially higher earnings, the market will on occasion underreact to this change. Although the company is profitable, its stock price does not rise because investors assume, because of the bias, that the change in earnings is only temporary; they remain anchored to their previous view of the company's potential profitability because they have underreacted to the new, positive information.

This does not mean that investors will never move away from their initial reference point, the "anchor". Similar to representativeness, as investors get better information about the company over time, they will eventually overcome mental mistakes caused by anchoring. They will realise that the company is likely to continue to be more profitable in the future and that its stock is probably an attractive potential investment.

The flip side of underreaction is overreaction. It is important to note that different heuristics (anchoring and overconfidence) cause underreaction and the conditions under which investors are vulnerable to these heuristics are different from the conditions that cause investors to be vulnerable to overreaction.

If the analysts are overconfident and also anchored to their most recent estimate, they may be reluctant to give as much weight as they should to the latest information. However, if the change in earnings that caused the surprise is permanent, over time the analysts will figure this out, they slowly raise their estimates and the stock price will drift upward after the earnings announcement, generating the well-documented post-announcement effect associated with earnings surprises.

Thus, the keys to exploiting the true source of the earnings surprise are determining under what conditions analysts are likely to be overconfident and anchored, and whether the earnings change associated with the surprise is permanent in nature.

Empirical objections to Rational Expectations Model
Rationality in finance theory requires impossible powers of calculations
The evidence we possess does not support rational behaviour
It is to easy to go "theory fishing" for factor structures and market imperfections to match the data ex post
Irrational traders should arbitrage away efficient pricing
Irrational investors will bear more risk and get richer
Accurate investors will learn their way to bad decisions
Apparent return predictability is spurious, so rational models of predictability are misguided

Active investing by exploiting biases and behavioural strategies

Behavioural finance seeks to identify market conditions in which investors are likely to overreact or underreact to new information. These mistakes can cause mispriced securities. And the goal of behavioural finance strategies is to invest in these securities before most investors recognise their error - and tend to benefit from the subsequent jump in price once they do.

A particularly attractive feature of behavioural finance investment strategies is that they have an advantage over most traditional approaches to investing. Most investors use information-based strategies. They try to generate good returns by acquiring better information about companies or by processing information better than their peers through a unique, quantitative strategy.

But gaining advantages trough these methods is becoming increasingly difficult. A combination of the internet and the increasing power of personal computers and communications is making information more readily available and easier to process for all investors, however unreliable this information may be.

Behavioural finance strategies, however, take advantage of human behaviour - and human behaviour changes very slowly. The brain has evolved over centuries. Its approach to solving complex problems, and the tools it uses to solve them, are unlikely to change in the near future; behavioural finance investors are expected to take advantage when mistakes are made.

One of the behavioural literature contribution in economics is that value investment strategies work because most investors are captive to judgmental errors and biases which influence their decisions.

When the facts of reality contradict their biased view of the world, investors often over-react, sending market prices to extreme highs and lows. Successful value investors can exploit these over-reactions by adhering to investment policies and procedures that circumvent bias and over-reaction.

CONCLUSIONS

Exploring rational behaviour is of paramount importance in economic theory.

Cognitive economics suggests a series of methodological and instrumental innovations about the realization of experiments and the analysis of the data, and some of these innovations can also be applied to economic decision making.

In particular, our research showed a definite heterogeneity among individuals, which partly depends on the fact that experiments are artificial tests carried out in a laboratory, so subjects react to this environment in different ways, for example in term of motivations and aims (this heterogeneity can be considered as a background "noise" in the experiment and should be eliminated). However, this heterogeneity also highlights differences in people's beliefs, as it depends on each person's characteristics, knowledge, ideas, and experience.

The experimental approach main limit is that laboratory is not a neutral place and this is why several researchers question the validity of the "ceteris paribus" condition in doing economic and psychological tests. The same subject, indeed, reacts in different ways under identical experimental conditions. If we put theory to the test, we may decide to neglect this aspect, but we cannot ignore it if we want to explain people's different behaviours.

What we surprisingly found during our experiments was how interesting some "noises" could be; the "satisfying optimiser"'s behaviour (Simon, 1979) shown by some subjects could cause some problems but showed the validity of Simon's model.

Apparently strange choices made it difficult to understand and read data, but were sometimes the result of original approaches caused by boredom of repetitive behaviour.

We have seen that an individual analysis of data, especially if guided by the psychological tests and post-experiment interviews, can help to understand these phenomena, but these new methodologies need to be further studied, fully understood and developed. More analyses and specific tests, will probably be necessary to make these novelties accepted by the scientific community, but their indications are already so strong that they cannot be neglected.

We started working on agents' heterogeneity drawing inspiration from the Rational Belief Theory by Mordecai Kurz. We observed that the simulative approach succeeded in reproducing the USA stock markets data but that it was inadequate in describing psychological determinants of the agents' heterogeneity.

Since cognitive sciences can be a very important source of inspiration for economists interested in better understanding investors' behaviours, we chose to change the research perspective by adopting the experimental approach; we applied psychology and experimental economics to understanding the most representative heuristics and biases driving human behaviours.

We focused our analysis on hindsight bias and its effects on learning processes in economics and on several biases related to the concept of bounded memory.

We designed tests through which we gathered data describing the subjects' behaviours; we recognised the existence of several biases and we hypothesised their effects on market dynamics.

Our initial aim was to calibrate Kurz's model by taking into account the subject's psychological profiles in updating beliefs and making actions. However, we were very surprised at the data we gathered and at the behavioural dynamics we observed; we recognised that the agents' psychological characteristics could not be directly applied to Kurz's rational beliefs model, as this was designed to produce simulations results involving a huge amount of parameters through which it controlled the simulations.

By using an "ad hoc" model we adopted a different approach: we observed and described the way subjects updated their beliefs whenever they were affected by hindsight bias. Besides, we followed Andrea Wilson's bounded memory model to illustrate the most frequent biases and we hypothesised some calibrations on that model by using our tests results.
Indeed, we obtained interesting data, but new research efforts will be required in order to refine the results. We aim to repeat the tests in more than 2 session in order to analyse, more in details, the relationships between the biases and the learning process.

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Hindsight Bias in Investment Decision-Making Processes

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I am responsible for all errors.

1 Introduction

The way individuals make investment decisions involves their personal ability to remember and manage a huge amount of data.

In real life, before choosing how to invest money, investors look at the investment prices in order to know the rich information they contain. By doing so, they try to realise the ongoing economic situation and to evaluate the future scenarios in order to estimate the investment expected returns.

Investors must consider many data and transform them into useful information before choosing how to allocate money.

These activities ask a considerable cognitive effort and they have a significant impact on the overall portfolio performance as well[1].

Classic economic literature assumes that economic agents may perform all these tasks in a very efficient way, by managing and remembering perfectly, without omissions or errors, all the important information they need over time.

Standard rational choice theory further assumes that investors are able to identify relevant information and discriminate against irrelevant information as well as weight and process it accurately.

While the use of standard finance theory for the purpose of characterising optimal behaviour is undisputed, the validity of using it to describe actual behaviour may not.

"The representative investor is assumed to understand the economy and the process determining asset prices; the individual investor frequently does not", Odean (1998)[2].

Psychologists and experimental economists observed that objective irrelevant[3] data and selectively presented information[4] influence individual trading behaviour.

A promising approach to describe and, possibly, explain investment decisions may be the explicit consideration of psychological factors, in particular, heuristics and biases.

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[1] We are considering financial markets where with imperfect information.

[2] Brennan 1995, p.61

[3] Kircher et al. 2001

[4] Dittrich et al. 2002

Therefore, we analyse, through an experimental approach, the investors' behaviour to detect the importance of one of the most relevant biases, the hindsight bias. The key motivation for our research is to understand the hindsight bias implications for economics as a strong and robust psychological phenomenon.

Fischhoff (1975) was the first to study what he called *hindsight bias*. Hindsight bias is a person's tendency to distort a previous judgement in the direction of the new information after learning the real outcome of a situation or the correct answer to a question. It is based on empirical evidence indicating that individuals, after receiving outcome information, claim to have "known it all along"[5], that is, once past, events seem more comprehensible and also more predictable than before.

Hindsight and foresight perspectives differ formally in the information available to the observer. The hindsightful judge possesses outcome knowledge, that is, he knows how things turn out. The foresightful judge does not.

Economic studies consider the agent's foresight perspective only, without taking into account the hindsight bias possible effects in the decision-making process.

Since information processing strongly characterises the individual's decisions, it follows that, the "supposed" ability to compare new information to previous expectations is fundamental for an effective learner. Hindsight bias is a very relevant phenomenon because it compromises the above ability and it makes the individual confuse the prior expectations with the new information.

Consistently in economics, hindsight bias alters the investor's errors perception. It reduces the learning dynamics efficiency by rendering unexpected results less effective, and therefore, it may compromise the investment performance over time.

Differently, a rational Bayesian decision-maker[6] simply updates his knowledge after having received new information, being able to distinguish his original estimates (the priors), the outcome information, and his updated estimates (the posteriors). He recognises the possible estimate error and therefore he updates his decision weights for the process next step.

[5] Fischhoff, 1975

[6] Updated References

Looking at our experimental results we claim that hindsight biased investors do not follow Bayes' rule in updating their knowledge. Because of the bias, they find it very difficult to distinguish between a bad decision (the priors) and a bad outcome (outcome information), since an unlucky exogenous event will be considered, in hindsight, to have been predictable.

They rather reveal overconfidence, believing they are better forecasters than they really are.

This paper aims to shed light on the hindsight bias in the investment decision-making process, trying to better understand the psychological explanations (Gigerenzer 2002, Kahneman & Tversky 1982) and the evidence in experimental economics.

We first refer to a rich psychological literature dealing with hindsight bias. In the first and fundamental research paper "Hindsight different from foresight: the effect of outcome knowledge on the judgement under uncertainty", Baruch Fischhoff, 1975, the discoverer of the hindsight bias recognises, for the first time, its power "to restrict one's ability to judge or learn from the past". In "Outcome feedback: hindsight and information", Stephen j. Hoch and George Loewenstein (1989) present experimental evidence for what it will be later called the "I knew it all along" reaction induced by hindsight bias. Another pillar paper is "Hindsight: Biased judgements of past events after the outcomes are known", Hawkins, S. A., and R Hastie, 1990. The authors' results postulate that outcome information affects the selection of evidence to make a judgement, the evidence evaluation and the manner in which evidence is integrated and the response generation process.

The hindsight bias evolutionary valence is described by two studies: "Simple heuristics that make us smart", Gigerenzer Gerd, Todd Peter M., 1999 and "Hindsight bias : A price worth paying for fast and frugal memory", Hoffrage Ulrichm Hertwig, Ralph, 1999.

In "Learning from experience: coping with the hindsight bias and ambiguity", Baruch Fischhoff renew the idea that "forecasts are made with foresight but evaluated in hindsight". He presents de-biasing strategies to reduce the hindsight cognitive distortions. In line with this research is "When debiasing backfires: accessible content and accessibility experiences in debiasing hindsight". The authors, Sanna, Schwarz, Stocker discover that attempts to debias hindsight by thinking about alternative outcomes may backfire and they are far from guarantying success. An inferential and metacognitive explanation of the bias is presented by "The hindsight bias: a meta-analysis", Jay J Christensen, 1991, Daniel Kahneman, Paul Slovic, and Amos Tversky, ed.: Judgement Under Uncertainty: Heuristics and Biases (Cambridge University Press, Cambridge) and "Strength of hindsight bias as a consequence of metacognitions", Schwarz and Atahlberg, 2003.

An advanced study dealing for the first time with hindsight bias in economics is "The curse of knowledge in economic settings: An experimental analysis", Camerer, Loewenstein, Weber, Journal of Political Economy, 97(5): 1232-1254; the curse of knowledge and hindsight bias are deeply analysed in an experimental market setting. In the paper "Hindsight bias and individual risk. Attitude within the context of experimental asset market", the author investigates whether traders are prone to hindsight bias in remembering their price predictions: by adopting an experimental approach they observe how individuals choose to participate to lotteries.

The paper, "Decision maker hindsight bias after receiving favourable and unfavourable feedback", Therese A. Louie, 1999

presents the bias as consistent with a self-serving interpretation: experimental investors whose outcomes matched favourably with their decisions showed strongest hindsight bias. The authors, Holzl, Kirchler, Rodler in "Hindsight bias in economic expectations: I knew it all along what I want to hear" analyse the bias during the time of the euro change-over finding that self-serving tendencies can influence the information reconstruction selectively.

The research presented here asks three main questions about the judgemental differences between hindsight and foresight perspectives in an investment decision-making process:

1. How does the acquisition of outcome information affect the investor's decisions?
2. What is the hindsight bias effect in the investment portfolio management?
3. What is the role of metacognitions[7] in a decision-making process?

[7] Metacognition is the process of thinking about thinking. We will present more details in the next section.

In order to answer to the previous questions and inspired by the psychological literature, we explore three possible hypotheses in an economic framework:

1. Reporting an outcome's occurrence increases its perceived probability of occurrence. It derives from a fundamental result in the psychological research (Hawkins and Hastie, 1990) whose potential has not been still explored in economics.
2. Hindsight bias induces individuals to be overconfident and to overreact to new information (Camerer, Loewenstein, Weber, 1989).
3. An individual's high confidence level in his a priori estimates (those made before knowing the outcome information) and a low confidence level in his recalled estimates (those recalled after having received the outcome information) induce the subject to become affected by a strong hindsight bias (Werth, Strack, Forster, 2002).

Our study approaches hindsight bias in economics by adopting a customised Fischhoff's "between subjects"[8] test design; in particular, we introduce the following novelties:

1. we bring subjects into a simulated real life investment situation by using a narrative technique inspired by economic articles;
2. We analyse hindsight bias by directly collecting both cognitive and metacognitive variables;
3. We analyse the individuals' psychological profile and their behavioural tendencies to discover possible relations with the exhibited hindsight bias;
4. We involve two different kinds of subjects, Phd/Master students and finance managers, to investigate the role of expertise on hindsight bias.

Our results confirm the importance of the hindsight bias in an economic decision-making context. In particular, we discover that the test group investor tends to exhibit hindsight bias once asked to recall his economic predictions (65% of financial managers) and this fact limits his ability to conduct efficient learning. If we consider that investing coincides with a long lasting learning process, we can realise the hindsight bias effect consequences.

[8] See the details in the next chapter.

Our research is mainly inspired by Werth, Strack, Forster's paper: "Certainty and uncertainty: the two faces of hindsight bias", 2002.
We adopt a similar approach to assess the participants' hindsight bias; in particular we design a narrative technique to realise a test divided in two phases.
In the first phase the subject, after reading an economic article[9], is asked to answer three sets of questions; he has to:

1. to estimate the future scenarios of the economy;
2. to evaluate the expected investment returns;
3. to decide the portfolio allocation.

In the second phase, two weeks later, following a "between subjects" design, we split up the subjects in two groups. Half of the subjects (Test Group Subjects), after having read the final part of the article where they found the journalist's comments about the economy developments, are asked to recall their original estimates.
The remaining subjects (Control Group Subjects) are simply asked to recall their original estimates without receiving any further outcome information.
We study the hindsight bias by comparing the answers from the two groups of subjects.
In particular, we expect hindsight bias to shift the answers provided by the test group's subjects toward the outcome information. Indeed, we observe that hindsight bias induces individuals to be excessively influenced by this new information and to recall ex ante predictions too close to the realisation.
The following section of this paper discusses the hindsight bias. Section 2 describes the experiment design and section 3 presents the model. Section 4 gives a concise conclusion.

[9] See the appendix for the test questionnaires.

2 Previous Research and Background

In this section we present a more detailed description of the hindsight bias and a survey on the several interpretations psychological literature offers. We evaluate different real life memory strategies, discovering the hindsight bias evolutionary valence as an optimised and parsimonious memory process; the final result of a knowledge updating dynamics.

We also describe the most important test designs followed by the researchers. Then, we introduce a survey in economics analysing the hindsight bias effects.

Hindsight bias can occur when people fail to remember their original (a priori) beliefs once they have observed additional information. If, in the interim, they are told what the correct judgments should have been, the memories of their own judgements tend to become biased toward the new information.

Hindsight bias has been demonstrated experimentally in a variety of settings, including politics, games and medicine.

Fischhoff (1975), whose early experimental studies carved out this new topic for memory researchers, stressed that hindsight bias is not only robust and difficult to eliminate[10] (Fischhoff and Byeth (1975), Fischhoff (1982a)), but also has potentially harmful consequences[11]:

"When we attempt to understand past events, we implicitly test the hypotheses or rules we use both to interpret and to anticipate the world around us. If, in hindsight, we systematically underestimate the surprises that the past held and holds for us, we are subjecting those hypotheses to inordinately weak test and, presumably, finding little reason to change them. Thus, the very outcome knowledge which gives us the feeling that we understand what the past was all about may prevent us from learning anything from it. (Fischhoff, 1982b, p.343).

Hindsight bias: an evolutionary process by-product.

Rather than stressing the harmful consequences of hindsight bias, others (e.g. Campbell & Tesser, 1982) have pointed out its potentially adaptive aspects. Presenting ourselves as wiser after the fact may enable us to appear intelligent, knowledgeable, or perspicacious.

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[10] Fischhoff, B. & Beyth, R. (1975). "I knew it would happen": Remembered probabilities of once-future things. *Organizational Behavior and Human Performance* 13, 1-16.

[11] For further psychological explanations and an extended review see Hawkins and Hastie (1990).

In addition to hindsight potential benefits in social interaction, hindsight bias may play an important role in creating and maintaining a coherent conception of oneself, very important in decision making dynamics.

Therefore, hindsight bias can be considered a by-product of an adaptive process rather than an adaptation itself (Campbell, 1959). To introduce this view, we first address the question: what are the alternatives that human memory is unbounded in its capacity?

Some current conceptions of human memory seem to assume that we do in fact keep a record of every discrete event we have experienced and that, when we retrieve information or classify an object, we compare a probe with all our existing memory traces.

The psychological plausibility of these models has been questioned, both for the extensive similarity computation, as well as for the vast memory resources they require: "it is too expensive to maintain access to an unbounded number of items", Anderson and Schooler's (1991).

In addition, a stockpile of memories may interfere with the only information that is relevant right now.

In this sense, forgetting may be necessary for memory to maintain its function, insofar as it prevents us from using old and possibly outdated information (Bjork, 1978; Ginzburg et al., 1996). Forgetting should most likely occur once the usefulness of some information has passed.

Hindsight bias: a parsimonious memory process.

An alternative to a memory system that includes an immense, continuously expanding long-term storage is a system that maintains access primarily to the information that is most likely or is correct. For such a memory system, it is crucial to update information constantly and automatically. This process would avoid the problems of exploding number of items, and the increasing retrieval time required if memory probes were compared with stored traces in a serial manner.

A boundedly rational memory system would make it possible, keeping available only those items that are most likely to be needed. Such a process of information updating is consistent with Bartlett's[12] (1932/1995) classical finding that schemata are constantly changing and being updated.

[12]Bartlett is best know for his studies of memory and social psychology. His most significant and influential work is Remembering (1932), which examines the influence of social factors on memory in an experimental setting. Instead of traditional nonsense

"Remembering is not the re-excitation of innumerable fixed, lifeless and fragmentary traces. It is an imaginative reconstruction or construction..."

Sir Frederic Bartlett[13]

And, therefore, according to Stahlberg & Maass (1998), we recognise that memories may not be precise reproductions of past events but they may involve a reconstruction that replaces or supplements episodic recollection.

Alternatives to memory retrieval.

Besides the fact that for most experiences there is not need for later recall (Anderson & Schooler, 1992), there is another reason why it is not necessary to maintain a memory trace for everything we have thought, said, or experienced in the past. When something needs to be recalled, there are alternatives to memory retrieval.

For example, imagine that you own 250 shares of stock in a company, which are listed in the newspaper as being worth €78.50 each. To calculate their total value, you multiply 250 times 78.5. A week later, you want to know this value again. Can you remember €19.625? Probably not. However this is not a problem, because you can compensate for your failure to retrieve this information from memory by performing the same calculation again.

Recalling today is not the same experience than recalling tomorrow.

Recall can be compensated for by performing the same calculation.

We posit that the same sort of recalculation can be done - and, in fact, is done - when a past judgement, such as the prediction of a stock future value, needs to be recalled. If it cannot be recalled, going through the same process that led to the original judgement can provide a good

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syllables, Bartlett used meaningful materials to study the effects of past experience on the assimilation of materials. He showed how individuals, instead of merely reproducing the materials, reworked them in the light of their past experience.

[13] From 1922 to 1952, Bartlett was reader in experimental psychology, director of psychology laboratory, and first professor of experimental psychology, at Cambridge University. He was also editor of the British Journal of Psychology from 1924 to 1948. He discovered that when he asked people to repeat an unfamiliar story they had read, they changed it to fit their existing knowledge, and it was this revised story which then became incorporated into their memory. Bartlett's findings led him to propose 'schema' - the cultural and historical contextualisation of memory, which has important implications for eyewitness testimony and false memory syndrome, and even for artificial intelligence.

approximation, and perhaps even a perfect substitute. There is, however, an important difference between multiplication and a judgment.

Performing arithmetic computations is a technical skill and we are trained to do it reliably. Therefore, performing the same multiplication a second time should yield the same result.

In contrast, making a judgment often implies drawing knowledge-based inferences.

If knowledge is constantly updated, as suggested above, inferences based on the updated knowledge may be different from those based on past knowledge.

Hindsight bias: an updating knowledge by-product.

Updating knowledge is the key assumption underlying the concept of hindsight bias we examine.

It applies to situations where the original judgment was a knowledge-based inference. If the attempt to remember this original judgment directly fails, it will be reconstructed by repeating the same process that led to this judgment. However, knowledge about the outcome of an event, or feedback on whether an inference was correct, leads to an updating of relevant knowledge.

As a consequence, the reconstruction based on the updated knowledge can be systematically different from the construction based on the original knowledge. This difference is what characterises hindsight bias. According to this approach (Hoffrage and Heetwig, 2002), hindsight bias is a by-product of an adaptive process, namely knowledge updating.

The research approach evolution to hindsight bias.

Psychological research presents different causes supposed to be responsible for hindsight bias. Memory impairment, inferential process results and metacognitions play a fundamental role in explaining experimental observations.

Hindsight bias: a memory impairment result.

It is generally assumed that the hindsight bias reflects a memory impairment (Fischoff, 1975). Hell, Gauggel, Mall, & Muller, 1988 claim that the information that participants receive about the correct estimate partially replaces and overwrites the memory trace of the original answer, leading to uncertainty. Uncertainty, in this case, can be seen as the result of the partial efficacy of the process, where the two different memory cues, the recent information and the old estimates, can be nebulously overlapping, causing an ambiguous recall.

Hindsight bias: an inferential process result in a judgment under uncertainty.

Another approach sees hindsight bias as the outcome of a feeling of uncertainty in an inferential process (Einhorn & Hogarth, 1981), which can derive from two main sources: a lack of knowledge and /or a weak recollective experience.

The above causes induce the individual to try to infer the forgotten information by using the outcome information as an anchor. Following this approach, hindsight bias can be described as the result of a memorial as well as an inferential process.

We must note that the meaning of the word "uncertainty" in psychology is rather different from the one used in economics.

Recent studies on hindsight bias considers certainty/uncertainty as two opposite feelings dealing with the subject's metacognition[14].

Hindsight bias: the result of a metacognitively driven inferential process.

Werth, Strack and Förster in their further investigations argue that the previous conceptualization of hindsight bias as "judgement under uncertainty" product is one-sided in that uncertainty is seen as its sole determinant.

However, according to the inferential approach, whereby the individual tries to remember the original information by using the outcome knowledge, the hindsight bias seems to be also dependent on metacognitions, that is, a combination of feelings of certainty as well as uncertainty perceived by the individual during the inferential and recollective processes.

In particular the subject can feel certain about his ex-ante estimates but uncertain about his ex-post recalled estimates[15].

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[14] Process of thinking about thinking.

[15] Meta-cognitions are strictly related to certainty and uncertainty feelings and pertain to two separate aspects of the decision making process.

Whereas certainty about the recollection is memorial and concerns the recollective experience, certainty at the time of judgment is inferential and concerns the individual's meta-knowledge ("I know that I knew that").

By adopting Werth, Strack and Förster's approach in psychology, we experimentally investigate the role of metacognitions in an investment decision-making process. We differentiate our research by asking directly the subjects their metacognitions while they were performing economic decisions.

Metacognition.

Metacognition is the process of thinking about thinking, that is the knowledge about cognitive phenomena.

"Metacognition refers to one's knowledge concerning one's own cognitive processes or anything related to them", Flavell (1976).

For example, an individual is engaging in metacognition if he is noting that he is having more trouble learning A than B.

What is basic to the concept of metacognition is the notion of thinking about one's own thoughts.

Those thoughts can concern:

- what one knows (i.e., metacognitive knowledge);
- what one is currently doing (i.e., metacognitive skill);
- one's current cognitive or affective state (i.e., metacognitive experience).

Metacogmitions in economics.

John Maynard Keynes was the first to introduce the idea of metaknowledge in economics; he also recognised that the "feeling of knowing something" plays *a key role in investment decisions.*

In "The General Theory of Employment, Interest and Money", chapter 12, "The State of Long-Term Expectation", he wrote: *"The state of long term expectation, upon which our decisions are based, does not solely depend on the most probable forecast we can make. It also depends on the confidence in which we make this forecast".*

According to Keynes, the future scenario, upon which investors will make their decisions is inferred from a state of confidence.

This state of confidence depends on what he called "conventions"; we can replace the Keynes' term "conventions" with the behavioural "heuristics and biases".

Hindsight bias in economics.

What is the hindsight bias relevance for economic decisions?

All the previously mentioned researches have shown that the hindsight bias arises in a large variety of contexts and even subjects conscious of this bias cannot avoid its effects.

Camerer, Loewenstein and Weber (1989) demonstrated that learning processes and market competition reduce but do not eliminate this bias. Since investment decisions are very complex and long lasting tasks, we decide to analyse the overall investment process by focusing on the constituting cognitive activities. Our results show that hindsight bias may cause investors overconfidence and this phenomenon seems not to be easy to prevent, exposing the biased investor to an higher degree of risk than he would have probably preferred.

3 Experimental Setting Design

Hypotheses.

Accordingly to the questions presented in the introduction, the experiment examines three occurrences:

1. the hindsight bias plays a corrective role in an investment decision-making process by partially hiding subject's estimate errors;
2. the individual's awareness as an hindsight bias mitigating factor
3. the hindsight bias depends on the individual's expertise
4. the hindsight bias magnitude is a joint function of:
   - increased certainty in the judgment task;
   - increased uncertainty in the memory task.

Method

Partecipants.

We first collected data from 25 Master and PhD students and then, to assess the robustness of our results, we involve 35 financial managers of a big Italian Bank. Students are all coming from the Master in Finance and the PhD in Economics at the Bocconi University; they all have an economic background. Bankers are all financial advisors in the Unicredit Bank. They assist investors in their economic decisions and in the elaboration of portfolio allocation. On average they have a 3 years experience in investments management.

Test description.

The paper-and-pencil experiment takes place at the Master in Finance at Bocconi University and at the Unicredit Bank headquarters in Milan. When subjects arrive they are seated at tables and separated from each other for the duration of the experiment. They are then given a set of instructions which are read out loud to them after they have had a chance to read them individually.

Subjects are not informed about the aim of the test.

Each experimental session lasts about 40 minutes.

We offer subjects (students) an incentive to answer questions truthfully; they earn on average approximately 6€ in each session according to their performance. No payments were given to financial managers.

Stimulus material.

In each phase of the experiment, participants read an economic article[16], whose content is customised for our purposes, and they worked on practice and test questions.

The article deals with the Portuguese Economy and presents some details about the financial markets. The reader is involved in a real life investment decision and he is asked to meditate about several economic options.

In order to investigate the subjects' decision-making processes we adopt two different questionnaires.

First Questionnaire

It is composed of 3 different sets of questions and it collects information about the subject's economic estimates/forecasts and his investment preferences.

The first set of questions asks the subject's to estimate how the economy will develop in the future. The subject is asked to estimate the likelihood of each of the four presented scenarios: the economy will develop with low inflation, the economy will develop with high inflation, the economy will stagnate with low inflation and the economy will stagnate with high inflation.

The second set asks the subject to estimate the expected returns he will obtain from different forms of investments: stocks, bonds and real estates.

The third set asks the subject to indicate his investment preferences, therefore his preferred allocations.

After each block, the subject is asked to indicate the confidence level in his answers (metacognition) by answering the following question: "Please write down your confidence level in your estimate (1=min ; 10=MAX)".

[16] See the article in the appendix.

Second Questionnaire

It is composed of 62 questions[17] divided in seven sets. It collects information about the subject-investor's psychological profile.

The first set asks the subject to define what kind of investor he is.

The second set asks the subject to define his personal experience in managing money.

The third set asks the subject to define his investment goals.

The fourth set asks the subject to define the criteria followed in choosing his investments.

The fifth set asks the subject to describe his attitude towards information about financial markets.

The sixth set asks the subject to define how he generally behaves in making decisions.

The seventh set asks the subject questions about himself.

Procedure.

We use an original approach to test the hindsight bias. Most of the psychological literature dealing with hindsight bias presents questionnaires made of almanac questions, and the researcher usually selects issues unknown to the subjects by using a pre-test; for example: "How many paintings does Picasso realise?".

Instead, we collect estimates and recalls made by the subjects about known economic issues on which they are expected to base their decisions. This is a novelty we introduce in respect to the previous experimental literature.

[17] See the questionnaires in the appendix.

The test.

The test is composed by two phases: the estimate phase and the memory phase;

In the Estimate Phase the subject receives a brief article describing the state of the economy. After reading the text, the subject is asked to answer some questions dealing with his investment predictions and decisions.

In the Memory Phase, two weeks later, following Fischhoff's "between subjects design", we randomly divide the subjects in two groups, the Test and Control Group.

The members of the two groups receive different information depending on the group they belonged to.

The Test Group subject is asked to read the second and final part of the article dealing with the developments of the previously described economy. Therefore, he is informed about what we previously called the outcome information. After that, he is asked to remember the original estimates (his priors) he gave in the first part of the test.

The Control Group subject is instead simply asked to try to remember the original estimates he has given in the first part of the test <u>without</u> receiving any information about the real outcome.

Design.

Dependent measures.

In order to detect the extent of the hindsight bias we compute for each group of subjects:

- the differences between the original estimates and the outcome information, in order to obtain the estimate errors;
- the differences between the recalled estimates and the original answer, in order to obtain the memory errors;
- the correlation between the two errors and their composition.

We consider hindsight bias that memory error which partially compensates the estimate error.

In order to investigate its origin we analyse the correlation between the observed hindsight bias and the subject' confidence in his original and recalled estimates.
We also analyse the relation between the observed hindsight bias and the subjects' psychological profile.

Empirical results.
Discovering the hindsight effect.

We expect that the hindsight bias could affect the test group subjects' perceived performance in two different ways: by shifting the recalled estimates toward the outcome information and/or by making the recalled estimates more concentrated around the outcome value.

Table 1 presents some descriptive statistics about the answers provided by the control and the test group subjects in the first phase of the experiment. It is divided in panels, each corresponding to a set of questions. The first panel is about the future developments of the economy (scenarios), the second panel is about the expected investment revenues and the last panel is about the investment allocations.
The distance between the group's average estimate and the outcome represents the group's average estimate error; accordingly to that, the last columns reports the best performing group.
As data show, the control group, on average, performs better or, at least, as equally accurate estimates as the test group do.
The control group is the best performer group in the 60% of cases in the estimate phase of the test.

Table 1

Panel A							
Scenarios Original Estimates	Control Group		Scenarios Outcome	Test Group		Best Performer Group
	Statistics	%	%	Statistics	%	
Growth & Low Infl.	Mean	45,29	100%	Mean	55,87	TG
	St.Dev	17,98		St.Dev	18,78	CG
Growth & High Infl.	Mean	21,75	0%	Mean	25,31	CG
	St.Dev	14,14		St.Dev	18,66	CG
Stagnation & Low Infl.	Mean	21,47	0%	Mean	13,13	TG
	St.Dev	14,55		St.Dev	15,48	CG
Stagnation & High Infl.	Mean	11,18	0%	Mean	4,68	TG
	St.Dev	6,96		St.Dev	5,9	TG
Panel B						
Revenues Original Estimates	Control Group		Revenues Outcome	Test Group		Best Performer Group
	Statistics	%		Statistics	%	
Stocks	Mean	9,44	10	Mean	9,16	CG≈TG
	St.Dev	4,51		St.Dev	4,61	CG≈TG
Bonds	Mean	2,22	3	Mean	3,11	TG
	St.Dev	1,03		St.Dev	1,07	CG≈TG
Real Estates	Mean	3,27	7	Mean	3,11	CG≈TG
	St.Dev	5,48		St.Dev	3,25	TG
Panel C						
Investment Original Estimates	Control Group		Investment Outcome	Test Group		Best Performer Group
	Statistics	%		Statistics	%	
Bonds	Mean	38,24	40	Mean	38,24	CG≈TG
	St.Dev	13,8		St.Dev	13,34	TG
Stocks	Mean	33,53	20	Mean	37,06	CG
	St.Dev	12,22		St.Dev	19,93	CG
Real Estates	Mean	28,24	40	Mean	24,71	CG
	St.Dev	19,76		St.Dev	16,25	TG

Table 2 presents some descriptive statistics about the answers provided by the control and the test group subjects in the second phase of the experiment, the memory phase.

We repeat the same analysis to detect which group, on the basis of its recalled average estimates, could think to have performed at best. The distance between the group's average recalled estimate and the outcome represents the group's average memory error; accordingly to that, the last columns reports the group that thinks to have performed at best.

We expect that the test group subjects, those who receive the outcome information, reveal recalled estimates shifted toward the feedback. Indeed, the test group subjects, (deceptively) believe to be good performers in the 70% of cases. In table2, as expected, we see that the test group subjects' remembered estimates are closer to the outcome than the original estimates (see Table 1). The data clearly reveal that the memory distortion which operates on the recalled estimates enhances the test group subjects' perceived performance by a 75% ratio.

Table 2

Panel A						
Scenarios Recalled Estimates	Control Group		Scenarios Outcome	Test Group		Best Performer Group
	Statistics	%		Statistics	%	
Growth & Low Infl.	Mean	40	100	Mean	54,376	TG
	St.Dev	24,75		St.Dev	23,16	TG
Growth & High Infl.	Mean	17,65	0	Mean	15,69	TG
	St.Dev	13,36		St.Dev	21,87	CG
Stagnation & Low Infl.	Mean	21,35	0	Mean	17,69	TG
	St.Dev	15,22		St.Dev	18,125	CG
Stagnation & High Infl.	Mean	15,29	0	Mean	8,2	TG
	St.Dev	13,86		St.Dev	5,93	TG
Panel B						
Revenues Recalled	Control Group		Revenues Outcome	Test Group		Best Performer Group
	Statistics	%		Statistics	%	
Stocks	Mean	16,06	10	Mean	12,22	TG
	St.Dev	18,57		St.Dev	6,16	TG
Bonds	Mean	7,86	3	Mean	7,79	CG≈TG
	St.Dev	11,54		St.Dev	14,5	CG
Real Estates	Mean	4,47	7	Mean	5,83	TG
	St.Dev	5,37		St.Dev	3,85	TG
Panel C						
Investment Recalled	Control Group		Investment Outcome	Test Group		Best Performer Group
	Statistics	%		Statistics	%	
Bonds	Mean	36,25	40	Mean	46,76	TG
	St.Dev	19,07		St.Dev	19,11	CG≈TG
Stocks	Mean	38,75	20	Mean	31,47	CG
	St.Dev	22,24		St.Dev	16,5	TG
Real Estates	Mean	25	40	Mean	22,35	CG
	St.Dev	18,25		St.Dev	14,37	TG

Example.

If we consider for example the second answer in the panel A of table 1, we see that the control group's estimate (21,75%) is closer to the outcome (0%) than the test group's one (25,31%). If we look at the remembered estimates in table 2, we discover that now, the test group's recalled estimate (15,69%) is closer to the outcome (0%) than the control group's one (17,65). The same phenomenon realises for the first and third answer of panel B and for the first answer of panel C.

We can easily recognise the hindsight bias effect; in particular, we see that this memory bias compensate the test group subjects' estimate error reducing their perceived overall error.

The 67% of test group (financial managers) subjects reveal hindsight biased answers while the 45% of the cases for the students. Within these "hindsight biased" subjects we observe that the 57% reports at least 3 biased answers, the 36% reports 6 biased answers, the 7% reports all biased answers.

After having considered the hindsight bias distribution over the test subjects we must analyse the hindsight bias average intensity.

We define hindsight bias intensity its capacity to partially compensate and hide the subject's estimate error; for example, let's suppose that the subject's prediction for the scenario number 2 is 25%. Let him discover that the outcome is 0% (that scenario does not realise). The individual has committed an estimate error equal to 25%. If we observe that the subject's recalled estimate for that scenario is 5% we can clearly distinguish the "I knew it all along effect". In fact, the subject has, may be unconsciously, updated his memory by recalling an "adjusted" estimate from 25% to 5%: the hindsight bias has improved the subject performance by making his recalled-perceived estimate more accurate. In this case the hindsight bias has significantly adjusted the subject's estimate error by a ratio of 80%.

Our data show that, on average, the hindsight bias intensity between the test group subjects is 62%, highlighting its role in the decision-making process.

Descriptive statistics

Now we analyse the relationships between the two kinds of errors constituting the overall perceived task error, M^{o}_{t+1}: the estimate error, M^{e}_{t}, and the memory error, M^{r}_{t+1}.

Descriptive statistics	*Me*	Descriptive statistics	*Mr*
Mean	28,51833	Mean	13,51852
Standard deviation	15,38061	Standard deviation	7,624541

Within the hindsight biased subjects (students), we observe that:

$Corr(M^{e}_{t};M^{r}_{t+1})=0.591$ if we consider a "between answers" analysis

$Corr(\bar{M}^{e}_{t};\bar{M}^{r}_{t+1})=0.757$ if we consider a "**between subjects**" analysis.

1 We calculate the average errors for each subject $(\bar{M}^{e}_{t};\bar{M}^{r}_{t+1})$

2 We calculate the correlations

While, within the hindsight biased subjects (financial managers), we observe that:

$Corr(M^{e}_{t};M^{r}_{t+1})=0.612$ if we consider a "between answers" analysis

$Corr(\bar{M}^{e}_{t};\bar{M}^{r}_{t+1})=0.921$ if we consider a "**between subjects**" analysis.

1 We calculate the average errors for each subject $(\bar{M}^{e}_{t};\bar{M}^{r}_{t+1})$

2 We calculate the correlations

Hindsight bias and metacognitive variables.

We analyse the relations between the hindsight bias effect and the metacognitive variables, confidence in estimates and confidence in recall by adopting a linear regression.

In particular, we try to find a link between the overall perceived error, M^{o}_{t+1}, the one lightened by the hindsight bias, and the two metacognitive variables, C^{e} and C^{r}:

$$M^{o}_{t+1} = aC^{e} + bC^{r} + \varepsilon$$

The statistical support for the above relationship is weak. In fact, the adjusted r-square is just 0.345)[18]. However, the signs of the regressor coefficients are as expected. Indeed, the coefficient of C^{e} is positive, while the coefficient of C^{r} is negative. Therefore, the more confident is the subject in his estimates and the more unconfident is in his recalls, the stronger will be the hindsight bias effect.

Once informed about the real outcome, subjects with a high level of confidence in their original estimate and a low level of confidence in their memories perceive the outcome as their estimate.

Hindsight bias effect and self-awareness.

We investigate the links between the confidence in estimate and the estimate error in order to describe the subject's self-awareness. We agree on the fact that, the higher is the confidence in estimate, the smaller we expect to be the estimate error.

We also investigate the links between the confidence in recall and the memory error.

$Corr(C^{e}_{t};M^{e}_{t}) = -0.466$ if we consider a "**between answers**" analysis

$Corr(\bar{C}^{e}_{t};\bar{M}^{e}_{t}) = -0,674$ if we consider a "**between subjects**" analysis.

1 We calculate the average values for each subject $(\bar{C}^{e}_{t};\bar{M}^{e}_{t})$

2 We calculate the correlations

$Corr(C^{r}_{t+1};M^{r}_{t+1}) = -0.688$ if we consider a "**between answers**" analysis

[18] Low r-square are quite common in the experimental economic literature.

$Corr(\bar{C}'_{t+1};\bar{M}'_{t+1}) = -0{,}904$ if we consider a "**between subjects**" analysis.

1 We calculate the average errors for each subject $(\bar{C}'_{t+1};\bar{M}'_{t+1})$

2 We calculate the correlations

The level of expertise and the hindsight bias.

In order to investigate the role of expertise in determining the hindsight bias phenomenon we compare the results obtained from the two populations of subjects involved in our tests: the students and the financial managers. Overall we observe that on average the hindsight bias appears in the 65% of cases within the financial managers population while in the 45% between the students. The other remarkable result is that the financial managers' recalled estimates are more concentrated around their mean values. By looking at the psychological profiling questionnaires we discover that the motivational incentives to appear "right" are stronger for the managers than for the students. This can be one reason for their stronger hindsight bias.

How to predict hindsight bias: the psychological profiles analysis.

In order to correlate the hindsight bias to a specific subject's profile, we investigate the links between the tests results and the individual psychological characteristics and attitudes that can be used to proxy the hindsight bias phenomenon.

We report the subject's average values for each single answer:

1 = min, 10 = MAX

N°	Question	Score
5	I think I'm competent in managing my money	8
23	I keep myself updated over the latest investment opportunities and financial markets	8
24	Before taking an investment decision I need detailed information	8
58	When you put forward what you feel is a good idea, can you strongly support it?	7.7
56	Do you think you have a good memory?	7.5
60	Are you usually sure about how you feel?	7.4

50	I think it appropriate to diversify my investments over different industrial fields and / or different countries	7.3
52	The recent developments in the financial markets have made me more cautious in the way I invest	7.1
15	I don't care about losses in the short run if I can gain high revenues in the long run	6.6
25	The more I'm informed on a investment. the more I'm ready to invest my money in it	6.6
18	I usually invest my money in companies that I know very well or that are quite famous	6.4
49	I prefer not to sell assets at a loss	6.2
48	I have already lost money because I sold some assets too late	5.9
44	After losing money. I tend to be more careful in my investments	5.6
17	I prefer to invest in my country's companies	5.3
14	I appreciate in particular the level of certainty it offers. even if I receive lower returns	4.9
35	New information makes me immediately buy or sell an asset	4.9
30	I usually invest in what the press recommends	4.6
46	A great part of my success in stocks investment is due to chance	4.6
47	I have already lost money because I sold some assets too soon	4.6
34	I very often say that, maybe unconsciously, I ignore those information that contradict my opinion about my investment	4.3
43	I have already lost some money invested in stocks simply because I was lazy and didn't "update" my investment strategy	4.2
29	My financial decisions are strongly influenced by a professional adviser	4.1
22	I usually choose an investment whose value has recently decreased considerably	4
36	I believe I am more successful than the others because I invest wisely	3.8
16	I usually choose investments with high expected returns even if I must accept higher risks	3.3
21	I usually choose an investment whose value has recently grown up considerably	3.2
57	Are you self-confident?	3.1

33	I get angry if, after selling a stock, I learn positive information about the stock I have just sold	2.6
53	Do you often forget what are you saying?	2.5
31	I usually invest in what the press doesn't recommend	2
42	My financial losses depend largely on misfortune	0,9

On average, the hindsight-biased subject is a wise investor who keeps himself updated on financial markets and carefully collects a lot of information before taking a decision. He also cares about long run revenues and he diversifies his investments.

If we analyse in details his psychological answers we discover that he presents some relevant contradictions. In particular, even if he is strongly convinced to be very competent in managing his portfolio, he generally feels insecure as a person (see question n°57).

Moreover, he says that he does not care about losses in the short run if he can gain high revenues in the long run, but he declares that he strongly prefers safe investments with low revenues.

He thinks that he has a good memory but he declares a low level of confidence in his recalled estimates, etc.

All these implicit contradictions can represent a warning factor about the probable hindsight bias of the investor.

In particular we can project a specific questionnaire containing a mixed set of differently phrased questions with similar contents in order to detect psychological conflicts or incongruent investment aims. By engineering a sort of "contradiction index" we can transform a subject's implicit information into a visible and explicit measure. It can be done by taking into account the number of similar content questions answered in opposite ways and by measuring the distance between the answers scores[19].

The higher will be the contradiction index the more probable will be the hindsight bias occurrence. Financial advisors can use this tool to develop more reliable relationships with their customers being able to detect their real needs and to fine tune their activities on. To be able to predict and avoid the hindsight bias will improve the a posteriori investor's satisfaction and, consequently, his trust in his investment advisor.

[19] See the model section.

Model

The model punctually reflects the experimental variables we identify in designing the tests.

Suppose that at time[20] t, a subject could estimate the future value of an economic variable x_{t+1}, by employing I_t, the available information at his disposal at that time: $E_t(x_{t+1}|I_t)$[21] represents the individual's estimate of the economic variable.

At time[22] $t+1$, the subject[23] is asked to remember his past estimate after having observed the realisation of the variable x_{t+1}; $R_{t+1}[E_t(x_{t+1})] \; |x_{t+1}$[24] represents the recalled estimate.

The subject is expected to have just in mind the recalled estimate and the outcome information.

Overall cognitive error.

We call overall cognitive error the overall perceived error in judging and remembering, $M^o_{t+1} = |R_{t+1}[E_t(x_{t+1})] - x_{t+1}|$.

It represents the distance between the recalled estimate, $R_{t+1}[E_t(x_{t+1})]$, and the true realisation, x_{t+1}, at time $t+1$.

We suppose it to be the result of two different components, the estimate error, M^e_t, and the memory error, M^r_{t+1}:

$$M^o_{t+1} = M^e_t + M^r_{t+1}$$

Where:

$M^e_t = |E_t(x_{t+1}) - x_{t+1}|$ is the estimate error at time t, considered in the foresight perspective;

$M^r_{t+1} = |R_{t+1}[E_t(x_{t+1})] - E_t(x_{t+1})|$ is the memory error at time $t+1$, considered in the hindsight perspective;

[20] It corresponds to the "estimate phase" in the test.

[21] Conditional estimate.

[22] It corresponds to the "memory phase" in the test.

[23] Test group subject: he receives outcome information before being asked to recall his original estimate.

[24] In this case the sign "$|Info$" means that the subject knows the outcome information.

The estimate error.

The estimate error, $M_t^e = \left| E_t(x_{t+1}) - x_{t+1} \right|$, represents the distance between the subject's estimate and its true realization in the foresight perspective. It characterises the subject's ability to produce reliable estimates and it depends on the specific heuristics he employed in managing information.

The estimate error does not adequately describe all the errors that may affect the subject's cognitive tasks; this is why we introduced the memory error.

The memory error[25].

The memory error, $M_{t+1}^r = \left| R_{t+1}[E_t(x_{t+1})] - E_t(x_{t+1}) \right|$, represents the distance between the subject's recalled estimate and the original estimate. It qualifies the subject's ability to store and retrieve information correctly; it may depend on several variables such as the information encoding level, the issue familiarity, the subject's age, and many other variables mapped by cognitive sciences. In particular, the memory error is totally invisible to those subjects who reveal hindsight bias.

Foresight Perspective Errors	**Hindsight Perspective Errors**
Estimate Error $M_t^e = \left\| E_t(x_{t+1}) - x_{t+1} \right\|$	Overall Perceived Error $M_{t+1}^o = \left\| R_{t+1}[E_t(x_{t+1})] - x_{t+1} \right\|$
	Memory Error $M_{t+1}^r = \left\| R_{t+1}[E_t(x_{t+1})] - E_t(x_{t+1}) \right\|$

The hindsight bias definition.

The hindsight bias is that memory error which reduces the perception of the overall cognitive error. In particular, it induces the subject to perceive the recalled estimates closer to the realisation than they should be.

$$HB: \quad R_{t+1}[E_t(x_{t+1})] \rightarrow x_{t+1}$$

[25] Usually the memory error is not considered in the investment decisison-making models in Economics.

The individual presents an overreaction to the outcome information that alters the memory of his priors (original estimates).

Hence, the hindsight bias effect is to render the perceived distance between the recalled estimate and the realisation smaller than the true distance between the original estimate and the realisation.

$$\left|R_{t+1}\left[E_t(x_{t+1})\right]\middle|I_t \quad -x_{t+1}\right|<\left|E_t(x_{t+1})-x_{t+1}\right|$$

This effects determines the individual's overconfidence in his estimates and decisions.

Hindsight bias effects

Since hindsight bias partially compensates the estimate error, tending to minimise the subject's overall perceived error, $M^{o}_{t+1}=\left|R_{t+1}[E_t(x_{t+1})]-x_{t+1}\right|$, therefore, we can represent its effects by the following relation:

$$M^{r}_{t+1}=-\delta\left|M^{e}_{t}\right|, \text{ with } 0\leq\delta\leq 1$$ [26]

Where δ measures the hindsight bias intensity: it captures the relationship between the memory error and the estimate error.

In particular, it means that the bigger is the estimate error, the stronger the hindsight bias corrective role will be, affecting the overall learning process efficiency.

$$\left|\frac{\delta M^{r}_{t+1}}{\delta M^{e}_{t}}\right|>0$$

We distinguish two cases where Hindsight Bias clearly reveals its effects in the Test Group subjects:

Case A: hindsight bias compensates the individual's underestimation

If the subject's estimate is lower than the realization, the hindsight bias effect shifts the value of the recalled estimate so that it will be perceived between the original estimate and the realization.

if originally, $E_t(x_{t+1})<x_{t+1} \quad \Rightarrow \quad H.B. \quad effect: \quad E_t(x_{t+1})<R_{t+1}[E_t(x_{t+1})]<x_{t+1}$

[26] δ measures the hindsight bias intensity

Case B: hindsight bias compensates the individual's overestimation

If the subject's estimate is higher than the realization, the hindsight bias effect shifts the value of the recalled estimate so that it will be perceived between the realization and the original estimate.

$$\textit{if}\ \text{originally},\quad E_t(x_{t+1}) > x_{t+1} \quad \Rightarrow \quad H.B. \quad \textit{effect}: \quad x_{t+1} < R_{t+1}[E_t(x_{t+1})] < E_t(x_{t+1})$$

Hindsight bias determining variables.

We model hindsight bias, $HB_{t+1} = \left| R_{t+1}[E_t(x_{t+1})] - E_t(x_{t+1}) \right|$, by using two different classes of variables:

- cognitive variables
- meta-cognitive variables[27].

Cognitive variables describe the result of a cognitive task, such as the estimates that the subjects are to produce in the first phase of the experiment, $[E_t(x_{t+1})]$, or the memories, $R_{t+1}[E_t(x_{t+1})]$, they are to recall during the second part of the test.

Meta-cognitive variables, in this case, represent the variables characterising the subject's self-confidence in solving a specific task.

In particular, we suppose that hindsight bias may depend on the two metacognitive variables, C_t^e and C_{t+1}^r, as represented by the following relation: $HB = f(C^e, C^r)$, where:

C_t^e denotes the confidence level in judging, the confidence that the individual has in his original estimates;

C_{t+1}^r denotes the confidence level in remembering, the confidence that the individual has in his recalled estimates.

[27] See the definition at page 58.

Meta-cognitive Variables:	Cognitive Variables:
Confidence on estimate, C_t^e	Original estimates, $E_t(x_{t+1})$
Confidence on recall, C_{t+1}^r	Recalled estimates, $R_{t+1}[E_t(x_{t+1})]$

Hindsight Bias and Inferential Processes

A test group subject who fails to remember directly his original estimate tries to infer his recalled estimate, $R_{t+1}[E_t(x_{t+1})]$, from the outcome information, x_{t+1}, in the hindsight perspective, moving backwards in time from $(t+1)$ to (t).

Since the agent is informed about the real outcome before trying to recall his original estimates, we observe that the subject's information set changes over time and the recalled estimates change as well.

The subject, may unconsciously use the outcome information as an anchor in a reconstructive-inferential process.

By looking at the test data we observed the following relation, where the HB depends on the subject's confidence in estimates and recalls:

$$HB = f\left(C^e, C^r\right)$$

We claim that the hindsight bias, HB, can depend positively on the subject's confidence in the estimate and negatively on the subject's confidence in recalling (meta-cognitive information). Whenever the subject showed a low level of confidence in his memory but a high level of confidence in his previous judgement, he was expected to exhibit a marked hindsight bias.

Awareness

1 **Confidence in estimate and estimate error relation.**

We verify that the relationship between the subject's perceived issue-familiarity, C_t^e, and the estimate error, M_t^e, are inversely proportional:

$$C_t^e = \left(\frac{1}{M_t^e}\right)$$

2 **Confidence in memory and memory error relation.**

Since the subject is supposed to be partially aware of his errors, we verify a negative relationship between the memory error and his confidence in recalling and:

$$C_{t+1}^r = \left(\frac{1}{M_{t+1}^r}\right)$$

3 **Confidence in memory and hindsight bias.**

We analyse the relationship between the confidence level in remembering, C_{t+1}^r and the HB; we observe a higher hindsight bias the lower the confidence level in remembering will be.

$$\left|\frac{\delta HB}{\delta C_t^r}\right| < 0$$

How can we foresee the subject's hindsight bias?

We recognise that the hindsight bias can play a crucial role in changing the subject's recalls and probably his future decisions. In order to detect and to avoid such undesired phenomenon we try to discover relationships between the subject's psychological profile and his attitude towards hindsight bias.

To predict the hindsight bias effects we proceed with a cluster analysis on the subjects' psychological profiles and revealed bias. We discover that the hindsight biased is positive related to the individual's degree of conflicts dealing with his investment goals and his personal abilities and tendencies.

By adopting an "ad hoc" psychological questionnaire mixing similar content but differently phrased questions, it is possible to forecast the subject's hindsight bias. In particular we design a contradiction index,

I^c which is a function of both the number of similar content questions answered in opposite ways and the distance (gap) between the answers scores.

$$I^c = f(N^i, Gap^i)$$

Where:

N^i identifies the number of similar content questions

Gap^i identifies the distance between the answers' score

The higher is the index the more probable will be the hindsight bias occurrence.

Conclusions

This study provides support for cognitive explanations of the individual's behaviour in an investment decision-making process. We analyse the subject's overall perceived error by focusing on the causal relations between the estimate and memory errors in determining the hindsight bias. We test PhD students in economics and financial advisors in an experimental setting. We ask the participants to estimate future economic scenarios and to decide accordingly how to invest their money after reading an economic article. They reveal hindsight bias whenever, after being informed about the real economic outcomes, their remembered estimates appear shifted toward the feedback. We clearly observe that investment estimates and decisions may be distorted by the hindsight bias. Indeed, half of the student subjects and two thirds of the financial managers involved in our tests tend to confuse their original predictions with the observed realisations. The hindsight bias can be so strong that it does not allow investors to recognise their errors; this fact may induce them to exhibit an excessive overconfidence in their predictions and decisions, revealing the "I knew it all along" distortion. Therefore, hindsight bias can have significant consequences on the investor's portfolio performance by altering his allocation perception and risk exposure. Thus, the way financial advisors and investors interact and communicate becomes increasingly important. Our research offers a significant empirical evidence to motivate all the subjects involved in an investment decision-making process to take into account the influence of hindsight bias.

We go one step beyond the existing literature by designing an experimental technique to elicit and to assess the investor's hindsight bias in an economic context and by inventing a useful tool to detect potentially biased investors.

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Estimate Phase		The Test time-table		
To be presented and repeated for each question				
Step	**Activity**	**Question**	**Subjects' Answer**	**Comment**
1	Read the experiment instructions			
2	Answer the question.	Prob. (Scenario 1)? Prob. (Scenario 2)? Prob. (Scenario 3)? Prob. (Scenario 4)?	$E_t(x_{t+1})$	Subject's original estimates about the expected economic scenario
3	Answer the question.	Confidence Level on Scenarios?	Flag on a graded scale from 1 to 10 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 1=min; 10=MAX Metacognition: c_t^e	Subject's degree of confidence in his original answer / estimate
4	Answer the question.	Stock returns? Bond returns? Real Estate returns?	$E_t(x_{t+1})$	Subject's original estimates about the expected investment returns

Step	Activity	Question	Subjects' Answer	Comment
5	Answer the question.	Confidence Level on returns?	Flag on a graded scale from 1 to 10 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 Metacognition: c_e	Subject's degree of confidence in his original answer / estimate
6	Answer the question.	Investment in stocks? Investment in Bonds? Investment in Real Estate?	$E_t(x_{t+1})$	Subject's original estimates about his investment decisions
7	Answer the question.	Confidence Level on Portfolio?	Flag on a graded scale from 1 to 10 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 Metacognition: c_e	Subject's degree of confidence in his original answer / estimate

Memory Phase	**Test Group**: subjects answer the questions after knowing the outcome information **Control Group**: subjects answer the questions without knowing the outcome information			
To be presented and repeated for each question				
Step	**Activity**	**Question**	**Subjects' Answer**	**Comment**
8	Answer the question.	Recall Prob. Scenario 1 Recall Prob. Scenario 2 Recall Prob. Scenario 3 Recall Prob. Scenario 4	$R_{t+1}\left[E_t(x_{t+1})\right]$	Subject's recalled estimates about the expected economic scenarios
9	Answer the question.	Confidence Level on Recall	Flag on a graded scale from 1 to 10 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 Metacognition: c_c	Subject's degree of confidence in his ability to recall

Step	Activity	Question	Subjects' Answer	Comment
10	Answer the question.	Recall your Confidence Level on Scenarios	Flag on a graded scale from 1 to 10 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 Metacognition: c_e	Subject's recalled degree of confidence in his original answer / estimate
11	Answer the question.	Recall stock returns Recall bond returns Recall real estate returns	$R_{t+1}[E_t(x_{t+1})]$	Subject's recalled estimates about the expected investment returns
12	Answer the question.	Confidence Level on Recall	Flag on a graded scale from 1 to 10 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 Metacognition: c_e	Subject's degree of confidence in his ability to recall
11	Answer the question.	Recall your Confidence Level on Returns?	Flag on a graded scale from 1 to 10 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 Metacognition: c_e	Subject's recalled degree of confidence in his original answer / estimate

Step	Activity	Question	Subjects' Answer	Comment
13	Answer the question.	Recall investment in stocks Recall investment in Bonds Recall investment in Real Estates	$R_{t+1}\left[E_t(x_{t+1})\right]$	Subject's recalled estimates about his investments decisions
10	Answer the question.	Confidence Level on Recall	Flag on a graded scale from 1 to 10 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 Metacognition: c_e	Subject's degree of confidence in his ability to recall
14	Answer the question.	Recall Confidence Level on Portfolio Investments?	Flag on a graded scale from 1 to 10 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 Metacognition: c_e	Subject's recalled degree of confidence in his original answer / estimate

time (t) → time (t+1)

At time (t)	→ delay time →	At time t+1)
Questions:		Questions:
$E_t(x_{t+1})$ Estimates	→ delay time →	$R_{t+1}[E_t(x_{t+1})]$ Recalled Estimates
C_e Confidence on Estimates 1 2 3 4 5 6 7 8 9 10	→ delay time →	C_r Confidence on Recalls 1 2 3 4 5 6 7 8 9 10
	→ delay time →	$R_{t+1}[c_e]$ Recalled Confidence on Estimates 1 2 3 4 5 6 7 8 9 10

Foresight → → → → → → → → → → → → → → → →

Is different from

Appendix 1

Estimate Phase Questionnaire

For the Test and Control Group

Subjects: Financial Managers

Name:____

Degree:_______________________________________

Introduction

The aim of the test is to understand some dynamics in decision-making.
The test will take place in two sessions: one today and a shorter one (5 minutes) in two weeks.

Today you'll be asked to read a passage where you will find some information dealing with your bank investor who needs your advices in order to decide what to do with his money.
You are asked to identify yourself in the story, to produce some economic estimates and to reveal your confidence in them.
Please try to think and to act as you usually do in your work.
It is very important for the test that you are very focused on your task.

If you help me in my research, I'll be glad to share my results with you at the end of the experiment.

Experiment Instructions:

Please fill in the form in all its parts by ticking the appropriate boxes.
You will be paid depending on the overall test performance: please write your name at the top of the questionnaire in order to receive the payment for the experiment.
The information provided in the test will remain strictly confidential.

Please do not communicate with any other student during the experiment.
This experiment is conducted for research purposes only.

It is very important that you understand these instructions.

Please, raise your hand if you have any question.

Thank you

It's the beginning of 2006 and you are 31 years old. You graduated six years ago in economics and now you work in a big information technology company. You live in Lisbon, Portugal, and have been engaged for three years with Anna, who works as a nurse.

Your salary increased considerably in the first years, but it has only slightly increased in the last two years.

Your deep ambition is to become an independent advisor.

One month ago you inherited € 50,000 from a distant relative you hardly ever visited.

Now your savings amount to € 80.000 and you know you can save approximately € 7,000 a year by working hard.

Before choosing how to invest your savings, you decide to look for some information on some investment opportunities: stocks, bonds and real estate.

You read a few articles in some financial papers and learn that:

- the economy growth rate rose from 0.5% in the first quarter of 2004 to 4% in the first quarter of 2005. The average rate of inflation in the same period was about 2.5%.
- In the last 15 months the reference rate has grown from 2% to 3.75% at the beginning of 2005 and it has almost reached what is assumed to be a neutral rate. The European Central Bank has announced that, in case of slowdown signals, it will not increase interest rates. The inflation scenario remains moderated, with very anchored expectations.
- In the labour market there are still unused resources and the economic growth rate is set to reach 3.5% in the second semester of 2005. The confidence indexes are however negatively affected by the high oil price.

- In the 2006 budget the government has announced a supply-side effort to increase competitiveness, especially for small and medium-sized enterprises.

You really want to make the best choice for your investments, so you decide to visit a friend you met at college, who now works as an economic advisor for a financial institution.

He tells you that:

- the real estate market in Lisbon has grown a lot in the last years, but now the prospects are unclear.
- The stock market is showing a positive trend accompanied by a high volatility due to the turbulent international events.

In the following days you carefully consider the several investment possibilities, taking into account the information you collected, and you try to estimate the attended real revenues (net of inflation), in order to optimally invest your savings.

- **Based on all the information you have been given, please assign a probability to each of these scenarios:**

	Possible Scenarios	%
1	the economy will grow with low inflation	%
2	the economy will grow with high inflation	%
3	the economy will stagnate with low inflation	%
4	the economy will stagnate with high inflation	%
		100%

Please write down your confidence level in your scenarios (0=min ; 10=MAX)

1	2	3	4	5	6	7	8	9	10

- **Please write down the real revenues you expect from:**

Stocks

	-30%	-20%	-10%	-5%	-1%	+1%	+5%	+10%	+20%	+30%	

Bonds

	-10%	-7%	-5%	-3%	-1%	+1%	+3%	+5%	+7%	+10%	

Real Estate

	-10%	-7%	-5%	-3%	-1%	+1%	+3%	+5%	+7%	+10%	

Please write down your confidence level in your real revenues estimates (1=min ; 10=MAX)

1	2	3	4	5	6	7	8	9	10

- **Please write down the quotas (%) of your savings you reserve for each type of investment (You must invest all your savings → 100%) :**

Bonds

10%	20%	30%	40%	50%	60%	70%	80%	90%	100%

Stocks

10%	20%	30%	40%	50%	60%	70%	80%	90%	100%

Real Estate

10%	20%	30%	40%	50%	60%	70%	80%	90%	100%

Please write down your confidence level in your portfolio (1=min ; 10=MAX)

1	2	3	4	5	6	7	8	9	10

What kind of investor are you?

In order to better understand your point of view, please give us some information about your investments experience. Please answer **Yes** or **No.**

		Yes	**No**
1	I have already invested in the stock market, therefore I can answer all the questions on the basis of my real experience.		
2	I do not have any real experience in the stock market, but I am interested in stock market issues. I will answer the questions only on the basis of my theoretical knowledge		
3	I don't have any real experience in the stock market, and I'm not interested in it.		

What is your experience in managing your money?

Please answer ticking the appropriate boxes (1=min (NO) ; 10=MAX(YES))

4	I'm very interested in economic issues	1	2	3	4	5	6	7	8	9	10	I don't Know
5	I think I'm competent in managing my money	1	2	3	4	5	6	7	8	9	10	I don't Know

6	**In the last five years I have decided to invest my money in:**			
	a. Savings account	Yes	No	I don't Know
	b. Deposits	Yes	No	I don't Know
	c. Single bonds	Yes	No	I don't Know
	d. Bonds funds	Yes	No	I don't Know
	e. Single shares	Yes	No	I don't Know
	f. Equity funds	Yes	No	I don't Know
	g. Mixed funds	Yes	No	I don't Know
	h. Index certificates	Yes	No	I don't Know
	i. Options, futures	Yes	No	I don't Know
	j. Real estate	Yes	No	I don't Know
	k. Real estate funds	Yes	No	I don't Know
	l. Others	Yes	No	I don't Know

What are your investments goals?

Please answer ticking the appropriate boxes (1=min (NO) ; 10=MAX(YES))

I invest my money in order:

7	to preserve my capital	1	2	3	4	5	6	7	8	9	10	I don't know
8	to increase my capital	1	2	3	4	5	6	7	8	9	10	I don't know
9	to increase my personal income with the investment revenues	1	2	3	4	5	6	7	8	9	10	I don't know
10	to save and buy something (car, house, etc.)	1	2	3	4	5	6	7	8	9	10	I don't know
11	to bequeath my money	1	2	3	4	5	6	7	8	9	10	I don't know

12	to "play" with some of my money buying and selling stocks	1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10	I don't know
13	Others (list here please):	1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10	I don't know

What criteria do you follow in choosing your investments?

Please answer ticking the appropriate boxes (1=min (NO) ; 10=MAX(YES))

14	I appreciate in particular the level of certainty it offers, even if I receive lower returns	1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10	I don't know
15	I don't care about losses in the short run if I can gain high revenues in the long run	1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10	I don't know
16	*I usually choose investments with high expected returns even if I must accept higher risks	1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10	I don't know
17	*I prefer to invest in my country's companies	1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10	I don't know
18	*I usually invest my money in companies that I know very well or that are quite famous	1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10	I don't know

19	*I choose above all to invest in companies that care about social commitment	1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10	I don't know
20	*I think the environment issue is very important in terms of a company's responsibilities	1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10	I don't know
21	*I usually choose an investment whose value has recently grown up considerably	1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10	I don't know
22	*I usually choose an investment whose value has recently decreased considerably	1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10	I don't know

How do you process information about financial markets?

Please answer ticking the appropriate boxes (1=min (NO) ; 10=MAX(YES))

23	I keep myself updated over the latest investment opportunities and financial markets	1	2	3	4	5	6	7	8	9	10	I don't know
24	Before taking an investment decision I need detailed information	1	2	3	4	5	6	7	8	9	10	I don't know
25	The more I'm informed on a investment, the more I'm ready to invest my money in it	1	2	3	4	5	6	7	8	9	10	I don't know
26	Before investing I read brochures available at the bank	1	2	3	4	5	6	7	8	9	10	I don't know
27	My investments are based on my partner's recommendations	1	2	3	4	5	6	7	8	9	10	I don't know
28	I trust my colleagues' financial advice	1	2	3	4	5	6	7	8	9	10	I don't know
29	*My financial decisions are strongly influenced by a professional adviser	1	2	3	4	5	6	7	8	9	10	I don't know
30	*I usually invest in what the press recommends	1	2	3	4	5	6	7	8	9	10	I don't know
31	*I usually invest in what the press doesn't recommend	1	2	3	4	5	6	7	8	9	10	I don't know

32	*After buying a stock, I later notice all the information that support my decision	1	2	3	4	5	6	7	8	9	10	I don't know
33	*I get angry if, after selling a stock, I learn positive information about the stock I have just sold	1	2	3	4	5	6	7	8	9	10	I don't know
34	*I very often say that, maybe unconsciously, I ignore those information that contradict my opinion about my investment	1	2	3	4	5	6	7	8	9	10	I don't know
35	*New information makes me immediately buy or sell an asset	1	2	3	4	5	6	7	8	9	10	I don't know

How do you generally behave?

Please answer ticking the appropriate boxes (1=min (NO) ; 10=MAX(YES))

36	I believe I am more successful than the others because I invest wisely	1 2 3 4 5 6 7 8 9 10	I don't know
37	I avoid those investments that could involve big financial losses	1 2 3 4 5 6 7 8 9 10	I don't know
38	I believe that women invest their money more wisely than men do	1 2 3 4 5 6 7 8 9 10	I don't know
39	I believe that women invest their money as wisely as men do	1 2 3 4 5 6 7 8 9 10	I don't know
40	I believe that women and men should learn a lot from each other on managing their money	1 2 3 4 5 6 7 8 9 10	I don't know
41	*My financial resources can sustain the last two years stock market crisis	1 2 3 4 5 6 7 8 9 10	I don't know
42	*My financial losses depend largely on misfortune	1 2 3 4 5 6 7 8 9 10	I don't know
43	*I have already lost some money invested in stocks simply because I was lazy and didn't "update" my investment strategy	1 2 3 4 5 6 7 8 9 10	I don't know
44	*After losing money, I tend to be more careful in my investments	1 2 3 4 5 6 7 8 9 10	I don't know
45	*I often buy single bonds (not mutual funds)	1 2 3 4 5 6 7 8 9 10	I don't know
46	*A great part of my success in stocks investment is due to chance	1 2 3 4 5 6 7 8 9 10	I don't know
47	*I have already lost money because I sold some assets too soon	1 2 3 4 5 6 7 8 9 10	I don't know
48	*I have already lost money because I sold some assets too late	1 2 3 4 5 6 7 8 9 10	I don't know
49	*I prefer not to sell assets at a loss	1 2 3 4 5 6 7 8 9 10	I don't know

50	*I think it appropriate to diversify my investments over different industrial fields and / or different countries	1 2 3 4 5 6 7 8 9 10	I don't know
51	I have lost a lot of money because of financial markets turmoil	1 2 3 4 5 6 7 8 9 10	I don't know
52	*The recent developments in the financial markets have made me more cautious in the way I invest	1 2 3 4 5 6 7 8 9 10	I don't know
53	Do you often forget what are you saying?	1 2 3 4 5 6 7 8 9 10	I don't know
54	Do you like playing sports?	1 2 3 4 5 6 7 8 9 10	I don't know
55	Do you often forget tour stuff around?	1 2 3 4 5 6 7 8 9 10	I don't know
56	Do you think you have a good memory?	1 2 3 4 5 6 7 8 9 10	I don't know
57	Do you like staying alone?	1 2 3 4 5 6 7 8 9 10	I don't know
58	When you put forward what you feel is a good idea, can you strongly support it?	1 2 3 4 5 6 7 8 9 10	I don't know
59	Do you like working in team?	1 2 3 4 5 6 7 8 9 10	I don't know
60	Are you usually sure about how you feel?	1 2 3 4 5 6 7 8 9 10	I don't know
61	Are you self-confident?	1 2 3 4 5 6 7 8 9 10	I don't know
62	Do you believe in destiny?	1 2 3 4 5 6 7 8 9 10	I don't know

Appendix 2

Memory Phase Questionnaire

For the Test Group

Subjects: Financial Managers

Name:____

Degree:______________________________________

Introduction

Today you'll be asked to read the second and final part of the passage and then try to remember the original estimates you gave in the first part of the test when you identified yourself in the story.

It is very important for the test that you are very focused on your task.

If you help me in my research, I'll be glad to share my results with you at the end of the experiment.

Experiment Instructions:

Please fill in the form in all its parts by ticking the appropriate boxes.
You will be paid depending on the overall test performance: please write your name at the top of the questionnaire in order to receive the payment for the experiment.
The information provided in the test will remain strictly confidential.

Please do not communicate with any other student during the experiment.
This experiment is conducted for research purposes only.

It is very important that you understand these instructions.

Please, raise your hand if you have any question.

Thank you

Financial Managers version:

January 2007: your client meets you in the bank after having read some articles dealing with the recent performances of the economy.

You realize that he desirers to better understand his investments performance and to talk with you about. He shows you that he knows that :

- the government's announcement of a supply-side effort to increase competitiveness, especially for small and medium-sized enterprises, played a good role in keeping the economy growing
- at the same time the Central Bank did not have to increase interest rates too much; they remain stable at around 4%. The inflation scenario remained quite moderate, with an average rate of 2.75%.
- You decide to read together some brief articles from the most famous financial newspaper and you discover that:
- the stock market has continued its steady upward trend (+10% a year);
- the real estate prices have finally reached a peak, and have now stopped increasing (+7% a year);
- bonds revenues are performing around the inflation rate (+3%).

Financial journalists say that the best performance investment portfolios for the last 12 months were composed of 40% Stocks, 20% Bonds and 40% Real Estate.

- **Please try to remember your original estimates on each of these scenarios:**

	Possible Scenarios	%
1	the economy will grow with low inflation	%
2	the economy will grow with high inflation	%
3	the economy will stagnate with low inflation	%
4	the economy will stagnate with high inflation	%
		100%

- **How sure are you about your ability to remember your original estimates? (1=min ; 10=MAX)**

1	2	3	4	5	6	7	8	9	10

- **Try to remember your confidence level in evaluating the scenarios**

(0=min ; 10=MAX)

1	2	3	4	5	6	7	8	9	10

- **Try to remember the original estimates you gave on the revenues you expected from:**

Stocks

	-30%	-20%	-10%	-5%	-1%	+1%	+5%	+10%	+20%	+30%	

Bonds

	-10%	-7%	-5%	-3%	-1%	+1%	+3%	+5%	+7%	+10%	

Real Estate

	-10%	-7%	-5%	-3%	-1%	+1%	+3%	+5%	+7%	+10%	

- **How confident are you about your ability to remember?**

1	2	3	4	5	6	7	8	9	10

- **Try to remember your confidence level in evaluating the real revenues**

(1=min ; 10=MAX)

1	2	3	4	5	6	7	8	9	10

- **Try to remember the quotas (%) of the savings you reserved for each type of investment:**

Bonds

10%	20%	30%	40%	50%	60%	70%	80%	90%	100%

Stocks

10%	20%	30%	40%	50%	60%	70%	80%	90%	100%

Real Estate

10%	20%	30%	40%	50%	60%	70%	80%	90%	100%

- **How confident are you about your ability to remember?**

1	2	3	4	5	6	7	8	9	10

- **Try to remember your confidence level in evaluating your portfolio (1=min ; 10=MAX)**

1	2	3	4	5	6	7	8	9	10

Appendix 3

Memory Phase Questionnaire

For the Control Group

Subjects: Financial Managers

Name:____________________________________

Degree:___________________________________

Introduction

Today you'll be asked to try to remember the original estimates you gave in the first part of the test when you identified yourself with the character of the story.

It is very important for the test that you are very focused on your task.

Experiment Instructions:

Please fill in the form in all its parts by ticking the appropriate boxes.
You will be paid depending on the overall test performance: please write your name at the top of the questionnaire in order to receive the payment for the experiment.
The information provided in the test will remain strictly confidential.

Please do not communicate with any other student during the experiment.
This experiment is conducted for research purposes only.

It is very important that you understand these instructions.

Please, raise your hand if you have any question.

Thank you

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- **Please try to remember your original estimates on each of these scenarios:**

	Possible Scenarios	%
1	the economy will grow with low inflation	%
2	the economy will grow with high inflation	%
3	the economy will stagnate with low inflation	%
4	the economy will stagnate with high inflation	%
		100%

- **How sure are you about your ability to remember your original estimates? (1=min ; 10=MAX)**

1	2	3	4	5	6	7	8	9	10

- **Try to remember your confidence level in evaluating the scenarios**

(0=min ; 10=MAX)

1	2	3	4	5	6	7	8	9	10

- **Try to remember the original estimates you gave on the revenues you expected from:**

Stocks

	-30%	-20%	-10%	-5%	-1%	+1%	+5%	+10%	+20%	+30%	

Bonds

	-10%	-7%	-5%	-3%	-1%	+1%	+3%	+5%	+7%	+10%	

Real Estate

	-10%	-7%	-5%	-3%	-1%	+1%	+3%	+5%	+7%	+10%	

- **How confident are you about your ability to remember?**

1	2	3	4	5	6	7	8	9	10

- **Try to remember your confidence level in evaluating the real revenues**

(1=min ; 10=MAX)

1	2	3	4	5	6	7	8	9	10

- **Try to remember the quotas (%) of the savings you reserved for each type of investment:**

Bonds

10%	20%	30%	40%	50%	60%	70%	80%	90%	100%

Stocks

10%	20%	30%	40%	50%	60%	70%	80%	90%	100%

Real Estate

10%	20%	30%	40%	50%	60%	70%	80%	90%	100%

- **How confident are you about your ability to remember?**

1	2	3	4	5	6	7	8	9	10

- **Try to remember your confidence level in evaluating your portfolio (1=min ; 10=MAX)**

1	2	3	4	5	6	7	8	9	10

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A New Experimental Approach to Bounded Memory and Biases in Information Processing

* I thank Andrea Beltratti, Alberto Bisin, Paolo Legrenzi for insightful discussions on this issue. I am responsible for all errors.

A New Experimental Approach to Bounded Memory and Biases in Information Processing

ABSTRACT

Drawing inspiration from A.Wilson's research paper[1], we investigate the connection between the results obtained by adopting a simulated bounded memory device and the psychological dynamics experimentally observed in an information processing framework.

Our aim is to empirically test the model hypotheses and verify their plausibility in a psychological and cognitive perspective.

The test participants are involved into economic decision-making processes; in particular, they are asked to make inferences basing on the information they receive. We provide them with several sequences of signals that are partially informative about the true "state of the economy". When this process is over, they are asked to take an action by revealing their inference. We offer them economic incentives: they are informed that their decision payoffs depend on the matching between their inferences and the true state of the economy.

We discover significant differences between the simulation results and the experimental observations that can be imported for a new version of the theoretical model.

[1] Bounded memory and biases in information processing.

1 INTRODUCTION

Throughout its history, finance theory has made simplifying assumptions regarding human behaviour. More recently, however, a big effort has been devoted to the experimental investigation of these assumptions, and the resultant behavioural finance has been introducing more realistic hypotheses in the current research paradigm. Indeed, recent research in cognitive science present new interesting results where the mind seems to work quite differently than traditional finance assumes. Knowledge and knowing are likely to be profoundly different from the forms in which they were presented by some extant models.

Within this perspective, the way individuals process the information at their disposal represent a fundamental issue for a better understanding of these dynamics.

In fact, every day individuals are asked to make choices and to decide over a large amount of issues. In order to do so, they collect data looking for the most reliable sources, trying to transform all the gathered signals into useful information.

All these tasks involve several cognitive mechanisms which constitute the information processing.

Over the time, philosophers, psychologists, physicists and game theorists were interested in this complex issue and, thanks to an interdisciplinary approach, the scientific literature has significantly grown up.

By focusing on cognitive psychology, information processing is an approach to the goal of understanding human thinking. It arose in the 1940s with Herbert Simon and the essence of its approach is to see cognition as being essentially computational in nature, with mind being the software and the brain being the hardware.

The information processing approach in psychology is closely allied to cognitivism[2] in psychology and functionalism[3] in philosophy although the

[2] In psychology, cognitivism is a theoretical approach to understanding the mind, which argues that mental function can be understood by quantitative, positivist and scientific methods, and that such functions can be described as information processing models.

[3] Functionalism is the dominant theory of mental states in modern philosophy: its core idea is that the mental states can be accounted for without taking into account the underlying physical medium (the neurons), instead attending to higher-level functions such as beliefs, desires, and emotions. According to functionalism, the mental states that make up consciousness can essentially be defined as complex interactions between different functional processes. Because these processes are not limited to a particular physical state or physical medium, they can be realized in multiple ways, including, theoretically, within non-biological systems.

terms are not quite synonymous.

Thanks to the great contribution of Claude Shannon, the parallelism with the artificial intelligence and with the computer science was suddenly realised. The main result of his theory is the noisy-channel coding theorem, which states that reliable communication is possible over unreliable channels. This means that it is possible to surround a noisy channel with appropriate encoding and decoding systems, such that messages can be communicated with an arbitrarily small probability of error.

Taking inspiration from this perspective, we try to identify a transposed meaning for this theorem in a simulated economic context as the one depicted by Andrea Wilson.

By considering financial markets as the noisy channel, composed by independent and heterogeneous atomised agents and, by realising that, "reliable" communications must be for sure possible, what can we consider the appropriate encoding and decoding information systems? What kind of "algorithms" do individuals use to manage the information? How do they agree or disagree on the information content?

We try to investigate the previous questions by starting from a very simple test. In particular, we design an experimental setup through which we analyse the way individuals extract information from a series of simple signals in an economic context. Being inspired by Andrea Wilson's research[4], we verify how realistically the presented theoretical model is able to capture the overall individual's information processing.

Beliefs and Probabilities.

To make predictions precise, individuals are usually assumed to use the laws of probability in structuring and revising beliefs about uncertainties. Recent evidence, mostly gathered by psychologists, suggests that probability theories might be inadequate descriptive models of individual choice.

Evidence of judgment bias, reported by experimental researches, poses an implicit challenge to economic theory based on rationality. The most recent research programs are addressed to understand the causes of the empirical findings.

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[4] See the references.

Therefore, more interest has been shown in experimental investigation of these assumptions, and the resultant behavioural finance has been presented as a significant departure from the current research paradigm.

Since our cognitive resources are limited, we cannot analyse the data the environment provides us with optimally. Instead, natural selection has designed minds that implement rules-of-thumb (algorithms, heuristics, or mental modules) selectively to a subset of cues (see Simon 1956). Such heuristics are effective when applied to appropriate problems. But their inevitable biases can become flagrant when used outside their ideal domain of applicability.

Economists often argue that errors are independent across individuals, and therefore cancel out in equilibrium. However, people share similar heuristics, those that worked well in our evolutionary past. So on the whole, we should be subjected to similar biases. Systematic biases (common to most people, and predictable based upon the nature of the decision problem) have been confirmed in a vast literature in experimental psychology.

There is much debate about exactly how good a job heuristics do. While psychologist such as Daniel Kahneman and Amos Tversky have made clear that heuristics can play a positive role, in the last decade, evolutionary (Darwinian) psychologists have strongly emphasised the adaptiveness of cognitive processes. In many cases biases diminish but do not vanish. Indeed, there is no guarantee that financial decision problems will be presented to individuals in a manner that favors the most accurate decisions[5].

The modern environment differs greatly from the prehistoric environment of evolutionary adaptation for which human cognitive mechanisms were designed by natural selection. Modern humans deal with new abstractions such as securities, money, impersonal markets, probabilities, etc.

This framework can provide guidance as to which biases identified in experiments represent general mechanisms, and which are conditional side-

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[5] See the experiment results.

effects[6].

Gigerenzer presenting one of his famous research "Fast and frugal heuristics that make us smart" refers to cognitive resources frugality as an essential evolutionary aspect.

Information processing and beliefs updating.

The information processing main result is the "coherent" creation of new beliefs and conjectures. Neuroscientists describe this phenomenon as the final effect of the brain plasticity; the capacity of transforming the synapse circuitries depending on the new information received.

There is a strong relation between the way individuals perceive new information and the way they develop new beliefs. The opposite relation is also true: the individuals' a priori beliefs may condition the way information is received and managed.

The aim of our research is to analyse how individuals change their beliefs over the time depending on the signal they receive.

If we consider that rational beliefs are a necessary condition for rational behaviour, while irrational beliefs are a sufficient condition for irrational behaviour, we can figure out how important is the information processing and its impact in the beliefs updating process.

Belief revision and belief update

During the tests, we analyse how belief revision and belief update can realise in an experimental framework.

We focus on the dynamics individuals update their beliefs as a result of receiving new information. Two instances of this general phenomenon have been studied in detail. Belief revision (Alchourr Gardenfors, & Makinson,1985; Gardenfors,1988) focuses on how an agent should change her set of beliefs when she adopts a particular new belief.

Belief update (Katsuno&Mendelzon,1991a), on the other hand, focuses on how an individual may change her beliefs when she realizes that the world has

[6] Explanations based upon cognitive adaptiveness are subject to the objection that it is too easy to come up with "just-so" stories that fit the data ex post. However, our goal is not to make the case that the evidence supports the adaptiveness approach (see Barkow, Cosmides, and Tooby (1992)). Rather, our point is that it is hard to make sense out of biases without conceptual framework and we retain that adaptiveness is the most significant.

changed.

Both approaches attempt to capture the intuition that an agent should make minimal changes in her beliefs in order to accommodate the new belief. The difference is that belief revision attempts to decide what beliefs should be discarded to accommodate a new belief, while belief update attempts to decide what changes in the world led to the new observation[7].

In the following sections we will focus on how belief revision and belief update can be represented.

Belief adoption and its constraints

It is a fact of life that we cannot believe everything we observe or that we are told. We accept a given information or datum as a belief on the basis of our previous beliefs, of its evidences, supports and sources, and of other psychological factors.

Here we will present some crucial points of these cognitive mechanisms.

Our knowledge base is not a file where one can introduce new data or eliminate a file-card without altering the other data. Our beliefs are integrated, interconnected and mutually supported: to drop a belief or to add a new one entails checking its coherence with other beliefs and revising previous knowledge. The belief-belief coherence and support is quite a well studied problem in philosophy and artificial intelligence (truth maintenance systems; belief revision and updating; argumentation) and in some cognitive agent architectures.

There are in fact two schools in belief revision (Harman 1986; Gärdenfors 1988; Doyle 1992): the "foundations approach" stressing the importance of supports and justifications of beliefs, and the "coherence approach" modelling logical compatibility and coherence.

[7] 'Decision' to believe is a strange 'decision; it is not a true voluntary deliberation, based on computation of advantages and disadvantages. On the contrary (Castelfranchi 1995) it is impossible to consciously decide (and to induce somebody) to believe something on the basis of rewards and advantages or on the basis of threats. We cannot consciously arrive to believe just because it is convenient to believe (Pascal). 'Decision' to believe is more a procedural decision. Otherwise it would be better to assume as a principle: Believe something if you do not have reasons for rejecting it. We cannot prevent ourselves from believing in a source, unless we suspect that there is "something wrong" in it (Castelfranchi 1997). In order to reject the information of a source (even of a social one) we must believe that there is "something wrong" in that source.

Source reliability and belief credibility

On the one side, believing in a source depends on its assumed reliability.

On the other side, the credibility of a piece of knowledge (a candidate belief) is a function of its sources.

The basic principles governing credibility can be summarised as follows:

1 if the source is reliable, its information is credible and it is believed. If it is not reliable, its information is not credible and it is not believed. In quantitative terms: the more reliable the source the more credible the information provided.

2 The bigger is the number of converging (independent) sources, the more credible is the information provided. Any convergent source of knowledge 'confirms' the other (in particular: S2 confirms S1, when S2 is a new source for a previous item whose source was S1).

Confirmation.

Confirmation is a fundamental cognitive 'integration' among sources. It consists on the fact that after the arrival of a 'confirming source', the belief results more stable, safe, and certain, and the subject is more sure and convinced about it; not only the belief is more 'credible', but also the confirmed source is more reliable, trusted.

Confirmation is a very important psychological phenomenon: when we control or check something, we are just looking for confirmatory sources; proactive behaviour, expectations, and goals imply some 'confirmation' mechanism. There is a very well studied 'confirmatory bias' in our cognition which is not only due to the cost of revising our knowledge (economic motivation) but also to our need for control and to our need to trust our knowledge and our ability to make predictions[8] (Bandura 1982).

Importance

The literature on belief revision presents a big difference between two properties of integrated beliefs (Cantwell 1996; Castelfranchi 1996; Paglieri 2004): their importance and their credibility.

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[8] apart from self-deception and defence mechanisms.

They are two distinct dimensions and notions: a belief could be very important but not very credible; or very credible but absolutely marginal.

By the term 'important' we mean that it will explain a lot; it will be very useful for understanding and integrating other information; 'Credible' means that it presents a lot of evidence, sources, supports to believe it.

The two aspects are distinct but interconnected: an integrated belief in a belief network is in fact both supported and supporting.

We call "credibility" the new belief capacity to be supported by external sources; we consider "plausibility" its possibility to be supported by internal sources. We call belief "importance" the new belief explanatory power.

When we are asked to decide whether to accept a new belief or to maintain an older one, both 'credibility' and 'importance' features play a significant role in the decision process. The fact that a belief is highly 'important', hence it explains and supports a lot of other beliefs, can be a strong reason for accepting it, and a strong reason for not changing / abandoning it. In fact, abandoning an important belief entails a lot of expensive revisions in our mental map.

Plausibility

To be believed, something should be 'plausible'. Often we resist or reject a new belief just because it is not 'plausible'.

What is the basis of this kind of evaluation? Why is it a basis for rejection? Plausibility is the credibility value assigned to the incoming signal from an inside perspective. It is evaluated just on the basis of the individual's previous knowledge, i.e. the knowledge it has to be integrated with.

Thus, we have two credibility values, one based on external sources, the other based on the niche that has to accept the new item. Metaphorically, one is the value provided by the offering agent that 'gives' the item, the other is the value attributed by the accepting (or refusing) agent. When there is a conflict and a difficulty to accept the new belief from outside, the conflict is between its 'external credibility' and its 'plausibility'. To accept a new belief the plausibility, i.e. the internal source, should 'confirm' the external source, and converge with external credibility; or, at least, external credibility should be stronger that implausibility.

All the presented aspects are extremely important in understanding how information is expected to spread out through the markets. We adopt an experimental approach in order to directly investigate a simple but very interesting phenomenon, the human information processing in an economic framework.

In particular, our results reveal how different can be considered the mechanisms determining the human behaviour from the algorithms employed in the theoretical simulations.

Introducing the model

Before presenting the results of our experimental approach we need to introduce and comment the main features of the theoretical model we are going to test. In particular we analyse in detail the information processing which drives the simulated economic agent in pursuing his aim: to discover the true state of the economy by using all the available signals.

Therefore, we will present the available signals, the bounded memory and the updating process. Hence, we will discover the psychological biases the model is able to simulate and its conditionings for the agent's decisions.

The states of the economy

The model assumes that there are two ex ante equally likely states of the economy. They can be "High" or "Low", depending if the economy is growing or if it is in recession: $S \in \{L, H\}$.

The state of the economy is unknown to the decision-maker and remains the same throughout the development of the model and hence, the test.

The signals

The model considers a decision-maker who receives a sequence of binary signals: there are two possible signals, a low signal, l, or a high signal, h: $s \in \{l, h\}$.

Signals are independent and independently distributed over time (i.i.d.). Each signal provides partial information about the state of the economy, S.

The signal information-content depends on the reliability condition represented by the following conditional probability:

$$\Pr(l|L) = \Pr(h|H) = \rho > \frac{1}{2}$$

The previous expression states that the "high" signal, h, is more likely when the economy is growing, therefore, in state H; while the "low" signal, l, is more likely when the economy is in recession, therefore, in state L. The model assumes that signals are partially informative of the states of the economy.

The memory device

The model describes a decision-maker whose memory can be represented by a set of N memory states/cells[9], $\mathrm{N}=\{1,2,....,\mathrm{N}\}$. The number of signals that the decision-maker can recall and process strictly depends on the memory capacity, hence on the number of the available memory cells. Therefore, the N memory cells may correspond to a set of N different beliefs that the decision-maker can have.

We represent the memory device by the following representation:

L H

1	2	3	4	5	6	7	8	...	N

The memory updating process

The model assumes that the memory updating process is characterised by a simple "rule of thumb" the agent is expected to use to store and to manage information in the overall decision-making process. Because of the bounded memory, the decision-maker cannot recall all the information he receives and therefore, he cannot perform as a Bayesian agent.

In particular, we can consider the decision maker's memory system like a fact-based memory: this means that in memory state i, the agent behaves as if he remembers $(i-1)$ $high$-signals, and $(\mathrm{N}-i)$ low-signals.

After receiving a new h-signal, he moves up to state $(i+1)$, where his new beliefs replace one of the $(\mathrm{N}-i)$ signals. So, if he receives new information when his memory is "full", he can either ignore it, or replace a previously stored fact with the new signal.

A standard Bayesian decision-maker is instead able to base his decisions on the entire sequence of signals and he is not influenced by their order.

[9] By the term "memory states" we mean "memory cells".

In detail, the memory updating process is characterised by:

- a set of N available memory states;
- an initial distribution $g_0(i)$ which describes the memory cell from which the information process begins in period zero.
- a transition rule σ, which indicates the direction the memory updating process follows, depending on the type of incoming signal and the previously occupied memory cell;
- an action rule $a(i)$, which indicates the decision-maker's final choice. When the memory updating process ends, the decision-maker must take a decision in relationship to his current memory state. Therefore, the final memory state characterises the decision-maker's decision which is represented by the inference about the state of the economy[10].

Memory transition rule

The transition rule $\sigma(i,s)(j)$ describes the way the memory processes the incoming information. Whenever the decision-maker receives a new signal, the memory updates itself by moving to the next memory state. This transition is described by the stationary signal processing rule:

$$\sigma(i,s)(j): \qquad (\mathrm{N} \quad x \quad \mathrm{K}) \qquad \rightarrow \qquad \Delta(\mathrm{N})\sigma$$

Where, $\sigma(i,s)(j)$ processes: (Memory States x Signals) $\rightarrow$ Memory States. It indicates the probability that the agent moves from the memory state i to the memory state j after receiving the signal s.

Transition probability

For a given transition rule, $\tau_{i,j}^{S}$ denotes the transition probability between the memory states conditional on the real state of the economy, $S \in \{L, H\}$, where:

$$\tau_{i,j}^{H} = (1-\rho)\sigma(i,l)(j) + \rho\sigma(i,h)(j)$$

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[10] The final output of the overall decision-making process is the inference about the state of the economy.

represents the transition rule between memory states i and j when the economy is growing, H.

$$\tau^{L}_{i,j} = \rho\sigma(i,l)(j) + (1-\rho)\sigma(i,h)(j)$$

represents the transition rule between memory states i and j when the economy is in recession, L.

Transition matrix

Define T^S to be a N x N transition matrix with (i,j)th entry $\tau^{S}_{i,j}$.

Let $g_t(i|S)$ show the probability that the decision-maker will be in memory state i at the beginning of period t, conditional on the state of the economy S. This notation suppresses dependence on the choice of (σ, g_0).

Let $g_t^S \in \Delta(\mathrm{N})$ be the vector with ith component $g_t(i|S)$, then:

$$g_t^S = g_{t-1}^S T^S = g_0 \left(T^S\right)^t$$

It describes the overall beliefs updating dynamics.

The beliefs updating process length.

At the beginning of each sequence of signals, there is a probability η that the memory updating process terminates and therefore, the decision-maker is asked to take an action by revealing his inference about the state of the economy. We focus on the case where η is close to zero, as presented by the model; this approximates the situation in which the agent expects to receive a long sequence of signals, but is not exactly sure when the process will end.

On the contrary, with probability $(1-\eta)$ the decision-making process continues and the agent will be exposed to further signals.

The parameter η is important for the existence of the agent's optimal strategy.

Action rule

Let $a : \mathrm{N} \to [0,1]$ indicate the action rule, where $a(i)$ represents the agent's probability of choosing action H, therefore, to infer that the economy is growing while he is occupying the i-th memory cell. The action rule strictly depends on the occupied memory cell, it does not take into account any other explanatory variables.

Payoffs

The payoffs depend on the correct decisions, and they are strictly related to the agent's ability to make the right inference about the true state of the economy. In particular, the decision-maker obtains a payoff of 1 if his inference is right, so when his decision matches the state, or 0 if it does not.

The model also assumes that the expected payoff is strictly increasing in the signal information content, therefore:

- a completely uninformative signal, $\left(\rho=\frac{1}{2}\right)$, yields an expected payoff of $\frac{1}{2}$ for all N,
- a perfectly informative signal, $(\rho \rightarrow 1)$, yields expected payoff $\rightarrow 1$.

Optimal strategy

The simulated agent's optimal strategy can be described by the following procedure. First, the memory device orders the memory states by increasing payoffs, therefore, $j<i$ implies that $\pi(j)<\pi(i)$.

Then, if (g_0,σ,a) represents the optimal memory updating rule, hence, the payoff corresponding to this strategy must be the highest for all the possible strategies with $\Pi(g_0,\sigma,a) \geq \Pi(g_0^{'},\sigma^{'},a^{'})$ for all $(g_0^{'},\sigma^{'},a^{'})$.

Thus, the decision-maker is convinced that the economy is in recession, state L, when he is using the memory cell number 1, while he is convinced that the economy is growing, state H, when is using the memory cell number N.

In sum, the model assumes that $\Pi^{*}(\eta,\mathrm{N})$ represents the decision-maker's optimal payoff; it is a function of η,N. In particular, it is increasing in N, and decreasing in η: hence, the bigger is the memory capacity and the shorter is the time devoted to the decision-making process, the highest will be the payoffs.

With these features the model satisfies the need for a parsimonious and prompt memory device representation.

1.1 Model hypotheses

The model presents several assumptions that the optimal memory process must satisfy. In the following sections we will present them with more details.

1. Polarization

The longer is the length of the overall decision-making process ($\eta \to 0$), the lower is the probability of moving away from the two extreme polarising memory states, 1 and N:

$$1 > \hat{\tau}^{S}_{1,1} > 1-\varepsilon \qquad \text{and} \qquad 1 > \hat{\tau}^{S}_{N,N} > 1-\varepsilon$$

This means that the two extreme memory states represent poles of attraction in the memory updating dynamics.

Since the probability of moving from the extreme memory states 1 *and* N is positive for $\eta > 0$, but tends to zero as $\eta \to 0$, therefore, one implication is that for small η, so for long lasting decision-making processes, the decision-maker will spend most of his time in these two extreme memory states.

In these cases, the decision-maker ignores new information with high probability.

Of course, outsiders cannot observe the decision-maker's memory states: this means that from their point of view, the decision-maker will appear as simply unresponsive to new signals most of the time.

This model assumption will also contribute in representing the confirmatory bias and the overconfidence / underconfidence biases that will be presented in the next sections.

It is important to recognise another key difference between the bounded memory agents and a Bayesian one: the former starts to ignore new information more quickly when he reaches the extreme states than the latter does.

2. Beliefs Updating Dynamics and Optimal solution.

The updating memory process moves only toward adjacent memory states after new information is received. In particular, at any memory state $i \in N \setminus \{1, N\}$, the updating memory process moves with probability 1 to:

- the next highest state, $(i+1)$, after an h-signal;
- the next lowest state $(i-1)$, after an l-signal.

This occurs for each memory states belonging to the memory device, therefore, for all $i \in N \setminus \{1, N\}$:

$$\hat{\sigma}^S(i,h)(i+1) = \hat{\sigma}^S(i,l)(i-1) = 1$$

3. Neutral a-priori belief

The model assumes that the decision-maker holds neutral a-priori belief, hence, the memory process starts in the central memory cell, $\frac{N+1}{2}$, with certainty, as represented by the following equation:

$$\hat{g}_0\left(\frac{N+1}{2}\right) = 1$$

4. Deterministic action rule

The model assumes that the agent chooses his inference according to a deterministic rule. The decision maker can choose both the inferences L or H while is using the central memory cells, while he must choose action L in all lower cells and action H in all higher cells.

$$\hat{a}(i) = \left\{ \begin{array}{lll} 0 & if & i \leq \frac{N-1}{2} \\ & & and \quad \hat{a}\left(\frac{N+1}{2}\right) \in \{0,1\} \\ 1 & if & i \geq \frac{N+3}{2} \end{array} \right\}$$

The following theorem summarises the model features.

Theorem.

The theorem states that adopting a memory device whose capacity is bigger than three units/cells, there exists a time interval $\bar{\eta}$, such that an optimal memory updating process can realise.

Let $N \geq 3$. For every $\varepsilon > 0$, there exists $\bar{\eta}$ such that whenever $0 < \eta < \bar{\eta}$, the memory updating process reveals:

1. Polarization

$$1 > \hat{t}^{S}_{1,1} > 1 - \varepsilon \qquad \text{and} \qquad 1 > \hat{t}^{S}_{N,N} > 1 - \varepsilon$$

2. Deterministic Beliefs Updating Dynamics

$$\text{For all } i \in N \setminus \{1, N\}, \qquad \hat{\sigma}^{S}(i,h)(i+1) = \hat{\sigma}^{S}(i,l)(i-1) = 1$$

3. Neutral a-priori Belief

$$\hat{g}_0\left(\frac{N+1}{2}\right) = 1$$

4. Deterministic Action Rule

$$\hat{a}(i) = \left\{ \begin{array}{lll} 0 & \textit{if} & i \leq \dfrac{N-1}{2} \\ & & \textit{and} \quad \hat{a}\left(\dfrac{N+1}{2}\right) \in \{0,1\} \\ 1 & \textit{if} & i \geq \dfrac{N+3}{2} \end{array} \right\}$$

Simulated Biases

Now we analyse more in details the psychological biases the model represents.

Confirmatory bias.

An individual tends to display a confirmatory bias whenever his initial belief induces him to pay too much attention to self-confirming signals and, therefore, to pay too little attention to opposing evidence: he simply ignores it, or even interprets it as supporting evidence. As he becomes more convinced of his initial hypothesis, he will probably disregards any information which contradicts that hypothesis.

The model represents the confirmatory bias by showing that once the decision-maker goes beyond a "threshold of confidence", therefore, whenever he occupies the extreme memory state 1 or N, he starts ignoring any opposing evidence with probability close to 1. This event can also occur at the beginning of the information processing, may be because the agent holds a non neutral a priori initial belief which induces him to reach very soon the polarising extreme memory cells 1 or N.

In the intermediate state $N \setminus \{1, N\}$, the agent cannot ignore evidence, and this implies that an initial string of h-signals can promptly convince him that state H is true. Moreover, if his initial belief is closer to state N and he receives a sequence of information which contains enough h-signals, he will probably think that the state N is true in a shorter time, therefore that the economy is growing.

It is important to note that for a Bayesian decision-maker, the order of the signals does not matter at all, while, on the contrary, for the bounded memory decision-maker, the order of the signals has a big impact on his belief updating process.

By manipulating the parameters η, T, τ, the model can replicate behavioural biases, such as the first impression bias and the last impression bias.

First impression bias: at the beginning of the information processing, the agent exhibits a fast beliefs updating dynamics by promptly reacting to each new signal. Information is processed in a way that puts too much weight on early signals.

Last impression bias.

In the long run the simulated polarization bias slows down the beliefs updating process. Note that since states 1 *and* N are essentially absorbing, the agent, provided with any long enough sequence of signals, will probably either get stuck in state N (if he reaches state N before 1), or get stuck in state 1 (if he reaches state 1 before N).

Then for finite sequences of information, we can choose a value for η[11] small enough that once the decision-maker reaches an extreme state, the probability of leaving this memory state before the end of the process is close to zero.

The model assumes that for fixed η, it is possible to choose a time interval T large enough that last impressions dominate first impressions. This means that the memory extreme states polarise the beliefs updating process. This is true by the ergodic theorem: for T sufficiently large, the probability distribution over memory states g_t is independent of the initial distribution.

In other words, the first impression effect/bias wears off after sufficiently long sequences. Instead, the last impression effect/bias is independent of the length of the sequence (because of the previously mentioned transition matrix ergodicity property).

Overconfidence / Underconfidence bias

The simulated bounded memory decision-maker will display two types of overconfidence/underconfidence biases:

- one based on the signals sample size;
- one based on the quality of the information that he receives.

The decision maker will typically be overconfident after short sequences of signals since he promptly reacts to each new single signal before

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[11] It represents the probability that the decision-making process ends and the agent must reveal his inference.

reaching the extreme polarising memory states 1 and N. Therefore, the model assumes that, at the beginning of the process, beliefs adjust very rapidly after each new signal.

On the contrary, the decision maker will be underconfident after long sequences of signals because of the polarisation bias. The agent is not expected to promptly react after each signal while he is occupying the extreme memory states; in this case, the agent will reveal complete unresponsiveness to new information.

Also the quality of the information may characterise the beliefs updating process; in particular, the more reliable are the signals, the more dynamic should be the overall process.

2 FROM THE THEORETICAL TO THE EXPERIMENTAL APPROACH

Neural networks, artificial intelligence techniques and agent-based simulations usually offer great insights in studying complex "issues" like human heuristics and biases.

Testing the simulation results trough an experimental approach represents an other complementary activity which is very important in a research program. In fact, it allows to reconcile the "normative behaviour" predicted by the model, with the observational data gathered during the experiments.

Hence, we design an experimental setup to evaluate the theoretical model hypotheses by analysing the subjects' behaviour in different contexts.

In particular, we test the following biases assumed or predicted by the model:

- the neutral a priori belief;
- the deterministic beliefs updating dynamics;
- the deterministic action rule;
- the polarisation bias;
- the confirmatory bias;
- the over/under confidence biases;
- the first/last impression biases.

The information processing dynamics.

The following steps summarise the information processing followed by the subject:

1. he receives the test instructions;
2. he is asked to reveal his initial a priori belief;
3. he is informed about the reliability of the signals he is going to receive;
4. he receives a sequence of signals;
5. he is asked to reveal his inference about the true state of the economy[12].

[12] All these steps, occur for each sequence of signals.

2.1 Testing the model hypotheses: the empirical approach.

1 **The neutral a-priori belief**

Before presenting any signals, we ask the subject to reveal his a-priori belief about the state of the economy.

This information allows us to observe the way the subject's a-priori initial belief can actively condition his further information selection process. In fact, we can observe whether:

- the same sequence of signals, provided to the subjects with different initial beliefs, can move their beliefs in opposite directions;
- the same sequence of signals, provided to the subjects with identical initial beliefs, can move their beliefs in the same directions.

According to the model hypotheses, we would expect to observe the agents to choose, at the beginning of each sequence, a neutral a priori belief, therefore, the central cells of the graded scale.

2 **The deterministic Beliefs Updating Dynamics**

Our aim is to collect information about the process followed by the individuals in updating their beliefs. To do so, we ask each subject to reveal his belief about the inferred state of the economy, after being exposed to each new single signal, by ticking a graded scale (see the following table).

By ticking box 1, the subject shows that he is strongly convinced that the economy is in recession, he expects state L, while, by ticking box 10, the subject shows that he is strongly convinced that the economy is growing, he expects state H. In order to collect independent data over time, a new graded scale is provided to the subject, whenever a new signal appears on the screen; in this manner, we do not offer any external support to keep track of his previous inferences.

The graded scale adopted for the experiment

L									H
1	2	3	4	5	6	7	8	9	10

According to the model hypotheses, we would expect to observe that the subjects update their beliefs after having received any new signal.

In particular, the model predicts that they would select an adjacent higher cell after having received a high signal or, an adjacent lower cell, after a low signal.

Testing the biases simulated by the model

1 The polarization bias

First, we inform the subject that he will be exposed to a sequence of signals[13] without telling him anything about the length of the sequence.
By looking at the relations between the submitted sequences of signals and the beliefs revealed by the subjects, we look for the possible existence of a persistent inference, therefore, the beliefs polarization bias.
We then compare the inference revealed by the subjects during the tests with those simulated by the model.

2 The confirmatory bias

We study the relation between the subject's a priori initial belief and his tendency to pay stronger attention to the self-confirming (identical) signals he will receive. In particular, we ascertain whether:

- subjects, with identical a priori initial beliefs, are induced to identical inferences once exposed to the same sequence of signals;
- subjects, with different a priori initial beliefs, are induced to different inferences once exposed to the same sequence of signals.

We also analyse whether an individual, who does not reveal neutral a priori beliefs, is more sensitive to those signals which are compatible with his a priori belief.
According to the model hypotheses, we would expect that, subjects with non neutral a priori beliefs would differently weight the incoming signals, depending on their a priori initial beliefs (priors).

3 The overconfidence / underconfidence biases

This simulated bias is the result of a two-parameter interaction in the model: the memory capacity and the signals quality.
Hence, a two-dimension experimental analysis is required.

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[13] According to the model parameter $\eta \rightarrow 0$ which characterises a long information processing, we inform the subject that he will be exposed to different sequences of signals during the test.

We develop several tests where we manipulate both the signals sample size and the signals reliability.

By varying the number of signals composing each independent sequence[14], we change the signals sample size[15].

By altering the quality of the signals, we vary their reliability about the true state of the economy[16].

At the beginning of each new sequence, the subjects are informed about the reliability of the signals. We provide them the following information:

$$\Pr(l|L) = \Pr(h|H) = X\%$$

By manipulating the reliability coefficient, the value of X, we present different sequences of signals. For example, if X>50%, it means that signal h is more likely in state H, and signal l is more likely in state L.

According to the model hypotheses we would expect that the more reliable is the signal, the more dynamic is the beliefs updating process; that is, the more informative is the signal about the state of the economy, the less polarized will be the subject's beliefs.

4 First impression biases

We analyse whether the rate in the beliefs updating process changes depending on the length of the sequence of signals. Moderated by the signals reliability effect, we expect to observe a more dynamic beliefs updating process at the beginning of the sequence and a slowing down rate, the longer the sequence will be.

5 Last impressions bias

As for the previous bias, we test whether the rate in the beliefs updating process changes, depending on the length of the sequence of signals. Moderated by the signals reliability effect, we expect to observe a less dynamic beliefs updating process at the end of the sequence, as predicted by the model.

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[14] The first, the third and the fifth sequences are composed by 21 signals, while the second by 12 signals and the fourth by 16.

[15] We also investigate the relations between the signals sample size and their presentation order to detect the polarization bias; see the tests results.

[16] The fourth and the fifth sequences are composed by signals reliable at 20%; the second and the third sequences are composed by signals reliable at 55%; the first sequence is composed by signals reliable at 80%.

3 The test

Hypotheses

Accordingly to our research aims, we test the following hypotheses presented by the theoretical model:

1. the neutral a priori beliefs;
2. the deterministic belief updating dynamics;
3. the polarization;
4. the confirmatory bias;
5. the overconfidence / underconfidence biases;
6. the first and last impression biases.

3.1 Method

Participants

The participants were 16 master students majored in Economics from Bocconi University.

Test Description.

The experiment takes place at the Master in Finance at Bocconi University in Milan. When subjects arrive, they are seated at the tables and separated from each other for the duration of the experiment. They are then given a set of instructions which are read out loud to them after they have had a chance to read them individually.

They are informed that five independent sequences of signals were projected on a big screen in the classroom. Each signal in the sequence is informative about the state of the economy they are asked to infer. Signals are represented by up or downward sloped arrows.

Subjects are not informed about the aim of the test.

Each experimental session lasts about 25 minutes.

We offer an economic incentive to make participants pay attention to the signals and reveal the right inference. The participants knew that they would be compensated according to their ability to infer the true state of the economy, by receiving €0.20 for each right inference after each signal.

We built 5 different sequences of signals, in order to verify the model assumptions and simulated biases. The sequences differ in length, in signals composition and in signals reliability.

Stimulus materials

We design 5 different and independent sequences of signals on a big screen. Each signal is represented by a graphic image, an upward or downward oriented arrow[17].

At the beginning of each sequence, we ask the subject to reveal his a priori initial belief about the state of the economy.

After that, we inform him about the reliability of the sequence of signals he is going to receive. In particular, we ask him to pay attention to each new single signal appearing on the screen and, to reveal his inference about the state of the economy by ticking the corresponding cell on the graded scale. The subjects is informed that, by ticking the cell number 1 he retains that the economy is in recession, while by ticking the cell number 10 he retains that the economy is growing.

The graded scale

L ... H

1	2	3	4	5	6	7	8	9	10

We collect the subject's inference after having projected each single signal.

At the end of each sequence, we distract the subject with some music in order to let him relax and "wash out" his memory.

Procedure and design

In order to experimentally test the model assumptions, we randomise the state of the economy (H or L) by using a generator of random numbers. After that, we produce five sequences of signals, according to the designed signal reliability. Hence, we use, once again, a generator of random numbers according to a binomial distribution, to produce the signals composing each sequence.

[17] See the appendix for the details about the experiment materials.

The test

The following table summarises the features characterising each sequence of signals. They differ in length, signals composition and signals reliability.

We choose the length of the sequences in order to keep high the subject's attention and, hence, not boring him during the task.

We also decide the reliability coefficients in order to design two opposite cases: one in which signals are significantly reliable (the first, the fourth and the fifth sequence) and one in which they are not (the second and the third).

Furthermore, by adopting two different coefficients for the reliable signals, 80% and 20%, we also investigate the role of the information format and its impact in the overall information process[18].

The following table reports, in the first column, the sequence under analysis, in the second, its length, in the third, the randomly drown state of the economy and in the fourth, its composing signals reliability.

As we can see, the second and the third sequence present identically reliable signals, but they differ in length. They allow us to identify the role of the sequence length in conditioning the subject's information processing.

The first, the third and the fifth sequence are composed by the same number of signals, but they differ in their signals reliability. They allow us to identify the role of the signal reliability in conditioning the subject's information processing, no matters how many identical signals he will receive.

The first, the fourth and the fifth sequence present reliable signals but they differ in the format trough which this information is provided[19]. They allow us to identify the role of the information format in conditioning the subject's information processing.

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[18] This occurs in the fourth and in the fifth sequence, as summarised by the table.

[19] See the tests results for a more detailed analysis.

Sequence N°	Sequence Length	State of the Economy	Signals Reliability $\Pr(h\|H)=\Pr(l\|L)$
1	21	H	80%
2	12	L	55%
3	21	L	55%
4	16	H	20%
5	21	H	20%

Design.

Dependent measures

In order to investigate the biases simulated by the model, we study the impact of the following variables:

- the subject's a priori initial belief;
- the reliability of the signals;
- the format of the information;
- the order of the signals;
- the length of the sequence;
- the inference revealed by the subject;
- the shifts in the beliefs updating process conditional on the previously occupied cell and the incoming signal.

3.2 Empirical Results

The neutral a priori beliefs

The instructions provided at the beginning of the experiment, clearly informed the participants that the two states of the economy (H = the economy is growing and L = the economy in recession) are ex ante equally likely. Therefore, we were expecting that each subject would have reported a neutral a priori initial belief.

By looking at the data we discover that, the average value depicting the subjects' initial belief is 5.34.

At first sight, it appears that the model "a priori neutral initial belief" hypothesis can be supported by the empirical evidence.

Nevertheless, the average of the observed values hides the fact that, only 57% of the subjects selected the central cells of the graded scales. If we also analyse the standard deviation, we discover that the answers are widely dispersed around their means, revealing a consistent heterogeneity in the subjects' answers.

Hence, we can note that, the observed empirical evidence does not allow us to sustain the model "neutral a priori belief" hypothesis.
On the contrary, we observe that a significant number of subjects, 43%, reveal non neutral initial beliefs, neglecting or misunderstanding the information provided by the test instructions.
The following table reports, in the first column the sequence of signals under analysis, in the second and the third columns the subjects' average initial belief and its standard deviation, in the last two columns the number of subjects who choose the neutral central cells.
In the last row, it presents the experiment data descriptive statistics.

Sequence	Initial Belief		N° of subjects choosing the central cells	% of subjects choosing the central cells
	Mean Value	Standard Deviation		5 | 6
1st	5.76	1,48	6/13	46%
2nd	5.06	1,48	9/15	60%
3rd	5.64	1,15	9/14	64%
4th	5.78	1,80	8/14	57%
5th	4.78	1,92	8/14	57%
Overall Mean	5,40	1,56		57%

We also check whether the subjects, who reveal neutral a priori belief in the first sequence of signals, they continue presenting neutral initial beliefs in the following sequences as well.
We observe that, the remaining subjects who do not reveal neutral a priori belief in the first sequence of signals, they continue presenting non neutral initial beliefs in all the other sequences.

The following table reports in details the subjects' a priori initial belief at the beginning of each sequence of signals; the last two columns show the average initial belief and its standard deviation for each subject.

Subject N°	Initial Beliefs In the Series					Mean Initial Belief	Initial Belief St.Dev.
	1st	2nd	3rd	4th	5th		
1	3	5	7	3	6	4,8	1,7
2	4	4	7	5	5	5	1,2
3	5	5	5	5	5	5	0
4	5	5	5	5	5	5	0
5	5	5	5	5	5	5	0
6	5	5	5	5	5	5	0
7	6	4	5	6	4	5	1
8	6	3	7	9	1	5,2	3,1
9	6	5	5	5	5	5,2	0,4
10	7	2	6	7	4	5,2	2,1
11	7	6	5	9	10	7,4	2,0
12	8	8	5	5	5	6,2	1,6
13	8	6	4	4	3	5	2
14		7	8	8	4	6,75	1,8

Belief updating dynamics

In order to test whether the model predictions were coping with the test observations, we compare the beliefs updating process simulated by the model with those revealed by the subjects.

We start our analysis by studying the correlation between the simulated results and the empirical data.

The following table summarises, on top, the main features of each sequence of signals, therefore, the signal reliability and its length. In the second part, it presents the correlation coefficients between the simulated and observed behaviours, calculated for each single subject.

The label "N.A" stands for "not available", while the asterisk represents what we called, "super rational" subjects, subjects who answered by presenting a constant choice, corresponding to the signal reliability coefficient. For example, the subject number 2 answered by constantly choosing the cell number 5 which corresponds to the signals reliability coefficient for that sequence (55%)[20].

	Sequences				
	Sequence N° 1	Sequence N° 2	Sequence N° 3	Sequence N° 4	Sequence N° 5
Reliability	80%	55%	55%	20%	20%
Lenght	21	12	21	16	21
Subjects	Correlation Coefficients				
s1	0,133428	0,772619	0,073332	0,894864	0,827727
s2	0,172675	*	*	0,215271	0,260178
s3	0,285776	-0,47045	-0,27144	0,05886	0,5844
s4	0,030131	0,415221	0,234091	-0,65267	-0,61636
s5	0,014364	0,385333	*	0,215271	0,260178
s6	0,193942	*	*	0,25505	0,287481
s7	0,708382	0,966	0,857431	-0,0292	-0,60401
s8	0,357174	0,946877	0,87634	-0,00794	0,913092
s9	0,318131	-0,2715	-0,28213	-0,70763	0,231572
s10	0,655522	0,909371	0,33322	0,899406	0,929704
s11	N.A.	-0,20587	0,571262	-0,72669	-0,95201
s12	0,142455	0,689787	0,294534	-0,41527	0,121591
s13	0,190989	N.A.	0,592421	-0,74732	-0,84182
s14	N.A.	N.A.	N.A.	-0,77866	-0,23374
s15	0,09384	N.A.	N.A.	-0,11299	0,109099
s16		0,030475	N.A.		

In order to better understand the predictive efficacy of the theoretical model, we proceed with a more detailed analysis, by splitting the collected data in two categories. In particular, as reported by the next table, on the left we group the positive correlation coefficients, those which support the model predictions, while on the right, we group the negative correlation coefficients, those which do not support the model predictions.

By doing so, we can clearly highlight the data in favour of the model.

[20] See the appendix for the tables reporting the tests results.

	Sequences						Sequences				
	N° 1	N° 2	N° 3	N° 4	N° 5		N° 1	N° 2	N° 3	N° 4	N° 5
Reliability	80%	55%	55%	20%	20%	Reliability	80%	55%	55%	20%	20%
Length	21	12	21	16	21	Length	21	12	21	16	21
Subjects	Positive Correlation Coefficients					Subjects	Negative Correlation Coefficients				
s1	0,133	0,773	0,073	0,895	0,828	S1					
s2	0,173	*	*	0,215	0,26	S2					
s3	0,286			0,059	0,584	S3		-0,4704	-0,271		
s4	0,03	0,415	0,234			S4				-0,653	-0,616
s5	0,014	0,385	*	0,215	0,26	S5					
s6	0,194	*	*	0,255	0,287	S6					
s7	0,708	0,966	0,857			S7				-0,029	-0,604
s8	0,357	0,947	0,876		0,913	S8				-0,008	
s9	0,318				0,232	S9		-0,2715	-0,282	-0,708	
s10	0,656	0,909	0,333	0,899	0,93	s10					
s11	N.A.		0,571			s11		-0,2059		-0,727	-0,952
s12	0,142	0,69	0,295		0,122	s12				-0,415	
s13	0,191	N.A.	0,592			s13				-0,747	-0,842
s14	N.A.	N.A.	N.A.			s14				-0,779	-0,234
s15	0,094	N.A.	N.A.		0,109	s15				-0,113	
s16		0,03	N.A.			s16					
N° of observ.	13	8	8	6	10	N° of observ.	0	3	2	9	5
% of observ.	100%	72,7%	80%	40%	66,6%	% of observ.	0%	27,3%	20%	60%	33,3%
Descriptive statistics											
Mean	0,254	0,639	0,479	0,423	0,453	**Mean**	/	-0,3159	-0,277	-0,464	-0,65
St. Deviation	0,316	0,325	0,315	0,419	0,455	**St. Deviation**	/	0,3341	0,4898	0,5055	0,583
Range	0,694	0,936	0,803	0,841	0,821	**Range**	/	0,2646	0,0107	0,7707	0,718
Min	0,014	0,03	0,073	0,059	0,109	**Min**	/	-0,4704	-0,282	-0,779	-0,952
Max	0,708	0,966	0,876	0,899	0,93	**Max**	/	-0,2059	-0,271	-0,008	-0,234

Quantitatively, the data supporting the model[21] represent 70,31% of the overall data set, while the remaining 29,68% of the data do not confirm the model predictions.

Qualitatively, the data supporting the model present, on average, a correlation coefficient equals to 0.43, while the observations which do not support the model present, on average, a coefficient equal to -0.49. Therefore, even if the data supporting the model are more numerous than those which don't, their strength is lower. Nonetheless, the empirical evidence does not allow us to sustain the model "deterministic beliefs updating rule" assumption, since the average correlation coefficient calculated on the overall data set is 0,16%.

To further investigate other possible determinants of the beliefs updating process, we analyse the impact of the signal reliability, the perceived signal reliability and the sequence length.

[21] We refer to the positive correlation coefficients.

Signal reliability analysis

We continue our analysis by studying the relation between the correlation coefficients and the signal reliability. We discover that the more reliable are the signals, the less efficient is the theoretical model in predicting the empirical data.

To investigate this relation we choose two sequences, the first and the third, both constituting by the same number of signals (21 items) but with a different signal reliability (80% and 55%). Then, we correlate their coefficients with their signal reliability: the result is -0,33.

We also replicate this analysis by using another couple of sequences with identical length (21 items) but different signal reliability (20% and 55%): the fifth and the third. Even in this case, we correlate their coefficients with their signal reliability: the result is -0,12.

Therefore, we note that, the model reveals its inefficiency in predicting the human beliefs updating process with reliable signals since it does not allow the simulated agent to process, in a different manner, differently reliable signals. We also note that, in the second case, few subjects correctly realise that signals reliable at 20% are very reliable (the complementary event signal is, in particular, reliable).

In particular, we discover that the signal reliability may characterise different beliefs updating dynamics, depending on the way this information is provided to the subject, therefore, depending on its format.

Indeed, the format through which this information is provided plays a significant role. In fact, if we consider for example the expression $\Pr(h|H)=\Pr(l|L)=X\%$, with values of X greater than 50%, the subjects clearly and easily understand the meaning of the conditional probability. It may be because the subjects usually look for data that confirm their expectations; in particular, we note that this kind of format helps them in doing their job.

But when we consider values of X lower than 50%, $\Pr(h|H)=\Pr(l|L)=20\%$, the subjects seem not to correctly interpret this information. They are unable to transform the provided reliability condition into a more comfortable information for their self-confirming research purpose. They do not succeed in recognising that the previous conditional probability,

$\Pr(h|H)=\Pr(l|L)=20\%$, can be read as $\Pr(l|H)=\Pr(h|L)=80\%$. In particular, they fail in realising the conditional probability for the complementary event; they do not recognise that the complementary event is still a reliable signal at 80%.

We observe that while processing the fourth sequence of signals, only 40% of the subjects seem to clearly understand the meaning of the signals reliability condition and hence, to react accordingly. Our result is also sustained by the fact that, the standard deviation is more than double in respect to the previous case whenever $x=80\%$; this fact reveals a higher degree of heterogeneity in the information processing among the individuals whenever they observe signals reliable at 20%.

This is a fundamental result: subjects reveal to better understand explicit and simple information, those requiring the lowest cognitive effort. The proof is that, even if the required effort is negligible in comparison with the economic incentives they receive, the subjects are not induced to transform and to decode the provided information into an easier format in order to take the right decision.

Sequence length analysis

We proceed our analysis by investigating the role of the sequence length in the beliefs updating process. Hence, we further study the relation between the correlation coefficients and the length of the sequence. We decide to compare couples of sequences constituting by equally reliable signals but different in their length.

By comparing the second and the third sequence, we discover that, the longer is the sequence of signals, the less efficient is the theoretical model in predicting the empirical data: (corr=-0,55); this occurs for signals reliable at 55% because of the polarisation bias simulated by the model but not identically observed in the reality.

On the contrary, whenever the signals are not perceived as clearly reliable (20%), as in the fourth and fifth sequence, the longer is the sequence of signals, the more efficient is the theoretical model in predicting the empirical data: (corr=(0,46).

In this case, we observe that subjects reveal a big heterogeneity in their answers and data are more dispersed around their mean. Data are more noisy in this case than before.

To summarise we report the average correlation coefficient for each sequence of signals, between the simulated and observed beliefs updating processes. The following table reports the results.

Series	Signals Reliability $\Pr(h\|H)=\Pr(l\|L)$	Sequence Length	Correlations between simulated and observed beliefs updating processes
1st	80%	21	0,2959
2nd	55%	12	0,6554
3rd	55%	21	0,2553
4th	20%	16	-0,364
5th	20%	21	0,3273

Cluster analysis

Since none of the observed subjects reveal an overall identical beliefs updating path to the one predicted by the model, we decide to investigate whether the model can correctly predict, at least, a series of single shifts in the beliefs updating process revealed by the subjects. Therefore, we decide to move from a global analysis approach, toward a local analysis of the data.

In order to better evaluate the way subjects update their beliefs, we adopt a method inspired by the cluster analysis. In particular, we invent an inequality measure that helps us in transforming the data revealed by the subjects[22] into a binary value[23]. Our aim is to group the tests observations according to the characteristics we are interested in. Each object within the cluster will be similar to every other object, and different from objects in other clusters. In other words, homogeneity is maximized within clusters and heterogeneity is maximized between them.

[22] Values from 1 to 10 on a graded scale.

[23] The binary measure is represented by 0 or 1 values: 1 if the observation respects the required condition, 0, vice versa.

Our cluster is composed by observations which respect the model's predictions in order to detect whether the subject follows the model prescriptions in updating his beliefs. To do so, we substitute[24] the subject's original inference with:

1 if the subject chooses a one step higher value on the graded scale after having received a high signal;
0 if the subject chooses a one step lower value on the graded scale after having received a high signal;

1 if the subject chooses a one step lower value on the graded scale after having received a low signal;
0 if the subject chooses a one step higher value on the graded scale after having received a low signal;

With this method we are allowed to understand how many time each subject behaves as the model predicts.
By calculating the mean of the previously transformed data for each subject, we verify whether if the value is greater than 0.5, the subject tends to confirm the predictions presented by the model, or, on the contrary, whether it does not occur.

Our research for locally coherent beliefs updating dynamics gives us the following results:
The model correctly predicts on average:
the 31% of the steps in the beliefs updating processes for the first sequence of signals;
the 28% of the steps in the beliefs updating processes for the third sequence of signals;
the 21% of the steps in the beliefs updating processes for the fifth sequence of signals;

Therefore, since the model efficacy in predicting the human beliefs updating process is always lower than one third, no matters the

[24] We proceed with this substitution in those sequences with perceived reliable signals.

reliability coefficient of the signals, we can't find support for the model prediction.

We also note another significant result: the subjects present a marginal decreasing response sensitivity to a series of identical signals as predicted by Shannon's Information Theory.
Subjects slowly update their beliefs when they are shown a series of identical signals but, surprisingly, they promptly react to the first different signal they receive.

Furthermore, subjects clearly keep in mind the signals reliability information during the overall test; in fact, if for example they are informed that the signals are reliable at 80%, they never choose the extreme graded scale cell number 10 as hypothesised by the model, but at least, they choose on average the cell number 8.
This is another significant difference with the model predictions, whose decision-maker is expected to reveal the beliefs polarisation bias, while, he is reaching the highest or lowest memory cells, without taking into account the signals reliability.
Even in this case, the experimental data differ from the theoretical model predictions.

Beliefs polarization.
The model assumes that polarization occurs when the beliefs updating process reaches the extreme memory "cells"; in that case, the decision maker is expected to exhibit a persistent and stable inference over the time, whatever the signal he may receive.
The experimental data do not reveal the same kind of beliefs polarisation; we must introduce two important distinctions.
The first is that the subject presents a persistent inference when he receives a series of identical and reliable signals, while the simulated decision-maker presents the bias after just a few and, mostly important, not necessarily reliable signals.
The second difference is that, the longer is the sequence of identical signals the subject receives, the slower is the beliefs updating process.

Instead, the model predicts a constant rate in the beliefs updating process, no matter what is the sequence of signals.
During the experiment we detect a sort of "marginal decreasing sensitivity" to identical signals. Only at the beginning of the process, the subject seems to behave as a Bayesian agent but only when, for example, he is receiving a sequence of identical signals. On the contrary, once the subject receives a new and different signal, he strongly reacts to it by revealing, on average, a significant shift[25] in the beliefs updating process while the model predicts a persistent inference due to the strong beliefs polarisation bias.
The observed phenomenon in compatible with the process called adaptation in neuroscience; neurons reveals slower and slower reactions when they are receiving identical signals; instead, they promptly react by producing a much more dynamic spikes train when the incoming signals change.

We can summarise the empirical results as follows:

- The subjects reveal to be more and more unresponsive, the longer is the sequence of identical signals, showing a marginal decreasing sensitivity;
- the longer is the sequence of reliable signals, the bigger is the shift in the belief updating process revealed by the subject once he receives a new different signal.

These phenomena occur symmetrically for both the high and low signals.
The following table summarises the experimental results.

Our method consist in detecting, within each sequence of signals, one or more subsequences of identical signals in order to observe the subject's behaviour in these occurrences. In particular we analyse how does a subject react to differently long sequences of identical signals.
Through the analysis of each single sequence of signals we detect whether there were sub-sequences of identical signals; the following table second column reports the length of each subsequence. Then we look for the subject's most frequently chosen graded scale after being exposed to the

[25] By the term "belief shift" we mean that the subjects promptly react to the new signals by jumping to a more distant graded scale than the adjacent one.

sub-sequence of identical signals. For example, the first sequence is composed by 21 signals; we can recognise 3 sub-sequences of identical signals composed by 3, 4 and 8 signals. The fourth column reveals, for each single subsequence, the most frequent inference preferred by the subject. We can easily recognise that once the subject is presented even a short series of reliable signals he reveals a level of confidence similar to the signal reliability.

Sequence	Signals Reliability $\Pr(h\|H)=\Pr(l\|L)$	Number of following identical signals	Most frequent inference	Average Inference	Average shift in the beliefs process
1st	80%	3	8 (1,46)[26]	7,62 (1,35)	3 (2,56)
		4	8 (1,39)	8,07 (1,16)	3,46 (2,55)
		8	8 (1,50)	8,39 (1,04)	N.A.[27]
3rd	55%	2	5[28] (1,39)	3,93 (1,41)	1 (1,30)
		4	5 (1,51)	4,21 (1,49)	0,86 (0,99)
5th	20%	4	8 (2,52)	6,2 (2,2)	2,6 (2,69)
		11	8	8 (3,22)	2,33 (2,58)

- **Confirmatory bias**

We analyse the relation between the subject's a-priori initial belief and his sensitivity to identical/compatible signals. In particular, we relate the subject's beliefs updating dynamics with the provided sequence of signals, taking into account his revealed initial belief. We detect

[26] Standard deviations are reported in parentheses.

[27] The data are "Not Available" since the sequence ends with the subsequence of identical signals and the subject had not the possibility to receive a different signal before the end of the subsequence.

[28] Six over fifteen subjects do not allow us to calculate the mode value since they were not presenting identical inferences.

whether the individual reveals an higher sensitivity to signals compatible with his a priori initial belief or, whether he reacts in a similar way to any signal.
For reliable sequences of signals[29], we note that subjects who reveal an initial belief in favour for example of state H (they expect that the economy is growing) are more sensitive to high signals.

In particular, if we consider all the participants involved in the test, we do not observe a significant correlation (0,379) between the a priori initial belief and the average sensitivity to identical/compatible signals. It means that, on average, the subjects do not react differently to the signals depending on his initial belief.
But, if we consider the subjects who reveals a non neutral initial belief, we note an interesting correlation (0,695) between their a-priori beliefs and the average sensitivity to identical/compatible signals.

- **Overconfidence / Underconfidence Biases**

We also test the overconfidence/underconfidence model simulated bias. We observe that subjects reveal a more dynamic beliefs updating at the beginning of the process; it slows down only when they are exposed to a sequence of identical signals.

As for the previous test[30] we observe that, even after a short sequence of identical signals, the subjects become very convinced of their inferences by revealing levels of confidence close to the signals reliability coefficients. Therefore they "jump" on their (supposed) correct inference after a few signals showing a very dynamic beliefs updating process.

The data also reveal that the subjects, after having received a series of identical signals, they tend to overreact to a new different signal.
This phenomenon remains totally unpredicted by the model. In fact, the model does not take into consideration different beliefs updating dynamics for series of identical or diverse signals.

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[29] Even in this case the reliability condition must be expressed in a easily comprensibile form.
[30] the model deterministic belief updating dynamics.

Another observation dealing with overconfidence concerns the reliability level of the signals. The more reliable the signals are, the more reactive the subject will be in updating his beliefs. Indeed, the average gap/jump between adjacent cells is about 1.1 for reliable signals, and 0.3 for unreliable ones.

First and Last Impression Bias

As we mentioned before, the most important dynamics driving force is represented by alternating signals, whenever they are significantly reliable. The sequence length seems not to play an important role even though it is very difficult to identify it in comparison with the signals order effect. Our results showed that on average, the subjects continue reacting to signals in the same way, no matter the sequence length.

How to model the results

Even if the sample of observations is quite small, we identify some explanatory variables for the subjects' behaviour.

We observe that, a subject i, at time t, reveals a level of confidence in his inferences, C_t^i which is positively related to both the signal reliability and to the simplicity of its format:

$$C_t^i = f(signal \quad reliability, \qquad signal \quad comprehensibility)$$

Therefore, the more reliable is the signal and the more comprehensible is its format, the more convinced will be the subject about his inference.

In fact, we observe that the subject, after having received a set of n identical signals he reveals a confidence level in his inference that tends to be equal to the signal reliability coefficient:

$$C_t^i \rightarrow signal \quad reliability$$

The number of identical signals that induces the subject to feel himself convinced about his inference is negatively related to both, the signal reliability and, the simplicity of its format.

$n = f(signal\ \ reliability,\ \ \ signal\ \ comprehensibility)$

In this case, the more reliable is the signal and the more comprehensible is its format, the smaller will be the number of identical signals needed by the subject to become convinced about his inference.

We note that, on average, even a reliable signal represented in an unclear format renders its information content not comprehensible, and therefore, it results to be misunderstood.

In fact, the more complex is perceived the signal, the bigger is the effort required by the subject to decode it, and therefore, the lower will be its perceived reliability.

We introduce the term "perceived reliability" to indicate a new measure which takes into account the objective signal reliability and its subjective comprehensibility.

$$\text{perceived reliability} = \phi^{I}\ .\ \ signal\ \ reliability \cdot signal\ \ comprehensibility$$

Where ϕ^{I} represents the subject's idiosyncratic ability in detecting the information.

Furthermore, we can identify the polarisation bias by observing that once the subject receives a number s greater than n[31] of identical signals[32], he will update his beliefs at a lower rate, becoming less and less sensitive to further identical signals. We call this phenomenon the subject's decreasing marginal sensitivity to identical signals.

Therefore we can expect that:

$$if\ \ s < n\ \ \Rightarrow fast\ \ beliefs\ \ updating\ \ process$$
$$if\ \ s > n\ \ \Rightarrow slow\ \ beliefs\ \ updating\ \ process$$

We also note that the polarisation bias occurs not necessarily in the extreme memory cells, as predicted by the model, but in others cells of the graded scale, depending on the reliability of the signals perceived by the subject.

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[31] n represents the number of identical signal that induces the subject to reveal a confidence in his inferences which tends to be equal to the signal reliability coefficient.

[32] Identically reliable and comprehensible.

In fact, when the signals are perceived as highly reliable, the most frequently chosen cells are the extreme ones, while when the signals are not reliable, the middle cells are preferred.

We discover another important result: the subject's impulse - response function, R_t^I. It specifies the subject's beliefs updating process whenever the subject, after being previously exposed to a series s of identical signals, receives the first different signal.
We note that the longer is the sequence of identical signals, the stronger will be his reaction in updating his beliefs once he receives a new and different signal.

$$R_t^I = f(s)$$

Steps	Activity Description	Questions and Tasks	Received Data	Comment
1	Read the experiment set up.			
	Answer the question.	What do you think about the state of the economy?	| L | H |	Ex-ante (initial) belief – before being exposed to any signals
2	See the signal (item)	(h) or (l)		
3	Answer the question. To be repeated after each signal	What do you think about the state of the economy?	Flag on a graded scale L H | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |	Subject's inference about the state of the economy (Low or High)

4 CONCLUSIONS

The agent's rationality is one of the most important features of classical economic literature, but it seems to be more the result of the models "ad hoc" assumptions than the result of real observations.

With this research we try to better define the relationship between the model simulations and the test observations in the information processing framework.

We pursue to check whether the bounded rational agent, described by the theoretical model, can cope with the results we obtain in our tests; therefore, we focus on the way individuals process economic information in a very simple setup.

Subjects were involved in a series of test, where, they have to infer the real state of the economy, after having being exposed to sequences of differently reliable signals.

The data we gather demonstrate the existence of common heuristics and, therefore, biases, between the individuals.

By relating the empirical evidence with the theoretical model predictions, we discover that the subjects involved in our tests behave quite differently from the model predictions.

The first aspect we note is that, individuals hold non neutral beliefs when they are processing new information. This means that, may be unconsciously, the subjects select and react to new signals in different ways. We recognise that individuals' a priori beliefs play an active role even when they are testing their assumptions. This means that, individuals give more weight to self confirming signals in respect to signals which do not sustain their a priori initial belief.

Another important observation is that, the subjects do not follow a deterministic beliefs updating rule, as predicted by the model, but they present interesting common path in managing the information process. In particular, we observe that they pay more attention to the more reliable signals; in particular they pay more attention to those signals which are reliable and easy to understand, so whenever the format of the information is easy to process. Another important observation is that, the participants pay a decreasing attention to the nth identical signal, therefore, the longer is the sequence of identical signals they receive, the lower is their frequency in updating their beliefs. They do not follow an identical "rule of thumb" in processing the signals over the time, as prescribed by the model; instead, they reveal a beliefs updating dynamics which is very reactive; it only slows down when they are presenting identical signals.

We can reconsider all the test results by sharing the Gigerenzer perspective according to which, human being developed fast and frugal heuristics over the centuries, and our minds are embedded with this "software". Individuals, even in the information processing, try to economise the effort required for the task in order to reach a satisfying level of confidence in their assessments.

All these aspects highlight the importance of the information processing as one of the main constituting task of the decision making process. With our research we succeed in verifying the model assumptions by Andrea Wilson, and thanks to our results we can present new model upgrades to render "the machinery" closer to the human behaviour. Therefore, by identifying statistically significant strategies followed by our testers in managing the information, we are able to offer new insights to renew the model by taking into account the subjects' cognitive tendencies.

All these results have been possible by the adoption of a simple but efficient cluster analysis which helped us in making comparison between the simulated agent and the human subjects.

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Appendix

The experimental data

	First sequence of signals: State of the Economy = H; Signals Reliability = 80%																					
Signal Number:	Ex Ante	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
Signal Type:		1	1	1	0	1	1	1	1	0	0	1	1	0	1	1	1	1	1	1	1	1
Bounded	5	6	7	8	7	8	9	10	10	10	9	10	10	10	10	10	10	10	10	10	10	10
Mean	5,5833	7	8	8	5	8	8	8	9	5	4	7	8	4	7	8	8	8	9	9	9	9
St.Dev	1,379	1	2	1	3	2	1	1	1	3	2	2	1	2	2	1	2	1	1	1	1	1
Mod.Dist.	0,5833	1	1	0	2	0	1	2	1	5	5	3	2	6	3	2	2	2	1	1	1	1
Correlation	0,2875																					
s1	5	8	8	9	10	8	9	10	10	7	6	6	7	5	6	7	8	9	9	10	10	8
s2	5	8	8	8	2	8	8	8	8	2	2	8	8	2	8	8	8	8	8	8	8	8
s3	7	7	8	8	8	9	9	9	10	8	7	7	7	7	6	7	9	9	10	9	10	9
s4	6	8	10	10	9	7	9	10	10	6	4	4	6	5	7	9	10	10	10	8	8	10
s5	8	8	8	8	2	8	8	8	8	2	2	8	8	2	8	8	8	8	8	8	8	8
s6	5	9	9	9	2	9	9	9	9	2	2	9	9	2	9	9	9	9	9	9	9	9
s7	5	8	10	10	8	10	10	10	10	9	8	9	10	9	10	10	10	10	10	10	10	10
s8	3	4	4	5	4	4	5	5	6	5	3	3	4	3	3	4	4	5	6	6	7	7
s9	6	7	8	6	5	6	7	8	9	4	3	6	7	5	8	9	10	8	9	10	10	7
s10	4	5	6	7	4	6	7	7	8	5	4	7	8	5	8	9	9	9	10	10	10	10
s11																						
s12	6	8	8	8	2	8	8	9	9	2	2	8	8	2	8	8	8	8	8	8	8	8
s13	7	8	8	8	8	9	7	7	7	7	7	7	8	8	8	9	9	9	9	8	9	9
s14																						
s15		8	8	8	2	8	8	8	8	2	2	8	8	2	8	8	8	8	8	8	8	8
s16																						

Second sequence of signals: State of the Economy = L; Signals Reliability = 55%													
Signal Number:		1	2	3	4	5	6	7	8	9	10	11	12
Signal Type:		1	0	0	0	0	1	0	0	0	1	1	0
Bounded	5	6	5	4	3	2	3	2	1	1	1	2	1
Mean	4,9231	5	5	4	4	4	4	4	3	3	4	5	5
St.Dev	1,5525	2	1	1	1	2	1	2	2	2	2	2	2
Mod.Dist.	0,0769	1	0	0	1	2	1	2	2	2	3	3	4
Correlation	0,6784												
s1	5	6	5	4	2	2	4	3	2	1	3	4	4
s2	5	5	5	5	5	5	5	5	5	5	5	5	5
s3	2	2	2	2	1	3	4	4	1	3	6	5	4
s4	4	5	6	4	5	6	5	4	4	3	3	4	5
s5	8	5	5	5	5	5	5	5	5	5	5	5	5
s6	5	5	5	5	5	5	5	5	5	5	5	5	5
s7	5	8	7	5	4	3	4	3	2	1	2	3	1
s8	5	5	5	4	4	3	3	3	2	2	2	3	3
s9	3	3	3	4	4	3	5	1	3	2	5	8	9
s10	4	5	3	3	2	2	3	2	2	1	2	2	1
s11	7	8	5	6	5	8	6	7	7	7	6	8	8
s12	5	6	5	5	4	4	5	5	4	4	4	5	5
s13													
s14													
s15													
s16	6	5	4	4	3	5	4	4	3	5	6	5	5

Third sequence of signals: State of the Economy = L; Signals Reliability = 55%; Sequence Length = 21																						
Signal Number:		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
Signal Type:		0	0	0	0	1	0	0	1	1	0	1	0	0	1	0	1	0	1	1	0	1
Bounded	6	5	4	3	2	3	2	1	1	2	1	1	1	1	2	1	1	1	2	3	2	3
Mean	5,7692	5	4	4	4	4	4	4	5	5	5	5	5	4	5	4	5	5	5	5	5	5
St.Dev	1,0919	1	1	2	2	1	1	2	1	2	2	2	2	2	2	2	2	1	2	2	2	2
Mod.Dist.	0,2308	0	0	1	2	1	2	3	4	3	4	4	4	3	3	3	4	4	3	2	3	2
Correlation	0,3238																					
s1	5	3	3	2	1	3	2	3	5	5	4	5	3	2	3	3	3	3	3	3	3	3
s2	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
s3	6	6	5	8	7	5	5	6	7	6	8	8	7	8	9	7	5	6	8	8	5	5
s4	5	5	3	3	1	1	2	1	3	3	3	4	4	2	3	3	6	5	6	6	3	5
s5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
s6	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
s7	5	4	3	2	1	2	1	1	2	2	1	2	1	1	2	1	1	1	2	2	1	1
s8	7	7	6	6	5	5	5	4	4	5	5	5	5	4	4	4	4	4	5	5	5	5
s9	7	2	1	1	1	4	4	2	7	9	7	10	6	7	8	7	9	6	8	9	9	10
s10	7	6	6	5	5	6	5	5	6	6	5	6	7	5	5	6	5	6	5	6	6	5
s11	8	5	5	5	4	5	5	5	5	6	5	5	5	5	5	4	5	5	6	6	6	6
s12	5	5	4	4	4	4	4	4	4	5	5	5	5	4	4	4	4	4	4	5	5	5
s13	5	5	4	4	4	4	3	3	4	3	3	4	4	3	3	3	4	4	4	4	3	3
s14																						
s15																						
s16																						

Fourth sequence of signals: State of the Economy = H; Signals Reliability = 20%; Sequence Length = 16																	
Signal Number:		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Signal Type:		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
Bounded	6	7	8	9	#	#	10	10	10	10	10	10	10	10	10	10	10
Mean	5,7857	7	7	7	7	6	6	6	6	6	6	6	6	6	5	6	4
St.Dev	1,8051	2	2	2	2	3	3	3	3	3	3	4	3	3	3	3	3
Mod.Dist.	0,2143	0	1	2	3	4	4	4	4	4	4	4	4	4	5	4	6
Correlation	-0,364																
s1	5	6	7	8	8	9	9	10	10	10	9	9	8	9	8	9	9
s2	5	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	2
s3	7	9	10	10	9	9	10	8	7	9	10	10	10	8	7	7	7
s4	6	7	7	7	5	5	5	2	3	2	1	1	4	4	2	1	1
s5	5	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	2
s6	5	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	2
s7	5	8	7	8	7	7	7	7	7	8	7	8	5	5	5	5	5
s8	3	3	3	2	2	1	1	1	1	2	3	3	4		5	6	6
s9	9	9	10	8	7	6	3	1	5	4	4	2	5	5	4	7	6
s10	5	6	7	8	8	9	10	10	10	9	10	10	10	9	10	9	8
s11	8	3	3	3	3	2	2	2	1	5	2	2	2	1	1	1	1
s12	5	8	8	7	7	6	6	6	6	5	5	5	5	5	5	5	5
s13	9	9	8	8	7	6	5	4	3	2	1	1	1	1	1	1	1
s14	4	4	3	3	3	3	2	2	2	2	2	2	2	2	1	3	
s15		8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	2
s16																	

Fifth sequence of signals: State of the Economy = H; Signals Reliability = 20%; Sequence Length = 21																							
Signal Number:		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	
Signal Type:		0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	
Bounded	5	6	7	8	9	8	9	10	#	10	10	10	10	10	10	10	10	10	10	10	10	10	
Mean	4,7857	6	6	6	6	4	6	6	6	6	6	6	6	6	6	6	6	4	6	6	6	6	
St.Dev	1,9287	2	2	3	3	3	3	3	3	3	3	3	4	4	3	3	3	3	3	3	3	3	
Mod.Dist.	0,2143	0	1	2	3	4	3	4	4	4	4	4	4	4	4	4	4	6	4	4	4	4	
Correlation	0,3273																						
s1	5	7	8	8	8	7	8	8	9	9	9	9	10	9	10	10	9	8	8	9	9	9	
s2	5	8	8	8	8	2	8	8	8	8	8	8	8	8	8	8	8	2	8	8	8	8	
s3	4	5	6	10	8	7	8	10	7	8	8	6	10	9	10	10	9	6	7	8	10	6	
s4	4	5	5	3	3	2	3	1	2	1	3	3	1	3	3	2	2	3	4	3	3	2	
s5	5	8	8	8	8	2	8	8	8	8	8	8	8	8	8	8	8	2	8	8	8	8	
s6	5	9	9	9	9	2	9	9	9	9	9	9	9	9	9	9	9	2	9	9	9	9	
s7	5	4	5	5	4	5	5	4	4	3	3	3	4	4	4	4	4	4	4	4	4	4	
s8	6	7	7	8	8	8	8	9	9	10	10	10	10	10	10	9	9	9	10	10	10	10	
s9	1	2	5	5	7	8	5	3	1	4	4	3	2	1	6	5	8	6	4	8	9		
s10	5	6	7	8	8	7	8	9	9	9	10	9	10	10	9	9	9	8	9	9	9	9	
s11	4	3	3	2	2	2	2	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	
s12	5	8	8	8	8	2	2	8	8	8	8	8	8	7	7	6	6	6	6	6	6	5	
s13	10	9	1	1	1	3	2	1	1	1	1	1	1	1	1	1	1	2	1	1	1	1	
s14	3	3	3	3	5	5	5	4	3	3	3	3	2	2	2	2	4	3	2	2	2		
s15		8	8	8	8	2	8	8	8	8	8	8	8	8	8	8	8	2	8	8	8	8	
s16																							