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Abstract

Chapter 1: This Chapter estimates a no-arbitrage term structure model of U.S. Treasury yields and corporate bond spreads with both economic factors and latent factors as drivers of term structure dynamics. We consider two sets of economic factors: macro factors consisting of inflation and real activity, and financial market factors consisting of funding liquidity and market volatility. We show that financial market factors have limited effects on the Treasury yield curve but substantial impacts on the credit spread term structure. In particular, negative liquidity shocks widen credit spreads, and this effect is more pronounced for short-term corporate bonds. We also find that out-of-sample forecasts for credit spreads improve when financial market factors are incorporated and when no-arbitrage restrictions are imposed. We also propose a minimum-chi-square method for estimating the term structure models of interest rate and credit spreads, which is more efficient and accurate than the widespread maximum-likelihood estimation.

Chapter 2: This chapter examines the relationship between funding liquidity and market liquidity using daily data of the S&P 500 Index options market covering January 2003 - January 2012. We find that options market liquidity is positively correlated with funding liquidity during the periods of high market uncertainty. Besides, this paper finds a positive relationship between the options market liquidity and VIX. We also split the whole sample into two sub-samples of pre- and post- August 2007. Consistent with the theoretical predictions of Brunnermeier and Pedersen (2009), we find that when funding liquidity is low, option market liquidity becomes sensitive to changes in funding liquidity and tends to decline with funding liquidity. This chapter also uncovers several important features of the index options market liquidity. First, the impact of funding liquidity on puts liquidity is much higher than that on calls. Second, investors are inclined to trade short maturity options during the recent financial crisis and their liquidity is sensitive to funding liquidity.

Chapter 3: This chapter investigates whether commodity futures provide the ability to hedge against inflation. We adopt a Markov-switching vector error correction model approach for a sample period ranging from January 1983 to December 2012. Over the full sample period, commodity futures show significant ability to hedge US inflation. In particular, the subindices of energy, industrial metals, and precious metals are the best inflation hedges. However, the hedging capacity exhibits substantial variation over time. Most of the inflation hedging power emerges under relatively longer and more common regimes covering the Great Moderation in which the long-run equilibrium correction speed coefficients are highly significant. We do not find any evidence for a significant hedging ability of agriculture and livestock subindices.

Chapter 1

A Gaussian Affine Term Structure Model of Interest Rates and Credit Spreads

We estimate a no-arbitrage term structure model of U.S. Treasury yields and corporate bond spreads with both economic factors and latent factors as drivers of term structure dynamics. We consider two sets of economic factors: macro factors consisting of inflation and real activity, and financial market factors consisting of funding liquidity and market volatility. We show that financial market factors have limited effects on the Treasury yield curve but substantial impacts on the credit spread term structure. In particular, negative liquidity shocks widen credit spreads, and this effect is more pronounced for short-term corporate bonds. We also find that out-of-sample forecasts for credit spreads improve when financial market factors are incorporated and when no-arbitrage restrictions are imposed. We also propose a minimum-chi-square method for estimating the term structure models of interest rate and credit spreads, which is more efficient and accurate than the widespread maximum-likelihood estimation.

1.1 Introduction

In this study, we propose a novel framework to identify what drives the relationship between the dynamics of economic factors, including macroeconomic factors and financial market factors, and the term structure of interest rates and credit spreads. Numerous empirical studies show that credit spreads depend crucially on the state of the macroeconomy and the state of the financial market. A partial list of studies along this line of research includes Carey (1998), Collin-Dufresne, Goldstein, and Martin (2001), Elton, Gruber, Agrawal, and Mann (2001), Altman, Brady, Resti, and Sironi (2005), Longstaff, Mithal, and Neis (2005), Giesecke, Longstaff, Schaefer, and Strebulaev (2011), Lin, Wang, and Wu (2011), Acharya, Amihud, and Bharath (2010), and Giesecke, Longstaff, Schaefer, and Strebulaev (2011). Though rich in economic intuitions, the results of these studies depend on the specific choices of the explanatory variables and the characteristics of credit spreads, such as maturity and credit-rating. Previous literature has mainly focused on explaining changes in credit spreads using regression analysis.

This study, by contrast, investigates credit spreads in a multi-factor (macro factors and financial market factors) term structure model subject to restrictions imposed by no-arbitrage assumptions. Using a no-arbitrage framework not only facilitates the interpretation of the estimation results, but it also provides insights into how yields of maturities not included in the regressions move. In our setup, defaultable corporate bonds are valued as if they were risk-free by replacing the short rate with a default adjusted rate, where the default-adjusted rate for a certain credit-rating class is an affine function of the underlying factors (Duffie and Singleton (1999) and Duffie, Pedersen, and Singleton (2003)). One innovation of our framework is that the underlying factors are comprised of both observable economic variables—macroeconomic variables and financial market variables—and unobserved latent factors. This study seeks to extend previous studies by drawing additional insights from the inclusion of macroeconomic variables, which consist of inflation and real activity and financial market variables, such as funding liquidity and market volatility.

Our main results are as follows. Macroeconomic factors display large explanatory power for Treasury yields. Positive shocks to both inflation and real activity increase Treasury yields, and these effects are stronger for short-term Treasury bonds. In addition, macroeconomic factors are also important determinants of credit spreads. Declines in inflation and real activity lead to higher credit spreads. In contrast, financial market factors have limited effects on the Treasury yield curve but substantial impacts on the credit spread term structure. Positive volatility shocks increase the BBB credit spreads, and this effect is stronger at the long end of the credit spreads curve. Their impacts on the A credit spreads are different over the whole credit spreads term structure, as they are negative at the short end but become positive in the middle. More importantly, liquidity factor has the strongest effects on the short-term credit spreads among the four economic factors. Funding liquidity shrinkage remarkably widens credit spreads, providing an explanation for the run-up in credit spreads in the recent global financial crisis. The impulse response functions show that the effects of macroeconomic factors on both Treasury yields and credit spreads are generally more persistent than the effects of financial market factors. Moreover, to investigate the role of financial market factors and no-arbitrage assumptions in forecasting credit spreads, we perform a comparison of out-of-sample forecasts for credit spreads. It turns out that adding financial market factors substantially improves forecasts for credit spreads. This improvement in forecasting performance is more noticeable for short-term credit spreads. Imposing no-arbitrage restrictions further helps in out-of-sample forecasts.

This work builds on recent studies of affine term structure models of default-free bonds with macroeconomic variables. In a seminal work, Ang and Piazzesi (2003) have introduced two macro factors, inflation and economic growth, as priced risk factors in a Gaussian affine term structure model of U.S. Treasury yields by using a factor representation of the pricing kernel. Inspired by their work, a rich literature has emerged that explores the importance of incorporating macro factors into no-arbitrage term structure models (see, for example, Gallmeyer, Hollifield, and Zin (2005), Diebold, Rudebusch, and Boragan Aruoba (2006), Hördahl, Tristani, and Vestin (2006), Wachter (2006), Bekaert, Cho, and Moreno (2010), Bik-

bov and Chernov (2010)). However, little has been done to expand this literature beyond the pricing of Treasury bonds. As a natural extension to Ang and Piazzesi (2003), this work incorporates corporate bonds into analysis and aims at pricing both default-free Treasury bonds and defaultable corporate bonds in a uniformed no-arbitrage framework with macroeconomic variables and financial market variables as drivers of term structure dynamics.

A large strand of literature uses either reduced-form or structural models to describe the term structure of interest rates and credit spreads. Prominent examples include Longstaff, Mithal, and Neis (2005), Longstaff and Schwartz (1995), Duffee (1999), Collin-Dufresne, Goldstein, and Martin (2001), Eom, Helwege, and Huang (2004), Driessen (2005), Collin-Dufresne, Goldstein, and Helwege (2010) and Joslin, Singleton, and Zhu (2011). However, most of these studies are solely based on latent factors, which are directly derived from the yield curve and credit spread term structure. The economic meanings of these latent factors are unclear. Some studies consider observable variables, but the number of these observable variables is small due to tractability reasons. In contrast, we use two macro factors, inflation and real activity, and two financial market factors, funding liquidity and market volatility, to summarize the information and suppress the noises in many observable macroeconomic and financial times series.

There is a growing literature that jointly estimates the term structure of Treasury yields and corporate yields in linear affine models with macroeconomic variables, including Luisi and Amato (2006), Mueller (2008), and Wu and Zhang (2008). Luisi and Amato (2006) estimate a version that combines macroeconomic variables and latent factors. By using a macro-finance term structure model, Mueller (2008) tries to capture the joint dynamics of GDP, inflation, Treasury yields, and credit spreads. Another important paper in this research area is Wu and Zhang (2008), who links the dynamics and market pricing of three risk dimensions (inflation, real output growth, and financial market volatility) to the term structure of U.S. Treasury yields and corporate bond credit spreads. Instead of identifying three risk dimensions, we go one step further in identifying and estimating risk dimensions using macroeconomic factors, financial market factors, and the traditional latent factors in a novel

Gaussian affine term structure framework. This approach uses linear regression to reduce the dimension of the numerical optimization problem, and it improves the numerical behavior of estimation by eliminating parameters from the objective function that cause problems for conventional maximum-likelihood estimation (MLE) and quasi-maximum likelihood estimate (QMLE) methods.

Gaussian affine term structure models (ATSM) have become fundamental tools for empirical research in macroeconomics and finance. However, tremendous numerical challenges emerge in estimating linear affine models using conventional MLE due to highly non-linear and badly behaved likelihood surfaces. Ang and Piazzesi (2003) find difficulties in estimating models with many factors by using the MLE and try to achieve the global maximum by using multiple starting values. Their results were recently criticized by Hamilton and Wu (2012), who show that Ang and Piazzesi's parameter estimates in fact correspond to a local maximum of the likelihood surface. Mueller (2008) evaluates the likelihood for two billion sets of starting values and then optimizes them by using the best twenty thousand points as starting values. The number of the starting points itself demonstrates that the methodology used in this framework is not reliable. Luisi and Amato (2006) estimate the parameters using the MLE, a method similar to that used by Ang and Piazzesi (2003). Thus, previous studies estimating affine term structure models of Treasury yields and corporate yields suffer the same problem as Ang and Piazzesi (2003). Following Hamilton and Wu (2012, 2014), we develop a minimum-chi-square estimation (MCSE) method for estimating the term structure models of interest rate and credit spreads, which turns out to be much more efficient and accurate than the MLE.

The effects of aggregate liquidity or liquidity risk on asset pricing have been recently highlighted in the literature. Brunnermeier and Pedersen (2009) provide a model that elaborates on the relationship between funding liquidity and market liquidity (FL-ML) and show that the two notions are mutually reinforcing, leading to liquidity spirals. Garleanu and Pedersen (2011) show that a funding liquidity crisis gives rise to a price gap between securities with identical cash flows but different margins. Dick-Nielsen, Gyntelberg, and Lund (2013) find

that the ease of obtaining term funding in the money markets determines the liquidity in the bond market. The relationship between illiquidity shocks and returns of corporate bonds have been documented by De Jong and Driessen (2012), Acharya, Amihud, and Bharath (2010), and Lin, Wang, and Wu (2011). Acharya, Amihud, and Bharath (2010) show that the impact of liquidity shocks on asset prices is conditional and is significantly stronger in adverse economic times. Lin, Wang, and Wu (2011) establish that liquidity risk is a key determinant of corporate bond returns. Given the evidence that funding liquidity is positively correlated with corporate bond market liquidity, we adopt it in our framework and study its effects on corporate bond pricing in a no-arbitrage framework.

The rest of the paper is organized as follows. Section 1.2 describes the data. Section 1.3 specifies the no-arbitrage term structure model. Section 1.4 discusses the estimation and identification strategy. Section 1.5 presents the estimation results both with and without arbitrage restrictions and discusses the implied impulse responses and forecasting results. Section 1.6 concludes.

1.2 Data

This section provides a detailed description of the data used in this study. Treasury yields and corporate bond yields are the observable data. The starting point of the sample period is determined by the first month in which the corporate bond yields are available.

1.2.1 Economic Variables

We consider four groups of economic variables: inflation-related series, real activity-related series, volatility-related series, and liquidity-related series. In the group of inflation-related series, we have five inflation measures: the consumer price index (CPI), the core CPI, the personal consumption expenditure (PCE) deflator, the core PCE deflator, and the producer price index (PPI). The CPI measures the average changes in the price level of a basket of goods

and services bought by a typical household. The PCE deflator measures the average changes in the price level of a basket of goods and services bought by a typical consumer. The core measures of CPI and PCE exclude food and energy, because their prices are highly volatile. The PPI measures average changes in prices received by domestic producers for their output. The second group consists of three output and employment series. UE (Unemployment Rate) measures the percentage of people who are without work and are actively seeking work compared to all individuals in the labor force. GEMP (Growth Rate of Employment) measures the growth rate of the percentage of remaining employed people in the labor force. GIP (Growth Rate of Industrial Production) measures the growth rate of the production of goods. Overall, the first two groups of variables are used to characterize the economy as a whole and are called "macro variables."

In addition to macro variables, we also consider the economic variables that measure the dynamics of the financial market, which are called "financial market variables". On the one hand, to capture the liquidity level of the financial market, we adopt the TED spread as a proxy for the level of funding liquidity, which is defined as the difference between the 3-month LIBOR and the 3-month U.S. Treasury bill rate.¹ This measure has been suggested by Hameed, Kang, and Viswanathan (2010), Boyson, Stahel, and Stulz (2010), and Brunnermeier and Pedersen (2009). Other liquidity measures are 6MTED (6-month LIBOR minus T-bill spread) and 3MCPMFFR (3-month commercial paper minus federal funds rate). Following Wu and Zhang (2008), we incorporate the dynamics of the financial market volatility. Specifically, we use two volatility indexes from option market: the VIX index and the VXO index measure the one-month at-the-money Black and Scholes (1973) implied volatility on the S&P 500 index and the S&P 100 index, respectively. They take the yearly moving average of the daily volatility series and use the last day of each month to obtain their monthly data. Both series are available daily from the Chicago Board of Options Exchange, but the VIX series starts on a later date in January 1990. We augment the VIX data with the estimated data from December 1988 to December 1989.

¹An alternative proxy for funding liquidity in this paper is the LIBOR-OIS spread. The results based on this alternative spread are quantitatively similar and are available upon request.

Following Ang and Piazzesi (2003) and Wu and Zhang (2008), to reduce the dimensionality of the system, we first normalize each series separately to have zero mean and unit variance and then extract the first principal component of each group of series separately. This leaves us with four economic variables, which are referred to as "inflation," "real activity," "volatility," and "liquidity." In what follows, we use the inflation and real activity factors to capture the information about the macroeconomic economy and volatility and liquidity factors to capture the information about the financial market.

Figure 1.1 plots the inflation and real activity factors. The inflation factor peaks in early 1991, which corresponds to the inflation pressure generated by oil shocks during the first Gulf War. In the subsequent period, the inflation factor shows a downward trend and stays low. As expected, most of the movement in the real activity factor is displayed at business cycle frequencies. For example, the real activity factor steadily grows in the period before 2008, but slumps at the onset of the recent global financial crisis. Figure 1.2 plots the financial market liquidity and volatility factors. The volatility factor has a couple of spikes over the sample period in response to financial market turbulence, such as the Asian crisis in 1997, the dot-com bubble burst in 2001, and the subprime mortgage crisis in 2008. Similarly, the liquidity factor fluctuates considerably over the sample period. Its troughs coincide with the spikes of the volatility factor, which together characterize major financial crises in the sample period. Table 1.1 shows the summary statistics of the four groups of economic factors that we consider. An important stylized fact is that all series are highly autocorrelated. This is consistent with the slow mean reversion observed in Figure 1.1 and Figure 1.2.

1.2.2 Government Bonds

The Treasury yields data are monthly continuously compounded spot rates at maturities 1 and 3 months and 2, 5, and 10 years from December 1988 to May 2013. The starting point of the sample period is determined by the availability of the corporate yields data, which will be discussed below. The sources of the data are twofold. The long maturity rates of 2, 5, and

10 years are from the dataset provided by Guerkeynak, Sack, and Wright,² while the short maturity rates of 1 and 3 months are from the Fama CRSP Treasury Bill files. The reason that we use the latter data is that the shortest yield maturity in the Guerkeynak, Sack, and Wright's dataset is one year and they advise against using their estimated yield curve parameters to calculate arbitrary maturity rates.

Figure 1.3 plots the time series of the Treasury yields at three maturities: 2, 5, and 10 years. The three long-term rates exhibit a downward trend over the sample period, largely matching the downward trend of the inflation factor observed in Figure 1.1. There are three periods of low short-term interest rates and high-term spreads around 1993, 2003, and 2008, each of which corresponds to a trough of the real activity factor in Figure 1.1. These comparisons suggest that the movement in the Treasury yields is related to macro factors.

1.2.3 Corporate Bonds

Following Wu and Zhang (2008), we construct continuously compounded spot rates for corporate bonds in the A and BBB rating classes, using month-end prices on corporate bonds that are either in the Merrill Lynch U.S. High Yield Index or the Merrill Lynch U.S. Corporate Master Index. The Merrill Lynch data covers the period from January 1997 to May 2013. To construct a longer sample, we augment the Merrill Lynch data by the Lehman Brothers Fixed Income database from December 1988 to December 1996. Therefore, we estimate our model based on data from December 1989 to May 2013.

To construct the spot rates, we estimate the Nelson and Siegel (1987) model on month-end prices of corporate bonds. The estimation chooses senior unsecured bonds with price quotes, with a fixed coupon schedule, and with maturities between 1 and 35 years, but without option features. Then, we calculate credit spread at three selected maturities, that is, 2, 5 and 10 years, for the A and BBB rating classes as the difference between the spot rate of the rating

²Gurkeynak, Sack, and Wright (2007) estimate the Treasury yield curve using a simple smoothing method and make their data publicly available on the Federal Reserve Board website (last update: December 22, 2011, <http://www.federalreserve.gov/pubs/feds/2008/200805/200805abs.html>).

class and the Treasury yield with the same maturity.

Figure 1.4 plots the time series of credit spreads in percentage at three maturities (2, 5, and 10 years) for the A and BBB rating classes. The credit spreads are higher for A rating class than for BBB rating class. Despite of the magnitude difference, the time series of different credit spreads exhibit strong common movements. We observe three common periods of high spreads, corresponding to the three recessions in the sample period. In particular, the spikes around 2008 highlight the periods of the highest spreads that occurred in the recent global financial crisis.

Table 1.2 reports the summary statistics of Treasury yields and credit spreads. There are some noteworthy stylized facts that characterize our bond yields data. The average Treasury yield curve is upward sloping, while the average credit spread curve is downward sloping; all series are highly autocorrelated. For a certain maturity, credit spread is higher for the lower rating class.

1.3 A No-Arbitrage Term Structure Model

1.3.1 State Variables

Let H_t be an $N_h \times 1$ vector of state variables at time t and ϵ_{t+1} be an $N_h \times 1$ vector of independent standard normal shocks. The observable vector H_t^m contains macroeconomic variables and financial market variables, while H_t^l only contains latent factors. We write the dynamics of H_{t+1} as a first order VAR:

$$H_{t+1} = \Theta_0 + \Theta_1 H_t + \Sigma_H \epsilon_{t+1}, \quad (1.1)$$

where Θ_0 is $N_h \times 1$, Θ_1 is $N_h \times N_h$, and Σ_H is $N_h \times N_h$. The risk-free rate, r_t^f is assumed to be general affine functions of the underlying state vector:

$$r_t^f = \alpha_0 + \alpha_1 H_t, \quad (1.2)$$

where α_0 is a scalar and α_1 is an $N_h \times 1$ vector. We use monthly data, so we adopt the one-month yield as the risk-free rate r_t^f . Following Ang and Piazzesi (2003), the stochastic discount factor (SDF) is given by

$$M_{t+1} = \exp(-r_t^f - \frac{1}{2}x_t'x_t - x_t'\epsilon_{t+1}), \quad (1.3)$$

where the market price of risk, x_t , follows the affine specification:

$$x_t = \xi_0 + \xi_1 H_t, \quad (1.4)$$

in which ξ_0 is an $N_h \times 1$ vector, and ξ_1 is an $N_h \times N_h$ matrix. Here the SDF is a quadratic function of x_t . As a result, the total gross return R_{t+1} of any nominal asset satisfies

$$E_t(M_{t+1}R_{t+1}) = 1. \quad (1.5)$$

For

$$\Theta_0^Q = \Theta_0 - \Sigma_H \xi_0, \quad (1.6)$$

$$\Theta_1^Q = \Theta_1 - \Sigma_H \xi_1, \quad (1.7)$$

risk-neutral investor believed that the factors are characterized by a Q-measure VAR given by

$$H_{t+1} = \Theta_0^Q + \Theta_1^Q H_t + \Sigma_H \epsilon_{t+1}^Q, \quad (1.8)$$

with ϵ_{t+1}^Q a vector of independent standard Normal distribution under the Q-measure. There are three sets of parameters that go into an affine term structure model:

$$(a) \ \Theta_0^Q, \Theta_1^Q, \Sigma_H; \quad (1.9)$$

$$(b) \ \Theta_0, \Theta_1; \quad (1.10)$$

$$(c) \ \xi_0, \xi_1. \quad (1.11)$$

If we know any two of these sets of parameters, we could calculate the third using (1.6) and (1.7).

1.3.2 Treasury Yields

Let $P^{TB}(H_t, \tau)$ denote the price of a τ -period default-free zero coupon bond at time t . That is, $P^{TB}(H_t, \tau)$ denotes the time- t price of an asset with a fixed payoff of one at time $t + \tau$. Because this asset has no intermediate payoffs, its return between t and $t + 1$ is given as

$$R^{TB}(H_{t+1}, \tau) = \frac{P^{TB}(H_{t+1}, \tau - 1)}{P^{TB}(H_t, \tau)}. \quad (1.12)$$

Then, equation (1.5) allows bond prices to be computed recursively by

$$P^{TB}(H_t, \tau) = E_t [M_{t+1} P^{TB}(H_{t+1}, \tau - 1)], \quad (1.13)$$

with boundary condition $P^{TB}(H_t, 0) = 1$. Therefore, Treasury bond prices are exponential affine functions of the state variables (Ang and Piazzesi (2003)):

$$P^{TB}(H_t, \tau) = \exp\{-A(\tau) - B(\tau)^T H_t\}, \quad (1.14)$$

where

$$B(\tau + 1) = \alpha_1 + B(\tau) \Theta_1^Q, \quad (1.15)$$

$$A(\tau + 1) = \alpha_0 + A(\tau) + B(\tau)^T \Theta_0^Q - \frac{1}{2} B(\tau)^T \Sigma_H^T \Sigma_H B(\tau), \quad (1.16)$$

with $B(0) = 0_{1 \times N_h}$ and $A(0) = 0$. The continuously compounded spot rates are affine functions of the underlying states,

$$y(H_t, \tau) = -\frac{\log P^{TB}(H_t, \tau)}{\tau} = \frac{A(\tau)}{\tau} + \frac{B(\tau)^T}{\tau} H_t. \quad (1.17)$$

The observed yields on zero coupon Treasury bonds are given by

$$y(t, \tau) = y(H_t, \tau) + e(t, \tau), \quad (1.18)$$

where $e(t, \tau)$ denotes the portion of the spot rate that is not explained by the underlying dynamic factors.

1.3.3 Corporate Bond Spreads

Duffie and Singleton (1999), and Duffie, Pedersen, and Singleton (2003) show that defaultable bonds can be valued as if they were risk-free by replacing the short rate r_t^f with a default adjusted rate $r_t^f + s_t^i$, where s_t^i for a certain credit-rating class i is an affine function of the underlying factors,

$$s_t^i = \eta_0^i + \eta_1^i H_t + \epsilon_t^i, \quad (1.19)$$

where ϵ_t^i denotes the portion of the credit spread that is not explained by the underlying factors. In discrete time, the same is true under the recovery of market value assumption.³ Therefore, we can write the time-t price of a zero-coupon defaultable bond for a certain credit-rating class

³Previous studies make discretization of continuous-time models, which have served as the building blocks of numerous theoretical models established in the literature. See, for example, Mueller (2008) and Wu and Zhang (2008).

i with time to maturity τ as

$$P_i^{CB}(H_t, \tau) = \exp(-A_i(\tau) - B_i(\tau)^T H_t), \quad (1.20)$$

with

$$B_i(\tau + 1) = \alpha_1 + \eta_1^i + B_i(\tau) \Theta_1^Q, \quad (1.21)$$

$$A_i(\tau + 1) = \alpha_0 + \eta_0^i + A_i(\tau) + B_i(\tau)^T \Theta_0^Q - \frac{1}{2} B_i(\tau)^T \Sigma_H^T \Sigma_H B_i(\tau), \quad (1.22)$$

subject to the boundary conditions $B_i(0) = 0_{1 \times N_h}$ and $A_i(0) = 0$.⁴ The continuously compounded spot rates on the defaultable bond is an affine function of the underlying states,

$$y_i(H_t, \tau) = -\frac{\log P_i^{CB}(H_t, \tau)}{\tau} = \frac{A_i(\tau)}{\tau} + \frac{B_i(\tau)^T}{\tau} H_t. \quad (1.23)$$

The observed yields on zero coupon defaultable bond are given by

$$y_i(t, \tau) = y_i(H_t, \tau) + e_i(t, \tau), \quad (1.24)$$

where $e_i(t, \tau)$ denotes the portion of the spot rate that is not explained by the underlying dynamic factors. Credit spreads can then be calculated as the difference between the yields on defaultable and default-free bonds:

$$\begin{aligned} CS_i(t, \tau) &= y_i(t, \tau) - y(t, \tau) \\ &= \left[\frac{A_i(\tau) - A(\tau)}{\tau} \right] + \left[\frac{B_i(\tau) - B(\tau)}{\tau} \right]^T H_t + e_i(t, \tau) - e(t, \tau). \end{aligned} \quad (1.25)$$

This model provides insights into the determinants of the Treasury yields, corporate bond yields, and the credit spreads.

⁴For the recursive equations, see Appendix A.

1.4 Econometric Methodology

1.4.1 Basic Framework

In this setup, the risk-free rate, r_t^f is assumed to be general affine functions of the underlying state vector:

$$r_t^f = \alpha_0 + \alpha_1 H_t, \quad (1.26)$$

where α_0 is a scalar and α_1 is an $N_h \times 1$ vector. As a result,

$$r_t^f = \alpha_0 + \alpha_{1m} H_t^m + \alpha_{1l} H_t^l. \quad (1.27)$$

Since H_t^m is independent of H_t^l , the values of α_0 , α_{1m} can be obtained by OLS estimation of

$$r_t^f = \alpha_0 + \alpha_{1m} H_t^m + v_t. \quad (1.28)$$

To determine the lags of the VAR for the state variables in Eq. (1.1), we follow Ang and Piazzesi (2003) and regress the short rate on economic factors and 12 lags of economic factors, respectively. The motivations for the two specifications with and without lags in Ang and Piazzesi (2003) stem from the original Taylor rule and the forward-looking version of the Taylor rule. Analogously, our two regressions can be regarded as being based on an augmented Taylor rule with financial market factors and its forward-looking version. As shown in panel A of Table 1.3, the coefficients on inflation, real activity, and liquidity factors are significant and positive. The coefficient on volatility is insignificant, which suggests that the volatility factor might not be a determinant of the short rate. In contrast, panel B shows that most parameter estimates for the looking-forward version of the augmented Taylor rule are not significant, except for the first lag on real activity and liquidity factors. This outcome suggests that the inclusion of many lags may lead to an overcomplicated and poorly behaved regression model. Therefore, we will use a VAR (1) for both the observable macro variables and the latent variables.

To estimate the model, we first solve for the unobservable factors from the joint dynamics of the economic factors, Treasury yields, and corporate bond yields. We follow Chen and Scott (1993), Ang and Piazzesi (2003), and Hamilton and Wu (2012) and assume that as many yields as unobservable factors are treated as measured without error, and the remaining yields are measured with error. In particular, we assume that the 1- and 12-month Treasury yields and 24-month BBB-rated corporate bond yields are priced without error, while the 3-, 24-, and 36-month Treasury yields and 60- and 120-month corporate bond yields are measured with error. Let Y_t^1 denote the $(N_l \times 1)$ vector consisting of yields without measurement error and Y_t^2 the remaining $(N_e \times 1)$ yields with measurement error. Then, the measurement specification is given as

$$\begin{bmatrix} Y_t^1 \\ Y_t^2 \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} + \begin{bmatrix} D_{1m} & D_{1l} \\ D_{2m} & D_{2l} \end{bmatrix} \begin{bmatrix} H_t^m \\ H_t^l \end{bmatrix} + \begin{bmatrix} 0 \\ \Sigma_e \end{bmatrix} u_t^e, \quad (1.29)$$

where Σ_e is assumed to be diagonal.

We adopt a VAR(1) for both the observable macroeconomic factors, financial market factors, and the latent factors:

$$H_{t+1}^m = \Theta_{0m} + \Theta_{mm}H_t^m + \Theta_{ml}H_t^l + \Sigma_{mm}\epsilon_{t+1}^m, \quad (1.30)$$

$$H_{t+1}^l = \Theta_{0l} + \Theta_{lm}H_t^m + \Theta_{ll}H_t^l + \Sigma_{lm}\epsilon_{t+1}^m + \Sigma_{ll}\epsilon_{t+1}^l. \quad (1.31)$$

Ang and Piazzesi (2003) propose that the macro dynamics are orthogonal to the unobserved latent factors, so that terms such as Θ_{ml} and Θ_{lm} are set to zero. In our setup, the macro dynamics have mean zero by construction, which means Θ_{0m} is set to zero. Ang and Piazzesi (2003) further assumed that:

$$\Theta_{0l} = 0, \quad (1.32)$$

$$\Sigma_{lm} = 0, \quad (1.33)$$

$$\Sigma_{ll} = I_{N_l}, \quad (1.34)$$

and

$$\Sigma_{mm} \text{ is lower triangular,} \quad (1.35)$$

$$\Theta_{ll} \text{ is lower triangular.} \quad (1.36)$$

Therefore, the structure model is given as

$$H_t^m = \Theta_{mm} H_{t-1}^m + \Sigma_{mm} \epsilon_t^m \quad (1.37)$$

$$Y_t^1 = C_1 + D_{1m} H_t^m + D_{1l} H_t^l \quad (1.38)$$

$$Y_t^2 = C_2 + D_{2m} H_t^m + D_{2l} H_t^l + \Sigma_e u_t^e. \quad (1.39)$$

1.4.2 Identification

Following Hamilton and Wu (2012, 2014), we change the structure model to the reduced model.⁵ The reduced form is given as

$$H_t^m = \phi_{mm}^* H_{t-1}^m + u_{mt}^* \quad (1.40)$$

$$Y_t^1 = C_1^* + \phi_{1m}^* H_{t-1}^m + \phi_{11}^* Y_{t-1}^1 + \psi_{1m}^* H_t^m + u_{1t}^* \quad (1.41)$$

$$Y_t^2 = C_2^* + \phi_{2m}^* H_t^m + \phi_{21}^* Y_t^1 + u_{2t}^*, \quad (1.42)$$

with

$$\phi_{mm}^* = \Theta_{mm} \quad (1.43)$$

$$u_{mt}^* = \Sigma_{mm} \epsilon_t^m \quad (1.44)$$

⁵For the details, see Appendix B.

$$C_1^* = C_1 - D_{1l}\Theta_{ll}D_{1l}^{-1}C_1 \quad (1.45)$$

$$\phi_{1m}^* = -D_{1l}\Theta_{ll}D_{1l}^{-1}D_{1m} \quad (1.46)$$

$$\phi_{11}^* = D_{1l}\Theta_{ll}D_{1l}^{-1} \quad (1.47)$$

$$\psi_{1m}^* = D_{1m} \quad (1.48)$$

$$u_{1t}^* = D_{1l}\epsilon_t^l \quad (1.49)$$

$$C_2^* = C_2 - D_{2l}D_{1l}^{-1}C_1 \quad (1.50)$$

$$\phi_{2m}^* = D_{2m} - D_{2l}D_{1l}^{-1}D_{1m} \quad (1.51)$$

$$\phi_{21}^* = D_{2l}D_{1l}^{-1} \quad (1.52)$$

$$u_{2t}^* = \Sigma_e u_t^e \quad (1.53)$$

$$Var \begin{bmatrix} u_{mt}^* \\ u_{1t}^* \\ u_{2t}^* \end{bmatrix} = \begin{bmatrix} \Omega_m^* & & \\ & \Omega_1^* & \\ & & \Omega_2^* \end{bmatrix} = \begin{bmatrix} \Sigma_{mm}\Sigma'_{mm} & & \\ & D_{1l}D'_{1l} & \\ & & \Sigma_e\Sigma'_e \end{bmatrix}. \quad (1.54)$$

Because u_{mt}^* , u_{1t}^* and u_{2t}^* are independent, full information maximum likelihood estimation is obtained by treating the three blocks separately, and with each block implemented by OLS equation by equation.

$$\hat{\Omega}_m^* = T^{-1} \sum_{t=1}^T \left[\left(H_t^m - \hat{\phi}_{mm}^* H_{t-1}^m \right) \cdot \left(H_t^m - \hat{\phi}_{mm}^* H_{t-1}^m \right)^T \right] \quad (1.55)$$

$$\hat{\Omega}_1^* = T^{-1} \sum_{t=1}^T \left[\begin{pmatrix} Y_t^1 - \hat{C}_1^* - \hat{\phi}_{1m}^* H_{t-1}^m - \hat{\phi}_{11}^* Y_{t-1}^1 - \hat{\psi}_{1m}^* H_t^m \\ Y_t^1 - \hat{C}_1^* - \hat{\phi}_{1m}^* H_{t-1}^m - \hat{\phi}_{11}^* Y_{t-1}^1 - \hat{\psi}_{1m}^* H_t^m \end{pmatrix}^T \right] \quad (1.56)$$

$$\hat{\Omega}_2^* = T^{-1} \sum_{t=1}^T \left[\begin{pmatrix} Y_t^2 - \hat{C}_2^* - \hat{\phi}_{2m}^* H_t^m - \hat{\phi}_{21}^* Y_t^1 \\ Y_t^2 - \hat{C}_2^* - \hat{\phi}_{2m}^* H_t^m - \hat{\phi}_{21}^* Y_t^1 \end{pmatrix}^T \right] \quad (1.57)$$

Table 1.4 summarizes the mapping between reduced-form and structural parameters. It is worth noting that α_0 cannot be estimated separately from the OLS regression (1.28), because the risk-free rate serves a dependent variable not only in the regression (1.28), but also in the regression (1.38). Another important observation is that the 9 elements of α_0 , ξ_0 and η_0 can be

inferred only from the 8 elements of C_1 and C_2 . This implies that α_0 and ξ_0 are unidentified.

We will discuss how to impose restrictions necessary for identification in Section 1.4.4.

1.4.3 Minimum-Chi-Square Estimation

We denote by π the vector consisting of all reduce-form parameters, by $\mathcal{L}(\pi; Y)$ the log likelihood for the entire sample, and by $\hat{\pi} = \arg \max \mathcal{L}(\pi; Y)$ the vector of full information maximum likelihood (FIML) estimates. We assume that $\hat{R}$ is a consistent estimate of the information matrix, which satisfies

$$R = -\frac{1}{T} E \left[\frac{\partial^2 \mathcal{L}(\pi; Y)}{\partial \pi \partial \pi'} \right]. \quad (1.58)$$

Based on estimate of the information matrix, we could then use the Wald test to examine the hypothesis that $\pi = g(\theta)$, where θ is a known vector of parameters. The Wald statistic is calculated as

$$T [\hat{\pi} - g(\theta)]' \hat{R} [\hat{\pi} - g(\theta)], \quad (1.59)$$

and its asymptotic distribution under the null hypothesis follows $\chi^2(q)$, where the degree of freedom q is the dimension of π . As mentioned in Rothenberg (1973), one could also use (1.59) to choose an estimate $\hat{\theta}$, which minimizes the chi-square statistic for the estimation. We could obtain asymptotic standard errors by considering an linear approximation

$$g(\theta) \simeq g(\theta_0) + \frac{\partial g(\theta)}{\partial \theta'} \Big|_{\theta=\theta_0} (\theta - \theta_0), \quad (1.60)$$

with

$$\begin{aligned} \gamma &\equiv g(\theta_0) - \Gamma \theta_0, \\ \Gamma &\equiv \frac{\partial g(\theta)}{\partial \theta'} \Big|_{\theta=\theta_0}, \end{aligned}$$

where $\hat{\pi} \rightarrow \pi$, and we assume there exists a value of θ_0 in the true model such that $g(\theta_0) = \pi_0$. We denote by $\hat{\theta}^*$ the linearized minimum-chi-square estimator solving the optimization problem

$$\min_{\theta} T [\hat{\pi} - \gamma - \Gamma\theta]' R [\hat{\pi} - \gamma - \Gamma\theta]. \quad (1.61)$$

In other words, $\hat{\theta}^*$ satisfies

$$\Gamma' R (\hat{\pi} - \gamma - \Gamma\hat{\theta}^*) = 0, \quad (1.62)$$

or equivalently,

$$\hat{\theta}^* = (\Gamma' R \Gamma)^{-1} \Gamma' R (\hat{\pi} - \gamma). \quad (1.63)$$

Then we have

$$\sqrt{T} (\hat{\pi} - \pi_0) \rightarrow N(0, R^{-1}), \quad (1.64)$$

and it implies that

$$\sqrt{T} (\hat{\theta}^* - \theta_0) \rightarrow N\left(0, [\Gamma' R \Gamma]^{-1}\right). \quad (1.65)$$

Therefore, Hamilton and Wu (2012) propose to approximate the variance of $\hat{\theta}$ with $T^{-1} [\hat{\Gamma}' \hat{R} \hat{\Gamma}]^{-1}$, in which $\hat{\Gamma} = \frac{\partial g(\theta)}{\partial \theta'}|_{\theta=\hat{\theta}}$. They show that this is identical to the usual asymptotic variance for the MLE approach. Put differently, the MCSE and MLE are asymptotically equivalent.

When a model is just identified, the minimum value attainable for (1.59) is zero. Then the problem in (1.59) can be simplified as

$$\min_{\theta} [\hat{\pi} - g(\theta)]' [\hat{\pi} - g(\theta)]. \quad (1.66)$$

It is important to note that in the case where the objective function in (1.66) is equal to zero, estimators resulting from two approaches, i.e., the MCSE and MLE, are asymptotically equivalent. As argued in Hamilton and Wu (2012), the minimum-chi-square algorithm has two major advantages over the traditional brute-force maximization of the likelihood function, although $\hat{\theta}_{MCSE}$ is identical to $\hat{\theta}_{MLE}$. First, checking whether (1.66) equals zero yields an observation on whether $\hat{\theta}$ corresponds to a global maximum of the original likelihood sur-

face. However, using the traditional approach requires hundreds of starting values to ensure a global maximum is indeed achieved. A second advantage is that the new optimization problem is computationally much simpler than the original likelihood function.

1.4.4 Estimation for Macro Finance Model

In this section, we discuss how to estimate our system and how to impose additional restrictions to facilitate the estimation. As discussed in Section 1.4.2, we have three independent blocks in the reduced-form equations (1.40)-(1.42). We assume that $Y_t^m = H_t^m$, and then the structure of block i for $i = 1, 2, m$ can be written in a compact as

$$Y_t^m = \Pi_i' x_{it} + u_{it}^* \quad (1.67)$$

$$u_{it}^* \sim N(0, \Omega_i^*). \quad (1.68)$$

Following Magnus and Neudecker (1988), the information matrix for all reduced-form parameters takes the form as

$$\hat{R} = \begin{bmatrix} \hat{R}_m & 0 & 0 \\ 0 & \hat{R}_1 & 0 \\ 0 & 0 & \hat{R}_2 \end{bmatrix} \quad (1.69)$$

where

$$\hat{R}_i = \begin{bmatrix} \hat{\Omega}_i^{*-1} \otimes T^{-1} \sum_{t=1}^T x_{it} x_{it}' & 0 \\ 0 & \frac{1}{2} D_{qi}' \left(\hat{\Omega}_i^{*-1} \otimes \hat{\Omega}_i^{*-1} \right) D_{qi} \end{bmatrix} \quad (1.70)$$

for D_N the $N^2 \times N(N+1)/2$ duplication matrix satisfying $D_N \text{vech}(\Omega) = \text{vec}(\Omega)$. The structural parameters Σ_e introducing measurement errors only exist in the block Ω_2^* , and thereby R_2 , and no other parameters appear in this block. The 5 structural parameters capturing measurement errors in Σ_e are just-identified by the 5 diagonal elements of $\hat{\Omega}_2^*$. Using the MCSE approach yields the estimates of diagonal elements of Ω_2^* . It is then easy to obtain the estimates of Σ_e by taking the square roots of the above diagonal elements of $\hat{\Omega}_2^*$. As proposed by Ang and Piazzesi (2003), $\hat{\alpha}_{1m}$ can be obtained by implementing the OLS method based on

the assumption that macro variables and latent factors are independent (see equation (1.28)). Then our system can be estimated by the minimum-chi-square method. The objective function in (1.59) is given by

$$\hat{\pi} = \left(\left[\text{vec} \left(\hat{\Pi}_1 \right) \right]', \left[\text{vech} \left(\hat{\Omega}_1^* \right) \right]', \left[\text{vec} \left(\hat{\Pi}_2 \right) \right]' \right)' \quad (1.71)$$

$$\hat{R} = \begin{bmatrix} \hat{\Omega}_1^{*-1} \otimes T^{-1} \sum_{t=1}^T x_{1t} x_{1t}' & 0 & 0 \\ 0 & \frac{1}{2} D_3' \left(\hat{\Omega}_1^{*-1} \otimes \hat{\Omega}_1^{*-1} \right) D_3 & 0 \\ 0 & 0 & \hat{\Omega}_2^{*-1} \otimes T^{-1} \sum_{t=1}^T x_{2t} x_{2t}' \end{bmatrix} \quad (1.72)$$

$$x_{1t} = (1, H_{t-1}^{m'}, Y_{t-1}^{1'}, H_t^{m'})' \quad (1.73)$$

$$x_{2t} = (1, H_t^{m'}, Y_t^{1'})' \quad (1.74)$$

$$\hat{\Pi}_i' = \left(\sum_{i=1}^T Y_t^i x_{it}' \right) \left(\sum_{i=1}^T x_{it} x_{it}' \right)^{-1} \text{ for } i = 1, 2 \quad (1.75)$$

$$\hat{\Omega}_1^* = T^{-1} \sum_{i=1}^T \left(Y_t^1 - \hat{\Pi}_1' x_{1t} \right) \left(Y_t^1 - \hat{\Pi}_1' x_{1t} \right)' \quad (1.76)$$

$$\hat{\Omega}_2^* = T^{-1} \sum_{i=1}^T \begin{bmatrix} [\hat{u}_{2t}(1)]^2 & \dots & 0 \\ \dots & \dots & \dots \\ 0 & \dots & [\hat{u}_{2t}(N_e)]^2 \end{bmatrix}, \quad (1.77)$$

with $\hat{u}_{2t}(j)$ the j th element of $Y_t^2 - \hat{\Pi}_2' x_{2t}$.

Following Ang and Piazzesi (2003) and Hamilton and Wu (2012), we impose further restrictions on parameters to achieve identification. The parameters fixed at zero include the (2,1) and (3,1) elements of Θ_{ll} (which was already lower triangular), the (1,2), (2,2), (3,2) and (1,3) elements of ξ_{ll} , all four elements in ξ_{0m} , and the second and third elements of ξ_{0l} . In order to optimize the objective function, we need to calculate the estimates of 37 remaining unknown parameters, 1 in Θ_0^Q , 16 in Θ_{mm}^Q , 5 in Θ_{ll}^Q , 4 in Θ_{ll} , 1 in η_0^{BBB} , 7 in η_1^{BBB} , and 3 in α_{1l} .⁶ After getting the parameters of (a) and (b) for the affine structure model, we could

⁶Following Ang and Piazzesi (2003) and Hamilton and Wu (2012), we change the parameterization to improve the numerical performance of the MCSE algorithms. Particularly, we impose the restrictions such that Θ_{ll}

calculate the parameters of the market price of risk, 1 in ξ_{0l} , 16 in ξ_{1m} , 5 in ξ_{1l} , using 1.6 and 1.7. Based on the parameters of (a), (b), and (c), we can numerically solve for the unknown parameters of rated-A corporate bond, 1 in η_0^{BBB} , 7 in η_1^{BBB} , by using equations (1.21) and (1.22).

1.5 Estimation Results

1.5.1 Unrestricted Estimation

For a preliminary view of how Treasury yields and credit spreads respond to macro factors, we run some unrestricted OLS regressions. Table 1.5 reports the estimation results from the regressions of Treasury yields with three different maturities (2, 5, and 10 years) on economic factors. Two types of specifications are considered: one only with inflation and real activity as the regressors, and the other one with all four macro factors. The adjusted R^2 of all the estimated models exceeds 35%. These numbers suggest that economic factors should help explain the dynamics of Treasury yields, which is consistent with previous studies. This explanatory power might be lower for long-term bonds, because the adjusted R^2 decreases with the bond maturity. While the coefficients on inflation and real activity factors are significant in all specifications, the coefficient on volatility factor is never significant, and neither is the liquidity factor in the estimation for 10-year Treasury yields. Moreover, adding the latter two regressors only marginally improves the model fit as reflected by the adjusted R^2 in the bottom panel of Table 1.5. These facts together imply that the explanatory power of the financial market factors for Treasury yield movements should be limited.

Table 1.6 and Table 1.7 present the unrestricted OLS estimation results for the A and BBB credit spreads, respectively. In contrast to the Treasury yield regression results, the inclusion of financial market factors gives rise to a substantial improvement in the goodness of fit, and

$$\text{and } \Theta_{ll}^Q \text{ take the following forms: } \Theta_{ll} = \begin{bmatrix} \theta_1 & 0 & 0 \\ 0 & \theta_2 & 0 \\ 0 & \theta_3 & \theta_4 \end{bmatrix}, \text{ and } \Theta_{ll}^Q = \begin{bmatrix} \theta_5 & 0 & 0 \\ \theta_6 & \theta_2 & \theta_7 \\ \theta_8 & \theta_3 & \theta_9 \end{bmatrix}.$$

this effect is more pronounced for the A credit spread than for the BBB credit spread. This highlights the vital role of financial market factors in describing the dynamics of credit spreads, particularly the dynamics of higher-rated credit spreads. The coefficients on the liquidity and volatility factors are highly significant. The coefficient on the liquidity factor is negative, while that on the volatility factor is positive. This is consistent with the previous literature that credit spreads widen as the aggregate market liquidity condition deteriorates and the market as a whole becomes more volatile.⁷

1.5.2 Estimation with No-Arbitrage Restrictions

Now we turn to the model with no-arbitrage restrictions imposed. The results of the MCSE of Treasury yields and corporate bond yields for 100 different starting values are shown in Table 1.8. Another advantage of the MCSE is that the value for the objective function itself gives us an immediate test of the various overidentifying restrictions. We have 82 known parameters in the reduced form model in Eq.(1.71) and 37 unknown parameters in ATSM. As a result, we do have 45 degree of freedom. The 1% critical value for a $\chi^2(45)$ variable is 70. Thus, the observed minimum value (11.25) provides evidence that the restrictions imposed by the model are consistent with the observed data.

The autocorrelations of the three latent factors are large, implying that they are highly persistent. The intercept estimate on the Treasury yields measures the long-term means of the instantaneous Treasury yields, which is estimated at 0.27% and significant. The intercept estimates on the corporate yields measure the fixed component of the instantaneous credit spread, which are 0.36% for A-rated corporate bonds and 0.44% for B-rated corporate bonds. As expected, the compensation for credit risk is higher for higher-rated bonds.

The slope estimates on Treasury yields measure the initial response of the instantaneous Treasury yields to unit shocks on the seven factors. The estimates for the macro factors are positive and are larger than those for the financial market factors, suggesting that the latter

⁷See Wu and Zhang (2008) for the effect of financial market volatility on credit spreads and Lin, Wang, and Wu (2011) for the effect of liquidity.

variables only play a marginal role in the dynamics of the short Treasury yields. The absolute magnitude of the slope estimates on the short spread gets much higher for financial market factors. In particular, the liquidity factor even becomes the most important driver of the short spread, and this variation is more pronounced for the A-rated bonds. Thus, in contrast to the finding of Wu and Zhang (2008) that financial market volatility factor is a major determinant of the credit spread term structure, our results show that the liquidity factor is more important than the volatility factor for capturing the dynamics of credit spreads.

1.5.3 Factor Loadings and Impulse Responses

Figure 1.5 illustrates how Treasury yields and corporate yields respond to a one standard deviation change in macro factors. As shown in panel (a), the loadings on inflation are positive for Treasury yields but negative for corporate yields, implying that positive inflation shocks lead to higher Treasury yields but lower corporate yields. For a certain maturity, the absolute magnitudes of inflation loadings are larger for corporate yields than for Treasury yields. Thus, the former is contemporaneously more sensitive to inflation change than the latter. Furthermore, it declines with maturity for both Treasury and corporate yields and exhibits a convergence to the long-run mean. Hence, long yields are less subject to inflation change than short yields. Similar patterns are observed for the real activity loadings in panel (b). One primary distinction is that the real activity loadings of the BBB corporate yields become larger than those of the A corporate yields and intensify with maturity.

Figure 1.6 shows the loadings of Treasury yields and corporate yields on financial market factors. Treasury yields have rather small loadings on both the liquidity and volatility factors. This is consistent with the unrestricted estimation results that Treasury yields are more influenced by macro factors than by financial market factors. The loadings on liquidity for both the BBB and A corporate yields are negative and decline in absolute value with maturity. Moreover, the liquidity loadings are larger in absolute value for the A corporate yields than for the BBB corporate yields, which suggests that the initial reaction to funding liquidity shocks

is stronger for higher-rated bonds. As shown in panel (b), the loadings on the volatility factor differ between the two corporate yields; whereas the volatility loadings of the BBB corporate yields are largely flat and remain positive across bond maturities, those of the A corporate yields are negative when the maturity is short and display an upward trend. This implies that for A-rated bonds, short and long yields react to the volatility change in different directions.

With factor loadings of Treasury yields and corporate yields, one can easily derive the factor loadings of credit spreads using equation (1.25). The results are shown in Figure 1.7 and Figure 1.8, which include economic factor loadings and latent factor loadings, respectively. The inflation, real activity and liquidity loadings of both the A and BBB credit spreads are high when the maturity is short and converge fast to their long term means, while the volatility loadings increase with maturity. Thus, the contemporaneous reaction to economic factors is generally larger for short spreads than for long spreads, except for the volatility factor. In contrast to Treasury yields, credit spreads load heavily on both the liquidity and volatility factors. This is consistent with the unrestricted regression results and justifies the importance of those factors for explaining credit spreads. The liquidity loadings are negative and higher in absolute value for the A credit spreads. This implies that funding liquidity shrinkage widens the credit spreads and this effect is more pronounced for those at a high-rating class. The observation that the volatility factor generally exerts a positive effect on the credit spread term structure is consistent with Wu and Zhang (2008). However, this effect is reversed for short A credit spreads. As shown in Figure 1.8, the loadings on the three latent factors behave in a similar fashion for both the A and BBB credit spreads.

Figure 1.9 illustrates the impulse response functions of 24-, 60- and 120-month Treasury yields. The impulse responses for all factors are large in absolute value at short horizon and level off slowly towards zero. The impulse response for inflation and real activity remains positive even after 12 months, whereas those for liquidity and volatility quickly fade away within one year; this effect is more pronounced for long-term Treasury yields. This implies that macroeconomic shocks have a more persistent impact on Treasury yields than do financial market shocks. The transient effects of financial market shocks are quite large: shocks

to the volatility factor have even larger impacts on long-term Treasury yields than those to macro factors at the very short horizon. Figure 1.10 and Figure 1.11 plot the impulse response functions of 24-, 60- and 120-month A and BBB credit spreads, respectively. Liquidity has larger instantaneous effects on the A credit spreads than on the BBB credit spreads. For all corporate bonds under consideration, the initial response of their credit spreads to financial market factors is much larger than that to macro factors. However, in general, this relationship is reversed after one year, implying that macro factors have more persistent effects on the credit spread term structure compared to financial market factors.

1.5.4 Out-of-Sample Forecasts

A true out-of-sample analysis would be to estimate the model based on data up to and including today, construct a forecast of tomorrow's value at $T + 1$, wait until tomorrow, record the forecast error at $T + 1$, re-estimate the model, make a new forecast of true value at $T + 2$, and so forth. At the end of this exercise, one would have a sample of forecast errors which would be truly out-of-sample and would give a very realistic picture of the model's performance. Since this procedure is very time-consuming, we instead resort to "pseudo," or "simulated," out-of-sample analysis, which means to mimic the procedure described above, using some historical date $T_0 < T$, rather than today's date T , as a starting point. The resulting forecasting errors are then used to assess the model's out-of-sample forecasting ability.

We perform a comparison of out-of-sample forecasts for credit spreads for three models. First, we investigate out-of-sample forecasts for two VAR(1) models without cross-equation restrictions induced by no-arbitrage condition. The first VAR involves two macro factors (inflation and real activity), and the second incorporates both macro factors and financial market factors (liquidity and volatility). Our last model is a VAR(1) model with four economic factors and cross-equation restrictions. We forecast over the last 36 months of our sample, which is chosen to avoid forecasting abnormal yields in recent financial crisis. We record the root mean square error (RMSE) and the mean absolute deviation (MAD) of the forecast versus the actual

value as criteria for comparing the forecasting abilities of different models.

Table 1.9 shows the comparisons of the out-of-sample forecasts. Lower RMSE and MAD denote better forecasts. The best forecasts are highlighted in bold. Two important points are noteworthy. First, incorporating financial market variables improves forecasts. In terms of both the RMSE and MAD criteria, the models with four factors uniformly outperformed the model with only macro factors (except for the 120-month A credit spreads). The improvement in forecasting performance is much more noticeable for short-term credit spreads, which is consistent with the previous result that the explanatory power of financial market factors for credit spreads emerges primarily at the short end of the yield curve. Second, imposing the cross-equation restrictions induced by no-arbitrage improves forecasts. This is strongly supported by the results of the RMSE: the no-arbitrage model outperforms the other two unrestricted models (except for 24-month A credit spreads). However, less evidence is found in results of the MAD, as the no-arbitrage model provides the best forecast only for the 24-month BBB credit spreads. This mismatch is likely due to the fact that the RMSE penalizes large errors more than the MAD does. Given that the MADs for the no-arbitrage model are close to those for the four-factor model and large errors are undesirable in our context, we can still conclude that adding term structure restrictions helps in forecasting.

1.6 Conclusion

This paper presents and estimates a no-arbitrage Gaussian affine term structure model of interest rate and credit spreads with observable economic variables and traditional latent variables. In addition to macro variables, financial market variables are included in the set of observable variables. The estimation is performed using a minimum-chi-square method, which is much more efficient and reliable than the widespread MLE. By estimating the model, we quantify the effects of economic variables on the whole term structure of interest rates and credit spreads and explore their role in forecasting.

We find that macro factors are important determinants of both Treasury yields and credit

spreads. Positive inflation and real activity shocks increase Treasury yields but decrease credit spreads. In contrast, financial market factors only have marginal effects on the Treasury yield curve but exert substantial impacts on the credit spread term structure. The volatility factor has positive effects on the BBB credit spreads. Its effects on the A credit spreads are negative at the short end of the credit spreads curve but turn positive at the middle. Funding liquidity shrinkage remarkably widens credit spreads, and this effect is strongest at the short end of the credit spread term structure. We show that adding financial market factors substantially improves forecasts for credit spreads. Imposing no-arbitrage restrictions further helps in out-of-sample forecasts.

1.7 Appendix

1.7.1 Discretization

State Variables

Following Wu and Zhang (2008), we consider a flexible specification, under which the factor dynamics under the risk-neutral Q-measure,

$$dH_t = \kappa^Q (\theta^Q - H_t) dt + \Sigma_H dW_t^Q, \quad (1.78)$$

where H_t is an $N_h \times 1$ vector of state variables at time t . Under this specification, the variable that determines the price of risk, x_t , is assumed to be general affine functions of the underlying state vector:

$$x_t = \xi_0 + \xi_1 H_t, \quad (1.79)$$

where ξ_0 is a scalar.

As a result,

$$\begin{aligned} H_{t+1} - H_t &= \kappa^Q (\theta^Q - H_t) + \Sigma_H \epsilon_{t+1}^Q, \\ H_{t+1} &= \kappa^Q \theta^Q + (I - \kappa^Q) H_t + \Sigma_H \epsilon_{t+1}^Q. \end{aligned}$$

Let $\kappa^Q \theta^Q \equiv \Theta_0^Q$, $(I - \kappa^Q) \equiv \Theta_1^Q$, then

$$H_{t+1} = \Theta_0^Q + \Theta_1^Q H_t + \Sigma_H \epsilon_{t+1}^Q. \quad (1.80)$$

Term Structure of Treasury Yields

In our model, the risk-free rate, r_t^f , is assumed to be a general affine functions of the underlying state vector:

$$r_t^f = \alpha_0 + \alpha_1 H_t + \epsilon_t^r, \quad (1.81)$$

where α_0 is a scalar and ϵ_t^r denotes movements in the instantaneous interest rate that are not explained by the underlying dynamic factors. We can write the time- t price of a default-free zero coupon bond with time to maturity τ as

$$\begin{aligned} P^{TB}(t, \tau) &= E_t^Q \left[\exp \left(- \int_t^{t+\tau} r_s^f ds \right) \right] \\ &= E_t^Q \left[\exp \left(- \int_t^{t+\tau} (\alpha_0 + \alpha_1 H_s) ds \right) \right] \cdot E_t^Q \left[\exp \left(- \int_t^{t+\tau} \epsilon_s^r ds \right) \right] \\ &= P^{TB}(H_t, \tau) E(t, \tau), \end{aligned} \quad (1.82)$$

where $E_t^Q[\cdot]$ denotes the expectation under Q-measure conditional on time- t filtration F_t .

Under this specification, $P^{TB}(H_t, \tau)$ is the exponential affine function of the state variables (Ang and Piazzesi (2003)):

$$P^{TB}(H_t, \tau) = \exp\{-A(\tau) - B(\tau)^T H_t\}, \quad (1.83)$$

where the coefficients $A(\tau)$ and $B(\tau)$ are solutions to the following ordinary differential equations:

$$B'(\tau) = \alpha_1 - (\kappa^Q)^T B(\tau), \quad (1.84)$$

$$A'(\tau) = \alpha_0 + B(\tau)^T \kappa^Q \theta^Q - \frac{1}{2} B(\tau)^T \Sigma_H^T \Sigma_H B(\tau), \quad (1.85)$$

with $B(0) = 0_{1 \times N_h}$ and $A(0) = 0$.

As a result,

$$\begin{aligned} B(\tau + 1) - B(\tau) &= \alpha_1 - (\kappa^Q)^T B(\tau), \\ B(\tau + 1) &= \alpha_1 + [I - (\kappa^Q)^T] B(\tau), \\ B(\tau + 1) &= \alpha_1 + B(\tau) \Theta_1^Q. \end{aligned}$$

Therefore,

$$\begin{aligned} B(\tau + 1) &= \alpha_1 \left((\Theta_1^Q)^\tau + (\Theta_1^Q)^\tau + \dots + \Theta_1^Q + 1 \right) \\ &= \alpha_1 \left(\frac{(\Theta_1^Q)^{\tau+1} - 1}{\Theta_1^Q - 1} \right), \end{aligned} \quad (1.86)$$

and

$$\frac{B(\tau + 1)}{\tau + 1} = \frac{\alpha_1}{\tau + 1} \left(\frac{(\Theta_1^Q)^{\tau+1} - 1}{\Theta_1^Q - 1} \right). \quad (1.87)$$

Also, we have

$$\begin{aligned} A(\tau + 1) - A(\tau) &= \alpha_0 + B(\tau)^T \kappa^Q \theta^Q - \frac{1}{2} B(\tau)^T \Sigma_H^T \Sigma_H B(\tau), \\ A(\tau + 1) - A(\tau) &= \alpha_0 + B(\tau)^T \Theta_0^Q - \frac{1}{2} B(\tau)^T \Sigma_H^T \Sigma_H B(\tau). \end{aligned} \quad (1.88)$$

Indeed,

$$A(1) = \alpha_0, \quad (1.89)$$

$$A(2) = \alpha_0 + A(1) + B(1)^T \Theta_0^Q - \frac{1}{2} B(1)^T \Sigma_H^T \Sigma_H B(1), \quad (1.90)$$

$$\begin{aligned} A(3) &= \alpha_0 + A(2) + B(2)^T \Theta_0^Q - \frac{1}{2} B(2)^T \Sigma_H^T \Sigma_H B(2) \\ &= 3\alpha_0 + (B(1)^T + B(2)^T) \Theta_0^Q \\ &\quad - \frac{1}{2} [B(1)^T \Sigma_H^T \Sigma_H B(1) + B(2)^T \Sigma_H^T \Sigma_H B(2)], \end{aligned} \quad (1.91)$$

therefore,

$$A(\tau + 1) = (\tau + 1)\alpha_0 + \left(B(1)^T + \dots + B(\tau)^T \right) \Theta_0^Q - \frac{1}{2} \left[B(1)^T \Sigma_H^T \Sigma_H B(1) + \dots + B(\tau)^T \Sigma_H^T \Sigma_H B(\tau) \right], \quad (1.92)$$

and

$$\frac{A(\tau + 1)}{\tau + 1} = \alpha_0 + \frac{1}{\tau + 1} \left(B(1)^T + \dots + B(\tau)^T \right) \Theta_0^Q - \frac{1}{2} \frac{1}{\tau + 1} \left[B(1)^T \Sigma_H^T \Sigma_H B(1) + \dots + B(\tau)^T \Sigma_H^T \Sigma_H B(\tau) \right]. \quad (1.93)$$

The continuously compounded spot rates are affine functions of the underlying states:

$$y(H_t, \tau) = -\frac{\log P^{TB}(H_t, \tau)}{\tau} = \frac{A(\tau)}{\tau} + \frac{B(\tau)^T}{\tau} H_t. \quad (1.94)$$

The observed yields on zero coupon Treasury bonds are given by

$$y(t, \tau) = y(H_t, \tau) + e(t, \tau), \quad (1.95)$$

where $e(t, \tau) = -\ln(E(t, \tau))/\tau$ denotes the portion of the spot rate that is not explained by the underlying dynamic factors.

Term Structure of Corporate Yields and Credit Spreads

Duffie and Singleton (1999), and Duffie, Pedersen, and Singleton (2003) show that defaultable bonds can be valued as if they were risk-free by replacing the short rate r_t^f with a default adjusted rate $r_t^f + s_t$, where s_t^i for a certain credit-rating class i is an affine function of the underlying factors,

$$s_t^i = \eta_0^i + \eta_1^i H_t + \epsilon_t^i, \quad (1.96)$$

where ϵ_t^i denotes the portion of the credit spread that is not explained by the underlying factors. Therefore, we can write the time- t price of a zero-coupon defaultable bond for a certain credit-rating class i with time to maturity τ as

$$\begin{aligned} P_i^{CB}(t, \tau) &= E_t^Q \left[\exp \left(- \int_t^{t+\tau} (r_u^f + s_u^i) du \right) \right] \\ &= E_t^Q \left[\exp \left(- \int_t^{t+\tau} (\alpha_0 + \alpha_1 H_u + \eta_0^i + \eta_1^i H_u) du \right) \right] \cdot E_t^Q \left[\exp \left(- \int_t^{t+\tau} (\epsilon_u^r + \epsilon_u^i) du \right) \right] \\ &= P_i^{CB}(H_t, \tau) E_i(t, \tau), \end{aligned} \quad (1.97)$$

with

$$P_i^{CB}(H_t, \tau) = \exp(-A_i(\tau) - B_i(\tau)^T H_t),$$

where the coefficients $A_i(\tau)$, and $B_i(\tau)$ are solutions to the ordinary differential equations:

$$B_i'(\tau) = \alpha_1 + \eta_1^i - (\kappa^Q)^T B_i(\tau), \quad (1.98)$$

$$A_i'(\tau) = \alpha_0 + \eta_0^i + B_i(\tau)^T \kappa^Q \theta^Q - \frac{1}{2} B_i(\tau)^T \Sigma_H^T \Sigma_H B_i(\tau), \quad (1.99)$$

subject to the boundary conditions $B_i(0) = 0_{1 \times N_h}$ and $A_i(0) = 0$. As a result,

$$\begin{aligned} B_i(\tau + 1) - B_i(\tau) &= \alpha_1 + \eta_1^i - (\kappa^Q)^T B_i(\tau), \\ B_i(\tau + 1) &= \alpha_1 + \eta_1^i + \left[I - (\kappa^Q)^T \right] B_i(\tau), \\ B_i(\tau + 1) &= \alpha_1 + \eta_1^i + B_i(\tau) \Theta_1^Q. \end{aligned}$$

Therefore,

$$\begin{aligned} B_i(\tau + 1) &= (\alpha_1 + \eta_1^i) \left((\Theta_1^Q)^\tau + (\Theta_1^Q)^{\tau-1} + \dots + \Theta_1^Q + 1 \right) \\ &= (\alpha_1 + \eta_1^i) \left(\frac{(\Theta_1^Q)^{\tau+1} - 1}{\Theta_1^Q - 1} \right), \end{aligned} \quad (1.100)$$

and

$$\frac{B_i(\tau+1)}{\tau+1} = \frac{(\alpha_1 + \eta_1^i)}{\tau+1} \left(\frac{(\Theta_1^Q)^{\tau+1} - 1}{\Theta_1^Q - 1} \right). \quad (1.101)$$

Also, we have

$$\begin{aligned} A_i(\tau+1) - A_i(\tau) &= \alpha_0 + \eta_0^i + B_i(\tau)^T \kappa^Q \theta^Q - \frac{1}{2} B_i(\tau)^T \Sigma_H^T \Sigma_H B_i(\tau), \\ A_i(\tau+1) - A_i(\tau) &= \alpha_0 + \eta_0^i + B_i(\tau)^T \Theta_0^Q - \frac{1}{2} B_i(\tau)^T \Sigma_H^T \Sigma_H B_i(\tau). \end{aligned} \quad (1.102)$$

Indeed,

$$A_i(1) = \alpha_0 + \eta_0^i, \quad (1.103)$$

$$A_i(2) = \alpha_0 + \eta_0^i + A_i(1) + B_i(1)^T \Theta_0^Q - \frac{1}{2} B_i(1)^T \Sigma_H^T \Sigma_H B_i(1), \quad (1.104)$$

$$\begin{aligned} A_i(3) &= \alpha_0 + \eta_0^i + A_i(2) + B_i(2)^T \Theta_0^Q - \frac{1}{2} B_i(2)^T \Sigma_H^T \Sigma_H B_i(2) \\ &= 3(\alpha_0 + \eta_0^i) + (B_i(1)^T + B_i(2)^T) \Theta_0^Q \\ &\quad - \frac{1}{2} [B_i(1)^T \Sigma_H^T \Sigma_H B_i(1) + B_i(2)^T \Sigma_H^T \Sigma_H B_i(2)]. \end{aligned} \quad (1.105)$$

Therefore,

$$\begin{aligned} A_i(\tau+1) &= (\tau+1)(\alpha_0 + \eta_0^i) + (B_i(1)^T + \dots + B_i(\tau)^T) \Theta_0^Q \\ &\quad - \frac{1}{2} [B_i(1)^T \Sigma_H^T \Sigma_H B_i(1) + \dots + B_i(\tau)^T \Sigma_H^T \Sigma_H B_i(\tau)], \end{aligned} \quad (1.106)$$

and

$$\begin{aligned} \frac{A_i(\tau+1)}{\tau+1} &= (\alpha_0 + \eta_0^i) + \frac{1}{\tau+1} (B_i(1)^T + \dots + B_i(\tau)^T) \Theta_0^Q \\ &\quad - \frac{1}{2} \frac{1}{\tau+1} [B_i(1)^T \Sigma_H^T \Sigma_H B_i(1) + \dots + B_i(\tau)^T \Sigma_H^T \Sigma_H B_i(\tau)] \end{aligned} \quad (1.107)$$

The continuously compounded spot rates on the defaultable bond is an affine function of the underlying states,

$$y_i(H_t, \tau) = -\frac{\log P_i^{CB}(H_t, \tau)}{\tau} = \frac{A_i(\tau)}{\tau} + \frac{B_i(\tau)^T}{\tau} H_t. \quad (1.108)$$

The observed yields on zero coupon defaultable bond are given by

$$y_i(t, \tau) = y_i(H_t, \tau) + e_i(t, \tau), \quad (1.109)$$

where $e_i(t, \tau) = -\ln(E_i(t, \tau))/\tau$ denotes the portion of the spot rate that is not explained by the underlying dynamic factors. Credit spreads can then be calculated as the difference between the yields on defaultable and default-free bonds:

$$\begin{aligned} CS_i(t, \tau) &= y_i(t, \tau) - y(t, \tau) \\ &= \left[\frac{A_i(\tau) - A(\tau)}{\tau} \right] + \left[\frac{B_i(\tau) - B(\tau)}{\tau} \right]^T H_t + e_i(t, \tau) - e(t, \tau) \end{aligned} \quad (1.110)$$

This model provides insights into the determinants of the Treasury yields, corporate bond yields, and the credit spreads.

1.7.2 Estimation

In our model setup, the structure model is given as

$$H_t^m = \Theta_{mm} H_{t-1}^m + \Sigma_{mm} \epsilon_t^m \quad (1.111)$$

$$Y_t^1 = C_1 + D_{1m} F_t^m + D_{1l} H_t^l \quad (1.112)$$

$$Y_t^2 = C_2 + D_{2m} F_t^m + D_{2l} H_t^l + \Sigma_e u_t^e \quad (1.113)$$

Then we have

$$H_t^m = \phi_{mm}^* H_{t-1}^m + u_{mt}^* \quad (1.114)$$

with

$$\phi_{mm}^* = \Theta_{mm}, \quad (1.115)$$

$$u_{mt}^* = \Sigma_{mm}\epsilon_t^m. \quad (1.116)$$

We substitute $H_t^l = D_{1l}^{-1} (Y_t^1 - C_1 - D_{1m}H_t^m)$ into the equation (1.113) for Y_t^2 ,

$$\begin{aligned} Y_t^2 &= C_2 + D_{2m}H_t^m + D_{2l}H_t^l + \Sigma_e u_t^e \\ &= C_2 + D_{2m}H_t^m + D_{2l}D_{1l}^{-1} (Y_t^1 - C_1 - D_{1m}H_t^m) + \Sigma_e u_t^e \\ &= C_2 - D_{2l}D_{1l}^{-1}C_1 + (D_{2m} - D_{2l}D_{1l}^{-1}D_{1m}) H_t^m + D_{2l}D_{1l}^{-1}Y_t^1 + \Sigma_e u_t^e. \end{aligned} \quad (1.117)$$

That is,

$$Y_t^2 = C_2^* + \phi_{2m}^* H_t^m + \phi_{21}^* Y_t^1 + u_{2t}^* \quad (1.118)$$

$$C_2^* = C_2 - D_{2l}D_{1l}^{-1}C_1 \quad (1.119)$$

$$\phi_{2m}^* = D_{2m} - D_{2l}D_{1l}^{-1}D_{1m} \quad (1.120)$$

$$\phi_{21}^* = D_{2l}D_{1l}^{-1} \quad (1.121)$$

$$u_{2t}^* = \Sigma_e u_t^e / \quad (1.122)$$

Also, we have

$$\begin{aligned} Y_t^1 &= C_1 + D_{1m}H_t^m + D_{1l}H_t^l \\ &= C_1 + D_{1m}H_t^m + D_{1l}(\Theta_{ll}H_{t-1}^l + \Sigma_{ll}\epsilon_t^l) \\ &= C_1 + D_{1m}H_t^m + D_{1l}(\Theta_{ll}D_{1l}^{-1} (Y_{t-1}^1 - C_1 - D_{1m}H_{t-1}^m) + \Sigma_{ll}\epsilon_t^l) \\ &= C_1 - D_{1l}\Theta_{ll}D_{1l}^{-1}C_1 + D_{1m}H_t^m + D_{1l}\Theta_{ll}D_{1l}^{-1}Y_{t-1}^1 - D_{1l}\Theta_{ll}D_{1l}^{-1}D_{1m}H_{t-1}^m + D_{1l}\epsilon_t^l. \end{aligned}$$

That is,

$$Y_t^1 = C_1^* + \phi_{1m}^* H_{t-1}^m + \phi_{11}^* Y_{t-1}^1 + \psi_{1m}^* H_t^m + u_{1t}^* \quad (1.123)$$

$$C_1^* = C_1 - D_{1l} \Theta_{ll} D_{1l}^{-1} C_1 \quad (1.124)$$

$$\phi_{1m}^* = -D_{1l} \Theta_{ll} D_{1l}^{-1} D_{1m} \quad (1.125)$$

$$\phi_{11}^* = D_{1l} \Theta_{ll} D_{1l}^{-1} \quad (1.126)$$

$$\psi_{1m}^* = D_{1m} \quad (1.127)$$

$$u_{1t}^* = D_{1l} \epsilon_t^l. \quad (1.128)$$

As a result, the reduced form is given as

$$H_t^m = \phi_{mm}^* H_{t-1}^m + u_{mt}^* \quad (1.129)$$

$$Y_t^1 = C_1^* + \phi_{1m}^* H_{t-1}^m + \phi_{11}^* Y_{t-1}^1 + \psi_{1m}^* H_t^m + u_{1t}^* \quad (1.130)$$

$$Y_t^2 = C_2^* + \phi_{2m}^* H_t^m + \phi_{21}^* Y_t^1 + u_{2t}^* \quad (1.131)$$

$$Var \begin{bmatrix} u_{mt}^* \\ u_{1t}^* \\ u_{2t}^* \end{bmatrix} = \begin{bmatrix} \Omega_m^* & & \\ & \Omega_1^* & \\ & & \Omega_2^* \end{bmatrix} = \begin{bmatrix} \Sigma_{mm} \Sigma'_{mm} & & \\ & D_{1l} D'_{1l} & \\ & & \Sigma_e \Sigma'_e \end{bmatrix}. \quad (1.132)$$

Because u_{mt}^* , u_{1t}^* and u_{2t}^* are independent, full-information-maximum likelihood estimation is obtained by treating the three blocks separately, and wit each block implemented by OLS equation by equation.

$$\hat{\Omega}_m^* = T^{-1} \sum_{t=1}^T \left[\left(H_t^m - \hat{\phi}_{mm}^* H_{t-1}^m \right) \cdot \left(H_t^m - \hat{\phi}_{mm}^* H_{t-1}^m \right)^T \right] \quad (1.133)$$

$$\hat{\Omega}_1^* = T^{-1} \sum_{t=1}^T \left[\begin{pmatrix} Y_t^1 - \hat{C}_1^* - \hat{\phi}_{1m}^* H_{t-1}^m - \hat{\phi}_{11}^* Y_{t-1}^1 - \hat{\psi}_{1m}^* H_t^m \\ Y_t^1 - \hat{C}_1^* - \hat{\phi}_{1m}^* H_{t-1}^m - \hat{\phi}_{11}^* Y_{t-1}^1 - \hat{\psi}_{1m}^* H_t^m \end{pmatrix}^T \right] \quad (1.134)$$

$$\hat{\Omega}_2^* = T^{-1} \sum_{t=1}^T \left[\begin{pmatrix} Y_t^2 - \hat{C}_2^* - \hat{\phi}_{2m}^* H_t^m - \hat{\phi}_{21}^* Y_t^1 \\ Y_t^2 - \hat{C}_2^* - \hat{\phi}_{2m}^* H_t^m - \hat{\phi}_{21}^* Y_t^1 \end{pmatrix}^T \right]. \quad (1.135)$$

1.7.3 Log Likelihood for the Macro Model

The P dynamics can be written as a special case of a first-order VAR by using the companion form $H_t = [H_t^{m'}, H_t^{l'}]'$, $\Theta_0 = [0'_{4 \times 1}, \Theta'_{0l}]'$, and

$$\Theta_1 = \begin{bmatrix} \Theta_{mm} & 0 \\ (4 \times 4) & \\ 0 & \Theta_{ll} \\ & (3 \times 3) \end{bmatrix} \quad (1.136)$$

$$\Sigma_H = \begin{bmatrix} \Sigma_{mm} & 0 \\ (4 \times 4) & \\ 0 & \Sigma_{ll} \\ & (3 \times 3) \end{bmatrix}. \quad (1.137)$$

Ang and Piazzesi (2003) and Hamilton and Wu (2012) imposed the restriction in the market price of risk, x_t , that the parameters in the affine specification are characterized by $\xi_0 = [\xi'_{0m}, \xi'_{0l}]'$ and

$$\xi_1 = \begin{bmatrix} \xi_{1m} & 0 \\ (4 \times 4) & \\ 0 & \xi_{1l} \\ & (3 \times 3) \end{bmatrix}. \quad (1.138)$$

From

$$\Theta_0^Q = \Theta_0 - \Sigma_H \xi_0, \quad (1.139)$$

$$\Theta_1^Q = \Theta_1 - \Sigma_H \xi_1, \quad (1.140)$$

we can have $\Theta_0^Q = [\Theta_{0m}^{Q'}, \Theta_{0l}^{Q'}]'$ and

$$\Theta_1^Q = \begin{bmatrix} \Theta_{mm}^Q & 0 \\ (4 \times 4) & \\ 0 & \Theta_{ll}^Q \\ & (3 \times 3) \end{bmatrix}. \quad (1.141)$$

In our setup, we use $N_l = 3$ and $N_e = 5$, assuming that the 1-, and 12-month Treasury yields and 24-month corporate bond yields are priced without error, while the 3-, 24-, and 36-month Treasury yields and 60- and 120-month corporate bond yields are priced with error, so that the D matrices can be written in partitioned form as

$$\begin{bmatrix} D_{1m}^0 & D_{1l} \\ (3 \times 4) & (3 \times 3) \\ D_{2m}^0 & D_{2l} \\ (5 \times 4) & (5 \times 3) \end{bmatrix} = \begin{bmatrix} B(1)^T / 1 \\ B(12)^T / 12 \\ B_i(24)^T / 24 \\ B(3)^T / 3 \\ B(24)^T / 24 \\ B(36)^T / 36 \\ B_i(60)^T / 60 \\ B_i(120)^T / 120 \end{bmatrix}. \quad (1.142)$$

The conditional density for this case is given by

$$f(H_t^m, Y_t | H_{t-1}^m, Y_{t-1}) = \frac{1}{|\det(J)|} f(H_t^m, H_t^l, u_t^e | H_{t-1}^m, H_{t-1}^l, u_{t-1}^e). \quad (1.143)$$

where

$$f(H_t^m, H_t^l, u_t^e | H_{t-1}^m, H_{t-1}^l, u_{t-1}^e) = f(H_t^m | H_{t-1}^m) f(H_t^l | H_{t-1}^l) f(u_t^e), \quad (1.144)$$

$$f(H_t^m | H_{t-1}^m) = \phi(H_t^m; \Theta_{mm} H_{t-1}^m, \Sigma_{mm} \Sigma_{mm}'), \quad (1.145)$$

$$f(H_t^l | H_{t-1}^l) = \phi(H_t^l; \Theta_{ll} H_{t-1}^l, \Sigma_{ll} \Sigma'_{ll}), \quad (1.146)$$

$$f(u_t^e) = \phi(u_t^e; 0, I_{N_e}), \quad (1.147)$$

$$\phi(y; \mu, \Omega) = \frac{1}{(2\pi)^{M/2} |\Omega|^{1/2}} \exp \left[-\frac{(y - \mu)' \Omega^{-1} (y - \mu)}{2} \right] \quad (1.148)$$

$$H_t^l = D_{1l}^{-1} (Y_t^1 - C_1 - D_{1m} H_t^m) \quad (1.149)$$

$$u_t^e = \Sigma_e^{-1} (Y_t^2 - C_2 - D_{2m} H_t^m - D_{2l} H_t^l) \quad (1.150)$$

$$J = \begin{bmatrix} D_{1l} & 0 \\ D_{2l} & \Sigma_e \end{bmatrix}. \quad (1.151)$$

The traditional approach (Ang and Piazzesi (2003)) is to arrive at estimates of these parameters by numerical maximization of

$$L(\theta; Y) = \sum_{t=1}^T \log f(H_t^m, Y_t | H_{t-1}^m, Y_{t-1}). \quad (1.152)$$

Table 1.1: Summary statistics of economic variables

Variables	Central moments				Autocorrelations		
	Mean	Stdev	Skew	Kurt	Lag 1	Lag 2	Lag 3
CPI	1.193	0.552	-0.263	4.515	0.950	0.862	0.775
PPI	1.056	1.112	-0.321	3.541	0.938	0.848	0.749
PCEde	0.967	0.449	0.002	3.936	0.963	0.899	0.828
GEMP	0.391	0.620	-1.648	6.104	0.971	0.945	0.908
GIP	0.840	1.831	-2.062	8.606	0.976	0.940	0.890
UE	6.064	1.595	0.887	2.840	0.995	0.989	0.980
3MTED	0.562	0.395	2.363	12.896	0.877	0.756	0.692
6MTED	0.582	0.358	1.789	7.971	0.928	0.859	0.806
3MCPMFFR	0.127	0.264	2.346	17.316	0.797	0.678	0.579
VXO	1.286	0.163	0.454	3.072	0.856	0.771	0.731
VIX	1.277	0.153	0.547	3.383	0.829	0.735	0.692

The table reports the summary statistics of economic variables. The inflation measures CPI, PPI, and PCEde are CPI inflation, PPI inflation, and PCE deflator respectively. The real activity measures GEMP, GIP, and UE are the growth rate of employment, the growth rate in industrial production, and the unemployment rate, respectively. The liquidity measures 3MTED, 6MTED, and 3MCPMFFR are 3-month LIBOR minus T-bill spread, 6-month LIBOR minus T-bill spread, and 3-month commercial paper minus federal funds rate, respectively. The volatility measures VIX and VXO are the VIX index and VXO index, respectively. The sample period is 1988:12 to 2013:05.

Table 1.2: Summary statistics of Treasury yields and credit spreads

	Central moments				Autocorrelations		
	Mean	Stdev	Skew	Kurt	Lag 1	Lag 2	Lag 3
Treasury Yields							
1 month	3.335	2.313	0.061	2.063	0.990	0.981	0.971
3 month	3.503	2.421	0.071	2.074	0.996	0.989	0.980
24 month	4.101	2.394	-0.079	2.081	0.993	0.984	0.973
60 month	4.735	2.128	-0.111	2.275	0.991	0.980	0.968
120 month	5.458	1.794	-0.048	2.453	0.989	0.977	0.967
A-rated Spreads							
24 month	1.220	1.204	4.127	22.261	0.961	0.903	0.843
60 month	1.129	0.867	3.222	16.064	0.962	0.906	0.844
120 month	1.074	0.770	2.885	13.673	0.965	0.920	0.871
BBB-rated Spreads							
24 month	1.871	1.431	2.974	14.088	0.965	0.913	0.855
60 month	1.750	1.100	3.152	16.337	0.964	0.905	0.832
120 month	1.642	0.937	2.750	13.763	0.968	0.914	0.851

The table reports the summary statistics of Treasury yields and credit spreads. The 1-, 3-, 24-, 60-, and 120-month Treasury yields are annual zero coupon bond yields. The 1-, 3-, 24-, 60-, and 120-months credit spreads under the A and BBB rating classes are calculated as the difference between the spot rate of the rating class and the Treasury yield with the same maturity. The sample period is 1988:12 to 2013:05.

Table 1.3: The dependence of the short rate on economic factors

	Inflation		Real Activity		Liquidity		Volatility	
Panel A:	Regressing short rate on economic factors (adjusted $R^2 = 0.52$)							
t	0.043	(9.600)	0.059	(9.974)	-0.018	(-2.944)	0.001	(0.177)
Panel B:	Regressing short rate on 12 lags of economic factors (adjusted $R^2 = 0.77$)							
t	0.023	(1.627)	0.098	(3.672)	0.028	(3.149)	0.012	(1.539)
t-1	-0.008	(-0.315)	0.022	(0.603)	0.006	(0.563)	0.003	(0.325)
t-2	0.005	(0.180)	-0.021	(-0.561)	-0.003	(-0.307)	0.011	(1.171)
t-3	0.003	(0.118)	0.002	(0.057)	-0.007	(-0.623)	0.006	(0.681)
t-4	-0.009	(-0.375)	-0.014	(-0.379)	0.003	(0.278)	-0.002	(-0.197)
t-5	0.004	(0.164)	-0.007	(-0.189)	0.002	(0.177)	-0.007	(-0.774)
t-6	0.008	(0.332)	0.000	(0.003)	-0.007	(-0.610)	-0.002	(-0.188)
t-7	0.000	(0.018)	0.010	(0.266)	-0.007	(-0.676)	-0.002	(-0.195)
t-8	0.011	(0.429)	0.002	(0.055)	-0.005	(-0.453)	-0.006	(-0.671)
t-9	0.000	(0.000)	-0.017	(-0.456)	-0.007	(-0.641)	0.004	(0.457)
t-10	-0.003	(-0.129)	-0.008	(-0.208)	-0.012	(-1.070)	-0.008	(-0.891)
t-11	0.014	(0.965)	0.043	(1.641)	-0.037	(-4.311)	-0.010	(-1.295)

The table reports the dependence of the short rate on economic factors. Panel A shows the results of regressing the 1-month yield on economic factors. Panel B shows the results of regressing the 1-month yield on 12 lags of economic factors. Standard errors are shown in parentheses.

Table 1.4: Mapping between structural and reduced-form parameters

VAR parameters	No. of elements	Σ_e 5	Σ_{mm} 10	Θ_{mm} 16	$\xi_{1m,1l}$ 16+9	α_1 7	Θ_{ll} 6	η_0 1	η_1 7	α_0 1	ξ_0 7
Ω_2^*	5	×									
Ω_m^*	10		×								
ϕ_{mm}^*	16			×							
ψ_{1m}^*	12			×	×	×			×		
ϕ_{21}^*	15				×	×	×		×		
Ω_1^*	6				×	×	×		×		
ϕ_{11}^*	9				×	×	×		×		
ϕ_{2m}^*	20			×	×	×	×		×		
ϕ_{1m}^*	12			×	×	×	×		×		
C_2^*	5		×	×	×	×	×	×	×	×	×
C_1^*	3		×	×	×	×	×	×	×	×	×

Table 1.5: Regressing Treasury yields on economic factors

Maturity	2 year	t-Statistic	5 year	t-Statistic	10 year	t-Statistic
Intercept	4.101	(40.733)	4.735	(50.020)	5.458	(65.536)
Inflation	0.599	(11.085)	0.571	(11.241)	0.511	(11.434)
Real activity	0.636	(8.806)	0.403	(5.926)	0.183	(3.066)
Adjusted R^2	0.480		0.418		0.366	
Intercept	4.101	(41.501)	4.735	(50.238)	5.458	(65.509)
Inflation	0.532	(9.254)	0.531	(9.692)	0.490	(10.117)
Real activity	0.655	(8.764)	0.408	(5.730)	0.188	(2.991)
Liquidity	-0.273	(-3.521)	-0.156	(-2.114)	-0.085	(-1.297)
Volatility	-0.068	(-0.807)	-0.055	(-0.680)	-0.023	(-0.323)
Adjusted R^2	0.499		0.423		0.366	

The table reports the parameter estimates from OLS regressions of Treasury yields on economic factors. The sample period is 1988:12 to 2013:05.

Table 1.6: Regressing A credit spreads on economic factors

Maturity	2 year	t-Statistic	5 year	t-Statistic	10 year	t-Statistic
Intercept	1.220	(23.438)	1.129	(27.203)	1.074	(28.453)
Inflation	0.029	(1.044)	0.077	(3.434)	0.029	(1.417)
Real activity	-0.569	(-15.237)	-0.358	(-12.003)	-0.298	(-11.016)
Adjusted R^2	0.450		0.327		0.294	
Intercept	1.220	(42.634)	1.129	(48.559)	1.074	(48.310)
Inflation	-0.061	(-3.681)	0.011	(0.814)	-0.024	(-1.862)
Real activity	-0.463	(-21.392)	-0.268	(-15.221)	-0.214	(-12.715)
Liquidity	-0.461	(-20.525)	-0.349	(-19.117)	-0.295	(-16.918)
Volatility	0.128	(5.250)	0.127	(6.427)	0.135	(7.145)
Adjusted R^2	0.834		0.789		0.755	

The table reports the parameter estimates from OLS regressions of A credit spreads on economic factors. The sample period is 1988:12 to 2013:05.

Table 1.7: Regressing BBB credit spreads on economic factors

Maturity	2 year	t-Statistic	5 year	t-Statistic	10 year	t-Statistic
Intercept	1.871	(35.221)	1.750	(37.138)	1.642	(38.393)
Inflation	0.002	(0.082)	0.034	(1.340)	0.010	(0.445)
Real activity	-0.765	(-20.057)	-0.527	(-15.588)	-0.409	(-13.311)
Adjusted R^2	0.595		0.460		0.388	
Intercept	1.871	(52.842)	1.750	(53.870)	1.642	(57.735)
Inflation	-0.039	(-1.878)	-0.009	(-0.487)	-0.022	(-1.349)
Real activity	-0.634	(-23.678)	-0.420	(-17.084)	-0.303	(-14.077)
Liquidity	-0.305	(-10.958)	-0.286	(-11.194)	-0.244	(-10.930)
Volatility	0.280	(9.267)	0.214	(7.715)	0.228	(9.384)
Adjusted R^2	0.820		0.743		0.729	

The table reports the parameter estimates from OLS regressions of Treasury yields on the BBB credit spreads. The sample period is 1988:12 to 2013:05.

Table 1.8: Model estimates

Θ_{1l}	0.9386 (76.934)	0	0	Θ_{1l}^Q	0.6660 (34.426)	0	0
	0	0.9934 (764.15)	0		-0.2778 (-10.289)	0.9934 (764.15)	-0.0255 (-6.8919)
	0	0.0016 (2.2857)	0.8262 (42.588)		-0.0298 (-2.0694)	0.0016 (2.2857)	0.9793 (489.65)
ξ_{0l}	-0.2893 (-9.3322)	0	0				
ξ_{1m}	0.6408 0.1536 -0.0160 0.1826	0.3906 0.5547 -0.0208 0.1283	0.1242 0.1788 0.1864 0.2158	-0.4270 0.3376 0.1446 -0.0809	ξ_{1l} 0.2726 0.2778 0.0298	0 0 0	0 0.0255 -0.1531
α_0	0.0027 (27.000)						
α_1	0.0004 (8.8300)	0.0006 (10.187)	-0.0002 (-3.2733)	4.94E-06 (0.0744)	1.46E-04 (12.586)	1.71E-04 (21.242)	6.51E-05 (5.9724)
η_0^{BBB}	0.0044 (42.718)						
η_1^{BBB}	-0.0007 (-0.7928)	-0.0002 (-0.2488)	-0.0008 (-9.0293)	0.0002 (3.3113)	3.72E-04 (4.9799)	1.84E-04 (16.140)	-1.75E-04 (-11.218)
η_0^A	0.0036 (32.143)						
η_1^A	-0.0011 (-1.4056)	-0.0008 (-2.162)	-0.0020 (-20.243)	-0.0008 (-5.2980)	-6.73E-05 (-2.6289)	1.48E-04 (11.128)	-9.08E-05 (-11.792)
Σ_m	0.4305 0.0042 0.0017 -0.0453	0 0.2272 -0.0295 0.0599	0 0 0.6905 -0.2044	0 0 0 0.7256			

The table reports the model estimates and standard errors (in parentheses). The parameters are estimated with the minimum-chi-square methods. The sample period is 1988:12 to 2013:05.

Table 1.9: Forecast comparisons

	RMSE criteria			MAD criteria		
	Two Factors	Four Factors	No-Arbitrage	Two Factors	Four Factors	No-Arbitrage
A24	0.0390	0.0192	0.0246	0.0268	0.0126	0.0146
A60	0.0380	0.0357	0.0277	0.0321	0.0198	0.0216
A120	0.0357	0.0367	0.0312	0.0302	0.0191	0.0223
BBB24	0.0442	0.0263	0.0211	0.0276	0.0215	0.0117
BBB60	0.0492	0.0411	0.0410	0.0430	0.0307	0.0317
BBB120	0.0462	0.0458	0.0453	0.0358	0.0298	0.0303

The table reports the comparisons of the out-of-sample forecasts for credit spreads. The forecasts are performed over the last 36 months of our sample, and the root mean square error (RMSE) and the mean absolute deviation (MAD) of the forecast versus the actual values are calculated. Lower RMSE and MAD denotes better forecasts. The best forecasts are highlighted in bold. We first estimate models on the in-sample, and update the estimations at each observation in the out-sample. Three models are considered: unrestricted VAR with two macro factors, unrestricted VAR with two macro factors, and two financial market factors and VAR with four factors and cross-equation restrictions.

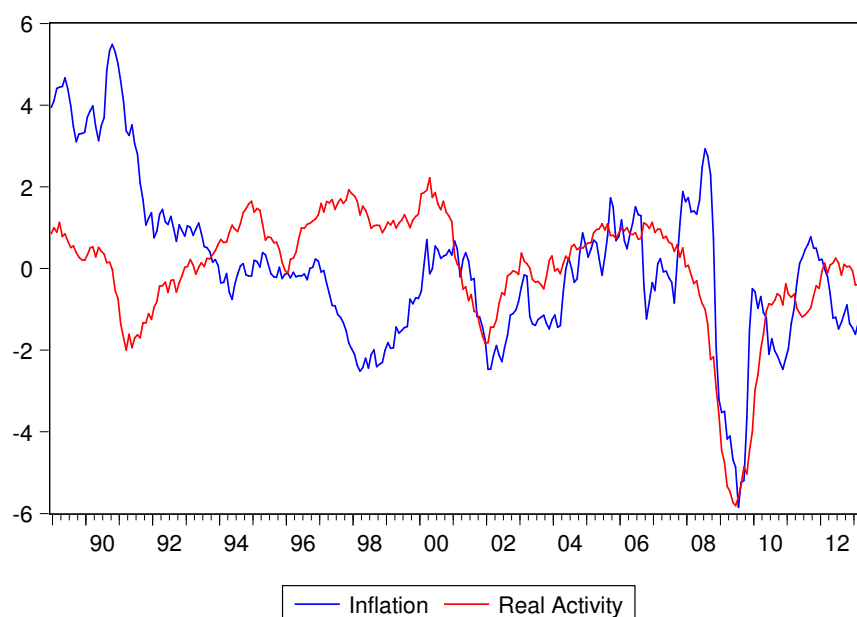


Figure 1.1: Macro factors: inflation and real activity. The figure illustrates the two macro factors representing inflation (blue) and real activity (red). The sample period is 1988:12 to 2013:05.

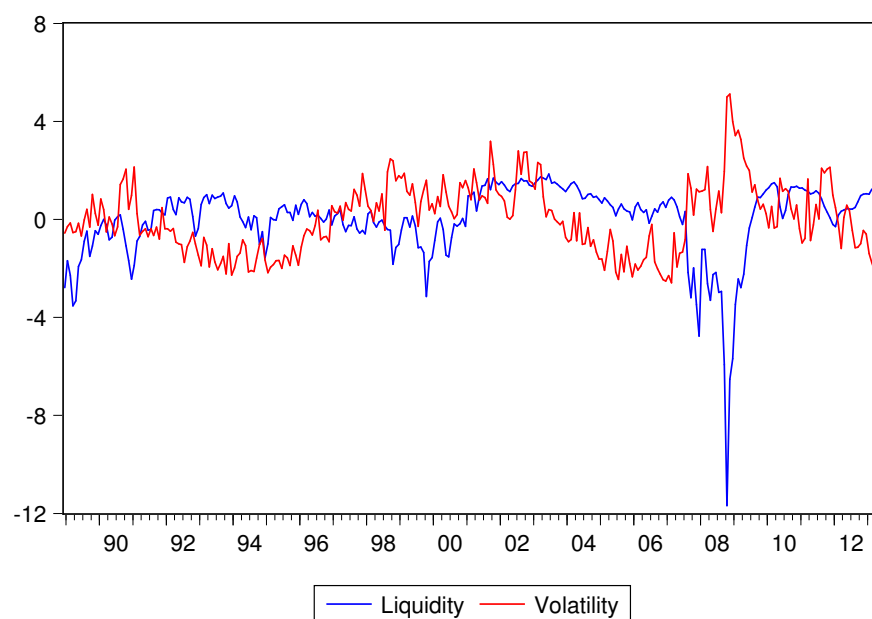


Figure 1.2: Financial market factors: liquidity and volatility. The figure illustrates the two financial market factors representing liquidity (blue) and volatility (red). The sample period is 1988:12 to 2013:05.

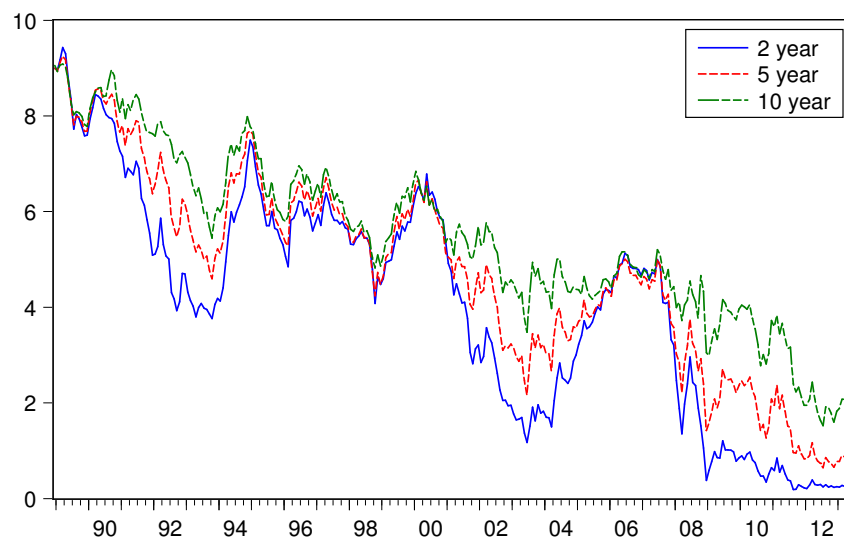
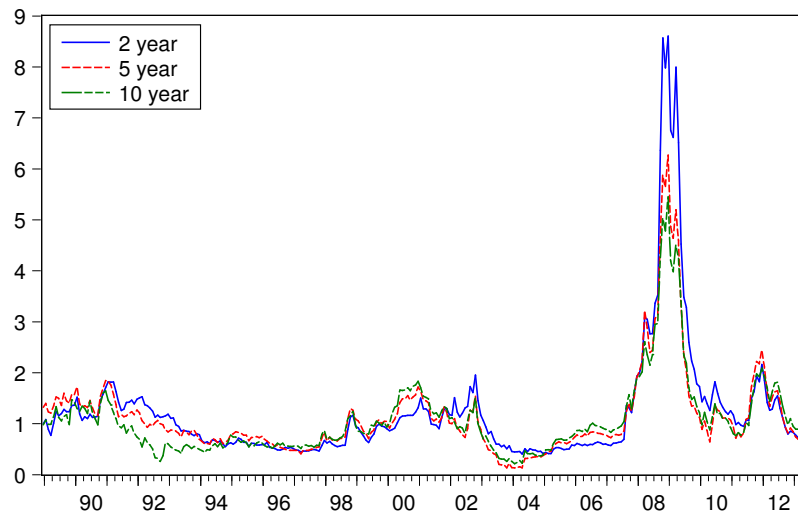
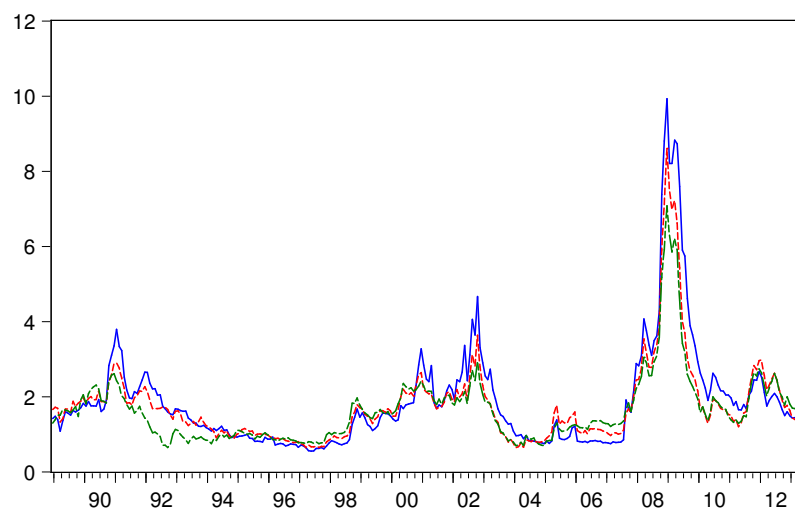


Figure 1.3: Treasury yields. This figure illustrates (annualized) monthly zero coupon bond yields of maturity 2 years (blue), 5 years (red), and 10 years (green). The sample period is 1988:12 to 2013:05.

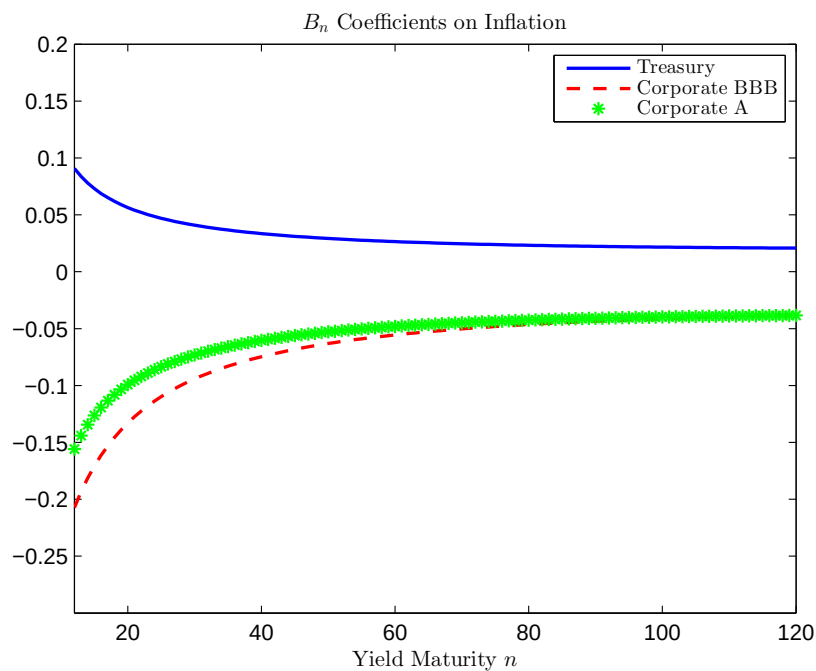


(a) A credit spreads

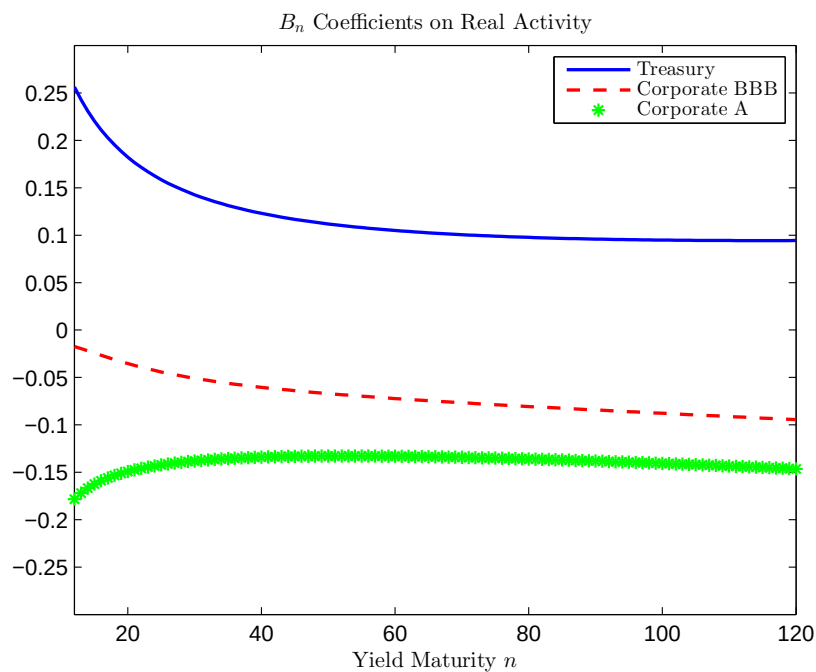


(b) BBB credit spreads

Figure 1.4: Credit spreads: A and BBB rating classes. Panel (a) illustrates the credit spreads at maturity 2 years, 5 years, and 10 years under A rating class. Panel (b) illustrates the credit spreads at maturity 2 years, 5 years, and 10 years under the BBB rating class.

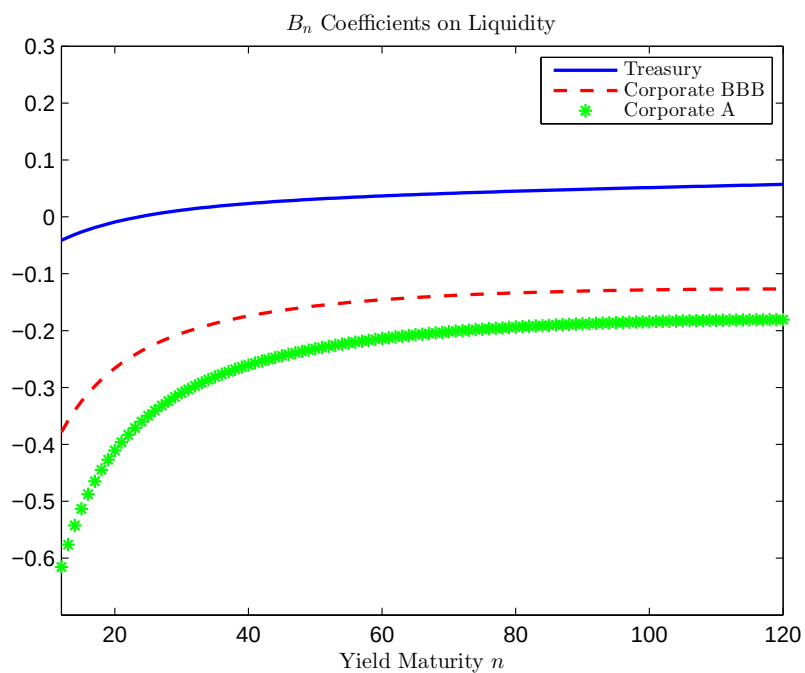


(a) Inflation factor

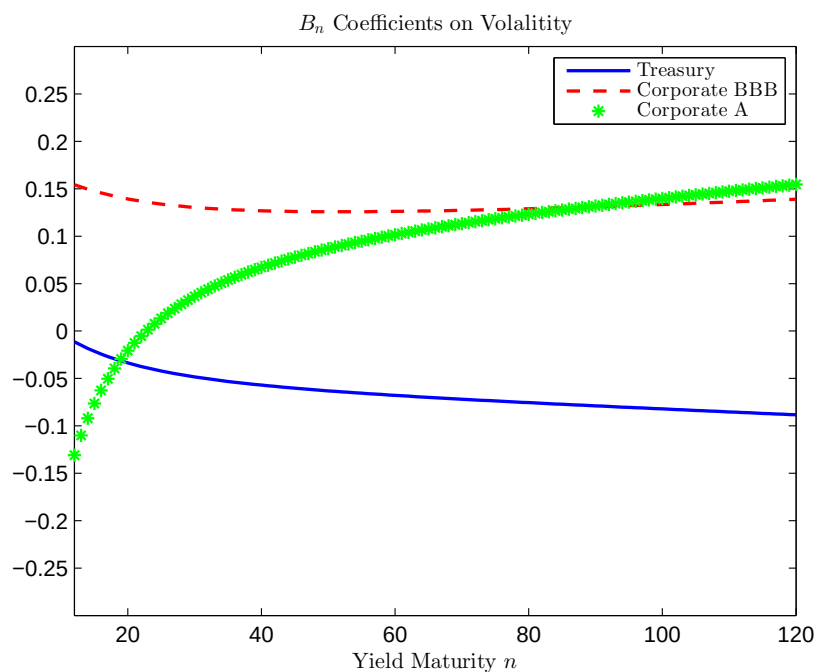


(b) Real activity factor

Figure 1.5: Loadings on macro factors. The figure shows how Treasury yields and corporate yields change respond to a one standard deviation change in macro factors. Panel (a) shows the loadings on the inflation factor and Panel (b) shows the loadings on real activity factor. Yields maturity is expressed in months.

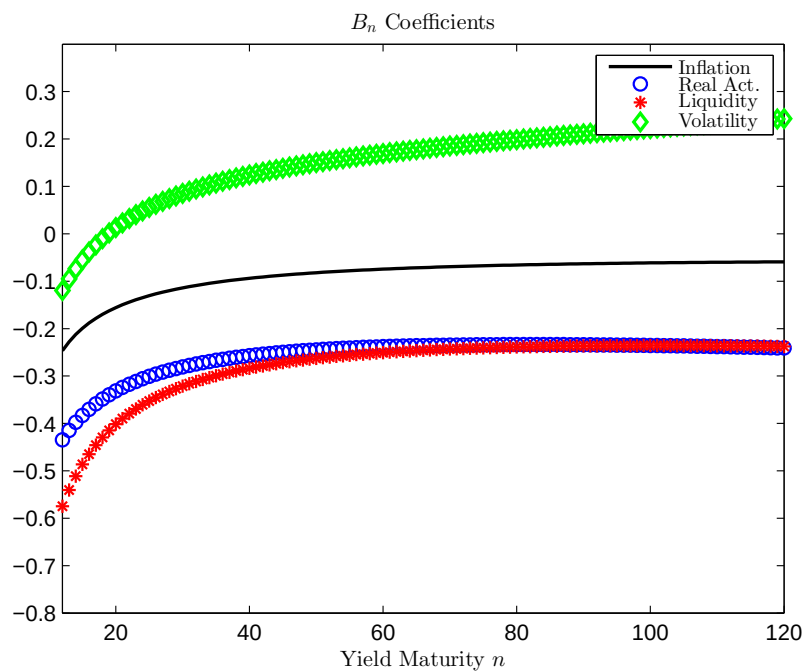


(a) Liquidity factor

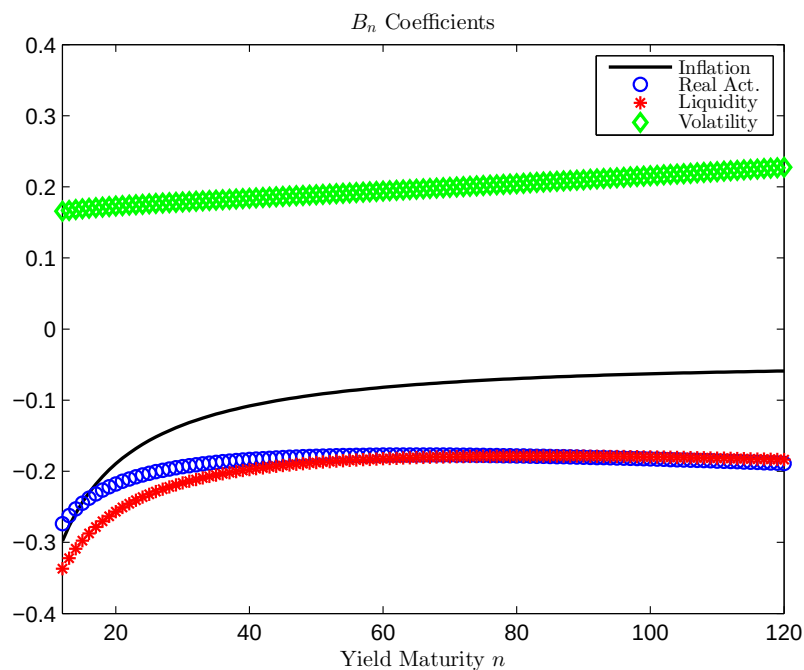


(b) Volatility factor

Figure 1.6: Loadings on financial market factors. The figure shows how Treasury yields and corporate yields respond to a one standard deviation change in financial market factors. Panel (a) shows the loadings on liquidity factor and panel (b) shows the loadings on the volatility factor. Yields maturity is expressed in months.

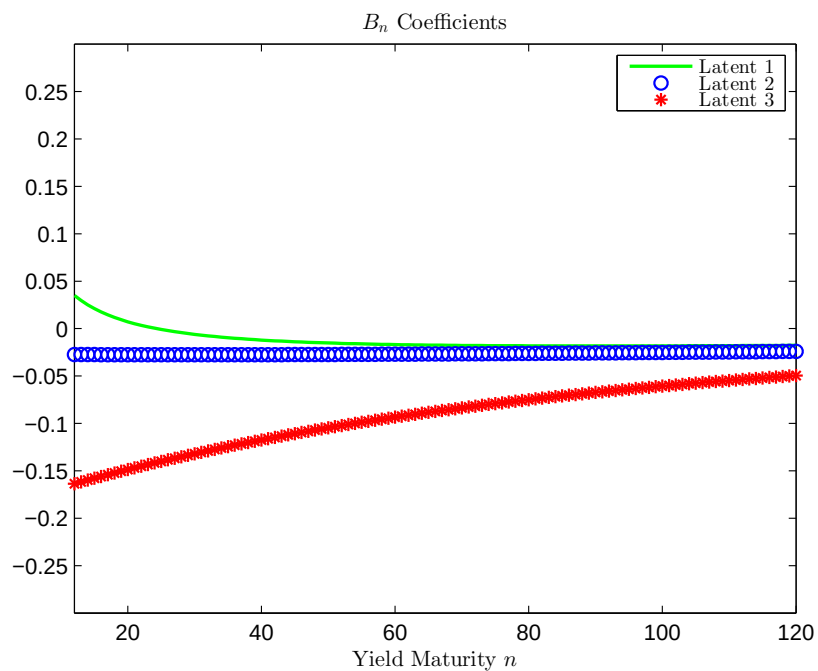


(a) A credit spreads

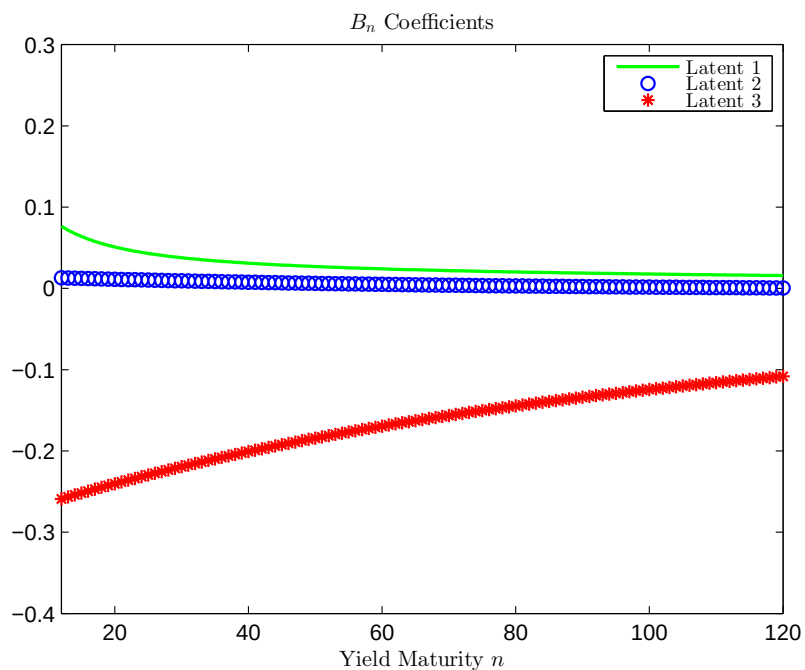


(b) BBB credit spreads

Figure 1.7: Loadings of credit spreads on economic factors. The figure shows how credit spreads respond to a one standard deviation change in economic factors. Panel (a) shows the loadings of the A credit spreads, and panel (b) shows the loadings of the BBB credit spreads. Yields maturity is expressed in months.



(a) A credit spreads



(b) BBB credit spreads

Figure 1.8: Loadings of credit spreads on latent factors. The figure shows how credit spreads respond to a one standard deviation change in latent factors. Panel (a) shows the loadings of the A credit spreads and panel (b) shows the loadings of the BBB credit spreads. Yields maturity is expressed in months.

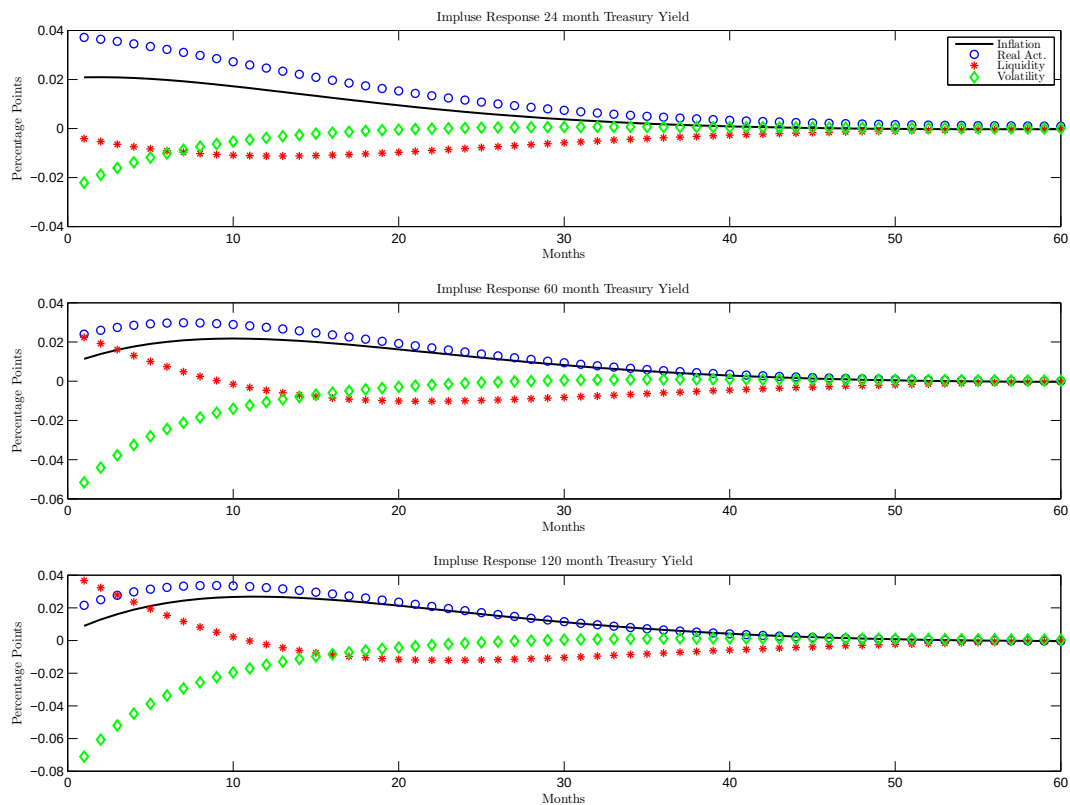


Figure 1.9: Impulse responses functions for Treasury yields. The figure shows impulse responses for 24-months (top), 60-months (middle), 120-months (bottom) Treasury yields. Only the impulse responses from the economic factors are shown. All impulse responses are from a one standard deviation shock.

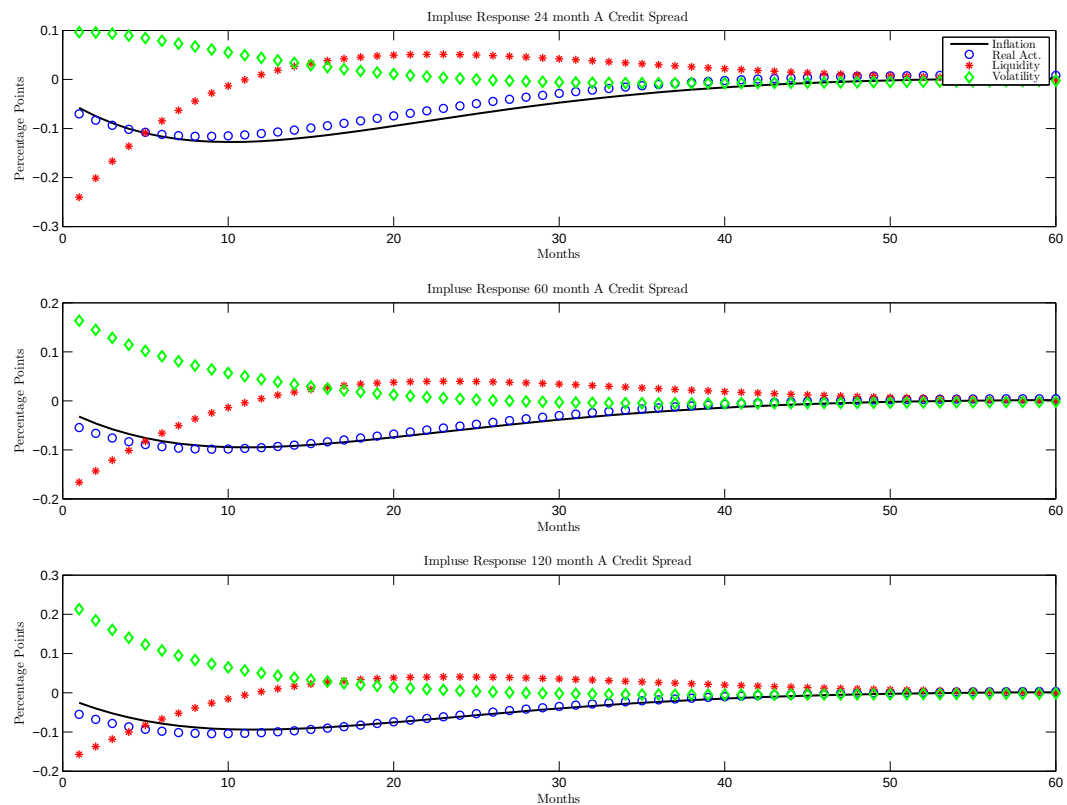


Figure 1.10: Impulse responses functions for A credit spreads. The figure shows impulse responses for 24-months (top), 60-months (middle), 120-months (bottom) A-rated bond yields. Only the impulse responses from the economic factors are shown. All impulse responses are from a one standard deviation shock.

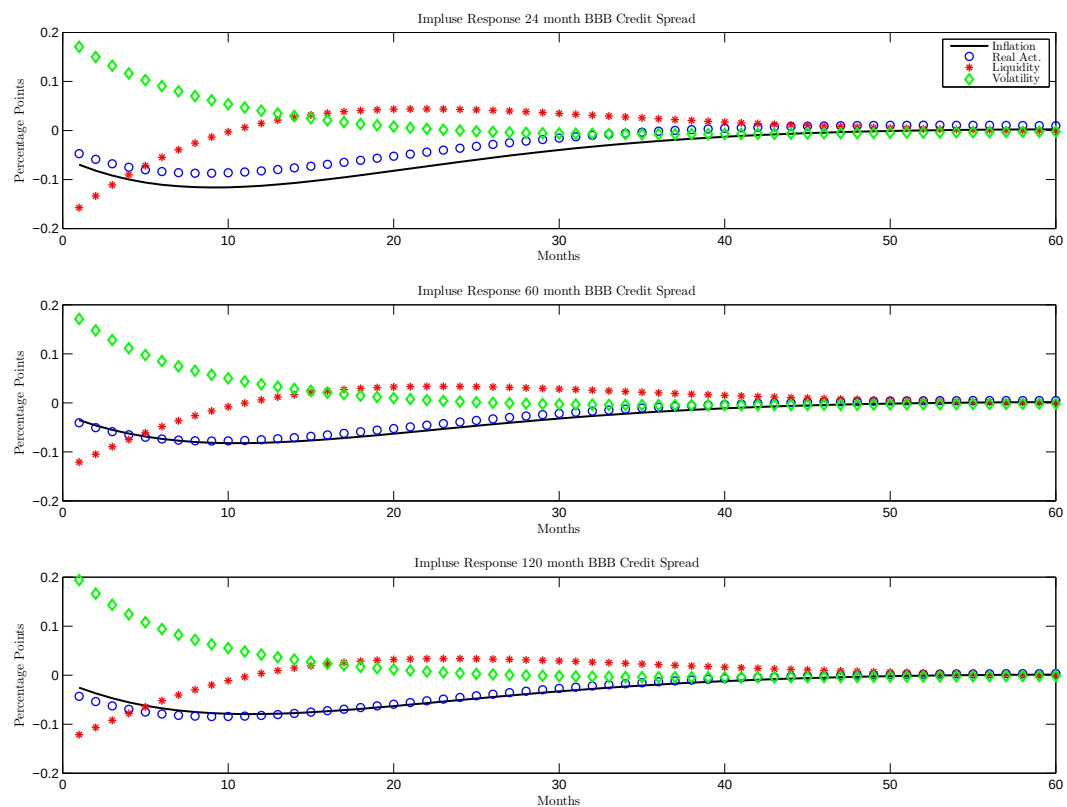


Figure 1.11: Impulse responses functions for BBB credit spreads. The figure shows impulse responses for 24-months (top), 60-months (middle), 120-months (bottom) BBB-rated bond yields. Only the impulse responses from the economic factors are shown. All impulse responses are from a one standard deviation shock.

Chapter 2

From Funding Liquidity to Market

Liquidity: Evidence from Index Options Market

This chapter examines the relationship between funding liquidity and market liquidity using daily data of the S&P 500 Index options market covering January 2003 - January 2012. We find that options market liquidity is positively correlated with funding liquidity during the periods of high market uncertainty. Besides, this paper finds a positive relationship between the options market liquidity and VIX. We also split the whole sample into two sub-samples of pre- and post- August 2007. Consistent with the theoretical predictions of Brunnermeier and Pedersen (2009), we find that when funding liquidity is low, option market liquidity becomes sensitive to changes in funding liquidity and tends to decline with funding liquidity. This chapter also uncovers several important features of the index options market liquidity. First, the impact of funding liquidity on puts liquidity is much higher than that on calls. Second, investors are inclined to trade short maturity options during the recent financial crisis and their liquidity is sensitive to funding liquidity.

2.1 Introduction

During the 2008 crisis, and especially the periods when Lehman Brothers and other important financial institutions failed, funding available to banks and non-financial firms was in short supply. In that period, a number of institutions failed because they had difficulties in raising funds in illiquid markets. It is thus timely and fitting to examine the dynamic changes in market liquidity in regards to changes in funding liquidity. This paper shows empirically that options market liquidity was strongly influenced by funding liquidity during the periods of high market uncertainty. More specifically, we find that liquidity in the S&P 500 Index options market is positively correlated with funding liquidity, after controlling VIX.

From a theoretical point of view, the idea that market declines cause asset illiquidity has received much attention. Brunnermeier and Pedersen (2009) provide a model that elaborates on the relationship between funding liquidity and market liquidity (FL-ML) and show that the two notions are mutually reinforcing, leading to liquidity spirals. They argue that a huge market-wide decline in prices reduces the ease with which market makers can obtain funding, which feeds back as higher comovement in market liquidity during the recessions. Garleanu and Pedersen (2007) focus on a feedback effect and argue that tighter risk management reduces liquidity, which in turn leads to tighter risk management. Their results help explain sudden drops in liquidity and in prices in connection with increased volatility or decreased risk-bearing capacity. More recently, Garleanu and Pedersen (2011) show that a funding liquidity crisis gives rise to a price gap between securities with identical cash-flows but different margins. In his 2010 AFA presidential address, Duffie (2010) argues that financial crisis and slow movement of investment capital increase the cost of intermediation and thus lead to increases in trading spreads. Moreover, Duffie (2012) points out that the 2008 financial crisis not only affected banks' lending function, but also had a major impact on market liquidity. He further argues that investors and issuers of securities would find it more costly to borrow, raise capital, invest, hedge risks, and obtain liquidity for their existing positions during the recent financial crisis.

The implications of these recent important theoretical findings have not been fully investigated from an empirical point of view and to date, to the best of our knowledge, there has not been a thorough empirical analysis of the relationship between market liquidity and funding liquidity over a long period of time. Chordia, Sarkar, and Subrahmanyam (2005) explore liquidity movements in stock and Treasury bond markets over a period of more than 1800 trading days and establish a link between macro liquidity, or money flows, and micro or transactions liquidity. Using a dummy variable to proxy for the period of low funding liquidity, Hameed, Kang, and Viswanathan (2010) test the relationship between funding liquidity and market liquidity in the stock market and their sample period is before the crisis. The Great Recession provides us with a laboratory to fully understand the dynamic changes in market liquidity in regards to changes in funding liquidity. Hu, Jain, and Jain (2013) provide the first empirical evidence of a non-linear relationship between FL-ML and show that the relationship weakens after the enactment of the Volcker Rule. Dick-Nielsen, Gyntelberg, and Lund (2013) examine how funding liquidity is driving the bond market liquidity in Denmark. They find that the ease of obtaining term funding in the money markets determines the liquidity in the bond market, for both long and short term bonds. Coffey, Hrungr, and Sarkar (2009) investigate how funding liquidity conditions have affected deviations from Covered Interest Parity (CIP) in the major currency markets during the financial crisis. Mancini, Ranaldo, and Wrampelmeyer (2013) use intraday trading and order data to measure liquidity in the foreign exchange (FX) market and show that negative shocks in funding liquidity lead to significantly lower FX market liquidity and systematic FX liquidity comoves with equity liquidity. However, none of the previous literature has studied the dynamics of funding liquidity and options market liquidity during the crisis.

This paper presents one of the first systematic empirical studies of liquidity in the S&P 500 Index options market and analyzes the impact of funding liquidity on the index options market liquidity during the recent financial crisis. We measure liquidity in the index option market on a daily basis, relate index options market liquidity to measures of funding liquidity as well as liquidity of equity markets, and provide solid evidence to support the theoretical

predictions of Brunnermeier and Pedersen (2009), namely higher comovement in liquidity and higher impact of funding liquidity on market liquidity during the crisis.

We compute options liquidity using a comprehensive dataset. Ranging from January 2003 to January 2012, our sample includes the financial crisis and is thus highly relevant for analyzing liquidity. Following Chordia, Roll, and Subrahmanyam (2000) and Cao and Wei (2010), we use proportional bid-ask (PBA) spread as our measure of index options liquidity in this paper. We compute the proportional bid-ask spread by dividing the difference between ask and bid quotes by the midquote. We adopt TED spread ¹ (difference between the 3-month LIBOR and the 3-month U.S. Treasury bill rate) as a proxy for the level of funding liquidity, suggested by leading scholars in Hameed, Kang, and Viswanathan (2010), Boyson, Stahel, and Stulz (2010), Brunnermeier, Nagel, and Pedersen (2008) and Brunnermeier and Pedersen (2009).

Using an *ARMAX* model to link the liquidity in the options market to funding costs and VIX over the entire sample period, we find a positive relationship between PBA spread and TED spread. And the coefficient is statistically significant at the 1% level. In particular, one standard deviation increase in TED spread can be translated into an increase in bid-ask spread as large as 0.95 basis point, which is about 22% of the standard deviation. Our empirical findings lend support to the hypothesis that market liquidity declines when liquidity providers face high funding costs. Moreover, we find that the liquidity of puts and calls responds asymmetrically to downward market movements. For instance, there is a statistically significant negative relationship between PBA spread and VIX for the subsample of puts over the entire sample period, while the coefficient of VIX for calls is not significant at all. Our empirical results illustrate that puts are favoured by informed traders to realize their information value during the recessions and are the investors' choice of trade in response to downward trends.

Next, to study the impact of funding liquidity on options market liquidity, we further split the whole sample according to maturity, forward-moneyness, and implied volatility of each

¹An alternative proxy for funding liquidity in this paper is the LIBOR-OIS spread. The results based on this alternative spread are similar and are available upon request.

option. We maintain the same specification of ARMAX model, linking the option market liquidity to funding liquidity and VIX. First, we find a positive relationship between PBA spread of short maturity option and TED spread. Thus, a reduction of funding liquidity are followed by lower short maturity option liquidity. This result might be due to the fact that trading in short-maturity options is the informed investors' choice. Second, the results show that TED spread is positively related to PBA spread and invariably significant in explaining the liquidity of options with different forward-moneyness. Therefore, the liquidity of options with different moneyness declines when investors face high funding costs consistent with predictions of Brunnermeier and Pedersen (2009). However, when we split the sample into five quintiles based on the implied volatility of each option, we cannot find such evidence.

Finally, to show that our results are not driven by the specific measure or sample period, we conduct several robustness test. First, we use alternative measures of investors' uncertainty, VXO, VXN and VXD provided by the CBOE. The results based on alternative measures are similar. Second, we split the whole sample into two sub-samples of pre- and post- August 2007 and add into our analysis two alternative measures for options market liquidity, namely transaction volume and total dollar volume. We also find similar results. Interestingly, the TED spread is strongly negatively related to the dollar trading volume for the sub-period of post-crisis, while its coefficient is not significant for the sub-period of pre-crisis. It means that a reduction of funding liquidity is followed by lower dollar trading volume, matching with the theoretical predictions of Brunnermeier and Pedersen (2009), who point out that when funding liquidity is low, market liquidity becomes sensitive to changes in funding liquidity and declines with a decline in funding liquidity.

Our empirical analysis is also related to the literature dealing with liquidity in options markets. While an extensive literature studying liquidity in equity markets exists, liquidity in the options market has mostly been neglected, although the options market is by far one of the most important markets. Cetin, Jarrow, Protter, and Warachka (2006) study the pricing of options in an extended Black-Scholes economy in which the underlying asset is not perfectly liquid and the liquidity risk is modelled as a stochastic supply curve. They show that liquidity

costs account for a significant portion of the option price. Using Ivy DB's OptionMetrics data, Cao and Wei (2010) illustrate the commonality among various measures based on the bid-ask spread, volumes and price impact. They provide convincing evidence that the options liquidity responds asymmetrically to upward and downward market trends, with calls reacting more in up markets and puts reacting more in down markets.

The remainder of this paper is organized as follows. In section 2, we describe the data, define the liquidity measures and report the summary statistics. Section 3 presents the main results concerning the dynamics of market liquidity and funding liquidity during the crisis. Some additional robustness tests are provided in Section 4. The final section concludes.

2.2 Data and variables

2.2.1 Data

Our data consist of daily closing bid and ask quotes, daily volume and open interest on S&P 500 options traded on the CBOE market. We cover the period January 17th 2003 - January 31st 2012, for a total of 2,265 trading days. We extract the CBOE data from the OptionMetrics IVY DB, which also includes daily closing price, option strike price, implied volatility and actual days to expiration. In the S&P 500 sample, we have 223,447 observations, of which 104,502 are calls and 118,945 are puts. Hence, out of the 2,265 trading days we end up with an average of about 99 options per day.

Following Bakshi, Cao, and Chen (1997), we set up three option classes by time-to-maturity, measured in actual days to expiration ($ADTE_{i,t}$):

1. Short-term if $ADTE_{i,t} < 60$;
2. Medium-term if $60 \leq ADTE_{i,t} \leq 180$;
3. Long-term if $180 \leq ADTE_{i,t}$.

Following Goncalves and Guidolin (2006), we set up five option classes by forward-moneyness($m_{i,t}$):

1. Deep-out-of-the-money (DOTM) if the contract is a call and $m_{i,t} < 1.06$ or if the contract is a put and $m_{i,t} < 0.94$;
2. Out-of-the-money (OTM) if the contract is a call and $1.01 < m_{i,t} \leq 1.06$ or if the contract is a put and $0.94 \leq m_{i,t} < 0.99$;
3. At-the-money (ATM) if $0.99 \leq m_{i,t} \leq 1.01$ for either puts or calls;
4. In-the-money (ITM) if the contract is a call and $0.94 \leq m_{i,t} < 0.99$ or if the contract is a put and $1.01 < m_{i,t} \leq 1.06$;
5. Deep-in-the-money (DITM) if the contract is a call and $m_{i,t} < 0.94$ or if the contract is a put and $m_{i,t} > 1.06$.

Following Cao and Wei (2010), we set up five option classes by implied volatility ($LogIV_{i,t}$):

1. LogIV1 if $LogIV_{i,t} < -2.0112$;
2. LogIV2 if $-2.0112 \leq LogIV_{i,t} < -1.7787$;
3. LogIV3 if $-1.7787 \leq LogIV_{i,t} < -1.5716$;
4. LogIV4 if $-1.5716 \leq LogIV_{i,t} < -1.3543$;
5. LogIV5 if $-1.3543 \leq LogIV_{i,t} \leq -0.1716$.

We adopt the option liquidity measures used in Cao and Wei (2010). We use the bid-ask spread (PBA) as the order-based measure and the dollar trading volume (DVOL) as the transaction-based measure. Since our data is a pooled sample, we cannot include Amihud's ILLIQ as the price impact measure. Appendix I includes the definitions of the option liquidity measures.

Our primary funding liquidity measures: (1) the TED spread from the Federal Reserve Bank in St. Louis, (2) net acquisition of financial assets by security brokers and dealers from

the flow of funds account provided by Federal Reserve Statistical Release. Adrian and Shin (2010) argue that the change of the aggregate balance sheet of financial intermediaries reflects the aggregate liquidity. We collect the net acquisition of financial assets by security brokers and dealers which is used in Hu, Jain, and Jain (2013) when studying the relation between funding liquidity and equity market liquidity, (3) return on security brokers, dealers and flotation companies, as in Hameed, Kang, and Viswanathan (2010).

Figure 2.1 depicts the evolution of options liquidity, the TED spread and VIX from January 2003 to January 2012. Both TED spread and VIX shoot up during the financial crisis. However, the options market bid-ask spread seems to reach its lowest level in the crisis. Before the crisis when both the TED and VIX stay at low levels, the options bid-ask spread is almost twice as high as its level during the crisis. The “cooling down” in boom and “heating up” in crisis of options market transactions point directly to the distinctive feature of this market. A similar pattern can also be observed in figure 2.2 which displays the options market liquidity along with equity market liquidity. Figure 2.2 illustrates that these two liquidity measures are negatively correlated.

2.2.2 Summary statistics

Table 2.1 presents the summary statistics of both the various options market bid-ask spread and the key explanatory variables. During our sample period, the mean bid-ask spread for all options is 12 basis points. It is consistent with the calculation in Cao and Wei (2010) who find a 13-bps bid-ask spread during the period from 1996 to 2004. Since our sample period has spanned the financial crisis when the options market was relatively more liquid, the options market liquidity measure should be higher in this study². The standard deviation is also quite close to the one in Cao and Wei (2010). Compared with call options, put options have a smaller bid-ask spread and a larger trading volume in terms of both the mean and the median. The higher liquidity for puts in our paper might be attributed to the high transaction activities

²The transaction and dollar volume should be higher, while the bid-ask spread should be lower correspondingly.

during the financial crisis. In the rest of the rows in Panel A, all options are classified into several categories based on different characteristics, such as maturity, moneyness and implied volatility. Consistent with the stylized fact, the liquidity of options decreases when moneyness fades away. Options that are deep in the money have a bid-ask spread as small as 3 bps while those deep out of the money ones are traded with a 20-bps bid-ask spread. The monotonic pattern cannot be found in options categorized by implied volatility. Out of the 5 quintiles, options with implied volatility in the second quintile are most liquid and the fifth quintile has the highest bid-ask spread.

Panel B contains the mean, median and standard deviation for the following sub-periods: the pre-crisis (01/2003 - 07/2007), within-crisis (08/2007 - 06/2009) and post-crisis period (07/2009 - 01/2012). Interestingly, in the crisis sub-period, the options market becomes most liquid. The options bid-ask spread is 3 bps lower than before the crisis and 2 bps lower than after the crisis. While options volume is not that different during the crisis, the total dollar volume is much higher, which is caused by the fact that the bid-ask spread declines in the crisis sub-period. Both TED and VIX increase dramatically as the crisis unfolds. The TED spread is more than ten times higher during the crisis, indicating that the funding liquidity suddenly drops in this period. We also notice that the VIX is twice as high as that before the crisis, implying that the market uncertainty perceived by investors increases during the crisis.

2.3 Empirical results

To model the relationship between options market liquidity and funding liquidity as well as market uncertainty, we first have to test for the stationarity of these several time series. The Augmented Dickey Fuller (ADF) test result is shown in table 2.2, revealing that all of the variables of interest are stationary. The null hypothesis of unit root is rejected at 1% for all of our series. Therefore, we choose *ARMAX* to model the effect of funding liquidity and

market uncertainty on options market liquidity. An $ARMAX(p, q)$ is specified as follows.

$$BASpread_t = \alpha + \sum_{i=1}^p \pi_i BASpread_{t-i} + \sum_{j=1}^q \theta_j \epsilon_{t-j} + \beta TED_{t-1} + \gamma VIX_{t-1} + \epsilon_t \quad (2.1)$$

where p and q are the number of lags for autoregression and moving average terms. β and γ are the coefficient of TED spread and the Chicago Board Options Exchange Volatility Index (VIX), respectively. The VIX is frequently used as a proxy for investors' fear and uncertainty in financial markets. We use AIC and BIC to determine the optimal lags.

2.3.1 Main results

Table 2.3 shows results of ARMAX regressions linking the liquidity in the options market to funding costs and VIX over the entire sample period from January 17th 2003 to January 31st 2012. After controlling for two lags of the proportional bid-ask (PBA) spread, we first regress PBA spread on TED spread and VIX using simple OLS. The results are presented in column (1). Interestingly, we find a negative relationship between PBA spread and VIX. When the market uncertainty increases by one standard deviation (9.88) yesterday, current option market bid-ask spreads respond by declining 1.08 basis points which is about 25% of one standard deviation(4.34). This effect is essentially different from what Mancini, Rinaldo, and Wrampelmeyer (2013) find in the currency market where increase in market uncertainty is followed by a decline in FX market liquidity. We interpret this finding as the distinctive function of options, which are primarily used to hedge or speculate during the crisis. Given that the demand for hedging or speculation is higher when investors are more uncertain about the future, any increase in implied volatility will lead to higher options market transactions and thus higher options market liquidity. Note that this effect is the net of controlling for two lags of proportional bid-ask spread. The magnitude of this effect estimated from OLS regressions without controlling for lags of PBA spread is actually doubled. In line with our hypothesis, we find a positive relationship between the current PBA spread and the previous day's TED spread. And the coefficient is statistically significant at the 1% level. One-standard

deviation increase in funding liquidity (0.5) can be translated into an increase in options bid-ask spread as high as 0.46, which is about 7.5% of the standard deviation of proportional bid-ask spread. Therefore, after controlling VIX, options market liquidity declines when liquidity providers face higher funding costs, consistent with the theoretical prediction of Brunnermeier and Pedersen (2009).

In column (2), we estimate an ARMAX(4,3) with four autoregressive and three moving average terms. VIX and TED spread lagged for one day are included as exogenous variables. The results are similar to those in the OLS estimation. However, the magnitude of the effect of TED on bid-ask spread is more than twice as high as in the first column, highlighting the necessity of taking into account the moving average part when we fit the process of bid-ask spread. Here one standard deviation increase in TED spread can be translated into an increase in bid-ask spread as large as 0.76 basis point, which is about 18% of the standard deviation of the PBA spread. Column (3) and (4) add two additional variables into the regression, maintaining initial number of autoregressive and moving average terms. *Acq2* captures the funding liquidity of dealers and brokers by measuring the amount of asset acquisition each quarter by these market participants. Although its coefficient is not significant in either of these two models, it has the expected sign. In the fourth column, we also add into a recession dummy (NBER recession) to capture any systematic difference in options bid-ask spread during recession periods from December 2007 to June 2009. However, the coefficient on the dummy variable is not significant³.

In Column (5) and (6), we further distinguish between call and put options to see if there is any difference in the pattern of how funding liquidity and market uncertainty affect options liquidity for calls and puts. As in previous regressions, we find a statistically significant negative relationship between the PBA spread and VIX, in both the call and the put sample. This suggests that rising market-wide uncertainty contributes to a higher bid-ask spread in both the call and the put options market, despite the distinctive functions of these two markets. The

³Note that PBA spreads during the recession is 9.7 basis points, lower than the spreads during non-recession times by 3 basis points, and the difference is significant at 1% level.

magnitude of VIX's effect on the PBA in terms of standard deviation is higher in the put sample. For call options, one-standard-deviation change in VIX is followed by a decrease in the call PBA spread by 0.24 standard deviation. For puts, the magnitude is slightly higher and is equal to 0.32. It illustrates that puts are favoured by informed traders to realize their information value during recessions and are the investors' choice of trade in response to downward trends. These results are consistent with empirical findings of Cao and Wei (2010). Strikingly, we can only find a positive relationship between the PBA spread and TED spread for the subsample of puts. A one-standard-deviation increase in TED spread is followed by an increase in the PBA spread as large as 1.43 basis points, which is equivalent to 0.3 of the standard deviation. This effect is much higher than that for options in the whole sample. It again lends support to Brunnermeier and Pedersen (2009). Overall, the results using daily options market liquidity support our hypothesis that market liquidity deteriorates when the supply of capital is tight during the periods of high market uncertainty.

2.3.2 Subsample analysis

To study the dynamic changes in options market liquidity in regards to changes in funding liquidity, we further split the sample three ways. In table 2.4, we split the sample according to maturity (Short-term if $ADTE_{i,t} < 60$; Medium-term if $60 \leq ADTE_{i,t} \leq 180$; Long-term if $180 \leq ADTE_{i,t}$). In table 2.5, we set up five options classes by forward-moneyness (DOTM if the contract is a call and $m_{i,t} < 1.06$ or if the contract is a put and $m_{i,t} < 0.94$; OTM if the contract is a call and $1.01 < m_{i,t} \leq 1.06$ or if the contract is a put and $0.94 \leq m_{i,t} < 0.99$; ATM if $0.99 \leq m_{i,t} \leq 1.01$ for either puts or calls; ITM if the contract is a call and $0.94 \leq m_{i,t} < 0.99$ or if the contract is a put and $1.01 < m_{i,t} \leq 1.06$; DITM if the contract is a call and $m_{i,t} < 0.94$ or if the contract is a put and $m_{i,t} > 1.06$). In table 2.6, we split the sample into 5 quintiles according to implied volatility.

We first categorize the whole options sample into 3 subsamples based on the maturity. Table 2.4 shows results of ARMAX regressions linking the liquidity of short, medium and

long options to funding costs and VIX over the entire sample period. We maintain the same specification of the ARMAX model except the lags of autoregressive and moving average terms. Samples for all maturities are fitted by the ARMAX(4,3) model which is selected on the basis of information criteria. Again, VIX is strongly negatively related to short and medium maturity options. However, its coefficient is not significant for long maturity options. Also, the magnitude of the effect of VIX on bid-ask spread for short maturity options is more than twice as high as that for medium maturity options. As a result, the liquidity of different maturity options responds asymmetrically to downward market movements. It illustrates that short maturity options are favoured by informed traders to realize their information value and are the investors' choice of trade during periods of high market uncertainty. We also find a positive relationship between the PBA spread of short maturity option and TED spread which is significant at the 10% level. Thus, a reduction of funding liquidity is followed by lower short maturity options liquidity. But the coefficients are not significant for options with medium and long maturity.

Next, we split our sample according to forward-moneyness. The options are divided into 5 categories based on the forward-moneyness of each option, namely deep out of the money (DOTM), out of the money (OTM), at the money (ATM), in the money (ITM) and deep in the money (DITM). Table 2.5 shows results of ARMAX regressions linking the liquidity of DOTM, OTM, ATM, ITM, DITM to funding liquidity. TED spread is positively related to the PBA spread. And in most cases, it is significant in explaining the liquidity of options with different moneyness. Therefore, the liquidity of options with different moneyness declines when investors face high funding costs consistent with predictions of Brunnermeier and Pedersen (2009). Interestingly, there is a monotonic decline on magnitude of coefficient from out-of-the-money options to in-the-money options. VIX is negatively correlated with market liquidity of these three types of options, as displayed from column (1) to (3). It means that increase in market uncertainty is followed by an increase in market liquidity of OTM, ATM and ITM. Surprisingly, the estimated coefficients become significantly positive for DITM options and positive but insignificant for DOTM. It might be due to the fact that investors have

to afford a high cost in transactions of DOTM and DITM and therefore the liquidity of these options declines during the periods of high market uncertainty.

We further split the sample into 5 quintiles based on the implied volatility of each option, namely IV1, IV2, IV3, IV4, and IV5. Table 2.6 shows results of ARMAX regressions linking the liquidity of these types of options to funding cost and market uncertainty. Unfortunately, we cannot find significant relationships between funding liquidity and market liquidity, except for the IV2 and IV3 quintile. There is an almost monotonic decline on magnitude of TED spread coefficients from IV1 to IV 5. Interestingly, we also find a monotonic decline of VIX coefficients from IV1 to IV5. When the market uncertainty increases, the PBA spread of options with low implied volatility increases while that of options with high implied volatility decreases. One possible explanation would be that investors would like to sell options with high implied volatility levels during the periods of high market uncertainty. As a result, the liquidity of options with high implied volatility increases.

2.3.3 Relation to liquidity of the US equity market

Even though we document a negative relationship between options market liquidity and funding liquidity, we cannot rule out the possibility that this effect is driven by the third confounding factor. A plausible candidate is the stock market liquidity, which is related to the liquidity of options. In this section, we control for stock market liquidity to investigate whether funding liquidity still survives. There are a number of reasons to expect a connection between equity and index options market liquidity. First, the market-wide liquidity is closely linked to the movements of the overall stock market (Cao and Wei (2010)). Second, an interdependence between liquidity in the two markets is consistent with the interaction of the market and funding liquidity during liquidity spirals predicted by Brunnermeier and Pedersen (2009). We choose three similar variables to proxy for aggregate daily stock market liquidity, namely the bid-ask spread, the trading volume and the dollar volume. The data is from the CRSP. The way that we calculate the bid-ask spread of stocks can be found in appendix I.

Table 2.7 shows results of an *ARMAX* model linking the options market liquidity, stock market liquidity, and funding liquidity. We use equally- and value-weighted stock market bid-ask spread as equity liquidity measures. The first two columns use bid-ask spread derived from the high ask and low bid prices, which are from CRSP. Consistent with the results of Cao and Wei (2010), the liquidity in the options market is closely linked to the liquidity of the equity market. The coefficient of stock market liquidity is highly significant and positive, implying the effectiveness in explaining the options market liquidity. When there is an one-standard-deviation increase in the equally-weighted (value-weighted) stock bid-ask spread, the liquidity of the options market responds by decreasing 18% (20%) of a one standard deviation. The coefficients on lagged TED spread are not significant, though their signs are still positive. In the next two columns, we adopt alternative equity bid-ask spreads measured at the closing spot each trading day. The coefficients on TED spread are marginally significant and their signs are positive in both column (3) and (4), implying that the effect of the funding liquidity on options liquidity remains stable. After controlling for stock market liquidity, we still find that options market liquidity is positively correlated with funding liquidity. In column (5) and (6), we use volume and dollar volume and obtain the same results. After controlling the VIX and the equity market liquidity, there is still a positive relationship between options market liquidity and funding liquidity.

2.4 Robustness

To show that our results are not driven by the specific measure or sample period, we conduct several robustness tests in this section. In table 2.8, we use alternative measures of investors' uncertainty, VXO, VXN and VXD. The results based on alternative measures are similar. Given that the demand for hedging or speculation is higher when investors are more uncertain about the future, any increase in volatility of the market will lead to an increase in options market transactions and higher options market liquidity.

2.4.1 Split sample pre-post financial crisis

Table 2.9 shows the results for different options liquidity measures for sub-samples of pre- and post- August 2007. The sample preceding the financial crisis spans from January 2003 to July 2007 and the post financial crisis sample spans from August 2007 to January 2012. We also add into our analysis two alternative measures for options market liquidity, transaction volume and total dollar volume. To facilitate the comparison among different sample periods, we also display the baseline results of the whole sample period. In column (1)-(3), we use proportional bid-ask spread as the dependent variable and find that TED is not significant in each sub-period even though its sign is in line with our expectation. Both proportional bid-ask spread of pre-crisis and that of post-crisis are strongly negatively related to VIX. In column (4)-(6), we adopt dollar trading volume as the dependent variable and find significantly positive relationship between VIX and dollar trading volume of pre-crisis and post-crisis. Thus, an increase in investors' uncertainty is followed by significantly higher options market liquidity. This is in line with what we find using PBA spread as the proxy for options market liquidity. Interestingly, the TED spread is strongly negatively related to the dollar trading volume for the sub-period of post-crisis, while its coefficient is not significant for the sub-period of pre-crisis. It means that a reduction of funding liquidity is followed by lower dollar trading volume, matching with the theoretical predictions of Brunnermeier and Pedersen (2009); namely higher comovement in liquidity and higher impact of funding liquidity on market liquidity during the crisis. Using volume as the proxy for options market liquidity, we get similar results in column (7)-(9).

2.4.2 Weekly data sample

Table 2.10 uses weekly data and shows the results of the ARMAX regressions linking the liquidity in the options market to funding costs and VIX over the entire sample period. The results are similar to that of the daily sample. Consistent with the results in column (2) of Table 2.3, increase in market uncertainty in week $t-1$ is followed by a decrease in the PBA spread,

i.e. an increase in options market liquidity. Also, the TED spread is positively related to the PBA spread, lending support to Brunnermeier and Pedersen (2009). However, the magnitude of the effect of TED spread on the PBA spread is much less than that of daily data sample, reflecting that option market is a highly sensitive market. In column (3) and (4), we split our sample into calls and puts. Consistent with the empirical findings in table 2.3, TED spread is significantly positively related to the PBA spread of puts, while its coefficient is marginally significant for the subsample of calls. Moreover, we find a negative relationship between the PBA spread and VIX for both calls and puts. However, the magnitude of the effect of VIX on the PBA spread of puts is twice as high as that of calls, revealing that the options market liquidity responds asymmetrically to market uncertainty.

2.5 Conclusion

Funding liquidity and its impact on market liquidity have become a major focus in the academic literature. Most studies investigate the relationship between FL-ML from a theoretical point of view. For instance, Brunnermeier and Pedersen (2009) explain that a huge market-wide decline in prices reduces the ease with which market makers can obtain funding, which feeds back as higher comovement in market liquidity during the recessions. Recently, some studies have emerged that examine the relationship between funding liquidity and market liquidity in stock, corporate bond, and foreign exchange markets. However, none of the previous literature studies the relationship of funding liquidity and options market liquidity during the crisis. This paper presents one of the first empirical analysis of liquidity in the S&P 500 Index options market, and studies the relationship between FL-ML.

Using data of the S&P 500 Index options traded on the CBOE market covering the period from January 17th 2003 to January 31st 2012, we establish convincing evidence of a positive relationship between funding liquidity and options market liquidity during the periods of high market uncertainty. More specifically, we find a positive relationship between the PBA spread of and TED spread and the coefficient is statistically significant at the 1% level. In particular,

one standard deviation increase in TED spread can be translated into an increase in the bid-ask spread as large as 0.95 basis point, which is about 22% of the standard deviation. These empirical findings lend support to the hypothesis that market liquidity declines when liquidity providers face high funding costs during the periods of high market uncertainty.

Moreover, we split the whole sample into two sub-samples of pre- and post- August 2007 and add into our analysis two alternative measures for options market liquidity, namely transaction volume and total dollar volume. We also find similar results. Interestingly, the TED spread is strongly negatively related to the dollar trading volume for the sub-period of post-crisis, while its coefficient is not significant for the sub-period of pre-crisis. It means that a reduction of funding liquidity is followed by lower dollar trading volume, matching with the theoretical predictions of Brunnermeier and Pedersen (2009), who point out that when funding liquidity is low during the recessions, market liquidity becomes sensitive to changes in funding liquidity and declines with a decline in funding liquidity.

Aside from documenting the positive relationship between funding liquidity and options market liquidity during the periods of high market uncertainty, this study also uncovers several important features of the index options market liquidity. First, information asymmetry plays an important role as a driving force of market liquidity. This paper shows the evidence that the liquidity of puts and calls responds asymmetrically to downward market movements and the impact of funding liquidity on puts liquidity is much higher than that on calls. The results illustrate that puts are favoured by informed traders to realize their information value during the recessions and are the investors' choice of trade in response to downward trends. These empirical findings support the previous finding that informed traders may choose to trade in the options market, suggested by Easley, O'hara, and Srinivas (1998), Pan and Poteshman (2006), and Cao and Wei (2010). Second, when we split the sample according to maturity of each option, we find a positive relationship between the PBA spread of short maturity options and TED spread. Thus, a reduction of funding liquidity is followed by lower short maturity options liquidity. This result shows that short maturity options are the informed investors' choice of trade during recessions and their liquidity is sensitive to changes in funding liquidity.

This paper serves as a first step toward understanding the relationship of funding liquidity and index options market liquidity during periods of high market uncertainty. It opens up several avenues for future research. One natural extension would be the in-depth examination of the relationship of funding liquidity and options market liquidity using a panel data sample. Another area of future research would be to investigate the effect of funding constraints on the pricing of index options.

Appendix I: Variable Definitions

Variable	Definition
Dependent variables	
Proportional Bid-ask spread (PBA)	$\frac{\sum_{j=1}^J VOL_j * \frac{ask_j - bid_j}{(ask_j + bid_j)/2}}{\sum_{j=1}^J VOL_j}$
Trading Volume (VOL)	where j is one specific trade $\sum_{j=1}^J VOL_j$
Dollar trading volume (DVOL)	$\sum_{j=1}^J VOL_j (ask_j + bid_j) / 2$
Independent variables	
Bid-ask spread of stocks (equally weighted)	$\frac{\sum_{i=1}^N (ask_i - bid_i)}{N}$
Bid-ask spread of stocks (volume weighted)	where i is one specific stock traded on a certain day $\frac{\sum_{i=1}^N VOL_i (ask_i - bid_i)}{\sum_{i=1}^N VOL_i}$
TED Spread	The difference between three-month LIBOR and the three-month U.S. Treasury bill rate
VIX	CBOE volatility index
VXO	CBOE S&P 100 volatility index
VXN	CBOE NASDAQ volatility index
VXD	CBOE DJIA volatility index
Asset acquisition	The amount of financial assets acquired by brokers and dealers in any quarter
d_recession	Recession period defined by NBER, from December 2007 to June 2009

Table 2.1: Summary statistics of daily option market liquidity, VIX and TED spread

This table shows summary statistics for the main variables used in this study. Panel A gives a summary of daily option market liquidity which is measured by bid-ask spread, volume and dollar volume. The bid-ask spread is in basis points. Aside from showing the liquidity measures of call and puts, this table also reports the basic statistics of liquidity measures for options with different maturity, moneyness and implied volatility. Panel B provides the basic statistics for option and stock liquidity, as well as VIX and TED spread, with the sample period being divided into three sub-periods. Specifically, the three sub-sample periods are defined as the pre-crisis (01/2003-07/2007), within-crisis (08/2007-06/2009) and post-crisis period (07/2009-01/2012).

<i>Panel A: Option Liquidity</i>									
	<i>Bid-ask spread</i>			<i>Volume</i>			<i>Dollar Volume</i>		
Statistics	Mean	Med.	Std.	Mean	Med.	Std.	Mean	Med.	Std.
All options	11.84	11.40	4.34	2.86	2.71	1.24	7.03	5.46	5.87
Call option	12.67	11.99	5.21	2.54	2.37	1.15	6.32	4.99	5.02
Put options	11.19	10.44	4.62	3.14	2.94	1.47	7.58	5.60	6.94
Short-maturity	13.64	13.09	5.10	3.35	3.07	1.72	5.83	4.02	5.90
Medium-maturity	7.82	7.61	2.87	2.27	1.97	1.29	8.57	6.06	7.73
Long-maturity	4.96	4.76	1.81	1.22	0.99	0.89	8.16	5.80	7.80
Out of the money	12.60	11.88	5.08	2.88	2.67	1.42	5.42	3.75	5.41
At the money	6.92	6.88	2.11	3.92	3.49	2.18	12.91	9.98	10.79
In the money	4.96	4.84	1.56	1.91	1.40	1.60	10.38	6.97	9.97
Deep out of the money	19.94	18.29	8.52	2.62	2.19	2.01	2.62	1.83	2.86
Deep in the money	2.69	2.47	1.08	1.38	0.64	2.10	14.09	6.29	21.24
1st quintile of implied vol.	18.39	14.42	14.44	2.28	2.08	1.32	2.61	2.18	3.70
2nd quintile of implied vol.	15.80	12.47	10.94	2.38	2.00	1.57	3.68	2.47	3.73
3rd quintile of implied vol.	17.35	12.89	13.50	3.37	2.33	5.69	4.46	2.67	6.19
4th quintile of implied vol.	17.98	12.45	17.06	3.14	2.49	4.34	6.07	3.84	7.63
5th quintile of implied vol.	19.55	12.67	20.16	3.28	2.99	2.49	7.48	4.61	8.63

<i>Panel B: The Level of Key Variables in Sub-Periods</i>									
	<i>Jan. 2003 - Jul. 2007</i>			<i>Aug. 2007 - Jun. 2009</i>			<i>Jul. 2009 - Jan. 2012</i>		
Statistics	Mean	Med.	Std.	Mean	Med.	Std.	Mean	Med.	Std.
Option bid-ask spread	12.82	12.37	4.75	9.69	9.18	3.52	11.68	11.54	3.46
Option volume	2.28	1.90	1.18	3.62	3.46	1.05	3.32	3.21	0.94
Option dollar volume	3.46	2.87	2.08	13.51	11.45	7.58	8.58	7.57	3.93
TED spread	0.10	0.10	0.18	1.09	0.91	0.56	0.70	0.61	0.17
VIX	15.42	14.32	4.61	31.50	26.22	13.06	23.46	22.22	6.21
Stock bid-ask spread (HL)	0.80	0.76	0.16	1.48	1.34	0.58	0.82	0.75	0.28
Stock bid-ask spread (close)	0.02	0.02	0.01	0.04	0.03	0.02	0.01	0.01	0.00
Stock volume	4.29	4.18	0.96	8.66	8.33	2.54	8.10	7.82	1.89
Stock dollar volume	128.74	118.05	49.11	273.21	260.80	70.62	234.06	225.97	57.03

Table 2.2: Stationarity test for key variables

This table shows results of stationarity test for key variables used in this paper, namely proportional bid-ask spread, dollar volume, TED spread, VIX and several stock market liquidity measures. The Augmented Dickey-Fuller test statistics and the 1% critical value is reported in column (3) and (4). And the corresponding p-value is shown in the last column. ***, ** and * denote significance level at 1 %, 5 % and 10 %.

Variable	# observations	Dickey-Fuller test statistic	1% value	p-value
Bid-ask spread	2263	-21.678***	-3.430	0.000
Volume	2263	-19.657***	-3.430	0.000
Dollar volume	2263	-12.075***	-3.430	0.000
TED spread	2263	-3.873***	-3.430	0.002
VIX	2263	-4.557***	-3.430	0.000
BAClose_ew	2263	-46.758***	-3.430	0.000
BAHL_ew	2263	-16.498***	-3.430	0.000
Stock_Volume	2263	-11.009***	-3.430	0.000
Stock_Dollar	2263	-11.038***	-3.430	0.000

Table 2.3: Option liquidity, TED and VIX

This table shows results of *ARMAX* regressions linking the liquidity in the option market to funding costs and VIX. For each *ARMAX* regression, we choose the optimal lags of AR and MA terms according to BIC and AIC information criterion. *VIX* is the CBOE S&P 500 volatility index. TED spread is the difference between three-month LIBOR rate and U.S. Treasury bill with the same maturity. Both the VIX and TED spread are lagged for one period (day). *Acq2* is the total amount of assets acquired by dealers and brokers measured on a quarterly basis which partially reflects funding liquidity. Below the exogenous regressors are several autoregressive terms for each *ARMAX* model. For brevity, the moving averaging terms are not displayed. T-statistics are shown below the coefficient estimates inside parentheses. ***, ** and * denote significance level at 1 %, 5 % and 10 %.

Dependent var. Sample	(1) bid-ask All	(2) bid-ask All	(3) bid-ask All	(4) bid-ask All	(5) bid-ask Call	(6) bid-ask Put
L.vix	-0.109*** (-8.47)	-0.117*** (-4.31)	-0.124*** (-4.49)	-0.125*** (-4.50)	-0.099*** (-2.73)	-0.132*** (-4.15)
L.TED_Spread	0.649*** (2.76)	1.527** (1.98)	1.358* (1.71)	1.215 (1.49)	0.644 (0.61)	2.036** (2.40)
Acq2			-0.001 (-0.80)	-0.001 (-0.86)		
d_recession				0.871 (0.81)		
<i>ARMA</i>						
L.ar	0.330*** (16.03)	2.385*** (33.52)	2.385*** (33.51)	2.392*** (33.99)	1.524*** (16.86)	2.114*** (13.78)
L2.ar	0.209*** (10.21)	-2.226*** (-15.50)	-2.227*** (-15.49)	-2.239*** (-15.71)	-0.525*** (-5.84)	-1.572*** (-6.26)
L3.ar		0.914*** (8.05)	0.916*** (8.04)	0.921*** (8.16)		0.457*** (4.56)
L4.ar		-0.074** (-2.24)	-0.075** (-2.25)	-0.074** (-2.26)		
Model	OLS	ARMAX(4,3)	ARMAX(4,3)	ARMAX(4,3)	ARMAX(2,2)	ARMAX(3,3)
N	1302	2264	2264	2264	2264	2264

Table 2.4: Option liquidity, funding liquidity and VIX: Maturity

This table shows results of *ARMAX* regressions on the liquidity of options with different maturities. An option is considered to be short-term if $ADTE_{i,t} < 60$, medium-term if $60 \leq ADTE_{i,t} \leq 180$, long-term if $180 \leq ADTE_{i,t}$. T-statistics are shown below the coefficient estimates inside parentheses. Below the exogenous regressors are several autoregressive terms for each *ARMAX* model. For brevity, the moving averaging terms are not displayed. ***, ** and * denote significance level at 1 %, 5 % and 10 %.

	(1) Short	(2) Medium	(3) Long
L.TED_Spread	1.572* (1.72)	-0.149 (-0.30)	-0.249 (-0.82)
L.vix	-0.154*** (-4.63)	-0.062*** (-2.88)	-0.016 (-1.24)
<i>ARMA</i>			
L.ar	2.366*** (32.60)	-0.375 (-0.58)	-0.617*** (-7.77)
L2.ar	-2.222*** (-15.78)	0.945*** (12.74)	0.490*** (8.19)
L3.ar	0.946*** (8.69)	0.295 (0.50)	0.785*** (10.36)
L4.ar	-0.091*** (-2.77)	-0.030 (-0.95)	0.076*** (2.60)
<i>N</i>	2263	2263	2257
Model	ARMAX(4,3)	ARMAX(4,3)	ARMAX(4,3)

Table 2.5: Option liquidity, funding liquidity and VIX: Moneyness

This table shows results of *ARMAX* regressions on the liquidity of options with different extent of moneyness. An option is considered to be deep-out-of-the-money (DOTM) if the contract is a call and $m_{i,t} < 1.06$ or if the contract is a put and $m_{i,t} < 0.94$, out-of-the-money (OTM) if the contract is a call and $1.01 < m_{i,t} \leq 1.06$ or if the contract is a put and $0.94 \leq m_{i,t} < 0.99$, at-the-money (ATM) if $0.99 \leq m_{i,t} \leq 1.01$ for either puts or calls, in-the-money (ITM) if the contract is a call and $0.94 \leq m_{i,t} < 0.99$ or if the contract is a put and $1.01 < m_{i,t} \leq 1.06$, deep-in-the-money (DITM) if the contract is a call and $m_{i,t} < 0.94$ or if the contract is a put and $m_{i,t} > 1.06$. Below the exogenous regressors are several autoregressive terms for each *ARMAX* model. For brevity, the moving averaging terms are not displayed. T-statistics are shown below the coefficient estimates inside parentheses. ***, ** and * denote significance level at 1 %, 5 % and 10 %.

	(1) OTM	(2) ATM	(3) ITM	(4) DOTM	(5) DITM
L.TED_Spread	3.136** (2.16)	1.682* (1.85)	0.490 (1.40)	0.101 (0.43)	0.410*** (3.30)
L.vix	-0.275*** (-4.53)	-0.123*** (-3.76)	-0.002 (-0.15)	0.016 (1.53)	0.032*** (5.94)
<i>ARMA</i>					
L.ar	1.451*** (4.21)	2.484*** (43.11)	1.740*** (42.00)	0.956*** (23.41)	0.979 (1.03)
L2.ar	-0.686 (-1.24)	-2.426*** (-19.55)	-0.741*** (-18.06)	-0.952*** (-29.33)	-0.035 (-0.04)
L3.ar	0.345 (1.40)	1.030*** (10.36)		0.955*** (27.52)	
L4.ar	-0.112*** (-2.78)	-0.089*** (-3.04)		-0.087*** (-3.16)	
Model	ARMAX(2,3)	ARMAX(2,1)	ARMAX(1,2)	ARMAX(1,1)	ARMAX(3,2)
N	2264	2264	2262	2262	1639

Table 2.6: Option liquidity, funding liquidity and VIX: Implied Volatility

This table shows results of *ARMAX* regressions on the liquidity of options with different levels of implied volatility. Following Cao and Wei (2010), we set up five implied volatility classes with $LogIV_{i,t}$ lying in the interval $(-\infty, -2.0112)$, $[-2.0112, -1.7787)$, $[-1.7787, -1.5716)$, $[-1.5716, -1.3543)$ and $[-1.3543, -0.1716)$ respectively. T-statistics are shown below the coefficient estimates inside parentheses. Below the exogenous regressors are several autoregressive terms for each *ARMAX* model. For brevity, the moving averaging terms are not displayed. ***, ** and * denote significance level at 1 %, 5 % and 10 %.

	(1)	(2)	(3)	(4)	(5)
	IV1	IV2	IV3	IV4	IV5
L.TED_Spread	10.008* (1.70)	9.518*** (3.67)	9.790*** (3.63)	3.427 (0.95)	-4.226 (-1.03)
L.vix	1.109*** (5.36)	0.608*** (7.23)	-0.021 (-0.13)	-0.447* (-1.84)	-0.762*** (-4.22)
<i>ARMA</i>					
L.ar	0.994*** (3.79)	1.160*** (33.95)	1.702*** (40.80)	1.599*** (24.79)	-0.841*** (-58.03)
L2.ar	-0.017 (-0.07)	-0.208*** (-7.35)	-0.704*** (-17.07)	-0.492*** (-5.34)	0.398*** (16.94)
L3.ar				-0.108*** (-3.70)	0.786*** (59.74)
Model	ARMAX(2,3)	ARMAX(2,1)	ARMAX(2,2)	ARMAX(3,2)	ARMAX(3,2)
<i>N</i>	1166	1648	1919	1788	1279

Table 2.7: Option liquidity and stock market liquidity

This table shows results of *ARMAX* regressions linking the liquidity in the option market to stock market liquidity. *BAHL_ew* and *BAHL_vw* are equally- and value-weighted stock market bid-ask spread using high ask and low bid of all the stocks in NYSE, NASDAQ and AMEX. *BAClose_ew* and *BAClose_vw* are equally- and value-weighted closing stock bid-ask spread for the whole market. The other two exogenous variables are *Stock_Volume* and *Stock_Dollar* which denote the trading volume and dollar volume of the whole stock market, both scaled down by 1 billion. T-statistics are shown below the coefficient estimates inside parentheses. Below the exogenous regressors are several autoregressive terms for each *ARMAX* model. For brevity, the moving averaging terms are not displayed. ***, ** and * denote significance level at 1 %, 5 % and 10 %.

Liquidity measure	(1)	(2)	(3)	(4)	(5)	(6)
Weight	High-Low equal	High-Low volume	Close equal	Close volume	Volume	Dollar
L.vix	-0.155*** (-5.38)	-0.160*** (-5.28)	-0.117*** (-4.31)	-0.128*** (-4.76)	-0.138*** (-4.54)	-0.139*** (-4.62)
L.TED_Spread	1.093 (1.31)	1.244 (1.44)	1.527* (1.94)	1.450* (1.77)	1.391* (1.66)	1.587* (1.90)
BAHL_ew	1.982*** (9.14)					
BAHL_vw		2.075*** (8.94)				
BAClose_ew			0.504 (0.74)			
BAClose_vw				25.011*** (3.15)		
Stock_Volume					0.280*** (4.95)	
Stock_Dollar						0.010*** (5.86)
<i>ARMA</i>						
L.ar	2.373*** (33.09)	2.394*** (35.57)	2.384*** (33.50)	2.396*** (36.61)	2.373*** (33.89)	2.379*** (34.56)
L2.ar	-2.206*** (-15.25)	-2.250*** (-16.42)	-2.225*** (-15.49)	-2.256*** (-16.90)	-2.204*** (-15.69)	-2.216*** (-15.97)
L3.ar	0.904*** (7.87)	0.933*** (8.51)	0.913*** (8.04)	0.937*** (8.76)	0.904*** (8.12)	0.910*** (8.25)
L4.ar	-0.073** (-2.17)	-0.079** (-2.43)	-0.074** (-2.23)	-0.078** (-2.46)	-0.074** (-2.23)	-0.074** (-2.27)
ARMAX(p,q)	(4,3)	(4,3)	(4,3)	(4,3)	(4,3)	(4,3)
N	2263	2263	2263	2263	2263	2263

Table 2.8: Robustness I: Alternative volatility measures

This table shows results of *ARMAX* regressions linking the liquidity in the option market to funding costs and VIX. *VIX* is the CBOE S&P 500 volatility index. *VXO* is CBOE S&P 100 volatility index. *VXN* is volatility of NASDAQ and *VXD* measures the volatility of DJIA. Below the exogenous regressors are several autoregressive terms for each *ARMAX* model. For brevity, the moving averaging terms are not displayed. T-statistics are shown below the coefficient estimates inside parentheses. ***, ** and * denote significance level at 1 %, 5 % and 10 %.

	(1) VIX	(2) VXO	(3) VXN	(4) VXD
L.vix	-0.122*** (-4.31)			
L.vxo		-0.117*** (-4.58)		
L.vxn			-0.190*** (-7.02)	
L.vxd				-0.136*** (-4.36)
L.TED_Spread	1.506* (1.86)	1.532* (1.87)	0.055 (0.10)	1.471* (1.82)
<i>ARMA</i>				
L.ar	1.493*** (18.49)	1.390*** (9.47)	-0.036 (-0.10)	1.494*** (18.51)
L2.ar	-0.495*** (-6.15)	-0.302 (-1.60)	0.672** (2.25)	-0.495*** (-6.16)
L3.ar		-0.089** (-1.99)		
Model	ARMAX(2,3)	ARMAX(3,2)	ARMAX(2,2)	ARMAX(2,3)
<i>N</i>	2263	2263	2261	2263

Table 2.9: Option liquidity, TED and VIX: Sub-periods

This table shows results of *ARMAX* regressions linking the liquidity in the option market to funding costs and VIX. The whole sample period is divided into two sub-periods, the pre-crisis period (01/2003-07/2007) and crisis onward (07/2007-01/2012). T-statistics are shown below the coefficient estimates inside parentheses. Below the exogenous regressors are several autoregressive terms for each *ARMAX* model. For brevity, the moving averaging terms are not displayed. ***, ** and * denote significance level at 1 %, 5 % and 10 %.

Sample Period	<i>Bid-ask spread</i>			<i>Dollar volume</i>			<i>Volume</i>		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	All	Pre-Crisis	Crisis onward	All	Pre-Crisis	Crisis onward	All	Pre-Crisis	Crisis onward
vix	-0.131*** (-5.13)	-0.374*** (-5.28)	-0.104*** (-4.17)	0.533*** (38.67)	0.386*** (17.29)	0.567*** (22.40)	0.049*** (6.85)	0.066*** (4.40)	0.055*** (7.35)
TED_Spread	1.901** (2.33)	0.701 (0.44)	0.023 (0.03)	0.960* (1.95)	0.619 (0.97)	-2.306*** (-3.08)	0.321 (1.61)	0.316 (0.72)	-0.362* (-1.76)
<i>ARMA</i>									
L.ar	1.676*** (30.93)	1.162*** (33.69)	0.785 (1.12)	0.950*** (219.85)	1.283*** (28.00)	1.448*** (12.96)	0.946*** (122.66)	1.578*** (69.49)	1.781*** (22.58)
L2.ar	-0.677*** (-12.53)	-0.167*** (-4.97)	0.041 (0.07)	-0.291*** (-6.59)	-0.291*** (-6.59)	-0.513*** (-5.07)	-0.578*** (-25.52)	-0.578*** (-25.52)	-0.785*** (-10.40)
ARMAX(p,q)	(2,3)	(2,1)	(2,3)	(1,1)	(2,1)	(2,2)	(1,1)	(2,1)	(2,2)
N	2264	1140	1124	2264	1140	1124	2264	1140	1124

Table 2.10: Option liquidity, funding liquidity and VIX: Weekly Data

This table uses weekly data and shows results of OLS and *ARMAX* regressions linking the liquidity in the option market to funding costs and VIX. The first two columns use bid-ask spread for all of the options as the dependent variable. Column (3) and (4) examine call and put option liquidity respectively. T-statistics are shown below the coefficient estimates inside parentheses. Below the exogenous regressors are several autoregressive terms for each *ARMAX* model. For brevity, the moving averaging terms are not displayed. ***, ** and * denote significance level at 1 %, 5 % and 10 %.

Option Type	(1) All	(2) All	(3) Call	(4) Put
TED_Spread	-0.502** (-2.32)	0.896** (2.07)	0.873* (1.74)	0.990** (2.00)
week_VIX		-0.134*** (-5.08)	-0.096*** (-3.65)	-0.207*** (-6.49)
<i>ARMA</i>				
L.ar	0.623*** (17.41)	-0.804*** (-27.51)	1.196*** (72.37)	0.373** (2.13)
L2.ar		0.833*** (33.63)	-1.187*** (-59.18)	-0.388** (-1.97)
L3.ar		0.877*** (31.99)	0.967*** (58.99)	0.007 (0.03)
L4.ar				0.607*** (3.84)
Model	OLS	ARMAX(3,3)	ARMAX(3,3)	ARMAX(4,4)
N	470	471	471	471

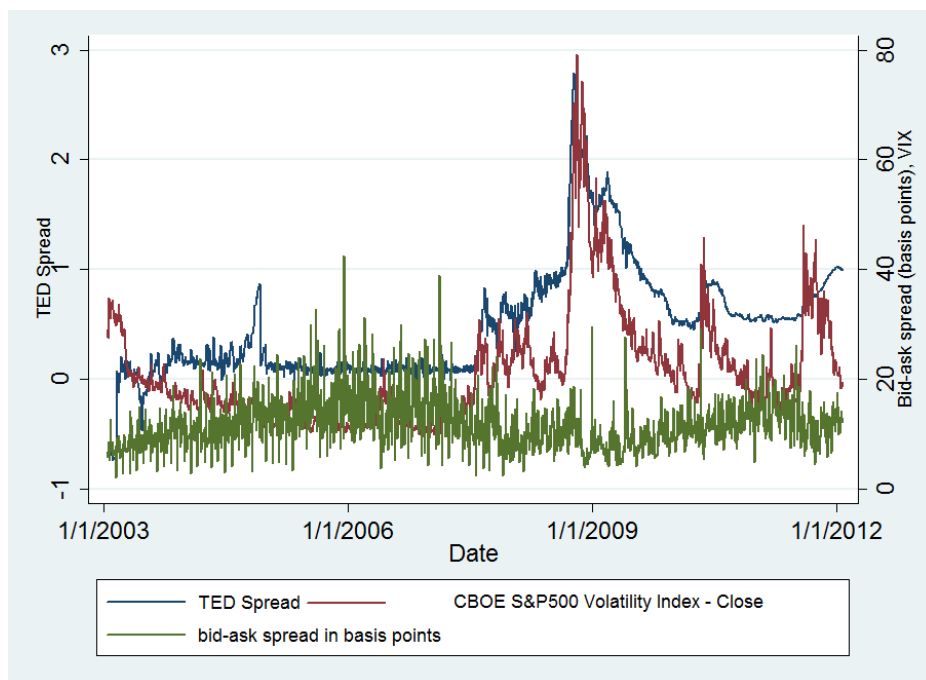


Figure 2.1: **The evolution of option market liquidity, TED spread and VIX.** This figure illustrates the evolution of option liquidity, the TED spread and VIX from January 2003 to January 2012. The way that we measure option market liquidity can be found in appendix I.

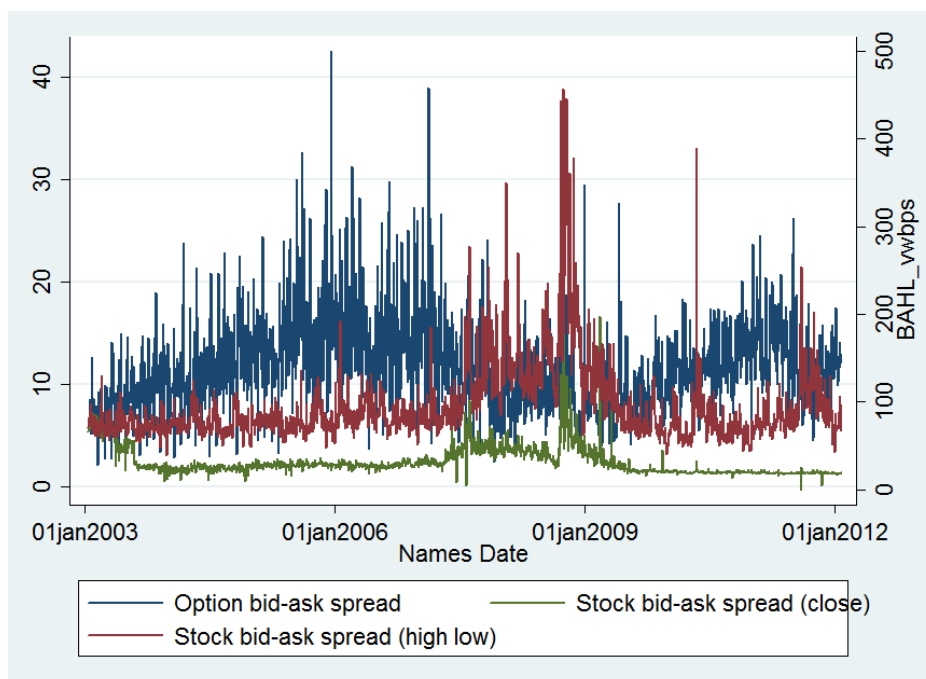


Figure 2.2: **The evolution of option market bid-ask spread, TED spread and VIX.** This figure illustrates the evolution of option and stock market liquidity from January 2003 to January 2012. The bid-ask spread is used as the proxy for liquidity and is defined in appendix I.

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di ZHOU ZHIPING

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Sono comunque fatti salvi i diritti dell'università Commerciale Luigi Bocconi di riproduzione per scopi di ricerca e didattici, con citazione della fonte.

Chapter 3

Commodity Futures as a Hedge against Inflation: An Application of Markov-Switching Cointegration and Error-Correction Modeling

This chapter investigates whether commodity futures provide the ability to hedge against inflation. We adopt a Markov-switching vector error correction model approach for a sample period ranging from January 1983 to December 2012. Over the full sample period, commodity futures show significant ability to hedge US inflation. In particular, the subindices of energy, industrial metals, and precious metals are the best inflation hedges. However, the hedging capacity exhibits substantial variation over time. Most of the inflation hedging power emerges under relatively longer and more common regimes covering the Great Moderation in which the long-run equilibrium correction speed coefficients are highly significant. We do not find any evidence for a significant hedging ability of agriculture and livestock subindices.

"The latest round of increases in energy prices has added to the upside risks to inflation and inflation expectations. The Federal Open Market Committee will strongly resist an erosion of longer term inflation expectations, as an unanchoring of those expectations would be destabilizing for growth as well as for inflation."

Ben Bernanke, June 9, 2008

3.1 Introduction

To circumvent recent financial crises, policy makers and regulators have been experimenting with unconventional tools, such as quantitative easing, which might help stimulate the economy but also is likely to simultaneously increase the monetary base and instigate inflation. As a result, inflation hedging has become particularly relevant in light of the current global recession. There exists voluminous literature on the topic of the inflation protection ability of different asset classes, including stocks, bonds, commodities, and real estate. Our study contributes to this literature by analyzing the inflation-hedging ability of commodity futures.

Commodity futures represent a bet on commodity prices, where the latter have played an important role in driving inflation rates. The 1973 oil crisis and the 1979 energy crisis, caused by interruptions in exports from the Middle East, remain the most common explanation for the Great Inflation of the 1970s. Moreover, Bernanke (2009) and Hamilton (2009) argue that the energy price run-up of 2007-2008 can account for much of the early stages of the Great Recession. However, the evidence linking commodity price shocks to inflation rates is not limited to energy prices. Commodity futures, such as agriculture and livestock futures, are directly linked to inflation rates; however, a broad-based commodity futures index does not contain all components of the consumer price index (for simplicity, CPI).

Researchers in finance have paid increased attention to the investigation of the hedge inflation effectiveness of commodity futures. Gorton and Rouwenhorst (2004); Erb and Harvey (2006); Hoevenaars, Molenaar, Schotman, and Steenkamp (2008); Gorton, Hayashi, and Rouwenhorst (2013); and Spierdijk and Umar (2013) are among recent papers that have focused on an aggregate commodity or commodity futures index, while a specific commodity

market, such as precious metals, has been investigated by Worthington and Pahlavani (2007); Hoevenaars, Molenaar, Schotman, and Steenkamp (2008); Bekaert and Engstrom (2010); Bruno and Chincarini (2010); Baur and McDermott (2010); and Wang, Lee, and Thi (2011). Most studies confirm the positive hedging ability of the commodities futures index.

Researchers have employed a variety of empirical techniques to investigate the inflation protection ability of different asset classes. Fama and Schwert (1977) try to verify the Fisher Effect and use a linear regression model to check the explanatory power of inflation in determining the returns of different asset classes. Bodie (1982) and Hoevenaars, Molenaar, Schotman, and Steenkamp (2008) investigate the Pearson correlation between inflation and nominal asset returns. Mahdavi and Zhou (1997) tackle the question whether gold and other commodity prices are leading indicators of inflation based on the estimation of a conventional vector error correction model (for simplicity, VECM) for the USA. Mahdavi and Zhou (1997) use the Johansen (1991) framework for a quarterly sample that covers from 1970 to 1994 and conclude that the CPI and the price of gold are not cointegrated. On the contrary, Levin, Montagnoli, and Wright (2006) also estimate a conventional VECM for the USA over a sample period from 1976 to 2005 and identify a long-run relationship between the gold price and the CPI. In a recent paper, Spierdijk and Umar (2013) apply a vector autoregressive (VAR) model to specify the relation between the CPI and commodity futures returns. In their framework, a rolling window and subsample analysis is performed to deal with structural changes in inflation rates and commodity futures prices (Chinn and Coibion (2013)). However, all these papers cannot reflect the different regimes present in the economy. The Markov-switching framework, a comprehensive framework to describe the relationship that changes from state to state, has been used in the past but not in the area of inflation hedging by commodity futures.

In this study, in order to analyze the changing nature of the long-run inflation hedging properties of commodity futures, we adopt a Markov-switching vector error correction model (MS-VECM) approach; this approach is able to distinguish between regimes in which long-run inflation hedging is statistically significant and relatively fast in the pace of adjustment, and regimes when no inflation hedging can be observed or even regimes when asset prices

diverge from inflation. The data used in this study are monthly data of returns on the S&P GSCI Total Return Index, ranging from January 1983 to December 2012. We choose the US seasonally adjusted CPI for all urban consumers as our inflation index. We also used the CRSP U.S. Treasury 10-Year Bond Index and the CRSP value-weighted US market index, a comprehensive proxy for US stocks that are publicly traded on the NYSE, NASDAQ, and AMEX. This study analyzes long-run relationships within MS-VECM models in eight sets of variables.

The main findings of this study are as follows. Over the full sample period, the aggregate S&P GSCI Total Return Index showed some ability to hedge US inflation, even though inflation hedging is slow in pace and it is of relatively low statistical significance. According to the speed coefficients related to the error-correction mechanism, energy, industrial metals, and precious metals are the best inflation hedges. However, the hedging capacity exhibits substantial variation over time. Particularly, in the model of the CPI and energy, the sign and the related t-statistic of the long-run error-correction coefficient in the energy index indicate that energy prices diverge from the inflation index and the divergence is very fast in regime 1. However, this regime regime has an expected duration of slightly more than 4 months. Under the common regime of this model, the positive sign of the speed coefficient on the energy index shows that this divergence disappears. In the model of inflation and industrial metals, inflation hedging is relatively fast in pace and is of relatively high statistical significance in regime 2. Regime 2 was the most persistent regime prevailing during 77.41 % of months and covered a relatively stable inflationary period, such as the Great Moderation. Under such conditions, low inflation tends to go hand-in-hand with low industrial metals futures returns. The results of the model comprising inflation and precious metals revealed that the precious metals index was a good hedge against inflation during the period 1983-2006 (with a short period of regime 1 in 1986, 1990, and 2005). After the Great Recession, the precious metals index was again a good hedge against inflation. As a result, most of the actual inflation hedging was obtained under relatively longer and more common regimes in which the equilibrium correction speed coefficients are highly significant. However, the speed coefficients on the agriculture

and livestock index are not statistically significant, suggesting that we cannot use these two indices to hedge inflation risk.

The remaining two relationships tested under the multivariate framework consist of the relationship between inflation, common stocks, government bonds, and commodity as well as the one between inflation, common stocks, government bonds, and energy. In the model of inflation, common stocks, government bonds, and commodity, both the sign of the speed coefficient and its corresponding t-statistic (-5.005) indicate that there is a significant long-run equilibrium adjustment within the inflation index in regime 1. In regime 2, however, the speed coefficient becomes positive, implying an appearance of error-correction in inflation rates. In the model of inflation, common stocks, government bonds, and energy, during the periods when the return on the energy index affects the bond market, it is the government bond that can hedge inflation risks; however, during the periods when the return on energy index influences the stock market, it is the common stock that provides a better long-run against inflation.

The remainder of this paper is organized as follows. Section 2 shows the econometric methodology adopted in this paper. In section 3, we describe the data and report the summary statistics. In section 4, we estimate the autoregressive orders of the considered vector autoregressions (VARs) and implement the Johansen's cointegration test. Section 5 presents the main results concerning the Markov-switching multistate vector error-correction analysis. The final section concludes.

3.2 Methodology

The review of the most relevant literature in the area of inflation hedging characteristics of different asset classes shows that single-state models used in previous research may not be the best description of the dynamic relationships in the data. This paper allows for regime shifts in autoregressive parameters and error-correction speed coefficients. The model specification in this paper is an M-regime, p-th order autoregressive with r cointegrating vectors, Markov-

switching vector error-correction model.

$$\Delta X_t = \mu(s_t) + \sum_{i=1}^p A_i(s_t) \Delta X_{t-i} + \sum_{j=1}^r \alpha_j(s_t) Z_{t-1} + \epsilon_t, \quad \epsilon_t | s_t \sim NID(0, \Sigma_{s_t}), \quad (3.1)$$

where X_t denotes the time t column vector of observations, $s_t = 1, 2, \dots, M$ represents the regime in time t , $\mu(s_t)$ collects the regime-dependent intercepts, $A_i(s_t)$ is a row vector of i -th order autoregressive parameters in regime s_t , $\alpha_j(s_t)$ measures the speed of error correction in regime s_t , and Z_t is the column vector containing the residuals from the error-correction equation, i.e., $Z_t = \widehat{C}X_t - \widehat{C}\bar{X}_t$, where $\widehat{C}$ is the estimated cointegration matrix. These unobservable states are generated by a discrete-state, irreducible and, ergodic first-order Markov chain: $Pr(s_t = j \mid s_{t-1} = i) = p_{ij}$, where p_{ij} is the generic $[i, j]$ element of the $M \times M$ transition matrix P .

In order to provide a regime-specific equilibrium correction, α_j should be negative and significantly different from zero. However, long-run error correction and cointegration is explained as a mixture of M different regime-specific equilibrium adjustments.

$$\Delta X_t | F_{t-1} \sim \begin{bmatrix} N \left(\mu(s_{1,t}) + \sum_{i=1}^p A_i(s_{1,t}) \Delta X_{t-i} + \sum_{j=1}^r \alpha_j(s_{1,t}) Z_{t-1}, \Sigma_1 \right), \text{ with prob. } q_{s_{1,t}} \\ N \left(\mu(s_{2,t}) + \sum_{i=1}^p A_i(s_{2,t}) \Delta X_{t-i} + \sum_{j=1}^r \alpha_j(s_{2,t}) Z_{t-1}, \Sigma_2 \right), \text{ with prob. } q_{s_{2,t}} \\ \vdots \\ N \left(\mu(s_{M,t}) + \sum_{i=1}^p A_i(s_{M,t}) \Delta X_{t-i} + \sum_{j=1}^r \alpha_j(s_{M,t}) Z_{t-1}, \Sigma_M \right), \text{ with prob. } q_{s_{M,t}} \end{bmatrix}, \quad (3.2)$$

where F_{t-1} is information content at time $t-1$ and $q_{s_{j,t}}$ is the predicted probability of being in regime j at time t and $q_{s_{j,t}} = e_j' P \xi_{t-1|F_{t-1}}$. Here, e_j is a unit column vector with a unit value at the j -th position, $\xi_{t|F_t}$ is a column vector of the filtered regime probabilities at time t , and $\bar{\xi}$ is defined as ergodic probabilities of the long-run equilibrium

$$\xi_{t|F_t} = \begin{bmatrix} \Pr(s_t = 1 | F_t) \\ \Pr(s_t = 2 | F_t) \\ \vdots \\ \Pr(s_t = M | F_t) \end{bmatrix} \quad \text{with } \bar{\xi}_t = \begin{bmatrix} \Pr(s_t = 1) \\ \Pr(s_t = 2) \\ \vdots \\ \Pr(s_t = M) \end{bmatrix}. \quad (3.3)$$

As a result, it can be concluded that unconditionally the MS(M)-VECM(p,r) has a long-run equilibrium and provides an unconditional cointegration if the long-run error-correction coefficients are negative and statistically different from zero.

Krolzig (1997) provides an exhaustive overview of Markov-switching vector autoregressions. In the first stage, the Johansen (1991) cointegration test is applied in order to test if cointegration exists and to determine the number of cointegrating vectors. The trace test and the L-Max test of Johansen's procedure¹ are provided in this study. The second stage of the MS model estimation is performed in Ox with the MS-VAR package and the MS-VECM code reported in Krolzig (1998). In particular, estimation and inferences are based on the expectation-maximization algorithm described by Dempster, Schatzoff, and Wermuth (1977) and Hamilton (1989). This algorithm is a filter that enables the iterative calculation of the one-step ahead forecast of the state vector $\xi_{t|t-1}$ given the information set and the construction of the log-likelihood function of the data. The cointegration matrix estimated in the first stage is used in order to calculate the error-correction residuals, which enter equation 1 as Z_t .

3.3 Data

We used monthly data of returns on the S&P GSCI Total Return Index, an index of commodity sector returns that represents a broadly diversified and long-term investment in commodity futures. This index has been adopted in prior literature assessing the hedging potential of commodities such as Hoevenaars, Molenaar, Schotman, and Steenkamp (2008) and Spierdijk and Umar (2013). The aggregate S&P GSCI Total Return Index includes 24 commodity futures contracts, categorized into five groups, namely energy, industrial metals, precious metals, agriculture, and livestock. We adopted the US seasonally adjusted CPI for all urban consumers as our inflation index. The monthly returns on the S&P GSCI Total Return Index were obtained from Thomson Reuters Datastream, and the inflation index was provided by the Bureau of Labor Statistics. We also used the CRSP U.S. Treasury 10-Year Bond Index and the CRSP value-

¹The trace test tests the null hypothesis that the number of cointegrating vector is equal to r and the L-Max test has a null hypothesis that the number of cointegrating vectors is no greater than r .

weighted US market index, a comprehensive proxy for US stocks that are publicly traded on NYSE, NASDAQ, and AMEX. Our sample covers the period January 1983 - December 2012 because of limited data availability of the subsector indices.

Table 3.1 provides summary statistics on returns on the S&P GSCI Total Return Subsector Indices, Stock Index, Bond Index, and CPI. Commodity futures returns exhibit substantially more volatility than the slowly moving process of inflation. The average monthly return on the S&P GSCI Total Return Index is 0.72%, with a standard deviation of 5.72%. Among the five subsector indices, energy has the highest average monthly return of 1.09% and the highest volatility of 9.09%, while agriculture has the lowest average monthly return of 0.26% and a low standard deviation of 5.30%. The average monthly inflation rate is 0.24%, and volatility equals 0.26%. The kurtosis of the returns on the S&P GSCI Total Return Subsector Indices covers from 3.67% to 7.26%. By contrast, the inflation index has some excess kurtosis, which implies departures from normality.

In order to perform the augmented Dickey-Fuller (ADF) unit root test, an assumption on the most appropriate maximum lag of an autoregressive (AR) representation of each of the time series needs to be made. Table 3.2 presents the Akaike and Schwarz information criterion (AIC and SIC, respectively) of each series. Due to the fact that the AIC tends to choose models that include relatively more parameters and the SIC information chooses more parsimonious models, whenever there is conflicting evidence from the two information criteria, the preference is given to the Schwarz criterion with which we can keep the saturation ratio high. The results show that the most appropriate AR order for the CPI is equal to 3, indicating that the inflation index has some persistence. The AR orders for energy and precious metals are 2 and 3, respectively, while the AR orders for industrial metals, precious metals, agriculture, and livestock are equal to 1. Table 3.3 shows the results of the ADF for all the nine considered series. It includes three different types: an unrestricted one with a trend and with an intercept, as well as two restricted ones, one without the trend and the other without the trend and without the intercept. With reference to three different types of the ADF test on a level series, the null hypothesis of a unit root cannot be rejected for all the nine indices at a 1% significant

level. When the first differences of time series are considered, the null hypothesis of a unit root is significantly rejected.

3.4 Single-state cointegration analysis

In this section, we first estimate the autoregressive orders of the considered VARs and then we implement the Johansen's cointegration test.

3.4.1 Selection of the autoregressive order of the VAR multivariate models

Table 3.4 presents the 12 analyzed VARs with the values of AIC and SIC information criteria corresponding to different autoregressive orders. In the model of inflation and commodity, AIC indicates an autoregressive order equal to 4, while SIC suggests an order equal to 2. In order to keep the saturation ratio high, we chose SIC information and used VAR model with an autoregressive order of 2. Moreover, the autoregressive orders revealed by SIC are equal to 2 for the rest of the models, except for the model of inflation and industrial metals, which have an autoregressive order of 3. With reference to the VAR models with two series, AIC suggests autoregressive orders of 4 for the models of inflation and commodity, energy, and industrial metals, while showing autoregressive orders of 3 for the rest of the models. When we add stocks and bonds series into these VAR models, AIC indicates an autoregressive order of 3 for the model of inflation, bonds, stocks and agriculture, while it indicates orders of 4 for the rest.

3.4.2 Johansen's cointegration test

An application of Johansen's cointegration test requires that the VAR models are changed into their corresponding vector error-correction form. This means that lag p of a vector autoregressive model corresponds to lag $p - 1$ of a vector error-correction model. Table 3.5 summarizes the results of Johansen's cointegration test applied to the VAR models and shows both L-Max and trace test statistics.

In the first model, comprising the inflation index and the aggregate S&P GSCI Total Return Index, results of Johansen's trace test suggest the existence of one cointegrating vector. In particular, Mackinnon, Haug, and Michelis (1999) p-values of the trace test imply that the null hypothesis of no cointegration is rejected at a 1% significance level. Moreover, the trace test cannot reject the null hypothesis of one cointegrating vector at the 5% level. Based on the p-values of the L-Max test, we obtained the same results. It can, therefore, be assumed that there is one cointegrating vector in the model and hence a long-run relationship between the inflation index and the commodity index. The estimation of the cointegrating vector, normalized with respect to the inflation index, is presented in Table 3.6 together with the corresponding t-statistics. Interestingly, the coefficients attached to the inflation index and the commodity index have opposite signs, indicating that the larger difference between the two series has a larger impact on the error-correction mechanism. An intercept in the cointegrating vector allows for a long-run relationship in which the series do not need to reach the same level for the error-correction mechanism to revert its direction of influence. In the second model of the CPI index and energy subsector index, the Johansen's cointegration test gives evidence of one cointegrating relationship. Mackinnon, Haug, and Michelis (1999) p-values of both the trace test and the L-Max test indicate one cointegrating equation at the 5% level. Table 3.9 shows the cointegrating equation of the CPI and energy index. The statistical significance of the coefficients imply a long-run relationship between these two series. The coefficient corresponding to energy is -0.163. This implies that the larger the difference between the CPI index and the energy subsector index, the stronger the influence of the error-correction mechanism. In the third model of the CPI and industrial metals index, Mackinnon, Haug, and Michelis (1999) p-values of both the trace test and the L-Max test imply one cointegrating vector at the 5% level. The results of estimating the cointegrating equation of the CPI and industrial metals are presented in table 3.12. The t-statistics indicate a long-run relationship because they show that the coefficients are statistically significant. The coefficient corresponding to industrial metals is -0.200, implying that the larger the difference between CPI index and industrial metals index, the stronger the influence of the error-correction mechanism.

In the fourth model of the CPI index and precious metals index, Mackinnon, Haug, and Michelis (1999) p-values of both the tract test and the L-Max test indicate one cointegrating vector at the 1% level. Table 3.15 presents estimates of the cointegrating relationship between the two considered time series. The low value of the t-statistic corresponding to the precious metals indicates that the coefficient is not significantly different from zero. The negative coefficient that is attached to precious metals implies that the larger the difference between the inflation index and the precious metals index, the greater the influence on the error-correction mechanism. In the fifth model of the CPI and the agriculture index, the Johansen's cointegration test gives evidence of one cointegrating relationship because Mackinnon, Haug, and Michelis (1999) p-values of both the tract test and the L-Max test indicate one cointegrating equation at the 5% level. However, the coefficient on the agriculture index in Table 3.18 is not statistically significant and the sign of it is positive. In the sixth model of the CPI and livestock index, both the trace test and the L-Max test imply one cointegrating equation at the 5% level. The t-statistic related to the livestock index in Table 3.21 equals -0.774, which suggests some statistical significance. In the last three models, we find that the intercepts are significantly different from zero.

3.5 Markov-switching multistate vector error-correction analysis

3.5.1 Inflation and commodity

Based on on AIC and SIC, Table 3.7 shows a comparison between different specifications of linear and Markov-switching vector error-correction models of the inflation index and the aggregate S&P GSCI Total Return Index. Both AIC and SIC imply that Markov-switching models are better than the linear models. In particular, the AIC shows that the best model specification is an MSIAH(3)-VECM(1,1), while the SIC suggests that MSIAH(2)-VECM(1,1) is ranked first. Because SIC is known to suggest more parsimonious models than AIC does, the former was chosen as the decisive one in our analysis. As a result, MSIAH(2)-VECM(1,1), the

model with two regimes, heteroscedastic errors, and an autoregressive order of 1, is included in further analysis.

Table 3.8 presents the results of the estimation of an MSIAH(2)-VECM(1,1) for the CPI and commodity. The Krolzig (1997) algorithm converged after 27 iterations. The number of observations in the system is 716, and the number of parameters in Markov switching system equals 24, while the corresponding linear model has 11 parameters. With two nuisance parameters specified in the multi-state model, the likelihood ratio linearity test should be implemented under a chi-square with 11 degrees of freedom. The p-values of both the LRT and the Davies (1977) test, which avoids the estimation of nuisance parameters, are less than 0.01%, indicating the rejection of the null hypothesis at a 1% level. As a result, the tests confirm that the Markov-switching model is better than the linear model.

Figure 3.1 depicts the two regimes identified by the MS model, showing the filtered and smoothed probabilities of being in regime 1 and regime 2. Regime 2 prevailed from 1983 through 1986 (with a short regime 1 in 1983); from 1987 through 1989; from 1991 through 1998; from 2002 through 2004; and from 2010 through 2012. Regime 1 prevailed from 1999 through 2001 and from August of 2007 through December 2009. Between January 2005 and July 2007, both regimes seem to have been present. The estimated transition matrix and ergodic probabilities suggest that regime 2 predominates, characterizing 74.86 % of months with spells lasting 11.84 months on average; regime 1 occurs during only 25.14 % of months, with spells lasting just 3.98 months on average.

The long-run relationship is explained by the coefficients corresponding to the error-correction component. In regime 1, there is no significant adjustment in the inflation index since the corresponding coefficient is not statistically significant. In regime 2, the coefficient equals -0.003 and its t-statistic is -2.896, implying a significant error-correction in the inflation index. The positive sign of the coefficient on the error-correction term for the commodity index in regime 2 is the correct one for the error-correction mechanism to work. Therefore, this coefficient implies that inflation hedging is slow in pace and is of relatively low statistical significance (t-statistic is 1.404). As a result, we find some inflation hedging properties of the

aggregate S&P GSCI Total Return Index in this regime.

3.5.2 Inflation and energy

Different specifications of the linear and Markov-switching vector error-correction models of the CPI index and the energy index are presented in Table 3.10. SIC suggests that the best model specification is an MSIA(2)-VECM(1,1), which is a model with two distinct regimes and a homoscedastic error covariance matrix. Table 3.11 presents the results of the estimation of an MSIA(2)-VECM(1,1) for the CPI and energy. The number of observations in the system is 716, which corresponds to approximately 34 observations per estimated parameter because there are 21 parameters in total. The p-values of both the LRT and the Davies (1977) test imply that the Markov-switching model is better than the linear model.

Figure 3.2 shows the filtered and smoothed probabilities of being in regime 1, which prevailed during 1983, 1986, 2001, 2005, 2006, and 2008, and regime 2, which prevailed during 1984, 1985, 1987-2000, 2002-2004, 2007, and 2009-2012, covering the Great Moderation. In regime 2 there is a 97.9% probability of no shift within one month, while in regime 1 this probability equals 75.3%. Both regimes are strongly persistent, with regime 2 lasting 47.01 months on average and prevailing 92% of the time, while regime 1 lasts 4.05 months on average and prevails during the remaining 8% of months.

An analysis of the long-run dependence, which is showed by the speed coefficients related to the error-correction mechanism, implies that the long-run relationship between the CPI and the energy index changes from regime to regime. In regime 1, the sign and the related t-statistic of the long-run error-correction coefficient in the energy index indicate that energy prices diverge from the inflation index, thus the opposite of an equilibrium adjustment is the case. Taking into account that the estimated speed coefficient is high, the divergence under regime 1 would be very fast. However, regime 1 lasts a relatively short time because its expected duration is slightly above 4 months. During regime 2, the positive sign of the speed coefficient on the energy index shows that this divergence disappears. Moreover, this implies some inflation hedging properties of the energy index. In regime 2, there is a significant long-

run equilibrium adjustment in inflation, which is confirmed by the negative error-correction coefficient and the related t-statistic of -4.460. Moreover, the short-term dependence is explained by the lagged autoregressive coefficients in the results of the MSIA(2)-VECM(1,1). In regime 1, the coefficient on the lagged energy changes equals 0.053 and its t-statistic is 12.943, implying a significant dependence of changes in the inflation index on the lagged energy changes. Regime 2 estimation also shows that there is a significant dependence of changes in the CPI on the lagged energy changes. However, the magnitude of the coefficient in regime 2 is less than that in regime 1. As a result, in regime 2 low inflation rates tend to go hand in hand with low energy futures return in the short run.

3.5.3 Inflation and industrial metals

Based on the SIC, Table 3.13 shows that the best model specification for the CPI and industrial metals is an MSIAH(2)-VECM(1,1), which has two regimes, heteroscedastic errors, and an autoregressive order of 1. Table 3.14 summarizes the results of estimation of an MSIAH(2)-VECM(1,1) for the CPI index and the industrial metals index. The estimation algorithm converged after 24 iterations. The number of observations in the system is 716, and the number of parameters that were estimated equals 24. With 11 parameters of the corresponding linear model and two nuisance parameters specified in the multi-state model, the LRT should be tested by the chi-square with 11 degrees of freedom. This rejects the null hypothesis at the 1% level. Also, the Davies (1977) test imply that the Markov-switching model is superior to the corresponding linear model.

With respect to the long-run relationship that is evidenced in the model, the emphasis has to be made on the error correction coefficients. In regime 1, the speed coefficient on inflation indicates an error-correction in inflation because it has a negative sign, but the equilibrium adjustment is not statistically significant. At the same time, the negative sign on industrial metals provides an indication that industrial metals diverge from the inflation index. In regime 2, there is a significant error-correction in inflation and a significant long-run adjustment in industrial metals, confirmed by the negative error correction coefficient on the inflation index

with the related significant t-statistic of -2.489 and by the positive speed coefficient on industrial metals with its t-statistic of 2.320. Both regimes are highly persistent, with regime 1 lasting a relatively short time, namely 4.74 months on average, and prevailing 22.59% of the time, while regime 2 lasts 16.23 months on average and prevails during 77.41% of months. Figure 3.3 depicts the filtered and smoothed probabilities of being in regime 1, which prevailed during 1983, 1986, 1988-1990, 2001, 2005-2006, and 2008-2009, and regime 2, which prevailed during in the rest of the time in our sample periods and covers the Great Moderation. The results of the MS model show that industrial metals provide a good long-run hedge against inflation for most of the time. The regime-specific speed coefficients imply that under regime 2, the most persistent regime prevailing during 77.41% of months, inflation hedging is relatively fast in pace and is of relatively high statistical significance. It is also worth noting that regime 2 covers a relatively stable inflationary period, such as the Great Moderation. Under such conditions, low inflation tends to go hand in hand with low industrial metals futures returns.

3.5.4 Inflation and precious metals

Table 3.16 presents a comparison between different linear and Markov-switching vector error-correction models. The SIC information shows that MSIAH(2)-VECM(1,1) is ranked first. The estimation of the MS model is presented in Table 3.17. The Krolzig (1997) algorithm converged after 32 iterations. The number of observations in the system is 716, which corresponds to approximately 30 observations per estimated parameter because there are 24 parameters in total. The fact that the MS model is better than the corresponding linear model is implied by the LRT and the Davies (1977) test.

The remaining parameters presented in Table 3.17 provide a description of the interactions between the inflation index and the precious metals index. In regime 1, we do not find significant equilibrium adjustment in the inflation index because the t-statistic of the coefficient on the error-correction term is -0.340. At the same time, there is no significant adjustment in precious metals. However, regime 1 lasts a relatively short time, 2.63 months on average, and

occurs once in every five observations. The filtered and smoothed probabilities of being in regime 1 are depicted in Figure 3.4. This also shows that regime 2 prevails during most of our sample period and lasts 13.69 months on average. In regime 2, there is significant equilibrium adjustment in the inflation index because the coefficient has a negative sign and its t-statistic is strongly significant. The inflation hedging properties of precious metals are evident in regime 2 when the pace of equilibrium adjustment of precious metals is fast and the speed coefficient is statistically significant. In Figure 3.4, the precious metals index is a good hedge against inflation during the period 1983-2006 (with a short period of regime 1 in 1986, 1990, and 2005). In the period between 2007 and 2009, both regimes seem to have been present. After the Great Recession, the precious metals index is again a good hedge against inflation.

3.5.5 Inflation and agriculture

In the fifth model of the CPI index and agriculture index, Johansen's cointegration test shows that the coefficient on the agriculture index in Table 3.18 is not statistically significant. Table 3.19 presents a comparison between different linear and Markov-switching vector error-correction models. The SIC, with which we can keep the saturation ratio high, suggests MSIAH(2)-VECM(1,1) as the best model. The estimation of this model is presented in Table 3.20. The estimation algorithm converged after 26 iterations. The number of observations in the system is 716, and the number of parameters that were estimated equals 24. The p-values of both the LRT and the Davies (1977) test confirm that the Markov-switching model is better than the linear model.

The interactions between the inflation index and the agriculture index are described by the remaining parameters in Table 3.20. In regime 2, the speed coefficient on the CPI with a t-statistic of -3.954 implies a significant error correction in inflation. However, we cannot find such evidence in regime 1. Moreover, the speed coefficients on the agriculture index in both regimes are not statistically significant. It can be concluded that the agriculture index cannot provide a hedge against inflation. Interestingly, the inflation index significantly depends on its own lagged returns in both regimes; it also depends on the past return of the agriculture

index, but only in regime 1. However, regime 1 lasts a relatively short period, namely 7.33 months on average, and prevails 37.36% of the time, while regime 2 lasts 12.29 months on average and prevails during the remaining time of our sample period. The filtered and smoothed probabilities of being in regime 1 and regime 2 are depicted in Figure 3.5. The results in the correlation matrix indicate a different correlation under different regimes. The contemporaneous correlation between the inflation index and the agriculture index is 0.055 in regime 1, whereas this correlation becomes negative in regime 2, suggesting a reversion of the direction of the correlation between the series from regime to regime.

3.5.6 Inflation and livestock

In this model, we investigate the relationship between the CPI index and the livestock index. Table 3.22 summarizes AIC and SIC information for different linear and Markov-switching vector error correction models. The SIC information shows that the best model specification is an MSIAH(2)-VECM(0,1), the model with two regimes, heteroscedastic errors, and without lagged terms. The estimation of this model is presented in Table 3.23. The Krolzig (1997) algorithm converged after 23 iterations. The number of observations in the system is 716, which corresponds to approximately 45 observations per estimated parameter because there are 16 parameters in total. Both the LRT and the Davies (1977) test reject the null hypothesis at the 1% level, providing evidence that the Markov-switching model is better than the linear model.

Table 3.23 provides a description of the interactions between the inflation index and the livestock index. The results in the transition matrix indicate that both regimes are persistent. The probability that regime 2 does not switch to another within one month is 95.24%, while the probability that regime 1 does not switch to regime 2 within one month equals 75.86%. The expected duration of regime 2 is 21.03 months, which is higher than that of regime 1. Figure 3.6 shows the filtered and smoothed probabilities of being in regime 1 and regime 2. In regime 2, the negative sign of the speed coefficient on the CPI with its t-statistic -3.710 provides an indication that there is a significant error correction in inflation. However, we

cannot find such evidence in regime 1. Moreover, in both regimes, the t-statistics on the speed coefficients of the livestock index are not statistically significant, suggesting that we cannot use the livestock index to hedge inflation risk.

3.5.7 Inflation, commodity, common stocks, and government bonds

Based on Johansen's test in Table 3.5, this model consisting of inflation, commodity, stocks, and bonds gives evidence of one cointegrating vector among these four time series. By providing coefficients related to these series and their corresponding t-statistics, Table 3.24 shows the estimation of the cointegrating relationship in the model. An analysis of the t-statistics indicates that the coefficient on the bond index is statistically significant. The intercept coefficient is also significantly different from zero.

A comparison between different specifications of linear and Markov-switching vector error-correction models is presented in Table 3.25. The SIC information, the one that gives preference to more parsimonious models, suggests MSIA(2)-VECM(1,1) as the best model. The estimation of this model is presented in Table 3.26. The estimation algorithm converged after 45 iterations. The number of observations in the system is 1432, which corresponds to approximately 24 observations per estimated parameter because there are 60 parameters in our MS model. The fact that the MS model is better than the corresponding linear model is implied by the LRT and the Davies (1977) test, which reject the null hypothesis at the 1% level.

Because the MSIA(2)-VECM(1,1) is homoscedastic, the estimated correlation matrix does not switch from regime to regime. The contemporaneous correlation coefficients between the CPI and commodity and stocks are positive, while that between the CPI and bonds is negative. Notice that the coefficient of the correlation between the CPI and commodity is the highest one. The short-term dependence is further explained by the lagged autoregressive coefficients in the results of the MSIA(2)-VECM(1,1). In regime 1, there is a significant dependence of changes in the inflation index on the lagged inflation index changes as well as on the recent changes in the bond index and in the stock index. However, regime 2 shows

a significant dependence of the inflation index only on the lagged commodity returns, without providing evidence to support the dependence on the lagged changes of the stock index and the bond index. The error-correction speed coefficient explains the direction and pace of the long-run adjustment mechanism. In regime 1, both the sign of the speed coefficient and its corresponding t-statistic (-5.005) indicate that there is a significant long-run equilibrium adjustment within the inflation index. In regime 2, the speed coefficient becomes positive, implying an appearance of error correction in inflation rates, and the coefficient is equal to 0.005, indicating a relatively low pace. The estimated transition matrix and ergodic probabilities suggest that regime 2 predominates, characterizing 95.5% of months with spells lasting 32.18 months on average; regime 1 occurs during only 4.6% of months with spells lasting just 1.54 months on average.

3.5.8 Inflation, energy, common stocks, and government bonds

Johansen's test in Table 3.5 implies that the model for inflation, energy, stocks, and bonds has two cointegrating vectors. The estimation of the two cointegrating vectors is presented in Table 3.27. The first vector is normalized with respect to the inflation index, while the second one is normalized with respect to the energy index. Under the first cointegrating relationship, both coefficients are statistically significant but they have different signs. The second cointegrating relationship implies that the larger the difference between inflation and each of the other two series, the stronger the influence of the error-correction mechanism.

Different Markov-switching vector error-correction models and the corresponding linear models are compared in Table 3.28. The SIC information shows that MSIA(2)-VECM(1,2) is ranked first. The estimation of the MS model is presented in Table 3.29. The Krolzig (1997) algorithm converged after 40 iterations. The number of observations in the system is 1432. As a result, we have approximately 21 observations per estimated parameter because there are 68 parameters in total. The p-values of both the LRT and the Davies (1977) test are less than 1%, providing evidence that the MS model is better than the corresponding linear model.

In this model, we have two error-correction mechanisms. The first one is related to the re-

lationship between inflation and returns on stocks and bonds, whereas the second one explains the relationship between return on the energy index and returns on stocks and bonds. Both regimes are highly persistent, with regime 1 lasting 2.01 months on average and prevailing 7.7% of the time, while regime 2 lasts 23.95 months on average and prevails during 92.3% of months. Figure 3.8 depicts the filtered and smoothed probabilities of being in regime 1 and regime 2. In regime 1, there is a significant long-run equilibrium adjustment in the bond index. In particular, the error-correction coefficient related to the first cointegrating relationship is positive and its t-statistic equals -5.605, whereas the coefficient related to the second cointegrating relationship is negative and its t-statistic is -4.505. In regime 1, the speed coefficients for the stock index are statistically insignificant. In regime 2, there is a change in the direction of error-correction mechanisms. The adjustment within the stock index becomes significant in the second regime. It can be concluded that in regime 1 the bond assets, and not the stocks, provide a long-run hedge against inflation, while in regime 2 the common stocks can hedge inflation risk.

Because the MSIA(2)-VECM(1,2) is homoscedastic, the estimated correlation matrix is constant over the two regimes. An analysis of the correlation matrix shows that the contemporaneous correlation of inflation and energy as well as of inflation and stocks is positive (0.165 and 0.085, respectively), while the correlation of inflation and bonds is negative (-0.074). The short-term relationships between these variables can be further explained by the coefficients on the lagged term of changes on these time series. In regime 1, there is a short-run dependence of inflation on the lagged inflation, energy index returns, and stock returns, whereas there is no significant interaction with the lagged bond returns. In regime 2, inflation exhibits autocorrelation and interacts with the lagged changes in energy but it does not relate to the lagged changes in the stock index or the bond index. In regime 1, the change of the return on the energy index influences the long-run equilibrium in the bond market but it does not influence the stock market. However, in regime 2, it affects the stock market, but it does not influence the bond market. As a result, in regime 1 when the return on the energy index affects the bond market, it is the government bond that can hedge inflation risks, while in regime 2

when the return on the energy index influences the stock market, it is the common stocks that provide a better long run against inflation.

3.6 Conclusion

During the Great Recession, policy-makers makers experimented with unconventional tools, such as quantitative easing to help stimulate the economy but also likely to simultaneously increase the monetary base and instigate inflation. As a result, inflation hedging has become particularly relevant in light of the current global recession. This study analyzes the question whether commodity futures provide the ability to hedge against inflation. Markov-switching framework, a comprehensive framework to describe the relationship that changes from state to state, has been used in the past but not in the area of inflation hedging by commodity futures. Because of the heterogeneity of the sample period in terms of inflationary regimes and commodity futures prices (Chinn and Coibion (2013)), we adopted an MS-VECM approach in order to analyze the changing nature of the long-run inflation hedging properties of commodity futures.

In this study, we found that commodity futures showed some ability to hedge US inflation over a sample period ranging from January 1983 to December 2012. By analyzing the speed coefficients related to the error-correction mechanism, we showed that commodities futures in the markets of energy, industrial metals, and precious metals are the best inflation hedges. However, the hedging capacity exhibits substantial variation over time. Most of the actual inflation hedging was obtained under the relatively longer and more common regimes covering the Great Moderation, in which the long-run equilibrium correction speed coefficients were highly significant. However, the speed coefficients on the agriculture and livestock indices were not statistically significant, suggesting that we cannot use these two indices to hedge inflation risk.

The analysis of the regime-specific inflation hedging properties of commodity futures, common stocks, and government bonds provides some interesting findings. In the model of

inflation, common stocks, government bonds, and commodity, both the sign of the speed coefficient and its corresponding t-statistic indicate that there is a significant long-run equilibrium adjustment within the inflation index in regime 1. In regime 2, the speed coefficient is equal to 0.005, implying an appearance of error correction in inflation rates but at a relatively low pace. In the model of inflation, common stocks, government bonds, and energy, during the periods when the return on the energy index affects the bond market, it is the government bond that can hedge inflation risks. However, during the periods when the return on the energy index contributes to the long-run equilibrium in the stock market, stocks provide a better long-run against inflation.

Table 3.1: Sample statistics for CPI, bonds, stocks and commodity subsector indices

	CPI	BOND	STOCK	COMM	EN	IM	PM	AGR	LC
Mean	0.24	0.70	0.94	0.72	1.09	0.96	0.46	0.26	0.39
Median	0.24	0.79	1.41	0.74	0.85	0.45	0.07	-0.12	0.38
Maximum	1.38	8.54	12.85	22.94	37.71	38.44	15.58	17.81	17.17
Minimum	-1.77	-6.68	-22.54	-28.20	-31.20	-26.66	-18.61	-18.97	-15.76
Std. Dev.	0.26	2.19	4.54	5.73	9.09	6.84	4.78	5.30	4.14
Skewness	-1.52	0.05	-0.88	-0.22	0.41	0.66	0.01	0.18	-0.14
Kurtosis	15.64	3.63	5.67	5.41	4.91	7.26	4.40	4.60	3.67

- Note: This table displays sample statistics (in %) for the nominal returns on the S&P GSCI Total Return Index and subsector indices during the period January 1983 - December 2012. COMM stands for the aggregate index, EN for energy, IM for industrial metals, PM for precious metals, AGR for agriculture, LC for livestock. All sample statistics are on a monthly basis.

Table 3.2: Univariate time series AR order selection

<i>Series</i>	<i>Lag</i>	<i>AIC</i>	<i>SIC</i>	<i>Series</i>	<i>Lag</i>	<i>AIC</i>	<i>SIC</i>	<i>Series</i>	<i>Lag</i>	<i>AIC</i>	<i>SIC</i>
CPI	1	-9.120	-9.098	Bonds	1	-4.821	-4.799*	Stocks	1	-3.315	-3.293*
	2	-9.295	-9.263		2	-4.822	-4.789		2	-3.318*	-3.285
	3	-9.327	-9.283*		3	-4.826	-4.782		3	-3.312	-3.269
	4	-9.331*	-9.277		4	-4.827*	-4.773		4	-3.308	-3.254
COMM	1	-2.867	-2.846	EN	1	-1.984	-1.963	IM	1	-2.559	-2.538*
	2	-2.893	-2.861*		2	-2.012*	-1.980*		2	-2.566	-2.533
	3	-2.887	-2.844		3	-2.008	-1.965		3	-2.557	-2.514
	4	-2.887*	-2.832		4	-2.004	-1.950		4	-2.576*	-2.522
PM	1	-3.237	-3.215	AGR	1	-3.047*	-3.025*	LS	1	-3.541*	-3.519*
	2	-3.302	-3.270		2	-3.040	-3.008		2	-3.535	-3.502
	3	-3.315*	-3.272*		3	-3.042	-2.999		3	-3.529	-3.485
	4	-3.309	-3.255		4	-3.035	-2.980		4	-3.529	-3.475

Table 3.3: Results of the ADF stationarity test

<i>Series</i>	<i>Lag</i>	<i>Level series</i>			<i>First differences</i>		
		<i>I</i>	<i>II</i>	<i>III</i>	<i>I</i>	<i>II</i>	<i>III</i>
CPI	2	-1.553	-3.010	8.651	-12.482	-12.034	-5.431
Bonds	0	-2.360	-1.674	5.604	-17.827	-17.789	-16.443
Stocks	0	-1.721	-1.747	3.122	-16.998	-16.954	-16.512
COMM	2	-2.268	-1.877	1.301	-15.722	-15.684	-15.581
EN	1	-2.626	-1.867	0.935	-15.648	-15.632	-15.576
IM	0	-1.436	-1.205	1.861	-16.756	-16.761	-16.606
PM	2	-0.599	1.501	1.983	-16.786	-22.299	-22.153
AGR	0	-2.298	-2.327	0.317	-18.892	-18.887	-18.902
LS	0	-1.296	-2.733	1.140	-18.281	-17.890	-17.830
Critical Values							
1%		-3.984	-3.448	-2.571	-3.984	-3.448	-2.571
5%		-3.422	-2.869	-1.942	-3.422	-2.869	-1.942
10%		-3.134	-2.571	-1.616	-3.134	-2.571	-1.616

- Note: I - ADF test with a trend and an intercept, II - ADF test without trend and with an intercept, III - ADF test without trend and without intercept.

Table 3.4: VAR order selection

<i>Series</i>	<i>Lag</i>	<i>AIC</i>	<i>SIC</i>	<i>Series</i>	<i>Lag</i>	<i>AIC</i>	<i>SIC</i>	<i>Series</i>	<i>Lag</i>	<i>AIC</i>	<i>SIC</i>
CPI	1	-12.12	-12.05	CPI	1	-11.22	-11.15	CPI	1	-11.73	-11.67
COMM	2	-12.73	-12.62*	EN	2	-11.78	-11.67*	IM	2	-11.92	-11.81
	3	-12.76	-12.61		3	-11.80	-11.65		3	-11.96	-11.81*
	4	-12.76*	-12.57		4	-11.81*	-11.61		4	-11.97*	-11.77
	5	-12.74	-12.50		5	-11.79	-11.55		5	-11.95	-11.71
CPI	1	-12.38	-12.32	CPI	1	-12.16	-12.10	CPI	1	-12.66	-12.60
PM	2	-12.67	-12.56*	AGR	2	-12.38	-12.27*	LS	2	-12.83	-12.72*
	3	-12.70*	-12.55		3	-12.41*	-12.25		3	-12.87*	-12.71
	4	-12.69	-12.49		4	-12.39	-12.19		4	-12.86	-12.67
	5	-12.67	-12.43		5	-12.38	-12.14		5	-12.84	-12.60
CPI	1	-20.35	-20.14	CPI	1	-19.43	-19.21	CPI	1	-20.04	-19.83
COMM	2	-20.97	-20.58*	EN	2	-20.02	-19.63*	IM	2	-20.22	-19.83*
Stocks	3	-20.99*	-20.43	Stocks	3	-20.04*	-19.47	Stocks	3	-20.25*	-19.69
Bonds	4	-20.99	-20.25	Bonds	4	-20.03	-19.29	Bonds	4	-20.25	-19.51
	5	-20.97	-20.06		5	-20.00	-19.08		5	-20.20	-19.28
CPI	1	-20.62	-20.40	CPI	1	-20.41	-20.20	CPI	1	-20.88	-20.66
PM	2	-20.91	-20.52*	AGR	2	-20.62*	-20.23*	LS	2	-21.05	-20.66*
Stocks	3	-20.91*	-20.35	Stocks	3	-20.61	-20.05	Stocks	3	-21.05*	-20.49
Bonds	4	-20.87	-20.13	Bonds	4	-20.58	-19.84	Bonds	4	-21.02	-20.28
	5	-20.83	-19.92		5	-20.54	-19.63		5	-20.97	-20.06

Table 3.5: Results of the Johansen cointegration test

<i>System</i>	<i>r</i>	<i>L-Max</i>	<i>p-value</i>	<i>Trace</i>	<i>p-value</i>	<i>System</i>	<i>r</i>	<i>L-Max</i>	<i>p-value</i>	<i>Trace</i>	<i>p-value</i>
CPI	0	136.624	0.000	143.311	0.000	CPI	0	72.011	0.000	83.081	0.000
COMM	1	6.686	0.144	6.686	0.144	PM	1	11.070	0.022	11.070	0.022
CPI	0	126.596	0.000	134.264	0.000	CPI	0	69.909	0.000	75.588	0.000
Energy	1	7.668	0.095	7.668	0.095	AGR	1	5.679	0.217	5.679	0.217
CPI	0	98.487	0.000	102.025	0.000	CPI	0	67.477	0.000	75.865	0.000
IM	1	3.538	0.485	3.538	0.485	LS	1	8.388	0.070	8.388	0.070
CPI	0	152.697	0.000	183.441	0.000	CPI	0	100.077	0.000	143.406	0.000
COMM	1	17.400	0.210	30.744	0.140	PM	1	25.026	0.020	43.329	0.005
Stocks	2	8.485	0.491	13.344	0.337	Stocks	2	9.699	0.363	18.303	0.091
Bonds	3	4.859	0.299	4.859	0.299	Bonds	3	8.604	0.064	8.604	0.064
CPI	0	137.454	0.000	170.166	0.000	CPI	0	99.387	0.000	129.782	0.000
Energy	1	17.761	0.191	32.712	0.090	AGR	1	17.224	0.220	30.395	0.150
Stocks	2	10.440	0.296	14.951	0.229	Stocks	2	7.070	0.661	13.171	0.350
Bonds	3	4.511	0.341	4.511	0.341	Bonds	3	6.101	0.183	6.101	0.183
CPI	0	101.079	0.000	129.875	0.000	CPI	0	97.764	0.000	132.485	0.000
IM	1	19.098	0.132	28.797	0.208	LS	1	22.328	0.050	34.721	0.056
Stocks	2	6.618	0.716	9.699	0.669	Stocks	2	7.471	0.612	12.393	0.414
Bonds	3	3.081	0.566	3.081	0.566	Bonds	3	4.922	0.292	4.922	0.292

Table 3.6: Cointegrating equation of CPI and prices of commodity index

<i>CPI</i>	<i>Commodity</i>	<i>Const.</i>
1	-0.252 [-5.875]	-4.096 [-15.621]

Table 3.7: Comparison of VECMs of CPI and prices of commodity index

<i>VECM lag = 1</i>	<i>AIC</i>	<i>SIC</i>	<i>VECM lag = 0</i>	<i>AIC</i>	<i>SIC</i>
Linear-VECM(1,1)	-12.705	-12.586	Linear-VECM(0,1)	-12.087	-12.011
MSIA(2)-VECM(1,1)	-12.938	-12.711	MSIA(2)-VECM(0,1)	-12.419	-12.278
MSIA(3)-VECM(1,1)	-12.949	-12.591	MSIA(3)-VECM(0,1)	NA	NA
MSIAH(2)-VECM(1,1)*	-13.032	-12.772	MSIAH(2)-VECM(0,1)	-12.612	-12.439
MSIAH(3)-VECM(1,1)	-13.091	-12.669	MSIAH(3)-VECM(0,1)	-12.702	-12.410

Table 3.8: Estimates of a MSIAH(2)-VECM(1,1) for CPI and prices of commodity index

<i>Iterations until convergence:</i>	27	<i>No. of parameters in linear system:</i>	11		
<i>No. of observations in the system:</i>	716	<i>No. of restrictions:</i>	11		
<i>No. of parameters in MS system:</i>	24	<i>No. of nuisance parameters:</i>	2		
<i>Log-likelihood of MS system:</i>	2356.80	<i>Log-likelihood of linear system:</i>	2285.17		
<i>LR linearity test statistic:</i>	143.26				
<i>LRT Chi(11) p-value:</i>	<0.0000	<i>Davies test p-value:</i>	<0.0000		
<i>LRT Chi(13) p-value:</i>	<0.0000				
<i>Transition matrix</i>		<i>Persistence of Regimes</i>			
	<i>Regime 1</i>	<i>Regime 2</i>	<i>Observations</i>	<i>Ergodic Prob.</i>	<i>Duration</i>
<i>Regime 1</i>	0.7485	0.2515	90.3	0.2514	3.98
<i>Regime 2</i>	0.0845	0.9155	267.7	0.7486	11.84
<i>Parameter estimates</i>					
	<i>Regime 1:</i>		<i>Regime 2:</i>		
	$\Delta CPI(t)$	$\Delta COMM(t)$	$\Delta CPI(t)$	$\Delta COMM(t)$	
<i>Const.</i>	0.002	-0.002	0.002	0.016	
	[5.153]	[-0.175]	[10.330]	[2.264]	
$\Delta CPI(t-1)$	0.131	-1.581	0.237	-1.632	
	[1.609]	[-0.642]	[3.907]	[-0.649]	
$\Delta COMM(t-1)$	0.038	0.389	0.015	-0.028	
	[7.203]	[2.742]	[6.231]	[-0.420]	
<i>EC</i>	-0.004	-0.071	-0.003	0.050	
	[-1.253]	[-0.717]	[-2.896]	[1.404]	
<i>SE</i>	0.003	0.082	0.001	0.042	
<i>Correlation matrix</i>					
	<i>Regime 1:</i>		<i>Regime 2:</i>		
	<i>CPI</i>	<i>COMM</i>	<i>CPI</i>	<i>COMM</i>	
<i>CPI</i>	1.000	0.246	1.000	0.055	
<i>COMM</i>	0.246	1.000	0.055	1.000	

Table 3.9: Cointegrating equation of CPI and prices of energy subindex

<i>CPI</i>	<i>Energy</i>	<i>Const.</i>
1	-0.163	-4.696
	[-3.769]	[-16.672]

Table 3.10: Comparison of VECMs of CPI and prices of energy subindex

<i>VECM lag = 1</i>	<i>AIC</i>	<i>SIC</i>	<i>VECM lag = 0</i>	<i>AIC</i>	<i>SIC</i>
Linear-VECM(1,1)	-11.751	-11.632	Linear-VECM(0,1)	-11.178	-11.102
MSIA(2)-VECM(1,1)*	-12.070	-11.842	MSIA(2)-VECM(0,1)	-11.470	-11.330
MSIA(3)-VECM(1,1)	-12.099	-11.741	MSIA(3)-VECM(0,1)	NA	NA
MSIAH(2)-VECM(1,1)	-12.085	-11.824	MSIAH(2)-VECM(0,1)	-11.668	-11.495
MSIAH(3)-VECM(1,1)	-12.134	-11.712	MSIAH(3)-VECM(0,1)	-11.731	-11.438

Table 3.11: Estimates of a MSIA(2)-VECM(1,1) for CPI and prices of energy subindex

<i>Iterations until convergence:</i>	34	<i>No. of parameters in linear system:</i>	11
<i>No. of observations in the system:</i>	716	<i>No. of restrictions:</i>	8
<i>No. of parameters in MS system:</i>	21	<i>No. of nuisance parameters:</i>	2
<i>Log-likelihood of MS system:</i>	2181.45	<i>Log-likelihood of linear system:</i>	2114.4
<i>LR linearity test statistic:</i>	134.09		
<i>LRT Chi(8) p-value:</i>	<0.0000	<i>Davies test p-value:</i>	<0.0000
<i>LRT Chi(10) p-value:</i>	<0.0000		

<i>Transition matrix</i>			<i>Persistence of Regimes</i>		
	<i>Regime 1</i>	<i>Regime 2</i>	<i>Observations</i>	<i>Ergodic Prob.</i>	<i>Duration</i>
<i>Regime 1</i>	0.753	0.247	30.7	0.08	4.05
<i>Regime 2</i>	0.021	0.979	327.3	0.92	47.01

<i>Parameter estimates</i>				
	<i>Regime 1:</i>		<i>Regime 2:</i>	
	$\Delta CPI(t)$	$\Delta Energy(t)$	$\Delta CPI(t)$	$\Delta Energy(t)$
<i>Const.</i>	0.003	-0.019	0.002	0.019
	[6.939]	[-0.841]	[15.445]	[2.313]
$\Delta CPI(t-1)$	0.022	-4.911	0.122	-3.324
	[0.328]	[-1.415]	[2.790]	[-1.291]
$\Delta Energy(t-1)$	0.053	0.412	0.011	0.123
	[12.943]	[2.082]	[10.278]	[2.218]
<i>EC</i>	0.004	-0.572	-0.003	0.021
	[0.890]	[-2.135]	[-4.460]	[0.534]
<i>SE</i>	0.001	0.085	0.001	0.085

<i>Correlation matrix</i>				
	<i>Regime 1:</i>		<i>Regime 2:</i>	
	<i>CPI</i>	<i>Energy</i>	<i>CPI</i>	<i>Energy</i>
<i>CPI</i>	1.000	0.145	1.000	0.145
<i>Energy</i>	0.145	1.000	0.145	1.000

Table 3.12: Cointegrating equation of CPI and prices of industrial metals subindex

<i>CPI</i>	<i>IM</i>	<i>Const.</i>
1	-0.200	-4.226
	[-7.910]	[-26.021]

Table 3.13: Comparison of VECMs of CPI and prices of industrial metals subindex

<i>VECM lag = 2</i>	<i>AIC</i>	<i>SIC</i>	<i>VECM lag = 1</i>	<i>AIC</i>	<i>SIC</i>
Linear-VECM(2,1)	-11.948	-11.785	Linear-VECM(1,1)	-11.902	-11.783
MSIA(2)-VECM(2,1)	-12.016	-11.701	MSIA(2)-VECM(1,1)	-11.993	-11.765
MSIA(3)-VECM(2,1)	-12.348	-11.859	MSIA(3)-VECM(1,1)	-12.207	-11.850
MSIAH(2)-VECM(2,1)	-12.303	-11.955	MSIAH(2)-VECM(1,1)*	-12.294	-12.034
MSIAH(3)-VECM(2,1)	-12.466	-11.912	MSIAH(3)-VECM(1,1)	-12.296	-11.873

Table 3.14: Estimates of a MSIAH(2)-VECM(1,1) for CPI and prices of industrial metals subindex

<i>Iterations until convergence:</i>	24	<i>No. of parameters in linear system:</i>	11		
<i>No. of observations in the system:</i>	716	<i>No. of restrictions:</i>	11		
<i>No. of parameters in MS system:</i>	24	<i>No. of nuisance parameters:</i>	2		
<i>Log-likelihood of MS system:</i>	2224.62	<i>Log-likelihood of linear system:</i>	2141.47		
<i>LR linearity test statistic:</i>	166.31				
<i>LRT Chi(11) p-value:</i>	<0.0000	<i>Davies test p-value:</i>	<0.0000		
<i>LRT Chi(13) p-value:</i>	<0.0000				
<i>Transition matrix</i>		<i>Persistence of Regimes</i>			
	<i>Regime 1</i>	<i>Regime 2</i>	<i>Observations</i>	<i>Ergodic Prob.</i>	<i>Duration</i>
<i>Regime 1</i>	0.7888	0.2112	81.4	0.2259	4.74
<i>Regime 2</i>	0.0616	0.9384	276.6	0.7741	16.23
<i>Parameter estimates</i>					
	<i>Regime 1:</i>		<i>Regime 2:</i>		
	$\Delta CPI(t)$	$\Delta IM(t)$	$\Delta CPI(t)$	$\Delta IM(t)$	
<i>Const.</i>	0.001	0.014	0.002	0.002	
	[2.195]	[1.009]	[10.136]	[0.405]	
$\Delta CPI(t-1)$	0.397	2.075	0.259	0.420	
	[3.724]	[0.700]	[4.167]	[0.203]	
$\Delta IM(t-1)$	0.009	0.246	0.003	-0.080	
	[1.991]	[1.842]	[1.720]	[-1.256]	
<i>EC</i>	-0.006	-0.069	-0.003	0.078	
	[-1.457]	[-0.597]	[-2.489]	[2.320]	
<i>SE</i>	0.004	0.103	0.001	0.047	
<i>Correlation matrix</i>					
	<i>Regime 1:</i>		<i>Regime 2:</i>		
	<i>CPI</i>	<i>IM</i>	<i>CPI</i>	<i>IM</i>	
<i>CPI</i>	1.000	0.103	1.000	0.099	
<i>IM</i>	0.103	1.000	0.099	1.000	

Table 3.15: Cointegrating equation of CPI and prices of precious metals subindex

<i>CPI</i>	<i>PM</i>	<i>Const.</i>
1	-0.115	-5.405
	[-0.847]	[-8.647]

Table 3.16: Comparison of VECMs of CPI and prices of precious metals subindex

<i>VECM lag = 1</i>	<i>AIC</i>	<i>SIC</i>	<i>VECM lag = 0</i>	<i>AIC</i>	<i>SIC</i>
Linear-VECM(1,1)	-12.668	-12.548	Linear-VECM(0,1)	-12.374	-12.299
MSIA(2)-VECM(1,1)	-12.842	-12.615	MSIA(2)-VECM(0,1)	-12.590	-12.450
MSIA(3)-VECM(1,1)	-12.970	-12.613	MSIA(3)-VECM(0,1)	NA	NA
MSIAH(2)-VECM(1,1)*	-12.965	-12.705	MSIAH(2)-VECM(0,1)	-12.803	-12.630
MSIAH(3)-VECM(1,1)	-13.023	-12.600	MSIAH(3)-VECM(0,1)	-12.885	-12.593

Table 3.17: Estimates of a MSIAH(2)-VECM(1,1) for CPI and prices of precious metals subindex

<i>Iterations until convergence:</i>	32	<i>No. of parameters in linear system:</i>	11		
<i>No. of observations in the system:</i>	716	<i>No. of restrictions:</i>	11		
<i>No. of parameters in MS system:</i>	24	<i>No. of nuisance parameters:</i>	2		
<i>Log-likelihood of MS system:</i>	2344.81	<i>Log-likelihood of linear system:</i>	2278.51		
<i>LR linearity test statistic:</i>	132.61				
<i>LRT Chi(11) p-value:</i>	<0.0000	<i>Davies test p-value:</i>	<0.0000		
<i>LRT Chi(13) p-value:</i>	<0.0000				
<i>Transition matrix</i>		<i>Persistence of Regimes</i>			
	<i>Regime 1</i>	<i>Regime 2</i>	<i>Observations</i>	<i>Ergodic Prob.</i>	<i>Duration</i>
<i>Regime 1</i>	0.6198	0.3802	58	0.1611	2.63
<i>Regime 2</i>	0.073	0.927	300	0.8389	13.69
<i>Parameter estimates</i>					
	<i>Regime 1:</i>		<i>Regime 2:</i>		
	$\Delta CPI(t)$	$\Delta PM(t)$	$\Delta CPI(t)$	$\Delta PM(t)$	
<i>Const.</i>	0.001	0.015	0.002	0.005	
	[0.831]	[0.915]	[10.825]	[0.931]	
$\Delta CPI(t-1)$	0.634	-4.413	0.128	0.067	
	[3.962]	[-2.772]	[2.114]	[0.040]	
$\Delta PM(t-1)$	0.028	-0.290	0.004	-0.126	
	[2.204]	[-1.528]	[1.690]	[-1.788]	
<i>EC</i>	-0.002	-0.020	-0.002	0.039	
	[-0.340]	[-0.195]	[-3.578]	[2.010]	
<i>SE</i>	0.004	0.051	0.001	0.043	
<i>Correlation matrix</i>					
	<i>Regime 1:</i>		<i>Regime 2:</i>		
	<i>CPI</i>	<i>PM</i>	<i>CPI</i>	<i>PM</i>	
<i>CPI</i>	1.000	0.438	1.000	0.202	
<i>PM</i>	0.438	1.000	0.202	1.000	

Table 3.18: Cointegrating equation of CPI and prices of agriculture subindex

<i>CPI</i>	<i>AGR</i>	<i>Const.</i>
1	0.005	-6.242
	[0.014]	[-3.313]

Table 3.19: Comparison of VECMs of CPI and prices of agriculture subindex

<i>VECM lag = 1</i>	<i>AIC</i>	<i>SIC</i>	<i>VECM lag = 0</i>	<i>AIC</i>	<i>SIC</i>
Linear-VECM(1,1)	-12.358	-12.238	Linear-VECM(0,1)	-12.138	-12.062
MSIA(2)-VECM(1,1)	-12.503	-12.275	MSIA(2)-VECM(0,1)	-12.368	-12.227
MSIA(3)-VECM(1,1)	-12.746	-12.389	MSIA(3)-VECM(0,1)	NA	NA
MSIAH(2)-VECM(1,1)*	-12.724	-12.464	MSIAH(2)-VECM(0,1)	-12.595	-12.422
MSIAH(3)-VECM(1,1)	-12.794	-12.371	MSIAH(3)-VECM(0,1)	-12.684	-12.392

Table 3.20: Estimates of a MSIAH(2)-VECM(1,1) for CPI and prices of agriculture subindex

<i>Iterations until convergence:</i>	26	<i>No. of parameters in linear system:</i>	11
<i>No. of observations in the system:</i>	716	<i>No. of restrictions:</i>	11
<i>No. of parameters in MS system:</i>	24	<i>No. of nuisance parameters:</i>	2
<i>Log-likelihood of MS system:</i>	2301.57	<i>Log-likelihood of linear system:</i>	2222.99
<i>LR linearity test statistic:</i>	157.15		
<i>LRT Chi(11) p-value:</i>	<0.0000	<i>Davies test p-value:</i>	<0.0000
<i>LRT Chi(13) p-value:</i>	<0.0000		

<i>Transition matrix</i>			<i>Persistence of Regimes</i>		
	<i>Regime 1</i>	<i>Regime 2</i>	<i>Observations</i>	<i>Ergodic Prob.</i>	<i>Duration</i>
<i>Regime 1</i>	0.8635	0.1365	135.1	0.3736	7.33
<i>Regime 2</i>	0.0814	0.9186	222.9	0.6264	12.29

<i>Parameter estimates</i>				
	<i>Regime 1:</i>		<i>Regime 2:</i>	
	$\Delta CPI(t)$	$\Delta AGR(t)$	$\Delta CPI(t)$	$\Delta AGR(t)$
<i>Const.</i>	0.002	0.008	0.002	-0.011
	[3.894]	[0.956]	[9.182]	[-2.049]
$\Delta CPI(t-1)$	0.470	-3.540	0.194	6.327
	[6.000]	[-2.060]	[2.828]	[3.074]
$\Delta AGR(t-1)$	0.016	-0.013	0.002	0.003
	[3.808]	[-0.137]	[0.719]	[0.045]
<i>EC</i>	-0.002	-0.020	-0.002	0.013
	[-1.196]	[-0.679]	[-3.954]	[0.859]
<i>SE</i>	0.003	0.069	0.001	0.037

<i>Correlation matrix</i>				
	<i>Regime 1:</i>		<i>Regime 2:</i>	
	<i>CPI</i>	<i>AGR</i>	<i>CPI</i>	<i>AGR</i>
<i>CPI</i>	1.000	0.055	1.000	-0.002
<i>AGR</i>	0.055	1.000	-0.002	1.000

Table 3.21: Cointegrating equation of CPI and prices of livestock subindex

<i>CPI</i>	<i>LS</i>	<i>Const.</i>
1	-0.140	-5.280
	[-0.774]	[-4.957]

Table 3.22: Comparison of VECMs of CPI and prices of livestock subindex

<i>VECM lag = 1</i>	<i>AIC</i>	<i>SIC</i>	<i>VECM lag = 0</i>	<i>AIC</i>	<i>SIC</i>
Linear-VECM(1,1)	-12.818	-12.699	Linear-VECM(0,1)	-12.651	-12.575
MSIA(2)-VECM(1,1)	-13.026	-12.799	MSIA(2)-VECM(0,1)	-12.892	-12.751
MSIA(3)-VECM(1,1)	-13.070	-12.713	MSIA(3)-VECM(0,1)	NA	NA
MSIAH(2)-VECM(1,1)	-13.112	-12.852	MSIAH(2)-VECM(0,1)*	-13.033	-12.860
MSIAH(3)-VECM(1,1)	-13.192	-12.769	MSIAH(3)-VECM(0,1)	-13.092	-12.800

Table 3.23: Estimates of a MSIAH(2)-VECM(0,1) for CPI and prices of livestock subindex

<i>Iterations until convergence:</i>	23	<i>No. of parameters in linear system:</i>	7		
<i>No. of observations in the system:</i>	716	<i>No. of restrictions:</i>	7		
<i>No. of parameters in MS system:</i>	16	<i>No. of nuisance parameters:</i>	2		
<i>Log-likelihood of MS system:</i>	2355.35	<i>Log-likelihood of linear system:</i>	2277.79		
<i>LR linearity test statistic:</i>	155.12				
<i>LRT Chi(7) p-value:</i>	<0.0000	<i>Davies test p-value:</i>	<0.0000		
<i>LRT Chi(9) p-value:</i>	<0.0000				
<i>Transition matrix</i>			<i>Persistence of Regimes</i>		
	<i>Regime 1</i>	<i>Regime 2</i>	<i>Observations</i>	<i>Ergodic Prob.</i>	<i>Duration</i>
<i>Regime 1</i>	0.7586	0.2414	59.2	0.1646	4.14
<i>Regime 2</i>	0.0476	0.9524	299.8	0.8354	21.03
<i>Parameter estimates</i>					
	<i>Regime 1:</i>		<i>Regime 2:</i>		
	$\Delta CPI(t)$	$\Delta LS(t)$	$\Delta CPI(t)$	$\Delta LS(t)$	
<i>Const.</i>	0.002	-0.003	0.002	0.004	
	[2.834]	[-0.392]	[24.980]	[1.723]	
<i>EC</i>	-0.005	-0.040	-0.002	-0.022	
	[-1.378]	[-1.308]	[-3.710]	[-1.819]	
<i>SE</i>	0.005	0.034	0.002	0.042	
<i>Correlation matrix</i>					
	<i>Regime 1:</i>		<i>Regime 2:</i>		
	<i>CPI</i>	<i>LS</i>	<i>CPI</i>	<i>LS</i>	
<i>CPI</i>	1.000	0.196	1.000	-0.009	
<i>LS</i>	0.196	1.000	-0.009	1.000	

Table 3.24: Cointegrating equation of CPI and prices of commodity, stocks, and bonds

<i>CPI</i>	<i>Commodity</i>	<i>Stocks</i>	<i>Bonds</i>	<i>Const.</i>
1	0.107	-0.030	-0.534	-1.988
	[1.430]	[-0.334]	[-4.410]	[-9.497]

Table 3.25: Comparison of VECMs of CPI and prices of commodity, stocks, and bonds

<i>VECM lag = 1</i>	<i>AIC</i>	<i>SIC</i>	<i>VECM lag = 0</i>	<i>AIC</i>	<i>SIC</i>
Linear-VECM(1,1)	-20.903	-20.534	Linear-VECM(0,1)	-20.255	-20.061
MSIA(2)-VECM(1,1)*	-21.316	-20.665	MSIA(2)-VECM(0,1)	-20.544	-20.241
MSIA(3)-VECM(1,1)	-21.287	-20.334	MSIA(3)-VECM(0,1)	NA	NA
MSIAH(2)-VECM(1,1)	-21.420	-20.661	MSIAH(2)-VECM(0,1)	-20.952	-20.541
MSIAH(3)-VECM(1,1)	-21.452	-20.281	MSIAH(3)-VECM(0,1)	-20.990	-20.341

Table 3.26: Estimates of a MSIA(2)-VECM(1,1) for CPI and prices of commodity, stocks, and bonds

<i>Iterations until convergence:</i>	45	<i>No. of parameters in linear system:</i>	34
<i>No. of observations in the system:</i>	1432	<i>No. of restrictions:</i>	24
<i>No. of parameters in MS system:</i>	60	<i>No. of nuisance parameters:</i>	2
<i>Log-likelihood of MS system:</i>	3875.5	<i>Log-likelihood of linear system:</i>	3775.62
<i>LR linearity test statistic:</i>	199.80		
<i>LRT Chi(24) p-value:</i>	<0.000	<i>Davies test p-value:</i>	<0.000
<i>LRT Chi(26) p-value:</i>	<0.000		

<i>Transition matrix</i>				<i>Persistence of Regimes</i>		
	<i>Regime 1</i>	<i>Regime 2</i>		<i>Obser.</i>	<i>E-Prob.</i>	<i>Duration</i>
<i>Regime 1</i>	0.349	0.651	<i>Regime 1</i>	16.6	0.046	1.540
<i>Regime 2</i>	0.031	0.969	<i>Regime 2</i>	341.4	0.955	32.180

<i>Parameter</i>								
	<i>Regime 1:</i>				<i>Regime 2:</i>			
	$\Delta CPI(t)$	$\Delta Comm(t)$	$\Delta Stock(t)$	$\Delta Bond(t)$	$\Delta CPI(t)$	$\Delta Comm(t)$	$\Delta Stock(t)$	$\Delta Bond(t)$
<i>Const.</i>	0.006	0.007	-0.090	0.005	0.002	0.023	0.017	0.008
	[9.616]	[0.343]	[-5.428]	[0.539]	[13.505]	[5.036]	[4.737]	[4.456]
$\Delta CPI(t-1)$	-0.062	-11.704	-7.526	-4.387	0.166	-4.708	-0.918	-0.492
	[-0.424]	[-2.382]	[-2.010]	[-2.193]	[4.093]	[-3.503]	[-0.895]	[-0.914]
$\Delta Comm(t-1)$	0.053	1.049	0.463	0.166	0.021	0.022	-0.121	-0.015
	[7.620]	[4.612]	[2.589]	[1.812]	[13.219]	[0.412]	[-2.953]	[-0.694]
$\Delta Stock(t-1)$	0.048	0.153	-0.265	-0.297	0.001	-0.113	-0.004	-0.061
	[5.825]	[0.562]	[-1.187]	[-2.695]	[0.346]	[-1.769]	[-0.073]	[-2.396]
$\Delta Bond(t-1)$	-0.047	-1.853	0.496	-0.583	-0.005	-0.337	-0.045	0.063
	[-2.238]	[-2.653]	[0.930]	[-2.043]	[-1.285]	[-2.558]	[-0.443]	[1.193]
<i>EC</i>	-0.025	0.190	0.077	-0.122	0.005	-0.015	0.004	0.051
	[-5.005]	[1.172]	[0.557]	[-1.856]	[4.660]	[-0.434]	[0.149]	[3.572]
<i>SE</i>	0.002	0.051	0.039	0.020	0.002	0.051	0.039	0.020

<i>Correlation</i>								
	<i>Regime 1:</i>				<i>Regime 2:</i>			
	<i>CPI</i>	<i>Comm</i>	<i>Stock</i>	<i>Bond</i>	<i>CPI</i>	<i>Comm</i>	<i>Stock</i>	<i>Bond</i>
<i>CPI</i>	1.000	0.217	0.078	-0.108	1.000	0.217	0.078	-0.108
<i>Comm</i>	0.217	1.000	0.118	-0.050	0.217	1.000	0.118	-0.050
<i>Stock</i>	0.078	0.118	1.000	-0.023	0.078	0.118	1.000	-0.023
<i>Bond</i>	-0.108	-0.050	-0.023	1.000	-0.108	-0.050	-0.023	1.000

Table 3.27: Cointegrating equation of CPI and prices of energy subindex, stocks, and bonds

<i>CPI</i>	<i>Energy</i>	<i>Stocks</i>	<i>Bonds</i>	<i>Const.</i>
1	0	0.084 [2.172]	-0.483 [-9.388]	-2.720 [-29.634]
0	1	-1.206 [-0.912]	-1.113 [-0.629]	13.078 [4.150]

Table 3.28: Comparison of VECMs of CPI and prices of energy subindex, stocks, and bonds

<i>VECM lag = 1</i>	<i>AIC</i>	<i>SIC</i>	<i>VECM lag = 0</i>	<i>AIC</i>	<i>SIC</i>
Linear-VECM(1,2)	-19.968	-19.556	Linear-VECM(0,2)	-19.347	-19.109
MSIA(2)-VECM(1,2)*	-20.354	-19.617	MSIA(2)-VECM(0,2)	-19.754	-19.365
MSIA(3)-VECM(1,2)	-20.371	-19.287	MSIA(3)-VECM(0,2)	-19.881	-19.319
MSIAH(2)-VECM(1,2)	-20.438	-19.592	MSIAH(2)-VECM(0,2)	-19.965	-19.468
MSIAH(3)-VECM(1,2)	-20.422	-19.121	MSIAH(3)-VECM(0,2)	-19.998	-19.220

Table 3.29: Estimates of a MSIA(2)-VECM(1,2) for CPI and prices of energy subindex, stocks, and bonds

<i>Iterations until convergence:</i>	40	<i>No. of parameters in linear system:</i>	38
<i>No. of observations in the system:</i>	1432	<i>No. of restrictions:</i>	28
<i>No. of parameters in MS system:</i>	68	<i>No. of nuisance parameters:</i>	2
<i>Log-likelihood of MS system:</i>	3711.44	<i>Log-likelihood of linear system:</i>	3612.18
<i>LR linearity test statistic:</i>	198.52		
<i>LRT Chi(28) p-value:</i>	<0.000	<i>Davies test p-value:</i>	<0.000
<i>LRT Chi(30) p-value:</i>	<0.000		

<i>Transition matrix</i>				<i>Persistence of Regimes</i>			
	<i>Regime 1</i>	<i>Regime 2</i>			<i>Obser.</i>	<i>E-Prob.</i>	<i>Duration</i>
<i>Regime 1</i>	0.502	0.498		<i>Regime 1</i>	28	0.077	2.010
<i>Regime 2</i>	0.042	0.958		<i>Regime 2</i>	330	0.923	23.950

<i>Parameter</i>								
	<i>Regime 1:</i>				<i>Regime 2:</i>			
	$\Delta CPI(t)$	$\Delta EN(t)$	$\Delta Stock(t)$	$\Delta Bond(t)$	$\Delta CPI(t)$	$\Delta EN(t)$	$\Delta Stock(t)$	$\Delta Bond(t)$
<i>Const.</i>	0.004	-0.001	-0.090	0.004	0.002	0.038	0.020	0.008
	[6.480]	[-0.030]	[-6.647]	[0.721]	[12.945]	[4.840]	[4.956]	[4.034]
$\Delta CPI(t-1)$	0.431	5.520	-2.882	-3.744	0.168	-9.011	-0.870	-0.443
	[3.494]	[0.611]	[-0.666]	[-2.893]	[3.890]	[-3.842]	[-0.847]	[-0.796]
$\Delta EN(t-1)$	0.024	0.396	0.189	0.117	0.013	0.080	-0.074	-0.023
	[4.853]	[1.238]	[1.276]	[2.268]	[12.372]	[1.463]	[-2.973]	[-1.683]
$\Delta Stock(t-1)$	0.032	0.267	-0.019	-0.191	0.002	-0.210	-0.105	-0.067
	[6.083]	[0.945]	[-0.140]	[-2.966]	[1.129]	[-1.872]	[-1.771]	[-2.431]
$\Delta Bond(t-1)$	0.022	-0.284	0.971	-0.423	-0.007	-0.607	-0.015	0.075
	[1.404]	[-0.257]	[1.698]	[-2.446]	[-1.647]	[-2.716]	[-0.148]	[1.399]
<i>EC1</i>	-0.064	-0.328	0.275	0.256	0.002	0.033	-0.144	0.088
	[-5.605]	[-0.516]	[0.931]	[1.954]	[0.890]	[0.250]	[-2.235]	[2.722]
<i>EC2</i>	0.000	0.039	-0.009	-0.024	0.000	-0.001	0.002	0.003
	[0.384]	[1.710]	[-0.775]	[-4.505]	[4.159]	[-0.212]	[0.776]	[2.581]
<i>SE</i>	0.002	0.081	0.036	0.020	0.002	0.081	0.036	0.020

<i>Correlation</i>								
	<i>Regime 1:</i>				<i>Regime 2:</i>			
	<i>CPI</i>	<i>EN</i>	<i>Stock</i>	<i>Bond</i>	<i>CPI</i>	<i>EN</i>	<i>Stock</i>	<i>Bond</i>
<i>CPI</i>	1.000	0.165	0.085	-0.074	1.000	0.165	0.085	-0.074
<i>EN</i>	0.165	1.000	0.078	-0.028	0.165	1.000	0.078	-0.028
<i>Stock</i>	0.085	0.078	1.000	0.029	0.085	0.078	1.000	0.029
<i>Bond</i>	-0.074	-0.028	0.029	1.000	-0.074	-0.028	0.029	1.000

Figure 3.1: Regime probabilities of a MSIAH(2)-VECM(1,1) for CPI and prices of commodity index

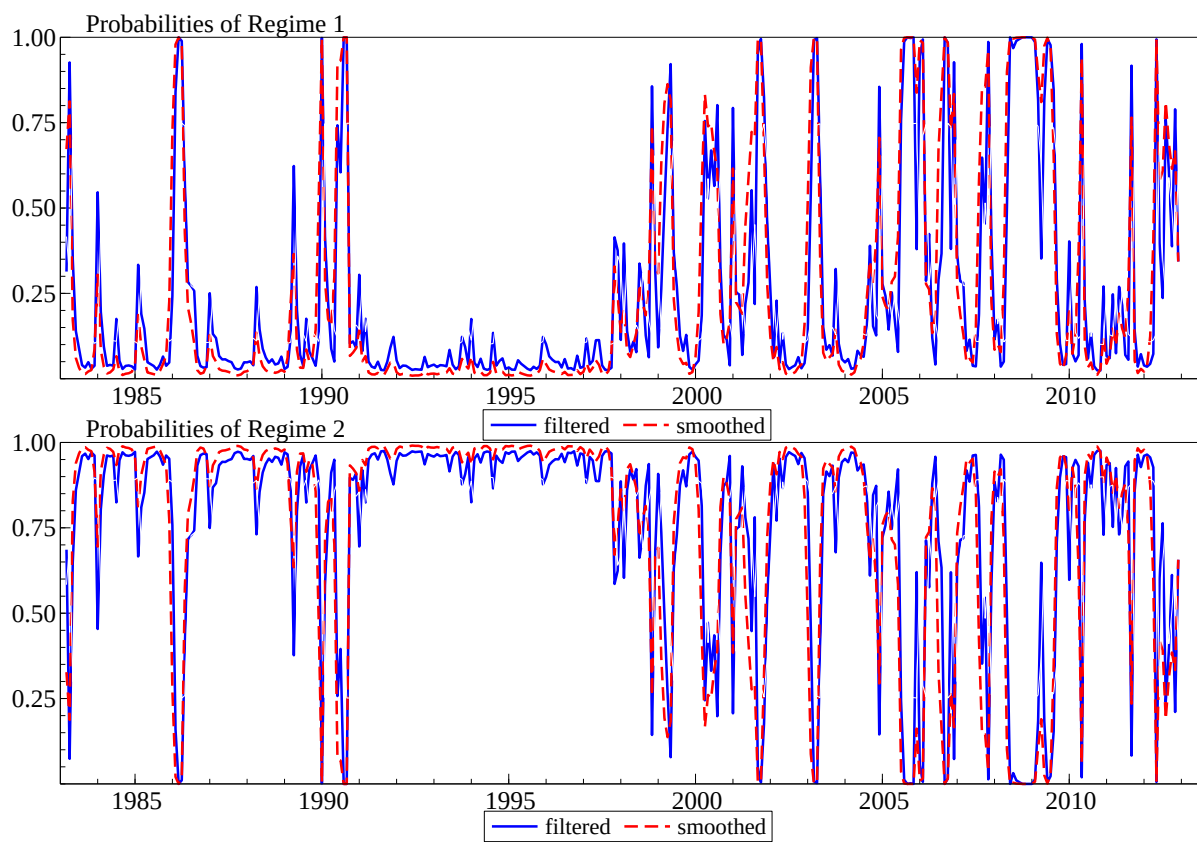


Figure 3.2: Regime probabilities of a MSIA(2)-VECM(1,1) for CPI and prices of energy subindex

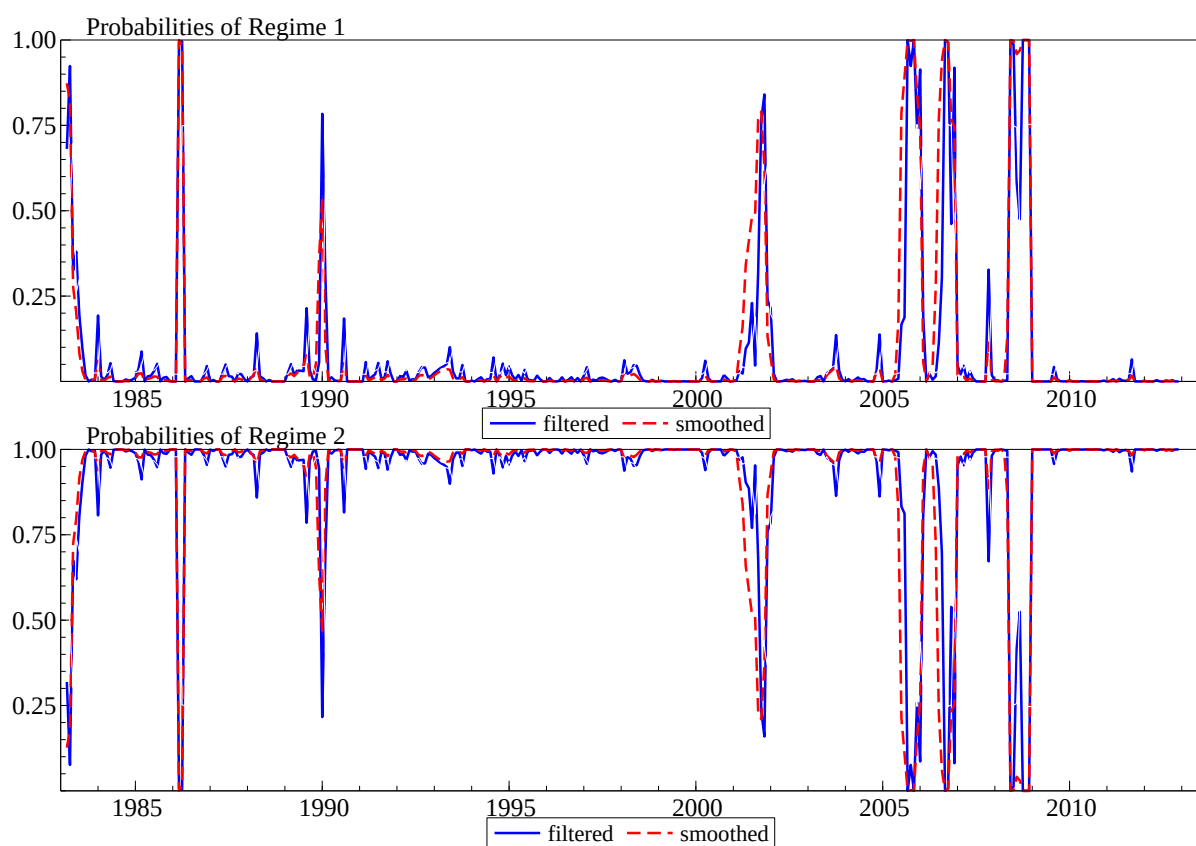


Figure 3.3: Regime probabilities of a MSIAH(2)-VECM(1,1) for CPI and prices of industrial metals subindex

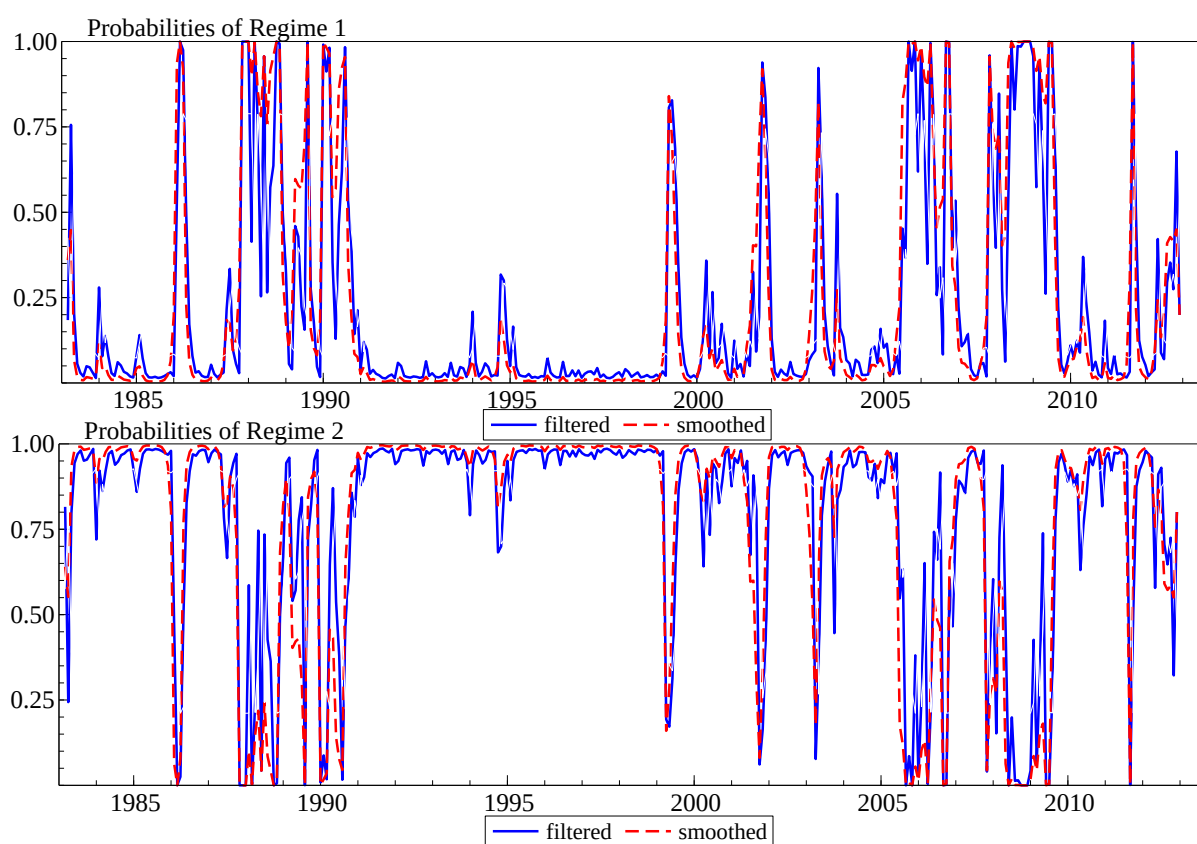


Figure 3.4: Regime probabilities of a MSIAH(2)-VECM(1,1) for CPI and prices of precious metals subindex

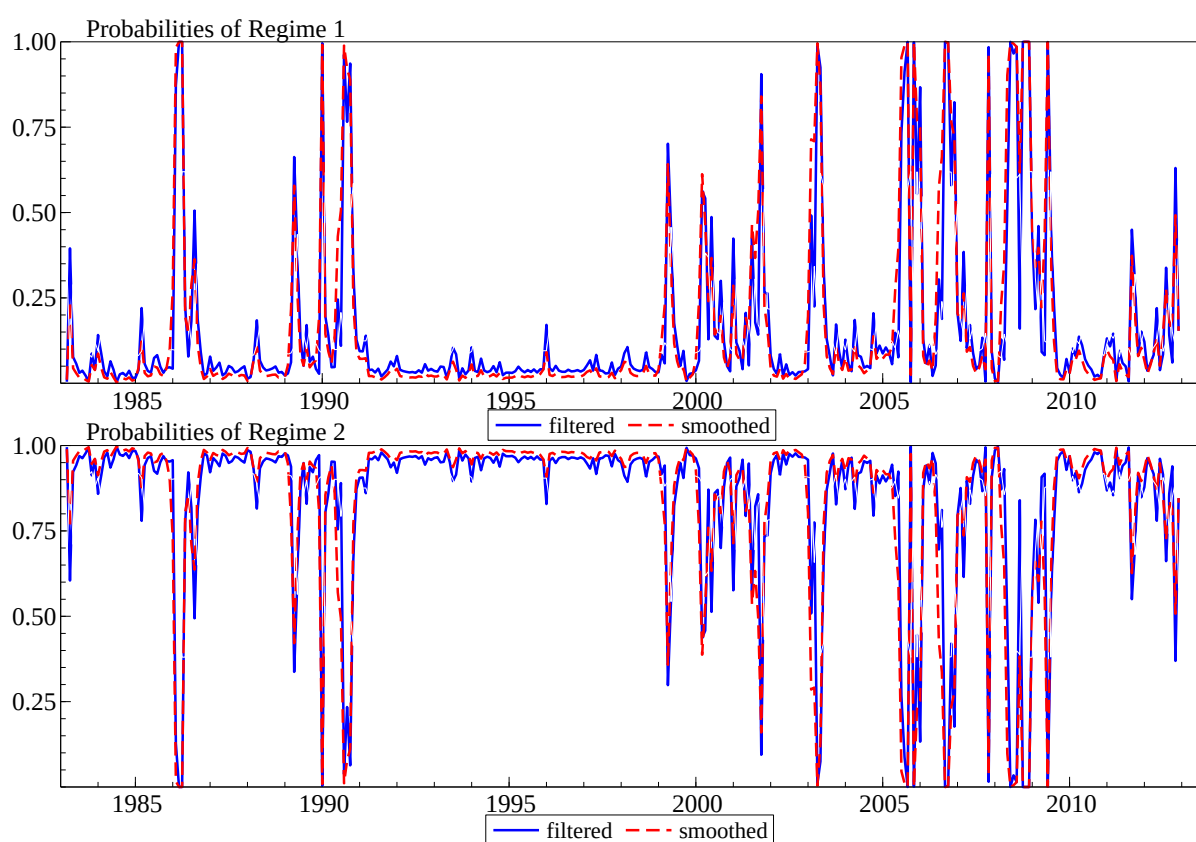


Figure 3.5: Regime probabilities of a MSIAH(2)-VECM(1,1) for CPI and prices of agriculture subindex

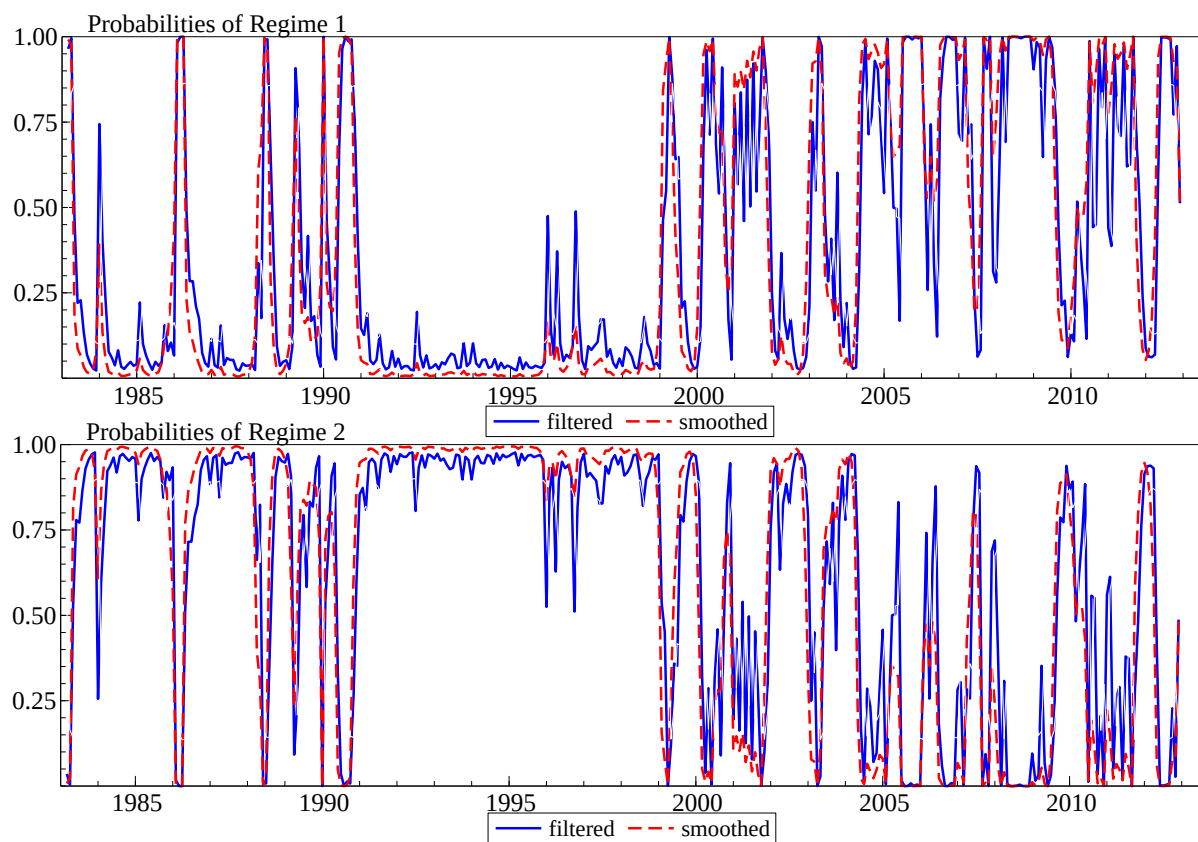


Figure 3.6: Regime probabilities of a MSIAH(2)-VECM(0,1) for CPI and prices of livestock subindex

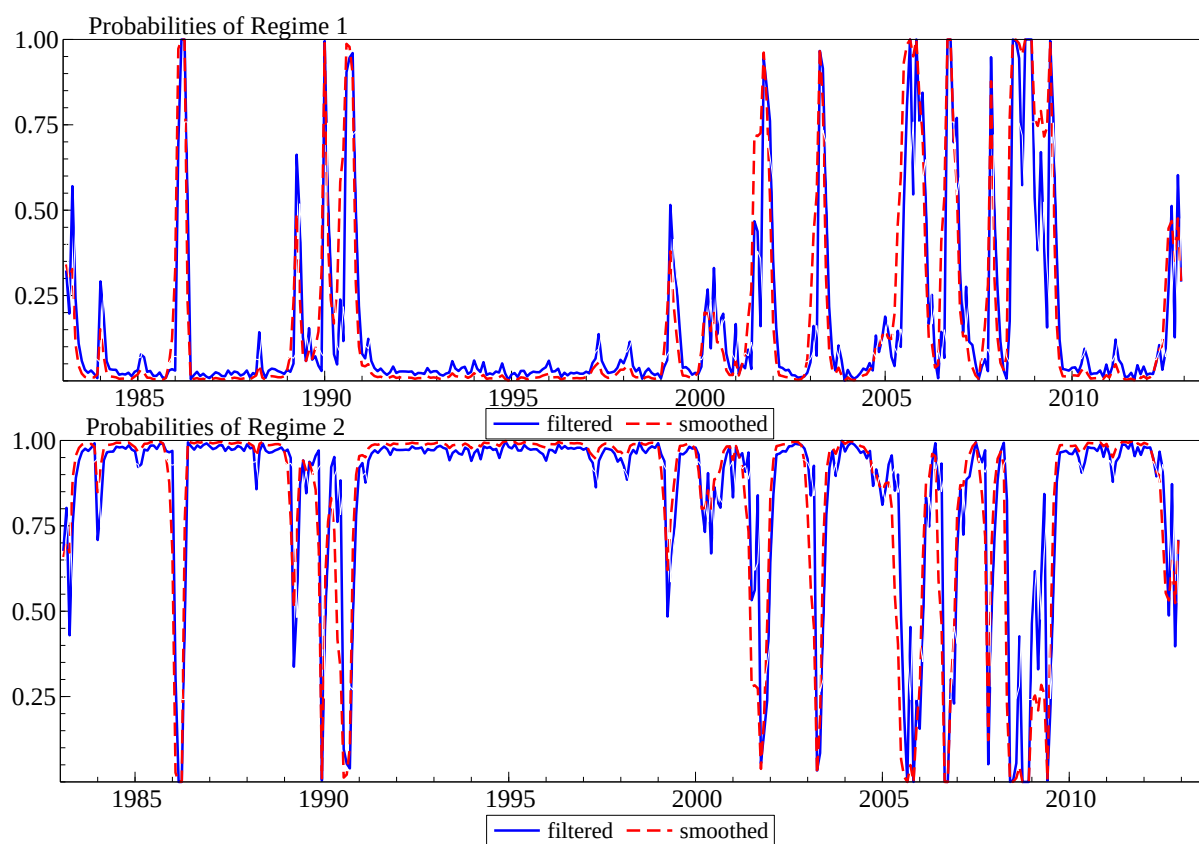


Figure 3.7: Regime probabilities of a MSIA(2)-VECM(1,1) for CPI and prices of commodity, stocks, and bonds

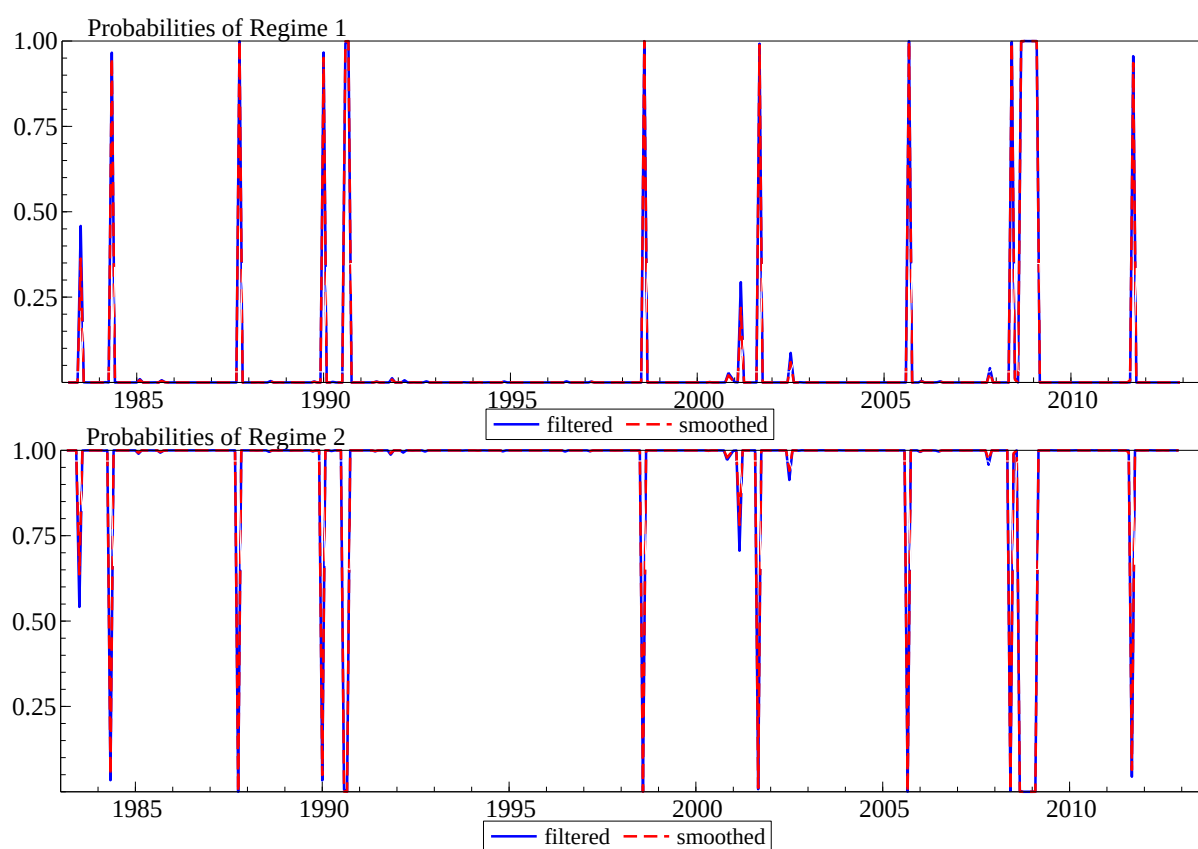


Figure 3.8: Regime probabilities of a MSIA(2)-VECM(1,2) for CPI and prices of energy subindex, stocks, and bonds

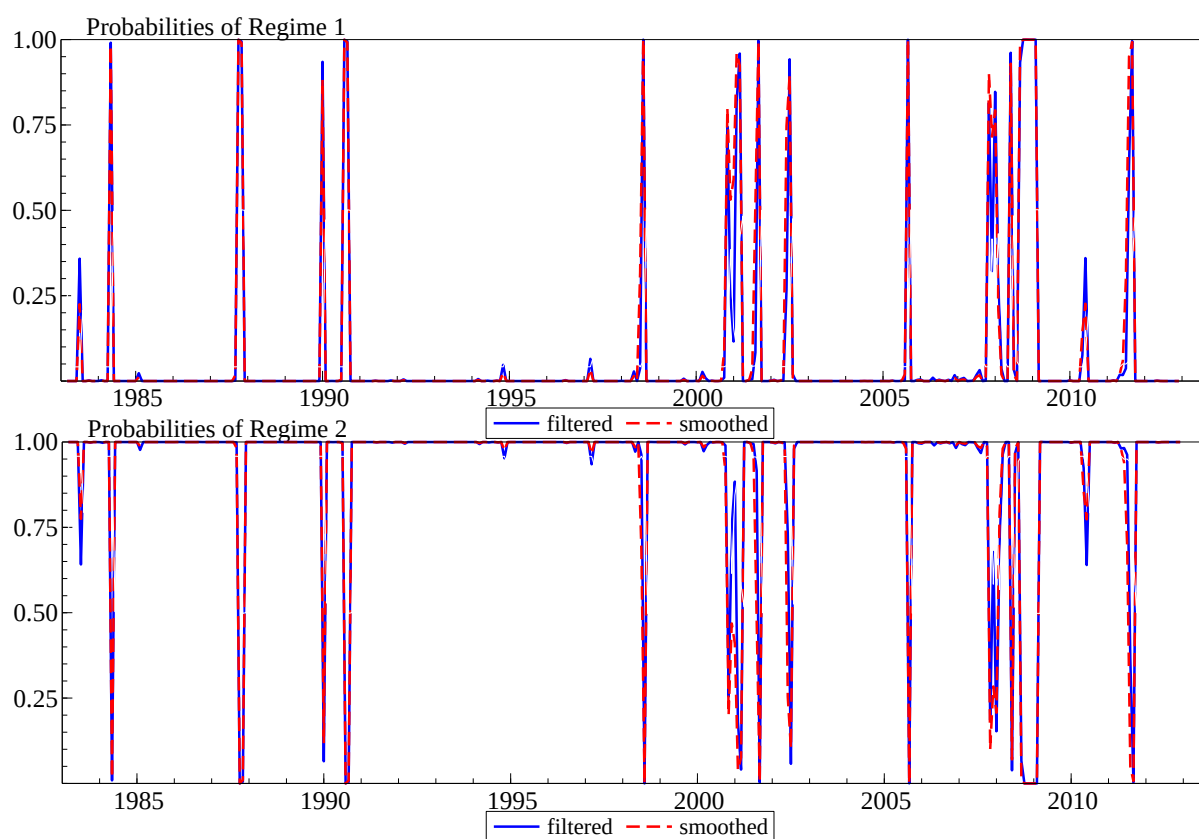


Table 3.30: Cointegrating equation of CPI and prices of industrial metals subindex, stocks, and bonds

<i>CPI</i>	<i>IM</i>	<i>Stocks</i>	<i>Bonds</i>	<i>Const.</i>
1	0.304	0.136	-1.236	0.449
	[2.074]	[0.485]	[-3.035]	[0.645]

Table 3.31: Comparison of VECMs of CPI and prices of industrial metals subindex, stocks, and bonds

<i>VECM lag = 1</i>	<i>AIC</i>	<i>SIC</i>	<i>VECM lag = 0</i>	<i>AIC</i>	<i>SIC</i>
Linear-VECM(1,1)	-20.154	-19.786	Linear-VECM(0,1)	-19.964	-19.770
MSIA(2)-VECM(1,1)	-20.353	-19.703	MSIA(2)-VECM(0,1)	-20.206	-19.903
MSIA(3)-VECM(1,1)	-20.646	-19.693	MSIA(3)-VECM(0,1)	NA	NA
MSIAH(2)-VECM(1,1)	-20.568	-19.809	MSIAH(2)-VECM(0,1)*	-20.575	-20.164
MSIAH(3)-VECM(1,1)	-20.715	-19.544	MSIAH(3)-VECM(0,1)	-20.677	-20.028

Table 3.32: Estimates of a MSIAH(2)-VECM(0,1) for CPI and prices of industrial metals subindex, stocks, and bonds

<i>Iterations until convergence:</i>	24	<i>No. of parameters in linear system:</i>	18
<i>No. of observations in the system:</i>	1436	<i>No. of restrictions:</i>	18
<i>No. of parameters in MS system:</i>	38	<i>No. of nuisance parameters:</i>	2
<i>Log-likelihood of MS system:</i>	3731.18	<i>Log-likelihood of linear system:</i>	3601.58
<i>LR linearity test statistic:</i>	259.22		
<i>LRT Chi(18) p-value:</i>	<0.000	<i>Davies test p-value:</i>	<0.000
<i>LRT Chi(20) p-value:</i>	<0.000		

<i>Transition matrix</i>					<i>Persistence of Regimes</i>			
	<i>Regime 1</i>	<i>Regime 2</i>				<i>Obser.</i>	<i>E-Prob.</i>	<i>Duration</i>
<i>Regime 1</i>	0.602	0.398			<i>Regime 1</i>	63.9	0.177	2.510
<i>Regime 2</i>	0.086	0.914			<i>Regime 2</i>	295.1	0.823	11.680
<i>Parameter</i>	<i>Regime 1:</i>				<i>Regime 2:</i>			
	$\Delta CPI(t)$	$\Delta IM(t)$	$\Delta Stock(t)$	$\Delta Bond(t)$	$\Delta CPI(t)$	$\Delta IM(t)$	$\Delta Stock(t)$	$\Delta Bond(t)$
<i>Const.</i>	0.002	0.015	-0.027	0.013	0.003	0.008	0.017	0.005
	[3.278]	[0.967]	[-2.768]	[3.642]	[23.887]	[2.418]	[7.359]	[4.034]
<i>EC1</i>	0.006	0.175	0.058	-0.022	0.002	-0.043	-0.015	0.018
	[2.185]	[2.532]	[1.289]	[-1.420]	[4.545]	[-3.460]	[-1.705]	[3.634]
<i>SE</i>	0.005	0.107	0.065	0.025	0.002	0.050	0.035	0.020

<i>Correlation</i>								
	<i>Regime 1:</i>				<i>Regime 2:</i>			
	<i>CPI</i>	<i>IM</i>	<i>Stock</i>	<i>Bond</i>	<i>CPI</i>	<i>IM</i>	<i>Stock</i>	<i>Bond</i>
<i>CPI</i>	1.000	0.187	-0.055	-0.353	1.000	0.070	-0.076	-0.051
<i>IM</i>	0.187	1.000	0.266	-0.162	0.070	1.000	0.286	-0.124
<i>Stock</i>	-0.055	0.266	1.000	0.025	-0.076	0.286	1.000	0.132
<i>Bond</i>	-0.353	-0.162	0.025	1.000	-0.051	-0.124	0.132	1.000

Table 3.33: Cointegrating equation of CPI and prices of precious metals subindex, stocks, and bonds

<i>CPI</i>	<i>PM</i>	<i>Stocks</i>	<i>Bonds</i>	<i>Const.</i>
1	0	0.189 [0.910]	-0.504 [-1.810]	-3.848 [-7.658]
0	1	-7.429 [-0.455]	-3.044 [-0.139]	109.422 [2.766]

Table 3.34: Comparison of VECMs of CPI and prices of precious metals subindex, stocks, and bonds

<i>VECM lag = 1</i>	<i>AIC</i>	<i>SIC</i>	<i>VECM lag = 0</i>	<i>AIC</i>	<i>SIC</i>
Linear-VECM(1,2)	-20.868	-20.456	Linear-VECM(0,2)	-20.558	-20.320
MSIA(2)-VECM(1,2)	-21.190	-20.453	MSIA(2)-VECM(0,2)	-20.754	-20.365
MSIA(3)-VECM(1,2)	-21.312	-20.228	MSIA(3)-VECM(0,2)	-20.769	-20.206
MSIAH(2)-VECM(1,2)	-21.235	-20.390	MSIAH(2)-VECM(0,2)*	-21.094	-20.596
MSIAH(3)-VECM(1,2)	-21.366	-20.065	MSIAH(3)-VECM(0,2)	-21.256	-20.477

Table 3.35: Estimates of a MSIAH(2)-VECM(0,2) for CPI and prices of precious metals subindex, stocks, and bonds

<i>Iterations until convergence:</i>	28	<i>No. of parameters in linear system:</i>	22
<i>No. of observations in the system:</i>	1436	<i>No. of restrictions:</i>	22
<i>No. of parameters in MS system:</i>	46	<i>No. of nuisance parameters:</i>	2
<i>Log-likelihood of MS system:</i>	3832.31	<i>Log-likelihood of linear system:</i>	3712.10
<i>LR linearity test statistic:</i>	240.42		
<i>LRT Chi(22) p-value:</i>	<0.000	<i>Davies test p-value:</i>	<0.000
<i>LRT Chi(24) p-value:</i>	<0.000		

<i>Transition matrix</i>				<i>Persistence of Regimes</i>		
	<i>Regime 1</i>	<i>Regime 2</i>		<i>Regime 1</i>	<i>Regime 2</i>	
<i>Regime 1</i>	0.730	0.270		73.9	0.204	3.710
<i>Regime 2</i>	0.069	0.931		285.1	0.796	14.470

<i>Parameter</i>								
	<i>Regime 1:</i>				<i>Regime 2:</i>			
	$\Delta CPI(t)$	$\Delta PM(t)$	$\Delta Stock(t)$	$\Delta Bond(t)$	$\Delta CPI(t)$	$\Delta PM(t)$	$\Delta Stock(t)$	$\Delta Bond(t)$
<i>Const.</i>	0.002	0.010	-0.031	0.015	0.003	0.003	0.016	0.005
	[2.794]	[1.034]	[-2.607]	[3.416]	[25.432]	[1.105]	[7.618]	[3.738]
<i>EC1</i>	0.012	-0.150	-0.558	0.085	0.005	-0.095	-0.156	0.086
	[0.934]	[-0.814]	[-2.662]	[1.047]	[1.700]	[-1.310]	[-2.722]	[2.706]
<i>EC2</i>	0.000	-0.001	-0.007	0.001	0.000	-0.002	-0.001	0.001
	[1.114]	[-0.400]	[-2.255]	[1.000]	[2.567]	[-2.550]	[-2.278]	[3.499]
<i>SE</i>	0.005	0.058	0.067	0.026	0.002	0.043	0.032	0.019

<i>Correlation</i>								
	<i>Regime 1:</i>				<i>Regime 2:</i>			
	<i>CPI</i>	<i>PM</i>	<i>Stock</i>	<i>Bond</i>	<i>CPI</i>	<i>PM</i>	<i>Stock</i>	<i>Bond</i>
<i>CPI</i>	1.000	0.054	-0.055	-0.361	1.000	0.204	0.001	-0.019
<i>PM</i>	0.054	1.000	0.112	0.146	0.204	1.000	-0.004	-0.039
<i>Stock</i>	-0.055	0.112	1.000	-0.104	0.001	-0.004	1.000	0.221
<i>Bond</i>	-0.361	0.146	-0.104	1.000	-0.019	-0.039	0.221	1.000

Table 3.36: Cointegrating equation of CPI and prices of agriculture subindex, stocks, and bonds

<i>CPI</i>	<i>AGR</i>	<i>Stocks</i>	<i>Bonds</i>	<i>Const.</i>
1	0.025	0.037	-0.498	-2.239
	[0.397]	[0.480]	[-4.876]	[-6.062]

Table 3.37: Comparison of VECMs of CPI and prices of agriculture subindex, stocks, and bonds

<i>VECM lag = 1</i>	<i>AIC</i>	<i>SIC</i>	<i>VECM lag = 0</i>	<i>AIC</i>	<i>SIC</i>
Linear-VECM(1,1)	-20.553	-20.184	Linear-VECM(0,1)	-20.342	-20.147
MSIA(2)-VECM(1,1)	-20.799	-20.149	MSIA(2)-VECM(0,1)	-20.587	-20.284
MSIA(3)-VECM(1,1)	-20.950	-19.996	MSIA(3)-VECM(0,1)	NA	NA
MSIAH(2)-VECM(1,1)	-21.059	-20.300	MSIAH(2)-VECM(0,1)*	-20.979	-20.568
MSIAH(3)-VECM(1,1)	-21.071	-19.900	MSIAH(3)-VECM(0,1)	-21.153	-20.504

Table 3.38: Estimates of a MSIAH(2)-VECM(0,1) for CPI and prices of agriculture subindex, stocks, and bonds

<i>Iterations until convergence:</i>	16	<i>No. of parameters in linear system:</i>	18
<i>No. of observations in the system:</i>	1436	<i>No. of restrictions:</i>	18
<i>No. of parameters in MS system:</i>	38	<i>No. of nuisance parameters:</i>	2
<i>Log-likelihood of MS system:</i>	3803.72	<i>Log-likelihood of linear system:</i>	3669.38
<i>LR linearity test statistic:</i>	268.68		
<i>LRT Chi(18) p-value:</i>	<0.000	<i>Davies test p-value:</i>	<0.000
<i>LRT Chi(20) p-value:</i>	<0.000		

<i>Transition matrix</i>				<i>Persistence of Regimes</i>		
	<i>Regime 1</i>	<i>Regime 2</i>		<i>Obser.</i>	<i>E-Prob.</i>	<i>Duration</i>
<i>Regime 1</i>	0.783	0.217	<i>Regime 1</i>	89.8	0.249	4.610
<i>Regime 2</i>	0.072	0.928	<i>Regime 2</i>	269.2	0.751	13.910

<i>Parameter</i>								
	<i>Regime 1:</i>				<i>Regime 2:</i>			
	$\Delta CPI(t)$	$\Delta AGR(t)$	$\Delta Stock(t)$	$\Delta Bond(t)$	$\Delta CPI(t)$	$\Delta AGR(t)$	$\Delta Stock(t)$	$\Delta Bond(t)$
<i>Const.</i>	0.002	-0.004	-0.029	0.012	0.003	0.000	0.015	0.005
	[3.680]	[-0.373]	[-2.815]	[3.231]	[23.576]	[0.077]	[6.851]	[3.742]
<i>ECI</i>	0.014	-0.198	-0.463	0.076	0.006	0.034	-0.017	0.071
	[1.449]	[-1.050]	[-2.736]	[1.213]	[3.243]	[0.815]	[-0.489]	[3.410]
<i>SE</i>	0.004	0.080	0.064	0.025	0.002	0.039	0.033	0.020

<i>Correlation</i>								
	<i>Regime 1:</i>				<i>Regime 2:</i>			
	<i>CPI</i>	<i>AGR</i>	<i>Stock</i>	<i>Bond</i>	<i>CPI</i>	<i>AGR</i>	<i>Stock</i>	<i>Bond</i>
<i>CPI</i>	1.000	0.031	0.008	-0.366	1.000	-0.048	-0.044	-0.023
<i>AGR</i>	0.031	1.000	0.347	0.108	-0.048	1.000	0.075	-0.107
<i>Stock</i>	0.008	0.347	1.000	-0.267	-0.044	0.075	1.000	0.286
<i>Bond</i>	-0.366	0.108	-0.267	1.000	-0.023	-0.107	0.286	1.000

Table 3.39: Cointegrating equation of CPI and prices of livestock subindex, stocks, and bonds

<i>CPI</i>	<i>LS</i>	<i>Stocks</i>	<i>Bonds</i>	<i>Const.</i>
1	0	0.100 [1.020]	-0.394 [-2.993]	-3.601 [-15.243]
0	1	-4.485 [-0.544]	-3.883 [-0.352]	69.548 [3.518]

Table 3.40: Comparison of VECMs of CPI and prices of livestock subindex, stocks, and bonds

<i>VECM lag = 1</i>	<i>AIC</i>	<i>SIC</i>	<i>VECM lag = 0</i>	<i>AIC</i>	<i>SIC</i>
Linear-VECM(1,2)	-21.018	-20.606	Linear-VECM(0,2)	-20.833	-20.595
MSIA(2)-VECM(1,2)	-21.314	-20.577	MSIA(2)-VECM(0,2)	-21.201	-20.812
MSIA(3)-VECM(1,2)	-21.408	-20.324	MSIA(3)-VECM(0,2)	-21.216	-20.653
MSIAH(2)-VECM(1,2)	-21.352	-20.506	MSIAH(2)-VECM(0,2)*	-21.338	-20.840
MSIAH(3)-VECM(1,2)	-21.509	-20.209	MSIAH(3)-VECM(0,2)	-21.405	-20.626

Table 3.41: Estimates of a MSIAH(2)-VECM(0,2) for CPI and prices of livestock subindex, stocks, and bonds

<i>Iterations until convergence:</i>	30	<i>No. of parameters in linear system:</i>	22
<i>No. of observations in the system:</i>	1436	<i>No. of restrictions:</i>	22
<i>No. of parameters in MS system:</i>	46	<i>No. of nuisance parameters:</i>	2
<i>Log-likelihood of MS system:</i>	3876.09	<i>Log-likelihood of linear system:</i>	3761.56
<i>LR linearity test statistic:</i>	229.06		
<i>LRT Chi(22) p-value:</i>	<0.000	<i>Davies test p-value:</i>	<0.000
<i>LRT Chi(24) p-value:</i>	<0.000		

<i>Transition matrix</i>				<i>Persistence of Regimes</i>		
	<i>Regime 1</i>	<i>Regime 2</i>		<i>Obser.</i>	<i>E-Prob.</i>	<i>Duration</i>
<i>Regime 1</i>	0.766	0.234	<i>Regime 1</i>	80.7	0.223	4.270
<i>Regime 2</i>	0.067	0.933	<i>Regime 2</i>	278.3	0.777	14.890

<i>Parameter</i>								
	<i>Regime 1:</i>				<i>Regime 2:</i>			
	$\Delta CPI(t)$	$\Delta LS(t)$	$\Delta Stock(t)$	$\Delta Bond(t)$	$\Delta CPI(t)$	$\Delta LS(t)$	$\Delta Stock(t)$	$\Delta Bond(t)$
<i>Const.</i>	0.002	-0.002	-0.029	0.013	0.003	0.004	0.016	0.005
	[3.041]	[-0.229]	[-2.596]	[3.224]	[24.602]	[1.572]	[7.479]	[3.649]
<i>EC1</i>	0.008	-0.215	-0.685	0.087	0.006	-0.116	-0.168	0.148
	[0.554]	[-1.361]	[-2.761]	[0.883]	[1.559]	[-1.349]	[-2.277]	[3.393]
<i>EC2</i>	0.000	-0.002	-0.010	0.001	0.000	0.000	-0.002	0.002
	[0.897]	[-0.601]	[-2.404]	[0.830]	[2.537]	[-0.468]	[-2.014]	[4.102]
<i>SE</i>	0.004	0.044	0.067	0.026	0.002	0.040	0.032	0.019

<i>Correlation</i>								
	<i>Regime 1:</i>				<i>Regime 2:</i>			
	<i>CPI</i>	<i>LS</i>	<i>Stock</i>	<i>Bond</i>	<i>CPI</i>	<i>LS</i>	<i>Stock</i>	<i>Bond</i>
<i>CPI</i>	1.000	0.141	-0.048	-0.345	1.000	0.007	-0.019	-0.022
<i>LS</i>	0.141	1.000	0.206	0.149	0.007	1.000	-0.058	-0.042
<i>Stock</i>	-0.048	0.206	1.000	-0.140	-0.019	-0.058	1.000	0.247
<i>Bond</i>	-0.345	0.149	-0.140	1.000	-0.022	-0.042	0.247	1.000

Figure 3.9: Regime probabilities of a MSIAH(2)-VECM(0,1) for CPI and prices of industrial metals subindex, stocks, and bonds

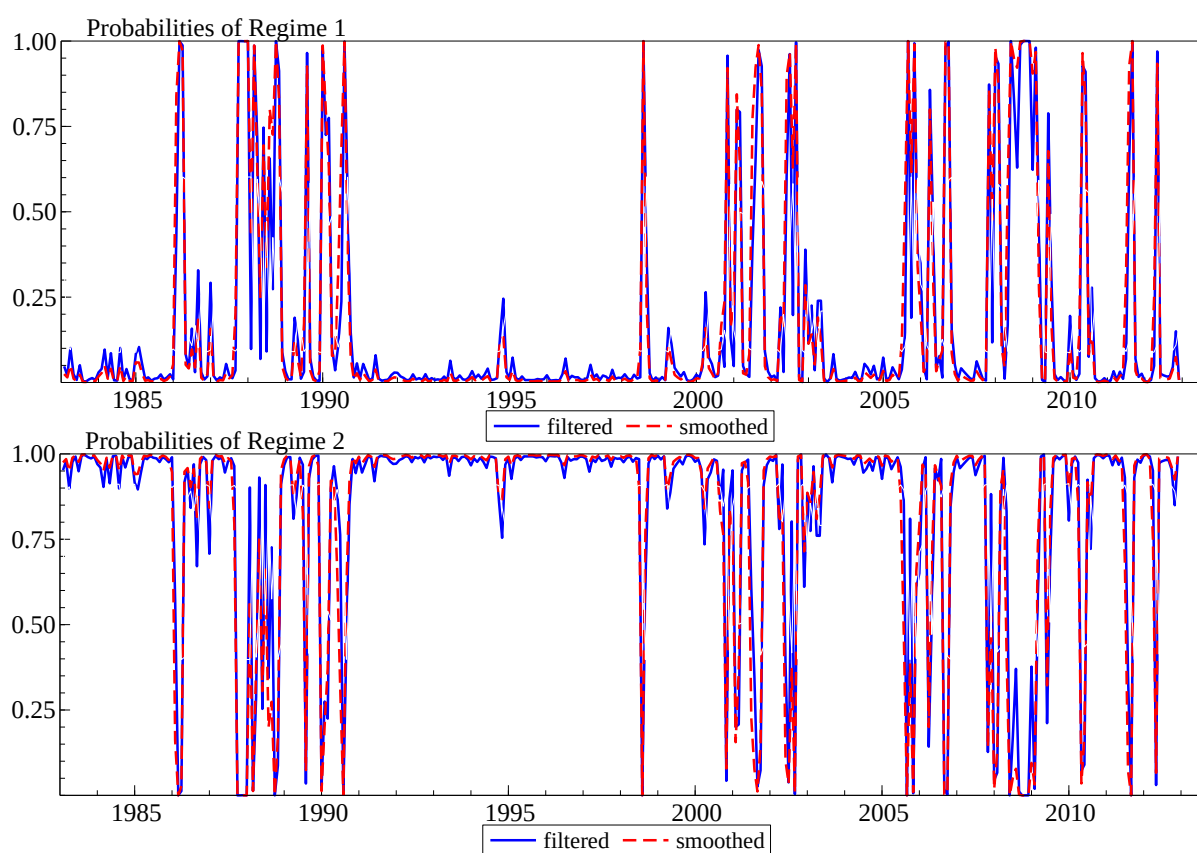


Figure 3.10: Regime probabilities of a MSIAH(2)-VECM(0,2) for CPI and prices of precious metals subindex, stocks, and bonds

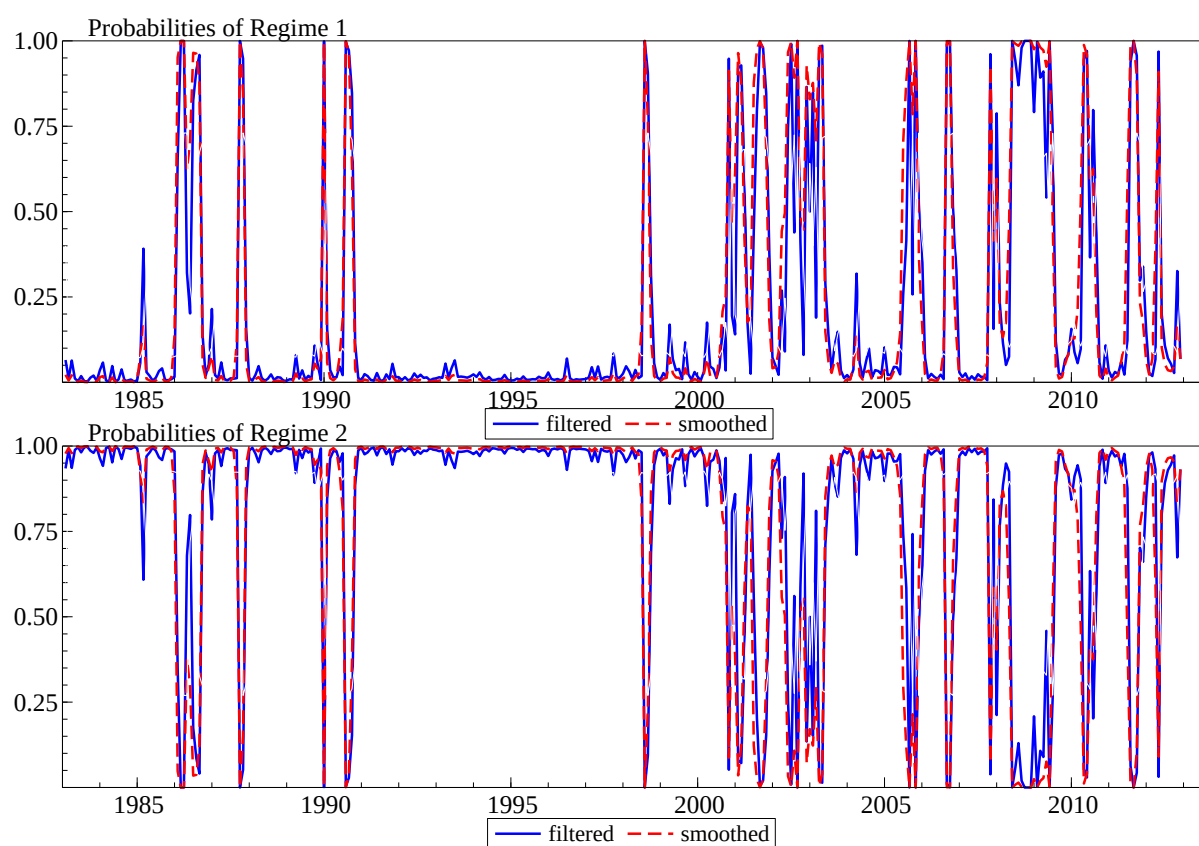


Figure 3.11: Regime probabilities of a MSIAH(2)-VECM(0,1) for CPI and prices of agriculture subindex, stocks, and bonds

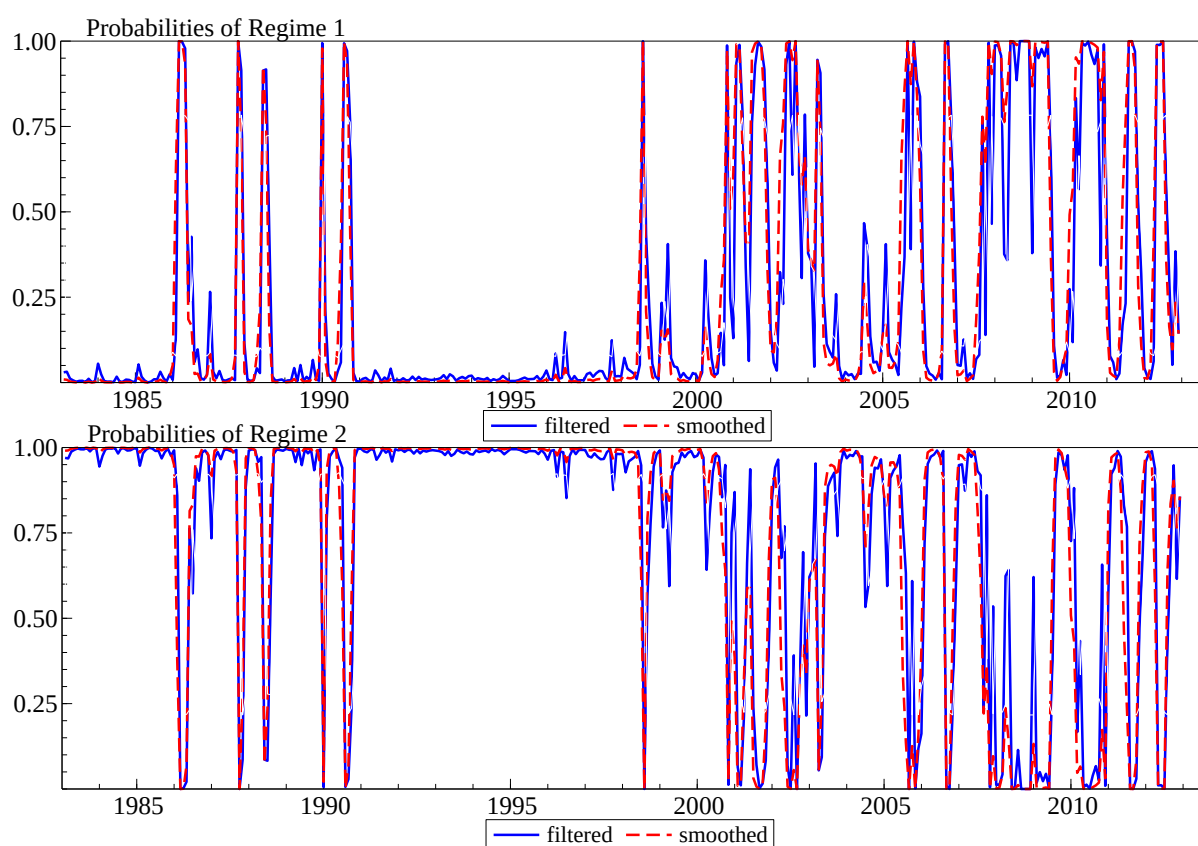
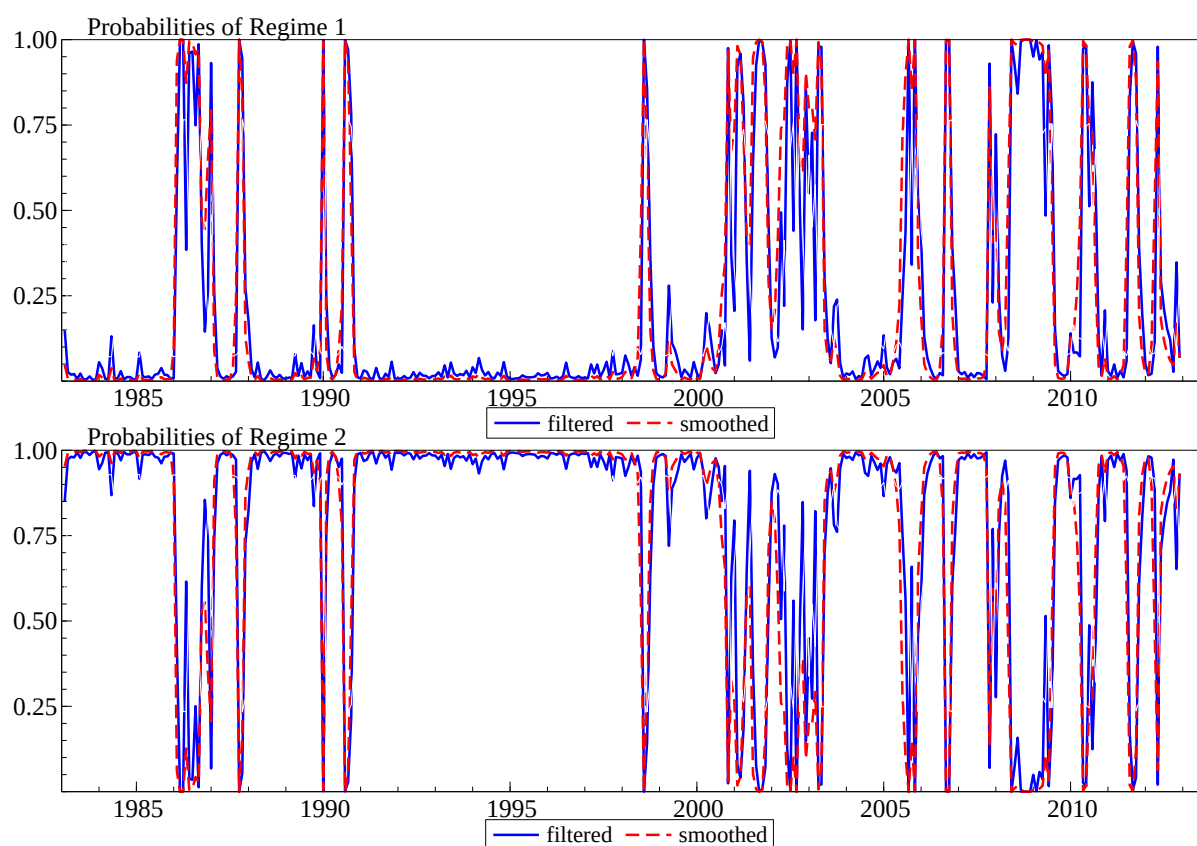


Figure 3.12: Regime probabilities of a MSIAH(2)-VECM(0,2) for CPI and prices of livestock subindex, stocks, and bonds



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