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Thesis on Macroeconomic Fundamentals and Exchange Rate Dynamics

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Contents:

Introduction

There are three chapters in my thesis. All of them focus on the explanation of exchange change rate dynamics using macroeconomic fundamentals. I adopt a no-arbitrage macro finance approach to carry out the studies. First of all, I explor the in-sample model performance of this no-arbitrage exchange rate model. Then I study the out-of-sample model forecasting performance. Finally, in order to investigate more than one exchange rate dynamics simultaneously, I extended the model from the general adopted two-country framework into a multi-country framework.

The first chapter investigates the relationship between the exchange rate dynamics and macroeconomic fundamentals by proposing an arbitrage-free stochastic discount factor model that jointly prices bond yields and exchange rates. Based on empirical analysis using data on exchange rates, yields of zero-coupon bonds, and macroeconomic variables of the US and the Euro area, this chapter finds a close link between macroeconomic fundamentals and the exchange rate dynamics. The model-implied monthly exchange rate changes can explain about 57% variation of the observed data. The macroeconomic innovations can help capture large volatility of exchange rate changes, and the foreign exchange risk premium can largely alleviate the forward premium anomaly.

Based on the encouraging in-sample model performance on exchange rate dynamics by the no-arbitrage exchange rate model, I move forward into inspecting the out-of-sample forecasting performance of macroeconomic fundamentals on exchange rate returns in the second chapter. Exchange rate movements are endogenously determined by ratios between domestic and foreign stochastic discount factors, through which the macroeconomic fundamentals nonlinearly model exchange rate dynamics. Testing on three floating nominal exchange rates, i.e. DEM(EUR)/USD, GBP/USD and JPY/USD observed at monthly as well as quarterly time frequencies, this chapter has the following findings. First, five out of the six model-implied time-varying foreign exchange risk premiums satisfy the Fama conditions (Fama, 1984). Second, comparing to the random walk model, this no-arbitrage macro-finance model reduces forecasting root mean square errors, especially for the data observed at quarterly time frequency.

Finally, I generalize the exchange rate model from a well adopted two-country setup into a multi-country setup in the third chapter. This chapter investigates the joint dynamics of multi bilateral nominal exchange rates simultaneously under a Multi-Country framework. We introduce the macroeconomic fundamental information to model exchange rate dynamics by adopting a no-arbitrage macro-finance approach. We allow macroeconomic fundamentals to be determined by global (common) factors as well as country-idiosyncratic factors under this multi-country open economy. The empirical study focuses an open economy including four countries, i.e., Germany, the UK, Japan and the US, where the US is taken as the numerarie (home) country. Empirical results show that exchange rate dynamics are better modeled by this multi-country no-arbitrage model comparing to previous studies. This model explains 53%, 34% and 32% variations of the observed exchange rate changes of DEM(EUR)/USD, GBP/USD and JPY/USD, respectively. Moreover, the global macroeconomic factors do exist.

Chapter 1

Macroeconomic Fundamentals and the Exchange Rate Dynamics: A No-Arbitrage Macro-Finance Approach

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Abstract

This paper investigates the relationship between the exchange rate dynamics and macroeconomic fundamentals by proposing an arbitrage-free stochastic discount factor model that jointly prices bond yields and exchange rates. Based on empirical analysis using data on exchange rates, yields of zero-coupon bonds, and macroeconomic variables of the US and the Euro area, the paper finds a close link between macroeconomic fundamentals and the exchange rate dynamics. The model-implied monthly exchange rate changes can explain about 57% variation of the observed data. The macroeconomic innovations can help capture large volatility of exchange rate changes, and the foreign exchange risk premium can largely alleviate the forward premium anomaly.

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Keywords: Exchange rate movements; Macroeconomic fundamentals; Forward premium anomaly; Stochastic discount factor; Term structure of interest rates; Unscented Kalman filter.

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I. Introduction

The nominal floating exchange rate has often been regarded as an asset price in exchange rate modeling since the 1970s. According to the standard asset pricing theory, its current price should reflect market's expectations concerning present and future economic conditions (Frenkel and Mussa, 1985; Obstfeld and Rogoff, 1996; Cochrane, 2004). However, a long-standing puzzle in international economics and finance is the disconnection between exchange rate movements and macroeconomic fundamentals such as output, inflation and monetary policy tools.

A variety of models have been proposed to relate exchange rates to macroeconomic fundamentals. Monetary models (Frenkel, 1976, 1979; Mussa, 1976; Bilson, 1978; Dornbusch, 1976) states the existence of a long-run equilibrium relationship among relative money supplies, relative income levels and the nominal exchange rate. New open economy macroeconomics models (Obstfeld and Rogoff, 2003) attempts to explain exchange rate movements by incorporating imperfect competition and nominal rigidities in a general equilibrium open economy. However, these models can not find empirical evidence on a close relationship between short-run exchange rate movements and macroeconomic fundamentals (Meese, 1990; Frankel and Rose, 1995; Engel and West, 2005). Furthermore, they fail to capture the volatile foreign exchange risk premium, implied by the well documented forward premium anomaly in foreign exchange markets (Fama, 1984; Hodrick, 1989; Backus, Gregory and Telmer, 1993; Bansal et al., 1995; Bekaert, 1996).

In this paper, we investigate interactions between the exchange rate dynamics and macroeconomic fundamentals by proposing an arbitrage-free stochastic discount factor model that jointly prices bond yields and exchange rates. Under a two-country world, the exchange rate of these two economies is governed by the ratio of their stochastic discount factors, which are modeled by a factor representation under the no-arbitrage condition. We take outputs, inflations and short-term interest rates as fundamental macroeconomic factors, which are assumed to drive the joint dynamics of two economies. Real output growth directly governs the aggregate consumption of an economy and should be a key element of the stochastic discount factor. Inflation can also enter into the stochastic discount factor via its dynamic interactions with the real production (Piazzesi and Schneider, 2006). The short-term interest rate is typically viewed as a macroeconomic variable reflecting monetary policy (Duffee, 2007). We extend macro-finance term structure models (Ang and Piazzesi, 2003; Diebold, Rudebushch and Aruoba, 2005; Ang, Dong and Piazzesi, 2007) to a two-country framework in order to improve identification of the time-varying market prices of risks, which in turn amplify roles of macroeconomic innovations on exchange rate movements. This is important since ignoring risk premia or assuming constant market prices of risks may mislead to a conclusion that exchange rates are not linked to macroeconomic fundamentals.

Under the above modeling set-ups, the exchange rate has a nonlinear relation with macroeconomic fundamentals. In contrast to uncovered interest parity, our model indicates that the expected exchange rate changes are determined by both the interest rate differential of two countries and the foreign exchange risk premium and that the unexpected exchange rate changes are also driven by the fundamental innovations, whose roles are amplified by the time-varying market prices of economic risks.

The paper is related to previous studies on joint dynamics of exchange rates and yield curves (Bansal, 1997; Backus, Foresi and Telmer, 2001; Dong, 2006; Engel and West, 2006; Han, Hammound and Ramezani, 2010; Marcello and Taboga, 2009). However, it is different in the following critical respects. First, we explicitly introduce macroeconomic fundamentals to modeling exchange rates without relying on any latent factors. As a result, our study is more economically meaningful. Second, we do not impose any restrictions on interdependence between two economies and use a more flexible approach. We do find evidence that real outputs of different economies co-move tightly and strongly impact each other. Third, our study sheds new lights on the relationship between macroeconomic fundamentals and the exchange rate dynamics. Macroeconomic fundamentals enter into the exchange rate dynamics in a nonlinear form, through which the unexpected macroeconomic innovations play a crucial role in driving exchange rates and in generating large volatility of exchange rate changes. Fourth, we propose an efficient likelihood-based estimation method by using the unscented Kalman filter, which is recently developed in the field of engineering and can efficiently handle highly nonlinear state-space models (Julier and Ulman, 1997, 2004).

Using monthly data of the US and the Euro area (EA) ranging from January, 1999 to December, 2008, we find a close link between macroeconomic fundamentals and the exchange rate dynamics. The model-implied monthly exchange rate changes can explain 57% variation of the observed data. We also find that both economies are highly interdependent. These findings are in stark contrast to previous studies using monetary and new open economy macroeconomics models and to a recent study of Dong (2006), which follows a similar modeling approach to this paper. The former finds that the models can only explain at most 10% variation of the data (Lubik and Schorfheide, 2005; Engel and West, 2005), and the latter, assuming no impact of the foreign country on the home country and using latent factors, finds that his model can explain about 38% variation of exchange rate changes between the US dollar and the German Mark.

The time-varying foreign exchange risk premium plays an important role in explaining the forward premium anomaly and in remedying the failure of uncovered interest parity. For example, if we run a regression of exchange rate changes only on the interest rate differential, the estimated coefficient is negative and far away from unity, and the R^2 is very tiny. However, if we introduce the risk premium term, as suggested by our model and Fama (1984), this estimate is positive and close to unity, and the parameter estimate of the risk premium term is positive and highly significant. The R^2 is improved dramatically.

Macroeconomic fundamentals enter into the exchange rate dynamics through the time-varying market prices of risks, and their shocks have time-varying effects on exchange rate movements. If we impose the constant market prices of risks in the model, the exchange rate changes become time-homogeneous, and the model performance is dramatically deteriorated. For example, the exchange rate changes implied from the model with the constant market prices of risks can only explain 25% variation of the observed data, less than half of the explanation power of our model. By decomposing the exchange rate changes into three parts: the interest rate differential, the foreign exchange risk premium, and the macro-shock driving part, we find that the first two components are much smoother and that a large fraction of variation of exchange rate changes is explained by the last component. For example, the last component, which is the product of the differential of market prices of risks and the fundamental innovations, can explain 37% variation of the observed data, taking 76% of the total explanation power of the model. To deeply explore sources of the explanation power, we also investigate whether it is enough to explain the exchange rate dynamics only using the yield curve information under the same modeling framework without relying on any macro variables. We find that in this case, the resulted model-implied exchange rate changes only explain 13% variation of the data.

The rest of paper is organized as follows. Section II presents data and implements a preliminary analysis. Section III introduces a no-arbitrage macro-finance modeling approach for the exchange rate dynamics. Section IV proposes a likelihood-based estimation method relying on the unscented Kalman filter. Section V presents the empirical results and discusses their economic implications. Section VI concludes the paper.

II. Data and Preliminary Analysis

A. Data

We use monthly data of the United State (US) and the Euro area (EA). The US is taken as the home country, and the EA as the foreign country. There are three types of data: the macroeconomic data (outputs and inflations), the yields data, and the exchange rate data. Data range from January 1999 to December 2008 in monthly frequency, in total, 120 months.

The home and foreign output gaps and inflations are proxied by their own Industrial Production Index's and CPI's, respectively. The US data are downloaded from the Federal Reserve, St. Louis, and the EA data are downloaded from the European Central Bank (ECB). The raw macroeconomic data are seasonally adjusted. Inflation rates are measured by the 12-month changes of the log CPI's, and output gaps are constructed by applying the HP filter (Hodrick and Prescott, 1997) to Industrial Production Index's with the smoothing parameter being set to 129,600 (Ravn and Uhlig, 2002).

The US yields data are those with maturity 1, 3, 12, 24, 36, 48, and 60 months. The 1-month and 3-month rates are directly from the CRSP Fama-Bliss rate file. While the yields of 12-month and longer maturity are derived from the respective bond prices obtained from the CRSP Fama-Bliss discount bond file using the formula $y_t^{(n)} = -\log(P_t^{(n)}/100)/n$, where $P_t^{(n)}$ is the bond price at time t with maturity n-year, and $y_t^{(n)}$ is the corresponding yield. Because the EA yields data from the ECB are available only from December 2006 and are not long enough to implement estimation, they are proxied by German data, which are obtained from BBK Statistics-Deutsche Bundesbank. The EA yields have maturity 1-month, 3-month, 12-month, 36-month, and 60-month.

The exchange rate data are those of the-last-day US-Dollar/Euro spot exchange rates of each month, downloaded from the Federal Reserve, St. Louis.

The upper panels of Figure 1 plot the annualized macroeconomic data used in estimation. We can see that two series of inflation rates are very persistent and output gaps are more volatile. The co-movement between them is very clear. During the sample period, the US output gap reaches a high level in the mid of 2000, just before the collapse of the dot-com bubble, and has a dramatic decline during the 2008-2009 financial crisis. Similar movements can also be observed in the EA output gap. The lower panels of Figure 1 present the annualized yields of the US and of the EA. The US yields are more volatile than the EA ones.

— Figure 1 around here —

B. A Counter-Factual Study

In this subsection, we implement a counter-factual study on interactions between exchange rate movements and macroeconomic fluctuations. This study provides an empirical support to our modeling framework in the next section.

Let Z_t be a vector of macroeconomic variables with the last element being the exchange rate

$$Z_t = [g_t^{(h)},\ \pi_t^{(h)},\ y_{1t}^{(h)},\ g_t^{(f)},\ \pi_t^{(f)},\ y_{1t}^{(f)},\ \Delta s_t]', \tag{1}$$

where the home variables are denoted with superscripts (h), and the foreign ones are denoted with superscripts (f). $g_t^{\prime}s$ denote output gaps, $\pi_t^{\prime}s$ inflation rates, $y_{1t}^{\prime}s$ yields of the zero-coupon bond with maturity one month, which represent the short-term interest rates, and Δs_t is the log difference of the spot exchange rates (the exchange rate changes).

We use data of the US and the EA described in subsection A. Assume that Z_t follows a vector autoregression process. According to both the Akaike information criterion (AIC) and the Bayesian information criterion (BIC), VAR(1) is the best specification for the data. We estimate this VAR(1) and identify innovations by the Cholesky decomposition, which implies that innovations of the exchange rate changes have the lagged impacts on the macroeconomic variables, whereas the macroeconomic innovations have both the contemporaneous and lagged impacts on the exchange rate changes.

Using the parameter estimates and the identified innovations from the VAR(1) model, we implement a counter-factual simulation to construct artificial time series for all variables in (1) through simulating the VAR(1) model over the sample period conditionally upon setting innovations of the exchange rate changes to zero. Figure 2 plots the actual and the simulated time series of the macro variables in Z_t, and the upper panel of Figure 4 plots the actual and the simulated time series of the exchange rate changes.

— Figure 2 around here —

It becomes clear that the exchange rate is the only variable strongly affected by its own innovations and the idiosyncratic fluctuations in exchange rate changes are not significant at all in explaining fluctuations of macroeconomic variables. This finding is consistent with Lubik and Schorfheide (2005), Pericoli and Taboga (2009) and De Santis and Favero (2009). Lubik and Schorfheide (2005) empirically find that there is no role of the exchange rate change on monetary policy rules. Pericoli and Taboga (2009) find that the exogenous shocks to the exchange rate have a negligible impact on the yield curve. De Santis and Favero (2009) find that the exchange rate is irrelevant to determine co-movements of macroeconomic variables. We find that the simulated exchange rate changes can only explain a small proportion of variation of the data (18%). This counter-factural study by the reduced-form VAR model implies that innovations of the exchange rate changes are irrelevant to determine fluctuations of macroeconomic variables and that the contemporaneous macroeconomic innovations may have potential power to explain exchange rate movements through a nonlinear form.

III. The Modeling Framework

Consider a two-country world, a home country and a foreign country, each with its own currency. Under the absence of arbitrage, the exchange rate between these two currencies is governed by the ratio of their stochastic discount factors. In this section, we firstly discuss how to model stochastic discount factors in subsection A. We then proceed to model the exchange rate dynamics in subsection B. Subsection C introduces a two-country affine term structure model for pricing zero-coupon bonds.

A. Macroeconomic Fundamentals and Stochastic Discount Factors

In each country, we take the output gap $\tilde{g}$, the inflation $\tilde{\pi}$ and the short-term interest rate $\tilde{r}$ as main macroeconomic fundamentals. Putting the home and foreign factors together, we have a state vector X_t in a two-country open economy,

$$X_t = \left[\tilde{g}_t^{(h)},\ \tilde{\pi}_t^{(h)},\ \tilde{r}_t^{(h)},\ \tilde{g}_t^{(f)},\ \tilde{\pi}_t^{(f)},\ \tilde{r}_t^{(f)}\right]', \tag{2}$$

where the home factors are denoted with superscripts (h), and the foreign factors with superscripts (f). Tildes are used upon factors to distinguish them from the market observed macroeconomic variables, which are assumed to be collected with measurement errors. We assume that the state X_t determines the dynamics of two-country open economy and follows a Gaussian vector autoregression process,

$$X_t = \mu + \Phi X_{t-1} + \Sigma \varepsilon_t, \tag{3}$$

where μ is a constant 6×1 vector, Φ a constant 6×6 matrix, ε_t an i.i.d Gaussian white noise $N(0, I_6)$, and Σ a low-triangular matrix such that $\Sigma'\Sigma$ captures the variance-covariance of ε_t.

In this two-country world, assume that no-arbitrage holds. Then, in each country, there exists at least one almost surely positive process M_t with $M_0 = 1$ such that the discounted gains process associated with any admissible trading strategy is a martingale (Harrison and Kreps, 1979). M_t is called the stochastic discount factor (SDF). In a Lucas-type exchange economy (Lucas 1982), the stochastic discount factor is also often interpreted as the representative agent's intertemporal marginal rate of substitution. We denote the home SDF as $M_t^{(h)}$ and the foreign one as $M_t^{(f)}$. In what follows, whenever a relation holds for both countries, we suppress the superscript (h) or (f) unless otherwise specified.

For absence of a generally accepted equilibrium model for asset pricing, many studies use flexible factor models under the no-arbitrage condition (Cochrane, 2004). In this paper, we also use a factor representation for the SDF's, based on which the exchange rate and the term structure of interest rates are modeled. For each of the home and foreign stochastic discount factors ($M_t^{(h)}$ and $M_t^{(f)}$), assume that it has an exponential form

$$\begin{aligned} M_{t+1} &= \exp(m_{t+1}) \\ &= \exp\Big(- \tilde{r}_t - \frac{1}{2}\lambda_t' \lambda_t - \lambda_t' \varepsilon_{t+1} \Big), \end{aligned} \tag{4}$$

where $\tilde{r}_t$ is the short-term interest rate of that country, λ_t is the time-varying market prices of risks assigned by investors in that country, and ε_t is the shock to the state X_t, which is the only common term for both the home and foreign SDF's. Of course, we have $\lambda_t = \Big(\lambda_t^{\tilde{g}^{(h)}}, \lambda_t^{\tilde{\pi}^{(h)}}, \lambda_t^{\tilde{r}^{(h)}}, \lambda_t^{\tilde{g}^{(f)}}, \lambda_t^{\tilde{\pi}^{(f)}}, \lambda_t^{\tilde{r}^{(f)}}\Big)'$, the market price of risk for each factor in the state vector X_t, respectively.

Denote the market prices of risks assigned in the home country as $\lambda_t^{(h)}$ and those assigned in the foreign country as $\lambda_t^{(f)}$. We use the state X_t to summarize uncertainties in this two-economy world and assume that market prices of risks assigned in each country are affine functions of X_t (Dai and Singleton, 2002; Duffee, 2002)

$$\lambda_t = \lambda_0 + \lambda_1 X_t, \tag{5}$$

where λ_0 is a constant 6×1 vector, and λ_1 a constant 6×6 matrix. The specification (5) implies that investors of each country may assign different market prices for these risks contained in the state X_t if λ_0 and λ_1 are different across these two countries and that if $\lambda_t^{(h)}$ and $\lambda_t^{(f)}$ comove tightly, the two SDF's could be highly correlated.

B. Exchange Rate Dynamics and Forward Premium Anomaly

Define the nominal spot exchange rate $\mathcal{S}_t$ at time t as the domestic currency price of one unit of the foreign currency. No-arbitrage dictate that the ratio of the stochastic discount factors between the home and foreign economies determines the dynamics of their exchange rate (Backus et al., 2001; Bekaert, 1996; Brandt and Santa-Clara, 2002; Brandt, Cochrane and Santa-Clara, 2006). We thus have

$$\frac{\mathcal{S}_{t+1}}{\mathcal{S}_t} = \frac{M_{t+1}^{(f)}}{M_{t+1}^{(h)}}. \tag{6}$$

The above relation formally defines the link between the stochastic discount factors of two economies and exchange rate movements between them. In complete markets, the stochastic discount factors in both economies are unique, and they uniquely determine the dynamics of their exchange rate. When markets become incomplete, there may exist many different stochastic discount factors that can guarantee the absence of arbitrage. In this case, introduction of extra instruments, i.e. zero-coupon bonds, can help identify market prices of risks.

Taking natural logarithms for both sides of equation (6) and using specifications of the SDF's (4), we obtain the following exchange rate dynamic equation

$$\begin{aligned}\Delta s_{t+1} &\equiv s_{t+1} - s_t = m_{t+1}^{(f)} - m_{t+1}^{(h)} \\ &= \left(\tilde{r}_t^{(h)} - \tilde{r}_t^{(f)}\right) + \frac{1}{2}\left(\lambda_t^{(h)\prime}\lambda_t^{(h)} - \lambda_t^{(f)\prime}\lambda_t^{(f)}\right) + \left(\lambda_t^{(h)} - \lambda_t^{(f)}\right)'\varepsilon_{t+1}, \end{aligned} \tag{7}$$

which shows that macroeconomic fundamentals X_t are imparted to the exchange rate dynamic via market prices of risk in a nonlinear form and that shocks to output gaps, inflations and interest rates also drive variation of the exchange rate changes. This is in contrast to the traditional models that often assume linear relation between the exchange rate and macroeconomic fundamentals or/and that only use latent factors and do not have this economically meaningful interpretations.

The time-varying conditional mean, $\mu_t^s \equiv (\tilde{r}_t^{(h)} - \tilde{r}_t^{(f)}) + \frac{1}{2}(\lambda_t^{(h)\ \prime} \lambda_t^{(h)} - \lambda_t^{(f)\ \prime} \lambda_t^{(f)})$, captures predictable variation of returns in foreign exchange markets. Equation (7) shows that market prices of risks not only are important in determining the conditional mean of exchange rate changes, but also directly affect the conditional volatility of exchange rate changes through $\sigma_t^s \equiv \lambda_t^{(h)} - \lambda_t^{(f)}$. Exposure of the exchange rate to macroeconomic innovations is amplified by the difference of the time-varying market prices of risks between two economies. Therefore, the exchange rate changes are heteroskedastic in our model.

Equation (7) clearly shows that uncovered interest parity does not hold in our model. Uncovered interest parity states that the currency with higher interest rate is expected to depreciate against the one with lower interest rate and thus the expected change of exchange rate is equal to the interest rate differential of two countries. However, in our model, the expected change of exchange rate is composed of two parts, the interest rate differential and a term called the foreign exchange risk premium rp_t,

$$rp_t \equiv \frac{1}{2}(\lambda_t^{(h)\prime} \lambda_t^{(h)} - \lambda_t^{(f)\prime} \lambda_t^{(f)}). \tag{8}$$

The importance of the time-varying foreign risk premium is also argued by Fama (1984) who points out that the departure from uncovered interest parity should be attributed to a time-varying risk premium.

C. Term Structure of Interest Rates

Having specified the stochastic discount factors for the home and foreign countries, we can model the short rates and price zero-coupon bonds. Introduction of bonds in our modeling framework is important for identifying market prices of risks. Because short rates of the home and foreign countries have been included in the state X_t as factors, the affine short rate equations can be easily specified as

$$\tilde{r}_t = \delta_0 + \delta_1' X_t, \tag{9}$$

where $\delta_0 = 0$, and $\delta_1^{(h)} = (0, 0, 1, 0, 0, 0)'$ for the home country and $\delta_1^{(f)} = (0, 0, 0, 0, 0, 1)'$ for the foreign country.

No-arbitrage guarantees that a zero-coupon bond with maturity n-year in each country can be priced at time t by using the following Euler equation

$$\tilde{P}_t^{(n)} = E_t[M_{t+1} \tilde{P}_{t+1}^{(n-1)}] \tag{10}$$

with the initial condition $\tilde{P}_t^{(0)} = 1$. Again, tilde indicates the true value. Under specifications of the state (3), the short rate (9), and the SDF (4), we can show that the bond price is an exponential linear function of the state X_t

$$\tilde{P}_t^{(n)} = \exp(A_n + B_n' X_t), \tag{11}$$

where A_n and B_n solve the following difference equations

$$A_{n+1} = A_n + B_n'(\mu - \Sigma\lambda_0) + \frac{1}{2} B_n' \Sigma\Sigma' B_n - \delta_0, \tag{12}$$

$$B_{n+1} = (\Phi - \Sigma\lambda_1)' B_n - \delta_1, \tag{13}$$

with $A_1 = -\delta_0$ and $B_1 = -\delta_1$ being the initial conditions. Accordingly, the yield is also an affine function of the state

$$\tilde{y}_t^{(n)} \equiv -\frac{\log P_t^{(n)}}{n} = a_n + b_n' X_t, \tag{14}$$

where $a_n = -A_n/n$ and $b_n = -B_n/n$.

From the difference equations (12) and (13), we can see that the constant market price of risk parameter λ_0 only affects the constant yield coefficient a_n, whereas the parameter λ_1 affects the factor loading b_n. This implies that the parameter λ_0 affects average term spreads and average expected bond returns, whereas the parameter λ_1 governs time variation in term spreads and expected bond returns.

IV. Econometric Methodology

Because we assume that the real macroeconomic factors are unobservable and that the econometrician observed macroeconomic variables are contaminated with measurement errors, we first cast the model into a state-space representation and then use a Bayesian filtering approach to estimate the model.

At each period t, we can observe the exchange rate change Δs_t, the yields of zero-coupon bonds in the home and foreign countries ($y_t^{(h)}$ and $y_t^{(f)}$), and the output gaps and inflations of the home and foreign countries ($V_t^{(h)} = (g_t^{(h)}, \pi_t^{(h)})'$ and $V_t^{(f)} = (g_t^{(f)}, \pi_t^{(f)})'$). We assume that each of these variables is collected with the normal i.i.d measurement errors. Thus, we have the following measurement equations

$$
\begin{aligned}
\Delta s_t &= (\tilde{r}_{t-1}^{(h)} - \tilde{r}_{t-1}^{(f)}) + \frac{1}{2}(\lambda_{t-1}^{(h)'}\lambda_{t-1}^{(h)} - \lambda_{t-1}^{(f)'}\lambda_{t-1}^{(f)}) \\
&\quad + (\lambda_{t-1}^{(h)'} - \lambda_{t-1}^{(f)'})\Sigma^{-1}(X_t - \mu - \Phi X_{t-1}) + \eta_t^{\Delta s} \\
y_t^{(h)} &= a^{(h)} + b^{(h)'}X_t + \eta_t^{y^{(h)}}, \\
y_t^{(f)} &= a^{(f)} + b^{(f)'}X_t + \eta_t^{y^{(f)}}, \\
V_t^{(h)} &= (I_2 \ \ 0_{2\times 4})X_t + \eta_t^{V^{(h)}}, \\
V_t^{(f)} &= (0_{2\times 3} \ \ I_2 \ \ 0_{2\times 1})X_t + \eta_t^{V^{(f)}},
\end{aligned}
\tag{15}
$$

where we use $\varepsilon_t = \Sigma^{-1}(X_t - \mu - \Phi X_{t-1})$ in the exchange rate dynamic equation, $y_t^{(h)}$ is a 7×1 vector containing yields of all maturity considered in the home country, $y_t^{(f)}$ is a 5×1 vector containing yields of all maturity considered in the foreign country, and η_t's capture measurement errors with distinct variances for different variables/series and are assumed to be mutually independent.

We have the latent factor X_t, which follows a first-order VAR with its dynamic (3). From the measurement equations, we note that observations depend on the current and lagged macroeconomic factors X_t and X_{t-1}, both of which should be taken as states and can be written in the following compact form

$$\begin{pmatrix} X_t \\ X_{t-1} \end{pmatrix} = \begin{pmatrix} \mu \\ 0_{6\times 1} \end{pmatrix} + \begin{pmatrix} \Phi & 0_{6\times 6} \\ I_6 & 0_{6\times 6} \end{pmatrix} \begin{pmatrix} X_{t-1} \\ X_{t-2} \end{pmatrix} + \begin{pmatrix} \Sigma \\ 0_{6\times 6} \end{pmatrix} \varepsilon_t. \tag{16}$$

Given the state-space model representation (15) and (16) with Gaussian noises, we can implement model estimation using Bayesian filtering approaches. We have noted that the exchange rate dynamic equation is a highly non-linear function of states, which makes the standard Kalman filter inapplicable. Instead, we can use the nonlinear Kalman filters. The usually used nonlinear Kalman filter is the extended Kalman filter, which linearizes the nonlinear system around the current state estimate using the first-order Taylor approximation. However, for the highly nonlinear system, the extended Kalman filter is computationally demanding and performs very poorly. An alternative is the unscented Kalman filter (UKF), recently developed in the field of engineering (Julier and Ulman 1997, 2004). The idea behind this approach is that in order to estimate the state information after a nonlinear transformation, it is favorable to approximate the probability distribution directly instead of linearizing the nonlinear functions. The unscented Kalman filter overcomes to a large extent pitfalls inherent to the extended Kalman filter and improves estimation accuracy and robustness without increasing computational cost.

To implement the unscented Kalman filter, we firstly concatenate the state variables $x_{t-1} = [X_{t-1}, X_{t-2}]'$, the observation noise η_{t-1}, and the state noise ε_{t-1} at time $t-1$

$$x^e_{t-1} = \begin{bmatrix} x'_{t-1} & \eta'_{t-1} & \varepsilon'_{t-1} \end{bmatrix}', \tag{17}$$

whose dimension is $L = L_x + L_\eta + L_\varepsilon$ and whose mean and covariance are

$$\hat{x}^e_{t-1} = \begin{bmatrix} E[x_{t-1}] & 0 & 0 \end{bmatrix}', \qquad P^e_{t-1} = \begin{bmatrix} P^x_{t-1} & 0 & 0 \\ 0 & \Sigma^2_\eta & 0 \\ 0 & 0 & I_6 \end{bmatrix}.$$

We then form a set of $2L+1$ sigma points

$$\chi^e_{t-1} = \begin{bmatrix} \hat{x}^e_{t-1} & \hat{x}^e_{t-1} + \sqrt{(L+\lambda)P^e_{t-1}} & \hat{x}^e_{t-1} - \sqrt{(L+\lambda)P^e_{t-1}} \end{bmatrix} \tag{18}$$

and the corresponding weights

$$w_0^{(m)} = \frac{\lambda}{L+\lambda}, \quad w_0^{(c)} = \frac{\lambda}{L+\lambda} + (1 - \alpha^2 + \beta), \tag{19}$$

$$w_i^{(m)} = w_i^{(c)} = \frac{1}{2(L+\lambda)}, \quad i = 1, 2, \ldots, 2L, \tag{20}$$

where superscripts (m) and (c) indicate that weights are for construction of the posterior mean and covariance, respectively, $\lambda = \alpha^2(L + \bar{\kappa}) - L$ is a scaling parameter, the constant α determines the spread of sigma points around $\bar{x}$ and is usually set to be a small positive value, $\bar{\kappa}$ is a second scaling parameter with value set to 0 or $3 - L$, and β is a covariance correction parameter and is used to incorporate prior knowledge of the distribution of x.

With these sigma points, we implement the UKF as follows: for the time update

$$\chi^x_{t|t-1} = F(\chi^x_{t-1}, \chi^\varepsilon_{t-1}), \qquad \hat{x}^-_t = \sum_{i=0}^{2L} w_i^{(m)} \chi^x_{i,t|t-1},$$

$$P^-_{x_t} = \sum_{i=0}^{2L} w_i^{(c)} (\chi^x_{i,t|t-1} - \hat{x}^-_t)(\chi^x_{i,t|t-1} - \hat{x}^-_t)',$$

and for the measurement update

$$
\begin{aligned}
\mathcal{Y}_{t|t-1} &= H(\chi^x_{t|t-1}, \chi^\eta_{t|t-1}), \quad \hat{Y}_t^- = \sum_{i=0}^{2L} w_i^{(m)} \mathcal{Y}_{i,t|t-1}, \\
P_{Y_t}^- &= \sum_{i=0}^{2L} w_i^{(c)} (\mathcal{Y}_{i,t|t-1} - \hat{Y}_t^-)(\mathcal{Y}_{i,t|t-1} - \hat{Y}_t^-)', \\
P_{x_t Y_t} &= \sum_{i=0}^{2L} w_i^{(c)} (\chi^x_{i,t|t-1} - \hat{x}_t^-)(\mathcal{Y}_{i,t|t-1} - \hat{Y}_t^-)', \\
\hat{x}_t &= \hat{x}_t^- + P_{x_t Y_t}(P_{Y_t}^-)^{-1}(Y_t - \hat{Y}_t^-), \\
P_{x_t} &= P_{x_t}^- - (P_{x_t Y_t}(P_{Y_t}^-)^{-1}) P_{Y_t}^- (P_{x_t Y_t}(P_{Y_t}^-)^{-1})',
\end{aligned}
$$

where Y_t is the observation vector containing all the observed variables, $\hat{Y}_t^-$ its predicted values, $P_{Y_t}^-$ its conditional variance-covariance matrix, $\hat{x}_t$ the filtered state vector, and P_{x_t} its variance-covariance matrix.

Assuming that the predictive errors are normally distributed, we can construct the log likelihood function at time t as follows

$$
\mathcal{L}_t(\Theta) = -\frac{1}{2} \ln |P_{Y_t}^-| - \frac{1}{2}(Y_t - \hat{Y}_t^-)'(P_{Y_t}^-)^{-1}(Y_t - \hat{Y}_t^-), \tag{21}
$$

where Θ is a vector of model parameters. Parameter estimates can be obtained by maximizing the joint log likelihood

$$
\hat{\Theta} = \arg \max_{\Theta \in \Xi} \sum_{t=1}^{T} \mathcal{L}_t(\Theta), \tag{22}
$$

where Ξ is a compact parameter space, and T is the length of the observations. Because the log likelihood function is misspecified for the non-Gaussian model, a robust estimate of variance-covariance of parameter estimates can be obtained using the approach of White (1982)

$$
\hat{\Sigma}_\Theta = \frac{1}{T} \left[A B^{-1} A \right]^{-1}, \tag{23}
$$

where

$$A = -\frac{1}{T}\sum_{t=1}^{T}\frac{\partial^2 \mathcal{L}_t(\hat{\Theta})}{\partial\Theta\partial\Theta'}, \qquad B = \frac{1}{T}\sum_{t=1}^{T}\frac{\partial \mathcal{L}_t(\hat{\Theta})}{\partial\Theta}\frac{\partial \mathcal{L}_t(\hat{\Theta})}{\partial\Theta'}. \tag{24}$$

With these parameter estimates $\hat{\Theta}$, the latent macroeconomic factors $\hat{X}_t$ can be extracted using the unscented Kalman filter.

The number of parameters in our model is large. Maximization of the likelihood (21) may involve a large number of likelihood evaluations. Therefore, we adopt a sophisticated quasi-Newton approach with the inverse Hessian of the likelihood function updated by the BFGS algorithm. The initial values are carefully selected by the following way. We first run the Nelder-Mead optimization algorithm for 100 feasible sets of starting values and stop them after 100 iterations. Then the best 10 parameter estimate sets (in terms of the likelihood) are selected among these 100 runnings as the initial values for the quasi-Newton algorithm. The parameter estimates are those resulting in the largest likelihood among these 10 runnings of the quasi-Newton method.

V. Empirical Results and Discussions

A. State Dynamics and Macroeconomic Factors

We firstly take look at the parameters governing the state dynamics. Table 1 presents their estimates and corresponding t-ratios (in brackets). The diagonal elements of the matrix Φ determine the persistence of the macroeconomic factors, and the off-diagonal ones of Φ govern their dynamic interactions. We note that the diagonal estimates are all larger than 0.77 and highly statistically significant, indicating that macroeconomic factors are very persistent. In particular, for both the US and the EA, the inflation rate is more persistent than the output gap. This can also be observed from estimates of Σ, where the output gap estimates are larger than the inflation rate estimates and in Figure 1, where the inflation rate is much smoother than the output gap. The US output gap is more persistent than the EA one, whereas its inflation rate is slightly less persistent than that of the EA. The short-term interest rates in both economies are also highly persistent.

— Table 1 around here —

From the off-diagonal estimates, we observe a weak link between output gap and inflation in both countries. For each country, we find that the short rate responds positively to shocks to output gap and inflation, suggesting that the short-term interest rate increases with both inflation and real output growth. We split the matrix Φ into four 3×3 sub-matrices. The off-diagonal sub-matrices control the interdependence between two countries. We can see that both economies are mutually dependent since a number of elements in the off-diagonal sub-matrices are statistically significant. In particular, the US output gap has a positively significant impact on the EA output gap, and vice versa. This implies a co-movement of the business cycles in these two countries. This finding is in contrast to previous studies that assume that the US economy has a leading impact on the foreign economy, but not vice versa.

The dark solid lines in Figure 3 plot the macroeconomic factors extracted from the data using the unscented Kalman filter. The left panels are for the US factors, and the right panels for the EA factors. To compare with the observed data, the estimated factors evolve similarly to the observed ones, but they have smaller variations, indicating that the real data are noisy and really contaminated by measurement errors.

— Figure 3 around here —

B. Market Prices of Economic Risks

Table 2 reports the parameter estimates of market prices of macroeconomic risks. Most of estimates in $\lambda_0^{(h)}$ and $\lambda_0^{(f)}$ and in $\lambda_1^{(h)}$ and $\lambda_1^{(f)}$ not only have the same signs, but also are very close each other. This implies that the SDF's of two countries should be highly correlated. Indeed, the correlation between the model-implied SDF's is as high as 99%. Brandt et al. (2006) show that volatility of the exchange rate and volatility of the SDF's based on asset markets imply that SDF's must be highly correlated across countries. Dong (2006) also finds a very high correlation between the US SDF and the German SDF. Figure 3 also plots the market price of risk of each macroeconomic factor assigned by the home and foreign investors (dashed line and bold line, respectively). We can see that market prices between these two markets are almost indistinguishable and highly correlated. If we think of risks as goods, the prices of these goods should be highly equalized in these two markets.

— Table 2 around here —

Because the estimate of μ in the state dynamics is very small, the estimate of λ_0 approximately captures the average market prices on the macroeconomic factors. The matrix λ_1 measures how the market price varies with respect to the risk level. All estimates in $\lambda_0^{(h)}$ and $\lambda_0^{(f)}$ are negative, but some of them are not statistically significant. We note that all the diagonal estimates of $\lambda_1^{(h)}$ and $\lambda_1^{(f)}$ are highly statistically significant and that a number of off-diagonal estimates there are not statistically different from zero, indicating that in each country, the market price on each macroeconomic factor varies mainly with its own risk level.

For each country, the diagonal output gap estimates in λ_1 are positive, whereas those for inflations and interest rates are negative, indicating that market prices of output gaps become less negative when the real outputs are high, but market prices of inflation factors and short-rate factors become more negative when the inflation rates and the short-rates are high. From Figure 3, we can clearly see these tendencies. The figure also shows that all the market prices of risks in these two economies are negative. Consistent to parameter estimates in λ_1, the market price of each risk is highly correlated with the corresponding macroeconomic factor. Investors demand higher compensation during the recession and during the time when inflation is high. When the short interest rate goes higher, its market price becomes even negatively smaller. Previous studies attribute the upward-sloping mean interest rate term structure to the negative market price of the interest rate risk (Backus et al., 1998).

C. Model Performance Analysis

Table 3 presents estimates of standard deviations of the observation measurement errors. For both the US yields and the EA yields, their standard deviations of the measurement errors are very small, ranging from 0.6 basis point to 5.8 basis point. The standard deviations of the measurement errors for the macroeconomic data are also small and are from 0.7 basis point to 3.5 basis point. These results indicate that the model can effectively capture the term structure of interest rates and the dynamics of macroeconomic fundamentals.

— Table 3 around here —

The standard deviation of the exchange rate measurement errors is large (238 basis point) with comparison to those of yields and macro variables as shown in Table 3. However, the correlation between the data and the model-implied exchange rate changes is as high as 76%! If we run a regression of the data (Δs) on the model-implied values ($\Delta \hat{s}$) with a constant, we have the following result

$$\Delta s_t = \underset{(0.0769)}{-0.0001} + \underset{(12.4744)}{1.2289} \Delta \hat{s}_t + e_t, \tag{25}$$

where in brackets presents the absolute values of t-ratios. The constant term is very small and not statistically significant, and the coefficient of $\Delta \hat{s}$ is about 1.2 and highly statistically significant. The resulted R^2 of this regression is 57%! Therefore, a reasonable proportion of exchange rate movements can be explained by our model. In contrast, empirical studies based on monetary models and/or new open economy macroeconomics models can only explain at most 10% variation of exchange rate changes. For example, Lubik and Schorfheide (2005) investigate the USD/Euro exchange rate and find that their estimated model explains 10% variation of the one-quarter exchange rate changes in data. A recent study by Dong (2006), which follows a simular approach to this paper, finds that 38% variation of the data can be explained.

The lower Panel of Figure 4 plots the model-implied exchange rate changes $\hat{\Delta s}$ and the observed data. We can see that the data is more volatile than the model-implied exchange rate changes, but the estimated values by our model can capture exchange rate movements reasonably well. In contrast, the linear VAR model of Section II is much less capable to capture the volatile exchange rate changes as shown in the upper panel of Figure 4.

— Figure 4 around here —

As comparisons, I also investigate the other two nested models. One assumes that the market price of risk parameter λ_1 is diagonal, and the other assumes that it is zero. Table 4 presents explained variances by the model-implied exchange rate changes and correlations between the observed and model-implied values. We note that our general model has the largest explained variance and correlation, whereas the constant case has the smallest explained variance and correlation. The likelihood ratio tests reject these two nested models.

— Table 4 around here —

D. Foreign Exchange Risk Premium and Forward Premium Anomaly

One of the most notable puzzles in foreign exchange markets is the forward premium anomaly, which finds the tendency for high interest rate currencies to appreciate. Fama (1984) attributes this departure from uncovered interest parity (UIP) to a time-varying risk premium. Our model also suggests that the expected exchange rate change is equal to the sum of the interest rate differential and the risk premium. Table 2 shows that the diagonal estimates of $\lambda^{(h)}$ and $\lambda^{(f)}$ are huge, and this results in a substantial foreign exchange risk premium, which can also been seen in Figure 5. Here I study an augmented UIP, which takes into account not only the interest rate differential but also the foreign exchange risk premium,

$$\Delta s_{t+1} = \alpha_0 + \alpha_1(r_t^{(h)} - r_t^{(f)}) + \alpha_2 rp_t + e_{t+1}, \tag{26}$$

where rp_t is the foreign exchange risk premium defined in equation (8), and e_{t+1} is a noise term. Using our data and the estimated risk premium, we obtain an estimate of α_1 0.63 with t-ratio 0.17 and an estimate of α_2 0.81 with t-ratio 1.97. The resulted R^2 is 0.097. α_2 is statistically significant and its value is not far away from unity, and although α_1 is not significant, its value is positive. However, if we impose zero on α_2 and estimate the UIP regression, the estimated α_1 is negative and far away from unity (-1.88 with t-ratio 0.97) and R^2 is only 0.008. The above regressions indicate that the UIP puzzle can be (partially) solved by introducing the foreign exchange risk premium term.

Fama (1984) argues that the implied risk premium should be negatively correlated with and have larger variance than the interest rate differential. They are usually termed as Fama's conditions. Our model implied risk premium (rp_t) does negatively correlate with the interest rate differential ($r^{(h)} - r^{(f)}$) with a correlation about -46% and have a larger variance (0.996 vs. 0.02). The top panel of Figure 5 plots the foreign exchange risk premium and the interest rate differential. It clearly shows a negative correlation between them and a greater variance of the risk premium.

The middle panel of Figure 5 presents the output gap differential $(g^{(f)} - g^{(h)})$. We find that the risk premium is positively correlated to the output gap differential with a correlation about 31%. This positive correlation implies that when the foreign output gap is higher than the domestic one, people in the market anticipate the foreign currency to appreciate while the domestic currency to depreciate. When one country is in a better economic situation than the other, the market becomes more confident to that country's currency and thus people would like to hold it, leading to its currency to appreciate. The lower panel depicts the risk premium and the inflation rate differential $(\pi^{(f)} - \pi^{(h)})$. They also have a positive correlation (80%). If the current inflation of the foreign country is high, people may expect the central bank to increase its insterest rate in the future. This results in a decreased interest rate differential and an increased risk premium.

— Figure 5 around here —

E. Macroeconomic Shocks and the Exchange Rate Dynamics

Previous studies find that exchange rate movements are largely disconnected to macroeconomic fundamentals. In monetary models and/or new open economy macroeconomic models, the exchange rate is a linear function of contemporaneous macroeconomic variables. Since the residuals are usually serially correlated in these models, the estimation is always implemented using the first-order differences of relevant variables

$$\Delta s_t = \beta_0 + \beta_1^{(h)} \Delta r_t^{(h)} + \beta_1^{(f)} \Delta r_t^{(f)} + \beta_2^{(h)} \Delta g_t^{(h)} + \beta_2^{(f)} \Delta g_t^{(f)} + \beta_3^{(h)} \Delta \pi_t + \beta_3^{(f)} \Delta \pi_t^{(f)} + v_t, \quad (27)$$

where v_t is a noise term. In these models, coefficients are typically constrained by $\beta_i^{(h)} = -\beta_i^{(f)}$, for $i = 1, 2, 3$. When estimating this linear model on the data used in this paper, we find a R^2 of 4.7% for the unconstrained regression and a R^2 of 2.8% for the constrained regression. Even though macroeconomic factors in our model can account for 57% variation of exchange rate movements, the linear model (27) cannot capture this link between macroeconomic factors and exchange rates.

What exact roles do macroeconomic fundamentals play in our model? We rewrite the exchange rate dynamic equation (7) as follows

$$\begin{aligned}\Delta\hat{s}_{t+1} &= (\hat{r}_t^{(h)} - \hat{r}_t^{(f)}) + \frac{1}{2}(\hat{\lambda}_t^{(h)\prime}\hat{\lambda}_t^{(h)} - \hat{\lambda}_t^{(f)\prime}\hat{\lambda}_t^{(f)}) + (\hat{\lambda}_t^{(h)\prime} - \hat{\lambda}_t^{(f)\prime})\hat{\varepsilon}_{t+1} \\ &\equiv \Delta\hat{s}_{1,t+1} + \Delta\hat{s}_{2,t+1} + \Delta\hat{s}_{3,t+1}, \end{aligned} \tag{28}$$

where $\Delta\hat{s}_{1,t+1} = \hat{r}_t^{(h)} - \hat{r}_t^{(f)}$ is the estimated differential of short term interest rates between the US and the EA, $\Delta\hat{s}_{2,t+1} = \frac{1}{2}(\hat{\lambda}_t^{(h)\prime}\hat{\lambda}_t^{(h)} - \hat{\lambda}_t^{(f)\prime}\hat{\lambda}_t^{(f)})$ is the estimated foreign exchange rate risk premium, and $\Delta\hat{s}_{3,t+1} = (\hat{\lambda}_t^{(h)\prime} - \hat{\lambda}_t^{(f)\prime})\hat{\varepsilon}_{t+1}$ is the estimated unexpected exchange rate changes related to macroeconomic shocks.

Figure 6 presents these three components of the exchange rate changes. The first component ($\Delta\hat{s}_{1,t+1}$) is very smooth. The second one ($\Delta\hat{s}_{2,t+1}$) becomes volatile in comparison to the first one, but it still has much smaller variation than the model-implied exchange rate changes. This implies that the third component ($\Delta\hat{s}_{3,t+1}$) must be more volatile and should play more important role in explaining exchange rate movements. This is true from the figure that $\Delta\hat{s}_{3,t+1}$ is very volatile and mimics fluctuations of exchange rate changes. The regression of the data on the unexpected exchange rate changes ($\Delta\hat{s}_{3,t+1}$) and a constant results in a R^2 of 37%, taking 76% of the total explained variance.

— Figure 6 around here —

The unexpected exchange rate change $\Delta\hat{s}_{3,t+1}$ is a product of the differential of market prices of risks ($\hat{\lambda}_t^{(h)\prime} - \hat{\lambda}_t^{(f)\prime}$) and the macro innovations ($\hat{\varepsilon}_{t+1}$), both of which are macro-dependent. When we regress the data on the model-implied macroeconomic innovations $\hat{\varepsilon}_{t+1}$ with a constant, the R^2 is 15%. This is close to that obtained in Section II. However, in VAR approach, macroeconomic shocks on exchange rate changes are time-homogeneous. In our model, the role of the macro innovations is further amplified by the time-varying differential of market prices of risks, and hence the exchange rate dynamic is heteroskedastic. The importance of heteroskedasticity can also be see from the third row in Table 4, where by setting the market prices of risks constant, only 25% variation of the data can be explained. Macroeconomic innovations are always regarded as "news" to macroeconomic fundamentals.

Their importance has also been investigated by Engel, Mark and West (2007) and Andersen et al. (2003).

F. Is Yield Curve Information Enough?

We have noticed that in our model, macroeconomic fundamentals enter into the exchange rate dynamics in a nonlinear form and that information contained in the term structure of interest rates play very important role in identifying market prices of risks. This modeling approach results in a big explanation improvement with comparison to the previous linear approach. However, people may think that this success of modeling exchange rates may be largely from nonlinear transformation of information contained in the yield data but not from introducing macroeconomic fundamentals. In this subsection, we investigate this issue by keeping the same modeling framework of jointly investigating exchange rate dynamic and two-country interest rate term structures but shutting down macroeconomic information.

The term structure of interest rates can be empirically captured by the "level", the "slope" and the "curvature" (Nelson and Siegel, 1987). We construct the empirical yield curve factors as follows. The "slope" (s_t) is defined as the spread between yields with longest and shortest maturity ($y_t^{(60)} - y_t^{(1)}$). The "curvature" (c_t) is defined as two times yields with medium maturity minus the sum of yields with longest and shortest maturity, $2y_t^{(24)} - (y_t^{(60)} + y_t^{(1)})$ for the US and $2y_t^{(12)} - (y_t^{(60)} + y_t^{(1)})$ for the EA. And the empirical "level" factor (r_t) is simply proxied by yields with shortest maturity.

Under this construction, we have a new state vector that includes the home and foreign yield curve factors only $X_t = \left(\tilde{s}_t^{(h)},\ \tilde{c}_t^{(h)},\ \tilde{r}_t^{(h)}, \tilde{s}_t^{(f)},\ \tilde{c}_t^{(f)},\ \tilde{r}_t^{(f)}\right)$. We estimate this model ("L-S-C" model) using the same econometric method discussed before. The last row of Table 4 reports the explained variance and the correlation between the data and the model-implied exchange rate changes under the "L-S-C" model. This model can capture the yield curve dynamics pretty well, consistent with previous studies. For exchange rate changes, we find that it still performs better than the linear model since it can explain 13% variation of the data and the model-implied exchange rate changes has 36% correlation with the observe values. However, its explained variance and correlation are much lower than those implied by our model and its nested models, indicating that yield curve information is not enought to explain the exchange rate dynamics and that macroeconomic fundamentals do really play important roles.

VI. Conclusion

This paper investigates relationship between the short-run nominal exchange rates changes and macroeconomic fundamentals by adopting a no-arbitrage macro-finance approach under a two-country framework, where macroeconomic information enters into the exchange rate dynamics through different channels in a non-linear form. Based on empirical analysis using an enriched dataset including exchange rates, yields of zero-coupon bonds, and macroeconomic variables of the US and the Euro area, the paper finds a close link between macroeconomic fundamentals and exchange rate dynamics. The model-implied exchange rate changes can explain about 57% variation of the observed data. This is in stark constrast to previous studies using monetary and new open economy macroeconomics models, which can explain only around 10% variation of exchange rates. Having been amplified by the time-varying market prices of risks, the innovations of macroeconomic fundamentals are the driving engine for generating large volatility of exchange rate changes.

The model in this paper has a fairly good fit to monthly exchange rate changes. However, there is still nearly 40% variation that cannot be explained. This is because there may be other missing factors such as current account (Hooper and Morton, 1978), market incompleteness (Brandt and Santa-Clara, 2002), government deficit, "news" from the stock market, default risk and so on. Therefore, it would be interesting to investigate these factors and to see whether they can help explain the exchange rate dynamics in the future.

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Table 1: **Parameter Estimates of the State Dynamics**

		Φ						$\Sigma(\times 10^3)$					
	$\mu(\times 10^3)$	$\tilde{g}^{(h)}$	$\tilde{\pi}^{(h)}$	$\tilde{r}^{(h)}$	$\tilde{g}^{(f)}$	$\tilde{\pi}^{(f)}$	$\tilde{r}^{(f)}$	$\tilde{g}^{(h)}$	$\tilde{\pi}^{(h)}$	$\tilde{r}^{(h)}$	$\tilde{g}^{(f)}$	$\tilde{\pi}^{(f)}$	$\tilde{r}^{(f)}$
$\tilde{g}^{(h)}$	-0.298	0.834	0.000	0.003	0.010	0.016	-0.002	0.637	0	0	0	0	0
	(0.94)	(3.49)	(0.65)	(0.50)	(2.19)	(0.83)	(0.53)	(3.77)	–	–	–	–	–
$\tilde{\pi}^{(h)}$	0.149	-0.001	0.918	0.023	-0.007	0.001	0.011	-0.018	0.142	0	0	0	0
	(2.50)	(1.24)	(4.48)	(1.81)	(1.16)	(1.75)	(0.38)	(1.43)	(1.94)	–	–	–	–
$\tilde{r}^{(h)}$	-0.083	0.048	0.051	0.941	-0.043	0.007	0.002	0.101	-0.241	0.344	0	0	0
	(0.35)	(2.05)	(1.98)	(2.56)	(1.02)	(0.61)	(0.95)	(1.21)	(0.44)	(1.96)	–	–	–
$\tilde{g}^{(f)}$	-0.331	0.025	0.014	-0.008	0.776	0.008	0.036	0.424	-0.166	0.014	0.796	0	0
	(2.14)	(2.03)	(1.07)	(1.41)	(2.28)	(1.92)	(0.40)	(1.99)	(0.48)	(1.95)	(2.03)	–	–
$\tilde{\pi}^{(f)}$	0.369	0.006	0.015	0.018	0.008	0.930	0.002	-0.008	-0.005	-0.041	-0.060	0.146	0
	(0.83)	(1.15)	(1.06)	(1.11)	(1.65)	(2.67)	(2.02)	(0.81)	(0.54)	(2.16)	(2.15)	(2.27)	–
$\tilde{r}^{(f)}$	-0.029	-0.008	0.011	0.022	0.050	0.095	0.896	-0.113	-0.080	0.252	0.042	-0.192	0.220
	(1.85)	(1.73)	(0.49)	(2.07)	(1.97)	(2.12)	(2.84)	(1.11)	(1.05)	(1.23)	(0.95)	(3.46)	(2.35)

Note: The table reports the parameter estimates for the state dynamics that follow a VAR(1) process. In parentheses, the absolute value of t-ratio of each estimate is reported. The sample period for estimation is from February 1999 to December 2008 and the data is in monthly frequency.

Table 2: **Parameter Estimates of Market Prices of Risks**

	US	$\boldsymbol{\lambda_1}^{(h)}$						*EA*	$\boldsymbol{\lambda}_{\mathbf{1}}^{(\mathbf{f})}$					
	$\boldsymbol{\lambda}_{\mathbf{0}}^{(\mathbf{h})}$	$\tilde{g}^{(h)}$	$\tilde{\pi}^{(h)}$	$\tilde{r}^{(h)}$	$\tilde{g}^{(f)}$	$\tilde{\pi}^{(f)}$	$\tilde{r}^{(f)}$	$\boldsymbol{\lambda}_{\mathbf{0}}^{(\mathbf{f})}$	$\tilde{g}^{(h)}$	$\tilde{\pi}^{(h)}$	$\tilde{r}^{(h)}$	$\tilde{g}^{(f)}$	$\tilde{\pi}^{(f)}$	$\tilde{r}^{(f)}$
$\tilde{g}^{(h)}$	-0.361	105.1	0.015	-0.001	0.031	-0.012	-0.006	-0.369	95.49	0.004	0.009	0.091	0.012	0.013
	(1.65)	(6.25)	(0.35)	(0.86)	(2.03)	(1.74)	(0.83)	(1.65)	(4.36)	(0.60)	(1.22)	(1.99)	(1.05)	(1.78)
$\tilde{\pi}^{(h)}$	-0.205	0.006	-30.66	0.013	-0.004	0.002	-0.005	-0.223	0.004	-27.10	0.153	0.076	0.005	-0.004
	(1.06)	(0.70)	(3.79)	(3.05)	(0.60)	(1.12)	(0.72)	(2.06)	(0.53)	(2.83)	(1.12)	(0.49)	(1.76)	(0.79)
$\tilde{r}^{(h)}$	-0.313	0.015	-0.047	-106.6	-0.009	-0.018	-0.008	-0.306	0.011	0.098	-107.5	0.050	0.026	0.011
	(2.37)	(1.07)	(1.48)	(3.36)	(1.20)	(1.89)	(2.42)	(2.03)	(2.12)	(0.44)	(3.82)	(0.70)	(1.80)	(0.75)
$\tilde{g}^{(f)}$	-0.211	-0.008	-0.011	0.078	78.25	0.013	0.009	-0.222	0.015	0.035	0.086	84.60	-0.017	0.045
	(0.69)	(0.41)	(0.48)	(1.65)	(4.04)	(1.62)	(0.45)	(0.74)	(2.31)	(0.65)	(1.00)	(3.74)	(0.50)	(1.80)
$\tilde{\pi}^{(f)}$	-0.360	0.022	0.018	-0.022	0.082	-46.49	0.007	-0.414	0.012	-0.101	0.022	-0.029	-6.511	-0.015
	(1.63)	(0.91)	(0.43)	(0.45)	(1.05)	(4.45)	(1.99)	(1.91)	(2.60)	(1.34)	(0.51)	(0.59)	(1.91)	(0.91)
$\tilde{r}^{(f)}$	-0.349	-0.006	-0.004	-0.055	0.011	0.002	-111.5	-0.310	0.012	-0.076	-0.027	-0.008	0.017	-124.4
	(1.74)	(0.42)	(0.99)	(2.04)	(1.87)	(1.99)	(2.87)	(2.04)	(1.53)	(0.80)	(2.04)	(1.11)	(0.60)	(4.48)

Note: The table presents the parameter estimates for affine equation of market prices of risk. In parentheses, the absolute value of t-ratio of each estimate is reported. The sample period for estimation is from February 1999 to December 2008 and the data is in monthly frequency.

Table 3: **Standard Deviations of Measurement Errors**

Yields	$n=1$	$n=3$	$n=12$	$n=24$	$n=36$	$n=48$	$n=60$
$\sigma_\eta^{(h)}$	5.76	4.30	2.58	0.58	3.13	2.37	4.63
	(4.83)	(3.10)	(3.33)	(1.95)	(3.44)	(3.08)	(2.03)
$\sigma_\eta^{(f)}$	2.47	3.27	1.11	–	2.26	–	5.27
	(1.91)	(3.07)	(2.02)	–	(2.16)	–	(2.06)
Macro & Exchange Rate	σ_η^{gh}	$\sigma_\eta^{\pi h}$	σ_η^{gf}	$\sigma_\eta^{\pi f}$	–	–	$\sigma_\eta^{\Delta s}$
	3.50	1.19	2.25	0.69	–	–	238
	(3.52)	(1.97)	(2.56)	(2.07)	–	–	(3.07)

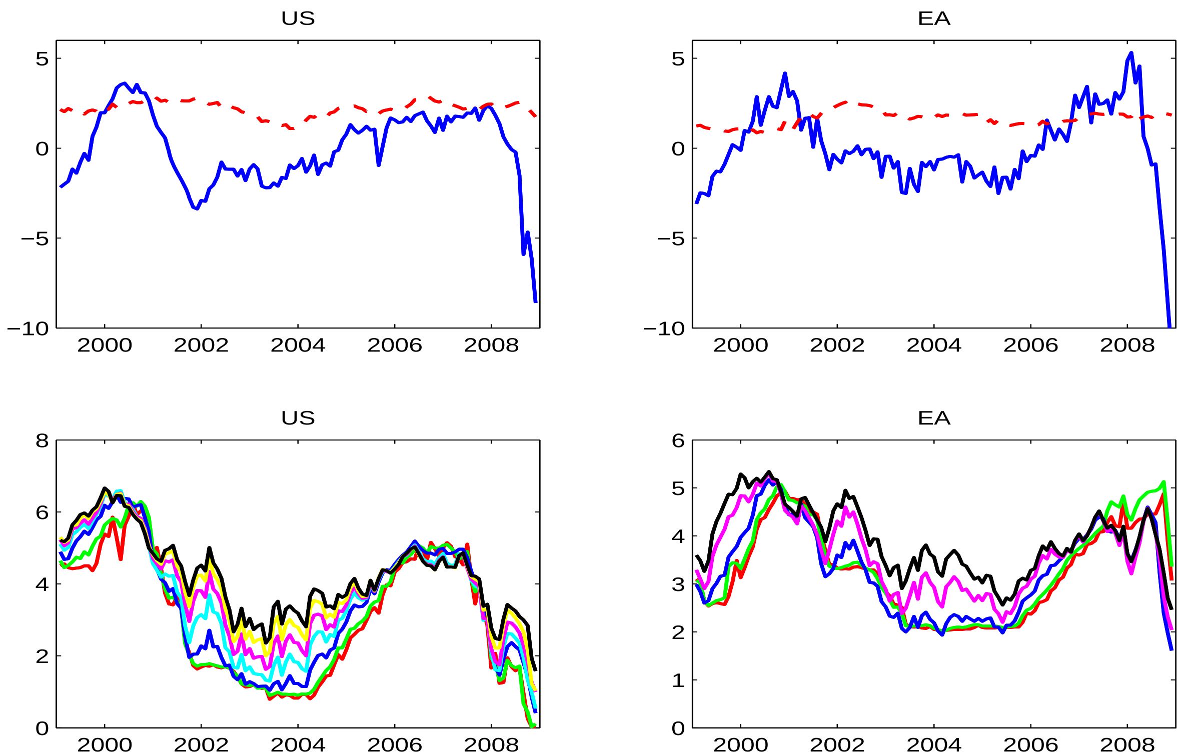
Note: The table reports the parameter estimates of standard deviations of measurement errors (in basis point). In parentheses, the absolute value of t-ratio of each estimate is reported. The sample period for estimation is from February 1999 to December 2008 and the data is in monthly frequency

Table 4: **Model Performance Comparison**

	Explained Variance	Correlation(Δs, $\Delta \hat{s}$)	LR Test
Full	57 %	76 %	–
Diagonal	34 %	58 %	92
Constant	25 %	50 %	173
L-S-C	13 %	36 %	–

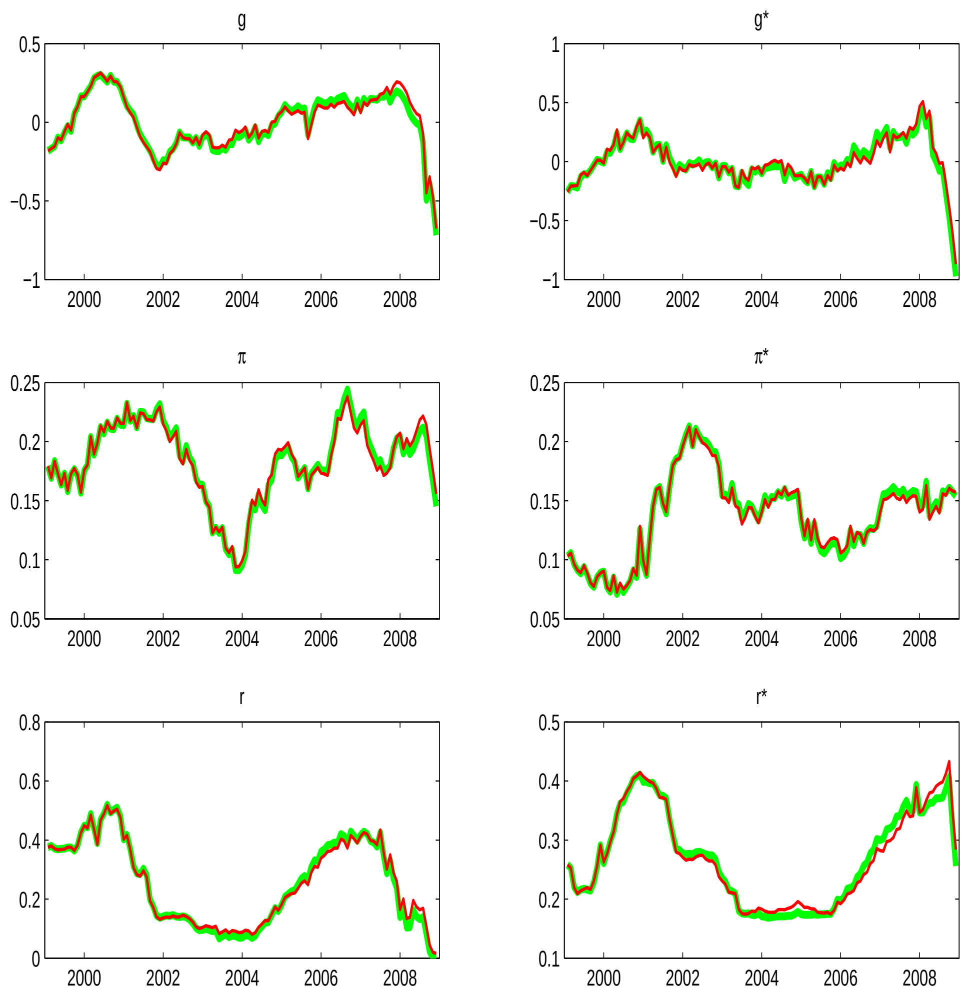
Note: The table reports the explained variance of the model-implied value and correlation between the model-implied values and the observed data for each of four models examined in the paper. The Full refers to our general model where λ_1 is a full matrix. The Diagonal refers to the model where λ_1 is a diagonal matrix. The Constant represents the model where λ_1 is a zero matrix. The L-S-C is the model where only the yield curve information is used without any macroeconomic variables. Explained variance is the adjusted R^2 of a regression of the observed exchange rate changes on the model-implied values with a constant.

Figure 1: Macroeconomic and Yield Data



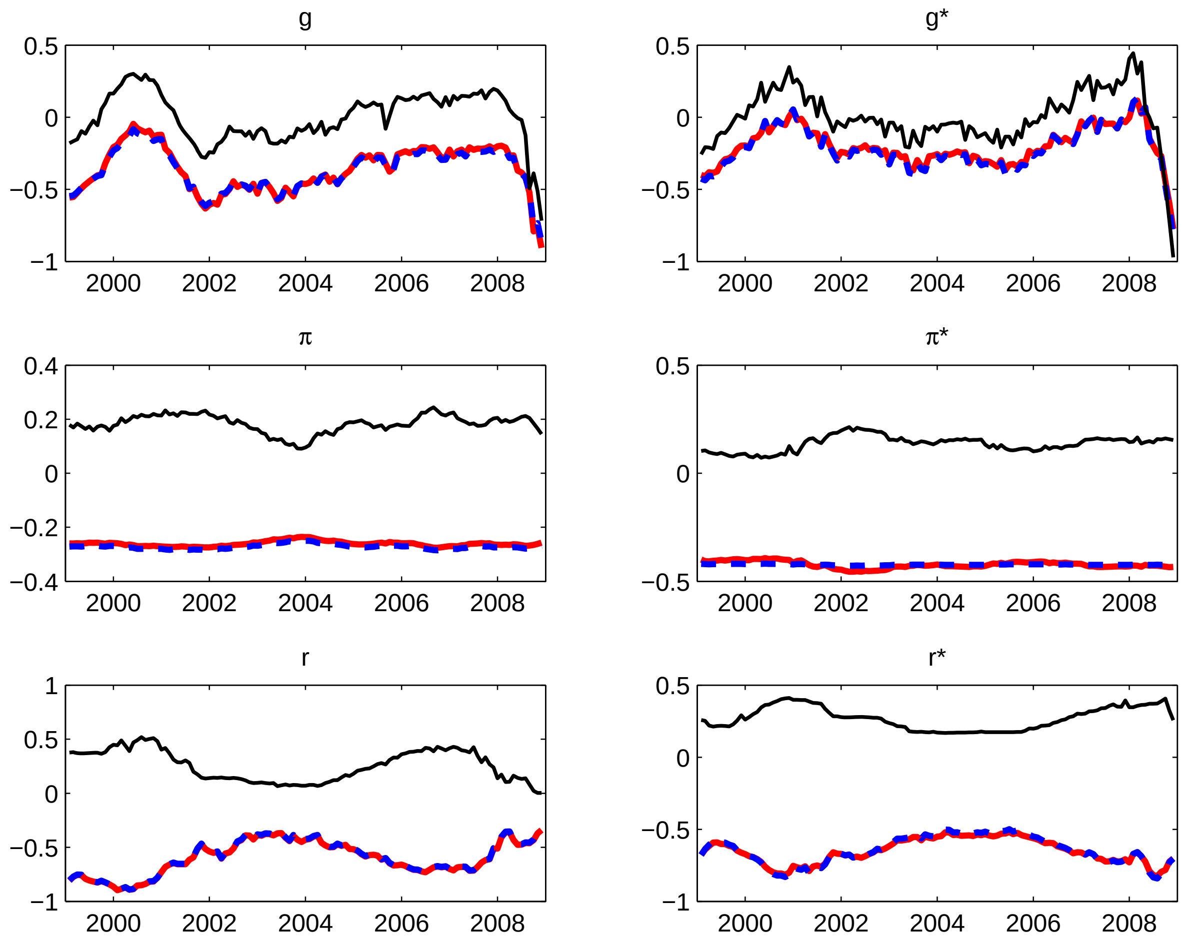
Note: The upper panels of the figure plot the annualized macroeconomic data used in the estimation in the upper panels. The output gaps are in solid lines and inflation rates in dish lines. The lower panels plot the annualized yields. The US yields have maturity 1-month, 3-month, 1-year, 2-year, 3-year, 4-year and 5-year, and the EA yields have maturity 1-month, 3-month, 1-year, 3-year, and 5-year. The left panels are for the US and the right ones for the EA. The unit on the vertical axis is in percentage.

Figure 2: The Role of Exchange Rate Innovations on Macro Variables



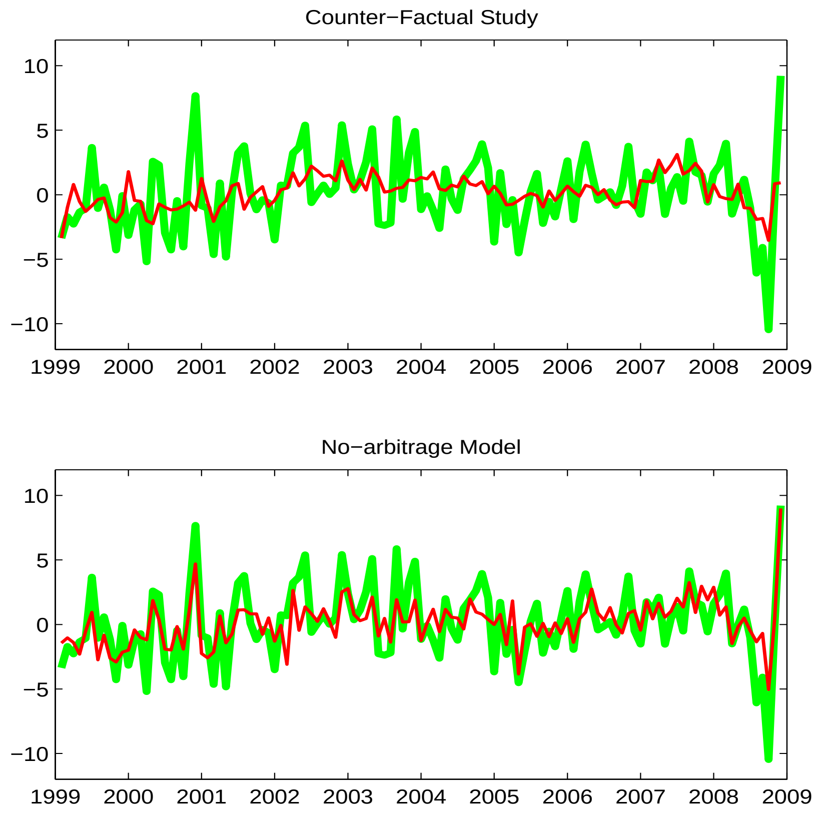
Note: The figure compares the actual and the counter-factual time series $[g_t^{(h)}\ \pi_t^{(h)}\ y_{1t}^{(h)}\ g_t^{(f)}\ \pi_t^{(f)}\ y_{1t}^{(f)}]'$. The counter-factual time series is simulated from the VAR(1) model using the estimated parameter in Section II by setting innovations of exchange rate changes to zero.

Figure 3: Macro Factors and Market Prices of Risks



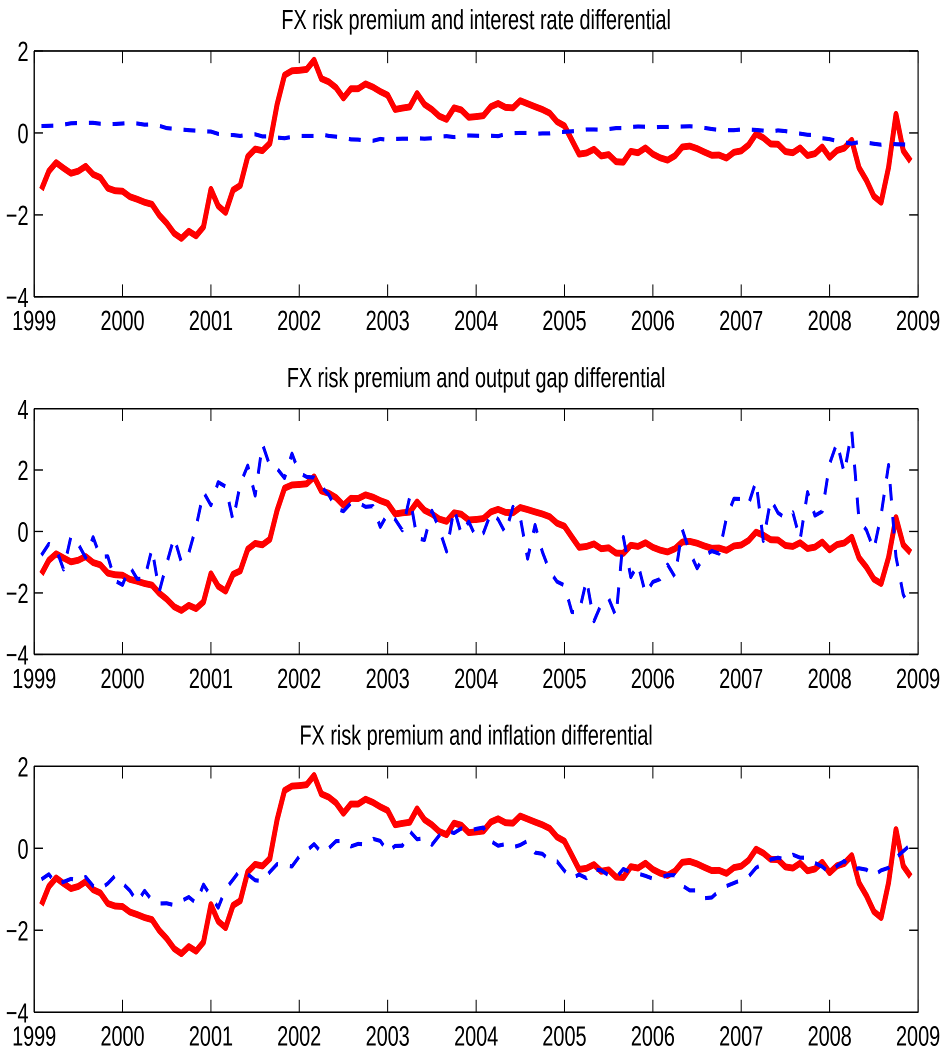
Note: Each panel of the figure plots the extracted macroeconomic factor (dark solid line) and the corresponding time-varying market price of risk assigned by the US market (dashed line) and the EA market (bold line).

Figure 4: Exchange Rate Dynamics



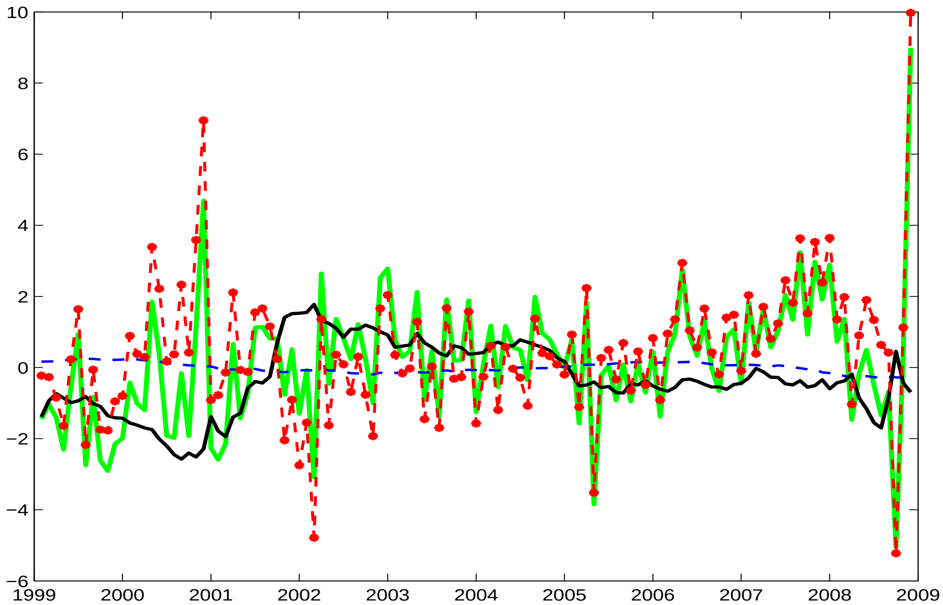
Note: The upper panel plots the simulated exchange rate changes (thin line) from the linear VAR model studied in Section B and the observe data (bold line). The lower Panel plots the model-implied exchange rate changes (thin line) from the no-arbitrage model and the observe data (bold line). The correlation between the model-implied exchange rate changes from the no-arbitrage model and the observed data is 76%, and the model-implied exchange rate changes can explain 57% variation of the data. The correlation between the simulated exchange rate changes and the observed data is 42%, and the model-implied values only explain 18% variation of the data. The unit on the vertical axis is in percentage.

Figure 5: The Foreign Exchange Risk Premium



Note: The top panel of the figure plots the foreign exchange risk risk premium and the interest rate differential $(r^{(h)} - r^{(f)})$; The middle panel plots the foreign exchange risk premium and the output gap differential $(g^{(f)} - g^{(h)})$. The bottom panel plots the foreign exchange risk premium and the inflation rate differential $(\pi^{(f)} - \pi^{(h)})$. The risk premium is in dark line and the others are in shallow line.

Figure 6: Components of Model-implied Exchange Rates



Note: The figure plots the model-implied exchange rate changes (bold solid line) and its components: $\Delta\hat{s}_{1,t+1} = (\hat{\hat{r}}_t^{(h)} - (\hat{\hat{r}})_t^{(f)})$, $\Delta\hat{s}_{2,t+1} = \frac{1}{2}(\hat{\lambda}_t^{(h)\prime}\hat{\lambda}_t^{(h)} - \hat{\lambda}_t^{(f)\prime}\hat{\lambda}_t^{(f)})$, and $\Delta\hat{s}_{3,t+1} = (\hat{\lambda}_t^{(h)\prime} - \hat{\lambda}_t^{(f)\prime})\hat{\varepsilon}_{t+1}$. The first component is in dashed line, the second in dark solid line and the third in dashed line with mark.

Chapter 2
On the Forecasting Performance of Macroeconomic Fundamentals on Exchange Rate Movements

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Abstract

This paper investigates the forecasting performance of macroeconomic fundamentals on exchange rate returns in the short run using a macro-finance approach. Exchange rate movements are endogenously determined by ratios between domestic and foreign stochastic discount factors, through which the macroeconomic fundamentals nonlinearly model exchange rate dynamics. Testing on three floating nominal exchange rates, i.e. DEM(EUR)/USD, GBP/USD and JPY/USD observed at monthly as well as quarterly time frequencies, this paper has the following findings. First, five out of the six model-implied time-varying foreign exchange risk premiums satisfy the Fama conditions (Fama, 1984). Second, comparing to the random walk model, this no-arbitrage macro-finance model reduces forecasting root mean square errors, especially for the data observed at quarterly time frequency.

Keywords: Exchange Rate Forecasting, Stochastic Discount Factor, Macroeconomic Fundamentals, Forward Premium Anomaly, Unscented Kalman Filter

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I. Introduction

The forecasting of exchange rates movements is important for decision makers in international trades, in international investments, in foreign exchange markets, as well as in central banks. However, the paper by Meese and Rogoff (1983) points out that the exchange rate models with macroeconomic fundamentals cannot beat the simple random walk in the out-of-sample forecasting performance in the seventies.

After the seminal work of Meese and Rogoff (1983), there are various studies on the forecasting performance of exchange rate dynamics. Most of the studies find that it is very hard for macroeconomic models to beat the simple random walk in out-of-sample forecasting performance for exchange rates dynamics. Even through most of studies bring us pessimistic results, there are several encouraging ones as well. For instance, Mark (1995) is one of these studies. He finds that macroeconomic models are helpful in predicting log exchange rates at long forecast horizons, such as 16-quarter and 20-quarter. Bacchetta and Wincoop (2006) is another encouraging one in examing the exchange rate determination puzzle. They introduce the information heterogeneity among agents about future economic fundamentals. They also find the exchange rate is closely related to observed fundamentals in the long run, but not in the short or medium run. Rime, Sarno and Sojli (2010) take a micro approach and find the order flow is the useful information during forecasting exchange rate movements.

In contrast, this paper focuses on the forecasting performance for short run (within one year) exchange rates movements under a macro-finance approach. Three nominal exchange rates have been studied, for data observed at both monthly and quarterly time frequencies.

The theoretical model is built via the factor representation of stochastic discount factor (SDF) approach under the no-arbitrage assumption. In the mean time, macroeconomic fundamentals are introduced into modeling the stochastic discount factors (SDF) through unobserved market price of risks. This model is coming from the combination of no-arbitrage and macro-finance literatures (Ahn, 2004; Ang and Piazzesi, 2003; Backus, Gregory and Telmer, 1993; Backus, Foresi and Telmer, 2001). Under no-arbitrage literatures from the asset pricing theory, exchange rate changes are endogenously determined by ratios of domestic and foreign

SDFs. The relative macroeconomic fundamentals, such as, differentials of short-term interest rates, output gaps and inflation rates, between domestic and foreign countries, are introduced as factors to modeling the exchange rate dynamics through the specification of SDFs. Following the most common used specification of SDF in macro finance literature (Ang and Piazzesi, 2003; And, Dong and Piazzesi, 2007; Diebold, Piazzesi and Rudebusch, 2005; Duffee, 2007), the model shows that exchange rate dynamics has a nonlinear relationship with macroeconomic fundamentals. Previous studies have mentioned that the nonlinearity may have some forecasting power on exchange rate movements (Engle and West, 2005; Engel, Mark and West, 2007).

This paper empirically tests this nonlinear model under a two-country open economy environment. We study three types of bilateral nominal exchange rates, currencies of Germany, UK and Japan, against US Dollar. Moreover, we exams these three exchange rates for data observed in both monthly and quarterly time frequencies. Hence, there are six cases have been studied in this paper. After the empirical study, we have following important findings. First, in five out of the six cases, model-implied time-varying foreign exchange risk premiums satisfy Fama conditions (1984). Therefore, this no-arbitrage exchange rates model can explain the forward premium anomaly for most cases. Second, forecasting performances for for data observed at quarterly time frequencies are better than the ones for data observed at monthly time frequency with forecasting horizons less than one year. For instance, this no-arbitrage model decreases about 1% forecasting root mean square errors comparing to the simple random walk model for all the three exchange rates observed at monthly time frequencies. In the meanwhile, for data observed at quarterly time frequency, forecasting root mean square errors reduced by 8.8%, 7.3% and 3.3%, respectively for 1-quarter ahead DEM(EUR)/USD, 2-quarter ahead JPY/USD and 1-quarter ahead GBP/USD, against the simple random walk model.

The rest of paper is organized as follows. Section II presents the data and implements a preliminary analysis. Section III introduces a SDF modeling approach for the exchange rate under the no-arbitrage assumption. Section IV discuss the details on the likelihood-based estimation relying on the unscented Kalman filter as well as the rolling out-of-sample forecasting. Section V presents the empirical results on both in-sample model fitting and out-of-sample model forecasting performance. Section VI concludes this paper. Section VII provides the details on how to carry out the unscented Kalman filter and Quasi-Maximum likelihood estimation.

II. Data and Preliminary Analysis

A. Data

In this paper, we test data observed at monthly as well as quarterly time frequencies. Hence, I can compare forecasting performances not only between the no-arbitrage model and the random walk model, but also between data observed at monthly and quarterly time frequencies. The whole sample period is from 1975M7 to 2007M8 (with 386 observations) for data observed at monthly time frequency, and from 1975Q3 to 2007Q2 (with 128 observations) for data observed at quarterly time frequency. The last ten years is taken as the forecast sample period while the rest is for the estimation sample. Therefore the forecasting period is from 1997M9 to 2007M8 for data observed at monthly time frequency, while is from 1997Q3 to 2007Q2 for data observed at quarterly time frequency.

This paper empirically studies the model under a two-country open economy. The United State (US) is taken as the domestic country and the foreign country taken into account is respectively Germany, United Kingdom (UK) and Japan. In short, we will studies three main floating nominal exchange rates, i.e. DEM(EUR)/USD, GBP/USD and JPY/USD. Exchange rates are defined as the prices of domestic currency per one unit of foreign currency. The macroeconomic data include short-term interest rates, output growth rates and inflations rates. Both exchange rate data and macroeconomic data are coming from International Financial Statistics (IFS) database, provided by International Monetary Fund (IMF). Downloading all the data from the same database is able to provide better compatibility among different countries than downloading from different databases.

Short-term interest rates are proxied by the Treasury Bill Rates (annual rate), line *60c*.

Output growth rates and inflation rates are the one-year percentage changes of Industrial Production Index, line *66* (seasonal adjusted) and Consumer Price Index, line *64*, respectively.

Exchange rate data are end-of-period market rates of US Dollar per National Currency, line *ag*. The exchange rate for Germany after 1999 is replaced by the exchange rate of Euro, which is the same method adopted in Corte, Sarno and Tsiakas (2009).

Relative macroeconomic fundamentals are the differentials of macroeconomic variables between home and foreign countries. There are differentials of interest rates, output growths and inflation rates. In order to match the unit of monthly (quarterly) changes of exchange rates, relative macroeconomic fundamentals are divided by 12 (4) into monthly (quarterly) equal quantities for estimation and forecasting.

B. Preliminary Analysis

The intuitive ideas on all the variables observed in monthly time frequency using in the model later on are provided by Figure 1. Here we plots the differentials of macroeconomic variables in monthly equal quantities and changes of exchange rates, for the whole sample from 1975M7 to 2007M8. The top, middle and bottom panel are the variables related to Germany, UK and Japan, respectively. The left panel are relative fundamentals, while the right one are monthly changes of exchange rates. The most volatile variables among relative macroeconomic fundamentals are differentials of output growths. However, changes of exchange rates are much more volatile comparing to differentials of output growths.

— Figure 1 around here —

The summary of these variables statistics can read from Table 1. In panel II, standard deviations of output growth differentials are about two times of the ones of interest rate differentials and inflation rate differentials for all the three countries. Comparing panel I with panel II, standard deviations for changes of exchange rates are around eight times of the one of interest rate differentials and inflation rate differentials. Autocorrelations of changes of exchange rates are very small, between 0.03 and 0.08. While relative fundamentals are very persistent, with autocorrelations almost one. These properties are consistent with the ones in Figure 1.

— Table 1 around here —

For the whole sample period, average monthly changes of exchange rates of US Dollar against German Mark and Japanese Yen are positive. This implies, on average, monthly US Dollar raises against these two currencies. However, US Dollar falls against British pound. The short term interest rate differentials show that, on average, the short interest rate in US is higher than in Germany and Japan, while lower than UK. Inflation rate differentials have roughly the similar pattern as short-term rate differentials. This implies that, comparing to US economy, countries with relative lower inflation rate have steady lower relative interest rates and steady appreciation against US Dollar. This fact shows that the expectation hypothesis (EH) works on average. These are consistent with previous studies (Engel, 1996; Hodrick, 2006; and Cochrane, 2005).

Figure 2 and Table 2 are for variables observed at quarterly time frequency. Among three relative fundamentals, output growth differentials are the most volatile one. Quarterly changes of exchange rates are much more volatile than relative fundamentals. These are the same patterns as for data observed at monthly time frequency.

— Figure 2 around here —

— Table 2 around here —

However, there are some different patterns between data observed at both monthly and quarterly time frequencies. First, correlations of quarterly changes of exchange rates are around twice higher than data observed at monthly time frequencies. Previous studies also find that the higher time frequency of exchange rate returns, the more volatile they are and hence the harder to forecast them (Bekaert and Hodrick, 1992). Second, on quarterly average, US Dollar appreciates against British Pound. This seems conflict to data observed at monthly time frequencies. But after we take into account that exchange rate data are notoriously volatile, there could be some differences on measurements on the average changes of end-of-period exchange rates in monthly and quarterly time frequencies.

One of the most puzzling properties of exchange rate dynamics is the forward premium anomaly. Here we also do some preliminary exercises to check this. The forward premium regression is,

$$
\begin{aligned}
s_{t+1} - s_t &= \alpha_1 + \alpha_2(f_t - s_t) + e_{t+1} \qquad (1)\\
&= \alpha_1 + \alpha_2(r_t^{(h)} - r_t^{(f)}) + e_{t+1},
\end{aligned}
$$

where s is log spot exchange rate of foreign currency against US Dollar, f is forward rate, $r_t^{(h)}$ is domestic (US) short-term interest rate, $r_t^{(f)}$ is the foreign one. The forward premium $(f_t - s_t)$ is equal to the interest differential $(r_t^{(h)} - r_t^{(f)})$ under the covered interest rate parity (CIP) assumption. According to the expectations hypothesis (EH), the values of coefficients α_1 and α_2 should be zero and unity, respectively. Table 3 reports the forward premium regression for data observed at both monthly and quarterly time frequencies, as well as for whole and estimation sample periods, respectively.

— Table 3 around here —

For both data observed at monthly and quarterly frequencies, the forward premium anomaly holds for mostly cases. Since all the values of coefficient α_2 are negative. Moreover, they are highly significant for GBP/USD and JPY/USD, while not significant for DEM(EUR)/USD. For data observed at monthly time frequency, values of R^2 are very small, around 0.01 to 0.03. These are similar as previous studies, Engel (1996) and Hodrick (2006). However, for data observed at quarterly time frequency, values of R^2 improve a lot, around 0.02 to 0.10 (vs 0.01 to 0.03, monthly). One possible reason for this result is that quarterly changes of exchange rates are more persistent than monthly ones (from Table 1 and Table 2), therefore, fits of forward premium regression increase for quarterly ones. Among these three exchange rates, regressions are with highest model fits for JPY/USD, while lowest for DEM(EUR)/USD.

III. Modeling Framework

Our model is built under a two-country open economy framework, with one domestic country and one foreign country, each with its own currency. Under the no-arbitrage assumption and the law of one price, exchange rate dynamics between these two currencies are governed by ratios of their stochastic discount factors. In this section, we firstly discuss how to model stochastic discount factors as well as how to relate them to the relative macroeconomic fundamentals in subsection A. We then proceed to model the exchange rate dynamics in subsection B.

A. Relative Macroeconomic Fundamentals and Stochastic Discount Factors

In a two-country open economy, suppose there are three relative macroeconomic fundamentals, which determine the exchange rates dynamics. They are short-term interest rates differentials $r_t^{(h)} - r_t^{(f)}$, output growth differentials $g_t^{(h)} - g_t^{(f)}$ and inflation rates differentials $\pi_t^{(h)} - \pi_t^{(f)}$. Stacking all the three fundamentals together, we have a fundamental vector V_t in this two-country open economy,

$$V_t = \left[g_t^{(h)} - g_t^{(f)}, \;\; \pi_t^{(h)} - \pi_t^{(f)}, \;\; r_t^{(h)} - r_t^{(f)} \right]', \tag{2}$$

We assume relative fundamentals are observed with measurement errors, which are independent across both time and variables. There are three factors corresponding to the three observed relative macroeconomic fundamentals, $f_{g,t}$, $f_{\pi,t}$ and $f_{r,t}$. Write them into a state vector X_t,

$$X_t = \left[f_{g,t}, \;\; f_{\pi,t}, \;\; f_{r,t} \right]', \tag{3}$$

In order to forecast, we assume that the state vector X_t which determines the dynamics of this two-country open economy system follows a Gaussian vector autoregression VAR(1) process,

$$X_t = \mu + \Phi X_{t-1} + \Sigma \varepsilon_t, \tag{4}$$

where μ is a constant 3×1 vector, Φ is a constant 3×3 matrix, ε_t is an i.i.d Gaussian white noise with its distribution of $N(0, I_3)$ and Σ a low-triangular matrix such that $\Sigma' \Sigma$ captures the variance-covariance matrix of state X_t.

Here, we take the relative macroeconomic fundamentals as the factors, instead of the each macroeconomic fundamentals as the factors in Yin (2010a) and Dong (2006). The reason is simply that this paper we are focus on the out-of-sample forecasting performance of the model. According to the parsimonious principle during forecasting, it is necessary to reduce the amount of parameters to a reasonable small number in order to decrease uncertainties brought by large amount of parameter estimates.

Assume that no-arbitrage holds in this two-country open economy. Then there exists at least one almost surely positive process M_t with $M_0 = 1$ for assets dominated in each economy's currency such that the discounted gains process associated with any admissible trading strategy dominated in that economy currency is a martingale (Harrison and Kreps, 1979). M_t is called the stochastic discount factor (SDF). We denote the home SDF as $M_t^{(h)}$ and the foreign one as $M_t^{(f)}$. In the following, whenever a relation holds both for the home and the foreign countries, we suppress the superscript (h) or (f).

For the absence of a generally accepted equilibrium model for asset pricing, many studies use flexible factor models under the no-arbitrage condition (Cochrane, 2004) from partial equilibrium. In this paper, we also use a factor representation for the SDF's, based on which exchange rate dynamics are modeled. For each of home and foreign stochastic discount factors ($M_t^{(h)}$ and $M_t^{(f)}$), assume that it has an exponential form

$$\begin{aligned} M_{t+1} &= \exp(m_{t+1}) \\ &= \exp\Big(- \tilde{r}_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\varepsilon_{t+1}\Big), \end{aligned} \tag{5}$$

where $\tilde{r}_t$ is the short-term interest rate of that country, λ_t is a time-varying market price vector of risks assigned by investors for assets dominated by the currency of that country and ε_t is the shock to state X_t. The tilde is used to distinguish with unobserved factor and the observed data, with the difference of measurement error between them.

This specification for SDF process is a very common used one in macro finance literatures, such as Ang and Piazzesi (2003) and Duffee (2007). In a Lucas-type exchange economy (Lucas (1982)), the stochastic discount factor is also often named as the representative agent's intertemporal marginal rate of substitution.

Denote the market prices of risks related to home currency as $\lambda_t^{(h)}$ and those related to foreign currency as $\lambda_t^{(f)}$. We use the state X_t to summarize uncertainties in this two-currency world and assume that the market prices of risks related to each country currency are affine functions of X_t (Dai and Singleton (2002); Duffee (2002))

$$\lambda_t = \lambda_0 + \lambda_1 X_t, \tag{6}$$

where λ_0 is a constant 3×1 vector and λ_1 is a constant 3×3 matrix. Both of these two parameter, λ_0 and λ_1 are different between these two countries. The specification (6) implies that the investors may assign different market prices related to different currency for these risks contained in the state X_t and that if $\lambda_t^{(h)}$ and $\lambda_t^{(f)}$ comove tightly, the two SDF's could be highly correlated.

B. Exchange Rate Dynamics

Define the nominal spot exchange rate $\mathcal{S}_t$ at the time-t as the domestic currency price of one unit of the foreign currency. No-arbitrage and law of one price dictate that the ratios of the stochastic discount factors between the home and foreign economies determines the dynamics of their exchange rate (Bachus, Foresi and Telmer, 2001; Bekaert, 1996; Brandt and Santa-Clara, 2002; Brandt, Cochrane and Santa-Clara, 2006). We thus have

$$\frac{\mathcal{S}_{t+1}}{\mathcal{S}_t} = \frac{M_{t+1}^{(f)}}{M_{t+1}^{(h)}}. \tag{7}$$

The above relation formally defines the link between the stochastic discount factors of two economies and exchange rate movements. In complete markets, the stochastic discount factors in both economies are unique and they uniquely determine dynamics of their exchange rate.

Taking natural logarithms for both sides of equation (7) and using specifications of the SDF's (equation 5), we obtain the following exchange rate dynamics equation

$$\begin{aligned}\Delta s_{t+1} &\equiv s_{t+1} - s_t = m_{t+1}^{(f)} - m_{t+1}^{(h)} \\ &= \left(\tilde{r}_t^{(h)} - \tilde{r}_t^{(f)}\right) + \frac{1}{2}\left(\lambda_t^{(h)\prime}\lambda_t^{(h)} - \lambda_t^{(f)\prime}\lambda_t^{(f)}\right) + \left(\lambda_t^{(h)} - \lambda_t^{(f)}\right)\varepsilon_{t+1}, \end{aligned} \tag{8}$$

which shows that macroeconomic fundamentals X_t are imparted to the change of exchange rates, via market prices of risk, in a nonlinear form of fundamentals. This is in contrast to the traditional models that often assume linear relation between the exchange rate and macroeconomic fundamentals or that only use latent factors and do not have this economically meaningful interpretations.

The expected exchange rate changes at time $t+1$ conditional on the information up to time t is,

$$\Delta s_{t+1|t}^{exp.} \equiv (\tilde{r}_t^{(h)} - \tilde{r}_t^{(f)}) + \frac{1}{2}(\lambda_t^{(h)\,\prime}\lambda_t^{(h)} - \lambda_t^{(f)\,\prime}\lambda_t^{(f)}), \tag{9}$$

it captures predictable variation of returns in foreign exchange markets. We can see that market prices of risks are important in determining the expected exchange rate changes. The uncovered interest rate parity does't hold for this model. Because the expected exchange rate changes are determined not only by the interest rate differentials between the two countries $(\tilde{r}_t^{(h)} - \tilde{r}_t^{(f)})$, but also by a foreign exchange risk premium term,

$$rp_t \equiv \frac{1}{2}(\lambda_t^{(h)\,\prime}\lambda_t^{(h)} - \lambda_t^{(f)\,\prime}\lambda_t^{(f)}) \tag{10}$$

The unexpected exchange rate changes at time $t+1$ conditional on the information up to time t is,

$$\Delta s_{t+1|t}^{unexp.} \equiv \left(\lambda_t^{(h)} - \lambda_t^{(f)}\right)\varepsilon_{t+1}, \tag{11}$$

from this equation tells us the unexpected exchange rate changes are determined by the shocks to the system one period ahead multiplied by the differential of market prices of risk between home and foreign country.

Hence we can rewrite exchange rate dynamics equation into,

$$\Delta s_{t+1} \quad = \quad \Delta s_{t+1|t}^{exp.} + \Delta s_{t+1|t}^{unexp.} \tag{12}$$

where the expected part depends on the previous period state vector X_t,

$$
\begin{aligned}
f(X_t) &\equiv \Delta s^{exp.}_{t+1|t}, \\
&= (\tilde{r}^{(h)}_t - \tilde{r}^{(f)}_t) + rp_t,
\end{aligned} \tag{13}
$$

This equation as well as the VAR process for state vector X_t will be the horse force of this paper during the forecasting. The details of forecasting will be discussed later on in the following section.

IV. Econometric Methodology

Since we assume that the true macroeconomic factors are unobservable and that the econometricians observe macroeconomic variables with measurement errors, we first transform the model into a state-space representation and then use a Bayesian filtering approach to estimate the model.

At each time period t, we can observe change of exchange rates Δs_t and macroeconomic fundamentals, V_t. We assume that each of these variables are collected with the normal i.i.d measurement errors. Thus, we have the following observation equations,

$$
\Delta s^{obs.}_t = (\tilde{r}^{(h)}_t - \tilde{r}^{(f)}_t) + \frac{1}{2}(\lambda^{(h)\,\prime}_t \lambda^{(h)}_t - \lambda^{(f)\,\prime}_t \lambda^{(f)}_t) + \Big(\lambda^{(h)}_t - \lambda^{(f)}_t\Big)\varepsilon_{t+1} + \eta^{\Delta s_t}_t \tag{14}
$$

$$
V_t = X_t + \eta^V_t, \tag{15}
$$

where η_t's capture measurement errors with distinct variances of different variables/series and are assumed to be mutually independent; and $(\tilde{r}^{(h)}_{t-1} - \tilde{r}^{(f)}_{t-1}) = (0\ 0\ 1)X_{t-1}$, from the definition of X_{t-1}.

Notice that both X_t and X_{t-1} are entered into the right hand of measurement equation. Hence, we have the state vector $[X_{t+1}, X_t]'$, which follows a first-order VAR with its dynamic

(4). The state equation in the state-space framework,

$$\begin{pmatrix} X_{t+1} \\ X_t \end{pmatrix} = \begin{pmatrix} \mu \\ 0_{3\times1} \end{pmatrix} + \begin{pmatrix} \Phi & 0_{3\times3} \\ I_3 & 0_{3\times3} \end{pmatrix} \begin{pmatrix} X_t \\ X_{t-1} \end{pmatrix} + \begin{pmatrix} \Sigma \\ 0_{3\times3} \end{pmatrix} \varepsilon_{t+1}. \tag{16}$$

Given the state-space model representation (14) and (16) with Gaussian noises, we can implement model estimation using Bayesian filtering approaches. We have noted that the exchange rates dynamic equation is a highly non-linear function of states, which makes the standard Kalman filter inapplicable. Instead, we can use the nonlinear Kalman filters. The usually used nonlinear Kalman filter is the extended Kalman filter (EKF), which linearizes the nonlinear system around the current state estimate using a Taylor approximation. However, for the highly nonlinear system, the extended Kalman filter is computationally demanding and performs very poorly. An alternative is the unscented Kalman filter (UKF), recently developed in the field of engineering (Julier and Ulman (1997), (2004)). The idea behind this approach is that in order to estimate the state information after a nonlinear transformation, it is favorable to approximate the probability distribution directly instead of linearizing the nonlinear functions. The unscented Kalman filter overcomes a large extent pitfalls inherent to the extended Kalman filter and improves estimation accuracy and robustness without increasing computational cost.

The details on unscented Kalman filter and quasi-maximum likelihood estimation for this model is in appendix.

The forecasting methodology we adopt in this paper is the rolling forecasting. Firstly, we estimate our model during the estimation sample period. Then we control the insignificant parameters into zero and reestimate the model again. This method is also using in Diebold and Li (2006), Singleton (2002) and Duffee (2002). According to the parsimonious principle, the less the number of the parameters, the better the forecasting performance of the model. This is simply because the uncertainty will increase when more parameters have to estimate, hence it will deteriorate the performance in out-of-sample forecasting.

According to the rolling forecasting, we estimate the model for each sample period $[t_0, t]$, where t belongs to the forecasting sample. With the parameter estimates for the sample period $[t_0, t]$, we dynamic forecast the spot changes of exchange rates for 1 period ahead and up to $t+12$ periods ahead for data observed at monthly time frequency, while up to $t+4$ periods ahead for data observed at quarterly time frequency). The details for the rolling forecasting are following. Because we have,

$$\begin{aligned} \Delta s_{t|t-1} &= \Delta s^{exp.}_{t|t-1}, \qquad (17)\\ &= f(X_{t-1}), \\ X_t &= \mu + \Phi X_{t-1} + \Sigma \varepsilon_t, \qquad (18) \end{aligned}$$

Therefore, after we estimate the model using data up to time t, the dynamic forecasts of the h-period ahead exchange rate changes at time t for $(s_{t+h} - s_t)|_t$ is,

$$\begin{aligned} for \;\; h = 1, \quad (s_{t+1} - s_t)|_t &= \Delta s_{t+1|t} \\ &= f(X_t) \qquad (19) \end{aligned}$$

and

$$\begin{aligned} for \;\; h \geq 2, \quad (s_{t+h} - s_t)|_t &= \Big[(s_{t+h} - s_{t+h-1}) + (s_{t+h-1} - s_{t+h-2}) + ... + (s_{t+1} - s_t)\Big]|_t \\ &= \Big[\Delta s_{t+h} + \Delta s_{t+h-1} + ... + \Delta s_{t+1}\Big]|_t \\ &= \Big[f(X_{t+h-1}) + f(X_{t+h-2}) + ... + f(X_t)\Big]|_t \\ &= \sum_{j=0}^{h-1} \Big[f(X_{t+j})\Big]|_t \\ &= \sum_{j=0}^{h-1} f(X_{t+j|_t}) \qquad (20) \end{aligned}$$

note that, from equation (18), we have,

$$X_{t+j|t} = \sum_{i=1}^{j} (\Phi^{i-1}\mu) + \Phi^j X_t, for \;\; j \geq 1 \qquad (21)$$

hence, we can rewrite both equation (19) and equation (20) into following expression,

$$\begin{aligned}(s_{t+h} - s_t)|_t &= f(X_t), && for \;\; h = 1,\\ &= \sum_{j=1}^{h-1} f\Big(\sum_{i=1}^{j} (\Phi^{i-1}\mu) + \Phi^j X_t \Big) + f(X_t), && for \;\; h \geq 2.\end{aligned} \tag{22}$$

We compare our model with simple random walk (RW) model. For random walk model, the levels of exchange rates are following a random walk process, hence the changes of exchange rates are white noises. Therefore the random walk model implies the expected changes of exchange rate are zero at any time-t, $(s_{t+h} - s_t)|_t = 0$, with any forecast horizon h. Random walk model is the well accepted benchmark model for evaluating exchange rates forecasting after the seminal work of Meese and Rogoff (1983a, 1983b). The forecasting accuracy criteria using are absolute mean forecasting error (AME) and root mean square error (RMSE) (Meese and Rogoff, 1983a).

V. Empirical Results and Discussions

A. Model Fit Performance Analysis

We study the two-country no-arbitrage model for three cases, Germany and the US, the UK and the US and Japan and the US. The US is taken as the domestic country as always among all these three cases.

Table 7 and Table 8 report data statistic summaries of the observed ones and the model-implied ones, and for data observed at both monthly and quarterly time frequencies. The differences of summary statistics between the observed and the model-implied relative macroeconomic fundamentals, such as the differentials of output growth, inflation rate and interest rate, are relatively small for all the three cases for data observed at both monthly and quarterly time frequencies.

— Table 7 around here —

— Table 8 around here —

For changes of exchange rates, there are some distance between model-implied and observed ones. However, due to the exchange rates movements are notoriously volatile and less persistent comparing to relative macroeconomic fundamentals, the model in this paper provides a fairly reasonable model for exchange rate dynamics.

It is common that in Table 7, for all the three types of changes of exchange rates, the model implied values have smaller standard deviations comparing to observed data, between 20% and 40% of the standard deviations of observed data. This implies the model can account for some part of the variation of the observed data. However, there are still more than half of the observed variations cannot be explained by the model. This maybe dues to other potential factors which affect exchange rates dynamics have not been included in the model in this paper, for instance, trade balances, current accounts, political events and government deficits. It is common that all of these model implied changes of exchange rates are much more consistent comparing to the observed data, about 0.9 for both GBP/USD and JPY/USD and around 0.6 for DEEM(EUR)/USD. This is not surprising when modeling exchange rates dynamics with macroeconomic variables, which are highly persistent.

These two properties are keeping true for data observed at quarterly time frequency as well, see Table 8. The model implied three types of changes of exchange rates can account for around 32% to 38% volatilities of the observed data. However, the model implied changes of exchange rates are less persistent comparing to the monthly ones, with autocorrelations of 0.61, 0.71 and 0.89 respectively (v.s. 0.59, 0.94 and 0.92). Hence the model has better model fit for data observed at quarterly time frequency than monthly quarterly time frequency during in-the-sample estimation.

B. Time-Varying Foreign Risk Premium

The validity of the estimated foreign risk premium can be tested by Fama conditions (Fama, 1984). The original Fama decomposition for forward premium is,

$$f_t - s_t = E_t\Big(s_{t+1} - s_t\Big) + \Big(f_t - E_t(s_{t+1})\Big), \tag{23}$$

where note that the forward premium, $p_t \equiv f_t - s_t = r_t^{(h)} - r_t^{(f)}$, by the covered interest rate parity (CIP) assumption; the expected change of exchange rates is $q_t \equiv E_t\Big(s_{t+1} - s_t\Big)$; and the foreign risk premium is $d_t \equiv f_t - E_t(s_{t+1})$. Corresponding to the notation is our model, the expected change of exchange rates is $q_t \equiv E_t(s_{t+1} - s_t) = (r_t^{(h)} - r_t^{(f)}) + rp_t$ and $d_t \equiv f_t - E_t(s_{t+1}) = -rp_t$. In order to match the forward premium anomaly, Fama (1984) proposed that q and d should satisfy the following two conditions, $cov(q,d) < 0$ and $var(d) > var(q)$.

— Table 4 around here —

The left panel in Table 4 reports the two Fama conditions. The first condition of the negative covariances between d_t and q_t are satisfied in each of the three types of foreign exchange rate risk premium, as well as for data observed at both monthly and quarterly time frequencies. Moreover, there are five cases out of six with larger variance of the foreign exchange risk premium comparing to the one of forward premium. These are consistent with the second Fama condition, which suggests that the foreign exchange risk premium are more volatile comparing with interest rate differentials.

In Section B, the empirical studies on the forward premium regression show there regressions are highly rejected by the expectation hypothesis for ten cases out of the twelve cases. However, if we run the forward premium regression including the estimated time-varying foreign risk premium $r\hat{p}_t$ as well, the results changes dramatically. The amended forward premium regression,

$$s_{t+1} - s_t = \alpha_1 + \alpha_2(r_t^{(h)} - r_t^{(f)}) + \alpha_3 r\hat{p}_t + e_{t+1} \tag{24}$$

In the right panel of Table 4 reports estimates for the above regression. At first sight, we find five out of six coefficients for interest rate differentials are with negative values. However, four out of the five standard errors are not significant from zero. Therefore, this proves that the estimated time-varying risk premia has the power to alleviate the forward premium anomaly in the sense that most coefficients before interest rate differential change from significantly negative values into less negative and insignificant values. Model fits for these two types of forward premium regressions can provide the same idea. There are three out of the six R^2 values have dramatic improvement after taken into account the time-varying foreign risk premium, for instance, for the quarterly observed DEM(EUR)/USD (12.8% vs 1.8%), GBP/USD (11.2% vs 5.0%) and for the monthly abserved GBP/USD (2.7% vs 2.0%).

Hence the time-vary foreign risk premium generated by the model in the paper show the importance in modeling exchange rate dynamics in not only the validity from the data (consistent with Fama's conditions), but also the alleviation on the forward premium anomaly.

C. Out-of-Sample Forecast Performance

In order to investigate the forecasting performance of the model in this paper, we choose the random walk model as the benchmark model (Meese and Rogoff, 1983a). The forecasting accuracy criteria are the absolute mean forecasting error (AME) as well as the root mean square error ($RMSE$). The $RMSE$ is the one of most popular forecasting accuracy measurements. However, AME measurement is more robust to the data processes with fat-tailed distribution. Therefore, both of these two measurements are presented in this paper. The The ratio of AME and $RMSE$ between the model in this paper and random walk model are reported in Table 5. The ratios with values less than one indicate our model outperformance the simple random walk model.

Table 5 represents the 1-, 3-, 6-, 9- and 12-month ahead forecasting performance for all the three exchanges rate for data observed at monthly frequency. The results show that the no-arbitrage model can outperformance the simple random walk model. The ratios are less than one, only besides four ratios out of thirty in total. Moreover, forecasting improves when forecasting period h increase. The forecasting performances for JPY/USD and DEM(EUR)/USD are much better than for GBP/USD.

— Table 5 around here —

Table 6 reports the 1-, 2-, 3- and 4-quarter ahead forecasting performance for all the three types of exchanges rate observed at quarterly frequency. The results show that the forecasting of the model in this paper are able to outperformance the simple random walk model for all the three types of exchange rates according to the *RMSE* ratios. According *AME* ratios, there are ten out of twelve ratios are less than one. Both these types of ratios decrease as the forecasting horizon increasing. For these three types of exchange rates, according to both *RMSE* ratios and *AME* ratios, JPY/USD has the best performance, followed by GBP/USD, and the last one is DEM(EUR)/USD. While for data observed at monthly time frequency, the best performance is for JPY/USD as well, but DEM(EUR)/USD has better performance than GBP/USD.

— Table 6 around here —

To conclude, our model in the paper can out performance the random walk model during the out-of-sample forecasting for data observed at both monthly and quarterly time frequencies according both the *RMSE* ratio and the *AME* ratio. As the forecasting period increasing, forecasting performance improves.

VI. Conclusion

This paper evaluates the no-arbitrage exchange rates model by comparing the out-of-sample forecasting performance with simple random walk model. The no-arbitrage model has nonlinear property on macroeconomic fundamentals under this modeling framework.

We exams the no-arbitrage model by six cases. There are three types of exchange rates, the currencies of Germany, the UK and Japan, against the US Dollar. For each exchange rates, we study for data observed at two kinds time frequencies, monthly and quarterly. After the empirical study, we have following findings. First, there are five out of the six cases whose model-implied time-varying foreign exchange risk premiums satisfy Fama conditions (1984). Moreover, the no-arbitrage exchange rates model can alleviate the forward premium anomaly by the time-varying foreign risk primium. Second, the no-arbitrage model has better forecasting performance comparing with random walk model, for all three types of exchange rates observed at both monthly and quarterly time frequencies. The forecasting performance improves as the forecasting horizon increasing, for all the three types of exchange rates observed at both monthly and quarterly time frequencies. The forecasting performance from no-arbitrage model comparing to the random walk model improves a lot for GBP/USD observed at quarterly time frequency comparing to monthly frequency (with *RMSE* ratios between 0.966 and 0.929, vs between 1.006 and 0.991), and has some improvement for DEM(EUR)/USD and JPY/USD (*RMSE* ratios decrease about 1%).

VII. Appendix: Unscented Kalman Filter and Quasi-Maximum Likelihood Estimation

The state-space framework of the model is,

$$
\begin{aligned}
&state\ \ equation: \qquad (25)\\
\Delta s_t &= (0\ 0\ 1)X_{t-1} + \frac{1}{2}(\lambda_{t-1}^{(h)'}\lambda_{t-1}^{(h)} - \lambda_{t-1}^{(f)'}\lambda_{t-1}^{(f)}) + \eta_t^{\Delta s}\\
V_t &= X_t + \eta_t^V,
\end{aligned}
$$

$$
observation\ \ equation:
$$

$$
\begin{pmatrix} X_t \\ X_{t-1} \end{pmatrix} = \begin{pmatrix} \mu \\ 0_{3\times 1} \end{pmatrix} + \begin{pmatrix} \Phi & 0_{3\times 3} \\ I_3 & 0_{3\times 3} \end{pmatrix} \begin{pmatrix} X_{t-1} \\ X_{t-2} \end{pmatrix} + \begin{pmatrix} \Sigma \\ 0_{3\times 3} \end{pmatrix} \varepsilon_t. \qquad (26)
$$

where η_t's capture measurement errors with distinct variances of different variables/series and are assumed to be mutually independent.

To implement the unscented Kalman filter, we firstly concatenate the state variables $x_{t-1} = [X_{t-1}, X_{t-2}]'$, the observation noises η_{t-1} and the state noises ε_{t-1} at time $t-1$

$$
x_{t-1}^e = \begin{bmatrix} x_{t-1}' & \eta_{t-1}' & \varepsilon_{t-1}' \end{bmatrix}', \qquad (27)
$$

whose dimension is $L = L_x + L_\eta + L_\varepsilon$ and whose mean and covariance are

$$
\hat{x}_{t-1}^e = \begin{bmatrix} E[x_{t-1}] & 0 & 0 \end{bmatrix}', \qquad P_{t-1}^e = \begin{bmatrix} P_{t-1}^x & 0 & 0 \\ 0 & \Sigma_\eta^2 & 0 \\ 0 & 0 & I_3 \end{bmatrix}.
$$

We then form a set of $2L+1$ sigma points

$$
\chi_{t-1}^e = \begin{bmatrix} \hat{x}_{t-1}^e & \hat{x}_{t-1}^e + \sqrt{(L+\lambda)P_{t-1}^e} & \hat{x}_{t-1}^e - \sqrt{(L+\lambda)P_{t-1}^e} \end{bmatrix} \qquad (28)
$$

and the corresponding weights

$$w_0^{(m)} = \frac{\lambda}{L+\lambda}, \quad w_0^{(c)} = \frac{\lambda}{L+\lambda} + (1-\alpha^2+\beta), \tag{29}$$

$$w_i^{(m)} = w_i^{(c)} = \frac{1}{2(L+\lambda)}, \quad i = 1, 2, \ldots, 2L, \tag{30}$$

where superscripts (m) and (c) indicate that weights are for construction of the posterior mean and covariance, respectively, $\lambda = \alpha^2(L+\bar{\kappa}) - L$ is a scaling parameter, the constant α determines the spread of sigma points around $\bar{x}$ and is usually set to be a small positive value, $\bar{\kappa}$ is a second scaling parameter with value set to 0 or $3-L$ and β is a covariance correction parameter and is used to incorporate prior knowledge of the distribution of x.

With these sigma points, we implement the non-linear Kalman filter as follows: for the time update

$$\begin{aligned}
\chi^x_{t|t-1} &= F(\chi^x_{t-1}, \chi^\varepsilon_{t-1}), \quad \hat{x}_t^- = \sum_{i=0}^{2L} w_i^{(m)} \chi^x_{i,t|t-1}, \\
P^-_{x_t} &= \sum_{i=0}^{2L} w_i^{(c)} (\chi^x_{i,t|t-1} - \hat{x}_t^-)(\chi^x_{i,t|t-1} - \hat{x}_t^-)',
\end{aligned}$$

and for the measurement update

$$\begin{aligned}
\mathcal{Y}_{t|t-1} &= H(\chi^x_{t|t-1}, \chi^\eta_{t|t-1}), \quad \hat{Y}_t^- = \sum_{i=0}^{2L} w_i^{(m)} \mathcal{Y}_{i,t|t-1}, \\
P^-_{Y_t} &= \sum_{i=0}^{2L} w_i^{(c)} (\mathcal{Y}_{i,t|t-1} - \hat{Y}_t^-)(\mathcal{Y}_{i,t|t-1} - \hat{Y}_t^-)', \\
P_{x_t Y_t} &= \sum_{i=0}^{2L} w_i^{(c)} (\chi^x_{i,t|t-1} - \hat{x}_t^-)(\mathcal{Y}_{i,t|t-1} - \hat{Y}_t^-)', \\
\hat{x}_t &= \hat{x}_t^- + P_{x_t Y_t}(P^-_{Y_t})^{-1}(Y_t - \hat{Y}_t^-), \\
P_{x_t} &= P^-_{x_t} - (P_{x_t Y_t}(P^-_{Y_t})^{-1}) P^-_{Y_t} (P_{x_t Y_t}(P^-_{Y_t})^{-1})',
\end{aligned}$$

where Y_t is the observation vector containing all the observed variables, $\hat{Y}_t^-$ its predicted values, $P^-_{Y_t}$ its conditional variance-covariance matrix, $\hat{x}_t$ the filtered state vector and P_{x_t} its variance-covariance matrix.

Assuming that the predictive errors are normally distributed, we can construct the log likelihood function at time t as follows

$$\mathcal{L}_t(\Theta) = -\frac{1}{2}\ln|P^-_{Y_t}| - \frac{1}{2}(Y_t - \hat{Y}^-_t)'(P^-_{Y_t})^{-1}(Y_t - \hat{Y}^-_t), \tag{31}$$

where Θ is a vector of model parameters. Parameter estimates can be obtained by maximizing the joint log likelihood

$$\hat{\Theta} = \arg\max_{\Theta\in\Xi} \sum_{t=1}^{T} \mathcal{L}_t(\Theta), \tag{32}$$

where Ξ is a compact parameter space and T is the length of total observations of data. Because the log likelihood function is misspecified for non-Gaussian models, a robust estimate of the variance-covariance matrix of parameter estimates can be obtained using White (1982)

$$\hat{\Sigma}_\Theta = \frac{1}{T}\left[AB^{-1}A\right]^{-1}, \tag{33}$$

where

$$A = -\frac{1}{T}\sum_{t=1}^{T}\frac{\partial^2\mathcal{L}_t(\hat{\Theta})}{\partial\Theta\partial\Theta'}, \qquad B = \frac{1}{T}\sum_{t=1}^{T}\frac{\partial\mathcal{L}_t(\hat{\Theta})}{\partial\Theta}\frac{\partial\mathcal{L}_t(\hat{\Theta})}{\partial\Theta'}. \tag{34}$$

With these parameter estimates $\hat{\Theta}$, the latent macroeconomic factors $\hat{X}_t$ can be extracted using the unscented Kalman filter.

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Table 1: **Data Statistics (Monthly Data)**

	Mean(%)	*Std. Dev.*(%)	*Skewness*	*Kurtosis*	*Autocorr.*
A. Changes of Exchange Rates					
DEM(EUR)/USD	0.056	1.366	-0.096	3.966	0.030
GBP/USD	-0.010	1.303	-0.103	4.878	0.062
JPY/USD	0.105	1.408	0.371	4.521	0.076
B. Relative Fundamentals					
1. Germany					
$(g^h - g^f)$	0.068	0.378	0.319	4.248	0.842
$(\pi^h - \pi^f)$	0.150	0.177	1.081	5.051	0.980
$(r^h - r^f)$	0.091	0.191	-0.438	3.533	0.969
2. UK					
$(g^h - g^f)$	0.134	0.313	-0.297	3.831	0.859
$(\pi^h - \pi^f)$	-0.151	0.290	-2.689	11.404	0.961
$(r^h - r^f)$	-0.197	0.189	-0.459	3.161	0.950
3. Japan					
$(g^h - g^f)$	0.020	0.406	0.523	3.371	0.918
$(\pi^h - \pi^f)$	0.199	0.180	-0.424	4.956	0.963
$(r^h - r^f)$	0.278	0.186	0.171	3.096	0.962

Note: This Table reports the summary of statistics of original data for estimation sample period with data observed at monthly frequency set. The change of exchange rates is the one-month change of log nominal exchange rate, where monthly nominal exchange rates are the end of period values. The relative fundamentals are the differentials of output growth rates, inflation rates and interest rates, which are US one less the foreign one. The superscript h stands for US and f stands for Germany, UK and Japan, respectively. Macroeconomic data are in monthly quantities, which are the one year percentage changes dividing 12. The sample period is July 1975 to August 2007 (386 observations) and the data is in monthly equal quantities.

Table 2: **Data Statistics (Quarterly Data)**

	Mean(%)	*Std. Dev.(%)*	*Skewness*	*Kurtosis*	*Autocorr.*
A. Changes of Exchange Rates					
DEM(EUR)/USD	0.554	5.935	0.102	2.750	0.051
GBP/USD	0.054	5.023	-0.111	3.294	0.131
JPY/USD	0.871	6.163	0.544	3.007	0.107
B. Relative Fundamentals					
1. Germany					
$(g^h - g^f)$	0.214	1.070	0.371	3.519	0.822
$(\pi^h - \pi^f)$	0.452	0.531	1.085	5.097	0.949
$(r^h - r^f)$	0.273	0.566	-0.564	3.406	0.935
2. UK					
$(g^h - g^f)$	0.403	0.886	-0.334	3.895	0.780
$(\pi^h - \pi^f)$	-0.454	0.870	-2.685	11.336	0.856
$(r^h - r^f)$	-0.591	0.555	-0.459	2.972	0.850
3. Japan					
$(g^h - g^f)$	0.062	1.186	0.541	3.286	0.878
$(\pi^h - \pi^f)$	0.597	0.535	-0.430	4.876	0.923
$(r^h - r^f)$	0.832	0.549	0.089	2.862	0.905

Note: This Table reports the summary of statistics of original data for estimation sample period with data observed at quarterly frequency set. The change of exchange rates is the one-quarter change of log nominal exchange rate, where quarterly nominal exchange rates are the end of period values. The relative fundamentals are the differentials of output growth rates, inflation rates and interest rates, which are US one less the foreign one. The superscript h stands for US and f stands for Germany, UK and Japan, respectively.Macroeconomic data are in monthly quantities, which are the one year percentage changes dividing 4. The sample period is 1975Q3 to 2007Q2 (128 observations) and the data is in quarterly equal quantities.

Table 3: **Forward Premium Regressions**

	α_1	α_2	$R^2(\%)$	α_1	α_2	$R^2(\%)$
	A. Monthly Data					
	A.1 Whole Sample			*A.2 Estimated Sample*		
DEM(EUR)/USD	0.118	-0.681	0.9	0.106	-0.551	0.7
	(0.077)	(0.363)		(0.101)	(0.412)	
GBP/USD	-0.169	-0.809	1.4	-0.274	-0.964	2.0
	(0.095)	(0.349)		(0.132)	(0.420)	
JPY/USD	0.445	-1.228	2.6	0.501	-1.279	3.3
	(0.128)	(0.381)		(0.147)	(0.428)	
	B. Quarterly Data					
	B.1 Whole Sample			*B.2 Estimated Sample*		
DEM(EUR)/USD	1.011	-1.675	2.6	0.977	-1.322	1.8
	(0.578)	(0.922)		(0.753)	(1.043)	
GBP/USD	-0.967	-1.726	3.6	-1.596	-2.050	5.1
	(0.641)	(0.792)		(0.891)	(0.959)	
JPY/USD	3.560	-3.231	8.3	4.027	-3.333	10.0
	(0.954)	(0.958)		(1.099)	(1.077)	

Note: Forward premium regression is $s_{t+1} - s_t = \alpha_1 + \alpha_2(f_t - s_t) + \epsilon_{t+1} = \alpha_1 + \alpha_2(r_t^{(h)} - r_t^{(f)}) + e_{t+1}$, where the forward premium is equal to the interest differential. s is log spot exchange rate of foreign currency against US Dollar, f is forward rate, $r_t^{(h)}$ is domestic (US) interest rate, $r_t^{(f)}$ is foreign interest rate. According to the expectations hypothesis, the value of α_2 should be positive and unity. The standard error is reported in the round parenthesis. In penal A, data are in monthly frequency and under monthly equal quantities. The whole sample is from 1975m7 to 2007m8 with 386 observations, while the estimated sample is from 1975m7 to 1997m8 with 266 observations. In penal B, data are in quarterly frequency and under quarterly equal quantities. The whole sample is from 1975Q3 to 2007Q2 with 128 observations, while the estimated sample is from 1975Q3 to 1997Q2 with 88 observations.

Table 4: **The Role of Time-varying Foreign Risk Premium**

	1. Fama Condition			*2. Augmented Forward premium regression*			
	$cov(q,d)$	$var(d)$	$var(q)$	α_1	α_2	α_3	$R^2(\%)$
A. 1975m7 – 1997m8 (266 obs.)							
DEM(EUR)/USD	-0.091	0.132	0.092	0.053	0.144	0.321	0.5
				(0.120)	(0.526)	(0.296)	
GBP/USD	-0.110	0.151	0.113	-0.280	-0.611	0.375	2.7
				(0.131)	(0.487)	(0.262)	
JPY/USD	-0.139	0.209	0.111	0.522	-1.501	-0.135	3.4
				(0.154)	(0.640)	(0.289)	
B. 1975Q3 – 1997Q2 (88 obs.)							
DEM(EUR)/USD	-6.988	7.258	7.133	0.999	-0.812	0.784	12.8
				(0.714)	(1.001)	(0.239)	
GBP/USD	-5.857	6.551	5.539	-0.599	-0.931	0.608	11.2
				(0.959)	(1.040)	(0.249)	
JPY/USD	-2.530	1.926	3.502	2.070	-1.632	0.645	11.0
				(2.327)	(2.083)	(0.676)	

Note: 1. The Fama decomposition is $f_t - s_t = E_t(s_{t+1} - s_t) + f_t - E_t(s_{t+1})$, where the forward premium is $p_t \equiv f_t - s_t = r_t^{(h)} - r_t^{(f)}$; the expected change of exchange rates is $q_t \equiv= E_t(s_{t+1} - s_t)$; and the foreign risk premium is $d_t \equiv f_t - E_t(s_{t+1})$. Corresponding to our model, the expected change of exchange rates is$q_t \equiv E_t(s_{t+1} - s_t) = (r_t^{(h)} - r_t^{(f)}) + rp_t$ and the foreign risk premium is $d_t \equiv f_t - E_t(s_{t+1}) = -rp_t$. The Fama conditions are, $cov(q,d) < 0$ and $var(d) > var(q)$.

2. The amended forward premium regression regression is $s_{t+1} - s_t = \alpha_1 + \alpha_2(r_t^{(h)} - r_t^{(f)}) + \alpha_3 r\hat{p}_t + e_{t+1}$, after taken into account the estimated time-varying foreign exchange risk premium $r\hat{p}_t$. According to the expectations hypothesis, we expect the value of α_2 will be closer to unity after amended UIP by time-varying risk premium. The standard error is reported in the round parenthesis.

In penal A, data are in monthly frequency and in monthly equal quantities. The sample period of estimation is from July 1975 to August 1997, with 268 observations. In penal B, data are in quarterly frequency and in quarterly equal quantities. The sample period of estimation is from 1975Q3 to 1997Q2, with 88 observations.

Table 5: **Forecasting Performance (Monthly Data)**

Horizon	$1-m$	$3-m$	$6-m$	$9-m$	$12-m$
A. DEM(EUR)/USD					
1. Random Walk Model					
MAE	0.930	1.810	2.530	3.162	3.888
RMSE	1.160	2.212	3.015	3.701	4.580
2. No-Arbitrage Model					
MAE ratio	1.009	0.973	0.928	0.890	0.889
RMSE ratio	1.010	0.979	0.942	0.916	0.917
B. GBP/USD					
1. Random Walk Model					
MAE	0.768	1.188	1.743	2.202	2.749
RMSE	0.937	1.476	2.177	2.714	3.265
2. No-Arbitrage Model					
MAE ratio	0.986	1.001	0.988	0.964	0.972
RMSE ratio	1.006	0.993	0.993	0.998	0.991
C. JPY/USD					
1. Random Walk Model					
MAE	0.981	1.780	2.624	2.979	3.395
RMSE	1.325	2.352	3.303	3.658	4.122
2. No-Arbitrage Model					
MAE ratio	1.007	0.998	0.935	0.863	0.852
RMSE ratio	0.995	0.970	0.924	0.867	0.844

Note: The table reports the model forecasting performance on the changes of exchange rates comparing with the random walk model for exchange rates. In random walk model for exchange rates, the forecasting changes of exchange rates are zeros with all horizons. In the model of this paper (No-Arbitrage Model), we take the rolling forecasting. *MAE* and *RMSE* stand for mean absolute value of forecasting error and root mean square forecasting error, respectively. The ratio value less than one means the SDFs model in this paper is prior to random walk model. The forecasting period is 1997m9 to 2007m8, totally 120 months.

Table 6: **Forecasting Performance (Quarterly Data)**

Horizon	$1-Q$	$2-Q$	$3-Q$	$4-Q$
A. DEM(EUR)/USD				
1. Random Walk Model				
MAE	4.221	6.228	7.386	9.127
RMSE	5.264	7.278	8.725	10.706
2. No-Arbitrage Model				
MAE ratio	1.020	0.911	0.932	0.866
RMSE ratio	0.990	0.954	0.931	0.894
B. GBP/USD				
1. Random Walk Model				
MAE	2.909	4.384	5.187	6.372
RMSE	3.445	5.105	6.343	7.630
2. No-Arbitrage Model				
MAE ratio	0.998	0.934	0.893	0.920
RMSE ratio	0.966	0.936	0.931	0.929
C. JPY/USD				
1. Random Walk Model				
MAE	4.276	6.665	7.326	8.927
RMSE	5.684	8.309	9.195	10.882
2. No-Arbitrage Model				
MAE ratio	1.041	0.981	0.871	0.839
RMSE ratio	0.976	0.933	0.856	0.832

Note: The table report the model forecasting performance on the changes of exchange rates comparing with the random walk model for exchange rates. In random walk model for exchange rates, the forecasting changes of exchange rates are zeros with all horizons. In the model of this paper (No-Arbitrage Model), we take the rolling forecasting. *MAE* and *RMSE* stand for mean absolute value of forecasting error and root mean square forecasting error, respectively. The ratio value less than one means the SDFs model in this paper is prior to random walk model. The forecasting period is 1997Q3 to 2007Q2, totally 40 quarters.

Table 7: **Summary Statistics of Data and Model Implied Data (Monthly Data)**

	Mean(%)		*Std. Dev.*(%)		*Skewness*		*Kurtosis*		*Autocorr.*	
	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
A. Germany and US										
Δs	0.0444	0.0156	1.4625	0.2774	-0.1821	2.0808	3.9372	27.0665	0.0007	0.5866
$g^h - g^f$	0.1168	0.1106	0.4066	0.3727	0.2427	0.2541	4.0119	3.2268	0.8372	0.9444
$\pi^h - \pi^f$	0.1753	0.1571	0.2044	0.1410	0.6997	-0.8319	3.6882	3.7128	0.9845	0.9775
$r^h - r^f$	0.1113	0.1113	0.2177	0.2177	-0.6324	-0.6324	3.1755	3.1755	0.9668	0.9668
B. UK and US										
Δs	-0.0495	-0.0125	1.4449	0.4451	-0.0768	-3.7497	4.4672	30.0523	0.0843	0.6950
$g^h - g^f$	0.1070	0.1078	0.3482	0.3302	-0.1624	-0.1151	3.3616	3.3252	0.8601	0.9093
$\pi^h - \pi^f$	-0.2120	-0.2120	0.3261	0.3261	-2.2616	-2.2616	8.4400	8.4401	0.9595	0.9595
$r^h - r^f$	-0.2329	-0.2315	0.2095	0.0814	-0.0916	-0.2316	2.6854	2.8923	0.9422	0.9265
C. Japan and US										
Δs	0.1485	-0.0236	1.4560	0.3243	0.1717	-0.6406	3.7647	3.2963	0.1022	0.9204
$g^h - g^f$	-0.0330	-0.0330	0.3934	0.3934	0.2309	0.2309	2.8428	2.8428	0.9015	0.9015
$\pi^h - \pi^f$	0.1870	0.1870	0.2051	0.2051	-0.2443	-0.2443	4.1034	4.1034	0.9651	0.9651
$r^h - r^f$	0.2759	0.2785	0.2061	0.2031	0.2426	0.1949	2.8717	2.6976	0.9558	0.9657

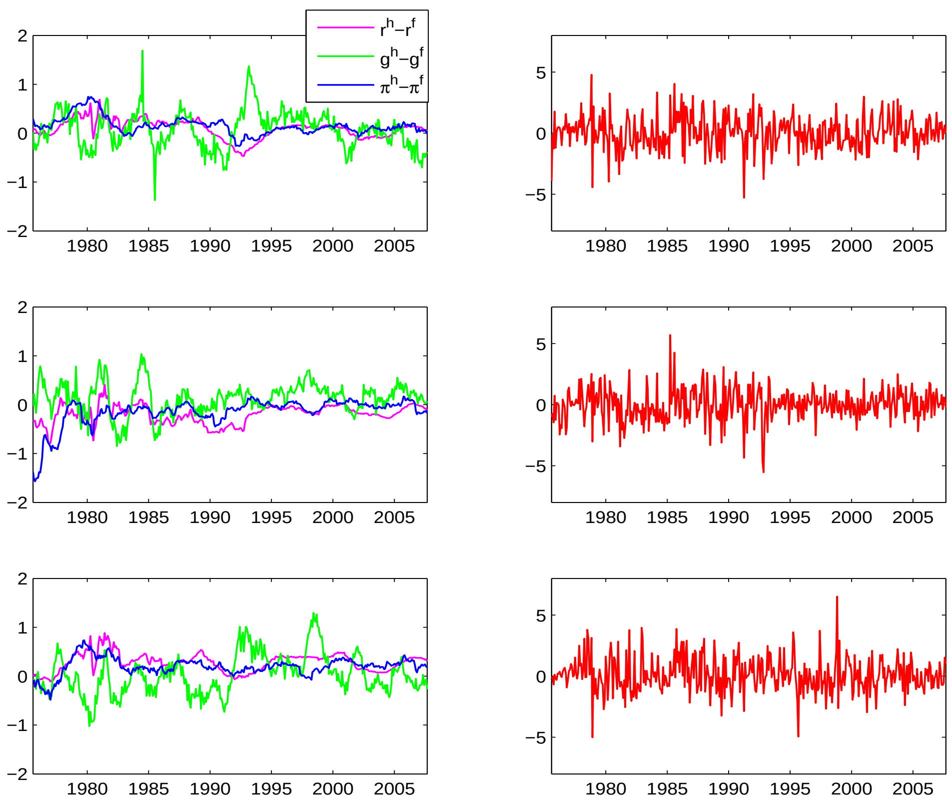
Note: The table reports summary statistics of the data and the model-implied values, under the data observed at monthly frequency set. The sample period of estimation is from July 1975 to August 1997, with 268 observations. Panel A reports the data of Germany and US. Panel B reports the data of UK and US. Panel C reports the data of Japan and US. The sample period is July 1975 to August 1997 and the data is in monthly frequency.

Table 8: **Summary Statistics of Data and Model Implied Data (Quarterly Data)**

	Mean(%)		*Std. Dev.*(%)		*Skewness*		*Kurtosis*		*Autocorr.*	
	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
A. Germany and US										
Δs	0.5370	0.1489	6.2908	2.4201	0.0203	-0.4596	2.6257	3.4327	0.0822	0.6074
$r^h - r^f$	0.3328	0.3328	0.6443	0.6443	-0.7520	-0.7520	3.1214	3.1213	0.9312	0.9312
$g^h - g^f$	0.3487	0.3487	1.1570	1.1566	0.3109	0.3108	3.1598	3.1589	0.8140	0.8142
$\pi^h - \pi^f$	0.5298	0.5298	0.6135	0.6135	0.6900	0.6900	3.6950	3.6950	0.9524	0.9524
B. UK and US										
Δs	-0.1596	-0.0409	5.6049	2.0369	-0.0890	-1.3034	2.9255	4.1689	0.1437	0.7055
$r^h - r^f$	-0.7008	-0.7008	0.6141	0.6141	-0.0898	-0.0898	2.5193	2.5193	0.8290	0.8290
$g^h - g^f$	0.3159	0.3159	0.9869	0.9868	-0.1666	-0.1666	3.4097	3.4097	0.7650	0.7650
$\pi^h - \pi^f$	-0.6384	-0.6383	0.9797	0.9797	-2.2455	-2.2455	8.3442	8.3440	0.8463	0.8463
C. Japan and US										
Δs	0.8714	0.8810	6.1625	1.9759	0.5443	-0.5147	3.0065	5.4868	0.1069	0.8855
$r^h - r^f$	0.8323	0.8323	0.5489	0.5489	0.0894	0.0894	2.8616	2.8616	0.9050	0.9050
$g^h - g^f$	0.0623	0.0623	1.1864	1.1864	0.5411	0.5411	3.2857	3.2857	0.8779	0.8779
$\pi^h - \pi^f$	0.5965	0.5965	0.5346	0.5346	-0.4301	-0.4301	4.8762	4.8762	0.9229	0.9229

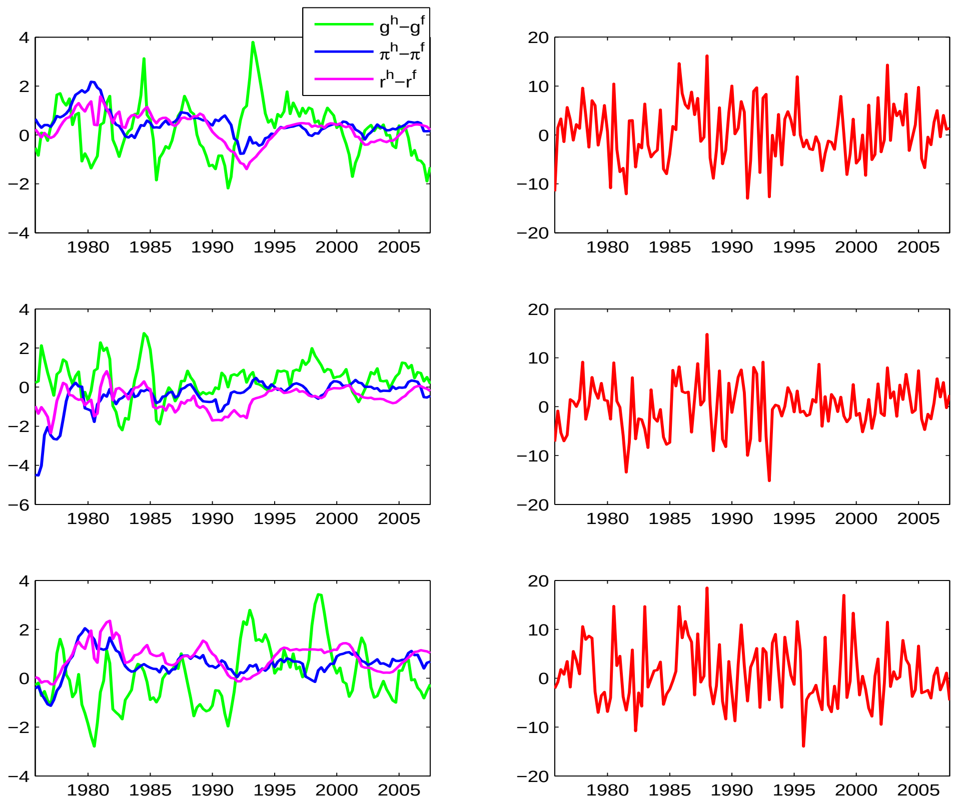
Note: The table reports summary statistics of the data and the model-implied values, under the data observed at quarterly frequency sets. The sample period of estimation is from July 1975 to August 1997. Panel A reports the data of Germany and US. Panel B reports the data of UK and US. Panel C reports the data of Japan and US. The sample period is July 1975 to August 1997 and the data is in quarterly frequency.

Figure 1: Macroeconomic Data and Exchange Rates Changes (Monthly)



*Note:*I plot the macroeconomic fundamentals and monthly changes of exchange rates used in the estimation. The upper, middle and bottom panels are for Germany, UK and Japan, respectively. The left three panel are the differential of interest rates, output gaps with solid line and inflation rates with dish line; and the right three panel are monthly changes of exchange rates. The units on the vertical axis are percent. I plot annualized quantities for macroeconomic differentials and monthly changes of exchange rates.

Figure 2: Macroeconomic Data and Exchange Rates Changes (Quarterly)



*Note:*I plot the macroeconomic fundamentals and quarterly changes of exchange rates used in the estimation. The upper, middle and bottom panels are for Germany, UK and Japan, respectively. The left three panel are the differential of interest rates, output gaps with solid line and inflation rates with dish line; and the right three panel are quarterly changes of exchange rates. The units on the vertical axis are percent. I plot annualized quantities for macroeconomic differentials and monthly changes of exchange rates.

Chapter 3

Macroeconomic Fundamentals and Exchange Rates Dynamics: A No-Arbitrage Multi-Country Model

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Abstract

This paper investigates the joint dynamics of multi bilateral nominal exchange rates simultaneously under a Multi-Country framework. We introduce the macroeconomic fundamental information to model exchange rate dynamics by adopting a no-arbitrage macro-finance approach. We allow macroeconomic fundamentals to be determined by global (common) factors as well as country-idiosyncratic factors. The empirical study focuses an open economy including four countries, i.e., Germany, the UK, Japan and the US, where the US is taken as the numerarie (home) country. Results show that exchange rate dynamics are better modeled by this multi-country no-arbitrage model comparing to previous studies. This model explains 53%, 34% and 32% variations of the observed exchange rate changes of DEM(EUR)/USD, GBP/USD and JPY/USD, respectively. Moreover, the global macroeconomic factors do exist.

Keywords: Multi-Country Model, Exchange Rate Dynamics, Macroeconomic Fundamentals, Global Factors, Country-idiosyncratic Factors

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I. Introduction

The floating nominal exchange rate is the market price of one currency converting into another. It is one of the most important factors in the international economic activities. For instance, international trades and international investments. But whether exchange rate dynamics are forced by macroeconomic fundamentals or not puzzles a great number of researchers after the seminal work by Meese and Rogoff (1983).

There are large amount of models proposed to try to explain exchange rate dynamics by macroeconomic fundamentals. For instance, monetary models (Frenkel (1976, 1979), Mussa (1976), Bilson (1978), Dornbusch (1976)) state that the existence of a long-run equilibrium for the nominal exchange rates is a function of the differentials of money supplies and income levels between home and foreign countries. Recent studies by New open economy macroeconomics models (Obstfeld and Rogoff (2003)) model exchange rate movements from optimization problem under the stochastic dynamic general equilibrium approach for open economy. However, these models can not find empirical evidence on a close relationship between short-run exchange rates movements and macroeconomic fundamentals (Meese (1990), Frankel and Rose (1995), Engel and West (2005)). It is also worth mentioning that most of the studies on nominal exchange rates are under a Two-Country model setting, which is the simplest one in the open economy studies.

In contract, the model in this paper has three main differences from the traditional exchange rate models.

First of all, this paper investigates interactions of exchange rates changes and macroeconomic fundamentals under a no-arbitrage macro-finance approach. Under this approach, the bilateral nominal exchange rate change is governed by the ratio of stochastic discount factors between that two countries. The stochastic discount factor, also named intertemporal marginal utilities substitution, is modeled by a factor representation under the no-arbitrage condition. We take outputs, inflations and short-term interest rates as macroeconomic fundamentals. Real output growth directly governs the aggregate consumption of a country and should be a key element in the stochastic discount factor. Inflation can also enter into the stochastic discount factor via its dynamic interactions with the real production (Piazzesi and Schneider (2006)). The short-term interest rate is typically viewed as a macro variable reflecting monetary policy (Duffee (2007)). Under the same specifications of stochastic discount factors, we extend macro-finance term structure models (Ang and Piazzesi (2003), Diebold, Rudebushch, and Aruoba (2005), Ang, Dong, and Piazzesi (2007)) into a multi-country open economy framework in order to help identify the time-varying market prices of risks, which in turn determine the foreign risk premium and amplify roles of macroeconomic innovations on exchange rate changes. This is important since ignoring foreign risk premium or assuming it is constant may mislead to a conclusion that exchange rates are not linked to macroeconomic fundamentals.

Second of all, the model is built under a Multi-Country Model (MCM) setting, hence it is able to simultaneously investigate the dynamics of more than one exchange rates. However it is impossible to do this under a Two-Country model setting. Moreover, under Two-Country model, in order to the studies more than one nominal exchange rates, it always separately studies on each single exchange rate. Therefore it potentially exists some inconsistent problem about the variables related to the numerarie country.

Third of all, we adopt the global and country-idiosyncratic macroeconomic factors in modeling the correlated macroeconomic fundamentals across countries. The beauty of doing this allows us to efficiently reduce the number of parameters which control the dynamics of the Multi-Country open economy system.

The empirical study focuses an open economy including four countries, i.e., Germany, the UK, Japan and the US, where the US is taken as the numerarie (home) country. Results show that exchange rate dynamics are better modeled by this multi-country no-arbitrage model comparing to previous studies. This model explains 53%, 34% and 32% variations of the observed exchange rate changes of DEM(EUR)/USD, GBP/USD and JPY/USD, respectively. Moreover, the macroeconomic global and country-idiosyncratic factors do exist.

The rest of the proposal is organized as follows. Section II presents the data. Section III introduces a Multi-Country no-arbitrage exchange rate model under the macro-finance literature. Section IV proposes the econometric methodology, the likelihood-based estimation combining with the unscented Kalman filter. Section V presents the empirical results and discusses their economic implications. Section VI concludes the paper.

II. Data and Preliminary Analysis

A. Data

Consider a Multi-Country world, where there exist $(N+1)$ countries. We take the last country (the $(N+1)^{th}$ country), as the domestic country, while the first N countries as foreign countries. For the empirical studies in this paper, we will consider about a $(3+1)$-country open economy case, with Germany/Euro area, the UK, Japan and the US. The first three are taken as foreign countries, while US is taken as domestic country. The data is in monthly time frequency, and the sample period is from 1985m9 to 2005m8.

The macroeconomic fundamentals taken into account in our studies are output growths, inflation rates, and short term interest rats. The nominal exchange rates are the market rate of the end of period values. Both the exchange rate and macroeconomic data are coming from International Financial Statistics (IFS) database, provided by International Monetary Fund (IMF), for all four countries. The same data source is able to provide better compatibility across different countries, comparing with more than one sources.

Short-term interest rates are proxied by the annual Treasury Bill Rates (line *60c*). Output growth rates and inflation rates are the one-year percentage changes of seasonal adjusted Industrial Production Indexes (line *66*) and Consumer Price Indexes (line *64*), respectively. Exchange rate data are the end of period market rate data of the US Dollar per National Currency (line *ag*). The exchange rate for German Mark after 1999 is replaced by the exchange rate of the Euro, and this is the same as Corte, Sarno, and Tsiakas (2009).

— Figure 1 around here —

In order to better identify the parameters which construct the market prices of risk, yield data are included as well. The zero-coupon bond yield data for German, the UK, Japan and the US, are generously provided by Francis Deibold, Canlin Li, and Vivian Yue, from 1985.09 to 2005.07 (beginning-of-month data). We take the yields with maturities of 1-month, 24-month and 60-month to stand for the short, medium and long yields. These three yields are commonly used to get the empirical 'level', 'slope' and 'curvature' components, which are sufficient to capture the term structure of interest rate.

Note that, in order to match the unit of monthly exchange rates changes, both the macroeconomic and yield data are divided by 12 into monthly equal quantities.

B. Preliminary Analysis

Firstly, let us have an intuitive idea on the cross-country relationships of macroeconomic fundamentals and exchange rates. The correlation matrix of these variables is in Table 7.

— Table 7 around here —

We can focus on the four diagonal submatrix from this 15×15 matrix. First of all, let us have a look at the correlations of the three groups of macroeconomic variables, output growths, inflations, and short-term interest rates. The first 4×4 block on the diagonal of this correlation matrix shows that the output growths are all positively correlated across these four countries. Among them, the highest two correlations are between Germany and Japan (53%), as well as UK and US (53%). The second 4×4 submatrix on the diagonal of this correlation matrix tells us that the inflation rates are also positively correlated, with highest two values of 78% between UK and US, and 70% between UK and Japan. The third 4×4 submatrix on the diagonal of this correlation matrix shows that short-term interest rates are positively correlated as well, with highest two correlations of 84% and 80%, for UK and Japan, and for UK and US, respectively. Macroeconimic variables are positively correlated across countries suggests that there may exit some global macroeconomic factors for output growth, inflation and short-term interest rates, driving the comovement of macroeconomic fundamental across countries.

Second of all, let us look at the correlations of three types of nominal exchange rates changes, which are in the last 3×3 submatrix on on the diagonal of this correlation matrix. It is interesting to notice that they are positively correlated as well. This is because that during the sample period, the US Dollar, took the role of international currency, during international payments in international trade and finance. Hence when US Dollar goes strong, the rest currencies become comparably weaker, vice versa.

From the above results, we explore deeper on the question whether there exist some common factors driving the comovements in macroeconomic fundamentals or not. We do three principal components analysis, for each group of macroeconomic fundamentals. The results are reported in Table 8. The first principle components associated with highest eigenvalues can explain 60%, 70% and 81% variations in the group of output growths, inflations and short-term interest rates, respectively. This tells us that there should exist some global factors, in each group, to determine the comovements of each macroeconomic variable across country. This is the evindence for the following model constructing strategy by adopting macroeconomic global and country-idiosyncratic factors.

— Table 8 around here —

III. The Modeling Framework

In this paper, we consider a $(N+1)$-country world, with N foreign countries and 1 domestic country. Because of the importance of the US economy and its currency in the global economy after the Bretton Woods system collapse, the US currency is taken as the numerical currency. Between these $(N + 1)$ countries' currency, the minimal number of bilateral nominal exchange rates relationships is N, while the rests can be induced by the triangle relationship from these N bilateral exchange rates. Therefore, we only focus on the N foreign exchange rates, for instance, the N foreign currencies against the US Dollar.

Under the no-arbitrage assumption and the law of one price, the exchange rate dynamics is governed by the ratio of stochastic discount factors related to these two countries. In this section, we firstly discuss macroeconomic global and country-specific factor setup under a Multi-country model, in subsection A. Then we introduce how to model stochastic discount factors related to macroeconomic fundamentals in subsection B. After that, we proceed to model the exchange rate dynamics in subsection C. In the end, we give the recursive relationship of the bond pricing for each country under the affine term structure modeling framework in subsection D.

A. Global, Country-Idiosyncratic and Country Factor Setup in a Multi-Country World

The setup of the state dynamics to drive a Multi-Country system is a tradeoff. It is better to include as much macroeconomic information as possible. Mean while, it is necessary to control the amount of the parameters as small as possible in order to carry out the estimation. Hence we introduce the macroeconomic global and country-idiosyncratic factor setup, which can well balance these two requirements.

In a $(N+1)$-country open economy, suppose there exists a global macroeconomic factor vector G_t, which can determine the co-movement of macroeconomic fundamentals across countries, such as, the global macroeconomic factors of output growth g_t^G, inflation π_t^G and short-term interest rate r_t^G. Write them into one vector G_t, where $G_t = \left(g_t^G,\ \ \pi_t^G,\ \ r_t^G\right)^T$.

For each economy i $(i = 1, 2, ..., N+1)$, we assume that its country macroeconomic factor vector $X_{i,t} = \left(\tilde{g}_{i,t},\ \ \tilde{\pi}_{i,t},\ \ \tilde{r}_{i,t}\right)^T$ loads on the global factor, $G_t = \left(g_t^G,\ \ \pi_t^G,\ \ r_t^G\right)^T$, as well as its country-idiosyncratic factor $F_{i,t} = \left(f_{i,t}^g,\ \ f_{i,t}^\pi,\ \ f_{i,t}^r\right)^T$. Note that the tilde is used to distinguish with the unobserved factor and the observed data, with the difference of measurement error between them. Hence for country factors $\tilde{g}_{i,t},\ \ \tilde{\pi}_{i,t},\ \ \tilde{r}_{i,t}$,

$$\begin{aligned}
\tilde{g}_{i,t} &= \alpha_i^g + \beta_i^g g_t^G + f_{i,t}^g, \\
\tilde{\pi}_{i,t} &= \alpha_i^\pi + \beta_i^\pi \pi_t^G + f_{i,t}^\pi, \qquad (1)\\
\tilde{r}_{i,t} &= \alpha_i^r + \beta_i^r r_t^G + f_{i,t}^r,
\end{aligned}$$

where $\{\alpha_i^g, \alpha_i^\pi, \alpha_i^r\}_{i=1,...,N+1}$ are constant terms, and $\{\beta_i^g, \beta_i^\pi, \beta_i^r\}_{i=1,...,N+1}$ are loadings on global or common factors $(g_t^G,\ \ \pi_t^G,\ \ r_t^G)$ for country i; and $\{f_{i,t}^g, f_{i,t}^\pi, f_{i,t}^r\}_{i=1,...,N+1}$ are country-idiosyncratic factors for country i. Write above equations into a matrix equation,

$$X_{i,t} = \alpha_i + \beta_i G_t + F_{i,t}, \qquad (2)$$

where $\{\alpha_i\}_{i=1,...,N+1}$ are constant 3×1 vectors, and $\{\beta_i\}_{i=1,...,N+1}$ are diagonal matrixes of the loading on global factor G_t.

About the two components in the above equation, firstly of all, we assume that this global factor G_t follows a Gaussian vector autoregression process,

$$G_t = \Phi^G G_{t-1} + \Sigma^G \upsilon_t^G, \qquad (3)$$

where Φ^G a constant 3×3 matrix; υ_t^G an i.i.d Gaussian white noise, with mean zeros, and identity variance-covariance matrix; and Σ^G a low-triangular matrix.

Secondly of all, assume that the country-idiosyncratic factors to have a Gaussian vector autoregression process,

$$F_{i,t} = \Phi^{F_i} F_{i,t-1} + \Sigma^{F_i} \upsilon^F_{i,t}, \tag{4}$$

where Φ^{F_i} is a constant 3×3 diagonal matrix; $\upsilon^F_{i,t}$ is country-idiosyncratic factor shock, with mean zero, and identity variance-covariance matrix; and we assume shocks in this equation are independent element by element, hence the shock's variance-covariance matrix of $\Sigma^{F_i}(\Sigma^{F_i})^T$ is diagonal. The idea of this setting up in a Multi-Country open economy with both global and country-idiosyncratic factors is also adopted in the paper Diebold, Li, and Yue (2008) for investigating the global yield curve.

By equation (2), (3), and (4), under the assumption that υ^G_t is independent to $\upsilon^F_{i,t}$, we can get the following process of the country factor $X_{i,t}$ for country i,

$$X_{i,t} = \alpha_i + \beta_i \Phi^G G_{t-1} + \Phi^{F_i} F_{i,t-1} + \Sigma^{X_i} \varepsilon^X_{i,t}, \tag{5}$$

with

$$\begin{aligned} \Sigma^{X_i} \varepsilon^X_{i,t} &= \beta_i \Sigma^G \upsilon^G_t + \Sigma^{F_i} \upsilon^F_{i,t}. \\ \Sigma^{X_i}(\Sigma^{X_i})^T &= \beta_i \Sigma^G (\beta_i \Sigma^G)^T + \Sigma^{F_i}(\Sigma^{F_i})^T, \end{aligned}$$

where assume $\varepsilon^X_{i,t}$ with zero means and identity variance-covariance matrix.

This equation captures the evolution of the country state factor $X_{i,t}$, for each country i. Note that the shock, or "uncertainty" to $X_{i,t}$ at time $t-1$ is $\Sigma^{X_i}\varepsilon^X_{i,t}$, which is comprised by two parts, the global loading (β_i) times of the shock on the global factor $\Sigma^G \upsilon^G_t$, and plus the country-idiosyncratic shock $\Sigma^{F_i}\upsilon^F_{i,t}$. Both of global and country-idiosyncratic shocks are i.i.d. normal distributions and independent with each other, hence $\Sigma^{X_i}\varepsilon^X_{i,t}$ is i.i.d. normal distribution with mean zero, and variance-covariance matrix $\beta_i \Sigma^G(\beta_i \Sigma^G)^T + \Sigma^{F_i}(\Sigma^{F_i})^T$. This equation is useful because it tells us how the shock to the country macroeconomic factor is pinned down by from the shocks of global and country-idiosyncratic shocks.

B. Relating Macroeconomic Fundamentals to Stochastic Discount Factors

In this Multi-Country world, assume that no-arbitrage holds. Then there exists at least one almost surely positive process M_t with $M_0 = 1$ for assets dominated in each economy currency such that the discounted gains process associated with any admissible trading strategy dominated in that economy currency is a martingale (Harrison and Kreps (1979)). M_t is called the stochastic discount factor (SDF). We denote the country i's SDF as $M_{i,t}$, for $i = 1, 2, ..., N+1$.

Without a generally accepted equilibrium model for asset pricing, many studies use flexible factor models under the no-arbitrage condition (Cochrane, 2004) from a partial equilibrium in the financial market. In this paper, we also use a factor representation for the SDF's, based on which the exchange rate and the term structure of interest rates are modeled. In this $(N+1)$-country open economy framework, for each country i's currency, there exists one unique stochastic discount factors $M_{i,t}$, for $i = 1, 2, ..., N+1$, and assume that the SDF for country i has an exponential form

$$\begin{aligned} M_{i,t+1} &= \exp(m_{i,t+1}) \\ &= \exp\Big(- \tilde{r}_{i,t} - \frac{1}{2}(\lambda_{i,t})^T \lambda_{i,t} - (\lambda_{i,t})^T \varepsilon^X_{i,t+1}\Big), \end{aligned} \tag{6}$$

where $\tilde{r}_{i,t}$ is the short-term interest rate of country i, $\lambda_{i,t}$ is the time-varying market prices of risks assigned by investors for assets dominated in country i's currency, and $\varepsilon^X_{i,t+1}$ is the "uncertainty" to the country i's state vector $X_{i,t+1}$ at time t, from equation (5).

This specification for SDF process is a very common used one in macro finance literatures, such as Ang and Piazzesi (2003), Duffee (2007), etc. In a Lucas-type exchange economy (Lucas (1982)), the stochastic discount factor is also named the intertemporal marginal rate of substitution from the representative agent's optimization problem.

Note that the market prices of risks related to country i's currency is $\lambda_{i,t}$. We use the country-specific state $X_{i,t}$ to summarize uncertainties in each economy and assume that the market prices of risks related to each country i's currency are affine functions of $X_{i,t}$, for $i = 1, ..., N+1$, (Dai and Singleton (2002); Duffee (2002))

$$\lambda_{i,t} = \lambda_{i,0} + \lambda_{i,1} X_{i,t}, \tag{7}$$

where $\lambda_{i,0}$ is a constant 3×1 vector, and $\lambda_{i,1}$ is a constant 3×3 matrix. It is also crucial to make some reasonable assumption for the coefficients of $\lambda_{i,0}$ and $\lambda_{i,1}$. Here we simplify the coefficient matrix $\lambda_{i,1}$ to be diagonal. This can reduce the amount of parameters in this multi-country model and without loss much efficiency to model market prices of risk on macroeconomic information.

C. Exchange Rate Dynamics

For foreign country j ($j = 1, ..., N$), its spot nominal bilateral exchange rate with the US $\mathcal{S}_{j,t}$ is defined as the price of the US dollar per one unit of the foreign country j's currency. No-arbitrage and law of one price dictate that the ratio of the stochastic discount factors between the home and foreign countries determines the dynamics of their exchange rate (Bachus, Foresi, and Telmer (2001); Bekaert (1996); Brandt and Santa-Clara (2002); Brandt, Cochrane, and Santa-Clara (2006)). We thus have

$$\frac{\mathcal{S}_{j,t+1}}{\mathcal{S}_{j,t}} = \frac{M_{j,t+1}}{M_{N+1,t+1}}. \tag{8}$$

The above relation formally defines the link between the stochastic discount factors of two economies and the exchange rate movements between them. In complete markets, the stochastic discount factors in both economies are unique, therefore they uniquely determine the dynamics of their exchange rate.

Taking natural logarithms for both sides of equation (8) and using specifications of the SDF's (6), we obtain the following exchange rate changes equation,

$$\begin{aligned} \Delta s_{j,t+1} &\equiv s_{j,t+1} - s_{j,t} = m_{j,t+1} - m_{N+1,t+1} \qquad (9)\\ &= \left(\tilde{r}_{N+1,t} - \tilde{r}_{j,t}\right) + \frac{1}{2}\left((\lambda_{N+1,t})^T \lambda_{N+1,t} - (\lambda_{j,t})^T \lambda_{j,t}\right) + \left((\lambda_{N+1,t})^T \varepsilon^X_{N+1,t+1} - (\lambda_{j,t})^T \varepsilon^X_{j,t+1}\right), \end{aligned}$$

which shows that macroeconomic fundamentals $X_{j,t}$ and $X_{N+1,t}$ are imparted to the exchange rate changes $\Delta s_{j,t+1}$, via market prices of risk, have have nonlinear form. This is in contrast to the traditional models that often assume a linear relation between the exchange rate dynamics and macroeconomic fundamentals or these only use latent factors and do not have any economically meaningful interpretations.

The exchange rate changes are determined by two parts, the expected and unexpected parts. The expected foreign exchange rate changes,

$$\Delta s^{exp.}_{j,t} \equiv E_{t-1}\Big(\Delta s_{j,t}\Big) = \Big(\tilde{r}_{N+1,t} - \tilde{r}_{j,t}\Big) + \frac{1}{2}\Big((\lambda_{N+1,t})^T \lambda_{N+1,t} - (\lambda_{j,t})^T \lambda_{j,t}\Big) \tag{10}$$

which captures predictable variation of returns in foreign exchange markets. We can see that market prices of risks are important in determining the expected part of exchange rate changes. The uncovered interest rate parity does not hold for this model. Because the expected exchange rate changes are determined not only by the interest rate differentials between the two countries $(\tilde{r}_{N+1,t} - \tilde{r}_{j,t})$, but also by a foreign exchange risk premium term, $rp_{j,t}$,

$$rp_{j,t} \equiv \frac{1}{2}\Big((\lambda_{N+1,t})^T \lambda_{N+1,t} - (\lambda_{j,t})^T \lambda_{j,t}\Big), \tag{11}$$

The unexpected exchange rate changes,

$$\Delta s^{unexp.}_{j,t} \equiv \Delta s_{j,t} - E_{t-1}\Big(\Delta s_{j,t}\Big) = (\lambda_{N+1,t})^T \varepsilon^X_{N+1,t+1} - (\lambda_{j,t})^T \varepsilon^X_{j,t+1}, \tag{12}$$

which implies that the unexpected exchange rate changes are the product of macroeconomic shock by its corresponding market price of risk. Note that the market price of risk is time-varying according to its linear relationship with macroeconomic variables.

In summary, we can rewrite the exchange rate dynamic equation in following ways,

$$\begin{aligned} \Delta s_{j,t+1} &= \Big(\tilde{r}_{N+1,t} - \tilde{r}_{j,t}\Big) + rp_{j,t} + \Delta s^{unexp.}_{j,t}, && (13) \\ &= \Delta s^{exp.}_{j,t} + \Delta s^{unexp.}_{j,t}, && (14) \end{aligned}$$

D. Bond Pricing

In the last part of our model, we give the recursive relationship of the bond pricing for each country i $(i = 1, ..., N+1)$ under the affine term structure modeling framework.

For each country i, having specified the stochastic discount factors $M_{i,t}$ and with itsshort rates, then we can price its zero-coupon bonds. Introduction of bonds in our modeling framework is important for identifying market prices of risks.

Because each country's short rate $\tilde{r}_{i,t}$ has been included in the state vector $X_{i,t}$ as one of elements, the affine short rate equations can be easily specified as

$$\tilde{r}_{i,t} = \delta_{i,0} + (\delta_{i,1})^T X_{i,t}, \tag{15}$$

with $\delta_{i,0} = 0$, and $\delta_{i,1} = \Big(0, 0, 1\Big)^T$.

In each country i, no-arbitrage guarantees that a zero-coupon bond with maturity n-year at time t can be priced by using the following Euler equation

$$\tilde{P}_{i,t}^{(n)} = E_t\Big[M_{i,t+1}\tilde{P}_{i,t+1}^{(n-1)}\Big] \tag{16}$$

with the initial condition $\tilde{P}_{i,t}^{(0)} = 1$. Again, tilde indicates the true value. Under specifications of the country state factor dynamics (5), the short rate (15) and the SDF (6), we can show that the country i's bond price is an exponential linear function of the glabal factor G_t as well as its country-idiosyncratic factors $F_{i,t}$,

$$\tilde{P}_{i,t}^{(n)} = \exp\Big(A_{i,n} + (B_{i,n})^T G_t + (C_{i,n})^T F_{i,t}\Big), \tag{17}$$

where $A_{i,n}, B_{i,n}$ and $C_{i,n}$ solve the following difference equations

$$\begin{aligned}
A_{i,n+1} &= A_{i,n} - \Big(\delta_{i,0} + (\delta_{i,1})^T \alpha_i\Big) + \frac{1}{2}(B_{i,n})^T \Sigma^G (\Sigma^G)^T B_{i,n} + \frac{1}{2}(C_{i,n})^T \Sigma^{F_i} (\Sigma^{F_i})^T C_{i,n} - K_{i,n}(\lambda_{i,0} + \lambda_{i,1}\alpha_i), \\
B_{i,n+1} &= \Big((B_{i,n})^T \Phi^G - K_{i,n}\lambda_{i,1}\beta_i - (\delta_1)^T \beta_i\Big)^T, \\
C_{i,n+1} &= \Big((C_{i,n})^T \Phi^{F_i} - K_{i,n}\lambda_{i,1} - (\delta_1)^T)^T,
\end{aligned} \tag{18}$$

with

$$K_{i,n} = \Big((\Sigma^{X_i})^{-1}\Big(\beta_i\Sigma^G(\Sigma^G)^T B_{i,n} + \Sigma^{F_i}(\Sigma^{F_i})^T C_{i,n}\Big)\Big)^T,$$

with $A_{i,1} = -\Big(\delta_{i,0} + (\delta_{i,1})^T\alpha_i\Big)$, $B_{i,1} = -(\beta_i)^T\delta_{i,1}$, and $C_{i,1} = -\delta_{i,1}$ being the initial conditions. Accordingly, the yield is also an affine function of the state

$$\tilde{y}_{i,t}^{(n)} \equiv -\frac{\log P_{i,t}^{(n)}}{n} = a_{i,n} + (b_{i,n})^T G_t + (c_{i,n})^T F_{i,t}, \tag{19}$$

where $a_{i,n} = -A_{i,n}/n$, $b_{i,n} = -B_{i,n}/n$, and $c_{i,n} = -C_{i,n}/n$.

From difference equations in (18), we can see that the constant market price of risk parameter $\lambda_{i,0}$ only affects the constant yield coefficient $a_{i,n}$, whereas the parameter $\lambda_{i,1}$ affects the loadings on global and country-idiosyncratic factors as well, $b_{i,n}$ and $c_{i,n}$. This implies that the parameter $\lambda_{i,0}$ affects average term spreads and average expected bond returns, whereas the parameter $\lambda_{i,1}$ governs time variation in term spreads and expected bond returns.

IV. Econometric Methodology

Since we assume that the macroeconomic factors $X_{i,t}$, yields $y_{i,t}$, and exchange rate changes $\Delta s_{j,t}$, are unobservable and that the econometrician observe the corresponding ones, $X_{i,t}^{obs.}$, $y_{i,t}^{obs.}$, and $\Delta s_{j,t}^{obs.}$, with measurement errors, $\eta_{i,t}^X$, $\eta_{i,t}^y$ and $\eta_{j,t}^{\Delta s}$. We first transform the model into a state-space representation and then use a Bayesian filtering approach to estimate the model.

A. State-Space Model Representation

At each period t, we can observe the depreciation rates, macroeconomic variables, and zero-coupon bond data. We assume that each of these variables is collected with the normal i.i.d measurement errors. Thus, we have the following measurement equations

$$\begin{aligned}
\Delta s_{j,t}^{obs.} &= \Big(\tilde{r}_{N+1,t-1} - \tilde{r}_{j,t-1}\Big) + \frac{1}{2}\Big((\lambda_{N+1,t-1})^T \lambda_{N+1,t-1} - (\lambda_{j,t-1})^T \lambda_{j,t-1}\Big) \\
&\quad + (\lambda_{N+1,t-1})^T (\Sigma^{X_{N+1}})^{-1} \Big(\beta_{N+1}(G_t - \Phi^G G_{t-1}) + (F_{N+1,t} - \Phi^{F_{N+1}} F_{t-1})\Big) \\
&\quad - (\lambda_{j,t-1})^T (\Sigma^{X_j})^{-1} \Big(\beta_j (G_t - \Phi^G G_{t-1}) + (F_{j,t} - \Phi^{F_j} F_{t-1})\Big) + \eta_{j,t}^{\Delta s}, \\
&\quad for\ j = 1,\ \ldots\ , N; \qquad (20) \\
X_{i,t}^{obs.} &= \alpha_i + \beta_i G_t + F_{i,t} + \eta_{i,t}^X,\ for\ i = 1, \ldots, N+1;, \qquad (21) \\
y_{i,t}^{obs.} &= a_i + b_i' G_t + c_i' F_{i,t} + \eta_{i,t}^y, \quad for\ i = 1,\ \ldots\ , N+1. \qquad (22)
\end{aligned}$$

where in the exchange rate changes equation, we use $\varepsilon_{i,t}^X = (\Sigma^{X_i})^{-1}(\beta_i \Sigma^G \upsilon_t^G + \Sigma^{F_i} \upsilon_{i,t}^F)$, and $\Sigma^G \upsilon_t^G = G_t - \Phi^G G_{t-1}$, $\Sigma^{F_i} \upsilon_{i,t} = F_{i,t} - \Phi^{F_i} F_{t-1}$ (for $i = 1, \ldots, N+1$), from equation (3) and (4); $\lambda_{i,t-1} = \lambda_0 + \lambda_1 X_{i,t} = \lambda_0 + \lambda_1(\alpha_i + \beta_i G_t + F_{i,t})$. η_t's capture measurement errors with distinct variances for different variables/series and are assumed to be mutually independent.

For the state vector in this system, we have the latent global factor G_t and country-idiosyncratic factor $F_{i,t}$, both following a first-order VAR with their dynamics in equation (3) and equation (4), respectively. From the measurement equations, we notice that observations depend on both current and lagged values of global and country-idiosyncratic factors, $G_t, F_{i,t}$ as well as $G_{t-1}, F_{i,t-1}$. Hence all of them should be taken as states and the state equations are,

$$\begin{pmatrix} G_t \\ G_{t-1} \end{pmatrix} = \begin{pmatrix} \Phi^G & 0_{3\times3} \\ I_3 & 0_{3\times3} \end{pmatrix} \begin{pmatrix} G_{t-1} \\ G_{t-2} \end{pmatrix} + \begin{pmatrix} I_3 \\ 0_{3\times3} \end{pmatrix} \Sigma^G \upsilon_t^G, \qquad (23)$$

$$\begin{pmatrix} F_{i,t} \\ F_{i,t-1} \end{pmatrix} = \begin{pmatrix} \Phi^{F_i} & 0_{3\times3} \\ I_3 & 0_{3\times3} \end{pmatrix} \begin{pmatrix} F_{i,t-1} \\ F_{i,t-2} \end{pmatrix} + \begin{pmatrix} I_3 \\ 0_{3\times3} \end{pmatrix} \Sigma^{F_i} \upsilon_{i,t}^F,\ for\ i = 1, \ldots, N+1. \qquad (24)$$

Therefore, the parameters needed to estimate in the Multi-Country model are,

$$\Theta = \Big(\{\alpha_i,\ \beta_i;\ \Phi^{F_i},\ \Sigma^{F_i};\ \lambda_{i,0}\ \lambda_{i,1};\ \Sigma^{\eta^{X_i}},\ \Sigma^{\eta^{y_i}}\}_{i=1,2,3,4};\ \{\sigma^{\eta_{\Delta s_j}}\}_{j=1,2,3};\ \Phi^G,\ \Sigma^G\Big), \qquad (25)$$

B. Quasi-Maximum Likelihood Estimation and Unscented Kalman Filter

Given the state-space model representation (20) and (23) with Gaussian noises, we can implement model estimation using Bayesian filtering approaches. We have noted that the depreciation rate equation is a highly non-linear function of states, which makes the standard Kalman filter inapplicable. Instead, we can use the nonlinear Kalman filters. The usually used nonlinear Kalman filter is the extended Kalman filter, which linearizes the nonlinear system around the current state estimate using a Taylor approximation. However, for the highly nonlinear system, the extended Kalman filter is computationally demanding and performs very poorly. An alternative is the unscented Kalman filter (UKF), recently developed in the field of engineering (Julier and Ulman (1997, 2004)). The idea behind this approach is that in order to estimate the state information after a nonlinear transformation, it is favorable to approximate the probability distribution directly instead of linearizing the nonlinear functions. The unscented Kalman filter overcomes to a large extent pitfalls inherent to the extended Kalman filter and improves estimation accuracy and robustness without increasing computational cost.

To implement the unscented Kalman filter, we firstly concatenate the state variables $x_{t-1} = [X_{t-1}, X_{t-2}]'$, the observation noises η_{t-1} and the state noises ε_{t-1} at time $t-1$

$$x^e_{t-1} = \begin{bmatrix} x'_{t-1} & \eta'_{t-1} & \varepsilon'_{t-1} \end{bmatrix}', \tag{26}$$

whose dimension is $L = L_x + L_\eta + L_\varepsilon$ and whose mean and covariance are

$$\hat{x}^e_{t-1} = \begin{bmatrix} E[x_{t-1}] & 0 & 0 \end{bmatrix}', \qquad P^e_{t-1} = \begin{bmatrix} P^x_{t-1} & 0 & 0 \\ 0 & \Sigma^2_\eta & 0 \\ 0 & 0 & I_6 \end{bmatrix}.$$

We then form a set of $2L+1$ sigma points

$$\chi^e_{t-1} = \begin{bmatrix} \hat{x}^e_{t-1} & \hat{x}^e_{t-1} + \sqrt{(L+\lambda)P^e_{t-1}} & \hat{x}^e_{t-1} - \sqrt{(L+\lambda)P^e_{t-1}} \end{bmatrix} \tag{27}$$

and the corresponding weights

$$w_0^{(m)} = \frac{\lambda}{L+\lambda}, \quad w_0^{(c)} = \frac{\lambda}{L+\lambda} + (1-\alpha^2+\beta), \tag{28}$$

$$w_i^{(m)} = w_i^{(c)} = \frac{1}{2(L+\lambda)}, \ \ i = 1, 2, \ldots, 2L, \tag{29}$$

where superscripts (m) and (c) indicate that weights are for construction of the posterior mean and covariance, respectively, $\lambda = \alpha^2(L+\bar{\kappa}) - L$ is a scaling parameter, the constant α determines the spread of sigma points around $\bar{x}$ and is usually set to be a small positive value, $\bar{\kappa}$ is a second scaling parameter with value set to 0 or $3 - L$, and β is a covariance correction parameter and is used to incorporate prior knowledge of the distribution of x.

With these sigma points, we implement the UKF as follows: for the time update

$$\begin{aligned}
\chi^x_{t|t-1} &= F(\chi^x_{t-1}, \chi^\varepsilon_{t-1}), \quad \hat{x}_t^- = \sum_{i=0}^{2L} w_i^{(m)} \chi^x_{i,t|t-1}, \\
P^-_{x_t} &= \sum_{i=0}^{2L} w_i^{(c)} (\chi^x_{i,t|t-1} - \hat{x}_t^-)(\chi^x_{i,t|t-1} - \hat{x}_t^-)',
\end{aligned}$$

and for the measurement update

$$\begin{aligned}
\mathcal{Y}_{t|t-1} &= H(\chi^x_{t|t-1}, \chi^\eta_{t|t-1}), \quad \hat{Y}_t^- = \sum_{i=0}^{2L} w_i^{(m)} \mathcal{Y}_{i,t|t-1}, \\
P^-_{Y_t} &= \sum_{i=0}^{2L} w_i^{(c)} (\mathcal{Y}_{i,t|t-1} - \hat{Y}_t^-)(\mathcal{Y}_{i,t|t-1} - \hat{Y}_t^-)', \\
P_{x_t Y_t} &= \sum_{i=0}^{2L} w_i^{(c)} (\chi^x_{i,t|t-1} - \hat{x}_t^-)(\mathcal{Y}_{i,t|t-1} - \hat{Y}_t^-)', \\
\hat{x}_t &= \hat{x}_t^- + P_{x_t Y_t}(P^-_{Y_t})^{-1}(Y_t - \hat{Y}_t^-), \\
P_{x_t} &= P^-_{x_t} - (P_{x_t Y_t}(P^-_{Y_t})^{-1}) P^-_{Y_t} (P_{x_t Y_t}(P^-_{Y_t})^{-1})',
\end{aligned}$$

where Y_t is the observation vector containing all the observed variables, $\hat{Y}_t^-$ its predicted values, $P^-_{Y_t}$ its conditional variance-covariance matrix, $\hat{x}_t$ the filtered state vector, and P_{x_t} its variance-covariance matrix.

Assuming that the predictive errors are normally distributed, we can construct the log likelihood function at time t as follows

$$\mathcal{L}_t(\Theta) = -\frac{1}{2}\ln|P_{Y_t}^{-}| - \frac{1}{2}(Y_t - \hat{Y}_t^{-})'(P_{Y_t}^{-})^{-1}(Y_t - \hat{Y}_t^{-}), \tag{30}$$

where Θ is a vector of model parameters. Parameter estimates can be obtained by maximizing the joint log likelihood

$$\hat{\Theta} = \arg\max_{\Theta\in\Xi}\sum_{t=1}^{T}\mathcal{L}_t(\Theta), \tag{31}$$

where Ξ is a compact parameter space, and T is the length of total observations of the data. Because the log likelihood function is misspecified for the non-Gaussian model, a robust estimate of the variance-covariance matrix of parameter estimates can be obtained using the approach proposed by White (1982)

$$\hat{\Sigma}_\Theta = \frac{1}{T}\left[AB^{-1}A\right]^{-1}, \tag{32}$$

where

$$A = -\frac{1}{T}\sum_{t=1}^{T}\frac{\partial^2\mathcal{L}_t(\hat{\Theta})}{\partial\Theta\partial\Theta'}, \qquad B = \frac{1}{T}\sum_{t=1}^{T}\frac{\partial\mathcal{L}_t(\hat{\Theta})}{\partial\Theta}\frac{\partial\mathcal{L}_t(\hat{\Theta})}{\partial\Theta'}. \tag{33}$$

With these parameter estimates $\hat{\Theta}$, the latent macroeconomic factors $\hat{X}_t$ can be extracted using the unscented Kalman filter.

V. Empirical Results and Discussions

A. Exchange Rate Dynamics

Exchange rates dynamics are the main focuses of this paper. First let us have a look at the model performance for exchange rate dynamics. Table 4 is the summary of statistics from the observed data and the model-implied data. It shows that the model can well capture not only the first moment of the data, but also the second moment as well as the third moment. However model implied exchange rate changes are with autocorrelations of around 50%, while only around 10% for the observed ones.

— Table 4 around here —

An intuitive picture for this model performance on exchange rate dynamics is in Figure 2. We plot the model implied exchange rates dynamics as well as the observed ones. Generally speaking, the model implied exchange rates dynamics well mimic the dynamics of the observed ones along the whole sample. The model-implied exchange rate dynamics can capture 53%, 34% and 32% variations of the observed exchange rate dynamics for DEM(EUR)/USD, GBP/USD and JPY/USD, repectively. Comparing with previous studies on exchange rate daynamics from macroeconomic fundamentals, this no-arbitrage Multi-Country model makes a big improvement.

— Figure 2 around here —

However, there are two exceptional time periods, where the model-implied exchange rate dynamics are distant from the observed ones. One is around 1992 and the other is around 2001. Note that these are two special time periods, cause some important macroeconomic events happened and with strong efforts on the downturn of output growths for global economy. For instance, around 1992, there was the Black Wednesday, when speculative attacked on currencies in the European Exchange Rate Mechanism. While the bursting of dot-com bubble happened around 2001.

B. Model Performance on Macroeconomic and Yield Variables

This no-arbitrage macro-finance model is able to not only model exchange rate dynamics but also macroeconomic and financial variables as well. Table 5 and Table 6 show the observed and model-implied statistic summaries for macroeconomic and yield data. It is different with the results for exchange rate dynamics. The model-implied macroeconomic and yield variables capture very well the statistics of observed ones, such as the mean, the standard error, the skewness, the kurtosis as well as the autocorrelation.

— Table 5 around here —

— Table 6 around here —

From Figure 3, this model is able to very well capture the dynamics of output growths, inflations and short-term interest rates for all the four countries. There are only two variables among twelve are with some distance between model-implied and observed data. One is the output for the US, the distance is around the time period of 1992. The other one is the interest rate for the UK, with the time period between 1985 and 1990.

— Figure 3 around here —

Figure 4 gives us intuitive information of model performance on yield variables. Here in order to have a complete picture of interest rate term structure, the short-term interest rates (1-month yields) are plotted again, as well as 24-month and 60-month yields. For each country, during the normal periods, the 1-month yields are the lowest and 60-month yields are the highest among these yields with short, medium and long maturities. However, this order changed completed around the crisis periods, such as the periods around 1992 and 2002. Because during normal time periods, the short maturity bond investment provides the investors more liquidity which brings more opportunities. Hence investors in long term bonds ask more compensation for their opportunity costs. While during crisis periods, investors believe the economy will go back to the normal level in the long run and long term investment is more safe. Hence the investors on short-term bonds ask for high return for the extra risk they take.

— Figure 4 around here —

Figure 4 shows that this model can capture the dynamics of interest rate term structures for Germany, the UK, Japan and the US. Besides the UK, for the rest of three countries, model-implied yields closely follow the observed ones. For the UK, the model-implied variables can capture the dynamics of observed ones, however, they are less volatile comparing to the observed ones.

C. Global and Country-Idiosyncratic Macroeconomic Factors

The above section shows that macroeconomic variables are fitted by this model. Moreover, the two components which determine country-level macroeconomic factors (in equation 2), are worthy investigating as well.

— Table 1 around here —

Table 1 reports the estimates of parameters related to the country, global and country-idiosyncratic factors. In the upper panel, report the factor loadings for country factors on global factors. Most coefficients of global loadings are significant from zero and with values around one. The middle panel report the global factor dynamics. From values in matrix Φ^G, we can see that each global macroeconomic factor is highly persistent, with the values on diagonal close to one. The statistic t-ratios in these two panels prove that global macroeconomic factors do exit. While the off-diagonal values are almost zero, and mostly not significant.

The bottom panel presents the dynamics of country-idiosyncratic factors. There are eleven out of twelve diagonal values in matrix Φ^{F_i} $(i = 1, 2, 3, 4)$ are significant from zero, hence the country-idiosyncratic factors are not ignorable in determining the country factor X_i. In addition, these values are close to one, especially coefficients for output growth rates. This implies these country-idiosyncratic factors are quite persistent.

— Figure 5 around here —

Figure 5 draws three global macroeconomic factors for the output growth, the inflation and the short-term interest rate. There are two big jumps for global output growth factor between 1992 and 1994 as well as between 2001 and 2003, when there were crises. The global inflation factor is positive all the time in our sample. The global short-term interest rate factor has the highest peak around 1991, and this is more or less the same time period for peak in short-term interest rates in Figure 1.

VI. conclusion

This paper investigates the joint dynamics of multi bilateral nominal exchange rates simultaneously under a Multi-Country framework. We introduce the macroeconomic fundamental information to model exchange rate dynamics by adopting a no-arbitrage macro-finance approach. We allow macroeconomic fundamentals to be determined by global (common) factors as well as country-idiosyncratic factors.

The empirical study focuses an open economy including four countries, i.e., Germany, the UK, Japan and the US, where the US is taken as the numerarie (home) country. Results show that exchange rate dynamics are better modeled by this multi-country no-arbitrage model comparing to previous studies. This model explains 53%, 34% and 32% variations of the observed exchange rate changes of DEM(EUR)/USD, GBP/USD and JPY/USD, respectively. Moreover, the global macroeconomic factors do exist.

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Table 1: **Estimates of the Country, Global and Country-Idiosyncratic Factor Parameters**

Factor Loadings ($X_{i,t} = \alpha_i + \beta_i G_t + F_{i,t}$)						
	$\alpha_i(\times 10^3)$			β_i		
	g^G	π^G	r^G	g^G	π^G	r^G
GM	0.01	0.03	-0.01	0.71	0.94	0.83
	(2.12)	(3.46)	(3.40)	(1.15)	(4.87)	(7.74)
UK	0.18	-0.44	-0.04	1.10	0.91	0.92
	(1.96)	(1.12)	(0.93)	(2.41)	(3.41)	(20.25)
JP	0.17	-0.43	-0.22	0.93	0.93	0.79
	(1.65)	(0.95)	(1.63)	(1.56)	(8.01)	(10.82)
US	0.21	0.24	-0.13	0.89	0.87	0.85
	(0.98)	(3.04)	(2.20)	(1.58)	(7.34)	(3.84)
Global Factor Dynamics ($G_t = \Phi^G G_{t-1} + \Sigma^G \upsilon_t^G$)						
	Φ^G			$\Sigma^G(\times 10^3)$		
	g^G	π^G	r^G	g^G	π^G	r^G
g^G	0.98	0.00	-0.00	0.73	0	0
	(73.79)	(2.07)	(1.61)	(2.45)	–	–
π^G	-0.00	0.99	-0.00	0.26	0.80	0
	(1.08)	(32.91)	(1.49)	(2.27)	(6.30)	–
r^G	-0.00	0.00	0.91	-0.34	-0.60	0.74
	(1.15)	(3.43)	(65.30)	(8.15)	(6.84)	(4.14)
Country-Idiosyncratic Factor Dynamics ($F_{i,t} = \Phi^{F_i} F_{i,t-1} + \Sigma^{F_i} \upsilon_{i,t}^F$)						
	Φ^{F_i}			$\Sigma^{F_i}(\times 10^3)$		
	$\tilde{g}_i$	$\tilde{\pi}_i$	$\tilde{r}_i$	$\tilde{g}_i$	$\tilde{\pi}_i$	$\tilde{r}_i$
GM	0.98	0.92	0.74	1.30	0.96	0.90
	(27.15)	(10.38)	(10.70)	(2.21)	(2.84)	(3.62)
UK	0.85	0.75	0.81	0.99	1.02	1.06
	(8.78)	(1.85)	(20.41)	(1.30)	(2.99)	(6.46)
JP	0.89	0.87	0.69	0.91	0.82	0.76
	(13.35)	(16.32)	(6.35)	(4.94)	(3.77)	(2.03)
US	0.99	0.79	0.80	1.05	0.88	0.97
	(33.86)	(15.54)	(10.25)	(1.24)	(3.30)	(2.15)

Note: This Table reports the estimates of the country, global and country-idiosyncratic factor parameters. In parentheses, the absolute value of t-ratio of each estimate is reported. The sample period is from 1985m08 to 2005m07 (with 240 observations).

Table 2: **Estimates of Market Prices of Risk Parameters**

	λ_0			λ_1		
	$\tilde{g}_i$	$\tilde{\pi}_i$	$\tilde{r}_i$	$\tilde{g}_i$	$\tilde{\pi}_i$	$\tilde{r}_i$
GM	-0.12	-0.41	-0.32	55.46	-4.72	-116.13
	(2.02)	(2.86)	(4.49)	(1.84)	(1.91)	(9.90)
UK	-0.26	-0.30	-0.21	72.86	-3.99	-96.78
	(1.95)	(1.95)	(2.76)	(3.15)	(1.69)	(10.22)
JP	-0.16	-0.57	-0.38	48.70	-3.86	-141.33
	(2.28)	(5.21)	(5.84)	(3.23)	(2.33)	(13.74)
US	-0.49	-0.21	-0.30	149.36	-34.50	-97.77
	(7.61)	(2.17)	(1.53)	(8.16)	(1.40)	(2.85)

Note: This Table reports the estimates of market prices of risk parameters. In parentheses, the absolute value of t-ratio of each estimate is reported. The sample period is from 1985m08 to 2005m07 (with 240 observations).

Table 3: **Estimates of Standard Deviation of Measurement Errors Parameters ($\times 10^4$)**

	g_i	π_i	r_i	$y_i^{(24)}$	$y_i^{(60)}$	Δs_j
GM	8.46	5.90	4.76	2.53	4.31	99.03
	(2.23)	(3.48)	(16.43)	(2.57)	(3.55)	(1.26)
UK	5.37	10.60	7.12	4.36	3.20	93.12
	(3.38)	(1.49)	(6.07)	(6.68)	(2.17)	(1.21)
JP	7.91	5.58	4.78	4.24	3.28	133.75
	(1.99)	(2.07)	(3.74)	(12.79)	(3.60)	(2.01)
US	11.25	4.46	3.48	2.90	3.10	
	(6.20)	(1.85)	(1.62)	(3.20)	(1.45)	

Note: This Table reports the estimates of standard deviation of measurement errors parameters. In parentheses, the absolute value of t-ratio of each estimate is reported. The sample period is from 1985m08 to 2005m07 (with 240 observations).

Table 4: **Model fit: Exchange Rate Dynamics**

	Mean(%)	*Std. Dev.(%)*	*Skewness*	*Kurtosis*	*Autocorr.*
			1. GEM(EUR)/USD		
Data	0.10	1.37	-0.20	3.39	0.10
Model	-0.05	1.82	0.69	4.76	0.45
			2. GBP/USD		
Data	0.04	1.25	-0.66	5.19	0.09
Model	0.00	1.95	1.36	7.14	0.55
			3. JPY/USD		
Data	0.14	1.45	0.36	4.43	0.08
Model	0.19	1.90	0.93	5.65	0.56

Note: This Table reports model fitting for exchange rate dynamics. The sample period is from 1985m08 to 2005m07 (with 240 observations).

Table 5: **Model Fit: Macro Data**

Maturities		*Mean(%)*	*Std. Dev.(%)*	*Skewness*	*Kurtosis*	*Autocorr.*
1. Germany						
output growth	Data	0.13	0.30	-1.11	4.71	0.85
	Model	0.13	0.28	-1.10	4.51	0.91
inflation	Data	0.16	0.11	0.88	4.05	0.97
	Model	0.16	0.11	0.86	3.97	0.98
interest rate	Data	0.37	0.17	0.85	2.59	0.99
	Model	0.35	0.16	0.75	2.73	0.99
2. UK						
output growth	Data	0.11	0.22	0.09	3.11	0.83
	Model	0.11	0.21	0.05	2.97	0.88
inflation	Data	0.31	0.18	1.49	4.80	0.98
	Model	0.29	0.15	1.57	5.29	0.99
interest rate	Data	0.61	0.26	0.81	2.50	0.99
	Model	0.57	0.21	0.49	2.47	0.98
3. Japan						
output growth	Data	0.11	0.42	-0.49	2.92	0.92
	Model	0.11	0.39	-0.52	2.91	0.96
inflation	Data	0.06	0.11	0.70	2.58	0.95
	Model	0.06	0.10	0.68	2.53	0.96
interest rate	Data	0.14	0.15	0.80	2.45	0.99
	Model	0.14	0.15	0.80	2.42	0.99
4. US						
output growth	Data	0.24	0.24	-0.53	3.14	0.97
	Model	0.18	0.23	-0.07	2.01	0.98
inflation	Data	0.25	0.09	0.64	3.13	0.97
	Model	0.25	0.09	0.68	3.36	0.97
interest rate	Data	0.39	0.17	-0.24	2.36	0.99
	Model	0.38	0.17	-0.17	2.34	0.99

Note: This Table reports model-implied and observed macroeconomic data statistic summary. The sample period is from 1985m08 to 2005m07 (with 240 observations).

Table 6: **Model Fit: Yield Data**

Maturities		*Mean(%)*	*Std. Dev.(%)*	*Skewness*	*Kurtosis*	*Autocorr.*
1. Germany						
24-m	Data	0.45	0.16	0.42	2.61	0.98
	Model	0.45	0.16	0.52	2.57	0.99
60-m	Data	0.46	0.14	0.48	2.60	0.98
	Model	0.47	0.14	0.59	2.67	0.98
2. UK						
24-m	Data	0.58	0.23	0.22	2.11	0.98
	Model	0.59	0.23	0.21	2.06	0.98
60-m	Data	0.61	0.22	0.00	2.00	0.98
	Model	0.61	0.23	0.20	1.94	0.99
3. Japan						
24-m	Data	0.22	0.19	0.56	2.02	0.99
	Model	0.24	0.17	0.40	1.94	0.99
60-m	Data	0.25	0.18	0.42	1.88	0.98
	Model	0.23	0.18	0.30	1.78	0.99
4. US						
24-m	Data	0.48	0.16	-0.19	2.33	0.97
	Model	0.49	0.15	-0.37	2.36	0.98
60-m	Data	0.50	0.16	-0.26	2.40	0.96
	Model	0.51	0.15	-0.25	2.30	0.98

Note: This Table reports model-implied and observed yield data statistic summary. The sample period is from 1985m08 to 2005m07 (with 240 observations).

Table 7: **Data Correlations: Macro and Exchange Rate Data**

	01	*02*	*03*	*04*	*05*	*06*	*07*	*08*	*09*	*10*	*11*	*12*	*13*	*14*	*15*
01	1.00	0.15	0.53	0.14	-0.25	0.49	0.20	0.34	-0.03	0.36	0.08	0.48	0.03	0.02	-0.05
02	0.15	1.00	0.32	0.53	-0.19	0.01	-0.07	0.04	-0.15	0.12	-0.01	0.35	0.03	0.01	0.10
03	0.53	0.32	1.00	0.30	-0.22	0.37	0.11	0.49	-0.12	0.24	0.09	0.36	-0.02	0.03	-0.01
04	0.14	0.53	0.30	1.00	-0.16	-0.17	-0.08	-0.12	-0.24	-0.18	-0.27	0.14	-0.05	0.02	0.00
05	-0.25	-0.19	-0.22	-0.16	1.00	0.20	0.55	0.35	0.76	0.19	0.44	-0.04	-0.02	-0.06	-0.03
06	0.49	0.01	0.37	-0.17	0.20	1.00	0.70	0.78	0.56	0.88	0.71	0.67	0.07	0.01	0.01
07	0.20	-0.07	0.11	-0.08	0.55	0.70	1.00	0.58	0.73	0.72	0.77	0.47	0.03	-0.01	0.03
08	0.34	0.04	0.49	-0.12	0.35	0.78	0.58	1.00	0.56	0.68	0.61	0.60	-0.04	-0.03	-0.02
09	-0.03	-0.15	-0.12	-0.24	0.76	0.56	0.73	0.56	1.00	0.65	0.82	0.42	0.03	-0.06	0.03
10	0.36	0.12	0.24	-0.18	0.19	0.88	0.72	0.68	0.65	1.00	0.84	0.80	0.10	0.02	0.05
11	0.08	-0.01	0.09	-0.27	0.44	0.71	0.77	0.61	0.82	0.84	1.00	0.54	0.10	0.01	0.12
12	0.48	0.35	0.36	0.14	-0.04	0.67	0.47	0.60	0.42	0.80	0.54	1.00	-0.00	-0.02	-0.01
13	0.03	0.03	-0.02	-0.05	-0.02	0.07	0.03	-0.04	0.03	0.10	0.10	-0.00	1.00	0.72	0.54
14	0.02	0.01	0.03	0.02	-0.06	0.01	-0.01	-0.03	-0.06	0.02	0.01	-0.02	0.72	1.00	0.44
15	-0.05	0.10	-0.01	0.00	-0.03	0.01	0.03	-0.02	0.03	0.05	0.12	-0.01	0.54	0.44	1.00

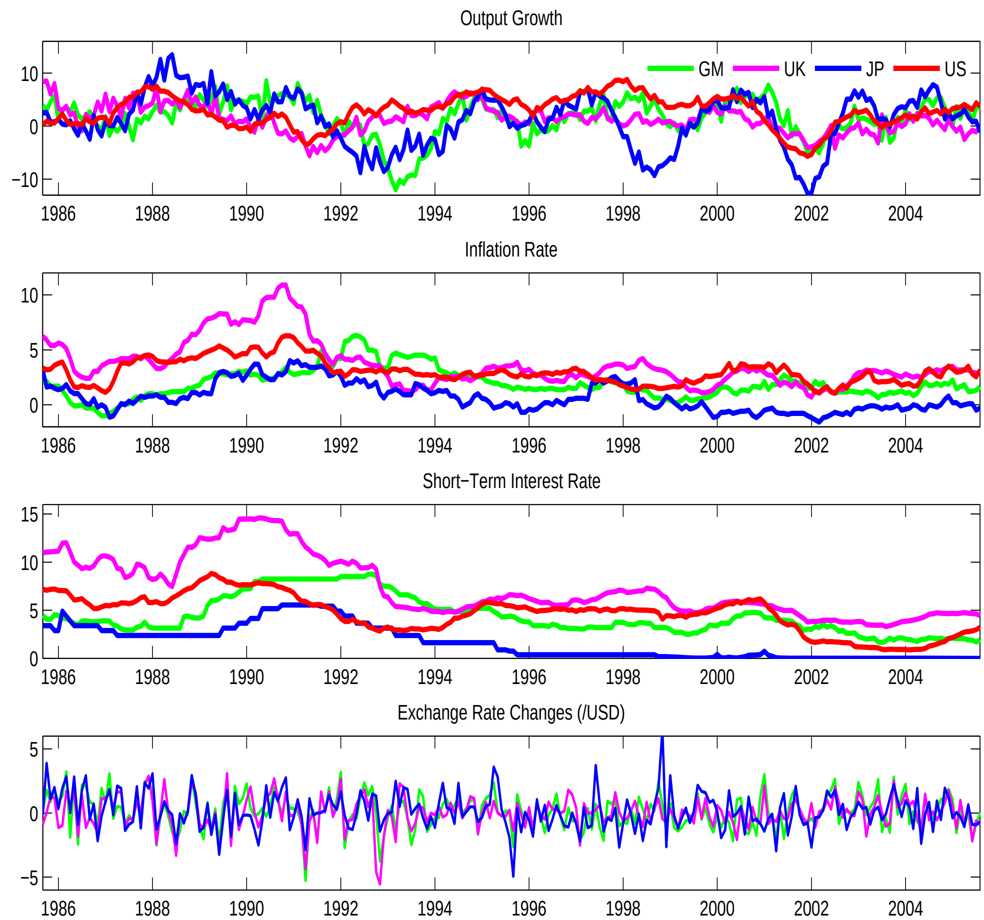
Note: This Table reports the correlations of original monthly macroeconomic variables and exchange rate changes. There are four time series of output growth rates (index of *1-4*), inflation rates (index of *5-8*) and short-term interest rates (index of *9-12*), for Germany, the UK, Japan and the US, respectively. Exchange rate changes data (index of *13-15*) are respectively German Mark/Euro, British Proud, Japanese Yen, against the US Dollar. The changes of exchange rate are the one-month changes of log nominal exchange rate, where monthly nominal exchange rates are the end-of-period values. The output growth rates, and inflation rates are the 12-month changes of IP and CPI, respectively. The sample period is from 1985m08 to 2005m07 (with 240 observations).

Table 8: **Principal Components Analysis for Macroeconomic Fundamentals**

I. Output Growth				
	PC1	*PC2*	*PC3*	*PC4*
Variance prop.	0.60	0.19	0.14	0.07
Cumulative prop.	0.60	0.80	0.93	1.00
II. Inflation Rates				
	PC1	*PC2*	*PC3*	*PC4*
Variance prop.	0.70	0.21	0.06	0.03
Cumulative prop.	0.70	0.91	0.97	1.00
III. Short-Term Interest Rates				
	PC1	*PC2*	*PC3*	*PC4*
Variance prop.	0.81	0.13	0.05	0.01
Cumulative prop.	0.81	0.94	0.99	1.00

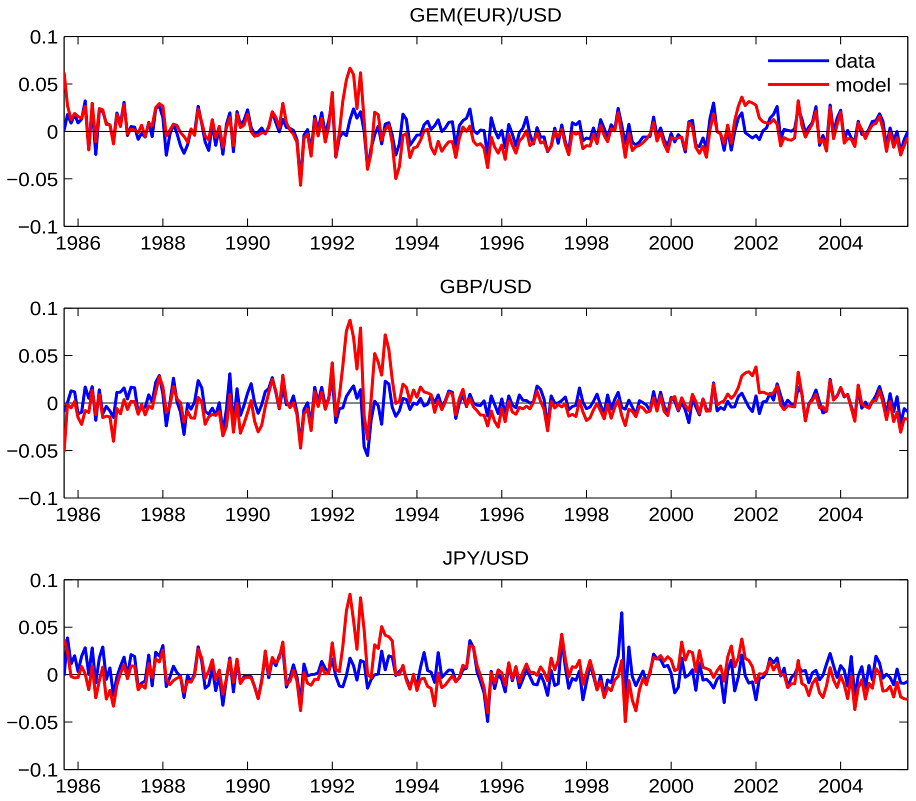
Note: This table reports the preliminary analysis of principle component analyses for each macroeconomic fundamental variable in different countries. For each group of output growth rates, inflation rates and short-term interest rates, I report the variance proportions and cumulative variance proportions associated with the four principal components, which is positioned with a descending order according to associated eigenvalues. The sample period is from from 1985m08 to 2005m07 (with 240 observations).

Figure 1: Macroeconomic Data and Exchange Rates Changes



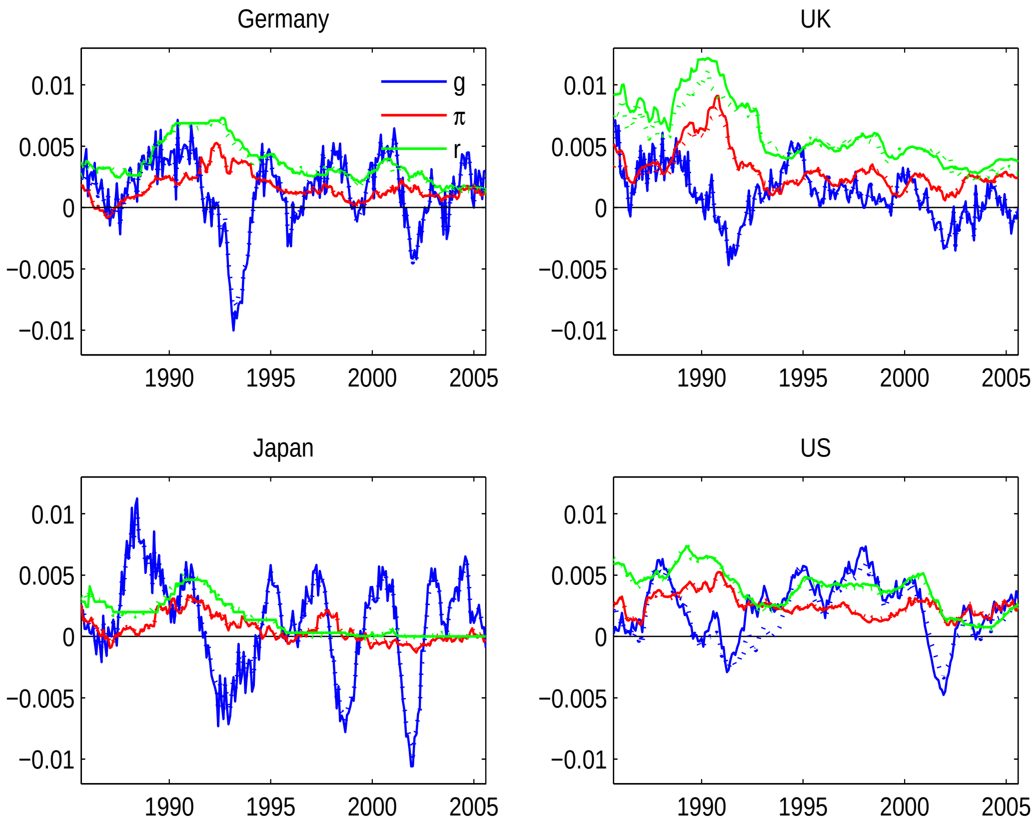
*Note:*I plot the macroeconomic fundamentals and monthly changes of exchange rates used in the estimation, for Germany, the UK, Japan, and the US. From up to bottom, they are output growth rates, inflation rates, short-term interest rates and exchange rate changes, respectively. I plot annualized quantities for macroeconomic differentials, and monthly changes of exchange rates. The sample period is from 1985m08 to 2005m07 (with 240 observations).

Figure 2: Exchange Rate Dynamics



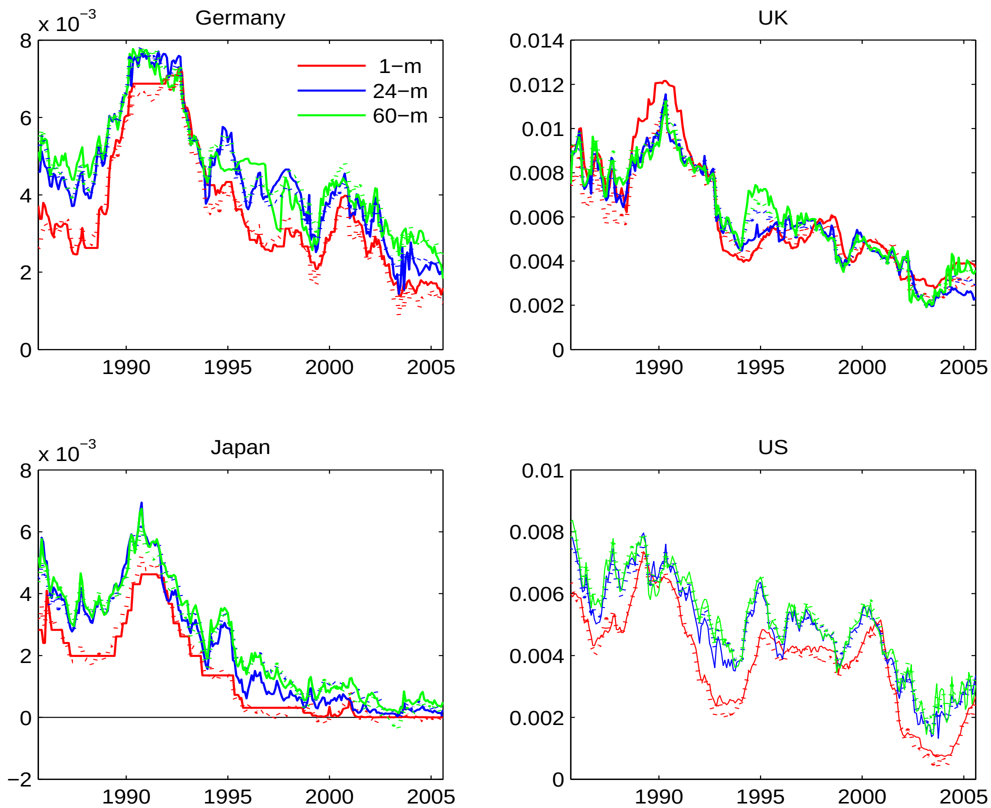
*Note:*I plot the monthly exchange rate changes for the foreign currencies of Germany, the UK, and Japan against the USD. The sample period is from 1985m08 to 2005m07 (with 240 observations).

Figure 3: Macroeconomic Fundamentals



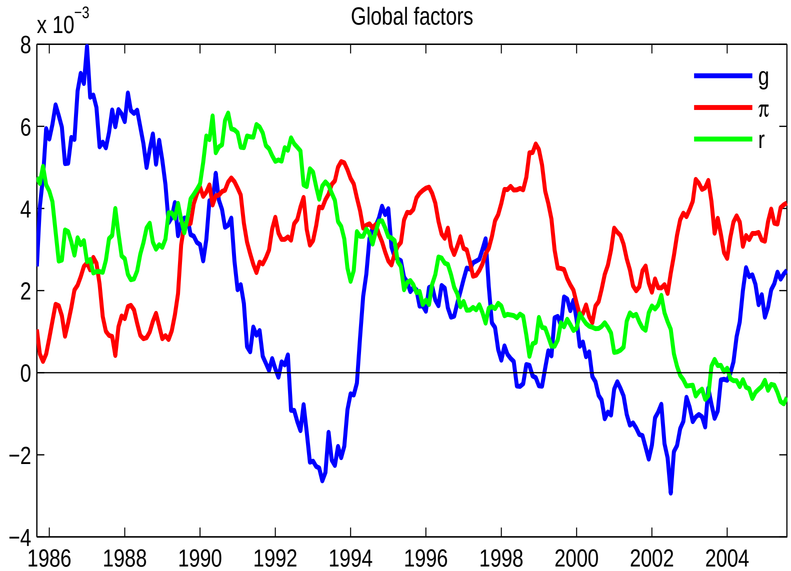
*Note:*I plot the monthly macroeconomic fundamentals, i.e., output growths, inflation rates, short-term interest rates, for Germany, the UK, Japan and the US. The solid lines are for observed data, while the dotted lines are for model-implied data. The sample period is from 1985m08 to 2005m07 (with 240 observations).

Figure 4: Yield Data



*Note:*I plot the monthly yield data with maturities of 1-month, 24-month and 60-month, for Germany, the UK, Japan and the US. The solid lines are for observed data, while the dotted lines are for model-implied data. The sample period is from 1985m08 to 2005m07 (with 240 observations).

Figure 5: Global Macroeconomic Factors



*Note:*I plot the macroeconomic global factors filtered from the no-arbitrage multi-country model. The sample period is from 1985m08 to 2005m07 (with 240 observations).

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